DESIGN OF ELECTRONIC ACTIVE FILTERS FROM LADDER NETWORKS USING LINEAR TRANSFORMATIONS

by

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ABSTRACT

A new approach to the design of active RC filters that simulate doubly terminated LC ladder filters is developed in this thesis. The approach is based on the Linear Transformation of the conventional port variable vectors $[V_i \quad I_i]^T$ of two-port subnetworks into which an LC ladder filter is decomposed. In their simplest form these subnetworks are taken to be the series and shunt arms of the LC ladder filter. Their transformation is then effected from the V-I description into a x-y domain where direct active RC implementation and interconnection is possible. The active RC filters so derived are referred to as the Linear Transformation Active (LTA) filters. The theoretical development as well as the effective implementation of the entire approach are analytically presented in such a manner to serve as a powerful active RC filter design tool. The flexibility of the new approach is such that new filter structures can be obtained having desirable features such as the R-Self-Dual LTA method and in addition many well known ladder simulation methods are interpreted as special cases of the general LTA method. The LTA approach is applied, both in its general form and also in its constrained forms, to several filter design problems to illustrate not only its flexibility but also its effectiveness in producing practical RC active filter simulations of doubly terminated LC ladder filters.

To

MY WIFE KALLIOPE

and

ALL THOSE WHO SUPPORTED ME

MORALLY AND FINANCIALLY

DURING THE RESEARCH FOR THIS THESIS

AKNOWLEDGEMENTS

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GLOSSARY OF SYMBOLS AND ABBREVIATIONS

$\mathfrak{a}_{1i}$: Nonsingular 2x2 left transformation matrix for the left port-variable vector of the two-port N_i .

$\mathfrak{a}_{2i}$: Nonsingular 2x2 right transformation matrix for the right port-variable vector of the two-port N_i.

T_i : The modified chain matrix of the two-port N_i .

$\mathfrak{a}^{-1}$: The inverse matrix of $\mathfrak{a}$.

$\mathfrak{a}^T$: The transpose of the matrix $\mathfrak{a}$.

$\mathfrak{a}_i^c$: The compatible matrix of $\mathfrak{a}_i$ under the cross-cascade connection.

$\wedge$: logic "and"

$\vee$: logic "or"

$\equiv$: "identically equal to"

$\Longrightarrow$: "implies that"

$\Longleftrightarrow$: "equivalent to"

LT : Linear Transformation, Linearly Transformed, Linear Transform.

TRE : Transfer Ratio Elimination.

TRI : Transfer Ratio Identification.

$\forall$: "for each and every"

CHAPTER 1

ACTIVE-RC FILTERS : A SURVEY

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1.1 SOME INTRODUCTORY COMMENTS ON FILTERS

Any communication system, by virtue of its very nature, involves signal processing operations and employs several filters at the various stages within its structure. These filters may be classified according to the following criteria:

(i) Frequency range of operation (audio, radio and microwave filters
(ii) Band selectivity (lowpass, highpass, bandpass, band-reject, all-pass).
(iii) Kind of elements employed (LC, distributed components, electromechanical, piezoelectric, magnetostrictive resonators).
(iv) Filter topology (ladder, lattice)
(v) External power requirements (passive, active).

The initial information given to the filter designer refers to the first two criteria (i.e. the frequency range of operation and the band selectivity) in the form of typical filter specifications.

These filter specifications define the limits which the response of the filter must not exceed. Typical specifications of a bandpass filter are shown in Fig. 1.1, where $T(j\omega)$ is the transfer function of the filter defined as $T(s) = \frac{Y(s)}{X(s)}$. $X(s)$ and $Y(s)$ are the Laplace transforms of the input signal $x(t)$ and $y(t)$ respectively.

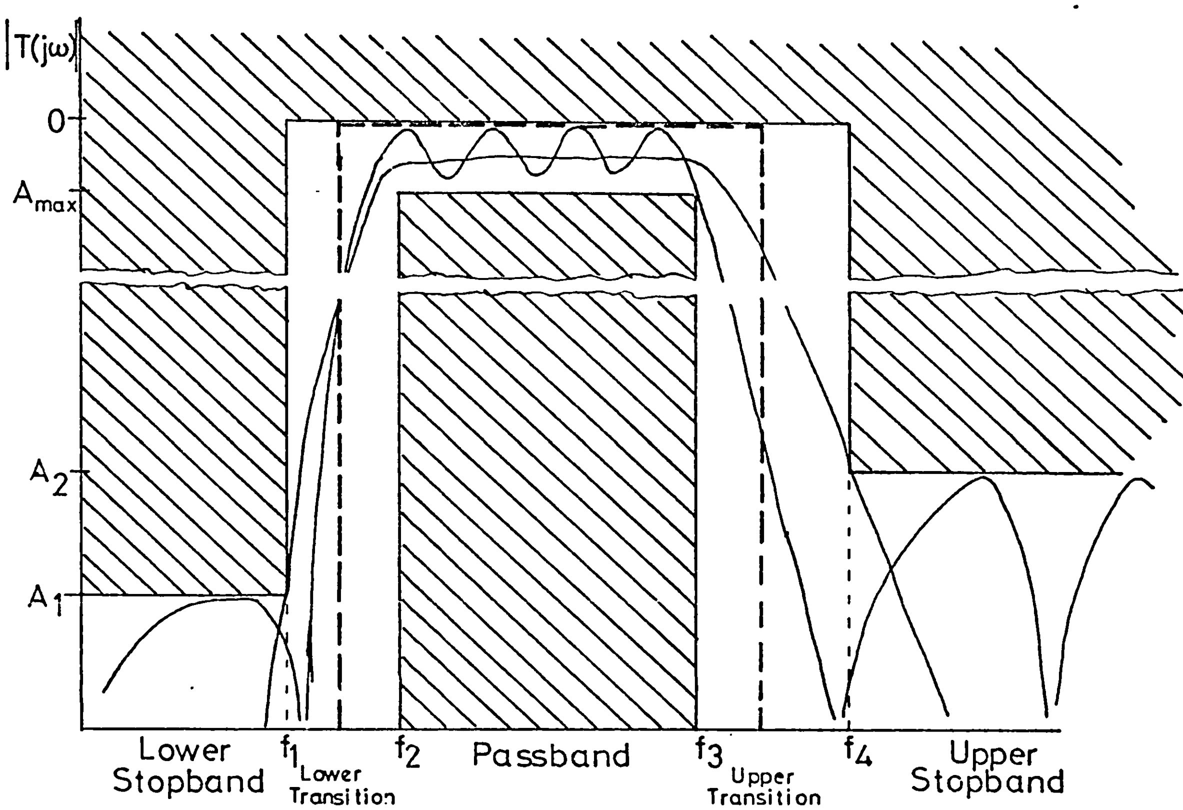

Fig. 1.1

The dotted line represents the ideal filter response to which the designer must approximate within the realistic specification margins. The approximation process consists of finding a function $T(s)$ which satisfies the specifications, i.e. the plot of $|T(j\omega)|$ lies in that region in Fig. 1.1 which is not cross-hatched. The approximation problem warrants and has received extensive treatment [1][2][3]. Nevertheless it is important to point out that the steepness in the transition region of the required filter (a measure of which is $\frac{A_1}{f_2-f_1}$ for the lower transition region and $\frac{A_2}{f_4-f_3}$ for the upper transition region) as well as the selectivity of the filter ($\frac{\sqrt{f_2 f_3}}{f_3-f_2}$) greatly affect the order of the transfer function $T(s)$ and its pole-Q values,

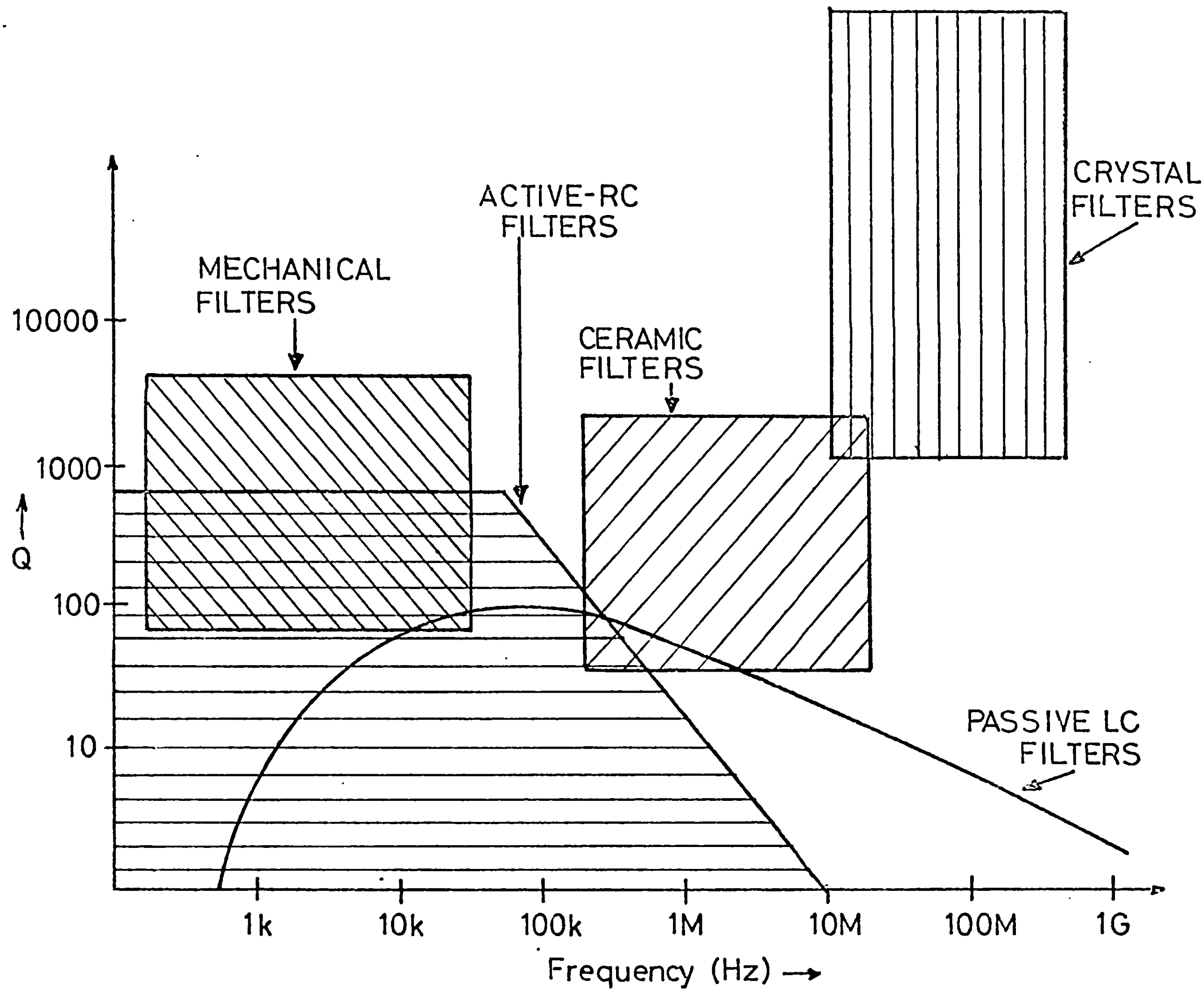

Fig. 1.2

in the sense that very steep and selective filters require high order functions with large pole Q values [4] .

Once a transfer function T(s) has been determined that meets the specifications, a synthesis procedure is then implicated for which the type of the filter, according to criteria (iii), (iv) and (v), must have been predetermined. This is done according to the capacity of each type of filter to provide the maximum pole-Q for the given transfer function at the required frequencies. This is discussed in detail in references [4] and [6] from which we have the plot of Q versus frequency of Fig. 1.2 above. The mechanical, ceramic and crystal filters are not within the scope of this thesis where we are entirely concerned with active RC filters and particularly with a special class of such filters, defined in this thesis, that simulate passive LC networks. Our attention will be, therefore, focused on the LC filters and active RC filter case of Fig. 1.2.

The weakness of the LC passive filters to operate at very low frequencies, even with very low pole-Q values, is due , among other reasons , to the fact that at these frequencies the inductors become very bulky and their inductive behaviour is dominated by the resistive behaviour due to their loss resistance R_l shown in Fig. 1.3 as a series loss resistance.

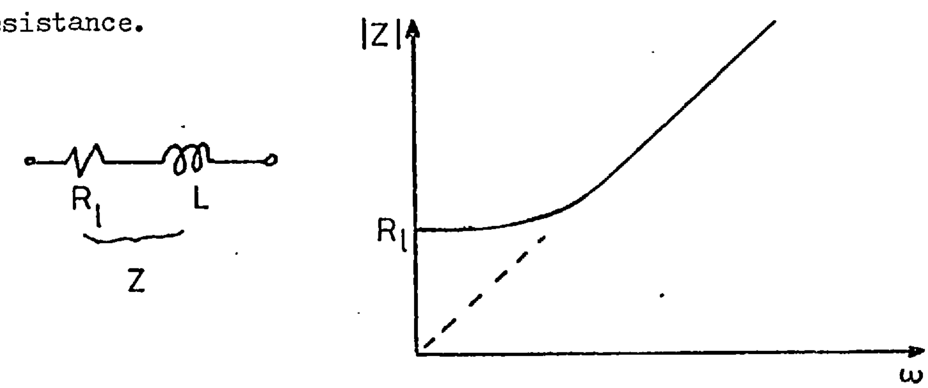

Fig. 1.3

Any attempt to reduce the value of R_l, increases the cost of the inductors and contributes rather negatively to the reduction of their size.

The case for active RC filters use, becomes immediately apparent from the above discussion and also from Fig. 1.2, in such a way that for low-to-medium pole-Q s at low-to-very-low frequencies appear without alternatives in the analogue domain. Moreover, within the voice frequency band, where in communication systems the demand on filter complexity may be considerable, the active RC filters are eminently suitable due to their being microelectronically realizable. In general, active RC filters can replace effectively LC passive filters for frequencies up to 100 kHz with pole-Q requirements of up to several hundreds, whilst at higher frequencies, where inductors are less bulky, LC filters are less objectionable.

The pole-Q and frequency limitations in active RC filters are mainly due to the active components available. The use, for instance, of commercial integrated operational amplifiers with a gain-bandwidth product 1 MHz does not allow satisfactory operation of the active RC filter at frequencies beyond 100 kHz approximately. However, many filtering applications are well within this frequency range. Active RC filters provide a very powerful, elegant and practicaly desirable solution to many of these filtering problems whilst the development of new design methods and the continuous improvement of active components could conceivably extend the frequency limits to much higher levels.

1.2 ACTIVE FILTER DESIGN - TRANSFER FUNCTION SIMULATION

Once the decision on the use of an active RC realization of the transfer function T(s) has been made, a synthesis procedure must be followed. The transfer function T(s) has, of course, been derived through some approximation procedure to satisfy the design specifications. This approximation procedure leads to a transfer function of the form:

$$T(s) = \frac{Y(s)}{X(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_o}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_o} \quad , m \leqslant n$$

which may be written as the product of second order functions as follows:

$$T(s) = \prod_{j=1}^{n/2} T_j(s) = \prod_{j=1}^{n/2} k_j \frac{(s-z_{j1})(s-z_{j2})}{(s-p_j)(s-p_j^*)}$$

where z_j are the zeroes of the transfer function (roots of the nominator) and p_j, p_j^* are the conjugate pole pairs of T(s) (i.e. the roots of the denominator). Odd polynomials, of course, have at least single negative real root in addition to the conjugate pole pairs p_j, p_j^*.

The above procedure is known as the decomposition of the transfer function T(s) into the second order subfunctions $T_j(s)$ which now can be realized individually by single-input-single-output second order active RC networks [6][8][27] which, when cascaded, form T(s).

This synthesis procedure is known as the transfer function simulation by cascading second order active RC building blocks. Several methods such as those presented in references [6] [7] [8] and [9], are concerned with the grouping of the poles and zeroes of T(s) so that the resulting cascade structure is optimized with respect to sensitivity, dynamic range and distortion. With the cascade structure as a starting network, several feedback paths may be added intending to reduce the sensitivity of the resulting structure. Such a modification, of course, will change the overall transfer function. The new transfer function may be re-adjusted and identified with the desirable T(s) and new element values are then determined so that the cascade structure with the additional feedback paths represents a realization of the initial T(s). Various active structures of this kind have appeared in the literature, of which the most successful hitherto have been the Follow-the-Leader structure [10] , [11] and the Shifted-Companion-Form structure [12].

A second approach to the synthesis problem is to use the analogue computer concept which involves solving the differential equation defined by the transfer function T(s) [8].

The use of Negative Impedance Converters (NIC s) for the synthesis of transfer impedances originated by Linvill [13] , was of great interest in the early days of active RC filter design (1950 - 1965) and many active RC filter synthesis methods using NICs have been proposed during that time. [14] , [15] , [16] , [17] . Considerable research has been done on the NIC active RC filters concerning their stability [18] [19] and sensitivity [20] [21] [22] . However, the NIC active RC filters based on transfer function simulation are no longer of potential practical significance due mainly to their large sensitivities with respect to element variations and also due to the excesive number of RC components needed for their realization [8].

In conclusion it must be said that active RC filters derivable through transfer function simulation, are not of minimum sensitivity and, as a rule, they are more sensitive than active RC structures obtainable through any ladder simulation method, as it will be seen in the following section [27] [57].

1.3 ACTIVE FILTER DESIGN - THE LADDER SIMULATION PRINCIPLE

A transfer function T(s) that meets the filter specifications, can be determined using the very familiar and systematic passive network synthesis techniques [1][2][3][41]. The familiarity of the majority of filter designers with those methods as well as the availability of LC filter design tables and catalogues[4][28] can be beneficially used for the design of active RC filters. Moreover, as it will be discussed later, active filters derived from passive LC networks by some form of simulation technique, present low sensitivity which is of vital importance in any filter structure.

The most general type of passive network is the lattice network which presents certain disadvantages over the most widely used passive filter circuit configuration, the ladder form. The most severe disadvantage of the lattice filter realization is that transmission zeroes are formed when a very sensitive bridge-balance condition is satisfied [23]. This leads to high sensitivity and poor performance in the stopband, a fact, among others, which makes ladder filters more desirable since in this case a transmission zero occurs only if either a series admittance or a shunt impedance becomes zero[23]. This transmission zero condition is easily satisfied and, therefore, the transmission zero sensitivities appear lower than in the lattice filter case.

Taking into account that the active RC simulation of passive filters is employed for it transfers desirable properties of the passive original to the active RC structure to copy its operation, the low sensitivity and satisfactory performance of the LC ladder in the stopband makes this kind of filter highly suitable for active RC simulation.

The sensitivity of a passive filter within the passband is extremely low since it appears to be proportional to the reflection coefficient ρ which assumes very low values within the passband [23]. Therefore, reactive ladders designed between two resistive terminations as to match them at certain frequencies (ρ=0, maximum power transfer points) will present zero sensitivity with respect to the reactive elements at these frequencies. This is known as the Orchard's Principle and states that the first order differential sensitivity of the transfer function of a passive LC doubly terminated network with respect to the reactive components assumes zero value at those frequencies for which the reactive ladder matches the source resistance to the load thereby allowing maximum power transfer to occur (ρ=0) [24]. A very elegant proof of this statement is given in references [6] and [23]. These sensitivity zeroes at the maximum power transfer points keep the sensitivity low within the entire passband.

Therefore, doubly terminated ladder LC filters with maximum power transfer points (referred to as well-designed doubly terminated LC ladder filters) can be used as a design vehicle for active RC filters for the following reasons :

(i) Their sensitivity performance within the passband is extremely low [23] [24] [25] [26] .

(ii) Their sensitivity in the stopband is lower than that of the corresponding lattice networks [23].

(iii) They are readily obtainable from design catalogues and tables [4] [28] [29].

The various active RC simulation methods are briefly described in the rest of this section.

1.3.1 Inductance Simulation Method

A doubly terminated LC ladder filter consists of resistors. capacitors and inductors (RLC network). It is the presence of the inductor, as explained earlier, that makes these filters very problematic at certain frequencies and very objectionable indeed at others. Therefore, if the inductive behaviour (i.e. a terminal relationship of the form $V = sL{\times}I$) can be performed by an active RC section, this simulated inductance can replace directly the inductors in the RLC ladder, turning it into an active RC ladder filter leaving the ladder topology unaltered. Several active RC simulated inductance circuits have been proposed [31][32] but the gyration concept is backed by a very sound theory.

A gyrator is an actively realizable two-port device that is capable of converting a capacitive load into a simulated inductance at its input terminals. Its schematic representation together with its Z-matrix is shown in Fig. 1.4a where port 2 is terminated in the perallel combination of a resistance R_1 and a capacitance C_1 producing the equivalent circuit as shown.

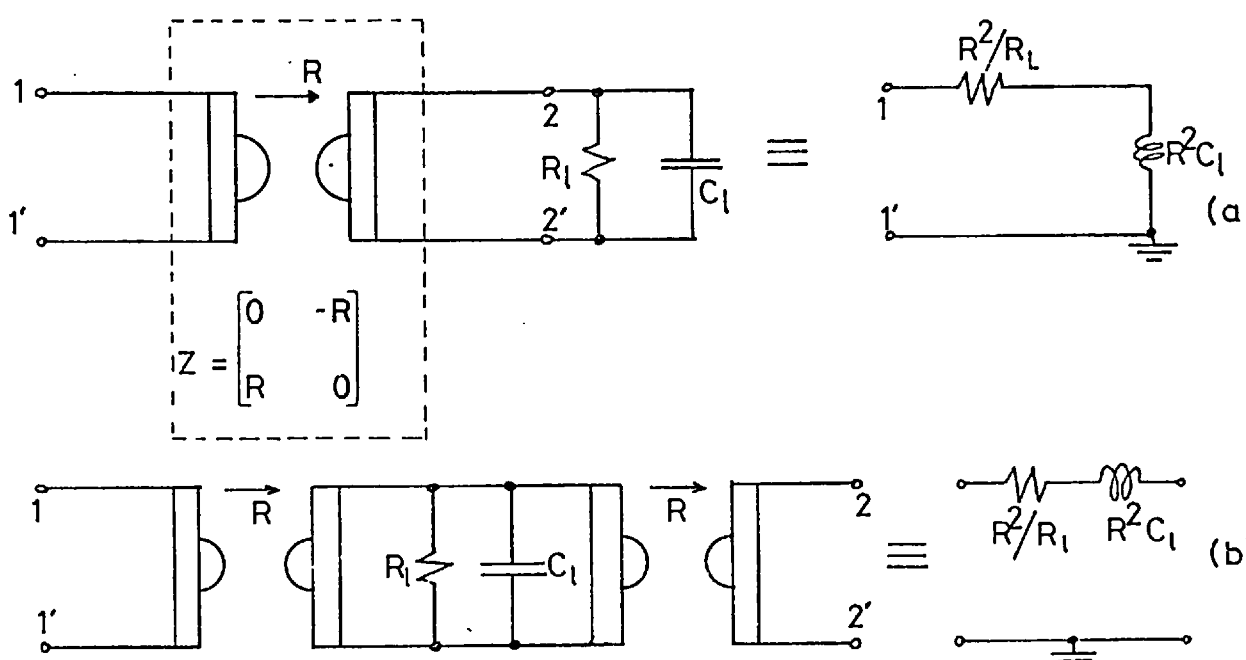

Fig. 1.4

It is observed that when $R_1 = \infty$, the input impedance of the terminated gyrator becomes purely inductive. The simulation of a floating inductor is more complicated requiring two gyrators in a back-to-back arrangement as shown in Fig. 1.4b. Several circuits for the active realization of the gyrator have been proposed in references [8][30][31].

1.3.2 The Impedance Scaling Method

On dividing all impedances of an LC doubly terminated ladder filter by a function of the complex frequency X(s), the transfer function $T(s) = \frac{Vo(s)}{E(s)}$, in accordance with Fig. 1.5 , remains unchanged.

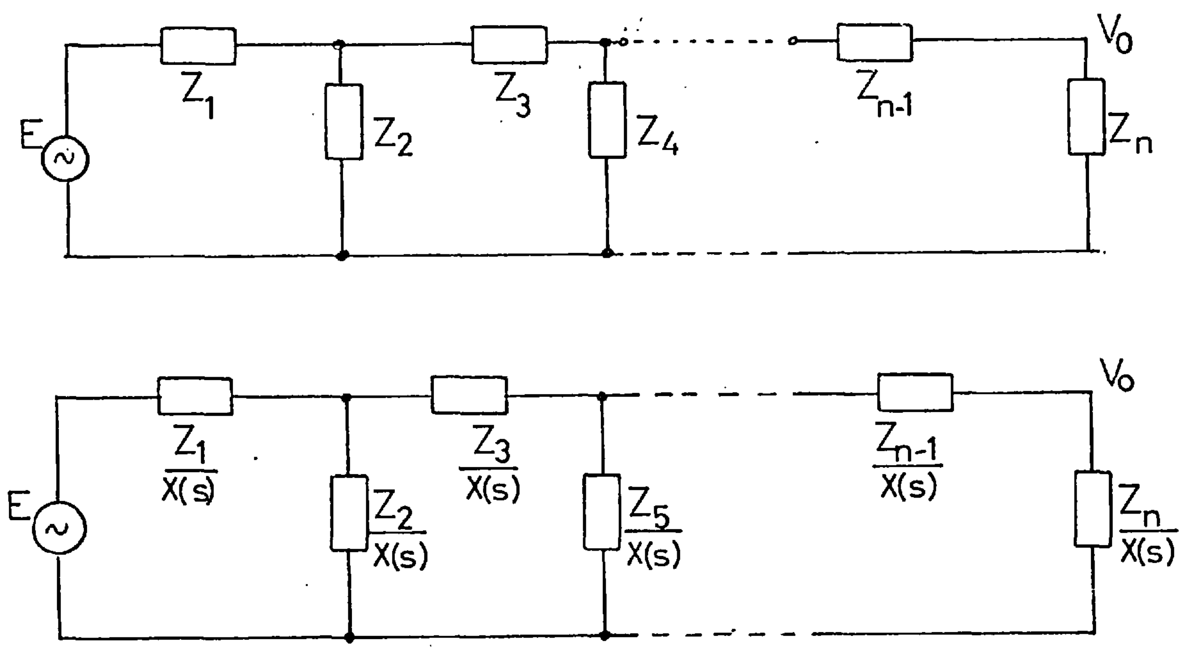

Fig. 1.5

This almost self evident statement , which revolutionized active filter design, was first presented , proved and applied to the simulation of LC ladder filters by L. Bruton [33], for X(s) = ks and has been generalized in reference [34]. Fig. 1.6 gives the scaled ladder impedances for X(s) =ks. The element with impedance of the form $1/s^2$ is referred to as supercapacitor or, rather missleadingly, as Frequency Dependent Negative Resistance (FDNR).

The realization of the earthed supercapacitor is shown in Fig. 1.6b for which we have:

$$Z_{in} = \frac{Z_1 Z_3 Z_5}{R_2 R_4}$$

Therefore, by making any two of the elements Z_1, Z_3, Z_5 capacitive and

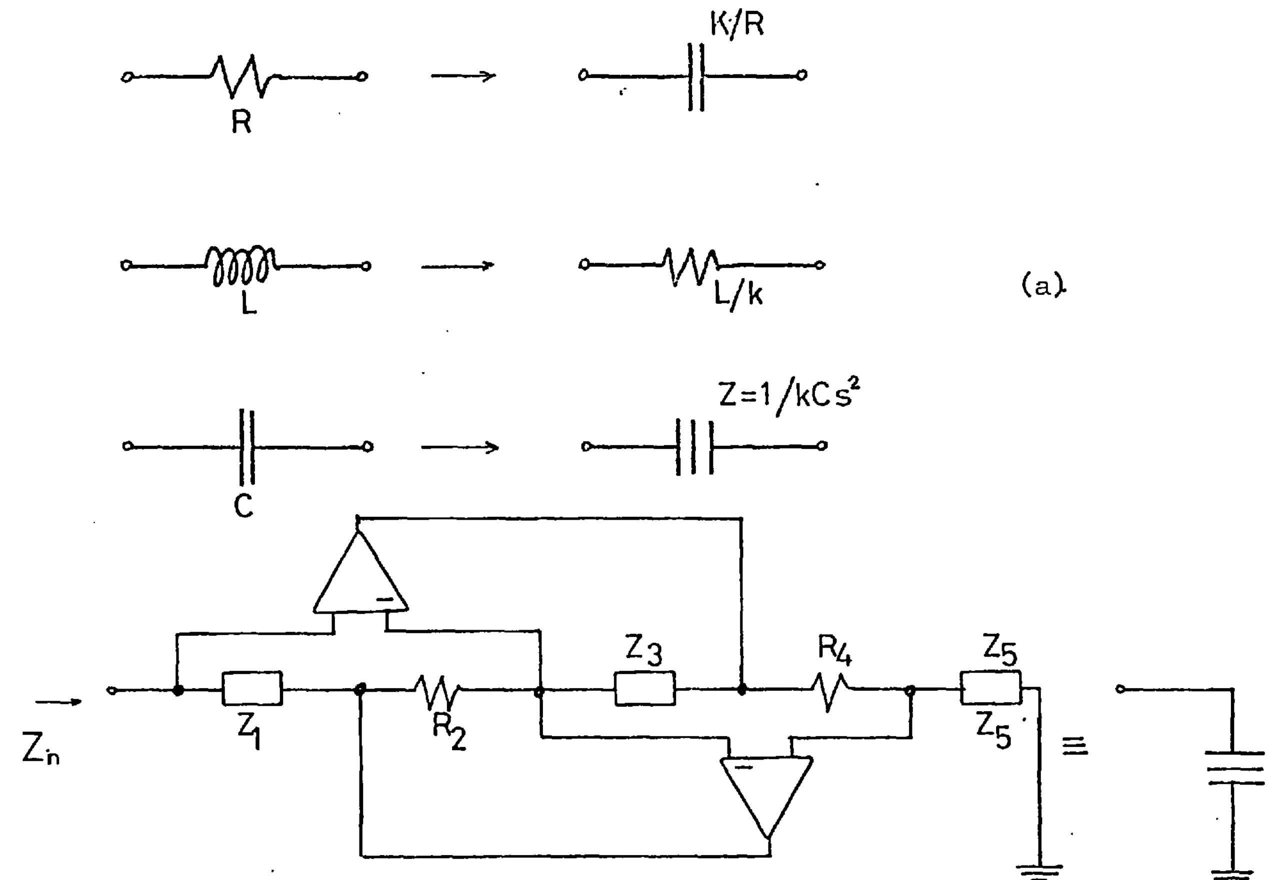

Fig. 1.6

the rest resistive, a supercapacitive behaviour is obtained. For minimum sensitivity, it has been proved that the choice $Z_1=Z_3=1/sC_D$, $Z_5=R_2=R_4$ is appropriate [35]. The floating supercapacitor that emerges from a floating inductor, requires two circuits similar to that of Fig. 1.6b connected in a back-to-back configuration [8] [33]. It must be noted that the circuit of Fig. 1.6b is a General Impedance Converter (GIC) and can be used for the realization of a simulated inductance as well as for the realization of other complex impedances by a proper choice of its impedances.

Some modifications of the impedance scaling method have been proposed in references [36] [37] [38] and [39] to enable practical simulation of narrowband bandpass filters at high frequencies, as well as to minimize the number of capacitors and supercapacitors involved in the active RC structures.

1.4 LINEAR TRANSFORMATION ACTIVE (LTA) FILTERS - A NEW APPROACH TO THE ACTIVE RC SIMULATION OF PASSIVE LADDER FILTERS [52][53].

In this thesis a new approach is presented for the design of active RC filters from doubly terminated LC ladder networks. The new approach is based on linearly transforming the port-variable vectors of a two-port network from the V-I domain to a new x-y domain, in which the two-port is then realized actively. By examining the ladder filter as a cascade connection of two-port subnetworks we obtain the Linearly Transformed active structure for each of these subnetworks and then interconnect them in an appropriate way to derive the Linear Transformation (LTA) filter simulating the original doubly terminated LC filter. The element-by-element linear transformation of the ladder filter ensures the low sensitivity of the derived structures due to the correspondence that exists between a ladder arm and the Linearly Transformed equivalent version of it. The new approach is systematically presented so as to yield a powerful active RC filter design technique which with the theoretical background provided can be very useful to active filter designers. The structure of this thesis is as follows :

In chapter 2 the fundamentals of the Linear Transformation theory as applied to the simulation of two port networks are presented.The necessary and sufficient conditions for the existence of the Linearly Transformed structures that correspond to a given two-port network are given explicitly. Moreover,analytical formulae for the Active realization of the corresponding LT structures for the constituent two-port subnetworks of a ladder filter (i.e. series and shunt arms) are given.

The interconnection problem of the so derived LT active RC structures is examined in chapter 3 and the necessary and sufficient conditions for the existence of such interconnection are given in terms of the elements of the adjacent transformation matrices. In addition, the input and the output termination constraints are studied in this chapter and some simplifications are made concerning the number of active elements required for the realization of the LT terminations.

Some rules for the efficient use of the LT concept in the simulation of passive ladder filters are given in chapter 4 concerning the complexity of th LT structures and the existence of a convenient output node which represents the output of the original ladder filter. Moreover, the minimum complexity interconnection is defined and it is proved that it can be used without any loss of generality. The above principles are practically applied in two design examples.

In chapter 5 the linear transformation procedures that can be derived from the general case are classified according to the relationshops imposed between the individual transformation pairs which are used for the LT simulation of a passive ladder filter. Moreover, some well-known ladder simulation techniques like the Wave Active Filters and the Leapfrog Synthesis, are interpreted as special cases of the general LTA approach.

The Self-Dual LTA filters presented in chapter 6 possess the self-duality property, as defined there, and the necessary and sufficient conditions for a transformation to be Self-Dual are established. In addition, by imposing a linear relationship between the transformed variables of the self-dual LT structures and by requiring them to possess the Rotation or R-Property, as defined in chapter 6, we obtain highly modular, simple and very insensitive LTA filters.

The LTA ladder simulation approach is put into practice in chapter 7, where several ladder filters of various orders and band selectivities are simulated using either the general acyclic LTA procedure or some special LTA cases to derive Active RC filters.

These filters are then constructed and their complexity, sensitivity and high frequency performance are examined and compared to those of the original passive ladder and to some conventional active RC structures so as to show the flexibility of the LTA approach and the importance of the practical results.

The final conclusions concerning the practicality, efficiency, flexibility and usefulness of the new approach, are drawn and discussed in chapter 8 where, in addition, some directions for further research along the line of the LTA simulation concept are given.

CHAPTER 2

THE LINEARLY TRANSFORMED TWO-PORT

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INTRODUCTION

The principles of Linear Transformations, as applied to passive two-ports, are defined and examined in this Chapter. The chain matrix description of a two-port network is used exclusively but in the slightly modified form:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = T_n \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad (2.1)$$

where the voltage and current variables are in accordance with Fig. 2.1

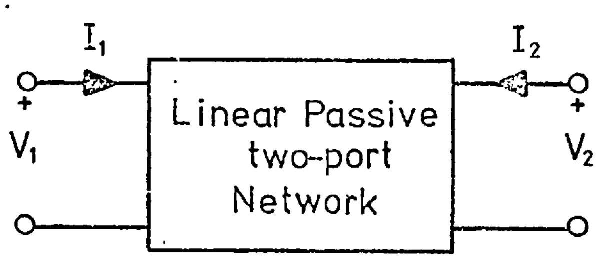


Fig. 2.1

The port vectors $\Lambda_1 = [V_1 \quad I_1]^T$ and $\Lambda_2 = [V_2 \quad I_2]^T$, are referred to as the left and the right port variable vectors respectively whilst T_n will be referred to as the modified chain or transmission matrix of the passive two-port N_n. The incorporation of the negative sign of $-I_2$ in B and D without changing the direction of the flow of I_2 allows uniform definitions of the transformed variable vectors without complicating the mathematical manipulations involved in this work.

The chain matrix description of a two-port network is a non-oriented terminal relationship [40] which describes the nonoriented two-port network N, in the sense that it does not provide any information as to which variables are the inputs (causes) and which are the outputs (effects).

On attempting to determine the input and output variables, the interdependence of V_1 and I_1 (V_2 and I_2) must be taken into account to ensure that they do not belong jointly to the input set or indeed the output set. Therefore, for an oriented description of a two-port network, the set of port variables

$$X=\{V_1, I_1, V_2, I_2\}$$

must be so divided into two sets that one set may then serve as the inputs (i.e. causes) whilst the other set may be taken as the outputs (i.e. effects). The interdependence of V_1 and I_1 (V_2 and I_2) implies that only two divisions of the set of port variables of a two-port network N_n into input and output sets are possible which are:

$$\text{(i)} \quad \{V_1, V_2 \mid I_1, I_2\}$$

$$\text{and} \qquad \text{(ii)} \quad \{V_1, I_2 \mid V_2, I_1\}$$

In division (i) $\{V_1, V_2\}$ may be taken as the inputs and consequently $\{I_1, I_2\}$ are the outputs or vice versa. Similar implications exist for division (ii) above.

2.1 ANALYTICAL CONSIDERATIONS

The left and the right port variable vectors that appear in the modified chain matrix description of a passive two-port network N_n, $[V_1 \quad I_1]^T$ and $[V_2 \quad I_2]^T$, can be linearly transformed as follows

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \mathfrak{a}_1 \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \mathfrak{a}_2 \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \tag{2.2}$$

where the left transformation matrix $\mathfrak{a}_1$ is defined as

$$\mathfrak{a}_1 = \begin{bmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix} \quad \text{with } \det \mathfrak{a}_1 \neq 0$$

whilst the right transformation matrix is

$$\mathfrak{a}_2 = \begin{bmatrix} \alpha_2 & \beta_2 \\ \gamma_2 & \delta_2 \end{bmatrix} \quad \text{with } \det \mathfrak{a}_2 \neq 0$$

The new port-vectors $[x_1 \ \ y_1]^T$ and $[x_2 \ \ y_2]^T$ are referred to as the left and the right Linearly Transformed port-vectors respectively. On substituting the port variable vectors in eqn. (2.1) by the new vectors, as defined by eqn. (2.2), we obtain the result:

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \mathfrak{a}_1 T_n \mathfrak{a}_2^{-1} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \tag{2.3}$$

Equation (2.3) constitutes a description of a nonoriented network N' , the terminal variables of which are x_1, x_2, y_1 and y_2. This network is referred to as the corresponding Linear Transform Network of the passive two-port N. This network can be represented schematically as in Fig. 2.2 below

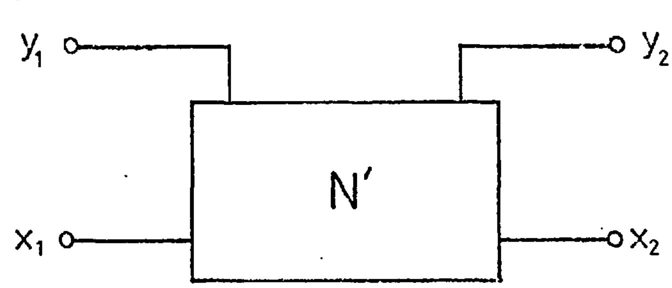


Fig. 2.2

The matrix description of the Linear-Transform nonoriented network provides the definition of the Linear - Transform chain matrix:

$$T' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \mathcal{Q}_1 T_n \mathcal{Q}_2^{-1}$$

as it can be seen from eqn. (2.3). This matrix relates the Linearly Transformed port-variable vectors $[x_i \quad y_i]^T$, i= 1, 2 in a manner similar to that achieved through the chain matrix of the normal V - I description of the passive two-port N_n. The elements of the Linear Transform chain matrix are determined from eqn. (2.3) to be as shown below:

$$\begin{aligned} a &= \frac{1}{\Delta_2}\left[\alpha_1(\delta_2 A - \gamma_2 B) + \beta_1(\delta_2 C - \gamma_2 D)\right] \\ b &= \frac{1}{\Delta_2}\left[\alpha_1(\alpha_2 B - \beta_2 A) + \beta_1(\alpha_2 D - \beta_2 C)\right] \\ c &= \frac{1}{\Delta_2}\left[\gamma_1(\delta_2 A - \gamma_2 B) + \delta_1(\delta_2 C - \gamma_2 D)\right] \\ d &= \frac{1}{\Delta_2}\left[\gamma_1(\alpha_2 B - \beta_2 A) + \delta_1(\alpha_2 D - \beta_2 C)\right] \end{aligned} \tag{2.4}$$

$$\Delta_2 = \alpha_2\delta_2 - \beta_2\gamma_2$$

It can be found that the determinant of T' is related to the determinants of $\mathcal{Q}_1$ and $\mathcal{Q}_2$ through the following equation

$$\det T' = ad - bc = -\frac{\det \mathcal{Q}_1}{\det \mathcal{Q}_2} = -\frac{\Delta_1}{\Delta_2} \neq 0$$

The dimensions of the elements of the Linear-Transform chain matrix depend on the dimensions of the elements of the transformation matrices $\mathcal{Q}_1$ and $\mathcal{Q}_2$ as it is apparent from eqn. (2.4). The transformation matrices evidently determine the dimensions of the transformed variables x and y. Indeed, the transformed variables can represent any electrical quantity depending on the dimensions of the elements of the transformation matrices . To illustrate this point it may be assumed that α and γ, for example, represent conductances whereas β and δ may be taken to be dimensionless quantities. In such a case x and y have the dimensions of electrical currents.

However, in practice the nature, and hence the dimensions of, the variables x and y is influenced by the active components to be used in the final implementation of the Linear Transform network. If operational amplifiers are employed it is reasonable to require the transformed variables to represent voltages. This implies that the a,b,c and d parameters of eqn. (2.3) and (2.4) represent voltage transfer ratios and hence they are dimensionless. This can be achieved by taking α and γ in the transformation matrices to be dimensionless (i.e. pure numbers or voltage transfer ratios) and β, δ to have dimensions of impedances.

The active realization of the LT netwotk requires an oriented description of the network of Fig. 2.2. It should be noted that the interdependence of V_1 and I_1 (V_2 and I_2) in the V-I domain is transferred to x_1 and y_1 (x_2 and y_2) through the linear transformation, implying that x_1 and y_1 (x_2 and y_2) can not be taken simultaneously as inputs or outputs of the LT network. Therefore, only two divisions of the set of the transformed variables into input and output sets are possible as previously, and they are

$$\text{(i)} \quad \{x_1, x_2, \mid y_1, y_2\}$$

$$\text{and} \qquad \text{(ii)} \quad \{x_1, y_2, \mid y_1, x_2\}$$

These two divisions, however, theoretically are not independent. It is apparent that by considering the first division and then interchanging the rows of $\mathcal{Q}_2$ we actually obtain the second division, since an interchange of x_2 and y_2 is effected thereby. Henceforth, only the first division will be considered without any loss of generality. Moreover in the first division x_1 and x_2 are considered to be the inputs and y_1 and y_2 the outputs and this is by no means restrictive since it can be observed that the case in which x_1 and x_2 are the outputs of the LT network for the excitations y_1 and y_2 is included by merely interchanging the rows of both $\mathcal{Q}_1$ and $\mathcal{Q}_2$. Therefore, the first division with x_1 and x_2 as inputs can be considered to be a general case.

