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IMPLICATIONS OF PHASE TRANSITIONS IN THE EARLY UNIVERSE

by

PIJUSHPANI BHATTACHARJEE

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Department of Physics
Blackett Laboratory
Imperial College
LONDON SW7 2BZ

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P. Bhattacharjee

ABSTRACT

Some cosmological aspects of vortex-strings (or simply strings) formed at a grand unification phase-transition are considered. The motivation comes from recent suggestions by some authors that vortex-strings could provide the initial density perturbations responsible for the formation of galaxies.

The nature of vortex-string as a topologically stable finite-energy solution of spontaneously broken gauge theory is discussed. The facts that stable strings are not allowed in the simplest SU(5) model of grand unified theory but allowed in the Spin(10) model are explained.

The evolution of a random configuration of string in an expanding universe is discussed. Numerical calculations simulating the behaviour of strings in an expanding universe are presented. It is found that an energetically consistent description requires the formation of closed loops of string from self-intersections of lengths of string.

The attention is then focussed on collapsing closed loops of string. It is argued that loops, on collapse, release their energy in the form of particles - mainly superheavy gauge and Higgs bosons, which subsequently decay, giving a significant contribution to the net baryon-number of the universe.

Finally, the process of "exchange of partners" which could give rise to closed loops when strings intersect themselves is considered. It is shown that under reasonable assumptions the process may indeed occur.

PREFACE

The work presented in this thesis was carried out in the Theoretical Physics Group, Physics Department, Imperial College, between October 1980 and November 1983, under the supervision of Professor T.W.B. Kibble, F.R.S. Except where otherwise stated, this work is original and has not been submitted for a degree of this or any other university. The work described in Chapter II was done in collaboration with N. Turok and is intended for publication in Phys. Rev. D. The material in Chapter III is based on work done in collaboration with T.W.B. Kibble and N. Turok, published in Phys. Lett. 119B, 95 (1982).

I must take this opportunity to express my gratitude to Professor T.W.B. Kibble for his valuable guidance, readiness for frequent discussions, helpful suggestions and constant encouragement throughout the course of this work. I also wish to thank Neil Turok for his collaborations and also for a number of illuminating discussions.

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I dedicate this thesis to the memory of my late grandfather

SRIJUKTA KAILASH CHANDRA SMRITITIRTHA

as a token of my deep respect for his great learning

and scholarship

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CHAPTER I

GENERAL INTRODUCTION

1.1 Introduction

The study of cosmological implications of Grand Unified Theories (GUTs [1]) has drawn much attention in recent years. In a GUT, the "standard model" of strong- [2] and electroweak [3] interactions described by the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ is treated as a part of a bigger theory. The gauge group of a GUT is generally a suitable simple Lie group which contains the 'standard model' group as a subgroup. Typically, in a GUT quarks and leptons are placed in the same multiplet which transforms under a certain representation of the GUT gauge group. In addition to the gauge vector bosons of the "standard model" there are new gauge vector bosons, generically called x-bosons, which mediate baryon- and lepton number violating interactions between quarks and leptons. The possibility of baryon-number violation suggests that proton, the lowest mass physical baryon, should actually decay. The experimental lower limit on the proton life time τ_p (presently $\tau_p > 10^{32}$ years [4]) puts strong constraints on m_x, the mass of the typical x-boson. Typically, m_x is expected to be of order 10^{15} GeV. This is enormous compared to the masses ($\sim 10^2$ GeV) of the vector bosons of the electroweak theory. Recently, there have been reports [5] of discoveries of the charged ($W^{\pm}$) [5a] as well as neutral ($Z°$)[5b] members of the gauge vector bosons of the electroweak theory. However, in contrast to the case in electroweak theory, any direct verification of the ideas of GUTs by creation and subsequent detection of the x-bosons in accelerators (as done for $W^{\pm}$ and $Z°$) is extremely difficult, if not impossible. An alternative way is, of course, to look for experimental signatures of proton decay events. Indeed, some candidate proton-decay events have also been reported [6]. Although these remain to be confirmed by other similar experiments currently in operation, most physicists believe that the general idea behind GUT is very likely to be true.

The emergence of GUT has opened up an exciting link between elementary particle physics and cosmology. A huge body of experimental data support (see, for example, Weinberg [7]) the "hot big-bang" model of cosmology according to which the universe was much hotter and smaller in the past than it is now. The two most crucial pieces of evidence in favour of the hot big-bang model are, of course, the well known observation of red-shift of distant galaxies [8] and the detection of the cosmic microwave background radiation [9]. The former is interpreted as the expansion of the universe on the very large scale (i.e. on the scale of galaxies or clusters of galaxies) while the latter is taken as evidence towards the notion of a very hot past history of the universe. The general picture [7] is that at very early times matter and radiation were in thermal equilibrium at a very high temperature. The universe cooled with the expansion of the universe and at a temperature of about 4000°K the electrons started to get bound in atoms, thereby destroying the process of photon-electron scattering which was earlier mainly responsible for maintaining the thermal equilibrium between matter and radiation. The 'decoupled' radiation then cooled separately until its present temperature of about 2.7°K, constituting the microwave background radiation. The hot past history of the universe suggests that the huge energy scales involved in

GUT presumably obtained in the very early universe. One may therefore expect that the application of the ideas of GUTs will provide new insight into the cosmology of the very early universe. For example, as is now well known, the observed excess of matter over antimatter in the universe, which could not be explained in pre-GUT cosmology, can now be accounted for [10] by attributing it to baryon number violating processes in GUT.

It is, however, clear that the existence of apparently distinct kinds of elementary particle interactions as we see today means that the full symmetry of the GUT is not directly manifest in the present day world. In other words, the GUT symmetry is broken. Specifically, this means that although the Lagrangian describing the elementary particle interactions is invariant under the full symmetry group of the GUT, the vacuum state is not.

An often cited example of a broken symmetry situation is a magnetised ferromagnet, where the relevant (broken) symmetry is the rotational symmetry. Here, however, we have the well-known fact that by raising the temperature above a certain critical temperature (the Curie temperature) the magnetized ferromagnet can be demagnetized where the full rotational symmetry is restored. This example of relationship between symmetry and temperature is particularly interesting in the context of the cosmological big-bang theory. Could it be possible that in a sufficiently hot environment in the past the symmetry of GUT was fully restored? The answer to this question obviously depends on the symmetry behaviour of gauge theories at finite temperature. Indeed, simple models of spontaneously broken gauge theories have been shown [11-13] to exhibit symmetry restoring phase transition above a certain critical temperature. If this is a general feature of all unified gauge theories, then it means that the universe in its past history must have passed through one or more phase transitions as it cooled through certain critical temperatures. Occurrence of these phase transitions may have had significant influence on the early evolution of the universe. It is the purpose of this thesis to discuss a particular aspect of symmetry breaking phase transitions in the early universe, namely, the appearance [14] of topological structures. Examples of topological structures related to phase-transitions are provided in more familiar settings by the domain structure in a ferromagnet, a vortex filament in a superfluid or a flux-tube in a type II superconductor. In the subsequent chapters of this thesis we shall deal only with one particular kind of stable topological objects, namely the vortex-strings [15] (similar to flux-tubes in type II superconductors [16]) which in some models of GUT are predicted to have appeared at certain stages of symmetry-breaking phase-transitions in the early universe. Apart from vortex strings, there are two other major kinds of topologically stable extended objects that could appear at phase-transitions in the early universe. These are domain walls [17,18,14] and magnetic monopoles [19]. The domain walls seem to have adverse gravitational effects [18] and their presence would lead to large-scale anisotropy of the cosmic background radiation. GUT models predicting domain walls, therefore, seem to be unacceptable [18,14] on cosmological grounds. The magnetic monopoles, on the other hand, seem to be an automatic consequence [20] of phase-transition in any model of GUT. Their presence leads to the so-called "monopole problem" , the problem being the fact that even conservative estimates seem to indicate the production of far too many [21] of these superheavy monopoles (mass $\sim 10^{16}$ GeV) to be compatible with the observed energy-density in the universe. In contrast, the vortex-strings, although optional, depending on the GUT model, seem to have certain positive features. In particular, they may have played an important role [20,22,23] in the context of a major unsolved problem in cosmology, namely, the formation of galaxies.

A very lucid description of consequences of phase-transitions in the early universe is contained in Kibble's original paper [14] and lectures [20,24]. In order to ease our later discussions of topics related to cosmological vortex-strings we shall first review, in section 1.2, the nature of vortex-string as a finite-energy topologically non-trivial solution of spontaneously broken gauge theories. The homotopy classification of these objects is elucidated by taking a specific model, namely, the abelian Higgs model, which was originally used by Nielsen and Olesen [15] to demonstrate the existence of vortex-solutions in field theory. The condition for vortices to arise in the general case of a spontaneous symmetry breaking from a group G to a group H $(H \subset G)$ is also discussed. The fact that vortex-solutions are not allowed in the simplest SU(5) model [1] of GUT is demonstrated. It is then briefly indicated how in the SO(10) model [25], the vortex-strings are allowed, as first shown by Kibble, Lazarides and Shafi [26].

In section 1.3 it is explained how the vortex-strings, given they are allowed by homotopy arguments, are expected to appear as 'defects' when the universe settles down to a broken symmetric phase after a phase transition.

Section 1.4 is concerned with a discussion of the possible role of these vortex-strings in the problem of formation of galaxies.

Section 1.5 outlines the contents of the succeeding chapters.

1.2 Nature of vortex-strings

Consider the abelian Higgs model described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (D_\mu \Phi)^* (D^\mu \Phi) - V(\Phi) \, , \tag{1.1}$$

where Φ is a complex scalar field - the Higgs field, $F_{\mu\nu}$ is the field strength tensor associated with the electromagnetic field A_μ,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \, ,$$

and $D_\mu \Phi$ is the covariant derivative,

$$D_\mu \Phi = (\partial_\mu + ie A_\mu) \Phi \, .$$

In (1.1), $V(\Phi)$ is the 'potential energy' for the scalar field. In general, $V(\Phi)$ can be a polynomial in Φ but for reasons of renormalizability of the quantum version of the theory, the polynomial is restricted to degree 4. Specifically, let us write

$$V(\Phi) = \frac{1}{4} h \left(|\Phi|^2 - \eta^2 \right)^2 \, . \tag{1.2}$$

The above form of $V(\Phi)$ ensures that the Lagrangian (1.1) is invariant under the gauge group U(1), the group of phase-rotations of the complex field Φ. Here h and η are constants.

In the full quantum theory, the vacuum state is determined by the minimum of what is called the "effective potential". In the "tree" approximation, the effective potential is the same as the classical potential given by (1.2) above. We thus see that the field Φ acquires a vacuum-expectation-value (v.e.v.) $\langle\Phi\rangle$ given by

$$|\langle\Phi\rangle|^2 = \eta^2 . \tag{1.3}$$

There is thus a whole continuum of degenerate vacuum states and the U(1) symmetry is spontaneously broken in the tree approximation. The symmetry is spontaneously broken because, although $V(\Phi)$ in (1.2) and the Lagrangian (1.1) are invariant under U(1), any chosen vacuum state (out of the whole manifold of them, the manifold in the present case being a circle) is always rotated into another by a U(1) transformation, and hence is not invariant.

The physical particles in the above spontaneously broken U(1) theory are a scalar particle of mass $m_S = \sqrt{h}\ \eta$ and a vector particle of mass $m_V = \sqrt{2}\, e\eta$.

The vacuum of the theory corresponds to the absolute minimum of the energy functional. The vacuum is thus characterized by

$$F_{\mu\nu} = 0 \ , \quad D_\mu \Phi = 0 \quad \text{and} \quad |\langle\Phi\rangle| = \eta . \tag{1.4}$$

As mentioned above, there is no unique vacuum of the theory.

The occurrence of a whole continuum of degenerate vacua can be exploited to construct topologically non-trivial stable finite-energy solutions (see, for example, Coleman [27] or Rajaraman [28]). The requirement of finite energy means that asymptotically the fields must approach a vacuum configuration given by (1.4) sufficiently fast. In other words, the spatial infinity must map on to the manifold of vacua. If this mapping is topologically non-trivial, we have a finite energy topologically non-trivial solution. What is relevant in determining whether or not topologically non-trivial stable finite-energy solutions are allowed [27,28] is the structure of the homotopy group [29] $\pi_{S_\infty}(M)$, where S_∞ represents the manifold formed by the points at spatial infinity and M the manifold of degenerate vacua. The elements of the group $\pi_{S_\infty}(M)$ are the homotopy-equivalent classes of maps from S_∞ to M, the identity element being the homotopy class containing the trivial mapping, namely, the one where the whole of S_∞ is mapped into a fixed single point in M. Thus topologically non-trivial stable finite-energy solutions are allowed if and only if the group $\pi_{S_\infty}(M)$ has non-trivial elements. Notice that the existence of a non-trivial mapping by itself is not enough - the mapping must, in addition, be non-homotopic to the trivial mapping; otherwise, since [30] any continuous dynamical evolution can be regarded as a homotopy, the configuration will, in the course of dynamical evolution, go over to the trivial configuration or, in other words, the solution would not be stable. It should also be mentioned here that the classification of non-trivial field configurations by distinct homotopy classes is gauge-invariant [27] - the field configurations related to each other by gauge transformations fall into the same homotopy class as it should be if the homotopy criterion of stability is to be a meaningful one. Another point to be noted is that, although gauge field does not seem to have featured much in the above discussion, it is in fact crucial for the topologically non-trivial static solutions to have finite energy. This is seen as follows: in the absence of the gauge field, the finite energy requirement would mean that $\underline{\nabla}\Phi$ vanish sufficiently fast as we go to spatial infinity. In D-spatial dimensions, the energy integral would involve

$$\int r^{D-1} (\underline{\nabla}\Phi)^2 dr$$

where r is the radial variable in D-dimension. The transverse components of $\underline{\nabla}\Phi$ can fall only as fast as $\frac{1}{r}$. Thus for spatial dimension D > 2, the energy integral would diverge. In the presence of gauge field, however, we have

$$\int r^{D-1} (\underline{D}\Phi)^2 dr$$

By adjusting the behaviour of the gauge field $\underline{A}$, we can now make $\underline{D}\Phi = \underline{\nabla}\Phi + ie\underline{A}\Phi$ vanish as fast as $r^{-\left(\frac{D+1}{2}\right)}$ to ensure the convergence of the integral for D > 2.

Now, returning to our U(1) model, M is S^1, the circle. If we consider D = 3, then $S_{\mathcal{A}}$ is S^2, the surface of a sphere. But all maps from S^2 to S^1 are homotopic to the trivial map (a closed loop on S^2 can be continuously shrunk to a point). There is thus no topologically non-trivial solution in the U(1) model if D = 3. This is also true for D > 3. If, however, we have D = 2, then $S_{\mathcal{A}}$ is also a circle, S^1. Maps from S^1 to S^1 fall into distinct homotopy classes characterized by the 'winding number' giving the number of times the second

S^1 is wound round as the first S^1 is once. In other words, $\pi_{S^1}(S^1) = \mathbb{Z}$, the additive group of integers. There is thus a topologically non-trivial stable finite-energy solution of the abelian Higgs model in 2 spatial dimensions. This is called a <u>vortex</u>. Imposing cylindrical symmetry along the Z-axis in 3 spatial dimensions (thereby reducing the problem to essentially 2 dimensions in the XY plane) Nielsen and Olesen [15] were able to construct explicit vortex solution valid for asymptotic regions. Because of the cylindrical symmetry, their solution is known as a <u>vortex-string</u> or simply a <u>string</u>. The asymptotic solutions of Nielsen and Olesen look like (with the gauge choice $A_0 = 0$)

$$\underline{A} \approx \hat{\theta}\left[\left(\frac{n}{er}\right) + \text{const.} \times e^{-m_V r}/\sqrt{r}\right] ,$$

$$\Phi \approx e^{in\theta}\left[\eta + \text{const.} \times e^{-m_S r}/\sqrt{r}\right] . \tag{1.5}$$

Here n is the 'winding number' of the map $S^1 \longrightarrow S^1$ and r and θ are the radial and angular coordinates in 2 dimensions, respectively. The magnetic field $\underline{B} = \underline{\nabla} \wedge \underline{A}$ is directed along the z-direction and is concentrated mainly in the cylindrical region extending to a radial distance $\lambda \sim \frac{1}{m_V}$ and the scalar field is similarly different from its vacuum expectation value in the region extending to a radial distance $\xi \sim \frac{1}{m_S}$. The energy associated with the solution is thus mostly concentrated in a tubular region of space which defines the string. The string is quite similar to flux-tubes [16] in superconductors. Borrowing terminology from superconductor theory, the lengths λ and ξ are referred to as 'penetration depth' and 'correlation length' respectively.

The interesting thing about the string is that the magnitude of the magnetic flux through the xy-plane is quantized [15] in units of $\frac{2\pi}{e}$. This is of course related to the fact that the 'winding number' of the map $S^1 \longrightarrow S^1$ is an integer. Although the solutions (1.5) describe a string lying along the z-direction, one may imagine a string to have any arbitrary configuration (in particular, an arbitrarily shaped closed loop, for example) in space. Of course, nobody has written down explicit solutions for strings having irregular configurations, but there is no reason why such solutions should not exist. As illustrated in Fig. 1.1, the phase of the Higgs field changes by an integral multiple of 2π as one goes along a closed curve around a point on the string (the closed curve is taken to lie on a plane orthogonal to the direction of the string at the given point). By continuity, there must be a point somewhere in the region bounded by the closed curve where the Higgs field passes through zero. Thus the configuration of the string may be said to be defined by the nodal line of the Higgs field. One may equivalently use the direction of the magnetic field to define the shape of the string. Notice that the original U(1) symmetry is unbroken ($\langle \Phi \rangle = 0$) at the centre ('core') of the string.

The above discussions were in context of a specific model where the symmetry (U(1)) was completely (spontaneously) broken. In the general case of a group G spontaneously broken to a group H, the manifold M may be identified with the coset space G/H (see, for example, [27,31]). In this case the condition for the string solution to be allowed is that the fundamental group of G/H be non-trivial, i.e. $\pi_1(G/H) \neq 1$ (the triviality is denoted by 1). Thus the knowledge of the structures of the groups G and H essentially determines whether or not strings may exist.

