

## Inclusive Production of the $X(4140)$ State in $p\bar{p}$ Collisions at D0

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We present a study of the inclusive production of the  $X(4140)$  state with the decay to the  $J/\psi\phi$  final state in hadronic collisions. Based on  $10.4 \text{ fb}^{-1}$  of  $p\bar{p}$  collision data collected by the D0 experiment at the Fermilab Tevatron collider, we report the first evidence for the prompt production of an  $X(4140)$  state and find the fraction of  $X(4140)$  events originating from  $b$  hadrons to be  $f_b = 0.39 \pm 0.07(\text{stat}) \pm 0.10(\text{syst})$ . The ratio of the nonprompt  $X(4140)$  production rate to the  $B_s^0$  yield in the same channel is  $R = 0.19 \pm 0.05(\text{stat}) \pm 0.07(\text{syst})$ . The values of the mass  $M = 4152.5 \pm 1.7(\text{stat})_{-5.4}^{+6.2}(\text{syst}) \text{ MeV}$  and width  $\Gamma = 16.3 \pm 5.6(\text{stat}) \pm 11.4(\text{syst}) \text{ MeV}$  are consistent with previous measurements.

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The  $X(4140)$  state [1] was first seen in 2009 as a narrow structure in the  $J/\psi\phi$  system near threshold. The CDF Collaboration reported the first evidence [2] for this state [then designated  $Y(4140)$ ] in the decay  $B^+ \rightarrow X(4140)K^+ \rightarrow J/\psi\phi K^+$  (charge conjugation is implied throughout this Letter) and measured the invariant mass  $M = 4143.0 \pm 2.9(\text{stat}) \pm 1.2(\text{syst}) \text{ MeV}$  and width  $\Gamma = 11.7_{-5.0}^{+8.3}(\text{stat}) \pm 3.7(\text{syst}) \text{ MeV}$ . The LHCb Collaboration found no evidence for the  $X(4140)$  state [3] in a 2.4 standard deviation disagreement with the CDF measurement. However, the presence of the  $X(4140)$  state in  $B^+$  decay was later confirmed by the CMS [4] and D0 [5] collaborations. The BABAR Collaboration searched for resonant production in the  $J/\psi\phi$  mass spectrum in  $B^{\pm,0}$  decays and obtained a significance below  $2\sigma$ , but it noted that the hypothesis that the events are distributed uniformly on the Dalitz plot gives a poorer description of the data [6]. The quantum numbers of the  $X(4140)$  state have not been measured. Since both the  $J/\psi$  and  $\phi$  mesons have  $J^G J^{PC} = 0^{-1} 1^{-}$ , the state has positive  $G$  and  $C$  parities.

A meson decaying into a charmed quark pair might be an excited charmonium state. However, the standard non-relativistic quark model of a single  $c\bar{c}$  pair does not predict a narrow hadronic state at this mass. Also, at masses above the open-charm threshold of 3740 MeV, such states are expected to decay predominantly to pairs of charmed mesons and to have a much larger width than is experimentally observed. It has been suggested that the  $X(4140)$  state could be a molecular structure made of two charmed

mesons, e.g.,  $(D_s, \bar{D}_s)$ . Other possible states are hybrids composed of two quarks and a valence gluon ( $q\bar{q}g$ ), four-quark combinations ( $c\bar{c}s\bar{s}$ ), or states with higher Fock components [7]. For details, see the reviews in Refs. [8] and [9] and the references therein. The Belle Collaboration found no evidence for the  $X(4140)$  state in the process  $\gamma\gamma \rightarrow J/\psi\phi$  [10], making its interpretation as a hadronic molecule with spin-parity  $J^P = 0^+$  or  $2^+$  unlikely.

