

Factoring Flexible Demand Non-Convexities in Electricity Markets

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Abstract—Uniform marginal pricing cannot generally support competitive equilibrium solutions in markets with non-convexities, yielding surplus sub-optimality effects. Previous work has identified non-convexities associated with the generation side of electricity markets and proposed different approaches to address surplus sub-optimality. This paper extends this concept to incorporate the demand side. Non-convexities of flexible demand (FD) are identified, including options to forgo demand activities as well as discrete and minimum power levels, and resulting surplus sub-optimality effects are demonstrated through simple examples and a larger case study. Generalized uplift and convex hull pricing approaches addressing these effects are extended to account for FD non-convexities. Concerning the former, generalized uplift functions for FD participants are proposed, and a new rule is introduced for equitable distribution of the total surplus loss compensation among market participants. Regarding the latter, it is demonstrated that convex hull prices are flattened at periods when FD is scheduled to eliminate surplus sub-optimality associated with the FD ability to redistribute energy requirements across time.

Index Terms—Convex hull pricing, electricity markets, flexible demand, generalized uplifts, surplus sub-optimality.

NOMENCLATURE

A. Indices and Sets

$t \in T$	Index and set of time periods.
$i \in I$	Index and set of generators.
$j \in J$	Index and set of FDs.
$J^c \subseteq J$	Subset of continuously-controllable FDs.
$J^f \subseteq J$	Subset of fixed-cycle FDs.
τ	Index of steps of the cycle of FDs $j \in J^f$, running from 1 to T_j^{dur} .
r	Index of iterations.
\mathcal{G}_i	Operating constraints set of generator i .
\mathcal{D}_j	Operating constraints set of FD j .

B. Variables

λ	Vector of electricity prices λ_t (£/MWh).
g_i	Vector of power outputs g_{it} of generator i (MW).

u_i	Vector of commitment statuses u_{it} of generator i .
d_j	Vector of power demands d_{jt} of FD j (MW).
v_j	Binary variable expressing whether the activity of FD j is forgone ($v_j = 0$ if it is forgone, $v_j = 1$ otherwise).
w_j	Vector of binary variables w_{jt} expressing whether FD $j \in J^c$ is active at t ($w_{jt} = 1$ if it is active, $w_{jt} = 0$ otherwise).
z_j	Vector of binary variables z_{jt} expressing whether the activity of FD $j \in J^f$ is initiated at t ($z_{jt} = 1$ if it is initiated, $z_{jt} = 0$ otherwise).
$\Delta\alpha_i^g$	Vector of uplift parameters $\Delta\alpha_{it}^g$ associated with the power output of generator i at t (£/MWh).
Δc_i^{on}	Vector of uplift parameters Δc_{it}^{on} associated with the “on” commitment status of generator i at t (£/h).
Δc_i^{off}	Vector of uplift parameters Δc_{it}^{off} associated with the “off” commitment status of generator i at t (£/h).
$\Delta\alpha_j^d$	Vector of uplift parameters $\Delta\alpha_{jt}^d$ associated with the power demand of FD j at t (£/MWh).
$\Delta\gamma_j^d$	Uplift parameter associated with forgoing the activity of FD j (£).

C. Constants

D	Vector of total inflexible demands D_t (MW).
B_j^0	Benefit associated with the demand activity of FD j (£).
d_{jt}^{min}	Minimum power limit of FD $j \in J^c$ at t (MW).
d_{jt}^{max}	Maximum power limit of FD $j \in J^c$ at t (MW).
E_j	Energy requirement of the activity of FD $j \in J^c$ (MWh).
t_j^{start}	Earliest initiation period of the activity of FD $j \in J^f$.
t_j^{end}	Latest termination period of the activity of FD $j \in J^f$.
T_j^{dur}	Duration of cycle of FD $j \in J^f$ (h).
d_j^{cy}	Power demand of FD $j \in J^f$ at step τ of its cycle (MW).

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D. Functions

C_i	Cost function of generator i (\mathcal{L}).
B_j	Benefit function of FD j (\mathcal{L}).
pro_i	Profit function of generator i (\mathcal{L}).
uti_j	Utility function of FD j (\mathcal{L}).
U_i^g	Generalized uplift function of generator i (\mathcal{L}).
U_j^d	Generalized uplift function of FD j (\mathcal{L}).

I. INTRODUCTION

IN markets with non-convexities, uniform marginal pricing cannot generally support *competitive equilibrium* solutions [1]–[10]. In other words, individual market participants' surplus maximizing self-schedule given the marginal prices determined by the centralized market clearing problem, is not generally consistent with the schedule calculated by the latter. In cases of such inconsistencies, the centralized schedules entail lower surpluses than self-scheduling, with this difference termed as *surplus loss* and the related effect as *surplus sub-optimality*. This effect is undesirable on the basis of allowing all self-interested market participants to determine independently their position given the prices, and not by central intervention.

A wide literature has identified non-convexities associated with the generation side of electricity markets, including binary (on/off) commitment decisions, fixed and start-up/shut-down costs, minimum stable generation constraints, and minimum up/down times [2]–[10]. As demonstrated in this literature, these non-convexities lead to generators' *schedules inconsistency* and *profit sub-optimality* effects.

Two general approaches have been explored to address such effects. The first one retains uniform pricing and attempts to minimize the extent of schedule inconsistency and profit sub-optimality. A primal-dual approach is proposed in [2] in order to determine the electricity prices minimizing the social welfare reduction caused by the schedules inconsistency, while ensuring non-negative profits for the generators. However, this approach does not achieve competitive equilibrium at the optimal solution of the centralized problem, and does not guarantee zero profit loss for the generators as it does not ensure recovery of opportunity costs. In [3]–[6], generators experiencing profit sub-optimality receive *lump-sum uplift payments* that compensate exactly their respective profit loss, and the uniform marginal prices are optimized so as to minimize the total profit loss and thus the total uplift payments. These minimum-uplift prices correspond to the *convex hull prices* and coincide with the Lagrangian multipliers optimizing the dual of the market clearing problem [4]–[6].

