TRANSIENT THERMAL STRESSES AND FATIGUE IN PRESSURE VESSEL NOZZLES

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ABSTRACT

The present work, which incorporates a theoretical and an experimental part, deals with the determination of thermal stresses and their influence on the fatigue life of pressure vessel nozzles. Numerical and computational methods are developed for the calculation of transient thermal stress distribution, in radial nozzles of various geometries and under specific heating conditions. A literature review is made of recent developments in pressure vessel technology and investigations relating to pressure vessel nozzles.

The heat transfer analysis makes use of the finite difference and finite element methods. The elastic thermal stress analysis is dealt with exclusively by the finite element method. The methods can be used to achieve more accurate design in nozzles and other parts of the vessel. The thermostructural computer programs developed in this work are flexible and can be applied to complex geometries.

The experimental part sets out to measure the thermal stresses for a particular nozzle configuration. When theoretical predictions and experimental results are compared the former are shown to be in more than adequate agreement for design purposes. The accuracy of the numerical methods in the measurements of stresses are also analysed.

With fatigue being an aspect of vital importance for safe and reliable design of pressure vessels and their components, a

theoretical investigation, dealing with crack initiation and propagation is made in Chapter 7 based on the boundary collocation analysis. The analysis determines the values of the stress intensity factor K along the axis of a hypothetical crack introduced in the structure and subsequently compares it with experimental values of the critical stress intensity factor K_c. This will determine if the applied level of thermal stress will be large enough to initiate a crack or, in case of an already existing one, cause it to propagate. The nozzle specimen was subjected to a thermal fatigue test of alternating heating and cooling cycles for a period of four months to investigate possible crack initiation and/or propagation.

ACKNOWLEDGMENTS

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GENERAL NOTATION

Symbol	Definition
a	crack length
c_s	specific heat of steel
c	depth of crack
c_p	specific heat of air at atmospheric pressure
d_1	first matrix coefficient
D_i	inside diameter of vessel
d_i	inside diameter of nozzle
E	modulus of elasticity
g	acceleration due to gravity
h	crack height
h_{av}	average heat transfer coefficient
h_c	convection heat transfer coefficient for nozzle
h_{COND}	conduction heat transfer coefficient for nozzle
h_N	heat transfer coefficient for nozzle
h_r	radiation heat transfer coefficient for nozzle
h_v	heat transfer coefficient for vessel
M	moment
Nu	Nusselt number

Symbol	Definition
k	thermal conductivity of steel
k_a	thermal conductivity of air
K	stress concentration factor
ℓ_v	characteristic length of vessel
Pr	Prandtl number
p	pressure
q_{rad}	heat transfer rate due to radiation
Re	Reynolds number
r	crack co-ordinate
S	allowable stress (code)
S_y	shear stress at the yield point
S_m	average tensile stress
S_b	bending stress
T_f	temperature of fluid
T_w	wall temperature
T_m	mean temperature
t_v	wall thickness vessel
t_n	nozzle thickness
U_f	velocity of fluid flow

GREEK SYMBOLS

α	thermal expansion coefficient
α'	thermal diffusivity, $\alpha' = \dfrac{k}{\rho c}$
β	volume coefficient of expansion
ϵ	emissivity
ϵ	true strain
$\epsilon_1, \epsilon_2, \epsilon_3$	principal strains
θ	crack co-ordinate
μ	dynamic viscosity
ν	Poisson's ratio
ν	Kinematic viscosity
ρ	density
σ'	Stefan-Boltzman constant
$\sigma_1, \sigma_2, \sigma_3$	principal stress
$\sigma_x, \sigma_y, \sigma_z, \sigma_L, \sigma_R$	direct stresses
σ_{maxp}	maximum principal stress
τ	time
$\tau_{max(12)}$	maximum shear stress in xy plane
$\tau_{max(23)}$	maximum shear stress in yz plane
$\tau_{max(31)}$	maximum shear stress in zx plane
χ	stress function
Δx	space increment in x direction
Δy	space increment in y direction
$\Delta \tau$	time increment

NOTE: Numbers in parentheses as (R.1-6) indicate references

in Appendix 8.

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INTRODUCTION

The design and analysis of modern pressure vessels and their components constitute two very important tasks which often require a broad understanding of a number of fields. Subjects like pressure vessel design criteria, fatigue fracture, elastic and plastic analysis, as well as the influence of high temperatures and external loads are of vital importance. As the design requirements become more and more strict the need for understanding of the above mentioned subjects grows stronger.

In recent years basic changes have taken place in the design of pressure vessels and pressure vessel components. With the introduction of high speed computers, analysis of localised stresses and strains is possible and more accurate results can be obtained. Present design criteria are based on a design which accounts for all possible modes of failure such as fatigue, brittle fracture, plasticity, shakedown, mechanical and thermal ratcheting. As a result of all these considerations today's design criteria are safer and more reliable.

Fatigue, both mechanical and thermal, continues to be a field of major concern in pressure vessels and components, especially nozzles. Nevertheless, as a result of major advances and studies which have been made in the atomic, microscopic and macroscopic aspects of various materials, fatigue can be considered as a problem for which sound theoretical and experimental analyses have been achieved, and results have been obtained which provide the ground for safer design.

The risk of fracture of components, like nozzles, increases as the characteristic length of the member increases. This is because there are always hidden flaws in the metal due to manufacturing or welding. It is impossible that any structural component of a pressure vessel including the vessel shell can be completely free of flaws. It is even more speculative to assume such a thing for welded regions. Of course there is always an attempt to keep the number of such flaws at a minimum through careful fabrication but this is an ideal rather than a practical target.

The advancement of the digital computer and computational techniques has made elastic-plastic analysis possible, although plastic analysis does tend to depend on plastic theories concerning the behaviour of the material after yielding. Computing has also made the consideration of more complicated geometries possible both in 2- and 3-dimensional cases.

1. DEVELOPMENTS IN PRESSURE —VESSEL TECHNOLOGY

1.1 Pressure Vessels

The main characteristic of a pressure vessel is the pressure it can hold. This however makes the vessel a potential source of danger if it fails, leading to uncontrolled release of the pressure. Therefore the study of the failure and of the general behaviour of pressure vessels has been of great value in preventing failures. This also leads of course to better design, which is not only safer, but also more economical. During the years of pressure vessel research and development attention was concentrated in various areas of the design and construction. In early steam and chemical vessels, rivet joints were the weakest link and were the subject of much investigation. The development of the welding process and its adoption switched attention to other areas, one of which was the improvement made possible by better shapes and reinforcements for the vessel. When the design of a particular vessel included the attachment of nozzles, attention had to be paid to the possibility of failure of such branches and also the way of welding these components on to the main body of the vessel.

Most vessels are built as cylinders due to ease of fabrication. Nevertheless their actual configuration may vary according to purpose, and spherical vessels for example are often used for storage purposes. Great attention is generally

attached to vessel heads enclosing the ends of cylindrical
vessels, and to nozzles. The nozzle and in particular the area
of attachment to the vessel is the area of study of the present
work.

In recent years the development of nuclear power has been
an important factor in the improvement of pressure vessel
design. Since safety was of significant concern and protection
against radiation hazards very important the structural analysis
of pressure vessels was crucial. A second equally important
factor was the desire to upgrade all the pressure vessel codes
to take advantage of the many advances that had been made in the
understanding of pressure vessel behaviour, thereby eliminating
potential weaknesses of existing codes and permitting safe, and
in some cases more economical, construction.

Also new criteria concerning the design of pressure vessels
were formulated in the 1950's. It was during this period that a
reasonable understanding of brittle fracture and low-cycle
fatigue was reached, the principles of limit design were estab-
lished, and the mechanics of thermal ratcheting, shakedown,
elastic-plastic action, and constrained plasticity were explained.
Of equal importance to the understanding of physical processes
were the advances made in computational methods. The electronic
computer made the routine solution of complex problems first
possible and then economical.

Since 1960 new criteria have been accepted and are based
on fundamental considerations of analysis and material behaviour.
An attempt has also been made to understand all possible modes of
failure and provide rational margins of safety against each type
of failure in a manner consistent with the consequences of what
type. As a result of this quest for fundamental understanding

several basic concepts have emerged. Thermal stresses have been the subject of a more detailed analysis and vessels built to the new criteria have a balanced design to better carry thermal and mechanical stresses. It is well known that stresses resulting from a given thermal gradient increase with section thickness, whereas for a given diameter, pressure stresses decrease with shell thickness.

The basis for design stress limits and structural evaluation has also been shifted from the maximum principal stress theory to the more accurate maximum shear stress theory. This is a fundamental change and is very important in regions where one principal stress is tensile and the other is compressive. Equally significant is the recognition that pressure vessels may be subject to changing thermal stresses, and specific procedures and criteria were introduced for evaluation of thermal fatigue damage. Incorporation of these methods was made possible by the understanding gained of low-cycle fatigue failure mechanisms in the 1950's. Equipment made of ductile material will normally fail only after some plastic action. In structural evaluation of equipment, in general, only elastic analysis is required, nevertheless, many concepts from plasticity theory have proved exceedingly useful in the development of design stress limits. The use of plastic theory is a result of the attempt to understand fundamental mechanical behaviour and is only possible because of major advances in applied mechanics theory and computational ability.

The most important feature of the change in normal design procedure is the concept of design by analysis. A complete and detailed analysis of the vessel is required and the calculated stresses must be compared to various allowable values. The

allowable values are established by consideration of the possible

modes of vessel failure. Thus, average membrane stresses are set

up by consideration of gross distortion and bursting, local

stresses are permitted to exceed yield but must "shake down" to

elastic action. Peak stresses are considered in conjunction with

specified cyclic service to ensure suitable fatigue life.

So far general considerations of vessel design and develop-

ments have been reviewed for the last two decades. But the

effort for future advance in the field of pressure vessels is

continuing. One of the more significant developments at present

is the preparation of new design criteria for use in the

temperature range above 800°F (427°C). High temperature design

criteria under development are expected to follow the "design by

analysis" philosophy, namely, consideration of all possible modes

of failure, and design criteria associated with each mode.

It is certain that methods and criteria will continue to

evolve and computer programs will be used extensively to calculate

and analyse cyclic, non-linear and time-dependent deformations.

The development of rational design criteria against which to

evaluate local and general creep and stress relaxation, fatigue

in the creep range, and other time-dependent structural behaviour

is already under development. As criteria are developed,

structures designed and built, and experience is gained the

understanding of 'in-service' behaviour will become clearer and

the fitness for purpose of any pressure vessel be more assured.

1.2 Criteria for Design Analysis

The determination of stresses in pressure vessels is the

primary objective of the investigator, but a calculated, or even

measured value of stress or strain means rather little until it

is associated with its location and distribution in the structure
and with the type of loading which produced it. This can be
illustrated by comparing the average stress and strain in the
wall of a pressure vessel with the stress and strain at the
root of a sharp notch in the wall. When the internal pressure
reaches a value which produces an average hoop stress in excess
of the yield strength, the wall becomes thinner, the diameter
becomes larger, and if it were not for strain hardening of the
material, the vessel would burst. Therefore the average stress
produced by internal pressure must be kept below the yield
strength. On the other hand, the material at the root of the
notch yields even before the working pressure is reached, but no
failure occurs. At the notch the local strain can be several
times the yield strain but this condition is perfectly safe as
long as the material is ductile and the load is not cycled often
enough to start a fatigue crack. There are also other stresses
whose calculated values can be safely allowed to exceed the
yield strength of a ductile material. A well known example is a
thermal stress, which is self equilibrating within the structure.
There is no fixed external load which must be balanced and
yielding helps the material to accommodate the imposed dis-
tortion pattern. Another example is the discontinuity stress
near the junction of a head and shell which is produced by
internal pressure. In this case the head and the shell want to
expand by different amounts and distortion is required to keep
them together at the junction. Usually stresses can be grouped
in three distinct categories. These can be designated as primary
stresses, secondary stresses and local or peak stresses. Primary
stresses are stresses developed by the imposed loading which is
necessary to satisfy the laws of equilibrium of external and

internal forces and moments. The basic characteristic of primary

stresses is that they are not self-limiting. If a primary stress

exceeds the yield strength of the material, the prevention of

failure is entirely dependent on the strain hardening properties.

Secondary stresses are developed by the self constraint of the

structure. The basic characteristic of secondary stresses is

that they are self-limiting since minor distortions can satisfy

the discontinuity conditions which cause the stresses to occur.

The local or peak stresses are the highest stresses in the

region under consideration. Both primary and secondary stresses,

as well as stress concentration effects, are considered. The

basic characteristic of local stresses is that they cause no

significant distortion and are objectionable only as a possible

source of fatigue failure, brittle fracture or stress corrosion

cracking. Nevertheless it can also be said that membrane

stresses, the average value across a section, and bending stresses

(linearly variable across the section) are stresses that can be

included within the above three groups of stresses.

If assessment has to be made of how close a given calculated

stress is to the danger point, it is helpful to consider the

various possible modes of failure. According to the International

Institute of Welding (R.1-1) these modes can be grouped as

follows:

1. Excessive plastic deformation, including plastic instability.

2. High strain fatigue.

3. Corrosion fatigue.

4. Stress corrosion.

5. Brittle fracture.

6. Excessive elastic deformation, including elastic instability.

The specification of allowable stresses is in fact a function of the code writing bodies rather than of research. Nevertheless the researcher must be able to produce concrete answers on what conditions will or will not produce failure so that designers can have a basis for choosing their margin of safety.

The analysis of stresses and strains must be sufficiently detailed to include multiaxiality. It is not sufficient to know only the magnitude of the largest principal stress at a given point. All three of the principal stresses should be known in both magnitude and direction, since it is well established that the distortion energy theory (also known as the Mises criterion and the octahedral shear theory) is a much better criterion of yielding and fatigue failure in ductile metals than is the maximum stress theory. The maximum shear stress theory is often used as a close and conservative approximation to the distortion energy theory, but this also requires knowledge of all three principal stresses. Knowledge of the stress distribution, as distinguished from knowledge of only the peak stress at a single point, is important. A stress which is uniform across the thickness of a section has a different significance from one which varies across the thickness because if the uniform stress exceeds the yield strength, the load must be transferred to other sections in other parts of the structure if collapse is to be avoided, and this may involve large distortions. On the other hand, if a member is subjected to a bending load which puts only the outer fibres of a section into a yielded condition the load is transferred to the inner fibres and a great load is required to produce collapse. Knowledge of stress distribution is also important in the study of fatigue phenomena. It is well

known that theoretical stress concentration factors frequently give an exaggerated picture of the real fatigue strength reduction in a member and one of the most important quantities is the stress gradient at the notch as well as the value of the peak stress.

In order to prevent failure of the pressure vessel by any of the failure modes that have already been mentioned certain limits have to be imposed on the value of stresses. The lowest of the allowable values must be assigned to primary stresses, which are the stresses required to satisfy the laws of equilibrium of internal and external forces. These stresses are not self-limiting and if the average value across a solid section exceeds the strain hardened yield strength, failure will occur. Consideration should also be given to the distribution of the primary stress across the thickness of a section, since the sum of membrane and bending components may be allowed to attain a higher value than the membrane component itself. The chosen allowable stress values must keep the imposed mechanical loading safely below the collapse load of the structure. Considerably higher values may be assigned to the calculated values of primary-plus-secondary stresses. If these stresses are calculated on an elastic basis, they do not need to be kept below yield strength, but must be kept low enough to insure that the elastic analysis is a valid measure of the strains in the structure. The primary-plus-secondary stresses must be limited to values which will prevent both cyclic and incremental collapse. Additionally, the sum of the primary and secondary stresses must be kept below the value which might produce brittle fracture, considering the temperature of operation and the size of the

possible defects. Finally a third set of allowable values must

be assigned to the peak stresses in the structure, since they

may form the nuclei for fatigue or stress corrosion cracking.

The allowable value to prevent fatigue depends on the threshold

stress for this phenomenon determined from tests. The establish-

ment of allowable stresses is based on the assumption that the

material being used has sufficient ductility to accommodate the

required plastic flow. Most of the materials commonly used for

pressure vessels do have enough ductility to accommodate

secondary and localised stresses as described above. When,

however, higher strength steels are used what degree of reduced

ductility can be accepted? When analysis assumes the idealised

stress-strain curve it is assumed that there is no limit to the

amount of plastic strain which can occur. Sharp corners and

defects such as might be found in many vessels can produce

theoretical stress concentration factors as high as 10 or even

more, even though the fatigue strength reduction factor is much

lower. Secondary stresses can produce strains as high as twice

the yield strain in relatively large volumes of material without

causing failure. Nevertheless sometimes loadings and stresses

do not affect the failure criteria directly. For instance, the

burst pressure is significant because it indicates how much

safety margin exists as protection against gross over-pressure,

but for protection against fatigue failure the burst pressure

has no significance at all. The true fatigue margin in the

design is the ratio between the highest strain range found at

any point and the strain range which the material can tolerate

for the required number of cycles. In establishing this true

safety margin for fatigue, the notch sensitivity of the material

is important, since the fatigue strength reduction factor, as

differentiated from the theoretical stress concentration factor, depends on both the notch sensitivity of the material and the stress gradient at the notch. For higher temperature when creep is significant, the differentiation between primary and secondary stresses is particularly important. Creep, like plastic flow, aggravates primary stresses but reduces secondary stresses. Therefore in the absence of notches, it is probably feasible to select values of allowable primary stress for use at high temperature which are controlled by the creep strength. Allowable values of secondary stress might still be based on the yield strength at the operating temperature. When notches are present, additional precautions are required to control the concentrated values of primary stresses, since the stress rupture strength at high temperature may be reduced by notches, whereas the yield strength at room temperature is not.

1.2.1 Strength Theories

The stress state at any point in a structure may be completely defined by giving the magnitudes and directions of the three principal stresses. When two or three of these stresses are different from zero, the proximity to yielding must be determined by means of a strength theory. The theories most commonly used are the maximum stress theory, the maximum shear stress theory (also known as the Tresca criterion), and the distortion energy theory (also known as the octahedral shear theory and the Mises criterion). It has been known for many years that the maximum shear stress theory and the distortion energy theory are both much better than the maximum stress theory for predicting yielding and fatigue failure in ductile metals. Most experiments show that the distortion energy theory is even more

accurate than the shear theory, but the shear theory is a little

more conservative, it is easier to apply, and it offers some

advantages in some applications of the fatigue analysis.

The maximum shear stress at a point is defined as one-half

of the algebraic difference between the largest and the smallest

of the three principal stresses. Thus, if the principal stresses

are σ_1, σ_2 and σ_3, and $\sigma_1 > \sigma_2 > \sigma_3$, the maximum shear stress is

$\frac{1}{2}(\sigma_1 - \sigma_3)$. The maximum shear stress theory of failure states

that yielding in a component occurs when the maximum shear stress

reaches a value equal to the maximum shear stress at the yield

point in a tensile test. In the tensile test, at yield,

$\sigma_1 = Sy$, $\sigma_2 = 0$ and $\sigma_3 = 0$. Sy is the shear stress at the yield

point. The maximum shear stress is Sy/2. Therefore yielding in

the component occurs when

$$\frac{1}{2}(\sigma_1 - \sigma_3) = \frac{1}{2} Sy \qquad (1.2.1.1)$$

In order to avoid dividing both the calculated and the allowable

stresses by two before comparing them, a new term called

"equivalent intensity of combined stress" or "stress intensity"

is used. The stress intensity is defined as twice the maximum

shear stress and is equal to the largest algebraic difference

between any two of the three principal stresses. Thus the stress

intensity is directly comparable to strength values found from

tensile tests. For simple analysis of low pressure vessel

stresses, it makes little difference whether the maximum stress

theory or the maximum shear stress theory is used. For example,

in the wall of a thin-walled cylindrical pressure vessel, remote

from any discontinuities, the hoop stress is twice the axial

stress and the radial stress on the inside is compressive and

equal to the internal pressure, p. If the hoop stress is σ, the

principal stresses are: $\sigma_1 = \sigma$, $\sigma_2 = \sigma/2$ and $\sigma_3 = -p$. According to the maximum shear stress theory, the controlling stress is the stress intensity, which is $(\sigma + p)$. Since p is small in comparison with σ for a thin-walled vessel, there is little difference between the two theories. In more complex stress situations, however, the difference between the two theories often becomes important.

Stress intensity limits are one of the major considerations in the design of pressure vessels. It has already been mentioned that stresses can be grouped in three basic categories of primary stresses, secondary stresses, and peak stresses. Limit design theory is based on certain assumptions and the choice of the stress intensity limits for the stress categories mentioned above is accomplished by the application of the limit design theory. The basic assumption which is made is that there is perfect plasticity with no strain-hardening. In a structure as simple as a straight bar in tension, a load producing yield stress, Sy, results in collapse. If the bar is loaded in bending, collapse does not occur until the load has been increased by a factor known as the shape factor of the cross section, at that time a plastic hinge is formed. The shape factor for a rectangular section, for instance, in bending is 1.5. When the primary stress in a rectangular section consists of a combination of bending and axial tension, the value of the limit load depends on the ratio between the tensile and bending loads. Fig.1.2.1.1 shows the value of the maximum calculated stress at the outer fibre of a rectangular section which would be required to produce a plastic hinge plotted against the average tensile stress across the section, both values expressed as multiples of the yield stress, S_y. When the average tensile stress, S_m, is zero,
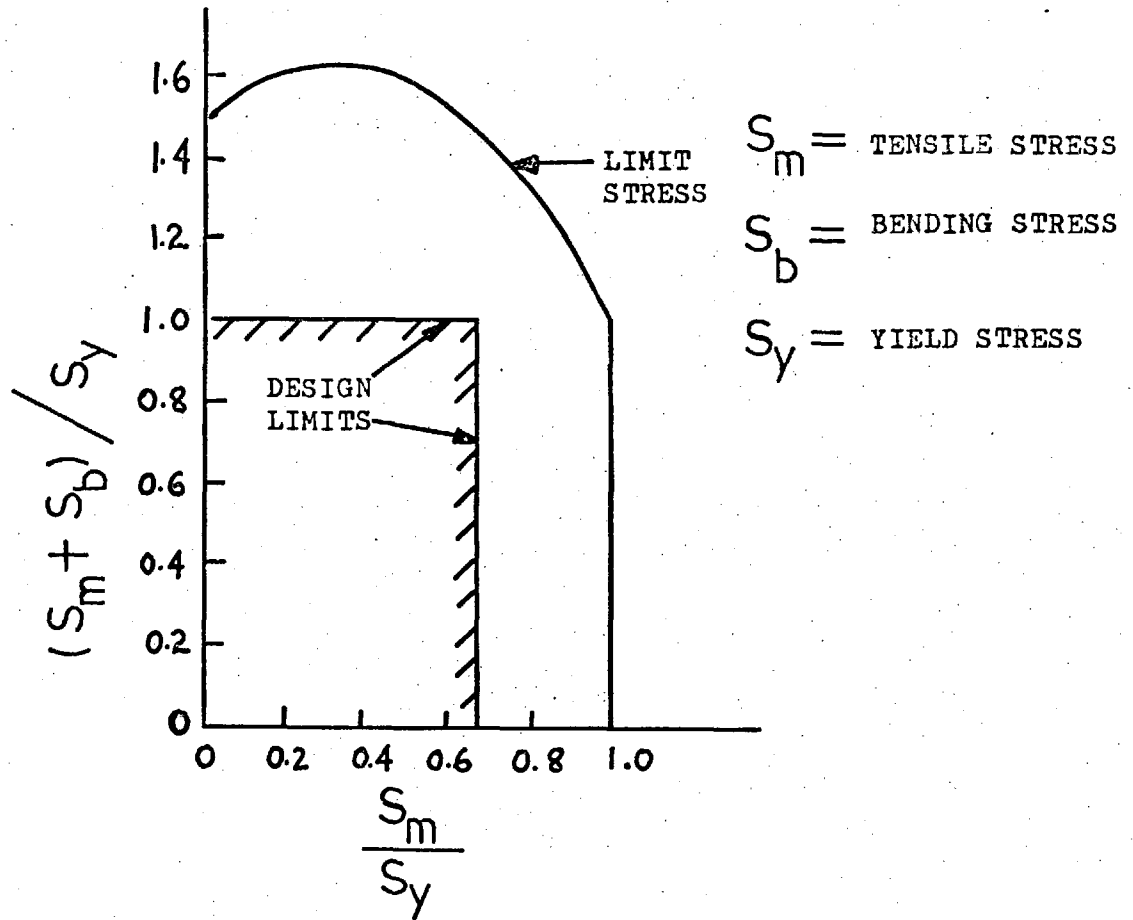
FIG. 1.2.1.1. Limit stress for combined tension and bending for rectangular section.

S_m = TENSILE STRESS

S_b = BENDING STRESS

S_y = YIELD STRESS

$S_y < S_1 < 2S_y$

(a)

$S_1 > 2S_y$

(b)
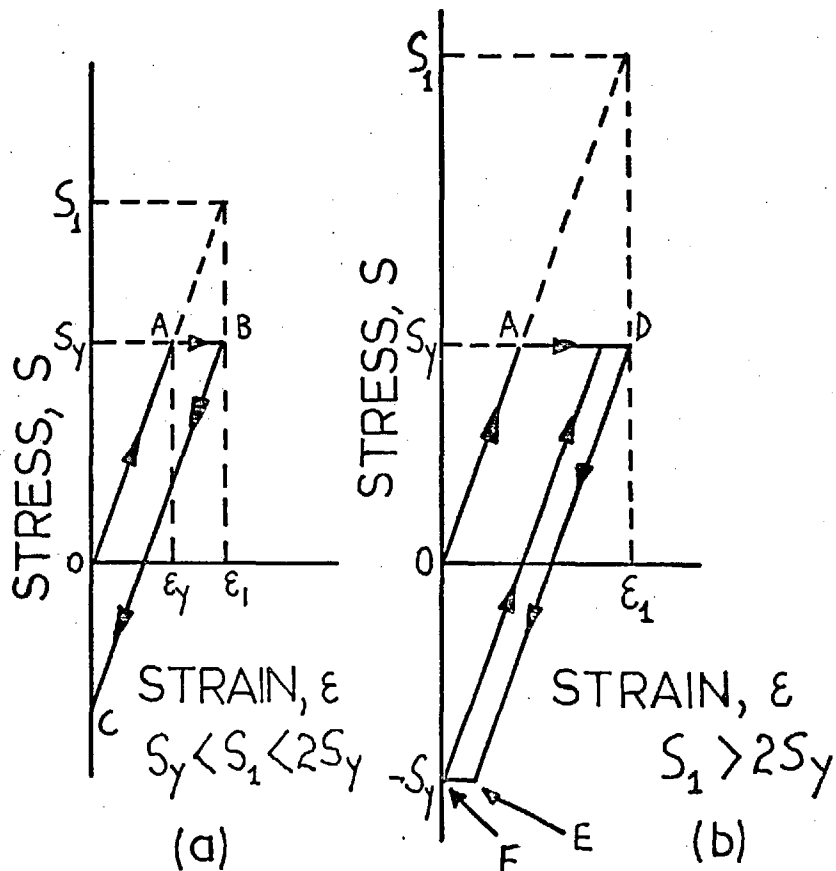
FIG. 1.2.1.2. Strain beyond yield.

the failure stress for bending is $1.5S_y$. When the average

tensile stress is S_y, no additional bending stress, S_b, may be

applied. Fig.1.2.1.1 can provide allowable values, in terms of

the yield stress, for general primary membrane stress, S_m, and

primary membrane plus bending stress, $S_m + S_b$. It may be seen

that limiting S_m to $(2/3)Sy$ and $S_m + S_b$ to S_y provides adequate

safety. Nevertheless the safety factor is not constant for all

combinations of tension and bending, but a design rule to pro-

vide a uniform safety factor would be unnecessary complicated.

In the study of allowable secondary stresses, a calculated

elastic stress range equal to twice the yield stress has a very

special significance. It determines the borderline between loads

which, when repetitively applied, allow the structure to "shake

down" to elastic action and loads which produce plastic action

each time they are applied. The theory of limit design provides

rigorous proof of this statement, but the validity of the concept

can easily be visualised. If, for example, the outer fibre of a

beam is considered which is strained in tension to a strain value

ε_1, somewhat beyond the yield strain as shown in Fig.1.2.1.2(a)

by the path OAB. The calculated elastic stress would be

$S = S_1 = E \varepsilon_1$. Since the case of secondary stress is considered

it will be assumed that the nature of the loading is such as to

cycle the strain from zero to ε_1 and back to zero, rather than

cycling the stress from zero to S_1, and back to zero. When the

beam is returned to its undeflected position, O, the outer fibre

has a residual compressive stress of magnitude $S_1 - S_y$. On any

subsequent loading, this residual compression must be removed

before the stress goes into tension and thus the elastic range

has been increased by the quantity $S_1 - Sy$. If $S_1 = 2Sy$, the

elastic range becomes $2Sy$, but if $S_1 > 2Sy$, the fibre yields in

compression, as shown by EF in Fig.1.2.1.1(b) and all subsequent

cycles produce plastic strain. Therefore, 2Sy is the maximum

value of calculated secondary elastic stress which will "shake

down" to purely elastic action.

The allowable stress intensity for austenitic steels and

some non-ferrous materials, at temperatures above 100°F, may

exceed (2/3)Sy and may reach 0.9Sy at temperature. Nevertheless

if Fig.1.2.1.1 is considered it will be seen that loads in

excess of the limit load are permitted. This is because these

materials have different stress strain diagram. Austenitic steels

that are usually employed for the construction of pressure

vessels have no well-defined yield point but have strong strain-

hardening capabilities so that their yield strength is effectively

raised as they are highly loaded. This is equivalent to choosing

a different definition of the "design yield strength" for those

materials which have no sharply defined yield point and which

have strong strain hardening characteristics. It can be con-

cluded that the primary criterion of the structural adequacy of a

design, is that the stresses, as determined by calculation or

experimental stress analysis, shall not exceed the specified

allowable limits. The third category of stresses, peak stresses,

are related only to fatigue and they will be discussed in

Chapter 7. In the case of fatigue analysis, plastic action can

actually persist throughout the life of the vessel and plastic

zones within the structure are responsible for initiation and

propagation of cracks. Repetitive plastic action occurs only

as the result of peak stresses in relatively localised regions

and these regions are intimately connected to larger regions of

the vessel which behave elastically. Typical examples of peak

stresses are those at the root of a notch in a fillet, at the

edge of a small hole, or at the junction of nozzle and vessel.
The material in these small regions is strain-cycled rather than
stress-cycled and the elastic calculations give numbers which
have the dimensions of stress but are really proportional to
the strain.

2. REVIEW OF RECENT INVESTIGATIONS IN PRESSURE VESSEL NOZZLES

2.1 Nozzle and Openings

It has generally been accepted that the design of nozzles and openings in pressure vessels is guided by the "replacement of area" rules. The rules stipulate that the area cut out of the vessel by the opening must be replaced within a prescribed zone around the opening. The area replacement rule may be considered as a crude limit pressure design. The nozzles and openings so designed are not likely to rupture due to a few applications of pressure up to the yield pressure of the unperforated vessel. This statement is made on the assumption that fabrication does not introduce cracks in the material and that brittle fracture does not occur. Nevertheless it is found that during fabrication, cracks are introduced and rupture of the component occurs below the expected limit of yield pressure. Additionally the area replacement rule does not give any assurance that performance will be satisfactory for cyclic pressure loading, for static or cyclic external loads such as piping systems which may impose forces and moments on the nozzles, for cyclic thermal gradients, or for combinations of those loads. Therefore, while design rules exist for nozzles and openings, a significant amount of research effort has been and is being directed toward improvement in their design and analysis. Elastic stresses in isolated, radial nozzles and circular openings in spherical shells can be analysed by means of axisymmetric shell theory. Waters

(R.2-1) developed a theory specifically for a uniform wall nozzle in a uniform wall sphere for internal pressure loading. Leckie and Penny (R.2-2) developed a similar theory, but more general in that it also covers moment, thrust, and shear loads applied to the nozzle. Calculated data on stresses at nozzles in spherical shells are given in reference (2-3). These data are for both internal pressure and external loads on the nozzle and cover a wide range of dimensional parameters, and they are useful for design at the design code level.

Computer programs have also been developed for various loading conditions but only a limited number of them will be referred to here. An axisymmetric shell computer program applicable to nozzles in spherical shells with variable wall thickness and for internal pressure, external loads on the nozzle and thermal gradients was developed by Kalnins (R.2-4). Axisymmetric finite element computer programs, such as that developed by Wilson (R.2-5), are also applicable to the same geometries and loadings as the shell computer programs but can be used to establish stresses in greater detail and with more accuracy than possible with shell programs.

Nozzles in cylindrical shells present a more difficult theoretical problem. Until a few years ago, analytical estimates of elastic stresses at small nozzles or openings were often obtained by reducing the problem to that of an opening or nozzle in a flat plate with edge loads. Papers by Beskin (R.2-6) and Waters (R.2-7) are examples of this sort of approximation. Solutions to the problem of a cylindrical shell with a circular opening are given in (R.2-8 and 2-9). Solutions to the problem of two normally intersecting cylindrical shells, with internal pressure loading is treated by A.C. Eringen (R.2-10). He solves

the boundary value problem by solving the differential equations of the two shells for a ratio of radii smaller than 0.3 and subjected to edge conditions along the intersection curve and at the ends of the cylinders. Bijlaard, Dohrman and Wang (R.2-11) formulated the problem for $d_i = D_i$ and indicated the form of the solution. Pan and Beckett (R.2-12) developed a theory for d_i/D_i up to unity and give a numerical example for $d_i = 0.5\ D_i$.

Several papers related to the subject of non-radial nozzles and non-circular holes are published in the Welding Research Council Bulletin No.153.

The overall problem of nozzles and openings in pressure vessels is quite complex because of the many dimensional parameters involved and because of the variety of loadings. The advances in the field of pressure vessel nozzles will now be examined from the theoretical and experimental sides. As will be seen, although a considerable amount of work has been done on calculating stresses in nozzles due to internal and/or external pressure and loadings the field of thermal stresses has been neglected and there are hardly any works that are directly concerned with thermal stresses exclusively. The behaviour of nozzles and openings at elevated temperatures, where creep and/or relaxation occurs, also needs further work in order to develop reliable design methods.

2.2 Theoretical Investigations of Nozzles

The theoretical work which has been done the last twenty years is concerned with elastic theory applied to nozzles, limit analysis of pressure and loads. Considerations on external loads have been met with difficulty and the work by Bijlaard (R.2-13, 2-14 and 2-15) estimates external loads on the nozzle

for distributed loads on the surface of cylindrical shells.
Bijlaard's theory has been completed with empirical modifications
but is essentially limited to small values of di/Di (up to 1/3
or at most ½). Pressure vessel nozzles can have a variety of
shapes and arrangements. Considerable amount of effort must be
spent to adapt straightforward solutions of the shell theory to
the mathematical idealisation of the local geometry. First
Beskin (R.2-6) studied the subject of reinforcement. He repre-
sented the reinforcement by a pair of pads on both sides of the
vessel wall, or a rim at the edge of the hole, or a combination
of both. He considered the vessel wall to be an infinite flat-
plate and therefore the results were thus strictly applicable
only to small openings. With larger opening-diameter to vessel-
diameter ratio, the effect of curvature increases the stress
concentration. Nevertheless it was found that the high predicted
stresses at the junction were sometimes much greater than that
observed. Agreement may be good along the nozzle and the vessel
wall, but the theoretical meridional bending-stress at the
junction does not represent adequately the actual load stress at
the junction and is, in many cases, much too high when the nozzle
is of the protruding type or when the nozzle is finished with
no sharp stress-raiser. This inaccuracy is due to the limitations
of the thin-shell theory which represents the shell as being
located at its mid-wall position, and thus cannot picture
accurately the local stresses at a shell discontinuity unless
additional correction is made there to a direct application of
the thin-shell theory.

Theoretical solutions of non-symmetric shells are harder to
achieve than shells of revolution. Nevertheless, recently, finite

element analysis and computer programs are used and can calculate
elastic and plastic stresses for nozzles in cylindrical shells.
The geometry of the configuration can be non-symmetric since
finite element analysis employs basic elements into which the
structure is broken down. Therefore finite element methods are
of course, applicable to variable wall thickness and many
possible nozzle or opening configurations. The significance of
the calculated elastic stresses in pressure vessel nozzles must
be assessed in terms of performance criteria such as excessive
deformation or rupture, due to either single loads applications,
or many load applications leading to fatigue failure, which is
the more normal operational condition.

2.2.1 Limit Pressure and Loads

Theoretical analysis of limit pressure for various types of
nozzles has attracted the attention of a great number of researchers
and S.S. Gill (R.2-16) gives an account of a theoretical analysis
for flush cylindrical nozzle in a spherical pressure vessel.
Membran forces in circumferential and meridional directions in
the shell and longitudinal membrane force in nozzle are used to
define a yield surface and the limit pressure was calculated for
a flush nozzle for a range of parameter ratios of nozzle radius
to sphere radius, and thickness ratios. The analysis considers
plastic behaviour and the method uses upper and lower bound
theorems. The nozzle is subjected to internal pressure, shear
force and moment at the junction. Using similar methods Dinno
and Gill (R.2-17) calculated the limit pressure for the case of
a spherical and cylindrical vessel with hemispherical ends inter-
sected by a nozzle. Various mechanisms of failure were considered
and the collapse pressure for the vessel was studied. It was

assumed that the effect of strain hardening and the effect of fillet welds at the junction are neglected. The junction hinge is assumed to form at the intersection of the centre lines of the plate thickness of the cylinder and sphere. Upper and lower bounds to the collapse pressure are calculated and these are for the assumed interaction surface which circumscribes the true interaction surface for a material which obeys the Tresca yield criterion. The effect of changes of geometry on the collapse pressure is also neglected. Cloud (R.2-18) has calculated limit pressure in a spherical shell intersected by a cylindrical nozzle and also for a pad reinforced shell. The analysis assumes a perfectly plastic material and the equations for equilibrium for the axisymmetric shell of revolution were used. Graphs of radial deflection of shell at nozzle junction for various internal pressures were provided. The theory was completed with deformation tests made on three steel models of cylindrical nozzles in hemi-spherical heads.

Coon, Gill and Kitching (R.2-19) obtained a lower bound of the limit pressure in a cylindrical pressure vessel with an unreinforced hole. The stress distribution was restricted to one in which the principal stress directions were in the circumferential and longitudinal directions of the cylinder. The results of the theoretical analysis were compared with experimental work on the limit pressure of cylindrical pressure vessels with branches. It was evident from the analysis that as the ratio t/T of the thickness of the branch (t) to the thickness of the cylinder (T) decreased a rapid fall of the limit pressure was observed. An approximate analysis of the plastic limit pressures of nozzles in cylindrical vessesl was made by Cloud and Rodabaugh (R.2-20) who estimated the limit

pressure from an upper bound analysis. The analysis was re-
stricted to nozzle/shell diameter ratios of ½ or less. They
also compared the theoretical analysis with other theoretical
approaches and the results of a single experiment. Design
graphs based on the analysis were prepared to indicate the
amount of nozzle or shell thickness required to make the limit
pressure of the nozzle-cylinder structure approximately equal to
the yield pressure of the unperforated cylinder. Calladine and
Goodall (R.2-21) studied the plastic behaviour of thin cylindrical
pressure vessels with circular openings and radial branches.

Shakedown pressures for cylinder-sphere shell intersections
were studied by Leckie (R.2-22). Lower bound estimates were
found of the shakedown pressure. This was achieved by applying
Melan's theorem and making use of elastic solutions already in
existence. The stress concentration and shakedown factors were
presented in graphical form for various ratios of nozzle/shell
thickness from 0.25 to 1.0. The same author together with
Penny (R.2-23) obtained further theoretical estimates of shake-
down values for pressure and thrust and moment loadings applied
through a radial nozzle in a spherical vessel. Again Melan's
theorem was used and by using available elastic solutions. The
results were obtained by using linear programming techniques and
have been presented in a useful graphical form. Elastic-plastic
behaviour of flush nozzles in spherical pressure vessels were
studied by Marcal and Turner (R.2-24) by developing a computer
program for the elasto-plastic analysis of axially symmetrical
shells of revolution. The program was modified appropriately to
allow interaction loads at nozzle junctions to be distributed
over bands of finite width, rather than the conventional con-
centrated lines of loading at the intersection of the shell centre

lines. They compared the results with previously published test

results for displacements, yield and collapse loads of flush

nozzles and showed that the above modification in the approach

improves the predictions from those of conventional shell theory.

Deflections of nozzle junctions were plotted against pressure

and meridional strain (maximum) were also plotted for various

values of pressure. This was done for nozzles with and without

fillet weld. It was found that the results were in broad agree-

ment with experiment.

2.2.2 Elastic Analysis

Stresses due to internal pressure loadings have been studied

on the basis of elastic theory as well as experimentally.

Bijlaard (R.2-25) developed a method where the stresses in

a juncture of a nozzle to cylindrical pressure vessel for equal

diameters can be determined. The application of the proposed

method included pipe lines, header nozzle junctures, process

reactors, etc. In the derivation of the methods utilised to

compute the stresses, the proposed shell theory takes into

account in fact that the shells are thick. This fact is neglected

in conventional thin shell theory. As part of the development in

the paper the shell shear deformation and the resulting stresses

are considered in addition to the thick shell stress distributions

caused by internal pressure loadings. The solution of the three

coupled partial differential shell equations is obtained through

the use of a series solution that satisfies the differential

equations as well as the continuity conditions at the juncture of

the two shells. In addition to the theoretical development, a

proposed method for the numerical solution of the equation is

given. Hasberry and Jones (R.2-26) made a theoretical study

into the elastic behaviour of a joint formed by the normal inter-
section of a right circular cylindrical shell with another of
larger diameter. The wall of the larger cylinder was assumed
to remain open inside the joint in order to given an arrangement
which is encountered frequently in pressure vessels or pipeline
intersections. An external bending moment which acts in the
plane of the joint was applied to the nozzle cylinder and is
equilibrated by moments of half this magnitude applied to either
end of the parent cylinder. A solution for this loading has been
obtained by assuming antisymmetric distributions of certain
stresses across a plane transverse to the joint. The analysis
presented is believed to be valid for nozzle to cylinder diameter
ratios of less than 1/3. Numerical results are given by a
number of cases having radius ratios of 1/10 and 1/4.

The problem of two normally intersecting cylindrical shells
subjected to internal pressure was considered by Pan and Beckett
(R.2-27). They solved the differential equations of shells sub-
ject to the boundary conditions imposed along the intersection
between the two cylinders. Numerical results for a radius ratio
of ½ are also given. They also discuss problems concerned with
the numerical computations and there is certain comparison with
some experimental results. Most of the analyses of elastic
stresses due to external loads are experimental and they are
discussed in Section 2.3.

2.2.3 Reinforcement of Nozzle Openings

Reinforcement of nozzle openings has so long been served by
the "area replacement rule". This has been proved to be a
satisfactory method. Although failure of pressure vessels is
relatively rare, many of the failures occur in the vicinity of

the openings due either to brittle fracture or fatigue (even

though the nature of the operation may not be cyclic in the

normal sense of the term). Although inadequate welding and

inspection are often factors in such failures, there is no

doubt that the high stress level existing in the vicinity of

the openings is a basic factor in the problem. Theoretical

and experimental procedures are quite difficult to be formulated

and this is mainly due to the large number of variables involved.

Nevertheless theoretical analyses have been made and an account

of some of them is given below.

2.2.3.1 Spherical Shells

A relatively rigorous analysis of the stresses in nozzle

connections in spherical shells have been made by

Professor E.O. Waters (R.2-28). The work indicated that there

were some significant differences under certain circumstances

between theoretical and experimental work. It has been found

that the outside fillet radius at the nozzle shell junction can

have a major influence on the resulting stress level. This is

rather apparent due to the fact that even a small fillet radius

can constitute an appreciable increase in effective nozzle wall

thickness at the most critical location, and is particular

important at small d_i/D_i ratios where the fillet radii used for

the experimental modes were sometimes relatively large in

relation to nozzle wall thickness.

Professor N.C. Lind (R.2-29) provided another more simpli-

fied analysis which gives only the hoop stress at the

intersection. Comparison of the results from Water's and Lind's

solutions with those of Penny and Leckie (R.2-30) indicated

that there is reasonable agreement for d_i/D_i ratios in the range

of 0.25-0.50, for D_m/t_v ratio of 100. However, for the same

range of diameter ratio and a materially greater D_i/t_v, the

Penny and Leckie results are low, and for a materially smaller

D_i/t_v ratio, the results are high. Rose and Thompson (R.2-31)

provided values of stress concentration and for small diameter

ratios they tend to agree with Penny's and Leckie's results but

both give results higher than Water's solution.

2.2.3.2 Cylindrical Shells

Theoretical investigations of stresses around holes in

cylindrical shells have been made by A.C. Eringen. Two

solutions were developed, the first is reported in reference

(2-32) and employs a perturbation technique. The second refer-

ence (2-33) employs an exact solution of thin shell theory and

satisfies the boundary conditions in a collocation sense, that

is in a definite number of points along the boundary. The

later work was later programmed for a digital computer, and

extensive data calculated for the case of a circular hole with

membrane closure $(t_n/t_v = 0)$, the results of which are reported

in references (2-34) and (2-35). The limitations of the

presently available data are as follows:

$$d_i/D_i \lessapprox 0.35$$

$$D_i/t_v \lessapprox 20.0$$

$$(d_i/D_i) \ \sqrt{(D_i/t_v)} - \text{between 0.155 and 3.90}$$

The works give graphs of diameter and thickness ratios against

stresses tangential to the edge of the opening on the longi-

tudinal and transverse axes of the cylinder. The distribution

of the tangential membrane stress around the edge of the hole is

also given and it is possible to locate points of maximum mem-

brane stress. Work is being done to extend the study to larger

diameter ratios.

2.3 Experimental Investigations of Nozzles

Experimental measurements of stresses due to various loads and internal pressure have been obtained using the strain gauge. Recent research on the experimental side of the stresses in pressure vessel nozzles has produced test data for various cases and an account of this research is given herein. The strain gauge is laid on and usually adhered on to the surface of the vessel. Any straining of the surface in the direction of the gauge causes a small change in the electrical resistance of the gauge wire, which is detected and quantitately measured by a sensitive amplifying instrument.

The other commonly used method for analysing stresses at a pressure vessel is photoelastic analysis. Stress freezing is used and it is a very convenient way for three-dimensional geometries. Vessel components forming shells of revolution simplify the analysis and reduce the work because of symmetry, but stress freezing is still required. The great advantage of stress freezing photoelastic analysis is that stress changes through the shell wall can also be measured. A complete picture can thus be made of the stresses if required. This is especially useful for thick-walled vessels, thin vessels are known to have a linear distribution of stresses through the thickness. Nevertheless there is always an unknown factor in the use of photoelastic results, this being the effect to the stress values caused by differences in the Poisson's ratio ν. Steel and other common pressure vessel materials have a value ν of about 0.3 while photoelastic plastics undergoing stress freezing have a value of 0.5. Studies on the nature of this effect have been made in references (2-36) and (2-37). The observations were

found to apply only to specific problems and could not be generalised, even extensions of observations to other similar geometries and loadings have to be treated with care.

Elastic stresses due to internal pressure have been investigated by a large number of researchers and Everett and McCutcham (R.2-38) have investigated the stress conditions in full-size branch connections under internal pressure. William, Morgan and Bizon (R.2-39) investigated the elastic stress distributions of two shell junctions and obtained measurements of the stresses. The junctions were incorporated in a single type of structure, a cylinder with a toriconical head. The data obtained from the tests were compared with theoretical curves determined by previously published analyses. It was established that one of the principal sources of experimental error was the variation in geometry from the dimensional values assumed for purposes of analysis.

In reference (2-40) experimental data were obtained by Hardenbergh and Zamrik to show the general state of stress in nozzles. This is one of the most complete works and includes studies of five hot-formed contoured nozzles, four insert nozzles, and one juncture reinforced by an annular pad. The nozzles vary in size, and in the amount and distribution of the reinforcing material around the outlets. The tests were con-ducted under internal pressure and various external loads to the branch pipe. Bending moments and effects of internal pressure on the elastic stress distributions in the vicinity of two transition rings used to connect cylindrical ducts with large spherical pressure vessel have been studied by Kitching and Jones (R.2-41). Experimental values of stresses were obtained near the weld where it was found that stresses attained

their maximum value. Riley measured stress distributions in thin walled cylindrical and spherical vessels with circular nozzles. Stress distributions were determined in a series of vessels for internal pressure, axial thrust on the nozzle, and also external moment forms of loading (R.2-41). Stresses due to axial loads and internal pressure on forged nozzles in spherical pressure vessels were studied by Kitching and Duffield for two different type forging rings which provide different degrees of reinforcement. It was found that there was a marginal reduction in maximum stress for the heavier specimen. Stress concentration factors due to pressure were also determined (R.2-43). Experimental investigations of plastic behaviour of nozzles were made by Dinno and Gill (R.2-44) and Cottam and Gill (R.2-45).

3. THE NATURE OF THE PRESENT PROBLEM OF THERMAL STRESSES

A great deal of research and investigation has been done in the field of pressure vessel nozzles, as described in the previous sections; but the research has been exclusively concentrated on evaluating stresses due to internal pressure and external loads. Thermal stresses, although very important to the design, have been treated rather qualitatively than quantitatively. Allowances for thermal stresses in design tend to take the form of an increase of the value of the safety factor based on predictions of thermal stresses at regions where they can be quite high and calculations have been far from detailed and precise. Temperatures across the thickness of nozzle and vessel walls were represented by mean values and thermal transients were treated by various analytical methods which were limited to simple geometries and boundary conditions and could not be applied easily to complex cases. Whenever the wall thickness is small the approximation of a mean temperature can be accepted and the error involved is comparatively small too, but this is not true when the vessel or nozzle wall is thick. Additionally, since the design of pressure vessel is greatly influenced by economic factors the precise and detailed determination of thermal stresses has become a desirable goal.

The theoretical part of this work deals with the following three main areas:-

(i)   The determination of time dependent temperature dis-

tributions within the nozzle and vessel walls during

warm-up for radial nozzles of various thicknesses

attached to the vessel by different junction con-

figurations.

(ii)   The calculation of transient thermal stresses due to

the effect of the thermal transients during warm-up and

cool-down and the determination of the areas of high

thermal stress concentrations.

(iii)   The determination of stress intensity factors K at areas

of high stress concentration in order to make predictions

about fatigue life and possible crack growth and pro-

pagation which could lead to fracture under certain

conditions.

Initially, the methods by which the above quantities have

to be evaluated are formulated. The equations describing the

various physical conditions are obtained and solved numerically

by writing the appropriate computer program and using the College

CDC6400 and the London University CDC6600 computers. Numerical

methods have the great advantage that they are immensely flexible

thus permitting them to be used over a wide range of various

geometries and boundary conditions. They are also fast and

accurate. Nevertheless it must be noted that whenever com-

putational methods are employed attention must be paid to the

evaluation of the errors involved in the numerical procedures.

This in turn will permit the necessary steps to be taken to

ensure that these, such as convergence and round-off errors, do

not affect the values of the quantities under investigation.

3.1 The Geometry of the Pressure Vessel Nozzle

There are various types of nozzles attached to pressure vessels but here only the radial type is considered. Radial nozzles are the most common ones and sections of a radial nozzle are shown in Fig.3.1.1. Nozzles can be of the protruding type or flush type. Protruding nozzles penetrate the vessel wall and are continued within the vessel. Alternatively, nozzles can be welded directly on to the vessel wall. In Fig.3.1.1 a radial nozzle of the protruding type and a flush nozzle are shown, together with a nozzle directly welded on the vessel. Variations of type (c) are the ones that are examined herein. The sections of the nozzles are through the longitudinal plane of the vessel as shown in Fig.3.1.2.

When a nozzle is directly welded on the vessel the junction can have different configurations. The form of the welding connection can also vary. The geometry of the junction affects the stresses directly and therefore is an area that requires investigation. The accurate determination of the stress levels near the junction on both nozzle and vessel sides inevitably leads to the designer being able to reinforce the area without having to speculate on the amount and type or reinforcement. The area replacement method as it has already been explained is rather a crude way of applying reinforcement. This is due to the fact that up to now stresses around the junction, as well as peak stresses at the junction, could not be evaluated accurately.

The configuration of Fig.3.1.1(c) is studied theoretically and experimentally. The advantage of this configuration is that the junction is now part of the main shell and the weld is further away from the junction. Therefore this contributes to

a higher strength since there are no discontinuities due to welding connections. Nevertheless the above design is more expensive because the vessel wall has to be machined down and this is costly. It is also limited to rather small pressure vessels and when the number of nozzles, inlet or outlet, is not large. The different cases of nozzle configurations which are investigated theoretically are described in the next section. Cases are not only distinguished for their particular geometry but also for the particular physical conditions which prevail with respect to heat transfer conditions across the nozzle-vessel boundaries.

The present analysis of thermal distributions and stresses applies not only to cylindrical vessels but to spherical vessels too under a main assumption: in Fig.3.1.3 a spherical vessel is shown; when the diameter of the vessel is large the curvature of the shell wall is small and can be neglected. In most practical cases spherical vessels do have large Di/di ratios and therefore the analysis can be used to determine stresses with the present methods. Appendix 2 of this work deals with a spherical vessel nozzle where the curvature is larger and is taken into account.

3.2 Nozzle-Vessel Cases Examined Theoretically

The cases examined theoretically are shown in Figs.3.2.1, 3.2.2, 3.2.3. Case A (Fig.3.2.1) represents a radial nozzle attached to the vessel and the geometry of the junction is as indicated, where the radius R and the rest of the dimensions can vary to any specified values. For all three cases the dimensions studied are shown in the figures. Case B (Fig.3.2.2) differs in the geometry of the junction and also in that heat is conducted only across the boundary of the nozzle while in

Case A heat is transferred across the vessel boundary as well.
Case C (Fig.3.2.3) is quite different from the other two. It
is the theoretical model of the experimental specimen. There
is no insulation on the outside and heat is conducted only
across the inside surface of the nozzle. The dimensions are
shown in the figure and the geometry of the junction is similar
to Case B. Heat conduction takes place when hot fluid passes
through the nozzle and enters the vessel. At an inlet nozzle
during the heating-up period stresses occur at the nozzle and
junction. The stresses are produced by thermal expansion in
the nozzle and by relative strain between nozzle and main vessel
shell, due to a difference in temperatures between the two com-
ponents. Inlet nozzles are subjected to higher thermal stresses
than outlet nozzles because of the higher thermal gradients set
up. In Case A and B the structure is thermally insulated on the
outside and the direct effect of this is that temperature inside
the wall increases more rapidly than in Case C where there is no
insulation.

The stresses produced are the combination of two effects:

(a) Thermal stresses developed due to variation of
temperature across the thickness of the nozzle and vessel walls.
Thermal stresses of this nature decrease as the thickness of the
wall decreases. In practical problems they can be neglected
completely if the thickness becomes very small, although their
values can still be estimated theoretically.

(b) Thermal discontinuity stresses which are produced due
to the difference in temperature between vessel and nozzle.
These stresses will be manifested immediately around the junction
because of the discontinuity of the structure in that area. The
combination of the stresses above produces an overall effect
which is studied in this work.

In certain pressure vessels inlet nozzles are thermally sealed on the inside and thermal stresses (a) above do not develop or are rendered minimal.

Wherever there is a discontinuity in a structure, such as the transition from nozzle to vessel, the variation in thickness of the corresponding walls will produce a differential strain. It is therefore desirable to make such discontinuities as smooth as possible in order to reduce the stresses. Thermal stresses develop along the outside surface of the nozzle and vessel as well as along the inside. The stresses reach maximum values at different areas, these areas being designated as high stress concentration areas. With the present theoretical analysis the stresses are calculated not only along the inside and outside surfaces of the structure, but also across the full thickness of the walls. This provides a complete picture of the stress field and decisions about reinforcement at the junction and position of the weld can be made for safer design.

3.3 Thermal Stresses and How They Develop

Often it is necessary to distinguish between thermal stress and thermal shock by saying that in thermal shock the thermal stresses are produced by transient temperature gradients, resulting from step changes in boundary temperature. For instance, if a body originally at a uniform temperature is suddenly immersed in a medium of different temperature, a condition of thermal shock is then introduced. At any instant the stresses are determined by the temperature distribution, and they are the same as they would be if this temperature distribution could be obtained in the steady-state condition. But the temperature gradient that can be established in the

'step change' initiated state are generally much higher than those that occur in the steady state. Hence, thermal shock is important relative to ordinary thermal stress because of the higher stress that can be induced. Another distinction between thermal stress due to finite rate transients and thermal shock is that the rate of application of stress is very rapid, and many materials are affected by the rate at which load is applied. Some materials are embrittled by rapid application of stress and therefore may not be able to withstand a stress in thermal shock which, if applied slowly, could readily be absorbed.

In dealing with thermal stresses it is thus desirable to distinguish between brittle and ductile materials. Brittle materials allow only a very small amount of strain prior to rupture. Ductile materials can undergo appreciable strain without rupture. Since thermal shock behaviour depends mainly on the ability of the material to absorb the induced strains necessary to maintain a continuous body upon the application of thermal strains, brittle materials generally exhibit inability to withstand these superimposed strains without rupture.

Ductile materials, on the other hand, can withstand these strains for many cycles, and in the absence of mechanical loads, ultimate fracture must involve exhaustion of the ductility through cyclic loading.

3.3.1 Thermal Shock of Ductile Material

It is the rapid step-change of heat that distinguishes thermal shock from ordinary conditions of time-dependent thermal stress. Such properties as specific heat and conductivity, which do not enter directly into consideration for thermal stresses under known conditions of temperature, become important in thermal shock applications because these properties determine the temperature, the temperature gradients, and the rate of

change of the gradients, which, in turn, govern strains and strain rates. Some materials are more sensitive than others to stress gradients or to strain rate, and therefore thermal shock may rate various materials differently than would slowly applied thermal stress.

Ductile materials cannot fail in one thermal cycle as brittle materials can. Therefore it is to be expected that considerations of failure will be limited to the mechanisms of distortion and fatigue rather than to fracture upon the application of a single cycle. The thermal fatigue aspects of such failure are examined later in this work. The thermal shock resistance of a material depends on several parameters. These include conductivity, thermal expansion coefficient and Poisson's Ratio. Thus for example the temperature distribution is governed to a considerable extent by the conductivity. Therefore good conductors will have an advantage over poor conductors. In a severe thermal shock, on the other hand, the induced strain is governed mainly by the parameter $\alpha\Delta T$ where α is the thermal expansion coefficient and ΔT the change in temperature of the structure, and is almost independent of the material conductivity. When the $\alpha\Delta T$ parameter is large and thus plastic strain is large, the ductile material will tend to have the highest cyclic life. When $\alpha\Delta T$ is small, on the other hand, the strain may be absorbed largely elastically and the material with the highest endurance limit will show a life advantage.

Thus when considering different materials it must be realised that their relative merits may interchange depending on the conditions. In addition, it is important to consider that the metallurgy of a material may change as its temperature increased. The properties of some materials deteriorate rapidly

as the temperature is raised, hence it is important to use the
properties at the temperatures at which the strains are imposed
in order to arrive at valid conclusions regarding relative
resistance to high temperature transients.

4. THEORETICAL DETERMINATION OF TRANSIENT THERMAL DISTRIBUTIONS FOR THE NOZZLE-VESSEL CASES

Here the three cases of nozzle-vessel configurations

mentioned in Section 3.2 are examined, and the thermal distri-

butions are determined at different times during the heating-up

period. Fluid enters and leaves the vessel through the nozzles;

in most cases the fluid is at a relatively high temperature and

the value of this temperature for the present investigations is

considered to be $T_f = 450^{\circ}F$. This temperature value is used

for both the theoretical and experimental parts. In order to

proceed to the evaluation of the temperature inside the material

in any practical problem involving fluid flow in nozzle and

vessel, the heat transfer coefficient must be determined. The

heat transfer coefficient affects the rate of heat flow across

the fluid/metal boundary and when the flow is low the heat

transfer coefficient is also low (see Section 4.1). In the case

of low flow there is a large convection boundary film thickness

and the rate of heat transfer from fluid to metal is decreased

proportionally. Therefore whenever the flow rate is small

thermal stresses are also small. Nevertheless in here a high

flow rate is considered for cases A and B which can be encountered

in practical problems. For Case C there is no flow of fluid.

This is because Case C is the theoretical model of the experi-

mentally examined specimen, where the fluid is simulated by a

resistance coil inserted in the nozzle. The resistance provides

heat at a rate necessary to maintain the inside surface of the

nozzle at $T_f = 450^{\circ}F$. Hence in the theoretical analysis of

Case C the temperature on the inside surface of the nozzle is

taken as $T_f = 450^{\circ}F$. As has already been said, there is no

heating along the vessel inside surface for Case C; thus the

theoretical model is equivalent to high flow in the nozzle, and

very low at the vessel wall.

4.1 Flow in Nozzle-Vessel and Forced Convection Heat Transfer Coefficients

The problem of forced convection heat transfer for a number

of physical problems can be solved analytically. In such cases

the principles of the convection processes and their relation to

fluid dynamics are employed to obtain an analytical solution.

However this is not always possible and experimental methods are

used to obtain empirical formulas or graphical charts, so that

they may be used with a maximum of generality.

The relationships used herein to obtain the values of heat

transfer coefficients for the vessel and nozzle are empirical

and the physical conditions are as follows:-

The flow within the nozzle and vessel is turbulent and this

is a realistic situation encountered in almost all cases of fluid

flow within pressure vessels. For the vessel the relationship

is approximated to that of turbulent forced convection past a

flat plate. This assumption is true when the diameter of the

vessel is large and therefore the curvature of the shell small.

The properties of the flow are evaluated at the mean film

temperature. This is the arithmetic mean of the temperature of

the surface of the wall, T_w and the fluid temperature T_f. Using

T_m to denote the mean temperature,

$$T_m = \frac{T_w + T_f}{2}$$

(4.1.1)

Sometimes in the case of internal flow a mean film temperature
which is the average of the surface temperature and the bulk
temperature is used in place of the mean temperature. The
analysis of the basic differential equations of motion, con-
tinuity, and energy of fluid mechanics (R.4-1) shows that forced
convection problems can be represented by a relation of the
form

$$Nu = f(Re, Pr) \qquad (4.1.2)$$

On the basis of this relation, experimental forced convection
data can be correlated to yield an empirical equation for a
certain set of conditions. The formulation above is based on
the assumption that the fluid properties involved in the
dimensionless parameters of Reynolds and Prandtl numbers are
constant. This assumption is not always true. For instance
the evaluation of all the properties at a mean temperature does
not produce a satisfactory correlation of the experimental data,
because, for example, of one strongly temperature-dependent
property (usually viscosity). In cases like this the introduction
of another dimensionless parameter which allows for a description
of the temperature dependence of one or more of the properties
will permit a satisfactory correlation of experimental data.
In general the experimental forced convection data correlations
can deviate up to ± 20%, and consequently even this order of
accuracy has to be considered quite satisfactory in the absence
of specific data on the configuration being considered.

Nusselt (R.4-2) has calculated the average heat transfer
coefficient h_{av} for a plate of length L as,

$$h_{av} = \frac{1}{L} \left[0.0292 (Pr)^{\frac{1}{2}} \left(\frac{u}{\nu} \right)^{0.8} k \int_{0}^{L} x^{-0.2} dx \right] \qquad (4.1.3)$$

where x is the distance of any point on the plate from the

leading edge. A Nusselt average number can be evaluated by

integration of the above equation.

$$h_{av} = \frac{1}{L} \left[\frac{5}{4} \, 0.0292 \, (Pr)^{1/3} \left(\frac{U}{\nu} \right)^{0.8} k \, L^{4/5} \right] \qquad (4.1.4)$$

hence

$$Nu = \frac{hL}{k} = 0.036 \, (Re)^{0.8} \, (Pr)^{1/3} \qquad (4.1.5)$$

where k is the thermal conductivity and ν the viscosity of the

fluid. The relation of 4.1.5 assumes that the boundary layer is

fully turbulent from the leading edge of the plate.

For fully developed turbulent flow in tubes Dittus and

Boelter (R.4-3) have proposed the following relation:

$$Nu = \frac{hd}{k} = 0.023 \, (Re)^{0.8} \, (Pr)^{0.4} \qquad (4.1.6)$$

Again the properties in the equation are evaluated at the

fluid mean temperature. The mean temperature T_m is normally

used when the wall temperature T_w is different from the fluid

free stream or bulk temperatures.

4.2 Evaluation of Heat Transfer Coefficient for Cases A,B and C

4.2.1 CASE A. Forced Convection across Nozzle and Vessel Walls

Referring to Fig.3.2.1 it can be seen that for Case A

there is heat transfer by forced convection across the nozzle

and vessel walls. Fluid of temperature $T_f = 450^\circ F$ flows through

the nozzle and along the inside wall of the main shell. The

fluid is assumed to have velocity $U_f = 2.00$ ft/sec. This is a

common velocity order of magnitude for pressure vessels where

transient thermal stresses become important. The velocity of

the fluid in the vessel and/or nozzle, and consequently the

vessel and nozzle heat transfer coefficients, can take any

value, but the computer program developed in here is able to determine the thermal transients for the various cases by simply taking the predetermined values of velocities. The nozzle and vessel inside surfaces are exposed to a convection boundary condition and the temperature of the wall does not automatically attain the temperature of the fluid. Nevertheless during warm-up the wall temperature increases fast due to the relatively high Reynold's number due to turbulent flow and the small film thickness. The effect of the convection boundary on the surface of the nozzle and vessel is determined later on by considering the heat transfer coefficients of nozzle and vessel sides which are calculated herein. The properties of the fluid are obtained at the initial mean temperature which is given by

$$T_m = \frac{T_f + T_w}{2} = \frac{450 + 60}{2} \approx 255°F$$

Fluid Properties for water at mean temperature $T_m = 255°F$.

$k = 0.306$ btu/ft.h.°F

$Pr = 1.405$

$\mu = 0.543$ lb/ft.h

$\rho = 62.428$ lb/ft³

Also the relation between dynamic and kinematic viscosities is:

$$\nu = \frac{\mu}{\rho} \qquad\qquad (4.2.1.1)$$

(i) Heat transfer coefficient h_v for the vessel

Characteristic length ℓ_v for the vessel = 7.00 in = 0.583 ft. The characteristic length is considered for half of the vessel-nozzle configuration above the centre line since the configuration is symmetrical about the xx' axis (see Fig.3.2.1).

Fluid velocity $U_f = 2.00$ ft/sec = 7200 ft/h.

$$Re = \frac{U_f \ell_v}{\nu} = \frac{U_f \ell_v \rho}{\mu} \qquad (4.2.1.2)$$

and by substituting the values for the above quantities the Reynolds number is obtained.

$Re = 0.4826 \times 10^6$

Now using Equation (4.1.5) and substituting for length ℓ_v we have,

$$Nu = \frac{h_v \ell_v}{k} = 0.036 \ (Re)^{0.8} (Pr)^{1/3}$$

hence

$$h_v = \frac{k}{\ell_v} \ 0.036 \ (Re)^{0.8} (Pr)^{1/3}$$

By substituting the values for k, ℓ_v, Re and Pr the heat transfer coefficient for the vessel is obtained h_v $\underline{964.8 \ Btu/ft^2 h.^\circ F}$.

(ii) <u>Heat transfer coefficient h_N for the nozzle</u>

The same fluid properties hold for the flow through the nozzle at fluid temperature of $450^\circ F$ and fluid velocity 2.00 ft/sec. The characteristic length is now the diameter of the nozzle

$d_i = 1.00 \ in = 0.083 \ ft.$

Reynolds number is,

$Re = \dfrac{U_f d_i \rho}{\mu}$ and by substituting the values for U_f, d_i, ρ, and μ

$Re = 0.06871 \times 10^6$

The heat transfer coefficient is then given by Equation (4.1.6) for forced convection within pipes

$$N_u = \frac{h_N d_i}{k} = 0.023 \ (Re)^{0.08} (Pr)^{0.4}$$

hence,

$$h_N = \frac{k}{d_i} \ 0.023 \ (Re)^{0.8} (Pr)^{0.4}$$

Substituting the appropriate values above for k, d_i, Re and Pr the coefficient is obtained.

$$h_N = \underline{906.1 \ Btu/ft^2 .h.^\circ F}$$

4.2.2 CASE B. Forced Convection across Nozzle Only

In this case there is no heat transfer across the vessel surface but only across the nozzle surface. The case is shown in Fig.3.2.2. The condition of heat transfer only across the nozzle is studied here because it is believed that the differential stresses developed are higher than those produced when heat transfer takes place across both the nozzle and vessel surfaces. This is because the thermal differential stresses set up between nozzle and vessel are higher due to larger difference in relative expansion between vessel and nozzle parts. In practice Case B can be considered as the one where fluid flows towards the vessel through a relatively long nozzle or pipe setting up differential stresses between nozzle and vessel while the fluid is not actually coming in contact with the inside surface of the vessel. This is particularly true when power plants involving pressure vessels are started-up, and hot 'slugs' of liquid enter vessels. It can also be said that Case B can be encountered in inlet nozzles while Case A corresponds more to outlet nozzles when there is a flow down the shell and out through the nozzle.

The flow conditions are the same as in Case A. The temperature of the fluid is $T_f = 450^\circ F$ and its velocity 2.00 ft/sec. Therefore all the other properties of the fluid are similar to the ones in Case A and the coefficient h_N is the same and equal to 906.1 Btu/ft²h.$^\circ$F. It must be noted that this is true because the nozzle inside diameter has the same value of $d_i = 1.00 in = 0.083$ ft.

4.2.3 CASE C. Radiation Heat Transfer in Nozzle

The configuration of this case, shown in Fig.3.2.3, is similar to the specimen which is tested experimentally and which

is shown in Fig.6.1.2. The fluid flow is simulated experiment-
ally by a heating cartridge element of 5/8in diameter which is
placed inside the nozzle and fixed so that it is concentric
with the nozzle as shown in the figure. More details of the
arrangement are given later on in the experimental part of this
work.

The heating cartridge heats the inside of the nozzle by
radiation and by means of a thermocouple attached on the inside
of the nozzle the surface temperature is maintained at 450°F.
The temperature of the sheath of the heater is allowed to reach
the value of 600°C (1112°F) which is less than the maximum of
700°C (1293°F) which can be obtained with clearances between
0.005in and 0.010in. Since the clearance between nozzle and
heater is 0.187in ($\frac{d_i - d_{el}}{2}$), the watt density is reduced by
means of an adjustable Variac which controls the voltage to the
heater and therefore the sheath temperature is limited to
1112°F (600°C).

It is desirable in this case to obtain theoretically the
heat transfer coefficient as in the previous two cases. In this
case heat is transferred by three modes; radiation, convection
and conduction. It is found that the conduction heat transfer
coefficient is the dominant one followed by the radiation heat
transfer coefficient. Since the clearance between the nozzle
and the heater is small (0.187in) and the air is almost stagnant
around the heater, the convection term is small compared to
the other two. Nevertheless, the convective heat transfer
coefficient is also obtained in order to determine the overall
coefficient.

The radiation problem is limited to that of determining the radiation heat transfer coefficient between two concentric cylinders of different surface temperatures.

When two concentric cylinders as shown in Fig.4.2.3.1 exchange heat by radiation the equation of heat transfer can be written in terms of certain parameters characteristic of the two cylinders. This is given by
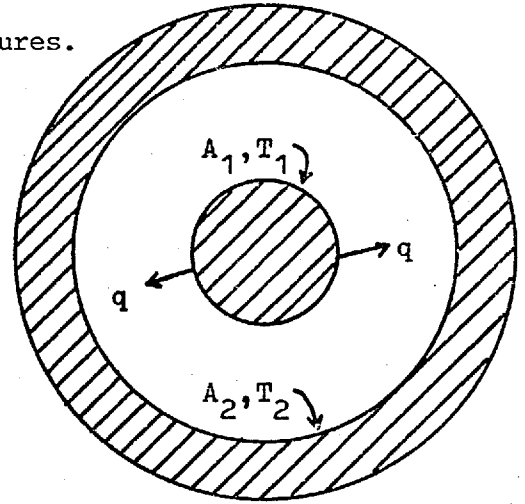
FIG.4.2.3.1

$$q_{rad} = \frac{\sigma' A_1 (T_1^4 - T_2^4)}{1/\varepsilon_1 + (A_1/A_2)(1/\varepsilon_2 - 1)} \qquad (4.2.3.1)$$

where the suffixes 1 and 2 refer to inner and outer cylinders respectively.

ε_1 and ε_2 = emissivities

A_1 and A_2 = total surface areas

T_1 and T_2 = temperature of the surfaces

σ' = Stefan-Boltzmann constant.

The derivation of the above relation is given in Ref.4-4. The ratio of areas A_1/A_2 can be replaced by the diameter ratio d_1/d_2.

It is now possible to evaluate the radiation heat transfer coefficient h_r for the nozzle of Case C as follows. From Equation (4.2.3.1)

$$\frac{q}{A_1} = \frac{\sigma' (T_1^4 - T_2^4)}{1/\varepsilon_1 + (A_1/A_2)(1/\varepsilon_2 - 1)} = h_r (T_1 - T_2)$$

and hence,

$$h_r = \frac{\sigma'(T_1^2 + T_2^2)(T_1 + T_2)}{1/\varepsilon_1 + (A_1/A_2)(1/\varepsilon_2 - 1)} \qquad (4.2.3.2)$$

and by putting $A_1/A_2 = d_1/d_2$

$$h_r = \frac{\sigma'(T_1^2 + T_2^2)(T_1 + T_2)}{1/\varepsilon_1 + (d_1/d_2)(1/\varepsilon_2 - 1)} \qquad (4.2.3.3)$$

Now the quantities in Equation (4.2.3.3) can be given the appropriate values in the case of nozzle heating.

Cartridge heater

The heater is the inner cylinder at a sheath temperature $T_1 = 1112°F$. The diameter $d_1 = 5/8in = 0.052ft$ and is manu-factured of stainless steel. The emissivity ε_1 of stainless steel at the temperature of $T_1 = 1112°F$ is obtained from Ref.(4-4) as 0.586.

Nozzle

The nozzle is the outer cylinder of Fig.4.2.3.1. The temperature T_2 is $450°F$ and this is recorded by a thermocouple attached on the inside surface of the nozzle (shown in Fig.6.1.2). The inside diameter of the nozzle is $d_2 = 1.00in = 0.083ft$. The emissivity ε_2 is that of stainless steel at temperature $T_2 = 450°F$ and is equal to 0.540 (R.4-4).

The above values are now substituted in Equation (4.2.3.3) where the Stefan-Boltzmann constant σ' is 0.171×10^{-8} Btu $ft^2 h.°R^4$ and the radiation heat transfer coefficient obtained as

$$h_r = 6.24 \text{ Btu/ft}^2 h.°F.$$

The convection heat transfer coefficient is determined by considering a relation for free convection from a heated hori-zontal cylinder given in Ref.(R.4-4). The relation is for

laminar flows and in the present case the air in the space between the cartridge heater and the nozzle can be considered still. The convective heat transfer coefficient h_c is given by

$$h_c = 0.27 \frac{\Delta T}{d}^{1/4} \qquad (4.2.3.4)$$

where $\Delta T = T_1 - T_2$; T_1 is the temperature at the surface of the cartridge heater, T_2 the temperature at the surface of the nozzle, and d the diameter of the heater in feet. For this problem:

$T_1 = 1112^\circ F$

$T_2 = 450^\circ F$

$d = 5/8 in = 0.052 ft.$

Therefore $h_c = 2.87$ Btu/ft^2 h.$^\circ$F.

Heat is transferred by conduction through the air between the inside cylinder (cartridge heater) and outside cylinder (nozzle). The conduction heat transfer coefficient is given by:

(R.4-4)

$$h_{cond} = \frac{2\pi k (T_1 - T_2)}{\ln(r_2/r_1)} \qquad (4.2.3.5)$$

where

k = thermal conductivity of air (at $\frac{450^\circ F + 1112^\circ F}{2} = 781^\circ F$)

$= 0.0340$ Btu/ft.h.$^\circ$F

$T_2 = 450^\circ F$ (see Fig.4.2.3.1)

$T_1 = 1112^\circ F$

$r_1 = 0.026 ft$

$r_2 = 0.0415 ft$

Hence the conduction heat transfer coefficient is found as

$h_{cond} = 302.44$ Btu/ft^2 h.$^\circ$F

Therefore the overall heat transfer coefficient due to the combined effects of radiation, convection and conduction is now equal to:

$$h_{TOTAL} = h_{RADIATION} + h_{CONVECTION} + h_{CONDUCTION}$$

$$= 311.55 \text{ Btu/ft}^2\text{h.}^{\circ}\text{F}$$

4.3 Two-Dimensional Transient Heat Flow in Nozzle-Vessel and Finite Difference Analysis

In all three cases heat is transferred inside the nozzle-vessel walls by conduction. The problem of heat conduction is a two-dimensional one, the heat being transferred in x and y directions across the section of the nozzle-vessel configuration in the xy plane.

The objective is to determine the temperature distribution within the walls at any particular time during warm-up. The thermal stresses can then be calculated at any time using the values of the obtained temperatures. The temperature of the fluid is considered steady with respect to time. In Cases A and B, where the outside of the nozzle and vessel is insulated, theoretically the temperature of the structure becomes everywhere the same and equal to that of the fluid. This happens after a period of prolonged exposure and under the assumption that the insulation is perfect and therefore there are no heat losses at the outside surface of the nozzle and vessel.

For all cases there are the following four basic heating-up phases.

(a) The nozzle-vessel section is initially at uniform room temperature $T_R = 60^{\circ}\text{F}$ and in thermal equilibrium with the surroundings. At this state the surface heat flux q' (heat rate per unit surface area) across the vessel and/or nozzle is zero.

(b) As fluids enters the nozzle the inside surface boundaries of the section are exposed at time t = 0 to the fluid

temperature $T_f = 450^\circ F$. There is now a surface heat flux

q' > 0 across the convection boundary film and the temper-

ature at the wall is proportional to the heat transfer

coefficient. The rest of the section is still at room

temperature.

(c) Heat is transferred by conduction and the temperature dis-

tribution within the nozzle-vessel walls is now a time-

dependent quantity.

(d) In Cases A and B where the outside of the nozzle-vessel

section is insulated and there is no heat flow across the

insulated boundary, the temperature of the structure reaches

everywhere the temperature of the fluid after a certain

prolonged exposure period. In Case C where there is no

insulation the thermal distribution within the section

becomes steady and the heat fluxes across the inside and

outside surfaces become steady too.

The problem of heat conduction through the nozzle-vessel

section is solved using the numerical method of finite differ-

ences. The method is analysed in the following section and in

detail in Appendix 1.

4.3.1 Finite Difference Analysis *

The differential equation which governs the heat flow with-

in a solid body in two dimensional unsteady-state conduction is

given by

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \rho c \frac{\partial T}{\partial \tau} \tag{4.3.1.1}$$

where T is the temperature at any point defined by (x,y) co-

ordinates at a particular time τ, ρ the density of the body,

k the thermal conductivity and c the thermal capacity.

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*See note on this paragraph and equation 4.3.1.1 in
page 83-N1 at the end of this chapter.

The solution to Equation (4.3.1.1) will give the temperature in a two-dimensional section of the solid as a function of two independent space coordinates x and y. The equation of heat flow can be solved analytically but only for certain specific boundary conditions and geometries.

Nevertheless the development of high speed digital computers enables us to obtain numerical solutions to many problems which cannot be solved by analytical methods.

In order to apply the finite difference method to solve the problem, the section is divided into equal increments in both the x and y directions. The grid thus produced has a number of nodal points as shown in Fig.4.4.1. Finite differences are then used to approximate differential increments in the temperature and space co-ordinates. The smaller the chosen finite increments the more closely the true temperature distribution is approximated.

The temperature gradients $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial y}$ at each nodal point are expressed by finite difference equations as is explained in detail in Appendix 1. Then the temperature at each nodal point after a time increment $\Delta \tau$ is calculated using the values of the temperatures of the nodes at the end of the previous time interval. The same procedure is repeated to obtain the temperature distribution after any desired number of time increments. This is done by computational methods described in the next sections.

The difference equations which are used to determine the conduction of heat through the solid cannot be used to calculate the temperature at the inside walls of nozzle and vessel. At the boundary of the section a convection resistance to heat flow exists which is a function of the heat transfer coefficients

already obtained. The equations governing the temperature change on the wall are also obtained in Appendix 1. Each convection boundary condition must be handled separately, depending on the particular geometric shape under consideration.

4.4 Solution and Results of Thermal Distributions for CASE A

The temperatures at the various time increments are determined by solving the equations obtained in Appendix 1. This is done by computer program TRAN1 which solves the transient thermal distributions for CASE A. The construction and operation of the program is described in this section. The program data designations and the listing is given in Appendix 3.

First the nozzle-vessel section of Case A is divided into the grid shown in Fig.4.4.1. The grid space increment Δx is equal to Δy and equal to 0.1in. This division produces a total of 3533 nodal points for the dimensions in Case A. Temperatures are determined for all the nodal points at selected time intervals. In the program the dimensions of the section are designated with variables I1, I2, I3, J1, J2 and J3 which can be given any values so that the dimensions of the section can be changed accordingly. This makes the program flexible for the study of different cases. There is heat flow across the inside boundaries of the nozzle and vessel but there is zero heat gradient $\frac{\partial q}{\partial t}$ on the outside insulated boundary. The nozzle and vessel heat transfer coefficients are the ones already determined in Section 4.2.

COMPUTER PROGRAM TRAN1 - CASE A

The program solves the difference equations of heat flow across the boundaries for all the boundary nodal points and for all the nodes inside the section for specified time intervals

during the transient heating-up period. It then prints the

numerical results of the temperatures for all the nodes for each

time interval as shown in Table 4.4.1. A subroutine named PLOT

is incorporated in the main program and plots the temperature

profiles for the corresponding times as shown in Figs.4.4.2(a)

and 4.4.2(b).

The main program and the subroutine were constructed by

the author in order to solve the case problems of nozzle-vessel

transients presented hereby. The plotting of the profiles

provides a more comprehensive picture than the numerical results

but the exact values of the temperatures are used later to obtain

the thermal stresses. The series of the main operations of the

program and the subroutine are given below and an attempt is

made to present the sequence of operations as comprehensively as

possible. This is as follows:

- The total number of nodal point temperatures is defined.

- The maximum number of time increment iterations is defined
  as well as the maximum number of iterations for the con-
  vergence of the temperature equations.

- The dimensions of the section are given including the
  radius RA of the junction, nozzle and vessel heat transfer
  coefficients, coefficient of thermal conductivity, initial
  temperature of the section and fluid temperature.

- The initial temperature of the section is defined by scanning
  all 3533 nodal points. This is set to $60^{o}F$.

