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Influence of Rock Heterogeneity on Fracture Pattern Formation

By

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A report submitted in partial fulfillment of the requirements for the MSc and/or the DIC.

September 2010

DECLARATION OF OWN WORK

I declare that this thesis *Influence of Rock Heterogeneity on Fracture Pattern Formation* is entirely my own work and that where any material could be construed as the work of others, it is fully cited and referenced, and/or with appropriate acknowledgement given.

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ABSTRACT

Fracture networks could significantly alter mechanical and hydraulic properties of the rock. Effect of rock heterogeneity on rock failure and fracture propagation is one of the most challenging questions nowadays. The objective of this paper is to quantify and characterize fracture pattern behavior in heterogeneous rocks. Heterogeneity realization is delivered by perturbing elasticity constants (Young's modulus and Poisson's ratio) across the model area within a practical range of values. Simulation is performed by means of finite-element based method, which allows propagating multiple cracks simultaneously. Generated fracture patterns exhibit similar propagation trends for various perturbation scenarios. Major fracture characteristics as spatial density, lengths, spacing, clustering are quantified.

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Influence of Rock Heterogeneity on Fracture Pattern Formation

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Abstract

Fracture networks could significantly alter mechanical and hydraulic properties of the rock. Effect of rock heterogeneity on rock failure and fracture propagation is one of the most challenging questions nowadays. The objective of this paper is to quantify and characterize fracture pattern behaviour in heterogeneous rocks. Heterogeneity realization is delivered by perturbing elasticity constants (Young's modulus and Poisson's ratio) across the model area within a practical range of values. Simulation is performed by means of finite-element based method, which allows propagating multiple cracks simultaneously. Generated fracture patterns exhibit similar propagation trends for various perturbation scenarios. Major fracture characteristics as spatial density, lengths, spacing, clustering are quantified.

Introduction

Important part of world's hydrocarbon reserves is contained in naturally fractured formations. This is largely because of the huge volume of remaining reserves concentrated in the Middle East, North and South America, North Africa, including reservoirs dominated by fracture flow. It is generally implied for fractured reservoirs that fractures provide main flow paths or enhance fluid transport while the rock matrix serves as storage capacity (especially in low permeability formations), but in some fields fractures can provide both essential storage and permeability (Nelson, 2001), (Lonergan et al., 2007). Fractures and faults occur in various types of rocks. The mineralisation degree, fracture aperture, tortuosity, connectivity and other properties can increase, decrease or even block fluid flow in the formation (Leckenby et al., 2005).

Numerous application fields express interest in simulating fracture growth, such as engineering construction materials (Cervenka, 2002), aeronautics and composite materials (Ingraffea & Wawrzynek, 2003), nuclear waste migration through the rock (Huysmans et al., 2006), hydraulic fracturing of hydrocarbon formations (Adachi et al., 2007), (Boone et al., 1986), flow transport models in fractured reservoirs (Feyen & Caers, 2006). One of the particular cases of interest from petroleum geoscience viewpoint is to understand the behaviour of fracture patterns while characterisation or modeling of the fractured reservoirs.

However, in reality, rock materials have a composite microstructure with some internal defects (voids, microcracks, planes of weakness). Although the distribution of such irregularities and stresses in the rock specimen may look homogeneous at macro scale, it shows highly disordered pattern at micro scale. Therefore, brittle failure behaviour of the rock and growth of cracks are directly dependant on the heterogeneous character of the material (Huet, 1997), (Liu et al., 2004),(Tang et al., 2000).

Heterogeneity is statistically bound to variability and uncertainty (Cesano et al., 2003). (Stagnitti et al., 1999) defined the heterogeneity of soils as a "non-random spatial and temporal variability of physical, chemical and biological components". Local stress variations widely range from the magnitude of applied stress because of such heterogeneous inclusions (Tang et al., 2007). In addition to "inclusion" concept, local stress direction perturbations can derive from effects of topography, local thermal effects (hydrothermalism or volcanism), anisotropy, small faults and effects of erosion (Zhang et al., 1994). In order to reasonably investigate the fracture behaviour, model should take into account such heterogeneous characteristics.

The early objective of fracture mechanics was to predict rock failure or identify the causes of fracture (Cotterell, 2002). Notable contributions into fracture theory were made by (Griffith, 1921) and Inglis (1913). Development of the equations for an elliptical shape in an elastic plate by Inglis led to the concept of stress concentrators which Griffith recognized as the reason for brittle material tensile strengths many times less than the theoretical maximum. Griffith realised that an increase in surface free energy is necessary to produce a fracture in elastic materials. Later, Irwin (1948) extended this concept and noted that fracture energy is two thousand times higher than the surface energy for low carbon steel. Independently to Griffith's unstable crack concept, (Obreimoff, 1930) conducted experiments on splitting mica specimens and noticed the reversibility of the cleavage. (Gurney & Hunt, 1967) formulated this as quasi-static growth and showed that "stress intensity to propagate geometrically similar cracks varies inversely as the square root of their size".

During the second half of the last century, with the development of computer power several methods of fracture representation were developed. (Tang, 1997), (Tang & Kaiser, 1998), (Blair & Cook, 1998), (Tang et al., 2000), (Fang & Harrison, 2002). Fig.1 represents simplified taxonomy of approaches in that field.

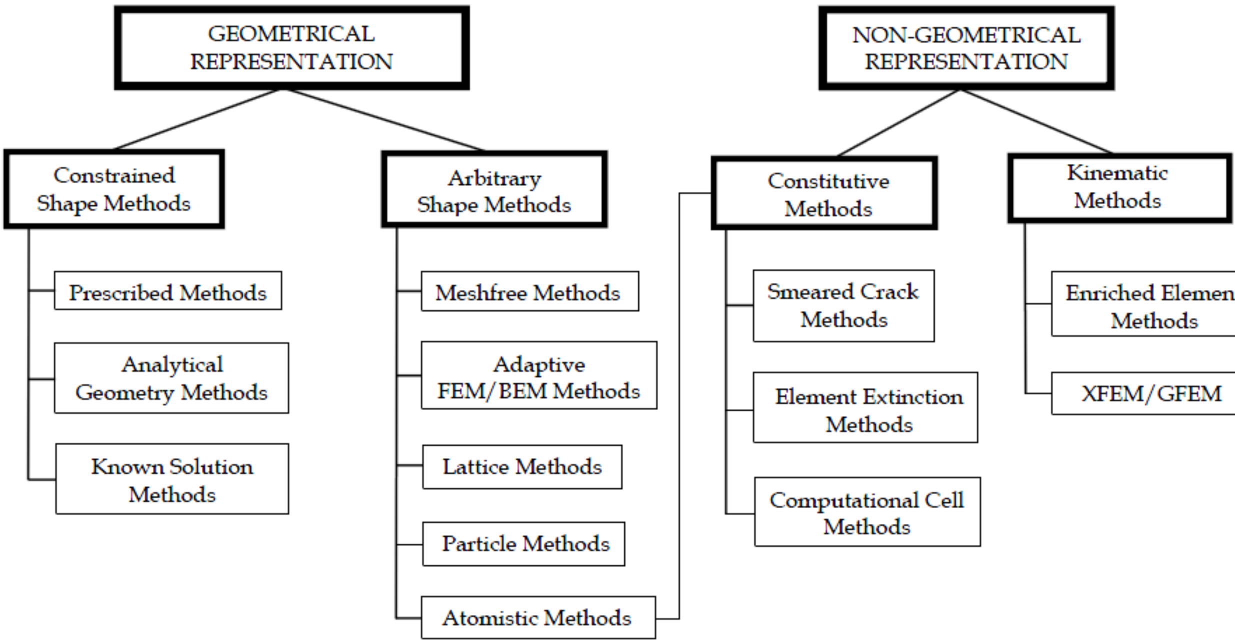

Fig. 1. Computational methods of fracture modelling (by Ingraffea&Wawrzynek, 2004).

