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THE INTEGRATION OF WIND POWER AND HYDROPOWER IN AN ELECTRICITY MARKET WITH A LARGE HYDROELECTRIC GENERATOR

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Abstract: This paper examines the effects of increased intermittency, caused by climate change, on strategic hydroelectric production when wind power is integrated with hydroelectricity. It is shown that increased uncertainty in water inflows caused by climate change raises hydroelectric output and increases the probability of overflow in future periods. The increased variability of wind speeds due to climate change will induce a rise in the price of electricity and more extreme electricity prices in the future.

Keywords: Hydroelectricity, Wind Integration, Climate Change, Energy storage, Electricity markets.
JEL: Q420, D430

HIGHLIGHTS:

- When integrated with wind power a strategic hydroelectric generator will store only some of the electricity generated by the wind turbines.
- Increased wind intermittency will lead to a rise in the price of electricity.
- Increased uncertainty in water inflow will lead to an increase in hydroelectric supply and the probability of overflows.

1. INTRODUCTION

Recent concerns over climate change have led to the retirement of fossil fuel generators. These generators are being replaced by generators that use renewable energy resources such as wind, solar energy, water etc. Once large capital investments have been made, these generators have the potential to generate electricity at little or no variable cost. Currently, the fastest growing renewable energy generators are wind turbines. Wind turbines convert the kinetic energy produced by wind into mechanical energy which is used to operate a generator. Using wind turbines guarantees that electricity can be produced without any added fuel costs or pollution. Wind speeds are however intermittent over time and this ensures that the electricity produced by wind turbines will be unpredictable and unreliable. One solution to the problem associated with intermittency is storage. Today, hydroelectric reservoirs offer the most viable form of storing wind power in large quantities. Although the capital costs associated with constructing a hydroelectric generator are large, the marginal costs are negligible because it involves only water, not any costly generating fuels. Furthermore, hydropower is a clean form of electricity: its production does not generate any greenhouse gases. Another advantage of hydropower is the ease and speed with which generation can be started or turned off to meet large fluctuation in demand or supply.

In recent years, countries in Europe have made significant steps towards increasing the use of renewable energy. These were prompted by the European Energy directive[1], which requires that 20% of the energy needs produced in the EU comes from renewable energy resources with Denmark even committing to targets of 50% by 2020 and 100% by 2050. To meet the European Energy directive, there has been significant investment in both onshore and offshore wind farms across Europe in recent years. Since wind power is intermittent, hydroelectric generators with reservoirs will be expected to play a big role in integrating the power produced by wind turbines. When wind turbines are integrated with hydroelectric generators, the intermittent nature of water inflows and wind speeds affects the stock of water over time. Furthermore, when a hydroelectric generator behaves strategically, the randomness of water inflows combined with the intermittency of wind speeds will have significant effect on the price of electricity in deregulated electricity markets. When too much water is left in the reservoir at the start of a period, and if the water inflow in the next period is large, the capacity of the reservoir will be exceeded, and spillage will occur. Since wasted water does not generate revenues, it might be better to release more water earlier. Thus the effects of intermittency can be complicated.

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[1] European commission, Energy 2020: A Strategy for competitive, sustainable and secure energy retrieved on 16th of May2015: https://ec.europa.eu/energy/sites/ener/files/documents/2011_energy2020_en_0.pdf

As the effects of climate change result in changing wind patterns, and increased uncertainty of water inflow rates, the effects of intermittency can be expected to be more prevalent in the coming years. The objective of this paper is to analyze the effect of intermittency caused by climate change on the strategic behaviour of a hydroelectric generator that integrates wind power. Specifically, we analyze how its strategic behavior is affected by increased intermittency of wind power and the randomness of water inflows caused by climate change. It is shown that increased intermittency of water inflows will increase the release of water by the hydroelectric generator while increased wind intermittency will increase the price of electricity in deregulated electricity markets.

There is an extensive literature on hydro scheduling in regulated markets, mainly in the field of operations research. The research in this area takes as given the demand for electricity in each period – an hour, a day, a week, or a month – and the objective is to find a hydro schedule that meets demand at minimum cost. The technique used to solve this type of problem is dynamic programming – deterministic or stochastic – according to whether inflows are assumed to be deterministic or random. There is a long history of using operations research to support regulatory policy and decision in the electricity industry. This approach is not appropriate in the present days of deregulated electricity markets in which generators with market power behave strategically. Some researchers in operations research have introduced strategic behavior of large electric generators into their models by allowing these producers to compete in quantities according to the Cournot model of competition. For example, Bushnell (2003) and Kelman et al. (2001), calibrate their models, using industry data, and then simulate these models. The results of the simulations indicate that some generators find it more profitable to direct more hydroelectric generation to off-peak periods than they would if they behave competitively. For models dealing with the integration of wind power, hydropower, and thermal power plants that burn fossil fuels, see Benitez et al (2008), and van Kooten (2010), who formulate a nonlinear programming problem to assign the share of electric generation to the various power plants so that their combined output meets demand at minimum cost. These models – which are typical of the operations research approach – are calibrated using industry data from Alberta, a province of Canada, and then simulated to gain insights about the impact of wind power penetration in this province. Using wind data and trade data, Green and Vasilakos (2012) have also analyzed the impact of wind power on trade patterns in integrated markets. Using data on the electricity trade in the Nordic region they argued that Denmark which has volume of wind turbines in its capacity mix was using hydroelectric reservoirs in other Nordic countries to store wind power.

In recent years, one of the arguments in favour of wind as a renewable source of electricity is its ability to reduce the emission of greenhouse gases by displacing high polluting fossil fuel generators. Recent empirical analysis has shown that the effect of wind turbines on emissions will depend on the capacity mix and the structure of the market being analysed. Therefore, subsidies paid to renewable energy generator should depend on the resulting emission saving attributable to the use of renewable energy. Using ERCOT (Electricity Reliability Council of Texas) data, Cullen (2013) uses an econometric analysis to measure the volume of generating capacity that is displaced by wind turbine production. His estimates show that 1 MWh of wind power produced will displace 0.72MWh of natural gas and 0.28 MWh of coal generators. Similarly, Kaffine et al (2013) conclude that that wind power will reduce more greenhouse gases when coal generators make up a larger share of the capacity mix than natural gas. They show that as the proportion of natural gas generators in the capacity mix grows, more natural gas generators will be displaced by wind power production. Finally, it is shown that as wind capacity rises the ability of natural gas turbines to accommodate wind will go down and wind power will then start to displace coal generators.

In contrast with the large empirical literature on the subject, the theoretical literature on the strategic behavior of large hydroelectric generators is sparse. Crampes and Moreau (2001) formulate a two-period model of competition between a hydroelectric generator and a firm that burns fossil fuels to generate electricity. In their model, the total volume of water available for exploitation over the two periods is known and already exists in the reservoir at the beginning, and there is no inflow in the second period. The game is solved for an open-loop equilibrium as well as a closed-loop equilibrium, and these solutions are shown to be different. The authors also solve the central planner’s problem, and show that the strategic behavior of the hydroelectric generator over the two periods is dynamic, and this induces a dynamic response from the generator that burns fossil fuels to generate electricity. Mathiesen et al. (2013) formulate a two-period model in the context of Norway in which all the firms are hydroelectric generators, and in which inflows are stochastic. The model is solved for two cases: (i) the producers behave competitively and (ii) the producers collude and behave like an integrated monopoly. Their analysis indicates that market power does not always lead to higher prices. They show that the outcome of prices will depend on the value of the elasticity of demand in both periods.

In the model we formalize, the supply side consists of a dominant firm and a competitive fringe. The dominant firm is a hydroelectric generator with a large reservoir capacity, while the competitive fringe consists of many small producers who burn fossil fuels to generate electricity. The dominant firm enters a power purchase agreement with wind power producers to buy all their wind power at a fixed price. Since wind power must be dispatched at the instant it is generated, and because the price it pays privately owned wind farms is fixed, the dominant firm can be thought of as a supplier of both wind power and hydroelectricity[2]. In the model, the output of wind power and water inflows are stochastic. The time horizon for the problem is the length of the water cycle, which is taken to be one year. We assume that the water cycle is made of two discrete periods. The problem faced by the dominant firm is to design a schedule of water release, taking into consideration the wind power generated in each period, to maximize profits over the water cycle. It is shown that only part of the power generated by wind turbines is stored as water while the remaining is sold in the market. In addition, wind power will displace the fossil fuel generator with the highest marginal cost. Finally, it is shown that the increased variability of wind speeds will increase the price of electricity while the increased uncertainty of inflows will increase the supply of hydropower.

