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A Dynamic Model for the Optimization of Oscillatory Low Grade Heat Engines

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Abstract. The efficiency of a thermodynamic system is a key quantity on which its usefulness and wider application relies. This is especially true for a device that operates with marginal energy sources and close to ambient temperatures. Various definitions of efficiency are available, each of which reveals a certain performance characteristic of a device. Of these, some consider only the thermodynamic cycle undergone by the working fluid, whereas others contain additional information, including relevant internal components of the device that are not part of the thermodynamic cycle. Yet others attempt to factor out the conditions of the surroundings with which the device is interfacing thermally during operation. In this paper we present a simple approach for the modeling of complex oscillatory thermal-fluid systems capable of converting low grade heat into useful work. We apply the approach to the NIFTE, a novel low temperature difference heat utilization technology currently under development. We use the results from the model to calculate various efficiencies and comment on the usefulness of the different definitions in revealing performance characteristics. We show that the approach can be applied to make design optimization decisions, and suggest features for optimal efficiency of the NIFTE.

Keywords: heat engine, thermofluidic oscillator, low grade heat, low temperature, linear model, electrical analogy, efficiency
PACS: 88.05.Bc, 88.05.De, 07.20.Pe, 05.70.-a, 45.30.+s

INTRODUCTION

With ever-increasing environmental concerns, including that of climate change, but also of energy security in the light of finite resources of common fossil fuels, it is becoming increasingly important to consider alternative clean, efficient and sustainable energy solutions. The efficiency and power density of appropriate thermodynamic systems are key quantities on which their usefulness and wider application rely. This is especially true for devices that operate with marginal energy sources and close to ambient temperatures. In this paper we present a modeling framework based on thermodynamic and fluid mechanical principles for the early stage development of oscillatory low grade heat (e.g. solar energy or waste heat) utilization systems. We demonstrate the approach by applying it to a promising new low grade heat utilization technology that has attracted attention recently as a result of potential reliability advantages as well as reduced capital and operating costs, on account of its few mechanical moving parts.

The "Non-Inertive-Feedback Thermofluidic Engine" (NIFTE), as proposed in Refs. [1,2], can be described as a "two-phase unsteady heat engine", in which persistent and reliable thermodynamic (p, T, etc.) oscillations are generated and sustained by *stationary* external temperature differences. These desirable oscillations are driven by and give rise to heat and fluid flows, which involve the evaporation-condensation of the working fluid. The NIFTE can also be considered a two-phase realization of a class of devices known as "thermofluidic oscillators", which includes thermoacoustic engines [3-5], liquid-piston (Fluidyne) engines [6], free-piston Stirling engines [7], pulsejets and pulse-tubes [8]. In common with many thermofluidic oscillators, the NIFTE is particularly well suited to the conversion of low grade heat into useful work for fluid pumping, heating/cooling and niche power generation applications. It is capable of operating across temperature differences down to 30 K between the heat source and sink.

Thermofluidic oscillators have many dynamic similarities with analogue electronic oscillator circuits, and thus, electrical analogies have been used to predict approximate stability/instability criteria and to estimate first order heat and work flows, and efficiencies. The analogies, pioneered by Backhaus and Swift [3], were extended to include a description of exergy flows and to allow for exergy losses due to heat transfer in order to model the NIFTE by Smith [1,2]. However, the NIFTE depends on a parasitic throttle valve (referred to as "feedback valve") to create a phase shift between the heat flow and the power stroke, which was assumed negligible in Refs. [1,2]. Here, we investigate the consequences of this assumption and show that it is not valid in certain regions of the parameter space. We use this result to construct a framework for including parasitic losses in future thermofluidic oscillator models.