The description of the oriented LT network, according to the previous discussion, can now be derived from eqn. (2.3) in the following form

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} K & L \\ M & N \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (2.5)$$

where the four transfer ratios K, L, M and N are given below in terms of the elements of the LT chain matrix .

$$\begin{aligned} K &= \frac{d}{b} \\ L &= -\frac{ad - bc}{b} \\ M &= \frac{1}{b} \\ N &= -\frac{a}{b} \end{aligned} \qquad (2.5a)$$

The mathematical procedure followed so far ensures the existence of such an LT network provided that $b \neq 0$. The representation of the oriented LT network described by eqn. (2.5) is shown in Fig. 2.3.a. In this figure the inputs x_1 and x_2 are denoted by a small circle, whilst the outputs y_1 and y_2 are denoted by a small triangle appropriately oriented. For the active realization , this representation is shown in Fig. 2.3b, where the signal flow graph of eqn. (2.5) is drawn.

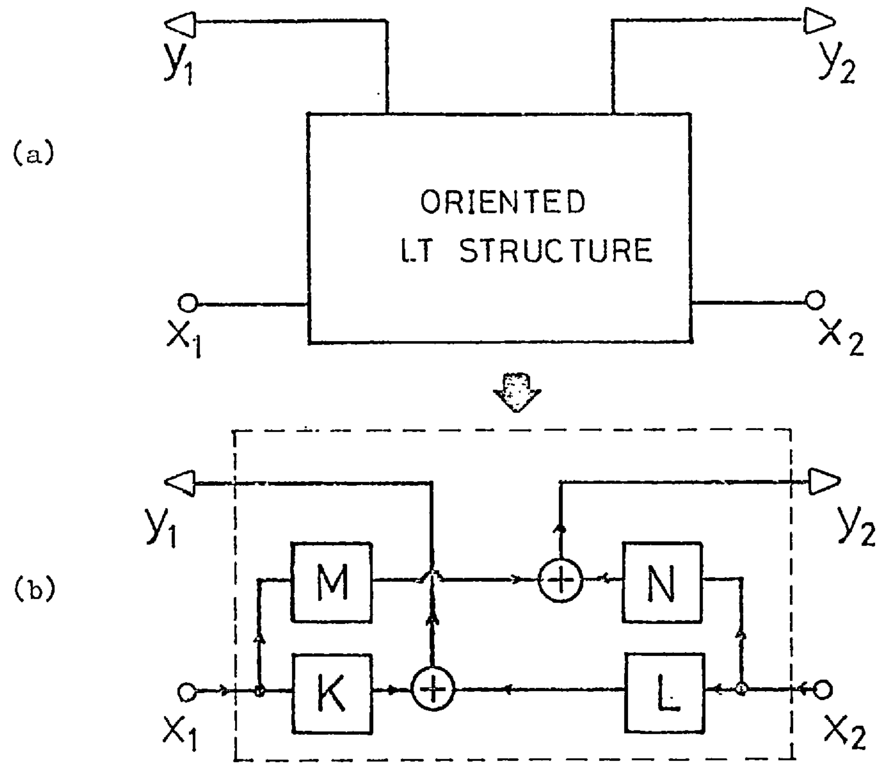


Fig. 2.3

The transfer ratios K,L,M and N must be practically realizable under the technology in mind (e.g. active RC), while their stability may or may not be necessary. This fact is not surprising if one were to re-examine the Impedance Scaling procedure of Bruton [33] [34] [37] where supercapacitors (i.e. one port elements having impedance of the form $1/ks^2$) are by themselves unstable but within an appropriate ladder connection are stable [42].

2.2 LADDER ELEMENT CONSIDERATIONS

Passive LC filters are composed of simple two-port networks which may be connected in series, in parallel or in cascade. These simple two-ports are in ladder LC filters of two kinds

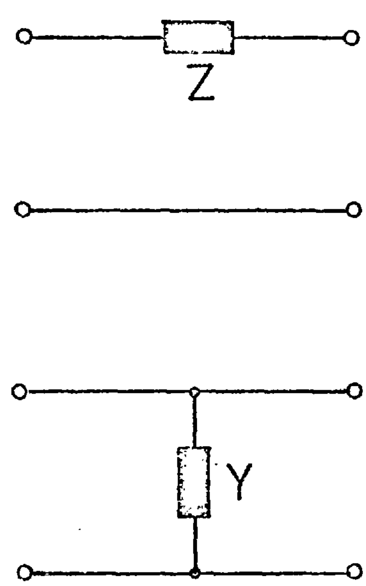
(i) Series arms (Fig. 2.4a)

and (ii) Shunt arms (Fig. 2.4b)

and are shown in Fig. 2.4 where also their modified chain matrices are included. It should be observed that for these elements $T_n = T_n^{-1}$.

(a)

(b)



$$T_Z = \begin{bmatrix} 1 & -Z \\ 0 & -1 \end{bmatrix} = T_Z^{-1}$$

$$T_Y = \begin{bmatrix} 1 & 0 \\ Y & -1 \end{bmatrix} = T_Y^{-1}$$

Fig. 2.4

This fact and the form of the chain matrix are very important contributing factors towards the simplification of the transform equations derived so far for the individual series and the shunt cases. Eqn. (2.3) which constitutes the nonoriented description of the LT network, gives the following relationships for the two cases taken individually

For the series arm Z

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \frac{1}{\det \mathcal{A}_2} \begin{bmatrix} \alpha_1(\delta_2 + \gamma_2 Z) + \beta_1\gamma_2 & -\alpha_1(\alpha_2 Z + \beta_2) - \beta_1\alpha_2 \\ \gamma_1(\delta_2 + \gamma_2 Z) + \delta_1\gamma_2 & -\gamma_1(\alpha_2 Z + \beta_2) - \delta_1\alpha_2 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \quad (2.6)$$

For the shunt arm Y

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \frac{1}{\det \mathcal{A}_2} \begin{bmatrix} \beta_1(\delta_2 Y + \gamma_2) + \alpha_1\delta_2 & -\beta_1(\beta_2 Y + \alpha_2) - \alpha_1\beta_2 \\ \delta_1(\delta_2 Y + \gamma_2) + \gamma_1\delta_2 & -\delta_1(\beta_2 Y + \alpha_2) - \gamma_1\beta_2 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \quad (2.7)$$

It can be observed that these equations can be simplified further by setting one or more of the elements of the transformation matrices equal to zero. This is discussed in the next chapter which deals not only with the simplification of the equations but also with the corresponding simplifications of the active realizations of the Linear Transform structures.

The transfer ratios of eqn. (2.5) can also be simplified by taking the series and the shunt arm cases separately as follows

<u>For the series arm Z</u>

$$\left.\begin{aligned} K &= \frac{\gamma_1(\alpha_2 Z + \beta_2) + \delta_1\alpha_2}{\alpha_1(\alpha_2 Z + \beta_2) + \beta_1\alpha_2} \\ L &= -\frac{\Delta_1}{\alpha_1(\alpha_2 Z + \beta_2) + \beta_1\alpha_2} \\ M &= -\frac{\Delta_2}{\alpha_1(\alpha_2 Z + \beta_2) + \beta_1\alpha_2} \\ N &= \frac{\alpha_1(\gamma_2 Z + \delta_2) + \beta_1\gamma_2}{\alpha_1(\alpha_2 Z + \beta_2) + \beta_1\alpha_2} \end{aligned}\right\} \quad (2.8)$$

For the shunt arm Y

$$K = \frac{\delta_1(\beta_2 Y + \alpha_2) + \gamma_1\beta_2}{\beta_1(\beta_2 Y + \alpha_2) + \alpha_1\beta_2}$$

$$L = -\frac{\Delta_1}{\beta_1(\beta_2 Y + \alpha_2) + \alpha_1\beta_2} \tag{2.9}$$

$$M = -\frac{\Delta_2}{\beta_1(\beta_2 Y + \alpha_2) + \alpha_1\beta_2}$$

$$N = \frac{\beta_1(\delta_2 Y + \gamma_2) + \alpha_1\delta_2}{\beta_1(\beta_2 Y + \alpha_2) + \alpha_1\beta_2}$$

In both cases $\Delta_i = \alpha_i\delta_i - \beta_i\gamma_i = \det \mathfrak{a}_i$.

K,L,M and N are in fact the transfer ratios to be actively realized within the LT structure to generate the outputs y_1 and y_2 from the inputs x_1 and x_2. Therefore ,as already pointed out, they must be realizable. This requirement sets some constraint on the choice of the transformation matrices as it will be discussed elsewhere in this work.

2.3 AN ILLUSTRATIVE EXAMPLE

An illustration of the ideas of this chapter is presented here using as an example the Linear Transformation of an inductance in the series arm. The nonoriented description of the corresponding LT network is given in eqn. (2.6). This equation for $Z= sL$ and $\mathfrak{a}_1$ and $\mathfrak{a}_2$ in the form

$$\mathfrak{a}_1 = \begin{bmatrix} \alpha_1 & \beta_1 \\ 0 & \delta_1 \end{bmatrix} , \quad \mathfrak{a}_2 = \begin{bmatrix} \alpha_2 & \beta_2 \\ 0 & \delta_2 \end{bmatrix}$$

yields the following expressions:

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \frac{1}{\alpha_2\gamma_2} \begin{bmatrix} \alpha_1\delta_2 & -\alpha_1(\alpha_2 Ls + \beta_2) - \beta_1\alpha_2 \\ 0 & -\delta_1\alpha_2 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

Considering x_1 and x_2 to be the inputs and y_1 and y_2 the outputs of the system, as it has been explained in the previous section, the oriented description of the LT structure for the series inductor under the linear transformation $\{\mathfrak{a}_1 , \mathfrak{a}_2\}$ as defined above, will be given by eqn. (2.5) as folows

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} K & L \\ M & N \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where, according to eqn. (2.8) we have

$$K(s) = \frac{\delta_1\alpha_2}{\alpha_1\beta_2 + \beta_1\alpha_2 + \alpha_1\alpha_2 Ls}$$

$$L(s) = -\frac{\alpha_1\delta_1}{\alpha_1\beta_2 + \beta_1\alpha_2 + \alpha_1\alpha_2 Ls}$$

$$M(s) = -\frac{\alpha_2\delta_2}{\alpha_1\beta_2 + \beta_1\alpha_2 + \alpha_1\alpha_2 Ls}$$

$$N(s) = \frac{\alpha_1\delta_2}{\alpha_1\beta_2 + \beta_1\alpha_2 + \alpha_1\alpha_2 Ls}$$

The parameters α_i, β_i and δ_i , $i = 1, 2$ may be, at this juncture, chosen freely.

Let $\alpha_1 = \alpha_2 = -1$, $\beta_1 = \beta_2 = R$ and $\delta_1 = \delta_2 = R$, so that

$$K(s) = -\frac{R}{sL - 2R}$$

$$L(s) = \frac{R}{sL - 2R}$$

$$R > 0$$

$$M(s) = \frac{R}{sL - 2R}$$

$$N(s) = -\frac{R}{sL - 2R}$$

The variables x_i and y_i represent voltages where the parameter R above has the dimensions of resistance. It can be seen that the four voltage ratios K,L,M, and N are unstable having a possitive real pole $s_p = 2R/L$. This is not important, however, as has been pointed out earlier. The problem in this case is that such voltage transfer ratios require complex structures for their realization with operational amplifiers . This can be avoided by choosing

$$\beta_1 = \beta_2 = -R \quad , \quad R > 0$$

in which case the voltage transfer ratios become stable and easily realizable. Moreover, it can be observed from the above expressions for the four voltage ratios, that for the new choice:

$$K(s) = -M(s) = -\frac{R}{sL + 2R}$$

and $L(s) = -N(s) = \frac{R}{sL + 2R}$

which yields $y_1 = -y_2 = K(s)x_1 + L(s)x_2$.

The ratio K(s) can be realized as shown in Fig. 2.5a whilst the ratio L(s) is realized as shown in Fig. 2.5b. For the realization of the output y_1 a circuit must be constructed to realize $y_1 = K(s)x_1 + L(s)x_2$ and an inverter must generate y_2 as $-y_1$ as it is shown in Fig. 2.5c. The final optimized version of this circuit is shown in Fig. 2.5d.

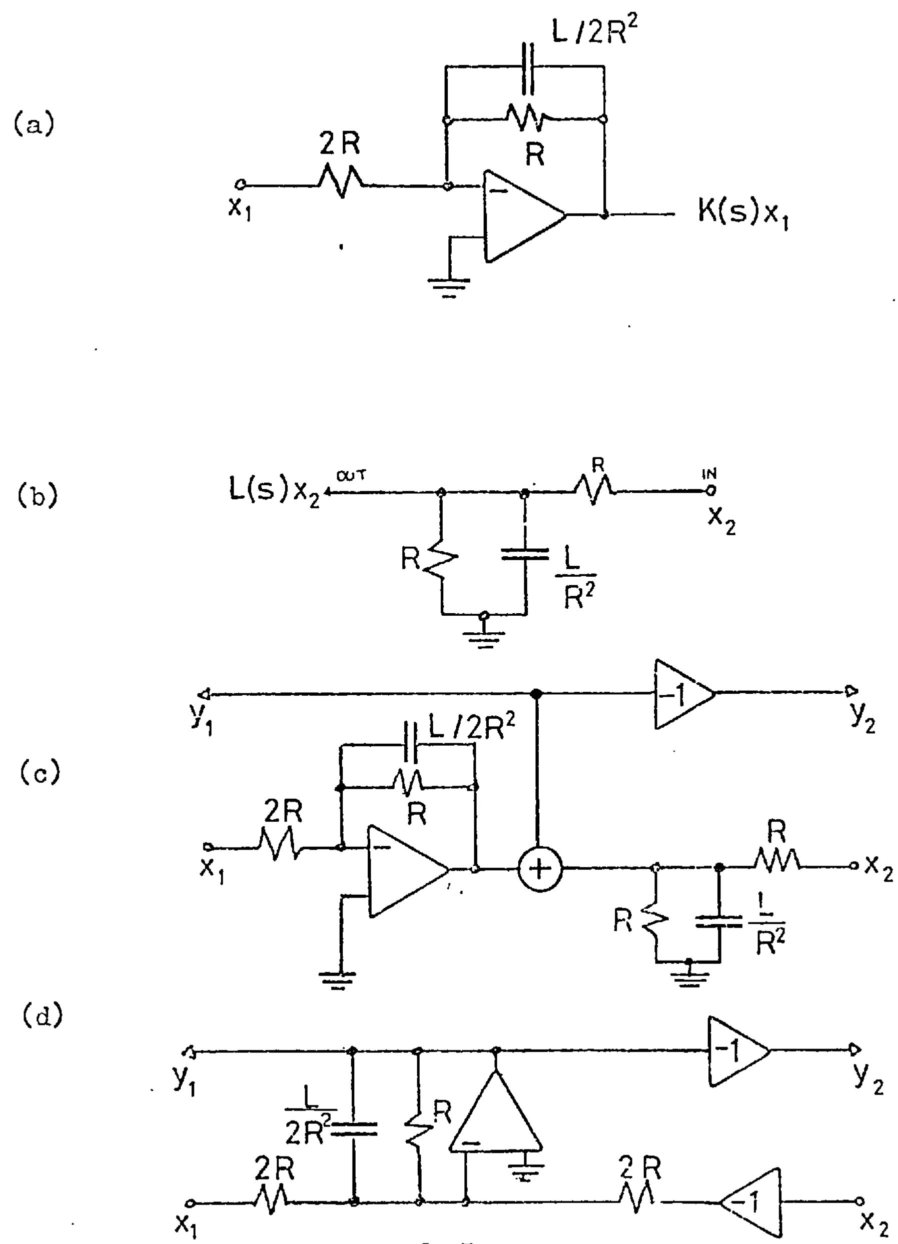


Fig. 2.5

An alternative and better choice of the elements of the transformation matrices, however, can lead to a different and simpler LT structure for the series inductor under consideration. For example when

$$\mathfrak{a}_1 = \begin{bmatrix} 1 & R \\ 0 & -R \end{bmatrix} \quad \text{and} \quad \mathfrak{a}_2 = \begin{bmatrix} -1 & 0 \\ 0 & R \end{bmatrix}$$

we obtain the following voltage transfer ratios

$$K(s) = L(s) = M(s) = N(s) = -\frac{1}{1 + sL/R}$$

The LT structure of the series inductor under the above linear transformation can be actively realized using only one operational amplifier as shown in Fig. 2.6 below.

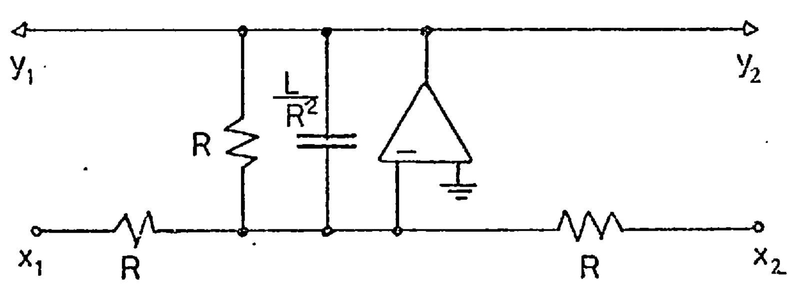


Fig. 2.6

Therefore, a successful choice of the transformation matrices cannot only lead to stable and easily realizable voltage transfer ratios (Fig. 2.5), but also to minimum operational amplifier realizations like the one of Fig. 2.6. The general rules for such a successful choice are given in chapter 4.

It must be observed, however, that although the LT structures obtained in this section do correspond to the series inductance in the x-y domain, they must not be taken as actual replacements of this element in the original V-I domain. This will become more apparent if we consider in the above R = sL. In such a case the voltage ratios become

$$K(s) = L(s) = M(s) = N(s) = -\frac{1}{2}$$

and the corresponding LT structure is as shown in Fig. 2.7 where no capacitors are involved (zero order network)

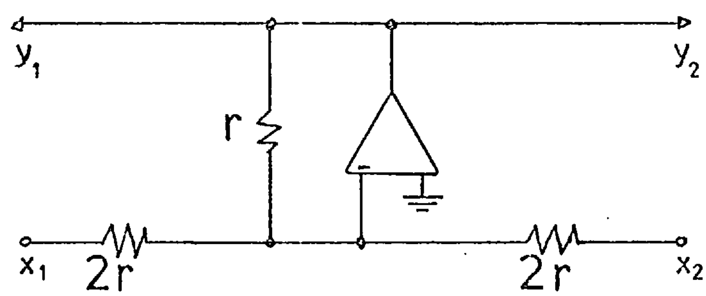


Fig. 2.7

This reduction of the order of the LT structure demonstrates the fact that the LT structures are not V-I simulations of the arm from which they were derived through a linear transformation of the original port variable vectors and they must be taken in the entire context of the LC filter transformation. (In fact the LT structure of Fig. 2.7 will be part of an LTA filter where its adjacent LT structures compensate for the reduction of the order in this section.)

CHAPTER 3

THE LINEARLY TRANSFORMED LADDER NETWORK

CHAPTER 3

THE LINEARLY TRANSFORMED LADDER NETWORK

INTRODUCTION

In this chapter we examine the manner in which ladder filters can be transformed. The ladder is examined from its constituent parts, that is, as being composed of series impedances and shunt admittances in addition to the resistive terminations at the input and output. These constituent parts are individually transformed and examined below.

The general form of a doubly terminated lossless ladder filter is shown in Fig. 3.1 where the two-port networks N_i, $i = 1, n$ are simple or composite series or shunt reactive arms connected in cascade to form the ladder filter.

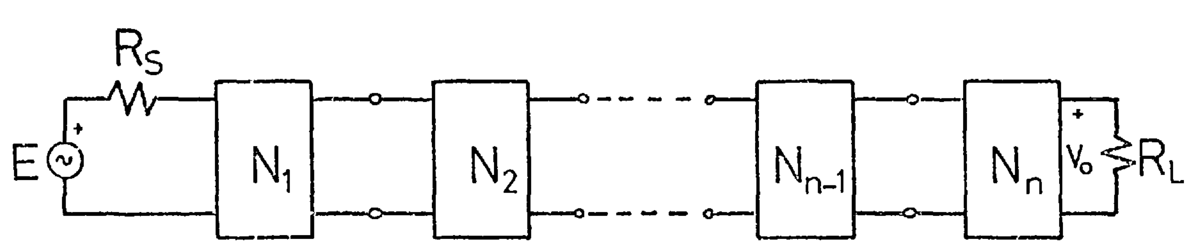


Fig. 3.1

In the previous chapter we dealt with transformations of such two-port networks and in this chapter the important question of their interconnection in the transformed domain is examined as in addition the transformation of the passive terminations (i.e. R_S in series with the voltage source E, and R_L) are considered so as to enable the complete formation of the Linear Transform Active (LTA) ladder filter.

3.1 INTERCONNECTION CONSTRAINTS

The interconnection (i.e. cascading) of two ladder elements (e.g. a series arm connected to a shunt arm) involves certain constraints at the interconnecting ports. The cascade connection of two passive two-ports in the V-I domain shown in Fig. 3.2 is described by eqn. (3.1) below

$$\begin{bmatrix} V_{2i} \\ I_{2i} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} V_{1j} \\ I_{1j} \end{bmatrix} \tag{3.1}$$

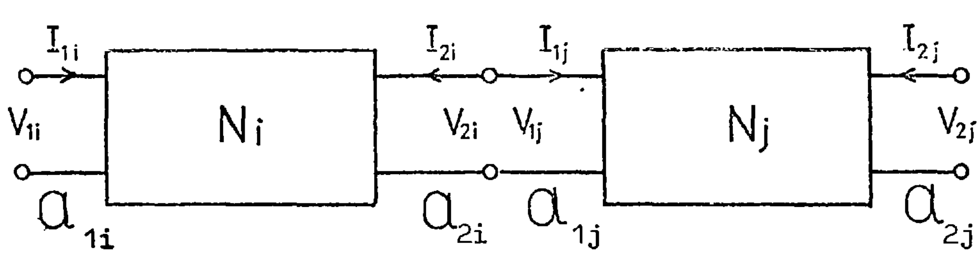


Fig. 3.2

The linear transformation $\{\alpha_{1i}, \alpha_{2i}\}$ is assumed to be applied to N_i whilst the transformation set $\{\alpha_{1j}, \alpha_{2j}\}$ is applied to N_j. The question of how the two LT structures so obtained must be interconnected in order that their connection represents the cascade connection of N_i and N_j is now explained. It is observed that eqn. (3.1) in conjunction with

$$\begin{bmatrix} V_{2i} \\ I_{2i} \end{bmatrix} = \alpha_{2i}^{-1} \begin{bmatrix} x_{2i} \\ y_{2i} \end{bmatrix} \tag{3.2}$$

and

$$\begin{bmatrix} V_{1j} \\ I_{1j} \end{bmatrix} = \alpha_{1j}^{-1} \begin{bmatrix} x_{1j} \\ y_{1j} \end{bmatrix}$$

from the definition of the transformed port-variable vectors, yields

$$\begin{bmatrix} x_{1j} \\ y_{1j} \end{bmatrix} = \alpha_{1j} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \alpha_{2i}^{-1} \begin{bmatrix} x_{2i} \\ y_{2i} \end{bmatrix} \qquad (3.3)$$

Equation (3.3), indicates the manner in which the two LT structures N_i' and N_j' are to be interconnected so that eqn. (3.1) is satisfied under the transformation matrices α_{2i} and α_{1j}. The matrix

$$I_{ij} = \alpha_{1j} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \alpha_{2i}^{-1} \qquad (3.3a)$$

of eqn. (3.3) is called the interconnection matrix which may be looked upon as describing the " interconnecting network " between the LT structures N_i' and N_j' as it is shown in Fig. 3.3 below.

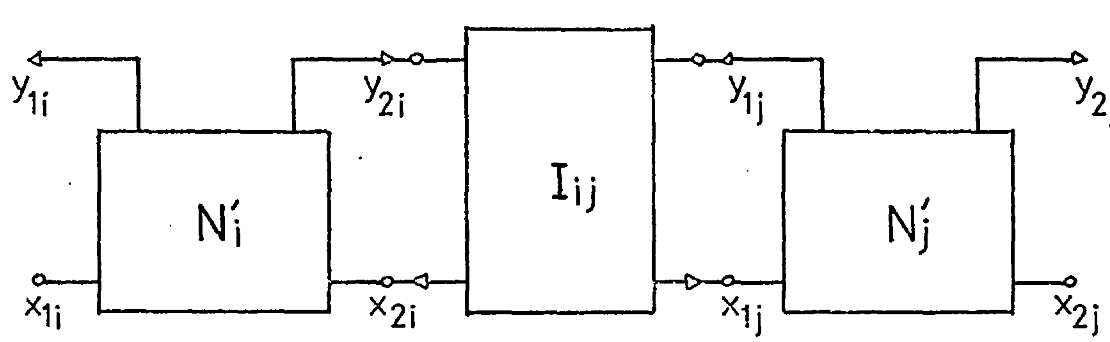


Fig. 3.3

The interconnection matrix I_{ij} can be written explicitlly in terms of the elements of the transformation matrices α_{2i} and α_{1j} as follows:

$$I_{ij}=\begin{bmatrix}\varepsilon & \zeta \\ \eta & \vartheta\end{bmatrix}=\begin{bmatrix}\dfrac{\alpha_{1j}\delta_{2i}+\beta_{1j}\gamma_{2i}}{\det \mathcal{Q}_{2i}} & \dfrac{-(\alpha_{1j}\beta_{2i}+\alpha_{2i}\beta_{1j})}{\det \mathcal{Q}_{2i}} \\ \dfrac{\gamma_{1j}\delta_{2i}+\delta_{1j}\gamma_{2i}}{\det \mathcal{Q}_{2i}} & \dfrac{-(\gamma_{1j}\beta_{2i}+\delta_{1j}\alpha_{2i})}{\det \mathcal{Q}_{2i}}\end{bmatrix} \qquad (3.3b)$$

Under the above notation eqn. (3.3) can be re-expressed as

$$\begin{bmatrix}x_{1j} \\ y_{1j}\end{bmatrix}=\begin{bmatrix}\varepsilon & \zeta \\ \eta & \vartheta\end{bmatrix}\begin{bmatrix}x_{2i} \\ y_{2i}\end{bmatrix} \qquad (3.3c)$$

The role of the interconnecting network is seen from Fig. 3.3 to be that of generating the inputs x_{1j} and x_{2i} from the outputs y_{1j} and y_{2i}. Consequently the system of equations of (3.3c) must be solvable with respect to the unknowns x_{1j} and x_{2i}. The necessary and sufficient condition for this is

$$\eta \neq 0 \iff \gamma_{1j}\delta_{2i}+\delta_{1j}\gamma_{2i}\neq 0 \qquad (3.4)$$

in which case eqn. (3.3c) yields

$$\begin{aligned} x_{1j} &= \frac{\varepsilon}{\eta}y_{1j} - \frac{\varepsilon\vartheta-\eta\zeta}{\eta}y_{2i} \\ x_{2i} &= \frac{1}{\eta}y_{1j} - \frac{\vartheta}{\eta}y_{2i} \end{aligned} \qquad (3.4a)$$

or in matrix form

$$\begin{bmatrix}x_{1j} \\ x_{2i}\end{bmatrix}=\begin{bmatrix}K_I & L_I \\ M_I & N_I\end{bmatrix}\begin{bmatrix}y_{1j} \\ y_{2i}\end{bmatrix} \qquad (3.5)$$

The condition $\gamma_{1j}\delta_{2i}+\delta_{1j}\gamma_{2i}\neq 0$ is the existence condition for the interconnecting network between two adjacent ports 2i and 1j under the transformation matrices $\mathcal{Q}_{2i}$ and $\mathcal{Q}_{1j}$ which are referred to as the adjacent transformation matrices. The physical meaning of the existence condition is that the outputs y_{1j} and y_{2i} are linearly independent (i.e. there is no k such that $y_{1j}=ky_{2i}$). This is violated in fact when $\gamma_{1j}\delta_{2i} + \delta_{1j}\gamma_{2i} = 0$ since eqn. (3.3c) implies $y_{1j} = \theta y_{2i}$.

The existence condition for the interconnecting network, imposes a constraint on the choice of the adjacent transformation matrices that may be used and it can be noticed directly from expression (3.4) that the following choices violate the existence condition.

$$\gamma_{1j} = \gamma_{2i} = 0$$

$$\delta_{1j} = \delta_{2i} = 0$$

The interconnecting network under eqn.(3.5) may be interpreted as a network generating two linear combinations of the two adjacent outputs y_{1j} and y_{2i} to generate each of the two inputs x_{1j} and x_{2i} as illustrated in Fig. 3.4 below.

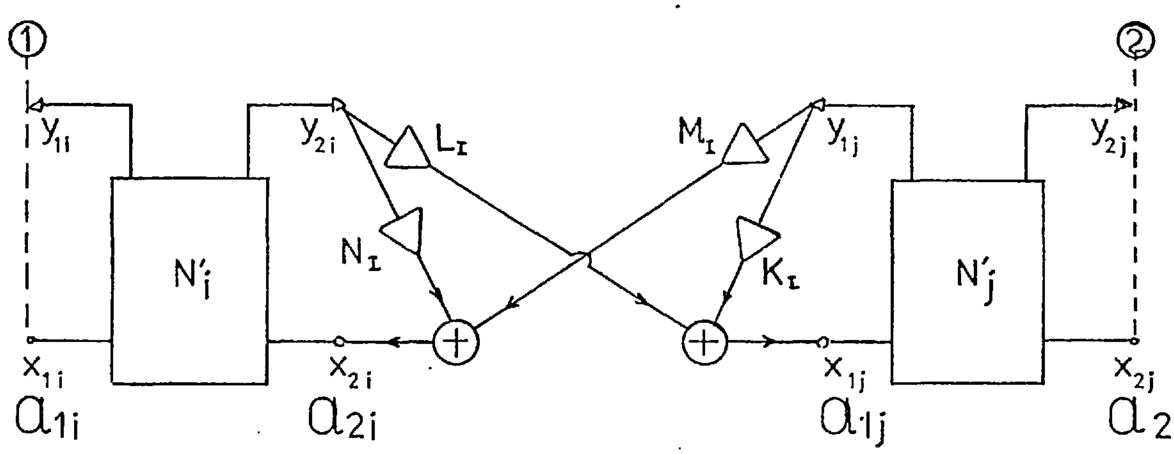


Fig. 3.4

The overall LT interconnection of Fig. 3.4 between the ports indicated by ① and ② is described by

$$\begin{bmatrix} x_{1i} \\ y_{1i} \end{bmatrix} = \alpha_{1i} T_i \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} T_j \alpha_{2j}^{-1} \begin{bmatrix} x_{2j} \\ y_{2j} \end{bmatrix}$$

i.e. the above equation represents the LT structure of the cascade connection of N_i and N_j under the transformation $\{\alpha_{1i}, \alpha_{2j}\}$ with

$$T_i \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} T_j$$

as the modified chain matrix of the passive cascade connection of N_i and N_j.

Eqn. (3.5) can be interpreted in two ways. (i) If the transformation matrices of the adjacent arms are assumed to be given then eqn. (3.5) dictates the manner in which the two LT structures must be interconnected. Moreover this interconnection depends strongly on the adjacent transformation matrices and hence the interconnection network will be different for different choices of the transformation matrices. (ii) Alternatively eqn. (3.5) may be interpreted as defining an interconnecting network to suit the design requirements thereby imposing a relationship, through eqn. (3.3), between the adjacent transformation matrices.

3.2 INPUT TERMINATION CONSTRAINTS

The resistive input termination of a ladder LC filter imposes certain constraints to the variables of the left port of the first reactive two-port of the ladder. The form of these constraints in the transformed x-y domain as well as their implications to the LT network of the first reactive arm of the ladder are examined in this section.

The first reactive arm N_1 of the doubly terminated lossless ladder filter of Fig. 3.1 is terminated by the source resistance R_s as shown in Fig. 3.5a below.

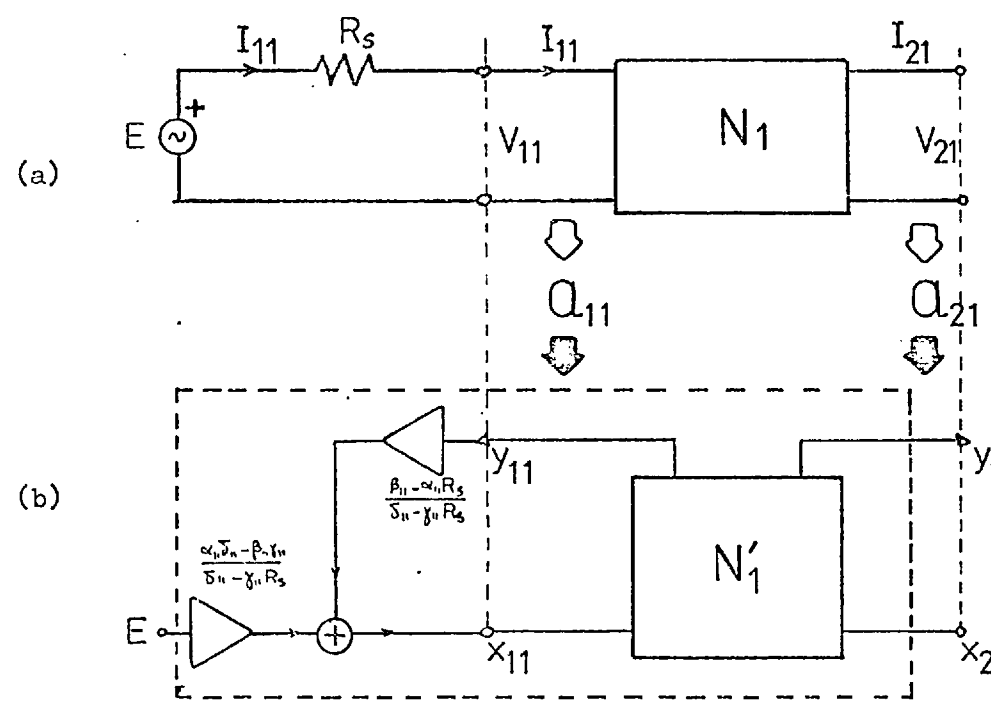


Fig. 3.5

Let N_1 be transformed using the linear transformation $\{a_{11}, a_{21}\}$ so that

$$\begin{bmatrix} x_{11} \\ y_{11} \end{bmatrix} = a_{11} T_1 a_{21}^{-1} \begin{bmatrix} x_{21} \\ y_{21} \end{bmatrix} \qquad (3.6)$$

Moreover, for the resistive termination we have $V_{11} = E - I_{11}R_s$ or equivalently

$$\begin{bmatrix} 1 & R_s \end{bmatrix} \begin{bmatrix} V_{11} \\ I_{11} \end{bmatrix} = E$$

so that when this equation is combined with the left transformed variable vector

$$\begin{bmatrix} x_{11} \\ y_{11} \end{bmatrix} = \mathcal{A}_{11} \begin{bmatrix} V_{11} \\ I_{11} \end{bmatrix}$$

we obtain the relationship

$$\begin{bmatrix} 1 & R_s \end{bmatrix} \mathcal{A}_{11}^{-1} \begin{bmatrix} x_{11} \\ y_{11} \end{bmatrix} = E \qquad (3.7)$$

Equation (3.7) is then said to describe the LT resistive termination of the filter. The explicit form of x_{11} in terms of E and y_{11} of eqn. (3.8) below is more illustrative since it describes the way in which the input x_{11} is expressed in terms of the other variables and in addition it shows the influence of the entries in the transformation matrix.

$$x_{11} = \frac{\alpha_{11}\delta_{11} - \beta_{11}\gamma_{11}}{\delta_{11} - \gamma_{11}R_s} E + \frac{\beta_{11} - \alpha_{11}R_s}{\delta_{11} - \gamma_{11}R_s} y_{11} \qquad (3.8)$$

The above relationship is illustrated in Fig. 3.5b from which it can be seen that the corresponding LT structure of the first reactive arm of the ladder when taken in conjuction with the source resistance, is in fact a two-input-one-output system; one of the two inputs being equal to the source voltage E. The description of this two-input-one-output LT structure which corresponds to the combination of the first reactive arm of the ladder and the source resistance can be derived from eqns (3.6) and (3.7) by eliminating the left linearly transformed vector $[x_{11} \quad y_{11}]^T$. The result is as given below.

$$\begin{bmatrix}1 & R_s\end{bmatrix} T_1\, \mathfrak{a}_{21} \begin{bmatrix}x_{21}\\ y_{21}\end{bmatrix} = E \qquad (3.9)$$

It is worth observing that this relationship depends only on the right transformation matrix $\mathfrak{a}_{21}$. The corresponding LT structure of the first reactive arm of the ladder under the above considerations is shown in Fig. 3.6 and it is denoted by N'_{10}.

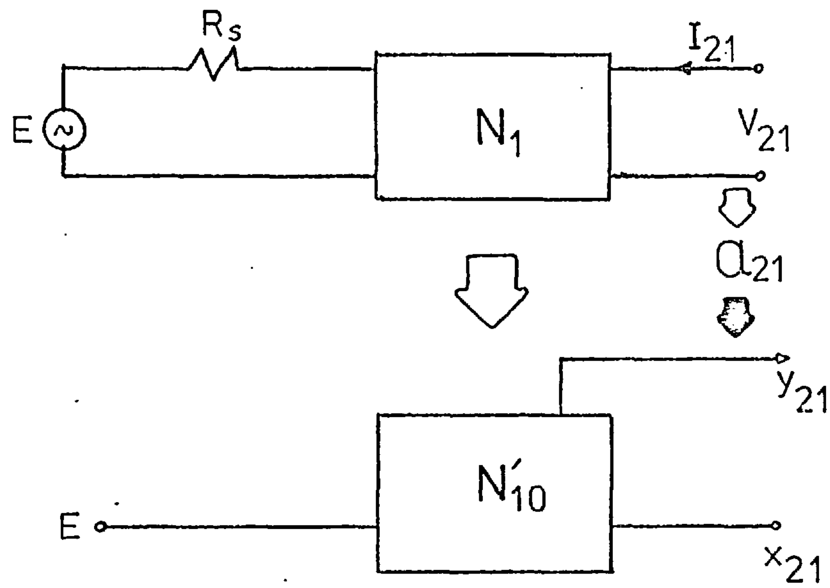


Fig. 3.6

The explicit forms of eqn. (3.9) are given separately below when N_1 is taken to be (i) a series arm of a ladder and (ii) a shunt arm of the ladder.

(i) For the series arm Z_1.

$$y_{21} = \frac{\beta_{21}\gamma_{21} - \alpha_{21}\delta_{21}}{\beta_{21} + \alpha_{21}(Z_1 + R_s)} E + \frac{\delta_{21} + \gamma_{21}(Z_1 + R_s)}{\beta_{21} + \alpha_{21}(Z_1 + R_s)} x_{21} \qquad (3.10a)$$

(ii) For the shunt arm Y_1

$$y_{21} = \frac{\beta_{21}\gamma_{21} - \alpha_{21}\delta_{21}}{\alpha_{21}R_s + \beta_{21}(1 + R_sY_1)} E + \frac{\gamma_{21}R_s + \delta_{21}(1 + R_sY_1)}{\alpha_{21}R_s + \beta_{21}(1 + R_sY_1)} x_{21} \qquad (3.10b)$$

3.3 OUTPUT TERMINATION CONSTRAINTS

The last reactive arm N_n of the ladder filter of Fig. 3.1 is shown in details in Fig. 3.7 together with the resistive termination which represents the load of the filter.

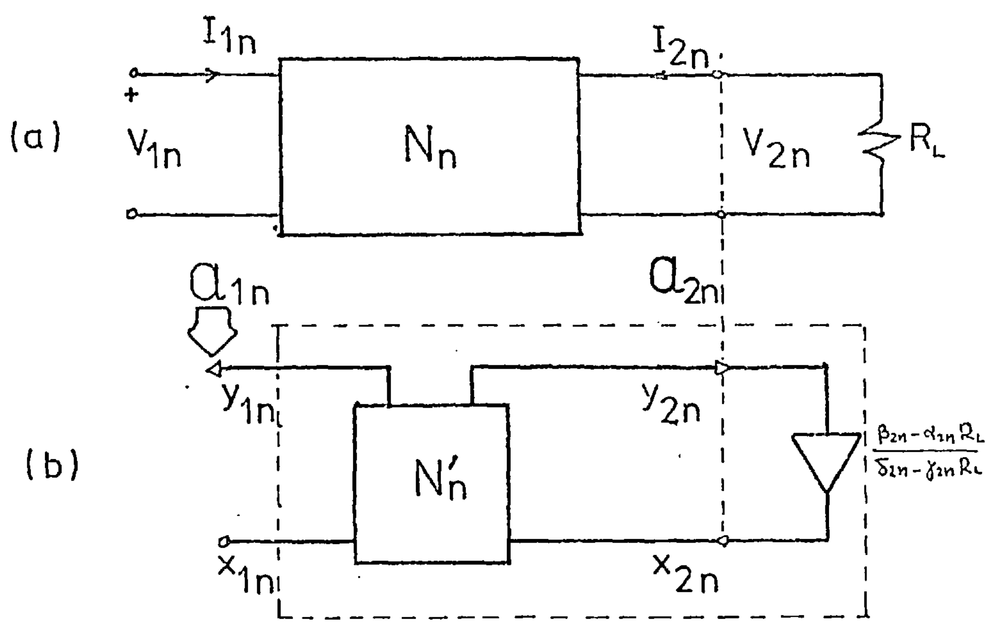


Fig. 3.7

On applying the linear transformation $\{\mathfrak{a}_{1n}, \mathfrak{a}_{2n}\}$ to N_n the LT structure N'_n can be derived described from the relationships

$$\begin{bmatrix} x_{1n} \\ y_{1n} \end{bmatrix} = \mathfrak{a}_{1n} \, T_n \, \mathfrak{a}_{2n}^{-1} \begin{bmatrix} x_{2n} \\ y_{2n} \end{bmatrix} \tag{3.11}$$

However, since

$$\begin{bmatrix} 1 & R_L \end{bmatrix} \begin{bmatrix} V_{2n} \\ I_{2n} \end{bmatrix} = 0$$

and moreover,

$$\begin{bmatrix} V_{2n} \\ I_{2n} \end{bmatrix} = \mathfrak{a}_{2n}^{-1} \begin{bmatrix} x_{2n} \\ y_{2n} \end{bmatrix}$$

from the definition of the right transformed variable vector of N_n, the following relationship between y_{2n} and x_{2n} can be derived

$$\begin{bmatrix}1 & R_L\end{bmatrix} \mathcal{Q}_{2n}^{-1} \begin{bmatrix} x_{2n} \\ y_{2n} \end{bmatrix} = 0 \tag{3.12}$$

In fact eqn. (3.12) represents the LT resistive termination of the filter and it shows the relationship between x_{2n} and y_{2n}, which can be written explicitly as

$$x_{2n} = \frac{\beta_{2n} - \alpha_{2n}R}{\delta_{2n} - \gamma_{2n}R} y_{2n} \tag{3.13}$$

The above relationship is illustrated in Fig. 3.7b where y_{2n} is considered to be the output of the LTA filter. If y_{2n} is expressed in terms of the output voltage V_{2n} of the original ladder filter, it can be shown that

$$y_{2n} = (\gamma_{2n} - \delta_{2n}G_L)V_{2n} \tag{3.14}$$

where $G_L = 1/R_L$. It should be noted that if y_{2n} is to be taken to represent the output voltage V_{2n} then the factor $(\gamma_{2n}-\delta_{2n}G_L)$ must be frequency independent and ideally equal to unity.

