Target Tracking with a Flexible UAV Cluster Array

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***Abstract*—Unmanned aerial vehicle (UAV) cluster applications, for tasks such as target localisation and tracking, are required to collect and utilise the data received on "flexible" sensor arrays, where the sensors, i.e. UAVs in this scenario, have time-variant positions. In this paper, using a parametric channel model, a UAV cluster mobility model and a kinematic model of the targets, an extended Kalman based state space model is proposed that tracks the unknown UAV positions and target parameters snapshot by snapshot. Simulation studies illustrating the tracking capabilities of the proposed technique have been presented.**

$Notation$

A, a	Scalar
$\underline{A}$, $\underline{a}$	Column vector
$\mathbb{A}$	Matrix
$(\cdot)^T$	Transpose
$(\cdot)^H$	Hermitian transpose
$\exp(\underline{A})$	Element by element exponential of vector $\underline{A}$
$\underline{1}_N$	Column vector of N ones
$\mathbb{I}_N$	$N \times N$ identity matrix
$\mathcal{R}$	Set of real numbers
$\mathcal{C}$	Set of complex numbers

I. Introduction

Clusters of unmanned aerial vehicles (UAVs) find applications in a variety of areas such as remote sensing, commercial aerial surveillance, domestic policing, oil and gas exploration, post battle surveillance for casualties, scientific research and synthetic aperture radar. As UAV capabilities improve, a cluster of UAVs has been proven to perform better as compared to a single UAV in terms of time and efficiency [1]. However, the major challenge with UAV clusters is twofold: tackling arbitrary and known movement since each UAV has its own propulsion system and arbitrary unknown movement due to turbulent sources such as gusts of wind and imperfections in the motors. Thus, the cluster of UAVs form a "flexible array" which is defined as a class of sensor arrays that have time variant sensor positions (i.e. time variant array geometry). Flexible array signal processing refers to the detection, estimation and reception of signals received on such "flexible arrays". The challenge presented by such scenarios is that the array manifold becomes a function of time and thus the resulting received signal covariance matrix that forms the basis of superresolution algorithms such as MUSIC is not time invariant. Hence, typical array signal processing algorithms fail since they are built on the premise of a constant, known and time-invariant array geometry.

A few techniques have been proposed in literature in order to tackle the problems arising with the detection, estimation and reception on flexible/time variant arrays. The most common approach employed is that of recursive localisation or repetitive localisation. However, such techniques do not account for source mobility and assume that there exists an interval over which the array geometry may be assumed to be static. Thus, these techniques do not address the true problem of a "flexible array". A category of techniques attempt to perform the task of source localization for static sources with time varying arrays using a maximum likelihood (ML) estimator that splits the estimation problem into optimization over the directions of arrival (DOAs) and signal/noise parameters [2] [3] [4]. However, such estimators require a multidimensional search and are often computationally very expensive. Alternatively, there are incoherent techniques in which the array is split into rigid subarrays where a simplified ML estimator could be utilised for each subarray [5]. However, these estimates are not accurate and they place a severe limitation on the number of sensors and types of perturbations. There are also techniques which tweak the traditional subspace approaches, such as MUSIC, towards time varying arrays. In incoherent subspace techniques, the array is split into subarrays such that the spatial covariance of the measurements of the subarray can be considered constant enabling the usage of the MUSIC algorithm whose spectra are subsequently combined [6]. This technique starts to face problems in effective combining when the signal-to-noise ratio (SNR) is low. There are also coherent subspace techniques such as array interpolation [2] and focussing matrices [2] that transform the time variant covariance matrix to a time invariant covariance matrix. The major drawback of this technique is that the transformation holds true only for signals within an interpolation sector and hence limits the observation space to a small region. Some alternative flexible array techniques are also discussed in [7], albeit towards a different flexible array application namely the underwater towed array. However, a majority of these approaches cannot support high mobility of the target and the array i.e. either the array and/or the target is assumed to be stationary within a specified observation interval. Also, the mobility model of the flexible array under consideration is not incorporated in the problem definition losing crucial information that may be exploited towards target localisation and tracking. Thus, in this paper, towards the problem of target tracking using a UAV cluster, a state space model that incorporates

- the flexible arrayed parametric model of the received

signal,
- mobility model of the UAV cluster, and
- the mobility model of the targets,

is employed in conjunction with an extended Kalman filter to simultaneously track the UAV locations and target parameters. Note that this is complementary to approaches that track the deviation in array geometry and employ a controller to correct these deviations. For example, in [8], decentralized controllers are employed where each UAV is assigned a linear kinematic model under the constraint that each vehicle (except the leading one) has state information about the vehicle ahead of it. On the other hand, in [9], distributed controllers are employed, where in addition to a local controller, each vehicle receives information about the state of a huge subset of vehicles in the cluster. However, such controllers are not perfect and are often quite expensive. In this paper, the proposed algorithm that simultaneously tracks the sensor/UAV positions, whilst tracking the target parameters, would enable the minimal usage/simplification of such controllers that attempt to continually measure (using physical sensors such as depth, height and pressure sensors) and maintain the UAV cluster's geometry.