The second approach addresses schedules inconsistency and profit sub-optimality by employing additional, differentiated (generator-specific) prices. In [7], after solving the initial mixed-integer centralized problem, authors solve a continuous version of the latter, with the binary commitment variables set equal to their optimal values. The dual variables of these equality constraints yield differentiated prices for the generators' commitment (*commitment tickets*), which along with the uniform energy prices support an equilibrium solution. However, approaches in [2]–[7] entail that the total compensation of profit loss is entirely charged to the demand side of the

market, which is thus treated inequitably. To this end, authors in [8]–[10] propose the use of *generalized uplift functions*, which include generator-specific linear and nonlinear terms and constitute additional revenues or payments for the generators. The parameters of these functions along with the electricity prices are adjusted to achieve consistency for every generator and an equitable distribution of the generators' profit loss compensation among the market participants.

Although the generalized uplift approach yields more equitable distribution of the total surplus loss compensation, it introduces price discrimination among the market participants that cannot be easily justified and may be considered nontransparent [2]–[4]. For this reason, the calculation of these differentiated prices in [9] is carried out through an optimization problem minimizing the extent of discrimination introduced. Since the approach developed in [9] could not efficiently deal with multi-period problems accounting for generators' time-coupling characteristics, an iterative cutting-plane algorithm for the calculation of uplift parameters and electricity prices is proposed in [10].

Recent developments have paved the way for the introduction of *flexible demand* (FD) in power systems, with significant economic, technical, and environmental benefits [11]. In the competitive environment, the realization of this potential is coupled with the integration of FD in electricity markets [12]. Authors in [13]–[19] have proposed different market clearing mechanisms considering FD participation and demonstrated the impact of FD on the market. However, previous work has not explored non-convexities and surplus sub-optimality effects associated with FD.

This paper identifies for the first time non-convexities associated with the operation of FD and demonstrates their relation with inconsistency and surplus sub-optimality effects through simple examples and a large case study with day-ahead horizon and hourly resolution. Both generalized uplift and convex hull pricing approaches are extended to account for FD participation in electricity markets. Concerning the former approach, generalized uplift functions for FD participants are proposed, and a new rule is introduced for equitable distribution of the total generators' profit loss and FDs' utility loss compensation among the market participants. Regarding the latter approach, it is demonstrated that convex hull prices are flattened at the periods that FD is scheduled by the market clearing and self-scheduling solutions to eliminate surplus sub-optimality associated with the FD ability to redistribute energy requirements across time.

The rest of this paper is organized as follows. Section II derives operational models of FD and formulates the centralized market clearing problem under FD participation. Section III identifies FD non-convexities and demonstrates the resulting schedules inconsistency and surplus sub-optimality effects through simple examples. Section IV and V detail the extension of generalized uplift and convex hull pricing approaches respectively to account for both generation and FD participation. Section VI presents the examined case study and Section VII discusses conclusions and future extensions of this work.

II. CENTRALIZED MARKET CLEARING UNDER FLEXIBLE DEMAND PARTICIPATION

Based on submitted bids and offers, the market operator solves the social welfare maximization problem (1), (2) to determine the clearing schedules $\xi_i^* \equiv [g_i^*, u_i^*]; \forall i \in I$,

$\psi_j^* \equiv [d_j^*, v_j^*, w_j^*]; \forall j \in J^c$ and $\psi_j^* \equiv [d_j^*, v_j^*, z_j^*]; \forall j \in J^f$. A continuous version of the same problem, with the binary variables set equal to their optimal values, is solved next to determine the electricity prices λ^* [values of dual variables associated with constraints (2) at the optimal solution]:

$$\max_{\substack{\xi_i \in \mathcal{G}_i; \forall i \in I \\ \psi_j \in \mathcal{D}_j; \forall j \in J}} \sum_{j \in J} B_j(\psi_j) - \sum_{i \in I} C_i(\xi_i) \quad (1)$$

$$\text{subject to : } \sum_{i \in I} g_{it} = D_t + \sum_{j \in J} d_{jt}; \forall t \in T. \quad (2)$$

In this paper, the generators' cost functions include variable, fixed, start-up and shut-down costs, and their operating constraints' sets include minimum stable and maximum generation limits, ramp rates, minimum-up and minimum-down times. The analytical formulation of these cost functions and operating constraints' sets follows the model presented in [20] and is not presented here for brevity reasons.

FD participants may generally correspond to large industrial/commercial consumers, participating individually in the market, or FD aggregators, representing a large number of smaller residential/commercial consumers [21]. Two different types of FDs are considered in this paper, capturing the largest part of flexible load models in the related literature: *continuously-controllable* (CCFD) and *fixed-cycle* (FCFD) [19]. The power demand of an FD of the first type can be continuously adjusted between a minimum and a maximum limit when the FD is active (i.e., its demand is not zero). FDs of the second type involve operating cycles which comprise a sequence of phases occurring at a fixed order, with fixed duration and fixed power consumption, which cannot be altered. Without loss of generality, it is assumed that a demand activity of a FCFD corresponds to one such fixed cycle, and that this cycle cannot be interrupted once it is initiated.

Furthermore, two different demand flexibility potentials are considered. The first is associated with the ability to completely forgo demand activities [22] (i.e., do not carry out the operation of an electrical load planned by the consumers) and the second is associated with the ability to redistribute the total electrical energy requirements of activities across time [11]. Without loss of generality, it is assumed that each FD participant carries out at most one activity over the market horizon. In order to account for the potential to forgo this activity, the benefit function associated with FD j is expressed by (3); the benefit of the consumers is zero if the activity is forgone, or B_j^0 otherwise:

$$B_j(\psi_j) = B_j^0 * v_j. \quad (3)$$

The operating constraints set D_j of a CCFD j includes the following constraints:

- 1) The total energy consumption is zero if the demand activity is forgone, or equal to the fixed energy requirement of the activity otherwise

$$\sum_{t \in T} d_{jt} * 1h = v_j * E_j. \quad (4)$$

- 2) When the CCFD is active, its power demand is bounded by a minimum and a maximum power limit:

$$w_{jt} * d_{jt}^{min} \leq d_{jt} \leq w_{jt} * d_{jt}^{max}; \forall t \in T. \quad (5)$$

The operating constraints set D_j of a FCFD j includes the following constraints:

- 1) The demand activity can be executed over the time window determined by t_j^{start} and t_j^{end} :

$$z_{jt} = 0; \forall t < t_j^{start} \text{ and } \forall t > t_j^{end} - T_j^{dur} + 1. \quad (6)$$

- 2) The demand activity is executed (and thus initiated) at most once during the above time window:

$$\sum_{t=t_j^{start}}^{t_j^{end}-T_j^{dur}+1} z_{jt} = v_j. \quad (7)$$

- 3) If the activity is not forgone, the demand at each period depends on the initiation time, T_j^{dur} and $d_{j\tau}^{cyc}$; $\forall \tau \in [1, T_j^{dur}]$:

$$d_{jt} = \sum_{\tau=1}^{T_j^{dur}} z_{j(t+1-\tau)} d_{j\tau}^{cyc}; \forall t \in T. \quad (8)$$

For the sake of generality, all FDs are assumed to exhibit both flexibility potentials. However, straightforward modifications can be carried out to model FDs with only one of these potentials. If for example the activity related to an FD cannot be forgone, its respective binary variable v_j will be forced equal to 1. Moreover, an FCFD that cannot redistribute its activity will be modeled with $t_j^{start} = t_j^{end} - T_j^{dur} + 1$.

