SHORT COMMUNICATION

Seismic hazard disaggregation in Performance-Based Earthquake Engineering: occurrence or exceedance?

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SUMMARY

Seismic hazard disaggregation is commonly used as an aid in ground-motion selection for the seismic response analysis of structures. This short communication investigates two different approaches to disaggregation related to the exceedance and occurrence of a particular intensity. The impact the different approaches might have on a subsequent structural analysis at a given intensity is explored through the calculation of conditional spectra. It is found that the exceedance approach results in conditional spectra that will be conservative when used as targets for ground-motion selection. It is however argued that the use of the occurrence disaggregation is more consistent with the objectives of seismic response analyses in the context of Performance-Based Earthquake Engineering.

KEY WORDS: seismic hazard disaggregation, seismic hazard deaggregation, ground-motion selection, Performance-Based Earthquake Engineering, conditional spectrum

1. INTRODUCTION

With the ongoing development of Performance-Based Earthquake Engineering (PBEE) there is an increasing need to use carefully selected ground motions for response history analysis in the seismic performance assessment of structures. Often seismic hazard disaggregation (or deaggregation) is used as an aid in the ground-motion selection process. In the simplest case, disaggregation may be used to identify which earthquake scenarios (e.g. magnitude and distance combinations) have contributed most significantly to the design ground motion level so that these scenarios can inform record selection. In a more sophisticated and PBEE focused case, disaggregation is used as tool required for the calculation of conditional spectra [1,2,3], which in turn are used as a target for ground-motion selection. Due to an increasing interest in conditional spectra they are used in this work as a proxy for evaluating the impact that different approaches to disaggregation might have on ground-motion selection and seismic performance assessment.

Two different broad approaches are commonly used in seismic hazard disaggregation: a more common 'exceedance' approach [4] and a less common 'occurrence' approach [5]. The difference between these two approaches (and the concept of seismic hazard disaggregation in general) is discussed in the following section, and the impact that using these different approaches has on seismic response analysis is the main focus of this short communication. This is investigated through a case study in section 3, which examines a hypothetical, but realistic, probabilistic seismic hazard analysis (PSHA) followed by disaggregation and calculation of conditional spectra. Discussion of the results and then conclusions are provided in sections 5 and 6 respectively.

2. BACKGROUND AND OPTIONS FOR SEISMIC HAZARD DISAGGREGATION

Within PSHA, exceedance contributions from a large number of causal earthquake scenarios (i.e. events with a given magnitude, distance and other parameters) are aggregated to form a hazard curve, which provides the total annual rate of exceeding at a particular intensity of ground shaking at the site of interest. Disaggregation is conceptually then the process of 'unravelling' which of the aggregated scenarios contribute (and in what proportions) to the hazard level of interest. This disaggregation usually breaks the total hazard level down into contributions in terms of magnitude, distance and epsilon, with epsilon being the number of standard deviations between a given spectral acceleration (or other intensity measure) and the mean value predicted by a ground-motion prediction equation (GMPE). However, disaggregation can also be carried out for other parameters, such as faulting style or even in terms of GMPEs in the case that more than one is used in the PSHA [3]. The annual rate of occurrence of ground motions in the range $x^L<Sa<x^U$, given that these motions arise from events in a particular magnitude, distance and epsilon bin can be determined from Equation (1):

$$\lambda\left(x^L < Sa < x^U \mid m, r, \varepsilon\right) = \sum_{s=1}^{n_s} v_s \left\{ \int_{\varepsilon^L}^{\varepsilon^U} \int_{r^L}^{r^U} \int_{m^L}^{m^U} I\left[x^L < Sa < x^U \mid m, r, \varepsilon\right] f_{M,R,E}(m, r, \varepsilon)\, dm\, dr\, d\varepsilon \right\}_s \qquad (1)$$

where v_s is the total number of events per year in source s (out of n_s sources), and subscripts U and L represent the 'upper' and 'lower' limits defining the variable bins, or the ground-motion range, respectively. The other terms are standard and mirror the terminology used by Bazzurro and Cornell [4].

