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DOMAIN WALLS AND THE UNIVERSE*

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$D = 11$ supergravity possesses $D = 5$ Calabi-Yau compactified solutions that may be identified with the vacua of the Hořava-Witten orbifold construction for M–theory/heterotic duality. The simplest of these solutions naturally involves two 3-brane domain walls, which may be identified with the orbifold boundary planes; this solution also possesses an unbroken $\mathbb{Z}_2$ symmetry. Consideration of nearby excited solutions, truncated to the zero-mode and $\mathbb{Z}_2$ invariant sector, yields an effective $D = 4$ heterotic theory displaying chirality and $N = 1$, $D = 4$ supersymmetry.

1 Introduction

One of the striking aspects of the ongoing reformulation of string theory is the extent to which supergravity effective field theories can provide important non-perturbative information about the underlying quantum theory. Various duality relations have been proposed to hold between the different perturbatively consistent string theories, and also between these theories and the anticipated quantum precursor to $D = 11$ supergravity, which has been called M–theory. We still lack, however, a satisfactory microscopic formulation of quantum M–theory. Thus, duality relations between M–theory and the perturbative string theories remain somewhat tentative. Accordingly, it is important to establish just which aspects of a duality relation are in fact already present at the effective field-theoretic level, and which truly involve quantum phenomena.

An important case in point is the Hořava-Witten relation between M–theory and $D = 10$ heterotic string theory.[1,2] The essential idea in this relation involves compactification of the $D = 11$ theory on an orbifold, causing a sensitivity to anomalies in an otherwise anomaly-free theory. Requiring cancellation of the anomalies proves to be the essential clue that reveals the presence of supersymmetric Yang-Mills modes propagating only in the two orbifold fixed planes that bound the $D = 11$ spacetime. In the limit where these boundary planes lie close together, these $D = 10$ zero modes are described by $D = 10$ heterotic string theory.

One aspect of the M–theory/heterotic duality that remains somewhat ob-

*Based upon a talk given at the Conference on Superfivebranes and Physics in 5+1 Dimensions, Trieste, Italy, 1-3 Apr 1998 scure in the Hořava-Witten scenario is whether the appearance of the orbifold can be viewed as a natural occurrence in M–theory, or whether this is being introduced from the outside as an extraneous ingredient. Given the absence of an underlying microscopic formulation of M–theory, this question is hard to answer at the quantum level. But one can still try to gain what insight one can into this question at the effective field-theoretic level of $D = 11$ supergravity.

Reduction of the $D = 10$ theory obtained from M–theory through the orbifold construction on down to $D = 4$ proceeds in a fairly standard fashion by further compactification on a Calabi-Yau manifold. Searching for a solution to the resulting $D = 4$ theory then reveals [3] a deformed Calabi-Yau vacuum solution, found originally at lowest order in a perturbative expansion in $\kappa^{\frac{2}{3}}$, where κ is the $D = 11$ gravitational coupling constant. The deformation of the Calabi-Yau solution involves, specifically, an evolution in the volume of the Calabi-Yau manifold as one moves in the bounded orbifold coordinate, originally taken to be x^{11}.

This behavior underlines another salient aspect of M–theory/heterotic duality. In modern Kaluza-Klein theory, considerable attention has been paid to the question of *consistency* of the dimensional reduction. By consistency, one means that solutions to the dimensionally reduced theory are still perfectly good, albeit rather specific, solutions to the original higher-dimensional theory. Variation of the Calabi-Yau volume as one moves across the orbifold indicates clearly that this $11 \to 10$ reduction is not consistent in this technical sense. Now, it is not automatically disastrous to get involved in a technically inconsistent Kaluza-Klein reduction like this. But what is required to treat such a case is a careful integrating out, or averaging, of the non-trivial higher dimensional modes in order to extract the effective lower dimensional physics. In consistent Kaluza-Klein reductions, by contrast, the non-trivial higher dimensional modes may simply be set equal to their vacuum values, which are typically vanishing. Such behavior may in fact be taken to give an alternative definition of a consistent reduction. For the present case of an inconsistent orbifold compactification, it seems more appropriate instead to retain the orbifold dimension within the lower-dimensional theory, leading thus to a $D = 5$ perspective on the Hořava-Witten construction.