In addition to the  $X(4140)$  state, the CDF Collaboration reported [2] seeing a second enhancement in the same channel, located near 4280 MeV. A similar structure is seen by the CMS Collaboration [4] at a slightly higher mass of  $4316.7 \pm 3.0(\text{stat}) \pm 7.3(\text{syst}) \text{ MeV}$ . Belle also reports [10] a new structure at  $M = 4350.6_{-5.1}^{+4.6}(\text{stat}) \pm 0.7(\text{syst}) \text{ MeV}$ .

In this Letter we present results of a search for the  $X(4140)$  resonance in the  $J/\psi\phi$  system produced inclusively in  $p\bar{p}$  collisions, either promptly, by pure QCD, or through weak decays of  $b$  hadrons. The measured production rates are normalized to the rate of the process  $B_s^0 \rightarrow J/\psi\phi$  measured with the same data set. The data sample corresponds to an integrated luminosity of  $10.4 \text{ fb}^{-1}$  collected with the D0 detector in  $p\bar{p}$  collisions at 1.96 TeV at the Fermilab Tevatron collider.

The D0 detector consists of a central tracking system, calorimeters, and muon detectors [11]. The central tracking system comprises a silicon microstrip tracker (SMT) and a central fiber tracker (CFT), both located inside a 1.9 T superconducting solenoidal magnet. The tracking system is designed to optimize tracking and vertexing for

pseudorapidities  $|\eta| < 3$ , where  $\eta = -\ln[\tan(\theta/2)]$ , and  $\theta$  is the polar angle with respect to the proton beam direction. The SMT can reconstruct the  $p\bar{p}$  interaction vertex (the primary vertex) for interactions with at least three tracks with a precision of 0.004 cm in the plane transverse to the beam direction. The muon detector, positioned outside the calorimeter, consists of a central muon system covering the pseudorapidity region  $|\eta| < 1$  and a forward muon system covering the pseudorapidity region  $1 < |\eta| < 2$ . Both central and forward systems consist of a layer of drift tubes and scintillators inside 1.8 T iron toroidal magnets with two similar layers outside the toroids [12].

Events used in this analysis are collected with both single-muon and dimuon triggers. Muon triggers require a coincidence of signals in trigger elements inside and outside the toroidal magnets. Dimuon triggers in the central rapidity region require at least one muon to penetrate the toroid. In the forward region, both muons are required to penetrate the toroid.

We study a wide range of the  $J/\psi\phi$  invariant mass, from threshold to 5.7 GeV, covering both the  $X(4140)$  state and the decay  $B_s^0 \rightarrow J/\psi\phi$ . Candidate events are required to include a pair of oppositely charged muons in the invariant mass range  $2.9 < M(\mu^+\mu^-) < 3.3$  GeV, consistent with  $J/\psi$  decay, accompanied by two additional particles of opposite charge, assumed to be kaons, with  $p_T > 0.4$  GeV and  $1.011 < M(K^+K^-) < 1.030$  GeV. In the event selection, both muons are required to be detected in the muon chambers inside the toroidal magnet, and at least one of the muons is required to be also detected outside the iron toroid [12]. Each muon candidate is required to match a track found in the central tracking system, and each of the four final-state tracks is required to have at least one SMT hit and at least one CFT hit. The  $J/\psi$  signal purity and the  $\phi$  signal purity in the selected sample are approximately 75% and 20%, respectively.

The dimuon invariant mass is constrained to the world-average  $J/\psi$  mass [1], and the four-track system is constrained to a common vertex. To reconstruct the primary vertex, tracks are selected that do not originate from the  $J/\psi\phi$  candidate, and a constraint is applied to the average beam position in the transverse plane. We require  $J/\psi\phi$  candidates to have  $5 < p_T < 20$  GeV and rapidity  $|y| < 2$ .

