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Approximation of system components for pump scheduling optimisation

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Abstract

The operation of pump systems in water distribution systems (WDS) is commonly the most expensive task for utilities with up to 70% of the operating cost of a pump system attributed to electricity consumption. Optimisation of pump scheduling could save 10-20% by improving efficiency or shifting consumption to periods with low tariffs.

Due to the complexity of the optimal control problem, heuristic methods which cannot guarantee optimality are often applied. To facilitate the use of mathematical optimisation this paper investigates formulations of WDS components. We show that linear approximations outperform non-linear approximations, while maintaining comparable levels of accuracy.

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1. Introduction

Water distribution systems (WDS) can account for up to 5% of a cities total electricity consumption, the majority of which is used to power the pump systems. The operation of these pump systems is a major cost factor for utilities and up to 70% of the operating cost of a pump system can be attributed to the electricity consumption (Bunn,2011).

Many systems use reservoir levels to determine the operating schedules of pumps. Significant savings can be made through the use of more advanced control methods such as pump scheduling. Optimal pump scheduling has been shown to reduce the energy cost of a system by 10 - 20%, shifting consumption to time periods with lower electricity cost or improving operational efficiency (Crawley and Dandy,1993,Boulos et al.,2001,Bunn and Reynolds,2009).

WDS pump schedules are optimised using either heuristic and evolutionary methods or mathematical optimisation. In the former methods, heuristics are used to generate schedules and evolutionary methods guide the search for optimality. While this can provide solutions for complex problems, it cannot guarantee that an optimal solution is found. Mathematical optimisation solves the problem by using information of the objective function to guide the search and does not rely on solving part of the problem outside of the optimisation solver. However, many complex

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problems, such as the optimisation of operating schedules for WDS are too complex to be solved in mathematical optimisation without approximations.

Nomenclature

T	Binary pump setting
h	Head at a node
q	Flow in link
λ	Binary variable for modelling pipe head loss with big-M constraints
Pe	Electricity Price
P_{i_p}	Electrical power of pump i_p
P_s	Pump switch penalty
a_{i_p}	1 st Polynomial factor of the pump curve of pump i_p
b_{i_p}	2 nd Polynomial factor of the pump curve of pump i_p
A_k	Area of reservoir at node k
M	Large constant
N_{con}	Number of constrains defining a convex set
N_{pieces}	Number of pieces in a piecewise approximation)

Past efforts that use heuristics include an application of a genetic algorithm (GA) to optimise pump scheduling by Mackle(1995). To improve the search performance Van Zyl et al.(2004) and Reis et al.(2006) used a GA together with an improvement for local searches, whereas Savic et al.(1997) and Marques et al.(2015) explored multi-objective optimisation capabilities. Other heuristic procedures include simulated annealing applied by Goldman and Mays(1999), which was combined with multi-objective optimisation capabilities by Peadarallu and Ozdamar(2008). To further reduce the CPU time by avoiding the computationally expensive hydraulic evaluations, Martínez et al.(2007) and Salomons et al.(2007) approximated the hydraulic model with an artificial neural network that represents the WDS response of the municipal supply networks of Haifa and Valencia (Jamieson et al.,2007). Commercial systems such as Tynemarch's MISER or Dorceto's Aquadapt use linear programming methods while others such as Innovyze's BalanceNet rely on a genetic algorithm to find a schedule for a mass balance model of the hydraulic network (Bunn and Helms,1999,Tynemarch Systems Engineering,1999,Innovyze,2015).

In mathematical optimisation the problem can be posed as an mixed integer problem (MIP), which can be solved using a branch and bound algorithm. Gleixner et al.(2012) used a detailed hydraulic description of the network to solve the mixed integer non-linear problem (MINLP) via a branch and bound algorithm. Burgschweiger et al.(2008) describe the MINLP method used for the optimisation of the Berlin water supply system. As with the heuristic approaches computational times are important and Ghaddar et al.(2015) showed that a naive approach to a larger problem does not yield an answer, as the computational effort scales badly and can become infeasibly large. Applying a Lagrangian decomposition yielded significant performance improvements. Biscos et al.(2003) solved an MINLP with simplified hydraulic descriptions to reduce computational effort. Other approaches to improve the scaling properties of the problem employ either iterative linear programming or divide the problem in two and then solve a non-linear problem (NLP), without integers, using dynamic programming (Price and Ostfeld,2013,Ulanicki et al.,2007).