III. FLEXIBLE DEMAND NON-CONVEXITIES AND IMPACT ON SURPLUS OPTIMALITY

In a competitive market, generation and FD participants act as self-interested surplus-maximizing entities, given the electricity prices and subject to their operating constraints sets. These self-scheduling optimization problems for a generator i and an FD j are expressed by (9) and (10), and the resulting optimal schedules by ξ_i^s and ψ_j^s respectively:

$$\max_{\xi_i \in \mathcal{G}_i} (pro_i \equiv (\lambda')' g_i - C_i(\xi_i)) \quad (9)$$

$$\max_{\psi_j \in \mathcal{D}_j} (uti_j \equiv B_j(\psi_j) - (\lambda')' d_j). \quad (10)$$

According to Section I, ξ_i^s and ψ_j^s given λ^* are not generally consistent with ξ_i^* and ψ_j^* , respectively. In cases of such inconsistencies, ξ_i^s and ψ_j^s generally entail lower surpluses than ξ_i^* and ψ_j^* , respectively, with this difference termed as *surplus loss*. The surplus loss for a generator i , an FD j and the total surplus loss are respectively expressed by

$$\Delta loss_i^g \equiv pro_i(\lambda^*, \xi_i^s) - pro_i(\lambda^*, \xi_i^*) \geq 0 \quad (11)$$

$$\Delta loss_j^d \equiv uti_j(\lambda^*, \psi_j^s) - uti_j(\lambda^*, \psi_j^*) \geq 0 \quad (12)$$

$$TotalLoss = \sum_{i \in I} \Delta loss_i^g + \sum_{j \in J} \Delta loss_j^d \geq 0. \quad (13)$$

A wide literature has identified non-convexities associated with the generation side, yielding inconsistency and surplus sub-optimality effects (Section I). In this section, FD participants are examined from the same perspective. The first FD non-convexity is associated with the potential to forgo demand activities and is mathematically captured by the binary decision variables v_j .

The relation of this non-convexity with inconsistency and surplus sub-optimality effects is demonstrated through a single-period problem, where the market includes 1) an FCFD j with $T_j^{dur} = 1$, $d_{j1}^{cyc} = 10$ MW, $B_j^0 = \text{£}150$ and only able to forgo its activity, and 2) a generator i with a cost function $C_i(g_{it}) = g_{it}^2$ and without any non-convex characteristics. Centralized market clearing involves carrying out the FCFD activity, since B_j^0 is higher than the generation cost incurred to satisfy the demand of this activity ($\text{£}100$), and thus yields $g_{it}^* = d_{jt}^* = 10$ MW and $\lambda_t^* = 20$ £/MWh. However, given this price, the FCFD would choose to forgo its activity, since its payments ($\lambda_t^* * d_{jt}^* = \text{£}200$) are higher than its benefit ($B_j = B_j^0 = \text{£}150$); therefore, under the centralized solution, the FCFD incurs a utility loss of $\text{£}50$.

Let us now neglect the potential to forgo the activity and only consider the ability to redistribute the activity across time. FCFDs still have a non-convex operating constraints' set, since their demand d_{jt} during their scheduling window $[t_j^{start}, t_j^{end}]$ can only take a set of discrete values, including 0 and the fixed power requirement $d_{j\tau}^{cyc}$ of each step of their cycle.

A two-period problem is considered here, where the market includes 1) an FCFD j with $T_j^{dur} = 1$, $d_{j1}^{cyc} = 12$ MW, $t_j^{start} = 1$ and $t_j^{end} = 2$, i.e., the FCFD can carry out its activity at either $t = 1$ or $t = 2$, 2) inflexible demands with $D_1 = 10$ MW and $D_2 = 20$ MW, and 3) the same generator i with the previous example. Centralized market clearing schedules the FCFD activity at $t = 1$ in order to flatten as much as possible the total demand profile and minimize the total generation costs, and thus yields $d_{j1}^* = 12$ MW, $d_{j2}^* = 0$, $\lambda_1^* = 44$ £/MWh and $\lambda_2^* = 40$ £/MWh. However, given these prices, the FCFD would choose to carry out its activity at $t = 2$ since this period exhibits a lower price; therefore, under the centralized solution, the FCFD incurs a utility loss of $\text{£}48$.

CCFDs with redistributing ability also exhibit a non-convex operating constraints' set, since their power demand can take the values $d_{jt} = 0$ and $d_{jt} = d_{jt}^{min}$ but not any value in the range $(0, d_{jt}^{min})$. A two-period problem is also considered here, where the market includes 1) a CCFD j with $E_j = 12$ MWh, $d_{j1}^{min} = d_{j2}^{min} = 5$ MW, $d_{j1}^{max} = d_{j2}^{max} = 15$ MW, 2) the same inflexible demands with the previous example, and 3) the same generator i with the previous examples. Centralized market clearing schedules the CCFD activity entirely at $t = 1$ in order to flatten as much as possible the total demand profile and minimize the total generation costs, and thus yields $d_{j1}^* = 12$ MW, $d_{j2}^* = 0$, $\lambda_1^* = 44$ £/MWh and $\lambda_2^* = 40$ £/MWh. However, given these prices the CCFD would choose to be scheduled entirely at $t = 2$ since this period exhibits a lower price; therefore, under the centralized solution, the CCFD incurs a utility loss of $\text{£}48$.

If the same problem is considered with $d_{j1}^{min} = d_{j2}^{min} = 0$, the optimal centralized solution flattens completely the total demand profile with $d_{j1}^* = 11$ MW, $d_{j2}^* = 1$ MW and $\lambda_1^* = \lambda_2^* = 42$ £/MWh. Given these prices, any feasible solution of the CCFD's self-scheduling problem—including its above centralized schedule—is an optimal one, and therefore the CCFD does not incur utility loss. It can be thus concluded that FDs'

non-convexities associated with their ability to forgo activities, as well as discrete and minimum power levels yield schedules' inconsistency and utility sub-optimality effects.