Analytical methodology by (Renshaw & Pollard, 1994) applied concept of stress field around multiple straight cracks. However, analytical methods had limitations with simulating arbitrary crack geometries and had to be simplified due to computational capacities (Liu 2002). Generally, crack growth is affected by coalescence of multiple cracks during their propagation and various attempts were made to simulate those patterns. Linear finite numerical method was one of the techniques that considered such classical driving forces as stress intensity factors (body's resistance to fracture) (Irwin, 1958), crack tip opening displacements and angles, energy release rates, and elastic and elasto-plastic crack front integrals (Ingraffea & Wawrzynek, 2003),(Ingraffea & Saouma, 1985). Arbitrary shapes of propagating cracks required modification of topology (Bouchard et al., 2003).

The advantage of the finite element-based modeling of deformation is the simplicity of the numerical discretisation of the solved equations. The idea is that by retaining an accurate representation of topology and material interfaces the numerical method is relieved of a sub-mesh representation of the geometry and there is more room to capture complex behaviour, such as heterogeneities, compaction, damage, and inelastic deformation.

Boundary element method is an alternative to the FEM/XFEM (Olson & Pollard, 1989). BEM numerically computes energy release rates, but initially was limited to grow a single set of straight cracks. Later BEM was advanced for linear (Aliabadi, 2003) for nonlinear methods (Cervenka, 2002).

Meshless ("mesh free" or "element-free") methods based on partition of unity concept and standard Galerkin procedure significantly simplified the meshing tasks (Belytschko et al., 1995, Belytschko et al., 1996), (Moes et al., 1999), (Belytschko & Black, 1999),(Yazid et al., 2009). Functional values at nodes are generated from adjacent nodes using various approximations, such as moving least square method. (Bordas et al., 2008) extended meshless method in 3D for non-linear materials. Currently this method is competing with FEM techniques, but increasing computer power and further researches make it attractive in 3D arena.

Another extreme FE realization of constitutive method is element extinction. When fracture propagates to the next element, the latter is simply removed and no longer sustains stresses (Beissel et al., 1998). Therefore fracture width and pattern is dependent on mesh.

Recent numerical studies show that fracture patterns can be realistically recreated by approximating mechanical behaviour using 2D simulations: a propagation methodology based on finite-element method was developed by (Paluszny & Matthäi, 2009). Possibility of introducing heterogeneous regions with different stress conditions and rock properties allows investigating various fracture patterns and attempt to quantify the effects: number of fractures, size, connectivity, density etc.

In this paper we investigate growth of fracture patterns with perturbed rock properties such as, variable Young's modulus, Poisson's Ratio; and initial flaw distribution.

The content of this paper is organized as follows. Section 1 describes the method with its governing equations, propagation algorithm and heterogeneity application. Section 2 discusses experimental setup with the simulation details and input values. Sections 3 investigate the impact of material heterogeneities on generated fracture pattern with quantified results of such implementations. Finally, Section 4 proposes further recommendations and concludes this work.

Methodology.

Fracture growth algorithm

We exploit the finite-element based method that is capable of stochastically generating multiple cracks all at once. Method implementation is iterative and applies sub-critical quasi-static fracture propagation along with adaptive remeshing. Adaptive mesh refinement allows economizing computational resources while capturing complex fracture shapes. Computation is simplified by using general equation of isotropic, homogeneous rock mass with linear elastic deformation behaviour (Cook et al., 1989) is:

$$\boldsymbol{\sigma} = \boldsymbol{D}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon_0}) + \boldsymbol{\sigma_0} \quad (1)$$

where ε is the strain vector, σ is the stress vector, while subscript "$_0$" denotes initial condition. $\boldsymbol{D}$ is the stiffness matrix of linear elastic material, but it could be adjusted to some specific elasticity behaviour of the solid. When certain boundary stress conditions applied, this equation comes to the force equilibrium: $\boldsymbol{\partial\sigma} + \boldsymbol{F} = \boldsymbol{0}$ (2)

The initial model area is broken into six-node isoparametric quadratic triangles (Taig, 1961) to numerically simulate fracture propagation. These triangles iteratively change size and orientation along propagated cracks. Mid-side nodes are shifted towards the fracture tips in order to correctly compute stress and displacement behaviour around it. Material properties, such as Young's modulus and Poisson's ratio, are determined at three Gauss integration points in every triangle (Fig. 2).

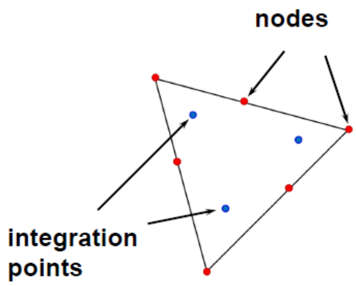

Fig. 2. Isoparametric quadratic triangle.

Sub-critical fracture growth is combined with propagation criterion, that allows to simulate multiple simultaneous cracks extending at different speeds. These different propagation speeds are weighed with energy release rate *G* of every crack. Fracture propagation is in the direction of maximum circumferential stress around the tip.

Model does not nucleate cracks by itself: fractures propagate from random initial flaws. Although, orientation, quantity and size of the flaws could be adjusted as desired.

Briefly the algorithm is as follows:

- Generating initial flaw set
- Meshing: subdivide model area with unstructured grid based on flaw set
- Applying displacement boundary conditions
- Solving partial differential equations of linear elastic deformation law
- Computing displacement, stress and strain fields
- Computing stress intensity factor, tip advance and growth direction of every crack
- Propagating fractures
- Remeshing of the model to capture deformation
- Recalculating of stresses after previous iteration
- Continuous iterative process until propagation stops: no growth for fixed boundary displacement

Generated fracture network is used to quantify and describe the pattern characteristics.

Numerical computations are performed by Complex System Modelling Platform (CSMP) (Matthai *et al.*, 2001) applying algebraic multigrid solver (Stüben, 2001). Detailed description and validation of the simulator could be found in Paluszny (2009) and Paluszny & Matthäi (2009).

Heterogeneity application

Hydrocarbons generally occur in underground traps formed by structural and/or stratigraphic features. Usual porous and permeable beds of hydrocarbon accumulations are mainly sands, sandstones, limestones and dolomites. Shales are believed to serve as the source rocks and cap rocks for hydrocarbon accumulations (Craft & Hawkins, 1959). Heterogeneous nature of rock matrix varies within the certain rock type and even within the same reservoir. Therefore it is more reasonable to talk about the ranges of values of rock properties than some "fixed" numbers.

There are several ways to measure mechanical rock properties: starting from various loading scenarios of the representative rock sample in the laboratory and up to seismic/acoustic testing of formation. Different methods could have different values of elastic constants. Early investigations (Zisman, 1933, Ide, 1936) defined some of the causes of such discrepancy to be due to microcracks, cavities, planes of weakness and dependance on the applied stress magnitude. Variation in rock bedding orientation, porosity, grain size distribution could give substantially diverse values. Table 1 provides elastic constants measured by static and dynamic methods:

Table 1. Elastic constants' values of some rocks.

	E_S (GPa)	E_D (GPa)	ν_S	ν_D	Source
Sandstone	6.6 - 53.2	-	0.09 - 0.28	-	Batugin, 1972
Sandstone	20.2 - 36.3	40.1 - 51.8	-	-	Nowakowski (maximum values), 2005
Limestone	24.8 - 60.45	-	0.2 - 0.28	-	Palchik & Hatzor, 2002
Chalcedonic limestone	55.16	46.89	0.18	0.25	US Bureau of Reclamation (1953)
Limestone (Saudi Arabia)	39.0 - 60.7	45.3 - 47.9	0.22 - 0.32	0.17 - 0.21	Al-Shayea, 2004
Dolomite	16.2 – 64.0	-	0.19 - 0.4	-	Palchik & Hatzor, 2002
Dolomite	37.2 - 51.0	-	0.26	-	Lee&Ehgartner, 2002
Shale	25.6 - 41.5	45.7 - 63.1	-	-	Nowakowski (maximum values), 2005
Coal (Zonguldak region)	-	-	0.15-0.49	-	Gercek, 2007 (from METU technical reports)
Mudstone-limestone (Italy)	-	68 - 82	-	0.26 - 0.30	Ciccotti & Mulargia, 2004

Foregoing table does not provide “handbook” values of elasticity constants, but mainly indicates ranges of them.

