IMPERIAL COLLEGE LONDON

Department of Earth Science and Engineering

Centre for Petroleum Studies

Impact of anisotropy and fracture density on the approximation of the effective permeability of a fractured rock mass using 2D models

By

Alimzhan Bekzatov

A report submitted in partial fulfilment of the requirements for the MSc and/or the DIC.

September 2010

DECLARATION OF OWN WORK

I declare that this thesis *Impact of anisotropy and fracture density on the approximation of the effective permeability of a fractured rock mass using 2D models* is entirely my own work and that where any material could be construed as the work of others, it is fully cited and referenced, and/or with appropriate acknowledgement given.

Signature:

Name of student: Alimzhan Bekzatov

Names of supervisors: Prof. R. W. Zimmerman and Dr. A. Paluszny

ABSTRACT

Proper modeling of a fractured reservoir is a main issue in reservoir optimization and characterization. In most cases simulation of discrete fractures are conducted in 2D due to computational constraints and lack of information, neglecting effects of interaction of fractures in the third dimension. The purpose of this study is to develop a methodology to approximate the effective permeability of a fractured porous medium with variable geometry and density using 2D slices of a 3D model.

The study involved the generation of 3D models and extraction of 2D slices (cut planes) of each model with subsequent numerical computation of effective permeability and fracture pattern properties. Stochastical method of the fracture model generation was applied in all simulations. Computation of the effective permeability was performed using discrete fracture and matrix model (DFM), based on finite element method. Generation of 3D models was conducted for different geometries (anisotropic and isotropic) and densities of fracture patterns. 2D cut planes captured geometry and flow at different locations of the original 3D model. Observed parameters are K_{eff}, fracture density, spacing, and length distribution. The study was analysed by measuring mean absolute deviation of all observed parameters.

The results of the study revealed that isotropic models require small variation of measured K_{eff} and fractures pattern properties as a function of cut planes locality, while approximation of these properties in anisotropic models is sensitive to location of cut planes. The results also showed that fracture density does not play a significant role in approximation of all observed parameters independently on geometry of the model. In terms of specific parameters, analysis concluded that spacing and length are more complex parameters to quantify. Spacing has the largest mean absolute deviation, whereas length is associated with the smallest mean deviation among all parameters.

ACKNOWLEDGEMENTS

It is a pleasure to thank those who made this thesis possible such as my supervisors, Prof. Zimmerman and Dr. Adriana Paluszny whose guidance and support from the initial to the final level enabled me to develop an understanding of the subject. I also would like give gratitude to my course director Prof. Gringarten for the intensive and interesting course. I would like to extend my thanks to my group mates and friends for their constant help and support through the thesis. Last but not least, my special thanks to my family for their encouragement and support through the all period of my studying.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Imperial College London

Impact of anisotropy and fracture density on the approximation of the effective permeability of a fractured rock mass using 2D models

Alimzhan Bekzatov

Imperial College supervisors: Prof. R. W. Zimmerman and Dr. A. Paluszny

Abstract

Proper modeling of a fractured reservoir is the main issue in reservoir optimization and characterization. In most cases simulation of discrete fractures are conducted in 2D due to computational constraints and lack of information, neglecting effects of interaction of fractures in the third dimension. The purpose of this study is to develop a methodology to approximate the effective permeability of a fractured porous medium with variable geometry and density using 2D slices of a 3D model.

The study involved the generation of 3D models and extraction of 2D slices (cut planes) of each model with subsequent numerical computation of effective permeability and fracture pattern properties. Stochastical method of the fracture model generation was applied in all simulations. Computation of the effective permeability was performed using discrete fracture and matrix model (DFM), based on finite element method. Generation of 3D models was conducted for different geometries (anisotropic and isotropic) and densities of fracture patterns. 2D cut planes captured geometry and flow at different locations of original 3D model. Observed parameters are K_{eff}, fracture density, spacing, and length distribution. The study was analysed by measuring mean absolute deviation of all observed parameters.

The results of the study revealed that isotropic models require small variation of measured K_{eff} and fractures pattern properties as a function of cut planes locality, while approximation of these properties in anisotropic models is sensitive to location of cut planes. The results also showed that fracture density does not play significant role in approximation of all observed parameters independently on geometry of the model. In terms of specific parameters, analysis concluded that spacing and length are more complex parameters to quantify. Spacing has the largest mean absolute deviation, whereas length is associated with the smallest mean deviation among all parameters.

Introduction

Fractured reservoirs contain more than half of the remaining conventional oil (BP, 2007). Reservoir optimization techniques based on understanding flow through fractured reservoir require proper modelling of the geometry of such reservoir. During the past 40 years there many studies have been performed to understand the impact of fracture patterns on the effective permeability of rocks (Renshaw, 1996). In order to better define the geologic history which has caused formation of natural fracture pattern geologists suggested different interpretations of understanding the behaviour of fracture systems (Pollard and Aydin, 1988). For instance, geometric and physical features of natural fracture patterns can be explained by the stress states during their formation (Cruikshank *et al.*, 1991*;* Olson and Pollard, 1989; Thomas and Pollard, 1993). Fracture networks have a significant effect on fluid flow, thus, many simulation techniques were developed to identify dependence fracture network's characteristics on the effective permeability of the rocks (e.g. Dershowitz and Einstein, 1988, Srivastav et al., 2005, Bogdanov et al 2003). However, these studies are often 2D or 3D realisations and the relationship between both is not frequently investigated.

There are several methods for the flow simulation in naturally fractured reservoirs. The equivalent porous medium (EPM) is a model that characterizes flow in a large scale area (Barenblatt and Zheltov, 1960; Bear, 1972). Fracture network and the porous blocks are systematized as distinct, and everything that overlapped is continua (Berkowitz *et al*, 1987). Barenblatt and Zheltov (1960) and Warren and Root (1963) introduced the dual porosity model, which assumed no flow through matrix blocks. Other assumptions are that grid blocks are isometric and each of them contains at least one fracture, and all fractures are interconnected. Thus, it is best suited to represent dense and well organized fracture patterns. In order to describe fracture networks discrete fracture network (DFN) model was developed by Witherspoon *et al*. (1980) and Dershowitz (1988). DFN is based on fracture properties and not on characterizing of the rock matrix properties. It is mostly used in single phase calculations to attain fracture network properties (Ganzer, 2002). Dual porosity models are not well suited for the simulation of a reservoir that might contain a small number of large-scale fractures, which control the flow. In contrast, discrete fracture models (DFM) represent fractures and model flow through matrix and fracture simultaneously (Matthai, 2005; Belayneh *et al*. 2006). The main idea of DFM is to reduce fracture dimensionality and represent them as lines in 2D or polygons in 3D (Geiger *et al*, 2009).

There are three main methods to build a geometric model. The first method is deterministic generation of model. This method requires precise information about the fracture system (fracture network and a single fault zone). The main problem of this method is that it requires derivation of 3D system out of 1D and 2D information, which are often missing (Hemminger *et al*, 1999). The second method is geostatistical, which provides systematic and observable approach that allocates data from many different sources (Srivastava *et al*, 2005). The third method is the stochastic generation of the fracture model. This method requires generation of models that comprise the statistical properties of natural fracture patterns. This technique generates sets of planar fractures of different size, orientation and location (Dershowitz and Einstein, 1988). Main advantage of this method is its speed and reliability.

Computation of this flow property is very important task in oil and gas industry, since it is considered as a basic step of the reservoir optimization. There are many numerical procedures to compute effective permeability of fractured reservoirs. Durlofsky (1991) developed a method in 2D to estimate effective permeability in a region where heterogeneity is distributed periodically. Using a continuum 2D model of simulation fractured reservoir Lough *et al.* (1996) suggested the boundary element method to compute effective permeability of grid blocks. Pickup *et al*. (1994) and Nakashima *et al.* (2000) analyzed advantages of periodic boundary condition in estimation of effective permeability for different sedimentary structures. Bogdanov *et al.* (2003) presented stochastically generated fracture dataset, and a finite element based effective permeability computation using 3D meshing of fractured porous media. Subsequently, Bogdanov *et al* (2007) measured effective permeability of fractured porous media where fracture sizes are distributed according to a power law. Using a 2D dimensional model Ronayne and Gorelick (2006) estimated effective permeability of a porous medium consisting of 'branching dendritic channels embedded in a low permeability matrix'. They concluded that 'averaging exponent' is a critical dimensionless parameter related to the degree of flow channeling'. Paluszny and Matthai (2010) used geomechanically generated 2D discrete fracture growth model to estimate influence of fracture development on the effective permeability of porous rocks. However, afore-mentioned studies focus on 2D or 3D measurements separately. Usually simulations of discrete fracture are performed in 2D due to constraints related to computational complexity and lack of information, ignoring effects of relations of fractures in the third dimension. Therefore, it would be useful to understand which characteristics have a first order effect on the approximation of K_{eff} and geometry properties of a 3D datasets using 2D slices. In other words, it is necessary to understand when approximations are better.

In this paper we study the impact of fracture geometry and density on the approximation of effective permeability of 2D slicesof 3D datasets. For the purpose of simulation we present discrete fracture and matrix model using finite element method approach. Fracture systems are generated stochastically. The aim is to improve the methodology to approximate the effective permeability of a 3D dataset using 2D datasets that capture geometry and flow at different locations. By comparing measurements of 2D datasets extracted from 3D model we define equivalence as a function of fracture density, spacing, and size distribution. We describe the aforementioned equivalence in terms of a quantifiable error dependant on the deviance of $K_{eff.}$.

This paper is organized as follows. First, we review the methods of fracture simulation, generation of datasets, and computation of $K_{eff.}$ Second, we describe the simulations of the study. A discussion follows on the effect of fracture density, spacing, and size distribution on K_{eff} in 2D and 3D fracture patterns, and how it can be quantified. Finally, we compare and analyse our results, and present some concluding remarks.

Methodology

Effective permeability calculation

Computation of effective permeability is performed to evaluate the impact of the fractures on the hydraulic conductivity of a porous medium. Study is also included the evaluation of the effects of fracture connectivity, density, spacing, apertures, and size distribution in order to reduce computation times.

Calculation of effective permeability is based on single-phase steady state flow. The specific discharge is given by Darcy's law:

$$q = \frac{k}{\mu} \nabla p \qquad (1)$$

where μ (Pa s) is the fluid viscosity and p (Pa) is the fluid pressure. By means of discretisation we find out constant matrix permeability k_m for matrix elements, and equivalent porous medium fracture permeability k_f in the fractures.