The paper is organized as follows. In section 2 we present some background to motivate our modelling strategy. In section 3, the model is presented. The equilibrium is analyzed in Section 4 while the impact of climate change is analysed in section 5. Section 6 contains some concluding remarks while the proofs of some technical results are given in the four appendixes.

2. BACKGROUND

The storage potential of a conventional hydroelectric reservoir depends on the flow rate of rivers and streams that fluctuate over time[3]. Hence the stock of water in a hydroelectric reservoir tends to fluctuate over time as water is released to produce electricity and as water flows into the reservoir from rivers and streams. As an example, the annual pattern of inflow in Brazil consists of two seasons: the dry season and the wet season. The dry season begins in May and ends in October, with an average inflow of 6000m3/s in August. The wet season begins in November and ends in April, with an average inflow of 16,000m3/s in February. The inflow variability is much higher in the wet season than in the dry season, with a standard deviation of $5000m^3/s$ in February and 2000 m^3/s in August. In Norway, the annual water cycle consists of four periods. Starting in spring, there is a large inflow of water between weeks 16 and 21 that is the result of snow melting. In the summer, there is little rain. Autumn is a rainy season, and the inflow rises. In winter, precipitation comes in the form of snow, which is not available for electricity generation until spring. This seasonal pattern repeats itself year

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[2] In some cases, hydroelectric generators provide balancing services which are usually regards as a form of operating reserves. These balancing services are offered to wind turbine generators and electric utilities. Since the operating reserve market and the wholesale electricity market are interdependent, the effects of operating reserves will have some effects on the wholesale price of electricity. Since this paper is concerned with the integration of wind turbines with a dominant hydroelectric generator the impact of operating reserves are ignored. Interested readers can consult chapter two of Williams (2015) for an economic analysis of operating reserves in a competitive market.

[3] Other hydroelectric generators like pumped storage generators are able to pump water from a lower reservoir to an upper reservoir and are therefore not affected by the inflow of water from rivers. On the other hand run of the river hydroelectric generators don't have reservoirs and hence their production is correlated to water inflows and hence they cannot store water. This paper will focus exclusively on conventional hydroelectric generators.

after year, and inflows vary considerably over the year.

Due to their renewable nature, wind power and hydropower are greatly impacted by the effects of climate change. Climate change affects hydroelectric generation through changes in river flows, evaporation, and dam safety while its effect on wind power comes in the form of shifts in the geographical distribution and the variability of wind speed. Beldring et al. (2006), who used atmospheric-ocean general circulation and regional climate models to map the future hydrological evolution on a time calendar for the Nordic region, have shown that in general there would be an overall increase in river flows and thus an increase in hydroelectric supply. However, the unstable winter climate would lead to infrequent and fast inflows that may strain the capacity of reservoirs. Analyzing the effect of climate change on wind power potential and wind speed distributions, Breslow and Sailor (2002) show that that continental USA is likely to experience decline in wind speed and consequently a decline in the potential for wind power. On the other hand, Pryor and Barthelmie (2010) find that climate change will result in a small increase in the wind energy resource in northern Europe. Since climate change has an impact on both wind speeds and river flows, the intermittency of wind farms and water inflow cannot be analyzed in isolation. Moreover, as the investment in renewable energy continues to grow, the impacts of climate change will become more pronounced in electricity market.

The large capital and strict geographical requirements necessary for building hydroelectric plants restrict the building of new hydroelectric capacity. Hence some large hydroelectric generators are government owned and available in a few electricity market. Fortunately, interconnections among power systems ensure that the benefits of hydroelectric generation can be accessed across power grids. It also implies that regulated utilities with surplus water in their reservoir can act as profit-maximizing suppliers of electricity in interconnected markets. One such example is Hydro Quebec (here after HQ), which is a government-owned public utility in the province of Québec, Canada. The company is mandated by law to provide a certain amount of power to the residents of this province. The residual power can then be sold in neighboring electricity markets. For this reason, HQ is an active supplier of electricity in the New York, Pennsylvania, New Jersey, and Maryland Interconnection as well as in Vermont. In the New York electricity market, it is specifically recognized that HQ has the ability to significantly influence the price of electricity in western New York. According to Patton et al. (2015):

"*Net imports from Hydro Quebec to New York accounted for 73 percent of net imports across all primary interfaces in 2014. In the past, flows from Hydro Quebec typically rose in the summer* *months and during periods of high natural gas prices, reflecting the flexibility of their hydroelectric generation to export power to New York when it was most valuable to do so. From 2013 to 2014, net imports from Hydro Quebec fell 12 percent. Most of the reduction occurred in the winter because of more frequent extreme cold weather conditions that led to limited hydro production and increased winter peaking load in Hydro Quebec. The reduction in the winter had significant price effects in Western New York"*

HQ's influence is as a result of the intertie that connects the power grid in Québec with that in New York. The low cost of the power sold by HQ and the high demand for electricity in New York have also encouraged plans to strengthen this interconnection by expanding its capacity in the coming years.

Recently, HQ has been diversifying its renewable energy portfolio by signing power purchase agreements with wind farms in and outside Quebec. These agreements are beneficial for both parties in different ways. HQ can better manage the water in its reservoir by storing wind power, while wind turbines are able to receive a guaranteed price for their power. Since HQ is required to sell a specified amount of power to consumers in Quebec, it is reasonable to expect that these power purchase agreements will not affect the quantity of power available for sale in Quebec but will affect HQ's export strategy. In addition, its market power in markets like New York implies that intermittency would have an impact on the price of electricity in such markets.

Another example is Bonneville Power Authority (BPA), a dominant hydroelectric generator in the Western US electricity market. BPA's total hydroelectric generating capacity as of 2015 was 22,458 MW. In this market there are two other large firms, PG&E and SCE, which are mainly in thermal generation. Although the BPA is a federal utility it now engages in commercial activities and has signed power purchase agreements with several privately owned wind farms. In addition, it also offers storage services to help other utilities integrate wind energy. It has long been recognized that the BPA has significant ability to influence the price of electricity in the western US. In addition, the divestiture of thermal assets by PG&E and SCE has increased the influence of the BPA in the western US electricity markets. Thus it is reasonable to assume that the large government-owned hydroelectric generators besides trying to fulfil its mandate also strive to maximize profits.

The flexibility of hydroelectric generators has also made wind power an attractive investment option. Due to their complementary nature some hydroelectric generators are investing in wind farms instead of building large capital intensive hydroelectric generators. In Norway for example, Statkraft, a government owned electric utility, supplies electricity in the Nord pool. Most of Statkraft's electricity generation comes from hydroelectric generators that it owns. Currently it has over 18,000 MW of hydroelectric generating capacity in its generating fleet with about 12,000MW of this capacity located in Norway. With its hydroelectric generators, it supplies 30% of the electricity produced in Norway. In addition to its hydroelectric capacity the company has investment in wind farms reaching a capacity of 800MW of wind turbine capacity as of 2015.

3. THE MODEL

The time horizon for the model is a water cycle of one year, which consists of two periods, called period 0 and period 1. In the management of water resources, the time horizon is between one and five years. For simplicity, we choose one year as the time horizon. The volume of water stored in the reservoir at any point in time depends on the inflow and the capacity of the reservoir. Precipitation varies from one part of the country to another and from season to season. For a country in the Northern hemisphere, inflows are highest during the spring when snow melts, and normally declines toward the summer. In autumn, inflows rise again due to rainfall. During the winter months, inflows are normally very low. The same pattern repeats itself year after year.

3.1. *Demand and Supply Conditions*

Consider a market for electricity in which the market demand curve in a period, say period t, is given by $D_t: p \to D_t(p), t = 0,1$, where p is the price and $D_t(p)$ is the market demand at price p. The supply side consists of a dominant firm and a competitive fringe. The dominant firm is a hydroelectric generator, who does not own any wind turbines, but purchases all the wind power output generated by privately owned wind farms at a stipulated price γ. The market power possessed by the dominant firm allows it to set the market price of electricity.