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METHODS

Problem Formulation: Heat, Power, Exergy and Efficiency Definitions

Consider an arbitrary, externally heated work producing device, within which power is produced by a working fluid that undergoes a thermodynamic cycle. Rather than dealing with thermodynamic states, we will deal with time-averaged flow rates of thermodynamic properties (e.g. "entropy flow rate" $\dot{s}$), so that irreversible processes may be quantified legitimately as increases in entropy flow rate. We use lower case letters to indicate specific quantities (per unit mass of working fluid). The temperature of the working fluid of the device that undergoes this "main cycle" is $T = T_o + T'$, where T' are fluctuations about the time-averaged value T_o. The working fluid is heated by an external source and cooled by an external sink, through contact with appropriate heat exchangers. The source and sink on the other side of the heat exchangers undergo separate processes. Let us denote by T_h the temperature of the source from which the main cycle obtains heat, and by T_c the temperature of the sink to which it rejects heat. Assuming that the main cycle is heated and cooled by purely irreversible heat transfer, i.e. conduction, T_h and T_c are bounded by lines of constant heat flow $\dot{q}$ that set a limit to the extent of the main cycle, wherein $T = \dot{q}/\dot{s}$. In addition, let $\Delta\dot{s}$ be the entropy change during the processes of heat addition and rejection that are equal in magnitude, but opposite in sign; and let v be the specific volume of the working fluid, such that $u = \dot{v}$ is the volumetric displacement flow rate per unit mass responsible for (hydraulic) power generation. Then, based on an arbitrary datum state (e.g. chose the time-averaged T_o, P_o), the rate of heat input to the main cycle, the *net* power produced, and the "*net* exergy flow rate" are,

$$\dot{q}_{in} = \int_{\Delta\dot{s}>0} T(\dot{s})d\dot{s} = T_o\Delta\dot{s} + \int_{\Delta\dot{s}>0} T'(\dot{s})d\dot{s} \ ; \ \dot{w} = \oint P(\dot{v})d\dot{v} = \oint P(u)du = \dot{q} = \oint T(\dot{s})d\dot{s}, \tag{1}$$

$$\dot{x}_{in} = \int_{\Delta\dot{s}>0}[T_h(\dot{s}) - T_o]d\dot{s} \ ; \ \dot{x}_{out} = \int_{\Delta\dot{s}<0}[T_c(\dot{s}) - T_o]d\dot{s} \ ; \ \dot{x} = \dot{x}_{in} - \dot{x}_{out} = \oint T_s(\dot{s})d\dot{s}\,, \tag{2}$$

where T_s is used to denote either T_h or T_c during the sequential heating and cooling parts of the cycle ($\Delta\dot{s} > 0$ and $\Delta\dot{s} < 0$, respectively). Hence, the power produced by the device $\dot{w} = \dot{q}$ may be described by the area inside a T-$\dot{s}$ diagram that represents the internally reversible part of the thermodynamic cycle. This area is enclosed by a larger area that represents the net exergy flow rate $\dot{x}$ of the full irreversible cycle, with the difference between these areas representing power loss through irreversible processes. Hence, $\dot{x}$ is the maximum power available to the cycle given the boundary conditions. It is clear from Equations (1) and (2) that changes in T_o will have no effect on $\dot{w}$ or $\dot{x}$. However, from Equation (1), we observe that $\dot{q}_{in}$ will depend on T_o, and in fact on the magnitude of T' relative to T_o.

Next we consider two classes of efficiency: (1) the thermal (first law) efficiency η_{th}, defined as $\eta_{th} = \dot{w}/\dot{q}_{in}$; and (2) the exergetic (second law) efficiency η_{ex}, defined as $\eta_{ex} = \dot{w}/\dot{x}$. In earlier models of the NIFTE viscous dissipation in parasitic components was assumed negligible [1,2]. We refer to efficiencies based on this assumption as (thermal or exergetic) "working fluid" efficiencies η_{wfl} and efficiencies for which parasitic dissipation is accounted for as (thermal or exergetic) "device" efficiencies η_{dev}. In this work we consider the effect of various device parameters on these efficiencies, with reference to the NIFTE which is introduced in more detail below.