In contrast to the orbifold reduction, the straightforward reduction of supergravity theories on Calabi-Yau manifolds may be considered to be "essentially consistent" in the sense that any corrections induced by integrating out the non-zero-modes can only be of higher order in derivatives than the leading order effective field theory.[4] The possibility of higher-derivative corrections to the effective theory presumably also corresponds to uncertainty about just what the explicit form of the Kaluza-Klein ansatz for a Calabi-Yau reduction ought to be, since the Calabi-Yau metrics are not known explicitly.

Here, we shall review from the above perspective another approach to the derivation of $D = 4$ physics from M–theory compactified on a Calabi-Yau manifold.[5,6] Taking a clue from the above discussion that the physics of the Hořava-Witten construction should really be viewed as five dimensional, we shall first dimensionally reduce $D = 11$ supergravity on a Calabi-Yau manifold down to $D = 5$. Finding the right vacuum solution, which will extend the solution of Ref.[3] to all orders in the gravitational coupling constant, will then reproduce for us the remaining salient features of the Hořava-Witten construction: the $\mathbb{Z}_2$ orbifold, chirality and $N = 1$, $D = 4$ supersymmetry, but this shall now occur within the natural context of solutions to $D = 11$ supergravity.

2 Generalized Calabi-Yau reduction of M–theory

Start from the $D = 11$ supergravity action, whose bosonic sector is

$$I_{11} = \int d^{11}x\sqrt{-g}(R - \tfrac{1}{48}G_{[4]}^2) + \tfrac{1}{6}G_{[4]} \wedge G_{[4]} \wedge A_{[3]} \ , \tag{1}$$

where $G_{[4]} = dA_{[3]}$ is the field strength for the 3-form gauge potential $A_{[3]}$. When making a standard compactification[7] $\mathcal{M}_{11} = \mathcal{M}_5 \times \mathcal{X}$ on a Calabi-Yau 3-fold $\mathcal{X}$ which is characterized by Hodge numbers $h^{(1,1)}$, $h^{(2,1)}$ and intersection numbers $d_{ijk} = \int_{\mathcal{X}} \omega_i \wedge \omega_j \wedge \omega_k$, one obtains $N = 1$, $D = 5$ supergravity theory coupled to $(h^{(1,1)} - 1)$ $D = 5$ vector multiplets, plus one universal hypermultiplet, plus $h^{(2,1)}$ additional hypermultiplets. Although there are a total of $h^{(1,1)}$ vector fields arising in the Kaluza-Klein reduction on $\mathcal{X}$, there are only $(h^{(1,1)} - 1)$ vector multiplets because one vector is required in the $N = 1$, $D = 5$ supergravity multiplet.

The universal hypermultiplet contains the modulus field $V = \frac{1}{6}d_{ijk}a^ia^ja^k$ corresponding to the volume of the Calabi-Yau manifold $\mathcal{X}$, where the a^i are general $h^{(1,1)}$ moduli. The a^i correspond to deformations of the complex structure: $\omega_{AB} = a^i\omega_{iAB}$, where the ω_{iAB}, $i = 1, \dots h^{(1,1)}$ form a basis of the (1,1) forms on the Calabi-Yau manifold (where $A = a, \bar{a}$ run over the six internal coordinate directions). In the form directly inherited from dimensional reduction, the universal hypermultiplet also contains the $D = 5$ 3-form gauge field $A_{\alpha\beta\gamma}$ and also a complex scalar ξ obtained from the purely Calabi-Yau components of the $D = 11$ 3-form: $A_{abc} = \frac{1}{6}\xi\Omega_{abc}$, where Ω_{abc} is the harmonic (3,0) form on the Calabi-Yau manifold.