We define the signed decay length of the  $J/\psi\phi$  system,  $L_{xy}$ , to be the vector pointing from the primary vertex to the

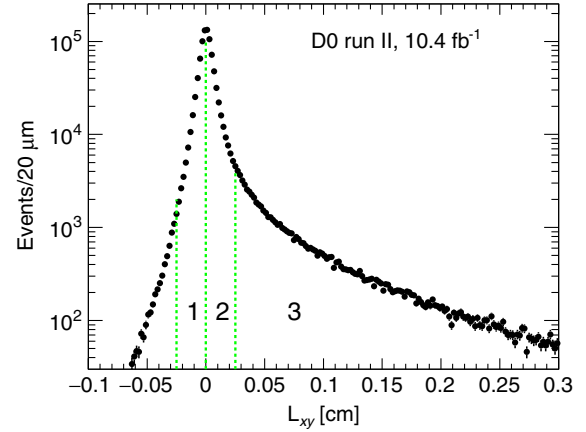


FIG. 1 (color online). Transverse decay length distribution of  $J/\psi\phi$  candidates. The vertical lines define the three regions discussed in the text.

decay vertex, projected onto the direction of the transverse momentum. The distribution of  $L_{xy}$  for the selected events is shown in Fig. 1.

We focus on two ranges of the  $J/\psi\phi$  invariant mass,  $M(J/\psi\phi) < 4.36$  GeV and  $4.8 < M(J/\psi\phi) < 5.7$  GeV. The low-mass range includes the  $X(4140)$  state. The high-mass range includes the reference decay process,  $B_s^0 \rightarrow J/\psi\phi$ . Background arises primarily from nonresonant pairs in the  $\phi$  mass window. At low  $L_{xy}$ , the background comes from  $J/\psi$  mesons directly produced in  $p\bar{p}$  collisions combined with random particles from the underlying event. At higher values of  $L_{xy}$ , the background consists of  $J/\psi$  mesons paired with random products of  $b$ -hadron decays.

We divide the data in each mass range into three independent subsamples according to the value of  $L_{xy}$ : (1)  $-0.025 \leq L_{xy} < 0$  cm, (2)  $0 \leq L_{xy} \leq 0.025$  cm, and (3)  $L_{xy} > 0.025$  cm. Region 1 includes half of the prompt events and almost no  $B$ -decay events (the fit result shown in Table I is  $37 \pm 26$  events). Region 2 includes the remaining half of all prompt events and a fraction of nonprompt events. The rest of the nonprompt events populate region 3. Given the average resolution of 0.006 cm in  $L_{xy}$ , we assume that the fraction of prompt events in region 3 is negligible. We perform binned maximum likelihood fits to the distributions of the  $J/\psi\phi$  invariant mass for events in the six subsamples

TABLE I. Summary of event yields in three  $L_{xy}$  regions and their sum for  $B_s^0$  and  $X(4140)$ . For regions 1 and 2 the mass of  $X(4140)$  is assumed to be 4152.5 MeV and the width is taken to be 16.3 MeV. Also shown are the deduced yields for the nonprompt and prompt production of  $X(4140)$ . The uncertainties are statistical.

Parent	$-0.025 < L_{xy} < 0$ cm	$0 < L_{xy} < 0.025$ cm	$L_{xy} > 0.025$ cm	Sum
$B_s^0$	$191 \pm 143$	$804 \pm 169$	$3166 \pm 81$	$4161 \pm 236$
$X(4140)$	$511 \pm 120$	$837 \pm 135$	$616 \pm 170$	$1964 \pm 248$
$X(4140)$ nonprompt	$37 \pm 26$	$156 \pm 54$	$616 \pm 170$	$809 \pm 175$
$X(4140)$ prompt	$474 \pm 123$	$681 \pm 149$	$\equiv 0$	$1155 \pm 193$

defined above. In the fits in the  $B_s^0$  mass region, the signal is described by a Gaussian function and the background is described by a second-order Chebychev polynomial. We also allow for the presence of the decay  $B^0 \rightarrow J/\psi\phi$ , where we set the mass to the world-average  $B^0$  mass, and we find no evidence of a signal. The fit for region 3 yields  $3166 \pm 81 B_s^0$  events.