The conditions for the existence of magnetic monopoles and domain walls are also given [14] in terms of the coset space G/H. The magnetic monopoles exist if $\pi_2(G/H)$ is non-trivial, whereas for domain walls to exist one requires $\pi_0(G/H) \neq 1$.

The condition $\pi_1(G/H) \neq 1$ for the existence of string implies that stable strings are not allowed in any symmetry breaking scheme in the simplest SU(5) model [1] of grand unification. This is easily seen as follows:

For any connected and simply-connected Lie group G (such as SU(5)), we have

$$\pi_0(G) = \pi_1(G) = \pi_2(G) = 1 .$$

A standard theorem in homotopy theory [29] then tells us that $\pi_1(G/H)$ and $\pi_0(H)$ are isomorphic to each other, i.e.,

$$\pi_1(G/H) = \pi_0(H) .$$

In the symmetry breaking scheme

$$SU(5) \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$$

which is achieved by using a set of Higgs field transforming under the 24-dimensional adjoint representation of SU(5), we have

$$G = SU(5) \text{ and } H = SU(3)_C \times SU(2)_L \times U(1)_Y$$

and so

$$\pi_1(G/H) = \pi_0(H) = \pi_0\left(SU(3)_c \times SU(2)_L \times U(1)_Y\right) = 1$$

(since each factor in H is a connected group), implying the non-existence of strings.

Kibble, Lazarides and Shafi [26] have shown that stable strings are possible in the SO(10) (or rather its simply connected covering group spin(10)) model [25] of GUT if a particular representation of Higgs field, namely 126, is used to break the spin(10) symmetry. In this case the symmetry breaking pattern one gets is

$$\text{Spin}(10) \xrightarrow[\underline{126}]{} SU(5) \times Z_2 \xrightarrow[\underline{45}]{} SU(3)_c \times SU(2) \times U(1) \times Z_2 \xrightarrow[\underline{10}]{} SU(3)_c \times U(1)_{em} \times Z_2,$$

where 126, 45 and 10 refer to the Higgs field representations required to achieve the indicated breakings. In the above scheme, Z_2 is the discrete multiplicative group $\{1, -1\}$. It is the appearance of this Z_2 factor that gives rise to vortex-strings at the first symmetry breaking because

$$\pi_1[\text{Spin}(10)/(SU(5) \times Z_2)] = \pi_0(SU(5) \times Z_2) = Z_2.$$

The topological quantum numbers of the string in this case are characterized by the elements of the group Z_2 (recall that the strings in the abelian Higgs model discussed earlier are associated with the group Z of integers). The Z_2 symmetry remains unbroken in the subsequent symmetry breakings and so the strings that arise at the first stage of symmetry breaking remain as stable objects. Moreover, because of the same reason (i.e. unbroken discrete Z_2 symmetry) no domain walls appear, making the model cosmologically desirable. The origin of the discrete symmetry Z_2 lies in the fact that the representations 126, 45 and 10 used above are all single-valued under spin(10), unlike the spinor 16 which is double valued. The single valuedness means that they are invariant under a 2π rotation contained in Spin(10), and so the unbroken symmetry group of the 126, for example, must include this 2π rotation as a group element. The element -1 in the group Z_2 is nothing but this 2π rotation belonging to the original spin(10). The group SU(5) by itself does not contain the element -1, and so the group Z_2 takes care of this. Interestingly enough, Olive and Turok [32] have shown that these so-called "Z_2-strings" also arise very naturally in GUT with any of the exceptional groups E_8, E_7 or E_6 as the gauge group.

In the above examples, the group G was taken to be a simply connected group - a criterion one may certainly wish to impose on a grand unifying group. In this case $\pi_1(G/H)$ and $\pi_0(H)$ were isomorphic to each other, which implied that the existence or otherwise of string could be inferred simply by examining whether or not H (the unbroken group) contained disconnected components. If, however, the group G is not simply connected the above isomorphism is not valid, and the structure of the unbroken group H by itself cannot tell us whether or not string is allowed - one now has to directly determine $\pi_1(G/H)$ by examining whether or not closed loops in the coset space G/H are shrinkable. Thus, for example, in the Salam-Weinberg model [3], the group G is SU(2) x $U(1)_Y$ which is not simply connected (because of the $U(1)_Y$ factor) and the group H is $U(1)_{em}$. The generator of $U(1)_{em}$ is a linear combination of the generator of $U(1)_Y$ and the generators of SU(2). By examining the closed loops in the coset space in this case, Schwarz and Tyupkin [33] and Everett [34] have shown that stable strings do not occur in Salam-Weinberg symmetry breaking, $SU(2) \times U(1)_Y \longrightarrow U(1)_{em}$.

Let us now see how vortex-strings (and, for that matter, any other topological objects) are expected to appear at a phase transition in the early universe.

1.3 Appearance of vortex-strings at a phase transition

In the finite temperature formulation [11-13] of spontaneously broken gauge theories, the 'effective potential' whose absolute minimum determines the vacuum state of the theory, is a temperature dependent object. Typically, above a certain critical temperature T_c, the absolute minimum of $V(\Phi)$ occurs at $<\Phi> = 0$. Below T_c, $V(\Phi)$ becomes such that its absolute minimum is at a non-zero $<\Phi>$. Thus in the hot big-bang model the universe starts out with exact grand unified symmetry ($<\Phi> = 0$). But as the universe expands and cools through T_c, there is a phase transition when the grand unified symmetry G is broken to the group H and $<\Phi>$ ($\neq 0$) lies on the manifold M. However, immediately below T_c, there are random thermal fluctuations in $<\Phi>$ and thus random regions of space may fluctuate back to the symmetric phase ($<\Phi> = 0$). Below a temperature T_G called the Ginzburg temperature [35] ($T_G < T_c$), the thermal fluctuations become negligible [14] and the universe tries to settle down to the asymmetric phase corresponding to $<\Phi>$ being any point on the manifold M (= G/H). There is, however, no reason for different regions of space to correspond necessarily to the same point on M. Indeed, it would be surprising if far separated regions of space get associated with the same vacuum state, especially if the regions are separated by more than the causal horizon distance. Thus, in general $<\Phi>$ is expected to vary from region to region.

There is then always a finite probability that while the universe is settling down to the asymmetric phase the spatial variation of $<\Phi>$ is such that points or regions of space in the symmetric phase ($<\Phi> = 0$) get trapped as 'defects'. Depending on the structure of the homotopy groups $\pi_2(G/H)$, $\pi_1(G/H)$ or $\pi_0(G/H)$ as discussed in the previous section, these 'defects' will appear as topologically stable objects, namely, magnetic monopoles [19], vortex strings [15] and domain walls [17,18,14]. There can also be more complicated objects composed basically of the above three kinds of defects. For example, a monopole and an antimonopole may get connected [36] by a string, a closed loop of string may form the boundary of a domain wall [37] etc. Each of these different kinds of topological objects has important cosmological implications which have been discussed in literature [36,37].

As already mentioned, the aim in this thesis is to investigate some cosmological aspects of pure vortex-strings. The strings might provide a viable theory of galaxy formation which we discuss briefly in the following section.

1.4 Vortex-strings as the 'seed' for galaxy formation

That strings might provide the essential mechanism for generating the initial density perturbation that is thought to lead to galaxy-formation (see, for example, Peebles [38]) was first suggested by Zeldovich [22] and by Kibble [20]. The origin of this initial density fluctuation has been a long-standing problem in the theory of galaxy formation. The strings can shed new light on this problem in the following way.

At the time of formation the strings are expected to be in a rather random tangled configuration. The typical length scale of the configuration at the time of formation is given by [14] the correlation length ξ_G of the Higgs field at the Ginzburg temperature. The general picture of evolution of such a random configuration of string in the early history of the universe has been discussed by Kibble [14,20,24]. Immediately after the formation of the strings at the grand unified phase transition, the motion of the strings is heavily damped by the dense surrounding medium. Due to tension in the string, the natural tendency of a bent piece of string is to straighten and shorten its length. Moreover, any closed loop will shrink and disappear, which adds to the shortening of the length. This equivalently means that the typical length scale in the tangled configuration of string has a tendency to increase. The heavy damping tends to enhance the process of straightening (the terminal velocity of a segment of string of radius of curvature r in presence of damping is proportional to 1/r, so that kinky portions will move faster than straighter sections). We see that during the period of heavy damping the scale-length L in the configuration of string will grow rapidly. Rough order-of-magnitude estimates indicate [23] that the length scale L grows and becomes about the same size as the causal horizon distance $\sim t$ at a time $t^* \sim 10^{-35}$ sec. This is quite early in the history of the universe. After this time, the effectiveness of damping in straightening of the strings is considerably reduced. There may, however, be another important process which may continue to contribute significantly to the straightening of strings.

This process is the formation of closed loops. The loops are likely to appear through the process of "change of partners" [14] when a string intersects itself. This is illustrated in Fig. 1.2. The closed loop finally disappears. This comes about because the loop either oscillates and radiates energy gravitationally and disappears finally, or it may collapse and release its energy in the form of particles. It is thus expected that L will continue to grow after the time t*. How fast it will grow will depend on the rate of formation of loops which in turn depends on the probability of "exchange of partners" when the string self-intersects. It is difficult to calculate the rate of loop formation from first principles. Vilenkin [23] makes an estimate of this rate by assuming that the rate is large enough to maintain the growth rate of L at the maximum possible value, i.e. $L \sim t$ (the scale length cannot be greater than the causal horizon distance $\sim t$). This gives [23]

$$\frac{dn}{dt} \sim \frac{1}{t^4}$$

where n is the number density of loops. This means approximately one horizon-size loop is formed in a horizon size volume per expansion time.

According to the above picture, we would expect to find, at any given time, approximately one length of string stretched across the visible universe, and also a number of closed loops, including those surviving from earlier times (if gravitational radiation is the dominant energy loss of oscillating loops, then these loops survive a long time - a loop formed at a time t of radius $\sim$ t has a life-time $\sim 10^6$ t [23,24].

Now consider a string of length $\sim$ t stretched across the universe. If μ is the mass per unit length of the string, then the mass density of the strings is

$$\rho_s \sim \mu t^{-2}.$$

This is to be compared with the density of the surrounding matter (see appendix A),

$$\rho \simeq \frac{.03}{G t^2}$$

(We are assuming radiation dominated universe). We thus get the ratio

$$\frac{\rho_s}{\rho} \approx 30\, G \mu .$$

Now μ, the mass per unit length of the string, is given by [14]

$$\mu \approx T_c^2 \approx |\langle \Phi \rangle|^2 ,$$

where T_c is the phase-transition temperature. We thus have

$$\frac{\rho_s}{\rho} \approx 30 \left(\frac{T_c}{M_P}\right)^2 ,$$

where $M_P = \sqrt{\frac{1}{G}}$ is the Planck mass.

Zel'dovich [22] argued that if $T_c \sim 10^{17}$ GeV, then $\frac{\rho_s}{\rho} \sim 10^{-3}$, which is roughly the value of the initial density perturbation needed [38] to start the galaxy formation process.

The above value of T_c ($\sim 10^{17}$ GeV) assumed by Zel'dovich is rather high compared to the usual (see Langacker [1]) values of T_c of the order of 10^{15} GeV or so. With a value of $T_c \sim 10^{15}$ GeV, we get $\frac{\rho_s}{\rho} \sim 10^{-7}$, which is too low to be of relevance to galaxy formation. Zel'dovich did not consider the effect of closed loops.

The situation is rather different, as first shown by by Vilenkin [23], if one considers the closed loops. With the rate of loop formation given by $\frac{dn}{dt} \sim \frac{1}{t^4}$, together with the fact that a loop takes a finite time to (gravitationally) radiate away all its energy, Vilenkin showed that

$$\frac{\rho_{loops}}{\rho} \sim \frac{T_c}{M_P} .$$

Thus with $T_c \sim 10^{15}$ GeV, one can get approximately the correct order of magnitude of the density perturbation. After the decoupling time $t_{dec} \sim 10^{12}$sec, the density perturbation grows essentially like $t^{2/3}$ [23]; before this time the high radiation pressure prevents much gravitational clustering of the loops. Vilenkin's arguments show that loops formed at $\sim 10^9$sec can bind objects of galactic mass-scales ($\sim 10^{12} M_\odot$, $M_\odot$ being the solar mass) at a time $\sim 10^{16}$sec.

It must be stressed here that there are two basic assumptions involved in Vilenkin's scenario. The first is that oscillating loops lose energy predominantly through gravitational radiation. Since the strings do not carry any electric or magnetic charge and so do not directly couple to electromagnetic radiation, this assumption seems to be reasonable. (There may be some indirect processes involving particle creation by the non-static field of an oscillating loop. The significance of these processes, however, remains to be studied properly). There is another related problem here. To see this, let us consider a loop of radius ℓ, of mass $M \sim 2\pi \ell \mu$ and oscillating with a characteristic frequency $\omega \sim 1/\ell$. The gravitational radiation from such a loop can be estimated from the quadrupole radiation formula (see, for example, [7])

$$\dot{M} \sim - G M^2 \omega^2 .$$

The life-time (τ) of the loop, calculated from this formula is [23,24]

$$\tau \sim 10^6 \ell .$$

Occasionally, the loop may intersect itself and break into smaller loops. Clearly these smaller loops have smaller life-times. Thus, if the loops self-intersect very frequently the mean life-time of loops is sharply reduced, which means that the loops may not survive long enough to produce the necessary density perturbation. For this reason, Vilenkin [23] makes the second assumption that the closed loops do not self-intersect frequently. Fortunately, Kibble and Turok [39] have shown that there indeed exists a large class of freely oscillating closed loop solutions which never self-intersect, thus strengthening the viability of Vilenkin's scenario.

The fact that closed loops of vortex-string may generate the initial density perturbations leading to galaxy-formation, provides the motivation for undertaking further investigations of the various cosmological implications of vortex-strings. With this in mind, some specific topics relating to cosmic vortex-strings are taken up in the following chapters of this thesis. The contents of these chapters are summarized in the next section.

1.5 Outlines of the following chapters

In Chapter II, the behaviour of energy of string in an expanding universe is studied. The results show that the energy-density of a random (Brownian) configuration of string would come to dominate the total energy-density in the early universe unless there was some efficient mechanism of energy-loss of string. Explicit numerical calculations of the energy of waves on strings are presented, which lead to the above conclusion. It is then shown that a possible energy-loss mechanism which could remove the inconsistency (of energy-density being dominated by string in the early universe) is the formation of closed loops. The energy-density of closed loops (of radius smaller than horizon distance), as we shall see, does not dominate the energy density in the early universe, essentially because of finite life-time of these loops, the finite life-time being due to gravitational radiation from these loops. We thus establish that formation of closed loop is a crucial condition for the consistency of the whole picture.

The subject of Chapter III is also closed loops. It is claimed that collapsing [39] closed loops of string in the early universe gives a significant contribution to the baryon-asymmetry of the universe. The baryon-asymmetry in this case arises from the decays of those superheavy bosons which are released when a loop collapses. Indeed, the estimated order of magnitude of baryon-asymmetry produced in this way is shown to be in reasonably good agreement with the observed value.

In Chapter IV, we address ourselves to the important question whether or not strings "exchange partners" [14] when they intersect. Recall that formation of closed loops requires an affirmative answer to this question. It is argued that what is relevant in this context is a proper understanding of the nature of interaction between strings. With this in mind, the interaction energy of a system of two parallel strings is first calculated. The result of this calculation then allows us to make certain qualitative comments which outline a possible mechanism through which "changing of partners" may occur when a string intersects itself.

CHAPTER II

EVOLUTION OF COSMIC STRINGS IN AN EXPANDING UNIVERSE

2.1 Introduction

The discussions in the previous chapter have shown that cosmic strings may have an important role to play in the problem of galaxy formation. It is, however, clear that the viability of this whole picture of galaxy formation from strings [22,23] depends upon a better understanding of how a network of strings formed at a grand unification phase transition would behave as the universe expanded. In this chapter some progress is made towards this goal by obtaining a quantitative picture of the process of energy exchange between the network of strings and the expanding universe they lie in.

In section 2.2 an approximate analysis of the energetics of lengths and loops of string in an expanding universe is given. Numerical results confirming this analysis will be presented in section 2.3. In section 2.4 the implications of the results for the consistency of the string picture are discussed and conclusions presented.

2.2 Energetics of strings in an expanding universe

The dynamics of the string will be described classically. The width of a string formed in grand unified symmetry breaking will be given by ${m_x}^{-1}$, where m_x is the superheavy gauge boson mass. For the cosmological scales we are interested in the width is utterly negligible and the string for all practical purposes can be regarded as essentially a one-dimensional object. Throughout this chapter the string will be treated as purely a mechanical object - the scalar and gauge fields 'constituting' the string will not explicitly enter into the discussions. In that case, the action functional describing the dynamics of the string can be taken to be simply proportional to the area of the two-dimensional world sheet swept out by the string in 4-dimensional space-time [40] (see Appendix B):

$$\begin{aligned} S &= -\mu \int dA \\ &= -\mu \int d\sigma\, d\tau \left[\left(\frac{\partial x}{\partial \sigma} \cdot \frac{\partial x}{\partial \tau} \right)^2 - \left(\frac{\partial x}{\partial \sigma} \right)^2 \left(\frac{\partial x}{\partial \tau} \right)^2 \right]^{1/2} \\ &\equiv \int d\sigma\, d\tau\, L \left\{ \frac{\partial x}{\partial \sigma}, \frac{\partial x}{\partial \tau} \right\}, \end{aligned} \qquad (2.1)$$

where $x^{\nu}(\sigma, \tau)$ is the space-time coordinate of the string, μ the mass per unit length of the string, and σ, τ the parameters describing the two-dimensional surface. The action (2.1) is analogous to that for a relativistic point particle in which case it is proportional to the invariant length of the world line of the particle.