IV. GENERALIZED UPLIFTS UNDER FLEXIBLE DEMAND PARTICIPATION

The *generalized uplift functions* U employed in [8]–[10] constitute additional revenues ($U > 0$) or payments ($U < 0$) for the generators and include generator-specific linear and non-linear terms. The parameters of these functions are adjusted so that generators' new or *augmented self-scheduling* is consistent with the centralized solution. Along with these parameters, the electricity prices are suitably adjusted to new values λ^N , to ensure that the (inflexible in [8]–[10]) demand side contributes at a desired level to the compensation of generators' profit loss. This adjustment is carried out through an optimization problem, minimizing the extent of discrimination introduced by the differentiated pricing terms (thus denoted as the *minimum discrimination problem* in the rest of this paper). In this section, this approach is extended to account for the FD non-convexities presented in Section III.

In [10], the generalized uplift function U_i^g (14) applying to generator i includes a set of adjustable generator-specific parameters $\Delta\pi_i^g = [\Delta\alpha_i^g, \Delta c_i^{on}, \Delta c_i^{off}]$ associated with the power output, the “on” commitment status and the “off” commitment status of generator i , respectively:

$$U_i^g(\xi_i, \Delta\pi_i^g) = \sum_{t \in T} [\Delta\alpha_{it}^g g_{it} + \Delta c_{it}^{on} u_{it} + \Delta c_{it}^{off} (1 - u_{it})]. \quad (14)$$

In the same vein, we propose a generalized uplift function U_j^d (15) applying to FD j . It includes a set of adjustable FD-specific parameters $\Delta\pi_j^d = [\Delta\alpha_j^d, \Delta\gamma_j^d]$ associated with the power input and forgoing the activity of FD j , respectively:

$$U_j^d(\psi_j, \Delta\pi_j^d) = \Delta\gamma_j^d (1 - v_j) + \sum_{t \in T} \Delta\alpha_{jt}^d d_{jt}. \quad (15)$$

Given the above uplift functions, the augmented self-scheduling optimization problems involve the maximization of the augmented profit of generator i (16) and the augmented utility of FD j (17):

$$pro_i(\lambda^N, \xi_i, \Delta\pi_i^g) \equiv (\lambda^N)' g_i - C_i(\xi_i) + U_i^g(\xi_i, \Delta\pi_i^g) \quad (16)$$

$$uti_j(\lambda^N, \psi_j, \Delta\pi_j^d) \equiv B_j(\psi_j) - (\lambda^N)' d_j + U_j^d(\psi_j, \Delta\pi_j^d). \quad (17)$$

The objective of minimizing the discrimination introduced by the differentiated pricing terms is expressed by the minimization of the square norm of uplift parameters [9]:

$$\min_{\substack{\Delta\pi_i^g; \forall i \in I \\ \Delta\pi_j^d; \forall j \in J}} \sum_{i \in I} \|\Delta\pi_i^g\|^2 + \sum_{j \in J} \|\Delta\pi_j^d\|^2. \quad (18)$$

The minimum discrimination problem includes the following constraints:

- 1) The solution of the centralized market clearing problem is identical to the solution of the augmented self-scheduling problems for all market participants (19), (20):

$$\xi_i^* = \xi_i^a \equiv \arg \max_{\xi_i \in \mathcal{G}_i} \text{pro}_i(\lambda^N, \xi_i, \Delta\pi_i^g); \forall i \in I \quad (19)$$

$$\psi_j^* = \psi_j^a \equiv \arg \max_{\psi_j \in \mathcal{D}_j} \text{uti}_j(\lambda^N, \psi_j, \Delta\pi_j^d); \forall j \in J. \quad (20)$$

- 2) Conservation of monetary flow within the market should be satisfied, meaning that the market operator is revenue neutral. This imposes that the total revenues collected by the consumers are equal to the total payments paid to the generators, including the uplifts (21). The combination of (21) with (2) implies that the sum of uplifts should be zero (22):

$$\begin{aligned} & \sum_{j \in J} [(\lambda^N)' d_j^* - U_j^d(\psi_j^*, \Delta\pi_j^d)] + (\lambda^N)' D \\ &= \sum_{i \in I} [(\lambda^N)' g_i^* + U_i^g(\xi_i^*, \Delta\pi_i^g)] \end{aligned} \quad (21)$$

$$\sum_{i \in I} U_i^g(\xi_i^*, \Delta\pi_i^g) + \sum_{j \in J} U_j^d(\psi_j^*, \Delta\pi_j^d) = 0. \quad (22)$$

- 3) The difference between a participant's surplus under augmented self-scheduling and self-scheduling without uplifts determines the contribution of this participant to the compensation for the total surplus loss [9] and is expressed by (23), (24), and (25) for generator i , FD j , and the inflexible demand,¹ respectively. This contribution should not be negative to ensure that participants do not derive exceptional surplus from the uplifts and the new electricity prices; in other words, under augmented self-scheduling, each participant should derive at most their maximum surplus under self-scheduling without uplifts [9]:

$$\Delta \text{cont}_i^g(\lambda^N, \Delta\pi_i^g) \equiv \text{pro}_i(\lambda^*, \xi_i^s, 0) - \text{pro}_i(\lambda^N, \xi_i^a, \Delta\pi_i^g) \geq 0 \quad (23)$$

$$\Delta \text{cont}_j^d(\lambda^N, \Delta\pi_j^d) \equiv \text{uti}_j(\lambda^*, \psi_j^s, 0) - \text{uti}_j(\lambda^N, \psi_j^a, \Delta\pi_j^d) \geq 0 \quad (24)$$

$$\Delta \text{cont}^{inf}(\lambda^N) \equiv (\lambda^N)' D - (\lambda^*)' D \geq 0. \quad (25)$$

The combination of (2), (11)–(13), (19), (20), and (22)–(25) yields (26), which expresses the fact that the total surplus loss is equal to the total compensation contribution by all participants:

$$\begin{aligned} \text{TotalLoss} &= \Delta \text{cont}^{inf}(\lambda^N) + \sum_{i \in I} \Delta \text{cont}_i^g(\lambda^N, \Delta\pi_i^g) \\ &+ \sum_{j \in J} \Delta \text{cont}_j^d(\lambda^N, \Delta\pi_j^d). \end{aligned} \quad (26)$$