From the conditional annual rates defined in Equation (1), the conditional probability that a contribution to the hazard arises from a given earthquake scenario is defined by normalising the conditional rate by the marginal rate, $\lambda(x^L<Sa<x^U)$, obtained for all magnitude, distance and epsilon scenarios as in Equation (2):

$$P\left(m, r, \varepsilon \mid x^L < Sa < x^U\right) = \lambda\left(x^L < Sa < x^U \mid m, r, \varepsilon\right) / \lambda\left(x^L < Sa < x^U\right) \qquad (2)$$

The expression in Equation (2) can be taken as a formal definition of disaggregation. As mentioned in the introduction, disaggregation is normally carried out in terms of 'exceedance' or 'occurrence'. The former considers all scenarios causing the exceedance of a particular value of spectral acceleration, which can be expressed as $Sa>x_i$, and uses $x^L=x$ and $x^U=\infty$ in Equation (2). This form of disaggregation is commonly available from organisations that provide seismic hazard information, such as the United States Geological Survey (USGS) [6] or the Italian National Institute of Geophysics and Volcanology (INGV) [7]. Perhaps due to its ready availability this form of disaggregation is often used directly as an aid in ground-motion selection in PBEE [8,9,10]. However, this is not consistent with the typical seismic analysis that follows, which is used to determine the response of a structure at a given intensity (i.e. for $Sa=x_i$ and not $Sa>x_i$, as required in the PEER PBEE framework [11]). It should be noted that the exceedance approach is consistent with conventional response history analyses, which are used to determine the expected response for ground motions defined by a certain return period for exceedance. Another example of where the exceedance approach might be preferable is in the case of a risk-based assessment carried out at a limited number of discrete intensities, each covering a wide intensity range Δx (e.g. a loss assessment in accordance with FEMA P-58 [9]). In this case it could be argued that the highest intensity should capture the likely response over a range from x-$\Delta x/2$ to infinity (but at all other intensities the occurrence approach would still be more appropriate).

In determining the seismic response of a structure at a given intensity level, a more consistent approach is to use seismic hazard disaggregation in terms of occurrence. For the occurrence case one must define values of x^L and x^U in Equation (2) that are sufficiently close to x such that a good approximation to the desired $P(m,k,\varepsilon|Sa=x)$ is obtained. However, this form of disaggregation is not commonly available. Note that as Sa is a continuous random variable the probability of $Sa=x_i$ is rigorously zero. Therefore, it is not possible to disaggregate hazard for the occurrence of $Sa=x_i$ exactly, but instead one must consider a range or ‘band’ of intensities about the intensity level of interest (i.e. $\Delta x=x^U-x^L$). This results in an ambiguous definition of ‘occurrence’ as the width and location of the band are at the analyst’s discretion. The width of the bands, Δx, should be carefully chosen to ensure that they are representative of the set of intensity levels for which structural analyses will be conducted.

The issue of exceedance or occurrence disaggregation has been briefly discussed by a number of researchers [3,12,13,14]; however, there does not appear to have been any detailed examination of the effects of disaggregation choices on spectral demands. Furthermore, it is noted that a large number of articles do not provide details of the disaggregation nor do they explicitly state whether exceedance or occurrence disaggregation is used.

3. CASE STUDY EXAMPLE

A case study is carried out to demonstrate the impact that different disaggregation options can have on the calculation of conditional spectra. PSHA is carried out for a hypothetical, but realistic (being very loosely based upon a location in central Italy), site, followed by disaggregation (using a number of different approaches) and calculation of conditional spectra.

3.1 Probabilistic seismic hazard analysis

The probabilistic seismic hazard analysis is performed for a fictitious site with an average shear-wave velocity over the upper 30m of 300m/s. The hazard at the site is assumed to be influenced by a single area source that contains the site and that extends beyond the distance bounds considered for the hazard calculations. A uniform depth distribution of shallow crustal seismicity is assumed and earthquake ruptures are generated according to [15]. The ground motions for the site are computed using the model of Campbell and Bozorgnia [16], and the default sediment depths are assumed to be appropriate in this model.

The ruptures are generated assuming a vertical dip and by considering the full range of possible strike angles at each considered epicentral location. The depth of the seismogenic layer is 15km and the seismicity follows a doubly-bounded exponential distribution with a *b*-value of 1.0 and a maximum magnitude of 7.5. The minimum magnitude considered for the integration is 5.0, while distances out to 200km are allowed to contribute.

The hazard curve is constructed from evaluations made for 18 levels of $Sa(1.0\text{s})$ that are logarithmically-spaced between 0.0894g and 0.876g. For each of these levels, disaggregation was also performed using all methods outlined in the following section.