Building capacity takes time and because the time horizon is short we assume that the capacity of thermal generators are fixed. Therefore using i to denote the type of fuel generator burns and j its coordinate among generators that burn fuel of type i we assume that the capacity of producer ij, say k_{ij}, is given. Also let m_{ij} denote the marginal cost of each generator ij. Thus, in each period, the supply of producer ij – as a function of the prevailing price of electricity p – is given by

$$s_{ijt}(p) = \begin{cases} 0 \; if \; p < m_{ijt}, \\ any \; value \; inside \; the \; interval \; [0, k_{ij}] if \; p = m_{ijt}, \\ k_{ij} \; if \; p > m_{ijt}. \end{cases}$$

The supply curve of the fringe in period $t, t = 0,1$, denoted by $S_t(p)$, is the horizontal sum of $S_{it}(p)$

over $ij, i = 1,2,3, j = 1, \ldots, n_i$. That is,

$$S_t(p) = \sum_{i=1}^{3} \sum_{j=1}^{n_i} S_{ijt}(p).$$

Observe that the supply curve of the fringe has the form of an ascending staircase, with the successive steps representing the marginal costs of the generators who are more inefficient. Furthermore, because the fringe consists of many small producers, the supply curve of the fringe has the appearance of a staircase with many small steps, and thus, for analytical simplicity, can be approximated by an upward-sloping smooth curve with infinite slope as the supply of the fringe approaches its total generating capacity. Therefore, we take this approach in our analysis.

The residual demand curve facing the dominant firm in period t is given by

(1) $Q_t: p \rightarrow Q_t(p) = D_t(p) - S_t(p).$

Let $\overline{p}_t$ be the value of p that solves $D_t(p) - S_t(p) = 0$. As defined, $\overline{p}_t$ is the choke price of the residual demand curve in period t. In what follows, we shall also express the residual demand curve in period t under its inverse form as $p_t(Q)$, with Q being the volume of electricity – wind power and hydropower – supplied by the dominant firm, and $p_t(Q)$ being the market price that must prevail to equate the market demand $D_t(p)$ and the market supply $Q + S_t(p)$. Since the marginal cost of hydroelectric generation is negligible, we will take for granted and assume that the marginal cost of the dominant firm is zero.

Let

(2) $\pi_t: Q \rightarrow \pi_t(Q) = Qp_t(Q)$

denote the total revenue curve associated with the residual demand curve in period t. We shall assume that $\pi_t(Q)$ is concave. Note that if the dominant firm sets the price in period t at $\overline{p}_t$, then residual demand is 0, and the revenue it obtains will be 0. When the dominant firm sets a price slightly below $\overline{p}_t$, residual demand, and a fortiori total revenue, will be positive. Thus, the price elasticity of residual demand will be greater than 1 if the supply – wind power and hydropower – of the dominant firm is rising from 0, even if the market demand $D_t(p)$ is inelastic. Furthermore, profit maximization dictates that the dominant firm only operates in the elastic region of the residual demand curve. Indeed, at a point on the residual demand curve where the price elasticity is less than unity, the dominant firm can obtain more revenues by cutting back hydroelectric generation.

Without loss of generality, we assume that the wind power generated in period t, which we denote by ω_t,[4] is a random variable with density $\phi_t(\omega_t), 0 \leq \omega_t \leq \overline{\omega}, t = 0,1$, where $\overline{\omega} > 0$ is the upper bound on the wind power output in each period. The mean and variance of ω_t are denoted, respectively, by μ_t and σ_t^2. Also, we assume that $\omega_t, t = 0,1$, are independent.

Let $\overline{X}$ denote the capacity of the reservoir owned by the dominant firm. For each $t = 0,1$, let u_t[5] be the stochastic water inflow in period t measured in units of electrical energy. The density function of u_t is denoted by $f_t(u_t), 0 \leq u_t \leq \overline{u}$, where $\overline{u} > 0$ is the upper bound on the inflows in both periods. We assume that $\overline{u} > \overline{X}$, i.e., the maximum possible inflow in each season exceeds the capacity of the reservoir. Also, we assume that $\omega_t, u_t, t = 0,1$, are independent random variables and that they are realized at the beginning of each period. The assumption that $\omega_t, u_t, t = 0,1$, are realized at the beginning of each period means in particular that the wind power output ω_0 and the inflow u_0 are known before the hydroelectric generator decides how much water to release in period 0.

Let X_{-1}, denote the volume of water that remains in the reservoir at the end of period -1 and carried over into period 0. One can think of X_{-1} as inventory at the start of period 0. If $X_{-1} + u_0 \leq \overline{X}$, then the total volume of water available for exploitation in period 0 is also $X_{-1} + u_0$. Otherwise, i.e., if $X_{-1} + u_0 > \overline{X}$, then overflow will occur, and the total volume of water that exists in the reservoir will be $\overline{X}$. Since X_{-1} and u_0 are known when the problem begins, we can assume without any loss in generality that $X_{-1} + u_0 \leq \overline{X}$, i.e., $X_{-1} + u_0$ is the volume of water in the reservoir that is available for exploitation in period 0.

Suppose that q_0 is the volume of water released in period 0, and u_1 is the inflow in period 1. If the inflow in period 1 does not exceed $\overline{X} - (X_{-1} + u_0 - q_0)$, then the total volume of water that is available for exploitation in period 1 is given by $X_{-1} + u_0 - q_0 + u_1$. On the other hand, if $u_1 > \overline{X} - (X_{-1} + u_0 - q_0)$, then overflow will occur, and the total volume of water that is available for exploitation in period 1 is given by $\overline{X}$. To preclude the possibility that the dominant firm might find it optimal to leave some water unexploited in the reservoir at the end of the time horizon, we shall assume that $\pi_t(Q)$ is increasing in the interval $0 \leq Q \leq \overline{X} + \overline{\omega}$.

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[4] For an extensive look at the derivation of wind power readers can refer to Pryor and Barthelmie (2010)

[5] For an extensive look at converting water into electricity, readers can refer to U.S. Department of the Interior, Bureau of Reclamation, Power Resources Office (July 2005): Reclamation Managing Water in The West Hydroelectric Power.

3.4. *Problem Statement*

Let ω_0 be the output of wind power realized at the beginning of period 0. If q_0 is the volume of water released in period 0, then the profit made by the dominant firm in period 0 is given by

(3) $\pi_0(\omega_0 + q_0) - \gamma\omega_0$.

In (3), the first term represents the revenue obtained from selling $(\omega_0 + q_0)$, the volume of electricity – wind power and hydropower – in period 0, and the second term, namely $\gamma\omega_0$, the cost of the wind power purchased from the wind farms, also in period 0.

The optimal decision for the dominant firm is to empty the reservoir in period 1. Thus, the profit made in period 1 is given by

(4) $\pi_1\left(\omega_1 + min\left\{X_{-1} + u_0 - q_0 + u_1, \overline{X}\right\}\right) - \gamma\omega_1$.

In (4), ω_1 is the output of wind power realized in period 1. Also, note that the expression $\pi_1\left(\omega_1 + min\left\{X_{-1} + u_0 - q_0 + u_1, \overline{X}\right\}\right)$ represents the total revenue obtained from selling the wind power output and the electricity generated by emptying the reservoir. The last term in (4), namely $\gamma\omega_1$, is the sum paid to the wind farms for the purchase of wind power in period 1.

Given the initial data X_{-1}, u_0, if q_0 is the volume of water released in period 0, then the profit made by the dominant firm over the two periods and under a particular realization (u_1, ω_1) of inflow and wind power in period 1is given by

(5) $\pi_0(q_0 + \omega_0) - \gamma\omega_0 + \pi_1\left(\omega_1 + min\left\{X_{-1} + u_0 - q_0 + u_1, \overline{X}\right\}\right) - \gamma\omega_1$[6].

