DESIGN OF FIBRE COMPOSITE STRUCTURES FOR LARGE WATER VALVES

Farid Mahmoud Hasan Bizzari B.Sc., M.Sc.

A thesis submitted for the degree of

Doctor of Philosophy

in the University of London

Department of Mechanical Engineering

Imperial College of Science and Technology

London S.W.7

July 1980

TO FADWA

S U M M A R Y

A brief introduction to the types of problem occurring in the stress analysis of Water Valves of the butterfly type is given to show the need for developing a design method for these structures.

The analytical work which has been undertaken for establishing a closed form solution to the problem is given, with comments.

A description of the anisotropic characteristics of the fibre composite material is presented in the form needed to allow the use of matrix algebra and practical examples of these materials are given.

A general finite element is derived in detail to show how it represents the structure in a plate or shell shape made of Isotropic, Orthotropic or Anisotropic material. Also a finite element method is suggested to investigate the structural strength and behaviour of different types of blades made of different materials and the advantages of the method are shown.

A method for the optimal design of fibre composite structures is suggested and recommendations for future work are given.

The computer program and the numerical results are presented.

CONTENTS

		Page No.
SUMMARY		3.
ACKNOWLEDGEMENTS		8.
LIST OF FIGURES		9.
LIST OF TABLES		17.
NOTATIONS		19.
CHAPTER ONE	INTRODUCTION	22.
	1.1. General	23.
	1.2. Description of the structure of main interest.	27.
	1.2.1. The disk or blade	28.
	1.2.2. The shaft	34.
	1.2.3. The body	34.
	1.2.4. Seating seals	36.
	1.2.5. Operating equipment	37.
	1.3. Review.	37.
	1.4. Materials	39.
	1.5. Limitation and scope of study.	40.
CHAPTER TWO	ANALYTICAL PROCEDURE FOR THE STRUCTURAL ANALYSIS OF THE BLADES OF ORDINARY BUTTERFLY VALVES.	41.
	2.1. The governing differential equation for circular plates.	42.
	2.2. The general solution of the governing differential equation.	47.
	2.3. The solution for stresses and deflections in a circular plate of uniform thickness subjected to a uniform normal pressure and supported at two points at opposite ends of a diameter.	49.
	2.3.1. Analytical work	49.
	2.3.2. Finite element analysis	68.

Page No.

CHAPTER TWO

2.4. The solution for stresses and deflections in a circular plate of uniform thickness subjected to a uniform normal pressure and supported at two short lengths of arcs at opposite ends of a diameter. 77.

2.4.1. Introduction 77.

2.4.2. Analytical work 77.

2.4.3. Finite element analysis 82.

2.5. Experimental Work. 88.

2.5.1. Introduction 88.

2.5.2. 4-Point bend testing of a perspex beam 96.

2.5.3. The testing equipment 106.

2.5.4. Testing a 12" diameter disk supported on two clamped short arcs subjected to a uniform pressure. 121.

2.5.5. Testing a 12" diameter disk supported on two clamped short arcs subjected to a point load. 122.

2.5.6. Testing a 12" diameter disk supported on two clamped short arcs and two short lengths of the diameter subjected to a uniform pressure. 123.

CHAPTER THREE RELEVANT PARTS OF THE THEORY OF ANISOTROPIC ELASTICITY 196.

3.1. Introduction 197.

3.2. The generalised Hooke's law for anisotropy. 199.

3.3. Elastic symmetry and orthotropic case. 202.

3.4. Transformation rules for an orthotropic body. 207.

		Page No.
CHAPTER THREE	3.4.1. Stress transformation	207.
	3.4.2. Strain transformation	210.
	3.4.3. Constants of elasticity	211.
CHAPTER FOUR	FINITE ELEMENT MODEL AND SOLUTION	213.
	4.1. Introduction	214.
	4.2. Geometry representation	214.
	4.3. Displacement field	216.
	4.4. Strain-displacement relations	220.
	4.5. Stress-strain relations	222.
	4.6. Derivation of the stiffness matrix.	223.
	4.7. Flow chart of solid finite element computer programme	225.
CHAPTER FIVE	IDEALIZATION OF FINITE ELEMENTS FOR STRUCTURAL DESIGN, NUMERICAL AND EXPERIMENTAL RESULTS.	227.
	5.1. Introduction	228.
	5.2. Cast iron blade of ordinary butterfly valve.	231.
	5.3. Test samples of composite materials.	233.
	5.3.1. Introduction	233.
	5.3.2. Preparation of test specimens, testing equipment and procedure.	240.
	5.3.3. Elastic constants of the specimens.	270.
	5.3.4. Finite element idealization of G.R.P. test beam.	271.
	5.3.5. Experimental work on G.R.P. test beam.	273.
	5.4. G.R.P. blade of ordinary butterfly valve.	291.

Page No.

CHAPTER SIX	CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK.	304.
	6.1. The analysis of ordinary blades	305.
	6.1.1. Conclusions	305.
	6.1.2. Further work	306.
	6.2. Fibre composite blades	307.
	6.2.1. Approach for optimal design.	307.
	6.2.2. Further work	308.
REFERENCES		311.
APPENDICES	APPENDIX 1	319.
	APPENDIX 2	338.
	APPENDIX 3	357.
	APPENDIX 4	373.
	APPENDIX 5	376.

ACKNOWLEDGEMENTS

The author wishes to express his sincere thanks to Professor J.M. Alexander for his valuable advice and kind encouragement while supervising this work.

He also wishes to thank the following, who have assisted with the experimental work and computing facilities. The staff of the University of London Computer Centre, especially the Advisory Department. The technicians and laboratory staff of the Dept. of Mechanical Engineering, University College Swansea. The librarian of the Dept. of Mechanical Engineering, Imperial College. Mr. M. Stone and Mr. J.M. Griffiths of Boving & Co. Ltd. Mr. R. Lovell of Tods of Weymouth. Dr. S.A. Abi-Shaheen and Mr. D. Lo of Stemmos Ltd.

Finally, I would like to thank my wife Fadwa for her encouragement and patience throughout the whole period of the work and Mrs. Mary Aldridge for typing the whole text.

LIST OF FIGURES

		Page No.
1.1.	Optimum approach to design with composites	25.
1.2.	A disk of constant thickness	29.
1.3a.	A tapered disk	30.
1.3b.	A hollow tapered disk	31.
1.4.	Shaft through the disk	32.
1.5.	Two shafts fitted in humps	32.
1.6.	Shafts as integral casting of the disk	32.
1.7.	Components of butterfly valve	33.
1.8.	Single flange body	35.
1.9.	"U" flangeless body	35.
1.10.	Double flanged body	35.
1.11.	Flangeless body	35.
2.1.	Expressions for bending moments and transverse shear in cartesian coordinates.	44.
2.2.	Polar coordinates.	44.
2.3.	Plate element in polar coordinates	46.
2.4.	Circular plates supported at two points	50.
2.5.	Pure bending on circular plate supported at two points.	52.
2.6.	Quadratic plate finite element	52.
2.7.	Finite element mesh	69.
2.8a.	Plate simply supported on two arcs.	78.
2.8b.	Reactive forces on plate simply supported on two arcs and subjected to uniform pressure.	78.
2.8c.	Plate supported on two clamped arcs.	78.

Page No.

2.9.	Plate supported on two short lengths of arc and short lengths of diameter.	89.
2.10.	Analysis of pure bending of a beam.	98.
2.11.	4-Point bend testing of a perspex beam.	98.
2.12.	4-Point bend testing equipment.	99.
2.13.	4-Point bend testing of a perspex beam.	99.
2.14.	A plot of total load vs central deflection.	101.
2.15.	A plot of strain vs deflection at the centre.	104.
2.16	Testing equipment.	107.
2.17.	Sample flange.	108.
2.18.	Testing equipment and control panel.	107.
2.19.	Perspex blade sample.	111.
2.20.	Sample blade with strain gauges connected fitted in testing flange.	111.
2.21.	Short flange for clamping.	112.
2.22.	Testing rig.	112.
2.23.	Dial gauges during testing.	113.
2.24.	Dial gauges frame.	113.
2.25.	Point load cell.	114.
2.26.	Point load cell attached to testing rig.	114.
2.27.	Point load testing	115.
2.28.	Line support.	115.
2.29.	Line support uniform pressure testing.	116.
2.30.	Dummy gauges.	116.
2.31.	Complete testing equipment.	117.
2.32.	Location of strain gauges.	118.
2.33.	U.V. Recorder output scale.	119.

Page No.

2.34.	Full scale U.V. strain output.	119.
2.35	Strain circle of a rosette.	120.
2.36a.	Dial gauge locations.	125.
2.36b	Dial gauge locations.	126.
2.37.	Comparison of deflections due to uniform distributed pressure along x & y axis - 5 psi	127.
2.38.	Comparison of deflections due to uniform distributed pressure along x & y axis - 10 psi.	128.
2.39.	Location of strain gauges on finite element mesh	129.
2.40.	Comparison of principal stresses - Gauge 1.	143.
2.41.	Comparison of principal stresses - Gauge 2.	144.
2.42.	Comparison of principal stresses - Gauge 3 & 7.	145.
2.43.	Comparison of principal stresses - Gauge 4.	146.
2.44.	Comparison of principal stresses - Gauge 5.	147.
2.45.	Comparison of principal stresses - Gauge 8.	148.
2.46.	Comparison of principal stresses - Gauge 9.	149.
2.47.	Dial gauge and point load locations.	150.
2.48.	Comparison of deflections due to point load at Node 6 along x-axis.	162.
2.49.	Comparison of deflections due to point load at Node 6 along x-axis.	163.
2.50a.	Comparison of deflections due to point load at Node 6 along y-axis.	164.
2.50b.	Comparison of deflections due to point load at Node 6 along y-axis.	164.
2.51.	Comparison of principal stresses - Gauge 1.	165.
2.52.	Comparison of principal stresses - Gauge 2.	165.
2.53.	Comparison of principal stresses - Gauge 4.	166.
2.54.	Comparison of principal stresses - Gauge 5.	167.
2.55.	Comparison of principal stresses - Gauge 7.	168.

		Page No.
2.56.	Comparison of principal stresses - Gauge 8.	169.
2.57.	Comparison of principal stresses - Gauge 9.	170.
2.58.	Dial gauge locations	171.
2.59.	Comparison of deflections due to uniform distributed pressure along x-axis. - 5 psi.	172.
2.60.	Comparison of deflections due to uniform distributed pressure along y-axis. - 10 psi.	172.
2.61.	Comparison of principal stresses - Gauge 1.	188.
2.62.	Comparison of principal stresses - Gauge 2.	189.
2.63.	Comparison of principal stresses - Gauge 3.	190.
2.64.	Comparison of principal stresses - Gauge 4.	191.
2.65.	Comparison of principal stresses - Gauge 5.	192.
2.66.	Comparison of principal stresses - Gauge 7.	193.
2.67.	Comparison of principal stresses - Gauge 8.	194.
2.68.	Comparison of principal stresses - Gauge 9.	195.
3.1.	A 3-dimensional solid of fibres embedded in matrix	198.
3.2.	Illustration of an orthotropic material in its laminated and lamina forms.	198.
3.3.	3-Dimensional state of stress	204.
3.4.	3-Dimensional element	204.
4.1.	8-Node first order hexahedron	217.
4.2.	16-Node element	217.
4.3a.) 4.3b.) 4.3c.)	General elements (natural coordinates.	217.
4.4.	20-Node quadratic hexahedron	218.

		Page No.
5.1a.	Blades of throughflow valves	229.
5.1b.	Blades of throughflow valves	229.
5.2.	A 72" dia. Cast iron butterfly valve - blade	232.
5.3a.	Finite element mesh of the cast iron blade	234.
5.3b.	Finite element mesh of the cast iron blade	234.
5.3c.	Finite element mesh of the cast iron blade	235.
5.4a.	Deflection test of the cast iron blade	236.
5.4b.	Deflection test of the cast iron blade	236.
5.4c.	Deflection test of the cast iron blade	236.
5.5a.	Actual dimensions of composite deep beam No. D1	241.
5.5b.	Composite deep beam specimen	241.
5.6a.	Composite deep beam specimen	242.
5.6b.	Composite deep beam specimen	242.
5.7.	Layout of multi G.R.P. layer samples.	243.
5.8.	Woven roving and unidirectional woven glass layers	244.
5.9.	Individual layer specimens	244.
5.10.	Individual layer specimens	245.
5.11.	Individual layer specimens	245.
5.12.	Individual layer specimens	246.
5.13.	4-Point testing of G.R.P. beam specimen	248.
5.14.	4-Point testing of G.R.P. beam specimen	248.
5.15.	Load vs. deflection in direction 1 - Layer A	249.
5.16.	Load vs. deflection in direction 2 - Layer A	250.
5.17.	Load vs. deflection in direction 1 - Layer B	251.
5.18.	Load vs. deflection in direction 2 - Layer B	252.
5.19.	Load vs. deflection in direction 1 - Layer C	253.
5.20.	Load vs. deflection in direction 2 - Layer C	254.

Page No.

5.21.	Longitudinal & transverse strain vs. deflection direction 1 - Layer A.	255. 255.
5.22.	Longitudinal & transverse strain vs. deflection direction 2 - Layer A.	256.
5.23.	Longitudinal & transverse strain vs. deflection direction 1 - Layer B.	257.
5.24.	Longitudinal & transverse strain vs. deflection direction 2 - Layer B.	258.
5.25.	Longitudinal & transverse strain vs. deflection direction 1 - Layer C.	259.
5.26.	Longitudinal & transverse strain vs. deflection direction 2 - Layer C.	260.
5.27.	4-Point bend testing of G.R.P. beam for ν_{13} & ν_{23}	261.
5.28.	4-Point bend testing of G.R.P. beam for ν_{13} & ν_{23}	261.
5.29.	4-Point bend testing of G.R.P. beam for ν_{13} & ν_{23}	262.
5.30.	Actual size specimen.	262.
5.31.	Longitudinal vs. vertical strains in direction 1 - Layer A.	263.
5.32.	Longitudinal vs. vertical strains in direction 2 - Layer A.	264.
5.33.	Longitudinal vs. vertical strains in direction 1 - Layer B.	265.
5.34.	Longitudinal vs. vertical strains in direction 2 - Layer B.	266.
5.35.	Longitudinal vs. vertical strains in direction 1 - Layer C.	267.
5.36.	Longitudinal vs. vertical strains in direction 2 - Layer C.	268.
5.37.	Minimum set of specimens.	269.
5.38.	Apparatus used for 4-Point bend testing of beams.	269.
5.39.	Finite element idealization of G.R.P. test beam.	272.

		Page No.
5.40a.) 5.40b.)	Location and numbers of strain gauges.	274.
5.41.	Experiment arrangement of G.R.P. test beam.	275.
5.42.	Experiment arrangement of G.R.P. test beam.	276.
5.43.	Experiment arrangement of G.R.P. test beam.	276.
5.44.	Gauge locations on the finite element mesh.	277.
5.45.	Comparison of stresses in direction 1 at Node 203	279.
5.46.	Comparison of stresses in direction 2 at Node 203	280.
5.47.	Comparison of stresses in direction 2 at Node 210	281.
5.48.	Comparison of stresses in direction 3 at Node 210	282.
5.49.	Comparison of stresses in direction 2 at Node 215	283.
5.50.	Comparison of stresses in direction 3 at Node 215	284.
5.51.	Comparison of stresses in direction 1 at Node 218	285.
5.52.	Comparison of stresses in direction 2 at Node 218	286.
5.53.	Comparison of stresses in direction 2 at Node 211	287.
5.54.	Comparison of stresses in direction 3 at Node 211	288.
5.55.	Comparison of stresses in direction 2 at Node 206	289.
5.56.	Comparison of stresses in direction 3 at Node 206	290.
5.57.	Load vs. deflections in direction 3 at Node 1	292.
5.58.	Load vs. deflections in direction 3 at Node 45	293.
5.59.	Load vs. deflections in direction 1 at Node 30	294.
5.60.	G.R.P. butterfly blade	295.
5.61.	G.R.P. butterfly blade	296.
5.62.	G.R.P. butterfly blade	296.
5.63.	G.R.P. butterfly blade	297.
5.64a.	Finite element idealization of G.R.P. blade	299.
5.64b.	Finite element idealization of G.R.P. blade	300.

		Page No.
5.65a.	Strain gauge locations on G.R.P. blade	301.
5.65b.	Strain gauge locations on G.R.P. blade	302
5.66.	Testing arrangement for deflections in G.R.P. blade.	303
A4.1.	Global, local and principal directions for thin shell element	375.
A5.1.	Three dimensional parallelo piped	378.

LIST OF TABLES

		Page No.
2.1.	Deflections and moments in a circular plate simply supported around the periphery and subjected to a uniform pressure.	70.
2.2.	Moments design coefficients for circular plate simply supported on two points at opposite ends of a diameter and subjected to a uniform pressure obtained from finite element method.	72.
2.3.	Comparison of deflection coefficients obtained by the 'closed-form' equation and finite element in circular plate simply supported on two points at opposite ends of a diameter and subjected to a uniform pressure.	73.
2.4.	Comparison of moments coefficients obtained by the 'closed-form' equation and finite element in circular plate simply supported on two points at opposite ends of a diameter and subjected to a uniform pressure.	76.
2.5.	Comparison of deflection coefficients obtained by 'closed-form' equation and finite element in circular plates simply supported on two short lengths of arcs at opposite ends of a diameter and subjected to a uniform pressure.	83.
2.6.	Moments design coefficients for circular plate simply supported on two short lengths of arcs at opposite ends of a diameter and subjected to uniform pressure obtained from finite element method.	84.
2.7.	Comparison of moments coefficients obtained by the 'closed-form' equation and finite element in circular plate simply supported on two short lengths of arcs at opposite ends of a diameter and subjected to a uniform pressure.	85.

		Page No.
2.8a.	Coefficients for deflections in circular plates simply supported on two short lengths of arcs at opposite ends of a diameter due to point load at the shown location.	86.
2.8b.	Coefficients for moments in circular plates simply supported on two short lengths of arcs at opposite ends of a diameter due to point load at the shown location.	87.
2.9.	Coefficients for deflections in circular plates clamped on two short lengths of arcs at opposite ends of a diameter due to point load at the shown location.	90.
2.10.	Coefficients for deflections in circular plate supported on two clamped short lengths of arc at opposite ends of a diameter and subjected to a uniform pressure.	91.
2.11.	Coefficients for moments in circular plate supported on two clamped short lengths of arc at opposite ends of a diameter and subjected to a uniform pressure.	92.
2.12a.	Coefficients for deflections in circular plates supported on two clamped arcs at the end of the diameter and two short lengths of the diameter as shown and subjected to uniform pressure.	93.
2.12b.	Coefficients for moments in circular plates supported on two clamped arcs at the end of the diameter and two short lengths of the diameter as shown and subjected to uniform pressure.	94.
2.12c.	Coefficients for deflections in circular plates supported on two clamped arcs at the end of the diameter and two short length of the diameter as shown due to point load at the shown location.	95.
3.1.	Direction cosines between two sets of coordinates	209.
5.1.	Material properties as given by manufacturers. Ref. (67).	237.

NOTATIONS

W W_p, W_c	Deflections, load during tests
a	Radius of a circle
r	Radial distance of a point
θ	Polar angle
p	Point loads
C_i	Constants, coefficients
∇	Differential operator as defined in Equation (2.8).
$x,y,z.$	Cartesian coordinates
ρ,θ,z	Polar coordinates
1,2,3.	Material coordinates
M_x, M_y, M_{xy}	Moments and twisting moments in cartesian coordinates per unit length.
M_r, M_t, M_{rt}	Moments and twisting moments in polar coordinates per unit length
ν	Poisson's ratio for an isotropic material
ν_{ij}	Poisson's ratio for an isotropic material giving the strain in the j direction caused by the strain in the i direction.
E	Young's modulus of elasticity.
E_x, E_y, E_z	Young's moduli of elasticity along x,y,z direction.
E_1, E_2, E_3	Young's moduli of elasticity along 1,2,3 the natural axis of material.
∂	Variation
q	Uniformly distributed load on pressure
D	Flexural rigidity
Q	Shear forces per unit length
t	Plate thickness

I		Second moments of area or integration value
F_i^n		Functions of n
n		Harmonic number
π	=	3.141592654
R		Reaction force
d, d'		Variations as defined in Equation (2.29)
β		Constant as defined in Equation (2.32), angle
α		Angle of arc support
A_i^n, B_i^n		Constant
V_r		Kirchhoff shear
K		As defined in Equation (2.43a).
σ, ε		Stresses and strain respectively
σ_{ij}, ε_{ij}		(i,j = 1,2,3) Stress and strain components in material natural axis. (i,j = x,y,z) Stress and strain components in global cartesian coordinates.
τ_{ij}, γ_{ij}		(i,j = 1,2,3) Shear stresses and strains in material axis. (i,j = x,y,z) Shear stresses and strain in global cartesian coordinates.
G_{ij}		(i,j = 1,2,3) Shear moduli associated with the material natural axis. (i,j = x,y,z) Shear moduli material associated with global axis.
$C_{ij\,kl}$		(i,j = 1,3 and k,l = 1,9) Elastic constants of of material.
$[C]$		Compliance or flexibility matrix
$[D]$, $[\bar{D}]$		Elasticity matrices related to global and natural axis respectively.
Ω		Strain energy per unit volume.
l_i, m_j, n_k		(i,j,k = 1,2,3) Direction cosines.
$[R]$		Stress transformation matrix
$[R_\varepsilon]$		Strain transformation matrix

$[I]$	Unit matrix
f_i	(i = 1,2) Interpolation functions
N_i	(i = 1,20) Shape function
$\xi\ \eta\ \zeta$	Isoparametric coordinates
α_i	(i = 1,20) Constants
u,v,w	Displacement components in the global x,y,z directions respectively.
u_i, v_i, w_i	(i = 1,20) Nodal displacements along the natural axis.
$[J]$	Jacobian matrix
$[B]$	Elasticity matrix
$[B^*]$	Defined in Equation (4.16)
$\{\sigma'\}$, $\{\varepsilon'\}$	Stress and strain vectors in material coordinates respectively.
$[k]$	Element stiffness matrix

CHAPTER ONE

INTRODUCTION

CHAPTER ONE

INTRODUCTION

1.1. General

The butterfly valve constitutes a very special case for structural analysis due to its geometrical shape which is determined by complex loading and boundary conditions during operation Ref. (1) which are very difficult to model mathematically.

In addition to this, its main function is as an engineering structure for sealing pipes, where deflections play an important role in defining the criteria for its efficient operation and possible failure.

Existing methods of analysis and design are based generally on approximate idealized "closed form" mathematical models for circular plates.

A long history of empirical formulae and experimental studies of butterfly valves exists, but as sizes of valves and industrial installations became larger with higher pressures, the desirable closing characteristic of the butterfly valve and new sealing methods led to its taking a greater share of the valve market and its replacing other types of valve. Ref. (2) (3) .

Due to the traditional methods of design which did not give confidence either to the buyers or to the manufacturers, very strict standards were demanded. Ref. (4) (5) (6) (7) . These specified minimum thicknesses for blades, bodies and shafts and required test pressures that seldom occur in practice.

As a consequence of this, very expensive test procedures had to be carried out in the manufacturing workshops rather than in the actual installations.

The use of butterfly valves in cooling systems employing sea water introduced corrosion as another major parameter to be considered in their design. This led to the use of coatings and linings and although this was satisfactory in temperate climates such as are found on European and North American coasts where sea water termperatures are low, it proved unsatisfactory in warm sea water Ref. (8) . In such environments the use of materials that provide good corrosion resistance, such as 18/8 3% molybdenum stainless steel, aluminium bronze or ni-resist cast iron becomes mandatory if metals are to be considered.

Fibre composite materials should therefore be seriously considered as alternatives, from the point of view of offering both corrosion resistance and cost savings.

Since such materials do not possess isotropic properties, analytical analysis becomes very complicated. More and more alternative combinations of fibre and matrix materials possessing high strength, high modulus and desirable corrosion properties (such as glass fibre reinforced plastic (G.R.P.), boron-epoxy, graphite-epoxy and boron-aluminium) have become available. Ref. (9) .

In contrast to the engineer who has traditionally designed a structure from a designated metal, in the field of composites the designer has freedom to design both the material and the structure as illustrated in Fig.(1.1)Ref. (10) .

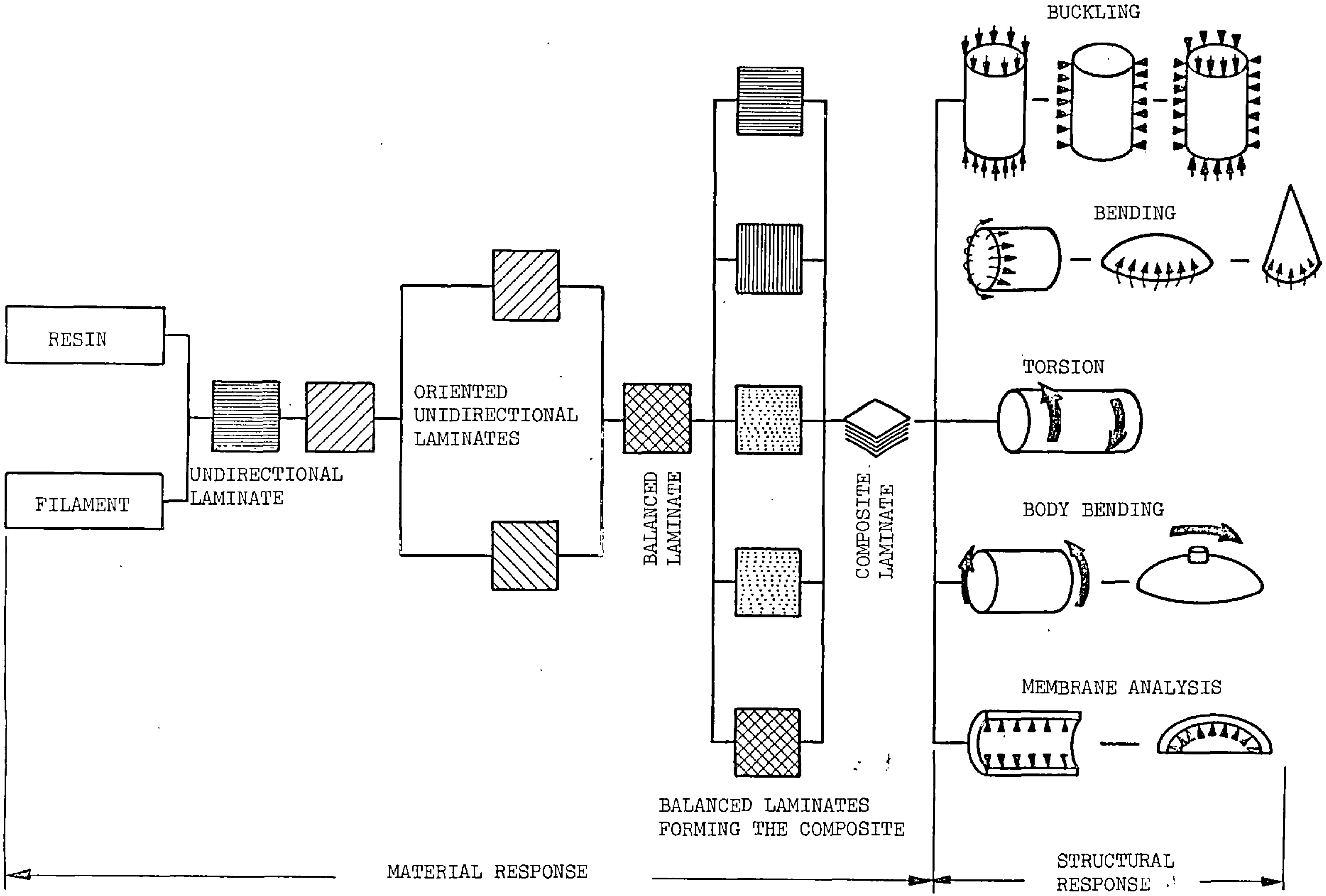


Fig. 1.1. Optimum Approach to Design with Composites

The need for numerical methods of analysis using modern computers to aid the design function becomes inevitable and the finite element method as applied in this study gives great confidence in such techniques. These techniques provide the necessary expertise and confidence to designers and their governing manufacturers, who have to take responsibility for guaranteeing efficient operation of their products, to promote and guarantee the application of butterfly valves in such applications. The overall concepts of product design and structural proportions have thereby been redefined and reviewed for the valve components and the possibility of using working stresses much closer to the yield point of the material, as in modern methods of designing other structures. This has consequently reduced the cross-sectional dimensions and weights of valves eventually. The reduction in size of the structural proportions of valve components is seen in the valve industry as the greatest single factor which can tilt the balance in favour of a better market share for products designed in this way. Ref. (11) .

1.2. Description of the Structure of Main Interest.

The butterfly valve essentially comprises a disk or blade which can rotate at right angles in a wing-like structure, about a diametrical axis within a pipe section body. A 90° rotation of the disk opens or closes the valve.

This basic simplicity provides a compact structure which has the least amount of body metal of any valve type, having few component parts (disk, shaft, body, seating seals and operating equipment).

Fully open the valve disk is the only obstruction which occupies the minimum amount of flow line space and causes very little head loss across the valve Ref. (18).

The valves are usually either resilient or metal to metal seated; the resilient seating may be in the body or attached to the periphery of the disk.

Butterfly valves are available in diameters from 2.5 cm to to 10 m and even larger sizes are presently being considered for tidal and ocean thermal energy conversion schemes Ref. (2) for pressures that range from vacuum to 300 psi full differential, depending on the valve size.

The 90° turn offers quick opening or closing with ease of operation and the pressure shut-off capabilities require modest actuator power compared to other types of valves, since the butterfly valve is balanced in the closed position against upstream and downstream line pressures. Dynamic flow torque does not reach a maximum until somewhere between 30° to 75° open. Thus the peak dynamic flow torque does not occur at the same time as the higher torque which is required to unseat the valve. Ref. (19).

Butterfly valves can be installed in systems with constant-head sources such as nearby reservoirs or in pumping systems where pressure falls off with valve opening. During the closing movement the rate of cut-off of the flow diminishes as the disk moves towards the closed position, making the valve well suited for flow regulation purposes, (e.g. throttling).

1.2.1. The Disk or Blade

The disk or blade of the valve is the most important component of the valve, as its configuration has the greatest effect on:-

(i) the head loss across the valve,
(ii) the torque applied on the shafts,
(iii) cavitation of the system,
(iv) the sealing efficiency,
(v) the noise.

The blade could be a simple flat disk of constant thickness Fig. (1.2) A Disk, or a solid or hollow tapered disk, Fig. (1.3) A Tapered Disk, or a lattice supported disk. As the size of the shaft varies according to the torque applied, the connection between the shaft and the blade varies from;
shaft through the disk, Fig. (1.4), two shafts fitted in humps on the disk Fig. (1.5), and shafts as integral casting of the disk Fig. (1.6) and (1.7).

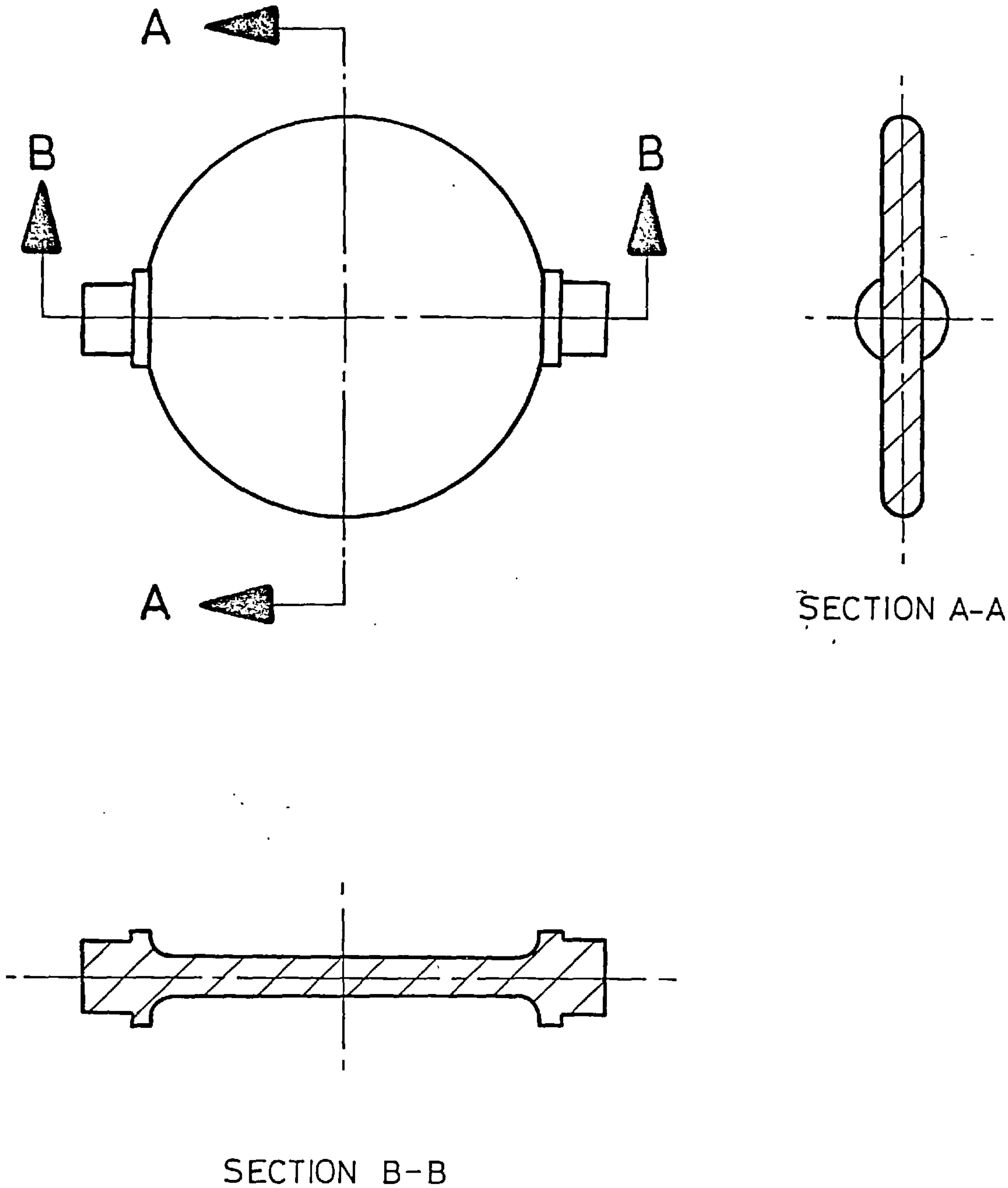


Fig. 1.2. A Disk of Constant Thickness

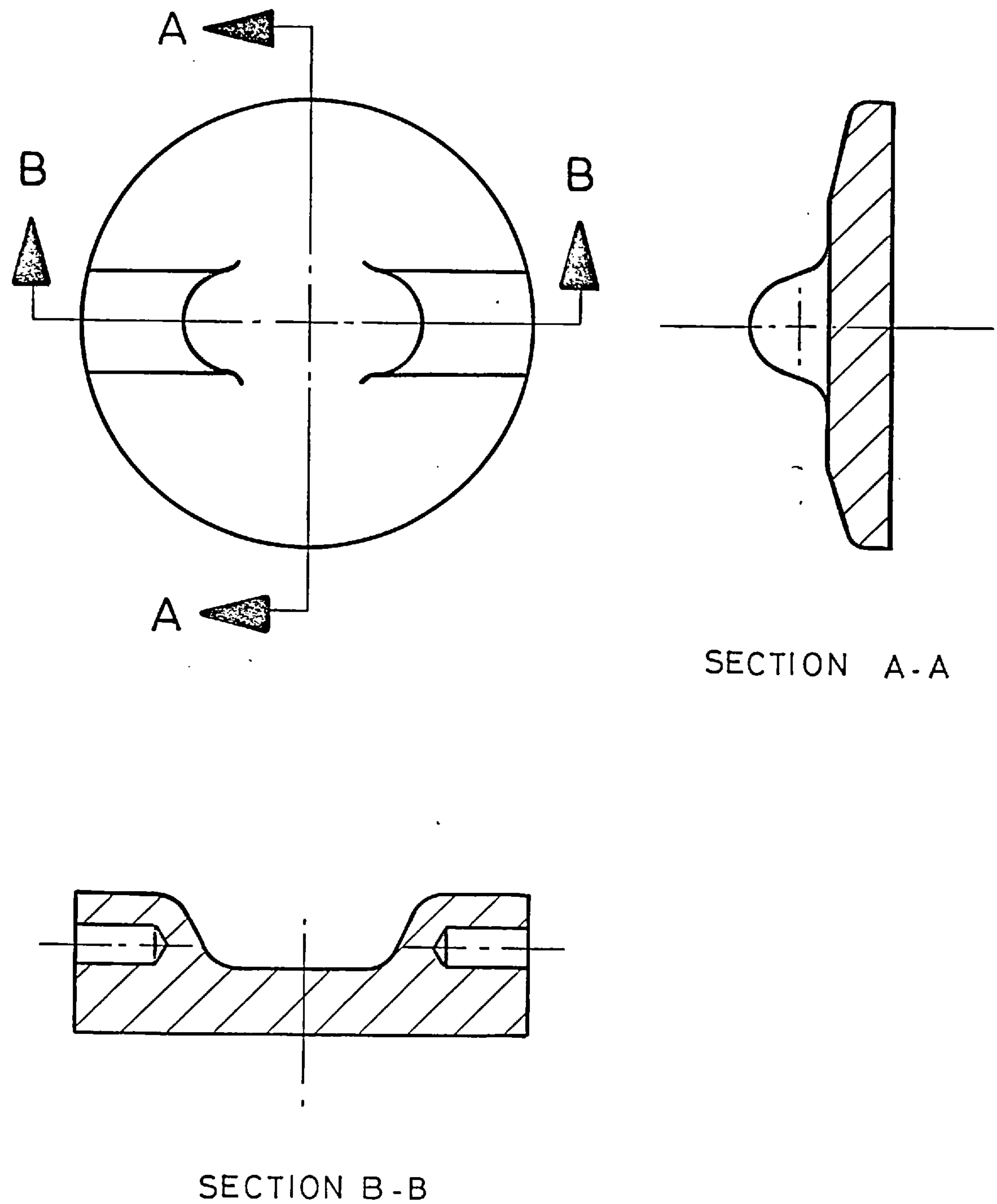


Fig. 1.3a. A Tapered Disk

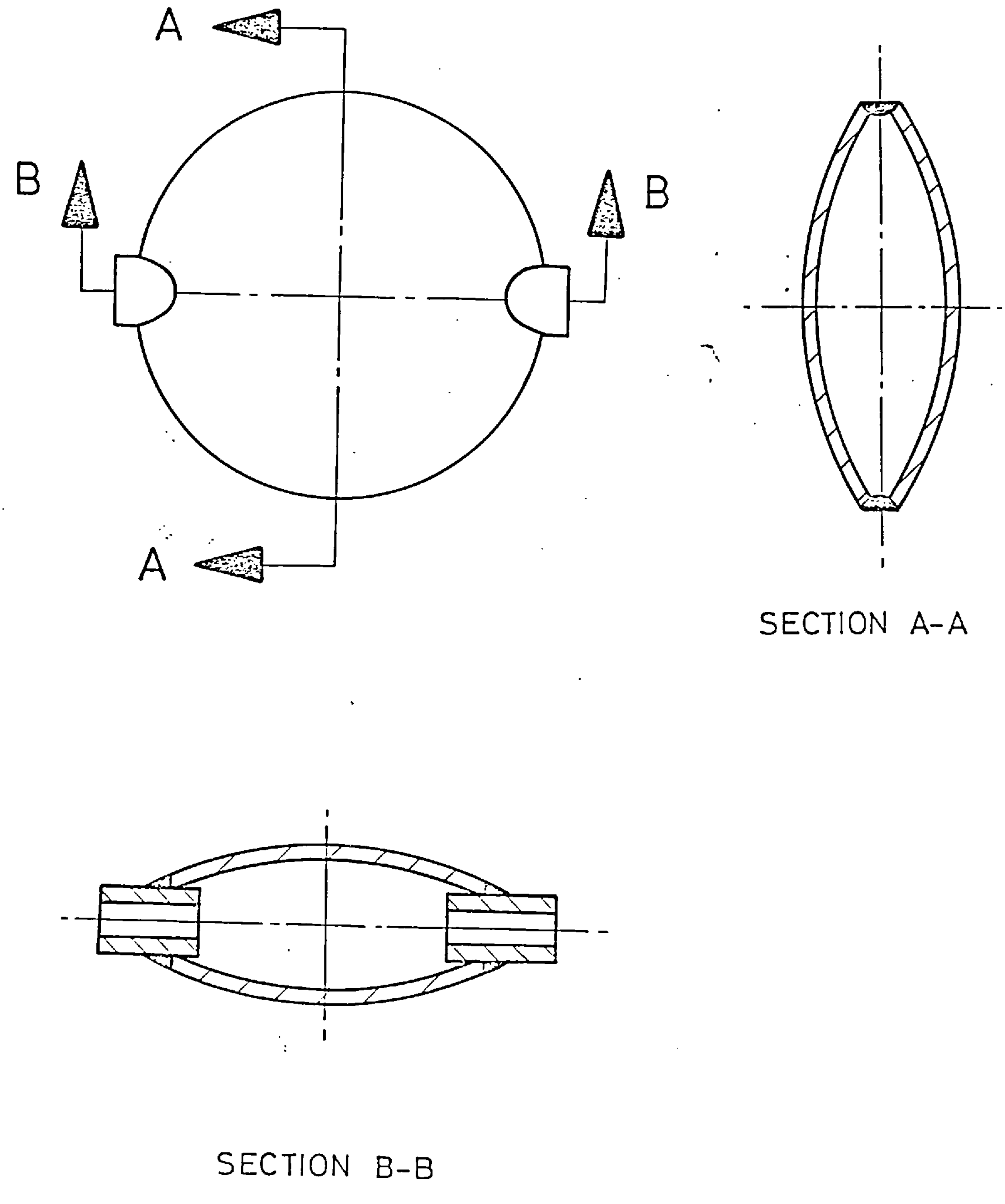


Fig. 1.3b. A Hollow Tapered Disk

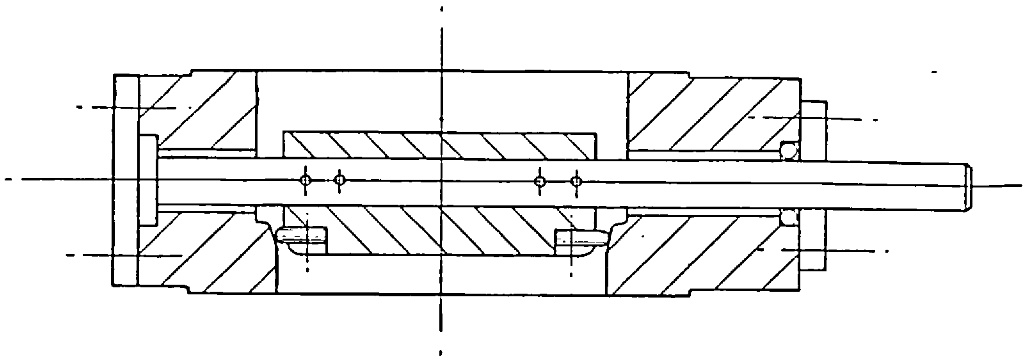


Fig. 1.4: Shaft through the disk

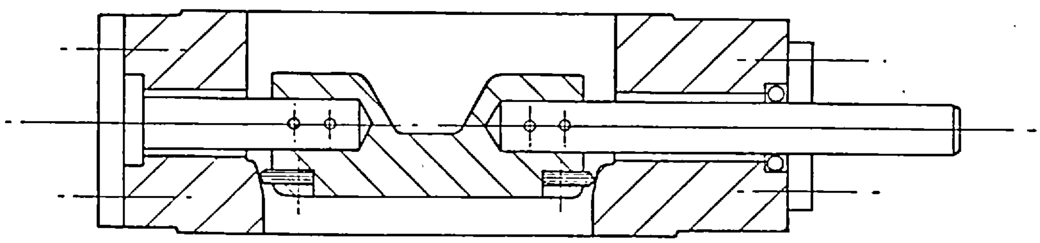


Fig. 1.5: Two shafts fitted in humps

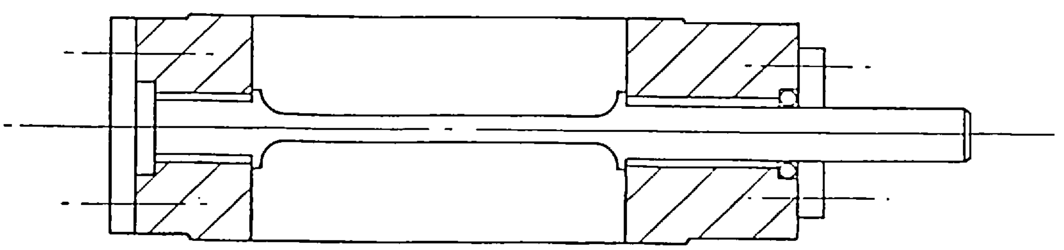


Fig. 1.6: Shafts as integral casting of the disk

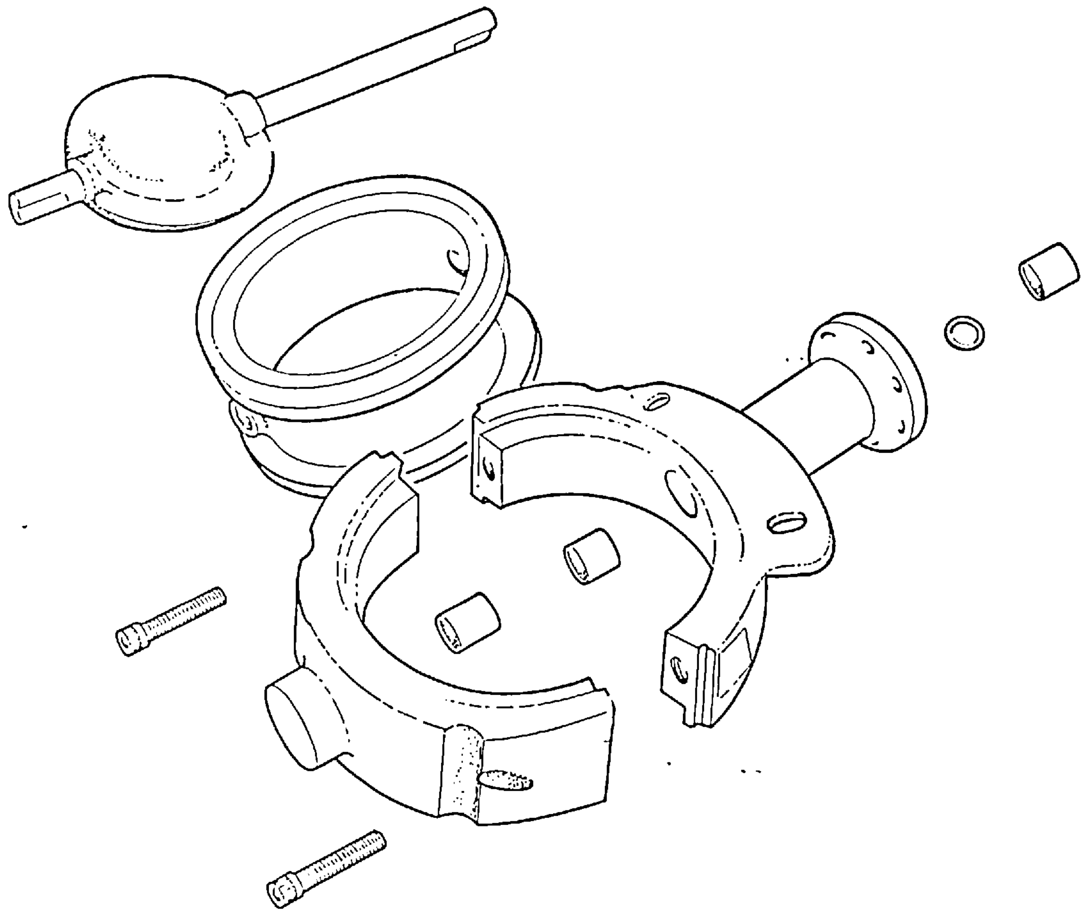


Fig. 1.7: Components of butterfly valve

1.2.2. The Shaft

The governing factor in the design of the shaft is the torque applied to it. The torque analysis of butterfly valves is a major subject which has received a fair amount of research and development in the past and has therefore not been tackled in this work.

One of the most significant observations in this study is the effect of the type of connection between shaft and blade or shaft and body, on the structural behaviour of the blade. No attempt to establish a criterion is made but it is left to the designer to assume the most suitable idealisation for the connection because this depends on working practices, allowable clearances, bearing arrangements and materials and empirical assumptions established by different design methods.

1.2.3. The Body

The body is the second most important component of the valve and it divides the valves mainly into two categories Ref. (2) .

Wafer: this is a valve for clamping between pipe flanges, using through bolting. The body may be single flange, flangeless or "U" section, as illustrated in Figs. (1.8), (1.9) and (1.11) respectively.

Double flanged: this is a valve having flanged ends for connection to pipe flanges by individual bolting Fig.(1.10). A wafer type with "U" section body will also come within this category if it is suitable for the

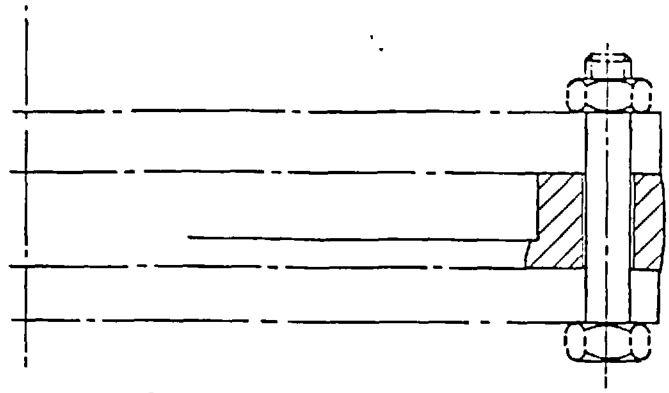


Fig. 1.8: Single flange body

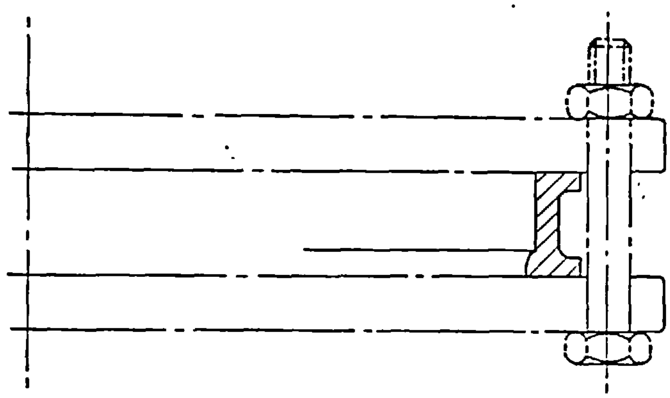


Fig. 1.9: U"U flangeless body

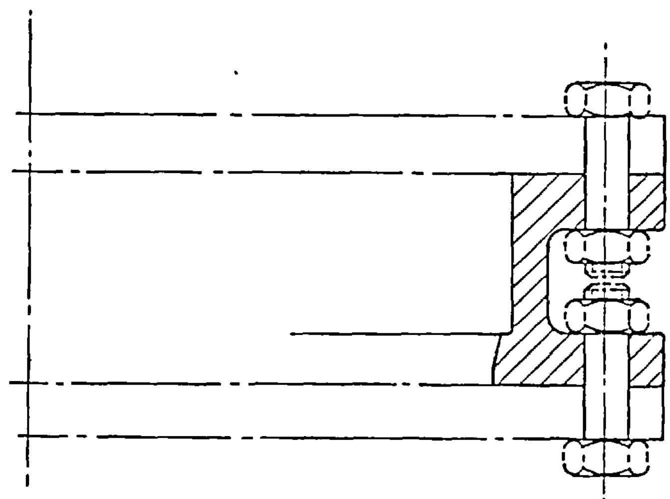


Fig. 1.10: Double flanged body

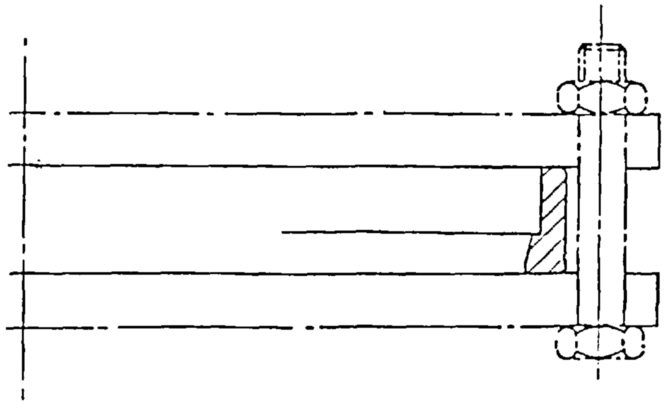


Fig. 1.11: Flangeless body

individual bolting of each flange to the pipework. In smaller valves the body could be in two sections. Fig. (1.7).

An unconventional body is the body of a 4-way butterfly valve which is the intersection of two pipes with an elliptical blade. The design of the body is governed by the piping system and the flange design. It is mainly a ring with two holes where a concentrated load causing shear, with or without correct installation and alignment, is another design factor to ensure non-interference between the blade and the piping system Ref. (19) . The design of the body is not part of this work.

1.2.4. Seating Seals

These are either resilient or metal to metal seated. The latter will normally provide longer life than resilient seated designs but are more difficult to make completely leak-tight. They are better suited for higher temperature duties as they do not have the temperature restrictions imposed by resilient seatings. Regarding the former, an extensive range of synthetic elastomer and plastic materials for use as resilient seatings have been developed. Positive shut-off with repeatability of performance is assured and the wide choice of seating materials provides the butterfly valve with a comprehensive range of service applications.

The mechanical interference between the metal and the elastomeric seat requires a torque to unseat the valve. This depends on the type of seat material, the seat shape and the shape of the disk edge profile. Without the interference, of course, the valve could not be made leak-tight against high line pressure. The design of seats for sealing is not part of this study.

1.2.5. Operating Equipments

These generally comprise a gear box connected to the shaft of the blade at one end (or both ends) coupled with a manual and/or electrical/hydraulic actuator. These equipments are very important, especially in large installations, for the safety and control of any process. They can be controlled from a console in the hall of the plant and at the moment are undergoing great developments, introducing microelectronics processors as detectors and remote control systems, which will make a great change in this field. Such developments are not covered by this study.

1.3 Review

Engineering literature in the English language on the topic under discussion is very sparse. There is only one book Ref. (12) , on the design of valves, to the knowledge of the writer and this does not mention butterfly valves at all.

Hydraulic handbooks Ref. (13), (14), (15), & (16), contain a few brief references to the butterfly valve, describing its geometrical shape, the characteristics of special installations and methods of operating the valve. They make no reference at all to the structural design of such valves.

Again the butterfly valve is described in most handbooks Refs. (2), (3) & (17), on choosing valves in a minor way with no mention at all of structural analysis.

Reviewing existing books on the design of turbines, pumps and piping systems, Ref. (15), it may be observed that the valve is considered mainly in respect of its hydraulic characteristics with very brief guidance to methods of calculating torque on shafts and no guidance is given about structural analysis or design.

The British Hydromechanics Research Association have published, Ref. (20) for their members a bibliography on butterfly valves, covering the period from 1929 to 1966. This has been reviewed by the present writer, who subsequently extended the bibliography to cover the years from 1967 to the end of 1979, covering all known relevant engineering publications in this field.

Nowhere in all that literature is there to be found any study about the structural analysis of the valve components generally or on the blade or disc of the valve, which is the component whose dimensions and strength are of most importance in the overall design of the valve.

Reviewing structural analysis handbooks for the analysis of rings, circular, and other plates, Ref. (33),(34), it became evident that there is a great need to develop a body of knowledge with which to guide designers and structural analysisists devoted to this task. Such a body of knowledge has been developed by the writer and is described in Chapter 2.

In considering new anisotropic materials and the more complex shapes of butterfly valve disc now required, the need to develop finite element programs for the purpose became evident. This has not been done before and has therefore been developed by the writer, as described in Chapters 3, 4, 5 and 6.

1.4. Materials

Valves are manufactured from a wide range of materials chosen to meet different operating requirements such as high or low temperature, corrosion, impact of particles carried in the flow, vibration characteristics, etc., subject to their availability. The materials to be discussed in this thesis are metallic alloys, glass reinforced plastic and perspex (for the experimental work). It is known that blades are made of polymers and fibres other than glass for the reinforcement of the matrix material with the result that Young's Modulus and Poisson's ratio can have virtually any value. In the analytical work (to be discussed in Chapter 2) Poisson's ratio has been varied from 0.05 to 0.5 to allow the designer to consider all possible alternative materials. In glass reinforced plastic Poisson's ratio is in the vicinity of 0.1, for polymers it is 0.35 - 0.4 and for metals it is 0.25 - 0.35.

1.5. Limitations and Scope of Study

The study is concerned with the structural analysis of the blades of butterfly valves, simulation of the loading applied and all possible boundary conditions occurring at the supports. It does not cover the very important subjects listed below since there is already an extensive literature in existence covering them.

1. Hydraulic analysis.
2. Torque analysis and design of shafts. Ref. (21) & (22).
3. Cavitation analysis. Ref. (23) & (24).
4. Analysis and design of the valve body. Ref. (25).
5. Noise analysis. Ref. (2) & (26).
6. Design of sealing, interference provisions and methods of installation and alignment. Ref. (27).
7. Vibration analysis. Ref. (28).
8. Temperature effects. Ref. (29).

CHAPTER TWO

ANALYTICAL PROCEDURE FOR THE STRUCTURAL ANALYSIS OF THE BLADES OF ORDINARY BUTTERFLY VALVES

CHAPTER TWO

ANALYTICAL PROCEDURE FOR THE STRUCTURAL ANALYSIS OF THE BLADES OF ORDINARY BUTTERFLY VALVES

The search of an approximate "closed-form" solution for estimating the deflections and stresses in such blades is based on the general solution Equation (2.9) given by Clebsch Ref. (1) to be derived later. Now:

$$w(r,\theta) = w_p + w_c \tag{2.9}$$

in the governing differential equation of a circular plate, namely:

$$\nabla^4 w = \frac{q(r,\theta)}{D} \tag{2.8}$$

which has to be solved with suitably defined boundary conditions and loading.

In Equation (2.9) w_p represents a known particular integral solution of the governing differential equation which is added to a complementary function w_c which is the solution of the simple (Poisson) homogeneous equation

$$\nabla^4 w_c = 0 \tag{2.10}$$

to satisfy the case under consideration.

The procedure is developed in detail as follows

2.1 The Governing Differential Equation for Circular Plates

To obtain the differential equation of a circular plate the starting point is to use the expressions for bending moment and transverse shear which are known for a cartesian system of co-ordinates, as illustrated in Fig. (2.1) (from Szilard Ref. (31)).

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) \qquad 2.1a$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) \qquad 2.1b$$

$$M_{xy} = M_{yx} = -(1-\nu)\, D \frac{\partial^2 w}{\partial x \partial y} \qquad 2.1c$$

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} \qquad 2.1d$$

$$Q_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} \qquad 2.1e$$

In the polar co-ordinates system shown in Fig. (2.2) the relationships between cartesian and polar co-ordinates are as follows:-

$$x = r \cos \theta \qquad 2.2a$$

$$y = r \sin \theta \qquad 2.2b$$

$$r = \sqrt{x^2 + y^2} \qquad 2.2.c$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \qquad 2.2d$$

The derivatives of $w(r,\theta)$ with respect to x can be derived from the derivatives with respect to r & θ as follows:-

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial w}{\partial \theta}\frac{\partial \theta}{\partial x} \qquad 2.3$$

$$\frac{\partial r}{\partial x} = \cos\theta \quad \text{and} \quad \frac{\partial \theta}{\partial x} = -\frac{1}{r}\sin\theta \qquad 2.4$$

$$\frac{\partial r}{\partial y} = \sin\theta \quad \text{and} \quad \frac{\partial \theta}{\partial y} = +\frac{1}{r}\cos\theta \qquad 2.5$$

The expressions for internal moments and shear forces in Equation (2.1) can be converted into polar co-ordinates using Equations (2.2, 2.3, 2.4 & 2.5) thus,

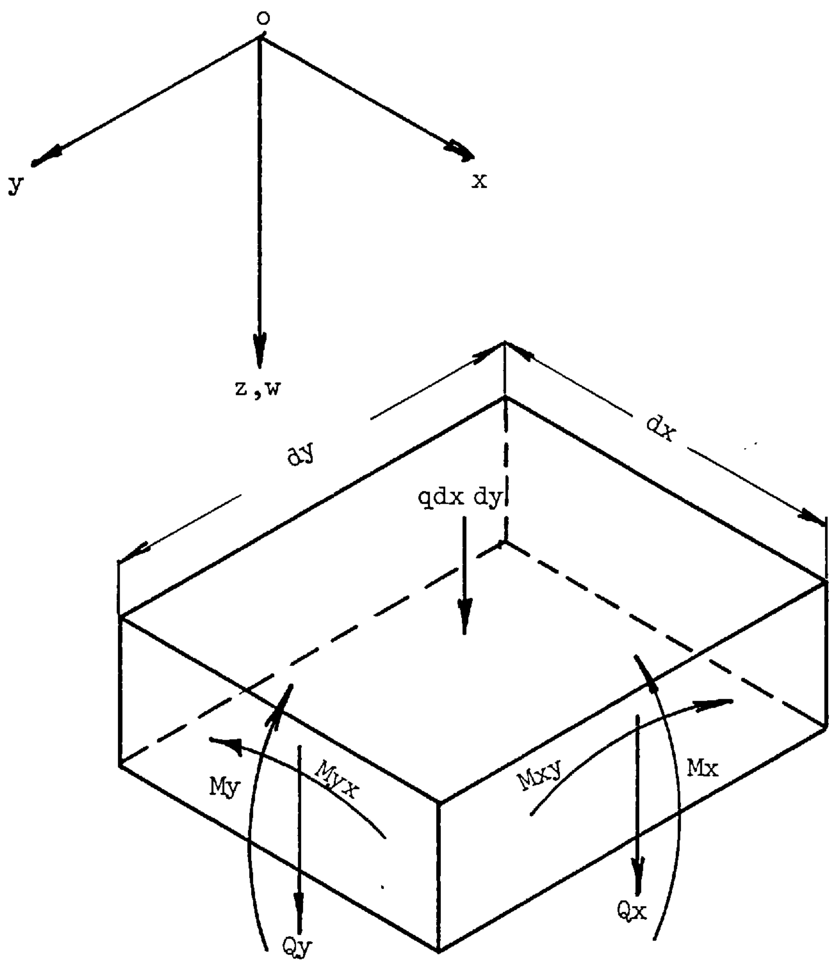


Fig. 2.1: Expressions for bending moments and transverse shear in cartesian coordinates.

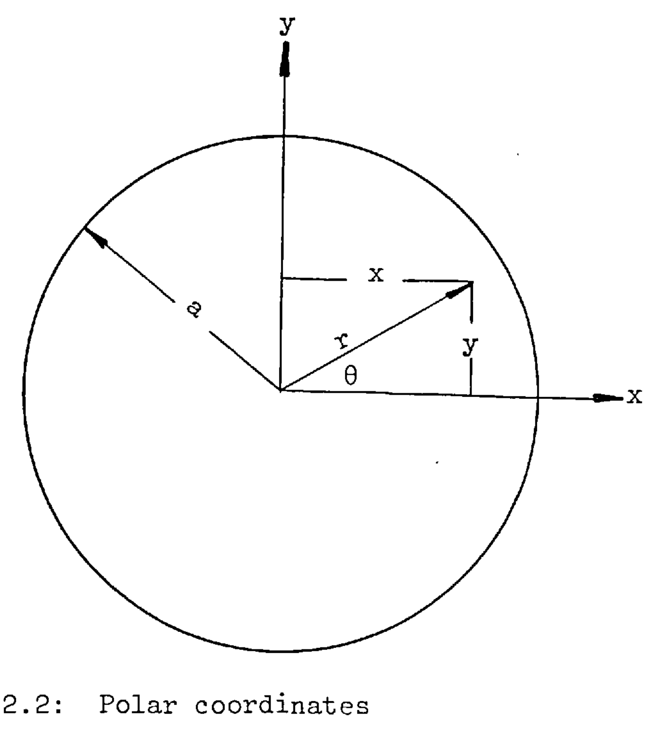


Fig. 2.2: Polar coordinates

$$M_r = -D\left[\frac{\partial^2 w}{\partial r^2} + \nu\left(\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right)\right] \qquad 2.6a$$

$$M_t = -D\left[\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \nu\frac{\partial^2 w}{\partial r^2}\right] \qquad 2.6b$$

$$M_{rt} = M_{\theta r} = -(1-\nu)\, D\left[\frac{1}{r}\frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial w}{\partial \theta}\right] \qquad 2.6c$$

$$Q_r = -D\frac{\partial}{\partial r}\nabla_r^2 w \qquad 2.6d$$

$$Q_\theta = -D\frac{1}{r}\frac{\partial}{\partial \theta}\nabla_r^2 w \qquad 2.6e$$

Considering an infinitesimally small plate element in the polar co-ordinates of Fig. (2.3) summing the moments (with correct signs) and neglecting moments due to the external load on the element as small quantities of higher order, we obtain the equilibrium equation in the 'r' direction of the element as:-

$$(M_r + \frac{\partial M_r}{\partial r}dr)\,(r + dr)\,d\theta - M_r\,r\,d\theta - M_\theta\,dr\,d\theta$$

$$+ Q_r\,r\,d\theta\,dr = 0 \qquad 2.7a$$

hence

$$M_r + \frac{\partial M_r}{\partial r}r - M_\theta + Q_r = 0 \qquad 2.7b$$

Substituting Equation (2.6) for $M_r - M_\theta$ and Q_r and taking the derivatives with respect to r, Equation (2.7b) becomes

$$\nabla_r^2\,(\nabla_r^2 w) = \frac{q\,(r,\theta)}{D} \qquad 2.8$$

where

$$q(r,\theta) = \frac{\partial Q_r}{\partial r}$$

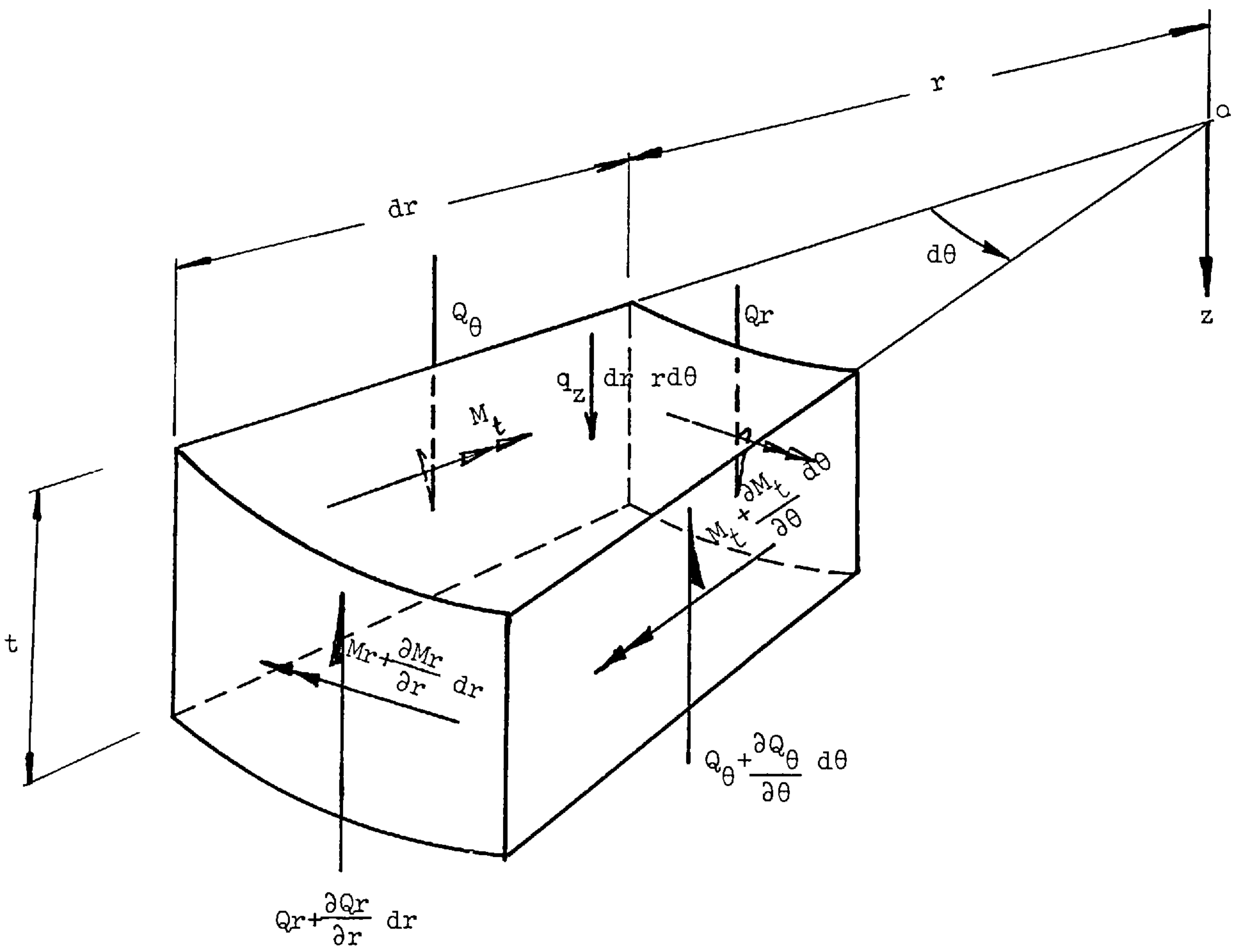


Fig. 2.3: Plate element in polar coordinates

and

$$\nabla_r^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

Equation (2.8) is the governing differential equation for a thin circular plate of isotropic material.

2.2. The General Solution of the Governing Differential Equation

The general solution of this biharmonic equation

$$\nabla_r^4 \; w = \frac{q(r,\theta)}{D} \qquad 2.8$$

can be taken as the sum of the two functions on the right-hand side of this equation:-

$$w(r,\theta) = w_p + w_c \qquad 2.9$$

in which w_p is a known particular integral solution of Equation (2.8) and w_c is a compl mentary function which is the solution of the homogeneous equation

$$\nabla_r^4 \, w_c = 0 \qquad 2.10$$

The solution of Equaticn(2.10) for a circular plate was obtained by Clebsch Ref. (30) in the form of the following series:

$$w_c = F_o + \sum_{n=1}^{\infty} F_n \cos n\theta + \sum_{n=1}^{\infty} F'_n \sin n\theta \qquad 2.11$$

Where each term of the series is a harmonic of order n. Each summation is for a particular value of n and extends over as many terms as are necessary for a proper representation of the loading.

The F_1, F_2.. F_n are functions of r only, which is the radial distance to any point on the disc Fig. (2.2), and they represent the symmetric components of the loading system. The F'_1, F'_2.. F_n are functions of r only and apply to the antisymmetric components of the loading system. F_o is a function of r only and is completely independant of the angle θ unlike the other functions which are symmetric or antisymmetric. Substituting these series into Equation (2.10) leads for each of these functions to an ordinary differential equation of the following kind.

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{n^2}{r^2}\right)\left(\frac{\partial^2 Fn}{\partial r^2} + \frac{1}{r}\frac{\partial Fn}{\partial r} - \frac{n^2 Fn}{r^2}\right) = 0 \qquad 2.12$$

the solutions of which are:

$$F_o = C_{10} + C_{20}\, r^2 + C_{30}\, \mathrm{Ln}\,\frac{r}{a} + C_{40} \quad r^2 \mathrm{Ln}\,\frac{r}{a} \text{ (for } n = 0)$$

$$F_1 = C_{11}\, r + C_{21}\, r^3 + C_{31}\, r^{-1} + C_{41} r\, \mathrm{Ln}\,\frac{r}{a} \quad \text{(for } n = 1) \qquad 2.13$$

\- - - - - - - - -

\- - - - - - - - -

\- - - - - - - - -

The general solution of n > 1 is therefore

$$F_n = C_{1n}\, r^n + C_{2n}\, r^{2+n} + C_{3n}\, r^{-n} + C_{4n}\, r^{-n+2}$$

Similar expressions can be written for the functions F'_n. Substituting these expressions for the functions F_n and F'_n into the series Equation (2.11) we obtain the general solution of Equation (2.10). The constants C_{1n}, C_{2n}, C_{3n} and C_{4n} in each particular case must be determined so as to satisfy the given boundary conditions.

2.3. The Solution for Stresses and Deflections in a Circular Plate of Uniform Thickness Subjected to a Uniform Normal Pressure and Supported at two points at Opposite ends of a Diameter.

2.3.1. Analytical Work

In this case a circular disc of uniform thickness "t" is assumed, subjected to a uniform normal pressure "q" and supported at two points at opposite ends of a diameter.

The reaction of this loading, which develops at each of the support points (trunnions in the case of a valve blade), depends on the degree of fixity of the shafts, as will be discussed later.

If the support reaction is simulated in this analysis by a triangularly distributed load (illustrated in Fig. (2.4a)) which is replaced by a reaction R acting at the centroid of the triangle so that the point of simple support is assumed to be at a distance a + a' from the centre of the disc Fig. (2.4b).

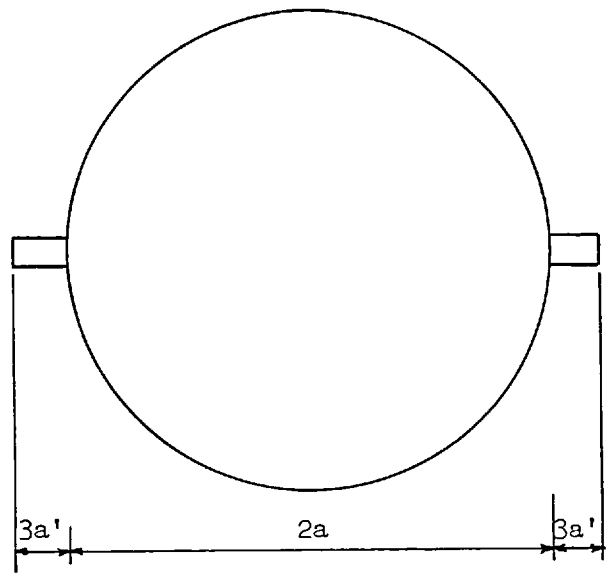
This system of loading can then be replaced by two systems, the effects of which are added together, as follows:

(i) Two diametrically opposed moments as illustrated in Fig. (2.4c) with

$$M = R\,a' \qquad 2.14$$

(These moments act in a plane perpendicular to the disc and pass through the points of support).

(ii) A uniformly distributed load of intensity q acting all over the area of one face of the circular disc, supported by two diametrically opposed equal reactions R acting at the periphery of the disc Fig. (2.4d) where



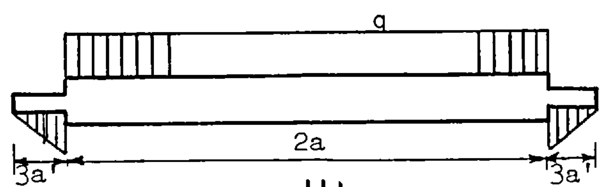


Fig. 2.4a

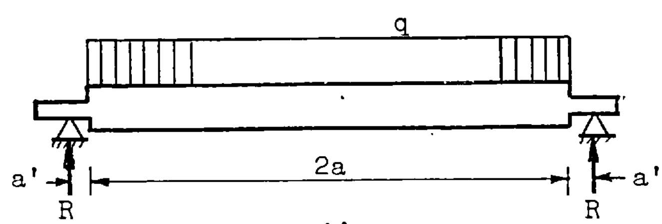


Fig. 2.4b

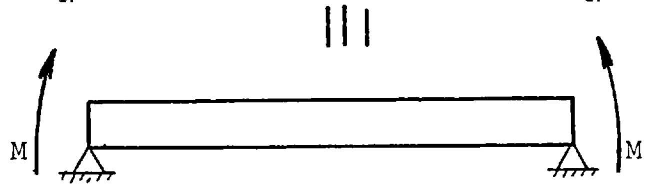


Fig. 2.4c

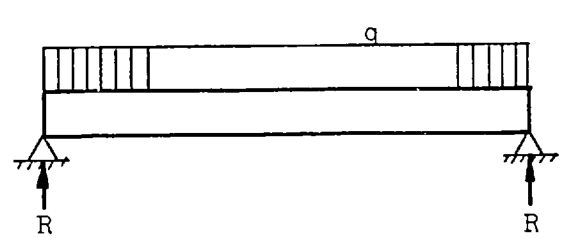


Fig. 2.4d

Fig. 2.4: Circular plates supported at two points

$$R = q\,\frac{\pi a^2}{2} \qquad 2.15$$

This loading occurs only when the valve is totally closed of course.

The effect of the first case of loading (pure bending) on a plate of non-uniform section Fig. (2.5) will now be considered.

The radial moment per unit length of any section can be obtained by dividing M (given by Equation (2.14)) by the appropriate chord length. The deflections along the diameter between the supports can be found as follows:-

$$\frac{\partial^2 w}{\partial x^2} = -\frac{M}{EI} \quad \text{(as given by Szilard Ref. (31))} \qquad 2.16$$

where

$$I = \frac{2yt^3}{12(1-\nu^2)} \qquad 2.17$$

Substituting Equation (2.17) in Equation (2.16) gives:

$$\frac{\partial^2 w}{\partial x^2} = \frac{-6(1-\nu^2)}{Et^3}\,\frac{M}{y} \qquad 2.18$$

Since $x = a \cos\theta$ and $y = a \sin\theta$, (from Equations (2.2a) and (2.2b)).

$$dx = -a \sin\theta \, d\theta.$$

Integrating Equation (2.18) with respect to x leads to

$$\int \frac{\partial^2 w}{\partial x^2}\,dx = C_1 + \frac{6(1-\nu^2)M}{Et^3}\int_{\frac{\pi}{2}}^{\theta} \frac{a \sin\theta \, d\theta}{a \sin\theta}$$

$$\frac{\partial w}{\partial x} = C_1 + \frac{6(1-\nu^2)M}{Et^3}\int_{\frac{\pi}{2}}^{\theta} d\theta$$

$$= C_1 - \frac{6(1-\nu^2)M}{Et^3}\left(\frac{\pi}{2} - \theta\right) \qquad 2.19$$

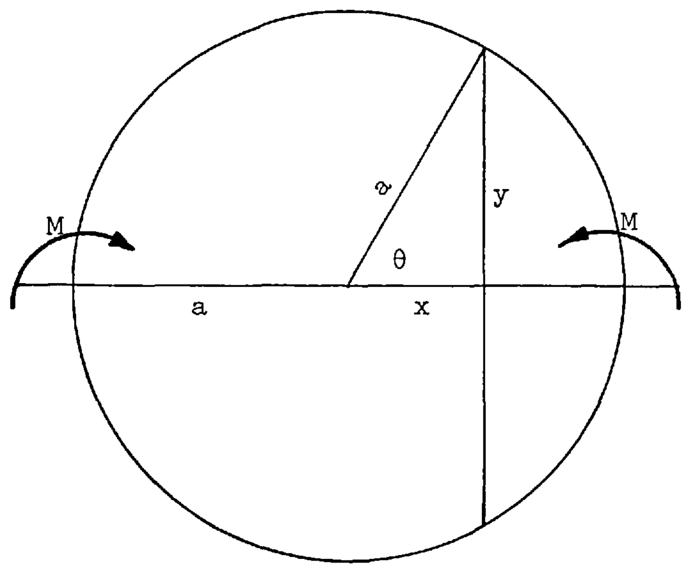


Fig. 2.5: Pure bending on circular plate supported at two points

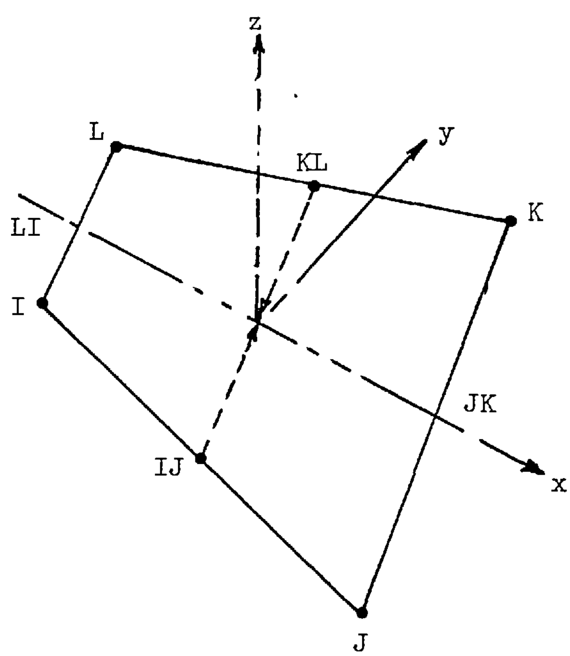


Fig. 2.6: Quadratic plate finite element

At $\theta = \frac{\pi}{2}$, $\frac{\partial w}{\partial x} = 0$ so C_1 must be zero

Integrating Equation (2.19) along x,

$$w = C_2 + \frac{6(1-\nu^2)M}{Et^3} a \int_{\frac{\pi}{2}}^{\theta} (\frac{\pi}{2} - \theta) \sin\theta \, d\theta$$

$$= C_2 + \frac{6aM(1-\nu^2)}{Et^3} \left[(-\frac{\pi}{2} \cos\theta + \theta\cos\theta - \sin\theta) \right]_{\frac{\pi}{2}}^{\theta}$$

$$= C_2 - \frac{6aM(1-\nu^2)}{Et^3} \left[-1 + (\frac{\pi}{2} - \theta)\cos\theta + \sin\theta \right] \qquad 2.20$$

At $\theta = 0$, $w = 0$ thus

$$C_2 = \frac{6aM(1-\nu^2)}{Et^3} (\frac{\pi}{2} - 1) = \frac{Ma}{2D} (\frac{\pi}{2} - 1)$$

and

$$w = \frac{Ma}{2D} \left[\frac{\pi}{2} - (\frac{\pi}{2} - \theta)\cos\theta - \sin\theta \right] \qquad 2.21$$

Maximum w occurs at $\theta = \frac{\pi}{2}$ so that

$$w_{max} = \frac{Ma}{2D} (\frac{\pi}{2} - 1) = 0.28548 \frac{Ma}{D} \qquad 2.22$$

The deflection of the disc at any point due to the second case of loading, Fig (2.4d) can be determined as the sum of the deflection of the disc simply supported along its entire periphery (as a particular solution) and the deflection due to a system of forces distributed along it periphery to satisfy the loading of Fig (2.4d) with the boundary conditions of the disc being also fulfilled.

In what follows the complementary function and the particular integral have been solved together and dealt with in the various derivations as the single function w of Equation (2.8).

In Equation (2.9) w_p (the particular integral solution for the deflection of a disc simply supported along its entire periphery) given by Timoshenko et al. Ref. (32) as

$$w_p = \frac{qa^4}{64D} (1-\rho^2) \left(\frac{5+\nu}{1+\nu} - \rho^2\right) \qquad 2.23$$

where

$$\rho = \frac{r}{a} \qquad 2.24$$

This is a symmetrical case of loading and deflections are measured from the diameter on which the two supports are positioned, i.e. the condition of geometrical symmetry is satisfied so that sin terms of the series Equation (2.11) may be omitted and the complimentary function may be written as

$$w_c = F_o + \sum_{n=1}^{\infty} F_n \cos n\theta \qquad 2.25$$

Substituting Equation (2.25) into Equation (2.10) and solving we obtain Equation (2.13) which must be satisfied by the constants C_{1n}, C_{2n}, C_{3n} and C_{4n} for this case. Since, at the centre of the plate the deflection, the slope and the internal moment are finite, at r = 0

$$\begin{array}{ccccc} C_{30} & = & C_{40} & = & 0 \\ C_{31} & = & C_{41} & = & 0 \\ - & & - & & - \\ - & & - & & - \\ C_{3n} & = & C_{4n} & = & 0 \end{array} \qquad 2.26$$

and Equation (2.13) can be written as

$$F_n = \quad (C_{1n}\, r^n + C_{2n}\, r^{2+n}). \qquad 2.27$$

Using Equation (2.25) and multiplying the first part by

$\frac{a^n}{a^n}$ and the second part by $\frac{a^{n+2}}{a^{n+2}}$ there results the equation

$$F_n = \quad (C^1_{1n} + C^1_{2n}\, \rho^2)\rho^n$$

where

$$C^1_{1n} = a^n C_{1n} \text{ (a new constant)}$$

and

$$C^1_{2n} = a^{n+2} C_{2n} \text{ (a new constant)}$$

Substituting these into Equation (2.25) and multiplying by

$\frac{D}{qa^4}$ gives:

$$\frac{D}{qa^4} w_c = \sum_{n=o}^{\infty} (A_n + B_n\, \rho^2)\rho^n \cos n\theta \qquad 2.28$$

where A_n and B_n are constants and $\rho = r/a$

At the boundary of the disc the radial bending moment is zero. Assuming the following relationships:-

$$d = a \frac{\partial}{\partial r}$$

$$\frac{\partial}{\partial r} = \frac{d}{a} \qquad 2.29a$$

$$\frac{\partial^2}{\partial r^2} = \frac{d^2}{a^2}$$

and from Equation (2.25) $r = a\rho$

$$\frac{1}{r} \frac{\partial}{\partial r} = \frac{1}{r} \frac{d}{a} = \frac{1}{a\rho} \frac{d}{a} = \frac{d}{a^2\rho} \qquad 2.29b$$

$$d' = \frac{\partial}{\partial \theta} \qquad 2.29c$$

and using Equation (2.29) with Equation (2.6a) gives:

$$M_{r_{\rho=1}} = -D \left[\frac{\partial^2}{\partial r^2} + \nu \left(\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\right] (w)_{\rho=1} + 0$$

$$= -D \left[\frac{d^2}{a^2} + \nu \left(\frac{1}{a^2\rho^2} \quad d'^2 + \frac{1}{a\rho}\frac{d}{a}\right)\right] (w)_{\rho=1} = 0$$

hence

$$M_{r_{\rho=1}} = \frac{-D}{a^2} \left[d^2 + \nu \left(\frac{d}{\rho} + \frac{d'^2}{\rho^2}\right)\right] (w)_{\rho=1} = 0 \qquad 2.30$$

Applying Equation (2.30) with Equation (2.23) gives zeros for w_p and applying Equation (2.30) with Equation (2.28) for w_c gives:

$$B_0 = B_1 = 0 \qquad 2.31$$

and

$$n(n-1)\, A_n + (n+1)\left(\frac{n+2}{\beta}\right) B_n = 0 \qquad 2.32$$

$$n > 1$$

where

$$\beta = \frac{1 - \nu}{1 + \nu}$$

At the supports the deflections are zero, i.e. $w_p = 0$

At $\rho = 1$ for $\theta = 0$ & π

Putting $\theta = 0$, $\rho = 1$ and using Equation (2.31) in Equation (2.28) there results:

$$0 = \sum_{n=2,3}^{\infty} A_n + B_n$$

or

$$A_0 + A_1 + \sum_{n=2,3}^{\infty} (A_n + B_n) = 0 \qquad 2.33$$

Putting $\theta = \pi$, $\rho = 1$ and using Equation (2.31) with Equation (2.28) results in

$$A_0 + A_1 + \sum_{n=2,3}^{\infty} (A_n + B_n) \cos n\pi = 0 \qquad 2.34$$

Solving Equation (2.22) and Equation (2.34) with

$$\cos n\pi = +1 \quad \text{(for even n)}$$

$$\cos n\pi = -1 \quad \text{(for odd n)}$$

gives

$$A_0 = -\sum_{n=2,4}^{\infty} (A_n + B_n) \qquad 2.35$$

and

$$A_1 = -\sum_{n=3,5}^{\infty} (A_n + B_n) \qquad 2.36$$

The reactions R at the supports can be found from the Equations:

$$\frac{R}{\pi a} \left(\frac{1}{2} + \sum_{n=1}^{\infty} \cos n\theta\right)$$

and 2.37

$$\frac{R}{\pi a} \left(\frac{1}{2} + \sum_{n=1}^{\infty} \cos n(\theta - \pi)\right)$$

as given by Timoshenko et al. Ref (32).

The value of the intensity of the vertical reaction at the boundary is the sum of these expressions

$$(V_r)_{\rho=1} = \frac{-R}{\pi a} \left[\frac{1}{2} + \sum_{n=1}^{\infty} \cos n\theta + \frac{1}{2} + \sum_{n=1}^{\infty} \cos n(\theta - \pi)\right] \qquad 2.38$$

since $\cos n(\theta - \pi) = \cos n\theta \cos n\pi$

$= \cos n\theta$ (for even n)

$= -\cos n\theta$ (for odd n)

$$(V_r)_{\rho=1} = \frac{-R}{\pi a} \left(1 + 2 \sum_{2,4}^{\infty} \cos n\theta\right) \qquad 2.39$$

The differential equation for V_r as given by Timoshenko et al. Ref. (32) (p.284) is

$$(V_r)_{\rho=1} = (Q_r - \frac{\partial}{r}\frac{M_{r\theta}}{\partial\theta})_{\rho=1} = 0 \qquad 2.40$$

Using Equation (2.29a)

$$(V_r)_{\rho=1} = (Q_r - \frac{1}{\rho a} d' \, M_{r\theta})_{\rho=1} = 0$$

From Equation (2.6d) we get

$$Q_r = -D \frac{\partial}{\partial r} (\frac{\partial}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}) \; (w)_{\rho=1}$$

$$Q_r = -D \; (\frac{\partial^3}{\partial r^3} + \frac{1}{r}\frac{\partial^2}{\partial r^2} - \frac{1}{r^2}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\frac{\partial}{\partial r} - 2\,\frac{1}{r^3}\frac{\partial^2}{\partial\theta^2})(w)_{\rho=1}$$

Using Equation (2.29a)

$$Q_r = \frac{-D}{a^3} \; (d^3 + \frac{d^2}{\rho} - \frac{d}{\rho^2} + \frac{dd'^2}{\rho^2} - 2\,\frac{d'^2}{\rho^3}) \qquad 2.41$$

From Equation (2.6c)

$$-\frac{1}{\rho a} d' M_{r\theta} = -\frac{D}{a^3}\left[\frac{(1-\nu)}{\rho^2} dd'^2 - \frac{(1-\nu)}{\rho^3} d'^2\right] \qquad 2.42$$

Substituting Equation (2.41) and Equation (2.42) into Equation (2.40) gives:

$$(V_r)_{\rho=1} = \frac{-D}{a^3}\left[d^3 + \frac{d^2}{\rho} - \frac{d}{\rho^2} + \frac{(2-\nu)}{\rho^2} dd'^2 + \frac{(\nu-3)}{\rho^3} d'^2\right](w)_{\rho=1}$$

as $w = w_p + w_c$ 2.43

To carry the solution further we have to obtain the differentation of w_p and w_c from Equations (2.23) and (2.28)

Assuming $\frac{qa^4}{D}$ is a constant = k, then

$$\frac{qa^4}{D} = k \qquad 2.43a$$

$$d(w_p) = a\frac{\partial}{\partial r}(w_p) = a\frac{k}{64}\frac{\partial}{\partial r}\left[(1-\frac{r^2}{a^2})(\frac{5+\nu}{1+\nu}-\frac{r^2}{a^2})\right]$$

$$= \frac{ak}{64}\left[(1-\frac{r^2}{a^2})(-\frac{2r}{a^2})+(\frac{5+\nu}{1+\nu}-\frac{r^2}{a^2})(\frac{-2r}{a^2})\right]$$

$$= (\frac{ak}{64})(\frac{-2r}{a^2})\left[(1-\rho^2)(\frac{5+\nu}{1+\nu}-\rho^2)\right]$$

$$= \frac{-k\rho}{32}(1+\frac{5+\nu}{1+\nu}-2\rho^2)$$

$$= \frac{-k\rho}{32}(\frac{1+\nu+5+\nu}{1+\nu}-2\rho^2)$$

$$= \frac{-k\rho}{32}(\frac{6+2\nu}{1+\nu}-2\rho^2)$$

and

$$d(w_p) = \frac{-k\rho}{16}(\frac{3+\nu}{1+\nu}-\rho^2) \qquad 2.44$$

$$d^2(w_p) = a\frac{\partial}{\partial r}\ \frac{k}{16}\frac{r}{a}(\frac{3+\nu}{1+\nu}-\frac{r^2}{a^2})$$

$$= \frac{k}{16}\frac{\partial}{\partial r}\left[(\frac{3+\nu}{1+\nu})\,r-\frac{r^3}{a^2}\right]$$

$$= \frac{k}{16}(\frac{3+\nu}{1+\nu}-3\rho^2) \qquad 2.45$$

$$d^3(w_p) = a\frac{\partial}{\partial r}\frac{k}{16}(\frac{3+\nu}{1+\nu}-3\frac{r^2}{a^2})$$

$$d^3(w_p) = \frac{k}{16}(-6\frac{r}{a}) = \frac{3k}{8}\rho \qquad 2.46$$

and

$$d'(w_p) = d'^2(w_p) = dd'(w_p) = dd'^2(w_p) = d^2d'(w_p)$$

$$= d'^3(w_p) = 0 \qquad 2.47$$

Similarly, for (n=2,4,...∞) we get

$$d(w_c) = k \sum \left[n\ A_n + (n+2)\ B_n\rho^2\right]\rho^{n-1} \cos n\theta \qquad 2.48$$

$$d^2(w_c) = k \sum\left[n(n-1)\ A_n + (n+2)(n+1)B_n\rho^2\right]\rho^{n-2} \cos n\theta \qquad 2.49$$

$$d^3(w_c) = k \sum\left[n(n-1)\ A_n + (n+2)(n+1)n\ B_n\rho^2\right]\rho^{n-3} \cos n\theta \qquad 2.50$$

and

$$d'(w_c) = -k \sum(n\ A_n + n\ B_n\ \rho^2)\rho^n \sin n\theta \qquad 2.51$$

$$d'^2(w_c) = -k \sum(n^2\ A_n + n^2\ B_n\rho^2)\rho^n \cos n\theta \qquad 2.52$$

$$d'^3(w_c) = k \sum(n^3\ A_n + n^3\ B_n\rho^2)\rho^n \sin \theta \qquad 2.53$$

$$dd'(w_c) = -k \sum\left[n^2\ A_n + n(n+2)\ B_n\rho^2\right]\rho^{n-1} \sin n\theta \qquad 2.54$$

$$dd'^2(w_c) = -k \sum\left[n^3\ A_n + n^2(n+2)\ B_n\rho^2\right]\rho^{n-1} \cos n\theta \qquad 2.55$$

$$d^2d'(w_c) = -k \sum\left[n^2\ (n-1)\ A_n + n(n+2)(n+1)\right.$$

$$\left. B_n\rho^2\right]\rho^{n-2} \sin n\theta \qquad 2.56$$

Using Equations (2.44 to 2.56) in Equation (2.43) for $(w_p + w_c) = 1$ and using Equation (2.31) for B_0, B_1 we get

$$(V_r)_{\rho=1} = \frac{-D}{a^3}\, k \left\langle \frac{3}{8}\,\rho - \frac{1}{16}\left(\frac{3+\nu}{1+\nu} - 3\rho^2\right)\frac{1}{\rho} + \frac{1}{16}\left(\frac{3+\nu}{1+\nu} - \rho^2\right)\frac{1}{\rho}\right.$$

$$+ \sum_{n=2,3}^{\infty} \left\{\left[n(n-1)(n-2) + n(n-1) - n - (2-\nu)n^3\right.\right.$$

$$\left. - (\nu-3)n^2\right]A_n + \left[(n+2)(n+1)n + (n+2)(n+1) - (n+2)\right.$$

$$\left.\left.\left. - (2-\nu)n^2\ (n+2) - (\nu-3)n^2\right]\ B_n\rho^2\right\}\ \rho^{n-3} \cos n\theta \right\rangle_{\rho=1} \qquad 2.57$$

$$(V_r)_{\rho=1} = \frac{-D}{a^3} k \left\langle \frac{\rho}{2} + \sum_{n=2,3}^{\infty} \left\{ -n^2(n-1)(1-\nu)\, A_n + n(n+1) \right.\right.$$

$$\left.\left. \Big[4-n(1-\nu)\Big] B_n \rho^2 \right\} \rho^{n-3} \cos n\theta \right\rangle_{\rho=1} \qquad 2.58$$

$$(V_r)_{\rho=1} = \frac{-D}{a^3} k \left\langle \frac{1}{2} + \sum_{n=2,3}^{\infty} \left\{ -n^2(n-1)(1-\nu)\, A_n + n(n+1) \right.\right.$$

$$\left.\left. \Big[4-n(1-\nu)\Big] B_n \right\} \cos n\theta \right\rangle_{\rho=1}$$

From Equation (2.15) we have $2R = \pi a^2 q$ and $k = \frac{qa^4}{D}$

so

$$\frac{-D}{a^3} k = - \frac{qa^4}{D} \frac{D}{a^3} = - \frac{2R}{\pi a}$$

Using this in Equation (2.58) leads to

$$(V_r)_{\rho=1} = \frac{-R}{\pi a} \left[1+2 \sum_{n=2,3}^{\infty} \left\{ -n^2(n-1)(1-\nu)\, A_n + n(n+1) \right.\right.$$

$$\left.\left. \Big[4-n(1-\nu)\Big]\; B_n \right\} \cos n\theta \right]_{\rho=1} \qquad 2.59$$

The coefficients of cos nθ in Equations (2.59) and (2.39) should be equal, thus

$$-n^2(n-1)(1-\nu)\; A_n + n(n+1)\Big[4-n\; (1-\nu)\Big] B_n = 1$$

$$\text{for } n = 2, 4, 6 \ldots \qquad 2.60$$

and

$$-n^2(n-1)(1-\nu)\; A_n + n(n+1)\Big[4-n\; (1-\nu)\Big] B_n = 0$$

$$\text{for } n = 3, 5, 7 \ldots \qquad 2.61$$

Solving Equations (2.32) and (2.61) gives

$$A_n = B_n = 0 \quad \text{for } n = 3, 5, 7 \ldots \qquad 2.62$$

Hence from Equation (2.36) we get

$$A_1 = 0$$

Solving Equations (2.32) and (2.60) gives

$$A_n = \frac{-\ (n+2/\beta)}{2n^2(n-1)(3+\nu)} \qquad 2.62a$$

or as

$$A_n = \frac{-1}{2(3+\nu)}\left[\frac{1}{n(n-1)} + \frac{1}{\beta n^2(n-1)}\right] \qquad 2.62b$$

Also,

$$B_n = \frac{1}{2n(n+1)(3+\nu)} \qquad 2.63$$

Therefore,

$$A_n + B_n = \frac{-1}{3+\nu}\left[\frac{1}{n(n^2-1)} + \frac{1}{\beta n^2(n-1)}\right] \qquad 2.64$$

Equations (2.62a, 2.62b, 2.63 and 2.64) are for

$n = 2, 4, 6...$ etc.

Using Equation (2.64) in Equation (2.35) gives

$$A_0 = \frac{1}{3+\nu}\sum_{n=2,4}\left[\frac{1}{n(n^2-1)}+\frac{1}{\beta n^2(n-1)}\right]$$

Considering ln series,

$$A_0 = \frac{1}{3+\nu}\left(\ln 2 - \frac{1}{2}\right) + \frac{1}{\beta}\left(\ln 2 - \frac{\pi^2}{24}\right)$$

or

$$A_0 = \frac{1}{3+\nu}\left[\frac{2\ \ln\ 2}{1-\nu} - \frac{\pi^2}{24\beta} - \frac{1}{2}\right] \qquad 2.65$$

Hence, Equation (2.28) since $A_1 = B_0 = B_1 = 0$ can be written as

$$\frac{D}{qa^4} w_c = A_0 + \sum_{n=2,4}^{\infty} (A_n + B_n \rho^2)\rho^n \cos n\theta \qquad (2.28)$$

Using Equations (2.62b) and (2.65) gives

$$\frac{D}{qa^4}\ w_c = \frac{1}{3+\nu}\left[\left\{\frac{2\ \ln 2}{1-\nu} - \frac{\pi^2}{24} - \frac{1}{2}\right\} - \frac{1}{2}\sum_{n=2,4}^{\infty}\left\{\frac{1}{n(n-1)}\right.\right.$$

$$\left.\left. + \frac{2}{\beta n^2(n-1)} - \frac{\rho^2}{n(n+1)}\right\}\rho^n \cos n\theta\right] \qquad 2.66$$

and for the complete expression of deflection

$$w = w_p + w_c \qquad (2.9)$$

$$w = \frac{qa^4}{D}\frac{1}{64}(1-\rho^2)\left(\frac{5+\nu}{1+\nu} - \rho^2\right) + \frac{qa^4}{D}\frac{1}{2(3+\nu)}$$

$$\left[\left\{\frac{4\ \ln 2}{1-\nu} - \frac{\pi^2}{12\beta} - 1\right\} - \sum_{n=2,4}^{\infty}\left\{\frac{1}{n(n-1)} + \right.\right.$$

$$\left.\left.\frac{2}{\beta n^2(n-1)} - \frac{\rho^2}{n(n+1)}\right\}\rho^n \cos n\theta\right] \qquad 2.67$$

or

$$\frac{wD}{qa^4} = \left(\frac{1-\rho^2}{64}\right)\left(\frac{5+\nu}{1+\nu} - \rho^2\right) + \frac{1}{2(3+\nu)}\left\{2\ \ln 2 - 1\right.$$

$$+ \left(\frac{1+\nu}{1-\nu}\right)\left(2\ \ln 2 - \frac{\pi^2}{12}\right) - \sum_{n=2,4}^{\infty}\left[\frac{1}{n(n-1)}\right.$$

$$\left.\left. + \frac{2(1+\nu)}{(1-\nu)(n-1)n^2} - \frac{\rho^2}{n(n+1)}\right]\rho^n \cos n\theta\right\} \qquad 2.68$$

which is the complete solution.

The expression for the radial moment M_r, using Equations (2.44 to 2.56) with Equation (2.30) is

$$M_r = -k\frac{D}{a^2}\left\langle\left[\frac{-1}{16}\left(\frac{3+\nu}{1+\nu} - 3\rho^2\right) - \frac{\nu}{16}\left(\frac{3+\nu}{1+\nu} - \rho^2\right)\right]\right.$$

$$+ \sum_{n=2,4}^{\infty}\left\{\left[n(n-1) + \nu(n-n^2)\right]A_n + \left[(n+2)(n+1)\right.\right.$$

$$\left.\left.\left. + \nu(n+2) - \nu n^2\right]B_n\ \rho^2\right\}\rho^{n-2}\cos n\theta\right\rangle$$

$$= \frac{-Dk}{a^2}\left[\frac{-(3+\nu)}{16}(1-\rho^2) + \sum_{n=2,4}^{\infty}\left\{n(n-1)(1-\nu)\,A_n\right.\right.$$

$$\left.\left. + n(n+1)\left[(1-\nu) + \frac{2}{n}(1+\nu)\right] B_n\,\rho^2\right\}\rho^{n-2}\cos n\theta\right] \qquad 2.69$$

Substituting for k, A_n & B_n from Equations (2.42a), (2.62b) and (2.63) leads to

$$\frac{M_r}{a^2q} = \frac{1}{16}(3+\nu)(1-\rho^2) + \frac{1}{2}\left(\frac{1+\nu}{3+\nu}\right)\sum_{n=2,4}^{\infty}\left(\beta + \frac{2}{n}\right)$$

$$(1-\rho^2)\rho^{n-2}\cos n\theta \qquad 2.70$$

The expression for the tangential moment M_t, using Equation (2.43) to (2.52), Equation (2.29) and Equation (2.6b) is

$$M_t = -k\frac{D}{a^2}\left\langle\left[\frac{-1}{16}\left(\frac{3+\nu}{1+\nu} - \rho^2\right) - \frac{\nu}{16}\left(\frac{3+\nu}{1+\nu} - 3\rho^2\right)\right]\right.$$

$$+ \sum_{n=2,4}^{\infty}\left\{\left[n-n^2 + \nu n(n-1)\right] A_n + \left[(n+2) - n^2\right.\right.$$

$$\left.\left.\left. + \nu(n+2)(n+1)\right] B_n\,\rho^2\right\}\rho^{n-2}\cos n\theta\right\rangle$$

or

$$M_t = -k\frac{D}{a^3}\left[\frac{-1}{16}(3+\nu) - (1+3\nu)\rho^2\right] + \sum_{n=2,4}^{\infty}$$

$$\left\{n(n-1)(1-\nu)\,A_n + n(n+1)\left[(1-\nu)\frac{-2}{n}\right.\right.$$

$$\left.\left.(1+\nu)\right] B_n\,\rho^2\right\}\rho^{n-2}\cos n\theta \qquad 2.71$$

Substituting k, A_n & B_n from Equations (2.42a, 2.61b and 2.62) leads to

$$\frac{M_t}{a^2q} = \frac{1}{16}\left[(3+\nu) - (1+3\nu)\rho^2\right] - \frac{1}{2}\left(\frac{1+\nu}{3+\nu}\right)\sum_{n=2,4}^{\infty}$$

$$\left\{\beta(1-\rho^2) + \frac{2}{n}(1+\rho^2)\right\}\rho^{n-2}\cos n\theta \qquad 2.72$$

The expression for the twisting moment $M_{r\theta}$, using Equations (2.43) to (2.52), Equation (2.29) and Equation (2.6c)

$$M_{rt} = k\,\frac{D}{a^2}\,(1-\nu) \sum_{n=2,4}^{\infty} \left\{(-n^2+n)\,A_n + \left[-n(n+2)+n\right] B_n\,\rho^2\right\} \rho^{n-2} \sin n\theta$$

$$= k\,\frac{D}{a^2}(1-\nu) \sum_{n=2,4}^{\infty} \left\{-n(n-1)\,A_n - n(n+1)\,B_n\,\rho^2\right\} \rho^{n-2} \sin n\theta \qquad 2.73$$

Substituting for k, A_n & B_n from Equations (2.42a, 2.61b and 2.62) leads to

$$\frac{M_{rt}}{a^2 q} = \frac{1}{2}\,\left(\frac{1+\nu}{3+\nu}\right) \sum_{n=2,4}^{\infty} \left\{\beta(1-\rho^2) + \frac{2}{n}\right\} \rho^{n-2} \sin n\theta \qquad 2.74$$

The expression for the radial shear Q_r, using Equations (2.43) to (2.52), Equation (2.29) and Equation (2.6d)

$$Q_r = -\,k\,\frac{D}{a^3} \left\langle \frac{3}{8}\,\rho - \frac{1}{16}\left(\frac{3+\nu}{1+\nu} - 3\rho^2\right)\frac{1}{\rho} + \frac{1}{16}\left(\frac{3+\nu}{1+\nu} - \rho^2\right)\frac{1}{\rho} + \sum_{n=2,4}^{\infty} \left\{\left[n(n-1)(n-2) + n(n-1) - n - n^3 + 2n^2\right] A_n + \left[(n+2)(n+1)n + (n+2)(n+1) - (n+2) - n^2(n+2) + 2n^2\right] B_n \rho^2\right\} \rho^{n-3} \cos n\theta \right\rangle$$

$$Q_r = -\,k\,\frac{D}{a^3} \left[\frac{\rho}{2} + 4 \sum_{n=2,4}^{\infty} n(n+1)\,B_n\,\rho^{n-1} \cos n\theta\right] \qquad 2.75$$

$$\frac{Q_r}{qa} = \frac{-1}{2} - \frac{2}{(3+\nu)} \sum_{n=2,4}^{\infty} \rho^{n-1} \cos n\theta \qquad 2.76$$

The expression for the tangential shear Q_t, using Equations (2.43) to (2.52), Equation (2.29) and Equation (2.6e)

$$Q_t = -k\frac{D}{a^3}\sum_{n=2,4}^{\infty} \left[-n^2(n-1) - n^2 + n^3\right] A_n$$

$$+ \left[-n(n+2)(n+1) - n(n+2) + n^3\right] B_n\rho^2 \quad \rho^{n-3} \sin n\theta$$

$$Q_t = -k\frac{D}{a^3} 4 \sum_{n=2,4}^{\infty} -n(n+1)\, B_n\, \rho^{n-1} \sin n\theta \qquad 2.77$$

$$\frac{Q_t}{qa} = \frac{2}{(3+\nu)}\sum_{n=2,4}^{\infty} \rho^{n-1} \sin n\theta \qquad 2.78$$

In Equation (2.68) the right-hand side would give a constant coefficient for a defined Poisson's ratio ν at a defined location (ρ,θ) on the plate. This coefficient can be converted into a deflection due to the uniformly distributed load by multiplying it by

$$\frac{qa^4}{D}$$

This further lends itself to the development of a hand table of coefficients which is shown in Appendix (1). In Equations (2.70, 2.72 and 2.74) the right-hand side would give a constant coefficient for a defined Poisson's ratio ν at a defined location (ρ,θ) on the plate. This coefficient can be converted into stresses due to the moments by multiplying it by

$$\pm\, 6q\left(\frac{a}{t}\right)^2$$

as the value of the moments acting in the radial and tangential directions, taken with respect to the centre of the plate, are claculated per unit length. Thus in the radial and tangential directions along sections described by $\theta = 0^o$ and $\theta = 90^o$ this procedure produces stresses in the principal directions whilst the radial and tangential moments acting along any other section, as described by $0^o<\theta<90^o$, produce only stresses in the given directions, the twisting moment $M_{r\theta}$ not being zero. Along $\theta = 0^o$ and $\theta = 90^o$ the stresses caused by the couples in Fig. (2.4c) should be added directly to the stresses acting in the radial and tangential direction as appropriate.

In Equations (2.76) and (2.78) the right-hand side would give a constant coefficient for a defined Poisson's ratio at a defined location (ρ,θ) on the plate. These coefficients can be converted into shear stresses by multiplying by

qa

Equations (2.68, 2.70, 2.72, 2.74, 2.76 and 2.78) were written in two small computer programs.

(i) To produce a complete hand table of coefficients for deflections, moment and shears as shown in Appendix (1). A FORTRAN listing of the program is in Appendix (2.A).

(ii) To suit a top desk microcomputer Ref. (36) where the designer interacts directly through the computer screen, feeding in t, a, E, ν, ρ, θ and q in a free format and immediately obtaining the deflection at the point and all the stresses due to moments and shears.

A FORTRAN IV listing of the program is given in Appendix (2.B) This completes the analytical work of this case.

2.3.2. Finite Element Analysis

The first step was the analysis of a circular plate simply supported around the periphery and subjected to a uniformly distributed load which is a well documented case in many texts such as Ref. (31), (32) & (35).

An existing finite element program for the analysis of plates was used, Ref. (38). It employs a quadratic plate element as shown in Fig. (2.6).

A sufficiently fine mesh, as determined by convergence criteria consisting of 160 elements, 184 nodal points per half circular plate was adopted as adequate for this study as shown later and illustrated in Fig. (2.7). The degree of symmetry in this case calls for quarter of a circle but lesser degrees of symmetry, called for later in this work, required the use of half a circle.

A comparison of results for a typical example of this simply supported case is shown in Table (2.1). This leads to the conclusion that the finite element model is adequate and useful for predicting the deflections and stresses in circular plates.

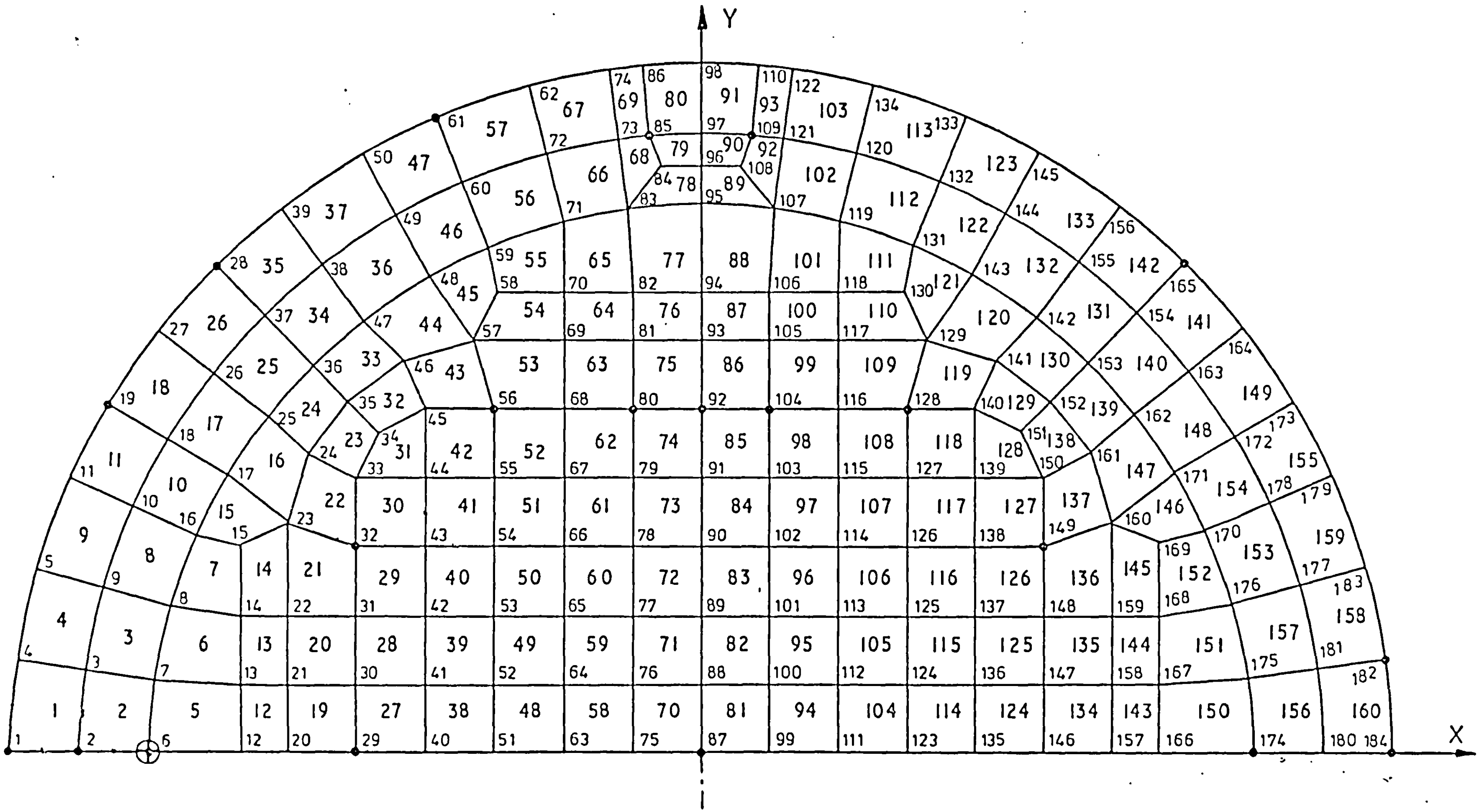


Fig. 2.7. Finite element mesh.

LOCATION		CENTRE POINT	$\rho = 0.5$
DEFLECTION	FINITE ELEMENT	$0.06544 \ \frac{qa^4}{D}$	$0.046147 \ \frac{qa^4}{D}$
	CLOSED FORM SOLUTION EQUATION	$0.065622 \ \frac{qa^4}{D}$	$0.04628 \ \frac{qa^4}{D}$
M_x, M_r	FINITE ELEMENT	$0.2014 \ q\,a^2$	$0.1586 \ q\,a^2$
	CLOSED FORM SOLUTION EQUATION	$0.203125 \ q\,a^2$	$0.1523 \ q\,a^2$
M_y, M_θ	FINITE ELEMENT	$0.2014 \ q\,a^2$	$0.17545 \ q\,a^2$
	CLOSED FORM SOLUTION EQUATION	$0.203125 \ q\,a^2$	$0.17578 \ q\,a^2$

Dimensions of example used for the finite element solution:

$a = 100, \quad t = 5, \quad E = 2 \times 10^6, \quad \nu = 0.25, \quad q = 2.$

Table 2.1. Deflections and moments in a circular plate simply supported around the periphery and subjected to a uniform pressure.

The second step was to adopt a boundary condition representing the case in hand, i.e. simply supported at two points at opposite ends of a diameter. Using the finite element results provides an independent method from the "closed form" theoretical solutions of 2.3.1. and also allows the production of design coefficient tables.

The use of these coefficients is valid when the plate is to be considered as a stiff plate which is also a thin plate, i.e. $t/2a < 1/8$ with flexural rigidity, carrying loads two-dimensionally mostly by internal (bending and torsional) moments and by transverse shears, as in Fig. (2.1).

Deflections can be assumed to be a function of the uniformly distributed load q, the flexural rigidity of the plate per unit length being expressed in the usual notation by

$$D = \frac{Et^3}{12(1-\nu^2)} \tag{2.79}$$

and a^4 (a being the radius) in the following form:-

$$W = C_1 \frac{qa^4}{D} \tag{2.80}$$

Similarly the bending moments per unit length can be written as:-

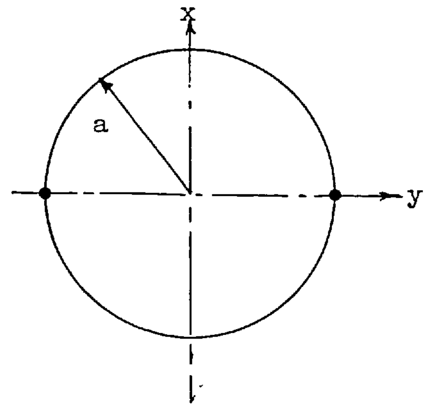
$$M_x = C_2 \, q \, a^2 \tag{2.81}$$

$$M_y = C_3 \, q \, a^2 \tag{2.82}$$

$$M_{xy} = C_4 \, q \, a^2 \tag{2.83}$$

A table of coefficients C_1 to C_4 at the 12 locations as indicated in Tables (2.2) and (2.3) can then be produced.

Element No.	Location		C_2	C_3	C_4
	x.a	y.a			
1	0.95	0.062	+0.0004	-0.3667	+0.0142
2	0.85	0.055	+0.0014	-0.3882	+0.0165
18	0.788	0.527	-0.2835	+0.0018	-0.1280
22	0.495	0.3675	+0.0351	-0.3835	+0.1088
27	0.45	0.05	+0.0422	-0.4649	-0.0076
35	0.5998	0.7128	-0.1228	-0.0745	-0.25
53	0.252	0.55	+0.1743	-0.3905	-0.0767
57	0.30475	0.8977	+0.1154	-0.0424	-0.2522
70	0.05	0.05	+0.0844	-0.5031	-0.0011
75	0.05	0.55	+0.2335	-0.4087	-0.0174
79	0.025	0.875	+0.6334	-0.2192	-0.0724



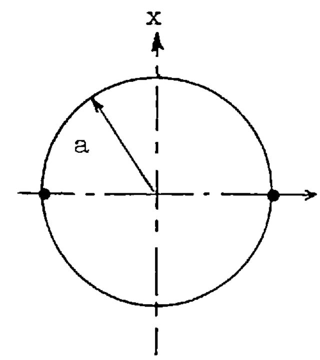
Symmetrical around x & y

$\nu = 0.378$

$M = C\,q\,a^2$

Table: 2.2. Moments design coefficients for circular plate simply supported on two points at opposite ends of a diameter and subjected to a uniform pressure obtained from finite element method.

F.E. Node No.	Location		C_1 From F.E.	C_1 From Equat.
	x/a	y/a		
1	1.0	0.0	0.43767	0.43948
2	0.9	0.0	0.41438	0.41596
6	0.8	0.0	0.39277	0.39420
19	0.866	0.5	0.35216	0.35346
28	0.707	0.707	0.2582	0.25786
29	0.5	0.0	0.3395	0.35638
32	0.5	0.3	0.3150	0.322487
56	0.3	0.5	0.2427	0.22729
80	1.0	0.5	0.2265	0.226427
85	0.0588	0.898	0.0585	0.06
87	0.0	0.0	0.30195	0.30245
92	0.0	0.5	0.2245	0.224809



Symmetrical around x & y

$\nu = 0.378$

$W = C_1 \frac{qa^4}{D}$

Table: 2.3. Comparison of deflection coefficients obtained by the 'closed-form' equation and finite element in circular plate simply supported on two points at opposite ends of a diameter and subjected to a uniform pressure.

The accuracy of the coefficients is directly dependent on the degree of refinement of the finite element mesh, i.e. the number of degrees of freedom in the considered plate. For this purpose, a number of meshes were used with increasing numbers of degrees of freedom until no significant change occurred to the resulting coefficients. This asymptotic convergence is an established procedure in finite element solutions.

Thus in the solution E is extracted from the answers through the flexural rigidity D as described above. The deflections and bending moments (M_x, M_y, M_{xy}) can be worked out using the coefficients for a specific value of say ν_1. Then the effects of another Poisson's ratio ν_2 can be estimated from:- Szilard Ref. (31).

$$W_2\ (x,y) \simeq \frac{1-\nu_2^2}{1-\nu_1^2} W_1\ (x,y) \qquad 2.84$$

$$(M_x)_2 \simeq \frac{1}{1-\nu_1^2} \left[(1-\nu_1\nu_2)(M_x)_1 + (\nu_2-\nu_1)(M_y)_1\right] \qquad 2.85$$

$$(M_y)_2 \simeq \frac{1}{1-\nu_1^2} \left[(\nu_2-\nu_1)(M_x)_1 + (1-\nu_1\nu_2)(M_y)_1\right] \qquad 2.86$$

$$(M_{xy})_2 \quad \frac{1-\nu_2^2}{1-\nu_1^2} (M_{xy})_1 \qquad 2.87$$

This case was run using a typical plate having values of $E = 0.46 \quad 10^6$ lb/in^2 and $\nu = 0.378$, a = 12in. The coefficients for deflections at twelve nodes are compared with the coefficients from Appendix (1) in Table (2.3). The coefficients for moments are compared in Table (2.4), the conversion from local x-y coordinates of the finite element mesh to global polar coordinates being described in Appendix (4).

The practical application of these coefficients is in day-to-day work, since they provide a reliable and simple technique for calculating deflections. In fact, they can predict to within 10% the measured deflections obtained in testing valves. However, it is not a good method for calculating the stresses especially in the vicinity of the support, because in reality the point support is a hypothetical point and can not be simulated in tests. Mathematically the coefficients obtained from the finite element solution would be a more accurate simulation of a point support for the stresses.

Element No.	Location		C_2 F.E.	C_2 Equ.	C_3 F.E.	C_3 Equ.	C_4 F.E.	C_4 Equ.
	x/a	y/a						
1	0.946	0.054	+0.0004	+0.000	-0.3667	0.39	+0.0142	0.00
2	0.846	0.056	+0.0014	+0.004	-0.3882	-0.40	+0.0165	0.00
18	0.788	0.527	-0.2835	-0.32	+0.0018	0.0018	-0.1280	-0.10
27	0.450	0.050	+0.0422	0.043	-0.4649	-0.467	-0.0076	-0.033
70	0.05	0.05	0.294	0.21	0.294	0.21	0.252	0.2935
75	0.05	0.55	+0.2334	0.24	-0.4087	-0.4087	-0.0174	0.0

Table: 2.4. Comparison of moments coefficients obtained by the 'closed-form' equation and finite element in circular plate simply supported on two points at opposite ends of a diameter and subjected to a uniform pressure.

2.4. The Solution for Stresses and Deflections in a Circular Plate of Uniform Thickness subjected to a Uniform Normal Pressure and supported along two short lengths of Arc at two opposite ends of a Diameter.

2.4.1. Introduction

This idealisation, shown in Fig. (2.8a), is a more practical one as the shafts occupy a short length of the periphery of any blade This becomes significant when the valve is used for regulating the flow and the torque loads are significant. This removes the singularity at the support of 2.3. and enables the calculation of the stresses and deflections at any point, including the points of attachment.

2.4.2. Analytical Work

The same analytical work done in 2.3.1. applies, from Figs. (2.8a) and (2.8b). The boundary conditions would be:

(i) At the boundary of the disc the radial bending moment is zero. Thus the work up to Equation (2.32) is applicable.

(ii) At the supports the deflections are zero at two points given by:-

$$\theta = \alpha \text{ and } \theta = \pi - \alpha$$

Putting $\theta = \alpha$, $\rho = 1$ and using Equation (2.31) in Equation (2.28) there results:-

$$A_o + A_1 \cos \alpha + \sum_{n=2,3}^{\infty} (A_n + B_n) \cos n\theta = 0 \qquad 2.88$$

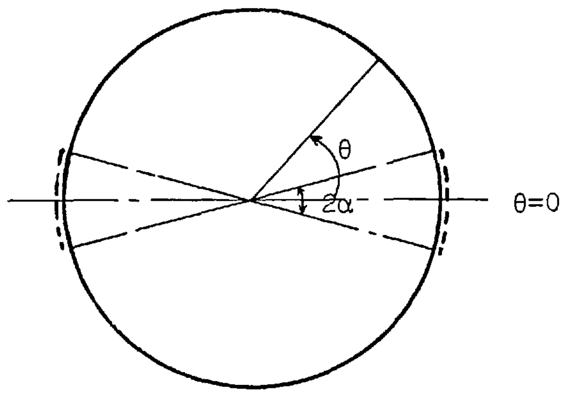


Fig. 2.8a. Plate simply supported on two arcs.

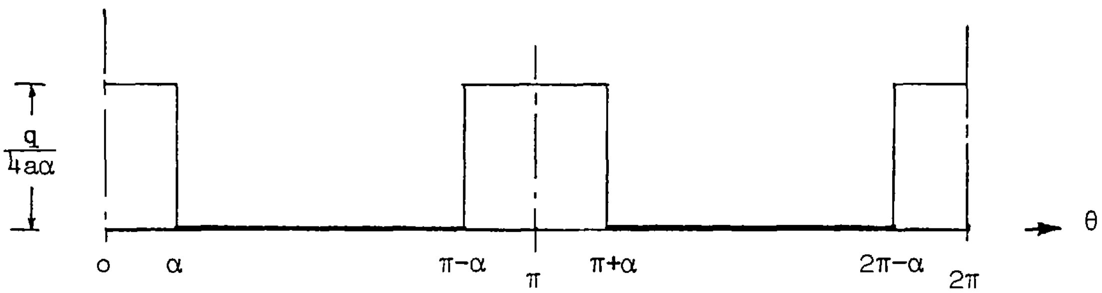


Fig. 2.8b. Reactive forces on plate simply supported on two arcs and subjected to uniform pressure.

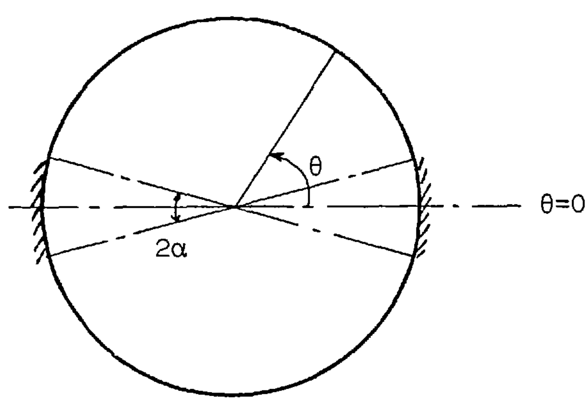


Fig. 2.8c. Plate supported on two clamped arcs.

Putting $\theta = \pi - \alpha$, $\rho = 1$ and using Equation (2.31) with Equation (2.28) gives:-

$$A_o - A_1 \cos \alpha + \sum_{n=2,3}^{\infty} (A_n + B_n) \cos n\alpha \text{ con } n\pi = 0 \qquad 2.89$$

Solving Equation (2.22) and Equation (2.93) with

$\cos n\pi \;=\; +1$ (for even n)

$\cos n\pi \;=\; -1$ (for odd n)

gives

$$A_o = -\sum_{n=2,4}^{\infty} (A_n + B_n) \cos n\alpha \qquad 2.90$$

And

$$A_1 \cos n\alpha = -\sum_{n=3,5}^{\infty} (A_n + B_n) \cos n\alpha \qquad 2.91$$

(iii) The intensity of the reactive forces on the boundary can be expressed by a fourier expansion, see Fig. (2.8b)

$$(V_r)_{\rho=1} = \frac{-R}{a} \left[\frac{1}{2} + \sum_{n=2,4}^{\infty} \left(\frac{\sin n\alpha}{n\alpha}\right) \cos n\theta\right]$$

$$= -qa \left[\frac{1}{2} + \sum_{n=2,4}^{\infty} \left(\frac{\sin n\alpha}{n\alpha}\right) \cos n\theta\right] \qquad 2.92$$

This is equal to Equation (2.39) and proceeding similarly using Equations (2.40) to (2.56) leads to:

$$(V_r)_{\rho=1} = -qa \left\langle \frac{1}{2} + \sum_{n=2,3}^{\infty} \left\{ -n^2(n-1)(1-\nu) A_n + n(n+1) \left[(4-n)(1-\nu)\right] B_n \right\} \cos n\theta \right\rangle_{\rho=1} \qquad 2.93$$

Equating the coefficients of $\cos n\theta$ in Equation (2.91) and Equation (2.97) gives:

$$-n^2(n-1)(1-\nu)\ A_n + n(n+1)\left[4-n\ (1-\nu)\right]\ B_n$$

$$= \frac{\sin n\alpha}{n\alpha} \quad \text{for } n = 2,4\ldots \qquad 2.94$$

$$-n^2(n-1)(1-\nu)\ A_n + n(n+1)\left[4-n(1-\nu)\right]\ B_n = 0$$

$$\text{for } n = 3,5 \qquad 2.95$$

The constants A_n & B_n are determined, as follows:-

Solving Equation (2.32) and Equation (2.95) gives

$A_n = B_n = 0$ for $n = 3,5,7$

Applying this in Equation (2.91) leads to

$A_1 = 0$

Solving Equation (2.32) and Equation (2.94) gives:

$$A_n = \frac{-(n+2/\beta)\ (\sin n\alpha/n\alpha)}{2n^2(n-1)\ (3+\nu)} \qquad 2.96$$

$$B_n = \frac{\sin n\alpha/n\alpha}{2n(n+1)\ (3+\nu)} \qquad 2.97$$

$$A_n + B_n = \frac{-\sin n\alpha}{2(3+\nu)n\alpha}\left[\frac{1}{n(n^2-1)} + \frac{2}{\beta n^2(n-1)} - \frac{1}{n(n+1)}\right] \qquad 2.98$$

Equations (2.96, 2.97, 2.98) are for $n = 2,4,6\ldots$ etc

Using Equation (2.98) in Equation (2.90) gives

$$A_o = \frac{1}{2(3+\nu)} \sum_{n=2,4}^{\infty} \left(\frac{\sin n\alpha}{n\alpha}\right)\left\{\frac{1}{n(n-1)} + \frac{2}{\beta n^2(n+1)}\right.$$

$$\left. - \frac{1}{n(n+1)}\right\} \cos n\alpha \qquad 2.99$$

Proceeding with the solution as in 2.3.1 we obtain similar equations as follows:

$$W = \frac{qa^4}{64D} \left\langle (1-\rho^2)\left(\frac{5+\nu}{1+\nu} - \rho^2\right) + \frac{1}{2\alpha(3+\nu)} \left[\sum_{n=2,4}^{\infty} \left(\frac{\sin n\alpha}{n}\right) \right.\right.$$

$$\left\{ \frac{1}{n(n-1)} + \frac{2}{\beta n^2(n-1)} - \frac{1}{n(n+1)} \right\} \cos n\alpha - \sum_{n=2,4}^{\infty} \left(\frac{\sin n\alpha}{n}\right)$$

$$\left.\left. \left\{ \frac{1}{n(n-1)} + \frac{2}{\beta n^2(n-1)} - \frac{\rho^2}{n(n+1)} \right\} \rho^n \cos n\theta \right] \right\rangle \qquad 2.100$$

$$M_r = qa^2 \left\langle \frac{1}{16}(3+\nu)(1-\rho^2) + \frac{1}{2\alpha}\left(\frac{1+\nu}{3+\nu}\right) \sum_{n=2,4}^{\infty} \left(\frac{\sin n\alpha}{n}\right) \right.$$

$$\left(\beta + \frac{2}{n}\right)(1-\rho^2)\, \rho^{n-2} \cos n\theta \qquad 2.101$$

$$M_t = qa^2 \left\langle \frac{1}{16}\left[(3+\nu) - (1+3\nu)\rho^2\right] - \frac{1}{2\alpha}\left(\frac{1+\nu}{3+\nu}\right) \sum_{n=2,4}^{\infty} \left(\frac{\sin n\alpha}{n}\right) \right.$$

$$\left\{ \beta(1-\rho^2) + \frac{2}{n}(1+\rho^2) \right\} \rho^{n-2} \cos n\theta \qquad 2.102$$

$$M_{rt} = \frac{qa^2}{2\alpha}\left(\frac{1+\nu}{3+\nu}\right) \sum_{n=2,4}^{\infty} \left(\frac{\sin n\alpha}{n}\right) \left\{ \beta(1-\rho^2) + \frac{2}{n} \right\} \rho^{n-2} \sin n \qquad 2.103$$

These equations were written in two small computer programs similar to those of 2.3.1. to produce hand tables of coefficients for deflections and moments. The program for the generation of the tables is listed in Appendix (2.C) and the relevant parts of the tables are given as a sample in Appendix (3).

