COMPARISON OF PUNCHING SHEAR REQUIREMENTS IN BS8110, EC2 AND MC2010

Soares LFS and Vollum RL

Department of Civil and Environmental Engineering

Imperial College London, London SW7 2AZ, United Kingdom

Corresponding author: Dr Robert L Vollum

Department of Civil and Environmental Engineering

Imperial College London, London SW7 2AZ, United Kingdom

Email: r.vollum@imperial.ac.uk

Phone +44 (0)20 75945992

Fax: +44(0)20 75945934

Abstract

This paper compares design provisions for punching shear at internal columns in the superseded British Standard BS8110, Eurocode 2 (EC2) and fib Model Code 2010 (MC2010). The latter is based on the Critical Shear Crack Theory (CSCT) of Muttoni which relates shear resistance to the width of the so called "critical shear crack" which depends on slab rotation. Parametric studies are presented which show BS8110 to require significantly less shear reinforcement within 1.5*d* (where *d* is the slab effective depth) of the loaded area than EC2 and MC2010 both of which have been extensively calibrated against test data. This raises the question of whether flat slabs designed with BS8110 have an adequate factor of safety against punching failure. This question is explored using nonlinear finite element modelling in conjunction with MC2010 Level IV. It is shown that punching resistance at internal columns can be increased significantly by restraint from the surrounding slab but the strength increase is variable and in the case of uniformly loaded slabs already largely included in BS8110 and EC2.

Notation

A_{sl} area of tensile reinforcement

A_{sw} area of shear reinforcement in each perimeter

c side length of square column

d average effective depth of slab

d_b shear reinforcement diameter

d_g maximum diameter of aggregate

d_v reduced effective depth of slab

E_s modulus of elasticity of reinforcement

F total load applied to each panel of flat slab

f_c compressive cylinder strength of concrete

f_{ck} characteristic compressive cylinder strength of concrete at 28 days

f_{ct} tensile strength of concrete

f_{cu} characteristic cube strength of concrete

f_y yield strength of reinforcement

f_{yw} yield strength of shear reinforcement

h slab thickness

k_{dg} effectiveness coefficient dependent on maximum aggregate size

k_e effectiveness coefficient for eccentric shear

k_{sys} efficiency factor for punching shear reinforcement system

k_ψ coefficient relating shear resistance to slab rotation

L span between column centrelines

m_R nominal moment capacity per unit width

m_s average bending moment per unit width

M_{sup} design support moment across panel width

r_c radius of equivalent circular column ($2c/\pi$ for square columns)

r_q radius of load introduction

r_s radius of an isolated slab element

s_o radial spacing to first perimeter of shear reinforcement from column face

s_r radial spacing of shear reinforcement

u length of basic control perimeter

u_{out} length of first control perimeter at which shear reinforcement is not required

V shear force

$V_{R,c}$ shear resistance of slab without shear reinforcement

$V_{R,cs}$ combined shear resistance of concrete and shear reinforcement

$V_{R,max}$ maximum possible shear resistance

$V_{R,out}$ shear resistance outside shear reinforced zone

$V_{R,s}$ shear resistance provided by shear reinforcement

V_{test} measured failure load

β coefficient to account for uneven shear

γ_c partial factor for concrete

γ_m BS8110 partial safety factor for shear

γ_s partial factor for reinforcement

ε_y reinforcement yield strain

ρ flexural tension reinforcement ratio

ρ_{xl}, ρ_{yl} flexural tension reinforcement ratio in x and y directions

σ_{sw} shear reinforcement stress

ψ slab rotation outside critical shear crack

Introduction

The paper considers design for punching shear at the internal columns of solid flat slabs. The relative economy of flat slabs depends on their thickness which is governed by either deflection limits or punching resistance. Thinner slabs not only save on direct material costs of the frame and supporting foundations but also reduce cladding costs due to reduced floor-to-floor heights. There is no generally accepted theoretical treatment of punching and design methods are calibrated with data from test specimens like that shown in Figure 1 in which the line of radial contraflexure is fixed and compressive membrane effects are minimal. Neither of these assumptions is realistic for flat slabs in which punching resistance can be significantly increased by restraint from the surrounding slab (Ockelston, 1955). The benefit of compressive membrane action on punching resistance has been demonstrated experimentally by researchers including Rankin and Long (1987), Chana and Desai (1992a) and Salim and Sebastian (2003). Regan (1986) carried out tests on cross shaped solid slab specimens which showed that punching resistance is increased by rotational restraint at slab edges. More recently, Choi and Kim (2012) tested three internal slab-column connections with a complex setup intended to provide zero rotation at the slab edges. The punching resistance was found to be almost independent of the percentage of moment redistribution used in the calculation of design moments contrary to the recommendations of BS8110 (BSI, 1997) and EC2 (BSI, 2004a) which relate shear resistance to the flexural reinforcement ratio over columns.

EC2 is the current UK standard for concrete structures having replaced BS8110:1997 (BSI, 1997) in 2010. Interestingly, BS8110: 1985 (BSI, 1985) could halve the area of shear reinforcement required by the previous UK code CP110 (BSI, 1972). Unsurprisingly, this caused concern which resulted in a test programme (Chana and Desai, 1992b), funded jointly by the Department of Environment and British Cement Association, that led to BS8110 being revised in 1992. EC2 and MC2010 (fib, 2012) typically require significantly more punching shear reinforcement within 1.5*d* of columns than the 1992 amendment to BS8110. Therefore, it is striking that there is no evidence of punching failure in flat slabs designed with BS8110. The most recent international design recommendations for punching are found in MC2010, which may influence future revisions of EC2. The MC2010 recommendations are based on the critical shear crack theory (CSCT) of Muttoni (2008), which relates punching resistance to the slab rotation, outside the so called critical shear crack. EC2 and BS8110 neglect any increase in punching resistance due to restraint from the surrounding slab but its effect can be modelled with the CSCT (Einpaul et al. 2015) using MC2010 Level IV (Muttoni and Fernandez-Ruiz, 2012).

Comparison of design methods for punching in BS8110, EC2 and MC2010

All three methods take the punching resistance of slabs with shear reinforcement as the least of $V_{R,cs}$ and $V_{R,out}$ but not greater than $V_{R,max}$ where:

i. $V_{R,cs} = \alpha V_{R,c} + V_{R,s} \geq V_{R,c}$ is the combined shear resistance of concrete $\alpha V_{R,c}$ where $\alpha \leq 1$ and shear reinforcement $V_{R,s}$.

ii. $V_{R,c} = v_c u d$ is the strength of an otherwise similar slab without shear reinforcement in which v_c is the design concrete shear stress, u is the basic control perimeter and d is the average slab effective depth.

iii. $V_{R,out} = v_c u_{out} d \geq V_{R,c}$ is the shear resistance provided by concrete along a perimeter of length u_{out} just outside the shear reinforcement.

iv. $V_{R,max}$ is the maximum possible punching resistance for given column size, slab effective depth and concrete strength.

BS8110 adopts rectangular perimeters for u and u_{out} that are located respectively at 1.5d from the column face and 0.75d from the outer perimeter of shear reinforcement. EC2 locates u at 2d from the column face and u_{out} at 1.5d from the outer perimeter of shear reinforcement. MC2010 adopts a similar approach but locates u at 0.5d from the column face, unless the loaded area is recessed into the slab, and u_{out} at 0.5d_v from the outer perimeter of shear reinforcement where d_v is the effective depth for shear which is taken as d-25 mm in the parametric studies of this paper (where 25 mm is the cover to the shear studs). BS8110 and EC2 multiply the design shear force by β to account for the effects of uneven shear due the support reaction being eccentric whereas MC2010 reduces the design shear resistance by the multiple k_e. At internal columns, in cases where lateral stability does not depend on frame action, β can be taken as 1.15 and k_e as 0.9. The maximum shear stress in the slab around the column is limited to $0.8\sqrt{f_{cu}}$ in BS8110 and $0.3(1-f_{ck}/250)f_{ck}/\gamma_c$ in the UK National Annex to

EC2 (BSI, 2004b) in which γ_c is the partial factor for concrete which EC2 and MC2010 take as 1.5. All three codes take the design yield strength of reinforcement as f_{yk}/γ_s where γ_s = 1.15.

BS8110

The punching resistance without shear reinforcement is given by:

$$V_{Rd,c}\ (BS8110) = v_{Rdc}ud = 0.27(100\rho f_{cu})^{\frac{1}{3}}(\frac{400}{d})^{\frac{1}{4}}ud/\gamma_m \tag{1}$$

where $\rho = \frac{A_{sl}}{bd} \leq 0.03$ is the flexural reinforcement ratio, f_{cu} is the characteristic compressive concrete cube strength and d is the effective depth. γ_m has a design value of 1.25. BS8110 limits f_{cu} to 40 Mpa in its design equations for shear but this limit is not applied in this paper as it is unduly restrictive.

BS8110 requires shear reinforcement to be provided in rectangular perimeters centred on the column. The required area of shear reinforcement is calculated in terms of the design shear stress v as follows:

$$\Sigma_{1.5d}A_{sw} \geq \frac{(v - v_{Rdc})ud}{f_{ywd}} \geq \frac{0.4ud}{f_{ywd}}\ for\ v \leq 1.6v_{Rdc} \tag{2}$$

$$\Sigma_{1.5d}A_{sw} \geq \frac{5(0.7v - v_{Rdc})ud}{f_{ywd}}\ for\ 1.6v_{Rdc} \leq v \leq 2v_{Rdc} \tag{3}$$

where $v = \frac{\beta V_{Ed}}{ud}$ and $v_{Rdc} = \frac{V_{Rdc}}{ud}$. The required shear reinforcement $\Sigma_{1.5d}A_{sw}$ is provided over at least two perimeters of which the first perimeter should not contain less than $0.4\Sigma_{1.5d}A_{sw}$. The first perimeter of shear reinforcement is provided at 0.5d from the column face with successive perimeters positioned at spacings of 0.75d. The spacing of shear reinforcement around a perimeter must not exceed 1.5d. Equations (2) and (3) are subsequently used to design the shear reinforcement on successive square perimeters spaced at multiples of 0.75d from the basic control perimeter.

EC2

$$V_{Rd,c} = 0.18(100\rho f_{ck})^{\frac{1}{3}}(1 + (200/d)^{0.5})ud/\gamma_c \tag{4}$$

where $\rho = \left(\rho_{xl}\rho_{yl}\right)^{0.5} \leq 0.02$ in which ρ_{xl} and ρ_{yl} are the flexural tension reinforcement ratios $\frac{A_{sl}}{bd}$ within a slab width equal to the column plus 3*d* to each side.

The required area of shear reinforcement is calculated as follows:

$$1.5A_{sw}\frac{d}{s_r} \geq \frac{\beta V_{Ed} - 0.75V_{Rdc}}{f_{ywd,ef}} \tag{5}$$

where A_{sw} is the area of shear reinforcement in each perimeter, s_r is the radial spacing of the shear reinforcement and $f_{ywd,ef} = (250 + 0.25d) \leq f_{ywd}$.

In addition to limiting the maximum shear stress around the column, EC2 limits the maximum possible design shear force to αV_{Rdc} where α is a Nationally Determined Parameter with a recommended value of 1.5 that is increased to 2 in the UK National Annex (BSI, 2004b).

Figure 6.22 of EC2 shows shear reinforcement being provided in radial or cross type configurations of which radial is most efficient. The radial spacing of the first perimeter of shear reinforcement from the column must lie between 0.3 and 0.5*d*. The maximum radial spacing of successive perimeters of shear reinforcement is 0.75*d*. The circumferential spacing of vertical legs of shear reinforcement should not exceed 1.5*d* within the first control perimeter and 2*d* outside where "that part of the perimeter is assumed to contribute to the shear capacity".