We investigate the computational cost of different approximations of system components and the loss in accuracy of the hydraulic solution caused by these approximations. Through a systematic investigation of non-linear, convex and piecewise linear approximations of pumps and pipes, we analyse the relative importance, with respect to performance and accuracy, of the individual approximations. We further investigate the computational cost of considering maintenance constraints and varying time step sizes to change the overall size of the problem.

In section 2.1 the objective function to minimise energy consumption is presented. It is adapted to also include a proxy for maintenance cost. In sections 2.2 three different methods to approximate the physical characteristics of pumps are described. These methods include an exact non-linear equation and an approximation by a convex set. In section 2.3 linear and non-linear formulations for the head loss in pipes are described. The results and discussion are found in sections 4 and 5, respectively.

2. Problem formulation

Three methods can be used to determine when to operate the pumps in a WDS. In the simplest form, low and high water levels in a tank will trigger the corresponding pump to switch on or off. The tank fill levels are set such that the water demand can always be satisfied with sufficient pressure. However, to benefit from the variation in energy cost throughout the day or to account for the daily water demand profile the desired level can be set in a time-varying fashion. This is known as implicit control. In explicit control the pump state is evaluated in the optimisation directly (Ormsbee and Lansey,1994). The optimisation problem solved here takes the form:

$$\begin{aligned} &\text{Minimise: Pumping cost} \\ &\text{subject to: Energy balance,} \\ &\hspace{10em} \text{Mass balance,} \end{aligned} \tag{1}$$

where the pumping cost models in the objective function are described below, and the energy equation for the pumps and pipes are described in sections 2.2 and 2.3, respectively. The mass balance constraints will be described in 2.4.

The use of integer or binary variables in the description of the pump state and the piecewise linear head difference approximations, leads to a formulation of the optimisation problem as a mixed integer problem (MIP). MIPs can be solved through a branch and bound algorithm, an algorithm that searches through the entire space of feasible solutions by using a search tree. By forming a continuous relaxation of the integer variables, some are fixed to their integer values as the algorithm branches along the tree and solves the relaxed form of the problem to generate upper and lower bounds for the value of the objective function of these branches. If the objective value's lower bound of branch *a* is larger than the upper bound of branch *b*, branch *a* can be discarded. This reduces computational effort by restricting the number of solutions in the search space that need to be evaluated (Garfinkel and Nemhauser,1972). Thus, two aspects of the problem affect computational complexity; the difficulty of solving the relaxed sub-problem is the first. The second depends on the number of integer variables and is caused by the need to solve more or fewer of such sub-problems as a result of branching in a larger or smaller integer variable space, respectively.

The computational effort in solving the bounding optimization sub-problems is governed by the convexity, non-linearity and size of the constraints resulting from the WDS component approximations used. On the other hand, the number of integer variables will depend on the number of time steps and, if a piecewise linear approximation is used, on the number of pieces needed for a sufficiently good hydraulic approximation.

2.1. Objective Function

The decision variable in pump switching of fixed speed pumps is the pumps state, ON or OFF, here described by $T_{i_p,j} \in \{0, 1\}$ for pump i_p at time step $j \in [0, N]$. With the power rating of the pump assumed fixed (i.e. independent of flow conditions for a fixed speed pump), the energy consumption by each pump during a 24h period and the associated energy cost for energy the linear function:

$$f_1(\cdot) := \sum_{i_p=1}^{i_p=N_p} \sum_{j=1}^{j=N} T_{i_p,j} P_{i_p,j} \tag{2}$$

where $P_{i_p,j}$ is the cost of energy in having pump i_p ON at time j .

Since pump switching can have a negative effect on the maintenance cost of a system due to the changing loads contributing to fatigue, penalising pump switching is often used to reduce this negative impact and account for maintenance cost (Savic et al.,1997,Lansey and Awumah,1994). A penalty function that approximates the switching cost can be added to the objective function to lower maintenance cost. Penalizing ON-to-OFF and OFF-to-ON states equally, we get the penalty function:

$$\begin{aligned} f_2(\cdot) &:= \sum_{i_p=1}^{i_p=N_p} P_s \sum_{j=1}^{j=N} |T_{i_p,j} - T_{i_p,j-1}| \\ &= \sum_{i_p=1}^{i_p=N_p} P_s \sum_{j=1}^{j=N} (T_{i_p,j} - T_{i_p,j-1})^2 \end{aligned} \tag{3}$$

where P_s is a switching penalty and the equality between the linear and quadratic terms holds because $T \in \{0, 1\}$.

2.2. Pump approximations

When a pump is OFF, the flow through it is zero and the status indicator variable is zero ($T = 0 \Rightarrow q = 0$). When a pump is ON, a relationship is enforced between the head difference across the pump and the flow, i.e. $T = 1 \Rightarrow h = f(q)$. This relationship is modelled by the characteristic curve supplied by the pump manufacturer or a simplification can be used as it may not always be necessary to model the entire operating range of a pump.

