

Short-term Ocean Wave Forecasting Using an Autoregressive Moving Average Model

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Abstract—In order to predict future observations of a noise-driven system, we have to find a model that exactly or at least approximately describes the behavior of the system so that the current system state can be recovered from past observations. However, sometimes it is very difficult to model a system accurately, such as real ocean waves. It is therefore particularly interesting to analyze ocean wave properties in the time-domain using autoregressive moving average (ARMA) models. Two ARMA/AR based models and their equivalent state space representations will be used for predicting future ocean wave elevations, where unknown parameters will be determined using linear least squares and auto-covariance least squares algorithms. Compared to existing wave prediction methods, in this paper (i) an ARMA model is used to enhance the prediction performance, (ii) noise covariances in the ARMA/AR model are computed rather than guessed and (iii) we show that, in practice, low pass filtering of historical wave data does not improve the forecasting results.

I. INTRODUCTION

A wave energy converter (WEC) is a device for capturing wave power directly from surface waves or from pressure fluctuations below the surface [3]. In 2011, renewable energy resources, including solar, wind, geothermal as well as biofuel, contributed about 8.2% of the world's total energy generation and the number is still increasing [12]. As a new renewable and sustainable energy resource and major competitor of offshore wind power, ocean waves have the highest energy density per unit area of all renewable resources [2]. The total wave power that can be generated around the coasts of the world is of the order of 1 TW, similar to current global electricity consumption [2], [4]. In 2008, the first wave power farm was opened in Portugal. Since then, many European countries, the United States and Russia have launched their own wave power farms to harvest energy from the ocean.

Real-time control of WECs requires knowledge of future incident wave elevations in order to approach optimal efficiency of wave energy extraction. The energy conversion in most WECs are based either on the relative oscillation between bodies or on oscillating pressure distributions within fixed or moving chambers [6]. Therefore, it is important to know the wave elevations before applying any future control techniques (i.e. latching or declutching control) in order to enable efficient power absorption over a wide range of wave conditions.

In [6] an autoregressive (AR) based ocean wave prediction model was introduced, which assumes that the current wave height depends linearly on a number of past wave heights. The linear relationship between the current and past wave heights is represented by the AR parameters, where the initial values are determined from low pass filtered historical data with a forgetting factor λ . When new data is received, the AR parameters are updated using the recursive least squares (RLS) method.

In order to improve the prediction accuracy of ocean waves, [6] tried to remove high frequency components in historical ocean waves data using a low-pass filter. The main problem of this approach is the delay caused by low-pass filtering. In signal processing, signal delays can be compensated by adjusting/shifting the original signal or using a zero-phase low-pass filter, so that the smoothed signal will be in phase and have the same number of data points as the original one. However, when we try to predict the smoothed signal, those delays significantly affected the performance of wave prediction. We are going to discuss this questionable approach in a later section.

Results in [6] show that the AR-RLS model with a forgetting factor is a promising way of forecasting ocean wave elevations. One main problem of the prediction model is that, if the measurements do not add new information to the system, then after a certain time the RLS gains may grow without bound. Hence, the estimated AR parameters can experience a very large growth, known as the phenomenon of blow-up [6]. Another problem is that there is no method for determining the value of the forgetting factor λ ; this is tuned based on historical data and is assumed not to change as new information comes in.

In order to overcome the disadvantages of using the AR-RLS model with a forgetting factor, [6] introduced an LTV state space model representation for AR processes, where the state dynamics matrix is an identity matrix, so that the evolution of AR parameters follow a random walk. The AR parameters can then be estimated and predicted as unknown system states using the Kalman filter and predictor, respectively. The main difficulty of predicting ocean wave heights using a Kalman filter and predictor is that the initial state, corresponding error covariance, process and output noise

statistics are all unknown.

In this paper, ocean waves are predicted based on original rather than low-pass filtered historical wave data. The instability issue in wave prediction is resolved by using a corrected RLS formulation. An ARMA based state space model is used to predict ocean waves, where all the unknown parameters and noise covariances are estimated by using linear least squares (LLS) algorithms. In addition, an AR based state space model introduced in [6] is also examined, where the AR parameters and noise covariances are estimated using the auto-covariance least squares (ALS) algorithm.