2.4.3. Finite Element Analysis

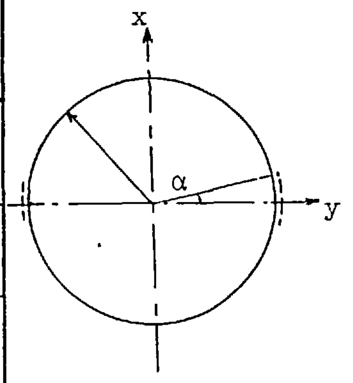
The same mesh as used in 2.3.2. of Fig. (2.7) was used to simulate a boundary condition of simply supported arc of angle $\alpha = 7.5^{\circ}$ by restraining nodes nos. 74, 86, 98, 110 and 122 in the z direction only and following the same procedure adopted in 2.3.2.. Similar coefficients were obtained. The coefficients for deflections at 12 locations are given in Table (2.5) compared with the equivalent ones obtained from Appendix (3). The coefficients for moments were obtained at 11 locations and are shown in Table (2.6). The coefficients for moments are compared with the equivalent ones obtained from Appendix (3) in Table (2.7). For comparing more coefficients the conversion from local x,y coordinates of the finite element mesh to the global polar coordinates must be observed as described in Appendix (4).

From this a method of simulating real boundary conditions and providing a more accurate value for the deflections and stresses is clearly possible using the finite element model. In this way coefficients can be obtained for cases which are very difficult or impossible to simulate mathematically such as:-

(i) Plates simply supported on two short lengths of arc and subjected to point load acting at a specified point. Table (2.8a) and (2.8b).

(ii) Plates which are clamped on a short length of arc at two opposite ends of a diameter Fig. (2.8c). and subjected to:

F.E. Note No.	Location		C_1 From F.E.	C_1 From Equat.
	x/a	y/a		
1	1.0	0.0	0.3715	0.403225
2	0.9	0.0	0.3509	0.381545
6	0.8	0.0	0.3318	0.361462
19	0.866	0.5	0.2915	0.320969
28	0.707	0.707	0.2043	0.229868
29	0.5	0.0	0.2856	0.311838
32	0.5	0.3	0.2625	0.290845
56	0.3	0.5	0.1976	0.212901
80	1.0	0.5	0.185	0.201193
85	0.0588	0.898	0.0415	0.045
87	0.0	0.0	0.2535	0.28376
92	0.0	0.5	0.1834	0.21253



Symmetrical around x & y

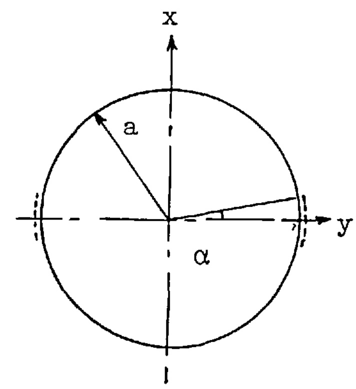
$\alpha = 7.5^{\circ}$

$\nu = 0.378$

$W = C_1 \frac{qa^4}{D}$

Table: 2.5. Comparison of deflection coefficients obtained by 'closed-form' equation and finite element in circular plates simply supported on two short lengths of arcs at opposite ends of a diameter and subjected to a uniform pressure.

F.E. Node No.	Location		C_2	C_3	C_4
	x/a	y/a			
1	0.95	0.062	-0.0002	-0.3543	+0.0141
2	0.85	0.055	-0.0008	-0.3745	+0.0163
18	0.7865	0.527	-0.2667	+0.0007	-0.1215
22	0.495	0.3675	+0.0229	-0.3607	+0.1097
27	0.45	0.05	+0.0261	-0.4416	0.0057
35	0.5988	0.7128	-0.0877	-0.0710	-0.2468
53	0.252	0.55	+0.1468	-0.3534	-0.0392
57	0.30475	0.8977	+0.2498	-0.0694	-0.2234
70	0.05	0.05	+0.0515	-0.4697	-0.0007
75	0.05	0.55	+0.1480	-0.3536	-0.0048
79	0.025	0.875	+0.0760	-0.0544	+0.0482



Symmetrical around x & y

$\alpha = 0.75^{o}$

$\nu = 0.378$

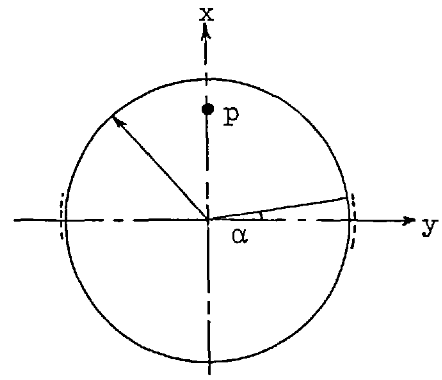
$M = C\ q\ a^2$

Table: 2.6. Moments design coefficients for circular plate simply supported on two short lengths of arcs at opposite ends of a diameter and subjected to uniform pressure obtained from finite element method.

Element No.	Location		C_2 F.E.	C_2 Equ.	C_3 F.E.	C_3 Equ.	C_4 F.E.	C_4 Equ.
	x/a	y/a						
1	0.946	0.054	0.003	0.008	-0.35	-0.658	0.03	0.0
18	0.788	0.527	0.031	0.03	0.294	0.0861	0.179	0.190
27	0.45	0.05	0.434	0.401	-0.02	-0.149	0.057	0.003
41	0.35	0.35	0.133	0.123	0.2044	0.233	0.240	0.264
70	0.05	0.05	0.211	0.2064	0.211	0.209	0.261	0.295
75	0.05	0.55	-0.153	-0.0279	0.323	0.449	0.050	0.0

Table: 2.7. Comparison of moments coefficients obtained by the 'closed-form' equation and finite element in circular plate simply supported on two short lengths of arcs at opposite ends of a diameter and subjected to a uniform pressure.

F.E. Node No.	Location		C_1
	x/a	y/a	
1	- 1.0	0.0	- 0.47975
2	- 0.9	0.0	- 0.43435
6	- 0.8	0.0	- 0.39045
19	- 0.866	0.5	- 0.3605
29	- 0.5	0.0	- 0.2668
32	- 0.5	0.3	- 0.2471
56	- 0.3	0.5	- 0.15095
85	- 0.0588	0.898	- 2.117
87	- 0.0	0.0	- 0.11275
109	0.0588	0.898	- 0.01485
128	0.3	0.5	- 0.0291
146	0.5	0.0	- 0.01175
149	0.5	0.3	- 0.06735
174	0.8	0.0	0.0372
182	0.9914	0.13	0.0671
184	1.0	0.0	0.0679



Symmetrical around x-axis

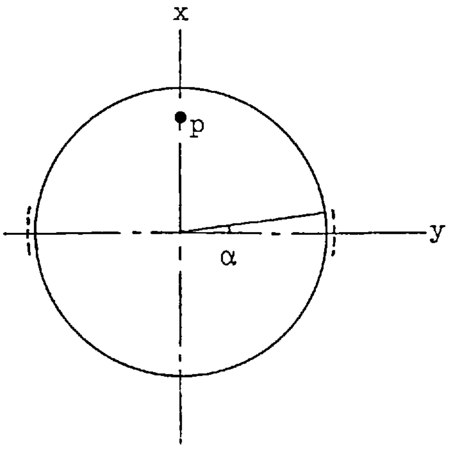
p at 0.8a

$\alpha = 7.5^{\circ}$

$W = C_1 \frac{qa^4}{D}$

Table: 2.8a. Coefficients for deflections in circular plates simply supported on two short lengths of arcs at opposite ends of a diameter due to point load at the shown location.

Element No.	Location		C_2	C_3	C_4
	x/a	y/a			
1	0.95	0.062	-0.0256	-0.473	+0.0196
2	0.85	0.055	-0.1086	-0.5193	+0.0057
18	0.788	0.527	-0.1058	+0.0021	-0.1446
22	0.495	0.3675	+0.0555	-0.1814	+0.1636
27	0.45	0.05	+0.2304	-0.3278	+0.0112
35	0.5998	0.7128	+0.1522	-0.0363	-0.1994
53	0.252	0.55	+0.2503	-0.1719	+0.1005
57	0.3047	0.8977	+0.4888	-0.0688	-0.1348
70	0.05	0.05	+0.1263	-0.1979	+0.0121
75	0.05	0.55	+0.1641	-0.1335	+0.1449
79	0.025	0.875	+0.1338	-0.0011	+0.3485
90	-0.025	0.875	-0.0185	-0.0068	+0.301
124	-0.45	0.05	+0.0391	-0.0924	+0.0068
137	-0.495	0.3675	+0.0272	-0.0784	+0.0014
142	-0.5998	0.7128	-0.0261	-0.0594	-0.0371
160	-0.95	0.062	+0.0161	-0.051	-0.0008



Symmetrical around x-axis

p at 0.8a

$\alpha = 7.5^{\circ}$

$M = C.p$

Table: 2.8b. Coefficients for moments in circular plates simply supported on two short lengths of arcs at opposite ends of a diameter due to point load at the shown location.

(a) Hydrostatic load (which is a very important case occurring during the use of the valve for throttling in large valves).

(b) Point load acting at a point perpendicular to the shaft line (which is used to unseat the valve with a screw) an example being given in Table (2.9).

(c) The all important uniform pressure. Table (2.10) and (2.11).

(iii) Plates which are supported on clamped or simply supported arcs plus short length of the diameter in simple or clamped manner as shown in Fig. (2.9) subjected to loading conditions (a), (b) & (c) above. A sample is given in Table (2.12a), (2.12b) and (2.12c).

(iv) Plates which have a non-circular configuration generally, in this application the elliptical blades of butterfly valves.

2.5. Experimental Work

Although the two theoretical approaches which have been discussed are sound, since the problem has been solved independently by closed-form mathematical solutions and by finite element methods, it was felt that experimental work was necessary as a confirmation especially in the cases where it was not possible to check the finite element solutions by "closed-form" mathematical solutions. A number of experiments was carried out and this section which follows is to describe the procedure and give a sample result for a plate supported on two short arcs at opposite ends of a diameter.

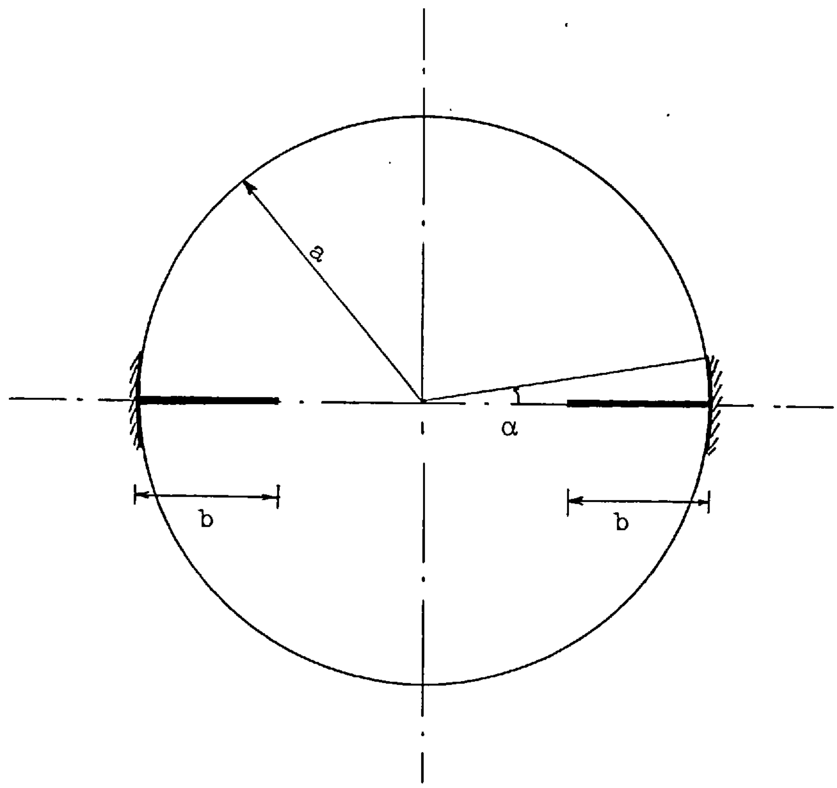
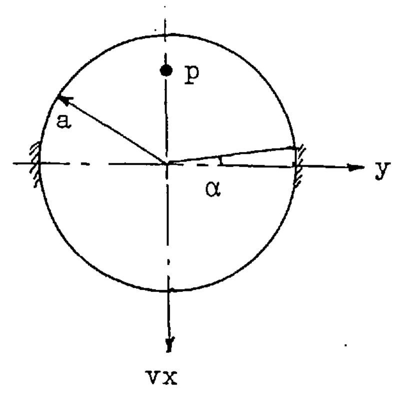


Fig. 2.9. Plate supported on two short lengths of arc and short lengths of diameter

F.E. Node No.	Location		C_1
	x/a	y/a	
1	- 1.0	0.0	0.33
2	- 0.9	0.0	0.294
6	- 0.8	0.0	0.2592
19	- 0.866	0.5	0.2362
29	- 0.5	0.0	0.16045
32	- 0.5	0.3	0.14615
56	- 0.3	0.5	0.0753
85	- 0.0588	0.898	0.00025
87	- 0.0	0.0	0.04155
109	0.0588	0.898	0.0001
128	0.3	0.5	- 0.00965
146	0.5	0.0	- 0.03055
149	0.5	0.3	- 0.0309
174	0.8	0.0	- 0.0661
182	0.9914	0.13	- 0.08825
184	1.0	0.0	- 0.08945



Symmetrical around x

$W = C_1 \frac{Pa^2}{D}$

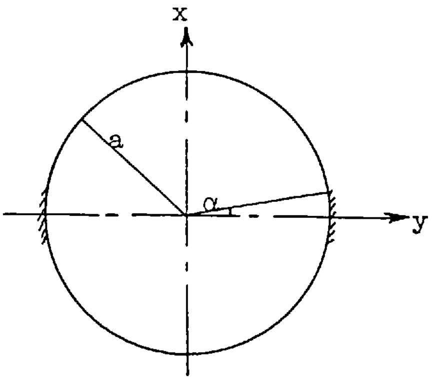
$\alpha = 7.5^{\circ}$

$\nu = 0.378$

p at 0.8a

Table: 2.9. Coefficients for deflection in circular plates clamped on two short lengths of arcs at opposite ends of a diameter due to point load at the shown location.

F.E. Node No.	Location		C_1
	x/a	y/a	
1	1.0	0.0	0.1744
2	0.9	0.0	0.1606
6	0.8	0.0	0.1475
19	0.866	0.5	0.1300
28	0.707	0.707	0.0834
29	0.5	0.0	0.1143
32	0.5	0.3	0.1029
56	0.3	0.5	0.0659
80	1.0	0.5	0.0560
85	0.0588	0.898	0.0041
87	0.0	0.0	0.0896
92	0.0	0.5	0.5474



Symmetrical around x & y axies.

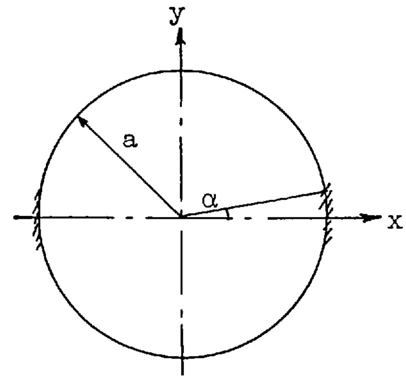
$$W = C_1 \frac{qa^4}{D}$$

$\alpha = 7.5^\circ$

$\nu = 0.378$

Table: 2.10 Coefficients for the deflection in circular plate supported on two clamped short lengths of arc at opposite ends of a diameter and subjected to a uniform pressure.

Element No.	Location		C_2	C_3	C_4
	x/a	y/a			
1	0.95	0.062	-0.0003	-0.1789	+0.0077
2	0.85	0.055	+0.0007	-0.1873	+0.0086
18	0.788	0.527	-0.1002	+0.0011	-0.0597
22	0.495	0.3675	+0.0413	-0.1413	+0.0528
27	0.45	0.05	+0.0494	-0.2092	-0.0036
35	0.5988	0.7128	+0.0516	+0.0031	-0.0855
53	0.252	0.55	+0.1676	-0.0425	-0.0223
57	0.30475	0.8977	+0.3230	+0.0882	+0.0669
70	0.05	0.05	+0.1014	-0.2176	-0.0006
75	0.05	0.55	+0.2264	-0.0128	-0.0056
79	0.025	0.875	+0.3475	+0.5795	+0.0275



Symmetrical around x & y axis

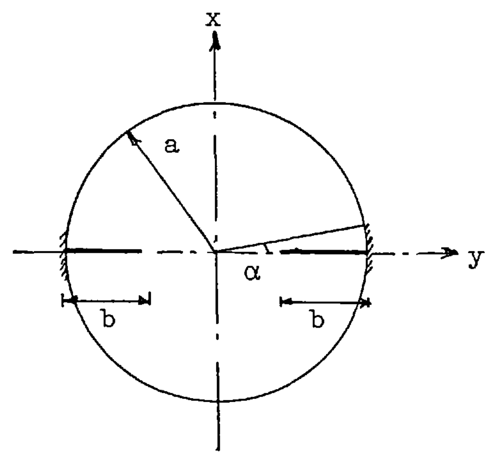
$\alpha = 7.5^{\circ}$

$\nu = 0.378$

$Mx = C_2\, qa^2$, $My = C_3\, qa^2$, $Mxy = C_4\, qa^2$

Table: 2.11. Coefficients for moments in circular plate supported on two clamped short lengths of arc at opposite ends of a diameter and subjected to a uniform pressure.

F.E. Node No.	Location		C_1
	x/a	y/a	
1	1.0	0.0	- 0.1084
2	0.9	0.0	- 0.0963
6	0.8	0.0	- 0.0845
19	0.866	0.5	- 0.0299
28	0.707	0.707	- 0.0505
29	0.5	0.0	- 0.0528
32	0.5	0.3	- 0.0473
56	0.3	0.5	- 0.0213
80	1.0	0.5	- 0.0096
85	0.0588	0.898	- 0.0000
87	0.0	0.0	- 0.0266
92	0.0	0.5	- 0.0079



Symmetrical around x & y

$b = \frac{2a}{3}$

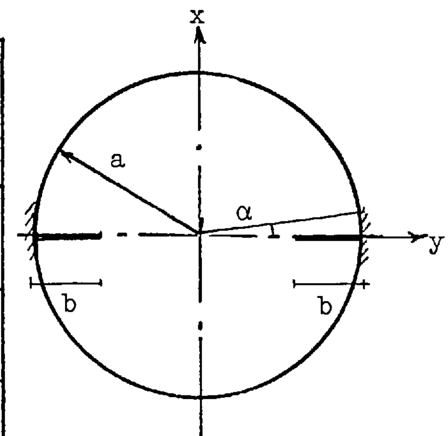
$\nu = 0.378$

$\alpha = 7.5^{\circ}$

$W = C_1 \frac{qa^4}{D}$

Table: 2.12a. Coefficients for deflections in circular plates supported on two clamped arcs at the end of the diameter and two short lengths of the diameter as shown and subjected to uniform pressure.

F.E. Node No.	Location x	y	C_2	C_3	C_4
1	0.95	0.062	-0.0002	-0.0879	+0.0044
2	0.85	0.055	+0.0022	-0.09	+0.0047
18	0.788	0.527	-0.0248	+0.0012	-0.0311
22	0.495	0.3675	+0.0497	-0.0323	+0.0261
27	0.45	0.05	+0.075	-0.0847	-0.0032
35	0.5988	0.7128	+0.0863	+0.0143	-0.0302
53	0.252	0.55	+0.1772	+0.1190	-0.0053
57	0.30475	0.8977	+0.21	+0.0146	+0.0247
70	0.05	0.05	+0.1736	-0.0838	-0.0011
75	0.05	0.55	+0.4250	+0.2617	-0.0323
79	0.025	0.875	+0.0753	+0.0318	+0.0319



Symmetrical around x & y

$b = \frac{2a}{3}$

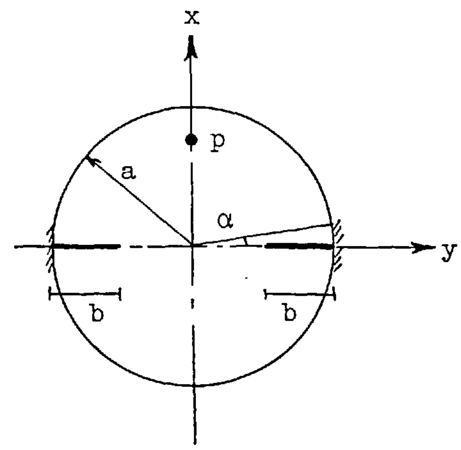
$\nu = 0.378$

$\alpha = 7.5^{\circ}$

$Mx = C_2\, qa^2$, $My = C_3\, qa^2$, $Mxy = C_4\, qa^2$

Table: 2.12b. Coefficients for moments in circular plates supported on two clamped arcs at the end of the diameter and two short lengths of the diameter as shown and subjected to uniform pressure.

F.E. Node No.	Location		C_1
	x/a	y/a	
1	- 1.0	0.0	- 0.30065
2	- 0.9	0.0	- 0.2654
6	- 0.8	0.0	- 0.23115
19	- 0.866	0.5	- 0.2139
29	- 0.5	0.0	- 0.13305
32	- 0.5	0.3	- 0.12145
56	- 0.3	0.5	- 0.05545
85	- 0.0588	0.898	- 0.00095
87	- 0.0	0.0	- 0.01355
109	0.0588	0.898	0.00085
128	0.3	0.5	0.02945
146	0.5	0.0	0.05795
149	0.5	0.3	0.0556
174	0.8	0.0	0.0941
182	0.9914	0.13	0.11715
184	1.0	0.0	0.11875



Symmetrical around x-axis

$b = \frac{2a}{3}$

$\alpha = 7.5^{\circ}$

$\nu = 0.378$

p at 0.8a

$W = C_1 \, pa^2$

Table: 2.12c. Coefficients for deflections in circular plates supported on two clamped arcs at the end of the diameter and two short lengths of the diameter as shown due to point load at the shown location.

2.5.1. Introduction

As the number of experimental tests required in this programme of checking was large, a great deal of consideration went into the design of the test arrangements, so as to simulate as many cases of loading as possible with minimum delays and maximum flexibility, to give as many cases of supporting conditions within the time available without damaging or replacing many samples.

A one inch thick 'perspex'* sheet simulating the blade was used because of ease of handling, availability and economy.

2.5.2. Four Point Bend Testing of a Perspex Beam

i Aims

i.1 To determine the Young's modulus of perspex and Poisson's ratio.

i.2 To examine the effect of creep on the properties of perspex.

i.3 To assess the reliability of using strain gauges to measure strains in perspex.

i.4 To determine the effect of low strain rates on the behaviour of perspex and to see whether it is possible to simulate the behaviour of a metal using perspex.

---

* 'Perspex' is a trade name for polymethylmethacrylate (p.m.m.a.)

ii The Test Beam

The dimensions of the perspex bar were as shown in Fig. (2.11) This was machined from a large sheet of 1 in. thick perspex and the rest of the sheet is being used to machine a circular diaphragm which will be used to simulate the blade of a large butterfly valve. (Fig. (2.19).

All dimensions of the beam were accurate to within ± 1.0%

iii The Testing Equipment

The beam was supported on two mild steel rollers which were 10 ins. apart as shown in Fig. (2.13). These rollers were screwed on to a mild steel plate to prevent them from slipping. The load was applied to the beam through a load cell and another pair of rollers to give 4-point loading with a constant bending moment.

Two strain gauges (PL-10 made by Tokyo Sokki Kenkyoso Co. Ltd. (T.S.K. Ltd.) gauge resistance 120 ± 0.3Ω, gauge length 10mm) were stuck longitudinally and another two perpendicular gauges to measure the transverse strain were stuck to each face of the centre of the beam as also shown in Fig. (2.12). The adhesive used was CN Adhesive made by T.M.L. Ltd. of T.S.K. Ltd.

The load cell was used solely for applying the load to the metal plate. The load readings were obtained from the scale of the 60,000 lb. maximum capacity Avery testing machine used for the experiments. The scale used for these experiments was 0-3,000 lbs. with each small division being equal to 5-lbs.

The two longitudinal strain gauges were arranged in a half bridge and fed with 3.V through a Bruel and Kjaer strain gauge amplifier, type 1526. The output was fed back to the amplifier which indicated the total strain in 'micro strains'*

---

* Note:- 1 microstrain = 1 x 10^{-6} strain

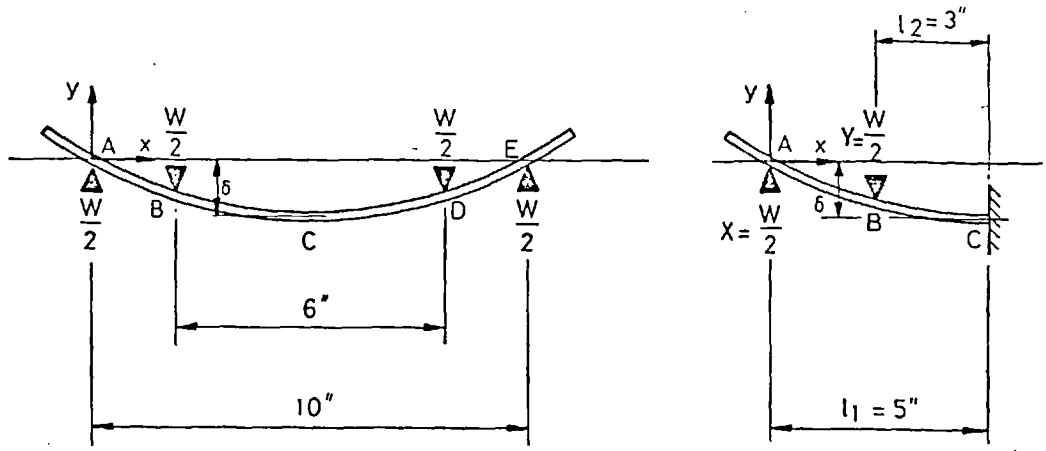


Fig. 2.10: Analysis of pure bending of a beam

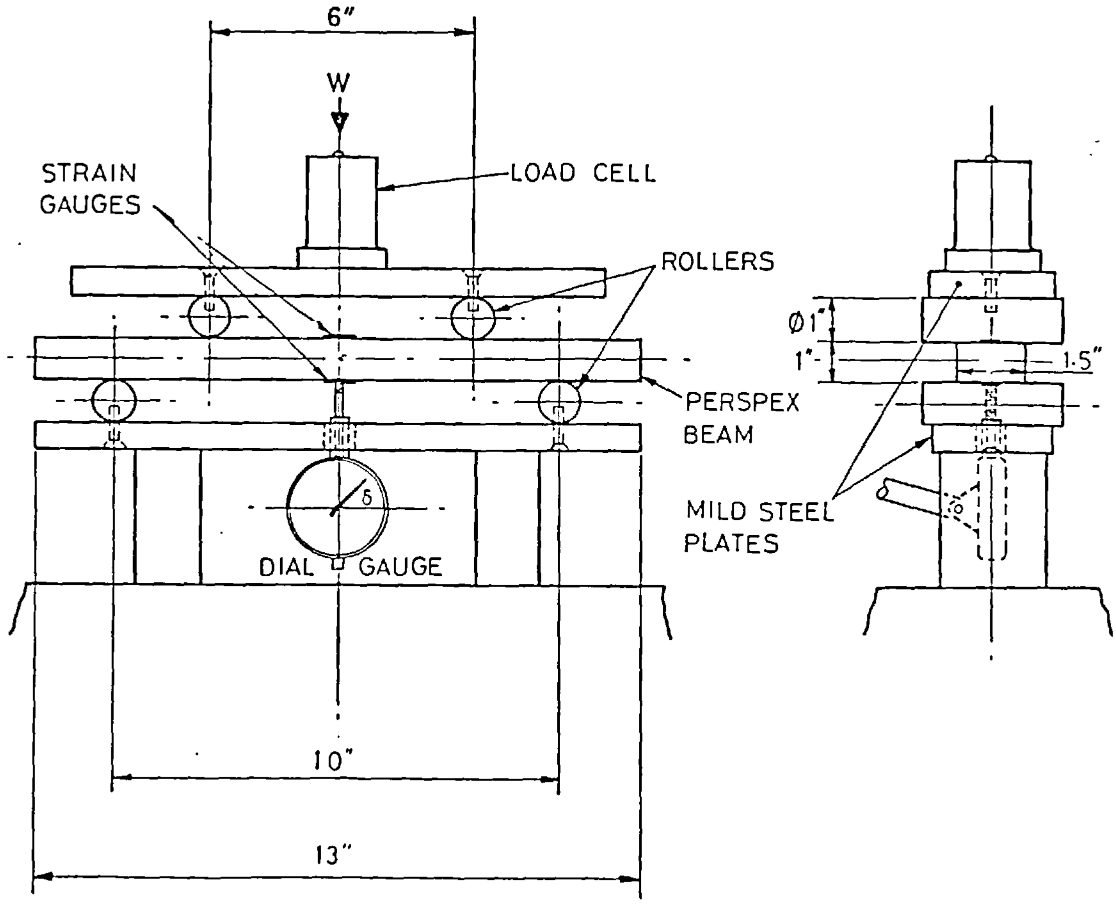


Fig. 2.11: 4-Point bend testing of a perspex beam

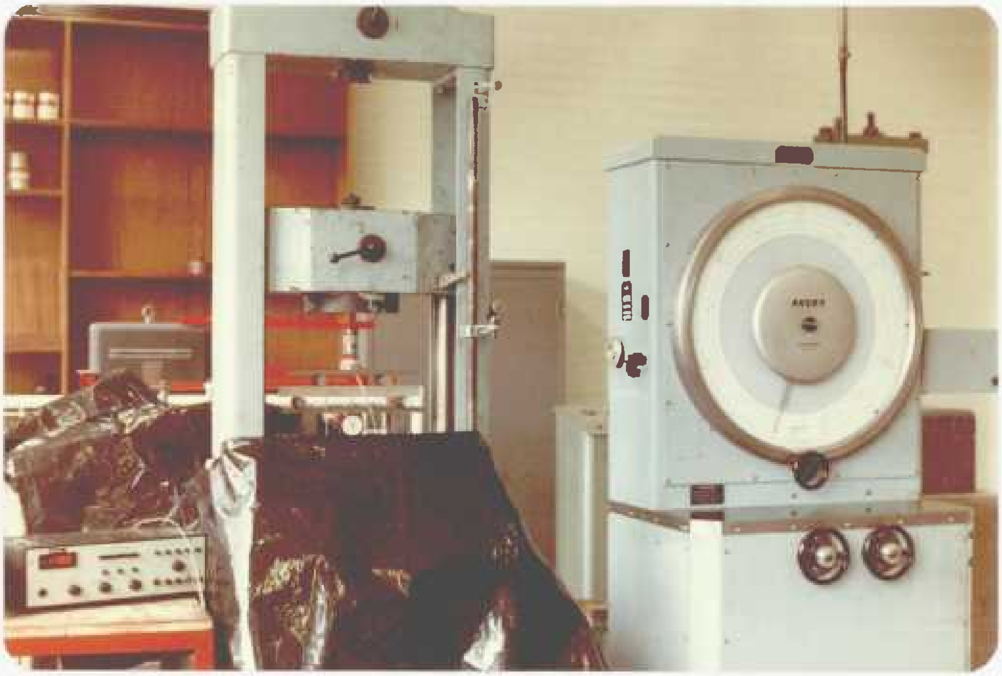


Fig. 2.12. 4-Point bend testing equipment

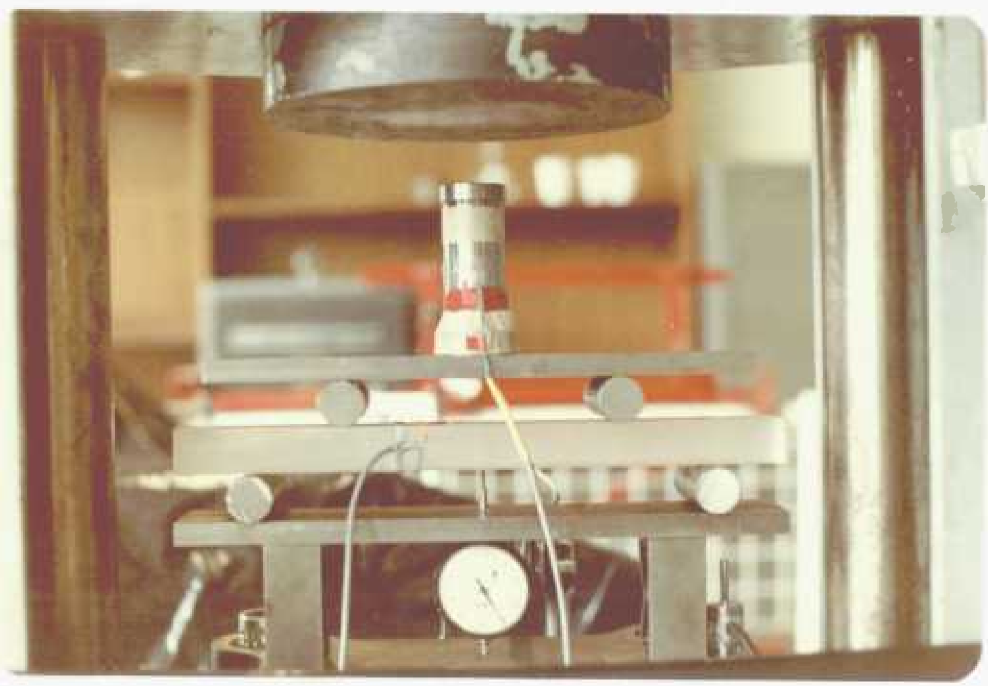


Fig. 2.13. 4-Point bend testing of a perspex beam

in a L.E.D. (Light Emitting Diode) display. (Although the gauge factor quoted by the manufacturer was 2.07, it was found that to obtain the theoretically correct readings on the display unit the gauge factor on the amplifier had to be set at 1.84 with the same arrangement for the two transverse strain gauges).

The deflection of the beam was measured by means of a 0.0001 in. smallest division dial gauge placed directly underneath the centre of the beam, as shown in Fig. (2.14).

iv Theory

From Fig. (2.10) it can be seen that the loading of the beam is equivalent to a cantilever subjected to two point loads. This similarity can be used to calculate the central deflection of the beam (δ) when it is subjected to a total load of W.

Therefore,

$$\delta = \left[\frac{\frac{W}{2} \times \ell_1^3}{3EI}\right] - \left[\frac{\frac{W}{2} \times \ell_2^3}{3EI} + \frac{\frac{W}{2}\ell_2^2}{2EI}(\ell_1 - \ell_2)\right]$$

$$= \frac{W}{EI}\left[\frac{\ell_1^3}{6} - \frac{\ell_2^3}{6} - \frac{\ell_2^2(\ell_1-\ell_2)}{4}\right] \qquad 2.104$$

However, $\ell_1 = 5$ in. and $\ell_2 = 3$ in.

Therefore,

$$\delta = \frac{W}{EI}\left[\frac{125}{6} - \frac{27}{6} - \frac{18}{4}\right]$$

and

$$W = \frac{6\ EI}{71}\delta \qquad 2.105$$

Referring to Fig. (2.10) the bending moment in the region BD is constant and is equal to $\frac{W}{2}.\ (\ell_1 - \ell_2)$ which is equal to W, if ℓ_1 and ℓ_2 are measured in inches.

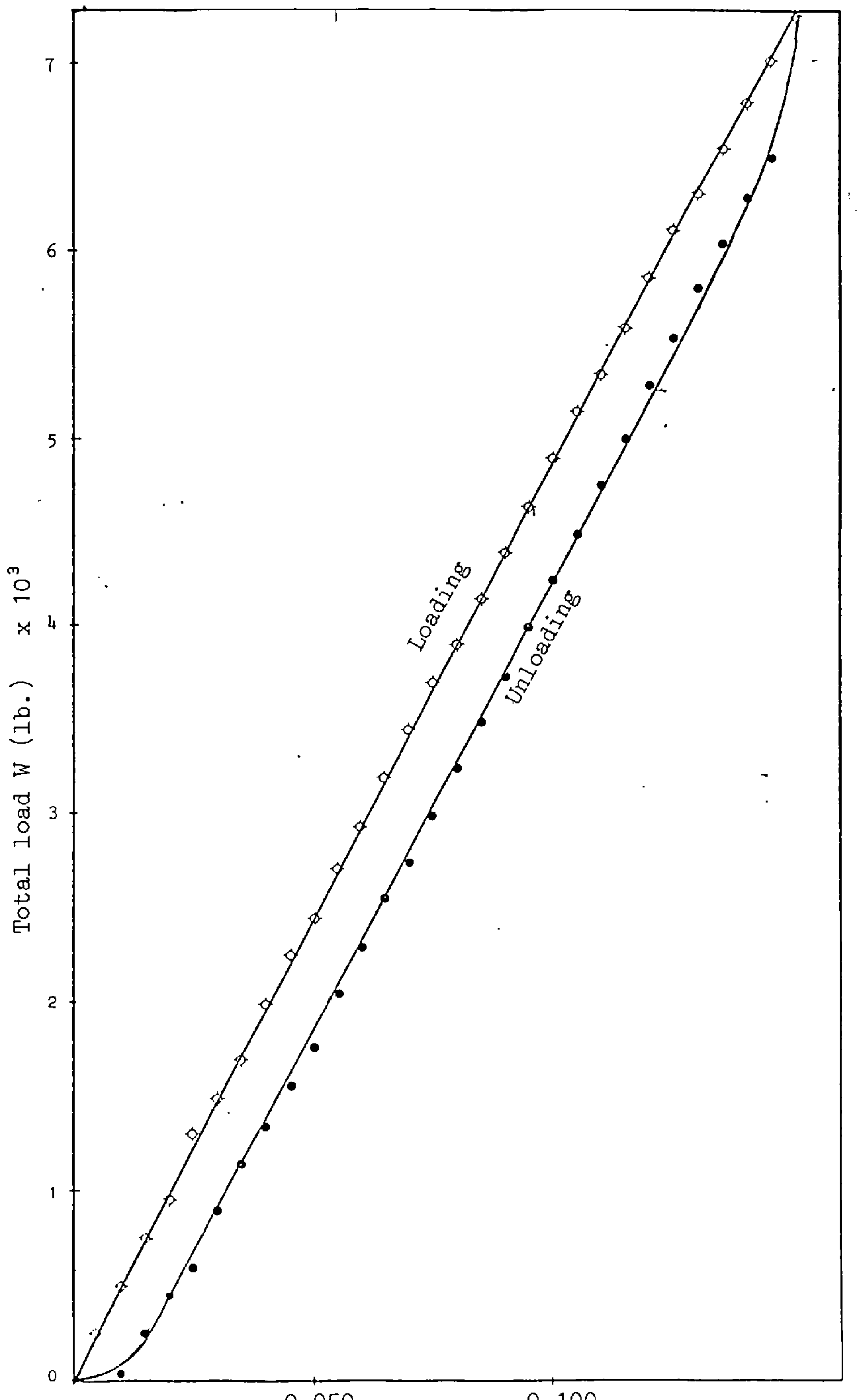


Fig. 2.14. A plot of total load vs central deflection

$$M = \frac{W}{2}(\ell_1 - \ell_2) = W \text{ (lbf.in. if W is in lbf. for example)} \qquad 2.106$$

Using

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y} \qquad 2.107$$

Where y is the distance from the neutral axis.

thus

$$\frac{W}{I} = \frac{E}{R} = \frac{\sigma_{max}}{d/2}, \qquad 2.108$$

at the surface of the beam where d is the depth of the beam.

Therefore $\sigma_{max} = \frac{Wd}{2\ell}$

However, $\varepsilon_{max} = \frac{\sigma_{max}}{E}$

Therefore $\varepsilon_{max} = \frac{Wd}{2EI}$

Substituting for $\frac{W}{EI}$ using Equation (2.105)

$$\varepsilon_{max} = \frac{3}{71}\delta.d$$

However, d = 1 in.

Therefore $\varepsilon_{max} = \frac{3\delta}{71}$ (if δ is in inches). 2.109

For the beam

$$I = \frac{1}{12} \times b \times d^3 \text{ in}^4. \qquad 2.110$$

where b is the width and equals 1.5 ins. Hence

$$I = \frac{1}{12} \times 1.5 \times 1^3 \text{ in}^4. = 0.125 \text{ in}^4.$$

v Experimental Procedure

The beam was loaded to a maximum deflection of 0.150 in. (assuming $\sigma_{allowable} = \sigma_{yield}$ for perspex is 3000 lbf./in^2 and E for perspex is 0.4×10^6 lbf./in^2), $\varepsilon_{allowable} = 0.0075$. Therefore δ_{max}, using Equation (2.109) is approximately 0.180 in. All tests were carried out at the very low speed of 12.5×10^{-3} in. central deflection/min. to avoid strain-rate effects (approximately equal to 500 µ-strain/min.). The load and the total strain were recorded at 0.005 in. intervals of the central deflection. Then, the beam was unloaded at the same speed and similar readings were taken. Figs. (2.14) and (2.15) show the results obtained.

The experiments were repeated at different speeds up to strain rates of .002/min., but no significant difference was observed. However, these results are not valid for suddenly applied loads. The error in such a case can be about ± 5%

vi Calculations

Using the gradient of Fig. (2.14)

$$\frac{W}{\delta} = \frac{6EI}{71} = \frac{6}{71} E \times 0.125 = 4850$$

Therefore

$$\underline{E = 0.459 \times 10^6 \text{ lbf./in}^2}$$

The gradient of Fig. (2.15) $= \varepsilon_{max}/\delta = \underline{0.0424 \frac{1}{\text{in.}}}$

The theoretically correct gradient (from 2.109)

$$= \frac{3}{71} \frac{1}{\text{in.}} = \underline{0.0423 \frac{1}{\text{in.}}}$$

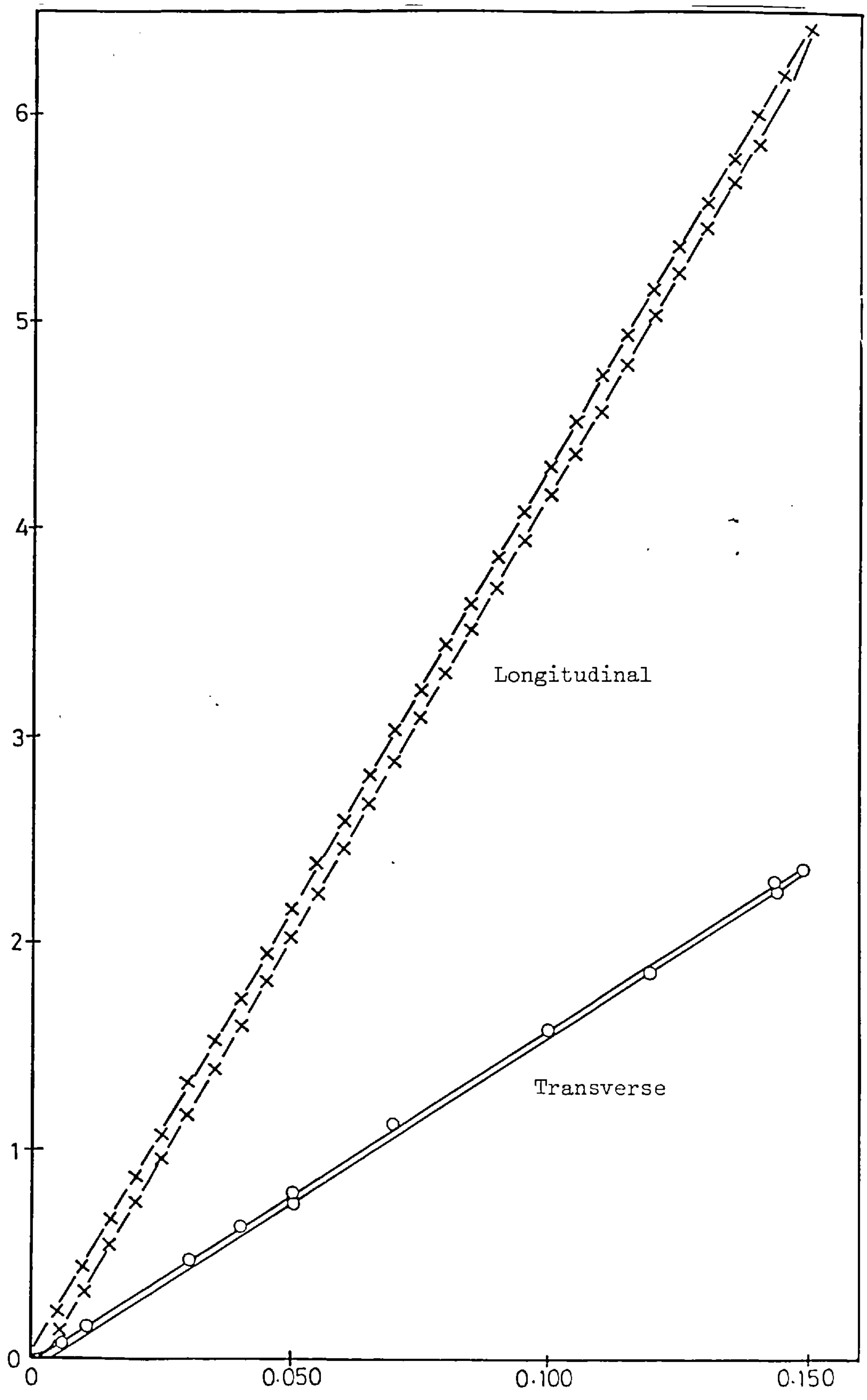


Fig. 2.15. A plot of strain vs. deflection at the centre.

Therefore the gauge factor should be set at 1.84.

The ratio between the transverse and longitudinal strains is Poisson's ratio and this was measured by taking the ratio of the gradients shown in Fig. (2.15).

vii Discussion and Conclusions

(1) The Young's modulus of perspex is approximately 0.46×10^6 lbf./in.^{2}

(2) The Poisson's ratio of perspex is approximately 0.378.

(3) The experiments have shown that perspex creeps at room temperature.

Therefore, for comparing experimental results the readings have to be taken either instantaneously or after fixed time delays. Provided this is remembered, perspex can be used to simulate metals or other elastic materials.

(4) It is possible to stick strain gauges on perspex and obtain reliable results.

(5) Perspex is not seriously affected by low strain rates - i.e. up to about .002/min. -. However, unlike metals, the sudden application of a load can lead to unreliable results, even if sufficient time is given to reach steady state conditions.

2.5.3. The Testing Equipment

To simulate a uniformly distributed pressure a $6\frac{3}{4}$" length of 12" diameter pipe with 3/8" wall thickness was used as the basic pressure container. A $\frac{3}{4}$" thick plate at the bottom and a $\frac{3}{4}$", 18" nominal diameter flange were welded to the pipe with a rubber O ring fitted at the upper side. Small pipes were welded to the bottom for air bleed, water drain, water or air inlet and a connection to the pressure gauge Fig. (2.16) and (2.28).

A 1" thick, 18" nominal diameter and 12" diameter flange was machined to fit on top of the pipe flange Fig. (2.17) and (2.20). Two short lengths of flange were provided to clamp the supports of the specimen in location as shown in Fig. (2.17) and (2.21). The whole assembly was bolted to a table 30" x 30" wide and 30" high from the ground, which can be levelled by screws fitted to its legs.

The frame was made up of $1\frac{1}{2}$" steel angles and the top was made of 30" x 30" x $\frac{1}{2}$" steel plate bolted to the angles. A 4" diameter hole was provided at the centre of the plate for the small pipes which come from the panel attached to the table. The panel carries a pressure gauge, air and water valves and regulator for the air supply Fig. (2.18) and (2.22).

The dial gauges (with 0.0001 in. division) could be placed over the sample to measure the deflection by fixing them to a three-legged frame which can stand above the diaphragm Fig. (2.22), (2.23) and (2.24). As the relative strength of the steel to the

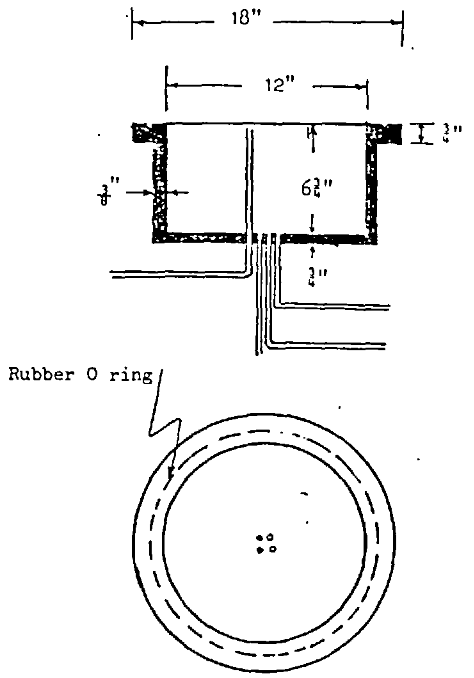


Fig. 2.16 Testing Equipment

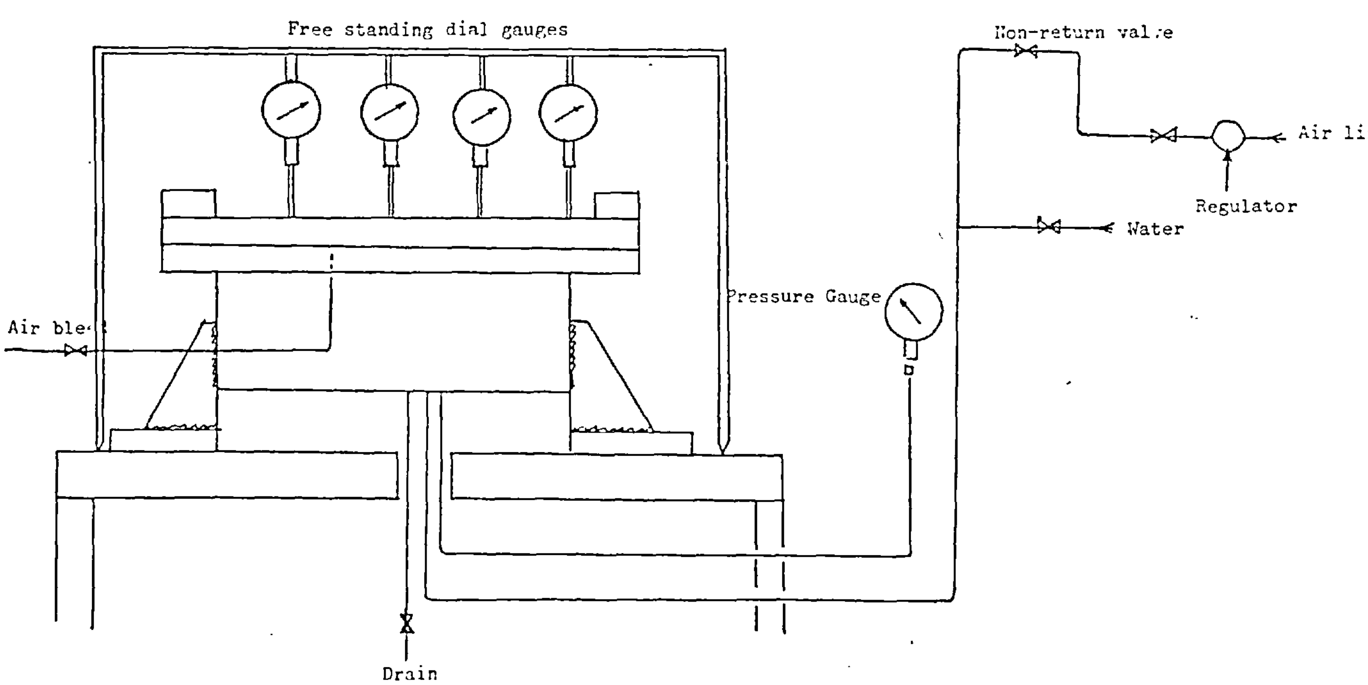


Fig. 2.18. Testing equipment and control panel

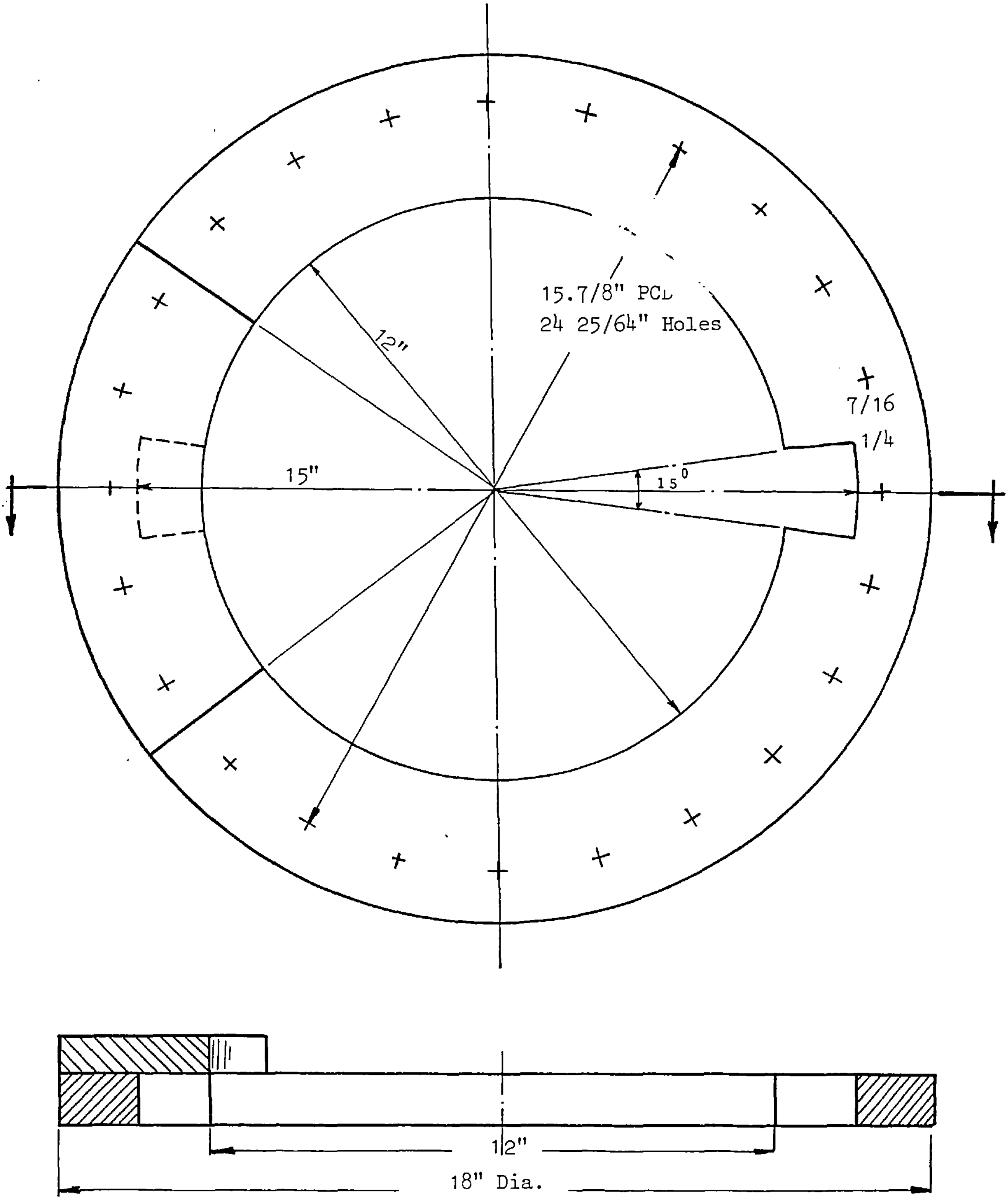


Fig. 2.17: Sample flange

perspex is high, there was no need to measure the uplift of the flange due to the pressure as it would be insignificant. This is not the case if the body and blade of a real valve are made from the same material.

The point load is applied through a calibrated load cell by turning a screw when it is attached to a frame which can be fixed to the flange of the rig as in Fig. (2.25), (2.26) and (2.27). The location of the load can be changed by turning the frame or using different holes in it.

The line support for a circular plate is provided by two short sharp-edged pieces of metal of known lengths attached to a plate which can be fitted to the two clamps at the support. The contact between the supports and the surface of the specimen can be controlled by the screws shown in Figs. (2.28) and (2.29).

Nine rosette strain gauges type FRA-6-11 made by T.S.K. Ltd. Gauge resistance 120 ± 0.5 Ω, gauge length 6mm were stuck to the upper surface of the perspex plate as shown in Figs. (2.20) and (2.32). Another nine dummy gauges of the same type were stuck to a beam of perspex from the same sheet. Fig. (2.30). Each gauge and the corresponding dummy gauge was connected to a channel of two Brueland and Kjaer switching units. An automatic selector, type 1542, which can switch from one gauge to the other at varying speeds down to 0.5 second and a twenty point panel type 1543 was connected to the other Fig. (2.31). This arrangement made it possible to read the 27 gauges in 13.5 seconds (to avoid creep effects).

The active gauge and the dummy one were arranged in a half bridge and fed with 3.0V. through a Brueland and Kjaer strain gauge amplifier, type 1526. The output was fed back to the amplifier which indicated the total strain in micro strains on a L.E.D. display. Because of the speed of switching this had to be fed in to S & P twelve-twelve U.V. recorder. Fig. (2.33) shows a plot at two known strains which was obtained at the beginning to provide the scale. Fig. (2.34) shows a typical full-scale result. All gauges were balanced within ± 50 micro strains at the beginning of every experiment and a 'no-load' plot was obtained for corrections as shown on page

The results were converted into maximum and minimum principal stresses using the Mohr strain circle principle as in Fig. (2.35) where:-

$$x = \frac{\varepsilon_x + \varepsilon_y}{2}$$

$$y = \frac{\varepsilon_x - \varepsilon_y}{2}$$

$$z = x - \varepsilon_{45}$$

$$R = \sqrt{y^2 + z^2}$$

$$\varepsilon_1 = x + R$$

$$\varepsilon_2 = x - R$$

and the principal stresses

$$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2)$$

$$\sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

Fig. 2.19. Perspex blade sample

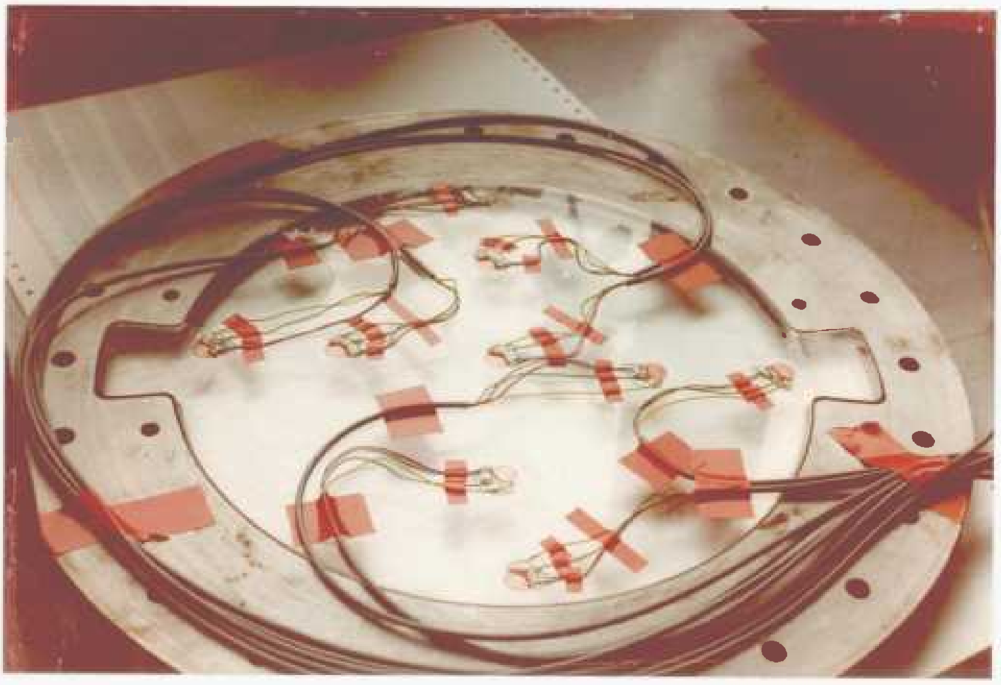


Fig. 2.20. Sample blade with strain gauges connected fitted in testing flange

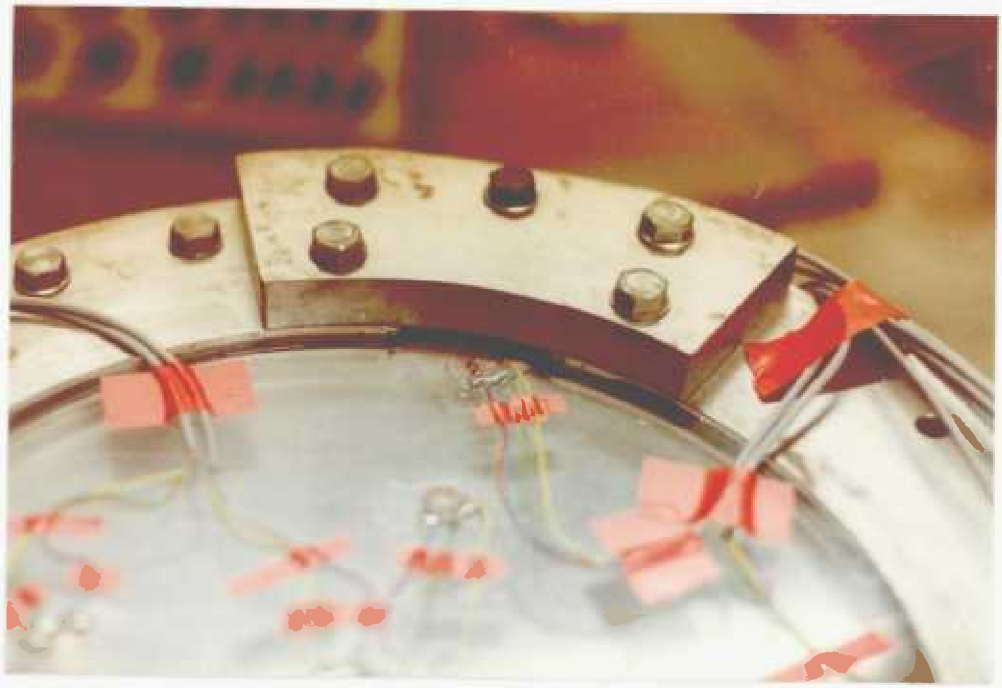


Fig. 2.21. Short flange for clamping

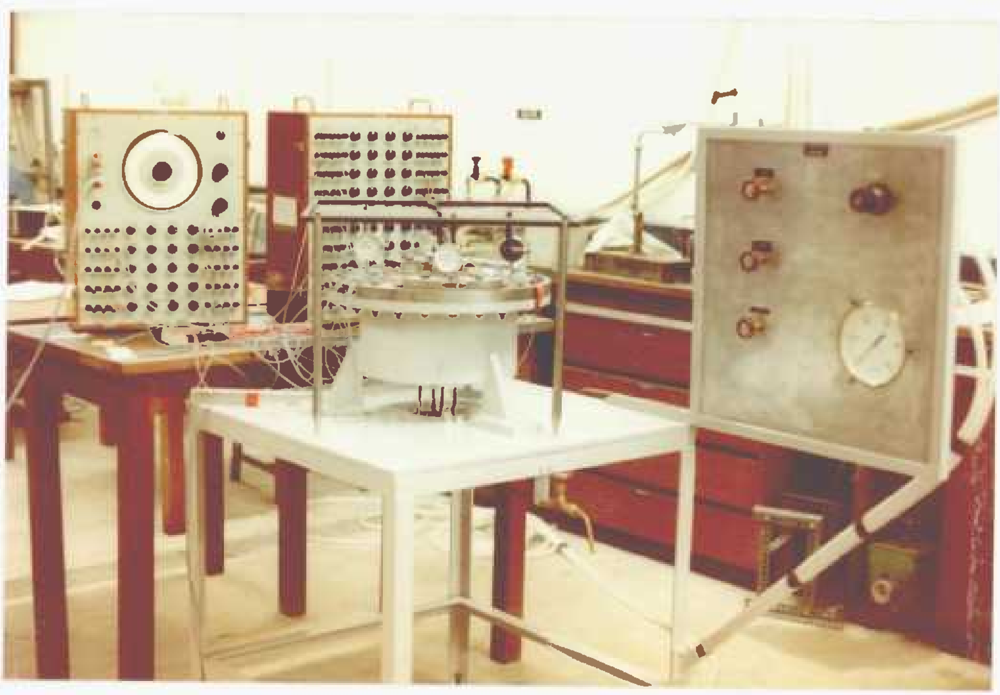


Fig. 2.22. Testing rig.

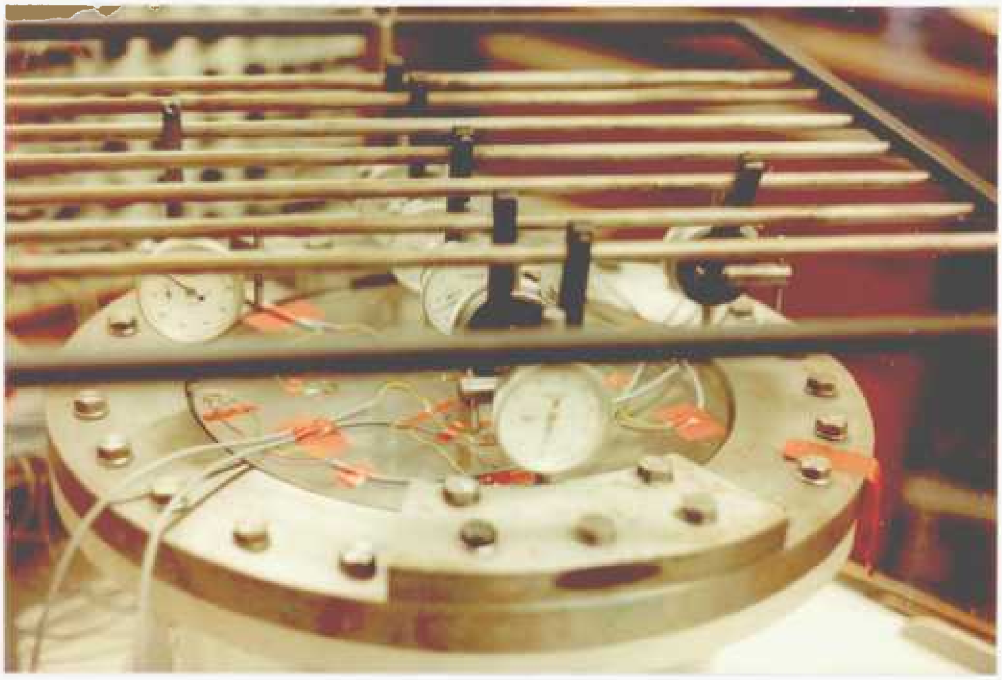


Fig. 2.23: Dial gauges during testing

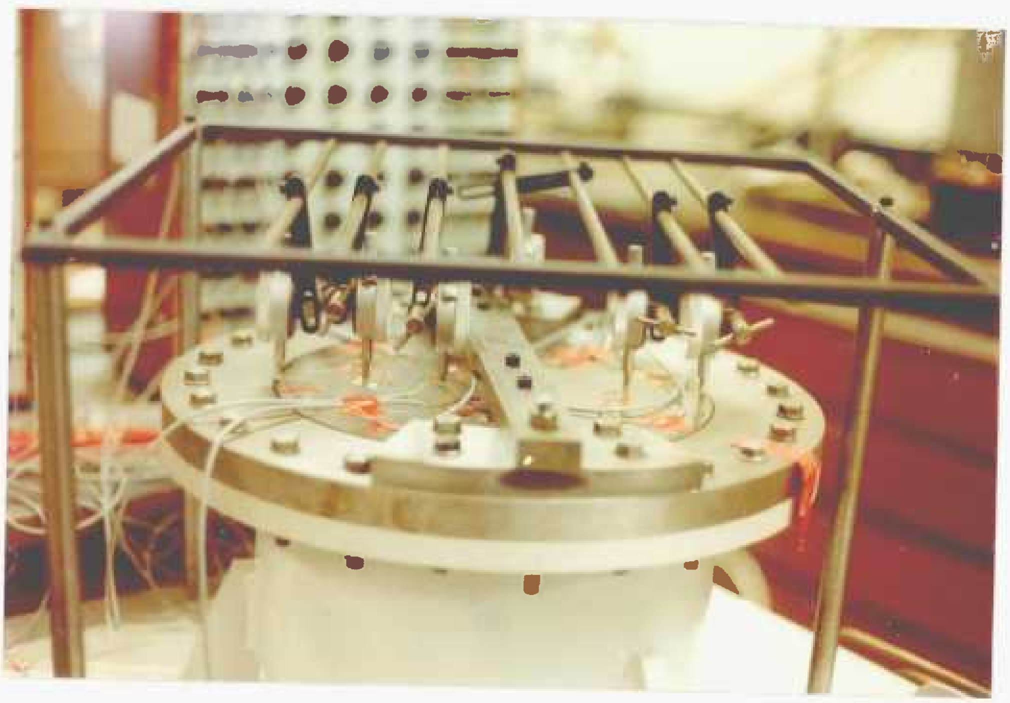


Fig. 2.24: Dial gauges frame

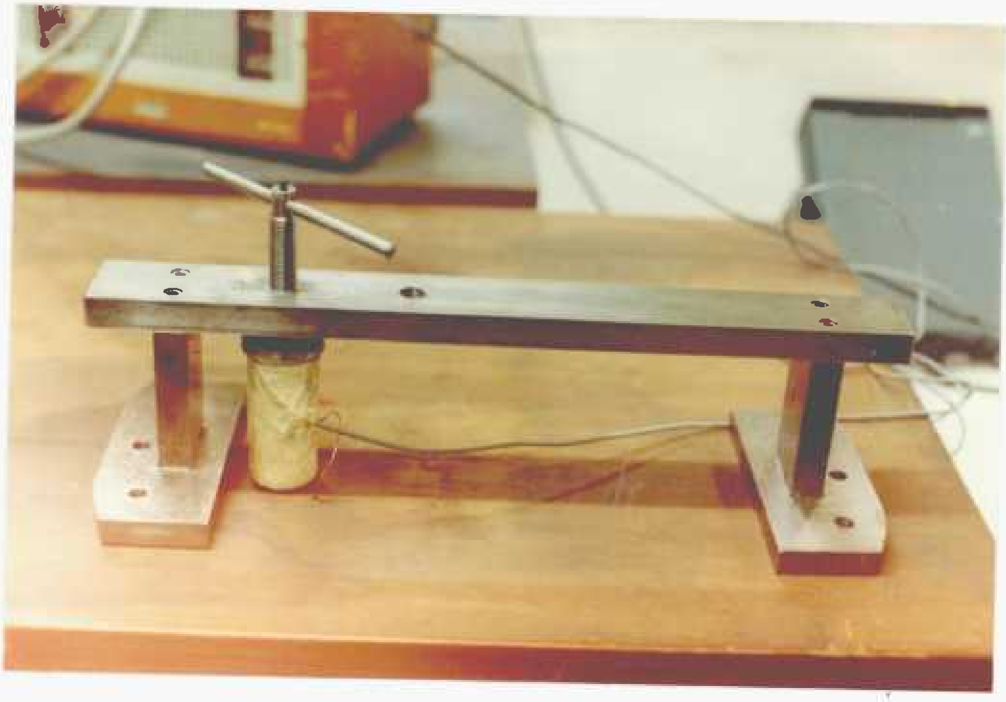


Fig. 2.25. Point load cell

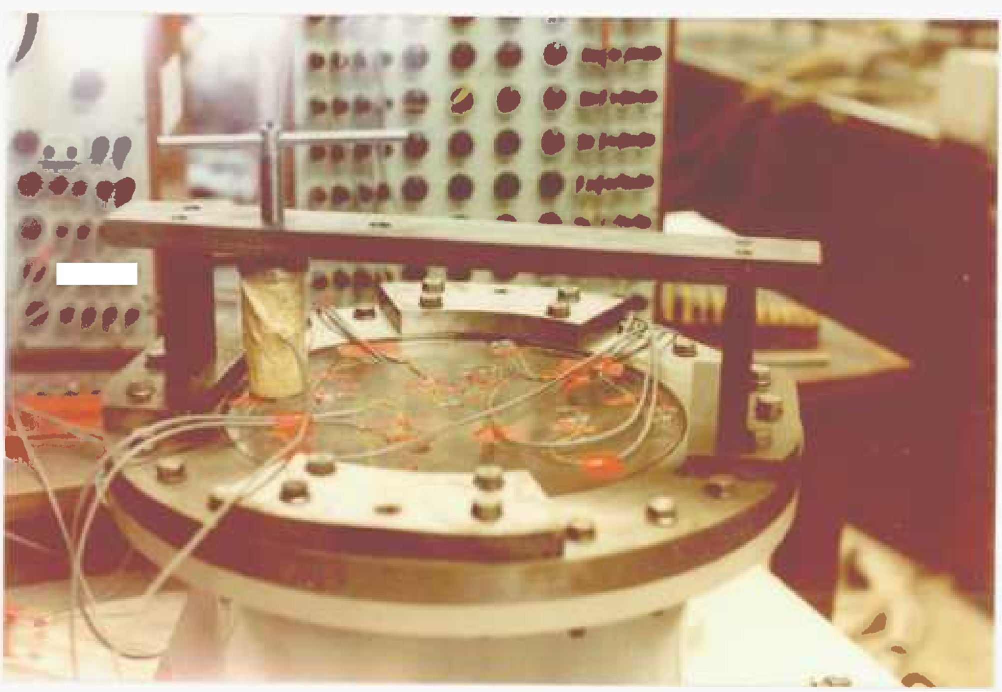


Fig. 2.26. Point load cell attached to testing rig.

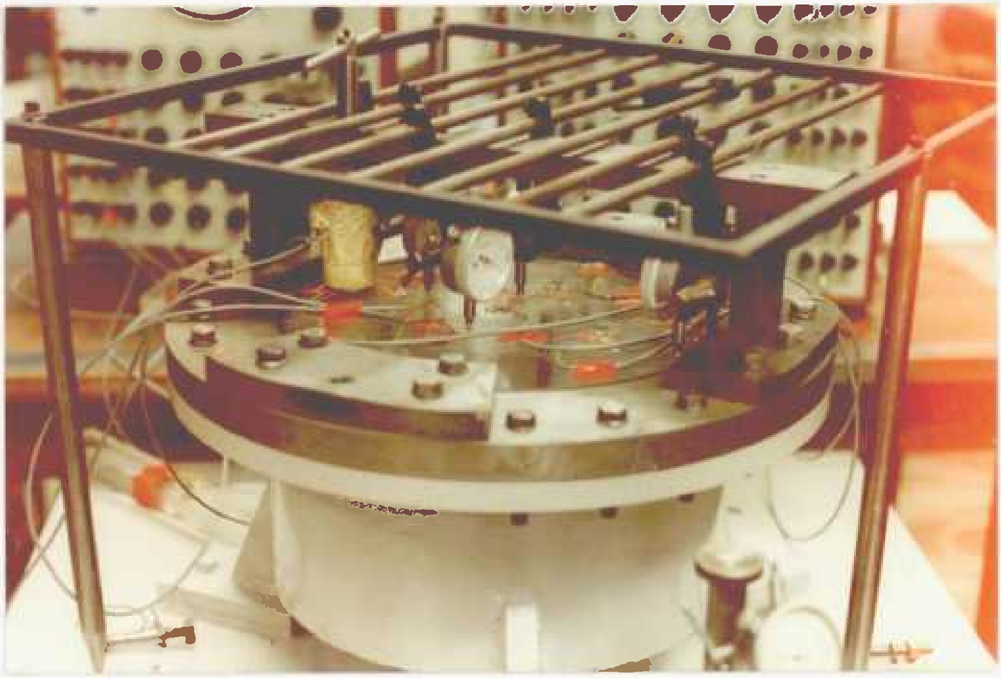


Fig. 2.27. Point load testing.

Fig. 2.28. Line support

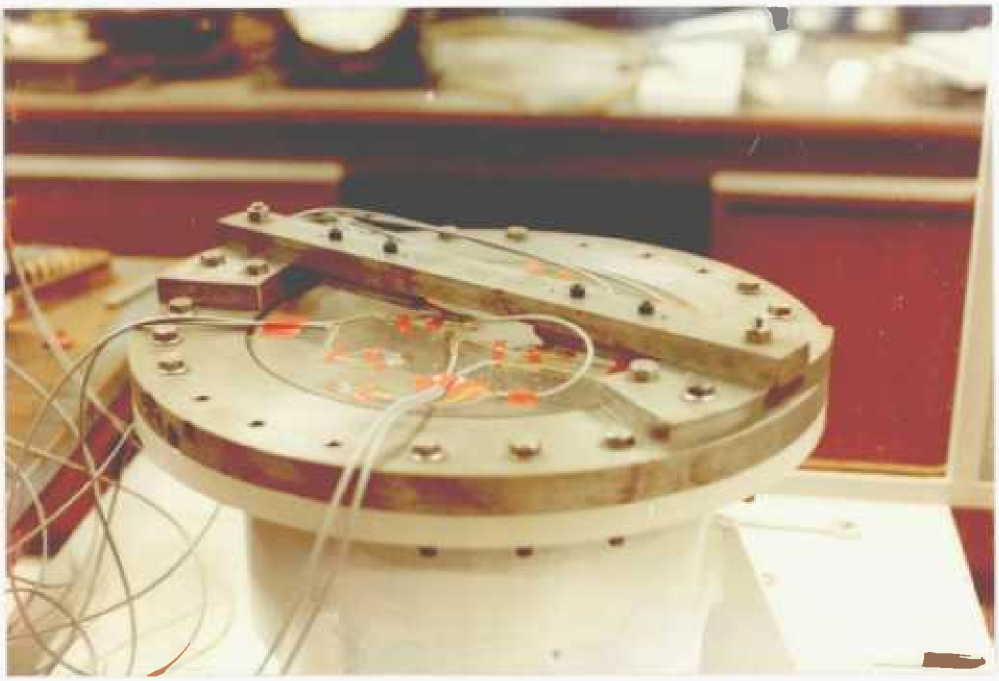


Fig. 2.29. Line support uniform pressure testing

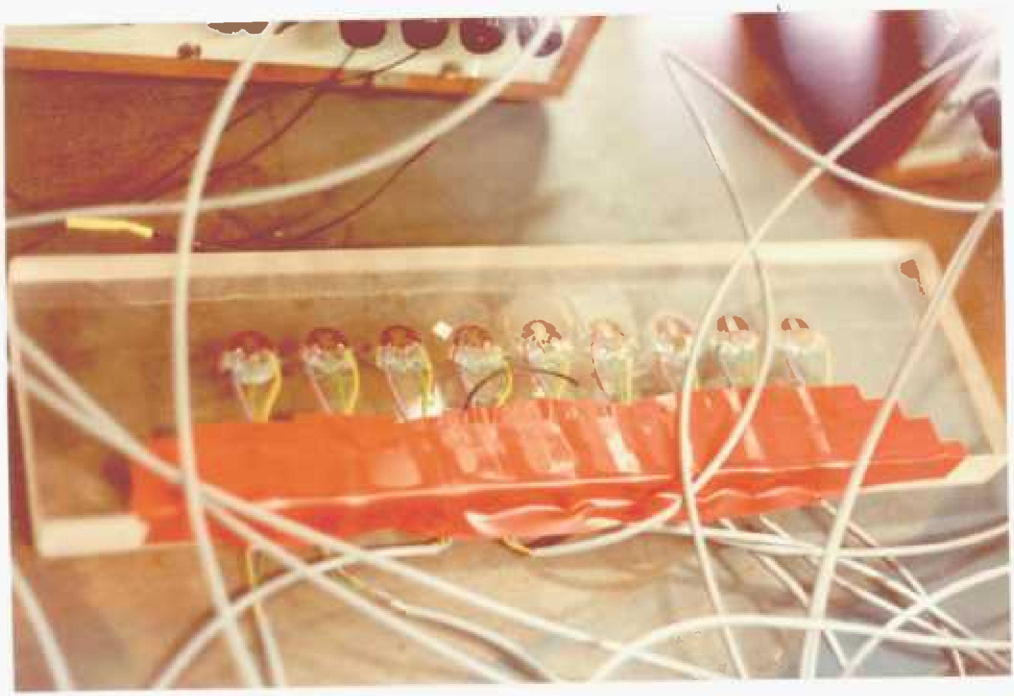


Fig. 2.30. Dummy gauges

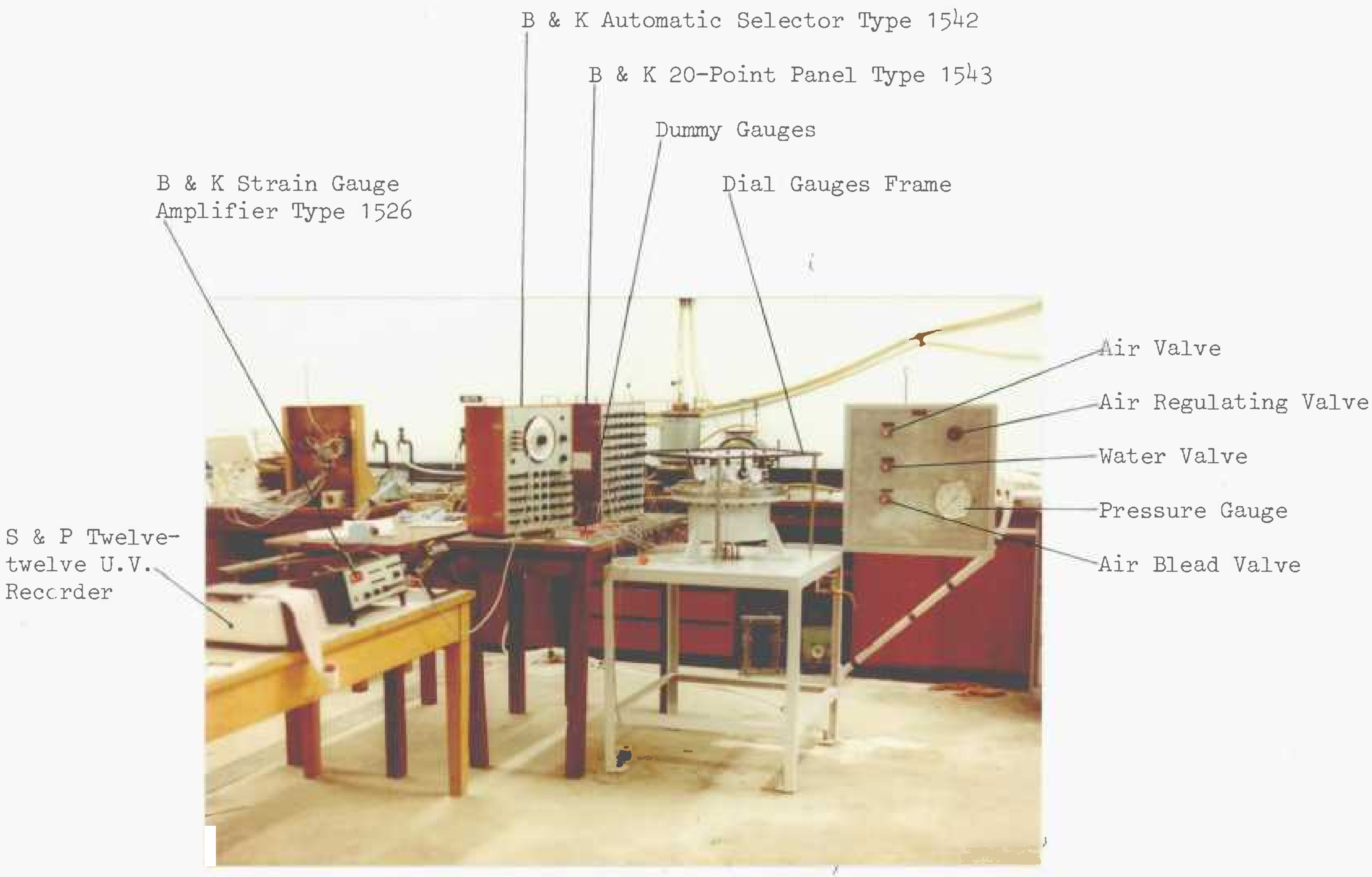


Fig. 2.31: Complete testing equipment

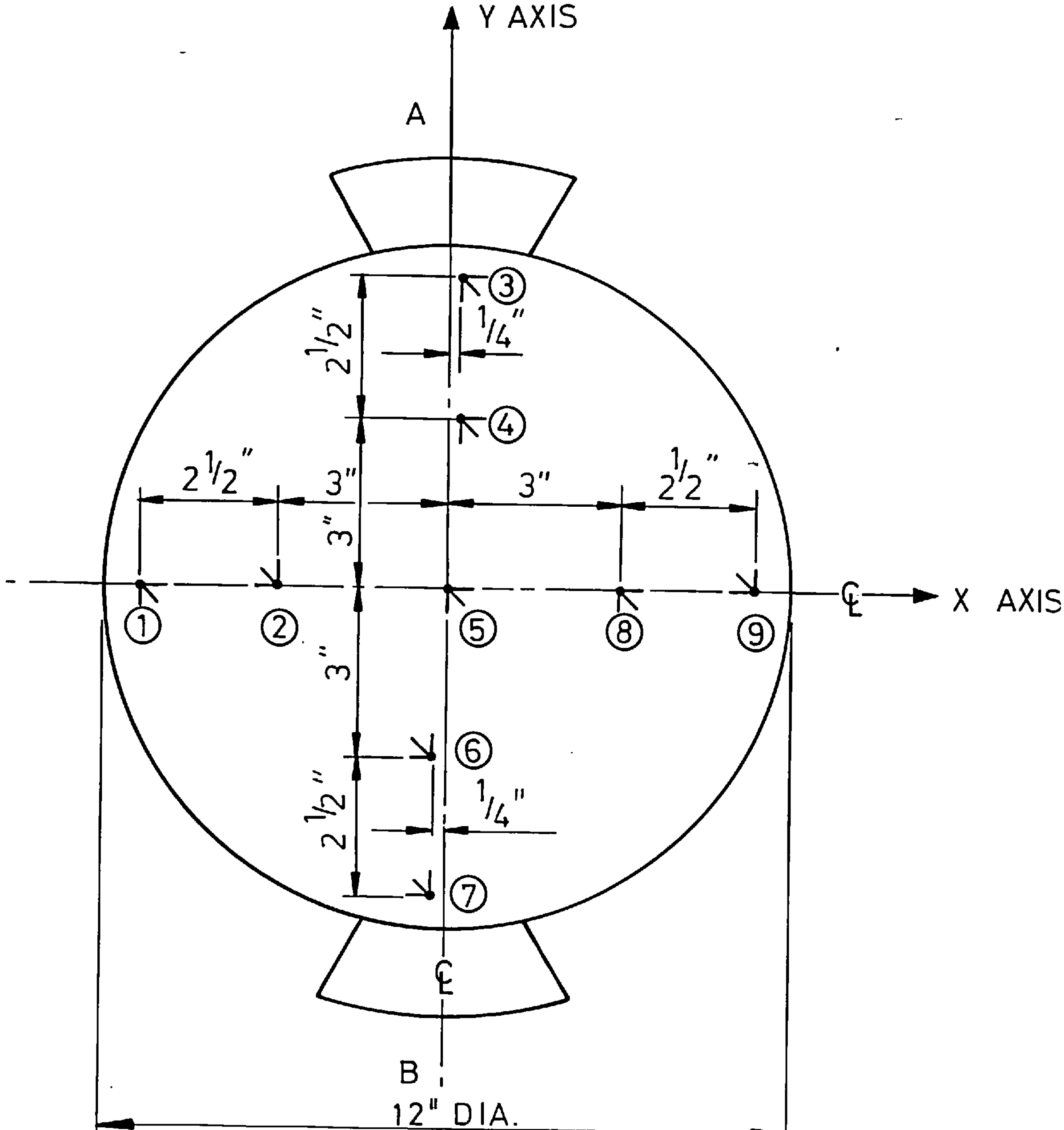


Fig. 2.32. Location of strain gauges.

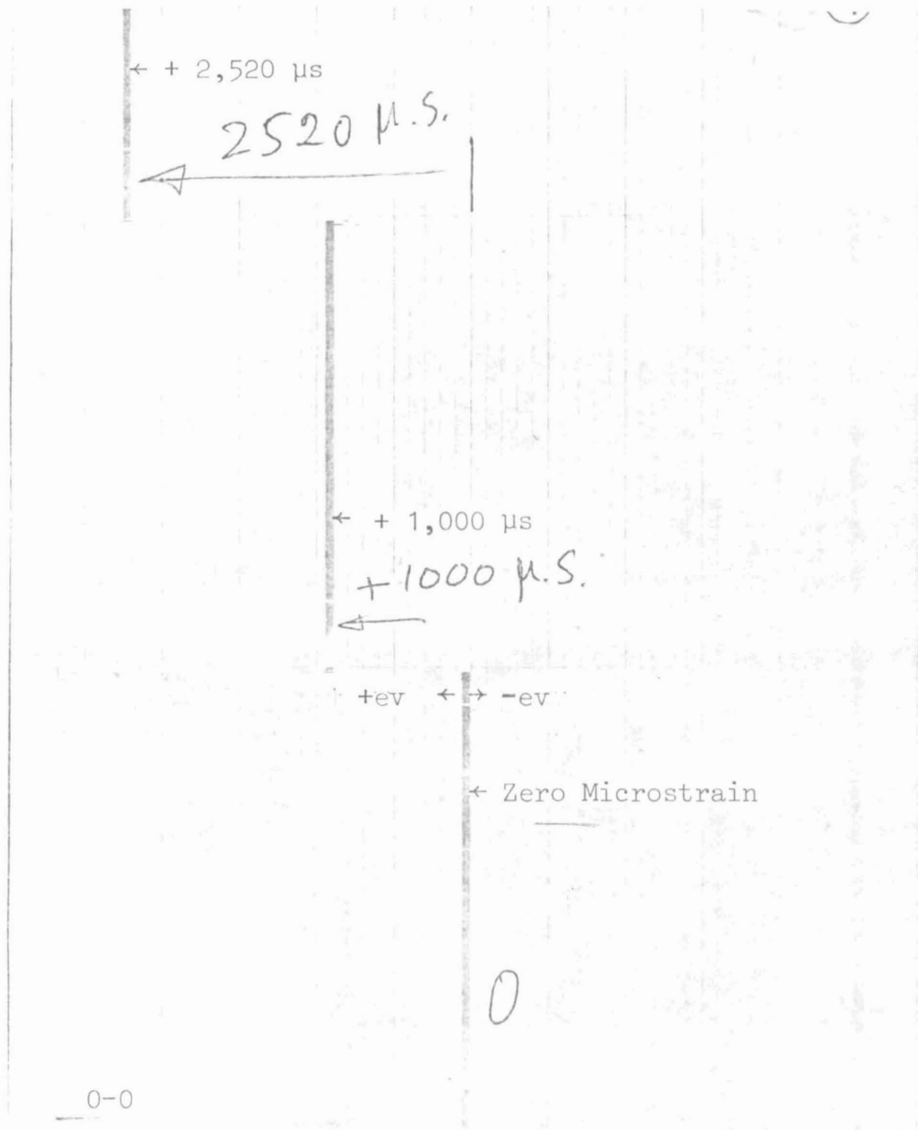


Fig. 2.33. U.V. recorder output

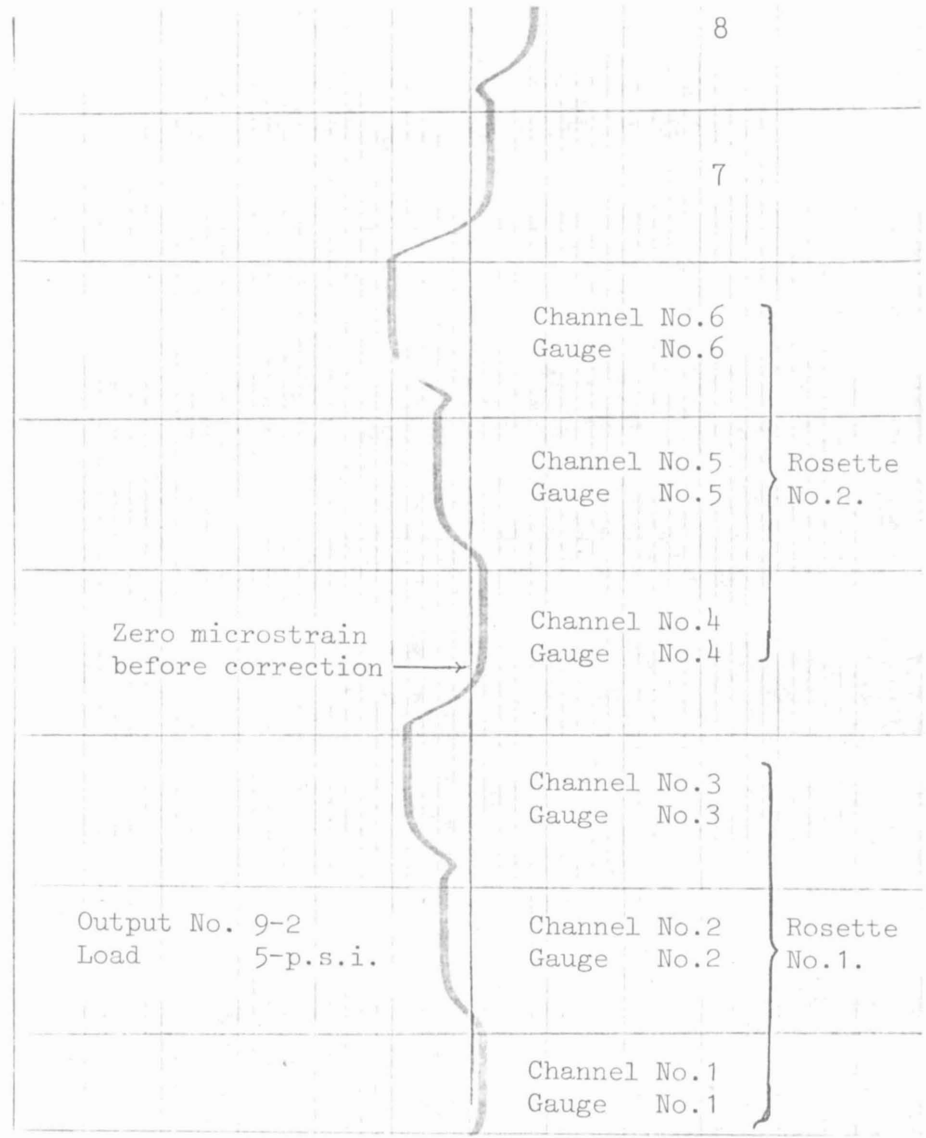


Fig. 2.34. Full-scale u.v. strain output

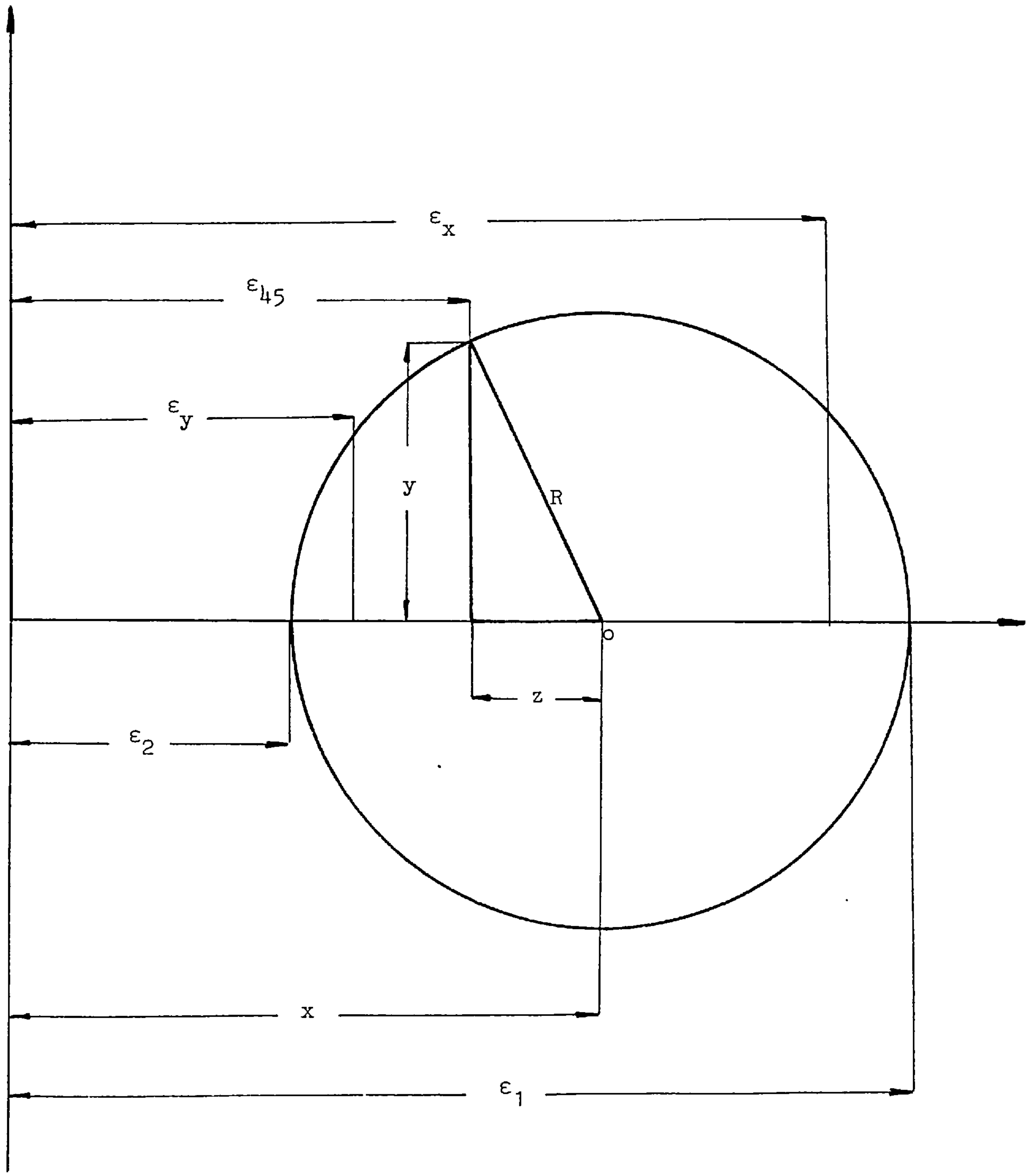


Fig. 2.35: Strain circle of a rosette

Where ε_x, ε_y, ε_{45} are the strains read from the plottings to the nearest 0.25mm multiplied by the scale of the u.v. recorder and fed into a small computer program listed in Appendix (2.D) which converted them from micro strains to strains and prints out the rosette number, the strains, the maximum and minimum strains ε_1 and ε_2 and the principal stresses σ_1 and σ_2.

2.5.4. Testing a 12" Diameter Disc supported on two clamped short arcs subjected to a uniform pressure.

(i) Deflections

Measuring the deflections at five locations as shown in Fig. (2.36) and (2.23) initial loading and unloading readings were taken and the results plotted on Fig. (2.37) and (2.38) against a plot of the deflections obtained from the finite element solution for a clamped and simply supported arc. The similarity between the two results is clear and satisfactory.

(ii) Stresses

The strain gauges were read in 13.5 seconds after the load was reached to avoid the creep effect on the reading. The creep would stop in 5 to 6 hours if the load was kept constant but this proved to be an impractical procedure.

The results plotted on the u.v. recorder are given on pages 130 to 136 reduced photographically. A full scale copy of the beginning of these records is shown in Fig. (2.34). The computer outputs for the principal stresses at the rosette locations is given on pages 137 to 142. The maximum stresses are plotted for each gauge against the maximum principal stresses obtained from the finite element solution for clamped arc support and simply supported arc. The program provides the principal stresses at the centre of the elements. The location of the strain gauges on the finite element mesh is shown in Fig. (2.39). Gauges 3 and 4 are shown on the equivalent location due to the symmetry. This case is symmetrical around x axes and y axes.

The results in Figs. (2.40), (2.41), (2.42), (2.43), (2.44), (2.45) and (2.46) show good agreement with the finite element solution and therefore with the solution based on the coefficients derived from it.

As in real valves it can be seen that the support was not 100% clamped. The degree of fixity is a major quality control problem in the clearances between the shaft and the body and the type of bearing used.

2.5.5. Testing a 12" Diameter Disc supported on two Clamped short arcs subjected to a point load.

Increasing loads of 200, 400 and 500 lbs. were applied through the pressure cell and then reduced similarly. The location of the applied load is shown in Fig. (2.47) and (2.26).

(i) Deflections

The deflections were measured at six locations as shown in Fig. (2.47) and (2.27). The results are plotted on Fig. (2.48) . (2.49), (2.50a) and (2.50b) against a plot of the deflections obtained from the finite element solution for a clamped and a simply supported arc. The similarity between the two is clear and satisfactory.

(ii) Stresses

The strains were read in the same manner as before, the results plotted on the u.v. recorder are given on pages 151 to 156.

The computer outputs for the principal stresses at the rosette locations is given on pages 157 to 161.

The maximum stresses are plotted for each gauge against the maximum principal stresses obtained from the finite element solution for clamped arc support and simply supported arc, as shown in Figs. (2.51), (2.52), (2.53), (2.54), (2.55), (2.56) and (2.57).

The results show good agreement in the vicinity of the applied load and give higher stresses than expected by the finite element solution. This is considered satisfactory because in practice this load is small in comparison with the uniformly dibtributed pressure on the valves.

2.5.6. Testing 12" Diameter Disc supported on two clamped short arcs and two short lengths of the diameter subjected to a uniform pressure.

(i) Deflections

The deflections were measured at the locations shown in Fig. (2.58). The results are plotted on Figs. (2.59) and (2.60) against the finite element solution. The results are self-explanatory. From the finite element solution it can be seen that the deflections are governed by the diameter support and the effect of the arc being clamped or simply supported is insignificant

(ii) Stresses

The strains were read in the same manner as before, the results plotted on the u.v. recorder are given on pages 173 to 180.

The computer outputs for the principal stresses at the rosette locations is given on pages 181 to 187.

The maximum stresses are plotted for each gauge against the maximum principal stresses obtained from the finite element solution for clamped and simply supported arcs. This is shown in Figs. (2.61), (2.62), (2.63), (2.64) (2.65), (2.66), (2.67) and (2.68).

The results show good agreement with the finite element solution and therefore with the solution based on the coefficients derived from it.

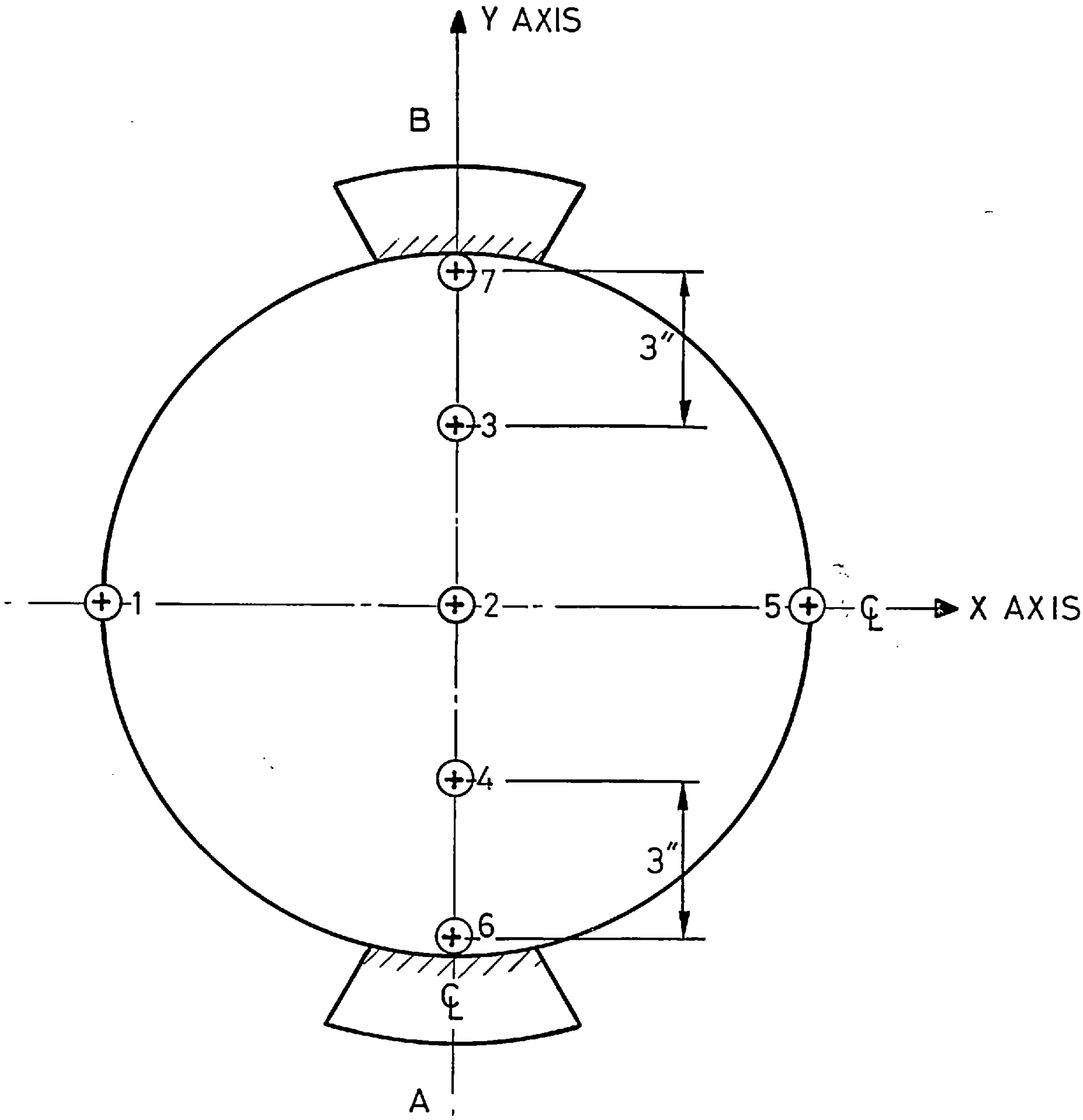


Fig. 2.36a Dial Gauge Locations

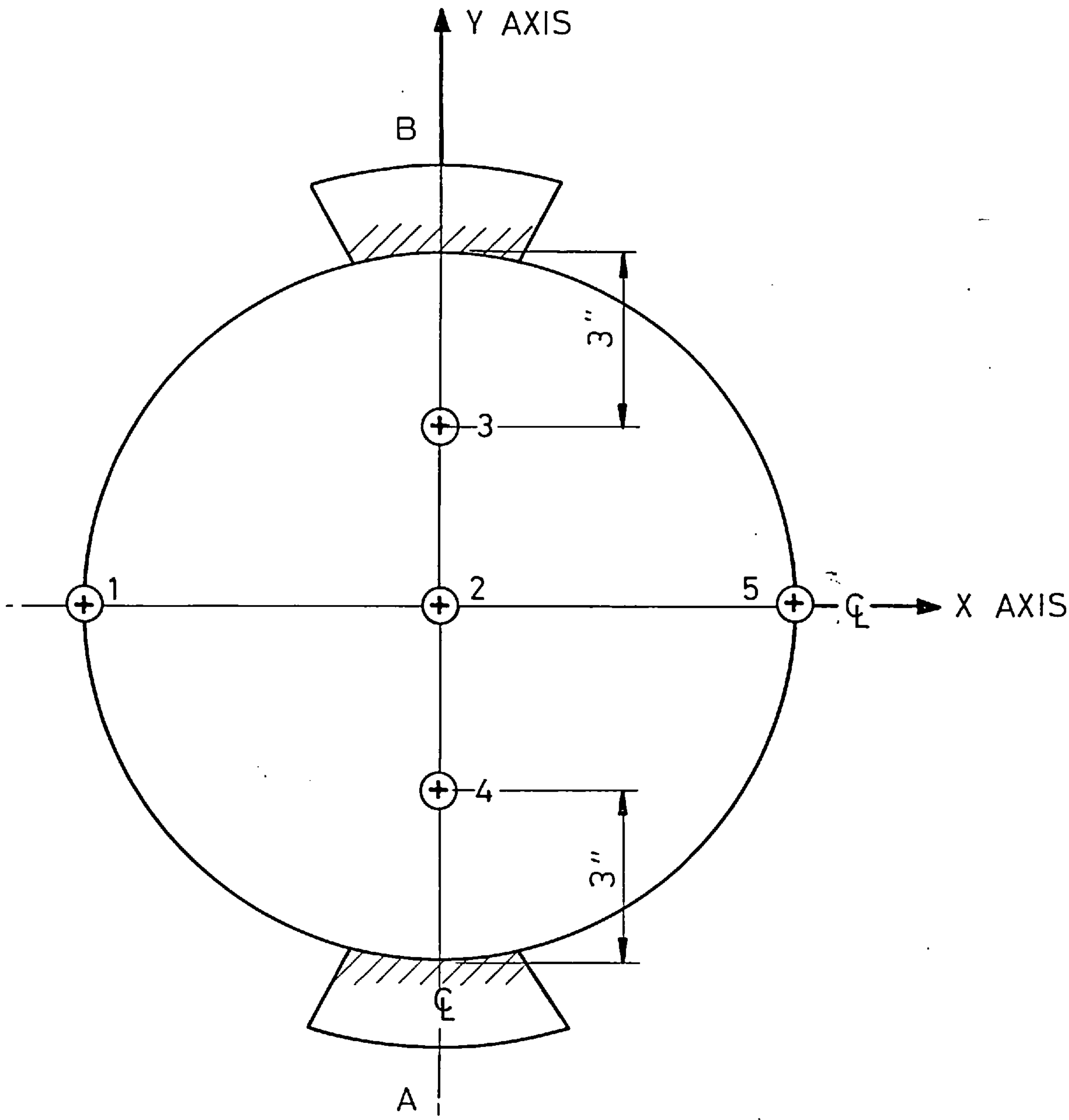


Fig. 2.36b : Dial gauge locations

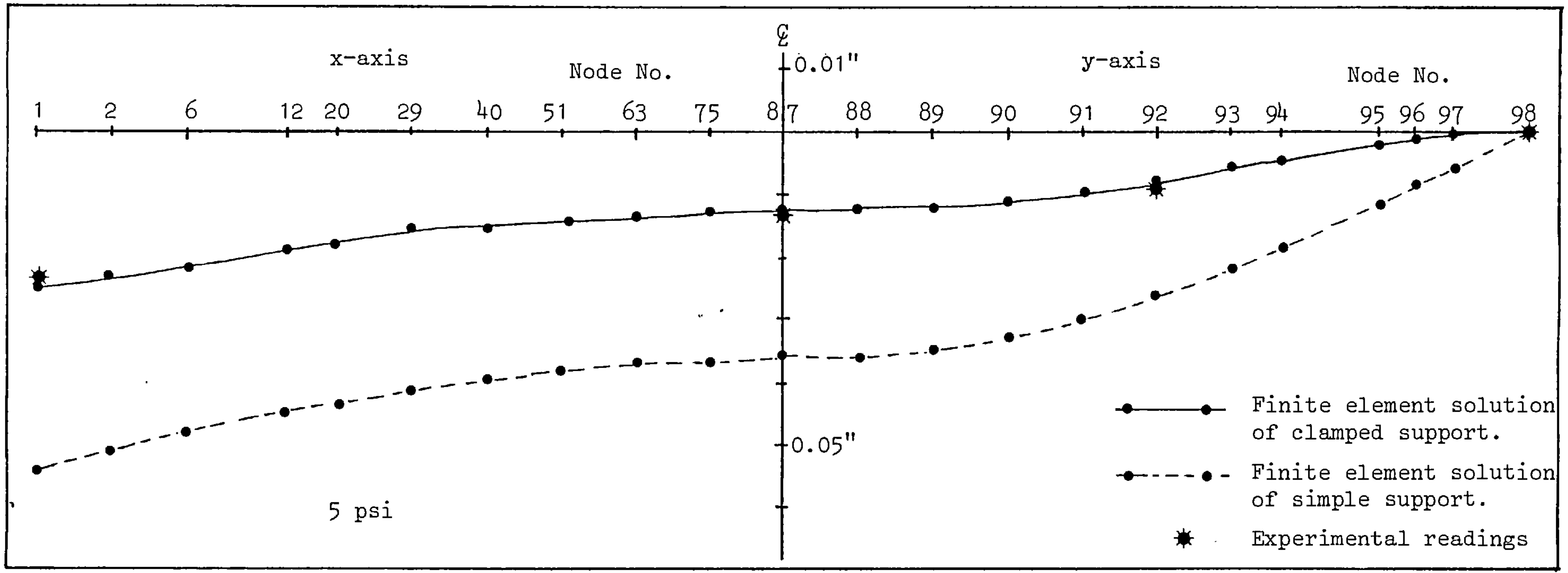


Fig. 2.37. Comparison of deflections due to uniform distributed pressure along x & y-axis.

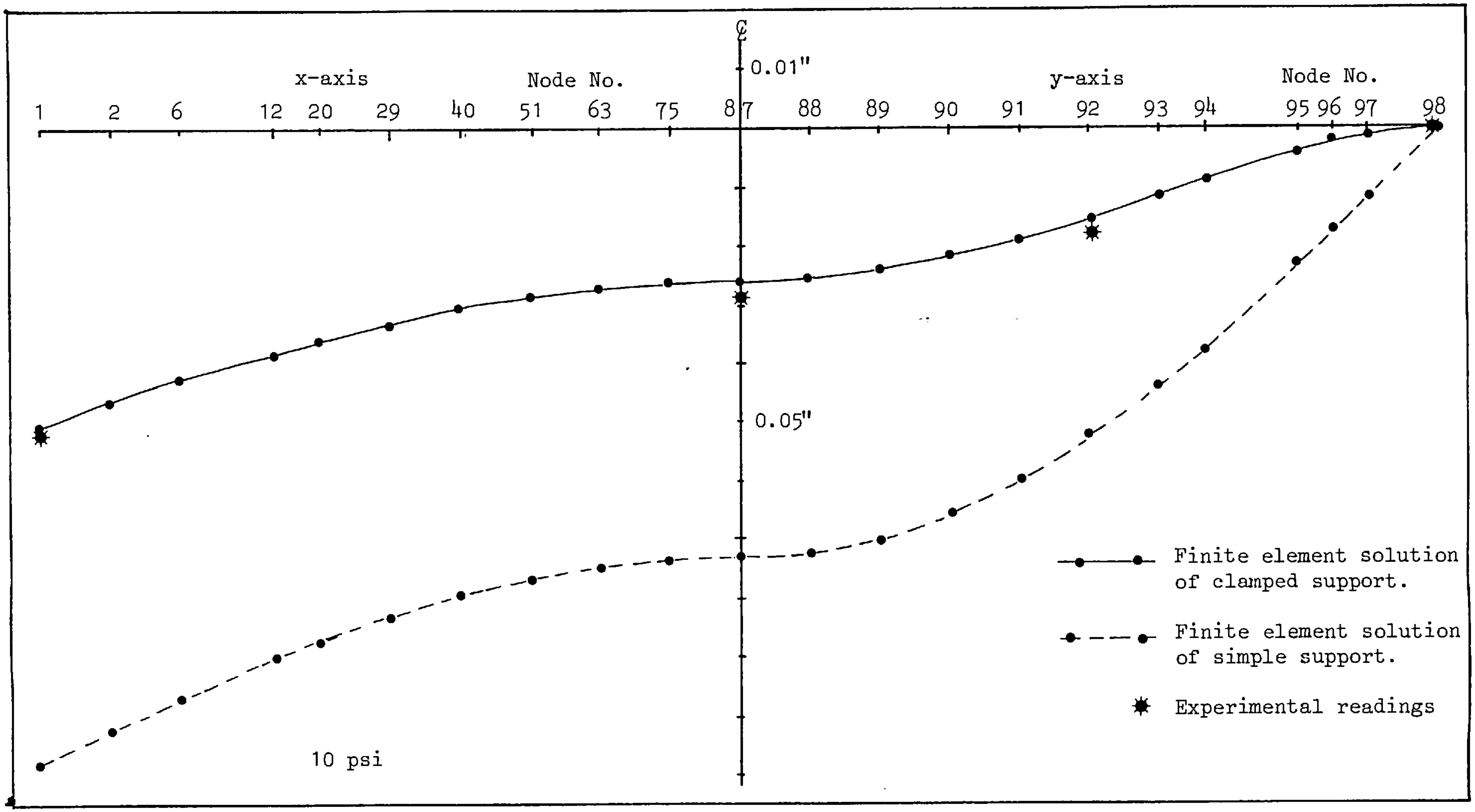


Fig. 2.38. Comparison of deflections due to uniform distributed pressure along x & y-axis

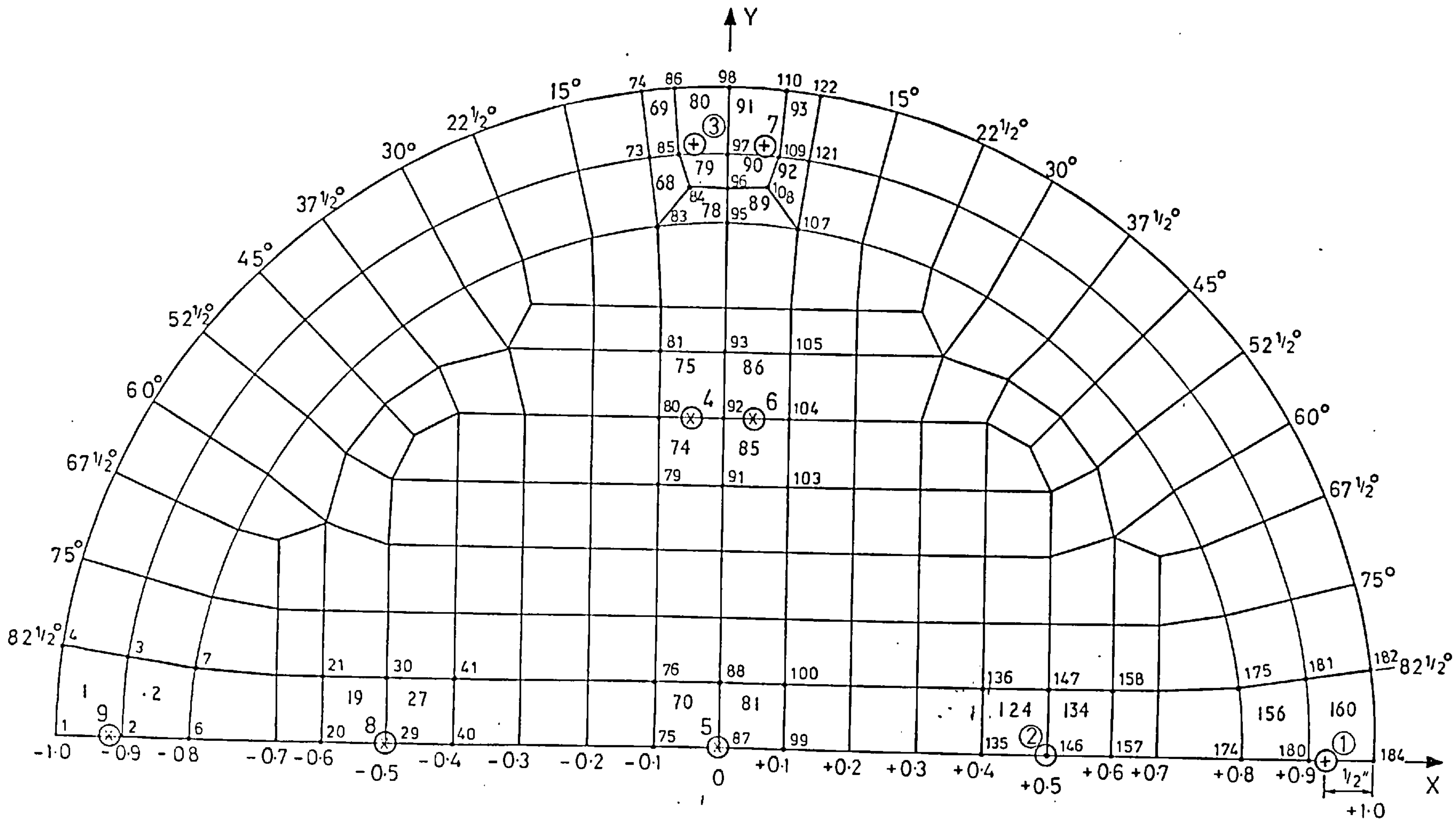
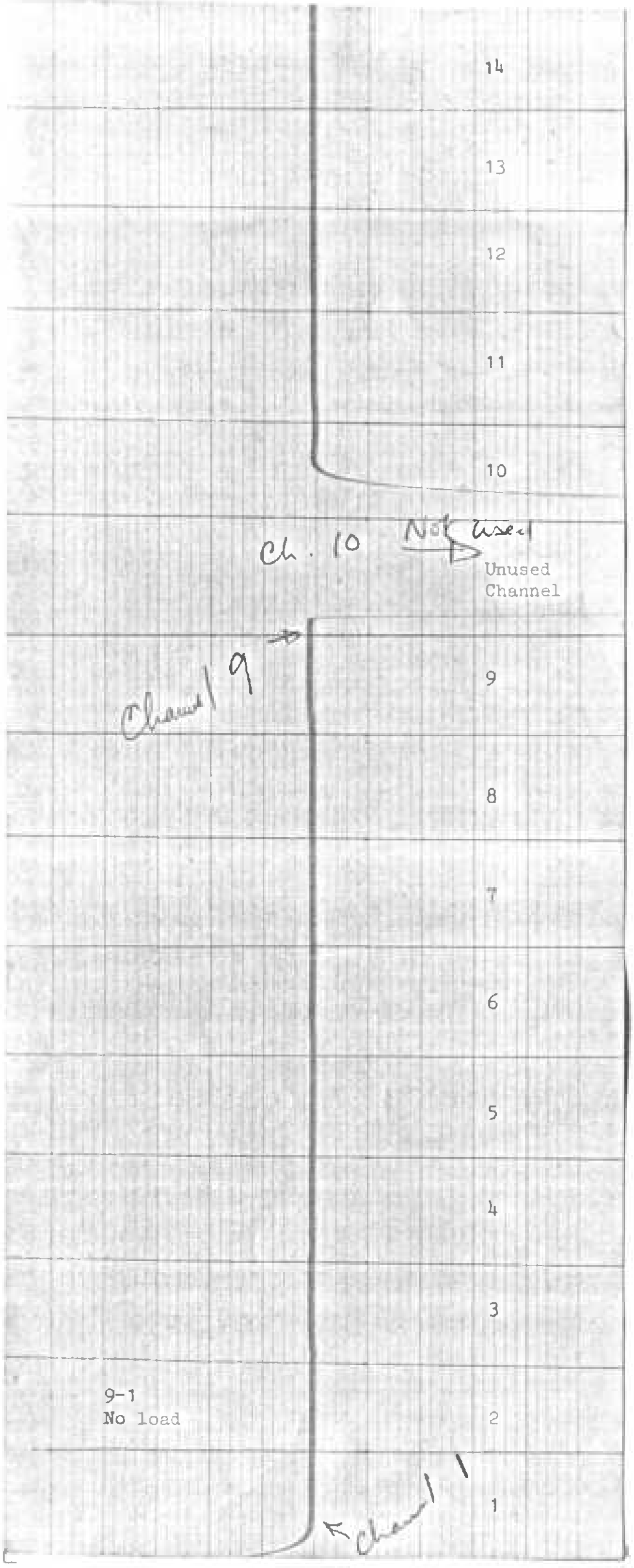
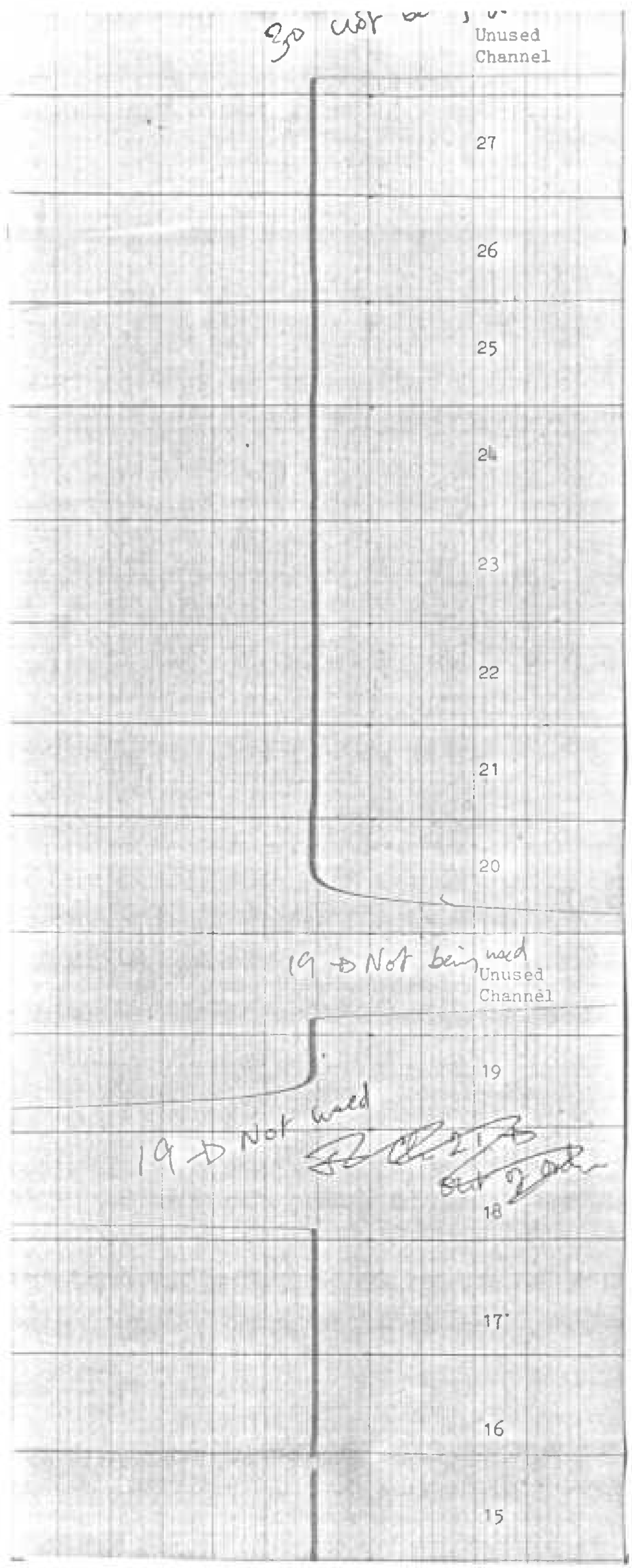
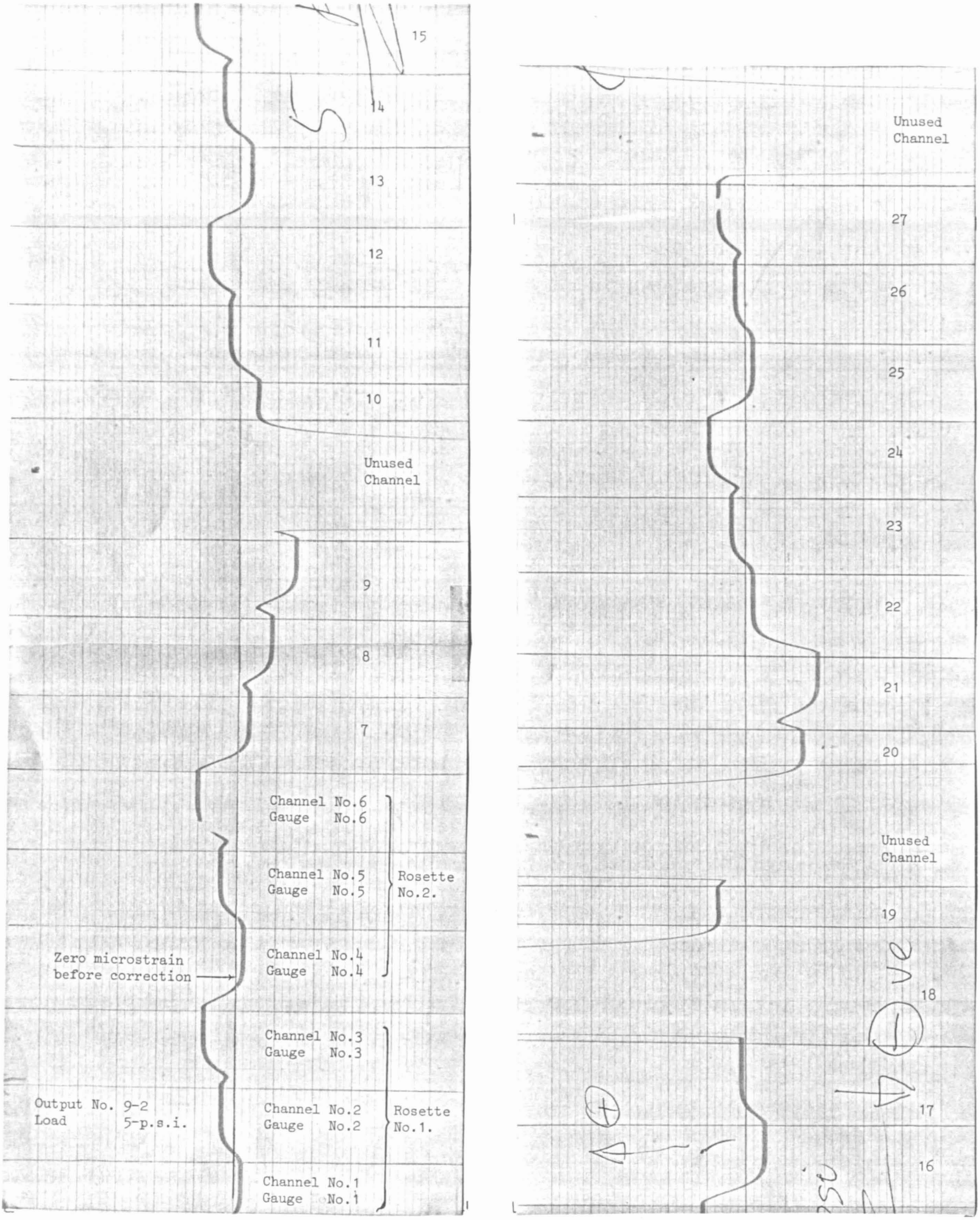
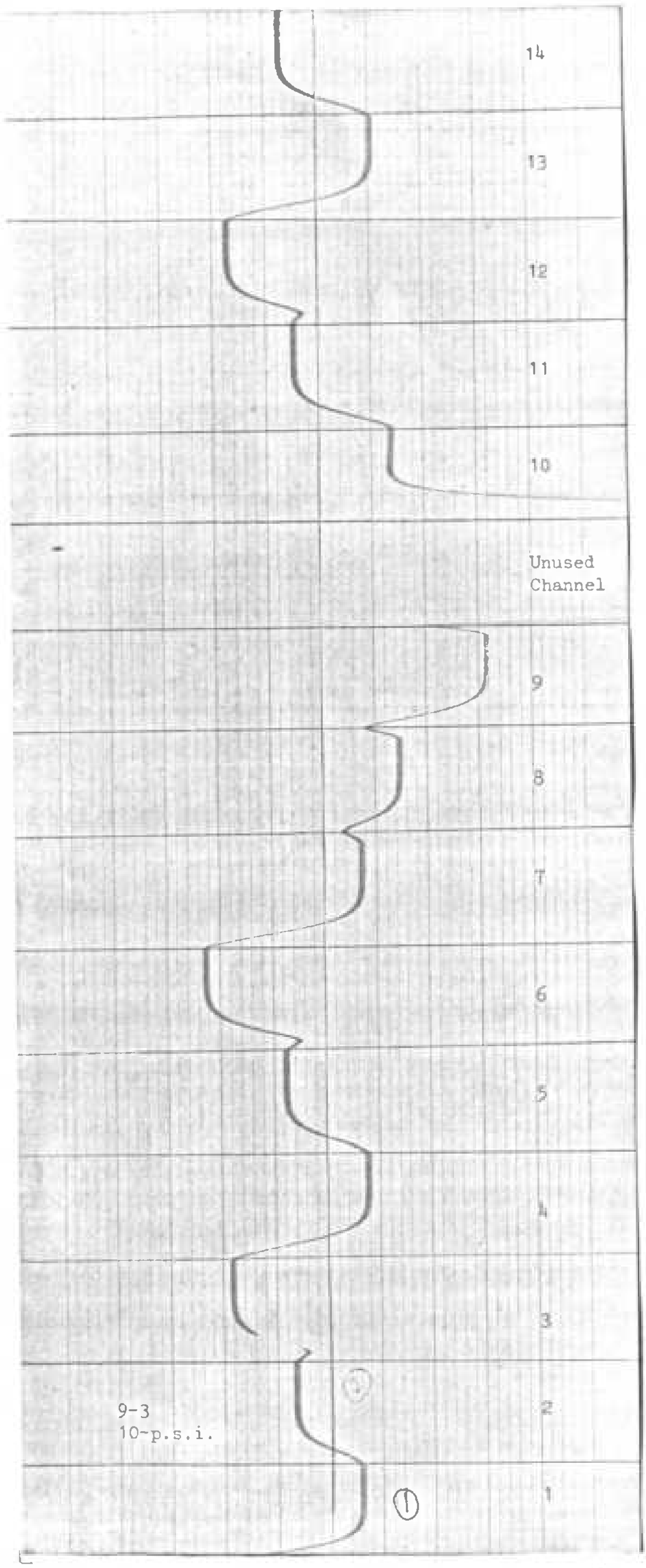


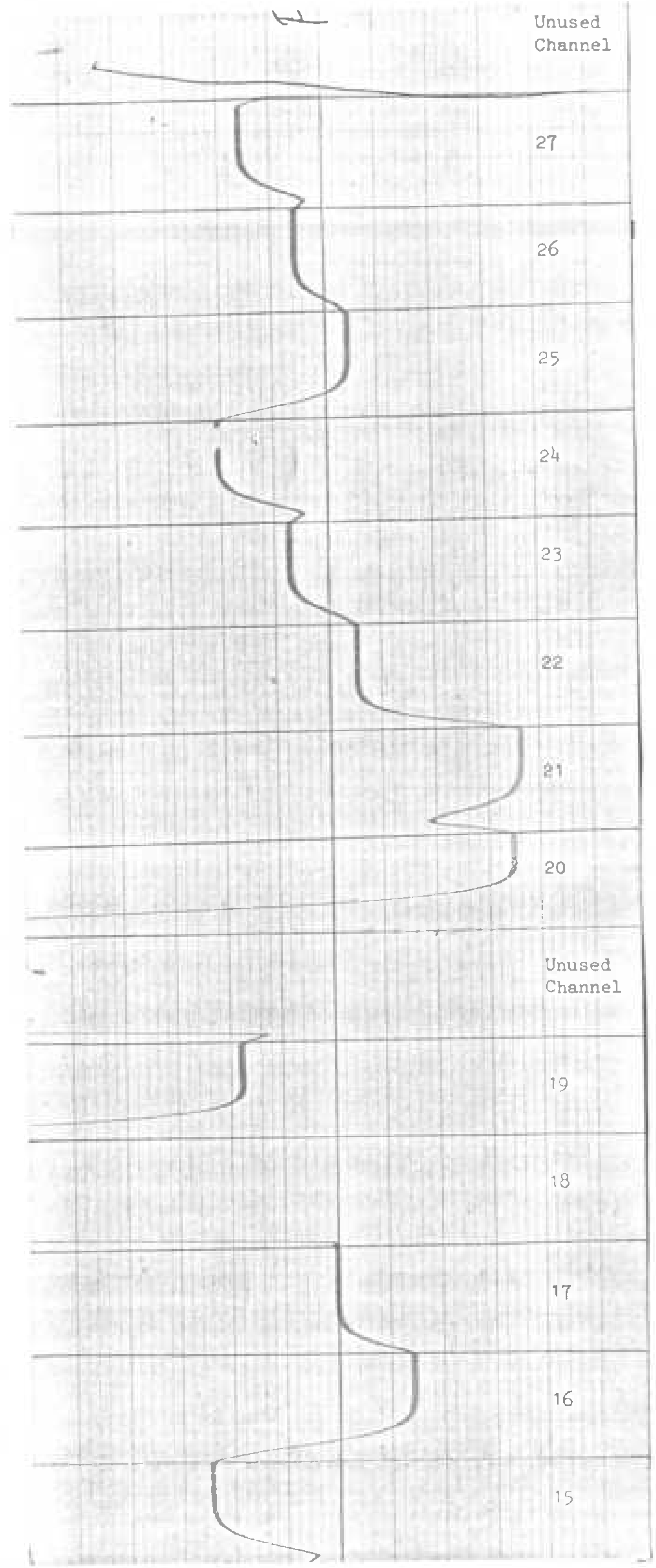
Fig. 2.39. LOCATION OF STRAIN GAUGES ON F.E. MESH.