We compute fracture permeability using the parallel plate law from local aperture. It means that fracture has smooth, parallel walls with a local separation of h (Kranz *et al.,* 1979; Witherspoon *et al.*, 1980)

$$k_f = \frac{h^2}{12} \qquad (2)$$

where h is the local fracture aperture. For a flow simulation we use special body that has triangle elements which represents the matrix and sub-dimensional line elements which represent the fracture (see **Fig. 1**). By defining a different conductivity

and capturing the varying aperture of the fracture we can compute a local permeability at any given point. We define variable k_f for a single fracture:

$$k_f = (k_{fo}| \dots |k_{fI}| \dots |k_{fN}) \quad (3)$$

where fracture f has n points along its centreline.

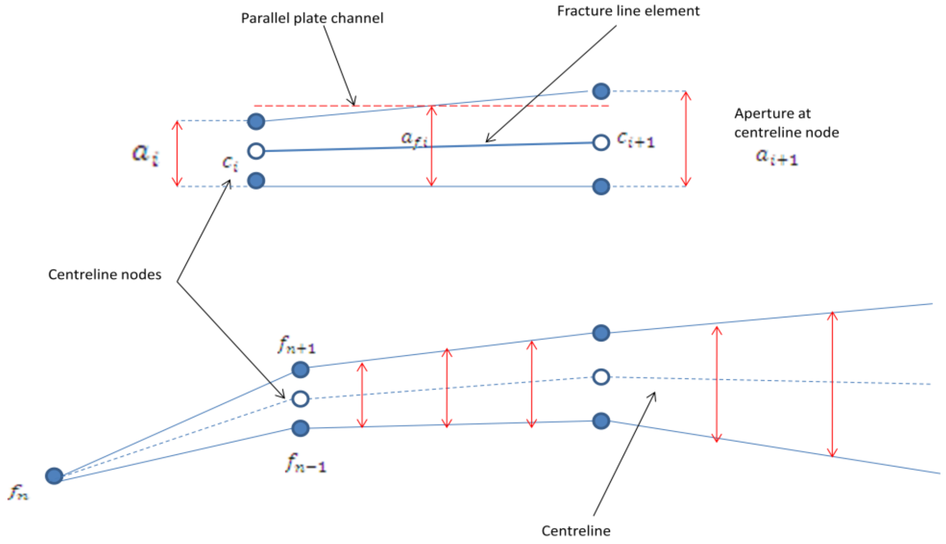


Fig. 1─Fracture centreline and aperture approximation. Figure represents skeleton of the fracture shape. c_i and c_{i+1} (white dots) represent centreline nodes . a_i is aperture of the fracture at c_i . f_n is a fracture tip. Green line represents fracture line element. f_{n+1} and f_{n-1} indicates nodes of the fracture walls.

Integration of the boundary fluxes allows as estimation of the total model throughput q. For a given fluid flow gradient we approximate k_{eff} as

$$k_{eff} = \frac{q\mu L}{A[p(u)-p(d)]} \quad (4)$$

where L (m) is the length of the model in the direction of flow, in 3D A (m^2) is the area of a cross section perpendicular to the flow, whereas in 2D A is a length, and p(u) and p(d) (Pa) are the fluid pressures applied to the upstream and downstream model boundaries (see **Fig. 2**). Effective permeability value in our analysis is presented dimensionless (Philip *et al.* 2005), as

$$K_{eff(d)} = \frac{k_{eff}}{k_m} \quad (5)$$

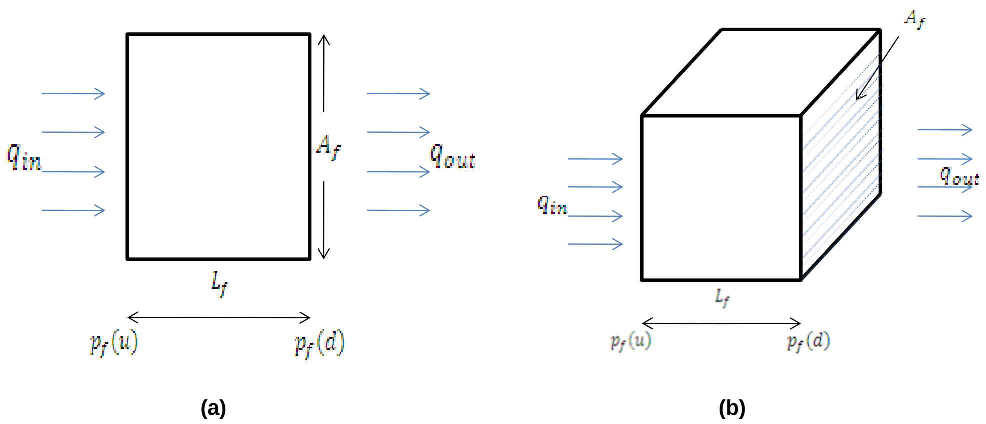


Fig. 2─Effective permeability boundary conditions in 2D (a) and 3D (b). q_{in} is total in flux, which is equal to q_{out} - total out flux. Area of length L can be observed by using fluid pressure gradient p(u), p(d).

Fracture Characterisation

Two common methods of fracture characterisation are outcrop measurement and core fracture characterisation. Outcrop fracture observation is very essential to understand 3-D fracture orientation and lithologies. Outcrop measurement is useful tool to detect the length and aperture size, which are impossible to obtain by extracting core plug from the subsurface. Core fracture observation is the fundamental of fracture research and also way of understanding subsurface fracture. Core fracture characterisation allows us to obtain information about aperture distribution. Knowledge of fracture aperture is very important because it directly affects oil and gas percolation (Song *et al.* 2000). The following is related to the definitions of the main parameters and to various techniques used to study extensional system of the generated fracture rock.

Aperture: state-dependent parameter defined as the closest distance between two fracture walls at any point along its centerline.

Density: specifically, the volume density of a 2D fracture set is defined as

$$d = \frac{1}{A}\sum_{i=1}^{n}\left(\frac{l_i}{2}\right)^2 \quad (6)$$

where n is the number of fractures in the set, A is the flaw area, and l_i is the length of fracture i (Budiansky and O'Connell 1976).

Length : defined as the length of the centreline defining a single fracture

Spacing: the area method defines spacing (m) as

$$s = \frac{A}{(l_s + \sum_{i=1}^{n} l_i)} \quad (7)$$

where l_s is the height of the specimen (Wu and Pollard, 1995). By means of this method we can properly estimate the saturation of different fracture sets.

Simulations

All simulations include generation of 3D patterns and their 2D slices, which are conducted in three cycles. Different scenarios of localisation of fracture patterns in the rock are expected in all simulations. Simulations conducted with respect to two types of fractured models: isotropic and anisotropic. First cycle requires generation of parallel fractures, after that we generate two sets of fractures and randomly orientated fractures in second and third cycles respectively. All cycles require low and high fracture density cases. Different methods of fractures localisation are explained by necessity of recognition how fracture orientation and network density affect effective permeability. 3D and 2D models are generated using Imperial College in-house software. **Table 1** summarizes the simulations that will be conducted in our analysis.

Table 1—List of simulations

Simulations	Cases of different fracture density	2D slices			
		Cut planes parallel to xy axes (Z cut plane)	cut planes parallel to yz axes (X cut plane)	cut planes parallel to xz axes (Y cut plane)	cut planes at arbitrary angles (Random cut planes)
3D Model: Parallel fractures	125 fractures (low density)	1(Z125)	*	1(Y125)	1(R125)
	250 fractures (high density)	1(Z250)	*	1(Y250)	1(R250)
3D Model: two sets at 60	125/125 fractures	2(Z)	*	2(Y)	2(R)
3D Model: randomly oriented fractures	250 fractures	3(Z250)	3(X250)	3(Y250)	3(R250)
	500 fractures	3(Z500)	3(X500)	3(Y500)	3(R500)

*Note: For the first two simulations X cut plane is aligned with the direction of the sets.

Material properties of the rock in our simulations are based on the samples of the typical fractured structures in Somerset, UK. Model size is determined as 10×10×10m in order to simulate estimation of the effective permeability of entire volumetric reservoirs. **Table 2** summarizes material properties that will be used for generation and simulation of models.

Table 2─Material Properties

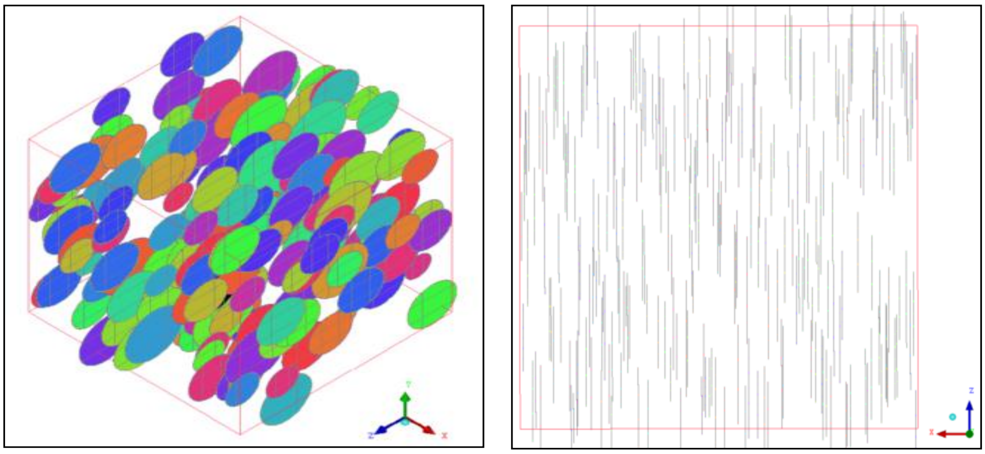
Fracture Shape	Disc
Fracture size	Normal distribution; Mean 0.5m, Std. Dev 0.1m, Max 1m, Min 0.25m
Box size	10×10×10m
Spacing	0.08m (10/125) - 0.04m (10/250)
Aperture size	0.004m
Matrix permeability	$1\times10^{-15}m^2$
Porosity	0.25
Viscosity	water at 25ºC (0.89cp)

First cycle of simulations: One set of parallel fractures. As a first step, we generate a 3D dataset and then project one set of random parallel fractions (anisotropic model) onto a 2D plane (see **Fig. 3**). All fractures are planar. The model has 125 and 250 fractures in low and high density cases, respectively.