Outside the universal hypermultiplet, there are the $(h^{(1,1)} - 1)$ remaining shape-determining metric moduli that are independent of V. These may be denoted by $b^i = V^{-\frac{1}{3}}a^i$. These modulus fields are members of the $(h^{(1,1)}-1)$ $D =$

5 vector multiplets arising from the dimensional reduction. For the fields b^i, one obtains a non-linear sigma model action with metric $G_{ij} = -\frac{1}{2}V^{\frac{2}{3}}\frac{\partial}{\partial a^i}\frac{\partial}{\partial a^j}\ln V$.

Ordinary dimensional reduction of $D = 11$ supergravity on a Calabi-Yau manifold [7] in this way produces a massless theory of $N = 1$, $D = 5$ supergravity coupled to $N = 1$, $D = 5$ supermatter. The natural subsequent dimensional reduction of this theory down to $D = 4$ on a circle would then give rise to an $N = 2$, $D = 4$ non-chiral theory. While interesting, this reduction path does not lead one to realistic-looking $D = 4$ physics. So now one may take a clue from another aspect of the known vacuum solution [3] to the Hořava-Witten construction: the 4-form field strength G_{ABCD} takes non-vanishing values in the internal Calabi-Yau directions.[5,6] Normally, letting a differential quantity such as a field strength take non-vanishing values in compactification directions might be thought to lead to inconsistency of the reduction. This apparent inconsistency would arise from forcing the underlying gauge potential to have non-trivial dependence on the internal Calabi-Yau coordinates, in contrast to the standard procedure of Kaluza-Klein reduction which suppresses all such internal coordinate dependence. However, in certain cases one may permit such dependence without endangering consistency, provided one uses an extension of the idea of Scherk-Schwarz reduction, known as generalized dimensional reduction.[8]

Generalized Kaluza-Klein reduction has a topological reformulation [9] as follows. One may admit dependence on the internal coordinates in a Kaluza-Klein reduction provided that such dependence is based upon elements of the homology groups of the internal manifold $\mathcal{X}$. Specifically, for an (n-1) form gauge potential $A_{[n-1]}$ in a space with internal coordinates z^A and with retained lower-dimensional coordinates x^μ, one may make the consistent reduction ansatz

$$A_{[n-1]}(x^\mu, z^A) = \omega_{[n-1]}(z^A) + A_{[n-1]}(x^\mu) \tag{2}$$

provided that $\omega_{[n-1]}$ satisfies

$$d\omega_{[n-1]} = \Omega_{[n]} \in H^n(\mathcal{X}, \mathbb{R}) \ . \tag{3}$$

Simple examples of such reductions [8] take a circle $\mathcal{S}^1$ as the reduction manifold with $\Omega_{[1]} = m dz \in H^1(\mathcal{S}^1, \mathbb{R})$, in which case it is an underlying axion field $A_{[0]}$ that acquires the z dependence: $A_{[0]}(x^\mu, z) = \omega_{[0]}(z) + A_{[0]}(x^\mu)$. This is cohomologically non-trivial because, although solving $\omega_{[0]} = mz$ is OK locally, this does not satisfy globally the required $\mathcal{S}^1$ periodicity condition.

Now let us apply the generalized reduction procedure to the Calabi-Yau reduction of M-theory. The Pontryagin class of the Calabi-Yau manifold $\mathcal{X}$ is characterized by a set of integers β_i:

$$\beta_i = \frac{-1}{8\pi^2} \int_{\mathcal{C}^i} \mathrm{tr}\, R \wedge R \ , \tag{4}$$

where the 4-cycle $\mathcal{C}^i$ is related to the harmonic (2,2) form ν_i dual to the basis (1,1) form ω^i: $\int_{\mathcal{C}^i} \nu^j = \delta_i^j$, $\int_{\mathcal{X}} \omega_i \wedge \nu^j = \delta_i^j$. Then, in analogy to (2), a generalized Kaluza-Klein reduction on the manifold $\mathcal{X}$ may be given by requiring the Calabi-Yau components of $G_{[4]}$ to take cohomologically nontrivial values

$$G_{ABCD} = \alpha_i \, \nu^i_{ABCD} \ , \tag{5}$$

where the coefficients α_i are given by rescaling the topological integers β_i by a factor: $\alpha_i = \frac{\pi}{\sqrt{2}} \left(\frac{\kappa}{4\pi}\right)^{2/3} \beta_i$. One may characterize this generalized reduction as turning on $G_{[4]}$–instantons in the Calabi-Yau dimensions.