MC2010

MC2010 has four levels of design of which Levels I to III are intended for design and Level IV for assessment. Level III is recommended for slabs with irregular geometry. The shear resistance is calculated in terms of the slab rotation ψ which Level II calculates as follows:

$$\psi = 1.5\frac{r_s}{d}\frac{f_{yd}}{E_s}(\frac{m_{Ed}}{m_{Rd}})^{1.5} \quad (6)$$

where r_s denotes the position where the radial bending moment is zero with respect to the column axis, m_{Ed} is the average bending moment per unit width in the support strip, which is assumed to be of width $1.5r_s$ where $r_s = 0.22L$, and m_{Rd} is the design average flexural strength per unit width of the support strip. For concentrically loaded inner columns, $m_{Ed} = V_d/8$ for Level II.

The punching resistance is calculated as $V_{Rd} = V_{Rd,c} + V_{Rd,s}$ where:

$$V_{Rd,c} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} u d_v \quad (7)$$

in which f_{ck} is in MPa and d_v is the shear resisting effective depth which is taken as d at u in this paper.

The parameter k_ψ depends on the rotations of the slab around the support region and is calculated as:

$$k_\psi = \frac{1}{1.5 + 0.9k_{dg}\psi d} \leq 0.6 \quad (8)$$

$$k_{dg} = \frac{32}{16 + d_g} \geq 0.75 \quad (9)$$

where d_g is the maximum aggregate size.

The shear resistance provided by transverse reinforcement is calculated as:

$$V_{Rds} = \Sigma_{d_v} A_{sw} k_e \sigma_{sw} \quad (10)$$

where $\Sigma_{d_v} A_{sw}$ is the cross-sectional area of all shear reinforcement within the zone bounded by $0.35d_v$ and d_v from the border of the support region. σ_{sw} is the stress that can be mobilized in the shear reinforcement and is taken as:

$$\sigma_{sw} = \frac{E_s\psi}{6}(1 + \frac{f_b}{\gamma_c f_{ywd}}\frac{d}{d_b}) \leq f_{ywd} \quad (11)$$

where d_b is the shear reinforcement diameter and f_b is the bond strength which is taken as 4.5 MPa in this paper as allowed by MC2010.

The maximum punching resistance is limited by crushing of concrete struts near the support region such that:

$$V_{Rd,max} = k_{sys} k_{\psi} \frac{\sqrt{f_{ck}}}{\gamma_c} u d_v \leq \frac{\sqrt{f_{ck}}}{\gamma_c} u d_v \tag{12}$$

The coefficient k_{sys} accounts for the performance of punching shear reinforcing systems and is taken as 2.4 for stirrups and 2.8 for studs provided the radial spacing to the first perimeter of shear reinforcement from the column face s_o is less than or equal to $0.5d_v$ and the spacing of successive perimeters of shear reinforcement is less than $0.6d_v$. The spacing of vertical legs of shear reinforcement around a perimeter should not exceed $3d_v$ where that part of the perimeter is assumed to contribute to the shear capacity.

Evaluation of BS8110, EC2 and MC2010 design methods for punching with test data

It is well established that EC2 and the CSCT (Muttoni, 2008), on which MC2010 is based, give similar and reasonable predictions of the punching resistance of test specimens like Figure 1. For example, Ferreira et al. (2014) compared the predictions of EC2 and the CSCT, with partial factors of 1.0, for 45 tests with shear reinforcement. They found the mean (μ) and covariance (COV) of V_{test}/V_{calc} to be μ =1.19:COV=0.136 for EC2 and μ =1.16:COV=0.121 for the CSCT. Closer inspection shows the similarity of these statistics to be misleading since in 14 cases EC2 falsely predicts failure to occur outside the shear reinforcement compared with only 5 cases for the CSCT. This is concerning because the introduction of partial factors γ_c = 1.5 and γ_s = 1.15 into EC2 makes failure outside the shear reinforcement unlikely because it causes a 50% increase in u_{out} which is not matched by a corresponding increase in punching resistance within the shear reinforcement V_{Rcs} as discussed by Vollum et. al. (2010).

The BS8110 design provisions are not strictly applicable to the test specimens considered by Ferreira et al. (2014) as the detailing of shear reinforcement did not comply with the onerous requirement of BS8110 that the transverse spacing of vertical legs should not exceed 1.5*d*. Nevertheless, an analysis was carried out to determine the accuracies of BS8110, EC2 and MC2010 at predicting the punching resistance inside the shear reinforcement V_{in}. Each method was used to calculate V_{test}/V_{in} for 40 specimens that failed within the shear reinforcement. The analysis considered 25 specimens of Ferreira et al. (2014), specimens PL6 to PL12 inclusive of Lips et al. (2012) and specimens 2 to 9 of Chana and Desai (1992b) which formed the basis of the 1992 amendment to BS8110. All the specimens were reinforced with studs except those of Chana and Desai which were reinforced with stirrups. In the case of MC2010, slab rotations were calculated as follows (Muttoni, 2008):

$$\psi = 1.5\frac{r_s}{d}\frac{f_y}{E_s}\left(\frac{V}{V_{flex}}\right)^{3/2} \tag{13}$$

V_{flex} was calculated in accordance with the recommendations of Muttoni (2008) as:

$$V_{flex} = 2\pi m_R \frac{r_s}{r_q - r_c} \tag{14}$$

in which r_s is the radius of the slab, r_q is the radius of load introduction and r_c is the radius of a circular column with the same perimeter as the column under consideration. The coefficient k_{sys} was taken as 2.8 in the calculation of V_{Rdmax} with equation (12).

Results are shown without and with partial factors in Figures 2a and b which also show mean and lower characteristic (5%) values of V_{test}/V_{in}. MC2010 gives conservative estimates of V_{in} with $\gamma_c = \gamma_s = 1.0$ unlike BS8110 and EC2 of which BS8110 is least safe. The BS8110 5% values of V_{test}/V_{in} increase from 0.60 to 0.72 for $\gamma_c = \gamma_s = 1.0$ and from 0.74 to 0.89 for $\gamma_c = 1.5$ and $\gamma_s = 1.15$ when the limit on $V_{Rd,max}$ is omitted due to reduction in scatter.

Modelling of restraint from surrounding slab with MC2010 Level IV

MC2010 Level IV was used to investigate whether restraint from the surrounding slab is sufficient to explain the satisfactory performance of flat slabs designed to BS8110. Consideration of equations (6) to (12) shows that the calculated shear resistance is independent of the axial force in the slab which implies increases in strength from compressive membrane action result from reductions in rotation. Justification for this assumption is provided by analysis of tests on prestressed slabs (Clément et al., 2014). The first step involved the development of a nonlinear finite element (NLFEA) procedure for calculating slab rotations. The procedure was calibrated with data from the internal slab-column punching tests of Guandalini et al. (2009) and Lips et al. (2012). All the slabs measured 3 m square on plan. The flexural reinforcement ratios ranged between 0.33% and 1.63% as shown in Table 1 which gives details of the test specimens including geometry, material properties, failure loads and ultimate rotations. The rotations were measured with inclinometers positioned at 1.38m from the column centreline at the positions depicted with small triangles in Figure 1.

The slabs were modelled with four-node quadrilateral isoparametric curved shell elements incorporating embedded reinforcement bars. A 2 × 2 × 9 integration scheme was adopted for the curved shell elements, where 9 denotes the number of integration points through the slab thickness, as recommended by Vollum and Tay (2007). Following a mesh sensitivity study, the element size was chosen to be around 50 mm square, with the exact dimensions dependent on the column size over which the nodes of the slab were vertically restrained.

The concrete was modelled with the 'total strain fixed crack model' in DIANA which evaluates stress-strain relationships in the directions of the principal axes at first cracking. A linear tension softening stress-strain relationship was used for concrete. Following the recommendations of Vollum and Tay (2007), the tensile stress was assumed to reduce from a peak value of $0.5f_{ct}$, where f_{ct} is the mean indirect tensile strength calculated in accordance with EC2 (see Table 1), to zero at half the reinforcement yield strain ε_y. The Thorenfeldt model (Thorenfeldt et al., 1987) was used to model concrete in compression. The reduction in concrete compressive strength due to lateral cracking was modelled as recommended by Vecchio and Collins (1993). A sensitivity study showed the calculated rotations to be almost independent of the shear retention factor β which was taken as 0.9. The concrete elastic modulus was calculated in accordance with EC2.

The NLFEA includes the effect of compressive membrane action unlike equation (6) (Muttoni, 2008). The measured and calculated rotations agreed well up to around 50% of the failure load when calculated with the full short term concrete elastic modulus but the measured slopes were significantly underestimated at failure. Figure 3 shows that much better estimates were obtained of the ultimate rotations when the concrete elastic modulus was reduced to half its short term value.

Assessment of Chana and Desai (1992a) punching tests with membrane action

The tests of Chana and Desai (1992a) are particularly pertinent to this investigation. They tested five 9 m square by 250 mm thick slabs which were supported at their centre on a 400 mm square plate and by block walls along all four edges. All the slabs had the same flexural reinforcement and the cube strengths were similar at around 40 MPa. Four slabs had shear reinforcement. The slabs were loaded at eight equally spaced points which were centred on the loading plate at a radius of 1.2 m. The tests showed that restraint from the surrounding slab increased the punching resistance by *30-50%* compared with Chana and Desai's (1992b) punching specimens of the type shown in Figure 1.

The punching resistances of Chana and Desai's (1992a) slabs FPS1 (without shear reinforcement) and FPS5 (with shear reinforcement) were evaluated with MC2010 Level IV.

Comparisons were also made with the shear resistance of Chana and Desai's (1992b) 3 m square punching specimens. Rotations were calculated with NLFEA, using the procedure described previously. The 3 m square panels were modelled with 50 mm square elements and the 9 m square panels with 100 mm square elements. Figure 4a shows that the measured and calculated deflections agree well which is significant as MC2010 attributes the increase in punching resistance from restraint to the reduction in rotation and hence deflection. The resulting load rotation responses are shown in Figure 4b along with the MC2010 punching resistances for slabs FPS1 and FPS5. Rotations are shown along the slab centreline at the loading radius, as measured in the tests of Lips et al. (2012), and additionally in the 9 m square slab at 0.7 m from the column centreline where rotations were greatest. For comparison, Figure 4b also shows rotations calculated with equation (13) which is applicable to the 3 m square panels. The calculated failure load is given by the intersection of the rotation and resistance curves. When the effect of continuity is included, the ratio V_{test}/V_{calc} for maximum rotations is 1.12 for FPS1 and 1.41 for FPS5. A measure of the influence of continuity is the ratio of the shear resistances given by the Level IV analysis with continuity and equation (13). This ratio is 1.52 for FPS1, which is close to the measured ratio of 1.4, and 1.07 for FPS5 which is significantly less than the measured ratio of 1.4. The underestimate in strength of FPS5 is a consequence of MC2010 neglecting shear deformation in the calculation of shear reinforcement stress.

Parametric studies to compare shear reinforcement requirements of BS8110, EC2 and MC2010

A parametric study was undertaken to investigate how the required areas of shear reinforcement vary at the internal columns of flat slabs according to BS8110, EC2 and MC2010 Level II. The span L between the column centrelines was taken as 7.5m and the internal columns as 450 mm square. The superimposed dead load was taken as 1.5 kN/m^2 and the superimposed live load was varied between 2.5 kN/m^2 and 10 kN/m^2. Dead and imposed load factors of 1.35 and 1.5 were used with EC2 and MC2010 and corresponding load factors of 1.4 and 1.6 with BS8110. Characteristic material strengths of f_{ck} = 30 MPa and f_{yk} = 500 MPa were adopted in conjunction with code recommended material partial factors. The slab thickness was related to the design imposed loading in accordance with Goodchild's (2009) recommendations for economic frame construction. The resulting slab thicknesses and mean effective depths are given in Table 2.