Second cycle of simulations: Two sets of fracture generation. Another simulation related to anisotropic model requires generation of 3D randomly located two sets of parallel fractures with different orientation (see **Fig. 4**). One set of fractures is parallel to the yz axes, while another is at 60º angle from the yz axes. Each set contains 125 fractures.

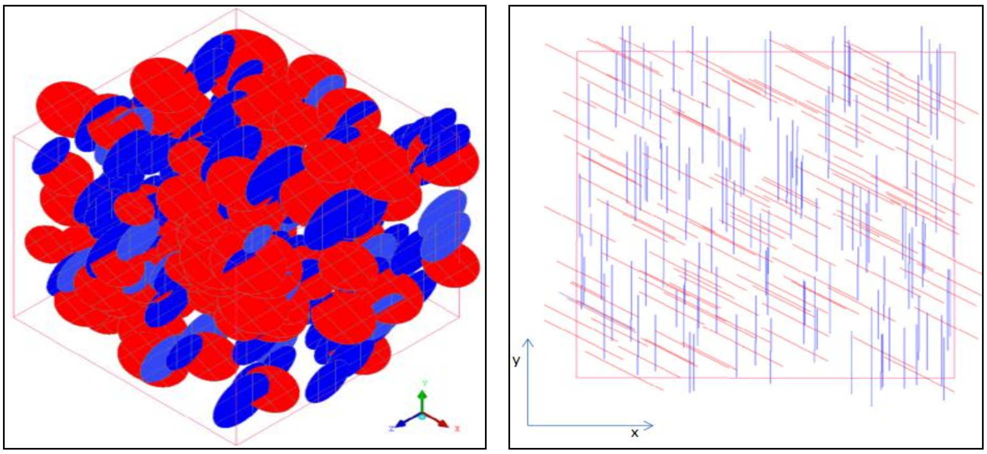
Third cycle of simulations: Randomly oriented fractures. This simulation refers to the generation of an isotropic model with randomly located and oriented fractures. Isotropic models are often used in numerical simulations as proxies of reservoir geometries because they can be generated using stochastical fracture population engines. This model is built with randomly oriented disc-shaped fractures with a power law length distribution based on the model of Paluszny and Matthai (2010). Fracture radii range from 0.25 m to 1 m; dimensions of the box are 10 × 10 × 10m. The 3D model contains 250 fractures for the low density case and 500 fractures for the high density case (see **Fig. 5**).

The following steps are common for all three cycles of simulations. After the generation of the 3D datasets we generate and extract 2D slices using cut planes which intersect with fractures in the 3D pattern. Cut planes are parallel to x, y or z axes, and at arbitrary angles (see **Fig. 6**). Cut planes at arbitrary angles are not aligned with the traversal direction of the sets. The number of cut planes for each simulation is 20. Subsequently, we compute the effective permeability of 2D slices of 3D fracture patterns. We also compute densities, spacing and length of the 2D equivalent datasets. Results are compared for different scenarios. We study how the geometric characteristics of the input 3D model affect the ability of 2D models to reproduce the effective permeability of the 3D source model. Comparisons are illustrated in the form of measurements of the effective permeability as a function of parameters such as density, spacing and length. Variation of effective permeability, length distribution, density and spacing is compared in relation to different cut planes. Simulations with three different models are compared between each other in order to quantify the quality of the K_{eff} approximations. In our study X, Y and Z cut planes are parallel to the YZ, XZ, and XY axes, respectively.



(a) (b)

Fig. 3─a) 3D model representing a rock with projected disc planes parallel to yz axes indicating fractures. Number of fractures is 250. b) 2D view of cutplane parallel to xz axes.



(a) (b)

Fig. 4─a) Three-dimensional view of the model with two sets of fractures. One set is parallel to yz axes, second set is on 60º angle from yz axes. Number of fracture is 125 for each set. b) 2D view of cut plane parallel to xy axes.

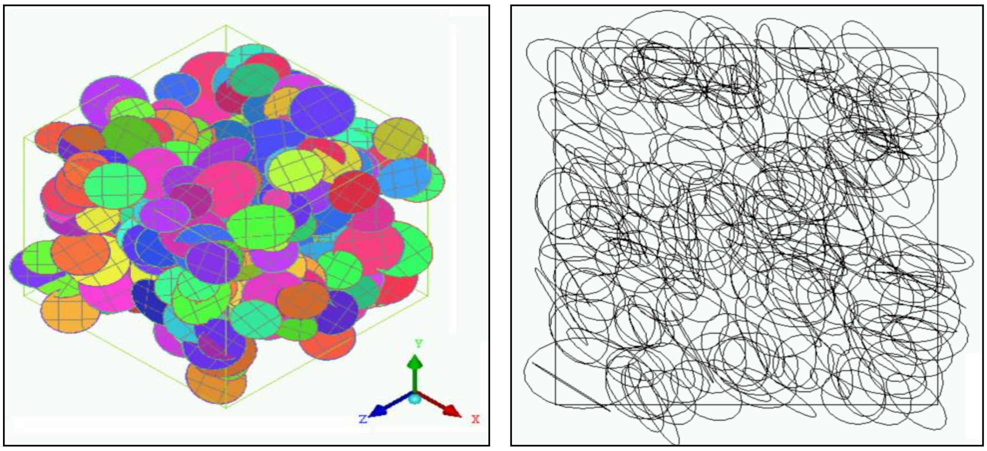
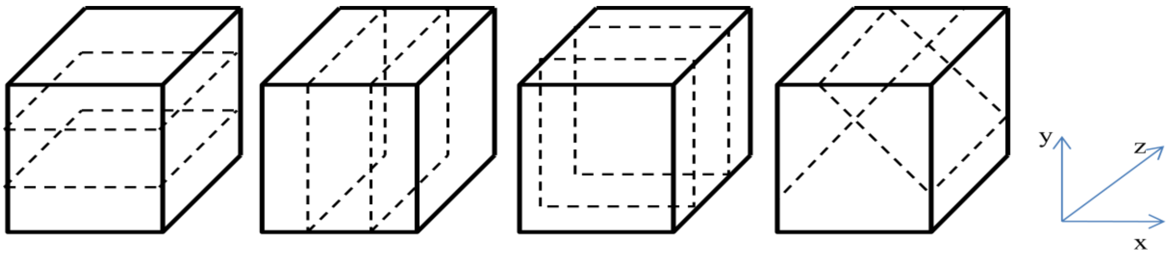

(a) **(b)**

Fig. 5─a) Three-dimensional view of randomly generated fractures, number of fractures is 250 fractures, b) 2d view of cut plane parallel to xy axes.



(a) **(b)** **(c)** **(d)**

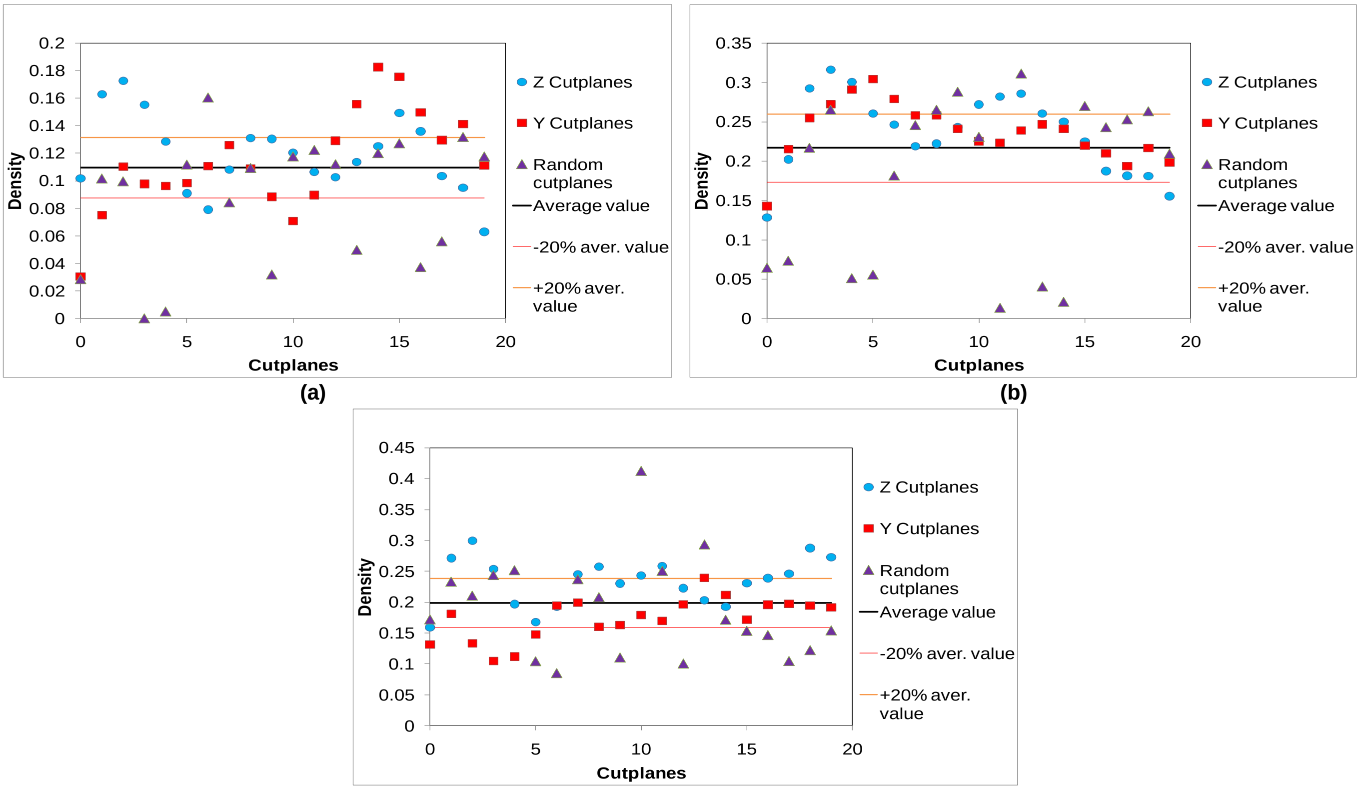
Fig. 6─Cut planes: a) Cut planes parallel to xz axes (Y cut plane), b) Cut planes parallel to yz axes (X cut plane), c) Cut planes parallel to xy axes (Z cut plane), d) Cut planes at arbitrary angles (Arbitrary cut planes).