Two ALS-based noise covariance estimation methods were introduced in [7], [8] and [11]. The ALS method establishes a linear relationship between the unknown noise covariances and covariance of innovation sequence that is obtained by using guessed noise covariances. Covariances can be determined by solving a constrained (positive-definite) linear least squares problem. The differences between the ALS method in [7], [8] and [11] are that the ALS method in [7], [8] does not involve any approximations and is able to estimate the initial state error covariance. Thus, in this paper, we are going to use the ALS method [7], [8] for noise covariance estimation.

This paper is organized as follows: In Section II, we introduce the models and formulations for ocean wave prediction. In Section III we discuss and investigate the influence of data smoothing on wave forecasting. A numerical example is given in Section IV. Finally, we draw conclusions in Section V.

$x \sim \mathcal{N}(\mu, P)$ denotes a random vector variable x with a normal distribution with mean μ and covariance matrix P . $\|w\|_W^2 := x^\top W x$ denotes weighted least squares of vector x . $\lceil x \rceil$ denotes the largest integer closest to x , i.e. $\lceil 3.5 \rceil = 4$ and $\lceil 3.4 \rceil = 3$. A^\dagger denotes the Moore-Penrose generalized inverse of a matrix A , such that $XX^\dagger X = X$.

II. AUTOREGRESSIVE MOVING AVERAGE MODEL

Suppose we have a sufficient amount of historical wave data $(y_k)_{k=1}^M$, and assume historical wave data can be fitted into a stationary stochastic model, namely an autoregressive moving average (ARMA) model, such that

$$y_{k+1} := \sum_{i=1}^p \phi_i y_{k+1-i} + \sum_{i=1}^q \theta_i w_{k+1-i} + w_{k+1}, \quad (1)$$

where $\phi_i \in \mathfrak{R}$ and $\theta_i \in \mathfrak{R}$ are the parameters of the AR and MA model, respectively, and $w_k \sim \mathcal{N}(0, Q)$. Stationary models assume that the process remains in statistical equilibrium with probabilistic properties (mean and variance) that do not change over time [1].

The ARMA model (1) can be written as the following state space representation [9]

$$\begin{aligned} x_{k+1} &:= Ax_k + Gw_{k+1}, \\ y_k &:= Cx_k \end{aligned} \quad (2)$$

where $C := [1 \ \theta_1 \ \theta_2 \ \cdots \ \theta_{n-1}]$

$$A := \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_{n-1} & \phi_n \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix}, \quad G := \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad (3)$$

$n := \max\{p, q+1\}$, $\phi_i = 0$ for $i > p$ and $\theta_i = 0$ for $i > q$.

Because ϕ_i , θ_i , w_k and Q are all unknown, in order to determine them given $(y_k)_{k=1}^M$, one has to solve the maximum likelihood estimation (MLE) problem, such that [9]

$$\begin{aligned} \max p(y_{1:M} | \phi, \theta, Q) &= \max \prod_{k=2}^M p(y_k | y_{k-1}, \phi, \theta, Q), \\ &= \max \prod_{k=2}^M (2\pi\Sigma)^{-\frac{1}{2}} e^{-\frac{y_k - Cx_k}{\Sigma}} \end{aligned} \quad (4)$$

where Σ is the innovation covariance. There are two main disadvantages of solving the MLE (4) problem: firstly, because (4) is a nonlinear optimization problem, finding a global optimal is not guaranteed; secondly, for higher order ARMA models, solving (4) could be computationally expensive.

Alternatively, ϕ_i , θ_i and Q can be approximated by solving two LLS problems. Firstly, we assume that the white Gaussian noise term w_k can be estimated by

$$\underbrace{\begin{bmatrix} \hat{w}_1 \\ \vdots \\ \hat{w}_{M-h} \end{bmatrix}}_{\hat{w}_{1, M-h}} = \underbrace{\begin{bmatrix} y_{h+1} \\ \vdots \\ y_M \end{bmatrix}}_{y_{h+1, M}} - \underbrace{\begin{bmatrix} y_h & \cdots & y_1 \\ \vdots & \ddots & \vdots \\ y_{M-1} & \cdots & y_{M-h} \end{bmatrix}}_{Y_{1, M-1}} \underbrace{\begin{bmatrix} \tilde{\phi}_1 \\ \vdots \\ \tilde{\phi}_h \end{bmatrix}}_{\tilde{\phi}_{1, h}}, \quad (5)$$