- The temperature of the fluid is set and the temperature of
  the boundary nodal points on the vessel and nozzle walls
  are determined for the first time increment using Equations
  (A1.12) and (A1,13) respectively (Appendix 1). These are

$$T_{i,j}^{n+1} = \frac{\alpha' \Delta \tau}{(\Delta x)^2} \left\{ 2 \frac{h_v \Delta x}{k} T_f + 2T_{i,j+1}^n + T_{i+1,j}^n + T_{i-1,j}^n \right.$$

$$\left. + \left[\frac{(\Delta x)^2}{\alpha' \Delta \tau} - 2 \frac{h_v \Delta x}{k} - 4 \right] T_{i,j}^n \right\}$$

for the vessel, where h_v is the vessel heat transfer coefficient.

And

$$T_{i,j}^{n+1} = \frac{\alpha' \Delta \tau}{(\Delta x)^2} \left\{ 2 \frac{h_N \Delta x}{k} T_f + 2T_{i+1,j}^n + T_{i,j+1}^n + T_{i,j-1}^n \right.$$

$$\left. + \left[\frac{(\Delta x)^2}{\alpha' \Delta \tau} - 2 \frac{h_N \Delta x}{k} - 4 \right] T_{i,j}^n \right\}$$

for the nozzle, where h_N is the heat transfer coefficient for the nozzle.

- The nodal temperatures are calculated for the first time increment $\Delta \tau$ inside the section for all nodal points using Equation (A1.7) in Appendix 1 and temperatures in the insulated boundaries of vessel junction and nozzle using Equations (A1.8), (A1.10) and (A1.9) respectively.

- For the second time increment the boundary values of the nodal temperatures are determined using the values of the temperatures at the end of the previous time increment. The temperatures of the inside of the section are also determined using the same equations described above and the temperatures at the end of the first time increment.

The procedure is the same for subsequent time increments until the temperature of the section is uniform.

- The values of the temperatures are printed every 0.5 minutes (30 seconds) and calculated for every 0.3 minutes for the first 12 minutes and printed for every 3 minutes thereafter.

The printing time can be changed to any value desired.

In Table 4.4.1 the printing of the temperature for selected

value of the index I at the time interval of 9 min 30 secs

is shown. Each nodal point is identified by two indices I

and J. The value of I changes at the top of the page and

the value of J across. Every temperature is then identified

as T_{ij}.

The temperatures for every time increment printed are

plotted by subroutine PLOT in a comprehensive way as it is ex-

plained below.

SUBROUTINE PLOT-CASE A

The thermal profiles for the section at the time intervals

of 9 min 30 secs and 54 min are shown in Figs.4.4.2(a) and

4.4.2(b) as they are plotted by the subroutine. Every letter

represents a band of 10 degrees F. Each temperature is trans-

lated into a blank or a letter according to the table provided

with the figure. The subroutine incorporates the following

steps.

-   Define the temperature bands by designating a letter to

    each band of 10 degrees ^{o}F.

-   Put index I = I1 and plot the temperatures for every value

    of the index J along the row.

-   The junction boundary is determined as follows.

The junction is a quarter circle. In the Fig.4.4.3 the

co-ordinates of the quarter circle at points A and B are I2,

J2-R and I2-R,J2.

The equation will then be:

$$I = (- \sqrt{R^2 - (J - J2)^2}) + I2 \qquad\qquad (4.4.1)$$

and

$$J = (- \sqrt{R^2 - (I - I2)^2}) + J2 \qquad\qquad (4.4.2)$$

where R = RA/Δx and RA the radius of the junction. These two

equations give the corresponding values of i and j for any values

of the radius of the junction. Since the grid elements are

finite, the nodal points on the quarter circle are rounded by

computation according to the decimal part of their value to

give a smooth curve through the points.

−   When the plotting of the temperatures for the entire

      nozzle-vessel section is completed for the first time

      interval, the operation is repeated for the subsequent

      time intervals.

Data designations and listing of the subroutine are given

in Appendix 3.

Thermal Results − CASE A

The temperatures for each time interval of 30 seconds for

the first 12.0 minutes, and each time interval of 1 minute

thereafter of the heating-up period were obtained by the main

program and plotted by the subroutine in order to study the

behaviour of thermal distributions in the section. The exact

values of the temperatures are subsequently used to determine

the thermal stresses. This is done for only one time interval

when the developed stresses are maximum, and it is explained

below. In Fig.4.4.4(a) to Fig.4.4.4(c) the thermal distributions

are shown for the time intervals τ = 3 min, 9 min 30 secs,

18 min, 27 min, 36 min, 45, 63, 81, 126, 144, 162, 200 minutes.

It can be seen that the temperature rises faster in the nozzle

because the thickness of the wall is smaller than that of the

vessel. At early time intervals the temperature variation

across the walls is larger. Later the heat flow becomes

steadier and it can be noticed that the profiles become more

distinct. In Fig.4.4.4(c) it is quite clear that temperature

variation across the section is decreasing appreciably. The
bounds of $10°F$ isothermals are getting wider and heat flow is
now more steady. As there is no heat flow across the insulated
boundary the temperature of the nozzle first becomes uniform at
time τ = 2 hr 06 min (Fig.4.4.4(c)). The temperature of the
entire vessel-nozzle configuration becomes uniform after 3 hours
and 20 minutes (Fig.4.4.4(c)). Thermal differential stresses
are present during the whole period of the transient heating-up
as the mean temperature of the nozzle is always higher than the
mean temperature of the vessel wall. The thermal differential
stresses are due to different expansion rates of nozzle and
vessel and are the most important ones but not the only ones as
has already been pointed out. Thermal stresses also develop
across the thickness of the vessel and nozzle walls due to the
variation in temperature. These are complementary to the
differential stresses and they contribute to the increase of the
overall value of the thermal stress field.

At time τ = 9 min 30 secs the variation of mean temperature
between nozzle and vessel $\Delta T_m = T^*_{m(nozzle)} - T^*_{m(vessel)}$ is
maximum. In Fig.4.4.5 the value of ΔT_m is plotted against time.
The value of ΔT_m increases continuously until the time of
9 min 30 secs, when it attains its maximum value of $193.1°F$ and
then decreases steadily to zero at the time of 3 hours 20 min when
the temperature of the section becomes uniform. It is evident
that maximum thermal stresses will develop after 9 min 30 secs of
the heating-up period. It is these maximum thermal stresses that
are examined later on in Section 5.

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* $T_{m(nozzle)}$ is the mean temperature in the nozzle
calculated as the average of all the nodal temperatures in
the nozzle. The same holds for the vessel $T_{m(vessel)}$.

4.5 Solution and Results of Thermal Distributions for CASE B

In Case B heat flows only across the boundary of the nozzle. The variation of temperature between nozzle and vessel will therefore be higher and differential expansion between nozzle and vessel will produce higher differential stresses.

The temperatures are determined using the same method of finite differences and program TRAN2. The nozzle-vessel section is divided into the grid shown in Fig.4.5.1. The physical conditions are also shown. There is a rate of heat flow $\partial q/\partial t$ across the inside boundary of the nozzle and there is no heat flow at the outside insulated boundary. The temperature of the fluid is $T_f = 450^{\circ}F$ and the heat transfer coefficient of the nozzle is $h_v = 906.1$ Btu/ft^{2}hr.$^{\circ}$F. This was calculated in Section 4.2.1. The temperature grid comprises of a total of 3712 nodal points with a grid spacing $\Delta x = \Delta y = 0.1$in. The data designations for TRAN2 are the same as for TRAN1 and are given in Appendix 3 together with the listing of TRAN2. The steps in program TRAN2 are the same as in Case A with the following differences.

PROGRAM TRAN2 - CASE B

- The temperature of the fluid is set and the temperature of the boundary nodal points on the nozzle walls are determined for the first time interval using Equation A1.13 (Appendix 1). This is

$$T_{ij}^{n+1} = \frac{\alpha' \Delta\tau}{(\Delta x)^2} \left\{ 2 \frac{h_N \Delta x}{k} T_f + 2 T_{i+1,j}^n + T_{i,j+1}^n + T_{i,j-1}^n \right. $$
$$\left. + \left[\frac{(\Delta x)^2}{\alpha' \Delta\tau} - 2 \frac{h_N \Delta x}{k} - 4 \right] T_{i,j}^n \right\}$$

- There is no heating across the vessel inside boundary.

- For the second time interval the boundary values along the nozzle are determined using the temperatures at the end of

the first interval. The interior nodal temperatures are

—then determined using the Equations (A1.1), (A1.8), (A1.9),

and (A1.10) of Appendix 1. Equation (A1.10) for the junction

is valid for the geometry of this case.

- Temperatures are determined for subsequent time intervals

using the values at the end of the previous times.

The temperatures profiles are plotted by subroutine PLOT.
The junction geometry is a straight line inclined at 45° in

respect of the vessel and nozzle. (See Fig.3.2.2.)

Thermal Results - CASE B

The temperatures of the section are calculated by the program

and plotted by the subroutine for time increments of 30 seconds

for the first 15 minutes and 1 minute thereafter. The time incre-

ments at which the temperatures are printed can be changed to any

desirable value of 18 seconds or more. The minimum value of 0·18

seconds is limited by the value of the parameter $\frac{(\Delta x)^2}{\alpha \Delta T}$ which

must be equal to 4 so that it can be eliminated from Equation A1.7

(Appendix 1). Table 4.5.1 gives the values of the nodal tempera-

tures of the entire section for time τ = 10 min. In Figs.4.5.2(a)

and 4.5.2(b) the thermal profiles are shown as they are plotted

by the subroutine for times τ = 10 min and τ = 54 min. In Figs.

4.5.3(a) and 4.5.3(b) the thermal distributions are shown for

time τ = 0.0 to τ = 54 minutes.

The isothermal lines obtained in this case are different to

the ones obtained in the previous one. It can be seen that heat

flows across the nozzle inside surface, changing the nozzle wall

temperature first and then as time proceeds the temperature of

the vessel wall. The nozzle is heated to a considerable higher

temperature than the vessel. This will introduce differential

stresses between nozzle and vessel. Generally it can be seen

that in Case B the section is heated-up slower than in Case A, and the variation temperature between nozzle and vessel is higher. This makes the differential stresses in Case B generally higher than in Case A but the overall stresses are higher in Case A due to higher temperatures. The variation in mean temperature between nozzle and vessel ΔT_m is shown in Fig.4.5.4. The maximum value is 302.4°F and it is attained after 10 minutes.

As there is no heating across the vessel boundary in Case B and the vessel wall is insulated, comparison of the two curves for ΔT_m for Case A and B indicates that maximum temperature variation for Case B contributes to higher differential stresses than in Case A, where the maximum value of ΔT_m is considerably lower (see Fig.4.5.4).

4.6   Solution and Results   of Thermal Distributions for CASE C

Case C examines theoretically the thermal distributions and thermal stresses of the nozzle-vessel geometry of the specimen tested experimentally. This makes possible the comparison of theoretical and experimental results and provides a criterion of how accurately the theory can approximate experimentation. The correlations of stress results between the theoretical Case C and the experiments are described in Chapter 6. In here only the thermal distributions are obtained, as for the two previous cases, and these distributions are used in Section 5 to obtain the thermal stresses.

Case C has the dimensions shown in Fig.3.2.3. The nozzle-vessel section is a section in the xy plane (longitudinal plane) of the specimen which is shown in Figs.6.1.1 and 6.1.2. The section is heated by a cartridge heater which is inserted inside the nozzle. This simulates the fluid of the previous cases.

Heat fow across the nozzle boundary is now a combined result of radiation, natural convection and conduction. This mode of heat transfer is different from the forced convection in the two previous CASES A and B. The combined radiation, convection and conduction heat transfer coefficient was obtained in Section 4.2.3 and its value is h_{TOTAL} = 311.55 Btu/ft^2.hr.$^\circ$F. The heat transfer coefficient h_{TOT} can be used to obtain the temperature of the wall when it is not known. In this case the cartridge heater provides the necessary amount of heat to maintain the inside surface of the nozzle at a temperature of 450°F. There is no insulation on the outside and therefore there is a heat loss dq/dt by natural convection from the outside walls to the surrounding air. The heat flow inside the wall by conduction and the heat loss to the air by natural convection are described in Appendix 1. Program TRAN 3 (Appendix 3) determines the temperature distributions for CASE C.

PROGRAM TRAN3

- Temperatures of nodal points inside the wall are calculated by Equation (A1.7):

$$T_{i,j}^{n+1} = \frac{1}{4} (T_{i,j}^n + T_{i-1,j}^n + T_{i,j+1}^n + T_{i,j-1}^n) \quad \text{(A1.7)}$$

- The heat transfer coefficient for the vessel outside wall and the junction outside wall are determined by Equation (A1.14):

$$\frac{hL}{k_a} = 0.13 \left[\left(\frac{L^3 g\beta (T_w - T_\infty)}{\nu^2} \right) (Pr) \right]^{0.33}$$

- The heat transfer coefficient for the nozzle outside wall

is determined by Equation (A1.15):

$$\frac{hL}{k_a} = 0.54 \left[\frac{L^3 g \beta (T_w - T_\infty)}{\nu^2} (Pr) \right]^{0.25}$$

- The temperature grid comprises a total of 1387 nodal points

and is shown in Fig.4.6.1.

Temperatures are obtained at 1 minute intervals for up to

18 minutes. In Figs.4.6.2(a) to 4.6.2(c) the thermal profiles

are shown for times = 0,4,6,10,12,14 minutes during heating-up.

In Table 4.6.1 the values of temperatures are shown for time

τ = 11 min. The mean temperature variation ΔT_m between nozzle

and vessel attains its maximum value of $236.9^\circ F$ at time

τ = 11 min. In Fig.4.6.3 ΔT_m is plotted against time. After

the eleventh minute the value of ΔT_m decreases with time. Never-

theless the value of ΔT_m will never be zero since there is no

insulation on the outside and heat flows across the boundaries.

Note on paragraph 4.3.1 (page 71)

The differential equation of heat conduction which governs the heat flow within a solid body in three-dimensional time-dependent conduction in cylindrical coordinates, specified as r (radial), z (axial) θ (circumferential), is given by

$$k\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right] = \rho c\,\frac{\partial T}{\partial \tau} \tag{N.1}$$

If a cylinder (Fig.N.1) whose axis coincides with the axis of z is heated and the initial and boundary conditions are independent of the coordinates
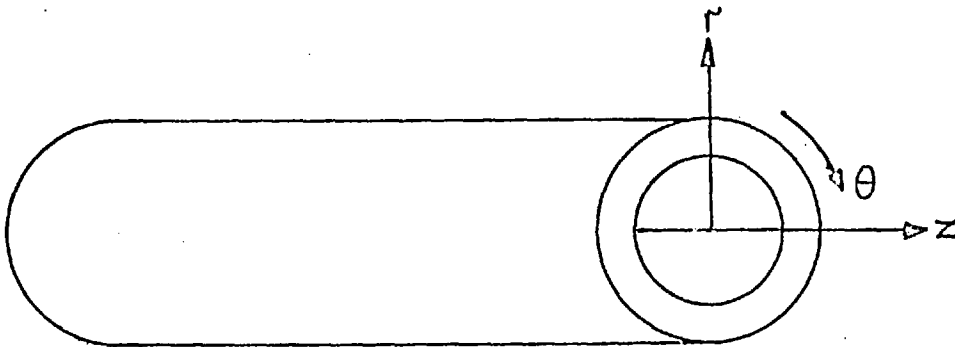
Figure N.1

θ and z the temperature will be a function of r and time only and the equation reduces to:

$$k\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] = \rho c\,\frac{\partial T}{\partial \tau} \tag{N.2}$$

In this case the flow of heat takes place in planes perpendicular to the axis and the lines of flow are radial.

When the initial and boundary conditions do not contain θ, the flow of heat takes place in planes through the axis and the equation of conduction becomes

$$k \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right] = \rho c \, \frac{\partial T}{\partial \tau} \qquad\qquad (N.3)$$

Nevertheless, when two axially symmetric bodies intersect as in the problem of this work the above equation N.3 cannot be used to analyse the transient temperature distribution in both cylinders and especially around the junction area due to lack of symmetry of the cylinders. Therefore an approximation is made in order to overcome this difficulty which results in using Equation 4.3.1.1.

$$k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = \rho c \, \frac{\partial T}{\partial \tau}$$

whose solution still involves three independent variables, but it treats the problem in two-dimensional plane wall cartesian coordinates. This is an approximation which works accurately (the error is between 1% and 2% for a vessel of thick wall and large diameter $D_i/t \simeq 20$) and quite accurately for the nozzle. The plane wall approximation for the nozzle of the present problem produces results where the error averages at about 11.5%.

In Appendix 9 (page 244) the temperature distributions which were obtained in the nozzle by using the two-dimensional time dependent plane wall equation

$$k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = \rho c \, \frac{\partial T}{\partial \tau} \qquad\qquad (4.3.1.1, \ p.71)$$

away from the junction for Case C are compared to the temperature distribution obtained in the same nozzle configuration using Equation N.2.

It has not been possible to compare the results with analytical solution data for the cylinder time-dependent equation with the above boundary conditions or with insulated outside boundary due to lack of any such data. However comparisons are made to the results of

temperature distribution for the cylinder obtained by solving the

Equation N.2. numerically as is shown in Appendix 9 (p.244). The

plane wall approximation for the nozzle of the present problem pro-

duces results in which the differences average at about 11.5%.

5. THEORETICAL DETERMINATION OF THERMAL STRESSES

Elastic thermal stresses are calculated for all three cases by computational analysis using the temperatures obtained in the previous section. Maximum thermal stresses develop at times when ΔT_m is maximum. It has been shown that for Case A, ΔT_m becomes maximum at time $\tau = 9$ min 30 secs, for Case B at $\tau = 10$ min and for Case C at $\tau = 11$ min. Only maximum thermal stresses are determined for Cases A and B. For Case C stresses are determined for time intervals of very two minutes up to 18 minutes. This makes possible the comparison between theoretical values of stresses and experimental results.

The analysis and solution of the stress problem is based on the method of finite elements. An account of the method is given in Appendix 4. The finite element method does provide a tool for solution of problem with complex geometries but it also has its limitations. The analysis of the present problem for example assumes a linear displacement field across each of the triangular elements into which the nozzle-vessel section is divided. This assumption tends to be true for small area triangles. The concept of imaginary forces at nodal points representing the distributed stresses actually acting on the element boundaries is also another assumption. This is acceptable to a certain extent when the stress field is more or less uniform. But in places where stresses change abruptly the

elements must be small to allow for the possible variation of stress field within the elements. The combination of tri- angular elements is called a MESH. Fine meshes are required at places where peak stresses are expected, or where the stress concentrations are high.

5.1 Computer Program STREN

The analysis given in Appendix 4 presents the mathematical procedure used for the determination of stresses. The equations are solved by computer program STREN which analyses the displace- ments in the structure due to thermal expansion. The programme calculates the direct stresses σ_x, σ_y, σ_z, where σ_x and σ_y are in the xy plane of the section (Fig.5.1.4) and σ_z is in a direction perpendicular to the plane. (See Fig.5.1.4 for stress notation)

The two principal stresses σ_1 and σ_2 in the x-y plane are also determined as well as the shear stress τ_{xy}. The angle of the principal stress σ_1 is determined in respect to the x axis. Stresses are calculated for every element and node of the mesh and they are printed in such a way that they can be read for each node and element.

The general analysis and the computational procedure con- sists of the following steps.

(i) Initially the physical data of the material are defined. The material used is Austenitic stainless steel of the EN58J type. The values of Young's modulus E, the coefficient of thermal expansion α and Poisson's ratio ν are all temperature dependent. A relation is therefore established for each of the above quantities with respect to the temperature. Values are obtained from reference (R.5-1) and are plotted against temper- ature. In Fig.5.1.1 the modulus of elasticity is shown for the

temperature range $0^{\circ}C$ ($32^{\circ}F$) to $800^{\circ}C$ ($1472^{\circ}F$). In Fig.5.1.2

the coefficient of thermal expansion α is plotted against the

temperature. The coefficient behaves linearly up to about $500^{\circ}C$

($932^{\circ}F$). Finally in Fig.5.1.3 Poisson's ratio ν is shown for a

temperature range of $400^{\circ}C$ ($720^{\circ}F$). The temperature of the

nozzle-vessel section at the various time intervals is not the

same everywhere. Most of the elements and nodes are at a

temperature generally different to that of other elements or

nodes. It would be extremely difficult, even with computational

techniques, to use different values of the above quantities for

each element and node according to their temperature. This would

of course be theoretically an ideal situation but practically it

involves an unnecessary amount of calculations which is not

justified by the degree of accuracy obtained. Therefore the

mean temperature of the section at each time interval concerned

is that at which the quantities above are obtained.

(ii) The number of elements and nodes of the mesh is defined

together with the x and y co-ordinates for every node. The

initial displacements of each node in x and y directions are set

to zero and the boundary conditions are defined as unrestrained.

The temperatures of all elements are given for the particular

time interval. The temperatures of the elements is the average

value, of the~~three~~ temperatures of the three surrounding nodes, and

are fed as data into the program.

(iii) The program calculates the areas of the elements and takes

into account any forces present at the nodes. These forces are

zero in the present problem since no external reactions are con-

sidered. The stiffness coefficients for the nodes are summed

and the stiffness matrices for each element are subsequently

formed. The displacement of each node is calculated due to

internal thermal forces set up by thermal expansion and by inversion of the main diagonal stiffness submatrix the flexibility submatrix at each node is formed.

A Gauss-Seidel iteration on the stiffness matrix for the entire section is performed and a test for convergence is used to check if a predetermined tolerance has been achieved. The stress components for each element are then calculated from the equations shown in Appendix 4. The stresses are also calculated for all nodes. Finally the direct stresses σ_x, σ_y and σ_z are printed for each node and element together with the principal stresses σ_1 and σ_2 in the xy (longitudinal) plane, the shear stress τ_{xy}, and the angle θ (in degrees) that σ_1 forms with respect to the x-axis. The displacements u and v of each node in x and y directions are also determined.

5.2 Thermal Stress Results for Case A and B

The finite element mesh for Case A is shown in Fig.5.2.1. The mesh has 377 elements and 221 nodes and it is finer at the junction where stress concentration is high. The stress results for the mesh are obtained by program STREN1 which is given in Appendix 5. Stresses are obtained for the time interval τ = 9 min 30 secs during heating-up when their values are maximum. For each element and node three direct stresses σ_x, σ_y and σ_z are determined. The two principal stresses σ_1 and σ_2 are also determined and the maximum principal stress is selected and plotted for the outside boundary of the structure. In Fig.5.2.2 the three direct stresses are plotted for time τ = 9 min 30 sec. Stress σ_L is compressive for the vessel and its value increases as the junction is approached. At node 7 it attains its maximum value of -21371 lb/in^2 and then gradually decreases to -2638 lb/in^2 at node 18. Then the σ_L becomes tensile for the rest of the

junction reaching a peak value at node 23 of +11.089 lb/in^2.
It then decreases along the outside of the nozzle but its value
remains tensile for the nozzle. The transition from compression
to tension for the σ_L stress occurs between nodes 17 and 18 in
the junction region. The two peak values also occur at the
junction, with the compressive maximum being on the vessel side
of the junction (node 7) and the tensile highest value on the
nozzle side of the junction (node 23). The exact values of the
stresses for the nodes along the outside of the structure are
given in Table 5.2.1

The physical explanation of thermal stress results and
their interpretation for complicated sections like the one under
investigation, are not always as straightforward as for instance
the results obtained under internal pressure loads. This is
because thermal loads affect the structure in more ways than
one. For example expansion of one part of the structure sets
up tensile stresses in the fibres of the metal. At the same
time this same part is under compression by the rest of the
structure surrounding it as its expansion is restricted by the
cooler metal around it. Again, during cooling compressive
forces are set up in the metal due to the drop in temperature
but since contraction is restricted by the rest of the structure
which is at a higher temperature tension is also applied on it.
Therefore the net effect eventually will be determined by the
magnitude of the different stresses, compressive and/or tensile,
that are present as the result of the specified thermal loads.

It can be seen from Fig.5.2.2 that the other two direct
stresses σ_z follow almost the same pattern as the stress
σ_L.   It has a compressive maximum at the junction node
(7) and (8) respectively and a tensile maximum at junction

node (23). Their values are compressive for the vessel, increasing gradually as the junction is approached, then they decrease along the junction and they become tensile between nodes (17) and (18). Along the nozzle part of the section they are tensile and they decrease as we move away from the junction. It can also be seen that the value of the stress σ_L is higher throughout the structure than that of stress σ_z. This clearly indicates that higher direct stresses are set up in the longitudinal plane of the section than in the transverse plane.

The compressive values of σ_L, σ_R and σ_z for the vessel from node (1) to node (17) are due to the expansion of the inner part of the vessel section which is at a higher temperature than the outer part, therefore setting up compressive forces on the outside boundary. These compressive stresses become higher as the junction is approached due to the discontinuity at node (7). For the vessel and part of the nozzle, up to node (17), the compressive stresses are higher than the tensile therefore producing an overall compressive result. Compressive forces for this part of the vessel and junction are also increased due to the effect of the expansion of the nozzle which as it expands compresses the vessel and the area around the junction.

The maximum principal stress for every node on the outside is plotted in Fig.5.2.3. The shear stress τ_{xy} is determined by the program but from the design point of view the maximum shear stress τ_{max} (12) $= | \frac{1}{2}(\sigma_1 - \sigma_2) |$, where σ_1 and σ_2 are the principal stresses in the xy plane, is more important. The maximum value of τ_{max}(12) depends of course on the magnitude of the principal stresses and also their signs. When σ_1 and σ_2

are of the same sign, τ_{max}(12) is equal to their difference and

when σ_1 and σ_2 are of opposite signs τ_{max}(12) is equal to their

sum. Additionally the other two maximum principal shear

stresses τ_{max}(23) and τ_{max}(31) are equal to $\frac{1}{2} |\sigma_2 - \sigma_3|$ and

$\frac{1}{2} |\sigma_3 - \sigma_1|$ respectively. It is found (Fig.5.2.3) that τ_{max}(12)

(acting in the xy plane) is considerably larger than the other

two maximum shear stresses. From Fig.5.2.3 it can also be seen

that the maximum principal stress graph is similar to the graphs

of the three direct stresses but with considerably higher stress

values. There is a maximum compressive stress at node (8) of

the junction of -23,665 lb/in^2 and a tensile maximum at node (23)

of the junction of +11.091 lb/in^2. The maximum shear stress

τ_{max}(12) graph also has a maximum value at node (8) of 10579 lb/in^2

and at node (23) of 5104 lb/in^2. It is clear that high stress

concentrations at the junction points (7), (8) and (23) are

present and it is at these regions where failure is possible.

The smooth geometry of the junction decreases the magnitude of

the peak stresses around that area, but maximum peak stresses

still develop at the points where the vessel and nozzle join

up with the junction (points (7)-(8), and (23)). This leads

to the conclusion that the welding of nozzles on vessels has

to be undertaken with great care to avoid sharp transitions

from one point to another. Even if the overall transition

appear to be smooth it is possible to have small discontinuities

and corners within the weld which will unavoidably produce peak

stresses at these areas. Nevertheless by providing a smooth

overall junction high stress concentrations can be avoided and

discontinuities within the junction region can be prevented

from acting as nuclei for very high peak stresses which could

lead to crack initiation. A method of machining down the

vessel wall and making the area of the junction part of the
vessel is thought to be more effective in combating failure at
the junction. The nozzle is then welded on further away from
the junction. This method of construction places the weld away
from the region where stresses are high and at the same time it
renders the junction region stronger since the junction is now
part of the vessel. This method is used in the experimental
part of this project.

Stresses are equally important on the inside surface of
the nozzle and vessel which is in contact with the fluid
(Fig.5.2.1 nodes (206) to (75)). It is found that
direct stresses σ_x, and σ_z are compressive with a maximum
compressive value at nodes (156) for σ_x and (157) for
σ_z (Fig.5.2.4). Again stress σ_x is greater than σ_z
especially between nodes (192) to (126) but both stresses
have approximately the same values for nodes further along the
nozzle. The thermal stresses for the inside surface are also
given in Table 5.2.2.

The maximum principal stress graph for the inside surface
is shown in Fig.5.2.5 and it can be seen that a maximum com-
pressive value is obtained at node (157). The maximum shear
stress also attains its maximum value of 5173 lb/in^2 at node
(156). The stresses at this part of the vessel are partly the
result of a bending moment produced by the relative expansion
of the nozzle. As the expansion of the nozzle is restricted a
bending moment is set up with a direct effect on the stresses
developed on the vessel side.

The finite element mesh for Case B is shown in Fig.5.2.6.
The three direct stresses are again plotted for the outside
surface and are shown in Fig.5.2.7. Table 5.2.3 gives the

values of the computed stresses. Both stress components are compressive for the vessel and they increase to a maximum at node (5) and then decrease gradually to become tensile at node (9). There is a tensile maximum for the stresses at node (11). The maximum principal stress and the maximum shear stress are shown in Fig.5.2.8. The highest compressive value for the principal stress is again at node (5) and the highest tensile stress at node (11). For the shear stress a maximum of 15,245 lb/in² occurs at node (5) and at node (11) a value of 1354 lb/in² is obtained which is relatively higher than the rest of the values along the nozzle. These high values of the stresses at the points where the junction forms sharp corners with the vessel and nozzle shows how discontinuities of this kind affect stress concentrations.

Stresses developed in Case A are generally higher than those in Case B. Comparison of the temperatures in the structure at time $\tau = 9$ min 30 sec for Case A and at time $\tau = 10$ min for Case B indicates that the temperatures are higher for Case A. This is because heat flows across both the vessel and nozzle inside surfaces thus producing a greater overall thermal load in Case A. Nevertheless it has been explained that the difference between the mean temperature of the nozzle and vessel is greater in Case B and therefore thermal differential stresses will generally be higher than in Case A. The net effect of the thermal expansion stresses and the thermal differential stresses eventually determines the magnitude of the resulting stresses.

The situation on the inside surface in Case B (Fig.5.2.9 and Table 5.2.4) is similar to that in Case A. The maximum principal stress is compressive at the vessel, junction and

nozzle and its value increases from -1362 lb/in^2 at node (149)
to -10,442 lb/in^2 at node (102). It then decreases along the
nozzle. The maximum shear stress develops a similar pattern
with the highest value of 4924 lb/in^2 at node (102).

Wherever the maximum shear stress develops peak values, a
crack can be initiated or an existing crack can propagate under
certain conditions of stress level. This could lead to failure
of the structure when cyclic loading is applied, as for instance
under continuous heating and cooling. The thermal fatigue
problem is considered later on when crack propagation is also
investigated.

5.3 Thermal Stress Results. Case C

The finite element mesh is shown in Fig.5.3.1. The mesh
is geometrically identical to the experimental specimen. The
theoretical results obtained are then compared to the experi-
mental results. This is done in the next chapter. The stresses
for Case C are presented for the time intervals of τ = 4, 8 and
11 minutes of the heating up period. At time 11 minutes the
maximum thermal stresses are developed. Temperatures for each
of the 549 elements and for each of the 340 nodes are taken from
the temperature results obtained by program TRAN3 for each of
the three time intervals. The temperatures are fed as data in
the program STREN3 which calculates the stresses at the re-
spective time intervals. Program STREN3 is similar to the
previous two programs which are used for Cases A and B. The
listing is given in Appendix 5.

The overall stress results obtained in this case indicate
that the magnitude of the stresses is lower than that of Case A
or B. This is partly due to a lower ΔT_m between nozzle and

vessel and partly due to the smaller dimensions of the section
which makes the thermal transients across the thickness of the
vessel and nozzle considerably lower in magnitude. The stress
σ_L is shown in Fig.5.3.2 and Table 5.3.1 gives the exact values
for the three time intervals. Stress σ_L is plotted for the out-
side boundary of the section from node (1) to node (77). The
stress is comparable for the vessel with a maximum at node (21).
It becomes positive at the middle of the junction and remains
positive along the nozzle with a maximum value at node (43) of
the nozzle. Graphs are plotted for the three time intervals
of the heating-up period. In Fig.5.3.3 and Fig.5.3.4 the other
two direct stresses σ_R^* and σ_z are plotted respectively and their
values are also given in Table 5.3.1. Generally the pattern of
the graphs is similar. Both σ_R and σ_z have a maximum compressive
value at node (21) for all the three time intervals, and a
tensile maximum value at node (43) of the nozzle. Stresses
reduce considerably near the ends of the nozzle and vessel. Again
Figs.5.3.5 and 5.3.6 show the maximum principal stress σ_{maxp} and
the maximum shear stress $\tau_{max}(12)$. Their values are given in
Table 5.3.2. A maximum compressive principal stress of
-8247 lb/in^2 is recorded at time 11 minutes at node (21) com-
pared to -14897 lb/in^2 for Case B and -23665 lb/in^2 in Case A.
The shear stress $\tau_{max}(12)$ has a maximum of 7872 lb/in^2 at
node (21) and time 11 min compared to 15245 lb/in^2 for Case B
and 10579 lb/in^2 for Case A.

Stresses are calculated for every element and node within
the section and the results are used to determine the stress
field which is used in the fatigue section of this project.

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* Values of σ_R are at the centroids of the elements near the
boundary. The values of σ_R is zero at the boundary.

6. EXPERIMENTAL DETERMINATION OF THERMAL STRESSES

In this experimental part of the project an attempt is made to determine the thermal stresses developed at the vessel, nozzle and junction areas using a specimen which was constructed to specifications described (Section 6.1). By determining the stresses experimentally, it is possible to determine how accurately we can predict the stresses by finite element analysis, and also by approximate theoretical analyses.

Strain gauges are used to measure the stresses. It has been found that thermal stress measurements pose considerable difficulties and special strain gauges are used. The situation is made more difficult by the strain gauges having to operate at relatively high temperatures.

The second objective of the experimental part is to discover whether, under continuous heating and cooling cycles, it is possible to initiate any cracks at regions of high stress concentration or make existing cracks propagate to the extent of causing failure of the structure. This second aim is considered in Chapter 7 together with a theoretical analysis on crack propagation and fatigue.

6.1 Geometry and Characteristics of the Specimen

The specimen used for the experimental part of the project simulates the part of the vessel where the nozzle is attached and the curvature in the tangential (hoop) direction is

neglected. This assumption is quite realistic, as has already been explained, since when the diameter of the vessel is large curvature is small compared to nozzle wall curvature and it can be neglected over a small arc of the vessel wall. A stereogram of the specimen is shown in Fig.6.1.1 and a section of it through the longitudinal plane is shown in Fig.6.1.2 with the exact dimensions. The specimen is constructed of austenitic stainless steel of the EN58J type and its physical characteristics are obtained from the graphs plotted for Young's modulus E, thermal coefficient α, thermal conductivity k, and Poisson's ratio ν.

The specimen was machined down from an original block of 4.5" x 8" x 8" to produce the result shown in Figs.6.1.1 and 6.1.2 where the junction is part of the vessel. The nozzle is then electrically welded on the vessel and the weld is well away from the junction where stress concentrations are high. This procedure is more expensive than welding the nozzle straight on to the vessel wall but it is generally considered stronger. The nozzle is made of the same steel EN58J and therefore has the same physical characteristics. In theory the weld is a region of discontinuity but if its dimensions are not large and if it is made of the same material as the specimen its effect can be neglected. This is not strictly correct since welding processes may change the properties of the material and thus produce discontinuities between the weld and the structure material which eventually influences the stress pattern. Nevertheless this aspect is not the object of the investigation. The specimen is shown in the picture of Fig.6.1.3.

6.2 The Design and Construction of the Rig

The general mode of operation of the rig consists of a number of alternative heating and cooling cycles. There are two objectives:

(a) To record the stresses on the outside of the specimen during heating and cooling.

(b) To subject the specimen to a continuous thermal cycling for fatigue investigations.

When measurements of thermal stresses are recorded, the specimen is heated for 20 minutes by means of a cartridge heater inserted into the nozzle as described below, and subsequently cooled by air for another 24 minutes (see figure below).

CYCLE FOR MEASURING STRESSES
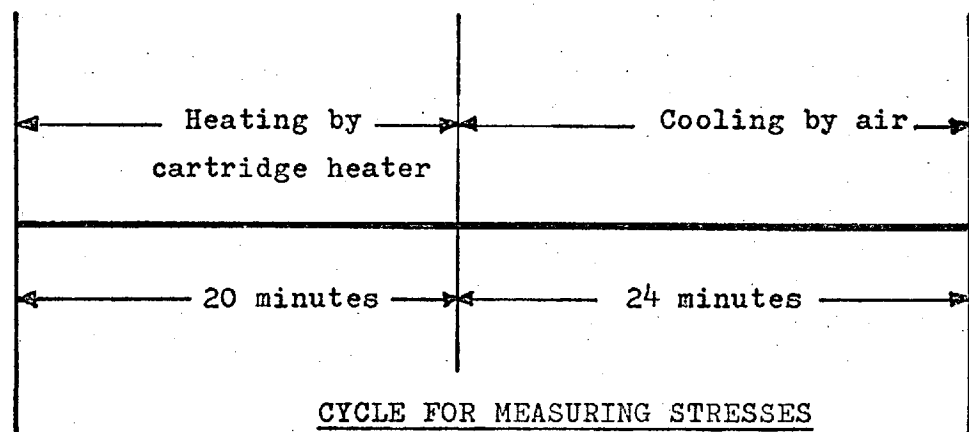
For the fatigue experiment the cycle consists of 12 minutes heating (enough for the maximum stresses to develop) by means of the cartridge and 6 minutes cooling by water (see figure in next page).

```
|<————————— Heating by ————————>|<———— Cooling by ————>|
           cartridge heater      |        water
|————————————————————————————————|——————————————————————|
|<——————————— 12 minutes ————————>|<———— 6 minutes ———————>|
```
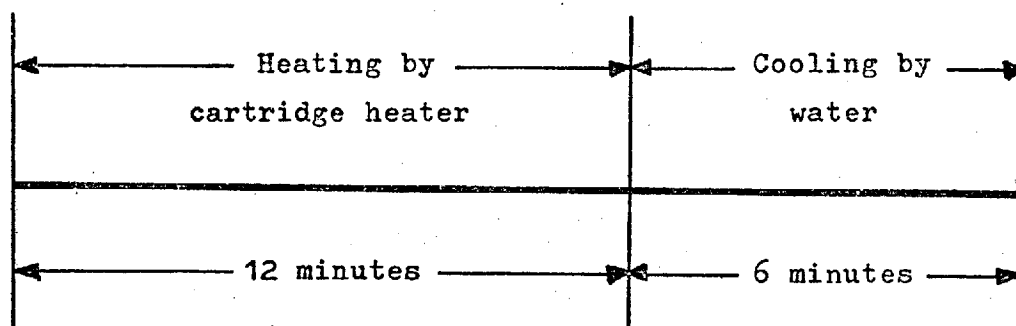
CYCLE FOR FATIGUE EXPERIMENT

Details of the heating and cooling cycles are described

in this section and Section 7.4.

In order to record the thermal stresses on the outside

surface of the specimen, the specimen is heated by means of a

cartridge heater inserted within the nozzle as shown in Fig.6.1.2.

The diameter of the heater is $^5/8$" and is mounted concentrically

within the nozzle with a clearance of 0.187". The element con-

sists of fine gauge resistance wire which is wound around a

refractory former and then inserted in a stainless steel sheath.

The total length of the heater is 12" with a cold length at one

end of $^3/4$" which allows the heater to be clamped in the hori-

zontal position. The maximum sheath temperature is $700^\circ C$ and

the maximum watt density 100 watts/sq.in (or 960 watts total).

The inside surface of the nozzle/vessel is maintained at $450^\circ F$

$(233^\circ C)$ by means of a thermocouple which is spot welded at one

end of the nozzle as shown in Fig.6.1.2. An alternative way of

positioning the thermocouple is to drill a hole through the

nozzle or vessel and insert the thermocouple through the hole.

This arrangement has basically two disadvantages: a hole would

introduce stress concentration on the outside surface where the

stress field is recorded and it would be difficult in determining

the influence of this stress concentration on the overall stress

field. Second the thermocouple lead would be subjected to a

considerable temperature gradient as it passed through the

nozzle or vessel and thus could affect the controlling temperature.

In welding the thermocouple on the inside surface of the nozzle care is taken to place the tip of the thermocouple within a small groove and cover it with a thin layer of high thermal resistance solder. This ensures that the thermocouple is recording the temperature of the surface without being affected by direct radiation from the cartridge heater. The voltage to the heater was reduced to 190 volts (maximum operating voltage 220 volts) by means of an adjustable transformer. This reduces the sheath temperature of the heater to $1112^{\circ}F$ ($600^{\circ}C$) since the maximum sheath temperature of $1292^{\circ}F$ ($700^{\circ}C$) can only be applied for clearance between 0.005" and 0.010". The reduction of the voltage, and consequently of the watt density and sheath temperature, prevents the heater from burning out. The cartridge heater is shown in Fig.6.2.1(a).

The heating-up and cooling-down processes of the specimens are controlled by a series of devices which are mounted on an instrument panel shown in the photograph of Fig.6.1.4. The instrument panel has dimensions 4'11" x 2' x 1'4" and is constructed of wood with metal legs. The sides are protected with expanding wire mesh. The following switches, fuse holders and instruments are mounted on the panel:

1. Main power switch (13A rating) and "orange" power indicating light.

2. Relay (AC 220/250 volts, 10A rating) with eleven contact points.

3. A cycle counter.

4. Electronic timer and timer switch (13A).

5. High temperature controller and switch (13A).

6. Low temperature controller and switch (13A).

7. Reset switch to isolate the high temperature controller from the lower temperature controller.

8. Cooling cycle switch (13A) and "green" cooling indicating light.

9. Heating cycle switch (13A) and "orange" indicating light.

On the panel the following fuses are mounted:

(a) low temperature controller fuse (5A)

(b) timer fuse (5A)

(c) high temperature controller fuse (15A)

(d) water valve fuse (5A)

(e) compressed air valve fuse (5A).

The operational procedures for the heating and cooling process are as follows: initially the high temperature controller is set at the desired maximum temperature at which the inside surface of the nozzle/vessel is to be maintained. This is 450°F (233°C) for the present experiments. The temperature is sensed by the Nickel Chromium/Nickel Aluminium thermocouple which is thermally insulated and it is connected to the high temperature controller. The controller is a thermic solid state one with a temperature range 0°C to 450°C (842°F). It is provided with a temperature deviation meter calibrated in degrees centigrade. It is also fitted with an over or under temperature alarm output (Fig.6.2.1 (e)) which is adjustable within the range of $\pm 15^\circ$C from set point. The alarm circuit provides a changeover output contact rated at 5A and 240 V AC. The high temperature controller is shown in Fig.6.2.1(e).

The cartridge heater is switched on through the high temperature controller and by means of the thermocouple the temperature of the inside surface of the nozzle/vessel is maintained at 450°F (233°C) for 20 minutes by means of the

timer (Fig.6.2.1(c)). The timer provides two time ranges of
1 to 120 seconds and 2 to 30 minutes by changing the range
sliding knob from position x1 to x20. At the end of the
specified time interval the timer, through the relay, switches
off the high temperature controller and consequently the heating
cartridge and at the same time it turns on the low temperature
controller and the cooling system. The low temperature con-
troller is similar to the high temperature controller. It has
a range from 0°C to 150°C (302°F). The controller is used to
control the cooling temperature of the specimen. This is done
with the same thermocouple which senses the high temperature.
The low temperature controller is connected to the high
temperature controller by Nickel Chromium/Nickel Aluminium com-
pensation leads. The connections are fitted with electronic
cold junction compensation to reduce the thermoelectric effects
in the thermocouple circuit. There are two cooling systems.
Compressed air is used to cool the specimen when strain gauge
readings are taken and water, which is more effective, is used
for the fatigue experiment. The compressed air is switched on
at the end of the heating-up period by the timer. This is done
by a two way spool valve which is connected to the relay. The
water cooling circuit operates by means of a pressure operated
solenoid valve shown in Fig.6.2.1(b). A series of pipes shown
in the picture of Fig.6.1.3 cool the nozzle and block. The
cooling temperature is set at 60°F (15.6°C) which is also the
value used for the theoretical analysis. The complete electrical
circuit diagram is shown in Fig.6.2.2.

6.3 Strain Gauge Measurements of Thermal Stresses

As the temperature on the outside of the nozzle reaches an average of $380^\circ F$ $(193.3^\circ C)$, special high thermal resistance strain gauges are used to record changes in strain. The gauges used are the 45° degree rosettes manufactured by the Tokyo Sokki Ltd. The dimensions of the rosette is 7.5 x 7.5mm with a gauge resistance of 120 ± 0.5 Ω. The rosette is a polyamide backing foil and it has a special grid configuration with good performance, especially in sensitivity, and creep characteristics at high temperature. The gauges are specially constructed for temperature up to $572^\circ F$ $(300^\circ C)$ and for locations of stress concentration. The maximum admissible current is 25mA and they have a fatigue life of 10^6 for a strain level of 1500×10^{-6}.

The strain gauge rosettes are bonded on the metal using a special adhesive made by the same manufacturer. The adhesive, NP-50, is a polyester with two components. The mixing ratio of the components is $2 \sim 4$ in weight. The surface of the metal is first smoothed with emery cloth and subsequently cleaned with acetone. Adhesive is applied to the surface of the metal and the back of the rosette foil and the rosette is then placed on the metal. A very fine line is grooved along the longitudinal and transverse planes so that rosettes are bonded with gauge number 1 in line with the planes. The accurate positioning of the gauges is very important since even small angles will affect the results. In order to be able to bond the rosettes with accuracy the rosettes are placed on a piece of cellotape and subsequently placed on the metal and held in position for one hour. The maximum operating temperature for the adhesive is the same as the maximum temperature for the gauges of $572^\circ F$ $(300^\circ C)$. The adhesive does not require any curing and has a

very good resistance to humidity. When the rosette is placed
on the metal, care must be taken to avoid any bubbles trapped
underneath the foil. Existing air bubbles are forced to escape
by pressing the rosette onto the metal working gradually from
the centre towards the edges taking care not to destroy the
gauges. Bubbles can produce false readings since the gauge is
not touching the surface of the metal, apart from the destructive
effect of the bubble expansion on heating.

Fifteen rosettes are placed on the specimen (Figs.6.3.1(a)
and (b)). Eleven of them are placed in the direction of the
longitudinal plane. Of these eleven three are on the vessel,
one on the junction and seven on the nozzle. The other four are
positioned in the transverse plane of the vessel as shown in
the figure. Due to symmetry it is expected that the gauges in
the transverse plane will indicate the same amounts of strain
as the corresponding ones in the longitudinal plane. Therefore
no gauges are placed in the transverse plane along the nozzle.

The centre of each rosette, that is the point where the
centre line of each of its three gauges meet which is near the
top right hand corner of the rectangular foil, is placed so
that it corresponds to the following nodes of the finite element
mesh of Fig.5.3.1.

Rosette	Gauge	Node
1	1,2,3	75 (Nozzle)
2	4,5,6	67 (Nozzle)
3	7,8,9	59 (Nozzle)
4	10,11,12	51 (Nozzle)
5	13,14,15	43 (Nozzle)
6	16,17,18	35 (Nozzle)
7	19,20,21	27 (Nozzle)
8	22,23,24	23 (Junction)
9	25,26,27	19 (Vessel)
10	28,29,30	12 (Vessel)
11	31,32,33	5 (Vessel)

For each of the forty five gauges a pair of connection terminals is used as junction point to connect the gauges to the instrumentation leads. This prevents the gauges being destroyed in the case of an accidental pull. An intermediate junction box is also used for the instrumentation leads before they are connected to the strain recorder. The intermediate box is held rigidly in position as shown in the photograph of Fig.6.1.3(b) to ensure that any pulling of the leads will not damage the gauges.

The strains are recorded using a multi-channel automatic strain recorder (Data-Log). The basic bridge-circuit is a direct current Whetstone bridge circuit with the electric potential drawn from a constant voltage electronic power pack. A digital voltmeter reads the out-of-balance voltage. Fifty bridges are provided in the recorder unit, each with its own

balancing unit, thus giving 50 available channels. A commutator unit selects the channels, either repeatedly or in turn, and the voltage reading is displaced each time on the digital voltmeter and printed on to paper tape through a fast printer for permanent record. The maximum selection speed of the unit is 10 channels per second. It is thus possible for a set of 50 readings to be recorded within 5 seconds from the moment of trigger. A slower speed of 4 channels per second was used throughout this series of experiments giving a time of 11.2 seconds for recording all 45 gauges.

The bridges in the recorder are separated from each other and each active gauge must be provided with a corresponding dummy gauge. However since thermal stresses are recorded and the gauges are temperature self compensating up to $356^{\circ}F$ ($180^{\circ}C$) the dummy gauges are replaced by 0.5 watts metal oxide resistors of the same resistance (120 Ω) as the active gauges. Attention is paid when selecting the resistors as an alternative to dummy gauges to ensure that their resistance does not change very much with temperature.

Voltage readings are taken every minute during the heating-up period for twenty minutes; this covers the necessary time for the maximum stresses to develop. Since transient heating is not a static but a dynamic process the time needed for the strain recorder to scan all 45 gauges has to be taken into account when comparison with theoretical results is made. It takes 11.2 seconds to record and print voltage changes for all 45 gauges at a scanning speed of 4 channels per second. Although the Data-Log maximum recording speed is 10 channels per second this cannot be used as the automatic printer can only record accurately at a maximum speed of 4 channels per second.

Therefore readings for every minute are actually recorded from approximately 6 seconds before the minute to 6 seconds after the minute. At the end of the twentieth minute the compressed air is switched on and cooling measurements are obtained again for every minute for 25 minutes of cooling.

A total of fourteen sets of readings are obtained under identical heating and cooling conditions. It is observed that a maximum of $\pm 0.27\%$ variation in voltage occurs between different sets of readings, which in turn produces a variation of direct and principal strains and stresses of $\pm 0.27\%$. The maximum variation occurs for strain gauges 1-21 along the nozzle where the temperature on the outside averages $380^{\circ}F$ ($193.3^{\circ}C$). At the beginning of the thirteenth test it was found that gauges 16, 18 and 19 were not working due to failure of the adhesive material. Careful examination indicated that the adhesive was cracked under the rosette foil and there was no proper contact between the gauges and the metal.

Special care is taken for the instrumentation leads to be kept away from the hot metal to prevent melting. The experimental results and comparisons with the theoretical analysis are presented and discussed in the next section.

6.4 Experimental Results and Comparisons with Theoretical Analysis

During heating-up measurements of stresses the strain gauges would under normal circumstances record changes in voltage due to strains set up in the structure plus a voltage change due to the expansion of each gauge. The apparent increase of the voltage due to the thermal expansion of the gauge is avoided by using temperature-compensating gauges. The gauge

exhibits a negative apparent strain due to heating (see

Appendix 6) up to a certain temperature and therefore the

indicated change in voltage is exclusively the result of the

stresses occurring in the material. However the strain gauges

used are temperature compensating only up to 180°C (356°F) and

therefore any strains recorded at points where the temperature

is higher than 180°C are apparent strains.

In the experiments conducted only the outside surface of

the nozzle at the time interval of 12 minutes is at temperatures

higher than 180°C. Adjustments of the values of the voltage,

and consequently strain and stresses, are performed for the

strain gauges of the following rosettes at the temperatures

indicated:

Rosette	Gauges	Temperature $^\circ$C	Apparent Strain
1 (Nozzle)	1,2,3	198.0	40×10^{-6}
2 (Nozzle)	4,5,6	197.9	33×10^{-6}
3 (Nozzle)	7,8,9	197.9	33×10^{-6}
4 (Nozzle)	10,11,12	197.8	28×10^{-6}
5 (Nozzle)	13,14,15	189.4	10×10^{-6}
6 (Nozzle)	16,17,18	188.4	7×10^{-6}
7 (Nozzle)	19,20,21	186.3	5×10^{-6}

The detail calculations are shown in Appendix 6. The value

of the voltage indicated is reduced by the determined value of

the observed strain which will give an adjusted value of strain,

and therefore stress, free from the apparent strain due to the

expansion of the gauge.

Measurements are obtained during the heating-up process for every one minute for all the fifteen rosettes (forty five gauges) up to twenty minutes. Cooling-down measurements are then recorded for another 24 minutes for every minute The Data-Log records changes in voltage which are printed on a tape by the printer and are identified by the number of the gauge. The observed values of the voltage are then converted into strain using the relationship

$$\varepsilon_{indicated}(in/in) = \frac{1}{250(F)} \times \frac{\Delta V}{V} \qquad (6.4.1)$$

where : F = the gauge factor

ΔV = voltage indicated in mV (10^{-3} volts)

V = voltage across the bridge in volts (this is set

to 6 volts)

The calculated strain from the above relationship is directly expressed in in/in and therefore is dimensionless. When it is used in relationships to calculate stresses, the units of stresses obtained will thus be unaffected. The above relationship is derived by considering the bridge circuits of the Data-Log strain recorder (R.A2-8).

The voltage readings are converted into strain and then stress by using Equation 6.4.1 and computer program EXPER2. The mathematical analysis of the equations and the program details are given in Appendix 6.

Basically the program performs the following calculations for each time interval of one minute during heating-up and cooling-down processes.

1. Converts voltage readings into strain for each gauge by using Equation (6.4.1).

2. Determines the three direct strains for each of the three

   gauges of each rosette and subsequently calculates the

   three direct stresses.

3. It then calculates the principal strains for each rosette,

   the principal stresses and the angle of the principal

   strain ε_I with respect to the longitudinal plane.

4. It then prints all the strain and stress results in a com-

   prehenisve form as shown in Table 6.4.1(a) and 6.4.1(b).

   The results obtained by the program are the indicated or

observed results. The corrections for the apparent strains at

time = 12 minutes during heating up for the rosettes along the

nozzle are made separately.

   First experimental results of stresses are plotted for the

time interval τ = 12 minutes when the stresses are maximum.

These are presented in the following graphs and tables:

Fig.6.4.1 and Table 6.4.2.   Stress σ_L.

   It is observed that for the vessel and junction gauges the

observed values of stress are also real values. This is because

the temperature at the corresponding positions of the rosettes

is below $180^{\circ}C$ and the gauges are self-compensating. Along the

nozzle the observed values are higher due to the apparent

strains for temperatures over $180^{\circ}C$ as has already been explained.

The difference between observed and real values of stresses in-

creases along the nozzle as the apparent strain increases due to

higher temperatures away from the junction.

   The theoretical graph is shown by the continuous line.

Generally there is good agreement at the vessel and the part of

the nozzle from nodes 35 to 75 with the experimental values

higher than the theoretical by an average of 16%. The possible

factors which account for the difference between the experimental

results are analysed at the end of this section. In the figure

there are two experimental values that deviate appreciably from

theory, at gauges (22) and (19). Gauge (22) is on the junction

and gauge (19) near the junction. Stress discontinuities at

this region produce this high amount of difference in value

between theory and experiment and it can be concluded that pre-

diction or measurement of stresses at or near the junction is

subject to a considerable degree of uncertainty. Nevertheless

the average between theoretical and experimental results at that

region can be considered as a realistic prediction.

Fig.6.4.2 and Table 6.4.2. Stress σ_z

Generally the situation for the σ_z stress is similar to

the previous figure with the theoretical values again higher

than the experimental. The observed values due to apparent

strain are also higher along the nozzle where the temperature on

the outside surface is higher than $180^\circ C$. There are two positions

along the nozzle (gauges 18 and 15) where the theoretical pre-

dictions are higher than the experimental.

Fig.6.4.3 and Table 6.4.3. MAXIMUM PRINCIPAL STRESS σ_{maxp}

The experimental readings for the maximum principal stress

at time $\tau = 12$ minutes during heating-up are calculated by

program EXPER2 from the direct stresses at each rosette. The

values presented are the real principal stresses after the

corrections for apparent strains. There is again generally good

agreement with theory. Experimental values are higher than

theoretical at the vessel and junction but for most of the

rosettes along the nozzle theoretical values are higher than

experimental with the exception of rosettes (2) and (1).

Fig.6.4.4 and Table 6.4.3.   MAXIMUM SHEAR STRESS $\tau_{max(12)}$

Maximum shear stress values are calculated using the values

of principal stresses obtained by program EXPER2 which analyses

the experimental readings of the direct stresses. Experimental

values are again in reasonable agreement with theory within

± 12% to ± 16%, with maximum differences again near the junction.

Figs.6.4.5, 6.4.6, 6.4.7.   Table 6.4.4.   STRESSES AT THE VESSEL

In the figures above the stress σ_y, the stress σ_z and the

maximum principal stress σ_{maxp} are plotted for the vessel for

every two minutes during heating-up and cooling down. Both

theoretical and experimental results are shown for node 12 on

the vessel where strain gauge rosette number 10 is positioned.

The strain gauge readings are the real ones since the temperature

on the outside of the vessel is below 180°C and therefore

there are no apparent strains for the gauges placed on the block.

It can be observed that experimental readings are again higher

than the theoretical values. Maximum experimental stresses

occur at time τ = 12 minutes during the heating-up and after

10 minutes from the beginning of the cooling-down process.

Theoretically the maximum stresses occur at time τ = 11 minutes

and therefore there is a difference of one minute between theory

and experiment. Variation of the actual value of k from that

listed (Ref.R.5-1) for this material could account for this.

Heat loss at the outside surface of the vessel and nozzle can

be higher than the theoretically estimated value and this also

delays the development of the maximum stresses in the structure.

Experimental stresses developed during cooling-down are

relatively higher than the equivalent stresses during heating-up.

Since cooling-down is performed by blowing a compressed air

stream of 55°F through the nozzle and around the specimen; no

theoretical predictions of stresses are made for the cooling-
down process. In real situations, pressure vessels are usually
cooled by natural convection which is a long process and there-
fore the stresses developed are generally lower than the ones
set up during the heating phase.

Figs.6.4.8, 6.4.9, 6.4.10, Table 6.4.5. STRESSES AT THE NOZZLE

Stresses at node 43 (rosette number 5) on the nozzle are
shown in the figures and values are recorded in Table 6.4.5.
Tensile stresses for this particular point on the nozzle increase
rapidly with time during the heating-up process to reach a
maximum of 5541 lb/in² for σ_x, 1890 lb/in² for σ_z and 5545 lb/in²
for σ_{maxp} after 12 minutes. These are strain gauge readings of
voltage converted to stress and again they represent real values.
At time 12 minutes the observed values have to be adjusted for
the effect of apparent strain as the temperature at that time
is over 180°C. The observed values are indicated on the graphs.

Theoretical values are lower than the experimentally recorded
ones by an average of 17% and theoretically the maximum stresses
develop one minute earlier at 11 minutes. Stresses during
cooling-down are compressive for the nozzle reaching a maximum
of -6240 lb/in² for σ_x, -2086 lb/in² for σ_z and -6377 lb/in²
for σ_{maxp}, after 10 minutes of cooling.

Account of errors involved in the theoretical and experimental
analysis

Assumptions made for the theoretical analysis and possible
errors involved in the experiments both influence the final
results. An attempt is made here to discuss all the possible
factors which the author can identify as important, without
implying that these errors are the only ones present.

1. Finite element analysis divides the structure into small
   parts and applies the theory of elastic analysis to each of
   them in order to find the collective result of the entire
   structure. In practice the structure consists of a much
   greater number of elementary particles which are also
   smaller than the theoretical triangular elements with a
   behaviour similar to that of the theoretical model but not
   identical.

2. Convergence of computational results is achieved to a very
   small tolerance but exact values would require a very large
   number of iterations which is impractical.

3. Coefficient of thermal expansion (α), modulus of elasticity
   (E), coefficient of thermal conduction (k), Poisson's
   ratio (ν) are all temperature dependent parameters. In
   practice their values change continuously during the heating-
   up process at all points within the metal. Theoretically
   an average value of each parameter for the entire structure
   is used at the temperature of each specific time interval.
   This is considerably more accurate than using an average
   value for each parameter for the entire structure but for
   an average temperature for the heating-up process.

4. The temperature of the inside surface of the nozzle is
   recorded by the thermocouple which is set at 450°F. In
   theory the above temperature is assumed to be acquired at
   time $\tau = 0$ but in experiment   there is a time lapse of
   about one minute until the temperature on the inside
   reaches the value of 450°F. During this time a heat transfer
   rate is taking place which is not accounted for in theory.

5. Apparent strains indicated by the strain gauges are estimated
   as has been explained in an earlier section. Nevertheless

there are additional apparent strains due to temperature

rise in the instrumentation leads. Every precaution is

taken to keep the leads away from the hot metal but there is

a certain amount of heat which is transferred to the leads

through the gauges. If the gauges were not self temperature

compensating and dummy gauges were used then the resistance

changes in the leads will cancel each other. However with

the self temperature compensated gauges only one pair of

lead wires is required and if they are connected in one leg

of the bridge variations in their resistance will be

indicated affecting the results. The problem is solved by

placing one lead in one bridge leg and the other lead in the

adjacent leg as shown below.

LEADS AT VARYING TEMPERATURE

SELF
TEMPERATURE
COMPENSATED
GAUGE

D

STRAIN
RECORDER

(EMF)
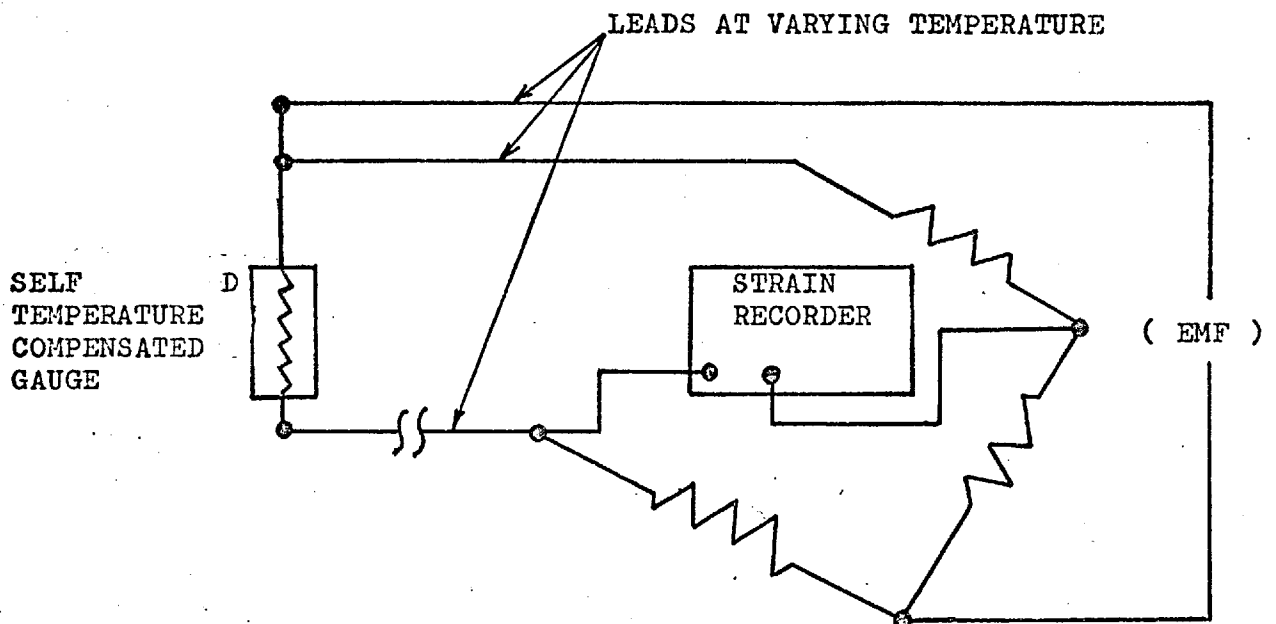
Nevertheless there is still a small percentage of error in-

volved due to small difference in length and temperatures

of the leads.

6. Gauge voltage readings and their recording for all 45 gauges

requires a total of approximately 12 seconds but when com-

parison to the theoretical results is made it is assumed

that experimental readings are instantaneous. Therefore

stresses for the first gauge recording (gauge No.1) and

the last gauge recording (gauge No.45) may vary to a maximum

of ± 2.8% from the assumed instantaneous reading at any

particular time interval given in the graphs. The above

percentage is estimated by determining stress changes

between successive readings of one minute and subsequently

allowing for the 12 seconds of recording time.

7. Finally changes in the resistance of resistors used for the

half bridge circuit introduce small amounts of error.

During the experimental measurements care is taken not to

produce any currents of air near the specimen during the heating

up process as heat losses from the outside surface of the

specimen are assumed to take place only by natural convection.

7. BASIC CONSIDERATIONS OF THERMAL FATIGUE EXPERIENCED BY VESSEL AND NOZZLE

7.1 Thermal Fatigue and Fracture Mechanics

Thermal fatigue of vessels and nozzles is a direct result of the cyclic thermal stresses applied during heating-up and cooling-down of the pressure vessel. Safety of a plant is therefore often connected directly to the endurance of the pressure vessel and its components under continuous cyclic stress. Thermal cyclic stresses, unlike other stresses, imply the presence of a liquid or gas which apart from causing thermal stresses it often acts as a corrosive agent. Corrosion can accelerate the propagation of cracks with all the associate consequences.

The problem of thermal fatigue in brittle and ductile materials has been investigated extensively and research is being carried out to produce solutions for unsolved questions. In this section no account of such works is given since this is not the object of the present project. However reference to certain thermal fatigue investigations directly connected to pressure vessels is given below.

The problem of thermal fatigue in structures is now associated with fracture mechanics as it is by crack initiation and propagation that failure eventually occurs. Basically the study of fracture can be divided into two main categories.

(a) Linear elastic fracture mechanics.

(b) Elastic-plastic fracture mechanics.

Additionally fracture mechanics must deal separately with brittle and ductile materials. In the case of thermal fatigue of pressure vessels ductile materials are used and considerations of crack growth are associated with the fracture mechanics of ductile materials. However brittle fracture can also occur but only under the influence of high internal pressure or excessive external loads. Low-cycle fatigue is associated with very high temperatures, usually above $1200^\circ F$ ($650^\circ C$) where plastic flow at the tip of the crack is mainly responsible for propagation. A very comprehensive account of low-cycle fatigue is given by Manson (R.7-1). Some fatigue studies of pressure vessel and nozzles are given in references (R.7-2,7-3 and 7-4).

7.2 Mechanics of Fracture and Crack Propagation

A load operating on a structure produces normal and shear components of stress. Failure occurs when the sum of external and any internal stresses reaches a critical value. Elastic failure (yielding) occurs when the maximum shear stress or shear strain energy of distortion reaches a critical value. Since the shear stresses are related to the difference between the principal components of stress, a larger principal stress exists when yielding begins under triaxial tension or compression than under uniaxial tension or compression. Whenever a discontinuity exists in a structure stress concentrations will occur near it and local yielding can occur in the vicinity of this discontinuity. Regions away from the discontinuity will remain elastic. The structure is then in an elastic-plastic state of deformation.

The concentration of elastic stress near the discontinuity is expressed analytically in terms of an elastic stress

concentration factor

$$K = \frac{\sigma \text{ local}}{\sigma \text{ nominal}} \tag{7.2.1}$$

which is the ratio of the local stress near the discontinuity to the nominal stress acting across the gross cross section. The elastic stress distribution depends on the shape of discontinuity. Around a discontinuity when a load is increased to a certain amount (which is usually expressed as a fraction of the load that would cause yield in the absence of the discontinuity) a plastic zone appears which will spread further as the load increases. The rest of the structure will behave elastically until the plastic deformation becomes large and the plastic zones spread over sufficiently large distances to cause a significant distortion.

Fracture is an inhomogeneous form of deformation which can be viewed on different levels. On an atomistic level it occurs by the breaking of atomic bonds, either perpendicular to a plane (cleavage) or across a plane (shear). On a microscopic level cleavage occurs by the formation and propagation of microcracks and the separation of grains along cleavage planes. Shear fracture (rupture) usually occurs by the formation of voids by intense shear. On a macroscopic level cleavage occurs when a cleavage crack spreads essentially perpendicular to the axis of maximum tensile stress. Shear fracture occurs when a fibrous crack advances essentially perpendicular to the axis of maximum tensile stress (normal rupture) or along a plane of maximum shear stress (shear rupture). Fracture is said to be transgranular when microcrack propagation and void coalescence occur through the grains and intergranular when they occur along grain boundaries. More than one mode of crack propagation can

contribute to the fracture of a structure. In general, cleavage fracture is favoured by low temperatures.

In order to analyse the fracture process under various types of stress systems it is necessary to establish a co-ordinate system with respect to both the fracture plane, the direction of crack propagation and the applied stress system. At the present time there is no standard type of notation and symbols are explained in the text as they occur. The system is shown in Fig.7.2.1(a). The crack whose length = a, height = 2h, and tip radius = ρ lies in the xz plane of a plate of thickness t and width w. The propagation of the crack is treated as a two-dimensional process since this introduces a relatively small error in computation and considerably simplifies the treatment of the problem. In practice a crack will also pro-pagate in one plane as long as that plane is the one on which maximum principal stresses occur. The direction of the crack propagation is along the x axis and x = 0 at the centre of the crack. The leading edge of the crack is parallel to the z axis. At the centre of the plate z = 0 and equals $\pm$ t/2 at the outer surfaces of the plate; (r, θ) are the co-ordinates of a point in the xy plane near the crack tip. Three distinct modes of separation at the crack tip can occur, as shown in Fig.7.2.1(b). Mode 1. The tensile component of stress is applied in the y direction, normal to the faces of the crack, either under plane strain (thick plate, t large) or plane-stress (thin plate, t small) conditions.

Mode 2. The shear component of stress is applied normal to the leading edge of the crack either under plane-strain or plane-stress conditions.

Mode 3. The shear component of stress is applied parallel to
the leading edge of the crack (antiplane strain).

7.2.1 Crack propagation

Over a wide range, the growth of fatigue cracks in both
fully plastic and elastic-plastic situations is by a continuum
controlled process. Direct observations at high applied strain
levels indicates a mechanism which depends on the crack tip low
field and results fractographically in striation formation.
The growth of a fatigue crack from its nucleation to final un-
stable propagation involves amounts of crack extension per
cycle which vary from the atomic to macroscopic scale.

In general fatigue failure consists of at least three
distinct stages. These involve the initiation or production of
a microcrack, steady crack growth at a measurable rate and
catastrophic fracture in one cycle. In a simple polished
specimen for instance the initiation time occupies most of the
life to fracture. However in most welded structures crack-like
defects will be present before the structure enters service or,
failing this, high stress concentration regions will probably
provide cracks, running into low stress regions, soon after
service loading commences.

Griffith (R.7-5) and Irwin (R.7-6) were the first to in-
vestigate the crack propagation in materials. They consider the
necessary conditions for the propagation of the elliptical,
elastic crack and the stress concentration around an elastic
crack. Irwin noticed that the local stresses near a flaw depend
on the product of the nominal stress σ and the square root of
the flaw depth 2c and that they are proportional to a term K
which is called the stress intensity factor and has dimensions
$[\text{stress } \sqrt{\text{length}}]$. For an infinitely sharp elastic crack, in an

infinitely wide plate, the stress intensity factor K is defined as

$$K = \sigma \sqrt{\pi c}$$

and the local stresses near the crack can also be expressed in terms of the stress intensity factor.

Since the introduction of the stress intensity factor the approach to crack growth has been expressed in the form of the law

$$\frac{da}{dN} = M \ (\Delta K)^n \qquad\qquad (7.2.1.1)$$

where M and n are material constants.

The exponent n was originally taken (R.7-7) to be equal to four but in recent work (R.7-8) it has been shown to vary widely, although this could be due to the limited range of data used. Coffin (R.7-9) has shown that plastic strain range considerations could produce something much closer to a universal criterion. Propagation laws have been examined and presented in a paper by Paris and Erdogan (R.7-7) and a comprehensive work on fracture is given by Liebowitz (R.7-10). In equation above $\underline{a}$ is the amount of crack growth and N the number of cycles. Therefore $\frac{da}{dN}$ is the fatigue crack growth per cycle and ΔK is the stress intensity range.

There is a threshold level ΔT_c below which growth does not occur. For most materials the threshold range of stress intensity factor is determined experimentally for the various modes of failures described in Section 7.2. It is found that aggressive environments, especially corrosive agents, tend to decrease the stress intensity threshold range and hence encourage crack growth at lower values of the stress intensity factor K.

The range ΔK is in fact the alternating stress intensity, given by the difference between the maximum and minimum values for each cycle. Therefore $\Delta K = K_{max} - K_{min}$. This equation has

been found to represent the behaviour of a large number of

materials (R.7-7), but it is now recognised that it is only valid

for the intermediate range of growth rates (10^{-5}- 10^{-3} mm/cycle).

The complete relationship between stress intensity and growth

rate (R.7-11) is shown schematically in Fig.7.2.2, where it can

be seen that Eqn.7.2.1.1 does not hold at very low values of ΔK,

where K approaches the threshold value K_c (Region A), nor for

very high values which approach the material's fracture toughness

K_{IC} (region C). For the mid-range of growth rates (region B),

values of the exponent '$\underline{n}$' , i.e. the slope of this "crack

growth-rate" curve, have been found to lie mainly in the range 2

to 4. These values are consistent with most analytical theories

of fatigue (R.7-12 to R.7-14) which are modelled on a mechanism

involving one growth increment per cycle. This pertains to the

generally accepted metallurgical theories which involve the pro-

duction of ductile striations, by alternate blunting and re-

sharpening of the crack tip, and where growth is controlled by

the local crack tip alternating plastic deformation. In such

situations Eqn.(7.2.1.1) can be used with confidence to predict

growth rates in service components and to provide the basis for

designs against failure by fatigue.

However, there is recent evidence that, in medium and high

strength steels, much higher values of the exponent n (up to 10)

can be obtained for a particular material or heat-treated

condition (R.7-15 to R.7-17).

7.3 A Theoretical Investigation of Crack Growth for Case C

A theoretical analysis is made for Case C in order to deter-

mine the stress intensity factor K under the stress loads during

heating-up of the structure. This is done for a region of high

stress concentration where crack propagation is most likely to occur under cyclic thermal loading. The boundary collocation method is used for the determination of K. The method is used as an alternative to the finite element analysis and it has the advantage of being able to calculate the stress intensity factor at any point including the tip of the crack.

No attempt is made to determine the constants M and n in Eqn.(7.2.1.1) since this requires experimental work directly concerned with the material used. However from the designer's point of view the determination of K is very important in order to predict if an existing crack will propagate and, according to the magnitude of K, if this crack propagation will be such as to cause extensive damage to the structure.

7.3.1 Regions of high stress concentration for Case C

Cracks already existing in the structure will propagate under the condition that the value of the stress intensity factor K is above the threshold value K_c. During heating-up the maximum value of the factor K_{max}, which will be attained when the maximum thermal stresses are reached, has to be determined in order to find out if it is sufficient to cause crack growth.

Initially it is necessary to determine the regions of the section where the maximum principal stresses develop. Cracks propagate under Mode I perpendicular to the lines of maximum principal stress, the rate of propagation being determined by K. Maximum principal stresses have been determined for Case C by program STREN3 and in Fig.7.3.1.1 the lines of maximum principal stresses are plotted for the section of Case C.

It can be seen that the area surrounded by the square is a region of stress concentration. The area is defined by the square whose corners are the nodes 285, 283, 200, 180 of the mesh

of Fig.5.3.1 for Case C. It is assumed that a small crack, similar to the ones present in the material due to machining and welding, exists at node 284. The crack length is taken as a = 0.04in which is a typical magnitude of a crack produced due to fabrication of the structure. During heating-up the nozzle expands and bending at the junction will cause growth of the crack assuming of course that the stress intensity factor K reaches the required value. In fact growth will occur for that period of heating-up during which the thermal stresses developed are large enough to produce a stress intensity factor K greater than K_c. On the other hand during cooling-down existing cracks could propagate from nodes 21 or 25 towards the interior of the structure.

7.3.2 Determination of stress intensity factor K by boundary collocation theory

If the state of stress in a two-dimensional solid is considered, it is found (R.7-18) that it is specified by three stress components σ_x, σ_y, τ_{xy} which satisfy the equilibrium equations

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \rho X = 0 \qquad (7.3.2.1)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \rho Y = 0 \qquad (7.3.2.2)$$

where X and Y are the components of the body force per unit mass. If X and Y are zero then the expressions

$$\sigma_x = \frac{\partial^2 \chi}{\partial y^2}, \quad \tau_{xy} = -\frac{\partial^2 \chi}{\partial x \partial y}, \quad \sigma_y \frac{\partial^2 \chi}{\partial x^2} \qquad (7.3.2.3)$$

satisfy the equilibrium equations for an arbitrary function χ. If the body is elastic then the components of stress satisfy the compatibility relations of the form

$$\frac{\partial^2}{\partial y^2}\{\sigma_x - \nu(\sigma_x + \sigma_y)\} + \frac{\partial^2}{\partial x^2}\{\sigma_y - \nu(\sigma_x + \sigma_y)\} = 2\frac{\partial^2 \tau_{xy}}{\partial x \partial y}$$

(7.3.2.4)

Substituting Eqn.(7.3.2.3) into Eqn.(7.3.2.4) it is seen that χ must satisfy the fourth-order linear partial differential equation

$$\frac{\partial^2 \chi}{\partial x^4} + 2\frac{\partial^4 \chi}{\partial x^2 \partial y^2} + \frac{\partial^2 \chi}{\partial y^4} = 0$$

(7.3.2.5)

which can be written as

$$\nabla^4 \chi = 0$$

(7.3.2.6)

which is the two-dimensional biharmonic equation and χ in the stress function.

In Fig.7.3.2.1 the area where the crack is introduced is shown. The method of analysis consists in finding a stress function χ that satisfies the biharmonic equation $\nabla^4 \chi = 0$ and also the boundary conditions at a finite number of stations along the boundary BC and AD. The biharmonic equation and the boundary conditions along the crack are satisfied by the Williams stress function (R.7-19), therefore

$$\chi(r,\theta) = \sum_{n=1,2\ldots}^{\infty}\left\{ (-1)^{n-1}d_{2n-1}\, r^{n+(\frac{1}{2})}\left[-\cos(n - \frac{3}{2})\theta \right.\right.$$
$$\left. + \frac{2n-3}{2n+1}\cos(n+\frac{1}{2})\theta \right] + (-1)^n d_{2n}\, r^{n+1}\left[-\cos(n-1)\theta \right.$$
$$\left.\left. + \cos(n+1)\theta \right]\right\}$$

(7.3.2.7)

The remaining boundary conditions to be satisfied are as follows (Derivations in Appendix 7).

Boundary BC:

$$\chi = -\frac{12M}{y^3}\left(\frac{a^3}{6} + \frac{a^2 y}{2} + \frac{ay^2}{2} + \frac{y^3}{6}\right) + \frac{6M}{y^2}\left(\frac{y^2}{2} + ay + \frac{a^2}{2}\right)$$
$$+ \sigma_x\left(\frac{y^2}{2} + ay + \frac{a^2}{2}\right),$$

where the first term is the moment effect and the second term is the stress effect. M is the moment of the sum of the components of σ_x along boundaries BC and AD.

Also along BC

$$\frac{\partial \chi}{\partial \chi} = 0$$

Boundary CD:

$$\chi = M/V \; , \; \frac{\partial \chi}{\partial \chi} = 0 \quad \text{(Moment effect)}$$

$$\chi = \frac{\sigma_x V^2}{2} \; , \; \frac{\partial \chi}{\partial y} = \sigma_x V \quad \text{(x-stress effect)}$$

The total effect will then be

$$\chi = \frac{M}{V} + \frac{\sigma_x V^2}{2}$$

$$\frac{\partial \chi}{\partial \chi} = 0$$

Boundary AB: The moment effect is

$$\chi = 0 \text{ and } \frac{\partial \chi}{\partial y} = 0, \text{ and the stress effect is}$$

$$\chi = 0, \; \frac{\partial \chi}{\partial y} = 0$$

therefore the total effect will be:

$$\chi = 0, \; \frac{\partial \chi}{\partial y} = 0$$

From here the procedure consists of solving 2m simultaneous algebraic equations corresponding to the value of the stress function χ and $\frac{\partial \chi}{\partial y}$ at m selected boundary stations. In this way the values for the first 2m coefficients in the Williams stress function are obtained, the remaining terms are neglected. Only the value of the first coefficient d_1 is needed for the present purpose because this is directly proportional to the stress-intensity factor K. According to Irwin (R.7-20), the stress component σ_y in the immediate vicinity of the crack tip (as r approaches zero) is given by

$$\sigma_y = \frac{K}{\sqrt{2nr}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right)$$

while in terms of the Williams stress function, as r approaches zero

$$\sigma_y = \frac{-d_1}{\sqrt{r}} \cos \frac{\theta}{2} (1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2})$$

and similarly for the other stress components, hence

$$K = -\sqrt{2\pi} \; d_1 \tag{7.3.2.8}$$

Once the value of d_1 has been determined for values of r

near the tip of the crack (where $\theta = 0$) the stress intensity

factor at the tip of the crack can be calculated. Now Eqn.7.3.2.7

by neglecting coefficients of higher order than 1 we have

$$\chi(r,\theta) = (-1)^{n-1} d_{2n-1} \; r^{n+(\frac{1}{2})} \left[- \cos(n - \frac{3}{2}) + \frac{2n-3}{2n+1} \cos(n+\frac{1}{2})\theta \right]$$

where $\quad n = 1$

hence $\quad \chi(r,\theta = 0) = - d_1 r^{3/2} (-\frac{4}{3})$

and

$$\chi(r,\theta = 0) = -\frac{4}{3} r^{3/2} d_1 \tag{7.3.2.9}$$

In order to determine the stress function χ for the area A of

Fig.7.3.1.1 a finite element mesh is constructed shown in

Fig.7.3.2.2. This enables the calculation of stresses at a much

smaller scale than they were calculated before for Case C.

Program STREN4 is used to solve this finer mesh. It does not

use the temperature differences to calculate the thermal stresses

but it sets as boundary conditions the stresses obtained by

STREN3 for Case C around the boundaries of area A. Program

STREN4 is listed in Appendix 7.

When the stresses are determined for the area, program FUNSTR

is used to determine the stress function χ for a total of 2025

nodal points (a grid of 45 x 45). Program FUNSTR performs the

following calculations:

Using finite difference analysis it determines the stress function χ which satisfies the biharmonic equation at the prescribed boundary points and at all interior mesh points. The grid is shown in Fig.7.3.2.3.

The following steps are performed by the program:

1. Read constants and dimensions parameters such as: mesh points, crack length, cell width and height, boundary co-ordinates.