The paper is organised as follows. In Section II, three crucial models are presented namely the parametric model of the received signal at the flexible array, mobility model of the UAV cluster and the mobility model of the targets. Following this, in Section III, the proposed state space model based algorithm in conjunction with an extended Kalman filter is detailed. This is followed by Section IV wherein a discussion on the simulation studies to evaluate the performance of the proposed algorithm is presented. Finally, the paper is concluded in Section V.

II. Flexible Array System Model for a UAV Cluster

Consider the UAV cluster as a "flexible" antenna array consisting of N antennas (where each UAV is treated as an antenna of the array) with time varying array geometry $\mathbf{r}(t) \in \mathcal{R}^{N\times 3}$ at a time instant t. Figure 1 is an illustration of the problem under consideration with a flexible UAV cluster array and unknown targets whose parameters need to be estimated. In this section, three important models are described namely: (i) the mathematical model of the signal received at the flexible array incorporating all the channel parameters of interest and the time varying array geometry, (ii) the mobility model of the flexible UAV array and (iii) the mobility model of the targets.

A. Received Signal Model

Assume that the UAV cluster, whose locations at time instant t are given by $\mathbf{r}(t) = \left[\underline{r}_x(t), \underline{r}_y(t), \underline{r}_z(t)\right] \in \mathcal{R}^{N\times 3}$, operates in the presence of M narrowband far field sources. Each source is assumed to be moving with an unknown constant radial velocity. The signal $\underline{x}(t) \in \mathcal{C}^{N\times 1}$ received at the UAV cluster array at a time instant t can be modelled[1] as follows

$$\underline{x}(t) = \sum_{i=1}^{M} \beta_i(t)\, \underline{S}_i(t)\, m_i(t) \exp\left(j2\pi\mathcal{F}_i t\right) + \underline{\mathrm{n}}(t), \tag{1}$$

where for the i-th user, $\underline{S}_i(t) \in \mathcal{C}^{N\times 1}$ denotes the time varying manifold vector, $\beta_i(t) \in \mathcal{C}^{1\times 1}$ represents the path fading coefficient, $m_i(t) \in \mathcal{C}^{1\times 1}$ represents the delayed message received at the antenna array, $\mathcal{F}_i$ denotes the Doppler frequency and $\underline{\mathrm{n}}(t) \in \mathcal{C}^{N\times 1}$ represents the channel noise. In this paper, $\underline{\mathrm{n}}(t)$ is modelled as an additive white Gaussian noise whose covariance matrix $\mathbb{R}_{\mathrm{nn}}$ is given by

$$\mathbb{R}_{\mathrm{nn}} = \sigma_{\mathrm{n}}^2 \mathbb{I}_N. \tag{2}$$

However, this is not a constraint as long as an estimate of the second order statistics may be estimated. The array manifold vector is given by

$$\begin{aligned}\underline{S}_i(t) &\triangleq \underline{S}\left(\mathbf{r}(t), \theta_i(t), \phi_i(t)\right)\\ &= \exp\left(-j\frac{2\pi F_c}{c}\mathbf{r}(t)\,\underline{u}_i(t)\right),\end{aligned} \tag{3}$$

where F_c denotes the carrier frequency, c denotes the speed of the transmitted signal in the transmission medium and the wavenumber vector $\underline{u}_i(t)$ is given by

$$\begin{aligned}\underline{u}_i(t) &\triangleq \underline{u}\left(\theta_i(t), \phi_i(t)\right)\\ &= \left[\cos\theta_i(t)\cos\phi_i(t), \sin\theta_i(t)\cos\phi_i(t), \sin\phi_i(t)\right]^T,\end{aligned} \tag{4}$$

where $(\theta_i(t), \phi_i(t))$ denotes the azimuth and elevation respectively. In this paper, without any loss of generality, the array elements (i.e. UAVs) are assumed to lie in the same plane, i.e. $\underline{r}_z(t) = \underline{0}_N$, and the elevation is assumed to be known (i.e. the altitude of the UAV array from the targets is known) and is set to zero. Consider that the received data is sampled at a sampling rate F_s (sampling period T_s). Therefore, the discretised version of Eq. 1 is given by

$$\underline{x}(t_l) = \sum_{i=1}^{M} \beta_i(t_l)\, \underline{S}_i(t_l)\, m_i(t_l) \exp\left(j2\pi l\mathcal{F}_i T_s\right) + \underline{\mathrm{n}}(t_l), \tag{5}$$

where $t_l = lT_s$ denotes the l-th snapshot.

B. UAV Cluster Mobility Model

Choosing an appropriate mobility model is crucial to vehicular cluster applications. Models such as random way-point and Gauss-Markov model have been widely used towards modelling the distributions of vehicles. However, these models fail to capture all the constraints of aeronautical applications such as the inability of UAVs to make sharp turns and changes in velocity. Thus, taking these factors into account, mobility models for UAVs may be classified into three broad categories. The first class of mobility models follows a semi-random distribution of UAVs, i.e. the variation in the UAV array geometry can be split into the sum of a known and an unknown perturbation in array geometry. An example of such a model is the semi-random circular mobility model proposed in [10] and

[1]It is assumed that there exists one path that is of much higher power than the other multipaths of the target.