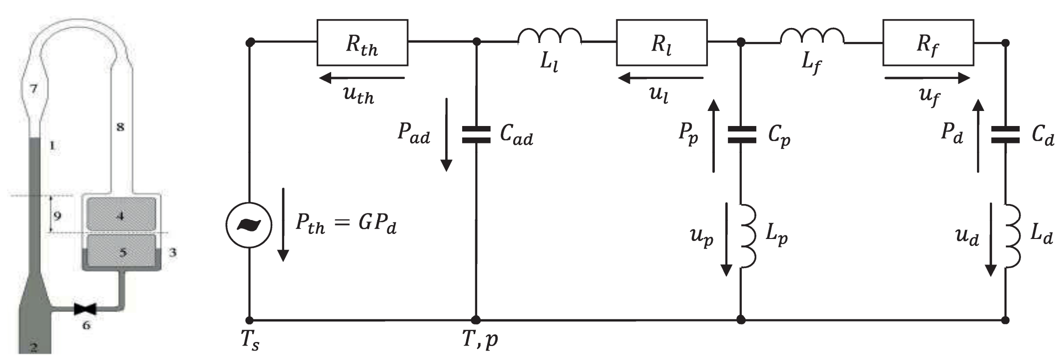

FIGURE 1. Left: NIFTE schematic taken from Ref. [1]. Component 2 is connected to the load line (denoted by subscript 'l'), 4 (hot) and 5 (cold) are heat exchanger blocks ('th'), 6 is the feedback line and valve ('f'), 7 is the power cylinder ('p') and 8 is the displacer cylinder ('d'). Levels 1 and 3 are the working fluid vapor-liquid interfaces in the power (left) and displacer (right) cylinders. The combined vapor space above Levels 1 and 3 is assumed to be an adiabatic vapor chamber ('ad'). Right: NIFTE model circuit diagram, where G is the feedback gain, R_i is a resistance, C_i is a capacitance, L_i is an inductance, P_i is a pressure and u_i is a volumetric flow rate (per unit mass). Subscript 'th' denotes the thermal domain (4,5 on the left), 'ad' the adiabatic domain (above Levels 1 and 3), 'l' the load (below 2), 'p' and 'd' the power (7) and displacer (8) cylinders, and 'f' the feedback valve (6).

TABLE 1. Expressions for the components appearing in the NIFTE as modeled in Fig. 1. The subscript 'fg' refers to phase change and 'o' to a time-averaged value; Δh_{fg} and Δv_{fg} denote the specific enthalpy and specific volume changes associated with phase change; k is a relevant thermal conductance; T_o and P_o are the time-averaged temperature and pressure; V_{ad} is the mean total volume in the vapor phase; μ and ρ are the viscosity and density of the liquid; and l_i and d_i are pipe lengths and diameters.

Thermal-Fluid Effect	Electrical Component Analogy	Expression
Heat exchanger thermal resistance	Resistor	$R_{th} = (\Delta h_{fg}/\Delta v_{fg})^2/kT_o$
Feedback valve & Load flow drag (viscous/pressure)	Resistor	$R_l = 128\mu l_l/\pi d_l^4$ & $R_f = 128\mu l_f/\pi d_f^4$
Power & Displacer cylinder hydrostatic pressure	Capacitor	$C_p = \pi d_p^2/4\rho g$ & $C_d = \pi d_d^2/4\rho g$
Adiabatic vapor compressibility	Capacitor	$C_{ad} = V_{ad}/\gamma P_o$
Power & Displacer cylinder inertia (liquid mass)	Inductor	$L_p = 4\rho l_p/\pi d_p^2$ & $L_d = 4\rho l_d/\pi d_d^2$
Feedback line & Load pipe inertia	Inductor	$L_f = 4\rho l_f/\pi d_f^2$ & $L_l = 4\rho l_l/\pi d_l^2$

Mathematical Modeling of the Non-Inertive-Feedback Thermofluidic Engine (NIFTE)

A NIFTE schematic can be seen in Fig. 1 (left). Further specifics of the construction and operation of the NIFTE can be found elsewhere [2], though these details are not central to the objective of the current investigation. Here, we need only regard the NIFTE as a device comprising a number of interconnected components (chambers, tubes and heat exchangers), that interact by exchanging heat and fluid so as to transform heat into fluid displacement (i.e. hydraulic power). We treat the case where the working fluid and power transmitting fluid are the same. A linear approximation of the NIFTE can be described by an LRC circuit, that is, an electrical circuit with inductors (*L*), resistors (*R*) and capacitors (*C*), as explained in the following paragraph.

