Non-linearity of gravelly soils under seismic compressional deformation based on KiK-net downhole array observations

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Abstract. In this paper the nonlinear behaviour of gravelly soils under seismic compressional deformation is investigated based on KiK-net downhole array earthquake observations in Japan. By comparing the amplification spectra between the vertical response at the ground surface and at the bottom of downholes subjected to strong motions with those subjected to weak motions, empirical curves for constrained modulus degradation are obtained considering different levels of vertical confining pressure. Results show that the nonlinearity associated with the compressional deformation can be as significant as that of the shear deformation, for gravelly soils above water tables. The proposed curves provide satisfactory predictions for the compressional soil nonlinearity investigated in previous studies. Furthermore, the proposed curves are formulated by a modified cyclic nonlinear model, which can account for constrained modulus degradation under a variety of confining pressure conditions and therefore extends the application of the proposed reference curves to nonlinear numerical analysis of geotechnical structures under multi-directional seismic loads.

Key words: nonlinearity, gravelly soils, compressional deformation, constrained modulus degradation, KiK-net

1 Introduction

Nonlinearity is one of the key characteristics of soils. Soil nonlinear behaviour is usually interpreted by some form of stiffness degradation at different deformation levels (Kramer, 1996). Both laboratory and in-situ tests have been conducted to investigate such nonlinear behaviour. Based on such experimental evidence, various empirical curves have been proposed to predict the variation of soil stiffness in a wide deformation range, e.g. Ishibashi and Zhang (1993), Vucetic (1994) and Darendeli (2001). However, most of prior research concentrates only on the soil nonlinearity related to shear deformation (i.e. shear modulus degradation). There is currently very limited investigation of the nonlinear soil behaviour associated with compressional deformation. Due to the repeatedly observed strong vertical ground motions and compressional damage of engineering structures (Papazoglou and Elnashai, 1996; Bradley, 2011), there is an increasing need to carry out comprehensive multi-directional site response analyses when performing the seismic design of critical structures (e.g. nuclear power plants, high dams). However, currently there are very few investigations of the seismic ground response in the vertical direction, where in particular the understanding of soil nonlinearity under compressional deformation, in terms of constrained modulus degradation, is still limited. Beresnev et al. (2002) is one reference in which the compressional soil nonlinearity was investigated by back-analysing the recorded motions at five KiK-net sites. In particular, constrained modulus degradation ratios were calculated by comparing the back-analysed fundamental frequencies of soil deposits subjected to strong motions, to the reference one subjected to a weak motion. As a result, constrained modulus showed a non-negligible degradation manner when large strains are mobilised. However, compressional soil nonlinearity was only observed in three cases, among which a scatter point was also observed. This suggested that more data should be involved in the investigation to better constrain the degradation curve. Therefore, in this paper, the nonlinearity of gravelly soils, which predominate in the KiK-net site profiles, under seismic compressional deformation is investigated based on KiK-net seismic observations. By analysing the response amplification spectra at 29 sites under more than 200 earthquakes, empirical curves for constrained modulus degradation are proposed considering different vertical confining pressures. The objective is to extend the investigation of compressional soil nonlinearity and further improve the accuracy of the existing studies (i.e. Beresnev et al., 2002) through comparisons with additional measurements. Furthermore, the effect of vertical confining pressure on constrained modulus degradation is also investigated by analysing the proposed degradation curves. Finally, the proposed curves are formulated as a modified hyperbolic model, which can account for constrained modulus degradation under a wide range of confining pressure conditions.

2 Methodology

The KiK-net system is a well-established downhole array monitoring system in Japan, which includes 659 stations equipped with three-directional seismometers both at the ground surface and base layer in a vertical array of a borehole. The monitored data are archived in the NIED database for academic investigations (Okada et al., 2004).

Downhole array monitoring data have been widely used for different geotechnical studies, e.g. the investigation of site amplification effects, the validation of the numerical modelling for site response analysis, the study of soil nonlinearity, etc. In particular, soil nonlinearity is usually studied by analysing the spectral ratios in the frequency-domain between the dynamic response at the ground surface and at the bottom of boreholes. The degree of nonlinearity is evaluated by comparing the back-analysed fundamental frequencies subjected to strong motions to the reference one calculated based on weak motion amplifications, e.g. in Sato et al. (1996), Huang et al. (2005) and others.