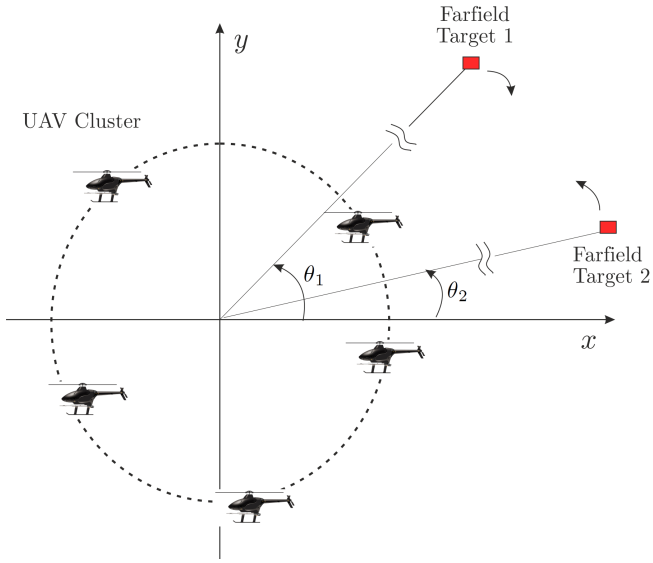


Fig. 1. Illustration of the problem formulation with the UAV cluster and the unknown targets in the x-y plane

such models are well suited for search and rescue missions. A second class of mobility models are based on a predetermined flight plan and are more suited for cargo and transportation applications [11]. The third class of mobility models that are employed for patrolling applications resemble swarms of vehicles and are closer in structure to the random way-point models in vehicular adhoc networks (VANETs). In this paper, the first group of mobility models is considered. Semi-random mobility models may be further classified as group mobility models and entity mobility models. In this paper, the group random mobility model will be considered, where the UAVs move as a group in formation around a fixed reference point, with each UAV permitted an element of random motion whose statistical properties are known. Therefore, in this paper, with no loss of generality, the group motion is assumed to be split into two terms namely a known fixed rotation about the reference point and an unknown random component associated with each sensor whose statistics are known. Therefore, the mobility model for the UAV array may be written as

$$\begin{bmatrix} \underline{r}_x(t_l) \\ \underline{r}_y(t_l) \end{bmatrix} = \mathbb{F} \begin{bmatrix} \underline{r}_x(t_{l-1}) \\ \underline{r}_y(t_{l-1}) \end{bmatrix} + \begin{bmatrix} \underline{\widetilde{r}}_x(t_l) \\ \underline{\widetilde{r}}_y(t_l) \end{bmatrix}, \quad (6)$$

where

$$\mathbb{F} = \begin{bmatrix} \cos\omega T_s, & -\sin\omega T_s \\ \sin\omega T_s, & \cos\omega T_s \end{bmatrix} \otimes \mathbb{I}_N, \quad (7)$$

where ω represents the angular velocity (deg/sec) of the array about the reference point. Here $\underline{r}_x(t_l), \underline{r}_y(t_l) \in \mathcal{R}^{N\times 1}$ denote the instantaneous x and y array positions respectively. The known transition matrix $\mathbb{F} \in \mathcal{R}^{2N\times 2N}$ is determined by the known forces and the physics of the medium. The errors in the array positions namely $\underline{\widetilde{r}}_x(t_l), \underline{\widetilde{r}}_y(t_l) \in \mathcal{R}^{N\times 1}$ arise from approximations in the mobility model and other external disturbances such as drifts/winds not modelled by the transition matrices and the driver terms. It is assumed that the first and second order statistics of these noise vectors is available. However, please note that although a relatively simple mobility model is chosen here, it may be modified to suit other class of mobility models as well. For example, for the third class of random mobility models, the transition matrix would be a random matrix and this may be further rewritten as a known transition matrix (obtained from the mean of the random transition matrix) and an error component whose second order statistics would be state dependent. This can be handled with an additional recursive update step of the extended Kalman filter to update the covariance matrix.

