



University of Dundee

Measurements of nonlinear lensing in a semiconductor disk laser gain sample under optical pumping and using a resonant femtosecond probe laser

Quarterman, A. H.; Mirkhanov, S.; J. C. Smyth, C.; Wilcox, Keith

Published in: **Applied Physics Letters**

DOI: 10.1063/1.4963352

Publication date: 2016

Document Version Publisher's PDF, also known as Version of record

Link to publication in Discovery Research Portal

Citation for published version (APA):

Quarterman, A. H., Mirkhanov, S., J. C. Smyth, C., & Wilcox, K. G. (2016). Measurements of nonlinear lensing in a semiconductor disk laser gain sample under optical pumping and using a resonant femtosecond probe laser. Applied Physics Letters, 109(12), [121113]. DOI: 10.1063/1.4963352

General rights

Copyright and moral rights for the publications made accessible in Discovery Research Portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from Discovery Research Portal for the purpose of private study or research.

You may not further distribute the material or use it for any profit-making activity or commercial gain.
You may freely distribute the URL identifying the publication in the public portal.

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.





Measurements of nonlinear lensing in a semiconductor disk laser gain sample under optical pumping and using a resonant femtosecond probe laser

A. H. Quarterman, S. Mirkhanov, C. J. C. Smyth, and K. G. Wilcox

Citation: Applied Physics Letters **109**, 121113 (2016); doi: 10.1063/1.4963352 View online: http://dx.doi.org/10.1063/1.4963352 View Table of Contents: http://scitation.aip.org/content/aip/journal/apl/109/12?ver=pdfcov Published by the AIP Publishing

Articles you may be interested in

Z-scan measurements of the nonlinear refractive index of a pumped semiconductor disk laser gain medium Appl. Phys. Lett. **106**, 011105 (2015); 10.1063/1.4905346

Absorber and gain chip optimization to improve performance from a passively modelocked electrically pumped vertical external cavity surface emitting laser Appl. Phys. Lett. **104**, 121115 (2014); 10.1063/1.4870048

Subpicosecond quantum dot saturable absorber mode-locked semiconductor disk laser Appl. Phys. Lett. **94**, 251105 (2009); 10.1063/1.3158960

Femtosecond time-resolved optical pump-probe spectroscopy at kilohertz-scan-rates over nanosecond-timedelays without mechanical delay line Appl. Phys. Lett. **88**, 041117 (2006); 10.1063/1.2167812

Blue light emission from an organic nonlinear optical crystal of 4-aminobenzophenone pumped by a laser diode Appl. Phys. Lett. **70**, 562 (1997); 10.1063/1.118208



Leuse of AIP Publishing content is subject to the terms at: https://publishing.aip.org/authors/rights-and-permissions. Download to IP: 134.36.50.219 On: Fri, 23 Sep 2016



Measurements of nonlinear lensing in a semiconductor disk laser gain sample under optical pumping and using a resonant femtosecond probe laser

A. H. Quarterman,^{a)} S. Mirkhanov, C. J. C. Smyth, and K. G. Wilcox School of Science and Engineering, University of Dundee, Dundee DD1 4HN, United Kingdom

(Received 29 June 2016; accepted 13 September 2016; published online 23 September 2016)

Accurate characterizations of the nonlinear refractive index of semiconductor disk laser (SDL) gain samples are of critical importance for understanding the behavior of self-mode-locked SDLs. Here we describe measurements of nonlinear lensing in an SDL gain sample for a wide range of optical pump intensities and using a probe which is on resonance with the quantum wells in the SDL gain sample and whose intensity, pulse duration, and spot size are chosen to be similar to those reported in self-mode-locked SDLs. Under these conditions, we determine an effective value of the nonlinear refractive index, $n_2 = -6.5 \times 10^{-13} \text{ cm}^2/\text{W}$ at zero pump intensity, and find that the value of n_2 changes by less than 25% over the range of pump intensities studied. The nonlinear refractive index is measured using a variation on the well-established z-scan technique, which was modified to make it better suited to the measurement of optically pumped samples. © 2016 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/). [http://dx.doi.org/10.1063/1.4963352]