In eq. (2.1), $a.b = g_{\mu\nu}$, $a^{\mu}b^{\nu}$, $g_{\mu\nu}$ being the metric to be specified. We are interested in the very early history of the universe, before the decoupling of radiation from matter. At such times, the spatial curvature effects may be ignored (see appendix A) and the universe described by a flat-space cosmological model with the metric

$$ds^2 = dt^2 - R^2(t)\, d\underline{x}^2 , \qquad (2.2)$$

where t is the cosmic time and R(t) the cosmological factor.

We may always choose [40] a parametrization of the surface such that

$$x^0 = t = \tau$$

and

$$\frac{\partial \underline{x}}{\partial \tau} \cdot \frac{\partial \underline{x}}{\partial \sigma} = 0 . \qquad (2.3)$$

With this parametrization, the energy of the string is obtained as (see Appendix B, eq. (B.20)),

$$E = \int d^3x \sqrt{-g}\, T^{00} = \mu R \int d\sigma\, \epsilon(\sigma,\eta) \,, \tag{2.4}$$

where

$$\epsilon(\sigma,\eta) = \left[\underline{x}'^2 / (1-\dot{\underline{x}}^2)\right]^{1/2} \,, \tag{2.5}$$

with $\underline{x}' = \partial \underline{x}/\partial\sigma$ and $\dot{\underline{x}} = \partial\underline{x}/\partial\eta$, and we have introduced the conformal 'time' η defined by dt = R$d\eta$. The proper length of the string is thus $R\int \epsilon\, d\sigma$. Now, to obtain the equation of motion, we can write L in eq. (2.1) as

$$L = -\mu R \left[\underline{x}'^2 (1-\dot{\underline{x}}^2)\right]^{1/2} ,$$

where we have used (2.3). The equation of motion (see Appendix B, eq. (B.18)) is then

$$\frac{\partial}{\partial\eta}(\epsilon\dot{\underline{x}}) + 2\frac{\dot{R}}{R}(\epsilon\dot{\underline{x}}) = \frac{\partial}{\partial\sigma}\left(\frac{\underline{x}'}{\epsilon}\right) , \tag{2.6}$$

where all dots (·) are with respect to the variable η . We are interested in how the energy changes with time. For that, we have to find $\dot{\epsilon}$. From (2.5) and (2.6) it follows, on using the condition $\dot{\underline{x}}.\underline{x}' = 0$, that

$$\dot{\epsilon} = -2\frac{\dot{R}}{R}\epsilon\dot{\underline{x}}^2 . \tag{2.7}$$

This enables us to rewrite eq. (2.6) as

$$\ddot{\underline{x}} + 2\frac{\dot{R}}{R}\dot{\underline{x}}(1-\dot{\underline{x}}^2) = \frac{\underline{x}''}{\epsilon^2} - \frac{\epsilon'}{\epsilon^3}\underline{x}' . \tag{2.8}$$

From eq. (2.8) it is seen that the expansion of the universe has the effect of damping the string's motion. Clearly, on a given curvature scale for the string, the damping term will dominate at very early times and the string's comoving velocity will be very small.

To be able to be a little more precise, let us introduce some terminology. A closed loop of string can always be described by the coordinates $\underline{x}(\sigma, t)$ which may be expanded as a Fourier series in the length along the string $\int \epsilon \, d\sigma$, with the Fourier coefficients dependent on t. We shall be interested in theories where the strings have no ends - so they are either closed loops or are infinitely long. In any case we can always consider a given length of string as a part of a much larger closed loop. Thus the periodicity in $\int \epsilon \, d\sigma$ always has a well defined spectrum. We shall call the periodicity in $\int \epsilon \, d\sigma$ the 'length in wave' since it is measured along the string, as opposed to the wavelength which is measured in a straight line.

Now, consider a general wavy string with a given 'length in wave' ℓ and of amplitude $A(\ell)$. Quite generally, the curvature ($\underline{x}''$) term is of order $A(\ell)/\ell^2$. At very early times the damping term will dominate over other terms in eq. (2.8) and the velocity $|\dot{\underline{x}}|$ is roughly of order $\frac{R}{\dot{R}} \frac{A(\ell)}{\ell^2}$ which is of course less than $\frac{R}{\dot{R}} \frac{1}{\ell} \sim \eta/\ell$ (since $A(\ell)$ has to be less than ℓ). Here we have used the fact that for a radiation dominated universe $R \propto t^{\frac{1}{2}} \propto \eta$. We thus see that if ℓ is very much greater than η , damping dominates and the terminal velocity is very small, i.e. $|\dot{\underline{x}}| \ll 1$. From eq. (2.7) we then see that the magnitude of $\dot{\epsilon}/\epsilon$ is much smaller than $\frac{\dot{R}}{R}$ and so from eq. (2.4) the behaviour of the energy is dominated by the expansion factor R. In other words, the string is conformally stretched by the expansion of the universe as long as the 'length in wave' is larger than the particle horizon distance.

Due to conformal stretching, the 'length in wave' grows like η, whereas the horizon grows like $t \propto \eta^2$. So, the horizon will soon catch up with ℓ. When ℓ falls inside the horizon, all the terms in eq. (2.8) become operational and it is not directly obvious how the energy in a 'length in wave' behaves with the expansion of the universe. One has to solve eq. (2.8) numerically. The numerical results for some specific cases are given in the next section. We can, however, perform an approximate perturbation analysis to first order in $\dot{R}/R$ in the regime where the 'length in wave' has fallen well within the horizon. From eq. (2.4) and (2.7) we can write

$$\frac{\partial}{\partial \eta}\left(\frac{E}{R}\right) = -2\,\frac{\dot{R}}{R}\,\mu \int d\sigma\ \dot{\underline{x}}^2\,\epsilon \ . \tag{2.9}$$

If we are doing only a first-order analysis in $\frac{\dot{R}}{R}$, we can treat ϵ on the r.h.s. of (2.9) as essentially a constant: $\epsilon = C$, say. For periods of motion less than the expansion time scale (i.e. 'length in wave' smaller than the horizon) we can average eq. (2.9) over a period T to give

$$\frac{\partial}{\partial \eta}\left(\frac{\bar{E}}{R}\right) \approx -2\,\frac{\dot{R}}{R}\,\mu C\,\frac{1}{T}\int_0^T d\eta \int d\sigma\ \dot{\underline{x}}^2 . \tag{2.10}$$

We can perform the integration in (2.10) by using the equation of motion (2.8). Again, to first order in $\frac{\dot{R}}{R}$, this amounts to effectively using the equations

$$\ddot{\underline{x}} = \frac{1}{c^2}\underline{x}'' \quad , \quad \dot{\underline{x}}^2 + \frac{1}{c^2}\underline{x}'^2 = 1 \quad , \quad \dot{\underline{x}}\cdot\underline{x}' = 0 \tag{2.11}$$

in (2.10). Integrating by parts, we have

$$\frac{\partial}{\partial\eta}\left(\frac{\bar{E}}{R}\right) \approx -2\,\frac{\dot{R}}{R}\,\frac{\mu}{2}\,C\left\{L - \frac{1}{TC^2}\int_0^T \left[\underline{x}\cdot\underline{x}'\right]_0^L d\eta + \frac{1}{T}\int_0^L \left[\underline{x}\cdot\dot{\underline{x}}\right]_0^T d\sigma\right\} , \tag{2.12}$$

where L corresponds to the length of the string involved. Now, for a closed loop, the boundary terms vanish and $\bar{E} \approx \mu RC\int_0^L d\sigma = \mu RCL$ from (2.4). Then (2.12) gives

$$\frac{\partial}{\partial\eta}\left(\frac{\bar{E}}{R}\right) \approx -\frac{\dot{R}}{R^2}\,\bar{E} \ ,$$

which implies

$$\bar{E} \approx \text{constant} . \tag{2.13}$$

This is just as well - if protons are made of strings (as current theories suggest) they had better not expand with the universe (size of the proton is, of course, less than the horizon size!)!

We have thus seen that the energy of a closed loop remains constant once the loop falls well within the horizon. As a slightly more complex example, consider a first order solution to (2.8) in the form of a spiral standing wave,

$$\underline{x} = \left(\lambda C\sigma,\ a\alpha\cos\frac{C\sigma}{a}\cos\frac{\eta}{a}\ ,\ a\alpha\sin\frac{C\sigma}{a}\cos\frac{\eta}{a}\right) \tag{2.14}$$

with $\lambda^2 + \alpha^2 = 1$ (so as to make $\epsilon = \left(\frac{\underline{x}'^2}{1-\dot{\underline{x}}^2}\right)^{1/2} = C$).
From eq. (2.12) we now find

$$\frac{\partial}{\partial\eta}\left(\frac{\bar{E}}{R}\right) \approx -\frac{\dot{R}}{R^2}\,\bar{E}\left(1-\lambda^2\right) ,$$

or

$$\bar{E} \propto R^{\lambda^2}. \tag{2.15}$$

Clearly, (2.13) is a limiting case of (2.15) when $\lambda = 0$. Notice that for a straight length of string ($\alpha = 0$, $\lambda = 1$) the energy increases like R.

If we now allow λ , α , a and C (= ϵ) to vary with time (the cylindrical symmetry tells us $\epsilon' = 0$ always), we can follow the evolution of a wave like (2.14).

The constraint $\dot{\underline{x}}.\underline{x}' = 0$ gives, from (2.14),

$$\sigma\lambda\epsilon\,\frac{\partial}{\partial\eta}(\lambda\epsilon) + \sigma\epsilon a\alpha^2 \cos^2\frac{\eta}{a}\,\frac{\partial}{\partial\eta}\left(\frac{\epsilon}{a}\right) = 0 . \tag{2.16}$$

The equations of motion preserve the periodicity in σ and so

$$\frac{\epsilon}{a} = \text{constant} . \tag{2.17}$$

The second term on the r.h.s. of (2.16) then drops out and we have

$$\lambda\,\epsilon = \text{constant}. \tag{2.18}$$

Combining (2.17) and (2.18) we have

$$\lambda\, a = \text{constant} . \tag{2.19}$$

Eq. (2.19) means that the comoving wavelength remains constant (i.e. proper wavelength increases like R). In other words, the spiral actually gets stretched out. If we start with a tightly bound spiral ($\lambda \ll 1$), eq. (2.15) tells us that initially the energy remains almost constant, as in the case of a closed loop. However, we know from (2.7) that ϵ decreases monotonically, so λ must increase (by (2.18)). Asymptotically, λ approaches 1 as the spiral is completely stretched out. As this happens, the energy (2.15) increases more and more nearly in proportion to R. The transition between almost constant energy ($\lambda^2 << 1$) and energy scaling like R ($\lambda^2 \sim 1$) occurs when λ, the ratio of the wavelength ($2\pi\lambda a$) to the 'length in wave' ($2\pi a$) becomes nearly unity. Finally, as the wave is almost completely stretched out, the energy per proper 'length in wave' should approach $\mu R 2\pi \lambda_0 a_0$, where λ_0, a_0 refer to the initial values of λ and a.

We next examine the results obtained by directly solving the equation of motion numerically and we shall see that the above first-order analysis is a good description of what actually happens to the energy of the string as the universe expands.

2.3 Numerical solution

Equation (2.8) for $\underline{u} = \dot{\underline{x}}$ and $\underline{v} = \underline{x}'$ can be solved by a simple finite difference algorithm using the derivatives

$$\frac{\partial f(\sigma,t)}{\partial t} = \frac{1}{\Delta t}\left[f(\sigma,t+\Delta t) - \frac{1}{2}\left\{f(\sigma+\Delta\sigma,t) + f(\sigma-\Delta\sigma,t)\right\}\right]$$

(2.20)

$$\frac{\partial f(\sigma,t)}{\partial \sigma} = \frac{1}{2\Delta\sigma}\left[f(\sigma+\Delta\sigma,t) - f(\sigma-\Delta\sigma,t)\right].$$

The wave equation

$$\ddot{\underline{x}} = \frac{1}{\epsilon^2} \underline{x}'' \qquad (2.21)$$

can be checked to be stable when approximated by (2.20) for $\frac{\Delta\sigma}{\Delta t} \geqslant \frac{1}{\epsilon}$. This can be understood as a reflection of the need to supply adequate information to fill the backward light-cone. Further, the lowest order errors (proportional to $\dddot{\underline{x}}$ and $\dot{\underline{x}}''$), introduced in (2.21) when we replace the partial derivatives by finite differences as defined in (2.20), can be shown to cancel when $\Delta\sigma = \frac{1}{\epsilon}\Delta t$. Now, in general, we expect the damping term in eq. (2.8) to only add to the stability of the integration, but if ϵ varies with σ (as is the case for an arbitrarily shaped wave) we have the problem of how to select the same value for the next time step Δt for different points along the string. Clearly, one needs to use the minimal value of ϵ to maintain stability, but then we sacrifice some accuracy at other points along the string where ϵ are greater than the minimal value. For this reason, the cylindrically symmetric standing waves ($\epsilon' = 0$) described in the previous section are the easiest to deal with - the results for them are presented in figure 2.1.

The initial conditions were

$$\underline{x}(\sigma, t=0) = (\lambda\sigma, \alpha \cos\sigma, \alpha \sin\sigma), \quad \alpha^2 + \lambda^2 = 1,$$

so that the 'length in wave' was always 2π initially. The graph shows how the energy grows in proportion to R initially, comoving length ℓ staying constant at 2π until $\eta \sim \ell/2\pi$ and the wave falls inside the horizon. Small amplitude waves have $\lambda^2 \sim 1$ and so their energy grows, soon approaching the asymptote $E = 2\pi\mu\lambda_0 R$. Larger amplitude waves ($\lambda^2 \ll 1$) initially have almost constant energy ($E \propto R^{\lambda^2}$) but as $2\pi R\lambda_0$ (the <u>proper</u> wavelength) approaches E/μ (the <u>proper</u>

'length in wave'), the energy rises faster, again approaching the asymptote E = $2\pi\mu\lambda_0 R$. On shorter time scales, it is seen that the energy oscillates. This is due to the energy exchange between the string and the space around it - when a wave is collapsing inwards, it gains energy from the expansion damping whereas when it is expanding outwards it loses energy.

The evolution of closed loops can be followed similarly. Initially static loops are straightened out by the expansion and tend to rapidly approach the velocity of light after they fall within the horizon. This does not mean they necessarily intersect themselves, but it does mean ϵ becomes large and the error terms due to the right hand side of eq. (2.8) grow rapidly.

Loops started in an initially non-static configuration with radius of order the horizon distance [23] (loops like these might be formed by the collision of waves travelling on string [41]) behave rather similarly to the very large-amplitude waves, small amplitude perturbations on the loops being smoothed out, but their average energy remaining constant (see Fig. 2.2).

2.4 Implications of the results

Let us now discuss the significance of the above results. The energetics of the network of strings in an expanding universe is crucial to the consistency of the string picture in the following way.

As we have seen, 'length in waves' larger than the horizon are conformally stretched by the expansion. Naively, if this were true on all scales, the energy of a length of string would scale as R and the volume it lies in as R^3, so the string energy density would scale as R^{-2}, compared to matter energy density scaling as R^{-3} and radiation as R^{-4}. It would thus rapidly come to dominate the energy density of the universe - a situation clearly incompatible with observation.

We have seen that things are not quite as bad as this - the energy of large-amplitude waves was almost constant. Nevertheless, the energy always increases, albeit slowly (e.g. eq. (2.15)). This means that in the absence of any mechanism of energy-loss, the energy density of string scales more slowly than that of matter (the string energy increases, and the volume increases like R^{-3}). This is an important result, for it shows that if there were no mechanism for energy-loss the string density would rapidly (in the radiation-dominated era) come to dominate the total energy density of the universe. It is independent of the density spectrum of strings at formation.

The most obvious mechanism of energy-loss is, of course, via gravitational radiation - when a string starts to move it radiates away energy via gravitational waves. The rate of gravitational energy loss of an oscillating loop of mass M is [23,24] of order $\dot{M} \sim -G\mu^2$. This gives loops a finite lifetime $L/(G\mu)$, where L is their length when they start to move freely, i.e. fall inside the horizon. Is the gravitational energy loss efficient enough for string energy not to dominate the total energy density of the universe? To answer this question we have to be a little more quantitative about the energy-density contribution from all strings on different scales at a given time. The initial spectrum for strings is expected [42] to be scale-free and recent numerical simulations have indeed confirmed [43] this. What this means is that, although there is a length scale in the problem, namely the correlation length at the time the strings are formed, the large-scale distribution of string (i.e. the distribution observed at lower resolution) should be independent of this length. In particular, let us work in a closed universe (i.e. periodic boundary conditions) since we can always obtain the infinite universe case as a large volume limit of this case. Then all strings form closed loops (strings have no ends in the models we consider). Now consider the number density n(a)da of loops of "radius" between a and a+da. Since "radius" is a large-scale (low resolution) property n(a) should be independent of the correlation length. Then, on dimensional grounds, we must have

$$n(a)\,da \sim \frac{da}{a^4} \, . \qquad (2.22)$$

This spectrum cannot in itself be used to find the total energy density of string. To do this we need to know the length L of a loop of "radius" a. A first guess would be that the string describes an approximately Brownian trajectory so $L \propto a^2/\xi$, where ξ is the persistence length.

Now let us make the most optimistic assumption about the behaviour of string beneath the horizon; we shall assume that waves are straightened out up to the horizon distance $\sim t$ so that the persistence length grows as t and the length of any loop of "radius" larger than the horizon is given by $L \propto a^2/t$. Then the energy-density contribution from loops of "radius" greater than the horizon is approximately (ignoring all dimensionless constants)

$$\rho(a \gtrsim t) \sim \mu \int_t^\infty \frac{da}{a^4} \frac{a^2}{t} = \mu t^{-2}, \tag{2.23}$$

which scales as radiation in a radiation dominated universe.

