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Analysis of Performance Indices for Simulated Skeleton Descents

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Abstract

In the winter Olympic sport of Skeleton, sliders sprint and load themselves onto the sled facing head forwards. The slider uses primarily their shoulders and torso to apply control to the direction of the sled as it progressively gains speed during its descent. These small control course keeping maneuvers alongside more severe use of toe tapping onto the ice will help determine the eventual trajectory of the sled. It is therefore of interest to consider for a possible trajectory what control actions will determine the fastest descent time and in particular what metrics should be examined. In this paper a three degree-of-freedom simulation has been developed to analyse the influence of different control strategies on the descent time of a bob-skeleton. A proportional-derivative (PD) controller is used to steer the simulation down a representation of the Igls ice-track. Parametric variations of the simulation's performance were analysed and compared to identify possible correlations for controllers assist the design of an optimal controller. Analysis of the results have identified positive correlations between descent time, transverse distance travelled and energy dissipation establishing that the fastest descent time is achieved by minimising the energy lost through the descent.

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Keywords: Skeleton;Optimisation;Race time simulation; Proportional-derivative Control; Trajectory evaluation

Nomenclature

α_c	longitudinal position of the particle representing the sled
β_c	transverse position of the particle representing the sled
F_{act}	total active forces on the particle representing the sled
r_α, $r_{\alpha\alpha}$	spatial derivative in the longitudinal direction
r_β, $r_{\beta\beta}$	spatial derivative in the transverse direction
$r_{\alpha\beta}$	spatial derivative in both longitudinal and transverse direction
m	mass of particle representing athlete and sled
g	acceleration due to gravity
h_0, h_f	initial and final height of the particle representing the sled
v_0, v_f	initial and final speed of the particle representing the sled
u(t)	control input
e(t)	error between reference position and actual position
K_p	proportional gain
K_d	derivative gain

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1. Introduction

The sport of skeleton involves the headfirst descent of an ice track performed by athletes on a sled. The goal is to achieve the fastest descent time. During a race, the sled can reach a maximum speed of around 130km/h; with the G-force reaching 5G through high banking corners and constant high levels of vibration due to uneven ice surfaces [1]. It would be of significant benefit to both coaches and athletes if an *a priori* 'optimum' trajectory could be predicted for the descent of a given track.

Through simulation, [4, 5] have shown that it is possible to find the 'optimum' descent trajectory for the Bobsleigh using optimal control algorithms. Due to the similarities between the underlying dynamics of the two sports, it is reasonable to believe a comparable approach could be used to find the 'optimum' descent trajectory for skeleton. An important element in designing any optimal control algorithm is the selection of the cost function to be minimised. In [5] it is stated that different costing strategies (i.e. minimum length path vs minimum energy dissipated path) have produced consistent difference in results, deducing that the final decent time will reduce if the energy dissipated is minimised. Though the results are promising, it cannot be trivially generalised to skeleton, since there exists discrepancies within the fundamental dynamics of the sports.

The objective of the work reported here is to identify the optimal cost function strategy for the skeleton descents by analysing the performance in simulation of a three degree-of-freedom skeleton dynamics model. Results from the simulation are used to identify feasible cost function for onward analysis.

2. Methodology

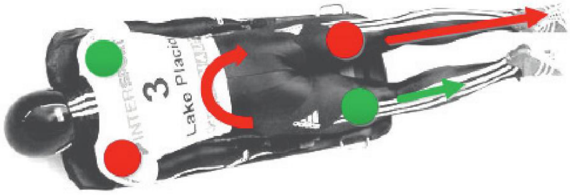

Figure 1 The principles of steering a skeleton [1]. This figure represents a right turn. The athlete applies pressure with left shoulder and right knee. This creates a frictional imbalance between left and right runners resulting in a turning moment.