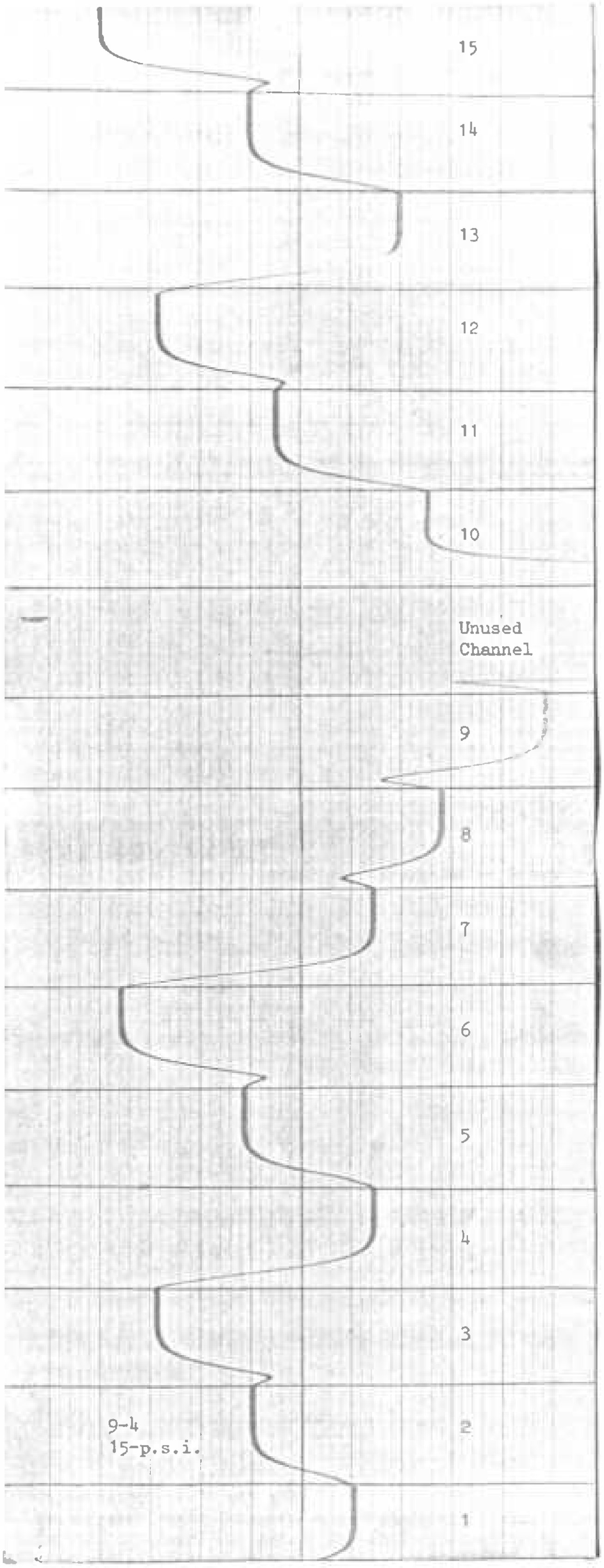

14
13
12
11
10
Ch. 10 Not used
Unused
Channel
Channel 9
9
8
7
6
5
4
3
9-1
No load
2
1
Channel 1

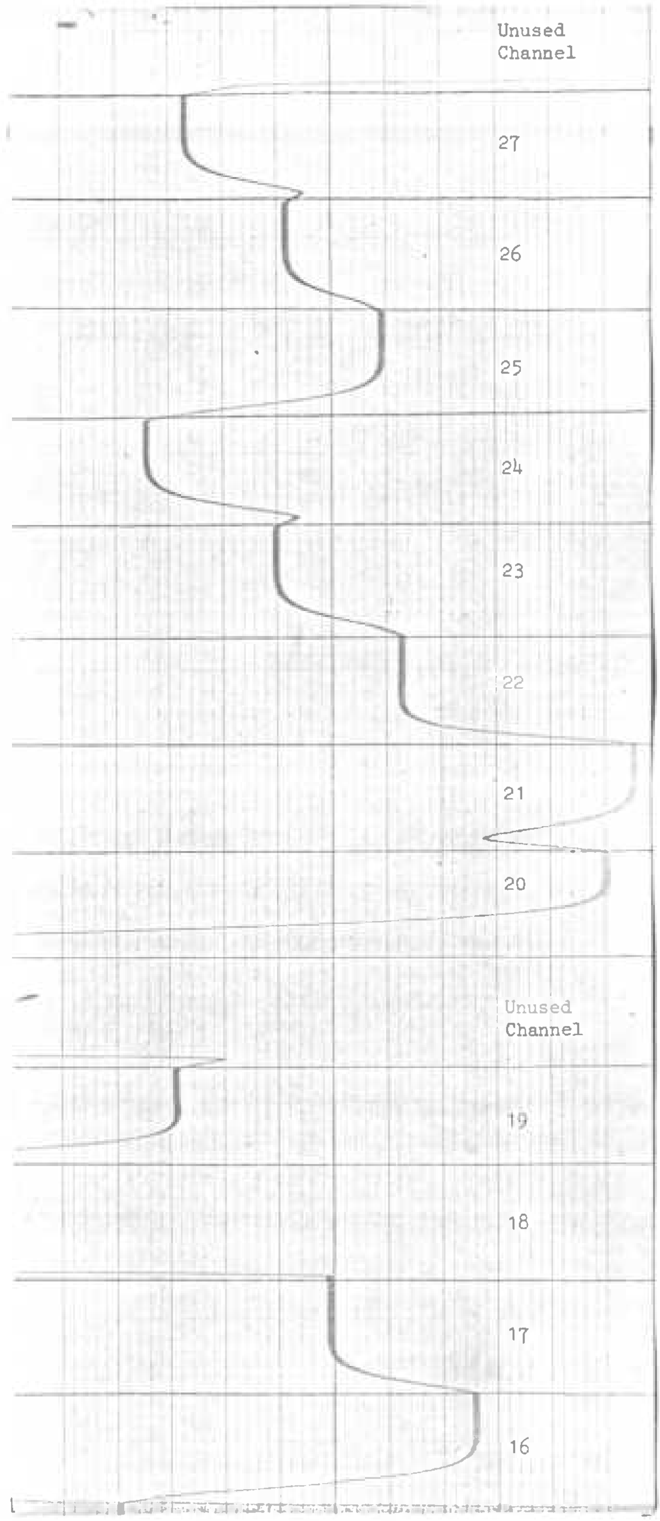

Unused
Channel
27
26
25
24
23
22
21
20
19 Not being used
Unused
Channel
19
19 Not used
18
17
16
15

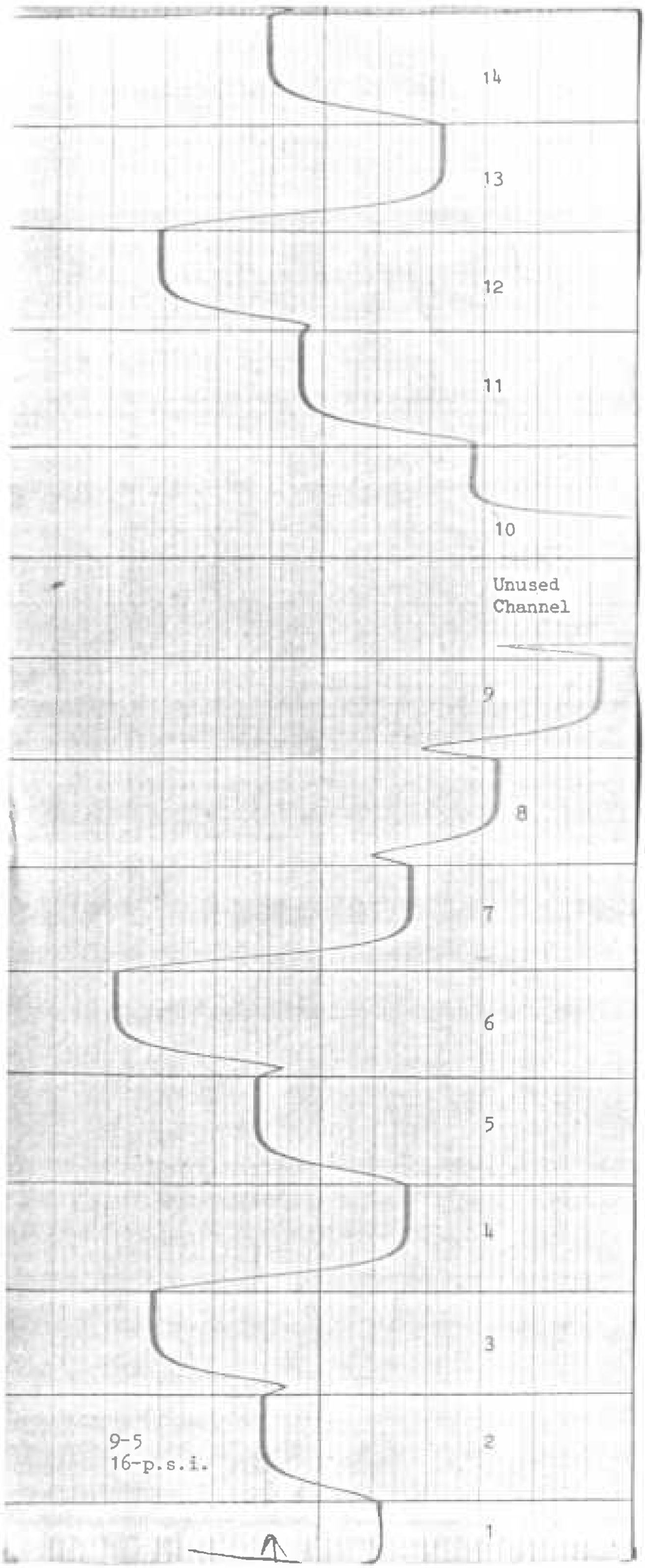

15
14
13
12
11
10
Unused Channel
9
8
7
Channel No.6 Gauge No.6
Channel No.5 Gauge No.5
Channel No.4 Gauge No.4
Rosette No.2.
Zero microstrain before correction
Channel No.3 Gauge No.3
Channel No.2 Gauge No.2
Channel No.1 Gauge No.1
Rosette No.1.
Output No. 9-2
Load 5-p.s.i.
Unused Channel
27
26
25
24
23
22
21
20
Unused Channel
19
18
17
16

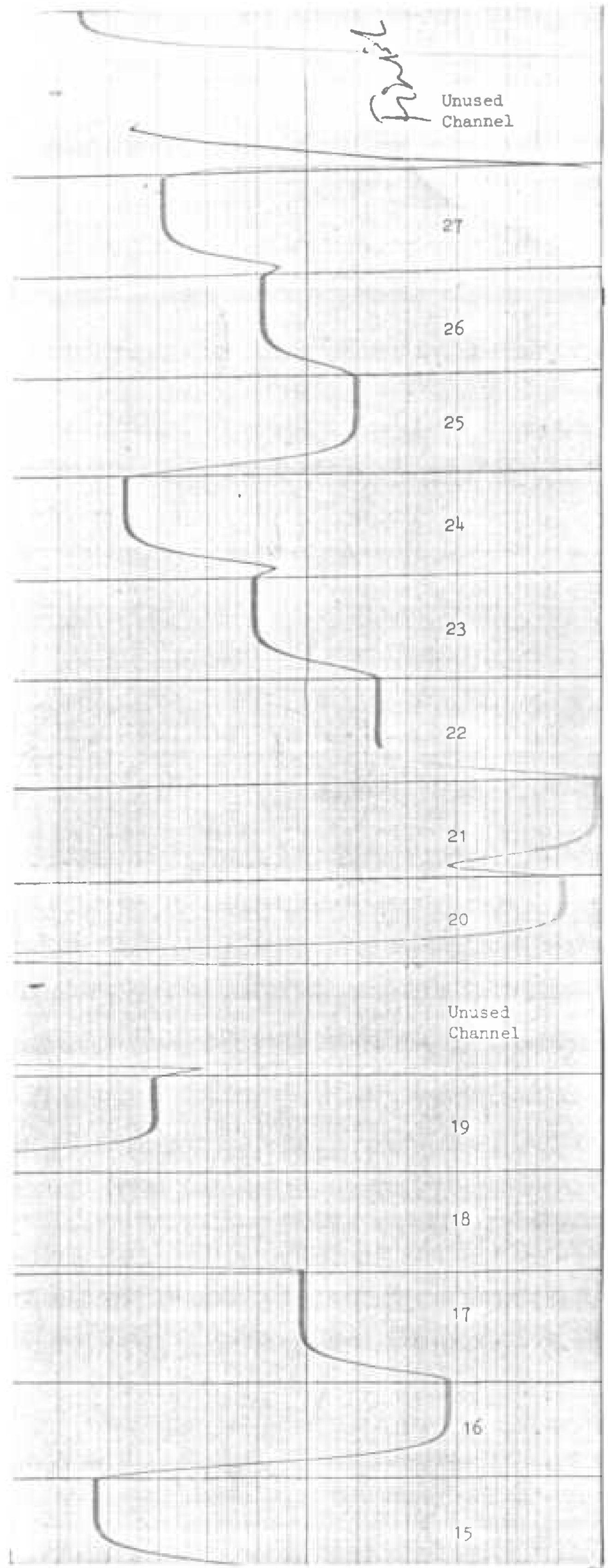

14
13
12
11
10
Unused Channel
9
8
7
6
5
4
3
2
1
9-3
10-p.s.i.
1

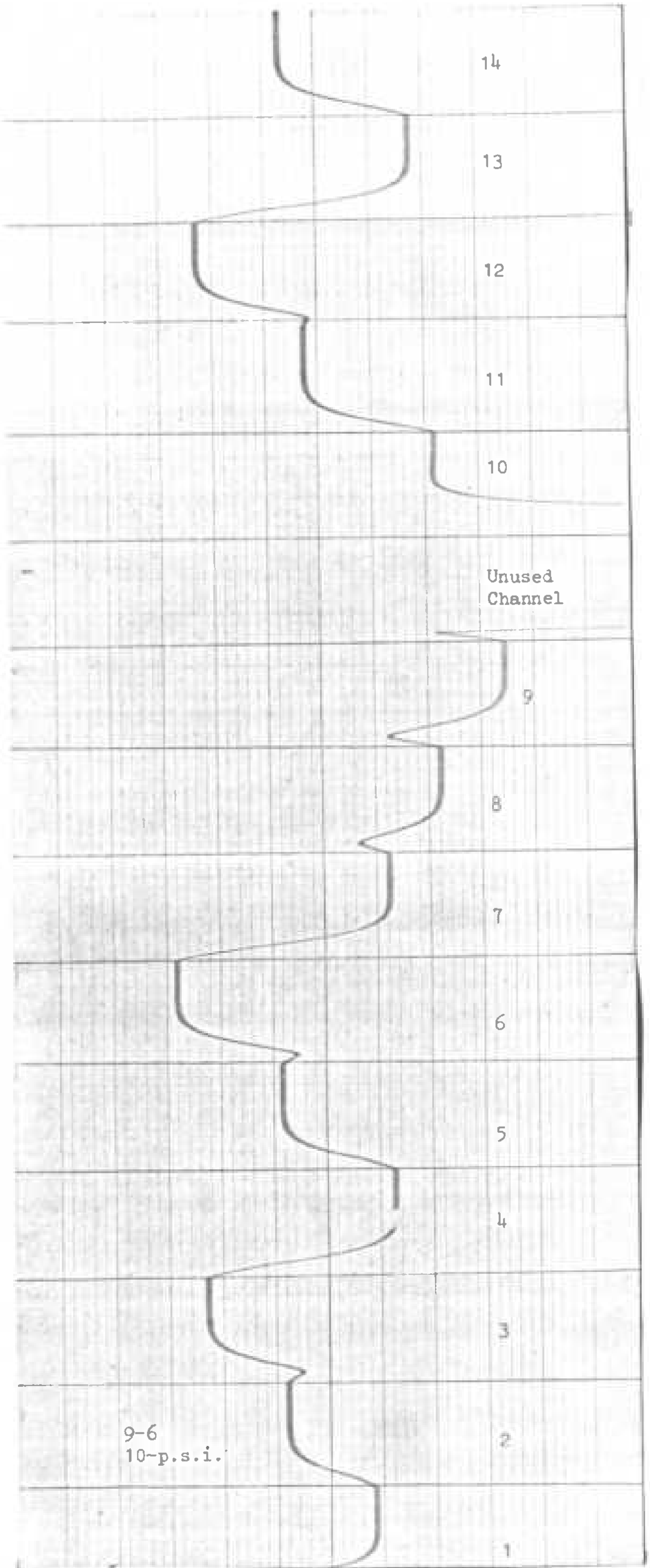
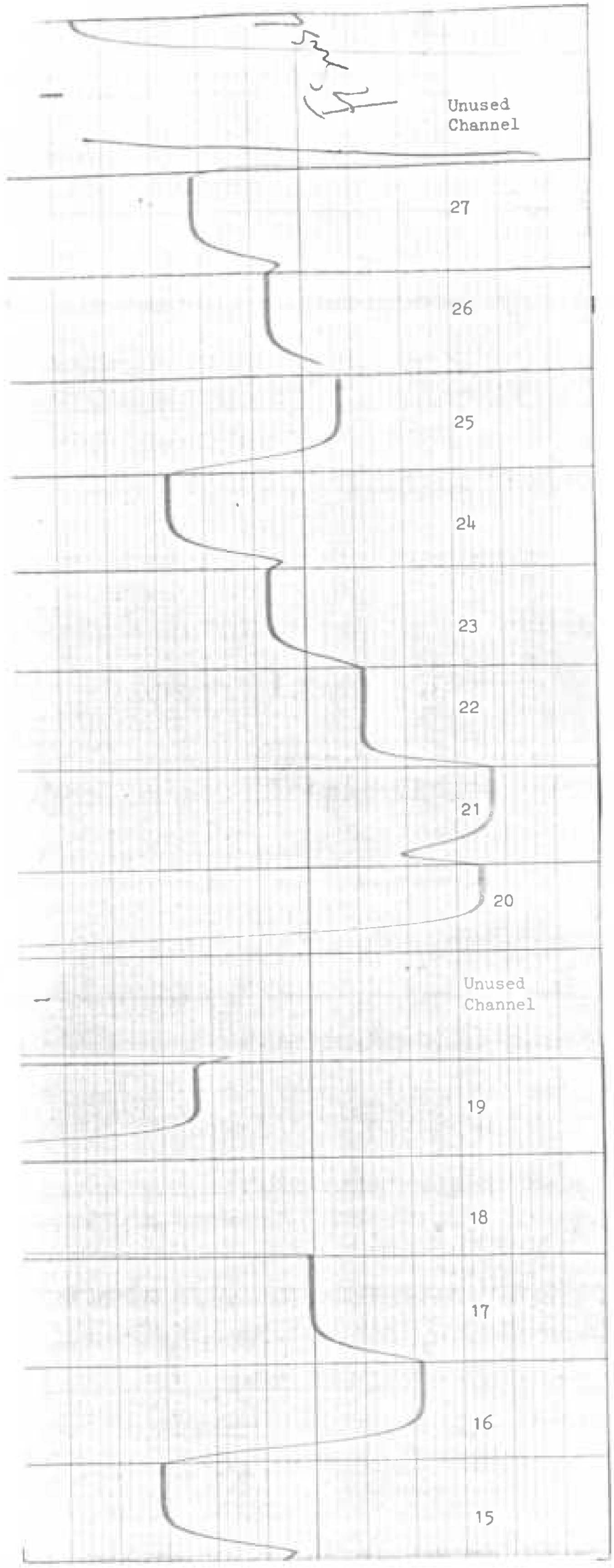

Unused Channel
27
26
25
24
23
22
21
20
Unused Channel
19
18
17
16
15

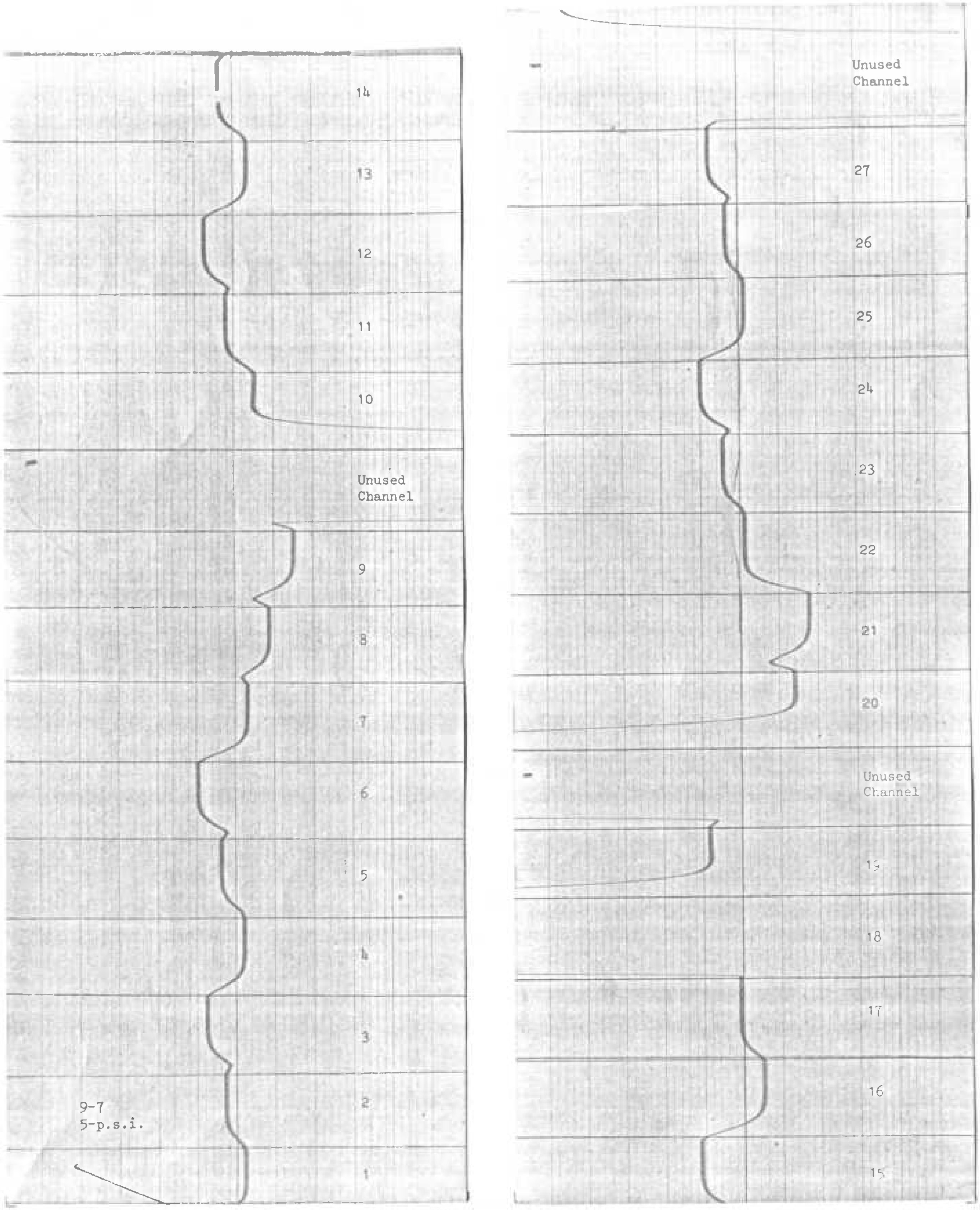

15
14
13
12
11
10
Unused
Channel
9
8
7
6
5
4
3
2
1
9-4
15-p.s.i.


Unused
Channel
27
26
25
24
23
22
21
20
Unused
Channel
19
18
17
16


14
13
12
11
10
Unused
Channel
9
8
7
6
5
4
3
9-5
16-p.s.i.
2
1


Finish
Unused
Channel
27
26
25
24
23
22
21
20
Unused
Channel
19
18
17
16
15


14
13
12
11
10
Unused Channel
9
8
7
6
5
4
3
9-6
10-p.s.i.
2
1

Unused Channel
27
26
25
24
23
22
21
20
Unused Channel
19
18
17
16
15


14
13
12
11
10
Unused Channel
9
8
7
6
5
4
3
2
1
9-7
5-p.s.i.
Unused Channel
27
26
25
24
23
22
21
20
Unused Channel
19
18
17
16
15

```
STRESS
9

    THE TOTAL NUMBER OF READINGS =     9

          INPUT THE STRAINS OF READING NO.     1
-83.43, 208.57, 458.87

   1   -.00008343 .00020857 .00045887 .00048491-.00035977      187.26    -94.71

          INPUT THE STRAINS OF READING NO.     2
-111.24, 236.39, 570.11

   2   -.00011124 .00023639 .00057011 .00059905-.00047390      225.36   -132.81

          INPUT THE STRAINS OF READING NO.     3
-139.05, -472.77, -889.92

   3   -.00013905-.00047277-.00088992 .00030147-.00091329      -23.48   -428.99

          INPUT THE STRAINS OF READING NO.     4
-278.10, 152.96, 458.87

   4   -.00027810 .00015296 .00045887 .00050166-.00062680      142.08   -234.62

          INPUT THE STRAINS OF READING NO.     5
-166.86, 236.39, 639.63

   5   -.00016686 .00023639 .00063963 .00067235-.00060282      238.55   -187.13

          INPUT THE STRAINS OF READING NO.     6
0.0, 0.0, 0.0

   6   0.000000000.000000000.000000000.000000000.00000000        0.00      0.00

          INPUT THE STRAINS OF READING NO.     7
347.63, -876.01, -1056.78

   7    .00034763-.00087601-.00105678 .00073707-.00126545      138.86   -529.62

          INPUT THE STRAINS OF READING NO.     8
-111.24, 208.57, 542.30

   8   -.00011124 .00020857 .00054230 .00056755-.00047022      209.20   -137.22

          INPUT THE STRAINS OF READING NO.     9
-111.24, 152.96, 403.25

   9   -.00011124 .00015296 .00040325 .00042542-.00038370      150.48   -119.62
 STOP
```

```
>STRESS
9

     THE TOTAL NUMBER OF READINGS =     9

          INPUT THE STRAINS OF READING NO.     1
-333.72, 375.44, 1020.63

   1   -.00033372 .00037544 .00102063 .00108165-.00103993        369.54   -338.68

          INPUT THE STRAINS OF READING NO.     2
-444.96, 444.96, 1279.27

   2   -.00044496 .00044496 .00127927 .00135445-.00135445        452.14   -452.14

          INPUT THE STRAINS OF READING NO.     3
-403.25, -834.30, -1863.27

   3   -.00040325-.00083430-.00186327 .00064424-.00188179        -36.00   -879.23

          INPUT THE STRAINS OF READING NO.     4
-792.59, 278.10, 973.35

   4   -.00079259 .00027810 .00097335 .00108475-.00159924        257.74   -638.23

          INPUT THE STRAINS OF READING NO.     5
-597.92, 411.59, 1404.41

   5   -.00059792 .00041159 .00140441 .00148719-.00167352        458.65   -596.45

          INPUT THE STRAINS OF READING NO.     6
0.0, 0.0, 0.0,

   6   0.000000000.000000000.000000000.000000000.00000000          0.00      0.00

          INPUT THE STRAINS OF READING NO.     7
973.35, -2016.22, -2057.94

   7    .00097335-.00201622-.00205794 .00162221-.00266508        329.96  -1101.21

          INPUT THE STRAINS OF READING NO.     8
-305.91, 444.96, 1181.93

   8   -.00030591 .00044496 .00118193 .00124358-.00110453        443.34   -340.50

          INPUT THE STRAINS OF READING NO.     9
-250.29, 305.92, 903.83

   9   -.00025029 .00030592 .00090383 .00094691-.00089128        327.38   -286.24
 STOP
```

```
>STRESS
9

    THE TOTAL NUMBER OF READINGS =      9

         INPUT THE STRAINS OF READING NO.     1
-584.01, 556.20, 1612.99

   1    -.00058401 .00055620 .00161299 .00170999-.00173780        565.18   -585.75

         INPUT THE STRAINS OF READING NO.     2
-806.49, 653.54, 2030.14

   2    -.00080649 .00065354 .00203014 .00215304-.00230599        687.69   -800.81

         INPUT THE STRAINS OF READING NO.     3
-806.49, -1585.17, -2808.81

   3    -.00080649-.00158517-.00280881 .00046347-.00285513      -330.47  -1438.28

         INPUT THE STRAINS OF READING NO.     4
-1446.12, 278.10, 1612.98

   4    -.00144612 .00027810 .00161298 .00177607-.00294409        355.93  -1219.74

         INPUT THE STRAINS OF READING NO.     5
-1140.21, -556.20, -2252.61

   5    -.00114021-.00055620-.00225261 .00058624-.00228265      -148.45  -1106.13

         INPUT THE STRAINS OF READING NO.     6
0.0, 0.0, 0.0

   6    0.000000000.000000000.000000000.000000000.00000000          0.00      0.00

         INPUT THE STRAINS OF READING NO.     7
1585.17, -2892.24, -3142.53

   7     .00158517-.00289224-.00314253 .00269414-.00400121        634.19  -1600.83

         INPUT THE STRAINS OF READING NO.     8
-736.97, 584.01, 1654.70

   8    -.00073697 .00058401 .00165470 .00177642-.00192938        561.97   -675.09

         INPUT THE STRAINS OF READING NO.     9
-528.39, 458.87, 1529.55

   9    -.00052839 .00045887 .00152955 .00160559-.00167511        521.87   -573.28
 STOP
```

```
>STRESS
9

     THE TOTAL NUMBER OF READINGS =      9

          INPUT THE STRAINS OF READING NO.      1
-625.73, 611.82, 1765.94

   1   -.00062573 .00061182 .00176594 .00187082-.00188473        621.69   -631.98

          INPUT THE STRAINS OF READING NO.      2
-917.73, 681.35, 2155.28

   2   -.00091773 .00068135 .00215528 .00229177-.00252815        717.08   -891.89

          INPUT THE STRAINS OF READING NO.      3
-945.54, -1863.27, -3003.48

   3   -.00094554-.00186327-.00300348 .00025920-.00306801       -483.29  -1593.97

          INPUT THE STRAINS OF READING NO.      4
-1599.08, 236.39, 1752.03

   4   -.00159908 .00023639 .00175203 .00191934-.00328203        364.26  -1372.04

          INPUT THE STRAINS OF READING NO.      5
-1268.14, 645.19, 2419.47

   5   -.00126814 .00064519 .00241947 .00258218-.00320513        735.60  -1196.30

          INPUT THE STRAINS OF READING NO.      6
0.0, 0.0, 0.0

   6   0.000000000.000000000.000000000.000000000.00000000          0.00      0.00

          INPUT THE STRAINS OF READING NO.      7
1710.32, -2933.95, -3267.68

   7    .00171032-.00293395-.00326768 .00291606-.00413969        725.20  -1630.13

          INPUT THE STRAINS OF READING NO.      8
-806.49, 611.82, 2071.85

   8   -.00080649 .00061182 .00207185 .00218483-.00237950        689.84   -833.81

          INPUT THE STRAINS OF READING NO.      9
-556.20, 500.59, 1646.35

   9   -.00055620 .00050059 .00164635 .00172776-.00178337        565.47   -606.60
 STOP
```

```
>STRESS
9

    THE TOTAL NUMBER OF READINGS =      9

         INPUT THE STRAINS OF READING NO.     1
-486.68, 444.96, 1279.27

   1    -.00048668 .00044496 .00127927 .00136020-.00140192       445.59   -476.45

         INPUT THE STRAINS OF READING NO.     2
-723.06, 472.77, 1571.27

   2    -.00072306 .00047277 .00157127 .00167356-.00192385       507.89   -692.99

         INPUT THE STRAINS OF READING NO.     3
-695.25, -1251.45, -2002.32

   3    -.00069525-.00125145-.00200232 .00009254-.00203924      -364.03  -1075.65

         INPUT THE STRAINS OF READING NO.     4
-1223.64, 139.05, 1307.07

   4    -.00122364 .00013905 .00130707 .00142859-.00251318       256.86  -1058.97

         INPUT THE STRAINS OF READING NO.     5
-1001.16, 389.34, 1779.84

   5    -.00100116 .00038934 .00177984 .00189266-.00250448       507.69   -960.16

         INPUT THE STRAINS OF READING NO.     6
0.0, 0.0, 0.0,

   6    0.000000000.000000000.000000000.000000000.00000000         0.00      0.00

         INPUT THE STRAINS OF READING NO.     7
1265.36, -2016.22, -2113.56

   7     .00126536-.00201622-.00211356 .00201482-.00276568       520.26  -1075.56

         INPUT THE STRAINS OF READING NO.     8
-667.44, 431.06, 1557.37

   8    -.00066744 .00043106 .00155737 .00164510-.00188148       501.21   -676.02

         INPUT THE STRAINS OF READING NO.     9
-417.15, 375.44, 1237.55

   9    -.00041715 .00037544 .00123755 .00129848-.00134019       424.99   -455.84
 STOP
```

```
>STRESS
9

    THE TOTAL NUMBER OF READINGS =     9

         INPUT THE STRAINS OF READING NO.    1
-125.15, 166.86, 403.25

   1   -.00012515 .00016686 .00040325 .00043018-.00038847       152.06   -121.21

         INPUT THE STRAINS OF READING NO.    2
-139.05, 208.58, 528.40

   2   -.00013905 .00020858 .00052840 .00055811-.00048858       200.41   -148.99

         INPUT THE STRAINS OF READING NO.    3
-180.77, -500.58, -903.83

   3   -.00018077-.00050058-.00090383 .00024474-.00092609       -56.52   -447.37

         INPUT THE STRAINS OF READING NO.    4
-305.91, 125.15, 431.06

   4   -.00030591 .00012515 .00043106 .00047385-.00065461       121.51   -255.19

         INPUT THE STRAINS OF READING NO.    5
-194.67, 208.58, 639.63

   5   -.00019467 .00020858 .00063963 .00067098-.00065707       226.81   -216.52

         INPUT THE STRAINS OF READING NO.    6
0.0, 0.0, 0.0

   6   0.000000000.000000000.000000000.000000000.00000000         0.00      0.00

         INPUT THE STRAINS OF READING NO.    7
417.15, -848.20, -1001.16

   7    .00041715-.00084820-.00100116 .00079319-.00122424       177.33   -496.12

         INPUT THE STRAINS OF READING NO.    8
-69.52, 250.29, 570.11

   8   -.00006952 .00025029 .00057011 .00059606-.00041529       235.65   -101.96

         INPUT THE STRAINS OF READING NO.    9
-83.43, 166.87, 431.06

   9   -.00008343 .00016687 .00043106 .00045068-.00036724       167.37   -105.66
 STOP
```

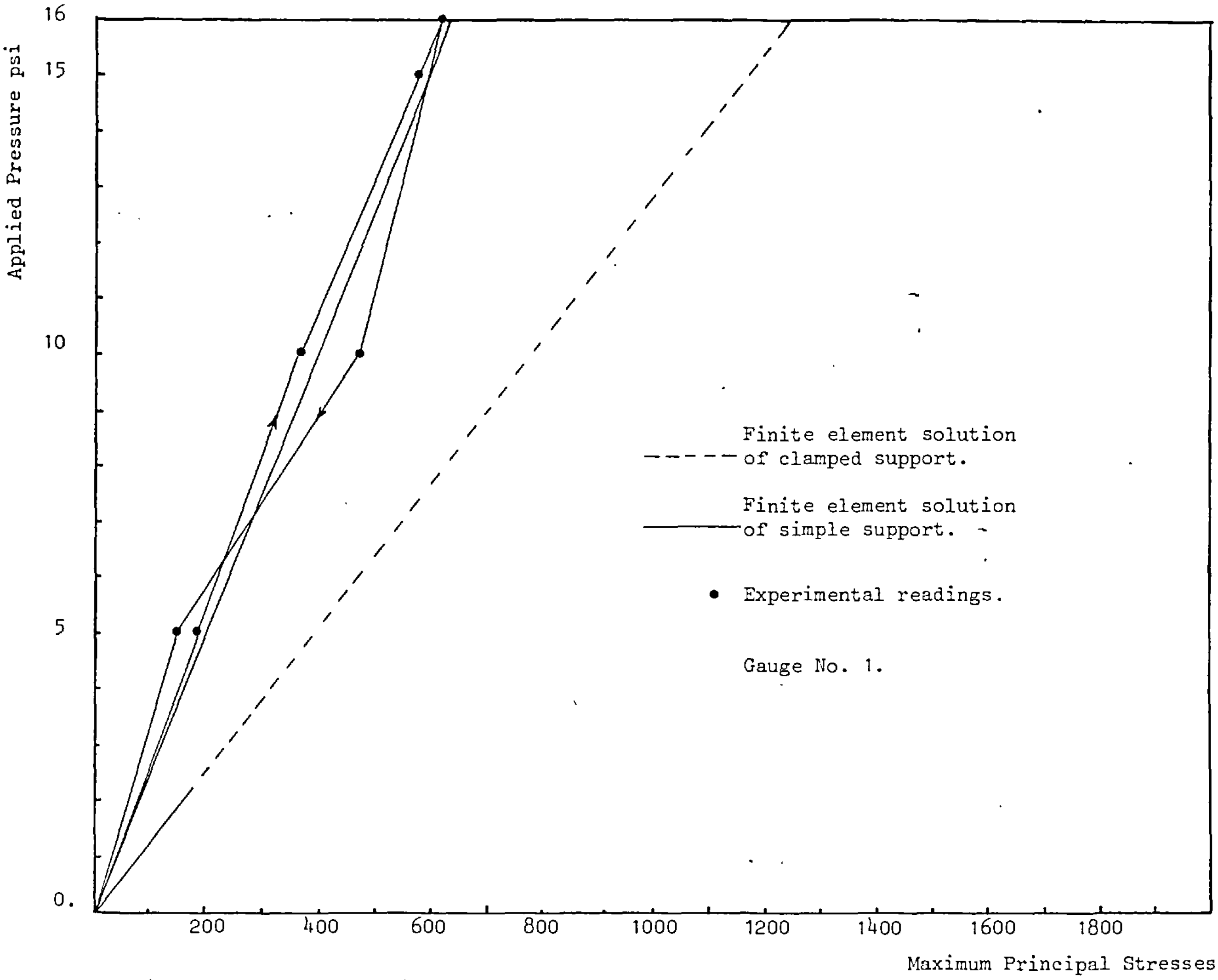


Fig. 2.40. Comparison of principal stresses.

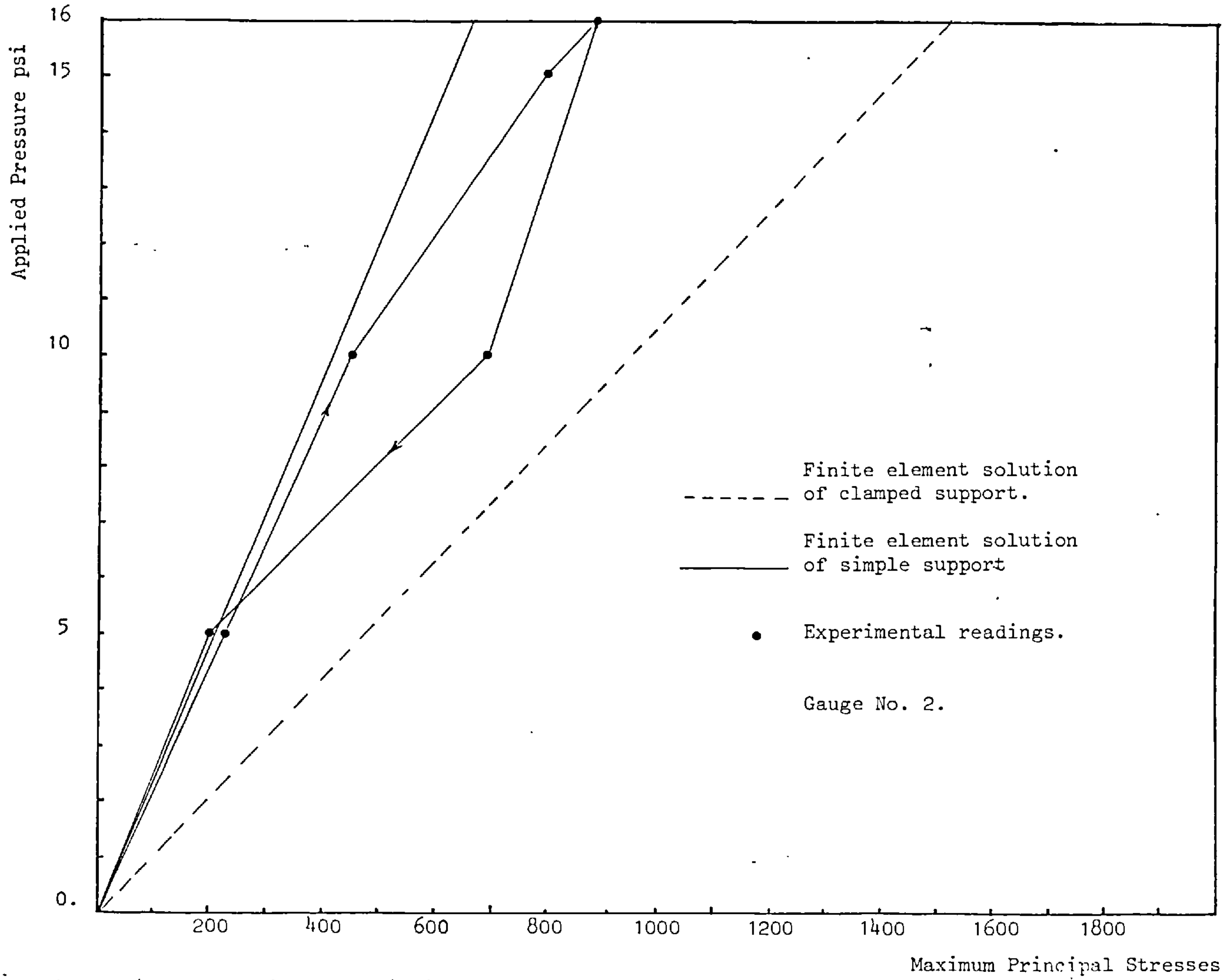


Fig. 2.41. Comparison of principal stresses.

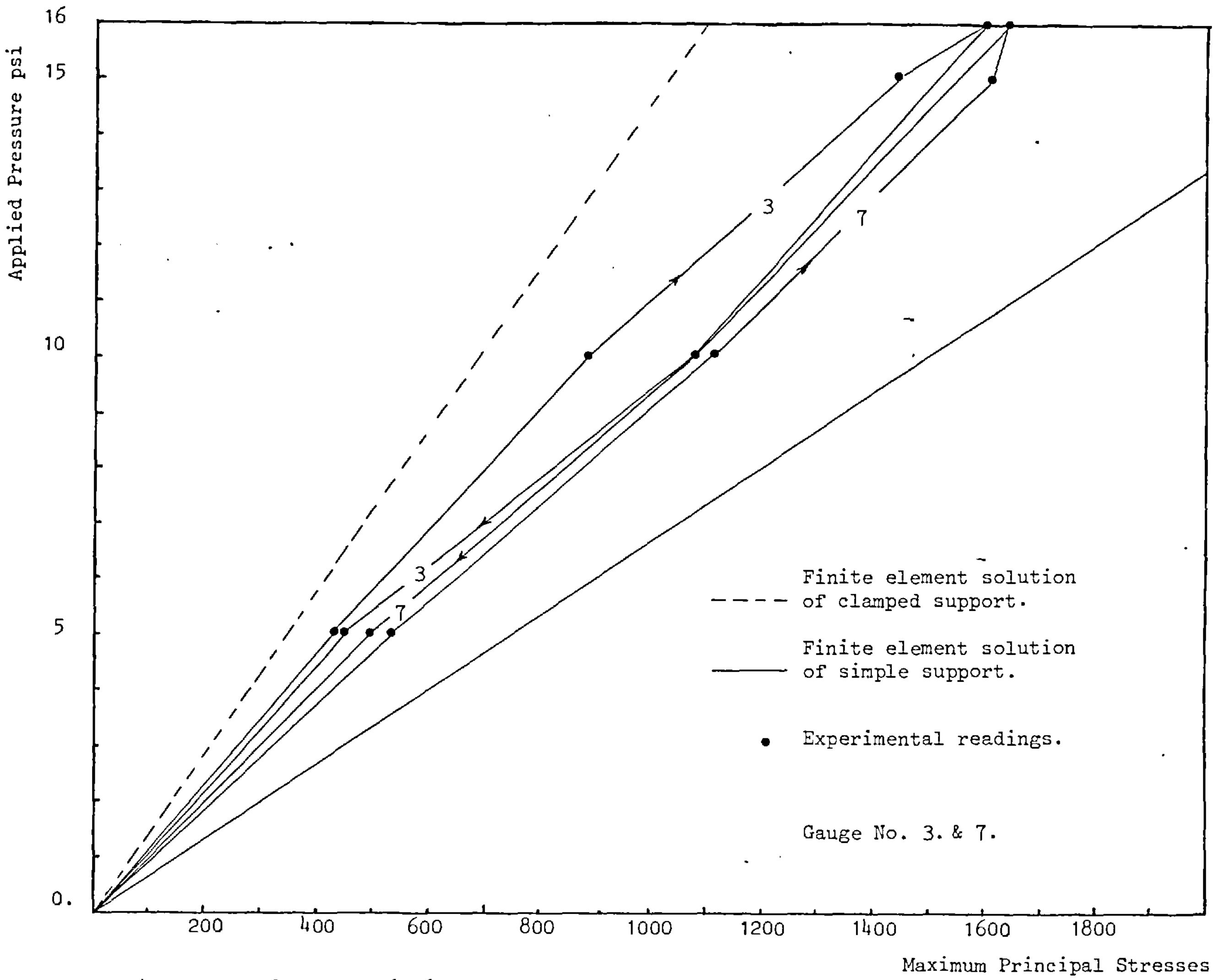


Fig. 2.42. Comparison of principal stresses.

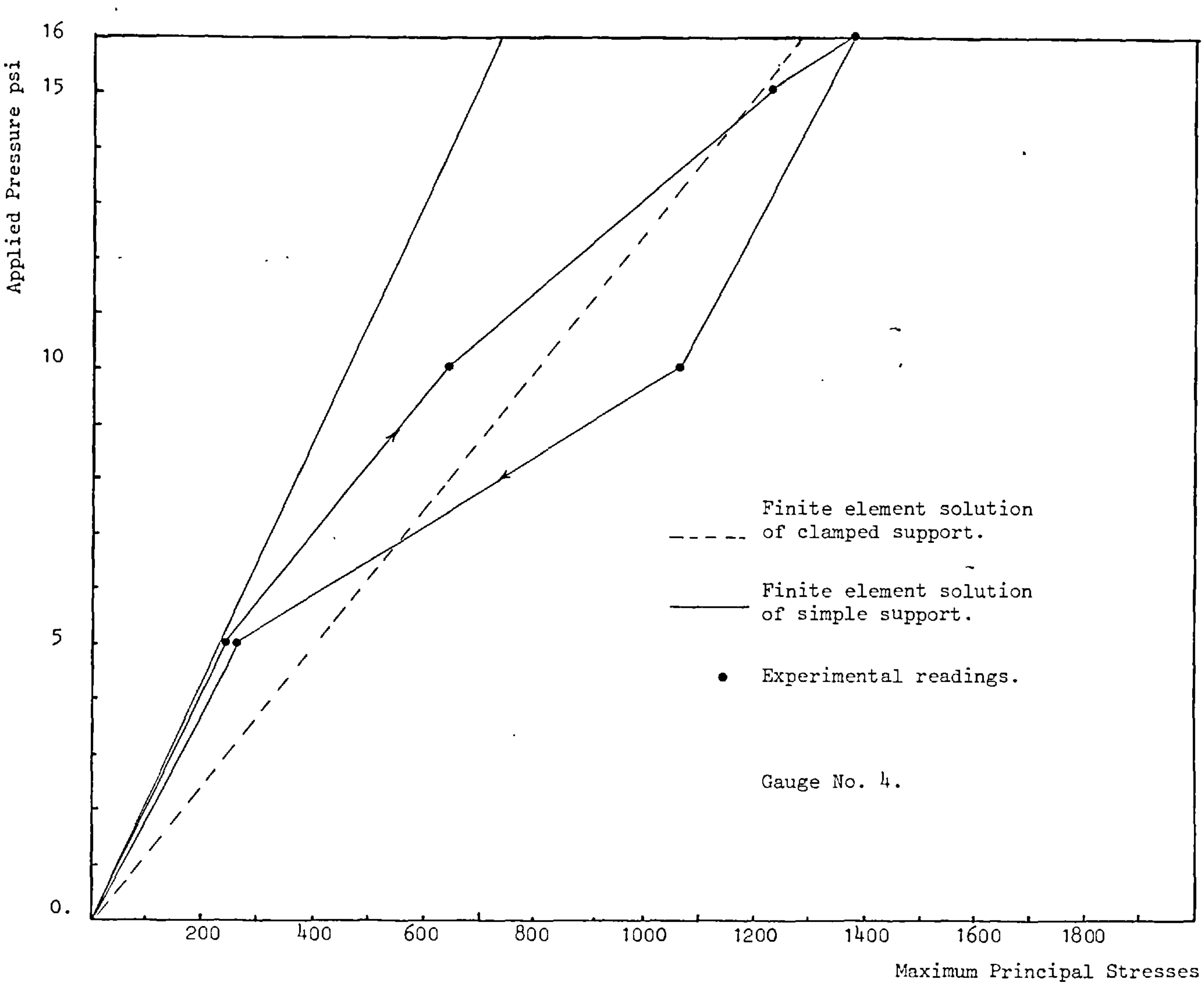


Fig. 2.43. Comparison of principal stresses.

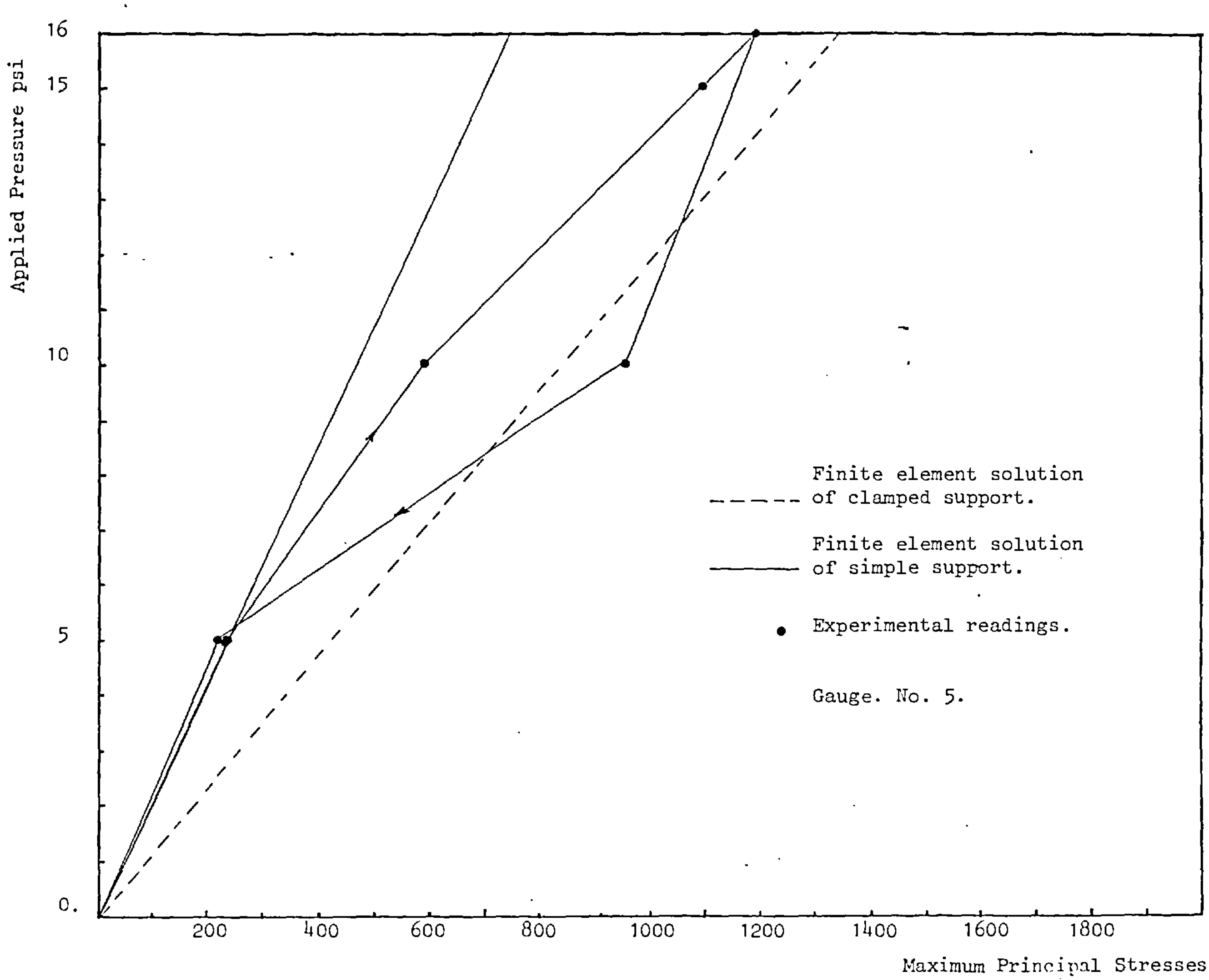


Fig. 2.44. Comparison of principal stresses

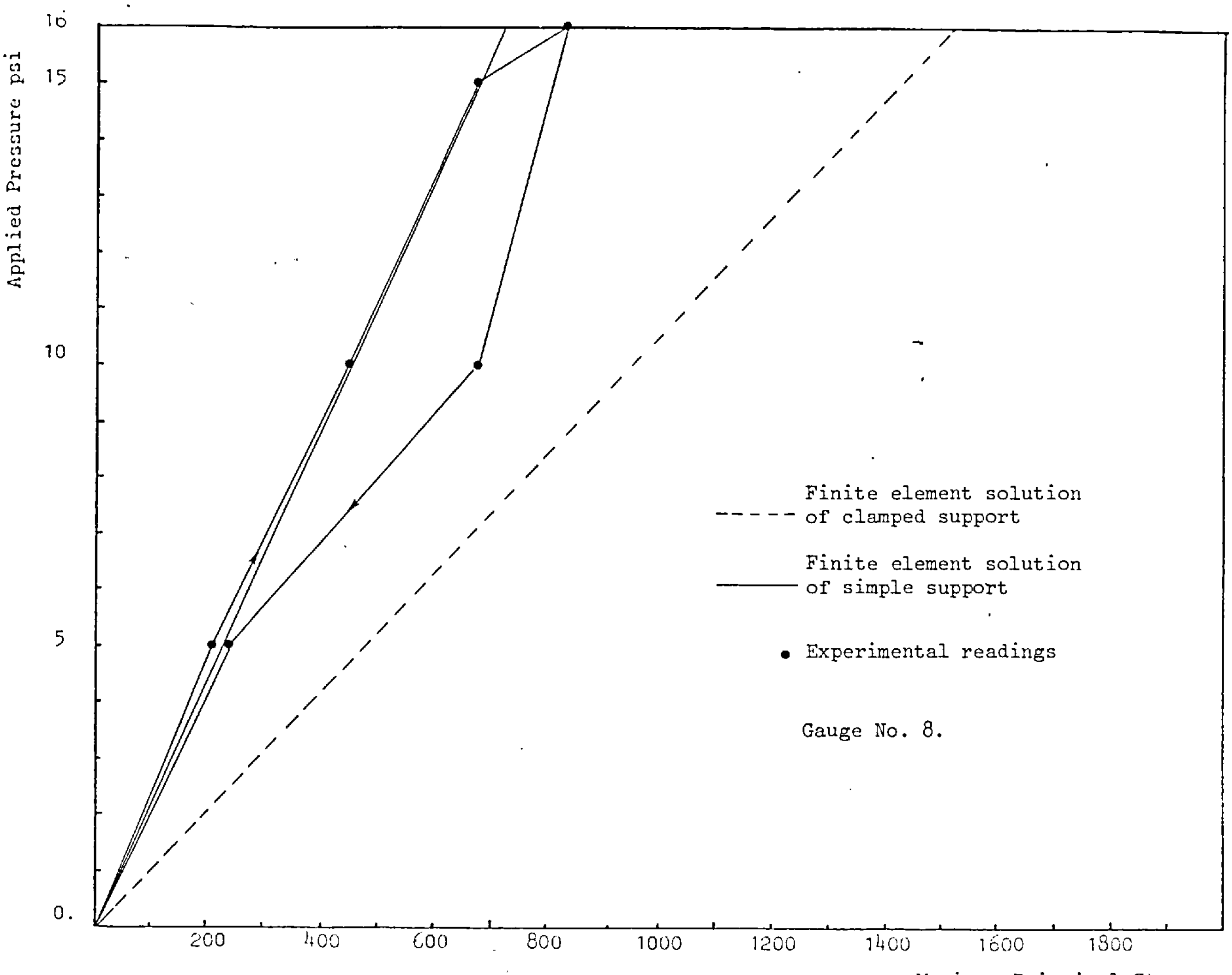


Fig. 2.45. Comparison of principal stresses

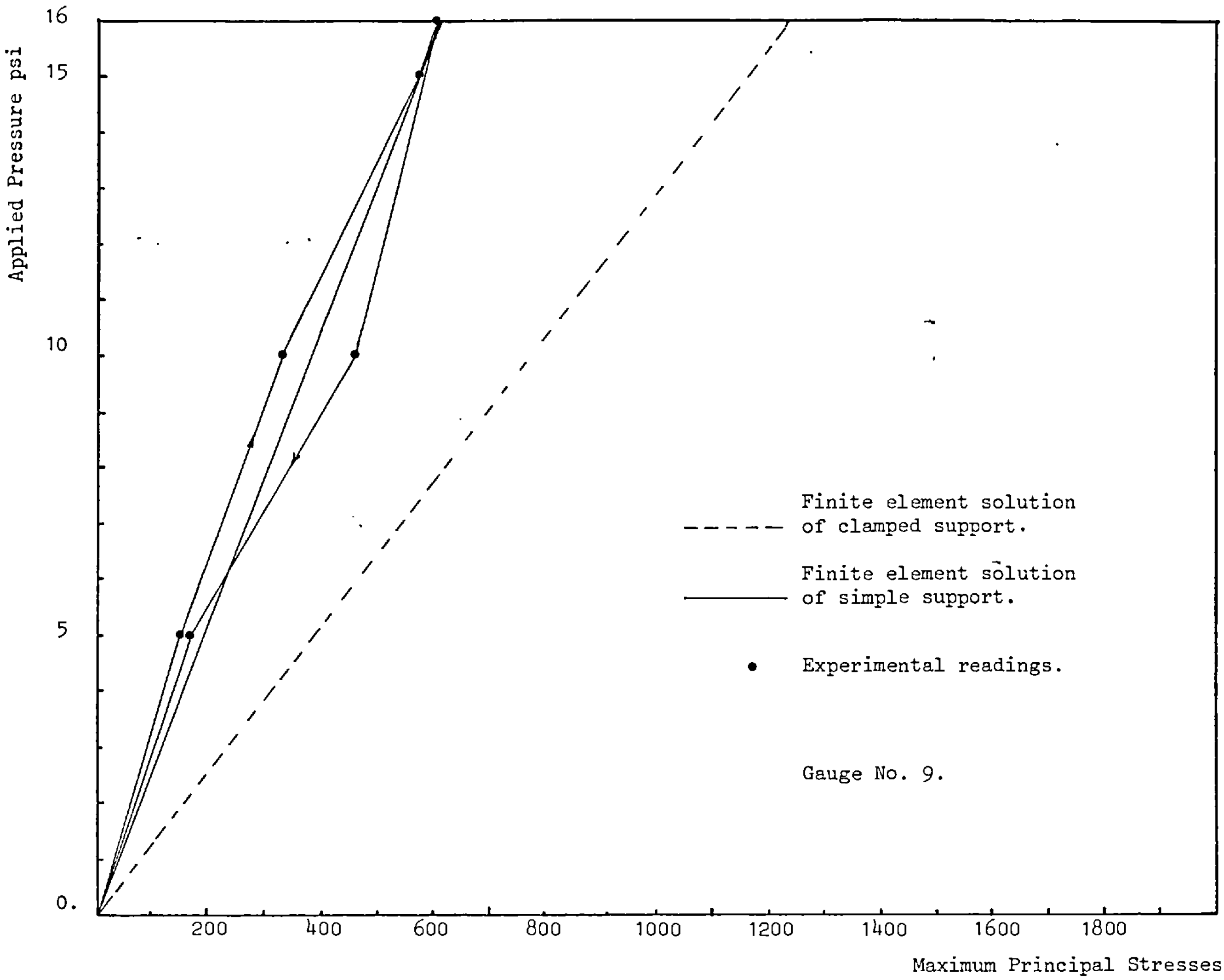


Fig. 2.46. Comparison of principal stresses

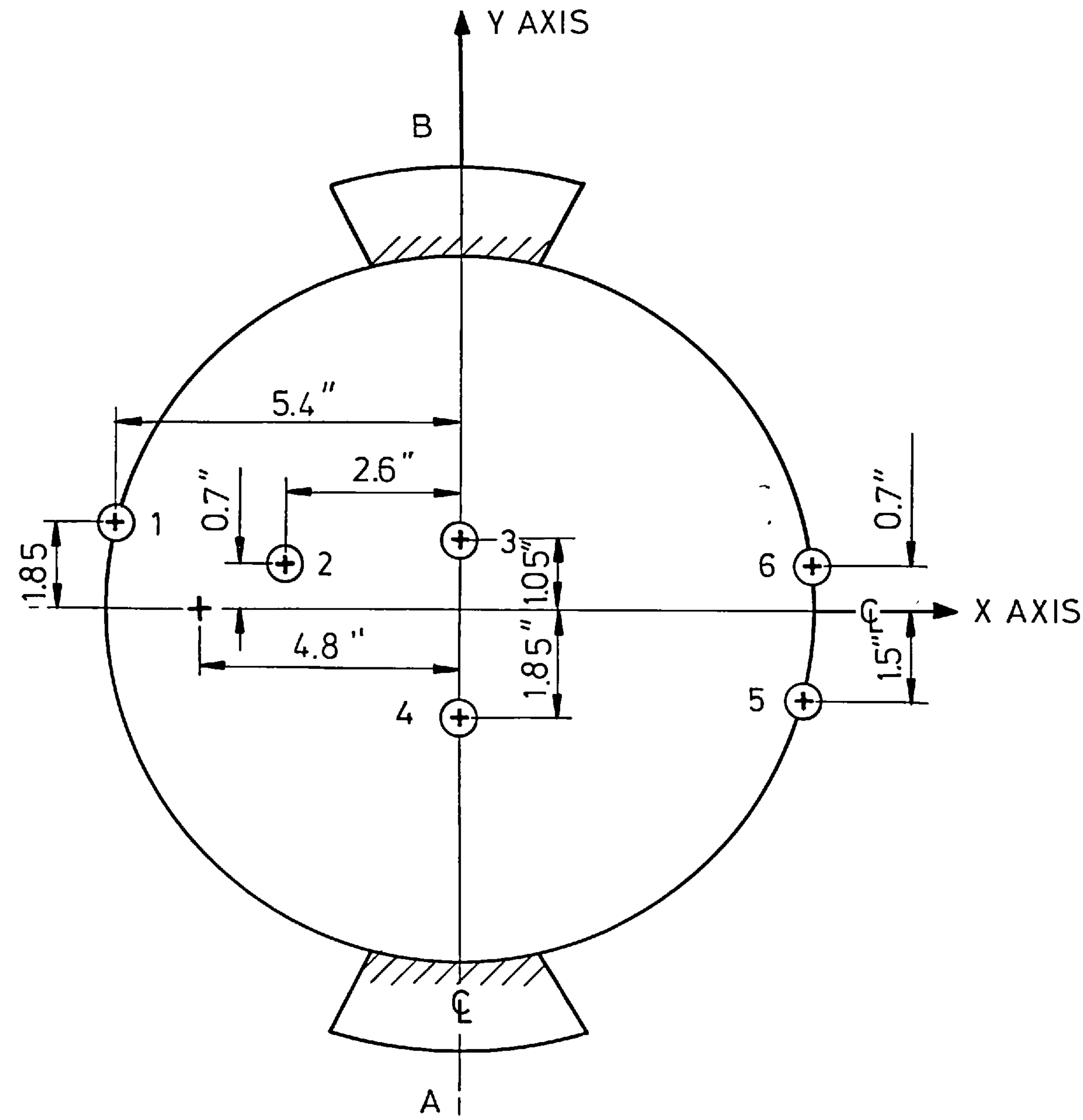
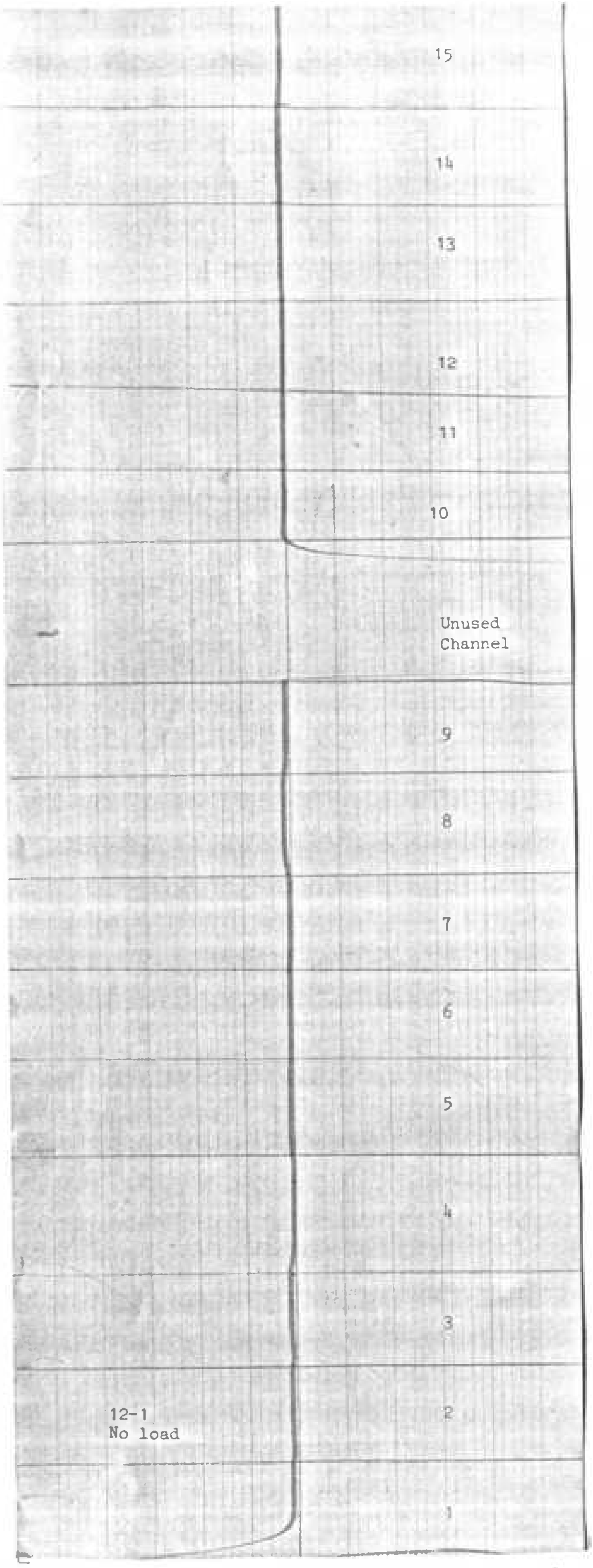
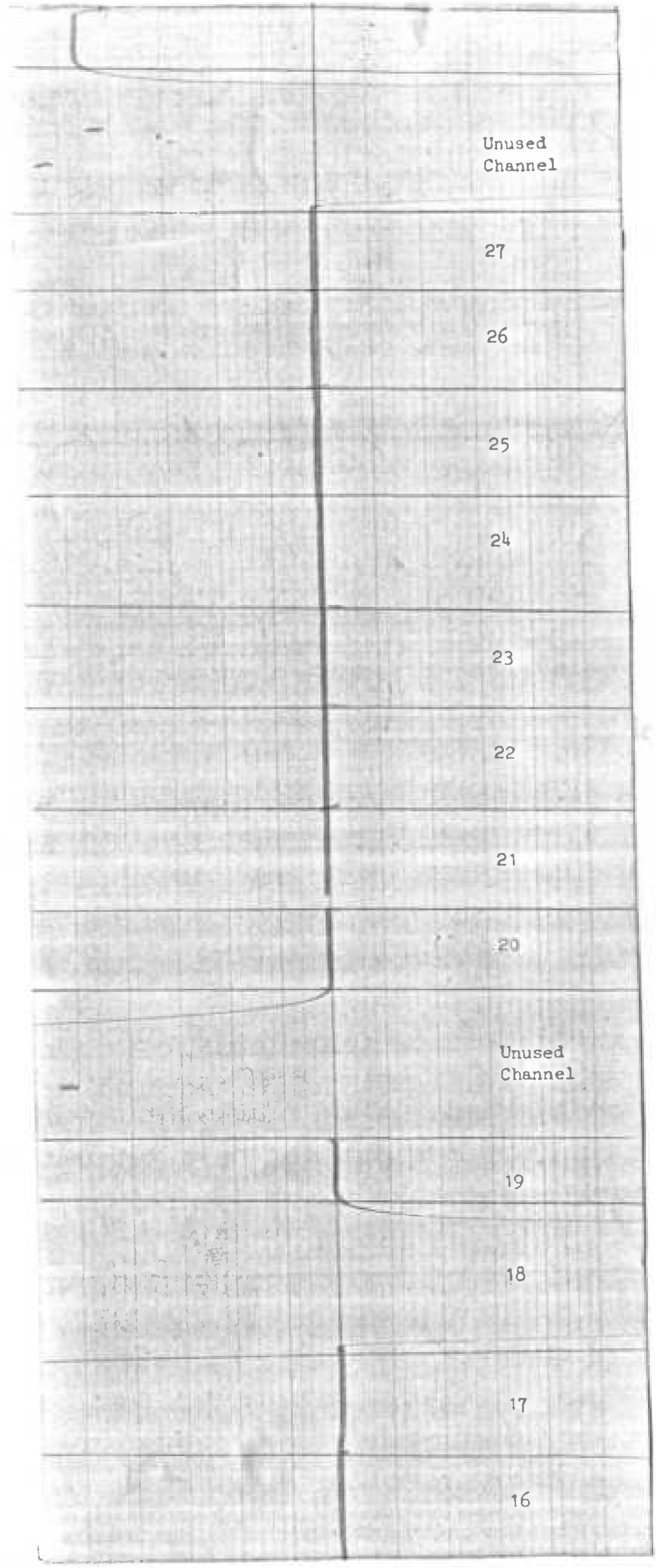
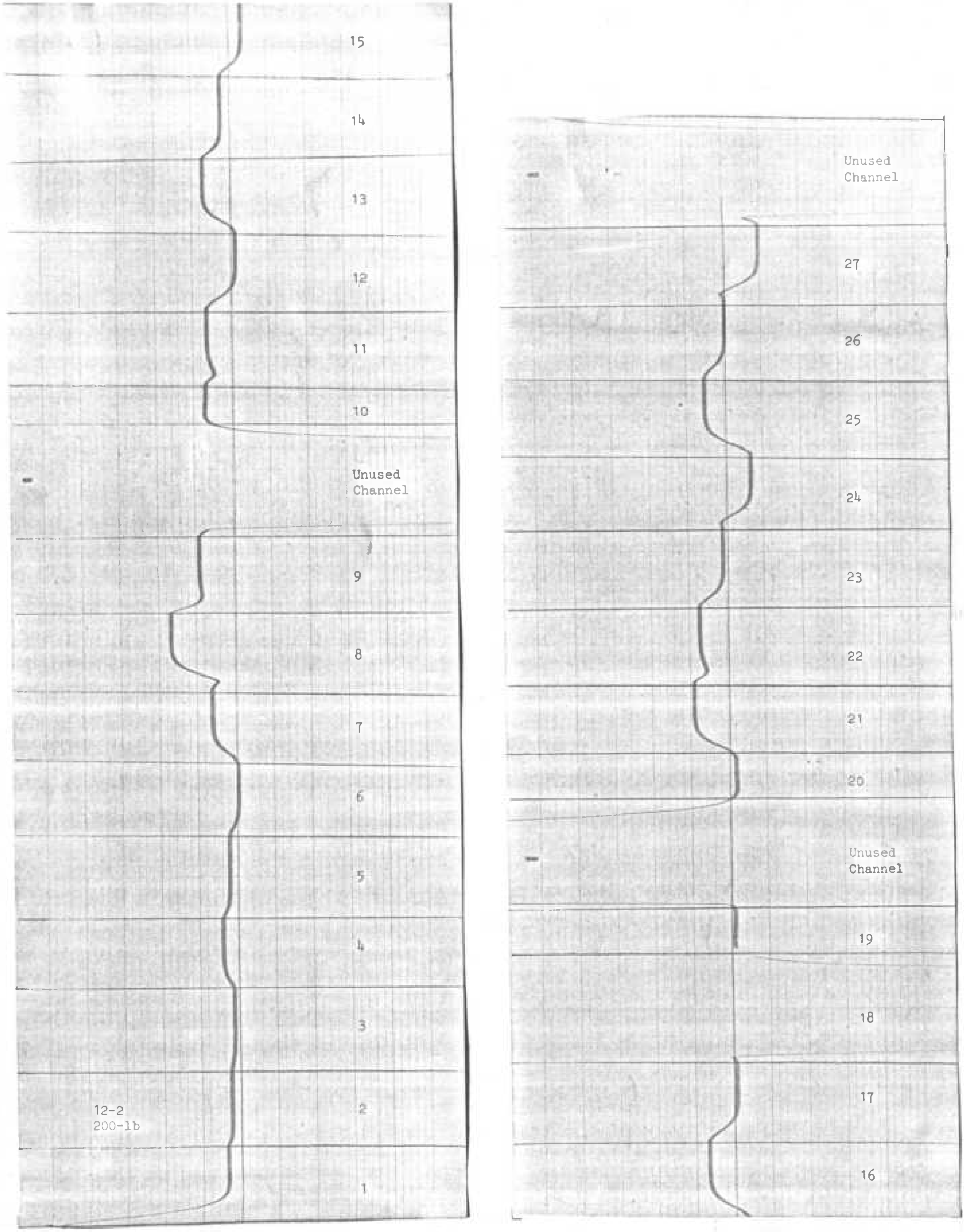
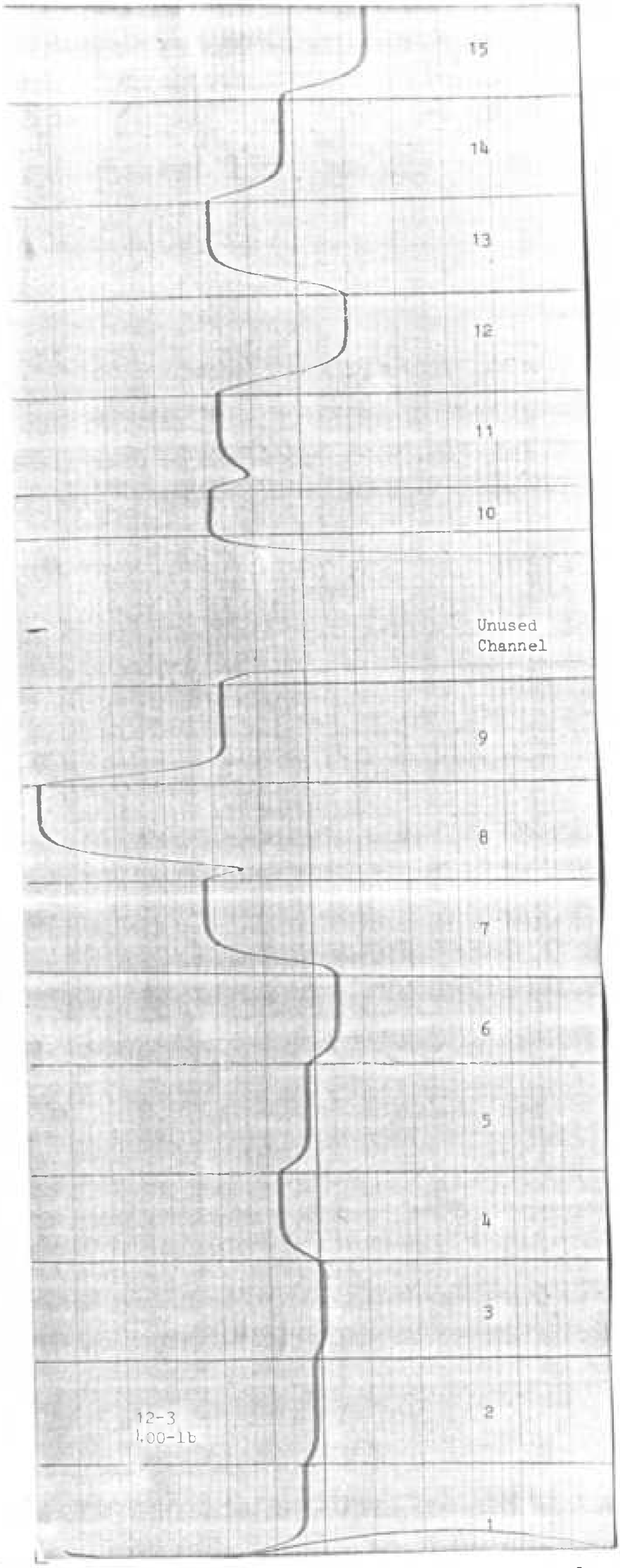


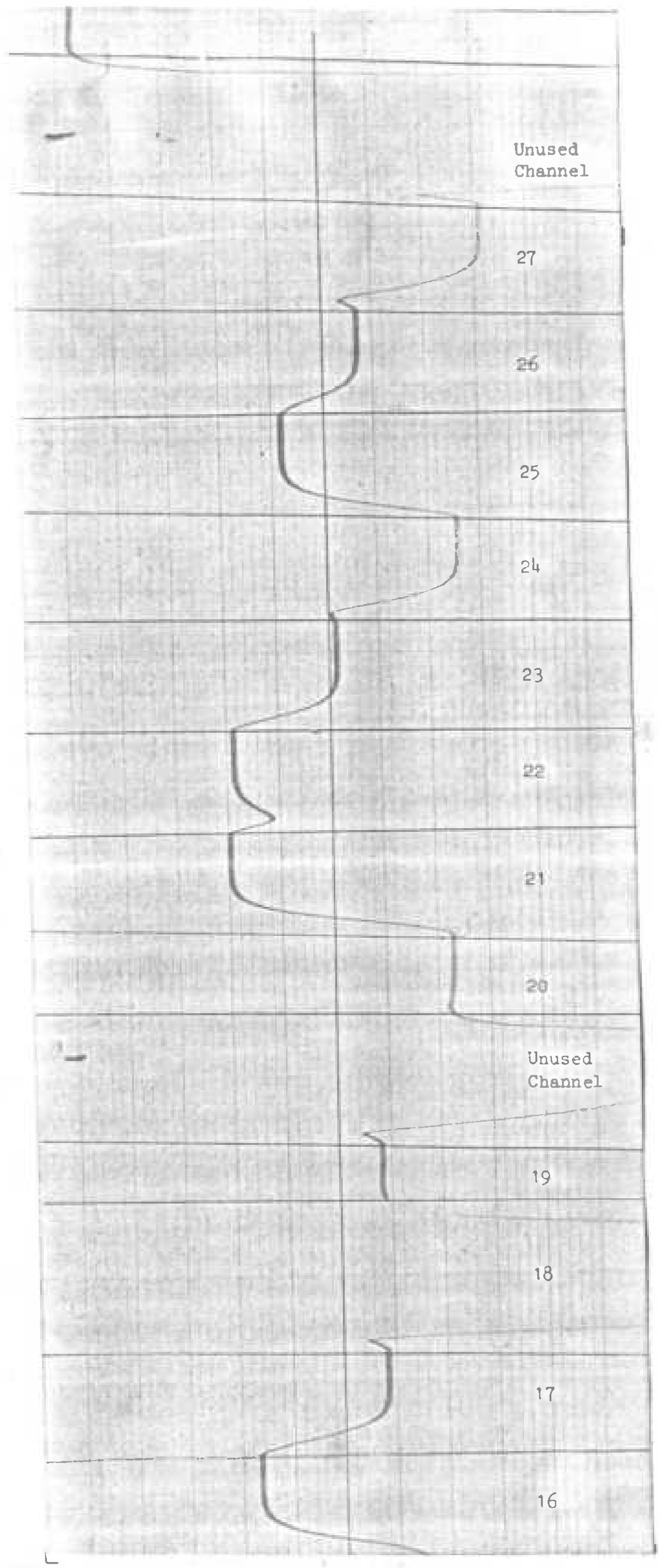
Fig. 2.47. Dial gauge and point load locations.

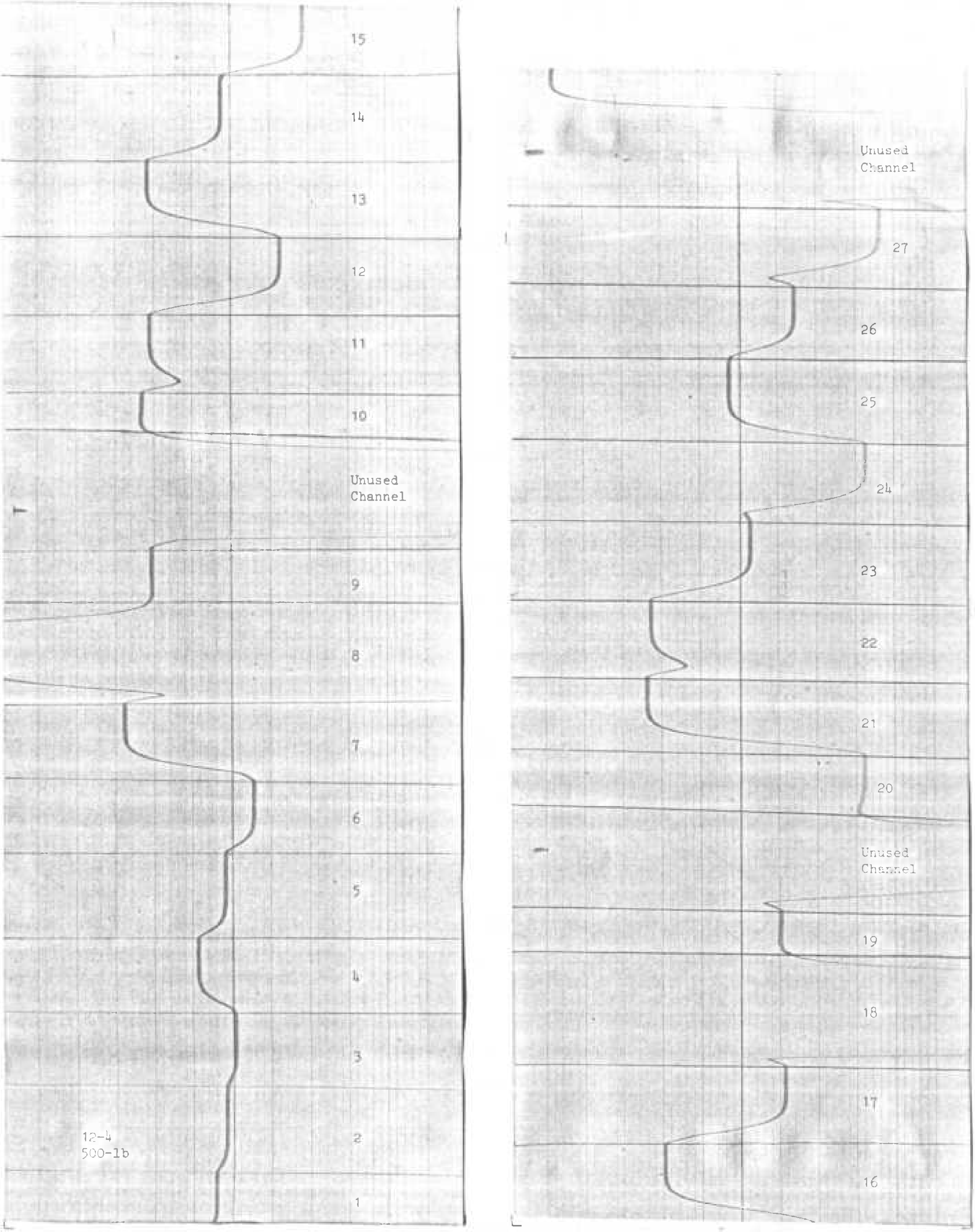

15
14
13
12
11
10
Unused Channel
9
8
7
6
5
4
3
2
1
12-1
No load

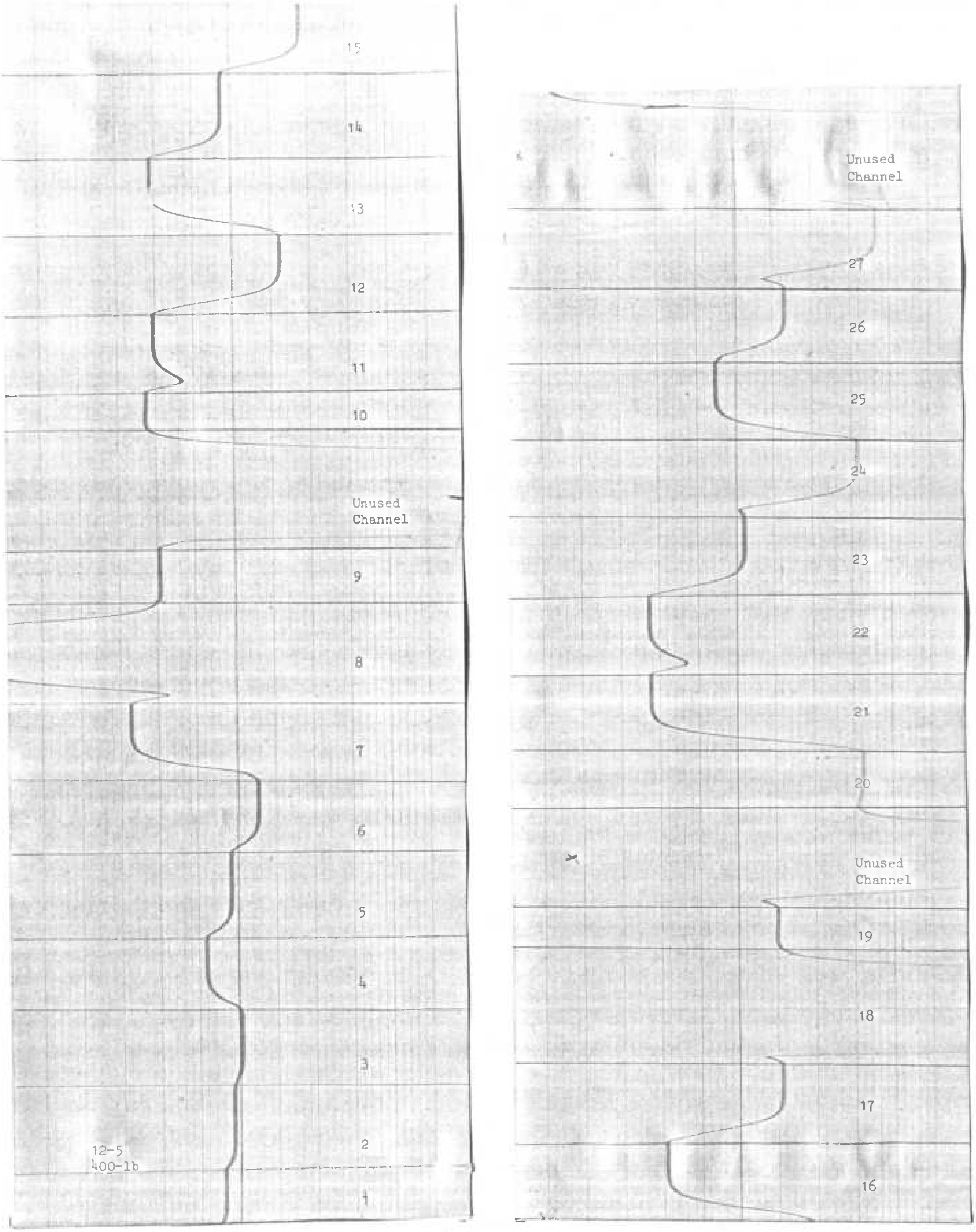

Unused Channel
27
26
25
24
23
22
21
20
Unused Channel
19
18
17
16

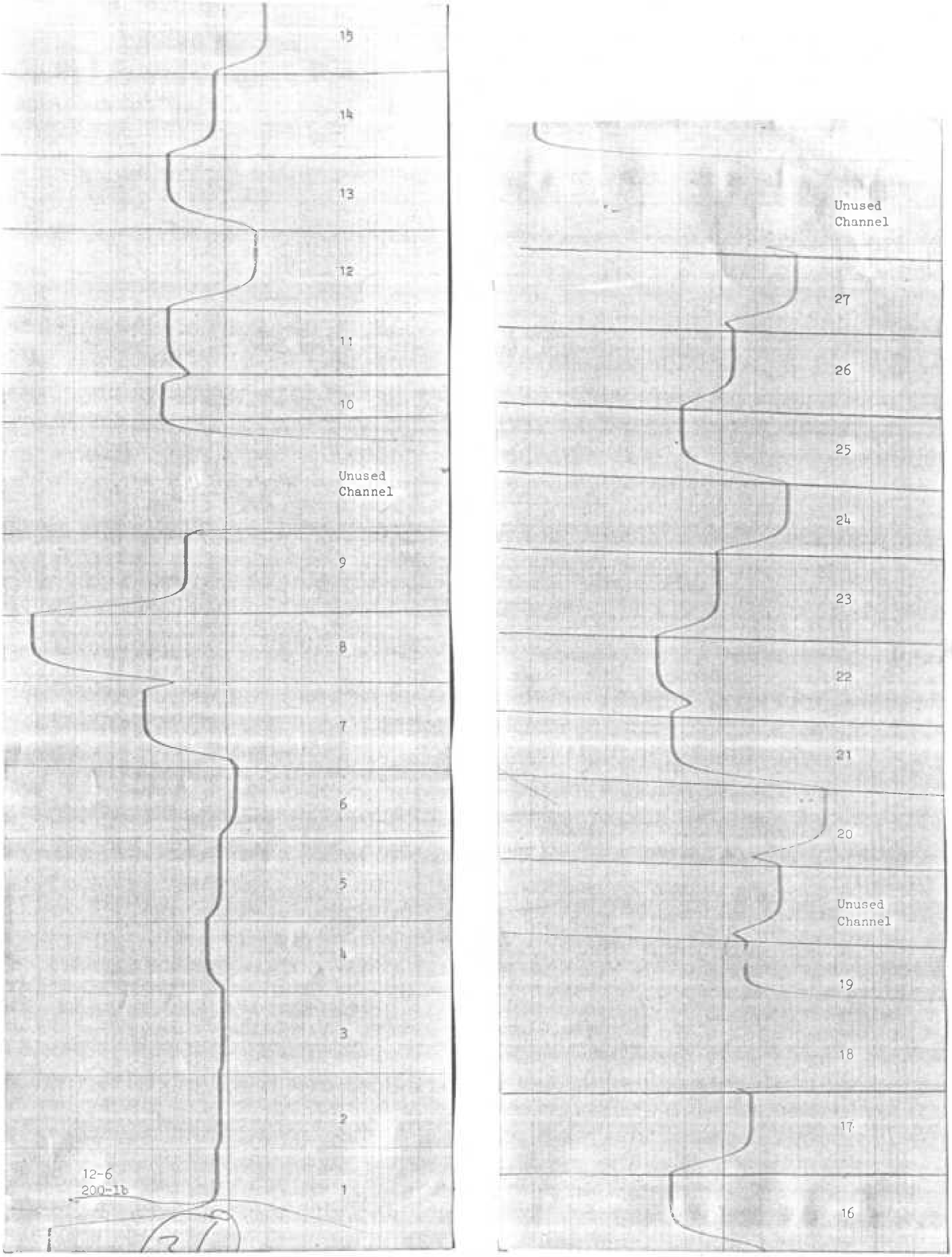

15
14
13
12
11
10
Unused Channel
9
8
7
6
5
4
3
2
1
12-2
200-lb
Unused Channel
27
26
25
24
23
22
21
20
Unused Channel
19
18
17
16


15
14
13
12
11
10
Unused
Channel
9
8
7
6
5
4
3
2
1
12-3
1.00-1b


Unused
Channel
27
26
25
24
23
22
21
20
Unused
Channel
19
18
17
16


15
14
13
12
11
10
Unused Channel
9
8
7
6
5
4
3
2
1
12-4
500-1b
Unused Channel
27
26
25
24
23
22
21
20
Unused Channel
19
18
17
16


15
14
13
12
11
10
Unused
Channel
9
8
7
6
5
4
3
2
1
12-5
400-1b
Unused
Channel
27
26
25
24
23
22
21
20
Unused
Channel
19
18
17
16


15
14
13
12
11
10
Unused Channel
9
8
7
6
5
4
3
2
1
12-6
200-lb
Unused Channel
27
26
25
24
23
22
21
20
Unused Channel
19
18
17
16

```
STRESS
9


   THE TOTAL NUMBER OF READINGS =     9

          INPUT THE STRAINS OF READING NO.     1
97.34, 83.43, 27.81

   1    .00009734 .00008343 .00002781 .00015335 .00002742       87.86     45.83

          INPUT THE STRAINS OF READING NO.     2
236.29, 139.05, 0.0

   2    .00023639 .000139050.00000000 .00038165-.00000621      203.56     74.09

          INPUT THE STRAINS OF READING NO.     3
361.53, 889.92, 528.39

   3    .00036153 .00088992 .00052839 .00090728 .00034417      556.74    368.77

          INPUT THE STRAINS OF READING NO.     4
417.15, 361.53, -55.62

   4    .00041715 .00036153-.00005562 .00083517-.00005649      436.76    139.11

          INPUT THE STRAINS OF READING NO.     5
417.15, 152.96, -194.67

   5    .00041715 .00015296-.00019467 .00078263-.00021252      376.91     44.71

          INPUT THE STRAINS OF READING NO.     6
0.0, 0.0, 0.0,

   6   0.000000000.000000000.000000000.000000000.00000000        0.00      0.00

          INPUT THE STRAINS OF READING NO.     7
-55.62, -83.43, 514.49

   7   -.00005562-.00008343 .00051449 .00051466-.00065371      143.59   -246.43

          INPUT THE STRAINS OF READING NO.     8
417.15, 55.62, -389.34

   8    .00041715 .00005562-.00038934 .00088770-.00041493      392.24    -42.60

          INPUT THE STRAINS OF READING NO.     9
305.91, -55.62, -556.20

   9    .00030591-.00005562-.00055620 .00083006-.00057977      327.86   -142.76
 STOP
```

```
>STRESS
9

     THE TOTAL NUMBER OF READINGS =     9

            INPUT THE STRAINS OF READING NO.     1
139.05, 27.81, -83.43

   1     .00013905 .00002781-.00008343 .00025932-.00009246        120.41       2.99

            INPUT THE STRAINS OF READING NO.     2
444.96, 97.34, -264.20

   2     .00044496 .00009734-.00026420 .00083401-.00029171        388.42      12.64

            INPUT THE STRAINS OF READING NO.     3
1237.55, 3031.29, 1001.16

   3     .00123755 .00303129 .00100116 .00357964 .00068920       2060.95   1096.07

            INPUT THE STRAINS OF READING NO.     4
1070.69, 945.54, -556.20

   4     .00107069 .00094554-.00055620 .00257368-.00055745       1268.16    222.94

            INPUT THE STRAINS OF READING NO.     5
1015.07, 139.05, -848.21

   5     .00101507 .00013905-.00084821 .00206812-.00091400        924.50    -70.98

            INPUT THE STRAINS OF READING NO.     6
0.0, 0.0, 0.0,

   6    0.000000000.000000000.000000000.000000000.00000000          0.00      0.00

            INPUT THE STRAINS OF READING NO.     7
-472.77, -1293.17, 1112.4,

   7    -.00047277-.00129317 .00111240 .00115413-.00292007         27.02  -1333.02

            INPUT THE STRAINS OF READING NO.     8
1084.59, -83.43, -1446.12

   8     .00108459-.00008343-.00144612 .00253299-.00153183       1048.66   -308.25

            INPUT THE STRAINS OF READING NO.     9
472.77, -500.58, -1752.03

   9     .00047277-.00050058-.00175203 .00179107-.00181888        592.25   -612.81
 STOP
```

```
>STRESS
9

     THE TOTAL NUMBER OF READINGS =      9

          INPUT THE STRAINS OF READING NO.     1
166.86, 55.62, -83.43

   1     .00016686 .00005562-.00008343 .00031370-.00007122       149.85     14.68

          INPUT THE STRAINS OF READING NO.     2
500.58, 111.24, -305.91

   2     .00050058 .00011124-.00030591 .00094795-.00033613       440.56     11.91

          INPUT THE STRAINS OF READING NO.     3
0.0, 0.0, 0.0

   3    0.000000000.000000000.000000000.000000000.00000000         0.00      0.00

          INPUT THE STRAINS OF READING NO.     4
1307.07, 1195.83, -695.25

   4     .00130707 .00119583-.00069525 .00319894-.00069604      1575.62    275.40

          INPUT THE STRAINS OF READING NO.     5
1237.55, 139.05, -1042.88

   5     .00123755 .00013905-.00104288 .00250452-.00112792      1115.32    -97.25

          INPUT THE STRAINS OF READING NO.     6
0.0, 0.0, 0.0

   6    0.000000000.000000000.000000000.000000000.00000000         0.00      0.00

          INPUT THE STRAINS OF READING NO.     7
-556.20, -1807.65, 1362.69

   7    -.00055620-.00180765 .00136269 .00143849-.00380234          .65  -1748.83

          INPUT THE STRAINS OF READING NO.     8
1307.07, -152.96, -1835.46

   8     .00130707-.00015296-.00183546 .00309760-.00194349      1268.16   -414.64

          INPUT THE STRAINS OF READING NO.     9
166.86, -862.11, -2044.04

   9     .00016686-.00086211-.00204404 .00142509-.00212034       334.68   -848.85
 STOP
```

```
>STRESS
9

     THE TOTAL NUMBER OF READINGS =     9

            INPUT THE STRAINS OF READING NO.     1
139.05, 27.81, -83.43

   1     .00013905 .00002781-.00008343 .00025932-.00009246       120.41       2.99

            INPUT THE STRAINS OF READING NO.     2
472.77, 83.43, -305.91

   2     .00047277 .00008343-.00030591 .00089370-.00033750       411.17        .17

            INPUT THE STRAINS OF READING NO.     3
1557.36, -83.43, 1112.40

   3     .00155736-.00008343 .00111240 .00163918-.00016525       846.20     243.85

            INPUT THE STRAINS OF READING NO.     4
1307.07, 1168.02, -695.25

   4     .00130707 .00116802-.00069525 .00317159-.00069650      1560.84     269.61

            INPUT THE STRAINS OF READING NO.     5
1195.83, 139.05, -1015.07

   5     .00119583 .00013905-.00101507 .00243097-.00109609      1082.30     -95.09

            INPUT THE STRAINS OF READING NO.     6
0.0, 0.0, 0.0,

   6    0.000000000.000000000.000000000.000000000.00000000         0.00       0.00

            INPUT THE STRAINS OF READING NO.     7
-584.01, -1835.46, 1279.26

   7    -.00058401-.00183546 .00127926 .00135671-.00377618       -37.94   -1751.38

            INPUT THE STRAINS OF READING NO.     8
1293.17, -139.05, -1779.84

   8     .00129317-.00013905-.00177984 .00304035-.00188623      1249.05    -395.52

            INPUT THE STRAINS OF READING NO.     9
305.91, -806.49, -2057.94

   9     .00030591-.00080649-.00205794 .00164099-.00214157       446.24    -816.45
 STOP
```

```
>STRESS
9

     THE TOTAL NUMBER OF READINGS =      9

          INPUT THE STRAINS OF READING NO.     1
83.43, 55.62, 13.91

   1     .00008343 .00005562 .00001391 .00012685 .00001220        70.55      32.28

          INPUT THE STRAINS OF READING NO.     2
305.91, 111.24, -125.15

   2     .00030591 .00011124-.00012515 .00055620-.00013905       270.30      38.21

          INPUT THE STRAINS OF READING NO.     3
1223.64, 2850.53, 597.92

   3     .00122364 .00285053 .00059792 .00369023 .00038394      2058.37     954.68

          INPUT THE STRAINS OF READING NO.     4
931.64, 862.11, -458.87

   4     .00093164 .00086211-.00045887 .00225307-.00045932      1116.00     210.56

          INPUT THE STRAINS OF READING NO.     5
862.11, 166.86, -597.92

   5     .00086211 .00016686-.00059792 .00167994-.00065097       769.54      -8.56

          INPUT THE STRAINS OF READING NO.     6
0.0, 0.0, 0.0

   6   0.000000000.000000000.000000000.000000000.00000000         0.00       0.00

          INPUT THE STRAINS OF READING NO.     7
-444.96, -1501.74, 723.06

   7   -.00044496-.00150174 .00072306 .00080345-.00275015      -126.72   -1312.97

          INPUT THE STRAINS OF READING NO.     8
653.54, 0.0, -1028.97

   8     .000653540.00000000-.00102897 .00172133-.00106779       707.19    -223.87

          INPUT THE STRAINS OF READING NO.     9
556.20, -305.91, -1168.02

   9     .00055620-.00030591-.00116802 .00148826-.00123797       547.58    -362.48
 STOP
```

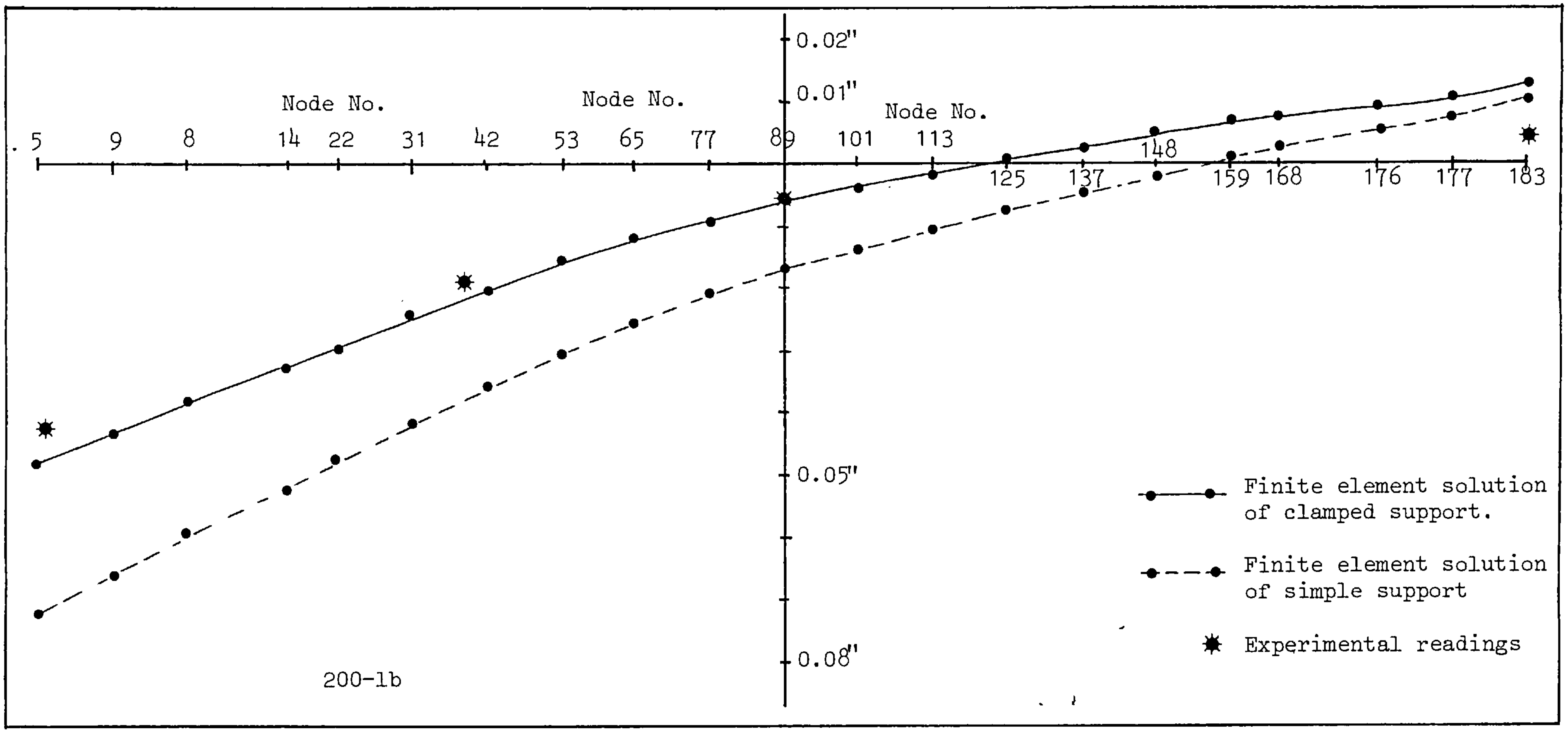


Fig. 2.48. Comparison of deflections due to point load at Node 6 along x-axis.

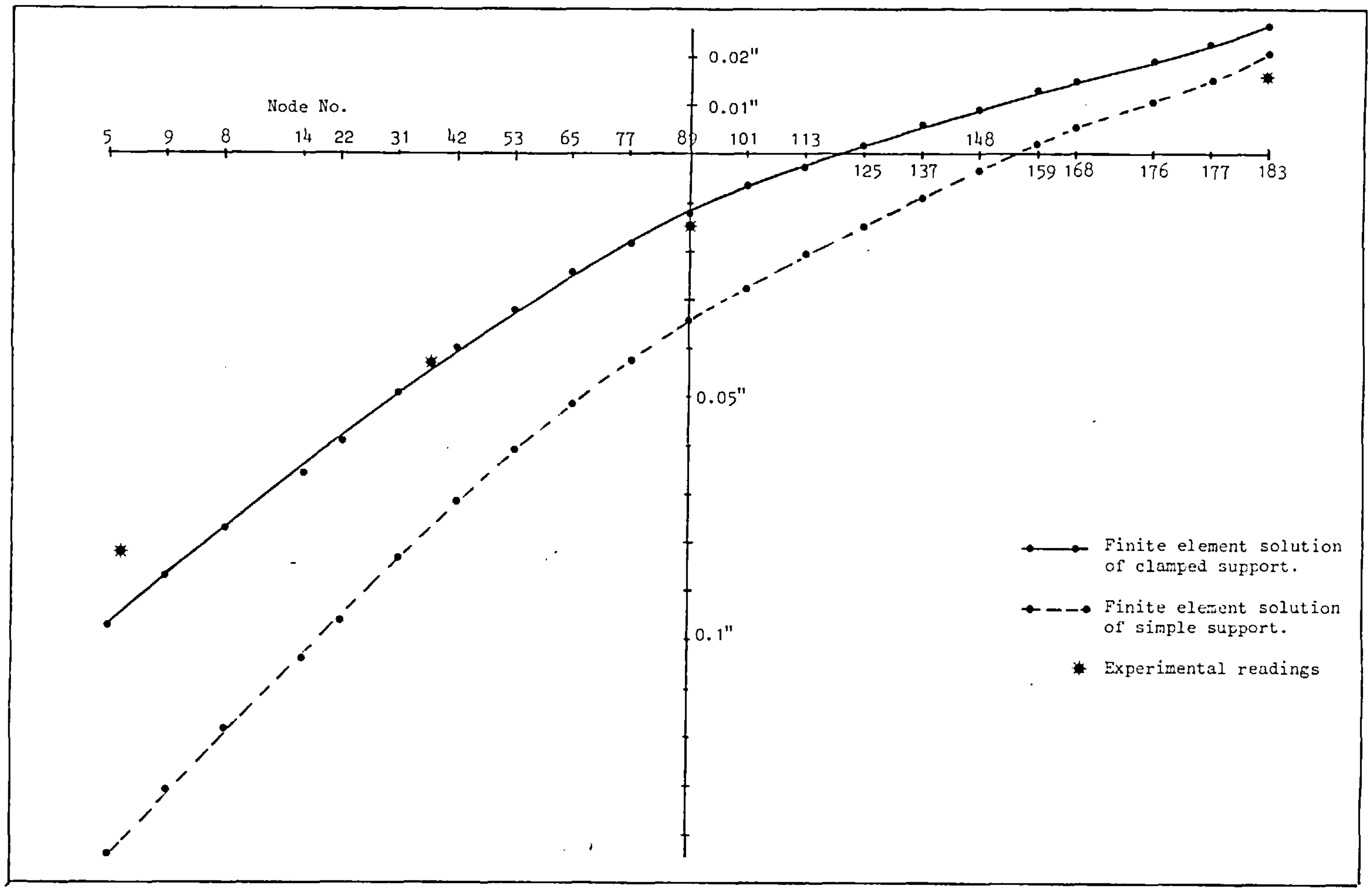


Fig. 2.49. Comparison of deflections due to point load at Node 6 along x-axis.

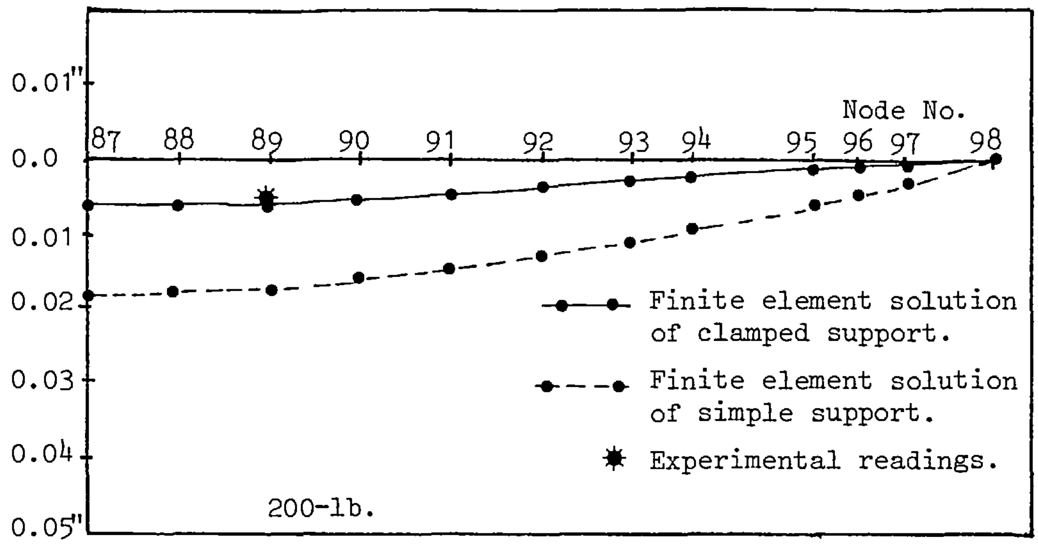


Fig. 2.50a. Comparison of deflections due to point load at Node 6 along y-axis.

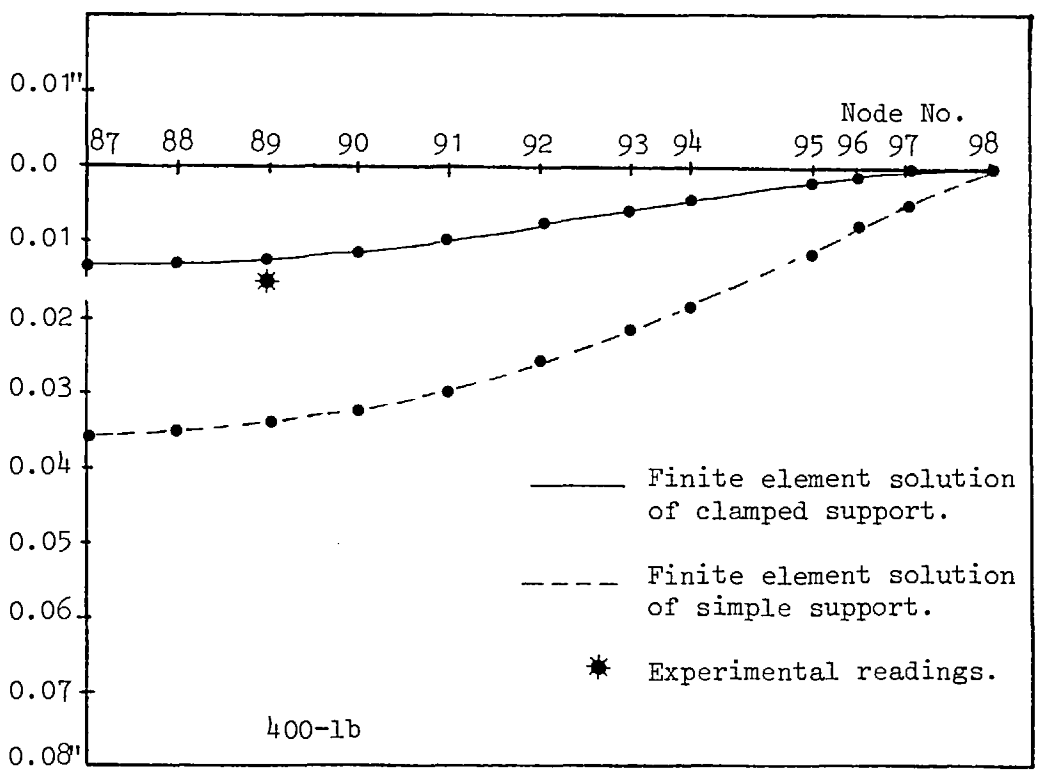


Fig. 2.50b. Comparison of deflections due to point load at Node 6 along y-axis.

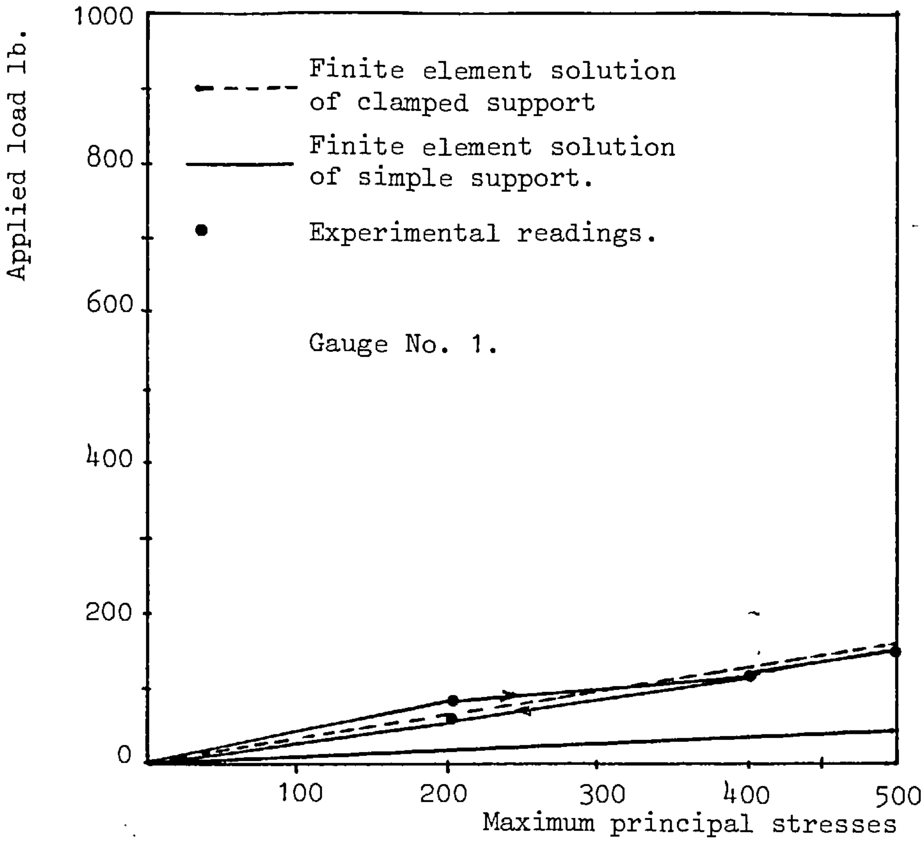


Fig. 2.51. Comparison of principal stresses

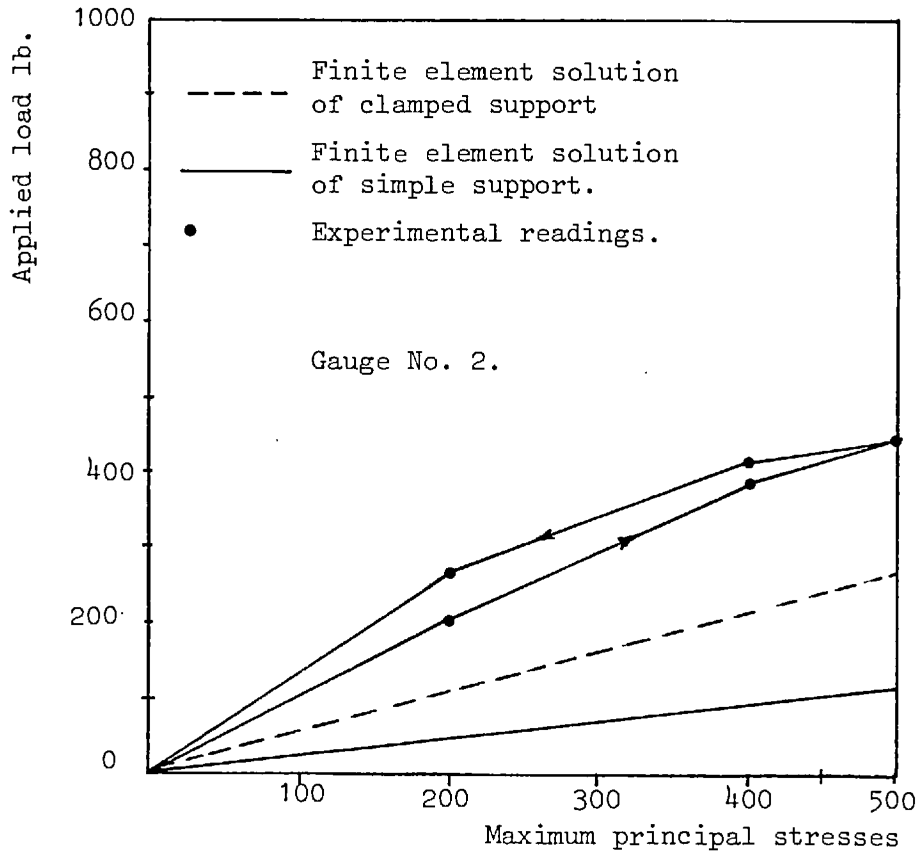
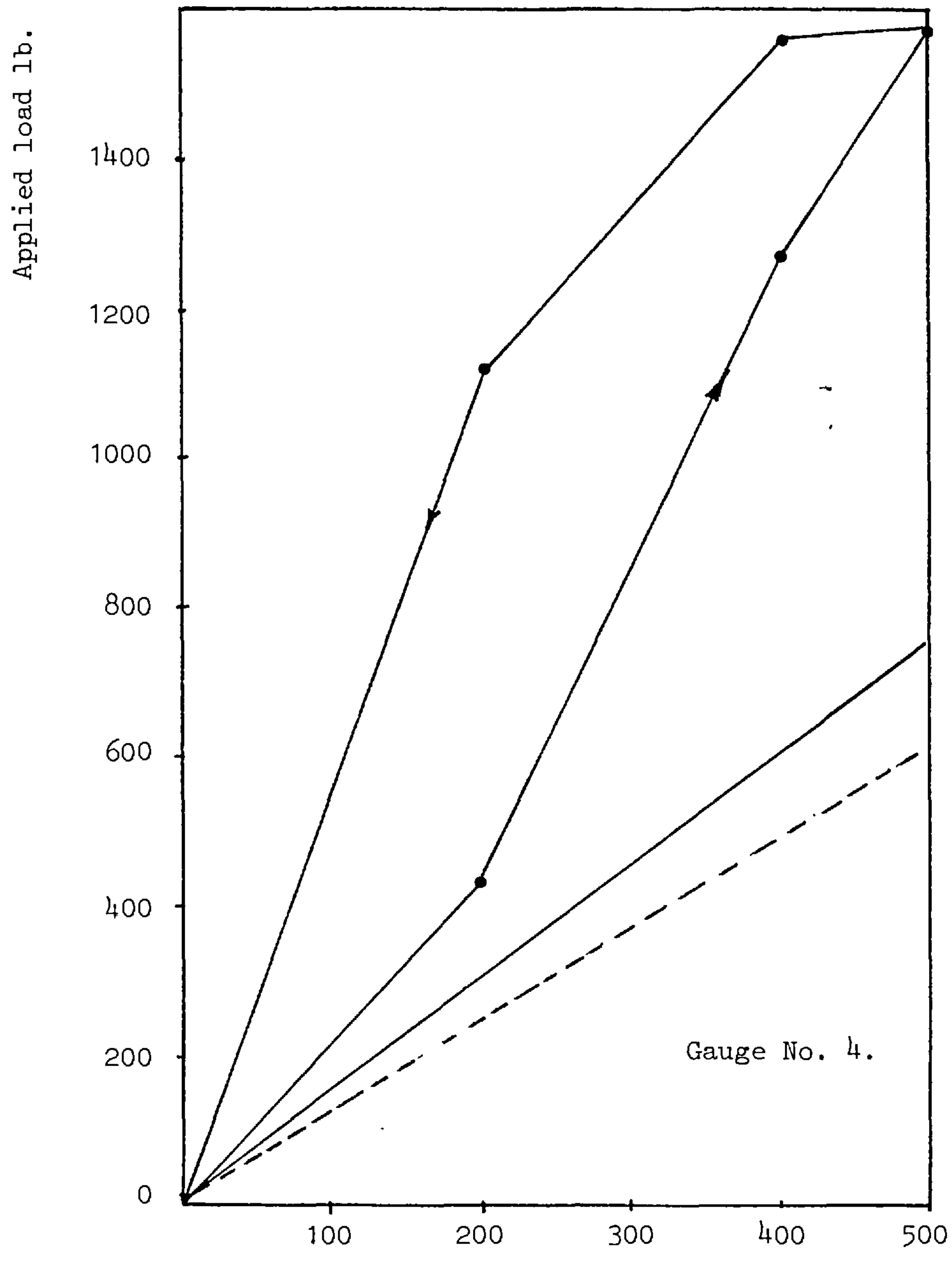


Fig. 2.52. Comparison of principal stresses



Finite element solution of clamped support.