In fitting the low-mass range, we assume a signal described by an  $\mathcal{S}$ -wave relativistic Breit-Wigner function convolved with a Gaussian resolution of  $\sigma(M) = 4$  MeV. The background is parametrized by the function  $f(m) \propto m(m^2/m_{\text{thr}}^2 - 1)^{c_1} e^{-mc_2}$  where  $m_{\text{thr}}$  is the kinematic threshold, and  $c_1$  and  $c_2$  are free parameters. For events in the

$L_{xy}$  region 3, we allow the signal mass and width parameters to vary. The fit yields  $616 \pm 170$  signal events, a mass of  $4152.5 \pm 1.7$  MeV, and a width of  $16.3 \pm 5.6$  MeV. The statistical significance of the signal, based on the increase of the likelihood with respect to the fit with no signal,  $-2\Delta \ln \mathcal{L} = 42.5$  for 3 degrees of freedom, is 5.9 standard deviations. For the fits in  $L_{xy}$  regions 1 and 2, we set the mass and width to the region 3 values.

The mass distributions with superimposed fits for both mass regions and for all three  $L_{xy}$ 's are shown in Fig. 2. The  $X(4140)$  and  $B_s^0$  yields are presented in Table I. We also show the expected number of  $X(4140)$  events originating from  $b$ -hadron decays in the two low  $L_{xy}$  regions, assuming