The LT structure for the last reactive ladder arm N_n of the ladder in conjunction with the resistive termination in Fig. 3.7b can be considered as a single-input-single-output network in a manner similar to the input termination case. The description of this network can be derived by eliminating the right transformed vector $[x_{2n} \quad y_{2n}]^T$ between eqn. (3.11) and eqn (3.12). This yields the result

$$\begin{bmatrix}1 & R_L\end{bmatrix} T_n^{-1} \ \mathcal{Q}_{2n}^{-1} \begin{bmatrix} x_{1n} \\ y_{1n} \end{bmatrix} = 0 \tag{3.15}$$

which equation again involves only one transformation matrix ($\mathcal{Q}_{1n}$)

However, if this model for the terminated last reactive ladder arm is employed the y_{2n} output which is directly associated with the output V_{2n} of the original ladder filter is no longer available explicitly. In such a case the output node of the LTA filter must be located from equation (3.11) which can be written as

$$\begin{bmatrix} x_{1n} \\ y_{1n} \end{bmatrix} = \alpha_{1n} \, T_n \begin{bmatrix} V_{2n} \\ I_{2n} \end{bmatrix} = \alpha_{1n} \, T_n \begin{bmatrix} 1 \\ -G_L \end{bmatrix} V_{2n} \qquad (3.16)$$

with a proper choice for α_{1n} as it will be discussed in the following chapter.

3.4 SOME ILLUSTRATIVE EXAMPLES

The series arm inductance of the example of the previous chapter will again be used with the same transformation matrices as shown in Fig. 3.8a.

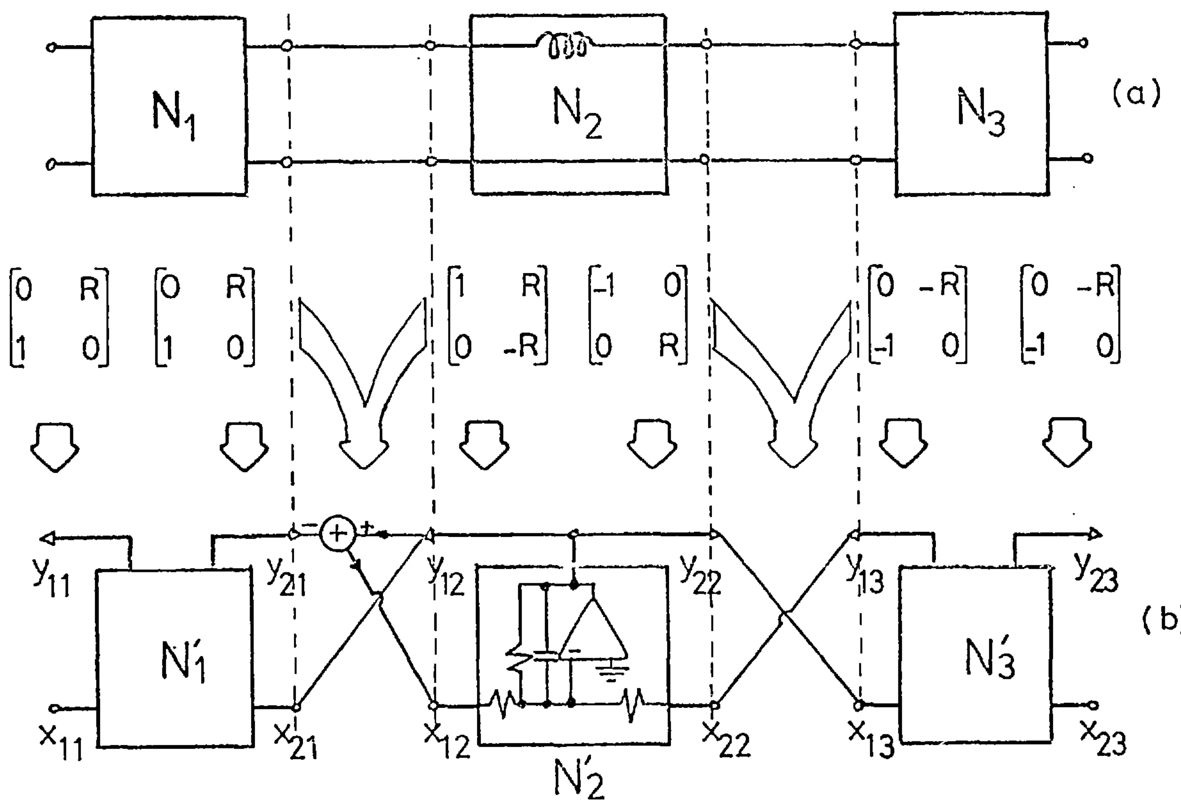


Fig. 3.8

The LT structure of the series inductance under these transformation matrices is already known and let us suppose that N_1' and N_3' are also known under the transformations shown in Fig. 3.8a. From these transformation matrices the interconnecting network between N_1' and N_2' as well as that between N_2' and N_3' can be derived using eqn. (3.3) from which we obtain for the first

$$\begin{bmatrix} x_{12} \\ y_{12} \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_{21} \\ y_{21} \end{bmatrix}$$

or equivalently

$$x_{21} = y_{12}$$

$$x_{12} = y_{21} - y_{12}$$

The above interconnection between N_1' and N_2' is shown in Fig. 3.8b from which it can be observed that it requires at least one active component for its implementation to perform the operation $y_{21} - y_{12}$ to generate x_{12}. Eqn. (3.3) will also give the interconnection between N_2' and N_3' as shown below

$$\begin{bmatrix} x_{13} \\ y_{13} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_{22} \\ y_{22} \end{bmatrix}$$

or equivalently

$$x_{13} = y_{22}$$

$$x_{22} = y_{13}$$

which as it can be observed from Fig. 3.8b it does not require any extra passive or active components.

Let us now suppose that N_1 is a shunt capacitor terminated by the source resistor as shown in Fig. 3.9a. According to eqn. (2.7) the corresponding LT structure can be found and the use of eqn. (3.8) will give the resistive termination. The LT structure for the input terminated capacitor is given in Fig. 3.9b.

However, the use of eqn. (3.9) for a two-input-two-output LT structure for the first reactive arm produces the relationship

$$y_{21} = \frac{R_s}{R(1 + sR_sC_1)} x_{21} + \frac{1}{R(1 + sR_sC_1)} E$$

which for $R = R_s$ can be realized as shown in Fig. 3.9c.

This example shows the advantage of using the two-input-two-output model for the first reactive arm of the ladder in conjunction with the resistive input termination.

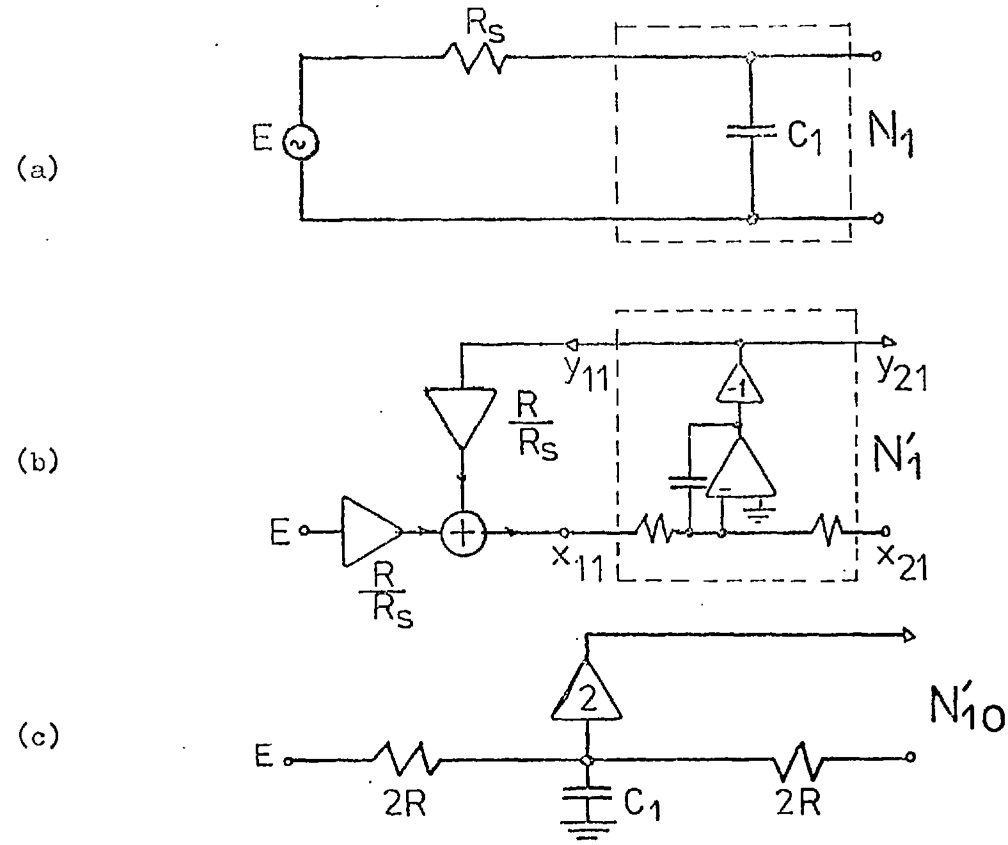


Fig. 3.9

Consider now N_3 of Fig. 3.8a to be a shunt capacitor C_3 and let it be terminated by the load resistance R_L as shown in Fig. 3.10a below.

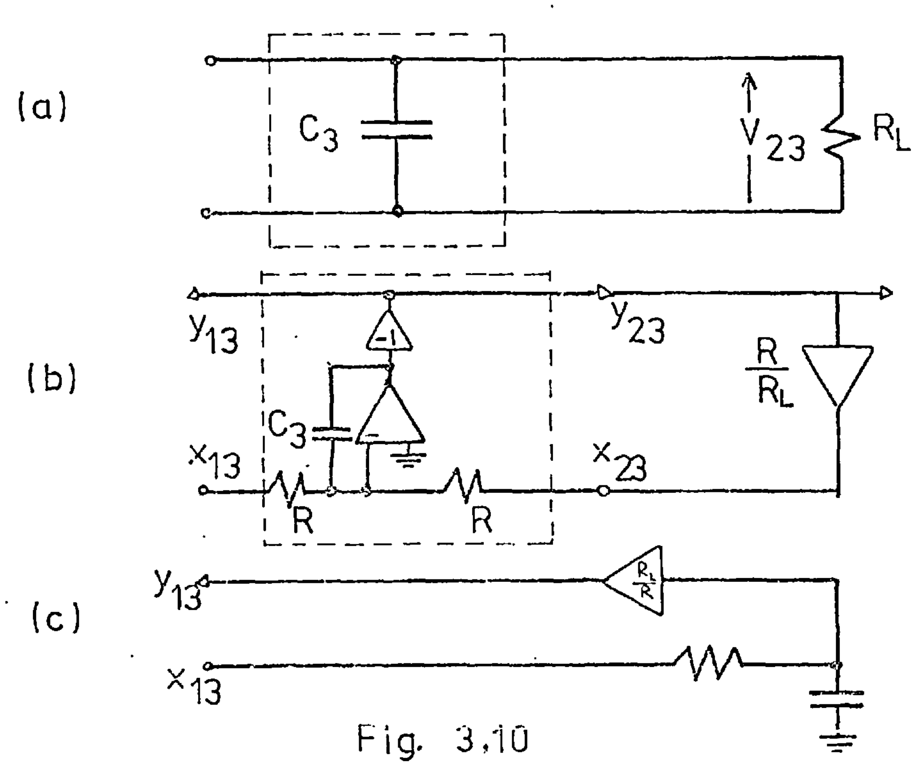


Fig. 3,10

When eqn. (2.7) is used for the transformation of the shunt capacitor C_3 with the transformation matrices shown under N_3 in Fig. 3.9a and moreover, eqn. (3.13) is used for the resistive termination, it is found that

$$y_{13} = y_{23} = \frac{1}{sRC_3}(x_{13} + x_{23}) \quad .$$

$$\text{and} \qquad x_{23} = \frac{R}{R_L} y_{23}$$

which yields the realization of Fig. 3.10b. The use of eqn. (3.15), however, would give the equation

$$y_{13} = \frac{R_L}{R(1 + sRC_3)} x_{13}$$

which has been realized in Fig. 3.10c

From the definition of the right transformed variable vector $[x_{23} \quad y_{23}]^T$ we have that

$$x_{23} = -RI_{23}$$

$$\text{and} \qquad y_{23} = -V_{23}$$

where V_{23} is the output of the original passive ladder filter. Henceforth y_{23}, of the LTA filter, can represent its output with 180° phase shift. In the realization of the single-input single-output model of Fig. 3.10c, however, the location of the output node is performed according to eqn. (3.16) as it has been mentioned in section 3.3, from which

$$\begin{bmatrix} x_{13} \\ y_{23} \end{bmatrix} = \begin{bmatrix} 0 & -R \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ sC_3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -\frac{1}{R_L} \end{bmatrix} V_{23}$$

$$\begin{bmatrix} x_{23} \\ y_{23} \end{bmatrix} = \begin{bmatrix} \frac{R}{R_L}(1 + sR_LC_3) \\ -1 \end{bmatrix} V_{23} \Rightarrow \begin{cases} x_{23} = -\frac{R}{R_L}(1 + sR_LC_3)V_{23} \\ y_{23} = -V_{23} = \text{OUTPUT} \end{cases}$$

Fig. 3.11 summarizes the LTA simulation procedure followed for this particular example of a third order polynomial lowpass filter.

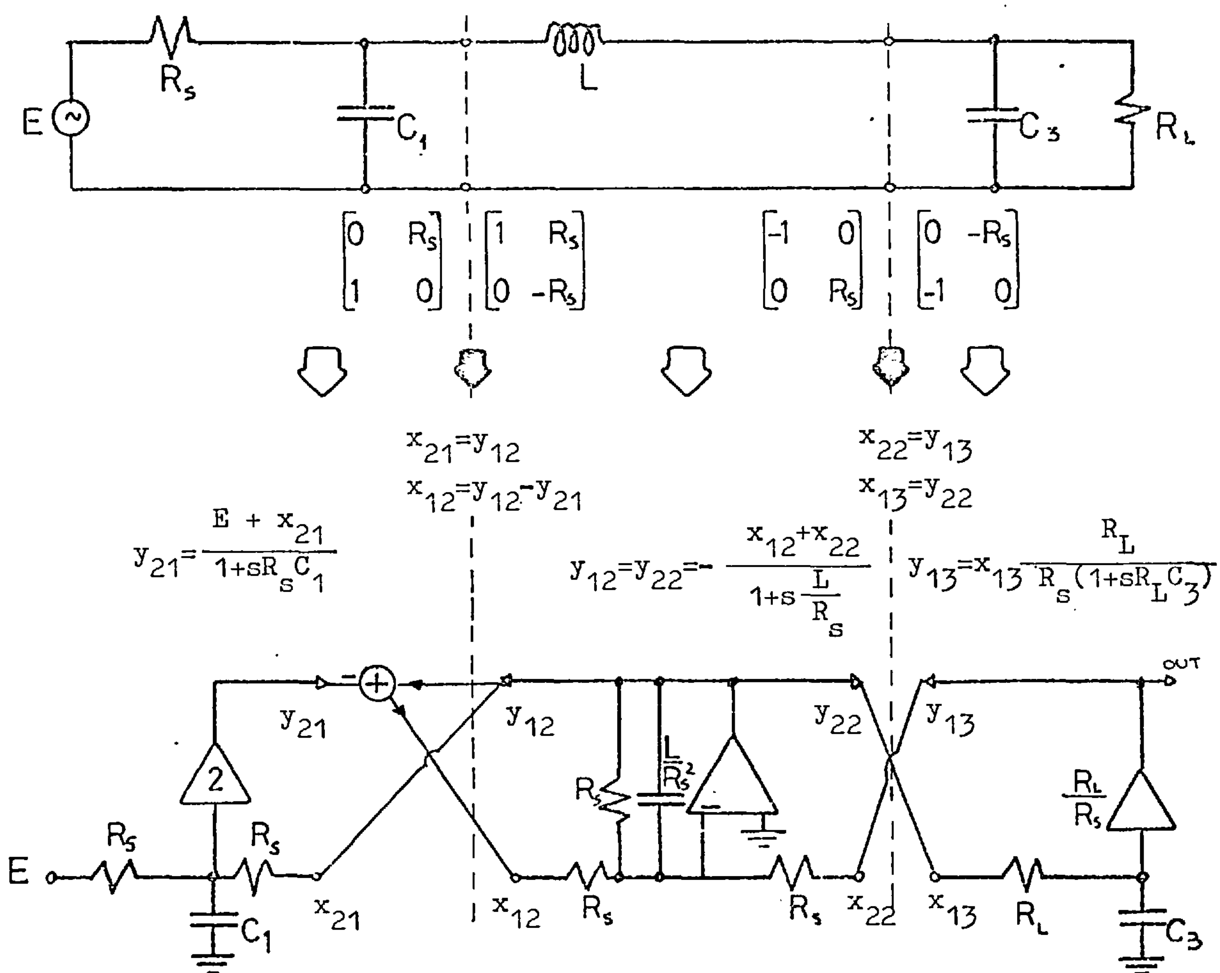


Fig. 3.11

3.5 THE CROSS-CASCADE CONNECTION

As it has been mentioned earlier, the adjacent transformation matrices (i.e. the right transformation matrix of one ladder arm and the left transformation matrix of its adjacent arm, α_{1j} and α_{2i} in Fig. 3.2), can be freely chosen provided that they do not violate the existence condition of the interconnecting network $\gamma_{1j}\delta_{2i}+\gamma_{2i}\delta_{1j} \neq 0$. Such an arbitrary choice defines an interconnecting network which generally requires for its realization several passive and active components. The simplest possible interconnecting network is defined by the following interconnection matrix

$$I_{ij} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{i.e.} \begin{cases} x_{1j} = y_{2i} \\ x_{2i} = y_{1j} \end{cases} \tag{3.17}$$

and this will be referred to as the cross-cascade interconnection. The above cross-cascade interconnection matrix yields the interconnecting network of Fig. 3.12 below.

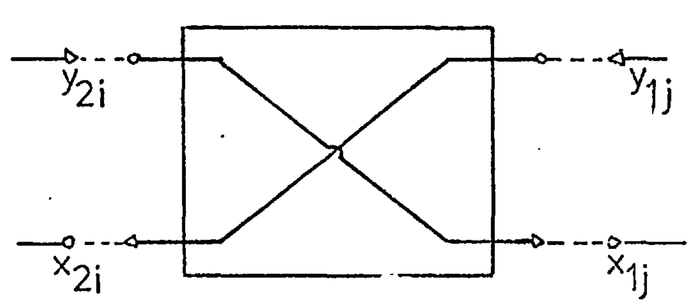


Fig. 3.12

According to section 3.1 of this chapter, by requiring the cross-cascade interconnection at some position, a relationship between the two adjacent matrices is automatically set since the general inter - connection matrix of eqn. (3.3) is restricted to the form of eqn. (3.17).

Eqn. (3.3) and eqn. (3.17) yield the following relationship between the adjacent transformation matrices under the cross-cascade interconnection:

$$\mathfrak{a}_{1j} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \mathfrak{a}_{2i}^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad (3.17a)$$

This relationship implies that in order that the interconnection of two LT structures be cross-cascade, each of the adjacent transformation matrices must be derivable from the other by merely interchanging rows and changing the sign of the second column of one of the matrices. Eqn. (3.17a) will be referred to as the compatibility relationship for cross cascade interconnection.

Under the cross-cascade connection the general interconnecting network can be considered then as the corresponding LT structure to the zero impedance series arm for specific transformation matrices. Referring to Fig. 3.13 the zero impedance series arm can be linearly transformed using the linear transformation $\{\mathfrak{a}^{c}_{2i}, \mathfrak{a}^{c}_{1j}\}$ where $\mathfrak{a}^{c}_{k\lambda}$ represents the compatible matrix of $\mathfrak{a}_{k\lambda}$ under the cross cascade connection as described by eqn. (3.17a).

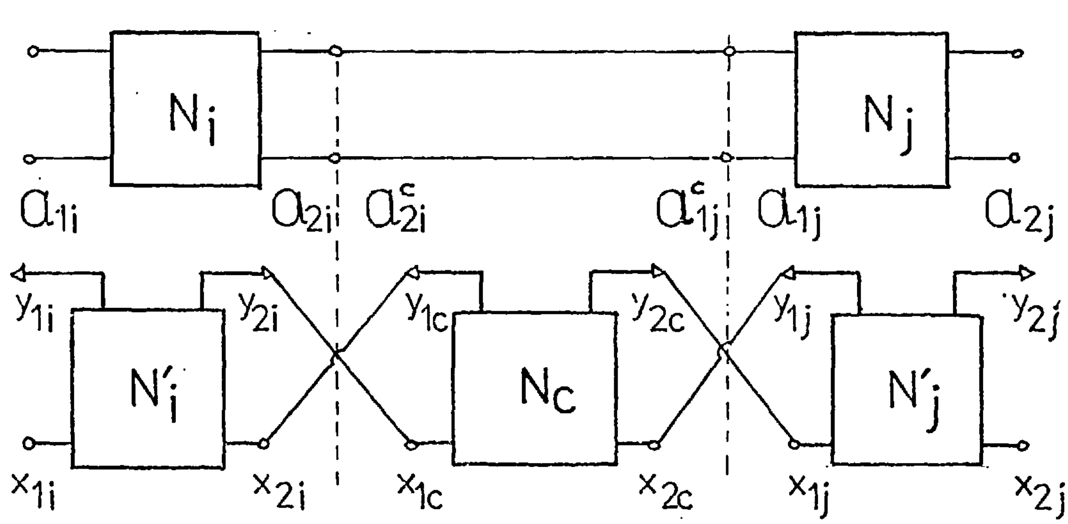


Fig. 3.13

This ensures the cross-cascade connection shown in Fig. 3.13. Moreover, the network N_c (the LT structure for the series arm with $Z = 0$) is described by

$$\begin{bmatrix} x_{1c} \\ y_{1c} \end{bmatrix} = \mathcal{a}^{c}_{2i} \, T_c \, \mathcal{a}^{-1}_{1j} \begin{bmatrix} x_{2c} \\ y_{2c} \end{bmatrix} \qquad (3.18)$$

where

$$\mathcal{a}^{c}_{2i} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \mathcal{a}_{2i} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\mathcal{a}^{c}_{1j} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \mathcal{a}_{1j} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\mathbf{T}_c = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

and

$$\begin{bmatrix} x_{1c} \\ y_{1c} \end{bmatrix} = \begin{bmatrix} y_{2i} \\ x_{2i} \end{bmatrix} \quad ; \quad \begin{bmatrix} x_{2c} \\ y_{2c} \end{bmatrix} = \begin{bmatrix} y_{1j} \\ x_{1j} \end{bmatrix}$$

Substituting these equations into eqn. (3.18) it yields

$$\begin{bmatrix} x_{1j} \\ y_{1j} \end{bmatrix} = \mathcal{a}_{1j} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \mathcal{a}^{-1}_{2i} \begin{bmatrix} x_{2i} \\ y_{2i} \end{bmatrix}$$

which is indeed the description of the general interconnecting network as given in eqn. (3.3).

The cross-cascade interconnection does not only contribute to the low complexity of the overall LTA structure, but in addition, it can be considered as a step towards the sensitivity improvement of the LTA filter realization. This can be explained by observing that in the general interconnecting network, changes in the passive or active elements do not correspond to any changes of the reactive elements of the original ladder and hence extra sensitivity contributions are introduced. Moreover, the cross-cascade interconnection does not have any extra elements in its realization and hence there will be no extra sensitivity contributions in this case.

CHAPTER 4

LINEAR TRANSFORMATIONS FOR LOW COMPLEXITY PRACTICAL FILTERS

CHAPTER 4

LINEAR TRANSFORMATIONS

FOR LOW COMPLEXITY PRACTICAL LTA FILTERS

INTRODUCTION

The application of Linear Transformations to the elementary two-ports of a series impedance Z or a shunt admittance Y from which a ladder filter is made up, as well as their interconnection and terminations have been presented in their general form in the previous two chapters.

It is worth recalling that the choice of the transformation matrices for each individual two-port is relatively free limited only by the constraint that they must be nonsingular and satisfy the existence condition $b \neq 0$ for the LT structure (section 2.1) Moreover, the necessary and sufficient condition for the existence of the interconnecting network between the ports 2i and 1j given in chapter 3, imposes a constraint to the adjacent transformation matrices α_{2i} and α_{1j}.

However, these requirements leave a great degree of freedom in the choice of the transformation matrices which seem to be almost arbitrary. It is this freedom that makes the LTA approach so general but at the same time so problematic to the filter designer. The problem lies in the fact that each arbitrary set of transformations, so chosen to satisfy the above mentioned basic requirements, leads always to an LTA filter structure, leaving the designer wondering whether another transformation would have given a better result, since the complexity of the individual LT structures as well as that of the interconnecting networks depend heavily on the Linear Transformations chosen.

It is, therefore, absolutely necessary to have in hand some rules for the choice of the transformation matrices so as to ensure the low complexity and practicality of the resulting LTA filters. The aim of this chapter is to give such rules and to discuss several matters concerning not only the complexity but also the performance of the overall LTA structures.

4.1 FREQUENCY INDEPENDENT LINEAR TRANSFORMATIONS FOR LOW COMPLEXITY REALIZATIONS.

The description of the oriented LT structure corresponding to a two-port N under the linear transformation set $\{\mathfrak{a}_1, \mathfrak{a}_2\}$ given in section 2.1 (eqn. (2.5)) is repeated here below for convenience.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} K & L \\ M & N \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{4.1}$$

K,L,M and N are the transfer ratios required to be actively realized. Explicit formulae for these ratios are given in section 2.2 (eqns(2.8) and (2.9)) for the two ladder cases of series and shunt arms separately.

Eqn. (4.1) implies that two of these transfer ratios are needed for the generation of each of the outputs y_1 and y_2 in addition to a summing network that may be required as shown in Fig. 2.3b of chapter 2. In practice, this implies that somewhat complex active circuits will be required in the realization of y_1 and y_2 and it is desirable to reduce their complexity. The way this may be approached is by utilizing the degrees of freedom that exist in the ratios K,L,M,N. There are two direct ways by means of which this reduction may be achieved. These ways are obtained by considering the general expressions for K,L,M and N of eqns (2.8) and (2.9) as follows

(i) By choosing the transformation matrices in such a way that one or more of the transfer ratios becomes zero. This will be referred to as the Transfer Ratio Elimination approach (TRE)

(ii) By choosing the transformation matrices so that some or all of the transfer ratios of eqn. (4.1) become identical. This will be referred to as the Transfer Ratio Identification (TRI) approach.

4.1.1 The Transfer Ratio Elimination (TRE) Approach.

It is apparent from the expressions of the transfer ratios of eqn. (2.8) and eqn. (2.9), that the ratios L and M cannot be made equal to zero since this would require either $\det \mathcal{Q}_1$ or $\det \mathcal{Q}_2$ to be equal to zero and so violating the condition for nonsingularity of the transformation matrices. Therefore, only two TRE cases are possible in this approach.

CASE 1 : K = 0

The necessary and sufficient conditions for K =0 can be found for the series and shunt arm cases as follows.

For the series arm Z

From eqn. (2.8)

$$K = \frac{\gamma_1\alpha_2 Z + \gamma_1\beta_2 + \delta_1\alpha_2}{\alpha_1\alpha_2 Z + \alpha_1\beta_2 + \alpha_2\beta_1} \equiv 0 \iff$$

$$\iff \{\gamma_1\alpha_2 Z + \gamma_1\beta_2 + \delta_1\alpha_2 \equiv 0\} \iff \{\gamma_1\alpha_2 = 0 \wedge \gamma_1\beta_2 + \delta_1\alpha_2 \equiv 0\}$$

$$\iff \{\gamma_1 = \delta_1 = 0 \quad \vee \quad \alpha_2 = \beta_2 = 0 \quad \vee \quad \gamma_1 = \alpha_2 = 0\}$$

The first two solutions violate the nonsingularity of the transformation matrices. Henceforth, for the series arm Z:

$$K \equiv 0 \iff \gamma_1 = \alpha_2 = 0$$

Under this condition the LT structure of the series arm Z will be described by

$$y_1 = -\frac{\delta_1}{\beta_2} x_2$$

and
$$y_2 = \frac{\gamma_2}{\alpha_1} x_1 + \frac{\alpha_1\gamma_2 Z + \alpha_1\delta_2 + \beta_1\gamma_1}{\alpha_1\beta_2} x_2$$

For the shunt arm Y

From eqn. (2.9)

$$K = \frac{\delta_1\beta_2 Y + \delta_1\alpha_2 + \gamma_1\beta_2}{\beta_1\beta_2 Y + \beta_1\alpha_2 + \alpha_1\beta_2} \equiv 0$$

$$\Longleftrightarrow \{\delta_1\beta_2 Y + \delta_1\alpha_2 + \gamma_1\beta_2 = 0\} \Longleftrightarrow \{\delta_1\beta_2 = 0 \wedge \delta_1\alpha_2 + \gamma_1\beta_2 = 0\}$$

$$\Longleftrightarrow \{\delta_1 = \gamma_1 = 0 \vee \beta_2 = \alpha_2 = 0 \vee \delta_1 = \beta_2 = 0\}$$

The first two solutions violate the nonsingularity condition of the transformation matrices leaving for the shunt arm Y :

$$K \equiv 0 \Longleftrightarrow \delta_1 = \beta_2 = 0$$

in which case

$$y_1 = \frac{\gamma_1}{\alpha_2} x_2$$

and

$$y_2 = -\frac{\delta_2}{\beta_1} x_1 + \frac{\beta_1\delta_2 Y + \beta_1\gamma_2 + \alpha_1\delta_2}{\beta_1\alpha_2} x_2$$

CASE 2 : N = 0

It can be found in a similar way that

For the series arm Z

$$N \equiv 0 \Longleftrightarrow \alpha_1 = \gamma_2 = 0$$

in which case

$$y_1 = \frac{\gamma_1\alpha_2 Z + \gamma_1\beta_2 + \delta_1\alpha_2}{\beta_1\alpha_2} x_1 + \frac{\gamma_1}{\alpha_2} x_2$$

and

$$y_2 = -\frac{\delta_2}{\beta_1} x_1$$

For the shunt arm Y

$$N = 0 \Longleftrightarrow \beta_1 = \delta_2 = 0$$

$$y_1 = \frac{\delta_1\beta_2 Y + \delta_1\alpha_2 + \gamma_1\beta_2}{\beta_1\beta_2} x_1 - \frac{\delta_1}{\delta_2} x_2 , \quad y_2 = \frac{\gamma_2}{\alpha_1} x_1$$

The conditions found thus far simplify the LT structure of an arm by making one of the outputs directly proportional or indeed equal to one of the inputs to the structure and leaving only one output to be realized RC-actively.

4.1.2 The Transfer Ratio Identification (TRI) Approach.

If in eqn. (4.1) we require that

$$K = \lambda M$$
$$\text{and} \qquad L = \lambda N \tag{4.2}$$

it yields $y_1 = \lambda y_2 = K\,x_1 + L\,x_2$ which in fact requires only one of the outputs y_1 and y_2 to be RC-actively realized the other being proportional or indeed equal (for $\lambda = 1$) to this. The necessary and sufficient conditions for the validity of eqn. (4.2) will be found below for the two ladder cases.

For the series arm Z

$$\text{eqn. (4.1)} \Longleftrightarrow \left\{ \begin{array}{l} \dfrac{\gamma_1\alpha_2 Z + \gamma_1\beta_2 + \delta_1\alpha_2}{\alpha_1\alpha_2 Z + \alpha_1\beta_2 + \beta_1\alpha_2} = \lambda \, \dfrac{\beta_2\gamma_2 - \alpha_2\delta_2}{\alpha_1\alpha_2 Z + \alpha_1\beta_2 + \beta_1\alpha_2} \\[2ex] \dfrac{\beta_1\gamma_1 - \alpha_1\delta_1}{\alpha_1\alpha_2 Z + \alpha_1\beta_2 + \beta_1\alpha_2} = \lambda \, \dfrac{\alpha_1\gamma_2 Z + \alpha_1\delta_2 + \beta_1\gamma_2}{\alpha_1\alpha_2 Z + \alpha_1\beta_2 + \beta_1\alpha_2} \end{array} \right\}$$

$$\Longleftrightarrow \left\{ \begin{array}{l} \gamma_1\alpha_2 Z + \gamma_1\beta_2 + \delta_1\alpha_2 + \lambda\alpha_2\delta_2 - \lambda\beta_2\gamma_2 = 0 \\ \lambda\alpha_1\gamma_2 Z + \lambda\alpha_1\delta_2 + \lambda\beta_1\gamma_2 + \alpha_1\delta_1 - \beta_1\gamma_1 = 0 \end{array} \right\}$$

$$\Longleftrightarrow \left\{ \begin{array}{l} \gamma_1\alpha_2 = 0 \wedge \alpha_1\gamma_2 = 0 \wedge \alpha_2(\delta_1 + \lambda\delta_2) + \beta_2(\gamma_1 - \lambda\gamma_2) = 0 \\ \qquad \wedge\ \alpha_1(\delta_1 + \lambda\delta_2) - \beta_1(\gamma_1 - \lambda\gamma_2) = 0 \end{array} \right\}$$

$$\Longleftrightarrow \left\{ \begin{array}{l} \alpha_1 = \gamma_1 = 0 \vee \alpha_2 = \gamma_2 = 0 \vee \alpha_1 = \alpha_2 = 0 \vee \gamma_1 = \gamma_2 = 0 \\[1ex] \alpha_2(\delta_1 + \lambda\delta_2) + \beta_2(\gamma_1 - \lambda\gamma_2) = 0 \quad \wedge \quad \alpha_1(\delta_1 + \lambda\delta_2) - \beta_1(\gamma_1 - \lambda\gamma_2) = 0 \end{array} \right\}$$

Since $\alpha_1 = \gamma_1 = 0$ and $\alpha_2 = \gamma_2 = 0$ violate the nonsingularity of the transformation matrices and $\alpha_1 = \alpha_2 = 0$ violates the existence condition of the LT structure (b = 0 in eqn. 2.5a) for the series arm the above system simplifies to

$$\left\{\begin{array}{l} \gamma_1 = \gamma_2 = 0 \\ \alpha_2(\delta_1 + \lambda\delta_2) = 0 \\ \alpha_1(\delta_1 + \lambda\delta_2) = 0 \end{array}\right\} \Longleftrightarrow \left\{\begin{array}{l} \gamma_1 = \gamma_2 = 0 \\ \delta_1 = -\lambda\delta_2 \end{array}\right\} \Longleftrightarrow \left\{\begin{array}{l} K = \lambda M \\ L = \lambda N \end{array}\right\}$$

Under the above conditions, the description of the LT structure for the series arm Z is given below

$$y_1 = -\frac{\delta_1}{\delta_2} y_2 = \lambda y_2$$

$$y_2 = -\frac{\alpha_2\delta_2}{\alpha_1\beta_2 + \alpha_2\beta_1 + \alpha_1\alpha_2 Z}(x_1 - \frac{\alpha_1}{\alpha_2} x_2)$$

<u>For the shunt arm Y</u>

In a similar manner it can be found for the shunt arm Y that

$$\left\{\begin{array}{l} K = \lambda M \\ L = \lambda N \end{array}\right\} \Longleftrightarrow \left\{\begin{array}{l} \delta_1 = \delta_2 = 0 \\ \gamma_1 = \lambda\gamma_2 \end{array}\right\}$$

in which case

$$y_1 = \frac{\gamma_1}{\gamma_2} y_2 = \lambda y_2$$

$$y_2 = \frac{\beta_2\gamma_2}{\beta_1\beta_2 Y + \alpha_1\beta_2 + \beta_1\alpha_2}(x_1 + \frac{\beta_1}{\beta_2} x_2)$$

The conditions for $K = \lambda M$ and $L = \lambda N$ simplify the LT structures by making one of the outputs proportional or even equal to the other which has to be realized actively.

Table 4.1 below summarizes the TRE and TRI conditions for low comlexity LT structures for both the series and the shunt arm cases. In the last column of this table some conditions for further reduction of the complexity of the corresponding structures are given explicitly.

TABLE 4.1

	a_1 a_2	LT STRUCTURE DESCR.	FURTHER SIMPLIFICATION
SERIES ARM Z	TRI $\begin{bmatrix}\alpha_1 & \beta_1\\ 0 & \delta_1\end{bmatrix}\begin{bmatrix}\alpha_2 & \beta_2\\ 0 & \delta_2\end{bmatrix}$	$y_1 = -\frac{\delta_1}{\delta_2} y_2$ $y_2 = \frac{-\alpha_2\delta_2}{\alpha_1\beta_2 \pm \beta_1\alpha_2+\alpha_1\alpha_2 Z}(x_1 - \frac{\alpha_1}{\alpha_2}x_2)$	$\delta_1=-\delta_2$, $\alpha_1=-\alpha_2$ for $y_1 = y_2$ $y_2=\frac{-\delta_2(x_1+x_2)}{\beta_1-\beta_2+\alpha_1 Z}$ $\alpha_1(\beta_1-\beta_2) \geqslant 0$
	TRE $N \equiv 0$ $\begin{bmatrix}0 & \beta_1\\ \gamma_1 & \delta_1\end{bmatrix}\begin{bmatrix}\alpha_2 & \beta_2\\ 0 & \delta_2\end{bmatrix}$	$y_1=\frac{\gamma_1\alpha_2+\gamma_1\beta_2+\delta_1\alpha_2 Z}{\beta_1\alpha_2}x_1+\frac{\gamma_1}{\alpha_2}x_2$ $y_2= -\frac{\delta_2}{\beta_1}x_1$	$\beta_1=-\delta_2$, $\gamma_1=\alpha_2$ for $y_2=x_1$ $y_1=\frac{\alpha_2 Z+\delta_1+\beta_2}{\beta_1}x_1+x_2$
	TRE $K \equiv 0$ $\begin{bmatrix}\alpha_1 & \beta_1\\ 0 & \delta_1\end{bmatrix}\begin{bmatrix}0 & \beta_2\\ \gamma_2 & \delta_2\end{bmatrix}$	$y_1= -\frac{\delta_1}{\beta_2}x_2$ $y_2=\frac{\gamma_2}{\alpha_1}x_1+\frac{\alpha_1\gamma_2 Z+\alpha_1\delta_2+\beta_1\gamma_2}{\alpha_1\beta_2}x_2$	$\delta_1=-\beta_2$, $\gamma_2=\alpha_1$ for $y_1= x_2$ $y_2=\frac{\alpha_1 Z+\delta_2+\beta_1}{\beta_2}x_2+x_1$
	$\begin{bmatrix}0 & \beta_1\\ \gamma_1 & \delta_1\end{bmatrix}\begin{bmatrix}0 & \beta_2\\ \gamma_1 & \delta_1\end{bmatrix}$	INADMISSIBLE	

TABLE 4.1 (continued)

	a_1 a_2	LT STRUCTURE DESCR.	FURTHER SIMPLIFICATION
SHUNT ARM Y	TRI $\begin{bmatrix}\alpha_1 & \beta_1\\ \gamma_1 & 0\end{bmatrix}\begin{bmatrix}\alpha_2 & \beta_2\\ \gamma_2 & 0\end{bmatrix}$	$y_1=\frac{\gamma_1}{\gamma_2}y_2$ $y_2=\frac{\beta_2\gamma_2}{\alpha_1\beta_2+\alpha_1\beta_2+\beta_1\beta_2 Y}(x_1+\frac{\beta_1}{\beta_2}x_2)$	$\gamma_1=\gamma_2$, $\beta_1=\beta_2$ for $y_1=y_2$ $y_2=\frac{\gamma_2(x_1+x_2)}{\alpha_1+\alpha_2+\beta_1 Y}$ $\beta_1(\alpha_1+\alpha_2)\geqslant 0$
	TRE $N\equiv 0$ $\begin{bmatrix}\alpha_1 & 0\\ \gamma_1 & \delta_1\end{bmatrix}\begin{bmatrix}\alpha_2 & \beta_2\\ \gamma_2 & 0\end{bmatrix}$	$y_1=\frac{\delta_1\beta_2 Y+\delta_1\alpha_2+\gamma_1\beta_2}{\alpha_1\beta_2}x_1-\frac{\delta_1}{\beta_2}x_2$ $y_2=\frac{\gamma_2}{\alpha_1}x_1$	$\gamma_2=\alpha_1$, $\delta_1=-\beta_2$ for $y_2=x_1$ $y_1=\frac{\delta_1 Y+\gamma_1-\alpha_2}{\alpha_1}x_1+x_2$
	TRE $K\equiv 0$ $\begin{bmatrix}\alpha_1 & \beta_1\\ \gamma_1 & 0\end{bmatrix}\begin{bmatrix}\alpha_2 & 0\\ \gamma_2 & \delta_2\end{bmatrix}$	$y_1=\frac{\gamma_1}{\alpha_2}x_2$ $y_2=-\frac{\delta_2}{\beta_1}x_1+\frac{\beta_1\delta_2 Y+\beta_1\gamma_2+\alpha_1\delta_2}{\beta_1\alpha_2}x_2$	$\gamma_1=\alpha_2$, $\delta_2=-\beta_1$ for $y_1=x_2$ $y_2=x_1+\frac{\delta_2 Y+\gamma_2+\alpha_1}{\alpha_2}x_2$
	$\begin{bmatrix}\alpha_1 & 0\\ \gamma_1 & \delta_1\end{bmatrix}\begin{bmatrix}\alpha_2 & 0\\ \gamma_2 & \delta_2\end{bmatrix}$	INADMISSIBLE	

The two inadmissible conditions violate the existence condition for the LT structure ($b=0$ in eqn. (2.5a)).