C. *Target Constant Velocity Mobility Model*

In this paper, the targets are assumed to move along a path with constant angular velocity. However, perturbations in angular velocity are also modelled as perturbations in the angular acceleration. Thus, the unknown target parameters for the i-th user may be grouped into a state vector $\underline{b}_i(t_l) \in \mathcal{R}^{2\times 1}$ given by

$$\underline{b}_i(t_l) = [\theta_i(t_l), v_{\theta_i}(t_l)]^T, \quad (8)$$

where $\theta_i(t_l)$ denotes the azimuth of the i-th user and $v_{\theta_i}(t_l)$ denotes the azimuthal angular velocity for the i-th user. As per the constant velocity motion model, the discrete kinematic model for the i-th target is given by

$$\underline{b}_i(t_l) = \mathbb{G}\underline{b}_i(t_{l-1}) + \underline{\widetilde{b}}_i(t_l), \quad (9)$$

where

$$\mathbb{G} = \begin{bmatrix} 1, & T_s \\ 0, & 1 \end{bmatrix}, \quad (10)$$

and the perturbations $\underline{\widetilde{b}}_i(t_l)$ are modelled as zero mean with the covariance matrix $\mathbb{Q}_i$ given by

$$\mathbb{Q}_i = \sigma^2_{v_{\theta_i}} \begin{bmatrix} \frac{T_s^3}{3}, & \frac{T_s^2}{2} \\ \frac{T_s^2}{2}, & T_s \end{bmatrix}, \quad (11)$$

where $\sigma^2_{v_{\theta_i}}$ denotes the continuous time model intensity in the azimuthal acceleration of the i-th target. Thus, the composite state vector $\underline{b}(t_l)$ consisting of the unknown parameters of M targets is given by

$$\underline{b}(t_l) = \left[\underline{b}_1^T(t_l), \underline{b}_2^T(t_l), \ldots, \underline{b}_M^T(t_l)\right]^T \in \mathcal{R}^{2M\times 1}, \quad (12)$$

and the composite kinematic model may be written as

$$\underline{b}(t_l) = (\mathbb{I}_M \otimes \mathbb{G})\,\underline{b}(t_{l-1}) + \underline{\widetilde{b}}(t_l), \quad (13)$$

where the perturbations $\underline{\widetilde{b}}(t_l)$ are zero mean with the covariance matrix $\mathbb{Q} \in \mathcal{R}^{2M\times 2M}$ that is a block diagonal matrix consisting of the individual covariance matrices $\mathbb{Q}_i$ of each of the users.

III. UAV Cluster Array Positions and Target Tracking using a State Space Model for Flexible Arrays

The signal $\underline{x}(t_l)$ received at the flexible array at an instant t_l in the observation interval, as given by Eq. 5, can be written as

$$\underline{x}(t_l) = \mathbb{H}(t_l)\,\underline{m}(t_l) + \underline{n}(t_l), \tag{14}$$

where $\mathbb{H}(t_l) \in \mathcal{C}^{N\times M}$ is the channel matrix given by

$$\mathbb{H}(t_l) = \left[\underline{S}_1(t_l), \underline{S}_2(t_l), \ldots, \underline{S}_M(t_l)\right], \tag{15}$$

and $\underline{m}(t_l) \in \mathcal{C}^{M\times 1}$ encompasses the message, path fading and Doppler coefficients and is given as

$$\underline{m}(t_l) = \begin{bmatrix} \beta_1(t_l)\, m_1[t_l] \exp(j2\pi l \mathcal{F}_1 T_s) \\ \beta_2(t_l)\, m_2[t_l] \exp(j2\pi l \mathcal{F}_2 T_s) \\ \vdots \\ \beta_M(t_l)\, m_M[t_l] \exp(j2\pi l \mathcal{F}_M T_s) \end{bmatrix}. \tag{16}$$

The next stage of building the state space model is to group all the unknowns of the problem under consideration into a state vector $\underline{z}(t_l)$ given by

$$\underline{z}(t_l) = \left[\underline{r}_x^T(t_l), \underline{r}_y^T(t_l), \underline{b}^T(t_l)\right]^T \in \mathcal{R}^{2(N+M)\times 1}. \tag{17}$$

Thus, Eqs. 6 and 13 may be combined to yield an evolution equation for the state vector $\underline{z}(t_l)$ as follows

$$\underline{z}(t_l) = \mathbb{F}_{\text{all}}\underline{z}(t_{l-1}) + \underline{\tilde{z}}(t_l), \tag{18}$$

where the state transition matrix $\mathbb{F}_{\text{all}} \in \mathcal{R}^{2(N+M)\times 2(N+M)}$ is given by

$$\mathbb{F}_{\text{all}} = \begin{bmatrix} \mathbb{F}, & 0_{2N\times 2M} \\ 0_{2M\times 2N}, & \mathbb{I}_M \otimes \mathbb{G} \end{bmatrix}. \tag{19}$$

The term $\underline{\tilde{z}}(t_l) \in \mathcal{R}^{2(N+M)\times 1}$ incorporates all the uncertainties that arise due to approximations in the mobility model or due to the presence of forces unaccounted for in the mobility model, such as gusts of wind, and can be represented as follows

$$\underline{\tilde{z}}(t_l) = \left[\underline{\tilde{r}}_x^T(t_l), \underline{\tilde{r}}_y^T(t_l), \underline{\tilde{b}}^T(t_l)\right]^T. \tag{20}$$