However, this estimate is only a lower bound and we can see that it is indeed too low for the following reason. As we have seen in section 2.2, the energy, and hence the length, of a loop of size bigger than horizon always grows while $L \propto a^2/t$ suggests it remains constant (the "radius" a would grow as the scale factor $R \propto t^{\frac{1}{2}}$ in a radiation-dominated universe). Thus we should rather have $L \propto a^2/\left(\xi_0 \left(\frac{t}{\xi_0}\right)^\alpha\right)$ with ξ_0 the original correlation length at formation of string and $\alpha < 1$. This yields

$$\rho(a \gtrsim t) \sim \mu \int_t^\infty \frac{da}{a^4} \frac{a^2}{\xi_0 \left(\frac{t}{\xi_0}\right)^\alpha}$$

$$= \mu t^{-(1+\alpha)} \xi_0^{-(1-\alpha)}, \tag{2.24}$$

and since $\alpha < 1$, we see that the energy density of string falls at a slower rate than that of radiation. For $\rho \propto t^{-2}$ we need $\alpha = 1$. It is easily checked that if we take account of the gravitational energy-loss, given by the formula [23,24] $dM/dt \sim -G\mu^2$ or equivalently $dL/dt \sim -G\mu$, of these loops, the effect is to subtract from L a term $G\mu(t-t_0)$ where t_0 is the time at formation. However, the (negative) contribution of this term to ρ scales as t^{-2} for $t \gg t_0$ and so cannot cancel the original term discussed above if $\alpha \neq 1$.

It is clear from the above analysis that some energy-loss mechanism is needed which increases the correlation or persistence length of the string to scale with the horizon distance $\sim t$ (so $\alpha = 1$). The only plausible mechanism would seem to be the self-intersection of these large loops and the 'exchange of partners' [14] to form smaller closed loops. This would have to happen on a scale $\sim t$ and would thus give rise to loops of "radius" of order of horizon distance. To yield $\rho \propto t^{-2}$ it would have to occur at the maximum possible rate, producing a constant number of such loops per horizon volume per expansion time, i.e. $dn/dt \sim 1/t^4$ where n is the number of loops per unit volume.

Loops of "radius" smaller than the horizon do not present an energy problem as is shown below. Their radius and length, as we saw in section 2.2, remain constant once they fall inside the horizon. Thus for loops falling inside the horizon, or those produced by self-intersection, as discussed above, the constancy of energy means that the energy-density decreases as R^{-3} (R is the scale factor) so that the energy-density spectrum obeys ($\rho = \int da\, \nu(a)$)

$$\nu(a)\, da \sim \mu \left(\frac{t_h}{t}\right)^{3/2} \frac{da_h}{a_h^4}\, t_h \,, \qquad (2.25)$$

where we have used $R \propto t^{\frac{1}{2}}$ and the fact that the length of these loops is of order t_h where t_h is the time when they fall inside the horizon (see the discussions in the previous paragraph); a_h is their "radius" at this time, and $t_h \sim a_h$, and $a \approx \text{constant} \approx a_h$ thereafter. However, these loops have a finite life-time due to gravitational radiation - a loop formed at a time t_f disappears at the time $\sim (G\mu)^{-1} t_f$ so that loops formed at times earlier than $G\mu t$ will not any more be present at the time t and hence will not contribute to the energy-density at time t. As explained above, the loops formed at the time $G\mu t$ have radius $a_h \sim G\mu t$. We thus have

$$\rho_{\substack{Loops \\ (a \leq t)}} \sim \frac{\mu}{t^{3/2}} \int_{G\mu t}^{t} \frac{da}{a^{3/2}} \sim \frac{\mu}{(G\mu)^{1/2}} t^{-2} \qquad (2.26)$$

which scales like radiation.

We are left with the conclusion that in order to render the string picture energetically consistent, there must occur production of closed loops of radius $\sim t$ at a rate $\frac{dn}{dt} \sim 1/t^4$. Turok [41] has shown that this might indeed be possible, but more detailed studies are needed to establish whether it actually occurs.

In fact, it is not as outlandish a requirement as it might seem at first sight, for what determines the production of such loops on this scale is the spectrum of waves on the same scale along larger lengths of string. Since the spectrum at least on scales larger than the horizon is 'scale-free', i.e. independent of the correlation length, the only possible scale entering the problem must be the scale on which waves start to move, namely, the horizon scale $\sim t$.

It should be stressed that the above arguments only dealt with the form of $\rho(t)$ and not its exact value. It is unlikely but quite conceivable that some dimensionless constants like π or 2 etc. occur in such a way as to make the density contribution of string in (2.23) or (2.26) much greater than all other matter. This would also rule out the string picture as a realistic model for galaxy formation.

To conclude, detailed numerical calculations are needed to check whether closed loops are formed by a network of strings in an expanding universe at a rate $dn/dt \sim 1/t^4$. Knowledge of the actual rate could be used to give a better value for the density perturbation spectrum produced by strings and to see more quantitatively the relevance of this spectrum for galaxy formation.

CHAPTER III

COLLAPSING CLOSED LOOPS OF VORTEX-STRING AND THE BARYON-ASYMMETRY OF THE UNIVERSE

3.1 Introduction

We have seen in the previous chapter that an energetically consistent picture of the evolution of a random configuration of string in the early universe seems to require the formation of closed loops of "radius" of the order of horizon distance from self-intersections of larger lengths of string. For consistency, the loops are required to be formed at a rate given by

$$\frac{dn}{dt} \sim \frac{1}{t^4} , \qquad (3.1)$$

where n is the number density of closed loops formed. The purpose of this chapter is to point out an interesting effect associated with a class of these closed-loops of vortex-string in the early universe.

Once a closed loop is formed, its ultimate fate is to disappear. It was shown by Kibble and Turok [39] that an initially static closed loop collapses to a point, or more generally, to a doubled loop (a configuration in which the string winds twice round the same loop) after a time L/4, where L is its initial length (and c = 1). The collapse is also the fate of the lowest frequency mode non-static loop. There is, however, a wide class of initially non-static solutions differing infinitesimally from the collapsing ones which correspond to loops oscillating back and forth and which never self-intersect. These oscillating loops would, however, lose energy by gravitational radiation and finally disappear. If gravitational radiation is the only mechanism of energy loss, then these non-self-intersecting loops live a long time ($\sim 10^6$L, where L is the initial length) before disappearing. These non-self-intersecting loops are the principal ingredient of the Vilenkin [23] scenario of galaxy formation from strings.

In this chapter, we shall only concentrate on the collapsing loop solutions and show that these may also have a positive feature. Recall that the strings which would arise [26,32] in some of the realistic grand unified models would have their topological quantum number (the 'magnetic' flux quantum number) associated with the elements of the group Z_2, the group of integer modulo 2. We shall refer to these strings simply as Z_2 strings. Since there is only one non-trivial element in the group Z_2, it follows that two superimposed Z_2-strings would be associated with the identity element. The latter corresponds to the situation of having no stable topological objects. In other words, when a closed loop of Z_2 string collapses to a doubled loop [39], the situation corresponds to having no stable string at all - the original string presumably simply annihilates itself. The corresponding situation in the case of a Z_3 string might be that the topological quantum numbers 'add' to form a string in the opposite direction. For other types of strings yet more complex processes could occur.

Now, a string may be thought to be a coherent state consisting of quanta of superheavy gauge- and Higgs bosons (the superheavy masses determine the width of the string). Thus, when the string annihilates, the superheavy bosons would be released. These bosons would then decay with CP and baryon-number violation, as in the standard mechanism for generating baryon asymmetry in GUTs. We shall claim that this process may well have been, in the early universe, an important contributor to the observed net baryon-number of the universe.

At very early times, the motion of the strings is very heavily damped by the friction with the surrounding matter [14]. Thus initially the loops' velocity would be very small and they would take a long time to collapse, during which they would lose most of their energy to the dense surrounding matter, producing very few superheavy bosons when they finally collapse. Thus, in the friction dominated era, the process of collapsing loops would give negligible baryon number. However, as the universe expands, the density of surrounding matter falls, reducing the friction. At the end of the friction dominated period, the loops collapse freely under their own tensions. Exactly circular initially static loops would tend to shrink symmetrically to a point, presumably producing black-holes when they enter the Schwarzchild radius. However, the loops, when formed, are expected to be in random configurations, and it therefore seems unlikely that very large numbers of exactly circular closed loops would be formed by chance (Black hole formation from circular closed loops has been studied by Vilenkin [42]). More irregular shaped loops would collapse to doubled loops and almost all the initial mass-energy of the loop would be released in the form of superheavy bosons. Thus, the baryon number generation from collapsing loops would effectively start only at the end of the friction dominated era.

In the next section, we first make a simple non-relativistic analysis to estimate the time t_e at which the friction dominated period effectively ends. We then numerically calculate the energy loss of collapsing closed loops due to friction and quantitatively show that at very early times the loops do indeed lose most of their energy in friction. In section 3.3 we estimate the contribution of the collapsing closed loops of cosmic strings to the net baryon number of the universe. Conclusions are presented in section 3.4.

3.2 Friction of collapsing loops with the surrounding medium

The mass per unit length of the string (equal to the tension) μ is given by [14]

$$\mu \sim T_c^2 \, , \tag{3.2}$$

where T_C ($\sim \langle \Phi \rangle$, the vacuum expectation value of the Higgs field) is the critical temperature at which the phase transition occurs. A segment of radius of curvature r experiences an accelerating force μ/r per unit length. This is opposed by the frictional force due to the surrounding matter. The force of friction per unit length is roughly $\sigma \rho \vartheta$, where ρ is the matter density, ϑ the velocity of the string segment, and σ the effective cross-section for string-particle scattering per unit length. Thus, the string reaches a terminal velocity, ϑ_{ter}, given by

$$\vartheta_{ter} \sim \frac{\mu}{\sigma \rho r} \, . \tag{3.3}$$

Any small scale irregularities or kinks on the loop will tend to straighten out in time (since $\vartheta_{ter} \propto \frac{1}{r}$) and a closed loop can then be effectively described by an average 'radius' r. The friction dominated period ends at $t = t_e$ when $\vartheta \sim 1$. The largest relevant loops at time t are those of radius $r \sim t$ [23]. For these to acquire relativistic speeds, we require $\frac{\mu}{\sigma \rho t} \sim 1$, which occurs at

$$t - t_e \sim \frac{\mu}{\sigma \rho} \; . \tag{3.4}$$

The string-particle scattering cross-section per unit length, σ, at temperatures below the transition temperature, is essentially determined by the typical wavelength of a particle of the medium. Following Everett [34], we can roughly take (ignoring a slowly varying [34] logarithmic factor),

$$\sigma \sim \frac{1}{T} \; , \tag{3.5}$$

where T is the temperature. Using the standard expressions (see Appendix A) for temperature and density in a radiation dominated universe, viz.,

$$T \approx (\cdot 03 \; M_P)^{1/2} \; t^{-\frac{1}{2}} \; ,$$

$$\rho \approx \frac{\cdot 03}{G t^2} \; ,$$

we see that

$$t_e \sim .03 \; \frac{M_P^3}{\mu^2} \tag{3.6}$$

$$\text{or} \quad t_e \sim 2 \times 10^{-29} \text{ sec.,}$$

corresponding to a temperature of around 10^{11} GeV. Before this time, the loops are expected to dissipate most of their energy to the surroundings. We demonstrate this below by direct numerical calculations.

To deal with closed loops of arbitrary shape is obviously difficult. The problem is simplified by considering a circular closed loop. Consider a small segment of the loop subtending an angle δ which is assumed to remain constant as the string collapses (the circular loop collapses symmetrically). The force of friction, f_F, on this segment when the loop has collapsed to a radius r is given by [34]*

$$f_F = -r\delta \cdot 2n\gamma \vartheta^2 \frac{\pi^2}{\ln^2(\gamma \xi T)} , \qquad (3.7)$$

where ϑ is the instantaneous inward velocity, $\gamma = (1-\vartheta^2)^{-\frac{1}{2}}$, n the number density of particles in the surrounding medium, T is the temperature and $\xi \sim m_H^{-1}$ the string thickness (m_H is the superheavy Higgs boson mass). The fraction of initial mass-energy dissipated due to friction during collapse is then given by

$$\frac{\Delta E}{E} = \frac{1}{\mu R \delta} \int_R^0 f_F \, dr$$

$$= \frac{1}{R\mu} \int_0^R 2nr\vartheta^2 \gamma \frac{\pi^2}{\ln^2(\gamma \xi T)} . \qquad (3.8)$$

Here R is the initial radius of the loop and μ is the mass per unit length of string. To be able to perform the integral in (3.8), we need to know the velocity ϑ as a function of the radius r. For this, we have to look at the equation of motion of the circular loop in the presence of friction. What is responsible for the collapse of the loop is the force of

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* Everett [34] has missed out a factor of ϑ in his eq. (42). The mistake first occurs in writing the relation " $\bar{p}' = \gamma \bar{E}$" on the 2nd line from and above his eq. (38a). The correct relation should, of course, be $\bar{p}' = \gamma \vartheta \bar{E}$, which gives an extra factor of ϑ .

tension. The relativistic force of tension, f_T, on the loop segment is given by [34]

$$f_T = \frac{\mu\delta}{\gamma} . \tag{3.9}$$

Combining equations (3.7) and (3.9), we have the force equation

$$\frac{dp}{dt} = \frac{\mu\delta}{\gamma} - r\delta . 2n v^2 \gamma \frac{\pi^2}{\ln^2(\gamma \xi T)} . \tag{3.10}$$

The relativistic momentum p of the loop segment is

$$p = \mu\delta . r v \gamma . \tag{3.11}$$

Using (3.11) in (3.10), we have

$$\frac{dv}{dt} = \frac{1}{r}\left(1-v^2\right)\left[1 - 2 n r v^2 \frac{\pi^2}{\mu \ln^2(\gamma \xi T)}\right] ,$$

$$\frac{dr}{dt} = -v . \tag{3.12}$$

The system of coupled non-linear differential equations (3.12) can be numerically solved to follow the progress of collapse of the loop. As Vilenkin [23] has argued, the loops at formation are expected to be of about horizon size and so for our actual calculation we start with loops of initial radius $R = t_o$, where t_o is the time of formation of a loop. Thus the initial conditions are

$$v(t = t_o) = 0$$

$$r(t = t_o) = t_o . \tag{3.13}$$

With a given value of t_o, the integration of (3.12) is continued up to a time when r and ϑ reach, within a certain pre-assigned accuracy, the respective limiting values 0 and 1. The intermediate steps of integration yield the numerical relationship between ϑ and r. With these data, the integral in eq. (3.8) is then performed to give $\frac{\Delta E}{E}$ as a function of t_o.

The results are shown in figure 3.1. The smallest value of t_o is the time of phase transition. If the superheavy gauge boson mass is $\sim 10^{14}$ GeV, then the time of phase transition is $\sim 10^{-37}$ sec. From the graph we see that loops formed before time $\sim 10^{-29}$ sec. lose more than 60% of their initial mass-energy in overcoming the friction due to surrounding medium. For $t_o > 10^{-27}$ sec, the energy loss in friction is negligible. We thus see that the time t_e obtained in eq. (3.6) is a reasonable estimate. In the following we shall only be interested in order of magnitude estimates. Therefore, in determining the baryon-number we shall neglect the effect of the loops collapsing before the time t_e.

3.3 Net baryon-number produced by collapsing loops

A closed loop may be said to have 'formed' when an already existing loop enters the horizon. As already mentioned, the loops may also be formed by intersection of lengths of string in which a "change of partners" [14] occurs. It is, however, not at all clear what fraction of these loops may be formed in initially static or near-static configurations, but here we shall make what seems to be a not unreasonable assumption that the fraction is significant. The static ones will then collapse and release their constituent superheavy boson quanta. The exact number of superheavy bosons released per unit invariant length of string is difficult to calculate, but a rough estimate is simply $\sim \mu/m_X$, m_X being the typical mass of the superheavy bosons. A typical loop created at time t is expected [23] to have a radius of order t. Thus a single loop after collapse gives rise to a net baryon number of order $\Delta B . \alpha . t . \frac{\mu}{m_X}$, where ΔB is the mean net baryon number (i.e. baryon number minus antibaryon number) produced in the decay of a single superheavy boson, and αt is the total length of the loop ($\alpha \sim 2\pi$).

We can now estimate the total baryon asymmetry produced by all loops collapsing after the damping period. The entropy density is

$$s = \frac{4}{3}\,\frac{\rho}{T} \approx 50\,T^3 . \qquad (3.14)$$

During the expansion of the universe, the ratio of n_B (the net baryon-number density) to s is constant except for the contribution from collapsing loops (or other baryon-number generating processes).

Thus,

$$\frac{d}{dt}\left(\frac{n_B}{s}\right) = \left[\frac{d}{dt}\left(\frac{n_B}{s}\right)\right]_{\text{expansion}} + \left[\frac{d}{dt}\left(\frac{n_B}{s}\right)\right]_{\text{loop collapse}}$$

$$= \left[\frac{d}{dt}\left(\frac{n_B}{s}\right)\right]_{\text{loop collapse}} . \qquad (3.15)$$

Since the number of bosons decaying is small compared to the number of particles already present, their contribution to the entropy density is negligible. Hence,

$$\left[\frac{d}{dt}\left(\frac{n_B}{s}\right)\right]_{\text{loop collapse}} \approx \frac{1}{s}\left[\frac{d n_B}{dt}\right]_{\text{loop collapse}} \qquad (3.16)$$

Now, recall [39] that the loops take a finite time to collapse - a loop formed at a time t and of length $\alpha \cdot t$ collapses at the time $t(1 + \frac{\alpha}{4})$. Therefore, in calculating $\frac{dn_B}{dt}$ due to loops collapsing at the time t, we must consider those loops that were formed at the time $t/(1+\frac{\alpha}{4})$.

We thus have

$$\left[\frac{dn_B}{dt}\right]_{\substack{\text{loop collapse}\\ \text{at time } t}} \approx \left(1+\frac{\alpha}{4}\right)^{-3/2} \frac{\mu}{m_X} \alpha \frac{t}{(1+\frac{\alpha}{4})} \Delta B \cdot f \left[\frac{dn}{dt}\right]_{\substack{\text{at time}\\ t(1+\frac{\alpha}{4})^{-1}}}, \tag{3.17}$$

where $\frac{dn}{dt}$ is the rate of formation of loops and f is the fraction of loops produced that collapse in this way. The factor $(1+\frac{\alpha}{4})^{-3/2}$ on the right hand side of (3.17) is the density-depleting factor coming from the fact that during the time of collapse the volume of the universe expands by a factor $(1 + \frac{\alpha}{4})^{3/2}$ (we are considering a radiation-dominated universe where the scale-factor R goes like $t^{\frac{1}{2}}$).