Results and Analysis

Analysis and results of 2D slices include: 1) variation of observed parameters such as fracture density (d), spacing (s), mean length distribution (μL) and effective permeability ($K_{eff(d)}$) as a function of cut plane locality, 2) how the amount of fractures in the models affect the latter, and 3) impact of geometry of the model on these parameters. Our measurements are summarized in **Fig. 7-14** which plot d, s, μL, and $K_{eff(d)}$ as a function of cut plane location. Each graph represents a different case of the fracture orientation and the amount of fractures. Cut planes are represented as numbers from 0 to 19. To present the results in our analysis we average (μ) 2D values and compute for the each case an absolute deviation (D).

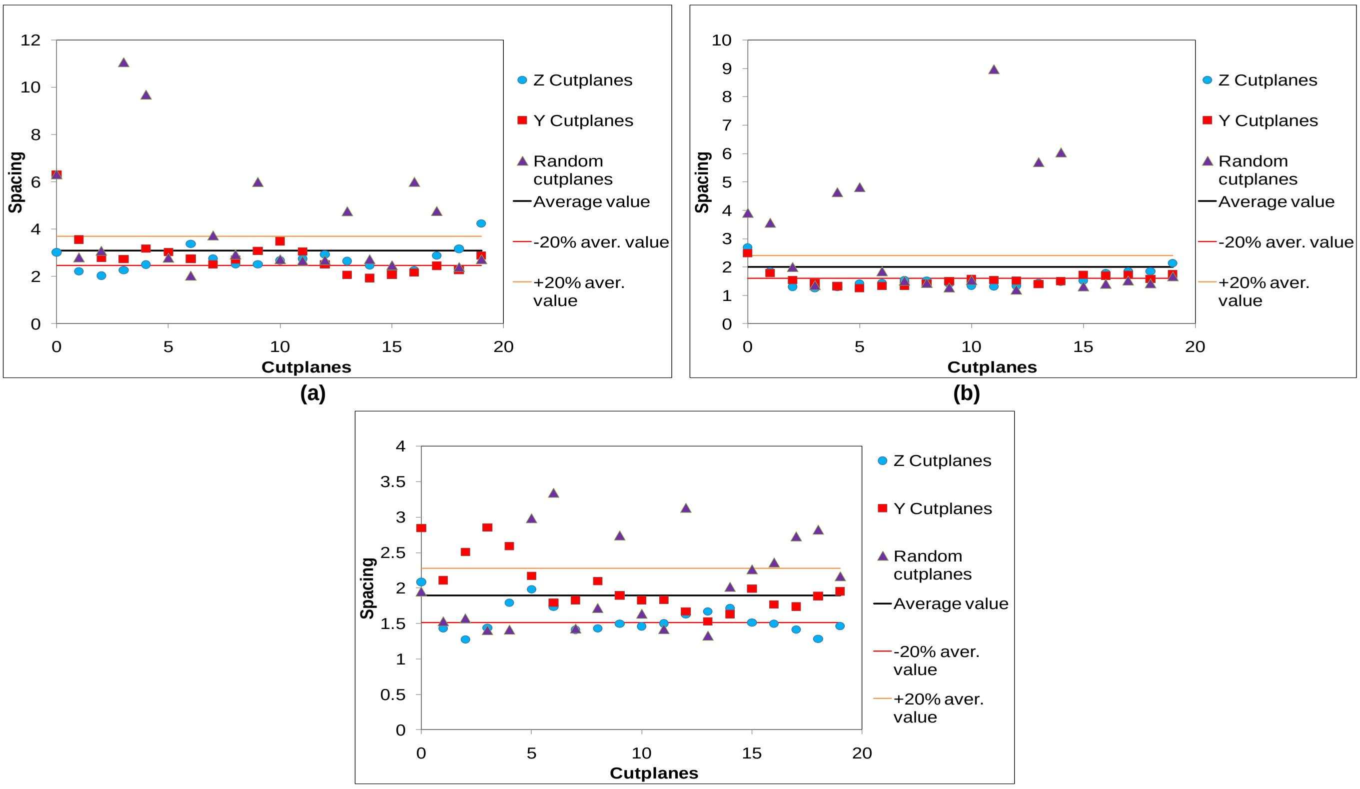
$D=|x_i- \mu(x)|$ ....(8)

where x_i is the measured data element, $\mu(x)$ is the average value of measured data. Mostly, we refer to the mean absolute deviation (D(av)), which averages D across the cut planes.



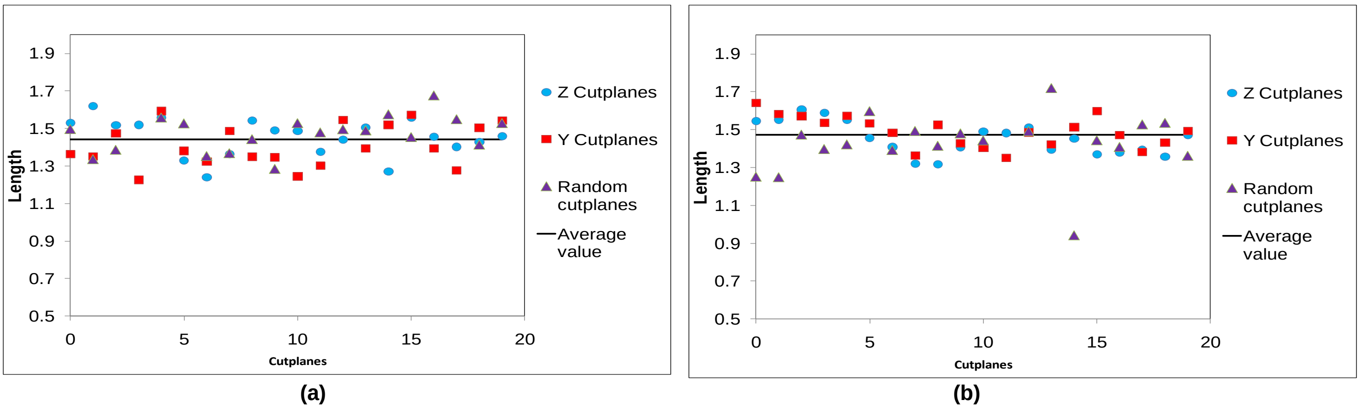
(c)

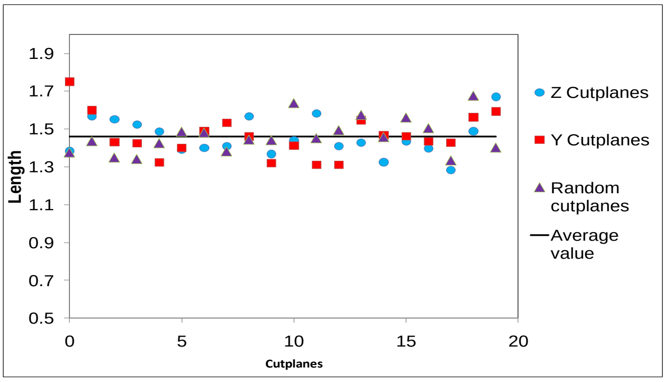
Fig. 7─Density measurements with different orientation of cut planes for anisotropic models. a) 125 parallel fractures, b) 250 parallel fractures, c) Two sets of fractures at 60º angle from each other.



(c)

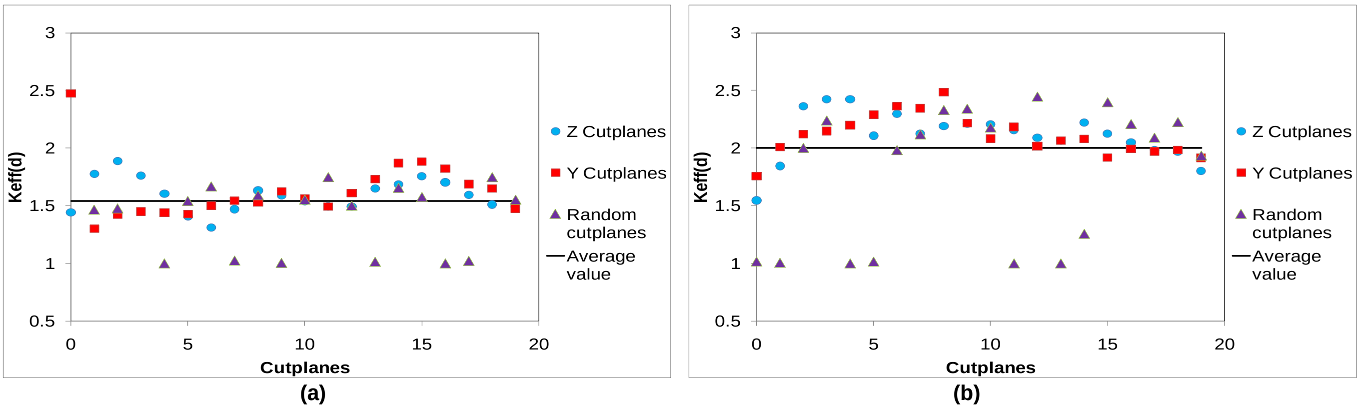
Fig. 8─Spacing measurements with different orientation of cut planes for anisotropic models. a) 125 parallel fractures, b) 250 parallel fractures, c) Two sets of fractures at 60º angle from each other.