Semiconductor disk lasers (SDLs), also known as Vertical-External-Cavity Surface-Emitting Lasers (VECSELs),¹ were conceived as a means of making use of the flexibility enabled by a semiconductor gain medium within the well-established external-cavity architecture of a conventional solid state laser.² While the initial motivation was to enable power scaling of a semiconductor laser while maintaining high beam quality, it quickly became clear that SDLs could be passively modelocked, and that their mode-locked performance had the potential to exceed that of monolithic semiconductor lasers in terms of both pulse duration and peak power. The first demonstration of a mode-locked SDL used a semiconductor saturable absorber mirror (SESAM) and produced pulses with 22 ps duration at a wavelength of 1030 nm and a peak power of 0.2 W.³ Since then, there have been a range of developments which have brought about improvements in performance. Advances in gain sample and SESAM design have enabled several demonstrations of low power mode-locked VECSELs with pulse durations below 300 fs,^{4,5} while improvements in gain sample thermal management enabled power scaling to average powers of several Watts, albeit at slightly longer pulse durations.⁶ Correspondingly, peak powers in the kiloWatt range have been demonstrated, with the record to date being 4.35 kW.⁷ These have been sufficient to enable demonstrations of applications requiring high peak powers, for example, two photon microscopy⁸ and supercontinuum generation.⁹ Furthermore, the compatibility of $10 \times \times$ nm SDLs with Yb-doped fiber amplifiers has allowed SDL-seeded systems to reach average powers of up to 200 W,¹⁰ and to generate coherent octave-spanning continuua.5

While saturable absorbers based on carbon nanotubes¹¹ and graphene¹² have both been demonstrated, SESAM mode-locking has dominated the field of high performance

SDLs (whether using an independent SESAM or an integrated one as used in Mode-locked Integrated External Cavity Surface Emitting Lasers (MIXSELs)^{13–15}). Several demonstrations exist describing the so-called self-modelocked SDLs,¹⁶⁻²⁴ but as of yet none of these reports have provided an unambiguous explanation of the mechanism causing the reported behaviour. A large part of the problem is that nonlinear lensing in SDL gain samples is poorly understood. Nonlinear lensing in semiconductor heterostructures is complex at photon energies above the bandgap energy of any of the sample's constituents,²⁵ and especially so if a high density of carriers is present as is the case in SDL gain samples. In addition to contributions from twophoton absorption and the linear and quadratic Stark effects, any perturbation to the carrier distribution will result in a nonlinear change of refractive index via the linewidth enhancement factor. This situation is further complicated by the fact that an SDL gain sample has coupled carrier populations in the barrier layers and the quantum wells, and that transfer of carriers between these populations and carrier scattering within the individual populations both occur on timescales similar to typical mode-locked SDL pulse durations. A theoretical treatment of the problem is therefore challenging, meaning that experimental characterizations are a more direct route to understanding any possible nonlinear lensing.

The first study of nonlinear lensing in an SDL gain sample found that a nonlinear lens was present with magnitude sufficient to perturb a typical SDL cavity, and that its sign depended on the level of carrier injection,²⁶ results which were broadly consistent with studies of optical nonlinearities in electrically pumped semiconductor optical amplifiers,^{27–29} and with values predicted from theory.²⁵ Unfortunately, the 1064 nm probe laser available for this first study was out of resonance with the 1040 nm quantum wells in the sample,

0003-6951/2016/109(12)/121113/4

109, 121113-1

 \odot

^{a)}a.h.quarterman@dundee.ac.uk

and its 10 ps pulse duration was long compared with typical mode-locked SDL pulses. More recently, a separate study examined the behavior of the same gain sample using a probe laser which was on resonance with the quantum wells and with a pulse duration of 230 fs, but without studying the effects of pumping.³⁰ This study found an unpumped n_2 value which was in the range between 2.7 and 5.0 times smaller than that measured in Ref. 26.