Equations 14 and 18 constitute an arrayed state space model that describes the signal received at the flexible array i.e. the UAV cluster in terms of the unknown target parameters and unknown UAV positions encompassed in the state vector. Thus, the state space model can be written as

$$\boxed{\begin{aligned} \underline{z}(t_l) &= \mathbb{F}_{\text{all}}\underline{z}(t_{l-1}) + \underline{\tilde{z}}(t_l) \\ \underline{x}(t_l) &= \mathbb{H}(\underline{z}(t_l))\,\underline{m}(t_l) + \underline{n}(t_l) \end{aligned}}. \tag{21}$$

Also, the mean of the state vector $\underline{z}(t_l)$ is given by

$$\begin{aligned} \mathcal{E}\{\underline{z}(t_l)\} &= \mathbb{F}_{\text{all}}\mathcal{E}\{\underline{z}(t_{l-1})\} \\ &= \mathbb{F}_{\text{all}}^l \underline{z}(t_0), \end{aligned} \tag{22}$$

assuming $\mathcal{E}\{\underline{\tilde{z}}(t_l)\} = \underline{0}_{2(N+M)}$. Also, the covariance matrix of $\underline{z}(t_l)$ denoted by $\mathbb{R}_{zz}(t_l) \in \mathcal{R}^{2(N+M)\times 2(N+M)}$ is given by

$$\begin{aligned} \mathbb{R}_{zz}(t_l) &= \mathcal{E}\left\{(\underline{z}(t_l) - \mathcal{E}\{\underline{z}(t_l)\})(\underline{z}(t_l) - \mathcal{E}\{\underline{z}(t_l)\})^T\right\} \\ &= \mathbb{F}_{\text{all}}\mathbb{R}_{zz}(t_{l-1})\mathbb{F}_{\text{all}}^T + \mathcal{E}\left\{\underline{\tilde{z}}(t_l)\,\underline{\tilde{z}}^T(t_l)\right\} \\ &= \mathbb{F}_{\text{all}}\mathbb{R}_{zz}(t_{l-1})\mathbb{F}_{\text{all}}^T + \mathbb{R}_{\tilde{z}\tilde{z}}, \end{aligned} \tag{23}$$

where $\mathbb{R}_{\tilde{z}\tilde{z}} \in \mathcal{R}^{2(N+M)\times 2(N+M)}$ is given as

$$\mathbb{R}_{\tilde{z}\tilde{z}} = \begin{bmatrix} \sigma_{r_x}^2 \mathbb{I}_N, & 0_{N\times N}, & 0_{N\times 2M} \\ 0_{N\times N}, & \sigma_{r_y}^2 \mathbb{I}_N, & 0_{N\times 2M} \\ 0_{2M\times N}, & 0_{2M\times N}, & \mathbb{Q} \end{bmatrix}, \tag{24}$$

where the perturbations in the UAV cluster array geometry are modelled as white Gaussian noise with zero mean and intensity of $\sigma_{r_x}^2$ and $\sigma_{r_y}^2$ along the x and y axes respectively. Note that Eq. 24 has been assumed for simplicity and is not a constraint. For instance, Eq. 24 can be suitably modified for the case of correlated noise in the state vector across the x and y antenna elements. Thus, the state space model presented by Eq. 21 forms the input to an arrayed extended Kalman filter (EKF) that tracks and adaptively estimates the state vector $\underline{z}(t_l)$ is presented. An extended Kalman filter is employed since the transition matrix $\mathbb{H}(\underline{z}(t_l))$ is non-linear. In order to estimate $\underline{z}(t_l)$ adaptively in the extended Kalman framework, the algorithm summarised in Table I may be employed where the notations are simplified as $\underline{z}_l \triangleq \underline{z}(t_l)$, $\mathbb{H}_l \triangleq \mathbb{H}(\underline{z}(t_l))$, $\underline{\widehat{m}}_l \triangleq \underline{\widehat{m}}(t_l)$ and $\underline{x}_l \triangleq \underline{x}(t_l)$.

TABLE I
Summary of steps to estimate target parameters and the UAV array positions using the extended Kalman filter