Taking the expectation of (5) with respect to the distribution of inflow and wind power output in period 1, we obtain the following expression for the expected profit made by the dominant firm over the two periods, as a function of the hydroelectric generation q_0 in period 0:

(6) $$\pi_0(\omega_0 + q_0) + \int_0^{\overline{\omega}} \left(\int_0^{\overline{u}} \pi_1\left(\omega_1 + min\left\{X_{-1} + u_0 - q_0 + u_1, \overline{X}\right\}\right) f_1(u_1) du_1\right) \phi_1(\omega_1) d\omega_1 - \gamma(\omega_0 + \mu_1)$$

$$= \pi_0(\omega_0 + q_0) + \int_0^{\overline{\omega}} \int_0^{\overline{X}-(X_{-1}+u_0-q_0)} \pi_1(\omega_1 + X_{-1} + u_0 - q_0 + u_1) f_1(u_1) du_1\, \phi_1(\omega_1) d\omega_1 +$$

$$\int_0^{\overline{\omega}} \int_{\overline{X}-(X_{-1}+u_0-q_0)}^{\overline{u}} \pi_1\left(\omega_1 + \overline{X}\right) f(u_1) du_1\, \phi_1(\omega_1) d\omega_1 - \gamma(\omega_0 + \mu_1)$$

Note that the right-hand side of (6) has been obtained by splitting the integral

$$\int_0^{\overline{u}} \pi_1\left(\omega_1 + min\left\{X_{-1} + u_0 - q_0 + u_1, \overline{X}\right\}\right) f_1(u_1) du_1$$

into two integrals, with the first integral, namely,

$$\int_0^{\overline{X}-(X_{-1}+u_0-q_0)} \pi_1(\omega_1 + u_0 - q_0 + u_1) f_1(u_1) du_1,$$

capturing the expected revenue associated with the event that the inflow in period 1 is not large

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[6] To reduce notations without losing important insights, discounting is ignored.

enough to cause an overflow of the excess water, and the second integral, namely,

$$\int_{\overline{X}-(X_{-1}+u_0-q_0)}^{\overline{u}} \pi_1\big(\omega_1+\overline{X}\big) f_1(u_1)du_1,$$

capturing the expected revenue associated with the event that the inflow in period 1 is large, and an overflow occurs. The problem of the dominant firm is to find q_0

$$(7)\qquad v(u_0,\omega_0) = max_{0\le q_0\le X_{-1}+u_0}\pi_0(\omega_0+q_0) + \int_0^{\overline{\omega}}\left(\int_0^{\overline{X}-(X_{-1}+u_0-q_0)} \pi_1(\omega_1+u_0-q_0+u_1)f_1(u_1)du_1\right)\phi_1(\omega_1)d\omega_1 + \int_0^{\overline{\omega}}\left(\int_{\overline{X}-(X_{-1}+u_0-q_0)}^{\overline{u}} \pi_1\big(\omega_1+\overline{X}\big)f_1(u_1)du_1\right)\phi_1(\omega_1)d\omega_1 - \gamma(\omega_0+\mu_1)$$

4. THE OPTIMAL SOLUTION

Differentiating the objective function of the maximization problem in (7) with respect to q_0, and then setting the result equal to 0, we obtain the following first-order condition:[7]

$$(8)\qquad \pi_0{}'(\omega_0+q_0) - \int_0^{\overline{\omega}}\left(\int_0^{\overline{X}-(X_{-1}+u_0-q_0)} \pi_1{}'(\omega_1+u_0-q_0+u_1)f_1(u_1)du_1\right)\phi_1(\omega_1)d\omega_1 = 0.$$

The first-order condition (8) asserts that the volume of water released for hydroelectric generation in period 0 should be at the level where the marginal revenue in this period is equal to the expected marginal revenue in period 1. The second-order condition is

$$(9)\ \pi_0''(\omega_0+q_0) + \int_0^{\overline{\omega}}\int_0^{\overline{X}-(X_{-1}+u_0-q_0)} \pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\phi_1(\omega_1)d\omega_1 - \int_0^{\overline{\omega}} \pi_1{}'(\omega_1+\overline{X})f\left(\overline{X}-(X_{-1}+u_0-q_0)\right)\phi_1(\omega_1)d\omega_1 < 0,$$

which is automatically satisfied due to the assumed concavity of the total revenue curve associated with the residual demand and the fact that the dominant firm will not operate in the region of the residual demand curve where marginal revenue is negative.

PROPOSITION 1: *Suppose that the volume of water available for exploitation in period 0 is below the capacity of the reservoir, i.e.,* $X_{-1}+u_0<\overline{X}$*. We have*

$$\frac{\partial v(u_0,\omega_0)}{\partial\omega_0} + \gamma = \frac{\partial v(u_0,\omega_0)}{\partial u_0}\ ;$$

that is, in period 0, water inflow is more valuable to the dominant firm than wind power at the margin.

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[7] It is simple to show that if the volume of water available in the reservoir in period 0 is small, and if the market demand in period 1 is not considerably higher in period 1 than in period 0, then the dominant firm will empty the reservoir in period 0. On the other hand, if demand is much higher in period 1 than in period 0, then the dominant firm might find it more profitable not to release any water in period 0 to take advantage of higher market demand in period 1. These corner solutions are not of interest to us.

PROOF: Apply the envelope theorem to the maximization problem in (7), first with respect to u_0, and then with respect to ω_0, we obtain, respectively,

$$\frac{\partial v(u_0,\omega_0)}{\partial u_0} = \int_0^{\overline{\omega}} \left(\int_0^{\overline{X}-(X_{-1}+u_0-q_0)} \pi_1{}'(\omega_1 + X_{-1} + u_0 - q_0 + u_1) f_1(u_1) du_1 \right) \phi_1(\omega_1) d\omega_1,$$

and

$$\frac{\partial v(u_0,\omega_0)}{\partial \omega_0} = \pi_0'(\omega_0 + q_0) - \gamma = \int_0^{\overline{\omega}} \left(\int_0^{\overline{X}-(X_{-1}+u_0-q_0)} \pi_1{}'(\omega_1 + X_{-1} + u_0 - q_0 + u_1) f_1(u_1) du_1 \right) \phi_1(\omega_1) d\omega_1 - \gamma.$$

Note that the last line has been obtained with the help of the first-order condition (8). Thus, we have $\frac{\partial v(u_0,\omega_0)}{\partial \omega_0} + \gamma = \frac{\partial v(u_0,\omega_0)}{\partial u_0}$.■

A more intuitive proof of Proposition 1 can be given as follows. Suppose that in period 0 the wind power output was ω_0; the inflow was u_0; and the volume of water released was q_0. Now let $\epsilon > 0$ be such that $\epsilon < \overline{X} - (X_{-1} + u_0)$, and then consider two scenarios. In the first scenario, the wind power realized rises by ϵ, while in the second scenario the inflow at the beginning of period 0 is higher than its original value also by ϵ. Under the first scenario, the volume of wind power sent to the market is $\omega_0 + \epsilon$. Suppose that the volume of water released under the first scenario is $\hat{q}_0$. The amount of electricity – wind power and hydropower – supplied by the dominant firm in period 0 is $\omega_0 + \epsilon + \hat{q}_0$, and the volume of water left in the reservoir at the end of period 0 is $u_0 - \hat{q}_0$. Now if under the second scenario $\epsilon + \hat{q}_0$ is the volume of water that the dominant firm releases in period 0, then $\omega_0 + \epsilon + \hat{q}_0$ is also its electricity supply in this period, and the volume of water left in the reservoir is $X_{-1} + u_0 + \epsilon - (\epsilon + \hat{q}_0) = X_{-1} + u_0 - \hat{q}_0$, which is the same as that under the first scenario. Such an action, which is feasible, yields the same expected profit over the time horizon as the action taken under the first scenario. Furthermore, under the first scenario, the dominant firm has to pay for the extra wind power generated, while the extra water inflow is free under the second scenario. Thus, the optimal solution under the second scenario yields a higher expected profit over two periods than the optimal solution under the first scenario.

In the literature on the integration of wind power and hydroelectric generation, it is presumed that when more wind power is generated, the extra wind power produced will be stored in the form of water in the reservoir. The following proposition asserts that part of the extra wind power is consumed and part of it is stored.

PROPOSITION 2: *A rise in the realized output of wind power in the first period, ceteris paribus, (a) lowers the volume of water released in this period, but (b) raises the volume of electricity – wind power and hydropower – that the dominant firm supplies to the market. Therefore, only part of the* *extra wind power generated is consumed and the remaining part is stored under the form of water in the reservoir.*

PROOF: The proof of Proposition 2 involves the computations of some derivatives, and is given in Appendix A.