4.2 LINEAR TRANSFORMATIONS FOR MINIMUM COMPLEXITY INTERCONNECTIONS

The complexity of the overall LTA filter structure depends not only on the complexity of the individual LT structures corresponding to the simple ladder two-ports but also on the complexity of the interconnecting network which in turn is dictated upon by the adjacent transformation matrices employed. Fig. 4.1a below shows such an LT connection where N_c is the interconnecting network described by eqn.(3.3).

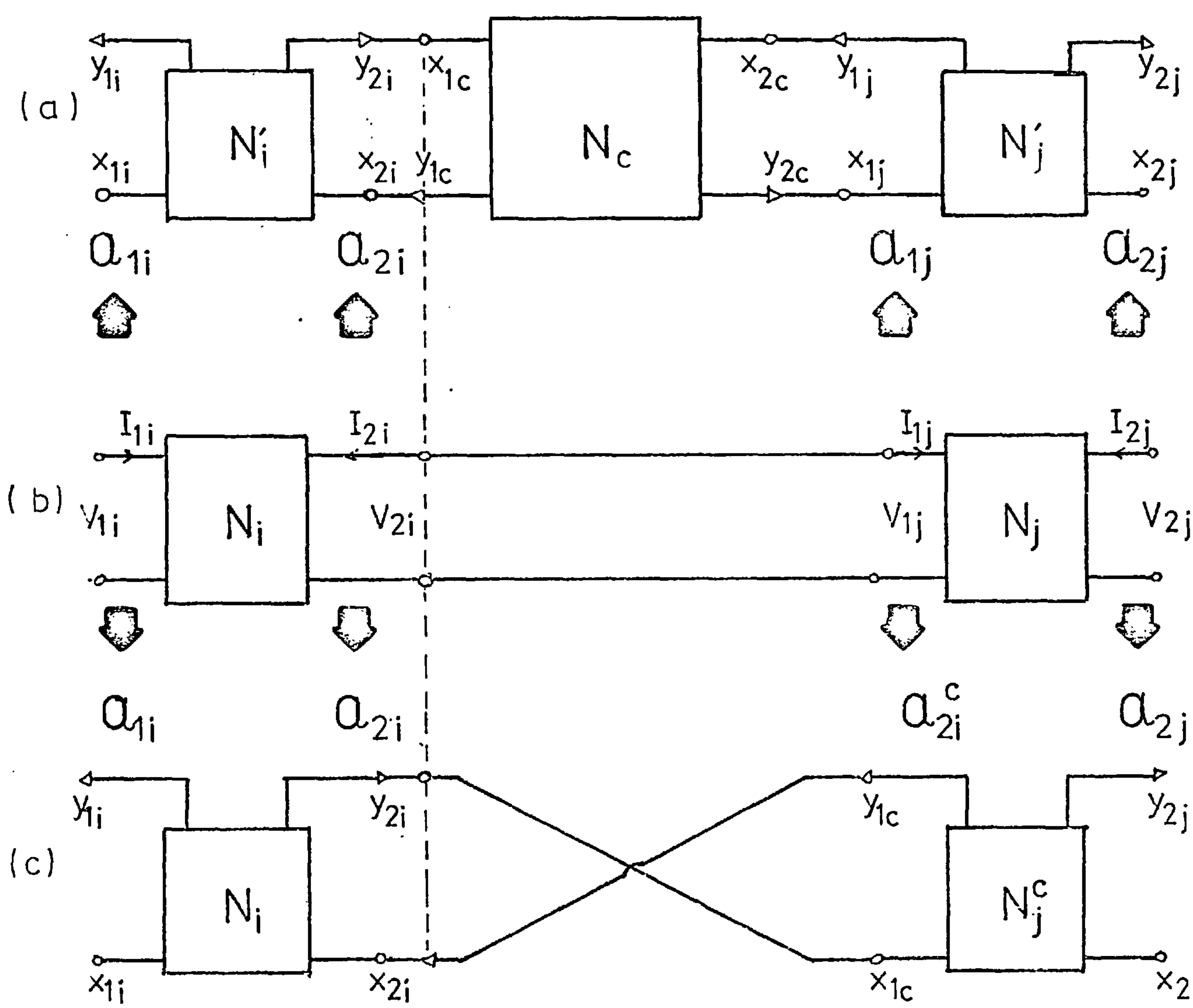


Fig. 4.1

Equation (3.3) is repeated here for convenience

$$\begin{bmatrix} x_{1j} \\ y_{1j} \end{bmatrix} = \mathcal{a}_{1j} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \mathcal{a}_{2i}^{-1} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_{1c} \\ y_{1c} \end{bmatrix} \qquad (4.3)$$

in accordance with Fig. 4.1a where $y_{1c} = x_{2i}$ and $x_{1c} = y_{2i}$. Substituting in eqn. (4.3) $[x_{1j} \quad y_{1j}]^T$ from the LT description of N'_j given below

$$\begin{bmatrix} x_{1j} \\ y_{1j} \end{bmatrix} = \mathcal{a}_{1j} \, T_j \, \mathcal{a}_{2j}^{-1} \begin{bmatrix} x_{2j} \\ y_{2j} \end{bmatrix}$$

and solving for $[x_{1c} \quad y_{1c}]^T$ we obtain

$$\begin{bmatrix} x_{1c} \\ y_{1c} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \mathcal{a}_{2i} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} T_j \, \mathcal{a}_{2j}^{-1} \begin{bmatrix} x_{2j} \\ y_{2j} \end{bmatrix} \qquad (4.4)$$

In this equation it is readily observed that

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \mathcal{a}_{2i} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \mathcal{a}_{2i}^{c}$$

where $\mathcal{a}_{2i}^{c}$ is the matrix compatible with $\mathcal{a}_{2i}$ under the cross-cascade interconnection as defined in chapter 3. This implies that the interconnecting network N_c together with the LT structure N'_j under the linear transformation set $\{\mathcal{a}_{1j}, \mathcal{a}_{2j}\}$ is equivalent to the LT structure N''_j under the transformation set $\{\mathcal{a}_{2i}^{c}, \mathcal{a}_{2j}\}$ as illustrated in Fig. 4.1c.

This equivalence may be considered as a validation of the fact that the cross-cascade interconnection is the general interconnection for LT structures which correspond to two two-ports connected in cascade in the V-I domain. The general interconnecting network N_c

which emerges when $a_{1j} \neq a^c_{2i}$ together with the corresponding LT structure for this case may be considered as an alternative way of realizing the LT structure which will emerge under the cross-cascade compatibility relationship, i.e. for $a_{1j} = a^c_{2i}$.

The final conclusion drawn from the above discussion is that the cross-cascade interconnection, which is the simplest way for connecting the LT structures derived from two two-ports connected in cascade in the V-I domain, may be used without any loss of generality. Therefore, the compatibility relationship between two adjacent linear transformation matrices under the cross-cascade interconnection (eqn. (3.17)), may be considered as a constraint on the choice of the transformation matrices, which contributes to the simplicity of the overall LTA filter structure.

4.3 LINEAR TRANSFORMATIONS FOR LOW COMPLEXITY TERMINATIONS

The choice of the linear transformation set $\{\mathcal{a}_{1n}, \mathcal{a}_{2n}\}$ for the last reactive arm N_n of the ladder is of vital importance to the LTA filter structure as a whole since it must enable an output node to appear, the voltage of which with respect to ground to be directly related to the actual output of the original ladder filter.

If either eqn. (3.12) or eqn (3.13) are used to describe the output model of Fig. 3.7 then we shall have

$$y_{2n} = (\gamma_{2n} - \delta_{2n}G_L)V_{2n}$$

where $G_L = 1/R_L$ and V_{2n} is the voltage output of the original passive ladder filter. The quantity $(\gamma_{2n}-\delta_{2n}G_L)$ must, therefore, be frequency independent and moreover, it must be equal to unity if the output of the LTA filter is to be exactly equal to that of the original passive ladder.

However, if the single-input-single-output model described by eqn. (3.15) is employed then y_{2n} is no longer available explicitly. For low complexity structures, therefore, the nodal voltages representing x_{1n} or y_{1n} must be required to provide an output nodal voltage since these variables are readily available in this component saving model. For this purpose it is necessary to express $[x_{1n} \quad y_{1n}]^T$ in terms of $[V_{2n} \quad I_{2n}]^T$ and from the obtained relationship to set the necessary conditions for one of the quantities x_{1n}, y_{1n} to be a suitable output node. The definition of the left transformed variable vector

$$\begin{bmatrix} x_{1n} \\ y_{1n} \end{bmatrix} = \mathcal{a}_{1n} \begin{bmatrix} V_{1n} \\ I_{1n} \end{bmatrix}$$

combined with

$$\begin{bmatrix} V_{1n} \\ I_{1n} \end{bmatrix} = T_n \begin{bmatrix} V_{2n} \\ I_{2n} \end{bmatrix}$$

and $V_{2n} = -I_{2n}R_L$ give the relationship

$$\begin{bmatrix} x_{1n} \\ y_{1n} \end{bmatrix} = \mathfrak{a}_{1n} T_n \begin{bmatrix} 1 \\ \frac{1}{-R_L} \end{bmatrix} V_{2n} \qquad (4.5)$$

from which the output node conditions can be derived for the two ladder cases separately as follows.

For the series arm Z_n eqn. (4.5) becomes

$$\begin{aligned} x_{1n} &= [\alpha_{1n}(1 + Z_n G_L) + \beta_{1n} G_L] V_{2n} \\ y_{1n} &= [\gamma_{1n}(1 + Z_n G_L) + \delta_{1n} G_L] V_{2n} \end{aligned} \qquad (4.5a)$$

from which it can be seen that either α_{1n} or γ_{1n} must be equal to zero making either x_{1n} or y_{1n} suitable as output respectively.

For a shunt arm Y_n eqn. (4.5) can be writen as

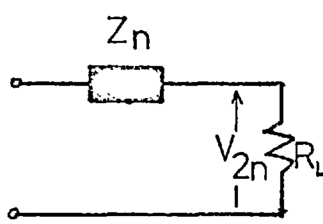
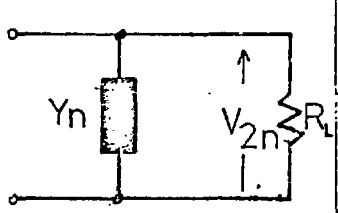
$$\begin{aligned} x_{1n} &= [\alpha_{1n} + \beta_{1n}(Y_n + G_L)] V_{2n} \\ y_{1n} &= [\gamma_{1n} + \delta_{1n}(Y_n + G_L)] V_{2n} \end{aligned} \qquad (4.5b)$$

Similarily either β_{1n} or δ_{1n} must be equal to zero to make either x_{1n} or y_{1n} respectively suitable as output.

It is, therefore, essential that there must exist a zero entry in the $\mathfrak{a}_{1n}$ transformation matrix in order that the single-input-single-output model be used effectively without the need for extra passive and active components to generate an output node other than the already available x_{1n} and y_{1n}. In Table 4.2 all zero entries are examined and the corresponding output node is given as well as the

LT description of the single-input-single-output LT structure of the resistively terminated last reactive arm Z_n or Y_n of the passive ladder.

TABLE 4.2

TERMINATED ARM	a_{1n}	LT STRUCTURE DESCRIPTION
	$\begin{bmatrix} 0 & \beta_{1n} \\ \gamma_{1n} & \delta_{1n} \end{bmatrix}$	$y_{1n} = \dfrac{\delta_{1n} + \gamma_{1n}(Z_n + R_L)}{\beta_{1n}} x_{1n}$ $x_{1n} = \beta_{1n} G_L V_{2n}$
	$\begin{bmatrix} \alpha_{1n} & \beta_{1n} \\ 0 & \delta_{1n} \end{bmatrix}$	$y_{1n} = \dfrac{\delta_{1n}}{\beta_{1n} + \alpha_{1n}(Z_n + R_L)} x_{1n}$ $y_{1n} = \delta_{1n} G_L V_{2n}$
	$\begin{bmatrix} \alpha_{1n} & 0 \\ \gamma_{1n} & \delta_{1n} \end{bmatrix}$	$y_{1n} = \dfrac{\gamma_{1n} R_L + \delta_{1n}(Y_n R_L + 1)}{\alpha_{1n} R_L} x_{1n}$ $x_{1n} = \alpha_{1n} V_{2n}$
	$\begin{bmatrix} \alpha_{1n} & \beta_{1n} \\ \gamma_{1n} & 0 \end{bmatrix}$	$y_{1n} = \dfrac{\gamma_{1n} R_L}{\alpha_{1n} R_L + \beta_{1n}(Y_n R_L + 1)} x_{1n}$ $x_{1n} = \gamma_{1n} V_{2n}$

4.4 OPERATIONAL AMPLIFIER CONSIDERATIONS

The operational amplifier is lately used almost exclusively [43] [44] for the realization of RC-active filters despite its limitations due to the finite gain bandwidth product. Although the LTA filters can be realized using some other active elements, the operational amplifier has been used exclusively in this thesis, because of its versatility. However, some operations involved in realizing the LT structures become very problematic when they are performed by the operational amplifiers due not only to their nature but also due to the gain bandwidth product limitations of the operational amplifiers and they should be avoided.

The major operation to be avoided within the LT structures is that of differentiation. This is due, among other reasons, to the fact that the gain of the differentiator is required to increase with frequency thereby lowering the useable bandwidth of the operational amplifier. Differentiation can be avoided systematically by inspecting the expressions for y_1 and y_2 in every case. It can be observed from Table 4.1 that whenever Z appears only in the numerator of an expression for y_1 or y_2 this particular transformation should be avoided for Z = sL (inductor in the series arm) or indeed $\frac{s^2LC+1}{sC}$ (series tuned circuit in the series arm). Similar conclusions can be drawn for the shunt arm cases. Non-inverting integration operation in the realizations should also be avoided for in such cases a non-inverting integrator is required which can be realized using the sensitive Deboo [45] [46] non-inverting integrator or the Miller-inverter non-inverting integrator which employs two operational amplifiers. Unfortunately, it is not always possible to avoid positive integration in which case the number of the non-inverting integrators must be minimized. Such a minimization is presented in the next chapter.

4.5 A DESIGN EXAMPLE

The third order polynomial lowpass filter of Fig. 4.2 will be simulated using the LTA technique taking into account the conditions for low complexity and effective circuit realizations presented in this chapter.

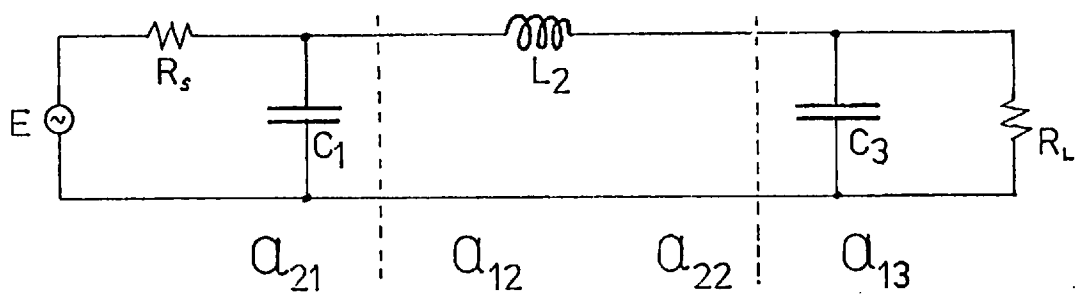


Fig. 4.2

Starting from C_3 taken in conjuntion with the load resistance R_L, one transformation matrix $\mathcal{A}_{13}$ is required for the single-input-single-output model as indicated in Fig. 4.2. This matrix is of the general form

$$\mathcal{A}_{13} = \begin{bmatrix} \alpha_{13} & \beta_{13} \\ \gamma_{13} & \delta_{13} \end{bmatrix}$$

For a convenient output node to exist however and in accordance with the previous considerations, either β_{13} or δ_{13} must be equal to zero. If β_{13} is equal to zero then according to Table 4.2 we shall have

$$y_{13} = \frac{\gamma_{13} + sC_3\delta_{13} + \frac{\delta_{13}}{R_L}}{\alpha_{13}} x_{13}$$

The active realization of this expression however, involves an undesirable differentiation and hence the case $\delta_{13} = 0$ must be considered for which we have from Table 4.2

$$y_{13} = \frac{\gamma_{13}}{\alpha_{13} + \beta_{13}/R_L + sC_3\beta_{13}} x_{13}$$

Choosing $\gamma_{13} = -1$ and $\beta_{13} = R_L$ with α_{13} possitive the above expression becomes

$$y_{13} = -\frac{1}{\alpha_{13} + 1 + sR_LC_3} x_{13}$$

which does not need a differentiator and can be realized in fact using an inverting lossy integrator as shown in Fig. 4.3 below.

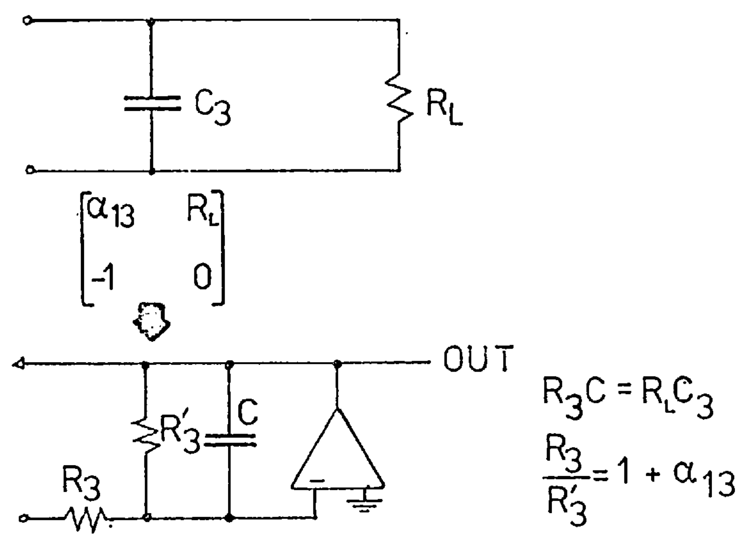


Fig. 4.3

From the $\mathfrak{a}_{13}$ matrix above the right transformation matrix of the series inductor can now be derived from the compatibility relationship yielding

$$\mathfrak{a}_{22} = \begin{bmatrix} \alpha_{22} & \beta_{22} \\ \gamma_{22} & \delta_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ \alpha_{13} & -R_L \end{bmatrix}$$

It can be observed from Table 4.1 that for a low complexity LT structure for the series inductor L_2, either α_{22} or γ_{22} must be taken as zero. But since $\alpha_{22} = \gamma_{13} = -1 \neq 0$ the only choice left in $\mathfrak{a}_{22}$ along the line of low complexity LT structure is $\gamma_{22} = \alpha_{13} = 0$ which makes

$R_3 = R_3'$ in the LT structure for the terminated capacitor C_3 of Fig. 4.3. Again, from Table 4.1 the left transformation matrix $\mathcal{a}_{12}$ for the series inductor L_2 must have either $\alpha_{12} = 0$ or $\gamma_{12} = 0$ for a low complexity realization of the corresponding LT structure. For $\alpha_{12} = 0$, however, the realization of the LT structure requires differentiation as it can be seen from the expression for y_{22} from Table 4.1. Consequently, γ_{12} must be equal to zero in which case the LT structure for the series inductor L_2 is described by

$$y_{12} = \frac{\delta_{12}}{R_L} y_{22}$$

$$y_{22} = \frac{R_L}{\beta_{12} + s\alpha_{12}L_2}(x_{12} + \alpha_{12}x_{22})$$

To reduce still the complexity of the realization we make $y_{12} = y_{22}$ so that $\delta_{12} = R_L$, and we set $\alpha_{12} = 1$. Under these assumptions the above description of the LT structure for the series inductor L_2 becomes

$$y_{12} = y_{22}$$

$$y_{22} = \frac{1}{\frac{\beta_{12}}{R_L} + \frac{sL_2}{R_L}}(x_{12} + x_{22})$$

For $\beta_{12} > 0$ the above expressions can be realized using an operational amplifier whilst for $\beta_{12} = 0$ a non-inverting integrator will be necessary. According to the last section this should be avoided. Therefore we choose $\beta_{12} = R > 0$ in which case the LT structure of the inductor L_2 can be realized as shown in Fig. 4.4.

Once the $\mathcal{a}_{12}$ matrix is defined, its adjacent matrix $\mathcal{a}_{21}$ is also defined from the compatibility relationship of eqn. (3.17a) from which

$$\mathcal{a}_{21} = \begin{bmatrix} 0 & -R_L \\ 1 & -R \end{bmatrix}$$

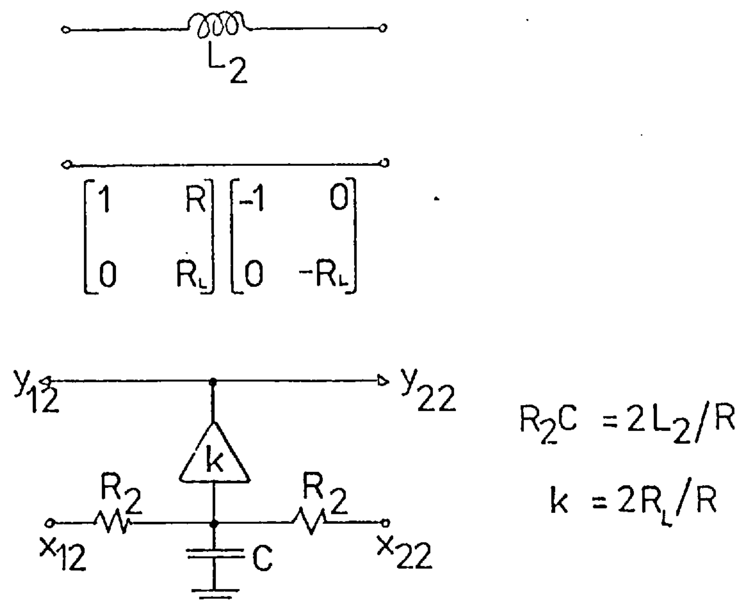


Fig. 4.4

The two-input-two-output LT structure corresponding to the shunt arm capacitor C_1 when taken in conjunction with the source resistance R_s, is derivable from eqn. (3.10b) and is given by

$$y_{21} = \frac{1}{1 + sR_sC_1} E + \frac{-R_s + R(1 + sR_sC_1)}{(1 + sR_sC_1)R_L} x_{21}$$

For $R = R_s$ the above expression becomes

$$y_{21} = \frac{1}{1 + sR_sC_1} E + \frac{R_s}{R_L} \frac{sR_sC_1}{1 + sR_sC_1} x_{21}$$

which can be realized as shown in Fig. 4.5 when $R_s = R_L$

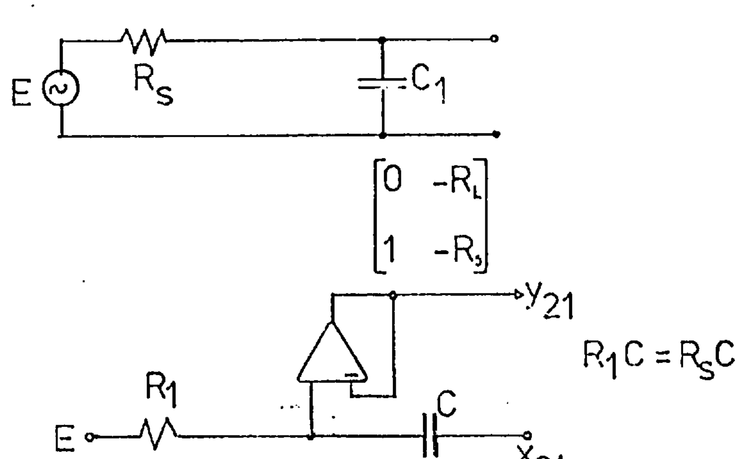


Fig. 4.5

The overall LTA filter structure which simulates the passive ladder filter of Fig. 4.2 together with the relevant transformations and the corresponding descriptions of the LT structures, is shown in Fig. 4.6 below. The frequency response and the sensitivity of this structure is examined elsewhere in this thesis.

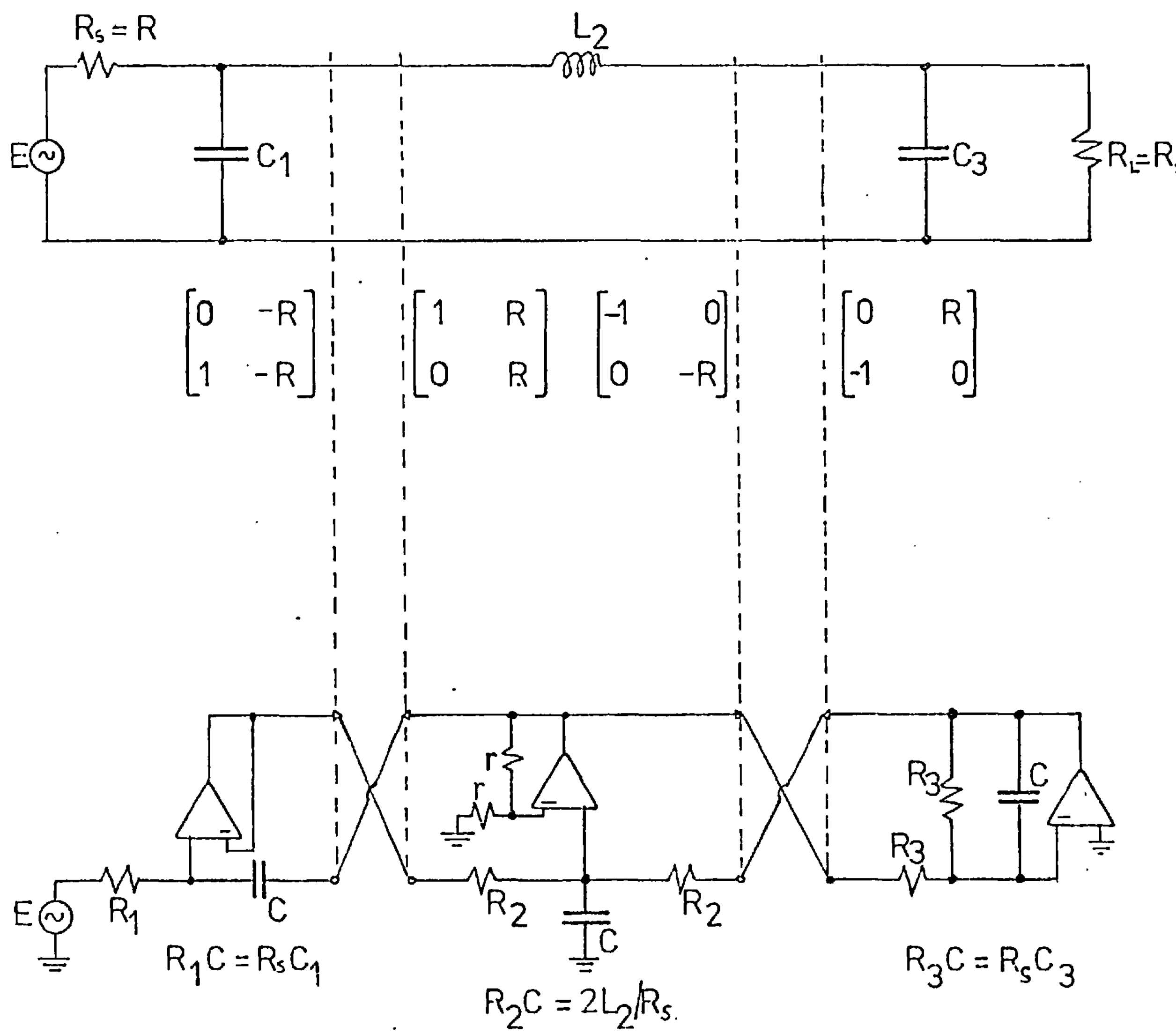


Fig. 4.6

CHAPTER 5

THE LTA FILTER STRUCTURES

CHAPTER 5

THE LTA FILTER STRUCTURES

INTRODUCTION

The flexibility of the LTA method, due to the almost arbitrary choice of the transformation matrices, may be used to derive new active filter structure from passive LC ladder networks. The LTA structures that can be derived from a given passive ladder filter are so many that there is space for reducing their complexity by means of an appropriate transformation choice, as already discussed in chapter 4.

In this chapter a classification of the LTA filters is attempted based on both the manner of selecting the transformation matrices as well as on some distinct and desirable characteristics of the LTA structures themselves. In addition, some well known ladder simulation methods are interpreted as special cases of the general LTA procedure.

The general ladder network, consists of series and shunt arms connected in cascade and it will be denoted by

$$\{N_1, N_2, \ldots, N_i, \ldots, N_n\}$$

where N_i is a series or a shunt arm of the ladder which, for the purposes of this work, will be transformed linearly using the LT matrices pair $\{\mathbf{a}_{1i}, \mathbf{a}_{2i}\}$. The transformation of the ladder, as a whole, requires a linear transformation set which consists of the n individual LT pairs and is denoted by the following set of ordered pairs :

$$\{a_{11}, a_{21}\}, \cdot \cdot, \{a_{1i}, a_{2i}\}, \cdot \cdot \cdot, \{a_{1n}, a_{2n}\}$$

Such a set of linear transformation pairs is said to be compatible if all the adjacent matrices satisfy the compatibility relationship for cross-cascade interconnections (see chapter 3).

Several families of LTA methods are obtained in this chapter by imposing certain relationships between the individual transformation pairs of a compatible transformation set.

5.1 THE SERIES - SHUNT COMPATIBLE TRANSFORMATIONS

DEFINITION 5.1 : A compatible set of linear transformation pairs

$$\{\mathcal{a}_{11},\mathcal{a}_{21}\},\ldots,\{\mathcal{a}_{1i},\mathcal{a}_{2i}\},\ldots,\{\mathcal{a}_{1n},\mathcal{a}_{2n}\}$$

used to transform a ladder $\{N_1, \ldots, N_i, \ldots, N_n\}$ is called series-shunt compatible if and only if the following conditions are satisfied

$$\mathcal{a}_{1i}=\mathcal{a}_{1(i+1)}$$
$$\mathcal{a}_{2i}=\mathcal{a}_{2(i+1)}$$
$$\forall \quad i = 1, 2, 3, \ldots, (n-2) .$$

These conditions imply that all series arms are transformed using the same linear transformation pair $\{\mathcal{a}_{1Z}, \mathcal{a}_{2Z}\}$, whilst all shunt arms are transformed using the linear transformation pair $\{\mathcal{a}_{1Y}, \mathcal{a}_{2Y}\}$ where $\mathcal{a}_{1Y}$ is compatible with $\mathcal{a}_{2Z}$ and $\mathcal{a}_{1Z}$ is compatible with $\mathcal{a}_{2Y}$. The LTA filter structures obtained by using a series-shunt compatible transformation set are referred to as the series-shunt compatible LTA filters.

Under definition 5.1, if for all series arms we set

$$\mathcal{a}_{1Z}=\begin{bmatrix}\alpha_1 & \beta_1\\ \gamma_1 & \delta_1\end{bmatrix} \quad \text{and} \quad \mathcal{a}_{2Z}=\begin{bmatrix}\alpha_2 & \beta_2\\ \gamma_2 & \delta_2\end{bmatrix}.$$

then as a consequence of the compatibility relationship we shall have

$$\mathcal{a}_{1Y}=\begin{bmatrix}\gamma_2 & -\delta_2\\ \alpha_2 & -\beta_2\end{bmatrix} \qquad \mathcal{a}_{2Y}=\begin{bmatrix}\gamma_1 & -\delta_1\\ \alpha_2 & -\beta_2\end{bmatrix} \tag{5.1}$$

and for the determinants , $\det\mathcal{a}_{1Y}=\det\mathcal{a}_{2Z}=\Delta_2$, $\det\mathcal{a}_{2Y}=\det\mathcal{a}_{1Z}=\Delta_1$

The transfer ratios K,L,M and N in the series-shunt compatible case, therefore, will be given as shown below.

Series arm Z

$$K_Z = \frac{\gamma_1\alpha_2 Z + \gamma_1\beta_2 + \delta_1\alpha_2}{\alpha_1\alpha_2 Z + \alpha_1\beta_2 + \beta_1\alpha_2} \qquad L_Z = \frac{-\Delta_1}{\alpha_1\alpha_2 Z + \alpha_1\beta_2 + \beta_1\alpha_2}$$

$$M_Z = \frac{-\Delta_2}{\alpha_1\alpha_2 Z + \alpha_1\beta_2 + \beta_1\alpha_2} \qquad N_Z = \frac{\alpha_1\gamma_2 Z + \alpha_1\delta_2 + \beta_1\gamma_2}{\alpha_1\alpha_2 Z + \alpha_1\beta_2 + \beta_1\alpha_2} \tag{5.2a}$$

Shunt arm Y

$$K_Y = \frac{\delta_1\beta_2 Y - \gamma_1\beta_2 - \delta_1\alpha_2}{\delta_1\delta_2 Y - \gamma_1\delta_2 - \delta_1\gamma_2} \qquad L_Y = \frac{-\Delta_2}{\delta_1\delta_2 Y - \gamma_1\delta_2 - \delta_1\gamma_2}$$

$$M_Y = \frac{-\Delta_1}{\delta_1\delta_2 Y - \gamma_1\delta_2 - \delta_1\gamma_2} \qquad N_Y = \frac{\beta_1\delta_2 Y - \alpha_1\delta_2 - \beta_1\gamma_2}{\delta_1\delta_2 Y - \gamma_1\delta_2 - \delta_1\gamma_2} \tag{5.2b}$$

In the above relationships no constraints have been placed in the α_i, β_i, γ_i and δ_i, $i=1, 2$ parameters other than the compatibility condition. When these parameters are constraint to assume specific values then different structures are derivable depending on these actual values. This is examined below.

5.1.1 The lowpass Leapfrog– LTA Approach

The well-known leapfrog Synthesis [47] emerges from the general LTA procedure as a shunt-series compatible LT approach by considering the transformation matrices to be, for the series arm case:

$$\mathbf{a}_{1Z} = \begin{bmatrix} \alpha & 0 \\ 0 & \delta \end{bmatrix} \qquad \mathbf{a}_{2Z} = \begin{bmatrix} -\alpha & 0 \\ 0 & \delta \end{bmatrix}$$

and consequently, for the shunt arm case we shall have:

$$\mathbf{a}_{1Y} = \begin{bmatrix} 0 & -\delta \\ -\alpha & 0 \end{bmatrix} \qquad \mathbf{a}_{2Y} = \begin{bmatrix} 0 & -\delta \\ \alpha & 0 \end{bmatrix}$$

where α is a real dimensionless number and δ has resistive dimensions but it may be positive or negative.

Such a scheme for the LT structure of the series arm Z yields the following relationships

$$y_1 = -y_2$$
$$y_2 = -\frac{\delta}{\alpha Z}(x_1 + x_2) \qquad (5.3a)$$

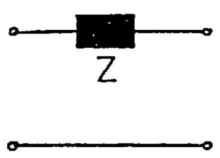
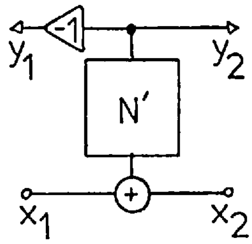
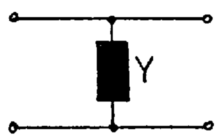
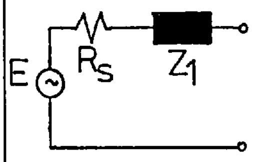
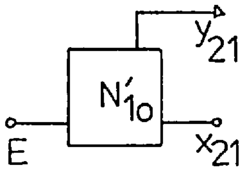
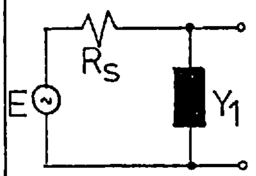
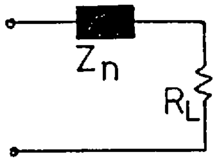
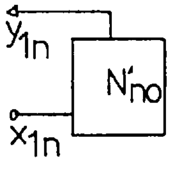
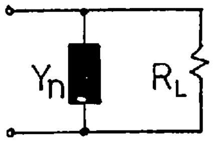
and for the shunt arm Y we have

$$y_1 = -y_2$$
$$y_2 = -\frac{\alpha}{\delta Y}(x_1 + x_2) \qquad (5.3b)$$

Table 5.1 gives the description of the LT structures for the series and shunt arms as well as the input and output terminations. The two-input-one output model has been employed for the input termination, whilst the appropriate zero entries in the chosen left transformation matrices allowed, for both ladder cases, the use of the single-input-single-output model for the last reactive arm of the ladder when taken in conjunction with the resistive termination , as shown in this Table.

The original Girling and Good leapfrog structures for lowpass polynomial ladder filters [47] are derivable for α=1 and $\delta=R_s$. The last column of Table 5.1 gives the LT descriptions of the corresponding LT structures for this case. It is obvious from this table that for the lowpass ladder elements (i.e. series inductor and shunt capacitor) the realizations of all LT structures involve only integrators which is very much desirable as it has been explained earlier. However, for highpass ladder arms (i.e. series capacitors and shunt inductors) this transformation is not of particular interest since it leads to LTA filters requiring differentiators for their realizations.

TABLE 5.1

LADDER ARM	LT MODEL	LT DESCRIPTION	$\alpha = 1$, $\delta = R_s$
		$y_1 = -y_2$ $y_2 = -\frac{\delta}{\alpha Z}(x_1+x_2)$	$y_1 = - y_2$ $y_2 = -\frac{R_s}{Z}(x_1 + x_2)$
		$y_1 = -y_2$ $y_2 = -\frac{\alpha}{\delta Y}(x_1+x_2)$	$y_1 = - y_2$ $y_2 = -\frac{1}{R_s Y}(x_1 + x_2)$
		$y_{21} = -\frac{\delta(x_{21}+\alpha E)}{\alpha(Z_1+R_s)}$	$y_{21} = -\frac{R_s}{Z_1+R_s}(x_{21}+E)$
		$y_{21} = -\frac{\alpha(\frac{R_s}{\delta} x_{21}-E)}{1 + R_s Y_1}$	$y_{21} = -\frac{x_{21}-E}{1 + R_s Y_1}$
		$y_{1n} = \frac{\delta x_{1n}}{\alpha(Z_n+R_L)}$ $y_{1n} = \frac{\delta}{R_L} V_{2n}$	$y_{1n} = \frac{R_s}{Z_n+R_L} x_{1n}$ $y_{1n} = \frac{R_s}{R_L} V_{2n}$
		$y_{1n} = \frac{\alpha R_L x_{1n}}{\delta(1+R_L Y_n)}$ $y_{1n} = -\alpha V_{2n}$	$y_{1n} = \frac{R_L}{R_s(1+R_L Y_n)} x_{1n}$ $y_{1n} = -V_{2n}$

$$a_{1Z} = \begin{bmatrix} \alpha & 0 \\ 0 & \delta \end{bmatrix}, \quad a_{2Z} = \begin{bmatrix} -\alpha & 0 \\ 0 & \delta \end{bmatrix}, \quad a_{1Y} = \begin{bmatrix} 0 & -\delta \\ -\alpha & 0 \end{bmatrix}, \quad a_{2Y} = \begin{bmatrix} 0 & -\delta \\ \alpha & 0 \end{bmatrix}$$

The leapfrog-LTA simulation of a 5th order lowpass polynomial filter is illustrated in Fig. 5.1. The integrators have been realized by the very effective Miller inverting integrator circuit.

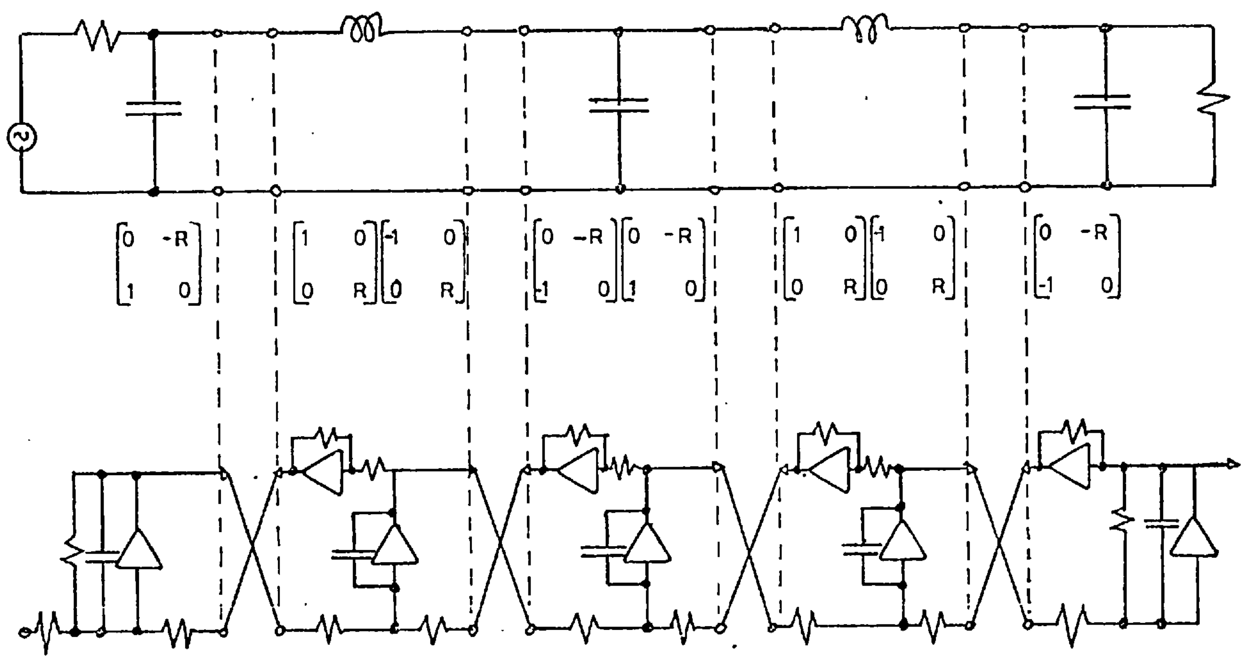


Fig. 5.1

5.2 SELF-COMPATIBLE TRANSFORMATIONS

DEFINITION 5.2 A compatible set of linear transformation pairs

$$\{\mathfrak{a}_{11},\mathfrak{a}_{21}\}, \ldots, \{\mathfrak{a}_{1i}, \mathfrak{a}_{2i}\}, \ldots, \{\mathfrak{a}_{1n},\mathfrak{a}_{2n}\}$$

used for the transformation of a ladder $\{N_1, \ldots, N_i, \ldots, N_n\}$ is defined as a self-compatible if and only if

$$\mathfrak{a}_{1i} = \mathfrak{a}_1 \wedge \mathfrak{a}_{2i} = \mathfrak{a}_2 \quad , \forall \quad i = 1,2, \ldots, n$$

where $\mathfrak{a}_1$ and $\mathfrak{a}_2$ are two nonsingular compatible square matrices.