2. Read boundary stresses in y-direction. These are provided as data from the results of STREN4.

3. Calculate moment for x-stress component.

4. Using finite difference equation solve for interior mesh points. The finite difference approximation for the biharmonic equation $\nabla^4\chi = 0$ is derived from the cell shown in Fig.7.3.2.3 and is given below as:

$$\nabla^4\chi \doteqdot 20\chi_{jk} - 8(\chi_{j,k+1} + \chi_{j,k-1} + \chi_{j+1,k}$$

$$+ \chi_{j-1,k}) + 2(\chi_{j+1,k+1} + \chi_{j+1,k-1}$$

$$+ \chi_{j-1,k+1} + \chi_{j-1,k-1}) + (\chi_{j+2,k} + \chi_{j-2,k}$$

$$+ \chi_{j,k+2} + \chi_{j,k-2}) = 0$$

5. Perform convergence test until predetermined tolerance.

6. Print the stress function χ for all nodal points.

The listing and designations of program FUNSTR are given in Appendix 7.

Having determined χ the first matrix coefficient d_1 is calculated by subroutine DMATRX which is linked to program FUNSTR. The subroutine calculates d_1 using Eqn.(7.3.2.9). The matrix coefficient d_1 is a function of χ and the distance r from the tip of the crack. The subroutine prints d_1 for each value of χ and r at every nodal point. Intermediate points are obtained by

specifying the co-ordinates x,y between two nodal points. The subroutine is also listed in Appendix 7.

Stress Intensity Factor K Results

The theoretical values of K obtained by the analysis are those developed at time τ = 11 minutes when the maximum stresses are set up for Case C. In Table 7.3.2.1 the values of the first matrix coefficient d_1 and the equivalent values of K are shown for distances r away from the tip of the crack. The values are calculated by subroutine DMATRX using the values of the stress function χ along the y axis of the crack where θ = 0. It can be seen from the table that K increases as the tip of the crack is approached. It attains its maximum value of 2112.3 at the tip of the crack. This is the maximum value which will be attained during heating-up at that particular point under the present conditions of thermal loading.

From reference R.7-21 the critical or threshold value of K for austenitic steel estimated by experimental analysis is of the order of 5.5×10^3. This value is considerably higher than the maximum value of 2112.3 obtained above, and it can be safely assumed that an existing crack will not propagate under the applied thermal load. In practice corrosion of the metal by the fluid responsible for the thermal stresses may lower the critical value of K, but such an effect is not likely to alter the value more than 10%. The critical value of K mentioned above is valid under the condition that a crack already exists and is not less than 0.02in in length. If the crack length is very small (0.001-0.010in) the critical value of K will tend to be lower. However as the crack grows K must be increased for further propagation and if this does not happen then the crack growth will be arrested permanently at a particular stress level. Therefore

in order to have a catastrophic crack growth the value of K must
be higher than the critical or threshold value at every stage of
the growth until the length of the crack is such to cause
fracture of the structure or the component. The theoretical
analysis can estimate K at every stage of the growth by taking
into account the various lengths of the crack and by determining
the stress function χ from the existing stress field.

If a crack does not exist at all, the stress intensity factor
must be very high in order to initiate one. This is not possible
unless the loads applied on the structure are quite large and
generally at least 50% of the tensile strength of the material,
if cyclic loading is applied.

Summarizing, the theoretical analysis for the determination
of the stress intensity factor K consists of the following steps:

(i)   Determine the stresses at the area where the crack is
        located.

(ii)   Using the calculated stresses determine the stress function
        χ by solving the biharmonic equation $\nabla^4 \chi = 0$. Use the
        boundary collocation analysis to define boundary conditions.

(iii)   The calculated values of χ are then used to determine the
        first matrix coefficient d_1 at the tip of the crack and
        at various distances r from the tip of the crack.

(iv)   The stress intensity factor K is a function of the first
        matrix coefficient d_1 (Eqn.7.3.2.8) and therefore it can
        be calculated at any point including the tip of the crack
        using the values of d_1.

(v)   The calculated value of K at the tip of the crack can
        then be compared to the threshold value K_c determined by
        experiments, and if $K > K_c$ then crack growth will take
        place.

(vi) By experimentally determining the values of M and n in the equation $\frac{da}{dN} = M(\Delta K)^n$ for the material considered, the rate of crack growth per cycle can be assessed.

7.4 Experimental Thermal Cycling of Specimen

In order to check if any cracks would develop in the specimen under investigation by subjecting it to thermal cycling, the experimental rig is slightly modified. Heating is still achieved by means of the heater but for cooling water is used which flows over the nozzle and the block as shown in the photograph of Fig.6.1.3.

Rapid cooling by water ensures that reverse stresses will develop of approximately the same order like the ones developed during the heating up process.

The inside of the nozzle is kept at $450^{\circ}F$ for twelve minutes, enough time for the maximum thermal stresses to develop. Cooling is subsequently switched on for approximately six minutes until the inside of the nozzle is cooled down to $65^{\circ}F$ $(18.3^{\circ}C)$. Therefore the specimen is subjected to a thermal cycle of 18 minutes and an alternating maximum principal stress which ranges from $\pm$ 8247 lb/in² at node (21) to $\pm$ 4583 lb/in² at node (43). The thermal cycling of the specimen at maximum stresses continued for four months at 12 hours daily to produce a total of 4783 cycles. Then the water valve failed and therefore it was decided to examine the specimen for possible cracks without inducing further cycles. This was decided in the light of the theoretical analysis which has predicted that the thermal stresses and therefore the stress intensity factors were not high enough to initiate cracks or cause growth of possible existing ones.

The specimen was cut in half through the longitudinal plane as shown in the photographs of Fig.7.4.1. It was then polished and examined under the microscope for possible cracks. The area shown within the square in Fig.7.4.1 was given special attention since it is in this area of high stress concentration and high stress intensity factors where existing cracks would most probably propagate. The results were negative and no cracks of detectable length were revealed.

It is therefore concluded that under the above thermal loadings the specimen will not fail within its possible service life unless the thermal stresses increase appreciably.

8. CONCLUSIONS

8.1 General Conclusions

To conclude the present work the following questions are answered in this section:

(1) Are the numerical and computational methods produced and developed in the theoretical part of the work satisfactory and reliable for the determination of thermal stresses and fatigue aspects in pressure vessel nozzles? Can they be used for design purposes and can they be applied to other parts of the vessel?

(2) What is the nature and magnitude of thermal stresses which develop in the various cases of vessel nozzles examined and how do they compare?

(3) Are the experimental techniques for measuring thermal strains (and consequently stresses) accurate enough, and how do theoretical predictions agree with experimental results?

The transient thermal distributions are determined for all three cases (A, B and C) by using the finite difference method to solve the heat convection and conduction equations at various time intervals during heating-up. The advantage of the method is that any heating conditions can be treated including transient convection at the boundary. The thermal programs developed can be used for other geometries by simply changing the input data. The method is reliable and converges fast. Truncation and round-off errors are present but are greatly reduced for small time

intervals and pre-determined tolerance. The heat transfer coefficient at the convective boundaries is a source of error since it is computed separately by using approximate relationships described in Section 4.1. Nevertheless this only affects the accuracy of the method indirectly and the values of heat transfer coefficients can be improved at any stage without disturbing the general structure of the program.

In Appendix 2 the finite element method is used to obtain thermal distributions with curvature taken into consideration. Its main advantage lies in the fact that the temperatures can be obtained at any time interval for every point within a square element. With the finite difference method the temperatures are obtainable at the nodes. However the method is not as fast as the finite difference method and it converges much slower.

The thermal stresses are calculated by the developed elastic stress-strain programs using the finite element method, and the thermal results obtained by the temperature distribution programs. Again stresses can be determined at any time and at any point of the structure where the temperature is known. The programs can be used extensively for other parts of the vessel and stresses can therefore be assessed for design purposes.

It is found that in all three cases the thermal stress pattern during heating-up is generally similar for both the outside and inside boundaries and for direct as well as principal and shear stresses. The outside vessel wall is in compression with the nozzle in tension. On the inside the wall is in compression. Additionally the maximum shear stress τ_{max} (12) develops in the xy plane for all three cases.

In CASE A, where the heating takes place from both the inside boundary of the vessel and nozzle, the magnitude of the stresses is the highest, and this is due to higher temperatures in the structure. Stresses in CASE B are generally higher than in CASE C although the rate of heating is the same. This is because in CASE B the inside vessel wall and the outside of the structure are insulated and this contributes to higher temperatures for CASE B at the corresponding time intervals during heating-up. In CASE C heat is transferred to the surroundings by natural convection. A considerable number of vessels are not insulated on the outside and this makes the analysis of CASE C useful with respect to the determination of thermal distributions when natural convection conditions prevail around the vessel.

In the experimental part of the project the temperature self compensating strain gauge is proved to be the most appropriate device to record strains under continuously changing temperature. The automatic strain recorder (Data-Log) is adequate for recording transient strains, but the printer's selection speed is limited and it takes 11.2 seconds to scan all 45 gauges. The experimental results are generally in good agreement with the theoretical model of CASE C with the experimental values higher than the theoretical by an average of 16%. The agreement indicates that both methods of finite difference and finite elements for the determination of temperatures and stresses can be used reliably for design assessments.

8.2 Conclusion to Chapter 7

The determination of the stress intensity factor K along the crack axis by the boundary collocation analysis for a structure other than a plate of thickness t poses a number of problems

directly associated with the loading at the boundaries. However
it is thought that boundary collocation analysis <u>can</u> be used to
determine K in a region of a structure as long as the stress
loading at the boundary can be determined.

To achieve this the stress field of the entire structure
determined in CASE C is used and region A (Fig.7.3.1.1) is iso-
lated with the stress loading on the boundary uniformly
distributed as shown in Fig.7.3.2.1. It is found that the value
of K at various distances from the tip of the crack can be
obtained from the stress function χ. The method is mathematically
accurate and as long as the maximum principal stress σ_{maxp} is
in one plane, and in this case the xy plane, the values of K are
reliable and the crack will propagate, if $K > K_c$. perpendicular
to the lines of maximum principal stress. The value of K for
CASE C at the tip of the crack is found to be 2112.3 $lb\sqrt{in^{-3}}$,
which is less than the critical value $K_c \approx 5500 \ lb\sqrt{in^{-3}}$ (determined
(R.7-21)
by experiments for the same material. Although for this low
value of K no cracks are detected in the region A of the specimen,
it should be emphasised that more experimental work is required
in order to establish fully that the theoretical model predictions
coincide with experimental results. However it is believed that
the theoretical analysis developed in Chapter 7 can be used
reliably in conjunction with experiments to assist the designer
of pressure vessels in his effort to improve his ability to predict
fatigue crack initiation and propagation.

9. DISCUSSION OF NUMERICAL METHODS

9.1 Finite Difference and Finite Element Methods for the Determination of Temperatures and How They Compare

The solution of the time-dependent heat transfer equation for the heat flow within a solid body in two-dimensional unsteady-state conduction has been solved by both finite difference and the finite element method. An account of the relative merits and disadvantages of each method thrown up by this work will be given in this section.

(a) Finite Difference Method

As is shown in Appendix 1, to solve the partial differential equation of heat flow the derivatives are replaced by finite difference approximations. The entire region of interest for the solution is divided into a mesh or grid over which the approximate solution is obtained. The solution at each point is equal to the average value of the quantity at the four adjacent points. In steady state problems the difference equations set up are solved by the iterative procedure over the entire grid until no point changes by more than the prescribed error; that is, the solution converges to the pre-determined accuracy. This accuracy depends on the problem and the available computing facilities. Now when time is one of the independent variables a sequence of solutions corresponding to each time increment must be obtained. The solution of the time dependent equation is an explicit solution, that is the expression used uses forward difference representation to give the future temperature T_i at node i in terms of the

current temperatures of its surrounding nodes. From the computational point of view the method is stable for small time increments $\Delta\tau$ and very fast.

Now the space increments Δx and Δy must be small enough to give the required accuracy. Of course the size of Δx, Δy depends on the overall dimensions of the body. Once the value of Δx, Δy (and in this case $\Delta x = \Delta y$) has been established, two other quantities are automatically fixed as well. The first is the time increment $\Delta\tau$ which is now fixed and the second is the total number of nodal points. It is clear that the larger the value of Δx and $\Delta\tau$ the more rapidly the solution will proceed. On the other hand, the smaller the value of these increments the more accuracy will be obtained. At first glance one might assume that small distance increments could be used for greater accuracy in combination with large time increments to speed the solution. This is not the case since the finite-difference equation (A 1.13, Appendix 1) limits the value of $\Delta\tau$ which may be used once Δx is chosen. It can also be seen that when Δx is increased $\Delta\tau$ is increased and when Δx decreases $\Delta\tau$ also decreases. This dependency of $\Delta\tau$ on Δx poses a computational problem. If for example the temperatures of a large structure were to be determined, the distance increment Δx cannot be taken as very small, which will otherwise be desirable for accuracy, because it will result in a very large number of nodal points which may be outside the storage capacity of the available computer facilities. At the same time, a small Δx means an equally small time increment $\Delta\tau$. A small time increment will unavoidably lead to a large number of time iterations to cover a fairly long heating-up process and therefore a considerable amount of computing time will be necessary. On the other hand if Δx is chosen to be large in order to make $\Delta\tau$ large and thus speed the solution the accuracy of the results will be limited.

It is therefore highly desirable under the above circumstances for the method of finite difference to converge rapidly. Rapid convergence saves computing time and it has been proved that finite difference method converges fast.

(b) Finite Element Method

In the Finite Element Method the temperature is approximated within the element by using shape functions which are defined piecewise element by element. Since the values of temperatures are known at time τ_o, values at time $\tau_1 = \tau_o + \Delta\tau$ can be obtained by summing around the nodes at this latter step and then solving the system of simultaneous linear algebraic equations that results. At each new step an identical procedure is used until a required time is reached. The accuracy of the results still depends on the size of the element but for the same Δx, Δy coordinate spacing the finite element method is more accurate than the finite difference method. This is mainly due to the fact that the temperature is defined better by the shape functions for the isoparametric elements used. On the other hand the finite element method is not as easily programmed as the finite difference and it does not converge to the exact solution as fast as the finite difference. The finite element method has the advantage that the determination of temperature at any point x, y within an element at time τ is obtained relatively more easily than for the finite difference method where the interpolation is time-consuming and not as accurate. The geometry of the domain under investigation often favours the use of the finite element method. For example in the case of the spherical vessel where the boundary shape is curved it is difficult to fit a finite difference grid to the boundary and therefore the finite element method is more appropriate for this particular geometric configuration. The finite element method is also unconditionally stable for any time

step $\Delta\tau$ although in certain cases where $\Delta\tau$ is very large some condition between the space increment Δx and the time increment $\Delta\tau$ must be imposed to obtain meaningful results.

Finally we can conclude in general that :

(1) The finite difference method is less accurate than the finite element method for large areas (greater than about 50,000 nodal points) and for relatively long times (greater than about 10,000 $\Delta\tau$) but it is more economical and computationally faster. On the other hand the finite element method is generally unconditionally stable for any time step $\Delta\tau$.

(2) For smaller areas and shorter times than those mentioned above the finite difference method should be preferred since the accuracy obtained is comparable to that of the finite element and at the same time it is faster. It must be emphasized that the area and time dimensions given above are based on the available computing facilities and the figures mentioned are of the computer systems used for this work (namely the CDC 6400 and CDC 6600 computers). For other computer systems the storage capacity and the central peripheral time must be taken into account in order to assess the relative value of the two methods in relation to any particular problem.

9.2 Comparisons of Finite Difference and Finite Element Methods for the Determination of Stresses

In this work the finite element method was used to calculate the thermal stresses and the finite difference method was used to solve the biharmonic equation $\nabla^4 \chi = 0$ in order to determine the stress function χ.

An advantage of the finite difference method, as has already been mentioned in Section 9.1, is the ease with which it may be formulated and programmed for solution by a digital computer. This is mainly due to the use of regular grids. In almost all finite difference methods the grid lines are parallel to the coordinate axes and therefore the

method is best suited to solving problems where the rows of points can terminate with points on the boundary (see Fig.7.3.2.3, p.337 for the finite difference solution of $\nabla^4 \chi = 0$).

Nevertheless, a disadvantage of finite difference method is a lack of geometric flexibility in fitting irregular boundary shapes and in concentration points in regions of the solution domain where the stresses for example change most rapidly (although it is possible to use irregular grids with varying distances between successive rows, this increases the complexity of the formulation).

The finite difference method is applied to the governing partial differential equations for the problem concerned while the finite element is formulated directly from the physical arguments used to derive such equations and therefore by using shape functions to describe the displacements the finite element method becomes more accurate.

Generally it can be concluded that:

(a) The finite difference method can be formulated and programmed more easily than the finite element and is faster.

(b) The finite element method is more accurate and has greater geometric flexibility, especially at irregular boundaries and at regions where the stresses change most rapidly. Computationally it is more complex and expensive than using finite differences.

APPENDICES

APPENDIX 1

Transient Heat Conduction and Convection

1.1 Transient Heat Conduction Finite Difference Equations

inside the Walls

The difference equations derived below apply only within the walls where heat is transferred by conduction. The equations across the convection boundary between fluid and nozzle-vessel section are derived in 1.2.

Consider the ⌐ozzle-vessel section divided into a two-dimensional net of space increments Δx and Δy, Δx being equal to Δy. The co-ordinates of the nodal points are represented by increments of i and j.
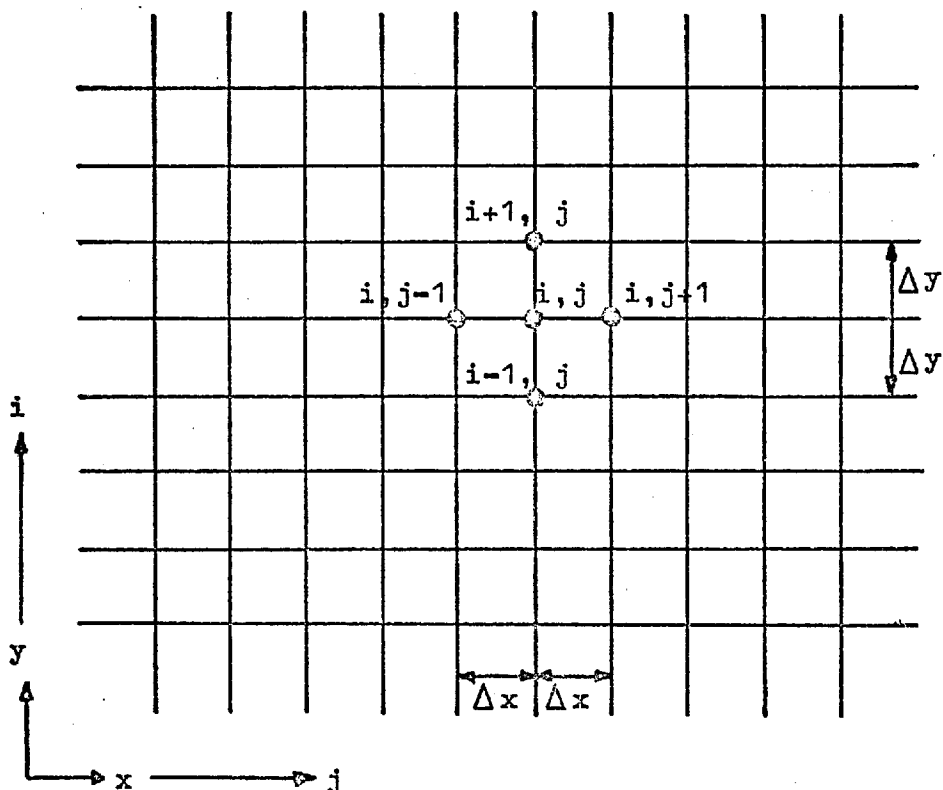
Finite differences are used to approximate differential increments in the temperature and space co-ordinates. The temperature gradients are written as follows:

$$\left.\frac{\partial T}{\partial x}\right)_{i+\frac{1}{2},j} \approx \frac{T_{i+1,j} - T_{i,j}}{\Delta x}$$

$$\left.\frac{\partial T}{\partial x}\right)_{i-\frac{1}{2},j} \approx \frac{T_{i,j} - T_{i-1,j}}{\Delta x}$$

$$\left.\frac{\partial T}{\partial y}\right)_{i,j+\frac{1}{2}} \approx \frac{T_{i,j+1} - T_{i,j}}{\Delta y}$$

$$\left.\frac{\partial T}{\partial y}\right)_{i,j-\frac{1}{2}} \approx \frac{T_{i,j} - T_{i,j-1}}{\Delta y}$$

The second partial derivatives will be:

$$\left.\frac{\partial^2 T}{\partial x^2}\right)_{i,j} \approx \frac{\left.\frac{\partial T}{\partial x}\right)_{i+\frac{1}{2},j} - \left.\frac{\partial T}{\partial x}\right)_{i-\frac{1}{2},j}}{\Delta x}$$

$$= \frac{T_{i+1,j} + T_{i-1,j} - 2T_{i,j}}{\Delta x^2} \qquad (A1.1)$$

and,

$$\frac{\partial^2 T}{\partial y^2} \approx \frac{\left.\frac{\partial T}{\partial y}\right)_{i,j+\frac{1}{2}} - \left.\frac{\partial T}{\partial y}\right)_{i,j-\frac{1}{2}}}{\Delta y^2}$$

$$= \frac{T_{i,j+1} + T_{i,j-1} - 2 T_{i,j}}{\Delta y^2} \qquad (A1.2)$$

The differential equation of transient heat conduction is:

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \rho c \frac{\partial T}{\partial \tau} \qquad (A1.3)$$

Substituting the gradients $\frac{\partial^2 T}{\partial x^2}$ and $\frac{\partial T}{\partial y^2}$ in equation (A1.1) above the time derivative equations is approximated by

$$\frac{\partial T}{\partial \tau} \approx \frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta \tau} \qquad (A1.4)$$

where n is the time increment. By combining the equations (A1.1), (A1.2), (A1.3) and (A1.4) the equation of the temperature T at a

particular time is derived in terms of the temperatures at the surrounding nodes at the beginning of the time increment.

$$\frac{T_{i+1,j}^{n} + T_{i-1,j}^{n} - 2T_{ij}^{n}}{(\Delta x)^2} + \frac{T_{i,j+1}^{n} + T_{i,j-1}^{n} - 2T_{i,j}^{n}}{(\Delta y)^2}$$

$$= \left[\frac{T_{i,j}^{n+1} - T_{i,j}}{\Delta \tau} \right] \left[\frac{1}{\alpha} \right] \qquad (A1.5)$$

where α is the thermal diffusivity in ft^2/hr and is equal to $\frac{k}{\rho c}$.

Since the space increments are equal, that is $\Delta x = \Delta y$ the equation (A1.5) above becomes

$$T_{i,j}^{n+1} = \frac{\alpha' \Delta \tau}{(\Delta x)^2} (T_{i,j}^{n} + T_{i-1,j}^{n} + T_{i,j+1}^{n} + T_{i,j-1}^{n})$$

$$+ \left[1 - \frac{4\alpha' \Delta \tau}{(\Delta x)^2} \right] T_{i,j}^{n} \qquad (A1.6)$$

By choosing the distance and time increment such as

$$\frac{(\Delta x)^2}{\alpha' \Delta \tau} = 4$$

the last term of the equation disappears and the temperature of the node (i,j) after a time increment is the arithmetic average of the four surrounding nodal temperatures at the beginning of the time increment.

Therefore,

$$T_{i,j}^{n+1} = \frac{1}{4} (T_{i,j}^{n} + T_{i-1,j}^{n} + T_{i,j+1}^{n} + T_{i,j-1}^{n}) \qquad (A1.7)$$

For the insulated boundaries, but not the junction, the energy balance gives the equation for the temperature T^{n+1}.
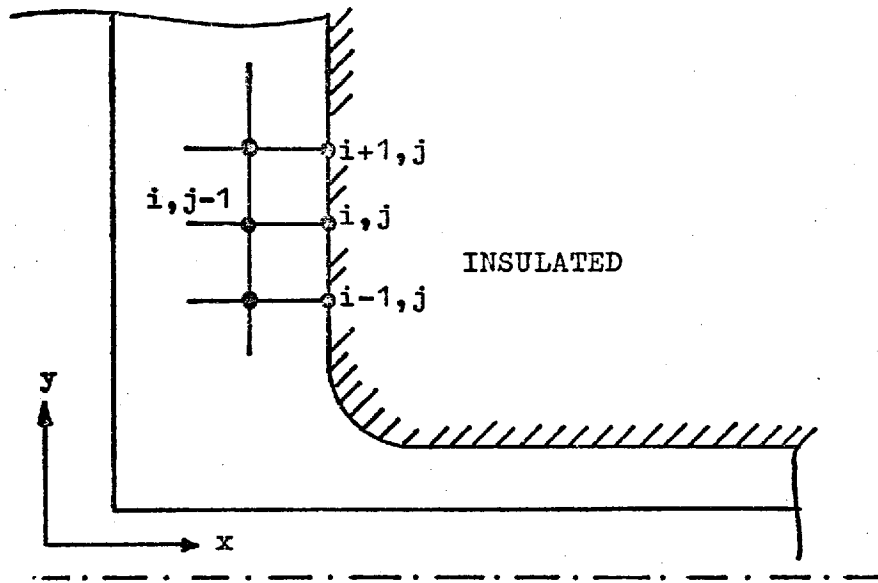
For the vertical nodal points

$$T_{i,j}^{n+1} = \frac{T_{i,j-1}^{n}}{2} + \frac{T_{i-1}^{n} + T_{i+1,j}^{n}}{4}$$ (A1.8)

For the horizontal nodal points

$$T_{i,j}^{n+1} = \frac{T_{i-1,j}^{n}}{2} + \frac{T_{i,j-1}^{n} + T_{i,j+1}^{n}}{4}$$ (A1.9)

The insulated boundary on the outside at the junction when

the junction is curved the energy balance gives,
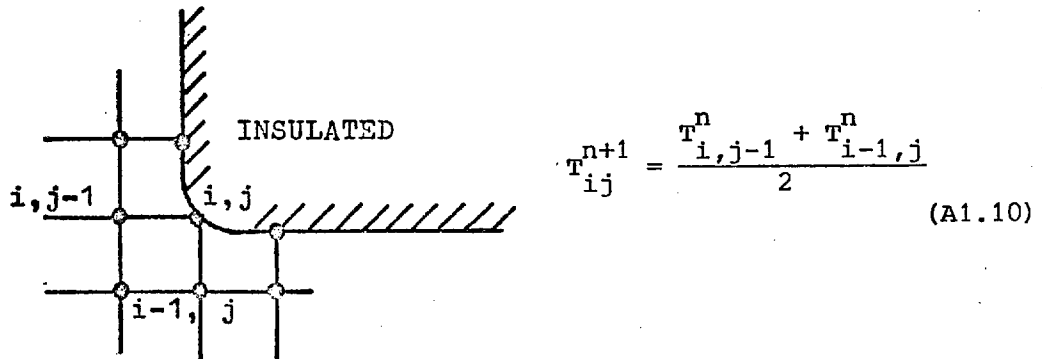
$$T_{ij}^{n+1} = \frac{T_{i,j-1}^{n} + T_{i-1,j}^{n}}{2}$$ (A1.10)

1.2 Difference Equations for Convective Boundary Condition
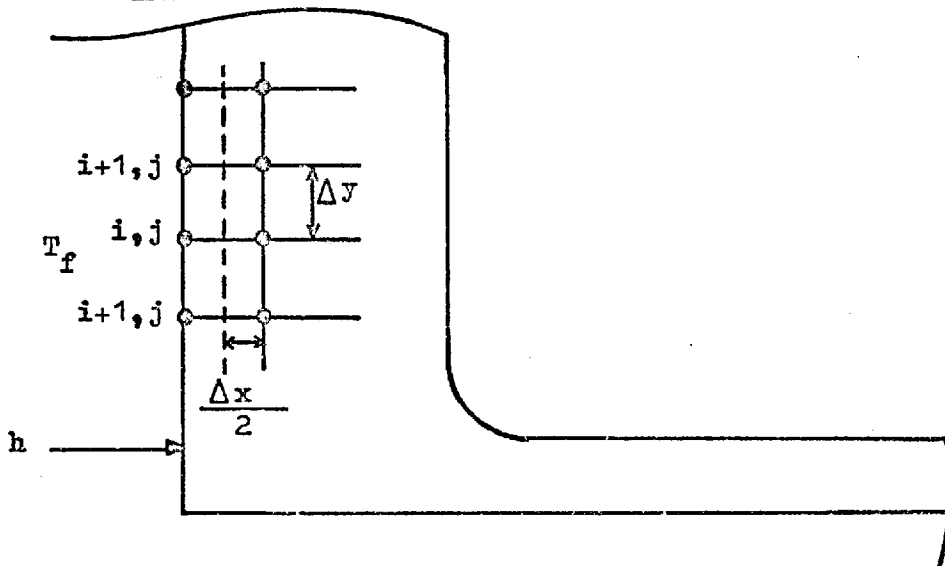
The boundary of the inside walls of the vessel and nozzle is exposed to a convection boundary condition. The transient energy balance on the node (i,j) is made by setting the sum of the energy conducted and converted into the node equal to the increase in the internal energy of the node. Therefore

$$k\Delta y \; \frac{T_{i,j+1}^n - T_{i,j}^n}{\Delta x} + k \; \frac{\Delta x}{2} \; \frac{T_{i+1,j}^n - T_{i,j}^n}{\Delta y}$$

$$+ k \; \frac{\Delta x}{2} \; \frac{T_{i-1,j}^n - T_{i,j}^n}{\Delta y} + h\Delta y \; (T_f - T_{i,j}^n) \qquad (A1.11)$$

$$= \rho c \; \frac{\Delta x}{2} \; \Delta y \; \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta \tau}$$

For $\Delta x = \Delta y$ the above equation for $T_{i,j}^{n+1}$ becomes

$$T_{i,j}^{n+1} = \frac{\alpha' \Delta \tau}{(\Delta x)^2} \left\{ 2 \; \frac{h\Delta x}{k} \; T_f + 2 \; T_{i,j+1}^n + T_{i+1,j}^n \right.$$

$$\left. + T_{i-1,j}^n + \left[\frac{(\Delta x)^2}{\alpha' \Delta \tau} - 2 \; \frac{h\Delta x}{k} - 4 \right] T_{i,j}^n \right\} \qquad (A1.12)$$

No attempt to choose the value of $\frac{(\Delta a)^2}{\alpha' \Delta \tau}$ to make the coefficient of T_j^n or $T_{i,j}^n$ zero, since the heat transfer coefficient influences the choice and also the time interval has been defined appropriately to give the value of $\frac{(\Delta x)^2}{\alpha' \Delta \tau}$ equal to 4 for the interior difference equations.

The equation for the nozzle inside convection boundary will be the same with the i and j subscripts interchanged and a different heat transfer coefficient. Thus,

$$T_{i,j}^{n+1} = \frac{\alpha' \Delta \tau}{(\Delta x)^2} \left\{ 2 \; \frac{h\Delta x}{k} \; T_f + 2 \; T_{i+1,j}^n + T_{i,j+1}^n \right.$$

$$\left. + T_{i,j-1}^n + \left[\frac{(\Delta x)^2}{\alpha' \Delta \tau} - 2 \; \frac{h\Delta x}{k} - 4 \right] T_{ij}^n \right\} \qquad (A1.13)$$

1.3 Natural Convection to Air for Case C

In Case C there is no insulation on the outside of the nozzle and vessel (see Fig.3.2.3) and therefore when the heat wave reaches the outside boundaries, heat is lost to the surrounding air by natural convection. Every precaution is taken during the experimental measurements of stresses not to create any air draughts near the specimen which will upset the condition of natural convection. During the transient period the heat lost to the gas is less than the heat input and the temperatures of the nodal points inside the wall will keep rising until steady state is reached. At steady state the heat input is equal to the heat lost to the gas plus the heat used to maintain the nodal temperatures of the structure at their steady state values. In order to determine the heat loss to the air the heat transfer coefficient h is determined for the vessel, junction and nozzle parts of the outside wall at the end of each time interval.

The equation of heat transfer for the vessel and junction wall for laminar flow (R.4-4) is given by:

$$\frac{hL}{k_a} = 0.13 \left[\frac{L^3 g \beta (T_w - T_\infty)}{\nu^2} (Pr) \right]^{0.33} \tag{A1.14}$$

where, L is the length of the vessel or junction wall, T_w is the wall temperature at the end of each time interval, and T_∞ is the air temperature ($60°F$). The temperature of the wall T_w is taken as the mean of all the nodal temperatures along the wall at the end of each time interval and the heat transfer coefficient thus obtained is the mean for the particular section of the wall under consideration.

For the vessel outside wall the equation is

$$\frac{hL}{k_a} = 0.54 \left[\left(\frac{L^3 g\beta \ (T_w - T_\infty)}{\nu^2} \right) (Pr) \right]^{0.25} \tag{A1.15}$$

Again T_w is the mean temperature along the wall and h the mean coefficient for each particular time interval. Once the heat transfer coefficient h has been determined for each section of the outside wall the heat lost to the air by convection is calculated by considering the heat balance equation at the wall. This has as follows (figure below).
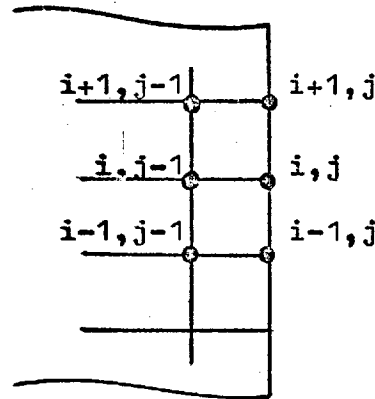
At the time interval n+1 the heat conducted from point i,j-1 to point i,j is equal to the heat used to raise the temperature of the wall at the point i,j plus the heat lost to the gas by convection. This happens for every nodal point of the structure near the inside wall.

Now the heat conducted from point i,j-1 to point i,j =

$- k \frac{\Delta x}{\Delta y} (\quad T_{i,j} - T_{i,j-1})$. The heat which is used to raise the temperature of the wall by $\Delta T = \rho_s C_s \Delta x \Delta y \Delta T$. The heat lost to the air $= h\Delta x (T_{i,j} - T_\infty)$. Therefore the equation of heat balance will be:

$$- k \frac{\Delta x}{\Delta y} (T_{i,j} - T_{i,j-1}) = \rho_s C_s \Delta x \Delta y \Delta T + h\Delta x (T_{i,j} - T_\infty)$$

or since $\Delta x = \Delta y$

$$- k (T_{i,j} - T_{i,j-1}) = \rho_s C_s (\Delta x)^2 \Delta T + h\Delta x (T_{i,j} - T_\infty) \tag{A1.16}$$

From this last equation ΔT can be calculated for the time interval n+1. Nevertheless in the computation of the temperatures at the nodal points on the outside wall an approximation has to be made. The temperature of each node on the outside wall $T_{i,j}$ at the time interval n is taken as equal to the temperature of the previous node $T_{i,j-1}$ of the same time interval. This is an approximation which is necessary in order to establish some value for the wall temperature which will enable us to calculate h. The real value of the wall temperature at the new time interval n+1 will therefore be

$$T_{i,j}^{n+1} = T_{i,j}^{n} - \Delta T \qquad\qquad (A1.17)$$

This is done for all the nodal points on the outside wall. The new heat transfer coefficient is then calculated at n+1 time using the new wall temperature $T_{i,j}^{n+1}$. This new coefficient is used to obtain the wall temperature at time n+2 and so on. A truncation error exists here since the wall temperature at any time increment is calculated using the heat transfer coefficient which is obtained by using the wall temperature at the previous time increment. This error is unavoidable because in the unsteady state, when temperature changes continuously with time, the heat transfer coefficient is a function of the wall temperature and the wall temperature cannot be calculated if the heat transfer coefficient is not known. Nevertheless the error is very small for small time increments used in this analysis.

APPENDIX 2

Transient Thermal Analysis for Spherical Vessels

2.1 Introduction

It was explained in Chapter 4 that the analysis for the determination of thermal distributions and stresses applied in the case of cylindrical vessels may be equally applied for spherical vessels. This is when the diameter of the spherical vessel is large and the curvature can be neglected. In here a method of determining the thermal distributions for spherical vessel where the curvature is taken into account is developed. The two-dimensional transient heat conduction problem is solved by the finite element method.

The finite element method was first applied by Wilson and Nickell (R.A2-1). Their method for analysing the unsteady flow of heat was based on the variational principle of Gurtin (R.A2-2). Richardson and Shunn (R.A2-3) used the same variational principle and the finite element method to solve transient heat conduction problems involving non-linear boundary conditions. Emery and Carson (R.A2-4) and Visser (R.A2-5) used the variational form-ulations in their finite element method applied to transient heat conduction problems. Zienkiewicz (R.A2-6), Desai and Abel (R.A2-7) have also used the finite element method for determination of two-dimensional heat conduction problems.

2.2 Theoretical Formulation of the Finite Element Method

The two-dimensional heat conduction equation which describes the unsteady temperature distribution in a solid which is in a domain P (Fig.A2.2.1) is given by the equation below:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + Q = \rho c \frac{\partial T}{\partial \tau} \qquad (A2.2.1)$$

The equation subject to the conditions on the boundary surface S_1

$$T = T_s \text{ on } S_1 \qquad (A2.2.2)$$

$$-k_x \frac{\partial T}{\partial x}\ell_x - k_y \frac{\partial T}{\partial y}\ell_y = q \text{ on } S_2 \qquad (A2.2.3)$$

$$-k_x \frac{\partial T}{\partial x}\ell_x - k_y \frac{\partial T}{\partial y}\ell_y = h(T - T_\alpha) \text{ on } S_3 \qquad (A2.2.4)$$

The initial condition is

$$T(x,y,o) = T_o(x,y) \qquad (A2.2.5)$$

where S $= S_1 + S_2 + S_3$

S_1 = the part of the boundary on which T is prescribed

S_2 = the part of the boundary on which q is prescribed (the intensity of heat input)

S_3 = the part of the boundary on which $h(T - T_\alpha)$ is prescribed

$T(x,y,\tau)$ = temperature in the solid

k_x & k_y = thermal conductivities

x,y = principal directions of the conductivity tensor

c = specific heat

ρ = density

Q = externally applied heat flux

T_s = boundary surface temperature

T_o = initial temperature

$$
\left. \begin{array}{l} \ell_x \\ \ell_y \end{array} \right\} \quad = \text{direction cosines of the outward normal to the boundary surface}
$$

h = heat transfer coefficient

T_α = temperature of the surrounding
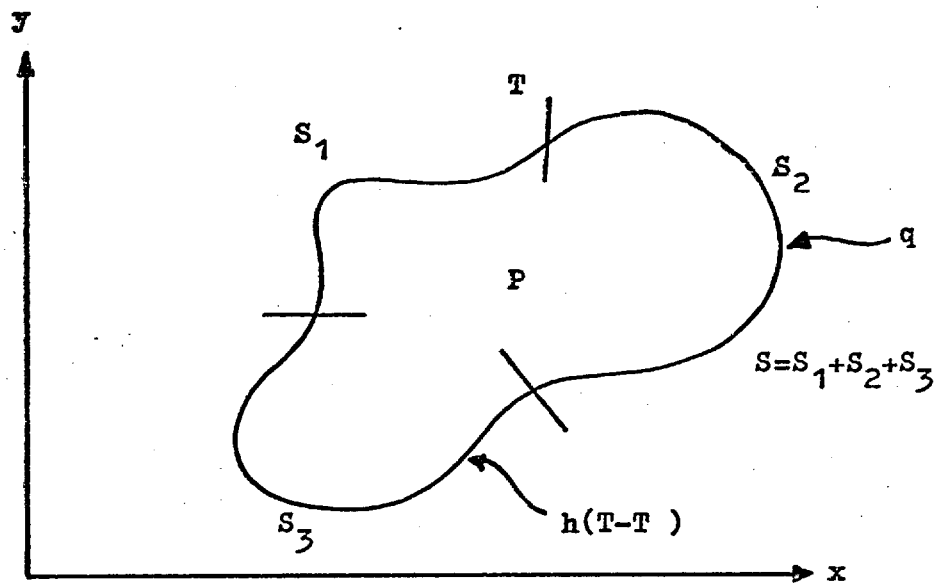
FIG.A2.2.1   P domain governed by Equation (A2.2.1)

Fig.A2.2.2 shows the P domain divided into finite elements in space and time.
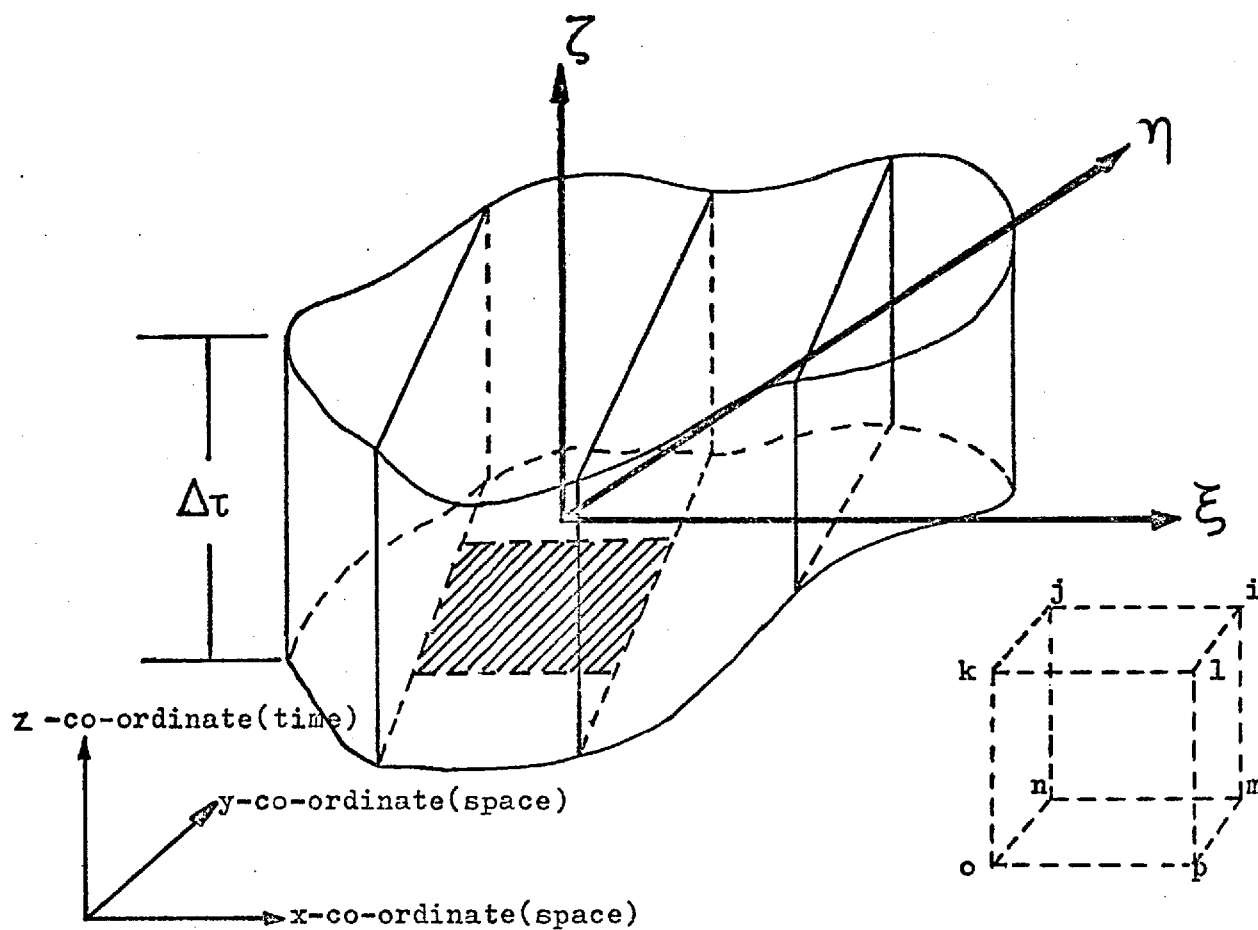
FIG.A2.2.2

The temperature within each element is approximated by

$$T(x,y,\tau) = \sum_{1}^{nnp} N_I (x,y,\tau) \, T_I$$

where,

N_I = the shape functions defined element by element

I = summation subscript

T_I = nodal temperature representation of $T(x,y,\tau)$

nnp = number of nodes in an element

Using the weighted residual process in which the weighting function is equal to the shape function defining the approximation within the elements, the heat conduction equation (A2.2.1) becomes,

$$\int_P N_I \left[\frac{\partial}{\partial x} \left(k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial T}{\partial y} \right) + Q - \rho c \frac{\partial T}{\partial y} \right] dP = 0$$

(A2.2.6)

where, P is the finite element domain and dP = dx dy dτ. Integrating by parts on the first and second terms in the equation above is simplified to

$$-\int \left[k_x \frac{\partial N_I}{\partial x} \frac{\partial T}{\partial x} + k_y \frac{\partial N_I}{\partial y} \frac{\partial T}{\partial y} - N_I Q + N_I \rho c \frac{\partial T}{\partial \tau} \right] dP$$

$$-\int_{S_2} N_I q \, dS_2 - \int_{S_2} N_I h (T - T_\alpha) dS_3 = 0 \qquad (A2.2.7)$$

where S_2 and S_3 are defined in Fig.A2.2.1.

Using the equation for the temperature approximation within each element the Ith equation of the system that will allow the solution for the n values of T_I is

$$-\int_{P}\left[k_x \frac{\partial N_I}{\partial x}\frac{\partial}{\partial x}(\sum_1^n N_I T_I) + k_y \frac{\partial N_I}{\partial y}\frac{\partial}{\partial y}(\sum_1^n N_I T_I)\right.$$

$$\left. - N_I Q + N_I \rho c \frac{\partial}{\partial t}(\sum_1^n N_I T_I) \right] dxdyd \qquad (A2.2.8)$$

$$-\int_{S_2} N_I q dS_2 - \int_{S_3} N_I (\sum_1^n N_I T_I - T_\alpha) S_3 = 0$$

Therefore the system consists of n linear algebraic equations in n unknowns.

2.3 Rectangular Prism (x,y,τ). Element Shape Functions

The rectangular prism is used to represent the three dimensional element (Fig.A2.2.2). Co-ordinates x,y are space co-ordinates while τ is the time co-ordinate. In order to establish the shape functions for a rectangular prism, the rectangular 12345678 is first considered shown below.

Let the shape function ϕ be expressed in a polynomial form in x and y. In order to ensure inter-element continuity of ϕ along the top and bottom sides the variation must be linear. The eight nodal points are to be determined and thus:
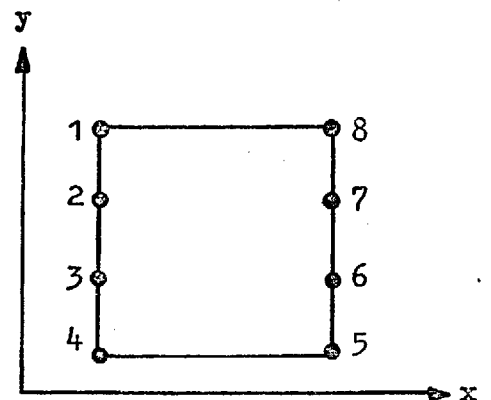
$$\phi = a_1 + a_2 + a_3 y + a_4 \, x \, y + a_5 \, y^2 + a_6 \, xy^2 + a_7 \, y^3 + a_8 \, xy^3$$

Substituting co-ordinates of the various nodes a set of simultaneous equations is written in a matrix form

$$\begin{bmatrix} \phi_1 \\ \vdots \\ \vdots \\ \vdots \\ \phi_8 \end{bmatrix} = \begin{bmatrix} 1, x_1, y_1, x_1 y_1, y_1^2, x_1 y_1^2, y_1^3, x_1 y_1^3 \\ \text{------------} \\ \\ \\ \text{--------------} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \vdots \\ \vdots \\ \alpha_8 \end{bmatrix} \qquad (A2.3.1)$$

or

$$[\phi]^e = [c][\alpha] \qquad \text{(A2.3.2)}$$

Equation (A2.3.1) can then be written

$$\phi = [P][\alpha] = [P][c]^{-1}[\phi]^e \qquad \text{(A2.3.3)}$$

where $[P] = [1, x, y, xy, y^2, xy^2, y^3, xy^3]$ (A2.3.4)

Therefore the shape functions for the elements defined by

$$\phi = [N][\phi]^e = [N_1 N_2 ---- N_8][\phi]^e \qquad \text{(A2.3.5)}$$

and hence $[N] = [P][c]^{-1}$ (A.2.3.6)

It is more convenient to use normalised co-ordinates and these are so chosen that on the faces of the rectangle their values are ± 1.

$$\xi = (x - x_c)/a \quad , \quad d\xi = dx/a$$

$$\eta = (y - y_c)/b \quad , \quad d\eta = dy/b \qquad \text{(A2.3.7)}$$

where a and b, the length of the sides of the rectangle. Once the shape functions are known in the normalised co-ordinates translation into actual co-ordinates can be carried out.

2.4 Three Dimensional Element. Rectangular Prism

As it has already been explained the three dimensional rectangular prism is used for the element discretization of the structure. This is shown in the figure below (linear element with 8 nodes).

(8 nodes)
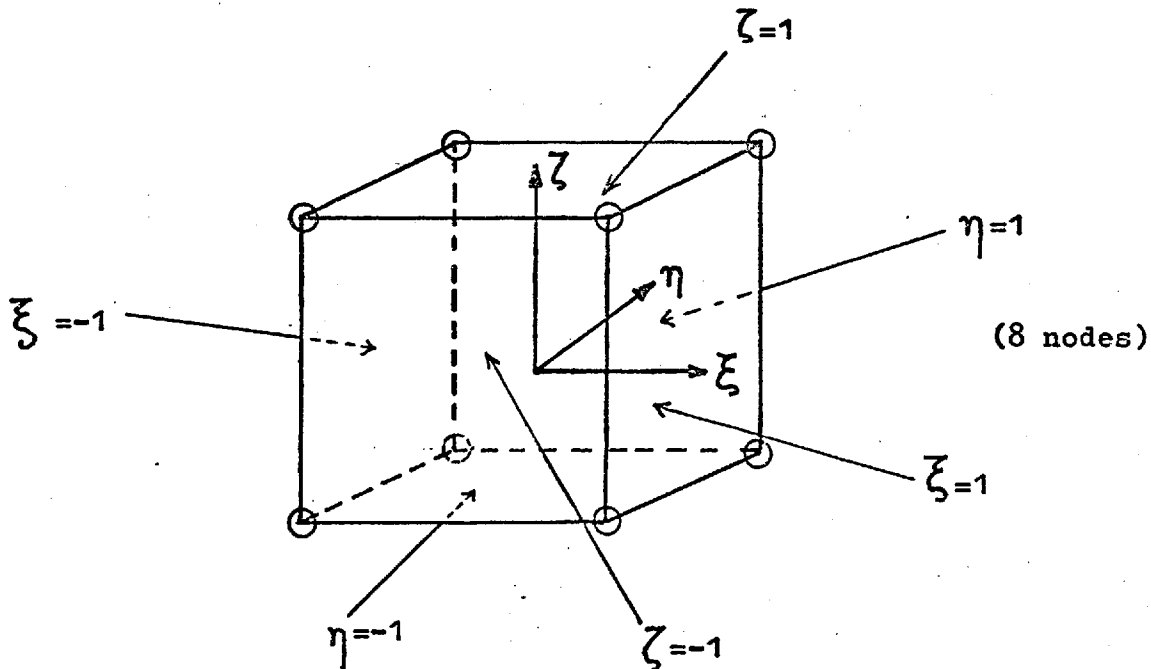
The shape function is thus given by

$$N_i = \frac{1}{8}(1 + \xi_o)\ (1 + \eta_o)\ (1 + \zeta_o) \qquad\qquad (A2.4.1)$$

where

$$\xi_o = x_c/a$$

$$\eta_o = y_c/b$$

$$\zeta_o = z_c/c$$

and x_c, y_c, z_c the co-ordinates of the centroid and a,b,c the dimensions of the element.

2.5 Numerical Solution of the Thermal Distribution

The section of nozzle-vessel is shown in Fig.A2.5.1(a). The solid is symmetrical about axis xx' and yy'. In Fig.A2.5.1(b) the finite element grid is shown. This divides the structure into 426 elements. Since values of the element temperatures are known at τ_o values at $\tau_1 = \tau_o + \Delta\tau$ can be obtained by summing around the nodes at this latter step, an identical procedure is used until a required time is reached.

The temperature approximation for each element using rectangular prisms as the elements and linear shape functions as it was explained in Section 2.4, is as follows.

$$T(x,y,\tau) = \frac{1}{8} (1 + \xi)(1 + \eta)(1 + \zeta)T_i$$
$$+ \frac{1}{8} (1 - \xi)(1 + \eta)(1 + \zeta)T_j$$
$$+ \frac{1}{8} (1 - \xi)(1 - \eta)(1 + \zeta)T_k$$
$$+ \frac{1}{8} (1 + \xi)(1 - \eta)(1 + \zeta)T_\ell$$
$$+ \frac{1}{8} (1 + \xi)(1 + \eta)(1 - \zeta)T_m$$
$$+ \frac{1}{8} (1 - \xi)(1 + \eta)(1 - \zeta)T_o$$
$$+ \frac{1}{8} (1 + \xi)(1 - \eta)(1 - \zeta)T_p$$

$$= N_i T_i + N_j T_j + N_k T_k + N_m T_m + N_n T_n + N_o T_o + N_p T_p$$

where $\xi = 2(x - x_c)/(\Delta x)$

$\eta = 2(y - y_c)/(\Delta y)$

$\zeta = 2(\tau - \tau_c)/(\Delta \tau)$

x_c, y_c, τ_c = co-ordinates of the centroid of each element.

$T_i, T_j, T_k, T_\ell, T_m, T_n, T_o, T_p$ = values of temperatures at the appropriate nodal points.

$N_i, N_j, N_k, N_\ell, N_m, N_n, N_o, N_p$ = shape functions

x = x co-ordinate spacing

y = y co-ordinate spacing

$i, j, k, \ell, m, n, o, p$ = node numbers

2.6 Boundary Conditions

At time $\tau = 0$ the structure is at room temperature of $60°F$. Boundaries BCDE and AG are insulated (Fig.A2.5.2), thus making the heat gradient zero.

Fluid enters at temperature $T_f = 450°F$ and heat input q is provided along the boundary GF. Temperature of fluid is maintained steady at $450°F$ along GF. Small fluctuations will not affect the thermal distribution within the solid and the temperature of the inside surface of the nozzle is assumed to be at $450°F$.
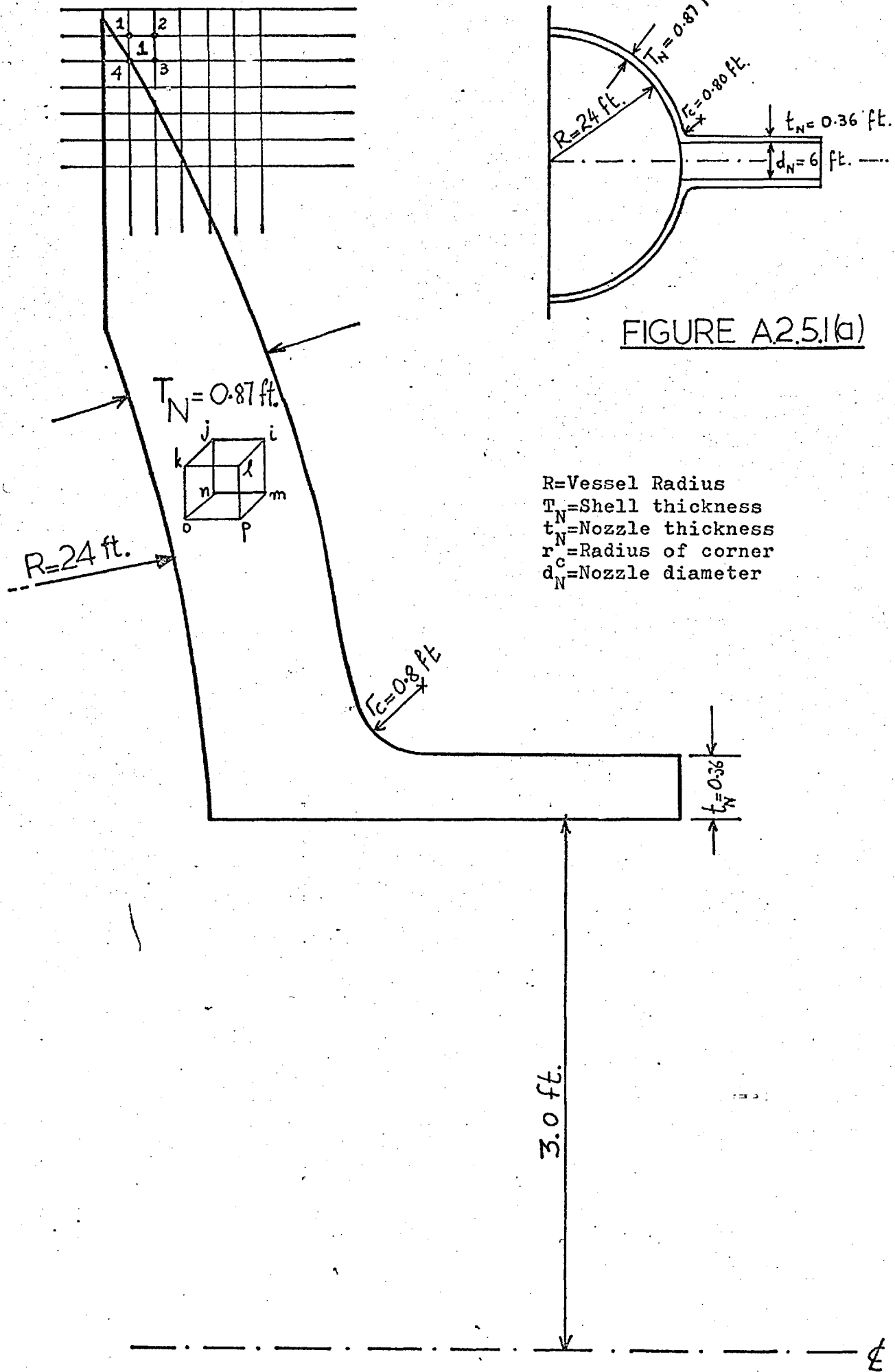
FIGURE A2.5.1(a)

R=Vessel Radius
T_N=Shell thickness
t_N=Nozzle thickness
r_c=Radius of corner
d_N=Nozzle diameter

FIGURE A2.5.1(b) FINITE ELEMENT MESH

B

$\dfrac{dq}{dt}$

A

Originally
at T=60°F

INSULATED BOUNDARY BCDE

INSULATED
BOUNDARY
AG

$\dfrac{dq}{dt}$ = 0 along BCDE

$\dfrac{dq}{dt}$ =0 ALONG AG

C

D   E

HEAT INPUT q̊

G ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ F   ↑ ↑

T_f=450°F ⟵

⊄

FIGURE A2.5.2

DIMENSIONS

$R = 24.0$ ft.
$T_S = 0.87$ ft.
$t_N = 0.36$ ft.
$r_c = 0.80$ ft
$d_N = 6.0$ ft.

2.60 ft.

3.0 ft.
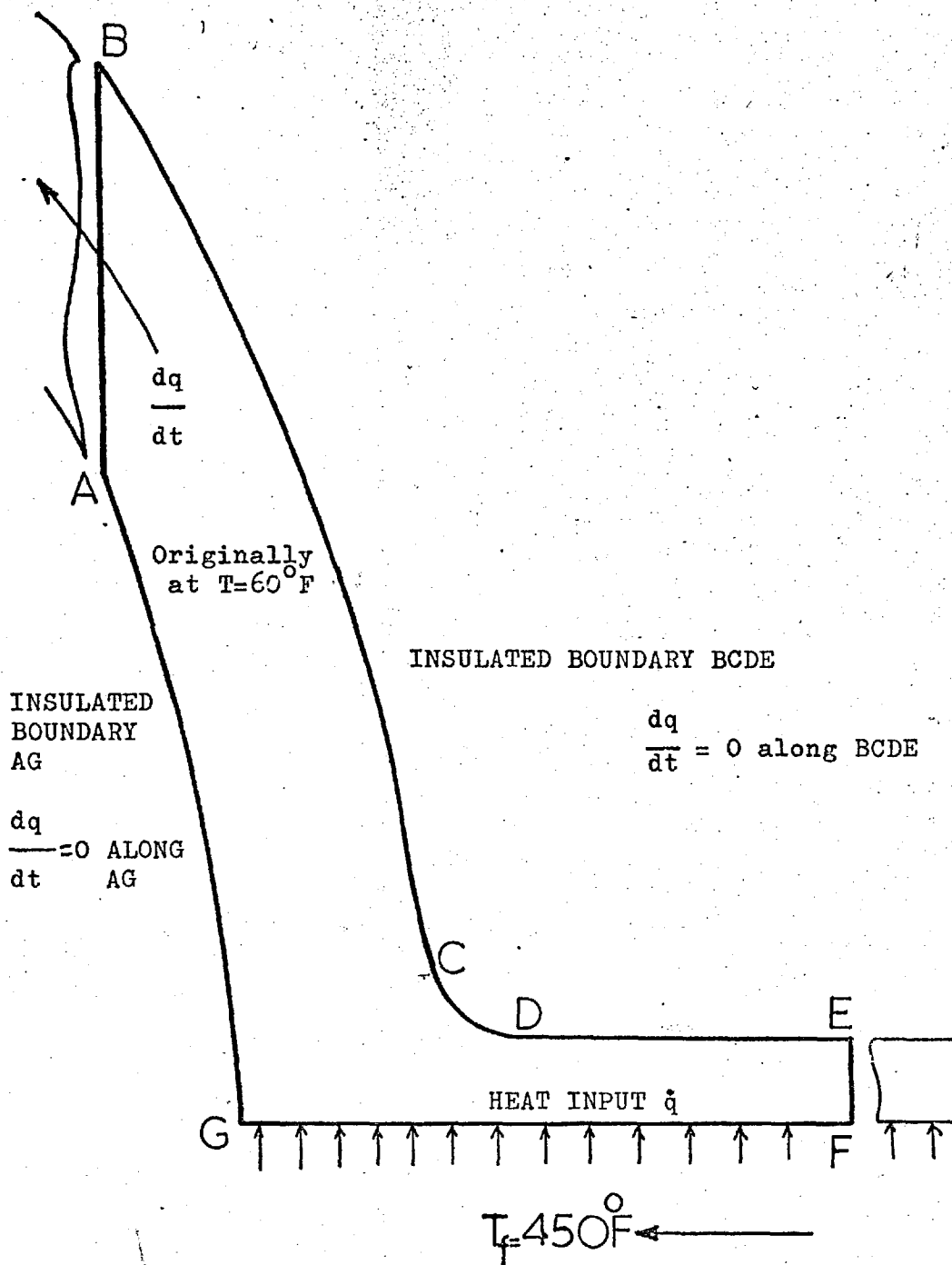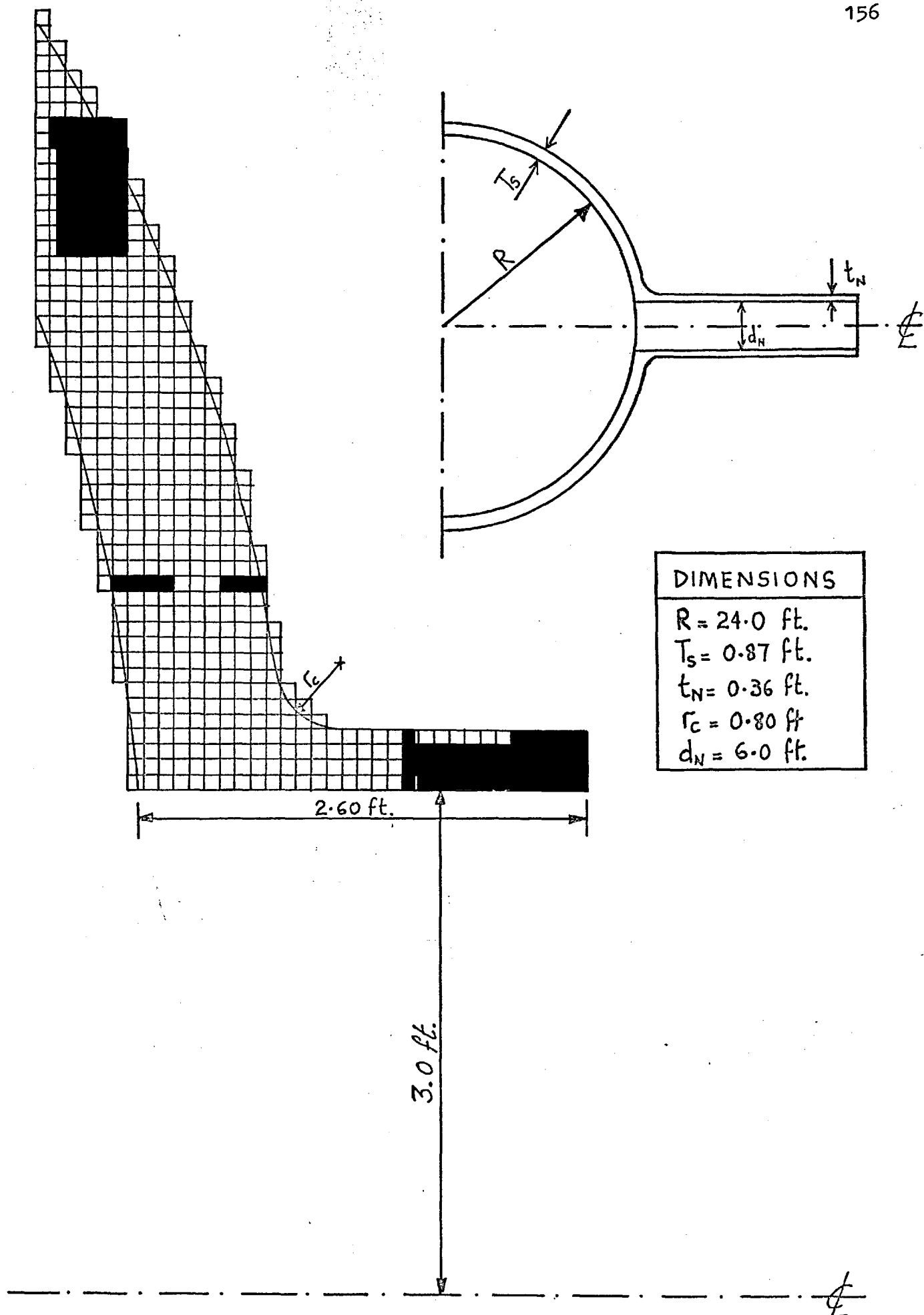
FIGURE A2.5.3 FINITE ELEMENT MESH

2.7 Computer Program TEFEL

The program is determining the temperature distribution at prescribed time intervals. Element and nodal temperatures are recorded every 3.0 minutes. The time increments may be increased or decreased to any value.

The variable data and main parameters in program TEFEL are designated as follows:

is   integer single

ia   integer array

rs   real single

ra   real array

Dimensions of the sections are in feet.

TEL   ra   temperature of the element

TEN   ra   temperature of nodes

NPI   ia ⎫

NPJ   ia ⎬ index of node adjacent to element

NPK   ia

NPM   ia ⎭

X   ra   x-co-ordinate of node

y   ra   y-co-ordinate of node

TFL   rs   temperature of fluid

TR   rs   temperature of structure at time = 0.0

MAXIT is   number of gaussian iterations

TIME   rs   time interval

TT   rs   time spacing

NEL   ia   number of elements

NNP   ia   number of nodes

L1 ⎫

L2 ⎬ is   working variables referring to elements

L3 ⎭

N1
N2 } is   working variables referring to nodes
N3

XC   rs   X co-ordinate of the centroid of element

YC   rs   Y co-ordinate of the centroid of element

X1   rs   X co-ordinate of point within element

Y1   rs   Y co-ordinate of point within element

XXX
YYY } rs   working variables

XX   X-spacing (Δy)

YY   rs   Y-spacing (Δy)

SNI
SNJ
} rs   shape functions
SNK
SNM

ITN   rs   iteration index

SUM   rs   iteration summation index

G   rs   Gaussian iteration variable

SFT   rs   time shape function

L8
} is   working variables referring to input data
L9

NNI
NNJ
} ia   index of node adjacent to node
NNK
NNM

The listing of the program is given in the next page.

```
      PROGRAM TEFEL(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      COMMON     TEL(451), TEN(515),
     1NPI(451),NPJ(451),NPK(451),NPM(451),
     2X(515),Y(515),X1(451),Y1(451),
     3NNI(515),NNJ(515),NNK(515),NNM(515) ,
     4SYMBOL(51),GG(41),TELL(22,41)
      COMMON BLANK
C     READ GRAPHING SYMBOLS AND BLANKS
      READ (5,708) (SYMBOL(J), J=1,51),BLANK
  708 FORMAT(52A1)
      DO 232  I=1,22
      DO 232  J=1,41
  232 TELL(I,J)=0.0
C     DEFINE CONSTANTS AND PARAMETERS
      TFL=450.
      TR=60.
      XX=0.5
      YY=0.5
      MAXIT=50
      TIME=0.0
      TT=0.0
      NEL=451
      NNP=515
C     READ ELEMENTS
      READ(5,8)    (LE,NPI(LE),NPJ(LE),NPK(LE),NPM(LE),L=1,NEL)
    8 FORMAT(5I5)
C     READ NODES
C     READ NODE CO-ORDINATES
      READ(5,10)(NP,NNI(NP),NNJ(NP),NNK(NP),NNM(NP),X(NP),Y(NP),N=1,NNP)
   10 FORMAT(5I5,2F5.4)
C     PERFORM INITIAL DISTRIBUTION OF TEMPERATURE AT TIME=0.0 MIN.
C     DEFINE BOUNDARY AND INTERIOR NODES AND ELEMENTS
C     ELEMENTS EXTERIOR
C     L1=17
C     L2=24
C     DO 2 L=L1,L2
C     TEL(L)=TFL
    2 CONTINUE
C     ELEMENTS INTERIOR
     -L3=1
      DO 3 L=L3,NEL
      TEL(L)=TR
    3 CONTINUE
C     NODES EXTERIOR
      N1=64
      N2=93
      DO 4 N=N1,N2
      TEN(N)=TFL
    4 CONTINUE
C     NODES INTERIOR
      DO 511 N=1,63
      TEN(N)=TR
  511 CONTINUE
      N3=94
      DO 5 N=N3,NNP
      TEN(N)=TR
    5 CONTINUE
      GO TO 33
  555 CONTINUE
      CALL DIN(TIME)
      WRITE (6, 82)
   82 FORMAT (1H1 //////
     1          57X, 18HINPUT ELEMENT DATA /
     2          57X, 18H------------------ /////
     3          3X, 40HELMT NPI    NPJ    NPK    NPM    TEMP
     4          5X, 40HELMT NPI    NPJ    NPK    NPM    TEMP
     5          6X, 29HELMT NPI    NPJ    NPK    NPM    TEMP /)
      L9 = (NEL + 2) / 3
      DO 33  L8 = 1, L9
      L3 = MIN0 (L8 + 2 * L9, NEL)
   83 WRITE(6,84)    (L,NPI(L),NPJ(L),NPK(L),NPM(L),TEL(L),L=L8,L3,L9)
   84 FORMAT(1H ,5I5,F6.1,15X,5I5,F6.1,15X,5I5,F6.1)
      WRITE (6, 85) (N, NNI(N),NNJ(N),NNK(N),NNM(N),TEN(N),X(N),Y(N),
     1N=1,NNP)
   85 FORMAT (1H1 //////
     1          58X, 15HINPUT NODE DATA /
     2          58X, 15H--------------- /////
     313X, 56HNODE  NNI  NNJ  NNK  NNM  TEMP              X              Y /
     4          (I16,X,4I5,F7.1,2E16.5))
C     START PERFORMING TIME STEPS.
C     TIME SHAPE FUNCTION IS DEFINED.
```

```
C       PARABOLIC FORM CF DISTRIBUTION IS ASSUMED IN EVERY ELEMENT
        Q=0.025
        TIME=3.0
        TT=3.0
    80  SFT=1.+Q*(TIME/TT)
C       CALCULATE THE ELEMENT TEMPERATURES
        DO 2  L=1,MEL
        I=NPI(L)
        J=NPJ(L)
        K=NPK(L)
        M=NPM(L)
C       CALCULATE THE CO-ORDINATES OF ELEMENT CENTROIDS
        XC   =(X(I)+X(J)+X(K)+X(M))/4
        YC   =(Y(I)+Y(J)+Y(K)+Y(M))/4.
        X1   =XC
        Y1   =YC
C       CALCULATE DIRECTION SHIFTS OF AXIS TO CENTROIDS
        XXX=0.0
        YYY=0.0
C       CALCULATE SHAPE FUNCTIONS
        SNI=(1./4.)*(1.+XXX   )*(1.+YYY   )
        SNJ=(1./4.)*(1.-XXX   )*(1.+YYY   )
        SNK=(1./4.)*(1.-XXX   )*(1.-YYY   )
        SNM=(1./4.)*(1.+XXX   )*(1.-YYY   )
C       SET ITERATION COUNTER TO 1
        ITN=1
    21  SUM=0.0
        A=SNI*TEN(I)+SNJ*TEN(J)+SNK*TEN(K)+SNM*TEN(M)
        IF(A.EQ.TEL(L)) GO TO 20
        G=1.84*(A-TEL(L))
        TEL(L)=TEL(L)+G
        SUM=SUM+ABS(G)
        ITN=ITN+1
        IF(ITN.GT.MAXIT)  GO TO  22
        GO TO 21
    22  CONTINUE
        TEL(L)=TEL(L)*SFT
        IF(TEL(L).GT.450)    TEL(L)=450.
    20  CONTINUE
C       CALCULATE THE NODES TEMPERATURES
        DO  25  N=1,NNP
        I=NNI(N)
        J=NNJ(N)
        K=NNK(N)
        M=NNM(N)
C       SET ITERATION COUNTER TO ONE
        ITN=1
    27  SUM=0.0
        B=(TEN(I)+TEN(J)+TEN(K)+TEN(M))/4.
        IF(B.EQ.TEN(N)) GO TO 30
        G=1.84*(B-TEN(N))
        TEN(N)=TEN(N)+G
        SUM=SUM+ABS(G)
        IF(ITN.GT.MAXIT)  GO TO  30
        ITN=ITN+1
        GO TO 27
    30  CONTINUE
        TEN(N)=TEN(N)+SFT
        IF(TEN(N).GT.450)    TEN(N)=450.
    25  CONTINUE
    33  CONTINUE
        IF(TIME.EQ.0) GO TO 555
        CALL DIN(TIME)
        TIME=TIME+TT
        IF(TEN(1).EQ.450)   GO TO 66
        GO TO 80
    66  CONTINUE
        STOP
        END
        SUBROUTINE DIN(TIME)
        COMMON      TEL(451), TEN(515),
       1NPI(451),NPJ(451),NPK(451),NPM(451),
       2X(515),Y(515),X1(451),Y1(451),
       3NNI(515),NNJ(515),NNK(515),NNM(515)  ,
       4SYMBOL(51),GG(41),TELL(22,41)
        COMMON BLANK
C
C
C       PLOT TEMPERATURES
C
C
        WRITE(6,900)
   900  FORMAT(1H1 ///////////)
        WRITE(6,47)      TIME
    47  FORMAT(58X,5HTIME=,F5.1)
```

```
      WRITE(6,477)
  477 FORMAT(5(/))
C
C
      I=22
  882 CONTINUE
      DO  883 J=1,41
      IF(TELL(I,J).EQ.0) GO TO 889
      K1=TELL(I,J)/9 +1
      IF(K1.GT.51) GO TO 889
C     PLACE APPROPRIATE SYMBOL IN APPROPRIATE POSITION
C
C
      GG(J)=SYMBOL(K1)
      GO TO 883
  889 GG(J)=BLANK
  883 CONTINUE
  891 CONTINUE
C     PRINT PROFILES
C
C
      WRITE(6,893)  GG
  893 FORMAT(45X,41A1)
      I=I-1
      IF(I.NE.0)  GO TO 882
      RETURN
      END
```

2.8 Thermal Distribution Results

The thermal profiles are plotted for the section for every 3.0 minutes. Temperatures are in degrees $^{\circ}F$. The method of finite element is flexible and the temperatures can be calculated at any point within the elements by defining the x and y co-ordinates. This is not possible with the finite difference method where values can only be obtained at the nodal points of the grid, unless an extrapolation procedure is performed which is not very accurate. On the other hand finite difference method is computationally faster and converges faster too.

Once the temperatures are determined at any point defined by x-y, the stress can be calculated by the methods described in Chapter 5.

In the next pages the temperature distributions are shown for specified time intervals of 3.0 minutes for the first 15 minutes and at various time intervals up to 100 minutes when the structure attains uniform temperature of $450^{\circ}F$.

INSULATED BOUNDARY

$$\frac{dq}{dt} = 0$$

60 °F

TEMPERATURES IN DEGREES °F

time = 0.00 hr.

€

time = 0.050 hr. (3 min.)

70

T_f = 450 ^{o}F

time = 0.010 hr. (6 min.)

70

time = 0.15 hr. (9 min.)