Using eq. (3.1) for the rate of formation of loops, we have,

$$\left[\frac{dn_B}{dt}\right]_{\substack{\text{loop}\\ \text{collapse}}} \approx f\alpha \left(1+\frac{\alpha}{4}\right)^{3/2} \Delta B \; t^{-3} \frac{T_c^2}{m_X} . \tag{3.18}$$

Integrating from $t = t_e (1 + \frac{\alpha}{4})$ onwards (effectively to ∞) we find

$$\left[\frac{n_B}{s}\right]_{\text{final}} \approx \alpha \cdot \left(1+\frac{\alpha}{4}\right)^{3/2} f \; \Delta B \; \frac{T_c^2}{m_X} \int_{t_e(1+\frac{\alpha}{4})}^{\infty} dt \; \frac{1}{s t^3}$$

or,

$$\left[\frac{n_B}{s}\right]_{\text{final}} \approx 2\alpha\left(1+\frac{\alpha}{4}\right) f\,\Delta B\,\frac{T_c^2}{m_X}\,\frac{t_e^{-\frac{1}{2}}}{50\,(.03\,M_P)^{3/2}}$$

$$\approx 700\,\frac{f\,\Delta B}{\alpha_G^2}\left(\frac{m_X}{M_P}\right)^3 , \qquad (3.19)$$

where we have used (3.6) for t_e and $m_X \sim \alpha_G^{1/2}\langle\Phi\rangle \sim \alpha_G^{1/2} T_c$, α_G being the characteristic grand unification coupling strength. If f is of order 10^{-1}, and $m_X \sim 5 \times 10^{14}$ GeV (see Langacker [1]), we get typically,

$$\frac{n_B}{s} \sim 10^{-8}\,\Delta B . \qquad (3.20)$$

The quantity ΔB depends on the CP violation parameter of the particular GUT model under consideration and its exact value is uncertain. If, however, following Nanopoulos and Weinberg [44], we assume that ΔB lies in the range between 10^{-2} and 1, we obtain a value in good agreement with the present observational bound [45] $\frac{n_B}{s} \sim 10^{-9.8 \pm 1.7}$.

This is of course in addition to any baryon asymmetry created earlier. One interesting feature of the mechanism is that baryon number is not generated uniformly throughout space, but in clumps around the collapsing loops. However, it remains to be seen in detail whether the scale of these clumps is big enough to be of relevance to galaxy formation.

3.4 Conclusions

In the standard mechanism [10] of generating a net baryon-number in the early universe, one has to assume, in addition to the requirement of CP violation, that the baryon number violating interactions fell out of thermal equilibrium at some stage soon after the initial phase transition. This is needed in order to provide "an arrow for time" for the development of baryon asymmetry. In this context, we may note that the process described in the previous section starts operating well after the grand unification phase transition at $T_c \sim 10^{15}$ GeV. It is obviously an irreversible process, so that the requirement that the system be out of thermal equilibrium is automatically satisfied.

There of course remain several uncertainties in the above theoretical prediction. First, the number of superheavy bosons released per unit length of collapsing string is uncertain. It should be possible to calculate it, but a deeper understanding of the quantum or at least semi-classical theory of strings is needed. Second, we have assumed that the process of baryon-number generation effectively begins at a certain time t_e. In reality, there is no sharp beginning - the process is a continuous one. The inclusion of loops collapsing before the time t_e would, however, need a large amount of numerical work because we would have to calculate, for each time, the quantity $\Delta E/E$ (the fraction of initial mass energy lost in friction) and use only the balance $(E - \Delta E)$ to determine the number of superheavy bosons released. Moreover, given the time of formation (of the loop) t_o, the time of collapse in each case would have to be obtained from the solution of eqs. (3.12). All this is an unnecessary complication at this stage when, simply, the inclusion of these loops is expected only to increase our estimate of $\frac{n_B}{s}$ by a slight amount. Third, the parameter ΔB is highly model-dependent and cannot at present be calculated from first principles. Lastly, there remains the question whether or not closed loops are formed in sufficient numbers. The occurrence of the process of "change of partners" in the intersection of lengths of strings, which can give rise to closed loops, depends on the details of interaction between strings. This question has not yet been settled. We make a beginning towards understanding this question in the next chapter.

To conclude, then, what has been shown in this chapter is that the process of collapsing cosmic strings may be a significant contributor to the net baryon-number of the universe. Certainly, in those GUTs that predict the appearance of stable strings, it cannot be ignored.

CHAPTER IV

"EXCHANGE OF PARTNERS" IN THE INTERSECTION OF VORTEX-STRINGS

4.1 Introduction

It has been stressed in the previous chapters that closed loops (of vortex-string) play a crucial role in Vilenkin's scenario [23] of galaxy-formation from vortex-strings. The discussions in Chapter II showed that one needs to impose the requirement of closed loop formation as a consistency condition if one considers the behaviour of energy of a system of string in an expanding universe. A possible mechanism of closed loop formation is the process of "change of partners" [14] in the intersection of lengths of string. The question is, does this process happen in practice? Or is it inhibited? If the process does happen, what are the possible various different stages involved in the process? In the present chapter, we make an attempt to provide some plausible answers to these questions.

A direct way to go about answering the above questions would be to follow the time evolution of a system of two intersecting strings. The spatial configuration of string may be taken to be specified by the position of the nodal line of the Higgs field, i.e. the curve along which the Higgs field takes the value zero. Thus, starting with a specified initial configuration which describes two intersecting strings, the relevant dynamical equations of motion would have to be solved to determine the final configuration of the strings after the intersection and see whether or not a "change of partner" occurs. Unfortunately, the relevant equations of motion are non-linear and the problem would, in general, involve all three spatial dimensions in addition to time. This makes any analytic treatment of the problem extremely difficult. It seems one has to take recourse to detailed numerical calculations. This is a rather long and extensive programme which, ultimately, seems to be unavoidable if one wants definite answers to the questions raised above. However, before embarking on such a programme, it may be worthwhile to make a beginning by concentrating on the essentials and thereby try to obtain a general understanding of the problem. This is the task we restrict ourselves to in the present chapter.

The problem may be formulated in terms of the interaction energy of a system of two intersecting strings. The changing of partners would be favourable if the overall energy of the system is lowered by doing so. Thus, ideally, what one needs to do is calculate the interaction energy of a given configuration of two intersecting strings. Unfortunately again, it seems to be extremely difficult to calculate the interaction energy except in the case of very simple configurations of strings. A very special case which does allow us to calculate the interaction energy, albeit approximately, is the case of two straight and parallel strings. In the next section, we show how the interaction energy can be calculated in this case. We present this calculation because it provides us with useful information regarding certain general features of interaction between vortex-strings. In fact, as we shall see, an understanding of these general features allows us to discuss, on a qualitative level, a possible mechanism through which intersecting lengths of string may indeed find it favourable to change their partners after intersection.

The main calculation of the interaction energy is presented in the next section. Some related matters are discussed in section 4.3. The implications for the process of changing of partners are discussed in section 4.4.

For simplicity, we shall here consider only the vortex-strings of Nielsen-Olesen [15] type arising in the Abelian Higgs model which was discussed in Chapter I.

4.2 Interaction energy of two straight, parallel, infinitely long vortex-strings

We start with the Lagrangian density describing the Abelian Higgs model:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \Phi)^* (D^\mu \Phi) - \frac{1}{4} \lambda (\Phi^* \Phi - \eta^2)^2 , \qquad (4.1)$$

where, as usual,

$$D_\mu \Phi = \partial_\mu \Phi + ie A_\mu \Phi ,$$

and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

The quantities λ, e, η are constants. We shall be interested in the static case and we shall choose $A_o=0$. The Hamiltonian density in this case is just the negative of the Lagrangian density and so the energy is given by

$$E = \int d^3\underline{x} \left[\frac{1}{2} (\underline{\nabla} \wedge \underline{A})^2 + (\underline{D}\Phi)^* \cdot (\underline{D}\Phi) + \frac{1}{4} \lambda (\Phi^* \Phi - \eta^2)^2 \right] . \qquad (4.2)$$

Let us write

$$\Phi(\underline{x}) = \varphi(\underline{x})\, e^{i\alpha(\underline{x})} , \qquad (4.3)$$

where

$$\varphi(\underline{x}) = |\Phi(\underline{x})| .$$

Clearly, for finite energy,

$$\varphi(\underline{x}) \xrightarrow[|\underline{x}| \to \infty]{} \eta ,$$

$$\underline{D}\Phi \xrightarrow[|\underline{x}| \to \infty]{} 0 , \qquad (4.4)$$

and

$$\underline{B} = \underline{\nabla} \wedge \underline{A} \xrightarrow[|\underline{x}| \to \infty]{} 0 .$$

From (4.3) and (4.4), we have

$$\underline{A}(\underline{x}) \xrightarrow[|\underline{x}| \to \infty]{} -\frac{1}{e} \underline{\nabla}\alpha(\underline{x}) . \qquad (4.5)$$

We shall consider the direction of the magnetic flux-lines to be the z-direction and use cylindrical co-ordinates (r, θ, z). Consider a circle C in the xy-plane of very large radius and centered at the origin. The total magnetic flux F passing along the z-direction through the xy-plane is

$$F = \int \underline{B} \cdot d\underline{s} = \oint_C \underline{A} \cdot d\underline{l} , \qquad (4.6)$$

where d$\underline{s}$ and d$\underline{l}$ are surface- and line elements respectively. Using the single-valuedness of the field Φ (so that $\alpha(\theta = 2\pi) - \alpha(\theta = 0) = 2\pi n$, $n \in \mathbb{Z}$) we have from (4.5) and (4.6),

$$|F| = \frac{2\pi}{e} n \qquad , \quad n \in \mathbb{Z} . \qquad (4.7)$$

The magnetic flux is thus quantized in units of $\frac{2\pi}{e}$. Here Z is the additive group of positive and negative integers including zero. The physical meaning is that if we have two separate vortices with individual flux quantum numbers n_1 and n_2, the composite system is equivalent to a single vortex with flux quantum number $n_1 + n_2$. However, the energy of a vortex can be shown to be a quadratic function of the flux contained in the vortex and so it is energetically favourable to have two vortex-strings, each carrying a single unit of flux (n=1), rather than one vortex with two units of flux (n=2). Therefore, in studying the interaction between vortex-strings, we shall consider strings, each carrying only one unit of flux. Each such string is, of course, stable due to topological reasons.

With these preliminaries, let us now proceed to our main task, i.e. to calculate the interaction energy of two infinitely long straight strings lying along, say, the z-direction and separated by a fixed distance R. In calculating the interaction energy, our strategy is simple and straightforward: we shall calculate the energy given by (4.2) corresponding to the solutions $\{\Phi, \underline{A}\}$ which describe the composite system of two separated strings, subtract the energies of two individual strings and thus isolate the part which explicitly depends on the separation R which gives the required interaction energy. Of course, the solution corresponding to two separated strings is not known. Indeed, for that matter, the single-string solution that is known [15] is valid only in the asymptotic regions where Φ and $\underline{A}$ are very near their asymptotic values given by (4.4) and (4.5). We shall, however, see that the known asymptotic single-string solution can be exploited to derive an approximate expression for the interaction energy of two separated strings. We shall consider the situation when the separation R between the strings is very much greater than the width of the strings ($\sim \frac{1}{m_V}$, m_V being the vector boson mass). In this circumstance, the use of asymptotic solutions is justified.

It is convenient to absorb the phase of $\Phi(\underline{x})$ in (4.3) in the gauge-transformed vector field

$$\underline{\mathcal{A}} = \underline{A} + \frac{1}{e} \underline{\nabla} \alpha . \tag{4.8}$$

From now on we shall work with the real scalar field $\varphi(\underline{x})$ and the vector field $\underline{\mathcal{A}}(\underline{x})$. The Lagrangian written in terms of φ and $\underline{\mathcal{A}}$ is

$$\mathcal{L} = -\frac{1}{2}\left(\underline{\nabla} \wedge \underline{\mathcal{A}}\right)^2 - \left[\left(\underline{\nabla}\varphi\right)^2 + e^2\varphi^2\underline{\mathcal{A}}^2\right] - \frac{1}{4}\lambda\left(\varphi^2 - \eta^2\right)^2 . \tag{4.9}$$

The equations of motion obtained from (4.9) are

$$\begin{aligned} &\underline{\nabla} \wedge \underline{\nabla} \wedge \underline{\mathcal{A}} + 2e^2\varphi^2\underline{\mathcal{A}} = 0 , \\ &\left(\underline{\nabla}^2 - e^2\underline{\mathcal{A}}^2\right)\varphi = \frac{1}{2}\lambda\left(\varphi^2 - \eta^2\right)\varphi . \end{aligned} \tag{4.10}$$

The energy integral (4.2) can be rewritten as

$$\epsilon = \int d^2x \left[\frac{1}{2}\left(\underline{\nabla} \wedge \underline{\mathcal{A}}\right)^2 + \left(\underline{\nabla}\varphi\right)^2 + e^2\varphi^2\underline{\mathcal{A}}^2 + \frac{1}{4}\lambda\left(\varphi^2 - \eta^2\right)^2\right] , \tag{4.11}$$

where ϵ is the energy per unit length along the z-direction. Here we have taken account of the z-independence of the system. The centres of the vortices are located at x=0, y=0 and at x=R, y=0, respectively. Note that $\underline{\mathcal{A}}$ and φ above are solutions corresponding to two separated vortices and so they will depend on both r and θ in the xy plane. (For an isolated vortex at the origin the solutions would be functions only of the radial variable r). Similarly $\underline{A}$ will have both $\hat{r}$ and $\hat{\theta}$ components. (Again for an isolated vortex at the origin $\underline{A}$ would have only the $\hat{\theta}$ component).

Let us write

$$\varphi(r,\theta) = \eta - f(r,\theta) \; . \tag{4.12}$$

Then eqs. (4.10) and (4.11) become

$$\underline{\nabla}\wedge\underline{\nabla}\wedge\underline{A} + 2e^2\left(\eta^2+f^2-2\eta f\right)\underline{A} = 0 \; , \tag{4.13}$$

$$\underline{\nabla}^2 f + e^2\eta\underline{A}^2 - e^2 f\underline{A}^2 - \lambda\eta^2 f = \frac{1}{2}\lambda f^3 - \frac{3}{2}\lambda\eta f^2 . \tag{4.14}$$

The energy integral (4.11) becomes

$$\epsilon = \int d^2x\left[\left\{\frac{1}{2}\left(\underline{\nabla}\wedge\underline{A}\right)^2 + e^2\eta^2\underline{A}^2\right\} - 2e^2\eta f\underline{A}^2 + \left(\underline{\nabla}f\right)^2 + e^2f^2\underline{A}^2 + \lambda\eta^2 f^2 - \lambda\eta f^3 + \frac{1}{4}\lambda f^4\right] . \tag{4.15}$$

The term linear in f in (4.15) can be eliminated by using eq. (4.14). We thus have

$$\epsilon = \int d^2x\left[\left\{\frac{1}{2}\left(\underline{\nabla}\wedge\underline{A}\right)^2 + e^2\eta^2\underline{A}^2\right\} + \left(\underline{\nabla}f\right)^2 - e^2f^2\underline{A}^2 - \lambda\eta^2 f^2 + 2f\,\underline{\nabla}^2 f + 2\lambda\eta f^3 - \frac{3}{4}\lambda f^4\right] \tag{4.16}$$

To proceed further, we shall use the asymptotic solutions of eqs. (4.13) and (4.14) for f and $\underline{A}$. Far away from the centres of both the vortices, f and $\underline{A}$ are small. We can then linearize eqs. (4.13) and (4.14) and thereby obtain the decoupled equations

$$\underline{\nabla} \wedge \underline{\nabla} \wedge \underline{A} + 2e^2\eta^2 \underline{A} = 0 \quad , \tag{4.17}$$

$$\underline{\nabla}^2 f - \lambda \eta^2 f = 0 \,. \tag{4.18}$$

These are the equations for a vector- and a scalar particles of masses $m_V = \sqrt{2}\, e\eta$ and $m_s = \sqrt{\lambda}\, \eta$, respectively.

Since eqs. (4.17) and (4.18) are linear, we can write the two-string solution simply as a superposition of the corresponding solutions for the individual strings. Thus we write,

$$\begin{aligned} \underline{A}(\underline{r}) &= \underline{A}_1(\underline{r}) + \underline{A}_2(\underline{r}) \, , \\ f(\underline{r}) &= f_1(\underline{r}) + f_2(\underline{r}) \, , \end{aligned} \tag{4.19}$$

where the subscript 1 and 2 refer to vortex 1 and vortex 2, respectively, and $\underline{r}$ is of course the position vector of a point in the xy plane. The isolated vortex solutions have rotational symmetry about their respective centres in the xy-plane. Thus let us write

$$\underline{A}_1(\underline{r}) = \hat{\theta}\, \frac{a_1(r)}{r} \quad , \tag{4.20a}$$

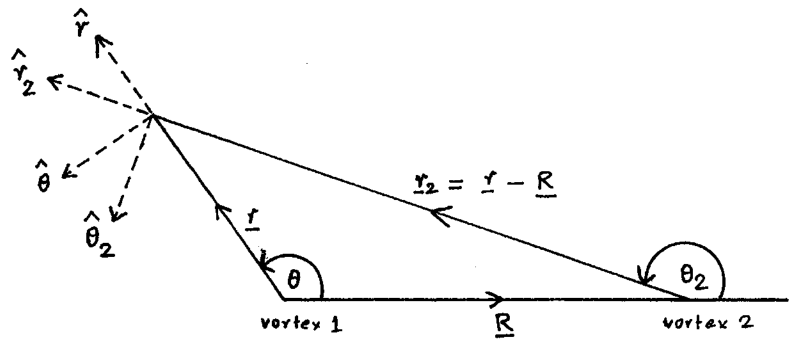
$$\underline{A}_2(\underline{r}) = \hat{\theta}_2\, \frac{a_2(r_2)}{r_2} \quad , \tag{4.20b}$$

$$f_1(\underline{r}) = f_1(r) \, , \tag{4.20c}$$

$$f_2(\underline{r}) = f_2(r_2) \, , \tag{4.20d}$$

where $r = |\underline{r}|$, $r_2 = |\underline{r} - \underline{R}| = (r^2 + R^2 - 2rR\cos\theta)^{1/2}$, and $\hat{\theta}_2$

is the unit vector as indicated in the diagram below:



For later convenience, we note that

$$\hat{\theta}_2 = \hat{r}\left(\frac{-R\sin\theta}{r_2}\right) + \hat{\theta}\left(\frac{r - R\cos\theta}{r_2}\right) . \tag{4.21}$$

Using (4.20a) in (4.17), we have

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}a_1(r)\right] - m_V^2 \frac{a_1(r)}{r} = 0 . \tag{4.22}$$

The solution of this equation is the modified Bessel function K_1 [46]. Thus

$$\underline{A}_1(\underline{r}) = \hat{\theta}\, C_a K_1(m_V r) , \tag{4.23}$$

where C_a is a constant.