Initialization	
$\underline{\widehat{z}}_{0\|0} =$	$\mathcal{E}\{\underline{z}_0\}$
$\mathbb{P}_{0\|0} =$	$\mathcal{E}\left\{\left(\underline{z}_0 - \underline{\widehat{z}}_{0\|0}\right)\left(\underline{z}_0 - \underline{\widehat{z}}_{0\|0}\right)^T\right\}$
For	$l = 1, \ldots, L$
$\underline{\widehat{z}}_{l\|l-1} =$	$\mathbb{F}_{\text{all}}\underline{\widehat{z}}_{l-1\|l-1}$
$\mathbb{P}_{l\|l-1} =$	$\mathbb{F}_{\text{all}}\mathbb{P}_{l-1\|l-1}\mathbb{F}_{\text{all}}^T + \mathbb{R}_{\tilde{z}\tilde{z}}$
$\underline{\widehat{m}}_l =$	$\mathbb{H}_{l\|l-1}^H\left(\mathbb{R}_{\text{nn}} + \mathbb{H}_{l\|l-1}\mathbb{H}_{l\|l-1}^H\right)^{-1}\underline{x}_l$
$\mathbb{D}_{l\|l-1} =$	$\nabla_{\underline{z}_l}\left(\mathbb{H}_{l\|l-1}\underline{\widehat{m}}_l\right)\big\|_{\underline{\widehat{z}}_{l\|l-1}}$
$\mathbb{K}_l =$	$\mathbb{P}_{l\|l-1}\mathbb{D}_{l\|l-1}^H \times$
	$\left(\mathbb{D}_{l\|l-1}\mathbb{P}_{l\|l-1}\mathbb{D}_{l\|l-1}^H + \mathbb{R}_{\text{nn}}\right)^{-1}$
$\underline{\widehat{z}}_{l\|l} =$	$\underline{\widehat{z}}_{l\|l-1} + \text{Re}\left\{\mathbb{K}_l\left(\underline{x}_l - \mathbb{H}_{l\|l-1}\underline{\widehat{m}}_l\right)\right\}$
$\mathbb{P}_{l\|l} =$	$\left(I_{4M} - \mathbb{K}_l\mathbb{D}_{l\|l-1}\right)\mathbb{P}_{l\|l-1}$

Thus the predicted state vector $\underline{\widehat{z}}_l$ consists of all the parameters to be estimated namely the unknown UAV cluster locations and the parameters of the targets. Note that the initial covariance matrix $\mathbb{P}_0$ can be tuned according to the level of uncertainty in the initial estimate $\underline{\widehat{z}}_0$. For instance, for high levels of uncertainty in the initial estimates, $\mathbb{P}_0$ may be set to $\mu\mathbb{I}_{2(N+M)}$ where μ is an arbitrarily large real number indicating the confidence level in the estimate. Note that an estimate of the unknown message is obtained at each step. At low SNR, this may be further refined by substituting the

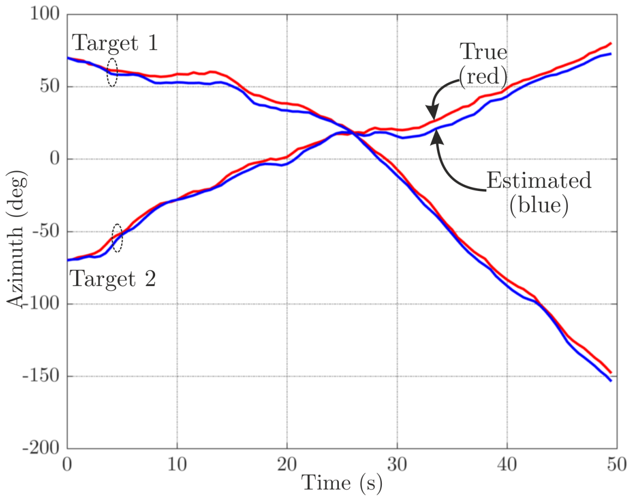


Fig. 2. True (red) and tracked (blue) azimuth trajectories of the two targets in the environment by the proposed algorithm.

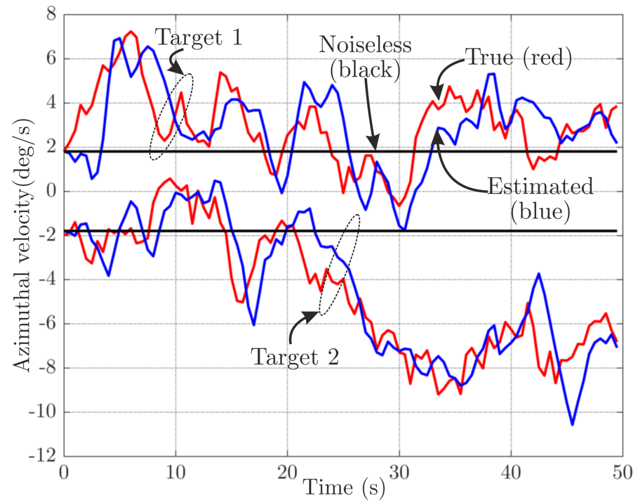


Fig. 3. True (red) and tracked (blue) azimuthal velocity trajectories of the two targets in the environment. The black line indicates the azimuthal velocity if no process noise were present in the mobility model.

updated estimate at the end of any iteration and may be repeated as many times as desired.

IV. Simulation Studies

For the purpose of simulations, the UAV cluster is assumed to consist of $N = 7$ elements. As described in Section II-B, the UAV cluster is assumed to begin with a circular geometry $\mathbf{r}$ given as follows in metres

$$\mathbf{r}^T = \begin{bmatrix} 3.2, & 1.1, & -1.9, & -3.4, & -2.4, & 0.4, & 2.9 \\ 1.2, & 3.2, & 2.9, & 0.3, & -2.7 & -3.4, & -1.8 \\ 0 & 0 & 0 & 0, & 0, & 0, & 0 \end{bmatrix}. \tag{25}$$