3.2 Seismic hazard disaggregation

The results of the disaggregation carried out for $Sa(1.0)=0.586\text{g}$, which approximately corresponds to a 10% probability of exceedance in 50 years is now presented. Four different approaches to disaggregation are considered. The first disaggregation is in terms of exceedance ($Sa>x_i$) while the others are in terms of occurrence ($Sa\approx x_i$). For the occurrence cases, three different width bands are considered: 'wide' ($x_i\leq Sa<1.1x_i$), 'narrow' ($x_i\leq Sa<1.01x_i$), and 'very narrow' ($x_i\leq Sa<1.0001x_i$). For all cases the bin spacing used for disaggregation is linear, with bin widths of 0.1 and 5km for magnitude and distance respectively. Plots of the four different cases of disaggregation are shown in Figure 1. It can be observed that the exceedance disaggregation has a greater contribution from events with larger magnitudes and shorter distances when compared to the occurrence cases. This is to be expected given that the exceedance case includes events that cause values of $Sa(1.0)$ that are much larger than 0.586g. There is almost no difference between the occurrence disaggregation plots with different bandwidths. In fact, in Figure 1 the differences are visually indiscernible and can only be identified through review of the numerical data.

While the differences among the disaggregation distributions for different occurrence bandwidths are very small, the difference between these occurrence distributions and the exceedance distribution is also not particularly strong in this case. However, the difference does still impact upon the conditional spectra in an important way, as will be shown in the following section. The differences between exceedance and occurrence distributions will vary with return period. For relatively long return periods the vast majority of scenarios require positive epsilon values in order to contribute to the hazard in both the exceedance and occurrence cases and this effectively means that the hazard is controlled by similar regions of the exponential tail of the ground motion distribution. However, when shorter return periods are considered, lower epsilon value scenarios contribute more significantly and greater differences appear between the occurrence and exceedance distributions. An example of this effect is shown in Figure 2 in which the mean magnitude, distance and epsilon triplet for both the exceedance and occurrence cases are shown for a large range of intensity measure levels. The annotation of each point by the relevant intensity measure level allows one to appreciate that these mean triplets tend to lie along the same 'path' for most positive epsilon values, but that the particular scenarios for magnitude and distance tend to lower magnitudes and greater distances for the occurrence case. For small levels of the intensity measure the mean epsilon values continue to push into the negative range for the occurrence scenarios, but saturate at epsilon of zero for the exceedance case. The implication of these differences in epsilon value between exceedance and occurrence is discussed in the context of conditional spectra in the following section. It should be noted that the differences in the exceedance and occurrence paths is not restricted to any particular return period range and will be a function of local activity rates. Based on initial investigations it does appear though that the most significant differences will relate to structural performance associated with serviceability, and minor to moderate damage, which can contribute significantly to economic losses.

To demonstrate further the differences that can arise between exceedance disaggregations and occurrence disaggregations, Figure 3 shows disaggregation paths in the same vein as those in Figure 2. However, in this case the PSHA has been adjusted to include a fault source that can generate events with magnitudes in the range 6-7.5 and that is located approximately 50Km away from the site at its closest point. By increasing the complexity of the source model it can be appreciated that greater differences in the disaggregation paths arise between the exceedance and occurrence cases. This adjusted hazard example also reinforces the idea that greater differences in these disaggregation paths occur for shorter rather than longer return periods. That said, it is still important to note that there are important differences in the mean magnitude and epsilon values in these disaggregation triplets between the exceedance and occurrence cases. This is particularly important to note given that it is these two parameters that exert the greatest influence upon the shape of conditional mean spectra.

3.3 Conditional spectra

Conditional spectra are now calculated using the disaggregation data obtained previously. At periods other than the conditioning period of T*=1.0s, spectral displacements are determined from conditioning on $Sd(1.0)=0.146$m (or equivalently $Sa(1.0)=0.586$g) and the magnitude distance pairs found from disaggregation, whilst taking account of the degree of correlation between spectral ordinates at different periods. Calculations are carried out using the MATLAB [17] code provided by Jayaram *et al.* [2]; however, this has been modified to account for all causal earthquakes (magnitude and distance pairs) rather than just a single mean or modal event. The consideration of all causal earthquakes is carried out as per Lin *et al*. [3]. All parameters are set to be consistent with the PSHA, including the use of the Campbell and Bozorgnia-2008 GMPE [16].