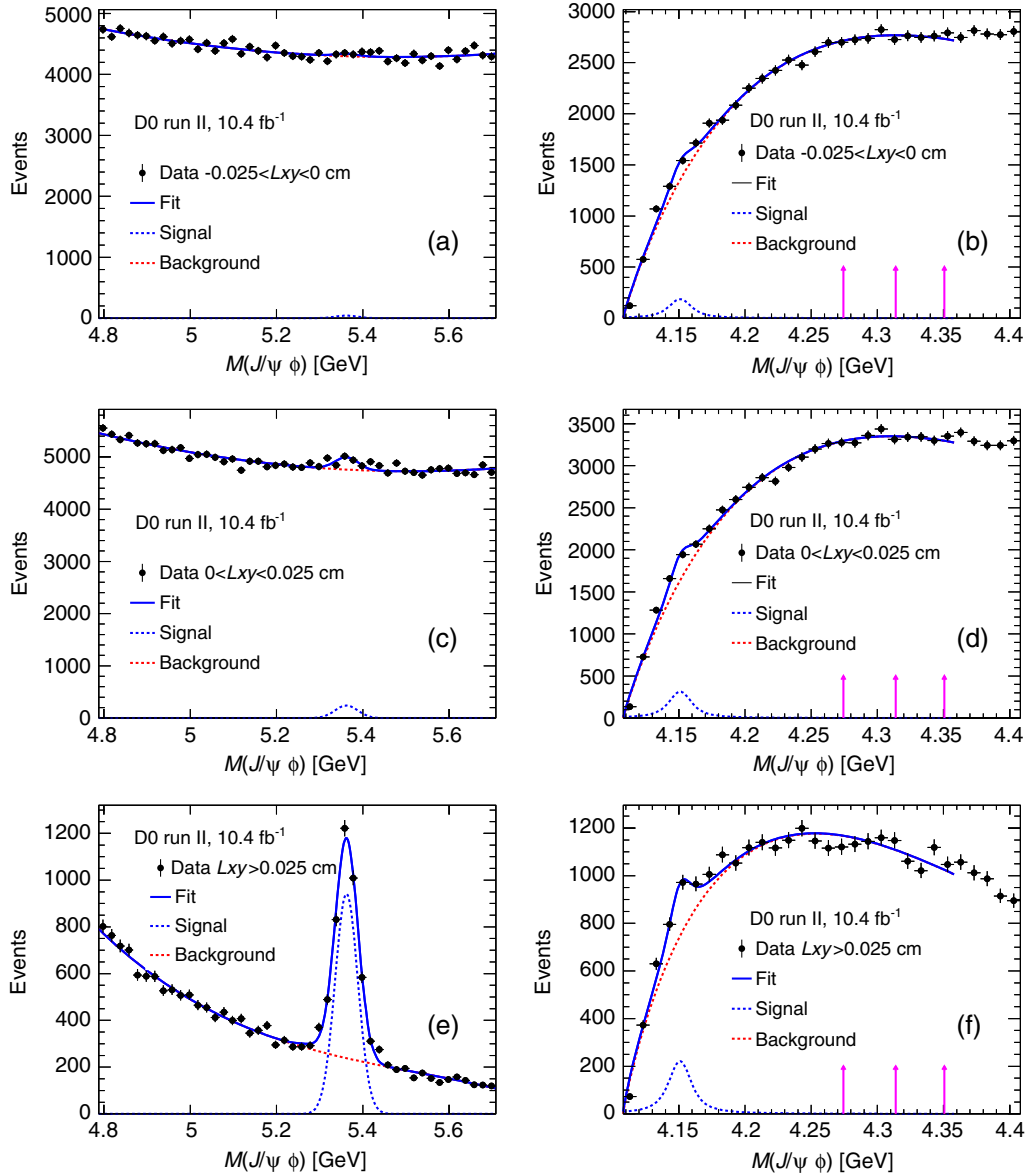


FIG. 2 (color online). Invariant mass distribution of the  $J/\psi\phi$  candidates in the mass window around (left panels)  $B_s^0$  and (right panels)  $X(4140)$ , for events with (a),(b)  $-0.025 < L_{xy} < 0$  cm, (c), (d)  $0 < L_{xy} < 0.025$  cm and (e),(f)  $L_{xy} > 0.025$  cm. The arrows indicate the structures seen by CDF [2], CMS [4], and Belle [10]. The signal and background models are described in the text.

that the  $L_{xy}$  distribution of the “nonprompt”  $X(4140)$  state is similar to that of  $B_s^0$ . For the regions 1 and 2, we find an excess of signal events, indicating prompt production of the  $X(4140)$  state. For events in region 2, the increase in the likelihood between the fit with a free signal yield and the fit with the expected nonprompt contribution only,  $-\Delta \ln \mathcal{L} = 23.6$ , corresponds to a statistical significance of  $4.9\sigma$  for the net prompt signal. The statistical significance of the total signal in this  $L_{xy}$  region is  $6.2\sigma$ . For region 1, the corresponding values of statistical significance are  $3.9\sigma$  and  $4.2\sigma$ . If the mass and width parameters are allowed to vary, the fit for region 2 gives the total yield  $N = 932 \pm 216$ ,  $M = 4146.8 \pm 2.4$  MeV, and  $\Gamma = 15.8 \pm 3.8$  MeV. The data in region 1 do not yield a stable fit. Fixing the  $X(4140)$  mass at 4152.5 MeV in this region, as obtained in region 3, we fit a total yield of  $N = 601 \pm 205$  and  $\Gamma$  of  $19.8 \pm 5.9$  MeV. A common fit to all three regions yields  $N = 2312 \pm 343$ ,  $M = 4149.6 \pm 1.2$  MeV, and  $\Gamma = 17.7 \pm 2.5$  MeV.

There are several uncertainties that may affect measurements of the  $X(4140)$  yield, mass, and width, the ratio  $R$  of the yields of nonprompt  $X(4140)$  and  $B_s^0$  events, and the fraction  $f_b$  of all  $X(4140)$  events that originate from weak decays of  $b$  hadrons.

The mass resolution of 4.0 MeV obtained in the simulations is in agreement with an approximately linear rise with the released kinetic energy for decays with a similar topology:  $\psi(2S) \rightarrow J/\psi\pi^+\pi^-$ ,  $X(3872) \rightarrow J/\psi\pi^+\pi^-$ , and the decay  $B_s^0 \rightarrow J/\psi\phi$ . We assign an uncertainty of  $\pm 0.1$  MeV to the resolution at the  $X(4140)$  mass.