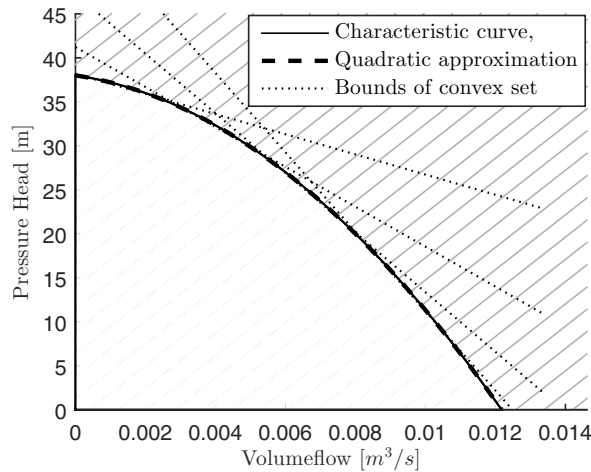


Fig. 1. Pump characteristics and representation of approximations

An example characteristic curve is shown in Figure 1 by the dashed arc. For a pump i_p that connects nodes J_1 and J_2 with flow q_{i_p} , the characteristic curve can be represented by a polynomial, for example by a **quadratic** function:

$$\begin{aligned}
 h_{J1} - h_{J2} &= a_{i_p} q_{i_p}^2 + b_{i_p} q_{i_p} + c_{i_p} \text{ if: } T_{i_p} = 1, \\
 q_{i_p} &= 0 \text{ if: } T_{i_p} = 0,
 \end{aligned}
 \tag{4}$$

where a_{i_p} , b_{i_p} & c_{i_p} are the polynomial coefficients of the pump and h_j the head at node j . This results in quadratic equality constraints, which would make 1 a mixed-integer non-linear program (MINLP).

We can also consider a **set of linear constraints** describing a convex set to approximate the characteristic curve can be used. The constraints enforced if $T_{i_p} = 1$, shown in Figure 1 as dotted lines, are described by:

$$\begin{aligned}
 h_{J1} - h_{J2} &\leq m_{i_p,1} q_{i_p} + c_{i_p,1}, \\
 h_{J1} - h_{J2} &\leq m_{i_p,2} q_{i_p} + c_{i_p,2} \dots \\
 h_{J1} - h_{J2} &\leq m_{i_p,N_{con}} q_{i_p} + c_{i_p,N_{con}}
 \end{aligned}
 \tag{5}$$

where $m_{i_p,1} \dots m_{i_p,N_{con}}$ and $c_{i_p,1} \dots c_{i_p,N_{con}}$ are the linear coefficients and N_{con} is the number of such constraints. If $T_{i_p} = 0$, $q_{i_p} = 0$, as for the quadratic approximations.

2.3. Pipe approximations

Pipes connect nodes in the network and the head loss in a pipe is a function of the flow. Head loss can be described with the Hazen-Williams or Darcy-Weisbach friction factors in hydraulic simulations, but is simplified for pump scheduling. Figure 2 shows the approximations for a flow through a pipe. Since both the Hazen-Williams and Darcy-Weisbach equation both contain a power of ≈ 2 , a quadratic approximation provides a close fit. A **quadratic** constraint for a pipe P_j , connecting nodes J_3 & J_4 is given below and implemented using big-M constraints as (see Gleixner et al.(2012) for details):.

$$h_{J3} - h_{J4} = \begin{cases} a_{P_j} q_{P_j}^2 + b_{P_j} q_{P_j} + c_{P_j}, & \text{if } q_{P_j} \geq 0 \\ -a_{P_j} q_{P_j}^2 + b_{P_j} q_{P_j} - c_{P_j}, & \text{if } q_{P_j} \leq 0, \end{cases}
 \tag{6}$$

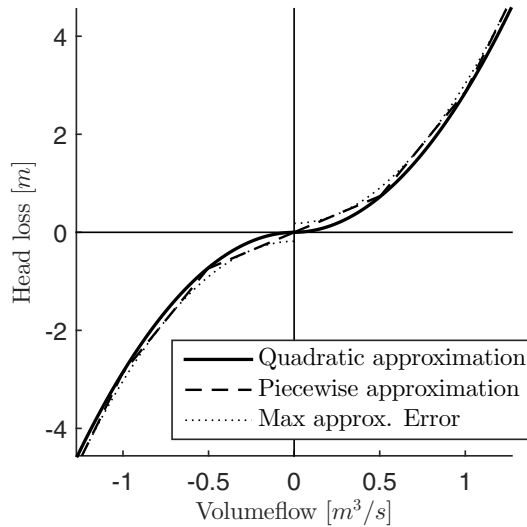


Fig. 2. Head loss approximations used

where, h_j is the head at node j and a_{P2} , b_{P2} & c_{P2} are the fitted coefficients for the pipe, c_{P2} represents the elevation difference. This again results in quadratic equality constraints and is thus renders (1) an MINLP.