BS8110 and the UK National Annex to EC2 allow design moments for slabs to be calculated using a single load case in which all spans are fully loaded provided the support moments are redistributed downwards by 20% and span moments increased accordingly. The parametric study investigates the effect of this moment redistribution on the amount of shear reinforcement required by each code. Consequently, the hogging moment at the column centreline was taken as either its elastic value of 0.083*FL* or 0.063*FL* from Table 3.12 of BS8110 which includes the 20% moment redistribution mentioned above (*F* is the total load on each panel). In each case the design span moment was taken as 0.063FL as given in Table 3.12 of BS8110 for interior panels. However, the same areas of hogging and sagging reinforcement were provided in the panels designed for M_{sup} = 0.083*FL* to simulate the common practice of adding surplus flexural reinforcement in the span to control deflection (Vollum, 2009). To maximise the difference between the two cases, the design hogging moment for flexural reinforcement was taken at the centreline of the column for elastically designed slabs and at $h_c/3$ from the column centreline for slabs designed for M_{sup} = 0.063*FL*. The latter moments satisfy the BS8110 requirement that the sum of the maximum span moment and average support moments across the panel width should exceed $F(L\text{-}2h_c/3)^2/8$. Additionally, the calculated areas of flexural reinforcement were increased by 4% in the slabs designed for M_{sup} = 0.083*FL* to allow for rationalisation of the reinforcement arrangement.

The design hogging moment was proportioned between the column and middle strips in the ratio 75:25 with two thirds of the column strip reinforcement placed in its central half in accordance with the requirements of BS8110 and EC2. The design shear force was multiplied by β = 1.15 in accordance with the recommendations of BS8110 and EC2. In the case of MC2010, the control perimeter u was multiplied by k_e = 0.9 and the maximum possible shear resistance was calculated with k_{sys} = 2.8 in equation (12) as recommended for studs.

The punching shear reinforcement was arranged radially in the EC2 and MC2010 designs but in square perimeters for the BS8110 designs. In the BS8110 and EC2 designs, the spacing of perimeters of shear reinforcement was taken as 0.5d, 1.25d, 2.0d etc. from the column face in accordance with UK practice. The perimeter spacing was reduced to 0.5d in the MC2010 designs as the required area of shear reinforcement doubles for spacings of 0.5d, 1.25d, 2.0d etc. since only one perimeter crosses the critical shear crack.

Figure 5 shows the variation in $V/ V_{Rmax\ EC2}$ (where $V_{Rmax\ EC2} = 2V_{Rdc\ \mathrm{EC2}}$) with design imposed load, and hence slab thickness, according to BS8110, EC2 and MC2010 for design hogging moments of 0.063FL and 0.083FL. The economic slab thicknesses of Goodchild (2009) are seen to comply with the BS8110 and UK National Annex to EC2 restrictions on $V_{Rmax} = 2V_{Rdc}$ but not the recommended code limit of $V_{Rmax} = 1.5V_{Rd,c}$ which is intended for stirrups. In the case of MC2010, V_{Rmax} is critical for all slabs with M_{sup} = 0.063FL.

Figure 6a compares the total areas of shear reinforcement required by each code within 1.5d of the column face neglecting the limit on V_{Rmax} which invalidates the MC2010 designs with M_{sup} = 0.063FL. BS8110 requires the least area of shear reinforcement and MC2010 the most. The difference between BS8110 and the other codes is greatest for slabs designed for M_{sup} = 0.083FL as the design shear force is less than 1.6$V_{Rd,c}$ making equation (2) of BS8110 applicable.

The shear reinforcement installation time depends on the total number of shear studs, which is governed by u_{out}, and spacing rules rather than A_{sw} which determines the stud diameter for a given shear reinforcement arrangement. Therefore, the required normal distances from the column face to the outer row of shear reinforcement are compared for each method in Figure 6b which shows remarkable disparities between the extents of shear reinforcement required by each code particularly for $M_{sup} = 0.063FL$ where the extent of shear reinforcement required by MC2010 is much greater than for BS8110 or EC2. The difference is in part due to MC2010 basing shear resistance at u_{out} on the critical shear crack width around the column which is particularly unrealistic once reinforcement yields over the column.

The minimum possible slab thickness can be limited by V_{Rmax} in thin slabs with edge and corner columns being most critical. In this case, V_{Rmax} can be increased by providing surplus hogging flexural reinforcement as shown in Figure 7a for a 265 mm thick slab. The shear resistances in Figure 7a are normalised by $V_{Rdc\ EC2}$ calculated with $A_{sprovided}$=$A_{srequired}$ for design support moments of $0.063FL$ and $0.083FL$ respectively. Figure 7a shows that MC2010 (with k_{sys} = 2.8) gives significantly lower maximum possible shear resistances than BS8110 or the UK National Annex to EC2 which limit V_{Rmax} to $2V_{Rdc}$. Figure 7b compares the areas of shear reinforcement required by each code, within 1.5d of the column face, for a design imposed load of 5 kN/m^2. EC2 and MC2010 require greatly more shear reinforcement than BS8110 particularly in cases where increasing $A_{sprovided}/A_{srequired}$ makes equation (2) govern.

Influence of restraint from surrounding slab

MC2010 Level IV was used to assess the influence of restraint from surrounding bays on the punching resistance of the slabs designed in the previous section. Rotations were calculated with NLFEA using the procedure described previously. The boundary conditions were varied as shown in Figure 8 of which 8a represents a conventional punching shear specimen of width 0.44L. Figure 8b represents 1/4 of an internal panel of a flat slab of span L

with rotational restraint at mid-span and 8c a slab with in-plane and rotational restraint at mid-span. The isolated slab of Figure 8a was loaded at eight points around its perimeter to simulate a conventional punching test whereas the continuous slabs were loaded uniformly. The rotations were extracted from the NLFEA along the slab centreline at around 0.2L from the column centre as measured by Lips et al. (2012). This position was chosen because it was used in the calibration of the NLFEA and is close to the position of maximum rotation as shown in Figure 9 for M_{sup} =0.063FL and q_k = 2.5 kN/m^2. Figures 10a to d present the calculated load versus rotation responses for slabs designed for M_{sup} = 0.063FL and 0.083FL with design imposed loads of 2.5 kN/m^2 and 7.5 kN/m^2. Additionally to NLFEA, rotations were calculated with equations (6), (13) and Muttoni's (2008) quadrilinear moment-curvature relationship which includes tension stiffening. The NLFEA rotations are denoted as follows in Figures 9 and 10:

- **Conventional**: NLFEA of isolated slab (see Figure 8a);
- **Continuous:** NLFEA of continuous slab with rotational restraint (see Figure 8b);
- **Continuous + Axial:** NLFEA of continuous slab with rotational and in-plane restraint (see Figure 8c).

Figure 10 also shows the punching resistances according to MC2010 for the areas of shear reinforcement required by BS8110, EC2 and MC2010 (respectively denoted "BS8110 resistance", "EC2 resistance" and "MC2010 resistance") as well as $V_{Rmax} = 2.8V_{R,dc}$ where critical. The punching resistances were calculated with $\gamma_c = \gamma_s = 1.0$, $k_e = 0.9$ and $d_b = 12$ mm. Failure loads are given by the intersection of the resistance and rotation curves. The design ultimate shear force $V_{Ed} = F$ is shown for comparison as is the flexural failure load of a comparable conventional test specimen which is denoted "PYL test specimen". The rotations given by Muttoni's quadrilinear relationship compare well with those given by NLFEA of isolated slabs up to around 70% of the flexural capacity given by equation (14). Figure 10 shows that rotational restraint at the panel edges increases punching resistance above that given by Muttoni's quadrilinear relationship or equation (13). Even greater resistances are obtained with rotational and full in-plane restraint but the latter is not generally available in flat slabs. Equation (6) is seen to conservatively estimate the benefit of rotational restraint from surrounding panels.

The calculated punching resistances are summarized in Table 3 which also shows the areas of shear reinforcement included in the calculation of punching resistance with MC2010. Strikingly when rotational restraint is included, V_{calc}/F is only around 20% greater for MC2010 than BS8110 despite MC2010 requiring 2.25 to 4 times the area of shear reinforcement within *d* of the column face. The ratio of resistances calculated with rotational restraint (continuous) and equation (13) gives an upper bound to the increase in strength due to rotational restraint. Table 3 shows this ratio varies between 1.12 and 1.49 and is typically less than the 30-50% found by Chana and Desai (1992a). Despite this, rotational restraint appears sufficient to explain the satisfactory performance of internal slab-column connections designed with BS8110 though according to MC2010, which tends to underestimate strength, the factor of safety is close to 1.0. The EC2 designs appear optimum from the view of economy and safety.

Influence of continuity on punching resistance of nine panel flat slab

The effect of structural continuity on the punching resistance of uniformly loaded flat slabs can be seen by analysing the 3/4 scale 9 panel flat slab tested by Guralnick and Fraugh (1963). The slab was 133 mm thick, with a mean effective depth of 109.5 mm, and spanned 4.57 m between column centrelines. The internal columns were 457 mm square. No shear reinforcement was provided. Deformed reinforcement bars were used with mean yield strength of 276 MPa. The concrete cylinder strength was 32.5 MPa. The slab was loaded to failure under a uniform load which was simulated by applying 25 concentrated loads to each panel. Punching failure occurred at an interior column at an estimated internal column load of 399 kN (Guralnick and Fraugh, 1963). The uniformly distributed failure load was 1.05 times that given by yield line analysis. Immediately before failure, the average recorded steel strain at the four faces of the critical column was around seven times the yield strain. The corresponding maximum strains in the span reinforcement were around ¾ of the yield strain. The load deflection response also indicates that the slab was close to flexural failure even though it failed in punching.

The shear resistance was calculated with BS8110, EC2 and MC2010 with $\gamma_c = \gamma_s = \gamma_m = 1.0$. The effect of moment transfer to the column was included in the BS8110 and EC2 strength assessments by multiplying the applied shear force by $\beta = 1.15$. In the case of MC2010, k_e was taken as 0.9. Slab rotations were calculated with equations (6), (13) and from the measured slab deflections as follows:

$$\theta \sim 3w/L \tag{15}$$

where w is the mean mid-span deflection in the 4 panels surrounding the critical column and L is the span.

Equation (15) was derived from NLFEA of the continuous slabs with rotational restraint considered in the previous section. The equation is remarkably accurate as shown in Figure 11a for a range of design support moments and column sizes. The 750 mm square columns are 1/10 of the span as in the Guralnick and Fraugh test. Figure 11b shows the calculated rotations as well as the punching resistance according to MC2010. Equation (13) gives the rotation of a conventional isolated punching specimen with the same hogging reinforcement as the tested slab and $r_s = r_q = 0.22L$ for which V_{flex} = 299 kN. The shear resistances corresponding to equations (13), (6) and (15) are 299 kN, 325 kN and 393 kN respectively. The latter agrees well with the measured strength of 399 kN and illustrates the benefit of flexural continuity which is only partly included in equation (6) which is used in MC2010 Level II. The shear resistances given by BS8110 and EC2 are 413 kN and 377 kN respectively which suggests that punching resistance was increased by rotational restraint but not compressive membrane action due to the high utilisation of flexural reinforcement in surrounding panels.

Conclusion

The paper compares the relative safety of the design rules for punching shear in BS8110, EC2 and MC2010. Both BS8110 and EC2 are shown to overestimate strength within the shear reinforced zone with BS8110 being least conservative. MC2010 performs noticeably better in this respect which is significant because failure outside the shear reinforcement is unlikely in practice since analysis shows that the introduction of partial factors tends to make failure inside the shear reinforced zone critical.