where $\tilde{\phi}_{1, h} = Y_{1, M-1}^\dagger y_{h+1, M}$ and h can be determined using the Akaike information criterion (AIC) [6]. Now we could establish a linear relationship between ϕ_i , θ_i , w_k and y_k

$$\hat{w}_{r+1, M-h} = y_{h+r+1, M} - H\rho \quad (6)$$

where $r = \max\{p, q\}$, $\rho = [\hat{\phi}_1 \ \cdots \ \hat{\phi}_p \ \hat{\theta}_1 \ \cdots \ \hat{\theta}_q]^\top$ and

$$H := \begin{bmatrix} y_{h+r} & \cdots & y_{h+r-p+1} & \hat{w}_r & \cdots & \hat{w}_{r-q} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ y_{M-1} & \cdots & y_{M-p} & \hat{w}_{M-h-1} & \cdots & \hat{w}_{M-h-q} \end{bmatrix}.$$

Hence, $\rho = H^\dagger y_{h+r+1, M}$ and $\hat{Q} \approx \frac{\hat{w}_{r+1, M-h} \hat{w}_{r+1, M-h}^\top}{M-h-r}$.

Once ϕ_i , θ_i and Q are all determined, the state sequence $(x_k)_{k=1}^M$ can be obtained using a steady state Kalman filter, where the steady state error covariance P_∞ can be obtained by solving the discrete algebraic Riccati equation (DARE) [13, p. 194]

$$P_\infty = AP_\infty A^\top + GQG^\top - AP_\infty C^\top (CP_\infty C^\top)^{-1} CP_\infty A^\top. \quad (7)$$

Hence, the steady state Kalman gain L_∞ is given by [13, p. 195]

$$L_\infty = (P_\infty C^\top) (CP_\infty C^\top)^{-1}. \quad (8)$$

After the state \hat{x}_M is determined, we could recursively predict future observations $\{y_k\}_{k=M+1}^{M+T_f}$ (T_f is a positive integer) by using

$$y_{M+i} = CA^i \hat{x}_M, \quad i = 1, 2, \dots, T_f. \quad (9)$$

When the new observation y_k , $k > M$, becomes available, the state vector \hat{x}_k is updated using an steady state Kalman filter based only on the most recent observation data y_k .

An AR-based state space model was introduced in [6], which assumes the AR parameters ϕ are slowly varying with time. Let the state vector $x_k := [\phi_{k,1} \quad \phi_{k,2} \quad \dots \quad \phi_{k,p}]^\top$ and time-varying output matrix

$$C_k := [y_{k-1} \quad y_{k-2} \quad \dots \quad y_{k-p}].$$

Suppose the evolution of the state vector x_k follows a random walk, so that

$$\begin{aligned} x_{k+1} &:= x_k + w_k \\ y_k &:= C_k x_k + v_k \end{aligned} \quad (10)$$

where $w_k \sim \mathcal{N}(0, Q)$, $v_k \sim \mathcal{N}(0, R)$ are two uncorrelated random variables and the order p of the AR model can be determined using the Akaike information criterion (AIC) [6]. Since y_k is a scalar, the state in (10) can be estimated using the following simplified Kalman filter equations:

$$L_k = P_{k|k-1} C_k^\top (C_k P_{k|k-1} C_k^\top + R)^{-1}, \quad (11a)$$

$$\begin{aligned} P_{k+1|k} &= P_{k|k-1} + Q \\ &\quad - P_{k|k-1} C_k^\top (C_k P_{k|k-1} C_k^\top + R)^{-1} C_k P_{k|k-1} \\ &= P_{k|k-1} + Q - \frac{P_{k|k-1} C_k^\top C_k P_{k|k-1}}{C_k P_{k|k-1} C_k^\top + R}. \end{aligned} \quad (11b)$$