The market arrangements should generally facilitate an equitable distribution of the compensation for the total surplus loss among the participants. In this context, authors in [9] proposed two market rules: 1) the total compensation is divided equally among generators and (inflexible) demand and 2) the ratio between profit under augmented self-scheduling and self-scheduling without uplifts is set equal for all generators. Given that an equitable compensation distribution between generators, FDs

and inflexible demand cannot be determined unambiguously, in this paper we propose an extended version of the latter market rule, where the ratio between surplus under augmented self-scheduling and self-scheduling without uplifts is set equal to a common value R ($0 < R < 1$) for all generators, FDs and inflexible demand² (27)–(29):

$$\text{pro}_i(\lambda^N, \xi_i^a, \Delta\pi_i^g) = \text{pro}_i(\lambda^*, \xi_i^s, 0) * R; \forall i \in I \quad (27)$$

$$\text{uti}_j(\lambda^N, \psi_j^a, \Delta\pi_j^d) = \text{uti}_j(\lambda^*, \psi_j^s, 0) * R; \forall j \in J \quad (28)$$

$$(\lambda^*)' D = (\lambda^N)' D * R. \quad (29)$$

Substituting (27)–(29) into (26) yields

$$\begin{aligned} \overbrace{\text{TotalLoss}}^A &= \overbrace{\left[\sum_{i \in I} \text{pro}_i(\lambda^*, \xi_i^s, 0) + \sum_{j \in J} \text{uti}_j(\lambda^*, \psi_j^s, 0) \right]}^B \\ &\quad * (1 - R) + \underbrace{(\lambda^*)' D}_C * \left(\frac{1}{R} - 1 \right). \end{aligned} \quad (30)$$

Along with the condition $0 < R < 1$, R is calculated as

$$R = \frac{-(A + C - B) + \sqrt{(A + C - B)^2 + 4BC}}{2B}. \quad (31)$$

Equation (29), along with the assumption that the electricity price is uniformly increased across all periods of the market horizon [9], fixes the new electricity prices according to (32):

$$\lambda_t^N = \lambda_t^* + \frac{(\frac{1}{R} - 1) \sum_{t \in T} \lambda_t^* D_t}{\sum_{t \in T} D_t}; \forall t \in T. \quad (32)$$

In order to solve the minimum discrimination problem, the optimal solutions ξ_i^a ; $\forall i \in I$ and ψ_j^a ; $\forall j \in J$ of the augmented self-scheduling problems need to be analytically expressed in terms of the uplift parameters and new electricity prices, so as to enforce the equal schedule conditions (19), (20). As discussed in [10], such analytical derivations are impractical for multi-period market clearing problems accounting for participants' time-coupling characteristics. In order to address this challenge, authors in [10] proposed an iterative cutting-plane algorithm for the solution of the minimum discrimination problem, and proved its convergence and optimality. This algorithm iteratively restricts the feasible set of uplift parameters through the sequential generation of surplus cutting planes, to impose indirectly schedules consistency. This algorithm has been extended in this work to compute the uplift parameters of both generation and FD participants; since this extension has not created particular challenges, the algorithm is not presented here for brevity reasons.

V. CONVEX HULL PRICING UNDER FLEXIBLE DEMAND PARTICIPATION

As discussed in Section I, the convex hull prices λ^{CH} minimizing the total surplus loss coincide with the Lagrangian multipliers optimizing the dual problem (33) of the market clearing

¹Given that the benefit of inflexible demand is not considered in this paper, strictly speaking, its contribution is not expressed by the difference between its surplus under augmented self-scheduling and self-scheduling without uplifts, but by the difference between its respective payments.

²In the same line as the previous footnote, strictly speaking, the ratio R does not relate the surplus under augmented self-scheduling and self-scheduling without uplifts in the case of inflexible demand, but its respective payments.

problem, where φ is the dual function and L is the Lagrangian function (34) of the problem:

$$\max_{\lambda} \varphi(\lambda) = \max_{\lambda} \min_{\substack{\xi_i \in \mathcal{G}_i, \forall i \in I \\ \psi_j \in \mathcal{D}_j, \forall j \in J}} L \quad (33)$$

$$L = \sum_{i \in I} C_i(\xi_i) - \sum_{j \in J} B_j(\psi_j) + (\lambda)' \left(\sum_{j \in J} d_j + D - \sum_{i \in I} g_i \right). \quad (34)$$

Given that the Lagrangian function constitutes an additive combination of the individual participants' surpluses, the dual problem is decomposed to the independent surplus maximization sub-problems (9), $\forall i \in I$ and (10), $\forall j \in J$ coordinated iteratively by a λ update algorithm until φ is maximized. The penalty-bundle method [23] is employed in this paper for this update, due to its favorable convergence performance with respect to alternative, sub-gradient methods.

In [5] and [6], where the case studies are carefully designed such that the duality gap of the market clearing problem is zero, this iterative algorithm terminates when the norm of the demand-supply imbalances $|\sum_{j \in J} d_j^* + D - \sum_{i \in I} g_i^*|$ is lower than a pre-specified tolerance. In order to address the general case where the duality gap is not necessarily zero and the optimal dual solution does not necessarily satisfy the demand-supply balance constraints [24], the algorithm terminates when the absolute difference in the dual function value between two consecutive iterations is lower than a pre-specified tolerance ε :

$$|\varphi^r - \varphi^{r-1}| < \varepsilon. \quad (35)$$

According to [3]–[6], after the convex hull prices λ^{CH} have been determined by the above dual optimization problem, generator i and FD j experiencing surplus sub-optimality receive a lump-sum uplift $U_i^{CH,g}$ and $U_j^{CH,d}$, respectively, exactly compensating their respective surplus loss:

$$U_i^{CH,g} \equiv \text{pro}_i(\lambda^{CH}, \xi_i^s) - \text{pro}_i(\lambda^{CH}, \xi_i^*) \geq 0 \quad (36)$$

$$U_j^{CH,d} \equiv \text{uti}_j(\lambda^{CH}, \psi_j^s) - \text{uti}_j(\lambda^{CH}, \psi_j^*) \geq 0. \quad (37)$$

The total compensation for the surplus loss is entirely charged to the inflexible demand side through a negative lump-sum uplift $U^{CH,inf}$ balancing the positive uplifts received by the rest of the participants:

$$U^{CH,inf} \equiv - \sum_{i \in I} U_i^{CH,g} - \sum_{j \in J} U_j^{CH,d}. \quad (38)$$