The principle of these investigations is attributed to the analytical solution for site response analysis employing the transfer function (Kramer, 1996). The transfer function determines how much the spectral input motion is amplified by a soil layer at each frequency component. The fundamental frequency of the transfer function (f_s) is related to the shear wave velocity of the material (v_s), i.e. $f_s = v_s/4H$ (H being the depth of the soil layer). Therefore, soil nonlinearity in terms of shear modulus degradation ratios ($G/G_{\max}$) can be calculated by comparing the fundamental frequencies of the soil layer subjected to strong motions with the ones obtained from weak motions, i.e. $G/G_{\max} = \left(v_{s\ strong} / v_{s\ weak} \right)^2 = \left(f_{s\ strong} / f_{s\ weak} \right)^2$. This procedure can be repeated for a variety of earthquake events of different intensities. Therefore, shear modulus degradation ratios can be presented against the corresponding seismically induced shear strains, to interpret the nonlinear characteristic of geotechnical materials.

However, the analytical solution employing the transfer function was only proposed for horizontal excitations (shear waves). Han (2014) showed that a similar solution is applicable for predicting site response subjected to vertical excitations (compressional waves). Therefore, the spectral ratio method can be also employed to investigate the soil nonlinearity related to compressional deformation. In particular, the degree of compressional soil nonlinearity, in terms of constrained modulus degradation ratios

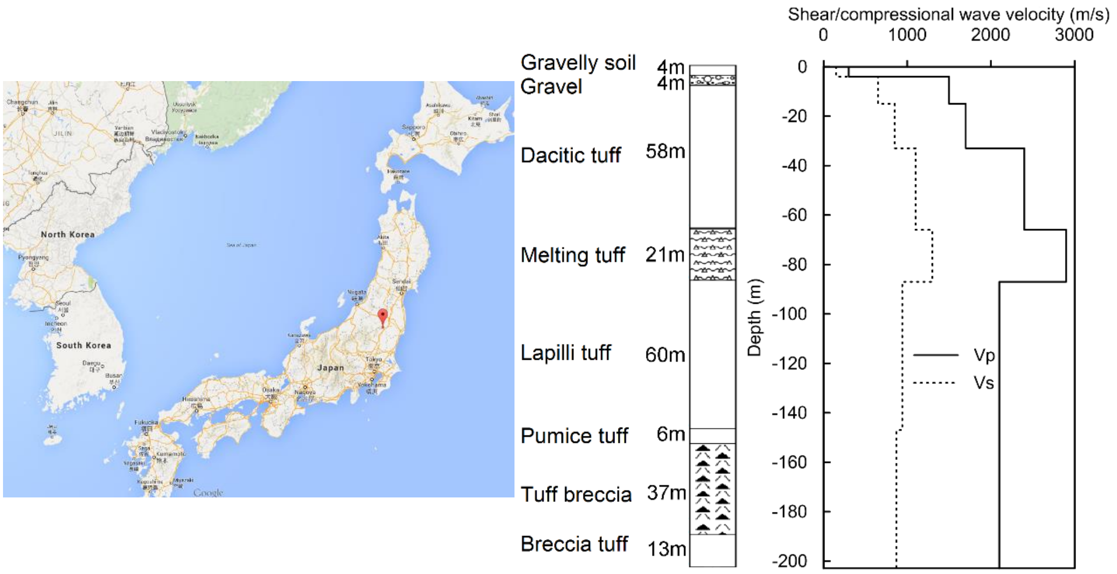
($M/M_{\max}$), can be evaluated by comparing the back-analysed fundamental frequencies of the soil deposit subjected to strong vertical motions, to the reference ones obtained from weak vertical motions, i.e. $M/M_{\max} = \left(v_{p\ strong}/v_{p\ weak}\right)^2 = \left(f_{p\ strong}/f_{p\ weak}\right)^2$. The resulting degradation ratios can be presented against corresponding seismically induced compressional strains, to interpret the compressional soil nonlinearity.