The one degree-of-freedom skeleton dynamic model built by [2] has been extended to two degrees-of-freedom following similar underlying equations of motion and surfacing mapping techniques as [3]. The model is steered by the empirical based steering model from [1], which produces yaw moments, in the same manner by which the athlete achieves a steer on the sled as shown in Figure 1. The new model is therefore extended to three degrees-of-freedom to include the influence of sled yaw relative to its track. Equation (1) relates to the transverse sled motion in the model to the actuation F_{act} generated from the empirical steering model.

$$s^2\beta_c(s) = \frac{\begin{array}{c}\left(F_{act}\cdot r_\alpha + \left(\left(s^2\alpha_c(s)^2 r_{\alpha\alpha} + s^2\beta_c(s)^2 r_{\beta\beta} + s^2\alpha_c(s)\beta_c(s) r_{\alpha\beta}\right)m\cdot r_\alpha\right)\right)(r_\alpha\cdot r_\beta) \\ -\left(F_{act}\cdot r_\beta + \left(\left(s^2\alpha_c(s)^2 r_{\alpha\alpha} + s^2\beta_c(s)^2 r_{\beta\beta} + s^2\alpha_c(s)\beta_c(s) r_{\alpha\beta}\right)m\cdot r_\beta\right)\right)\cdot r_\alpha^2\end{array}}{m\left((r_\alpha\cdot r_\beta)^2 - r_\alpha^2 r_\beta^2\right)} \tag{1}$$

The track surface, following the convention of [3,6] and illustrated in Figure 2 uses a parametric representation of the actual modelled three dimensional surface where the position $\boldsymbol{r}$, is given as a function of a distance parameter α and transverse parameter β. In (1) the first and second derivatives of position $\boldsymbol{r}$ in directions α, β are written using the appropriate subscript. The forces acting on the sled, F_{act} (i.e. aerodynamic drag and lift are calculated using similar fundamental physical equations as [3,6], runner ice friction is forces as computed in [2]) and those due to steering control or wall collision will act to counter the gain in kinetic energy as the potential energy of the mass of the slider and sled is converted into kinetic energy. The air drag assumes a constant value of drag coefficient based on typical projected area of sled and slider. Likewise a semi-empirical value for ice friction coefficient is used.

$$\boldsymbol{r} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = f(\alpha, \beta) \tag{2}$$

The track geometry, Figure 2(a), used for this simulation is a representation of the Olympic Bob track in Innsbruck, Igls, Austria. The model was built as reconstructions of track topography and a combination of straights and corners with estimated cross-section. Igls was selected as it contains a board range of corner geometries, making it suitable for testing the robustness of the dynamic model and to ensure that results obtained on this track has a fair degree of generality.

The track model was constructed by estimating the track cross-sections from physical measurements and digital images at key locations along the track. The track's centerline, Fig. 2(a) was estimated using imagery from Google Earth. 14 points were equally distributed along each cross-section (Fig. 2(b)). A cubic spline was then fitted through the respective points in each cross-section, generating 14 new splines along the length of the track. New cross-sections were interpolated using the 14 splines at 1.5m intervals along the track. This created a 3x14x890 matrix of points that could be used by the simulation as an approximation of the Igls track.

The simulation was built in the real-time Matlab-Simulink environment. A PD controller was used to steer the simulation and is designed with the objective of following a descent trajectory (β_{ref}) pre-defined by subject matter experts to be the best line of descent. The PD controller treats the dynamic model as a 'black box' and asserted control inputs $(u(t))$ in the form of nominal left and right steering forces. Said steering forces were determined by the error $(e(t))$ between current sled centre position β_c and the desired reference position β_{ref}. The magnitude and rate of these steering forces were further controlled by tuning the controller gains (i.e. proportional gain K_p and derivative gain K_d), equation (3) illustrates the basic principles of the PD controller. A 2^{nd} order Butterworth low pass filter was implemented at the output of the PD controller to represent the steering input responses of a human athlete. The controller gains were tuned over a region ($K_p \in [0.015, 0.05]$, $K_d \in [0.04, 0.07]$) where the dynamic model were able to complete the entire simulated descent, this region of controller gains was named the 'stable region' and the corresponding steering forces were recorded. It was shown that the steering forces were directly proportional to the magnitude of the controller gains.

$$u(t) = K_p e(t) + \frac{K_d de(t)}{dt} \tag{3}$$

The intention of the simulation is that the relative proportional magnitude of ice friction and air drag forces should be tuned so that the simulated descent time agrees to a reasonable level with a typical human descent in Igls. This gives confidence to the subsequent analysis examining the course keeping actions performed by the PD controller and the impact of tuning the controller gains. Said simulations were performed over the 'stable region', the performance indices (i.e. descent time, distance travelled & energy dissipation) were recorded and their correlation were used to design the cost strategies for the optimal controller.