VI. CASE STUDY

A. Input Data

The examined case study involves the demonstration of schedules inconsistency and surplus sub-optimality effects associated with FD non-convexities, as well as the application of both generalized uplift and convex hull pricing approaches to a market with both generation and FD participants, day-ahead

TABLE I
GENERATION PARTICIPANTS' CHARACTERISTICS

Generator i	1	2	3	4	5	6	7
a_i (£/h)	18,431	17,005	13,755	9,930	9,900	8,570	7,530
b_i (£/MWh)	5.5	30	35	60	80	95	100
c_i (£/MW ² h)	0.0002	0.0007	0.0010	0.0064	0.0070	0.0082	0.0098
C_i^u (£)	4,000,000	325,000	142,500	72,000	55,000	31,000	11,200
C_i^d (£)	800,000	28,500	18,500	14,400	12,000	10,000	8,400
g_i^{min} (MW)	3,292	2,880	1,512	667	650	288	275
g_i^{max} (MW)	6,584	5,760	3,781	3,335	3,252	2,880	2,748
RU_i (MW/h)	1,317	1,152	1,512	1,334	1,951	1,728	2,198
RD_i (MW/h)	1,317	1,152	1,512	1,334	1,951	1,728	2,198
UT_i (h)	24	20	16	10	8	5	4
DT_i (h)	24	20	16	10	8	5	4
u_{i0}	1	1	1	1	1	0	0
g_{i0} (MW)	5,268	4,608	3,025	2,668	2,602	0	0

TABLE II
CCFD PARTICIPANTS' CHARACTERISTICS

CCFD j	1	2	3	4
B_j^0 (£ mil)	0.188	0.418	0.197	0.144
E_j (MWh)	2,589	2,784	1,315	958
d_j^{min} (MW)	1,262	1,253	394	144
d_j^{max} (MW)	1,942	2,088	986	719
Scheduling period (h)	19 – 8	18 – 7	9 – 17	11 – 18

TABLE III
FCFD PARTICIPANTS' CHARACTERISTICS

FCFD j	5	6	7	8
B_j^0 (£ mil)	0.518	0.418	0.134	0.400
T_j^{dur} (h)	4	3	1	2
d_{jt}^{cyc} (MW)	$\tau = 1$	1,285	581	1,183
	$\tau = 2$	692	628	0
	$\tau = 3$	844	1,575	0
	$\tau = 4$	630	0	0
t_j^{start} (h)	18	19	9	9
t_j^{end} (h)	7	9	17	16

horizon and hourly resolution. This study was implemented in FICO Xpress [25] on a computer with a 6-core, 3.47 GHz Intel(R) Xeon(R) X5690 processor and 192 GB of RAM. The market includes 7 generation participants, with fixed costs a_i , linear b_i and quadratic c_i parameters of their variable cost functions, start-up C_i^u and shut-down C_i^d costs, minimum stable g_i^{min} and maximum g_i^{max} generation limits, ramp-up RU_i and ramp-down RD_i rates, minimum-up UT_i and minimum-down DT_i times, and initial commitment status u_{i0} and output g_{i0} given in Table I.

Furthermore, the market includes 4 CCFD participants and 4 FCFD participants able to forgo and redistribute their demand activities, with parameters given in Table II and III respectively. Half of the FD participants of each type can be scheduled during night/morning hours, representing domestic FD, and the other half during midday hours, representing commercial/industrial FD. The minimum and maximum power limits of each CCFD are assumed identical at every hour of their scheduling period (and are thus denoted by d_j^{min} and d_j^{max} respectively) and zero at the rest of the hours.

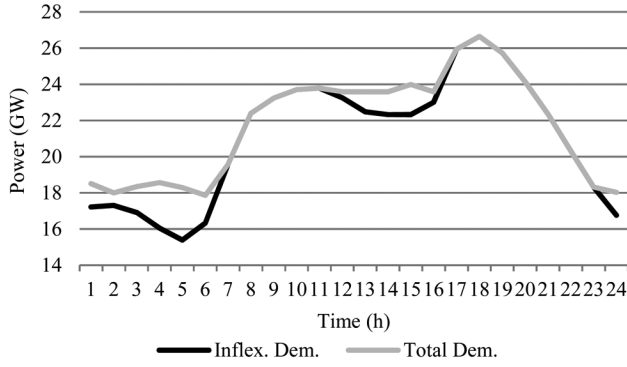


Fig. 1. Inflexible demand and total demand under centralized market clearing.