Finite element solution of simple support.

Experimental readings

Fig. 2.53. Comparison of principal stresses

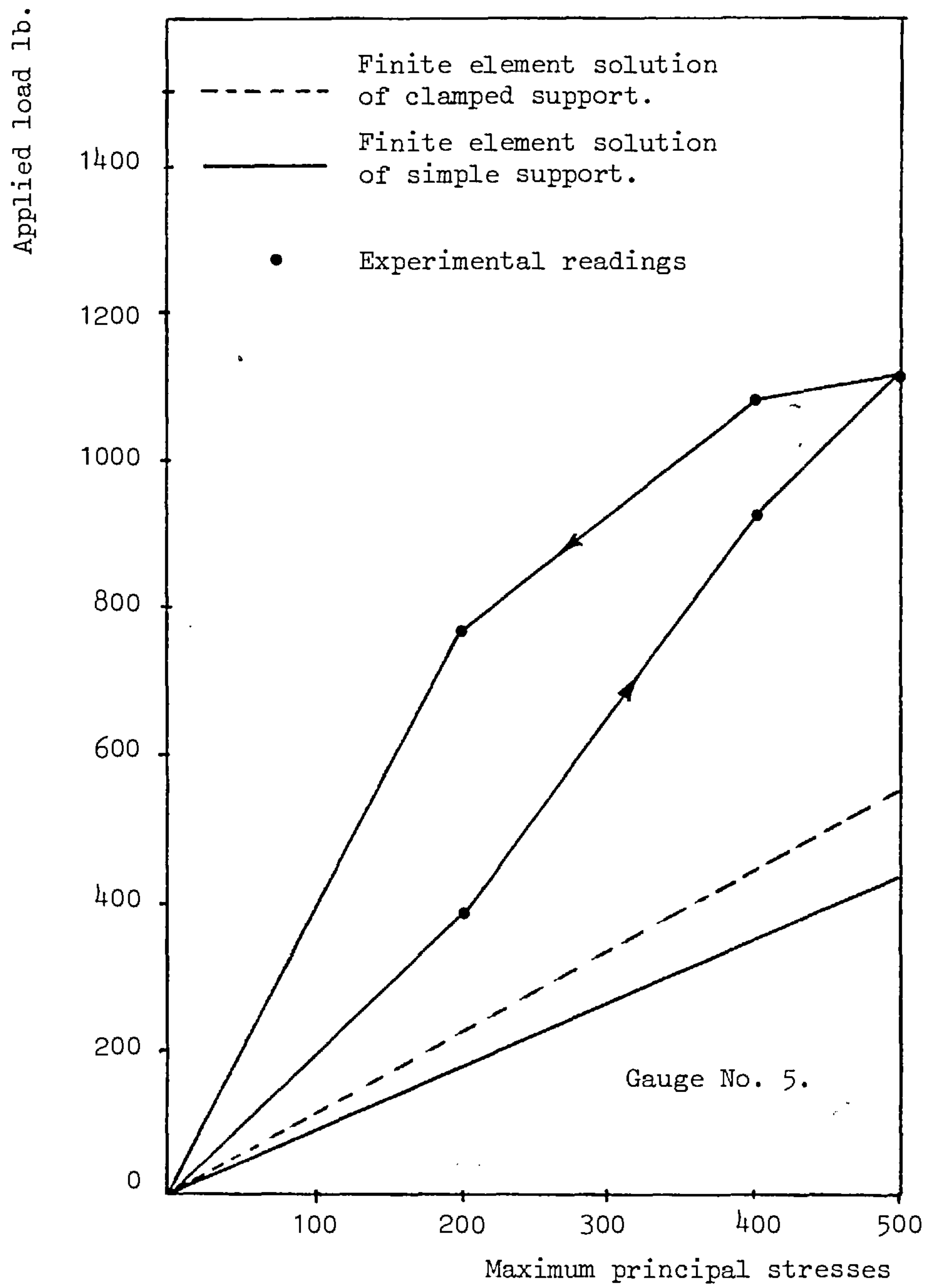


Fig. 2.54. Comparison of principal stresses

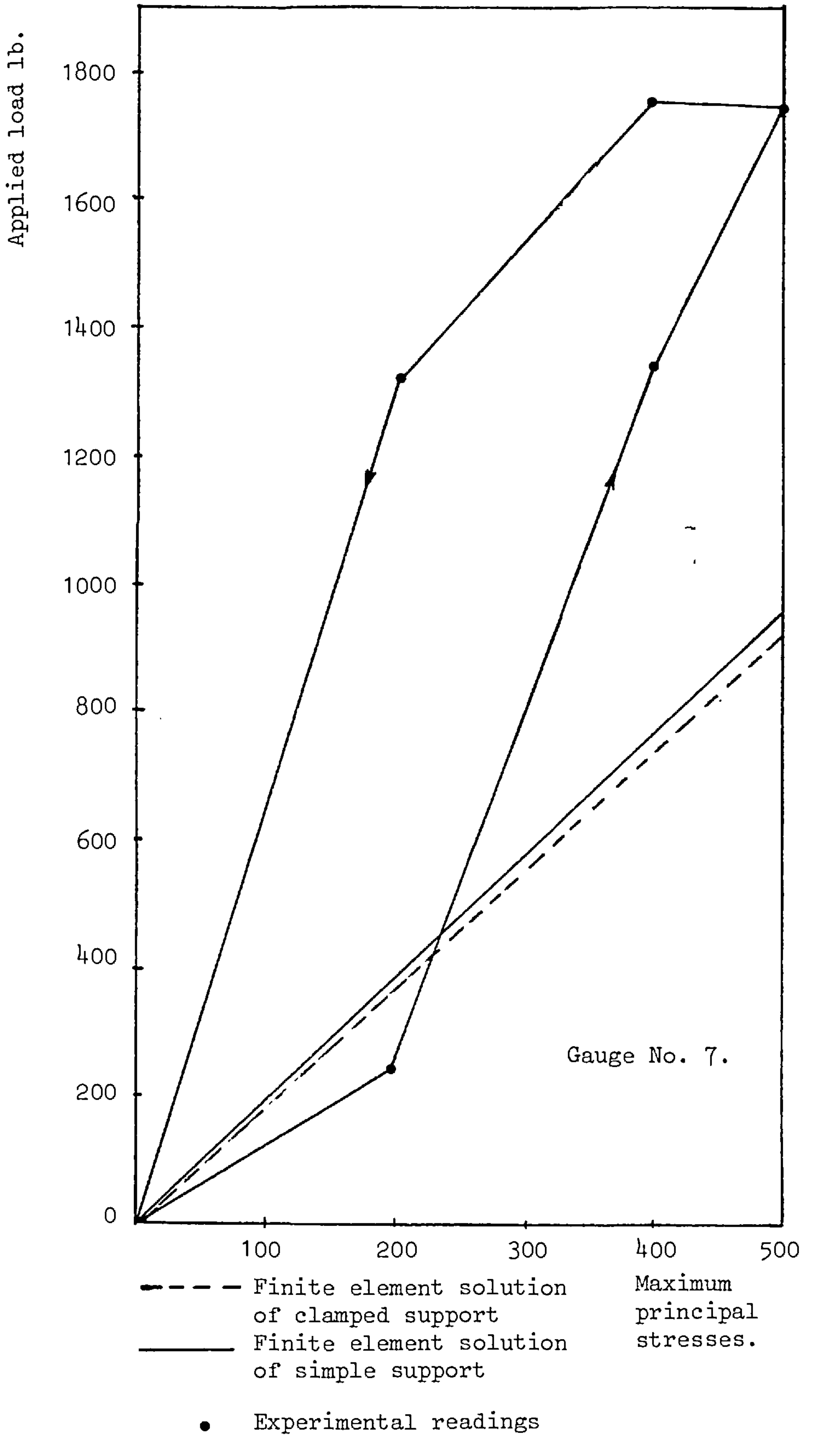


Fig. 2.55. Comparison of principal stresses.

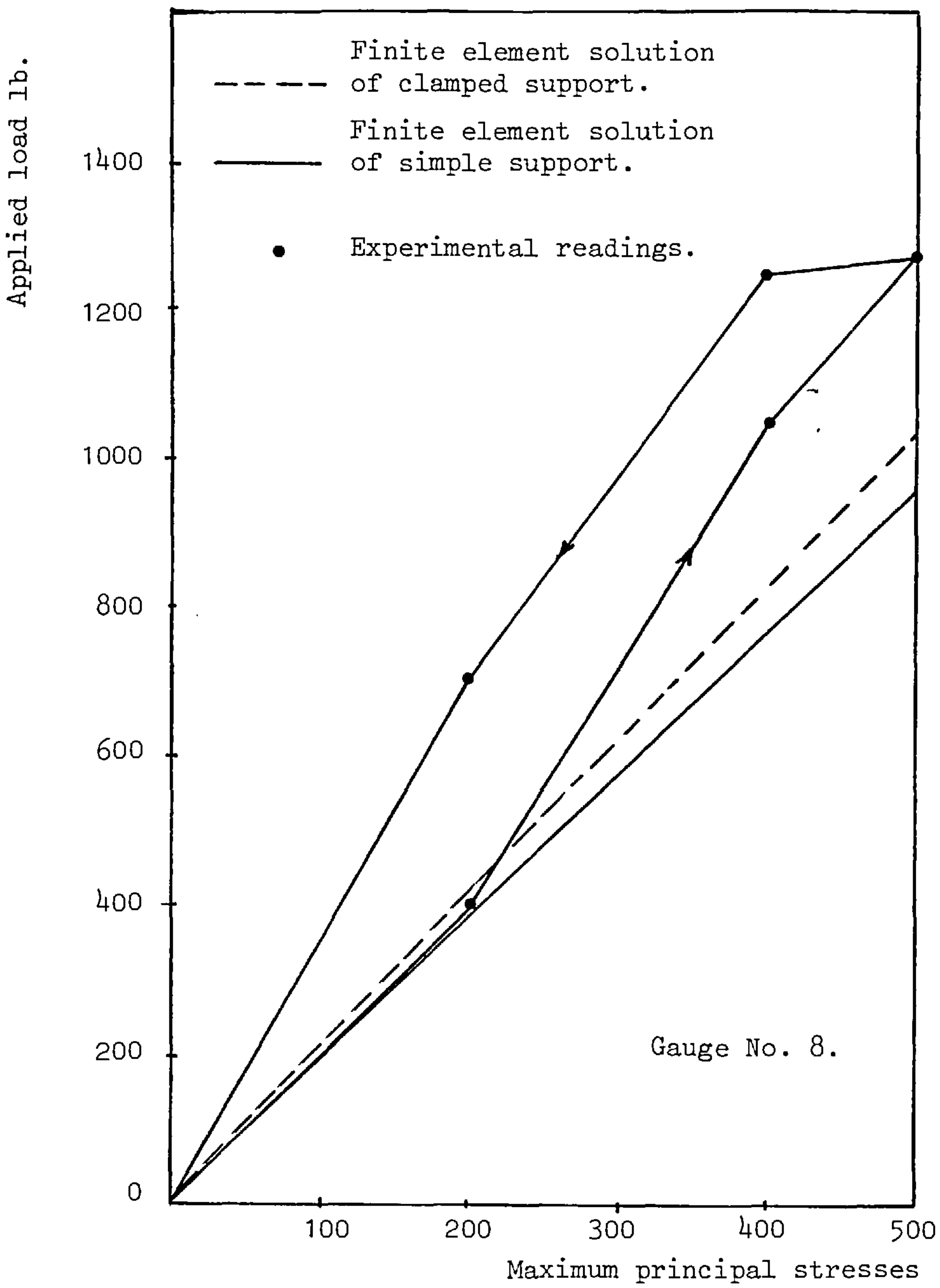


Fig. 2.56. Comparison of principal stresses.

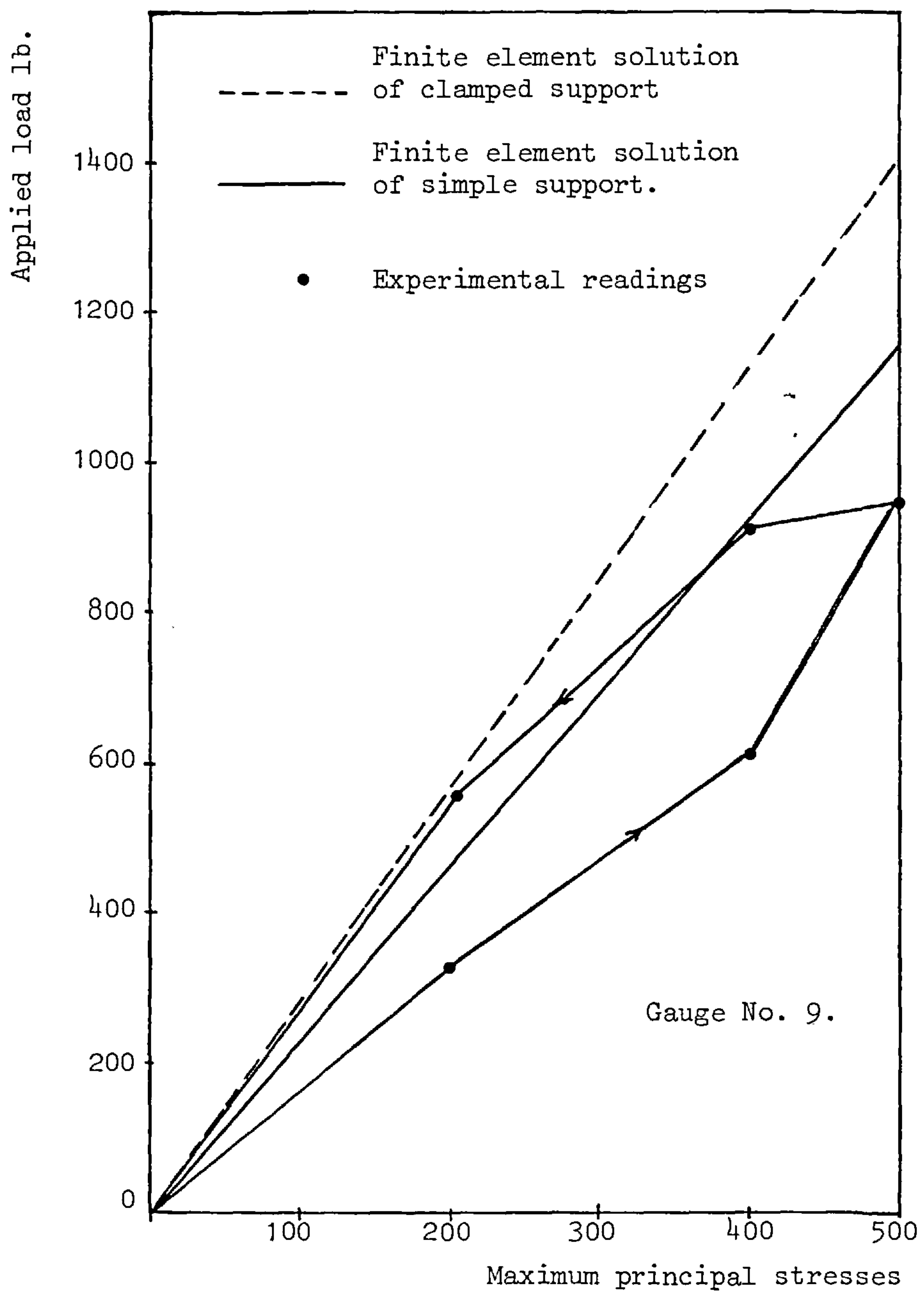


Fig. 2.57. Comparison of principal stresses

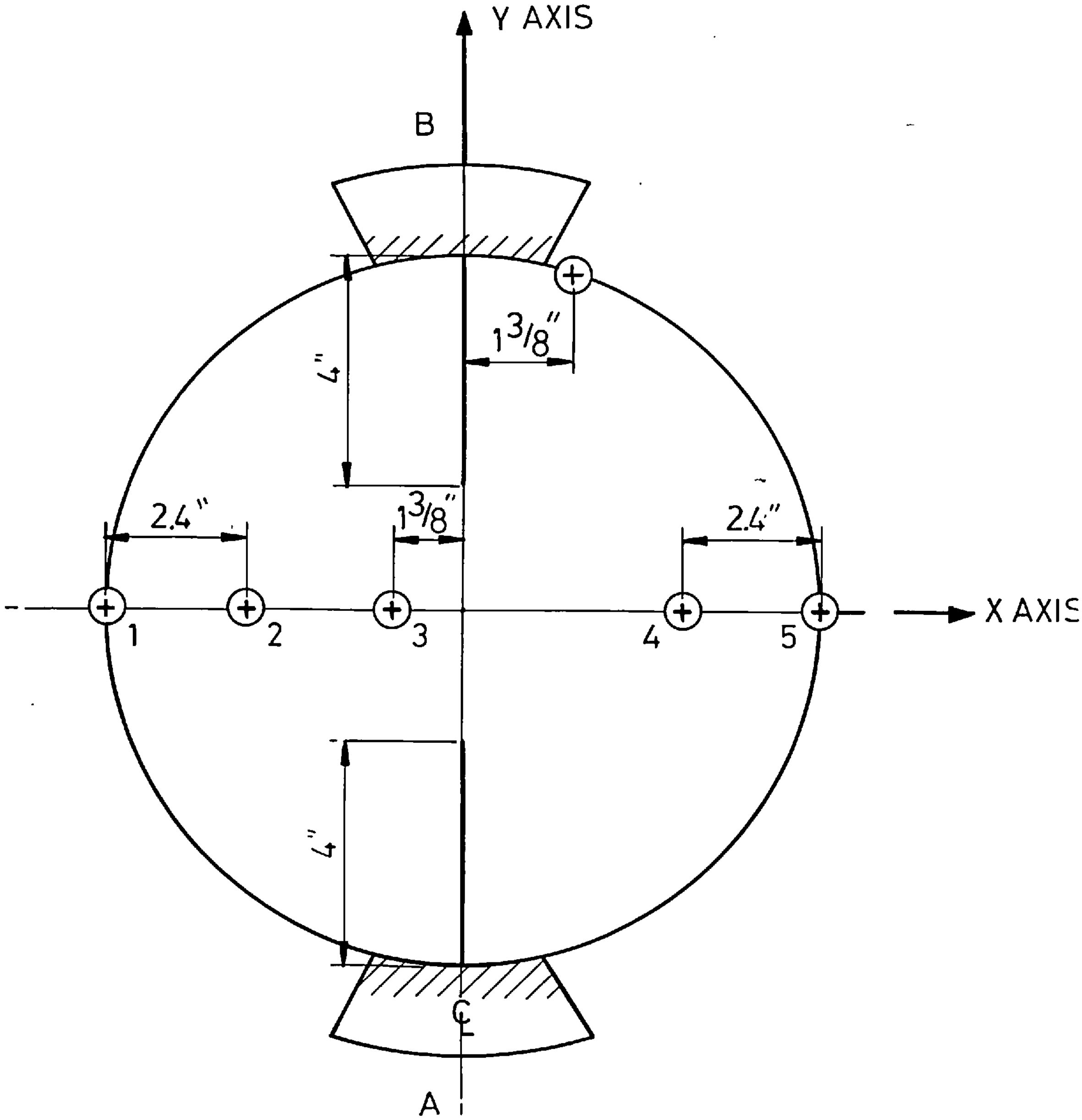


Fig. 2.58. Dial gauge locations.

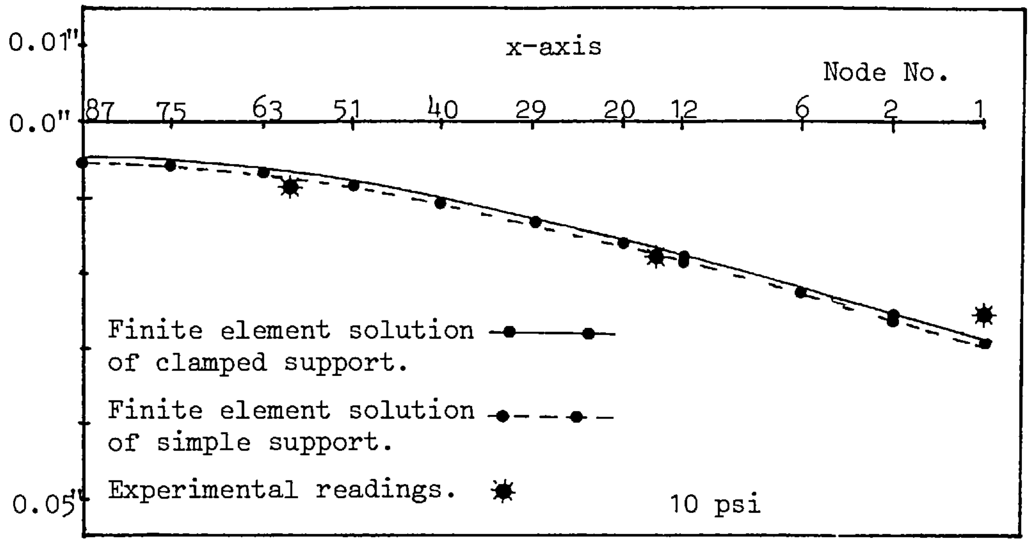


Fig. 2.60. Comparison of deflections due to uniform distributed pressure along x-axis.

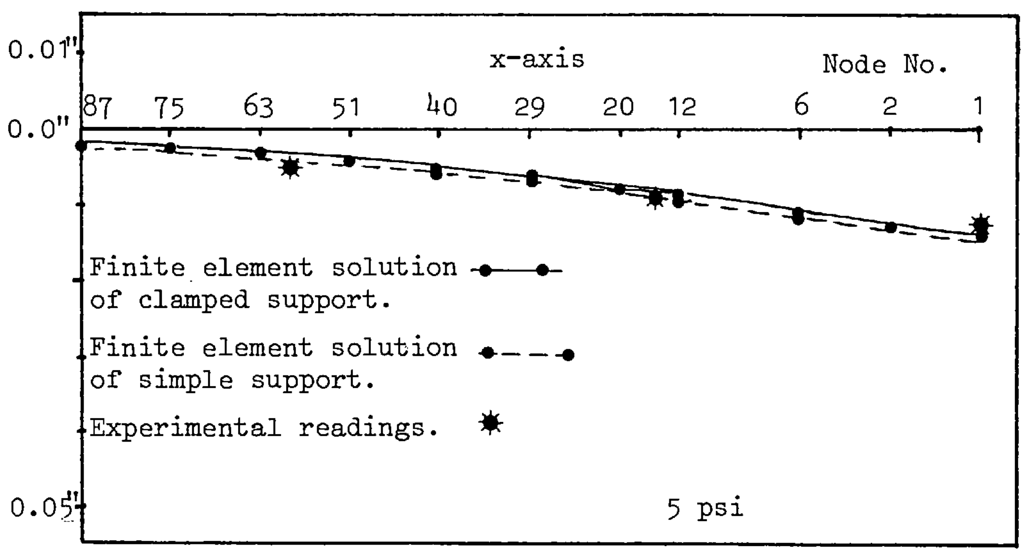
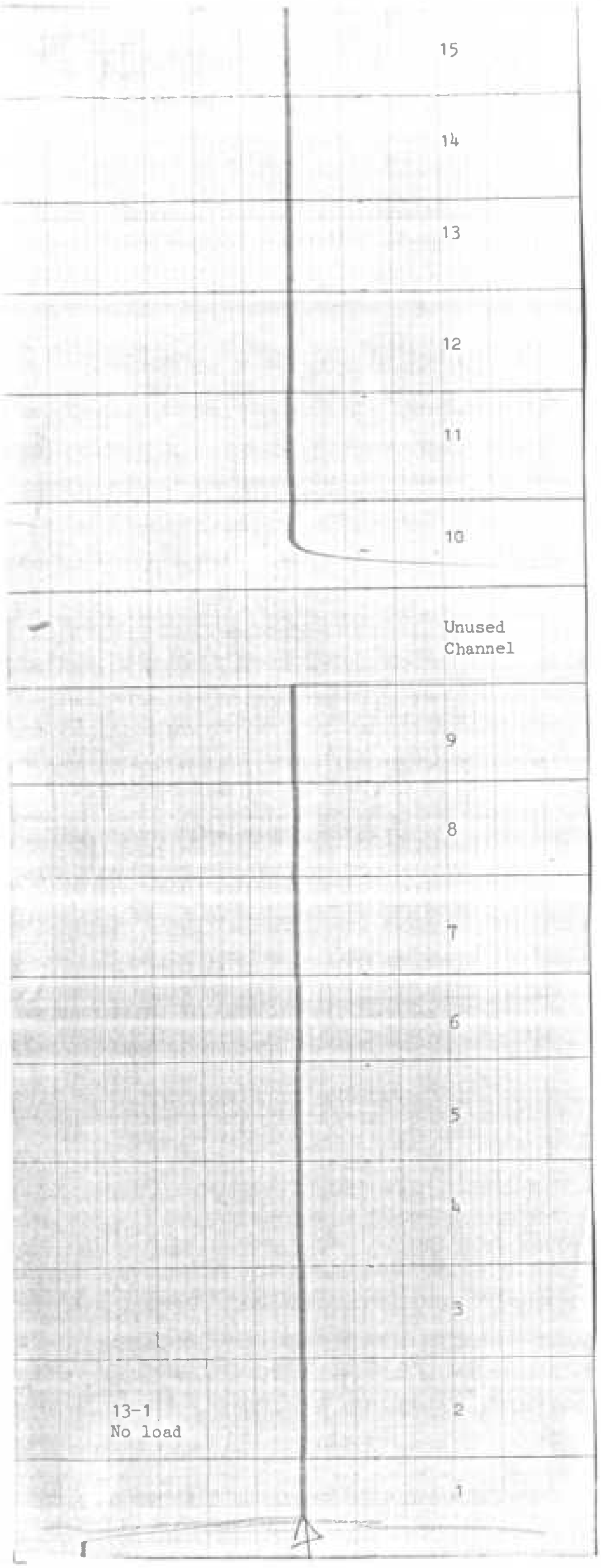
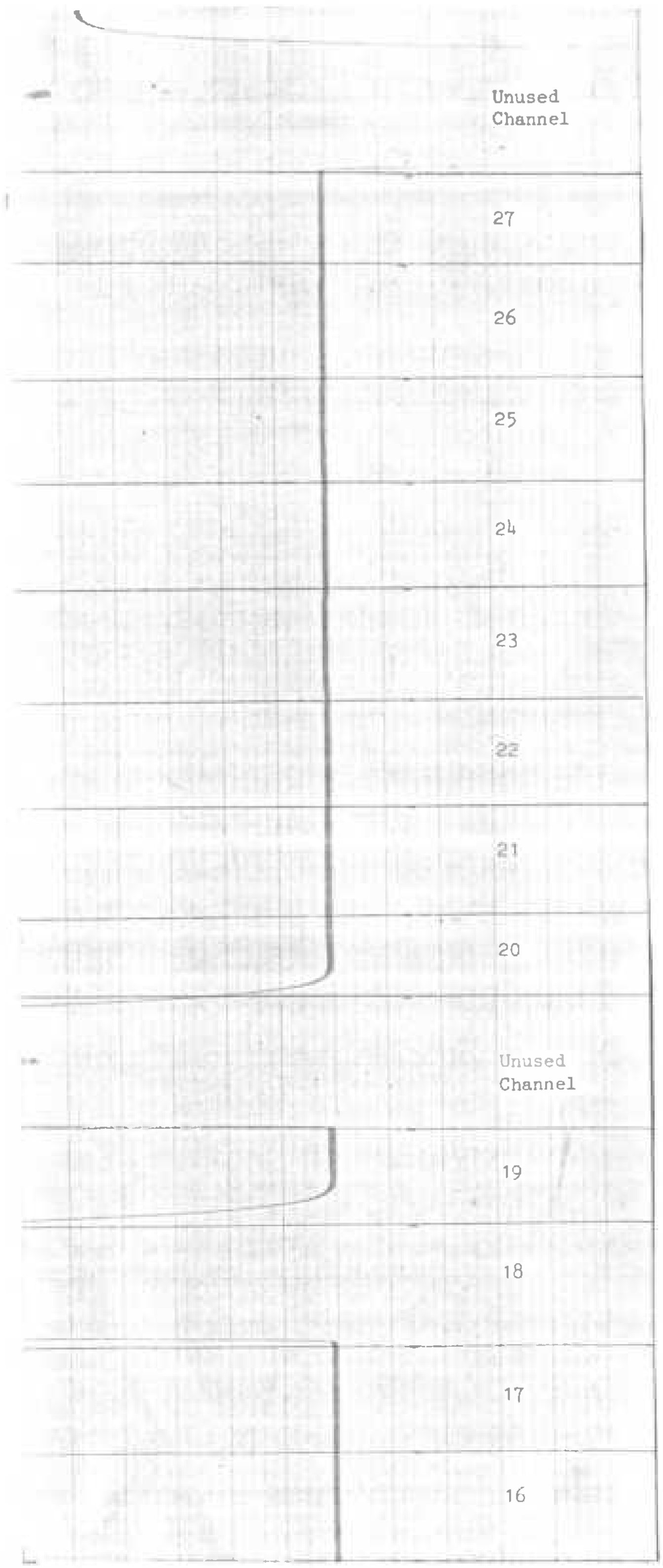
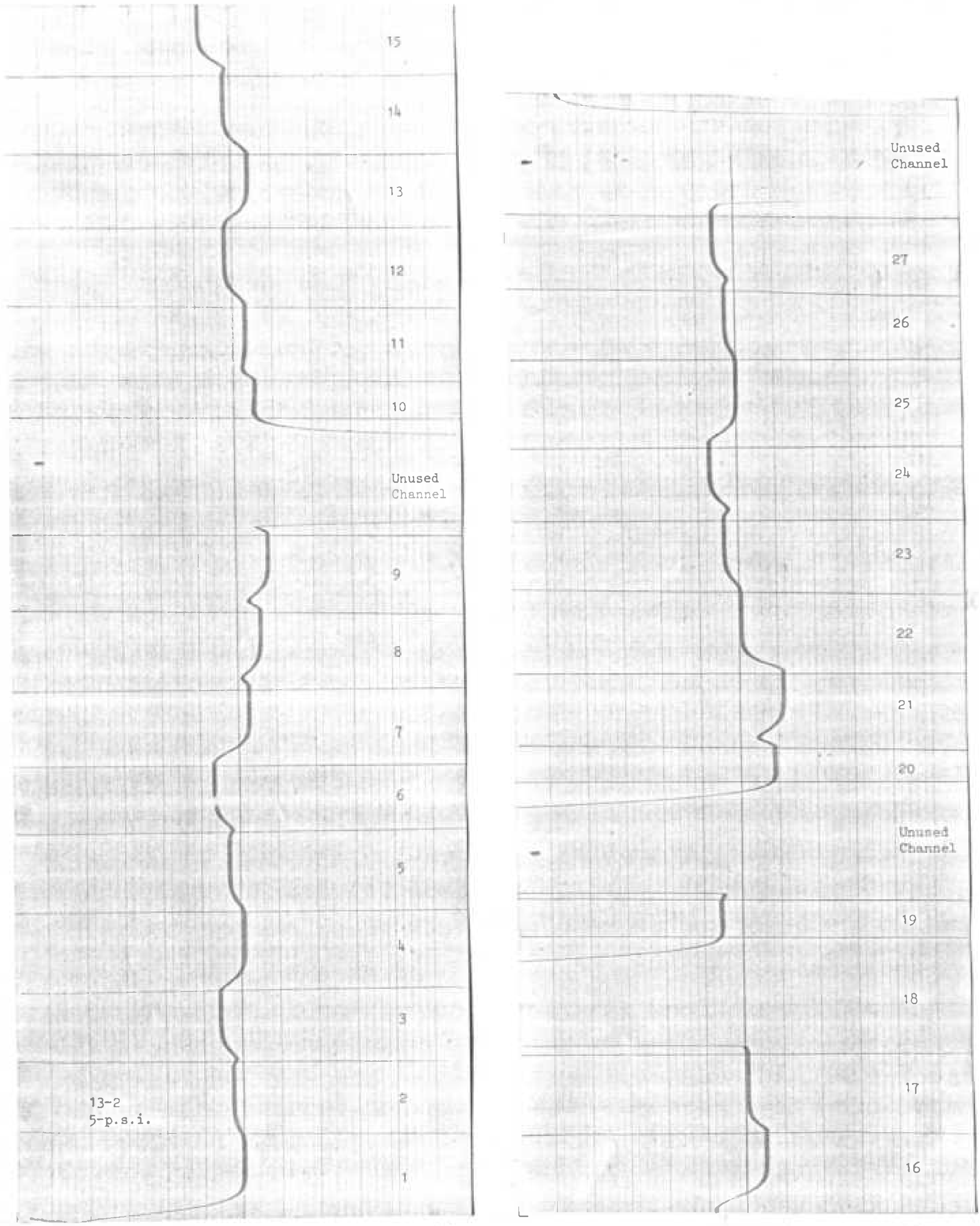
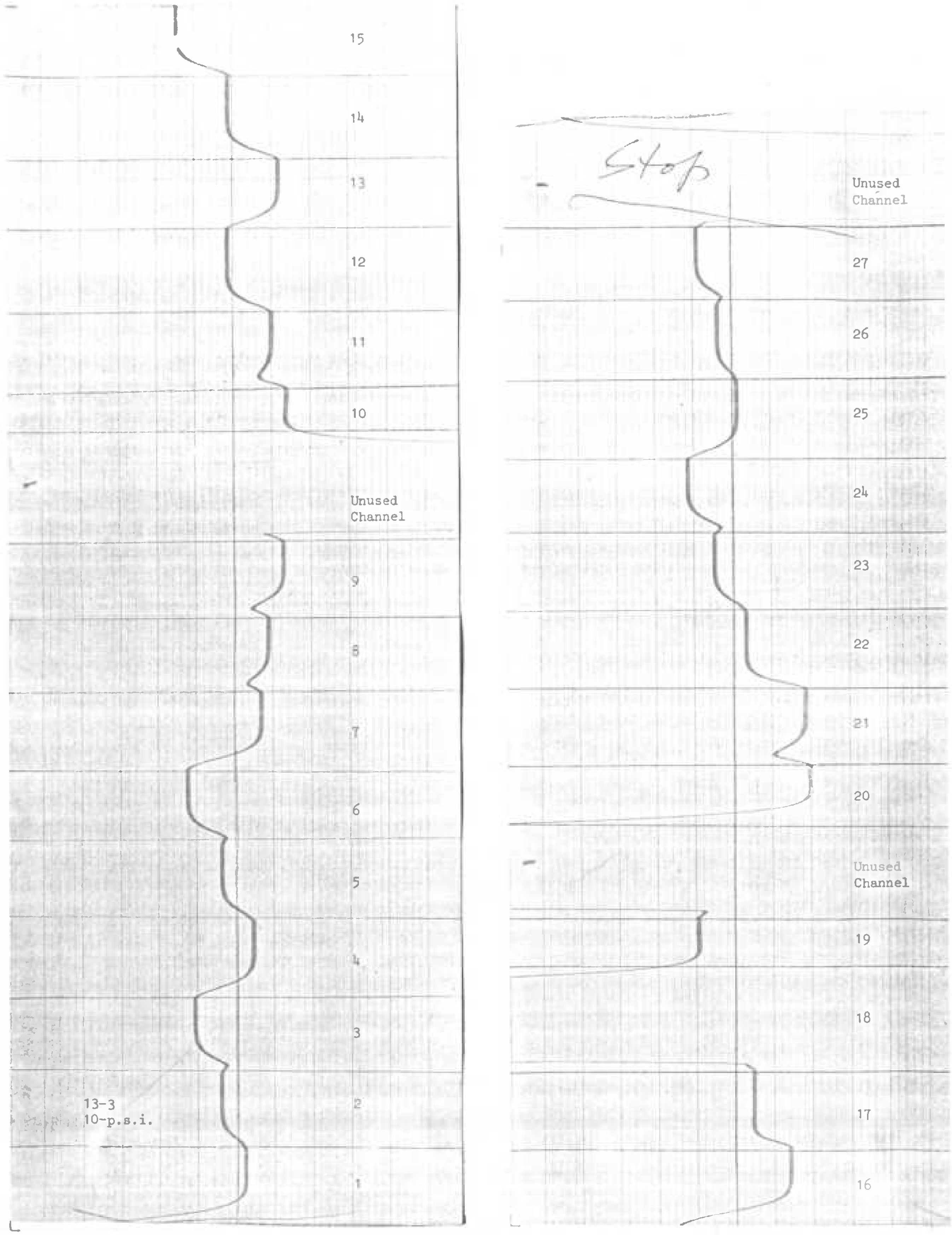


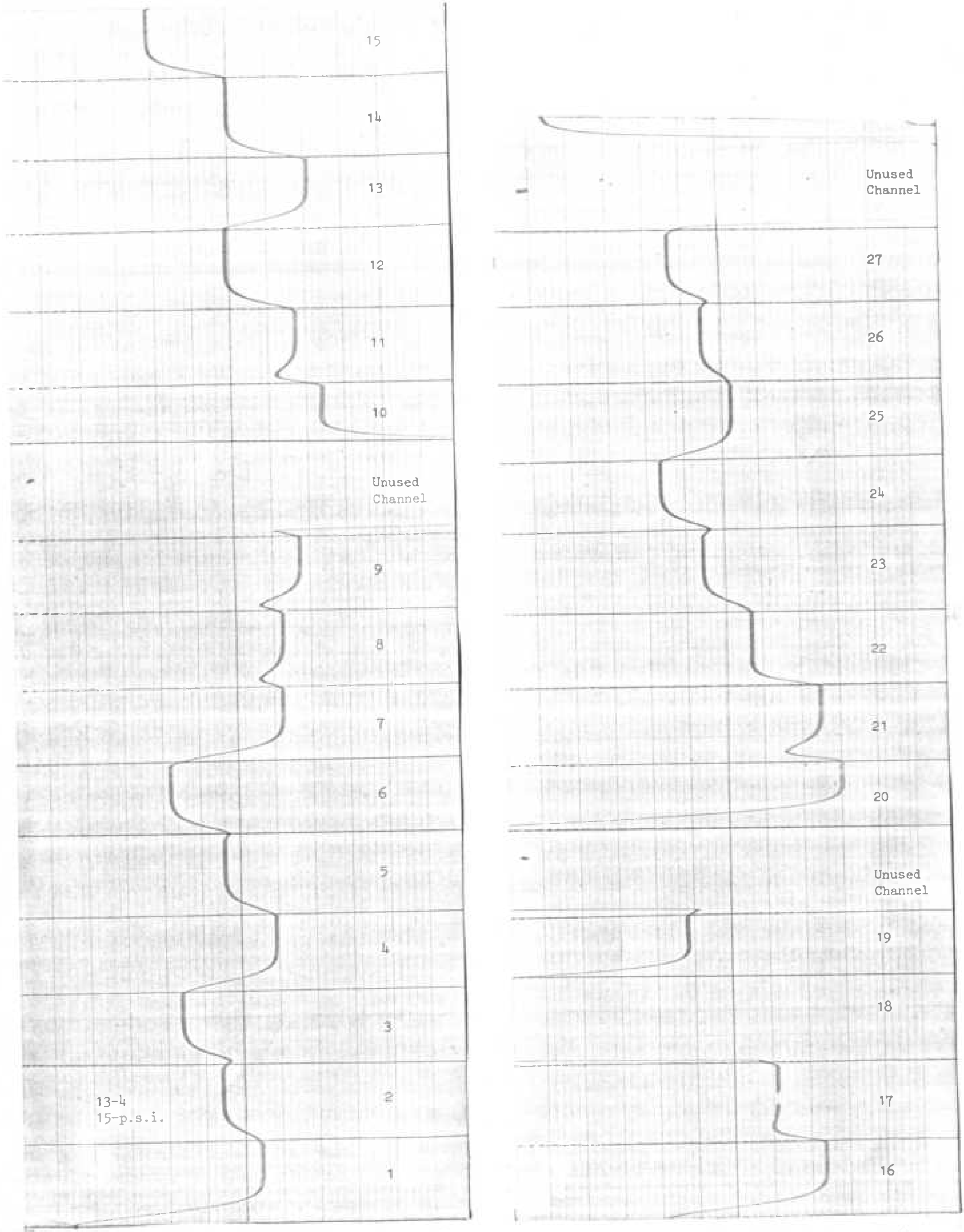
Fig. 2.59. Comparison of deflections due to uniform distributed pressure along x-axis.

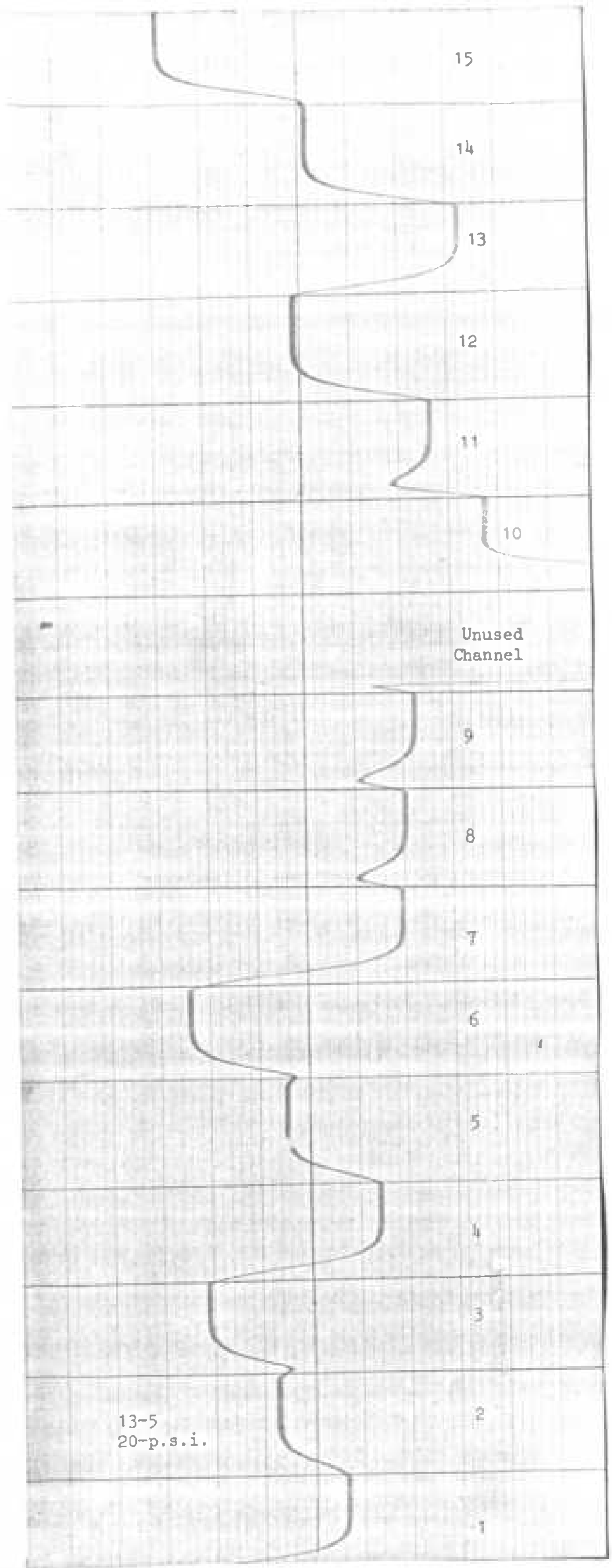

15
14
13
12
11
10
Unused
Channel
9
8
7
6
5
4
3
2
1
13-1
No load

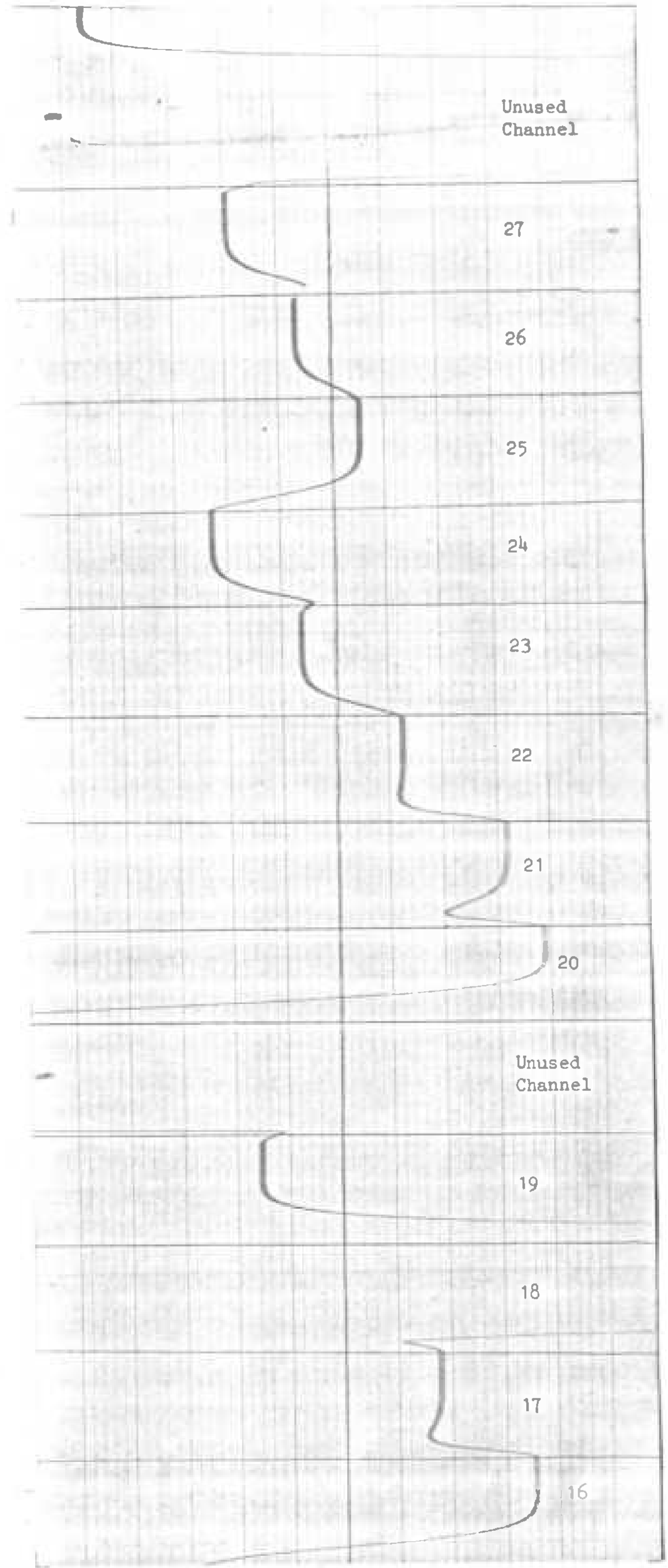

Unused
Channel
27
26
25
24
23
22
21
20
Unused
Channel
19
18
17
16

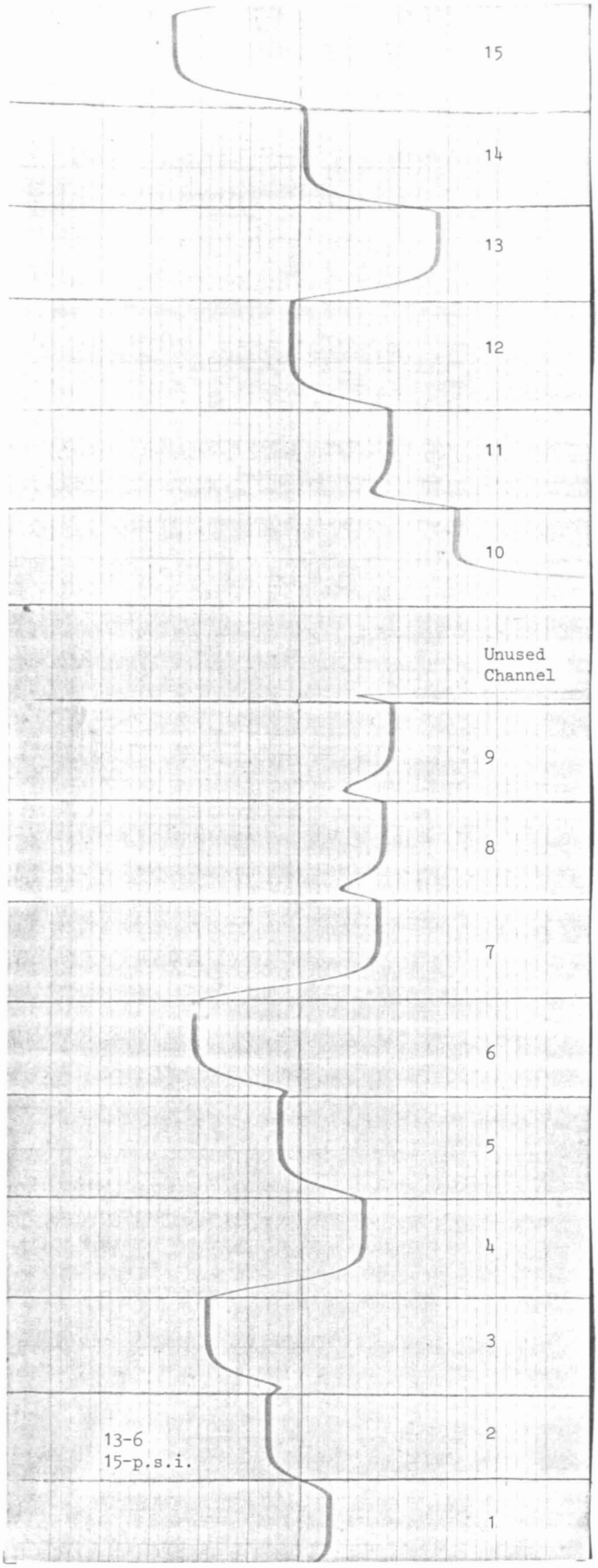
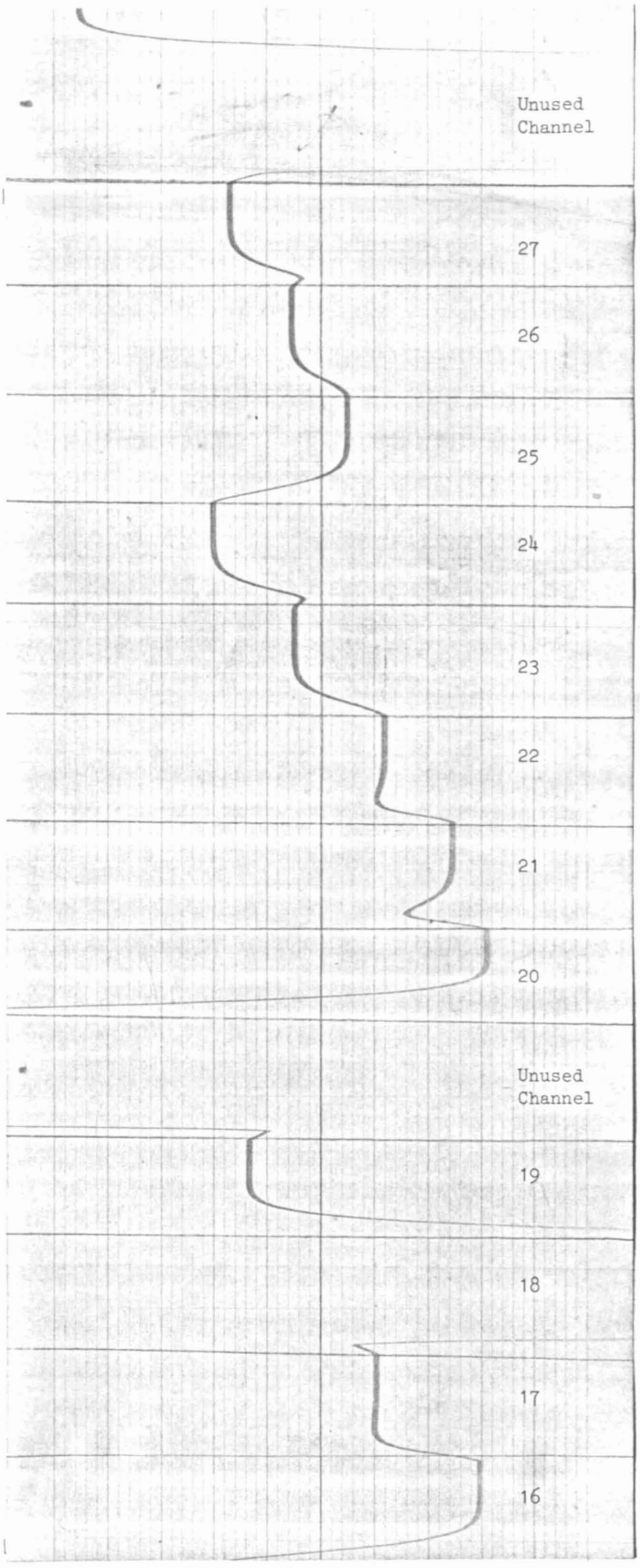

15
14
13
12
11
10
Unused
Channel
9
8
7
6
5
4
3
2
1
13-2
5-p.s.i.
Unused
Channel
27
26
25
24
23
22
21
20
Unused
Channel
19
18
17
16

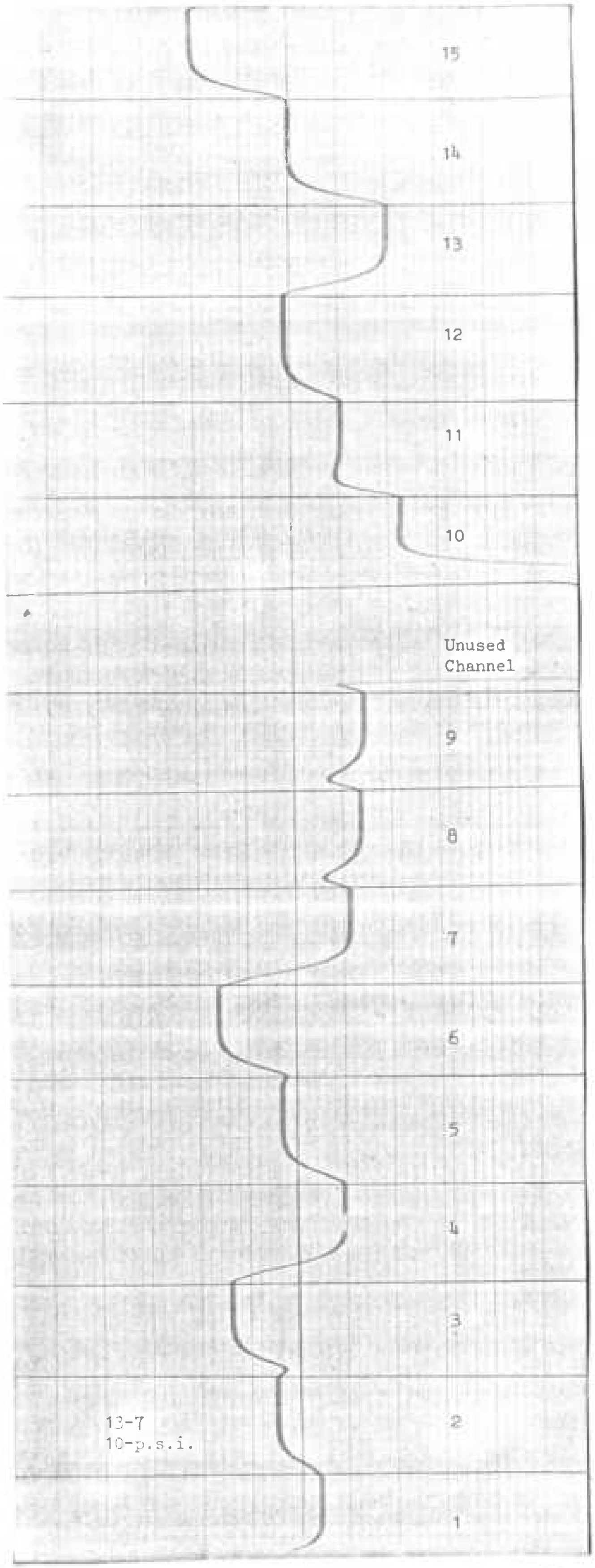

15
14
13
12
11
10
Unused Channel
9
8
7
6
5
4
3
2
1
13-3
10-p.s.i.
Stop
Unused Channel
27
26
25
24
23
22
21
20
Unused Channel
19
18
17
16

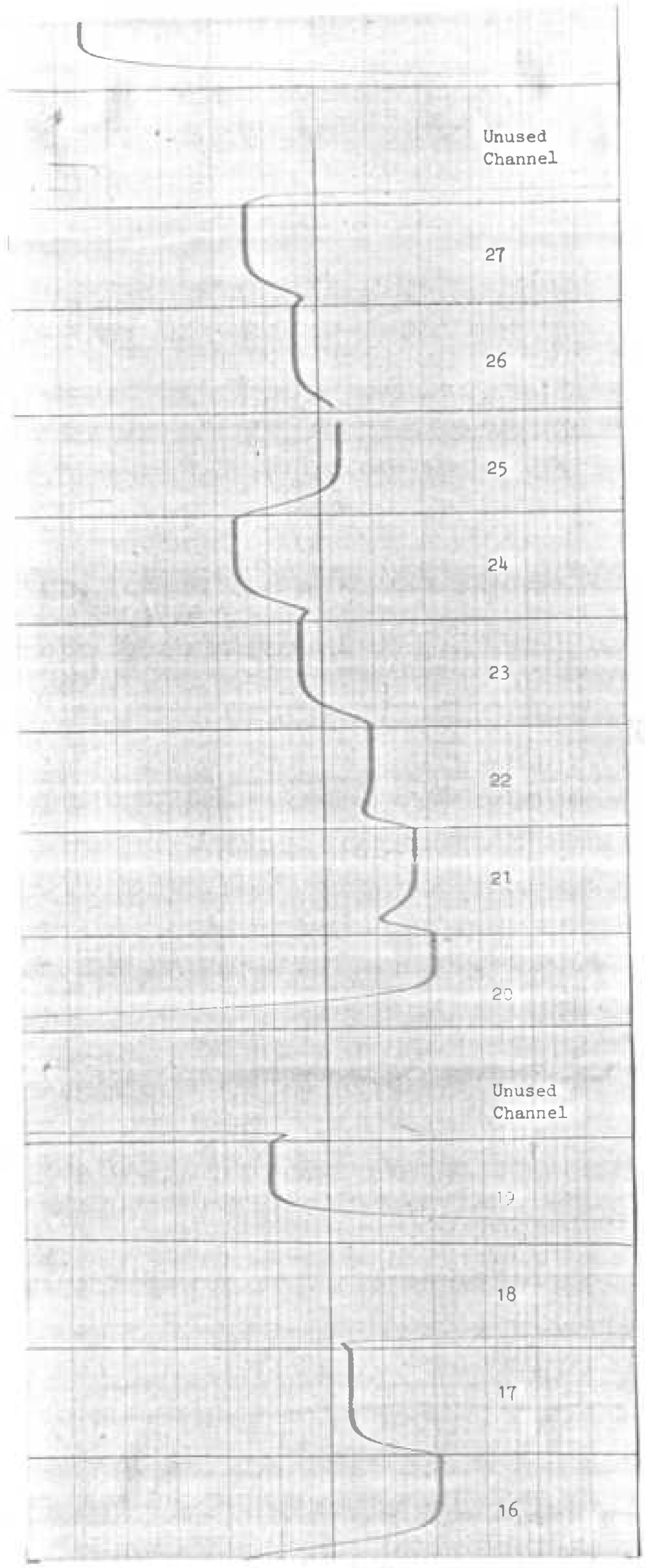

15
14
13
12
11
10
Unused Channel
9
8
7
6
5
4
3
13-4
15-p.s.i.
2
1
Unused Channel
27
26
25
24
23
22
21
20
Unused Channel
19
18
17
16

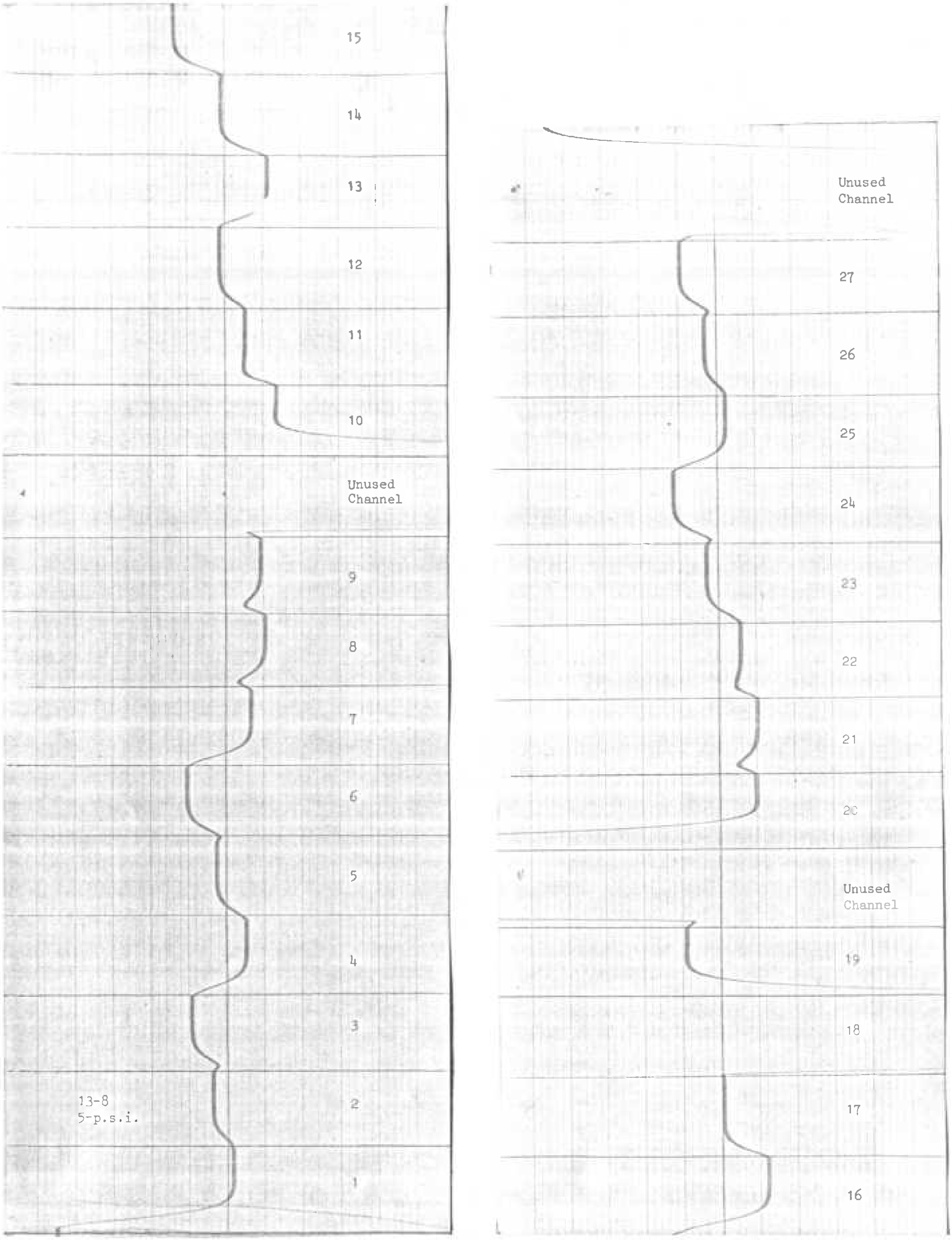

15
14
13
12
11
10
Unused Channel
9
8
7
6
5
4
3
13-5
20-p.s.i.
2
1


Unused Channel
27
26
25
24
23
22
21
20
Unused Channel
19
18
17
16


15
14
13
12
11
10
Unused
Channel
9
8
7
6
5
4
3
2
13-6
15-p.s.i.
1

Unused
Channel
27
26
25
24
23
22
21
20
Unused
Channel
19
18
17
16


15
14
13
12
11
10
Unused Channel
9
8
7
6
5
4
3
2
1
13-7
10-p.s.i.


Unused Channel
27
26
25
24
23
22
21
Unused Channel
18
17
16



15
14
13
12
11
10
Unused Channel
9
8
7
6
5
4
3
2
1
13-8
5-p.s.i.
Unused Channel
27
26
25
24
23
22
21
20
Unused Channel
19
18
17
16

```
STRESS
9

     THE TOTAL NUMBER OF READINGS =     9

          INPUT THE STRAINS OF READING NO.     1
0.0, 139.05, 333.72

   1    0.00000000 .00013905 .00033372 .00034271-.00020366        142.61    -39.78

          INPUT THE STRAINS OF READING NO.     2
44.06, 155.74, 389.34

   2     .00004406 .00015574 .00038934 .00039468-.00019488        172.28    -24.52

          INPUT THE STRAINS OF READING NO.     3
164.02, -372.65, -486.67

   3     .00016402-.00037265-.00048667 .00036280-.00057143         78.79   -233.08

          INPUT THE STRAINS OF READING NO.     4
-319.82, -211.35, 111.24

   4    -.00031982-.00021135 .00011124 .00011512-.00064629        -69.33   -323.50

          INPUT THE STRAINS OF READING NO.     5
-250.29, 111.24, 472.77

   5    -.00025029 .00011124 .00047277 .00050210-.00064115        139.40   -242.24

          INPUT THE STRAINS OF READING NO.     6
0.0, 0.0, 0.0

   6    0.000000000.000000000.000000000.000000000.00000000          0.00      0.00

          INPUT THE STRAINS OF READING NO.     7
319.82, -500.58, -653.54

   7     .00031982-.00050058-.00065354 .00060634-.00078710        165.74   -299.42

          INPUT THE STRAINS OF READING NO.     8
-27.81, 194.67, 406.03

   8    -.00002781 .00019467 .00040603 .00042467-.00025781        175.61    -52.21

          INPUT THE STRAINS OF READING NO.     9
-13.90, 155.74, 333.72

   9    -.00001390 .00015574 .00033372 .00034707-.00020523        144.63    -39.73
 STOP
```

```
>STRESS
9


     THE TOTAL NUMBER OF READINGS =     9

          INPUT THE STRAINS OF READING NO.     1
-111.24, 222.48, 611.82

   1    -.00011124 .00022248 .00061182 .00063631-.00052507       234.98    -152.71

          INPUT THE STRAINS OF READING NO.     2
-289.22, 211.36, 695.25

   2    -.00028922 .00021136 .00069525 .00073674-.00081460       230.14    -287.72

          INPUT THE STRAINS OF READING NO.     3
-400.46, -511.70, -736.96

   3    -.00040046-.00051170-.00073696-.00016975-.00074241      -241.71    -432.88

          INPUT THE STRAINS OF READING NO.     4
-792.59, -586.79, 97.34

   4    -.00079259-.00058679 .00009734 .00010404-.00148342      -245.10    -775.02

          INPUT THE STRAINS OF READING NO.     5
-681.35, 83.43, 834.30

   5    -.00068135 .00008343 .00083430 .00089708-.00149500       178.16    -620.35

          INPUT THE STRAINS OF READING NO.     6
0.0, 0.0, 0.0

   6    0.000000000.000000000.000000000.000000000.00000000         0.00       0.00

          INPUT THE STRAINS OF READING NO.     7
528.39, -1112.40, -1056.78

   7     .00052839-.00111240-.00105678 .00082957-.00141358       158.45    -590.35

          INPUT THE STRAINS OF READING NO.     8
-194.67, 250.29, 656.32

   8    -.00019467 .00025029 .00065632 .00069453-.00063891       243.13    -202.00

          INPUT THE STRAINS OF READING NO.     9
-69.52, 211.36, 500.58

   9    -.00006952 .00021136 .00050058 .00052295-.00038111       203.34     -98.45
 STOP
```

```
>STRESS
9

    THE TOTAL NUMBER OF READINGS =      9

          INPUT THE STRAINS OF READING NO.     1
-250.29, 333.72, 917.73

   1    -.00025029 .00033372 .00091773 .00096512-.00088169        339.10    -277.40

          INPUT THE STRAINS OF READING NO.     2
-539.51, 225.27, 1056.78

   2    -.00053951 .00022527 .00105678 .00111558-.00142982        308.65    -541.05

          INPUT THE STRAINS OF READING NO.     3
-692.47, -706.37, -1001.16

   3    -.00069247-.00070637-.00100116-.00039760-.00100124       -416.50    -618.01

          INPUT THE STRAINS OF READING NO.     4
-1112.4, -1003.94, 55.62

   4    -.00111240-.00100394 .00005562 .00005694-.00217328       -410.33   -1154.81

          INPUT THE STRAINS OF READING NO.     5
-1195.83, 0.0, 1195.83

   5    -.001195830.00000000 .00119583 .00129286-.00248869        188.98   -1073.36

          INPUT THE STRAINS OF READING NO.     6
0.0, 0.0, 0.0

   6    0.000000000.000000000.000000000.000000000.00000000          0.00       0.00

          INPUT THE STRAINS OF READING NO.     7
681.35, -1696.41, -1418.31

   7     .00068135-.00169641-.00141831 .00099012-.00200518        124.60    -875.29

          INPUT THE STRAINS OF READING NO.     8
-417.15, 292.01, 923.52

   8    -.00041715 .00029201 .00092352 .00098533-.00111047        303.53    -396.08

          INPUT THE STRAINS OF READING NO.     9
-180.76, 266.90, 778.68

   9    -.00018076 .00026690 .00077868 .00081198-.00072584        288.53    -224.82
 STOP
```

```
>STRESS
9

     THE TOTAL NUMBER OF READINGS =     9

          INPUT THE STRAINS OF READING NO.      1
-389.34, 403.25, 1209.74

   1   -.00038934 .00040325 .00120974 .00127334-.00125943      427.89   -417.60

          INPUT THE STRAINS OF READING NO.      2
-817.61, 239.17, 1362.69

   2   -.00081761 .00023917 .00136269 .00144514-.00202358      365.07   -792.85

          INPUT THE STRAINS OF READING NO.      3
-1123.52, -1151.33, -1293.16

   3   -.00112352-.00115133-.00129316-.00098107-.00129378     -788.99   -893.38

          INPUT THE STRAINS OF READING NO.      4
-2071.85, -1518.42, 69.53

   4   -.00207185-.00151842 .00006953 .00008995-.00368022     -698.32  -1956.87

          INPUT THE STRAINS OF READING NO.      5
-1835.46, -83.43, 1626.89

   5   -.00183546-.00008343 .00162689 .00177122-.00369011      201.99  -1621.10

          INPUT THE STRAINS OF READING NO.      6
0.0, 0.0, 0.0

   6   0.000000000.000000000.000000000.000000000.00000000        0.00      0.00

          INPUT THE STRAINS OF READING NO.      7
889.92, -2183.09, -1793.75

   7    .00088992-.00218309-.00179375 .00127092-.00256409      161.91  -1118.28

          INPUT THE STRAINS OF READING NO.      8
-723.06, 347.63, 1268.14

   8   -.00072306 .00034763 .00126814 .00136345-.00173888      378.98   -656.63

          INPUT THE STRAINS OF READING NO.      9
-319.81, 378.22, 1126.31

   9   -.00031981 .00037822 .00112631 .00118049-.00112208      405.92   -362.72
 STOP
```

```
>STRESS
9


     THE TOTAL NUMBER OF READINGS =     9

          INPUT THE STRAINS OF READING NO.     1
-347.63, 361.53, 1028.97

   1    -.00034763 .00036153 .00102897 .00108873-.00107483        366.26   -355.98

          INPUT THE STRAINS OF READING NO.     2
-761.99, 197.46, 1195.83

   2    -.00076199 .00019746 .00119583 .00127173-.00183626        310.00   -727.50

          INPUT THE STRAINS OF READING NO.     3
-914.95, -970.57, 1098.49

   3    -.00091495-.00097057 .00109849 .00109868-.00298420        -15.75  -1378.69

          INPUT THE STRAINS OF READING NO.     4
-1807.65, -1045.65, 97.34

   4    -.00180765-.00104565 .00009734 .00014424-.00299754       -530.69  -1579.47

          INPUT THE STRAINS OF READING NO.     5
-1585.17, -55.62, 1168.02

   5    -.00158517-.00005562 .00116802 .00131002-.00295081        104.45  -1317.89

          INPUT THE STRAINS OF READING NO.     6
0.0, 0.0, 0.0

   6    0.000000000.000000000.000000000.000000000.00000000          0.00      0.00

          INPUT THE STRAINS OF READING NO.     7
764.78, -1752.03, -1390.50

   7     .00076478-.00175203-.00139050 .00105168-.00203893        150.79   -880.91

          INPUT THE STRAINS OF READING NO.     8
-681.35, 264.20, 1156.90

   8    -.00068135 .00026420 .00115690 .00123643-.00165358        328.12   -636.62

          INPUT THE STRAINS OF READING NO.     9
-292.00, 294.79, 973.35

   9    -.00029200 .00029479 .00097335 .00101667-.00101388        339.95   -337.88
 STOP
```

```
>STRESS
9

     THE TOTAL NUMBER OF READINGS =     9

            INPUT THE STRAINS OF READING NO.      1
-236.39, 264.20, 778.68

   1    -.00023639 .00026420 .00077868 .00081860-.00079079        278.90    -258.34

            INPUT THE STRAINS OF READING NO.      2
-567.32, 169.65, 889.92

   2    -.00056732 .00016965 .00088992 .00095059-.00134826        236.65    -530.74

            INPUT THE STRAINS OF READING NO.      3
-650.75, -789.8, -834.30

   3    -.00065075-.00078980-.00083430-.00058673-.00085382       -488.10    -577.26

            INPUT THE STRAINS OF READING NO.      4
-1279.26, -600.69, 69.53

   4    -.00127926-.00060069 .00006953 .00012502-.00200497       -339.64   -1050.67

            INPUT THE STRAINS OF READING NO.      5
-1140.21, 0.0, 1140.21

   5    -.001140210.00000000 .00114021 .00123273-.00237294        180.19   -1023.44

            INPUT THE STRAINS OF READING NO.      6
0.0, 0.0, 0.0

   6    0.000000000.000000000.000000000.000000000.00000000          0.00       0.00

            INPUT THE STRAINS OF READING NO.      7
625.73, -1195.83, -1001.16

   7     .00062573-.00119583-.00100116 .00087354-.00144364        175.95    -597.57

            INPUT THE STRAINS OF READING NO.      8
-528.39, 208.58, 934.42

   8    -.00052839 .00020858 .00093442 .00099479-.00131460        267.20    -503.72

            INPUT THE STRAINS OF READING NO.      9
-222.48, 239.17, 778.68

   9    -.00022248 .00023917 .00077868 .00081252-.00079583        274.62    -262.28
 STOP
```

```
>STRESS
9

     THE TOTAL NUMBER OF READINGS =      9

          INPUT THE STRAINS OF READING NO.     1
-83.43, 208.58, 528.39

   1   -.00008343 .00020858 .00052839 .00055074-.00042559       209.23   -116.68

          INPUT THE STRAINS OF READING NO.     2
-303.13, 141.84, 597.92

   2   -.00030313 .00014184 .00059792 .00063346-.00079475       178.74   -298.02

          INPUT THE STRAINS OF READING NO.     3
-372.65, -595.13, -542.29

   3   -.00037265-.00059513-.00054229-.00035825-.00060953      -315.92   -399.80

          INPUT THE STRAINS OF READING NO.     4
-750.87, -308.69, 83.43

   4   -.00075087-.00030869 .00008343 .00012207-.00118163      -174.20   -609.40

          INPUT THE STRAINS OF READING NO.     5
-639.63, 69.53, 778.68

   5   -.00063963 .00006953 .00077868 .00083622-.00140632       163.49   -585.11

          INPUT THE STRAINS OF READING NO.     6
0.0, 0.0, 0.0,

   6   0.000000000.000000000.000000000.000000000.00000000         0.00      0.00

          INPUT THE STRAINS OF READING NO.     7
514.49, -556.20, -584.01

   7    .00051449-.00055620-.00058401 .00075615-.00079786       243.95   -274.80

          INPUT THE STRAINS OF READING NO.     8
-333.72, 139.05, 656.32

   8   -.00033372 .00013905 .00065632 .00069252-.00088719       191.68   -335.65

          INPUT THE STRAINS OF READING NO.     9
-126.14, 183.55, 542.30

   9   -.00012614 .00018355 .00054230 .00056513-.00050772       200.30   -157.84
 STOP
```

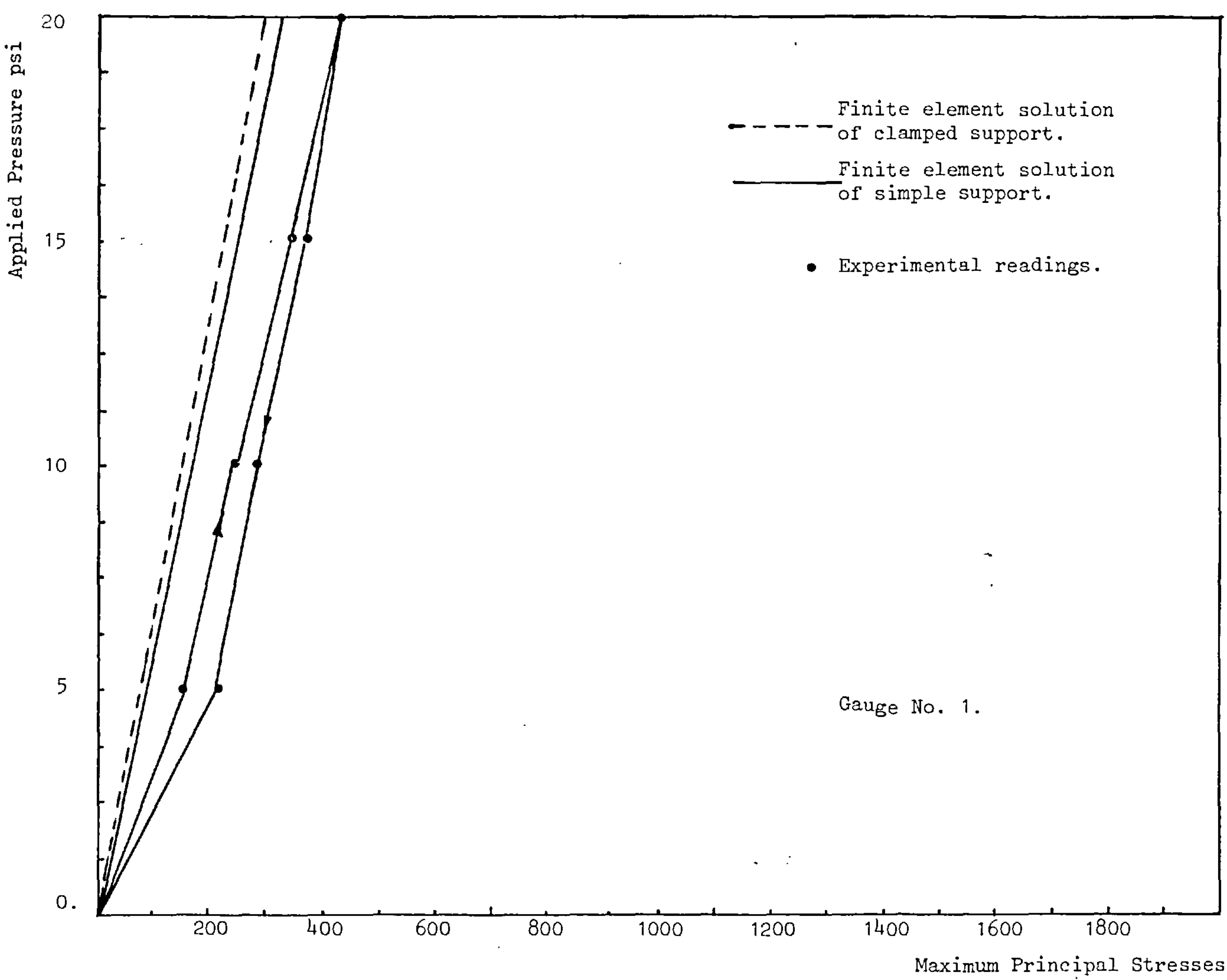


Fig. 2.61. Comparison of principal stresses

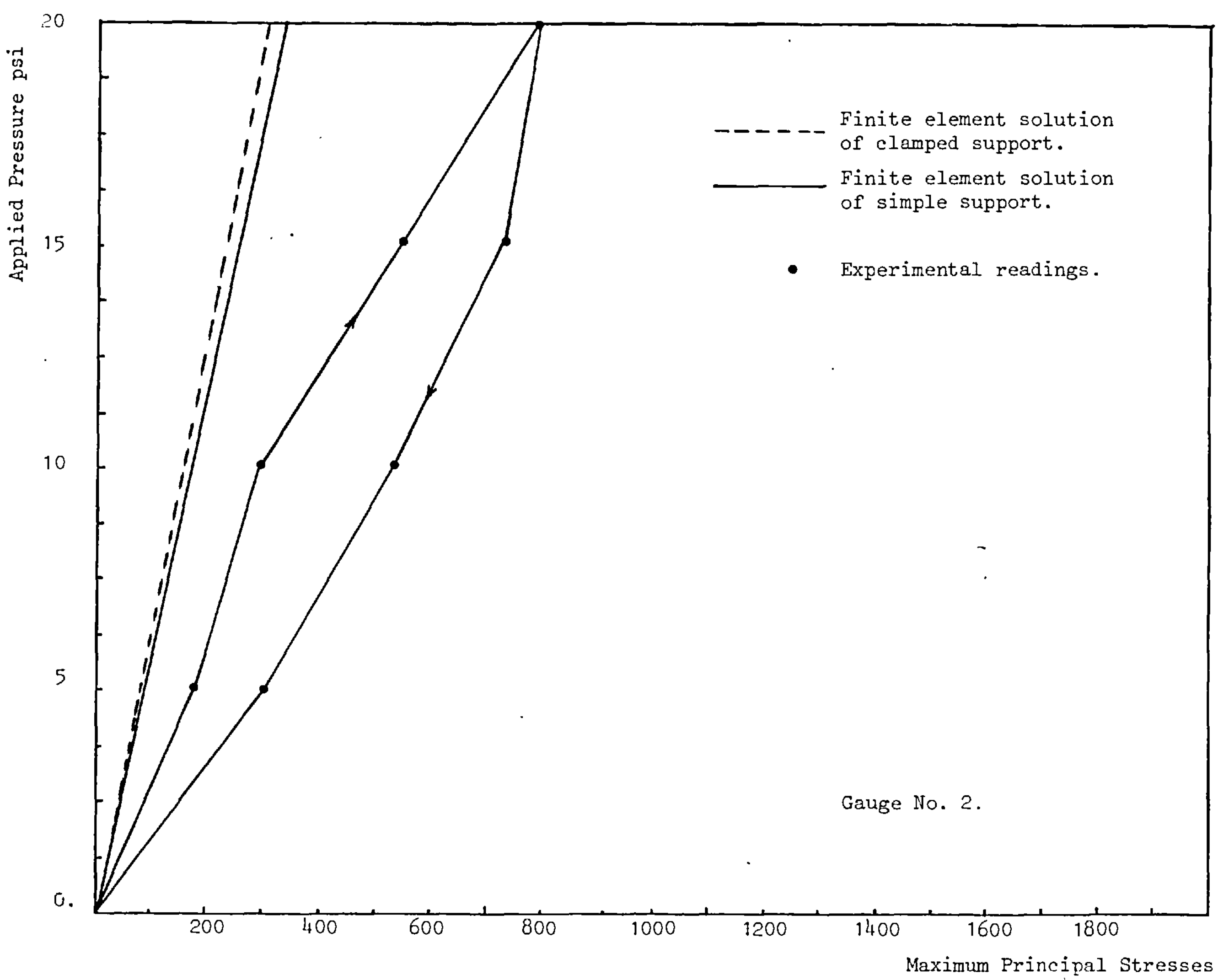


Fig. 2.62. Comparison of principal stresses

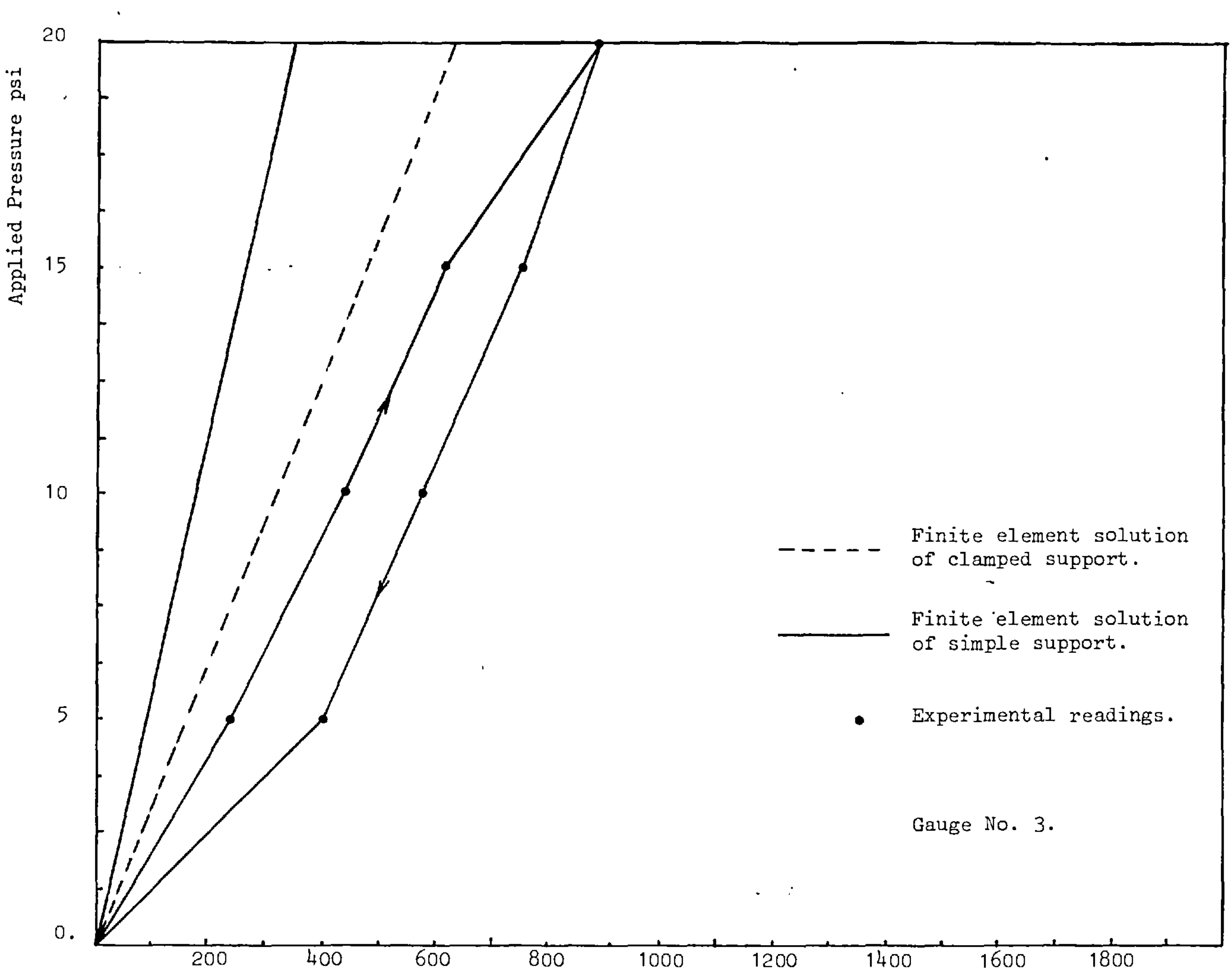


Fig. 2.63. Comparison of principal stresses

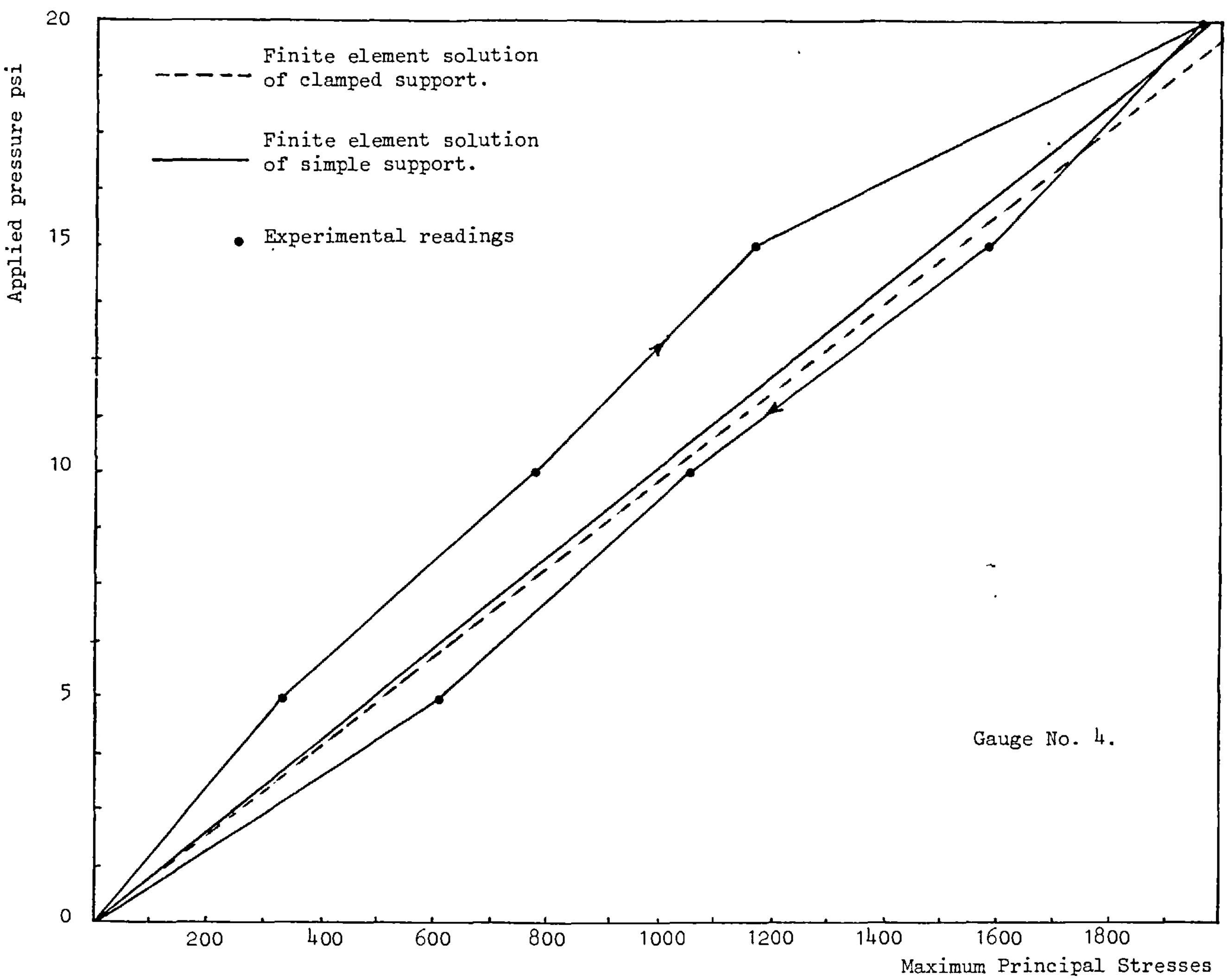


Fig. 2.64. Comparison of principal stresses

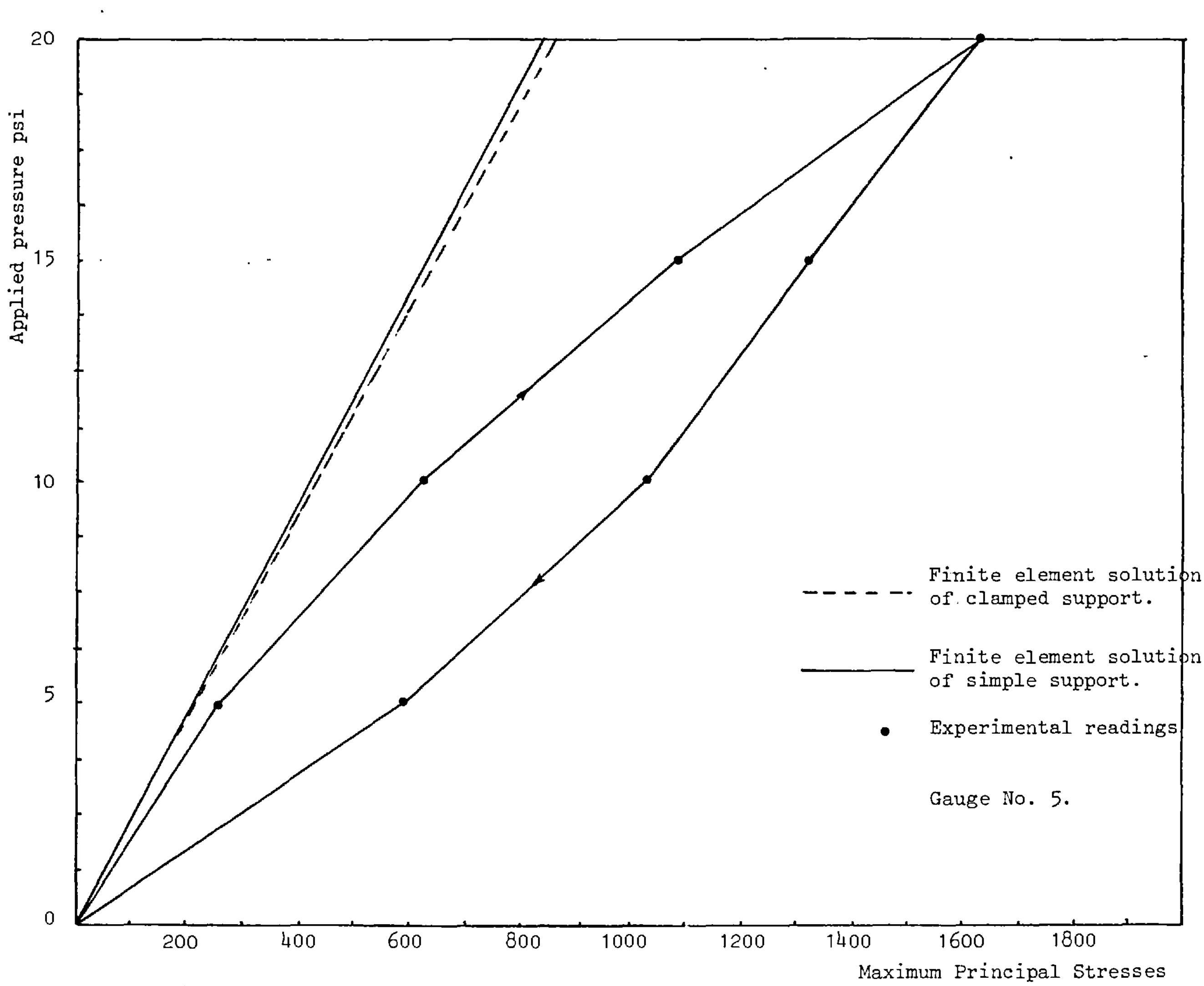


Fig. 2.65. Comparison of principal stresses

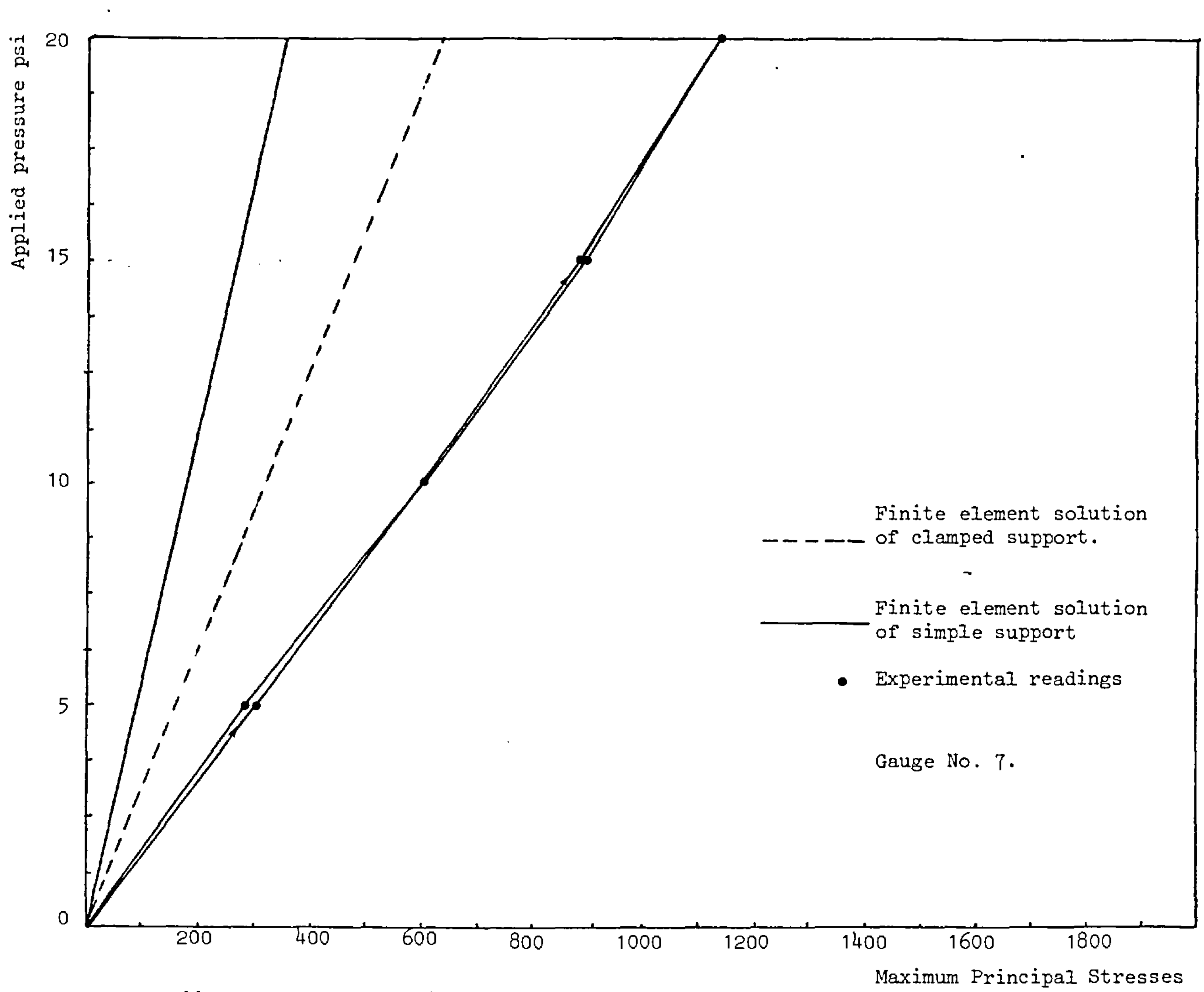


Fig. 2.66. Comparison of principal stresses

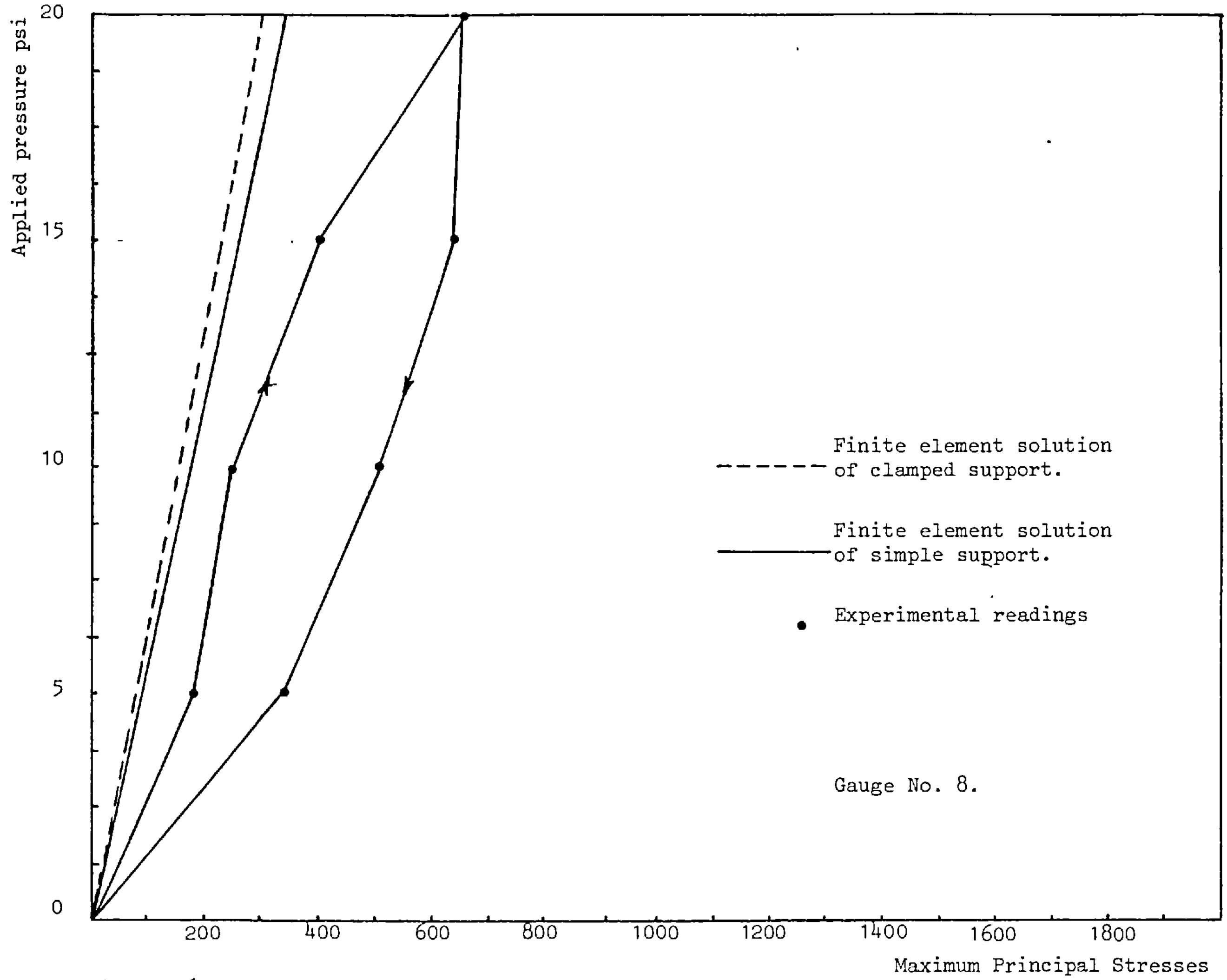


Fig. 2.67. Comparison of principal stresses

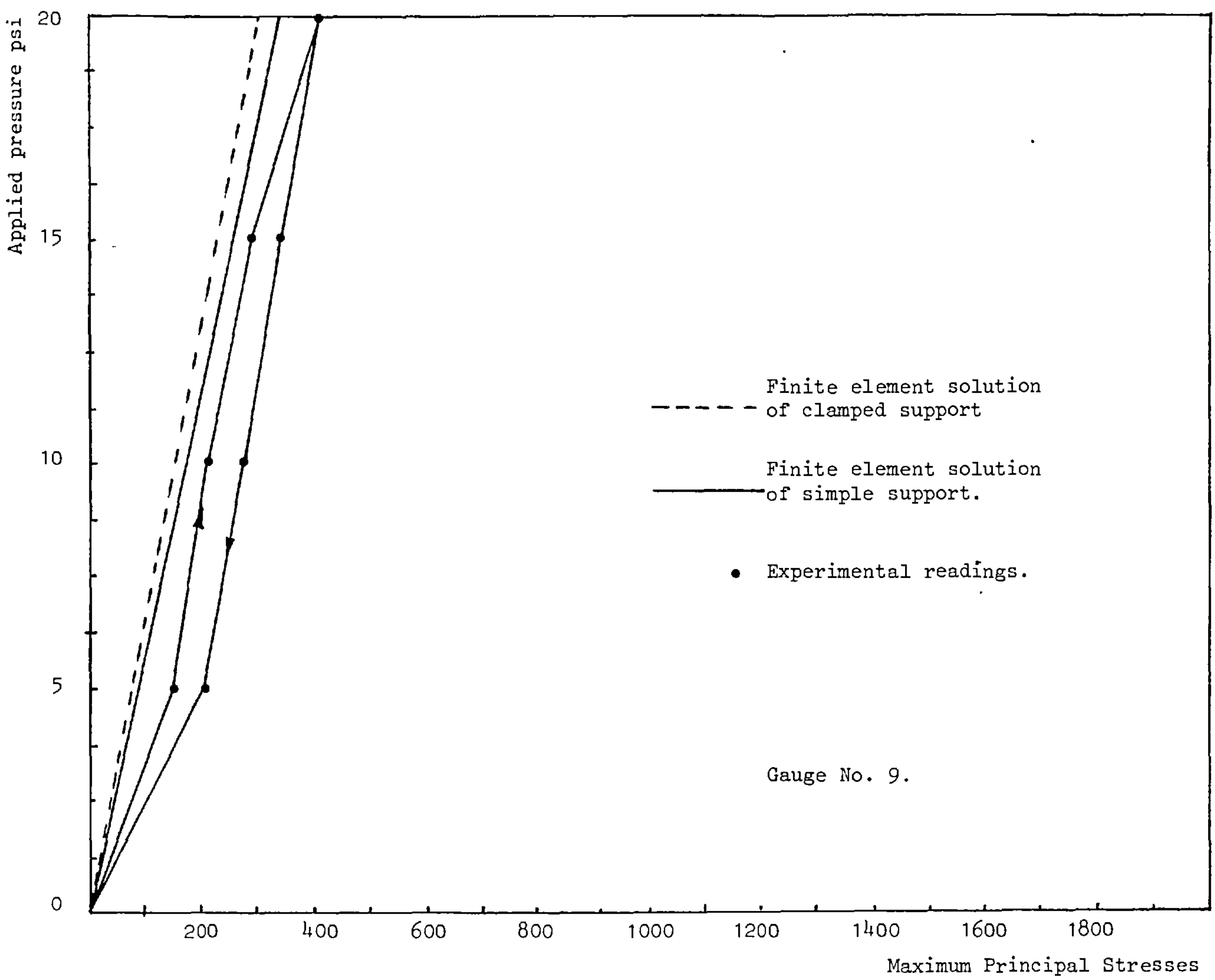


Fig. 2.68. Comparison of principal stresses

CHAPTER THREE

RELEVANT PARTS OF THE THEORY OF ANISOTROPIC ELASTICITY

CHAPTER THREE

RELEVANT PARTS OF THE THEORY OF ANISOTROPIC ELASTICITY

3.1. Introduction:-

It is proposed to consider a general class of complex materials i.e. composites in which non-woven fibres are deliberately orientated in a matrix in such a way as to increase its structural efficiency. An imaginary general square solid with fibres embedded in a matrix in three directions, coinciding with its axes is shown diagrammatically in Fig. (3.1).

In practice a filamentary composite is normally made up of several plies or laminae. One lamina of a filamentary composite consists of one row of parallel filaments surrounded by the matrix. Considering this case, as illustrated in Fig. (3.2), the laminae are stacked with various orientations of the filament directions between laminae to obtain a laminate which has the desired stiffness or strength properties.

It is evident that each individual ply or lamina of a filamentary composite has three mutually perpendicular planes of symmetry for the material constants. The intersection of the three planes forms the axes of the co-ordinates system as shown. Since the lamina has three perpendicular axes it can be considered to be an orthotropic material on the macroscopic level. Also, the lamina may be considered homogeneous on the macro scale. The set of axes which are parallel and perpendicular to the filament directions are termed the material natural principal axes. These orthotropic properties if

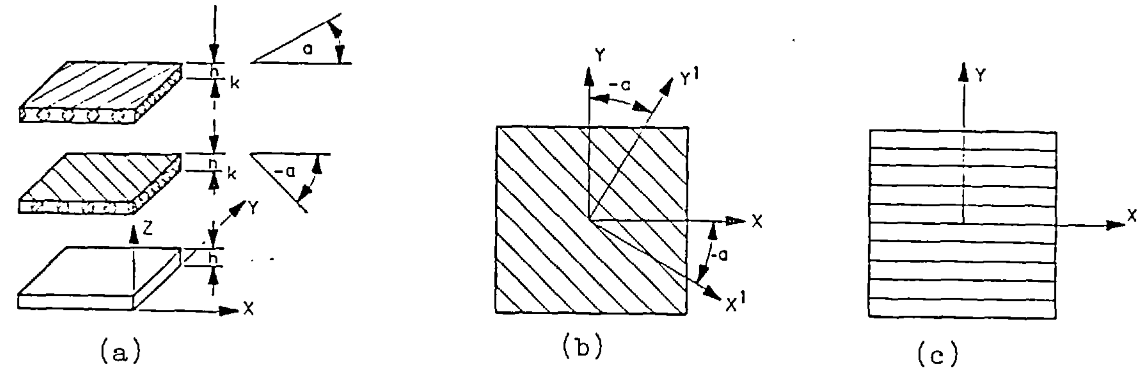


Fig. 3.2. Illustration of an orthotropic material in its laminated and lamina forms.

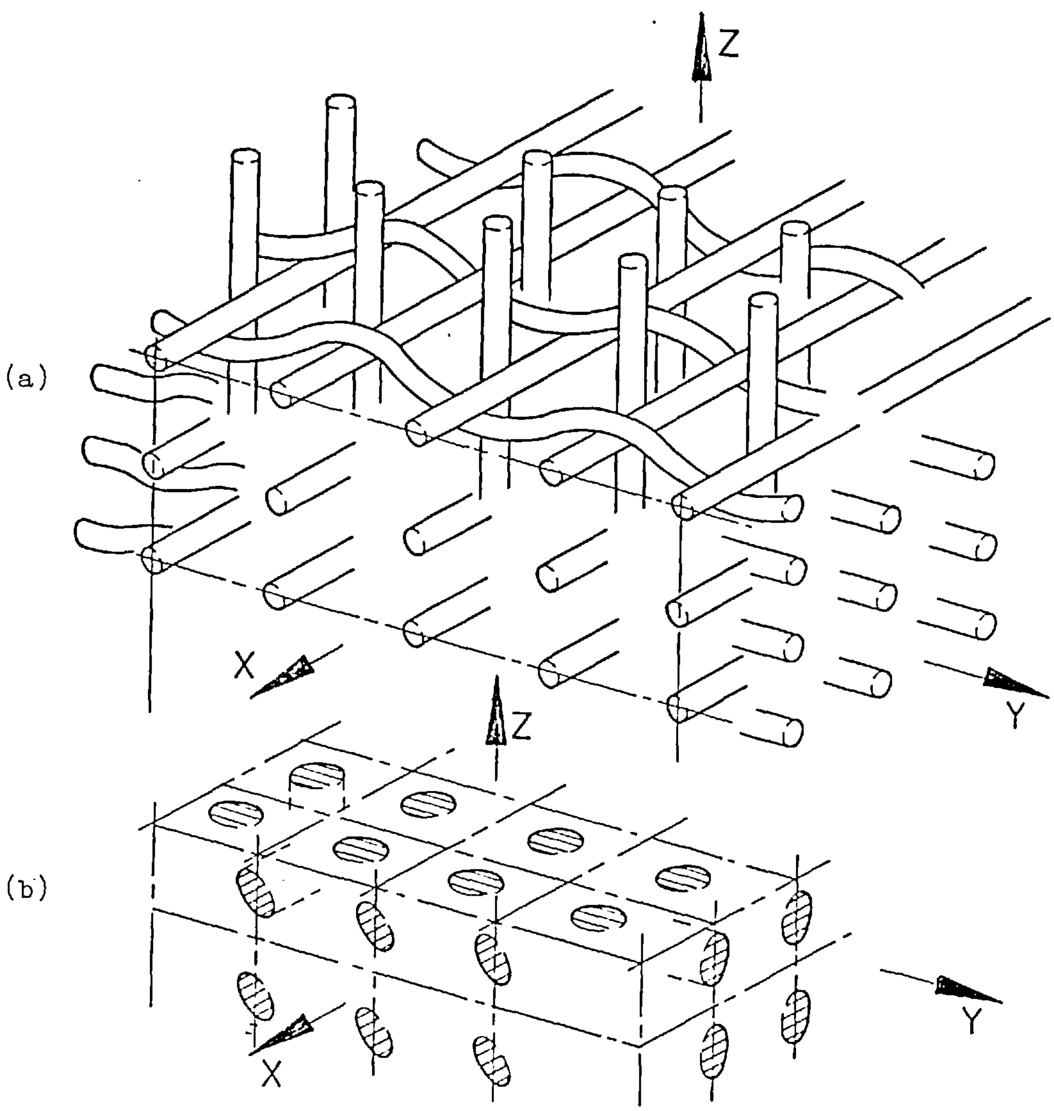


Fig. 3.1. A 3-dimensional solid of fibres embedded in matrix

translated into other convenient axes will produce an anisotropic solid in the new axes. To describe the mechanical constitution of the material a generalised Hooke's Law for the stress strain relationships is required. Ref. (39), (40), (41) & (42).

3.2. The Generalised Hooke's Law for Anisotropy

To obtain the stress strain relationships in an elastic body of this type the generalised Hooke's Law for anisotropic material will be derived first, then simplified so as to apply to orthotropic material. First consider the three-dimensional stress state illustrated in Fig. (3.3), in which it is assumed that the components of strain are linear functions of the components of stress. The equation which expresses the generalised Hooke's Law in rectangular orthogonal cartesian co-ordinates x_1 x_2 x_3 (or has xyz) the generalised form

$$\varepsilon_{ij} = C_{ijkl} \; \sigma_{ij} \qquad 3.1a$$

in tensor notation, with the repeated suffixes i and j taking values 1 to 3, k and l 1 to 9 and Einstein's summation convention applying. This equation may be expressed in matrix form, using engineering shear strains such as τ_{xy} etc.

thus:-

$$
\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{yx} \\ \gamma_{zy} \\ \gamma_{xz} \end{bmatrix}
=
\begin{bmatrix}
C_{1111} & C_{2233} & C_{1133} & C_{1123} & C_{1131} & C_{1112} & C_{1132} & C_{1113} & C_{1121} \\
C_{2211} & C_{2222} & C_{2233} & C_{2223} & C_{2231} & C_{2212} & C_{2232} & C_{2213} & C_{2221} \\
\cdot & & \cdot & & & & & & \cdot \\
\cdot & & & \cdot & & & & & \cdot \\
\cdot & & & & \cdot & & & & \cdot \\
\cdot & & & & & \cdot & & & \cdot \\
\cdot & & & & & & \cdot & & \cdot \\
\cdot & & & & & & & \cdot & \cdot \\
C_{2111} & C_{2121} & C_{2133} & C_{2123} & C_{2131} & C_{2112} & C_{2132} & C_{2113} & C_{2131}
\end{bmatrix}
\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{yx} \\ \tau_{zy} \\ \tau_{xz} \end{bmatrix}
\qquad 3.1b
$$

or

$$\{\varepsilon\} = [C]\ \{\sigma\} \qquad 3.1c$$

The $[C]$ matrix is the 'compliance' or 'flexibility' matrix which gives the strain stress relations for the material. The inverse of the flexibility matrix is the stiffness matrix, and it gives the stress-strain relations.

Since there are only six independent stresses and strains, because the stress and strain tensors are symmetric, i.e.

$$\gamma_{xy} = \gamma_{yx} \quad , \quad \tau_{xy} = \tau_{yx}$$

$$\gamma_{yz} = \gamma_{zy} \quad , \quad \tau_{yz} = \tau_{zy}$$

$$\gamma_{zx} = \gamma_{xz} \quad , \quad \tau_{zx} = \tau_{xz}$$

The 81 constants in the flexibility matrix are reduced to 36 elastic constants and Equation (31c) can be written as:

$$\underset{6x1}{\{\varepsilon\}} = \underset{6x6}{[C]} \underset{6x1}{\{\sigma\}} \qquad 3.2$$

Where the shear components of $\{\varepsilon\}$ are the three engineering shear strains rather than the tensor shear strains and the components of σ are σ_x, σ_y and σ_z (normal stresses) and τ_{xy} τ_{yx} and τ_{zx} (shear stresses).

The solution of the above equations will give an equivalent form for the equations of the generalised Hooke's Law, namely:

$$\{\sigma\} = [D] \{\varepsilon\} \qquad 3.3$$

where $[D] = [C]^{-1}$, the stiffness matrix.

Assuming that an elastic potential exists equal to the potential strain energy density (strain energy per unit volume) (Ω) of deformation i.e. Ω is a function of strain then:

$$\Omega = \frac{1}{2} (\sigma_{ij} \varepsilon_{ij}) \qquad 3.4$$

in tensor notation

$$i,j = 1 \rightarrow 3$$

with the property that

$$\sigma_{ij} = \frac{\partial \Omega}{\partial \varepsilon_{ij}} \qquad 3.5$$

using Equation (3.2a) we have

$$\sigma_{ij} = \frac{\partial \Omega}{\partial \varepsilon_{ij}} = C_{ijkl} \varepsilon_{kl} \qquad 3.6$$

$$k,l = 1 \rightarrow 3$$

Differentiating this equation with respect to ε_{kl} gives

$$\frac{\partial^2 \Omega}{\partial \varepsilon_{ij} \partial \varepsilon_{kl}} = C_{ijkl} \qquad 3.7$$

in the same way we can prove that

$$\frac{\partial^2 \Omega}{\partial \varepsilon_{kl} \partial \varepsilon_{ij}} = C_{klij} \qquad 3.8$$

but

$$\frac{\partial^2 \Omega}{\partial \varepsilon_{ij} \partial \varepsilon_{kl}} = \frac{\partial^2 \Omega}{\partial \varepsilon_{kl} \partial \varepsilon_{ij}} \qquad 3.9$$

hence

$$C_{ij} = C_{kl\,ij}$$

This symmetrical property of the elements of the elasticity matrix reduces the unknowns to a total of 21 elastic constants. The most general form of this matrix for anisotropic materials may be written as:

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ & & & & & \\ & & & & & \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \qquad 3.10$$

where all the elements are populated.

3.3. Elastic Symmetry and Orthotropic Case

Considering one layer Fig.(3.1b), if the density of fibre is uniform, then the assumption that the three orthogonal planes of elastic symmetry can pass through any point of the body is a valid assumption.

Considering an element of the body in the form of a rectangular parallelopiped with sides parallel to the planes of elastic symmetry, after a deformation due to the action of the stress σ_z, it will remain a rectangular parallelopiped as shown in Fig. (3.4).

As the three principal directions apply to each point of the body, we can assume that the normal and shear stresses are all uncoupled, i.e. C_{14}, C_{15}, C_{16}, C_{24}, C_{25}, C_{26}, C_{34} and C_{36} are all equal to zero.

By essentially the same reasons the coupling between the shear stresses will also vanish, i.e. C_{45}, C_{46} and C_{56} are all equal to zero. The coupling between normal stresses, however, are expected to remain Tsai Ref. (39).

By directing the axes of the co-ordinates perpendicular to these planes, we obtain the following equations for the generalised Hooke's Law.

$$\varepsilon_x = C_{11}\sigma_x + C_{12}\sigma_y + C_{13}\sigma_z \qquad 3.11a$$

$$\varepsilon_y = C_{12}\sigma_x + C_{22}\sigma_y + C_{23}\sigma_z \qquad 3.11b$$

$$\varepsilon_z = C_{13}\sigma_x + C_{23}\sigma_y + C_{33}\sigma_z \qquad 3.11c$$

$$\gamma_{xy} = C_{44}\tau_{xy} \qquad 3.11d$$

$$\gamma_{yz} = C_{55}\tau_{yz} \qquad 3.11e$$

$$\gamma_{zx} = C_{66}\tau_{zx} \qquad 3.11f$$

and the compliance (flexibility) matrix becomes

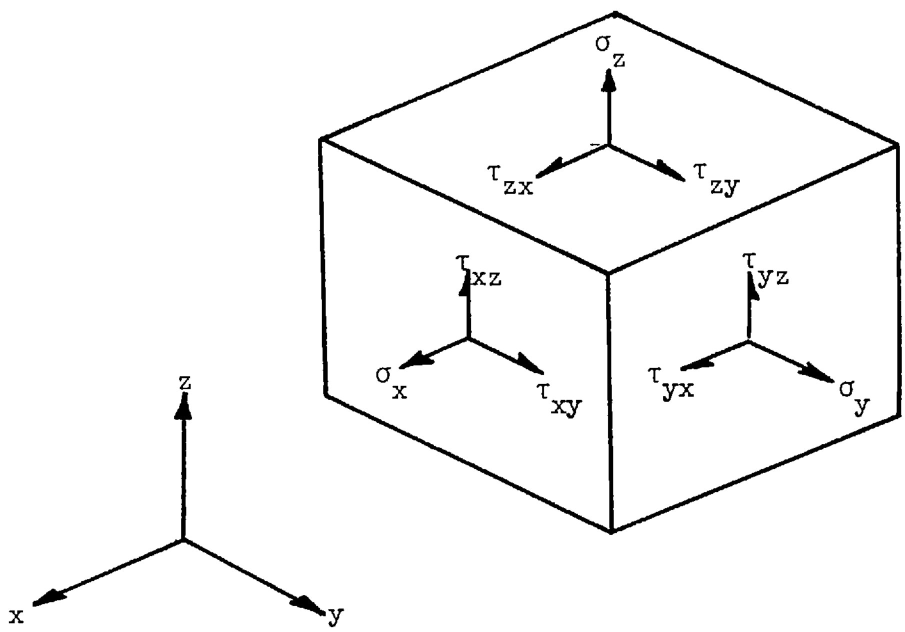


Fig. 3.3. 3-Dimensional state of stress

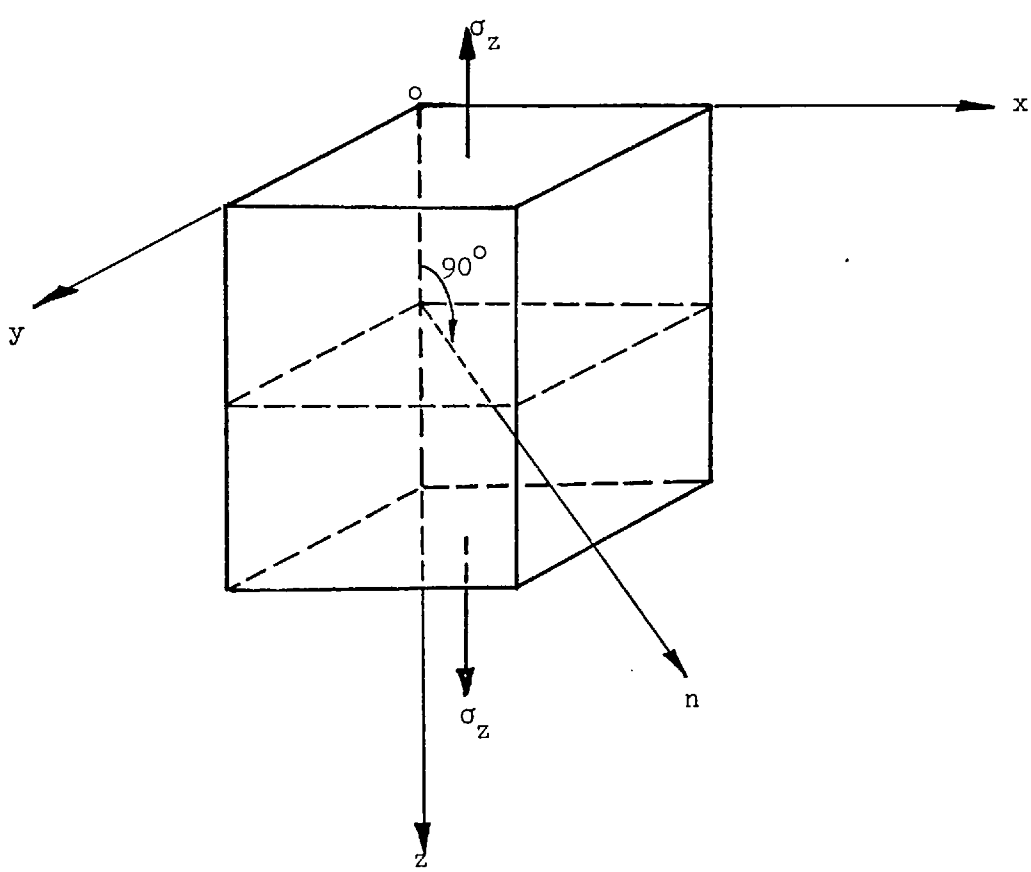


Fig. 3.4. 3-Dimensional element

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ & C_{22} & C_{23} & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & \text{Symmetrical} & & C_{44} & 0 & 0 \\ & & & & C_{55} & 0 \\ & & & & & C_{66} \end{bmatrix} \qquad 3.12$$

and nine independent elastic constants remain. They are called the principal constants and can be expressed in terms of "technical constants". Adopting the notation E_x, E_y, E_z (Young's moduli), G_{xy}, G_{yz}, G_{zx} (Shear moduli) and ν_{12}, ν_{21}, ν_{13}, ν_{31}, ν_{23}, ν_{32} (Poisson's ratios)

Equation (3.11) may be written in the form:

$$\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{21}}{E_y}\sigma_y - \frac{\nu_{31}}{E_z}\sigma_z \qquad 3.13a$$

$$\varepsilon_y = \frac{\nu_{12}}{E_x}\sigma_x + \frac{1}{E_y}\sigma_y - \frac{\nu_{32}}{E_z}\sigma_z \qquad 3.13b$$

$$\varepsilon_z = -\frac{\nu_{13}}{E_z}\sigma_x - \frac{\nu_{23}}{E_y}\sigma_y + \frac{1}{E_z}\sigma_z \qquad 3.13c$$

$$\gamma_{xy} = \frac{1}{G_{xy}}\tau_{xy} \qquad 3.13d$$

$$\gamma_{yz} = \frac{1}{G_{yz}}\tau_{yz} \qquad 3.13e$$

$$\gamma_{zx} = \frac{1}{G_{zx}}\tau_{zx} \qquad 3.13f$$

where

$$E_x \; \nu_{21} = E_y \; \nu_{12} \qquad 3.14a$$

$$E_y \; \nu_{32} = E_z \; \nu_{23} \qquad 3.14b$$

$$E_z \; \nu_{13} = E_x \; \nu_{31} \qquad 3.14c$$

and

$$C_{11} = \frac{1}{E_x} \qquad 3.14d$$

$$C_{12} = -\frac{\nu_{21}}{E_y} \qquad 3.14e$$

$$C_{13} = -\frac{\nu_{31}}{E_z} \qquad 3.14f$$

$$C_{22} = \frac{1}{E_y} \qquad 3.14g$$

$$C_{23} = \frac{\nu_{32}}{E_y} \qquad 3.14h$$

$$C_{33} = \frac{1}{E_2} \qquad 3.14i$$

$$C_{44} = \frac{1}{G_{xy}} \qquad 3.14j$$

$$C_{55} = \frac{1}{G_{yz}} \qquad 3.14k$$

$$C_{66} = \frac{1}{G_{zx}} \qquad 3.14l$$

The body which has three orthogonal planes of elastic symmetry at each point is called orthogonally-anisotropic, or for brevity, orthotropic. This form of elastic symmetry is very important because it occurs in fibre composites. If such a material is considered in any other co-ordinates it becomes anisotropic and since the properties will be identified together with the finite element locations in eleven local axes the orthotropic properties are all that is required.

3.4. Transformation Rules for an Orthotropic Body

3.4.1. Stress Transformation

Knowing the stresses in three mutually perpendicular planes or directions, which are referred to as the material natural axes (1, 2, 3), the stress which acts on any plane passing through the same point can be determined. Hence

$$(\sigma_x)_n = \sigma_{11} \cos(n,1) + \tau_{12} \cos(n,2) + \tau_{13} \cos(n,3)$$

$$(\sigma_y)_n = \tau_{12} \cos(n,1) + \sigma_{22} \cos(n,2) + \tau_{23} \cos(n,3)$$

$$(\sigma_z)_n = \tau_{13} \cos(n,1) + \tau_{23} \cos(n,2) + \sigma_{33} \cos(n,3)$$

where $(\sigma_x)_n$, $(\sigma_y)_n$ and $(\sigma_z)_n$ are components of stress which act on any plane with the arbitrary normal direction n and cos (n,1) is the direction cosine between the n-direction and the 1-direction, for example. By projection, using equations above, it is possible to find the normal and tangential components of stress acting on any arbitrary plane.

Considering, for convenience, the reference cartesian system x, y, z, the position of this system with respect to the 1, 2, 3 system is determined by the Table (3.1) of direction cosines, where $l_1 = \cos(n,1)$, $m_1 = \cos(n,2)$ etc. Equation (3.14).