Another realization of rock heterogeneity is bound to small cracks in the solid matrix. Such imperfections naturally occur in almost every rock. Their distribution and scale depends on rock grain sizes, shapes, mineralization degree and crustal conditions. It is well accepted that such microscale flaws could initiate cracks and therefore affect developing possible fracture networks.

Experimental setup

Simulation is carried out using a finite element based 2D fracture growth code (Paluszny & Matthäi, 2009). This module grows a set of fractures and outputs the crack geometry at each iteration step, while computing essential quantitative pattern characteristics: fracture density (*d*), spacing, length (*l*) and connectivity. The objective of current fracture propagation simulation is to investigate the impact of fracture heterogeneity on crack pattern formation. Still, heterogeneity is a very wide term to operate and we will be running a scenario of heterogeneities that is *elastic constants' perturbation*.

First, it is necessary to cover viable interval of elasticity constants' values while keeping in mind their substantial range. We stopped on the reference values of:

Young's modulus=40 GPa, maximum perturbation of ±80% (values between 8-72 GPa);

Poisson's ratio=0.25, maximum perturbation of ±80% (values between 0.05-0.45).

These ranges are covering most of the hydrocarbon-bearing rocks, except maybe some highly disordered and auxetic solids (materials with negative Poisson's ratio that become thicker when stretched). Therefore, we get a chance to visualize fracture propagation patterns within fictitious hydrocarbon reservoir. Simulation cases will consist of:

- Constant values of both parameters (E=40 GPa and ν=0.25, 0% perturbation from reference values);
- Small perturbation (E=32-48 GPa and ν=0.2-0.3, 20% perturbation from reference values);
- Medium perturbation (E=24-56 GPa and ν=0.15-0.35, 40% perturbation from reference values);
- Substantial perturbation (E=16-64 GPa and ν=0.1-0.4, 60% perturbation from reference values);
- Large perturbation (E=8-72 GPa and ν=0.05-0.45, 80% perturbation from reference values);

Model area dimensions as 5m×5m square, with the initial flaw area of 80% of both X and Y axis. The fracture propagation area is assumed to be 90% of the model dimensions, in order to eliminate computational boundary effects.

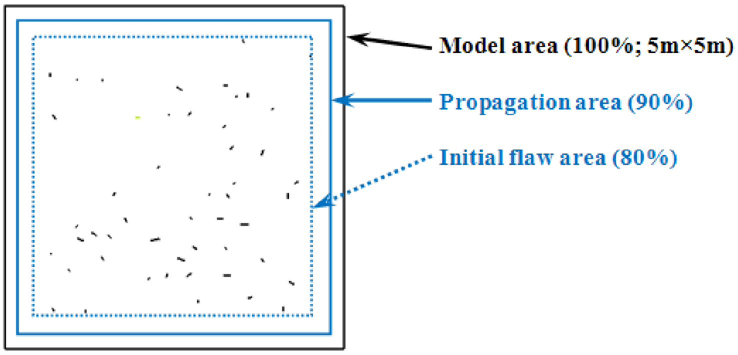

Fig. 3. Model area schematic for elastic constants' variation scenario.

It is expected that fracture tips will advance faster in the regions of low Young's Modulus and low Poisson's Ratio and vice versa. Moreover, since heterogeneous regions serve as stress concentrators it is reasonable to notice crack pattern distortion and coalescence in/around such regions.

The obtained results will give us an idea of crack propagation behaviour and allow to quantify some parameters: spacing, length, density, connectivity under purely random heterogeneous conditions.

Other generic model options are as follows: initial flaw number is to be 50, flaw angle is random, displacement for the boundaries is kept default at 2×10^{-5}m, flaw size of 0.05m, flaw spacing is three times the flaw size, growth index is 0.35.

Initial differential boundary stress conditions are imposed as a vertical tensile force with additional compression at the sides, this allows to establish so called “large differential stress” simulator boundary conditions for the model area.

Results and Analysis

The model was simulated from the same initial flaw set with total 80 iteration steps. It gives us a chance to compare perturbed realizations with constant elastic properties scenario.

Propagated fracture patterns exhibit similar trends on macroscopic scale. Dominating horizontal direction of fracture propagation is governed by large differential stresses, Fig. 4. Results confirming similar trends were noted by Paluszny (2009) for different input values of model and rock matrix.

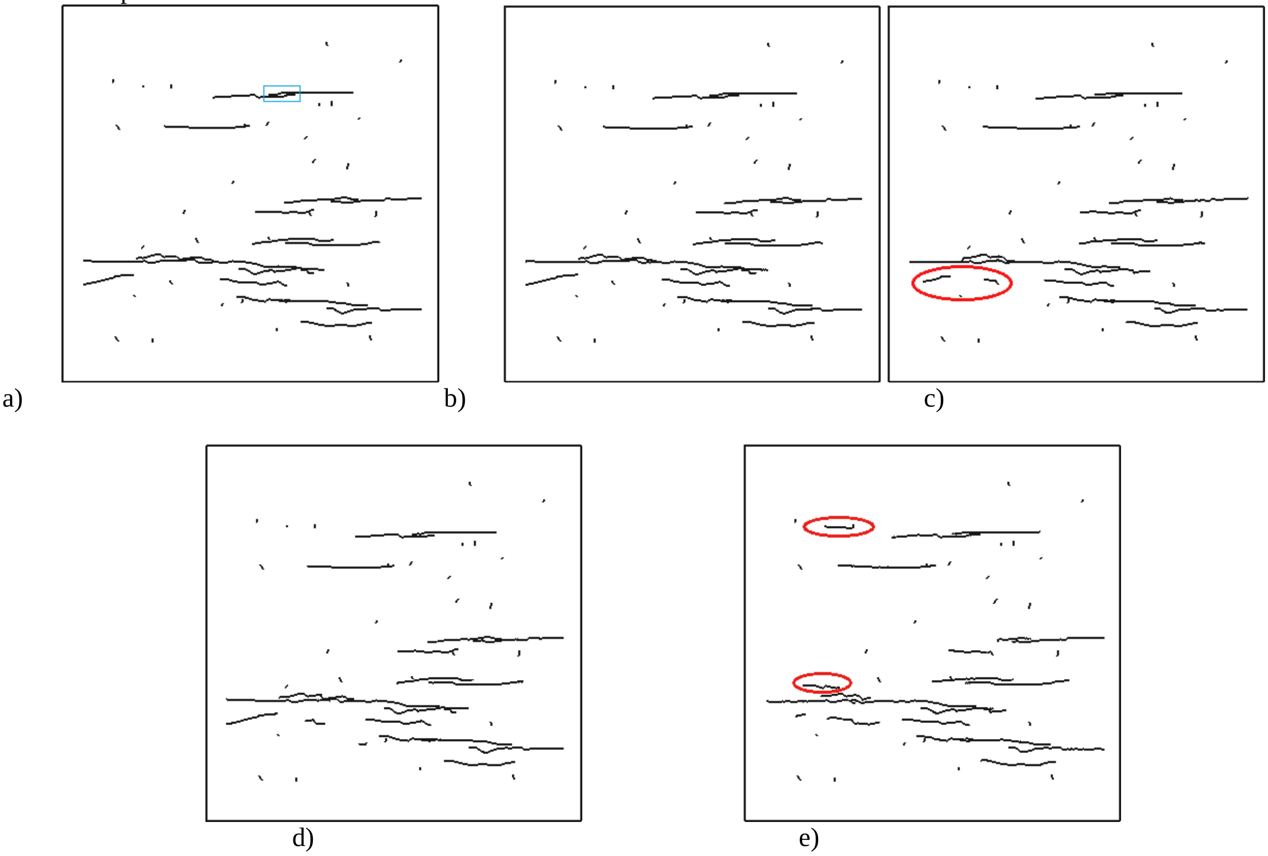

Fig. 4. Simulated fracture patterns for a)0%, b)20%, c)40%, d)60%, e)80% perturbation cases.