We assign an asymmetric uncertainty of 3 MeV to the  $J/\psi\phi$  mass scale in the vicinity of  $X(4140)$ , based on the range of the mass deficit between 1 and 5 MeV compared to world-average values, found in several channels with this topology. We assign the uncertainty in the signal model, taken from the range of results obtained with relativistic and nonrelativistic Breit-Wigner shapes and a relativistic  $P$ -wave Breit-Wigner shape. Simulations show the event reconstruction and selection efficiency to be independent of the  $M(J/\psi\phi)$  invariant mass, with a possible variation of  $\pm 10\%$  [5]. The possible variation of the efficiency within the  $X(4140)$  mass range affects the mass, width, and yield of the signal. We have assessed the effects of the fitting procedure and the background size and shape by lowering the upper edge to 4.28 GeV and raising it to 4.4 GeV, and by using 8-MeV instead of 10-MeV bins. For each of the measured quantities, we assign a symmetric uncertainty equal to one half of the difference between the extreme results.

Some of the single-muon triggers include a trigger term requiring a presence of tracks with a nonzero impact parameter. Events recorded solely by such triggers constitute approximately 5% of all events. Assuming that such triggers are 100% efficient for events originating from weak decays of  $b$  hadrons and reject all prompt events, we apply a 5% correction to the prompt yield. We assign a systematic

uncertainty of  $\pm 5\%$  on the fraction  $f_b$  due to this correction. Finally, our assumption of the equality of the relative rates in regions 1–3 for the nonprompt  $X(4140)$  and  $B_s^0$  events is based on an expectation of the equality of the average lifetime of  $b$ -hadron parents of the  $X(4140)$  event and that of the  $B_s^0$  event in the  $J/\psi\phi$  channel. The world average of the  $B_s^0$  lifetime is 6% lower than the lifetime averaged over all  $b$  hadron species [1]. We assign an asymmetric uncertainty in the ratio  $R$  and the fraction  $f_b$  based on this difference. The systematic uncertainties are summarized in Table II.

We test the stability of the results of the event selection by changing the  $\phi$  mass window to  $1.012 < M(K^+K^-) < 1.029$  GeV. As additional cross-checks, we perform fits to subsamples corresponding to the transverse momentum ranges  $5 < p_T < 10$  GeV and  $10 < p_T < 20$  GeV, to early and late data-taking periods, and to events in the central ( $|y| < 1$ ) and forward rapidity regions. In each case, the background shape in the two subsamples is well described by the same functional form, although it requires different values of the parameters. In all cases the sums of the resulting signal yields agree with the total yield within a few events.

Our measured values of the mass and width of the  $X(4140)$  state are compared with earlier measurements in Table III. The ratio of the  $X(4140)$  to  $B_s^0$  yield for events with  $L_{xy} > 0.025$  cm is  $R = 0.19 \pm 0.05(\text{stat}) \pm 0.07(\text{syst})$ . After correcting for the efficiency of this  $L_{xy}$  cut and for the trigger bias, we find the fraction of  $X(4140)$  events originating from  $b$  hadrons to be  $f_b = 0.39 \pm 0.07(\text{stat}) \pm 0.10(\text{syst})$ . The yield for the  $X(4140)$  state at  $L_{xy} > 0.025$  cm can also be compared with the yield of  $52 \pm 19$  events of  $X(4140)$  from the decay process  $B^+ \rightarrow J/\psi\phi K^+$  obtained by D0 [5] for the same data set. After correcting for a factor of  $2.5 \pm 0.5$  for the efficiency of the full reconstruction of the  $B^+$  decay and lower kaon  $p_T$  threshold, we expect the yield from the  $B^+$  decay to be  $\approx 130 \pm 60$  events in this analysis. Our observed yield of  $616 \pm 170$  events exceeds this estimate, suggesting that decays of  $b$  hadrons other than the decay  $B^+ \rightarrow J/\psi\phi K^+$  contribute to the nonprompt production of an  $X(4140)$  state.

TABLE II. Summary of systematic uncertainties.