By projecting $(\sigma_x)_n$, $(\sigma_y)_n$, $(\sigma_z)_n$ in the direction of reference axes (i.e. on to the planes normal to the x, y, z axes), we obtain

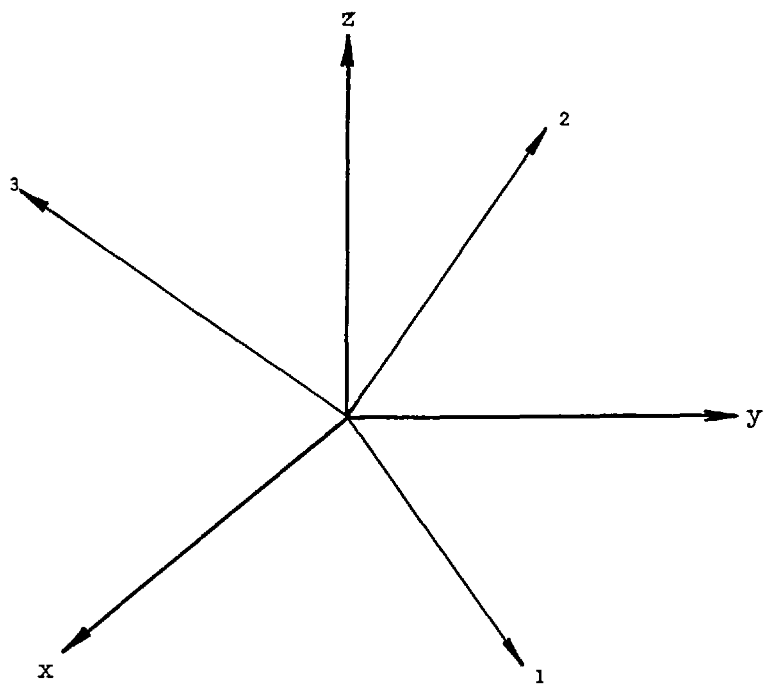
$$\sigma_x = \sigma_1 l_1^2 + \sigma_2 m^2 + \sigma_3 n^2 \; 2 \tau_{12} l_1 m_1 + 2 \tau_{13} l_1 n_1 + 2 \tau_{13} n_1 m_1 \qquad 3.15$$

and

$$\tau_{xy} = \sigma_1 l_1 m_1 + \sigma_2 m_1 m_1 + \sigma_3 n_1 n_1 + \tau_{12} (l_1 m_2 + l_2 m_1) + \tau_{23} (m_1 n_2 + m_2 n_1) + \tau_{31} (n_1 l_2 + n_2 l_1) \qquad 3.16$$

The expression for σ_y and σ_z are obtained by means of cyclic permutation of the suffixes of l m and n in Equation (3.15). Similarly, τ_{yz} and τ_{xz} may be obtained from Equation (3.16). The general formula for the transformation from 1, 2, 3 to x, y, z systems, in matrix form, will be:-

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & 2l_1m_1 & 2m_1n_1 & 2n_1l_1 \\ l_2^2 & m_2^2 & n_2^2 & 2l_2m_2 & 2m_2n_2 & 2n_2l_2 \\ l_3^2 & m_3^2 & n_3^2 & 2l_3m_3 & 2m_3n_3 & 2n_3l_3 \\ l_1m_1 & m_1n_1 & n_1l_1 & (l_1m_2+l_2m_1) & (m_1n_2+m_2n_1) & (n_1l_2+n_2l_1) \\ l_2m_2 & m_2n_2 & n_2l_2 & (l_2m_3+l_3m_2) & (m_2n_2+m_3n_2) & (n_2l_3+n_3l_2) \\ l_3m_3 & m_3n_3 & n_3l_3 & (l_3m_1+l_1m_3) & (m_3n_1+m_1n_3) & (n_3l_1+n_1l_3) \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{bmatrix}$$



	x	y	z
1	l_1	m_1	n_1
2	l_2	m_2	n_2
3	l_3	m_3	n_3

Table: 3.1. Direction cosines between two sets of coordinates or

$$\{\sigma\} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \{\sigma\} \qquad 3.17a$$

$$\{\sigma\} = [R] \quad \{\bar{\sigma}\} \qquad 3.17b$$

3.4.2. Strain Transformation

Let $\bar{C}_{ij}$ be the coefficients of strain for the system 1, 2, 3 of the elements. It is required to determine the coefficients for the new global system x, y, z which will be denoted by C_{ij}. For this new system the equations of the generalised Hooke's law are:

$$\{\varepsilon\} = [C] \{\sigma\} \qquad 3.18$$

The elastic potential, which is considered as a function of the components of stress, is determined by Equation (3.4), and its expression is:

$$\Omega = \frac{1}{2} [C] \{\sigma\}^T \{\sigma\}, \qquad 3.19$$

whilst the elastic potential for the natural axes of the original element is given by the expression:

$$\Omega = \frac{1}{2} [\bar{C}] \{\bar{\sigma}\} \quad \{\bar{\sigma}\} \qquad 3.20$$

Equating the two expressions for the elastic potential we obtain the following:

$$\frac{1}{2} [\bar{C}] \{\sigma\}^T \{\bar{\sigma}\} = \frac{1}{2} [C] \{\sigma\}^T \{\sigma\}. \qquad 3.21$$

since

$$\{\bar{\sigma}\} = [\bar{C}] \quad \{\bar{\varepsilon}\}$$

and

$$\{\sigma\} = [C] \{\varepsilon\}$$

Equation (3.21) gives:

$$\{\bar{\varepsilon}\}^T \{\bar{\sigma}\} = \{\varepsilon\}^T \{\sigma\}$$

Using Equation (3.17b) we get:

$$\{\bar{\varepsilon}\}^T \{\bar{\sigma}\} = \{\varepsilon\}^T [R] \{\bar{\sigma}\}$$

$$\{\varepsilon\} = [R^T]^{-1} \{\bar{\varepsilon}\}$$

$$\{\varepsilon\} = [R_\varepsilon] \{\bar{\varepsilon}\} \qquad 3.22$$

This gives the strain transformation rule. The matrices $[R]$ & $[R_\varepsilon]$ each have the property that the inverse of one equals the transpose of the other, i.e.

$$[R]^T [R_\varepsilon] = [I]$$

and

$$[R_\varepsilon] = \left[\begin{array}{c|c} R_{11} & \frac{1}{2}R_{12} \\ \hline 2R_{21} & R_{22} \end{array}\right] \qquad 3.23$$

3.4.3. Constants of Elasticity

The stress-strain relations may be written in either coordinate system as:-

$$\{\sigma\} = [D] \{\varepsilon\} \qquad 3.24$$

in one and

$$\{\bar{\sigma}\} = [\bar{D}] \{\bar{\varepsilon}\} \qquad 3.25$$

in the other

Suppose $[\bar{D}]$ is the elasticity matrix related to the material natural axes, and $[D]$ is required.

We are dealing with orthotropic materials where the elastic properties are known in the principal directions 1, 2, 3 but these properties are needed in global coordinates x, y, z to generate the element stiffness matrix as will be discussed.

Transformation of $[\bar{D}]$ is also needed for the isoparametric shell element of Chapter Four.

The transformation is derived from the arguments that during any loading process the resulting strain energy density must be the same regardless of the coordinate system in which it is computed, thus from Equations (3.19) to (3.22):

$$\Omega = \frac{1}{2} \{\varepsilon\}^T \{\sigma\} = \frac{1}{2} \{\bar{\varepsilon}\}^T \{\bar{\sigma}\} \qquad 3.26$$

Substituting from Equations (3.24) and (3.25) leads to:

$$\{\varepsilon\}^T \{\sigma\} = \{\varepsilon\}^T [R_\varepsilon]^T [\bar{D}] \{\bar{\varepsilon}\}$$

$$\{\varepsilon\}^T \left[\{\sigma\} - [R_\varepsilon]^T [\bar{D}] [R_\varepsilon] \{\varepsilon\} \right] = 0 \qquad 3.27$$

As the latter relation must be true for any $\{\varepsilon\}$, the coefficients of $\{\varepsilon\}$ must vanish. Thus we obtain:

$$\{\sigma\} = [D] \{\varepsilon\}$$

where the coefficients of D are as follows:

$$[D] = [R_\varepsilon]^T [\bar{D}] [R_\varepsilon] \qquad 3.28$$

CHAPTER FOUR

FINITE ELEMENT MODEL AND SOLUTION

CHAPTER FOUR

VARIABLE NUMBER NODE THICK ANISOTROPIC SHELL ELEMENT

4.1. INTRODUCTION

In this chapter a general formulation for a quadratic, isoparametric, anisotropic element of a thick shell is developed which permits the analysis of any arbitrarily layered curved shell or solid structure.

The use of general shell theory to derive doubly curved elements involves geometric complexities that make shell theory difficult. These difficulties are avoided with gained economy by appropriately specialising a three-dimensional solid element which has suitable doubly curved surfaces.

To achieve this, an isoparametric element is chosen, Ref. (45) with the same interpolation functions to define the element shape as those used to define displacements within the element.

4.2. Geometrical Representation of Various Elements in Natural and Global Coordinates.

In this work, all elements will be geometrically represented according to its global nodal coordinates. On the other hand each one of them should be represented according to the orthotropic properties of its material. Therefore, it is necessary to define the natural axes of each material with respect to the global axes to obtain a general representation.

Consider the linear interpolation function

$$f_1 = \alpha_1 + \alpha_2\xi + \alpha_3\eta + \alpha_4\zeta + \alpha_5\xi\eta + \alpha_6\eta\zeta$$

$$+ \alpha_7\xi\zeta + \alpha_8\xi\eta\zeta \qquad 4.1$$

To represent a linear 8-node hexahedron Fig. (4.1), where ξ, η, ζ are the isoparametric (natural) coordinates of the elements, then the boundary conditions are as follows:-

$\xi, \eta, \zeta = 0$ (at the centroid of the element)

and $\xi, \eta, \xi = \pm 1$ (at the surface). See Appendix (5). 4.2

A more warped element with doubly curved surfaces Fig. (4.2) could be adequately represented by the quadratic interpolation function

$$f_2 = f_1 + \alpha_9\xi^2 + \alpha_{10}\eta^2 + \alpha_{11}\zeta^2 + \alpha_{12}\xi^2\eta$$

$$+ \alpha_{13}\xi^2\zeta + \alpha_{14}\eta^2\xi + \alpha_{15}\eta^2\zeta + \alpha_{16}\zeta^2\xi \qquad 4.3$$

$$+ \alpha_{17}\zeta^2\eta + \alpha_{18}\xi^2\eta\zeta + \alpha_{19}\eta^2\xi\zeta + \alpha_{20}\zeta^2\xi\eta$$

The global coordinates of any nodal point (x, y, z) would then be

$$x = \sum_{i=1}^{20} N_i\, x_i$$

$$y = \sum_{i=1}^{20} N_i\, y_i \qquad 4.4$$

$$z = \sum_{i=1}^{20} N_i\, z_i$$

Where (N_i) the mapping or shape functions are

$$N_i = \frac{1}{8}(1+\xi\xi_i)(1+\eta\eta_i)(1+\zeta\zeta_i)(\xi\xi_i+\eta\eta_i+\zeta\zeta_i-2)$$

for i = 1 to 8 4.5a

$$N_i = \frac{1}{4}(1-\xi^2)(1+\eta\eta_i)(1+\zeta\zeta_i)$$

for i = 9, 11, 13, 15 4.5b

$$N_i = \frac{1}{4}(1-\eta^2)(1+\xi\xi_i)(1+\zeta\zeta_i)$$
for $i = 10, 12, 14, 16$ 4.5c

$$N_i = \frac{1}{4}(1-\zeta^2)(1+\xi\xi_i)(1+\eta\eta_i)$$
for $i = 17, 18, 19, 20$ 4.5d

This mapping is an interpolation scheme that yields the global xyz coordinates of any point in the element when the corresponding natural $\xi\eta\zeta$ coordinates are given.
The natural coordinate axes $\xi\eta\zeta$ are in general not orthogonal. They are orthogonal, of course, when the surfaces are planes and the element is the basic rectangular parallelepiped. Fig. (4.3a).

4.3. Displacement Fields

The displacements within the element are defined by the same interpolation (shape) functions as are used to define the element shape. Thus

$$u = \sum_{i=1}^{20} N_i\, u_i$$
$$v = \sum_{i=1}^{20} N_i\, v_i \qquad 4.6$$
$$w = \sum_{i=1}^{20} N_i\, w_i$$

In which u, v and w are measured along the global coordinate axes x, y and z and the u_i, v_i, w_i are measured along the natural coordinate directions $\xi\eta\zeta$

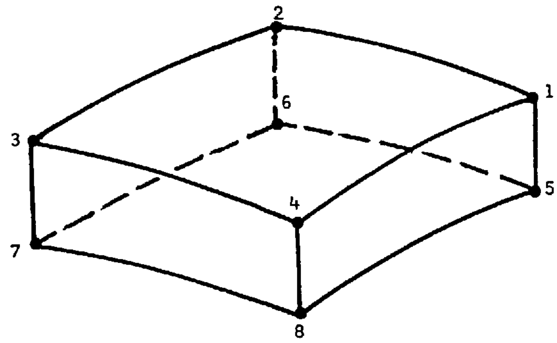


Fig. 4.1. 8-Node first order hexahedron

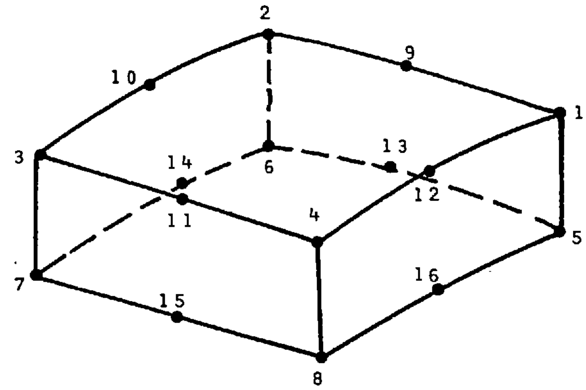
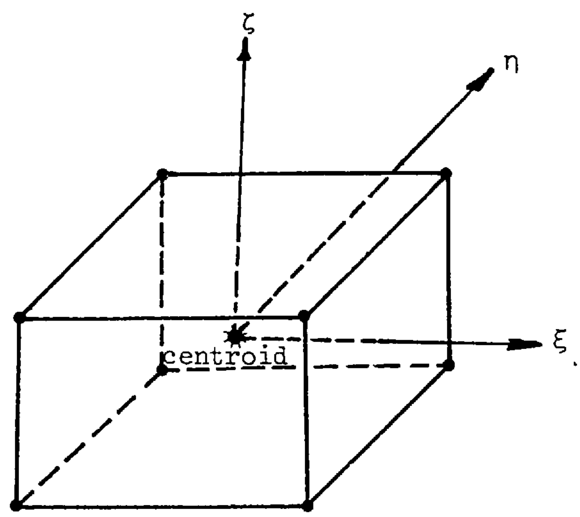
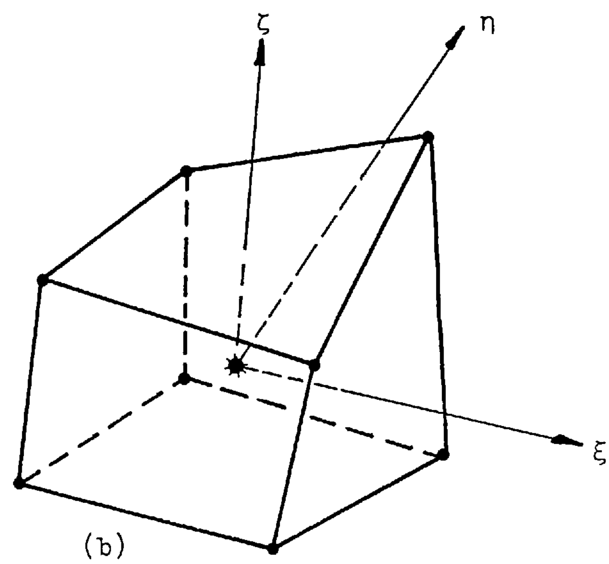


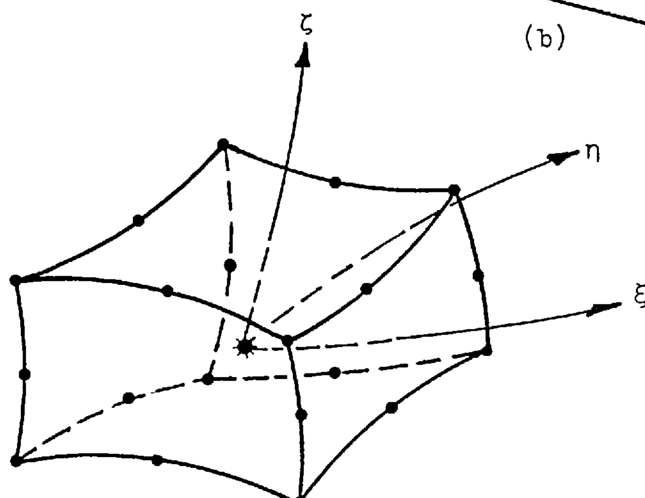
Fig. 4.2. 16-Node Element



(a)



(b)



(c)

Figs. 4.3a., 4.3b & 4.3c General elements (natural coordinates).

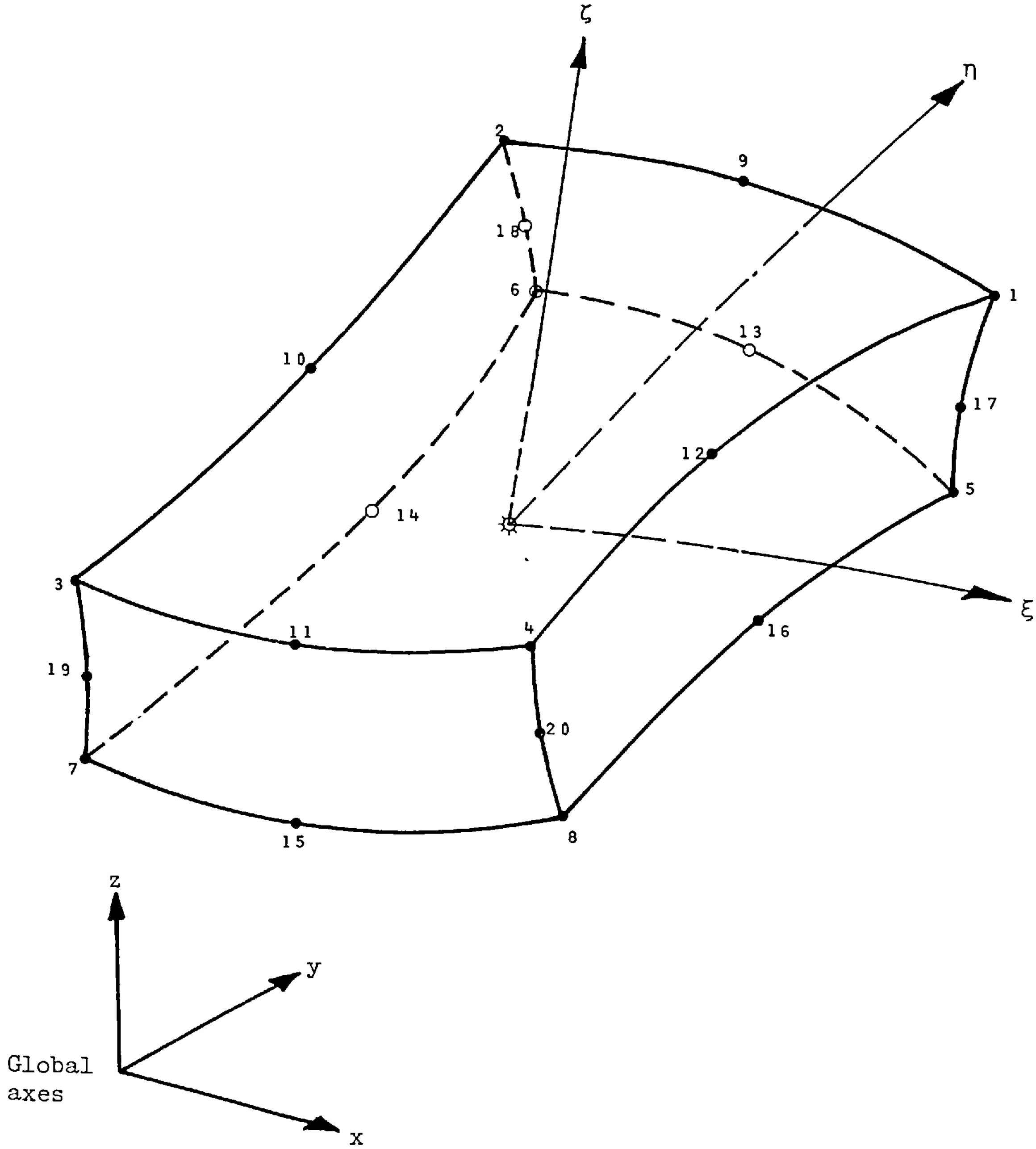


Fig. 4.4. Twenty node quadratic hexahedron

The relationships between derivatives in the two cordinate systems are established by the chain rule of differentiation as follows, for example (where, e.g., $x_{,\xi}$ means $\frac{\partial x}{\partial \xi}$)

$$\begin{Bmatrix} (\ )_{,\xi} \\ (\ )_{,\eta} \\ (\ )_{,\zeta} \end{Bmatrix} = \begin{bmatrix} X_{,\xi} & Y_{,\xi} & Z_{,\xi} \\ X_{,\eta} & Y_{,\eta} & Z_{,\eta} \\ X_{,\zeta} & Y_{,\zeta} & Z_{,\zeta} \end{bmatrix} \begin{Bmatrix} (\ )_{,x} \\ (\ )_{,y} \\ (\ )_{,z} \end{Bmatrix} = [J] \begin{Bmatrix} (\ )_{,x} \\ (\ )_{,y} \\ (\ )_{,z} \end{Bmatrix} \qquad 4.7$$

Where $[J]$ is the summed Jacobian matrix, defined by the following:-

$$[J] = \sum_{i=1}^{20} \begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \\ \frac{\partial N_i}{\partial \zeta} \end{Bmatrix} [x_i \ y_i \ z_i] \qquad 4.8$$

The differential volume of the whole element transforms to

$$d\ (\text{volume}) = \left|\det\ [J]\right|\ d_\xi\ d_\eta\ d_\zeta \qquad 4.9$$

in which $\left|\det\ [J]\right|$ = the absolute value of the determinant of the summed Jacobian matrix.

Since $[J]$ is a variable, numerical integration is unavoidable. Three dimensional gaussian quadrature is used.

The nodal variables are translations (without derivatives) and the coordinate transformations have the same form as those of the displacements.

To ensure convergence

$$\sum_{i=1}^{20} N_i = 1 \qquad 4.10$$

4.4. Strain Displacement Relationships

For the three dimensional solid the field of total strain is expressed as:

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial x} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \end{bmatrix} \qquad 4.11$$

Substituting Equation (4.6) into (4.11) gives:

$$\begin{aligned}
\frac{\partial u}{\partial x} &= \sum_{i=1}^{20} \frac{\partial N_i}{\partial x} u_i \\
\frac{\partial v}{\partial y} &= \sum_{i=1}^{20} \frac{\partial N_i}{\partial y} v_i \\
\frac{\partial w}{\partial z} &= \sum_{i=1}^{20} \frac{\partial N_i}{\partial z} w_i \\
\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} &= \sum_{i=1}^{20} \frac{\partial N_i}{\partial y} u_i + \sum_{i=1}^{20} \frac{\partial N_i}{\partial x} v_i \\
\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} & \quad \sum_{i=1}^{20} \frac{\partial N_i}{\partial z} v_i + \sum_{i=1}^{20} \frac{\partial N_i}{\partial y} w_i \\
\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} &= \sum_{i=1}^{20} \frac{\partial N_i}{\partial x} w_i + \sum_{i=1}^{20} \frac{\partial N_i}{\partial z} u_i
\end{aligned} \qquad 4.12$$

Thus

$$\{\varepsilon\} = \sum_{i=1}^{20} \begin{Bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} \end{Bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} \qquad 4.13$$

or

$$\{\varepsilon\} = \sum_{i=1}^{20} [B]_i \, \{\delta\}_i \qquad 4.14$$

The interpolation functions N_i are defined in terms of the natural curvilinear coordinates ξ, η, ζ, so that a change in the derivatives of $[B]_i$ is required to relate global to natural derivatives thus:-

$$\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \\ \frac{\partial}{\partial \zeta} \end{bmatrix} \qquad 4.15$$

Where $[J]$ is as defined in Equation (4.8).

Hence

$$\{\varepsilon\} = \sum_{i=1}^{20} [J]^{-1} [B^*]_i \, \{\delta\} \qquad 4.16$$

Where $[B^*]$ is given by the Equation.

$$
[B^*] = \begin{bmatrix} \frac{\partial N_i}{\partial \xi} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial \eta} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial \zeta} \\ \frac{\partial N_i}{\partial \eta} & \frac{\partial N_i}{\partial \xi} & 0 \\ 0 & \frac{\partial N_i}{\partial \zeta} & \frac{\partial N_i}{\partial \eta} \\ \frac{\partial N_i}{\partial \zeta} & 0 & \frac{\partial N_i}{\partial \xi} \end{bmatrix}
$$

Equation (4.16) can be written as:

$$
\{\varepsilon\} = [B] \{\delta\} \qquad 4.17
$$

Which is the required strain-displacement relationship

4.5. Stress-Strain Relations

For an orthotropic material whose natural axes are 1-2-3, the stress strain relations are: Ref. (46).

$$
\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ & D_{22} & D_{23} & 0 & 0 & 0 \\ & & D_{33} & 0 & 0 & 0 \\ & & & G_{12} & 0 & 0 \\ & \text{Symmetrical} & & & G_{23} & 0 \\ & & & & & G_{31} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{31} \end{Bmatrix}
$$

or

$$
\{\sigma'\} = [D'] \{\varepsilon'\} \qquad 4.18
$$

4.6. Derivation of the Stiffness Matrix

The stiffness matrix of the element is

$$[k] = \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} [B]^T [D] [B] \left| \det[J] \right| d\xi \, d\eta \, d\zeta \qquad 4.19$$

The Jacobian matrix and its determinant are computed directly from Equation (4.8), whilst the integration of the above Equation is carried out numerically.

Gr uss quadrature with respect to ξ η and ζ completes the integration according to the required degree using two, three or four integration points, which are controlled in the present program.

For thick shells the through thickness interpolations, in ζ can be evaluated less accurately than those in-plane (i.e. in the ξ and η plane), for which the number of points may be 4.

Integration can be achieved by integrating first with respect to one coordinate and then with respect to the other two. Thus, integration is as follows:

$$I = \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} f(\xi,\eta,\zeta) \, d\xi \, d\eta \, d\zeta$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} W_i \, W_j \, W_k \, f(\xi_i, \eta_j, \zeta_k)$$

where n = the number of integration points and W = their associated 'weights'.

It is possible to use the same number of points in each direction, but it is not essential to do so, for the reasons explained above.

Where

$$D_{11} = E_1/C_3$$

$$D_{12} = (C_1/C_2)\ D_{11}$$

$$D_{13} = C_4\ D_{11}$$

$$D_{22} = E_2/C_1 + (C_2/C_1)\ D_{12}$$

$$D_{23} = \nu_{32}\ E_2/C_1 + (C_2/C_1)\ D_{13}$$

$$D_{33} = E_3 + \nu_{31}\ D_{13} + \nu_{32}\ D_{23}$$

where

$$C_1 = 1 - \nu_{32}^2\ (E_2/E_3)$$

$$C_2 = \nu_{21} + \nu_{31}\ \nu_{32}\ (E_2/E_3)$$

$$C_3 = 1 - (E_1/E_3)\left[\nu_{31}^2 + ((E_3/E_2)\ (C_2^2/C_1)\right]$$

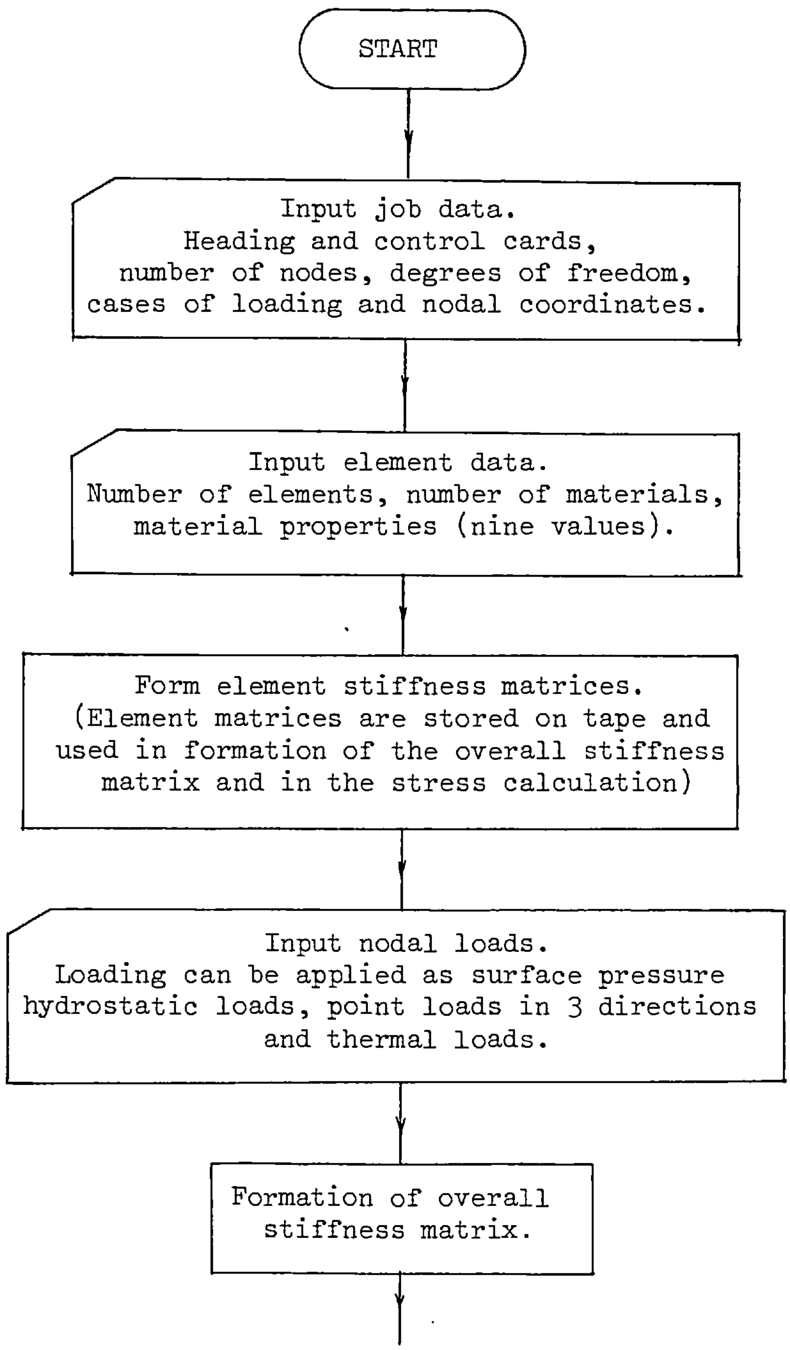
$$C_4 = \nu_{31} + \nu_{32}\ (C_2/C_1)$$

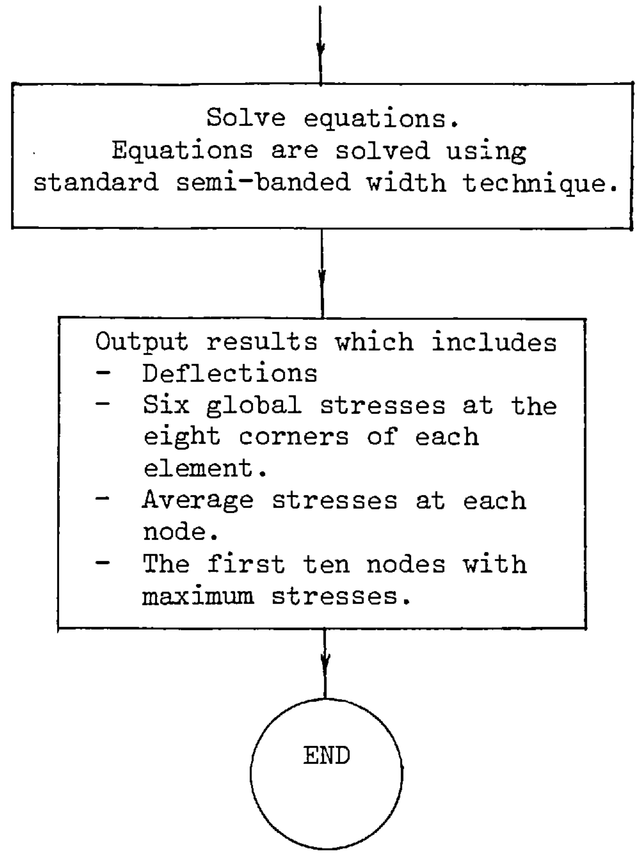
in which E_1, E_2, E_3 are Young's moduli of the material in the directions of its natural axes 123, ν_{12}, ν_{23}, ν_{13} are the Poisson's ratios and G_{12}, G_{23} and G_{31} are the moduli of Rigidity Constants.

If the directions of symmetry of the material do not coincide with the global reference directions the matrix $[D]$ has to be transformed, as discussed in Chapter 3.

4.7. Flow chart of solid finite element computer programme

The equations in Chapter 3 and the work done in this chapter were written in a computer programme (BFSOLID). The following is a flow chart and the listing in FORTRAN IV is given in APPENDIX 2E.




Solve equations.
Equations are solved using
standard semi-banded width technique.
Output results which includes
- Deflections
- Six global stresses at the eight corners of each element.
- Average stresses at each node.
- The first ten nodes with maximum stresses.
END

CHAPTER FIVE

IDEALIZATION OF FINITE ELEMENTS FOR STRUCTURAL DESIGN, NUMERICAL AND EXPERIMENTAL RESULTS

CHAPTER FIVE

IDEALIZATION OF FINITE ELEMENTS FOR STRUCTURAL DESIGN, NUMERICAL AND EXPERIMENTAL RESULTS

5.1. Introduction

The finite element program (BFSOLID) was used to solve the challenging problems of large water valves more accurately, especially when the blade dimensions were such as to exclude the work done in Chapter Two, i.e. outside the assumption of thin plate theory, such as when the thickness-diameter ratio is more than 1/8. An example of an ordinary cast iron blade is discussed in 5.2.

The program, or any equivalent isotropic solid finite element program, proved to be the only accurate method for structural analysis and design when the large diameter and high pressure necessitated the blade to take the form of a thick plate supported on a lattice which could take different complicated configurations as shown in Figs.(5.1a) and (5.1b). Ref. (50).

For anisotropic material of the fibre composite type, the various elastic constants have to be inserted into the computer program according to Equations (3.14a) to (3.14l). For isotropic material the same equations are used but with only three elastic constants E, ν and G to which all others are equal as explained later in 5.2.

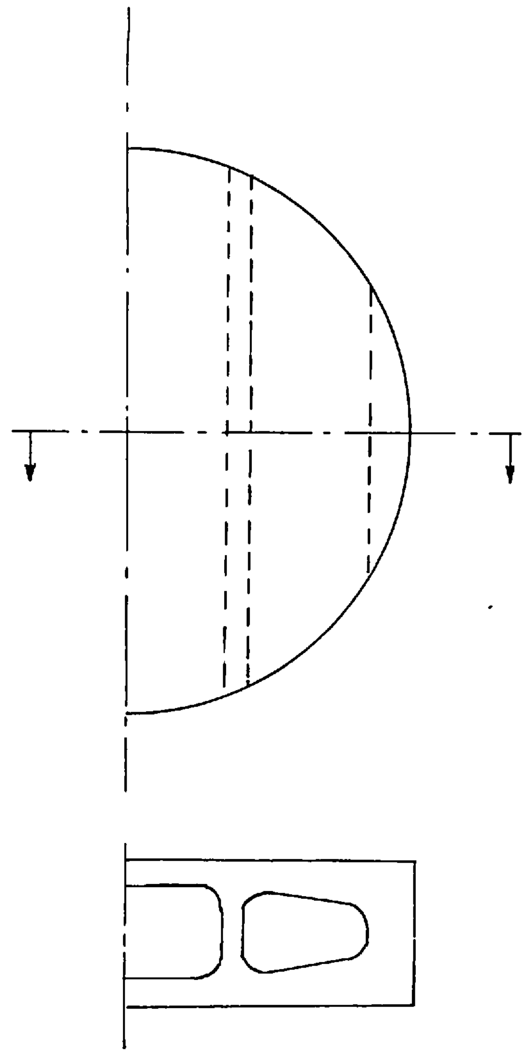


Fig. 5.1a. Blades of throughflow valves

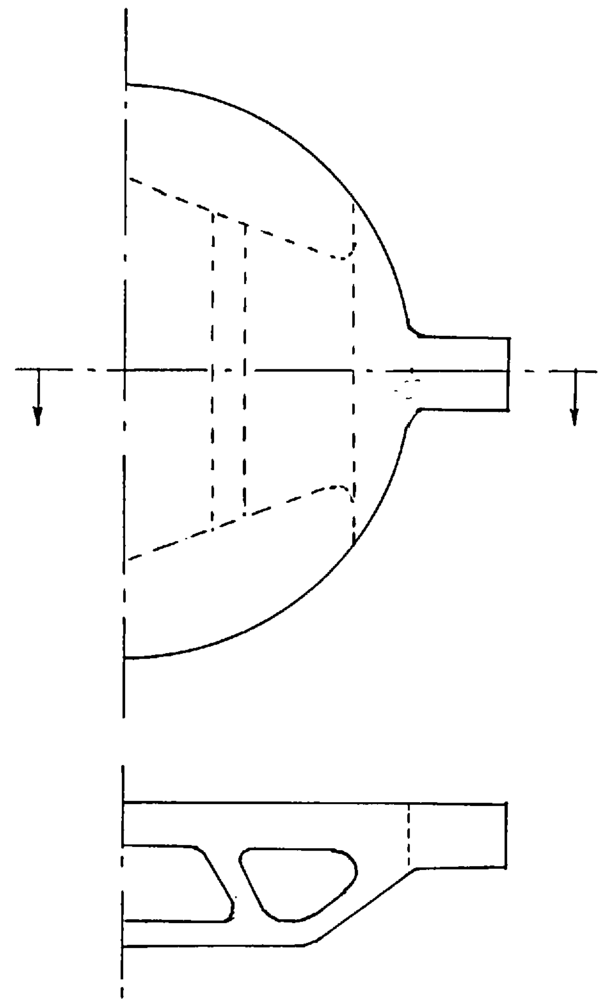


Fig. 5.1b. Blades of throughflow valves

The analysis of fibre composite material is a vast subject of a complex nature and theories have been developed whereby, knowing the properties of the fibres the matrix and their volume fractions in a composite, one can estimate the elastic constants and strength properties of unidirectional laminates, unidirectional laminates with orientated fibres, balanced laminates and multilayer, multidirectional composites.

The ability to predict the elastic properties of various types of laminates and composites has been verified experimentally. A great deal of theoretical and experimental work on predicting strength properties and failure modes in composite materials subjected to various types of loading has also been done. Refs. (42), (43), (51), (52), (53), (54), (55), (56) and (57), give a good impression of the underlying difficulties of the subject.

Various factors of manufacturing technology, as well as temperature, chemical and environmental effects that influence the strength and modes of failure have also been covered extensively in Refs. (58), (59), (60), (61) and (62).

To develop a practical approach to analyse stresses and strains in the elastic region where the environmental problems of temperature, chemical and time dependent factors are covered by complying with standards, Ref. (63), only partial experimental verification of the elastic properties of such composites has been obtained up to the present time. A great deal of work is still in progress to overcome this problem where, by specifying the fibre content and alignment of known types it is possible to predict the elastic constants of the end product as though it were an anisotropic continuum.

An attempt to give a practical approach for stress analysts and designers is discussed in 5.3.

The program was also used to analyse a 60" butterfly valve made up of three materials, namely; woven roving g.r.p., chopped strand mat g.r.p. and stainless steel shaft inserts. This is described in 5.4.

5.2. Cast Iron Blade of an Ordinary Butterfly Valve.

A 72" diameter, 6" thick blade of a butterfly valve, shown in Fig. (5.2), made of cast iron GR17 which is fitted in a body made of the same material, is to be checked for a design requirement of a maximum deflection of 0.125" for sealing under a test pressure of 75 psi. Using Appendix 3:

$$\nu = 0.26$$

$$C_1 = 0.386$$

$$q = 75 \text{ psi}$$

$$a = 71.2''/2$$

$$t = 6''$$

$$W = \frac{0.386 \times 75 \times 71.2^4 \times 12(1-0.26^2)}{16 \times 18 \times 10^6 \times 6^3}$$

$$= 0.134''$$

Deflection due to the reaction couple

$$W = \frac{MR}{2D}\left(\frac{\pi}{2} - 1\right) \qquad M = R \times \frac{t}{3}$$

$$2R = 298614 \text{ lb.}$$

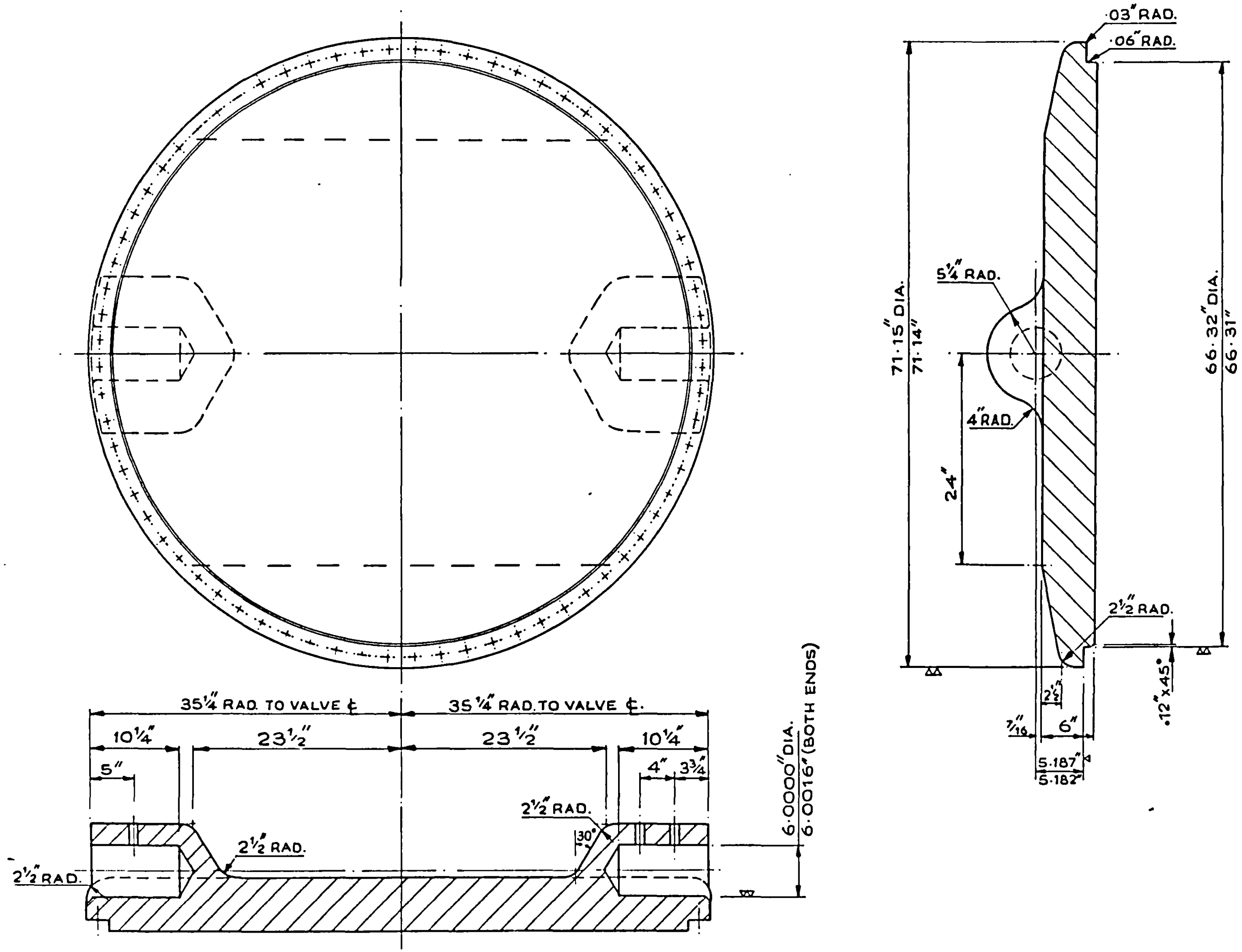


Fig. 5.2. 72" dia. Cast iron butterfly valve - blade

$$W = \frac{298614 \times 71.2 \times 12(1-\nu^2)(\frac{\pi}{2} - 1)}{2 \times 2 \times 18 \times 10^6 \times 6^3}$$

Total $W = 0.134 + 0.008 = 0.142''$

Idealizing the blade into 34 21-node elements with a total of 94 nodes as shown in Figs. (5.3a), (5.3b) and (5.3c) and giving $E_{11} = E_{22} = E_{33} = 18 \times 10^6$ psi and $\nu_{12} = \nu_{13} = \nu_{31} = 0.26$ and $G_{12} = G_{13} = G_{23} = \frac{E}{2(1+\nu)} = 7.14 \times 10^6$ psi the maximum deflection occurring at node 92 or 89 was - .10387".

The valve was pressurised up to 75 psi and the deflections measured at locations shown on Fig. (5.4a) relative to the body. The total deflection was 0.095" as shown in Figs. (5.4b) and (5.4c) which also shows the deflected shape in the two perpendicular planes of interest.

5.3. Test Samples of Composite Materials.

5.3.1. Introduction

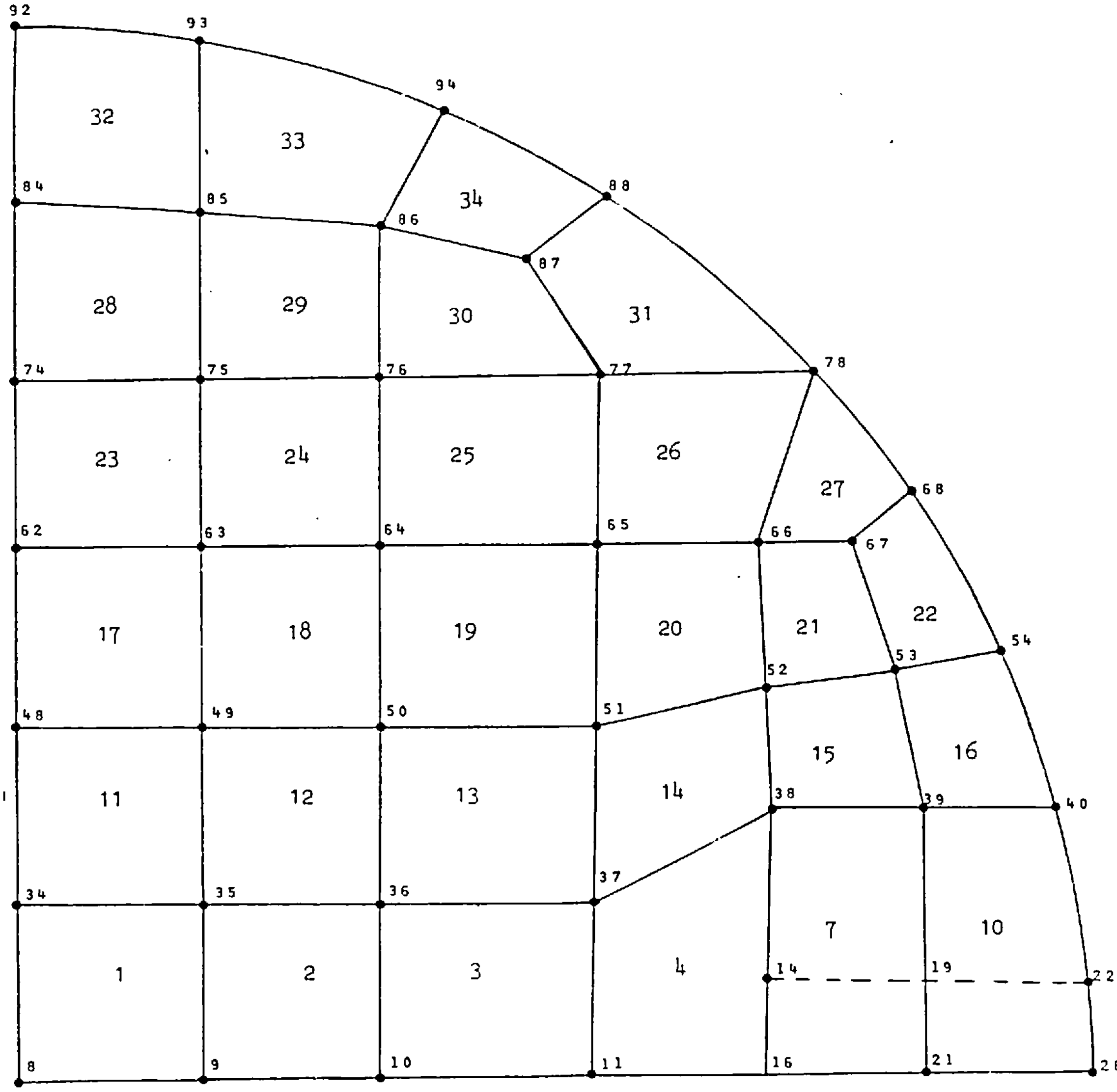
G.R.P. can be taken as a good example of fibre composite material which is widely used in an increasing number of manufacturing industries. One major use is for manufacturing pipes and pipe fittings which can combat the corrosive effect of sea water. Ref. (64).

A great deal of literature on G.R.P. is available. Ref. (61), (62), (65) and (66) are given as samples of this literature.

Many G.R.P. manufacturers are now prepared to give reliable basic mechanical properties as shown in Table (5.1). Ref. (67).

In these tables the flexural (elastic) modulus of a composite consisting of continuous, aligned fibres in a softer matrix is given by simple elasticity theory using

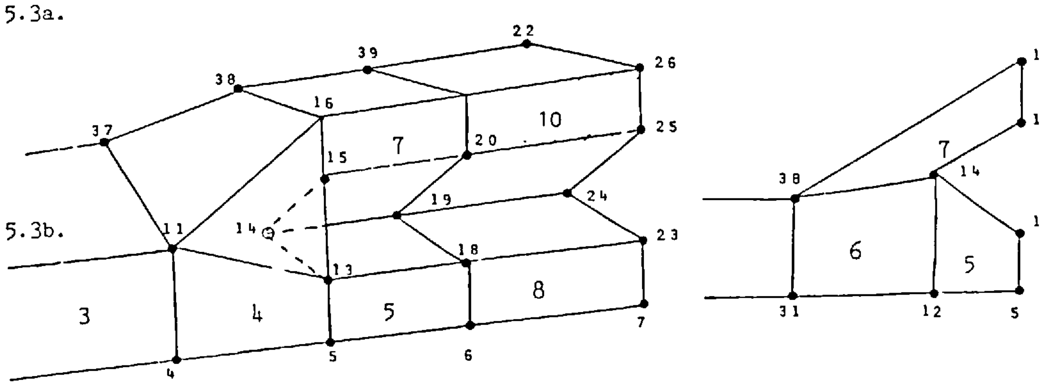
$$E_c = E_c V_f + E_m(1-V_f)$$



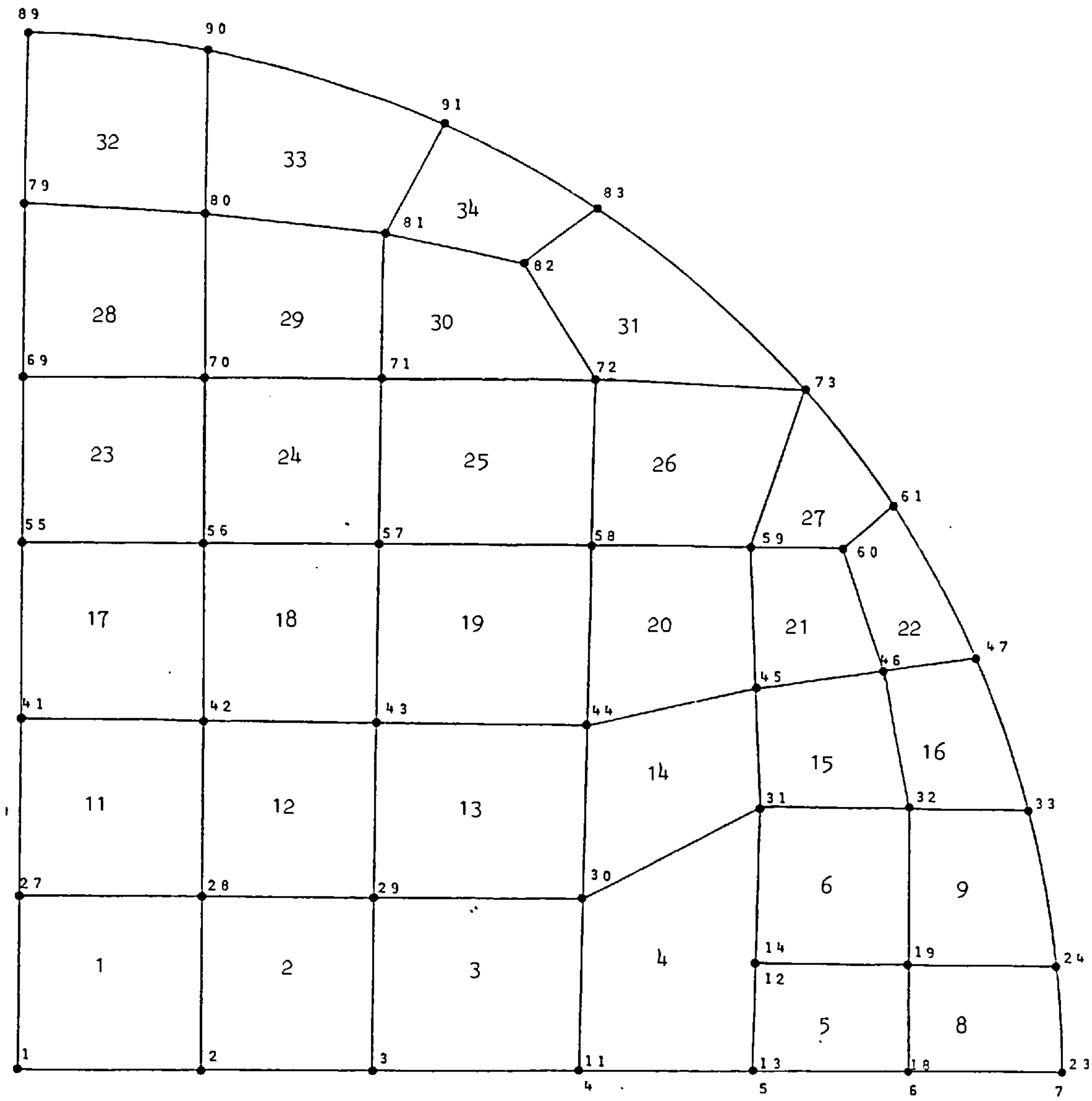
Upper face

Fig. 5.3a.

Fig. 5.3b.



Figs. 5.3a & 5.3b. Finite element mesh of the cast iron blade



Lower Face

Fig. 5.3c. Finite element mesh of the cast iron blade.

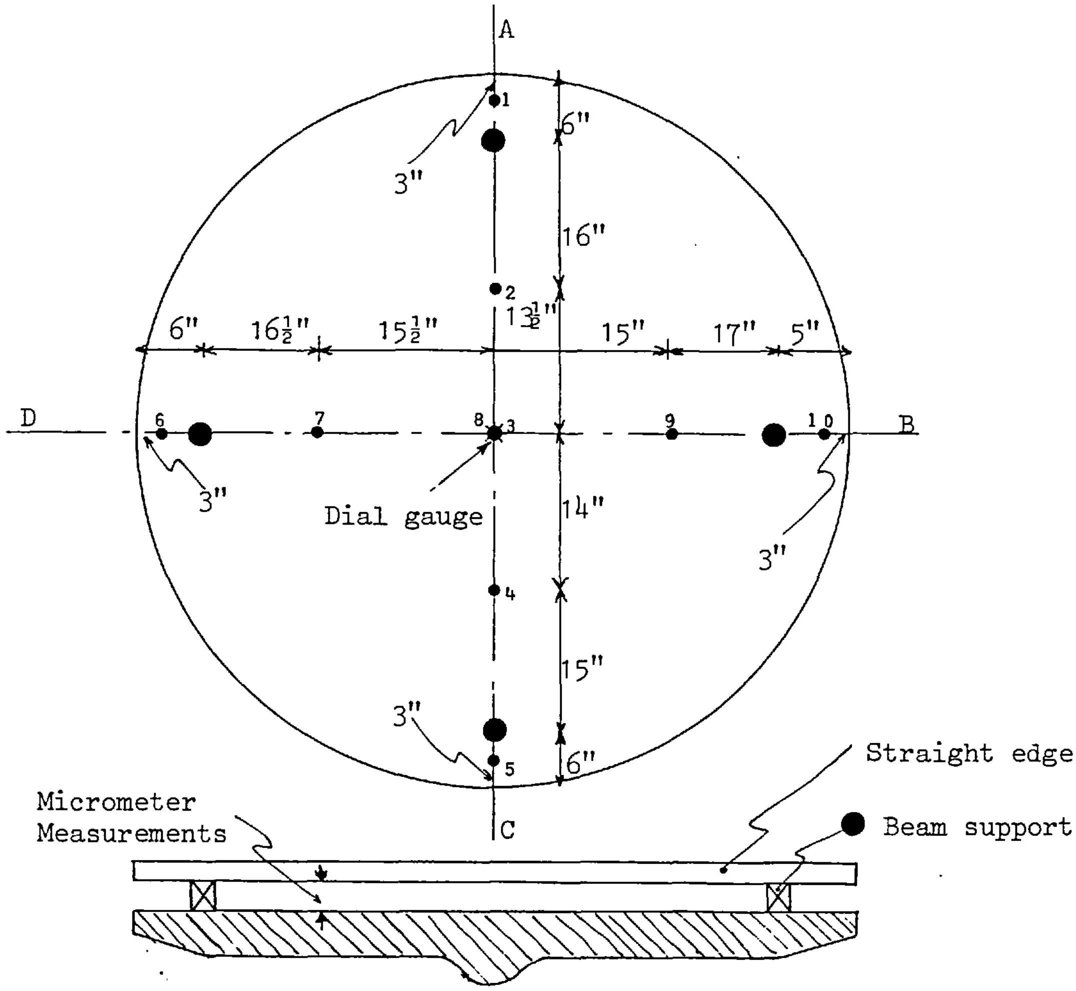


Fig. 5.4a. Deflection test of cast iron blade.

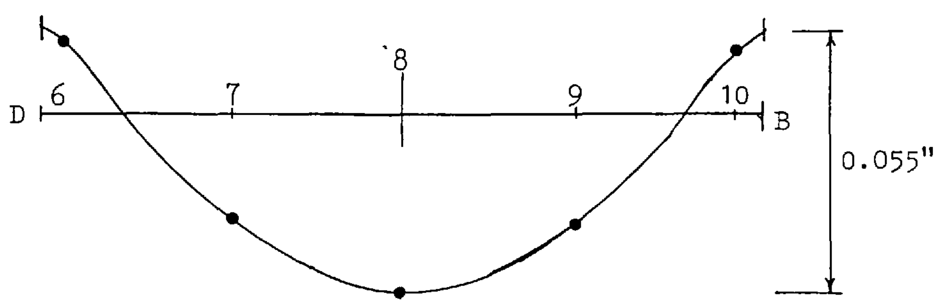


Fig. 5.4b. Deflection at centre relative to bearings.

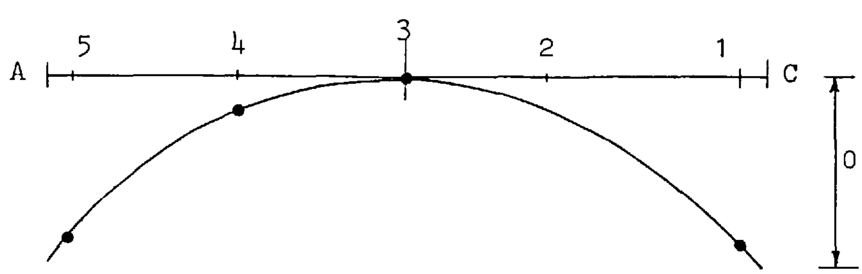


Fig. 5.4c. Deflection at wings relative to centre.

Material	Fibre Content by Weight %	Density Mg/m^2	Tensile Strength GN/m^2	Tensile Modulus GN/m^2	Compressive Strength MN/m^2	Flexural Strength MN/m^2	Flexural Modulus GN/m^2
Uni-directional glass							
Wound epoxide	60-90	1.7-2.2	530-1730	28-62	310-480	690-1860	34-48
Uni-directional polyester	50-75	1.6-2.0	410-1180	21-41	210-480	690-1240	27-41
Bi-directional glass							
Satin weave polyester	50-70	1.6-1.9	250-400	14-25	210-280	207-450	17-23
Woven roving polyester	45-60	1.5-1.8	230-340	13-17	98-140	200-270	10-17
Random glass mat							
Preform polyester	25-50	1.4-1.6	70-170	6-12	130-160	70-240	
Hand & spray up polyester	25-40	1.4-1.5	63-40	6-12	130-170	140-250	5-8
Moulding compounds							
DMC polyester	10-40	1.8-2.0	34-70	12-14	140-180	40-140	
SMC polyester	20-35	1.8-1.85	50-90	9	240-310	140-210	9-14
Glass filled nylon	20-40	1.3-1.5	120-200	6-14	110-170		

$1\ GN/m^2 = .14508 \times 10^6$ psi

Table: 5.1. Material properties as given by manufacturers. Ref. (67).

where the subscripts c, f and m refer to composite, fibre and matrix and V_f is the fibre volume fraction. This expression is strictly true only for the case where both fibre and matrix are isotropic and it assumes that the two components are constrained to deform together. This elementary "Rule of mixtures" is in fact a lower bound.

Such moduli will generally be exceeded by an amount depending upon the difference in Poisson's ratio of the two constituents, as a result of the lateral elastic constraint which the two phases exert on one another. The equivalent theoretical expression for the shear modulus can be written as follows:-

$$\frac{G_c}{G_m} = \left[\frac{(\frac{G_f}{G_m} + 1) + (\frac{G_f}{G_m} - 1)\ V_f}{(\frac{G_f}{G_m} + 1) - (\frac{G_f}{G_m} - 1)\ V_f}\right]$$

Which shows that the ratio $\frac{G_c}{G_m}$ is a function of V_f depending on the value of stiffness ratio, $\frac{G_f}{G_m}$, after Tsai, Adams and Doner Ref. (68).

As discussed in Chapter 3, an aligned composite is highly anisotropic. A lot of work is based on the assumption that the material is transversely isotropic. However, if the material is being used in plate form, as is frequently the case with laminates, the assumption of a plane stress state reduces the number of elastic constants to four, E_{11}, E_{22}, ν_{12}/E_{11} or ν_{21}/E_{22} and G_{12}, which can be easily established by a simple testing procedure for the first three carried out on a test coupon supplied by the manufacturer. This would be an improvement on the manufacturer's information and is covered in the present work.

To test the solid finite element experimentally an attempt to establish as many elastic constants as possible to satisfy Equations (3.14a) to (3.14l) was made. In many texts, Ref. (69), $E_{33} \simeq E_{22}$ where E_{22} is $< E_{11}$. From the tests ν_{13} and ν_{23} can be easily obtained and, using Equation (3.14b) and Equation (3.14c), most of the remaining elastic constants can be estimated.

The shear modulus proved a more critical factor to assume without the difficult and expensive experimental measurement which would otherwise be necessary. One of the most serious limitations of fibre composite materials is that the shear strength parallel with the fibres can be as low as that of the matrix or the fibre resin interface, either of which may be an order of magnitude (or more) lower than the maximum tensile strength of the composite. Consequently the composite may fail at low loads if the stress system is such as to cause a high shear stress on these planes of weakness. A bar loaded in flexure will fail in shear at the neutral plane before it breaks by tensile failure of the outer fibres if the ultimate shear stress, τ_{max} is reached before the outer fibre failure stress σ_{max}. Thus a short composite beam was used as a test sample to ensure that no shear failure occurred in the elastic region and the shear modulus assumption is a lower bound one. This assumes the material to be homogeneous and isotropic in all directions which, although not true, if observed with the guide lines laid down in BS.4994 would produce a structurally sound product.

From this simple beginning a sample of each distinctive layer or solid can be specified and the general behaviour of the complete structure can be predicted as discussed below.

5.3.2. Preparation of test specimens, testing equipment and procedure

(i) The Samples

A number of deep short beams, samples (D1, D2, D3, D4) made of three distinctive layers are cut from a block which is layered up from three distinctive layers, as shown in Figs. (5.5a), (5.5b), (5.6a) and (5.6b).

The block is made during the laying of the layers by installing plastic insulation sheets after the completion of each layer, as shown in Fig. (5.7), the layers being made from woven roving and uni-directional glass polyester material as shown in Fig. (5.8). Each layer is then cut into specimens and carefully marked to show the principal natural axis of the material as shown in Figs. (5.9), (5.10), (5.11) and (5.12).

(ii) Test procedure

Two longitudinal and two transverse strain gauges of two rosette strain gauges type FRA-6-11 made by T.S.K. Ltd. gauge resistance 120 ± 0.5Ω, gauge length 6mm were stuck at the top and bottom surfaces. The adhesive used was CN adhesive made by T.M.L. Ltd. of T.S.K. Ltd.

The two longitudinal strain gauges were arranged as a 'half-bridge' and fed with 3.V through a Brueland and Kjaer strain gauge amplifier, type 1525. The output was fed back to the amplifier which indicated the total strain (microstrain).

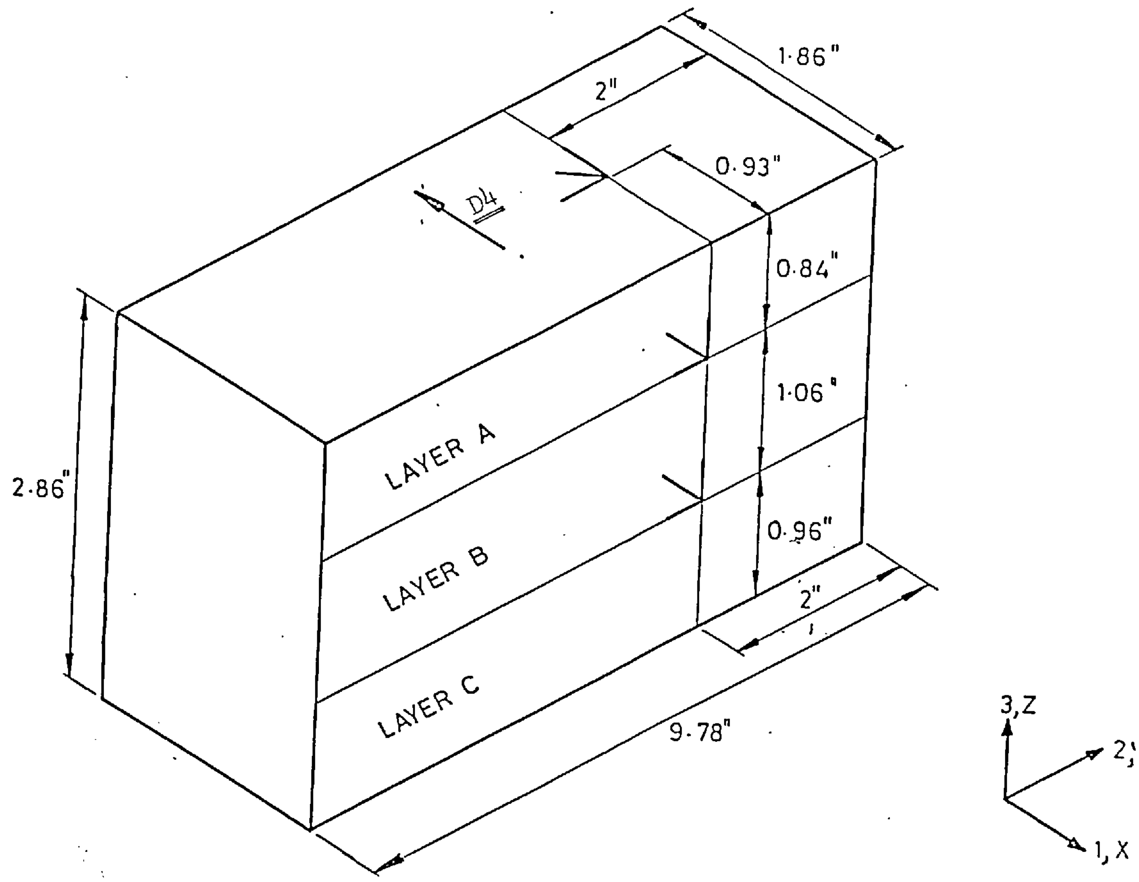


Fig. 5.5a. Actual dimensions of composite deep beam No. D1.

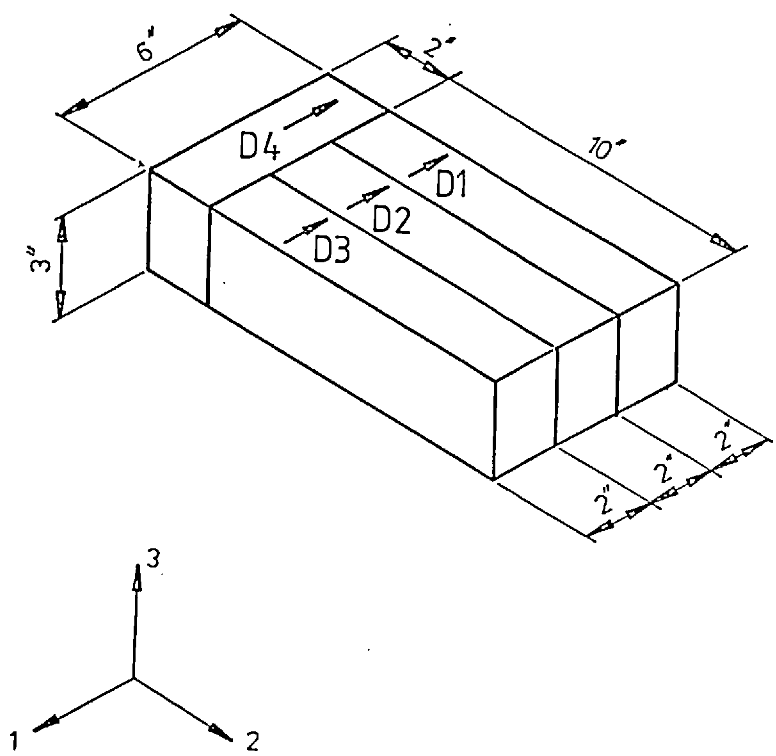


Fig. 5.5b. Composite deep beam specimen

Fig. 5.6a. Composite deep beam specimen

Fig. 5.6b. Composite deep beam specimen

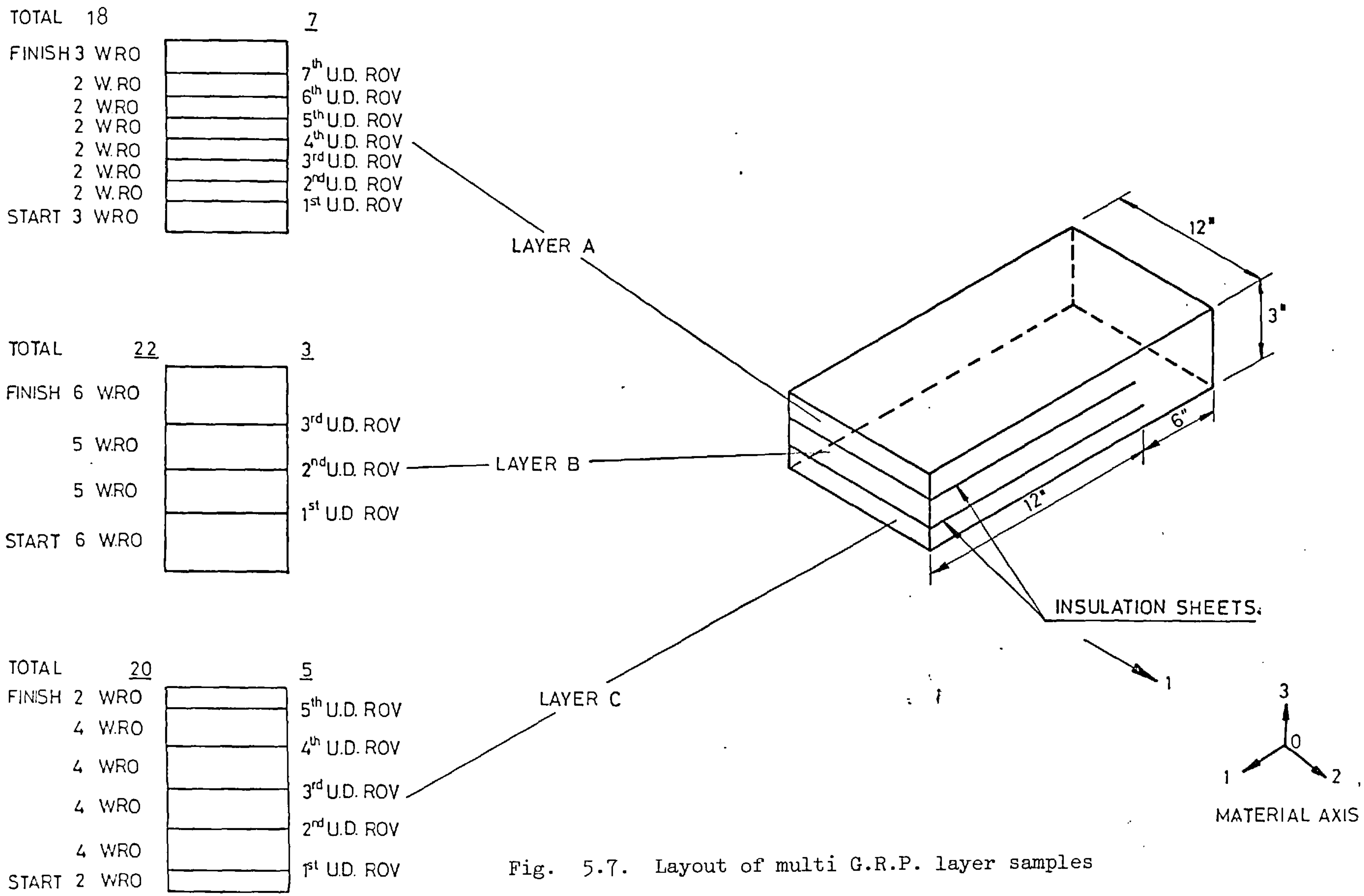


Fig. 5.7. Layout of multi G.R.P. layer samples

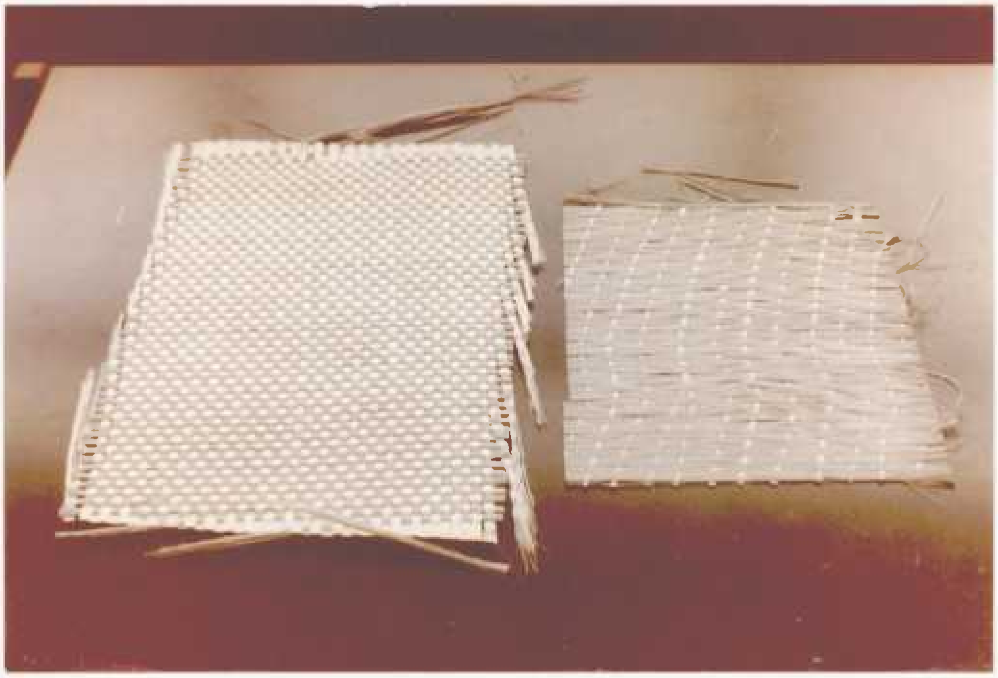


Fig. 5.8. Woven roving and unidirectional glass layers.

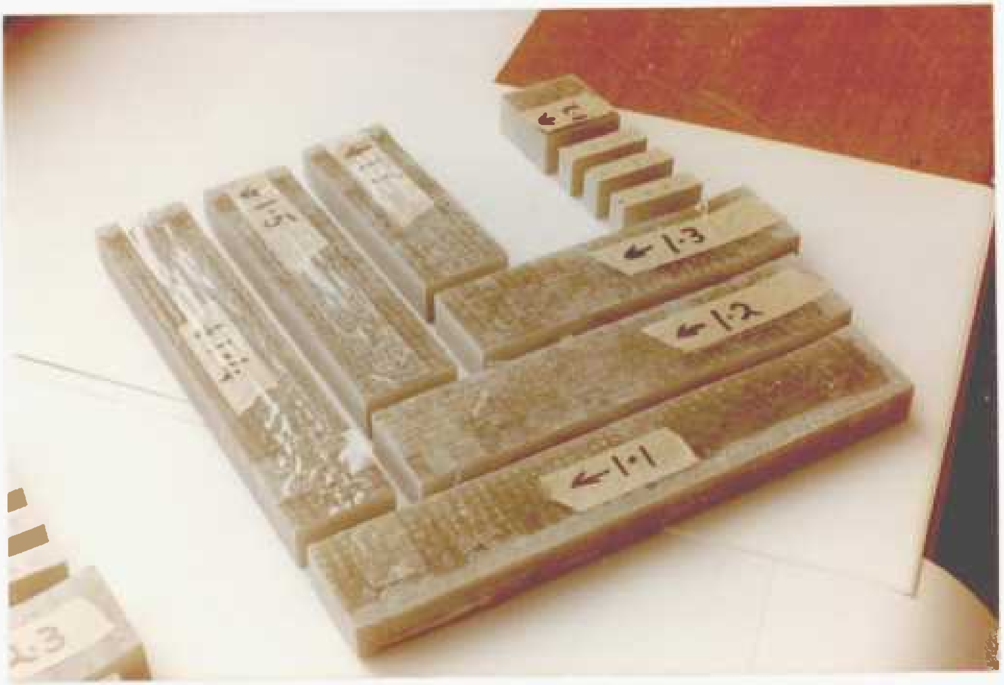


Fig. 5.9. Individual layer specimens

Fig. 5.10. Individual layer specimens.

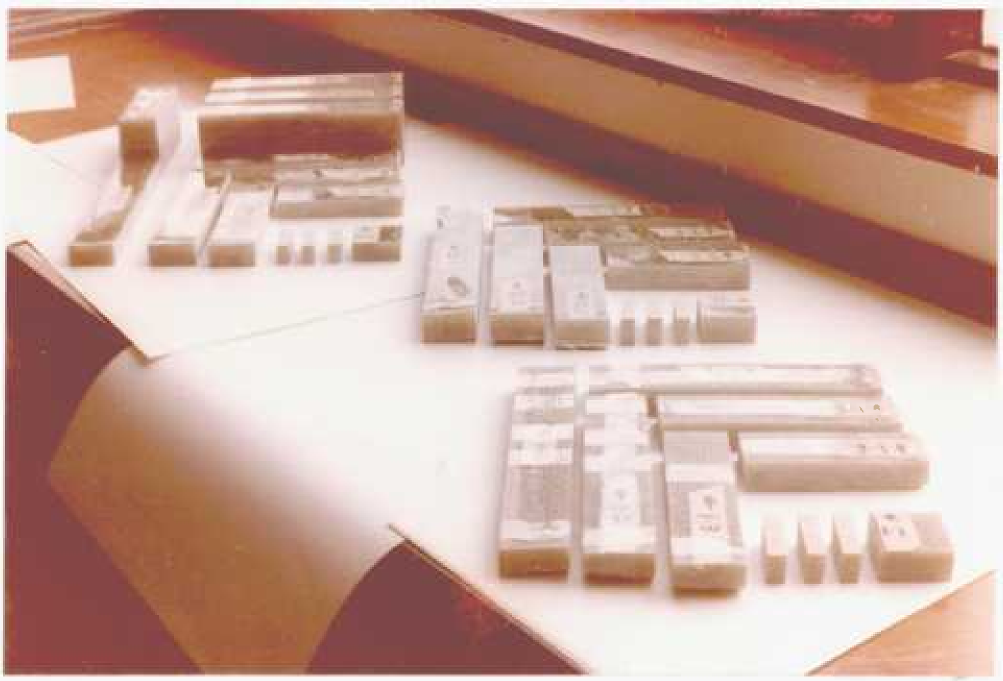


Fig. 5.11. Individual layer specimens.

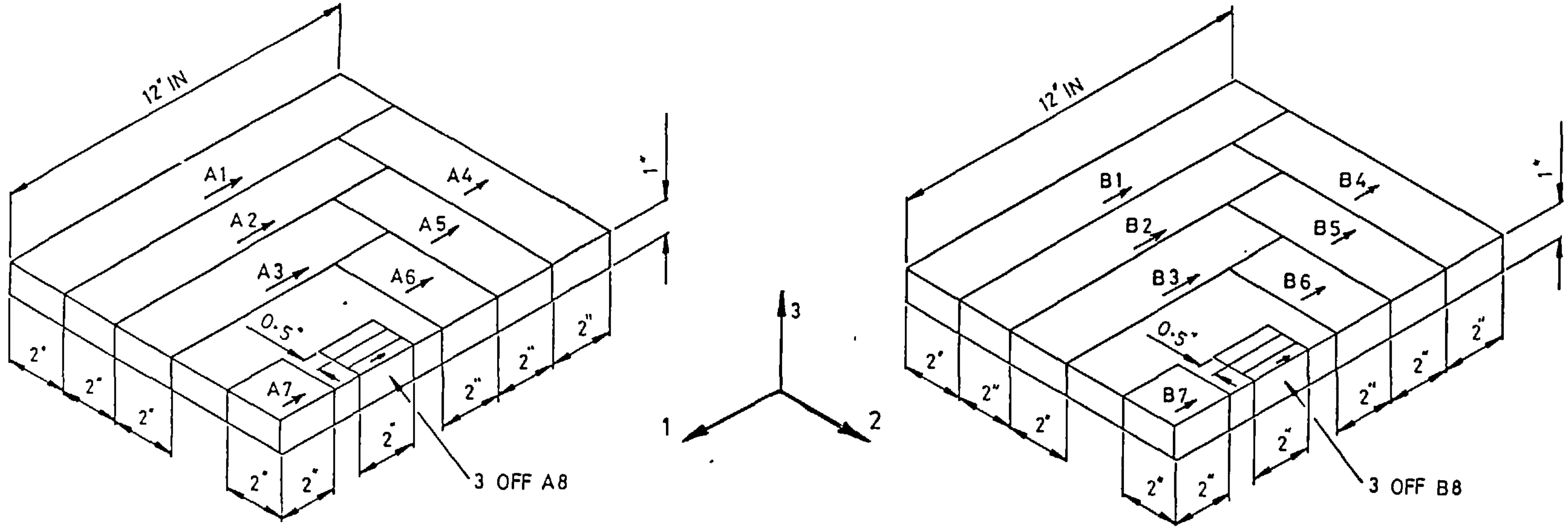
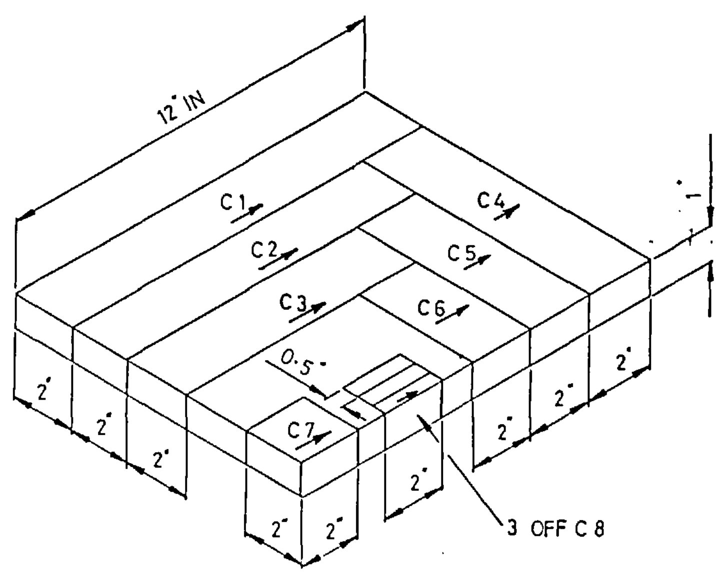


Fig. 5.12. Individual layer specimens.



The transverse strain gauges were arranged in the same manner using another amplifier of the same type. Four point bend testing of the G.R.P. beams was carried out to establish the elastic moduli by applying the load through a load cell as shown in Figs. (5.13) and (5.14). Figs. (5.15) and (5.16) are a sample of the results for layer A. Figs. (5.17) and (5.18) are a sample of the results for layer B. Figs. (5.19) and (5.20) relate to layer C. The two strains were plotted against the central deflections and Figs. (5.21) and (5.22) are a sample of the results for layer A. Figs. (5.23) and (5.24) relate to layer B and Figs. (5.25) and (5.26) to layer C.

Two more strain gauges, one longitudinal and one vertical, of the same type of rosette were stuck to the sides of the test specimens at approximately one third of the beam depth using the same adhesive. Two dummy gauges of the same type were stuck on another specimen and arranged with the active ones into two half-bridges using two amplifiers of the same type as before. This is shown in Figs. (5.27), (5.28), (5.29) and (5.30). The two readings were plotted against one another. Figs. (5.31) and (5.32) are examples of the results for layer A. Figs. (5.33) and (5.34) relate to layer B and Figs. (5.35) and (5.36) to layer C. Figs. (5.37) shows a minimum group of samples, i.e. two per layer and the apparatus used is shown in Fig. (5.38).

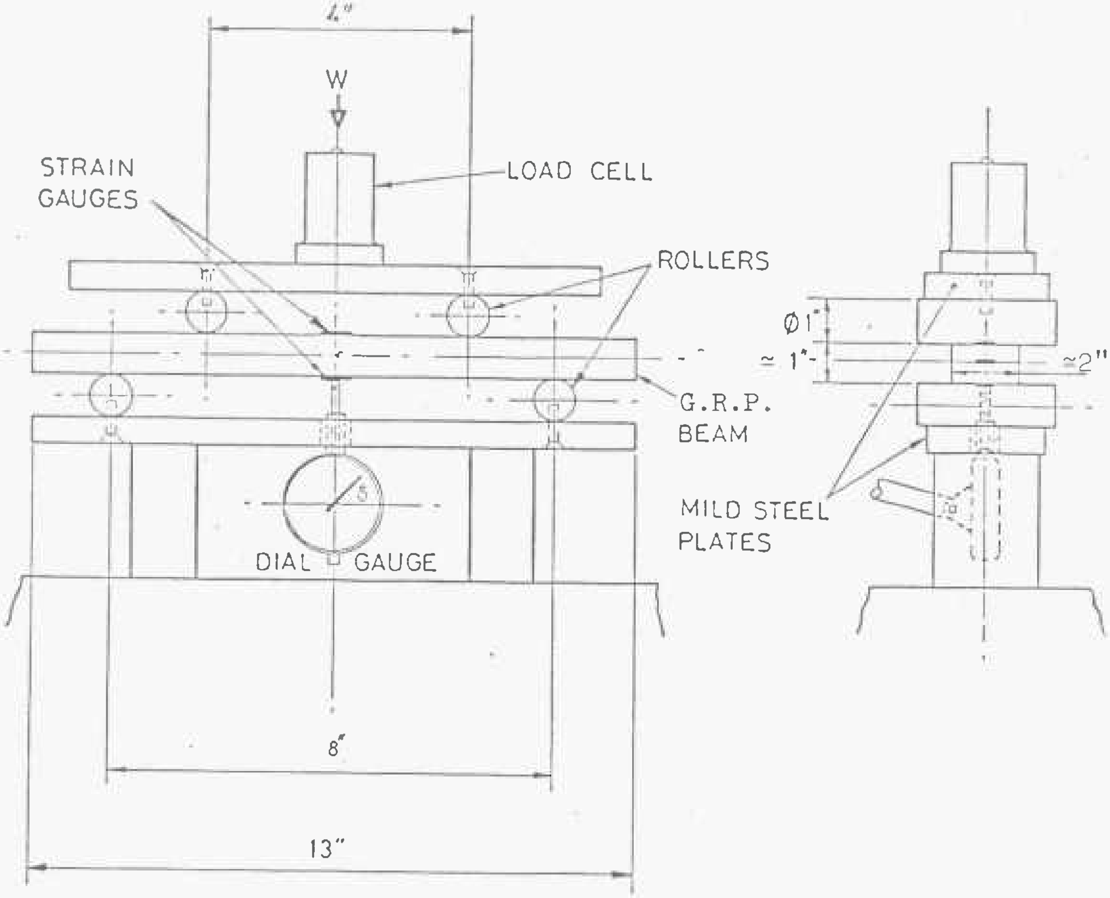


Fig. 5.13. 4-Point testing of G.R.P. beam specimens.

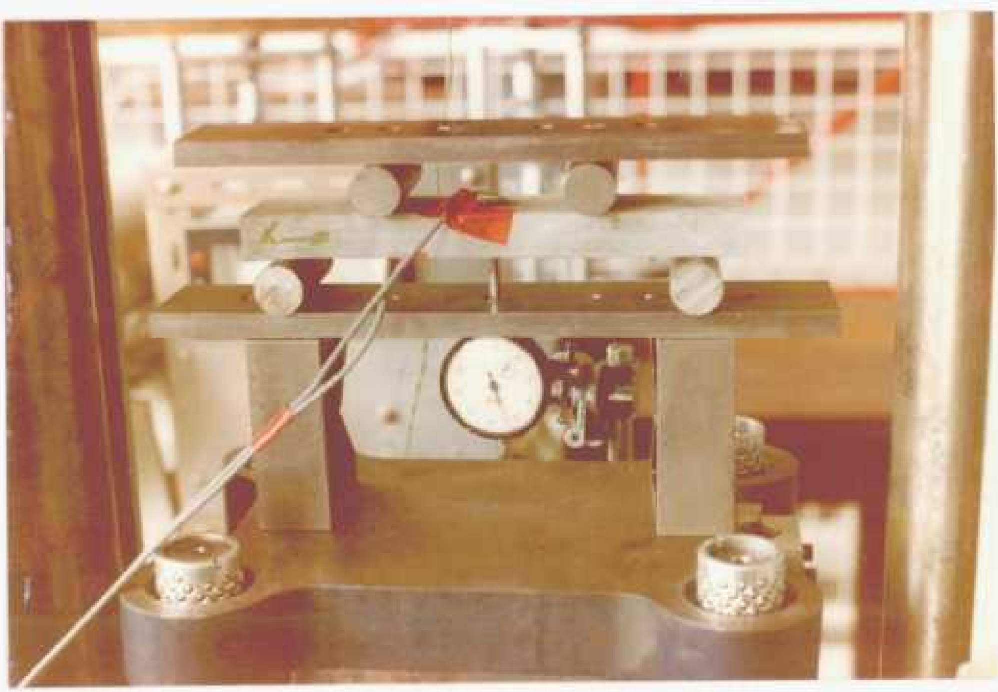


Fig. 5.14. 4-Point testing of G.R.P. beam specimens.

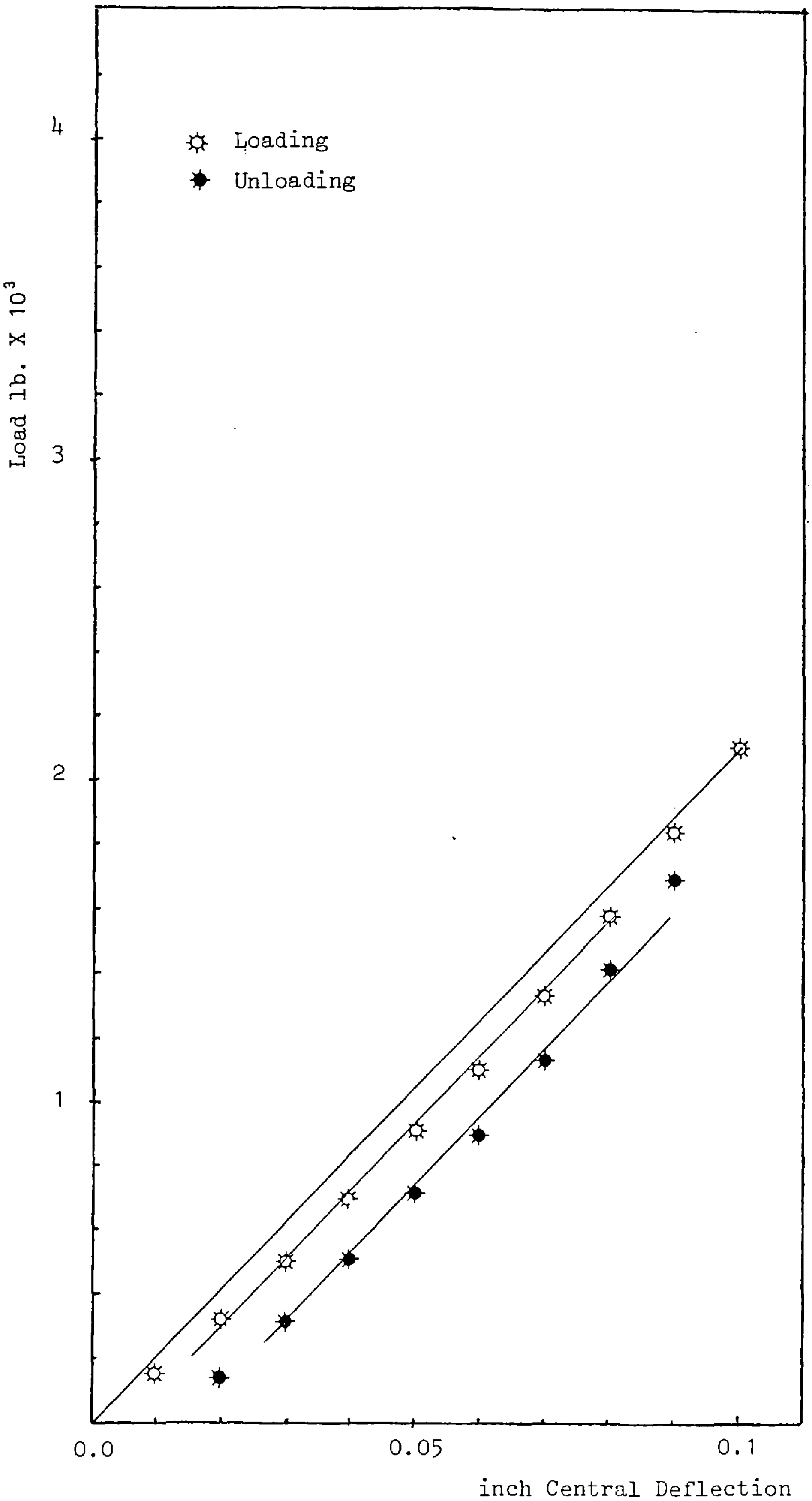


Fig. 5.15. Load vs. deflection in direction 1 - Layer A

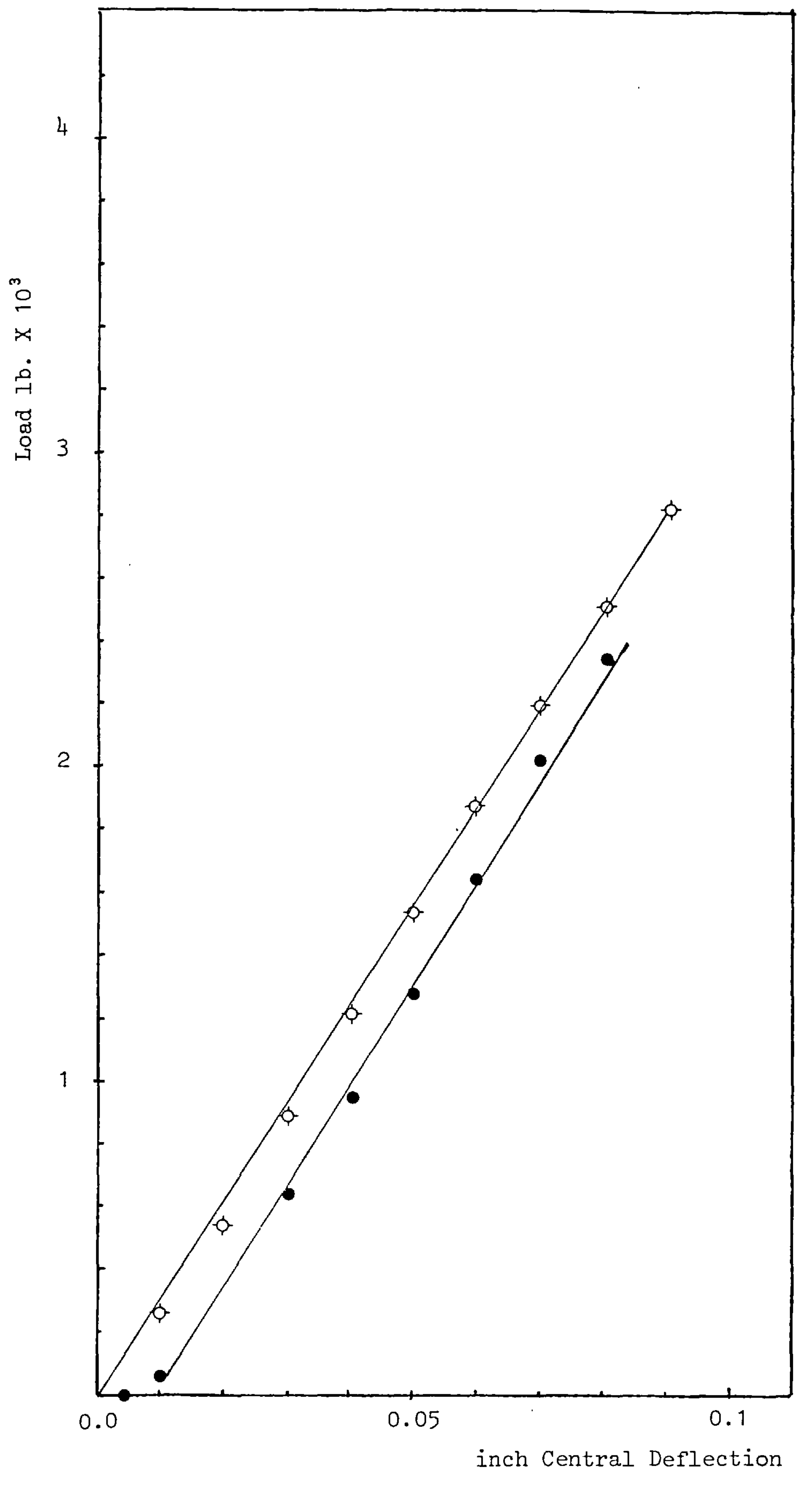


Fig. 5.16. Load vs. deflection in direction 2 - Layer A

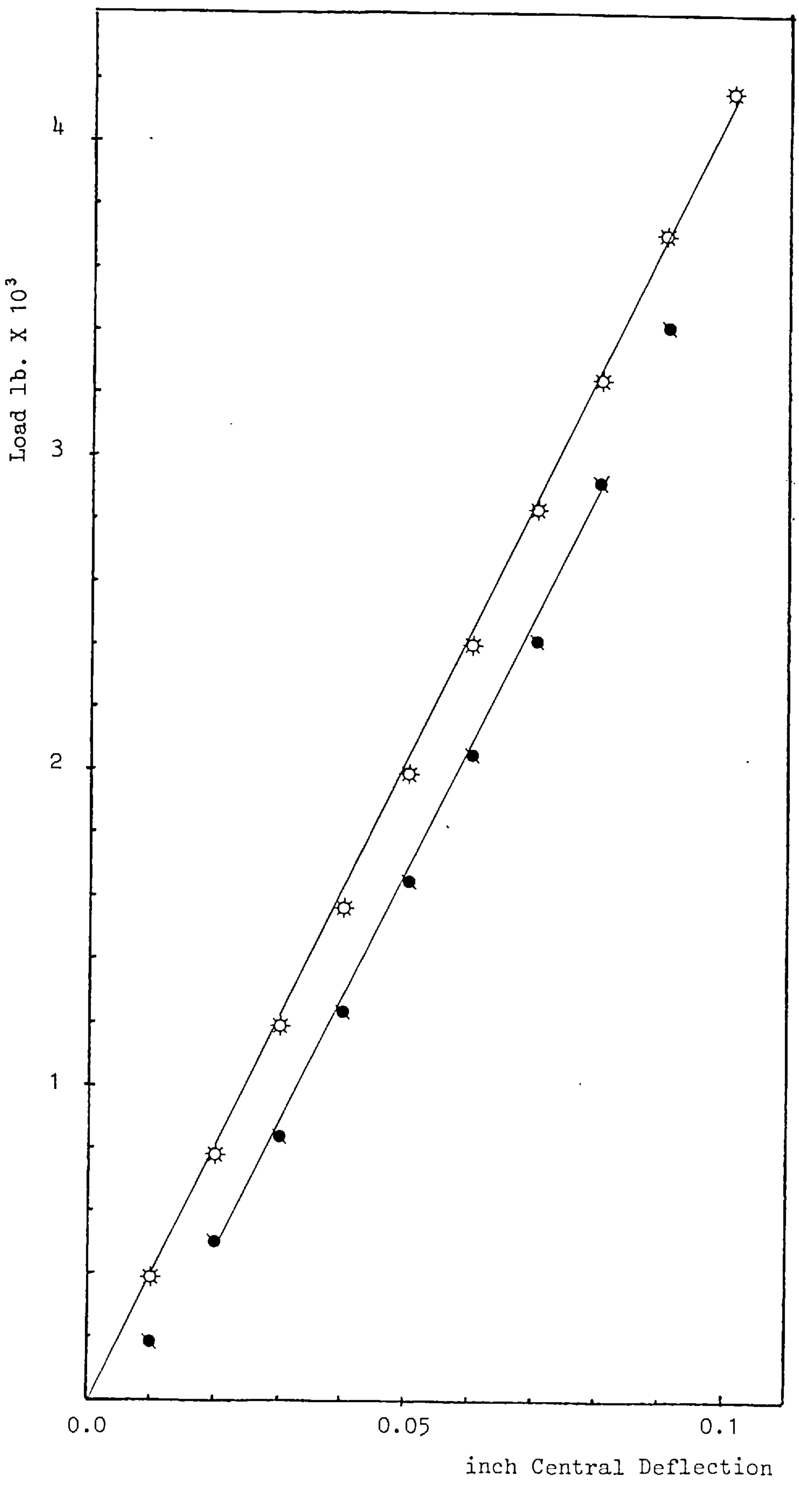


Fig. 5.17. Load vs. deflection in direction 1 - Layer B

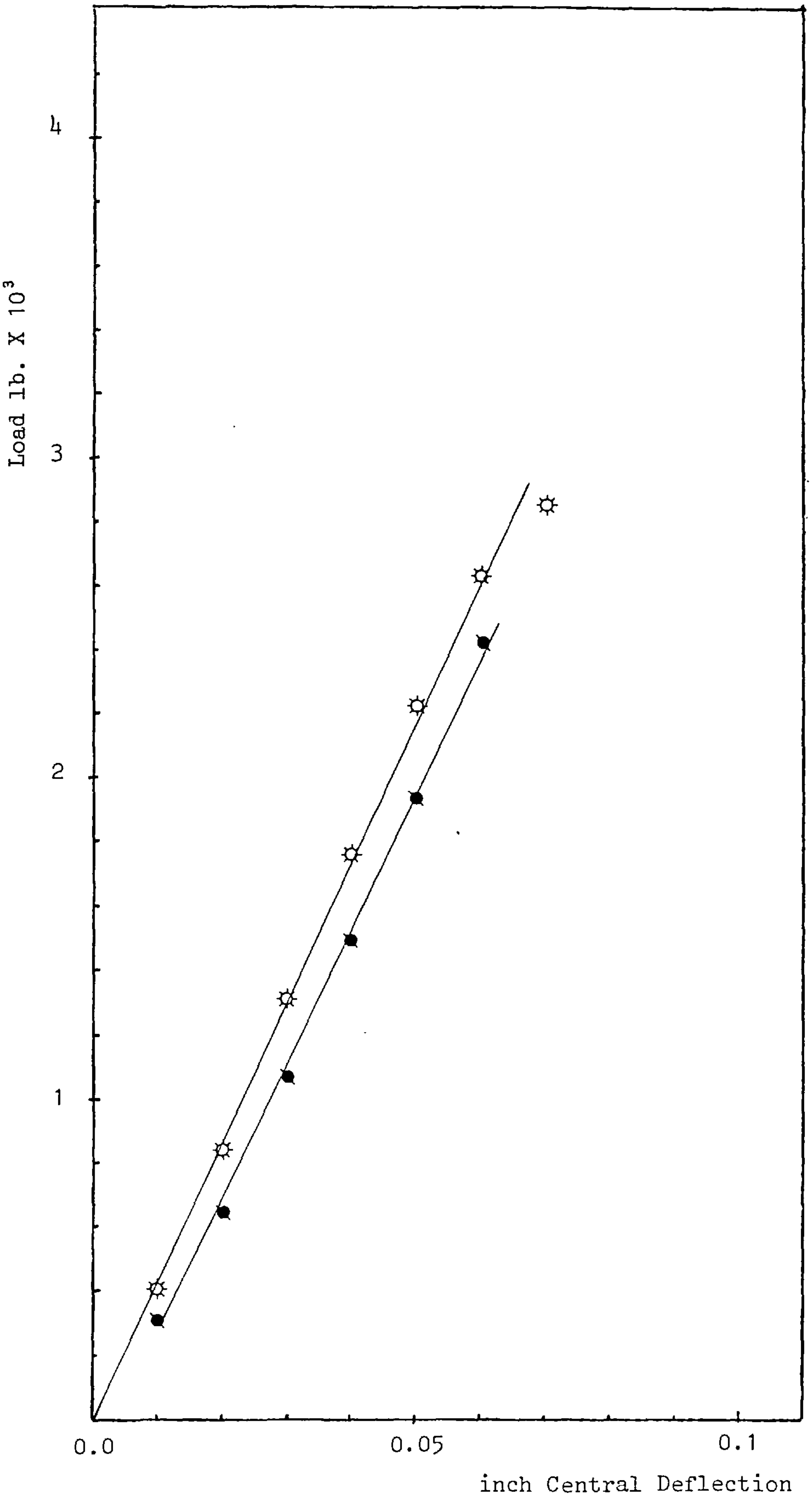


Fig. 5.18. Load vs. deflection in direction 2 - Layer B

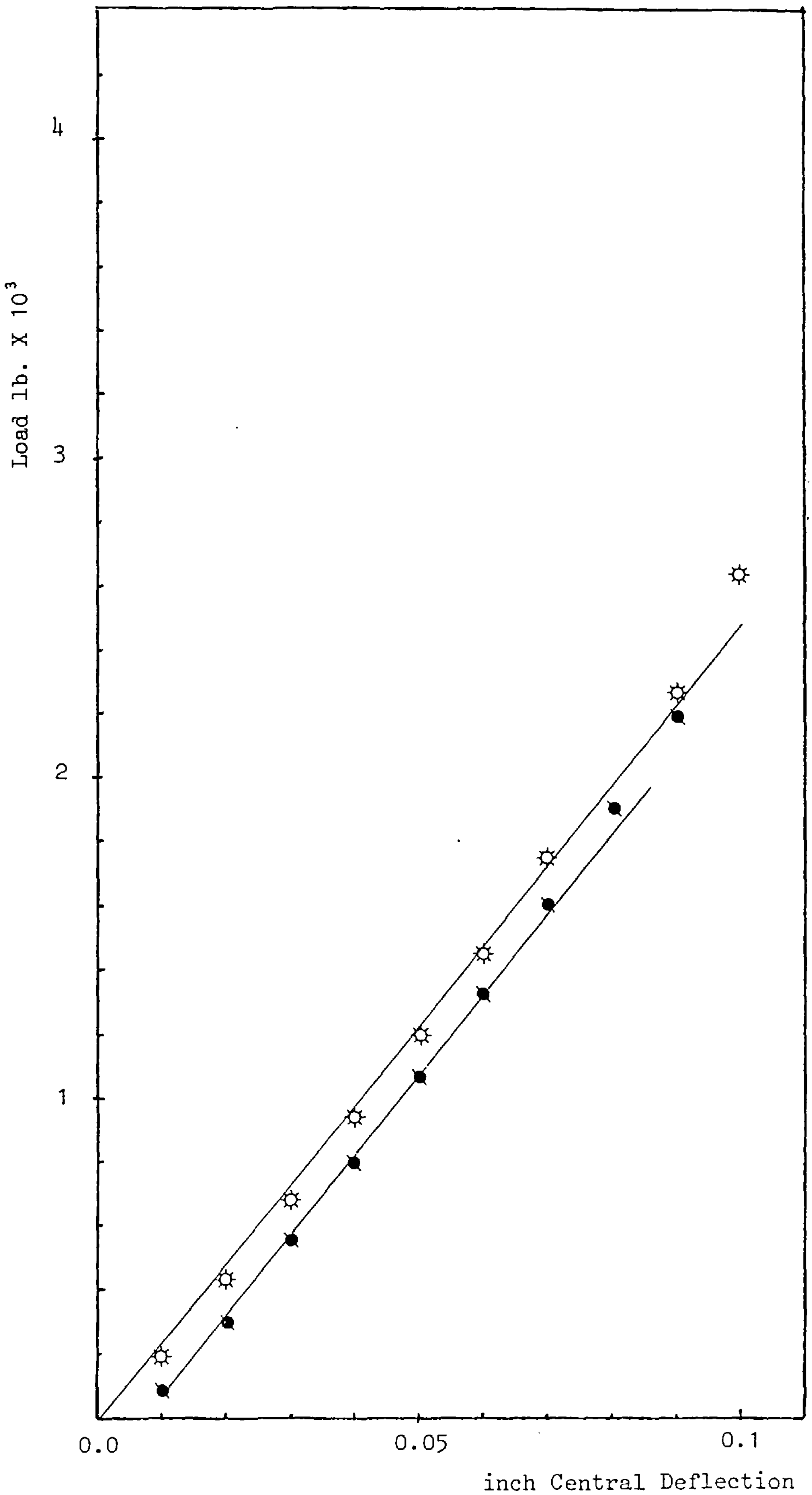


Fig. 5.19. Load vs. deflection in direction 1 - Layer C

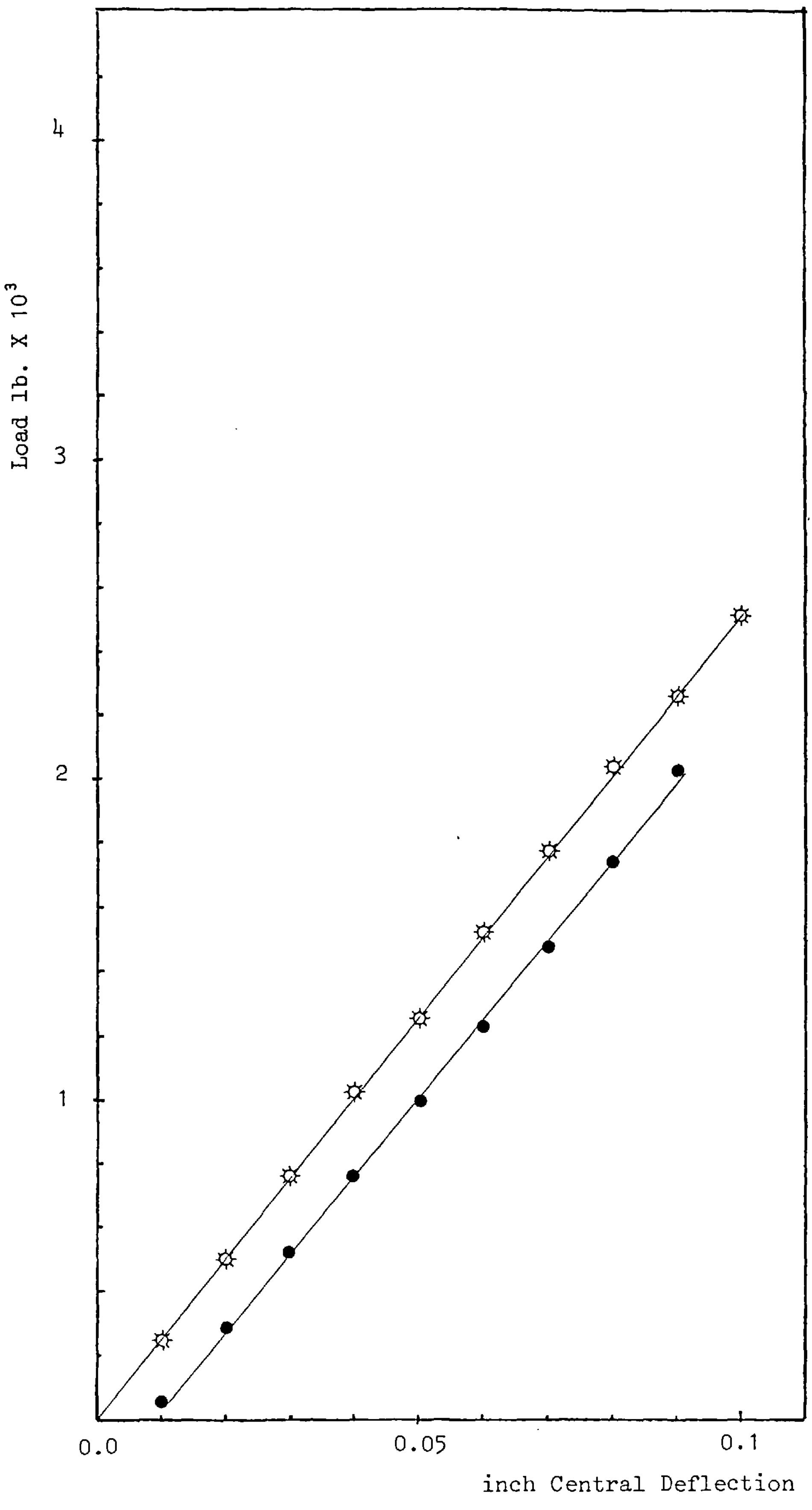


Fig. 5.20. Load vs. deflection in direction 2 - Layer C

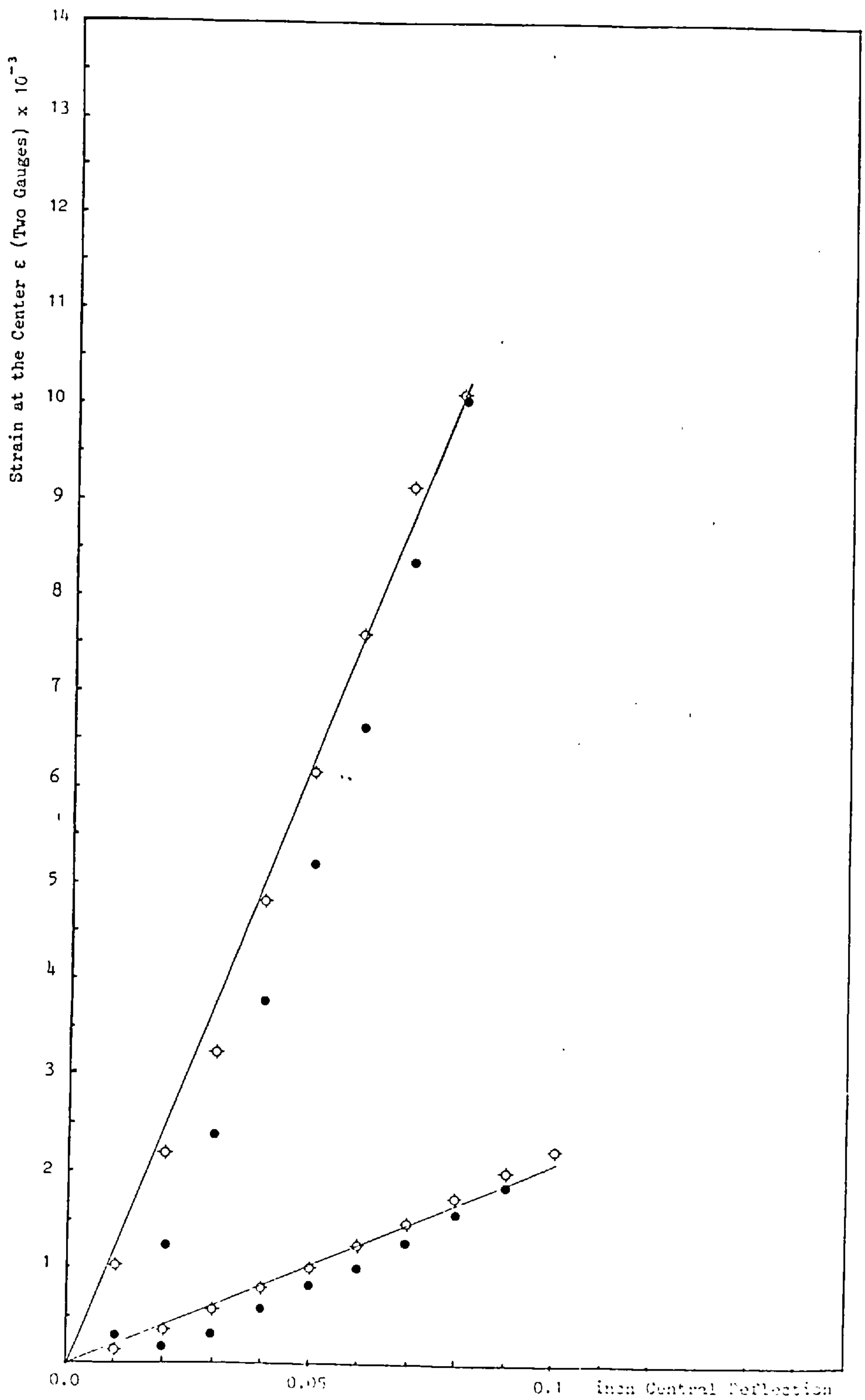


Fig. 5.21. Longitudinal & transverse strain vs. deflection direction 1 - Layer A.

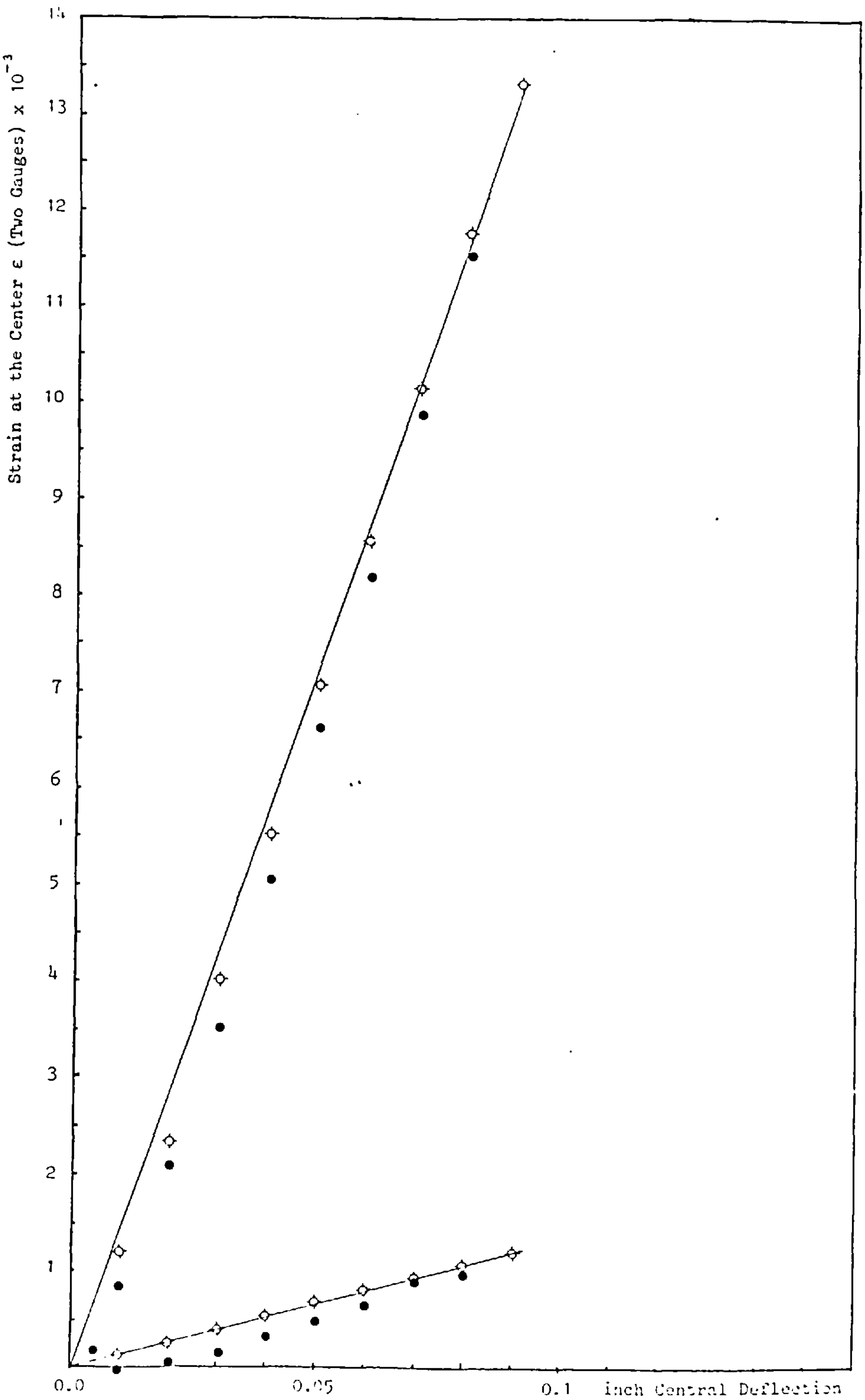


Fig. 5.22. Longitudinal & transverse strain vs. deflection direction 2 - Layer A.

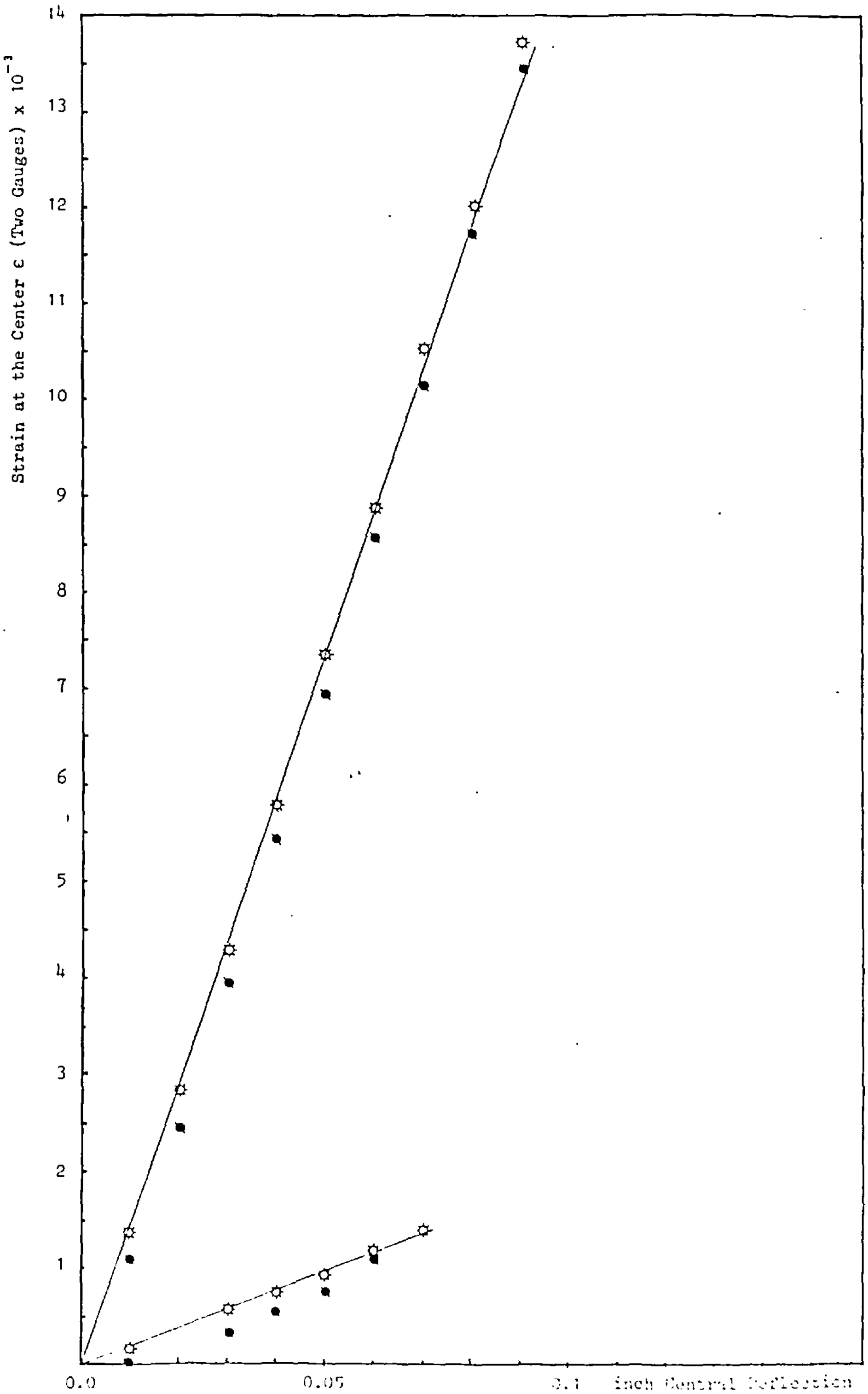


Fig. 5.23. Longitudinal & transverse strain vs. deflection direction 1 - Layer B.

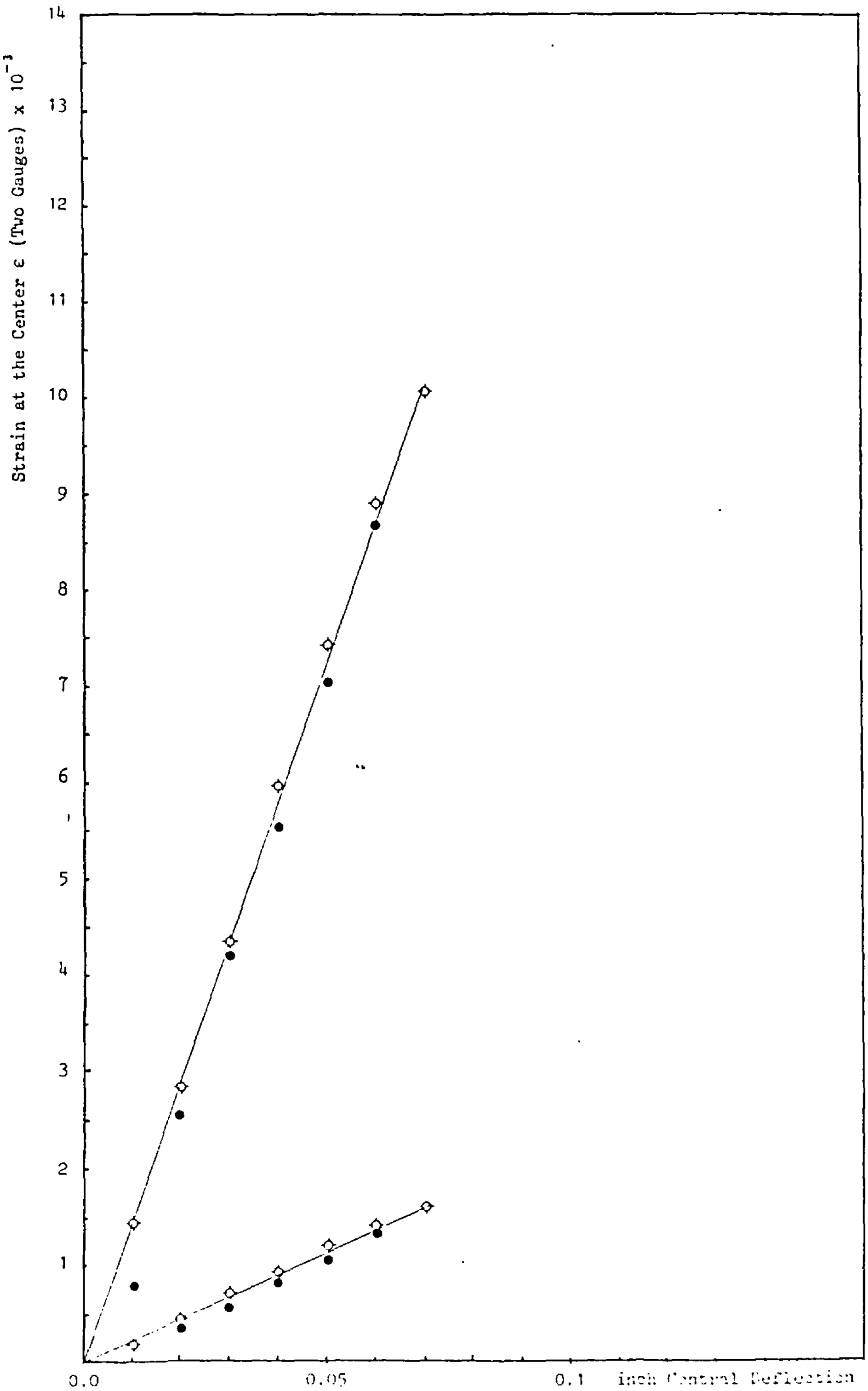


Fig. 5.24. Longitudinal & transverse strain vs. deflection direction 2 - Layer B

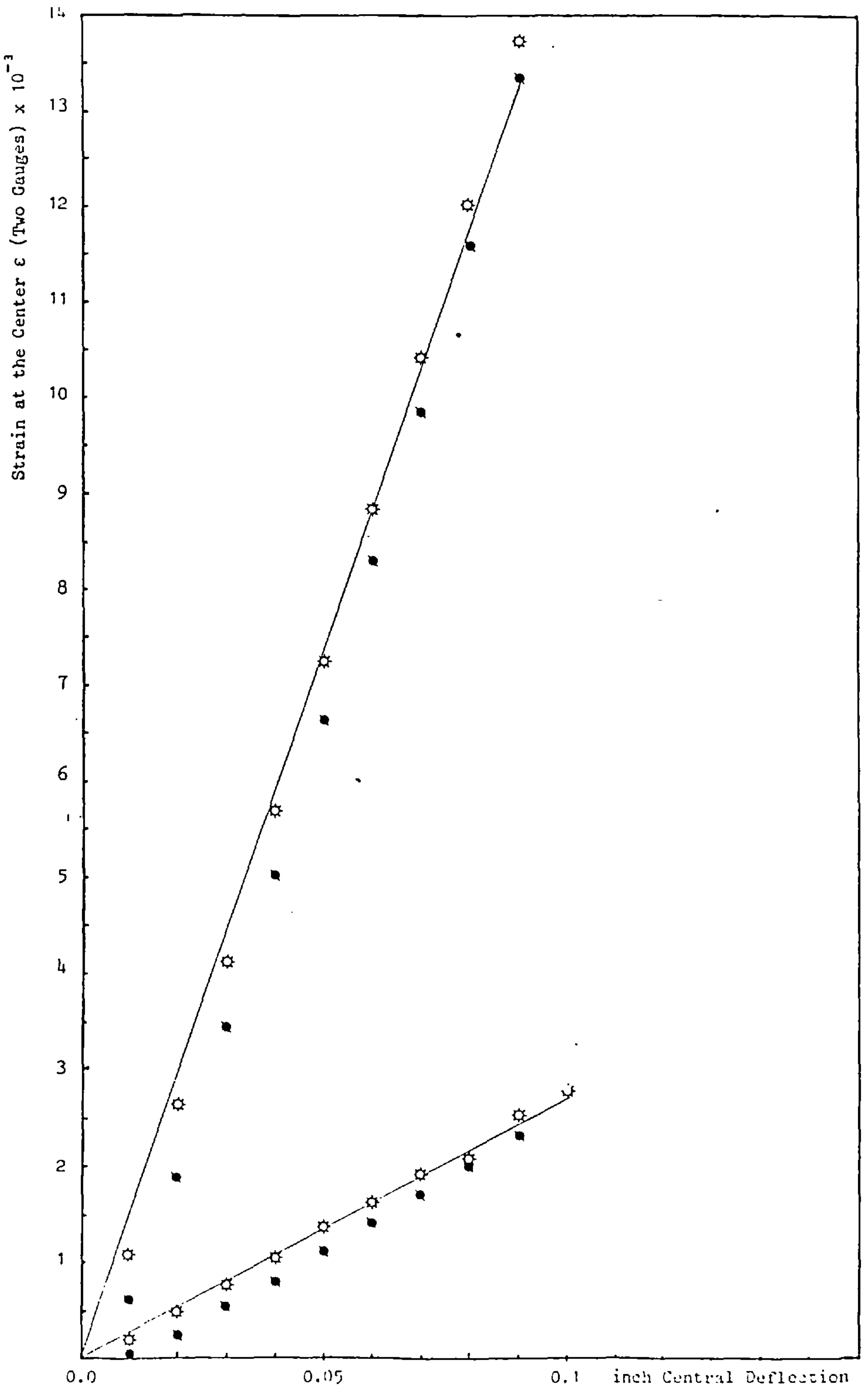


Fig. 5.25. Longitudinal & transverse strain vs. deflection direction 1 - Layer C.