In [6], x_M is estimated by solving a weighted LLS problem

$$\hat{x}_M := \arg \min_{x_M} \|y_{1,M} - Y_{1,M-1} x_M\|_\Lambda^2,$$

where the weight matrix $\Lambda \in \mathfrak{R}^{M \times M}$ is defined as $\Lambda := \bigoplus_{k=0}^{M-n-1} \lambda^k$ and $\lambda \in [0.97, 0.995]$ is the forgetting factor, so that more weight is given to recent observations according to an exponential law [6]. Hence, if $Y_{1,M-1}$ is a full column rank matrix, then

$$\hat{x}_M = (Y_{1,M-1}^\top \Lambda^{-1} Y_{1,M-1})^{-1} Y_{1,M-1}^\top \Lambda^{-1} y_{n+1,M}.$$

Once \hat{x}_M is determined, future wave heights can be predicted using $C_k \hat{x}_M$. When the new observation y_k becomes available, the AR parameters \hat{x}_k will be updated by [6]

$$L_k = \tilde{P}_{k+1} C_k^\top, \quad (12a)$$

$$\tilde{P}_{k+1} = \tilde{P}_k (C_k \tilde{P}_k C_k^\top + \lambda)^{-1}, \quad (12b)$$

$$\hat{x}_k = \hat{x}_{k-1} + L_k (y_k - C_k \hat{x}_{k-1}), \quad (12c)$$

with state error covariance $\tilde{P}_m = I$ and $\tilde{P}_M = \hat{x}_M \hat{x}_M^\top$.

There are two mistakes with the approach in [6]. Firstly, incorrect RLS equations (12) are used. The correct formulation should be

$$L_k = \tilde{P}_{k+1} C_k^\top \lambda^{-1}, \quad (13a)$$

$$\tilde{P}_{k+1} = \lambda \tilde{P}_k (C_k \tilde{P}_k C_k^\top + \lambda)^{-1}, \quad (13b)$$

Algorithm 1 Noise Covariance Estimation with ALS [7], [8]

- 1: Given κ , $\kappa \leq M$, $(y_k)_{k=1}^\kappa$, $\hat{x}_{1|0}$, initial guesses of P_1 , Q and R , determine the smoothed initial state $\hat{x}_{1|\kappa}$ by using the Kalman smoother for $\bar{k} = 1, \dots, \kappa$.
 - 2: Set $\hat{x}_{1|0} = \hat{x}_{1|\kappa}$, then recursively determine the filtered states $(\hat{x}_k)_{k=1}^M$ using the Kalman filter.
 - 3: Construct \mathcal{A} using (5) in [7], [8]. Calculate the innovation sequence $(y_k - C \hat{x}_k)_{k=1}^M$.
 - 4: Determine the lag N by looking at the auto-correlation plot of the innovation sequence.
 - 5: Estimate P_1 , Q and R using the ALS algorithm in [7], [8].
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thus (12) is only correct when $\lambda = 1$; secondly, an inappropriate state error covariance \tilde{P}_M is applied, because

$$\begin{aligned} \tilde{P}_M &= \mathbb{E} \left\{ (x_M - \hat{x}_M) (x_M - \hat{x}_M)^\top \right\} \\ &= \mathbb{E} \left\{ x_M x_M^\top \right\} - \hat{x}_M \hat{x}_M^\top \neq \hat{x}_M \hat{x}_M^\top. \end{aligned}$$

Our choice of \tilde{P}_m may not be correct either, but \tilde{P}_M will become accurate after several iterations. Most of the time, these two mistakes may not lead to a serious problem, because [6] limits the choice of λ within 0.97 and 0.995, which makes (12) approximately equal to (13). P_M is a positive definite matrix, which may only have a limited effect on system stability and prediction accuracy. However, these two mistakes will cause a robustness problem if the measurements do not add new information to the system [6]. We will discuss more details in Section IV-B.

In [6], covariances Q and R in (11) are all user-defined matrices. Inappropriate choices could result in poor or even unstable wave height predictions. In order to improve the prediction performance, one could use Algorithm 1 to provide optimal covariance matrices for estimation model (10).

III. CAN SMOOTHED HISTORICAL DATA HELP THE FORECASTING?

Figure 1 is the wave spectrum that was recorded at Galway at 01:20 on 11 March 2005 with a sampling frequency of $f_s = 2.56$ Hz and total data length $L = 3072$ [10].