Red regions indicate specific fracture network areas with different fracture propagation behaviour. Variance in fracture propagation is caused by changing stress fields with regards to elastic properties. However, example magnification of blue rectangle region (Fig. 4a) shows that fractures have more tortuous shapes in the cases with higher perturbations, Fig. 5.

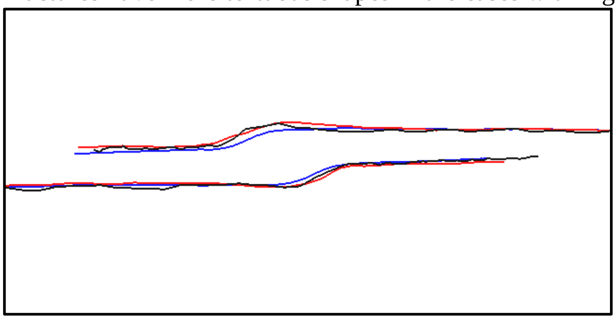

Fig. 5. Detailed image of fracture patterns for 0% (blue), 40% (red) and 80% (black) perturbation scenarios (50x25 cm area).

Obviously, tortuous behaviour of cracks is due to more substantial stress fluctuations in close proximity along the cracks. Simulator assigns elastic properties of the media at three Gaussian integration points in every triangle of the mesh. Such effect could be noticed while comparing fractures in composite materials or rocks with various grain size sorting.

Visualization of Young’s moduli distribution on Fig. 6 shows that resolution of nodes is comparable for all cases.

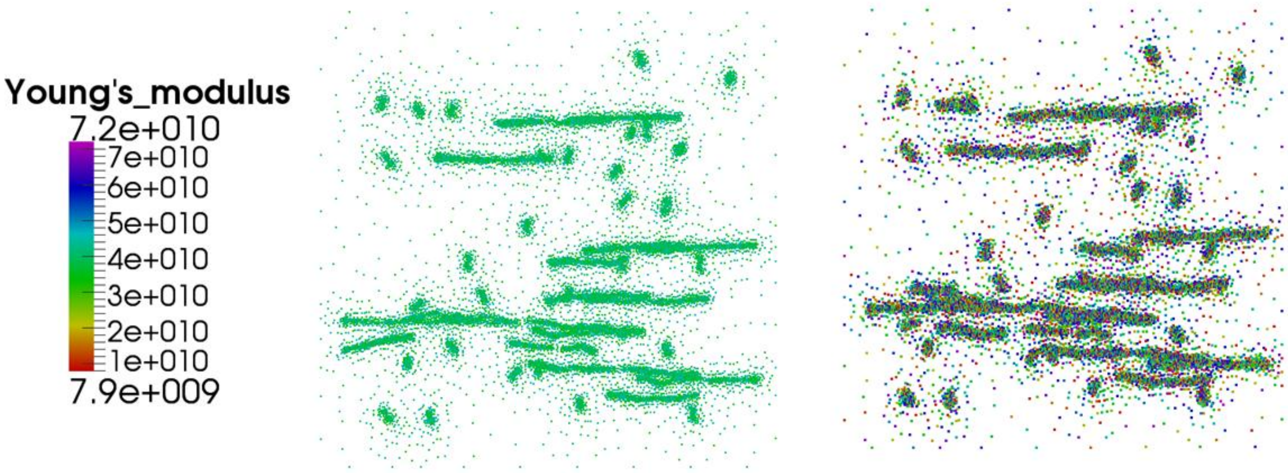

Fig. 6. Young's moduli resolution for 20% (left) and 80% (right) cases.

Quantification of the fracture pattern was based on typical fracture properties, as density, spacing, lengths and connectivity.

Step-by-step visualization of fracture growth shows that random orientation of initial flaws prevented some of them from propagating. Physical count of "active fractures" that contributed to a network results are in the Table 2. Difference in "active" fracture quantity between 0% and 80% perturbation scenarios is purely due to influence of variations in the stress fields of the model.

Table 2. Number of fractures that propagated during simulation.

	Iteration steps							
	10	**20**	**30**	**40**	**50**	**60**	**70**	**80**
0% case	17	17	18	18	18	18	18	18
20% case	17	17	18	18	18	18	18	18
40% case	17	18	19	19	19	19	19	19
60% case	18	19	20	20	20	20	20	20
80% case	19	21	21	21	21	21	21	21

Absolute fracture count results are in Fig. 7:

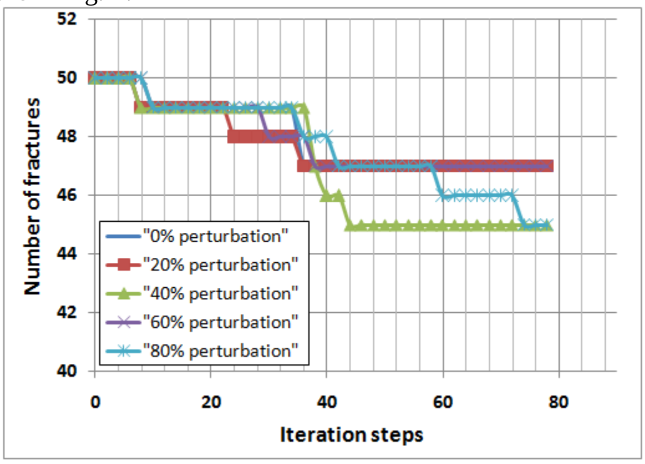

Fig. 7. Number of fractures for each perturbation case.

When two fractures merge, it is counted as one object. Number of fractures range between 45-50, including initial flaws. Reduction in the fracture quantity is due to forming clusters, which include 2-4 cracks in our cases. Cluster extensions in the direction perpendicular to the applied boundary conditions are in the range of 212-280 cm (the longest crack extensions are 280 and 278 cm for 40% and 80% perturbation cases repectively). However, final average lengths for all cases are between 45-48.3 cm, which means comparable fracture propagation scenarios among all cases (Fig. 8 and 9).

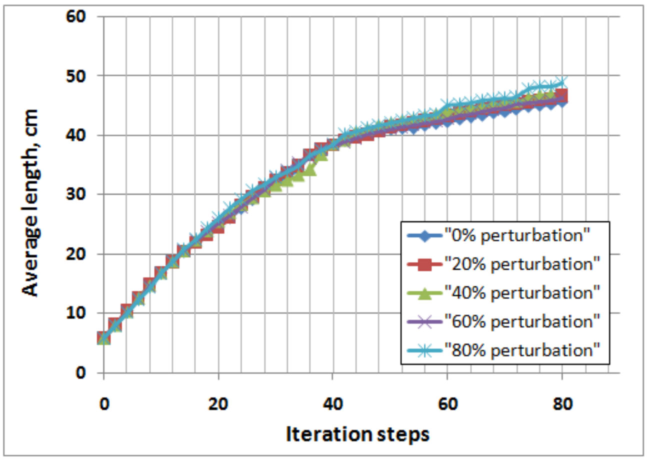

Fig. 8. Average fracture lengths for each perturbation case.