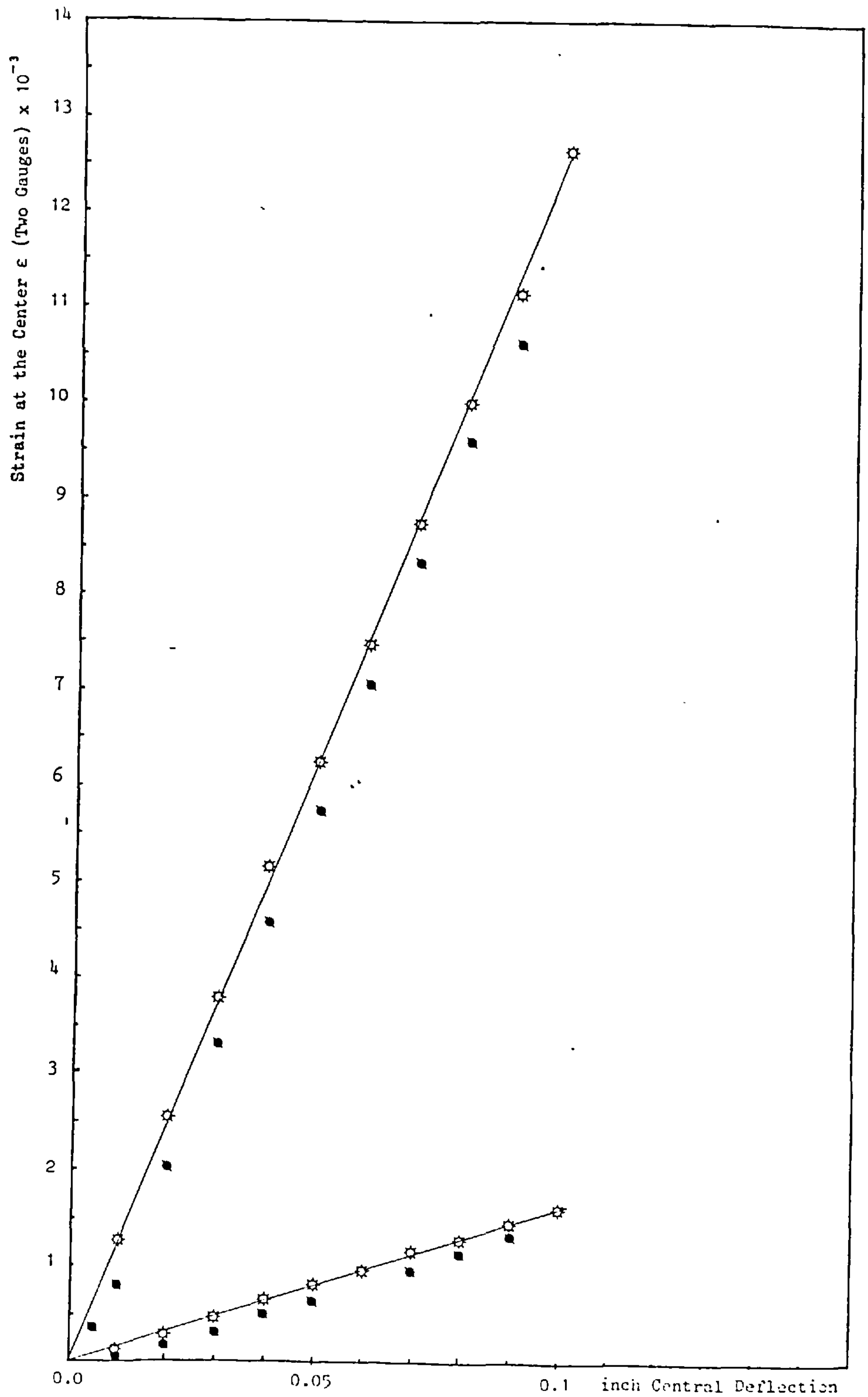


Fig. 5.26. Longitudinal & transverse strain vs. deflection direction 2 - Layer C.

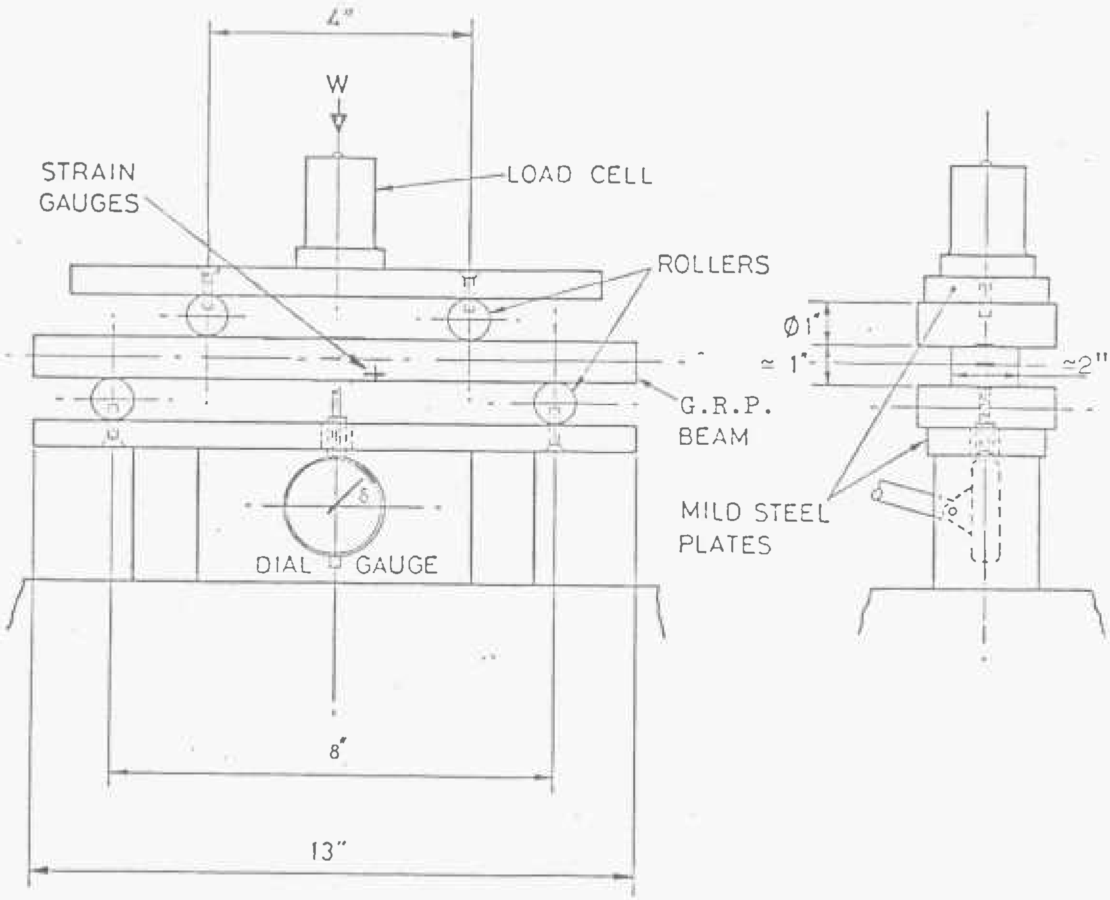


Fig. 5.27. 4-Point bend testing of G.R.P. beam for ν_{13} & ν_{23}

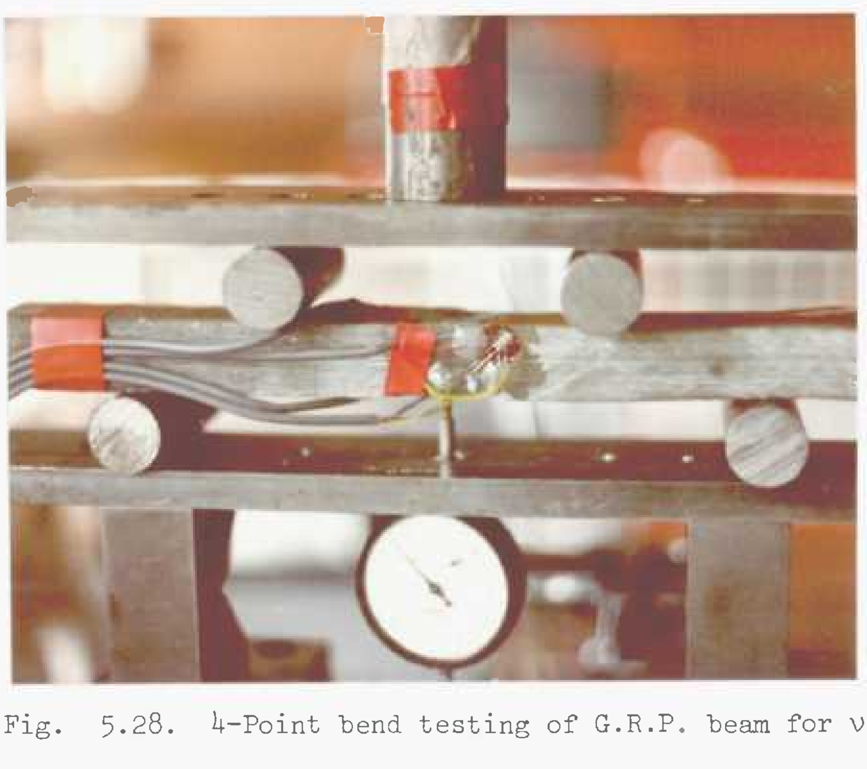


Fig. 5.28. 4-Point bend testing of G.R.P. beam for ν_{13} & ν_{23}

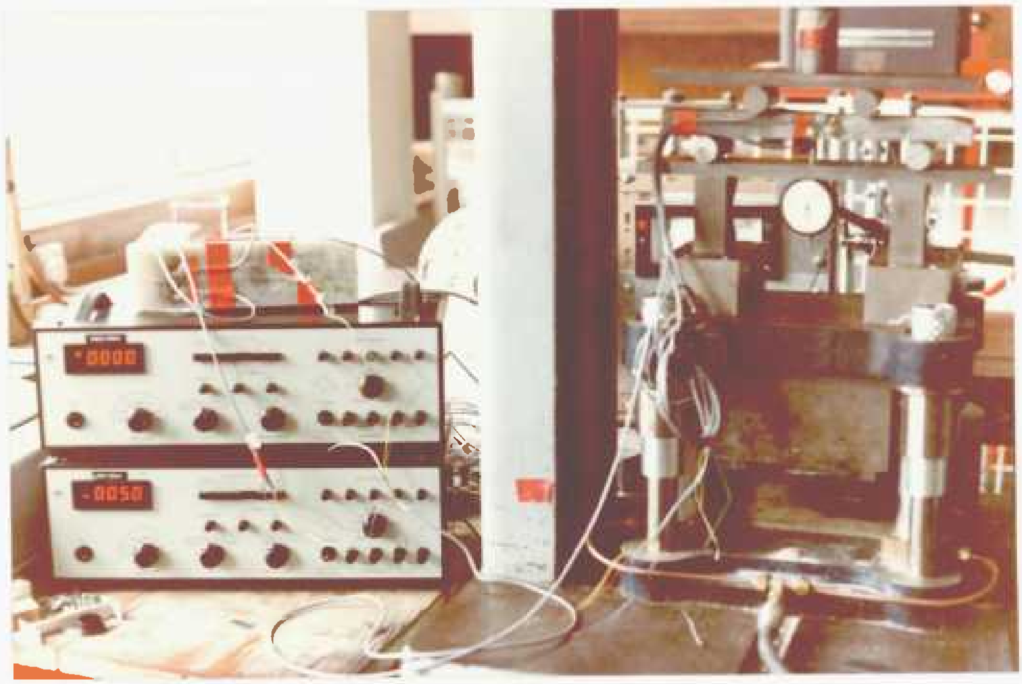


Fig. 5.29. 4-Point bend testing of G.R.P. beam for ν_{13} & ν_{23}

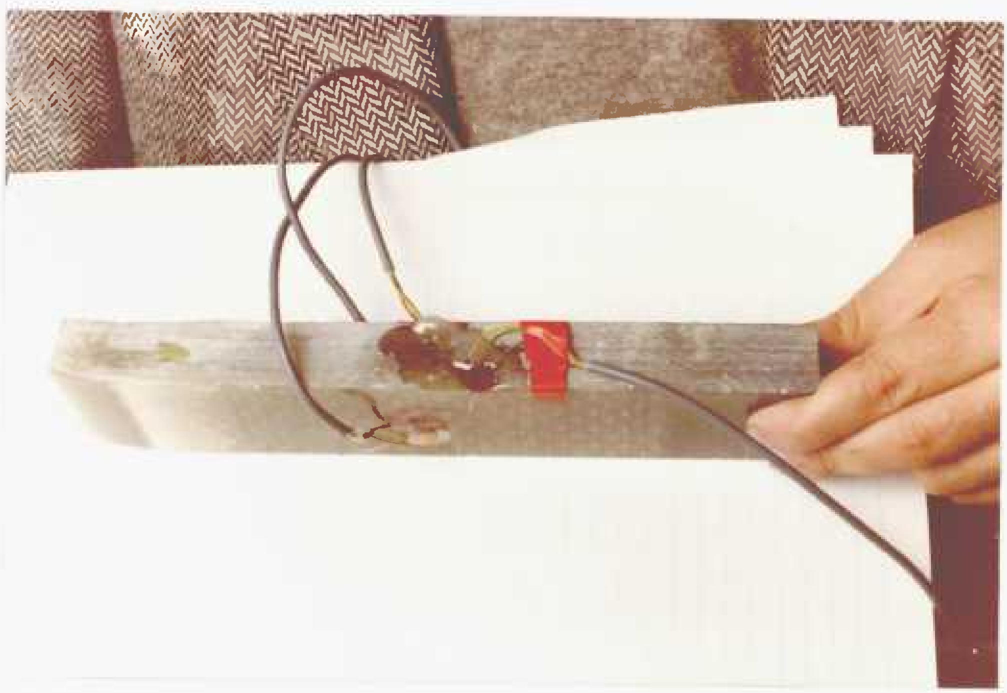


Fig. 5.30. Actual size of specimen.

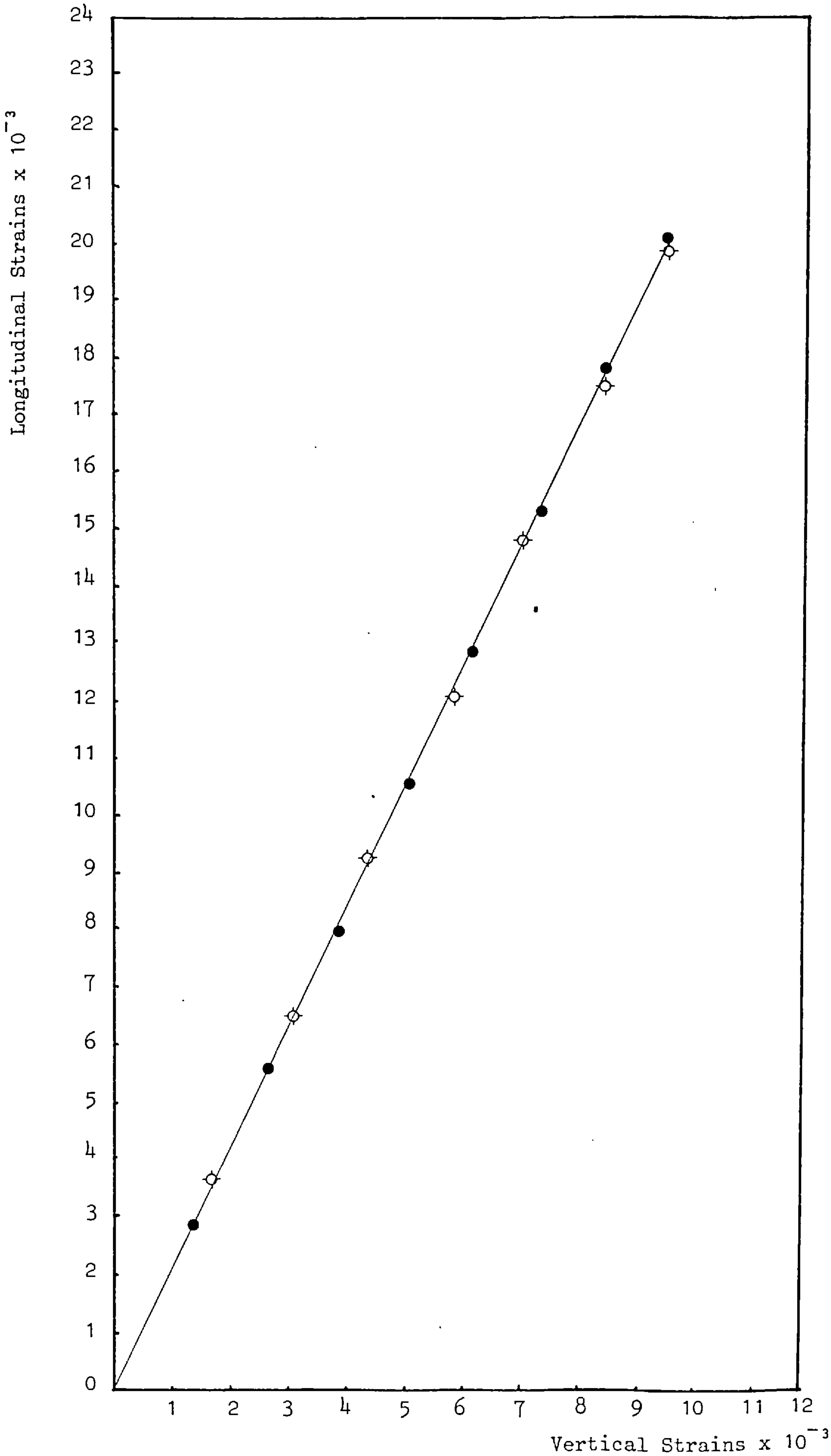


Fig. 5.31. Longitudinal vs. vertical strains in direction 1 - Layer A

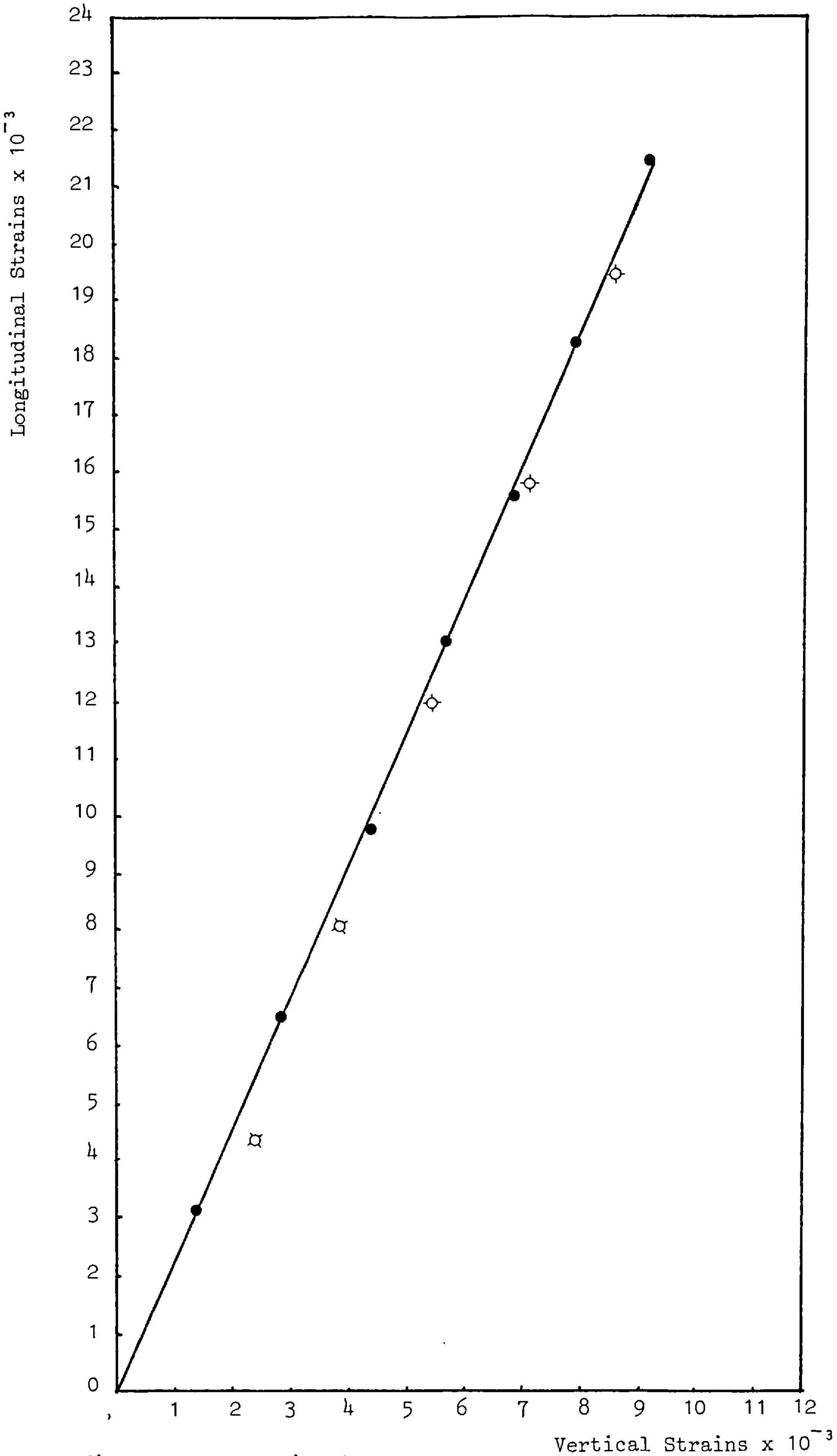


Fig. 5.32. Longitudinal vs. vertical strains in direction 2 - Layer A

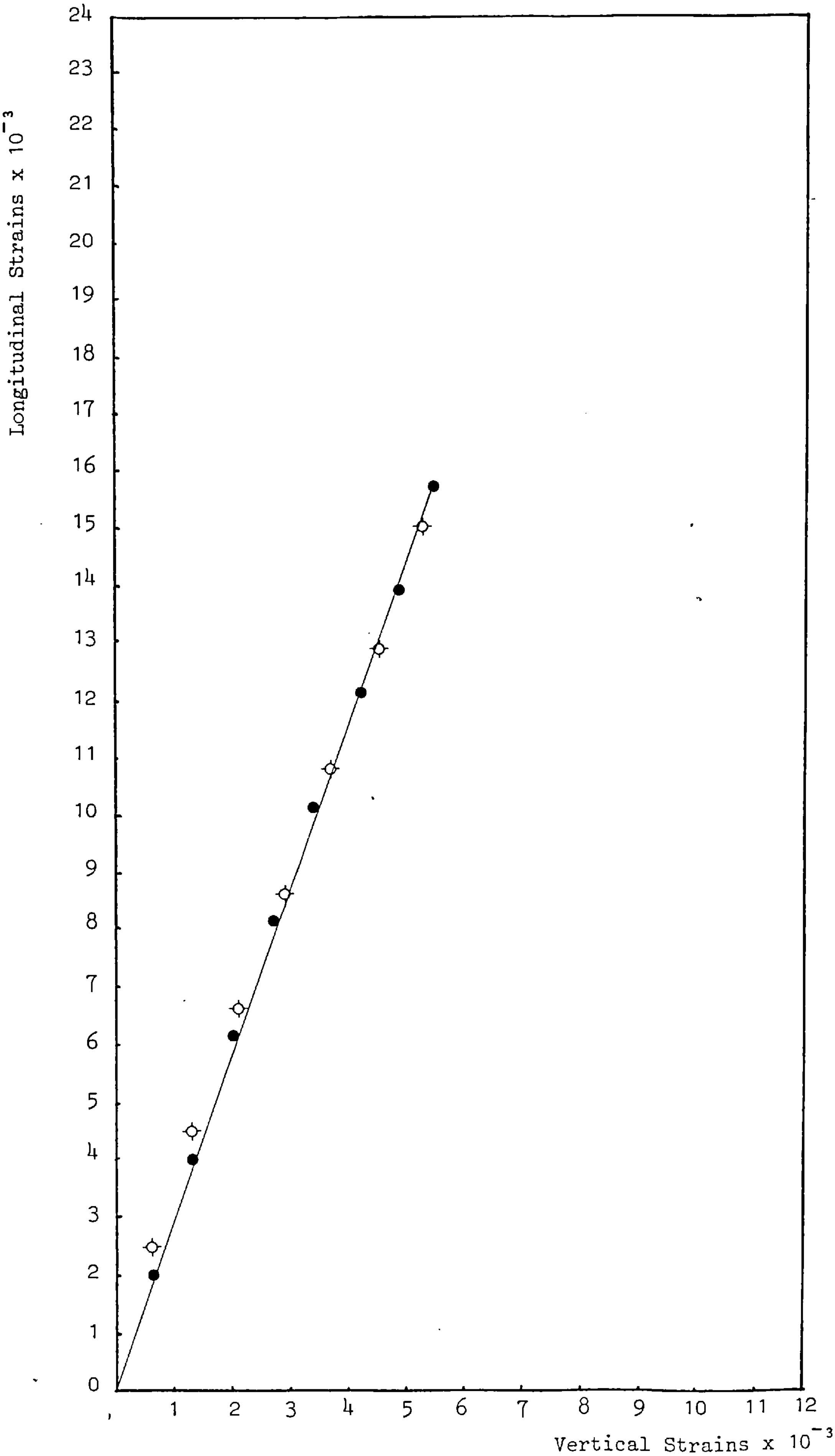


Fig. 5.33. Longitudinal vs. vertical strains in direction 1 - Layer B

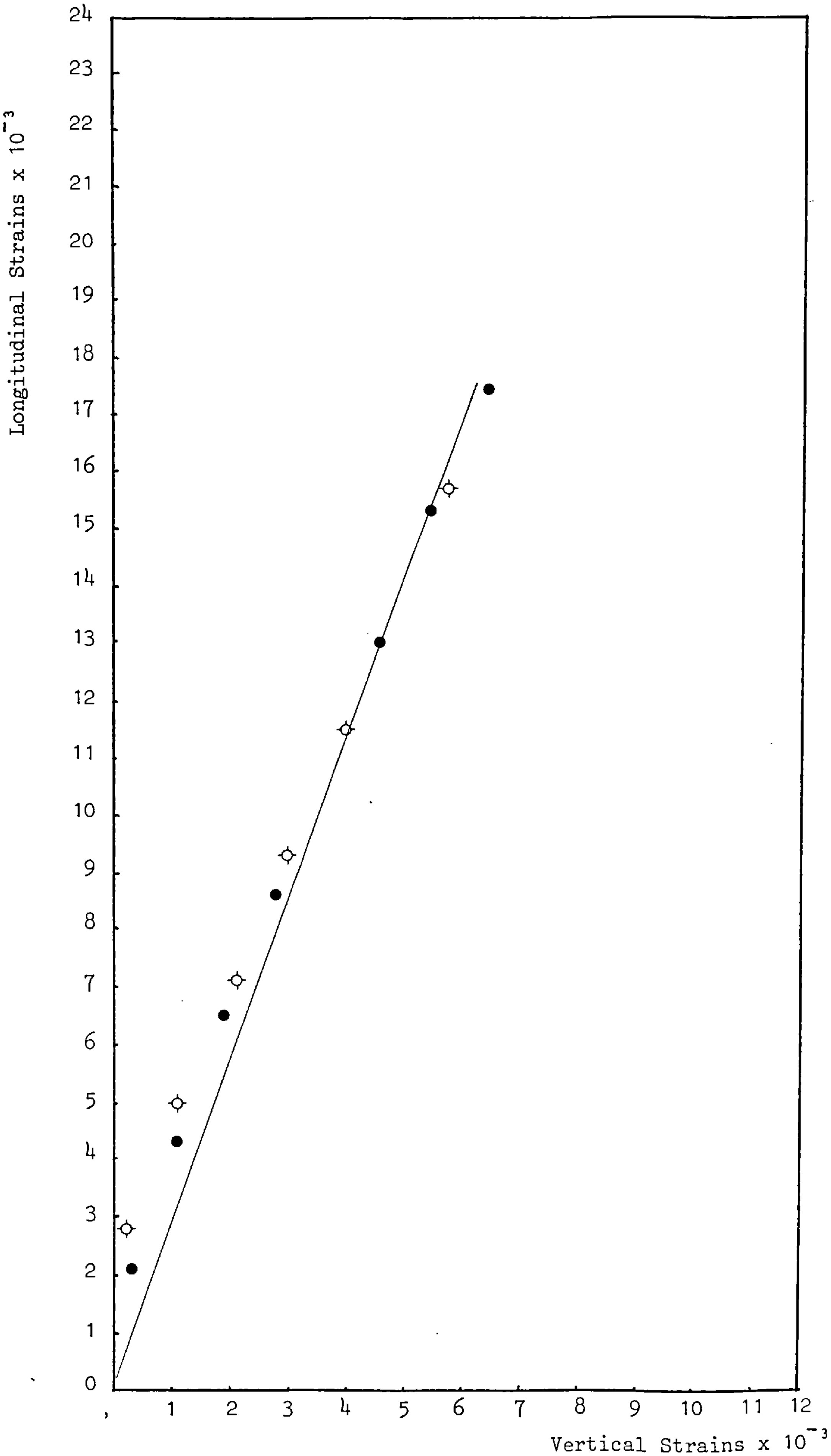


Fig. 5.34. Longitudinal vs. vertical strains in direction 2 - Layer B

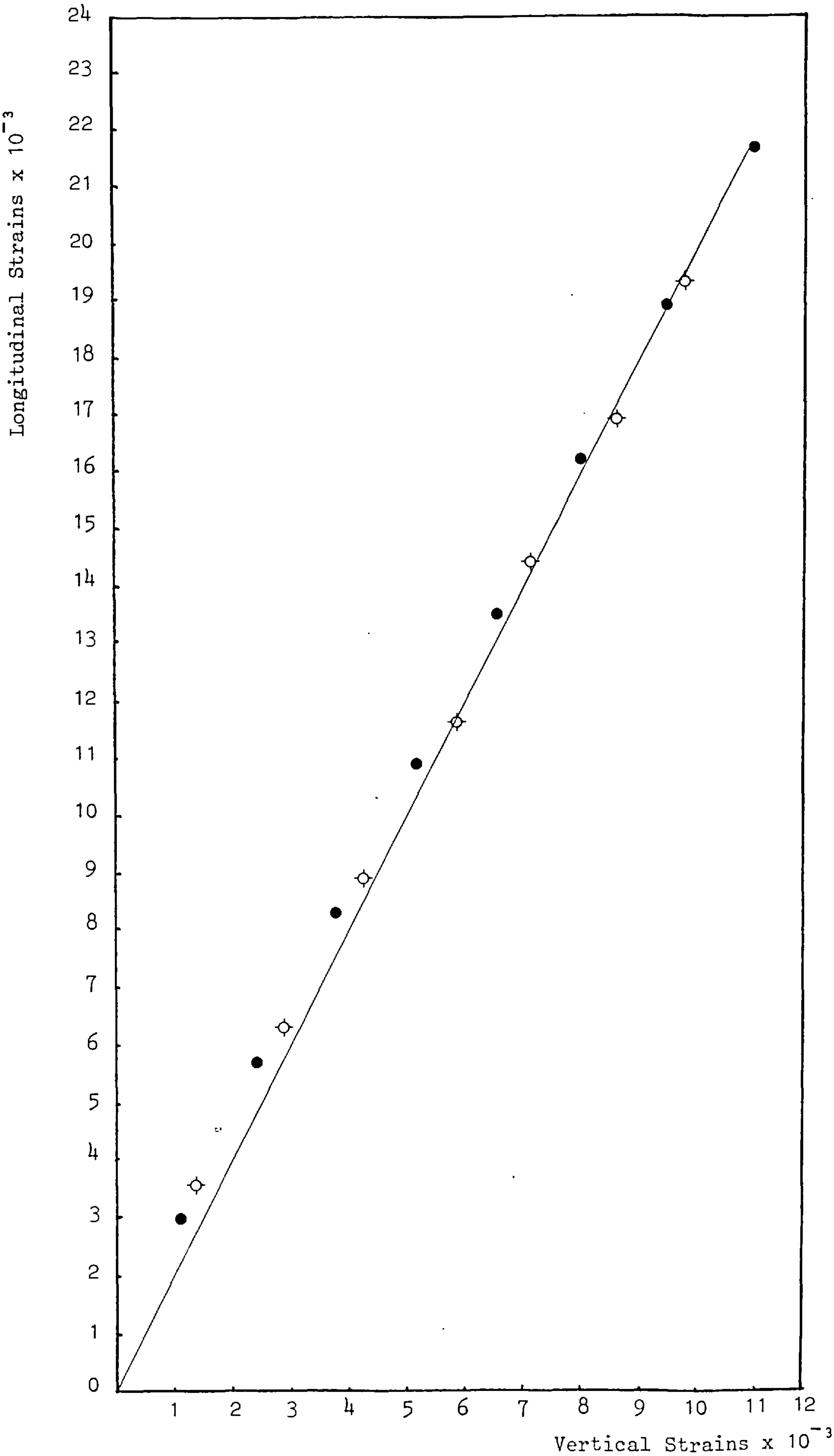


Fig. 5.35. Longitudinal vs. vertical strains in direction 1 - Layer C

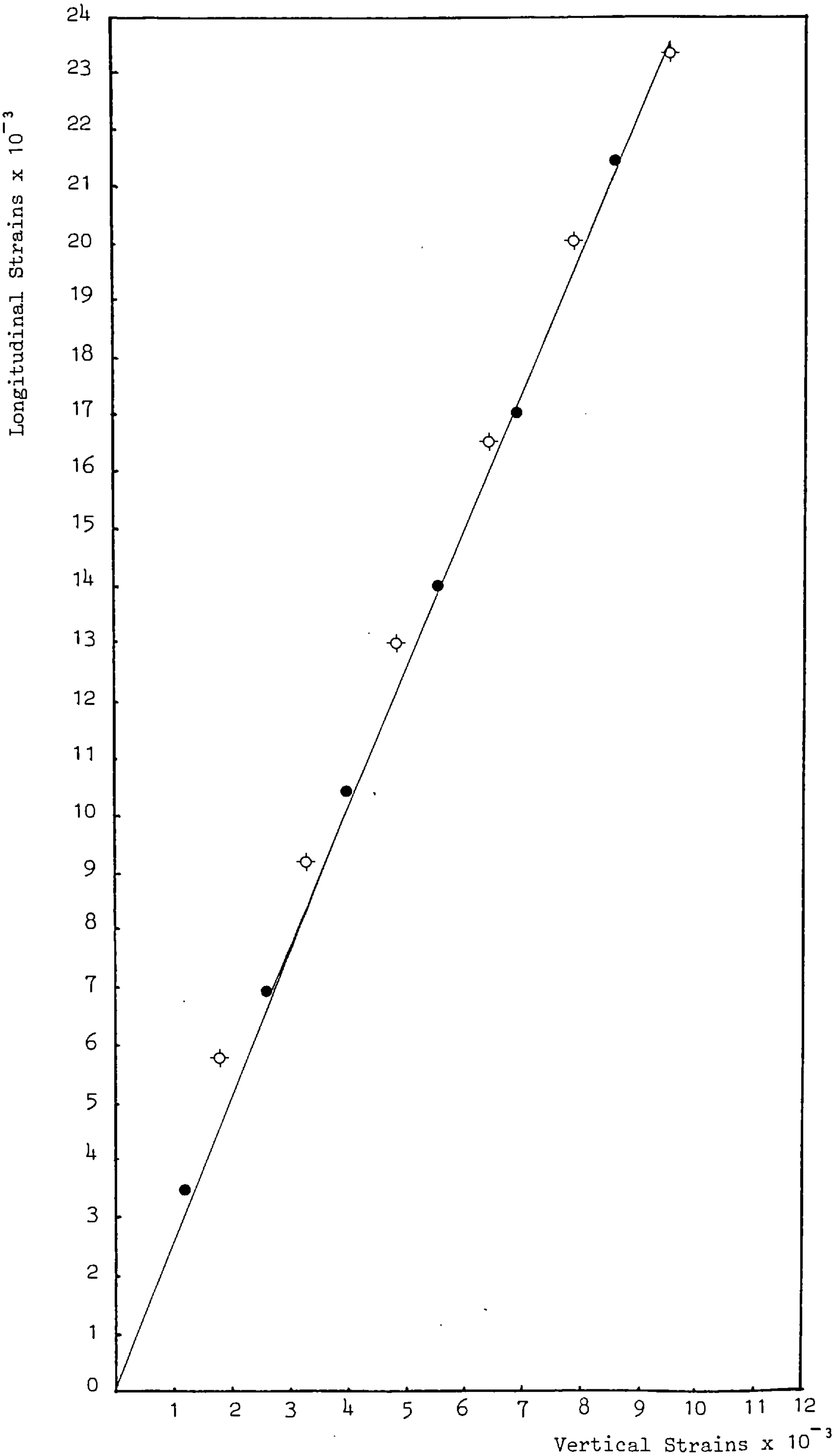


Fig. 5.36. Longitudinal vs. vertical strains in direction 2 - Layer C

Fig. 5.37. Minimum set of specimens

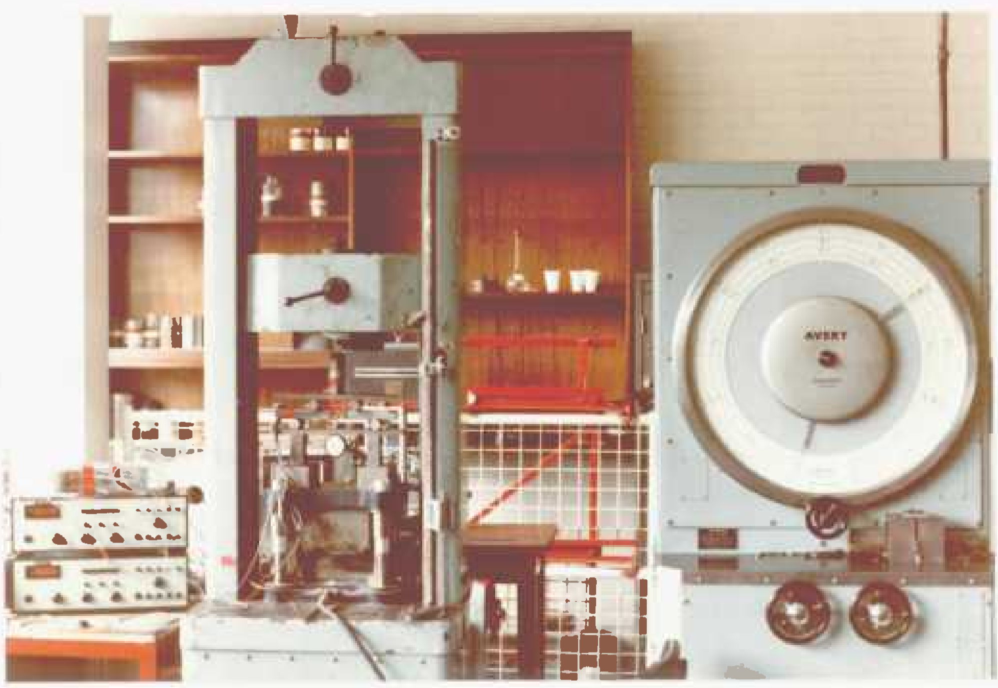


Fig. 5.38. Apparatus used for 4-point bend testing of beams.

5.3.3. Elastic constants of the specimens

Using Equation (2.104) with $\ell_1 = 4"$ and $\ell_2 = 2"$ as shown in Figs. (5.13) and (5.27) and Equation (2.105) gives:-

$$E = \frac{W}{\delta} \times \frac{22}{3} \times \frac{12}{b\ d^3} \text{ lb. in.}$$

and measuring all specimens accurately to ± 0.001", E_{33} was assumed to be lower than E_{11} and E_{22} and taken to be 1×10^6 psi as suggested by a reliable manufacturer. G_{12} was assumed to be 2×10^5 psi and G_{13} and G_{23} were assumed to be the same proportions as the relevant elastic moduli.

(i) For Layer A

From Fig. (5.15) $E_{11} = 1.274 \times 10^6$ psi ± $7\frac{1}{2}\%$

From Fig. (5.16) $E_{22} = 1.565 \times 10^6$ psi ± $7\frac{1}{2}\%$

From Fig. (5.22) $\nu_{12} = .163 \pm 5\%$

From Fig. (5.23) $\nu_{21} = 0.99 \pm 4\%$

From Fig. (5.31) $\nu_{13} = .48 \pm 2\%$

From Fig. (5.32) $\nu_{23} = .438 \pm 2\%$

E_{33} was assumed to be 1×10^6 psi,

G_{12} was assumed to be 2×10^5 psi,

G_{13} was assumed to be 1.3×10^5 psi,

G_{23} was assumed to be 1.6×10^5 psi.

(ii) For Layer B

From Fig. (5.17) $E_{11} = 1.62 \times 10^{-6}$ psi ± $7\frac{1}{2}\%$

From Fig. (5.18) $E_{22} = 1.8 \times 10^{-6}$ psi ± $7\frac{1}{2}\%$

From Fig. (5.23) $\nu_{12} = .159 \pm 3\%$

From Fig. (5.24) $\nu_{21} = .128 \pm 2\%$

From Fig. (5.33) $\nu_{13} = .325 \pm 2\%$

From Fig. (5.34) $\nu_{23} = .29 \pm 2\%$

E_{33} was assumed to be 1×10^6 psi,

G_{12} was assumed to be 2×10^5 psi,

G_{13} was assumed to be 1.11×10^5 psi,

G_{23} was assumed to be 1.68×10^5 psi.

(iii) For Layer C

From Fig. (5.19) $E_{11} = 1.62 \times 10^6$ psi $\pm$ 6%

From Fig. (5.20) $E_{22} = 1.66 \times 10^6$ psi $\pm$ 6%

From Fig. (5.25) $\nu_{12} = .128 \pm 1\%$

From Fig. (5.26) $\nu_{21} = .128 \pm 1\%$

From Fig. (5.35) $\nu_{13} = .49 \pm 1\%$

From Fig. (5.36) $\nu_{23} = .40 \pm 2\%$

E_{33} was assumed to be 1×10^6 psi,

G_{12} was assumed to be 2×10^5 psi,

G_{13} was assumed to be 1.2×10^5 psi,

G_{23} was assumed to be 1.95×10^5 psi.

5.3.4. Finite Element Idealization of G.R.P. Test Beam

A finite element mesh as shown in Fig. (5.39) was arranged to suit the different average thicknesses of all three layers, to suit the supporting arrangement of the test, the location of the strain gauges on the test beam and to allow the application of a load off the centre so as to cause a complicated case of torsional bending as well as pure bending. The mesh consisted of 144 elements and 260 nodes. The properties of each element were read individually as E_{11}, E_{22}, E_{33}, ν_{12}, ν_{13}, ν_{23}, G_{12}, G_{13}, and G_{23}. ν_{21} was used to check the accuracy of the experimental results.

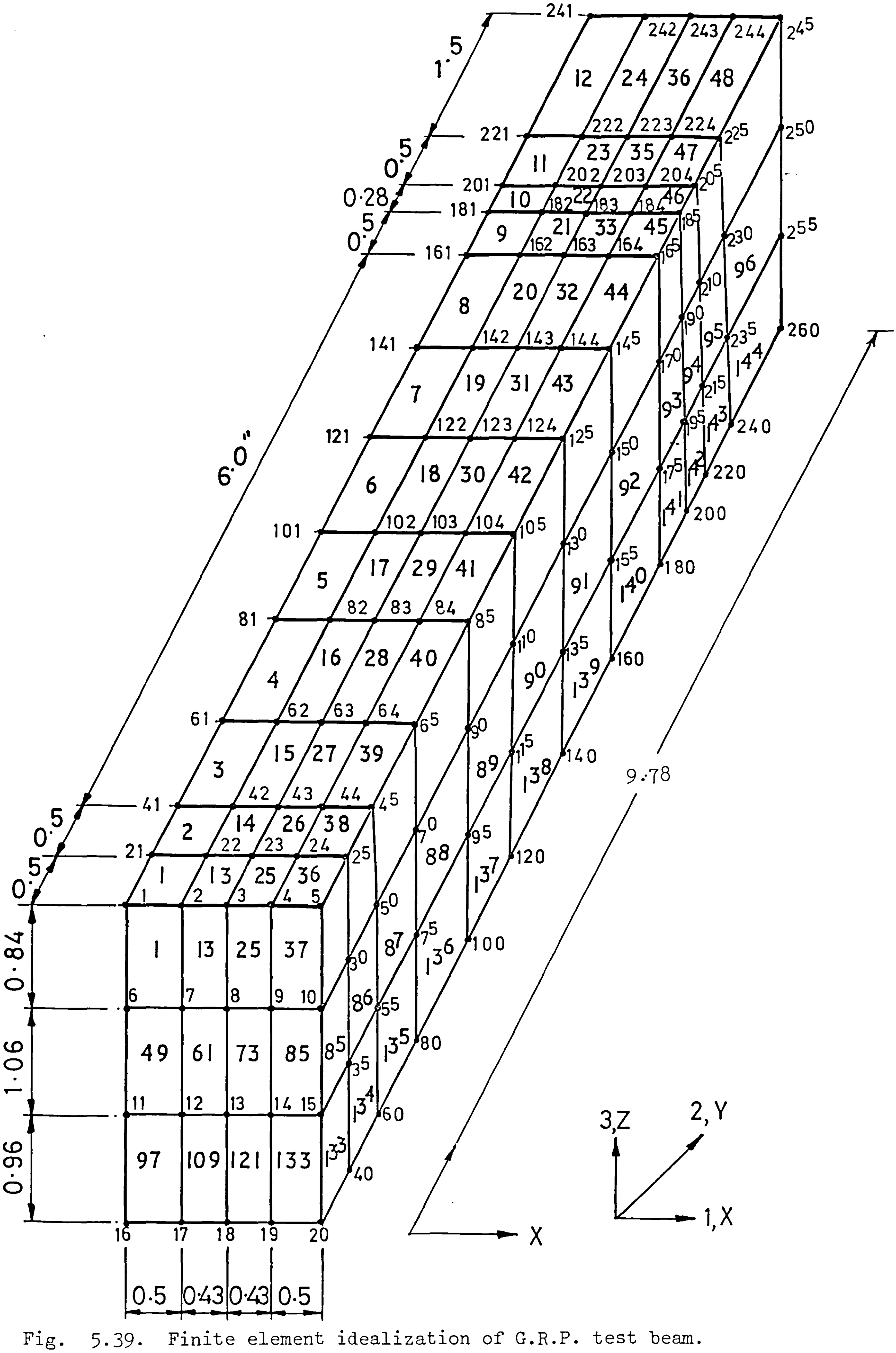


Fig. 5.39. Finite element idealization of G.R.P. test beam.

The program would give values for ν_{21}, ν_{31}, ν_{32} using Equations (3.14a) (3.14b) and (3.14c). In calculating stresses the average of the values from all elements adjacent to the node is used. If the node is at the interface of two layers, the average of the elements in one layer is calculated separately from those elements meeting at the node from another layer.

5.3.5. Experimental Work on G.R.P. Test Beam

(i) Test specimen

The test on the multilayered beam from the end Block D, No. D4 is described in this section. Its measured dimensions were 2.86" x 1.86" x 9.78", considering the average thickness of Layer A to be 0.84", Layer B - 1.06" and Layer C - 0.98".

Six rosette strain gauges type FRA-6-11 made by T.S.K. Ltd. Gauge resistance 120 ± 0.5Ω, gauge length 6mm were stuck 0.5" away from the support as shown in Figs. (5.40a), (5.40b) and (5.41) and their location on the finite element mesh is shown in Fig. (5.44). The gauges were located near the support and a cantilever arrangement was adopted to give higher and more readable strains from the equipment available.

The upper surface of the specimen had to be filed smooth to stick rosette No. 1 which reduced the thickness of Layer A by 0.14" for a length of 0.78".

The specimen was clamped to a heavy table as shown in Figs. (5.41), (5.42) and (5.43). The loads were applied through a free hook into a shallow dent in the upper surface of the

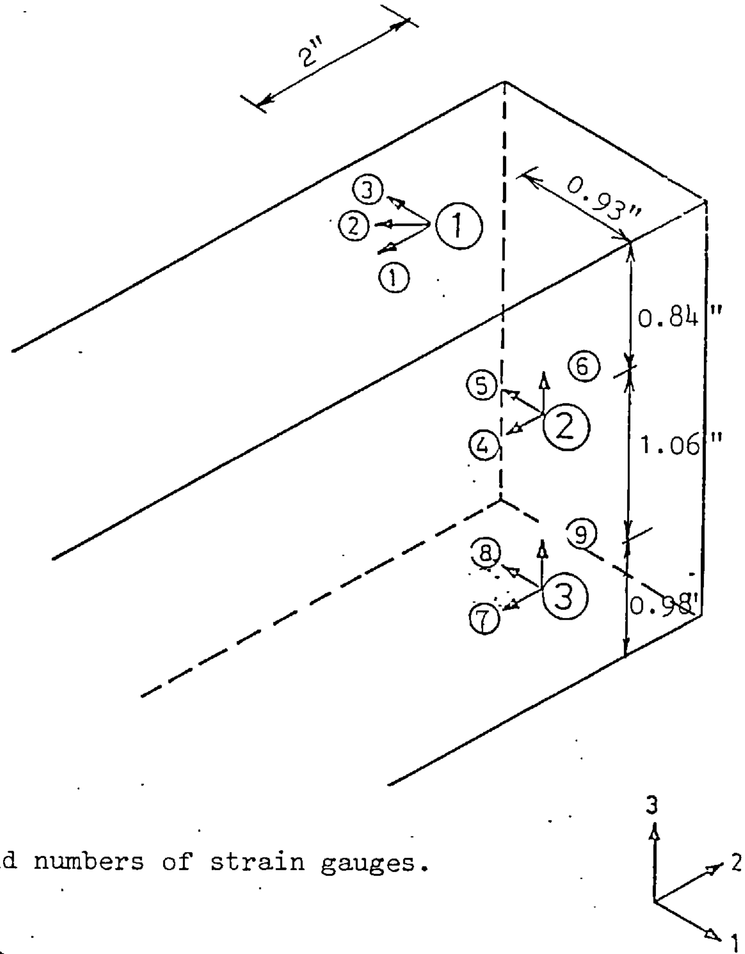


Fig. 5.40a. Location and numbers of strain gauges.

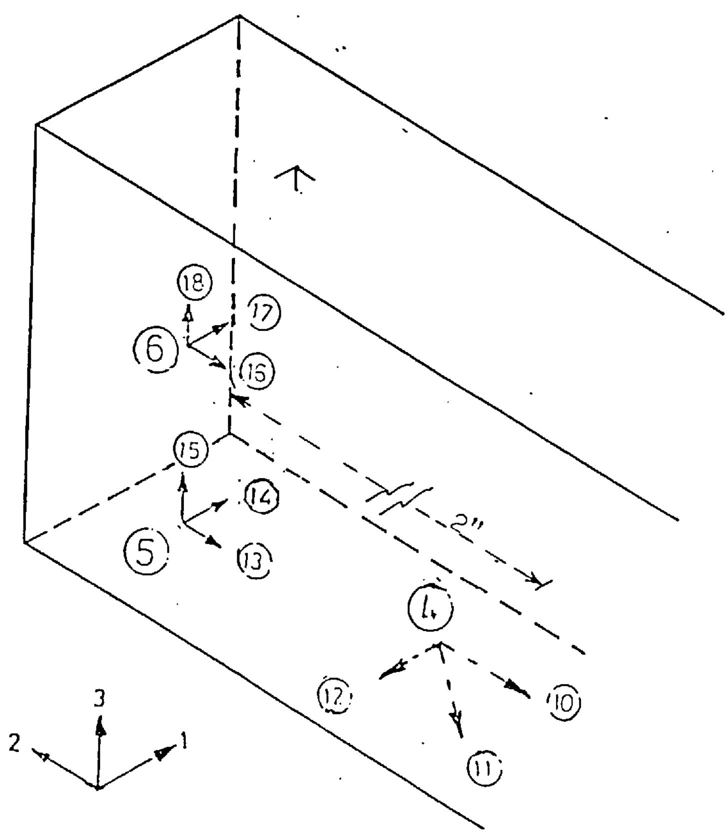


Fig. 5.40b. Location and numbers of strain gauges.

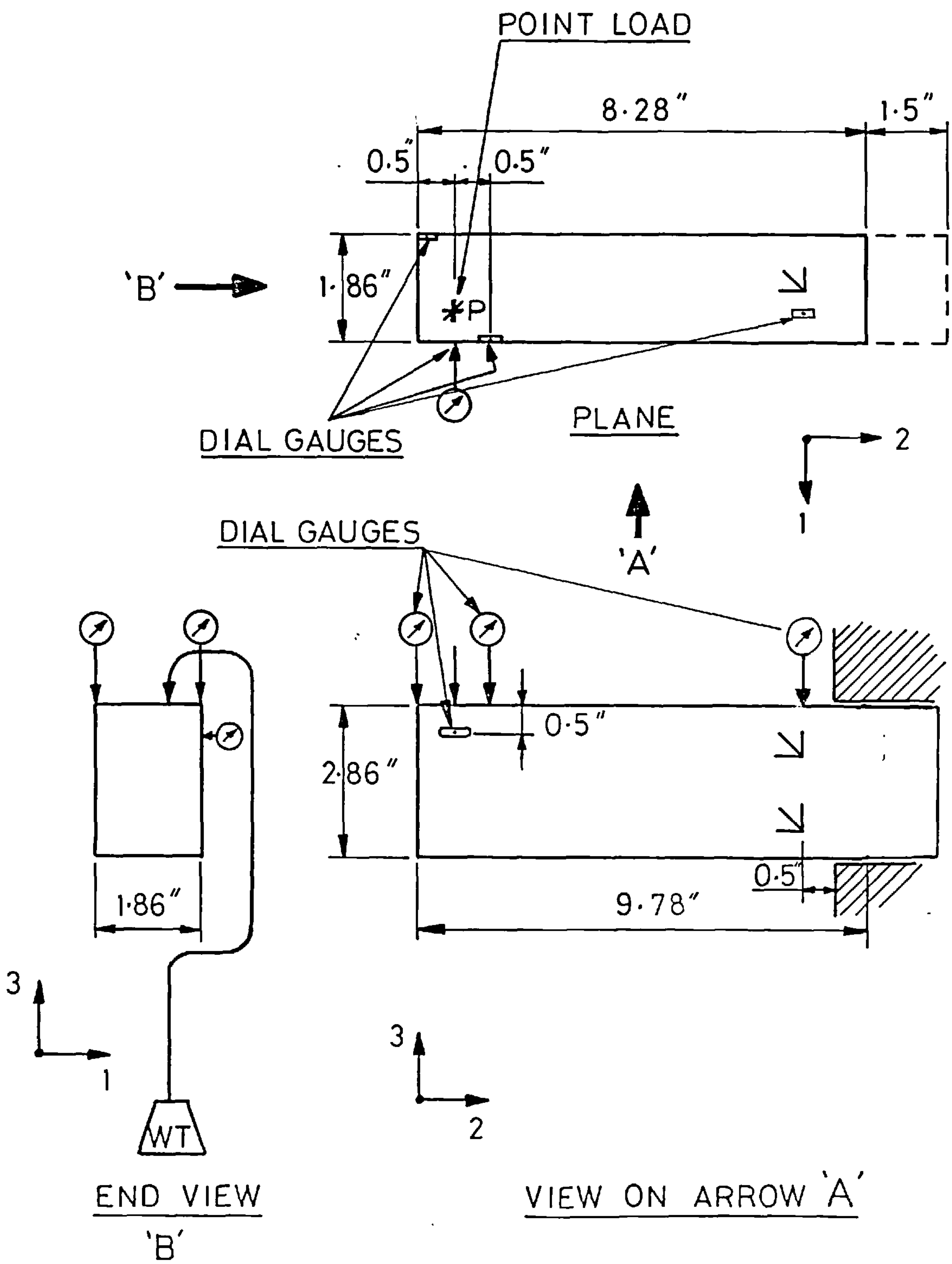


Fig. 5.41. Experiment arrangement of G.R.P. test beam.

Fig. 5.42. Experiment arrangement of G.R.P. test beam.

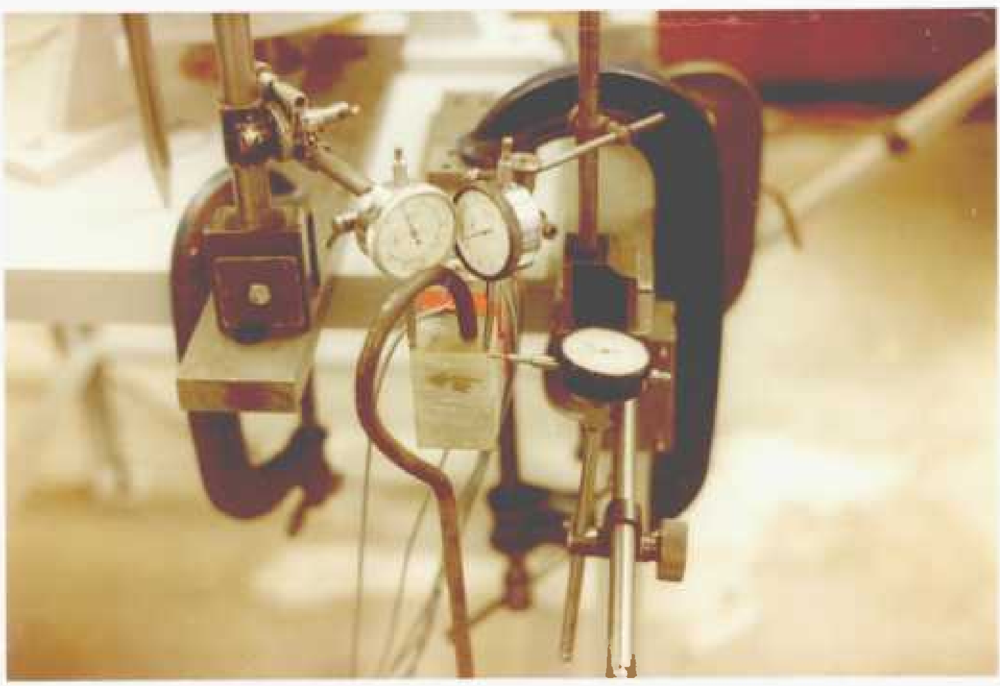


Fig. 5.43. Experiment arrangement of G.R.P. test beam.

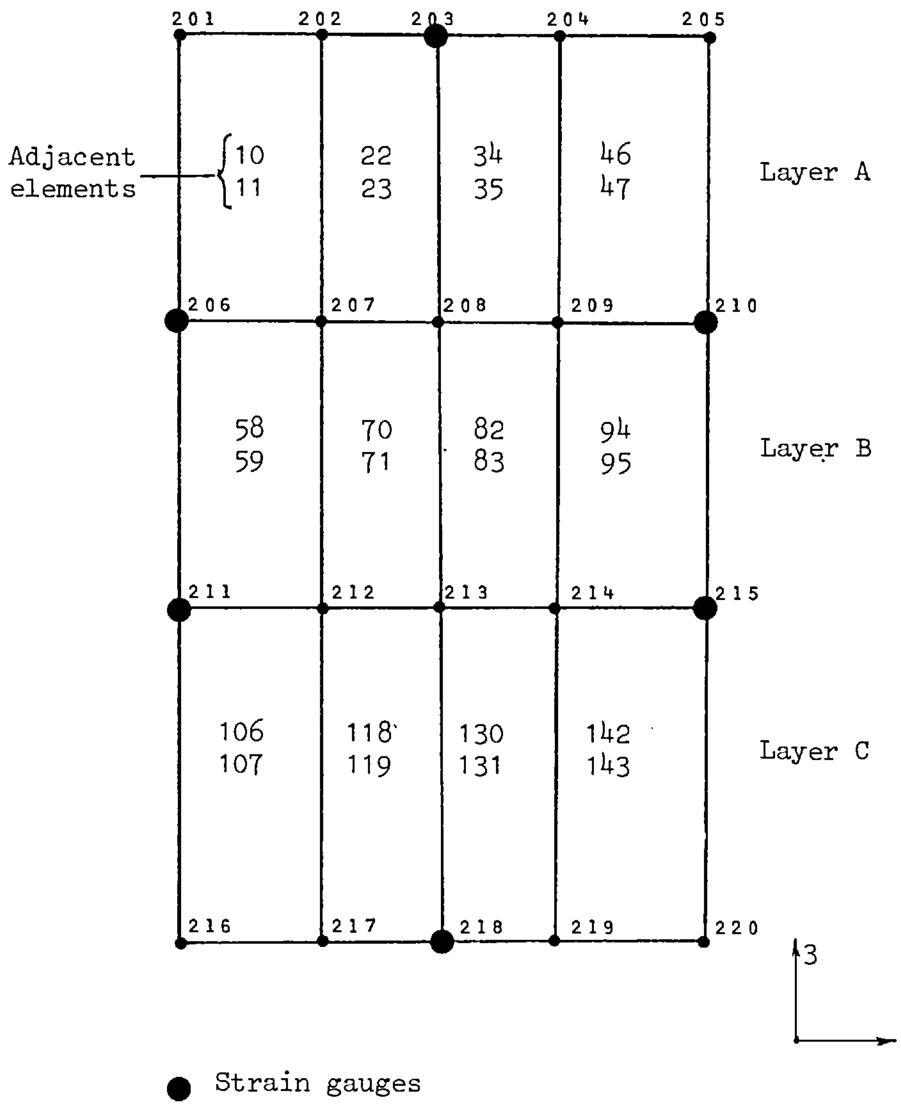


Fig. 5.44. Gauge locations on the finite element mesh.

specimen 0.5" from both edges equivalent to Node 24 on the finite element mesh, so as to introduce combined torsion and bending.

(ii) Testing procedure

(a) Stresses

The 18 gauges from the six rosettes were arranged with 18 dummy gauges from six similar gauges stuck on other specimens of the same material so that each active gauge and the corresponding dummy gauge were connected to a channel of a Brueland and Kjaer switching unit type 1542 from which they were arranged in a half bridge and fed with 3.0V through a Brueland and Kjaer strain gauge amplifier type 1526. The output was fed back to the amplifier which indicated the total strains in microstrains on a L.E.D. display. The highest strain recorded was 630 microstrains with the accuracy of the equipment of ± 30 microstrains.

The stresses were calculated using the first terms of Equations (3.13a), (3.13b) and (3.13c).

The equivalent stresses to those occurring at the corner of the relevant elements in the relevant layer were plotted on Figs. (5.45), (5.46), (5.47), (5.48), (5.49), (5.50), (5.51), (5.52), (5.53), (5.54), (5.55) and (5.56) against the results from the finite element solution. As can be seen the results were very satisfactory.

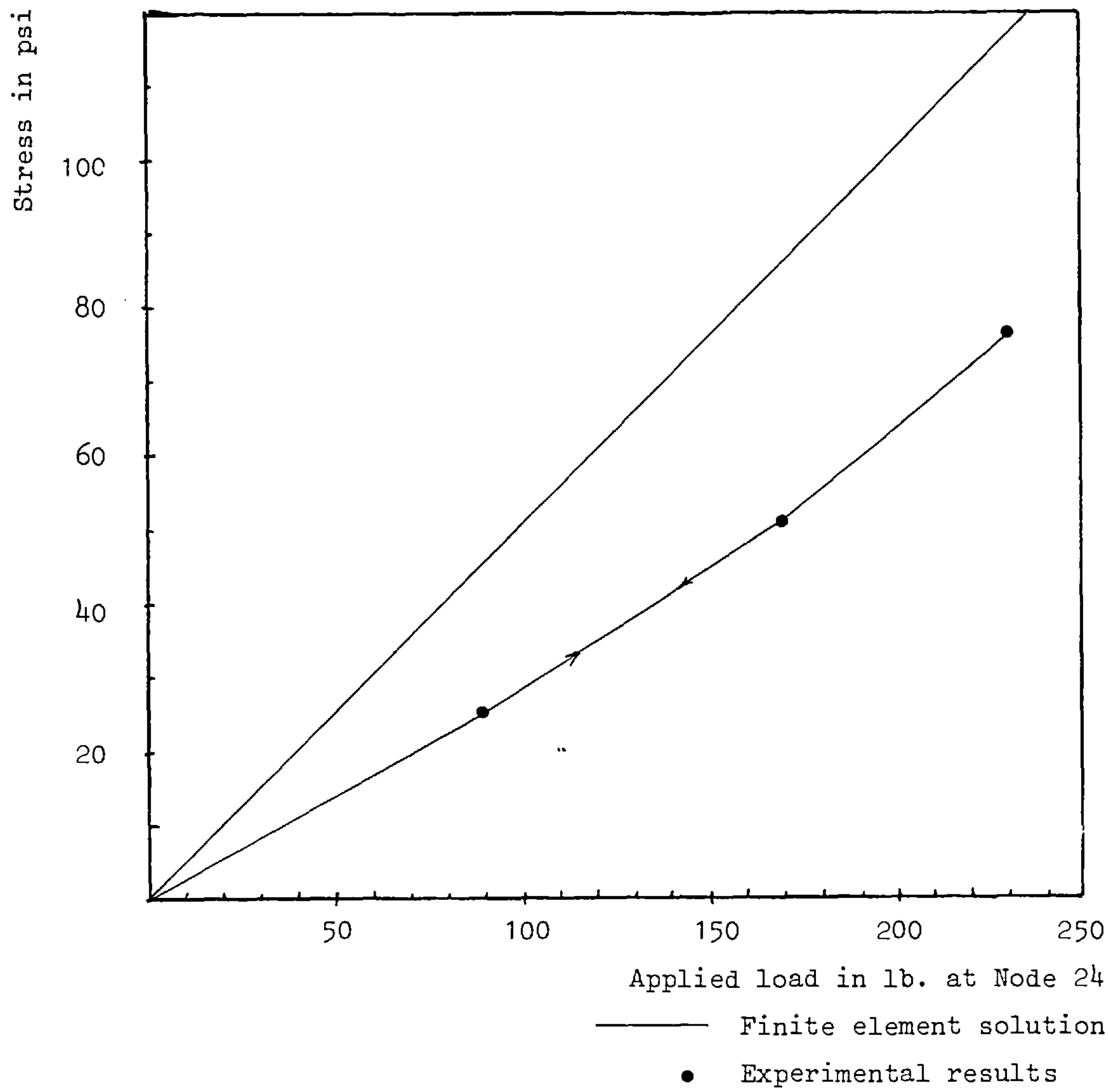


Fig. 5.45. Comparison of stresses in direction 1 at Node 203

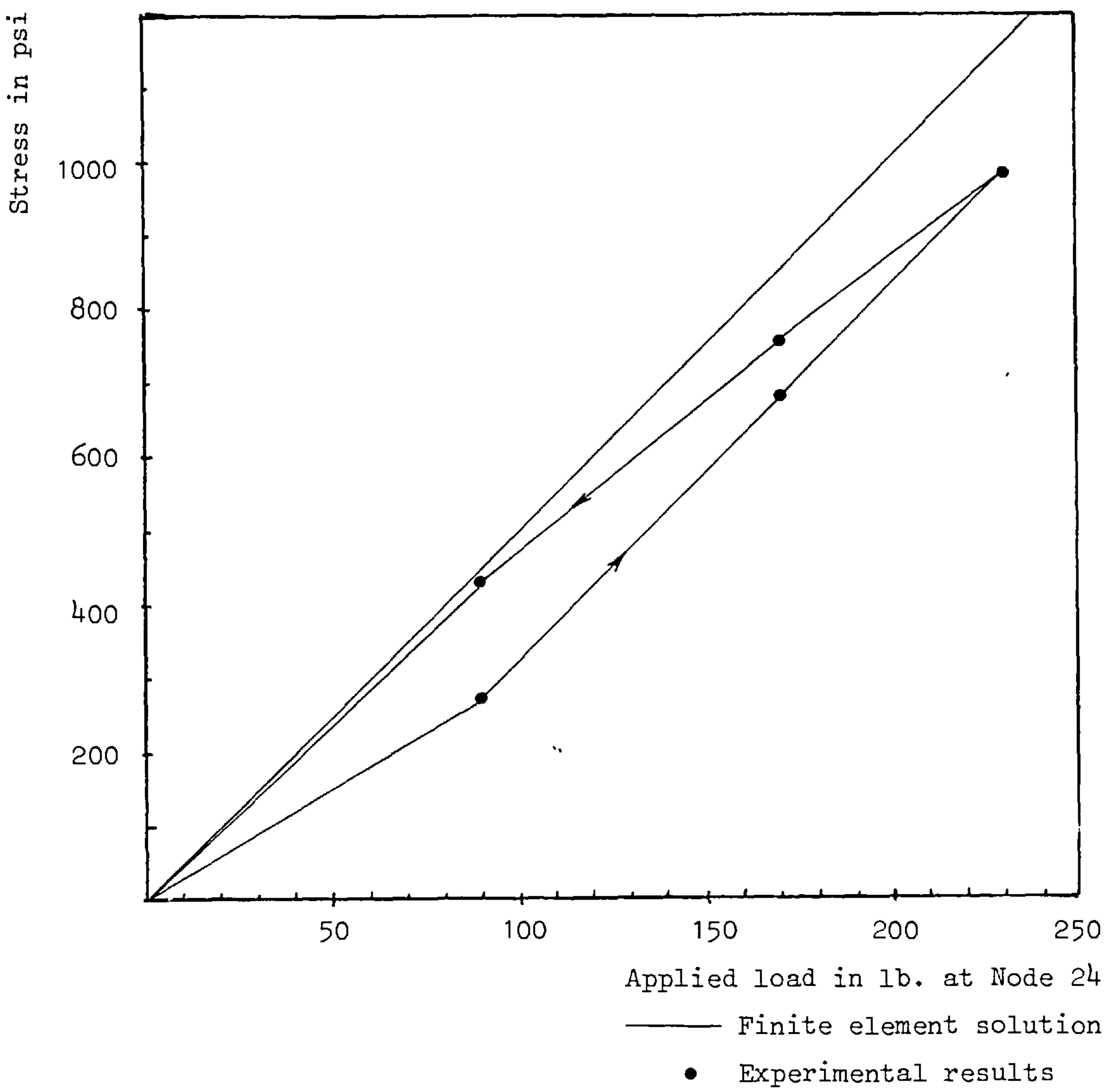


Fig. 5.46. Comparison of stresses in direction 2 at Node 203

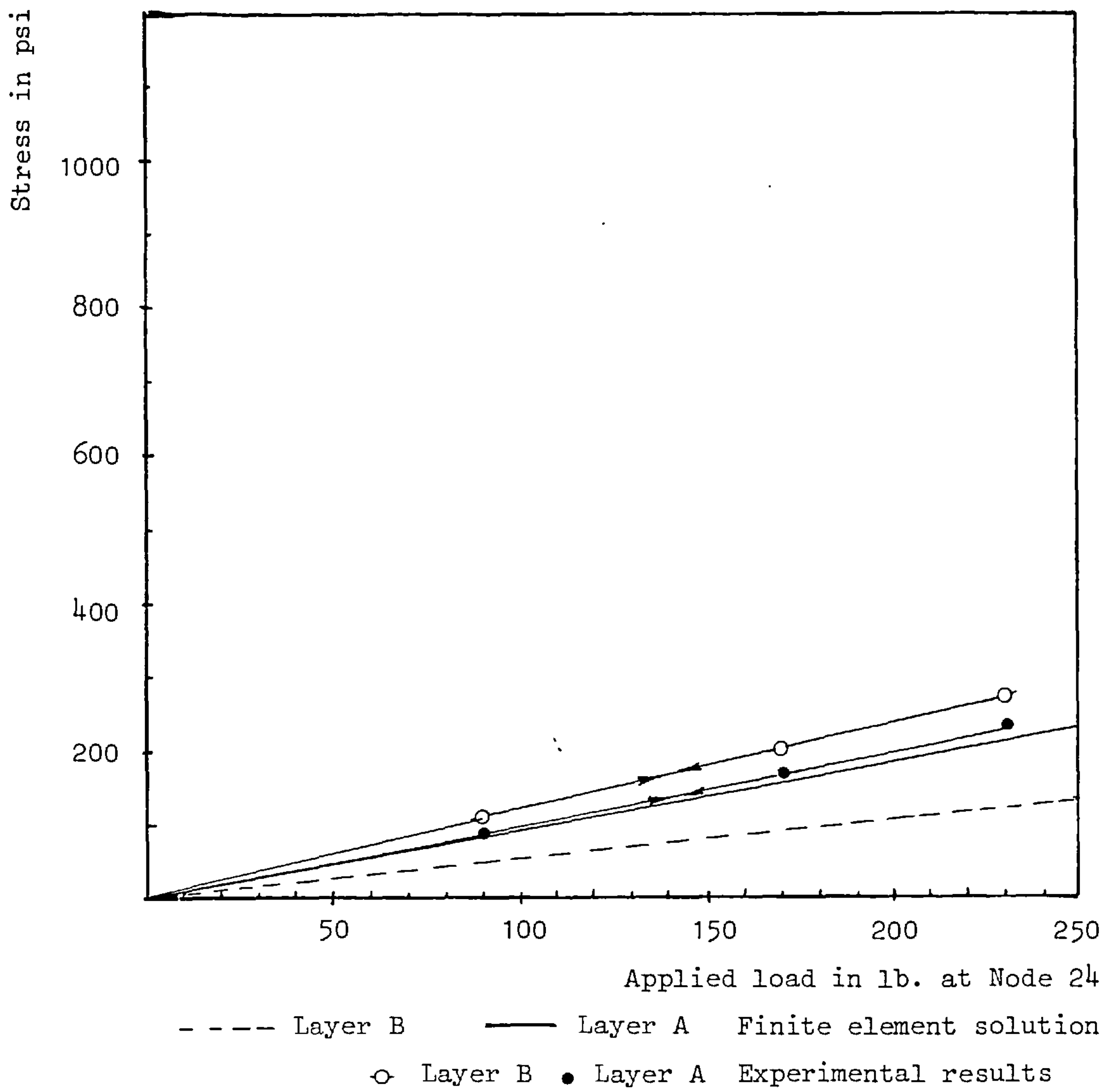


Fig. 5.47. Comparison of stresses in direction 2 at Node 210

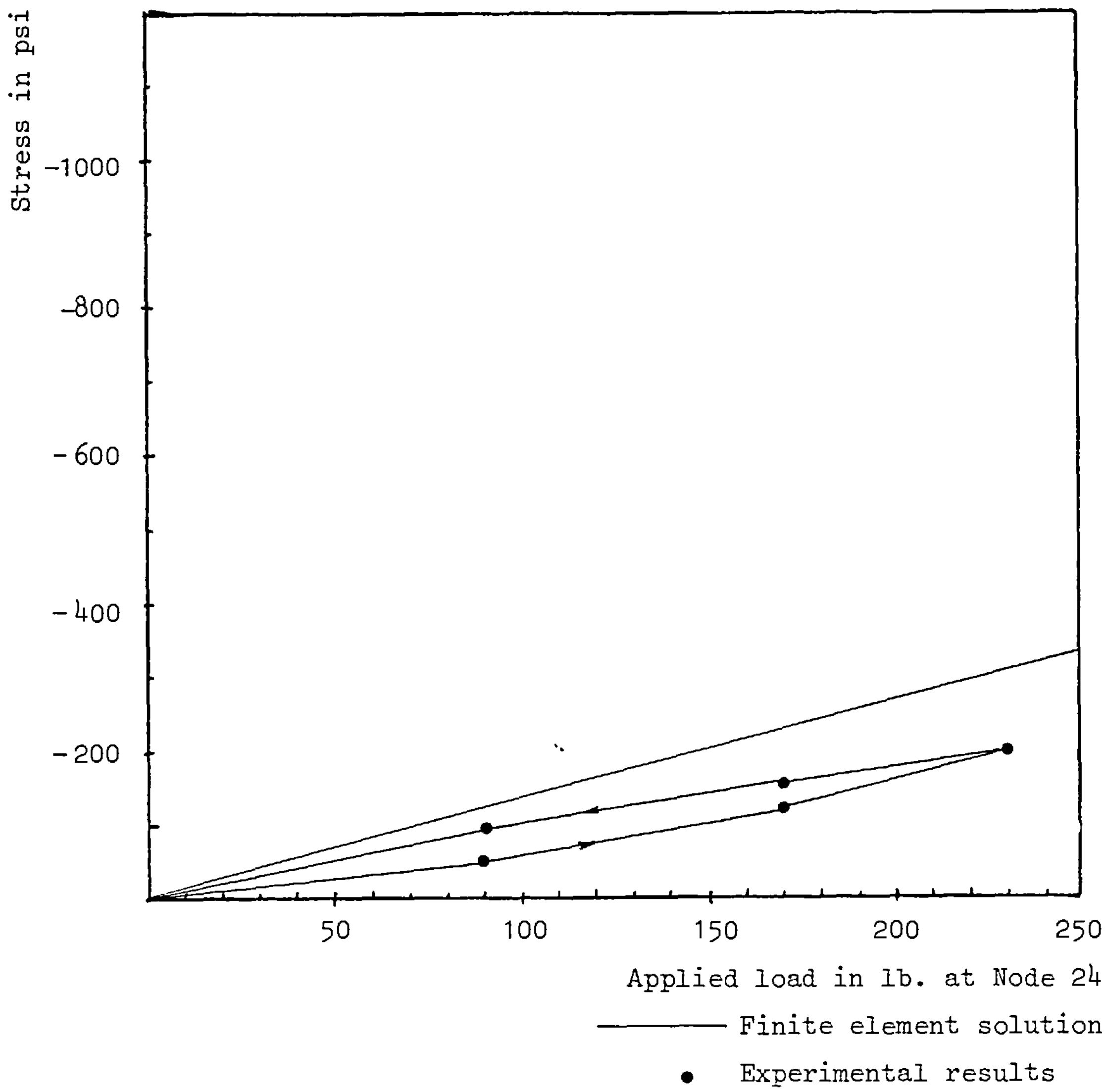


Fig. 5.48. Comparison of stresses in direction 3 at Node 210

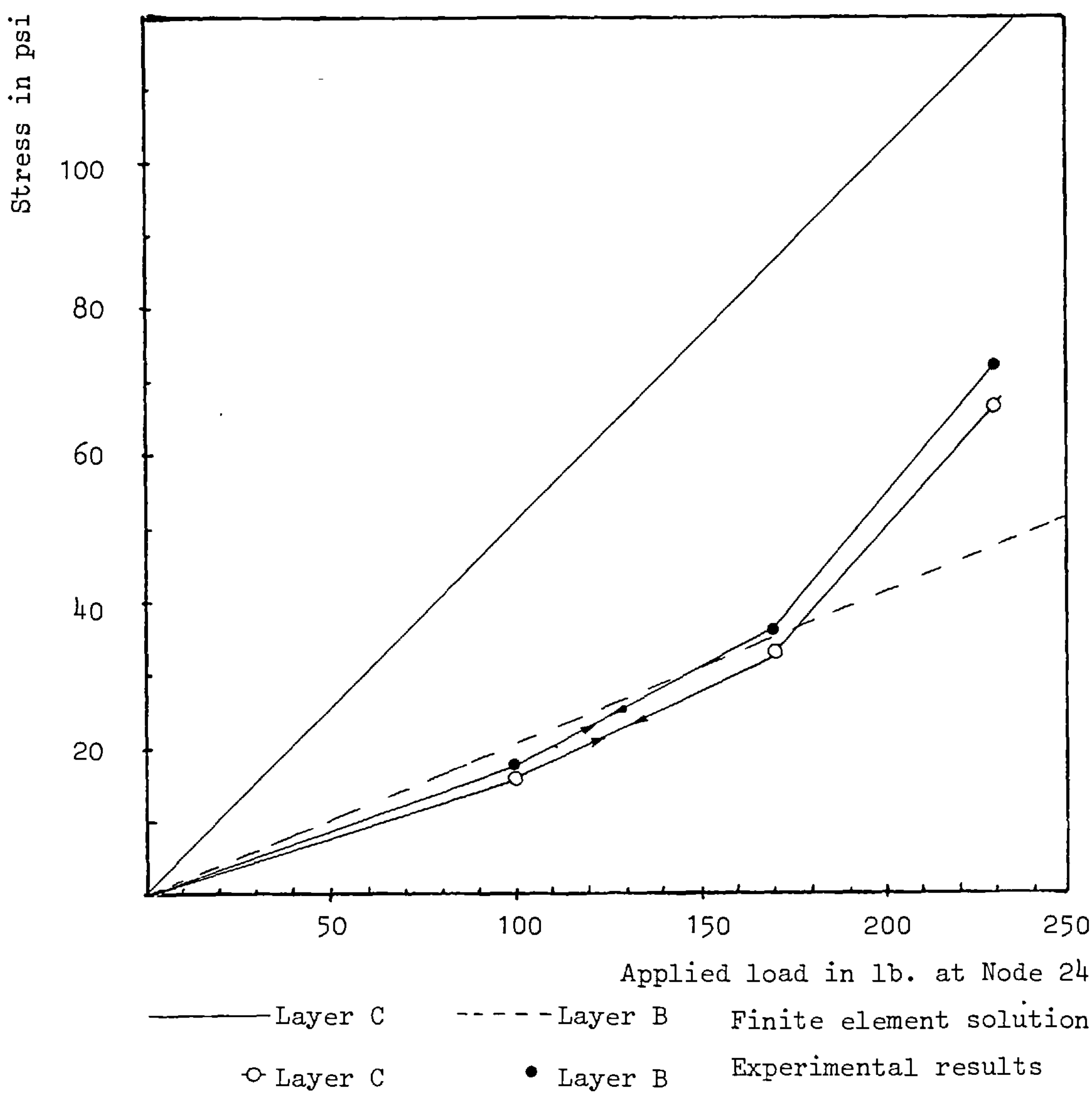


Fig. 5.49. Comparison of stresses in direction 2 at Node 215

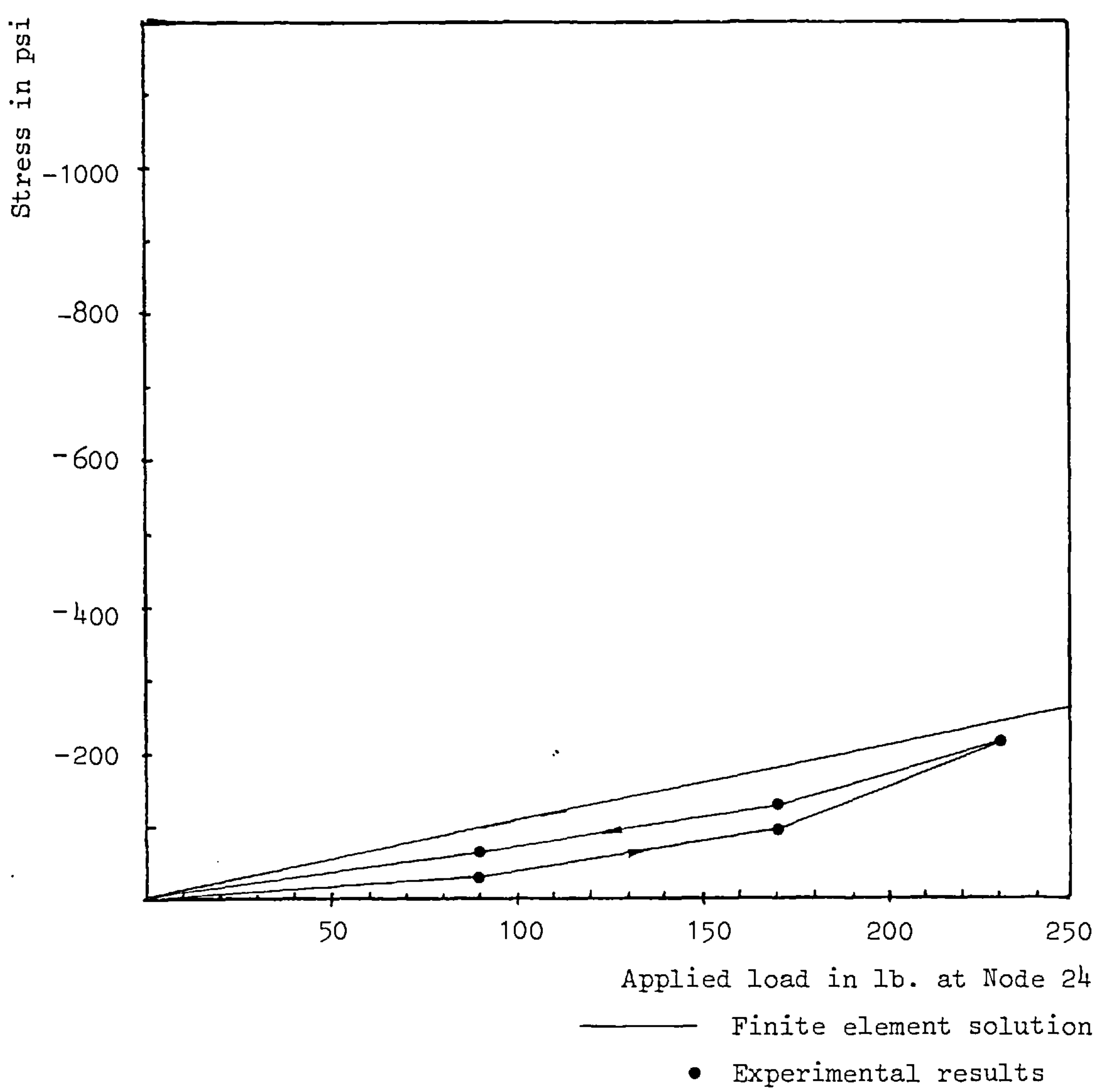


Fig. 5.50. Comparison of stresses in direction 3 at Node 215

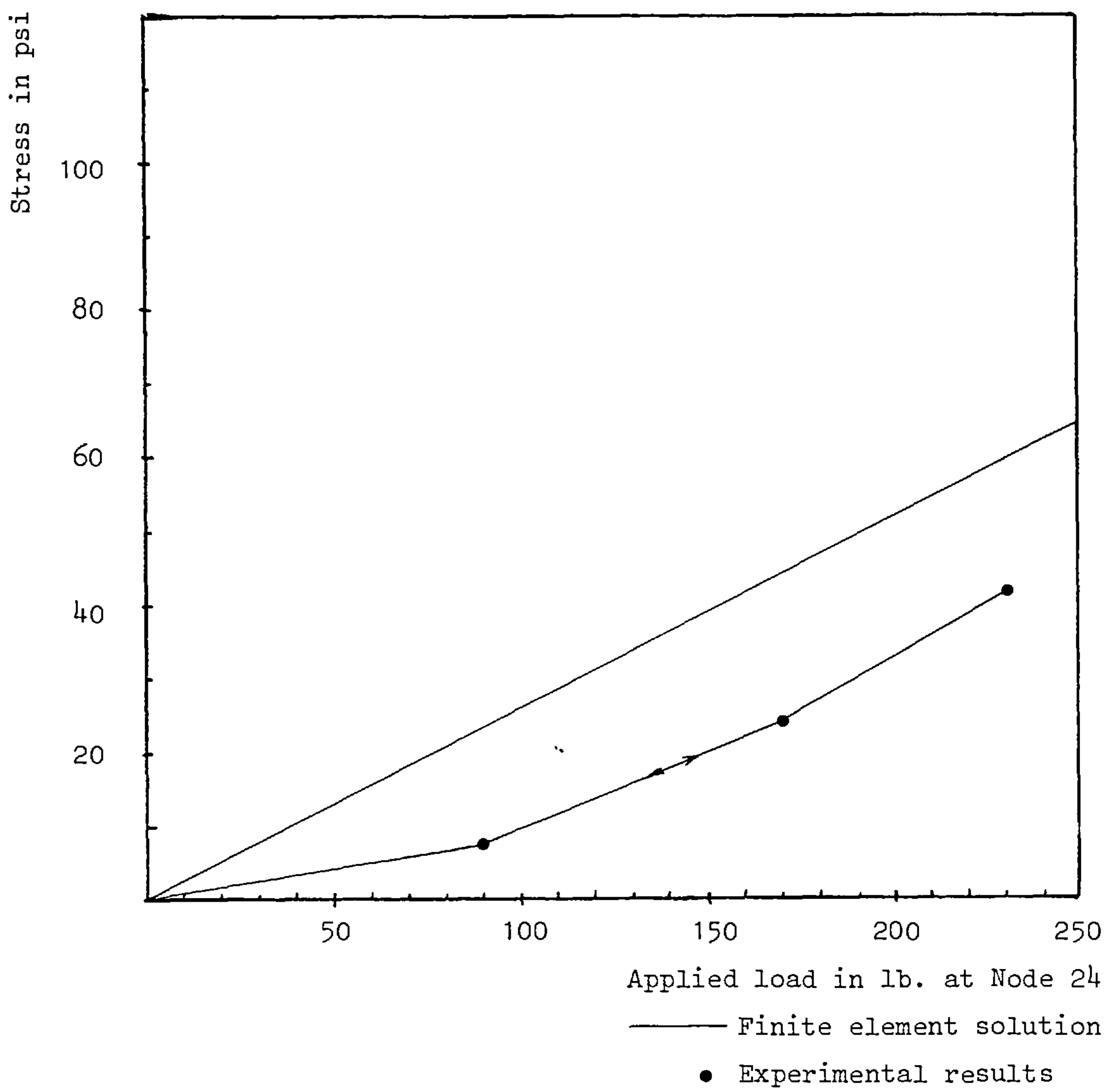


Fig. 5.51. Comparison of stresses in direction 1 at Node 218

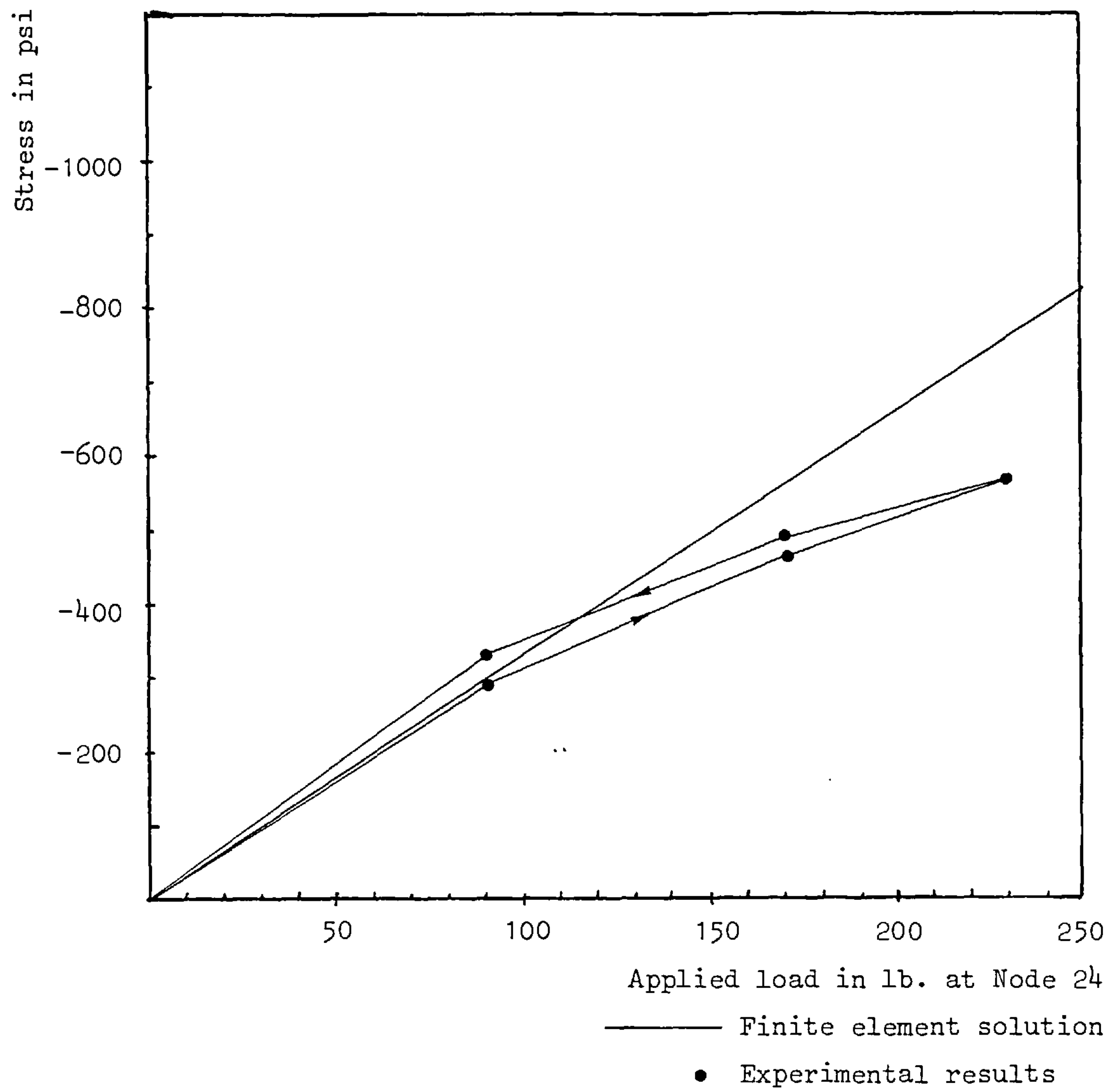


Fig. 5.52. Comparison of stresses in direction 2 at Node 218

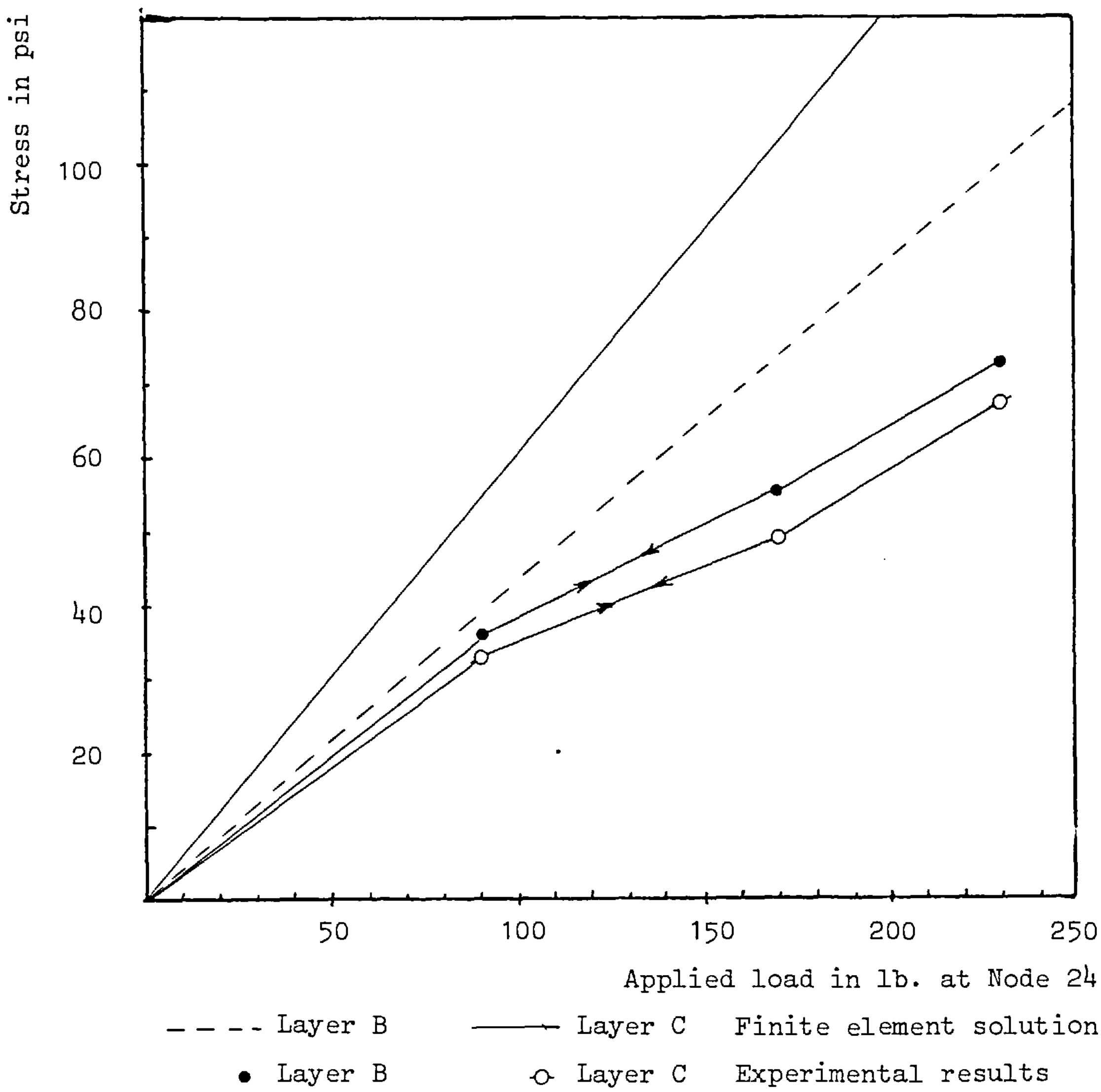


Fig. 5.53. Comparison of stresses in direction 2 at Node 211

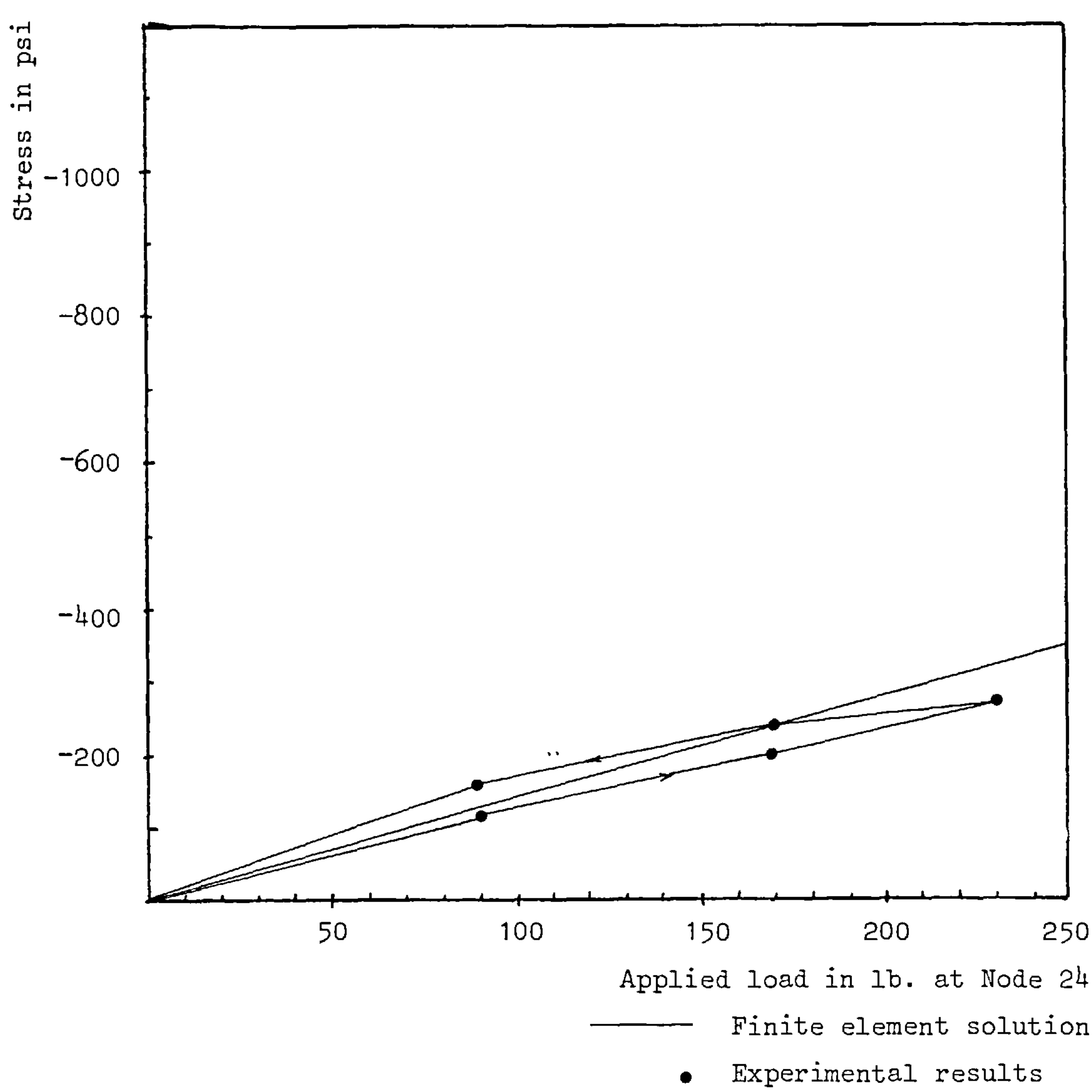
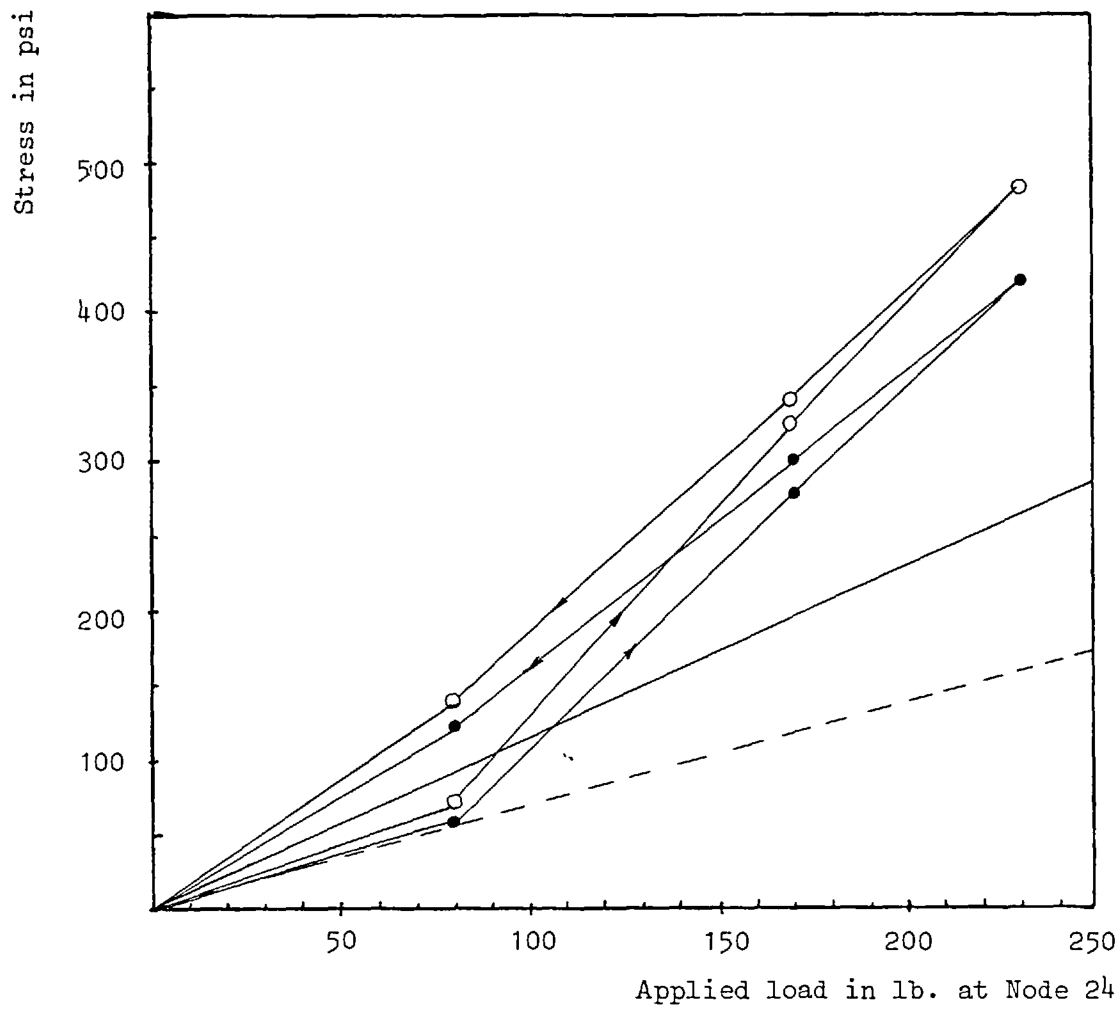


Fig. 5.54. Comparison of stresses in direction 3 at Node 211



———Layer A   - - - -Layer B   Finite element solution

• Layer A   o Layer B   Experimental results

Fig. 5.55. Comparison of stresses in direction 2 at Node 206

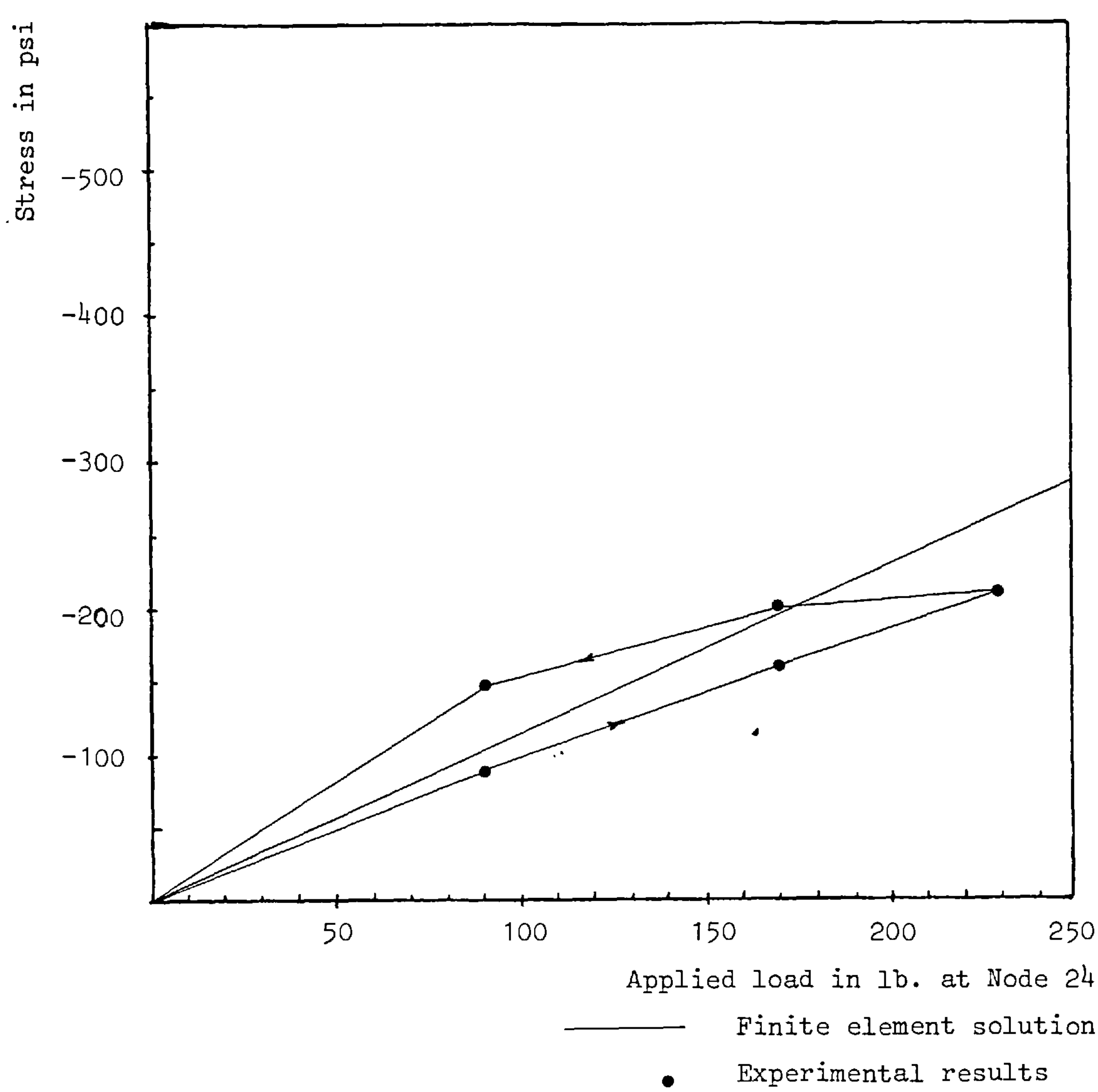


Fig. 5.56. Comparison of stresses in direction 3 at Node 206

(b) Deflections

Three dial gauges were set up to measure the deflections. Two vertical directions at the equivalent locations of Node 1 and 45 on the finite element mesh and one horizontal at the equivalent location of Node 30, as shown in Figs. (5.41), (5.42) and (5.43).

As the clamping length was short a fourth gauge was located 0.5" away from the support to indicate any rigid body rotation.

The deflections measured were plotted against the finite element predicted deflections in Figs. (5.57), (5.58) and (5.59). The relative deflections in the three directions corresponded closely to the theoretical predictions although the absolute magnitudes were somewhat different. It should be remembered that the deflections were small and difficult to measure accurately.

5.4. G.R.P. Blade of an Ordinary Butterfly Valve

A 60" nominal diameter G.R.P. butterfly valve as shown in Figs. (5.60), (5.61), (5.62) and (5.63), made of isophthalic polyester resin reinforced by woven-roving glass fibres as the main material and chopped strand mat as another material around the shaft hubs. The blade is retained and driven within the body of the valve, which is made from woven-rovings and chopped strand reinforced polyester resin of the same type, by stainless steel stub shafts located in the disc by stainless steel inserts.

The dimensions shown in Fig. (5.60) were calculated so as to satisfy the maximum allowable deflection for sealing the valve using the work done in Chapter 2.

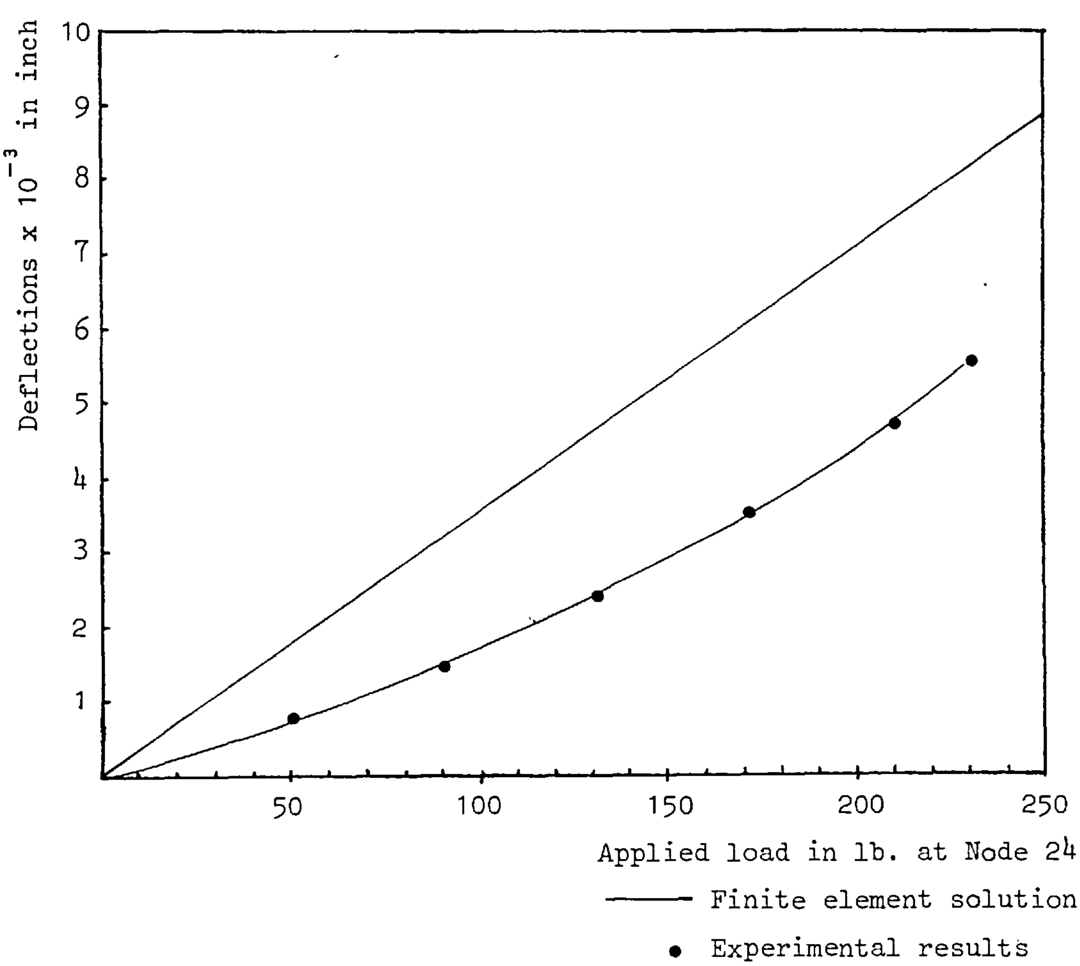


Fig. 5.57. Load vs. deflections in direction 3 at Node 1

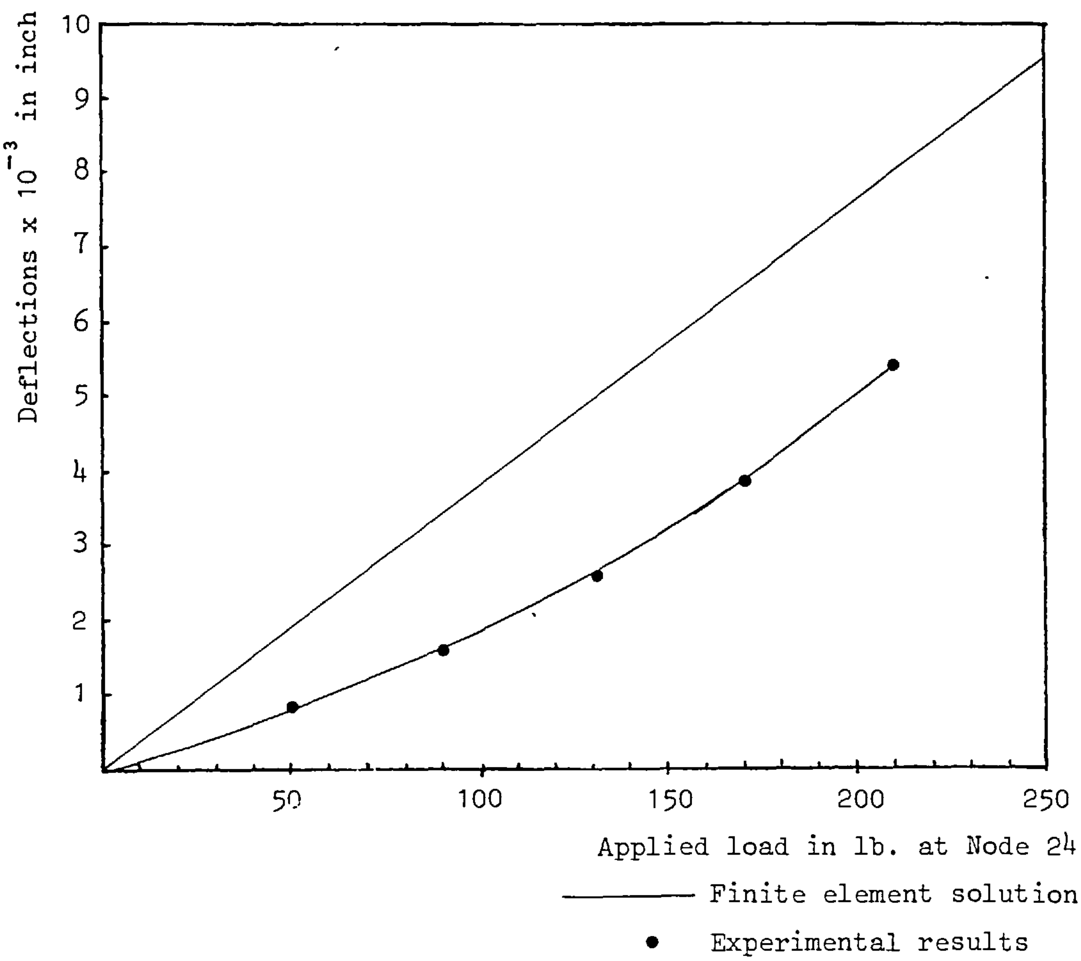


Fig. 5.58. Load vs. deflections in direction 3 at Node 45.

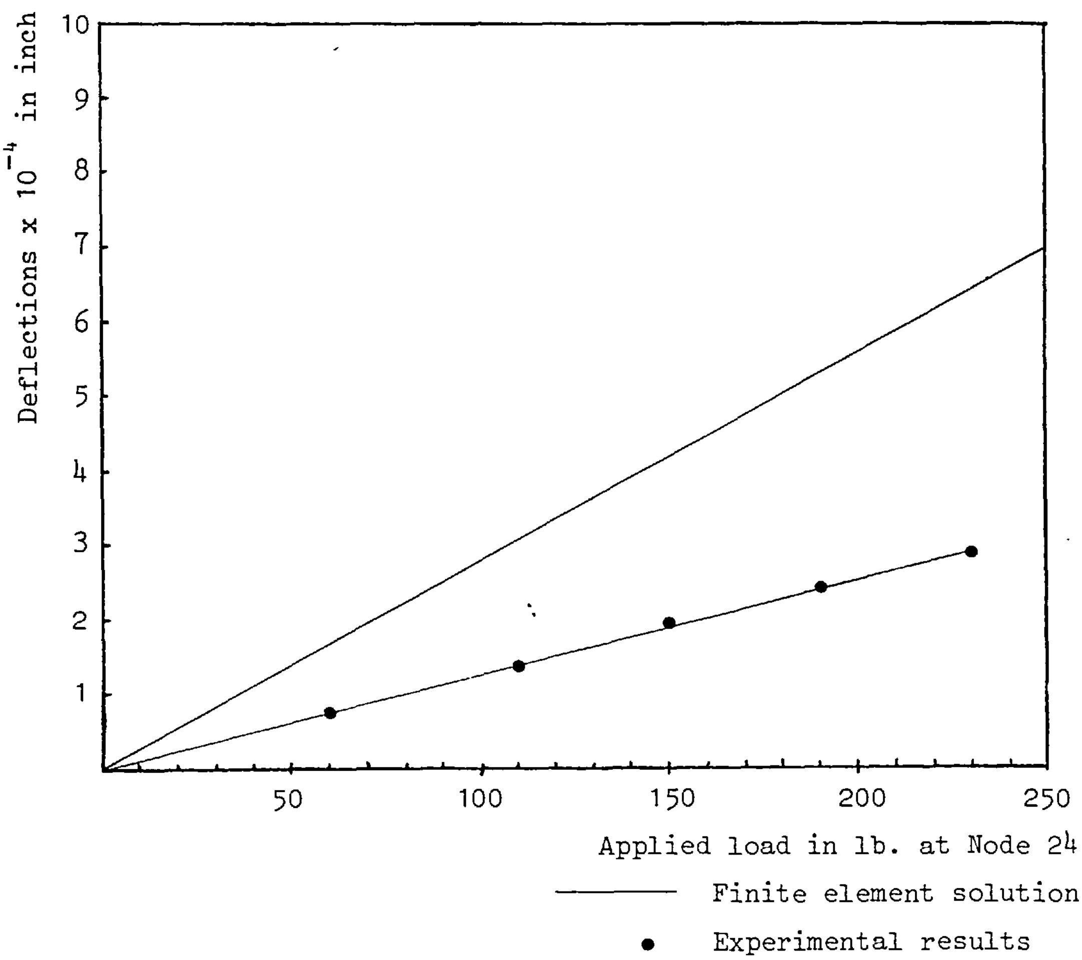


Fig. 5.59. Load vs. deflections in direction 1 at Node 30

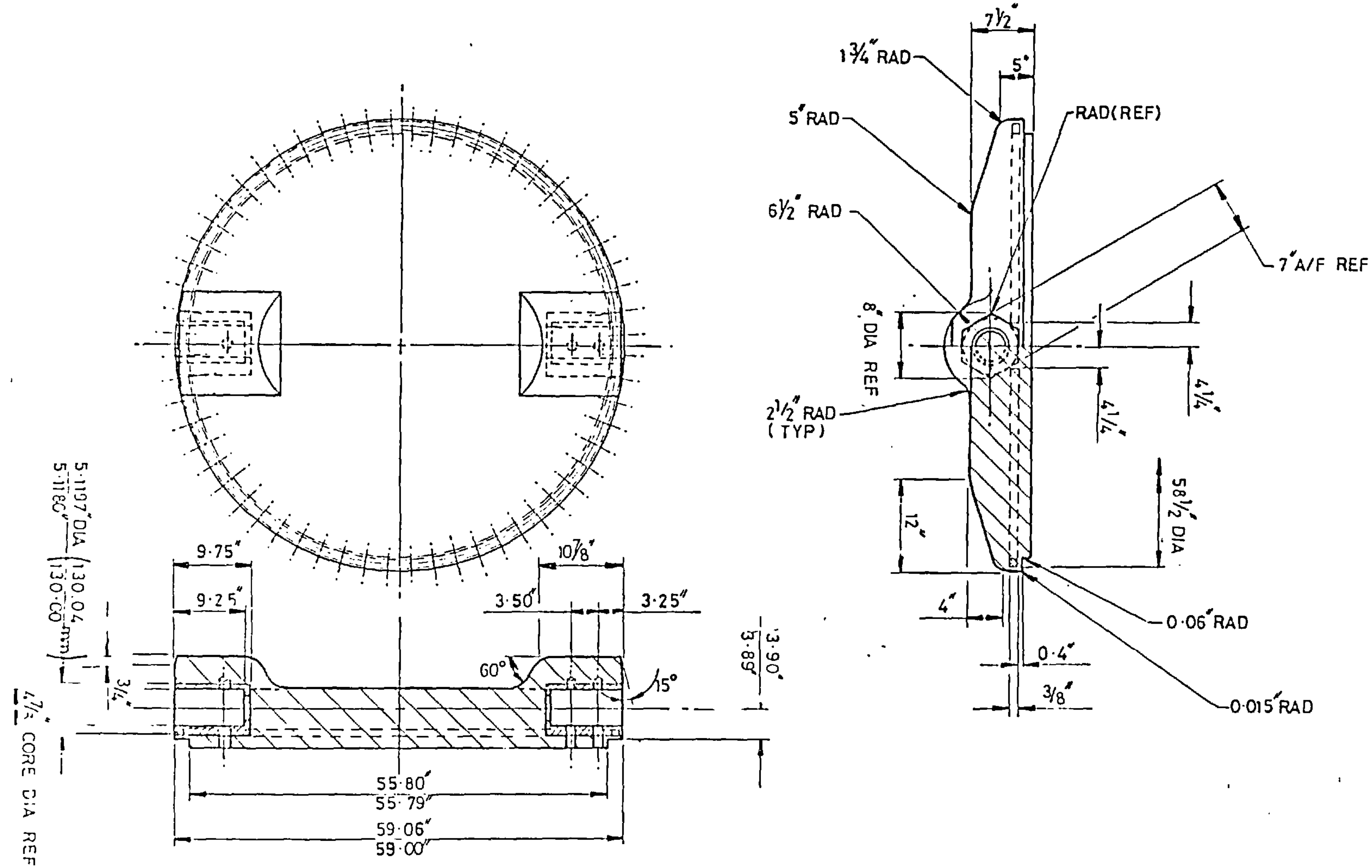


Fig. 5.60. G.R.P. butterfly blade

Fig. 5.61. G.R.P. butterfly blade.

Fig. 5.62. G.R.P. butterfly blade

Fig. 5.63. G.R.P. butterfly blade.

A finite element analysis using (BFSOLID) was carried out. The blade was idealized into 44 elements, 9 of which were given the stainless steel mechanical properties, 11 were given the chopped strand woven glass reinforced plastic mechanical properties as a homogenous material as shown in Figs. (5.64a) and (5.64b). The deflections predicted were very satisfactory.

For the allowable design stresses the factor of safety was calculated in accordance with BS.4994, Ref. (63) as follows:-

Factor relating to method of manufacturing $k_1 = 1.6$

Factor relating to long term behaviour $k_2 = 1.2$

Factor relating to temperature $k_3 = 1.1$

Factor relating to cyclic loading $k_4 = 1.1$

Factor relating to curing procedure $k_5 = 1.3$

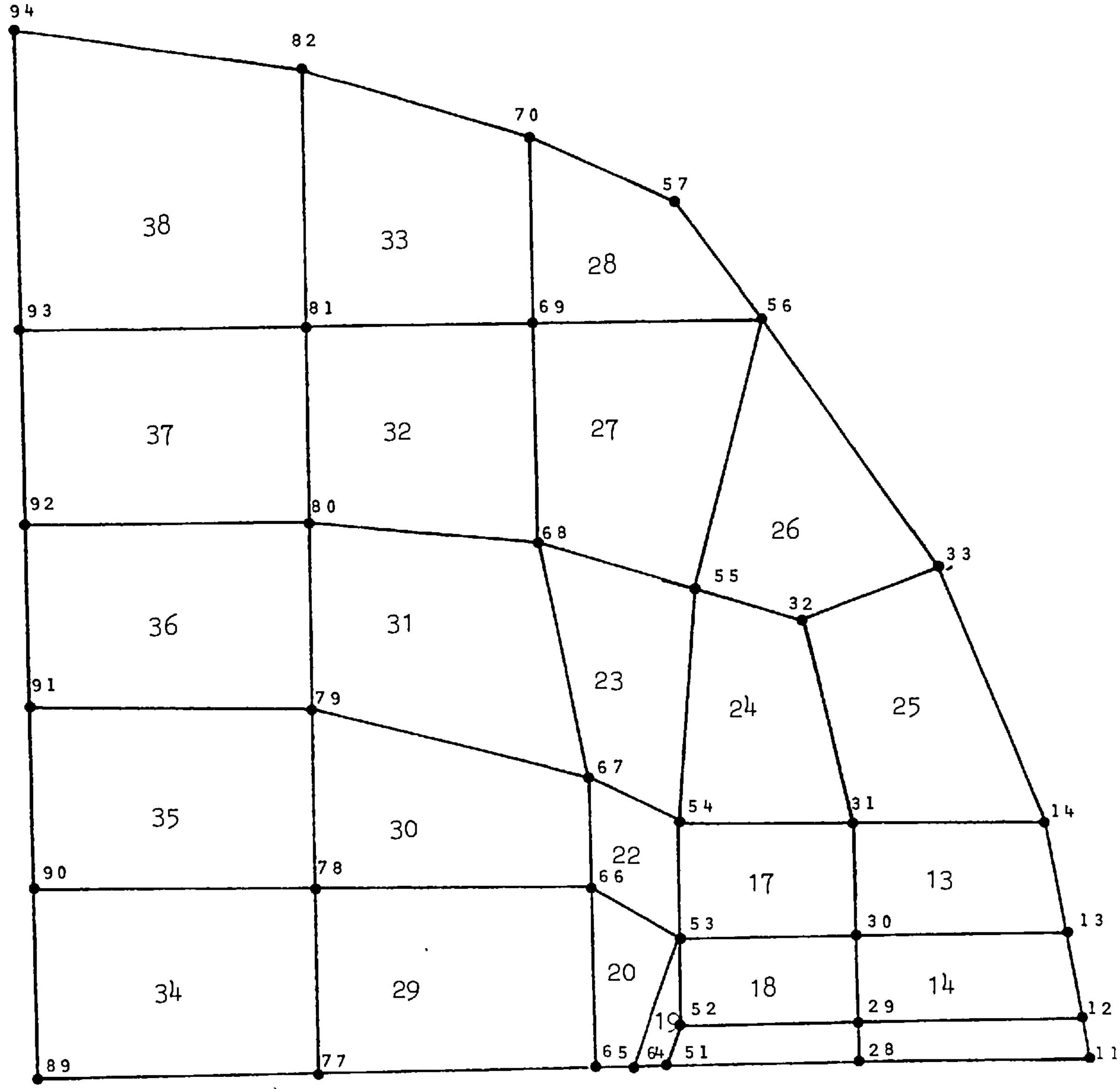
Overall design factor:-

$$k = 3 \times k_1 \times k_2 \times k_3 \times k_4 \times k_5 = 9.06$$

Adopted factor = 10.