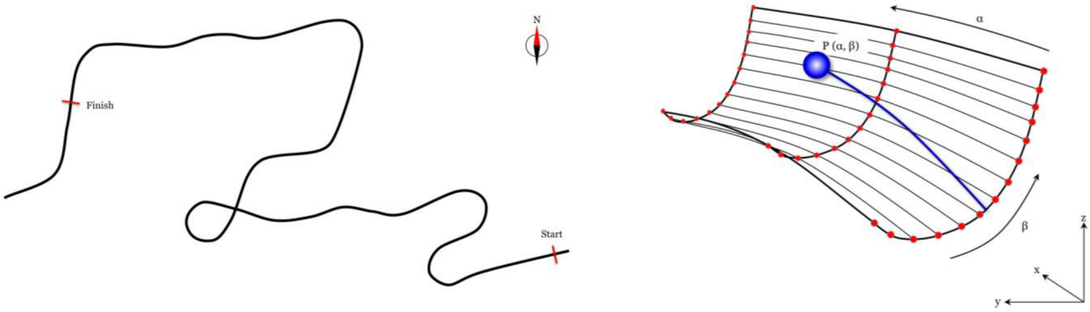

Figure 2 (a) Overview of the Igls Bob track. (b) Schematic of track cross-section coordinates adapted from [6].

3. xSimulation Results

3.1 Descent Time

Descent time is the most self-evident performance index to investigate as it ties in directly to the outcome of the race. Although the descent time is *ad hoc* to the dynamic model and track geometry, it can however adequately provide an understanding of general system behaviour and be utilised as a performance reference to gauge against other possible indices.

Fig. 3 (a) indicates that the descent time exhibits a quadratic structure initially decreasing proportionally with the value of K_d and K_p. However, once K_d is sufficiently small, the descent time increases as K_p approaches its stable boundaries. This is not truly representative of the index behaviour for the following reason:

When K_d is small, a large K_p will induce aggressive control actions and lead to instability in the system therefore leading to undesired oscillations, which causes a slower descent time. This is a defect caused by design of the controller and will skew the simulation data. In Fig. 3 (b), the x and y axes represent K_d and K_p respectively, the colour of the contour represent descent time. To avoid using data resulted from the aforementioned defect, a 'Representative Region' was put in place, indicated by the dashed box. The region cuts off at $K_p = 0.04$, which is the minimum value of K_d therefore within the region, the PD controller remains D dominant and should provide useful data. In Fig. 3 (b), the x and y axes represent K_d and K_p respectively, the colour of the contour represent the general trend of descent time with respect to steering forces: Increments in steering force leads to increments in descent time. To keep the analysis relevant, only data from the representative region will be used for the other two performance indices.

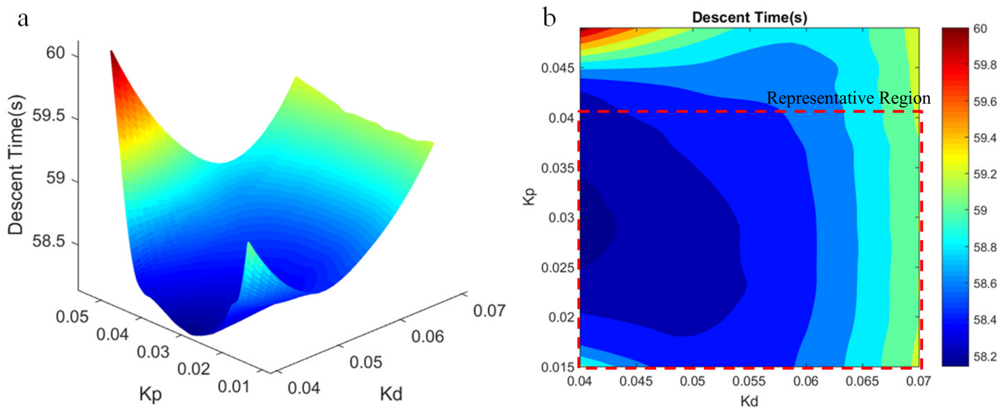

Fig. 3. (a) Surface plot of descent time vs steering force; (b) Contour plot of descent time vs steering force.