Of course we can consider replacing quadratic constraints by **piecewise linear approximations** that can provide an approximation of the head loss in a pipe using linear big-M constraints:

$$h_{J3} - h_{J4} = \begin{cases} m_{P2,1}q_{P2} + c_{P2,1}, & \text{if } q_{lim1} \leq q_{P2} \leq q_{lim2} \\ m_{P2,2}q_{P2} + c_{P2,2}, & \text{if } q_{lim2} \leq q_{P2} \leq q_{lim3} \dots \\ m_{P2,N_{piece}}q_{P2} + c_{P2,N_{piece}}, & \text{if } q_{limN_{piece}-1} \leq q_{P2} \leq q_{limN_{piece}} \end{cases} \quad (7)$$

where $\lambda_{P2,(1,2\dots N_{piece})}$, $\lambda \in \{0, 1\}$ is a switch for each corresponding linear section given by $m_{P2,(1,2\dots N_{piece})}q_{P2} + c_{P2,(1,2\dots N_{piece})}$.

A formulation with N_{pieces} pieces introduces N_{pieces} of integer variables per pipe at each time step. Thus approximations with closer fits to the head loss curve can scale badly for larger network models. Furthermore, while it is continuous around flow reversal it introduces discontinuities where the linear sections meet, which may lead to numerical issues.

2.4. Mass balance at network nodes

Since water can be considered incompressible, the mass flow is equal to the volume flow. Thus for a network node J_3 joining components $P_1, P_2, \dots P_n$, the mass must balance at each time step j as:

$$q_{P1,j} + q_{P2,j} + \dots + q_{Pn,j} = 0. \quad (8)$$

Demand at a node is considered in the mass balance and must always be met in feasible solutions. To further ensure feasibility of the solutions, a minimum hydraulic head can be enforced at each node.

Tanks provide storage in the network. For a tank at J_4 with flows q_{in} & q_{out} the mass balance for time steps $j = 1 \dots N - 1$ is given by:

$$q_{in,j} + q_{out,j} = (h_{J4,j+1} - h_{J4,j}) \times A_{J4}, \quad (9)$$

where the surface area of the tank is given by A_J . Since demand patterns are similar from day to day, we ensure that schedules are repeatable (reasonably similar) by enforcing the constraint that final levels in tanks do not differ much from their initial conditions:

$$(h_{J4,1} - h_{J4,N}) \times A_{J4} \leq \delta_V, \quad (10)$$

Table 1. Formulation combinations used to investigate the influence of the component approximations. For each optimisation problem defined the constraints apply to all relevant components: (4 or 5) would apply to all pumps in the network, (7 or 6) would apply to all pipe in the network and (8) to all nodes and (9) to all tanks.

No.:	Formulation	Details
MINLP problems solved with SCIP:		
AN1	min.: $f_1(\cdot) + f_2(\cdot)$ s.t.: (4),(6), (8),(9)	MINLP with the fewest number of integer variables, (1 per pipe and 1 per pump) but most non-linear constraints.
AN2	min.: $f_1(\cdot) + f_2(\cdot)$ s.t.: (4),(7), (8),(9)	MINLP with a piecewise linear pipe approximation and thus less non-linear constraints than AN1, but more integer variables ($N_{piece} = 7$).
AN3	min.: $f_1(\cdot) + f_2(\cdot)$ s.t.: (4),(7),(8),(9)	MINLP with the same non-linear constraints as AN2, but a less close piecewise linear approximation and thus fewer integer variables ($N_{piece} = 3$).
MIQP / MILP problems solved with CPLEX:		
AN4	min.: $f_1(\cdot) + f_2(\cdot)$ s.t.: (5),(7),(8),(9)	MIQP with the closest piecewise linear and convex set approximation and only linear constraints ($N_{piece} = 7, N_{con} = 7$).
AN5	min.: $f_1(\cdot) + f_2(\cdot)$ s.t.: (5),(7),(8),(9)	MIQP like AN3, but with a less close piecewise linear approximation and thus fewer integer variables but a similar number of constraints ($N_{piece} = 3, N_{con} = 7$).
AN6	min.: $f_1(\cdot) + f_2(\cdot)$ s.t.: (5),(7),(8),(9)	MIQP like AN3, but a less close convex set approximation and thus less constraints but the same number of integer variables variables ($N_{piece} = 7, N_{con} = 3$).
AN7	min.: $f_1(\cdot) + f_2(\cdot)$ s.t.: (5),(7),(8),(9)	MIQP like AN3, but significantly less integer variables and constraints through the most relaxed approximations ($N_{piece} = 3, N_{con} = 3$).
AN8	min.: $f_1(\cdot)$ s.t.: (5),(7),(8),(9)	MILP like AN7, but a linear objective function, used to identify the performance effect of the switching penalty ($N_{piece} = 3, N_{con} = 3$).