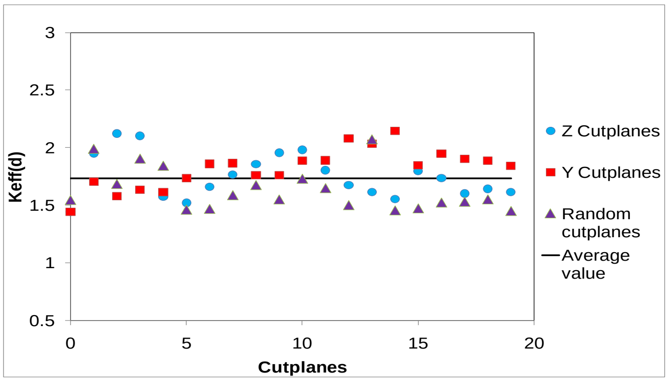




(c)

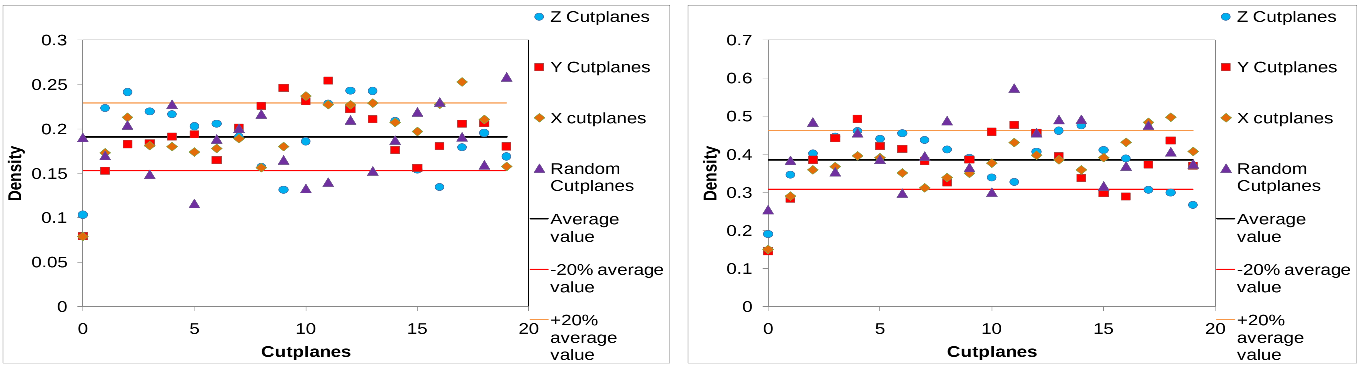
Fig. 9─Length measurements with different orientation of cut planes for anisotropic models. a) 125 parallel fractures, b) 250 parallel fractures, c) Two sets of fractures at 60º angle from each other.





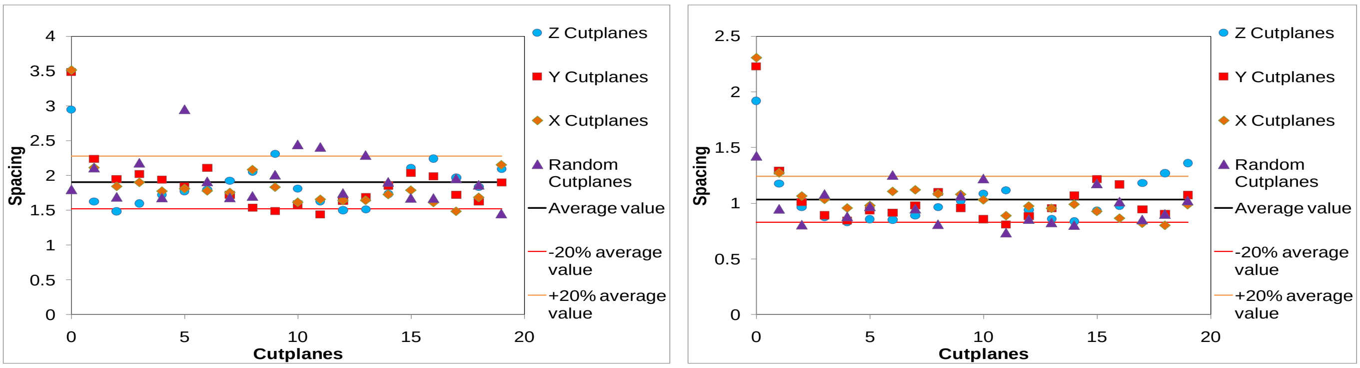
(c)

Fig. 10─$K_{eff(d)}$ measurements with different orientation of cut planes for anisotropic models. a) 125 parallel fractures, b) 250 parallel fractures, c) Two sets of fractures at 60º angle from each other.



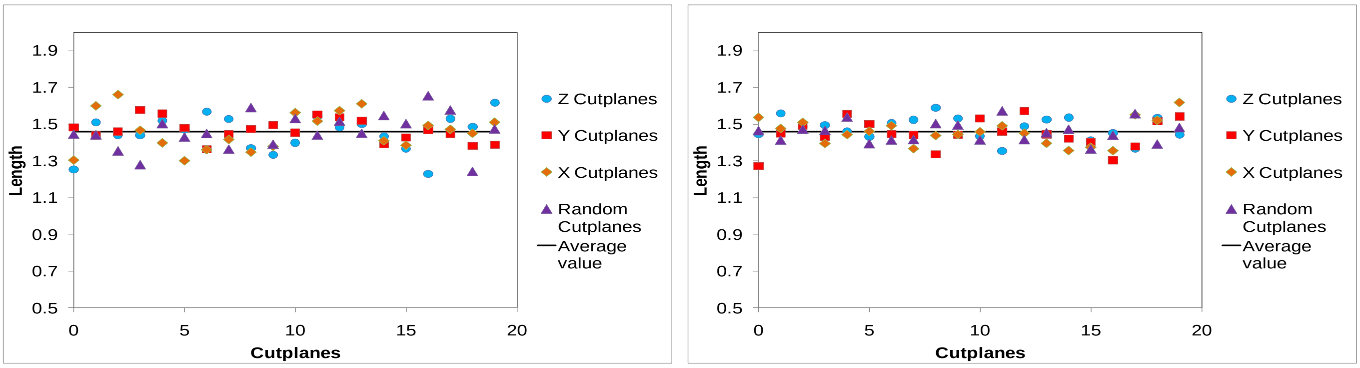
(a) (b)

Fig. 11─Density measurements with different orientation of cut planes for anisotropic models. a) 250 parallel fractures, b) 500 parallel fractures.



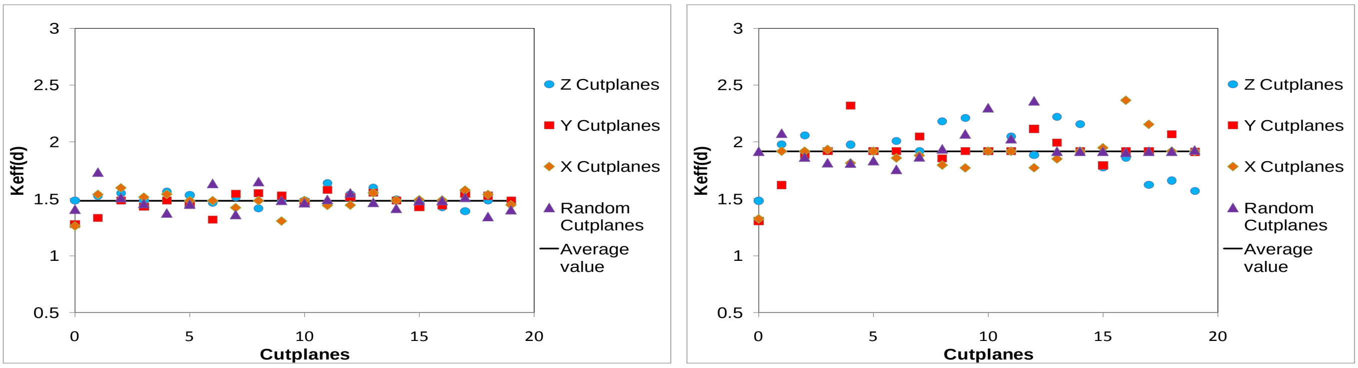
(a) (b)

Fig. 12─Spacing measurements with different orientation of cut planes for anisotropic models. a) 250 parallel fractures, b) 500 parallel fractures.



(a) (b)

Fig. 13─Length measurements with different orientation of cut planes for anisotropic models. a) 250 parallel fractures, b) 500 parallel fractures.



(a) (b)

Fig. 14─$K_{eff(d)}$ measurements with different orientation of cut planes for anisotropic models. a) 250 parallel fractures, b) 500 parallel fractures.

Impact of cut plane orientation on approximation of observed parameters

From **Fig.7, 8, 9, 10** we can see that in the anisotropic models (first and second simulation) extraction of 2D slices by cutting the model using arbitrary cut planes (AC) leads to a big range of the variation in all measured properties comparing to X, Y, Z cut planes. We measure a mean absolute deviation of 19% for the $K_{eff\,(d)}$ property in the case of random cut planes (see **Fig13 a**). Y and Z cut planes show better approximations of effective permeability, with *D(av)* of 20%. Differences in the mean absolute deviation between AC and Y, Z cut planes for *s, μL, d* are 52%>21%, 8%>6% and 36%>20%, respectively (see **Fig. 13 b,c,d**). High values of *D(av)* in (AC) case can be explained by complexity of random cut planes. Due to the anisotropy of the model it leaves possibility to cross space of the model where small amount of fractures are concentrated. This generates non-representative low density 2D slices, which bring the mean value down.