3 Compressional soil nonlinearity

3.1 Investigation procedures

29 sites in the KiK-net system are selected to investigate compressional soil nonlinearity, which all experienced at least three strong earthquake events (PGA>1m/s^2). Among these 29 sites, the FKSH10 site (see Figure 1) is chosen herein to illustrate the investigation procedures. The superficial materials in the top 10m are mainly gravelly soils, underlain by rocks. It should be noted that the similar stratigraphy conditions are observed for all studied KiK-net sites, i.e. mainly involving shallow gravelly soils and deeper weathered rocks. The water table at the FHSK10 site is assumed at 4m below ground level (b.g.l.) due to the more significant increase of compressional wave velocity at this depth compared to shear wave velocity (Beresnev et al., 2002). 9 strong motions (PGA>1m/s^2 including the 2011 Tohoku earthquake) are selected to investigate the compressional soil nonlinearity at this site (listed in Table 1) and 6 weak motions (PGA<0.1m/s^2) are selected to back-analyse the small strain soil properties. Response amplification spectra of the vertical motions are calculated for all the strong and weak motions, by dividing the recorded response spectra at the ground surface by the spectra at the bottom of the borehole over the frequency range (assuming 5% damping for single degree of freedom system).

The response amplification spectra of the selected weak and strong motions are plotted in Figure 2a and b respectively. The average amplification spectra of all the studied weak motions are calculated and plotted in Figure 2a. Based on this, the representative fundamental frequency of the soil deposit subjected to weak motions ($f_{p\ weak}$) is estimated at 11.5Hz. Furthermore, by comparing the fundamental frequencies of the strong motions ($f_{p\ strong}$ in Figure 2b) with the adopted average $f_{p\ weak}$, the constrained modulus degradation ratios can be obtained, as shown in Figure 3. The corresponding seismically induced compressional strain (ε_v) for each strong motion is calculated based on the approximate equation of Beresnev and Wen (1996), a function of observed peak vertical acceleration (A), predominant frequency of surface motion ($f_{p\ surface}$) and near-surface P-wave velocity (v_p'), see Equation (1). This equation was employed by Beresnev and Wen (1996) to estimate the seismically induced peak strains of the SMART2 sites in Taiwan, achieving satisfactory predictions when compared with the results using McCall's (1994) model. In the current work, the near-surface P-wave velocity (v_p') is obtained by calculating the weighted average P-wave velocity for the materials above the water table. This is due to the fact that the analysed constrained modulus degradation only represents the compressional soil nonlinearity for the materials above the water table, as the soils under the water table are very difficult to degrade due to the large constrained modulus induced by saturation with water. Furthermore, the influence of vertical confining pressure on constrained modulus degradation is also investigated. In particular, the representative effective vertical confining pressure (σ_v') is calculated by multiplying the average depth of the soil deposit above the water table with the bulk unit weight of the material, i.e. σ_v'=0.5×4m×20kN/m^3=40kPa for the FKSH10 site.