where δ_V defines the volumetric difference. This allows us to not specify the final or initial tank levels as input data, which would limit the feasible search space and would potentially lead to a sub-optimal final solution. A similar method is used by Price and Ostfeld(2013) while Crawley and Dandy(1993) include a penalty for a water level below the original level or a specified target at the end of the operating period.

3. Methodology

3.1. Problems considered

The optimisation problem consists of an objective function, which is minimised to reduce operational cost, a set of constraints that approximate the behaviour of the network components. The MIP formulations AN1–AN7 in Table 1 were tested for computational effort, computed operational cost and accuracy of the resulting hydraulic solution. The accuracy in terms of the hydraulic solution and the calculated operational cost have to balance with the requirement to solve the system sufficiently quickly for operational purposes, where decision time is limited to a fraction of the time step. The computational cost of the maintenance penalty in the objective function is analysed using formulations AN7 and AN8.

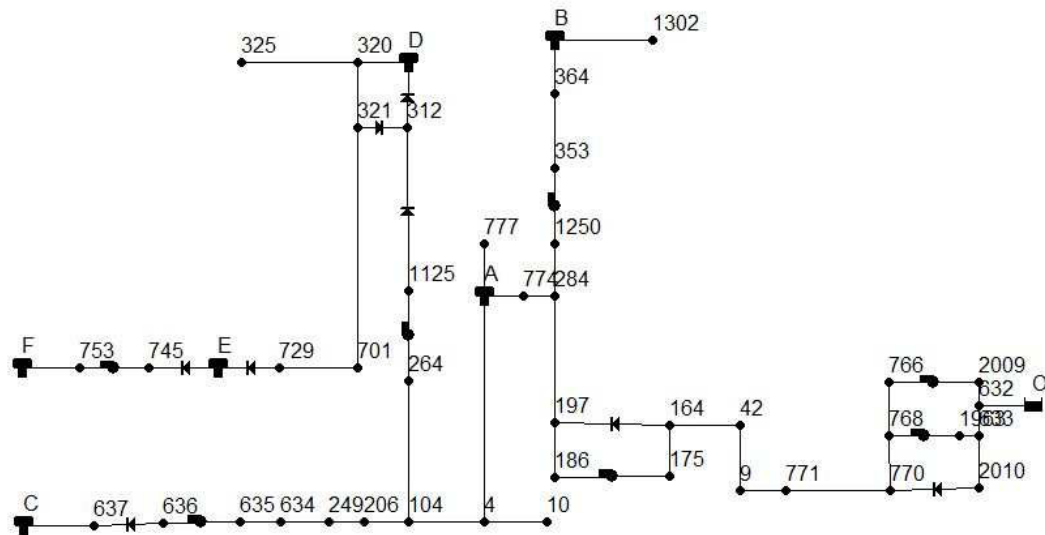


Fig. 3. Richmond skeleton network

3.2. Investigation procedure

The Richmond network¹ shown in Figure 3 is a network often considered when benchmarking operation optimisation. The network is a skeletonised version of a real supply network and consists of 44 pipes (7 pumps) and 41 nodes (6 storage tanks and 10 demand nodes). The computational effort and accuracy of the hydraulic solution computed are evaluated by the following procedure:

1. The problem as defined by the approximation number and number of time steps is generated and solved by the corresponding MIP solver.
2. The solver finds a solution for the MIP problem with a 5% optimality gap or terminates after ten minutes producing an operating schedule \mathbf{x} and a corresponding hydraulic solution \mathbf{u} ,
3. The minimal operating cost found by the solver, the time taken and the remaining optimality gap are recorded,
4. \mathbf{x} is passed to a nullspace Newton algorithm which solves the hydraulic equations to compute \mathbf{u}' , the hydraulic solution for the pump settings \mathbf{x} (Abraham and Stoianov,2015),
5. The difference between \mathbf{u}' and \mathbf{u} is computed and recorded.
6. The procedure is repeated 3 times to obtain averaged measures for the computational effort.