Similarly, using eq. (4.20c) in (4.18) it is easy to show that

$$f_1(\underline{r}) = C_f K_0(m_s r) , \tag{4.24}$$

where C_f is again a constant. The solutions $\underline{A}_2$ and f_2 are the same as in (4.23) and (4.24), except that $\hat{\theta}$ must be replaced by $\hat{\theta}_2$ and r by r_2.

With the form of $\underline{A}$ and f given as simple superpositions as in (4.19), the energies of the individual vortices can be easily isolated and we are left with the interaction energy ϵ_{int} which, to lowest order in the small quantities f's and $|\underline{A}|$'s, is

$$\epsilon_{int} \simeq \int d^2x \Big[(\underline{\nabla} \wedge \underline{A}_1) \cdot (\underline{\nabla} \wedge \underline{A}_2) + m_V^2 \underline{A}_1 \cdot \underline{A}_2 + 2 \underline{\nabla} f_1 \cdot \underline{\nabla} f_2 - 2 m_s^2 f_1 f_2 + 2 \left(f_1 \underline{\nabla}^2 f_2 + f_2 \underline{\nabla}^2 f_1 \right) \Big] \tag{4.25a}$$

$$= \int d^2x \Big[C_a^2 \left\{ m_V^2 K_0(m_V r) K_0(m_V r_2) + \underline{\nabla} K_0(m_V r) \cdot \underline{\nabla} K_0(m_V r_2) \right\} + 2 C_f^2 \left\{ \underline{\nabla} K_0(m_s r) \cdot \underline{\nabla} K_0(m_s r_2) - m_s^2 K_0(m_s r) K_0(m_s r_2) + K_0(m_s r) \underline{\nabla}^2 K_0(m_s r_2) + K_0(m_s r_2) \underline{\nabla}^2 K_0(m_s r) \right\} \Big] \tag{4.25b}$$

where we have used eqs. (4.23), (4.24) and the corresponding relations for $\underline{A}_2$ and f_2. We have also used the facts that

$$K_1 = -K_0'$$

and

$$\hat{\theta} \cdot \hat{\theta}_2 = \hat{r} \cdot \underline{\nabla} r_2 \ .$$

The Bessel function K_0 satisfies [46] the equation,

$$\left(\underline{\nabla}^2 - m^2 \right) K_0(mr) = -2\pi \, \delta^{(2)}(\underline{r}) \ . \tag{4.26}$$

Using (4.26) in (4.25b) we have,

$$\epsilon_{int} = \int d^2x \Big[C_a^2 \Big\{ K_0(m_V r_2) \left(\underline{\nabla}^2 K_0(m_V r) + 2\pi \delta^{(2)}(\underline{r}) \right) + \underline{\nabla} K_0(m_V r) \cdot \underline{\nabla} K_0(m_V r_2) \Big\}$$

$$+ 2C_f^2 \Big\{ K_0(m_s r_2)\, \underline{\nabla}^2 K_0(m_s r) - K_0(m_s r)\, 2\pi\, \delta^{(2)}(\underline{r}_2) + \underline{\nabla} K_0(m_s r) . \underline{\nabla} K_0(m_s r_2) \Big\} \Big] .$$

The integrals involving δ-functions can be immediately performed and we get

$$\begin{aligned} \epsilon_{int} = {} & C_a^2\, 2\pi\, K_0(m_V R) - C_f^2\, 4\pi\, K_0(m_s R) \\ & + C_a^2 \int d^2x\ \underline{\nabla} \cdot \Big[K_0(m_V r_2)\ \underline{\nabla} K_0(m_V r) \Big] \\ & + 2C_f^2 \int d^2x\ \underline{\nabla} \cdot \Big[K_0(m_s r_2)\ \underline{\nabla} K_0(m_s r) \Big] . \end{aligned} \qquad (4.27)$$

Now, the last two terms involving the integrals of total divergences can be easily shown to vanish :

$$\begin{aligned} & \int d^2x\ \underline{\nabla} \cdot \Big[K_0(m_V r_2)\ \underline{\nabla} K_0(m_V r) \Big] \\ & = \int_0^{2\pi} d\theta \int_0^{\infty} r\, dr\ \frac{1}{r} \frac{\partial}{\partial r} \Big[r K_0(m_V r_2) \frac{\partial}{\partial r} K_0(m_V r) \Big] \\ & = \int_0^{2\pi} d\theta \Big[r K_0(m_V r_2) \frac{\partial}{\partial r} K_0(m_V r) \Big] \Bigg|_{r=0}^{r=\infty} \\ & = 0 . \end{aligned}$$

We thus finally get

$$\epsilon_{int} = C_a^2\, 2\pi\, K_0(m_V R) - C_f^2\, 4\pi\, K_0(m_s R) . \qquad (4.28)$$

4.3 Discussions

We see that the interaction energy separates into two parts. The first term on the right-hand side of (4.28) is due to the electromagnetic repulsion between the two strings (oriented in the same direction), whereas the second term reflects the attractive nature of interaction between two sources of scalar fields. Note the short range nature of the interactions. This is of course related to the fact that in the above model the scalar- and the vector bosons are massive.

The constant C_a first appearing in eq. (4.23) can be determined from the condition that the magnetic flux carried by an isolated vortex should be $\frac{2\pi}{e}$. From (4.23) the magnetic field is

$$\underline{B} = \underline{\nabla} \wedge \underline{A} = \underline{\nabla} \wedge \underline{\mathcal{A}} = C_a \, m_V \, K_0(m_V r) \, \hat{z} \; . \qquad (4.29)$$

This satisfies the equation (see eq. (4.26)),

$$\underline{\nabla} \wedge \underline{\nabla} \wedge \underline{B} + m_V^2 \, \underline{B} = 2\pi C_a \, m_V \, \delta^{(2)}(\underline{r}) \, \hat{z} \; . \qquad (4.30)$$

Integrating both sides over the whole of xy plane we get

$$\int \underline{\nabla} \wedge \underline{\nabla} \wedge \underline{B} \cdot d\underline{s} + m_V^2 \int \underline{B} \cdot d\underline{s} = 2\pi C_a m_V \int \delta^{(2)}(\underline{r}) \, \hat{z} \cdot d\underline{s}$$

or,

$$\oint_C \underline{\nabla} \wedge \underline{B} \cdot d\underline{l} + m_V^2 \cdot \text{Flux} = 2\pi C_a \, m_V \, .$$

The first term on the left hand side vanishes for a circle of large radius C. Then the condition Flux = $\frac{2\pi}{e}$ gives

$$C_a = \frac{m_V}{e} \; . \qquad (4.31)$$

The constant C_f, however, cannot be determined by any simple argument and has to be determined numerically from the condition that the full solution for the scalar field must be regular at the origin and that the solution near the origin must smoothly match on to the distant solution where $f \longrightarrow 0$.

The result (4.28) is also found in the calculation of interaction energy of flux-tubes in type II superconductors [47] in Ginzburg-Landau [48] theory. This is not surprising since the abelian Higgs model we have considered is just the relativistic analogue of the phenomenological Ginzburg-Landau theory. A superconductor, in the presence of an external magnetic field whose strength is greater than a certain critical value, is seen to be perforated by thin flux-tubes, each carrying one quantized unit of magnetic flux. The flux-lines form either a square- or a triangular array, and so in superconductor theory the interaction energy is derived from a consideration of the type of array structure formed. The method followed here is, however, quite general and simple.

An interesting situation arises [49] when the parameters in the model are chosen such that the masses of the scalar and vector bosons are equal. The condition $m_V = m_S$ gives

$$\lambda = 2\, e^2 \tag{4.32}$$

The equations of motion (4.13) and (4.14) for an isolated vortex described by the cylindrically symmetric fields

$$\underline{A} = \hat{\theta}\, \frac{a(r)}{r}$$

$$f = f(r)$$

can be explicitly written as

$$\frac{d^2}{dr^2} a(r) - \frac{1}{r} \frac{d}{dr} a(r) - 2e^2 \big(\eta - f(r)\big)^2 a(r) = 0 \;,$$

$$\frac{d^2}{dr^2} f(r) + \frac{1}{r} \frac{d}{dr} f(r) + \frac{e^2}{r^2} a^2(r)\big(\eta - f(r)\big) - \frac{1}{2}\lambda \eta^2 \big(\eta - f(r)\big) + \frac{1}{2}\lambda \big(\eta - f(r)\big)^3 = 0 \,. \tag{4.33}$$

When the condition (4.32) holds, the above two equations reduce to the following two first-order differential equations

$$\frac{1}{r}\frac{d}{dr}a(r) = \sqrt{\frac{\lambda}{2}}\left(f^2(r) - 2\eta f(r)\right)$$

$$\frac{d}{dr}f(r) = -\frac{e}{r}\,a(r)\left(\eta - f(r)\right). \tag{4.34}$$

Indeed, one can directly check that equations (4.34) give back eqs. (4.33), provided (4.32) holds. Eqs. (4.34) give, in the asymptotic regions where a(r) and f(r) are small, the linearized equations

$$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}a(r)\right) - m_V^2\,\frac{a(r)}{r} = 0\,, \tag{4.35a}$$

$$\frac{d}{dr}f(r) = -\frac{m_V}{\sqrt{2}}\,\frac{a(r)}{r}\,, \tag{4.35b}$$

where we have used (4.32).

The solution of (4.35a) is obtained as in (4.23), so that

$$\frac{a(r)}{r} = C_a\,K_1(m_V r)\,.$$

Substituting this in (4.35b) and using the fact that $K_0' = -K_1$, we get

$$f(r) = \frac{C_a}{\sqrt{2}}\,K_0(m_V r)\,.$$

Comparing with (4.24) we conclude that $C_f = \frac{C_a}{\sqrt{2}}$ when (4.32) holds. In this case, we find from (4.28) that the interaction energy vanishes. We thus see that in the special case $m_s = m_V$, two straight and parallel strings do not interact. Again, this is a result also true in Ginzburg-Landau theory in the special case when the so-called

Ginzburg-Landau parameter κ (the ratio of the coherence length to penetration depth) is unity. Interestingly, a similar thing happens [50] in the case of interaction between 't-Hooft-Polyakov monopoles [19] in the so-called Bogomolnyi-Prasad-Sommerfield (BPS) [51] limit $\lambda \to 0$. There is a long-range repulsive force between two magnetic monopoles (of like magnetic charge). This long range force is due to the massless vector field associated with the unbroken U(1) group in the symmetry breaking SO(3) $\longrightarrow$ U(1). However, in the BPS limit, the scalar field is also massless and the attractive long-range force due to it exactly cancels the repulsive long-range magnetic force, so that 't-Hooft-Polyakov monopoles do not interact in the BPS limit.

Coming back to strings, the constant C_f for arbitrary values of the coupling constants e and λ can only be obtained numerically. If we follow the case [47] of flux-tubes in superconductors, we can conclude that for $m_s > m_V$ the strings repel each other, whereas they attract if $m_s < m_V$.

4.4 Implications for the process of "changing of partners"

In the above discussions we have considered two strings which are parallel to each other. The important case we should consider and which is relevant to the question regarding change of partners is the situation of two straight strings with a relative angle α between them (see Fig. 4.1a). The difficulty in this case is, of course, that the solutions φ and $\underline{\mathcal{A}}$ are now functions of all three spatial co-ordinates and so the whole problem is now in 3 dimensions. The total interaction energy (not the interaction energy per unit length) will still be given by the right-hand-side of eq. (4.25a), except that $d^2\underline{x}$ will be replaced by $d^3\underline{x}$ and f_2 and $|\underline{\mathcal{A}}_2|$ at a given point in space are functions of the perpendicular distance of the point from the second string. (We take the first string to be along the z-axis and the second string tilted at an angle α with respect to the first). The solutions f_2 and $\underline{\mathcal{A}}_2$ will, as before, be given by Bessel functions. But now the arguments of the Bessel functions are themselves complicated functions of all three spatial variables. This makes it extremely difficult to carry out the integrals, unlike the case of parallel strings. This frustrates our attempt to obtain a simple expression for the interaction energy in the present case. However, guided by the results we have obtained for parallel strings, we can make some qualitative comments, as follows.

On general grounds, one expects that the magnetic part of the interaction will involve the component of the magnetic flux in one string along the direction of the other or, in other words, the magnetic part of the interaction will be proportional to $\cos\alpha$. This is not entirely obvious, but seems to be a reasonable guess as to what one should expect. Certainly the term $(\underline{\nabla} \wedge \underline{\mathcal{A}}_1).(\underline{\nabla} \wedge \underline{\mathcal{A}}_2)$ in eq. (4.25a) suggests a $\cos\alpha$ dependence for the magnetic part of the interaction energy. This is consistent with the result for parallel strings, namely, the magnetic force is repulsive if the fluxes are oriented in the same direction ($\alpha = 0$) and attractive if the fluxes are oriented in the opposite directions ($\alpha = \pi$). Other things being equal, the factor of $\cos\alpha$ should be absent in the corresponding term $-2m_s^2 f_1 f_2$ for scalar field interaction (see eq. (4.25a)). If we assume the above general behaviour for the vector-and scalar field interactions in the case of two straight tilted strings, then it is clear that a system of two strings will have the lowest interaction energy when they are exactly anti-parallel ($\alpha = \pi$).

Now, consider the self-intersection of a string. Let us concentrate on the two intersecting segments of the string so that the situation may be idealized by two straight strings intersecting at a point with an angle α (see Fig. 4.1(a)). Now, recall that both scalar- and vector field interactions are short-ranged. This implies that parts (of the strings) which are far away from the point of intersection do not feel much force of interaction. The parts which interact significantly are two small sections near the point of intersection (the sections ab and cd in Fig. 4.1(a)). In an attempt to lower the interaction energy, the sections ab and cd will tend to align themselves anti-parallel to each other so that we have an intermediate situation as in Fig. 4.1(b). Ultimately the ends b and c (and similarly d and a) will come together, so that the two sections ab and cd superimpose on each other with the fluxes running against each other (Fig. 4.1(c)). Presumably, then, the two superimposed anti-parallel sections will just annihilate each other, releasing the energy contained in these sections in the form of particles. This will thus lead to the final situation, as in Fig. 4.1(d), in which strings emerge in a new configuration having "exchanged their partners".

The above discussions, although certainly not constituting a proof, at least make it plausible that the process of change of partners may, in fact, occur when a string intersects itself. One way the process might be inhibited is if the sections ab and cd, after superimposing on one another (Fig. 4.1(c)), just pass through one another instead of permanently annihilating (note that the strings initially approach one another with a certain relative velocity). Such a peculiar behaviour is well known for the case of solitons in Sine-Gordon model [52]. The Sine-Gordon model is a particular scalar field-theory model in (1+1) dimension and has the remarkable property that it is an exactly soluble model. The model exhibits topologically stable finite-energy solutions (the 'solitons'). It is known [52] that there exists a soliton-antisoliton scattering solution whose properties are as follows: at time $t \rightarrow -\infty$, the scattering solution corresponds to a pair of solitons and an antisoliton far apart and approaching one another with a certain relative velocity. At t=0, the pair temporarily annihilates and the solution is zero everywhere. But then the pair re-emerges at positive t and finally at $t \rightarrow +\infty$, the solution corresponds to a soliton and an antisoliton moving away from one another with the same relative velocity and the same shape as before! The only effect of the interaction is a time-delay suffered by the soliton and the antisoliton when they re-emerge after their temporary 'annihilation' at t=0. There also exists a soliton-antisoliton 'bound state' solution which corresponds to the soliton and the antisoliton oscillating with respect to one another with a certain period. However, the above (1+1)-dimensional model is obviously unrealistic, and the above properties of the solitons appear to be very special indeed. It would be surprising if topological solutions (such as strings) of realistic theories show similar behaviour. Moreover, it is extremely hard to see how this kind of behaviour for strings (namely, the sections ab and cd in Fig. 4.1(c) just passing through one another) could lead to a lower energy state than if they (i.e. ab and cd) simply annihilated - certainly, as far as the string as a whole is concerned, the overall length and hence the mass-energy of the string, is lowered if the sections ab and cd simply annihilate and disappear. Nevertheless, uncertainties remain in the transition from Fig. 4.1(c) to 4.1(d). It seems the crucial question that has to be answered is whether or not two antiparallel strings annihilate one another. A final answer to this question, it seems, must wait until the full dynamical problem of scattering of two strings is solved.

Before closing this chapter, it is perhaps worthwhile to mention here that there are some experimental evidences [53] which indicate that the so-called "flux-line cutting" (intersection and subsequent cross-joining of singly quantized flux-lines) does indeed occur in superconductors. It is, therefore, not entirely unreasonable to expect such a "cutting" process to occur also for strings of our interest. However, one must also keep in mind that the strings which arise in grand unified theories are expected to have complicated internal structures involving several different length scales corresponding to different energy scales of successive symmetry breakings from the GUT group down to $SU(3)_c \times U(1)_{em}$, and the nature of interaction between these strings may be more complicated than what we have considered above. This clearly is something that will need further investigation.