In both Refs. 26 and 30, z-scan setups were used to measure the nonlinear lensing in SDL gain samples. In this type of experiment, a lens is used to focus a pulsed probe beam and the sample under examination is translated relative to the focus, varying the intensity on the sample. The beam transmitted (or reflected) by the sample is split using a beamsplitter, and the two components detected in two power meters. The "open" power meter detects the unobstructed beam and is used to measure the nonlinear transmission (or reflection) of the sample. The second beam is passed through a circular aperture with a diameter smaller than the beam diameter before being detected in the "closed" power meter, which is therefore sensitive to nonlinear changes in the beam diameter as well as power, and can be used to detect any effect from nonlinear lensing.^{31,32} The z-scan method is difficult to use in the case of an optically pumped gain sample due to the fact that the overlap between the pump and probe beams changes during the measurement unless the pump spot size is also changed to match the probe. In Ref. 26 this was addressed by arranging the experiment such that the pump and probe spot sizes on the sample were the same at the peak and the trough of the measured data, and using an extraction method that relied solely on the data taken at these two points.³³ In Ref. 30, the issue did not arise as the sample was not pumped, but this means that the dependence of n_2 on carrier density could not be measured.

In this report we follow a method that is based on the zscan method, but with several refinements designed to make it better suited to the measurement of reflective optically pumped samples in general. A conventional z-scan is used to identify the position of the focus of the probe beam (the z-position of the two-photon absorption dip in the data from the "open" power meter), from which the positions at which the peak and trough would occur can be calculated. These are hereafter referred to as the z-plus (peak) and z-minus (trough) positions (see Figure 1(a)). The sample is then placed at each of these positions in turn, and the normalized transmission (defined as the power transmitted through the "closed" power meter's aperture divided by the power measured by the "open" power meter) at each position is measured as the power of the probe beam is varied. For a nonlinear lens, the normalized transmission as a function of probe power should change symmetrically in opposite directions at the two positions (with the transmission at the z-plus position increasing (decreasing) for a defocussing (focussing) nonlinear lens and vice versa for the z-minus position). The gradient of a graph of normalized transmission vs. probe intensity can then be used to evaluate the nonlinear lensing coefficient, which can be treated as an empirical n₂ value provided that the change in transmission with probe intensity is linear (assuming that the change is small and that the sample is thin compared with the Rayleigh range). This



FIG. 1. Schematics of a transmission z-scan setup illustrating the z+ and zpositions relative to the probe beam focus and the peak and trough positions of a typical z-scan measurement (a), and of the reflection-type setup used for the measurements reported here (b).

procedure can be repeated with an optical pump overlapping with the probe spot in order to measure the dependence of the effective n_2 value on carrier injection. As the gain sample is a heterostructure containing materials of different compositions, and with a non-trivial intensity distribution inside the sample given by the microcavity enhancement factor, it should be emphasized that the value of n_2 is an *effective* n_2 for the sample when treated as a "black box" optical component.

The basic setup used, shown in Figure 1(b), is that of a reflection-type z-scan setup with the sample and a mirror arranged on a translation stage such that the beam reflected from the sample is redirected to follow a path parallel to the incident beam. In this way the optical path length between the focussing lens and the iris before the "closed" aperture power meter is kept constant regardless of the position of the translation stage. Pumping optics are also placed on the translation stage so that the pump spot on the SDL sample is also independent of the translation stage position. Both the pump and probe spot diameters were $100 \,\mu\text{m}$ at the z-plus and z-minus positions. The pump laser was a fiber-coupled 35 W, 808 nm diode laser and the probe laser was an Ekspla FF3000 mode-locked fiber laser, producing up to 2.5 W average power at 1035 nm and 7.64 MHz repetition rate. The probe power on the sample was adjusted using a polarising beam splitter and a zero-order half wave plate in a motorised rotation mount. The pulse duration was measured after the polarising beam splitter and was found to be 340 fs for all half wave plate angles. The gain sample was an 11 QW antiresonant sample designed for operation at 1040 nm. It was grown upside down and flip-chip bonded to a 300 μ m thick diamond, which was mounted on a water cooled copper block which was kept at 18 °C. For further details of the sample see Ref. 26 and references within.

The normalized transmission as a function of incident probe intensity at the two z positions at zero pump power is shown in Figure 2(a). At high probe intensities, the curves at the two z positions diverge symmetrically with increasing probe intensity, which is equivalent to the difference in 121113-3 Quarterman et al.