As the problem is either posed as MILP, MIQP or MINLP as outlined in Table 1, the MINLP are solved with SCIP. For the MIQP and MILP problems CPLEX was used due to significantly better performance for this class of problems. The problems are parsed using the Matlab toolbox OPTI or the CPLEX MATLAB API (Currie and Wilson,2012,Achterberg,2009,IBM,2009). The simulations are performed on an Intel Xeon E5-2665 with 2.4Ghz and 32GB RAM and Matlab 2014b.

Branch and bound provides a lower bound on the relaxed underlying problem and a solution taken before convergence can be tested for sub-optimality. In an operational setting near optimal will often be sufficient. After initial analysis a 5% optimality gap between the feasible solution and the best bound was considered sufficient as further evaluations did not improve the objective function but tightens the best bound with further iterations.

Table 2. Mean of the relative hydraulic error

Approximation	1	2	3	4	5	6	7	8
Error	1.74 %	3.22 %	3.99 %	2.78 %	4.00 %	3.65 %	3.3 %	3.3 %

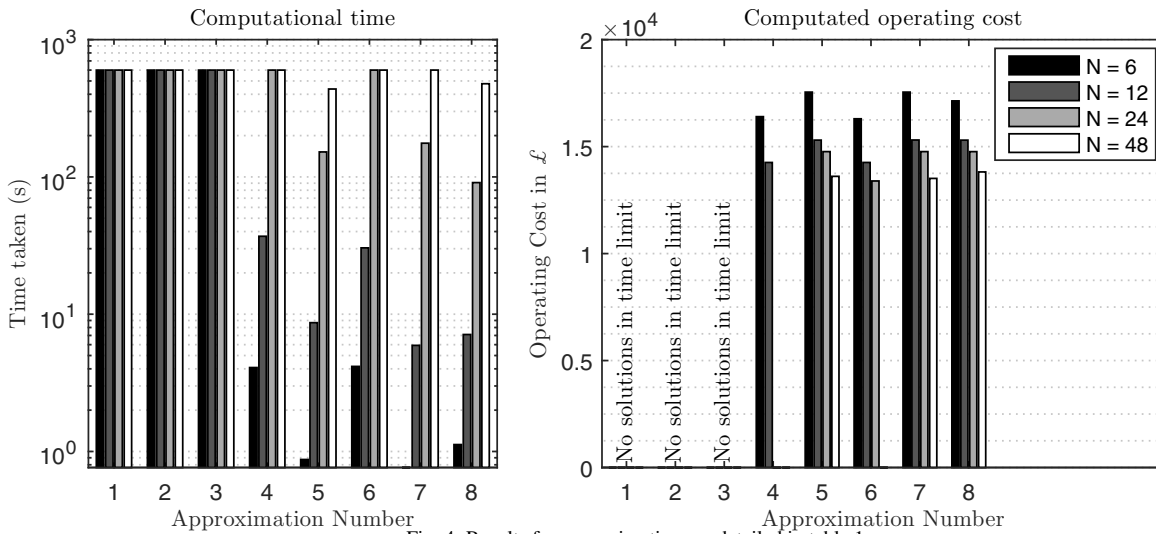


Fig. 4. Results for approximations as detailed in table 1

4. Results

Figure 4 shows the time taken by the MIP solvers to achieve the 5% optimality gap and the operating cost of the computed schedule. It clearly shows that the non-linear approximations did not find a solution in the time limit. The approximations AN4 and AN6 did not yield schedules for the larger problems either, while approximations AN5, AN7 and AN8 all reach the specified optimality gap of 5% within the time limit.

The differences in computed schedules are small. There is no difference between the schedules computed by approximations AN5 and AN7, while approximations AN4 and AN6 do provide schedules of lower cost, when they converge to solution. The 24 time step case of approximation 6 only converged to an optimality gap of 8.09% at the time limit, thus the schedule computed within the time limit was used.

The relative mean errors are compiled in Table 2, showing the mean of the relative error $\frac{\|\mathbf{u} - \mathbf{u}'\|}{\|\mathbf{u}\|}$. Due to the failure of approximations AN1–AN3 to converge in the given time limit, the mean errors of the analysis on the Van Zyl network are considered in the analysis.²

5. Discussion

The performance of the approximations in terms of computational time and hydraulic accuracy differs significantly between MINLPs and MIQP/LPs and different component approximations and time step sizes. The number of time steps and thus the overall size of the problem has a very clear effect on the computational effort. The time steps are doubled three times from 6 to 48, and the increase in computational effort is exponential as shown in Figure 4. The operating cost drop slightly as the size of the time steps decrease. This is to be expected as smaller time steps provide a closer control over the pumping, but the gains are clearly not as significant as the increase in computational time.