PROPOSITION 3: *A rise in the realized output of wind power in the first period, ceteris paribus, induces a fall in the market price of electricity in each period. The end result is a displacement of the output of the producers in the fringe with higher marginal costs as well as a lower level of greenhouse gas emissions.*

PROOF: According to (b) of Proposition 2, a rise in ω_0 induces a rise in $(\omega_0 + q_0)$, and this means a lower price of electricity set by the dominant firm in period 0. The fall in the price set by the dominant firm in turn induces a fall in the supply of the fringe and a fortiori a fall in the fossil fuel inputs used by the fringe to generate electricity in period 0.
According (a) of Proposition 2, a rise in ω_0 induces a fall in q_0, and this means there is more water left in the reservoir at the end of period 0. For a given level of inflow and a given realized wind power output – both in period 1 – there will be more electricity supply coming from the dominant firm, with the ensuing fall of the market price in period 1. Again, as in the preceding paragraph, the end result is a lower volume of fossil fuels burned by the competitive fringe to generate electricity in period 1.■

Proposition three confirms the common intuition that wind power will reduce pollution by displacing fossil fuel generators. The volume of greenhouse gases that will be reduced will depend on the type of fossil fuel generators that are displaced. Since the marginal cost of fossil fuel generators depends on the price of fossil fuels, the set of generators that will be displaced will also depend on the fuel prices. Normally, we expect that oil-fired plants will be the first to be displaced due to their consistently high prices, while the next set of generators will fluctuate between coal generators and natural gas generators depending on the price of natural gas. In electricity markets with large amounts of natural gas generators compared to coal plants, it can be expected that wind power will more often than not, displace natural gas generators instead of coal generators. This is as a result of the high demand by natural gas generators and consumers who use natural gas for heating purposes. Therefore in such markets, wind turbines can be expected to displace natural gas plants leading to a lower reduction in emissions. An example of this type of market can be the New York electricity market where natural gas generators set the price of electricity a majority of the time[8]. On the other hand when natural gas generators don't make up a large share of the capacity mix compared to coal generators the demand for natural gas can be expected to be lower which will reduce gas prices below coal prices. This will lead to more coal generators being displaced by wind leading to higher reduction in emissions by wind turbines in such markets.

5. IMPACT OF CLIMATE CHANGE

Propositions 1 to 3 present the results of our analysis on the integration of wind power and hydroelectric generation as a component of a program to mitigate the negative impacts of climate change. The following propositions – Propositions 4 through 7 – close the circle with the results of our analysis on how climate change affects the exploitation of the water resources.

Proposition 4 deals with the case of a country where climate change induces higher and more variable inflows in period one. One way to parameterize the upward shift in the inflows in period one is to write $\tilde{u}_1 = \alpha u_1$ as the new period one inflow, with u_1 being the period one inflow before the climate change, and $\alpha \geq 1$ as the shift parameter. This specification captures the idea that the new inflow has a higher mean and a higher variance than the original inflow. With a rise in period one inflows, the dominant firm now solves the following version of the maximization problem in (7):

$$(10) \quad max_{0 \leq q_0 \leq X_{-1}+u_0} \pi_0(\omega_0+q_0) + \int_0^{\overline{\omega}} \left(\int_0^{\frac{\overline{X}-(X_{-1}+u_0-q_0)}{\alpha}} \pi_1(\omega_1 + X_{-1} + u_0 - q_0 + \alpha\, u_1) f_1(u_1) du_1 \right) \phi_1(\omega_1) d\omega_1 + \int_0^{\overline{\omega}} \left(\int_{\frac{\overline{X}-(X_{-1}+u_0-q_0)}{\alpha}}^{\overline{u}} \pi_1(\omega_1 + \overline{X}) f_1(u_1) du_1 \right) \phi_1(\omega_1) d\omega_1 - \gamma(\omega_0 + \mu_1)$$

The following version of the first-order condition (8) now characterizes an interior solution of the maximization problem in (10).

$$(11) \quad \pi_0{}'(\omega_0+q_0) - \int_0^{\overline{\omega}} \left(\int_0^{\frac{\overline{X}-(X_{-1}+u_0-q_0)}{\alpha}} \pi_1{}'(\omega_1 + X_{-1} + u_0 - q_0 + \alpha u_1) f_1(u_1) du_1 \right) \phi_1(\omega_1) d\omega_1 = 0.$$

Differentiating (11) with respect to α, we obtain

$$(12) \quad \pi_0''(\omega_0+q_0)\frac{\partial q_0}{\partial \alpha} - \int_0^{\overline{\omega}} \int_0^{\frac{\overline{X}-(X_{-1}+u_0-q_0)}{\alpha}} \left(u_1 - \frac{\partial q_0}{\partial \alpha} \right) \pi_1''(\omega_1 + X_{-1} + u_0 - q_0 + \alpha u_1) f_1(u_1) du_1\, \phi_1(\omega_1) d\omega_1 - \int_0^{\overline{\omega}} \left(\frac{\frac{\partial q_0}{\partial \alpha}\alpha - \left(\overline{X}-(X_{-1}+u_0-q_0)\right)}{\alpha^2} \right) \pi_1{}'(\omega_1+\overline{X}) f_1\left(\frac{\overline{X}-(X_{-1}+u_0-q_0)}{\alpha} \right) \phi_1(\omega_1) d\omega_1 = 0.$$

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[8] This intuition has been confirmed in competitive markets by Cullen (2013) and Kaffine et al (2013) in the ERCOT market in Texas which is made up of mainly natural gas generators and wind turbines.

Solving (12) for $\frac{\partial q_0}{\partial \alpha}$, and then setting $\alpha = 1$ in the result, we obtain

(13) $\left.\frac{\partial q_0}{\partial \alpha}\right|_{\alpha=1} =$

$$\frac{\int_0^{\overline{\omega}} \int_0^{\overline{X}-(X_{-1}+u_0-q_0)} u_1 \pi_1''(\omega_1+X_{-1}+u_0-q_0+\ u_1) f_1(u_1) du_1 \phi_1(\omega_1) d\omega_1 - \int_0^{\overline{\omega}} \left(\overline{X}-(X_{-1}+u_0-q_0)\right) \pi_1'(\omega_1+\overline{X}) f_1\left(\overline{X}-(u_0-q_0)\right) \phi_1(\omega_1) d\omega_1}{\pi_0''(\omega_0+q_0) + \int_0^{\overline{\omega}} \int_0^{\overline{X}-(X_{-1}+u_0-q_0)} \pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1) f_1(u_1) du_1 \phi_1(\omega_1) d\omega_1 - \int_0^{\overline{\omega}} \pi_1'(\omega_1+\overline{X}) f_1\left(\overline{X}-(X_{-1}+u_0-q_0)\right) \phi_1(\omega_1) d\omega_1} > 0$$

The numerator of the expression on the right-hand side of (13) is negative because $\pi_1' > 0, \pi_1'' < 0$, and $\left(\overline{X} - (u_0 - q_0)\right) > 0$. The denominator of the same expression is negative because $\pi_1' > 0, \pi_1'' < 0$. We have just proved the following proposition.

PROPOSITION 4: *Suppose that climate change induces a rise in period one inflows so that* $\tilde{u}_1 = \alpha u_1, \alpha \geq 1$, *now represents the period one inflows after the climate change. We have* $\left.\frac{\partial q_0}{\partial \alpha}\right|_{\alpha=1} > 0$. *That is, the rise in the period one inflows due to climate change leads to more water being released in period 0.*

With a rise in the period one inflows, the probability of overflow in period 1 is given by

(14) $\quad 1 - \int_0^{\overline{\omega}} \left(\int_0^{\frac{\overline{X}-(X_{-1}+u_0-q_0)}{\alpha}} f_1(u_1) du_1 \right) \phi_1(\omega_1) d\omega_1.$

According to Proposition 4, a rise in the period 1 inflows due to climate change leads to more water being released in period 0, and a lower volume of water left in the reservoir at the end of period 0. Ceteris paribus, this result reduces the chance of overflows in period 1. However, the larger inflows in period 1 due to climate change raise the probability of overflows. The net impact of climate change on the probability of overflows, as represented by (14), depends on which effect dominates. A definite answer to this question is given in Proposition 5.