Parametric studies (see Figure 5) show that limiting V_{Rmax} to $k_{sys}V_{R,dc}$ in MC2010 prevents 20% downwards moment redistribution over the columns of flat slabs, which has been allowed in the UK for many years. Assessment with MC2010 Level 4 shows that punching resistance is increased by rotational continuity at mid-span and even more so by combined in-plane and rotational restraint. The increase in calculated punching resistance due to rotational continuity is best seen by comparing strengths calculated with rotations from equation (13) and NLFEA of a complete panel with rotational restraint at mid-span. The increase in strength due to rotational continuity is partially included in MC2010 Level II if rotations are calculated with equation (6).

It is necessary to invoke flexural continuity to explain the observed strength of the Guralnick and Fraugh slab with MC2010, but not BS8110 or EC2, as it increases punching resistance by reducing rotations below those in comparable isolated punching specimens with which MC2010 is calibrated. The wide variation in calculated failure loads of identical slabs evident in Figure 10 suggests that the adoption of a rotational based failure criterion could lead to disagreements between designers and checking engineers. Consequently, the more empirical design methods of BS8110 and EC2 seem better suited for normal design though MC2010 is useful for assessment. Rotational restraint from surrounding panels, along with rounding up of calculated areas of reinforcement, seem sufficient to explain the satisfactory performance of flat slabs designed to BS8110.

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List of Tables

Table 1: Properties of slabs used in calibration of NLFEA

Specimen	**h (mm)**	**c[a] (mm)**	**d (mm)**	**f_c (MPa)**	**f_{ct} (MPa)**	**ρ (%)**	**f_y (MPa)**	**$V_{r,test}$ (kN)**	**ψ (‰)**
PL1	250	130 x 130	193	36.2	2.272	1.63	583	682	6.0
PV1	250	260 x 260	210	34.0	2.16	1.50	709	974	7.6
PL4	320	340 x 340	267	30.5	2.02	1.58	531 ø20 580 ø26	1625	6.5
PL5	400	440 x 440	353	31.9	2.076	1.5	580	2491	4.7
PG10	250	260 x 260	210	28.5	1.94	0.33	577	540	22.3
PG11	250	260 x 260	210	31.5	2.06	0.75	570	763	10.0

Notes: [a] Dimension of loaded area in mm

Table 2: Properties of slabs used in parametric studies

Design Hogging Moment		**0.063FL**				**0.083FL[a]**			
Design imposed live load [kN/m^2]		2.5	5.0	7.5	10.0	2.5	5.0	7.5	10.0
Slab thickness [mm]		239	265	324	363	239	265	324	363
F [kN]		779	1039	1362	1647	779	1039	1362	1647
Average effective depth [mm]		194	220	279	318	194	220	279	318
M_{hog}	A_s Centre column strip [mm^2]	1917	2259	2316	2451	3255	3845	3897	4111
	A_s Edge column strip [mm^2]	959	1130	1158	1226	1627	1922	1949	2056
M_{hog} - A_s Middle strip [mm^2]		959	1130	1158	1226	1627	1922	1949	2056
M_{span}	A_s Column strip [mm^2]	2498	2943	3017	3193	3580	4229	4287	4523
	A_s Middle strip [mm^2]	2043	2408	2469	2613	2929	3460	3508	3700
L = 7.5m, columns 450 mm square, f_{ck} = 30 MPa, design imposed dead load 1.5 kN/m^2									

Note: [a] $A_{sprovided}/A_{srequired}$ =1.04 to allow for rationalisation of reinforcement arrangement.

Table 3: Influence of continuity on shear strengths calculated with MC2010 with $\gamma_c = \gamma_s = 1.0$ and $k_e = 0.9$

		V_{calc}/F			
Design method for shear reinforcement	Boundary conditions	**M_{sup} = 0.063FL**		**M_{sup} = 0.083FL**	
		q_k = 2.5 kN/m^2	**q_k = 7.5 kN/m^2**	**q_k = 2.5 kN/m^2**	**q_k = 7.5 kN/m^2**
BS8110	$\Sigma_{d_v} A_{sw}$ [mm^2]	433[b]	1200[c]	368[d]	534[c]
	Equations 13 &14	0.74	0.79[e]	0.80	0.94
	Equation 6	0.77	0.83	0.87	0.96
	Conventional [a]	0.82	0.85	0.96	1.05
	Continuous [a]	**0.95**	**0.99**	**1.01**	**1.10**
	Continuous/Eq 13[f]	**1.28**	**1.25**	**1.26**	**1.17**
	Continuous + axial[a]	1.22	1.17	1.16	1.21
EC2	$\Sigma_{d_v} A_{sw}$ [mm^2]	878	1538	758	1365
	Equations 13 &14	0.79 [e]	0.79[e]	0.93	1.01
	Equation 6	0.92	0.90	0.99	1.03
	Conventional [a]	0.90	0.89	1.08	1.11
	Continuous [a]	**1.06**	**1.05**	**1.14**	**1.13**
	Continuous/Eq 13[f]	**1.34**	**1.33**	**1.23**	**1.12**
	Continuous + axial[a]	1.36	1.20	1.25	1.25
MC2010	$\Sigma_{d_v} A_{sw}$ [mm^2]	1508	2710	1161	2156
	Equations 13 &14	0.79 [e]	0.79[e]	1.07	1.18
	Equation 6	1.04	0.99	1.13	1.20
	Conventional [a]	0.93	0.92	1.19	1.26
	Continuous [a]	**1.18**	**1.16**	**1.26**	**1.34**
	Continuous/Eq 13[f]	**1.49**	**1.47**	**1.18**	**1.14**
	Continuous + axial[a]	1.68	1.38	1.39	1.31

Note: [a] Rotation calculated with NLFEA, [b] 50:50 split of $\Sigma_{1.5d} A_{sw}$ between 1st and 2nd perimeters, [c] 40:60 split of $\Sigma_{1.5d} A_{sw}$ between 1st and 2nd perimeters, [d] $A_{sw\ BS8110min}$, [e] flexural failure, [f] upper bound of enhancement in strength of isolated test specimen due to rotational restraint.

List of Figures

Figure 11: Calculation of MC2010 shear resistance of Guralnick and Fraugh slab (1963) a) calibration of equation (15) (columns 750 square unless noted otherwise) and (b) calculation of MC2010 shear resistance.

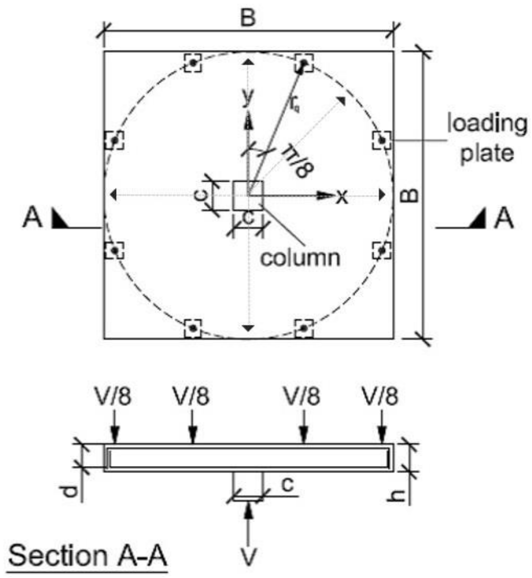
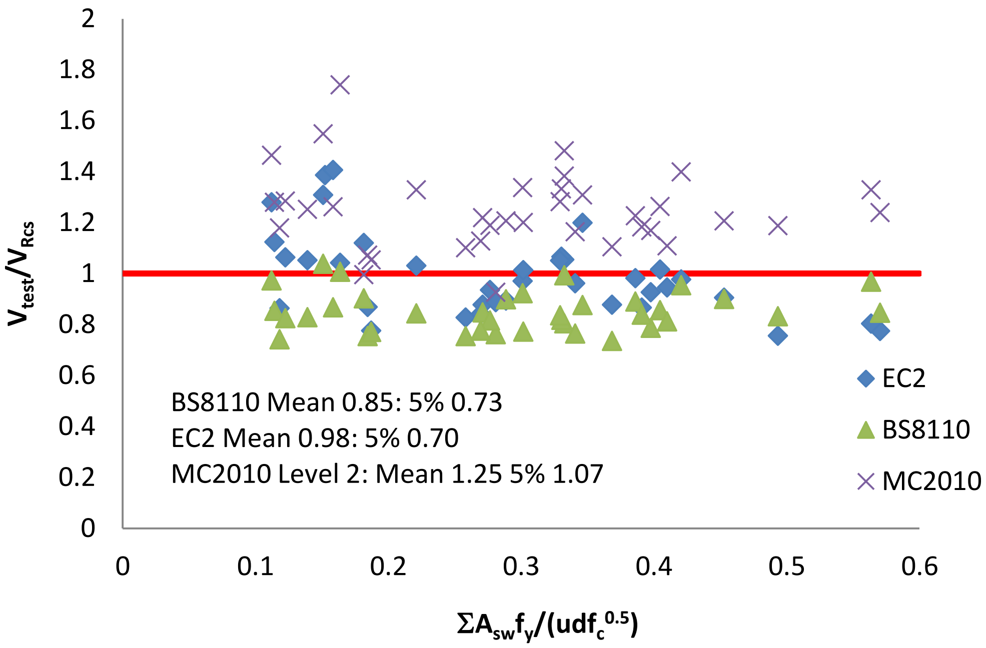
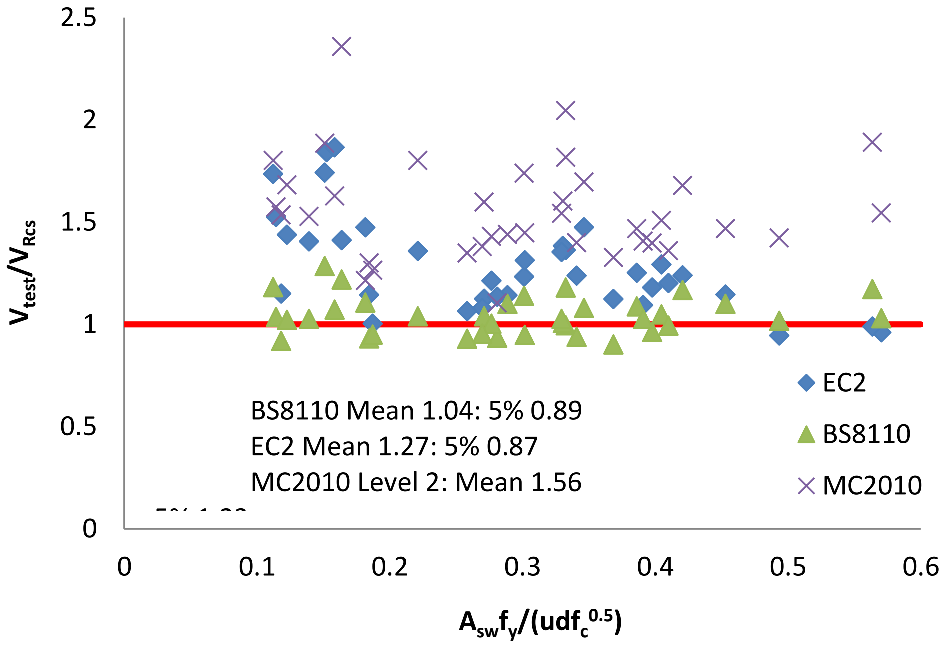


Figure 1: Conventional punching shear specimen



a)



b)

Figure 2: Influence of shear reinforcement ratio on $V_{test}/V_{Rcs\ calc}$ for BS8110, EC2 and MC2010 Level 2 with a) $\gamma_c = \gamma_s = 1.0$ [5% = μ-1.64sd] and b) with code recommended partial factors γ_m, γ_c and γ_s.