Figure 4 show that the MIP with a linear objective function requires more computational effort to solve, in particular for the smaller size problems with larger time steps. The switching penalty provides a gradient or regularisation to improve the search; over all coarseness in time steps, however, the two have similar performance in computational

¹ The network was taken from: http://emps.exeter.ac.uk/media/universityofexeter/emps/research/cws/downloads/Richmond_skeleton.inp

² Van Zyl network taken from: Lopez-Ibanez(2009)

time. The computed schedules and thus objective function values are marginally worse for the LP because it does not consider the cost of switching that need to be added after the optimisation. The hydraulic error is not affected by the approximation of the pump cost and thus remains unchanged.

The error in demand estimate for 15 minute time steps is $\sim 5 - 10\%$ (Bakker et al., 2013). This is the same order as the error introduced due to the component approximations, with the error for the NLPs clearly smaller than that of the less detailed linear component approximations. While these results are in line with expectations, they do show that a piecewise approximation with only a few pieces will often have sufficient detail to provide a schedule with a hydraulic error similar in magnitude to the error in the demand forecast. This is achieved while simultaneously significantly reducing the computational effort.

The linear approximations only have small differences in computed operating cost between them and the produced schedules are similar. The component approximations over estimating head loss in pipes and underestimating the head difference a pump can generate. Thus the closer approximations provide schedules of lower operating cost. For the Richmond Network evaluated the closeness of the pipe approximation has a stronger effect on the schedule produced than the closeness of the pump approximation. For 1 hour time steps approximation AN6 provides a schedule with operating cost similar to that for approximation AN5 and AN7 at half hour time steps, despite not reaching a sub-optimality of 5% at the time limit. For the same pipe approximation, the closeness of the pump approximation does not provide a schedule with lower operating cost but does not lead to an increase on computational effort either. On the contrary for the smallest time step it was even decreased slightly.

5.1. Further work

The preliminary results reported here indicate trends that need to be confirmed for a range of networks to investigate their dependence on network properties. In particular, the proposed non-linear approximations were not suitable to find a solution for the Richmond network in the short time frame required. Further development to reduce the computational effort could make these methods more competitive.

Finally, a detailed analysis of the computed and actual operating cost due to the schedules is needed to further compare the different approximation methods. As a result of this more or less detailed approximations for the pump cost can be formulated to either improve the accuracy of the computed operating cost or reduce the computational effort further.

6. Conclusions

We show that piecewise linear component approximations require significantly lower computational effort to solve the MIP compared to the non-linear constraints considered here. Although this may result in a small increase in error of the hydraulic solution computed by the MIP solver, the mean error in flow rates is only $\sim 4\%$, which is well within the modelling uncertainties of a WDS. By using smaller time steps, the computed operation cost improved slightly (by 5 – 10%), while the computational time increased exponentially as the number of inter variables increased. Similarly, more detailed approximations of pipes, which also introduce more integer variables, lead to a small decrease in computed operating cost while their computational effort increases significantly.

References

- S. Bunn, Reducing the GHG footprint at water and wastewater utilities in the US and the UK, Report, Derceto, Inc, 2011.
- P. D. Crawley, G. C. Dandy, Optimal Operation of Multiple Reservoir System, *Journal of Water Resources Planning and Management* 119 (1993) 1–17.
- P. F. Boulous, M. Moore, P. Hsiung, D. Thomas, Optimal Pump Operation of water distribution systems using genetic algorithms, *Distribution System Symposium* (2001).
- S. M. Bunn, L. Reynolds, The energy-efficiency benefits of pump-scheduling optimization for potable water supplies, 2009.
- G. Mackle, Application of genetic algorithms to pump scheduling for water supply, in: *1st International Conference on Genetic Algorithms in Engineering Systems: Innovations and Applications (GALESIA)*, volume 1995, IET, IEE, 1995, pp. 400–405.
- J. E. Van Zyl, D. A. Savic, G. A. Walters, Operational optimization of water distribution systems using a hybrid genetic algorithm, *Journal of Water Resources Planning and Management* 130 (2004) 160–170.

