

**IMPERIAL COLLEGE LONDON**

**Department of Earth Science and Engineering**

**Centre for Petroleum Studies**

**Forchheimer Model for Non-Darcy Flow in Porous Media and Fractures**

by

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**A report submitted in partial fulfilment of the requirements for the MSc and/or the DIC**

**September 2011**

## DECLARATION OF OWN WORK

I declare that this thesis *Forchheimer Model for Non-Darcy Flow in Porous Media and Fractures* is entirely my own work and that where any material could be construed as the work of others, it is fully cited and referenced, and/or with appropriate acknowledgement given.

Signature:

Name of student: Dastan Takhanov

Name of supervisor: Prof. R. W. Zimmerman

## ABSTRACT

Darcy's law, which states that a fluid flow rate is directly proportional to the pressure gradient, is shown to be accurate only at low flow velocities, both in liquid and gas phase systems. At higher flow rates, Darcy's law is usually replaced by the Forchheimer equation, which includes a term that is quadratic in the flow rate. However, there is no clear or simple derivation of the Forchheimer equation. Moreover, there are no simple and accurate expressions available to predict the "non-Darcy" flow coefficient that appears in the Forchheimer equation. The purpose of this study is to assess the validity of the Forchheimer model (i.e. is the new term that should be added to Darcy's law actually quadratic in respect with the flow rate?) Another aim is to develop a better understanding of the relationship between the numerical value of the Forchheimer coefficient and other rock properties such as permeability and porosity.

In this study, a literature review on the theory behind the Forchheimer model and the available correlations of the Forchheimer coefficient was performed. The data from literature for unconsolidated and consolidated porous media, and fractured gas systems, were analyzed, and the validity of the Forchheimer model was assessed. Furthermore, correctness of the raw experimental data, analysis used and validity of the correlations proposed for the estimation of the Forchheimer coefficient in both single and multiple phase systems, was analysed. The results of the study revealed the existence of a weak inertia regime for the lowest Reynolds numbers in the range of 8-500, where the additional non-Darcy pressure drop is proportional to the cube of the mass flux for fractures with large widths. High velocity flow in porous media and fractures can be modelled by the Forchheimer equation, in which the non-Darcy pressure drop is quadratic in the mass flux. This flow regime corresponds with the higher values of Reynolds numbers. Geertsma's (1974) correlation gives a reasonable estimate of the Forchheimer coefficient for porous media, whereas the Pascal et al. (1980) correlation possibly gives a better estimate of the Forchheimer coefficient for fractures.

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**Forchheimer Model for Non-Darcy Flow in Porous Media and Fractures**

Dastan Takhanov

Imperial College supervisor: Prof. R. W. Zimmerman

**Abstract**

Darcy's law, which states that a fluid flow rate is directly proportional to the pressure gradient, is shown to be accurate only at low flow velocities, both in liquid and gas phase systems. At higher flow rates, Darcy's law is usually replaced by the Forchheimer equation, which includes a term that is quadratic in the flow rate. However, there is no clear or simple derivation of the Forchheimer equation. Moreover, there are no simple and accurate expressions available to predict the "non-Darcy" flow coefficient that appears in the Forchheimer equation. The purpose of this study is to assess the validity of the Forchheimer model (i.e. is the new term that should be added to Darcy's law actually quadratic in respect with the flow rate?) Another aim is to develop a better understanding of the relationship between the numerical value of the Forchheimer coefficient and other rock properties such as permeability and porosity.

In this study, a literature review on the theory behind the Forchheimer model and the available correlations of the Forchheimer coefficient was performed. The data from the literature for unconsolidated and consolidated porous media, and fractured gas systems, were analyzed, and the validity of the Forchheimer model was assessed. Furthermore, the correctness of the raw experimental data, analysis used and validity of the correlations proposed for the estimation of the Forchheimer coefficient in both single and multiple phase systems, was analysed. The results of the study revealed the existence of a weak inertia regime for the lowest Reynolds numbers in the range of 8-500, where the additional non-Darcy pressure drop is proportional to the cube of the mass flux for fractures with large widths. High velocity flow in porous media and fractures can be modelled by the Forchheimer equation, in which the non-Darcy pressure drop is quadratic in the mass flux. This flow regime corresponds with the higher values of Reynolds numbers. Geertsma's (1974) correlation gives a reasonable estimate of the Forchheimer coefficient for porous media, whereas the Pascal et al. (1980) correlation possibly gives a better estimate of the Forchheimer coefficient for fractures.

**Introduction**

Flow regime in porous media and fractures is generally characterized by a dimensionless number, the Reynolds number (*Re*) (Skjetne 1995):

$$Re = \frac{\rho v l}{\mu} \dots\dots\dots (1)$$

where  $\rho$  is the fluid density [ $\text{kg m}^{-3}$ ],  $v$  is the seepage velocity (Darcy velocity) [ $\text{m s}^{-1}$ ],  $l$  is the characteristic length [m], and  $\mu$  is the fluid viscosity [Pa s]. The characteristic length is different for porous media and fractures, and it is normally accepted as the fracture width ( $w$ ) for parallel plates (Skjetne 1995), and the diameter of the grain ( $d_p$ ) (Fancher et al. 1933), respectively.

Recently, flow regimes in porous media were classified (Skjetne 1995) as shown in Figure 1. According to this classification, flow regime changes as follows, from small *Re* to large *Re*: 1 = Darcy, 2 = Weak inertia, 3 = Forchheimer (strong inertia), 4 = transition from Forchheimer to turbulence, and 5 = turbulence.

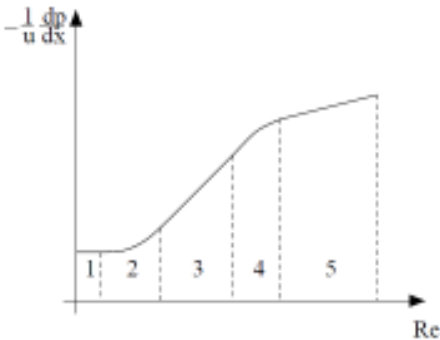


Fig.1-Flow regimes in a porous media. 1 = Darcy's law , 2 = Weak inertia, 3 = Forchheimer (strong inertia), 4 = transition from Forchheimer to turbulence, and 5 = turbulence. (after Skjetne 1995)

Darcy's law (Darcy, 1856) is valid to describe flow in porous media and fractures at low flow rates ( $Re < 1$ ), when the flow rate and the pressure gradient have a linear relationship. This law assumes that viscous forces dominate over inertial forces in porous media; hence, inertial forces can be neglected. For a one-directional steady state flow of an incompressible Newtonian fluid through a horizontal porous medium, it could be written as

$$-\frac{\partial P}{\partial x} = \frac{\mu}{k} V \dots\dots\dots (2)$$

where  $P$  is the pressure [Pa],  $V$  is the seepage velocity (Darcy velocity) [ $\text{m s}^{-1}$ ],  $x$  is the space coordinate [m],  $\mu$  is the fluid dynamic viscosity [Pa s], and  $k$  is the medium's permeability (Darcy permeability) [ $\text{m}^2$ ].

However, as the flow rate (Reynolds's number) increases, inertial forces become more significant and the relationship between pressure gradient and seepage velocity become non-linear. In such a case, Darcy's law is generally corrected by a cubic term in seepage velocity.

$$-\frac{\partial P}{\partial x} = \frac{\mu}{k} V + \frac{\gamma}{\mu} \rho^2 V^3 \dots\dots\dots (3)$$

where  $k$  is the medium's permeability (Darcy permeability) [ $\text{m}^2$ ],  $\gamma$  is the weak inertia factor or  $\gamma$ -factor [-]. Weak inertia is a regime in which the inertial force is of the same order as the viscous force. Firstly this deviation from Darcy's law was presented numerically by Barrere (1990) and later by Mei and Auriault (1991). Wodie and Levy (1991) derived this law analytically for homogeneous isotropic and spacially periodic porous media by double scale homogenization. The weak inertia equation was derived for both microscopic and macroscopic scale porous media. The existence of weak inertia regime numerically was confirmed by Rasoloarijaona and Auriault (1994) and Skjetne (1995) and experimentally by Skjetne and Auriault (1997).

As the flow rate (Reynolds number) increases further, the pressure loss transforms from weak inertia regime to Forchheimer (strong inertia) regime, where a pressure drop is proportional to the square of seepage velocity (Forchheimer 1901).

$$-\frac{\partial P}{\partial x} = aV + bV^2 \dots\dots\dots (4)$$

where  $a$  and  $b$  are constants. Muskat (1937), Green and Duwez (1951) and Cornell and Katz (1953) related coefficients  $a$  and  $b$  to fluid and rock properties and reformulated the Forchheimer equation:

$$-\frac{\partial P}{\partial x} = \frac{\mu}{k_{fh}} V + \beta \rho V^2 \dots\dots\dots (5)$$

where  $k_{fh}$  is the Forchheimer permeability [ $\text{m}^2$ ],  $\beta$  is the inertial resistance (coefficient), non-Darcy flow coefficient, or Forchheimer coefficient [ $\text{m}^{-1}$ ]. Eq. 5 is commonly known as the Forchheimer equation, where  $k \neq k_{fh}$  because of weak inertia transition regime. It is assumed that the main reason for this phenomena is viscous dissipation (conversion of kinetic energy into internal energy by work done against the viscous stress). At low  $Re$ , viscous dissipation is caused by the viscous force and remains constant. At higher  $Re$ , the viscous dissipation will rise because of strong inertial forces, following by change in apparent permeability of the media.