95

110

150
165
220
380

¢

time = 0.20 hr. (12 min.)

time = 0.25 hr. (15 min.)
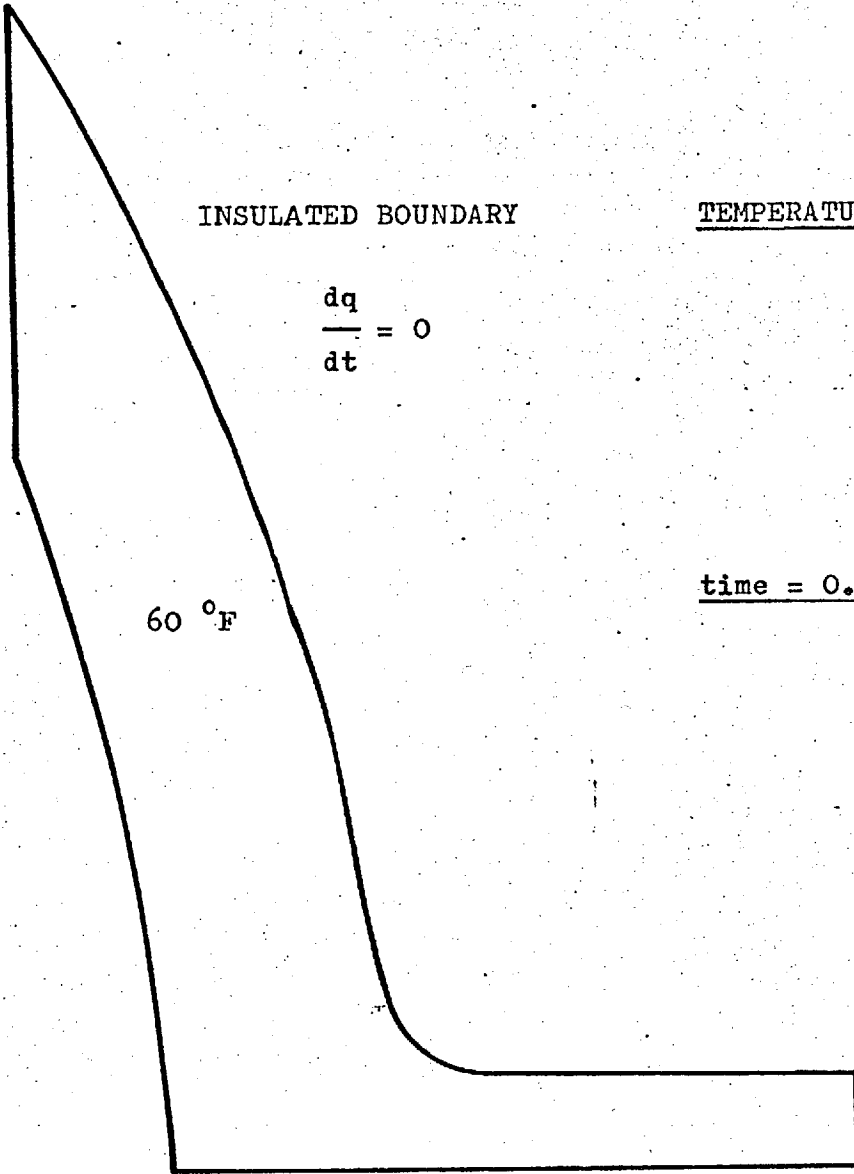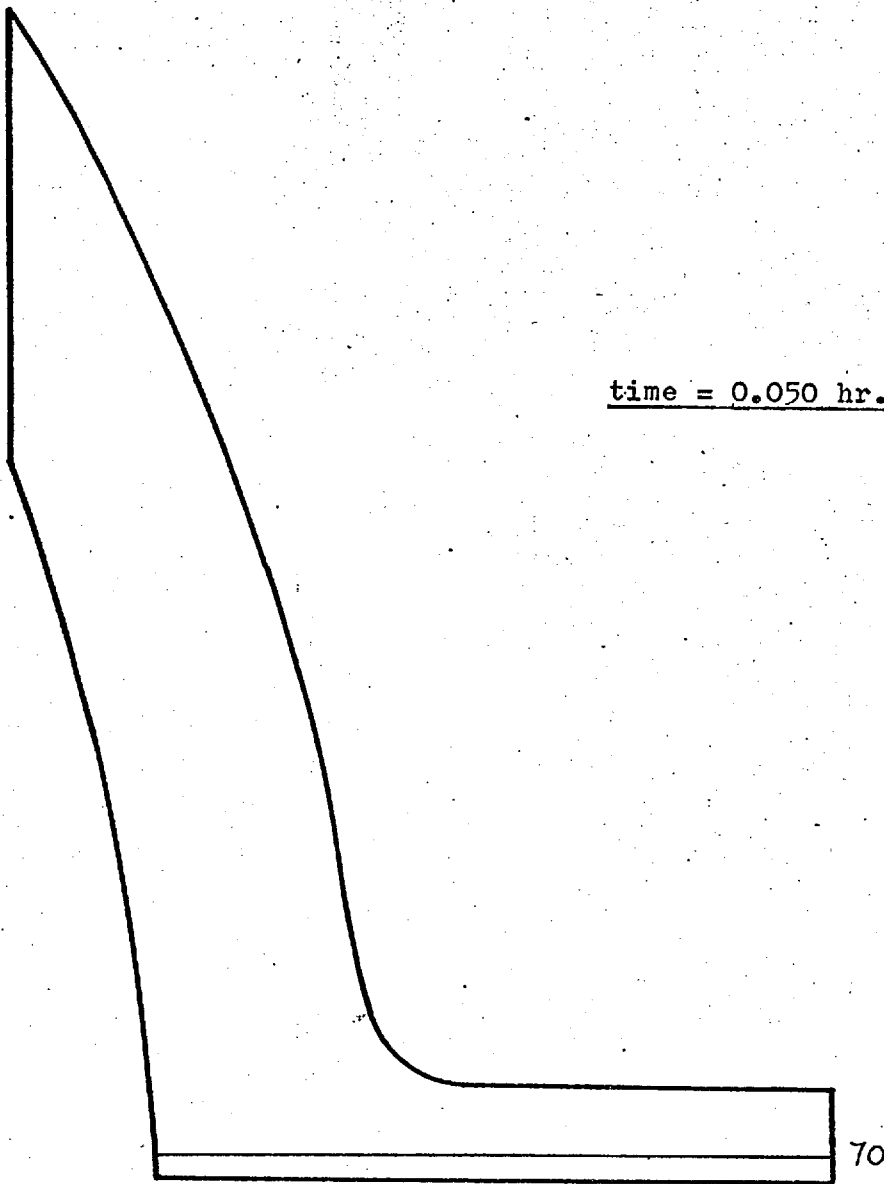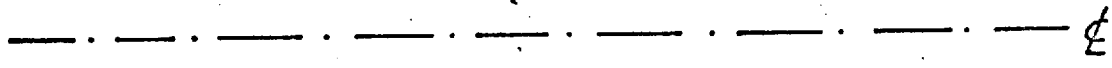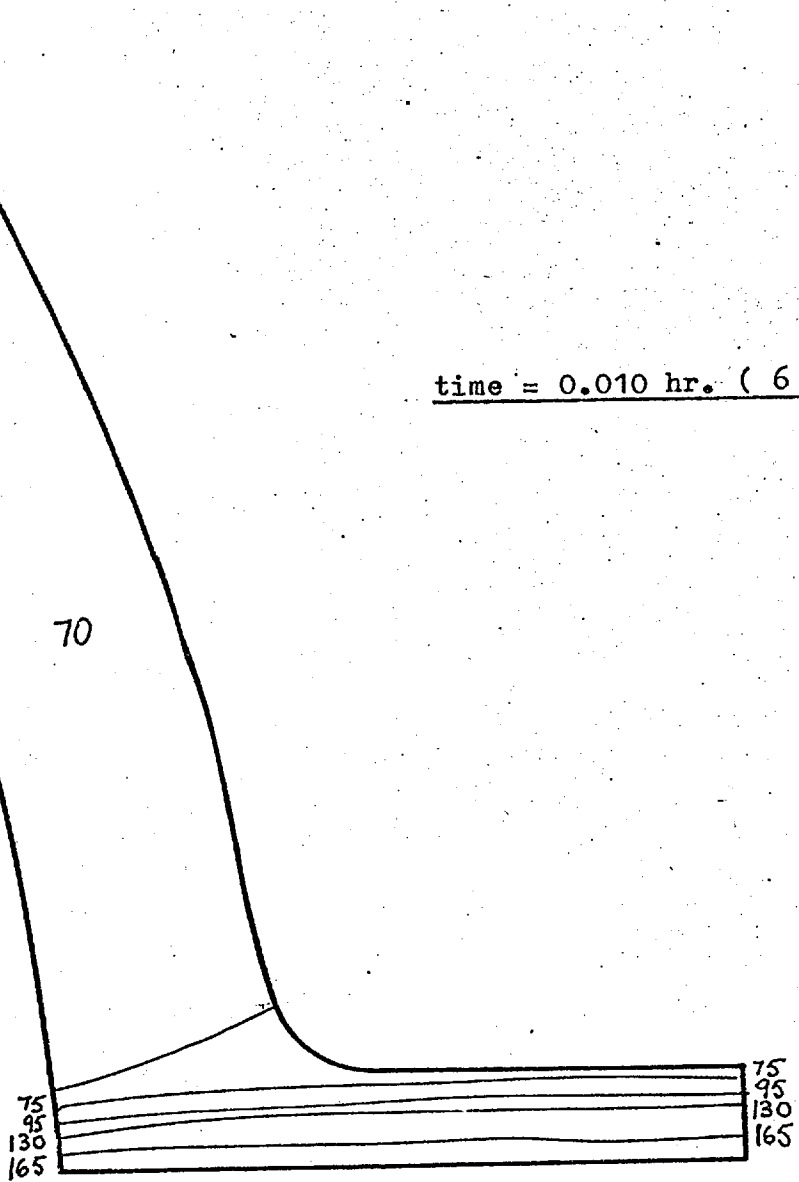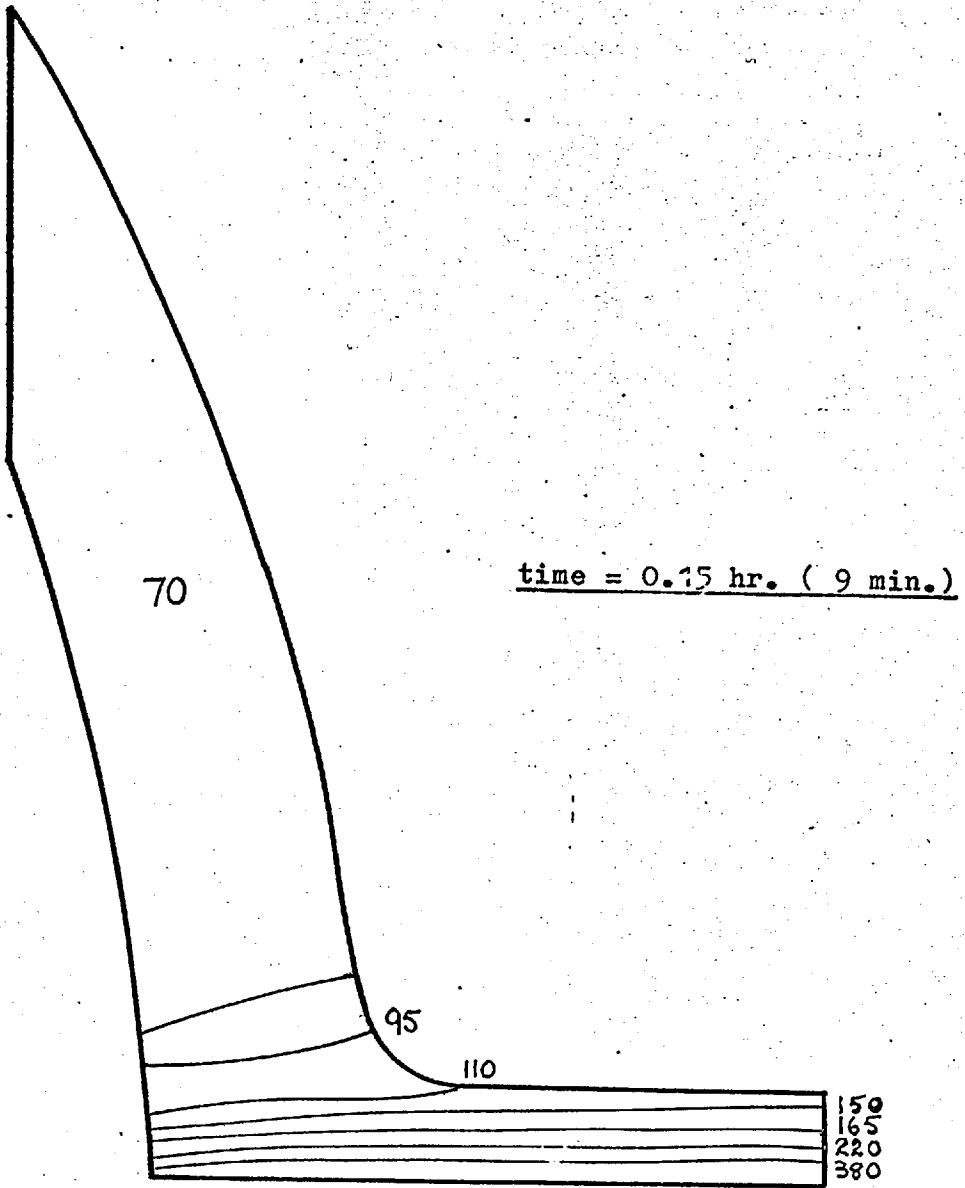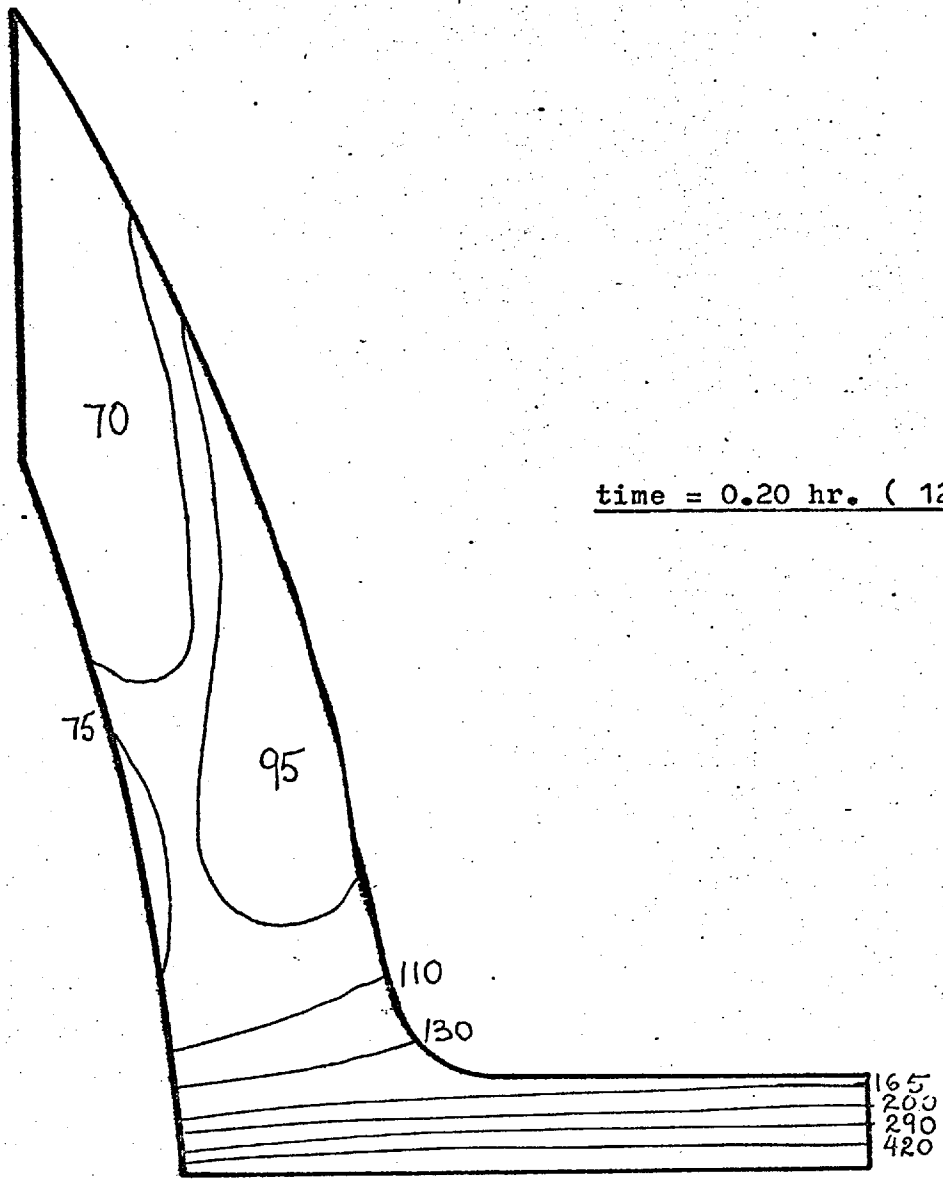
time = 0.30 hr. (20 min.)
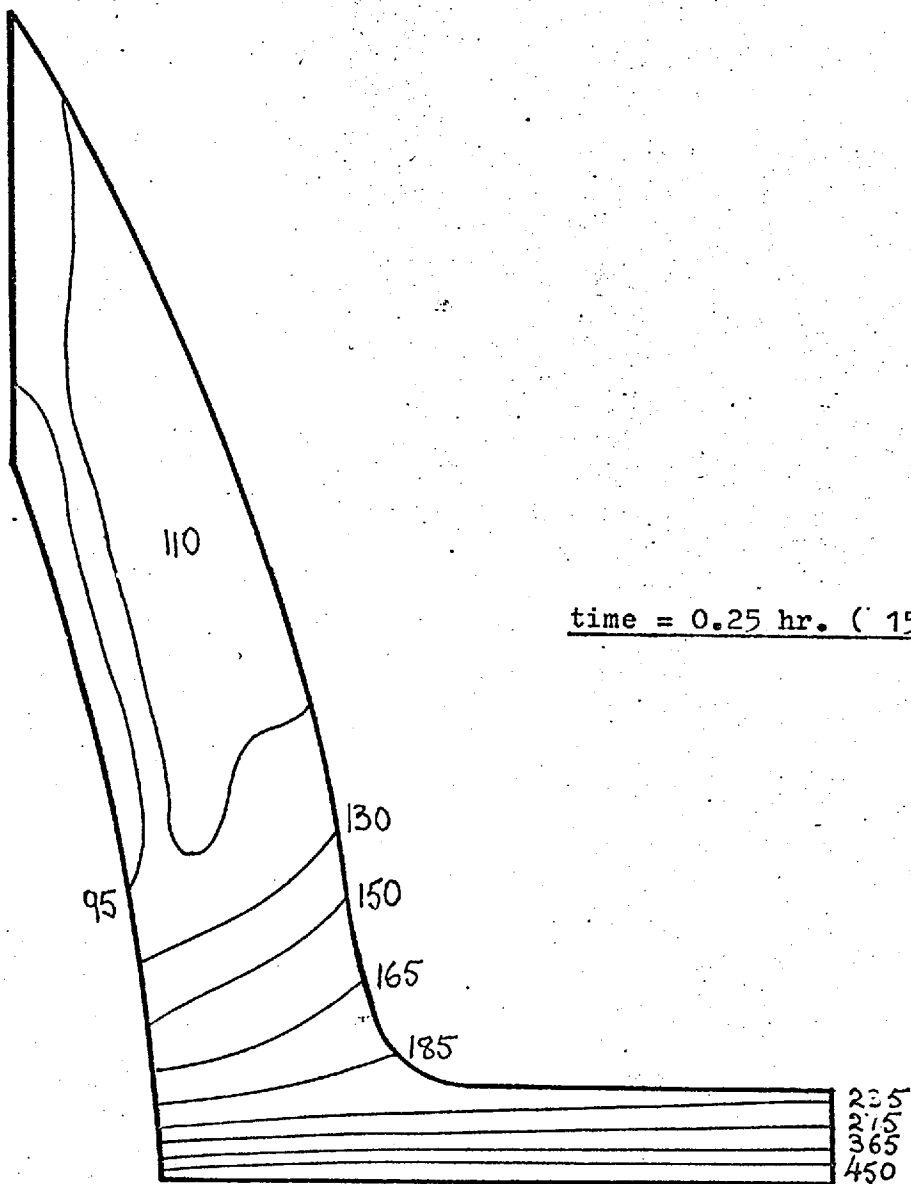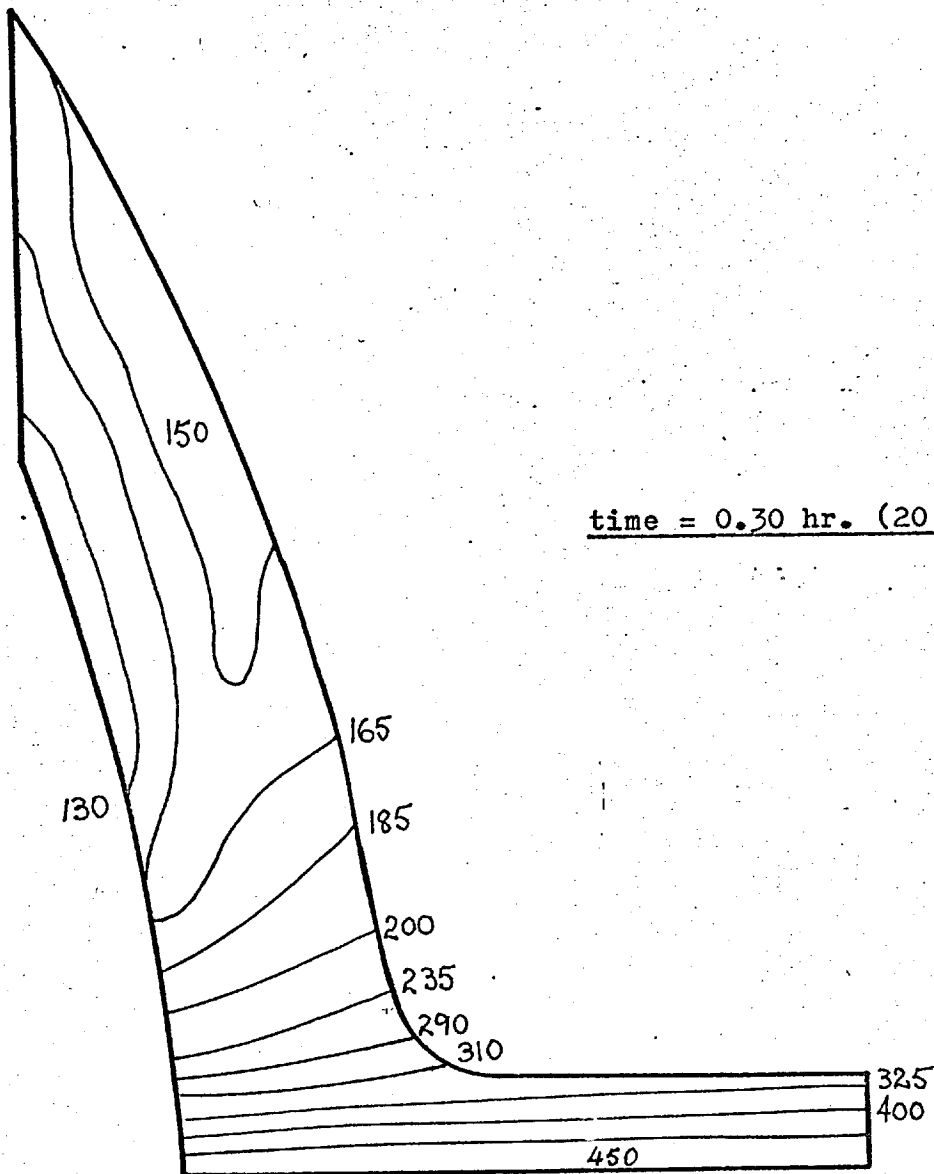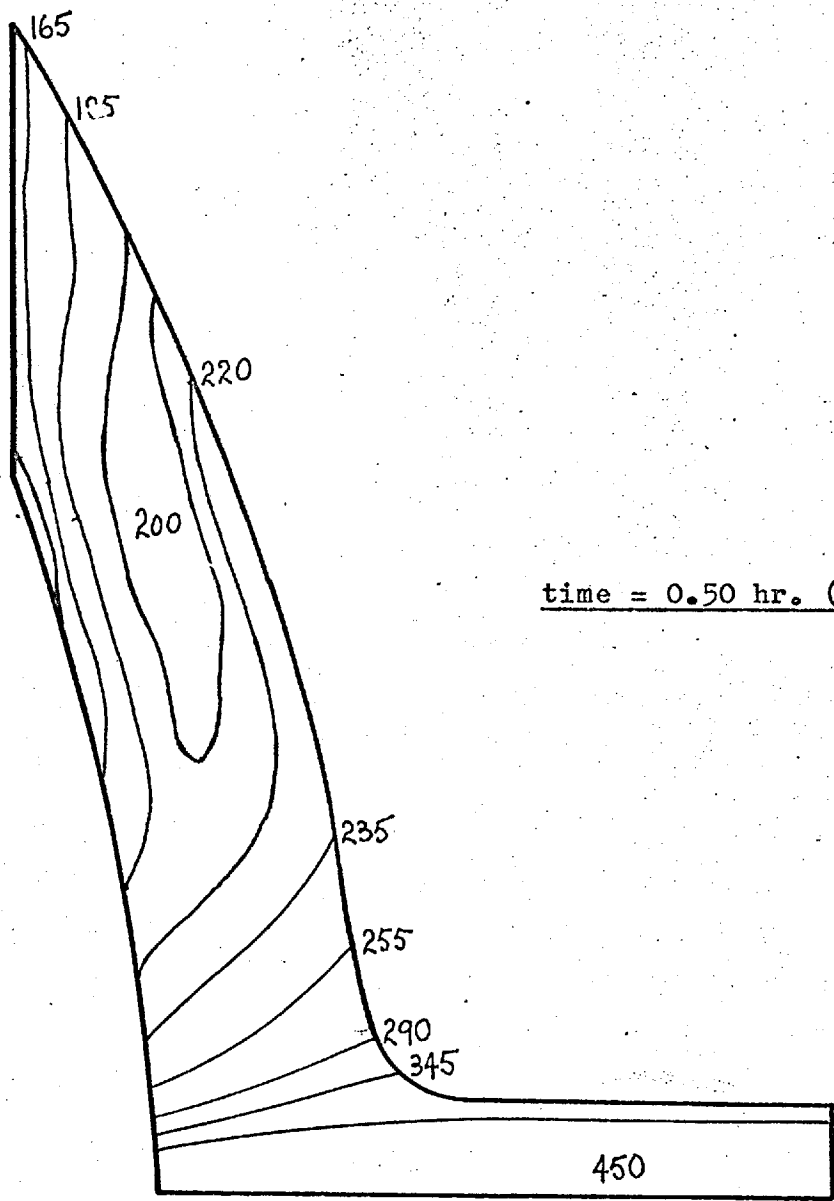
time = 0.50 hr. (30 min.)

time = 0.75 hr. (45 min.)

345
365
400
420
435

450

time = 1.16 hr. (70 min.)

₵

435

450

time = 1.50 hr. (90 min.)

₵
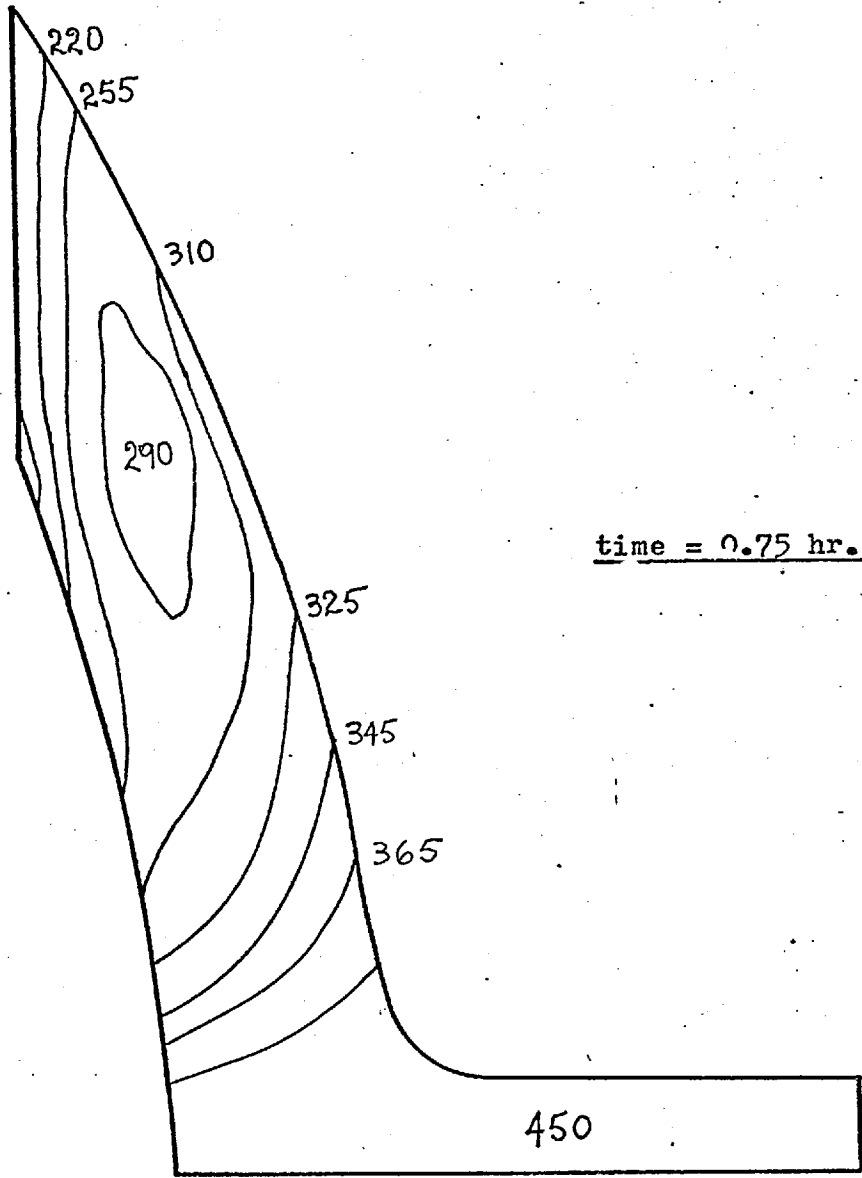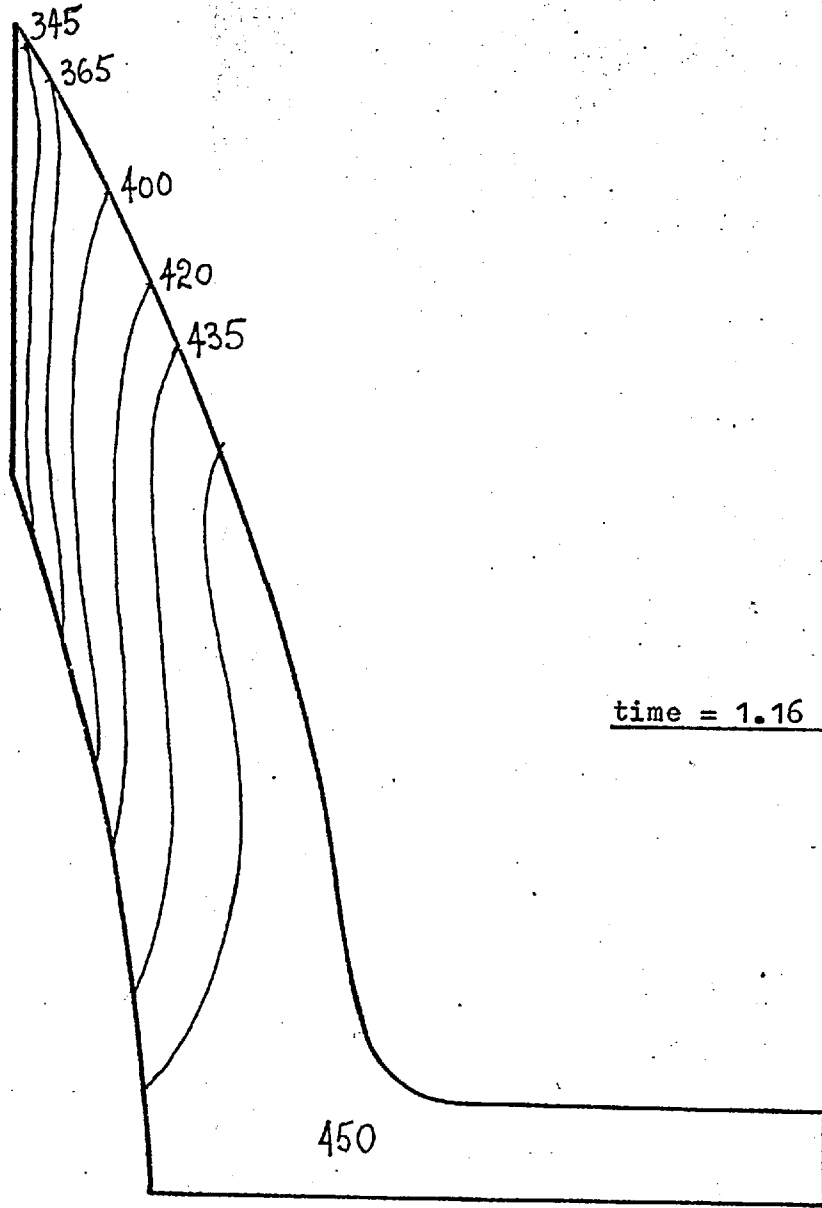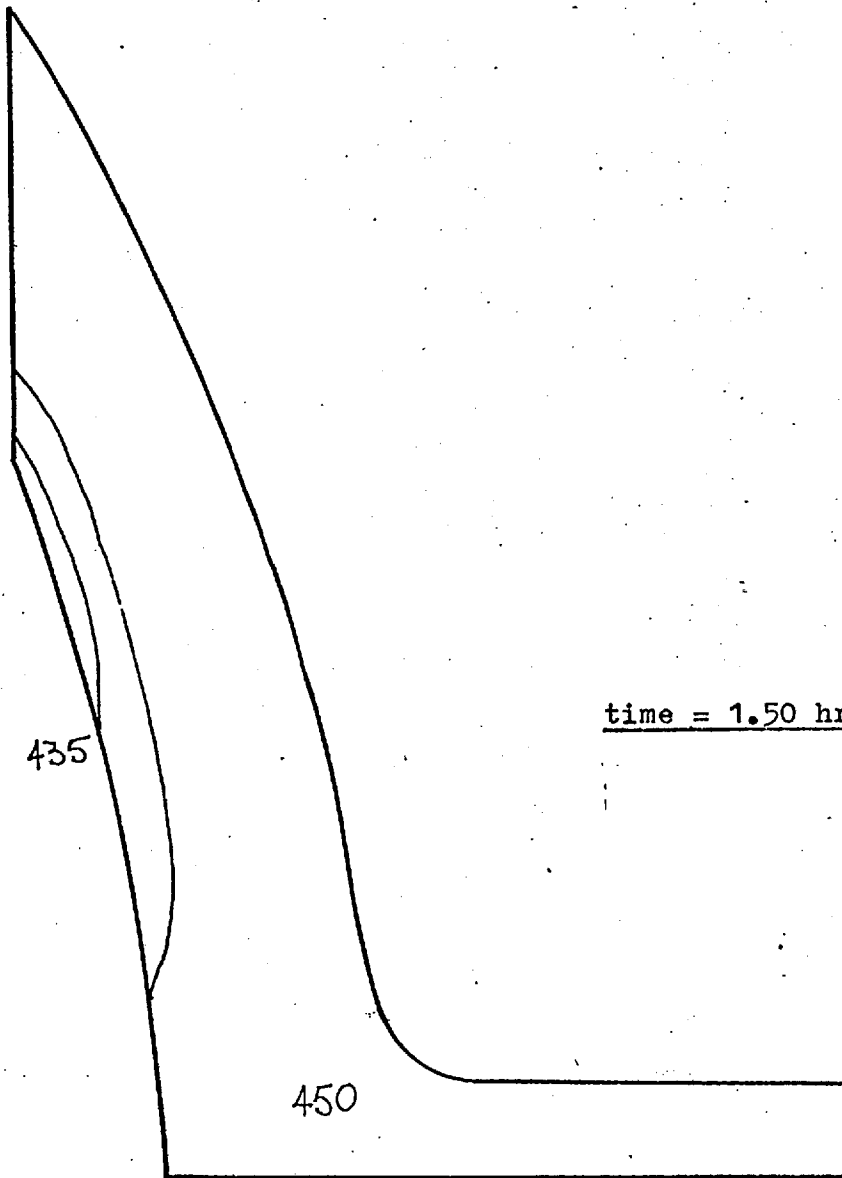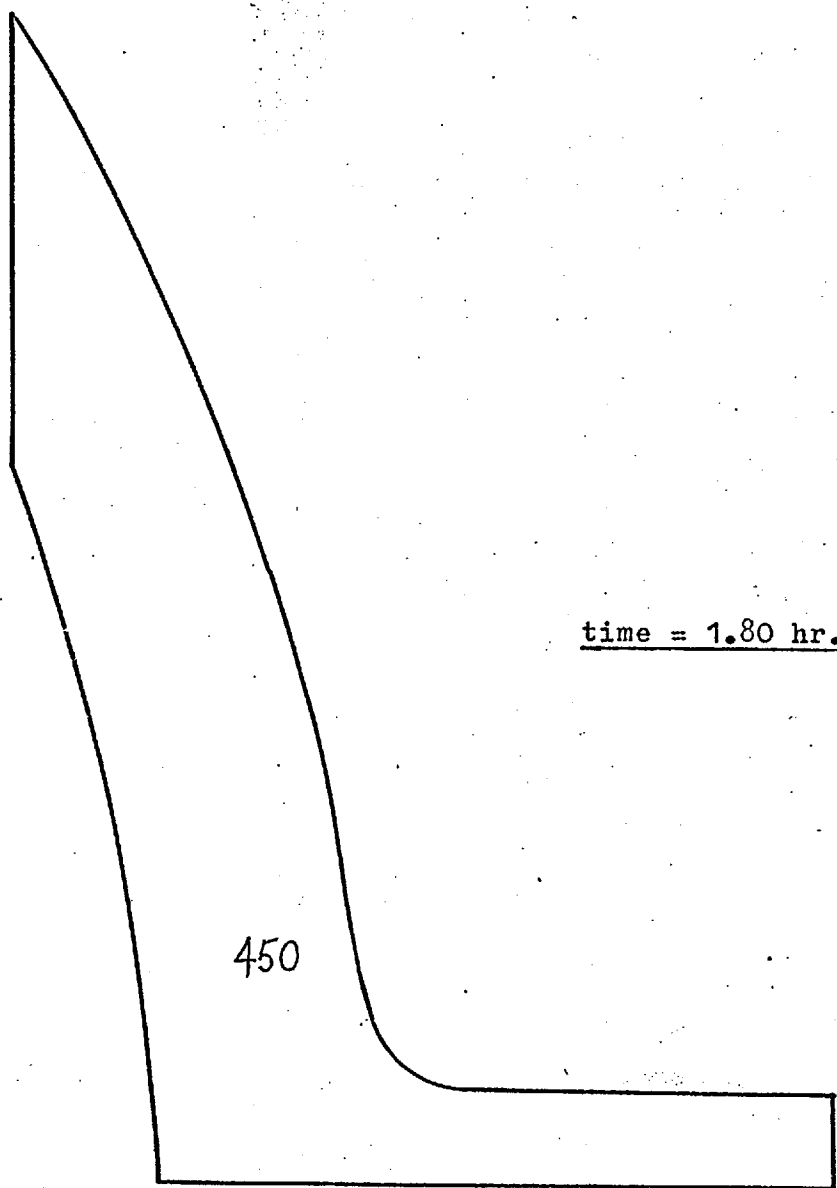
time = 1.80 hr. (108 min)

450

¢

APPENDIX 3

COMPUTER PROGRAMS: TRAN 1, TRAN 2, TRAN 3. SUBROUTINE PLOT

3.1 Data designations for Programs TRAN1, TRAN2, TRAN3 and Subroutine PLOT

The data are designated as follows:

INTEGER	SINGLE	is
INTEGER	ARRAY	ia
REAL	SINGLE	rs
REAL	ARRAY	ra

The main parameters have the following names:

MAXIT	is	number of iterations (for grid)
NMAX	is	number of iterations (for time)
W	rs	dimension of nozzle-shell
Z	rs	dimension of nozzle-shell
V	rs	dimension of nozzle-shell
RA	rs	dimension of nozzle-shell
GRID	rs	size of grid increments
R	rs	generalised corner radius
COCON	rs	coefficient of thermal conductivity of metal
HTRCOV	rs	heat transfer coefficient for vessel
HTRCON	rs	heat transfer coefficient for nozzle
TIN	rs	temperature of fluid ($^{\circ}$F)
TIS	rs	initial temperature of structure ($^{\circ}$F)
RO	rs	density of metal
U	ia	nodal temperatures

G	ia	symbol for subroutine PLOT
I1	ia	generalised co-ordinate
I2	ia	generalised co-ordinate
I3	ia	generalised co-ordinate
J1	ia	generalised co-ordinate
J2	ia	generalised co-ordinate
J3	ia	generalised co-ordinate
DIFF	ra	difference of nodal temperature
SUM	ra	sum of differences of nodal temperatures for iteration process
ITN	ra	number of iterations
OMEGA	rs	convergence constant
TIME	ra	time increments
PLOT	SUBROUTINE	this subroutine plots temperature profiles
BLANK	ra	symbol designations for subroutine PLOT
L10	ia	generalised co-ordinate for boundary calculations
L15	ia	generalised co-ordinate for boundary calculations
N	ia	time increments for all nodal points
L18	ia	generalised co-ordinate for boundary calculations

```
      JOB(UMEMM19,J13,T500,CM50C00)               OIKONOMIDES.C.N.      171
      MNF(T,D,R=2,F=1,P=5000)
         PROGRAM TRAN1(INPUT,OUTPUT,TAPE5=INPUT,TAPE6,OUTPUT)
      C
      C     PROGRAM TRAN1 DETERMINES THE TRANSIENT THERMAL DISTRIBUTIONS
      C     FOR THE NOZZLE-VESSEL SECTION FOR CASE A BY SOLVING THE
      C     TWO-DIMENSIONAL UNSTEADY-STATE CONDUCTION DIFFERENTIAL
      C     EQUATION AND BY APPLYING THE APPROPRIATE BOUNDARY CONDITIONS.
      C
         COMMON U(70,100),
        1SYMBOL(51),
        1G(100),
        1R,I1,I2,I3,J1,J2,J3,Z,GRID
         COMMON BLANK
      C     READ GRAPHING SYMBOLS AND BLANK
         READ(5,707) (SYMBOL(J), J=1,49),BLANK
      707 FORMAT(52A1)
      C
      C
      C
      C
         MAXIT=50
         NMAX=75
         MAX=10
         W=4.
         Z=10.
         V=7.
         S=1.
         RA=2.
         GRID=0.1
         R=RA/GRID
         EPS=3.
         SIGMA=1.0.
         COCON=9.75/(12.*3600.)
         HTRCCV= 964.3/(3600.*144.)
         HTRCON= 906.1/(3600.*144.)
         TIN=450.
         TIS=60.
         RO=490./(12.**3)
      C
      C
         I1=V/GRID
         I2=(S+RA)/GRID
         I3=S/GRID
         J1=Z/GRID
         J2=(W+RA)/GRID
         J3=W/GRID
      C
      C     AT INITIAL STATE THE TEMPERATURE OF THE MATERIAL IS DEFINED
      C     THIS MAY VARRY
      C
      C     VESSEL PART CALCULATIONS--BOUNDARY--
      C
      C
         J=1
         DO 11  I=1,I1
      11 U(I,J)=TIN
      C
         I=1
         DO 12  J=1,J1
      12 U(I,J)=TIN
      C
         DO 99 I=2,I1
         DO 99    J=2,J1
      99 U(I,J)=TIS
      C
      C     SET ITERATION COUNTER TO 1
         ITN=1
      C     START AN ITERATION BY SETTING SUM OF RESIDUES TO ZERO
      9 SUM=0.0
         J=1
         L15=I1-1
         DO 15   I=2,L15
         U(1,1)=U(2,1)
         U(I1,1)=U(I1-1,1)
         DIFF=HTRCO/*GRID*TIN/(HTRCO/*GRID+2.*COCON)
        1+COCON*(2.*U(I,J+1)+U(I+1,J)+U(I-1,J))
        1/(2.*HTRCO/*GRID+4.*COCON)-U(I,J)
         U(I,J)=U(I,J)+DIFF
      15 SUM=SUM+ABS(DIFF)
         IF(SUM.LE.(EPS*I1)) GO TO 16
         ITN=ITN+1
         IF(ITN.GT.MAXIT) GO TO 16
      C     NOT CONVERGED--GO BACK FOR ANOTHER ITERATION
         GO TO 9
```

```
   16 CONTINUE
      ITN=1
   17 SUM=0.0
      I=1
      L18=J1-1
      DO 18   J=2,L13
      U(1,J1)=U(1,J1-1)
      DIFF=HTRCON+GRIC*TIN/(HTRCON*GRIC+2.*COCON)
     1+COCON*(2.*U(I+1,J)+U(I,J-1)+U(I,J+1))
     1/(2.*HTRCON*GRID+4.*COCON)-U(I,J)
      U(I,J)=U(I,J)+DIFF
   18 SUM=SUM+ABS(DIFF)
      IF(SUM.LE.(EPS*J1)) GO TO 19
      ITN=ITN+1
      IF(ITN.GT.MAXIT)   GO TO 19
      GO TO 17
   19 CONTINUE
C
C
C
C
C
C      VESSEL AND NOZZLE WALL CALCULATIONS
C
C      THIS IS TIME ITERATION
      ITN=1
  300 SUM=0.0
      L1=I1-1
      L10=J3-1
      DO 100   I=2,L1
      DO 100   J=2,L18
      U(I1,J)=U(I1-1,J)
      U(I,J1)=U(I,J1-1)
      DIFF=1.8*((U(I-1,J)+U(I+1,J)+U(I,J+1)+U(I,J-1))/4.-U(I,J))
      U(I,J)=U(I,J)+DIFF
  100 CONTINUE
      ITN=ITN+1
      IF(ITN.GT.MAX) GO TO 101
      GO TO 300
  101 CONTINUE
      DO 120   J=J2,L18
      DO 120   I=I2,L1
  120 U(I,J)=U(J2,I2)
  130 CONTINUE
      N=1
      WRITE(5,650) N
  650 FORMAT(2X,*N=*,I2/)
      I=I1
  600 CONTINUE
      WRITE(6,2999) I
 2999 FORMAT(2X,*I=*,I2/)
      WRITE(6,3000)  (J,J=1,10)
 3000 FORMAT(2X,10(I2,10X))
      WRITE(6,3200)
 3200 FORMAT(1(/))
      WRITE(6,3300)  (U(I,J),J=1,J1)
 3300 FORMAT((1X,10(F5.1,7X)))
      WRITE(6,3400)
 3400 FORMAT(1(/))
      I=I-1
      IF(I.EQ.0) GO TO 1500
      GO TO 600
      CALL PLOT(N)
 1500 CONTINUE
      DO 1000 N=1,NMAX
      DO 112   I=1,I1
      J=1
  112 U(I,J)=TIN
      DO 113   J=1,J1
      I=1
  113 U(I,J)=TIN
      ITN=1
  310 SUM=0.0
      DO 200   I=2,L1
      DO 200   J=2,L18
      U(I1,J)=U(I1-1,J)
      U(I,J1)=U(I,J1-1)
      DIFF=1.8*((U(I-1,J)+U(I+1,J)+U(I,J+1)+U(I,J-1))/4.-U(I,J))
      U(I,J)=U(I,J)+DIFF
  200 CONTINUE
      ITN=ITN+1
      IF(ITN.GT.MAX) GO TO 201
      GO TO 310
  201 CONTINUE
      DO 250   J=J2,L18
      DO 250   I=I2,L1
```

```
  250 U(I,J)=U(I2,J2)
      IF((N+2)/3.NE.N/3) GO TO 1000
      WRITE(6,550) N
  550 FORMAT(2X,-N=-,I2/)
      I=I1
      JJ=J3
  500 CONTINUE
      WRITE(6,1999) I
 1999 FORMAT(2X,-I=-,I2/)
      WRITE(6,2000) (J,J=1,10)
 2000 FORMAT(2X,10(I2,10X))
      WRITE(6,2200)
 2200 FORMAT(1(/))
      WRITE(6,2300) (U(I,J),J=1,JJ)
 2300 FORMAT((1X,10(F5.1,7X)))
      WRITE(6,2400)
 2400 FORMAT(1(/))
      I=I-1
      IF(I.LT.I2.AND.I.GT.I3)    GO TO 670
      IF(I.LE.I3,   JJ=J1
      IF(I.EQ.0) GO TO 145
      GO TO 500
  670 C=FLOAT(J2)-SQRT(R**2-(FLOAT(I-I2)**2))
      JJ=INT(C+0.5)
      GO TO 500
  145 CONTINUE
  145 CALL PLOT(N)
 1000 CONTINUE
      STOP
      END
      SUBROUTINE PLOT(N)
      COMMON U(70,100),
     1SYMBOL(51),
     1G(100),
     1R,I1,I2,I3,J1,J2,J3,Z,GRID
      COMMON BLANK
C
C
C     FROM HERE THE TEMPERATURES ARE PLOTTED
C
C
C
C     MOVE TO A NEW PAGE
      WRITE(6,845)
  845 FORMAT(1H1)
      IF(N.GE.60) GO TO 852
      GO TO 853
  852 TIME=FLOAT(N)/60.
  853 WRITE(6,847) N,TIME
  847 FORMAT(*N=*,I2,10X,*TIME=*,F10.2)
C     SET ROW INDEX I TO I1 FOR TOP ROW
      I=I1
C     RUN THROUGH COMPLETE ROW
  802 CONTINUE
      IF(I.LT.I2.AND.I.GT.I3) GO TO 808
      GO TO 806
  808 CONTINUE
      C=FLOAT(J2)-SQRT(R**2-(FLOAT(I-I2)**2))
      J1=INT(C+0.5)
  806 CONTINUE
      DO 901 J=1,J1
C     CHECK IF POINT IS OUTSIDE LIMIT
      IF(I.LT.I2.AND.I.GT.I3.AND.J.GT.J3.AND.J.LT.J2) GO TO 809
      IF(I.GE.I2.AND.J.GT.J3) GO TO 880
  809 CONTINUE
C     NOT OUTSIDE--SELECT LETTER OR BLANK TO REPRESENT TEMPERATURE
      K1=U(I,J)/0 +1.
      IF(K1.GT.51) GO TO 880
C     PLACE APPROPRIATE SYMBOL IN APPROPRIATE POSITION IN LINE
      G(J)=SYMBOL(K1)
      GO TO 901
C     INSERT BLANK IF OUTSIDE LIMIT
  880 G(J)=BLANK
  901 CONTINUE
      IF(I.LT.I3) GO TO 807
      IF(I.GE.I2) GO TO 902
      L808=Z/GRID
      L809=INT(0+1.)
      DO 807  J=L808,L809
      G(J)=BLANK
  807 CONTINUE
      J1=Z/GRID
  902 CONTINUE
C     WRITE ENTIRE LINE
  905 WRITE(6,910) G
```

```
 C  910 FORMAT(1H,1X,100A1)
        MOVE TO THE NEXT ROW
        I=I-1
 C      CHECK FOR COMPLETION
        IF(I.NE.0) GO TO 802
    920 CONTINUE
        RETURN
        END
```

```
     JOB(UMEMM18,J10,T300,CM40000,LC3072)        OIKCNCMIDES.C.N
     MAP(PART)
     MNF(T,C,R=2,E=1,P=5000)                                           175
         PROGRAM TRAN2(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
     C
     C     PROGRAM TRAN2 DETERMINES THE TRANSIENT THERMAL DISTRIBUTIONS
     C     FOR THE NOZZLE-VESSEL SECTION FOR CASE 2 BY SOLVING THE
     C     TWO-DIMENSIONAL UNSTEADY-STATE CONDUCTION DIFFERENTIAL
     C     EQUATION AND BY APPLYING THE APPROPRIATE BOUNDARY CONDITIONS.
     C
         COMMON U(70,100),
        1SYMBOL(51),
        1G(100),
        1R,I1,I2,I3,J1,J2,J3,Z,GRID
         COMMON BLANK
     C   READ GRAPHING SYMBOLS AND BLANK
         READ(5,707)   (SYMBOL(J), J=1,51),BLANK
     707 FORMAT(52A1)
     C
     C
     C
         MAXIT=50
         NMAX=75
         MAX=10
         S=1.
         Z=10.
         V=7.
         W=4.
         GRID=0.1
         RA=1.5
         R=RA/GRID
         SIGMA=10.
         COCON=9.75/(12.*3600.)
         HTRCON=905.1/(3600.*144.)
         TIN=450.
         TIS=60.
         RO=490./(12.**3)
     C
     C
         I1=V/GRID
         I2=(S+RA)/GRID
         I3=S/GRID
         J1=Z/GRID
         J2=(W+RA)/GRID
         J3=W/GRID
     C
     C     AT INITIAL STATE THE TEMPERATURE OF THE MATERIAL IS DEFINED
     C     THIS MAY VARRY
     C
     C     VESSEL PART CALCULATIONS--BOUNDARY--
     C
     C
         J=1
         DO 11   I=2,I1
     11 U(I,J)=TIS
     C
         I=1
         DO 12   J=1,J1
     12 U(I,J)=TIN
     C
         DO 99 I=2,I1
         DO  99    J=2,J1
     99 U(I,J)=TIS
     C
     C
     C     SET ITERATION COUNTER TO 1
     C
     C     NOZZLE PART CALCULATIONS-BOUNDARY-
     C
         ITN=1
     17 SUM=0.0
         I=1
         L18=J1-1
         DO 18   J=2,L18
         U(1,J1)=U(1,J1-1)
         U(1,1)=U(1,2)
         DIFF=HTRCON*GRID*TIN/(HTRCON*GRID+2.*COCON)
        1+COCON*(2.*U(I+1,J)+U(I,J-1)+U(I,J+1))
        1/(2.*HTRCON*GRID+4.*COCON)-U(I,J)
         U(I,J)=U(I,J)+DIFF
     18 SUM=SUM+ABS(DIFF)
         ITN=ITN+1
         IF(ITN.GT.MAXIT)   GO TO 19
         GO TO 17
```

```
   1  CONTINUE
C
C
C
C     VESSEL-NOZZLE WALL CALCULATIONS
C
C
C     THIS IS TIME ITERATION
      ITN=1
  300 SUM=0.0
      L1=I1-1
      L10=J3-1
      DO 100   I=2,L1
      DO 100   J=2,L18
      U(I1,J)=U(I1-1,J)
      U(I,J1)=U(I,J1-1)
      U(I,1)=U(I,2)
      DIFF=1.8*((U(I-1,J)+U(I+1,J)+U(I,J+1)+U(I,J-1))/4.-U(I,J))
      U(I,J)=U(I,J)+DIFF
  100 CONTINUE
      ITN=ITN+1
      IF(ITN.GT.MAX) GO TO 101
      GO TO 300
  101 CONTINUE
      DO 120   J=J2,L18
      DO 120   I=I2,L1
  120 U(I,J)=U(J2,I2)
  130 CONTINUE
      N=1
      CALL PLOT(N)
      WRITE(6,650) N
  650 FORMAT(2X,-N=-,I2/)
      I=I1
  600 CONTINUE
      WRITE(6,2999) I
 2999 FORMAT(2X,-I=-,I2/)
      WRITE(6,3000)  (J,J=1,10)
 3000 FORMAT(2X,10(I2,10X))
      WRITE(6,3200)
 3200 FORMAT(1(/))
      WRITE(6,3300)  (U(I,J),J=1,J1)
 3300 FORMAT((1X,10(F5.1,7X)))
      WRITE(6,3400)
 3400 FORMAT(1(/))
      I=I-1
      IF(I.EQ.0) GO TO 1500
      GO TO 600
 1500 CONTINUE
      DO 1000 N=1,NMAX
      DO 113   J=1,J1
      I=1
  113 U(I,J)=TIN
      ITN=1
  310 SUM=0.0
      DO 200   I=2,L1
      DO 200   J=2,L18
      U(I1,J)=U(I1-1,J)
      U(I,J1)=U(I,J1-1)
      U(I,1)=U(I,2)
      DIFF=1.8*((U(I-1,J)+U(I+1,J)+U(I,J+1)+U(I,J-1))/4.-U(I,J))
      U(I,J)=U(I,J)+DIFF
  200 CONTINUE
      ITN=ITN+1
      IF(ITN.GT.MAX) GO TO 201
      GO TO 310
  201 CONTINUE
      DO 250   J=J2,L18
      DO 250   I=I2,L1
  250 U(I,J)=U(I2,J2)
      IF((N+2)/3.NE.N/3) GO TO 1000
      IF(N.EQ.12)  GO TO 1000
      WRITE(6,550) N
  550 FORMAT(2X,-N=-,I2/)
      I=I1
      JJ=J3
  500 CONTINUE
      WRITE(6,1999) I
 1999 FORMAT(2X,-I=-,I2/)
      WRITE(6,2000)  (J,J=1,10)
 2000 FORMAT(2X,10(I2,10X))
      WRITE(6,2200)
 2200 FORMAT(1(/))
      WRITE(6,2300)  (U(I,J),J=1,JJ)
 2300 FORMAT((1X,10(F5.1,7X)))
      WRITE(6,2400)
```

```
 2400 FORMAT(1(/))
      I=I-1
      IF(I.LT.I2.AND.I.GT.I3)    GO TO 670
      IF(I.LE.I3)      JJ=J1
      IF(I.EQ.0) GO TO 145
      GO TO 500
  670 C=65.-FLOAT(I)
      JJ=INT(C)
      GO TO 500
  145 CONTINUE
  145 CALL PLOT(N)
 1000 CONTINUE
      STOP
      END
      SUBROUTINE PLOT(N)
      COMMON U(70,100),
     1SYMBOL(51),
     1G(100),
     1R,I1,I2,I3,J1,J2,J3,Z,GRID
      COMMON BLANK
C
C
C
C       FROM HERE THE TEMPERATURES ARE PLOTTED
C
C
C
C       MOVE TO A NEW PAGE
      WRITE(6,845)
  845 FORMAT(1H1)
      IF(N.GE.60) GO TO 852
      GO TO 853
  852 TIME=FLOAT(N)/60.
  853 WRITE(6,847) N,TIME
  847 FORMAT(*N=*,I2,10X,*TIME=*,F10.2)
C       SET ROW INDEX I TO I1 FOR TOP ROW
      I=I1
C       RUN THROUGH COMPLETE ROW
  802 CONTINUE
      IF(I.LT.I2.AND.I.GT.I3) GO TO 808
      GO TO 806
  808 CONTINUE
      C=65.-FLOAT(I)
      J1=INT(C)
  806 CONTINUE
      DO 901 J=1,J1
C       CHECK IF POINT IS OUTSIDE LIMIT
      IF(I.LT.I2.AND.I.GT.I3.AND.J.GT.J3.AND.J.LT.J2) GO TO 809
      IF(I.GE.I2.AND.J.GT.J3) GO TO 360
  809 CONTINUE
C       NOT OUTSIDE--SELECT LETTER OR BLANK TO REPRESENT TEMPERATURE
      K1=U(I,J)/5.+1.
      IF(K1.GT.51) GO TO 680
C       PLACE APPROPRIATE SYMBOL IN APPROPRIATE POSITION IN LINE
      G(J)=SYMBOL(K1)
      GO TO 901
C       INSERT BLANK IF OUTSIDE LIMIT
  680 G(J)=BLANK
  901 CONTINUE
      IF(I.LT.I3) GO TO 807
      IF(I.GE.I2) GO TO 902
      L809=Z/GRID
      L808=INT(C+1.)
      DO 807 J=L808,L809
      G(J)=BLANK
  807 CONTINUE
      J1=Z/GRID
  902 CONTINUE
C       WRITE ENTIRE LINE
  905 WRITE(6,910) G
  910 FORMAT(1H ,1X,100A1)
C       MOVE TO THE NEXT ROW
      I=I-1
C       CHECK FOR COMPLETION
      IF(I.NE.0) GO TO 802
  920 CONTINUE
      RETURN
      END
```

```
.4 JOB(UMEHM18,J13,T620,CM50000,LC5120)          OIKONOMIDES.C.N.
   MNF(T,D,R=2,E=1,I=BIN)
   MAP(P)
   BIN.
        PROGRAM TRAN3(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
   C    PROGRAM TRAN3 DETERMINES THE TRANSIENT THERMAL DISTRIBUTION
   C    FOR THE NOZZLE-VESSEL SECTION FOR THE CASE C BY SOLVING THE
   C    TWO-DIMENSIONAL UNSTEADY STATE CONDUCTION DIFFERENTIAL
   C    EQUATION AND BY APPLYING THE APPROPRIATE BOUNDARY CONDITIONS.
   C
   C
   C
        COMMON U(35,100),
       1R,I1,I2,I3,J1,J2,J3,Z,GRID
   C    ----------------------------------------------------------------
   C
   C
   C    DATA DESIGNATIONS--DIMENSIONS--
   C    DIMENSIONS OF THE STRUCTURE IN INCHES.
   C    TEMPERATURES IN DEGREES FAHRENHEIT
   C
        TIN=450.
        TIS=60.
        S=0.5
        Z=10.0
        V=3.5
        W=3.0
        GRID=0.1
        RA=0.5
        RB=0.5
        R=RA/GRID
        TI=5.
        AMINUT=60.
        TIMEB=2.5
   C    ----------------------------------------------------------------
   C
   C
   C
   C    PROPERTIES OF STEEL-BRITISH UNITS--
   C
   C    THERMAL CONDUCTIVITY IN BTU/FT-HR-F
        CONDS=35.0
   C    DENSITY OF STEEL IN LB/CUBIC FEET
        DENST=491.0
   C    SPECIFIC HEAT OF STEEL IN BTU/LB-F
        SPEST=0.11
   C    CHARACTERISTIC LENGTHS IN FEET
        XCHAR1=2.5/12.
        XCHAR2=.757/12.
        XCHAR3=6.5/12.
   C    ----------------------------------------------------------------
   C
   C
   C    PROPERTIES OF AIR AT 60 DEGREES F-BRITISH UNITS-
   C
   C    THERMAL CONDUCTIVITY OF  AIR IN BTU/FT-HR-F
        GASK=0.0146
   C    SPECIFIC HEAT OF AIR IN BTU/LB-F
        SPHEAT=0.240
   C    DYNAMIC VISCOSITY OF AIR IN LB/FT-HR
        DYNVIS=0.0437
   C    KINEMATIC VISCOSITY OF AIR IN FEET SQUARE/HR
        VISCK=0.579
   C    VOLUME COEFFICIENT OF EXPANSION CF AIR PER/DEG.F
        B=0.00164
   C    ----------------------------------------------------------------
   C
   C
   C    DATA RELATED TO CONDITIONS-BRITISH UNITS-
   C
   C    ACCELARATION DUE TO GRAVITY IN FEET/HR.SQUARE
        ACCEG=4.173*(10.E+08)
   C    TEMPERATURE OF AIR -DEGREES F--
        TGAS=60.
   C    UNIT DIMENSION OF ELEMENT-FEET-
        RCY=1.0
   C    ----------------------------------------------------------------
   C
   C
   C
        I1=V/GRID
        I2=(S+RB)/GRID
        I3=S/GRID
        J1=Z/GRID
        J2=(W+RA)/GRID
        J3=W/GRID
   C
   C    AT INITIAL STATE THE TEMPERATURE OF THE MATERIAL IS DEFINED
   C    THIS MAY VARRY
```

```
C
C      VESSEL PART CALCULATIONS--BOUNDARY--
C

       J=1
       DO 11  I=1,I1
   11 U(I,J)=TIM
C
       I=1
       DO 12  J=2,J1
   12 U(I,J)=TIS
C
C------------------------------------------------------------
       L1=I1-1
       L18=J1-1
       I22=I2-1
       I33=I3+1
       J33=J3+1
C
C------------------------------------------------------------
C
       DO 95  I=2,I33
       DO 95  J=2,J1
   95 U(I,J)=TIS
       DO 96  I=I33,I22
C      THE EQUATION OF THE INCLINED INSIDE SURFACE OF THE VESSEL
C      WALL DETERMINES THE VALUE OF J CO-ORDINATE FOR EACH VALUUE OF I
       JJ=40-I
       JJJ=JJ+1
       DO 96  J=2,JJJ
   96 U(I,J)=TIS
       DO 97  I=I2,I1
       DO 97  J=2,J33
   97 U(I,J)=TIS
C
C------------------------------------------------------------
C
C
C
C
       VESSEL WALL CALCULATIONS
       THIS IS TIME ITERATION
       TIME=2.50
       NV=1
  299 CONTINUE
  300 CONTINUE
       DO 100  I=2,I3
       DO 100  J=2,L18
       U(1,J)=U(2,J)
       U(I,J)=(U(I-1,J)+U(I+1,J)+U(I,J+1)+U(I,J-1))/4.
       U(I,J1)=U(I,J1-1)
  100 CONTINUE
C
C------------------------------------------------------------
C
C
C      HEAT IS CONVECTED TO AIR BY NATURAL CONVECTION.
C      CALCULATE THE HEAT TRANSFER COEFFIECIENT AT THE WALL TEMPERATURE
C      OF THIS TIME INTERVAL
C      PRAND=(SPHEAT*DYNVIS)/GASK
C
C      CALCULATE GRASHOFF NUMBER AS A FUNCTION OF THE WALL
C      TEMPERATURE AT THE END OF THIS TIME INTERVAL----------
       UM=(U(I3,J2)+U(I3,J1))/2.
       GRASH=((XCHAR1**3)*ACCEG*B*(UM-TGAS))/(VISCK**2)
       ANSL=0.54*((GRASH*PRAND)**0.25)
       HEATCO=(GASK*ANSL)/XCHAR1
C
C      FROM THE HEAT BALANCE EQUATION ON THE INSIDE BOUNDARY
C      CALCULATE THE TEMPERATURE DROP OF THE WALL NODAL
C      TEMPERATURES DUE TO COVVECTION AT THIS TIME INTERVAL
C
       DO 480  J=J2,J1
       AA1=-CONDS*(GRID/12.)*RCY*(U(I3,J)-U(I3-1,J))
       IF(AA1.EQ.0.) GO TO 480
       BB1=HEATCO*(GRID/12.)*RCY*(U(I3,J)-TGAS)
       IF(BB1.EQ.0.) GO TO 480
       CC1=DENST*CP_ST*(GRID/12.)*(GRID/12.)*RCY
       DIFF=(-AA1-BB1)/CC1
       FDIFF=(DIFF*TIME)/3600.
       U(I3,J)=U(I3,J)-FDIFF
  480 CONTINUE
C
C------------------------------------------------------------
C
       DO 120  I=I33,I22
```

```
C         THE EQUATION OF THE INCLINED INSIDE SURFACE OF THE VESSEL
C.        WALL DETERMINES THE VALUE OF J CO-ORDINATE FOR EACH VALUUE OF I
          JJ=43-I
          DO 120  J=2,JJ
          U(I,JJ)=U(I,JJ-1)
          U(I,J)=(U(I-1,J)+U(I+1,J)+U(I,J+1)+U(I,J-1))/4.
          U(I,JJ+1)=U(I,JJ)
  120 CONTINUE
C
C
          UM=(U(I2,J2)+U(I3,J2))/2.
          GRASH=((XCHAR2**3)*ACCEG*B*(UM-TGAS))/(VISCK**2)
          ANSL=0.13*((GRASH*PRAND)**0.33)
          HEATCO=(GASCK*ANSL)/XCHAR2
          DO  481  I=I33,I22
          JJ=43-I
          AA2=-CONDS*(GRID/12.)*RCY*(U(I,JJ)-U(I,JJ-1))
          IF(AA2.EQ.0.)GO TO 481
          BB2=HEATCO*(GRID/12.)*RCY*(U(I,JJ)-TGAS)
          IF(BB2.EQ.0.)GO TO 481
          CC2=DENST*CPEST*(GRID/12.)*(GRID/12.)*RCY
          DIFF2=(-AA2-BB2)/CC2
          FDIFF2=(DIFF2*TIMEB)/3600.
          U(I,JJ)=U(I,JJ)-FDIFF2
  481 CONTINUE
C
C   ----------------------------------------------------------------
C
          DO 130  I=I2,L1
          DO 130  J=2,J3
          U(I,J)=(U(I-1,J)+U(I+1,J)+U(I,J+1)+U(I,J-1))/4.
          U(I1,J)=U(I1-1,J)
          U(I,J1)=U(I,J1-1)
  130 CONTINUE
          UM=(U(I1,J3)+U(I2,J3))/2.
          GRASH=((XCHAR3**3)*ACCEG*B*(UM-TGAS))/(VISCK**2)
          ANSL=0.13*((GRASH*PRAND)**0.33)
          HEATCO=(GASCK*ANSL)/XCHAR3
          DO  482  I=I2,I1
          AA3=-CONDS*(GRID/12.)*RCY*(U(I,J3)-U(I,J3-1))
          IF(AA3.EQ.0.) GO TO 482
          BB3=HEATCO*(GRID/12.)*RCY*(U(I,J3)-TGAS)
          IF(BB3.EQ.0.) GO TO 482
          CC3=DENST*CPEST*(GRID/12.)*(GRID/12.)*RCY
          DIFF3=(-AA3-BB3)/CC3
          FDIFF3=(DIFF3*TIMEB)/3600.
          U(I,J3)=U(I,J3)-FDIFF3
  482 CONTINUE
C
C   ----------------------------------------------------------------
C
          IF(TIME.EQ.32.5)  TI=10.
          IF(TIME.EQ.122.5)  TI=30.
          IF(TIME.EQ.702.5)  TI=60.
          IF(TIME.EQ.1802.5)  TI=120.
          IF(TIME.EQ.32.5)  NV=4
          IF(TIME.EQ.122.5)  NV=5
          IF(TIME.EQ.702.5)  NV=6
          IF(TIME.EQ.1802.5)  NV=16
          TINT=TI*FLOAT(NV)
          IF(TIME.EQ.TINT)  GO TO 101
          TIME=TIME+2.5
          GO TO 300
  101 CONTINUE
          NV=NV+1
C
C   ----------------------------------------------------------------
C
C
C
C         CALCULATE THE MEAN TEMPERATURES FOR THE SECTIONS OF THE VESSEL
C         WALL.
C
C
          SUM=0.0
          NNP=0
          DO 2000  I=I2,I1
          DO 2000  J=1,J3
          SUM=SUM+U(I,J)
          NNP=NNP+1
          THEAN1=SUM/FLOAT(NNP)
 2000 CONTINUE
C
C
          SUM=0.0
          NNP=1
          DO 2010  I=I33,I22
```

```fortran
      JJ=40-I
      DO 2010   J=1,JJ
      SUM=SUM+U(I,J)
      NNP=NNP+1
      TMEAN2=SUM/FLOAT(NNP)
2010  CONTINUE
C
C
C
      SUM=0.0
      NNP=0
      DO 2020   I=1,I3
      DO 2020   J=J2,J1
      SUM=SUM+U(I,J)
      NNP=NNP+1
      TMEAN3=SUM/FLOAT(NNP)
2020  CONTINUE
C
C
C     CALCULATE THE MEAN DIFFERENCES IN TEMPERATURE BETWEEN SECTIONS
C
      DELT12=TMEAN1-TMEAN2
      DELT23=TMEAN2-TMEAN3
      IF(TIME.GT.60.AND.TIME.LT.180.) GO TO 1000
      IF(TIME.GE.180.) GO TO 523
      TIME=TIME*TIMEB
C
C
C
C     -------------------------------------------------------------------
C
C     PRINT ALL NODAL TEMPERATURES AT SPECIFIED TIME INTERVALS
C
C
 523  CONTINUE
      STIME=TIME/60.
      WRITE(6,2400)
2400  FORMAT(1(/))
      WRITE(6,550) STIME ,TIME
 550  FORMAT(2X,*TIME IN MINUTES=*,F9.3,10X,*TIME IN SECONDS=*,F9.2)
      WRITE(6,2450)
2450  FORMAT(1(/))
      WRITE(6,2460)   DELT12, DELT23
2460  FORMAT(2X,*DIFFERENCE DELT12=*,F9.2,10X,*DIFFERENCE DELT23=*,F9.2)
      I=I1
      JJ=J3
 500  CONTINUE
      WRITE(6,1999) I
1999  FORMAT(2X,*I=*,I3)
      WRITE(6,2030)  (J,J=1,10)
2030  FORMAT(2X,10(I2,10X))
      WRITE(6,2300)  (U(I,J),J=1,JJ)
2300  FORMAT((1X,10(F5.1,7X)))
      I=I-1
      IF(I.LT.I2.AND.I.GT.I3)    GO TO 670
      IF(I.LE.I3)    JJ=J1
      IF(I.EQ.0) GO TO 1010
      GO TO 500
 670  C=40.-FLOAT(I)
      JJ=INT(C)
      GO TO 500
1010  IF(TIME.LE.150.) GO TO 528
      GO TO 1000
 528  TIME=TIME/TIMEC
1000  IF(STIME.EQ.AMINUT) GO TO 1001
      GO TO 299
C     -------------------------------------------------------------------
1001  CONTINUE
      STOP
      END
```

APPENDIX 4

Stress Analysis by Finite Elements

4.1 Finite Elements

The structure is considered as an assemblage of elements interconnected at a discrete number of nodal points. If the force displacement relationships for the individual elements are known it is possible to derive the properties and study the behaviour of the assembled structure. In an elastic continuum the real number of interconnection points is infinite. The concept of finite elements attempts to overcome the difficulty of an infinite number of points by assuming the real continuum to be divided into elements interconnected only at a finite number of nodal points at which some imaginary forces, representative of the distributed stresses actually acting on the element boundaries, were supposed to be introduced. With this idealisation the problem reduces to that of a conventional structural type which can be solved by numerical treatment.

4.2 Equations

In Cartesian co-ordinates (x,y,z) the stresses are:

Direct stresses: σ_{xx}, σ_{yy}, σ_{zz}

Shear stresses: τ_{xy}, τ_{yz}, τ_{zx}

First subscript denotes the direction of the stress and the second subscript the direction of the outward normal to the surface on which it acts. Tensile stresses are positive.

Strains are:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y} \quad , \quad \varepsilon_{zz} \frac{\partial w}{\partial z}$$

$$\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad , \quad \varepsilon_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad , \quad \varepsilon_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

where u,v,w are the corresponding displacements in x,y,z direction.

For the isotropic elastic solid with thermal strains

$$\varepsilon_{xx} = \frac{1}{E} \left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz}) \right] + \alpha.\Delta T$$

$$\varepsilon_{yy} = \frac{1}{E} \left[\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx}) \right] + \alpha.\Delta T$$

$$\varepsilon_{zz} = \frac{1}{E} \left[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy}) \right] + \alpha.\Delta T$$

$$\varepsilon_{xy} = \frac{\tau_{xy}}{G} = \frac{2(1+\nu)}{E} \sigma_{xy}$$

$$\varepsilon_{yz} = \frac{\tau_{yz}}{G} = \frac{2(1+\nu)}{E} \sigma_{yz}$$

$$\varepsilon_{zx} = \frac{\tau_{zx}}{G} = \frac{2(1+\nu)}{E} \sigma_{zx}$$

where   E = Young's modulus

    G = Shear modulus

    ν = Poisson's ratio

    α = Coefficient of thermal expansion

    ΔT = Temperature rise

For this particular project the elementary part in which the sections of vessel/nozzle are divided is the triangular element. The displacements are given by u and v in the direction of the x and y axes. Due to thermal expansion of the solid the stress in a direction perpendicular to the x-y plane is not zero. This is the case of plane strain. By definition the strain in the z-direction is zero, and therefore no contribution to internal work is made by the stress in this direction, which is evaluated from the three main components. The displacements within an element are given by two linear polynomials.

$$u = \alpha_1 + \alpha_2 x + \alpha_3 y$$

$$v = \alpha_4 + \alpha_5 x + \alpha_6 y$$

The six constants α can be evaluated by solving the two sets of three simultaneous equations which will arise if the nodal co-ordinates are inserted and the displacements equated to the appropriate nodal displacements.

$$u_i = \alpha_1 + \alpha_2 x_i + \alpha_3 y_i$$

$$u_j = \alpha_1 + \alpha_2 x_j + \alpha_3 y_j$$

$$u_m = \alpha_1 + \alpha_2 x_m + \alpha_3 y_m$$

Solving for α_1, α_2, α_3 in terms of the nodal displacements u_i, u_j, u_m and obtain

$$u = \frac{1}{2\Delta} \left[(a_i + b_i x + c_i y)u_i + (a_j + b_j x + c_j x) \right.$$

$$\left. + a_m + b_m x + c_m y)u_m \right] \qquad \text{(A4.2.1)}$$

where,

$$a_i = x_j y_m - x_m y_i$$

$$b_i = y_j - y_m = y_{im}$$

$$c_i = x_m - x_j = x_{mj}$$

The other coefficients can be obtained by a cyclic process of subscripts in the order i,j,m and where

$$2\Delta = \det \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_m & y_m \end{vmatrix} = 2(\text{area of triangular element ijm})$$

For the vertical displacement v

$$v = \frac{1}{2\Delta} \left[(a_i + b_i x + c_i y) v_i + (a_j + b_j x + c_j y) v_j \right.$$

$$\left. + a_m + b_m x + c_m y) v_m \right] \qquad (A4.2.2)$$

The displacement function guarantees continuity of displacements with adjacent elements because the displacements vary linearly along any side of the triangle and, with identical displacement imposed at the nodes, the same displacement will exist all along the interface. The total strain at any point within the element can be defined by its three components which contribute to internal work.

$$[\varepsilon] \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{bmatrix}$$

Using equations (A4.2.1) and (A4.2.2) we have

$$\varepsilon \begin{bmatrix} \frac{\partial N_i'}{\partial x} & 0 & \frac{\partial N_j'}{\partial x} & 0 & \frac{\partial N_m'}{\partial x} & 0 \\ 0 & \frac{\partial N_i'}{\partial y} & 0 & \frac{\partial N_j'}{\partial y} & 0 & \frac{\partial N_m'}{\partial y} \\ \frac{\partial N_i'}{\partial y} & \frac{\partial N_i'}{\partial x} & \frac{\partial N_j'}{\partial y} & \frac{\partial N_j'}{\partial x} & \frac{\partial N_m'}{\partial y} & \frac{\partial N_m'}{\partial x} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ u_j \\ vj \\ u_m \\ v_m \end{bmatrix}$$

$$= \frac{1}{2\Delta} \begin{bmatrix} b_i & 0 & b_j & 0 & b_m & 0 \\ 0 & c_i & 0 & c_j & 0 & c_m \\ c_i & b_i & c_j & b_j & c_m & b_m \end{bmatrix} \begin{bmatrix} \delta \end{bmatrix}^c$$

where

$$\delta = \begin{pmatrix} u_i \\ v_i \end{pmatrix}$$

$$N'_i = (a_i + b_i x + c_i y)/2$$

$$N'_j = (a_j + b_j x + c_j y)/2 \qquad \text{e.t.c.}$$

The matrix thus derived is independent of the position within the element and hence the strains are constant thoughout it.

In the case of plane strain for the case of isotropic thermal expansion σ_z exists and we have

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu\sigma_{yy}}{E} - \frac{\nu\sigma_{zz}}{E} + \alpha\Delta T^e$$

$$\varepsilon_{yy} = -\frac{\nu\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \frac{\nu\sigma_{zz}}{E} + \alpha\Delta T^e$$

$$\gamma_{xy} = \frac{2(1 + \nu)\ \tau_{xy}}{E}$$

but also

$$\varepsilon_{zz} = 0 = -\frac{\nu\sigma_{xx}}{E} - \frac{\nu\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E} + \alpha\Delta T^e$$

By elininating σ_{zz} and solving for the three remaining stresses the matrix $\begin{bmatrix} D \end{bmatrix}$ is obtained as

$$\begin{bmatrix} D \end{bmatrix} = \frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix} 0 & \nu/(1 - \nu) & 0 \\ \nu/(1 - \nu) & 1 & 0 \\ 0 & 0 & \frac{(1 - 2\nu)}{2(1 - \nu)} \end{bmatrix}$$

The direct stresses are then calculated from

$$\{\sigma\} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} - \{\varepsilon_o\} \end{bmatrix}$$

where

$$\{\varepsilon_o\} = \begin{Bmatrix} \alpha\Delta T_{xx} \\ \alpha\Delta T_{yy} \\ 0 \end{Bmatrix} \qquad \text{and}$$

σ_{zz} is determined from Equation (A4.2.3). The principal stresses are subsequently calculated from

$$\{\sigma_1\} = \tfrac{1}{2} \{\sigma_{xx} + \sigma_{yy}\} + \{\tfrac{1}{2} \sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4 \, \tau_{xy}^2}\}$$

$$\{\sigma_2\} = \tfrac{1}{2} \{\sigma_{xx} + \sigma_{yy}\} - \{\tfrac{1}{2} \sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4 \, \tau_{xy}^2}\}$$

As the condition of plane strain is valid for the present problem due to large thickness of the vessel in the z-direction (closed ring) and as $\varepsilon_z = 0$, the stress σ_z can be assumed to represent the principal stress σ_3 in the z-direction.

APPENDIX 5

COMPUTER PROGRAMS: STREN 1, STREN 2, STREN 3

5.1 Data Designations for Programs STREN1, STREN2, STREN3

The data are categorised by the following designations:

is	integer single
ia	integer array
rs	real single
ra	real array
hs	Hollerith single
ha	Hollerith array

The units can be any consistent set, with angles in degrees.