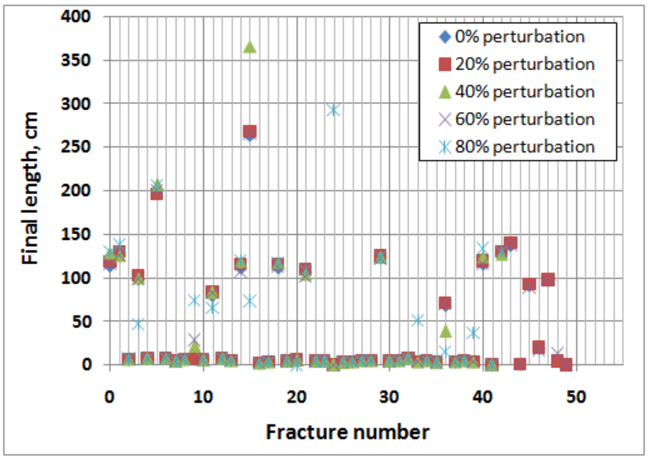

Fig. 9. Final lengths of every fracture for all perturbation cases.

Quantified results of spatial density on Fig. 10 show continuous increase with the iteration steps, which means that the model area was not fully saturated with fractures. The discrepancy between 40% case density and others is caused by formation of bigger cluster (4 fractures) during that case (Fig. 11).

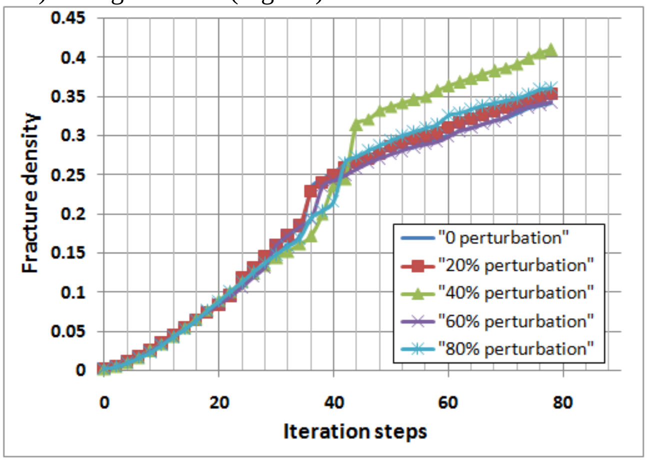

Fig. 10. Fracture densities for each perturbation case.

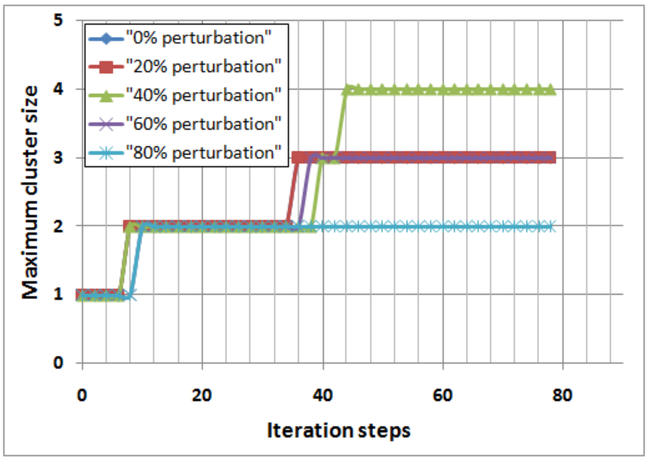

Fig. 11. Maximum cluster sizes.

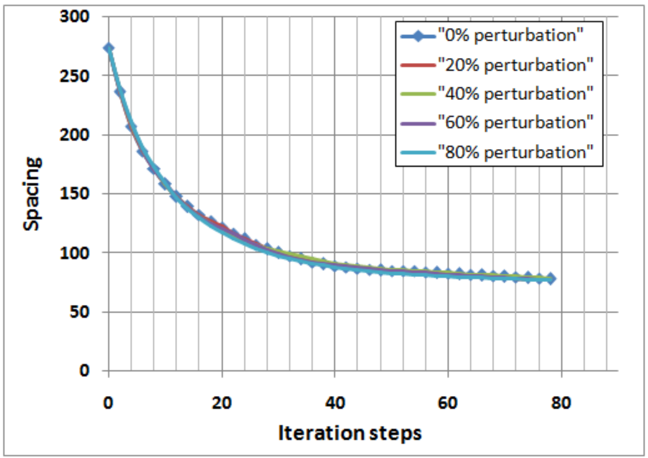

Fig. 12. Fracture spacing for each perturbation case.

It is worth noting that spacing (Fig. 12) decreases sharply during the first 30 iterations. This is due to independent crack growth in the beginning. Further simulation will result in stabilizing of the spacing. This is because stress fields that surround fractures interfere with neighbouring cracks and prevent them from growing. Almost perfect match of spacing trends between various cases mean that cracks extend almost similarly.

Conclusion and recommendations

Numerous inquiries are performed around such sensitive issue as heterogeneity. And yet, there is a range of definitions for heterogeneity across the fields. However, unquestionable importance of this subject brings up new insights into understanding of solid behavior. In this attempt we tried to quantify the results of the impact of rock heterogeneity on fracture pattern formation. Observations show that perturbation of elastic constants in the wide practical range of rocks does not crucially differentiate the fracture pattern, despite more tortuous surfaces of cracks. These results are consonant with composite material experiments of Arrea and Ingraffea (1982) on unreinforced single-notched mortar beam loading. The reason for such simulation results could be due to adaptive mesh refinement around cracks, which results in realization of fine-grain solid material.

Further recommendations may include several other realizations of heterogeneity:

- Observation of fracture patterns with various initial flaw sets: different flaw angles and spacing scenarios will affect fracture clustering and growth directions;
- Introducing regions of arbitrary shape into the model area, and investigating the stress distribution fields around the fractures and heterogeneities.

This will allow to get more deep insight into the fracture propagation in the naturally occurring rocks or other solids.