The LTA filters emerging from such transformation sets will be referred to as Self-Compatible LTA Filters.

Thus if we set the matrix $\mathfrak{a}_1$ to be

$$\mathfrak{a}_1 = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \tag{5.4a}$$

then from the compatibility relationship we find that

$$\mathfrak{a}_2 = \begin{bmatrix} \gamma & -\delta \\ \alpha & -\beta \end{bmatrix} \tag{5.4b}$$

Moreover, we have $\det\mathfrak{a}_1 = \det\mathfrak{a}_2 = \Delta$

The transfer ratios K,L,M and N can now be re-expressed under the above transformation matrices as follows.

For the series arm Z

$$K_Z = \frac{\gamma^2 Z}{\alpha\gamma Z - \Delta} \qquad L_Z = \frac{-\Delta}{\alpha\gamma Z - \Delta}$$

$$M_Z = \frac{-\Delta}{\alpha\gamma Z - \Delta} \qquad N_Z = \frac{\alpha^2 Z}{\alpha\gamma Z - \Delta} \tag{5.5a}$$

For the shunt arm Y

$$K_Y = \frac{\delta^2 Y}{\beta\delta Y + \Delta} \qquad L_Y = \frac{\Delta}{\beta\delta Y + \Delta} \tag{5.5b}$$

$$M_Y = \frac{\Delta}{\beta\delta Y + \Delta} \qquad\qquad N_Y = \frac{\beta^2 Y}{\beta\delta Y + \Delta} \tag{5.5b}$$

5.2.1 The Active-RC All-Integrator Highpass Filters [47]

The lowpass Leapfrog Linear Transformation which which can be used for deriving the leapfrog structures from lowpass polynomial ladder networks have been discussed in section 5.1.1 where it was observed that these particular transformations would, in the case of highpass filters, lead to leapfrog-LTA structures involving undesirable differentiations.

There are, however, two self compatible transformations leading to the Girling and Good [47] all-integrator active structures for the highpass filter case. The first of these transformations, is defined by the following self-compatible matrices

$$\mathcal{a}_1 = \begin{bmatrix} 0 & \beta \\ \gamma & 0 \end{bmatrix} \qquad \mathcal{a}_2 = \begin{bmatrix} \gamma & 0 \\ 0 & -\beta \end{bmatrix}$$

which is obtainable from the general self compatible matrices of eqn. (5.4) for $\alpha=\delta=0$. This transformation will be referred to as the HP-LT1 case. The voltage transfer ratios K,L,M and N for the HP-LT1 case are found from eqns (5.5) and are given below.

For the series arm Z

$$\left.\begin{matrix} K_Z = \frac{\gamma Z}{\beta} & L_Z = 1 \\ M_Z = 1 & N_Z = 0 \end{matrix}\right\} \Rightarrow \begin{matrix} y_1 = \frac{\gamma Z}{\beta} x_1 + x_2 \\ y_2 = x_1 \end{matrix} \tag{5.6a}$$

For the shunt arm Y

$$\left.\begin{matrix} K_Y = 0 & L_Y = 1 \\ M_Y = 1 & N_Y = -\frac{\beta}{\gamma} Y \end{matrix}\right\} \Rightarrow \begin{matrix} y_1 = x_2 \\ y_2 = x_1 - \frac{\beta}{\gamma} Y x_2 \end{matrix} \tag{5.6b}$$

The second of the self-compatible transformations which leads to all-integrator active highpass filters is defined by

$$\mathcal{A}_1 = \begin{bmatrix} \alpha & 0 \\ 0 & \delta \end{bmatrix} , \quad \mathcal{A}_2 = \begin{bmatrix} 0 & -\delta \\ \alpha & 0 \end{bmatrix}$$

This transformation will be referred to as the HP-LT2 case. The corresponding descriptions of the LT structures for the ladder arms are given below.

<u>For the series arm Z</u>

$$y_1 = x_2$$

$$y_2 = x_1 - \frac{\alpha Z}{\delta} x_2$$

<u>For the shunt arm Y</u>

$$y_1 = \frac{\delta Y}{\alpha} x_1 + x_2$$

$$y_2 = x_1$$

The corresponding LT structures for the highpass ladder arms for the HP-LT2 case as well as the terminations are given in Table 5.2

The use of this Table for the transformation of a highpass polynomial filter leads to the active structures of Girling and Good. This is illustrated in Fig.5.2 by means of the LTA simulation of a 5th order polynomial highpass ladder filter. In Fig. 5.2a the structure has not been yet simplified. The simplified version of this structure is shown in Fig. 5.2b. The simplification was based on the fact that when Miller integrators are used, they can perform addition at the virtual earth terminal in which case some of the adders are not necessary. Similar simplifications can be made using the virtual earth point of the inverters. The simplified LTA filter structure will ultimately need eight operational amplifiers connected as shown in this figure.

TABLE 5.2

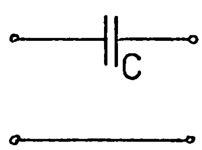
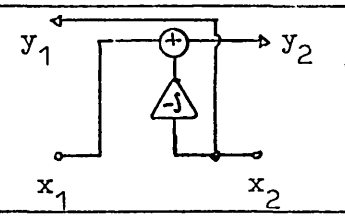
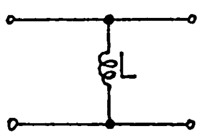
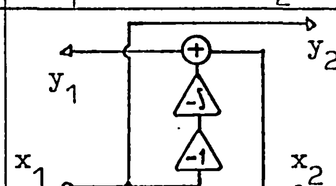
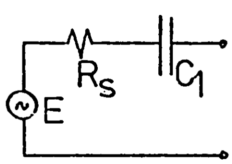
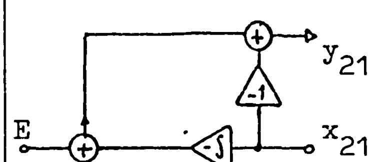
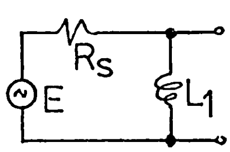
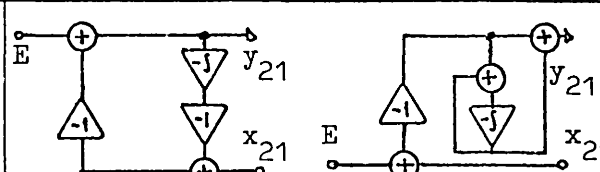
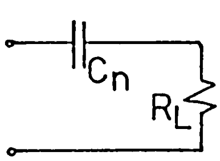
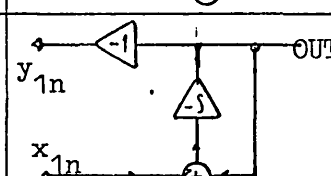
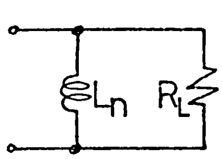
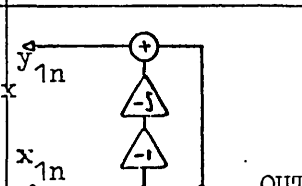
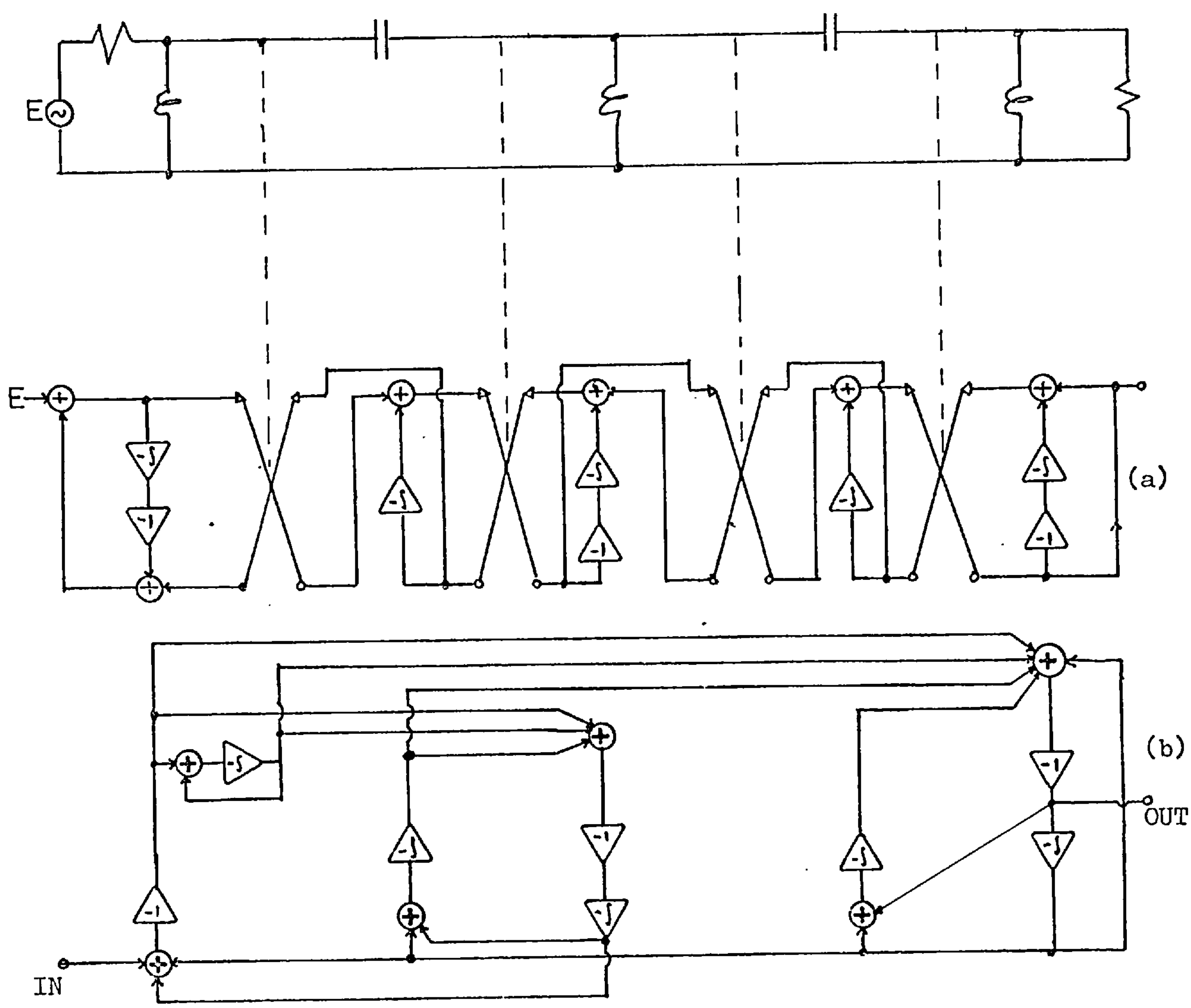
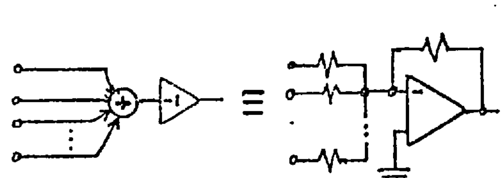
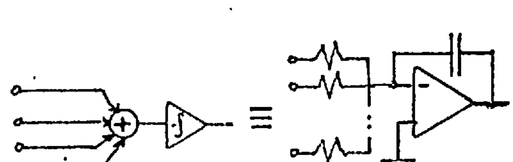
HP-LT1: $a_1=\begin{bmatrix}\alpha & 0\\ 0 & \delta\end{bmatrix}$, $a_2=\begin{bmatrix}0 & -\delta\\ \alpha & 0\end{bmatrix}$		$\alpha = 1$, $\delta = R_s$
LADDER ARM		LT MODEL
	$y_1=x_2$ $y_2=x_1-\frac{1}{sR_sC}x_2$	
	$y_1=\frac{1}{sL/R_s}x_1+x_2$ $y_2=x_1$	
	$y_{21}=-\frac{sR_sC_1+1}{sR_sC_1}x_{21}+E$	
	$y_{21}=\frac{sL/R_s}{1+sL/R_s}(E-x)$	
	$y_{1n}=\frac{sR_sC_n}{1+sR_sC_n}x_{1n}$	
	$y_{1n}=\frac{R_s}{R_L}(1+\frac{1}{sL/R_L})x$	

Fig. 5.2





Summing inverter



Summing Miller inverting integrator

5.2.2 The Wave Active Filters

An additional constraint within the self-compatible linear transformations can simplify the general equations and derive one very interesting special case. By considering $\mathcal{Q}_1 = \mathcal{Q}_2$, eqns (5.4) yield

$$\mathcal{Q}_1 = \mathcal{Q}_2 = \begin{bmatrix} \alpha & \beta \\ \alpha & -\beta \end{bmatrix}$$

and consequently for the series arm Z we have

$$K_Z = \frac{\alpha Z}{\alpha Z + 2\beta} \qquad L_Z = \frac{2\beta}{\alpha Z + 2\beta}$$

$$M_Z = \frac{2\beta}{\alpha Z + 2\beta} \qquad N_Z = \frac{\alpha Z}{\alpha Z + 2\beta} \tag{5.7a}$$

whilst for the shunt arm Y we shall have

$$K_Y = \frac{-\beta Y}{\beta Y + 2\alpha} \qquad L_Y = \frac{2\alpha}{\beta Y + 2\alpha}$$

$$M_Y = \frac{2\alpha}{\beta Y + 2\alpha} \qquad N_Y = \frac{-\beta Y}{\beta Y + 2\alpha} \tag{5.7b}$$

This self compatible LT procedure for $\alpha=1$ and $\beta=R$ was first presented by A.G.Constantinides and G.Haritantis under the name Wave Active Filters [48] [49] and independently by H.Wupper and K.Meerkötter under the name Scattering Parameter Active Filters [50] [51]. The Wave Active Filter terminology stems from the fact that under this specific self compatible transformation set, the transformed variables x and y can be identified with the incident and reflected voltage waves at the corresponding ports, whilst the voltage transfer ratios K, L, M and N, being equal to the elements of the scattering matrix of the corresponding ladder arms, thereby justify the second term.

As far as the input termination is concerned, for the wave active filter case eqn. (3.8) yields :

$$x_{11} = \frac{2R}{R + R_s} E$$

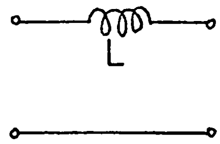
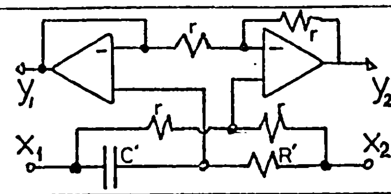
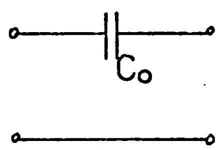
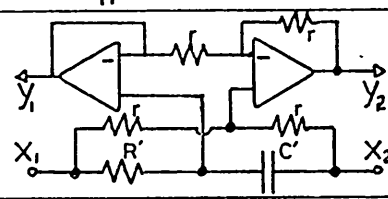
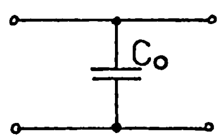
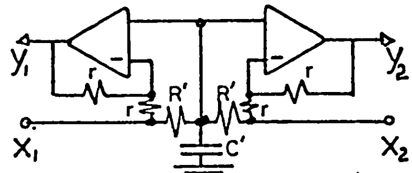
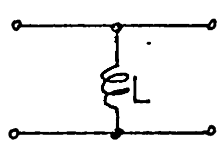
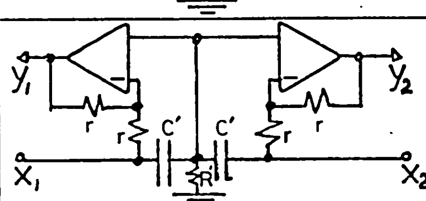
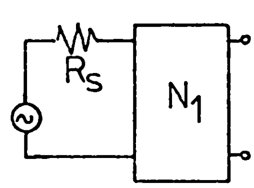
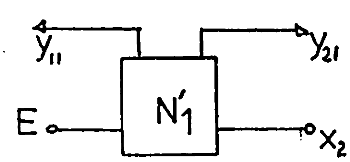
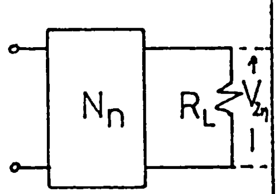
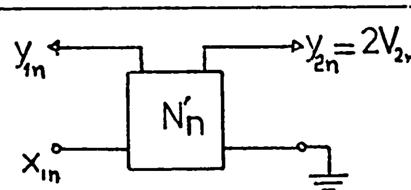
which becomes $x_{11} = E$ for $R = R_s$. This implies that the x_{11} input of the LT structure corresponding to the first reactive arm N_1 of the ladder represents the actual input of the Wave Active Filters. It is worth noting that in the Wave Active Filter case the output y_{11} represents the complementary output of the filter as pointed out in ref.[48][49] The output termination of the Wave Active filters can be found from eqn. (3.13) as follows

$$x_{2n} = \frac{R_L - R}{R + R_L} y_{2n}$$

which implies that for $R = R_L$ the input x_{2n} must be grounded in which case from eqn. (3.14) we find that $y_{2n} = 2V_{2n}$. This implies that the output y_{2n} of the Wave LTA structure represents the output of the original ladder filter with a gain of 2. The single-input-single-output model for the last reactive arm of the ladder when transformed in conjunction with the load resistance,cannot be used in the Wave Active Filter case due to the absence of the necessary for the existence of a convenient output node zero entries in the Wave Active transformation matrices.

Table 5.3 gives the Wave Active LT structures for all simple ladder arm cases. The series (parallel) tuned circuit in the series (shunt) arm is not shown in the table since due to the self compatibility of the transformations it can be formed by simply cross-cascading the Wave Active LT structures of its consistuent parts. The series (parallel) tuned circuit in the shunt (series) arm are more problematic. Their Wave Active LT structures are shown in Fig. 5.3a and Fig. 5.3b respectively whilst Fig. 5.3c and 5.3d show the operational amplifier saving (but non-canonic) Wave Active LT structures for the same arms, proposed in ref.[51].

TABLE 5.3

LADDER ARM	WAVE ACTIVE LT STRUCTURE	
		$R'C' = L/2R$
		$R'C' = RC_o/2$
		$R'C' = RC_o$
		$R'C' = L/R$
		$R = R_S$
		$R = R_L$

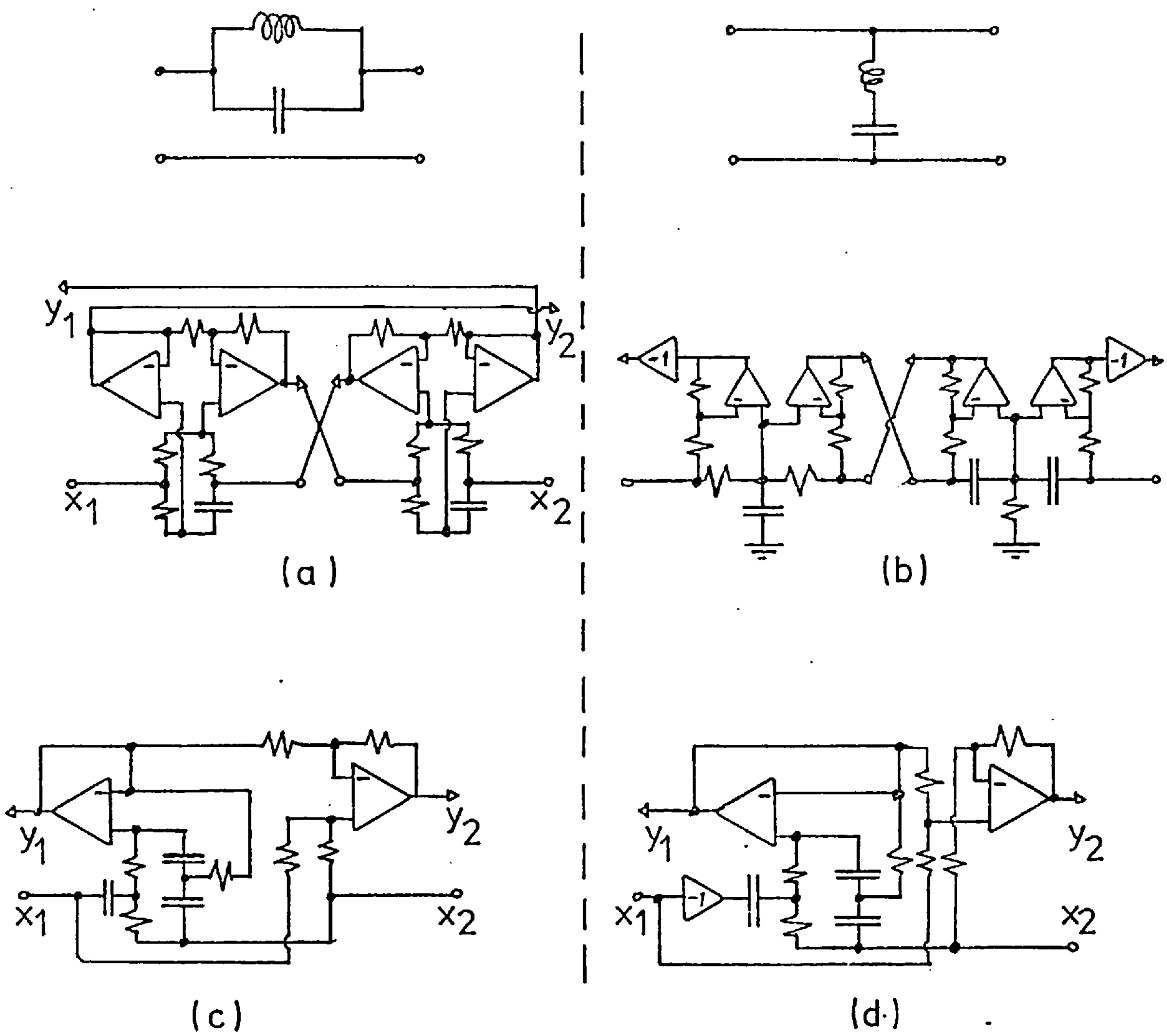


Fig. 5.3

5.3 FORM - COMPATIBLE TRANSFORMATIONS

DEFINITION 5.3 : A compatible set of linear transformation pairs

$$\{\mathfrak{a}_{11}, \mathfrak{a}_{21}\}, \ldots, \{\mathfrak{a}_{1i}, \mathfrak{a}_{2i}\}, \ldots, \{\mathfrak{a}_{1n}, \mathfrak{a}_{2n}\}$$

is said to be form-compatible if and only if

$$|\mathfrak{a}_{1i}| = |\mathfrak{a}_{1(i+2)}| \wedge |\mathfrak{a}_{2i}| = |\mathfrak{a}_{2(i+2)}|$$

$$\forall \quad i = 1, 2, \ldots, (n-2)$$

where by $|\mathfrak{a}|$ we denote a matrix the elements of which are the absolute values of those of $\mathfrak{a}$, i.e

$$\mathfrak{a} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \Rightarrow |\mathfrak{a}| = \begin{bmatrix} |\alpha| & |\beta| \\ |\gamma| & |\delta| \end{bmatrix}$$

From the above definition it becomes clear that all transformations for series arms will have the same zero entries. The same is true for the transformations of the shunt arms. It can also be observed that any series-shunt compatible transformation as well as any self compatible transformation is a form-compatible transformation under definition 5.3.

The form-compatible linear transformations may be used for the reduction of the number of the operational amplifiers in some self-compatible and series-shunt compatible cases as it is shown below.

5.3.1 Form-Compatible Transformations for minimum Operational Amplifier Leapfrog-LTA Structures.

We recall that the leapfrog structures for lowpass polynomial ladder filters can be derived using the series-shunt compatible set of transformation pairs defined by

$$\mathfrak{a}_{1Z} = \begin{bmatrix} 1 & 0 \\ 0 & R \end{bmatrix}, \quad \mathfrak{a}_{2Z} = \begin{bmatrix} -1 & 0 \\ 0 & R \end{bmatrix}, \quad \mathfrak{a}_{1Y} = \begin{bmatrix} 0 & -R \\ -1 & 0 \end{bmatrix}, \mathfrak{a}_{2Y} = \begin{bmatrix} 0 & -R \\ 1 & 0 \end{bmatrix}$$

The LT structures so derived for the simple lowpass ladder arms need one inverting Miller integrator for every simple lowpass arm and in addition to an inverter as it can be seen from Table 5.1.

The form-compatible leapfrog LT is defined by the following transformation matrices.

For the series arms

$$|a_{1Z}| = \begin{bmatrix} 1 & 0 \\ 0 & R \end{bmatrix} = |a_{2Z}|$$

whilst for the shunt arms

$$|a_{1Y}| = \begin{bmatrix} 0 & R \\ 1 & 0 \end{bmatrix} = |a_{2Y}|$$

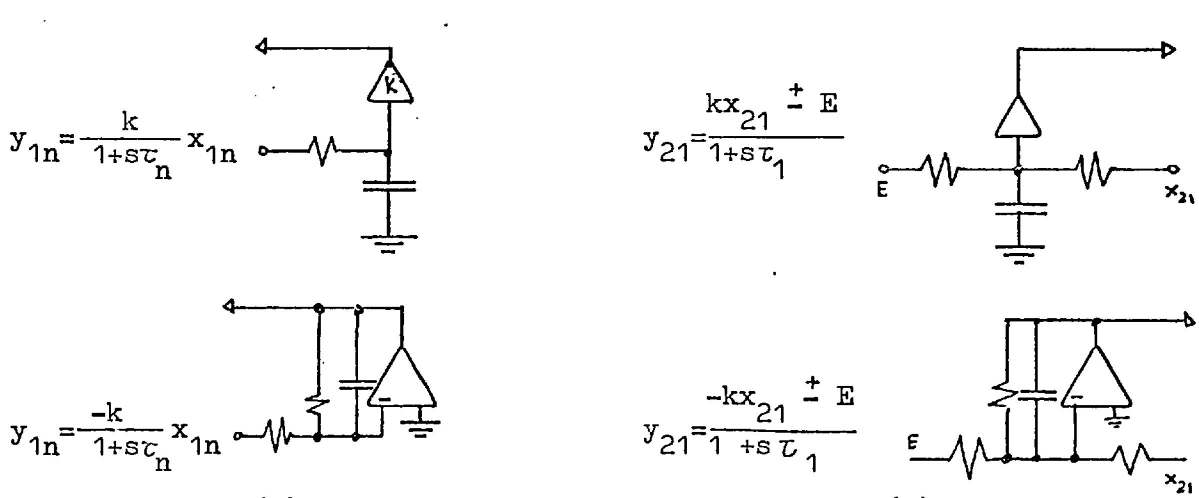
Under the above transformations the output terminated reactive lowpass arm will be described by an expression of the form (Table 5.1)

$$y_{1n} = \pm \frac{k}{1 + s\tau_n} x_n \quad , \quad k > 0$$

which may be realized as in Fig. 5.4a. Similarily the input terminated lowpass reactive arm will be described by (Table 5.1)

$$y_{21} = \frac{kx_{21} \pm E}{1 + s\tau_1} \quad , \quad k > 0$$

which can be realized as in Fig. 5.4b below.



(a) (b)

Fig. 5.4

Therefore, under any form-compatible leapfrog LT only one amplifier is needed for the input and one for the output lowpass reactive arms when transformed in conjunction with the terminations. Note that any amplifier used to scale the input E is not really necessary.

For the intermediate arms transformed using the leapfrog form-compatible linear transformations we have that for a series arm Z under the transformation

$$\mathfrak{a}_{1Z} = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \delta_1 \end{bmatrix}, \quad \mathfrak{a}_{2Z} = \begin{bmatrix} \alpha_2 & 0 \\ 0 & \delta_2 \end{bmatrix}$$

the LT structure description is given below

$$y_1 = -\frac{\delta_1}{\delta_2} y_2$$

$$y_2 = -\frac{\delta_2}{\alpha_1 Z} \left(x_1 - \frac{\alpha_1}{\alpha_2} x_2\right)$$

From these expressions it is evident that constraining y_1 to be equal to y_2 and removing inversions from the input variables, the following conditions must be held: $\alpha_2 = -\alpha_1$ and $\delta_2 = -\delta_1$. Therefore, the general form-compatible leapfrog transformations for minimum number of operational amplifiers for the LT structure of the series arms Z is as follows

$$\mathfrak{a}_{1Z} = \begin{bmatrix} \alpha & 0 \\ 0 & \delta \end{bmatrix}, \quad \mathfrak{a}_{2Z} = \begin{bmatrix} -\alpha & 0 \\ 0 & -\delta \end{bmatrix} \tag{5.8a}$$

yielding $y_{1Z} = y_{2Z} = \frac{\delta}{\alpha Z} (x_{1Z} + x_{2Z})$

Similarily, it can be found that the general form-compatible leapfrog transformations for the shunt arms Y for minimum number of operational amplifiers, is of the general form

$$\mathfrak{a}_{1Y} = \begin{bmatrix} 0 & \beta \\ \gamma & 0 \end{bmatrix} \qquad \mathfrak{a}_{2Y} = \begin{bmatrix} 0 & \beta \\ \gamma & 0 \end{bmatrix} \tag{5.8b}$$

yielding $y_{1Y} = y_{2Y} = \frac{\gamma}{\beta Y}(x_{1Y} + x_{2Y})$.

From the above relationships it can be observed that for a desirable negative integration operation for a series arm Z_i to exist, the parameters α and δ must be of opposite sign. The compatibility relationship between the two adjacent matrices $\mathfrak{a}_{2Zi}$ and $\mathfrak{a}_{1Y(i+1)}$ will make β and γ of the shunt arm $Y_{(i+1)}$ have the same sign and, of course, the sign of the corresponding integration positive

Therefore, the sign of the integration required within the form-compatible LT structures alternates. The minimization of the positive integrations in the intermediate stages is evidently possible only for odd order filters since in that case there is an odd number of intermediate LT structures which require an integrator, and by making the first (or the last) of them negative (inverting) we minimize the number of positive (non-inverting) integrators.

Once the transformation matrices for the first (or the last) of the intermediate reactive arms is chosen, with the help of eqns (5.8), to yield an inverting integrator, the rest of the leapfrog form-compatible linear transformations can be found from the same equations. Fig. 5.5 below illustrates the LTA simulation of a 5th

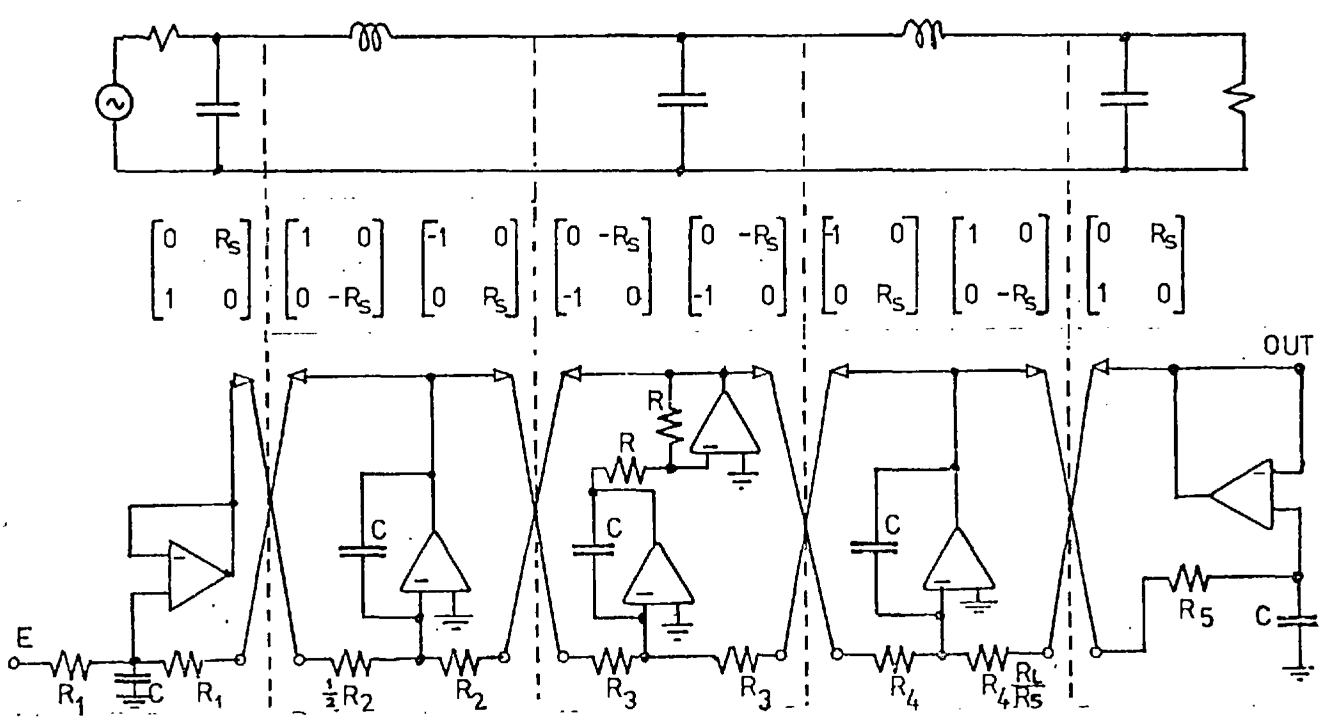


Fig. 5.5

order lowpass polynomial filter using the form-compatible linear transformations to minimize the number of the operational amplifiers. The resulting leapfrog LTA structure when compared to the original leapfrog structure of Fig.5.1 illustrates the benefitial use of these transformations. It is evident from these two Figures that in the original leapfrog case of Fig. 5.1, nine operational amplifiers are used whilst in the improved version of Fig. 5.5 only six operational amplifiers are employed two of which are used as buffers.

The three classes of linear transformations sets examined in this chapter together with their special cases are summarized in Table 5.4 and can be used as a quick reference table for design purposes.

TABLE 5.4

LT CASE	CONDITIONS	SPECIAL CASES
GENERAL (ACYCLIC)	NONE	ALL
SERIES-SHUNT COMPATIBLE	$\{a_{1i}, a_{2i}\} = \{a_{1(i+2)}, a_{2(i+2)}\}$ $\forall\ i=1,2,\ldots,(n-2)$	LEAPFROG STRUCTURES (Table 5.1) For all series arms $a_{1Z}=\begin{bmatrix}1 & 0\\ 0 & R\end{bmatrix}$, $a_{2Z}=\begin{bmatrix}-1 & 0\\ 0 & R\end{bmatrix}$ For all shunt arms Y $a_{1Y}=\begin{bmatrix}0 & -R\\ -1 & 0\end{bmatrix}$, $a_{2Y}=\begin{bmatrix}0 & -R\\ 1 & 0\end{bmatrix}$ Note : This transformation is suitable for lowpass polynomial ladders and the bandpass ladder that can be derived from them via LP to BP transformation.

TABLE 5.4 (continued)

SELF COMPATIBLE	$a_{1i} = a_{1(i+1)}$ $a_{2i} = a_{2(i+1)}$ $\forall\ i=1,2,\ldots(n-1)$	<u>ALL INTEGRATOR HIGHPASS FILTERS</u> For all series and shunt arms (see Table 5.2) $a_1 = \begin{bmatrix} 1 & 0 \\ 0 & R \end{bmatrix},\ a_2 = \begin{bmatrix} 0 & -R \\ 1 & 0 \end{bmatrix}$ <u>WAVE ACTIVE FILTERS</u> For all arms (see Table 5.3) $a_1 = a_2 = \begin{bmatrix} 1 & R \\ 1 & -R \end{bmatrix}$
FORM COMPATIBLE	$\lvert a_{1i}\rvert = \lvert a_{1(i+1)}\rvert$ $\lvert a_{2i}\rvert = \lvert a_{2(i+1)}\rvert$	<u>MINIMUM OP.AMP. LEAPFROG STRUCTURES</u> For the series arms $a_{1Zi} = \begin{bmatrix} \alpha_i & 0 \\ 0 & \delta_i \end{bmatrix},\ a_{2Zi} = \begin{bmatrix} -\alpha_i & 0 \\ 0 & -\delta_i \end{bmatrix}$ For the shunt arms $a_{1Yi} = \begin{bmatrix} 0 & \beta_i \\ \gamma_i & 0 \end{bmatrix},\ a_{2Yi} = \begin{bmatrix} 0 & \beta_i \\ \gamma_i & 0 \end{bmatrix}$ $\lvert\alpha_i\rvert = \lvert\gamma_i\rvert = 1,\ \lvert\beta_i\rvert = \lvert\delta_i\rvert = R_s$ The sign of $\alpha_i\delta_i$ and $\beta_i\gamma_i$ determines only the sign of the corresponding integrator which varies periodically. (see eqn. (5.8))

5.4 THE ACYCLIC TRANSFORMATIONS

DEFINITION 5.4 : A set of compatible linear transformation pairs

$$\{\mathfrak{a}_{11}, \mathfrak{a}_{21}\}, \ldots, \{\mathfrak{a}_{1i}, \mathfrak{a}_{2i}\}, \ldots, \{\mathfrak{a}_{1n}, \mathfrak{a}_{2n}\}$$

employed in the transformation of a ladder network $\{N_1, \ldots, N_i, \ldots, N_n\}$ is called acyclic when no constraints are imposed on these pairs apart from the necessary compatibility relationship.

This is, in other words, the general case of the LTA method leading to the acyclic LTA filter structures. This term is due to the fact that transformation matrices once used for transforming a ladder arm are not used again. Their re-deployment would enable the definition of a period or cycle of transformations as in the previous cases studied in this chapter. The use of such acyclic transformations is rather intuitive and heuristic and requires considerable experience on behalf of the designer who must be very familiar with the general LTA method, otherwise the resulting LTA structures will be very complex indeed.

However, one can draw some useful rules for using the acyclic case, to ensure that certain conditions are automatically met. Thus one should start from a matrix $\mathfrak{a}_{1n}$ which ensures the existence of a convenient output node in accordance with the results of section 4.3 so that a practical LT structure may be derived for the last reactive arm of the ladder taken in conjunction with the load resistance. With $\mathfrak{a}_{1n}$ specified, its adjacent right transformation matrix $\mathfrak{a}_{2(n-1)}$ is automatically defined by the compatibility relationship. Next, the matrix $\mathfrak{a}_{1(n-1)}$ can now be so chosen that the transformation pair $\{\mathfrak{a}_{1(n-1)}, \mathfrak{a}_{2(n-1)}\}$ leads to a practical and easily realizable LT structure for the reactive arm N_{n-1}. This can be checked using the general expressions for the transfer ratios K,L,M and N (see chapter 2) unless the transformation pair falls into one of the categories defined in this chapter in wich case the appropriate Tables can be used.

Once the matrix $\alpha_{1(n-1)}$ has been determined, its adjacent matrix $\alpha_{2(n-2)}$ is then automatically known and the acyclic procedure continues on this basis so that the LT structure relizations can be tailored to the needs of the designer. The realization of the final LTA structure, from such choices of transformation pairs, whether practical or not, will in any case be a novel one since all known structures result from cyclic LTA procedures.

5.5 FREQUENCY DEPENDENT LINEAR TRANSFORMATIONS (FDLT)

The dimensions of the elements of the transformation matrices have been considered so far to be such that α and γ are dimensionless whereas β and δ are taken to have dimensions of impedances. The simplest form of such a scheme, as illustrated so far, is to consider α and γ as numerical constants with β and δ set equal to some resistive values (i.e. frequency independent). In such a case the linear transformations become frequency independent since the elements of the transformation matrices do not vary with frequency.

However, if any one of the elements of the transformation matrices of a set of linear transformation pairs is taken to depend on frequency, then the transformation becomes frequency dependent and it is our aim in this section to study such transformations and to assess the way in wich they affect the corresponding LT structures.

The frequency dependent elements will be chosen in such a way, that the transformed variables will still represent voltages and hence α and γ can only be voltage transfer ratios whilst β and δ can be impedences (i.e. they can be measured in Ω). The TRE and TRI conditions for low complexity LT structures established in section 4.1 and summarized in Table 4.1 in the form of appropriate zero entries in the transformation matrices are still valid although they have been derived under the assumption that the transformation matrices are frequency independent. This is so, because these zero entries have been determined in both the TRE and TRI approaches from an expression of the form

$$\varphi(\alpha,\beta,\gamma,\delta)X(s) + \theta(\alpha,\beta,\gamma,\delta) = 0 \qquad (5.9)$$

where X(s) is the immittance of the arm under consideration, by considering $\varphi(\alpha,\beta,\gamma,\delta) = 0$. This condition then allowed a frequency independent relationship between the transformation parameters of the form

$$\theta(\alpha,\beta,\gamma,\delta) = 0 .$$

On considering the FDLT case with $\varphi(\alpha,\beta,\gamma,\delta) \neq 0$, eqn. (5.9) establishes a relationship between the elements of the transformation matrices and the immittance X(s) of the arm under consideration. This implies that at least one of the elements will be related to X(s). The compatibility relationship for cross cascade interconnection (eqn. 3.17a)) will then transfer this transformation matrix element to the adjacent matrix thus making the adjacent LT structure dependent on the immittance X(s) of the neighbouring arm. This will break the desirable one-to-one correspondence between the ladder arms and the LT structures wich contributes greatly to thelow sensitivity of the LTA simulated filters. Therefore, in the FDLT case $\varphi(\alpha,\beta,\gamma,\delta)$ must be equal to zero and from eqn. (5.9) we obtain $\theta(\alpha,\beta,\gamma,\delta) = 0$. This implies that for the FDLT case the TRE and TRI conditions will be derived in the same manner as already done for the frequency independent case. This , of course, will lead to the same zero entries as those of Table 4.1. The nonzero matrix elements shown there can then be frequency dependent. The secondary conditions for further simplifications of Table 4.1 can still be used as a guide to even lower complexity realizations of the corresponding LT structures. Moreover, all families of LTA procedures presented in this chapter can be made frequency dependent by considering one or more matrix elements to depend on the complex frequency s.