The resulting bosonic $D = 5$ reduced theory for the supergravity, universal hypermultiplet and (1,1) modulus fields has the form [6]

$$\begin{aligned} I_5 = & \\ & -\frac{1}{2\kappa_5^2} \int_{M_5} d^5x \sqrt{-g} \Big[R + G_{ij}(b) \partial_\alpha b^i \partial^\alpha b^j + G_{ij}(b) \mathcal{F}^i_{\alpha\beta} \mathcal{F}^{j\alpha\beta} \\ & \qquad + \frac{\sqrt{2}}{12} \epsilon^{\alpha\beta\gamma\delta\epsilon} d_{ijk} \mathcal{A}^i_\alpha \mathcal{F}^j_{\beta\gamma} \mathcal{F}^k_{\delta\epsilon} \Big] \\ & -\frac{1}{2\kappa_5^2} \int_{M_5} d^5x \sqrt{-g} \Big[\frac{1}{2} V^{-2} \partial_\alpha V \partial^\alpha V + 2V^{-1} \partial_\alpha \xi \partial^\alpha \bar{\xi} + \frac{1}{24} V^2 G_{\alpha\beta\gamma\delta} G^{\alpha\beta\gamma\delta} \\ & \quad + \frac{\sqrt{2}}{24} \epsilon^{\alpha\beta\gamma\delta\epsilon} G_{\alpha\beta\gamma\delta} \big(\mathrm{i}(\xi \bar{X}_\epsilon - \bar{\xi} X_\epsilon) + 2\alpha_i \mathcal{A}^i_\epsilon \big) + \frac{1}{2} V^{-2} G^{ij}(b) \alpha_i \alpha_j \Big] \ . \end{aligned} \tag{6}$$

In the reduced action (6), one may note the following salient features:

1) A *potential* $\frac{1}{2} V^{-2} G^{ij}(b) \alpha_i \alpha_j$ has appeared for the moduli.

2) The Kaluza-Klein vector fields $\mathcal{A}^i_\alpha$ participate in *gauge couplings* to the universal hypermultiplet.

3) The gauge coupling linear in α_i in the last line of (6) breaks the $x^5 \to -x^5$ parity $\mathbb{Z}_2$ invariance possessed by the original $D = 11$ action (1).

The action (6) may be cast into a more standard form by performing a duality transformation on the $D = 5$ 4-form field strength $G_{\alpha\beta\gamma\delta\epsilon}$: introduce a Lagrange multiplier σ in order to impose the Bianchi identity for this field strength as an equation of motion. Then, $G_{\alpha\beta\gamma\delta\epsilon}$ can be taken to be an independent field and can be eliminated by its now-algebraic field equation. The result for the universal hypermultiplet $(V,\sigma,\xi,\bar{\xi})$ is an action for an ${\rm SU}(2,1)/{\rm SU}(2)\times {\rm U}(1)$ nonlinear sigma model. The gauge coupling observed in 2) above acts on the U(1) factor in the denominator group. This gauge coupling allows the elimination of the σ field, with the consequent appearance of a mass term for the vector-field combination $\alpha_i\mathcal{A}^i_\alpha$. This structure is characteristic of a gauged coupling of $D=5$ supergravity to a nonlinear sigma model as was found in the original studies of $D=5$ supergravity.[10]

3 Domain walls and the $\mathbb{Z}_2$ orbifold in 4.5 dimensions

Observation 1) above on the structure of the reduced action (6) is characteristic of theories obtained by generalized Kaluza-Klein reduction.[8] The presence of a potential for the moduli has a dramatic effect on the vacuum structure of this dimensionally-reduced theory: flat space, and, more generally, any maximally symmetric spacetime does not occur as a solution to the resulting equations of motion. Instead, it appears that the solution displaying the highest degree of symmetry (including supersymmetry) for such a theory is actually a *domain wall.* In the absence of a maximally-symmetric solution, a system of domain walls would seem to be the best candidate for a "vacuum" solution to the theory. Thus, the theory (6) is one in which translation invariance appears to be spontaneously broken.