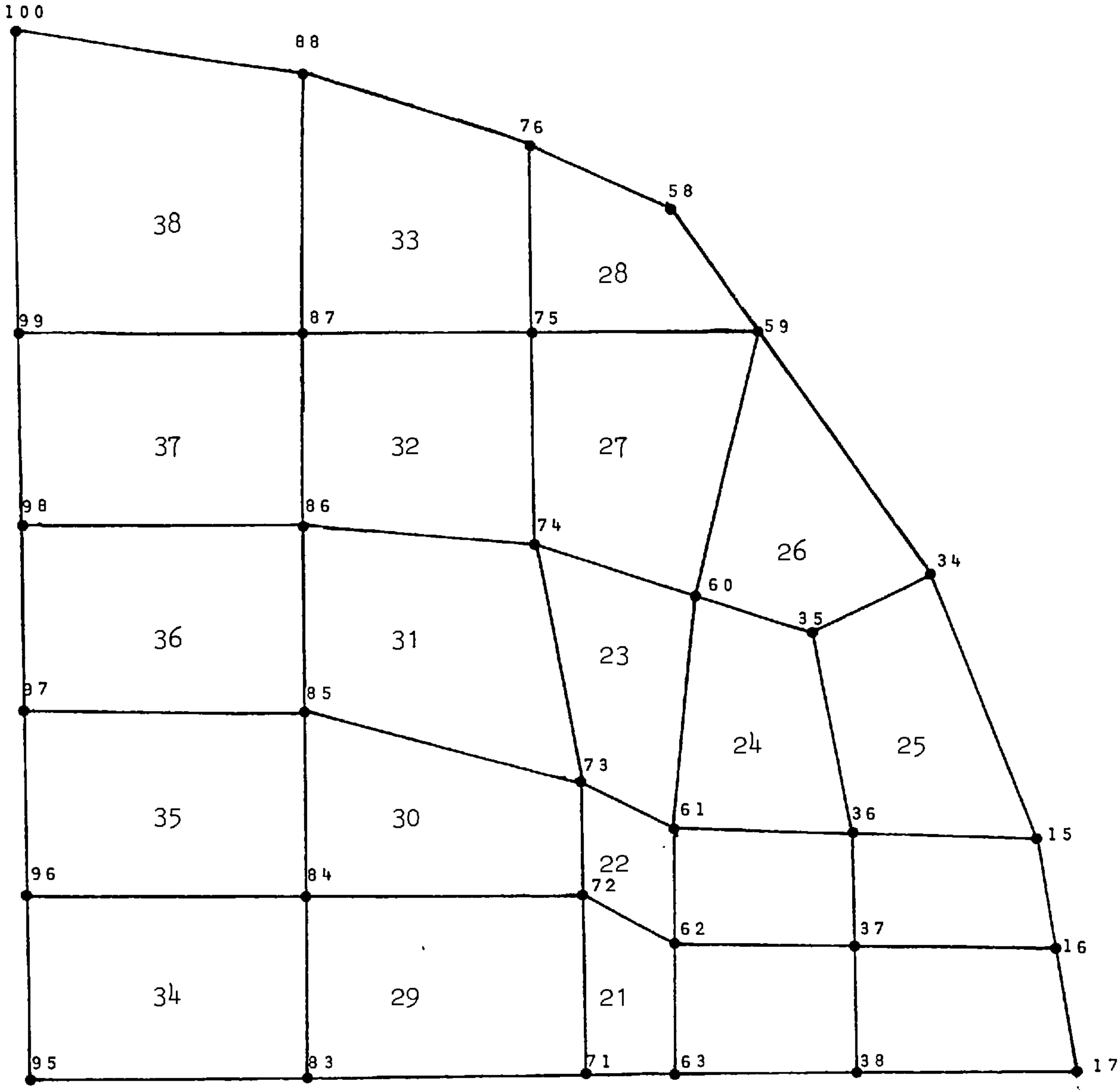
All the stresses predicted by the finite element analysis were lower than the allowable stresses.

An experimental programme for determining the stresses is planned in the near future. The location and direction of the strain gauges has been chosen to suit the finite element analysis, as shown in Figs. (5.65a) and (5.65b). The deflections will also be checked. The testing arrangement is shown in Fig. (5.66). A fair comment on the test results will be given in Ref. (70), by the eventual users of the valve, two of which will be put into actual service by the C.E.G.B. before the end of 1980.



Lower Face.

Fig. 5.64a. Finite element idealization of G.R.P. blade



Upper Face.

Fig. 5.64b. Finite element idealization of G.R.P. blade

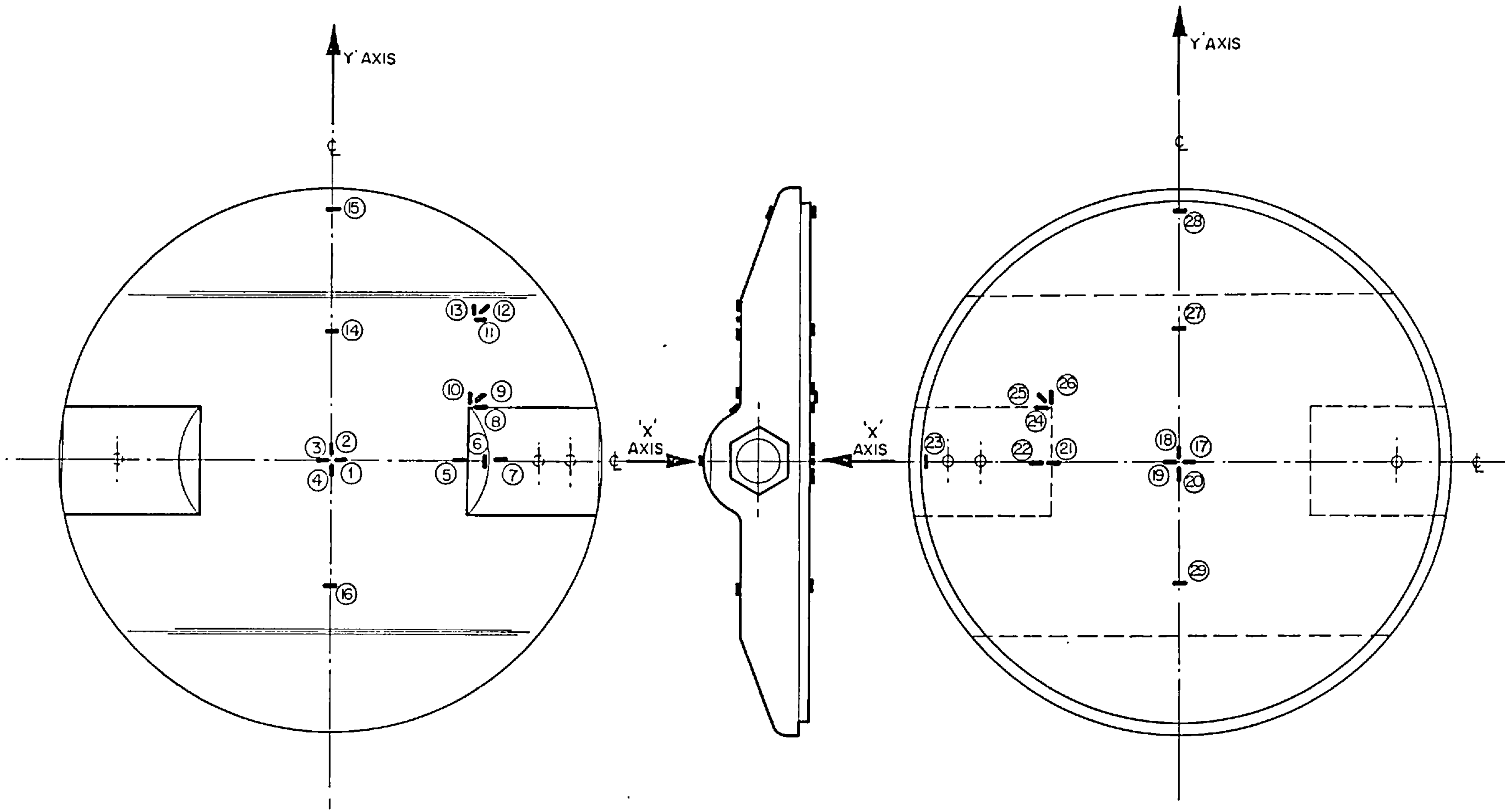


Fig. 5.65a. Strain gauge locations on G.R.P. blade.

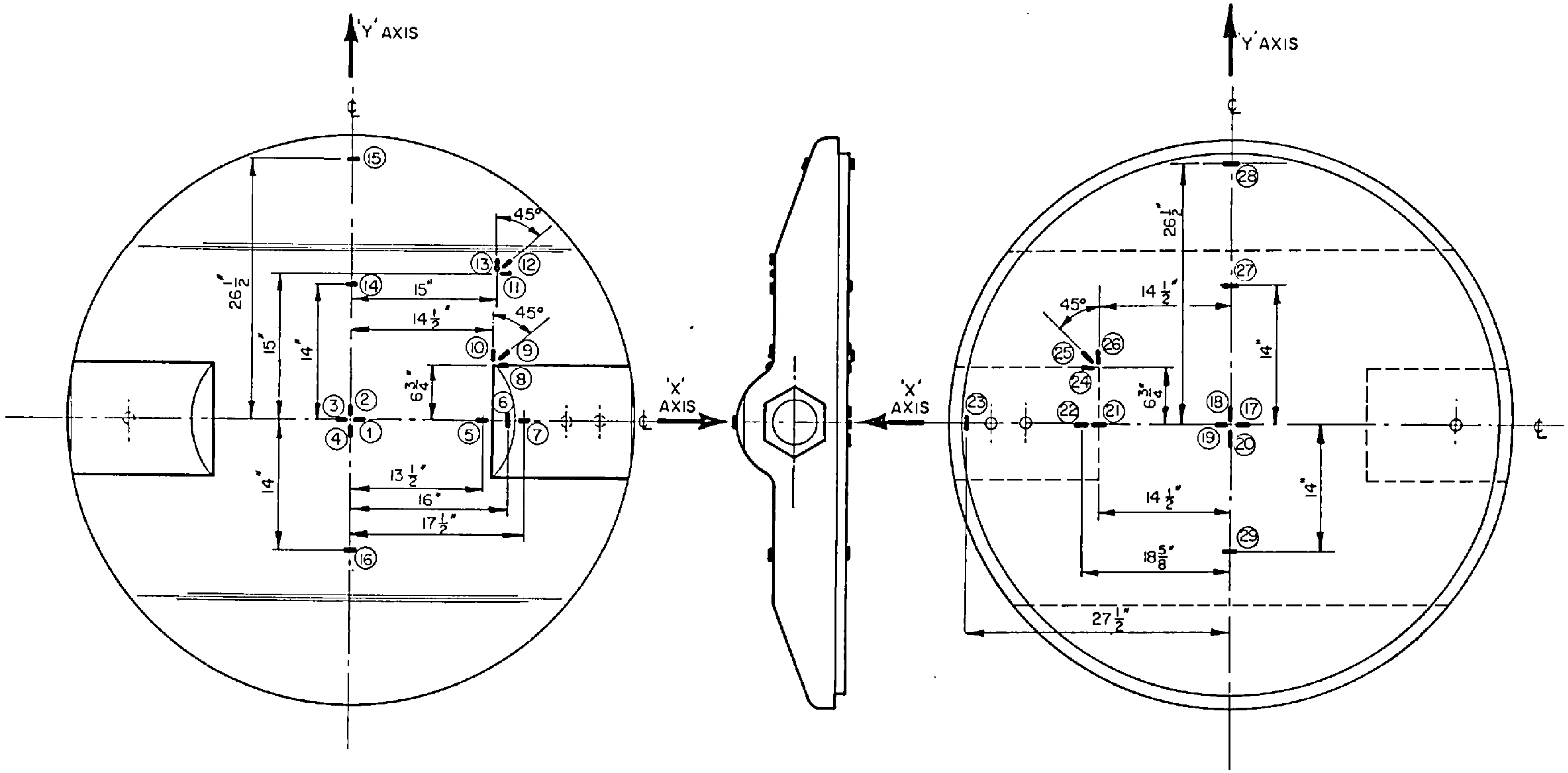


Fig. 5.65b. Strain gauge locations on G.R.P. blade.

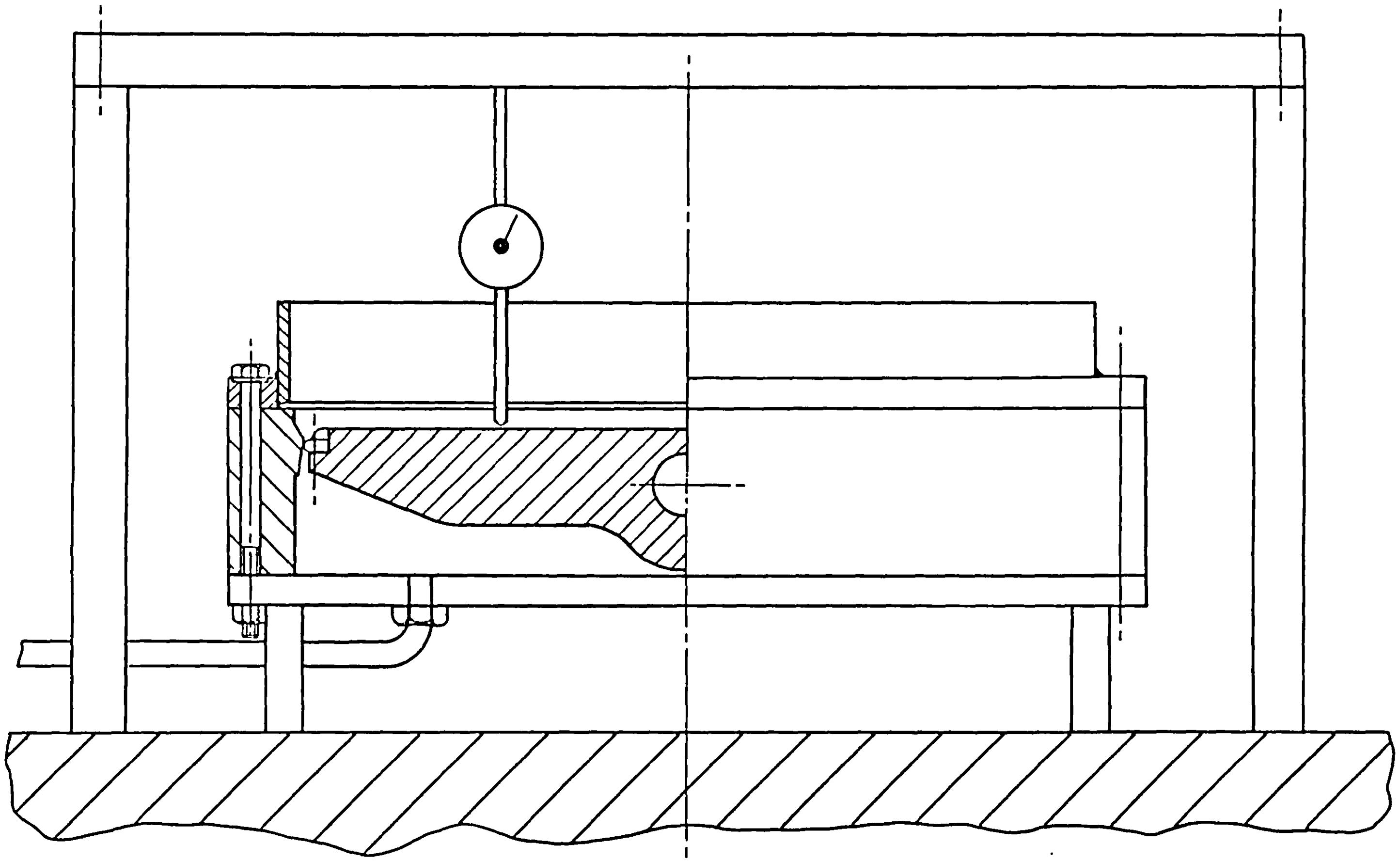


Fig. 5.66. Testing arrangement for deflections in G.R.P. blade.

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

6.1. The Analysis of Ordinary Blades

6.1.1. Conclusions

Prior to this research there has been no really serious attempt to study the structural problems of ordinary butterfly valve blades. The work described in Chapter 2 is considered by the author to be a major contribution to the advancement of knowledge in relation to the design and operation of valves of this type.

The use of the finite element method as a technique for obtaining design coefficients represents a major break through, since the majority of practical cases have no mathematical solutions. This is because there are a great number of combinations of complex loading and support conditions which are required in practice.

The very important problem of predicting the behaviour of blades can be solved by adopting this method. The tapering configuration must then be expressed as a function of the radius or alternatively as a fraction of the thickness of the theoretically embracing flat blade. This would lead to savings of material and also better meet the hydraulic requirements.

The finite element solution requires a large computer to carry the mathematics involved in inverting the stiffness matrices. On the other hand storing a large number of tables on the floppy discs of very small desk top computers is a much cheaper and more practical proposition, which justifies the approach and the use of coefficients which has been emphasised throughout this thesis. This work should contribute to possible relaxation of the standard minimum required thicknesses at present demanded by the various authorities and, with improved quality control, to the testing procedures.

Since the deflections could be predicted with greater accuracy by the methods described in this thesis, a significant saving in the cost of seals and the use of different materials for resilient seals should be possible.

The effect of the length of the arc of the periphery occupied by the shaft is clear from reading through the tables of Appendix 3. This knowledge can be used to optimize the cost of blades and their operating equipments and actuators.

Finally, this work could lead to a complete set of structural models to cover all possibilities of behaviour for the ordinary butterfly valve blade, on which some confidence can be placed in terms of the extensive experimental comparisons which have been made.

6.1.2. Further work

The major area of further work should be in the field of simulating real valves to the different structural models presented. In real valves the clearance between the shaft and the body is a parameter whose importance has not been sufficiently appreciated and identified in the past. A study of the effect of this clearance (generally specified in "thous per in." of shaft diameter) on the structural behaviour of the blade should be made.

In this study the following factors should be considered:-

(i) The length of the supported part of the shaft.

(ii) The type of bearings used.

(iii) The type of shaft used and the effect of the clearance between shaft and blade, bearing in mind the method of attachment.

In the present day, with the introduction of microprocessors and minicomputers for the control of machine tools (as can be seen from Ref. (71)) it is no longer difficult or expensive to attain near-perfect clearances.

Another factor which has not been covered in this study is the effect of the eccentricity of the shaft centre line from the middle surface of the blade. This is not important for thin blades. As the blade and shaft humps increase in size, eventually they clearly indicate when the blade should be considered as being a thick plate problem.

A criterion for the effect of the eccentricity has to be developed and the only clear method is the use of solid finite elements.

Finally it must be remembered by any structural analyst working on the subject that a close relationship exists between any structural changes and the flow characteristics of the valve, particularly in relation to the ever-increasingly important problem of noise and the requirements of the operating equipment.

6.2. Fibre Composite Blades

6.2.1. Approach for optimal design

An approach towards the design of fibre composite blades can be seen from the work done in Section 5.3. There, it was shown that it is possible to assess the stresses fairly accurately by using the measured values of the various moduli. The butterfly valve blade is an ideal structure to be made from fibre composite materials since it is governed by a failure criterion based on very low deflections (conditioned by the requirement for sealing). This means that high strains need not be considered as a criterion for failure and yielding of the structure is therefore not a factor to be considered.

The major cost in fibre composites is directly related to the fibre content, especially when the fibres or woven-roving layers have to be hand laid. Therefore, their distribution through the thickness of the blade should be optimised according to the particular requirements of the loading and geometrical arrangement of the blade. In this way, a blade which has the minimum deflection and adequate strength can be optimally designed so as to have minimum cost.

The main concern in fibre composite structures is the fear of failure due to shear or lap stress at interfaces and it is inadequate to assume that the material is homogeneous in order to estimate strains at these surfaces. This is true even for thin blades and it is always necessary to make a three-dimensional analysis, even if a homogeneous equivalent material is assumed for the purpose of obtaining a lower bound for the strains as, in a three dimensional analysis, the inter-laminar stresses or lap shear will be revealed.

Young's moduli and Poisson's ratios can easily and cheaply be obtained experimentally in a layer as has been demonstrated in this work and, although the through-thickness properties are more difficult to obtain experimentally, they can be estimated conservatively by using the properties of the resin matrix which are provided by the manufacturers.

The method of selecting samples for a multi-layered fibre composite demonstrated in this thesis, from which the various moduli and Poisson's ratios were estimated, coupled with their use in predicting the stresses and deflections of a structure (a cantilever) subjected to a complex loading situation, indicate the feasibility of the optimal designing of a blade using these methods. Also, with the use of the finite element program, the optimal combination of metallic and fibre composite materials can be achieved, provided that the interfacial conditions are adequately defined. In particular, the bond should be examined independently.

6.2.2. Further work

Two full-size fibre composite butterfly valves have been made and are being tested for use in service at the Fawley Power Station, by the Central Electricity Generating Board (C.E.G.B.). These valves have been designed, as regards their structural integrity, by the author and their behaviour in long-term use will be a final justification for the procedures described in this thesis. Further research is required in the more accurate determination of through-thickness properties and shear moduli of the various fibre composite layers used in such structures. In addition to this, much development is needed in the manufacturing techniques required for the laying-up of structures of this type. Laying-up manually is both costly and inaccurate and any methods which could be devised for automatic laying-up are highly desirable and should be actively pursued. Such investigations are at present being carried out under the supervision of the author and this will facilitate accurate determination of through-thickness and shear moduli at a minimum cost.

Some work of this type has been done in aero-space industries, mainly from the point of view of optimising strength-weight ratios. However, the present objectives are different, being mainly concerned with minimising corrosion, cost and maximising reliability. Therefore it would be very desirable to survey the literature already available from the aero-space industries, with these different objectives in mind.

REFERENCES

CHAPTER ONE

1. GRIFFITHS, J.M. "Proceedings of the International Valve Conference". Brighton, 1978.

2. British Valve Manufacturers Association Handbook, 1972.

3. "A guide to the Selection of Automatic Control Valves"; Engineering Equipment Users Association Handbook, No. 32. 1969.

4. British Standards Institution BS.5155, 1979.

5. Central Electricity Generating Board, C.E.G.B. Std. 568502.

6. American Water Works Association, AWWA. C.504-74.

7. British Standards Institution, BS.4504, BS.1560, BS.3290.

8. Boving Valves, Boving & Co. Ltd., 1978.

9. ROSATO, D.V., Editor, Polymer Engineering and Technology Series, John Wiley & Sons, N.Y.

10. GRESZCZUK, L.B., "Consideration of Failure Modes in the Design of Composite Structures", McDonnell Douglas Astronautics Co., 1975.

11. MAJMUDAR, A.A., "Proceedings of the International Valve Conference", Brighton, 1978.

12. PEARSON, G.A., "Valve Design", Mechanical Engineering Publications Ltd., London, 1972.

13. ADDISON, H. "A Treaties on Applied Hydraulics". Chapman & Hall Ltd., London.

14. CREAGER, W.P., JUSTIN, J.D., "Hydroelectric Handbook", John Wiley & Sons Ltd.,

15. DAVIES, C.V., "Handbook of Applied Hydraulics" McGraw-Hill, New York, 1942.

16. "a-Choosing Valves, Water Works and Pumping Practices", Glenfield and Kennedy Ltd., Kilmarnock, 1955.

17. "b, c-Choosing Valves, Water Works and Pumping Practices", Glenfield and Kennedy Ltd., Kilmarnock, 1955.

18. KURKJIAN, G.A., "Follow these simple rules to get long life from your large Butterfly Valves". Henry Pratt Co., Power V.118, No. 7, July, 1974.

19. FREEMAN, M.L., SPROUT, D.H., "Butterfly Valves-Correct Installation and Alignment", Masoneilan Ltd., London, Paper Technology, V. 15, No. 2, April, 1974.

20. "A Bibliography on Butterfly Valves", The British Hydromechanics Research Association, BIB24, April, 1966.

21. BOGER, H.W., "Low-torque Butterfly Valve Design", Masoneilan International, Inc., Instrument Technology, V. 17, No. 9, Sept., 1970.

22. HAMPHILL, J.E., "Development of a Low-torque Butterfly Control Valve", Advances in Instrumentation, V. 24, Part 1, Proceedings of 24th Annual ISA Conference, Houston, Oc-. 1969.

23. SHALNEV, K.K., ZHESTKOV, A.A., "Mechanism of Cavitation Vibrations in Butterfly Valves", Proceedings of 5th Conference on Fluid Mechanics, Budapest, 1975.

24. JAMES, B.W., TULLIS, P.J., "Cavitation in Butterfly Valves", ASCE Journal of the Hydraulics Division, V. 99, No. H.79, Sept., 1973.

25. BELAND, R., "New Standard Metric Valves", Proceedings of the International Valve Conference, Brithton, 1978.

26. CARLETON, R.L., "Noise Created by Control Valves in Compressible Service", Masoneilan Int., Inc., ISA Translations, V. 17, No. 1, 1978.

27. MILLEVILLE, B.J., "What every Engineer needs to know about Valve Sealing", Rockwell Int., Pittsburgh, Pensylvania, Specification Engineering, V. 34, No. 5, Nov. 1975.

28. EXEKOYE, L.I., "Simplified Method for Calculating the Natural Frequency of Valve Superstructures", Westinghouse Nuclear Energy Systems, Pittsburgh, Pensylvania, ASME, Paper No. 78 - PVP-4, June, 1978.

29. WAYNE, B., "Pressure Sensitive and Temperature Responsive Butterfly Valve for Cryogenic Service", SAME, Paper No. 78, Nov., 1978.

CHAPTER TWO

30 CLEBSCH, A. "Theorie der Elastizitat fester Körper". B.G. Teubner, Leipzig, 1962.

31. SZILARD, R. "Theory and Analysis of Plates" Prentice-Hall Inc., Englewood Cliffs, New Jersey.

32. TIMOSHENKO, S.P. and WOINOWSKY-KRIEGER, S., "Theory of Plates and Shells", McGraw-Hill Book Company Inc., 1959.

33. FLÜGGE, W., Editor, "Handbook of Engineering Mechanics", McGraw-Hill Book Company Inc., 1962.

34. BARES, R., "Tables for the Analysis of Plates, Slabs and Diaphragms Based on the Elastic Theory", Bauverlay GmbH., Wiesbaden, Germany 1969.

35. TURNER, C.E., "Introduction to Plate and Shell Theory", Longmans, Green & Co. Ltd., 1965.

36. The Billings Microsystem Model BC-12FD, Stemmos Ltd., London.

37. ZIENKIEWICZ, O.C., " The Finite Element Method in Engineering Science", 2nd Edition, McGraw-Hill, London, 1971.

38. "Shell", Finite Element Programme for the Analysis of Plates and Shells, Stemmos Ltd., London.

<u>CHAPTER THREE</u>

39. TSAI, S.W. "Mechanics of Composite Materials", Part II, Technical Report AFML-TR-66-149, November, 1966.

40. LEKHNITSKI, S.G., "Theory of Elasticity of an Anisotropic Elastic Body, Holden-Day, Inc., 1963.

41. ALEXANDER, J.M., "Strength of Material" To be published.

42. ASHTON, J.E., HALPIN, J.C. and PETIT, P.H. "Primer and Composite Materials Analysis", Technomic Publishing Co., Inc., 1969.

43. SCHWARTZ, R.T. and SCHWARTZ, H.S., Editors, "Fundamental Aspects of Fiber Reinforced Plastics Composites.

44. WILLIAMS, J.G., "Stress Analysis of Polymers" Longman Group Ltd., 1973.

<u>CHAPTER FOUR</u>

45. IRONS, B.M., "Quadrature Rules for Brick Based Finite Elements", Int. J. Num. Meth. Eng., 3, 1971.

46. BREBBIA, C.A., and CONNOR, J.J., "Fundamentals of Finite Element Techniques", Butterworths, 1973.

47. COOK, R.D., "Concepts and Applications of Finite Element Analysis", John Wiley & Sons, 1974.

48. ROBINSON, J., "Integrated Theory of Finite Element Methods", John Wiley & Sons, 1973.

49. FENNER, R.T., "Finite Element Methods for Engineers"., The MacMillan Press Ltd., 1975.

CHAPTER FIVE

50. BRAMHAM, H.T., "Throughflow Type Butterfly Valve Development", Proceedings of the International Valve Conference, Brighton, 1978.

51. TSAI, S.W., WU, E.M., "A General Theory of Strength for Anisotropic Materials". J. Composite Materials, V. 5, Jan., 1971.

52. AZZI, V.D., TSAI, S.W., "Elastic Moduli of Laminated Anisotropic Composites", Experimental Mechanics, V. 5, June, 1965.

53. TSAI, S.W., PAGANO, N.J., "Invariant Properties of Composite Materials", Composite Materials Workshop, Edited by Tsai, S.W., Halpin, J.C., Pagano, N.J., Technomic Publishing Company, Stamford, Connecticut, 1968.

54. ARGYRIS, J.H., "Three-Dimensional Anisotropic and Inhomogeneous Elastic Media Matrix Analysis for Small and Large Displacements", Sonderabdruck Aus, Ingenieur-Archiv, 34 Band, 1 Heft, S.33-55, Springer-Verlag, Berlin, 1965.

55. HUANG, Chi-Lung, KIRMSER, P.G., "A Criterion of Strength for Orthotropic Materials", Fibre Sci. Technol., V. 8, No. 2, April, 1975.

56. OWEN, D.R., PRAKASH, A., ZIENKIEWICZ, O.C., "Finite Element Analysis of Non-linear Composite Materials by use of Overlay Systems", Computers and Structures, V. 4, 1974.

57. TSAI, S.W., "Strength Characteristics of Composite Materials", NASA Report CR-224, 1965.

58. ISHAI, O., MOEHLERPAH, A.E., PREIS, A., "Yield and Failure of Glass-Epoxy Composites", J. Engineering Mechanics Div., Proc. ASCE., Oct., 1970.

59. AGARWAL, B.D., NARANG, J.N., "Strength and Failure Mechanism of Anisotropic Composites", Fibre Sci. Technol. V. 10, No. 1, Jan., 1977.

60. ECKOLD, G.C., LEADBETTER, D., SODEN, P.D., GRIGGS, P.R., "Lamination Theory in the Prediction of Failure Envelopes for Filament Wound Materials subjected to Biaxial Loading", Composites, Oct. 1978.

61. Symposium on "Reinforced Plastics in Anti-Corrosion Applications", National Engineering Laboratory, East Kilbride, Glasgow, Sept., 1979.

62. Symposium on "Reinforced Plastic Constructed Equipment in Chemical Process Industry", The North Western Branches of the Institution of Chemical and Mechanical Engineering, Manchester, March, 1980.

63. British Standards Institution, BS.4994, 1973.

64. BRYAN-BROWN, M.H., "Operational Experience of G.R.P. Structures in Power Stations". (Ref. (62) above).

65. CHEREMISINOFF, N.P., CHEREMISINOFF, P.N., "Fibre Glass Reinforced Plastics Deskbook", Ann Arbor Science Publishers Inc., Michigan, 1978.

66. PENN, W.S., "G.R.P. Technology Handbook to the Polyester/Glass Fibre Plastics Industry", MacLaren & Sons Ltd., London, 1966.

67. TODS of Weymouth, "Fibre Reinforced Composites", Dorset, 1980.

68. TSAI, S.W., ADAMS, D.F., DONER, D.R., NASA Report CR-620, Nov., 1966.

69. LANGLEY, M., Editor "Carbon Fibres in Engineering" McGraw-Hill Book Company (U.K.) Ltd., England, 1973.

70. BRYAN-BROWN, D.M., WALTER, O.M., WIGATT, R.C., "Advances in the use of G.R.P. for the Power Industry" to be submitted for Reinforced Plastic Congress 80, Brighton, November, 1980.

71. 21st International Machine Tool Design and Research Conference to be held 8-12th Sept., 1980, University of Swansea, (Private communication Prof. J.M. Alexander).

APPENDICIES

APPENDIX 1

Coefficients tables for the calculation of deflections and forces in circular plates simply supported on two points at opposite ends of a diameter.

$$W = C_1 \frac{qa^2}{D}$$

$$M_r = C_2 \, q \, a^2$$

$$M_t = C_3 \, q \, a^2$$

$$M_{rt} = C_4 \, q \, a^2$$

$$Q_r = C_5 \, q \, a^2$$

$$Q_t = C_6 \, q \, a^2$$

in the tables coefficients C_1 to C_6 are listed under W, MR, MT, MRT, QR and QT respectively.

POISSON,S RATIO= .050
THETA = 0/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.240636	.518494	-.137244	0	-.500000	[illegible]
.10	.238005	.515724	-.140559	0	-.566236	[illegible]
.20	.230138	.507379	-.150683	0	-.636612	[illegible]
.30	.217112	.493347	-.168190	0	-.716176	[illegible]
.40	.199062	.473405	-.194177	0	-.812223	[illegible]
.50	.176173	.447085	-.230480	0	-.936732	[illegible]
.60	.148692	.413204	-.279912	0	-1.111037	[illegible]
.70	.116937	.368472	-.346271	0	-1.374608	[illegible]
.80	.081329	.304038	-.433535	0	-1.800730	[illegible]
.90	.042488	.197918	-.543121	0	-2.523087	[illegible]
1.00	.001465	-.000000	-.667316	0	-3.778689	[illegible]

POISSON,S RATIO= .050
THETA = 15.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.240636	.474568	-.093318	.163934	-.500000	0
.10	.238237	.471019	-.095384	.165258	-.557116	.033361
.20	.231074	.460221	-.101534	.169322	-.616196	.070334
.30	.219252	.441683	-.111584	.176415	-.679135	.115418
.40	.202954	.414454	-.125075	.187017	-.747452	.175223
.50	.182446	.376948	-.141011	.201734	-.821256	.260489
.60	.158100	.326841	-.157492	.221055	-.896650	.389576
.70	.130410	.261438	-.171401	.244611	-.960062	.594501
.80	.100029	.179374	-.178564	.269430	-.977198	.931025
.90	.067770	.085270	-.175291	.286253	-.873285	1.494918
1.00	.034564	-.000000	-.162795	.272458	-.500002	2.447247

POISSON,S RATIO= .050
THETA = 30.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.240636	.354560	.026690	.283943	-.500000	0
.10	.238870	.349796	.027117	.284674	-.532452	.057356
.20	.233618	.335401	.028609	.286697	-.562737	.118113
.30	.225027	.311116	.031800	.289432	-.587850	.185563
.40	.213341	.276708	.037698	.291705	-.602998	.262420
.50	.198908	.232375	.047524	.291472	-.600538	.349383
.60	.182178	.179440	.062365	.285579	-.569038	.441775
.70	.163684	.121348	.082606	.269921	-.493069	.522621
.80	.144014	.064724	.107219	.240756	-.354801	.549752
.90	.123740	.019934	.133238	.198537	-.139063	.433458
1.00	.103312	-.000000	.156045	.156524	.155740	.000001

POISSON,S RATIO= .050
THETA = 45.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.240636	.190625	.190625	.327869	-.500000	0
.10	.239732	.186325	.192317	.326290	-.499344	.065567
.20	.237064	.173727	.197298	.321309	-.494763	.130938
.30	.232759	.153797	.205216	.312251	-.482439	.195141
.40	.227020	.128311	.215289	.298241	-.459107	.255752
.50	.220104	.099960	.226087	.278639	-.423156	.308658
.60	.212300	.072224	.235389	.253702	-.376717	.349061
.70	.203886	.048719	.240273	.225274	-.329085	.375268
.80	.195098	.031672	.237643	.196913	-.301781	.397730
.90	.186107	.019076	.225436	.172219	-.335605	.457003
1.00	.177042	-.000000	.204815	.149180	-.500002	.655737

POISSON,S RATIO= .050
THETA = 60.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.240636	.026691	.354559	.283943	-.500000	0
.10	.240592	.025248	.355107	.280514	-.466892	.056221
.20	.240476	.021302	.356464	.270398	-.432009	.109041
.30	.240326	.015886	.357847	.254157	-.394302	.155146
.40	.240195	.010360	.358196	.232834	-.353961	.191591
.50	.240140	.005896	.356550	.207954	-.312372	.216285
.60	.240203	.002913	.352535	.181297	-.270967	.228242
.70	.240412	.000736	.346850	.154421	-.228011	.226587
.80	.240789	-.001944	.341444	.128197	-.171943	.206425
.90	.241357	-.004781	.338842	.103125	-.069266	.148717
1.00	.242136	.000000	.339652	.081989	.155739	.000002

POISSON,S RATIO= .050
THETA = 75.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.240636	-.093318	.474568	.163935	-.500000	0
.10	.241221	-.091188	.472785	.161097	-.443539	.032226
.20	.242953	-.084948	.467486	.152980	-.389042	.061234
.30	.245769	-.075073	.458836	.140674	-.338426	.084504
.40	.249564	-.062389	.447160	.125679	-.293441	.100673
.50	.254207	-.048001	.432892	.109564	-.255589	.109645
.60	.259548	-.033065	.416353	.093708	-.226633	.112455
.70	.265429	-.018500	.397346	.079203	-.210854	.111396
.80	.271696	-.005171	.374795	.066881	-.221022	.111321
.90	.278206	.004311	.347073	.057144	-.291112	.123705
1.00	.284856	-.000000	.314229	.048853	-.500001	.175702

POISSON,S RATIO= .050
THETA = 90.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.240636	-.137244	.518494	.000001	-.500000	0
.10	.241451	-.133498	.515549	.000001	-.435075	.000000
.20	.243856	-.122784	.507037	.000001	-.373897	.000000
.30	.247742	-.106539	.493831	.000000	-.319521	.000000
.40	.252940	-.086790	.477136	.000000	-.273860	.000000
.50	.259246	-.065812	.458269	.000000	-.237449	.000000
.60	.266442	-.045910	.438611	.000000	-.208955	.000000
.70	.274322	-.029328	.419797	.000000	-.183233	.000000
.80	.282710	-.017849	.403957	.000000	-.145782	.000001
.90	.291484	-.010912	.393398	.000000	-.060253	.000001
1.00	.300562	.000000	.388422	.000000	.155738	.000003

POISSON,S RATIO= .100
THETA = 0/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.245898	.516331	-.128831	0	-.500000	0
.10	.243227	.513503	-.132319	0	-.565168	0
.20	.235240	.504984	-.142969	0	-.634409	0
.30	.222012	.490657	-.161372	0	-.712689	0
.40	.203669	.470295	-.188661	0	-.807187	0
.50	.180389	.443426	-.226738	0	-.929688	0
.60	.152407	.408882	-.278555	0	-1.101181	0
.70	.120021	.363472	-.348225	0	-1.360502	0
.80	.083625	.298684	-.440461	0	-1.779750	0
.90	.043799	.193474	-.558339	0	-2.490456	0
1.00	.001524	-.000000	-.697715	0	-3.725806	0

POISSON,S RATIO= .100
THETA = 15.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.245898	.473113	-.085613	.161290	-.500000	0
.10	.243468	.469540	-.087871	.162598	-.556195	.032823
.20	.236212	.458673	-.094606	.166615	-.614321	.069200
.30	.224234	.440038	-.105660	.173634	-.676246	.113556
.40	.207711	.412719	-.120622	.184142	-.743461	.172397
.50	.185906	.375191	-.138577	.198779	-.816074	.256287
.60	.162181	.325223	-.157731	.218111	-.890252	.383292
.70	.134025	.260225	-.175058	.241979	-.952641	.584912
.80	.103078	.178871	-.186338	.267893	-.969502	.916008
.90	.070147	.085525	-.187367	.287654	-.867265	1.470807
1.00	.036160	-.000000	-.177695	.280829	-.500002	2.407776

POISSON,S RATIO= .100
THETA = 30.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.245898	.355040	.032460	.279363	-.500000	0
.10	.244125	.350312	.032673	.280118	-.531929	.056431
.20	.238855	.336029	.033524	.282224	-.561725	.116208
.30	.230233	.311946	.035643	.285138	-.586433	.182570
.40	.218504	.277848	.040045	.287757	-.601337	.250187
.50	.204018	.233933	.047994	.288150	-.598917	.343747
.60	.187225	.181480	.060695	.283314	-.567924	.434649
.70	.168666	.123802	.078782	.269294	-.493181	.514191
.80	.148935	.067260	.101665	.242357	-.357143	.540885
.90	.128618	.021813	.127009	.202493	-.144885	.426467
1.00	.108189	-.000000	.150941	.161332	.145163	.000001

POISSON,S RATIO= .100
THETA = 45.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.245898	.193750	.193750	.322581	-.500000	0
.10	.245021	.189498	.195270	.321109	-.499355	.064510
.20	.242434	.177031	.199752	.316457	-.494847	.128826
.30	.238267	.157284	.206897	.307978	-.482722	.191993
.40	.232726	.131970	.216014	.294823	-.459767	.251627
.50	.226072	.103683	.225809	.276354	-.424395	.303680
.60	.218599	.075775	.234228	.252774	-.378706	.343431
.70	.210593	.051739	.238495	.225790	-.331842	.369215
.80	.202299	.033751	.235537	.198770	-.304978	.391315
.90	.193898	.019956	.223012	.175250	-.338257	.449632
1.00	.185524	-.000000	.201209	.153763	-.500002	.645160

POISSON,S RATIO= .100
THETA = 60.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.245898	.032460	.355040	.279363	-.500000	0
.10	.245914	.030998	.355535	.276093	-.467426	.055314
.20	.245979	.026979	.356752	.266444	-.433105	.107282
.30	.246133	.021399	.357954	.250941	-.396007	.152644
.40	.246433	.015568	.358139	.230562	-.356316	.188501
.50	.246936	.010621	.356388	.206742	-.315398	.212796
.60	.247691	.006978	.352331	.181164	-.274661	.224561
.70	.248730	.004008	.346620	.155298	-.232398	.222932
.80	.250079	.000471	.341158	.129959	-.177235	.203096
.90	.251769	-.003317	.338586	.105558	-.076213	.146318
1.00	.253830	.000000	.340188	.084508	.145163	.000002

POISSON,S RATIO= .100
THETA = 75.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.245898	-.085613	.473113	.161291	-.500000	0
.10	.246567	-.083599	.471413	.158572	-.444450	.031706
.20	.248554	-.077703	.466354	.150795	-.390832	.060246
.30	.251793	-.068384	.458075	.138991	-.341032	.083141
.40	.256183	-.056439	.446858	.124587	-.296773	.099050
.50	.261594	-.042936	.433080	.109077	-.259531	.107876
.60	.267874	-.028998	.417002	.093777	-.231042	.110642
.70	.274870	-.015526	.398375	.079735	-.215518	.109600
.80	.282425	-.003372	.376051	.067762	-.225521	.109526
.90	.290396	.004957	.348166	.058288	-.294481	.121709
1.00	.298679	-.000000	.313984	.050354	-.500001	.172868

POISSON,S RATIO= .100
THETA = 90.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.245898	-.128831	.516331	.000001	-.500000	0
.10	.246806	-.125247	.513528	.000001	-.436123	.000000
.20	.249492	-.114999	.505416	.000000	-.375931	.000000
.30	.253845	-.099473	.492802	.000000	-.322432	.000000
.40	.259698	-.080626	.476792	.000000	-.277507	.000000
.50	.266845	-.060657	.458599	.000000	-.241683	.000000
.60	.275069	-.041796	.439502	.000000	-.213649	.000000
.70	.284163	-.026216	.421045	.000000	-.188342	.000000
.80	.293951	-.015646	.405315	.000000	-.151495	.000001
.90	.304310	-.009592	.394773	.000000	-.067346	.000001
1.00	.315164	.000000	.390457	.000000	.145161	.000003

POISSON,S RATIO= .150
THETA = 0/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.252373	.514335	-.120585	0	-.500000	0
.10	.249652	.511451	-.124245	0	-.564133	0
.20	.241511	.502760	-.135413	0	-.632275	0
.30	.228024	.488143	-.154699	0	-.709313	0
.40	.209309	.467367	-.183269	0	-.802311	0
.50	.185536	.439957	-.223090	0	-.922867	0
.60	.156925	.404761	-.277249	0	-1.091639	0
.70	.123754	.358681	-.350164	0	-1.346843	0
.80	.086390	.293535	-.447258	0	-1.759437	0
.90	.045364	.189191	-.573215	0	-2.458862	0
1.00	.001590	-.000000	-.727348	0	-3.674603	0

POISSON,S RATIO= .150
THETA = 15.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.252373	.471804	-.078054	.158730	-.500000	0
.10	.249902	.468205	-.080501	.160022	-.555303	.032302
.20	.242524	.457269	-.087811	.163995	-.612507	.068101
.30	.230339	.438535	-.099851	.170941	-.673448	.111754
.40	.213522	.411123	-.116260	.181359	-.739596	.169661
.50	.192328	.373565	-.136197	.195917	-.811057	.252219
.60	.167114	.323720	-.157970	.215262	-.884058	.377208
.70	.138359	.259101	-.178645	.239431	-.945456	.575628
.80	.106693	.178420	-.193956	.266405	-.962049	.901468
.90	.072917	.085790	-.199201	.289011	-.861435	1.447461
1.00	.037954	-.000000	-.192320	.288933	-.500002	2.369557

POISSON,S RATIO= .150
THETA = 30.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.252373	.355605	.038145	.274929	-.500000	0
.10	.250587	.350910	.038149	.275707	-.531422	.055536
.20	.245278	.336732	.038370	.277893	-.560745	.114363
.30	.236592	.312841	.039437	.280981	-.585061	.179672
.40	.224774	.279035	.042369	.283935	-.599728	.254089
.50	.210176	.235515	.048473	.284934	-.597346	.338291
.60	.193250	.183518	.059069	.281121	-.566846	.427750
.70	.174543	.126228	.075033	.268687	-.493289	.506030
.80	.154660	.069751	.096195	.243907	-.359410	.532299
.90	.134203	.023650	.120837	.206323	-.150521	.419697
1.00	.113670	-.000000	.145800	.165988	.134922	.000001

POISSON,S RATIO= .150
THETA = 45.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.252373	.196875	.196875	.317460	-.500000	0
.10	.251521	.192667	.198226	.316092	-.499365	.063486
.20	.249009	.180326	.202216	.311759	-.494929	.126781
.30	.244969	.160751	.208596	.303841	-.482396	.188946
.40	.239610	.135596	.216767	.291512	-.460406	.247633
.50	.233199	.107362	.225566	.274141	-.425595	.298859
.60	.226036	.079277	.233097	.251876	-.380631	.337980
.70	.218416	.054715	.236726	.226290	-.334511	.363355
.80	.210593	.035800	.233405	.200568	-.308073	.385103
.90	.202758	.020827	.220523	.178185	-.340824	.442495
1.00	.195050	-.000000	.197520	.158201	-.500002	.634920

POISSON,S RATIO= .150
THETA = 60.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.252373	.038145	.355605	.274929	-.500000	0
.10	.252452	.036663	.356046	.271813	-.467943	.054436
.20	.252705	.032571	.357119	.262616	-.434167	.105579
.30	.253174	.026828	.358130	.247827	-.397658	.150221
.40	.253918	.020694	.358134	.228362	-.358597	.185509
.50	.254999	.015271	.356256	.205570	-.318329	.209419
.60	.256469	.010976	.352126	.181034	-.278238	.220997
.70	.258367	.007227	.346352	.156147	-.236645	.219394
.80	.260723	.002845	.340789	.131666	-.182358	.199872
.90	.263574	-.001880	.338195	.107913	-.082940	.143996
1.00	.266958	.000000	.340509	.086946	.134922	.000002

POISSON,S RATIO= .150
THETA = 75.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.252373	-.078053	.471803	.158731	-.500000	0
.10	.253132	-.076153	.470181	.156128	-.445332	.031203
.20	.255389	-.070594	.465344	.148679	-.392564	.059290
.30	.259078	-.061817	.457411	.137362	-.343556	.081821
.40	.264100	-.050594	.446617	.123530	-.299999	.097477
.50	.270323	-.037958	.433287	.108606	-.263348	.106164
.60	.277598	-.024997	.417623	.093844	-.235312	.108885
.70	.285770	-.012597	.399325	.080250	-.220033	.107860
.80	.294685	-.001595	.377175	.068615	-.229878	.107787
.90	.304196	.005602	.349083	.059395	-.297743	.119777
1.00	.314196	-.000000	.313549	.051807	-.500001	.170124

POISSON,S RATIO= .150
THETA = 90.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.252373	-.120585	.514335	.000001	-.500000	0
.10	.253381	-.117159	.511667	.000001	-.437137	.000000
.20	.256367	-.107366	.503935	.000000	-.377900	.000000
.30	.261219	-.092541	.491878	.000000	-.325250	.000000
.40	.267770	-.074575	.476511	.000000	-.261039	.000000
.50	.275814	-.055591	.458944	.000000	-.245784	.000000
.60	.285132	-.037749	.440357	.000000	-.218194	.000000
.70	.295515	-.023152	.422206	.000000	-.193289	.000000
.80	.306786	-.013478	.406539	.000000	-.157027	.000001
.90	.318825	-.008294	.395962	.000000	-.074214	.000001
1.00	.331555	.000000	.392229	.000000	.134921	.000003

POISSON,S RATIO= .200
THETA = 0/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.260214	.512500	-.112500	0	-.500000	0
.10	.257429	.509559	-.116328	0	-.563131	0
.20	.249098	.500699	-.128007	0	-.630208	0
.30	.235289	.485796	-.148163	0	-.706043	0
.40	.216114	.464613	-.177994	0	-.797588	0
.50	.191732	.436670	-.219531	0	-.916260	0
.60	.162349	.400831	-.275991	0	-1.082395	0
.70	.128221	.354090	-.352088	0	-1.333611	0
.80	.089683	.288583	-.453933	0	-1.739758	0
.90	.047219	.185060	-.587766	0	-2.428255	0
1.00	.001666	-.000000	-.756250	0	-3.625000	0

POISSON,S RATIO= .200
THETA = 15.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.260214	.470633	-.070633	.156250	-.500000	0
.10	.257692	.467009	-.073266	.157527	-.554439	.031798
.20	.250158	.456002	-.081142	.161456	-.610749	.067037
.30	.237711	.437168	-.094153	.168332	-.670738	.110087
.40	.220520	.409659	-.111983	.178663	-.735853	.167010
.50	.198837	.372063	-.133866	.193145	-.806197	.248278
.60	.173010	.322326	-.158210	.212501	-.878057	.371315
.70	.143508	.258062	-.182166	.236963	-.938496	.566634
.80	.110953	.178018	-.201426	.264963	-.954830	.887383
.90	.076137	.086066	-.210805	.290325	-.855788	1.424844
1.00	.039983	-.000000	-.206683	.296785	-.500002	2.332533

POISSON,S RATIO= .200
THETA = 30.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.260214	.356250	.043750	.270633	-.500000	0
.10	.258408	.351586	.043548	.271434	-.530931	.054668
.20	.253039	.337507	.043151	.273698	-.559796	.112576
.30	.244252	.313796	.043183	.276953	-.583732	.176865
.40	.232295	.280267	.044672	.280232	-.598170	.250119
.50	.217521	.237122	.048962	.281818	-.595825	.333005
.60	.200387	.185555	.057487	.278996	-.565802	.421066
.70	.181445	.128628	.071356	.268098	-.493394	.498123
.80	.161313	.072199	.090806	.245409	-.361607	.523982
.90	.140611	.025449	.114718	.210034	-.155982	.413140
1.00	.119864	-.000000	.140625	.170499	.125002	.000001

POISSON,S RATIO= .200
THETA = 45.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.260214	.200000	.200000	.312500	-.500000	0
.10	.259385	.195835	.201184	.311231	-.499375	.062494
.20	.256942	.183611	.204689	.307208	-.495008	.124800
.30	.253019	.164199	.210314	.299832	-.483262	.185994
.40	.247830	.139192	.217547	.288305	-.461024	.243764
.50	.241646	.110999	.225354	.271997	-.426758	.294190
.60	.234776	.082732	.231993	.251006	-.382496	.332699
.70	.227524	.057647	.234967	.226774	-.337097	.357677
.80	.220155	.037820	.231251	.202309	-.311072	.379086
.90	.212869	.021689	.217973	.181028	-.343311	.435581
1.00	.205813	-.000000	.193750	.162500	-.500002	.624999

POISSON,S RATIO= .200
THETA = 60.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.260214	.043750	.356250	.270633	-.500000	0
.10	.260359	.042248	.356636	.267667	-.468444	.053585
.20	.260809	.038083	.357559	.258907	-.435196	.103930
.30	.261608	.032175	.358371	.244810	-.399257	.147873
.40	.262820	.025742	.358181	.226231	-.360806	.182610
.50	.264508	.019849	.356152	.204434	-.321167	.206146
.60	.266731	.014913	.351919	.180909	-.281703	.217544
.70	.269531	.010394	.346045	.156970	-.240760	.215966
.80	.272944	.005181	.340342	.133319	-.187321	.196749
.90	.277013	-.000470	.337677	.110195	-.089457	.141746
1.00	.281787	.000000	.340625	.089309	.125001	.000002

POISSON,S RATIO= .200
THETA = 75.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.260214	-.070633	.470633	.156250	-.500000	0
.10	.261070	-.068844	.469082	.153760	-.446186	.030715
.20	.263617	-.063612	.464453	.146629	-.394243	.058363
.30	.267792	-.055367	.456838	.135783	-.346000	.080543
.40	.273492	-.044850	.446435	.122507	-.303124	.095954
.50	.280588	-.033062	.433512	.108149	-.267045	.104505
.60	.288931	-.021058	.418216	.093909	-.239447	.107184
.70	.298364	-.009709	.400199	.080749	-.224408	.106175
.80	.308734	.000162	.378174	.069442	-.234099	.106103
.90	.319891	.006245	.349831	.060468	-.300903	.117906
1.00	.331720	-.000000	.312932	.053215	-.500001	.167466

POISSON,S RATIO= .200
THETA = 90.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.260214	-.112500	.512500	.000001	-.500000	0
.10	.261331	-.109228	.509959	.000001	-.438119	.000000
.20	.264641	-.099878	.502586	.000000	-.379808	.000000
.30	.270034	-.085737	.491054	.000000	-.327981	.000000
.40	.277340	-.068630	.476289	.000000	-.284460	.000000
.50	.286352	-.050610	.459302	.000000	-.249756	.000000
.60	.296849	-.033767	.441177	.000000	-.222597	.000000
.70	.308620	-.020134	.423285	.000000	-.198082	.000000
.80	.321487	-.011343	.407635	.000000	-.162386	.000001
.90	.335326	-.007019	.396975	.000000	-.080867	.000001
1.00	.350065	.000000	.393750	.000000	.125000	.000003

POISSON,S RATIO= .250
THETA = 0/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.269626	.510817	-.104567	0	-.500000	0
.10	.266764	.507821	-.108562	0	-.562160	0
.20	.258200	.498734	-.120745	0	-.628205	0
.30	.243996	.483609	-.141758	0	-.702873	0
.40	.224259	.462024	-.172832	0	-.793010	0
.50	.199137	.433556	-.216058	0	-.909856	0
.60	.168817	.397084	-.274780	0	-1.073435	0
.70	.133534	.349690	-.353398	0	-1.320786	0
.80	.093587	.283817	-.460490	0	-1.720685	0
.90	.049409	.181074	-.602006	0	-2.398589	0
1.00	.001732	-.000000	-.784455	0	-3.576923	0

POISSON,S RATIO= .250
THETA = 15.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.269626	.469594	-.063344	.153846	-.500000	0
.10	.267040	.465945	-.066161	.155109	-.553602	.031308
.20	.259312	.454867	-.074594	.158995	-.609045	.066006
.30	.246540	.435931	-.088559	.165803	-.668112	.108315
.40	.228888	.408320	-.107787	.176050	-.732224	.164440
.50	.206600	.370679	-.131583	.190458	-.801486	.244459
.60	.180018	.321037	-.158449	.209825	-.872241	.365602
.70	.149602	.257104	-.185625	.234570	-.931750	.557916
.80	.115963	.177663	-.208754	.263566	-.947832	.873731
.90	.079886	.086351	-.222190	.291599	-.850314	1.402923
1.00	.042295	-.000000	-.220797	.304395	-.500002	2.296647

POISSON,S RATIO= .250
THETA = 30.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.269626	.356971	.049279	.266469	-.500000	0
.10	.267792	.352337	.048875	.267292	-.530455	.053827
.20	.262338	.338350	.047869	.269631	-.558876	.110844
.30	.253411	.314809	.046884	.273050	-.582443	.174144
.40	.241261	.281542	.046953	.276643	-.596660	.246271
.50	.226241	.238751	.049460	.278797	-.594351	.327882
.60	.208815	.187591	.055946	.276937	-.564789	.414588
.70	.189543	.131003	.067746	.267528	-.493495	.490459
.80	.169056	.074607	.085495	.246865	-.363736	.515921
.90	.147994	.027211	.108650	.213630	-.161275	.406784
1.00	.126915	-.000000	.135417	.174871	.115336	.000001

POISSON,S RATIO= .250
THETA = 45.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.269626	.203125	.203125	.307692	-.500000	0
.10	.268818	.199000	.204144	.306521	-.499385	.061532
.20	.266438	.186887	.207170	.302798	-.495085	.122880
.30	.262625	.167627	.212049	.295947	-.483519	.183132
.40	.257594	.142757	.218353	.285197	-.461624	.240014
.50	.251626	.114596	.225173	.269919	-.427885	.289664
.60	.245036	.086143	.230915	.250162	-.384304	.327580
.70	.238140	.060538	.233217	.227244	-.339603	.352175
.80	.231213	.039813	.229074	.203997	-.313979	.373254
.90	.224468	.022543	.215363	.183783	-.345722	.428880
1.00	.218058	-.000000	.189904	.166667	-.500002	.615384

POISSON,S RATIO= .250
THETA = 60.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.269626	.049279	.356971	.266470	-.500000	0
.10	.269841	.047757	.357301	.263648	-.468929	.052761
.20	.270501	.043517	.358071	.255313	-.436193	.102331
.30	.271654	.037446	.358676	.241886	-.400807	.145598
.40	.273364	.030715	.358276	.224165	-.362948	.179801
.50	.275702	.024358	.356076	.203333	-.323918	.202975
.60	.278727	.018789	.351711	.180788	-.285061	.214197
.70	.282490	.013513	.345704	.157767	-.244748	.212643
.80	.287031	.007479	.339820	.134921	-.192131	.193722
.90	.292400	.000916	.337038	.112407	-.095773	.139565
1.00	.298654	.000000	.340545	.091599	.115386	.000002

POISSON,S RATIO= .250
THETA = 75.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.269626	-.063344	.469594	.153847	-.500000	0
.10	.270588	-.061664	.468110	.151465	-.447014	.030242
.20	.273453	-.056754	.463673	.144642	-.395870	.057465
.30	.278156	-.049027	.456354	.134254	-.348369	.079303
.40	.284597	-.039202	.446308	.121514	-.306153	.094478
.50	.292644	-.028244	.433754	.107707	-.270629	.102897
.60	.302149	-.017179	.418784	.093971	-.243456	.105535
.70	.312954	-.006861	.401001	.081233	-.228648	.104541
.80	.324902	.001899	.379054	.070243	-.238190	.104471
.90	.337842	.006887	.350419	.061508	-.303966	.116092
1.00	.351651	-.000000	.312142	.054580	-.500001	.164890

POISSON,S RATIO= .250
THETA = 90.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.269626	-.104567	.510817	.000001	-.500000	0
.10	.270862	-.101445	.508397	.000001	-.439071	.000000
.20	.274529	-.092528	.501362	.000000	-.381657	.000000
.30	.280515	-.079055	.490325	.000000	-.330627	.000000
.40	.288648	-.062788	.476123	.000000	-.287776	.000000
.50	.298720	-.045711	.459673	.000000	-.253606	.000000
.60	.310506	-.029845	.441965	.000000	-.226865	.000000
.70	.323792	-.017160	.424286	.000000	-.202727	.000000
.80	.338397	-.009238	.408608	.000000	-.167580	.000001
.90	.354195	-.005764	.397818	.000000	-.087315	.000001
1.00	.371116	.000000	.395032	.000000	.115385	.000003

POISSON,S RATIO= .300
THETA = 0/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.280884	.509280	-.096780	0	-.500000	0
.10	.277928	.506230	-.100939	0	-.561218	0
.20	.269080	.497037	-.113619	0	-.626263	0
.30	.254398	.481575	-.135477	0	-.699799	0
.40	.233980	.459593	-.167777	0	-.788570	0
.50	.207962	.430608	-.212667	0	-.903646	0
.60	.176514	.393510	-.273613	0	-1.064746	0
.70	.139844	.345471	-.355895	0	-1.308350	0
.80	.098212	.279230	-.466936	0	-1.702190	0
.90	.051994	.177227	-.615951	0	-2.369823	0
1.00	.001851	-.000000	-.811995	0	-3.530303	0

POISSON,S RATIO= .300
THETA = 15.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.280884	.468682	-.056182	.151515	-.500000	0
.10	.278219	.465008	-.059179	.152764	-.552789	.030834
.20	.270254	.453857	-.068161	.156609	-.607393	.065006
.30	.257082	.434818	-.083066	.163351	-.665564	.106674
.40	.238865	.407102	-.103669	.173516	-.728705	.161949
.50	.215840	.369409	-.129345	.187852	-.796918	.240755
.60	.188339	.319847	-.158690	.207231	-.866601	.360063
.70	.156812	.256223	-.189022	.232250	-.925208	.549463
.80	.121862	.177353	-.215948	.262211	-.941047	.860492
.90	.084265	.086646	-.233365	.292835	-.845006	1.381667
1.00	.044948	-.000000	-.234672	.311774	-.500002	2.261850

POISSON,S RATIO= .300
THETA = 30.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.280884	.357765	.054735	.262432	-.500000	0
.10	.279013	.353158	.054132	.263275	-.529994	.053011
.20	.273447	.339259	.052528	.265688	-.557984	.109165
.30	.264335	.315877	.050542	.269264	-.581194	.171505
.40	.251928	.282858	.049214	.273163	-.595195	.242[illegible]
.50	.236583	.240402	.049967	.275869	-.592922	.322914
.60	.218770	.189626	.054443	.274940	-.563808	.408307
.70	.199060	.133354	.064201	.266975	-.493594	.483028
.80	.178098	.076976	.080258	.248277	-.365801	.508104
.90	.156548	.028937	.102630	.217117	-.166407	.400620
1.00	.135003	-.000000	.130177	.179110	.106062	.000001

POISSON,S RATIO= .300
THETA = 45.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.280884	.206250	.206250	.303030	-.500000	0
.10	.280095	.202163	.207107	.301953	-.499394	.060600
.20	.277775	.190155	.209660	.298520	-.495159	.121019
.30	.274063	.171038	.213800	.292180	-.483769	.180357
.40	.269182	.146294	.219184	.282183	-.462205	.236377
.50	.263419	.118155	.225020	.267905	-.428977	.285275
.60	.257100	.089510	.229863	.249344	-.386057	.322617
.70	.250552	.063390	.231475	.227699	-.342033	.346839
.80	.244064	.041779	.226877	.205634	-.316797	.367599
.90	.237860	.023390	.212698	.186455	-.348059	.422382
1.00	.232100	-.000000	.185985	.170707	-.500002	.606060

POISSON,S RATIO= .300
THETA = 60.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.280884	.054735	.357765	.262432	-.500000	0
.10	.281175	.053192	.358037	.259751	-.469400	.051962
.20	.282064	.048877	.358651	.251827	-.437150	.100780
.30	.283600	.042643	.359040	.239051	-.402310	.143392
.40	.285851	.035617	.358417	.222162	-.365024	.177077
.50	.288892	.028801	.356025	.202265	-.326586	.199900
.60	.292789	.022609	.351502	.180671	-.288318	.210951
.70	.297596	.016586	.345327	.158540	-.248616	.209421
.80	.303359	.009741	.339227	.136475	-.196796	.190787
.90	.310136	.002277	.336282	.114551	-.101897	.137450
1.00	.317993	.000000	.340278	.093820	.106062	.000002

POISSON,S RATIO= .300
THETA = 75.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.280884	-.056182	.468682	.151516	-.500000	0
.10	.281964	-.054607	.467259	.149240	-.447817	.029784
.20	.285180	-.050012	.463001	.142715	-.397448	.056595
.30	.290469	-.042793	.455953	.132770	-.350667	.078102
.40	.297729	-.033645	.446235	.120552	-.309090	.093047
.50	.306828	-.023501	.434012	.107278	-.274105	.101336
.60	.317616	-.013357	.419326	.094032	-.247343	.103936
.70	.329933	-.004050	.401735	.081702	-.232759	.102957
.80	.343619	.003618	.379821	.071020	-.242156	.102888
.90	.358519	.007527	.350854	.062516	-.306936	.114333
1.00	.374499	-.000000	.311187	.055903	-.500001	.162391

OK

POISSON,S RATIO= .300
THETA = 90.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.280884	-.096780	.509280	.000001	-.500000	0
.10	.282252	-.093804	.506974	.000000	-.439994	.000000
.20	.286316	-.085309	.500260	.000000	-.383450	.000000
.30	.292961	-.072489	.489688	.000000	-.333193	.000000
.40	.302014	-.057043	.476012	.000000	-.290992	.000000
.50	.313262	-.040888	.460057	.000000	-.257339	.000000
.60	.326477	-.025981	.442721	.000000	-.231004	.000000
.70	.341439	-.014227	.425212	.000000	-.207231	.000000
.80	.357965	-.007163	.409465	.000000	-.172617	.000001
.90	.375924	-.004530	.398501	.000000	-.093568	.000001
1.00	.395249	.000000	.396086	.000000	.106061	.000003

POISSON,S RATIO= .350
THETA = 0/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.294357	.507882	-.089132	0	-.500000	0
.10	.291287	.504778	-.093453	0	-.560305	0
.20	.282095	.495423	-.106623	0	-.624378	0
.30	.266834	.479686	-.129317	0	-.696817	0
.40	.245594	.457314	-.162824	0	-.784263	0
.50	.218495	.427817	-.209353	0	-.897621	0
.60	.185689	.390104	-.272488	0	-1.056317	0
.70	.147352	.341425	-.357779	0	-1.296285	0
.80	.103704	.274814	-.473275	0	-1.684247	0
.90	.055055	.173513	-.629613	0	-2.341915	0
1.00	.001967	-.000000	-.838899	0	-3.485075	0

POISSON,S RATIO= .350
THETA = 15.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.294357	.467890	-.049140	.149254	-.500000	0
.10	.291596	.464190	-.052315	.150489	-.552001	.030374
.20	.283341	.452967	-.061838	.154294	-.605790	.064036
.30	.269682	.433822	-.077669	.160972	-.663093	.105082
.40	.250776	.405998	-.099626	.171057	-.725232	.159532
.50	.226854	.368246	-.127151	.185325	-.792487	.237161
.60	.198238	.318753	-.158930	.204713	-.861129	.354689
.70	.165367	.255416	-.192363	.230000	-.918862	.541262
.80	.128834	.177086	-.223013	.260897	-.934464	.847649
.90	.089409	.086950	-.244339	.294033	-.839857	1.361045
1.00	.048021	-.000000	-.248320	.318933	-.500002	2.228091

POISSON,S RATIO= .350
THETA = 30.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.294357	.358629	.060121	.258515	-.500000	0
.10	.292438	.354047	.059323	.259379	-.529546	.052220
.20	.286729	.340230	.057130	.261862	-.557119	.107535
.30	.277379	.316999	.054159	.265592	-.579982	.168946
.40	.264641	.284212	.051457	.269787	-.593774	.238919
.50	.248880	.242073	.050482	.273027	-.591535	.318095
.60	.230570	.191660	.052979	.273003	-.562855	.402213
.70	.210295	.135683	.060718	.266439	-.493690	.475819
.80	.188718	.079307	.075091	.249646	-.367804	.500520
.90	.166532	.030630	.096656	.220501	-.171386	.394641
1.00	.144368	-.000000	.124907	.183223	.097017	.000001

POISSON,S RATIO= .350
THETA = 45.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.294357	.209375	.209375	.298507	-.500000	0
.10	.293586	.205324	.210071	.297521	-.499403	.059696
.20	.291321	.193415	.212157	.294371	-.495232	.119212
.30	.287704	.174433	.215568	.288525	-.484011	.177666
.40	.282966	.149803	.220039	.279259	-.462769	.232349
.50	.277402	.121678	.224896	.265950	-.430038	.281017
.60	.271348	.092837	.228834	.248551	-.387758	.317802
.70	.265146	.066204	.229741	.228140	-.344391	.341662
.80	.259100	.043720	.224659	.207223	-.319532	.362112
.90	.253445	.024228	.209978	.189047	-.350327	.416078
1.00	.248351	-.000000	.181996	.174627	-.500001	.597014

POISSON,S RATIO= .350
THETA = 60.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.294357	.060122	.358628	.258515	-.500000	0
.10	.294732	.058558	.358842	.255971	-.469857	.051186
.20	.295873	.054167	.359295	.248446	-.438098	.099276
.30	.297831	.047769	.359462	.236300	-.403768	.141252
.40	.300680	.040452	.358603	.220219	-.367039	.174434
.50	.304497	.033182	.356000	.201229	-.329175	.196916
.60	.309355	.026375	.351292	.180557	-.291477	.207803
.70	.315312	.019615	.344919	.159290	-.252368	.206296
.80	.322423	.011969	.338566	.137982	-.201321	.187940
.90	.330750	.003615	.335416	.116632	-.107839	.135399
1.00	.340372	.000000	.339832	.095974	.097016	.000002

POISSON,S RATIO= .350
THETA = 75.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.294357	-.049140	.467890	.149254	-.500000	0
.10	.295569	-.047670	.466524	.147081	-.448596	.029340
.20	.299180	-.043381	.462430	.140846	-.398979	.055750
.30	.305128	-.036661	.455632	.131331	-.352896	.076936
.40	.313308	-.028176	.446212	.119619	-.311939	.091658
.50	.323587	-.018830	.434286	.106862	-.277476	.099826
.60	.335812	-.009590	.419845	.094091	-.251114	.102385
.70	.349821	-.001277	.402402	.082157	-.236748	.101421
.80	.365449	.005319	.380478	.071774	-.246005	.101352
.90	.382536	.008166	.351142	.063495	-.309818	.112627
1.00	.400936	-.000000	.310073	.057186	-.500001	.159968

POISSON,S RATIO= .350
THETA = 90.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.294357	-.089132	.507882	.000000	-.500000	0
.10	.295875	-.086299	.505684	.000000	-.440890	.000000
.20	.300386	-.078217	.499272	.000000	-.385189	.000000
.30	.307775	-.066034	.489138	.000000	-.335683	.000000
.40	.317864	-.051391	.475953	.000000	-.294111	.000000
.50	.330436	-.036140	.460452	.000000	-.260961	.000000
.60	.345257	-.022173	.443447	.000000	-.235018	.000000
.70	.362102	-.011335	.426066	.000000	-.211600	.000000
.80	.380781	-.005117	.410211	.000000	-.177503	.000001
.90	.401160	-.003314	.399030	.000000	-.099634	.000001
1.00	.423170	.000000	.396922	.000000	.097015	.000003

POISSON,S RATIO= .400
THETA = 0/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.310546	.506618	-.081618	0	-.500000	0
.10	.307338	.503460	-.086098	0	-.559418	0
.20	.297728	.493944	-.099753	0	-.622549	0
.30	.281766	.477936	-.123270	0	-.693923	0
.40	.259529	.455178	-.157969	0	-.780083	0
.50	.231124	.425178	-.206114	0	-.891774	0
.60	.196676	.386856	-.271464	0	-1.048136	0
.70	.156332	.337545	-.359651	0	-1.284575	0
.80	.110262	.270560	-.479513	0	-1.666831	0
.90	.058703	.169925	-.643004	0	-2.314828	0
1.00	.002103	-.000000	-.865196	0	-3.441176	0

POISSON,S RATIO= .400
THETA = 15.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.310546	.467213	-.042213	.147059	-.500000	0
.10	.307667	.463488	-.045564	.148281	-.551237	.029927
.20	.299058	.452191	-.055619	.152047	-.604234	.063094
.30	.284806	.432940	-.072364	.158663	-.660695	.103536
.40	.265062	.405004	-.095654	.168671	-.721979	.157186
.50	.240048	.367186	-.124999	.182871	-.788185	.233674
.60	.210076	.317750	-.159171	.202270	-.855818	.349473
.70	.175577	.254680	-.195648	.227815	-.912702	.533303
.80	.137130	.176859	-.229954	.259621	-.928075	.835184
.90	.095498	.087262	-.255123	.295196	-.834859	1.341030
1.00	.051619	-.000000	-.261750	.325881	-.500002	2.195325

POISSON,S RATIO= .400
THETA = 30.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.310546	.359559	.065441	.254713	-.500000	0
.10	.308566	.355001	.064450	.255597	-.529112	.051452
.20	.302675	.341261	.061677	.258149	-.556279	.105954
.30	.293023	.318171	.057736	.262028	-.578806	.166461
.40	.279869	.285605	.053681	.266510	-.592395	.235406
.50	.263579	.243764	.051004	.270270	-.590189	.313417
.60	.244640	.193693	.051550	.271122	-.561931	.396298
.70	.223649	.137991	.057295	.265918	-.493782	.468822
.80	.201291	.081603	.069991	.250976	-.369748	.493160
.90	.178291	.032290	.090727	.223785	-.176218	.388837
1.00	.155325	-.000000	.119608	.187214	.088237	.000001

POISSON,S RATIO= .400
THETA = 45.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.310546	.212500	.212500	.294118	-.500000	0
.10	.309792	.208483	.213036	.293220	-.499412	.058818
.20	.307578	.196668	.214661	.290343	-.495302	.117459
.30	.304052	.177811	.217350	.284978	-.484246	.175053
.40	.299451	.153287	.220916	.276421	-.463317	.229425
.50	.294080	.125167	.224799	.264053	-.431066	.276884
.60	.288290	.096125	.227829	.247780	-.389408	.313128
.70	.282438	.068983	.228015	.228568	-.346679	.336638
.80	.276845	.045637	.222421	.208764	-.322186	.356787
.90	.271760	.025060	.207207	.191563	-.352528	.409959
1.00	.267362	-.000000	.177941	.178431	-.500001	.588234

POISSON,S RATIO= .400
THETA = 60.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.310546	.065441	.359559	.254714	-.500000	0
.10	.311015	.063856	.359713	.252302	-.470300	.050433
.20	.312438	.059390	.360002	.245164	-.439008	.097816
.30	.314872	.052829	.359938	.233631	-.405183	.139175
.40	.318390	.045221	.358831	.218333	-.368994	.171869
.50	.323079	.037503	.355998	.200224	-.331687	.194020
.60	.329015	.030088	.351080	.180446	-.294544	.204747
.70	.336264	.022602	.344479	.160018	-.256010	.203262
.80	.344885	.014165	.337840	.139445	-.205714	.185176
.90	.354949	.004932	.334444	.118651	-.113606	.133408
1.00	.366546	.000000	.339216	.098065	.088236	.000002

POISSON,S RATIO= .400
THETA = 75.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.310546	-.042213	.467213	.147059	-.500000	0
.10	.311908	-.040845	.465900	.144985	-.449351	.028908
.20	.315971	-.036858	.461957	.139032	-.400464	.054930
.30	.322669	-.030625	.455388	.129934	-.355059	.075805
.40	.331898	-.022791	.446237	.118713	-.314705	.090310
.50	.343521	-.014227	.434575	.106458	-.280749	.098358
.60	.357383	-.005874	.420342	.094148	-.254774	.100879
.70	.373315	.001463	.403007	.082599	-.240619	.099929
.80	.391149	.007004	.381032	.072505	-.249740	.099862
.90	.410715	.008804	.351290	.064444	-.312615	.110970
1.00	.431854	-.000000	.308808	.058432	-.500001	.157615

POISSON,S RATIO= .400
THETA = 90.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.310546	-.081618	.506618	.000000	-.500000	0
.10	.312235	-.078924	.504521	.000000	-.441759	.000000
.20	.317259	-.071244	.498394	.000000	-.386878	.000000
.30	.325499	-.059686	.488671	.000000	-.338099	.000000
.40	.336773	-.045828	.475944	.000000	-.297139	.000000
.50	.350858	-.031463	.460859	.000000	-.264476	.000000
.60	.367512	-.018419	.444144	.000000	-.238915	.000000
.70	.386504	-.008480	.426852	.000000	-.215842	.000000
.80	.407634	-.003098	.410850	.000000	-.182246	.000001
.90	.430764	-.002117	.399412	.000000	-.105521	.000001
1.00	.455825	.000000	.397549	.000000	.088235	.000003

POISSON,S RATIO= .450
THETA = 0/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.330142	.505480	-.074230	0	-.500000	0
.10	.326764	.502270	-.078369	0	-.558557	0
.20	.316646	.492595	-.093002	0	-.620773	0
.30	.299828	.476319	-.117332	0	-.691112	0
.40	.276378	.453180	-.153208	0	-.776024	0
.50	.246381	.422683	-.202946	0	-.886096	0
.60	.209940	.383760	-.270359	0	-1.040192	0
.70	.167162	.333824	-.361511	0	-1.273205	0
.80	.118160	.266463	-.485653	0	-1.649321	0
.90	.063088	.166458	-.656137	0	-2.288526	0
1.00	.002264	-.000000	-.890912	0	-3.398551	0

POISSON,S RATIO= .450
THETA = 15.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.330142	.466647	-.035397	.144927	-.500000	0
.10	.327118	.462895	-.038920	.146137	-.550494	.029493
.20	.318076	.451525	-.049502	.149866	-.602724	.062179
.30	.303097	.432165	-.067146	.156421	-.658366	.102036
.40	.282326	.404115	-.091750	.166355	-.718762	.154908
.50	.255978	.366225	-.122886	.180489	-.784009	.230287
.60	.224353	.316833	-.159412	.199898	-.850662	.344408
.70	.187869	.254011	-.198881	.225694	-.906721	.525573
.80	.147093	.176672	-.236778	.258382	-.921871	.823080
.90	.102783	.087582	-.265723	.296326	-.830006	1.321594
1.00	.055884	-.000000	-.274972	.332628	-.500002	2.163508

POISSON,S RATIO= .450
THETA = 30.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.330142	.360553	.070697	.251022	-.500000	0
.10	.328085	.356018	.069517	.251925	-.528690	.050706
.20	.321964	.342349	.066173	.254544	-.555463	.104418
.30	.311934	.319392	.061276	.258567	-.577664	.164049
.40	.298253	.287032	.055888	.263328	-.591056	.231994
.50	.281299	.245475	.051534	.267592	-.588882	.308875
.60	.261568	.195725	.050155	.269296	-.561033	.390554
.70	.239674	.140277	.053928	.265413	-.493872	.462027
.80	.216330	.083865	.064956	.252266	-.371636	.486012
.90	.192299	.033919	.084840	.226973	-.180911	.383202
1.00	.168309	-.000000	.114281	.191091	.079712	.000001

POISSON,S RATIO= .450
THETA = 45.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.330142	.215625	.215625	.289855	-.500000	0
.10	.329402	.211640	.216006	.289043	-.499420	.057965
.20	.327237	.199913	.217173	.286432	-.495370	.115757
.30	.323798	.181173	.219147	.281534	-.484475	.172516
.40	.319329	.156745	.221815	.273666	-.463849	.226100
.50	.314152	.128622	.224727	.262211	-.432065	.272871
.60	.308630	.099375	.226845	.247033	-.391011	.308590
.70	.303139	.071727	.226297	.228984	-.348901	.331759
.80	.298019	.047531	.220165	.210261	-.324763	.351616
.90	.293538	.025884	.204386	.194006	-.354665	.404017
1.00	.289883	-.000000	.173822	.182126	-.500001	.579709

POISSON,S RATIO= .450
THETA = 60.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.330142	.070698	.360552	.251022	-.500000	0
.10	.330717	.069091	.360646	.248738	-.470730	.049702
.20	.332465	.064548	.360767	.241977	-.439892	.096399
.30	.335441	.057824	.360466	.231038	-.406557	.137158
.40	.339726	.049928	.359100	.216502	-.370893	.169378
.50	.345411	.041767	.356019	.199248	-.334126	.191208
.60	.352581	.033752	.350867	.180338	-.297521	.201780
.70	.361306	.025548	.344009	.160725	-.259546	.200316
.80	.371652	.016330	.337051	.140866	-.209979	.182492
.90	.383699	.006227	.333370	.120612	-.119206	.131474
1.00	.397549	-.000000	.338436	.100095	.079711	.000002

POISSON,S RATIO= .450
THETA = 75.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.330142	-.035397	.466647	.144928	-.500000	0
.10	.331678	-.034128	.465382	.142951	-.450085	.028489
.20	.336265	-.030437	.461578	.137271	-.401907	.054134
.30	.343834	-.024682	.455217	.128577	-.357160	.074706
.40	.354279	-.017485	.446309	.117833	-.317390	.089001
.50	.367460	-.009690	.434878	.106065	-.283926	.096932
.60	.383215	-.002208	.420816	.094204	-.258328	.099417
.70	.401372	.004168	.403552	.083028	-.244378	.098481
.80	.421754	.008672	.381487	.073215	-.253367	.098415
.90	.444179	.009440	.351305	.065366	-.315330	.109362
1.00	.468472	-.000000	.307399	.059642	-.500001	.155331

POISSON,S RATIO= .450
THETA = 90.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.330142	-.074230	.505480	.000000	-.500000	0
.10	.332029	-.071673	.503480	.000000	-.442603	.000000
.20	.337650	-.064387	.497621	.000000	-.388517	.000000
.30	.346881	-.053439	.488284	.000000	-.340446	.000000
.40	.359533	-.040351	.475981	.000000	-.300079	.000000
.50	.375375	-.026853	.461277	.000000	-.267889	.000000
.60	.394158	-.014715	.444814	.000000	-.242699	.000000
.70	.415638	-.005663	.427573	.000000	-.219960	.000000
.80	.439606	-.001104	.411387	.000000	-.186851	.000001
.90	.465917	-.000938	.399654	.000000	-.111238	.000001
1.00	.494499	.000000	.397977	.000000	.079710	.000003

POISSON,S RATIO= .500
THETA = 0/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.354117	.504464	-.066964	0	-.500000	0
.10	.350532	.501202	-.071759	0	-.557720	0
.20	.339787	.491370	-.086365	0	-.619048	0
.30	.321917	.474830	-.111499	0	-.688382	0
.40	.296973	.451315	-.148536	0	-.772080	0
.50	.265022	.420326	-.199847	0	-.880580	0
.60	.226134	.380810	-.269350	0	-1.032475	0
.70	.180371	.330255	-.363360	0	-1.262159	0
.80	.127782	.262514	-.491700	0	-1.633493	0
.90	.068423	.163108	-.669022	0	-2.262976	0
1.00	.002458	-.000000	-.916071	0	-3.357143	0

POISSON,S RATIO= .500
THETA = 15.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.354117	.466186	-.028686	.142857	-.500000	0
.10	.350916	.462408	-.032380	.144055	-.549773	.029072
.20	.341338	.450963	-.043480	.147746	-.601256	.061291
.30	.325461	.431494	-.062012	.154243	-.656104	.100578
.40	.303424	.403327	-.087911	.164104	-.715637	.152695
.50	.275430	.365358	-.120811	.178175	-.779952	.226997
.60	.241768	.316000	-.159653	.197593	-.845652	.339488
.70	.202841	.253407	-.202063	.223634	-.900911	.518065
.80	.159205	.176522	-.243488	.257178	-.915644	.811321
.90	.111611	.087911	-.276148	.297423	-.825291	1.302714
1.00	.061015	-.000000	-.287995	.339183	-.500002	2.132601

POISSON,S RATIO= .500
THETA = 30.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.354117	.361607	.075893	.247436	-.500000	0
.10	.351964	.357093	.074525	.248357	-.528280	.049982
.20	.345555	.343491	.070618	.251041	-.554671	.102927
.30	.335045	.320659	.064780	.255205	-.576555	.161705
.40	.320703	.288495	.058078	.260237	-.589755	.228680
.50	.302911	.247203	.052072	.264991	-.587612	.304462
.60	.282181	.197756	.048792	.267523	-.560161	.384975
.70	.259149	.142544	.050615	.264922	-.493960	.455427
.80	.234559	.086095	.059983	.253520	-.373469	.479069
.90	.209221	.035519	.078994	.230071	-.185469	.377728
1.00	.183925	-.000000	.108929	.194856	.071430	.000001

POISSON,S RATIO= .500
THETA = 45.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.354117	.218750	.218750	.285714	-.500000	0
.10	.353393	.214795	.218976	.284986	-.499429	.057137
.20	.351273	.203151	.219692	.282633	-.495436	.114103
.30	.347917	.184521	.220958	.278188	-.484697	.170051
.40	.343580	.160180	.222735	.270989	-.464365	.222870
.50	.338598	.132045	.224679	.260421	-.433036	.268973
.60	.333353	.102590	.225882	.246306	-.392568	.304182
.70	.328244	.074438	.224586	.229389	-.351060	.327019
.80	.323633	.049404	.217892	.211715	-.327266	.346593
.90	.319807	.026702	.201519	.196379	-.356742	.398246
1.00	.316964	-.000000	.169643	.185714	-.500001	.571428

POISSON,S RATIO= .500
THETA = 60.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.354117	.075893	.361607	.247436	-.500000	0
.10	.354818	.074264	.361639	.245277	-.471149	.048992
.20	.356944	.069644	.361590	.238881	-.440751	.095022
.30	.360553	.062758	.361045	.228520	-.407892	.135199
.40	.365734	.054576	.359407	.214723	-.372737	.166958
.50	.372582	.045975	.356061	.198299	-.336496	.188477
.60	.381190	.037369	.350654	.180234	-.300414	.198897
.70	.391637	.028456	.343511	.161412	-.262981	.197454
.80	.403996	.018466	.336203	.142246	-.214122	.179885
.90	.418352	.007502	.332199	.122517	-.124646	.129596
1.00	.434825	-.000000	.337500	.102068	.071430	.000002

POISSON,S RATIO= .500
THETA = 75.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.354117	-.028686	.466186	.142857	-.500000	0
.10	.355860	-.027515	.464966	.140974	-.450739	.028082
.20	.361066	-.024113	.461288	.135559	-.403308	.053361
.30	.369666	-.018827	.455116	.127260	-.359200	.073639
.40	.381547	-.012257	.446426	.116979	-.319999	.087730
.50	.396566	-.005216	.435194	.105684	-.287013	.095547
.60	.414555	.001410	.421270	.094258	-.261780	.097997
.70	.435334	.006843	.404039	.083444	-.248030	.097074
.80	.458714	.010324	.381846	.073905	-.256890	.097009
.90	.484501	.010075	.351191	.066262	-.317969	.107800
1.00	.512497	-.000000	.305851	.060817	-.500001	.153112

POISSON,S RATIO= .500
THETA = 90.00/DEGREES

ROH	DEFLECTION	MR	MT	MRT	QR	Q T
0	.354117	-.066964	.504464	.000000	-.500000	0
.10	.356241	-.064541	.502555	.000000	-.443423	.000000
.20	.362569	-.057641	.496949	.000000	-.390110	.000000
.30	.372973	-.047289	.487973	.000000	-.342725	.000000
.40	.387257	-.034955	.476064	.000000	-.302935	.000000
.50	.405179	-.022307	.461705	.000000	-.271205	.000000
.60	.426477	-.011059	.445458	.000000	-.246375	.000000
.70	.450894	-.002880	.428231	.000000	-.223960	.000000
.80	.478211	.000864	.411827	.000000	-.191324	.000001
.90	.508269	.000225	.399760	.000000	-.116792	.000001
1.00	.540995	.000000	.398214	.000000	.071429	.000003

APPENDIX 2

Listing of computer programmes. All programmes are in FORTRAN IV computer language.

APPENDIX 2A.

Computer programme for the creation of tables of coefficients for the deflections and forces in circular plates simply supported on two points at opposite ends of a diameter.

APPENDIX 2B.

Computer programme for the deflections and forces in circular plates simply supported on two points at opposite ends of a diameter.

APPENDIX 2C

Computer programme for the creation of tables of coefficients for the deflections and bending moment forces in circular plates simply supported on two lengths of arcs at opposite ends of a diameter.

APPENDIX 2D

Computer programme to obtain principal stresses from micro-strains rosette strain gauge readings at a point on the surface of structures.

APPENDIX 2E

Finite element programme. BFSOLID listing.

```
      PROGRAM SLAB(INPUT,OUTPUT,TAPE5 = INPUT,TAPE6 = OUTPUT)
      PI = 3.141591
      NO = 6
      AA = 4.0 * ALOG(2.0)
      PR = 0.0
   30 PR = PR + 0.05
       PR3=(3.+PR)
       PR2=(1.+PR)/PR3
      PRR = (1. + PR) / (1. - PR)
      RR1=1./PRR
      WRITE(NO,11)
   11 FORMAT(1HQ)
      THETA = 0.0
   20 RO = -.1
      ATHETA = THETA * PI / 180.0
      WRITE(6,40) PR,THETA
   40 FORMAT(///10X,*POISSON,S RATIO=*F9.3/10X*THETA =*F9.2*/DEGREES*)
      WRITE(6,60)
   60 FORMAT(/12X,*ROH*8X*DEFLECTION*9X*MR*14X*MT*14X*MRT*14X*QR*14X*Q
     1T*/)
   10 RO = RO + 0.1
      R2 = RO * RO
      R21=(1.-R2)
      R22=1.+R2
      W1 = (1.0-R2)/64.0 * ((5.+PR)/(1.+PR) - R2)
      WI = AA/(1.0-PR) - PI*PI * PRR / 12.0 - 1.0
      RM1=(PR3*(1.-R2))/16.
      SUM = 0.0
      SUM1 = SUM2 = SUM3 = SUM4 = SUM5 = 0.0
      DO 1 I=2,10,2
      AI = I
      SUN = 1.0/(AI*(AI-1.))+ 2.0-PRR/(AI*AI*(AI-1.)) - R2/(AI*(AI+1.))
      SUN = SUN * RO**I * COS(AI*ATHETA)
      SUN1=(RR1+(2./AI))*R21*RO**(I-2)*COS(AI*ATHETA)
      SUN2=(RR1*R21+(2./AI)*R22)*RO**(I-2)*COS(AI*ATHETA)
      SUN3=(RR1*R21+2./AI)*RO**(I-2)*SIN(AI*ATHETA)
      SUN4=RO**(I-1)*COS(AI*ATHETA)
      SUN5=RO**(I-1)*SIN(AI*ATHETA)
      SUM = SUM + SUN
      SUM2=SUM2+SUN2
      SUM3=SUM3+SUN3
      SUM4=SUM4+SUN4
      SUM5=SUM5+SUN5
    1 SUM1 = SUM1 + SUN1
      W2 = 0.50 * (WI - SUM) / (3.0 + PR)
      W = W1 + W2
      RM2=0.5*PR2*SUM1
      TM1=(PR3-(1.+3.*PR)*R2)/16.
      TM2=0.5*PR2*SUM2
      RM=RM1+RM2
      TM=TM1-TM2
      RTM=0.5*PR2*SUM3
      QR=-.5-(2./PR3)*SUM4
      QT=(2./PR3)*SUM5
      WRITE(6,50) RO,W,RM,TM,RTM,QR,QT
      IF(RO.LT.1.0) GO TO 10
      THETA = THETA + 15.0
      IF(THETA.LE.90) GO TO 20
      IF(PR.LT.0.50) GO TO 30
   50 FORMAT(/10X,F5.2,6(6X,F10.6))
      STOP
      END
```

APPENDIX 2A.

BLADE.FORTRAN:PROGRAML

```
      PROGRAM PLATE
      WRITE(5,10)
   10 FORMAT(3X,50HINPUT YOUNG'S MODULUS, POISSON'S RATIO, ROH ,THE
     *TA,//,3X,30H THICKNESS, RADIUAS ,PRESSURE ,//)
      READ(5,20) E,V,RO,TH,T,R,P
   20 FORMAT(7F10.0)
      PI = 4.*ATAN(1.0)
      AA = ALOG(2.0)*4.
      PR3=3.+V
      PR2=(1.+V)/PR3
      PR=(1.+V)/(1.-V)
      RR=1./PR
      THITA=TH*PI/180.
      R2=RO *RO
      R21=1.-R2
      R22=1.+R2
      W1=(1.-R2)/64.*((5.+V)/(1.+V)-R2)
      WI=AA/(1.0-V)-PI*PI*PR/12.-1.
      RM1=(PR3*R21)/16.
      SUM=0.
      SUM1=0.
      SUM2=0.
      SUM3=0.
      SUM4=0.
      SUM5=0.
      DO 1 I=2,100,2
      AI=I
      SU=1./(AI*(AI-1.))+2.*PR/(AI*AI*(AI-1.))-R2/(AI*(AI+1.))
      SU=SU*RO**I*COS(AI*THITA)
      SU1=(RR+(2./AI))*R21*RO**(I-2)*COS(AI*THITA)
      SU2=(RR*R21+(2/AI)*R22)*RO**(I-2)*COS(AI*THITA)
      SU3=(RR*R21 +2./AI)*RO**(I-2)*SIN(AI*THITA)
      SU4=RO**(I-1)*COS(AI*THITA)
      SU5=RO**(I-1)*SIN(AI*THITA)
      SUM=SUM+SU
      SUM1=SUM1+SU1
      SUM2=SUM2+SU2
      SUM3=SUM3+SU3
      SUM4=SUM4+SU4
      SUM5=SUM5+SU5
 1    CONTINUE
      W2=.5*(WI-SUM)/(3.+V)
      W=W1+W2
      RM2=.5*PR2*SUM1
      TM1=(PR3-(1.+3.*V)*R2)/16.
      TM2=.5*PR2*SUM2
      RM=RM1+RM2
      TM=TM1-TM2
      RTM=.5*PR2*SUM3
      QR=-.5-(2./PR3)*SUM4
      QT=(2./PR3)*SUM5
      D=E*T**3/(1.-V*V)
      D=D/12.
      R4=R**4
      DEF=W*R4*P/D
C
      WRITE(3,70) RO,TH,DEF
   70 FORMAT(10X,22HTHE DEFLECTION AT ROH=,F5.3,3X,9H& THETA =,F7.2,
     *3X,9HDEGREES =,//20X,F12.5/)
      END
```

APPENDIX 2B.

```
TA FORTRAN COMPILER (VERSION 5.3 - 13/09/78) ON THE 6000 UNDER NOS/BE 1.2 ON
F.

      PROGRAM SLAB(INPUT,OUTPUT,TAPE5 = INPUT,TAPE6 = OUTPUT)
      PI = 3.141591
      NO = 6
      AALFA=2.
   70 AALFA=AALFA+.5
      PR = 0.0
   30 PR = PR + 0.05
      PR3=(3.+PR)
      PR2=(1.+PR)/PR3
      PRR = (1. + PR) / (1. - PR)
      PRP=1./PRR
      WRITE(NO,11)
   11 FORMAT(1H0)
      THETA = 0.0
   20 RO = -.1
      ALFA =AALFA*PI/180.
      ATHETA = THETA * PI / 180.0
      WRITE(6,40) PR,THETA,AALFA
   40 FORMAT(///10X,*POISSON,S RATIO=*F9.3/10X*THETA =*F9.2*/DEGREES*/10
     *X*ALFA =*F9.2*/DEGREES*)
      WRITE(6,60)
   60 FORMAT(/12X,*ROH*8X*DEFLECTION*9X*MR*14X*MT*14X*MRT*)
   10 RO = RO + 0.1
      R2 = RO * RO
      R21= (1.-R2)
      R22=1.+R2
      W1 = (1.0-R2)/64.0 * ((5.+PR)/(1.+PR) - R2)
      RM1=(PR3*(1.-R2))/16.
      SUI1=SUI2=SUI3=SUI4=SUI5=0.
      DO 1 I=2,10,2
      AI = I
      AI1= 1./AI
      AI2=AI1*AI1
      AI3=1./(AI-1)
      AI4=1./(AI+1.)
      SUL1 =SIN(AI*ALFA)*AI1*(AI1*AI3+2.*PRR*AI2*AI3-AI1*AI4)*COS(AI*ALF
     *A)
      SUL2=SIN(AI*ALFA)*AI1*(AI1*AI3+2.*PRR*AI2*AI3-RO**2*AI1*AI4)*RO
     ***I*COS(AI*ATHETA)
      SUL3=SIN(AI*ALFA)*AI1*(2.*AI1+PRP)*R21*RO**(I-2)*COS(AI*ATHETA)
      SUL3=SUL3*.5*PR2/ALFA
      SUL4=SIN(AI*ALFA)*AI1*(PRP*R21+R22*2.*AI1)*RO**(I-2)*COS(AI*ATHETA
     *)
      SUL4=SUL4*.5*PR2/ALFA
      SUL5=SIN(AI*ALFA)*AI1*(PRP*R21+2.*AI1)*RO**(I-2)*SIN(AI*ATHETA)
      SUI1=SUI1+SUL1
      SUI2=SUI2+SUL2
      SUI3=SUI3+SUL3
      SUI4=SUI4+SUL4
    1 SUI5=SUI5+SUL5
      SUL=(SUI1-SUI2)*.5/(ALFA*PR3)
      W=W1+SUL
      RM=RM1+SUI3
      TM1=(PR3-(1.+3.*PR)*R2)/16.
      TM=TM1-SUI4
      RTM=SUI5*.5*PR2/ALFA
      WRITE(6,50) RO,W,RM,TM,RTM
      IF(RO.LT.1.0) GO TO 10
      THETA = THETA + 15.0
      IF(THETA.LE.90.)GO TO 20
      IF(PR.LT.0.50) GO TO 30
      IF(AALFA.LE.9.) GO TO 70
   50 FORMAT(10X,F5.2,4(6X,F10.6))
      STOP
      END
```

APPENDIX 2C.

STRESS.FORTRAN:PROGRAML

```
      PROGRAM STRES
      E=4.6E05
      V=.378
      E1=E/(1.-V*V)
      READ(3,1) NN
    1 FORMAT(I5)
      WRITE(5,2) NN
    2 FORMAT(/,5X,30HTHE TOTAL NUMBER OF READINGS =,I5,/)
      DO 11 N=1,NN
      WRITE(5,50) N
   50 FORMAT(10X,32HINPUT THE STRAINS OF READING NO.,I5,/)
      READ(3,4) EX,EY,EXY
      EX=EX*1.0E-06
      EY=EY*1.0E-06
      EXY=EXY*1.0E-06
      X=.5*(EX+EY)
      Y=.5*(EX-EY)
      Z=X-EXY
      R=SQRT(Y*Y + Z*Z)
      EE1=X+R
      EE2=X-R
      S1=E1*(EE1+V*EE2)
      S2=E1*(EE2+V*EE1)
      WRITE(5,10) N,EX,EY,EXY,EE1,EE2,S1,S2
   10 FORMAT(2X,I3,3X,5F10.8,3X,2F10.2,/)
  11  CONTINUE
    4 FORMAT(3F10.0)
      STOP
      END
```

APPENDIX 2D.