3.2 Transverse Distance Travelled

In the parametric surface representation each longitudinal spline, although varying slightly and not actually representing a viable trajectory would represent a minimal distance. The cross track error e.g. the distance away from the prescribed trajectory is referred to in this work as the transverse distance travelled and its total is calculated by integrating |β| over the duration of the simulated descent. α was not included in the calculation since all simulated descent start and end are at the same α values.

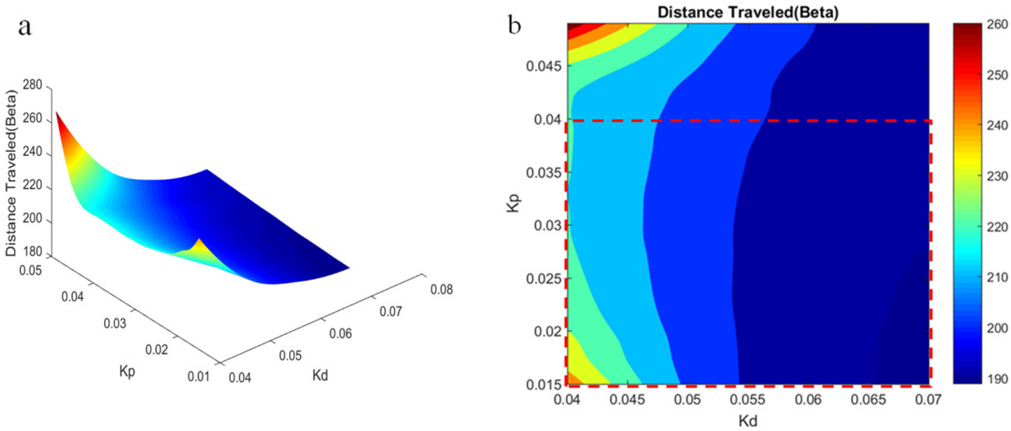

Fig. 4. (a) Surface plot of travel distance vs steering force; (b) Contour plot of travel distance vs steering force.

Fig. 4 (a) presents the simulation data in different axes orientation to provide a better view for the overall system behaviour. It is clear that distance travelled is also skewed by the controller defect. In Fig. 4 (b), the x and y axes represent K_d and K_p respectively, the colour of the contour represent the trend of distance travelled with respect to steering force; increments in steering force leads to decrements in travel distance.

3.3 Energy Dissipation

In this model, energy is lost through friction and drag. The dissipation is calculated by subtracting kinetic energy gained from the potential energy lost through the simulation. Kinetic energy gained is calculated via $\frac{1}{2}m(v-v_0)^2$ where v_0 is the initial velocity after the sprint phase and v_f is the terminal velocity, (usually at the track finish).

$$E_{lost} = m\left(g(h_f - h_0) - \frac{1}{2}(v_f^2 - v_0^2)\right) \tag{4}$$

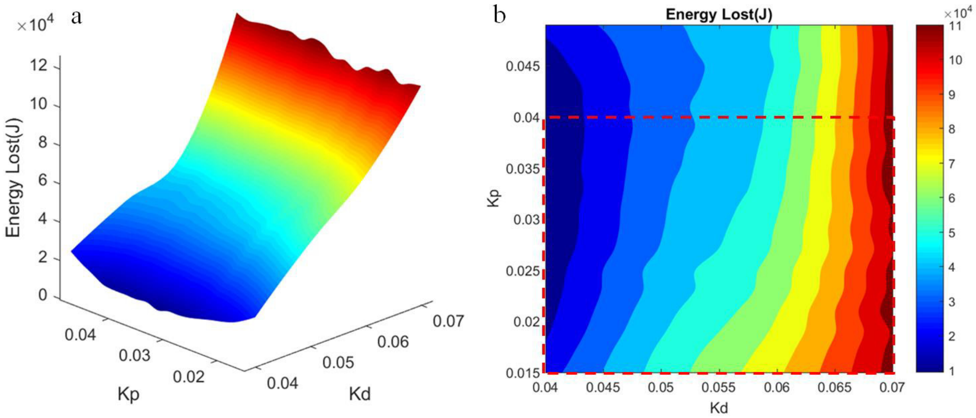

Fig. 5. (a) Surface plot of energy dissipation vs steering force; (b) Contour plot of energy dissipation vs steering force.