The reason for using FDLT is that the degree of the voltage transfer ratios K,L,M and N may be influenced, in contrast to the frquency independent case where the degree of an LT structure corresponding to a ladder arm is determined by its immittance. Consequently it would be possible to reduce the order of an LT structure for an arm

N_i of the ladder. However, due to the compatibility relationship the frequensy dependent element(s) of a linear transformation pair $\{a_{1i}, a_{2i}\}$ will be transmitted to the transformation matrices used for the adjacent arms so affecting the degree of the neighbouring LT structures. Therefore, the use of FDLT need not be used for the transformation of canonic ladders[1] in that, reduction of the order of an LT structure in such filters will be compensated for in the LT structures of the adjacent ladder arms.

However, it has been found that if the ladder to be transformed has a T or a Π of similar impedances (Fig. 5.6a) or indeed of impedances having a common pole (Fig. 5.6b), the FDLT procedure may be applied beneficially. This is due to the fact that if one or more elements of

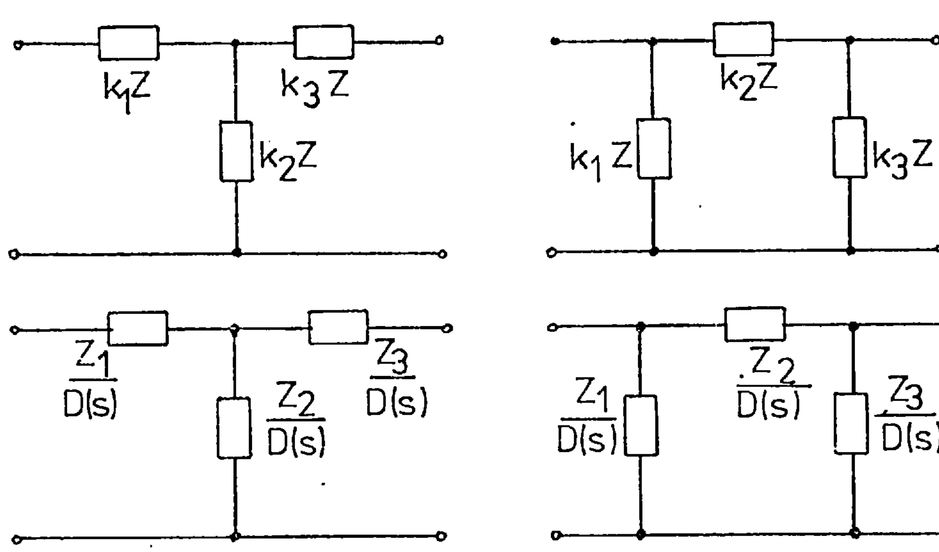


Fig. 5.6

the transformation matrices for the middle arm of the Π or T section are chosen to be frequency dependent so as to reduce the order of its corresponding LT structure, the compatibility relationship will transfer

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(1) Canonic in the sense that they employ the minimum number of reactive elements consistent with the order of the filter.

this frequency dependence to the adjacent matrices thereby reducing the degree of the LT structures of the adjacent arms. To illustrate the above points and also the manner in which the FDLT may be usefully employed, let us consider an example in some detail and also another design of practical significance.

A doubly terminated reactive ladder is shown in Fig. 5.7 [4], consisting of five reactive elements connected in such a way that an inductive T is formed. The straightforward application of the frequency

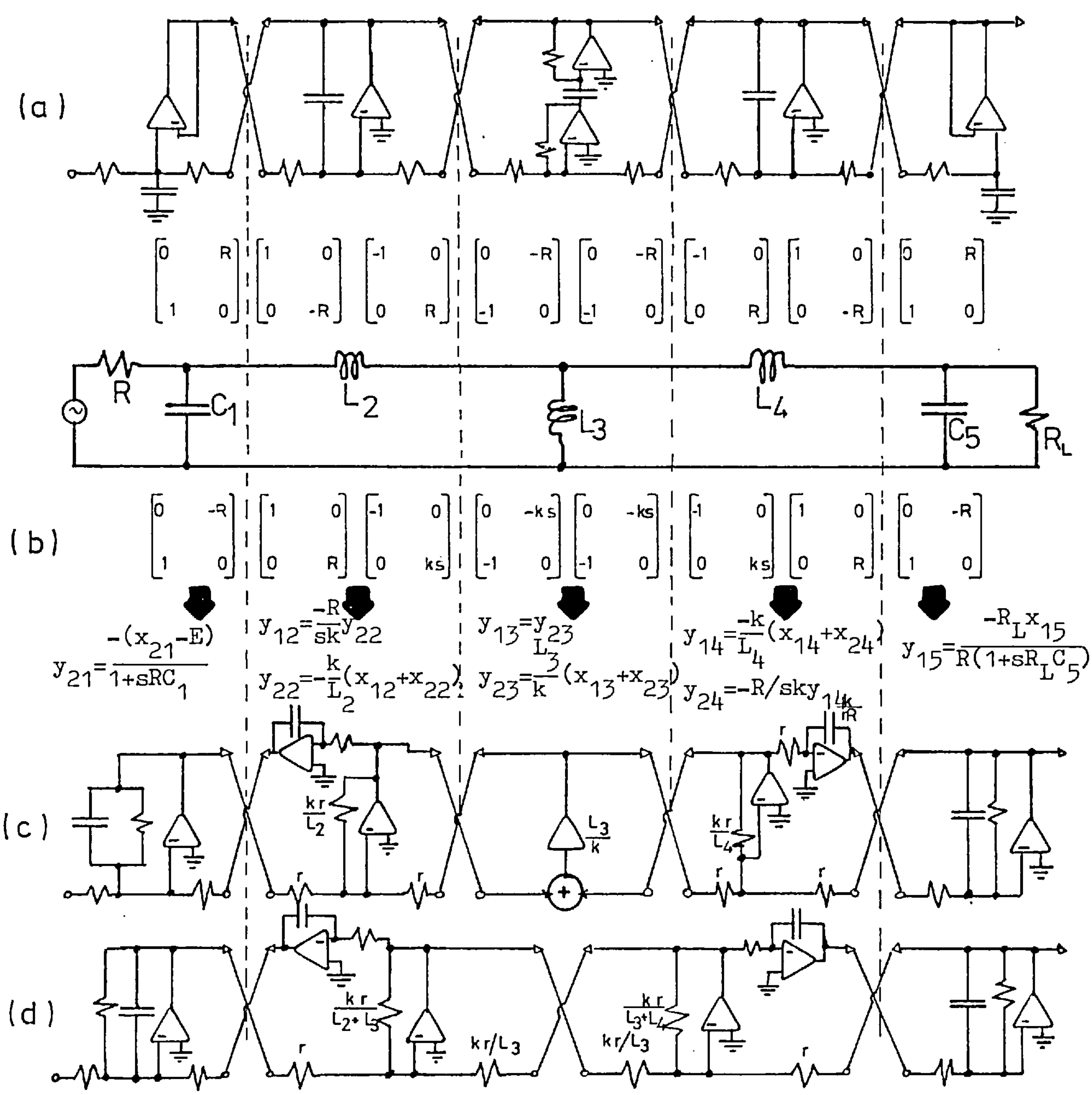


Fig. 5.7

independent form compatible transformation set, used for the transformation of the ladder of Fig. 5.5 would lead to an LTA filter with five capacitors and six operational amplifiers one of which is connected as a differentiator as shown in Fig. 5.7a. The use of the FDLT shown in Fig. 5.7b, with the appropriate frequency dependent matrices, yields the LTA structure of Fig. 5.7c with the evident simplification of Fig.5.7d. The final LTA structure employs only four rather than five capacitors and the same number of operational amplifiers as the structure of Fig. 5.7a, with the very noticable difference that no differentiation is involved in this case.

It is worth noting that not all transformation matrices are frequency dependent. In fact, frequency dependent transformations are used for transforming that part of the ladder which has a Π or T of the kind shown in Fig. 5.6. The FDLT procedure is very powerful in the very important case, from the practical point of view, of zig-zag bandpass filters which have arms of the kind shown in Fig.5.8 [4]. In such a case the filter is composed of Π and T networks of the form shown in Fig. 5.6b where the series arms have an impedance of the form

$$Z_z = \frac{1+s^2(C_1+C_2)L}{s(1+s^2LC_1)C_2}$$

and the shunt arms have admittances of the form $Y=1/Z_y$ where

$$Z_y = \frac{1+s^2LC_1}{s(C_1+C_2)(1+s^2LC_1C_2/(C_1+C_2))} .$$

Both arm impedances have a pole at s=0 which may be removed by using the FDLT approach. The detailed FDLT simulation of an 8th order zig-zag filter is presented in chapter 7.

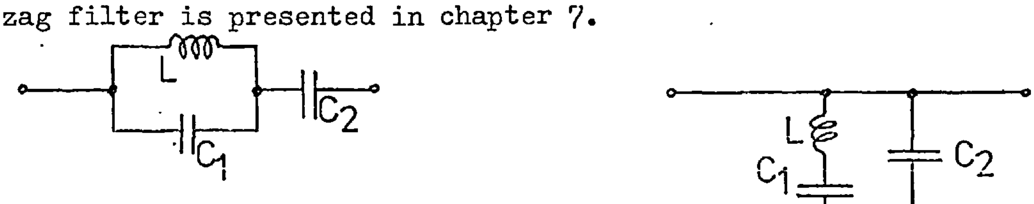


Fig. 5.8

CHAPTER 6

SELF-DUAL LTA FILTERS

CHAPTER 6

SELF-DUAL LTA FILTERS

INTRODUCTION

In the previous chapter a classification of the LTA structures was presented and some well known ladder simulation methods were interpreted as special cases of the general LTA procedure. This classification was based on imposed relationships between the individual linear transformations used for the LTA simulation of a given ladder network.

However, there are some more degrees of freedom within the presented classes of LTA procedures which can be effectively utilized to produce LTA filter structures possessing highly desirable features. In this chapter the series-shunt compatible case is so constrained to yield structures that possess the property of self-duality as defined in section 6.1. The introduction of two more constraints, one in the form of a simple linear relationship between the transformed variables and the other in the form of the so called "rotation" or R-property, yields even more practical and highly modular active RC realizations.

6.1 NECESSARY AND SUFFICIENT CONDITIONS FOR SELF DUALITY

DEFINITION 6.1 : A set of linear transformation pairs

$$\{\mathcal{Q}_{11},\mathcal{Q}_{21}\}, \ldots ,\{\mathcal{Q}_{1i},\mathcal{Q}_{2i}\}, \ldots ,\{\mathcal{Q}_{1n},\mathcal{Q}_{2n}\}$$

used for the LTA simulation of a ladder $\{N_1, \ldots ,N_i, \ldots ,N_n\}$ is said to be self-dual if and only if

(i) It is series-shunt compatible (see section 5.1)

(ii) The LT structures of a series arm Z_μ (shunt arm Y_λ) is identical to that of its dual shunt arm $Y_\mu = Z_\mu/R_o^2$ (series arm $Z_\lambda = Y_\lambda R_o^2$), where R_o is a convenient resistance value.

Such a transformation yields active RC filters which will be referred to as self-dual LTA filters.

The necessary and sufficient conditions for self-duality in terms of the elements of the transformation matrices are derived below. For the series-shunt compatible transformation case we have, according to the previous chapter, all series arms transformed using the same transformation pair below

$$\mathcal{Q}_{1Z}=\begin{bmatrix}\alpha_1 & \beta_1\\ \gamma_1 & \delta_1\end{bmatrix} \qquad \mathcal{Q}_{2Z}=\begin{bmatrix}\alpha_2 & \beta_2\\ \gamma_2 & \delta_2\end{bmatrix} \qquad (6.1a)$$

whilst all shunt arms are transformed using the transformation pair

$$\mathcal{Q}_{1Y}=\begin{bmatrix}\gamma_2 & -\delta_2\\ \alpha_2 & -\beta_2\end{bmatrix} \qquad \mathcal{Q}_{2Y}=\begin{bmatrix}\gamma_1 & -\delta_1\\ \alpha_1 & -\beta_1\end{bmatrix} \qquad (6.1b)$$

where the series-shunt compatibility has been taken into account. Under these conditions the LT-chain matrix descriptions of the LT structures for the two ladder cases (eqn. (2.6) and eqn. (2.7) respectively) are given below :

For the series arm Z we have

$$\begin{bmatrix} x_1 \\ \\ y_1 \end{bmatrix} = \begin{bmatrix} \dfrac{\alpha_1\gamma_2 Z + \alpha_1\delta_2 + \beta_1\gamma_2}{\Delta_2} & -\dfrac{\alpha_1\alpha_2 Z + \alpha_1\beta_2 + \beta_1\alpha_2}{\Delta_2} \\ \\ \dfrac{\gamma_1\gamma_2 Z + \gamma_1\delta_2 + \delta_1\gamma_2}{\Delta_2} & -\dfrac{\gamma_1\alpha_2 Z + \gamma_1\beta_2 + \delta_1\alpha_2}{\Delta_2} \end{bmatrix} \begin{bmatrix} x_2 \\ \\ y_2 \end{bmatrix} \qquad (6.2a)$$

For the shunt arm Y we have

$$\begin{bmatrix} x_1 \\ \\ y_1 \end{bmatrix} = \begin{bmatrix} \dfrac{\beta_1\delta_2 Y - \alpha_1\delta_2 - \beta_1\gamma_2}{\Delta_1} & -\dfrac{\delta_1\delta_2 Y - \gamma_1\delta_2 - \delta_1\gamma_2}{\Delta_1} \\ \\ \dfrac{\beta_1\beta_2 Y - \alpha_1\beta_2 - \beta_1\alpha_2}{\Delta_1} & -\dfrac{\delta_1\beta_2 Y - \gamma_1\beta_2 - \delta_1\alpha_2}{\Delta_1} \end{bmatrix} \begin{bmatrix} x_2 \\ \\ y_2 \end{bmatrix} \qquad (6.2b)$$

where $\Delta_1 = \det \mathcal{Q}_{1Z}$ and $\Delta_2 = \det \mathcal{Q}_{2Z}$.

The conditions for self-duality are derived by requiring the two LT-chain matrices of eqn. (6.2a) and eqn. (6.2b) to be equal when $Y=Z/R^2$, i.e. the two ladder arms are dual. This yields the following relations:

$$Z(\alpha_1\gamma_2 R^2\Delta_1 - \beta_1\delta_2\Delta_2) + R^2(\alpha_1\delta_2 + \beta_1\gamma_2)(\Delta_1 + \Delta_2) = 0$$

$$Z(\alpha_1\alpha_2 R\,\Delta_1 - \delta_1\delta_2\Delta_2) + R^2(\alpha_1\beta_2 + \beta_1\alpha_2)\Delta_1 + (\gamma_1\delta_2 + \delta_1\gamma_2)\Delta_2 = 0$$

$$Z(\gamma_1\gamma_2 R^2\Delta_1 - \beta_1\beta_2\Delta_2) + R^2(\gamma_1\delta_2 + \delta_1\gamma_2)\Delta_1 + (\alpha_1\beta_2 + \beta_1\alpha_2)\Delta_2 = 0$$

$$Z(\gamma_1\alpha_2 R^2\Delta_1 - \delta_1\beta_2\Delta_2) + R^2(\gamma_1\beta_2 + \delta_1\alpha_2)(\Delta_1 + \Delta_2) = 0$$

For the entries $\alpha_i, \beta_i, \gamma_i$ and δ_i to be independent of the arm impedance Z, we obtain from the above equations the following relationships which must hold even for the FDLT case:

$$\frac{R^2\Delta_1}{\Delta_2} = \frac{\beta_1\delta_2}{\alpha_1\gamma_2} = \frac{\delta_1\delta_2}{\alpha_1\alpha_2} = \frac{\beta_1\beta_2}{\gamma_1\gamma_2} = \frac{\delta_1\beta_2}{\gamma_1\alpha_2} \qquad (6.3a)$$

$$(\alpha_1\delta_2 + \beta_1\gamma_2)(\Delta_1 + \Delta_2) = 0 \quad , \quad (\gamma_1\beta_2 + \delta_1\alpha_2)(\Delta_1 + \Delta_2) = 0 \qquad (6.3b)$$

$$(\alpha_1\beta_2 + \beta_1\alpha_2)\Delta_1 + (\gamma_1\delta_2 + \delta_1\gamma_2)\Delta_2 = 0 \qquad (6.3c)$$

$$(\alpha_1\beta_2 + \beta_1\alpha_2)\Delta_2 + (\gamma_1\delta_2 + \delta_1\gamma_2)\Delta_1 = 0 \qquad (6.3d)$$

It can be found from eqn. (6.3a) that

$$\beta_1\alpha_2 = \delta_1\gamma_2 \quad \text{and} \quad \alpha_1\beta_2 = \gamma_1\delta_2 \tag{6.4}$$

under which, eqn. (6.3c) and eqn.(6.3d) become identical both yielding:

$$(\gamma_1\delta_2 + \delta_1\gamma_2)(\Delta_1 + \Delta_2) = 0 \tag{6.5}$$

From eqn. (6.3a), eqn. (6.3b) and eqn. (6.5) the necessary and sufficient conditions for self-duality follow. Mathematical manipulations taking into account that $\Delta_i \neq 0$, i=1,2 produce the following explicit conditions for self-duality :

$$\frac{\delta_1\delta_2}{\alpha_1\alpha_2} = \frac{\beta_1\beta_2}{\gamma_1\gamma_2} = \frac{\beta_1\delta_2}{\alpha_1\gamma_2} = \frac{\delta_1\beta_2}{\gamma_1\alpha_2} = -R^2 \quad , \quad \Delta_1 = -\Delta_2 \tag{6.6}$$

where R is a convenient resistance value used for the definition of duality between a series impedance Z and a shunt admittance $Y = Z/R^2$.

6.2 PRACTICAL SELF-DUAL LTA FILTERS

Any series-shunt compatible set of linear transformations satisfying the necessary and sufficient conditions of eqn. (6.6), is a self-dual set of transformations. In such a case the LT description of the series arm Z and that of the dual shunt arm $Y=Z/R^2$ will be identical and their general form will be as given below:

$$y_1 = K_s x_1 + L_s x_2$$
$$y_2 = M_s x_1 + N_s x_2 \qquad (6.7)$$

The simplification achieved , however, is not evident from such expressions since four voltage transfer ratios are still required to be realized in order to generate the outputs y_1 and y_2 from the inputs x_1 and x_2. Nevertheless, it can be noticed that the necessary and sufficient conditions for self-duality impose only five relationships on the eight elements of the two series-shunt compatible transformation matrices, leaving three degrees of freedom which may be effectively utilized to lead to more practical realizations. To this end, we require the following condition to hold between the transformed variables of each and every ladder arm:

$$y_1 + \varepsilon y_2 = \varepsilon_1 x_1 + \varepsilon_2 x_2 \qquad (6.8)$$
$$\varepsilon_1 = 1,0,-1 \quad , \quad \varepsilon_2 = 1,0,-1 \quad , \quad \varepsilon = \pm 1$$

The usefulness of the above constraint lies in the fact that under it, only one of the expressions of eqn. (6.7) is required to be realized, so as to generate one output, the other output being directly derivable from eqn. (6.8).

Consequently, the necessary and sufficient conditions for a self-dual transformation to satisfy the additional constraint of practical significance of eqn. (6.8) are as follows.

$$\gamma_1 = \varepsilon_1 \alpha_1$$

$$\gamma_2 = \frac{\varepsilon_2}{\varepsilon} \alpha_2$$

$$\alpha_1 \delta_2 (\varepsilon - \varepsilon_1 \varepsilon_2) = \Delta_1 \tag{6.9}$$

$$\alpha_2 \delta_1 (\varepsilon - \varepsilon_1 \varepsilon_2) = \varepsilon^2 \Delta_2$$

If the nonsingularity of the transformation matrices is to be preserved we also have:

$$\alpha_1 \alpha_2 \delta_1 \delta_2 \neq 0 \quad \text{and} \quad \varepsilon \neq \varepsilon_1 \varepsilon_2 \quad . \tag{6.9a}$$

The conditions of eqn. (6.9) are equivalent to the following:

$$\gamma_1 = \varepsilon_1 \alpha_1 \tag{6.10a}$$

$$\gamma_2 = \frac{\varepsilon_2}{\varepsilon} \alpha_2 \tag{6.10b}$$

$$\alpha_1 \delta_2 + \delta_1 \alpha_2 = 0 \tag{6.10c}$$

$$\alpha_1 \delta_2 (\varepsilon - \varepsilon_1 \varepsilon_2) = \Delta_1$$

from which it is found that under self-duality only three of these are independent, the fourth condition being derivable from these three and the self-duality conditions of eqn. (6.6). Therefore, the necessary and sufficient conditions for self duality with the linear constraint $y_1 + \varepsilon y_2 = \varepsilon_1 x_1 + \varepsilon_2 x_2$ are summarized below:

$$\frac{\delta_1 \delta_2}{\alpha_1 \alpha_2} = \frac{\beta_1 \beta_2}{\gamma_1 \gamma_2} = \frac{\beta_1 \delta_2}{\alpha_1 \gamma_2} = \frac{\delta_1 \beta_2}{\gamma_1 \alpha_2} = -R^2 \ ,$$

$$\Delta_1 = -\Delta_2 \tag{6.11}$$

$$\gamma_1 = \varepsilon_1 \alpha_1$$

$$\gamma_2 = \frac{\varepsilon_2}{\varepsilon} \alpha_2$$

$$\alpha_1 \delta_2 + \delta_1 \alpha_2 = 0$$

Solving these eight equations for the eight unknown entries of the transformation matrices, we find the following transformation matrices for the series arms :

$$\mathrm{a}_{1Z}=\begin{bmatrix} 1 & \frac{\varepsilon_2}{\varepsilon}\mu R \\ \varepsilon_1 & \mu R \end{bmatrix}, \quad \mathrm{a}_{2Z}=\begin{bmatrix} -\frac{1}{\varepsilon} & \frac{\varepsilon_1}{\varepsilon}\mu R \\ -\varepsilon_2 & \frac{1}{\varepsilon}\mu R \end{bmatrix} \tag{6.12a}$$

and consequently, for the shunt arms we have :

$$\mathrm{a}_{1Y}=\begin{bmatrix} -\varepsilon_2 & -\frac{1}{\varepsilon}\mu R \\ -\frac{1}{\varepsilon} & -\frac{\varepsilon_1}{\varepsilon}\mu R \end{bmatrix}, \quad \mathrm{a}_{2Y}=\begin{bmatrix} \varepsilon_1 & -\mu R \\ 1 & -\frac{\varepsilon_2}{\varepsilon}\mu R \end{bmatrix} \tag{6.12b}$$

In both cases the parameter μ assumes values ± 1.

The corresponding LT description of the series arm under this constrained self-dual transformation is given by:

$$y_1 = \frac{\varepsilon_1 Z + \mu R(1-\varepsilon_1^2)}{Z+\mu R(\varepsilon\varepsilon_2-\varepsilon_1)}x_1 + \frac{\mu R(\varepsilon-\varepsilon_1\varepsilon_2)}{Z+\mu R(\varepsilon\varepsilon_2-\varepsilon_1)}x_2$$

$$y_2 = \frac{-\mu R(\varepsilon-\varepsilon_1\varepsilon_2)}{Z+\mu R(\varepsilon\varepsilon_2-\varepsilon_1)}x_1 + \frac{\varepsilon\varepsilon_2 Z + \mu R(\varepsilon_2^2-1)}{Z+\mu R(\varepsilon\varepsilon_2-\varepsilon_1)}x_2 \tag{6.13a}$$

whilst the corresponding relationships for the shunt arm can be found from the above for $Z=YR^2$, as follows:

$$y_1 = \frac{\varepsilon_1 RY + \mu(1-\varepsilon_1^2)}{RY+\mu(\varepsilon\varepsilon_2-\varepsilon_1)}x_1 + \frac{\mu(\varepsilon-\varepsilon_1\varepsilon_2)}{RY+\mu(\varepsilon\varepsilon_2-\varepsilon_1)}x_2$$

$$y_2 = \frac{-\mu(\varepsilon-\varepsilon_1\varepsilon_2)}{RY+\mu(\varepsilon\varepsilon_2-\varepsilon_1)}x_1 + \frac{\varepsilon\varepsilon_2 RY + \mu(\varepsilon_2^2-1)}{RY+\mu(\varepsilon\varepsilon_2-\varepsilon_1)}x_2 \tag{6.13b}$$

Due to the self-duality of the transformation of eqn. (6.12), only one of the above expressions, either (6.13a) or (6.13b), is sufficient to describe the self dual structures.

All self dual cases for which $y_1 + \varepsilon y_2 = \varepsilon_1 x_1 + \varepsilon_2 x_2$, are tabulated in Table 6.1 for the various combinations of ε, ε_1 and ε_2. The parameter μ has been incorporated in R which can now assume either positive or negative resistance values. However, it can be observed from this Table that cases 2, 3 and 13 are not suitable for practical and convenient realizations. In the last column of Table 6.1, we indicate the output which can most conveniently be realized by an active RC network, the other output being of course realized using eqn. (6.8).

TABLE 6.1

No	ε_1	ε_2	ε	a_{1Z}	a_{2Z}	LT STRUCTURE DESCRIPTION		
1 *	-1	-1	-1	$\begin{bmatrix}1 & R\\-1 & R\end{bmatrix}$	$\begin{bmatrix}1 & R\\1 & -R\end{bmatrix}$ $y_1-y_2=-x_1-x_2$	$y_1=\frac{-Z}{Z+2R}x_1+\frac{-2R}{Z+2R}x_2$	$y_2=\frac{2R}{Z+2R}x_1+\frac{Z}{Z+2R}x_2$	y_1 y_2
	-1	-1	1	$\varepsilon=\varepsilon_1\varepsilon_2$		INADMISSIBLE		
	-1	1	-1	$\varepsilon=\varepsilon_1\varepsilon_2$		INADMISSIBLE		
2	-1	1	1	$\begin{bmatrix}1 & R\\-1 & R\end{bmatrix}$	$\begin{bmatrix}-1 & -R\\-1 & R\end{bmatrix}$ $y_1+y_2=x_1+x_2$	$y_1=\frac{-Z}{Z+2R}x_1+\frac{2R}{Z+2R}x_2$	$y_2=\frac{-2R}{Z+2R}x_1+\frac{Z}{Z+2R}x_2$	X
	1	-1	-1	$\varepsilon=\varepsilon_1\varepsilon_2$		INADMISSIBLE		
3	1	-1	1	$\begin{bmatrix}1 & -R\\1 & R\end{bmatrix}$	$\begin{bmatrix}-1 & R\\1 & R\end{bmatrix}$ $y_1+y_2=x_1-x_2$	$y_1=\frac{Z}{Z-2R}x_1+\frac{2R}{Z-2R}x_2$	$y_2=\frac{-2R}{Z-2R}x_1+\frac{-Z}{Z-2R}x_2$	X
4 *	1	1	-1	$\begin{bmatrix}1 & -R\\1 & R\end{bmatrix}$	$\begin{bmatrix}1 & -R\\-1 & -R\end{bmatrix}$ $y_1-y_2=x_1+x_2$	$y_1=\frac{Z}{Z-2R}x_1+\frac{-2R}{Z-2R}x_2$	$y_2=\frac{2R}{Z-2R}x_1+\frac{-Z}{Z-2R}x_2$	y_1 y_2 $R<0$
	1	1	1	$\varepsilon=\varepsilon_1\varepsilon_2$		INADMISSIBLE		
5 *	0	-1	-1	$\begin{bmatrix}1 & R\\0 & R\end{bmatrix}$	$\begin{bmatrix}1 & 0\\1 & -R\end{bmatrix}$ $y_1-y_2=-x_2$	$y_1=\frac{R}{Z+R}(x_1-x_2)$	$y_2=\frac{R}{Z+R}x_1+\frac{Z}{Z+R}x_2$	y_2
6 *	0	-1	1	$\begin{bmatrix}1 & -R\\0 & R\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\1 & R\end{bmatrix}$ $y_1+y_2=-x_2$	$y_1=\frac{R}{Z-R}(x_1+x_2)$	$y_2=\frac{-R}{Z-R}x_1+\frac{-Z}{Z-R}x_2$	y_1 $R<0$
7 *	0	1	-1	$\begin{bmatrix}1 & -R\\0 & R\end{bmatrix}$	$\begin{bmatrix}1 & 0\\-1 & -R\end{bmatrix}$ $y_1-y_2=x_2$	$y_1=\frac{R}{Z-R}(x_1-x_2)$	$y_2=\frac{R}{Z-R}x_1+\frac{-Z}{Z-R}x_2$	y_2 $R<0$
8 *	0	1	1	$\begin{bmatrix}1 & R\\0 & R\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\-1 & R\end{bmatrix}$ $y_1+y_2=x_2$	$y_1=\frac{R}{Z+R}(x_1+x_2)$	$y_2=\frac{-R}{Z+R}x_1+\frac{Z}{R+Z}x_2$	y_1
9 *	-1	0	-1	$\begin{bmatrix}1 & 0\\-1 & R\end{bmatrix}$	$\begin{bmatrix}1 & R\\0 & -R\end{bmatrix}$ $y_1-y_2=-x_1$	$y_1=\frac{-Z}{Z+R}x_1+\frac{-R}{Z+R}x_2$	$y_2=\frac{R}{Z+R}(x_1-x_2)$	y_1
10 *	-1	0	1	$\begin{bmatrix}1 & 0\\-1 & R\end{bmatrix}$	$\begin{bmatrix}-1 & -R\\0 & R\end{bmatrix}$ $y_1+y_2=-x_1$	$y_1=\frac{-Z}{Z+R}x_1+\frac{R}{Z+R}x_2$	$y_2=\frac{-R}{Z+R}(x_1+x_2)$	y_2
11 *	1	0	-1	$\begin{bmatrix}1 & 0\\1 & R\end{bmatrix}$	$\begin{bmatrix}1 & -R\\0 & -R\end{bmatrix}$ $y_1-y_2=x_1$	$y_1=\frac{Z}{Z-R}x_1+\frac{-R}{Z-R}x_2$	$y_2=\frac{R}{Z-R}(x_1-x_2)$	y_1 $R<0$
12 *	1	0	1	$\begin{bmatrix}1 & 0\\1 & R\end{bmatrix}$	$\begin{bmatrix}-1 & R\\0 & R\end{bmatrix}$ $y_1+y_2=x_1$	$y_1=\frac{Z}{Z-R}x_1+\frac{R}{Z-R}x_2$	$y_2=\frac{-R}{Z-R}(x_1+x_2)$	y_2 $R<0$
13	0	0	-1	$\begin{bmatrix}1 & 0\\0 & R\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & -R\end{bmatrix}$ $y_1-y_2=0$	$y_1=\frac{R}{Z}(x_1-x_2)$	$y_2=\frac{R}{Z}(x_1-x_2)$	X
14	0	0	1	$\begin{bmatrix}1 & 0\\0 & R\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\0 & R\end{bmatrix}$ $y_1+y_2=0$	$y_1=\frac{R}{Z}(x_1+x_2)$	$y_2=\frac{-R}{Z}(x_1+x_2)$	y_1 y_2

6.3 THE ROTATION OR R-PROPERTY IN SELF-DUAL LTA FILTERS

DEFINITION 6.3 : A self dual linear transformation is said to possess the rotation or R-property if the RC-actively realized output y_i of the LT structure for a series arm Z (shunt arm Y) being

$$y_i = A(s)x_1 + B(s)x_2 \quad ,$$

the RC-actively realized output y_i' of the LT structure of the series arm $Z' = \frac{R^2}{Z}$ (shunt arm $Y' = {}^1/_{YR^2}$) is

$$y_i' = A(s)x_2' + B(s)x_1' \; .$$

The self-dual LTA transformations possessing the rotation property will be referred to as R-Self-Dual transformations and the corresponding active filters as the R-Self-Dual LTA filters.

The definition of the R-property implies that the LT structures of the highpass arms can be realized from the LT structures of the corresponding lowpass arms by interchanging the inputs x_1 and x_2. A similar state of afairs exists between the band-reject and the bandpass ladder arms. This is very desirable indeed since the already high modularity of the Self-Dual LTA filter structures is greatly enhanced by enabling a larger variety of ladder arms to be actively realized using the same active module.

The necessary and sufficient conditions for a Self-Dual transformation to possess the rotation property can be found in terms of the signs $\varepsilon, \varepsilon_1$ and ε_2 since all eight parameters of the transformation matrices have been defined from eqn. (6.12a) and eqn. (6.12b).

If y_1 is the RC actively realized output in the LT structure of a series arm Z, then from eqn. (6.13a) and the definition of the rotation property we have

$$y_{1Z} = K(Z)x_1 + L(Z)x_2 = K({}^{R^2}/Z)x_2 + L({}^{R^2}/Z)x_1$$

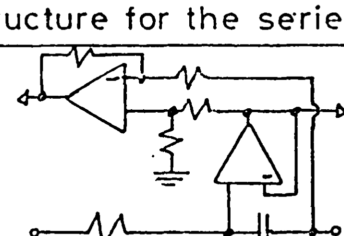
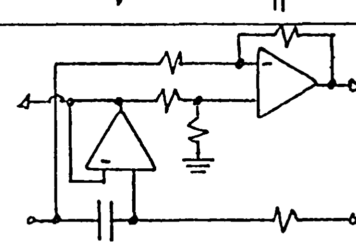
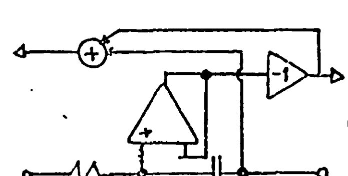
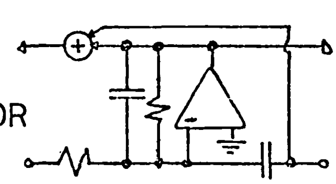
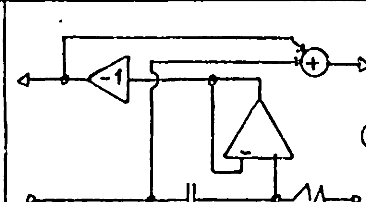
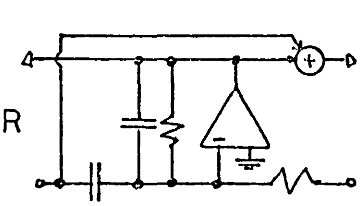
from which it can be seen that

$$\varepsilon_1 = \varepsilon = -1 \quad \text{and} \quad \varepsilon_2 = 0$$

corresponding to case 9 of Table 6.1. In addition cases 5,7 and 11 of this Table can also be seen to posses the rotation property, these four cases being the only R-. elf-Dual cases of Table 6.1.

Table 6.2 illustrates the four R-Self Dual cases and the corresponding LT structure for the series inductor L is given. This structure is identical to that of a shunt capacitance $C = {}^{L}/R^2$ and moreover by interchanging the inputs x_1 and x_2 it can provide the LT structures required for the simple highpass arms. Therefore, the LT structure of the series inductance is very illustrative for any R-Self-Dual case since it gives automatically the LT structures of four ladder arms.

TABLE 6.2

R Self Dual Case	LT Structure for the series inductor L
CASE 5 $\begin{bmatrix} 1 & R \\ 0 & R \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -R \end{bmatrix}$ $R>0$	
CASE 11 $\begin{bmatrix} 1 & 0 \\ 1 & R \end{bmatrix} \begin{bmatrix} 1 & -R \\ 0 & -R \end{bmatrix}$ $R<0$	
CASE 7 $\begin{bmatrix} 1 & -R \\ 0 & R \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & -R \end{bmatrix}$ $R<0$	 OR 
CASE 9 $\begin{bmatrix} 1 & 0 \\ -1 & R \end{bmatrix} \begin{bmatrix} 1 & R \\ 0 & -R \end{bmatrix}$ $R>0$	 OR 

The LT structures in Table 6.2 are realized using operational amplifiers. It can readily observed that the most economical cases are case 5 and case 11 because of the low number of active elements they require. Indeed it is apparent that the LT structures for the series inductance in cases 5 and 11 are symmetric. This is expected since they emerge from symmetric transformations in the sense that the left transformation matrix of one case is equal to the right transformation matrix of the other.

6.4 THE APPLICATION OF AN R-SELF-DUAL TRANSFORMATION

For reasons explained in the previous section, cases 5 and 11 of Table 6.1 and 6.2, are the most practical cases and since they are very similar, the study is concentrated in case 5 only.

The R-Self-Dual transformation of case 5 uses for all series arms the transformation pair

$$a_{1Z} = \begin{bmatrix} 1 & R \\ 0 & R \end{bmatrix} , \qquad a_{2Z} = \begin{bmatrix} 1 & 0 \\ 1 & -R \end{bmatrix}$$

and consequently for all shunt arms the transformation pair below:

$$a_{1Y} = \begin{bmatrix} 1 & R \\ 1 & 0 \end{bmatrix} , \qquad a_{2Y} = \begin{bmatrix} 0 & -R \\ 1 & -R \end{bmatrix}$$

For both series and shunt cases the output y_1 can be formed from the input x_2 and the RC actively realized output y_2 using the relationship

$$y_1 = y_2 - x_2$$

The output y_2 is given for the series arm Z case by the equation:

$$y_{2Z} = \frac{R}{R + Z} x_1 + \frac{Z}{R + Z} x_2$$

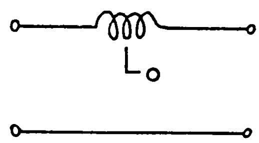
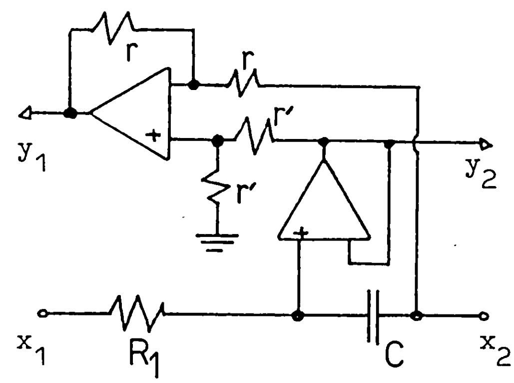
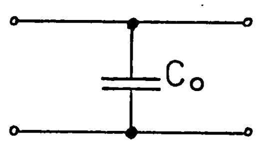
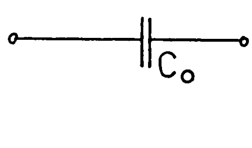
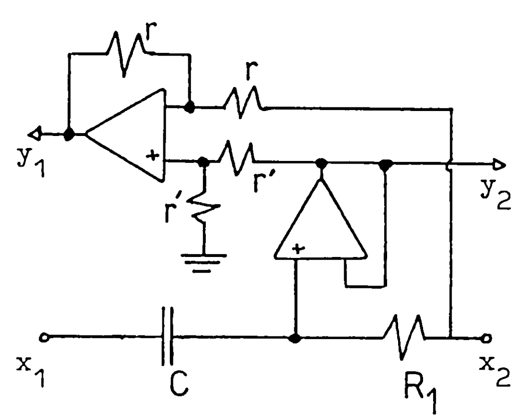
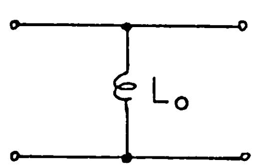
Whilst for the shunt arm Y case it is given by

$$y_{2Y} = \frac{1}{1 + RY} x_1 + \frac{RY}{1 + RY} x_2$$

The above expression for y_{2Z} becomes identical to that for y_{2Y} for $Y = {}^{Z}/R^2$, as expected from a Self-Dual transformation.

Table 6.3 gives the LT structures for this R-Self-Dual case for all simple ladder arms wherein both the self duality and the rotation properties are evident and the transformed terminations are included. Table 6.4 gives the LT structures for the tuned circuits.

TABLE 6.3

ARM CASE		LT STRUCTURE
	$R_1C = \frac{L_o}{R}$	
	$R_1C = RC_o$	
	$R_1C = RC_o$	
	$R_1C = \frac{L_o}{R}$	

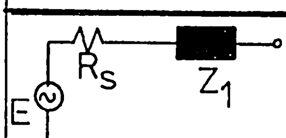
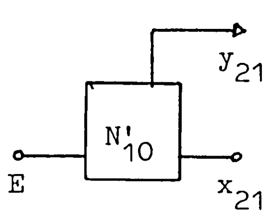
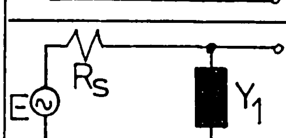
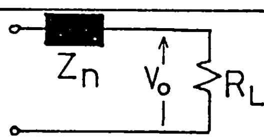
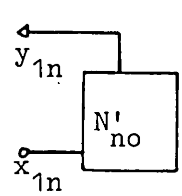
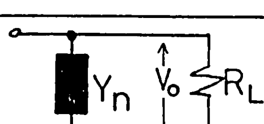
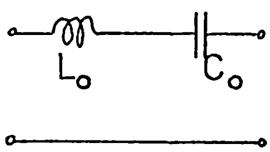
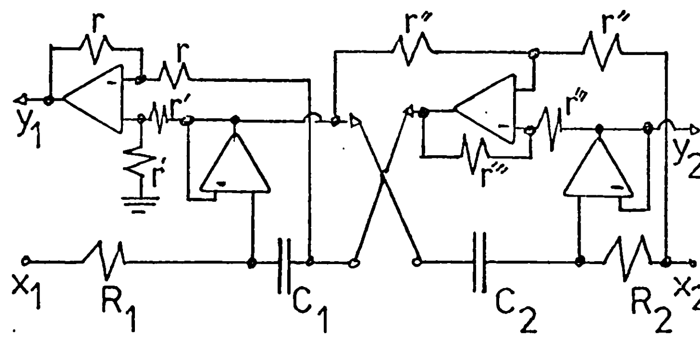
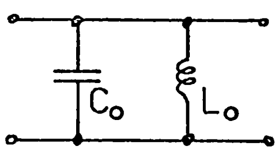
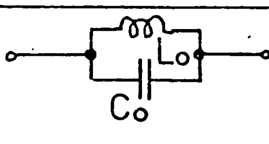
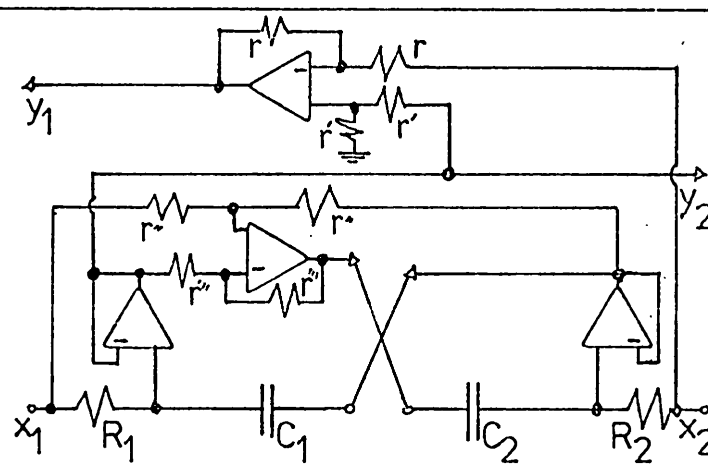
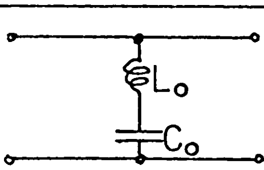
		$y_{21} = \frac{R}{R+Z_1}E + \frac{Z_1}{R+Z_1}x_{21}$ $R = R_s$
		$y_{21} = \frac{1}{1+RY_1}E + \frac{RY_1}{1+RY_1}x_{21}$ $R = R_s$
		$y_{1n} = \frac{R}{R + R_L + Z_n}x_{1n} = V_o$
		$y_{1n} = \frac{R_L}{1 + \frac{R_L}{R} + R_LY_n}x_{1n} = V_o$

TABLE 6.4

<table>
<tr><th>TUNED ARM</th><th>LT STRUCTURE</th></tr>
<tr><td>

$R_1C_1=\frac{L_o}{R}$, $R_2C_2=R\,C_o$</td><td rowspan="2">
</td></tr>
<tr><td>

$R_1C_1=R\,C_o, R_2C_2=\frac{L_o}{R}$</td></tr>
<tr><td>

$R_1\,C_1=\frac{L_o}{R}$, $R_2\,C_2=RC_o$</td><td rowspan="2">
</td></tr>
<tr><td>

$R_1C_1=RC_o$, $R_2C_2=\frac{L_o}{R}$</td></tr>
</table>

It is observed that in the circuits of Table 6.3 and 6.4 the LT structures for all ladder arms are constructuble using the active block shown in Fig. 6.1 below, consisting of a unity gain voltage follower (buffer) and a differential amplifier.