In Smith [1,2] a simple, but powerful model for the dynamic behavior of the NIFTE was proposed, following Backhaus and Swift [3], Ceperley [5] and Huang and Chuang [8]. This involved lumped (spatially averaged, and thus independent) and linearized sub-models for each NIFTE component that were interconnected to derive a complete model for the whole device. The linearization allows analogies to be drawn with analogue electrical components, thus enabling an electrical network to be constructed for the NIFTE. The network contains resistors (accounting for viscosity, fluid drag and thermal resistance), and capacitors (accounting for gravity and compressibility). Here we extend this approach, with the added inclusion of inductors to capture finite inertial effects. In more detail, the dominant physical process undergone inside each sub-component is firstly identified and modeled linearly. By drawing analogies between the physical variables of pressure (*P*) and voltage (*E*), volumetric flow rate (*u*) and current (*I*), temperature (*T*) and voltage (*E*), and entropy flow rate ($\dot{s}$) and current (*I*), the linear, lumped equations involving the raw thermal-fluid variables are transformed into a suitable network of electrical elements. Referring to Fig. 1 (right), resistance to thermal or fluid flow is represented by a resistor (*R*); hydrostatic pressure (i.e. gravitational potential energy) of the liquid and adiabatic compressibility of the vapor are both represented by capacitors (*C*); and fluid inertia is represented by an inductor (*L*). The dynamic equation of each element is $E = R.I$, $\dot{E} = 1/C\,.I$ and $E = L.\dot{I}$, respectively, where E is the potential difference across the component and I is the current through it. The physical quantities used to evaluate the value of each electrical component in Fig. 1 (right) are shown in Table 1, with a detailed explanation of all variables in the caption.

Since all aforementioned components are assumed linear, the complete NIFTE network is also linear. For simplicity, but without loss of generality, we treat the case of sinusoidal oscillations with a single angular frequency ω in all quantities. We limit ourselves to considering the (efficiency) performance of the NIFTE at marginal stability, i.e. in conditions in which it exhibits sustained oscillations of constant amplitude with minimal gain. The internal feedback process with gain *G*, that can be seen on the far left in Fig. 1 (right), is increased until this marginal stability condition is established. This gain is related directly to the spatial temperature gradient established in the source/sink heat exchanger configuration, and thus the difference between the hot and cold temperatures available externally to the device and the design of the heat exchanger configuration. At this point we note the relative amplitudes of oscillation of all variables. We can then evaluate the integrals for heat, power and exergy in Equations (1) and (2), which for single-frequency sinusoidal signals simplify to,

$$\dot{q}_{in} = T_o\Delta\dot{s} + \omega\hat{T}_s\hat{s}\cos\measuredangle\{T,\dot{s}\}/4\ ;\ \dot{w} = \hat{p}\hat{u}\cos\measuredangle\{p,u\}/2\ ;\ \dot{x} = \omega\hat{T}_s\hat{s}\cos\measuredangle\{T_s,\dot{s}\}/2\,, \tag{3}$$

where $\hat{T}$ and $\hat{T}_s$ are the temperature amplitudes of the main cycle (i.e. the working fluid) and its thermal environment (measured at the internal surface of the source/sink heat exchangers), $\hat{s} = \pi\Delta\dot{s}/\omega$ is the entropy flow amplitude of the main cycle, and $\measuredangle\{a,b\}$ is the phase angle between variables *a* and *b*. Finally, from the quantities in Equation (3) we can evaluate all required efficiencies defined in the previous section. For the two thermal efficiencies η_{th} we have used $\hat{T}/T_o = 0.05$ and $\hat{T}_s = \hat{T}$, i.e. a 30 K peak difference between hot and cold at $T_o = 300$ K.

RESULTS AND DISCUSSION

In Fig. 2 results are presented for the effects of various parameters (Rs, Cs, Ls in Fig. 1, Table 1) on the NIFTE efficiencies: thermal efficiency of the device $\eta_{th,dev}$, thermal efficiency of the working fluid system $\eta_{th,wfl}$, exergetic efficiency of the device $\eta_{ex,dev}$, and exergetic efficiency of the working fluid $\eta_{ex,wfl}$. We note that the two classes of device efficiency (thermal and exergetic) often lead to conflicting design interpretations, and that the device efficiency is not necessarily optimal when the working fluid cycle exhibits its maximum efficiency. For example, in Fig. 2(a) both η_{th} and η_{ex} experience a sharp drop as L_p is reduced, yet even though η_{th} recovers (up to 50% of its maximum) at very low values of L_p, this is not observed in η_{ex}. Also, in Fig. 2(b) the two η_{dev} are insensitive to the design of the displacer cylinder (i.e. choice of C_d and L_d), with $\eta_{th,dev} \sim 0.7\%$ and $\eta_{ex,dev} \sim 1\%$, even though η_{dev} can be almost doubled by increasing C_d within the investigated range. The plots highlight critical components that require careful design for the optimization of the NIFTE, namely the power cylinder, the adiabatic volume and the feedback line and valve. In Fig. 2(a) C_p and L_p can result in a significant increase in $\eta_{th,dev}$ (up to 5%) and in $\eta_{ex,dev}$ (almost 60%). In conclusion, an efficient NIFTE design would feature low C_p, high L_p, high C_{ad}, low R_f and low L_f.