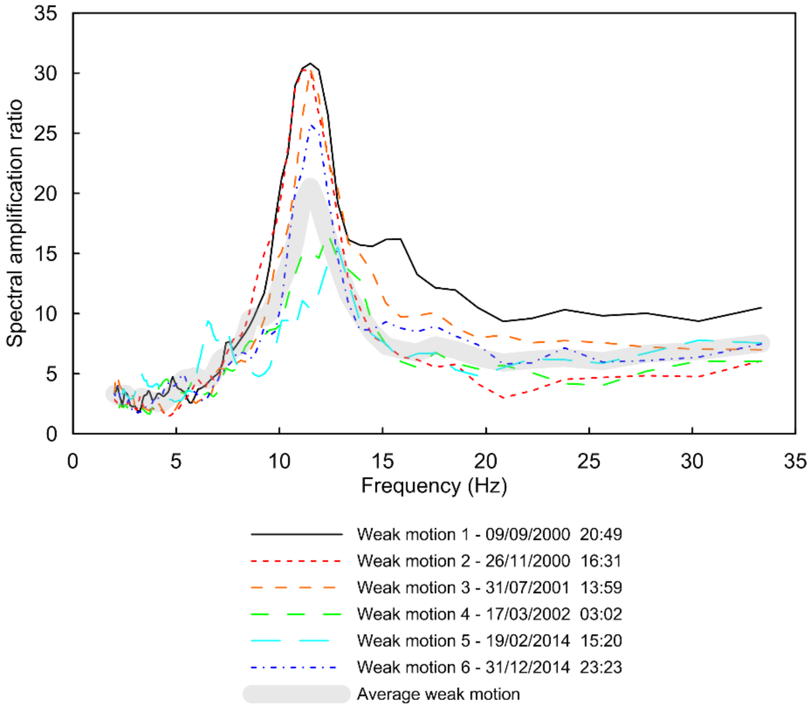
(a): Location

(b): Stratigraphy conditions

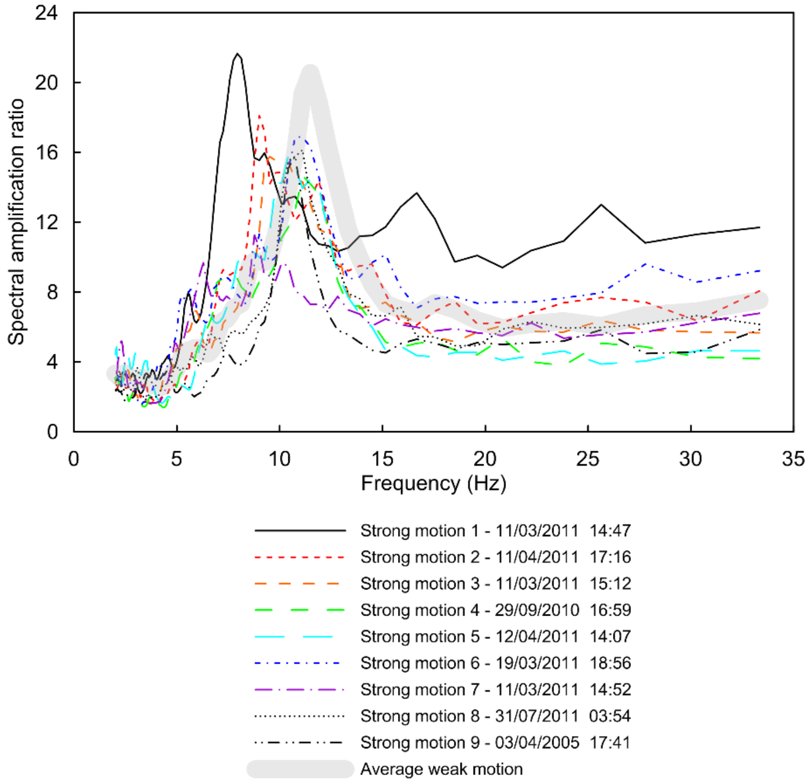
Figure 1:FKSH10 site

Table 1: Selected strong motions for FKSH10 site

Earthquakes Parameters	11/03/2011 14:47	11/04/2011 17:16	11/03/2011 15:12	29/09/2010 16:59	12/04/2011 14:07	19/03/2011 18:56	11/03/2011 14:52	31/07/2011 03:54	03/04/2005 17:41
PGA (m/s^2)	13.4	4.6	3.1	1.5	1.4	1.5	2.2	2.0	2.0
PGA in vertical direction (m/s^2)	10.2	3.2	1.2	1.1	1.0	1.0	0.9	0.9	0.6
Predominant frequency for surface motion (Hz)	7.9	9.0	9.8	11.1	11.5	10.8	10.1	11.1	10.8
Compressional strain (%)	6.8×10^{-2}	1.9×10^{-2}	6.5×10^{-3}	5.2×10^{-3}	4.6×10^{-3}	4.7×10^{-3}	4.9×10^{-3}	4.1×10^{-3}	2.8×10^{-3}



(a): Weak motions



(b): Strong motions

Figure 2: Response amplification spectra subjected to weak and strong motions at FKSH10 site