The Forchheimer equation was proven valid by experimental data given in Muskat (1937), Ahmed and Sunuda (1969) Scheidegger (1960), Chauveteau (1967), Wright (1968), Dulien and Azzam (1973), Geertsma (1974), Firoozabadi and Katz (1979) and Tiss and Evans (1989). Revision of Forchheimer equation was done by Savins (1970), Bear (1972), and Dullin (1992).

Moreover, the physical understanding of the Forchheimer law is not clear. According to different researchers' work, the source of the onset of the nonlinear behavior is different. Currently, there seems to be no clear or simple derivation of the Forchheimer equation. Moreover, there are no simple accurate expressions available to predict the "non-Darcy" flow coefficient that appears in the Forchheimer equation. In order to improve the understanding of the Forchheimer equation, we assessed the validity of the Forchheimer model (i.e., is the new term that should be added to Darcy's law actually quadratic in  $q$ ?) using the data from literature. The goal is to develop a better understanding of the relationship between the numerical value of the Forchheimer coefficient and other rock properties such as permeability and porosity.

This thesis is organized as follows. First, we review the literature on derivation of the Forchheimer equation and validate the Forchheimer equation for unconsolidated, consolidated porous media and fractures using experimental data obtained from the literature. Second, we critically review many correlations between the Forchheimer coefficient and rock properties available in the literature, and the assumptions that the correlation are based on, in order to select the most suitable correlation. Finally, we discuss the findings of this project, and present some concluding remarks.

## Literature review on derivation of the Forchheimer equation

This section presents revision of the theory behind the Forchheimer equation and the literature for flow in porous media and fractures. In addition, the Forchheimer equation is validated based on available data obtained from the literature.

There are macroscale and microscale approaches used to derive the Forchheimer equation. Macroscale approaches derive the Forchheimer equation from the Navier-Stokes equation at the continuum scale. Microscale approach represents a porous medium as a group of capillary tubes aligned in series or parallel and use simple analytical solutions.



**Macroscale approaches**

The Forchheimer equation was derived from the Navier-Stokes equation for the model of spheres of equal diameters for homogenous isotropic medium by Irmay (1958). This approach was based on an assumption that flow macroscopically is one dimensional. Similar research was carried by Bachmat (1965), Ahmed and Sunada (1969) and Dullien and Azzam (1973). Hassanizadeh and Gray (1987) derived the Forchheimer equation based on fundamental laws of continuum mechanics for high velocity isothermal flow through uniform isotropic porous media. Ruth and Ma (1992) derived the Forchheimer equation by means of the averaging theorem for highly idealized periodic porous media. Giorgi (1997) derived the Forchheimer law using matched asymptotic for a rigid porous medium. The Forchheimer equation was derived by Liu and Masliyah (1999) and Tsakiroglou (2002) for nonidealized media with a generalized Newtonian fluid, and by Chen et al. (2001) for compressible fluids. Fourar et. al. (2004) performed numerical simulations for a steady-state flow of an incompressible Newtonian fluid at intermediate and high Reynolds number through a two and a three-dimensional periodic porous medium. They showed that for 2-D porous media, flow goes through Darcy’s regime, the transition regime and the strong inertia regime, while for 3-D porous media there is a significantly reduced transition regime. Hence, flow in the non-Darcy regime can adequately described by the Forchheimer equation. Zimmerman et al. (2004) performed Navier-Stokes simulations and laboratory flow experiments on an epoxy cast of a rock fracture and confirmed the existence of a weak inertia and Forchheimer regimes. They concluded that ‘Forchheimer equation can probably be used over the entire range of  $Re$ , as it effectively reduces to Darcy’s law at low values of  $Re$ ’.

**Microscale approaches**

In this approach, idealized pore geometry was modelled through capillaries in series, capillaries in parallel, or packed beds of spheres to derive the Forchheimer equation by Scheidegger (1960), Rumer and Drinker (1966) and Blick (1966). Blick (1966) modelled a porous medium as a bundle of parallel tubes with orifice plates spaced along each other and filled with fluid. This approach was based on the assumption that the medium rigid, and fluid to be homogenous and Newtonian. Lucas et al. (2007) analysed the model of a crenulated channel and examined the influence of the flow asymmetry (non-periodicity) on flow law. This research shows that it is impossible and sometimes numerically very difficult to achieve quadratic term appearance in the periodic case. They concluded that non-periodicity is significant factor responsible for a quadratic term in Forchheimer model. Chai et al. (2010) performed a numerical investigation on the non-Darcy effect on incompressible flows through disordered porous media, and confirmed that quadratic term in the Forchheimer model may be a result of flow non-periodicity in disordered porous media.

The source of the onset of the non-linear behavior is not well understood. The earlier study by Muskat (1937) related the non-linear squared term in the Forchheimer equation to turbulence, based on turbulent flow in pipes. This theory was rejected by Scheidegger (1960), Bear (1972) and Geertsma’s (1974) experimental work, where the onset of non-linearity starts before the flow becomes turbulent. They concluded that flow was not turbulent because of large linear term and the transition was not sharp as in pipe flow. According to Scheneebeli (1955) and Beliashevsky (1974) inertia forces in a laminar flow are the source of the onset of the nonlinear behavior. Minsky (1951) relates non-linearity to a macro-roughness of pores, Houpeurt (1953) to a kinetic energy losses in restrictions and constrictions, and Chauveteau (1967) to a streamline pattern deformation by inertia forces. Geertsma (1974) relates it to convective acceleration and deceleration of the fluid particles in porous media. Hassanizadeh (1987) derived the Forchheimer equation based on fundamental laws of continuum mechanics and concluded that viscous forces (drag forces) at microscopic level are the source of non-linearity. According to Barak (1987) and Mei and Auriault (1991), the inertial effects is the source of the onset of the non-linearity. Ma and Ruth (1993) relate non-linearity to the microscopic inertial force which has been manifested in the interfacial drag force, while Hayes et al. (1995) relate it to channel curvature. Whitaker (1996) relate non-linearity to a viscous boundary layer, Panfilov et al. (2003) to a singularity of streamline pattern and/or microscale flow non-periodicity, Fourar et al. (2004) to a space dimension and Panfilov and Fourar (2006) to a pure inertia and an inertia-viscous cross effect. It is generally accepted that the quadratic term is related to inertia effects in the laminar regime.

**Methodology for validation of the Forchheimer equation**

Experimental data for flow in a consolidated Berea sandstone core (Firoozabadi et al. 1995), natural and artificial unconsolidated porous media (Macini et al. 2011), and sandstone fractures (Skjetne et al. 1999) were used to assess the validity of the Forchheimer equation. These data were analyzed separately, as they represent different types of porous media: consolidated porous media, unconsolidated porous media, and fractures.

Since mass flux ( $\rho V$ ) for gas remains constant in steady state flow even when the gas may be expanding, we express the pressure gradient through mass flux as:

$$\rho \left( -\frac{\partial P}{\partial x} \right) = \frac{\mu}{k_{fh}} \rho V + \beta (\rho V)^2 \dots\dots\dots (6)$$

As per gas law:

$$\rho = \frac{M}{ZRT} P \dots\dots\dots (7)$$

$$-\frac{M}{RT} P_r \int_{P_{r1}}^{P_{r2}} \frac{P_r}{Z} dP_r = \left[ \frac{\mu}{k_{fh}} \rho V + \beta (\rho V)^2 \right] \int_1^2 dL \dots\dots\dots (8)$$

$$-\frac{M}{RT} P_r^2 \left[ \int_{0.2}^{P_{r2}} \frac{P_r}{Z} dP_r - \int_{0.2}^{P_{r1}} \frac{P_r}{Z} dP_r \right] = \left[ \frac{\mu}{k_{fh}} \rho V + \beta(\rho V)^2 \right] L \dots\dots\dots (9)$$

where  $T$  is the temperature [K].

Assuming that  $\mu$ ,  $T$ ,  $Z$  are constant for small pressure gradients in the experiments, and ignoring the Klinkenberg slip effect, the following equation may be derived:

$$\frac{(P_1^2 - P_2^2)M}{2ZR TL} = \frac{\mu}{k_{fh}} \rho V + \beta(\rho V)^2 \dots\dots\dots (10)$$

Rearranging the above gives:

$$\frac{(P_1^2 - P_2^2)M}{2ZR TL \rho V} = \frac{\mu}{k_{fh}} + \beta \rho V \dots\dots\dots (11)$$

where  $P_1$  and  $P_2$  are the inlet and outlet pressures [Pa],  $M$  is the molecular weight of the gas [g/mol],  $Z$  is the gas compressibility [Pa<sup>-1</sup>],  $R$  is the universal gas constant [8.314 J/(molK)], and  $L$  is the core length [m]. Data were analyzed using equations (10) and (11). The validity of the the Forchheimer equation could be indicated by a straight line when plotted  $\frac{(P_1^2 - P_2^2)M}{2ZR TL \rho V}$  vs. mass flux  $\rho V$ . In this plot, the slope of the straight line equal to  $\beta$  (Forchheimer coefficient) and the intercept is equal to  $\mu/k_{fh}$ .

Forchheimer plots are presented with respect to Reynolds number ( $Re$ ), where the mass flux is replaced by an interstitial Reynolds number.