APPENDICES

APPENDIX A

For convenience, we list in this Appendix some relevant formulae frequently used in discussions of early universe (see, for example, Weinberg [7] for details).

The universe is assumed to be described by the homogeneous, isotropic, Robertson-Walker metric:

$$d\tau^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right], \qquad (A.1)$$

where τ is the proper time, t is the cosmic time, r, θ, φ are the spatial coordinates, R(t) is the scale-factor of the universe and k is a constant characterizing the curvature: k = +1, -1 or 0 for a 'closed', 'open' or a 'flat' universe, respectively. (By convention, r and R(t) have been rescaled so that k takes only one of these discrete values).

The evolution of the scale-factor is governed by Einstein equation, viz.,

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} G\rho - \frac{k}{R^2}, \qquad (A.2)$$

where ρ is the density and G is of course the gravitational constant. The quantity $H \equiv \frac{\dot{R}}{R}$ is the Hubble 'constant'. In (A.2) we have dropped a possible cosmological term Λ which is observationally very small.

In addition to (A.2) we have the equation expressing energy conservation:

$$\frac{d}{dt}\left(\rho R^3\right) = -p\,\frac{d}{dt}\left(R^3\right). \qquad (A.3)$$

In hot big-bang theory, it is assumed that the energy-density in the very early history of the universe is dominated by relativistic particles like photons described by the equation of state $p = \frac{1}{3}\rho$. This implies, from (A.3), that

$$\rho \propto R^{-4} \qquad \text{for} \quad p = \frac{1}{3}\rho . \qquad \text{(A.4)}$$

This is interesting because this means that at very early times the density term dominates over the curvature term ($\propto \frac{1}{R^2}$) in (A.2). In other words, at very early times the curvature effects are relatively unimportant and so the universe can be approximately treated as 'flat' (k = 0). This is slightly less true in a universe dominated by non-relativistic matter with negligible pressure, in which case (A.3) gives

$$\rho \propto R^{-3} \qquad \text{for} \quad p \ll \rho . \qquad \text{(A.5)}$$

If one assumes that in the very early universe thermal equilibrium prevailed at a temperature T much larger than all particle masses, then the material content of the early universe can be treated as an ideal relativistic gas undergoing adiabatic expansion, i.e. maintaining constant entropy. (The assumption of adiabatic expansion is abandoned in Guth's [54] "inflationary universe" model, in which the universe expands exponentially with time for a certain period immediately after the (grand-unified) symmetry-breaking phase transition before settling down to the standard adiabatic expansion. We will not consider this in the present thesis). It should be mentioned here that, owing to the property of asymptotic freedom [55] of elementary particle interactions, the assumption of ideal gas is not unjustified.

The expressions for energy-density (ρ), entropy-density (s) and number density (n) for ideal gas of relativistic particles are

$$\rho = 3p = \frac{\pi^2}{30} N^*(T)\, T^4 , \qquad (A.6)$$

$$s = \frac{2\pi^2}{45} N^*(T)\, T^3 , \qquad (A.7)$$

$$n = \frac{\zeta(3)}{\pi^2} N^{*\prime}(T)\, T^3 , \qquad (A.8)$$

where $\zeta(3)$ is the Riemann Zeta function, $\zeta(3) \simeq 1.202$, T is the temperature, and

$$N^*(T) = N_b^*(T) + \frac{7}{8} N_f^*(T) ,$$
$$N^{*\prime}(T) = N_b^*(T) + \frac{3}{4} N_f^*(T) , \qquad (A.9)$$

and N_b^* (N_f^*) is the number of effective massless bosonic (fermionic) degrees of freedom at temperature T.

In all the above formulae, we have chosen units in which

$\hbar = c = k_B = 1$ (k_B is the Boltzmann constant).

Some useful numbers in these units are

$$1\ \text{m} \simeq 5.068 \times 10^{15}\ \text{GeV}^{-1},$$
$$1\ \text{kg} \simeq 5.610 \times 10^{26}\ \text{GeV},$$
$$1\ \text{sec} \simeq 1.519 \times 10^{24}\ \text{GeV}^{-1}, \qquad (A.10)$$
$$1^\circ\text{K} \simeq 8.617 \times 10^{-14}\ \text{GeV},$$
$$M_P = \frac{1}{\sqrt{G}} \simeq 1.22 \times 10^{19}\ \text{GeV} \quad (M_P \text{ is the Planck mass})$$
$$t_P \simeq 5.4 \times 10^{-44}\ \text{sec.} \quad (t_P \text{ is the Planck time})$$

Time-temperature relation:

Adiabatic expansion (i.e. constant entropy) implies

$$RT = \text{constant} \tag{A.11}$$

In the radiation dominated universe, using (A.6) in (A.2) with k = 0, one has

$$\left(\frac{\dot{T}}{T}\right)^2 = \frac{4\pi^3 G N^*}{45} T^4 ,$$

which gives, assuming N* to be a constant (i.e. for T above any mass threshold),

$$T^2 = \left(\frac{45}{16\pi^3 N^*}\right)^{1/2} \frac{M_P}{t} . \tag{A.12}$$

The number N* is typically [20] of order 10^2. This gives

$$T \approx (\cdot 03\, M_P)^{1/2}\, t^{-1/2} .$$

(A.11) and (A.12) imply, in a radiation dominated universe,

$$R \propto t^{\frac{1}{2}} . \tag{A.13}$$

Similarly, from (A.6) and (A.12), we have

$$\rho \approx \cdot 03\, M_P^2\, t^{-2} . \tag{A.14}$$

Horizon distance

We take a 'flat' universe, k = 0.

Then, radial co-ordinate distance travelled by a light pulse ($d\tau^2 = 0$ in (A.1)) between time t_1 and t_2 is

$$r(t_2, t_1) = \int_{t_1}^{t_2} \frac{dt}{R(t)} . \tag{A.15}$$

The co-ordinate horizon length, $r_H(t)$, at time t is defined as

$$r_H(t) \equiv r\ (t,0)\ . \tag{A.16}$$

The <u>proper horizon distance</u>, $d_H(t)$, at time t is

$$d_H(t) = R(t)\ r_H(t)$$

or,

$$d_H(t) = R(t) \int_0^t dt'\ R^{-1}(t')\ .$$

Using (A.13), we see that in the radiation dominated universe,

$$d_H(t) = 2t\ . \tag{A.17}$$

APPENDIX B

The Action, Equation of Motion and Energy-Momentum Tensor of String (see, for example, [40])

A string in the course of its evolution traces out a two-dimensional world sheet (analogous to world-line for a point particle) in 4-dimensional space-time. Let σ and τ be respectively the space-like and time-like parameters which label the points on this two-dimensional surface and let $x^{\mu}(\sigma, \tau)$ denote the space-time coordinate of a point on this surface. The range of the parameter τ is $-\infty < \tau < +\infty$ and that of σ is $\alpha \leq \sigma \leq \beta$ for closed strings (we shall always consider closed strings). Here α and β are two fixed constants. The dynamics must be independent of particular forms of parameterization of the world sheet, i.e.,

$$x^{\mu}(\sigma, \tau) = x^{\mu'}(\sigma', \tau') , \tag{B.1}$$

where

$$\sigma' = \sigma'(\sigma, \tau) \quad , \quad \tau' = \tau'(\sigma, \tau) \quad \text{and} \quad J = \frac{\partial(\sigma', \tau')}{\partial(\sigma, \tau)} \neq 0 . \tag{B.2}$$

The action describing the dynamics of the string must be formed out of geometrical quantities which are intrinsic to the world-sheet. It (i.e. the action) must be invariant under the transformations (B.2) and also under general co-ordinate transformations. In addition, if we want equations of motion to involve not higher than second-order derivatives of x^{μ} (w.r.t. σ, τ), the action must be constructed from quantities that involve at most first derivatives of x^{μ}.

Analogous to Einstein-Hilbert action of general relativity, the action S for the string, which satisfies the above mentioned conditions, is

$$S \propto \int d\sigma \, d\tau \, (-\gamma)^{\frac{1}{2}} , \tag{B.3}$$

where

$$\gamma = det\left(\gamma_{ab}\right) , \tag{B.4}$$

γ_{ab} (a, b = 0, 1) being the metric defined on the world-sheet of the string, i.e.

$$\gamma_{ab} = g_{\mu\nu} \frac{\partial x^{\mu}}{\partial \lambda^{a}} \frac{\partial x^{\nu}}{\partial \lambda^{b}} \quad ; \quad \lambda^{0} = \tau \, , \, \lambda^{1} = \sigma \, , \tag{B.5}$$

and $g_{\mu\nu}$ is the space-time metric with signature (+,-,-,-). The requirement that σ and τ correspond respectively to spacelike and time-like extensions of the sheet means that

$$\gamma_{00} \geqslant 0 \quad , \quad \gamma_{11} \leqslant 0 \, . \tag{B.6}$$

The definition (B.3) is the same as requiring the action to be proportional to the area of the world-sheet of the string, i.e.,

$$S \propto \int dA \, , \tag{B.7}$$

where dA is the area-element of the world-sheet. This is seen as follows:

The (antisymmetric) area tensor element $dA^{\mu\nu}$ on the world-sheet formed from two infinitesimal displacements $\frac{\partial x^{\mu}}{\partial \sigma} d\sigma$ and $\frac{\partial x^{\mu}}{\partial \tau} d\tau$ is

$$dA^{\mu\nu} = \left(\frac{\partial x^{\mu}}{\partial \sigma} \frac{\partial x^{\nu}}{\partial \tau} - \frac{\partial x^{\mu}}{\partial \tau} \frac{\partial x^{\nu}}{\partial \sigma} \right) d\sigma \, d\tau \tag{B.8a}$$

$$= -\epsilon^{ab} \frac{\partial x^\mu}{\partial \lambda^a} \frac{\partial x^\nu}{\partial \lambda^b} \, d\sigma \, d\tau \,, \tag{B.8b}$$

where

$$\epsilon^{01} = -\epsilon^{10} = 1 \quad , \quad \epsilon^{00} = \epsilon^{11} = 0 \,. \tag{B.9}$$

The invariant area element dA on the world-sheet is then

$$dA = \left(\frac{1}{2} \left| dA^{\mu\nu} \, dA_{\mu\nu} \right| \right)^{1/2} . \tag{B.10}$$

From (B.8a) we find

$$dA^{\mu\nu} dA_{\mu\nu} = 2 (d\sigma \, d\tau)^2 \left[\left(\frac{\partial x}{\partial \sigma} \right)^2 \left(\frac{\partial x}{\partial \tau} \right)^2 - \left(\frac{\partial x}{\partial \sigma} \cdot \frac{\partial x}{\partial \tau} \right)^2 \right] \tag{B.11}$$

while (B.8b) gives

$$dA^{\mu\nu} dA_{\mu\nu} = \epsilon^{ab} \frac{\partial x^\mu}{\partial \lambda^a} \frac{\partial x^\nu}{\partial \lambda^b} \epsilon^{cd} \frac{\partial x_\mu}{\partial \lambda^c} \frac{\partial x_\nu}{\partial \lambda^d} (d\sigma \, d\tau)^2$$

$$= \epsilon^{ab} \epsilon^{cd} \gamma_{ac} \gamma_{bd} (d\sigma \, d\tau)^2 \,, \quad \text{using (B.5)}$$

$$= 2 (\gamma_{00} \gamma_{11} - \gamma_{01} \gamma_{10}) (d\sigma \, d\tau)^2$$

$$= 2 \gamma \, (d\sigma \, d\tau)^2 . \tag{B.12}$$

We thus have

$$dA = \left(\frac{1}{2} \left| dA^{\mu\nu} dA_{\mu\nu} \right| \right)^{1/2} = \left(|\gamma| \right)^{1/2} d\sigma \, d\tau$$

$$= (-\gamma)^{1/2} d\sigma \, d\tau \qquad (\because \gamma < 0 \text{ by (B.6)}) .$$

Thus (B.3) and (B.7) are equivalent. So, the action for the string can be written as

$$S = -\mu \int d\sigma d\tau \left[\left(\frac{\partial x}{\partial \sigma} \cdot \frac{\partial x}{\partial \tau} \right)^2 - \left(\frac{\partial x}{\partial \sigma} \right)^2 \left(\frac{\partial x}{\partial \tau} \right)^2 \right]^{1/2} \qquad \text{(B.13a)}$$

$$= -\mu \int d\sigma d\tau \left(-\gamma \right)^{1/2} , \qquad \text{(B.13b)}$$

where μ is the mass per unit length of the string.

Equations of motion

These are obtained from variational principle, i.e. by requiring that the action S be stationary under infinitesimal variation

$$x^\mu(\sigma,\tau) \longrightarrow x^\mu(\sigma,\tau) + \delta x^\mu(\sigma,\tau) , \qquad \text{(B.14)}$$

with conditions

$$\delta x^\mu(\tau = \pm\infty , \sigma) = 0 , \qquad \text{(B.15a)}$$

$$\delta x^\mu(\tau , \sigma = \alpha) = \delta x^\mu(\tau , \sigma = \beta) , \qquad \text{(B.15b)}$$

and

$$\left. \frac{\partial x^\mu}{\partial \sigma} \right|_{\sigma = \alpha} = \left. \frac{\partial x^\mu}{\partial \sigma} \right|_{\sigma = \beta} . \qquad \text{(B.15c)}$$

The conditions (B.15b) and (B.15c) are appropriate for a closed string, (B.15c) being the requirement that the string be smooth.

Let us rewrite (B.13a) as

$$S = \int d\sigma d\tau \, L \left\{ \dot{x}^\mu , x'^\mu \right\} , \qquad \text{(B.16)}$$

where

$$\dot{x}^\mu \equiv \frac{\partial x^\mu}{\partial \tau} , \qquad x'^\mu \equiv \frac{\partial x^\mu}{\partial \sigma} , \quad \text{and}$$

$$L = -\mu\left[\left(\frac{\partial x}{\partial\sigma}\cdot\frac{\partial x}{\partial\tau}\right)^2 - \left(\frac{\partial x}{\partial\sigma}\right)^2\left(\frac{\partial x}{\partial\tau}\right)^2\right]^{1/2} . \tag{B.17}$$

By demanding $\delta S = 0$ and by using integration by parts and conditions (B.15), one obtains the equations of motion

$$\frac{\partial}{\partial\tau}\frac{\delta L}{\delta\dot{x}^\mu} + \frac{\partial}{\partial\sigma}\frac{\delta L}{\delta x'^\mu} = 0 . \tag{B.18}$$

<u>The Energy-Momentum Tensor ($T^{\mu\nu}$):</u>

From (B.13a), using the definition (see, for example, Weinberg [7])

$$T^{\mu\nu} = \frac{-2}{(-g)^{1/2}}\frac{\delta S}{\delta g_{\mu\nu}} , \tag{B.19}$$

one obtains,

$$T^{\mu\nu}(X) = -\mu(-g)^{-\frac{1}{2}}\int d\tau\, d\sigma\left[\left(\frac{\partial x}{\partial\tau}\cdot\frac{\partial x}{\partial\sigma}\right)^2 - \left(\frac{\partial x}{\partial\tau}\right)^2\left(\frac{\partial x}{\partial\sigma}\right)^2\right]^{-1/2}$$
$$\cdot\left[\left(\frac{\partial x}{\partial\sigma}\right)^2\frac{\partial x^\mu}{\partial\tau}\frac{\partial x^\nu}{\partial\tau} - \left(\frac{\partial x}{\partial\sigma}\cdot\frac{\partial x}{\partial\tau}\right)\left(\frac{\partial x^\mu}{\partial\tau}\frac{\partial x^\nu}{\partial\sigma} + \frac{\partial x^\mu}{\partial\sigma}\frac{\partial x^\nu}{\partial\tau}\right)\right.$$
$$\left. + \left(\frac{\partial x}{\partial\tau}\right)^2\frac{\partial x^\mu}{\partial\sigma}\frac{\partial x^\nu}{\partial\sigma}\right]\delta^4\big(X - x(\sigma,\tau)\big) \tag{B.20}$$

Here X is any space-time point. The δ-function in (B.20) ensures that $T^{\mu\nu}$ vanishes everywhere except on the world-sheet of the string.

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FIGURE CAPTIONS

FIG. 1.1 An arbitrary configuration of a vortex-string. The phase of the Higgs field Φ changes by 2π in going once round the string.

FIG. 1.2 Schematic diagram illustrating the formation of a closed-loop when a string intersects itself. The arrows indicate the directions of the magnetic flux.

FIG. 2.1 The behaviour of energy of a spiral standing wave (energy $\propto \epsilon\eta$) in an expanding universe (see text for details).

FIG. 2.2 The behaviour of energy of an initially non-static closed-loop with radius of the order of horizon distance in an expanding universe (see text).

FIG. 3.1 Fractions ($\frac{\Delta E}{E}$) of initial mass-energies (E) of collapsing circular closed loops lost in friction with the surrounding matter. Here t_o is the time of formation of a loop and t_c is the time of phase transition.

FIG. 4.1 Schematic diagrams illustrating the possible stages in the process of "exchange of partners". The unbroken lines represent the position of the strings. The arrows indicate the directions of the magnetic flux.