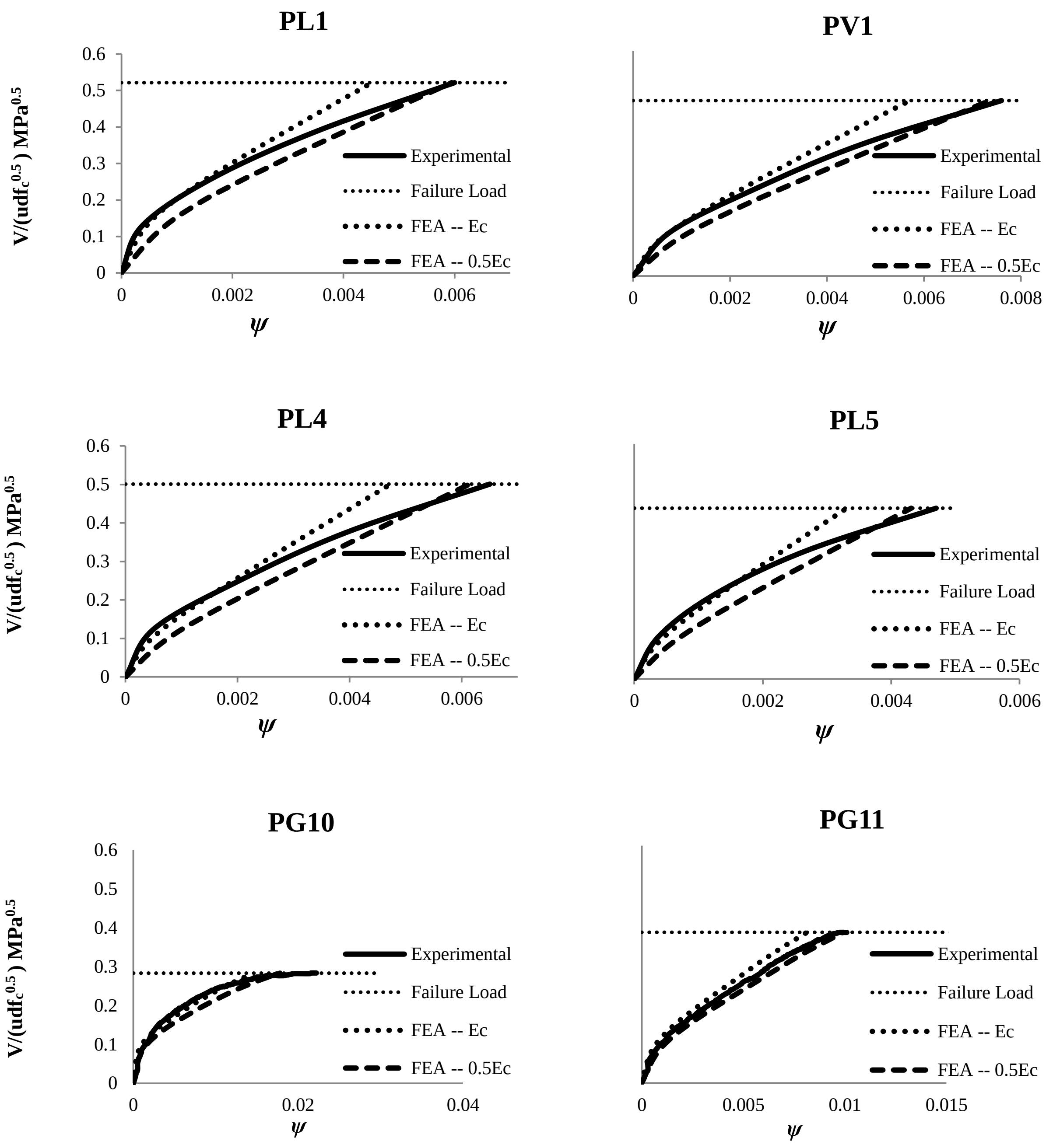
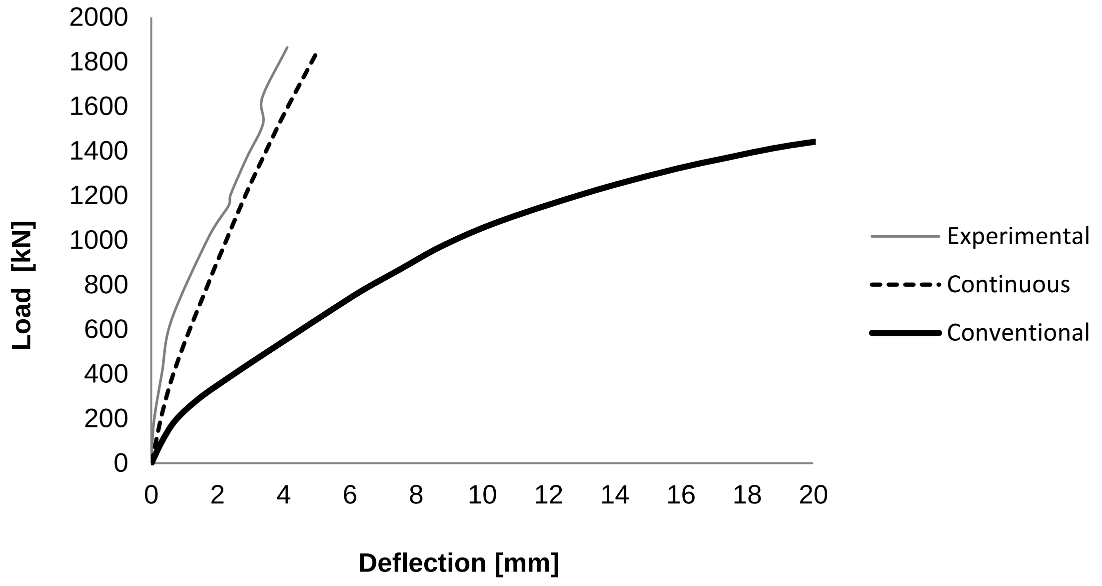
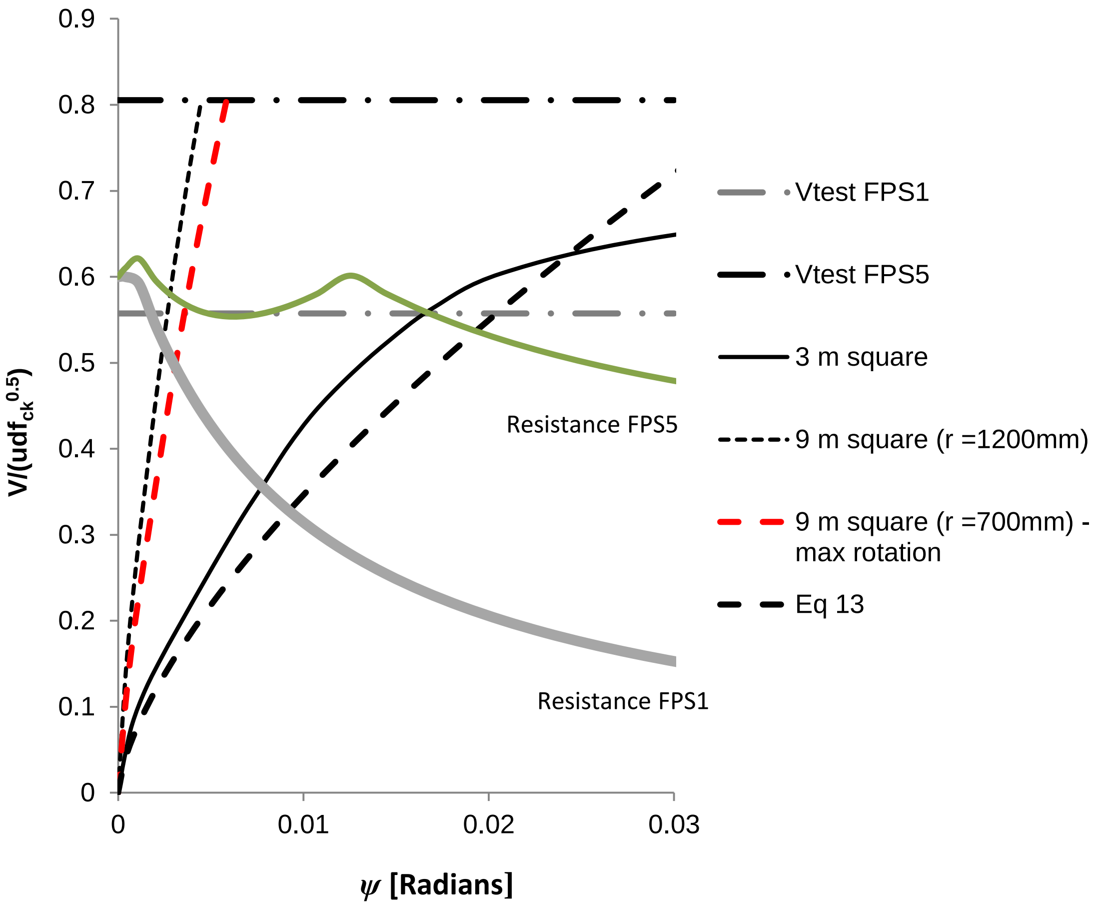


Figure 3: Comparison of measured and calculated load versus rotation response for slabs PL1, PV1, PL5, PL4, PG10 and PG11



Load [kN]
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0
2
4
6
8
10
12
14
16
18
20
Deflection [mm]
Experimental
Continuous
Conventional

a)



V/(udf$_{ck}^{0.5}$)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.01
0.02
0.03
ψ [Radians]
Resistance FPS5
Resistance FPS1
Vtest FPS1
Vtest FPS5
3 m square
9 m square (r =1200mm)
9 m square (r =700mm) - max rotation
Eq 13

b)

Figure 4: Analysis of Chana and Desai (1992a) slabs FPS1 (no stirrups) and FPS5 (with stirrups) a) comparison of measured and predicted deflections and b) calculation of resistance with MC2010.