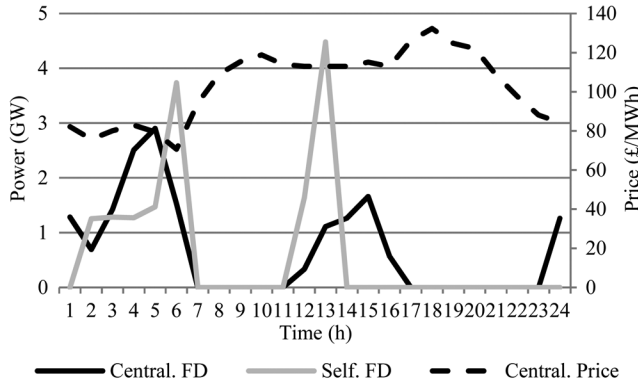


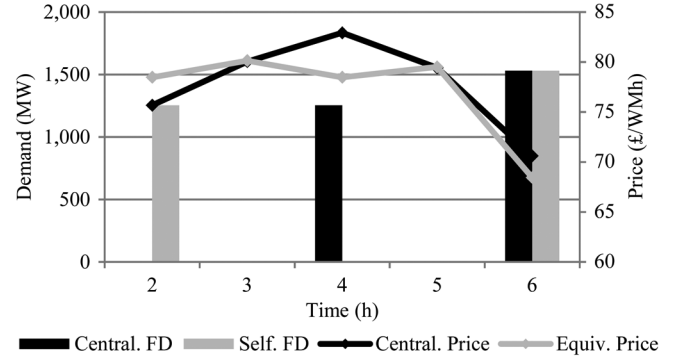
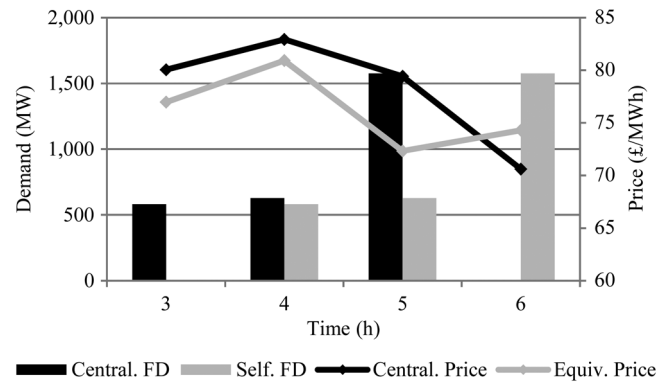
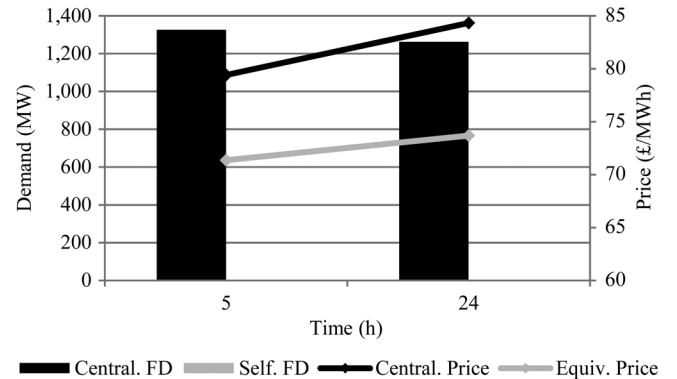
Fig. 2. Comparison of total FD under centralized market clearing and self-scheduling without uplifts.

B. Impact of Flexible Demand Non-Convexities

Under centralized market clearing, domestic and commercial/industrial FDs fill the inflexible demand's night and midday valleys, respectively, flattening significantly the total demand profile, as shown in Fig. 1. However, these valleys and subsequently the centralized prices at these periods are not completely flattened, due to the minimum power levels of CCFDs and the discrete power levels of FCFDs (Section III).

As illustrated in Fig. 2–5, FD participants' self-scheduling given the centralized market clearing prices is not consistent with the centralized market clearing schedule. These inconsistencies are associated with both demand flexibility potentials. Considering the ability to redistribute their activities across time, under self-scheduling, FDs concentrate at the lowest-priced hours within their scheduling period, which is not consistent with the centralized schedule involving an as-flat-as-possible total demand profile (Fig. 2). This effect is demonstrated in Figs. 3 and 4 for CCFD 2 and FCFD 6, respectively. Under self-scheduling, the former chooses to acquire its total energy requirements at the two lowest-priced hours within its scheduling period (2 and 6), while centralized market clearing schedules it at hours 4 and 6. Furthermore, under self-scheduling, FCFD 6 carries out its cycle at hours 4–6, since this leads to lower total payments than carrying it out at hours 3–5 according to centralized market clearing.

Considering the ability to forgo their demand activities, CCFD 1 chooses to do so under self-scheduling, since its benefit $B_1^0 = £0.188$ mil is lower than the lowest payment it could

Fig. 3. Comparison of demand of CCFD 2 under centralized market clearing and self-scheduling without uplifts, and equivalent prices $\lambda_{2t}^{eq} = \lambda_t^N - \Delta\alpha_{2t}^d$ faced by CCFD 2 given the calculated generalized uplift parameters.Fig. 4. Comparison of demand of FCFD 6 under centralized market clearing and self-scheduling without uplifts, and equivalent prices $\lambda_{6t}^{eq} = \lambda_t^N - \Delta\alpha_{6t}^d$ faced by FCFD 6 given the calculated generalized uplift parameters.Fig. 5. Comparison of demand of CCFD 1 under centralized market clearing and self-scheduling without uplifts, and equivalent prices $\lambda_{1t}^{eq} = \lambda_t^N - \Delta\alpha_{1t}^d$ faced by CCFD 1 given the calculated generalized uplift parameters.

achieve by optimally scheduling its activity within its feasible scheduling period (£0.189 mil). On the other hand, under centralized market clearing, its activity is not forgone since B_1^0 is higher than the extra generation cost incurred to satisfy the demand of this activity (£0.186 mil). This inconsistency is illustrated in Fig. 5. The opposite case holds for FCFD 7; under self-scheduling, FCFD 7 chooses to carry out its activity, since its benefit $B_7^0 = £0.134$ mil is higher than the lowest payment it could achieve by optimally scheduling its activity (£0.133 mil). Under centralized market clearing however, its demand

TABLE IV
RESULTS OF GENERALIZED UPLIFT APPROACH

Market Participants		Centralized surplus (£)	Self-scheduling surplus (£)	Augmented self-scheduling surplus (£)	Surplus loss (£)	Contribution to surplus loss (£)	Generalized Uplift (£)
Generator $i \in I$	1	14,788,400	14,788,400	14,775,100	0	13,300	-28,377
	2	9,188,328	9,188,328	9,180,060	0	8,268	-21,450
	3	5,604,222	5,604,222	5,599,180	0	5,042	-13,697
	4	1,662,733	1,662,733	1,661,240	0	1,493	-7,696
	5	502,784	502,784	502,331	0	453	-4,533
	6	79,703	83,089	83,014	3,386	75	1,518
	7	9,686	26,677	26,653	16,991	24	16,168
CCFD $j \in J^c$	1	-24,107	0	0	24,107	0	24,354
	2	205,627	214,712	214,518	9,085	194	9,157
	3	48,707	48,707	48,663	0	44	82
	4	35,490	35,492	35,460	2	32	62
FCFD $j \in J^f$	5	239,898	245,865	245,644	5,967	221	6,075
	6	193,956	208,369	208,182	14,413	187	14,491
	7	0	51	51	51	0	51
	8	95,153	98,783	98,694	3,630	89	3,795
Inflexible demand		53,529,257	53,529,257	53,577,467	0	48,210	0
Total					77,632	77,632	0

activity is forgone since B_7^0 is lower than the extra generation cost required to satisfy its demand (£0.138 mil).

The generation side also exhibits inconsistencies between centralized scheduling and self-scheduling, associated with generation non-convexities explored in [2]–[10]. All the above inconsistencies are translated into surplus losses, as demonstrated in Table IV.

C. Generalized Uplift Approach

In order to address these schedules inconsistency and surplus sub-optimality effects, the generalized uplift approach of Section IV was applied. The parameter R associated with the equitable distribution of the total surplus loss compensation was calculated as $R = 99.91\%$ according to (31). The new electricity prices were then set according to (32) as $\lambda_t^N = \lambda_t^* + 0.095 \text{ £/MWh}$; $\forall t \in T$. The generalized uplift parameters' iterative computation algorithm [10] converged after 63 iterations and 203 seconds of computational time. As shown in Table IV, the total compensation contribution by all participants exactly cancels out the total surplus loss, and the sum of all uplifts is zero.