It is stated in [6] that when predicting a signal from historical data using an AR based model, it is often the case that high frequency disturbances in the historical data may significantly affect the prediction performance. Thus, historical data needs to be smoothed using a low-pass filter before passing through the AR model for forecasting.

In order to verify this statement, we set up Forecasting Test 1 and 2 with the prediction horizon $T_f = \lfloor 4f_s \rfloor$ and prediction length $T_h = 250$.

Figure 2 shows the forecasting results given by Forecasting Test 1, 2 and ARMA model (1) without data smoothing. The accuracy of prediction are determined by the percentage of fitness [6]:

$$\text{Fit}\% := 100\% - \frac{\|y - \hat{y}\|_{\ell_2}}{\|y\|_{\ell_2}}. \quad (14)$$

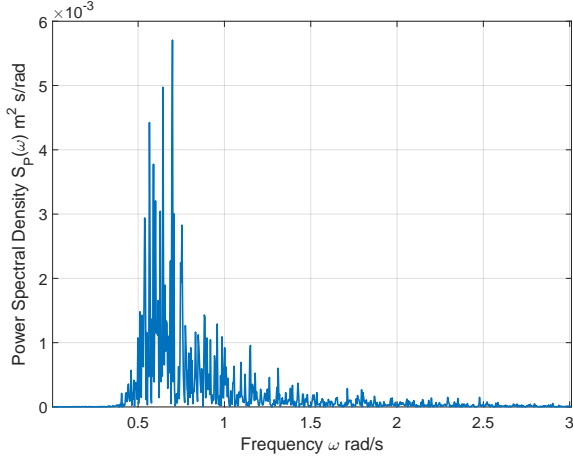


Fig. 1. Spectrum of measured ocean waves

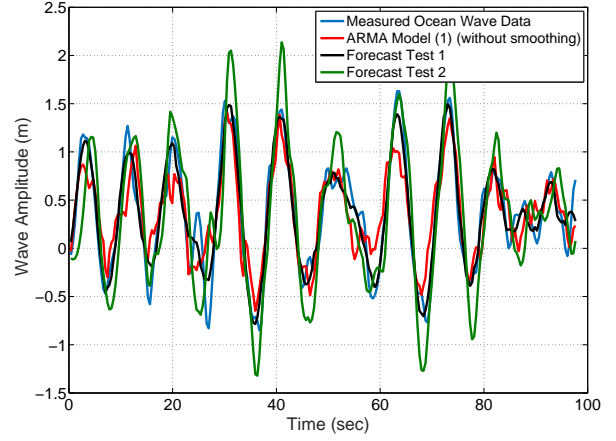


Fig. 2. Ocean waves forecasting tests

Forecasting Test 1

- 1: Given smoothed wave data $(\tilde{y}_k)_{k=1}^L$, let $M := 1500$.
- 2: Calculate ρ and Q based on $(\tilde{y}_k)_{k=1}^M$ using (5) and (6).
- 3: Estimate \hat{x}_M in (2) using the steady state Kalman filter.
- 4: Predict future wave $(\hat{y}_k)_{k=M+T_f}^{M+T_h}$:
- 5: **for** $t = 1, \dots, T_h$ **do**
- 6: $\hat{y}_{M+T_f+t-1} = CA^{T_f} \hat{x}_{M+t-1}$.
- 7: Given new data \tilde{y}_{M+t} , estimate \hat{x}_{M+t} using a Kalman filter.
- 8: **end for**

where y is the measured wave heights, \hat{y} is the predicted wave heights.

Based on the recording at 5:20 on 10 February 2005, Figure 2 clearly indicates that the best result is produced by Test 1 with Fit% = 71%, followed by the result using ARMA model without data smoothing with Fit% = 57.78%, the worst result is given by Test 2 with Fit% = 40%.