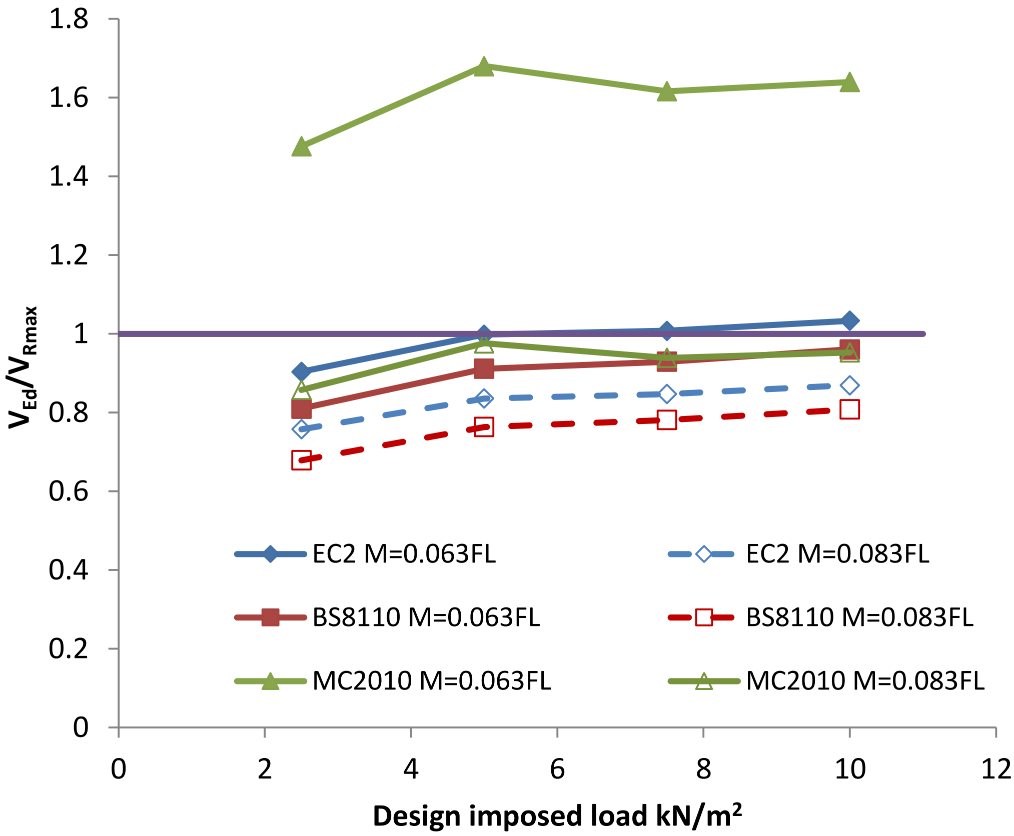
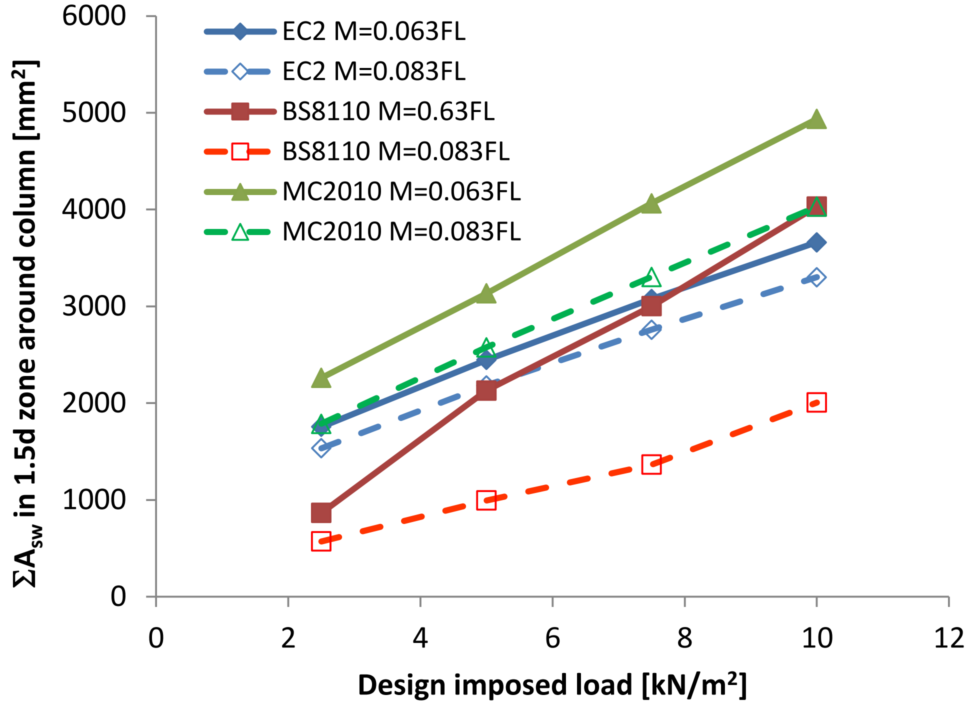
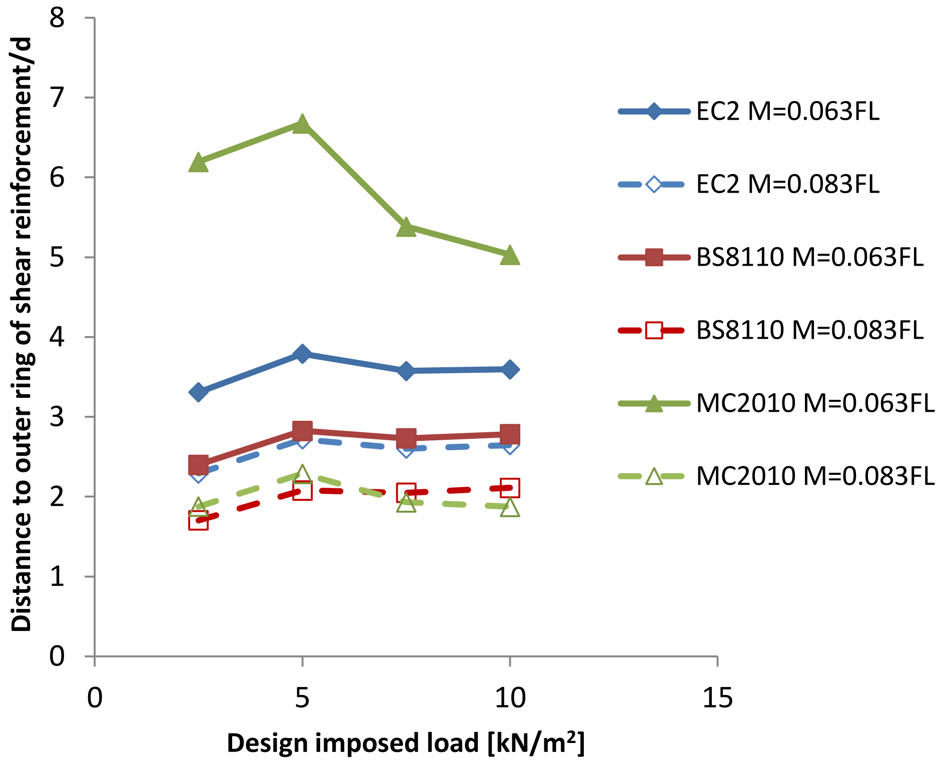


Figure 5: Influence of design imposed load on V_{Ed}/V_{Rmax} for design support moments of 0.063FL and 0.083FL

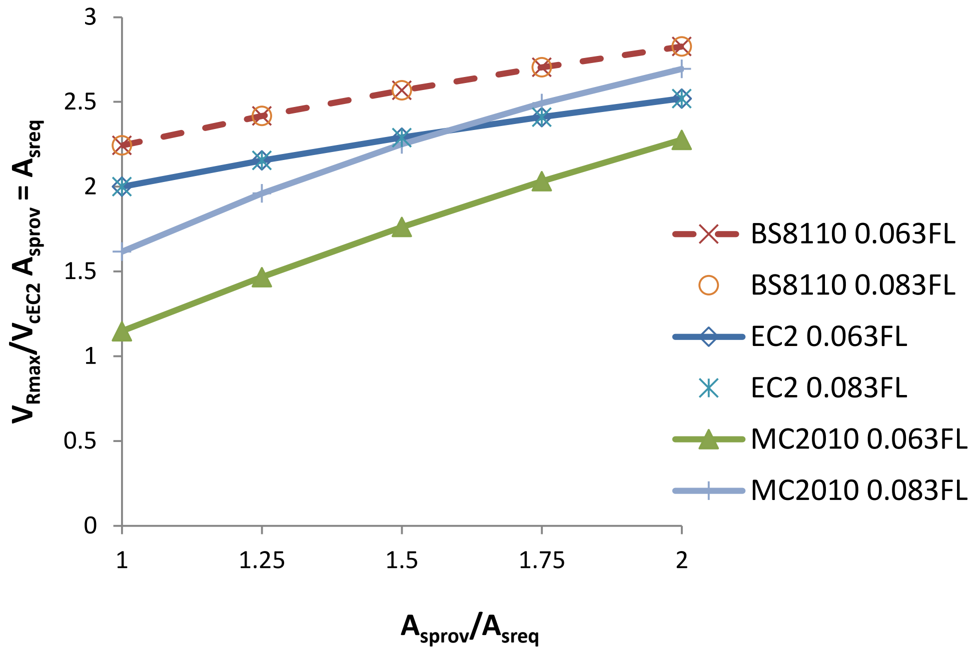

ΣA_sw in 1.5d zone around column [mm²]
6000
5000
4000
3000
2000
1000
0
0
2
4
6
8
10
12
Design imposed load [kN/m²]
EC2 M=0.063FL
EC2 M=0.083FL
BS8110 M=0.63FL
BS8110 M=0.083FL
MC2010 M=0.063FL
MC2010 M=0.083FL

a)

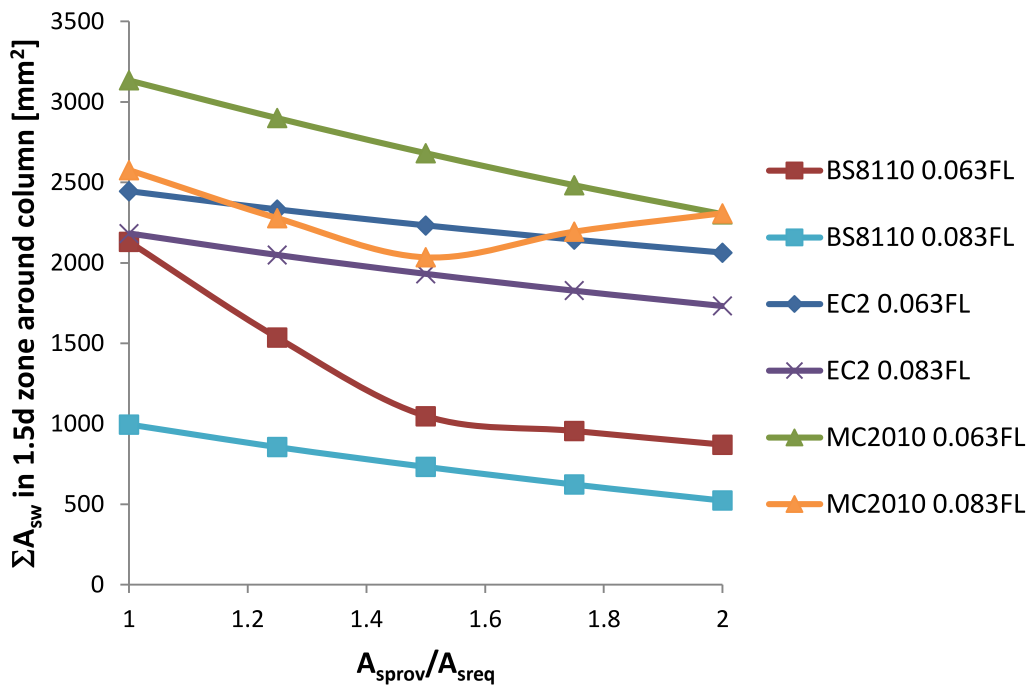

Distannce to outer ring of shear reinforcement/d
8
7
6
5
4
3
2
1
0
0
5
10
15
Design imposed load [kN/m²]
EC2 M=0.063FL
EC2 M=0.083FL
BS8110 M=0.063FL
BS8110 M=0.083FL
MC2010 M=0.063FL
MC2010 M=0.083FL

b)

Figure 6: Comparison of a) areas of shear reinforcement required within 1.5d of column and b) required minimum distance to outer shear reinforcement for design support moments of 0.063FL and 0.083FL


3
2.5
2
1.5
1
0.5
0
1
1.25
1.5
1.75
2
V_{Rmax}/V_{cEC2} $A_{sprov} = A_{sreq}$
A_{sprov}/A_{sreq}
BS8110 0.063FL
BS8110 0.083FL
EC2 0.063FL
EC2 0.083FL
MC2010 0.063FL
MC2010 0.083FL

a)


3500
3000
2500
2000
1500
1000
500
0
1
1.2
1.4
1.6
1.8
2
ΣA_{sw} in 1.5d zone around column [mm^2]
A_{sprov}/A_{sreq}
BS8110 0.063FL
BS8110 0.083FL
EC2 0.063FL
EC2 0.083FL
MC2010 0.063FL
MC2010 0.083FL

b)

Figure 7: Influence of surplus flexural reinforcement for 265 mm thick slab with 450 mm square column on a) maximum possible shear resistance and b) area of shear reinforcement for F = 1039 kN.