,PRESSURE ,//)

```
      OVERLAY(TSHE,0,0)
      PROGRAM SOLID(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
     1                TAPE1 , TAPE2 , TAPE3 , TAPE4 ,
     2                TAPE7 , TAPE8 , TAPE9 , TAPE10 , TAPE11 )
      COMMON /JUNK / HED(12),JUK(210)
      COMMON /ELPAR/ NPAR(14),NUMNP,MBAND,NELTYP,N1,N2,N3,N4,N5,MTOT,NEQ
      COMMON /EM/ QQQ(4044)
      COMMON /DYN/   DUS(8),NDYN
      COMMON /TAPES/ NQQ(6)
      COMMON /EXTRA/ MODEX,NT8,N10SV,NT10,KEQB,NUMEL,T(10)
      COMMON /SOL/   NBLOCK,NEQB,LL,NF,IDUM,NEIG,NAD,ANORM,NVV,NFO
      COMMON A(12401)
      MTOT=12400
      NT8 = 8
      REWIND NT8
      NT10= 10
      REWIND NT10
      N1=1
    5 CALL SECOND(T(1))
      READ (5,100) HED,NUMNP,NELTYP,LL,NF,NDYN,MODEX,NAD,KEQB,N10SV
      IF(MODEX.GT.0) MODEX = 1
      IF (NUMNP.EQ.0) STOP
      WRITE (6,200) HED,NUMNP,NELTYP,LL,NF,NDYN,MODEX,NAD,KEQB,N10SV
      IF(KEQB.LT.2( KEQB = 99999
      IF (NDYN.NE.0) LL=1
      IF(LL.GE.1) GO TO 10
      WRITE (6,300)
      STOP
C***  DATA PORTHOLE SAVE
   10 IF(MODEX.EQ.1)
     *WRITE (NT8)     HED,NUMNP,NELTYP,LL,NF,NDYN
C
      KDYN = IABS(NDYN) +1
      IF(KDYN.LE.5) GO TO 14
      WRITE (6,310) NDYN
      STOP
   14 IF(NDYN.LT.0) GO TO 20
      N2=N1+6*NUMNP
      N3=N2+NUMNP
      N4=N3+NUMNP
      N5=N4+NUMNP
      N6=N5+NUMNP
      IF(N6.GT.MTOT) CALL ERROR(N6-MTOT)
      CALL INDATA(A(N1),A(N2),A(N3),A(N4),A(N5),NUMP,NEQ)
      CALL SECOND(T(2))
      MBAND=0
      NUMEL=0
      REWIND 1
      REWIND 2
      DO 900 M=1,NELTYP
      READ  (5,1001) NPAR
      IF(MODEX.EQ.1) WRITE (NT8) NPAR
      IF(MODEX.EQ.0) WRITE (1)   NPAR
      NUMEL=NUMEL+NPAR(2)
      MTYPE=NPAR(1)
      CALL THICKS(MTYPE)
  900 CONTINUE
      NEQB=(MTOT - 4*LL)/(MBAND + LL + 1)/2
      IF(KEQB.LT.NEQB) NEQB = KEQB
      GO TO  (690,700,700,700,730), KDYN
  690 CONTINUE
      NEQB1=(MTOT - MBAND)/(2*(MBAND+LL) + 1)
      NEQB2=(MTOT - MBAND - LL*(MBAND-2))/(3*LL + MBAND + 1)
      IF (NEQB1.LT.NEQB) NEQB=NEQB1
      IF (NEQB2.LT.NEQB) NEQB=NEQB2
      NBLOCK = (NEQ-1)/NEQB +1
      IF(NEQB.GT.NEQ) NEQB=NEQ
      GO TO 790
  700 IF (NEQB.LT.NEQ) GO TO 710
      NIM=3
      NC=NF + NIM
      NVM=6
      NCA= NEQ*MAX0(MBAND,NC)
      NTOT=NCA + 4*NEQ + 2*NVM*NEC + 5*NC
      NEIG=0
      IF(NTOT.LE.MTOT) GO TO 720
  710 NV=MIN0(2*NF,NF+8)
      IF (NAD.NE.0) NV=NAD
      NEQB1=(MTOT - MBAND)/(2*MBAND + 1)
      NEQB2=(MTOT - MBAND - 2*NV - NV*(MBAND-2))/(3*NV + MBAND + 1)
      NEQB3=(MTCT - 3*NV*NV - 3*NV)/(2*NV + 1)
      NEQB4=(MTOT - 5*NV)/(1 + MBAND)
      IF (NEQB1.LT.NEQB) NEQB=NEQB1
      IF (NEQB2.LT.NEQB) NEQB=NEQB2
      IF (NEQB3.LT.NEQB) NEQB=NEQB3
      IF (NEQB4.LT.NEQB) NEQB=NEQB4
      NEIG=1
  720 CONTINUE
      NBLOCK = (NEQ-1)/NEQB +1
      IF (NEQB.GE.NEQ) NEQB=NEQ
      KREM = 1000
      NTOT = NBLOCK*NEQB*NF + KREM
      IF(MTOT.LT.NTOT)
     *WRITE (6,320)
      GO TO 790
  730 CONTINUE
      NN2 = NEQ
      NN3 = NEQ
      NEQB1 = (MTOT-NN2-NN3-NEC-MBAND)/(2*MBAND+1)
      NEQB2 = (MTOT-MBAND-2*(NN2+NN3)-5*NEQ)/(MBAND+1)
      IF(NEQB1.LT.NEQB) NEQB = NEQB1
      IF(NEQB2.LT.NEQB) NEQB = NEQB2
      IF(NEQB.GT.NEQ)   NEQB = NEQ
      NBLOCK = (NEQ-1)/NEQB +1
      NN2 = 10
      NN3 = 40
      NN4 = 6*NUMNP + 2*NN2*NEQ
      IF(NN4.GT.MTOT)
     *WRITE (6,320)
      NN4 = NEQ*2*(NN2+1) + NN2*(1+2*NN3)
      IF(NN4.GT.MTOT)
     *WRITE (6,320)
  790 CONTINUE
      N3=N2+NEQB*LL
      N4=N3+6*LL
      WRITE (6,201) NEQ,MBAND,NEQB,NBLOCK
      CALL SECOND(T(3))
      CALL INL(A(N1),A(N2),A(N3),A(N4),NUMNP,NEQB,LL)
      CALL SECOND(T(4))
      NE2B=2*NEQB
      N2=N1+NEQB*MBAND
      N3=N2+NEQB*LL
      N4=N3+4*LL
      NN2=N1+NE2B*MBAND
      NN3=NN2+NE2B*LL
      NN4=NN3+4*LL
      CALL ADDSTF (A(N1),A(NN2),A(NN3),A(NN4),NUMEL,NBLOCK,NE2B,LL,MBAND
     1,ANORM,NVV)
      CALL SECOND(T(5))
   20 CONTINUE
   32 CALL OVERLAY(4HTSHE,12,0,6HRECALL)
      CALL SECOND  (T(6))
      DO 33 I=7,10
   33 T(I) = T(6)
   90 TT = 0.0
      DO 95 I=1,9
      T(I) = T(I+1)-T(I)
      TT = TT + T(I)
   95 CONTINUE
      WRITE (6,203)  (T(K),K=1,9),TT
      GO TO 5
  100 FORMAT (12A6/9I5)
  200 FORMAT(1H1,12A6///
     1  39H C O N T R O L   I N F O R M A T I O N, // 4X,
     2  27H NUMBER OF NODAL POINTS    =, I5 / 4X,
     3  27H NUMBER OF ELEMENT TYPES   =, I5 / 4X,
     4  27H NUMBER OF LOAD CASES      =, I5 / 4X,
     5  27H NUMBER OF FREQUENCIES     =, I5 / 4X,
     6  27H ANALYSIS CODE (NDYN)      =, I5 / 4X,
     7  16H    EQ.0,  STATIC,              / 4X,
     8  26H    EQ.1,  MODAL EXTRACTION,    / 4X,
     9  25H    EQ.2,  FORCED RESPONSE,     / 4X,
     A  27H    EQ.3,  RESPONSE SPECTRUM,   / 4X,
     *  28H    EQ.4,  DIRECT INTEGRATION,  / 4X,
     B  27H SOLUTION MODE (MODEX)     =, I5 / 4X,
     C  19H    EQ.0,  EXECUTION,           / 4X,
     D  20H    EQ.1,  DATA CHECK,          / 4X,
     E  19H NUMBER OF SUBSPACE,            / 4X,
     F  27H ITERATION VECTORS (NAD)   =, I5 / 4X,
     G  27H EQUATIONS PER BLOCK       =, I5 / 4X,
     H  27H TAPE10 SAVE FLAG (N10SV)  =, I5 / 4X)
  201 FORMAT (38H1E Q U A T I O N   P A R A M E T E R S, //
     *          34H  TOTAL NUMBER OF EQUATIONS          =,I5,
     1         /34H  BANDWIDTH                          =,I5,
     2         /34H  NUMBER OF EQUATIONS IN A BLOCK     =,I5,
     3         /34H  NUMBER OF BLOCKS                   =,I5)
```

```
203 FORMAT (1H1,31HO V E R A L L   T I M E   L O G, //
    1 5X,33HNODAL POINT INPUT                 =, F8.2 /
    2 5X,33HELEMENT STIFFNESS FORMATION       =, F8.2 /
    3 5X,33HNODAL LOAD INPUT                  =, F8.2 /
    4 5X,33HTOTAL STIFFNESS FORMATION         =, F8.2 /
    5 5X,33HSTATIC ANALYSIS                   =, F8.2 /
    6 5X,33HEIGENVALUE EXTRACTION             =, F8.2 /
    7 5X,33HFORCED RESPONSE ANALYSIS          =, F8.2 /
    8 5X,33HRESPONSE SPECTRUM ANALYSIS        =, F8.2 /
    * 5X,33HSTEP-BY-STEP INTEGRATION          =, F8.2 //
    9 5X,33HTOTAL SOLUTION TIME               =, F8.2 /)
300 FORMAT (// 48H ** ERROR.  (AT LEAST ONE LOAD CASE IS REQUIRED), )
310 FORMAT (// 33H ** ERROR.  ANALYSIS CODE (NDYN =,I3,9H) IS BAD., )
320 FORMAT (// 47H ** WARNING.  ESTIMATE OF STORAGE FOR A DYNAMIC,
    1                32H ANALYSIS EXCEEDS AVAILABLE CORE, // 1X)
1001 FORMAT (14I5)
     END
     SUBROUTINE INDATA(ID,X,Y,Z,T,NUMNP,NEQ)
     DIMENSION X(1),Y(1),Z(1),ID(NUMNP,6),T(1)
     COMMON /EXTRA/ MODEX,NT8
     DIMENSION IPRC(4)
     DATA IPRC/1H ,1HA,1HB,1HC/
     IPR = IPRC(1)
     RAD = ATAN(1.0)/ 45.0
     WRITE (6,2000)
     WRITE (6,2001)
     NOLD=0
  10 READ  (5,1000) IT,N,JPR,(ID(N,I),I=1,6),X(N),Y(N),Z(N),KN,T(N)
     WRITE (6,2002) IT,N,JPR,(ID(N,I),I=1,6),X(N),Y(N),Z(N),KN,T(N)
     IF(N.EQ.1) IPR = JPR
     IF(IT.NE.IPRC(4)) GO TO 15
     DUM = Z(N)* RAD
     Z(N) = X(N)* COS(DUM)
     X(N) = X(N)* SIN(DUM)
  15 CONTINUE
     IF(NOLD.EQ.0) GO TO 50
     DO 20 I=1,6
     IF(ID(N,I).EQ.0.AND.ID(NOLD,I).LT.0) ID(N,I)=ID(NOLD,I)
  20 CONTINUE
     IF(KN.EQ.0) GO TO 50
     NUM=(N-NOLD)/KN
     NUMN=NUM-1
     IF(NUMN.LT.1) GO TO 50
     XNUM=NUM
     DX=(X(N)-X(NOLD))/XNUM
     DY=(Y(N)-Y(NOLD))/XNUM
     DZ=(Z(N)-Z(NOLD))/XNUM
     DT=(T(N)-T(NOLD))/XNUM
     K=NOLD
     DO 30 J=1,NUMN
     KK=K
     K=K+KN
     X(K)=X(KK)+DX
     Y(K)=Y(KK)+DY
     Z(K)=Z(KK)+DZ
     T(K)=T(KK)+DT
     DO 30 I=1,6
     ID(K,I)=ID(KK,I)
     IF (ID(K,I).GT.1) ID(K,I)=ID(KK,I)+KN
  30 CONTINUE
C
  50 NOLD=N
     IF(N.NE.NUMNP) GO TO 10
     IF(IPR.EQ.IPRC(2) .OR. IPR.EQ.IPRC(4)) GO TO 52
     WRITE (6,2003)
     WRITE (6,2001)
     WRITE (6,2005) (N,(ID(N,I),I=1,6),X(N),Y(N),Z(N),T(N),N=1,NUMNP)
  52 CONTINUE
     NEQ=0
     DO 60 N=1,NUMNP
     DO 60 I=1,6
     ID(N,I)=IABS(ID(N,I))
     IF(ID(N,I)-1) 57,58,59
  57 NEQ=NEQ+1
     ID(N,I)=NEQ
     GO TO 60
  58 ID(N,I)=0
     GO TO 60
  59 ID(N,I)=-ID(N,I)
  60 CONTINUE
     IF(IPR.EQ.IPRC(3) .OR. IPR.EQ.IPRC(4)) GO TO 62
     WRITE (6,2004) (N,(ID(N,I),I=1,6), N=1,NUMNP)
  62 CONTINUE
     IF(MODEX.EQ.0) GO TO 70
     WRITE (NT8) ((ID(N,I),I=1,6),N=1,NUMNP)
     WRITE (NT8) (X(N),N=1,NUMNP)
     WRITE (NT8) (Y(N),N=1,NUMNP)
     WRITE (NT8) (Z(N),N=1,NUMNP)
     WRITE (NT8) (T(N),N=1,NUMNP)
     ENDFILE NT8
     REWIND 2
     WRITE (2) ID
     RETURN
  70 CONTINUE
     REWIND 8
     WRITE (8) ID
     RETURN
1000 FORMAT (2(A1,I4),5I5,3F10.0,I5,F10.0)
2000 FORMAT (//23H NODAL POINT INPUT DATA )
2001 FORMAT (5H0NODE 3X 24HBOUNDARY CONDITION CODES 11X
    . 23HNODAL POINT COORDINATES / 7H NUMBER 2X 1HX 4X 1HY 4X 1HZ 3X
    . 2HXX 3X 2HYY 3X 2HZZ12X 1HX 12X 1HY 12X 1HZ 12X 1HT )
2002 FORMAT (1X,A1,I4,A1,I3,5I5,3F13.3,I5,F13.3)
2003 FORMAT (//21H GENERATED NODAL DATA)
2004 FORMAT (//17H EQUATION NUMBERS/
    1 35H    N    X    Y    Z   XX   YY   ZZ /(7I5))
2005 FORMAT (I5,6I5,4F13.3)
     END
     SUBROUTINE THICKS(MTYPE)
   8 CALL SOT
     RETURN
     END
     SUBROUTINE SOT
     DIMENSION MAXN(10),SIGM(10)
     DIMENSION INCON(8),ASIG(700,6),SIG(700)
     COMMON /ELPAR/ NPAR(14),NUMNP,MBAND,NELTYP,N1,N2,N3,N4,N5,MTOT,NEQ
     COMMON /EM/ NS,ND,LM(63),B(43,63),SS(63,4)
     COMMON /JUNK / LT,LH,L,SIG(43),N6,N7,N8,N9,N10,N11,N12,N13,N14,
    1               N15,N16,N17
     COMMON /EXTRA/ MODEX,NT8,N10SV,NT10
     COMMON         A(1)
     DO 32 I=1,10
     MAXN(I)=0.0
     SIGM(I)=0.0
  32 CONTINUE
     DO 807 I=1,700
     DO 808 J=1,6
 808 ASIG(I,J)=0.0
     SIS(I)=0.0
 807 CONTINUE
     IF(NPAR(1).EQ.0) GO TO 500
     WRITE (6,1000)
     IF(NPAR(2).GT.0) GO TO 10
     WRITE (6,1001) (NPAR(K),K=1,10)
     WRITE (6,1002)
     CALL EXIT
  10 IF(NPAR(3).GT.0) GO TO 20
     WRITE (6,1001) (NPAR(K),K=1,10)
     WRITE (6,1003)
     CALL EXIT
  20 IF(NPAR(4).EQ.0) NPAR(4) = 1
     IF(NPAR(7).EQ.0) NPAR(7) = 21
     IF(NPAR(7).GE.8 .AND. NPAR(7).LE.21) GO TO 30
     WRITE (6,1001) (NPAR(K),K=1,10)
     WRITE (6,1004)
     CALL EXIT
  30 IF(NPAR(9).EQ.0) NPAR(9) = 2
     IF(NPAR(9).GE.2 .AND. NPAR(9).LE.4) GO TO 40
     WRITE (6,1001) (NPAR(K),K=1,10)
     WRITE (6,1005)
     CALL EXIT
  40 IF(NPAR(10).EQ.0) NPAR(10) = 2
     IF(NPAR(10).GE.2 .AND. NPAR(10).LE.4) GO TO 50
     WRITE (6,1001) (NPAR(K),K=1,10)
     WRITE (6,1005)
     CALL EXIT
  50 N6  = N5  + NUMNP
     N7  = N6  + NPAR(3)
     N8  = N7  + NPAR(3)
     N9  = N8  + NPAR(3)
     N10 = N9  + NPAR(3) * NPAR(4) * 13
     N11 = N10 + NPAR(5) * 9
     N12 = N11 + NPAR(6)
     N13 = N12 + NPAR(6)
     N14 = N13 + NPAR(6) * 7
     N15=N14+NPAR(8)*8
     N16 = N15 + NPAR(3)
     N17 = N16 + NPAR(7) * 139
     IF(N17.GT.MTOT) CALL ERROR(N17-MTOT)
     CALL OVERLAY(4HTSHC,7,0,6HRECALL)
```

```
      RETURN
  500 WRITE (6,2001)
      NUME = NPAR(2)
      DO 600 MM=1,NUME
      CALL STRSC (A(N1),A(N3),NEQ,0,INCON)
      IF(NDSV.EQ.1)
     *WRITE (NT10) NS
      IF(NS.EQ.1) GO TO 800
      WRITE (6,5000)
      DO 700 L=LT,LH
      CALL STRSC (A(N1),A(N3),NEQ,1,INCON)
      LOC = NS/6
      K1 = -5
      DO 600 N=1,LOC
      IAS=INCON(N)
      SNS(IAS)=SNS(IAS)+1.0
      K1 = K1 + 6
      K2 = K1 + 5
      IF(N.EQ.1) WRITE (6,3001) MM,L,N,(SIG(I),I=K1,K2)
      IF(N.GT.1) WRITE (6,4001)         N,(SIG(I),I=K1,K2)
      DO 602 IK=K1,K2
      I=IK-K1+1
      ASIG(IAS,I)=ASIG(IAS,I)+SIG(IK)
  602 CONTINUE
      IF(NDSV.EQ.1)
     *WRITE (NT10) MM,L,N,(SIG(I),I=K1,K2)
  600 CONTINUE
      WRITE (6,5000)
  700 CONTINUE
  800 CONTINUE
      DO 803 I=1,NUMNP
      DO 803 N=1,6
      IF(SNS(I).EQ.0) GO TO 803
      ASIG(I,N)=ASIG(I,N)/SNS(I)
  803 CONTINUE
      WRITE(6,804)
  804 FORMAT(   23H AVERAGE STRESS AT NODE,//
     221H     NODE   CONN,18X,6HSIG-XX,9X,6HSIG-YY,9X,6HSIG-ZZ,
     39X,6HSIG-XY,9X,6HSIG-YZ,9X,6HSIG-ZX,//1X)
      DO 806 I=1,NUMNP
      ISNS=SNS(I)
      WRITE(6,809)I,ISNS,(ASIG(I,N),N=1,6)
  809 FORMAT(I9,I6,9X,6E15.6)
  806 CONTINUE
      WRITE(6,11)
   11 FORMAT(32H PRINCIPAL STRESSES AT EACH NODE,//,6H   NODE,19X,
     17HPRIN-1 ,8X,7HPRIN-2 ,8X,7HPRIN-3 ,8X,9HMAX SHEAR)
      DO 21 I=1,NUMNP
      S1=ASIG(I,1)
      S2=ASIG(I,2)
      S3=ASIG(I,3)
      S4=ASIG(I,4)
      S5=ASIG(I,5)
      S6=ASIG(I,6)
      A9=-(S1+S2+S3)
      B9=S1*S2+S2*S3+S1*S3-S4**2-S5**2-S6**2
      C=-(S1*S2*S3+2.0*S4*S5*S6-S1*S6**2-S2*S5**2-S3*S4**2)
      X=0.0
  301 FX=X**3+A9*X**2+B9*X+C
      FPX=3.0*X**2+2.0*A9*X+B9
      X=X-FX/FPX
      IF(ABS(FX).LE.1.0E-1) GO TO 201
      GO TO 301
  201 CONTINUE
      F=A9+X
      G=B9+A9*X+X**2
      X2=(-F+SQRT(F**2-4.0*G))/2.0
      X3=(-F-SQRT(F**2-4.0*G))/2.0
      X01=ABS(X)-ABS(X2)
      X01=ABS(X01)
      X02=ABS(X)-ABS(X3)
      X02=ABS(X02)
      IF(X02.GE.X01) X01=X02
      X02=ABS(X2)-ABS(X3)
      X02=ABS(X02)
      IF(X02.GE.X01)X01=X02
      SXX=X01/2.0
      WRITE(6,31)I,X,X2,X3,SXX
   31 FORMAT(1X,I5,14X,4E15.6)
      PM=ABS(X)
      IF(ABS(X2).GE.PM) PM=ABS(X2)
      IF(ABS(X3).GE.PM) PM=ABS(X3)
      MAXN(I)=I
      SIGM(I)=PM
      DO 12 K=1,9
      MI=K+1
      DO 13 J=MI,10
      IF(SIGM(K).GE.SIGM(J)) GO TO 13
      IT=MAXN(K)
      SIT=SIGM(K)
      MAXN(K)=MAXN(J)
      SIGM(K)=SIGM(J)
      MAXN(J)=IT
      SIGM(J)=SIT
   13 CONTINUE
   12 CONTINUE
   21 CONTINUE
      WRITE(6,3005)
 3005 FORMAT(70H1 OUTPUT FIRST TEN MAXIMUM PRINCIPAL STRESSES   (IN DECE
     1NDING ORDER    // 63H                         NODE NO          MAX PR
     2INCIPAL STRESS    //)
      DO 44 I=1,10
      WRITE(6,3006)MAXN(I),SIGM(I)
   44 CONTINUE
 3006  FORMAT(15X,I5,25X,E12.4)
      RETURN
 1000 FORMAT (53H121 - N O D E   S O L I D   E L E M E N T   I N P U T,
     1 10H   D A T A,//38H0C O N T R O L   I N F O R M A T I O N,/ 1X)
 1001 FORMAT (43H0ERROR DETECTED WHILE PROCESSING MASTER ELEMENT,
     1 12H CONTROL CARD,//15X,1H(,10I5,1H),/1X)
 1002 FORMAT (32H NO 3/D SOLID ELEMENTS SPECIFIED,/1X)
 1003 FORMAT (23H NO MATERIALS REQUESTED, / 1X)
 1004 FORMAT (49H MAXIMUM NUMBER OF NODES MUST BE GE.8 .AND. LE.21,/1X)
 1005 FORMAT (42H INTEGRATION ORDER MUST BE GE.2 .AND. LE.4,/1X)
 2001 FORMAT (54H121 - N O D E   S O L I D   E L E M E N T   S T R E S ,
     1 3HE S,//
     2 23H ELEMENT  LOAD LOCATION,3X,6HSIG-XX,9X,6HSIG-YY,9X,6HSIG-ZZ,
     3 9X,6HSIG-XY,9X,6HSIG-YZ,9X,6HSIG-ZX,//1X)
 3001 FORMAT (I8,I6,I9,6E15.6)
 4001 FORMAT (  14X,I9,6E15.6)
 5000 FORMAT ( / )
      END
      SUBROUTINE INL(ID,B,TR,THASS,NUMNP,NEQB,LL)
C
      DIMENSION ID(NUMNP,6),B(NEQB,LL),TR(6,LL),THASS(NEQB)
      COMMON / JUNK / R(6),TXM(6)
      COMMON /EXTRA/ MODEX,NT8
      NT=8
      REWIND NT
      KSHF=0
      WRITE (6,2002)
      IF(MODEX.EQ.1) GO TO 50
      DO 750 I=1,NEQB
      TMASS(I)=0.
      DO 750 K=1,LL
  750 B(I,K)=0.0
   50 DO 900 NN=1,NUMNP
      DO 100 I=1,6
      TXM(I)=0.
      DO 100 J=1,LL
  100 TR(I,J)=0.0
      IF(NN.EQ.1) GO TO 300
  150 IF(N.NE.NN) GO TO 400
      DO 200 I=1,6
      IF (L) 180,180,190
  180 TXM(I)=R(I)
      GO TO 200
  190 TR(I,L)=R(I)
  200 CONTINUE
  300 READ (5,1001) N,L,R
      IF (N.EQ.0) GO TO 150
      WRITE(6,2001) N,L,R
      GO TO 150
  400 IF(MODEX.EQ.1) GO TO 900
      DO 800 J=1,6
      II=ID(NN,J)-KSHF
      IF (II) 800,800,500
  500 DO 600  K=1,LL
  600 B(II,K)=TR(J,K)
      TMASS(II)=TXM(J)
  610 IF(II.NE.NEQB) GO TO 800
      WRITE (NT) B,TMASS
      KSHF=KSHF+NEQB
      DO 700 I=1,NEQB
      TMASS(I)=0.
      DO 700 K=1,LL
  700 B(I,K)=0.0
  800 CONTINUE
  900 CONTINUE
      IF(MODEX.EQ.1) RETURN
```

```
      WRITE (NT) B,TMASS
      RETURN
 1001 FORMAT (3I5,7F10.4)
 2001 FORMAT (2(3X,I4),6E15.5)
 2003 FORMAT (47H1N O D A L   L O A D S   (S T A T I C)   O R ,
     4            29HM A S S E S   (D Y N A M I C), ///
     5            3X,4HNODE,3X,4HLOAD,
     1  3(3X,6HX-AXIS,9X,6HY-AXIS,9X,6HZ-AXIS), / 7H NUMBER,3X,4HCASE,
     2  3(10X,5HFORCE), 3(9X,6HMOMENT), / 1X)
      END
      SUBROUTINE ERROR(N)
      WRITE (6,2000) N
 2000 FORMAT (// 20H STORAGE EXCEEDED BY  I6)
      STOP
      END
      SUBROUTINE ADDSTF (A,B,STR,TMASS,NUMEL,NBLOCK,NE2B,LL,MBAND,ANORM,
     1NVV)
      DIMENSION A(NE2B,MBAND),B(NE2B,LL),STR(4,LL),TMASS(NE2B),SS(1)
      COMMON /DYN/   NT,NOT,ALFA,DT,BETA,NFN,NGM,NAT,NDYN
      COMMON /EM/    LRD,ND,LM(1)
      COMMON /EXTRA/ MODEX,NT8
      EQUIVALENCE (SS,ND)
      NEB=NE2B/2
      K=NEB+1
      X=NBLOCK
      MB=SQRT(X)
      MB=MB/2+1
      NEBB=MB*NE2B
      MM=1
      NDEG=0
      NVV=0
      ANORM=0.
      NSHIFT=0
      REWIND 3
      REWIND 4
      REWIND 9
      WRITE (6,2000)
      DO 50 L=1,LL
      READ  (5,1002)    (STR(I,L),I=1,4)
   50 WRITE (6,2002) L,(STR(I,L),I=1,4)
      IF(MODEX.EQ.0) WRITE (3) STR
      IF(NDYN.NE.4) GO TO 65
      READ  (5,1004) NFN,NGM,NAT,NT,NOT,DT,ALFA,BETA
      WRITE (6,2004) NFN,NGM,NAT,NT,NOT,DT,ALFA,BETA
      IF(NAT.EQ.0) NAT = 1
      IF(NOT.EQ.0) NOT = 1
      IF(DT.GT.1.0E-12) GO TO 55
      WRITE (6,3000)
      STOP
   55 TETA = 1.4
      DT1  = TETA*DT
      DT2  = DT1**2
      A0   = (6.+3.*ALFA*DT1)/(DT2+3.*BETA*DT1)
   65 IF(MODEX.EQ.1) RETURN
      DO 1000 M=1,NBLOCK ,2
      DO 100 I=1,NE2B
      DO 100 J=1,MBAND
  100 A(I,J)=0.
      READ (3) ((B(I,L),I=1,NEQB),L=1,LL),(TMASS(I),I=1,NEQB)
      IF  (M.EQ.NBLOCK) GO TO 200
      READ (3)  ((B(I,L),I=K,NE2B),L=1,LL),(TMASS(I),I=K,NE2B)
  200 CONTINUE
      REWIND 7
      REWIND 2
      NA=7
      NUME=NUM7
      IF  (M.NE.1) GO TO 75
      NA=2
      NUME=NUMEL
      NUM7 =0
   75 DO 700 N=1,NUME
      READ (NA) LRD,(SS(I),I=1,LRD)
      NDK=ND*(ND+1)/2
      DO 600 I=1,ND
      LMN=1-LM(I)
      II=LM(I)-NSHIFT
      IF  (II.LE.0.OR.II.GT.NE2B) GO TO 600
      KK=1+NDK+6*ND
      TMASS(II)=TMASS(II)+ SS(I+KK)
      DO 300 L=1,LL
      DO 300 J=1,4
      KK=1+NDK+J*ND
  300 B(II,L)=B(II,L)+SS(I+KK)*STR(J,L)
      DO 500 J=1,ND
      JJ=LM(J)+LMN
      IF(JJ) 500,500,395
  395 IF(J-I) 396,394,394
  394 KK=1+ND*I-(I-1)*I/2+J
      GO TO 400
  396 KK=1+ND*J-(J-1)*J/2+I
  400 A(II,JJ)=A(II,JJ)+SS(KK)
  500 CONTINUE
  600 CONTINUE
      IF (MM.GT.1) GO TO 700
      DO 650 I=1,ND
      II=LM(I) -NSHIFT
      IF(II.GT.NE2B.AND.II.LE.NEBB) GO TO 660
  650 CONTINUE
      GO TO 700
  660 WRITE (7) LRD,(SS(I),I=1,LRD)
      NUM7=NUM7+1
  700 CONTINUE
      DO 710 L=1,NEQB
      ANORM=ANORM + A(L,1)
      IF  (A(L,1).NE.0.) NDEG=NDEG + 1
      IF  (A(L,1).EQ.0.) A(L,1)=1.E+20
      IF  (TMASS(L).NE.0.) NVV=NVV + 1
  710 CONTINUE
      IF(NDYN.NE.4) GO TO 716
      DO 714 I=1,NEQB
  714 A(I,1) = A(I,1) + A0* TMASS(I)
      WRITE (4) ((A(I,J),I=1,NEQB),J=1,MBAND)
      GO TO 718
  716 WRITE (4) ((A(I,J),I=1,NEQB),J=1,MBAND),((B(I,L),I=1,NEQB),L=1,LL)
  718 WRITE (9)    (TMASS(I),I=1,NEQB)
      IF(M.EQ.NBLOCK) GO TO 1000
      DO 720 L=K,NE2B
      ANORM=ANORM + A(L,1)
      IF  (A(L,1).NE.0.) NDEG=NDEG + 1
      IF  (A(L,1).EQ.0.) A(L,1)=1.E+20
      IF  (TMASS(L).NE.0.) NVV=NVV + 1
  720 CONTINUE
      IF(NDYN.NE.4) GO TO 726
      DO 724 I=K,NE2B
  724 A(I,1) = A(I,1) + A0* TMASS(I)
      WRITE (4) ((A(I,J),I=K,NE2B),J=1,MBAND)
      GO TO 728
  726 WRITE (4) ((A(I,J),I=K,NE2B),J=1,MBAND),((B(I,L),I=K,NE2B),L=1,LL)
  728 WRITE (9)    (TMASS(I),I=K,NE2B)
      IF  (MM.EQ.MB) MM=0
      MM=MM+1
 1000 NSHIFT=NSHIFT+NE2B
      IF  (NDEG.GT.0) GO TO 730
      WRITE (6,1010)
      STOP
  730 ANORM=(ANORM/NDEG)*1.E-8
      RETURN
 1002 FORMAT (4F10.0)
 1004 FORMAT (5I5,3F10.0)
 1010 FORMAT (51H0STRUCTURE WITH NO DEGREES OF FREEDOM   CHECK DATA    )
 2000 FORMAT (/// 10H STRUCTURE,13X,7HELEMENT,4X,4HLOAD,4X,
     1 11HMULTIPLIERS,/ 10H LOAD CASE,12X,1HA,9X,1HB,9X,1HC,9X,1HD,/ 1X)
 2002 FORMAT (I6,7X,4F10.3)
 2004 FORMAT (45H1S T E P - B Y - S T E P   S O L U T I O N     ,
     1              37HC O N T R O L   I N F O R M A T I O N, ///
     2 5X, 35HNUMBER OF TIME VARYING FUNCTIONS  =, I5      //
     3 5X, 35HGROUND MOTION INDICATOR           =, I5      /
     4 9X, 15HEQ.0, NONE, /
     5 9X, 29HGT.0, READ ACCELERATION INPUT, //
     6 5X, 35HNUMBER OF ARRIVAL TIMES                =, I5      /
     7 9X, 26HEQ.0, ALL FUNCTIONS ARRIVE, /
     8 9X, 18H       AT TIME ZERO, //
     9 5X, 35HNUMBER OF SOLUTION TIME STEPS     =, I5      //
     A 5X, 35HOUTPUT (PRINT) INTERVAL           =, I5      //
     B 5X, 35HSOLUTION TIME INCREMENT           =, E14.4 //
     C 5X, 32HMASS-       PROPORTIONAL DAMPING, /
     D 5X, 35HCOEFFICIENT (ALPHA)               =, E14.4 //
     E 5X, 30HSTIFFNESS-PROPORTIONAL DAMPING, /
     F 5X, 35HCOEFFICIENT (BETA)                =, E14.4 /// 1X)
 3000 FORMAT (27H0*** ERROR   ZERO TIME STEP, / 1X)
      END
      SUBROUTINE PRINTD (ID,D,B,NEQB,NUMNP,LL,NBLOCK,NEQ,NT,MQ)
      DIMENSION ID(NUMNP,6),B(NEQB,LL),D(6,LL)
      REWIND 8
      READ (8) ID
      N=NEQ
      NN=NEQB*NBLOCK
      IF  (MQ.EQ.2) GO TO 50
      IF  (MQ.EQ.3) GO TO 55
      REWIND NT
```

```
      Q1 = 6H  LOAD
      Q2 = 6H  CASE
      GO TO 60
   50 Q1 = 6HEIGEN-
      Q2 = 6HVECTOR
      GO TO 60
   55 Q1 = 6H MODE
      Q2 = 6HNUMBER
      REWIND NT
      READ (NT)
   60 WRITE (6,2003) Q1,Q2
      N=NUMNP
      DO 500 KK=1,NUMNP
      I=6
      DO 250 II=1,6
      DO 100 L=1,LL
  100 D(I,L)=0.
      IF(M.GT.NN) GO TO 150
      IF (M.EQ.0) GO TO 150
      READ (NT) B
      NM=NN-NEQB
  150 IF(ID(N,I).LT.1) GO TO 250
      K=M-NM
      M=M-1
      DO 200 L=1,LL
  200 D(I,L)=B(K,L)
  250 I=I-1
      WRITE (6,2004) N,(L,(D(I,L),I=1,6),L=1,LL)
  500 N=N-1
      RETURN
 2003 FORMAT (1H1,38HN O D E   D I S P L A C E M E N T S / ,
     1          17HR O T A T I O N S,// 3X,4HNODE,2X,A6,2(12X,2HX-,12X,
     2          2HY-,12X,2HZ-), / 7H NUMBER,2X,A6,3(3X,11HTRANSLATION),
     3          3(6X,8HROTATION), / 1X)
 2004 FORMAT (1H0,I6,I8,6E14.5 / (7X,I8,6E14.5) )
      END
      SUBROUTINE STRESS(STR,B,D,NEQB,LB,LL,NEQ,NBLOCK)
      DIMENSION D(NEQ,LB),B(NEQB,LL),STR(4,LL)
      COMMON /ELPAR/ NPAR(14),NUMNP,MBAND,NELTYP,N1,N2,N3,N4,N5,MTOT,NEQ
      COMMON /JUNK/ LT,LH
      COMMON /EXTRA/ MODEX,NT8,N10SV,NT10
      READ (8) STR
      NT=(LL-1)/LB +1
      LH=0
      IF(N10SV.EQ.1)
     *WRITE (NT10) NELTYP,NT
      DO 1000 II=1,NT
      LT =LH+1
      LLT=1-LT
      LH=LT+LB-1
      IF(LH.GT.LL) LH=LL
      REWIND 2
      IF(N10SV.EQ.1)
     *WRITE (NT10) LT,LH
      NJ=NEQB*NBLOCK
      DO 200 NN=1,NBLOCK
      READ (2) B
      N=NEQB
      IF (NN.EQ.1) N=NEQ-NJ+NEQB
      NJ=NJ-NEQB
      DO 200 J=1,N
      I=NJ+J
      DO 200 L=LT,LH
      K=L+LLT
  200 D(I,K)=B(J,L)
      LK=LH-LT+1
      REWIND 1
      DO 1000 M=1,NELTYP
      READ (1) NPAR
      IF(N10SV.EQ.1)
     *WRITE (NT10) NPAR
      MTYPE=NPAR(1)
      NPAR(1)=0
      CALL THICKS(MTYPE)
 1000 CONTINUE
      RETURN
      END
      SUBROUTINE CALBAN (MBAND,NDIF,LM,XM,S,P,ND,NDM,NS)
      DIMENSION LM(1),XM(1),S(NDM,NDM),P(NDM,4)
      COMMON /EXTRA/ MODEX,NT8
      MIN=100000
      MAX=0
      DO 800 L=1,ND
      IF (LM(L).EQ.0) GO TO 800
      IF (LM(L).GT.MAX) MAX=LM(L)
      IF (LM(L).LT.MIN) MIN=LM(L)
  800 CONTINUE
      NDIF=MAX-MIN+1
      IF (NDIF.GT.MBAND) MBAND=NDIF
      IF(MODEX.EQ.1) GO TO 810
      LRD=1 + 6*ND + ND*(ND+1)/2
      WRITE (2) LRD,ND,(LM(I),I=1,ND),((S(I,J),J=I,ND),I=1,ND),
    1 ((P(I,J),I=1,ND),J=1,4),(XM(I),I=1,ND)
      RETURN
  810 WRITE (1) ND,NS,(LM(I),I=1,ND)
      RETURN
      END
      SUBROUTINE STRSC(STR,D,NEQ,NTAG,INCON)
      DIMENSION STR(4,1),D(NEQ,1),INCON(8)
      COMMON /JUNK/ LT,LH,L,SG(20),SIG(8),LXTRA(150)
      COMMON /EM/   NS,ND,LM(63),B(48,63),TI(63,4)
      IF (NTAG.EQ.0) GO TO 800
      LL=L-LT+1
      DO 300 I=1,NS
      SG(I)=0.0
      DO 300 J=1,4
  300 SG(I)=SG(I)+TI(I,J)*STR(J,L)
      DO 500 J=1,ND
      JJ=LM(J)
      IF(JJ.EQ.0) GO TO 500
      DO 400 I=1,NS
  400 SG(I)=SG(I)+B(I,J)*D(JJ,LL)
  500 CONTINUE
      GO TO 900
  800 READ  (1) ND,NS,(LM(I),I=1,ND),(( B(I,J),I=1,NS),J=1,ND),
     1((TI(I,J),I=1,NS),J=1,4),(INCON(I),I=1,8)
  900 RETURN
      END
      OVERLAY(TSHE,7,0)
      PROGRAM ELT8
      COMMON /ELPAR/ NPAR(14),NUMNP,K1,K2,N1,N2,N3,N4,N5,K3,K4
      COMMON /JUNK/ DUM(51),N6,N7,N8,N9,N10,N11,N12,N13,N14,N15,N16
      COMMON        A(1)
      CALL THDFE(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),
     1           A(N10),A(N11),A(N12),A(N13),A(N14),A(N15),A(N16),
     2           NPAR(2),NPAR(3),NPAR(4),NPAR(5),NPAR(6),NPAR(7),
     3           NPAR(8),NPAR(9),NPAR(10),NUMNP)
      RETURN
      END
      SUBROUTINE THDFE(ID,X,Y,Z,T,DEN,RHO,NTP,EE,
     1                 DCA,NFACE,LT,PHA,LOC,MAXPTS,SS,
     2                 NUME,NUMMAT,MAXTP,NORTHO,NDLS,MAXNOD,
     3                 NOPSET,INTRS,INTT,NUMNP)
      COMMON /JUNK/ DXM(6),XLF(4),YLF(4),ZLF(4),TLF(4),PLF(4),
     1              FILL1(22),V2(3),
     1              FILL2(12),LS(4),KLS(4),NOD(21),NODSM(13),KOD(21),
     2              TAG,NREAD,E(12)
      COMMON /ELPAR/ FILL3(15),MBAND
      COMMON/EM/ LM(63),ND,NS,RF(63,4),XM(63),SF(48,4),INEL,NEL,
     1           NDIS,KDIS,NXYZ,KXYZ,NMAT,KMAT,NAXES,KAXES,IOP,
     2           KIOP,TZ,TTZ,NRSINT,KRSINT,NTINT,KTINT,IREUSE,
     3           KREUSE,K1,K2,DT,RATIO,D(6,6),TEMP(6,6),DUM(6,6),
     4           ALPHA(6),XX(3,21),B(6,63),H(21),P(3,21),SIGOT(6),
     5           DELT(21),FT(63),DL(21),PL(63),LOCOP(8),VIS(6),
     6           SDT(48,63)
      COMMON /GAUSS/ XG(4,4),WGT(4,4),STPTS(27,3)
      COMMON /DYN  / FILL4(9),NDYN
      COMMON /EXTRA/ MODEX,NT8
      DIMENSION  ID(NUMNP,1),X(1),Y(1),Z(1),T(1),DEN(1),RHO(1),
     1           NTP(1),EE(MAXTP,13,1),DCA(3,3,1),NFACE(1),LT(1),
     2   PHA(7,1),LOC(8,1),MAXPTS(1),SS(63,1)
      DATA STPTS /  1.,-1.,-1., 1., 1.,-1.,-1., 1., 0.,-1., 0., 1., 0.,
     1         -1., 0., 1., 1.,-1.,-1., 1., 0., 1.,-1., 0., 0., 0., 0.,
     2              1., 1.,-1.,-1., 1., 1.,-1.,-1., 1., 0.,-1., 0., 1.,
     3         0.,-1., 0., 1., 1.,-1.,-1., 0., 0., 0., 1.,-1., 0., 0.,
     4              1., 1., 1., 1.,-1.,-1.,-1.,-1., 1., 1., 1., 1.,-1.,
     5         -1.,-1.,-1., 0., 0., 0., 0., 0., 0., 0., 0., 0., 1.,-1./
      DATA XG /           0.,                 0.,                 0.,                 0.,
     1 -.577350269189626, .577350269189626,                 0.,                 0.,
     2 -.774596669241483, .000000000000000, .774596669241483,                 0.,
     3 -.861136311594053,-.339981043584856, .339981043584856, .861136311594053/
      DATA WGT / 2.000,                   0.,                 0.,                 0.,
     1 1.000000000000000,1.000000000000000,                 0.,                 0.,
     2  .555555555555556, .888888888888889, .555555555555556,                 0.,
     3  .347854845137454, .652145154862546, .652145154862546, .347854845137454/
      NT8SV = MODEX
      DO 10 I=4,6
      DO 10 J=1,63
   10 D(I,J) = 0.0
      DO 14 I=1,48
```

```
      DO 14 J=1,4
   14 UF(I,J)=0.0
      WRITE (6,3001) NUME,NUMMAT,MAXTP,NORTHO,NDLS,MAXNOD,NOPSET,INTRS,
     1               INTT
      CALL INP21        (NUMMAT,MAXTP,NORTHO,NDLS,NOPSET,NTBSV,NUMNP,X,
     1                    Y,Z,DEN,RHO,NTP,EE,DCA,NFACE,LT,PWA,LOC,MAXPTS)
                        NREAD =  8
      IF(MAXNOD.GT.8) NREAD = 21
      WRITE (6,3014)  (I,I=1,8)
      IF(MAXNOD.GT.8)
     *WRITE (6,3016)  (I,I=9,21)
      NEL = 0
      READ  (5,1008) INEL,NDIS,NXYZ,NMAT,MAXES,IOP,TZ,KG,NRSINT,NTINT
     1,IREUSE,(LS(I),I=1,4)
      READ  (5,1009)  (NOD(I),I=1,NREAD)
      IREUSE = 0
      IF(INEL.EQ.1) GO TO 51
      WRITE (6,4014)  INEL
      WRITE (6,4014)
      CALL EXIT
   50 READ  (5,1008) INEL,NDIS,NXYZ,NMAT,MAXES,IOP,TZ,KG,NRSINT,NTINT
     1,IREUSE,(LS(I),I=1,4)
      READ  (5,1009)  (NOD(I),I=1,NREAD)
   51 IF(NDIS.EQ.0)  NDIS = MAXNOD
      IF(NDIS.LE.MAXNOD) GO TO 5051
      WRITE (6,3015) INEL,NDIS,NXYZ,NMAT,MAXES,IOP,TZ,KG,NRSINT,NTINT
     1,IREUSE,(LS(I),I=1,4)
      WRITE (6,4015) NDIS, MAXNOD
      CALL EXIT
 5051 IF(NDIS.GE.8) GO TO 52
      WRITE (6,4023) NDIS
      CALL EXIT
   52 IF(NXYZ.EQ.0)  NXYZ = NDIS
      IF(NXYZ.LE.NDIS) GO TO 5052
      WRITE (6,4016) NXYZ,NDIS
      WRITE (6,4099)
      MODEX = 1
      GO TO 53
 5052 IF(NXYZ.GE.8) GO TO 53
      WRITE (6,4024) NXYZ
      WRITE (6,4099)
      MODEX = 1
   53 IF(NMAT.GE.1 .AND. NMAT.LE.NUMMAT) GO TO 54
      WRITE (6,3015) INEL,NDIS,NXYZ,NMAT,MAXES,IOP,TZ,KG,NRSINT,NTINT
     1,IREUSE,(LS(I),I=1,4)
      WRITE (6,4017)
      WRITE (6,4099)
      MODEX = 1
   54 IF(MAXES.LE.NORTHO) GO TO 55
      WRITE (6,3015) INEL,NDIS,NXYZ,NMAT,MAXES,IOP,TZ,KG,NRSINT,NTINT
     1,IREUSE,(LS(I),I=1,4)
      WRITE (6,4011)
      WRITE (6,4099)
      MODEX = 1
   55 IF(IOP.GE.0 .AND. IOP.LE.NOPSET) GO TO 56
      WRITE (6,3015) INEL,NDIS,NXYZ,NMAT,MAXES,IOP,TZ,KG,NRSINT,NTINT
     1,IREUSE,(LS(I),I=1,4)
      WRITE (6,4013)
      WRITE (6,4099)
      MODEX = 1
   56 DO 57 I=1,4
      IF(LS(I).GE.0 .AND. LS(I).LE.NDLS) GO TO 57
      WRITE (6,3015) INEL,NDIS,NXYZ,NMAT,MAXES,IOP,TZ,KG,NRSINT,NTINT
     1,IREUSE,(LS(J),J=1,4)
      WRITE (6,4020) LS(I)
      WRITE (6,4099)
      MODEX = 1
   57 CONTINUE
      IF(KG.EQ.0) KG = 1
      IF(NRSINT.EQ.0) NRSINT = INTRS
      IF(NTINT.EQ.0) NTINT = INTT
      DO 58 I=1,8
      IF(NOD(I).GE.1 .AND. NOD(I).LE.NUMNP) GO TO 58
      WRITE (6,3015) INEL,NDIS,NXYZ,NMAT,MAXES,IOP,TZ,KG,NRSINT,NTINT
     1,IREUSE,(LS(J),J=1,4)
      WRITE (6,4022) I,NOD(I)
      CALL EXIT
   58 CONTINUE
      IF(MAXNOD.LT.9) GO TO 60
      II = 0
      DO 59 I=9,21
      IF(NOD(I).EQ.0) GO TO 59
      II = II + 1
      NODNM(II) = I
      IF(NOD(I).LE.NUMNP) GO TO 59
      WRITE (6,3015) INEL,NDIS,NXYZ,NMAT,MAXES,IOP,TZ,KG,NRSINT,NTINT
     1,IREUSE,(LS(J),J=1,4)
      WRITE (6,4022) I,NOD(I)
      CALL EXIT
   59 CONTINUE
      I = II + 8
      IF(I.EQ.NDIS) GO TO 60
      WRITE (6,4025) I,NDIS
      CALL EXIT
   60 NEL = NEL + 1
      ML = INEL - NEL
      IF(ML) 65,70,80
   65 WRITE (6,4022) INEL
      CALL EXIT
   70 KDIS = NDIS
      KXYZ   = NXYZ
      KMAT   = NMAT
      KAXES  = MAXES
      KIOP   = IOP
      TTZ    = TZ
      KKG    = KG
      KRSINT = NRSINT
      KTINT  = NTINT
      KREUSE = IREUSE
      DO 72 I=1,4
   72 KLS(I) = LS(I)
      DO 74 I=1,NREAD
   74 KOD(I) = NOD(I)
      TAG = 1H*
      GO TO 90
   80 DO 85 I=1,NREAD
      IF(KOD(I).LT.1) GO TO 85
      KOD(I) = KOD(I) + KKG
   85 CONTINUE
      TAG = 1H
   90 ND = 3 * KDIS
      TAV = 0.0
      DO 95 K=1,KXYZ
      I = KOD(K)
   95 TAV = TAV + T(I)
      TAV = TAV / KXYZ
      NT = NTP(KMAT)
      IF(NT.GT.1) GO TO 100
   97 DO 99 I=1,12
   99 E(I) = EE(1,I+1,KMAT)
      GO TO 112
  100 IF(TAV.GE.EE(1,1,KMAT)) GO TO 104
  102 WRITE (6,4030) TAV,NEL,KMAT
      CALL EXIT
  104 IF(TAV.GT.EE(NT,1,KMAT)) GO TO 102
      IF(TAV.EQ.EE(1,1,KMAT)) GO TO 97
      IF(MODEX.EQ.1) GO TO 112
      DO 106 K=2,NT
      K2 = K
      K1 = K-1
      IF(TAV.GT.EE(K1,1,KMAT) .AND. TAV.LE.EE(K2,1,KMAT)) GO TO 108
  106 CONTINUE
  108 DT = EE(K2,1,KMAT) - EE(K1,1,KMAT)
      RATIO = (TAV - EE(K1,1,KMAT)) / DT
      DO 110 I=1,12
  110 E(I) = EE(K1,I+1,KMAT) + RATIO * (EE(K2,I+1,KMAT)-EE(K1,I+1,KMAT))
  112 CONTINUE
      IF(MODEX.EQ.0)
     *CALL SOLAW (D,E,TEMP,DCA(1,1,KAXES),KAXES,KMAT,NEL,DUM,ALPHA)
      IF(MODEX.EQ.1) GO TO 410
      DO 130 I=1,KDIS
      II = KOD(I)
      IF(I.LT.9) GO TO 125
      JJ = NODNM(I-8)
      II = KOD(JJ)
  125 XX(1,I) = X(II)
      XX(2,I) = Y(II)
      XX(3,I) = Z(II)
  130 CONTINUE
      DO 170 I=1,63
      DO 170 J=1,4
  170 RE(I,J)=0.0
      IF(KREUSE.EQ.1) GO TO 300
      DO 180 I=1,KDIS
  180 DL(I)=0.0
      DO 190 I=1,ND
  190 FT(I)=0.0
      KTL = 0
      DMX = 0.0
      DO 200 I=1,4
```

```
210 DUX = DUX + ABS(TLF(I))
    IF(DUX.GT.1.0E-06) KTL = 1
    IF (NDYN.GT.0) KTL=0
    IF(KTL.EQ.0 .OR. NDYN.GT.0) GO TO 235
    DO 210 I=1,6
    SIGDT(I) = 0.0
    DO 205 K=1,6
205 SIGDT(I) = SIGDT(I) + D(I,K)* ALPHA(K)
210 CONTINUE
    DO 230 I=1,KDIS
    II = KOD(I)
    IF(I.LT.9) GO TO 220
    J = NOD9M(I-8)
    II = KOD(J)
220 DELT(I) = T(II) - TTZ
230 CONTINUE
235 DUX=0.0
    DO 250 I=1,4
250 DUX = DUX + ABS(XLF(I)) + ABS(YLF(I)) + ABS(ZLF(I))
    KGL = 0
    IF(DUX.GT.1.0E-6) KGL = 1
    IF (NDYN.GT.0) KGL=0
    KMS = 0
    IF(NDYN.GT.0) KMS = 1
    DO 270 K=1,ND
270 XM(K) = 0.0
    DO 280 I=1,ND
    DO 280 K=I,ND
280 SS(I,K) = 0.0
    CALL STP21 (D,B,SS,XX,NOD9M,H,P,SIGDT,DELT,FT,DL,XM,NEL,ND,KDIS,
   1          KXYZ,KTL,KGL,KMS,KRSINT,KTINT,DEN(KMAT),RHO(KMAT))
300 IF (KTL.EQ.0) GO TO 325
    DO 320 I=1,ND
    DO 310 K=1,4
310 RF(I,K) = FT(I)* TLF(K)
320 CONTINUE
325 IF (KGL.EQ.0) GO TO 350
    DO 340 I=1,KDIS
    K3 = 3*I
    K2 = K3-1
    K1 = K2-1
    DO 330 L=1,4
    RF(K1,L) = RF(K1,L) + XLF(L)* DL(I)
    RF(K2,L) = RF(K2,L) + YLF(L)* DL(I)
330 RF(K3,L) = RF(K3,L) + ZLF(L)* DL(I)
340 CONTINUE
350 IF(NDLS.LT.1.OR.NDYN.GT.0) GO TO 405
    DO 400 L=1,4
    IF(PLF(L).EQ.0.0) GO TO 400
    M = KLD(L)
    IF(M.LT.1) GO TO 400
    DO 360 K=1,ND
360 PL(K) = 0.0
    CALL FACEPR (NEL,KDIS,KXYZ,XX,NOD9M,H,P,PL,NFACE(M),LT(M),
   1          PHA(1,M),M)
    DO 370 I=1,ND
370 RF(I,L) = RF(I,L) + PL(I)* PLF(L)
400 CONTINUE
405 CONTINUE
410 K = -3
    DO 420 I=1,KDIS
    II = KOD(I)
    IF(I.LT.9) GO TO 415
    JJ = NOD9M(I-8)
    II = KOD(JJ)
415 K = K+3
    DO 420 L=1,3
    M = K+L
420 LM(M) = ID(II,L)
    IF(KIOP.GT.0) NS = 6*MAXPTS(KIOP)
    IF(KIOP.EQ.0) NS = 6
    IF(NDYN.GT.0) NS=48
    CALL CALBAN (MBAND,NDIF,LM,XM,SS,RF,ND,63,NS)
    IF (NDYN.LT.1) GO TO 425
    NOP=8
    DO 422 I=1,8
422 LOCOP(I)=I + 20
    GO TO 450
425 IF (KIOP.EQ.0) GO TO 440
    NOP = MAXPTS(KIOP)
    DO 430 I=1,NOP
430 LOCOP(I) = LOC(I,KIOP)
    GO TO 450
440 NOP = 1
    LOCOP(1) = 21
450 IF(MODEX.EQ.1) GO TO 510
    DO 500 L=1,NOP
    M = LOCOP(L)
    E1= STPTS(M,1)
    E2= STPTS(M,2)
    E3= STPTS(M,3)
    CALL DERSDS (NEL,XX,B,DET,E1,E2,E3,NOD9M,H,P,KDIS,KXYZ)
    DO 470 I=1,6
    N = 6*(L-1)+I
    DO 465 J=1,ND
    Q = 0.0
    DO 460 K=1,6
460 Q = Q + D(I,K)* B(K,J)
465 SDT(N,J) = Q
470 CONTINUE
    IF(KTL.EQ.0 .OR. NDYN.GT.0) GO TO 500
    Q = 0.0
    DO 480 K=1,KDIS
480 Q = Q + H(K)* DELT(K)
    DO 485 K=1,6
485 VIS(K) = -Q * SIGDT(K)
    DO 490 I=1,6
    N = 6*(L-1)+I
    DO 490 K=1,4
490 SF(N,K) = VIS(I)* TLF(K)
500 CONTINUE
510 CONTINUE
    IF(MODEX.EQ.0)
   1WRITE (1) ND,NS,(LM(I),I=1,ND),((SDT(I,J),I=1,NS),J=1,ND),
   2((SF(I,J),I=1,NS),J=1,4), (KOD(I),I=1,8)
    WRITE (6,3015) NEL,KDIS,KXYZ,KMAT,KAXES,KIOP,TTZ,KKG,KRSINT,KTINT,
   1          KREUSE,KLS
    WRITE (6,3017) (KOD(I),I=1,NREAD)
    IF(NTAPV.EQ.1)
   1WRITE (NT8)    NEL,KDIS,KXYZ,KMAT,KAXES,KIOP,TTZ,      KRSINT,KTINT,
   2               KREUSE,KLS, IREAD,
   3               (KOD(I),I=1,NREAD)
    IF(NUME-NEL) 65,600,530
530 IF(ML)        50, 50, 60
600 RETURN
1008 FORMAT (6I5,F10.0,4I5,4I2)
1009 FORMAT (16I5)
3011 FORMAT ( 7X,34HNUMBER OF 21-NODE ELEMENTS        = I6 //
   1         7X,34HNUMBER OF MATERIAL SETS            = I6 //
   2         7X,26HMAXIMUM NUMBER OF MATERIAL,        /
   3         7X,34HTEMPERATURE INPUT POINTS           = I6 //
   4         7X,13HNUMBER OF MATERIAL,                /
   5         7X,34HAXIS ORIENTATION SETS              = I6 //
   *         7X,34HNUMBER OF DISTRIBUTED LOAD SETS    = I6 //
   6         7X,34HMAXIMUM NUMBER OF ELEMENT NODES    = I6 //
   7         7X,34HNUMBER OF STRESS OUTPUT SETS       = I6 //
   8         7X,34HR,S COORDINATE INTEGRATION ORDER   = I6 //
   9         7X,34HT   COORDINATE INTEGRATION ORDER   = I6 // 1X)
3014 FORMAT (52H13 / D  B  T )  2 1  N O D E  S O L I D  E L E,
   1  18H M E N T  D A T A, //  8H ELEMENT 2(2X,5HNODES),2(2X,
   2  5HMATL.),2X,6HSTRESS,4X,6HSTRESS,2X,4HNODE,2(2X,5HGAUSS),6X,
   3  2HK-,5X,3HLSA,3X,3HLSB,3X,3HLSC,3X,3HLSD, /
   4  8H NUMBER,7H -NDIS-,7H -NXYZ-,2X,5HTABLE,3X,4HAXES,2X,6HOUTPUT,
   5  6X,4HFREE,2X,4HINC.,2(3X,4HPTS.),2X,6HMATRIX,2X,4(2X,4H-UP-), /
   6  26X,3HNO.,4X,3HSET,5X,3HSET,5X,5HTEMP.,2X,4H-KG-,2X,5H-R,S-,4X,
   7  3H-T-,2X,6HRE-USE,2X, 8(2X,2HN-,I2)  )
3015 FORMAT (I8,4I7,I3,F10.1,I6,2I7,I3,2X,4I6)
3016 FORMAT (84X,8(2X,2HN-,I2) / 84X,5(2X,2HN-,I2) )
3017 FORMAT (84X,8I6 / 84X,8I6,/ 84X,5I6)
4014 FORMAT (33HOERROR***   ENCOUNTERED ELEMENT (,I5,13H), BUT EXPECT,
   1        21H TO READ ELEMENT ONE., / 1X)
4015 FORMAT (42HOERROR***   NUMBER OF DISPLACEMENT NODES (,I5,4H) IS,
   1        33H LARGER THAN MAXIMUM ALLOWED (,I5,2H)., / 1X)
4016 FORMAT (43HOERROR***   NUMBER OF COORDINATE NODES (,I5,6H) MUST,
   1        39H BE .LE. NUMBER OF DISPLACEMENT NODES (,I5,2H).)
4017 FORMAT (36HOERROR***   ILLEGAL MATERIAL NUMBER. )
4018 FORMAT (44HOERROR***   ILLEGAL MATERIAL AXIS REFERENCE.  )
4019 FORMAT (41HOERROR***   ILLEGAL OUTPUT SET REFERENCE.  )
4020 FORMAT (41HOERROR***   PRESSURE LOAD SET REFERENCE (,I5,4H) IS,
   1        9H ILLEGAL. )
4021 FORMAT (15HOERROR***   THE ,I2,18H-TH ELEMENT NODE (,I5,4H) IS,
   1        9H ILLEGAL.,/ 1X)
4022 FORMAT (26HOERROR***   ELEMENT NUMBER (,I5,11H) IS OUT OF,
   1        10H SEQUENCE., / 1X)
4023 FORMAT (42HOERROR***   NUMBER OF DISPLACEMENT NODES (,I5,
   1        25H) MUST BE AT LEAST EIGHT.  )
4024 FORMAT (40HOERROR***   NUMBER OF COORDINATE NODES (,I5,
   1        25H) MUST BE AT LEAST EIGHT.  )
4025 FORMAT (34HOERROR***   NUMBER OF NON-ZERO NODES (,I3,5H) READ,
   1        53H DOES NOT EQUAL THE NUMBER OF DISPLACEMENT NODES (,
```

```
  2                 I3,2H),,/ 1X)
4033 FORMAT (13H0ERROR***       AVERAGE TEMPERATURE (,F10.2,5H) FOR,
     1              19H ELEMENT (,I5,29H) OUT OF RANGE FOR MATERIAL (,I3,
     2              2H).,  / 1X)
4034 FORMAT (12X,31HPROCEED IN DATA CHECK ONLY MODE. / 1X)
     END
     SUBROUTINE INP21  (NUMMAT,MAXTP,NORTHO,NOLS,NOPSET,NTHSV,NUMNP,X,
    1                    Y,Z,DEN,RHO,NTP,EE,DCA,NFACE,LT,PHA,LOC,MAXPTS)
     COMMON /JUNK/ DXM(6),XLF(4),YLF(4),ZLF(4),TLF(4),PLF(4),
    1                 FILL1(22),V2(3)
     COMMON /EXTRA/ MODEX,NT8
     DIMENSION        X(1),Y(1),Z(1),DEN(1),RHO(1),NTP(1),EE(MAXTP,13,1),
    1     DCA(3,3,1),NFACE(1),LT(1),PHA(7,1),LOC(8,1),
    2                 MAXPTS(1)
     DIMENSION        HED(6)
     WRITE (6,3000)
     DO 10 I=1,NUMMAT
     READ  (5,1001) M,NTP(I),DEN(I),RHO(I),(HED(N),N=1,6)
     IF(RHO(I).EQ.0.0) RHO(I) = DEN(I) / 386.4
     IF(NTP(I).EQ.0) NTP(I) = 1
     WRITE (6,3002) M,NTP(I),DEN(I),RHO(I),(HED(N),N=1,6)
     IF(I.EQ.M) GO TO 2
     WRITE (6,4001)
     CALL EXIT
   2 IF(NTP(M).LE.MAXTP) GO TO 4
     WRITE (6,4002) MAXTP
     CALL EXIT
   4 NT = NTP(M)
     DO 6 K=1,NT
     READ(5,1002)EE(K,1,M),F,VG,VP,GG,GP,EG,EP,(EE(K,L,M),L=11,13)
     EPP=EP/0.875
     EE(K,2,M)=F*EG+0.25*EPP
     EE(K,3,M)=EPP*EG/(F*EPP+0.25*EG*0.9375)
     EE(K,4,M)=EE(K,3,M)
     EE(K,5,M)=F*VG+0.25*VP
     EE(K,6,M)=EE(K,5,M)*EE(K,2,M)/EE(K,3,M)
     EE(K,7,M)=EE(K,6,M)
     EE(K,8,M)=GG*GP/(0.25*GG+VG*GP)
     EE(K,9,M)=EE(K,8,M)
     EE(K,10,M)=EE(K,8,M)
     WRITE (6,3003) (EE(K,L,M),L=1,13)
   6 CONTINUE
     IF(NT.EQ.1) GO TO 10
     DO 8 J=2,NT
     IF(EE(J,1,M).GT.EE(J-1,1,M)) GO TO 8
     WRITE (6,4003)
     CALL EXIT
   8 CONTINUE
  10 CONTINUE
     IF(NT8SV.EQ.0) GO TO 12
     DO 11 M=1,NUMMAT
     WRITE (NT8) M,NTP(M),DEN(M),RHO(M)
     NT = NTP(M)
     WRITE (NT8) ((EE(K,L,M),L=1,13),K=1,NT)
  11 CONTINUE
  12 IF(NORTHO.EQ.0) GO TO 21
     WRITE (6,3004)
     DO 20 M=1,NORTHO
     READ  (5,1003) N,NI,NJ,NK
     WRITE (6,3005) N,NI,NJ,NK
     IF(NT8SV.EQ.1)
    *WRITE (NT8)      N,NI,NJ,NK
     IF(N.EQ.M) GO TO 13
     WRITE (6,4004)
     CALL EXIT
  13 IF(NI.GT.0 .AND. NI.LE.NUMNP) GO TO 5015
     L = NI
5014 WRITE (6,4005) L
     CALL EXIT
5015 IF(NJ.GT.0 .AND. NJ.LE.NUMNP) GO TO 5016
     L = NJ
     GO TO 5014
5016 IF(NK.GT.0 .AND. NK.LE.NUMNP) GO TO 14
     L = NK
     GO TO 5014
  14 CONTINUE
     CALL VECTP2 (DCA(1,1,M),X(NI),Y(NI),Z(NI),X(NJ),Y(NJ),Z(NJ),IERR)
     IF(IERR.EQ.0) GO TO 16
     WRITE (6,4006)
     CALL EXIT
  16 CALL VECTP2 (V2,X(NI),Y(NI),Z(NI),X(NK),Y(NK),Z(NK),IERR)
     IF(IERR.EQ.0) GO TO 17
     WRITE (6,4007)
     CALL EXIT
  17 CALL CROSS2 (DCA(1,1,M),V2,DCA(1,3,M),IERR)
     IF(IERR.EQ.0) GO TO 18
     WRITE (6,4008)
     CALL EXIT
     WRITE (6,4008)
     CALL EXIT
  18 CALL CROSS2 (DCA(1,3,M),DCA(1,1,M),DCA(1,2,M),IERR)
     IF(IERR.EQ.0) GO TO 20
     WRITE (6,4009)
     CALL EXIT
  20 CONTINUE
  21 IF(NOLS.EQ.0) GO TO 31
     WRITE (6,3006)
     DO 30 M=1,NOLS
     READ  (5,1004) N,NFACE(M),LT(M)
     WRITE (6,3007) N,NFACE(M),LT(M)
     IF(N.EQ.M) GO TO 22
     WRITE (6,4010)
     CALL EXIT
  22 IF(NFACE(M).GE.1 .AND. NFACE(M).LE.6) GO TO 23
     WRITE (6,4011)
     CALL EXIT
  23 IF(LT(M).EQ.0) LT(M) = 1
     IF(LT(M).GE.1 .AND. LT(M).LE.2) GO TO 24
     WRITE (6,4012)
     CALL EXIT
  24 IF(LT(M).EQ.2) GO TO 26
     READ  (5,1005) (PHA(I,M),I=1,4)
     DO 25 I=2,4
  25 IF(PHA(I,M).EQ.0.0) PHA(I,M) = PHA(1,M)
     WRITE (6,3008) (PHA(I,M),I=1,4)
     GO TO 30
  26 READ  (5,1005) (PHA(I,M),I=1,7)
     WRITE (6,3009) (PHA(I,M),I=1,7)
  30 CONTINUE
     IF(NT8SV.EQ.0) GO TO 5031
     DO 5030 M=1,NOLS
     WRITE (NT8) NFACE(M),LT(M),(PHA(I,M),I=1,7)
5030 CONTINUE
5031 CONTINUE
  31 IF(NOPSET.EQ.0) GO TO 49
     WRITE(6,3010)(I,I=1,8)
     DO 40 M=1,NOPSET
     READ(5,1006)(LOC(I,M),I=1,8)
     WRITE(6,3011)M,(LOC(I,M),I=1,8)
     L = 0
     DO 35 J=1,8
     IF(LOC(J,M).EQ.0) GO TO 36
     L = L + 1
     IF(LOC(J,M).GE.1 .AND. LOC(J,M).LE.27) GO TO 35
     WRITE (6,4013) J
     MODEX = 1
     GO TO 36
  35 CONTINUE
  36 IF(L.GT.0) GO TO 37
     L = 1
     LOC(1,M) = 21
  37 MAXPTS(M) = L
  40 CONTINUE
     IF(NT8SV.EQ.1)
    *WRITE(NT8)((LOC(I,J),I=1,8),J=1,NOPSET)
  49 WRITE (6,3012)
     READ  (5,1007) XLF,YLF,ZLF,TLF,PLF
     WRITE (6,3013) XLF,YLF,ZLF,TLF,PLF
     IF(NT8SV.EQ.1)
    *WRITE (NT8)      XLF,YLF,ZLF,TLF,PLF
     RETURN
1001 FORMAT (2I5,2F10.0,6A6)
1002 FORMAT(8F10.0/3F10.0)
1003 FORMAT (4I5)
1004 FORMAT (3I5)
1005 FORMAT (7F10.0)
1006 FORMAT(8I5)
1007 FORMAT (4F10.0)
3000 FORMAT (//48H M A T E R I A L   P R O P E R T Y   T A B L E S )
3002 FORMAT (//23H0MATERIAL NUMBER      = (,I3,1H),/
    1             10H NUMBER OF,                        /
    2             23H TEMPERATURE POINTS = (,I3,1H),/
    3             23H WEIGHT DENSITY     = (,E12.4,1H),/
    4             23H MASS  /DENSITY     = (,E12.4,1H),/
    5             23H IDENTIFICATION     = (,6A6,1H),//
    6 1X,12HTEMPERATURE,4X,3HE11,9X,3HE22,9X,3HE33,4X,3HV12,4X,3HV13,
    7 4X,3HV23,8X,3HG12,8X,3HG13,8X,3HG23,3X,7HALPHA-1,3X,7HALPHA-2,
    8 3X,7HALPHA-3,/1X)
3003 FORMAT (F12.2,3F12.1,3F7.3,3F11.1,3E10.3)
3004 FORMAT (//50H M A T E R I A L   A X I S   O R I E N T A T I O N,
```

```
     1 3X,3HT A B L E,//
     2 38H     SET    NODE     NODE     NODE, /
     3 38H NUMBER      NI       NJ       NK, / 1X)
3005 FORMAT (4I7)
3006 FORMAT (//51H D I S T R I B U T E D   S U R F A C E   L O A D   ,
     1          11HT A B L E S,/1X)
3007 FORMAT (//7X,27HLOAD SET NUMBER              = I6 /
     1          7X,27HLOAD SURFACE ELEMENT FACE = I6 /
     1          7X,27HLOAD TYPE CODE             = I6 / 1X)
3008 FORMAT (12H DISTRIBUTED,   11X,4HP(1),11X,4HP(2),11X,4HP(3),11X,
     1          4HP(4), / 4X,8HPRESSURE,4F15.3)
3009 FORMAT (12H HYDROSTATIC,10X,5HGAMMA,11X,4HX(S),11X,4HY(S),11X,
     1          4HZ(S),11X,4HX(N),11X,4HY(N),11X,4HZ(N), /
     2          4X,8HPRESSURE, 7F15.3)
3010 FORMAT (//52H S T R E S S   O U T P U T   R E Q U E S T   T A B L,
     1 2H E,//
     2  8H    SET ,8(2X,5HPOINT), / 8H NUMBER ,8(4X,1H(,I1,1H)),/ 1X)
3011 FORMAT (I8,8I7)
3012 FORMAT (///34H E L E M E N T   L O A D   C A S E,3X,
     1 21HM U L T I P L I E R S,// 31X,6HCASE A,4X,6HCASE B,4X,6HCASE C,
     2 4X,6HCASE D,/1X)
3013 FORMAT (
     1 27H  X-DIRECTION GRAVITY =      , 4F10.2 /
     2 27H  Y-DIRECTION GRAVITY =      , 4F10.2 /
     3 27H  Z-DIRECTION GRAVITY =      , 4F10.2 /
     4 27H  THERMAL  LOADING    =      , 4F10.2 /
     5 27H  PRESSURE LOADING    =      , 4F10.2 // 1X)
4001 FORMAT (40H0ERROR***    MATERIAL CARDS OUT OF ORDER.,/1X)
4002 FORMAT (52H0ERROR***    NUMBER OF TEMPERATURE CARDS EXCEEDS USER,
     1 10H MAXIMUM (,I4,2H)., / 1X)
4003 FORMAT (51H0ERROR***    TEMPERATURES MUST BE INPUT IN ASCENDING,
     1 7H ORDER., / 1X)
4004 FORMAT (47H0ERROR***    AXIS ORIENTATION CARD OUT OF ORDER.,/1X)
4005 FORMAT (36H0ERROR***    UNDEFINED NODE NUMBER = ,I5 / 1X)
4006 FORMAT (38H0ERROR***    VECTOR IJ HAS ZERO LENGTH.,/1X)
4007 FORMAT (38H0ERROR***    VECTOR IK HAS ZERO LENGTH.,/1X)
4008 FORMAT (43H0ERROR***    IJ AND IK VECTORS ARE PARALLEL.,/1X)
4009 FORMAT (43H0ERROR***    F3 AND F1 VECTORS ARE PARALLEL.,/1X)
4010 FORMAT (50H0ERROR***    SET NUMBERS MUST BE IN ASCENDING ORDER,/1X)
4011 FORMAT (43H0ERROR***    INVALID SURFACE FACE NUMBER.,/1X)
4012 FORMAT (33H0ERROR***    INVALID LOAD TYPE.,/1X)
4013 FORMAT (42H0ERROR***    INVALID OUTPUT POINT NUMBER = , I6 / 1X)
      END
      SUBROUTINE CROSS2 (A,B,C,IERR)
      DIMENSION A(3),B(3),C(3)
      X = A(2) * B(3) - A(3) * B(2)
      Y = A(3) * B(1) - A(1) * B(3)
      Z = A(1) * B(2) - A(2) * B(1)
      XLN = SQRT(X*X+Y*Y+Z*Z)
      IERR = 1
      IF(XLN.LE.1.0E-08) RETURN
      XLN = 1.0 / XLN
      C(3) = Z * XLN
      C(2) = Y * XLN
      C(1) = X * XLN
      IERR = 0
      RETURN
      END
      SUBROUTINE VECTR2 (V,XI,YI,ZI,XJ,YJ,ZJ,IERR)
      DIMENSION V(3)
      IERR = 1
      X = XJ - XI
      Y = YJ - YI
      Z = ZJ - ZI
      XLN = SQRT(X*X+Y*Y+Z*Z)
      IF(XLN.LE.1.0E-08) RETURN
      XLN = 1.0 / XLN
      IERR = 0
      V(3) = Z * XLN
      V(2) = Y * XLN
      V(1) = X * XLN
      RETURN
      END
      SUBROUTINE SSLAW (D,E,TEMP,DCA,KAXES,KMAT,NEL,DUM,ALPHA)
      DIMENSION D(6,6),E(12),TEMP(6,6),DCA(3,3),IPRM(3),DUM(6,6),
     1          ALPHA(6)
      DATA IPRM / 2,3,1 /
      DO 20 I=1,3
      ALPHA(I)   = E(I+9)
      ALPHA(I+3) = 0.0
      IF(E(I).GT.1.0E-08) GO TO 15
      WRITE (6,3000) I,I,KMAT,NEL
      STOP
3000 FORMAT (23H0ERROR***    MODULUS  E(,2I1,16H) FOR MATERIAL (,I2,
     1          14H) IN ELEMENT (,I5,10H) IS ZERO., / 1X)
```

```
   15 TEMP(I,I) = 1.0/E(I)
   20 CONTINUE
      TEMP(1,2) = -E(4)* TEMP(2,2)
      TEMP(2,1) =        TEMP(1,2)
      TEMP(1,3) = -E(5)* TEMP(3,3)
      TEMP(3,1) =        TEMP(1,3)
      TEMP(2,3) = -E(6)* TEMP(3,3)
      TEMP(3,2) =        TEMP(2,3)
      DO 60 N=1,3
      X = 1.0/TEMP(N,N)
      DO 30 J=1,3
   30 TEMP(N,J) = - TEMP(N,J)* X
      DO 50 I=1,3
      IF(N.EQ.I) GO TO 50
      DO 40 J=1,3
      IF(N.EQ.J) GO TO 40
      TEMP(I,J) = TEMP(I,J) + TEMP(I,N)* TEMP(N,J)
   40 CONTINUE
   50 TEMP(I,N) = TEMP(I,N)* X
      TEMP(N,N) = X
   60 CONTINUE
      DO 70 I=1,6
      DO 70 J=1,6
   70 D(I,J) = 0.0
      DO 80 I=1,3
      DO 80 J=1,3
   80 D(I,J) = TEMP(I,J)
      D(4,4) = E(7)
      D(5,5) = E(9)
      D(6,6) = E(8)
      IF(KAXES.LT.1) RETURN
      DO 100 I1=1,3
      I2 = IPRM(I1)
      DO 90 J1 = 1,3
      J2 = IPRM(J1)
      TEMP(I1  ,J1  ) = DCA(J1,I1)*DCA(J1,I1)
      TEMP(I1+3,J1  ) = DCA(J1,I1)*DCA(J1,I2) * 2.0
      TEMP(I1  ,J1+3) = DCA(J1,I1)*DCA(J2,I1)
      TEMP(I1+3,J1+3) = DCA(J1,I1)*DCA(J2,I2) +
     1                  DCA(J2,I1)*DCA(J1,I2)
   90 CONTINUE
  100 CONTINUE
      DO 130 I=1,6
      DO 120 J=1,6
      X = 0.0
      DO 110 K=1,6
  110 X = X + D(I,K)*TEMP(K,J)
  120 DUM(I,J) = X
  130 CONTINUE
      DO 160 I=1,6
      DO 150 J=I,6
      X = 0.0
      DO 140 K=1,6
  140 X = X + TEMP(K,I)*DUM(K,J)
      D(I,J) = X
      D(J,I) = X
  150 CONTINUE
  160 CONTINUE
      DO 200 I=1,6
      X = 0.0
      DO 190 K=1,3
  190 X = X + TEMP(K,I)*E(K+9)
      IF(I.GT.3) X = X*2.0
  200 ALPHA(I) = X
      RETURN
      END
      SUBROUTINE ST8R21 (E,B,S,XX,NOD9,H,P,SIGDT,DELT,FT,DL,XM,NEL,ND,
     1                   IELD,IELX,KTL,KGL,KMS,NINT,NINTZ,WTDEN,MSDEN)
      DIMENSION  E(6,1),B(6,1),S(3,1),XX(3,1),NOD9(1),H(1),P(3,1),
     1           SIGDT(1),DELT(1),FT(1),DL(1),XM(1),U(9),SDT(6),
     2           BV(63),W(3,3),IPERM(3,3),KOX(3),LOX(3)
      COMMON /GAUSS/ XG(4,4),WGT(4,4)
      REAL MSDEN
      DATA IPERM / 1,4,6, 4,2,5, 6,5,3 /
      VOL   = 0.0
      DUM = 0.0
      DO 20 I=4,6
      J = I-1
      DO 20 K=1,J
   20 DUM = DUM + ABS(E(K,I))
      ISO = 1
      IF(DUM.GT.1.0E-6) ISO = 0
      IF(ISO.EQ.0) GO TO 24
      DO 22 I=2,3
      DUM = DUM + ABS(E(I  ,I  ) - E(I-1,I-1))
```

```
   22 DUM = DUM + ABS(E(I+3,I+3) - E(I+2,I+2))
      DUM = DUM + ABS(E(1  ,2  ) - E(2  ,3  ))
      DUM = DUM + ABS(E(2  ,3  ) - E(3  ,1  ))
      IF(DUM.GT.1.0E-6) ISO = 0
   24 CONTINUE
      DO 10 LX=1,NINT
      E1=XG(LX,NINT)
      DO 10 LY=1,NINT
      E2=XG(LY,NINT)
      DO 10 LZ=1,NINTZ
      E3=XG(LZ,NINTZ)
      WT=WGT(LX,NINT)*WGT(LY,NINT)*WGT(LZ,NINTZ)
      CALL DERIOS (NEL,XX,3,DET,E1,E2,E3,NOD9,H,P,IELD,IELX)
      FACT = WT* DET
      FACT2 = SQRT(FACT)
      DO 25 I=1,IELD
      K3 = 3*I
      K2 = K3-1
      K1 = K2-1
      BV(K1) = B(1,K1)* FACT2
      BV(K2) = B(2,K2)* FACT2
      BV(K3) = B(3,K3)* FACT2
   25 CONTINUE
      DO 30 I=1,ND
      DO 30 J=I,ND
   30 S(I,J) = S(I,J) + BV(I)* BV(J)
      VOL = VOL + FACT
      IF(KSL.EQ.0) GO TO 150
      DO 130 K=1,IELD
  130 DL(K) = DL(K) + H(K)* FACT* WTDEN
  150 IF(KTL.EQ.0) GO TO 190
      DT = 0.0
      DO 160 K=1,IELD
  160 DT = DT + H(K)* DELT(K)
      DT = DT* FACT
      DO 170 K=1,6
  170 SDT(K) = SIGDT(K) *DT
      DO 180 K=1,ND
      DO 175 I=1,6
  175 FT(K) = FT(K) + B(I,K)* SDT(I)
  180 CONTINUE
  190 CONTINUE
   10 CONTINUE
      DO 35 I=1,2
      IC = ND-I
      DO 35 J=1,IC
      M=J+I
   35 S(I,J) = S(J,M)
      IF(ISO.EQ.0) GO TO 75
      D1 = E(1,1)
      D2 = E(1,2)
      D3 = E(4,4)
      DO 60 I=1,IELD
      K3 = 3*I
      K2 = K3-1
      K1 = K2-1
      K0 = K1-1
      DO 60 J=I,IELD
      L3 = 3*J
      L2 = L3-1
      L1 = L2-1
      L0 = L1-1
      IC = 0
      DO 40 II=1,3
      M = II+ K0
      DO 40 JJ=1,3
      N = JJ+ L0
      IC = IC+ 1
      D(IC) = S(M,N)
   40 CONTINUE
      S(K1,L1) = D(1)* D1 + (D(5) + D(9))* D3
      S(K2,L2) = D(5)* D1 + (D(1) + D(9))* D3
      S(K3,L3) = D(9)* D1 + (D(5) + D(1))* D3
      S(K1,L2) = D(2)* D2 + D(4)* D3
      S(K2,L1) = D(4)* D2 + D(2)* D3
      S(K2,L3) = D(6)* D2 + D(8)* D3
      S(K3,L2) = D(8)* D2 + D(6)* D3
      S(K3,L1) = D(7)* D2 + D(3)* D3
      S(K1,L3) = D(3)* D2 + D(7)* D3
   60 CONTINUE
      GO TO 110
   75 DO 100 I=1,IELD
      K0 = 3*I-3
      DO 100 J=I,IELD
      L0 = 3*J-3
      DO 80 II=1,3
      M = II+ K0
      DO 80 JJ=1,3
      N = JJ+ L0
      W(II,JJ) = S(M,N)
   80 CONTINUE
      DO 100 K=1,3
      I1 = K0+K
      DO 82 IJ=1,3
   82 KDX(IJ)=IPERM(IJ,K)
      DO 95 L=1,3
      I2 = L0+L
      DO 83 IJ=1,3
   83 LDX(IJ)=IPERM(IJ,L)
      SUM=0.0
      DO 90 II=1,3
      K1 = KDX(II)
      DO 85 JJ=1,3
      K2 = LDX(JJ)
   85 SUM = SUM + W(II,JJ)*E(K1,K2)
   90 CONTINUE
      S(I1,I2) = SUM
   95 CONTINUE
  100 CONTINUE
  110 CONTINUE
      DO 200 I=1,ND
      DO 200 J=I,ND
  200 S(J,I) = S(I,J)
      IF(KMS.EQ.0) RETURN
      FACT = VOL* MSDEN/ IELD
      DO 220 K=1,ND
  220 XM(K) = FACT
      RETURN
      END
      SUBROUTINE FACEPR (NEL,KDIS,KXYZ,XX,NOD9,H,P,PL,NFACE,LT,PWA,KLS)
      DIMENSION      XX(3,1),NOD9(1),H(1),P(3,1),PL(1),PWA(1)
      DIMENSION      XJ(3,3),ETA(3),KFACE(6,8),KCRD(6),FVAL(6),IPRM(3),
     1               PR(8),NODES(8),IPR4(4)
      COMMON /GAUSS/ XK(4,4),WGT(4,4)
      DATA  KFACE  /  1,  2,  1,  4,  1,  5,
     1                4,  3,  5,  8,  2,  6,
     2                8,  7,  6,  7,  3,  7,
     3                5,  6,  2,  3,  4,  8,
     4               12, 10, 17, 20,  9, 13,
     5               20, 19, 13, 15, 10, 14,
     6               16, 14, 18, 19, 11, 15,
     7               17, 18,  9, 11, 12, 16/
      DATA  KCRD   /  1,  1,  2,  2,  3,  3/
      DATA  FVAL   / 1.,-1., 1.,-1., 1.,-1./
      DATA  IPRM   /  2,  3,  1/
      DATA  IPR4   /  2,  3,  4,  1/
      DO 2 I=1,4
      NODES(I  ) = KFACE(NFACE,I)
      NODES(I+4) = 0
    2 CONTINUE
      IF(KDIS.LT.9) GO TO 9
      NN9 = KDIS-8
      DO 8 K=5,8
      DO 4 I=1,NN9
      J = I
      IF(KFACE(NFACE,K).EQ.NOD9(I)) GO TO 6
    4 CONTINUE
      GO TO 8
    6 NODES(K) = J
    8 CONTINUE
    9 CONTINUE
      FACT = -FVAL(NFACE)
      GO TO (10,30), LT
   10 DO 25 K=1,8
      IF(NODES(K).EQ.0) GO TO 25
      IF(K.GT.4) GO TO 15
      PR(K) = PWA(K) * FACT
      GO TO 25
   15 J = K-4
      L = IPR4(J)
      PR(K) = (PWA(J) + PWA(L)) * 0.5 * FACT
   25 CONTINUE
      GO TO 75
   30 GAMMA = PWA(1)* FACT
      XLN = 0.0
      DO 35 K=1,3
      ETA(K) =  PWA(K+4) - PWA(K+1)
   35 XLN = XLN + ETA(K)**2
      XLN = SQRT(XLN)
      IF(XLN.GT.1.E-6) GO TO 4
```

```
      WRITE (6,3000) KLS,NEL
 3000 FORMAT (31H0ERROR***   PRESSURE LOAD SET (,I3,15H) FOR ELEMENT (,
     1        I5,43H) HAS UNDEFINED HYDROSTATIC SURFACE NORMAL., / 1X)
      STOP
   40 DO 45 K=1,3
   45 ETA(K) = ETA(K)/ XLN
      DO 70 N=1,8
      IF(NODES(N).EQ.0) GO TO 70
      XLN = 0.0
                  NOD = NODES(N)
      IF(N.GT.4) NOD = NOD + 8
      DO 50 I=1,3
   50 XLN = XLN + (XX(I,NOD) - PWA(I+1))* ETA(I)
      PR(N) = XLN* GAMMA
      IF(XLN.LT.0.0) PR(N) = 0.0
   70 CONTINUE
   75 CONTINUE
      ML = KCRD(NFACE)
      MM = IPRM(ML)
      MN = IPRM(MM)
      ETA(ML) = FVAL(NFACE)
      DO 300 LX=1,3
      ETA(MM) = XK(LX,3)
      DO 300 LY=1,3
      ETA(MN) = XK(LY,3)
      WT = WGT(LX,3)* WGT(LY,3)
      CALL FNCT (ETA(1),ETA(2),ETA(3),H,P,NOD9,XJ,DET,XX,KDIS,KXYZ,NEL)
      A1 =  XJ(MM,2)* XJ(MN,3)- XJ(MM,3)* XJ(MN,2)
      A2 =  XJ(MM,3)* XJ(MN,1)- XJ(MM,1)* XJ(MN,3)
      A3 =  XJ(MM,1)* XJ(MN,2)- XJ(MM,2)* XJ(MN,1)
      AA =  SQRT(A1**2 + A2**2 + A3**2)
      IF(AA.GT.1.0E-8) GO TO 100
      WRITE (6,3010) NFACE,NEL
 3010 FORMAT (36H0ERROR***   UNDEFINED NORMAL IN FACE (,I1,5H) FOR,
     1        10H ELEMENT (,I5,2H)., / 1X)
      STOP
  100 FACT = 1.0/AA
      A1 = A1* FACT
      A2 = A2* FACT
      A3 = A3* FACT
      AA = 0.0
      BB = 0.0
      CC = 0.0
      DO 120 I=1,3
      AA = AA + XJ(MM,I)**2
      CC = CC + XJ(MN,I)**2
  120 BB = BB + XJ(MM,I)* XJ(MN,I)
      C  = SQRT(AA*CC - BB**2)
      PRESS = 0.0
      DO 130 K=1,8
      IF(NODES(K).EQ.0) GO TO 130
                  NOD = NODES(K)
      IF(K.GT.4) NOD = NOD + 8
      PRESS = PRESS + H(NOD)* PR(K)
  130 CONTINUE
      FACT = WT* C* PRESS
      DO 160 L=1,8
      IF(NODES(L).EQ.0) GO TO 160
      IF(L.GT.4) GO TO 140
      N = NODES(L)
      K = 3*N
      GO TO 150
  140 J = NODES(L)
      N = J+8
      K = 3* NOD9(J)
  150 QQ = FACT* H(N)
      PL(K-2) = PL(K-2) + QQ* A1
      PL(K-1) = PL(K-1) + QQ* A2
      PL(K  ) = PL(K  ) + QQ* A3
  160 CONTINUE
  300 CONTINUE
      RETURN
      END
      SUBROUTINE DERBDS (NEL,XX,B,DET,R,S,T,NOD9,H,P,IELD,IELX)
      DIMENSION  XX(3,1),B(6,1),NOD9(1),H(1),P(3,1)
      DIMENSION  XJ(3,3),XJI(3,3)
      CALL FNCT (R,S,T,H,P,NOD9,XJ,DET,XX,IELD,IELX,NEL)
      DUM=1.0/DET
      XJI(1,1)=DUM*( XJ(2,2)*XJ(3,3) - XJ(2,3)*XJ(3,2))
      XJI(2,1)=DUM*(-XJ(2,1)*XJ(3,3) + XJ(2,3)*XJ(3,1))
      XJI(3,1)=DUM*( XJ(2,1)*XJ(3,2) - XJ(2,2)*XJ(3,1))
      XJI(1,2)=DUM*(-XJ(1,2)*XJ(3,3) + XJ(1,3)*XJ(3,2))
      XJI(2,2)=DUM*( XJ(1,1)*XJ(3,3) - XJ(1,3)*XJ(3,1))
      XJI(3,2)=DUM*(-XJ(1,1)*XJ(3,2) + XJ(1,2)*XJ(3,1))
      XJI(1,3)=DUM*( XJ(1,2)*XJ(2,3) - XJ(1,3)*XJ(2,2))
      XJI(2,3)=DUM*(-XJ(1,1)*XJ(2,3) + XJ(1,3)*XJ(2,1))
      XJI(3,3)=DUM*( XJ(1,1)*XJ(2,2) - XJ(1,2)*XJ(2,1))
      DO 130 K=1,IELD
      K2=K*3
      DO 115 L=1,3
      B(L,K2-2) = 0.0
      B(L,K2-1) = 0.0
  115 B(L,K2  ) = 0.0
      DO 120 I=1,3
      B(1,K2-2) = B(1,K2-2)  + XJI(1,I)* P(I,K)
      B(2,K2-1) = B(2,K2-1)  + XJI(2,I)* P(I,K)
  120 B(3,K2  ) = B(3,K2  )  + XJI(3,I)* P(I,K)
      B(4,K2-2) = B(2,K2-1)
      B(4,K2-1) = B(1,K2-2)
      B(5,K2-1) = B(3,K2  )
      B(5,K2  ) = B(2,K2-1)
      B(6,K2-2) = B(3,K2  )
  130 B(6,K2  ) = B(1,K2-2)
      RETURN
      END
      SUBROUTINE FNCT (R,S,T,H,P,NOD9,XJ,DET,XX,IELD,IELX,NEL)
      DIMENSION H(1),P(3,1),NOD9(1),IPERM(8),XJ(3,3),XX(3,1)
      DATA IPERM / 2,3,4,1,6,7,8,5 /
      IEL = IELD
      NND9= IELD-8
      RP=1.0 + R
      SP=1.0 + S
      TP=1.0 + T
      RM=1.0 - R
      SM=1.0 - S
      TM=1.0 - T
      RR=1.0 - R*R
      SS=1.0 - S*S
      TT=1.0 - T*T
      H(1)=0.125*RP*SP*TP
      H(2)=0.125*RM*SP*TP
      H(3)=0.125*RM*SM*TP
      H(4)=0.125*RP*SM*TP
      H(5)=0.125*RP*SP*TM
      H(6)=0.125*RM*SP*TM
      H(7)=0.125*RM*SM*TM
      H(8)=0.125*RP*SM*TM
      P(1,1)= 0.125*SP*TP
      P(1,2)=-P(1,1)
      P(1,3)=-0.125*SM*TP
      P(1,4)=-P(1,3)
      P(1,5)= 0.125*SP*TM
      P(1,6)=-P(1,5)
      P(1,7)=-0.125*SM*TM
      P(1,8)=-P(1,7)
      P(2,1)= 0.125*RP*TP
      P(2,2)= 0.125*RM*TP
      P(2,3)=-P(2,2)
      P(2,4)=-P(2,1)
      P(2,5)= 0.125*RP*TM
      P(2,6)= 0.125*RM*TM
      P(2,7)=-P(2,6)
      P(2,8)=-P(2,5)
      P(3,1)= 0.125*RP*SP
      P(3,2)= 0.125*RM*SP
      P(3,3)= 0.125*RM*SM
      P(3,4)= 0.125*RP*SM
      P(3,5)=-P(3,1)
      P(3,6)=-P(3,2)
      P(3,7)=-P(3,3)
      P(3,8)=-P(3,4)
      IF (IEL.EQ.8) GO TO 50
      I=0
    2 I=I + 1
      IF (I.GT.NND9) GO TO 40
      NN=NOD9(I) - 8
      GO TO (9,10,11,12,13,14,15,16,17,18,19,20,21) ,NN
    9 H(9) =0.25*RR*SP*TP
      P(1,9) =-0.50*R*SP*TP
      P(2,9) = 0.25*RR*TP
      P(3,9) = 0.25*RR*SP
      GO TO 2
   10 H(10)=0.25*RM*SS*TP
      P(1,10)=-0.25*SS*TP
      P(2,10)=-0.50*RM*S*TP
      P(3,10)= 0.25*RM*SS
      GO TO 2
   11 H(11)=0.25*RR*SM*TP
      P(1,11)=-0.50*R*SM*TP
      P(2,11)=-0.25*RR*TP
```

```
      P(3,11)= 0.25*RR*SM
      GO TO 2
   12 H(12)=0.25*RP*SS*TP
      P(1,12)= 0.25*SS*TP
      P(2,12)=-0.50*RP*S*TP
      P(3,12)= 0.25*RP*SS
      GO TO 2
   13 H(13)=0.25*RR*SP*TM
      P(1,13)=-0.50*R*SP*TM
      P(2,13)= 0.25*RR*TM
      P(3,13)=-0.25*RR*SP
      GO TO 2
   14 H(14)=0.25*RM*SS*TM
      P(1,14)=-0.25*SS*TM
      P(2,14)=-0.50*RM*S*TM
      P(3,14)=-0.25*RM*SS
      GO TO 2
   15 H(15)=0.25*RR*SM*TM
      P(1,15)=-0.50*R*SM*TM
      P(2,15)=-0.25*RR*TM
      P(3,15)=-0.25*RR*SM
      GO TO 2
   16 H(16)=0.25*RP*SS*TM
      P(1,16)= 0.25*SS*TM
      P(2,16)=-0.50*RP*S*TM
      P(3,16)=-0.25*RP*SS
      GO TO 2
   17 H(17)=0.25*RP*SP*TT
      P(1,17)= 0.25*SP*TT
      P(2,17)= 0.25*RP*TT
      P(3,17)=-0.50*RP*SP*T
      GO TO 2
   18 H(18)=0.25*RM*SP*TT
      P(1,18)=-0.25*SP*TT
      P(2,18)= 0.25*RM*TT
      P(3,18)=-0.50*RM*SP*T
      GO TO 2
   19 H(19)=0.25*RM*SM*TT
      P(1,19)=-0.25*SM*TT
      P(2,19)=-0.25*RM*TT
      P(3,19)=-0.50*RM*SM*T
      GO TO 2
   20 H(20)=0.25*RP*SM*TT
      P(1,20)= 0.25*SM*TT
      P(2,20)=-0.25*RP*TT
      P(3,20)=-0.50*RP*SM*T
      GO TO 2
   21 H(21)=RR*SS*TT
      P(1,21)=-2.0*R*SS*TT
      P(2,21)=-2.0*S*RR*TT
      P(3,21)=-2.0*T*RR*SS
      GO TO 2
   40 IN=0
   41 IN=IN + 1
      IF (IN.GT.NND9) GO TO 50
      II=IN + 7
      IF (II.EQ.IELX) GO TO 51
   42 IN=NOD9(IN)
      IF (IN.GT.16) GO TO 46
      I1=IN - 8
      I2=IPERM(I1)
      H(I1)=H(I1) - 0.5*H(IN)
      H(I2)=H(I2) - 0.5*H(IN)
      H(IN+8)=H(IN)
      DO 45 J=1,3
      P(J,I1)=P(J,I1) - 0.5*P(J,IN)
      P(J,I2)=P(J,I2) - 0.5*P(J,IN)
   45 P(J,IN+8)=P(J,IN)
      GO TO 41
   46 IF (IN.EQ.21) GO TO 30
      I1=IN - 16
      I2=I1 + 4
      H(I1)=H(I1) - 0.5*H(IN)
      H(I2)=H(I2) - 0.5*H(IN)
      H(IN+8)=H(IN)
      DO 47 J=1,3
      P(J,I1)=P(J,I1) - 0.5*P(J,IN)
      P(J,I2)=P(J,I2) - 0.5*P(J,IN)
   47 P(J,IN+8)=P(J,IN)
      GO TO 41
   30 IN=0
   31 IN=IN + 1
      IN=NOD9(IN)
      IF (IN.EQ.21) GO TO 35
      IF (IN.GT.16) GO TO 33
      I1=IN - 8
      I2=IPERM(I1)
      H(I1)=H(I1) + 0.125*H(21)
      H(I2)=H(I2) + 0.125*H(21)
      DO 32 J=1,3
      P(J,I1)=P(J,I1) + 0.125*P(J,21)
   32 P(J,I2)=P(J,I2) + 0.125*P(J,21)
      GO TO 31
   33 I1=IN - 16
      I2=I1 + 4
      H(I1)=H(I1) + 0.125*H(21)
      H(I2)=H(I2) + 0.125*H(21)
      DO 34 J=1,3
      P(J,I1)=P(J,I1) + 0.125*P(J,21)
   34 P(J,I2)=P(J,I2) + 0.125*P(J,21)
      GO TO 31
   35 DO 36 I=1,8
      H(I)=H(I) - 0.125*H(21)
      DO 36 J=1,3
   36 P(J,I)=P(J,I) - 0.125*P(J,21)
      NN=NND9 + 7
      IF (NN.EQ.8) GO TO 50
      DO 38 I=9,NN
      H(I)=H(I) - 0.25*H(21)
      DO 38 J=1,3
   38 P(J,I)=P(J,I) - 0.25*P(J,21)
      H(NND9+8)=H(21)
      DO 39 J=1,3
   39 P(J,NND9+8)=P(J,21)
   50 IF (IELX.LT.IELD) RETURN
   51 DO 100 I=1,3
      DO 100 J=1,3
      DUM=0.0
      DO 95 K=1,IELX
   95 DUM=DUM + P(I,K)*XX(J,K)
  100 XJ(I,J)=DUM
      DET = XJ(1,1)*XJ(2,2)*XJ(3,3)
     1    + XJ(1,2)*XJ(2,3)*XJ(3,1)
     2    + XJ(1,3)*XJ(2,1)*XJ(3,2)
     3    - XJ(1,3)*XJ(2,2)*XJ(3,1)
     4    - XJ(1,2)*XJ(2,1)*XJ(3,3)
     5    - XJ(1,1)*XJ(2,3)*XJ(3,2)
      IF(DET.GT.1.0E-8) GO TO 111
      WRITE (6,2000) NEL,R,S,T
      STOP
  111 IF (IELX.LT.IELD) GO TO 42
      RETURN
 2000 FORMAT (43H0ERROR***   NEGATIVE OR ZERO JACOBIAN DETERMINANT,
     1           23H COMPUTED FOR ELEMENT (,I5,1H), /
     2           12X,  3HR =, F10.5 /
     3           12X,  3HS =, F10.5 /
     4           12X,  3HT =, F10.5 / 1X)
      END
      OVERLAY(TSHE,14,0)
      PROGRAM SOLEQ
      COMMON A(1)
      COMMON /ELPAR/ NP(14),NUMNP,MBAND,NELTYP,N1,N2,N3,N4,N5,MTOT,NEC
      COMMON /SOL  / NBLOCK,NEQB,LL,NF
      DIMENSION TT(4)
      CALL SECOND (TT(1))
      NSB=(MBAND+LL)*NEQB
      NSBB=NEQB*LL*(2+(MBAND-2)/NEQB)
      IF(NSBB.LT.NSB) NSBB=NSB
      N4=N3+NSBB
      MI = MBAND + NEQB -1
      CALL SESOL (A(N1),A(N3),A(N4),NEQ, MBAND,LL,NBLOCK,NEQB,NSB,MI,
     1            4,3,2,7)
      CALL SECOND (TT(2))
      N2=N1+NUMNP*6
      N3=N2+6*LL
      CALL PRINTD (A(N1),A(N2),A(N3),NEQB,NUMNP,LL,NBLOCK,NEQ,2,1)
      CALL SECOND (TT(3))
      N2=N1+4*LL
      N3=N2+NEQB*LL
      LB=(MTOT-N3)/(NEQ +12)
      CALL STRESS(A(N1),A(N2),A(N3),NEQB,LB,LL,NEQ,NBLOCK)
      CALL SECOND (TT(4))
      DO 55 K=1,3
   55 TT(K) = TT(K+1)-TT(K)
      WRITE (6,2000) (TT(L),L=1,3)
 2000 FORMAT (//// 46H S T A T I C   S O L U T I O N   T I M E   L O G,
     1         //5X,21HEQUATION SOLUTION     =, F8.2 /
     2           5X,21HDISPLACEMENT OUTPUT  =, F8.2 /
     3           5X,21HSTRESS RECOVERY      =, F8.2 /)
      RETURN
```

```
      END
      SUBROUTINE SESOL (A,B,MAXA,NEQ,MA,NV,NBLOCK,NEQB,NAV,MI,NSTIF,
     1                  NRED,NL,NR)
      DIMENSION A(NAV),B(NAV),MAXA(MI)
      MI=1
      MA2=MA - 2
      IF(MA2.EQ.0) MA2=1
      INC=NEQB - 1
      NWA=NEQB*MA
      NTB=(MA-2)/NEQB + 1
      NEB=NTB*NEQB
      NEBT=NEB + NEQB
      NAV=NEQB*NV
      NAVV=NEBT*NV
      N1=NL
      N2=NR
      REWIND NSTIF
      REWIND NRED
      REWIND N1
      REWIND N2
      DO 600 NJ=1,NBLOCK
      IF (NJ.NE.1) GO TO 10
      READ (NSTIF) A
      IF(NEQ.GT.1) GO TO 100
      MAXA(1)=1
      WRITE(NRED) A,MAXA
      IF(A(1)) 1,174,3
    1 KK=1
      WRITE(6,1010) KK,A(1)
    3 DO 5 L=1,NV
    5 A(1+L)=A(1+L)/A(1)
      KR=1+NV
      WRITE(NL) (A(KK),KK=2,KR)
      RETURN
10    IF (NTB.EQ.1) GO TO 100
      REWIND N1
      REWIND N2
      READ (N1) A
100   KU=1
      KM=MIN0(MA,NEQB)
      MAXA(1)=1
      DO 110 N=2,MI
      IF (N-MA) 120,120,130
120   KU=KU + NEQB
      KK=KU
      MM=MIN0(N,KM)
      GO TO 140
130   KU=KU + 1
      KK=KU
      IF (N-NEQB) 140,140,136
136   MM=MM - 1
140   DO 160 K=1,MM
      IF (A(KK)) 110,160,110
160   KK=KK - INC
110   MAXA(N)=KK
      IF (A(1)) 172,174,176
174   KK=(NJ-1)*NEQB + 1
      IF (KK.GT.NEQ) GO TO 590
      WRITE (6,1000) KK
      STOP
172   KK=(NJ-1)*NEQB + 1
      WRITE (6,1010) KK,A(1)
176   DO 200 N=2,NEQB
      NH=MAXA(N)
      IF (NH-N) 200,200,210
210   KL=N + INC
      K=N
      D=0.
      DO 220 KK=KL,NH,INC
      K=K - 1
      C=A(KK)/A(K)
      D=D + C*A(KK)
220   A(KK)=C
      A(N)=A(N) - D
      IF (A(N)) 222,224,230
224   KK=(NJ-1)*NEQB + N
      IF (KK.GT.NEQ) GO TO 590
      WRITE (6,1000) KK
      STOP
222   KK=(NJ-1)*NEQB + N
      WRITE (6,1010) KK,A(N)
230   IC=NEQB
      DO 240 J=1,MA2
      MJ=MAXA(N+J) - IC
      IF (MJ-N) 240,240,250
250   KU=MIN0(MJ,NH)
      KN=N + IC
      C=0.
      DO 300 KK=KL,KU,INC
300   C=C + A(KK)*A(KK+IC)
      A(KN)=A(KN) - C
240   IC=IC + NEQB
      K=N + NWA
      DO 430 L=1,NV
      KJ=K
      C=0.
      DO 440 KK=KL,NH,INC
      KJ=KJ - 1
440   C=C + A(KK)*A(KJ)
      A(K)=A(K) - C
430   K=K + NEQB
200   CONTINUE
      DO 400 NK=1,NTB
      IF ((NK+NJ).GT.NBLOCK) GO TO 400
      NI=N1
      IF ((NJ.EQ.1).OR.(NK.EQ.NTB)) NI=NSTIF
      READ (NI) B
      ML=NK*NEQB + 1
      MR=MIN0((NK+1)*NEQB,MI)
      IF(MA.EQ.1) ML=MR
      MD=MI - ML
      KL=NEQB + (NK-1)*NEQB*NEQB
      N=1
      DO 500 M=ML,MR
      NH=MAXA(M)
      KL=KL + NEQB
      IF (NH-KL) 505,510,510
510   K=NEQB
      D=0.
      DO 520 KK=KL,NH,INC
      C=A(KK)/A(K)
      D=D + C*A(KK)
      A(KK)=C
520   K=K - 1
      B(N)=B(N) - D
      IF (MD) 580,580,530
530   IC=NEQB
      DO 540 J=1,MD
      MJ=MAXA(M+J) - IC
      IF (MJ-KL) 540,550,550
550   KU=MIN0(MJ,NH)
      KN=N + IC
      C=0.
      DO 575 KK=KL,KU,INC
575   C=C + A(KK)*A(KK+IC)
      B(KN)=B(KN) - C
540   IC=IC + NEQB
580   KN=N + NWA
      K=NEQB + NWA
      DO 610 L=1,NV
      KJ=K
      C=0.
      DO 620 KK=KL,NH,INC
      C=C + A(KK)*A(KJ)
620   KJ=KJ - 1
      B(KN)=B(KN) - C
      KN=KN + NEQB
610   K=K + NEQB
505   MD=MD - 1
500   N=N + 1
      IF (NTB.NE.1) GO TO 560
      WRITE (NRED) A,MAXA
      DO 570 I=1,NAV
570   A(I)=B(I)
      GO TO 600
560   WRITE (N2) B
400   CONTINUE
      N=N1
      N1=N2
      N2=N
590   WRITE (NRED) A,MAXA
600   CONTINUE
      DO 700 K=1,NAVV
700   B(K)=0.
      REWIND NL
      DO 800 NJ=1,NBLOCK
      BACKSPACE NRED
      READ (NRED) A,MAXA
      BACKSPACE NRED
      K=NT
```

```
      DO 810 L=1,NV
      DO 820 I=1,NEB
      B(K)=B(K-NEQB)
820   K=K - 1
810   K=K + NEBT + NEB
      KM=0
      KK=MA
      NDIF=NEQB
      IF (NJ.EQ.1) NDIF=NEQB - (NBLOCK*NEQB - NEQ)
      DO 855 L=1,NV
      DO 850 K=1,NDIF
850   B(KM+K)=A(KK+K)/A(K)
      KK=KK + NEQB
855   KM=KM + NEBT
     IF(MA.EQ.1) GO TO 915
      ML=NEQB + 1
      KL=NEQB
      DO 860 M=ML,MI
      KL=KL + NEQB
      KU=MAXA(M)
      IF (KU-KL) 860,870,870
870   K=NEQB
      KM=M
      DO 880 L=1,NV
      KJ=K
      DO 890 KK=KL,KU,INC
      B(KJ)=B(KJ) - A(KK)*B(KM)
890   KJ=KJ - 1
      KM=KM + NEBT
880   K=K + NEBT
860   CONTINUE
      N=NEQB
      DO 910 I=2,NEQB
      KL=N + INC
      KU=MAXA(N)
      IF (KU-KL) 910,920,920
920   K=N
      DO 930 L=1,NV
      KJ=K
      DO 940 KK=KL,KU,INC
      KJ=KJ - 1
940   B(KJ)=B(KJ) - A(KK)*B(K)
930   K=K + NEBT
910   N=N - 1
 915 KK=0
      KM=0
      DO 950 L=1,NV
      DO 960 K=1,NEQB
      KK=KK + 1
960   A(KK)=B(KM+K)
950   KM=KM + NEBT
      WRITE (NL) (A(K),K=1,NHV)
900   CONTINUE
1000 FORMAT (// 46H   STOP  ***  ZERO DIAGONAL ENCOUNTERED DURING, /
     1               18H EQUATION SOLUTION, /
     2          13X,16H EQUATION NUMBER =, I6 )
1010 FORMAT (/  50H WARNING ***  NEGATIVE DIAGONAL ENCOUNTERED DURING,
     1               18H EQUATION SOLUTION, /
     2          13X,16H EQUATION NUMBER =, I6, 5X, 7HVALUE =, E20.8 )
      RETURN
      END
```

APPENDIX 3

Part of coefficients tables for the calculation of deflections and bending moments forces in circular plates simply supported on two short lengths of arcs at opposite ends of a diameter.

$$W = C_1 \frac{qa^2}{D}$$

$$M_r = C_2 \, q \, a^2$$

$$M_t = C_3 \, q \, a^2$$

$$M_{rt} = C_4 \, q \, a^2$$

in the tables coefficients C_1 to C_4 are listed under W, MR, MT and MRT respectively.

POISSON,S RATIO= .050
THETA = 0/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.237805	.518078	-.136928	0
.10	.235176	.515300	-.141139	0
.20	.227316	.506929	-.150213	0
.30	.214302	.492847	-.167642	0
.40	.196268	.472821	-.193494	0
.50	.173402	.446365	-.229567	0
.60	.145952	.412270	-.278605	0
.70	.114238	.367240	-.344301	0
.80	.078684	.302509	-.430503	0
.90	.039914	.196454	-.538534	0
1.00	-.001022	-.000000	-.660799	0

POISSON,S RATIO= .050
THETA = 15.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.237805	.474208	-.092958	.153726
.10	.235408	.470656	-.095017	.155040
.20	.228250	.459852	-.101149	.169073
.30	.216439	.441306	-.111170	.176106
.40	.200154	.414078	-.124629	.186603
.50	.179664	.376601	-.140548	.201151
.60	.155338	.326585	-.157069	.220205
.70	.127673	.261374	-.171149	.243366
.80	.097316	.179605	-.178721	.267664
.90	.065080	.085733	-.176189	.283956
1.00	.031891	-.000000	-.164723	.270025

POISSON,S RATIO= .050
THETA = 30.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.237805	.354352	.026898	.283582
.10	.236040	.349596	.027318	.284305
.20	.230792	.335229	.028789	.286303
.30	.222205	.310996	.031935	.289000
.40	.210526	.276674	.037751	.291235
.50	.196100	.232462	.047441	.290992
.60	.179375	.179677	.062081	.285155
.70	.160884	.121725	.082058	.269719
.80	.141214	.065157	.105376	.240973
.90	.120934	.020250	.132122	.199329
1.00	.100496	-.000000	.154734	.157646

POISSON,S RATIO= .050
THETA = 45.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.237805	.190625	.190625	.327453
.10	.236901	.186337	.192305	.325877
.20	.234233	.173774	.197250	.320904
.30	.229927	.153895	.205112	.311871
.40	.224106	.128462	.215117	.297908
.50	.217268	.100143	.225856	.278378
.60	.209458	.072387	.235142	.253518
.70	.201037	.048784	.240098	.225123
.80	.192242	.031576	.237674	.196675
.90	.183244	.018859	.225828	.171747
1.00	.174175	-.000000	.205680	.148533

POISSON,S RATIO= .050
THETA = 60.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.237805	.026809	.354351	.283583
.10	.237761	.025460	.354893	.280166
.20	.237641	.021521	.356235	.270086
.30	.237485	.016109	.357602	.253900
.40	.237347	.010573	.357941	.232644
.50	.237281	.006085	.356295	.207831
.60	.237331	.003078	.352275	.181237
.70	.237527	.000907	.346541	.154428
.80	.237887	-.001716	.340978	.128305
.90	.238437	-.004502	.338044	.103390
1.00	.239195	.000000	.338353	.082362

POISSON,S RATIO= .300
THETA = 0/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.277258	.508896	-.096396	0
.10	.274305	.505839	-.100543	0
.20	.263464	.496625	-.113185	0
.30	.250797	.481123	-.134972	0
.40	.230401	.459076	-.167146	0
.50	.204411	.429983	-.211821	0
.60	.173002	.392721	-.272396	0
.70	.135383	.344454	-.354033	0
.80	.094820	.277993	-.463978	0
.90	.043694	.176060	-.611211	0
1.00	-.001328	-.000000	-.304539	0

POISSON,S RATIO= .300
THETA = 15.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.277258	.468349	-.055849	.151323
.10	.274595	.464673	-.058840	.152564
.20	.265637	.453517	-.067803	.156381
.30	.253478	.434474	-.082679	.163071
.40	.235277	.406760	-.103247	.173146
.50	.212275	.369092	-.128895	.187336
.60	.184801	.319604	-.158256	.206479
.70	.153305	.256134	-.183719	.231138
.80	.113388	.177501	-.216011	.260589
.90	.080822	.086935	-.234201	.290587
1.00	.041530	-.000000	-.236878	.308989

POISSON,S RATIO= .300
THETA = 30.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.277258	.357573	.054927	.262099
.10	.275388	.352973	.054320	.262935
.20	.269826	.339098	.052699	.265325
.30	.260721	.315760	.050675	.268867
.40	.248321	.282812	.049281	.272729
.50	.232985	.240458	.049916	.275417
.60	.215180	.189807	.054209	.274530
.70	.195475	.133552	.063708	.266730
.80	.174513	.077324	.079439	.248389
.90	.152958	.029195	.101456	.217799
1.00	.131400	-.000000	.124677	.180394

POISSON,S RATIO= .300
THETA = 45.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.277258	.206250	.206250	.302646
.10	.276469	.202173	.207096	.301570
.20	.274148	.190195	.209518	.298144
.30	.270436	.171121	.213710	.291823
.40	.265553	.146422	.219031	.281863
.50	.259786	.118313	.224808	.267643
.60	.253461	.089655	.229622	.249149
.70	.246904	.063461	.231279	.227538
.80	.240406	.041722	.226859	.205411
.90	.234192	.023233	.213055	.186015
1.00	.228426	-.000000	.185974	.169966

POISSON,S RATIC= .300
THETA = 60.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.277258	.054927	.357573	.262099
.10	.277547	.053387	.357839	.259429
.20	.278432	.049079	.358439	.251534
.30	.279961	.042847	.358810	.238804
.40	.282202	.035813	.358174	.221972
.50	.285230	.028976	.355775	.202134
.60	.289111	.022761	.351241	.180596
.70	.293899	.016736	.345019	.158527
.80	.299642	.009927	.338769	.136554
.90	.306394	.002495	.335469	.114788
1.00	.314222	.000000	.338791	.094247

POISSON,S RATIO= .300
THETA = 75.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.277258	-.055849	.468349	.151323
.10	.278335	-.054283	.466930	.149057
.20	.281544	-.049712	.462584	.142558
.30	.286822	-.042532	.455657	.132648
.40	.294067	-.033432	.445965	.120466
.50	.303148	-.023342	.433774	.107222
.60	.313915	-.013259	.419134	.093997
.70	.326209	-.004033	.401628	.081672
.80	.339872	.003527	.379903	.070967
.90	.354750	.007345	.351333	.062402
1.00	.370712	-.000000	.312404	.055704

POISSON,S RATIO= .300
THETA = 90.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTICN	MR	MT	MRT
0	.277258	-.096396	.508896	.000001
.10	.278623	-.093434	.506599	.000000
.20	.282679	-.084977	.499909	.000000
.30	.289311	-.072212	.489373	.000000
.40	.298347	-.056826	.475735	.000000
.50	.309575	-.040726	.459814	.000000
.60	.322767	-.025856	.442494	.000000
.70	.337704	-.014104	.424953	.000000
.80	.354203	-.006995	.409062	.000000
.90	.372132	-.004315	.397735	.000000
1.00	.391421	.000000	.394601	.000000

POISSCN,S RATIC= .350
THETA = 0/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTICN	MR	MT	MRT
0	.290494	.507504	-.088754	0
.10	.287427	.504393	-.093063	0
.20	.278244	.495018	-.106196	0
.30	.262998	.479243	-.128818	0
.40	.241781	.456808	-.162203	0
.50	.214712	.427210	-.208520	0
.60	.181946	.389340	-.271288	0
.70	.143665	.340447	-.355937	0
.80	.100090	.273629	-.470331	0
.90	.051540	.172400	-.624845	0
1.00	-.001419	-.000000	-.831271	0

POISSON,S RATIO= .350
THETA = 15.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.290494	.467562	-.048812	.149064
.10	.287735	.463860	-.051981	.150292
.20	.279488	.452633	-.061485	.154070
.30	.265842	.433484	-.077287	.160698
.40	.246955	.405663	-.099208	.170696
.50	.223056	.367934	-.126703	.184820
.60	.194468	.318512	-.158494	.203979
.70	.161632	.255323	-.192050	.228912
.80	.125134	.177218	-.223059	.259300
.90	.085742	.087266	-.245165	.291794
1.00	.044382	-.000000	-.250577	.316084

POISSON,S RATIO= .350
THETA = 30.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTICN	MR	MT	MRT
0	.290494	.358440	.060310	.258187
.10	.288576	.353865	.059508	.259043
.20	.282871	.340071	.057299	.261505
.30	.273528	.316882	.054293	.265201
.40	.260799	.284165	.051525	.269360
.50	.245047	.242124	.050436	.272581
.60	.226746	.191831	.052753	.272594
.70	.206476	.135967	.060235	.266186
.80	.184900	.079640	.074277	.249740
.90	.162707	.030877	.095472	.221163
1.00	.140529	-.000000	.123372	.184536

POISSON,S RATIO= .350
THETA = 45.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.290494	.209375	.209375	.298129
.10	.289723	.205334	.210061	.297144
.20	.287457	.193454	.212117	.294000
.30	.283840	.174513	.215489	.288172
.40	.279100	.149928	.219990	.278941
.50	.273531	.121832	.224687	.265688
.60	.267471	.092979	.228594	.248353
.70	.261260	.066276	.229541	.227978
.80	.255203	.043670	.224631	.207002
.90	.249538	.024082	.210329	.188613
1.00	.244437	-.000000	.193008	.173869

POISSON,S RATIO= .350
THETA = 60.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.290494	.060311	.358439	.258187
.10	.290867	.058750	.358647	.255653
.20	.292004	.054365	.359086	.248156
.30	.293955	.047970	.359235	.236055
.40	.296793	.040644	.358362	.220029
.50	.300596	.033354	.355751	.201097
.60	.305437	.026524	.351031	.180479
.70	.311375	.019761	.344611	.159273
.80	.318463	.012148	.338109	.138056
.90	.326764	.003822	.334600	.116863
1.00	.335354	.000000	.338312	.096411

POISSON,S RATIO= .350
THETA = 75.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.290494	-.048812	.467562	.149065
.10	.291703	-.047353	.466290	.146951
.20	.295307	-.043086	.462118	.140691
.30	.301243	-.036403	.455338	.131209
.40	.309406	-.027965	.445942	.119532
.50	.319666	-.018672	.434047	.106805
.60	.331869	-.009491	.419650	.094055
.70	.345853	-.001255	.402290	.082126
.80	.361457	.005237	.381552	.071721
.90	.378520	.007996	.351616	.063382
1.00	.396900	-.000000	.311318	.056984

POISSON,S RATIO= .350
THETA = 90.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.290494	-.088754	.507594	.000000
.10	.292009	-.085934	.505314	.000000
.20	.296512	-.077889	.498925	.000000
.30	.303887	-.065760	.488825	.000000
.40	.313958	-.051175	.475677	.000000
.50	.325508	-.035979	.463208	.000000
.60	.341305	-.022049	.443219	.000000
.70	.359123	-.011215	.425807	.000000
.80	.376773	-.004956	.409810	.000000
.90	.397119	-.003111	.398262	.000000
1.00	.419092	.000000	.395403	.000000

POISSON,S RATIO= .400
THETA = 0/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.306406	.506244	-.091244	0
.10	.303290	.503081	-.085714	0
.20	.293600	.493546	-.099332	0
.30	.277654	.477502	-.122779	0
.40	.255442	.454684	-.157357	0
.50	.227069	.424588	-.205293	0
.60	.192665	.386118	-.270219	0
.70	.152379	.336605	-.357828	0
.80	.106388	.269428	-.476582	0
.90	.054934	.160865	-.638209	0
1.00	-.001525	-.000000	-.857401	0

POISSON,S RATIO= .400
THETA = 15.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.306406	.466890	-.041890	.146872
.10	.303529	.463163	-.045234	.148087
.20	.294928	.451862	-.055272	.151827
.30	.280690	.432607	-.071936	.158394
.40	.260965	.404674	-.095240	.168318
.50	.235976	.366880	-.124553	.182378
.60	.206036	.317511	-.158733	.201553
.70	.171573	.254582	-.195326	.226751
.80	.133164	.176977	-.229984	.258050
.90	.091569	.087556	-.255937	.292966
1.00	.047719	-.000000	-.264056	.322971

POISSON,S RATIO= .400
THETA = 30.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.306406	.359372	.065628	.254390
.10	.304427	.354322	.064632	.255266
.20	.298540	.341103	.061845	.257797
.30	.288896	.318055	.057871	.261643
.40	.275751	.285555	.053752	.266089
.50	.259471	.243810	.050964	.269829
.60	.240541	.193854	.051333	.270715
.70	.219556	.138260	.056821	.265658
.80	.197199	.081921	.069182	.251051
.90	.174193	.032527	.089532	.224427
1.00	.151212	-.000000	.118042	.188557

POISSON,S RATIO= .400
THETA = 45.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.306406	.212500	.212500	.293744
.10	.305651	.208492	.213028	.292848
.20	.303437	.196705	.214623	.289977
.30	.299910	.177888	.217265	.284629
.40	.295307	.153407	.220770	.276105
.50	.289932	.125315	.224593	.263791
.60	.284135	.096264	.227589	.247581
.70	.278274	.069056	.227811	.228405
.80	.272669	.045594	.222385	.208546
.90	.267573	.024924	.207552	.191134
1.00	.263167	-.000000	.178975	.177657

POISSON,S RATIO= .400
THETA = 60.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.306406	.065628	.359372	.254390
.10	.306872	.064046	.359521	.251988
.20	.308292	.059585	.359795	.244877
.30	.310717	.053026	.359714	.233387
.40	.314224	.045410	.358592	.218143
.50	.318898	.037673	.355750	.200090
.60	.324816	.030235	.350819	.180366
.70	.332044	.022744	.344171	.159998
.80	.340641	.014337	.337384	.139514
.90	.350676	.005128	.333625	.118377
1.00	.362239	.000000	.337662	.098512

POISSON,S RATIO= .400
THETA = 75.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.306406	-.041890	.466890	.146873
.10	.307765	-.040530	.465581	.144808
.20	.311820	-.036566	.461648	.138878
.30	.318505	-.030370	.455096	.129813
.40	.327717	-.022581	.445969	.118626
.50	.339319	-.014070	.434335	.106400
.60	.353156	-.005775	.420143	.094112
.70	.369063	.001487	.402889	.082568
.80	.386870	.006929	.381098	.072454
.90	.406410	.008645	.351759	.064333
1.00	.427527	-.000000	.310089	.058225

POISSON,S RATIO= .400
THETA = 90.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.305406	-.081244	.506244	.000000
.10	.308091	-.078564	.504156	.000000
.20	.313106	-.070921	.498051	.000000
.30	.321332	-.059414	.488360	.000000
.40	.332587	-.045614	.475667	.000000
.50	.345648	-.031302	.460614	.000000
.60	.363277	-.018295	.443915	.000000
.70	.382239	-.008363	.426593	.000000
.80	.403338	-.002944	.410450	.000000
.90	.426433	-.001925	.398643	.000000
1.00	.451452	.000000	.395997	.000000

POISSON,S RATIO= .450
THETA = 0/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.325672	.505112	-.073862	0
.10	.322298	.501896	-.078490	0
.20	.312190	.492203	-.092587	0
.30	.295390	.475893	-.115848	0
.40	.271965	.452698	-.152605	0
.50	.242003	.422109	-.202137	0
.60	.205609	.383047	-.269189	0
.70	.162893	.332921	-.359706	0
.80	.113976	.265380	-.482735	0
.90	.059019	.155449	-.651315	0
1.00	-.001651	-.000000	-.882956	0

POISSON,S RATIO= .450
THETA = 15.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.325672	.466328	-.035078	.144744
.10	.322651	.462575	-.038596	.145946
.20	.313617	.451231	-.049159	.149649
.30	.298654	.431838	-.066773	.156157
.40	.277904	.403791	-.091339	.166008
.50	.251582	.365923	-.122442	.180008
.60	.219991	.316597	-.158972	.199198
.70	.183546	.253909	-.193550	.224652
.80	.142812	.176776	-.236791	.256835
.90	.098542	.087856	-.256527	.294104
1.00	.051675	-.000000	-.277326	.329658

POISSON,S RATIO= .450
THETA = 30.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.325672	.360369	.070881	.250703
.10	.323617	.355840	.069696	.251599
.20	.317501	.342193	.066338	.254197
.30	.307478	.319276	.061410	.258189
.40	.293808	.286981	.055961	.262913
.50	.276865	.245515	.051500	.267156
.60	.257144	.195876	.049946	.268890
.70	.235257	.140533	.053463	.265145
.80	.211914	.084169	.064151	.252323
.90	.187876	.034147	.083635	.227597
1.00	.163869	-.000000	.112681	.192461

POISSON,S RATIO= .450
THETA = 45.00/DEGREES
ALFA = 2.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.325672	.215625	.215625	.289487
.10	.324933	.211649	.215997	.288677
.20	.322767	.199949	.217136	.286071
.30	.319327	.181248	.219064	.281188
.40	.314856	.156862	.221673	.273352
.50	.309674	.128766	.224524	.261948
.60	.304144	.099511	.226606	.246331
.70	.298643	.071801	.226089	.228819
.80	.293511	.047495	.220121	[illegible]
.90	.289018	.025759	.204725	.193583
1.00	.285354	-.000000	.174878	.181335

POISSON,S RATIO= .250
THETA = 15.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MPT
0	.253739	.466561	-.060311	.152095
.10	.251173	.462895	-.063072	.153280
.20	.243507	.451773	-.071341	.156918
.30	.230837	.437792	-.085044	.163250
.40	.213331	.405199	-.103956	.172686
.50	.191232	.367779	-.127513	.185770
.60	.164880	.318795	-.154526	.203040
.70	.134728	.256241	-.182835	.224614
.80	.101368	.178916	-.209076	.249203
.90	.065555	.089325	-.228917	.272040
1.00	.028182	-.000000	-.238448	.280914

POISSON,S RATIO= .250
THETA = 30.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MPT
0	.253739	.355220	.051030	.263436
.10	.251917	.350652	.050581	.264188
.20	.246497	.336830	.049419	.266322
.30	.237624	.313745	.048099	.269431
.40	.225541	.281134	.047544	.272695
.50	.210595	.239776	.048989	.274692
.60	.193235	.189258	.053830	.273204
.70	.174006	.133748	.063335	.265246
.80	.153521	.077830	.078235	.247686
.90	.132409	.029621	.098312	.219258
1.00	.111227	-.000000	.122255	.185155

POISSON,S RATIO= .250
THETA = 45.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MPT
0	.253739	.203125	.203125	.304190
.10	.252931	.199094	.204049	.303033
.20	.250550	.187252	.206792	.299375
.30	.246730	.168389	.211218	.292697
.40	.241692	.143941	.216958	.282292
.50	.235681	.116052	.223240	.267556
.60	.229037	.087495	.228719	.248429
.70	.222058	.061224	.231409	.225857
.80	.215057	.039341	.228787	.202131
.90	.208231	.021155	.218226	.180158
1.00	.201762	-.000000	.197871	.160787

POISSON,S RATIO= .250
THETA = 60.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MPT
0	.253739	.051031	.355219	.263436
.10	.253942	.049536	.355499	.260708
.20	.254567	.045355	.356140	.252647
.30	.255660	.039309	.356589	.239647
.40	.257284	.032592	.356074	.222451
.50	.259511	.025951	.353921	.202162
.60	.262404	.020163	.349383	.180129
.70	.266012	.014848	.342995	.157644
.80	.270376	.009121	.335874	.135582
.90	.275540	.002828	.330205	.114354
1.00	.281552	.000000	.328404	.094983

POISSON,S RATIO= .250
THETA = 75.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MPT
0	.253739	-.060311	.466561	.152095
.10	.254681	-.058708	.465116	.149801
.20	.257487	-.054924	.460799	.143213
.30	.262094	-.046650	.453670	.133147
.40	.268406	-.037265	.443872	.120740
.50	.276298	-.026799	.431616	.107215
.60	.285626	-.016289	.417056	.093668
.70	.296240	-.005683	.400020	.080975
.80	.307393	.001119	.379663	.069799
.90	.320749	.005301	.354284	.060563
1.00	.334399	-.000000	.321826	.053011

POISSON,S RATIO= .250
THETA = 30.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MPT
0	.253739	-.101065	.507315	.000001
.10	.254952	-.098079	.504979	.000000
.20	.258551	-.089505	.498178	.000000
.30	.264428	-.076533	.487471	.000000
.40	.272418	-.060817	.473630	.000000
.50	.282319	-.044246	.457501	.000000
.60	.293913	-.028727	.439955	.000000
.70	.306988	-.016076	.422024	.000000
.80	.321365	-.007769	.405163	.000000
.90	.336910	-.003893	.391427	.000000
1.00	.353541	.000000	.383016	.000000

POISSON,S RATIO= .300
THETA = 0/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MPT
0	.264010	.505831	-.093331	0
.10	.261079	.502721	-.097395	0
.20	.252305	.493344	-.109730	0
.30	.237752	.477534	-.130951	0
.40	.217523	.454975	-.162147	0
.50	.191763	.425063	-.205155	0
.60	.159660	.386547	-.262863	0
.70	.124448	.336578	-.339569	0
.80	.083426	.258502	-.441194	0
.90	.038021	.167188	-.574964	0
1.00	-.011069	-.000000	-.747855	0

POISSON,S RATIO= .300
THETA = 15.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MPT
0	.264010	.455694	-.053194	.149790
.10	.261366	.452004	-.056136	.150964
.20	.253465	.450813	-.064954	.154567
.30	.240402	.431732	-.079596	.160848
.40	.222339	.404037	-.099877	.170225
.50	.199513	.366558	-.125297	.183274
.60	.172256	.317629	-.154749	.200505
.70	.141010	.255324	-.186151	.222510
.80	.106359	.178477	-.216118	.248080
.90	.069046	.089426	-.239989	.273351
1.00	.029965	-.000000	-.252752	.287724

POISSON,S RATIO= .300
THETA = 30.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MPT
0	.264010	.356040	.056460	.259444
.10	.262151	.351498	.055814	.260219
.20	.256621	.337807	.054062	.262431
.30	.247566	.314819	.051756	.265704
.40	.235230	.282432	.049827	.269276
.50	.219964	.240876	.049548	.271812
.60	.202223	.191201	.052410	.271219
.70	.182559	.135968	.059881	.264627
.80	.161602	.080057	.073041	.248932
.90	.140001	.031259	.092197	.222569
1.00	.118345	-.000000	.116596	.189643

POISSON,S RATIO= .300
THETA = 45.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MPT
0	.264010	.206250	.206250	.299581
.10	.263221	.202254	.207015	.298517
.20	.260899	.190508	.209293	.295145
.30	.257190	.171776	.212992	.288968
.40	.252281	.147440	.217822	.279300
.50	.246484	.119569	.223118	.265540
.60	.240109	.090832	.227677	.247582
.70	.233484	.064082	.229674	.226294
.80	.226906	.041367	.226511	.203786
.90	.220615	.022095	.215502	.182877
1.00	.214791	-.000000	.194146	.164685

POISSON,S RATIO= .300
THETA = 60.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MRT
0	.264010	.056460	.356040	.259445
.10	.264288	.054944	.356263	.256854
.20	.265140	.050685	.356747	.249193
.30	.266612	.044474	.356980	.236829
.40	.268773	.037374	.356235	.220448
.50	.271697	.030370	.353780	.201080
.60	.275452	.023961	.349173	.179987
.70	.280095	.017888	.342618	.158385
.80	.285670	.011319	.335290	.137091
.90	.292228	.004096	.329424	.116454
1.00	.299826	.000000	.327843	.097285

POISSON,S RATIO= .300
THETA = 75.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MPT
0	.264010	-.053194	.465694	.149791
.10	.265168	-.051695	.464309	.147599
.20	.268222	-.047320	.460161	.141302
.30	.273409	-.040445	.453291	.131568
.40	.280533	-.031727	.443805	.119774
.50	.289467	-.022064	.431865	.106777
.60	.300067	-.012461	.417572	.093720
.70	.312180	-.003843	.400704	.081441
.80	.325658	.002906	.380357	.070584
.90	.340359	.006042	.354674	.061586
1.00	.356168	-.000000	.321105	.054296

POISSON,S RATIO= .300
THETA = 90.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MPT
0	.264010	-.093331	.505831	.000000
.10	.265353	-.090479	.503605	.000000
.20	.269345	-.082327	.497111	.000000
.30	.275876	-.069994	.486853	.000000
.40	.284777	-.055088	.473522	.000000
.50	.295843	-.039430	.457874	.000000
.60	.308853	-.024870	.440697	.000000
.70	.323592	-.013167	.422945	.000000
.80	.339873	-.005755	.406033	.000000
.90	.357562	-.002754	.392096	.000000
1.00	.376575	.000000	.383778	.000000

POISSON,S RATIO= .350
THETA = 0/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MPT
0	.276339	.504484	-.085734	0
.10	.273295	.501324	-.089952	0
.20	.264182	.491791	-.102793	0
.30	.249058	.475724	-.124857	0
.40	.228018	.452802	-.157277	0
.50	.201193	.422426	-.201949	0
.60	.168752	.383369	-.261881	0
.70	.130902	.332872	-.341621	0
.80	.087994	.264544	-.447647	0
.90	.040123	.163937	-.588378	0
1.00	-.011928	-.000000	-.773287	0

POISSON,S RATIO= .350
THETA = 15.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MP	MT	MPT
0	.276339	.464947	-.046197	.147555
.10	.273600	.461232	-.049317	.148717
.20	.265413	.449971	-.058676	.152287
.30	.251870	.430790	-.074243	.158517
.40	.233127	.402988	-.095872	.167838
.50	.209417	.365444	-.123124	.180853
.60	.181061	.316558	-.154974	.198244
.70	.149491	.254483	-.189412	.220468
.80	.112278	.173095	-.223036	.246989
.90	.073161	.089541	-.250863	.274623
1.00	.032029	-.000000	-.256814	.294330

POISSON,S RATIO= .350
THETA = 30.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.276339	.356930	.061820	.255572
.10	.274432	.352411	.060982	.256369
.20	.268761	.338796	.058648	.258656
.30	.259471	.315946	.055371	.262088
.40	.246811	.283770	.052091	.265958
.50	.231133	.242498	.050113	.269019
.60	.212911	.193144	.051025	.269292
.70	.192678	.138171	.056496	.264026
.80	.171108	.082252	.067916	.250141
.90	.148869	.032848	.086131	.225782
1.00	.126585	-.000000	.111116	.193998

POISSON,S RATIO= .350
THETA = 45.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.276339	.209375	.209375	.295109
.10	.275567	.205412	.209982	.294135
.20	.273300	.193756	.211802	.291042
.30	.269677	.175146	.214782	.285350
.40	.264920	.150913	.218708	.276396
.50	.259319	.123051	.223024	.263584
.60	.253206	.094129	.226658	.246769
.70	.246923	.066902	.227868	.226718
.80	.240780	.043367	.224215	.205392
.90	.235032	.023025	.212726	.185516
1.00	.229868	-.000000	.190344	.168467

POISSON,S RATIO= .350
THETA = 60.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.276339	.061821	.356929	.255572
.10	.276700	.060282	.357094	.253114
.20	.277801	.055945	.357419	.245843
.30	.279692	.049570	.357427	.234094
.40	.282445	.042180	.356440	.218505
.50	.286138	.034727	.353763	.200030
.60	.290845	.027706	.348961	.179850
.70	.296627	.020886	.342209	.159104
.80	.303536	.013485	.334637	.138555
.90	.311628	.005343	.328533	.118491
1.00	.320969	-.000000	.327111	.099519

POISSON,S RATIO= .350
THETA = 75.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.276339	-.046197	.464947	.147555
.10	.277527	-.044800	.463616	.145463
.20	.281072	-.040727	.459624	.139448
.30	.286912	-.034341	.452991	.130234
.40	.294947	-.026276	.443789	.118837
.50	.305050	-.017400	.432130	.106352
.60	.317074	-.008687	.418065	.093770
.70	.330865	-.001041	.401324	.081892
.80	.346269	.004673	.380944	.071345
.90	.363140	.006778	.354918	.062578
1.00	.381355	-.000000	.320219	.055543

POISSON,S RATIO= .350
THETA = 90.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.276339	-.085734	.504484	.000000
.10	.277830	-.083022	.502362	.000000
.20	.282265	-.075273	.496158	.000000
.30	.289532	-.063566	.486322	.000000
.40	.299459	-.049451	.473465	.000000
.50	.311838	-.034689	.458260	.000000
.60	.326440	-.021969	.441408	.000000
.70	.343045	-.010297	.423794	.000000
.80	.361462	-.003767	.406790	.000000
.90	.381549	-.001632	.392611	.000000
1.00	.403225	.000000	.384331	.000000

POISSON,S RATIO= .400
THETA = 0/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MPT
0	.291188	.503269	-.078269	0
.10	.288007	.500058	-.082649	0
.20	.278483	.490373	-.095979	0
.30	.262667	.474050	-.118876	0
.40	.240643	.450769	-.152503	0
.50	.212330	.419936	-.198815	0
.60	.178473	.380345	-.260935	0
.70	.138649	.329322	-.343657	0
.80	.093274	.260734	-.453994	0
.90	.042648	.160799	-.601530	0
1.00	-.012716	-.000000	-.798154	0

POISSON,S RATIO= .400
THETA = 15.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MPT
0	.291188	.454314	-.039314	.145385
.10	.288333	.460574	-.042609	.146536
.20	.279797	.449243	-.052501	.150075
.30	.265668	.429958	-.068980	.156255
.40	.246098	.402047	-.091937	.165521
.50	.221311	.364431	-.120992	.178502
.60	.191619	.315578	-.155199	.195951
.70	.157442	.253713	-.192619	.218487
.80	.119342	.177737	-.229835	.245931
.90	.078046	.089671	-.261550	.275858
1.00	.034444	-.000000	-.280647	.300743

POISSON,S RATIO= .400
THETA = 30.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MPT
0	.291188	.357885	.067115	.251814
.10	.289221	.353388	.066086	.252632
.20	.283371	.339844	.063180	.254993
.30	.273784	.317123	.058948	.258579
.40	.260712	.285145	.054335	.262738
.50	.244513	.244142	.050685	.266308
.60	.225658	.195090	.049673	.267423
.70	.204725	.140355	.053147	.263443
.80	.182377	.084415	.062857	.251315
.90	.159322	.034417	.080113	.228899
1.00	.136226	-.000000	.105517	.198224

POISSON,S RATIO= .400
THETA = 45.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MPT
0	.291188	.212500	.212500	.290769
.10	.290433	.203568	.212952	.289883
.20	.288217	.196997	.214317	.287060
.30	.284684	.178500	.216587	.281839
.40	.280063	.154362	.219616	.273578
.50	.274654	.126499	.222956	.261686
.60	.268802	.097388	.225652	.245980
.70	.262853	.069686	.226111	.227130
.80	.257166	.045342	.221904	.206950
.90	.251979	.023945	.209900	.188076
1.00	.247504	-.000000	.186471	.172137

POISSON,S RATIO= .400
THETA = 60.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MPT
0	.291188	.067116	.357894	.251814
.10	.291642	.065555	.357990	.249485
.20	.293023	.061139	.358152	.242591
.30	.295384	.054599	.357928	.231440
.40	.298801	.046321	.356687	.216619
.50	.303356	.039025	.353769	.199012
.60	.309131	.031393	.348748	.179717
.70	.316191	.023841	.341769	.159801
.80	.324596	.015620	.333919	.139975
.90	.334406	.006571	.327577	.120469
1.00	.345697	-.000000	.326218	.101687

POISSON,S RATIO= .400
THETA = 75.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.291188	-.039313	.464313	.145385
.10	.292525	-.038017	.463032	.143389
.20	.296517	-.034239	.459184	.137648
.30	.303100	-.028333	.452768	.128842
.40	.312174	-.020909	.443821	.117928
.50	.323608	-.012805	.432411	.105940
.60	.337253	-.004966	.418537	.093819
.70	.352950	.001726	.401883	.082330
.80	.370541	.006421	.381430	.072084
.90	.389873	.007509	.355025	.063541
1.00	.410811	-.000000	.319176	.056753

POISSON,S RATIO= .400
THETA = 90.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MPT
0	.291188	-.078269	.503269	.000000
.10	.292849	-.075693	.501244	.000000
.20	.297791	-.068338	.495315	.000000
.30	.305900	-.057244	.485873	.000000
.40	.317002	-.043902	.473458	.000000
.50	.330879	-.030018	.458657	.000000
.60	.347297	-.017321	.442092	.000000
.70	.366030	-.007464	.424575	.000000
.80	.386878	-.001904	.407441	.000000
.90	.409693	-.000525	.392980	.000000
1.00	.434389	.000000	.384684	.000000

POISSON,S RATIO= .450
THETA = 0/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.309194	.502180	-.070930	0
.10	.305847	.498919	-.075469	0
.20	.295820	.489083	-.089283	0
.30	.279160	.472507	-.113001	0
.40	.255939	.448872	-.147820	0
.50	.226257	.417585	-.195750	0
.60	.190236	.377465	-.260024	0
.70	.148016	.325921	-.345676	0
.80	.099763	.257968	-.460241	0
.90	.045703	.157769	-.614429	0
1.00	-.013769	-.000000	-.822492	0

POISSON,S RATIO= .450
THETA = 15.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.309194	.463789	-.032539	.143278
.10	.306197	.460025	-.036008	.144419
.20	.297233	.448622	-.046426	.147926
.30	.282387	.429233	-.063803	.154059
.40	.261804	.401210	-.088068	.163271
.50	.235700	.363516	-.118899	.176220
.60	.204376	.314684	-.155424	.193725
.70	.168241	.253012	-.195777	.216563
.80	.127843	.177432	-.236520	.244904
.90	.083901	.083814	-.272056	.277057
1.00	.037307	-.000000	-.294261	.306969

POISSON,S RATIO= .450
THETA = 30.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.309194	.358903	.072347	.248164
.10	.307152	.354427	.071131	.249003
.20	.301076	.340948	.067650	.251435
.30	.291115	.318348	.062486	.255171
.40	.277324	.286558	.056562	.259611
.50	.260668	.245805	.051263	.263675
.60	.241029	.197037	.048353	.265607
.70	.219201	.142522	.049863	.262878
.80	.195871	.086549	.057862	.252454
.90	.171781	.035958	.074139	.231926
1.00	.147648	-.000000	.099899	.202328

POISSON,S RATIO= .450
THETA = 45.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.309194	.215625	.215625	.286555
.10	.308455	.211722	.215923	.285754
.20	.306288	.200231	.216840	.283192
.30	.302840	.181840	.218405	.278429
.40	.298352	.157786	.220544	.270842
.50	.293133	.129915	.222912	.259843
.60	.287545	.100611	.224687	.245214
.70	.281960	.072435	.224362	.227530
.80	.276728	.047291	.219576	.209464
.90	.272135	.024855	.207026	.190563
1.00	.268395	-.000000	.182529	.175701

POISSON,S RATIO= .450
THETA = 60.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.309194	.072348	.358902	.248164
.10	.309755	.070764	.358948	.245960
.20	.311457	.066269	.358943	.239433
.30	.314355	.059565	.358491	.228864
.40	.318530	.051601	.356973	.214787
.50	.324072	.043266	.353798	.198022
.60	.331067	.035043	.348534	.179588
.70	.339588	.026758	.341298	.160479
.80	.349699	.017726	.333139	.141356
.90	.351470	.007790	.326440	.122389
1.00	.374986	-.000000	.325169	.103793

POISSON,S RATIO= .450
THETA = 75.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.309194	-.032539	.463789	.143278
.10	.310705	-.031340	.462554	.141376
.20	.315215	-.027853	.458836	.135900
.30	.322660	-.022416	.452617	.127490
.40	.332938	-.015620	.443899	.117045
.50	.345914	-.008275	.432705	.105539
.60	.361435	-.001294	.418987	.093866
.70	.379337	.004459	.402383	.082756
.80	.399453	.008152	.381818	.072801
.90	.421622	.008237	.354998	.064477
1.00	.445695	-.000000	.317981	.057928

POISSON,S RATIO= .450
THETA = 90.00/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.309194	-.070330	.502180	.000000
.10	.311052	-.068487	.500247	.000000
.20	.316585	-.061518	.494575	.000000
.30	.325675	-.051022	.485503	.000000
.40	.338141	-.038439	.473498	.000000
.50	.353758	-.025414	.459065	.000000
.60	.372285	-.013624	.442749	.000000
.70	.393484	-.004668	.425290	.000000
.80	.417145	.000134	.407990	.000000
.90	.443113	.000569	.393208	.000000
1.00	.471297	.000000	.384846	.000000

POISSON,S RATIO= .500
THETA = 0/DEGREES
ALFA = 7.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.331258	.501212	-.063712	0
.10	.327706	.497901	-.068408	0
.20	.317060	.487915	-.082699	0
.30	.299360	.471090	-.107230	0
.40	.274665	.447104	-.143225	0
.50	.243055	.415369	-.192749	0
.60	.204623	.374725	-.259146	0
.70	.159466	.322663	-.347680	0
.80	.107692	.253539	-.466391	0
.90	.049437	.154843	-.627098	0
1.00	-.015935	-.000000	-.846293	0

POISSON,S RATIO= .250
THETA = 15.00/DEGREES
ALFA = 9.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.246052	.464737	-.058487	.151042
.10	.243498	.461062	-.061216	.152182
.20	.235868	.449914	-.069388	.155673
.30	.223260	.430908	-.082935	.161725
.40	.205841	.403326	-.101656	.170632
.50	.183856	.366027	-.125058	.183015
.60	.157643	.317408	-.152119	.199093
.70	.127649	.255612	-.180997	.218886
.80	.094460	.179450	-.208852	.241028
.90	.058812	.090827	-.232052	.261008
1.00	.021581	-.000000	-.247331	.267752

POISSON,S RATIO= .250
THETA = 30.00/DEGREES
ALFA = 9.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.246052	.354167	.052083	.261612
.10	.244236	.349638	.051608	.262322
.20	.238837	.335994	.050354	.264335
.30	.229996	.313099	.048838	.267261
.40	.217954	.280870	.047320	.270328
.50	.203052	.239558	.048749	.272217
.60	.185733	.190207	.052536	.270909
.70	.166532	.135332	.060800	.263711
.80	.146050	.079716	.074012	.247803
.90	.124911	.031062	.092214	.221945
1.00	.103672	-.000000	.114345	.190339

POISSON,S RATIO= .250
THETA = 45.00/DEGREES
ALFA = 9.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.246052	.203125	.203125	.302084
.10	.245243	.199150	.203993	.300936
.20	.242861	.187467	.206568	.297317
.30	.239037	.168841	.210725	.290740
.40	.233979	.144645	.216129	.280537
.50	.227958	.116926	.222081	.266120
.60	.221283	.088327	.227377	.247364
.70	.214269	.061696	.230236	.225060
.80	.207207	.039166	.228403	.201170
.90	.200331	.020446	.219525	.178362
1.00	.193821	-.000000	.201930	.157871

POISSON,S RATIO= .250
THETA = 60.00/DEGREES
ALFA = 9.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.246052	.052083	.354167	.261613
.10	.246248	.050605	.354416	.258940
.20	.246851	.046458	.354980	.251041
.30	.247908	.040427	.355336	.238295
.40	.249481	.033577	.354749	.221411
.50	.251641	.026910	.352466	.201447
.60	.254453	.020980	.347994	.179717
.70	.257969	.015609	.341425	.157534
.80	.262228	.010007	.333686	.135880
.90	.267271	.003831	.326544	.115301
1.00	.273143	.000000	.322007	.096652

POISSON,S RATIO= .250
THETA = 75.00/DEGREES
ALFA = 9.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.246052	-.058487	.464737	.151042
.10	.246982	-.056930	.463317	.148800
.20	.248751	-.052381	.459069	.142352
.30	.254300	-.045217	.452052	.132477
.40	.260535	-.036095	.442401	.120270
.50	.268333	-.025922	.431320	.106916
.60	.277555	-.015735	.415991	.093487
.70	.288054	-.006526	.399354	.080833
.80	.299638	.000759	.379823	.069576
.90	.312330	.004508	.356120	.060098
1.00	.325879	-.000000	.326550	.052239

POISSON,S RATIO= .250
THETA = 90.00/DEGREES
ALFA = 9.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.246052	-.098959	.505209	.000000
.10	.247250	-.096040	.502923	.000000
.20	.251808	-.087685	.496261	.000000
.30	.256620	-.075012	.485750	.000000
.40	.264524	-.059627	.472125	.000000
.50	.274322	-.043363	.456191	.000000
.60	.285800	-.028064	.438757	.000000
.70	.298749	-.015468	.420724	.000000
.80	.312999	-.006992	.403284	.000000
.90	.323384	-.002930	.389066	.000000
1.00	.344847	.000000	.376799	.000000

POISSON,S RATIO= .300
THETA = 0/DEGREES
ALFA = 9.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.255837	.503757	-.091257	0
.10	.252920	.500614	-.095250	0
.20	.244192	.491127	-.107398	0
.30	.229716	.475117	-.128242	0
.40	.209609	.452227	-.158795	0
.50	.183995	.421792	-.200717	0
.60	.153097	.382491	-.256581	0
.70	.117156	.331478	-.330155	0
.80	.076494	.262445	-.426553	0
.90	.031562	.161600	-.551936	0
1.00	-.016920	-.000000	-.712182	0

POISSON,S RATIO= .300
THETA = 15.00/DEGREES
ALFA = 9.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.255837	.463899	-.051399	.148753
.10	.253206	.460199	-.054308	.149883
.20	.246344	.448985	-.063028	.153344
.30	.232346	.429881	-.077514	.159354
.40	.214374	.402197	-.097601	.168274
.50	.191669	.364837	-.122956	.180583
.60	.164559	.316259	-.152334	.196750
.70	.133482	.254680	-.184268	.216903
.80	.099012	.178946	-.215811	.240032
.90	.061877	.090826	-.243062	.262358
1.00	.022949	-.000000	-.261849	.274243

POISSON,S RATIO= .300
THETA = 30.00/DEGREES
ALFA = 9.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.255837	.355004	.057496	.257648
.10	.253985	.350499	.056826	.258382
.20	.248476	.336932	.054985	.260476
.30	.239455	.314176	.052494	.263569
.40	.227163	.282159	.050215	.266945
.50	.211945	.241128	.049338	.269368
.60	.194247	.192095	.051263	.268933
.70	.174614	.137477	.057399	.263059
.80	.153662	.081860	.068844	.248961
.90	.132033	.032632	.086046	.225158
1.00	.110316	-.000000	.109594	.194953

POISSON,S RATIO= .300
THETA = 45.00/DEGREES
ALFA = 9.50/DEGREES

ROH	DEFLECTION	MR	MT	MRT
0	.255837	.206250	.206250	.297507
.10	.255047	.202308	.205960	.296451
.20	.252724	.190716	.209076	.293116
.30	.249001	.172212	.212514	.287033
.40	.244092	.148122	.217112	.277557
.50	.238274	.120417	.221979	.264104
.60	.231866	.091645	.226342	.246514
.70	.225194	.064554	.229444	.225484
.80	.218562	.041224	.226083	.202829
.90	.212216	.021437	.216756	.181103
1.00	.206348	-.000000	.198302	.161698

APPENDIX 4

Use of the coefficients for bending moments from finite element solution. The finite element solution using a quadratic plate element as shown in Fig. (2.7) produces the bending moments at the centroid of the elements and related to its own local axis.

The local axes of the elements are defined from the co-ordinates of the nodes of the element and they are dependent on the numbering directions. (The connection matrix as follows):

The x axis passes through the centroid of the element and the middle point of the fourth side of the element as it is numbered in the connection matrix of the elements, i.e. the side connecting node No. 4 with Node No. 1 as in Fig. (2.6).

The y axis is perpendicular to the x axis and passes through the first side of the element, i.e. the side connecting Node No. 1 with Node No. 2 as in Fig. (2.6),

as all elements must be as near as possible to a rectangular shape.

M_x, and M_y are positive when causing tension stresses on the top surface of the element as in Fig. (2.1). Thus the coefficients obtained from a finite element solution would produce M_x, M_y and M_{xy} when multiplied by qa^2. To transform these moments into global polar coordinates the following equations canbe used.

$$M_r = M_x \cos^2 \Psi + M_y \sin^2 \Psi + M_{xy} \sin 2\Psi$$

$$M_\theta = M_x \sin^2 \Psi + M_y \cos^2 \Psi - M_{xy} \sin 2\Psi$$

$$M_{r\theta} = -\frac{1}{2} (M_x - M_y) \sin 2\Psi + M_{xy} \cos 2\Psi$$

Where Ψ is the angle between the polar direction and the element local axes x-y. as shown in Fig. (A4.1).

For a given principal moment M_1 & M_2 the following equations can be used:-

$$M_r = M_x \cos^2\beta + M_y \sin^2\beta$$

$$M_\theta = M_x \sin^2\beta + M_y \cos^2\beta$$

$$M_{r\theta} = \frac{1}{2} (M_x - M_y) \sin^2\beta$$

Where β is the angle between the polar direction and the principal directions 1-2 as shown in Fig. (A4.1).

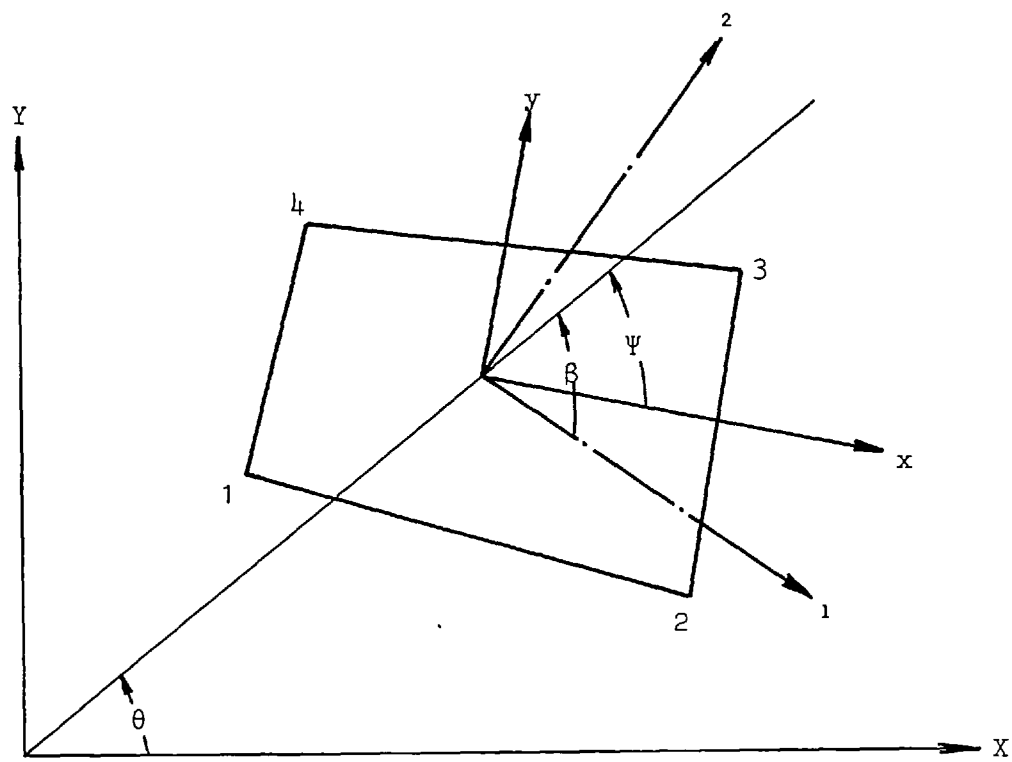


Fig. A4.1. Global, local and principal directions for a thin shell element.

APPENDIX 5

According to the following linear interpolation function

$$f_1 = \alpha_1 + \alpha_2\,\xi + \alpha_3\,\eta + \alpha_4\,\zeta + \alpha_5\,\xi\eta + \alpha_6\,\eta\zeta + \alpha_7\,\xi\zeta + \alpha_8\,\xi\eta\zeta \qquad \text{(A5.1)}$$

we can represent the shape of any three dimensional parallelopiped with lengths 2a, 2b and 2c. Fig. (A5.1).

Where the local dimensionless coordinates ξ η ζ referred to the centroid $(x_c,\ y_c,\ 3_c)$

$$= (x - x_c)/a$$

$$= (y - y_c)/b$$

$$= (3 - 3_c)/c$$

Solving for α's in the following way:-

$$x_1 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8$$

$$y_1 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8$$

$$\vdots$$

$$\therefore \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \\ \alpha_8 \end{bmatrix} \qquad (A\ 5.2)$$

Then replacing ι's by nodal coordinates using the substitutions $x = x_1$ at $\xi = \eta = 1$ etc. Equation (A5.2) is produced.

$$x = \sum_{i=1}^{\infty} N_i \, x_i$$

$$y = \sum_{i=1}^{\infty} N_i \, y_i$$

$$z = \sum_{i=1}^{\infty} N_i \, z_i$$

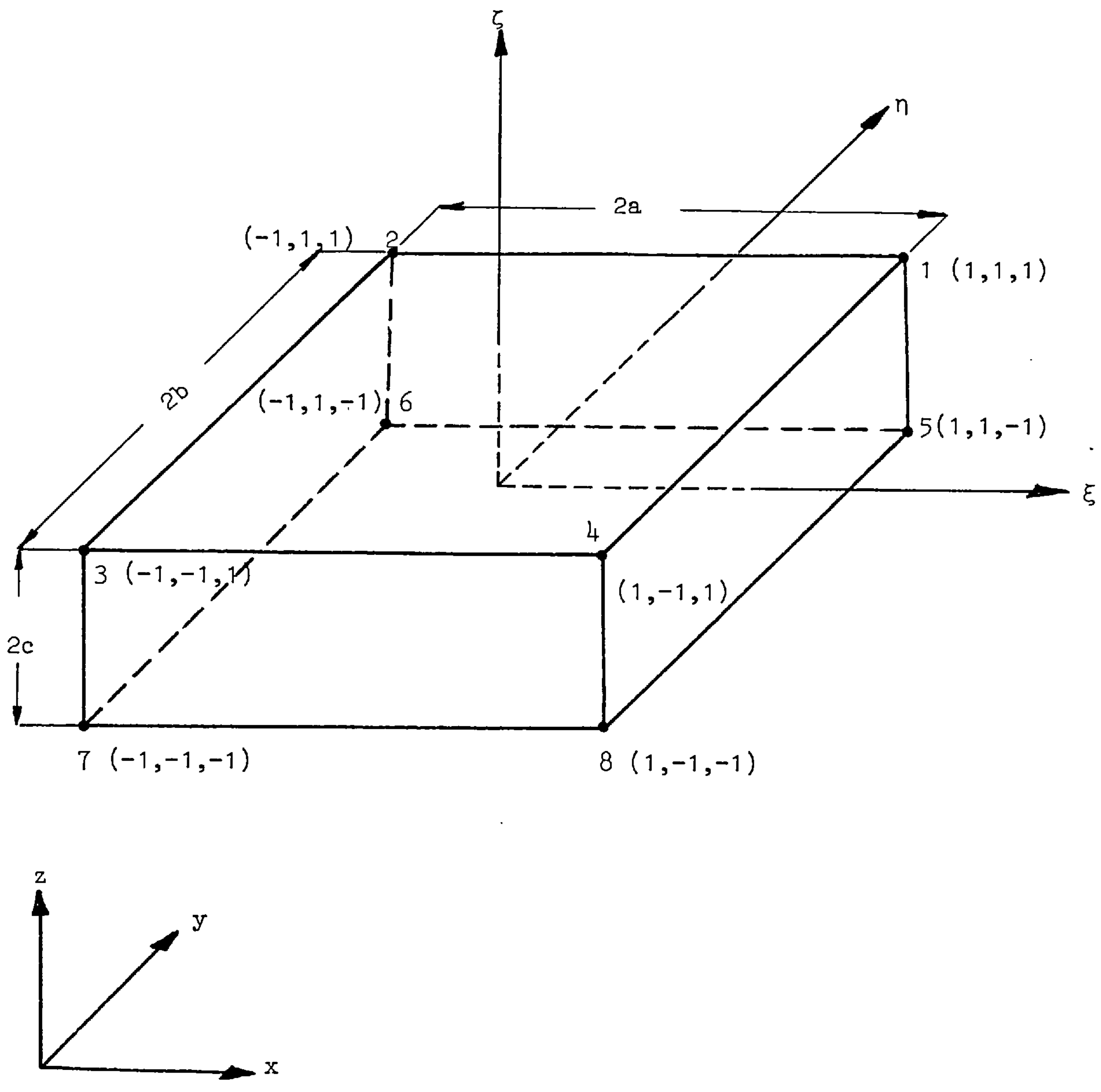


Fig. A5.1.