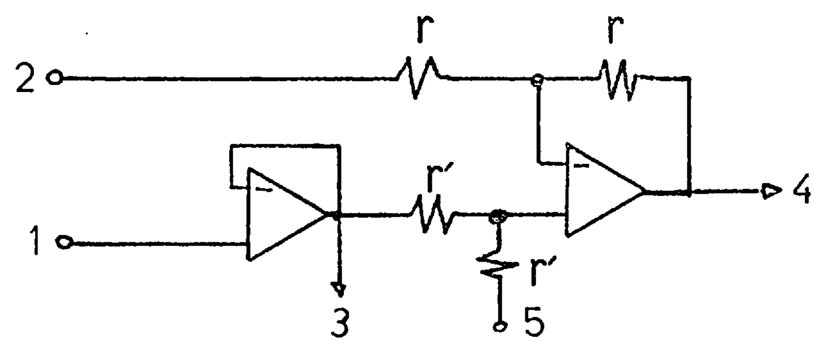
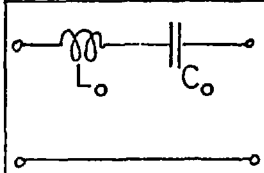
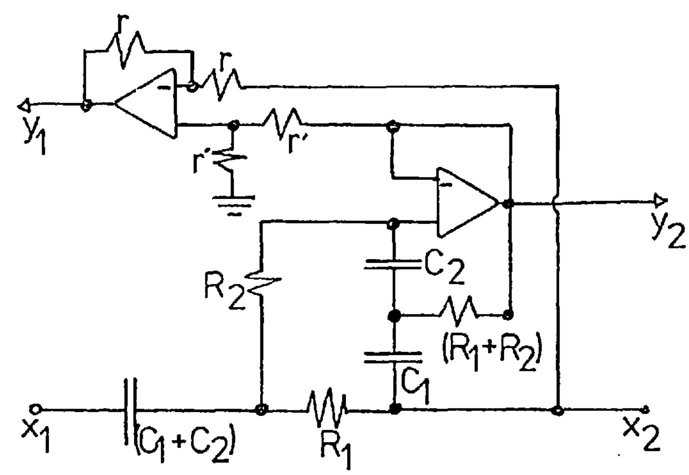
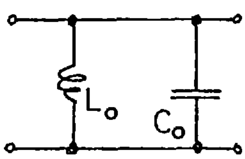
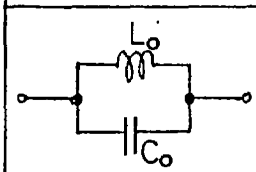
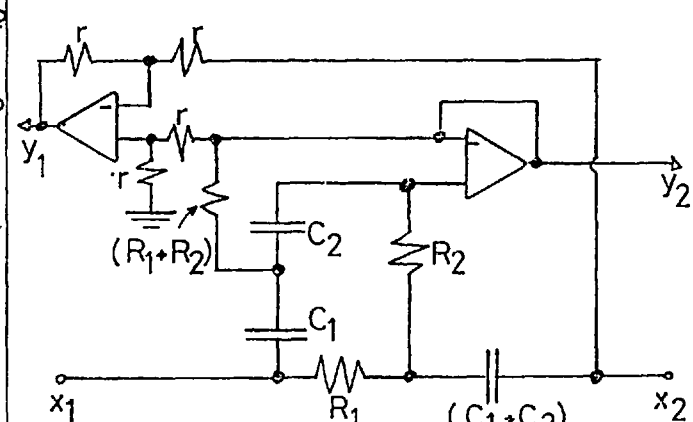
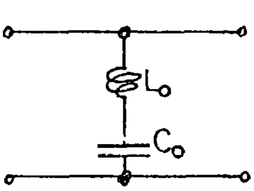


Fig. 6.1

This situation is highly desirable from the practical point of view since it makes the corresponding R-Self-Dual LTA filters highly modular and amenable to microelectronic realization.

Table 6.5 gives alternative realizations for the LT structures of the tuned circuits which involve a saving in the number of the operational amplifiers. These economic realizations are based on the twin-T network arangement [6] [27] and they also use the active building block of Fig. 6.1. Tables 6.3 , 6.4 and 6.5 can be used to obtain the R-Self-Dual LTA structure directly from the passive ladder. This has been done in the relevant section of the next chapter to simulate various ladder networks using the R-Self-Dual transformation concept.

TABLE 6.5

TUNED ARM		LT STRUCTURE
	$C_1=C_2=C$ $R_1R_2=\frac{L_oC_o}{C^2}$ $\frac{R_2}{R_1}=\frac{4L_o}{R^2C_o}$	
	$C_1=C_2=C$ $R_1R_2=\frac{L_oC_o}{C^2}$ $\frac{R_2}{R_1}=\frac{4R^2C_o}{L_o}$	
	$C_1=C_2=C$ $R_1R_2=\frac{L_oC_o}{C^2}$ $\frac{R_2}{R_1}=\frac{4R^2C_o}{L_o}$	
	$C_1=C_2=C$ $R_1R_2=\frac{L_oC_o}{C^2}$ $\frac{R_2}{R_1}=\frac{4L_o}{R^2C_o}$	

CHAPTER 7

PRACTICAL APPLICATIONS OF THE LTA METHOD

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INTRODUCTION

The general LTA approach to the simulation of passive ladder networks,has been presented in chapters 2 and 3, whilst in chapter 4 some useful rules for its effective application have been proposed. In chapter 5 we dealt with the classification of the LTA procedures based on the imposed constraints between the linear transformation pairs of the transformation set, necessary for the simulation of a ladder and we interpreted several existing simulation methods as special cases of the LTA approach ,as a result of which they were further developed. The FDLT case was also presented in chapter 5 whilst the Self Dual LTA filters were discussed in chapter 6.

It is our aim of this chapter to apply the above theory to practice by simulating several ladder filters. The frequency responses of the LTA structures so derived as well as their sensitivities are examined in each case and comparisons are carried out showing the advantages of LTA procedures over some existing ladder simulation methods.

7.1 A STUDY OF LOW ORDER LTA STRUCTURES

The LTA simulation of second and third order polynomial low-pass filters will be used as a basis for comparing the sensitivities and performance of various LTA structures derived from such polynomial filters using different transformation sets.

The reactive ladder filters used here are so designed, as to match the source resistance R_S to the load R_L at certain fraquencies within the passband, so providing maximum power transfer points at which the first order sensitivity of their amplitude characteristic with respect to a reactive component is zero (see chapter 1). This is known as the Orchard's Principle and is one of the main reasons why the simulation of these doubly terminated LC ladders is desirable. Ideally, RC-active simulation of such filters should possess this low sensitivity property with respect to most, if not all, of their passive (RC) elements. However, in practice, only some of the passive elements of the simulated RC-active structure obey the Orchard's Principle so that the sensitivity of the filter attenuation characteristic with respect to these elements will be very low within the passband due to the zero sensitivity at the maximum power transfer points.

It is possible in the LTA structures to make all capacitors present this low sensitivity due to their correspondance with the reactive elements of the original ladder filter. It is not seldom, though, to find in the LTA structures in addition to the capacitors some resistors obeying Orchard's Principle due to an RC-product correspondance with a reactive ladder element as it will be illustrated in this chapter.

7.1.1 Second Order Lowpass Ladder LTA Simulation

The second order Chebyshev ladder filter shown in Fig. 7.1a (R_S=1./1.3554 R_L= 1, L_o=1.2087 ,C_o= 1.6382 and passband ripple 0.1 dB) is simulated using several different LTA approaches. Since there are only two reactive arms in the passive ladder, only two transformation pairs are needed. However, both of these arms may be taken in conjunction with the appropriate termination resistance and hence only one matrix is significant for each transformation, according to sections 3.3 and 3.4, always provided that a_{12} is chosen to provide a convenient output node. The LTA simulations shown in Fig. 7.1 give the LT matrices employed, the LTA filter structure and the sensitivities $\left|S_{x_i}^{|T|}\right|$ (the absolute value of the gain sensitivity with respect to the passive components x_i of the LTA filter). The sensitivities of the original passive ladder are shown in Fig. 7.1b. In this figure the sensitivities of this LTA filter are shown under the LTA structure in continuous lines. It is observed that the sensitivity of the realization with respect to R_2 is constant and equal to unity and therefore this resistor can be used for output level adjustment. It is also observed, that the sensitivities with respect to the capacitors C_1,C_2 and the resistor R_4 are identical to those for L_o and C_o in the original passive ladder, presenting zero sensitivity at the maximum power transfer point as expected by Orchard's Principle. The sensitivity with respect to R_1 is identical to that corresponding to the dissipative elements in the passive ladder filter whilst the sensitivity with respect to R_3 (the only element of the LTA structure which does not correspond to any element of the passive ladder) is very low as shown.

It can be seen from Fig. 7.1b that the resistance R_1 can be removed to reduce the number of components but in so doing we shall no longer be simulating the LC filter of Fig. 7.1a. Moreover, a

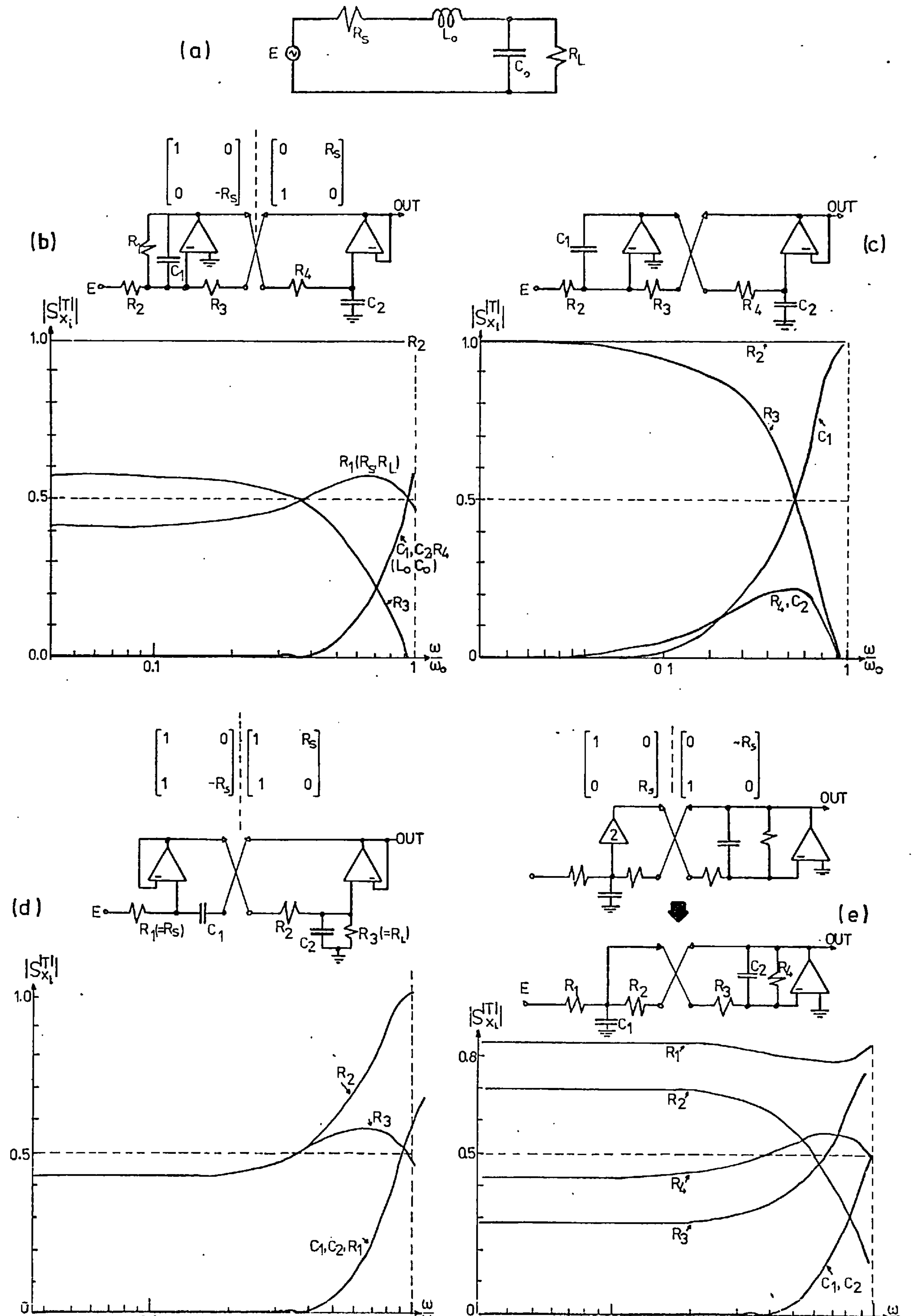

(a)
E
R_S
L_o
C_o
R_L
(b)
(c)
(d)
(e)
OUT
E
R_1
R_2
R_3
R_4
C_1
C_2
$|S_{x_i}^{|T|}|$
1.0
0.5
0.0
0.1
1
$\frac{\omega}{\omega_o}$
$R_1(R_S,R_L)$
C_1,C_2,R_4
$(L_o\,C_o)$
R_4,C_2
$R_1(=R_S)$
$R_3(=R_L)$
C_1,C_2,R_1
C_1,C_2
0.8
2

Fig. 7.1

readjustment of component values must be done in order to keep the transfer function the same. It is interesting, from the practical point of view, to examine the resultant circuit which is shown in Fig. 7.1c.

The sensitivity performance of this circuit, of course, is not expected to be similar to either that of the passive prototype or indeed to that of the LTA filter of Fig. 7.1b. In fact, we do not expect to have zero sensitivity at the frequency point corresponding to maximum power transfer. This can be verified from the sensitivity plots of Fig. 7.1c. Moreover, it can be seen from these sensitivity plots, that the sensitivities with respect to the capacitors have increased as well as that corresponding to R_3 which has been doubled in this case.

As a general conclusion, it can be seen that the sensitivity performance of the LTA filter is indeed lower than that of an active RC filter not simulating the doubly terminated LC ladder but merely having the same transfer function.

The transformation matrices used in the design of the LTA filter of Fig. 7.1d, are the R-Self-Dual transformations (case 5 of Table 6.1) The corresponding realization structure is seen to be very simple where both operational amplifiers are employed as buffers. This state of affairs is highly desirable for high frequency operation. Moreover, the elements C_1, C_2 and R_2 present very low sensitivity performance within the passband due to their correspondence with the reactive elements of the passive ladder, as it can be observed from the sensitivity plots of Fig. 7.1d.

In Fig. 7.1e the LTA filter is derived using the transformation matrices shown and is subsequently modified as indicated, keeping the modifications within the LC simulation limits. Thus, the first amplifier of the initial LTA structure employed to provide a gain of 2 and also to isolate its input node from the rest of the circuit is removed since both of these operations can be performed by passive elements when their values are appropriately modified. Consequently, the resulting single amplifier circuit with C_1 arbitrary and $R_1 = R_2' = R_3 = 3L_o/R_sC_1$, $R_4 = 9L_oR_L/R_s^2C_1$ and $C_2 = R_s^2C_oC_1/9L_o$ is equivalent to the two amplifier LTA filter and of course to the LC filter. The disadvantage of the single amplifier circuit is the wide spread of the component values that results in addition to the increased sensitivity performance of the resistors. The sensitivity of the capacitors, however, is identical to that of L_o and C_o components in the original passive ladder.

7.1.2 Third Order Lowpass LTA Simulation

A third order Chebyshev lowpass (ripple 0.5 dB) shown in Fig. 7.2a, is simulated using different linear transformation sets. Normally three transformation pairs are needed as follows:

$$\{a_{11}, a_{21}\}, \{a_{12}, a_{22}\}, \{a_{13}, a_{23}\}$$

but since the terminated arms are transformed in conjunction with the corresponding resistive termination, a_{11} and a_{23} are redundant.

The doubly terminated ladder exhibits maximum power transfer at zero frequency and at approximately $\frac{\omega}{\omega_o} = 0.85$. Therefore, zero sensitivity is expected at these points with respect to the reactive elements. Indeed, all capacitors of the LTA structures shown in Fig. 7.2 exhibit this property and in three cases one or more resistors are seen to behave similarily, as is evident from the figure.

In Fig. 7.2b a leapfrog structure is presented having four operational amplifiers and designed using the transformation set indicated. There are also three capacitors in this realization all

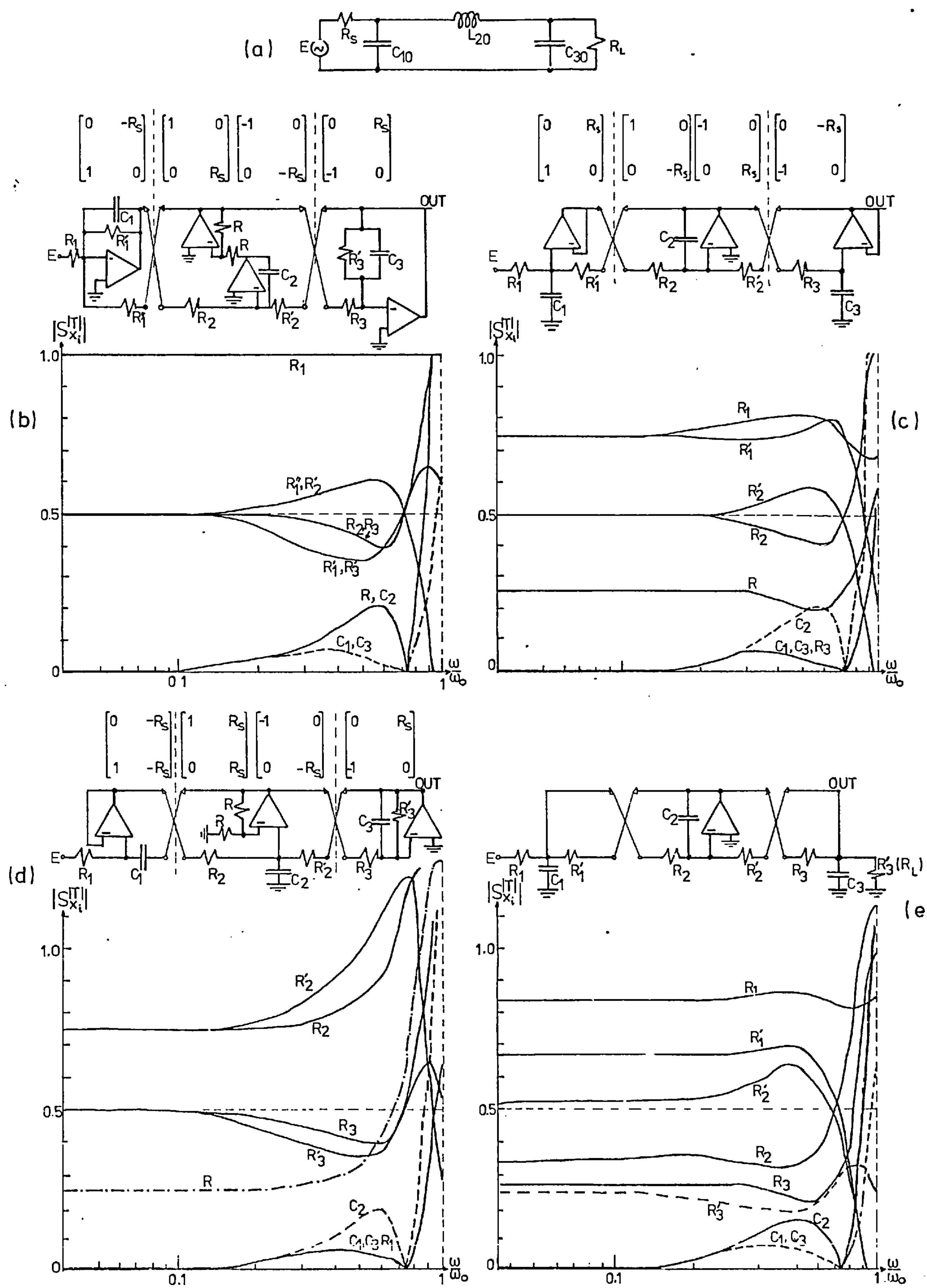

(a)
E
Rs
C10
L20
C30
RL
(b)
(c)
(d)
(e)
OUT
1.0
0.5
0
0.1
1
R1
R1', R2'
R2, R3
R1', R3'
R, C2
C1, C3
R1'
R2'
R2
R
C2
C1, C3, R3
R3
R3'
C1, C3, R1
R3'(RL)

Fig. 7.2

presenting zero sensitivity performance at the maximum power transfer points, owing to the one-to-one correspondence that exists between these capacitors and the reactive arms of the passive ladder. Moreover, it can be observed from the circuit that changes in the resistance R can be interpreted as changes in the capacitor C_2 since the stage containing R merely scales (nominaly by unity) the integrator employing C_2. Consequently, the resistance R in the circuit will have the same differential sensitivity performance as the capacitor C_2(and hence it will be zero at the maximum power transfer points). Element values for ladder equivalence are

$$C_1 = C_2 = C_3 = C$$
$$R_1 = R_1' = R_1'' = R_s C_{1o}/C$$
$$R_2 = R_2' = L_{2o}/R_s C$$
$$R_3 = R_3' = R_s C_{3o}/C$$

where C is a conveniently chosen but otherwise arbitrary capacitor value. Note that the capacitors in the active filter are all of equal value.

The form-compatible transformations have been used in deriving the LTA circuit of Fig. 7.2c, in accordance with section 5.3. The LTA structure shown in this figure employs only three operational amplifiers (as opposed to four amplifiers in the leapfrog structure) two of which act as buffers. The sensitivity performance is still observed to be very low for all passive elements, whilst elements C_1, C_2, C_3 and R_3 present zero sensitivity at the maximum power transfer points. In comparing this circuit with the previous one in Fig. 7.2b, it is seen that both have very low sensitivities, but the LTA filter shown in Fig. 7.2c has the following advantages over that of Fig. 7.2b :

(i) Only three amplifiers are employed(4 in Fig. 7.2b, i.e. reduction in active components)

(ii) Two, out of the three, amplifiers are buffers (i.e. ease of design and extension of frequency operation).

(iii) Only five resistors are used (9 in Fig. 7.2b, i.e. reduction of passive resistive components)

Element values for ladder equivalence must be chosen so that:

$$C_1 = C_2 = C_3 = C$$

$$R_1 = R_1' = 2R_s C_{1o}/C$$

$$2R_2 = R_2' = L_{2o}/R_s C$$

$$R_3 = R_L C_{3o}/C$$

Note that this LTA structure has equal value capacitors also since C is a conveniently chosen capacitor value.

In Fig. 7.2d alternative transformation matrices have been employed as shown, with the middle pair having a third entry. This has the effect, in conjunction with the other transformation matrices pairs, to produce amplifier connections having no frequency dependent gains. There are in fact three operational amplifiers in the circuit of Fig. 7.2d. The first is employed effectively as a buffer, whilst the second amplifier is connected to provide a gain of 2. The third amplifier may be interpreted as a lossy integrator the frequency performance of which is, of course, almost ideal even with amplifiers having gains very far from ideal since the amplifier in this lossy integrator is acting at low frequencies as a unity gain inverter. The frequency performance, therefore, of the entire filter is expected to be very good indeed. This was exemplified by constructing a filter with cutoff frequency of 100 kHz (rather high for normal RC active filters [44]) and there was no severe deviation from the ideal passive filter attenuation characteristic as it is the case with with the leapfrog filter structure of Fig. 7.2b at this high frequency.

The sensitivity performance of this filter with respect to the passive components is very low within the passband as expected. In addition the capacitors C_1, C_2 and C_3 as well as the resistor R_1 present the zero sensitivity at the maximum power transfer points. The element values for ladder equivalence are found directly from the descriptions for the individual LT structures as follows:

$$C_1 = C_2 = C_3 = C$$
$$R_1 = R_s C_{1o}/C$$
$$R_2 = R_2' = L_{2o}/R_s C$$
$$R_3 = R_3' = R_s C_{3o}/C$$

Again here we note that this filter is an equal value capacitor design.

The RC active structure of Fig. 7.2e has been derived from the structure corresponding to the form-compatible LTA filter of Fig. 7.2c. This derivation was effected by removing the first and the last operational amplifiers and altering the values of the passive elements in such a way as to incorporate the loading effects on the corresponding nodes that will result. These adjustments are all within the LC simulation limits and the element values of the active filter so derived are related to the LC filter elements as follows: The capacitor C_1 may be chosen arbitrarily whilst R_1, R_2 and R_1' are equal to $3C_{1o}R_s/C_1$. Additionally we have $R_2' = \frac{3}{2}R_1$, $C_2 = L_{2o}C_1/9R_s^2C_{1o}$, $R_3C_3 = 2R_LC_{3o}$ and $R_3' = R_L$.

The sensitivity performance of this structure with respect to all passive elements is shown in Fig. 7.2e and it can be observed that it is comparable to that of the corresponding LTA three amplifier filter of Fig. 7.2c. However, the wide spread of the element values, that results as a consequence of the adjustments above, is the price paid for the reduction in the number of operational amplifiers. Note that the sensitivity performance of the structure with respect to the capacitors C_1, C_2 and C_3 exhibits the expected zero value at the maximum power transfer points (and low value elsewhere in the pass-band) which is, of course, a validation of the adjustments carried out as being within the LC simulation framework.

Concluding this section it is noticed that different LTA structures with distinct properties and special features are derived from the same original LC ladder for different choices of the transformation matrices. However, the sensitivity performance of the various LTA structures is very low independently of the transformation used. Moreover, those elements in these structures which do not correspond to any reactive arms of the passive ladder filter, appear to have sensitivities of the same order as those which simulate the sensitivity behaviour of the reactive arms of the ladder.

7.2 APPLICATION OF THE FORM - COMPATIBLE TRANSFORMATIONS TO THE SIMULATION OF A 5th ORDER CHEBYSHEV FILTER

The form compatible transformations of Fig. 5.5 are applied to the simulation of a fifth order lowpass Chebyshev ladder filter (ripple o.5 dB) as shown in Fig. 7.3a along with the appropriate transformation matrices pairs. The resulting LTA filter structure, after denormalization for f_c=10 kHz and R = 1kΩ, was constructed with equal value polysterene capacitors (10 nF, 2% tolerance) and 5% tolerance resistors the values of which can be found from the design equations shown in Fig. 7.3 below.

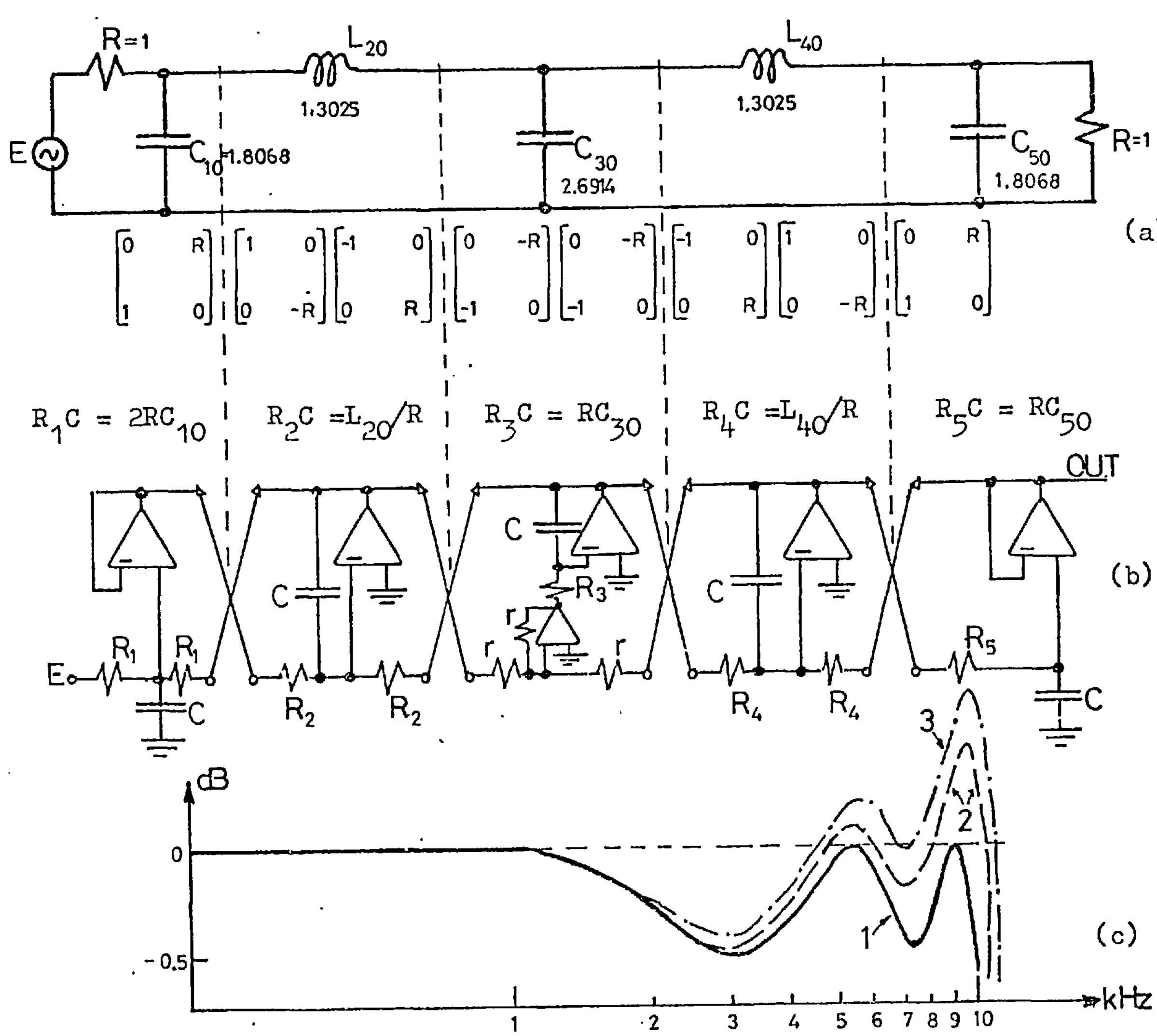


Fig. 7.3

The commercially available μA741 operational amplifier was employed for the realization of the LTA filter and the experimental response of the filter is shown in Fig. 7.3c (curve 2). The response of a leapfrog realization structure of the same filter is also shown in this figure (curve 3), from which it can be observed that the form-compatible design is favourably compared to the direct leapfrog structure. This becomes more evident at higher frequencies where in the form compatible case the performance of the filter is satisfactory for frequencies up to 50kHz.

It is interesting and of practical significance to remove the first and the last operational amplifiers in the LTA circuit designed above by compensating for the loading effects that will be produced at the appropriate nodes merely by modifying the element values of those elements connected to these nodes (a similar procedure was followed in the previous section, see Fig. 7.2e). The result of this procedure is shown in Fig. 7.4, where the values of the elements indicated with a prime have been suitably modified. The behaviour of this circuit was similar to that of Fig. 7.3b and indeed there exists an one-to-one correspondance between the reactive components of the original ladder and the capacitors contained in the realization of Fig. 7.4. Consequently a zero first order sensitivity at the maximum power transfer frequencies exists with respect to the capacitors of this realization.

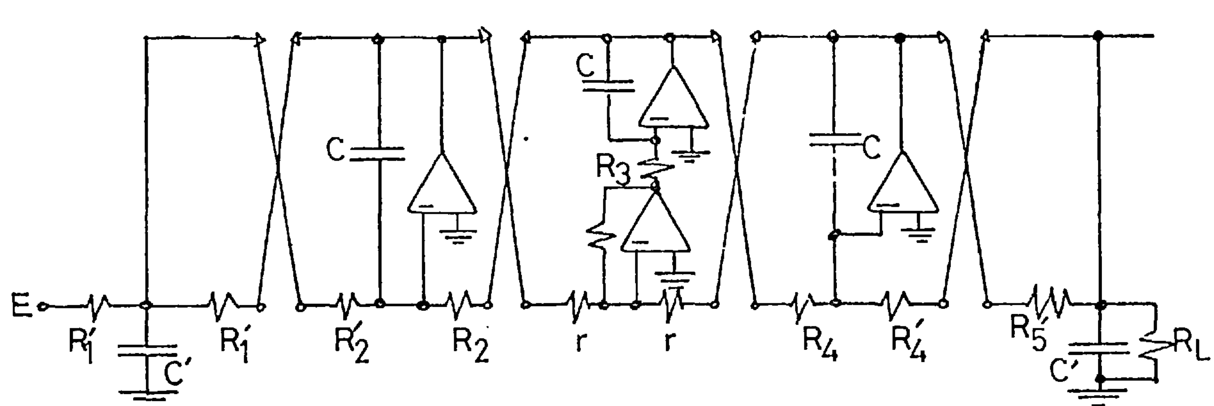


Fig. 7.4

7.3 APPLICATION OF THE ACYCLIC LTA PROCEDURE

The general acyclic procedure as described in section 5.4 is aplied in this section to two different ladder filters.

7.3.1 An Acyclic Simulation of a 5th Order Lowpass Chebyshev Filter.

The simulation procedure is illustrated in Fig. 7.5 where we started with the transformation matrix $\mathfrak{a}_{15}$ (the left transformation matrix for C_5 taken in conjunction with R_L). The choice of this matrix was so made that a very simple LT structure is obtainable with a convenient output node(see section 4.3). The matrix $\mathfrak{a}_{24}$ was then determined

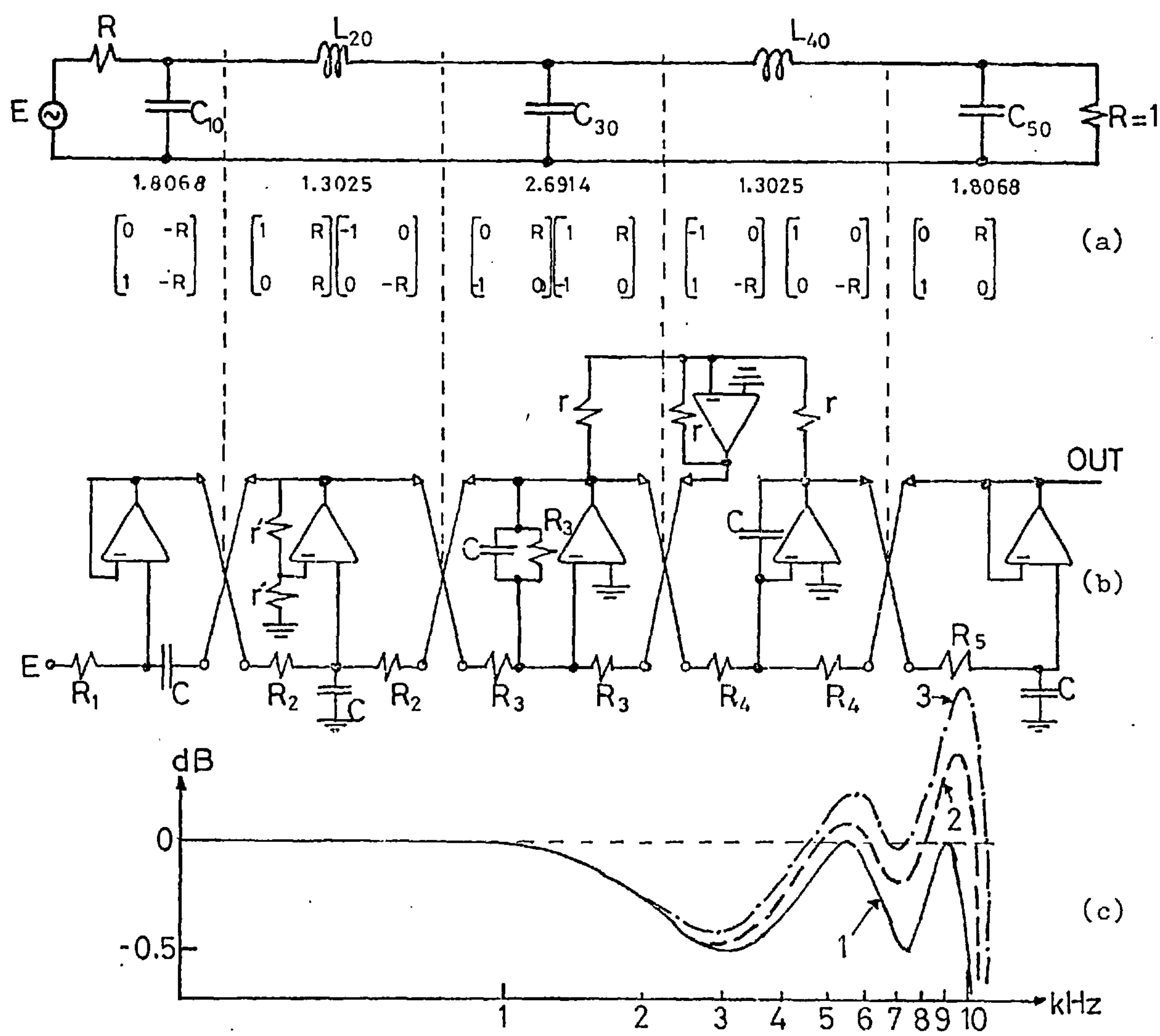


Fig. 7.5

from the compatibility relationship (eqn. 3.17a) and $\mathfrak{a}_{14}$ was chosen as to yield a simple LT structure for the inductor L_{40} avoiding the $\gamma_{14}=0$ case which would lead to a TRI structure (see Table 4.1). The right transformation matrix $\mathfrak{a}_{23}$ for the transformation of the capacitor C_{30} is automatically determined from the compatibility relationship and $\mathfrak{a}_{13}$ is so chosen as to yield a practically desirable inverting lossy integrator for the realization of the LT structure of C_{30}. The right transformation matrix $\mathfrak{a}_{22}$ for L_{20} is then automatically determined and $\mathfrak{a}_{12}$ is so chosen as to yield an LT structure which can be realized using a noninverting lossy integrator as shown in Fig. 7.5b. The transformation matrix $\mathfrak{a}_{21}$ for C_{10} taken in conjunction with the source resistance R is consequently found and the corresponding LT structure is very simple indeed, requiring its active component to serve the rôle of a buffer in its realization. The overall LTA structure requires six operational amplifiers as is the case for the form-compatible LTA structure of Fig. 7.3b. However, in this present structure only two operational amplifiers are involved in frequency dependent feedback arrangements, a fact which is desirable for good high frequency operation of the active RC filter.

The filter was constructed using equal value (10 nF, 2%) polysterene capacitors, µA741 operational amplifiers (six in total) and 5% resistors. The resistor values were determined from the following design equations which are directly derivable from the descriptions of the corresponding LT structures:

$R_1C = RC_{10}$, $R_2C = 2L_{20}/R$, $R_3C = RC_{30}$, $R_4C = L_{40}/R$ and $R_5C = RC_{50}$.

Denormalization was carried out for $R=1k\Omega$ and $f_c=10$ kHz. The measured frequency response is shown without any compensation in Fig. 7.5c(curve 2) A response very near to the ideal (curve 1) was easily achieved by adjusting the gain of the inverter (resistors r). Curve 3 in this figure represents the uncompensated response of the corresponding leapfrog structure of Fig. 5.1 when implemented in the same way as that of Fig. 7.5 (i.e. µA 741 operat. amplifiers, 2% polysterene capacitors and 5% resistors)

It must be emphasized here that in the structure of Fig. 7.5 all capacitors and, in addition, resistors R_1 and R_5 present zero sensitivity behaviour at the maximum power transfer frequencies due to their correspondence with the reactive elements of the ladder. The sensitivity performance of the rest of the resistors was found to be of the order of that of the elements of the original ladder filter.

7.3.2 An Acyclic LTA Simulation of a 6th Order Bandpass Ladder Derived from a 3rd Order Chebyshev Lowpass Prototype.

The passive bandpass ladder is shown in Fig. 7.6 together with its element values and passband characteristics.