The resulting conditional displacement spectra are shown in Figure 4 as conditional mean displacement spectra and the corresponding standard deviation. The conditional spectrum corresponding to the exceedance disaggregation has larger mean spectral displacements across all periods (except $T=T^*=1.0$s) when compared to the occurrence cases. This is a result of the exceedance disaggregation having a larger contribution from large and close earthquakes, which then leads to small values of ε, or in other words less ‘peakedness’ around T^*. The conditional mean spectra from the different occurrence cases are all similar, as would be expected following on from the similarities observed in disaggregation. Interestingly, between all cases there is only very minimal difference in the calculated standard deviations. This is to be expected however given that the conditional standard deviations are independent of epsilon and also have a relatively mild dependence upon magnitude and distance. To demonstrate further the difference between using exceedance or occurrence disaggregation, the ratios of spectral accelerations from the conditional mean spectra for exceedance and for occurrence ($1.01x_i>Sa>x_i$) are shown in Figure 5. Also included are the same ratios for a lower hazard level corresponding to $Sa=0.153$g. It can be seen that at the lower intensity the difference between exceedance and occurrence is even more severe, with ratios as high as 1.3. This result can be anticipated through consideration of the disaggregation paths shown in Figures 2 and 3.

4. DISCUSSION

From the case study investigation, it is clear that the use of exceedance or occurrence disaggregation can have a fairly significant effect on the calculation of conditional spectra. Using the conditional spectra as targets for ground-motion selection may then impact the results of seismic response analysis; however, how significant any differences may be for risk assessment is difficult to estimate as it will vary depending on how inelastic the response is and whether higher modes have a significant effect, amongst other factors. For discussion on the impact that conditional spectra might have on structural analysis results the reader is referred to Baker and Cornell [18] and Baker [1].It is difficult to generalise the results as different sites will be influenced by different seismic sources with different characteristics.

However, based on experience and initial parametric studies it is reasonable to expect that differences at least as large as those shown in Figure 4 could exist for more general hazard analyses. The reason for this is that the use of a single uniform seismicity area source essentially allows the joint distribution of magnitude, distance and epsilon to be smoothly varying over the full integration range of the PSHA. When more general source models are used there may be discontinuities in this joint distribution and this dictates that the disaggregation paths for exceedance and occurrence (like those contrasted between Figures 2 and 3) will be less similar. Differences in these paths, as well as differences in position along these paths (when the paths effectively overlap) control the shape of the conditional spectra – particularly for the magnitude-epsilon paths shown in the left panels of Figures 2 and 3. These aspects should be investigated in more detail in future research.

It has been shown that the width of the ‘band’ used for occurrence disaggregation has minimal effect on the disaggregation and subsequent conditional spectra. This is encouraging as it indicates that reasonable approximations of occurrence disaggregation may be obtained from relatively widely spaced exceedance disaggregations by subtracting the rates at the higher intensity from the rates at the lower intensity. This is demonstrated in Equation (3), which calculates the conditional probability of ‘occurrence’ of a scenario from the conditional and marginal exceedance rates:

$$P(m,r,\varepsilon \mid x_i < Sa < x_{i+1}) = \frac{\lambda(Sa > x_i \mid m,r,\varepsilon) - \lambda(Sa > x_{i+1} \mid m,r,\varepsilon)}{\lambda(Sa > x_i) - \lambda(Sa > x_{i+1})} \tag{3}$$

where $Sa=x_{i+1}$ corresponds to a higher intensity than $Sa=x_i$. This makes the exceedance disaggregations provided by seismological institutes much more useful to the analyst. In this work the band of hazard is always located immediately above the hazard level of interest. It is not expected that changing the location of this band to be below (e.g $0.9x_i \leq Sa < x_i$) or around (e.g $0.95x_i \leq Sa < 1.05x_i$) the intensity level of interest would have a significant effect.

5. CONCLUSIONS

The impact of using two different approaches to seismic hazard disaggregation, exceedance or occurrence, has been investigated by observing the effect that the different approaches have on the calculation of conditional spectra. In the context of PBEE the use of exceedance disaggregation as an aid in selecting ground motions, which appears to be relatively common, is not consistent with the objective of the subsequent structural analysis, which is to determine the seismic response of a structure at a specific intensity ($Sa=x_i$). For this form of assessment one should therefore use the occurrence disaggregation instead. This presents some difficulty as the width of the band used in the occurrence disaggregation can be chosen at the analyst’s discretion. It has been shown in this work that the width of the band was relatively unimportant; however, as this may not be the general case, it would be prudent for engineers and/or seismologists to clearly state how they disaggregate hazard for occurrence.