Two narrowband far-field targets are assumed to be present in the environment. The signals of the targets are assumed to be independent identically distributed Gaussian sources of zero mean and unity variance. The carrier frequency is assumed to be 50 MHz. The input SNR is assumed to be 20 dB. Tracking is assumed to be carried out over an interval of 50 s with a sampling interval $T_s = 0.01$ s (i.e. 5000 snapshots). Figures 2 and 3 illustrate the true and tracked azimuthal and azimuthal velocity trajectories of the two moving targets. It is clear from these figures that the target parameters are indeed tracked successfully. The continuous time model intensity $\sigma^2_{v_\theta}$ in the azimuth tracking model is chosen to be $10\ (\text{deg/s})^2$. Also, the intensities of the perturbations along the x and y axis, are set as follows $\sigma^2_{r_x} = \sigma^2_{r_y} = 1.8\ (\text{m/s})^2$. Furthermore, the angular velocity of the array ω is chosen to be 1 deg/s. The covariance matrix $\mathbb{P}_0$ is set to $\mu\mathbb{I}_{2(N+M)}$ where $\mu = 10^{-4}$ which indicates a reasonably high confidence in the accuracy of the initial estimates.

The proposed arrayed EKF algorithm also simultaneously tracks the sensor/UAV positions as shown in Fig. 4. The initial estimates of the array locations are set to the known circular geometry is shown in Fig. 4a at $t = 0T_s$. As dictated by the mobility model in Section II-B, this geometry evolves to the perturbed geometry illustrated in Figs. 4b, 4c and 4d at time $t = 500T_s$, $t = 1000T_s$ and $t = 3000T_s$ respectively. The black markers indicates the positions predicted by the rotation alone in the mobility model without accounting for the noise. The blue markers indicates the estimated positions of the UAVs while the red markers indicate the true positions. It is clear that the proposed algorithm indeed tracks the array locations iteratively even though the deviations from the mobility model is quite high. This illustrates that the algorithm is robust to errors in the mobility model due to factors such as winds or controller imperfections. Table II demonstrates the difference in the mean square error (MSE) in predictions from the noiseless mobility model and the tracked state estimates provided by the proposed algorithm. The results are obtained from Monte-Carlo simulations carried out over 100 iterations. For instance, a 20 dB suppression ability in azimuth angle implies that an azimuthal root mean square error of 10 degrees from a prediction made by the mobility model with no knowledge of the state and environmental noise would be brought down to about 1 degree by the proposed algorithm. Note that, to our knowledge, there exist

TABLE II

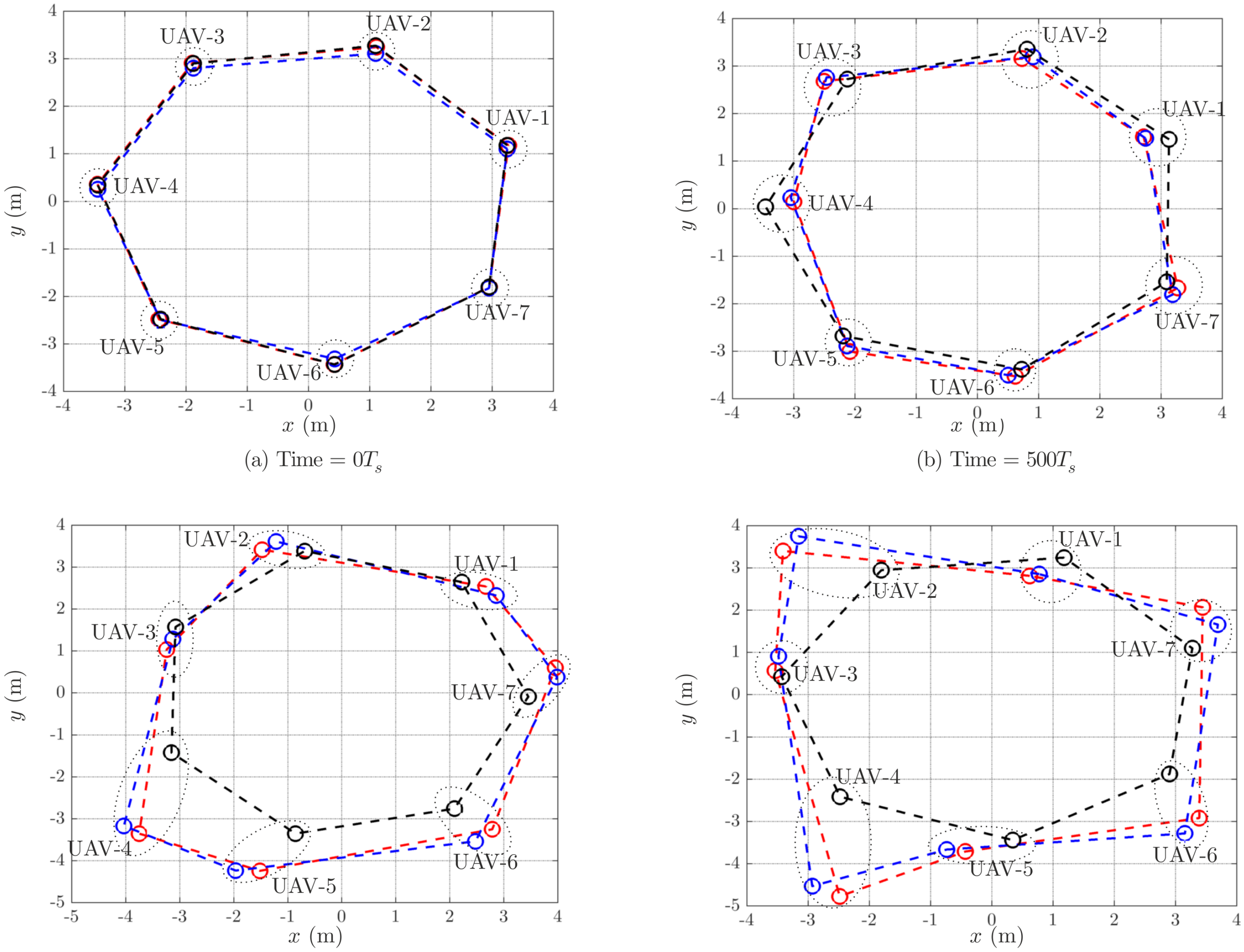
Difference in MSE of the predictions from the noiseless mobility model and the proposed algorithm