Nomenclature

E=Young's Modulus
N=Poisson's ratio
d =fracture density
l=mean fracture length

References

1. Adachi, J., Siebrits, E., Peirce, A. & Desroches, J. (2007) Computer simulation of hydraulic fractures. *International Journal of Rock Mechanics and Mining Sciences,* 44 (5), 739-757.
2. Aliabadi, M. H. (2003) Boundary Element Methods in Linear Elastic Fracture Mechanics. In: I. Milne, R. O. Ritchie & B. Karihaloo (eds.) *Comprehensive Structural Integrity.* Oxford, Pergamon, pp. 89-125.
3. Al-Shayea, Naser A. (2004) Effects of testing methods and conditions on the elastic properties of limestone rock. *Engineering Geology,* 74 (1-2), 139-156.
4. Arrea, M, Ingraffea, A. R. (1982) Mixed-mode crack propagation in mortar and concrete. Department of Structural Engineering, Cornell University.
5. Batugin S. A. and Nirenburg R. K. (1972) Approximate relation between the elastic constants of anisotropic rocks and the anisotropy parameters. Translated from *Fiziko-Tekhnicheskie Problemy Razrabotki Poleznykh Iskopaemykh, No. 1, pp. 7–12, January–February, 1972.* Kuzbass Polytechnic Institute, Kemerovo.
6. Beissel, S. R., Johnson, G. R. & Popelar, C. H. (1998) An element-failure algorithm for dynamic crack propagation in general directions. *Engineering Fracture Mechanics,* 61 (3-4), 407-425.
7. Belytschko, T. & Black, T. (1999) Elastic crack growth in finite elements with minimal remeshing. *International Journal for Numerical Methods in Engineering,* 45 (5), 601-20.
8. Belytschko, T., Krongauz, Y., Organ, D., Fleming, M. & Krysl, P. (1996) Meshless methods: An overview and recent developments. *Computer Methods in Applied Mechanics and Engineering,* 139 (1-4), 3-47.
9. Belytschko, T., Lu, Y. Y., Gu, L. & Tabbara, M. (1995) Element-free galerkin methods for static and dynamic fracture. *International Journal of Solids and Structures,* 32 (17-18), 2547-2570.
10. Blair, S. C. & Cook, N. G. W. (1998) Analysis of compressive fracture in rock using statistical techniques: Part II. Effect of microscale heterogeneity on macroscopic deformation. *International Journal of Rock Mechanics and Mining Sciences,* 35 (7), 849-861.
11. Boone, T. J., Wawrzynek, P. A. & Ingraffea, A. R. (1986) Simulation of the fracture process in rock with application to hydrofracturing. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts,* 23 (3), 255-265.
12. Bordas, S., Rabczuk, T. & Zi, G. (2008) Three-dimensional crack initiation, propagation, branching and junction in non-linear materials by an extended meshfree method without asymptotic enrichment. *Engineering Fracture Mechanics,* 75 (5), 943-960.
13. Bouchard, P. O., Bay, F. & Chastel, Y. (2003) Numerical modelling of crack propagation: automatic remeshing and comparison of different criteria. *Computer Methods in Applied Mechanics and Engineering,* 192 (35-36), 3887-3908.
14. Cervenka, V. (2002) Computer simulation of failure of concrete structures for practice. Cervenka Consulting, Prague, Czech Republic.
15. Cesano, D., Bagtzoglou, A. C. & Olofsson, B. (2003) Quantifying fractured rock hydraulic heterogeneity and groundwater inflow prediction in underground excavations: the heterogeneity index. *Tunnelling and Underground Space Technology,* 18 (1), 19-34.
16. Ciccotti, M. & Mulargia, F. (2004) Differences between static and dynamic elastic moduli of a typical seismogenic rock. *Geophysical Journal International,* 157 (1), 474-477.
17. Cook, R. D., Malkus, D. S. & Plesha, M. E. (1989) *Concepts and applications of finite element analysis.* New York [etc.], Wiley.
18. Cotterell, B. (2002) The past, present, and future of fracture mechanics. *Engineering Fracture Mechanics,* 69 (5), 533-553.
19. Craft, B. C. & Hawkins, M. F. (1959) *Applied petroleum reservoir engineering.* London, Constable.
20. Fang, Z. & Harrison, J. P. (2002) Application of a local degradation model to the analysis of brittle fracture of laboratory scale rock specimens under triaxial conditions. *International Journal of Rock Mechanics and Mining Sciences*, 39 (4), 459-476.
21. Feyen, Luc & Caers, Jef. (2006) Quantifying geological uncertainty for flow and transport modeling in multi-modal heterogeneous formations. *Advances in Water Resources*, 29 (6), 912-929.
22. Gercek, H. (2007) Poisson's ratio values for rocks. *International Journal of Rock Mechanics and Mining Sciences,* 44 (1), 1-13.
23. Griffith, A. A. (1921) The Phenomena of Rupture and Flow in Solids. *Philosophical Transactions of the Royal Society of London.Series A, Containing Papers of a Mathematical Or Physical Character,* 221, pp. 163-198.
24. Gurney, C. & Hunt, J. (1967) Quasi-Static Crack Propagation. *Proceedings of the Royal Society of London.Series A, Mathematical and Physical Sciences,* 299 (1459), 508-524.
25. Huet, C. (1997) An integrated micromechanics and statistical continuum thermodynamics approach for studying the fracture behaviour of microcracked heterogeneous materials with delayed response. *Engineering Fracture Mechanics,* 58 (5-6), 459-463.
26. Huysmans, M., Berckmans, A. & Dassargues, A. (2006) Effect of excavation induced fractures on radionuclide migration through the Boom Clay (Belgium). *Applied Clay Science,* 33 (3-4), 207-218.
27. Ide, J. M. (1936) Comparison of Statically and Dynamically Determined Young's Modulus of Rocks. *Proceedings of the National Academy of Sciences of the United States of America,* 22 (2), pp. 81-92.
28. Inglis, C. E. (1913). Stresses in a plate due to the presence of cracks and sharp corners. *Transactions of the Institute of Naval Architecture*, 55:219.
29. Ingraffea, A. R. & Saouma, V. (1985) Numerical modelling of discrete crack propagation in reinforced and plain concrete. In: *Fracture Mechanics of Concrete.* Martinus Nijhoff Publisher, pp. 171-225.
30. Ingraffea, A. R. & Wawrzynek, P. A. (2003) Finite Element Methods for Linear Elastic Fracture Mechanics. In: I. Milne, R. O. Ritchie & B. Karihaloo (eds.) *Comprehensive Structural Integrity.* Oxford, Pergamon, pp. 1-88.
31. Ingraffea, A. R. and Wawrzynek, P. A. (2004). Computational fracture mechanics: a survey of the field. In *European Congress on Computational Methods in Applied Sciences and Engineering*.
32. Irwin, G. R. (1958). *Fracture. Handbuch der Physik.*, volume 6. Springer.
33. Irwin, G. R. (1948). *Fracturing of metals,* chapter Fracture dynamics, page 147166. American Society of Metals.
34. Leckenby, R. J., Sanderson, D. J. & Lonergan, L. (2005) Estimating flow heterogeneity in natural fracture systems. *Journal of Volcanology and Geothermal Research,* 148 (1-2), 116-129.
35. Lee Y.M., Ehgartner B. L. (2002) *Laboratory Evaluation of Damage Criteria and Creep Parameters of Tioga Dolomite and Rock Salt from Cavern Well No. 1.* Sandia National Laboratories. Report number: SAND2001-0918.