For fractures (Skjetne et al., 1999)  $Re$  is obtained by replacing  $v$  and  $l$  in Eq.1 by interstitial velocity  $\tilde{V}$  and  $w$ .

$$\tilde{V} = \frac{v}{\phi} \dots\dots\dots (12)$$

where,  $\phi$  is the porosity [-]:

$$\phi = \frac{wh}{S} \dots\dots\dots (13)$$

where  $h$  is the height of the fracture [m], and  $S$  is the cross-sectional area of the core [m<sup>2</sup>]:

$$Re = \frac{\rho vl}{\mu} = \frac{\rho \tilde{V} w}{\mu} = \frac{\rho V w}{\mu \phi} = \frac{(\rho V) S}{\mu h} \dots\dots\dots (14)$$

From Skjetne et al. (1999)  $h = 0.03$  m,  $S = 0.0007065$  m<sup>2</sup> and  $\mu = 0.0000174$  Pa s. Combination of Eqs. 11 and 14 for fractures yields,

$$\frac{(P_1^2 - P_2^2)M}{2ZR TL \rho V} = \frac{\mu}{k_{fh}} + \beta \frac{\mu h}{S} Re \dots\dots\dots (15)$$

For unconsolidated (Macini et al. 2011) and consolidated (Firoozabadi et al. 1995) porous media,  $Re$  is obtained by replacing  $v$  and  $l$  in Eq.1 by interstitial velocity  $\tilde{V}$  and  $d_p$ :

$$Re = \frac{\rho vl}{\mu} = \frac{\rho \tilde{V} d_p}{\mu} = \frac{(\rho V) d_p}{\mu \phi} \dots\dots\dots (16)$$

Combination of Eqs. 11 and 16 yields

$$\frac{(P_1^2 - P_2^2)M}{2ZR TL \rho V} = \frac{\mu}{k_{fh}} + \beta \frac{\mu \phi}{d_p} Re \dots\dots\dots (17)$$

From Macini et al. (2011) average  $d_p = 0.000387$ m,  $\phi = 0.43$  and  $\mu = 0.0000176$  Pa s. From Firoozabadi et al. (1995) average  $d_p = 0.000200$ m,  $\phi = 0.22$  and  $\mu = 0.0000176$  Pa s.

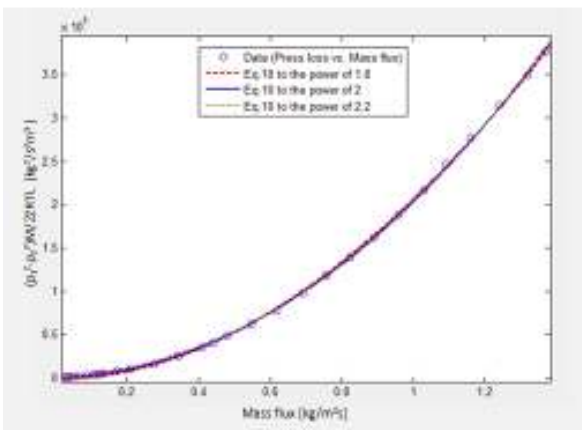


Fig.2-Darcy plot. Skjetne et al. (1999).  $w_a=50$   $\mu$ m. (Circles) experimental data, (red dashed line) Eq.10 to the power of 1.8 ( $R^2=0.9998$ ), (blue solid line) Eq. 10 to the power of 2 ( $R^2=0.9999$ ), (brown dotted line) Eq. 10 to the power of 2.2 ( $R^2=0.9997$ ).

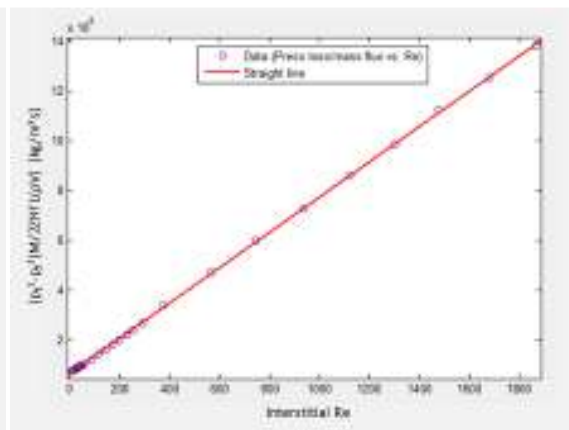


Fig.3-Forchheimer plot. Skjetne et al. (1999).  $w_a=50$   $\mu$ m. (Circles) experimental data, (red solid line) Eq.15.

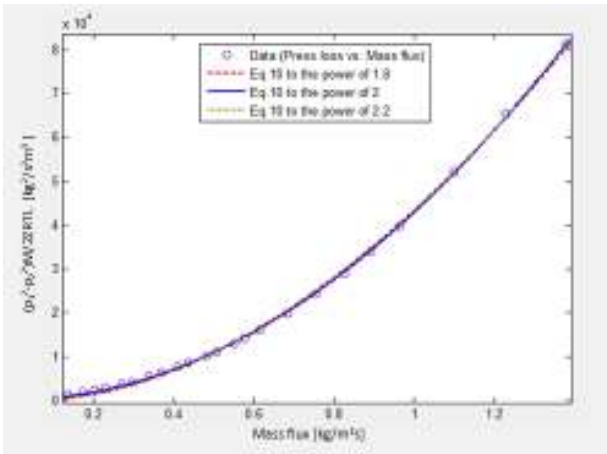


Fig.4-Darcy plot. Skjetne et al. (1999).  $w_a=800 \mu\text{m}$ . (Circles) experimental data, (red dashed line) Eq.10 to the power of 1.8 ( $R^2=0.9989$ ), (blue solid line) Eq.10 to the power of 2 ( $R^2=0.9997$ ), (brown dotted line) Eq. 10 to the power of 2.2 ( $R^2=0.9989$ ).

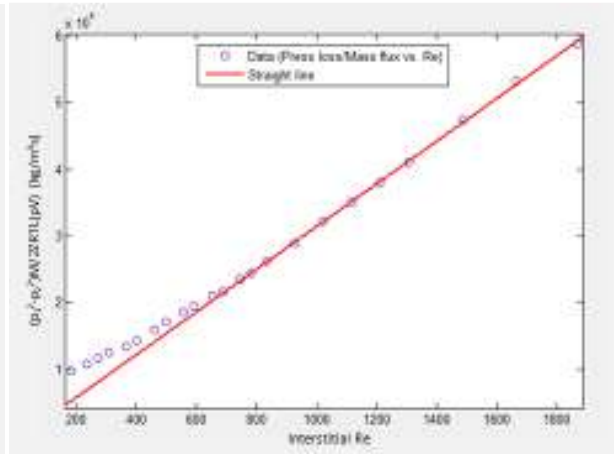


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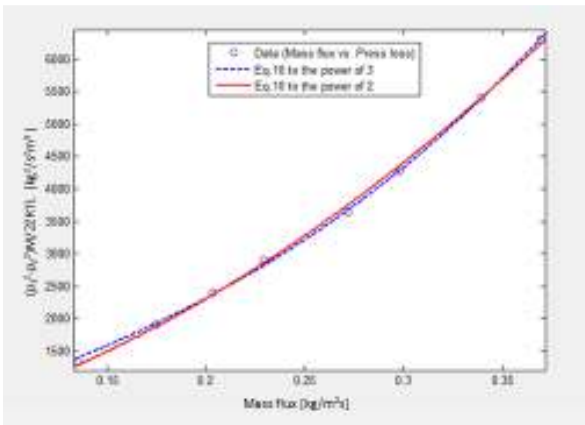


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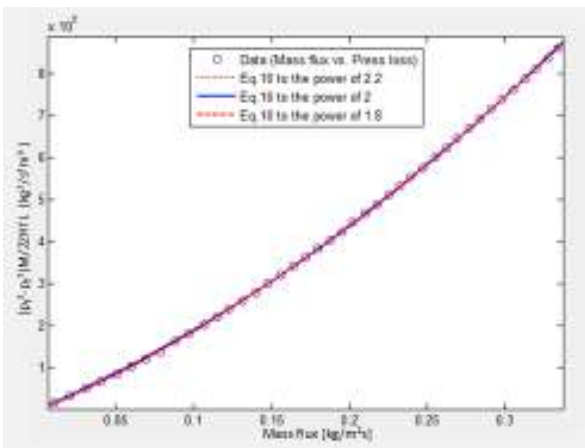


Fig.7-Darcy plot. Macini et al. (2011). (Circles) experimental data, (red dashed line) Eq.10 to the power of 1.8 ( $R^2=0.9999$ ), (blue solid line) Eq.10 to the power of 2 ( $R^2=0.9999$ ), (brown dotted line) Eq.10 to the power of 2.2 ( $R^2=0.9998$ ).

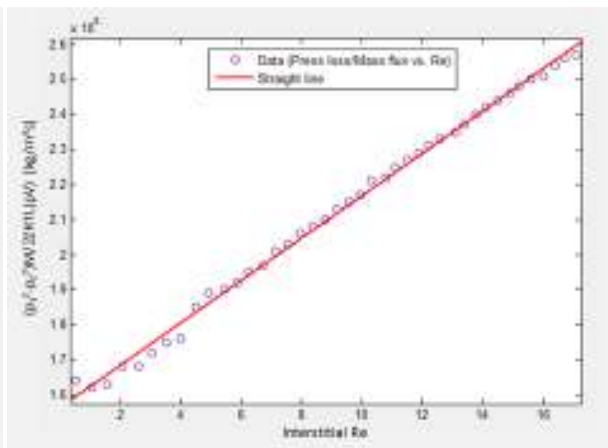


Fig.8-Forchheimer plot. Macini et al. (2011). (Circles) experimental data, (red solid line) Eq.15.