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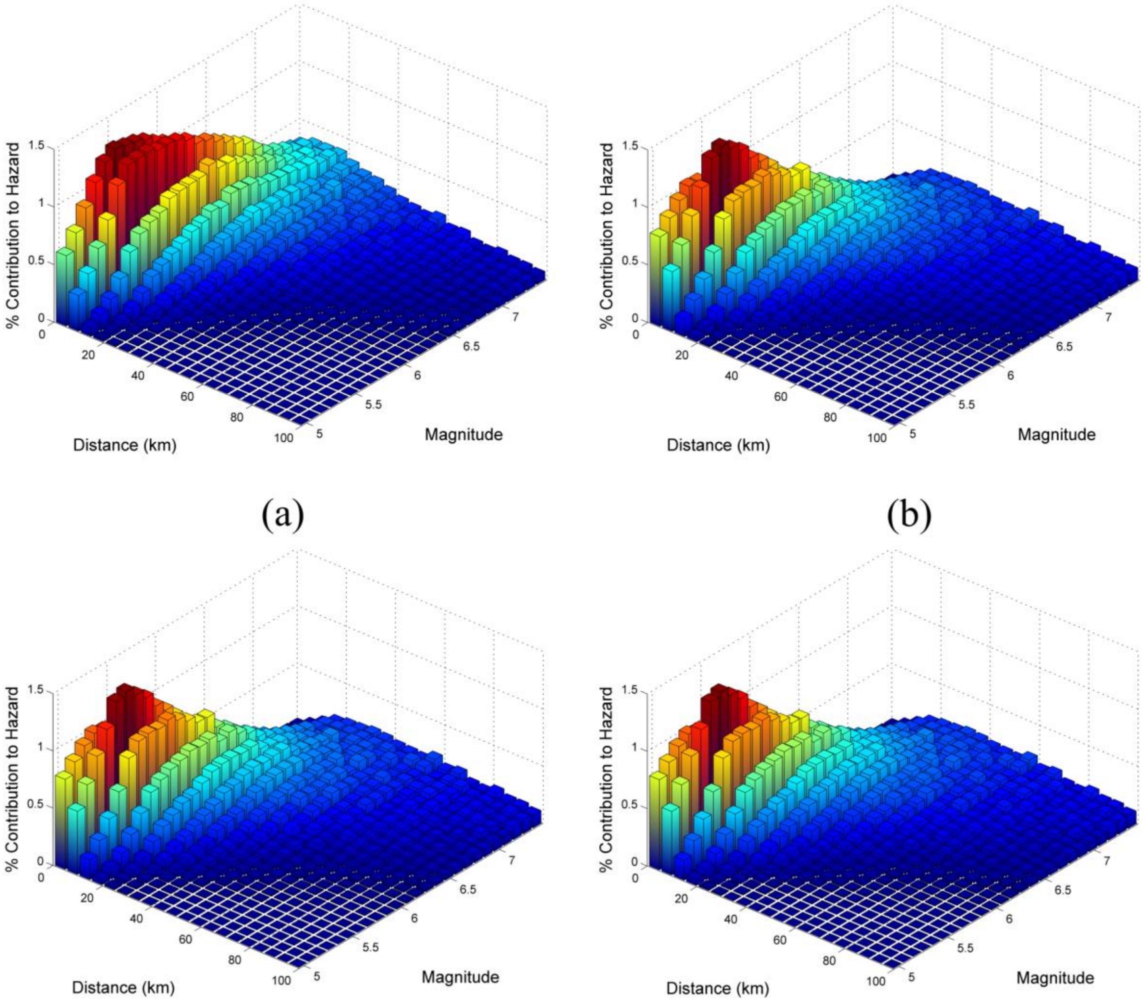


(a)

(b)

(c)

(d)

Figure 1. Seismic hazard disaggregation for $Sa(1.0)=0.586$ g: (a) $Sa>x_i$, (b) $x_i \leq Sa<1.1x_i$, (c) $x_i \leq Sa<1.01x_i$, and (d) $x_i \leq Sa<1.0001x_i$.