When a signal is smoothed by a low-pass filter, a constant delay will be introduced in the output signal. The length of the delay D can be pre-calculated, which depends on the cut-off frequency, order and the type of the filter. In order to

Forecasting Test 2

- 1: Let $M := 1500$, given original wave data $(y_k)_{k=1}^M$.
- 2: Obtain smoothed signal $(\tilde{y}_k)_{k=1}^{1500}$ by using a low pass filter with cut-off frequency $f_c = 1.2$ rad/s and filter delay D .
- 3: Calculate ρ and Q based on $(\tilde{y}_k)_{k=1}^M$ using (5) and (6).
- 4: Estimate \hat{x}_M in (2) using a steady state Kalman filter.
- 5: Predict future wave $(\hat{y}_k)_{k=M+T_f}^{M+T_h}$:
- 6: **for** $t = 1, \dots, T_h$ **do**
- 7: $\hat{y}_{M+T_f+t-1} = CA^{T_f} \hat{x}_{M+t-1}$.
- 8: Given new y_{M+t} , obtain smoothed signal $(\tilde{y})_{k=1}^{M+t}$.
- 9: Estimate $(\hat{x}_k)_{k=M+t-D}^{M+t}$ using a Kalman filter.
- 10: **end for**

compensate filter delays, one has to add the same amount of extra data at the end of the original signal before filtering, for example

$$[y_1 \ y_2 \ \dots \ y_K \ y_K \ \dots \ y_K] \in \mathbb{R}^{1 \times (K+D)}, \quad (15)$$

so that the filtered signal can be shifted in phase and still has the same data length as the original signal. The reason why Test 1 provides better results than Test 2 is that the smoothed data $(\tilde{y}_k)_{k=1}^K$, $M < K < L - D$ used in Test 1 is obtained using

$$[y_1 \ y_2 \ \dots \ y_K \ y_{K+1} \ \dots \ y_{K+D}],$$

rather than (15), and requires future observations $y_{K+1} \ \dots \ y_{K+D}$. Therefore, it is practically impossible to use Test 1 for wave prediction; hence, low-pass filtering the historical data will not help the ocean wave forecasting.

IV. PERFORMANCE OF WAVE PREDICTION

In this section, the performance of ocean wave prediction using models (1) and (10) will be examined and compared with existing methods. The ocean wave data was recorded at Galway with a sampling frequency of $f_s = 2.56$ Hz and total data length $L = 3072$ [10]; the first $M = 1500$ points will be used as the historical wave. We are going to use the remaining points as the reference signal to examine the performance of ocean wave prediction.

For the ALS-based covariance estimation algorithms in [7], [8], we set $M_e := 1000$; N is determined by plotting the auto-correlation function of the innovation sequence. The initial guesses of P_m , Q and R are set to $I_n \times 10^{-1}$, $I_n \times 10^{-7}$ and 1, respectively. In order to reduce the computational complexity, we assume that all covariance matrices are diagonal.

A. Prediction Horizon and Choice of p and q in ARMA Model

The prediction horizon T_f is one of the important parameters in ocean wave forecasting, which indicates how many seconds we would like to predict the wave elevations into the

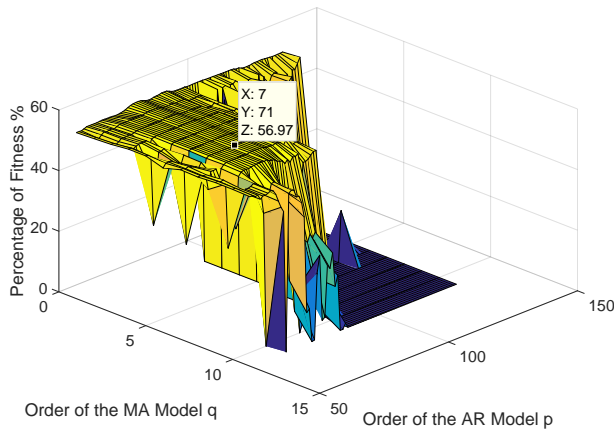


Fig. 3. Percentage of fitness orders of ARMA model

future; we define $T_f := NT_s$, where T_s is the sampling time and N is a positive integer. The prediction horizon should be long enough for the WEC to respond; typically this should be approximately equal to half of the typical period T_p [5]. Figure 1 indicates that the peak frequency ω_p is located at 0.6 rad/s, thus the prediction horizon $T_f \approx 4$ sec should be long enough.

In order to determine the orders p and q in the ARMA model (1), different combinations of p and q are tested using the first 250 predictions, Figures 3 show the prediction performance versus the orders of the ARMA model, based on the recording at 1:20 on 11 of March 2005.