All FD participants apart from FCFD 7 require some non-zero uplift parameters $\Delta\alpha_{jt}^d$ and do not require an uplift parameter $\Delta\gamma_j^d$ to address their surplus sub-optimality, as their demand activity is not forgone under centralized market clearing. On the other hand, FCFD 7 requires only a non-zero uplift parameter $\Delta\gamma_j^d$ since its demand activity is forgone under centralized market clearing. The elimination of FD surplus sub-optimality by the calculated uplift parameters is demonstrated by observing the new equivalent electricity prices $\lambda_{jt}^{eq} = \lambda_t^N - \Delta\alpha_{jt}^d$ faced by FD participants in Figs. 3–5.

For CCFD 2 (Fig. 3), the uplift parameters make the equivalent prices at hours 2 and 4 equal to incentivize CCFD 2 to follow the market clearing solution and self-schedule at hours 4 and 6, as now self-scheduling at hours 2 and 6 (according to the original self-schedule of Fig. 3) does not bring additional surplus. For FCFD 6 (Fig. 4), the uplift parameters make λ_{6t}^{eq} lower than λ_t^* at hours 3–5 and higher at hour 6 to incentivize FCFD 6

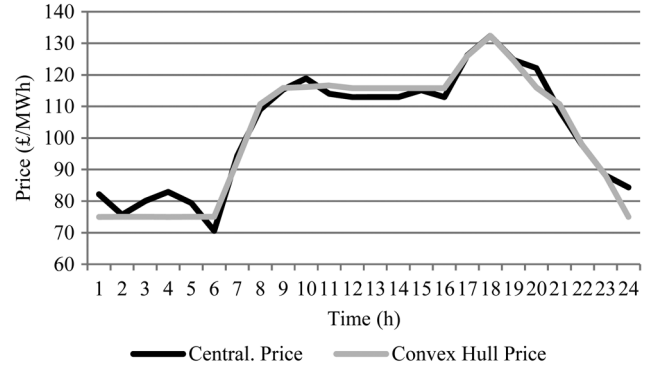


Fig. 6. Centralized price and convex hull price.

to follow the market clearing solution and self-schedule its cycle at hours 3–5, as now self-scheduling at hours 4–6 (according to the original self-schedule of Fig. 4) does not bring additional surplus. For CCFD 1 (Fig. 5), the uplift parameters reduce significantly λ_{1t}^{eq} with respect to λ_t^* at hours 5 and 24 to incentivize CCFD 1 to follow the market clearing solution and carry out its activity at these two hours, as now forgoing its activity (according to the original self-schedule of Fig. 5) does not bring additional surplus. Finally, for FCFD 7, the uplift $\Delta\gamma_7^d = 51\text{£}$ (reward for forgoing its activity) incentivizes FCFD 7 to follow the centralized market clearing solution and forgo its activity, as now carrying out its activity does not bring additional surplus.

D. Convex Hull Pricing Approach

The convex hull pricing approach of Section V was also applied with $\varepsilon = \text{£}1$ in (35). The penalty-bundle algorithm employed to solve the dual optimization problem (33) and thus determine the convex hull prices, converged after 50 iterations and 82 seconds of computational time.

Very interestingly, in contrast to the centralized market clearing prices, convex hull prices are flattened at the night and midday valleys where the FDs are scheduled by centralized market clearing and self-scheduling (Fig. 6). This flattening effect eliminates surplus sub-optimality associated with the FD

TABLE V
RESULTS OF CONVEX HULL PRICING APPROACH

Market Participants		Centralized surplus (£)	Self-scheduling surplus (£)	Augmented self-scheduling surplus (£)	Surplus loss (£)	Contribution to surplus loss (£)	Lump-Sum Uplift (£)
Generator $i \in I$	1	14,642,469	14,642,469	14,642,469	0	0	0
	2	9,060,677	9,060,677	9,060,677	0	0	0
	3	5,520,419	5,520,419	5,520,419	0	0	0
	4	1,636,037	1,645,527	1,645,527	9,490	0	9,490
	5	504,140	507,526	507,526	3,386	0	3,386
	6	85,005	87,805	87,805	2,800	0	2,800
	7	19,184	20,343	20,343	1,159	0	1,159
CCFD $j \in J^c$	1	-6,477	0	0	6,477	0	6,477
	2	208,840	208,863	208,863	23	0	23
	3	44,992	44,992	44,992	0	0	0
	4	32,782	32,785	32,785	3	0	3
FCFD $j \in J^f$	5	258,820	258,820	258,820	0	0	0
	6	208,838	208,850	208,850	12	0	12
	7	0	0	0	0	0	0
	8	91,238	91,245	91,245	7	0	7
Inflexible demand		53,205,642	53,205,642	53,228,999	0	23,357	-23,357
Total					23,357	23,357	0

ability to redistribute activities across time, as now self-scheduling at different valley hours than the ones determined by centralized market clearing does not bring additional surplus.

As shown in Table V, the positive uplift received by each generation and FD participant exactly cancels out its respective surplus loss, and the total compensation contribution is charged entirely to the inflexible demand, through a negative uplift balancing the positive uplifts received by the rest of the participants. It is worth noting that total surplus loss under convex hull prices (£23 357) is (minimum and) significantly lower than the respective loss under centralized market clearing prices (£77 632).

VII. CONCLUSIONS AND FUTURE WORK

This paper has identified for the first time non-convexities associated with different types of flexible demand, including options to forgo demand activities as well as discrete and minimum power levels. The relation of these non-convexities with schedules' inconsistency and surplus sub-optimality effects are demonstrated through simple one- and two-time period examples and a larger case study with day-ahead horizon and hourly resolution. Generalized uplift and convex hull pricing approaches addressing these effects have been extended to account for the FD non-convexities.

However, the critical analysis and comparison of these two approaches has been left out of the scope of this paper. Future work should comprehensively investigate the strengths and weaknesses of generalized uplift and convex hull pricing methodologies, considering both generation and FD market participation in different scenarios for the generation system composition and the penetration of flexible demand technologies in the future. Furthermore, the potential of combining the minimum surplus loss property of convex hull pricing with the equitable distribution property of generalized uplifts through a hybrid approach should be explored.

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