ALPHA	ra	coefficients of thermal expansion
ANGLEL	rs	angle of the major principal in plane stress on an element, counter-clockwise from the x-axis
ANGLEN	rs	angle of the major principal in plane stress at a node, counter-clockwise from the x-axis
ANP	ia	indices of the forced node and of the displaced nodes adjacent to it
AREA	rs	area of an element
B	ra	the B-matrix in the compatibility or in the equilibrium equation for an element
BI	ra	$y_j - y_m$ for the elements
BJ	ra	$y_m - y_i$ for the elements
BM	ra	$y_i - y_j$ for the elements
BODYF	rs	gravitational body force per unit thickness on an element taken to be in the -y direction

CASCTL	hs	constant with the value NFCASE used for testing for the termination of the job
CI	ra	$-(x_j-x_m)$ for the elements
CJ	ra	$-(x_m-x_i)$ for the elements
CM	ra	$-(x_i-x_j)$ for the elements
CØND	ia	specified conditions of restraint for boundary nodes with the three possible values
		0 fixed in both x and y directions
		1 fixed in the x direction only
		2 allowed to move along a line at an in plane angle ϕ measured counter-clockwise from the x-axis
D	ra	the elasticity D-matrix for an element
DELTAT	ra	temperature rise in the elements above their initial constant value
DELTAU	rs	x-component of the cyclic increment in the iteration to the displacement of a node
DELTAV	rs	y-component of the cyclic increment in the iteration to the displacement of a node
E	ra	moduli of elasticity in the elements
EEXX	rs	linear total strain in the x-direction in an element
EEXY	rs	total shear strain in the plane of an element
EEYY	rs	linear total strain in the y-direction in an element
ETH	rs	thermal strain in an element
EXX	rs	linear elastic strain in the x-direction in an element
EXY	rs	elastic shear strain in the plane of an element
EYY	rs	linear elastic strain in the y-direction in an element

FXX	ra	components for each node of flexibility times thickness in the x-direction due to a force per unit thickness on itself in the x-direction
FXY	ra	components for each node of flexibility times thickness in the x-direction due to a force per unit thickness on itself in the y-direction
FYX	ra	components for each node of flexibility times thickness in the y-direction due to a force per unit thickness on itself in the x-direction
FYY	ra	components for each node of flexibility times thickness in the y-direction due to a force per unit thickness on itself in the y-direction
I	is	working index of the first node of an element
J	is	working index of the second node of an element
K	ra	matrix of stiffness per unit thickness for an element
KXX	ra	components for each node of stiffness per unit thickness in the x-direction due to displacements in the x-direction of itself and of nodes adjacent to it
KXY	ra	components for each node of stiffness per unit thickness in the x-direction due to displacements in the y-direction of itself and of nodes adjacent to it
KYX	ra	components for each node of stiffness per unit thickness in the y-direction due to displacements in the x-direction of itself and of nodes adjacent to it
KYY	ra	components for each node of stiffness per unit thickness in the y-direction due to displacements in the y-direction of itself and of nodes adjacent to it

L	is	running index of the elements
L_1	is	working variable used in the output of element data
L_2	is	working variable used in the output of element data
L_3	is	working variable used in the output of element data
LE	is	index of particular element
LN	ia	working indices of the three nodes of an element
M	is	working index of the third node of an element
MAT	ia	indices of materials specified for the elements
MATN	is	index of a particular material
N	is	running index of the nodes
N_1	is	working variable used in the output of node displacements
N_2	is	working variable used in the output of node displacements
N_3	is	working variable used in the output of node displacements
NAP	ia	numbers of nodes adjacent to each node plus one added for itself
NAP1	is	number of nodes adjacent to a particular node plus one added for itself
NB	ia	indices of the restrained boundary nodes with values corresponding to those of NP
NBR	is	number of restrained boundary nodes
NC	is	cycle count in the iteration
NCC	is	cycle interval for writing the iteration progress
NCY	is	maximum number of cycles for the case
NEL	is	number of elements

NM	is	running index of materials
NMAT	is	number of materials
NNP	is	number of nodes
NP	is	index of a particular node
NPI	ia	indices of the first nodes of the elements
NPJ	ia	indices of the second nodes of the elements
NPM	ia	indices of the third nodes of the elements
NR	is	running index of restrained boundary nodes with values 1, 2, ...
NU	ra	Poisson's ratios in the elements
Ø	is	working row or column index used in matrix multiplication
ØMEGA	rs	over-relaxation factor
P	is	working row index of a matrix
PS	hs	control parameter specifying the type of deformation in the body with the two possible values
		STRESS plane stress
		STRAIN plane strain
Q	is	working column index of a matrix
QA	is	running index of a reference node and the nodes adjacent to it
RELDIF	rs	relative sum of absolute cyclic component increments in displacements
RHØ	ra	densities in the elements
RX	ra	x-components of forces per unit thickness acting on the nodes, external or total
RX1	rs	x-component of an out-of-balance force per unit thickness acting on a node plus the force per unit thickness acting on the node due to its own displacement

RY	ra	y-components of forces per unit thickness acting on the nodes, external or total
RY1	rs	y-component of an out-of-balance force per unit thickness acting on a node plus the force per unit thickness acting on the node due to its own displacement
SD	rs	sum of the absolute components of displacements
SDD	rs	sum of the absolute components of cyclic displacement increments
SIGLM	rs	the second invariant of stress in an element
SIGLXX	ra	normal stress in the x-direction in the elements
SIGLXY	ra	shear in plane components of stress in the elements
SIGLYY	ra	normal stress in the y-direction in the elements
SIGLZZ	ra	normal stress in the z-direction in the elements
SIGL1	ra	maximum principal in plane stress on an element
SIGL2	rs	minimum principal in plane stress on an element
SIGL3	rs	principal anti-plane stress on an element
SIGNM	rs	the second invariant of stress at a node
SIGNXX	rs	normal stress in the x-direction at a node
SIGNXY	rs	shear in plane component of stress at a node
SIGNYY	rs	normal stress in the y-direction at a node
SIGNZZ	rs	normal stress component in the z-direction at a node
SIGN1	rs	maximum principal in plane stress at a node
SIGN2	rs	minimum principal in plane stress at a node
SIGN3	rs	principal anti-plane stress at a node
SIGTH	rs	thermal stress in an element
SLØPE	ra	tan ϕ for the condition CØND - 2 at the restrained boundary nodes

SR	is	control parameter specifying stresses to be output, with the four possible values
		0 none
		1 elements only
		2 nodes only
		3 both elements and nodes
STRAIN	hs	constant with the value STRAIN used in testing which type of deformation is specified
STRESS	hs	constant with the value STRESS used in testing which type of deformation is specified
TITLE	ha	input text being the case title
TØLER	rs	cyclic convergence test constant
U	ra	x-components of displacements of the nodes, the input guessed values or the solution values
V	ra	y-components of displacements of the nodes, the input guessed values or the solution values
W	ra	working matrix
WI	is	control parameter specifying how much of the input data is to be output, with the two possible values
		0 case title and control parameters only
		1 all input data
W_1	rs	working variable
W_2	rs	working variable
W_3	rs	working variable
W_4	rs	working variable
W_5	rs	working variable
W_6	rs	working variable
W_7	rs	working variable
X	ra	abscissae of the nodes
Y	ra	ordinates of the nodes

5.2 Listing of Program STREN1

```
      PROGRAM STREN1(INPUT=1002,OUTPUT=1002,TAPE5=INPUT,TAPE6=OUTPUT)
C
C
C
C  0 SPECIFICATION AND DATA INITIALIZATION STATEMENTS.
      INTEGER   TITLE,      PS,         SR,          QA,         O,         NP,
     1          CASCTL,     STRESS,     WI,          ANP,        P,
     2                      STRAIN,     COND,                    Q
      REAL      NU,         K,          KXX,
     1                                  KXY,
     2                                  KYX,
     3                                  KYY
      COMMON    TITLE(13),  NPI(377),   X(221),
     1          E(5),       NPJ(377),   Y(221),
     2          NU(5) ,     NPM(377),   RX(221),
     3          RHO(5),     MAT(377),   RY(221),
     4          ALPHA(5),   DELTAT(377), U(221),
     5                                  V(221),
     1          BI(377),    KXX(221,9), ANP(221,9),  SIGLXX(377),
     2          BJ(377),    KXY(221,9), NAP(221),    SIGLYY(377),
     3          BM(377),    KYX(221,9), LN(3),       SIGLZZ(377),
     4          CI(377),    KYY(221,9), E(3,6),      SIGLXY(377),
     5          CJ(377),    FXX(221),   D(3,3),
     6          CM(377),    FXY(221),   W(3,6),
     7                      FYX(221),   K(6,6),
     8                      FYY(221)
      DATA  CASCTL, STRESS, STRAIN / 6HNFCASE, 6HSTRESS, 6HSTRAIN /
C
C
C
C  10 INPUT THE CASE TITLE BUT STOP IF THERE IS NO FURTHER CASE.
   11 READ (5, 12)  TITLE
   12 FORMAT (13A6)
      IF (TITLE (1) .EQ. CASCTL)   STOP
C
C
C  20 INPUT THE CONTROL PARAMETERS AND PREPARE TO INPUT THE ARRAY DATA.
      READ (5, 21)  NEL, NNP,      NMAT, NCY, NOC,
     1              TOLER, OMEGA, PS, SR, WI
   21 FORMAT (5I5, E13.5, F7.3, 2X, A6, 2I3)
      DO 22  NM = 1, NMAT
   22 E (NM) = -1.
      DO 23  L = 1, NEL
   23 NPI (L) = -1
      DO 24  N = 1, NNP
   24 X (N) = -3.123456789E35
C
C
C
C  30 INPUT AND CHECK THE ARRAY DATA.
      READ (5, 31)  (MATN, E (MATN), NU (MATN),
     1               RHO (MATN), ALPHA (MATN),  NM = 1, NMAT)
   31 FORMAT (I5, 4E13.5)
      DO 33  NM = 1, NMAT
      IF (E (NM) .NE. -1.)  GO TO 33
      WRITE(6,333) NM
  333 FORMAT(1X,7H E(NM)=, I5)
      WRITE (6, 32)  TITLE
   32 FORMAT (1H1 ////// 5H IN ', 13A6, 1H' /
     1         36H THERE IS A MISSING INPUT DATA CARD.,
     2         24H   THE JOB IS ABANDONED.)
      STOP
   33 CONTINUE
      READ (5, 34)  (LE, NPI (LE), NPJ (LE), NPM (LE),
     1               DELTAT(LE), MAT(LE),  L=1,NEL )
   34 FORMAT(4I5,F10.3,I5)
      DO 35  L = 1, NEL
      IF (NPI (L) .NE. -1)  GO TO 35
      WRITE(6,334)  L
  334 FORMAT(1X, 7H L=,  I5)
      WRITE (6, 32)  TITLE
      STOP
   35 CONTINUE
      DO 350 N=1,45
      READ(5,36)  A1,B1,NP1,A2,B2,NP2,A3,B3,NP3,A4,B4,NP4,A5,B5,NP5
   36 FORMAT(5(F4.0,1X,F4.0,I4,3X))
      WRITE(6,371) NP1,NP2,NP3,NP4,NP5
  371 FORMAT(1X,5(I20))
      X(NP1)=(A1-6848.)/254.5X(NP2)=(A2-6848.)/254.
      X(NP3)=(A3-6848.)/254.5X(NP4)=(A4-6848.)/254.
      X(NP5)=(A5-6848.)/254.
      Y(NP1)=(B1-811.)/254.5Y(NP2)=(B2-811.)/254.
      Y(NP3)=(B3-811.)/254.5Y(NP4)=(B4-811.)/254.
      Y(NP5)=(B5-811.)/254.
  350 CONTINUE
      DO 361  N=1,NNP
      RX(N)=0.
```

```
  361 CONTINUE
      DO 37  N = 1, NNP
      IF (X (N) .NE. -0.123456789E35)  GC TO 37
      WRITE(6,335) N
  335 FORMAT(1X,  3H N=, I5)
      WRITE (6, 32)  TITLE
      STCP
   37 CONTINUE
C
C
C
   40 OUTPUT THE TITLE AND THE CONTROL PARAMETERS.
      WRITE (6, 41)
   41 FORMAT (1H  //////
     1          45X, 41HTHE CASE TITLE AND THE CCNTROL PARAMETERS /
     2          45X, 41H---------------------------------------- //////)
      WRITE (6, 42)  TITLE
   42 FORMAT (1H , 13A6)
      WRITE (6, 43)  NEL, NNP, N3R, NMAT, NCY, NCC, TOLER, CMEGA, PS
   43 FORMAT (26H0THE NUMBER OF ELEMENTS IS, I5, 1H. /
     1          37H0THE NUMBER OF NODAL POINTS IS, I5, 1H. /
     2          43H0THE NUMBER CF RESTRAINED JCUNDARY NODES IS, I5, 1H. /
     3          37H0THE NUMBER CF DIFFERENT MATERIALS IS, I5, 1H. /
     4          23H0THE ITERATION LIMIT IS, I5, 8H CYCLES. /
     5          36H0THE INTERVAL FCR PROGRESS CUTPUT IS, I5, 8H CYCLES. /
     6          29H0THE CONVERGENCE TOLERANCE IS, E13.5, 1H. /
     7          30H0THE OVER-RELAXATION FACTOR IS, F7.3, 1H. /
     8          37H0THE CASE HAS THE CONDITICN CF PLANE , A6, 1H. /
     9          23H0STRESSES ARE FOUND FCR)
      IF (SR .EQ. 0)   WRITE (6, 44)
   44 FORMAT (1H+, 23X, 42HNEITHER THE ELEMENTS NOR THE NODAL POINTS.)
      IF (SR .EQ. 1)   WRITE (6, 45)
   45 FORMAT (1H+, 23X, 18HTHE ELEMENTS ONLY.)
      IF (SR .EQ. 2)   WRITE (6, 46)
   46 FORMAT (1H+, 23X, 22HTHE NODAL FCINTS CNLY.)
      IF (SR .EQ. 3)   WRITE (6, 47)
   47 FORMAT (1H+, 23X, 36HBCTH THE ELEMENTS AND THE NOBAL PCINTS.)
      IF (WI .EQ. 0)   WRITE (6, 48)
   48 FORMAT (37H0THE INPUT ARRAY DATA ARE NCT OUTPUT.)
      IF (WI .EQ. 1)   WRITE (6, 49)
   49 FORMAT (33H0THE INPUT ARRAY DATA ARE OUTPUT.)
C
C
C
   50 OUTPUT THE ARRAY DATA.
      NPI(376)=132 & NPJ(376)=181
      IF (WI .EQ. 0)  GC TO 61
      WRITE (6, 51)  (NM, E (NM), NU (NM), RHC (NM),
     1               ALPHA (NM),  NM = 1, NMAT)
   51 FORMAT (1H  //////
     1          56X, 19HINPUT MATERIAL DATA /
     2          56X, 19H----------------- /////
     3          23X, 42HMATERIAL    ELASTIC MODULUS  PCISSON'S RATIO,
     4          34H     DENSITY        TH'L EXPN CCEFF //
     5          (I33, X, 4E17.5))
      WRITE (6, 52)
   52 FORMAT (1H  //////
     1          57X, 18HINPUT ELEMENT DATA /
     2          57X, 18H---------------- /////
     3          3X, 45HELMT NPI   NPJ   NPM   MTRL   DELTA T
     4          48HELMT NPI   NPJ   NPM   MTRL   DELTA T
     5          33HELMT NPI   NPJ   NPM   MTRL   DELTA T /)
      L1 = (NEL + 2) / 3
      DO 53  L2 = 1, L1
      L3 = MINJ (L2 + 2 * L1, NEL)
   53 WRITE (6, 54)  (L, NPI (L), NPJ (L), NPM (L),
     1               MAT (L), DELTAT (L),  L = L2, L3, L1)
   54 FORMAT (1H , 5I5, F10.3, 13X, 5I5, F10.3, 13X, 5I5, F10.3)
      DO 540  N=1,NNP
      U(N)=0.
      V(N)=0.
  540 CONTINUE
      WRITE (6, 55)  (N, X (N), Y (N), RX (N), RY (N),
     1               U (N), V (N),  N = 1, NNP)
   55 FORMAT (1H1 //////
     1          53X, 15HINPUT NODE DATA /
     2          53X, 15H------------- /////
     3          13X, 49HNODE          X          Y
     4          31HRX              RY
     5          21H  U             V //
     6          (I15, X, 6E17.5))
C
C
C
   60 DO GEOMETRIC CALCULATIONS ON THE ELEMENTS.
   61 DO 72  L = 1, NEL
      I = NPI (L)
```

```
      M = NPM (L)
      BI (L) = Y (J) - Y (M)
      BJ (L) = Y (M) - Y (I)
      BM (L) = Y (I) - Y (J)
      CI (L) = - X (J) + X (M)
      CJ (L) = - X (M) + X (I)
      CM (L) = - X (I) + X (J)
      AREA = (BJ (L) * CM (L) - BM (L) * CJ (L)) / 2.
      IF (AREA .GT. 1.E-35) GO TO 71
      WRITE (6, 62) L
   62 FORMAT (1H1 ////// 8H ELEMENT, I5, 25H HAS A NON-POSITIVE AREA. /
     1          23H THE CASE IS ABANDONED.)
      GO TO 221
C
C
C
   70 SUM THE TOTAL NODAL POINT FORCES.
   71 MATN = MAT (L)
      IF (PS .EQ. STRESS) SIGTH = 1. - NU (MATN)
      IF (PS .EQ. STRAIN) SIGTH = (1. + NU (MATN))
     1                                    * (1. - 2. * NU (MATN))
      SIGTH = - E (MATN) * ALPHA (MATN) * DELTAT (L) / SIGTH
      BODYF = AREA * RHO (MATN) / 3.
      RX (I) = RX (I) - 0.5 * BI (L) * SIGTH
      RY (I) = RY (I) - 0.5 * CI (L) * SIGTH - BODYF
      RX (J) = RX (J) - 0.5 * BJ (L) * SIGTH
      RY (J) = RY (J) - 0.5 * CJ (L) * SIGTH - BODYF
      RX (M) = RX (M) - 0.5 * BM (L) * SIGTH
   72 RY (M) = RY (M) - 0.5 * CM (L) * SIGTH - BODYF
C
C
C
   80 PREPARE TO SUM THE STIFFNESS COEFFICIENTS.
      DO 82 N = 1, NNP
      DO 81 GA = 1, 9
      KXX (N, GA) = 0.
      KXY (N, GA) = 0.
      KYX (N, GA) = 0.
      KYY (N, GA) = 0.
   81 ANP (N, GA) = 0
      ANP (N, 1) = N
   82 NAP (N) = 1
C
C
C
   90 FORM THE STIFFNESS MATRIX FOR THE ELEMENT.
      DO 105 L = 1, NEL
      DO 92 P = 1, 3
      DO 91 Q = 1, 6
   91 B (P, Q) = 0.
      DO 92 Q = 1, 3
   92 D (P, Q) = 0.
      B (1, 1) = BI (L)
      B (1, 3) = BJ (L)
      B (1, 5) = BM (L)
      B (2, 2) = CI (L)
      B (2, 4) = CJ (L)
      B (2, 6) = CM (L)
      B (3, 1) = CI (L)
      B (3, 2) = BI (L)
      B (3, 3) = CJ (L)
      B (3, 4) = BJ (L)
      B (3, 5) = CM (L)
      B (3, 6) = BM (L)
      MATN = MAT (L)
      W1 = NU (MATN)
      W2 = 1. - W1
      W3 = 1. + W1
      W4 = 1. - 2. * W1
      IF (PS .EQ. STRAIN) GO TO 93
      W5 = E (MATN) / (W2 * W3)
      W6 = W1
      W7 = W2 / 2.
      GO TO 94
   93 W5 = E (MATN) * W2 / (W3 * W4)
      W6 = W1 / W2
      W7 = W4 / (2. * W2)
   94 AREA = (BJ (L) * CM (L) - BM (L) * CJ (L)) / 2.
      W5 = W5 / (4. * AREA)
      D (1, 1) = W5
      D (1, 2) = W5 * W6
      D (2, 1) = W5 * W6
      D (2, 2) = W5
      D (3, 3) = W5 * W7
      DO 95 P = 1, 3
      DO 95 Q = 1, 6
      DO 95 O = 1, 3
   95 W (P, Q) = W (P, Q) + D (P, O) * B (O, Q)
```

```
      DO 96  P = 1, 6
      DO 96  C = 1, 6
      K (P, C) = 0.
      DO 96  O = 1, 3
   96 K (P, C) = K (P, C) + B (O, P) * W (O, C)
C
C
C
C 100 SUM AND STORE FOR EACH (FORCED) NODE THE NON-ZERO STIFFNESS
C     COEFFICIENTS FOR THE ADJACENT (DISPLACED) NODES.
      LN (1) = NPI (L)
      LN (2) = NPJ (L)
      LN (3) = NPM (L)
      DO 105  P = 1, 3
      NP = LN (P)
      DO 105  Q = 1, 3
      QA = 0
  101 QA = QA + 1
      IF (QA .LE. 9)  GO TO 103
      WRITE (6, 102)  NP
  102 FORMAT (1H1 ////// 5H NODE, I5,
     1         36H HAS MORE THAN EIGHT ADJACENT NODES. /
     2         23H THE CASE IS ABANDONED.)
      GO TO 221
  103 IF (ANP (NP, QA) .EG. LN (Q))  GO TO 104
      IF (ANP (NP, QA) .NE. 0)  GO TO 101
      ANP (NP, QA) = LN (Q)
      NAP (NP) = QA
  104 KXX (NP, QA) = KXX (NP, QA) + K (2 * P - 1, 2 * Q - 1)
      KXY (NP, QA) = KXY (NP, QA) + K (2 * P - 1, 2 * Q)
      KYX (NP, QA) = KYX (NP, QA) + K (2 * P, 2 * Q - 1)
  105 KYY (NP, QA) = KYY (NP, QA) + K (2 * P, 2 * Q)
C
C
C
C 110 BY INVERSION OF THE MAIN DIAGONAL STIFFNESS SUBMATRIX, FIND THE
C     FLEXIBILITY SUBMATRIX AT EACH NODE DISPLACED BY FORCES AT ITSELF.
      DO 112  N = 1, NNP
      W1 = KXX (N, 1) * KYY (N, 1) - KXY (N, 1) * KYX (N, 1)
      FXX (N) =   KYY (N, 1) / W1
      FXY (N) = - KXY (N, 1) / W1
      FYX (N) = - KYX (N, 1) / W1
  112 FYY (N)= KXX(N,1) / W1
C
C
  125 CONTINUE
C
C
C 130 PERFORM A GAUSS - SEIDEL ITERATION ON THE STIFFNESS MATRIX FOR
C     THE ENTIRE STRUCTURE.
      IF (NCC .GT. NCY)  GO TO 132
      WRITE (6, 131)
  131 FORMAT (1H1 //////
     1         34X, 35HThE PROGRESS OF THE GAUSS - SEIDEL ,
     2         23HITERATION TOWARD CONVERGENCE /
     3         34X, 35H------------------------------------,
     4         23H-------------------------- /////
     5         33X, 39HCYCLE                 CONVERGENCE CRITERION,
     6         26H          CURRENT DIFFERENCE)
  132 NC = 0
  133 SD = 0.
      SDD = 0.
      DO 136  N = 1, NNP
      IF (FXX (N) + FYY (N) .EQ. 0.)  GO TO 136
      RX1 = RX (N)
      RY1 = RY (N)
      NAP1 = NAP (N)
      DO 134  QA = 1, NAP1
      NP = ANP (N, QA)
      RX1 = RX1 - KXX (N, QA) * U (NP) - KXY (N, QA) * V (NP)
  134 RY1 = RY1 - KYX (N, QA) * U (NP) - KYY (N, QA) * V (NP)
      IF (FXX (N) .EQ. 0.)  GO TO 135
      DELTAU = FXX (N) * RX1 + FXY (N) * RY1
      U (N) = U (N) + OMEGA * DELTAU
      SD = SD + ABS (U (N))
      SDD = SDD + ABS (DELTAU)
      IF (FYY (N) .EQ. 0.)  GO TO 136
  135 DELTAV=FYX (N)*RX1+FYY(N) *RY1
      V (N) = V (N) + OMEGA * DELTAV
      SD = SD + ABS (V (N))
      SDD = SDD + ABS (DELTAV)
  136 CONTINUE
      SD=AMAX1 (SD,1.E-35)
      RELDIF = SDD / SD
C
C
```

```
      NC = NC + 1
      IF (NCC .GT. NCY .OR. MOD (NC, NCC) .NE. 0)  GO TO 142
      WRITE (6, 141) NC, TOLER, RELDIF
141 FORMAT (1H , I36, 3X, 2E27.5)
142 IF (RELDIF .GT. TOLER)  GO TO 144
      WRITE (6, 143) NC
143 FORMAT (1H1 ////// 28H THE ITERATION HAS CONVERGED,
     1          6H AFTER, I5, 6H CYCLES.)
      GO TO 151
144 IF (NC .LT. NCY)  GO TO 133
      WRITE (6, 145) NC, RELDIF, TOLER
145 FORMAT (1H1 ////// 28H THE ITERATION HAS COMPLETED, I5, 5H THE ,
     1          52HMAXIMUM NUMBER OF CYCLES WITHOUT CONVERGING, BUT HAS /
     2          34H ACHIEVED A RELATIVE DIFFERENCE OF, E13.5,
     3          35H AGAINST A SPECIFIED TOLERANCE OF, E13.5, 1H. /
     4          43H THE UNCONVERGED RESULTS ARE WRITTEN BELOW.)
C
C
C 150 OUTPUT THE NODE DISPLACEMENTS.
151 WRITE (6, 152)
152 FORMAT (1H1 //////
     1          49X, 33HTHE CALCULATED NODE DISPLACEMENTS /
     2          49X, 33H-------------------------------- /////
     3          3X, 47HNODE            U              V
     4              47HNODE            U              V
     5              30HNODE            U              V /)
      N1 = (NNP + 2) / 3
      DO 153  N2 = 1, N1
      N3 = MINO (N2 + 2 * N1, NNP)
153 WRITE (6, 154) (N, U (N), V (N),  N = N2, N3, N1)
154 FORMAT (1H , I5, 2E16.5, I15, 2E16.5, I15, 2E16.5)
C
C
C 160 CALCULATE THE ELEMENT STRESS COMPONENTS.
      IF (SR .EQ. 0)  GO TO 221
      DO 184  L = 1, NEL
      I = NPI (L)
      J = NPJ (L)
      M = NPM (L)
      AREA = (BJ (L) * CM (L) - BM (L) * CJ (L)) / 2.
      EEXX = (BI (L) * U (I) + BJ (L) * U (J)
     1              + BM (L) * U (M)) / (2.*AREA)
      EEYY = (CI (L) * V (I) + CJ (L) * V (J)
     1              + CM (L) * V (M)) / (2. * AREA)
      EEXY = (CI (L) * U (I) + BI (L) * V (I)
     1       + CJ (L) * U (J) + BJ (L) * V (J)
     2       + CM (L) * U (M) + BM (L) * V (M)) / (2. * AREA)
      MATN = MAT (L)
      W1 = NU (MATN)
      W2 = 1. - W1
      W3 = 1. + W1
      W4 = 1. - 2. * W1
      IF (PS .EQ. STRAIN)  GO TO 161
      W5 = E (MATN) / (W2 * W3)
      W6 = W1
      W7 = W2 / 2.
      GO TO 162
161 W5 = E (MATN) * W2 / (W3 * W4)
      W6 = W1 / W2
      W7 = W4 / (2. * W2)
162 ETH = ALPHA (MATN) * DELTAT (L)
      EXX = EEXX - ETH
      EYY = EEYY - ETH
      EXY = EEXY
      SIGLXX (L) = W5 * (EXX + W6 * EYY)
      SIGLYY (L) = W5 * (W6 * EXX + EYY)
      IF (PS .EQ. STRAIN)  GO TO 163
      SIGLZZ (L) = 0.
      GO TO 164
163 SIGLZZ (L) = W1 * (SIGLXX (L) + SIGLYY (L))
164 SIGLXY (L) = W5 * W7 * EXY
C
C
C 170 CALCULATE THE ELEMENT PRINCIPAL AND MEAN STRESSES.
      IF (SR .EQ. 2)  GO TO 184
      W1 = (SIGLXX (L) + SIGLYY (L)) / 2.
      W2 = (SIGLXX (L) - SIGLYY (L)) / 2.
      W3 = SIGLXY (L)
      W4 = SQRT (W2 ** 2 + W3 ** 2)
      SIGL1 = W1 + W4
      SIGL2 = W1 - W4
      IF (ABS (W2) .GT. 1.E-35 .OR. ABS (W3) .GT. 1.E-35)  GO TO 171
      ANGLEL = 0.
      GO TO 172
171 ANGLEL = 28.647889756 * ATAN2 (W3, W2)
```

```
  172 SIGL3 = SIGLZZ (L)
      SIGLM = SQRT ((SIGL1 + SIGL2 + SIGL3) ** 2
     1          - 3. * (SIGL2 * SIGL3 + SIGL3 * SIGL1 + SIGL1 * SIGL2))
C
C
C 180 OUTPUT THE ELEMENT STRESSES.
      IF (L .NE. 1)  GO TO 182
      WRITE (6, 181)
  181 FORMAT (1H1 //////
     1          27X, 40HTHE CALCULATED ELEMENT STRESS COMPONENTS,
     2               40H AND THE PRINCIPAL AND THE MEAN STRESSES /
     3          27X, 40H----------------------------------------,
     4               40H---------------------------------------- /////
     5          1X, 45HELEMENT       SIGMXX              SIGMYY       ,
     6              45HSIGMZZ              SIGMXY          ANGLE1      ,
     7              37HSIGM1              SIGM2          SIGMM /)
  182 WRITE (6, 183) L, SIGLXX (L), SIGLYY (L), SIGLZZ (L),
     1               SIGLXY (L), ANGLEL, SIGL1, SIGL2, SIGLM
  183 FORMAT (1H , I5, 4E16.5, F12.2, X, 3E16.5)
  184 CONTINUE
      IF (SR .EQ. 1)  GO TO 221
C
C
C 190 FIND THE NODE STRESS COMPONENTS.
      DO 212 N = 1, NNP
      SIGNXX = 0.
      SIGNYY = 0.
      SIGNZZ = 0.
      SIGNXY = 0.
      QA = 1
      DO 191 L = 1, NEL
      IF (NPI (L) .NE. N .AND. NPJ (L) .NE. N .AND. NPM (L) .NE. N)
     1                                                  GO TO 191
      SIGNXX = SIGNXX + SIGLXX (L)
      SIGNYY = SIGNYY + SIGLYY (L)
      SIGNZZ = SIGNZZ + SIGLZZ (L)
      SIGNXY = SIGNXY + SIGLXY (L)
      QA = QA + 1
      IF (QA .EQ. NAP (N))  GO TO 192
  191 CONTINUE
  192 W1 = FLOAT (QA - 1)
      SIGNXX = SIGNXX / W1
      SIGNYY = SIGNYY / W1
      SIGNZZ = SIGNZZ / W1
      SIGNXY = SIGNXY / W1
C
C
C 200 CALCULATE THE NODE PRINCIPAL AND MEAN STRESSES.
  201 W1 = (SIGNXX + SIGNYY) / 2.
      W2 = (SIGNXX - SIGNYY) / 2.
      W3 = SIGNXY
      W4 = SQRT (W2 ** 2 + W3 ** 2)
      SIGN1 = W1 + W4
      SIGN2 = W1 - W4
      IF (ABS (W2) .GT. 1.E-35 .OR. ABS (W3) .GT. 1.E-35)  GO TO 202
      ANGLEN = 0.
      GO TO 203
  202 ANGLEN = 28.647889756 * ATAN2 (W3, W2)
  203 SIGN3 = SIGNZZ
      SIGNM = SQRT ((SIGN1 + SIGN2 + SIGN3) ** 2
     1          - 3. * (SIGN2 * SIGN3 + SIGN3 * SIGN1 + SIGN1 * SIGN2))
C
C
C 210 OUTPUT THE NODE STRESSES.
      IF (N .NE. 1)  GO TO 212
      WRITE (6, 211)
  211 FORMAT (1H1 //////
     1          29X, 37HTHE CALCULATED NODE STRESS COMPONENTS,
     2               40H AND THE PRINCIPAL AND THE MEAN STRESSES /
     3          29X, 37H-------------------------------------,
     4               40H---------------------------------------- /////
     5          3X, 43HNODE          SIGMXX              SIGMYY       ,
     6              45HSIGMZZ              SIGMXY          ANGLE1      ,
     7              37HSIGM1              SIGM2          SIGMM /)
  212 WRITE (6, 213) N, SIGNXX, SIGNYY, SIGNZZ, SIGNXY,
     1               ANGLEN, SIGN1, SIGN2, SIGNM
  213 FORMAT (1H , I5, 4E16.5, F12.2, X, 3E16.5)
C
C
C 220 RETURN TO THE BEGINNING OF THE PROGRAM FOR THE NEXT CASE.
  221 GO TO 11
      END
```

5.3 Listing of Program STREN2

```
      PROGRAM STREN2(INPUT=1002,OUTPUT=1002,TAPE5=INPUT,TAPE6=OUTPUT)
C   0 SPECIFICATION AND DATA INITIALIZATION STATEMENTS.
      INTEGER  TITLE,     PS,          SR,          QA,          O,          NP,
     1          CASCTL,    STRESS,      WI,          ANP,         P,
     2          STRAIN,    COND,                     Q.
      REAL     NU,        K,           KXX,
     1                                 KXY,
     2                                 KYX,
     3                                 KYY
      COMMON    TITLE(13),    NPI(277),       X(166),
     1          E(1),         NPJ(277),       Y(166),
     2          NU(1) ,       NPM(277),       RX(166),
     3          RHO(1),       MAT(277),       RY(166),
     4          ALPHA(1),     DELTAT(277),    U(166),
     5                                        V(166),
     1          BI(277),      KXX(166,9),     ANP(166,9),   SIGLXX(277),
     2          BJ(277),      KXY(166,9),     NAP(166),     SIGLYY(277),
     3          BM(277),      KYX(166,9),     LN(3),        SIGLZZ(277),
     4          CI(277),      KYY(166,9),     C(3,6),       SIGLXY(277),
     5          CJ(277),      FXX(166),       D(3,3),
     6          CM(277),      FXY(166),       W(3,6),
     7                        FYX(166),       K(6,6),
     8                        FYY(166)
      DATA  CASCTL, STRESS, STRAIN / 6HNFCASE, 6HSTRESS, 6HSTRAIN /
C
C
C  10 INPUT THE CASE TITLE BUT STOP IF THERE IS NO FURTHER CASE.
   11 READ (5, 12) TITLE
   12 FORMAT (13A6)
      IF (TITLE (1) .EQ. CASCTL) STOP
C
C
C  20 INPUT THE CONTROL PARAMETERS AND PREPARE TO INPUT THE ARRAY DATA.
      READ (5, 21) NEL, NNP,      NMAT, NCY, NCC,
     1             TOLER, OMEGA, PS, SR, WI
   21 FORMAT (5I5, E13.5, F7.3, 2X, A6, 2I3)
      DO 22 NM = 1, NMAT
   22 E (NM) = -1.
      DO 23 L = 1, NEL
   23 NPI (L) = -1
      DO 24 N = 1, NNP
   24 X (N) = -0.123456789E35
C
C
C  30 INPUT AND CHECK THE ARRAY DATA.
      READ (5, 31) (MATN, E (MATN), NU (MATN),
     1             RHO (MATN), ALPHA (MATN), NM = 1, NMAT)
   31 FORMAT (I5, 4E13.5)
      DO 33 NM = 1, NMAT
      IF (E (NM) .NE. -1.) GO TO 33
      WRITE(6,333) NM
  333 FORMAT(1X,7H E(NM)=, I5)
      WRITE (6, 32) TITLE
   32 FORMAT (1H1 ////// 5H IN ', 13A6, 1H' /
     1         36H THERE IS A MISSING INPUT DATA CARD.,
     2         24H    THE JOB IS ABANDONED.)
      STOP
   33 CONTINUE
      READ (5, 34) (LE, NPI (LE), NPJ (LE), NPM (LE),
     1             DELTAT(LE), MAT(LE), L=1,NEL )
   34 FORMAT(4I5,F10.3,I5)
      DO 35 L = 1, NEL
      IF (NPI (L) .NE. -1) GO TO 35
      WRITE(6,334) L
  334 FORMAT(1X, 3H L=, I5)
      WRITE (6, 32) TITLE
      STOP
   35 CONTINUE
      DO 350 N=1,34
      READ(5,36) A1,B1,NP1,A2,B2,NP2,A3,B3,NP3,A4,B4,NP4,A5,B5,NP5
   36 FORMAT(5(F4.0,1X,F4.0,I4,3X))
      WRITE(6,371) NP1,NP2,NP3,NP4,NP5
  371 FORMAT(1X,5(I20))
      X(NP1)=(A1-3607.)/505.$X(NP2)=(A2-3607.)/505.
      X(NP3)=(A3-3607.)/505.$X(NP4)=(A4-3607.)/505.
      X(NP5)=(A5-3607.)/505.
      Y(NP1)=(B1-1101.)/505. $ Y(NP2)=(B2-1101.)/505.
      Y(NP3)=(B3-1101.)/505. $ Y(NP4)=(B4-1101.)/505.
      Y(NP5)=(B5-1101.)/505.
  350 CONTINUE
      DO 361 N=1,NNP
      RX(N)=0.
      RY(N)=0.
  361 CONTINUE
      DO 37 N = 1, NNP
```

```
      IF (X (N) .NE. -0.123456789E35)  GO TO 37
      WRITE(6,335) N
  335 FORMAT(1X,  3H N=, I5)
      WRITE (6, 32)  TITLE
      STOP
   37 CONTINUE
C
C
C
   40 OUTPUT THE TITLE AND THE CONTROL PARAMETERS.
      WRITE (6, 41)
   41 FORMAT (1H1 //////
     1           45X, 41HTHE CASE TITLE AND THE CONTROL PARAMETERS /
     2           45X, 41H---------------------------------------- //////)
      WRITE (6, 42)  TITLE
   42 FORMAT (1H , 13A6)
      WRITE (6, 43)  NEL, NNP, NBR, NMAT, NCY, NCC, TOLER, OMEGA, PS
   43 FORMAT (26H0THE NUMBER OF ELEMENTS IS, I5, 1H. /
     1           31H0THE NUMBER OF NODAL POINTS IS, I5, 1H. /
     2           43H0THE NUMBER OF RESTRAINED BOUNDARY NODES IS, I5, 1H. /
     3           37H0THE NUMBER OF DIFFERENT MATERIALS IS, I5, 1H. /
     4           25H0THE ITERATION LIMIT IS, I5, 8H CYCLES. /
     5           36H0THE INTERVAL FOR PROGRESS OUTPUT IS, I5, 8H CYCLES. /
     6           29H0THE CONVERGENCE TOLERANCE IS, E13.5, 1H. /
     7           30H0THE OVER-RELAXATION FACTOR IS, F7.3, 1H. /
     8           37H0THE CASE HAS THE CONDITION OF PLANE , A6, 1H. /
     9           23H0STRESSES ARE FOUND FOR)
      IF (SR .EQ. 0) WRITE (6, 44)
   44 FORMAT (1H+, 23X, 42HNEITHER THE ELEMENTS NOR THE NODAL POINTS.)
      IF (SR .EQ. 1) WRITE (6, 45)
   45 FORMAT (1H+, 23X, 18HTHE ELEMENTS ONLY.)
      IF (SR .EQ. 2) WRITE (6, 46)
   46 FORMAT (1H+, 23X, 22HTHE NODAL POINTS ONLY.)
      IF (SR .EQ. 3) WRITE (6, 47)
   47 FORMAT (1H+, 23X, 39HBOTH THE ELEMENTS AND THE NODAL POINTS.)
      IF (WI .EQ. 0) WRITE (6, 48)
   48 FORMAT (37H0THE INPUT ARRAY DATA ARE NOT OUTPUT.)
      IF (WI .EQ. 1) WRITE (6, 49)
   49 FORMAT (33H0THE INPUT ARRAY DATA ARE OUTPUT.)
C
C
C
   50 OUTPUT THE ARRAY DATA.
      IF (WI .EQ. 0)  GO TO 61
      WRITE (6, 51)  (NM, E (NM), NU (NM), RHO (NM),
     1                ALPHA (NM), NM = 1, NMAT)
   51 FORMAT (1H1 //////
     1           56X, 19HINPUT MATERIAL DATA /
     2           56X, 19H------------------- /////
     3           28X, 42HMATERIAL   ELASTIC MODULUS  POISSON'S RATIO,
     4           34H       DENSITY        THML EXPN COEFF //
     5          (I33, X, 4E17.5))
      WRITE (6, 52)
   52 FORMAT (1H1 //////
     1           57X, 18HINPUT ELEMENT DATA /
     2           57X, 18H------------------ /////
     3           3X, 48HELMT NPI   NPJ   NPM   MTRL  DELTA T
     4           48HELMT NPI   NPJ   NPM   MTRL  DELTA T
     5           33HELMT NPI   NPJ   NPM   MTRL  DELTA T /)
      L1 = (NEL + 2) / 3
      DO 53 L2 = 1, L1
      L3 = MINO (L2 + 2 * L1, NEL)
   53 WRITE (6, 54)  (L, NPI (L), NPJ (L), NPM (L),
     1                MAT (L), DELTAT (L),  L = L2, L3, L1)
   54 FORMAT (1H , 5I5, F10.3, 13X, 5I5, F10.3, 13X, 5I5, F10.3)
      DO 540  N=1,NNP
      U(N)=0.
      V(N)=0.
  540 CONTINUE
      WRITE (6, 55)  (N, X (N), Y (N), RX (N), RY (N),
     1                U (N), V (N),  N = 1, NNP)
   55 FORMAT (1H1 //////
     1           58X, 15HINPUT NODE DATA /
     2           58X, 15H--------------- /////
     3           13X, 49HNODE             X                        Y
     4           31HRX                      RY
     5           21H  U                     V //
     6          (I16, X, 6E17.5))
C
C
C
   60 DO GEOMETRIC CALCULATIONS ON THE ELEMENTS.
   61 DO 72 L = 1, NEL
      I = NPI (L)
      J = NPJ (L)
      M = NPM (L)
      BI (L) = Y (J) - Y (M)
      BJ (L) = Y (M) - Y (I)
```

```
      BM (L) = Y (I) - Y (J)
      CI (L) = - X (J) + X (M)
      CJ (L) = - X (M) + X (I)
      CM (L) = - X (I) + X (J)
      AREA = (BJ (L) * CM (L) - BM (L) * CJ (L)) / 2.
      IF (AREA .GT. 1.E-35) GO TO 71
      WRITE (6, 62) L
   62 FORMAT (1H1 ////// 8H ELEMENT, I5, 25H HAS A NON-POSITIVE AREA. /
     1          23H THE CASE IS ABANDONED.)
      GO TO 221
C
C
C
   70 SUM THE TOTAL NODAL POINT FORCES.
   71 MATN = MAT (L)
      IF (PS .EQ. STRESS) SIGTH = 1. - NU (MATN)
      IF (PS .EQ. STRAIN) SIGTH = (1. + NU (MATN))
     1                      * (1. - 2. * NU (MATN))
      SIGTH = - E (MATN) * ALPHA (MATN) * DELTAT (L) / SIGTH
      BODYF = AREA * RHO (MATN) / 3.
      RX (I) = RX (I) - 0.5 * BI (L) * SIGTH
      RY (I) = RY (I) - 0.5 * CI (L) * SIGTH - BODYF
      RX (J) = RX (J) - 0.5 * BJ (L) * SIGTH
      RY (J) = RY (J) - 0.5 * CJ (L) * SIGTH - BODYF
      RX (M) = RX (M) - 0.5 * BM (L) * SIGTH
   72 RY (M) = RY (M) - 0.5 * CM (L) * SIGTH - BODYF
C
C
C
   80 PREPARE TO SUM THE STIFFNESS COEFFICIENTS.
      DO 82 N = 1, NNP
      DO 81 QA = 1, 9
      KXX (N, QA) = 0.
      KXY (N, QA) = 0.
      KYX (N, QA) = 0.
      KYY (N, QA) = 0.
   81 ANP (N, QA) = 0
      ANP (N, 1) = N
   82 NAP (N) = 1
C
C
C
   90 FORM THE STIFFNESS MATRIX FOR THE ELEMENT.
      DO 105 L = 1, 277
      DO 92 P = 1, 3
      DO 91 Q = 1, 6
   91 B (P, Q) = 0.
      DO 92 Q = 1, 3
   92 D (P, Q) = 0.
      B (1, 1) = BI (L)
      B (1, 3) = BJ (L)
      B (1, 5) = BM (L)
      B (2, 2) = CI (L)
      B (2, 4) = CJ (L)
      B (2, 6) = CM (L)
      B (3, 1) = CI (L)
      B (3, 2) = BI (L)
      B (3, 3) = CJ (L)
      B (3, 4) = BJ (L)
      B (3, 5) = CM (L)
      B (3, 6) = BM (L)
      MATN = MAT (L)
      W1 = NU (MATN)
      W2 = 1. - W1
      W3 = 1. + W1
      W4 = 1. - 2. * W1
      IF (PS .EQ. STRAIN) GO TO 93
      W5 = E (MATN) / (W2 * W3)
      W6 = W1
      W7 = W2 / 2.
      GO TO 94
   93 W5 = E (MATN) * W2 / (W3 * W4)
      W6 = W1 / W2
      W7 = W4 / (2. * W2)
   94 AREA = (BJ (L) * CM (L) - BM (L) * CJ (L)) / 2.
      W5 = W5 / (4. * AREA)
      D (1, 1) = W5
      D (1, 2) = W5 * W6
      D (2, 1) = W5 * W6
      D (2, 2) = W5
      D (3, 3) = W5 * W7
      DO 95 P = 1, 3
      DO 95 Q = 1, 6
      W (P, Q) = 0.
      DO 95 O = 1, 3
      IF (P.GT.3) P=3
      IF (Q.GT.6) Q=6
```

```
   95 W (P, Q) = W (P, Q) + D (P, O) * B (O, Q)
      DO 96   P = 1, 6
      DO 96   Q = 1, 6
      K (P, Q) = 0.
      DO 96   O = 1, 3
   96 K (P, Q) = K (P, Q) + B (O, P) * W (O, Q)
C
C
C 100 SUM AND STORE FOR EACH (FORCED) NODE THE NON-ZERO STIFFNESS
C     COEFFICIENTS FOR THE ADJACENT (DISPLACED) NODES.
      LN (1) = NPI (L)
      LN (2) = NPJ (L)
      LN (3) = NPM (L)
      DO 105   P = 1, 3
      NP = LN (P)
      DO 105   Q = 1, 3
      QA = 0
  101 QA = QA + 1
      IF (QA .LE. 9)   GO TO 103
      WRITE (6, 102)   NP
  102 FORMAT (1H1 /////// 5H NODE, I5,
     1          36H HAS MORE THAN EIGHT ADJACENT NODES. /
     2          23H THE CASE IS ABANDONED.)
      GO TO 221
  103 IF (ANP (NP, QA) .EQ. LN (Q))   GO TO 104
      IF (ANP (NP, QA) .NE. 0)   GO TO 101
      ANP (NP, QA) = LN (Q)
      NAP (NP) = QA
  104 KXX (NP, QA) = KXX (NP, QA) + K (2 * P - 1, 2 * Q - 1)
      KXY (NP, QA) = KXY (NP, QA) + K (2 * P - 1, 2 * Q)
      KYX (NP, QA) = KYX (NP, QA) + K (2 * P, 2 * Q - 1)
  105 KYY (NP, QA) = KYY (NP, QA) + K (2 * P, 2 * Q)
C
C
C 110 BY INVERSION OF THE MAIN DIAGONAL STIFFNESS SUBMATRIX, FIND THE
C     FLEXIBILITY SUBMATRIX AT EACH NODE DISPLACED BY FORCES AT ITSELF.
      DO 112   N = 1, NNP
      W1 = KXX (N, 1) * KYY (N, 1) - KXY (N, 1) * KYX (N, 1)
      FXX (N) =   KYY (N, 1) / W1
      FXY (N) = - KXY (N, 1) / W1
      FYX (N) = - KYX (N, 1) / W1
C
C
C
C 130 PERFORM A GAUSS - SEIDEL ITERATION ON THE STIFFNESS MATRIX FOR
C     THE ENTIRE STRUCTURE.
      IF (NCC .GT. NCY)   GO TO 132
      WRITE (6, 131)
  131 FORMAT (1H_ /////
     1          34X, 35HTHE PROGRESS OF THE GAUSS - SEIDEL ,
     2          28HITERATION TOWARD CONVERGENCE /
     3          34X, 35H-----------------------------------,
     4          28H---------------------------- /////
     5          33X, 39HCYCLE                    CONVERGENCE CRITERION,
     6          26H             CURRENT DIFFERENCE)
  132 NC = 0
  133 SD = 0.
      SDD = 0.
      DO 136   N = 1, NNP
      IF (FXX (N) + FYY (N) .EQ. 0.)   GO TO 136
      RX1 = RX (N)
      RY1 = RY (N)
      NAP1 = NAP (N)
      DO 134   QA = 1, NAP1
      NP = ANP (N, QA)
      RX1 = RX1 - KXX (N, QA) * U (NP) - KXY (N, QA) * V (NP)
  134 RY1 = RY1 - KYX (N, QA) * U (NP) - KYY (N, QA) * V (NP)
      IF (FXX (N) .EQ. 0.)   GO TO 135
      DELTAU = FXX (N) * RX1 + FXY (N) * RY1
      U (N) = U (N) + CMEGA * DELTAU
      SD = SD + ABS (U (N))
      SDD = SDD + ABS (DELTAU)
      IF (FYY (N) .EQ. 0.)   GO TO 136
  135 DELTAV = FYX (N) * RX1 + FYY (N) * RY1
      V (N) = V (N) + CMEGA * DELTAV
      SD = SD + ABS (V (N))
      SDD = SDD + ABS (DELTAV)
  136 CONTINUE
      SD = AMAX1 (SD, 1.E-35)
      RELDIF = SDD / SD
C
C
C 140 TEST FOR CONVERGENCE.
      NC = NC + 1
      IF (NCC .GT. NCY .OR. MOD (NC, NCC) .NE. 0)   GO TO 142
```

```
      WRITE (6, 141)  NC, TOLER, RELDIF
  141 FORMAT (1H0, I36, 3X, 2E27.5)
  142 IF (RELDIF .GT. TOLER)  GO TO 144
      WRITE (6, 143)  NC
  143 FORMAT (1H1 ////// 28H THE ITERATION HAS CONVERGED,
     1            6H AFTER, I5, 8H CYCLES.)
      GO TO 151
  144 IF (NC .LT. NCY)  GO TO 133
      WRITE (6, 145)  NC, RELDIF, TOLER
  145 FORMAT (1H1 ////// 28H THE ITERATION HAS COMPLETED, I5, 5H THE ,
     1            52HMAXIMUM NUMBER OF CYCLES WITHOUT CONVERGING, BUT HAS /
     2            34H ACHIEVED A RELATIVE DIFFERENCE OF, E13.5,
     3            33H AGAINST A SPECIFIED TOLERANCE OF, E13.5, 1H. /
     4            43H THE UNCONVERGED RESULTS ARE WRITTEN BELOW.)
C
C
C 150 OUTPUT THE NODE DISPLACEMENTS.
  151 WRITE (6, 152)
  152 FORMAT (1H1 //////
     1            49X, 33HTHE CALCULATED NODE DISPLACEMENTS /
     2            49X, 33H--------------------------------- /////
     3             3X, 47HNODE            U                V
     4                 47HNODE            U                V
     5                 30HNODE            U                V /)
      N1 = (NNP + 2) / 3
      DO 153  N2 = 1, N1
      N3 = MINO (N2 + 2 * N1, NNP)
  153 WRITE (6, 154)  (N, U (N), V (N), N = N2, N3, N1)
  154 FORMAT (1H , I5, 2E16.5, I15, 2E16.5, I15, 2E16.5)
C
C
C 160 CALCULATE THE ELEMENT STRESS COMPONENTS.
      IF (SR .EQ. 0)  GO TO 221
      DO 164  L = 1, NEL
      I = NPI (L)
      J = NPJ (L)
      M = NPM (L)
      AREA = (BJ (L) * CM (L) - BM (L) * CJ (L)) / 2.
      EEXX = (BI (L) * U (I) + BJ (L) * U (J)
     1               + BM(L) * U(M))/ (2.*AREA)
      EEYY = (CI (L) * V (I) + CJ (L) * V (J)
     1               + CM (L) * V (M)) / (2. * AREA)
      EEXY = (CI (L) * U (I) + BI (L) * V (I)
     1       + CJ (L) * U (J) + BJ (L) * V (J)
     2       + CM (L) * U (M) + BM (L) * V (M)) / (2. * AREA)
      MATN = MAT (L)
      W1 = NU (MATN)
      W2 = 1. - W1
      W3 = 1. + W1
      W4 = 1. - 2. * W1
      IF (PS .EQ. STRAIN)  GO TO 161
      W5 = E (MATN) / (W2 * W3)
      W6 = W1
      W7 = W2 / 2.
      GO TO 162
  161 W5 = E (MATN) * W2 / (W3 * W4)
      W6 = W1 / W2
      W7 = W4 / (2. * W2)
  162 ETH = ALPHA (MATN) * DELTAT (L)
      EXX = EEXX - ETH
      EYY = EEYY - ETH
      EXY = EEXY
      SIGLXX (L) = W5 * (EXX + W6 * EYY)
      SIGLYY (L) = W5 * (W6 * EXX + EYY)
      IF (PS .EQ. STRAIN)  GO TO 163
      SIGLZZ (L) = 0.
      GO TO 164
  163 SIGLZZ (L) = W1 * (SIGLXX (L) + SIGLYY (L))
  164 SIGLXY (L) = W5 * W7 * EXY
C
C
C 170 CALCULATE THE ELEMENT PRINCIPAL AND MEAN STRESSES.
      IF (SR .EQ. 2)  GO TO 184
      W1 = (SIGLXX (L) + SIGLYY (L)) / 2.
      W2 = (SIGLXX (L) - SIGLYY (L)) / 2.
      W3 = SIGLXY (L)
      W4 = SQRT (W2 ** 2 + W3 ** 2)
      SIGL1 = W1 + W4
      SIGL2 = W1 - W4
      IF (ABS (W2) .GT. 1.E-35 .OR. ABS (W3) .GT. 1.E-35)  GO TO 171
      ANGLEL = 0.
      GO TO 172
  171 ANGLEL = 28.647889756 * ATAN2 (W3, W2)
  172 SIGL3 = SIGLZZ (L)
      SIGLM = SQRT ((SIGL1 + SIGL2 + SIGL3) ** 2
```

```
     1                         - 3. * (SIGL2 * SIGL3 + SIGL3 * SIGL1 + SIGL1 * SIGL2))
C
C
C 180 OUTPUT THE ELEMENT STRESSES.
      IF (L .NE. 1)  GO TO 182
      WRITE (6, 181)
  181 FORMAT (1H1 //////
     1        27X, 40HTHE CALCULATED ELEMENT STRESS COMPONENTS,
     2             40H AND THE PRINCIPAL AND THE MEAN STRESSES/
     3        27X, 40H-----------------------------------------,
     4             40H----------------------------------------- /////
     5        1X, 45HELEMENT         SIGMXX              SIGMYY          ,
     6            45HSIGMZZ              SIGMXY              ANGLE1       ,
     7            37HSIGM1              SIGM2              SIGMM /)
  182 WRITE (6, 183)  L, SIGLXX (L), SIGLYY (L), SIGLZZ (L),
     1                SIGLXY (L), ANGLEL, SIGL1, SIGL2, SIGLM
  183 FORMAT (1H , I5, 4E16.5, F12.2, X, 3E16.5)
  184 CONTINUE
      IF (SR .EQ. 1)  GO TO 221
C
C
C 190 FIND THE NODE STRESS COMPONENTS.
      DO 212  N = 1, NNP
      SIGNXX = 0.
      SIGNYY = 0.
      SIGNZZ = 0.
      SIGNXY = 0.
      QA = 1
      DO 191  L = 1, NEL
      IF (NPI (L) .NE. N .AND. NPJ (L) .NE. N .AND. NPM (L) .NE. N)
     1                                                 GO TO 191
      SIGNXX = SIGNXX + SIGLXX (L)
      SIGNYY = SIGNYY + SIGLYY (L)
      SIGNZZ = SIGNZZ + SIGLZZ (L)
      SIGNXY = SIGNXY + SIGLXY (L)
      QA = QA + 1
      IF (QA .EQ. NAP (N))  GO TO 192
  191 CONTINUE
  192 W1 = FLOAT (QA - 1)
      SIGNXX = SIGNXX / W1
      SIGNYY = SIGNYY / W1
      SIGNZZ = SIGNZZ / W1
      SIGNXY = SIGNXY / W1
C
C
C 200 CALCULATE THE NODE PRINCIPAL AND MEAN STRESSES.
  201 W1 = (SIGNXX + SIGNYY) / 2.
      W2 = (SIGNXX - SIGNYY) / 2.
      W3 = SIGNXY
      W4 = SQRT (W2 ** 2 + W3 ** 2)
      SIGN1 = W1 + W4
      SIGN2 = W1 - W4
      IF (ABS (W2) .GT. 1.E-35 .OR. ABS (W3) .GT. 1.E-35)  GO TO 202
      ANGLEN = 0.
      GO TO 203
  202 ANGLEN = 28.647889756 * ATAN2 (W3, W2)
  203 SIGN3 = SIGNZZ
      SIGNM = SQRT ((SIGN1 + SIGN2 + SIGN3) ** 2
     1        - 3. * (SIGN2 * SIGN3 + SIGN3 * SIGN1 + SIGN1 * SIGN2))
C
C
C 210 OUTPUT THE NODE STRESSES.
      IF (N .NE. 1)  GO TO 212
      WRITE (6, 211)
  211 FORMAT (1H1 //////
     1        29X, 37HTHE CALCULATED NODE STRESS COMPONENTS,
     2             40H AND THE PRINCIPAL AND THE MEAN STRESSES /
     3        29X, 37H-------------------------------------,
     4             40H---------------------------------------- /////
     5        3X, 43HNODE            SIGMXX              SIGMYY          ,
     6            45HSIGMZZ              SIGMXY              ANGLE1       ,
     7            37HSIGM1              SIGM2              SIGMM /)
  212 WRITE (6, 213)  N, SIGNXX, SIGNYY, SIGNZZ, SIGNXY,
     1                ANGLEN, SIGN1, SIGN2, SIGNM
  213 FORMAT (1H , I5, 4E16.5, F12.2, X, 3E16.5)
                                      X, 3E16.5)
C
C
C
C 220 RETURN TO THE BEGINNING OF THE PROGRAM FOR THE NEXT CASE.
  221 GO TO 11
      END
```

```
      PROGRAM STREN3(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C    0 SPECIFICATION AND DATA INITIALIZATION STATEMENTS.
      INTEGER   TITLE,      PS,          SR,        OA,        O,        NF,
     1          CASCTL,     STRESS,      WI,        AMP,       P,
     2          STRAIN,     COND,                   Q
      REAL      NU,         K,           KXX,
     1                                   KXY,
     2                                   KYX,
     3                                   KYY
      COMMON    TITLE(13),    NPI(549),       X(340),
     1          E(1),         NPJ(549),       Y(340),
     2          NU(1)   ,     NPM(549),       RX(340),
     3          RHO(1),       MAT(549),       RY(340),
     4          ALPHA(1),     DELTAT(549),    U(340),
     5                                        V(340),
     1          BI(549),      KXX(340,9),     AMP(340,9),   SIGLXX(549),
     2          BJ(549),      KXY(340,9),     NAP(340),     SIGLYY(549),
     3          BM(549),      KYX(340,9),     LN(3),        SIGLZZ(549),
     4          CI(549),      KYY(340,9),     B(3,6),       SIGLXY(549),
     5          CJ(549),      FXX(340),       C(3,3),
     6          CM(549),      FXY(340),       W(3,6),
     7                        FYX(340),       K(6,6),
     8                        FYY(340)
      DATA  CASCTL, STRESS, STRAIN / 6HNFCASE, 6HSTRESS, 6HSTRAIN /
C
C
C   10 INPUT THE CASE TITLE BUT STOP IF THERE IS NO FURTHER CASE.
   11 READ (5, 12)  TITLE
   12 FORMAT (13A6)
      IF (TITLE (1) .EQ. CASCTL)  STOP
C
C
C   20 INPUT THE CONTROL PARAMETERS AND PREPARE TO INPUT THE ARRAY DATA.
      READ (5, 21)  NEL, NNP,     NMAT, NCY, NCC,
     1              TOLER, OMEGA, PS, SR, WI
   21 FORMAT (5I5, E13.5, F7.3, 2X, A6, 2I3)
      DO 22  NM = 1, NMAT
   22 E (NM) = -1.
      DO 23  L = 1, NEL
   23 NPI (L) = -1
      DO 24  N = 1, NNP
   24 X (N) = -0.123456789E35
C
C
C   30 INPUT AND CHECK THE ARRAY DATA.
      READ (5, 31)  (MATN, E (MATN), NU (MATN),
     1               RHO (MATN), ALPHA (MATN),  NM = 1, NMAT)
   31 FORMAT (I5, 4E13.5)
      DO 33  NM = 1, NMAT
      IF (E (NM) .NE. -1.)  GO TO 33
      WRITE(6,333) NM
  333 FORMAT(1X,7H E(NM)=, I5)
      WRITE (6, 32)  TITLE
   32 FORMAT (1H1 ////// 5H IN ', 13A6, 1H' /
     1            36H THERE IS A MISSING INPUT DATA CARD.,
     2            24H  THE JOB IS ABANDONED.)
      STOP
   33 CONTINUE
      READ (5, 34)  (LE, NPI (LE), NPJ (LE), NPM (LE),
     1               DELTAT(LE), MAT(LE),  L=1,NEL )
   34 FORMAT(4I5,F10.3,I5)
      DO 35  L = 1, NEL
      IF (NPI (L) .NE. -1)  GO TO 35
      WRITE(6,334)  L
  334 FORMAT(1X, 3H L=, I5)
      WRITE (6, 32)  TITLE
      STOP
   35 CONTINUE
      DO 350  N=1,68
      READ(5,36)  A1,B1,NP1,A2,B2,NP2,A3,B3,NP3,A4,B4,NP4,A5,B5,NP5
   36 FORMAT(5(F4.0,1X,F4.0,I4,3X))
      WRITE(6,371) NP1,NP2,NP3,NP4,NP5
  371 FORMAT(1X,5(I20))
      X(NP1)=(A1-1698.)/797.EX(NP2)=(A2-1698.)/797.
      X(NP3)=(A3-1698.)/797.EX(NP4)=(A4-1698.)/797.
      X(NP5)=(A5-1698.)/797.
      Y(NP1)=(B1-1477.)/797. E Y(NP2)=(B2-1477.)/797.
      Y(NP3)=(B3-1477.)/797. E Y(NP4)=(B4-1477.)/797.
      Y(NP5)=(B5-1477.)/797.
  350 CONTINUE
      DO 361  N=1,NNP
      RX(N)=0.
      RY(N)=0.
  361 CONTINUE
      DO 37  N = 1, NNP
```

```
      IF (X (N) .NE. -(.123456789E35)  GO TO 37
      WRITE(6,335) N
  335 FORMAT(1X,  3H N=, I5)
      WRITE (6, 32) TITLE
      STOP
   37 CONTINUE
C
C
C
   40 OUTPUT THE TITLE AND THE CONTROL PARAMETERS.
      WRITE (6, 41)
   41 FORMAT (1H2 //////
     1          45X, 41HTHE CASE TITLE AND THE CONTROL PARAMETERS /
     2          45X, 41H------------------------------------------ //////)
      WRITE (6, 42)  TITLE
   42 FORMAT (1H , 13A6)
      WRITE (6, 43)  NEL, NNP, NBR, NMAT, NCY, NCC, TOLER, OMEGA, PS
   43 FORMAT (26H0THE NUMBER OF ELEMENTS IS, I5, 1H. /
     1          39H0THE NUMBER OF NODAL POINTS IS, I5, 1H. /
     2          43H0THE NUMBER OF RESTRAINED BOUNDARY NODES IS, I5, 1H. /
     3          37H0THE NUMBER OF DIFFERENT MATERIALS IS, I5, 1H. /
     4          23H0THE ITERATION LIMIT IS, I5, 8H CYCLES. /
     5          36H0THE INTERVAL FOR PROGRESS OUTPUT IS, I5, 8H CYCLES. /
     6          29H0THE CONVERGENCE TOLERANCE IS, E13.5, 1H. /
     7          38H0THE OVER-RELAXATION FACTOR IS, F7.3, 1H. /
     8          37H0THE CASE HAS THE CONDITION OF PLANE , A6, 1H. /
     9          23H0STRESSES ARE FOUND FOR)
      IF (SR .EQ. 0) WRITE (6, 44)
   44 FORMAT (1H+, 23X, 42HNEITHER THE ELEMENTS NOR THE NODAL POINTS.)
      IF (SR .EQ. 1) WRITE (6, 45)
   45 FORMAT (1H+, 23X, 18HTHE ELEMENTS ONLY.)
      IF (SR .EQ. 2) WRITE (6, 46)
   46 FORMAT (1H+, 23X, 22HTHE NODAL POINTS ONLY.)
      IF (SR .EQ. 3) WRITE (6, 47)
   47 FORMAT (1H+, 23X, 39HBOTH THE ELEMENTS AND THE NODAL POINTS.)
      IF (WI .EQ. 0) WRITE (6, 48)
   48 FORMAT (37H0THE INPUT ARRAY DATA ARE NOT OUTPUT.)
      IF (WI .EQ. 1) WRITE (6, 49)
   49 FORMAT (33H0THE INPUT ARRAY DATA ARE OUTPUT.)
C
C
C
   50 OUTPUT THE ARRAY DATA.
      IF (WI .EQ. 0) GO TO 61
      WRITE (6, 51)  (NM, E (NM), NU (NM), RHO (NM),
     1                ALPHA (NM),  NM = 1, NMAT)
   51 FORMAT (1H1 //////
     1          56X, 19HINPUT MATERIAL DATA /
     2          56X, 19H------------------- /////
     3          28X, 42HMATERIAL   ELASTIC MODULUS   POISSON'S RATIO,
     4          34H    DENSITY        THML EXPN COEFF //
     5          (I33, X, 4E17.5))
      WRITE (6, 52)
   52 FORMAT (1H1 //////
     1          57X, 18HINPUT ELEMENT DATA /
     2          57X, 18H------------------ /////
     3          3X, 48HELMT NPI   NPJ   NPM   MTRL   DELTA T
     4          48HELMT NPI   NPJ   NPM   MTRL   DELTA T
     5          33HELMT NPI   NPJ   NPM   MTRL   DELTA T /)
      L1 = (NEL + 2) / 3
      DO 55  L2 = 1, L1
      L3 = MIN0 (L2 + 2 * L1, NEL)
   53 WRITE (6, 54)  (L, NPI (L), NPJ (L), NPM (L),
     1                MAT (L), DELTAT (L),  L = L2, L3, L1)
   54 FORMAT (1H , 5I5, F10.3, 13X, 5I5, F10.3, 13X, 5I5, F10.3)
      DO 540  N=1,NNP
      U(N)=0.
      V(N)=0.
  540 CONTINUE
      WRITE (6, 55)  (N, X (N), Y (N), RX (N), RY (N),
     1                U (N), V (N),  N = 1, NNP)
   55 FORMAT (1H1 //////
     1          53X, 15HINPUT NODE DATA /
     2          58X, 15H--------------- /////
     3          13X, 49HNODE           X              RY             Y
     4          31HRX                             V //
     5          21H    U
     6          (I16, X, 6E17.5))
C
C
C
   60 DO GEOMETRIC CALCULATIONS ON THE ELEMENTS.
   61 DO 72  L = 1, NEL
      I = NPI (L)
      J = NPJ (L)
      M = NPM (L)
      BI (L) = Y (J) - Y (M)
      BJ (L) = Y (M) - Y (I)
```

```
      BM (L) = Y (I) - Y (J)
      CI (L) = - X (J) + X (M)
      CJ (L) = - X (M) + X (I)
      CM (L) = - X (I) + X (J)
      AREA = (BJ (L) * CM (L) - BM (L) * CJ (L)) / 2.
      IF (AREA .GT. 1.E-35) GO TO 71
      WRITE (6, 62) L
   62 FORMAT (1H1 ////// 8H ELEMENT, I5, 25H HAS A NON-POSITIVE AREA. /
     1           23H THE CASE IS ABANDONED.)
      GO TO 221
C
C
C
   70 SUM THE TOTAL NODAL POINT FORCES.
   71 MATN = MAT (L)
      IF (PS .EQ. STRESS) SIGTH = 1. - NU (MATN)
      IF (PS .EQ. STRAIN) SIGTH = (1. + NU (MATN))
     1                          * (1. - 2. * NU (MATN))
      SIGTH = - E (MATN) * ALPHA (MATN) * DELTAT (L) / SIGTH
      BODYF = AREA * RHO (MATN) / 3.
      RX (I) = RX (I) - 0.5 * BI (L) * SIGTH
      RY (I) = RY (I) - 0.5 * CI (L) * SIGTH - BODYF
      RX (J) = RX (J) - 0.5 * BJ (L) * SIGTH
      RY (J) = RY (J) - 0.5 * CJ (L) * SIGTH - BODYF
      RX (M) = RX (M) - 0.5 * BM (L) * SIGTH
   72 RY (M) = RY (M) - 0.5 * CM (L) * SIGTH - BODYF
C
C
C
   80 PREPARE TO SUM THE STIFFNESS COEFFICIENTS.
      DO 82  N = 1, NNP
      DO 81  QA = 1, 9
      KXX (N, QA) = 0.
      KXY (N, QA) = 0.
      KYX (N, QA) = 0.
      KYY (N, QA) = 0.
   81 ANP (N, QA) = 0
      ANP (N, 1) = N
   82 NAP (N) = 1
C
C
C
   90 FORM THE STIFFNESS MATRIX FOR THE ELEMENT.
      DO 105  L = 1, NEL
      DO 92  P = 1, 3
      DO 91  Q = 1, 6
   91 B (P, Q) = 0.
      DO 92  Q = 1, 3
   92 D (P, Q) = 0.
      B (1, 1) = BI (L)
      B (1, 3) = BJ (L)
      B (1, 5) = BM (L)
      B (2, 2) = CI (L)
      B (2, 4) = CJ (L)
      B (2, 6) = CM (L)
      B (3, 1) = CI (L)
      B (3, 2) = BI (L)
      B (3, 3) = CJ (L)
      B (3, 4) = BJ (L)
      B (3, 5) = CM (L)
      B (3, 6) = BM (L)
      MATN = MAT (L)
      W1 = NU (MATN)
      W2 = 1. - W1
      W3 = 1. + W1
      W4 = 1. - 2. * W1
      IF (PS .EQ. STRAIN) GO TO 93
      W5 = E (MATN) / (W2 * W3)
      W6 = W1
      W7 = W2 / 2.
      GO TO 94
   93 W5 = E (MATN) * W2 / (W3 * W4)
      W6 = W1 / W2
      W7 = W4 / (2. * W2)
   94 AREA = (BJ (L) * CM (L) - BM (L) * CJ (L)) / 2.
      W5 = W5 / (4. * AREA)
      D (1, 1) = W5
      D (1, 2) = W5 * W6
      D (2, 1) = W5 * W6
      D (2, 2) = W5
      D (3, 3) = W5 * W7
      DO 95  P = 1, 3
      DO 95  Q = 1, 6
      H (P, Q) = 0.
      DO 95  O = 1, 3
      IF(P.GT.3)  P=3
      IF(O.GT.6)  O=6
      IF(O.GT.3)  O=3
```

```
   95 W (P, Q) = W (P, Q) + D (P, O) * B (O, Q)
      DO 96   P = 1, 6
      DO 96   Q = 1, 6
      K (P, Q) = 0.
      DO 96   O = 1, 3
   96 K (P, Q) = K (P, Q) + B (O, P) * W (O, Q)
C
C
C 100 SUM AND STORE FOR EACH (FORCED) NODE THE NON-ZERO STIFFNESS
C     COEFFICIENTS FOR THE ADJACENT (DISPLACED) NODES.
      LN (1) = NPI (L)
      LN (2) = NPJ (L)
      LN (3) = NPM (L)
      DO 105   P = 1, 3
      NP = LN (P)
      DO 105   Q = 1, 3
      QA = 0
  101 QA = QA + 1
      IF (QA .LE. 9)   GO TO 103
      WRITE (6, 102)   NP
  102 FORMAT (1H- ////// 5H NODE, I5,
     1          36H HAS MORE THAN EIGHT ADJACENT NODES. /
     2          23H THE CASE IS ABANDONED.)
      GO TO 221
  103 IF (ANP (NP, QA) .EQ. LN (Q))   GO TO 104
      IF (ANP (NP, QA) .NE. 0)   GO TO 101
      ANP (NP, QA) = LN (Q)
      NAP (NP) = QA
  104 KXX (NP, QA) = KXX (NP, QA) + K (2 * P - 1, 2 * Q - 1)
      KXY (NP, QA) = KXY (NP, QA) + K (2 * P - 1, 2 * Q)
      KYX (NP, QA) = KYX (NP, QA) + K (2 * P, 2 * Q - 1)
  105 KYY (NP, QA) = KYY (NP, QA) + K (2 * P, 2 * Q)
C
C
C 110 BY INVERSION OF THE MAIN DIAGONAL STIFFNESS SUBMATRIX, FIND THE
C     FLEXIBILITY SUBMATRIX AT EACH NODE DISPLACED BY FORCES AT ITSELF.
      DO 112   N = 1, NNP
      W1 = KXX (N, 1) * KYY (N, 1) - KXY (N, 1) * KYX (N, 1)
      FXX (N) =   KYY (N, 1) / W1
      FXY (N) = - KXY (N, 1) / W1
      FYX (N) = - KYX (N, 1) / W1
  112 FYY (N) = KXX (N, 1) / W1
C
C
C
C
C 130 PERFORM A GAUSS - SEIDEL ITERATION ON THE STIFFNESS MATRIX FOR
C     THE ENTIRE STRUCTURE.
      IF (NCC .GT. NCY)   GO TO 132
      WRITE (6, 131)
  131 FORMAT (1H- //////
     1          34X, 35HTHE PROGRESS OF THE GAUSS - SEIDEL ,
     2               28HITERATION TOWARD CONVERGENCE /
     3          34X, 35H-----------------------------------,
     4               28H-------------------------- /////
     5          33X, 39HCYCLE                 CONVERGENCE CRITERION,
     6               26H          CURRENT DIFFERENCE)
  132 NC = 0
  133 SD = 0.
      SDD = 0.
      DO 136   N = 1, NNP
      IF (FXX (N) + FYY (N) .EQ. 0.)   GO TO 136
      RX1 = RX (N)
      RY1 = RY (N)
      NAP1 = NAP (N)
      DO 134   QA = 1, NAP1
      NP = ANP (N, QA)
      RX1 = RX1 - KXX (N, QA) * U (NP) - KXY (N, QA) * V (NP)
  134 RY1 = RY1 - KYX (N, QA) * U (NP) - KYY (N, QA) * V (NP)
      IF (FXX (N) .EQ. 0.)   GO TO 135
      DELTAU = FXX (N) * RX1 + FXY (N) * RY1
      U (N) = U (N) + OMEGA * DELTAU
      SD = SD + ABS (U (N))
      SDD = SDD + ABS (DELTAU)
      IF (FYY (N) .EQ. 0.)   GO TO 136
  135 DELTAV = FYX (N) * RX1 + FYY (N) * RY1
      V (N) = V (N) + OMEGA * DELTAV
      SD = SD + ABS (V (N))
      SDD = SDD + ABS (DELTAV)
  136 CONTINUE
      SD = AMAX1 (SD, 1.E-35)
      RELDIF = SDD / SD
C
```

```
      NC = NC + 1
      IF (NCC .GT. NCY .OR. MOD (NC, NCC) .NE. 0)   GO TO 142
      WRITE (6, 141) NC, TOLER, RELDIF
  141 FORMAT (1H0, I36, 3X, 2E27.5)
  142 IF (RELDIF .GT. TOLER)   GO TO 144
      WRITE (6, 143) NC
  143 FORMAT (1H1 ////// 28H THE ITERATION HAS CONVERGED,
     1          6H AFTER, I5, 8H CYCLES.)
      GO TO 151
  144 IF (NC .LT. NCY)   GO TO 133
      WRITE (6, 145) NC, RELDIF, TOLER
  145 FORMAT (1H1 ////// 28H THE ITERATION HAS COMPLETED, I5, 5H THE ,
     1          52HMAXIMUM NUMBER OF CYCLES WITHOUT CONVERGING, BUT HAS /
     2          34H ACHIEVED A RELATIVE DIFFERENCE OF, E13.5,
     3          33H AGAINST A SPECIFIED TOLERANCE OF, E13.5, 1H. /
     4          43H THE UNCONVERGED RESULTS ARE WRITTEN BELOW.)
C
C
C 150 OUTPUT THE NODE DISPLACEMENTS.
  151 WRITE (6, 152)
  152 FORMAT (1H1 //////
     1          49X, 33HThE CALCULATED NODE DISPLACEMENTS /
     2          49X, 33H--------------------------------- /////
     3          3X, 47HNODE                 U                  V
     4          47HNODE                 U                  V
     5          30HNODE                 U                  V /)
      N1 = (NNP + 2) / 3
      DO 153 N2 = 1, N1
      N3 = MINO (N2 + 2 * N1, NNP)
  153 WRITE (6, 154) (N, U (N), V (N),  N = N2, N3, N1)
  154 FORMAT (1H , I5, 2E16.5, I15, 2E16.5, I15, 2E16.5)
C
C
C 160 CALCULATE THE ELEMENT STRESS COMPONENTS.
      IF (SR .EQ. 0)   GO TO 221
      DO 184 L = 1, NEL
      I = NPI (L)
      J = NPJ (L)
      M = NPM (L)
      AREA = (BJ (L) * CM (L) - BM (L) * CJ (L)) / 2.
      EEXX = (BI (L) * U (I) + BJ (L) * U (J)
     1          + BM (L) * U (M)) / (2.*AREA)
      EEYY = (CI (L) * V (I) + CJ (L) * V (J)
     1          + CM (L) * V (M)) / (2. * AREA)
      EEXY = (CI (L) * U (I) + BI (L) * V (I)
     1          + CJ (L) * U (J) + BJ (L) * V (J)
     2          + CM (L) * U (M) + BM (L) * V (M)) / (2. * AREA)
      MATN = MAT (L)
      W1 = NU (MATN)
      W2 = 1. - W1
      W3 = 1. + W1
      W4 = 1. - 2. * W1
      IF (PS .EQ. STRAIN)  GO TO 161
      W5 = E (MATN) / (W2 * W3)
      W6 = W1
      W7 = W2 / 2.
      GO TO 162
  161 W5 = E (MATN) * W2 / (W3 * W4)
      W6 = W1 / W2
      W7 = W4 / (2. * W2)
  162 ETH = ALPHA (MATN) * DELTAT (L)
      EXX = EEXX - ETH
      EYY = EEYY - ETH
      EXY = EEXY
      SIGLXX (L) = W5 * (EXX + W6 * EYY)
      SIGLYY (L) = W5 * (W6 * EXX + EYY)
      IF (PS .EQ. STRAIN)  GO TO 163
      SIGLZZ (L) = 0.
      GO TO 164
  163 SIGLZZ (L) = W1 * (SIGLXX (L) + SIGLYY (L))
  164 SIGLXY (L) = W5 * W7 * EXY
C
C
C 170 CALCULATE THE ELEMENT PRINCIPAL AND MEAN STRESSES.
      IF (SR .EQ. 2)   GO TO 184
      W1 = (SIGLXX (L) + SIGLYY (L)) / 2.
      W2 = (SIGLXX (L) - SIGLYY (L)) / 2.
      W3 = SIGLXY (L)
      W4 = SQRT (W2 ** 2 + W3 ** 2)
      SIGL1 = W1 + W4
      SIGL2 = W1 - W4
      IF (ABS (W2) .GT. 1.E-35 .OR. ABS (W3) .GT. 1.E-35)   GO TO 171
      ANGLEL = 0.
```

```
  172 SIGL3 = SIGLZZ (L)
      SIGLM = SQRT ((SIGL1 + SIGL2 + SIGL3) ** 2
     1            - 3. * (SIGL2 * SIGL3 + SIGL3 * SIGL1 + SIGL1 * SIGL2))
C
C
C 180 OUTPUT THE ELEMENT STRESSES.
      IF (L .NE. 1)  GO TO 182
      WRITE (6, 181)
  181 FORMAT (1H1 //////
     1            27X, 40HTHE CALCULATED ELEMENT STRESS COMPONENTS,
     2                 40H AND THE PRINCIPAL AND THE MEAN STRESSES /
     3            27X, 40H----------------------------------------,
     4                 40H----------------------------------------  //////
     5            1X, 45HELEMENT         SIGMXX              SIGMYY        ,
     6                45HSIGMZZ              SIGMXY              ANGLE1      ,
     7                37HSIGM1               SIGM2               SIGMM /)
  182 WRITE (6, 183)  L, SIGLXX (L), SIGLYY (L), SIGLZZ (L),
     1                SIGLXY (L), ANGLEL, SIGL1, SIGL2, SIGLM
  183 FORMAT (1H , I5, 4E16.5, F12.2, X, 3E16.5)
  184 CONTINUE
      IF (SR .EQ. 1)  GO TO 221
C
C
C 190 FIND THE NODE STRESS COMPONENTS.
      DO 212  N = 1, NNP
      SIGNXX = 0.
      SIGNYY = 0.
      SIGNZZ = 0.
      SIGNXY = 0.
      QA = 1
      DO 191  L = 1, NEL
      IF (NPI (L) .NE. N .AND. NPJ (L) .NE. N .AND. NPM (L) .NE. N)
     1                                                       GO TO 191
      SIGNXX = SIGNXX + SIGLXX (L)
      SIGNYY = SIGNYY + SIGLYY (L)
      SIGNZZ = SIGNZZ + SIGLZZ (L)
      SIGNXY = SIGNXY + SIGLXY (L)
      QA = QA + 1
      IF (QA .EQ. NAP (N))  GO TO 192
  191 CONTINUE
  192 W1 = FLOAT (QA - 1)
      SIGNXX = SIGNXX / W1
      SIGNYY = SIGNYY / W1
      SIGNZZ = SIGNZZ / W1
      SIGNXY = SIGNXY / W1
C
C
C 200 CALCULATE THE NODE PRINCIPAL AND MEAN STRESSES.
  201 W1 = (SIGNXX + SIGNYY) / 2.
      W2 = (SIGNXX - SIGNYY) / 2.
      W3 = SIGNXY
      W4 = SQRT (W2 ** 2 + W3 ** 2)
      SIGN1 = W1 + W4
      SIGN2 = W1 - W4
      IF (ABS (W2) .GT. 1.E-35 .OR. ABS (W3) .GT. 1.E-35)  GO TO 202
      ANGLEN = 0.
      GO TO 203
  202 ANGLEN = 28.647889756 * ATAN2 (W3, W2)
  203 SIGN3 = SIGNZZ
      SIGNM = SQRT ((SIGN1 + SIGN2 + SIGN3) ** 2
     1            - 3. * (SIGN2 * SIGN3 + SIGN3 * SIGN1 + SIGN1 * SIGN2))
C
C
C 210 OUTPUT THE NODE STRESSES.
      IF (N .NE. 1)  GO TO 212
      WRITE (6, 211)
  211 FORMAT (1H1 //////
     1            29X, 37HTHE CALCULATED NODE STRESS COMPONENTS,
     2                 40H AND THE PRINCIPAL AND THE MEAN STRESSES /
     3            29X, 37H-------------------------------------,
     4                 40H---------------------------------------- //////
     5            3X, 43HNODE            SIGMXX              SIGMYY        ,
     6                45HSIGMZZ              SIGMXY              ANGLE1      ,
     7                37HSIGM1               SIGM2               SIGMM /)
  212 WRITE (6, 213)  N, SIGNXX, SIGNYY, SIGNZZ, SIGNXY,
     1                ANGLEN, SIGN1, SIGN2, SIGNM
  213 FORMAT (1H , I5, 4E16.5, F12.2, X, 3E16.5)
                                      X, 3E16.5)
C
C
C 220 RETURN TO THE BEGINNING OF THE PROGRAM FOR THE NEXT CASE.
  221 GO TO 11
      END
```

APPENDIX 6

Experimental Determination of Stresses

6.1 Determination of Apparent Strain for Temperatures above $180^\circ C$

Strains conform to the folllowing designations:

ε_I = strain indicated by strain gauge.

ε_R = real strain due to thermal expansion of the material.

ε_A = apparent strain due to heating of the gauge (above 180°).

The indicated strain will then be:

$$\varepsilon_I = \varepsilon_R + \varepsilon_A \qquad\qquad (A6.1.1)$$

The relation is valid for the stresses

$$\sigma_I = \sigma_R + \sigma_A \qquad\qquad (A6.1.2)$$

Therefore the real strain and stress

$$\varepsilon_R = \varepsilon_I - \varepsilon_A \qquad\qquad (A6.1.3)$$

and

$$\sigma_R = \sigma_I - \sigma_A \qquad\qquad (A6.1.4)$$

The figure below gives the relation between apparent strain and temperature. It can be seen that the gauge is self compensating up to $180^\circ C$ since the apparent strain is negative and decreases with temperature. For temperatures over $180^\circ C$ the apparent strain increases, that is the gauge expands, and therefore the indicated strain is not the real strain but the combination of the real and apparent strains as it is described in Equation (A6.1.1).
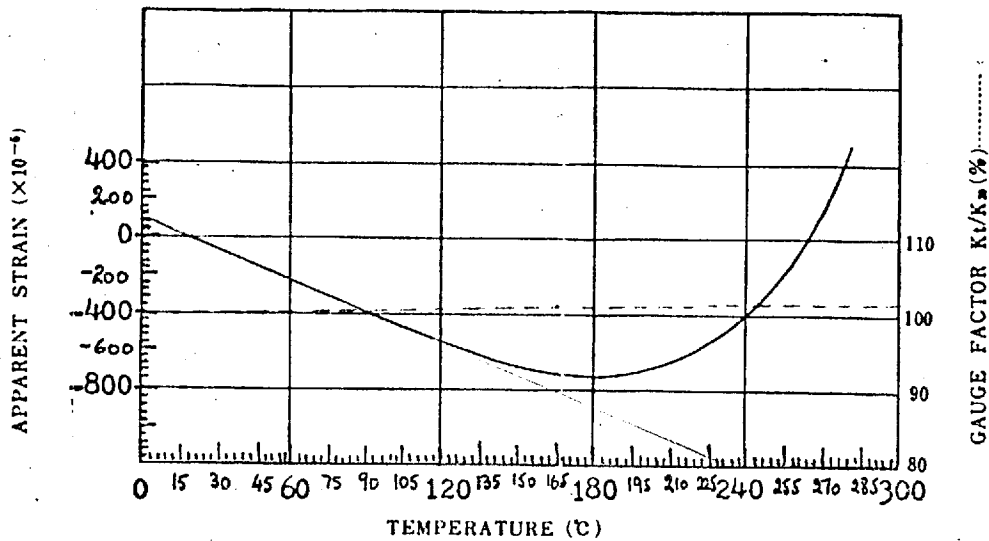
Tokyo Sokki Kenkyujo Co., Ltd.

TSK Co. Ltd advise that no correction needed up to 180°C for gauge apparent strain, due to self compensating feature, persumably non-linear at upper end.

FIG.A6.1.1

Calculations

At time interval 12 minutes during heating-up the following rosettes along the nozzle have temperatures above 180°C.

Rosette	Gauge	Temperature °C	Degrees above 180°C
1	1,2,3	198.0	18.0
2	4,5,6	197.9	17.9
3	7,8,9	197.9	17.9
4	10,11,12	197.8	17.8
5	13,14,15	189.4	9.4
6	16,17,18	188.4	8.4
7	19,20,21	186.3	6.3

The determination of the apparent strain can be calculated as follows:

For rosette (1) the temperature is 198.0°C. The apparent strain for 180°C, read from Fig.A6.1.1, is -720×10^{-6} and for 198.0°C is -680×10^{-6}. Therefore the apparent strain is:

$\varepsilon_A = -680 \times 10^{-6} - (-720 \times 10^{-6}) = 40 \times 10^{-6}$. For the rest of the rosettes the calculation is similar.

Rosette	Gauge	Apparent Strain
1	1,2,3	40×10^{-6}
2	4,5,6	33×10^{-6}
3	7,8,9	33×10^{-6}
4	10,11,12	28×10^{-6}
5	13,14,15	10×10^{-6}
6	16,17,18	7×10^{-6}
7	19,20,21	5×10^{-6}

6.2 Determination of Direct Stresses and Strains. Principal Strains and Stresses, Direction Angles and Maximum Shear Stresses

The determination of all the strain and stress components of the experimental results is performed by the computer program EXPER2 which was constructed for this purpose. The various processes and their mathematical analysis is as follows.

1. The program converts the millivolt readings recorded by the Data-Log into strain (in/in).

    This is done by using the relationship

$$\varepsilon_{indicated} = \frac{1}{250(F)} \times \frac{\Delta V}{V} \qquad (A6.2.1)$$

where:

F = the gauge factor

ΔV = voltage indicated in mV (10^{-3} volts)

V = voltage across the bridge in volts

(This is set to 6 volts.)