PROPOSITION 5: *Suppose that climate change induces a rise in the expected period one inflows so that* $\tilde{u}_1 = \alpha u_1, \alpha \geq 1$, *now represents the period one inflows induced by the climate change. The rise in period one inflows leads to a rise in the probability of overflows in period 1.*
PROOF: See Appendix B.

Together, Propositions 4 and 5 capture the intuitive idea that a rise in the second period inflows allows the dominant firm to generate more hydroelectric power in both periods. More hydroelectric generation is captured by Proposition 4, which asserts a rise in q_0, the volume of water released in the period 0, while more hydroelectric generation in the second period means more water in the reservoir in period 1, which manifests itself through a higher probability of overflow asserted by Proposition 5.

The impact of the intermittency of wind power can be analyzed by considering a mean-preserving spread of the distribution of wind power generation either in both periods or in only period 1, after the realization of ω_0. To this end, let ω_0 be the realized output of wind power in period 0. With more intermittency, the wind power generated in period 1 is represented by the random variable $\theta\omega_1 + (1-\theta)\mu_1$, with ω_1 being the random wind power output in period 1 before the rise in intermittency. In what follows, we shall refer to ω_1 as the wind power output in the base case, and $\theta\omega_1 + (1-\theta)\mu_1$ as the wind power output in the more intermittent wind case. The following version of the first-order condition (8) holds for the more intermittent case.

$$\pi_0'(\omega_0 + q_0) - \int_0^{\overline{\omega}} \left(\int_0^{\overline{X}-(X_{-1}+u_0-q_0)} \pi_1'(\theta\omega_1 + (1-\theta)\mu_1 + X_{-1} + u_0 - q_0 + u_1) f_1(u_1) du_1 \right) \phi_1(\omega_1) d\omega_1 = 0. \tag{15}$$

Differentiating (15) with respect to θ, we obtain

$$0 = \pi_0''(\omega_0 + q_0)\frac{\partial q_0}{\partial \theta} - \int_0^{\overline{\omega}} \int_0^{\overline{X}-(u_0-q_0)} \left(-\frac{\partial q_0}{\partial \theta} + \omega_1 - \mu_1 \right) \pi_1''(\theta\omega_1 + (1-\theta)\mu_1 + X_{-1} + u_0 - q_0 + u_1) f_1(u_1) du_1 \phi_1(\omega_1) d\omega_1 - \int_0^{\overline{\omega}} \frac{\partial q_0}{\partial \theta} \pi_1'\left(\theta\omega_1 + (1-\theta)\mu_1 + \overline{X}\right) \phi_1(\omega_1) d\omega_1.$$

Solving the preceding equation for $\frac{\partial q_0}{\partial \theta}$, and then setting $\theta = 1$, we obtain

$$\left.\frac{\partial q_0}{\partial \theta}\right|_{\theta=1} = \frac{\int_0^{\overline{\omega}} \left(\int_0^{\overline{X}-(u_0-q_0)} (\omega_1 - \mu_1) \pi_1''(\omega_1 + X_{-1} + u_0 - q_0 + u_1) f_1(u_1) du_1 \right) \phi_1(\omega_1) d\omega_1}{\pi_0''(\omega_0 + q_0) + \int_0^{\overline{\omega}} \left(-\pi_1'(\omega_1 + \overline{X}) + \int_0^{\overline{X}-(X_{-1}+u_0-q_0)} \pi_1''(\omega_1 + X_{-1} + u_0 - q_0 + u_1) f_1(u_1) du_1 \right) \phi_1(\omega_1) d\omega_1}. \tag{16}$$

The denominator of the expression on the right-hand side of the equality in (16) is negative. The sign of $\frac{\partial q_0}{\partial \theta}$ is thus the opposite of the sign of the numerator. To determine the sign of the numerator, let us define for any $Q \geq 0$,

$$r_1(Q) = -\frac{\pi_1''(Q)}{\pi_1'(Q)}. \tag{17}$$

As defined, $r_1(Q)$ has the appearance of the Arrow-Pratt measure of absolute risk aversion often encountered in the theory of expected utility.[9] If we consider $\pi_1(Q)$ as the utility function of the dominant firm, then $r_1(Q)$ can be interpreted as a measure of absolute aversion of the large

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[9] See Varian (1992): Microeconomic Analysis, Third Edition, p.178 and p. 184-186.

hydroelectric generator, and it is reasonable to expect that this measure of risk aversion is a decreasing function of the dominant firm's supply.

LEMMA 1: *If* $r_1(Q)$ *is a decreasing function of* Q*, then*

$$\int_0^{\overline{\omega}}\left(\int_0^{\overline{X}-(u_0-q_0)}(\omega_1-\mu_1)\pi_1''(\omega_1+X_{-1}+u_0-q_0+\ u_1)f_1(u_1)du_1\right)\phi_1(\omega_1)d\omega_1>0.$$

PROOF: See Appendix C.

The following proposition follows immediately from Lemma 1.

PROPOSITION 6: *If* $r_1(Q)$ *is a decreasing function of* Q*, then* $\left.\frac{\partial q_0}{\partial\theta}\right|_{\theta=1}<0$; *that is, if the wind in period one becomes more intermittent, then the dominant firm will release less water for hydroelectric generation in the first period.*

The price set by the dominant firm in period 1 when the wind in that period expected to be more intermittent is given by $p_1(\theta\omega_1+(1-\theta)\mu_1+X_{-1}+u_0-q_0+\ u_1)$. Differentiating this expression with respect to θ, and then setting $\theta=1$, we obtain

(18) $\left.\frac{d}{d\theta}p_1(\theta\omega_1+(1-\theta)\mu_1+X_{-1}+u_0-q_0+\ u_1)\right|_{\theta=1}=p_1{}'(\omega_1+X_{-1}+u_0-q_0+\ u_1)\left(\omega_1-\mu_1-\frac{\partial q_0}{\partial\theta}\right).$

Now given (ω_0,u_0,u_1), the term $p_1{}'(\omega_1+X_{-1}+u_0-q_0+\ u_1)$ is negative because it is the slope of the residual demand curve. As for the term $\left(\omega_1-\mu_1-\frac{\partial q_0}{\partial\theta}\right)$, it increases linearly with ω_1. Furthermore, when $\omega_1=\mu_1$, it is reduced to $-\frac{\partial q_0}{\partial\theta}$, which is positive under the assumption that $r_1(Q)$ is decreasing in Q. The following lemma gives more information about the behavior of the map $\omega_1\to\left(\omega_1-\mu_1-\frac{\partial q_0}{\partial\theta}\right),0\leq\omega_1\leq\overline{\omega}$.

LEMMA 2: *Suppose that* $r_1(Q)$ *is a decreasing function of* Q*. Then for each realization of* (ω_0,u_0,u_1)*, there exists a unique value of* ω_1*, say* $\omega_1=\varphi(\omega_0,u_0,u_1)$*, with* $0<\varphi(\omega_0,u_0,u_1)<\mu_1$*, that solves* $\omega_1-\mu_1-\frac{\partial q_0}{\partial\theta}=0$*. Furthermore,*

$$\begin{cases}\omega_1-\mu_1-\frac{\partial q_0}{\partial\theta}<0\ if\ \omega_1<\varphi(\omega_0,u_0,u_1),\\ \omega_1-\mu_1-\frac{\partial q_0}{\partial\theta}>0\ if\ \omega_1>\varphi(\omega_0,u_0,u_1).\end{cases}$$

PROOF: See Appendix D.

The following proposition describes the impact of climate change on the market prices of electricity.

PROPOSITION 7: *Suppose that* $r_1(Q)$ *is a decreasing function of* Q. *A rise in the intermittency of the wind in period 1*

(a) induces a rise in the price set by the dominant firm in period 0, and

(b) will leads to a more extreme variation in the market price of electricity in period 1. More specifically, for each realization of (ω_0, u_0, u_1), *if the market price of electricity in period 1 before the climate change is low (high), then the market price of electricity in period 1 after the climate change is even lower (higher).*

PROOF: To prove (a), note that the total amount of electricity – hydroelectric generation and wind power – supplied by the dominant firm in period 0 is $\omega_0 + q_0$. According to Proposition 6, $\frac{\partial q_0}{\partial \theta} < 0$. Hence $(\omega_0 + q_0)$ falls as the wind in period 1 becomes more intermittent. The fall in the supply of the dominant firm in period 0 means a rise in the price it sets in period 0.