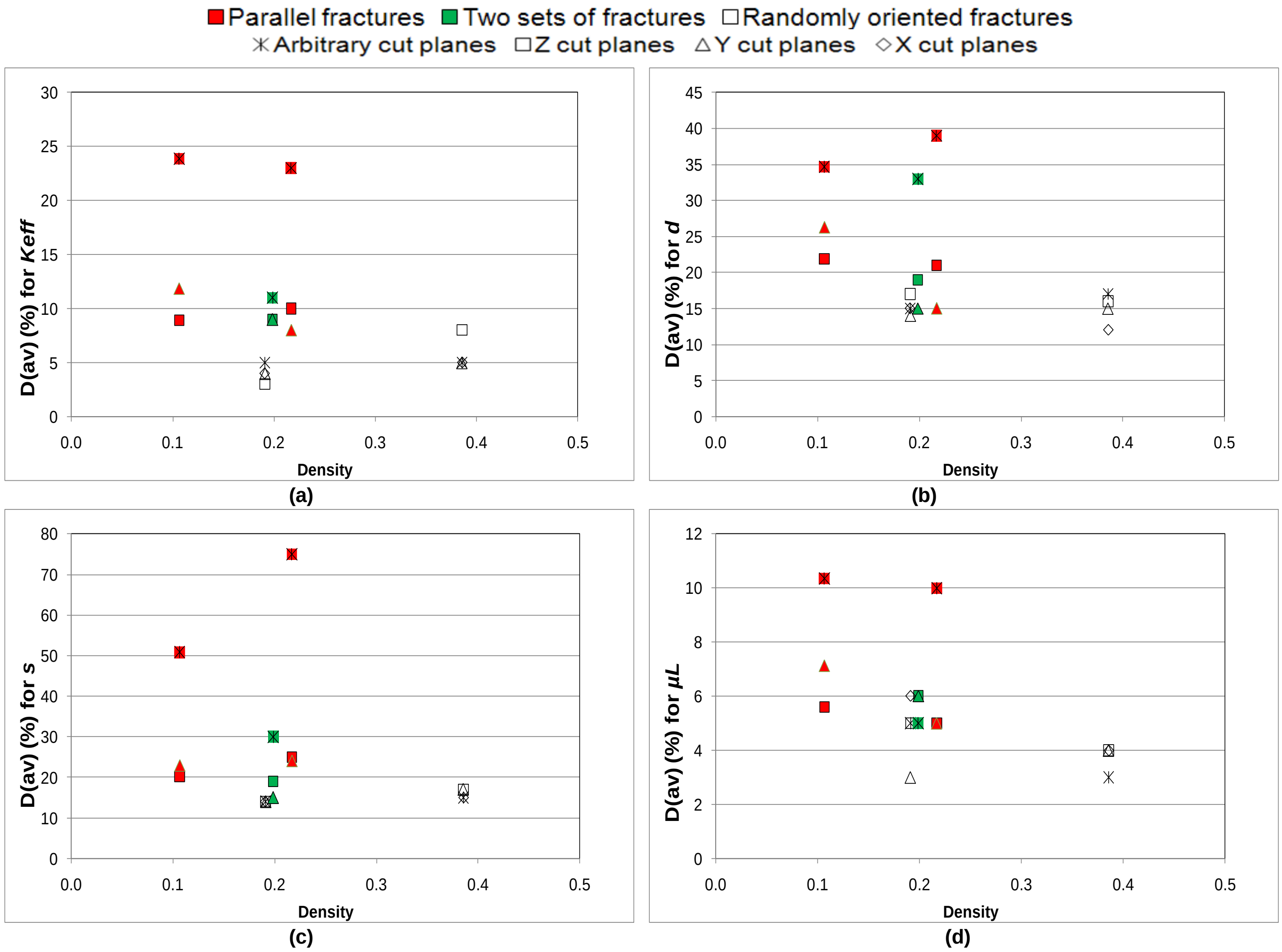


Fig. 15–Comparison of mean absolute deviation for effective permeability (k_{eff}), density (*d*), spacing (*s*), and mean length distribution (*μL*) with various oriented cut planes in anisotropic and isotropic models with different fracture densities.

For the isotropic models location of cut planes is of no importance (see **Fig. 11, 12, 13, 14**). Therefore, we obtained much closer values of *D*(*av*) for all cases of cutting in both cases of density (see **Fig. 15**). Mean absolute deviation for *d* for (AC) is 16%, whereas for X,Y,Z cut planes *D*(*av*) is 15%. Other flow parameters have the same tendency of difference in *D(av)* between RC and X,Y,Z cut planes: 14.5%≈15%, 4%≈4.3%, and 5%≈4.8% for spacing, length and effective permeability, respectively.

Impact of the amount of fractures on approximation of observed parameters

K_{eff} is a linear function of fracture density. However, according to our simulations these parameters are not proportional. The first and the third simulations include two cases of the fracture density: low and high. Both simulations show that the double increase in fracture density resulted in average increase by approximately 30% in measured $K_{eff\,(d)}$ (see **Fig. 16**). For both simulations changing the amount of fractures slightly affects mean absolute deviation of *d*, $K_{eff\,(d)}$, *s, and μL* (see **Fig. 17**). The mean absolute deviations of $K_{eff\,(d)}$, *d, μL* decreases from 15% to 14%, 28% to 25%, 8% to 7% with increasing the amount of fractures from 125 to 250 in anisotropic model. *D*(*av*) for spacing increased from 31% in case of 125 fractures to 35% in case of 250 fractures in parallel model. *s* has inverse proportion to *d*, thus, it decreases with increase of fracture density. However, the dependence is not direct, because *s* is also controlled by the length of the fractures. (Wu and Pollard, 1995).

In the case of random fractures, an increase in the amount of fractures from 250 to 500 also leads to a slight variance in *D*(*av*) observed parameters: $K_{eff\ (d\ )}$and s increases from 4% to 6% and 14% to 16%, respectively; *D*(*av*) for density remains the same for both density cases (15%), while length shows decrease in *D*(*av*) from 5% to 4% (see **Fig. 17**).

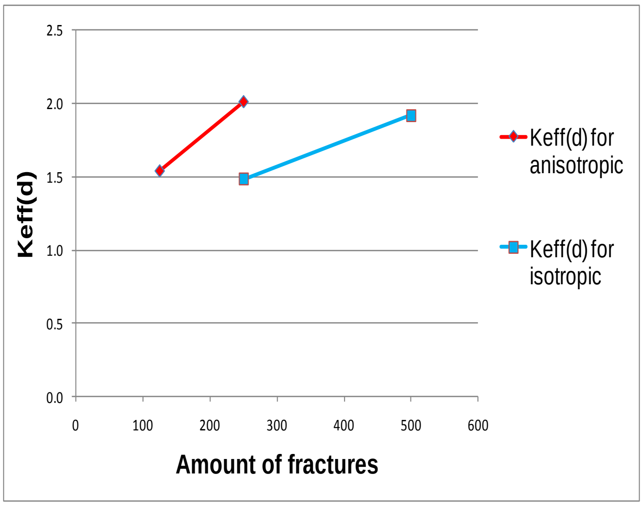


Fig. 16–$K_{eff(d)}$ measurements in anisotropic and isotropic models as a function of amount of fractures.

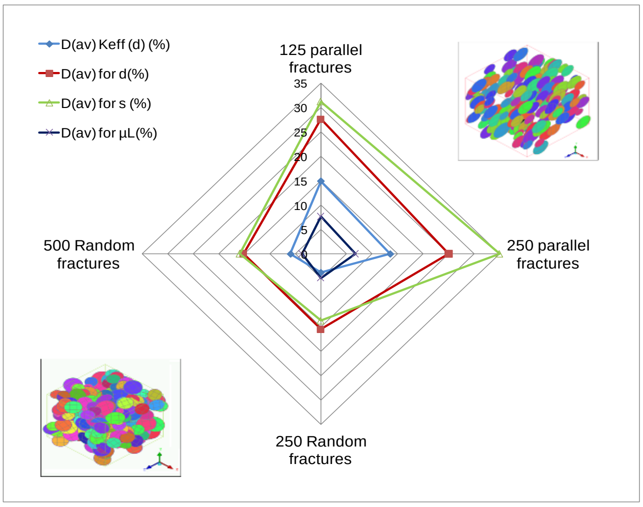


Fig. 17–Mean absolute deviation for effective permeability ($K_{eff(d)}$), density (*d*), spacing (*s*), and mean length distribution (*L*) in low and high dense cases for anisotropic and isotropic models.

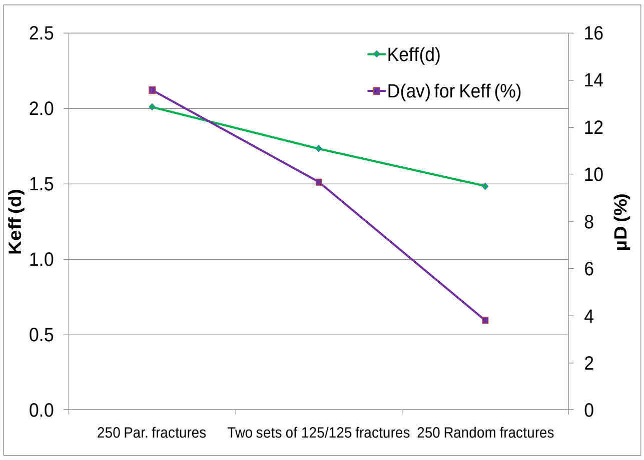


Fig. 18–Impact of different geometries of the model on effective permeability. Figure represents variation of measured $K_{eff(d)}$ and its mean absolute deviation.

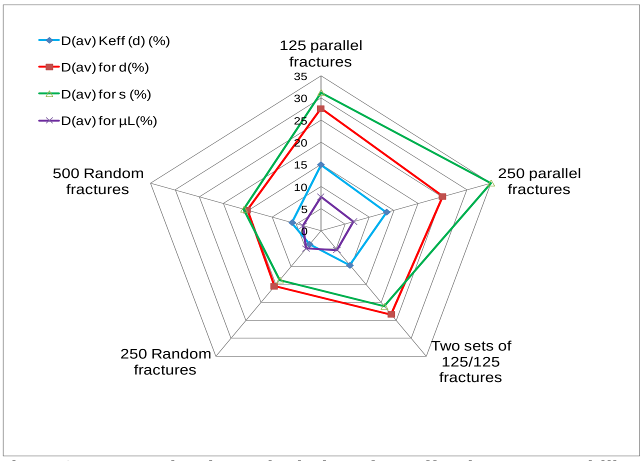


Fig. 19–Mean absolute deviation for effective permeability ($K_{eff(d)}$), density (*d*), spacing (*s*), and mean length distribution (*µL*) in all simulations.

Effective permeability approximation as a function of geometry of the model

In **Fig.17** one can see that in randomly located fractures (isotropic) mean absolute deviation for *d*, *s*, is much lower than parallel fractures (anisotropic) model with the same amount of fractures (250). Spacing shows the mean absolute deviation of 35% for parallel fractures, whereas for random fractures *D*(*av*) is 14%. For the density difference in *D(av)* between anisotropic and isotropic models is 25%>14%. *µL* is slightly affected by geometry of the model showing change of mean absolute deviation from 7% in parallel fractures model (250 fractures) to 5% in random fractures model (250 fractures). Compactness and ability of fractures cross each other in the isotropic (randomly located) model, which does not occur in the anisotropic models (first and second simulation), leads to smaller error in fracture pattern properties.

Three models with the same amount of fractures (250), but with the different geometry (parallel, two sets, and randomly located fractures) show different measurements of $K_{eff(d)}$. **Fig.18** shows that the most permeable is the model with parallel fractures, while least permeable is randomly located fractures. This figure also shows that the largest error of $K_{eff(d)}$ is related to parallel fractures model with 250 fractures (*D*(*av*) = 14%), whereas the smallest error is associated with randomly located fractures (*D*(*av*) =4%) model with the same amount of fractures. Two sets of fractures model shows middle value of *D*(*av*) for $K_{eff(d)}$ *(d)* which is 10%. More complicated geometry of fractures leads to smaller value of $K_{eff(d)}$ and at same time to smaller mean error of this parameter.

Generally, **Fig. 19** represents that spacing value shows the largest mean error among all parameters, especially in simulations with parallel fractures. Mean absolute deviation of *s* for all simulations is 24%. Length associated with the smallest *D*(*av*) of 6% for all simulation.