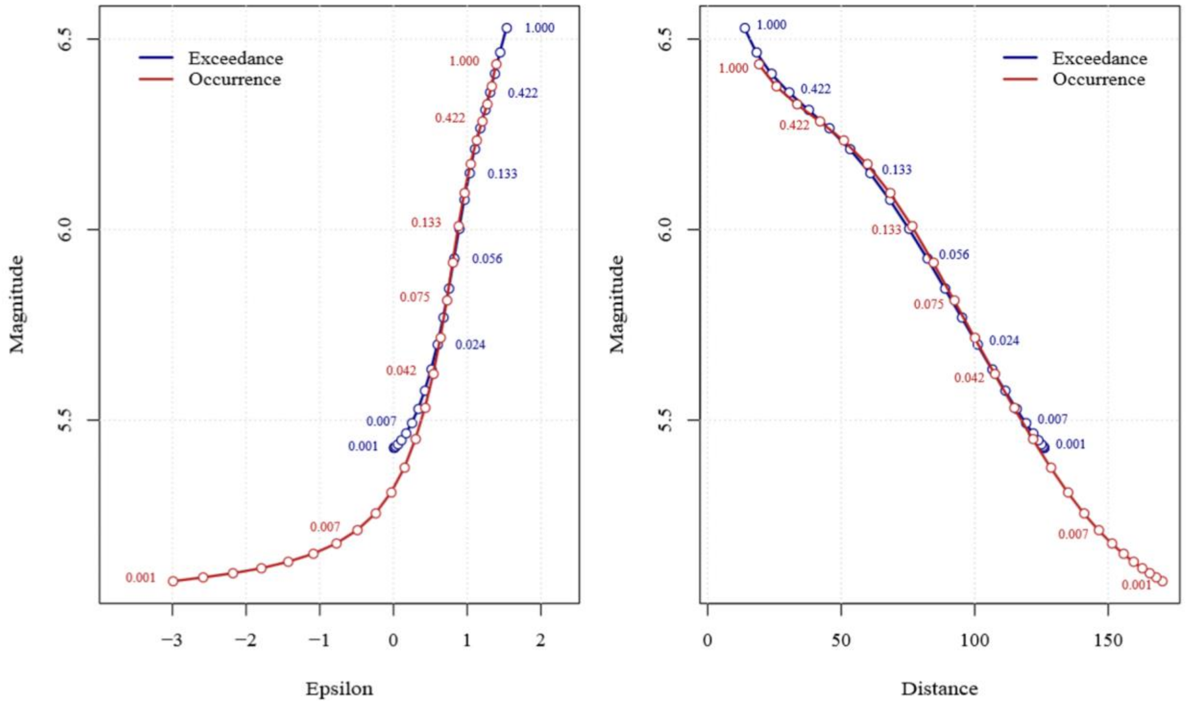


Figure 2. Mean disaggregation triplets for a large number of *Sa*(1.0s) levels (shown as annotations on the figures) for both exceedance and occurrence cases. The left panel shows the cross-section in magnitude-epsilon space, while the right panel shows the magnitude-distance space.