The conversion is performed for all 45 gauges for every one minute interval during heating-up and cooling-down.

2. Calculate the direct stresses for each gauge by using the relationship:

$$\sigma = E\varepsilon \qquad \text{(A6.2.2)}$$

3. Calculate principal strains for all fifteen rosettes.
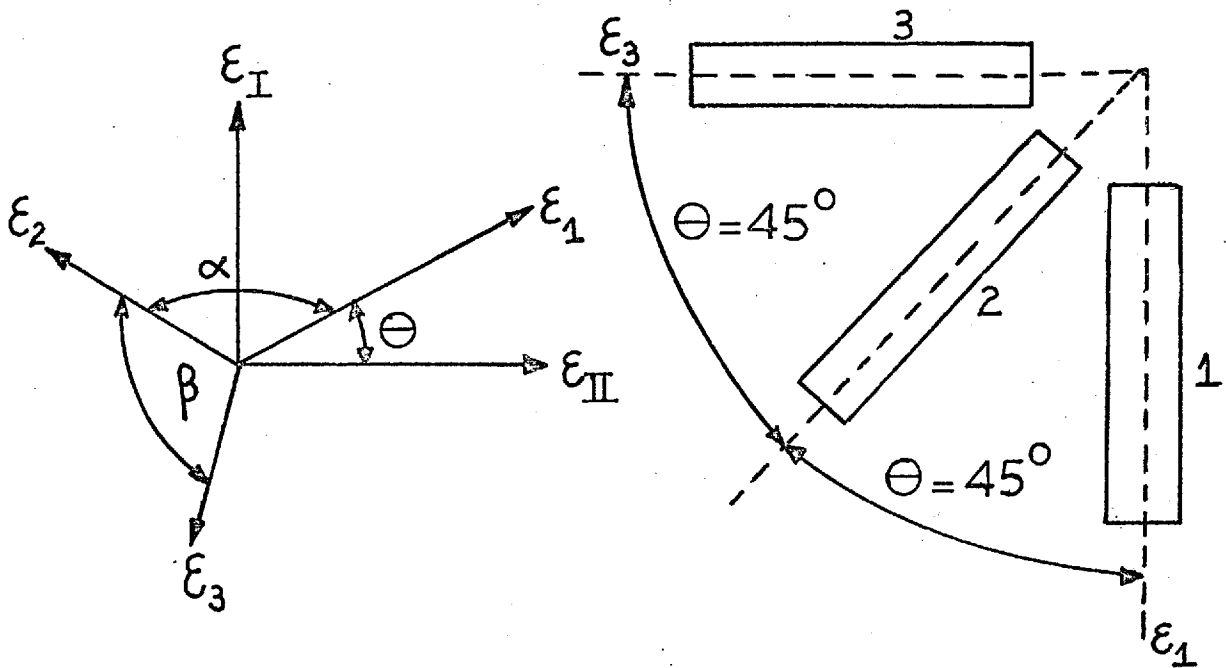
If the principal strains in a two-dimensional system are ε_I and ε_{II} then the direct strains ε_1, ε_2, ε_3 in directions inclined at θ, $(\theta + \alpha)$, $(\theta + \alpha + \beta)$ to ε_I are

$$\varepsilon_1 = \tfrac{1}{2}(\varepsilon_I + \varepsilon_{II}) + \tfrac{1}{2}(\varepsilon_I - \varepsilon_{II})\cos 2\theta$$

$$\varepsilon_2 = \tfrac{1}{2}(\varepsilon_I + \varepsilon_{II}) + \tfrac{1}{2}(\varepsilon_I - \varepsilon_{II})\cos 2(\theta + \alpha) \qquad \text{(A6.2.3)}$$

$$\varepsilon_3 = \tfrac{1}{2}(\varepsilon_I + \varepsilon_{II}) + \tfrac{1}{2}(\varepsilon_I - \varepsilon_{II})\cos 2(\theta + \alpha + \beta)$$

In practice the directions of the principal strains are not known usually but if the three direct strains ε_1, ε_2, ε_3 are measured in known directions, then the three unknowns in Equation (A6.2.3) are

$$\varepsilon_I, \ \varepsilon_{II} \ \text{and} \ \theta$$

Now if three strain gauges are arranged so that $\alpha = \beta = 45°$ they form a $45°$ rosette like the one in the figure which is used for this work. Equations (A6.2.3) become

$$\varepsilon_1 = \tfrac{1}{2}(\varepsilon_I + \varepsilon_{II}) + \tfrac{1}{2}(\varepsilon_I - \varepsilon_{II})\cos2\theta$$

$$\varepsilon_2 = \tfrac{1}{2}(\varepsilon_I + \varepsilon_{II}) - \tfrac{1}{2}(\varepsilon_I - \varepsilon_{II})\sin2\theta$$

$$\varepsilon_3 = \tfrac{1}{2}(\varepsilon_I + \varepsilon_{II}) - \tfrac{1}{2}(\varepsilon_I - \varepsilon_{II})\cos2\theta$$

On eliminating θ from these equations we find that ε_I, ε_{II} are the roots of the quadratic equation

$$\varepsilon^2 - (\varepsilon_1 + \varepsilon_3)\varepsilon - \tfrac{1}{4}(\varepsilon_1 + \varepsilon_3 - 2\varepsilon_2)^2 = 0 \qquad (A6.2.4)$$

and the two principal strains are given by

$$\varepsilon_I = \{(\varepsilon_1 + \varepsilon_3) + \sqrt{(\varepsilon_1 + \varepsilon_3)^2 + (\varepsilon_1 + \varepsilon_3 - 2\varepsilon_2)^2}\}/2$$

$$\varepsilon_{II} = (\varepsilon_1 + \varepsilon_3) - \sqrt{(\varepsilon_1 + \varepsilon_3)^2 + (\varepsilon_1 + \varepsilon_3 - 2\varepsilon_2)^2}\}/2$$

4. Calculate the principal stresses for all fifteen rosettes.

When the principal strains ε_I and ε_{II} have been estimated, the corresponding principal stresses are deduced from the relations

$$\sigma_I = \frac{E}{1 - v^2}(\varepsilon_I + v \ \varepsilon_{II}) \qquad (A6.2.5)$$

$$\sigma_{II} = \frac{E}{1 - v^2}(\varepsilon_{II} + v \ \varepsilon_I) \qquad (A6.2.6)$$

5. Determine the angle θ between the directions of ε_I and ε_1 measured clockwise from the direction of ε_I.

The equation for the angle θ is

$$\tan2\theta = \frac{2\varepsilon_2 - \varepsilon_1 - \varepsilon_3}{\varepsilon_3 - \varepsilon_1} \qquad (A6.2.7)$$

The angle is determined in degrees.

6. Print all the results for the first time interval and then
   repeat the above procedure for the second time interval.

   The millivolt readings for each gauge at each time interval
are given to the program as data. The data designation and the
listing of the program are given in the next pages.

PROGRAM EXPER2

DATA DESIGNATION

_____

is integer single

ia integer array

rs real single

ra real array

_____

MVOLTS	ra	millivolts readings of strain gauges
NU	rs	Poisson's ratio
SGFACT	ra	strain gauge factor
THETA	ra	angle of direction of principal stress
PSTRE1	ra	principal stress I
PSTRE2	ra	principal stress II
DSTRAN	ra	direct strain
DSTRES	ra	direct stress
PSTAN1	ra	principal strain I
PSTAN2	ra	principal strain II
NG	ra	number of gauge
ITINI	is	iteration parameter
ITMAX	is	maximum iteration parameter
NSG1	is	number of strain gauge
NSG2	is	
NRO1	is	number of rosette
NRO2	is	
VOLT	rs	voltage across the bridge

E	rs	modulus of elasticity
L1	is	working parameter
L2	is	working parameter
L3	is	working parameter
B		
C		
D	rs	working constants
F		

```
JOB(UME'M18,J1)                           OIKCNGHIDES.C,N
      PROGRAM EXPER2(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C
C
C     PROGRAM EXPER2 INITIALLY CALCULATES THE DIRECT STRAINS IN THE
C     LONGITUDINAL AND TRANSVERSE PLANES FOR ALL 45 STRAIN GAUGES
C     OF 15 ROSETTES BY CONVERTING VOLTAGE READINGS FROM A MULTI
C     CHANEL WHEASTON BRIDGE INTO DIRECT STRAIN FOR VARIOUS TIME
C     INTERVALS DURING HEATING AND COOLING CYCLES.IT DETERMINES FOR
C     THE SAME TIME INTERVALS THE DIRECT STRESSES,THE PRINCIPAL
C     STRESSES AS WELL AS THEIR DIRECTION
C
      REAL    MVOLTS, NU
      COMMON    MVOLTS(45),          DSTRAN(45),
     1          SGFACT(45),          DSTRES(45),
     2          THETA(15),           PSTAN1(15),
     3          PSTRE1(15),          PSTAN2(15),
     4          PSTRE2(15),          NG(45)
      ITINI=1
      ITMAX=16
      NSG1=1
      NSG2=45
      NRO1=1
      NRO2=15
C     READ DATA FOR SPECIFIED TIME INTERVALS
C     READ NUMBER OF STRAIN GAUGES AND ROSETTES
C     READ MILLIVOLTS OUTPUT AND STRAIN GAUGE FACTORS
C     NUMBER OF STRAIN GAUGES =45
      DO  100  IT=ITINI,ITMAX
      READ(5,10)(NG(N),MVOLTS(N),SGFACT(N),N=NSG1,NSG2)
  10  FORMAT(5(I4,F6.2,F5.2))
      T=FLOAT(IT)
C
C     DEFINE VOLTAGE(IN VOLTS) ACROSS THE BRIDGE
C     CONVERT VOLTAGE READINGS INTO DIRECT STRAIN FOR ALL GAUGES
      VOLT=6.00
      DO 15  N=NSG1,NSG2
      DSTRAN(N)=(0.004*MVOLTS(N))/(VOLT*SGFACT(N))
  15  CONTINUE
C
C     MODULUS OF ELASTICITY
      E=30.0*(10.**6)
C     CALCULATE DIRECT STRESSES FOR ALL GAUGES
      DO 20  N=NSG1,NSG2
      DSTRES(N)=E*DSTRAN(N)
  20  CONTINUE
C
C     CALCULATE PRINCIPAL STRAINS FOR ALL FIFTEEN ROSETTES
C
      N=0
      DO 25    M=NSG1,NSG2,3
      N=N+1
      L1=M
      L2=M+1
      L3=M+2
      B=DSTRAN(L1)+DSTRAN(L3)
      C=DSTRAN(L1)+DSTRAN(L3)-2.*DSTRAN(L2)
      PSTAN1(N)=(B+SQRT((B**2)+(C**2)))/2.
      PSTAN2(N)=(B-SQRT((B**2)+(C**2)))/2.
C
C     CALCULATE PRINCIPAL STRESSES FOR ALL FIFTEEN ROSETTES
C     POISSON'S RATIO  NU=0.3
      PSTRE1(N)=(E*(PSTAN1(N)+0.3*PSTAN2(N)))/0.91
      PSTRE2(N)=(E*(PSTAN2(N)+0.3*PSTAN1(N)))/0.91
C     CALCULATE THE DIRECTION OF PRINCIPAL STRESSES
C     THE ANGLE IS MEASURED IN DEGREES
      D=2.*DSTRAN(L2)-DSTRAN(L1)-DSTRAN(L3)
      F=DSTRAN(L3)-DSTRAN(L1)
      THETA(N)=57.29577794*(ATAN(D/F))/2.
  25  CONTINUE
C
C     PRINT ALL RESULTS
C
      WRITE(6,55)
  55  FORMAT(5(/))
      WRITE(6,56)  T
  56  FORMAT(1X,*CYCLE*,8X,*TIME=*,F5.2)
      WRITE(6,57)
  57  FORMAT(1X,24(*-*))
      WRITE(6,58)
  58  FORMAT(2(/))
      WRITE(6,59)
  59  FORMAT(1X,30HGAUGE NO          MVOLTS READINGS          DIR
```

```
     1CT STRAINS                  DIRECT STRESSES /
     21X,89H-------------------------------------------------------------
     3------------------------ //)
        WRITE(6,65)(NG(N),MVOLTS(N),DSTRAN(N),DSTRES(N),N=NSG1,NSG2)
  65 FORMAT(1X,I5,19X,F6.3,13X,E16.4,18X,F3.1)
        WRITE(6,70)
  70 FORMAT(5(/))
        WRITE(6,75)
  75 FORMAT(1X,97HROSETTE NO              PRINCIPAL STRAINS           PR
     1INCIPAL STRESSES          DIRECTION ANGLE /
     21X,97H-------------------------------------------------------------
     3------------------------ //)
        WRITE(6,80) (N,PSTAN1(N),PSTAN2(N),PSTRE1(N),
     1PSTRE2(N),THETA(N),N=NRO1,NRO2)
  80 FORMAT(1X,I6,5X,E16.4,E16.4,7X,F8.1,2X,F8.1,16X,F6.2)
        WRITE(6,90)
  90 FORMAT(1X,132(*-*))
 100 CONTINUE
     STOP
     END
```

APPENDIX 7

Boundary Collocation Analysis

7.1 Boundary Collocation Analysis.

Derivation of Boundary Conditions.

The Williams stress function is given by

$$\chi(r,\theta) = \sum_{n=1,2,\ldots}^{\infty} \left\{ (-1)^{n-1} d_{2n-1} r^{n+(\frac{1}{2})} \left[-\cos\left(n-\frac{3}{2}\right)\theta \right. \right.$$

$$\left. + \frac{2n-3}{2n+1} \cos\left(n+\frac{1}{2}\right)\theta \right] + (-1)^n d_{2n} r^{n+1} \left[-\cos(n-1)\theta \right.$$

$$\left. \left. + \cos(n+1)\theta \right] \right\}$$

The stress components are given by

$$\sigma_x = \frac{\partial^2 \chi}{\partial x^2} = \frac{\partial^2 \chi}{\partial r^2}\cos^2\theta - 2\frac{\partial^2 \chi}{\partial\theta\partial r}\frac{\sin\theta\cos\theta}{r} + \frac{\partial\chi}{\partial r}\frac{\sin^2\theta}{r}$$

$$+ 2\frac{\partial\chi}{\partial\theta}\frac{\sin\theta\cos\theta}{r^2} + \frac{\partial^2\chi}{\partial\theta^2}\frac{\sin^2\theta}{r^2} \tag{A7.1.1}$$

$$\sigma_y = \frac{\partial^2\chi}{\partial y^2} = \frac{\partial^2\chi}{\partial r^2}\sin^2\theta + 2\frac{\partial^2\chi}{\partial\theta\partial r}\frac{\sin\theta\cos\theta}{r} + \frac{\partial\chi}{\partial r}\frac{\cos^2\theta}{r} \tag{A7.1.2}$$

$$- 2\frac{\partial\chi}{\partial\theta}\frac{\sin\theta\cos\theta}{r^2} + \frac{\partial^2\chi}{\partial\theta^2}\frac{\cos^2\theta}{r^2}$$

$$-\tau_{xy} = \frac{\partial^2\chi}{\partial x\partial y} = \sin\theta\cos\theta\frac{\partial^2\chi}{\partial r^2} + \frac{\cos 2\theta}{r}\frac{\partial^2\chi}{\partial r\partial\theta}$$

$$- \frac{\sin\theta\cos\theta}{r^2}\frac{\partial^2\chi}{\partial r\partial\theta} - \frac{\sin\theta\cos\theta}{r}\frac{\partial\chi}{\partial r} \tag{A7.1.3}$$

$$- \frac{\cos 2\theta}{r^2}\frac{\partial\chi}{\partial\theta}$$

Therefore σ_x, σ_y, $-\tau_{xy}$ are all functions of χ, r, θ. The conditions at different points will be as follows:

Along the crack surface:

$\sigma_x = f(\cos\theta, \sin\theta)$ $\cos\theta = \cos(\pm\pi)$, $\sin\theta = \sin(\pm\pi) = 0$

hence $\sigma_x = 0$, and $-\tau_{xy} = f(\cos\theta\sin\theta)$

therefore $-\tau_{xy} = 0$. Hence the normal and shearing stresses are zero along the surface of the crack.

BOUNDARY $\vec{AB}$:

At boundary AB $\theta = + \pi/2$

$\chi = f(\cos\theta, r) = f(\cos(\frac{\pi}{2}), r) = 0$

and therefore $\chi = 0$, $\frac{\partial \chi}{\partial y} = 0$

BOUNDARY $\vec{BC}$:

Equation (A7.1.1) for σ_x for $\theta = 0$ becomes

$$\sigma_x = \frac{\partial^2 \chi}{\partial r^2}$$

Integrating

$$\frac{\partial \chi}{\partial r} = \sigma_x r + A \tag{A7.1.4}$$

$$\chi = \frac{\sigma_x r^2}{2} + Ar + B \tag{A7.1.5}$$

Using conditions on boundary AB to determine the constants,

for $r = -a$ (on boundary AB) $\chi = 0$, $\frac{\partial \chi}{\partial y} = 0$

Therefore Equation (A7.1.4) becomes

$$0 = - \sigma_x a + A$$

hence, $A = \sigma_x a$

Equation (A7.1.5) becomes $\chi = \frac{\sigma_x r^2}{2} + \sigma_x ar + B$

at $r = a$, $\chi = 0$ (at tip of the crack)

hence $\chi = 0 = \frac{\sigma_x a^2}{2} - \sigma_x a^2 + B$

and hence, $B = \sigma_x a^2 - \sigma_x \frac{a^2}{2} = \frac{\sigma_x a^2}{2}$

Therefore the force component of the stress function is given by

$$\chi = \sigma_x \left(\frac{y^2}{2} + ay + \frac{\alpha^2}{2}\right) \text{ along } \vec{BC} \qquad (A7.1.6)$$

BOUNDARY $\vec{CD}$:

In Equation (A7.1.6) above at $\vec{CD}$ $y = V, a = 0$

$$\therefore \chi = \frac{\sigma_x V^2}{2}$$

and $\quad \frac{\partial \chi}{\partial x} = 0 \ , \qquad \frac{\partial \chi}{\partial y} = \sigma_x V$

7.2 PROGRAM STREN4

Data designations for program STREN4 are the same as the ones given in Appendix 5. The listing of the program is given in the next page.

```
      PROGRAM STREN4(INPUT=1002,OUTPUT=1002,TAPE5=INPUT,TAPE6=OUTPUT)
C
C
C
C SPECIFICATION AND DATA INITIALIZATION STATEMENTS.
      INTEGER   TITLE,     PS,        SR,        QA,        O,
     1          CASCTL,    STRESS,    WI,        AMP,       P,
     2                     STRAIN,    COND,                 Q
      REAL      NU,        K,         KXX,
     1                                KXY,
     2                                KYX,
     3                                KYY
      COMMON    TITLE (13), NPI (430),     X (256),     NB(15),
     1          E  (5),     NPJ (430),     Y (256),     COND(15),
     2          NU (5),     NPM (430),     RX (256),    SLOPE(15),
     3          RHO (5),    MAT (430),     RY (256),
     4          ALPHA (5),  DELTAT (430),  U (256),
     5                                     V (256),
     1          RI (430),   KXX (256, 9),  ANP (256, 9),  SIGLXX (430),
     2          RJ (430),   KXY (256, 9),  NAP (256),     SIGLYY (430),
     3          RM (430),   KYX (256, 9),  LN (3),        SIGLZZ (430),
     4          CI (430),   KYY (256, 9),  B (3, 6),      SIGLXY (430),
     5          CJ (430),   FXX (256),     D (3, 3),
     6          CM (430),   FXY (256),     W (3, 6),
     7                      FYX (256),     K (6, 6),
     8                      FYY (256)
      DATA CASCTL, STRESS, STRAIN / 6HNFCASE, 6HSTRESS, 6HSTRAIN /
C
C
C
10 INPUT THE CASE TITLE BUT STOP IF THERE IS NO FURTHER CASE.
11    READ (5, 12)   TITLE
12    FORMAT (13A6)
      IF (TITLE (1) .EQ. CASCTL)   STOP
C
C
C
20 INPUT THE CONTROL PARAMETERS AND PREPARE TO INPUT THE ARRAY DATA.
      READ (5, 21)   NEL, NNP,NBR,   NMAT, NCY, NCC,
     1               TOLER, OMEGA, PS, SR, WI
21    FORMAT (6I5, E13.5, F7.3, 2X, A6, 2I3)
      DO 22  NM = 1, NMAT
22    E (NM) = -1.
      DO 23  L = 1, NEL
23    NPI (L) = -1
      DO 24  N = 1, NNP
24    X (N) = -0.123456789E35
C
C
C
30 INPUT AND CHECK THE ARRAY DATA.
      READ (5, 31)   (MATN, E (MATN), NU (MATN),
     1               RHO (MATN), ALPHA (MATN),  NM = 1, NMAT)
31    FORMAT (I5, 4E13.5)
      DO 33  NM = 1, NMAT
      IF (E (NM) .NE. -1.)  GO TO 33
      WRITE (6, 32)  TITLE
32    FORMAT (1H1 ////// 5H IN ', 13A6, 1H' /
     1          36H THERE IS A MISSING INPUT DATA CARD.,
     2          24H   THE JOB IS ABANDONED.)
      STOP
33    CONTINUE
      READ (5, 34)   (LE, NPI (LE), NPJ (LE), NPM (LE),
     1               MAT (LE), DELTAT (LE),  L = 1, NEL)
34    FORMAT (5I5, F10.5)
      DO 35  L = 1, NEL
      IF (NPI (L) .NE. -1)  GO TO 35
      WRITE (6, 32)  TITLE
      STOP
35    CONTINUE
      READ (5, 36)   (NP, X (NP), Y (NP), RX (NP), RY (NP),
     1               U (NP), V (NP),  N = 1, NNP)
36    FORMAT(I5,4E12.4,2F12.5)
      DO 37  N = 1, NNP
      IF (X (N) .NE. -0.123456789E35)  GO TO 37
      WRITE (6, 32)  TITLE
      STOP
37    CONTINUE
      READ (5, 38)   (NB (NR), COND (NR), SLOPE (NR),  NR = 1, NBR)
38    FORMAT (2I5, F10.5)
C
C
C
40 OUTPUT THE TITLE AND THE CONTROL PARAMETERS.
      WRITE (6, 41)
41    FORMAT (1H1 //////
     1          45X, 41HTHE CASE TITLE AND THE CONTROL PARAMETERS /
     2          45X, 41H------------------------------------------ /////)
      WRITE (6, 42)  TITLE
42    FORMAT (1H , 13A6)
```

```
      WRITE (6, 43)   NEL, NNP, NBR, NMAT, NCY, NCC, TOLER, OMEGA, PS
   43 FORMAT (26H0THE NUMBER OF ELEMENTS IS, I5, 1H. /
     1              33H0THE NUMBER OF NODAL POINTS IS, I5, 1H. /
     2              43H0THE NUMBER OF RESTRAINED BOUNDARY NODES IS, I5, 1H. /
     3              37H0THE NUMBER OF DIFFERENT MATERIALS IS, I5, 1H. /
     4              23H0THE ITERATION LIMIT IS, I5, 8H CYCLES. /
     5              36H0THE INTERVAL FOR PROGRESS OUTPUT IS, I5, 8H CYCLES. /
     6              29H0THE CONVERGENCE TOLERANCE IS, E13.5, 1H. /
     7              33H0THE OVER-RELAXATION FACTOR IS, F7.3, 1H. /
     8              37H0THE CASE HAS THE CONDITION OF PLANE , A6, 1H. /
     9              23H0STRESSES ARE FOUND FOR)
      IF (SR .EQ. 0)   WRITE (6, 44)
   44 FORMAT (1H+, 23X, 42HNEITHER THE ELEMENTS NOR THE NODAL POINTS.)
      IF (SR .EQ. 1)   WRITE (6, 45)
   45 FORMAT (1H+, 23X, 18HTHE ELEMENTS ONLY.)
      IF (SR .EQ. 2)   WRITE (6, 46)
   46 FORMAT (1H+, 23X, 22HTHE NODAL POINTS ONLY.)
      IF (SR .EQ. 3)   WRITE (6, 47)
   47 FORMAT (1H+, 23X, 39HBOTH THE ELEMENTS AND THE NODAL POINTS.)
      IF (WI .EQ. 0)   WRITE (6, 48)
   48 FORMAT (37H0THE INPUT ARRAY DATA ARE NOT OUTPUT.)
      IF (WI .EQ. 1)   WRITE (6, 49)
   49 FORMAT (33H0THE INPUT ARRAY DATA ARE OUTPUT.)
C
C
C
   50 OUTPUT THE ARRAY DATA.
      IF (WI .EQ. 0)   GO TO 61
      WRITE (6, 51)   (NM, E (NM), NU (NM), RHO (NM),
     1                 ALPHA (NM), NM = 1, NMAT)
   51 FORMAT (1H1 //////
     1              56X, 19HINPUT MATERIAL DATA /
     2              56X, 19H------------------- /////
     3              23X, 42HMATERIAL   ELASTIC MODULUS   POISSON'S RATIO,
     4              34H       DENSITY      THML EXPN COEFF //
     5              (I33, X, 4E17.5))
      WRITE (6, 52)
   52 FORMAT (1H1 //////
     1              57X, 18HINPUT ELEMENT DATA /
     2              57X, 18H------------------ /////
     3              3X, 48HELMT NPI   NPJ   NPM   MTRL   DELTA T
     4                  48HELMT NPI   NPJ   NPM   MTRL   DELTA T
     5                  33HELMT NPI   NPJ   NPM   MTRL   DELTA T /)
      L1 = (NEL + 2) / 3
      DO 53 L2 = 1, L1
      L3 = MIN0 (L2 + 2 * L1, NEL)
   53 WRITE (6, 54)   (L, NPI (L), NPJ (L), NPM (L),
     1                 MAT (L), DELTAT (L), L = L2, L3, L1)
   54 FORMAT (1H , 5I5, F10.3, 13X, 5I5, F10.3, 13X, 5I5, F10.3)
      WRITE (6, 55)   (N, X (N), Y (N), RX (N), RY (N),
     1                 U (N), V (N), N = 1, NNP)
   55 FORMAT (1H1 //////
     1              58X, 15HINPUT NODE DATA /
     2              58X, 15H--------------- /////
     3              13X, 49HNODE              X                    Y
     4              31HRX             RY              ,
     5              21H  U              V //
     6              (I16, X, 6E17.5))
      WRITE (6, 56)   (NB (NR), COND (NR), SLOPE (NR), NR = 1, NBR)
   56 FORMAT (1H1 //////
     1              48X, 35HINPUT RESTRAINED BOUNDARY NODE DATA /
     2              48X, 35H----------------------------------- /////
     3              33X, 28HRESTND BNDY NODE              ,
     4              37HRESTNT CONDITION               SLOPE //
     5              (I42, I27, F30.5))
C
C
C
   60 DO GEOMETRIC CALCULATIONS ON THE ELEMENTS.
   61 DO 72 L = 1, NEL
      I = NPI (L)
      J = NPJ (L)
      M = NPM (L)
      BI (L) = Y (J) - Y (M)
      BJ (L) = Y (M) - Y (I)
      BM (L) = Y (I) - Y (J)
      CI (L) = - X (J) + X (M)
      CJ (L) = - X (M) + X (I)
      CM (L) = - X (I) + X (J)
      AREA = (BJ (L) * CM (L) - BM (L) * CJ (L)) / 2.
      IF (AREA .GT. 1.E-35)   GO TO 71
      WRITE (6, 62)   L
   62 FORMAT (1H1 ////// 8H ELEMENT, I5, 25H HAS A NON-POSITIVE AREA. /
     1              23H THE CASE IS ABANDONED.)
      GO TO 221
C
C
```

```
C  70 SUM THE TOTAL NODAL POINT FORCES.
   71 MATN = MAT (L)
      IF (PS .EQ. STRESS)  SIGTH = 1. - NU (MATN)
      IF (PS .EQ. STRAIN)  SIGTH = (1. + NU (MATN))
C    1                               * (1. - 2. * NU (MATN))
      SIGTH = - E (MATN) * ALPHA (MATN) * DELTAT (L) / SIGTH
      BODYF = AREA * RHO (MATN) / 3.
      RX (I) = RX (I) - 0.5 * CI (L) * SIGTH
      RY (I) = RY (I) - 0.5 * CI (L) * SIGTH - BODYF
      RX (J) = RX (J) - 0.5 * BJ (L) * SIGTH
      RY (J) = RY (J) - 0.5 * CJ (L) * SIGTH - BODYF
      RX (M) = RX (M) - 0.5 * CM (L) * SIGTH
   72 RY (M) = RY (M) - 0.5 * CM (L) * SIGTH - BODYF
C
C
C  80 PREPARE TO SUM THE STIFFNESS COEFFICIENTS.
      DO 82  N = 1, NNP
      DO 81  QA = 1, 9
      KXX (N, QA) = 0.
      KXY (N, QA) = 0.
      KYX (N, QA) = 0.
      KYY (N, QA) = 0.
   81 ANP (N, QA) = 0
      ANP (N, 1) = N
   82 NAP (N) = 1
C
C
C  90 FORM THE STIFFNESS MATRIX FOR THE ELEMENT.
      DO 105  L = 1, NEL
      DO 92  P = 1, 3
      DO 91  Q = 1, 6
   91 E (P, Q) = 0.
      DO 92  Q = 1, 3
   92 D (P, Q) = 0.
      B (1, 1) = BI (L)
      L (1, 3) = BJ (L)
      B (1, 5) = BM (L)
      B (2, 2) = CI (L)
      B (2, 4) = CJ (L)
      E (2, 6) = CM (L)
      B (3, 1) = CI (L)
      B (3, 2) = BI (L)
      B (3, 3) = CJ (L)
      B (3, 4) = BJ (L)
      B (3, 5) = CM (L)
      B (3, 6) = BM (L)
      MATN = MAT (L)
      W1 = NU (MATN)
      W2 = 1. - W1
      W3 = 1. + W1
      W4 = 1. - 2. * W1
      IF (PS .EQ. STRAIN) GO TO 93
      W5 = E (MATN) / (W2 * W3)
      W6 = W1
      W7 = W2 / 2.
      GO TO 94
   93 W5 = E (MATN) * W2 / (W3 * W4)
      W6 = W1 / W2
      W7 = W4 / (2. * W2)
   94 AREA = (BJ (L) * CM (L) - BM (L) * CJ (L)) / 2.
      W5 = W5 / (4. * AREA)
      D (1, 1) = W5
      D (1, 2) = W5 * W6
      D (2, 1) = W5 * W6
      D (2, 2) = W5
      D (3, 3) = W5 * W7
      DO 95  P = 1, 3
      DO 95  Q = 1, 6
      W (P, Q) = 0.
      DO 95  O = 1, 3
      IF (O.GT.3)  O=2
   95 W (P, Q) = W (P, Q) + D (P, O) * B (O, Q)
      DO 96  P = 1, 6
      DO 96  O = 1, 6
      K (P, Q) = 0.
      DO 96  O = 1, 3
   96 K (P, Q) = K (P, Q) + B (O, P) * W (O, Q)
C
C
C 100 SUM AND STORE FOR EACH (FORCED) NODE THE NON-ZERO STIFFNESS
C     COEFFICIENTS FOR THE ADJACENT (DISPLACED) NODES.
      LN (1) = NPI (L)
      LN (2) = NPJ (L)
      LN (3) = NPM (L)
      DO 105  P = 1, 3
```

```
      NP = LN (P)
      DO 105  Q = 1, 3
      QA = 0
101   QA = QA + 1
      IF (QA .LE. 9)  GO TO 103
      WRITE (6, 102)  NP
102   FORMAT (1H1 /////// 5H NODE, I5,
     1          36H HAS MORE THAN EIGHT ADJACENT NODES. /
     2          23H THE CASE IS ABANDONED.)
      GO TO 221
103   IF (ANP (NP, QA) .EQ. LN (Q))  GO TO 104
      IF (ANP (NP, QA) .NE. 0)  GO TO 101
      ANP (NP, QA) = LN (Q)
      NAP (NP) = QA
104   KXX (NP, QA) = KXX (NP, QA) + K (2 * P - 1, 2 * Q - 1)
      KXY (NP, QA) = KXY (NP, QA) + K (2 * P - 1, 2 * Q)
      KYX (NP, QA) = KYX (NP, QA) + K (2 * P, 2 * Q - 1)
      KYY (NP, QA) = KYY (NP, QA) + K (2 * P, 2 * Q)
105   CONTINUE
C
C
C 110 BY INVERSION OF THE MAIN DIAGONAL STIFFNESS SUBMATRIX, FIND THE
C     FLEXIBILITY SUBMATRIX AT EACH NODE DISPLACED BY FORCES AT ITSELF.
      DO 112  N = 1, NNP
      W1 = KXX (N, 1) * KYY (N, 1) - KXY (N, 1) * KYX (N, 1)
      FXX (N) =   KYY (N, 1) / W1
      FXY (N) = - KXY (N, 1) / W1
      FYX (N) = - KYX (N, 1) / W1
112   FYY (N) =   KXX (N, 1) / W1
C
C
C 120 CALCULATE THE EFFECTIVE SELF FLEXIBILITY SUBMATRICES FOR THE
C     RESTRAINED BOUNDARY NODES.
      DO 125  NR = 1, NBR
      NP = NB (NR)
      IF (COND (NR) - 1)  123, 122, 121
121   FXX (NP) = (FXX (NP) * FYY (NP) - FXY (NP) * FYX (NP))
     1          / (FXX (NP) * SLOPE (NR) ** 2
     2          - (FXY (NP) + FYX (NP)) * SLOPE (NR) + FYY (NP))
      FXY (NP) = FXX (NP) * SLOPE (NR)
      FYX (NP) = FXY (NP)
      FYY (NP) = FXY (NP) * SLOPE (NR)
      GO TO 125
122   FYY (NP) = FYY (NP) - FYX (NP) * FXY (NP) / FXX (NP)
      GO TO 124
123   FYY (NP) = 0.
124   FXX (NP) = 0.
      FXY (NP) = 0.
      FYX (NP) = 0.
125   CONTINUE
C
C
C 130 PERFORM A GAUSS - SEIDEL ITERATION ON THE STIFFNESS MATRIX FOR
C     THE ENTIRE STRUCTURE.
      IF (NCC .GT. NCY)  GO TO 132
      WRITE (6, 131)
131   FORMAT (1H1 //////
     1          34X, 35HTHE PROGRESS OF THE GAUSS - SEIDEL ,
     2          23HITERATION TOWARD CONVERGENCE /
     3          34X, 35H----------------------------------- ,
     4          23H----------------------------------- /////
     5          33X, 39HCYCLE               CONVERGENCE CRITERION,
     6          26H           CURRENT DIFFERENCE)
132   NC = 0
133   SD = 0.
      SDD = 0.
      DO 136  N = 1, NNP
      IF (FXX (N) + FYY (N) .EQ. 0.)  GO TO 136
      RX1 = RX (N)
      RY1 = RY (N)
      NAP1 = NAP (N)
      DO 134  QA = 1, NAP1
      NP = ANP (N, QA)
      RX1 = RX1 - KXX (N, QA) * U (NP) - KXY (N, QA) * V (NP)
134   RY1 = RY1 - KYX (N, QA) * U (NP) - KYY (N, QA) * V (NP)
      IF (FXX (N) .EQ. 0.)  GO TO 135
      DELTAU = FXX (N) * RX1 + FXY (N) * RY1
      U (N) = U (N) + OMEGA * DELTAU
      SD = SD + ABS (U (N))
      SDD = SDD + ABS (DELTAU)
      IF (FYY (N) .EQ. 0.)  GO TO 136
135   DELTAV = FYX (N) * RX1 + FYY (N) * RY1
      V (N) = V (N) + OMEGA * DELTAV
      SD = SD + ABS (V (N))
      SDD = SDD + ABS (DELTAV)
```

```
  136 CONTINUE
      SD = AMAX1 (SD, 1.E-35)
      RELDIF = SDD / SD
C
C
C 140 TEST FOR CONVERGENCE.
      NC = NC + 1
      IF (NCC .GT. NCY .OR. MOD (NC, NCC) .NE. 0)   GO TO 142
      WRITE (6, 141)  NC, TOLER, RELDIF
  141 FORMAT (1H0, I36, 3X, 2E27.5)
  142 IF (RELDIF .GT. TOLER)   GO TO 144
      WRITE (6, 143)  NC
  143 FORMAT (1H1 /////// 28H THE ITERATION HAS CONVERGED,
     1            6H AFTER, I5, 8H CYCLES.)
      GO TO 151
  144 IF (NC .LT. NCY)  GO TO 133
      WRITE (6, 145)  NC, RELDIF, TOLER
  145 FORMAT (1H2 /////// 28H THE ITERATION HAS COMPLETED, I5, 5H THE ,
     1            52HMAXIMUM NUMBER OF CYCLES WITHOUT CONVERGING, BUT HAS /
     2            34H ACHIEVED A RELATIVE DIFFERENCE OF, E13.5,
     3            33H AGAINST A SPECIFIED TOLERANCE OF, E13.5, 1H. /
     4            43H THE UNCONVERGED RESULTS ARE WRITTEN BELOW.)
C
C
C 150 OUTPUT THE NODE DISPLACEMENTS.
  151 WRITE (6, 152)
  152 FORMAT (1H1 ///////
     1            49X, 33HTHE CALCULATED NODE DISPLACEMENTS /
     2            49X, 33H--------------------------------- /////
     3             3X, 47HNODE            U                   V
     4                 47HNODE            U                   V
     5                 30HNODE            U                   V /)
      N1 = (NNP + 2) / 3
      DO 153  N2 = 1, N1
      N3 = MIN0 (N2 + 2 * N1, NNP)
  153 WRITE (6, 154)  (N, U (N), V (N),  N = N2, N3, N1)
  154 FORMAT (1H , I5, 2E16.5, I15, 2E16.5, I15, 2E16.5)
C
C
C 160 CALCULATE THE ELEMENT STRESS COMPONENTS.
      IF (SR .EQ. 0)  GO TO 221
      DO 164  L = 1, NEL
      I = NPI (L)
      J = NPJ (L)
      M = NPM (L)
      AREA = (BJ (L) * CM (L) - BM (L) * CJ (L)) / 2.
      EEXX = (BI (L) * U (I) + BJ (L) * U (J)
     1                       + BM (L) * U (M)) / (2. * AREA)
      EEYY = (CI (L) * V (I) + CJ (L) * V (J)
     1                       + CM (L) * V (M)) / (2. * AREA)
      EEXY = (CI (L) * U (I) + BI (L) * V (I)
     1       + CJ (L) * U (J) + BJ (L) * V (J)
     2       + CM (L) * U (M) + BM (L) * V (M)) / (2. * AREA)
      MATN = MAT (L)
      W1 = NU (MATN)
      W2 = 1. - W1
      W3 = 1. + W1
      W4 = 1. - 2. * W1
      IF (PS .EQ. STRAIN)  GO TO 161
      W5 = E (MATN) / (W2 * W3)
      W6 = W1
      W7 = W2 / 2.
      GO TO 162
  161 W5 = E (MATN) * W2 / (W3 * W4)
      W6 = W1 / W2
      W7 = W4 / (2. * W2)
  162 ETH = ALPHA (MATN) * DELTAT (L)
      EXX = EEXX - ETH
      EYY = EEYY - ETH
      EXY = EEXY
      SIGLXX (L) = W5 * (EXX + W6 * EYY)
      SIGLYY (L) = W5 * (W6 * EXX + EYY)
      IF (PS .EQ. STRAIN)  GO TO 163
      SIGLZZ (L) = 0.
      GO TO 164
  163 SIGLZZ (L) = W1 * (SIGLXX (L) + SIGLYY (L))
  164 SIGLXY (L) = W5 * W7 * EXY
C
C
C 170 CALCULATE THE ELEMENT PRINCIPAL AND MEAN STRESSES.
      IF (SR .EQ. 2)  GO TO 184
      W1 = (SIGLXX (L) + SIGLYY (L)) / 2.
      W2 = (SIGLXX (L) - SIGLYY (L)) / 2.
      W3 = SIGLXY (L)
      W4 = SQRT (W2 ** 2 + W3 ** 2)
```

```
      SIGL1 = W1 + W4
      SIGL2 = W1 - W4
      IF (ABS (W2) .GT. 1.E-35 .OR. ABS (W3) .GT. 1.E-35)  GO TO 171
      ANGLEL = 0.
      GO TO 172
  171 ANGLEL = 28.647889756 * ATAN2 (W3, W2)
  172 SIGL3 = SIGLZZ (L)
      SIGLM = SQRT ((SIGL1 + SIGL2 + SIGL3) ** 2
     1            - 3. * (SIGL2 * SIGL3 + SIGL3 * SIGL1 + SIGL1 * SIGL2))
C
C
C 180 OUTPUT THE ELEMENT STRESSES.
      IF (L .NE. 1)  GO TO 182
      WRITE (6, 181)
  181 FORMAT (1H1 //////
     1            27X, 40HTHE CALCULATED ELEMENT STRESS COMPONENTS,
     2            40H AND THE PRINCIPAL AND THE MEAN STRESSES /
     3            27X, 40H----------------------------------------, /////
     4            40H----------------------------------------,
     5            1X, 45HELEMENT        SIGMXX              SIGMYY         /,
     6            45HSIGMZZ              SIGMXY              ANGLE 1
     7            37HSIGM1              SIGM2              SIGMM /)
  182 WRITE (6, 183)  L, SIGLXX (L), SIGLYY (L), SIGLZZ (L),
     1               SIGLXY (L), ANGLEL, SIGL1, SIGL2, SIGLM
  183 FORMAT (1H , I5, 4E16.5, F12.2, X, 3E16.5)
  184 CONTINUE
      IF (SR .EQ. 1)  GO TO 221
C
C
C 190 FIND THE NODE STRESS COMPONENTS.
      DO 212  N = 1, NNP
      SIGNXX = 0.
      SIGNYY = 0.
      SIGNZZ = 0.
      SIGNXY = 0.
      QA = 1
      DO 191  L = 1, NEL
      IF (NPI (L) .NE. N .AND. NPJ (L) .NE. N .AND. NPM (L) .NE. N)
     1                                                         GO TO 191
      SIGNXX = SIGNXX + SIGLXX (L)
      SIGNYY = SIGNYY + SIGLYY (L)
      SIGNZZ = SIGNZZ + SIGLZZ (L)
      SIGNXY = SIGNXY + SIGLXY (L)
      QA = QA + 1
      IF (QA .EQ. NAP (N))  GO TO 192
  191 CONTINUE
  192 W1 = FLOAT (QA - 1)
      SIGNXX = SIGNXX / W1
      SIGNYY = SIGNYY / W1
      SIGNZZ = SIGNZZ / W1
      SIGNXY = SIGNXY / W1
C
C
C 200 CALCULATE THE NODE PRINCIPAL AND MEAN STRESSES.
  201 W1 = (SIGNXX + SIGNYY) / 2.
      W2 = (SIGNXX - SIGNYY) / 2.
      W3 = SIGNXY
      W4 = SQRT (W2 ** 2 + W3 ** 2)
      SIGN1 = W1 + W4
      SIGN2 = W1 - W4
      IF (ABS (W2) .GT. 1.E-35 .OR. ABS (W3) .GT. 1.E-35)  GO TO 202
      ANGLEN = 0.
      GO TO 203
  202 ANGLEN = 28.647889756 * ATAN2 (W3, W2)
  203 SIGN3 = SIGNZZ
      SIGNM = SQRT ((SIGN1 + SIGN2 + SIGN3) ** 2
     1            - 3. * (SIGN2 * SIGN3 + SIGN3 * SIGN1 + SIGN1 * SIGN2))
C
C
C 210 OUTPUT THE NODE STRESSES.
      IF (N .NE. 1)  GO TO 212
      WRITE (6, 211)
  211 FORMAT (1H1 //////
     1            29X, 37HTHE CALCULATED NODE STRESS COMPONENTS,
     2            40H AND THE PRINCIPAL AND THE MEAN STRESSES /
     3            29X, 37H-------------------------------------,
     4            40H----------------------------------------, /////
     5            3X, 43HNODE        SIGMXX              SIGMYY         ,
     6            45HSIGMZZ              SIGMXY              ANGLE 1 /,  y
     7            37HSIGM1              SIGM2              SIGMM /)
  212 WRITE (6, 213)  N, SIGNXX, SIGNYY, SIGNZZ, SIGNXY,
     1               ANGLEN, SIGN1, SIGN2, SIGNM
  213 FORMAT (1H , I5, 4E16.5, F12.2, X, 3E16.5)
C
C


C 220 RETURN TO THE BEGINNING OF THE PROGRAM FOR THE NEXT CASE.
```

7.3 Program FUNSTR and Subroutine DMATRX

The data designations are as follows:

is   integer single

ia   integer array

rs   real single

ra   real array

PROGRAM FUNSTR

CRACKL   rs   Crack length

HMESH   rs   Height of cell

WIDTH   rs   Width of cell

EDIST   rs   Dimension

JJ
KK
KKE   is   Grid co-ordinates
JJE

SIGMYY   ra   Stresses in y-direction

SIGMXX   ra   Stresses in x-direction

SIGMTO   ra   Specified stress at boundary points

XX
A1
A2   rs   Working parameters
A3
A4

AMOM   rs   Moment

B1 ⎫
B2 ⎬ rs   Working parameters
B3 ⎭

CONVER   rs   Convergence constant

PREVIO   rs   Convergence criterion

SUBROUTINE DMATRX

The same as above and additionally:

MCR   is   Crack positions

DI   ra   First matrix coefficient

CONS   rs   Working constant

R   ra   Working constant

The listing of FUNSTR and DMATRX is given in the next page.

```
JOB(UMEMM18,J12,T1200,CM60000,LC9900)     OIKONOMIDES.C.N
MAP(PART)
MNF(T,0,E=2,R=2,P=5000)
      PROGRAM FUNSTR(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C
C     USING FINITE DIFFERENCE ANALYSIS PROGRAM FUNSTR
C     DETERMINES THE STRESS FUNCTION  X   WHICH SATISFIES
C     THE BIHARMONIC EQUATION AT THE PRESCRIBED BOUNDARY POITS
C     AND ALL THE INTERIOR MESH POINTS, THE MAGNITUDE
C     OF THE MESH DEPENDS ON THE SIZE OF THE REGION
C     UNDER CONSIDERATION
      COMMON  X(45,45), SIGMYY(43),SIGMXX(43),SIGMTO ,
     1D1(43),R(43),
     1CRACKL,HMESH,JJ
C
C     CONSTANTS AND DIMENSION PARAMETERS
C     MESH POINTS
      MAXIT=700
      EPS=0.1
      CRACKL=0.04
      HMESH=0.025
      WIDTH=1.00
      EDIST=0.5
      JJ=43
      KK=43
      KKE=KK+2
      JJE=JJ+2
C
C
C
C
      READ BOUNDARY STRESSES IN Y-DIRECTION
   90 READ(5,100)    J,SIGMYY(J),SIGMXX(J),NEXT
  100 FORMAT(I5,2F10.2,I1)
      IF(NEXT.EQ.0)  GO TO 90
      WRITE(6,105)
  105 FORMAT(1H1,////)
      WRITE(6,102)
  102 FORMAT(15X,33HJ          SIGMYY          SIGMXX ///)
      WRITE(6,104)   (J,SIGMYY(J),SIGMXX(J),J=1,JJ)
  104 FORMAT(15X,I3,7X,F8.2,7X,F8.2)
      WRITE(6,103)
  103 FORMAT(5(/))
C
C
C
C
C     CALCULATE MOMENT FOR X-STRESS COMPONENT
C
C
C
      DO 88   I=3,JJ
      IF(I.GT.3)  GO TO 92
      IF(I.EQ.3)  SIGMTO=SIGMXX(3)
      GO TO 88
   92 SIGMTO=SIGMTO+SIGMXX(I)
   88 CONTINUE
      AMOM=SIGMTO*EDIST
C
C
C     CLEAR ARRAYS
      DO  110  J=1,JJE
      DO  110  K=1,KKE
  110 X(J,K)=0.0
C     PRESCRIBE THE BOUNDARY VALUES FOR BOUNDARY 3-JJ
C
      DO  200  J=3,JJ
      K=3
      XX=FLOAT(J-3)*HMESH-CRACKL
      A2=((CRACKL**3)/6.)+(((CRACKL**2)*XX)/2.)+((CRACKL* (XX**2))/2.)
     1+((XX**3)/6.)
      A1=(-12.*AMOM)/(WIDTH**3)
      A4=((XX**2)/2.)+(CRACKL*XX)+((CRACKL**2)/2.)
      A3=(6.*AMOM)/(WIDTH**2)
C
      X(J,K)=(A1*A2)+(A4*(A3+SIGMYY(J)))
  200 CONTINUE
C
C     PRESCRIBE VALUES FOR BOUNDARY   3-KK
C
      DO  300  K=3,KK
      J=3
      X(J,K)=0.0
```

```
  300 CONTINUE
      WRITE(6,282)   SIGMTO
  282 FORMAT(15X,7HSIGMTO=,F12.2)
      WRITE(6,283)
  283 FORMAT(3(/))
      WRITE(6,465)
  465 FORMAT(1H1, /////)
C
C
C
C     USING FINITE DIFFERENCE EQUATION
C     SOLVE FOR INTERIOR MESH POINTS
C
      ITN=1
  290 CONTINUE
      DO 308  J=4,JJ
      DO 308  K=4,KK
      X(JJ-1,K)=X(JJ-2,K)
      X(J,KK-1)=X(J,KK-2)
C
C
      B1= 8.*(X(J,K+1)+X(J,K-1)+X(J+1,K)+X(J-1,K))
      B2=2.*(X(J+1,K+1)+X(J+1,K-1)+X(J-1,K+1)+X(J-1,K-1))
      B3= X(J+2,K)+X(J-2,K)+X(J,K+2)+X(J,K-2)
C
C
      DIFF=1.800  *(       (B1-B2-B3)/20.      -X(J,K)      )
      X(J,K)=X(J,K)+DIFF
      X(JJ,K)  =X(JJ-1,K)
      X(J,KK)  =X(J,KK-1)
  308 CONTINUE
      IF(ITN.EQ.1)  GO TO 325
      CONVER=X(JJ,KK)-PREVIO
      IF(ABS(CONVER).LE.EPS)  GO TO 400
  325 CONTINUE
      IF(ITN.EQ.MAXIT)  GO TO 400
      IF(ITN/ 50.EQ.(ITN+ 49)/ 50) GO TO 400
      PREVIO=X(JJ,KK)
      ITN=ITN+1
      GO TO 290
  400 CONTINUE
C
C
      ABSCON=ABS(CONVER)
      WRITE(6,500) ITN,ABSCON
  500 FORMAT(5X,11HITERATIONS=,I4,5X,22HCONVERGENCE CRITERION=,F16.6)
      WRITE(6,507)
  507 FORMAT(2(/))
      IF(ITN.EQ.MAXIT) GO TO 504
      IF(ABSCON.GT.EPS)  GO TO 505
  504 CONTINUE
      WRITE(6,508)
  508 FORMAT(1H1 /////)
      J=JJ
  509 WRITE(6,510)  J
  510 FORMAT(5X,  2HJ=, I3)
      WRITE(6,520)
  520 FORMAT(52X,18HSTRESS FUNCTION  X ///)
      WRITE(6,530) (X(J,K),K=1,KK)
  530 FORMAT((2X,10(F10.2,2X)))
      J=J-1
      IF(J.EQ.0)  GO TO 505
      WRITE(6,512)
  512 FORMAT(3(/))
      GO TO 509
  505 CONTINUE
      IF(ABSCON.LE.EPS)  GO TO 540
      IF(ITN.EQ.MAXIT) GO TO 540
      PREVIO=X(JJ,KK)
      ITN=ITN+1
      GO TO 290
  540 CONTINUE
      CALL DMATRX(ITN)
      STOP
      END
      SUBROUTINE DMATRX(ITN)
C
C
C
C     SUBROUTINE DMATRX DETERMINES THE VALUES
C     OF THE FIRST MATRIX COEFFICIENT  D1.
C     THE COEFFICIENT IS A FUNCTION OF THE STRESS FUNCTION  X
C     AND THE DISTANCE  R  FROM THE TIP OF THE CRACK.
C
      COMMON  X(45,45), SIGMYY(43),SIGMXX(43),SIGMTO ,
     101(43),R(43),
     1CRACKL,HMESH,JJ
```

```
C
C      AXIS OF THE CRACK
       MCR=23
C
C
C
C      CLEAR   D1

       DO 998   L=1,JJ
       D1(L)=0.
 998   CONTINUE
       DO 1000   I=1,JJ
       R(I)=(HMESH*FLOAT(I-3))-CRACKL
       IF(R(I).LE.0.0)   GO TO 1000
       CONS=(-4./3.)*(R(I)**(3./2.))
       D1(I)=X(I,MCR)/CONS
1000   CONTINUE
       WRITE(6,1100)
1100   FORMAT(1H1 ////
      120X, 51HJ                X                R                      D1 ////)
       J=JJ
1140   CONTINUE
       WRITE(6,1150)  J,X(J,MCR),R(J),D1(J)
1150   FORMAT(17X,I3,5X,F12.2,8X,F10.5,6X,F12.2)
       J=J-1
       IF(J.EQ.0) GO TO 1200
       GO TO 1140
1200   CONTINUE
       WRITE(6,1250)
1250   FORMAT(1H1)
       RETURN
       END
```

APPENDIX 8

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APPENDIX 9

Comparison of Temperature Distributions

A 9.1 Finite Difference Equation in a Cylindrical System

Consider the system in a three-dimensional cylindrical co-ordinates as shown in Fig.A 9.1.1. The energy balance equation gives the following equation for the temperature T_o at node $\underline{o}$.

$$k \frac{(r_i + \Delta r/2)\Delta\Theta\Delta z}{\Delta r} (T_2 - T_o) + k \frac{(r_i - \Delta r/2)\Delta\Theta\Delta z}{\Delta r} (T_4 - T_o)$$

$$+ k \frac{\Delta r\Delta z}{\Delta\Theta r_i} (T_1 + T_3 - 2T_o) + k \frac{r_i\Delta\Theta\Delta r}{\Delta z} (T_5 + T_6 - 2T_o) = 0 \qquad (A\ 9.1.1)$$

Now consider a part of a hollow cylinder subdivided into equal space increments $\underline{\Delta r}$ as shown in Fig.(A 9.1.2). The explicit difference equation for the node $\underline{m}$ is from Equation A 9.1.1 and for the one-dimensional (radial) time dependent conduction

$$\frac{k}{\Delta r} \left[(T_{m-1}^n - T_m^n)(r_m - \frac{\Delta r}{2}) + (T_{m+1}^n - T_n^n)(r_m + \frac{\Delta r}{2}) \right]$$

$$= \frac{\Delta r\rho c\, r_m}{\Delta\tau} \left[T_m^{n+1} - T_m^n \right] \qquad (A\ 9.1.2)$$

Δr = space increment in radial direction

$\Delta\tau$ = time increment, and $\underline{n}$ indicates temperature at time $(\tau + \Delta\tau)$.

In explicit solutions of time dependent equations the expressions used give the $\underline{future}$ temperature T_m at node $\underline{m}$ (say) in terms of the $\underline{current}$ temperatures at the surrounding nodes. The explicit formulation uses

forward—difference approximation for the non-steady conduction equation.

Now if the time and space increment is chosen so that $\dfrac{(\Delta r)^2}{\alpha \Delta \tau} = 2$ then equation A 9.1.2 above becomes:

$$T_m^{n+1} = \frac{1}{2}\left[T_{m-1}^n \left(1 - \frac{\Delta r}{2r_m}\right) + T_{m+1}^n \left(1 + \frac{\Delta r}{2r_m}\right) \right] \qquad (A\ 9.1.3)$$

A 9.2   Determination of Temperature Distribution

The temperature distribution is determined by program CYL1DT (Listing at the end of the Appendix) which solves the equation

$$k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) = \rho c\,\frac{\partial T}{\partial \tau} \qquad \left(N.2\ p.83\right)$$

numerically using the forward-difference expression A 9.1.3. Temperatures are determined along the radial direction (r) at specified time increments. The space increment Δr is chosen equal to 0.05 inches and this produces 11 nodal points along the radius of the cylinder as shown in Fig.A 9.2.1. The parameter $\dfrac{(\Delta r)^2}{\alpha \Delta \tau} = 2$ for $\Delta r = 0.05$ inches and $\alpha = 0.347\ ft^2/hr$ (for steel EN58J (R.5-1) gives $\Delta \tau = 0.09$ secs. This time increment will produce $\dfrac{60\ secs}{0.09\ secs} = 667$ time iterations for every minute.

The boundary conditions are identical to that of the nozzle in Chapter 4, that is, the inside circumference is maintained at the constant temperature of $T_f = 450^\circ F$ and there is heat loss by natural convection to the air on the outside boundary. The heat transfer coefficient is that given in Reference (R.4-4) for horizontal cylinders and pipes.

$$\frac{h_c D_o}{k_f} = 0.53 \left[\frac{D_o^3\, \rho_f\, g\, \beta_f\, \Delta t}{\mu_f^2}\quad \left(\frac{c_p \mu}{k_f}\right) \right]^{0.25} \qquad (A\ 9.2.1)$$

where h_c = heat transfer coefficient

D_o = outside diameter of cylinder

k_f = thermal conductivity of air

ρ_f = density of air

g = acceleration due to gravity

β_f = coefficient of volumetric expansion of air

$\Delta t = T_w - T_\infty$

T_w = temperature on the outside surface of cylinder

T_∞ = is the air temperature $(60^\circ F)$

μ_f = viscosity of air

C_p = specific heat of air (at atm.pressure)

The heat transfer coefficient is calculated at each time increment using the equivalent surface node temperature.

A 9.3 Comparison of Temperature Results

The temperature distribution for the nozzle of Case C away from the vessel (grid points 91 to 100 see Fig.4.6.1, page 289) which was obtained using the two-dimensional (x,y), time dependent (τ) equation for a plane wall

$$k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \rho c \frac{\partial T}{\partial \tau} \qquad \left(4.3.1.1, p\,71\right)$$

is compared to the results obtained by using the one-dimensional (radial) time dependent (τ) equation for the cylinder

$$k \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) = \rho c \frac{\partial T}{\partial \tau} \qquad \left(N.2, p.83\right)$$

for the times 1 minute (Fig.A 9.3.1, Table A 9.3.1), 2 minutes.(Fig. A 9.3.2, Table A 9.3.2), and 4 minutes (Fig. A 9.3.3, Table A 9.3.3). The cylinder results are given in Table A 9.3.4. The dimensions of both cylinder and nozzle are identical, with D_i = 1 in and D_o = 2 in, and the boundary conditions already referred to.

It can be seen from Fig. A 9.3.1 that at time 1 minute the temperature at r = 0.6 inches for the nozzle is higher by 5.3% of the value of the cylinder and by 19.0% on the outside boundary. The average deviation is 12.2%. For time 2 minutes the average deviation is 11.8% (Fig. A 9.3.2) and for time 4 minutes 11.6%. In general the temperatures obtained in the nozzle using the two-dimensional time dependent plane wall conduction equation are higher than the temperatures obtained for the cylinder by an average of 11-12%. Near the junction where the end effects (vessel) affect the conduction in the z (axial) direction the equation for the cylinder will in fact be two-dimensional (r and z)

$$k \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 r}{\partial z^2} \right) = \rho c \frac{\partial T}{\partial \tau} \qquad \left(N.3, \ p.\ 83-N2 \right)$$

It can be concluded that the approximation of the plane wall heat conduction in two-dimensions (x,y) and time (τ) gives reasonable results for the nozzle and can be used to overcome the difficulty in the analysis of the heat conduction at the intersection of two cylinders.

Of course the alternative of treating the problem in three-dimensional time dependent formulation will soon be possible with the expansion of the computing facilities in order to give acceptable accuracy. But even so, the economics and time consumption of three-dimensional unsteady analysis will still be subjected to considerable competition from the two-dimensional analysis especially whenever axisymmetric geometries are considered.
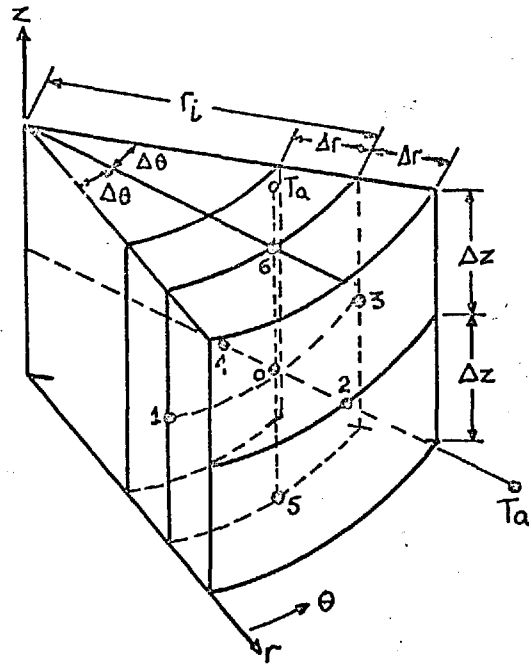
FIGURE A9.1.1

Three-dimensional cylindrical system
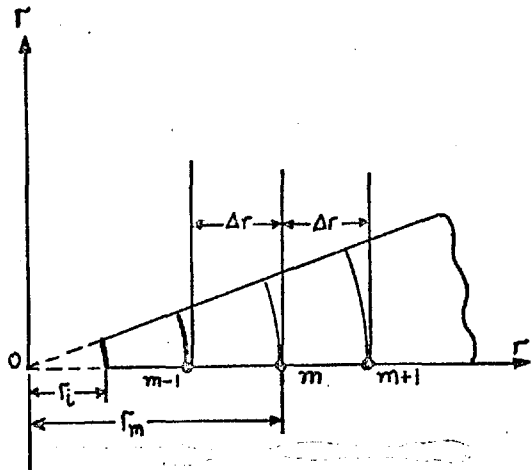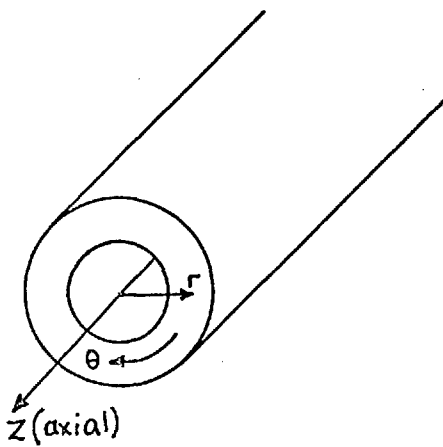
FIGURE A9.1.2

Subdivision of cylindrical wall

One-dimensional cylindrical system

Heat conduction(unsteady) in radial direction
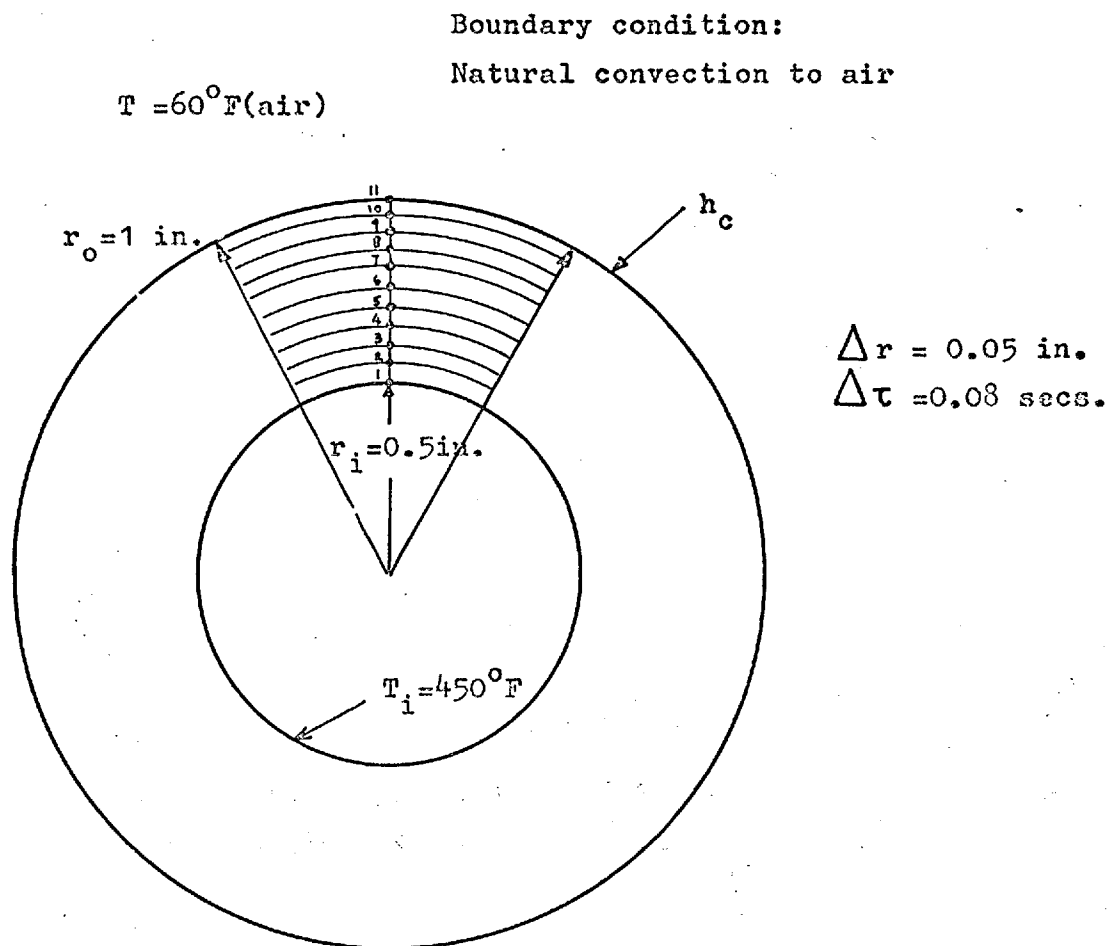
Finite-Difference Grid

FIGURE A9.2.1

FIGURE A9.3.1

Comparisons of temperatures for time = 1 minute.
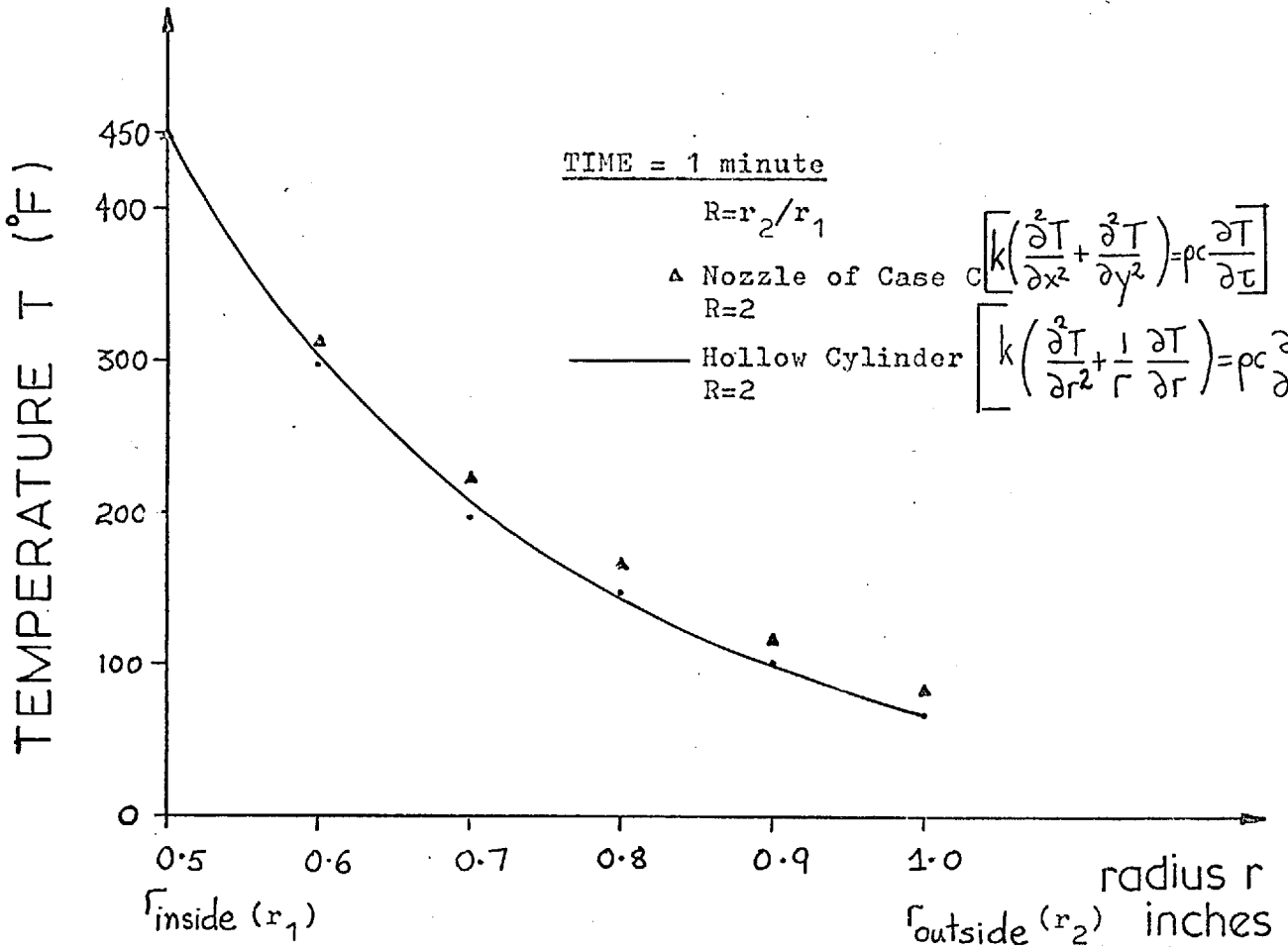
TIME = 1 minute

RADIUS r inches	TEMPERATURE (^{O}F) NOZZLE OF CASE C	TEMPERATURE (^{O}F) HOLLOW CYLINDER	DEVIATION %
0.5 (r_1)	450.0	450.0	
0.6	311.2	294.4	+5.7 %
0.7	220.4	195.6	+12.7 %
0.8	164.7	149.5	+10.2 %
0.9	116.8	101.7	+14.8 %
1.0 (r_2)	82.6	70.2	+17.6 %

AVERAGE

DEVIATION = 12.2 %

TIME = 2 minutes

▲ Nozzle of Case C (R=2)
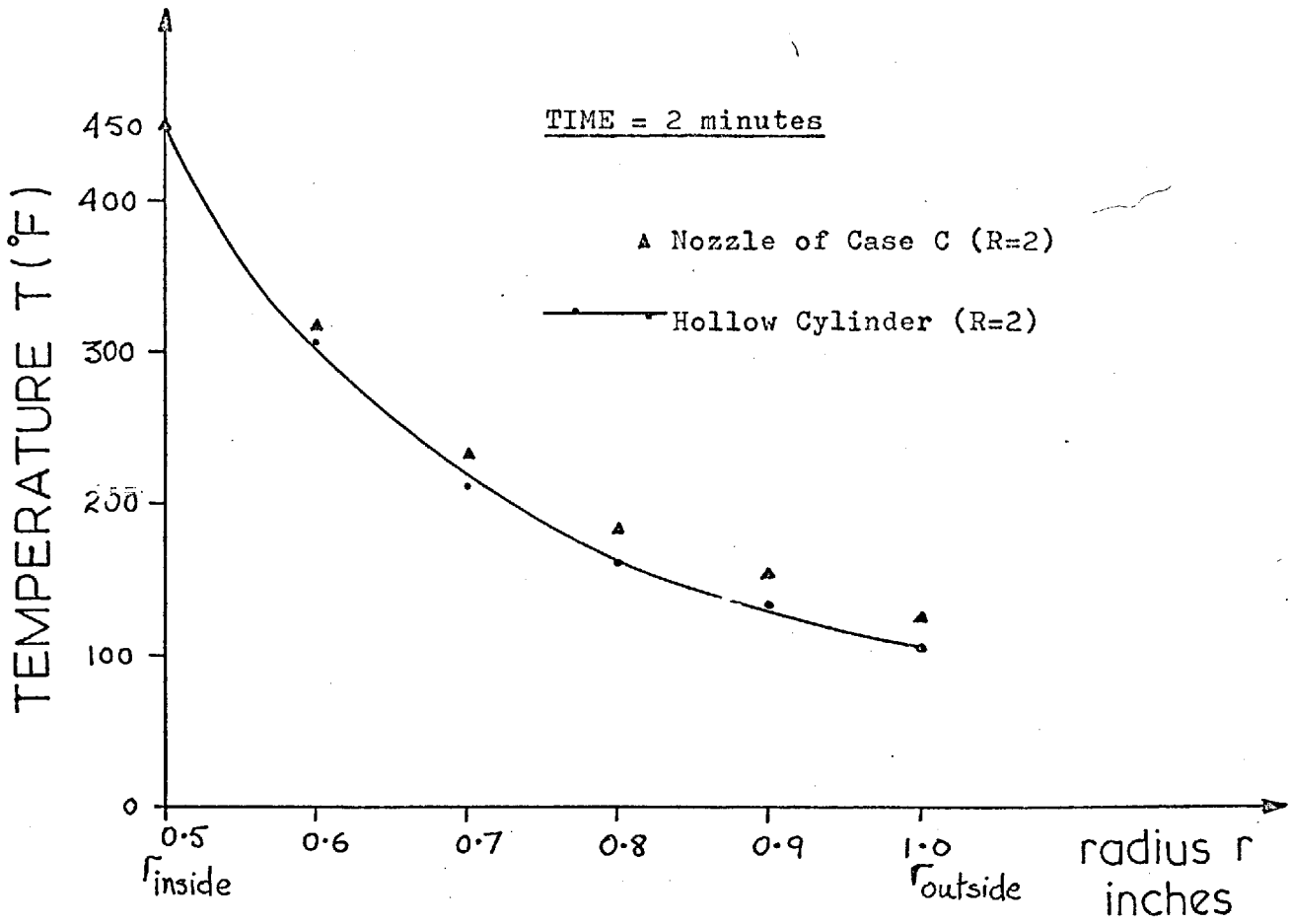
━•━ Hollow Cylinder (R=2)

TIME = 2 minutes			
RADIUS r inches	TEMPERATURE (^{O}F) NOZZLE CASE C	TEMPERATURE (^{O}F) HOLLOW CYLINDER	DEVIATION %
0.5 (r_1)	450.0	450.0	
0.6	316.7	306.3	+3.4 %
0.7	231.2	211.7	+9.2 %
0.8	183.1	160.2	+14.3 %
0.9	151.8	134.5	+12.9 %
1.0	125.2	105.2	+19.0 %

AVERAGE

DEVIATION= 11.8 %

FIGURE A9.3.2

Comparisons of temperatures for time = 2 minutes.

TIME = 4 minutes

▲ Nozzle of Case C (R=2)
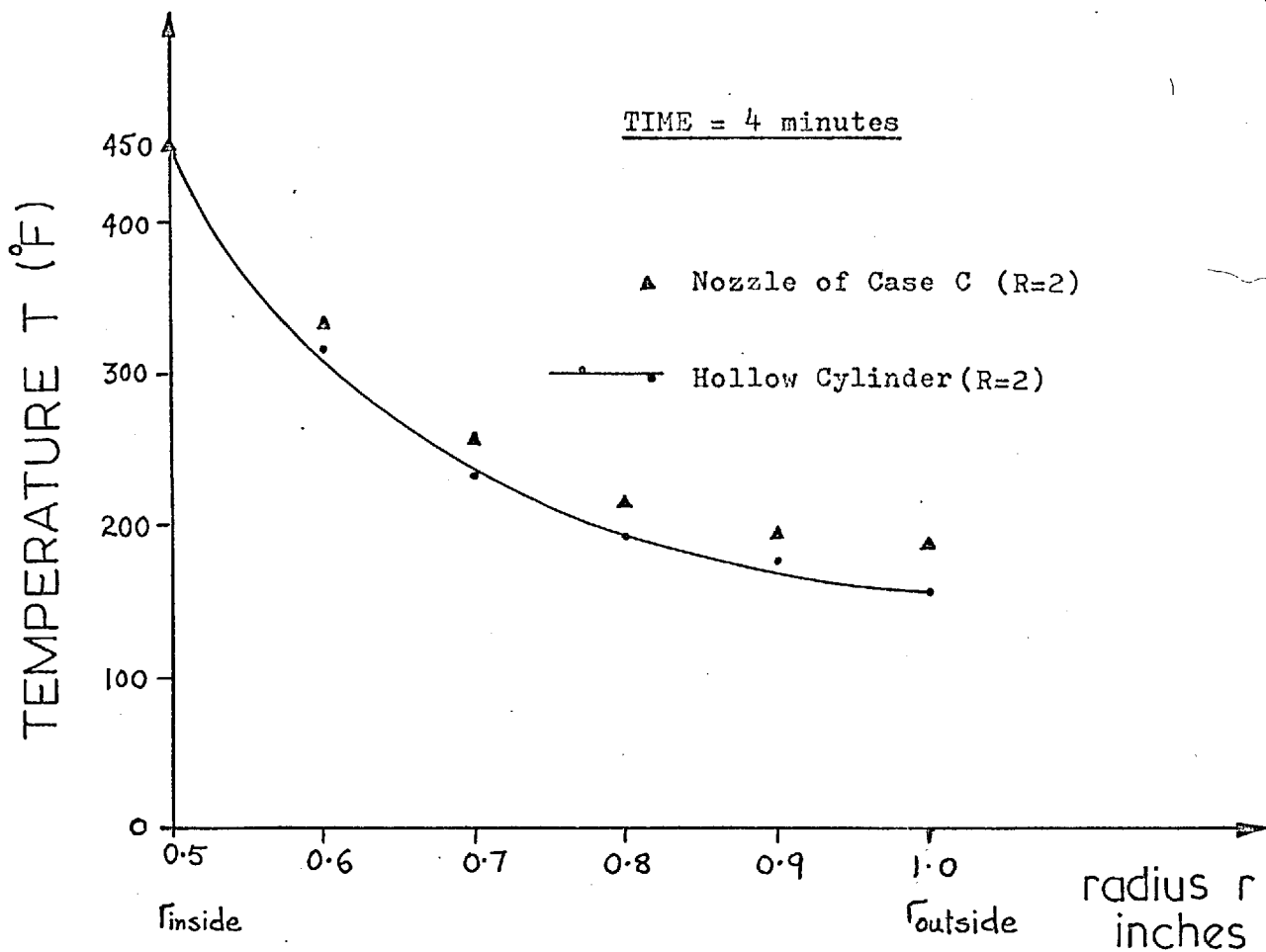
Hollow Cylinder(R=2)

TIME = 4 minutes			
RADIUS r inches	TEMPERATURE (^{O}F) NOZZLE OF CASE C	TEMPERATURE (^{O}F) HOLLOW CYLINDER	DEVIATION %
0.5 (r_1)	450.0	450.0	
0.6	333.4	316.6	+5.3 %
0.7	255.2	232.2	+9.9 %
0.8	215.7	191.4	+12.7 %
0.9	195.4	176.0	+11.0 %
1.0	186.3	156.4	+19.0 %

AVERAGE

DEVIATION = 11.6 %

FIGURE A9.3.3

Comparisons of temperatures for time = 4 minutes

TABLE A9.3.1

Nozzle Case C temperatures for time=1 minute.

I= 1									
91	92	93	94	95	96	97	98	99	100
450.0	453..	45..2	450.0	450.0	450.0	450.0	450.0	450.0	450.0
I= 2									
91	92	93	94	95	96	97	98	99	100
311.2	311..	311.2	311.2	311.2	311.2	311.2	311.2	311.2	311.2
I= 3									
91	92	93	94	95	96	97	98	99	100
220.4	220.4	22..4	220.4	220.4	220.4	220.4	220.4	220.4	220.4
I= 4									
91	92	93	94	95	96	97	98	99	100
164.7	164.7	164.7	164.7	164.7	164.7	164.7	164.7	164.7	164.7
I= 5									
91	92	93	94	95	96	97	98	99	100
116.8	116.8	116.8	116.8	116.8	116.8	116.8	116.8	116.8	116.8
I= 6									
91	92	93	94	95	96	97	98	99	100
82.6	82.6	82.6	82.6	82.6	82.6	82.6	82.6	82.6	82.6

TABLE A9.3.2

Nozzle Case C temperatures for time=2 minutes.

I= 1									
91	92	93	94	95	96	97	98	99	100
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
I= 2									
91	92	93	94	95	96	97	98	99	100
316.7	316.7	316.7	316.7	316.7	316.7	316.7	316.7	316.7	316.7
I= 3									
91	92	93	94	95	96	97	98	99	100
231.2	231.2	231.2	231.2	231.2	231.2	231.2	231.2	231.2	231.2
I= 4									
91	92	93	94	95	96	97	98	99	100
183.1	183.1	183.1	183.1	183.1	183.1	183.1	183.1	183.1	183.1
I= 5									
91	92	93	94	95	96	97	98	99	100
151.8	151.8	151.8	151.8	151.8	151.8	151.8	151.8	151.8	151.8
I= 6									
91	92	93	94	95	96	97	98	99	100
125.2	125.2	125.2	125.2	125.2	125.2	125.2	125.2	125.2	125.2

TABLE A9.3.3

Nozzle Case C temperatures for time=4 minutes.