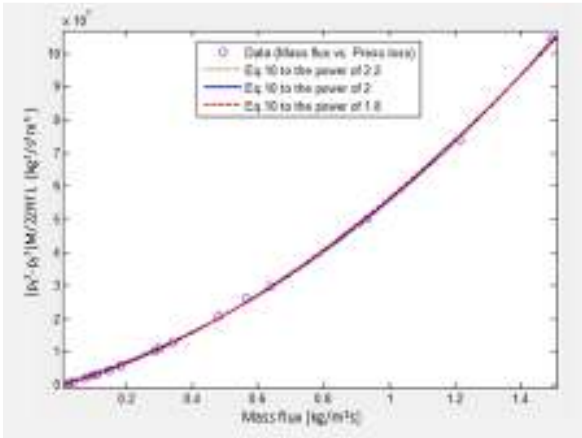


Fig.9-Darcy plot. Firoozabadi et al. (1995). (Circles) experimental data, (red dashed line) Eq.10 to the power of 1.8 ( $R^2=0.9995$ ), (blue solid line) Eq.10 to the power of 2 ( $R^2=0.9997$ ), (brown dotted line) Eq.10 to the power of 2.2 ( $R^2=0.9997$ ).

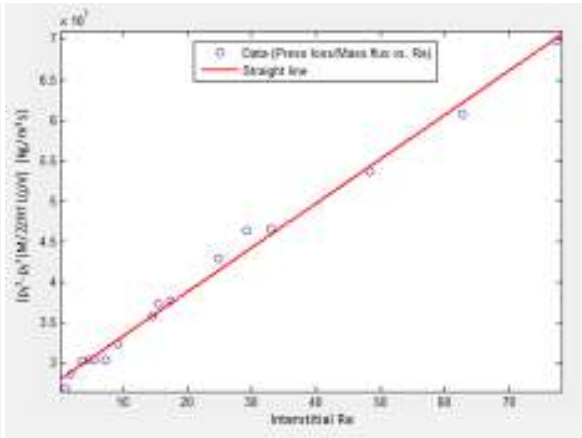


Fig.10-Forchheimer plot. Firoozabadi et al. (1995). (Circles) experimental data, (red solid line) Eq.15.

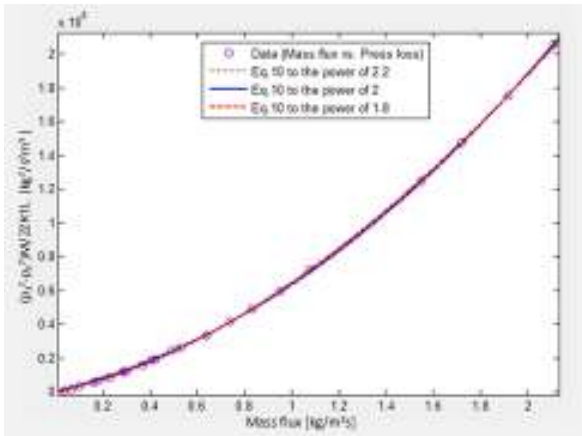


Fig.11-Darcy plot. Firoozabadi et al. (1995). (Circles) experimental data, (red dashed line) Eq.10 to the power of 1.8 ( $R^2=0.9999$ ), (blue solid line) Eq.10 to the power of 2 ( $R^2=0.9999$ ), (brown dotted line) Eq.10 to the power of 2.2 ( $R^2=0.9998$ ).

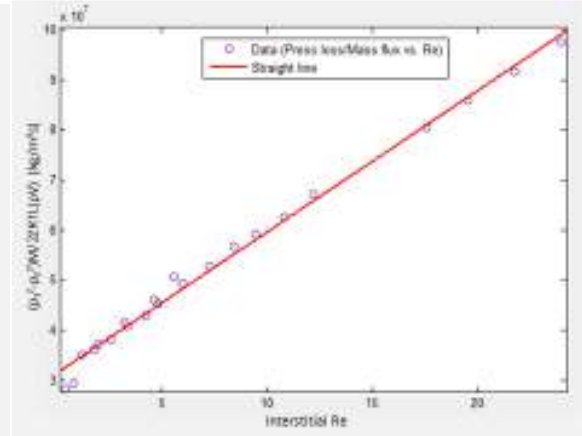


Fig.12-Forchheimer plot. Firoozabadi et al. (1995). (Circles) experimental data, (red solid line) Eq.15.

**Correlations between Forchheimer coefficient and Rock Properties**

The  $\beta$  coefficient is given several names in the literature, as inertial resistance (coefficient), non-Darcy flow coefficient, or Forchheimer coefficient. In this project we will use the name “Forchheimer coefficient”. It is accepted by the most researchers that  $\beta$  is a property of the porous medium, and it is usually determined from laboratory measurements and multi-rate well tests. However such data are not readily available, so many relations between Forchheimer coefficient and rock properties proposed in the literature. They generally could be divided as empirical correlations and theoretical equations (Li and Engler, 2001).

**Theoretical Equations**

Scheidegger (1953 & 1974) and Bear (1972) summarized many research on capillaries in series and in parallel models.

**Capillaries in parallel model.** Capillaries in parallel model assume that a porous medium is made of group of straight, parallel capillary tubes of uniform diameter. Li and Engler (2001) compared non-linear laminar flow equation by Ergun and Orning (1949) and Irmay (1958) to find Forchheimer coefficient as

$$\beta = \frac{c}{k^{0.5} \phi^{1.5}} \dots \dots \dots (18)$$

where  $c$  is a constant [-],  $k$  is the permeability [ $m^2$ ], and  $\phi$  is the porosity [-]. The main disadvantage of this model is that capillary tubes go through porous media without changing diameter.

**Capillaries in series models.** Capillaries in series model assume that capillary tubes of varying diameter are aligned in series. Li and Engler (2001) comparing non-linear laminar flow equation by Scheidegger (1953 & 1974) and Forchheimer (1901) equation find Forchheimer coefficient as:

$$\beta = \frac{c'' \tau}{k \phi} \dots \dots \dots (19)$$

where  $c''$  is a constant related to pore size distribution [-], and  $\tau$  is the tortuosity [-].

However, these simplified representations of porous media (capillary model) do not adequately describe the real process taking place, as the flow path is idealized. Studies on the range of applicability of the capillary model was done by Mauret and Renaud (1997) and Puncochar and Drahos (2000).

### Empirical Correlations

Li and Engler (2001) divided empirical correlations to one-phase and multi-phase systems. Results of these experiments are different, because of variety of porous media, fluids and analysis used in experiments. Some of the previous correlations may be invalid, due to the poor data quality, incorrect analysis, or the correlation may be weak.

**One-Phase Systems.** Janicek and Katz (1955) re-correlated data by Cornell and Katz (1953) and for natural porous media (limestone, dolomite and sandstone) suggest following,

$$\beta = 1.82 \times 10^8 k^{-5/4} \phi^{-3/4} \dots \dots \dots (20)$$

where  $k$  is the permeability [mD], and  $\beta$  is the Forchheimer coefficient[1/cm]. Gewers and Nichol (1969) re-evaluated the Janicek and Katz (1955) correlation for limestone cores, and found that values of the Forchheimer coefficient an order of magnitude higher than predicted by the Janicek and Katz (1955) correlation. This was the result of higher degree of inhomogeneity of limestone cores.

Tek et al. (1962) analyzed data by Cornell and Katz (1953) and correlation by Janicek and Katz (1955), and proposed

$$\beta = \frac{5.5 \times 10^9}{k^{5/4} \phi^{3/4}} \dots \dots \dots (21)$$

According to Geertsma (1974), this equation is dimensionally incorrect and poorly predicts for a large range of permeabilities.

Cooke (1973), based on experiments on for brines, reservoir oils, and gases in propped fractures under stress derived

$$\beta = bk^{-a} \dots \dots \dots (22)$$

where  $k$  is the permeability [Darcy], and  $a$  and  $b$  are constants, experimentally determined based on proppant size. This correlation predicts values 2-50 times higher than the correlation for natural porous media proposed by Janicek and Katz (1955). Crushed proppant sand increases the Forchheimer coefficient, for the same value of permeability, but it was not considered in this equation. Geertsma indicated that Eq. 22 dimensionally incorrect.

Geertsma (1974) carried out dimensional analysis on experimental data for unconsolidated sands with liquids and gases, consolidated sands with gases, together with data obtained by Green and Duwez (1951) and Cornell and Katz (1953) and presented,

$$\beta = \frac{0.005}{k^{0.5} \phi^{5.5}} \dots \dots \dots (23)$$

where  $k$  is the permeability [ $\text{cm}^2$ ] and  $\beta$  is the Forchheimer coefficient [1/cm]. This correlation proved to be almost always valid for sandstones, but not for limestone because of their complicated pore structure. Geertsma correlation gave the best results when compared with correlations available in the literature by Thauvin and Mohanty (1998).

Based on multi-rate tests data from low permeability hydraulically fractured wells, Pascal et al. (1980) expressed the Forchheimer coefficient as

$$\beta = \frac{4.8 \times 10^{12}}{k^{1.176}} \dots \dots \dots (24)$$

where  $k$  is the permeability [md] and  $\beta$  is the Forchheimer coefficient [1/m].