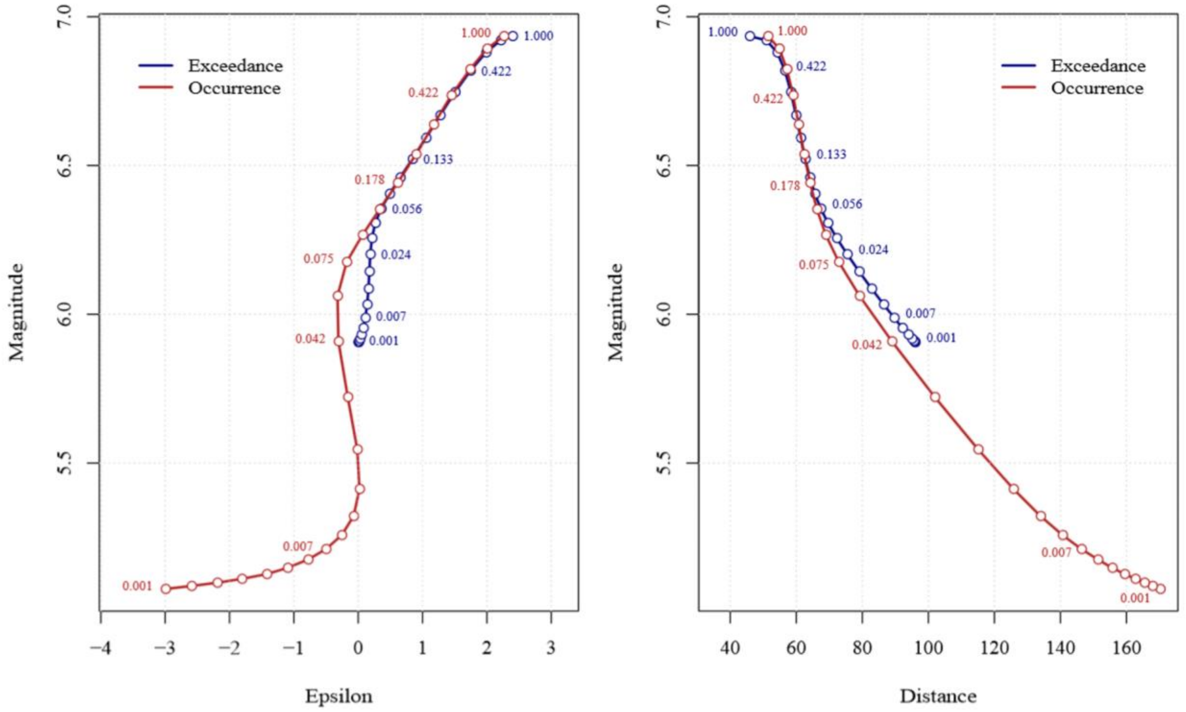
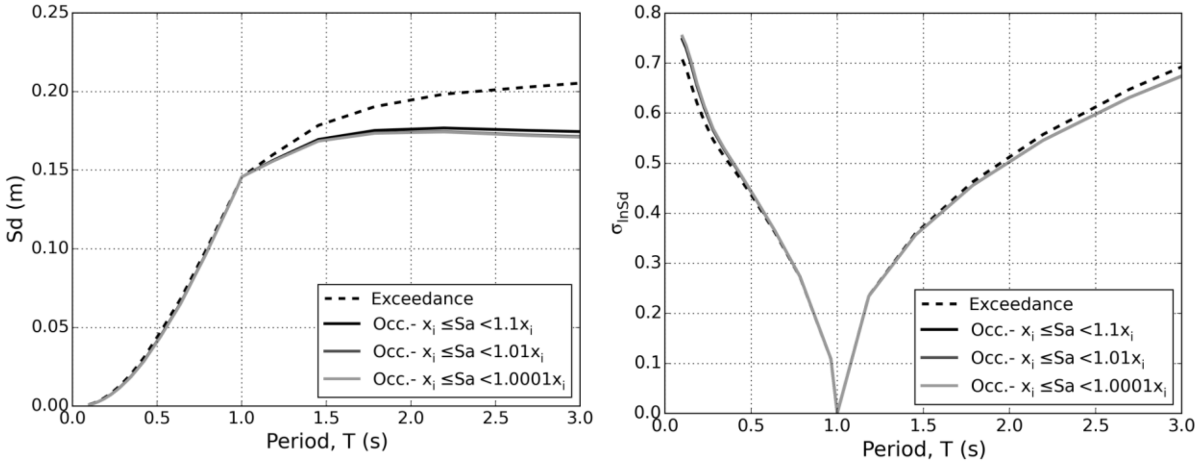


Figure 3. As for Figure 2, but with mean disaggregation triplets shown for the case where a fault source is added to the area source.



(a) (b)

Figure 4. (a) Conditional mean spectra, and (b) corresponding standard deviations for the different cases of disaggregation under consideration.

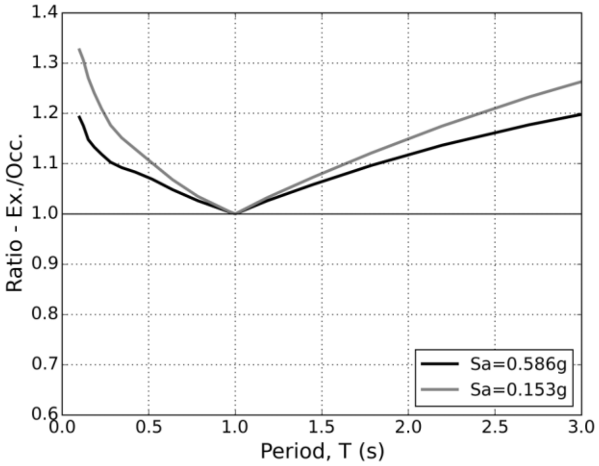


Figure 5. Ratios of spectral ordinates obtained from the conditional mean spectra corresponding to exceedance and occurrence.