36. Liu, H. Y. (2002) Numerical modelling of the rock fracture process under mechanical loading. Licentiate thesis, Luleå University of Technology, Luleå, Sweden.
37. Liu, H. Y., Roquete, M., Kou, S. Q. & Lindqvist, P. -A. (2004) Characterization of rock heterogeneity and numerical verification. *Engineering Geology,* 72 (1-2), 89-119.
38. Lonergan, L., Jolly, R. J. H., Rawnsley, K. & Sanderson, D. J. (eds.) (2007) *Fractured reservoirs.* , Geological Society.
39. Matthai, S. K., Geiger, S., and Roberts, S. G. (2001). The complex systems platform csp3d3.0: Users guide. Technical report, ETH Z□urich Research Reports.
40. Moes, N., Dolbow, J. & Belytschko, T. (1999) A finite element method for crack growth without remeshing. *International Journal for Numerical Methods in Engineering,* 46 (1), 131-50.
41. Nelson, R. A. (2001) Evaluating Fractured Reservoirs: Introduction. In: *Geologic Analysis of Naturally Fractured Reservoirs (Second edition).* Woburn, Gulf Professional Publishing, pp. 1-100.
42. Nowakowski A. (2005) The static and dynamic elasticity constants of sandstone and shales from the hard coal mine "Jasmos" determined in the laboratory conditions. In: *Eurock 2005 - Impact of Human Activity on the Geological Environment.* ed. by Pavel Konečný.
43. Obreimoff, J. W. (1930) The Splitting Strength of Mica. *Proceedings of the Royal Society of London.Series A, Containing Papers of a Mathematical and Physical Character,* 127 (805), pp. 290-297.
44. Olson, J. & Pollard, D. D. (1989) Inferring paleostresses from natural fracture patterns: A new method. *Geology,* 17 (4), 345-348.
45. Palchik, V. & Hatzor, Y. H. (2002) Crack damage stress as a composite function of porosity and elastic matrix stiffness in dolomites and limestones. *Engineering Geology,* 63 (3-4), 233-245.
46. Paluszny, A. (2009), Numerical modeling of fracture propagation and its implications for fluid flow, Ph.D. tesis, Imperial Coll., London.
47. Paluszny, A. & Matthäi, S. K. (2009) Numerical modeling of discrete multi-crack growth applied to pattern formation in geological brittle media. *International Journal of Solids and Structures,* 46 (18-19), 3383-3397.
48. Renshaw, C. E. & Pollard, D. D. (1994) Numerical simulation of fracture set formation: a fracture mechanics model consistent with experimental observations. *Journal of Geophysical Research,* (99), 9359-9372.
49. Stagnitti, F., Li, L., Allinson, G., Phillips, I., Lockington, D., Zeiliguer, A., Allinson, M., Lloyd-Smith, J. & Xie, M. (1999) A mathematical model for estimating the extent of solute- and water-flux heterogeneity in multiple sample percolation experiments. *Journal of Hydrology,* 215 (1-4), 59-69.
50. Stüben, K. (2001) A review of algebraic multigrid. *Journal of Computational and Applied Mathematics,* 128 (1-2), 281-309.
51. Taig, I. C. (1961) Structural analysis by the matrix displacement method, English Electric Aviation Report.
52. Tang, C. A. & Kaiser, P. K. (1998) Numerical Simulation of Cumulative Damage and Seismic Energy Release During Brittle Rock Failure - Part I: Fundamentals. *International Journal of Rock Mechanics and Mining Sciences,* 35 (2), 113-121.
53. Tang, C. A., Liu, H., Lee, P. K. K., Tsui, Y. & Tham, L. G. (2000) Numerical studies of the influence of microstructure on rock failure in uniaxial compression - Part I: effect of heterogeneity. *International Journal of Rock Mechanics and Mining Sciences,* 37 (4), 555-569.
54. Tang, C. A., Tham, L. G., Wang, S. H., Liu, H. & Li, W. H. (2007) A numerical study of the influence of heterogeneity on the strength characterization of rock under uniaxial tension. *Mechanics of Materials,* 39 (4), 326-339.
55. Tang, C. (1997) Numerical simulation of progressive rock failure and associated seismicity. *International Journal of Rock Mechanics and Mining Sciences,* 34 (2), 249-261.
56. US Bureau of Reclamation (1953). Physical properties of some typical foundation rocks. Concrete Laboratory Report No. SP-39, 50 pp.
57. Yazid, A., Abdelkader, N. & Abdelmadjid, H. (2009) A state-of-the-art review of the X-FEM for computational fracture mechanics. *Applied Mathematical Modelling,* 33 (12), 4269-4282.
58. Zhang, Y-Z., Dusseault, M. B. & Yassir, N. A. (1994) Effects of rock anisotropy and heterogeneity on stress distributions at selected sites in North America. *Engineering Geology, 37* (3-4), 181-197.
59. Zisman,W.A. (1933) A comparison of the statically and seismologically determined elastic constants of rock. Proc. *Natl. Acad. Sci. U.S.A.* 19, 680– 686.

Appendix

APPENDIX A: LITERATURE REVIEWS

International Journal of Solids and Structures 46 (2009) p. 3383-3397.
Numerical modelling of discrete multi-crack growth applied to pattern formation in geological brittle media

Authors: Adriana Paluszny, Stephan K. Matthai.

Contribution to the numerical modelling of fracture propagation: Describes finite-element method of numerical modelling multiple fractures simultaneously. Systematizes previous works on numerical simulators and develops sustainable code for multi-set fracture propagation subject to flexible input conditions.

Objective of the paper: Develop numerical algorithm that could propagate multiple fractures simultaneously.

Methodology used: Used finite-element method for fracture propagation with adaptive remeshing which allowed to refine fracture around the tip and coarsen it everywhere else. Fracture growth process is based on three criteria:

- Failure – Sub-critical crack growth: ($K_{IO} \leq K_I \leq K_{IC}$).
- Propagation – Restricts fracture growth by weighing the energy at the fracture tip with empirical velocity index α: ($l_{adv} = l_{max} \left(\frac{G}{G_{max}}\right)^{\alpha=0.35}$).
- Propagation angle: Determined by maximum circumferential stress.

Conclusion reached:

1. Close match between numerical and physical experiments obtained: realistic heterogeneous medium analogues could be generated.
2. Remeshing consumes <2% of computational time.
3. Model has capability to introduce heterogeneous regions in the grid and propagate fractures.

Comments: This paper summarizes pervious critical researches in fracture modelling field and demonstrates ready-to-use fracture propagation code that could be executed on current machines and give adequate match with physical rock analogues. 2D model could be used as starting point for 3D studies; the main challenge for 3D simulators is the lack of adaptive meshing module.

Journal of Geophsyical Research, Vol. 99, No. B5, p. 9359-9372, May 10, 1994.
Numerical simulation of fracture set formation: A fracture mechanics model consistent with experimental observations

Authors: Carl E. Renshaw, David D. Pollard.

Contribution to the understanding of numerical fracture simulators: Introduced stochastic network simulator for modelling single set 2D fractures while considering fracture mechanics principles. Fracture set depends on flaw geometry and velocity exponent. This advance allowed to generate fractures based on geological conditions of the region and reduce number of possible realizations by reducing physically unviable ones.

Objective of the paper: Introduce stochastic single-set fracture simulator with attempt to include fracture mechanics concepts.

Methodology used: Spatially random flaw distributions were generated by stochastic network simulator on a square grid of homogeneous, isotropic and linear elastic solid subject to uniform stress conditions. Fracture tips proportionally advance if stress intensity factor (SIF) exceeds some critical value.

Conclusion reached:

1. Numerical and experimental fracture sets were compared, visibly good match obtained.

2. Sensitivity analysis performed on flaw density (d_0) and velocity exponent (α) with varying duration of fracturing.

Comments: This paper describes advance in application of physical concepts in relatively simple numerical simulator. Velocity exponent α is unkown for the time of experiment which significantly affects range of generated fracture sets. Also, fracture model is single-set which could be viable only in very idealised conditions. Paper introduces key physical assumptions into the stochastic simulator, but the model could not be applied for heterogeneous rock mass.

Computational Materials Science 46 (2009), p. 667-671.

Influence of heterogeneity on fracture behaviour in multi-layered materials subjected to thermo-mechanical loading

Authors: L.C. Li, C.A. Tang, Y.F. Fu.

Contribution to the understanding of rock heterogeneity on fracture propagation behavior: Realisation of numerical model for layered material subject to thermo-mechanical-damage conditions. Identified relation between fracture pattern and material heterogeneity.

Objective of the paper: 1.Investigate influence of stress states between two adjacent fractures for three-layer model.
2. Investigate fracturing of mid-layer with no pre-assigned flaws subject to isothermal loading.
3. Observe relation between material heterogeneity and fracture propagation behaviour.

Methodology used: Numerical RFPA (Realistic Failure Process Analysis) code based on finite-element method with incorporated thermo-mechanical-damage model. Heterogeneity is defined by *m* parameter introduced into the distribution function.

Conclusion reached:

1. Stress analysis was investigated based on critical spacing to fractured layer thickness ratio, S/T_C, results show that this ratio is lower than 1 for thermal effects.

2. Rock heterogeneity directly control stress redistribution after initial fractures formed. Heterogeneity was introduced by homogeneity index *m*, and the fracture patterns are more irregular and tend to interfacial delamination.

Comments: Large differences in coefficients of thermal expansion of layers will result in severe stresses when temperature changes taken into account. This thermal stresses should be added to external mechanical loads. Although the FEM simulation was performed as three-layered 2D, it could serve as reasonable realisation of simplified 3D model.

Computer simulation of failure of concrete structures for practice. Cervenka Consulting, 2002.
Author: Vladimir Cervenka

Contribution to the numerical fracture simulators: Describes non-linear finite element analysis of failure that can contribute to better economy in design of structures.

Objective of the paper: Describe the code (ATENA software) that is capable of generating fracture models for 2D and 3D realisations and validate the model results with the mechanical experiment.