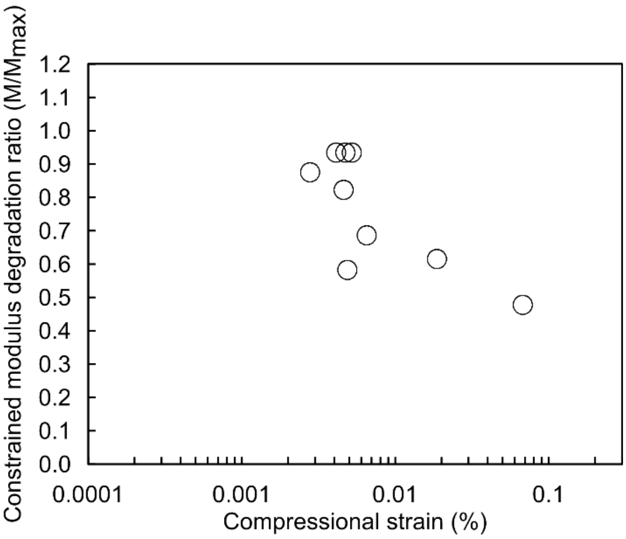


Figure 3: Constrained modulus degradation ratios at FKSH10 site

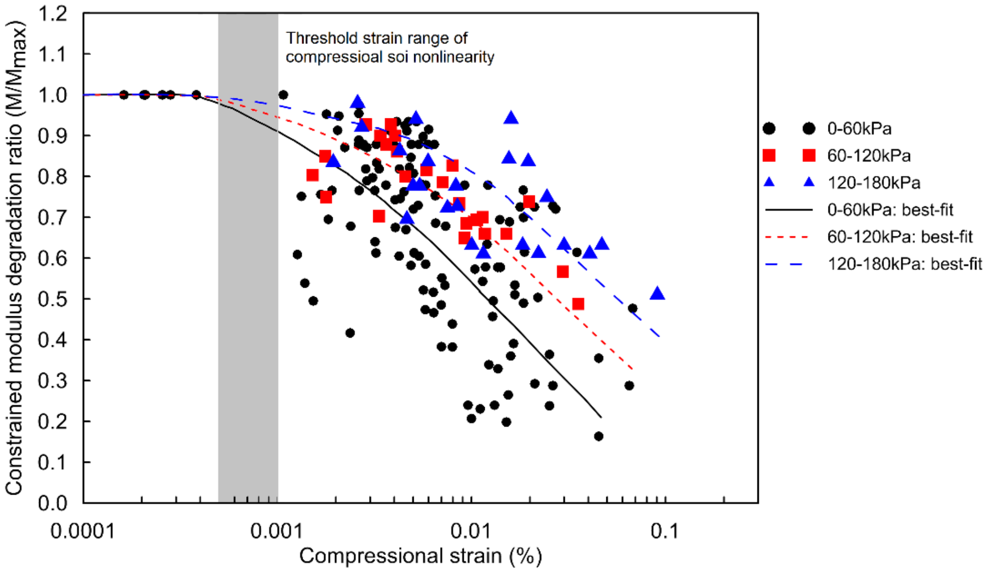
$$|\varepsilon_v| = \frac{A}{2\pi \cdot f_{p\ surface} \cdot v_p'} \tag{1}$$

3.2 Constrained modulus degradation curves

By employing the method presented in Section 3.1, the constrained modulus degradation ratios for gravelly soils at the 29 sites (listed in Appendix) subjected to 177 strong motions are calculated and shown in Figure 4. The obtained degradation ratios are categorised into three groups, based on the different confining pressure ranges, i.e. 0-60kPa, 60-120kPa and 120-180kPa. For each group, a best-fit line is generated to represent the compressional nonlinear soil behaviour, as shown in Figure 4a.

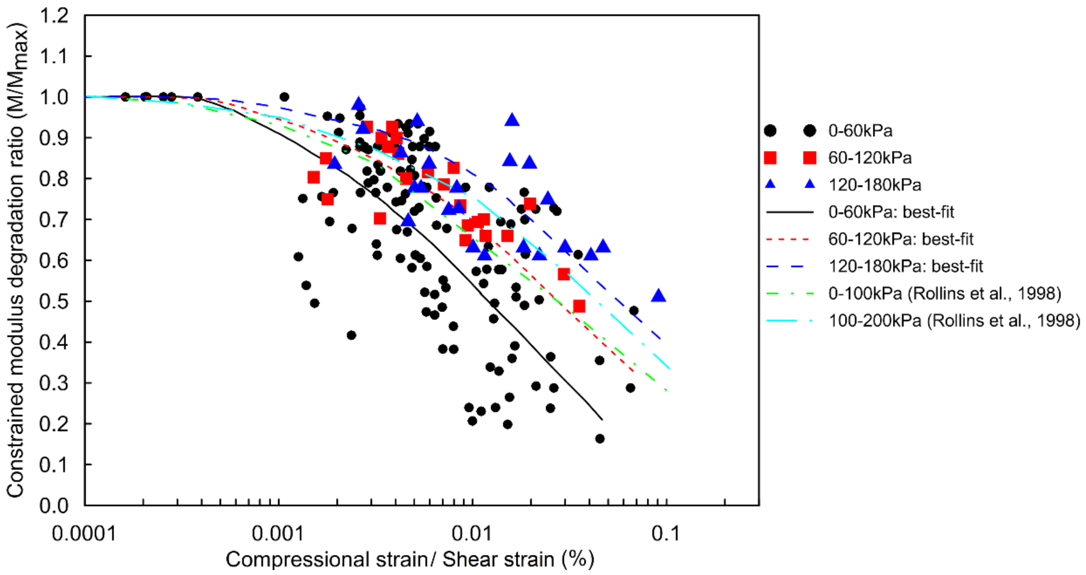
Based on the best-fit degradation curves, the threshold strain range for the onset of nonlinear degradation is approximately 0.5×10^{-3}% to 1.0×10^{-3}% vertical strain, depending on the level of confining pressure. A higher vertical confining pressure is shown to result in a delayed constrained modulus degradation in terms of the threshold vertical strain, while the rate of degradation appears to be independent of the stress level, for the gravelly soils considered in this study. Furthermore, the obtained curves are compared with the shear modulus degradation curves for gravelly soils proposed by Rollins et al. (1998) in Figure 4b. In particular, for confining pressures under 100kPa (i.e. 0-60kPa in present study and 0-100kPa in Rollins et al. (1998)), the constrained modulus has a wider linear plateau but steeper degradation compared to the shear modulus. This observation is in agreement with Han (2014) when back-analysing downhole array seismic data. However, for confining pressure above 100kPa (i.e. 120-180kPa in present study and 100-200kPa in Rollins et al. (1998)), the constrained modulus has a much wider linear plateau and smaller rate of degradation. Furthermore, it shows that the effect of confining pressure is more significant in the constrained modulus degradation than in the shear modulus degradation for the cases considered.