- L. F. R. Reis, F. T. Bessler, G. A. Walters, D. Savic, Water Supply Reservoir Operation by Combined Genetic Algorithm Linear Programming (GA-LP) Approach, *Water Resources Management* 20 (2006) 227–255.
- D. A. Savic, G. A. Walters, M. Schwab, Multiobjective genetic algorithms for pump scheduling in water supply, in: *Evolutionary Computing*, Springer, 1997, pp. 227–235.
- J. a. Marques, M. Cunha, D. A. Savić, Multi-objective optimization of water distribution systems based on a real options approach, *Environmental Modelling & Software* 63 (2015) 1–13.
- F. E. Goldman, L. W. Mays, The application of simulated annealing to the optimal operation of water systems, in: *29th Annual Water Resources Planning and Management Conference*.
- C. S. Pedamallu, L. Ozdamar, Investigating a hybrid simulated annealing and local search algorithm for constrained optimization, *European Journal of Operational Research* 185 (2008) 1230–1245.
- F. Martínez, V. Hernández, J. M. Alonso, Z. Rao, S. Alvisi, Optimizing the operation of the Valencia water-distribution network, *Journal of Hydroinformatics* 9 (2007) 65.
- E. Salomons, A. Goryashko, U. Shamir, Z. Rao, S. Alvisi, Optimizing the operation of the Haifa-A water-distribution network, *Journal of Hydroinformatics* 9 (2007) 51–64.
- D. Jamieson, U. Shamir, F. Martínez, M. Franchini, Conceptual design of a generic, real-time, near-optimal control system for water-distribution networks, *Journal of Hydroinformatics* 9 (2007) 3–14.
- S. Bunn, S. Helms, Application of an Expert System to Control Treated Water Distribution, in: *WRPMD'99*, American Society of Civil Engineers, Reston, VA, 1999, pp. 1–7.
- Tynemarch Systems Engineering, Optimising Water Resource and Supply Systems, *Wet News* 5 (1999).
- Innovyze, BalanceNet, 2015.
- A. Gleixner, H. Held, W. Huang, S. Vigerske, Towards globally optimal operation of water supply networks, *Numerical Algebra, Control and Optimization* 2 (2012) 695–711.
- J. Burgschweiger, B. Gnädig, M. C. Steinbach, Optimization models for operative planning in drinking water networks, *Optimization and Engineering* 10 (2008) 43–73.
- B. Ghaddar, J. Naoum-Sawaya, A. Kishimoto, N. Taheri, B. Eck, A Lagrangian decomposition approach for the pump scheduling problem in water networks, *European Journal of Operational Research* 241 (2015) 490–501.
- C. Biscos, M. Mulholland, M. V. Le Lann, C. A. Buckley, C. J. Brouckaert, Optimal operation of water distribution networks by predictive control using MINLP, *Water SA* 29 (2003) 393–404.
- E. Price, A. Ostfeld, Iterative Linearization Scheme for Convex Nonlinear Equations: Application to Optimal Operation of Water Distribution Systems, *Journal of Water Resources Planning and Management* 139 (2013) 299–312.
- B. Ulanicki, J. Kahler, H. See, Dynamic Optimization Approach for Solving an Optimal Scheduling Problem in Water Distribution Systems, *Journal of Water Resources Planning and Management* 133 (2007) 23–32.
- L. E. Ormsbee, K. E. Lansey, Optimal control of water supply pumping systems, *Journal of Water Resources Planning and Management* 120 (1994) 237–252.
- R. S. Garfinkel, G. L. Nemhauser, *Integer programming*, volume 4, Wiley New York, 1972.
- K. E. Lansey, K. Awumah, Optimal pump operations considering pump switches, *Journal of Water Resources Planning and Management* 120 (1994) 17–35.
- E. Abraham, I. Stoianov, An efficient null space inexact newton method for hydraulic simulation of water distribution networks, *Environmental Modelling & Software* (2015).
- J. Currie, D. I. Wilson, OPTI: Lowering the Barrier Between Open Source Optimizers and the Industrial MATLAB User, in: N. Sahinidis, J. Pinto (Eds.), *Foundations of Computer-Aided Process Operations*, Savannah, Georgia, USA.
- T. Achterberg, SCIP: solving constraint integer programs, *Mathematical Programming Computation* 1 (2009) 1–41.
- IBM, V12. 1: User's Manual for CPLEX, Technical Report, 2009.
- M. Lopez-Ibanez, *Operational Optimisation of Water Distribution Networks*, Thesis, Edinburgh Napier University, 2009.
- M. Bakker, J. Vreeburg, K. van Schagen, L. Rietveld, A fully adaptive forecasting model for short-term drinking water demand, *Environmental Modelling & Software* 48 (2013) 141–151.