Trying a domain-wall ansatz for a solution to the $D=5$ theory (6), one posits

$$\begin{aligned} ds_5^2 &= a(y)^2 dx^\mu dx^\nu \eta_{\mu\nu} + b(y)^2 dy^2 \\ V &= V(y) \qquad\qquad b^i = b^i(y)\ , \end{aligned} \tag{7}$$

where $\mu,\nu=0,1,\ldots,3$ will remain as a set of $D=4$ Lorentz coordinates and $y=x^5$ will play the rôle of the coordinate transverse to the domain wall. Analyzing the resulting equations, one finds [6] a solution with

$$V(y) = (\tfrac{1}{6}d_{ijk}f^if^jf^k)^2\ , \quad a(y) = \tilde{k}V^{\frac{1}{6}}\ , \quad b(y) = kV^{\frac{2}{3}}\ , \quad b^i(y) = V^{-\frac{1}{6}}f^i\ , \tag{8}$$

where $k,\tilde{k}$ are integration constants and the functions $f^i(y)$ are determined implicitly in terms of a set of $h^{(1,1)}$ harmonic functions $H_i(y)$:

$$d_{ijk}f^jf^k = H_i \qquad\qquad H_i(y) = 2\sqrt{2}k\alpha_i y + k_i\ , \tag{9}$$

where the k_i are a further set of $h^{(1,1)}$ integration constants.

The solution (7–9) is still not yet satisfactory, for it leads to singularities at the zeros of the harmonic functions $H_i(y)$. But this is a standard problem with codimension-one solutions, for which the harmonic functions are linear, as in (9). The solution is to replace the linear dependence on y by dependence on $|y|$, $H_i(y) = 2\sqrt{2}k\alpha_i|y| + k_i$. This causes a "kink" in the harmonic function at $y = 0$, corresponding to the location of the domain wall. This modification still does not fully cure the problem of the singularities, however: although analysis of the curvature shows the metric to go flat as $y \to \pm\infty$, the scalar field $V(y)$ still blows up in these limits. The cure to this remaining problem is to declare the $y = x^5$ coordinate to be on a compact $\mathcal{S}^1$ dimension by identifying points with $y = \pm\pi\rho$. After this identification, the harmonic function $H_i(y)$ now has a second kink, corresponding to a second domain wall located at $y = \pi\rho \leftrightarrow -\pi\rho$.

This construction of the "vacuum" resolves the singularity problems of the naïve "proto-vacuum" solution (8), but, on the other hand, it does not strictly speaking give a proper solution to the dimensionally reduced theory (6). This is because the replacement of y by $|y|$ in the harmonic function in (9) amounts to a $\mathbb{Z}_2$ transformation on (8) for $y < 0$ and, as we have noted in point 3) above, the reduced action (6) is not $\mathbb{Z}_2$ invariant. In fact, casual inspection of the generalized reduction ansatz (5) shows why this must be the case: one is setting a $\mathbb{Z}_2$ odd quantity G_{ABCD} to take a non-vanishing $\mathbb{Z}_2$ insensitive fixed value – thus it is clear that the $\mathbb{Z}_2$ symmetry must be broken in (6). What one has achieved in replacing y by $|y|$ in (9) is, strictly speaking, to patch together two non-overlapping regions of the $D = 11$ spacetime, with two different $G_{[4]}$ Kaluza-Klein ansätze related by a $\mathbb{Z}_2$ transformation. One may write the overall ansatz as

$$G_{ABCD} = \alpha_i \, \nu^i_{ABCD} \, \epsilon(y) \; , \tag{10}$$

where $\epsilon(y)$ is the function that steps between (-1) and 1 as y passes through zero.