Jones (1987) carried out experiments on 355 sandstone and 29 limestone cores, which include laminated, shaly, tight sandstones, micro-fractured samples, vuggy limestones, clean, fine- grained sandstones, very fine grained argillaceous sandstones, sucrosic dolomite, calcareous sands, and crystalline limestones containing inclusions of various sizes. After analysis of this data the Forchheimer coefficient was estimated as

$$\beta = \frac{6.15 \times 10^{10}}{k^{1.55}} \dots \dots \dots (25)$$

where  $k$  is the permeability [md] and  $\beta$  is the Forchheimer coefficient [1/ft]. Coles and Hartman (1998) noticed that for a given permeability there is a spread in the Forchheimer coefficient values of about one order of magnitude. Additionally, Forchheimer coefficients for limestone samples were higher than those obtained for sandstone samples for given permeability. The Forchheimer coefficient estimation for limestone reservoirs may be erroneous, as this correlation is mainly based on sandstone samples compared to limestone samples.

Liu et al. (1995) re-examined data acquired by Cornell and Katz (1953), Geertsma (1974), Evans et al. (1987), Evans and Evans (1988), and Whitney (1988) by including tortuosity and showed new correlation for consolidated and unconsolidated porous media,

$$\beta = 8.91 \times 10^8 k^{-1} \phi^{-1} \tau \dots \dots \dots (26)$$

where  $k$  is the permeability [md] and  $\beta$  is the Forchheimer coefficient [1/ft].

By comparing Ergun's (1952) empirical flow equation for bead pack and the Forchheimer equation, Thauvin and Mohanty (1998) identified,

$$\beta = ab^{-1/2} (10^{-8} k)^{-1/2} \phi^{-3/2} \dots \dots \dots (27)$$

where  $a = 1.75$ ,  $b = 150$ ,  $k$  is as the permeability [Darcy], and  $\beta$  is the Forchheimer coefficient [1/cm]. MacDonald et al. (1979) found  $b = 180$  and  $a$  between 1.8 and 4, when he generalized the Ergun (1952) equation for particles of different roughness. Thauvin and Mohanty (1998) developed microscopic network model for high velocity flow proposed following:

$$\beta = \frac{1.55 \times 10^4 \tau^{3.35}}{k^{0.98} \phi^{0.29}} \dots \dots \dots (28)$$

where  $k$  has units of [Darcy],  $\beta$  has units of [1/cm] and  $\tau$  is the tortuosity [-]. Weak correlation for all type of porous media.

Cooper et al (1999) performed research in anisotropic porous media at the pore level. They concluded that permeability and tortuosity have main influence in determining the Forchheimer coefficient:

$$\beta = \frac{10^{-3.25} \tau^{1.943}}{k^{1.023}} \dots \dots \dots (29)$$

where  $k$  has units of [cm<sup>2</sup>] and  $\beta$  has units of [1/cm].

Li et al. (2001) performed experiments on Berea sandstone core to simulate wafer non-Darcy flow and proposed correlation for heterogeneous porous media,

$$\beta = \frac{11500}{k\phi} \dots \dots \dots (30)$$

where  $k$  has units of [Darcy] and  $\beta$  has units of [1/cm].

Kollbotn and Bratteli (2005) analysed a sandstone cores from North Sea reservoirs, together with data obtained by Geertsma (1974), Firoozabadi (1979) and Katz et al.(1959) (Handbook of Natural Gas Engineering) and developed the following correlation:

$$\beta = \frac{\lambda F}{k} \dots \dots \dots (31)$$

where  $k$  has units of [Darcy] and  $\beta$  has units of [1/ft],  $\lambda$  is new turbulent flow correlation factor [ft], and  $F$  is formation resistivity factor [-]. Because of the nature of new turbulent flow correlation factor is not determined and the method how to measure it is not clear, this correlation needs further development.

Khaniaminjan and Goudarzi (2008) performed experiments on unconsolidated porous media made of sand packing of different grain size with liquids and proposed,

$$\beta = \frac{4.8 \times 10^{11}}{k^{1.8} \phi^{-0.48}} \dots \dots \dots (32)$$

where  $k$  has units of -[md] and  $\beta$  has units of [1/ft]. There was not enough data analyzed to propose this equation. Moreover, Eq. 32 is dimensionally incorrect.

Several researchers performed non-Darcy flow experiments in multiphase systems, and resulted empirical equations to estimate the Forchheimer coefficient presented below.

**Multi-Phase Systems.** Geertsma (1974) was the first to propose a correlation in a two-phase system with immobile water saturation. He concluded that in the two-phase system, permeability must be replaced by the gas effective permeability at a given water saturation, and the porosity must be replaced by the void fraction occupied by the gas.

$$\beta = \frac{0.005}{k^{0.5} \phi^{5.5}} \cdot \frac{1}{(1-S_{wr})^{5.5} k_{rel}^{0.5}} \dots \dots \dots (33)$$

where  $S_{wr}$  is the residual water saturation,  $k_{rel}$  is the gas relative permeability,  $k$  is in cm<sup>2</sup> and  $\beta$  is in 1/cm. The presence of a liquid phase in a gas flow increases the Forchheimer coefficient compared to a dry system.

Wong (1970) studied the effect of mobile-liquid saturation on the Forchheimer coefficient in limestone cores and found an eightfold increase in  $\beta$  when liquid saturation increased from 40 to 70%. Evans and Evans (1988) showed that even low mobile-liquid saturation could increase the Forchheimer coefficient by order of magnitude. Same results were presented by Al-Rumhy and Kalam (1996), Grigg and Hwang (1998), Coles and Hartman (1998) and Lombard et al. (2000). Noh and Firoozabadi (2008) studied the effect of wettability on the Forchheimer coefficient in two-phase gas/liquid flow and showed significant increase of mobile liquid phase on the Forchheimer coefficient (270-fold for strongly liquid wetting rock). But for intermediate a wetting rock liquid saturation increase results less increase in Forchheimer coefficient. However, Liu et al. in (1995) argued that in Eq. 33 is completely inaccurate even it is dimensionally correct. They suggest including tortuosity and that the effect of existence of second fluid can be incorporated with effective permeability only and there is no need in replacement of porosity by the void fraction occupied by the gas. Eq. 33 is not valid at high water saturation (40-70%), because it is based on the assumption that water is irreducible, hence boundary between liquid and gas phases is constant.

Kutasov (1993) derived correlation for obtaining Forchheimer coefficient based on experiments:

$$\beta = \frac{1432.6}{k_g^{0.5} [\phi(1-S_w)]^{1.5}} \dots \dots \dots (34)$$

where  $k_g$  is the gas effective permeability in Darcy,  $\beta$  is in 1/cm, and  $S_w$  is the water saturation.

Frederick and Graves (1994) re-examined experimental data acquired by Cornell and Katz (1953), Geertsma (1974), and Evan et al. (1986) together with his own experimental data and proposed

$$\beta = \frac{2.11 \times 10^{10}}{k_g^{1.55} [\phi(1-S_w)]} \dots \dots \dots (35)$$

$$\beta = \frac{1}{[\phi(1-S_w)]^2} e^{45 - \sqrt{407 + 81 \times \ln(k_g / [\phi(1-S_w)])}} \dots \dots \dots (36)$$

where  $k_g$  is the gas effective permeability in [mD],  $\beta$  is in [1/ft].

Coles and Hartman (1998) conducted experiments on cores with nitrogen and paraffin wax. Research shows an increase in  $\beta$ , with increase in paraffin saturation. For paraffin saturation up to 20%,  $\beta$  was estimated as

$$\beta = \beta_{dry} \exp(6.265 S_p) \dots \dots \dots (37)$$

where  $\beta_{dry}$  is the Forchheimer coefficient for one-phase, and  $S_p$  is the paraffin saturation. However correlation does not predict the Forchheimer coefficient accurately above 20% liquid saturation. Moreover it has not been validated for various rock types and a wider range of permeabilities. This analysis based on assumption that  $\beta$  obtained from transient analysis using solidified paraffin is comparable to transient method with immobile liquid. The assumption still has not been proven.

Li and Engler (2001) proposed following (Appendix C) procedure to choose among correlations for prediction Forchheimer coefficient.

## Discussion

**Fractures.** The fractures studied in this project are Berea sandstone fractures data presented by Skjetne et al. (1999). In these high-permeability media Darcy's law often fails, because the velocities can be outside the creeping flow regime. The Darcy plots in Fig. 2 and Fig. 4 show Eq. 10 (Forchheimer equation), where pressures drop proportionally to the power of 1.8 (red dashed line), 2.0 (blue solid line) and 2.2 (brown dotted line) of the mass flux. All three equations show the same trend and accurately fit the experimental data for fractures. Eq. 10 to the power of 2 (solid blue line) shows slightly better results for the flow in fractures compared to same Eq. 10 to the power of 1.8 and 2.2 (Table 1). Fig. 3 and Fig. 5 are Forchheimer plots that follow the straight-line behaviour (Eq.15) for the whole range of Reynolds numbers. However, for large fracture widths (Fig. 5), Eq. 15 does not fit the experimental data at low Reynolds numbers, where the experimental data deviates upward. Weak inertia regime could cause this deviation. Fig. 6 is a Darcy plot that shows Eq. 10, where pressures drop proportionally to the power of 2.0 (red solid line) and 3.0 (blue dashed line) at the low Reynolds numbers. Both quadratic and cubic equations confirm to fit with the data. Cubic equation (blue dashed line) shows better fit than quadratic (red solid line). This confirms that deviation is due to weak inertia regime. There is no indication of a weak inertia regime for small fracture widths (Fig. 3). Weak inertia from a fundamental perspective suggests that the quadratic Forchheimer equation is not universal in laminar flow and not really an "extension" to Darcy's law. Permeability from Forchheimer plot ( $k_{fh}$ ) is different from medium (Darcy) permeability ( $k$ ). Therefore, replacement of Darcy's equation with Forchheimer's equation at low velocities could be erroneous. However, from an engineering importance, the difference is quite small (Balhoff and Wheeler, 2007). Moreover, the cubic correction is mostly negligible. Even though Forchheimer's equation is considered as an empirical model, it has its own limitations. The easy application of this equation makes it useful and adequate in the range of velocities normally observed in oil and gas reservoirs. Linear regression can be used to find the intercept ( $\mu/k_{fh}$ ) and slope  $\beta$  from Fig.3 and Fig.5. The macroscopic parameters for this medium are estimated as  $k_{fh} = 2.68 \times 10^{-11} \text{ m}^2$  and  $\beta = 9.61 \times 10^6 \text{ m}^{-1}$  for  $\omega_a = 50 \text{ } \mu\text{m}$ .