To prove (b), note that according to Lemma 2, the right-hand side of (18) is negative if $\omega_1 > \varphi(\omega_0, u_0, u_1)$, and a rise in the intermittency of the wind in period 1 induces a fall in the price set by the dominant firm in this period. On the other hand, if $\omega_1 < \varphi(\omega_0, u_0, u_1)$, then the right-hand side of (18) is positive, and a rise in the intermittency of the wind in period 1 induces a rise in the price set by the dominant firm in period. ■

6. CONCLUSION

The objective of this paper was to analyse the impact of intermittency, caused by climate change, on the strategic behaviour of a hydroelectric generator that integrates wind power. To simplify the model, a two period, stochastic dynamic optimization model was developed. It has been shown that increased wind intermittency due to climate change will lead to an increase in the price of electricity. Furthermore, the increased uncertainty in water inflows due to climate change will lead to an increase in the output of hydroelectric generators and an increase in the probability of overflows in future periods.

In the procurement program that was considered, wind energy producers are paid a stipulated price for their output. Therefore, there is no incentive for wind power generators to move their output across time for a higher price. BPA, by offering to store wind energy in its reservoirs for a fee, now makes it possible for wind energy producers to move the power they generate across time periods for the highest price and in effect creates a market for energy storage. Thus this paper opens further research questions. First, if a hydroelectric generator offered storage as a separate service, how much would it charge for such a service? Second, when wind energy producers have rational expectations, what are the market prices of electricity? The answers to these questions are of great interest, given the expected growth of the world wind energy market and the importance of wind power as a component of the renewable energy resources needed for sustainable development.

A hydroelectric generator can also store other forms of energy besides wind power. Switzerland, with its extensive network of run-of-river power plants and pumped storage, imports cheap base-load nuclear power during off-peak hours from neighboring France and Germany to pump water uphill and store it in vast reservoirs. The energy thus stored can then be sold back to these countries during peak periods or to Italy, Europe’s largest electricity importer. The price differential between peak and off-peak periods can be interpreted as the storage fee that Switzerland charges for storing the surplus energy in periods of excess supply. Thus, the insights gained from the research on the economics of storing wind energy in reservoirs storage can shed light on the storage of other forms of energy under the form of water.

APPENDIX A: Proof of Proposition 2

Differentiating the first-order condition (8) with respect to ω_0, we obtain

$$\pi_0''(\omega_0+q_0)\left(1+\frac{\partial q_0}{\partial \omega_0}\right)+\frac{\partial q_0}{\partial \omega_0}\int_0^{\overline{\omega}}\int_0^{\overline{X}-(X_{-1}+u_0-q_0)}\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\,\phi_1(\omega_1)d\omega_1-\int_0^{\overline{\omega}}\frac{\partial q_0}{\partial \omega_0}\pi_1'\left(\omega_1+\overline{X}\right)f_1\left(\overline{X}-(X_{-1}+u_0-q_0)\right)\phi_1(\omega_1)d\omega_1=0.$$

Solving the preceding equation for $\frac{\partial q_0}{\partial \omega_0}$, we obtain

$$\frac{\partial q_0}{\partial \omega_0}=\frac{-\pi_0''(\omega_0+q_0)}{\pi_0''(\omega_0+q_0)+\int_0^{\overline{\omega}}\int_0^{\overline{X}-(X_{-1}+u_0-q_0)}\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\phi_1(\omega_1)d\omega_1-\int_0^{\overline{\omega}}\pi_1'(\omega_1+\overline{X})f_1\left(\overline{X}-(X_{-1}+u_0-q_0)\right)\phi_1(\omega_1)d\omega_1}<0.$$

The preceding inequality follows from the facts that (i) π_t is concave and (ii) $\pi_t' \geq 0$. We have just proved (a) of Proposition 2.

Next, note that

$$\frac{\partial(\omega_0+q_0)}{\partial \omega_0}=1+\frac{\partial q_0}{\partial \omega_0}$$

$$=1+\frac{-\pi_0''(\omega_0+q_0)}{\pi_0''(\omega_0+q_0)+\int_0^{\overline{\omega}}\int_0^{\overline{X}-(X_{-1}+u_0-q_0)}\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\phi_1(\omega_1)d\omega_1-\int_0^{\overline{\omega}}\pi_1'(\omega_1+\overline{X})f_1\left(\overline{X}-(u_0-q_0)\right)\phi_1(\omega_1)d\omega_1}$$

$$=\frac{\int_0^{\overline{\omega}}\int_0^{\overline{X}-(u_0-q_0)}\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\phi_1(\omega_1)d\omega_1-\int_0^{\overline{\omega}}\pi_1'(\omega_1+\overline{X})f_1\left(\overline{X}-(X_{-1}+u_0-q_0)\right)\phi_1(\omega_1)d\omega_1}{\pi_0''(\omega_0+q_0)+\int_0^{\overline{\omega}}\int_0^{\overline{X}-(u_0-q_0)}\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\phi_1(\omega_1)d\omega_1-\int_0^{\overline{\omega}}\pi_1'(\omega_1+\overline{X})f_1\left(\overline{X}-(u_0-q_0)\right)\phi_1(\omega_1)d\omega_1}>0.$$

The strict inequality is due to the facts that the numerator and the denominator of the left-hand side of the inequality are both negative. We have just proved (b) of Proposition 2.

Note that in part (a), the volume of water released in period 0 falls when more wind power is generated in this period, and this means some of the extra wind power generated is stored under the form of water in the reservoir. From part (b), the supply – wind power and hydropower – of the dominant firm in period 0 rises with the rise in wind power generated, and this result together with the preceding statement imply that part of the extra wind power is consumed.■

APPENDIX B: Proof of Proposition 5

The probability of overflows in period 1 is given by

$$1-\int_0^{\overline{\omega}}\left(\int_0^{\frac{\overline{X}-(X_{-1}+u_0-q_0)}{\alpha}} f_1(u_1)du_1\right)\phi_1(\omega_1)d\omega_1.$$

Differentiating the preceding expression with respect to α, and then setting $\alpha = 1$ in the result, we obtain

$$-\int_0^{\overline{\omega}}\left(\left(\frac{\partial q_0}{\partial\alpha}-\left(\overline{X}-(X_{-1}+u_0-q_0)\right)\right)f_1\left(\overline{X}-(X_{-1}+u_0-q_0)\right)\right)\phi_1(\omega_1)d\omega_1.$$

To establish the claim made by Proposition 5, we shall show that the preceding derivative is positive. Because the density $f_1(u_1)$ is positive, it is sufficient to show that

$$\frac{\partial q_0}{\partial\alpha}-\left(\overline{X}-(X_{-1}+u_0-q_0)\right)<0.$$