The corrected $G_{[4]}$ ansatz (10) actually restores the $\mathbb{Z}_2$ symmetry that was lost in the original ansatz (5), because a $\mathbb{Z}_2$ transformation combined with a simple rotation of the $\mathcal{S}^1$ coordinates by π (a special instance of the translations on $\mathcal{S}^1$ that are generically broken by the inhomogeneous domain wall solution (7–9)) brings the solution back to its original configuration. Thus, if one were to go beyond the static vacuum solution that we have been considering, nearby fluctuations could be separated into even and odd modes under this now-restored $\mathbb{Z}_2$ symmetry.

The most basic mechanism for ensuring consistency of a Kaluza-Klein truncation or reduction is to make a projection into the invariant sector with respect to some symmetry. This presents one with a final "half dimension" of Kaluza-Klein truncation that can be made consistently, at the classical level that we have been discussing, by projection into the even sector under the preserved

$\mathbb{Z}_2$ symmetry. This may be mnemonically considered to be reduction to "4.5 dimensions." After such a projection, the retained $D = 5$ modes may be considered to take their values on the spacetime $\mathcal{M}_4 \times \mathcal{S}^1/\mathbb{Z}_2$. Thus, at the level of the classical modes, the theory (6) may be viewed as having spontaneously created the $\mathcal{S}^1/\mathbb{Z}_2$ orbifold.

4 Magnetic charge and the cohomology condition

As a result of the step function in the corrected ansatz (10), the ordinary Bianchi identity for $G_{[4]}$ is no longer valid, but instead one has

$$(dG)_{5ABCD} = \frac{-1}{4\sqrt{2}\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}(\delta(y) - \delta(y - \pi\rho))\mathrm{tr}\,(R \wedge R)_{ABCD} \ , \qquad (11)$$

implying the presence of magnetic charges in the "vacuum" solution (7–9). These magnetic charges arise from the $G_{[4]}$ instantons in the Calabi-Yau directions that have been turned on by the choice of the topological integers β_i (4), and are occasioned by the $\mathbb{Z}_2$ sign flips as one crosses the domain walls.

Although the ordinary Bianchi identity for $G_{[4]}$ has been modified to (11) in the presence of the domain walls, there is still a constraint on the fields arising from the global structure of this equation. Demanding that $(dG)_{[5]}$ be an exact 5-form in the full $D = 11$ spacetime, one obtains a requirement that $\int_{5-\mathrm{cycle}} dG = 0$ for integration over an arbitrary 5-cycle. Picking in particular a 5-cycle $\mathcal{C}^i \times \mathcal{S}^1$, where $\mathcal{C}^i$ is one of the 4-cycles on the Calabi-Yau manifold, one obtains the *cohomology condition* [11]

$$\sum_{\mathcal{S}^1 \ \mathrm{patches}} \alpha_i = 0 \ . \qquad (12)$$

This is clearly satisfied for the choice of just two proto-vacuum patches with $\alpha_i^{(2)} = -\alpha_i^{(1)}$, as we have arranged in replacing y by $|y|$ in the harmonic functions $H_i(y)$ (9). This condition also governs the structure of solutions [11] with more than the minimal pair of domain walls distributed around the $\mathcal{S}^1$.

Since we have been at pains to maintain Kaluza-Klein consistency in our reduction from 11 to 5 dimensions, the $D = 5$ double-domain-wall "vacuum" solution may also be oxidized back up to $D = 11$. In $D = 11$, the $D = 5$ 3-brane is recognized as a 4-dimensional stack of $D = 11$ 5-branes wrapped around 2-cycles of the compactifying Calabi-Yau space, thus effecting a mixed "diagonal" reduction in two dimensions and "vertical" reduction in four dimensions of the compactifying Calabi-Yau manifold.[12]