Discussion

There have been studies on impact of anisotropy and fracture density on numerical computation of flow properties of porous medium. (Bogdanov *et al.*, 2003; Nakashima *et al.* 2000). However, uniqueness of current study is what we measure approximation of K_{eff} in 3D model using 2D slices as a function of geometry and density if the model. Consequently, results that were obtained in the present study can be considered as novel.

For isotropic datasets plane location does not play significant role in describing the 2D property. In nature, often fracture patterns are of strong anisotropic character. If we approximate these using isotropic models, the tendency of the 2D sliceswill be to reproduce the original 3D properties. However, if these are represented using anisotropic models, our results indicate that 2D slices will be poor reproductions of the original model. We cannot consider the latter result as unique and uncontradicted, as it gives opportunity to develop the way of study in terms of this issue. The main question is to choose optimal way to interpret the anisotropic fractured mass. More research needs to be conducted to identify the minimum number of cut planes which is necessary to obtain acceptable approximation of effective permeability in terms of 2D.

In terms of specific measurements, spacing is of poor approximation in both (isotropic and isotropic) models, showing the largest mean absolute deviation from average value among all observed parameters. Largest difference between mean absolute deviations for measured properties of anisotropic vs. isotropic models is associated with effective permeability. We also compared three different models with the same amount of fracture. Comparison concludes the inverse linear dependence of measured K_{eff} and mean absolute deviation for this parameter on the complexity of fractured network. Our study stipulates simulation of anisotropic and isotropic datasets with low and high dense cases. Fracture density has slight impact on the approximation of effective permeability in both anisotropic and isotropic models. Our findings suggest that independently on the fracture density K_{eff} show relatively equal error of approximation.

Concluding remarks

We have successfully presented a methodology to approximate K_{eff} and fracture geometric properties of 2D slices extracted from 3D model. Our study provides various observations, which can be summarised in the following concluding remarks:

- Isotropic model leads to less variation of the measured density, spacing, length, and effective permeability. Thus, randomly located fractures model has the smallest *D*(*av*) for all properties.
- Fracture density slightly affects the *D*(*av*) for all observed parameters in both models (isotropic and anisotropic).
- Spacing and density shows the largest error in approximation to average value, while length is associated with the smallest mean absolute deviation for all models.

Our results indicates that arbitrary cut planes lead to large mean absolute deviation for K_{eff} and all fracture pattern properties (*s*, *d*, *μL*) in anisotropic models. Consequently, it leaves possibilities to improve the way of approximations of flow properties in the 2D simulation, which requires defining the optimal number of cut planes.

Nomenclature

1D	= one dimension
2D	= two dimensions
3D	= three dimensions
A	= area cross section
a	= aperture size
d	= dimensionless fracture density
h	= local fracture aperture
k_{eff}	= effective permeability
$K_{eff(d)}$	= dimensionless effective permeability
k_f	= fracture permeability
k_m	= matrix permeability
l_s	= is the height of the specimen
l	= length of the fracture
μL	= mean length distribution
D	= absolute deviation
D(av)	= mean absolute deviation
μ(x)	= average value of the measured data
P	= pressure
P(u), *P(d)*	= fluid pressure gradients

q_{in} = total in flux
q_{out} = total out flux
AC = arbitrary cut planes

Subscripts

f = fracture
m = matrix
s = specimen
d = dimensionless

References

1. Barenblatt, G.I., Zheltov, I.P., and Kochina, I.N. 1960. Basic concepts in the theory of seepage of homogenous liquids in fissured rocks. Applied Mathematical Methods **24**: 1225–1303.
2. Bear, J. and Braester, C. 1972. On the Flow of Two Immscible Fluids in Fractured Porous Media. In: IAHR (ed.) *Developments in Soil Science.* Elsevier, pp. 177-202.
3. Belayneh, M., Geiger, S., and Matthai, S. K. 2006. Numerical simulation of water injection into layered fractured carbonate reservoir analogs. *American Association of Petroleum Geologists*, **90** (10): 1473-1493.
4. Berkowitz, B., Bear, J., and Braester, C. 1988. Continuum models for contaminant transport in fractured porous formations. *Water Resources Research* **24** (8): 1225–1236.
5. Bogdanov, I. I., Mourzenko, V. V., Thovert, J. F., and Adler, P. 2003. Effective permeability of fractured porous media in steady state flow. *Water Resources Research* **39** (1): 1023-1039. doi:10.1029/2001WR000756.
6. Bogdanov, I. I., Mourzenko, V. V., Thovert, J-F., and Adler, P. 2007. Effective permeability of fractured porous media with power-law distribution of fracture sizes. *Physical Review E* **76**(036309): 1-15.
7. BP. 2007. Statistical review of world energy. Technical report. British Petroleum p.l.c.
8. Budiansky, B., and O'Connell, R.J. 1976. Elastic moduli of a cracked solid. *Int. J. Solid structures* **12**:81-97
9. Cruikshank, K.M., Zhao, G., and Johnson, Arvid M. 1991. Analysis of minor fractures associated with joints and faulted joints. *Journal of Structural Geology* **13** (8): 865-886.
10. Dershowitz, W. S. and Einstein H.H. 1988. Characterizing rock joint geometry with joint system models. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts,* **26** (1): 21-51.
11. Durfolsky, L. J. 1991. Numerical calculation of equivalent grid block permeability tensors of heterogeneous porous media : *Water Resources Research* **27** (5): 299–708.
12. Ganzer, L.J., HOT Engineering 2002. Simulating Fractured Reservoirs Using Adaptive Dual Continuum. Paper SPE 75233 available from SPE, Richardson, Texas.
13. Geiger, S., Huangfu, Q., Reid, A., Matthai, S., Coumou, D., Belayneh, M., Fricke, C., and Schmid, K. 2009. Massively Parallel Sector Scale Discrete Fracture and Matrix. Paper SPE 118924 presented at the SPE Symposium on Reservoir Simulation, The Woodlands, Texas, USA, 2–4 February.
14. Hemminger, A., Neunhauserer, L., and Helmig, R. 1999. The reliability of a stochastic fracture generator. Proccedings of the modelCare 99 Conference, Zurich, Switzerland, September. IAHS publication no. 265
15. Kranz, R. L., Frankel, A. Engelder, D. T., and Scholz, C. H. 1979. The permeability of whole and jointed barre granite. *Int. J. Rock Mech. Min. Sci.*, **16**: 225– 234.
16. Lough, M. F., Lee, S. H. and Kamath, J. 1996. A New Method to Calculate the Effective Permeability of Grid Blocks Used in the Simulation of Naturally Fractured Reservoirs. Paper SPE 36730 presented at the SPE Annual Technical Conference and Exhibition, Denver, CO, USA, 6–9 October.
17. Matthai, S.K., Mezentsev, A., Belayneh, M., Centre for Petroleum Studies, Dept. of Earth Science & Engineering, Imperial College London 2005. Control-Volume Finite-Element Two-Phase Flow Simulations with Fractured Rock Represented by Unstructured 3D Hybrid Meshes. Paper SPE 93341 presented at the SPE Symposium on Reservoir Simulation, Houston, Texas, USA, 31 January–2 February.
18. Nakashima, T., Sato, K., Arihara, N., Waseda, N., and Yazawa, N. 2000. Effective Permeability Estimation for Simulation of Naturally Fractured Reservoirs. Paper SPE 64286 presented at the SPE Asia Pacific Oil and Gas Conference and Exhibition held in Brisbane, Australia, 16–18 October.
19. Olson, J. E. and Pollard, D. D. 1989. Inferring paleostresses from natural fracture patterns; a new method. *Geology* **17**:345-348.
20. Paluszny, A., and Matthai, S. K. 2010. Impact of fracture development on the effective permeability of porous rocks as determined by 2-D discrete fracture growth modelling. *Journal of Geophysical Research* 115(B02203), doi:10.1029/2008JB006236.
21. Phillip, Z. G., Jr., Jennings, J. W., Olson, J., Laubach, S. E., and Holder J. 2005. Modeling coupled fracture-matrix fluid flow in geomechanically simulated fracture networks. Paper SPE 77340 presented at the SPE Annual Technical Conference and Exhibition, San Antonio, Texas, USA, 29 September –2 October

22. Pickup, G.E., Ringrose, P.S., Jensen, J.L., and Sorbie, K.S. 1994. Permeability tensors for sedimentary structures. *Mathematical Geology* **26** (2): 227-250.
23. Pollard, D. D., and Aydin, A. 1988. Progress in understanding jointing over the past century. *Geol. Soc. Am. Bull.* **100** (8): 1181-1204.
24. Renshaw, C. E. 1996. Influence of subcritical fracture growth on the connectivity of fracture networks. *Water Resources Research* **32** (6), 1519–1530.
25. Ronayne, M. J. and Gorelick, M. 2006. Effective permeability of porous media containing branching channel networks. *Physical Review E* 73(026305).
26. Song, X., Shen, P., Yuan**, S.,** and Cao, H. 2000. The Integrated Characterization Techniques on Reservoir Fractures. Paper SPE 64766 presented at the SPE International Oil and Gas Conference and Exhibition, Beijing, China, 7–10 November.
27. Srivastava, R. M., Frykman, P., and Jensen, M. 2005. Geostatistical Simulation of Fracture Networks. Quantitative Geology and Geostatistics. Netherlands: Springer.
28. Thomas, A. L., and Pollard, D. D. 1993. The geometry of echelon fractures in rock: implications from laboratory and numerical simulations. *Journal of Structural Geology* **15**(5): 323-334.
29. Warren, J. E., and Root, P. J. 1963. The behavior of naturally fractured reservoirs, *J. Society of Petroleum Engineers* 245-255,.
30. Witherspoon, P. A., Wang, J. C. Y., Iway, K., and Gale, J. E. 1980. Validity of the cubic law for fluid flow in a deformable rock fracture. *Water Resources Research* **16** (6):1016-1024.
31. Wu, H., and Pollard, D. D. 1995. An expeimental study of the relationship between joint spacing and layer thickness, *Journal of Structural Geology* **17**(6): 887– 905.