I= 1									
91	92	93	94	95	96	97	98	99	100
450.0	450.1	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
I= 2									
91	92	93	94	95	96	97	98	99	100
333.4	333.4	333.4	333.4	333.4	333.4	333.4	333.4	333.4	333.4
I= 3									
91	92	93	94	95	96	97	98	99	100
255.2	255.2	255.2	255.2	255.2	255.2	255.2	255.2	255.2	255.2
I= 4									
91	92	93	94	95	96	97	98	99	100
215.7	215.7	215.7	215.7	215.7	215.7	215.7	215.7	215.7	215.7
I= 5									
91	92	93	94	95	96	97	98	99	100
195.4	195.4	195.4	195.4	195.4	195.4	195.4	195.4	195.4	195.4
I= 6									
91	92	93	94	95	96	97	98	99	100
186.3	186.3	186.3	186.3	186.3	186.3	186.3	186.3	186.3	186.3

```
FL REQJIRED TO LOAD    24400
FL REQJIRED TC RUN     17000
INITIAL TRANSFER TO CYLI1DT   -      4302


BLOCK ASSIGNMENTS.

BLOCK          ADDRESS     LENGTH    FILE

CYLI1DT           101       4262     BIN
ABORTE           4363         47     FORLIBB
CPUCPR           4432        201     FORLIBB
CPUCIO           4633         13     FORLIBB
CPUCPM           4646          5     FORLIBB
CPULFM           4653         10     FORLIBB
CPURDS           4663         42     FORLIBB
CPURDW           4725        133     FORLIBB
CPUSYS           5060         32     FORLIBB
CPUWTC           5112         13     FORLIBB
CPUWTS           5125         43     FORLIBB
CPUWTW           5170        104     FORLIBB
GETBA            5274         17     FORLIBB
 /CIOBJF./       5313        426
INOUTE           5741        143     FORLIBB
INPUTE           6104       1245     FORLIBB
 /FREEFOR/       7351          3
OUTPUTE          7354       1156     FORLIBB
RUNIOP          10532        324     FORLIBB
RUNSYS          11056       1106     FORLIBB
SYSTEM          12164        442     FORLIBB
TRACBAK         12626        277     FORLIBB
TRACEE          13125        750     FORLIBB
 //             14075       2623
```

TIME IN SECONDS= 60.0

450.0   366.5   294.4   245.1   195.6   170.1   149.5   114.1   101.7   75.3   70.2
_____   _____   _____   _____   _____   _____

TIME IN SECONDS=120.0

450.0   350.1   306.3   345.6   211.7   184.5   160.2   135.1   134.5   115.8   105.2
_____   _____   _____   _____   _____   _____

TIME IN SECONDS=240.0

450.0   355.2   316.6   260.4   232.2   205.3   191.4   176.9   176.0   163.2   156.4
_____   _____   _____   _____   _____   _____

TABLE A9.3.4

Cylinder temperatures for times=1,2, and 4 minutes.

PROGRAM CYL1DT

The program solves the one-dimensional time
dependent, heat conduction equation

$$k \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) = \rho c \frac{\partial T}{\partial \tau}$$

for a hollow cylinder explicitly using forward
difference formulation in time (τ) and space (radial)
increments. It assumes a constant temperature
around the inside circumference and along the
axis (z) of the cylinder and heat conduction by
natural convection on the outside boundary.

(The program was initially developed in
November 1976 for an investigation of heat conduction
in jacketed pressure vessels used by Ethylene
Plastique Co. Ltd.,France, for the storage of
ethylene gas at low temperatures; Internal Report
2-EP-5000L/C.N.Oikonomides,Applied Mechanics, and
in here the boundary conditions are changed for
natural convection to air.)

```
JOB(UHEIM19,J10,I7JJ,C440000,LC30772)        0173040 FIDES.C.H
MNF(T,D,R=2,F=1,L=BIN)
MAP(P)
BIN.
C     ----------------------------------------------------
C
C
C     ----------------------------------------------------
C
C
      PROGRAM CYL1DT(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C     PROGRAM CYL1DT SOLVES THE ONE DIMENSIONAL TIME DEPENDENT
C     DIFFERENTIAL EQUATION OF HEAT CONDUCTION  IN A HOLLOW
C     CYLINDER IN THE RADIAL R DIRECTION  USING THE CYLINDRICAL
C     FORWARD.DIFFERENCE EXPLICIT FORMULATION IN TIME AND SPACE
C     INCREMENTS.IT ASSUMES A CONSTANT TEMPERATURE AROUND THE
C     INSIDE CIRCUMFERENCE AND ALONG THE AXIS OF CYLINDER AND
C     HEAT CONDUCTION BY NATURAL CONVECTION ON THE OUTSIDE
C     BOUNDARY.
C
      COMMON U(500),
     1         IRINS,    IROUT,
     2         GRID,   A(300), B(300),X(300),
     3         INPUT(12),  OUTPUT(12)
C
C     DATA DESIGNATIONS DIMENSIONS
C
      READ(5,100)    TIN
      READ(5,110)    TCYL
      READ(5,120)    SPACIN
      READ(5,125)    ROUT
      READ(5,128)    RINS
      READ(5,130)    RADTH
      READ(5,140)    GRID
      READ(5,135)    T1
      READ(5,145)    AMINVT
      READ(5,150)    TIME
  100 FORMAT(1X,F12.6)
  110 FORMAT(1X,F12.6)
  120 FORMAT(1X,F12.6)
  130 FORMAT(1X,F12.6)
  140 FORMAT(1X,F12.6)
  125 FORMAT(1X,F12.6)
  128 FORMAT(1X,F12.6)
  135 FORMAT(1X,F12.6)
  145 FORMAT(1X,F12.6)
  150 FORMAT(1X,F12.6)
      IROUT=ROUT/GRID
      IRINS=RINS/GRID
C
C
C     PROPERTIES OF MATERIAL
C
      READ(5,160)    CONDS,
     1               DENST,
     2               SPEST,
     3               XCHART
  160 FORMAT(1X,4(F12.6))
C
C     PROPERTIES OF GAS ON THE OUTSIDE
C     WHERE NATURAL  CONVECTION TAKES PLACE
C
C
      READ(5,170)    GASK,
     1               SPHEAT,
     2               DYNVIS,
     3               VISCK,
     4               E,
     5               DENSG
  170 FORMAT(1X,6(F12.6))
C
C     DATA RELATED TO CONDITIONS
      READ(5,175)    ACCLG,
     1               TGAS
  175 FORMAT(1X,2(F12.6))
C
C     RCY=2*(3.141592654)*ROUT
      I1=(IROUT-IRINS)/GRID+1
C
C     INITIAL STATE  TIME=0. TEMPERATURE OF THE MATERIAL
C     IS  DEFINED
      DO 186 M=1,I1
  186 U(M)=TIN
C
C
```

```
      L1= I1-1
C
CC    RADIAL TEMPERATURE CALCULATION
C     THIS IS TIME ITERATION
      READ(5,187) TIME
  187 FORMAT(1X,F12.6)
      NV=1
  299 CONTINUE
  300 CONTINUE
      DO 196  M=2,I1
      U(I1)= U(I1-1)
      RM=RINS +FLOAT(M)*GRID
      G=U(M-1)*(1.-(GRID/(2.*RM)))
      C=U(M+1)*(1.+(GRID/(2*RM)))
      U(M) =   1./2.*(G+C)
  196 CONTINUE
C
C
CC    HEAT IS CONVECTED TO THE SURROUNDING GAS BY NATURAL CONVECTION
CC    CALCULATE TH  HEAT TRANSFER COEFFICIENT AT THE WALL
C     TEMPERATURE OF THIS  TIME INTERVAL
C
C     PRAND=(SPHEAT *DYNVIS)/GASK
C
CC    CALCULATE THE GRASHOFF  NUMBER AS A FUNCTION OF THE  WALL
C     TEMPERATURE  AT THE END OF THIS TIME  INTERVAL
      UM= (U(I1))
      GRASH=(((2.*ROUT)**3)*(DENSG**2)*ACC G*.*(UM-TGAS))/(VISCK**2)
      ANSL=0.53*((GRASH*PRAND)**0.25)
      HEATCO=(GASK*ANSL)/(2.*ROUT)
C
CC    FROM THE HEAT BALANCE EQUATION ON THE OUTSIDE  BOUNDARY
CC    CALCULATE THE TEMPERATURE DROP OF THE WALL NODAL TEMPERATURE
C     DUE  TO CONVECTION AT THIS TIME INTERVAL
C
      AA1=-CONDS*(GRID)*RCY*(U(I1)-U(I1-1))
      IF(AA1.EQ.0.)GO TO 480
      BB1=HEATCO*(GRID)*RCY*(U(I1)-TGAS)
      IF(BB1.EQ.0.)  GO TO 480
      CC1=DENST*SPEGT*(GRID**2)*RCY
      DIFF=(-AA1-BB1)/CC1
      FDIFF=(DIFF*TIME 3)/36 0.
      U(I1)=U(I1)*FDIFF
  480 CONTINUE
C
CC
C
      IF (TIME.E . 52.5)  T1=10.
      IF (TIME.E . 122.5) T1=30.
      IF (TIME.E . 302.5) T1=60.
      IF (TIME.E . 1802.5)T1=120.
      IF (TIME.E . 32.5)  NV=4
      IF (TIME.E . 122.5) NV=5
      IF (TIME.E . 302.5) NV=6
      IF (TIME.E . 1802.5)NV=16
      TINT=T1*FLOAT(NV)
      IF (TIME.E . TINT) GO TO 111
      TIME=TIME+ .08
      GO TO  301
  111 CONTINUE
      NV=NV+1
C
CC
CC    PRINT ALL NODAL TEMPERATURES AT SPECIFIED TIME INTERVALS
CC
C
  523 CONTINUE
      STIME=TIME/60.
      WRITE(6,2400)
 2400 FORMAT(1(/))
      WRITE(6,550)  STIME, TIME
  550 FORMAT(2X,* TI  IN MINUTES=*, F8.3,1 X, *TIME IN SECONDS=*,F9.2)
      WRITE(6,2450)
 2450 FORMAT(1(/))
      I=I1
  500 CONTINUE
      WRITE(6,1999) I
 1999 FORMAT(2X,*I=*,I3)
      WRITE(6,2300)  (U(I),I=1,I1)
 2300 FORMAT((1X,10(F5.1,7X)))
      I=I-1
      GO TO 500
 1010 IF(TIME.L .45 .) GO TO  528
  528 TIME=TIME/TIME3
 1000 IF(STIME.E . AMINUT) GO TO 1001
```

```
         GO TO 240
C    ------------------------------------------------------------------
      CALL DATA(N)
 1001 CONTINUE
      STOP
      END
C
      SUBROUTINE DATA(N)
      COMMON    U(500),IRINC,IROUT,GRID,
     1          A(750),B(750),X(30),
     2          INFMT(12),OUTFMT(12)
      DO 505 L=1,30
  505 X(L)=0.0
      READ(5,875)   INFMT
      READ(5,875)   OUTFMT
  875 FORMAT(6X,12A6)
      READ(5,105)  N,EPS,MAX
  105 FORMAT(I3,F10.0,I3)
      REWIND 1
      DO 605 I=1,N
      READ (5,INFMT)  (A(L),L=1,N)
  605 WRITE(1)  (A(M), M=1,N)
      READ(5,INFMT)  (B(L), L=1,N)
      ITER=1
  995 BIG=0.0
      DO 1005 I=1,N
  102 SUM=0.0
      READ(1)  (A(L),L=1,N)
      IF(I.EQ.1) GO TO 1035
      LAST=I-1
      DO 1065 J=1,LAST
 1065 SUM=SUM+A(J)*X(J)
      IF(I.EQ.N) GO TO 1035
 1055 INITL=I+1
      DO 1075  J=INITL,N
 1075 SUM=SUM+A(J)*X(J)
 1035 TEMP=(B(I)-SUM)/A(I)
      IF(ABS(TEMP-X(I)).GT.BIG) BIG=ABS(TEMP-X(I))
 1005 X(I)=TEMP
      IF  (BIG.LT.EPS) GO TO 752
      IF  (ITER.GE.MAX) GO TO 752
      ITER=ITER+1
      GO TO 995
  752 WRITE(6,147)  ITER,EPS,BIG
  147 FORMAT(1H0,I5,2E14.7)
      WRITE(6,OUTFMT) (X(L),L=1,N)
      RETURN
      END
C    CYL1DT-CYL1DT-CYL1DT-CYL1DT-CYL1DT-CYL1DT-CYL1DT-CYL1DT-CYL1DT
```

TABLES

I= 3									
1	2	3	4	5	6	7	8	9	10
450.0	442.0	445.0	444.0	442.2	440.4	435.7	437.1	435.6	434.1
432.3	431.0	430.1	428.1	428.0	427.0	426.0	425.1	424.3	423.5
422.8	422.0	421.0	421.1	420.5	420.0	419.5	419.1	418.7	418.4
418.1	417.0	417.2	417.3	417.1	416.9	416.7	416.5	416.4	416.2
416.1	415.5	415.9	415.8	415.8	415.7	415.6	415.6	415.5	415.2
415.4	415.3	415.3	415.2	415.3	415.3	415.2	415.2	415.2	415.1
415.2	415.1	415.1	415.1	415.1	415.1	415.1	415.1	415.1	415.1
415.1	415.1	415.1	415.1	415.1	415.1	415.1	415.1	415.1	415.1
415.1	415.1	415.1	415.1	415.1	415.1	415.1	415.1	415.1	415.1
415.1	415.1	415.1	415.1	415.1	415.1	415.1	415.1	415.1	414.6

I= 2									
1	2	3	4	5	6	7	8	9	10
450.0	448.0	447.0	447.0	446.0	445.1	445.3	443.5	442.7	441.9
441.0	440.0	441.0	437.0	438.8	438.3	437.4	437.4	437.0	436.6
436.0	435.0	435.0	435.1	435.1	434.8	434.6	434.4	434.2	433.0
433.0	433.0	433.0	433.0	433.1	433.3	433.2	433.1	433.0	432.6
432.0	432.5	432.6	432.7	432.7	432.7	432.6	432.6	432.6	432.4
432.5	432.4	432.5	432.5	432.5	432.5	432.5	432.5	432.5	432.4
432.4	432.4	432.4	432.4	432.4	432.4	432.4	432.4	432.4	432.4
432.4	432.4	432.4	432.4	432.4	432.4	432.4	432.4	432.4	432.4
432.4	432.4	432.4	432.4	432.4	432.4	432.4	432.4	432.4	432.4
432.4	432.4	432.4	432.4	432.4	432.4	432.4	432.4	432.4	432.1

I= 1									
1	2	3	4	5	6	7	8	9	10
450.0	450.0	451.0	451.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	451.0	451.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	451.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	451.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	451.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	451.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0

TABLE 4.4.1 CASE A. Thermal results

for time = 9min. 30 secs

(continued)

TABLE 4.4.1 (continued)

I= 5									
1	2	3	4	5	6	7	8	9	10
450.0	445.1	442.4	438.9	431.5	427.3	423.3	419.4	415.8	412.3
409.0	405.5	402.3	400.1	397.5	385.0	392.7	390.6	388.6	386.7
384.9	393.3	384.3	332.4	379.1	377.9	373.1	376.7	375.7	373.9
373.1	372.2	371.2	371.1	370.6	373.1	365.0	369.2	368.9	368.5
368.2	367.2	367.5	367.4	367.2	367.0	367.0	366.7	366.5	366.4
366.3	366.2	365.1	365.9	365.9	365.9	365.8	365.8	365.8	365.7
365.7	365.7	365.6	365.6	365.6	365.6	365.6	365.5	365.5	365.5
365.5	365.5	365.5	365.5	365.5	365.5	365.5	365.5	365.5	365.5
365.5	365.5	365.5	365.5	365.5	365.5	365.5	365.5	365.5	365.5
365.5	365.5	365.5	365.5	365.5	365.5	365.5	365.5	365.5	364.3

I= 5									
1	2	3	4	5	6	7	8	9	10
450.0	446.7	442.9	435.5	434.9	431.5	429.2	425.1	422.1	419.3
416.6	414.1	411.7	405.4	407.3	405.3	403.5	401.7	400.1	388.3
397.1	395.5	395.5	393.7	392.4	391.5	335.2	390.6	389.0	388.3
387.7	387.1	386.5	386.1	385.6	385.2	384.9	384.5	384.2	384.0
383.7	383.5	383.3	383.1	383.0	382.8	382.7	382.5	382.4	382.3
382.3	382.2	382.1	382.1	382.0	381.9	381.9	381.9	381.8	381.8
381.9	381.7	381.7	381.7	381.7	381.7	381.7	381.7	381.7	381.6
381.6	381.6	381.6	381.6	381.6	381.6	381.6	381.6	381.6	381.6
381.6	381.6	381.6	381.6	381.6	381.6	381.6	381.6	381.6	381.6
381.6	381.6	381.6	381.6	381.6	381.6	381.6	381.6	381.6	380.6

I= 4									
1	2	3	4	5	6	7	8	9	10
450.0	447.5	444.0	441.2	438.5	435.9	433.4	431.0	428.7	426.6
427.5	422.5	420.6	418.1	417.5	416.0	414.6	413.2	412.0	410.9
409.6	408.0	407.5	407.0	406.2	405.5	404.6	404.2	403.7	403.2
402.7	402.2	401.5	401.5	401.2	403.9	400.6	400.3	400.1	399.0
399.7	399.5	399.4	399.3	399.2	399.0	398.9	398.9	398.3	398.7
398.6	398.5	398.3	398.5	398.4	398.4	398.3	398.3	398.3	398.3
398.3	398.2	398.2	398.2	398.2	398.2	398.2	398.2	398.2	398.2
398.2	398.2	398.2	398.2	398.2	398.2	398.2	398.2	398.2	398.2
398.2	398.2	398.2	398.2	398.2	398.2	398.2	398.2	398.2	398.2
398.2	398.2	398.2	398.2	398.2	398.2	398.2	398.2	398.2	397.4

1	2	3	4	5	6	7	8	9	10
450.0	432.1	414.5	397.5	380.7	364.4	348.5	333.1	318.2	303.9
290.0	276.0	264.0	251.9	240.1	229.0	218.3	208.2	198.7	189.6
181.0	172.3	165.2	158.0	151.1	144.3	138.7	133.1	127.8	122.0
118.3	114.1	112.1	108.4	103.0	99.8	96.9	94.2	91.7	89.5
87.0	46.3	84.0	83.3	82.9	82.1	81.5	81.1	80.7	80.0
85.4	37.1	88.0	90.4	92.0	93.6	95.1	96.5	97.8	99.0
100.2	101.3	102.2	103.1	103.9	104.7	105.3	106.0	106.5	107.0
107.4	107.9	108.0	108.5	108.8	109.0	109.3	109.5	109.5	110.0
109.0	110.0	110.1	110.2	110.3	110.4	110.4	110.5	110.5	110.6
110.6	110.7	110.7	110.7	110.7	110.7	110.7	110.8	110.8	80.0

1	2	3	4	5	6	7	8	9	10
450.0	432.4	415.1	398.2	381.7	365.5	349.9	334.7	319.9	305.7
292.0	278.3	266.1	254.0	242.3	231.2	220.6	210.6	201.0	191.9
183.3	175.1	167.0	160.1	153.3	146.8	140.8	135.1	129.8	124.6
120.1	115.8	111.0	108.0	95.1	101.3	98.3	95.6	93.1	81.1
89.2	57.6	85.0	85.1	54.1	83.3	82.6	82.1	91.7	85.5
86.0	63.3	89.0	91.2	92.7	94.1	95.4	96.7	97.4	135.2
135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2
135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2
135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2
135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	110.8

1	2	3	4	5	6	7	8	9	10
450.0	432.4	415.1	398.2	381.7	365.5	349.9	334.7	319.9	305.7
292.0	278.3	266.1	254.0	242.3	231.2	220.6	210.6	201.0	191.9
183.3	175.1	167.4	160.2	153.3	146.9	140.8	135.1	129.8	124.6
120.2	115.8	111.0	108.1	85.1	101.4	98.4	82.6	95.6	81.1
89.3	37.7	86.3	85.0	84.1	83.3	82.6	82.1	81.7	135.2
86.9	95.3	89.0	91.2	92.7	94.1	95.4	96.7	93.0	135.2
135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2
135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2
135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2
135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	135.2	110.8

TABLE 4.4.1

247

I = 2

1	2	3	4	5	6	7	8	9	10
429.6	429.6	429.0	429.3	429.6	429.8	430.1	430.3	430.5	430.6
430.8	431.0	431.1	431.2	431.3	431.4	431.5	431.6	431.7	431.7
431.8	431.9	431.9	432.0	432.0	432.0	432.1	432.1	432.1	432.2
432.2	432.2	432.2	432.2	432.3	432.3	432.3	432.3	432.3	432.3
432.3	432.3	432.3	432.3	432.3	432.4	432.4	432.4	432.4	432.3
432.3	432.3	432.3	432.3	432.3	432.3	432.3	432.3	432.3	432.3
432.3	432.3	432.3	432.3	432.3	432.3	432.3	432.3	432.3	432.3
432.3	432.3	432.2	432.2	432.2	432.2	432.2	432.2	432.2	432.2
432.2	432.2	432.2	432.2	432.2	432.2	432.2	432.2	432.2	432.2
432.2	432.2	432.2	432.2	432.2	432.2	432.2	432.2	432.2	432.2

I = 1

1	2	3	4	5	6	7	8	9	10
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0

TABLE 4.5.1 CASE B. Thermal results sample for time = 10 minutes. (continued)

TABLE 4.5.1(continued)

I = 4

1	2	3	4	5	6	7	8	9	10
387.3	387.3	388.3	389.2	390.0	390.7	391.4	392.0	392.5	343.
393.5	393.9	394.3	394.7	395.0	395.3	395.5	395.8	396.0	396.2
396.4	396.6	396.7	396.8	397.0	397.1	397.2	397.3	397.4	397.4
397.5	397.6	397.6	397.7	397.7	397.8	397.8	397.8	397.9	397.9
397.9	397.9	398.0	398.0	398.0	398.0	398.0	398.0	397.9	397.9
397.9	397.9	397.9	397.9	397.8	397.8	397.8	397.8	397.8	397.9
397.8	397.8	397.8	397.8	397.8	397.7	397.7	397.7	397.7	397.7
397.7	397.7	397.7	397.7	397.7	397.7	397.7	397.7	397.7	397.7
397.7	397.7	397.7	397.7	397.7	397.7	397.7	397.7	397.7	397.7
397.7	397.7	397.7	397.7	397.7	397.7	397.7	397.7	397.7	395.9

I = 3

1	2	3	4	5	6	7	8	9	10
407.7	407.7	408.4	409.0	409.5	410.0	410.5	410.9	411.3	411.4
412.0	412.2	412.5	412.7	413.0	413.2	413.3	413.5	413.7	413.8
413.9	414.0	414.1	414.2	414.3	414.4	414.5	414.5	414.6	414.5
414.7	414.7	414.7	414.8	414.8	414.8	414.9	414.5	414.6	414.5
414.9	415.0	415.0	415.0	415.0	415.0	415.0	415.0	415.0	415.0
414.9	414.9	414.9	414.9	414.9	414.9	414.9	414.9	414.9	414.9
414.9	414.9	414.8	414.8	414.8	414.8	414.8	414.8	414.8	414.8
414.8	414.8	414.8	414.8	414.8	414.8	414.8	414.8	414.8	414.8
414.8	414.0	414.8	414.8	414.8	414.8	414.8	414.8	414.8	414.8
414.8	414.8	414.8	414.8	414.8	414.8	414.8	414.8	414.8	414.8

TABLE 4.5.1

I=70

1	2	3	4	5	6	7	8	9	10
60.0	60.4	60.4	60.5	60.5	60.6	60.7	60.7	60.8	60.8
60.9	60.9	61.0	61.1	61.1	61.2	51.2	61.3	61.3	61.4
61.5	61.5	61.6	61.6	61.7	61.7	61.7	61.8	61.8	61.9
61.9	61.9	62.0	62.0	62.0	62.9	63.6	64.4	65.3	65.3

I=69

1	2	3	4	5	6	7	8	9	10
60.4	60.4	60.5	60.6	60.6	60.7	50.7	60.8	60.9	60.9
61.0	61.0	61.1	61.2	61.2	61.3	61.4	61.4	61.5	61.5
61.6	61.6	61.7	61.7	61.8	61.8	61.9	61.9	62.0	62.0
62.0	62.1	62.1	62.1	62.9	63.5	64.2	65.0	65.9	65.9

I=68

1	2	3	4	5	6	7	8	9	10
60.5	60.5	60.5	60.6	60.6	60.7	60.8	60.8	60.9	60.9
61.0	61.1	61.1	61.2	61.3	61.3	61.4	61.4	61.5	61.6
61.6	61.7	61.7	61.8	61.8	61.9	61.9	62.0	62.0	62.1
62.1	62.1	62.2	62.2	62.9	63.5	64.2	65.0	65.9	65.9

I=67

1	2	3	4	5	6	7	8	9	10
60.5	60.5	60.5	60.6	60.7	60.7	60.8	50.9	60.9	61.0
61.1	61.1	61.2	61.3	61.3	61.4	51.4	61.5	61.6	61.6
61.7	61.7	61.8	61.9	61.9	62.0	62.0	62.0	62.1	62.1
62.2	62.2	62.2	62.3	63.0	63.6	64.3	65.1	66.0	67.0

I= 3									
1	2	3	4	5	6	7	8	9	10
407.7	407.7	408.4	409.0	409.5	410.0	410.5	410.9	411.?	411.6
412.?	412.2	412.5	412.7	413.0	413.2	413.3	413.5	413.?	413.66
413.?	414.1	414.1	414.2	414.3	414.4	414.5	414.5	414.6	414.6
414.7	415.?	415.?	415.3	415.5	415.6	415.7	415.?	416.1	416.2
416.?	417.?	416.?	416.8	416.9	417.3	417.9	418.?	417.?	417.?
417.?	418.?	417.7	417.7	417.8	418.2	417.9	418.2	417.?	418.1
418.?	418.?	418.1	418.2	418.2	418.2	418.2	418.2	418.?	418.3
418.3	418.?	418.4	418.3	418.3	418.3	418.3	418.3	418.?	418.3
418.4	418.4	418.4	418.4	418.4	418.4	418.4	418.4	418.4	417.?

I= 2									
1	2	3	4	5	6	7	8	9	10
429.8	429.?	429.0	429.3	429.6	429.8	430.1	430.3	430.5	430.6
430.?	431.?	431.1	431.2	431.3	431.4	431.5	431.6	431.7	431.7
431.?	431.?	431.?	432.0	432.0	432.0	432.1	432.?	432.1	432.2
432.?	432.?	432.?	432.2	432.6	432.6	432.7	432.?	432.?	432.?
433.?	433.1	432.?	433.2	433.3	433.7	433.4	433.?	433.?	433.?
433.?	433.?	433.?	433.7	433.7	433.7	433.9	433.?	433.?	433.?
434.?	434.3	434.00	434.0	434.0	434.0	434.0	434.?	434.0	434.0
434.0	434.3	434.0	434.0	434.0	434.0	434.0	434.0	434.0	433.7

I= 1									
1	2	3	4	5	6	7	8	9	10
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0

TABLE 4.6.1   CASE C. Thermal results sample
for time = 11 minutes. (continued)

TABLE 4.6.1 (continued)

I= 6

1	2	3	4	5	6	7	8	9	10
348.3	348.3	349.8	351.2	352.4	353.6	354.6	355.6	356.5	357.3
358.8	358.7	359.3	360.8	360.3	360.7	361.2	361.6	362.0	362.3
362.6	362.5	363.3	363.3	363.5	363.7	363.9	364.0	364.4	364.7
365.1	365.5	365.9	366.3	366.6	367.0	367.4	367.7	368.1	368.5
369.8	369.3	369.9	369.9	370.2	370.5	370.8	371.1	371.4	371.6
371.9	372.1	372.3	372.4	372.6	372.8	372.9	373.0	373.1	

I= 5

1	2	3	4	5	6	7	8	9	10
367.5	367.3	368.7	369.0	370.9	371.9	372.7	373.5	374.2	374.9
376.5	376.3	376.6	377.0	377.4	377.8	378.2	378.5	378.8	379.0
379.6	379.9	379.7	379.6	380.0	380.2	380.3	380.4	380.6	381.0
381.2	381.5	381.9	382.2	382.5	382.8	383.0	383.3	383.7	383.9
384.6	384.6	384.9	385.2	385.3	385.6	385.9	386.0	386.6	386.4
386.6	387.1	387.4	387.0	387.2	387.3	387.4	387.5	387.1	387.7
387.8	387.9	388.0	388.0	388.0	388.0	388.1	388.3	388.1	388.3
388.2	388.3	388.2	388.3	388.3	388.3	388.3	388.3	388.3	388.3
388.4	388.4	388.4	388.4	388.4	388.4	388.4	388.4	388.4	387.2

I= 4

1	2	3	4	5	6	7	8	9	10
387.3	387.3	388.3	389.2	390.0	390.7	391.4	392.0	392.5	393.0
394.3	394.3	394.7	394.7	395.0	395.3	395.5	395.8	396.0	396.2
396.4	396.3	396.7	396.8	397.0	397.1	397.2	396.4	397.4	396.9
397.8	398.1	398.5	398.6	399.8	399.0	399.2	399.4	399.6	399.9
400.1	400.3	400.5	400.7	400.9	401.1	401.2	401.4	401.6	401.7
401.8	402.0	402.1	402.2	402.3	402.4	402.5	402.6	402.6	402.8
402.7	402.8	402.8	402.8	402.9	402.9	402.9	403.0	403.0	403.0
403.0	403.1	403.0	403.1	403.1	403.1	403.1	403.1	403.1	403.1
403.1	403.1	403.1	403.1	403.1	403.1	403.1	403.1	403.1	403.1
403.1	403.1	403.1	403.1	403.1	403.1	403.1	403.1	403.1	402.3

TABLE 4.6.1

I=35									
1	2	3	4	5	6	7	8	9	10
60.0	95.7	97.7	99.6	101.5	103.2	104.9	106.5	108.1	109.5
110.9	112.3	113.4	114.6	115.7	116.8	117.8	119.0	120.1	121.2
122.4	123.3	124.8	126.1	127.5	129.0	130.7	132.6	134.5	136.5

I=34									
1	2	3	4	5	6	7	8	9	10
97.8	97.3	99.8	101.8	103.6	105.4	107.1	108.8	110.2	111.8
113.2	114.3	115.8	116.9	118.1	119.2	120.3	121.5	122.6	123.8
125.0	126.3	127.5	128.8	130.2	131.9	133.7	135.5	137.4	139.3

I=33									
1	2	3	4	5	6	7	8	9	10
98.4	98.1	100.4	102.3	104.2	106.0	107.7	109.3	110.9	112.4
113.8	115.1	116.3	117.5	118.7	119.8	120.9	122.1	123.2	124.4
125.6	126.3	128.1	129.4	130.8	132.4	134.2	136.0	137.9	139.8

I=32									
1	2	3	4	5	6	7	8	9	10
99.5	99.3	101.5	103.5	105.4	107.2	108.9	110.5	112.1	113.6
115.0	116.3	117.5	118.7	119.8	120.9	122.1	123.2	124.4	125.5
126.7	128.3	129.2	130.5	131.9	133.5	135.3	137.1	139.0	140.9

. I=31									
1	2	3	4	5	6	7	8	9	10
101.5	101.3	103.3	105.2	107.1	108.9	110.6	112.3	113.8	115.3
116.7	118.1	119.3	120.5	121.6	122.7	123.8	125.0	126.1	127.3
128.5	129.7	130.9	132.2	133.6	135.1	136.9	138.7	140.8	142.5

CASE A. OUTSIDE BOUNDARY

NODE	σ_L	σ_R^*	σ_Z	σ_{maxp}	$\tau_{max(12)}$
1		-963	-853		
2	-1195	-1185	-945	-4823	3468
3	-7598	-1568	-1661	-7687	4917
4	-11063	-2308	-3369	-11065	5449
5	-16438	-2981	-5828	-17921	8207
6	-18785	-3742	-6243	-17608	8528
7	-21371	-4211	-6850	-21602	10184
8	-18505	-3451	-7851	-23665	10579
9	-18274	-2708	-7753	-22560	10147
10	-15995	-1912	-6786	-20778	9902
11	-9137	-1583	-5612	-18765	8933
12	-8945	-1350	-4425	-14728	7957
13	-7142	-1002	-3779	-11605	6593
14	-6342	-953	-2691	-10471	5985
15	-4921	-810	-1920	-7813	5305
16	-3930	-792	-1668	-6198	3418
17	-2638	-543	-1119	-4725	2859
18	237	268	100	2646	2478
19	4782	1193	2029	7007	3624
20	6842	1542	3012	8067	4931
21	8239	1601	3496	10855	5029
22	10354	1921	3640	10905	5093
23	11089	1995	3766	11091	5104
24	9912	2005	2886	9515	4814
25	6756	1853	2743	6775	3335
26	6443	1648	2493	6461	2645
27	4369	1410	2158	4969	2421
28	2056	1180	1650	4420	2307
29		956	1594		

Columns σ_L and σ_R^*: 1 graph. Columns σ_{maxp} and $\tau_{max(12)}$: 1 graph.

* See note at the bottom of page 94

TABLE 5.2.1

CASE A. OUTSIDE BOUNDARY

NODES	$\tau_{max(12)}$	$\tau_{max(23)}$	$\tau_{max(31)}$
1		2024	587
2	3468	2688	780
3	4917	3811	1108
4	5449	4223	1226
5	8207	6361	1846
6	8528	6610	1918
7	10184	7893	2291
8	10579	8200	2379
9	10147	7865	2282
10	9902	7675	2227
11	8933	6924	2009
12	7957	6167	1780
13	6593	5110	1483
14	5985	4639	1346
15	5305	4112	1193
16	3418	2649	769
17	2859	2216	643
18	2478	1921	557
19	3624	2809	815
20	4931	3822	1109
21	5029	3898	1131
22	5093	3948	1145
23	5104	3956	1148
24	4814	3731	1083
25	3335	2585	750
26	2645	2050	595
27	2421	1876	525
28	2307	1788	519
29		1694	491

TABLE 5.2.1.1

CASE A. INSIDE BOUNDARY

| | 1 graph | | | 1 graph | |
NODE	σ_x	$\sigma_y{}^*$	σ_z	σ_{maxp}	$\tau_{max(12)}$
206		-155	-120		
200	-772	-278	-315	-1118	592
192	-2632	-293	-865	-3026	1443
186	-4875	-302	-1528	-5157	2610
172	-8099	-361	-2538	-8489	4258
165	-10572	-580	-3085	-10572	5142
157	-10921	-1573	-3748	-10986	5150
156	-10960	-1592	-3480	-10288	5173
129	-10219	-1383	-3103	-9094	4487
128	-9012	-1100	-2803	-5509	4423
127	-5458	-1080	-1583	-3425	2639
126	-3341	-920	-1028	-1579	1712
125	-1490	-810	-1001	-1078	1525
124	-1303	-703	-958	-1022	1403
123	-1120	-702	-940	-982	1321
122	-1002	-650	-857	-916	1201
78	-952	-610	-803	-893	1194
77	-677	-601	-763	-802	1072
76	-603	-582	-745	-710	956
75		-504	-701		

* See note at the bottom of page 94.

TABLE 5.2.2

CASE B. OUTSIDE BOUNDARY

NODE	σ_L	σ_R^*	σ_Z	$\sigma_{max\,p}$	$\tau_{max(12)}$
	1 graph			1 graph	
133		-376	-297		
132	-1358	-480	-263	-1728	1289
131	-2758	-653	-765	-2763	5605
1	-6574	-833	-890	-4815	6113
2	-8145	-940	-980	-6345	6270
3	-10803	-1007	-1102	-8109	7607
4	-11911	-2185	-3928	-12660	10397
5	-14287	-4230	-4373	-14897	15245
6	-5074	-2355	-2153	-7786	4197
7	-2255	-1100	-956	-4474	2879
8	-1164	-840	-493	-2758	1934
9	229	105	97	-1693	1855
10	866	642	368	1904	1290
11	3846	2613	1506	3865	1354
12	2923	2541	1494	3499	1317
13	2861	2119	1048	3062	1315
14	2480	1823	1003	2974	1307
15	2473	1174	1000	2615	1095
16	2404	1020	904	2613	1009
158	2034	810	848	2509	1008
160	1005	703	777	2498	990
162		641	634		

$*$ See note at the bottom of page 94.

TABLE 5.2.3

CASE B. INSIDE BOUNDARY

NODE	$\sigma_{max\,p}$	$\tau_{max(12)}$
149		
147	-1481	1332
124	-2297	1595
123	-3066	1871
122	-3350	2638
121	-7498	3165
102	-10442	4924
101	-9190	4743
96	-5946	3077
95	-4489	1558
94	-3718	1369
87	-3648	1358
88	-3154	1274
89	-2016	1203
90	-1739	1091
91	-1714	1047
92	-1688	970
93	-1631	957
159	-1232	792
161	-1061	772
163		

1 graph

TABLE 5.2.4

CASE C. OUTSIDE BOUNDARY

NODE	time=4 minutes			time=8 minutes			time=11 minutes		
	σ_L	σ_R^*	σ_z	σ_L	σ_R^*	σ_z	σ_L	σ_R^*	σ_z
1		-32	-137		-74	-319		-102	-438
4	-520	-38	-157	-1209	-87	-365	-1663	-120	-502
7	-1274	-118	-266	-2959	-274	-619	-4069	-377	-851
10	-1573	-150	-299	-3654	-349	-1060	-5024	-480	-1458
13	-2166	-217	-519	-5031	-505	-1426	-6918	-694	-1961
16	-2226	-256	-675	-5171	-594	-1569	-7110	-817	-2157
19	-2370	-277	-776	-5506	-644	-1803	-7571	-886	-2479
21	-2581	-347	-810	-5996	-806	-1881	-8245	-1108	-2587
22	-194	-72	-87	-452	-168	-201	-622	-231	-277
25	423	136	134	983	316	311	1351	435	427
28	705	157	259	1638	365	601	2252	502	826
31	831	175	302	1930	406	700	2654	558	963
34	1009	176	356	2345	409	826	3225	562	1136
37	1145	197	405	2660	457	940	3658	628	1292
40	1246	198	433	2895	459	1006	3980	631	1383
43	1434	204	489	3330	473	1136	4579	651	1562
46	1152	87	359	2676	203	833	3680	279	1146
49	1098	84	343	2551	194	798	3508	267	1097
52	1006	59	283	2336	138	657	3212	190	904
55	906	58	256	2104	136	594	2893	187	817
58	858	57	253	1993	132	588	2740	182	808
61	784	44	179	1820	101	415	2503	139	571
64	611	41	166	1419	95	385	1951	131	529
67	591	39	151	1372	91	351	1886	125	482
70	512	35	128	1190	81	298	1636	111	410
73	421	31	100	977	73	232	1344	100	319
76	255	23	63	591	54	147	813	74	202
77		20	61		47	141		65	194

* See note at the bottom of page 94

TABLE 5.3.1

CASE C. OUTSIDE BOUNDARY

NODE	time=4 minutes		time=8 minutes		time=11 minutes	
	$\sigma_{max\,p}$	$\tau_{max(12)}$	$\sigma_{max\,p}$	$\tau_{max(12)}$	$\sigma_{max\,p}$	$\tau_{max(12)}$
1						
4	-573	590	-1330	1370	-1829	1884
7	-1280	1423	-2972	3305	-4087	4545
10	-1576	1563	-3660	3631	-5033	4992
13	-2167	2011	-5033	4670	-6921	6421
16	-2246	2083	-5217	4838	-7174	6652
19	-2375	2162	-5516	5020	-7584	6903
21	-2582	2465	-5998	5725	-8247	7872
22	-550	809	-1276	1880	-1755	2585
25	433	420	1006	976	1383	1342
28	715	568	1661	1319	2284	1813
31	834	663	1938	1540	2665	2118
34	1012	838	2350	1945	3231	2675
37	1147	944	2663	2193	3662	3016
40	1253	1062	2910	2466	4001	3391
43	1435	1240	3333	2879	4583	3959
46	1159	1123	2692	2607	3702	3585
49	1099	1102	2553	2560	3511	3520
52	1009	1054	2343	2448	3221	3366
55	858	1053	1993	2447	2741	3365
58	849	962	1972	2234	2712	3072
61	795	746	1846	1732	2538	2382
64	615	677	1428	1573	1964	2163
67	601	580	1396	1347	1919	1852
70	513	557	1192	1294	1639	1779
73	445	196	1034	455	1422	625
76	160	79	372	183	512	251
77						

TABLE 5.3.2

TABLE 6.4.1(a)

GAUGE NO	MVOLTS READINGS	DIRECT STRAINS	DIRECT STRESSES
1	.019	.5919E-05	177.6
2	.009	.2857E-05	85.7
3	.019	.2804E-05	84.1
4	.024	.7477E-05	224.3
5	.015	.4762E-05	142.9
6	.012	.3738E-05	112.1
7	.033	.1028E-04	308.4
8	.023	.7302E-05	219.0
9	.011	.3427E-05	102.8
10	.034	.1059E-04	317.8
11	.021	.6667E-05	200.0
12	.017	.5296E-05	158.9
13	.036	.1121E-04	336.4
14	.018	.5714E-05	171.4
15	.019	.5919E-05	177.6
16	.039	.1215E-04	364.5
17	.027	.8571E-05	257.1
18	.015	.4673E-05	140.2
19	.031	.9657E-05	289.7
20	.018	.5714E-05	171.4
21	.014	.4361E-05	130.8
22	.011	.3427E-05	102.8
23	.009	.2857E-05	85.7
24	.006	.1869E-05	56.1
25	-.072	-.2243E-04	-672.9
26	-.030	-.9524E-05	-285.7
27	.028	.8723E-05	261.7
28	-.054	-.1682E-04	-504.7
29	-.029	-.9206E-05	-276.2
30	-.023	-.7165E-05	-215.0
31	-.033	-.1023E-04	-308.4
32	-.019	-.6032E-05	-181.0
33	-.013	-.4050E-05	-121.5
34	-.007	-.2181E-05	-65.4
35	-.008	-.2540E-05	-76.2
36	-.012	-.3738E-05	-112.1
37	-.029	-.9034E-05	-271.0
38	-.031	-.9841E-05	-295.2
39	-.074	-.2305E-04	-691.6
40	-.028	-.7788E-05	-233.6
41	-.029	-.9206E-05	-276.2
42	-.056	-.1745E-04	-523.4
43	-.015	-.4673E-05	-140.2
44	-.021	-.6667E-05	-200.0
45	-.035	-.1093E-04	-327.1

ROSETTE NO	PRINCIPAL STRAINS		PRINCIPAL STRESSES		DIRECTION ANGLE
1	.8975E-05	-.2521E-06	293.4	80.5	22.00
2	.1128E-04	-.6343E-07	371.2	109.5	12.17
3	.1372E-04	-.1463E-07	452.2	135.2	-3.72
4	.1599E-04	-.1023E-06	526.1	154.8	12.38
5	.1760E-04	-.4625E-06	575.5	156.3	23.57
6	.1682E-04	-.1526E-06	554.6	166.3	-1.23
7	.1414E-04	-.1186E-06	464.9	135.9	13.03
8	.5304E-05	-.3248E-06	174.6	52.2	-7.52
9	.5018E-06	-.1421E-04	-124.0	-463.5	-4.86
10	.3196E-06	-.2431E-04	-229.9	-798.2	15.00
11	.8908E-07	-.1442E-04	-139.7	-474.5	10.00
12	.5949E-05	-.2963E-07	-195.8	-57.9	-14.16
13	.5595E-06	-.1457E-04	-125.9	-474.9	5.01
14	.4528E-06	-.2569E-04	-239.1	-842.3	-17.62
15	.8033E-07	-.1586E-04	-152.2	-515.4	-9.90

TABLE 6.4.1(b)

EXPERIMENTAL RESULTS. STRESSES (lb/in^2)

Time=12 minutes. Heating-up.			
DIRECT STRESSES			
ROSETTE	GAUGE	CORRECTED	OBSERVED
1	1	1116	2316
	2	584	1211
	3	388	805
2	4	2254	3243
	5	1150	1655
	6	634	912
3	7	3212	4200
	8	2248	2940
	9	811	1061
4	10	3985	4824
	11	2502	3030
	12	1063	1287
5	13	3920	4218
	14	1943	2091
	15	904	973
6	16	4065	4275
	17	2444	2571
	18	993	1045
7	19	4308	4458
	20	2211	2288
	21	1098	1136
8	22	3192	-
	23	2543	-
	24	1451	-
9	25	-9122	-
	26	-4841	-
	27	-3002	-
10	28	-7345	-
	29	-4204	-
	30	-2248	-
11	31	-3231	-
	32	-2985	-
	33	-948	-

TABLE 6.4.2

EXPERIMENTAL RESULTS.

Time = 12 minutes. Heating-up.		
ROSETTE	MAXIMUM PRINCIPAL STRESS $\sigma_{maxp}(lb/in^2)$	MAXIMUM SHEAR STRESS $\tau_{max(12)} lb/in^2$
1	1567	1401
2	2311	2118
3	2419	2641
4	2478	2845
5	2463	3109
6	2853	3281
7	3841	3613
8	4202	5718
9	-8194	7402
10	-7218	6348
11	-3178	3312

TABLE 6.4.3

VESSEL (NODE 12). STRESSES lb/in^2.

Time(minutes)		Gauge 28	Gauge 30	Rosette 10
		σ_y	σ_z	σ_{maxp}
Heating	0	0	0	0
	2	-1186	-421	-1093
	4	-2243	-763	-2151
	6	-3418	-1140	-3502
	8	-5006	-1496	-5124
	10	-6414	-1794	-6591
	12	-7345	-2248	-7218
	14	-7096	-2018	-6940
	16	-6307	-1991	-6690
	18	-4844	-1488	-5923
	20	-3652	-1476	-4688
Cooling	22	1364	497	1257
	24	2579	874	2484
	26	3931	1379	4062
	28	5757	1765	5995
	30	8446	2585	8301
	32	8160	2341	7981
	34	7253	2353	7694
	36	5571	1771	6811
	38	4200	1727	5391
	40	3110	1452	4711
	42	1448	808	3642
	44	608	819	3233

TABLE 6.4.4

NOZZLE (NODE 43). STRESSES lb/in^2.

Time(minutes)		Gauge 13	Gauge 15	Rosette 5	OBSERVED		
		σ_x	σ_z	σ_{maxp}	σ_x	σ_z	σ_{maxp}
Heating	0	0	0	0			
	2	893	264	868			
	4	1754	508	1921			
	6	2614	911	3002	The same		
	8	3721	1483	3855			
	10	4891	1709	5002			
	12	5541	1890	5545			
	14	5426	1814	5294			
	16	4875	1551	4321			
	18	3762	1366	3717			
	20	2918	1077	3109			
Cooling	22	-1027	-304	-998			
	24	-2035	-584	-2209			
	26	-3045	-1048	-3452			
	28	-5674	-1705	-4433			
	30	-6372	-2174	-6377			
	32	-6240	-2086	-6088			
	34	-5606	-1784	-4969	The same		
	36	-4326	-1571	-4275			
	38	-3356	-1239	-3575			
	40	-2169	-763	-2653			
	42	-1478	-491	-1627			
	44	-724	-304	-1141			

TABLE 6.4.5

r (inches) Distance from the tip of the crack along y	d_1 First matrix coefficient	$K = -\sqrt{2\pi}(d_1)$
0.960	-279.8	702.6
0.910	-320.4	803.1
0.885	-332.1	832.5
0.735	-421.6	1056.8
0.660	-462.7	1159.8
0.585	-507.4	1271.9
0.460	-578.9	1451.1
0.360	-637.2	1597.2
0.210	-726.7	1821.6
0.110	-782.4	1961.2
0.085	-801.6	2009.3
0.060	-813.5	2039.1
0.035	-834.2	2091.0
0.010	-839.4	2104.1
(Tip of the crack)	-842.6	2112.3

TABLE 7.3.2.1

FIGURES
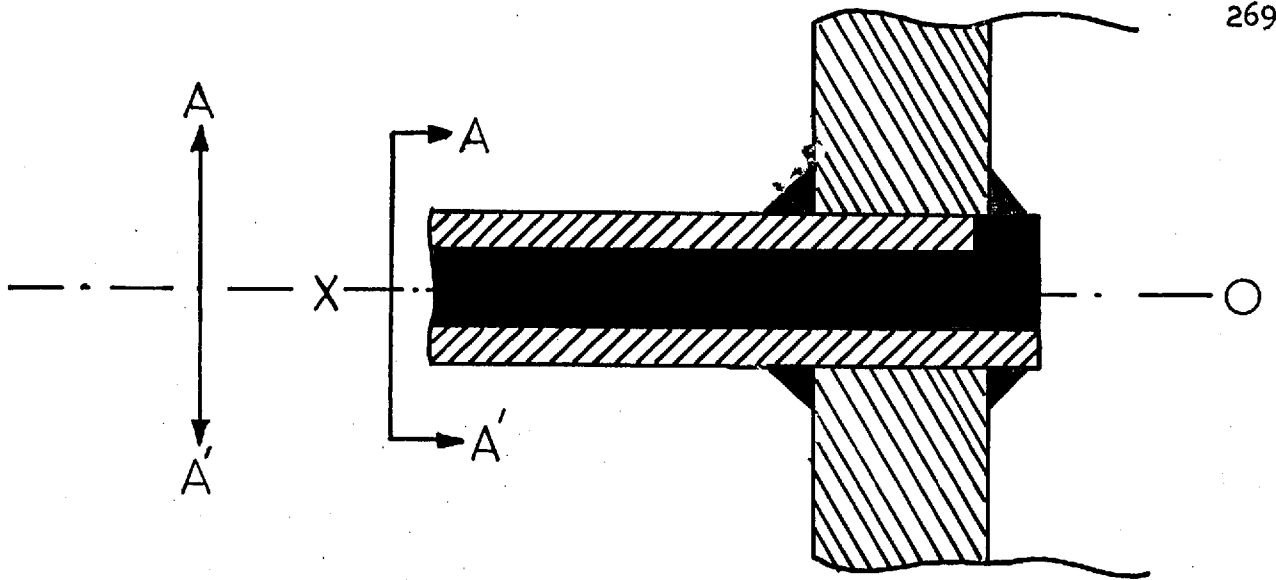
[a] Radial protruding nozzle
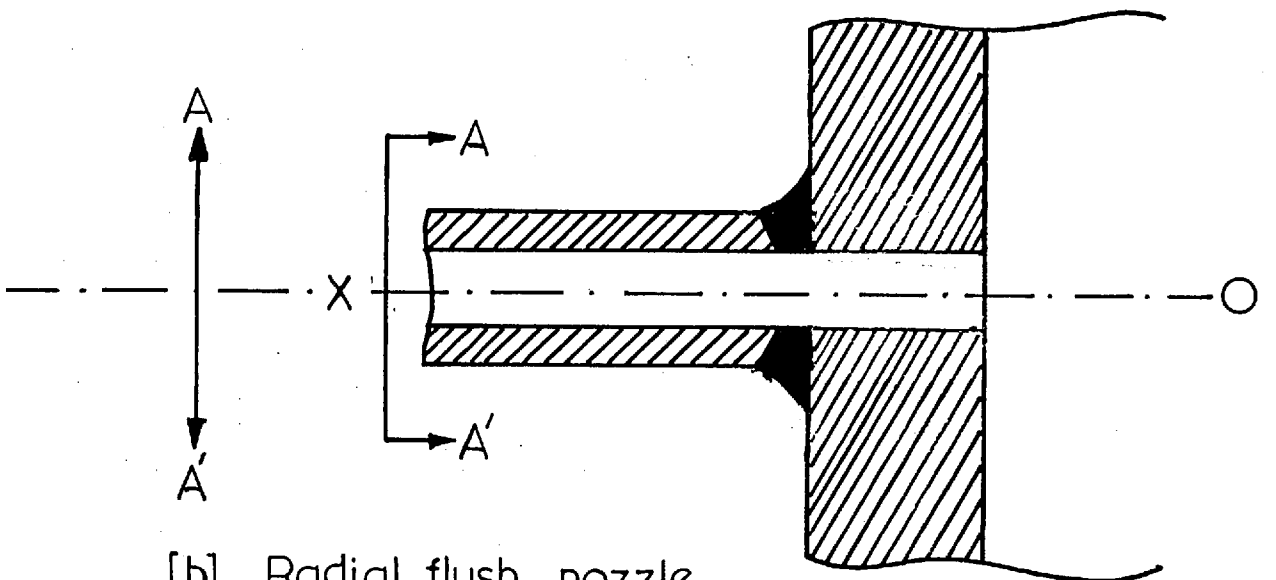
[b] Radial flush nozzle
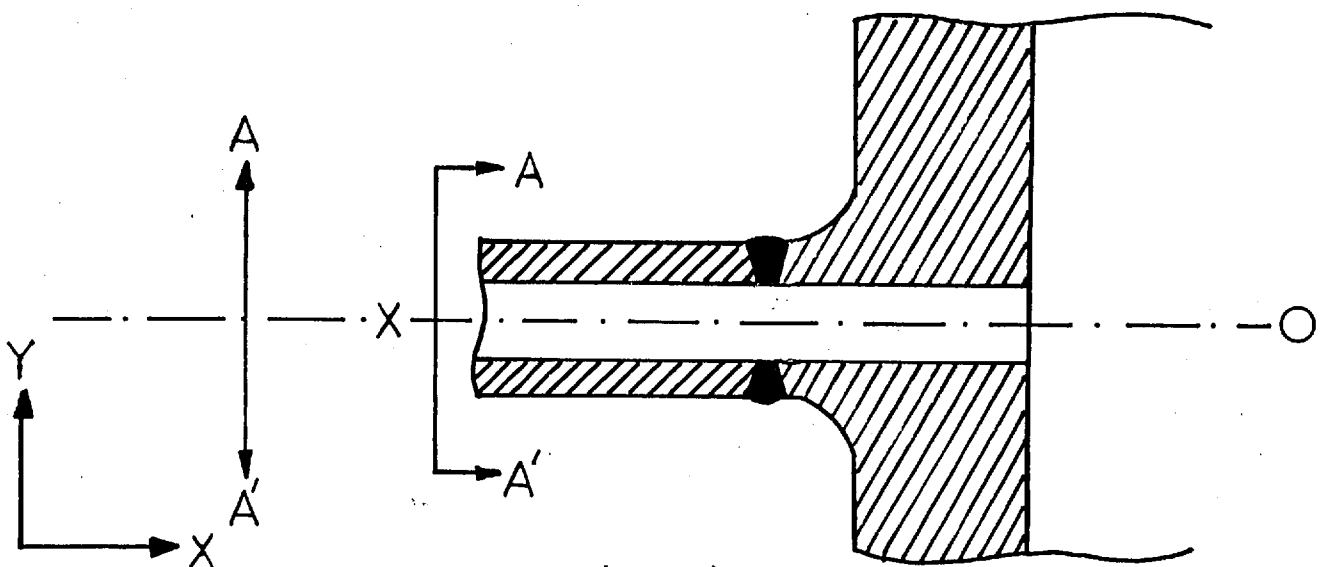
[c] Radial nozzle directly welded on the vessel

FIGURE 3.1.1

Y

X ———————————————————— X'

GENERAL SECTION
SHOWN IN FIGURE
3.1.1.c

Y

X

Y'
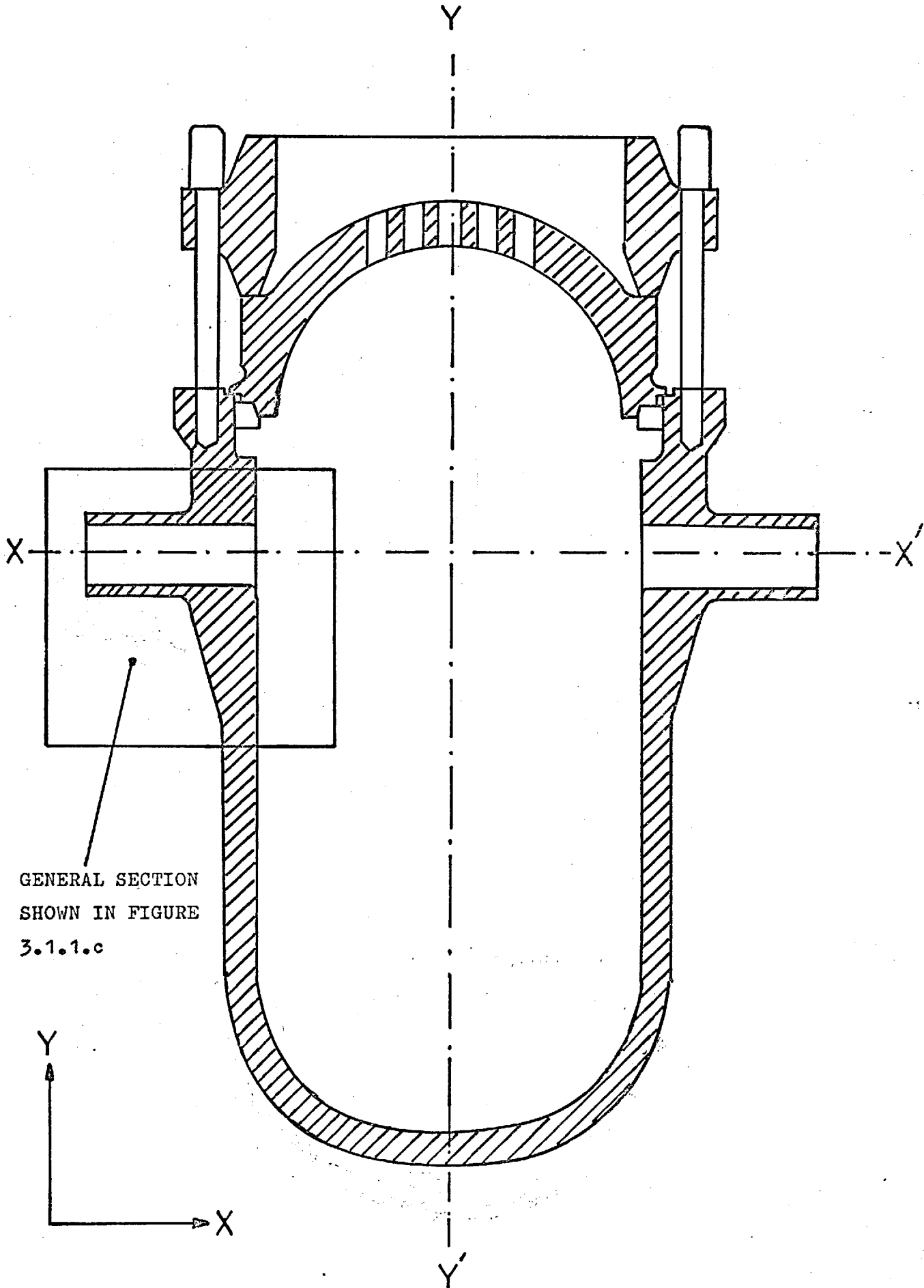
FIGURE 3.1.2 A PRESSURE VESSEL
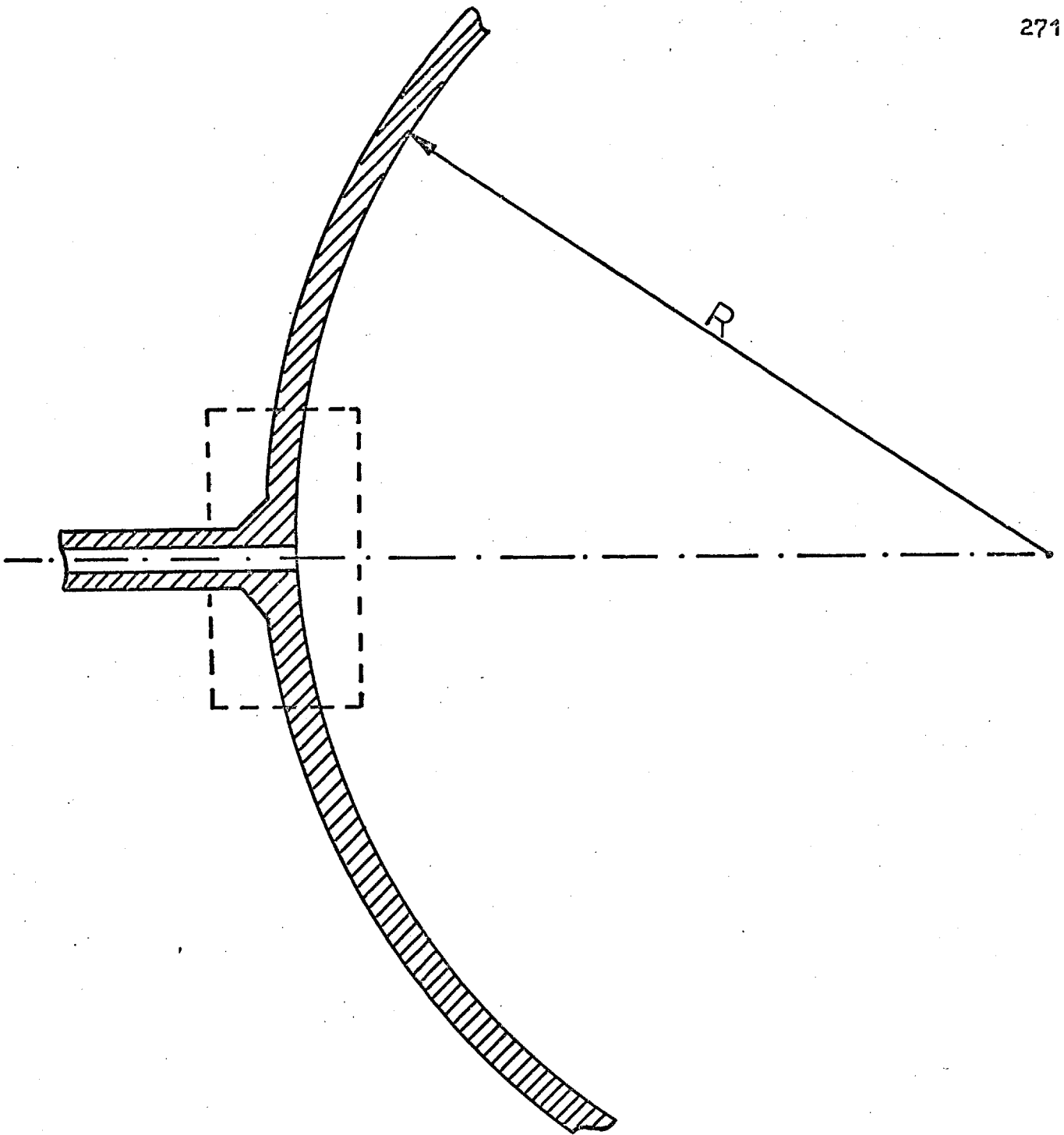
CURVATURE OF SHELL NEAR THE JUNCTION IS
SMALL COMPARED TO THAT OF NOZZLE.

FIGURE 3.1.3

FIGURE 3.2.1 CASE A.

FIGURE 3.2.2. CASE B.

FIGURE 3.2.3

CASE C. (EXPERIMENTAL SPECIMEN)

TEMPERATURE GRID FOR CASE A.

TEMPERATURE(F)	SYMBOL
60	D
70-80	E
90-100	F
110-120	G
130-140	H
150-160	J
170-180	K
190-200	L
210-220	M
230-240	N
250-260	O
270-280	P
290-300	Q
310-320	R
330-340	S
350-360	T
370-380	U
390-400	V
410-420	W
430-440	X
450	Y

```
YX     TS  QP O HM LL  K  JJ HH    GG       F
YX  UTS  Q  O N M L K   J  H  GJG   FFFF
YX  UTS  Q  O N M L K   J  H  GJG   FFFF
YX  UTS  Q  O N M L K   J  H  GJG   FFFF
YX  UTS  Q  O N M L K   J  H  GJG   FFFF
YX  UTS  Q  O N M L K   J  H  GJG   FFF
YX  UTS  Q  O N M L K   J  H  GJG   FFF
YX  UTS  Q  U N M L K   J  H  GJG   FFF
YX  UTS  Q  O N M L K   J  H   GG   FFFF
YX  UTS  Q  O N M L K   J  HH  GG   FFFF
YX  UTS  Q  O N M L K   J  HH  GG   FFFF
YX  UTS  Q  O N M L K   J  HH  GG   FFFF
YX  UTS  RQ O N H L K   J  HH  GG   FFFF
YX  UTS RQ PO N M L K   J  HH  GG   FFF
YX  UTS RQ PO N M L KK  J  HH  GGG  FFF
YX  UTS RQ PO N M L KK JJ HH  GGG  FFF
YX  UTS RQ PO N N L KK JJ HH  GGG  FFF
YX  UT  RQ PO N M L KK JJ HH   GG   FFF
YX  UT  RQ PO N M L KK JJ H    GG   FF
YX  UT  RQ PO N M L  K JJ  HH  GGG   FF
YX  UT  RQ PO N M L  K JJ  HH  GGG    F
YX  UT  RQ P  N H LL K  J  HH  GGG    F
YX  UT  RQ P  N H LL K  J  HH   GGG
YX  UT  RQ P  N M  L K JJ HH    GGG
YX  UT  RQ P OH M  L K  JJ  HH  GGGG
YX  UT  RQ P O   H L KK JJ  HH   GGG
YX  UT  R  P O N H L KK JJ  HH  GGJG
YX  UT  R  P O N H L  K  J   HH  GGGG
YX  UT  R  P O N H L  K JJ HH   GGGG
YX  UT  R  P O N H LL KK JJ HJH  GGGG
YX  UT SR QP O N H LL KK  JJ  HHH  GGG
YX  VUT SR QP O N M  L  K  JJ  HHH   GG
YX  VU  SR Q  O N HH L  KK JJJ  HHH
YX  VU  SR Q PO NN H LL KK JJ  HHH
YX  VU  S  Q P O N N  L K  JJ   HHHH
YX  VU  S  Q P U N M  L  KK JJJ  HHHH
Y   VU  S  Q P O N  H LL  KK JJJ  HHHH
Y   VU TS RQ P O HN H  L  KK  JJJ   HH
Y   VU TS R QP OO N HM LL KK   JJJJ
Y    V  TS R Q P O N  H  LL KK   JJJJ
Y   HV  T  R Q P O NN HH  LL  KKK  JJJJJ
Y   HV  T SR Q P OO N  H  LLL  KKK    JJ
Y   HV  T S  Q PP O HN H  LLL  KKKK
Y   HV UT S R Q P OO NN HH  LL  KKKKK
Y   HV U  S R Q P  O N  HH  LLL   KKK
Y   H  U TS R QQ P OO  N  HH LLLL
Y   H  U T S R Q PP OO NN  HMM  LLLLL
Y   H VU T S R QQ P  OO  NN  HHM  LLLLL
Y   H V  T S R RR Q  P OO HNH  HHMM
Y  XH  V U T S R  Q  P OOO HNNN  HHMMHHHH
Y  XH V U T  S  R QQ PP OOO  NHHNN      H
Y  X   V U T  S RR QQ PP  OOOO  NHNNHNHNN
Y  X  H V U T SS RR  1Q PPP  OOOOO      HN
Y  X  H V U TT SS RR  1QQ PPPP   OOOOOOOOOO
Y  X  H V UU T  SS  RR OOO  PPPPP         O
Y  X  H  V U  T  SS  RRR  OQQQQ   PPPPPPPPPPPP
Y  X  H V  U  TT  SS   RRR  OOQOOOQO
Y  XX H  V  U TT  SSS   RRRRR    OOQOOQQQQ
Y   X HH VV  UU  TTT  SSSSS    RRRRR RRRRRRRRRRRRRR
Y   X  H VV  UUU   TTTT    SSSJSSSSSS
```

```
YY XX  H   VV   UUU    TTTTTT      SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
YY XX   HH  VVV   UUUUU     TTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTT
YY XX   WWW  VVVV      UU JUJUUUUUUUJUJUJUUUUUJUUUUUUUJUJUUUUJUUUUUUUUUU...
YY   XXX  HWHW    VVVVVVJJJVVVVV
YYY   XXXXX   HWHWHWWWWW
YYYYYYYY   XXXXXXXXX            XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
YYYYYYYY  YYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYY
```

FIGURE. 442(a)

CASE A. TEMPERATURE PROFILES FOR TIME=9min.30secs.

FIGURE. 4.4.2(b)

CASE A. TEMPERATURE PROFILES FOR TIME=54 minutes.

JUNCTION BOUNDARY DETERMINATION

FIGURE 4.4.3

FIGURE 4.4 (a)

TEMPERATURE PROFILES FOR CASE A. TEMPERATURES IN DEGREES °F

time = 3 min.

time = 9 min. 30 sec.

time = 18 min.

time = 27 min.

FIGURE 4.4.4(b)

time = 36 min.

time = 45 min.

time = 63 min.

time = 81 min.

FIGURE 444 c

time = 126 min.

440

450

time = 144 min.

450

time = 162 min.

450

time = 200 min.

450

$$\Delta T_m = T_{mean\ NOZZLE} - T_{mean\ VESSEL}$$

$T_{mean\ NOZZLE}$ is the average of all the nodal temperatures in nozzle.

$T_{mean\ VESSEL}$ is the average of all the nodal temperatures in vessel.

ΔT_m IS MAXIMUM = 193.1 °F AT TIME = 9 min. 30 secs.
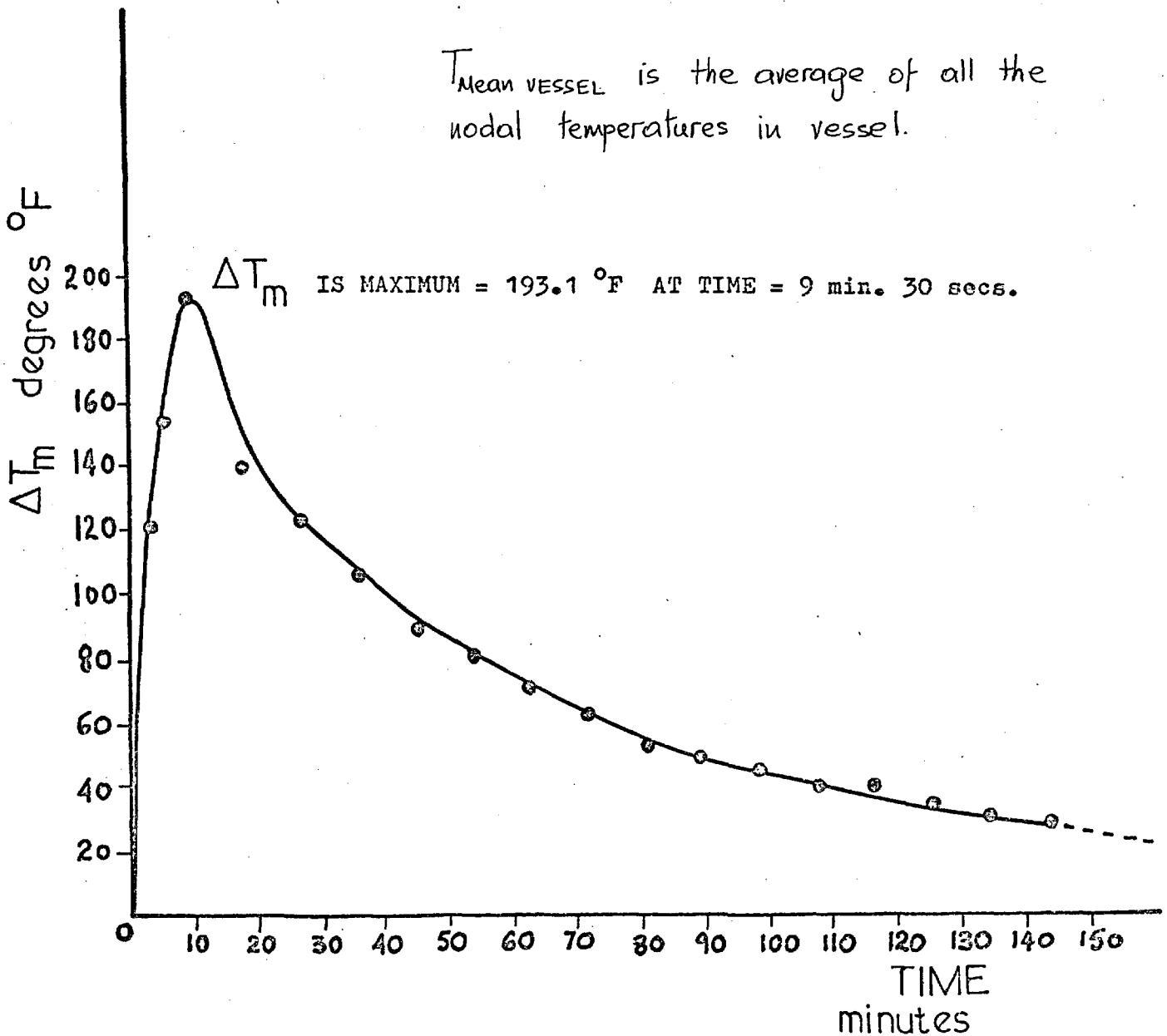
FIGURE 4.4.5. CASE A. MEAN TEMPERATURE VARIATION ΔT_m BETWEEN NOZZLE AND VESSEL.

FIGURE 4.5.1 TEMPERATURE GRID FOR CASE B.

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TEMPERATURE(°F)	SYMBOL
60	D
69-78	E
88-96	F
106-114	G
124-132	H
142-150	I
160-168	J
178-186	K
196-204	L
214-222	M
232-240	N
250-258	O
268-276	P
286-294	Q
304-312	R
322-330	S
339-348	T
358-366	U
376-384	V
394-402	W
412-420	X
430-438	Y
448	Z
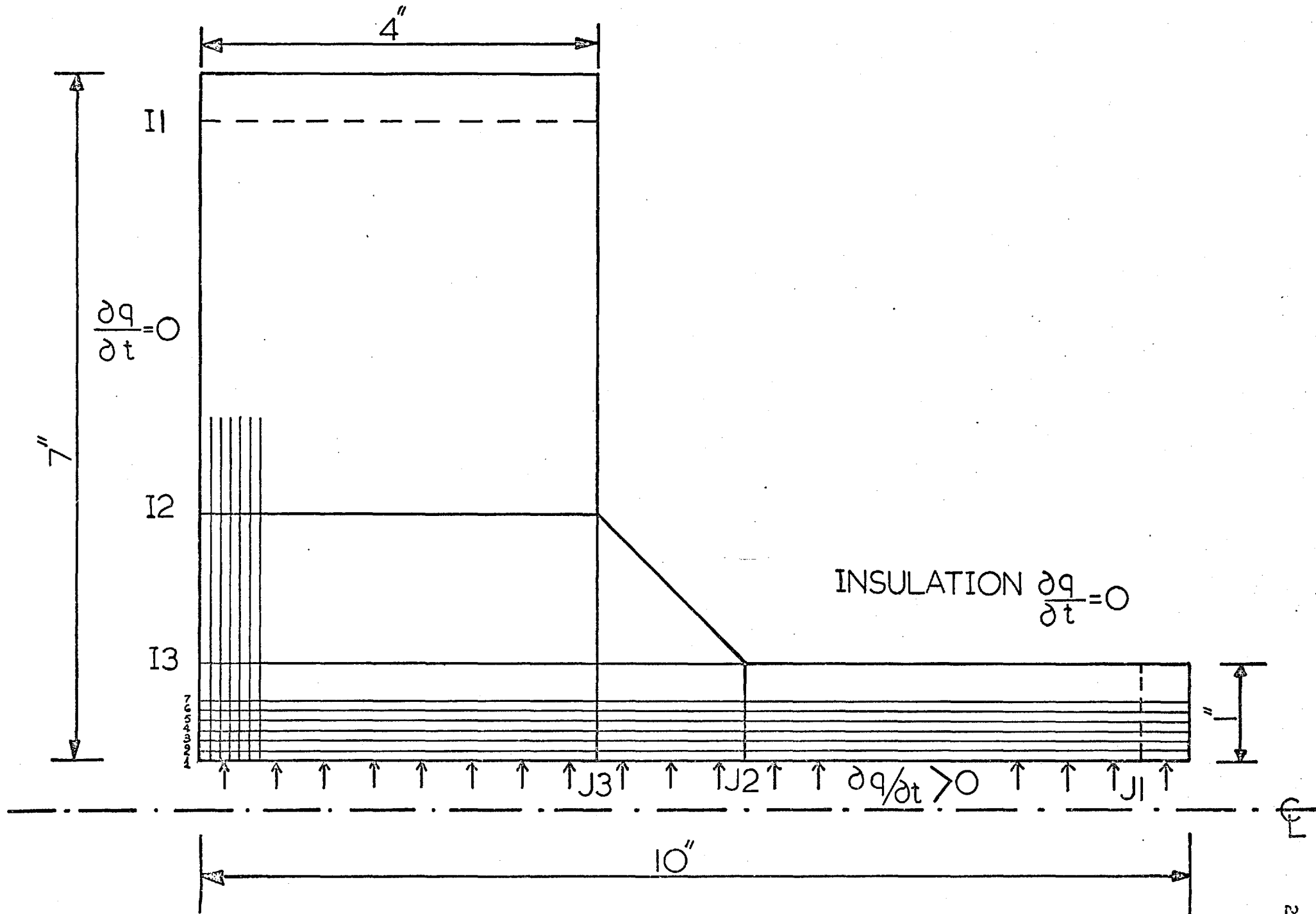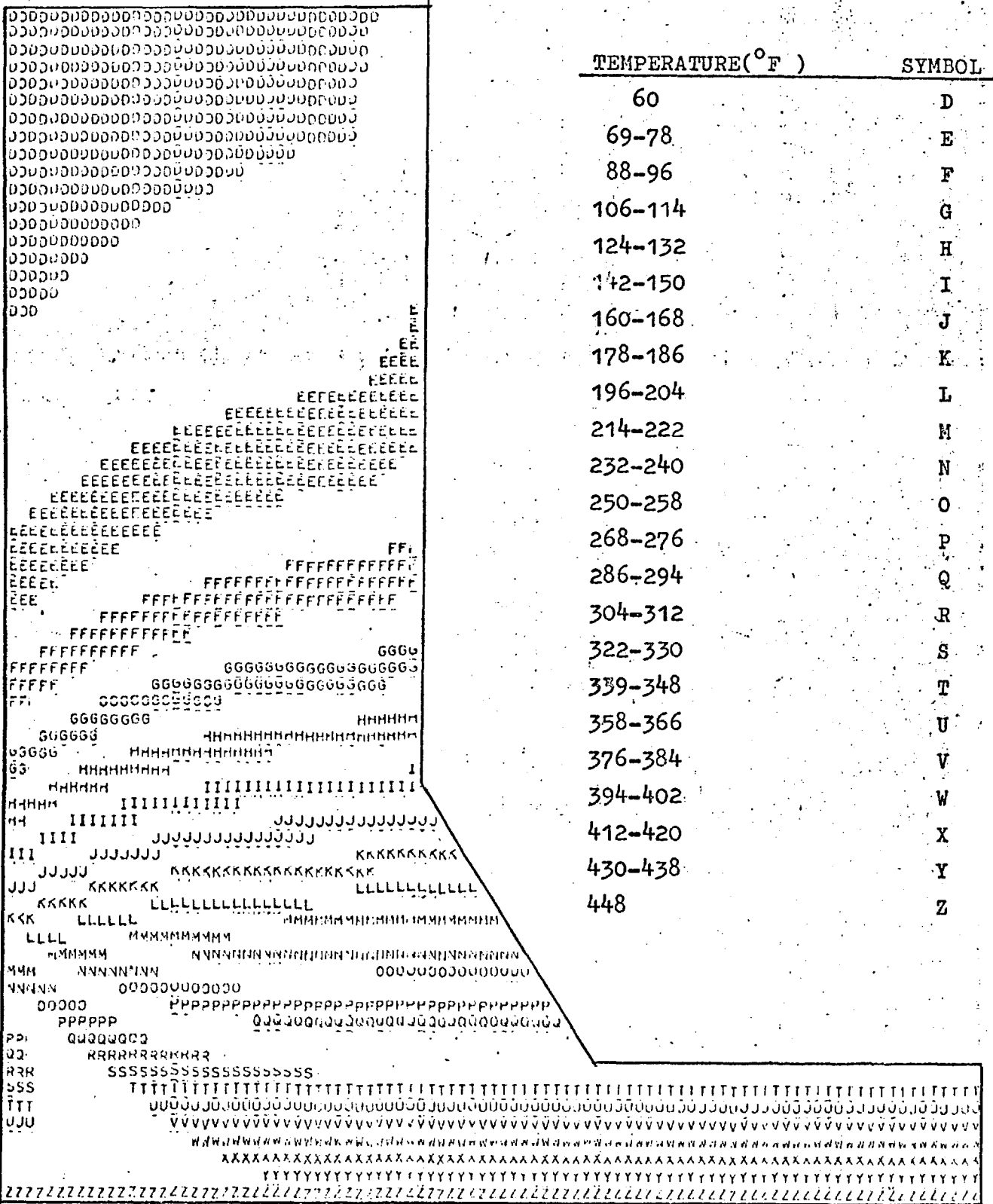
FIGURE 4.5.2 (a).

CASE B. TEMPERATURE PROFILES FOR TIME = 10 minutes.

TEMPERATURE($^{\circ}$F)	SYMBOL
	D
69-78	E
88-96	F
106-114	G
124-132	H
142-150	I
160-168	J
178-186	K
196-204	L
214-222	M
232-240	N
250-258	O
268-276	P
286-294	Q
304-312	R
322-330	S
339-348	T
358-366	U
376-384	V
394-402	W
412-420	X
430-438	Y
448	Z
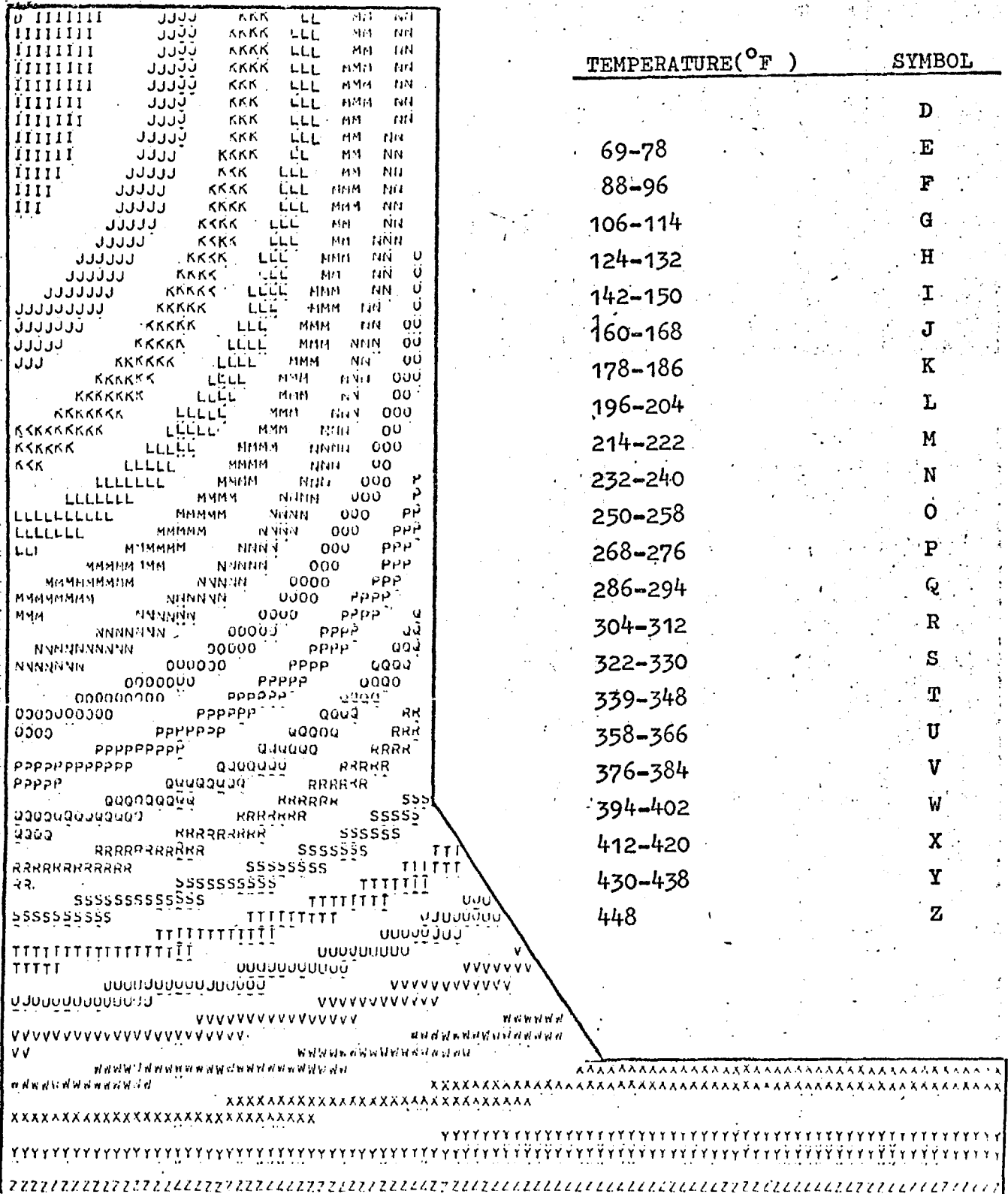
FIGURE 4.5.2 (b).