5 Chirality and Supersymmetry

In arriving at the above domain-wall solution, we have set the stage for a compactification to "4.5" dimensions, as explained above. The final half-dimension of truncation is implemented by projecting all $D=5$ fields into the $\mathbb{Z}_2$ invariant sector. Taking into account the parity oddness of the $D=11$ $A_{[3]}$ gauge potential as one can see from the action (1), this final truncation amounts to imposing conditions like $g_{\mu\nu}(x^\mu,y)=g_{\mu\nu}(x^\mu,-y)$, $g_{\mu 5}(x^\mu,y)=-g_{\mu 5}(x^\mu,-y)$, $A_{\mu\nu\rho}(x^\mu,y)=-A_{\mu\nu\rho}(x^\mu,-y)$, $\psi^i_\mu(x^\mu,y)=\Gamma_5\psi_\mu(x^\mu,-y)$, *etc.* $(\mu,\nu=0,1,\ldots,3)$. Now it becomes clear how the final reduction to $D=4$ causes chirality to appear. Even though this final step does not correspond to a consistent Kaluza-Klein reduction and thus one needs to properly integrate out the non-trivial $D=5$ modes, the chirality implications of the reduction may still be seen just by suppressing the y dependence in the $\mathbb{Z}_2$ truncation formulas, yielding chiral fields in $D=4$.

$N=1$, $D=4$ supersymmetry likewise emerges in an expansion of the theory about the background of the domain-wall "vacuum" (7–9). This degree of supersymmetry is reduced with respect to that possessed by the $D=5$ action (6) for a reason typical of brane solutions. Such BPS solutions typically display a reduced degree of supersymmetry with respect to the vacuum solution of the corresponding theory. In the present case, starting from a $D=5$ SU(2)-Majorana spinor parameter ϵ^i with 8 *a priori* independent components, the domain-wall background has a surviving supersymmetry requiring $\Gamma_5\epsilon^i=(\tau_3)^i{}_j\epsilon^j$, cutting the number of independent supersymmetries down to 4. The difference here with respect to other BPS brane solutions is that there is no other "vacuum" solution to the theory (6) to compare this with, and for want of another candidate, one might be inclined to consider the double-domain-wall solution itself as the vacuum. Thus, this solution for the generalized Calabi-Yau reduction of M–theory presents both a spontaneous appearance of $D=4$ chirality and also a spontaneous breakdown of supersymmetry to the phenomenologically interesting chiral $D=4$, $N=1$ supersymmetry.

6 Conclusion

We have seen that $D=11$ supergravity naturally possesses a solution compactified down to 4+1 dimensions that reproduces the Hořava-Witten $D=4$ ground state (and in fact which extends this solution to all orders in $\kappa^{\frac{2}{3}}$). This solution replaces the orbifold fixed planes of the Hořava-Witten construction with a pair of standard BPS domain walls. What has not yet been shown from this Kaluza-Klein perspective is just how all the fluctuation modes of the domain walls should appear, including in particular the supersymmetric Yang-

Mills multiplets. The outlines of how this should work can be stated, however. As on the $D = 11$ Hořava-Witten orbifold,[1,2] the resulting chiral zero-mode theory will be vulnerable to quantum-level anomalies, and such anomalies will be required to cancel between the $D = 5$ bulk action variations and variations of the $D = 3 + 1$ domain-wall zero-modes, through the mechanism of anomaly inflow.[13]

This anomaly-cancellation requirement may well allow a rather wide class of possibilities for the 3-brane zero modes. But at least one example of such a anomaly-canceling zero-mode set can be given: just reduce the known $D = 10$ Hořava-Witten orbifold fixed plane theories [1,2] down to $D = 5$ on the Calabi-Yau manifold. If one assumes the standard embedding of the spin connection into the gauge group, then one ends up [6] with an E_8 super Yang-Mills multiplet on one of the $D = 3+1$ domain walls and with an E_6 super Yang-Mills multiplet coupled to chiral supermatter in the **27** representation on the other, together with the $D = 5$ bulk multiplets discussed above. The general picture that emerges from these M–theory compactifications is likely to be much richer, however, with many possibilities for non-standard embeddings [11] and resulting $D = 4$ gauge groups. Could this be the way in which M–theory finally makes contact with ordinary $D = 4$ physics?

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