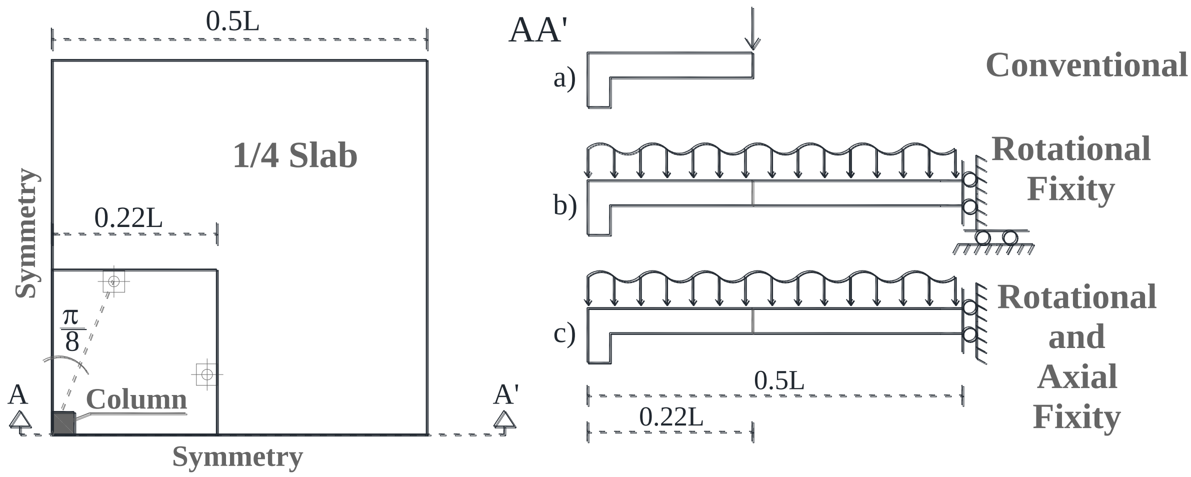


Figure 8: Boundary conditions for MC2010 Level 4 analysis of interior panels of flat slab

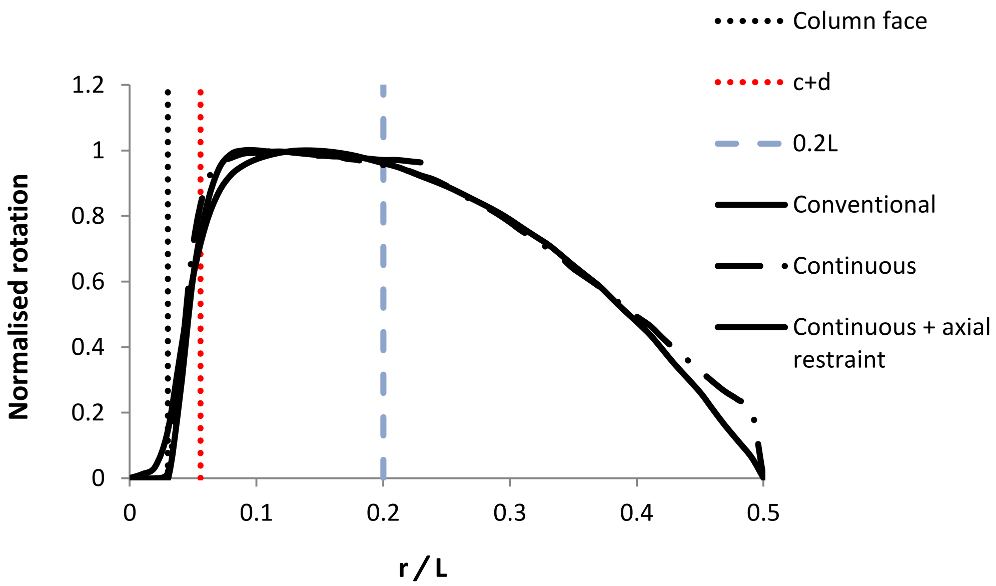
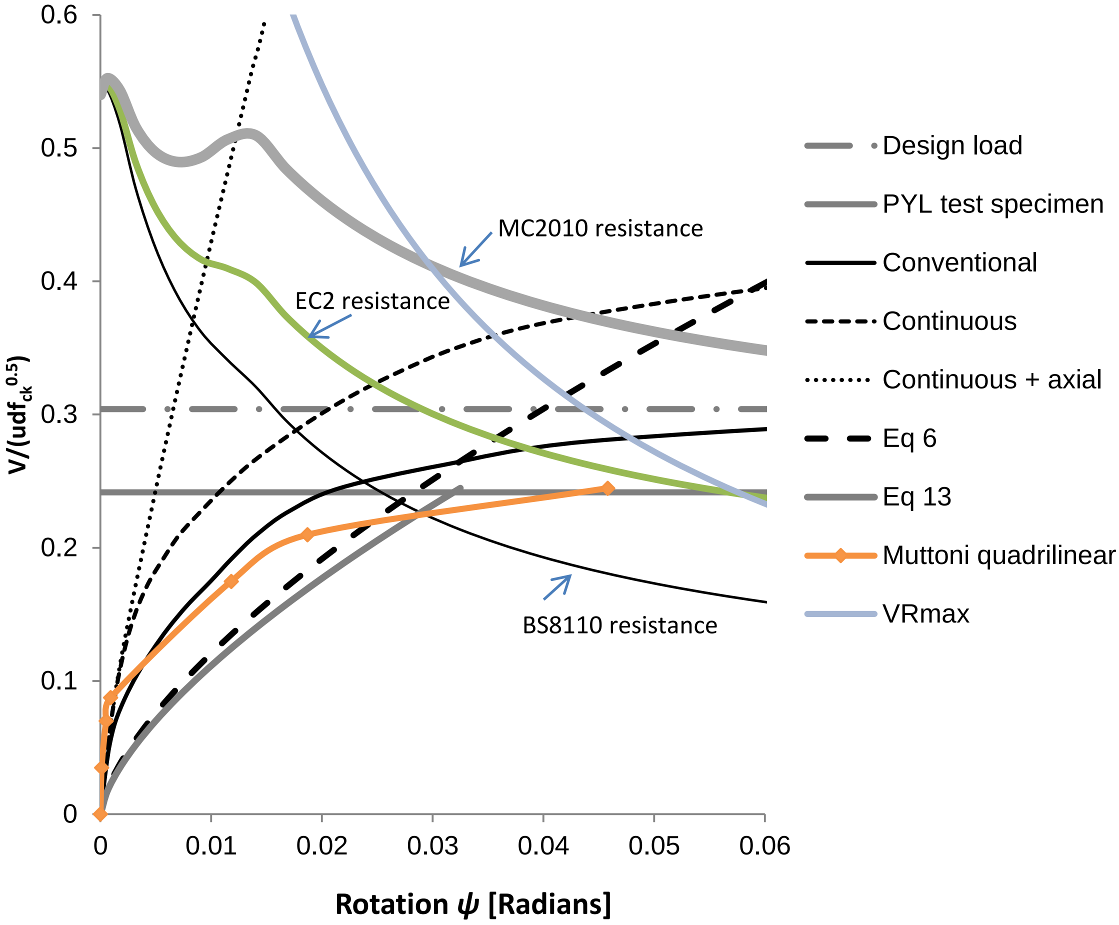
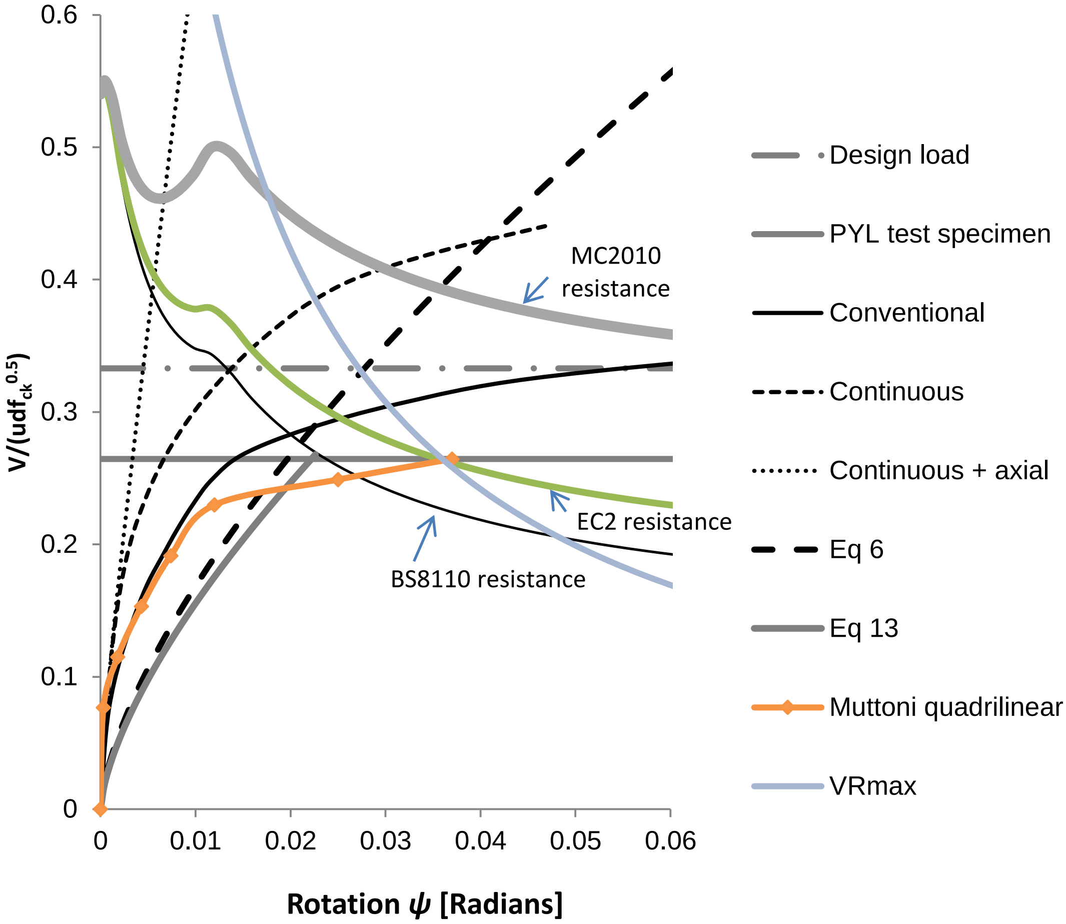