**Unconsolidated porous media.** The unconsolidated porous media studied in this project are natural and artificial sand packings data acquired by Macini et al. (2011). Average particle size  $d_p = 0.000387 \text{ m}$  and porosity = 43%. The Darcy plot in Fig. 7 shows Eq. 10 (Forchheimer equation), where pressures drop proportional to the power of 1.8 (red dashed line), 2.0 (blue solid line) and 2.2 (brown dotted line) of the mass flux. All three equations show same trend and accurately fit experimental data for unconsolidated porous media. Eq. 10 to the power of 2 (solid blue line) shows slightly better results for the flow in fractures compared to same Eq. 10 to the power of 1.8 and 2.2. Fig. 8 is Forchheimer plot that follow the straight line behaviour (Eq.15) for the whole range of Reynolds numbers. There is no indication of a weak inertia regime for unconsolidated porous media. As previously, linear regression can be used to find the intercept ( $\mu/k_{fh}$ ) and  $\beta$ . The macroscopic parameters for this medium are estimated as  $k_{fh} = 1.13 \times 10^{-11} \text{ m}^2$  and  $\beta = 3.09 \times 10^6 \text{ m}^{-1}$ .

**Consolidated porous media.** The consolidated porous media studied in this project is Berea sandstone data acquired by Firoozabadi et al. (1995). Average particle size  $d_p = 0.0002 \text{ m}$  and porosity = 22%. The Darcy plots in Fig. 9 and Fig. 11 show the Eq. 10 (Forchheimer equation) where pressures drop proportional to the power of 1.8 (red dashed line), 2.0 (blue solid line) and 2.2 (brown dotted line) of mass flux. All three equations show same trend and accurately fits experimental data for consolidated porous media. Eq. 10 to the power of 2 (solid blue line) shows slightly better results for the flow in fractures compared to same Eq.10 to the power of 1.8 and 2.2. Fig. 10 and Fig.12 are Forchheimer plots that follow the straight-line behaviour (Eq.15) for the whole range of Reynolds numbers. There is no indication of a weak inertia regime for consolidated porous media. As previously, linear regression can be used to find the intercept ( $\mu/k_{fh}$ ) and  $\beta$ . The macroscopic parameters for this medium are estimated as  $k_{fh} = 6.33 \times 10^{-13} \text{ m}^2$  and  $\beta = 2.83 \times 10^7 \text{ m}^{-1}$ . Nitrogen was used in high velocity flow experiments for all porous media.

Correlations between the Forchheimer coefficient and rock properties are presented in Eq. 18-37. In general, the Forchheimer coefficient decreases with the increase of permeability and porosity. Tortuous flow path in porous media is a major cause of non-Darcy flow (Liu et al. 1995). Hence, the Forchheimer coefficient increases with increase of tortuosity. The effect of overburden stress is included through porosity and permeability. Correctness of the raw experimental data, analysis used and validity of the correlations were commented. As per above comments, not all of the correlations give accurate prediction of the Forchheimer coefficient. Geertsma (1974) correlation seems to give reasonable estimate of the Forchheimer coefficient for porous media with one-phase and multi-phase fluids. Moreover, it is a dimensionally consistent correlation. For hydraulically fractured reservoirs, Pascal et al. (1980) correlation possibly gives a better estimate.

Table 1— Forchheimer equation

Eq.10 to the power of 'n'	Correlation coefficient $R^2$
Berea sandstone fractures. Skjetne et al. (1999). $w_a=50\mu\text{m}$	
Eq.10 to the power of 1.8	0.9998
Eq.10 to the power of 2	0.9999
Eq.10 to the power of 2.2	0.9997
Berea sandstone fractures. Skjetne et al. (1999). $w_a=500\mu\text{m}$	
Eq.10 to the power of 1.8	0.9996
Eq.10 to the power of 2	0.9998
Eq.10 to the power of 2.2	0.9995
Berea sandstone fractures. Skjetne et al. (1999). $w_a=800\mu\text{m}$	
Eq.10 to the power of 1.8	0.9989
Eq.10 to the power of 2	0.9997
Eq.10 to the power of 2.2	0.9989
Berea sandstone fractures. Skjetne et al. (1999). $w_a=950\mu\text{m}$	
Eq.10 to the power of 1.8	0.9994
Eq.10 to the power of 2	0.9999
Eq.10 to the power of 2.2	0.9997
Unconsolidated porous media. Macini et al. (2011)	
Eq.10 to the power of 1.8	0.9999
Eq.10 to the power of 2	0.9999
Eq.10 to the power of 2.2	0.9998
Consolidated porous media. Firoozabadi et al. (1995) (complete core)	
Eq.10 to the power of 1.8	0.9995
Eq.10 to the power of 2	0.9997
Eq.10 to the power of 2.2	0.9997
Consolidated porous media. Firoozabadi et al. (1995) (half a core)	
Eq.10 to the power of 1.8	0.9999
Eq.10 to the power of 2	0.9999
Eq.10 to the power of 2.2	0.9998

### Concluding remarks

Validation of the Forchheimer equation was performed for the gas systems, where the nonlinear flow is much more significant, due to the lower gas viscosity which will give high  $Re$  numbers for the same velocity as in liquid systems. We have presented a validation of the Forchheimer model for unconsolidated, consolidated porous media and fractures. The study provides various observations, which can be summarised in the following remarks:

- The analysis of fractures with large width confirms the existence of a weak inertia regime for the lowest Reynolds numbers in the range of 8-500, where the additional non-Darcy pressure drop is proportional to the cube of the mass flux. This is in agreement with the analysis of Mei & Auriault (1991) and Zimmerman et al. (2004). However, the weak inertia regime was not observed for small-width fractures and porous media.
- High velocity flow in porous media and fractures can be modelled by Forchheimer equation, in which the non-Darcy pressure drop is quadratic in the mass flux, for higher values of Reynolds numbers.
- The Forchheimer equation can probably be used over the entire range of mass fluxes, as it describes the flow in weak inertia regime with slightly less accuracy than cubic relationship.
- In a single phase systems, the Forchheimer coefficient decreases with the increase of permeability and porosity.
- The presence of a second phase drastically increases the Forchheimer coefficient compared to dry system.
- Geertsma (1974) correlation gives reasonable estimate of Forchheimer coefficient for porous media.
- Pascal et al. (1980) correlation possibly gives better estimate of Forchheimer coefficient for fractures.

### Nomenclature

$2D$	= Two dimensions
$3D$	= Three dimensions
$a, b, c$	= Constants
$c''$	= Constant related to pore size distribution
$L$	= Core length, L, m
$M$	= Molecular weight, m, g/mol
$R$	= Universal gas constant, 8.314 J/(molK)
$Re$	= Reynolds number
$S_w$	= Water saturation
$S_{wr}$	= Residual water saturation
$S_p$	= Paraffin saturation
$T$	= Temperature, K
$V$	= Seepage velocity, L/T, m/s



$Z$	= Gas compressibility factor
$k$	= Medium permeability, L <sup>2</sup> , m <sup>2</sup> , mD or Darcy
$k_{fh}$	= Forchheimer permeability, L <sup>2</sup> , m <sup>2</sup> , mD or Darcy
$k_{rel}$	= Gas relative permeability
$k_g$	= Gas effective permeability
$l$	= Characteristic length scale, L, m
$d_p$	= Diameter of the particle or grain, L, m
$P$	= Pressure, m/(T <sup>2</sup> L), Pa or bar
$P_1$	= Inlet pressure, Pa or bar
$P_2$	= Outlet pressure, Pa or bar
$v$	= Typical microscopic velocity, L/T, m/s
$w$	= Fracture width, L, m
$x$	= Space coordinate
$\rho V$	= Mass flux
$F$	= Formation resistivity factor

### Greek letters

$\beta$	= Forchheimer coefficient, inertial resistance (coefficient) or non-Darcy flow coefficient, 1/L, 1/m or 1/ft
$\beta_{dry}$	= Forchheimer coefficient for one-phase, 1/L, 1/m or 1/ft
$\phi$	= Porosity
$\gamma$	= Weak inertia factor
$\mu$	= Fluid viscosity, m/(TL), Pa×s
$\rho$	= Fluid density, m/L <sup>3</sup> , kg/m <sup>3</sup>
$\tau$	= Tortuosity
$\lambda$	= New turbulent flow correlation factor, ft

### Subscripts

$w$	= water
$w_r$	= residual water
$p$	= paraffin
$fh$	= Forchheimer
$rel$	= relative
$g$	= gas
$p$	= particle or grain

### Embellishment

~	=Interstitial (averaged over pore space)
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## Appendices

### Appendix A

#### Critical Milestones

Table A-1—MILESTONES IN FORCHHEIMER MODEL FOR NON-DARCY FLOW IN POROUS MEDIA AND FRACTURES.