FIG. 2. Normalized transmission as a function of probe intensity for the z-plus and z-minus positions at zero pump intensity, showing the symmetrical divergence of the two curves at high probe intensity (a). Normalized transmission as a function of probe intensity for a range of pump intensities and for the zminus position (b) and z-plus position (c). The pump does not change the gradients of the curves but does cause a symmetrical offset due to thermal lensing.

transmission between the peaks and the troughs of the corresponding z-scans increasing. In contrast, at low probe intensity the transmission drops at both positions. This drop cannot be explained by a lensing effect, as a lens should cause the transmission curves to go symmetrically in opposite directions at the two z positions. The drop at low intensity is interpreted as resulting from the change in the beam width on the sample due to saturable absorption. Figure 2(a) also shows the average of the z-plus and z-minus positions, illustrating that the effect causing the drop at low intensity saturates and the curves become symmetrical at intensities above a few GW/cm². It is therefore possible to isolate the effect of nonlinear lensing by taking linear fits to the z-plus and z-minus curves at high intensity and extracting an effective value of n₂ from the gradients of these fits.

In order to investigate the effect of pumping, data were taken for both z positions for a range of pump intensities in equal steps from zero up to 75.4 kW/cm², a range which covers the pump intensities used in mode-locked SDLs. The resulting transmission curves for the z-minus position are shown in Figure 2(b) and for the z-plus position in Figure 2(c). The major features of the pumped curves are similar to those of the unpumped curves shown in Figure 2(a), but there is also a symmetrical, pump-dependent offset between the curves which corresponds to a lens which depends on pump intensity but not on probe intensity. This is, therefore, attributed to a focusing thermal lens in the gain sample. Unfortunately, while the pump and probe intensities used here reflect typical SDL operating conditions, the thermal load on the gain sample will be quite different due to the much lower repetition rate of the probe laser compared with typical mode-locked SDLs. The thermal lensing shown here would therefore be expected to be more severe than that found in an actual laser and is consequently of little interest.

An effective value of n_2 for each pump intensity can be found from the gradients of the curves shown in Figures 2(b) and 2(c) using the method described in Ref. 33. These values are shown in Figure 3. In Ref. 26, the value of n_2 at zero pump intensity was found to be $-1.5(0.2) \times 10^{-12} \text{ cm}^2/\text{W}$ and to increase with pump intensity at a rate of $3.1(0.2) \times 10^{-17} \text{ cm}^4/\text{W}^2$. Here we find that the zero pump intensity value is smaller by a factor of 2.3, and that it changes by less than 25% across a similar range of pump intensities. That the zero intensity value is different is not surprising, as the two experiments used probe lasers with different wavelengths and pulse durations, and it is worth noting that the value of n_2 at zero pump power measured here is in agreement with that reported in Ref. 30, where the probe laser wavelength and pulse durations were similar.

It is likely that the difference in behavior with pump power between the results described here and those in Ref. 26 results from the difference in interaction strengths between the on-resonance and off-resonance cases. Off resonance, the probe field will not affect the quantum well carrier distributions strongly, and therefore the carrier density sampled will depend on the pump intensity, whereas on resonance the interaction is sufficiently strong and the probe sufficiently intense that the carrier density in the quantum wells will be reduced to transparency regardless of pump intensity, meaning that the probe field will sample similar carrier densities at all pump powers. This would imply that the behavior is dominated by effects arising from the interaction between the carriers and the probe laser rather than by material effects, and therefore that similar values of n_2 might be expected in any sample using the same material system and with similar gain. It would also imply a strong dependence of n₂ on probe pulse duration, as the interaction between carriers and probe would be depend on the pulse duration relative to the carrier scattering and gain recovery timescales.

While informative, the two previous measurements of nonlinear lensing in SDL gain samples both had serious



FIG. 3. Effective n_2 as a function of pump intensity extracted from the data shown in Figure 2.