Comparison against the constrained modulus degradation investigated by Beresnev et al. (2002) is shown in Figure 4c. In Beresnev et al. (2002), the reduced constrained moduli of the materials at five sites in the KiK-net system, NARH01, OKYH09, SMNH01, SMNH02 and TTRH02 (not included in the present study), were back-analysed from the monitored data in six strong earthquake events. It shows that the proposed degradation curve under the confining pressure of 120-180kPa is in agreement with the modulus reduction ratios at these site (σ_v'=150kPa). Therefore, the proposed degradation curves provide a satisfactory prediction for the compressional soil nonlinearity studied in Beresnev et al. (2002).

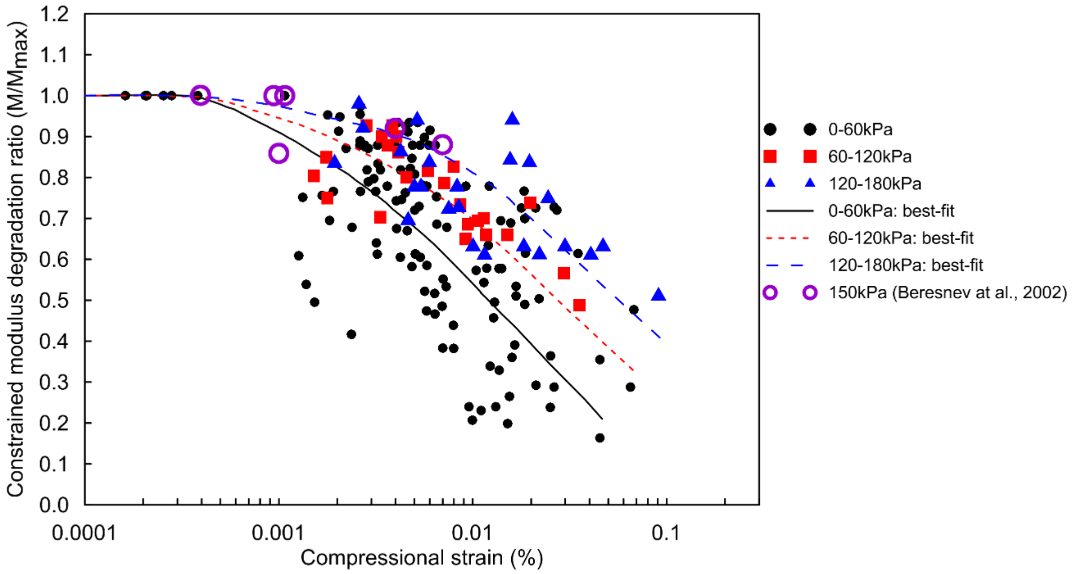


(a): Constrained modulus degradation ratios

Figure 4: Constrained modulus degradation curves of gravelly soils based on KiK-net data



(b): Comparison with Rollins et al. (1998)



(c): Comparison with Beresnev et al. (2002)

Figure 4 (continued): Constrained modulus degradation curves of gravelly soils based on KiK-net data

4 Constrained modulus degradation model

The obtained constrained modulus degradation curves can account for the effect of vertical confining pressure on compressional soil nonlinearity, but only for three discrete stress ranges. In this part, the proposed curves are formulated by a modified cyclic nonlinear model, in order to predict constrained modulus degradation under a variety of confining pressure conditions.