B. Robustness of AR Prediction Models

We proceed to test and compare the robustness of ARMA models (1) and the AR model of [6] when there is a period of time without any signal. Figure 4 illustrates that when the signal is resumed after approximately a 38 sec signal loss, wave prediction using the ARMA models shows a strong tracking ability and good prediction performance, while the wave prediction using the AR model in [6] tends to infinity, due to mistakes we discussed earlier.

C. Which Prediction Model is Better?

In this section, we compare prediction results and computation time based on model (1) and (10). Table I compares the forecasting performance between using ARMA model (1) (without smoothing) and Forecasting Test 2. Table I verifies the statement we made in Section III that low-pass filtering the historical data will not improve the accuracy of wave forecasting.

Tables I and II compares the wave prediction results and the computation time taken T_f between ARMA model (1) and AR model (10) using 10 different ocean wave data. Tables I and II show that prediction using an ARMA model is more accurate and much faster than the AR model (10). Figure 5 illustrates 140 sec of ocean wave prediction with approximately 4 sec

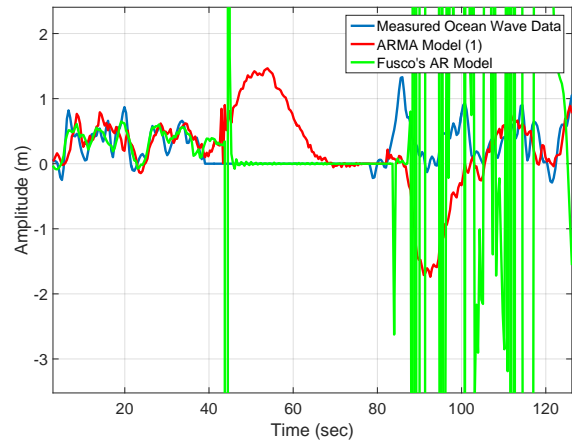


Fig. 4. Robustness tests for AR prediction with models

TABLE I
OCEAN WAVE PREDICTION RESULTS

Time, Date	p	q	ARMA Model (1)		Forecasting Test 2	
			Fit%	T sec	f_c rad/s	Fit%
13:20, 02/02/05	87	0	56	51.9	1	53
22:20, 30/11/04	98	0	52	51.5	0.9	27
08:20, 01/11/04	63	7	49	51.6	1	42
00:27, 16/02/05	82	0	53	51.9	1	49
17:20, 23/01/05	51	5	62	51.5	0.9	57
01:20, 11/03/05	71	7	58	51.6	1.5	48
00:22, 01/10/04	64	2	53	51.7	0.8	37
14:20, 24/10/04	87	2	53	51.7	1.2	34
01:20, 15/12/04	63	3	54	51.6	0.7	33
15:20, 31/12/04	51	5	51	51	1	39

TABLE II
OCEAN WAVE PREDICTION RESULTS WITH AR MODEL (10)

Time, Date	p (rad/s)	Fit%	T (sec)
13:20, 02/02/05	93	53	648
22:20, 30/11/04	84	45	535
08:20, 01/11/04	94	45	663
00:27, 16/02/05	82	50	512
17:20, 23/01/05	96	60	697
01:20, 11/03/05	71	55	394
00:22, 01/10/04	53	51	233
14:20, 24/10/04	94	49	667
01:20, 15/12/04	63	46	317
15:20, 31/12/04	57	40	264

prediction horizon using both ARMA model (1) and AR model (10), reference waves are based on the recording at 22:20 on 30 November 2004. Figure 6 shows the percentage of fitness versus the prediction horizon using both ARMA model (1) and AR model (10), based on the recording at 17:20 on 11 January 2005.