limitations in terms of matching the measurement conditions to the typical operating conditions in mode-locked SDLs. In Ref. 26, the major limitations were that the probe wavelength was out of resonance with the quantum wells in the sample and that the pulse duration was long compared with typical SDL pulses, whereas in Ref. 30 the sample was not pumped, and the spot sizes probed were smaller than typical SDL pump spots. In both cases the repetition rates of the probe lasers were much lower than the Gigahertz repetition rates of mode-locked SDLs. In contrast, in this study the probe is on resonance with the quantum wells in the sample, has a $100 \,\mu\text{m}$ spot diameter on the sample, and has a pulse duration (340 fs) in the range common in SESAM mode-locked SDLs. The probe pulse duration is somewhat shorter than those reported in self-mode-locked SDLs, which are generally in the range of 700-1000 fs, and while the difference in gain depletion with pulse duration would be expected to lead to a small change in the value of n_2 , we would expect its sign, magnitude and behavior with pumping to remain fundamentally similar. n_2 is measured from zero to 75 kW/cm² pump laser CW intensity and zero to 10 GW/cm² probe laser peak intensity, covering the ranges found in mode-locked SDLs. The only major deviation from typical operating conditions is the low repetition rate which leads to increased heating and therefore has the effect of exacerbating the thermal lensing as demonstrated in Figure 2. We have also used a method for measuring n_2 which is better suited to optically pumped samples that the techniques used in Refs. 26 and 30.

In terms of self-mode-locking of SDLs, this latest measurement agrees with the two previous studies in confirming that the nonlinear lensing coefficient of a SDL gain sample is large enough that it could, in principle, bring about a perturbation of an SDL cavity, and therefore that it may be possible to use nonlinear lensing as a mode-locking mechanism. Furthermore, that the value of n_2 only changes by a small amount over a large range of both pump and probe intensity implies that designing such an SDL may be more simple than in the case where the sign of the nonlinear lens changed with pump power as was previously thought.

Funding supporting this work was received from the UK Engineering and Physical Sciences Research Council (EPSRC), Grant No. EP/J017043/2. The data shown in the figures in this paper are openly available at doi 10.15132/10000113.

- ¹O. G. Okhotnikov, *Semiconductor Disk Lasers: Physics and Technology* (Wiley, 2010).
- ²M. Kuznetsov, F. Hakimi, R. Sprague, and A. Mooradian, IEEE Photonics Technol. Lett. **9**(8), 1063 (1997).
- ³S. Hoogland, S. Dhanjal, A. C. Tropper, J. S. Roberts, R. Haring, R. Paschotta, F. Morier-Genoud, and U. Keller, IEEE Photonics Technol. Lett. **12**(9), 1135 (2000).
- ⁴P. Klopp, U. Griebner, M. Zorn, and M. Weyers, Appl. Phys. Lett. **98**, 071103 (2011).