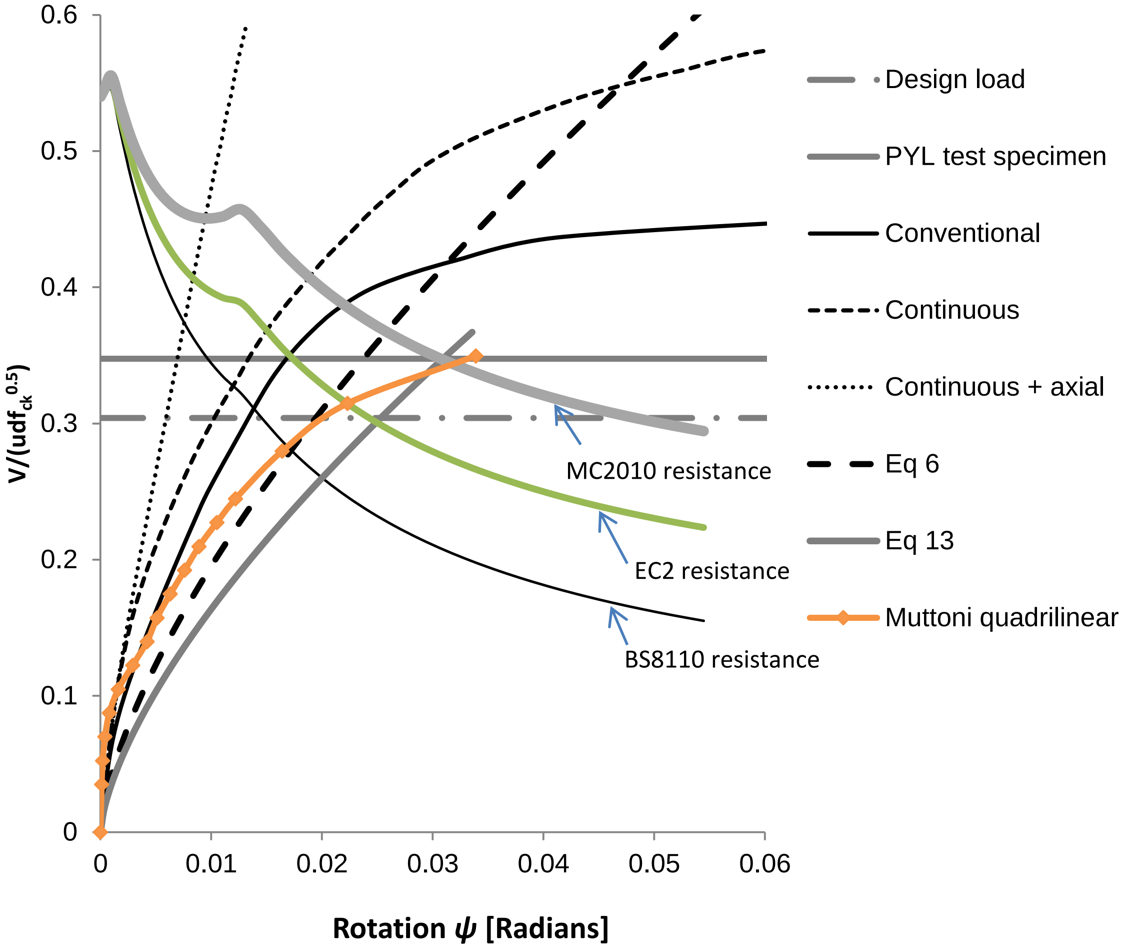
Figure 9: Variation in rotation along slab centreline


0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.01
0.02
0.03
0.04
0.05
0.06
V/(udf$_{ck}$$^{0.5}$)
Rotation ψ [Radians]
MC2010 resistance
EC2 resistance
BS8110 resistance
Design load
PYL test specimen
Conventional
Continuous
Continuous + axial
Eq 6
Eq 13
Muttoni quadrilinear
VRmax

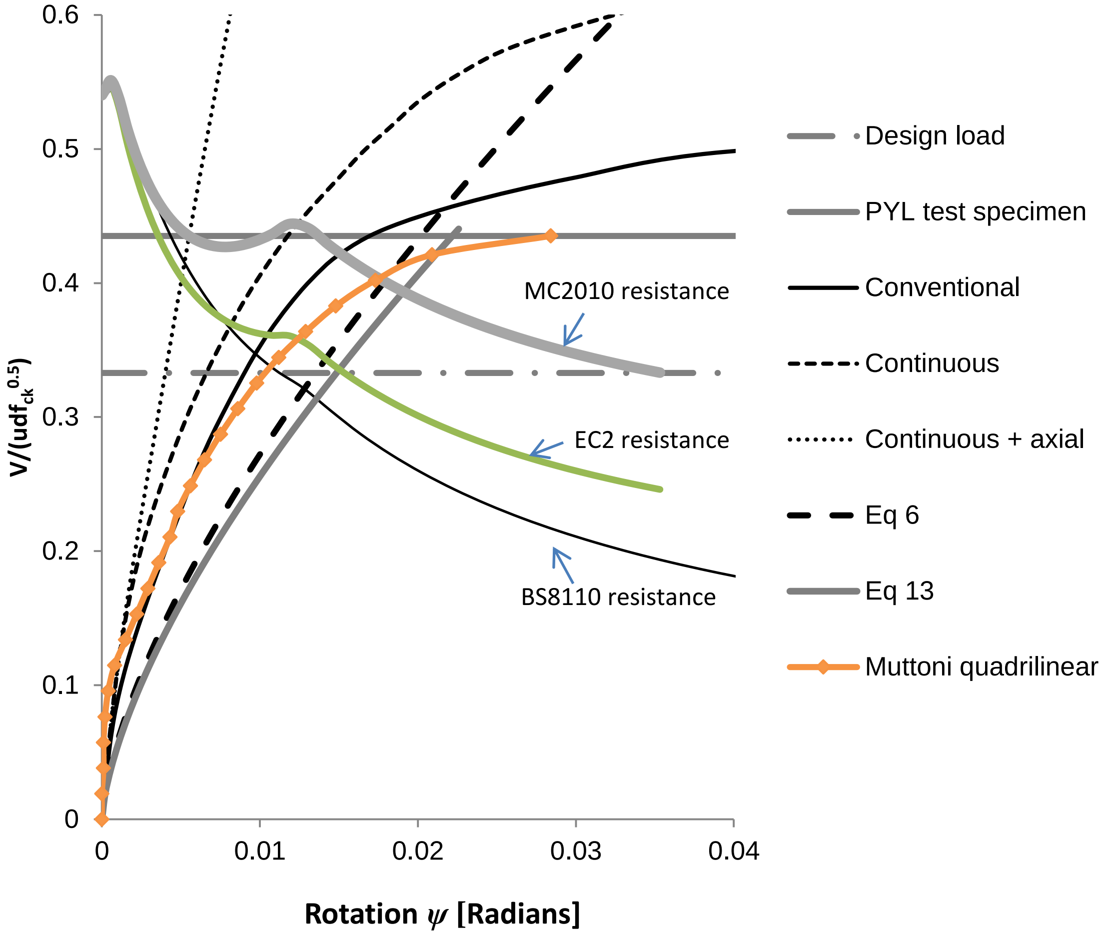
a)


0.6
0.5
0.4
0.3
0.2
0.1
0
V/(udf$_{ck}$$^{0.5}$)
0
0.01
0.02
0.03
0.04
0.05
0.06
Rotation ψ [Radians]
MC2010 resistance
EC2 resistance
BS8110 resistance
Design load
PYL test specimen
Conventional
Continuous
Continuous + axial
Eq 6
Eq 13
Muttoni quadrilinear
VRmax

b)

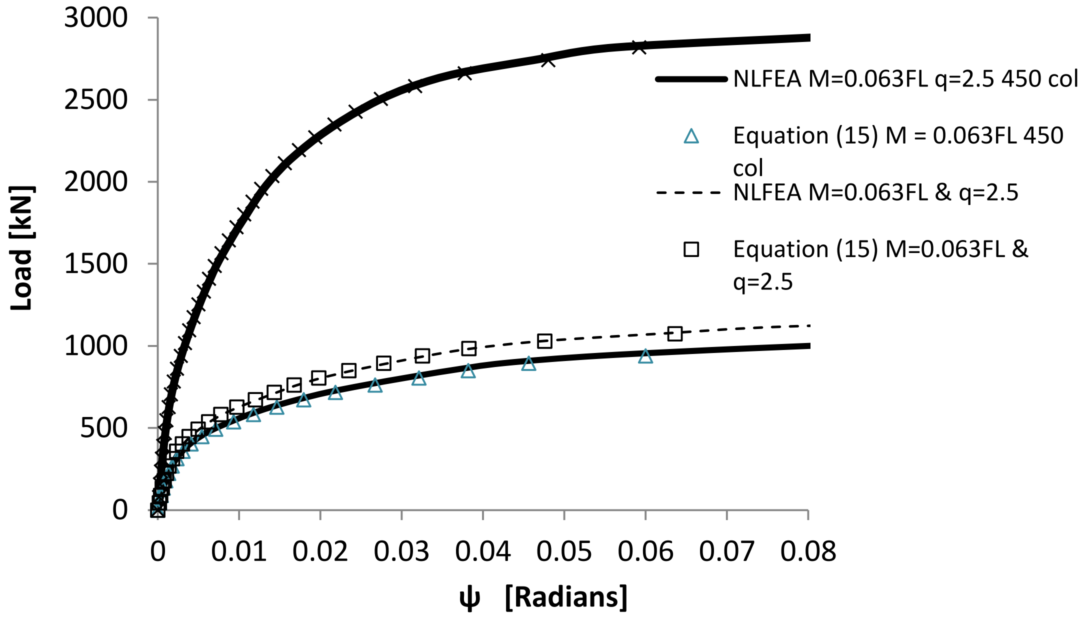

0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.01
0.02
0.03
0.04
0.05
0.06
V/(udf_ck^0.5)
Rotation ψ [Radians]
MC2010 resistance
EC2 resistance
BS8110 resistance
Design load
PYL test specimen
Conventional
Continuous
Continuous + axial
Eq 6
Eq 13
Muttoni quadrilinear

c)

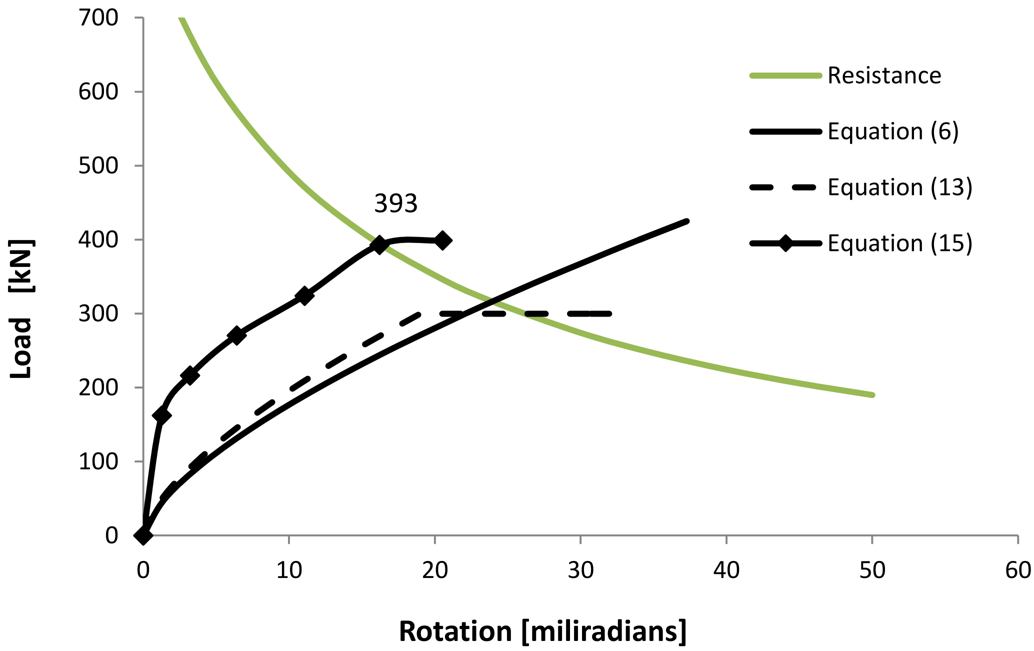


d)

Figure 10: Influence of slab continuity on shear resistance for a) M_{sup} = 0.063FL and q_k = 2.5 kN/m^2, b) M_{sup} = 0.063FL and q_k = 7.5 kN/m^2, c) M_{sup} = 0.083FL and q_k = 2.5 kN/m^2 and d) M_{sup} = 0.083FL and q_k = 7.5 kN/m^2.



a)



b)

Figure 11: Calculation of MC2010 shear resistance of Guralnick and Fraugh slab (1963) a) calibration of equation (15) (columns 750 square unless noted otherwise) and (b) calculation of MC2010 shear resistance.