SPE Paper n°	Year	Title	Authors	Contribution
	1901	“Wasserbewegung durch Boden”	Forchheimer, P.	First to suggest a nonlinear relationship between hydraulic gradient and flux at large Reynolds numbers
pp.89-91	1933		Lindquist, R.W.	First to find that the failure of Darcy’s law is due to internal forces in the flow through irregular pore space.
	1953	"Flow of Gases Through Consolidated Porous Media"	Cornell, D. Katz, D.L.	Attributed the non-Darcy effect (the nonlinearity between pressure gradient and velocity) to turbulence. Determined $\alpha$ and $\beta$ values of fairly wide variety of reservoir rocks.
702-707	1958	"On the Theoretical Derivation of Darcy and Forchheimer Formulas"	Irmay, S.	Derived theoretically Darcy and Forchheimer equations from the dynamic Navier-Stokes equations
JPT	1970	"Effect of Liquid Saturation on Turbulence Factors for Gas-Liquid Systems"	Wong, S.W.	Found that $\beta$ increased by eight times when liquid saturation increased from 40 to 70 percent
Book pp.176-184	1972	“Dynamics of Fluids in Porous Media”	Bear, J.	Summarized many researchers work on capillary models at large Reynolds number. Internal forces play vital role in deviation from Darcy’s law. Actual turbulence occur at Re values at least one order of magnitude higher than the Re at which deviation from Darcy’s law is observed.
445-450	1974	"Estimating the Coefficient of Inertial Resistance in Fluid Flow through Porous Media"	Geertsma, J.	Introduced an empirical, time-bounded relationship between inertial coefficient and permeability, porosity based on experimental data and dimensional analysis First to propose a $\beta$ correlation in a two-phase system
	1974	“The Physics of Flow through Porous Media”	Scheidegger, A.E.	Modified Parallel and serial type models of porous medium introduced by Kozeny (1927) Derived an equation to describe the non-Darcy flow
14207	1985	“Estimation of Coefficient of Inertial Resistance in High rate Gas Wells”	Norman, R. Shrimanker, N. Archer, J.S.	Estimation of Coefficient of Inertial Resistance in High rate Gas Wells (105 wells) from multi rate pressure tests & supported with experimental data.
PhD thesis, Universite’ Bordeaux I, France	1990	“Modelisation des e’coulements de Stokes et Navier– Stokes en milieux poreux”	Barrere, J.	First numerically found that initial deviation from linearity will be a cubic function of the flow rate, rather than quadratic (weak inertia).
JFM	1991	“The effect of weak inertia on flow through a porous-medium”	Mei, C. C. Auriault, J. L.	First analytically derived that initial deviation from linearity will be a cubic function of the flow rate, rather than quadratic (weak inertia). First found influence of medium anisotropy on the character of nonlinearity. Has not been universally accepted
20709	1990	“The Effects of Non-Darcy Flow in Propped Hydraulic Fractures”	Martins, J.P Milton-Taylor, D. Leung, H.K.	Laboratory tests to investigate non-Darcy flow effects in proppant packs.
24869	1995	“High Velocity Flow in Porous Media”	Firoozabadi, A. Thomas, L.K. Todd, B	Summarized literature on high velocity flow.
	1998	"Network Modelling of Non- Darcy Flow through Porous Media"	Thauvin, F. Mohanty, K.K.	Developed a pore-level network model to describe high velocity flow. Input pore size distributions and network coordination numbers into the model, and obtained outputs such as permeability, non-Darcy coefficient, tortuosity, and porosity. Finally obtained a correlation
Journal	1998	“Darcian, transitional and turbulent flow	Venkataraman, P.	Quadratic Forchheimer law has been confirmed

of Hydraulic Engineering (ASCE) 124:840–846		through porous media”	Rao, P.R.M.	based experimental data (Ward, Ahmed, Sunda) at high Reynolds numbers.
Transport in Porous Media 41:349–357, 2000.	2000	Resolution of a Paradox Involving Viscous Dissipation and Nonlinear Drag in a Porous Medium	Nield, D. A.	The reason that Forchheimer term not explicitly includes the viscosity term is that the viscosity cancels in the inertial term.
<i>TranspPorous Media</i> 44:325–35	2001	“Derivation of the Forchheimer law via Homogenization”	Chen, Z. X. Lyons, S. L. Qin, G.	Performed perturbation analysis of the Navier-Stokes equations. Found that the first deviations from linearity were of second-order in velocity, not third-order
C. R. Mecanique 331 (2003)	2003	“Singular nature of nonlinear macroscale effects in high-rate flow through porous media”	Panfilov, M. Oltean, C. Panfilova, I. Buès, M.	Justified Forchheimer equation suggesting a quadratic correction by mathematical way
Advances in Water Resources 27 (2004) 667	2004	“On the non-linear behavior of a laminar single-phase flow through two and three-dimensional porous media”	Fourar, M. Radilla, G. Lenormand, R. Moyne, C.	Based on the numerical analysis of flow structures and the evolution of the pressure and the viscous drags, three laminar flow regimes are identified: Darcy, transition and strong inertia. It is shown that the transition zone for the 3D-flow is very narrow in comparison with that of the 2D-flow. As a consequence, the non-Darcy 3D-flow is correctly modelled by Forchheimer’s equation. This explains the success of this equation in interpreting most of the experiments performed in real (3D) porous media.
International Journal of Rock Mechanics and Mining Sciences 41 (2004) 3	2004	“Non-linear regimes of fluid flow in rock fractures”	Zimmerman, R.W. Al-Yaarubi, A. Pain, C.C. Grattoni, C.A.	Performed Navier-Stokes simulations and laboratory flow experiments on an epoxy cast of a rock fracture. Confirmed the existence of a weak inertia regime ( $Re$ 1-10) & Forchheimer relation can probably be used over the entire range of $Re$ , as it effectively reduces to Darcy’s law at low values of $Re$ . Critical $Re = 10$
90406	2004	“Multiphase Non-Darcy Pressure Drop in Hydraulic Fracturing”	Olson, K. Haidar, S. Milton-Taylor, D. Olsen, E.	Evaluate the validity of the Forchheimer equation under very high rates. Straight line fit when $(dp/dl)/(\mu v)$ is plotted against $(\rho v/\mu)$
Advances in Water Resources 29 (2006) 3	2006	“Physical splitting of nonlinear effects in high-velocity stable flow through porous media”	Panfilov, M. Fourar, M.	Suggested to split the overall nonlinear macroscopic effects into two kinds of different physical origin: (1) a pure inertia effect produced by the convective term of Navier–Stokes equations and (2) an inertia–viscous cross effect representing a variation of the viscous dissipation due to a streamline deformation by inertia forces. An inertia–viscous cross effect is negligible for nonperiodic flow structure, while a pure inertia effect is negligible for periodic flow structure (almost zero).
European Journal of Mechanics B/Fluids 26 (2007) 295–303	2006	“High velocity flow through fractured and porous media: the role of flow non-periodicity”	Lucas, Y. Panfilov, M. Buès, M.	Studied periodic and non-periodic flows in order to identify ranges of the Reynolds number corresponding to deviation from Darcy’s law. Showed that quadratic deviation in a periodic flow caused as a result of asymmetry or non-periodicity and the main factor which may responsible for quadratic term appearance is non-periodicity. “The quadratic deviation appears at lower $Re$ if the non-periodicity degree increases”
Hydrol. Process., 21:554, doi:10.1002/6264	2007	“Determination of Forchheimer equation coefficients a and b”	Sidiropoulou, M. G. Moutsopoulos, K. N. Tshrintzis, V. A.	Analysed an extensive alternative data set from literature dating back to 1901 to find Forchheimer coefficient.
J. Hydraul. Eng., 134(9),	2008	“Approximate solutions for Forchheimer flow to a well”	Mathias, S. A. A. P. Butler H. Zhan	An exact solution for transient Forchheimer flow to a well does not currently exist. Research only interested in Forchheimer flow to wells for constant abstraction rates

1318– 1325, doi:10.1061 (CE)0733- 9429(2008), 1318)				
WATER RESOUR CES RESEAR CH, VOL. 46, W07514, doi:10.10 29/2009 WR0086 35, 2010	2009	“Stepdrawdown tests and the Forchheimer equation”	Mathias, S. A. Todman, L.C.	The main conclusions of this work are as follows: (1) The Forchheimer equation offers a parametrically simple method for interpreting SDTs where the step durations are insufficient to allow drawdowns to reach a quasi steady state. (2) Providing the step durations are sufficiently large, the Forchheimer equation is equivalent to the Jacob method. (3) It is possible to obtain field-scale Forchheimer parameters from Jacob well-loss coefficients (the B parameter) and vice versa.
WATER RESOUR CES RESEAR CH, VOL. 46, W07514, doi:10.10 29/2009 WR0086 35, 2010	2009	“Stepdrawdown tests and the Forchheimer equation”	Mathias, S. A. Todman, L.C.	The main conclusions of this work are as follows: (1) The Forchheimer equation offers a parametrically simple method for interpreting SDTs where the step durations are insufficient to allow drawdowns to reach a quasi steady state. (2) Providing the step durations are sufficiently large, the Forchheimer equation is equivalent to the Jacob method. (3) It is possible to obtain field-scale Forchheimer parameters from Jacob well-loss coefficients (the B parameter) and vice versa.