Appendices

Appendix A

Critical Milestones

Table A-1─MILESTONES IN NUMERICAL ESTIMATION OF THE EFFECTIVE PERMEABILITY OF FRACTURED RESERVOIR

Paper n°	Year	Title	Authors	Contribution
SPE 16011	1987	Computing absolute transmissibility in the presence o f fine-scale heterogeneity	White, C.D., Horne, R.N	Presented a numerical technique for the determination of equivalent permeability tensors that requires using of different sets of boundary conditions
WATER RESOURCES RESEARCH **27** (5): 299–708	1991	Numerical Calculation of Equivalent Grid Block Permeability Tensors for Heterogeneous Porous Media	Durlofsky, L. J.	Investigated a method in 2D model to estimate effective permeability in the region where heterogeneity was distributed periodically
SPE 36730	1996	A New Method to Calculate the Effective Permeability of Grid blocks Used in the Simulation of Naturally Fractured Reservoirs	Lough, M. F., Lee, S. H. and Kamath, J.	Developed the boundary element method to compute effective permeability of grid blocks
SPE 64286	2000	Effective Permeability Estimation for Simulation of Naturally Fractured Reservoirs	Nakashima, T., Sato, K., Arihara, N., Waseda, N., and Yazawa, N.	Identified advantages of periodic boundary condition in estimation of effective permeability for different sedimentary structures.
WATER RESOURCES RESEARCH **39** (1): 1023-1039	2003	Effective permeability of fractured porous media in steady state flow	Bogdanov, I. I., Mourzenko, V. V., Thovert, J. F., and Adler, P.	First, who provided systematic parametric investigation of the influence of fracture density and conductivity on the effective permeability of fractured media in 3D discrete fracture media.
PHYSICAL REVIEW E 73(026305)	2006	Effective permeability of porous media containing branching channel networks	Ronayne, M. J. and Gorelick, M.	Provided useful contributory model for studying dynamical processes in natural systems which contain branching channel form geometries.
PHYSICAL REVIEW E **76**(036309): 1-15	2007	Effective permeability of fractured porous media with power-law distribution of fracture size	Bogdanov, I. I., Mourzenko, V. V., Thovert, J-F., and Adler, P.	Gives idea of measurement of effective permeability of fractured porous media where fracture sizes are distributed according to a power law.

JOURNAL OF GEOPHYSICAL RESEARCH 115(B02203)	2010	Impact of fracture development on the effective permeability of porous rocks as determined by 2D discrete growth modelling	Paluszny, A., and Matthai, S. K	Developed discrete crack models that can be used to geomechanically generated 2D fracture patterns.

Appendix B

Critical Literature Review

SPE 16011(1987)

Computing absolute transmissibility in the presence o f fine-scale heterogeneity

Authors: White, C.D., Horne, R.N.

Contribution to the understanding of fractured reservoirs:
Presented a numerical technique for the determination of equivalent permeability tensors that requires using of different sets of boundary conditions.

Objective of the paper:
To describe a technique for the demonstration of the effects heterogeneities smaller than the grid-block scale, based on a fine grid single phase flow simulations.

Methodology used:
Tensor scaling algorithm:

- Fine scale reservoir description.
- Microscale simulation: Calculations of pressures and fluxes from several microsimulations with different boundary conditions.

Conclusion reached:
The arithmetic complications associated with a tensor treatment of transmissibility are not unnecessary. Without considering the off-diagonal elements of the permeability tensor methods to compute effective vertical permeability of sand/shale sequences fail.
The tensor scaling algorithm allows to obtain predictions, closed to the fine-grid results

Comments:
Technique is based on a finite difference method, which is not exactly the best method to treat geometrically cross-bedding.

WATER RESOURCES RESEARCH, VOL. 27, NO. 5, PAGES 699-708, (1991)

Numerical Calculation of Equivalent Grid Block Permeability Tensors for Heterogeneous Porous Media

Authors: Durlofsky, L. J.

Contribution to the understanding of fractured reservoirs:
Investigated a method in 2D model to estimate effective permeability in the region where heterogeneity was distributed periodically.

Objective of the paper:
To present a numerical procedure for computing effective permeability of a region with the fine scale permeability distribution within the region.

Methodology used:

- Determination of effective permeability is formulated by a two-scale approach
- Effective permeability calculations of porous medium containing periodically distributed heterogeneities.

Conclusion reached:
The use of triangle-based finite element method for the estimation of effective permeability leads to accurate modeling of the complex geometries.

Comments:
First provided a general framework for the numerical calculation of effective permeability. Previous studies are applicable for the certain situations.

SPE 36730 (1996)

A New Method to Calculate the Effective Permeability of Grid blocks Used in the Simulation of Naturally Fractured Reservoirs

Authors: Lough, M. F., Lee, S. H. and Kamath, J.

Contribution to the understanding of fractured reservoirs:
Developed the boundary element method to compute effective permeability of grid blocks.

Objective of the paper:
To present a new technique to estimate the effective permeability of gridblocks used in conventional simulators.

Methodology used:

- Integration discrete-fracture models with the complexity of the flow calculations offered by continuum method.
- Numerical code based on the boundary-element method.

Conclusion reached:
Only fractures and gridblock surfaces need to be descretized.
New technique allows describing general cases where the fracture permeability dominates the matrix permeability and other cases where it cannot be taken into account.

Comments:
First used a simple fracture system to demonstrate the validity of the method.

SPE 64286 (2000)

Effective Permeability Estimation for Simulation of Naturally Fractured Reservoirs

Authors: T. Nakashima and K. Sato,

Contribution to the understanding of fractured reservoirs:
Analyzed advantages of periodic boundary condition in estimation of effective permeability for different sedimentary structures.

Objective of the paper:
To describe a semi-analytical technique of estimation effective permeability for periodically or randomly fractured media.

Methodology used:

- The complex variable boundary element method, based on flow calculations for fracture distributions.
- Evaluation of effective permeability for discrete fracture systems in order to validate the method.
- Stochastic fracture generation method.

Conclusion reached:

- The larger each of the parameters (total length, mean length, and variation of length), the higher the effective permeability.

- In case of fractures parallel to the pressure gradient, the effective permeability has linear dependence on length, mean length and variation of length.

Comments:
Investigated the effect of diagonal and off-diagonal components on effective permeability.

WATER RESOURCES RESEARCH Vol: 39 No:5 : 1023-1039 (2003)

Effective permeability of fractured porous media in steady state flow

Authors: Bogdanov, I. I., Mourzenko, V. V., Thovert, J. F., and Adler, P,

Contribution to the understanding of fractured reservoirs:
First, who provided systematic parametric investigation of the influence of fracture density and conductivity on the effective permeability of fractured media in 3D discrete fracture media.

Objective of the paper:
To describe a discrete fracture model of 3D fracture network and embedding matrix.

Methodology used:

- 3D meshing of fractured media
- Finite volume method to discretized flow equations.
- Stochastic fracture generation method.

Conclusion reached:

- The importance of the percolation threshold of the fracture network has been proved.
- By means of excluded volume the shape of the fracture can be taken into account.

Comments:
The permeability of fractured media studied considering real geometry of the fracture network. Additionally, investigated the impact of fracture shape on effective permeability in 3D model.

PHYSICAL REVIEW E 73, 026305 (2006)

Effective permeability of porous media containing branching channel networks

Authors: Ronayne, M. J. and Gorelick, M.

Contribution to the understanding of fractured reservoirs:
Provided useful contributory model for studying dynamical processes in natural systems which contain branching channel form geometries.

Objective of the paper:
To estimate effective permeability of a porous medium consisting of 'branching dendritic channels embedded in a low permeability matrix'

Methodology used:
- Channel network generation (generation of synthetic channel deposits) based on the invasion percolation model with a 'nonlooping constraint to simulate drainage networks'.
- Detailed 2D numerical simulations which involves steady-state and laminar flow to determine the actual effective permeability for each channel matrix

Conclusion reached:
The averaging exponent is a useful dimensionless parameter related to the degree of flow channelling. Spatial power averaging method can give ideas to understand the nature of the flow dynamics, to provide predictive structure for permeability upscaling in structurally heterogeneous fields.

PHYSICAL REVIEW E 76, 036309 (2007)

Effective permeability of fractured porous media with power-law distribution of fracture sizes

Authors: Bogdanov, I. I., Mourzenko, V. V., Thovert, J-F., and Adler, P.

Contribution to the understanding of fractured reservoirs:
Gives idea of measurement of effective permeability of fractured porous media where fracture sizes are distributed according to a power law.

Objective of the paper:
To investigate numerically the effective macroscopic permeability of fractured porous media for parameters such as the fracture density, shape and size distribution, and the permeability, possibly size dependent, of the fractures.

Methodology used:

- Detection of the geometric model and estimation percolation properties
- Description of the flow in the matrix by the Darcy equations
- Numerical implementation of the geometrical and physical models using model identical to that of *Bogdanov* et al.
- The generation of the fracture network and the subsequent percolation tests

Conclusion reached:
For networks which do not percolate effective permeability can be evaluated by the quadratic expansion. Except for extremely permeable fractures, the percolation status does not significantly affects the average effective permeability.

Comments:
The present work is the extension and synthesis of the earlier studies (Huseby *et al.* Bogdanov *et al.* Mourzenko *et al.)* and it addresses ‘the full complexity of flow in permeable fractured media, by accounting for the matrix flow and for the size polydispersity of the fractures’.

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 115, B02203, 18 PP., (2010)

Impact of fracture development on the effective permeability of porous rocks as determined by 2D discrete growth modelling

Authors: Paluszny, A., and Matthai, S. K

Contribution to the understanding of fractured reservoir:
Presents discrete crack models that can be used to geomechanically generated 2D fracture patterns.

Objective of the paper:
To present a 2-D linear elastic finite element model that generates fracture patterns in incremental iterative steps and to stimulate quasi-static multiple crack propagation. Then, to study their impact on fluid flow as a function of fracture density.

Methodology used:

- Numerical solution the steady state single phase flow using the finite element method (FEM)
- Integration of the flux along a cross section of the dataset
- Computation the fracture-matrix flux ratio
- Computation of the pressure field and k_{eff}.
- Characterization of the generated patterns by measuring density, connectivity, and length and aperture distributions.

Conclusion reached:
The fixed apertures assume that topological connectivity implicates flow connectivity, thus they overpredict the k_{eff} of the system by up to six orders of magnitude.
For stress-dependent apertures, with various fracture permeability, the preferred flow path is determined not only by topology.
When an open path connects the flow boundaries, percolation causes a jump in the k_{eff} by several orders of magnitude. There is a steady linear increase in the effective permeability of the system occurs after percolation.

Comments:
All methods in this work are part of the Complex Systems Modeling Platform (CSMP++) developed at Imperial College and the ETH Zurich.