Fig. 5 shows that energy dissipation is affected by the controller defect to a much lesser degree. The reason for this is that energy lost is only proportional to friction caused by steering force (i.e. a simulated descent could take longer to complete but still have low energy lost). In Fig. 5, the *x* and *y* axes represent K_d and K_p respectively, the colour of the contour represents the trend of energy lost with respect to steering force; increments in steering force leads to increments in energy lost.

3.4 Correlation Analysis

To obtain suitable correlations for use in the optimal control designs, the values of each performance index were averaged across all the K_p values within the representative region. Fig. 6 presents the processed data compared against change in K_d, which can be perceived as steering forces at this point.

Characteristics identifications were carried out and the following correlations can be drawn from analysing the figures:

- Descent time exhibits a quadratically increasing characteristic with respect to the steering force.
- Transverse distance travelled exhibits a quadratically decreasing characteristic with respect to the steering force.
- Energy dissipation exhibits a quadratically increasing characterstic with respect to the steering force.

A clear correlation between performance indices can be drawn from above characteristics: Increments in energy dissipation → decrements in transverse distance travelled → Increments in descent time.

Although the simulation results demonstrated a clear correlation between the three performance indices, it however indicates that: ‘By maximising the transverse distance travelled, the athlete can minimise their descent time’. Without the context of energy dissipation, this correlation seems absurd. Analysing the above statement from another angle produces a more intuitive statement: ‘Instead of maximising the transverse distance travelled, athlete should not aim to reduce their travel distance in order minimise the energy lost.’ Therefore the distance travelled must be used in conjunction with energy lost for defining the cost minimisation strategy.

4. Conclusion

By analysing the correlation between performance indices, a feasible cost minimisation strategy could be determined. The relationship between energy dissipation and descent time is clear and makes logical sense. However, the effect of transverse distance travelled on the descent time is more complicated and further researche with different steering strategies are required to clarify this. Currently, to achieve the best descent time, the optimal controller should aim to minimise the energy lost through the race by limiting its steering actions. This is consistent with the strategy of the best sliders who aim to steer an optimum track with a minimum of course keeping interventions.

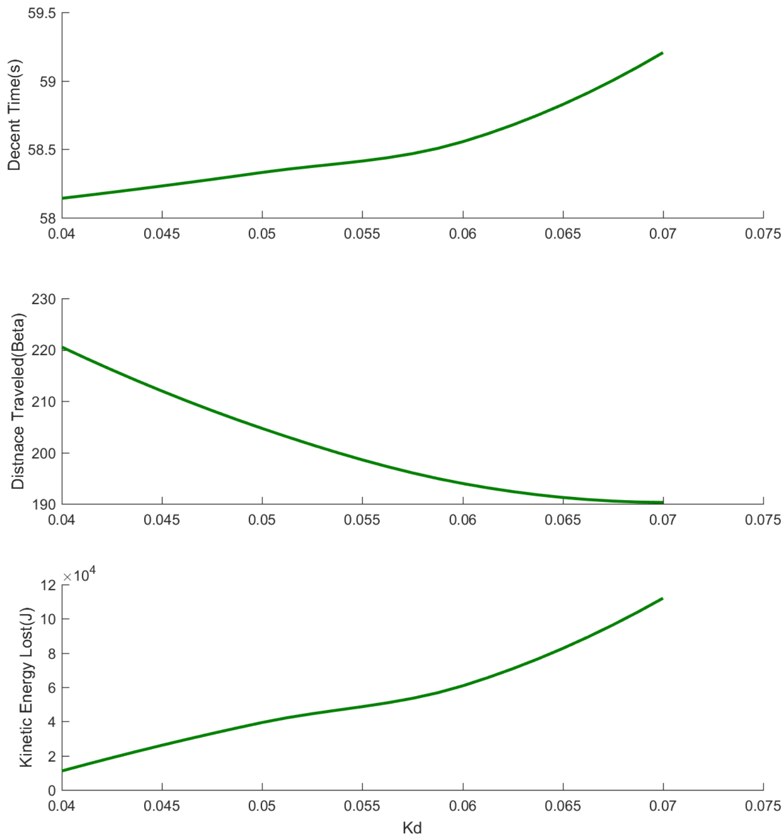

Fig. 6. Correlation graph between performance indices.

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