## Appendix B

### Critical Literature Review

#### SPE Journal 1974, pp.445-450

Estimating the coefficient of inertial resistance in fluid flow through porous media

Authors: Geertsma, J.

Contribution to the understanding of Forchheimer model:

Presented an empirical, time honoured relationship between Forchheimer coefficient and rock properties (permeability and porosity) for single-phase fluid flow and validated Forchheimer equation by experimental data.

Objective of the paper:

To obtain a correlation between Forchheimer coefficient and rock properties (permeability and porosity) for single-phase fluid flow.

Methodology used:

Used Forchheimer equation for gas:  $-\frac{P_L^2 - P_O^2}{2LcG\mu} = \alpha + \beta\left(\frac{G}{\mu}\right)$ , where  $c = \frac{RT}{M}$ ,  $G = \rho v$  is mass flow rate.

Validate the Forcheimer equation by plotting  $\frac{P_L^2 - P_O^2}{2LcG\mu}$  against  $G/\mu$ . Straight line indicated validity of the Forcheimer equation. In this plot, the slope of the straight line equal to  $\beta$  (Forchheimer coefficient) and intercept equal to  $\alpha$ .

Conclusion reached:

Empirical formula (Eq.17) predicts the Forchheimer coefficient better than other correlation available in literature. The correlation is only valid for one-phase fluid (100% liquid or gas).

In two-phase flow, when gas flows through porous media with residual water saturation  $\beta$  normally higher than predicted for dry state. In the two-phase system, permeability must be replaced by the gas effective permeability at a given water saturation, and the porosity must be replaced by the void fraction occupied by the gas (Eq.26).

Comments:

Eq.26 is not valid at high water saturation (more than 30%), because equation based on assumption that water is irreducible, hence boundary between liquid and gas phases is constant.

**Journal of Petroleum Technology 1979, pp.211-216.**

An analysis of high-velocity gas flow through porous media

Authors: Firoozabadi, A. and Katz, D. L

Contribution to the understanding of Forchheimer model:

Further improved the understanding of high velocity flow through porous media by performing experiment. Presented a correlation between Forchheimer coefficient and rock properties (permeability and porosity) for single-phase fluid flow and validated Forchheimer equation by experimental data.

Objective of the paper:

To obtain a correlation between Forchheimer coefficient and rock properties (permeability and porosity) for single-phase fluid flow and improve the understanding of high velocity flow through porous media.

Methodology used:

Used Forchheimer equation for gas:  $\frac{(P_1^2 - P_2^2)Mg_c}{2\mu ZRTGL} = \frac{1}{k} + \beta \left(\frac{u}{\mu}\right)$ . Validate the Forcheimer equation by plotting  $\frac{(P_1^2 - P_2^2)Mg_c}{2\mu ZRTGL}$  against  $u/\mu$ . Straight line indicated validity of the Forcheimer equation. In this plot, the slope of the straight line equal to  $\beta$  (Forchheimer coefficient) and intercept equal to  $1/k$ .

Conclusion reached:

Deviation of the data from straight line in lower portion is due to the slip effect (Klinkenberg effect).

Deviation of the data from straight line in upper portion is due to inadequacy of Forchheimer equation to describe the flow at high velocities.

Increased energy consumption at high velocities related to shear and tension stresses when flow passing through increasing and decreasing pore spaces.

Geertsma correlation for the Forchheimer coefficient is representative of the character of reservoir rock. More data only show modest improvement in correlation.

Comments:

Standardize the nomenclature used in high velocity flow in order to avoid future misunderstanding.



**Dover Publications Inc. 1972**

Dynamics of fluids in porous media

Authors: Bear, J.

Contribution to the understanding of Forchheimer model:

Summarized many researchers work on capillary models at large Reynolds number.

Showed that actual turbulence occur at Re values at least one order of magnitude higher than the Re at which deviation from Darcy's law is observed. Concluded that internal forces play vital role in deviation from Darcy's law.

Objective of the book:

Summarized many researchers work on capillary models at large Reynolds number.

Methodology used:

N/A

Conclusion reached:

Onset of non-linearity starts before flow becomes turbulent. Flow was not turbulent because of large linear term and not sharp transition as in pipe flow.

Comments:

**Transport in Porous Media 1987, 2, 521–531**

High velocity flow in porous media

Authors: Hassanizadeh, S.M. and Gray, W.G.

Contribution to the understanding of Forchheimer model:

Derived Forchheimer equation based on fundamental laws of continuum mechanics for high velocity isothermal flow through uniform isotropic porous media.

Objective of the paper:

Establish the upper boundary for validity of the Darcy's law and derive relationship to describe flow in porous media at high velocities.

Provide physical basis for derived relationship and identify the source of non-linearity.

Methodology used:

- Macroscopic balance of forces at starting point.
- An order-of-magnitude argument to show onset of non-linearity.
- Fundamental laws of continuum mechanics to derived equation of motion.

Conclusion reached:

General equation of motion for high velocity isothermal flow through uniform isotropic porous media (Forchheimer equation).

Viscous forces (drag forces) at microscopic level is the source of non-linearity.

Comments:

Barak (1987) argued about distinction between macroscopic and microscopic internal forces and empirically proposed Forchheimer equation and mathematically derived equation.

**Transport in Porous Media 1997, 29, 191–206.**

Derivation of the Forchheimer Law Via Matched Asymptotic Expansions

Authors: Giorgi, T.

Contribution to the understanding of Forchheimer model:

Derived a Forchheimer law using matched asymptotic for a rigid porous medium.

Objective of the paper:

Derivation of the Forchheimer Law Via Matched Asymptotic Expansions

Methodology used:

- Microscopic scale using a linearized version of the Navier-Stokes equations (Oseen approximation)
- An idea of Hassanizadeh and Gray is used along with some fundamental results on tensor-valued isotropic functions
- Use matched asymptotics to analyze the nonlinear laminar flow

Conclusion reached:

Darcy's law obtained using Stokes equation.

Forchheimer law obtained by series expansion for anisotropic materials for one-phase flow incompressible Newtonian fluid through a rigid porous medium.

Comments:

Deviation based on assumption that the microscopic velocity is high enough to cause non-negligible, nonlinear effects, but not so high as to produce turbulence.

**Int J Rock Mech Min Sci 2004, 41,163–169.**

Non-linear regimes of fluid flow in rock fractures

Authors: Zimmerman, R.W., Al-Yaarubi, A., Pain, C.C., Grattoni, C.A.

Contribution to the understanding of Forchheimer model:

Performed Navier-Stokes simulations and laboratory flow experiments on an epoxy cast of a rock fracture. Confirmed the existence of a weak inertia regime ( $Re$  1-10) & Forchheimer relation can probably be used over the entire range of  $Re$ , as it effectively reduces to Darcy's law at low values of  $Re$ .  
Critical  $Re = 10$

Objective of the paper:

Get algebraic form of the nonlinear relation between pressure drop and flow rate.  
Find “critical” value of  $Re$  at which nonlinear effects become appreciable.

Methodology used:

- Fracture surfaces made of epoxy casts.
- High-resolution Navier–Stokes simulations and laboratory measurements of fluid flow in a natural sandstone fracture.

Conclusion reached:

Confirmed existence of a “weak inertia regime” for Reynolds numbers in the range of 1–10, but effect is small.

Forchheimer model exists for Reynolds numbers above 20.

Comments:

Used dimensionless parameters in analysis

**Advances in Water Resources 2004, 27 (2004) 669–677**

On the non-linear behavior of a laminar single-phase flow through two and three-dimensional porous media

Authors: Fourar, M., Radilla, G., Lenormand, R, Moyne C.

Contribution to the understanding of Forchheimer model:

Analysed the effect of space dimensions on development of flow regimes.

Objective of the paper:

To examine the 3D effect on flows at high velocities through homogeneous porous media.

Methodology used:

- Numerical simulations for a steady flow of an incompressible Newtonian fluid at intermediate and high Reynolds number through a 2D and a 3D periodic porous media.
- Macroscopic momentum balance equation applied to each periodic cell

Conclusion reached:

- 2D porous media flow goes through Darcy's regime, the transition regime and the strong inertia regime.
- 3D porous media significantly reduce transition regime.
- Non-Darcy flow in 3D is correctly modelled by Forchheimer equation.

Comments:

This paper explains the success of Forchheimer equation in interpreting most of the experiments performed in real (3D) porous media.

**European Journal of Mechanics B/Fluids 26 (2007) 295–303**

High velocity flow through fractured and porous media: the role of flow non-periodicity

Authors: Lucas, Y., Panfilov, M., Buès, M.

Contribution to the understanding of Forchheimer model:

Presented that quadratic deviation in Forchheimer law appears as result of non-periodicity in porous media and fractures.

Objective of the paper:

To examine the influence of the flow periodicity or non-periodicity on macroscopic flow law.

Methodology used:

- Porous media and fractures represented by crenulated channel.
- Numerical analysis based on the Navier–Stokes equations.
- Stochastic fracture generation method.

Conclusion reached:

It is impossible and sometimes numerically very difficult to achieve quadratic term appearance in the periodic case.

The quadratic deviation in a periodic flow caused as a result of asymmetry or non-periodicity.

The main factor which may be responsible for quadratic term appearance is non-periodicity.

The quadratic deviation appears at lower  $Re$  if the non-periodicity degree increases.

Comments:

The quadratic deviation becomes all the more important as the non-periodicity degree is high.

Appendix C

**Procedure for selection one-phase correlation for the Forchheimer coefficient (after Li and Engler 2001)**