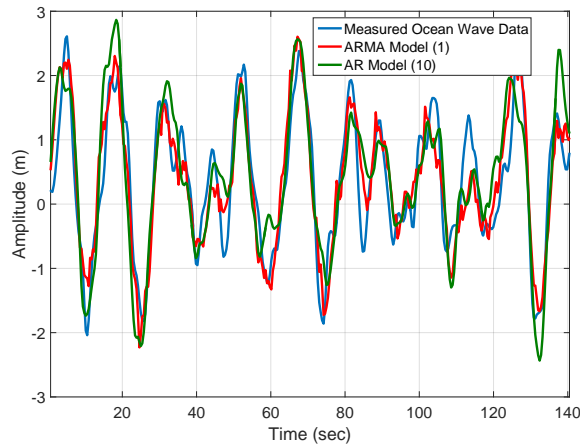


Fig. 5. Wave prediction performances for different prediction models

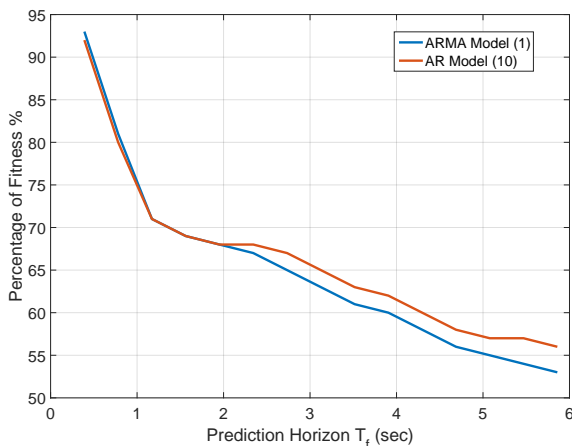


Fig. 6. Wave prediction performances for different prediction models

V. CONCLUSIONS

In this paper, we focused on forecasting ocean wave elevations for wave energy converters. Two existing ARMA/AR based state space wave prediction models were firstly reviewed, for which the noise covariances in the AR model were determined using ALS algorithm. This was followed by

discussions of the accuracy and stability issues of the wave prediction method of [6]. Two forecasting tests were used to investigate the influence of using smoothed data in wave forecasting. We found there is no evidence to indicate any improvements in prediction accuracy by using the smoothed data rather than original data.

Following this, we tested the performance and efficiency of both model (1) and (10) using 10 different ocean wave data files. Results have shown that ARMA model (1) gave the best prediction performance and is the fastest, compared to the AR model (10) with ALS method.

Future work could involve using the expectation maximization (EM) method to identify parameters in ARMA model. Moreover, instead of predicting ocean waves at one location from one measurement, one could predict ocean waves based on the reconstruction of the wave field from an array of distributed measurements.

REFERENCES

- [1] George E. P. Box, Gwilym M. Jenkins, Gregory C. Reinsel, and Greta M. Ljung. *Time Series Analysis: Forecasting and Control (5th Edition)*. WILEY, New Jersey, U.S., 2015.
- [2] J. Cruz. *Ocean Wave Energy Current Status and Future Perspectives*. Springer, 2008.
- [3] Joshua Earnest. *Wind Power Technology*. Prentice-Hall of India Pvt. Ltd, Delhi, India, 2014.
- [4] J. Falnes. A review of wave-energy extraction. *Marine Structures*, 20(4):185–201, 2007.
- [5] Zhe Feng and Eric C. Kerrigan. Declutching control of wave energy converters using derivative-free optimization. In *Proceeding of the 19th IFAC World Congress*, Cape Town, South African, August 2014.
- [6] Francesco Fusco. Short-term wave forecasting as a univariate time series problem. Technical Report 2009-03, National University of Ireland, December 2009.
- [7] Ming Ge. *Noise Covariance Identification for Filtering and Prediction*. PhD thesis, Imperial College London, 2016.
- [8] Ming Ge and Eric C. Kerrigan. Noise covariance estimation for time-varying and nonlinear systems. In *Proceeding of The 19th IFAC World Congress*, Cape Town, South Africa, August 2014.
- [9] James Douglas Hamilton. *Time Series Analysis*. Princeton University Press, New Jersey, U.S., 1994.
- [10] Marine Institute Ireland. Ocean wave height digital dataset. <http://marine.ie/Home/>, 2010.
- [11] M. R. Rajamani and J. B. Rawlings. Application of a new data-based covariance estimation technique to a nonlinear industrial blending drum. Technical Report 2007, University of Wisconsin-Madison, 2007.
- [12] Paris: REN21 Secretariat. *Renewables 2012: Global status report*. Worldwatch Institute Washington, DC, Paris, France, 2012.
- [13] Dan Simon. *Optimal State Estimation: Kalman, H_∞ and Nonlinear Approaches*. John Wiley & Son, Inc., New Jersey, U.S., 2006.