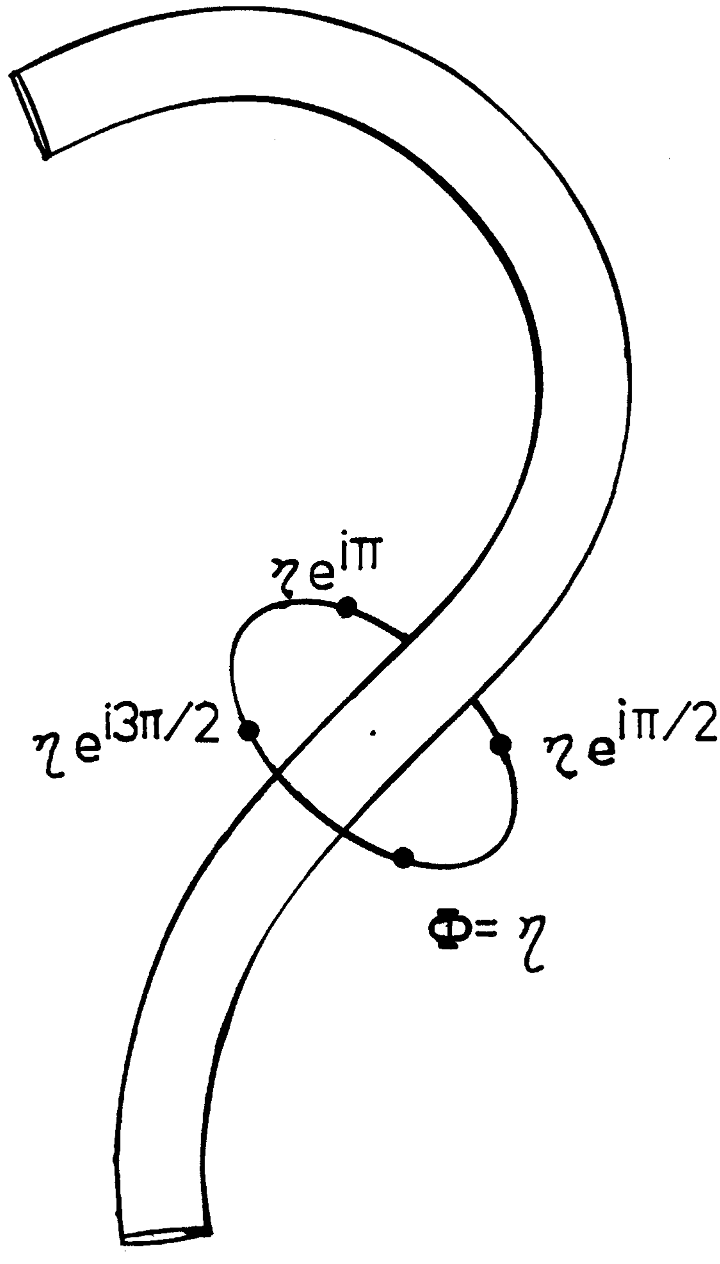


FIG.1.1.

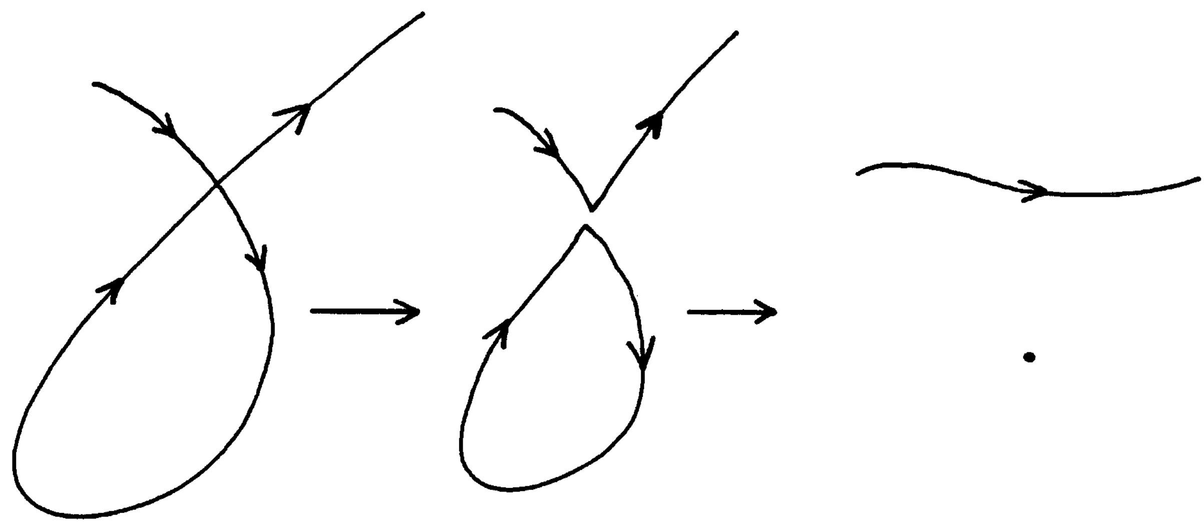
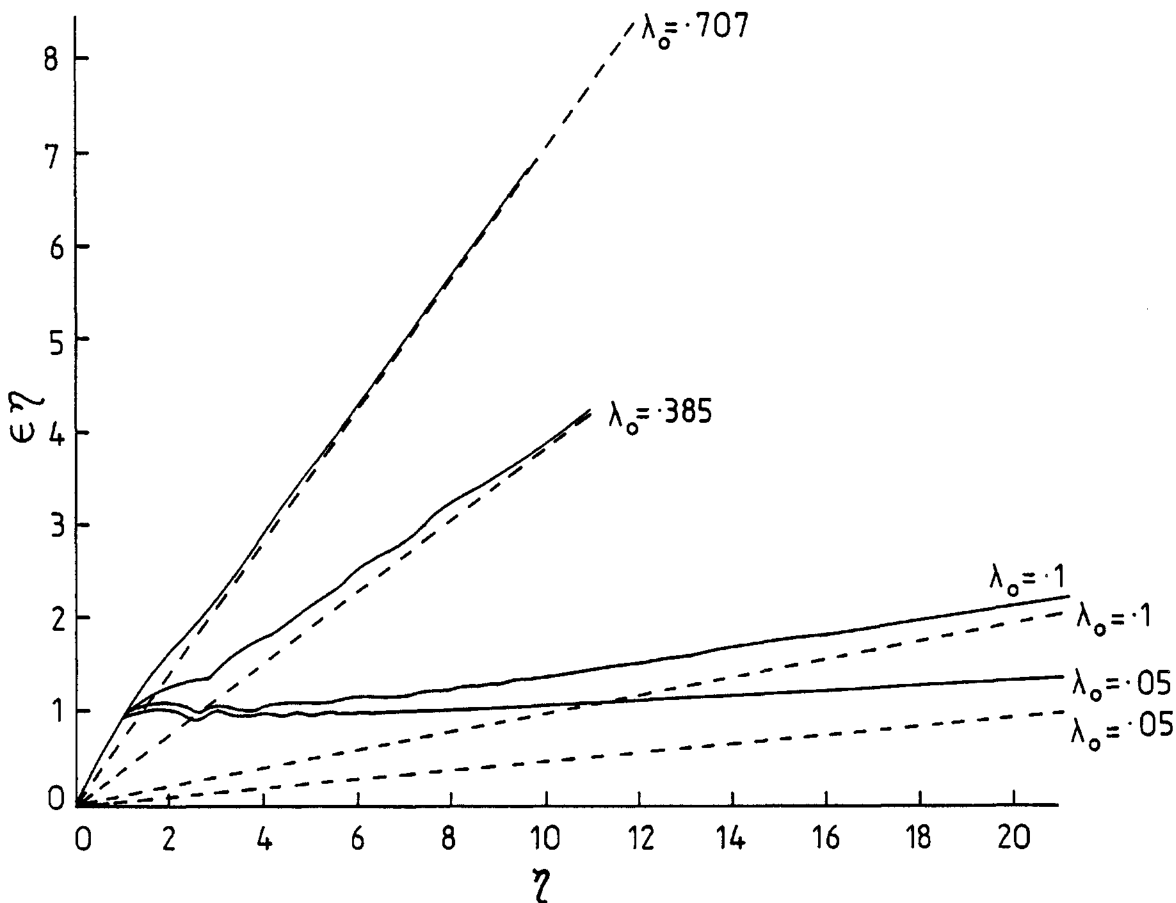


FIG. 1.2.



$\lambda_o = \cdot 707$
$\lambda_o = \cdot 385$
$\lambda_o = \cdot 1$
$\lambda_o = \cdot 1$
$\lambda_o = \cdot 05$
$\lambda_o = \cdot 05$
$\epsilon\eta$
η
0
1
2
3
4
5
6
7
8
0
2
4
6
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10
12
14
16
18
20

FIG.2.1.

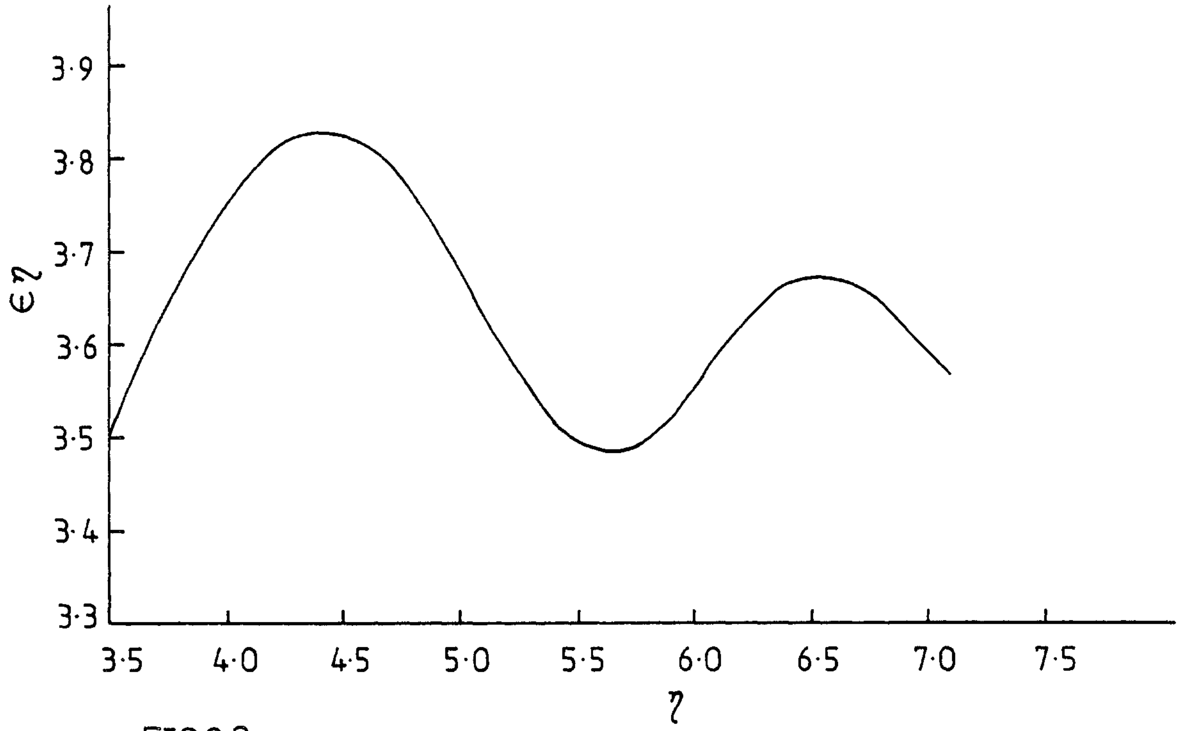

FIG.2.2.

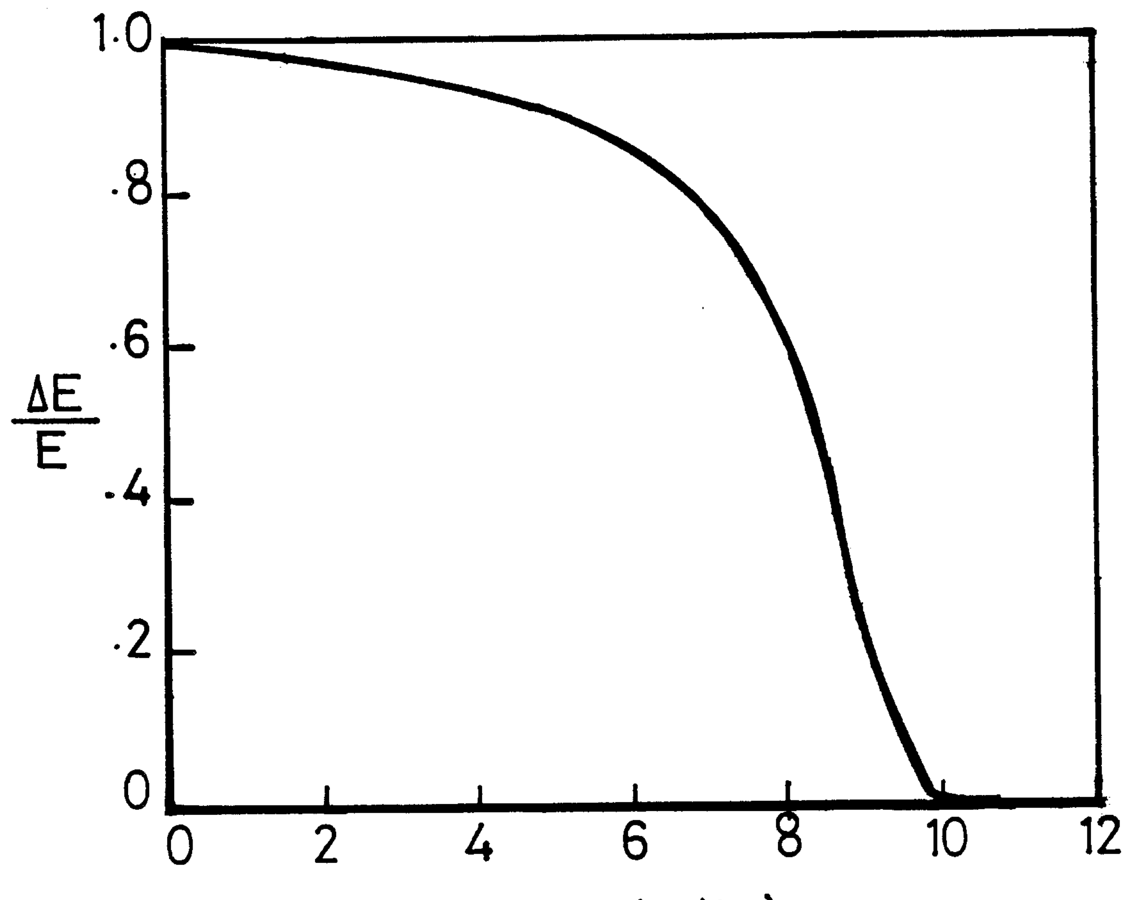

FIG.3.1.

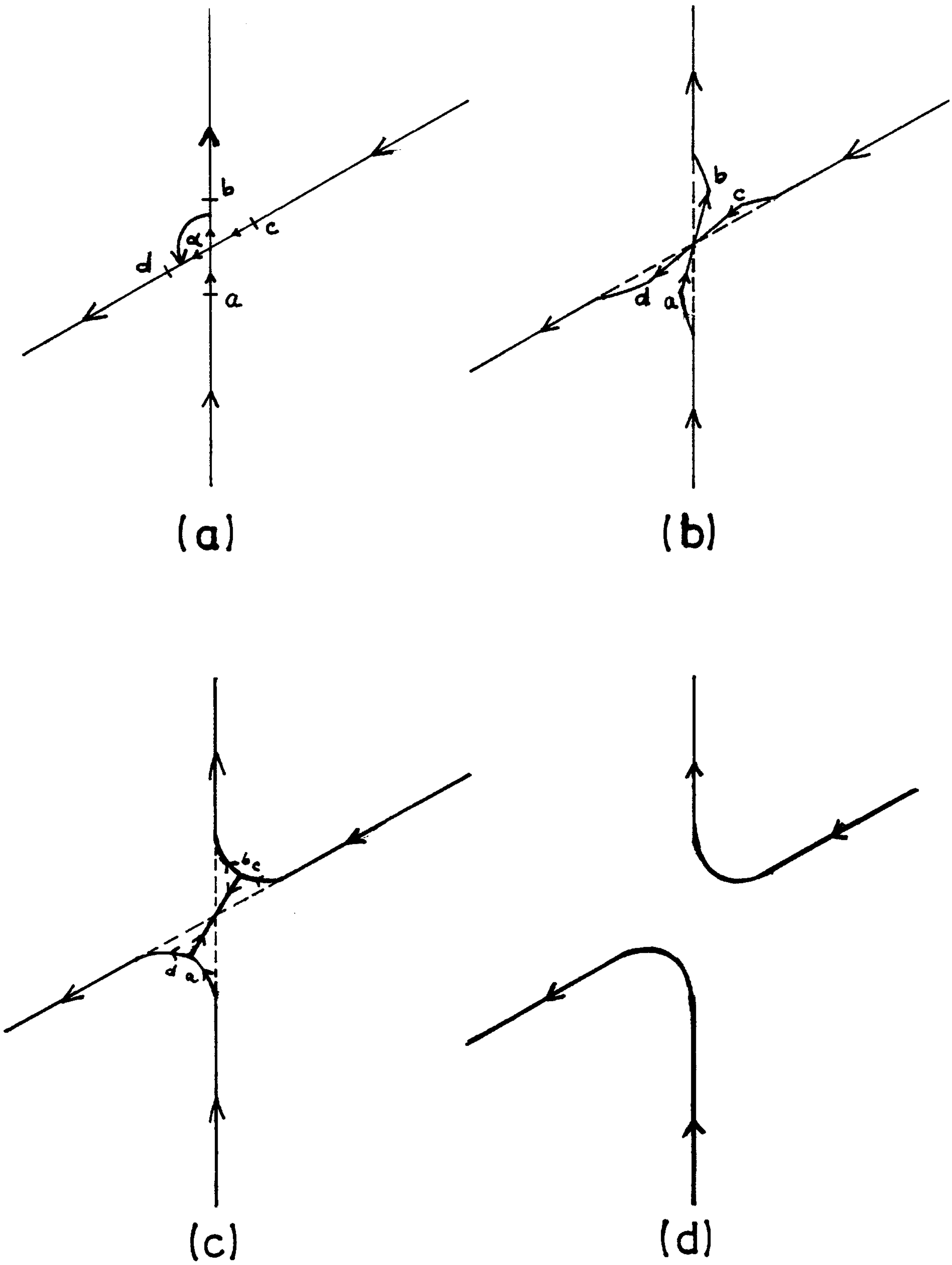

FIG. 4.1