Parameter	MSE Difference
Azimuth	20.2 dB
Azimuthal velocity	4.0 dB
UAV sensor positions	5.2 dB

no alternative algorithms that support these levels of mobility in the targets and variations in array geometry.

V. Conclusions

In this paper, an algorithm to track farfield fast-moving targets using a UAV cluster is presented. The proposed al-



(a) Time $= 0T_s$ (b) Time $= 500T_s$

(c) Time $= 1000T_s$ (d) Time $= 3000T_s$

Fig. 4. True (red), estimated (blue) and no-noise mobility model predictions (black) of the UAV cluster array's sensor locations at (a) $t = 0T_s$ (b) $t = 500T_s$ (c) $t = 1000T_s$ and (d) $t = 3000T_s$. The input SNR is set to 20 dB.

gorithm incorporates a fully parametric array channel model incorporating the flexible geometry of the array, mobility model of the UAV cluster and the mobility model of the targets in conjunction with an extended Kalman filter to simultaneously predict the unknown UAV locations and target parameters. Simulation studies to illustrate the performance of the proposed algorithm are also presented.

References

[1] S. H. Brehency, R. Andrea, and J. Miller, "Using airborne vehicle-based antenna arrays to improve communications with UAV clusters," in *Conference on Decision and Control*, Dec. 2003, pp. 4158–4162.

[2] B. Friedlander and A. Zeira, "Eigenstructure-based algorithms for direction finding with time-varying arrays," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 32, no. 2, pp. 689–701, Apr. 1996.

[3] G. Edelson and D. Tufts, "Maximum likelihood estimation of the location of a narrow-band acoustic source using a spatially-referenced towed array," in *OCEANS'92 Mastering the Oceans Through Technology*. IEEE, 1992, pp. 414–419.

[4] J. Sheinvald, M. Wax, and A. Weiss, "Localization of multiple sources with moving arrays," *IEEE Transactions on Signal Processing*, vol. 46, no. 10, pp. 2736–2743, Apr. 1998.

[5] W. Hinich, Melvin. J., Rule, "Bearing estimation using a large towed array," *Journal of Acoustical Society of America*, vol. 58, no. 5, pp. 1023–1029, Nov. 1975.

[6] D. W. Rieken and D. R. Fuhrmann, "Generalizing MUSIC and MVDR for multiple noncoherent arrays," *IEEE Transactions on Signal Processing*, vol. 52, no. 9, pp. 2396–2406, Sep. 2004.

[7] V. Sridhar, M. Willerton, and A. Manikas, "Towed Arrays: Channel Estimation, Tracking and Beamforming," in *Beamforming: Sensor Signal Processing for Defence Applications*. World Scientific, 2015, pp. 189–219.

[8] D. Stipanović, G. Inalhan, R. Teo, and C. Tomlin, "Decentralized overlapping control of a formation of unmanned aerial vehicles," in *Conference on Decision and Control*, Dec. 2002, pp. 2829–2835.

[9] S. Tonetti, M. Hehn, S. Lupashin, and R. D'Andrea, "Distributed control of antenna array with formation of UAVs," in *18th World Congress: The International Federation of Automatic Control*, Sep. 2011, pp. 7848–7853.

[10] W. Wang, X. Guan, B. Wang, and Y. Wang, "A novel mobility model based on semi-random circular movement in mobile ad hoc networks," *Information Sciences*, vol. 180, no. 3, pp. 399–413, Feb. 2010.

[11] Y. Wan and S. Roy, "A scalable methodology for evaluating and designing coordinated air-traffic flow management strategies under uncertainty," *IEEE Transactions on Intelligent Transportation Systems*, vol. 9, no. 4, pp. 644–654, Dec. 2008.