To this end, note that

$$\left.\frac{\partial q_0}{\partial\alpha}\right|_{\alpha=1}-\left(\overline{X}-(u_0-q_0)\right)=$$

$$\frac{\int_0^{\overline{\omega}}\int_0^{\overline{X}-(X_{-1}+u_0-q_0)} u_1\pi_1''(\omega_1+X_{-1}+u_0-q_0+\ u_1)f_1(u_1)du_1\phi_1(\omega_1)d\omega_1-\int_0^{\overline{\omega}}\left(\overline{X}-(X_{-1}+u_0-q_0)\right)\pi_1'(\omega_1+\overline{X})f_1\left(\overline{X}-(u_0-q_0)\right)\phi_1(\omega_1)d\omega_1}{\pi_0''(\omega_0+q_0)+\int_0^{\overline{\omega}}\int_0^{\overline{X}-(X_{-1}+u_0-q_0)}\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\phi_1(\omega_1)d\omega_1-\int_0^{\overline{\omega}}\pi_1'(\omega_1+\overline{X})f_1\left(\overline{X}-(X_{-1}+u_0-q_0)\right)\phi_1(\omega_1)d\omega_1}-$$

$$\left(\overline{X}-(X_{-1}+u_0-q_0)\right)$$

$$=$$

$$\frac{-\pi_0''(\omega_0+q_0)\left(\overline{X}-\left(X_{-1}+u_0-q_0\right)\right)+\int_0^{\overline{\omega}}\left(\int_0^{\overline{X}-(X_{-1}+u_0-q_0)}\left(u_1-\left(\overline{X}-\left(X_{-1}+u_0-q_0\right)\right)\right)\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\right)\phi_1(\omega_1)d\omega_1}{\pi_0''(\omega_0+q_0)+\int_0^{\overline{\omega}}\int_0^{\overline{X}-(X_{-1}+u_0-q_0)}\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\phi_1(\omega_1)d\omega_1-\int_0^{\overline{\omega}}\pi_1'(\omega_1+\overline{X})f_1\left(\overline{X}-(X_{-1}+u_0-q_0)\right)\phi_1(\omega_1)d\omega_1}<$$

$$0.$$

The strict inequality follows from the following facts: (i) $\pi_t''<0, t=0,1$, (ii) $\pi_t'>0$, (iii) $\overline{X}-(u_0-q_0)\geq 0$, and (iv) $u_1-\left(\overline{X}-(u_0-q_0)\right)<0, u_1\neq\left(\overline{X}-(u_0-q_0)\right)$.■

APPENDIX C: Proof of Lemma 1

Interchanging the order of integration of the numerator of the expression on the right-hand side of (16), we obtain

(C.1) $\int_0^{\overline{X}-(X_{-1}+u_0-q_0)}\left(\int_0^{\overline{\omega}}(\omega_1-\mu_1)\pi_1{}''(\omega_1+X_{-1}+u_0-q_0+\ u_1)\phi_1(\omega_1)d\omega_1\right)f_1(u_1)du_1.$

We shall now show that (C.1) is positive by showing that

(C.2) $\int_0^{\overline{\omega}}(\omega_1-\mu_1)\pi_1{}''(\omega_1+X_{-1}+u_0-q_0+\ u_1)\phi_1(\omega_1)d\omega_1>0.$

Using the definition of $r_1(Q)$, we can write

$$(\omega_1-\mu_1)\pi_1{}''(\omega_1+X_{-1}+u_0-q_0+\ u_1)\phi_1(\omega_1)$$
$$=-r_1(\omega_1+X_{-1}+u_0-q_0+\ u_1)(\omega_1-\mu_1)\pi_1{}'(\omega_1+X_{-1}+u_0-q_0+\ u_1)\phi_1(\omega_1).$$

For $\omega_1>\mu_1$, we have

(C.3) $\omega_1+X_{-1}+u_0-q_0+\ u_1>\mu_1+X_{-1}+u_0-q_0+\ u_1.$

Using (C.3) and the assumption that $r_1(Q)$ is decreasing, we can then assert that

$$r_1(\omega_1+X_{-1}+u_0-q_0+\ u_1)<r_1(\mu_1+X_{-1}+u_0-q_0+\ u_1).$$

Using (C.3) and the concavity of $\pi_1(Q)$, we can assert that

$$\pi_1{}'(\omega_1+X_{-1}+u_0-q_0+\ u_1)<\pi_1{}'(\mu_1+X_{-1}+u_0-q_0+\ u_1).$$

Thus for $\omega_1>\mu_1$, we have

(C.4) $(\omega_1-\mu_1)\pi_1{}''(\omega_1+X_{-1}+u_0-q_0+\ u_1)\phi_1(\omega_1)>-r_1(\mu_1+X_{-1}+u_0-q_0+\ u_1)(\omega_1-\mu_1)\pi_1{}'(\mu_1+X_{-1}+u_0-q_0+\ u_1)\phi_1(\omega_1).$

In the same manner, for $\omega_1<\mu_1$, inequality (C.4) also holds. Thus, inequality (C.4) holds when $\omega_1\neq\mu_1$, and integrating (C.4) with respect to ω_1, we obtain

$$\int_0^{\overline{\omega}}(\omega_1-\mu_1)\pi_1{}''(\omega_1+X_{-1}+u_0-q_0+\ u_1)\phi_1(\omega_1)d\omega_1>\int_0^{\overline{\omega}}-r_1(\mu_1+X_{-1}+u_0-q_0+\ u_1)(\omega_1-\mu_1)\pi_1'(\mu_1+X_{-1}+u_0-q_0+\ u_1)\phi_1(\omega_1)d\omega_1=0$$

which is the claim asserted by Lemma 1.■

APPENDIX D: Proof of Lemma 2

When $\omega_1 = 0$, the expression $\left(\omega_1 - \mu_1 - \frac{\partial q_0}{\partial \theta}\right)$ is reduced to

$$-\mu_1 - \frac{\partial q_0}{\partial \theta}$$

$$= -\mu_1 - \frac{\int_0^{\overline{\omega}}\left(\int_0^{\overline{X}-(X_{-1}+u_0-q_0)}(\omega_1-\mu_1)\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\right)\phi_1(\omega_1)d\omega_1}{\pi_0''(\omega_0+q_0)+\int_0^{\overline{\omega}}\int_0^{\overline{X}-(X_{-1}+u_0-q_0)}\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\phi_1(\omega_1)d\omega_1-\int_0^{\overline{\omega}}\pi_1'(\omega_1+\overline{X})\phi_1(\omega_1)d\omega_1}$$

$$= \frac{\begin{pmatrix}\mu_1\left(\pi_0''(\omega_0+q_0)+\int_0^{\overline{\omega}}\int_0^{\overline{X}-(X_{-1}+u_0-q_0)}\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\phi_1(\omega_1)d\omega_1-\int_0^{\overline{\omega}}\pi_1'(\omega_1+\overline{X})\phi_1(\omega_1)d\omega_1\right)\\ +\int_0^{\overline{\omega}}\left(\int_0^{\overline{X}-(X_{-1}+u_0-q_0)}(\omega_1-\mu_1)\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\right)\phi_1(\omega_1)d\omega_1\end{pmatrix}}{-\pi_0''(\omega_0+q_0)-\int_0^{\overline{\omega}}\int_0^{\overline{X}-(X_{-1}+u_0-q_0)}\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\phi_1(\omega_1)d\omega_1+\int_0^{\overline{\omega}}\pi_1'(\omega_1+\overline{X})\phi_1(\omega_1)d\omega_1}$$

$$= \frac{\mu_1\left(\pi_0''(\omega_0+q_0)-\int_0^{\overline{\omega}}\pi_1'(\omega_1+\overline{X})\phi_1(\omega_1)d\omega_1\right)+\int_0^{\overline{\omega}}\left(\int_0^{\overline{X}-(X_{-1}+u_0-q_0)}\omega_1\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\right)\phi_1(\omega_1)d\omega_1}{-\pi_0''(\omega_0+q_0)-\int_0^{\overline{\omega}}\left(-\pi_1'(\omega_1+\overline{X})+\int_0^{\overline{X}-(X_{-1}+u_0-q_0)}\pi_1''(\omega_1+X_{-1}+u_0-q_0+u_1)f_1(u_1)du_1\right)\phi_1(\omega_1)d\omega_1}$$

$$< 0.$$

Note that the strict inequality follows from the concavity of $\pi_t, t = 0,1$, and from $\pi_1'(\omega_1 + \overline{X}) > 0$. Finally, note that because the straight line $\omega_1 \to \omega_1 - \mu_1 - \frac{\partial q_0}{\partial \theta}, 0 \leq \omega_1 \leq \overline{\omega}$, begins at $\omega_1 = 0$ below the horizontal axis and rises above the horizontal axis at $\omega_1 = \mu_1$, it must cross the horizontal axis at a single point that we denote by $\varphi(\omega_0, u_0, u_1)$, with $0 < \varphi(\omega_0, u_0, u_1) < \mu_1$. Furthermore,

$$\begin{cases}\omega_1 - \mu_1 - \frac{\partial q_0}{\partial \theta} < 0 \; if \; \omega_1 < \varphi(\omega_0, u_0, u_1), \\ \omega_1 - \mu_1 - \frac{\partial q_0}{\partial \theta} > 0 \; if \; \omega_1 > \varphi(\omega_0, u_0, u_1).\end{cases}$$

■

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