Methodology used: The finite element technique in which the basic matrix equilibrium equation works with the nodal force vector P, nodal displacements vector U and the stiffness matrix K, $K\Delta U=P-R$. Material properties are described by the constitutive relations between stresses and strains $\sigma = F(\sigma,\varepsilon)$. These material relations for concrete are highly non-linear. Crack model is divided into fixed and rotating.

Conclusion reached:

1. Non-linear finite element method can be well used for the simulation of real behaviour of reinforced concrete structures.

2. Model results were validated with experimental fracture patterns.

3. Model seems to useful for assessment of the remaining structural capacity and investigating the causes of damage and failures.

Comments: Systematized advance in non-linear analysis of concrete failure. Model could be run in 2D and 3D and has capability of introducing individual truss bar elements in concrete mesh.

Mechanics of materials 39 (2007), p. 326-339.
A numerical study of the influence of heterogeneity on the strength characterization of rock under uniaxial tension

Authors: C.A. Tang, L.G. Tham, S.H. Wang, H. Liu, W.H. Li.

Contribution to the numerical fracture simulators and investigation of heterogeneity influence: Numerical Rock Failure Analysis Code (RFPA) was previously described by Tang (1997) and Tang et al. (2000), thus insignificant contribution into the code development. However, paper shows the influence of rock heterogeneity (various *m* values) on various out parameters.

Objective of the paper: Provide numerical results of RFPA code with different heterogeneity index values and investigate the relation with load-deformation characteristics, stress redistribution, acoustic emission and failure modes.

Methodology used: Numerical simulator by Tang (1997) is capable of generating finite element mesh subject to input conditions. Material properties (failure-strength rc and elastic module Ec) are assigned randomly to each mesh element in accordance with the Weibull distribution:

$$\varphi = \frac{m}{\sigma_0}\left(\frac{\sigma}{\sigma_0}\right)^{m-1}\exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^{m}\right],$$

Where *m* is so-called homogeneity index, larger value of *m* implying more homogeneous material.

Conclusion reached:

1. Fracture patterns were generated for all four parameters indicated in the paper objectives versus various *m* values.
2. Homogeneous rock specimens have uniformly distributed fractures throughout the rock mass, while in the heterogeneous rock once the crack nucleates, it can be taken as a precursor, and the further growth of the crack can be traced. The nucleation stage involves the localization of the relatively slower propagation and coalescence of the macro-cracks.
3. Failure modes are sensitive to heterogeneity localization.
4. Evidence of uniaxially stronger homogeneous specimens than heterogeneous ones.

Comments: Simulation is 2D, and the simulation results are more qualitative than quantitative.

Engineering geology 37 (1994) p. 181-197.
Effects of rock anisotropy and heterogeneity on stress distributions at selected sites in North America

Authors: Ying-Zhen Zhang, Maurice B. Dusseault, Najwa A. Yassir.

Contribution to the understanding of fracture pattern behaviour: Describes the influence of soft and stiff inclusions (heterogeneity) on stress patterns.

Objective of the paper: Introduce regions of heterogeneity and/or fault in the numerical simulator and investigate the relation with the change in stress direction.

Methodology used: 2D numerical model based on finite-element method.

Conclusion reached:

1. Simulate result of the fault model as a soft inclusion have almost similar stress rotation as observed on the field (Murre Fault in the Jeanne D'Arc basin)

2. Presence of strong tectonism or sedimentological heterogeneities significantly influences local stress redistribution, thus affecting fracture propagation direction.

Comments: There is still much uncertainty when identifying in-situ stresses in the rock mass. Careful seismic profiling or other geophysical approaches should be combined with confirmatory drilling and core analysis.

Engineering Geology 72 (2004), p. 89–119.
Characterization of rock heterogeneity and numerical verification

Authors: H.Y. Liu, M. Roquete, S.Q. Kou, P.-A. Lindqvist

Contribution to the understanding of fracture process of heterogeneous rock: Another paper with obtained evidences of heterogeneity impact on fracture propagation patterns; numerical results were validated with experimental rock images.

Objective of the paper: Improve understanding of rock heterogeneity; model and compare numerical outputs with lab data.

Methodology used: Representative volume rock piece was simulated by R-T (on the basis of RFPA code by Tang) that utilizes finite element method. Heterogeneity is imposed by m parameter that describes σ scatter.

Conclusion reached:

1. Weak minerals and grain boundaries play important role in the non-linear deformation of brittle media.

2. Different homogeneity indices represent different degrees of heterogeneity and could serve as rock specimen model.

Comments: Paper doesn't provide significant advance with respect to Tang (2000) but reports about applicable simulator and additional evidence of heterogeneity impact on fracture propagation pattern.

Journal of Structural Geology (2009), p. 1–13.
Effects of internal structure and local stresses on fracture propagation, deflection, and arrest in fault zones

Authors: Agust Gudmundsson, Trine H. Simmenes, Belinda Larsen, Sonja L. Philipp.

Contribution to the understanding the effects of heterogeneity on fracture propagation: Increased understanding of how fractures propagate and become arrested within fault zones, and how the fault zone thickness is confined at any particular time during its evolution.

Objective of the paper: Simulate local stress field in the rock mass and model fault zone as inclusion with different stress magnitudes and directions.

Methodology used: Output results of finite element and boundary element based methods were used to generated fracture planes and validate them with geologically extracted assumptions.

Conclusions reached:

1. A fault zone may be regarded as an elastic inclusion with mechanical properties that differ from those of the host rock.
2. The local stresses of the fault zone and its heterogeneities and interfaces and discontinuities (fractures, contacts) determine propagation, deflection, and arrest of the fractures in the fault zone.
3. Changes in local stresses within the fault zone may generate barriers to fracture propagation and contribute to fracture deflection and arrest at interfaces and discontinuities.
4. Analytical solutions on the material toughnesses of interfaces such as discontinuities and contacts between mechanically dissimilar layers within a fault zone show that fractures commonly become deflected into, and often arrested at, interfaces.

Comments: Heterogeneity is implemented as a region with different stress magnitude and direction. Comparison numerical simulator fracture pattern and geological image of the region was done, however the simulator methodology is not clearly detailed.

APPENDIX B: CRITICAL MILESTONES TABLE

Paper No.	Year	Title	Authors	Contribution
	1913	"Stresses in a plate due to the presence of cracks and sharp corners."	Inglis, C. E.	Investigated stress distribution for cracked and scratched plate. Established mathematical equations for elliptical hole in the solid.
	1921	The Phenomena of Rupture and Flow in Solids.	Griffith, A. A.	Identified crack tips as stress concentrators and attempt to relate surface free energy with the fracture failure.
	1930	"The splitting strength of mica."	I.W. Obreimoff	Defined quasi-static crack growth as a conceptual model that prescribes the slow and steady growth of cracks under equilibrium.
	1937	"Kerbspanmungslehre"	H. Neuber	First to relate stress concentration with the crack length and tip curvature
	1985	"Numerical modelling of discrete crack propagation in reinforced and plain concrete"	A.R. Ingraffea, V. Saouma	First geometric-based propagation method devised to simultaneously generate multiple fractures.
	1989	"Inferring paleostresses from natural fracture patterns: a new method"	J.E. Olson, D.D. Pollard.	First applied boundary element method for computation of energy release rates and estimate growth of a set of planar cracks.
	1977	"A numerical approach to the testing of the fission thesis.	L.B. Lucy	First introduced mesh-free method that does not require a connectivity of the nodes in the grid.

APPENDIX C: GENERIC FRACTURE PARAMETERS

Spacing: $s = \frac{A}{l_0 + \sum_{i=1}^{n} l_i} = \frac{A}{l_0 + L}$

Spatial density: $d = \frac{1}{A} \sum_{i=1}^{n} (\frac{l_i}{2})^2$ (by Budiansky and O'Connell)

where, *s*-spacing, *l*-fracture length, *d*-density, *A* is flow area, $l_i$is *i* fracture length.