Source	Mass (MeV)	Width (MeV)	Rate nonprompt (%)	Rate prompt (%)
Mass resolution	$\pm 0.1$	$\pm 0.2$	$\pm 1$	$\pm 1$
Mass bias	$^{+3}_{-0}$	$\dots$	$\dots$	$\dots$
Efficiency	$\pm 4$	$\pm 5$	$\pm 4$	$\pm 4$
Signal model	$\pm 1$	$\pm 2.7$	$\pm 13$	$\pm 15$
Fitting range	$\pm 3$	$\pm 7.0$	$\pm 20$	$\pm 6$
Bin size	$\pm 1.6$	$\pm 7.0$	$\pm 25$	$\pm 10$
Trigger bias	$\dots$	$\dots$	$\dots$	$\pm 5$
Mean lifetime	$\dots$	$\dots$	$-1.5$	$+1.5$
Total	$^{+6.2}_{-5.4}$	$\pm 11.4$	$\pm 35$	$\pm 19$

TABLE III. Summary of  $X(4140)$  measurements.

Experiment	Process	Mass (MeV)	Width (MeV)
CDF [2]	$B^+ \rightarrow J/\psi\phi K^+$	$4143.0 \pm 2.9 \pm 1.2$	$11.7^{+8.3}_{-5.0} \pm 3.7$
CMS [4]	$B^+ \rightarrow J/\psi\phi K^+$	$4148.0 \pm 2.4 \pm 6.3$	$28^{+15}_{-11} \pm 19$
D0 [5]	$B^+ \rightarrow J/\psi\phi K^+$	$4159.0 \pm 4.3 \pm 6.6$	$19.9 \pm 12.6^{+3.0}_{-8.0}$
D0 (this work)	$\bar{p}p \rightarrow J/\psi\phi + \text{anything}$	$4152.5 \pm 1.7^{+6.2}_{-5.4}$	$16.3 \pm 5.6 \pm 11.4$

The  $J/\psi\phi$  invariant mass distributions presented in Fig. 2 show no evidence for states in the mass region  $4250 < M(J/\psi\phi) < 4375$  MeV. Fits allowing for the states reported by CDF, CMS, and Belle, at  $L_{xy} > -0.025$  cm, yield  $351 \pm 268$ ,  $-283 \pm 468$ , and  $-382 \pm 247$  events, respectively. Using the  $CL_s$  method [13], we obtain the 95% upper limits of 808, 750, and 296 events for the three states. In this upper limit calculation we did not account for systematic uncertainties, as they were checked to have a negligible impact. The corresponding 95% upper limits on the rate ratios relative to the total yield of the  $X(4140)$  state are 0.48, 0.42, and 0.18, where the statistical and systematic uncertainties on the measured yield of the  $X(4140)$  state are taken into account.

In summary, we have carried out the first search for inclusive production of the  $X(4140)$  state in hadronic collisions. We find strong evidence for its direct, prompt production, and we observe its production in weak decays of  $b$  hadrons with a rate exceeding the expected rate for the known decay  $B^+ \rightarrow J/\psi\phi K^+$ . The significance of the prompt production, including systematic uncertainties, is  $4.7\sigma$ . This is the first evidence for the prompt production of the  $X(4140)$  state. The significance of the nonprompt production, including systematic uncertainties, is  $5.6\sigma$ . The nonprompt production rate of the  $X(4140)$  state relative to the  $B_s^0$  value observed in the same final state is  $R = 0.19 \pm 0.05(\text{stat}) \pm 0.07(\text{syst})$ . Assuming a relativistic Breit-Wigner line shape, we measure the mass and width of the  $X(4140)$  state to be  $M = 4152.5 \pm 1.7(\text{stat})^{+6.2}_{-5.4}(\text{syst})$  MeV and width  $\Gamma = 16.3 \pm 5.6(\text{stat}) \pm 11.4(\text{syst})$  MeV, consistent with previous measurements [2,4,5].

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