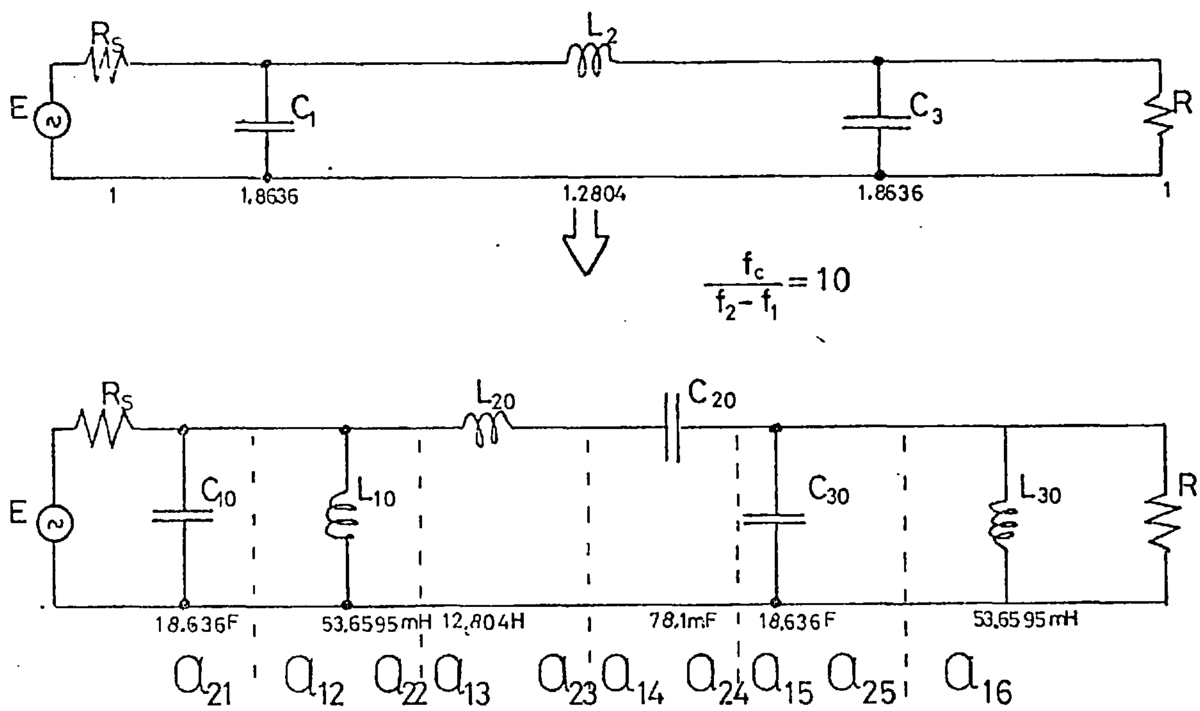


Fig. 7.7

For the LTA simulation we start from the output terminated inductor L_{30} by using the matrix $a_{16} = \begin{bmatrix} 1 & 0 \\ 2 & -2R \end{bmatrix}$, which yields for the LT structure of that arm when taken in conjunction with the load resistor:

$$y_{16} = -\frac{1}{s\frac{L_{30}}{2R}}x_{16}\ ,\quad x_{16} = V_o \text{(the output of the ladder)}$$

This relationship may then be realized as shown in Fig. 7.7 below

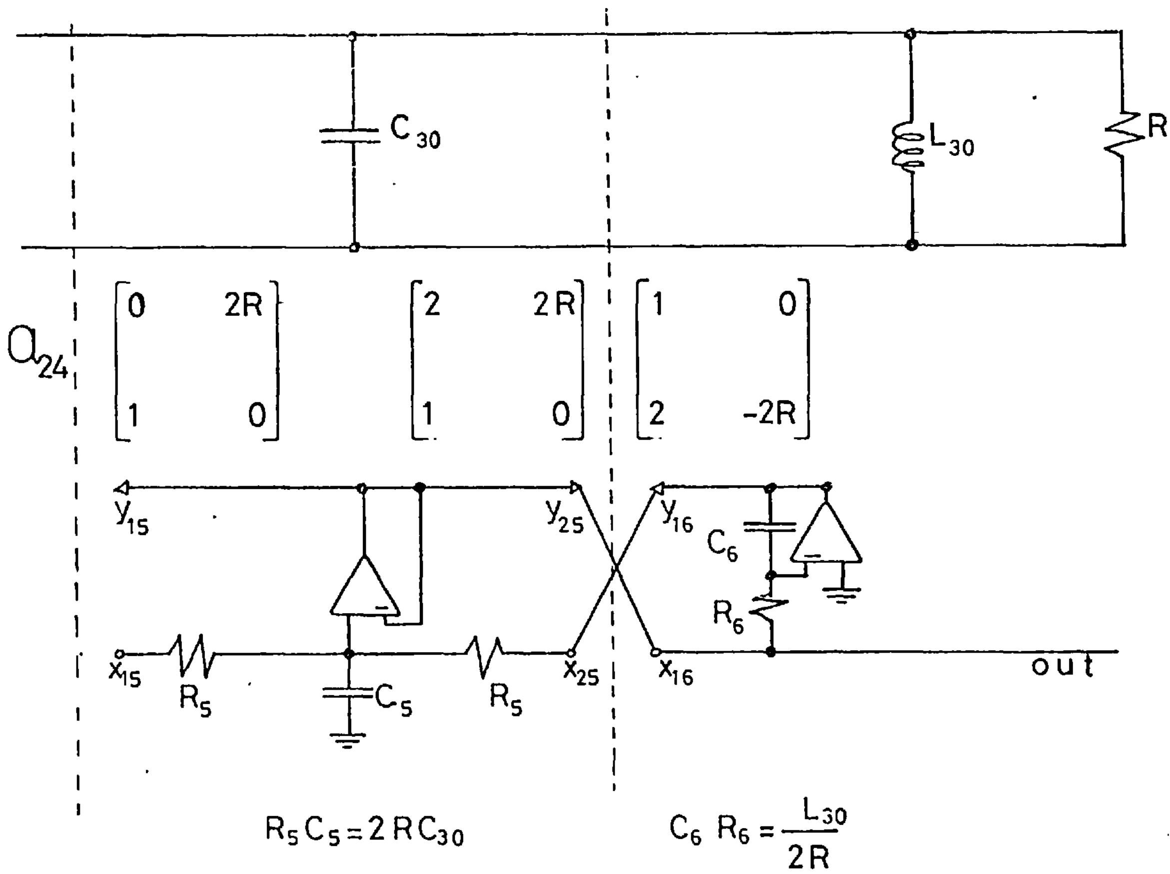


Fig. 7.7

The Q_{25} matrix for the transformation of the shunt capacitor C_{30} is readily found from Q_{16} by interchanging its rows and changing the sign of the second column (compatibility relationship). Q_{15} is then so chosen as to yield a noninverting lossy integrator for the realization of the LT structure for C_{30}. This is shown in Fig. 7.7 above. The matrix Q_{24} is then automatically defined by the matrix Q_{15}, and the matrix Q_{14} is taken to be $\begin{bmatrix} 0 & 2R \\ 1 & 2R \end{bmatrix}$ yielding

$$y_{24} = x_{14} \quad \text{and} \quad y_{14} = \frac{1 + s2RC_{20}}{s2RC_{20}}x_{14} + x_{24}$$

(which is a TRE case, see Table 4.1). The LT structuremay then be realized as shown in Fig. 7.8a. Once the matrix $\mathfrak{a}_{14}$ is known, its adjacent matrix $\mathfrak{a}_{23}$ is defined by the compatibility relationship, and $\mathfrak{a}_{13}$ is then so chosen as to yield an inverting integration for the LT structure of the series inductor L_{20}. Thus, the transformation pair $\mathfrak{a}_{13},\mathfrak{a}_{23}$ shown in Fig. 7.8 for L_{20} yields:

$$y_{13} = y_{23}$$

$$y_{23} = -\frac{1}{s\frac{L_{20}}{2R}}(x_{13}+x_{23})$$

the realization of which is shown in Fig. 7.8a. The virtual earth point of the inverting integrator involved in the LT structure for L_{20} can perform the summing operation of the adder that appears in the LT structure of the capacitor C_{20} and this has been done in Fig. 7.8b. Thus, a reduction of one active element takes place.

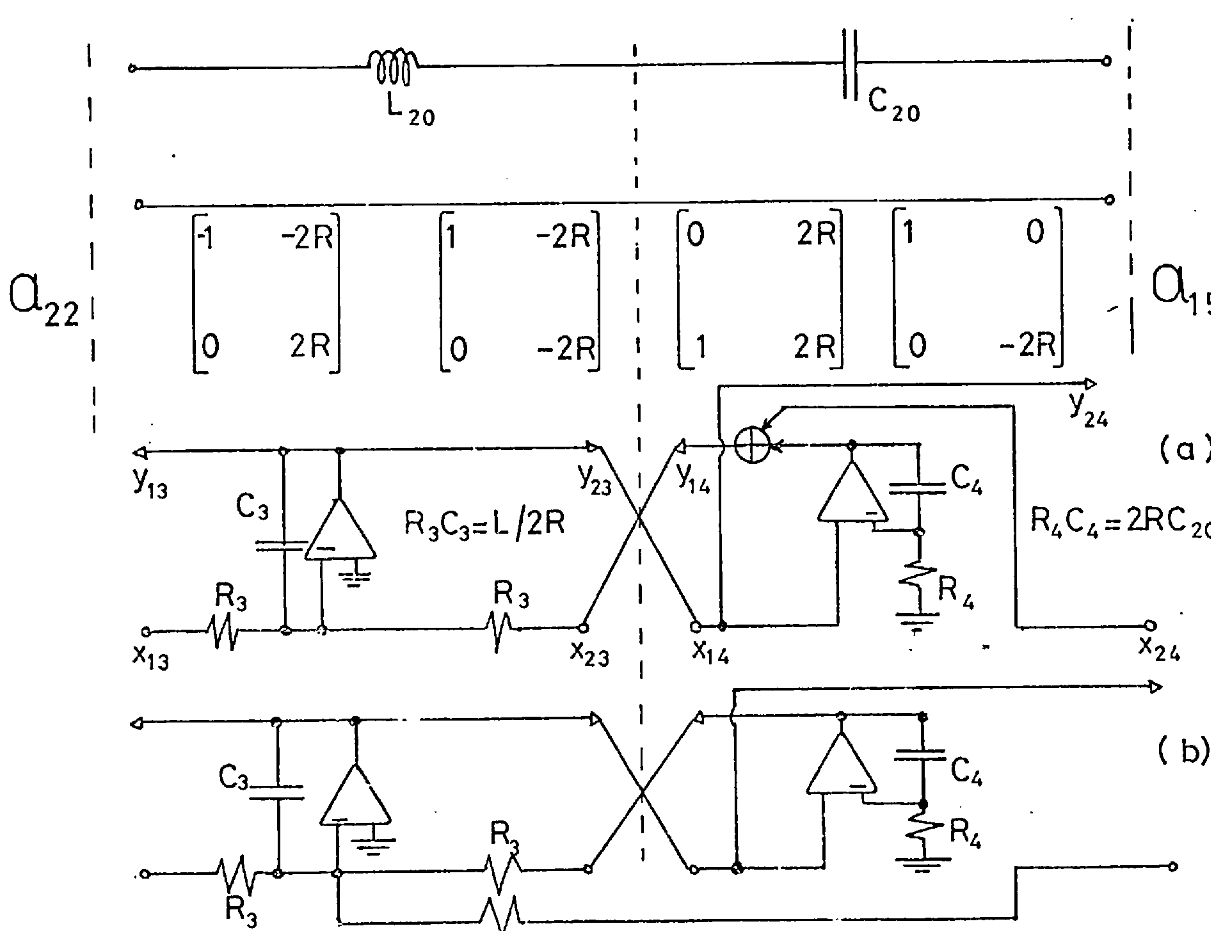


Fig. 7.8

Again from the matrix $\mathcal{a}_{13}$, we can find its adjacent matrix $\mathcal{a}_{22}$ for the transformation of L_{10} and then determine a matrix $\mathcal{a}_{12}$ for the transformation pair $\mathcal{a}_{12}, \mathcal{a}_{22}$ to yield a simple LT structure. The transformation matrices for L_{10} are shown in Fig. 7.9a which yield for the corresponding LT structure

$$y_{12} = \frac{s\frac{L_{10}}{2R} + 1}{s\frac{L_{10}}{2R}} x_{21} + x_{22} \quad , \quad y_{22} = - x_{12} - x_{22}$$

the realization of which is shown in Fig. 7.9a. Finally, the transformation of the capacitor C_{10} taken in conjunction with the source resistance çan be obtained from $\mathcal{a}_{12}$ and the compatibility relationship. The matrix $\mathcal{a}_{21}$ is shown in Fig. 7.9a from which we have $y_{21} = \frac{-1}{\frac{2R}{R_s} - 1 + s2RC_{10}}(x_{21} - \frac{2R}{R_s}E)$

In the filter under consideration $R_L = R_s = R$, and the above equation after this simplification may be realized as shown in Fig. 7.9 below.

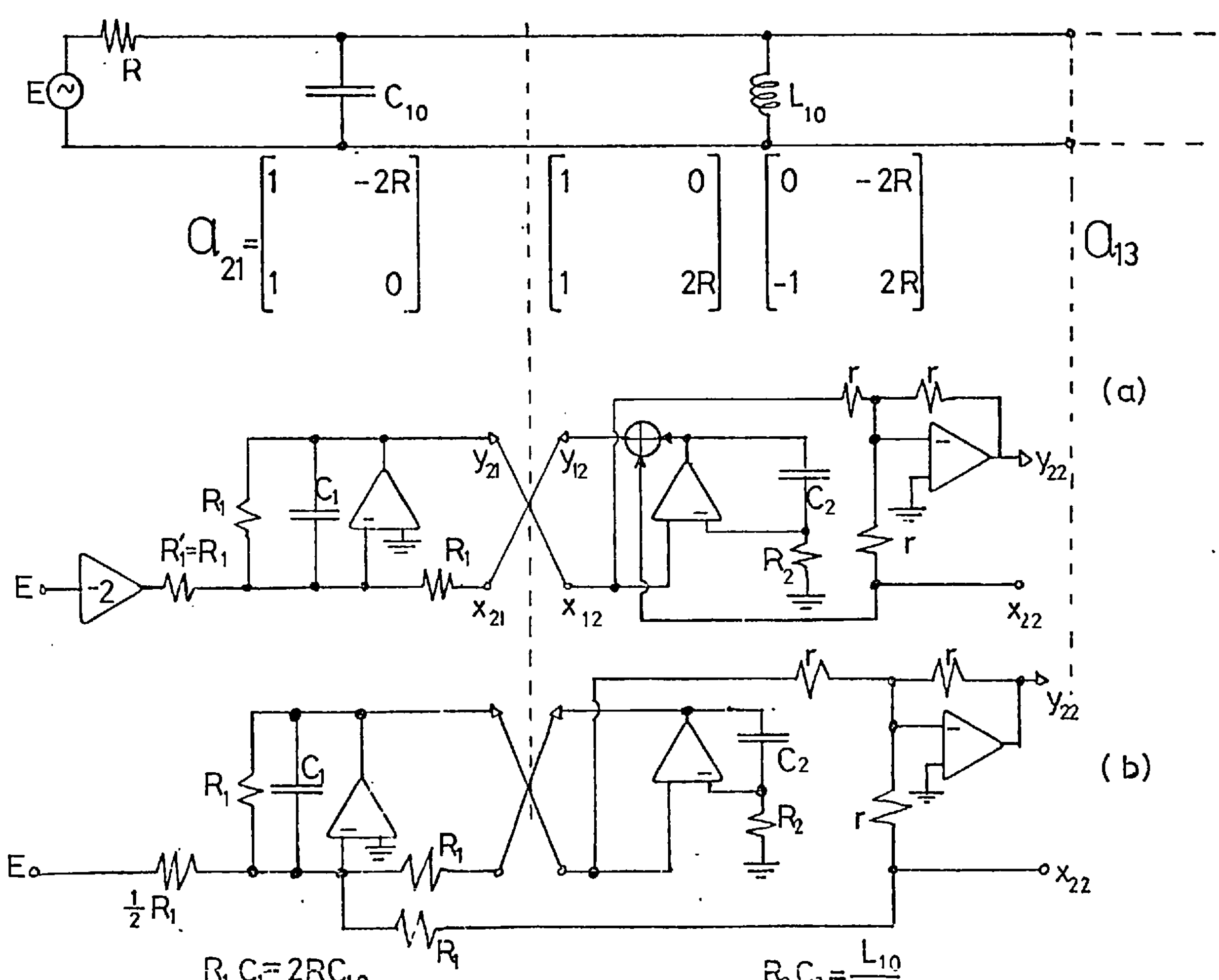
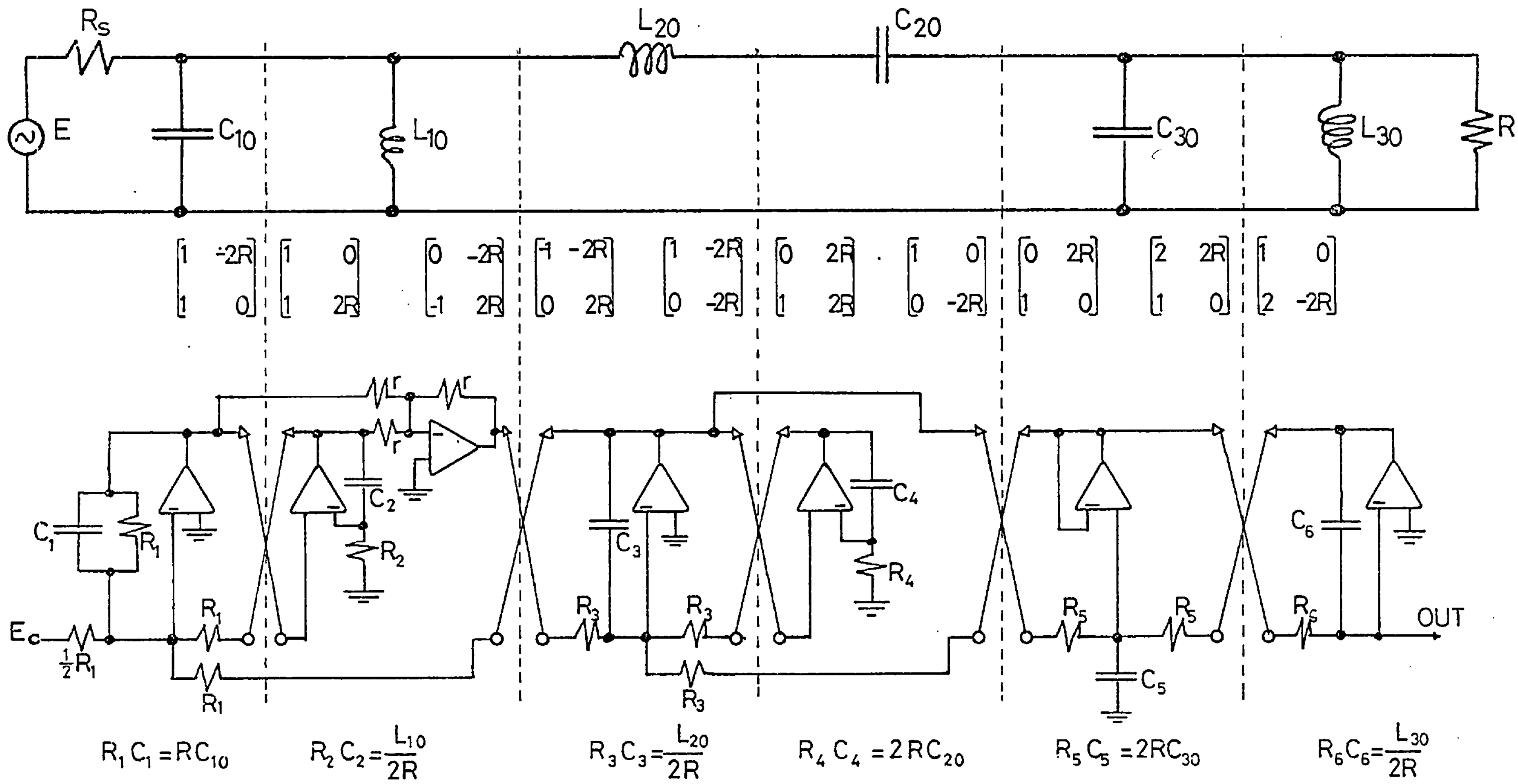


Fig 7.9

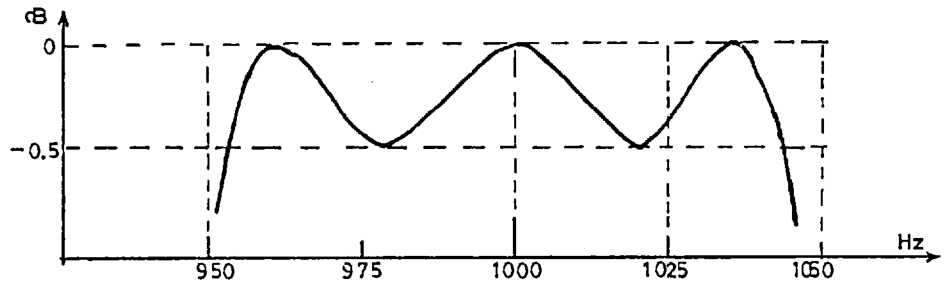
A simplified version of this structure is shown in Fig. 7.9b, where the input inverter has been removed introducing a phase shift of 180°, and the value of R'_1 has been halved to provide the required gain of 2. Moreover, the virtual earth point of the first amplifier is effectively used to perform the addition required in the LT structure for L_{10}.

Fig. 7.10 shows the overall LTA structure for the 6th order bandpass filter and gives the element values of the active RC structure so derived. The passband response of the LTA filter (constructed using two 4136 quad operational amplifier chips, 2% polysterene capacitors and 1% resistors) is shown in Fig. 7.10 from where it is evident that no measurable deviation from the ideal behaviour occurs. As far as the component sensitivities are concerned, all capacitors show zero sensitivity at the maximun power transfer points and, in addition, the resistors R_2, R_4 and R_6 (related to L_{10}, C_{20} and L_{30} respectively) possess this property. The sensitivities with respect to the remaining resistors were also examined using the sensitivity facility of the MINNIE interactive computer program [58], and they were found to be very low and comparable to those with respect to the capacitors (and therefore, comparable to those of the reactive elements of the passive ladder original).

Finally, it may be observed that in the LTA filter above, when two consecutive LT structures of elements in the same position (i.e. series or shunt) are taken together, they produce biquadratic sections. The three biquadratic sections that appear as a result of such a grouping are all of finite Q in contrast with other techniques of simulation that require at least the central biquadratic section to be of infinite Q (e.g. leapfrog, direct SFG simulation [55]).



Rs
L20
C20
E
C10
L10
C30
L30
R
C1
R1
C2
R2
r
C3
R3
C4
R4
R5
C5
R6
C6
Ec
½R1
OUT
R1C1 = RC10
R2C2 = L10/2R
R3C3 = L20/2R
R4C4 = 2RC20
R5C5 = 2RC30
R6C6 = L30/2R



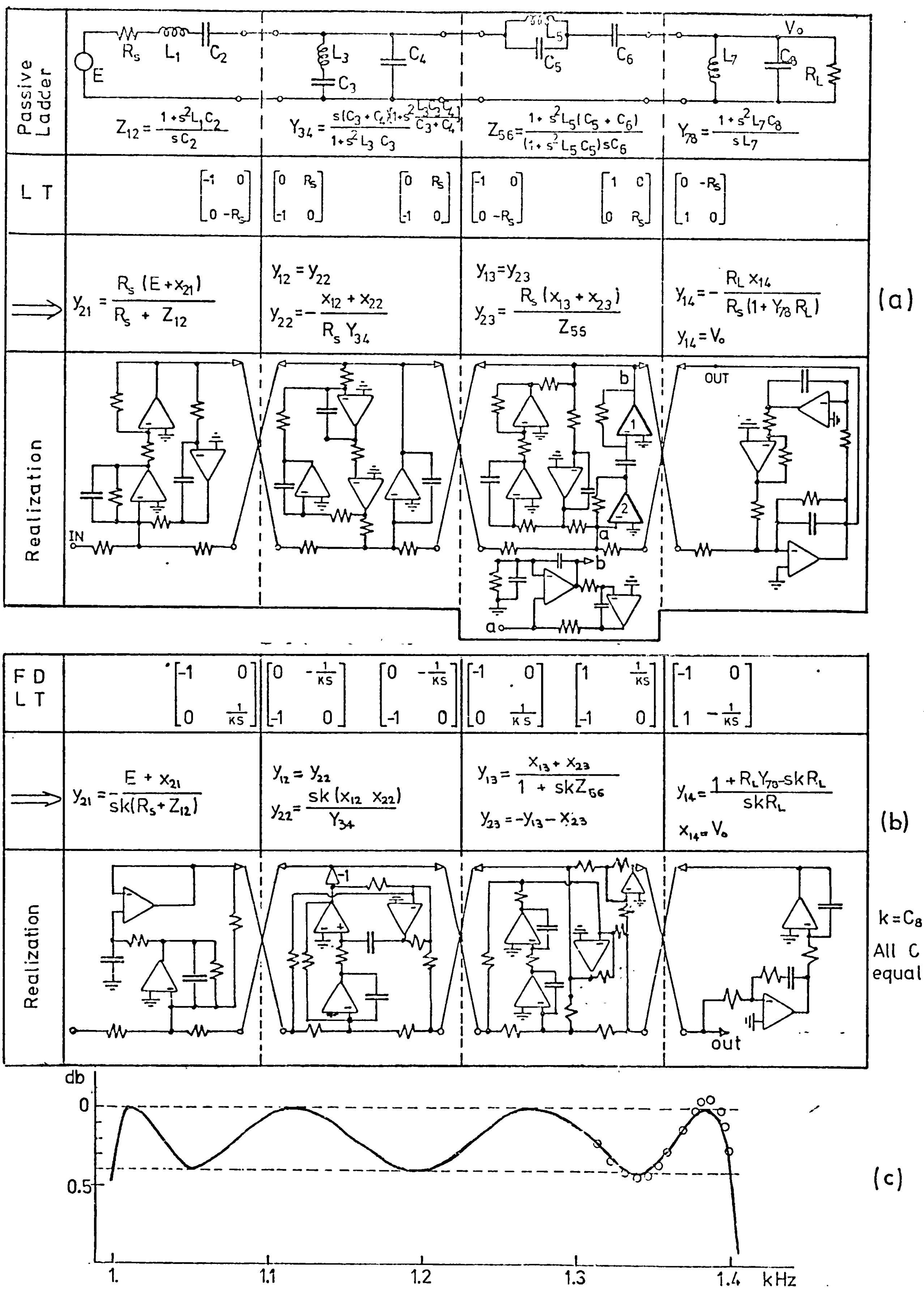
dB
0
-0.5
Hz
950
975
1000
1025
1050

Fig. 7.10

7.4 THE APPLICATION OF FDLT TO AN 8th ORDER ZIG - ZAG FILTER

The zig-zag filter shown in Fig 7.11a is transformed here using a frequency dependent linear transformation set for reasons explained in section 5.5. This filter was taken from reference[55]where it is simulated using the "Direct Signal Flow Graph" method. Before we apply the FDLT set it would be perhaps interesting to obtain the same structure for the zig-zag filter as that obtained in reference[55]using a suitable transformation set in order to show the flexibility of our methods. This is shown in Fig. 7.11b where a form-compatible transformation set has been employed. The"port reciprocator" used in ref.[55],which is necesary whenever an inductive T or a capacitive ⊓ appears in the LC filter, is here realized by a noninverting differentiator showing in fact that the "port reciprocator" is no more than an implicit differentiator. The active RC filter so derived employs 15 operational amplifiers and 10 capacitors. Precisely the same structure and the number of components are given in reference [55].

The same LC ladder has been simulated using the FDLT set shown in Fig. 7.11c. The resulting LTA structure employs only 12 operational amplifiers and 8 capacitors. The total number of capacitors is precisely equal to the order of the original zig-zag filter. This LTA filter was constructed using three 4136 quad operational amplifier chips,with 2% polystyrene capacitors and 5% resistors. The measured frequency response is shown in Fig. 7.11, which even before any compensations or adjustments is caried out,is indeed much closer to the theoretically expected response than that of the filter resulting from the methods of reference[55].



Passive Ladder
L T
Realization
(a)
F D L T
Realization
(b)
k = C8
All C equal
IN
OUT
out
db
0
0.5
1.
1.1
1.2
1.3
1.4
kHz
(c)

Fig. 7.11

7.5 APPLICATION OF THE R-SELF-DUAL TRANSFORMATIONS

The R-Self-Dual transformations prasented in section 6.4 will be used for the LTA simulation of a third order lowpass Chebyshef filter and a third order lowpass elliptic filter for which two alternative self dual realizations will be given.

7.5.1 The R-Self-Dual LTA 3rd Order Lowpass Filter

A third order Chebyshev lowpass filter is shown in Fig. 7.12 with the element values normalized. The corresponding R-Self-Dual LTA structure is also shown in this figure, derived directly from Table 6. The design equations are also shown in this figure. When the element values are denormalized for R_s = 1 kΩ and f_c = 10kHz, for equal capacitor design (10 nF) design we obtain:

$C_1 = C_2 = C_3 = C = 10nF$, $R_1 = 2966\Omega$, $R_2 = 2037\Omega$ and $R_3=2966\Omega$.

The R-Self-Dual LTA structure was constructed using a 4136 quad operational amplifier chip, with 5% resistors and 2% polysterene capacitors. The measured frequency response of the constructed prototype was found coincident with the theoretical curve. When the capacitor values are changed to 1 nF, thereby increasing the cutoff frequency f_c to a value of 100 kHz, the frequency response obtained is shown in Fig. 7.12c (curve 2). The maximum deviation from the ideal behaviour does not exceed 0.35 dB and was easily compensated for by adjusting resistor r' to earth (see Fig. 7.12). The frequency response of the corresponding leapfrog filter is also shown in Fig. 7.12c for comparison reasons. The obvious superiority of the R-Self-Dual filter as far as the high frequency response is concerned is mainly due to the fact that all its operational amplifiers are involved in frequency independent feedback arrangements. Moreover, all capacitors and resistors R_1, R_2 and R_3 are directly related

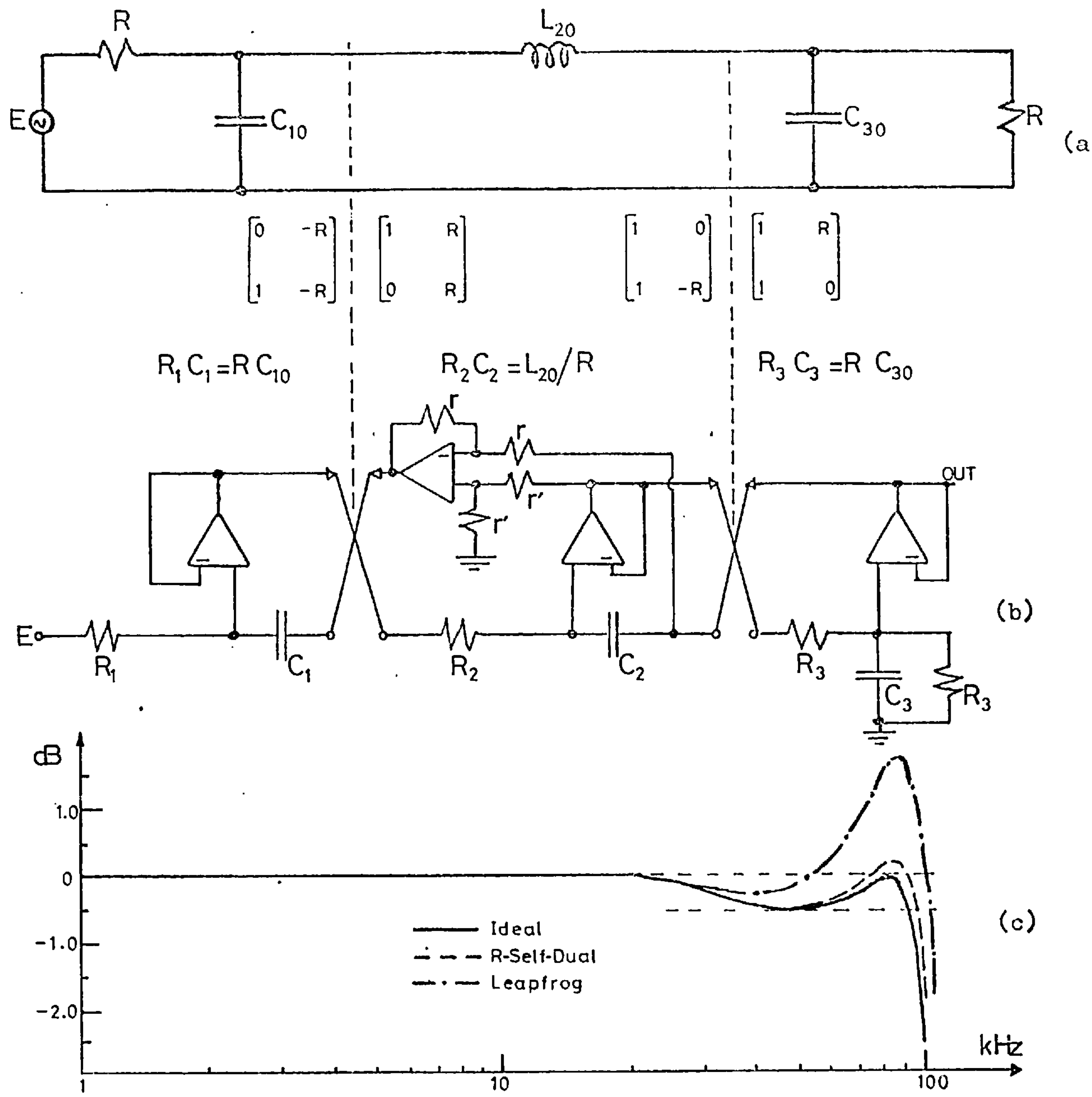


Fig. 7.12

to the reactive arms of the original passive ladder,thereby presenting the zero sensitivity at the maximum power transfer points. The sensitivity behaviour of the filter with respect to the resistors r and r' (which define the gains of the differential amplifier) are found to be very low indeed and in any case no larger than that of R_2 and R_3 in the leapfrog structure (see Fig. 7.2b).

From the above, therefore, it appears that the R-Self-Dual filter structure is preferable to that of the leapfrog case which employs the same number of passive and active components.

7.5.2 A Third Order Elliptic R-Self-Dual LTA Filter

The original passive ladder is shown in Fig. 7.13a together with the normalized element values. The R-Self-Dual structure is directly derivable from Table 6.3 and 6.4 for $R=R_s$, as shown in Fig. 7.13b together with the design equations. Denormalization of the element values for R_s $R_s = 1\ k\Omega$ and $f_c = 10$ kHz for equal value (10nF) capacitors gives the resistor values for this active RC structure. The LTA filter constructed using $1\frac{1}{2}$ 4136 quad operational amplifier chip, with 5% resistors and 2% polysterene capacitors, behaves ideally without any measurable deviation from the theoretically expected response.

On replacing the 10 nF capacitors by 1nF, the passband is extended to 100 kHz, in which case a 0.65 dB maximum deviation is observed as shown in Fig. 7.13c. However, this discrepancy was easily compensated for by adjusting the resistance R_x. The corresponding leapfrog structure which requires the realization of a transfer ratio of the form $\frac{s^2LC+1}{sL/R}$ (function which increases with frequency) could not match the response of the R-Self-Dual LTA structure at these high frequencies of up to 100 kHz, being very problematic even at the frequency of 10 kHz. This is mainly due to the differentiation involved in the realization of the above transfer ratio, and partialy due to the integrators employed in the leapfrog structures. The R-Self-Dual LTA structure requires operational amplifiers to have frequency independent gains with a maximum value of 2, thereby ensuring such good high frequency performance of the active RC filter.

The intermediate LT structure corresponding to the parallel tuned circuit in the series arm can be replaced by an equivalent twin-T configuration (see Table 6.5). The total number of amplifiers can then be reduced to 4, three of which act as buffers but the number of the capacitors increases by one resulting to a noncanonic structure. This replacement was carried out in practice and did not appear to affect the sensitivity of the overall R-Self-Dual LTA filter structure. The

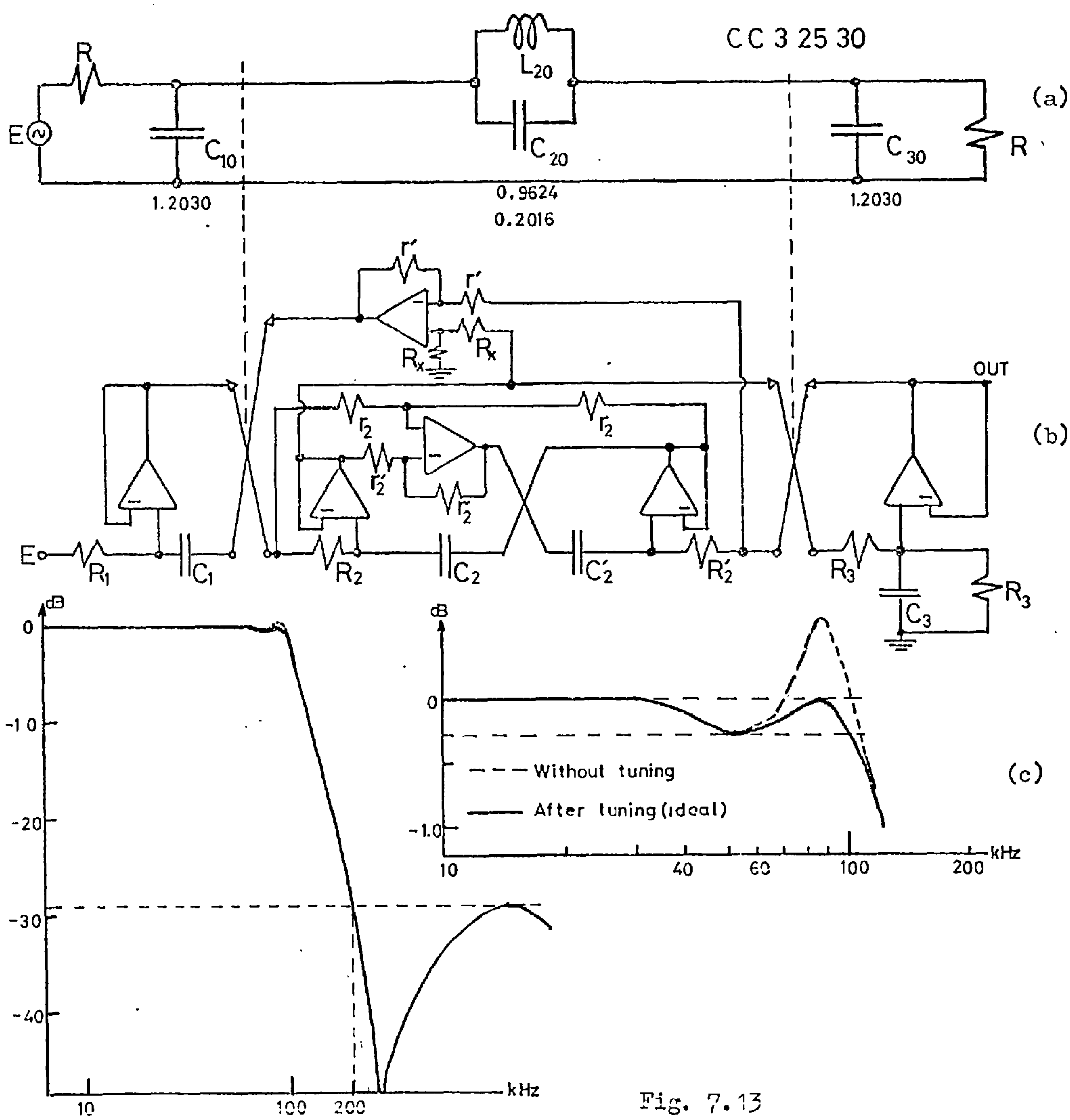


Fig. 7.13

behaviour of the modified filter was very near to the theoretically expected one and without any compensation or adjustment the filter operated satisfactorily with cutoff frequencies up to 50 kHz.
The transmission zero of the filter was very well behaved in both cases presenting an attenuation of more than 60 dB.

CHAPTER 8

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

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A new approach to the design of active RC filters (LTA filters) simulating doubly terminated passive LC ladder filters, has been developed and presented in this thesis. The capability of this approach to interpret existing simulation methods as special cases of the general LTA method combined with its inherent potential to derive novel practical active RC filter structures tailored to the design needs, make the LTA simulation more than a new active RC filter design method. Indeed, it could conceivably be labelled as a new design philosophy.

New structures have been derived at various stages in this work, with the derivation of the R-Self-Dual LTA filters serving as a striking example of the manner in which the general procedure may be so constrained to yield active RC filter structures with prescribed special features. In fact in the case of the R-Self-Dual LTA filters the general approach was constrained to yield highly modular structures whereby all ladder arms are realized using the same basic active building block of Fig. 6.1 (see Tables 6.3,6.4 and 6.5). This feature may be greatly appreciated not only by the active RC filter theorist who can study these structures more effectively, but also by the microelectronic industry since the realization of highly modular structures is more economical and space saving.

As far as the generality of the LTA approach is concerned, it must be emphasized that all the existing LC ladder simulation methods which involve transformations of the voltages and the currents of the original ladder filter may be seen as special LTA cases. This has been shown explicitly for the Wave Active Filter case in section 5.2.2 and for the Leapfrog Synthesis in section 5.1.1 in which case the conventional method has been improved as to derive more economical structures (section 5.3. The Direct Signal Flow Graph simulation has also been shown to be a form-compatible LTA case where the application of the FDLT proved to be more effective yielding more economical and, under certain conditions, canonic structures.

The Impedance Scaling method introduced by L. Bruton can be interpreted as a voltage-current LTA procedure in the sense that the transformed variables x and y do not both represent voltages but instead, one of them represents current and the other retains its voltage nature. The transformation which leads to the impedance scaling method structures is a self-compatible transformation defined by the matrices

$$\mathrm{a}_1 = \begin{bmatrix} 1 & 0 \\ 0 & X(s) \end{bmatrix} \quad , \mathrm{a}_2 = \begin{bmatrix} 0 & -X(s) \\ 1 & 0 \end{bmatrix}$$

which are used for all series and shunt arms of the ladder. The quantity X(s) is a dimensionless frequency dependent ratio which in the original FDNR[33] case is of the form X(s) = sk. The modifications of the original Impedance scaling method (see chapter 1) may be seen as voltage-current acyclic transformations where the scaling factor X(s) may differ from arm to arm according to its nature so as to give conveniently realized structures. Research into the voltage current linear transformations would not only give an insight to the variou impedance scaling techniques, but also would obtain some more interesting new methods.

Although the inductance simulation method for deriving active RC filter structures from doubly terminated passive ladders does not alter the port variables of the constituent subnetworks of the original ladder, the LTA method may be used as a tool for deriving new simulated inductance circuits as it has been done in references [48] and [49] from the Wave Active LTA method. The establishment of a general LTA procedure towards this end would be a very interesting research area since it could derive new, and perhaps better, simulated inductance circuits.

Research on the sensitivity of the general LTA structures in order to express explicit sensitivity formulae in terms of the parameters of the transformations would be of great theoretical and practical value since the sensitivities of the various LTA structures would then be compared and perhaps minimized so establishing optimum sensitivity performance structures within the LTA concept which, as already pointed out, it includes most of the existing ladder simulation methods.

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