- ⁵C. A. Zaugg, A. Klenner, M. Mangold, A. S. Mayer, S. M. Link, F. Emaury, M. Golling, E. Gini, C. J. Saraceno, B. W. Tilma, and U. Keller, Opt. Express 22(13), 16445 (2014).
- ⁶M. Scheller, T.-L. Wang, B. Kunert, W. Stolz, S. W. Koch, and J. V. Moloney, Electron. Lett. **48**(10), 588 (2012).
- ⁷K. G. Wilcox, A. C. Tropper, H. E. Beere, D. A. Ritchie, B. Kunert, B. Heinen, and W. Stolz, Opt. Express 21(2), 1599 (2013).
- ⁸R. Aviles-Espinosa, G. Filippidis, C. Hamilton, G. Malcolm, K. J. Weingarten, T. Südmeyer, Y. Barbarin, U. Keller, S. I. C. O. Santos, D. Artigas, and P. Loza-Alvarez, Biomed. Opt. Express 2(4), 739 (2011).
- ⁹C. R. Head, H.-Y. Chan, J. S. Feehan, D. P. Shepherd, S.-u. Alam, A. C. Tropper, J. H. V. Price, and K. G. Wilcox, IEEE Photonics Technol. Lett. **25**(5), 464 (2013).
- ¹⁰P. Dupriez, C. Finot, A. Malinowski, J. K. Sahu, J. Nilsson, D. J. Richardson, K. G. Wilcox, H. D. Foreman, and A. C. Tropper, Opt. Express 14(21), 9611 (2006).
- ¹¹K. Seger, N. Meiser, S. Y. Chio, B. H. Jung, D.-I. Yeom, F. Rotermund, O. Okhotnikov, F. Laurell, and V. Pasiskevicius, Opt. Express 21(15), 17806 (2013).
- ¹²C. A. Zaugg, Z. Sun, V. J. Wittwer, D. Popa, S. Milana, T. S. Kulmala, R. S. Sundaram, M. Mangold, O. D. Sieber, M. Golling, Y. Lee, J. H. Ahn, A. C. Ferrari, and U. Keller, Opt. Express 21(25), 31548 (2013).
- ¹³M. Mangold, M. Golling, E. Gini, B. W. Tilma, and U. Keller, Opt. Express 23(17), 22043 (2015).
- ¹⁴B. Rudin, V. J. Wittwer, D. J. H. C. Maas, M. Hoffmann, O. D. Sieber, Y. Barbarin, M. Golling, T. Südmeyer, and U. Keller, Opt. Express 18(26), 27582 (2010).
- ¹⁵S. M. Link, A. Klenner, M. Mangold, C. A. Zaugg, M. Golling, B. W. Tilma, and U. Keller, Opt. Express 23(5), 5521 (2015).
- ¹⁶Y. F. Chen, Y. C. Lee, H. C. Liang, K. Y. Lin, K. W. Su, and K. F. Huang, Opt. Lett. **36**(23), 4581 (2011).
- ¹⁷L. Kornaszewski, G. Maker, G. P. A. Malcolm, M. Butkus, E. U. Rafailov, and C. J. Hamilton, Laser Photonics Rev. 6(6), L20 (2012).
- ¹⁸K. G. Wilcox and A. C. Tropper, Laser Photonics Rev. 7(3), 422 (2013).
- ¹⁹L. Kornaszewski, G. Maker, G. Malcolm, M. Butkus, E. U. Rafailov, and C. Hamilton, Laser Photonics Rev. 7(4), 555 (2013).
- ²⁰H. C. Liang, Y. C. Lee, J. C. Tung, K. W. Su, K. F. Huang, and Y. F. Chen, Opt. Lett. **37**(22), 4609 (2012).
- ²¹A. R. Albrecht, Y. Wang, M. Ghasemkhani, D. V. Seletskiy, J. G. Cederberg, and M. Sheik-Bahae, Opt. Express 21(23), 28801 (2013).
- ²²M. Gaafar, C. Moller, M. Wichtmann, B. Heinen, B. Kunert, A. Rahimi-Iman, W. Stolz, and M. Koch, Electron. Lett. **50**(7), 542 (2014).
- ²³M. Gaafar, D. Al Nakdali, C. Moller, K. A. Fedorova, M. Wichtmann, M. K. Shakfa, F. Zhang, A. Rahimi-Iman, E. U. Rafialov, and M. Koch, Opt. Lett. **39**(15), 4623 (2014).
- ²⁴M. Gaafar, P. Richter, H. Keskin, C. Moller, M. Wichtmann, W. Stolz, A. Rahimi-Iman, and M. Koch, Opt. Express 22(23), 28390 (2014).
- ²⁵M. Sheik-Bahae and E. W. van Stryland, Phys. Rev. B 50(19), 14171 (1994).
- ²⁶A. H. Quarterman, M. A. Tyrk, and K. G. Wilcox, Appl. Phys. Lett. 106(1), 011105 (2015).
- ²⁷M. J. LaGasse, K. K. Anderson, C. A. Wang, H. A. Haus, and J. G. Fujimoto, Appl. Phys. Lett. **56**, 417 (1990).
- ²⁸K. L. Hall, A. M. Darwish, E. P. Ippen, U. Koren, and G. Raybon, Appl. Phys. Lett. **62**, 1320 (1993).
- ²⁹C. T. Hultgren and E. P. Ippen, Appl. Phys. Lett. **59**, 635 (1991).
- ³⁰E. A. Shaw, A. H. Quarterman, A. P. Turnbull, T. Chen-Sverre, C. R. Head, A. C. Tropper, and K. G. Wilcox, IEEE Photonics Technol. Lett. **28**(13), 1395 (2016).
- ³¹M. Sheik-Bahae, A. A. Said, and E. W. van Stryland, Opt. Lett. **14**(17), 955 (1989).
- ³²M. Martinelli, L. Gomes, and R. J. Horowicz, Appl. Opt. **39**(33), 6193 (2000).
- ³³E. W. van Stryland and M. Sheik-Bahae, in *Characterization Techniques and Tabulations for Organic Nonlinear Materials*, edited by M. G. Kuzyk and C. W. Dirk (Marcel Dekker, 1998), p. 655.