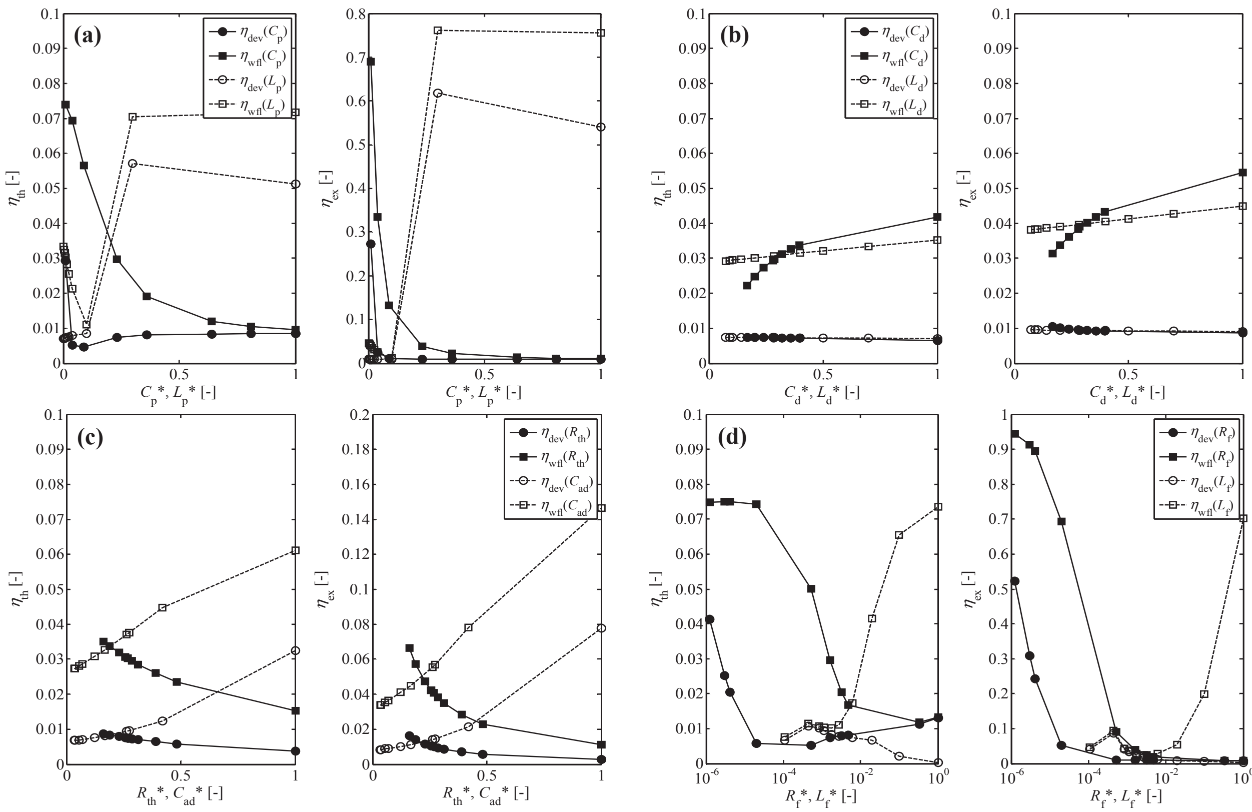

FIGURE 2. Effect on the NIFTE efficiencies of: (a) pressure and (b) displacer cylinder capacitances C and inductances L; (c) thermal resistance R_{th} and adiabatic capacitance C_{ad}; and (d) feedback valve resistance R_f and line inductance L_f. Parameters varied one at a time, with all others set to a 'default' value. Abscissas normalized by the largest value in the investigated range.

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