A variant version of the hyperbolic formulation (Kondner and Zelasko, 1963), which is also known as the hyperbolic cyclic nonlinear model in Imperial College Finite Element Program (Potts and Zdravkovic, 1999), is employed for this purpose. In particular, the maximum shear modulus G_{max}, shear stress τ and shear strain γ in the original backbone curve of the hyperbolic model are replaced with maximum constrained modulus M_{max}, compressional stress σ_v and compressional strain ε_v to predict compressional soil behaviour. After rearranging the modified backbone curve, compressional soil nonlinearity can be expressed in Equation (2), in terms of constrained modulus degradation ratios at different compressional strain levels. Therefore, Equation (2) can be employed to formulate the proposed constrained modulus degradation curves. In particular, the model parameter *b* is parametrically calibrated to reproduce the degradation curves for the three confining pressure ranges, as shown in Figure 5. Based on the calibrated values of *b* (see Table 2), a linear relation between the model parameter *b* and the average vertical confining pressure $\bar{\sigma}_v'$ of each group is obtained, as shown in Equation (3). Consequently, by substituting Equation (3) into (2), a modified hyperbolic model, as shown in Equation (4), can be employed to predict constrained modulus degradation under a variety of confining pressure conditions. As the confining pressure employed to derive the $\bar{\sigma}_v'$-b relation only ranges from 30-150kPa, the accuracy of the application of the modified hyperbolic model beyond this range may be biased.

$$\frac{M}{M_{\max}} = \frac{1}{1 + b \cdot \varepsilon_v} \tag{2}$$

where b is the model parameter.

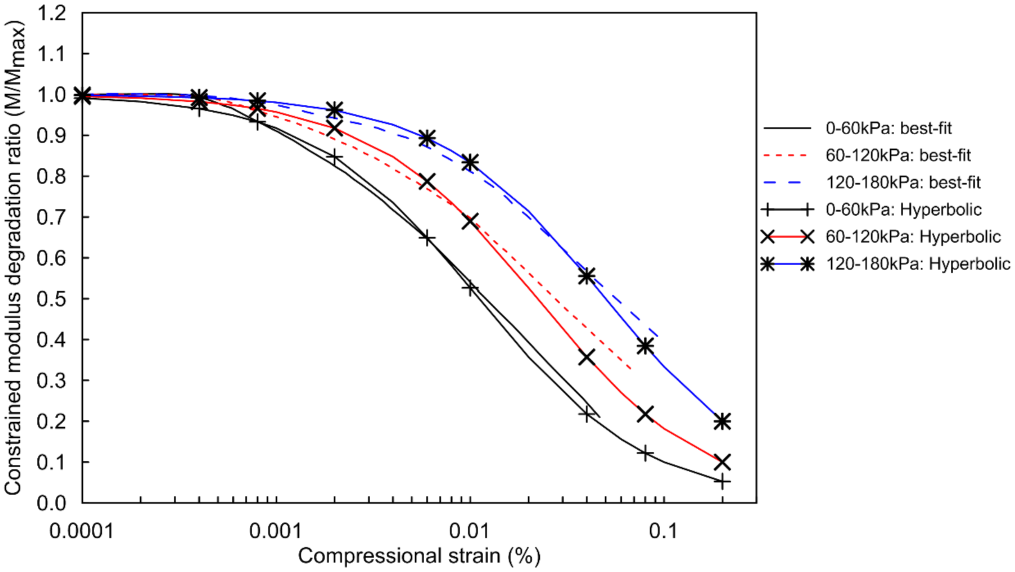


Figure 5: Formulated constrained modulus degradation curves

Table 2: Calibrated model parameters

Parameters \ Groups	30-60kPa	60-120kPa	120-180kPa
Average vertical confining pressure (kPa)	32.4	106.0	150.0
Model parameter b	9000	4500	2000

$$b = -60\sigma_v' + 10907 \tag{3}$$

$$\frac{M}{M_{\max}} = \frac{1}{1 + \left(-60\sigma_v' + 10907\right)\cdot \varepsilon_v} \tag{4}$$

5 Conclusions

In this paper, the nonlinear behaviour of gravelly soils under seismic compressional deformation is investigated based on KiK-net seismic monitoring data. By comparing the amplification spectra between the vertical response at the ground surface and bottom of downholes subjected to strong motions with those subjected to weak motions, empirical curves for constrained modulus degradation are obtained considering different levels of vertical confining pressure.

Results show that the obtained soil nonlinearity associated with compressional deformation can be as significant as that of shear deformation for gravelly soils above water tables. Comparison with the existing shear modulus degradation curves indicates that the effect of confining pressure is more significant on constrained modulus degradation than on shear modulus degradation. Furthermore, the proposed curves provide satisfactory predictions for the compressional soil nonlinearity investigated in previous studies.

Finally, the proposed curves are formulated as a modified hyperbolic model, which can account for constrained modulus degradation under a variety of confining pressure conditions and therefore extends the application of the proposed reference curves to the nonlinear numerical analysis of geotechnical structures under multi-directional seismic loads. It should be noted that the proposed model only represents average compressional soil nonlinearity under a specific confining pressure condition, i.e. actual degradation ratios may deviate from the degradation curve. Therefore, for the investigation of a specific geotechnical material, laboratory tests should be carried out to more accurately understand its compressional nonlinear behaviour.

6 Appendix

Table 3: List of investigated sites

0-60kPa	Site code	FKSH21	IBRH14	IWTH14	IWTH18	KSRH06	KSRH10	MYGH03
	Water table	2.0	2.0	2.0	2.0	2.0	3.0	2.6
	σ_v' (kPa)	20.0	20.0	20.0	20.0	20.0	30.0	26.0
	v_p' (m/s)	490.0	270.0	550.0	450.0	180.0	528.0	700.0
		NIGH09	IWTH21	MYGH10	FKSH10	FKSH12	IWTH02	IWTH26
		2.0	2.5	1.0	4.0	4.0	5.0	4.0
		20.0	25.0	10.0	40.0	40.0	50.0	40.0
		350.0	330.0	500.0	300.0	530.0	300.0	450.0
		IWTH27	IWTH28	KSRH05	MTGH04	MYGH05	NGNH18	TKCH08
		4.0	4.0	4.0	4.0	4.0	6.0	4.0
		40.0	40.0	40.0	40.0	40.0	60.0	40.0
		360.0	500.0	350.0	450.0	420.0	627.0	300.0
60-120kPa	Site code	IWTH22	NGNH29	FKSH18	IWTH05	IBRH16		
	Water table	10.0	10.0	12.0	9.0	12.0		
	σ_v' (kPa)	100.0	100.0	120.0	90.0	120.0		
	v_p' (m/s)	816.0	771.0	400.0	618.0	1042.0		
120-180kPa	Site code	IBRH13	IWTH04	IBRH18				
	Water table	16.0	15.0	14.0				
	σ_v' (kPa)	160.0	150.0	140.0				
	v_p' (m/s)	447.0	680.0	828.0				

Note: the location of water table is presented using b.g.l. in meter, σ_v is vertical confining pressure and v_p' is near-surface P-wave velocity.

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Notation:

A	Peak vertical acceleration
b	Model parameter
f_p	Fundamental frequency (P wave)
$f_{p\ strong}$	Fundamental frequency of strong motions (P wave)
$f_{p\ surface}$	Predominant frequency of surface motions
$f_{p\ weak}$	Fundamental frequency of weak motions (P wave)
f_s	Fundamental frequency (S wave)
$f_{s\ strong}$	Fundamental frequency of strong motions (S wave)
$f_{s\ weak}$	Fundamental frequency of weak motions (S wave)
$G/G_{\max}$	Shear modulus degradation ratio
G_{max}	Maximum shear modulus
H	Depth of the soil layer
$M/M_{\max}$	Constrained modulus degradation ratio
M_{max}	Maximum constrained modulus
v_p	P wave velocity
v_p'	Near-surface P-wave velocity
v_s	S wave velocity
σ_v and ε_v	Compressional stress and strain
σ_v'	Effective vertical confining pressure
$\bar{\sigma}_v'$	Average effective vertical confining pressure
τ and γ	Shear stress and shear