CASE B. TEMPERATURE PROFILES FOR TIME = 54 minutes.

TEMPERATURE PROFILES FOR CASE B. TEMPERATURES IN DEGREES °F

FIGURE 4.5.3(a)

time = 0

60

time = 3 min.

60

75
95
115
135
155
220

time = 10 min.

60
75
95
115
135
155
175
215
275
295
370
400
430
450

time = 36 min

75
95
115
135
155
195
215
255
295
355
400
430
450

FIGURE 4.5.3 (b)

time = 27 min

time = 36 min.

time = 45 min.

time = 54 min.

* For definition of ΔT_m see page 282

ΔT_m^* IS MAXIMUM = 302.4 AT TIME = 10 minutes.
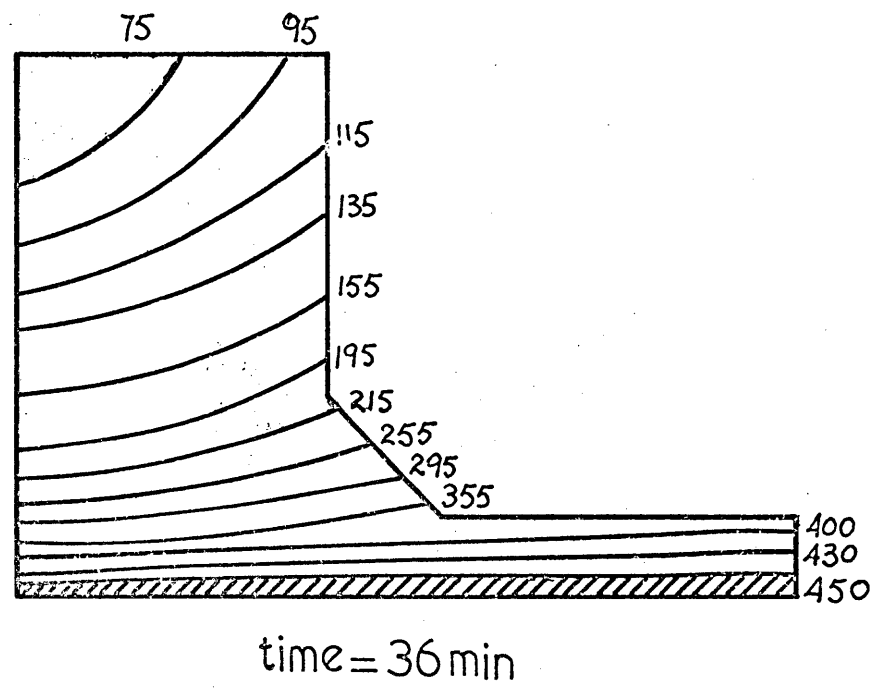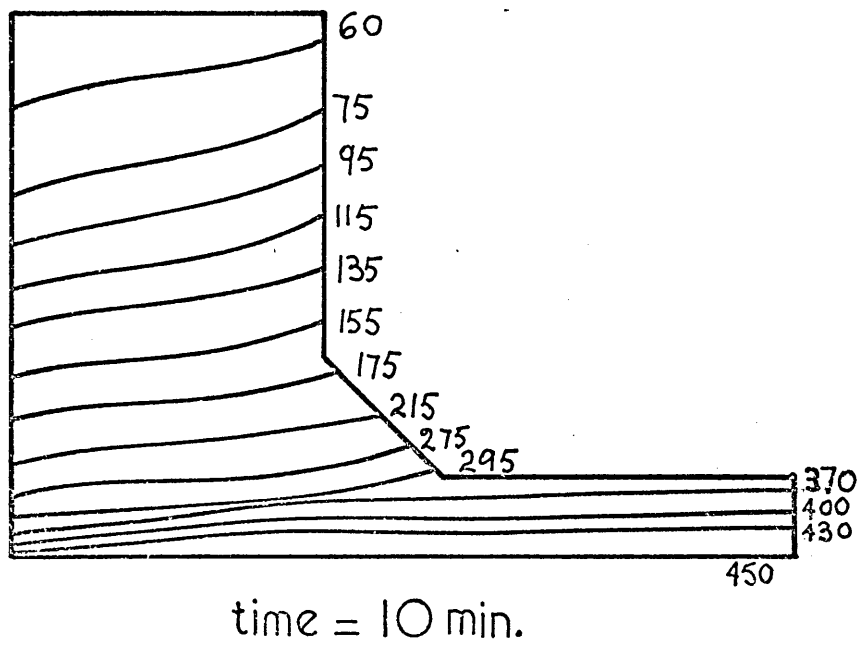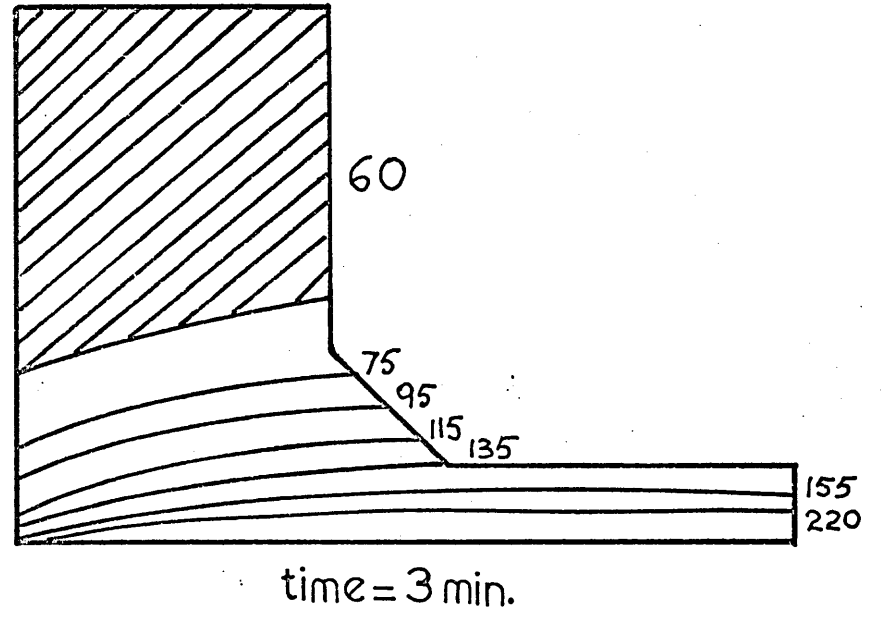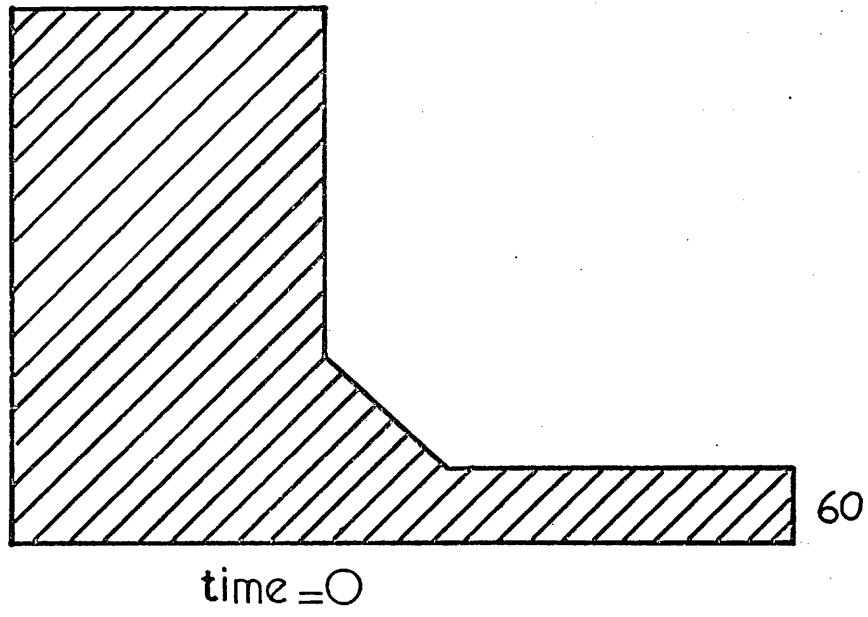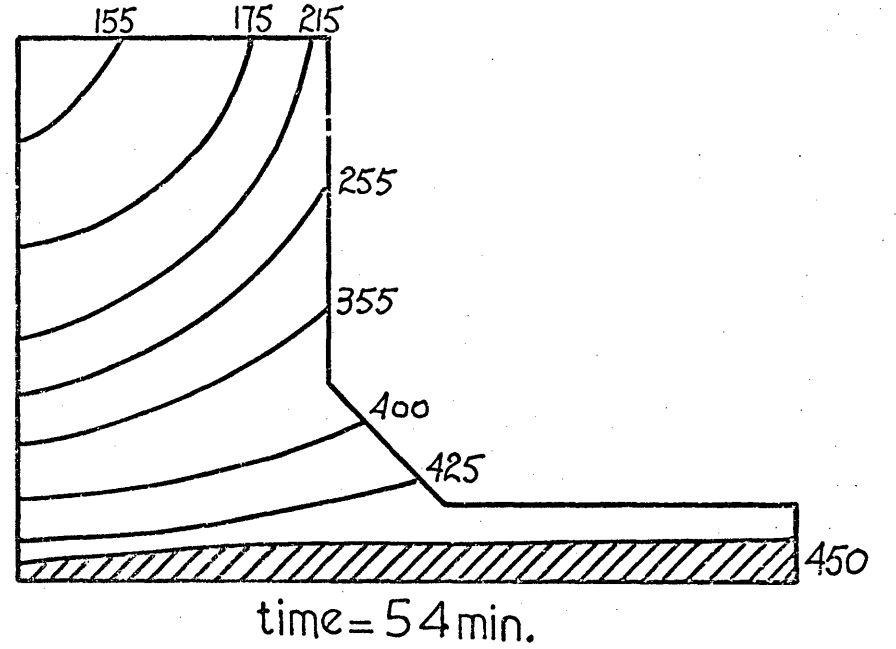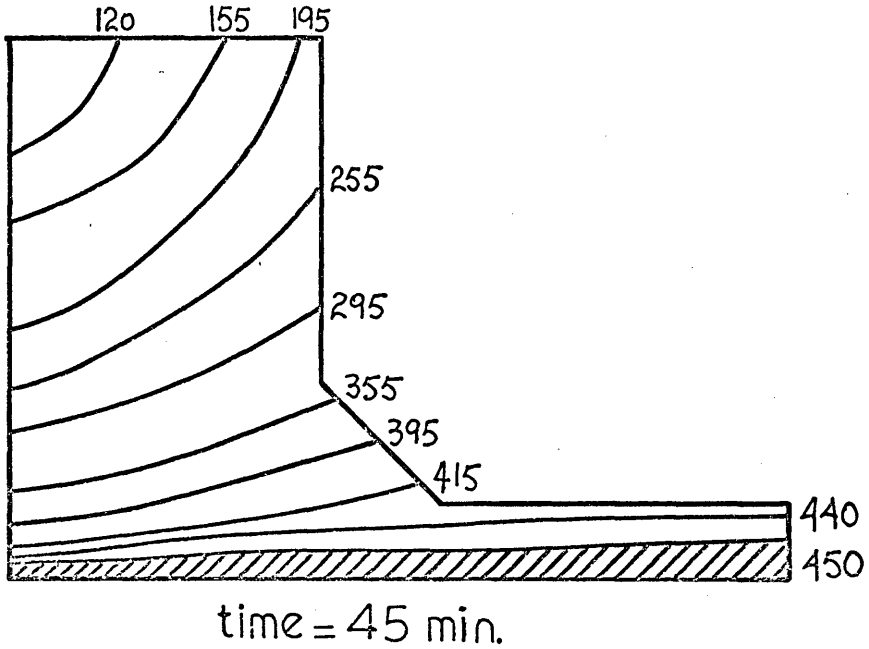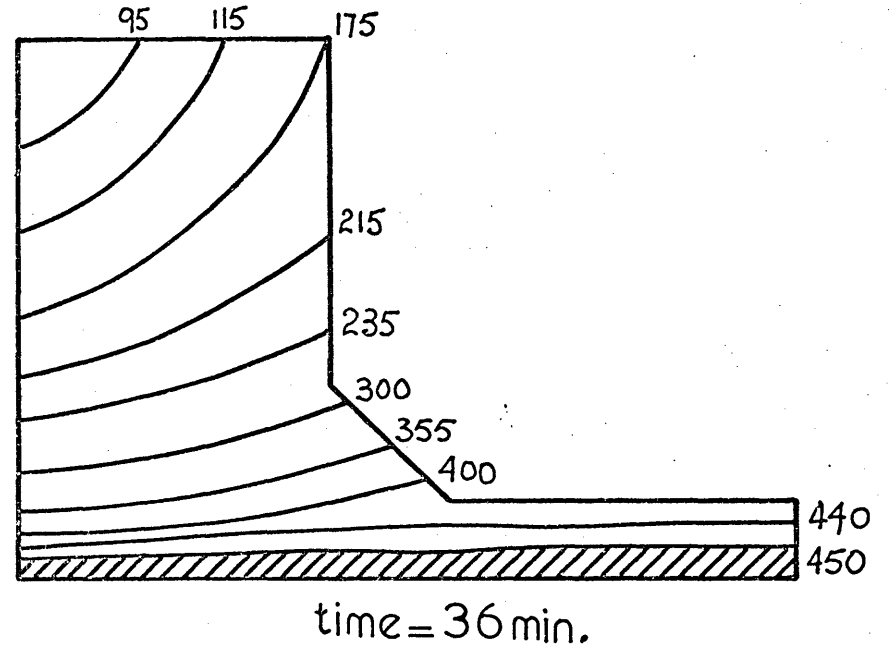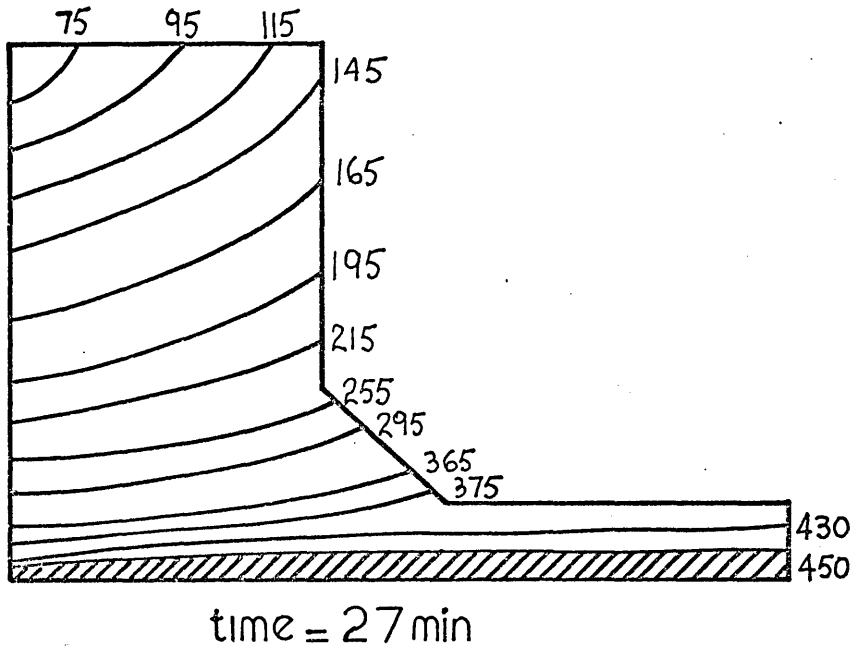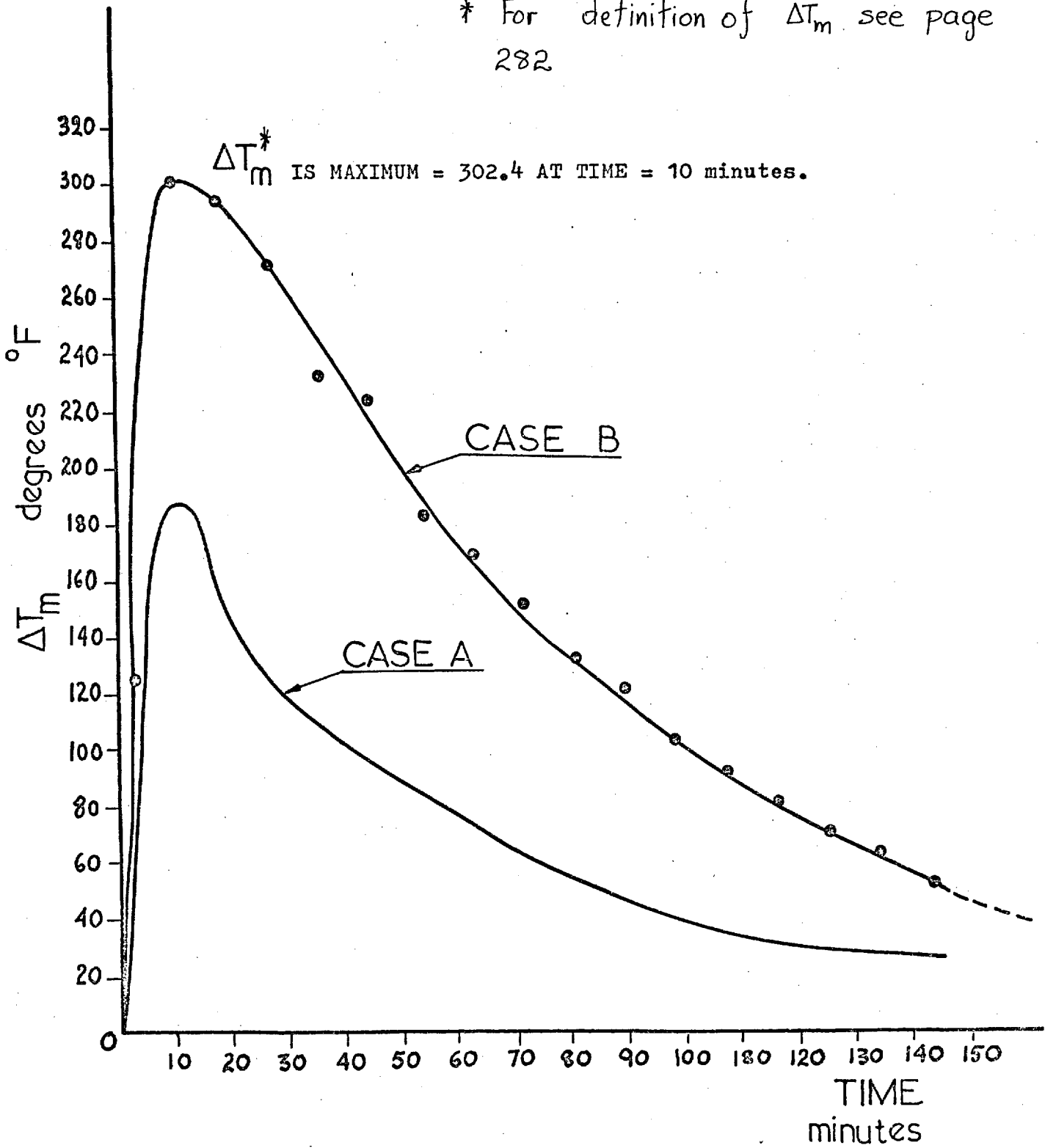
FIGURE 4.5.4 CASE B. MEAN TEMPERATURE VARIATION ΔT_m BETWEEN NOZZLE AND VESSEL.

$$h_T = h_{rad} + h_{NC} + h_{COND}$$

$$\frac{\partial q}{\partial t} > 0$$
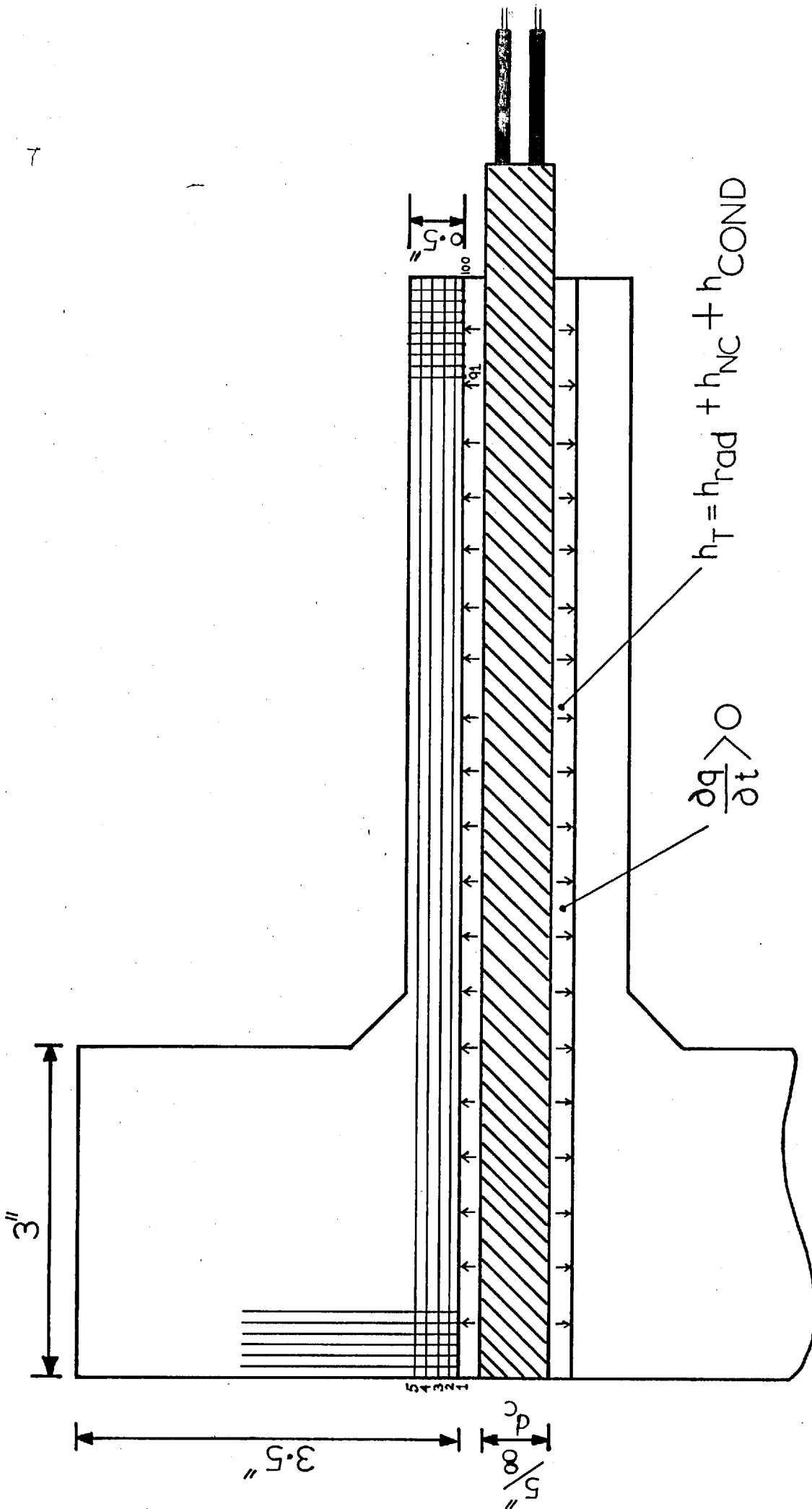
FIGURE 4.6.1 TEMPERATURE GRID FOR CASE C.

FIGURE 4.6.2(a)

time=0

60

time=4 min.

60 75 95

115
135
145
155
165
175
185
195
225
275
450

290

time = 6 min.

time = 10 min.

FIGURE 4.6.2 (b)

FIGURE 4.6.2(c)

time = 12 min.

time = 14 min.

* For definition of ΔT_m see page 282

ΔT_m^* IS MAXIMUM = 236.9 °F AT TIME = 11 minutes.
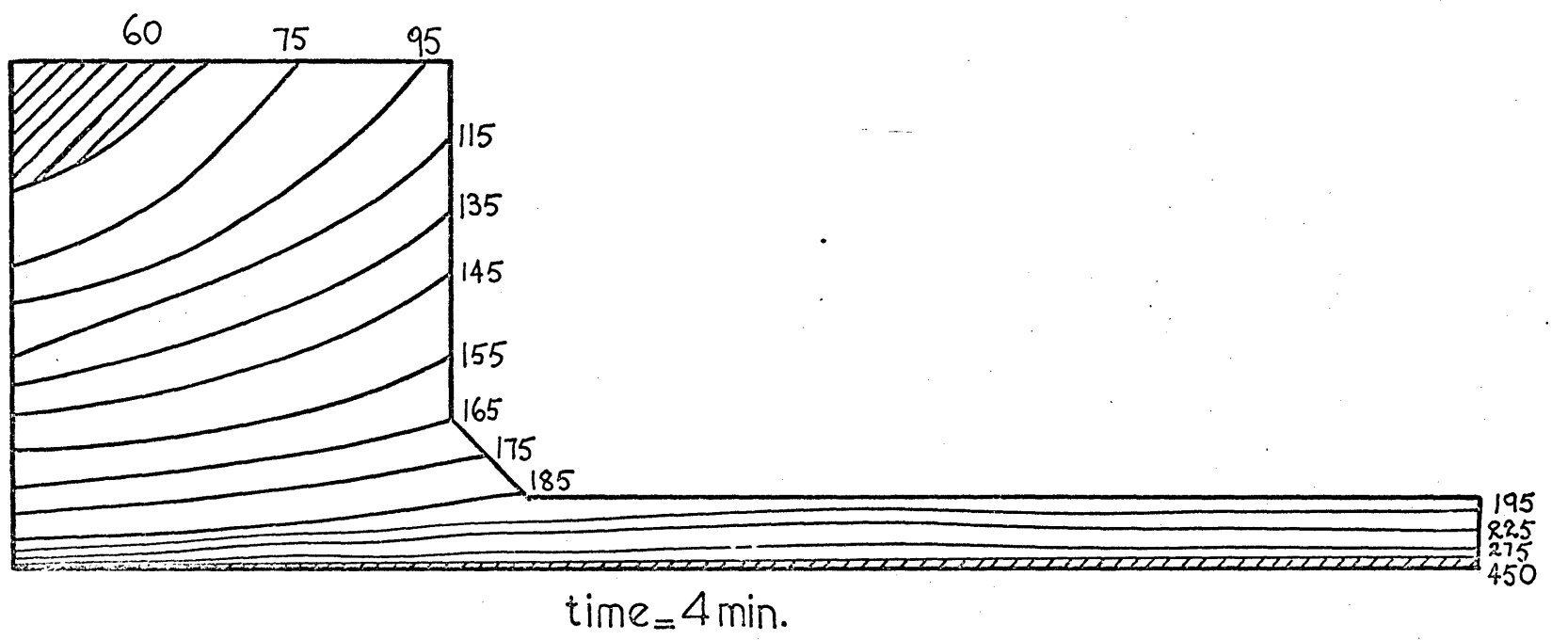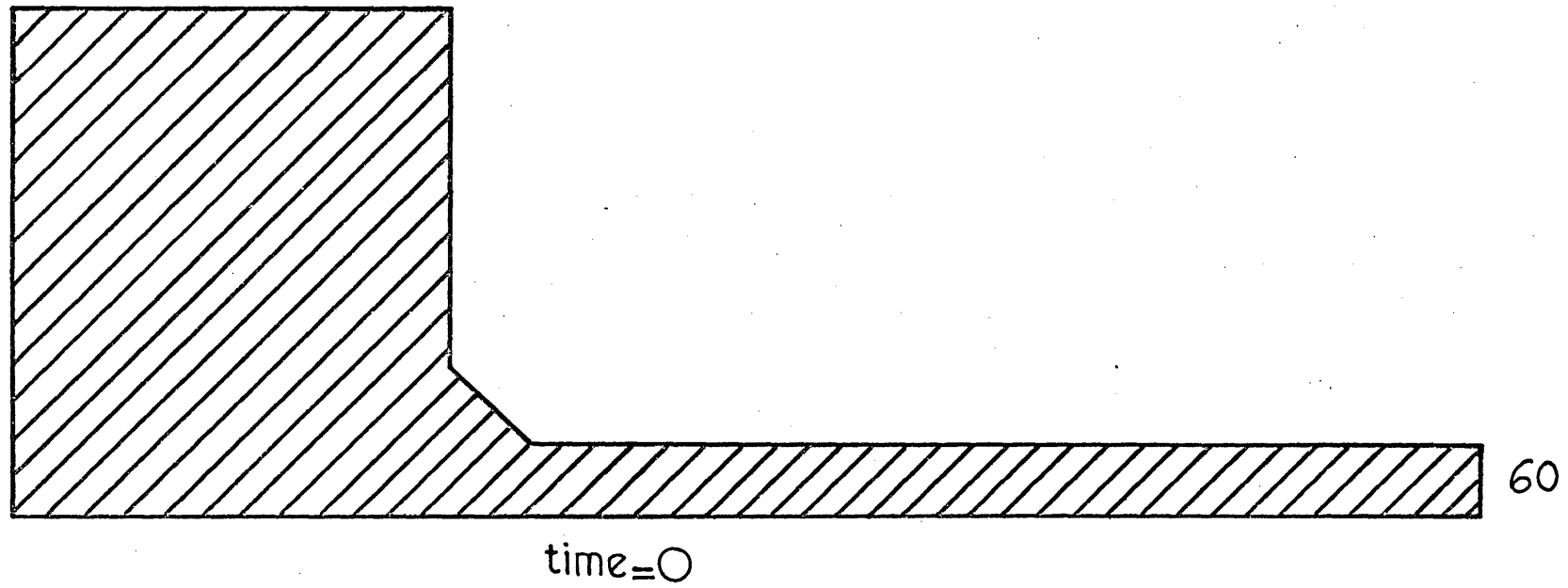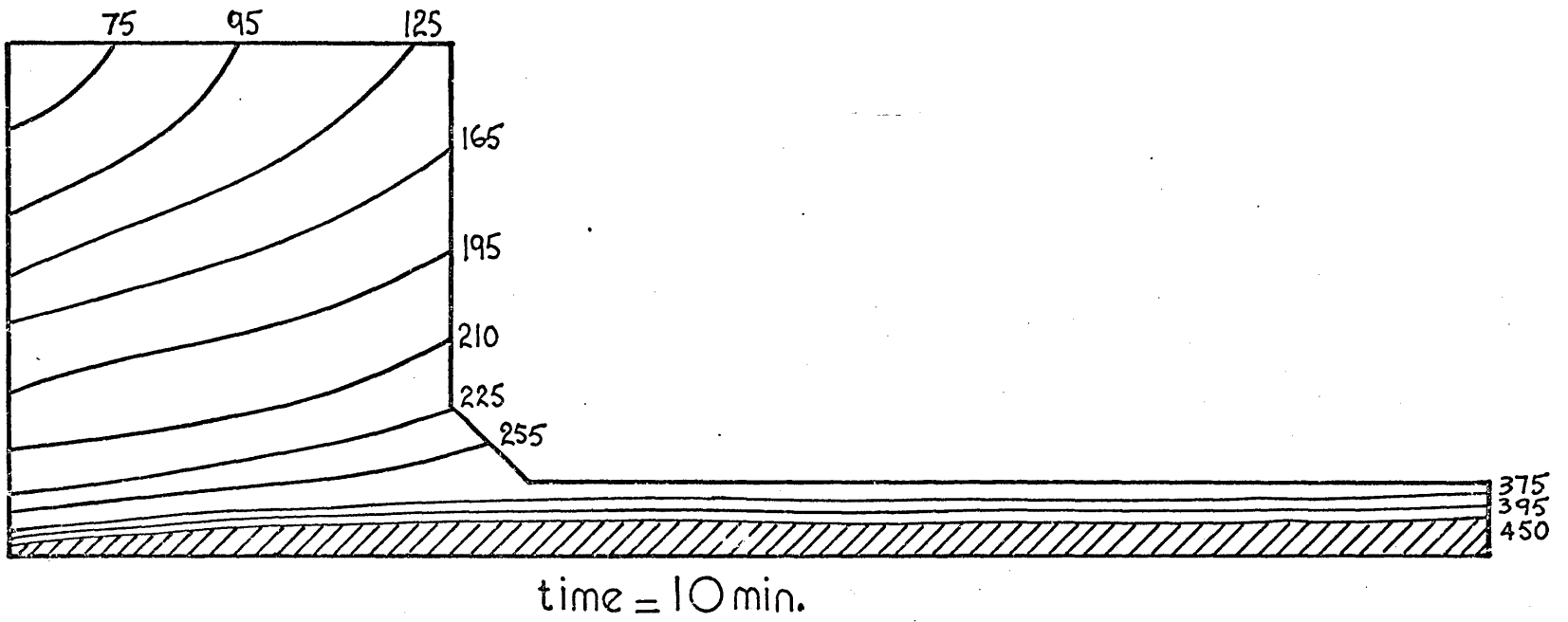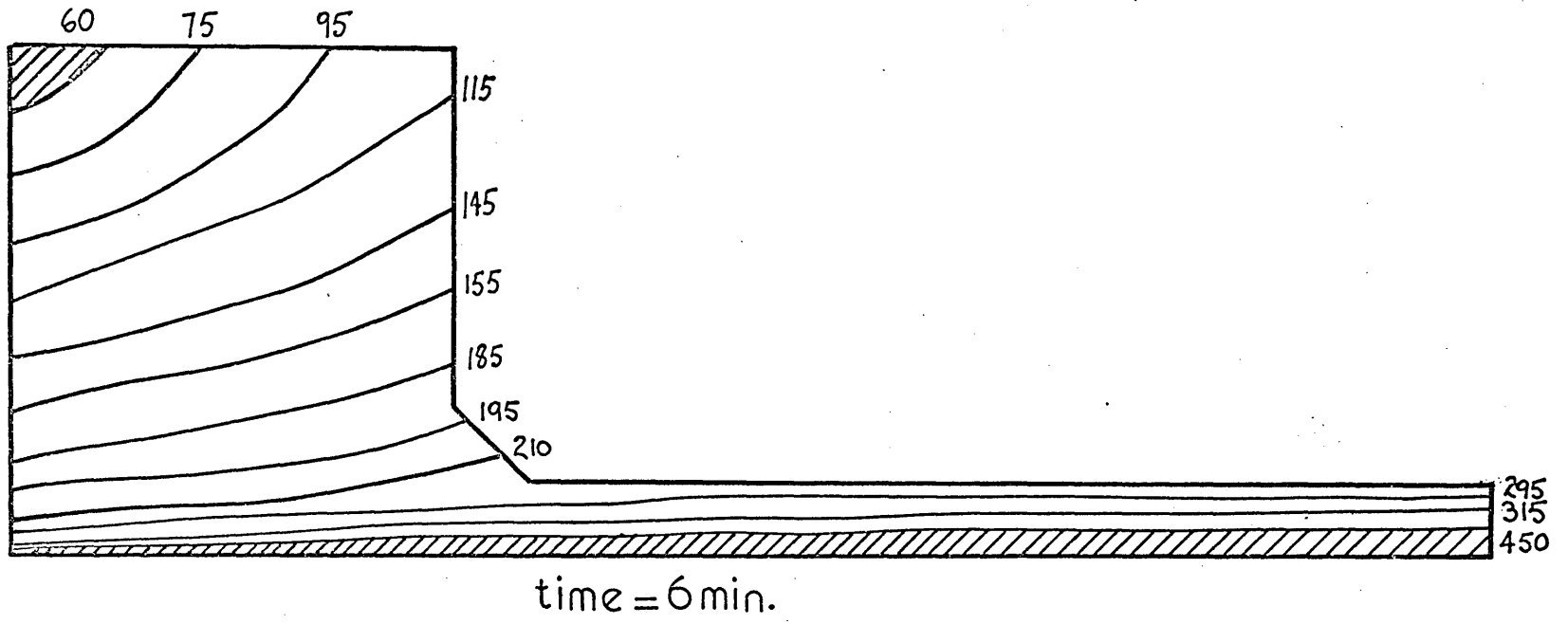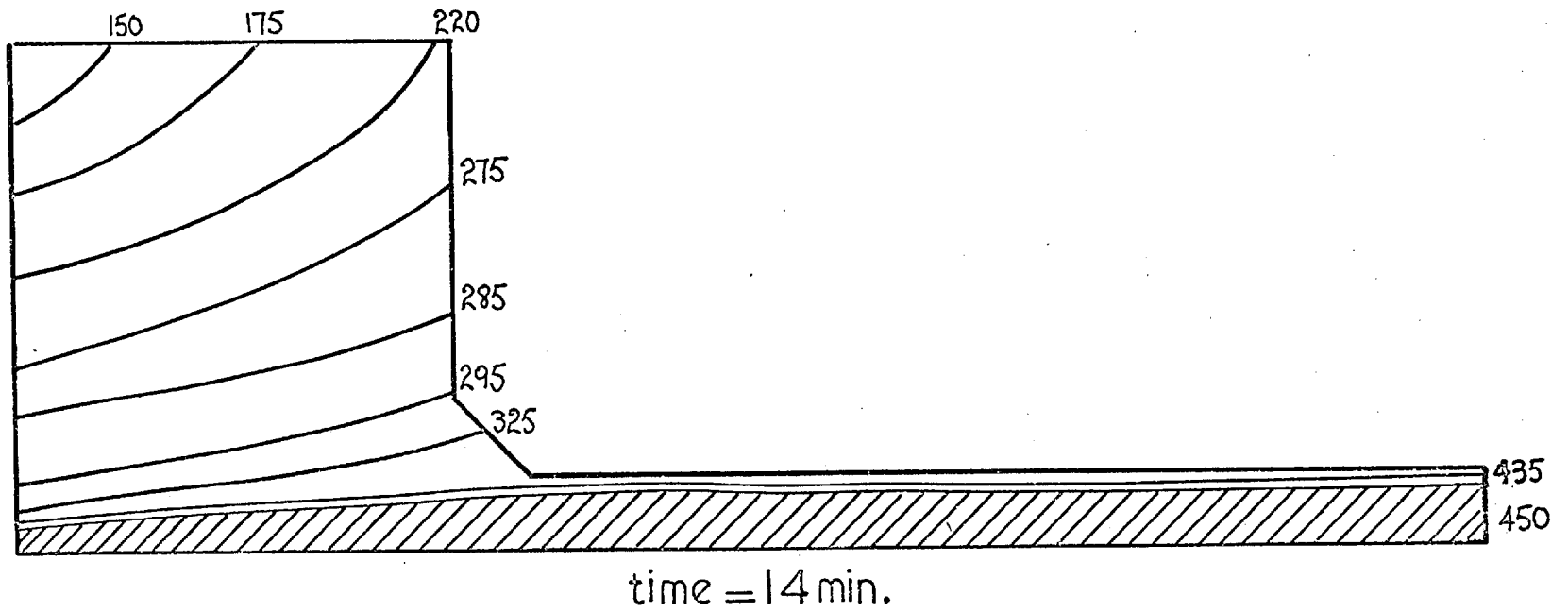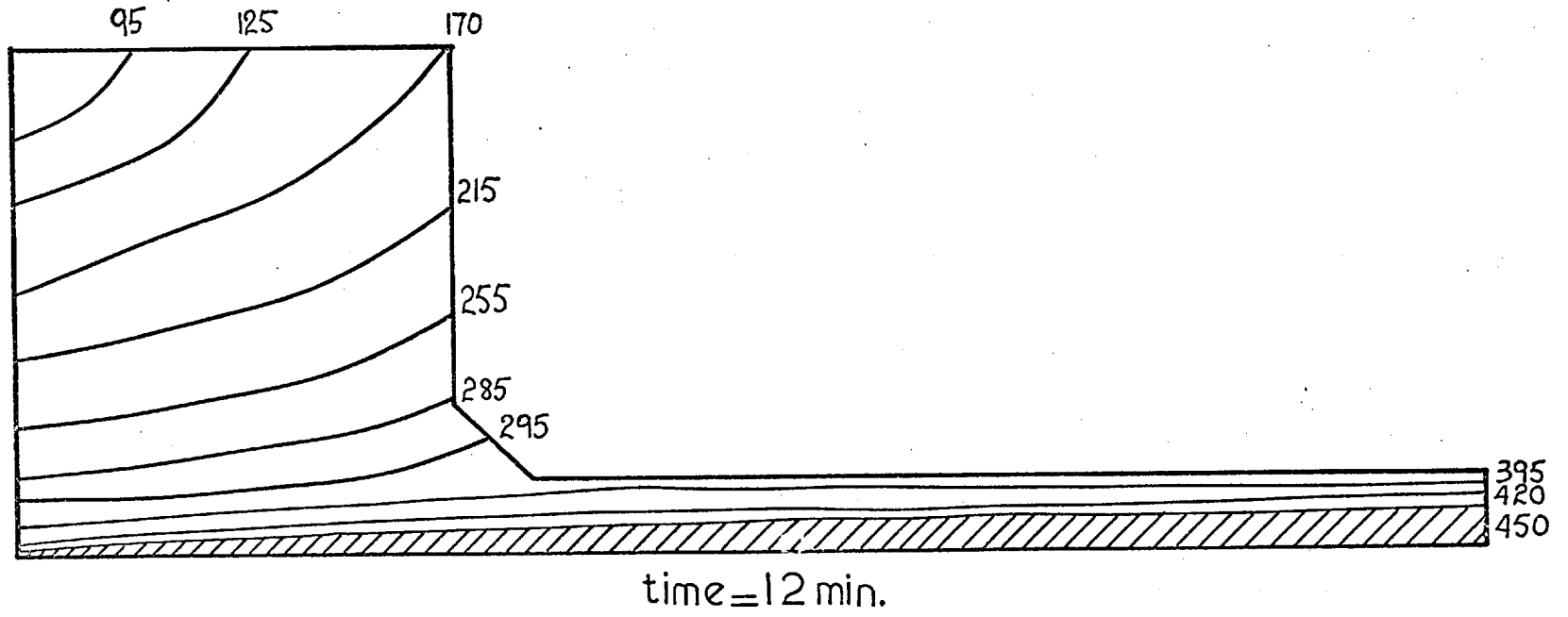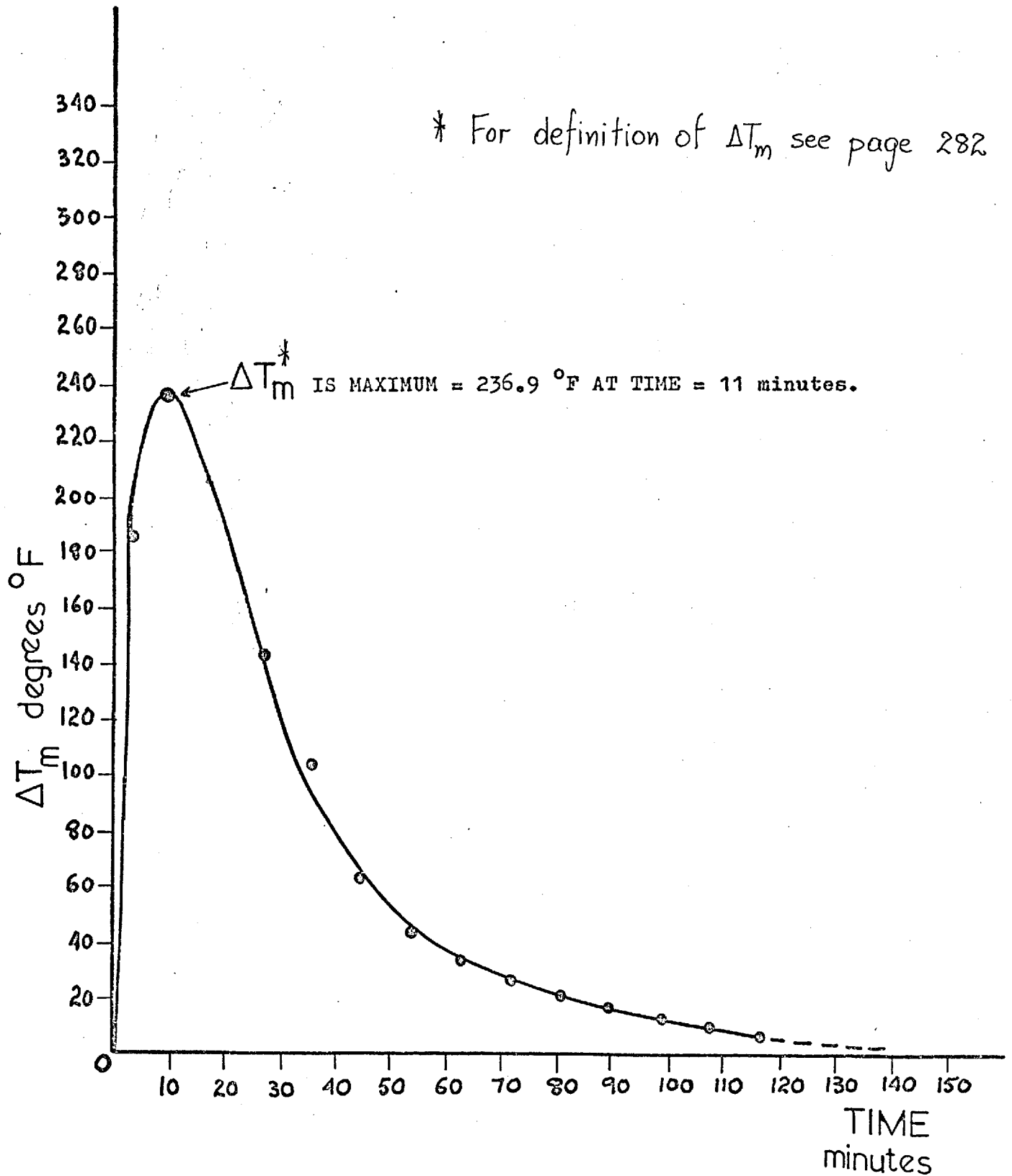
FIGURE 4.6.3 CASE C. MEAN TEMPERATURE VARIATION ΔT_m BETWEEN NOZZLE AND VESSEL.
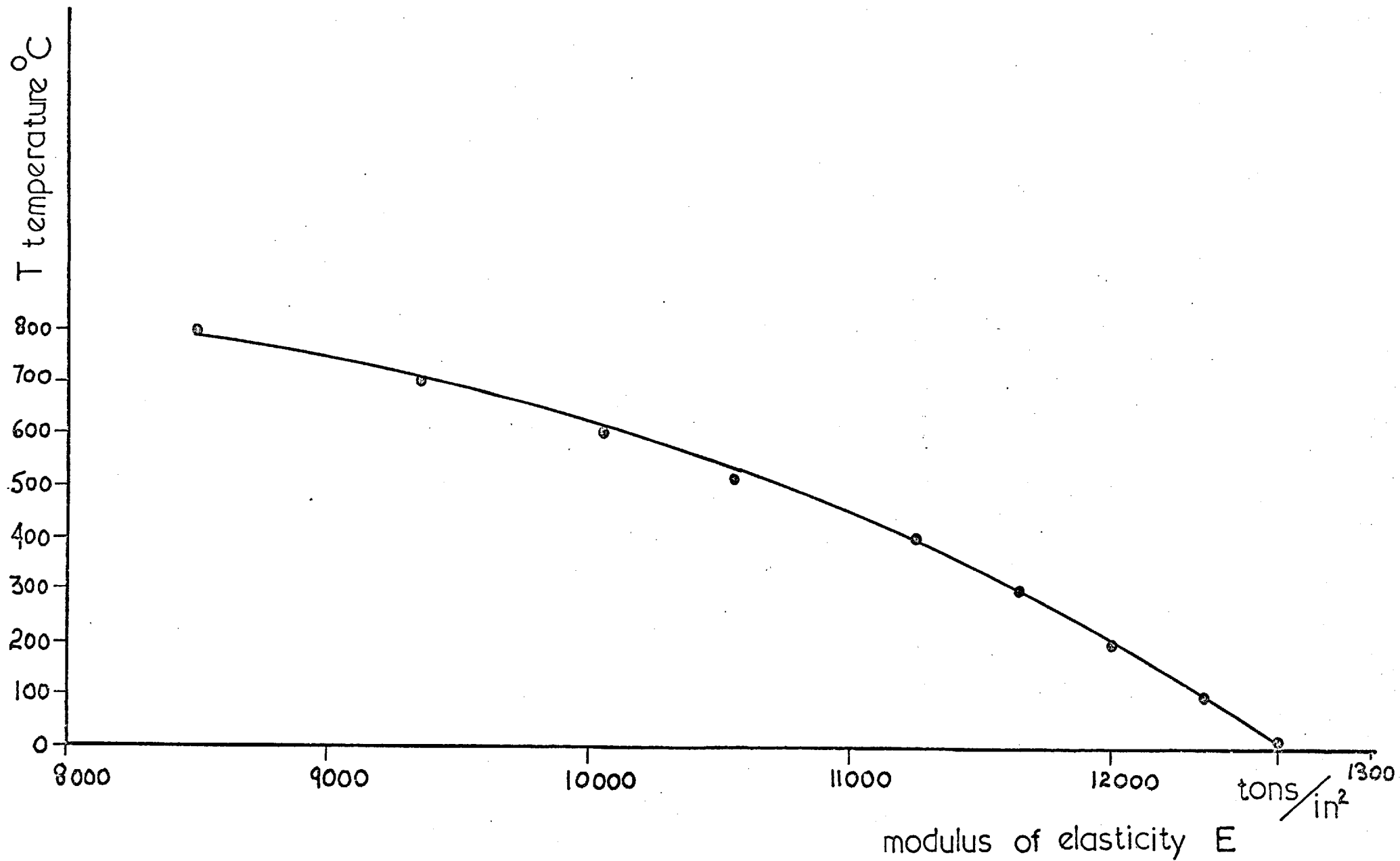
FIGURE 5.1.1
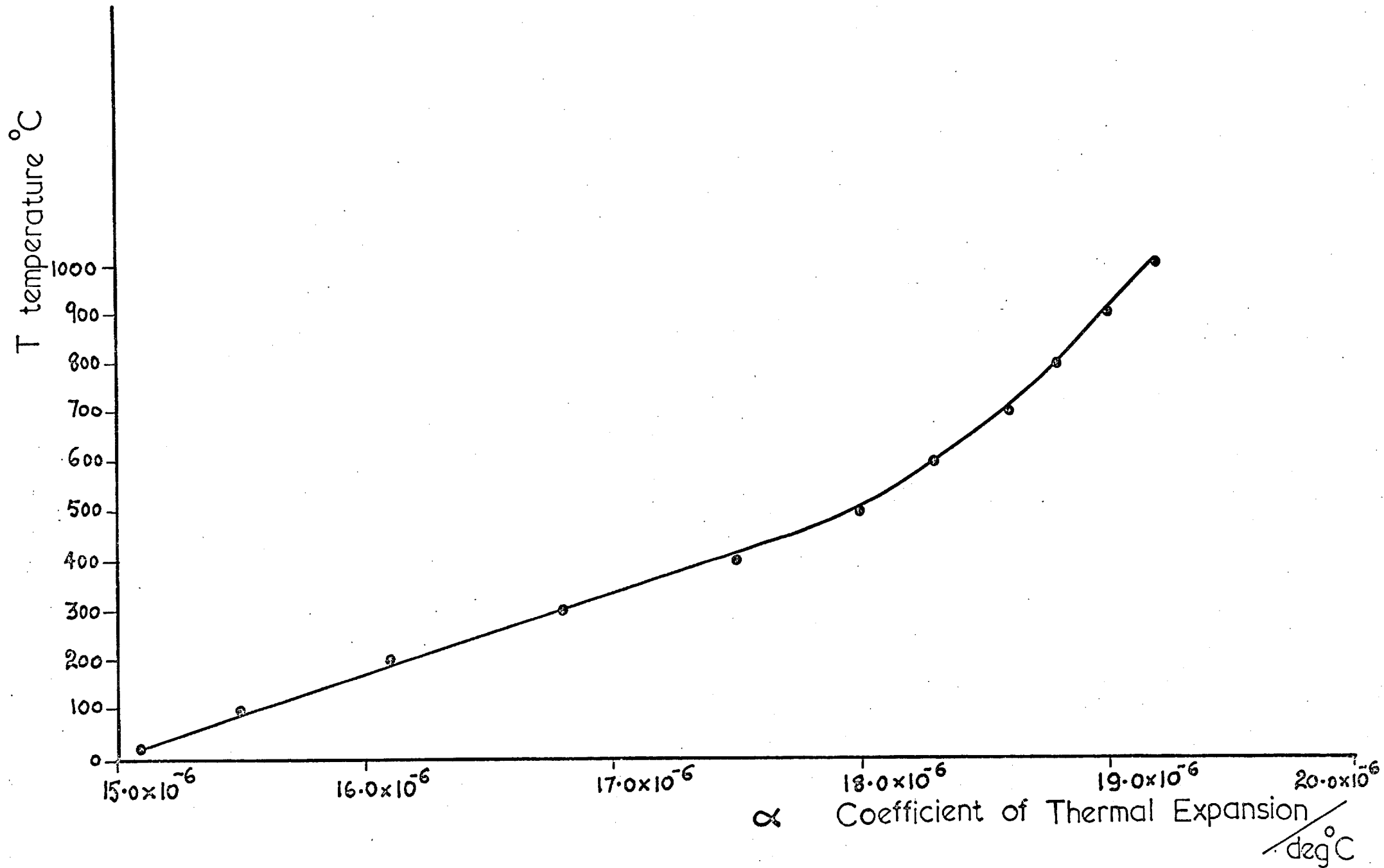
FIGURE 5.1.2
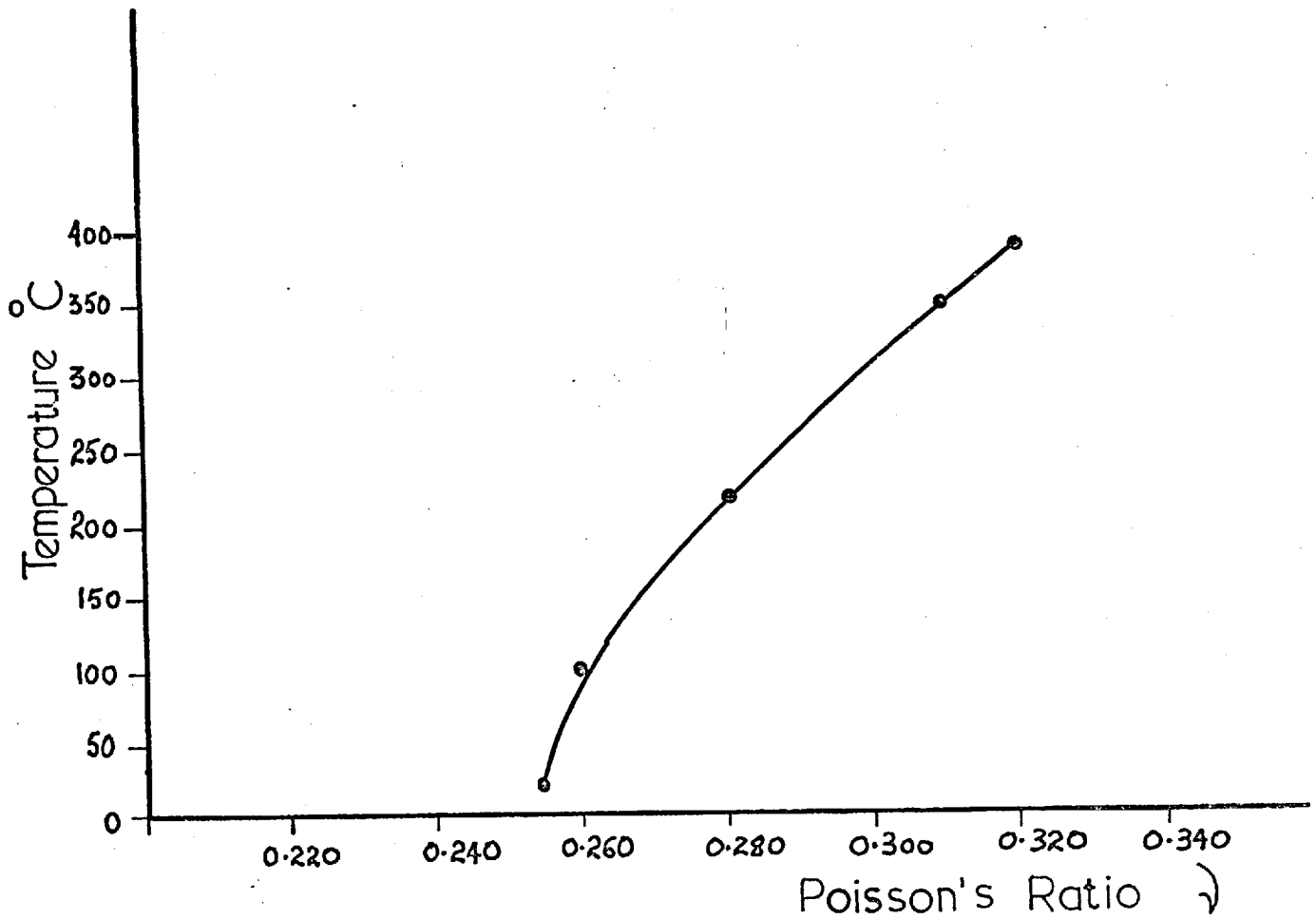
FIGURE 5.1.3   POISSON'S RATIO VERSUS TEMPERATURE.

STRESS NOTATION(See figure 5.1.4 in the next page)

In order to facilitate the comparison of the theoretical predictions with the experimental results on the outside of the vessel-junction-nozzle boundary, two stresses σ_L and σ_R are introduced into the system. Stress σ_L always lies along the boundary ABCD and σ_R is always perpendicular to the boundary ABCD at any point. Stress σ_L is in fact stress σ_y at the vessel part of the boundary (AB) and stress σ_x at the nozzle part (CD) and the stress component of σ_y and σ_x at the junction part BC. Similarly σ_R is in fact stress σ_x at the vessel part, σ_y at the nozzle part and the stress component of σ_y and σ_x at any point perpendicular to the junction part (BC).

Therefore the introduction of σ_L and σ_R, which are in fact the stresses σ_y and σ_x (or their stress component for the junction part), makes the representation of the stress field at the outside boundary more uniform and the comparison with the experimental results more comprehensive.
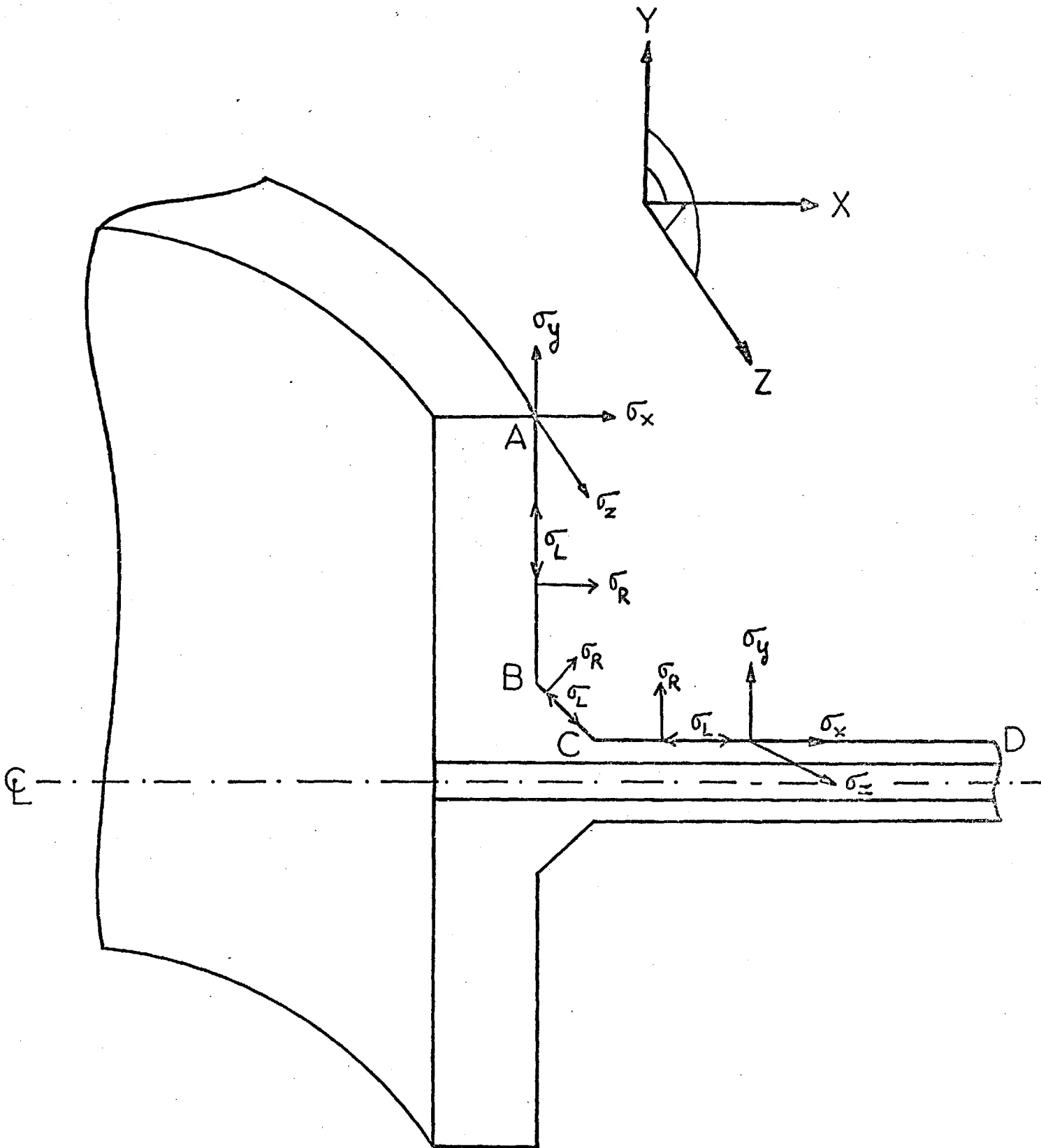
NOTE: STRESS σ_z IS PERPENDICULAR TO THE PLANE
OF THE PAPER (z DIRECTION).

FIGURE 5.1.4 DIRECTION OF STRESSES.

FIGURE 5.2.1

CASE A. FINITE ELEMENT MESH.

299

VESSEL   JUNCTION   NOZZLE

CASE A. OUTSIDE BOUNDARY (See fig. 5.2.1)

THERMAL STRESSES FOR
TIME t = 9 min. 30 sec.

× STRESS σ_z

△ STRESS σ_L

FIGURE 5.2.2

300
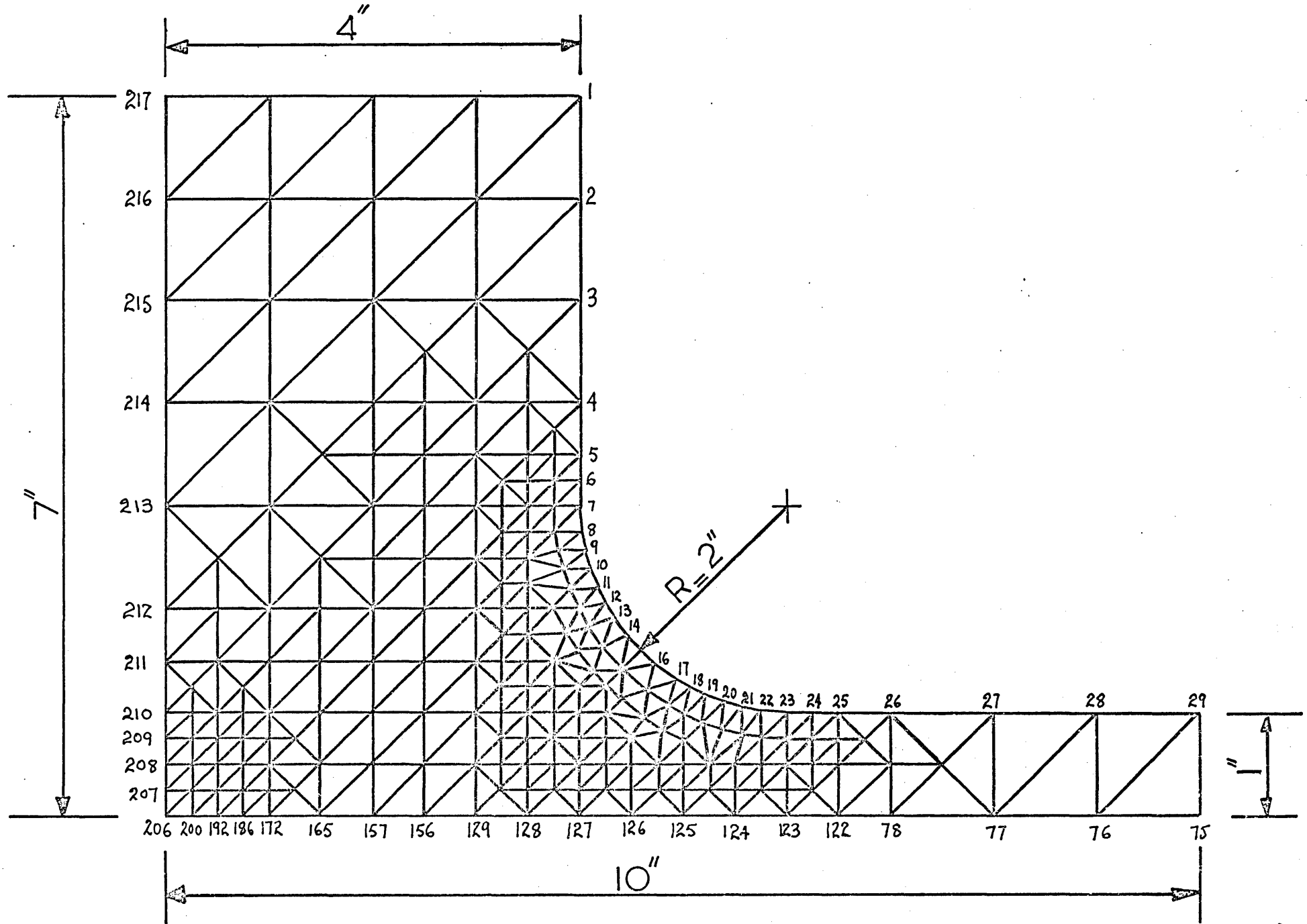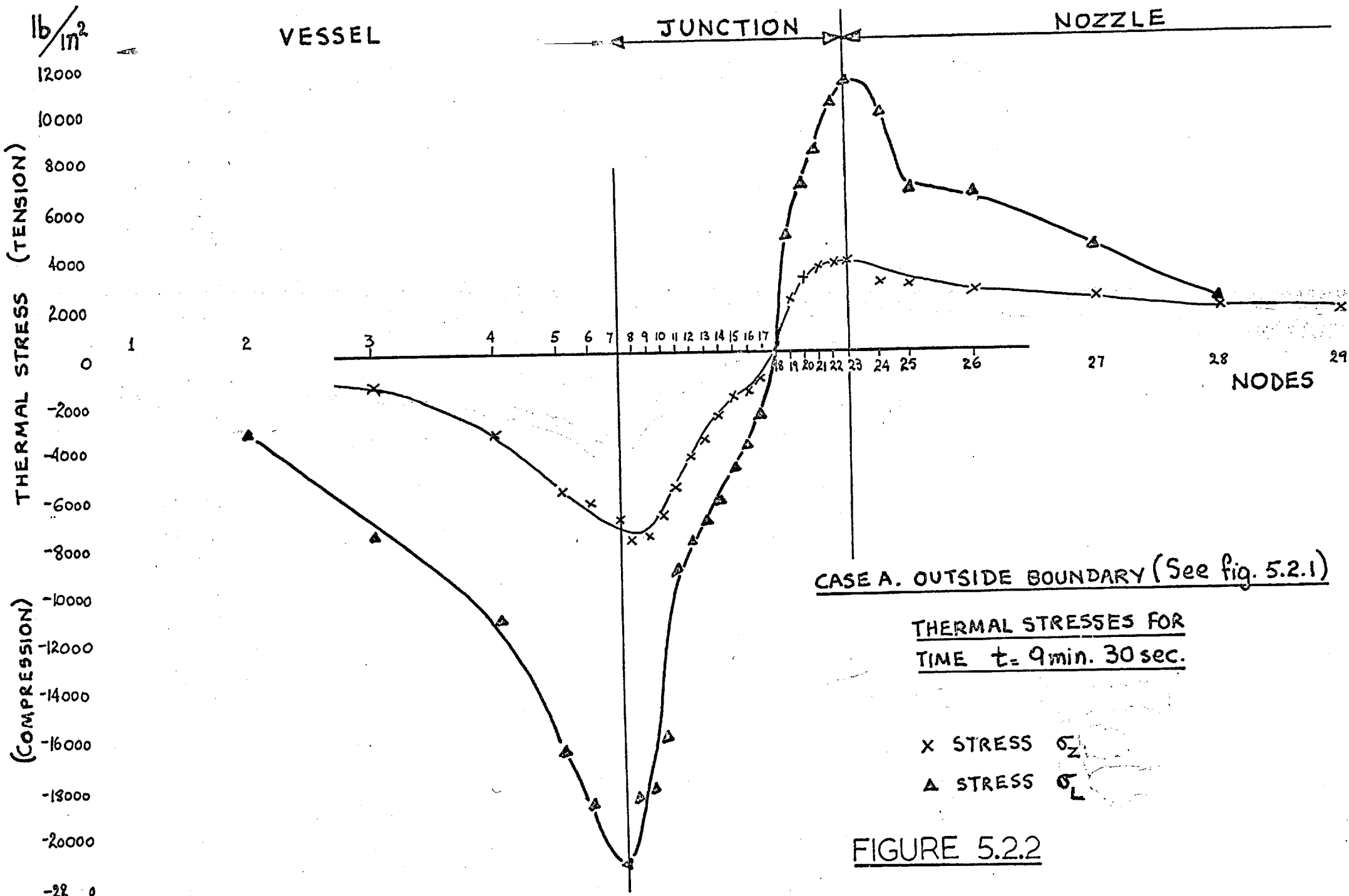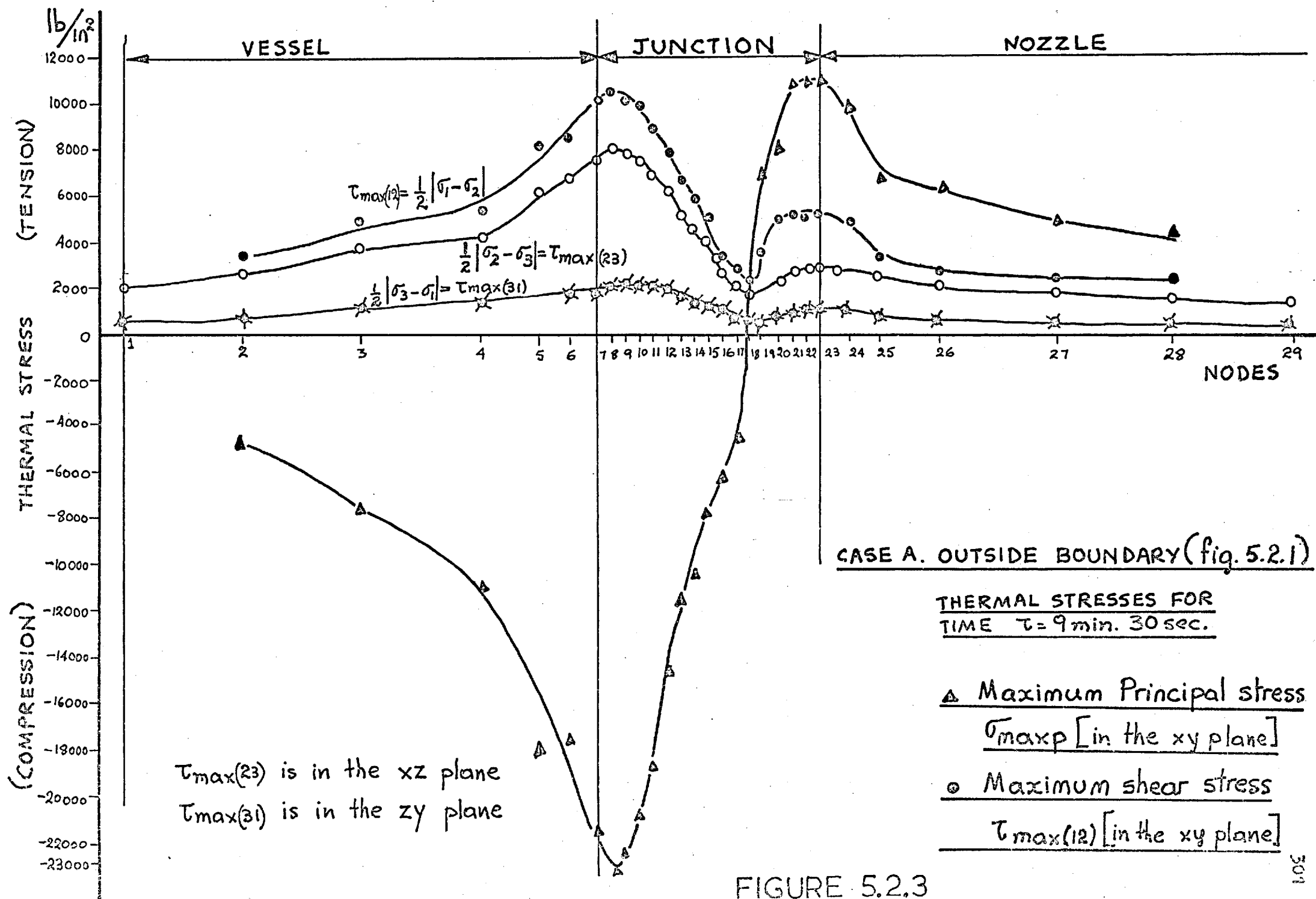
FIGURE 5.2.3

CASE A. INSIDE BOUNDARY ALONG THE NOZZLE (fig. 5.2.1)

THERMAL STRESSES AT TIME $\tau = 9$ min. 30 sec.

× STRESS σ_z

▲ STRESS σ_x
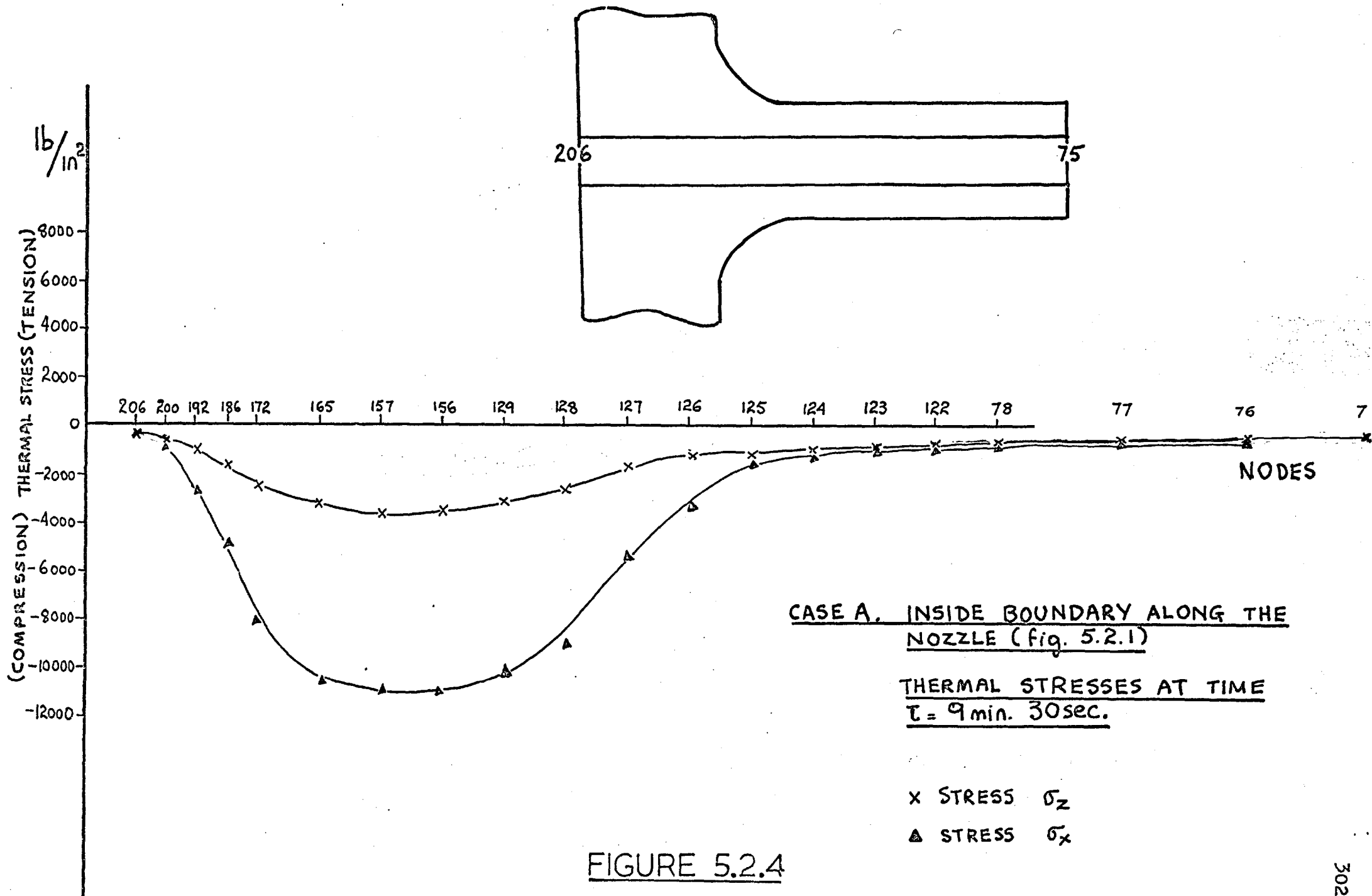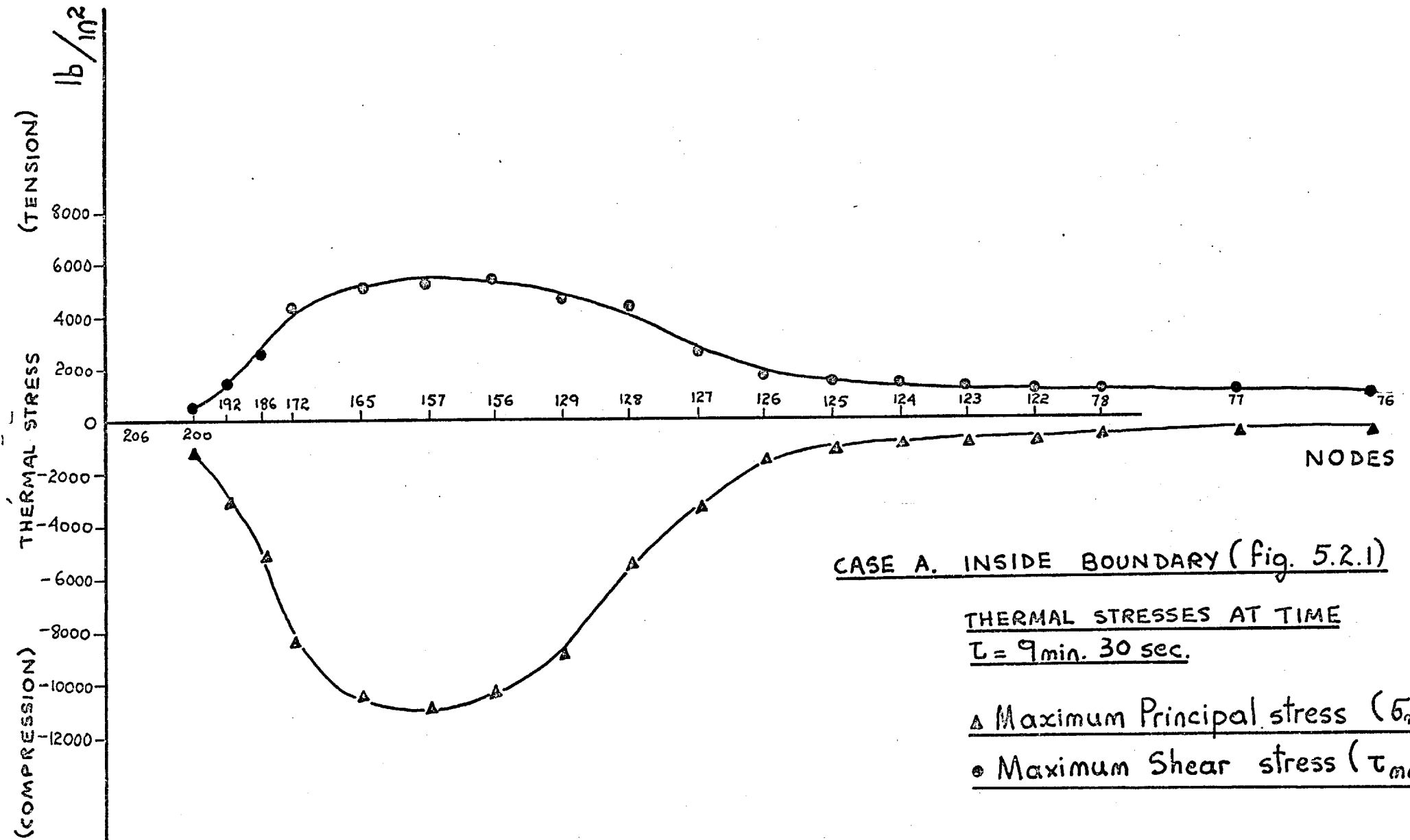
FIGURE 5.2.4

CASE A. INSIDE BOUNDARY (fig. 5.2.1)

THERMAL STRESSES AT TIME
$t = 9$ min. 30 sec.

△ Maximum Principal stress (σ_{maxp})
● Maximum Shear stress $(\tau_{max(12)})$

FIGURE 5.2.5

FIGURE 5.2.6

CASE B. FINITE ELEMENT MESH.
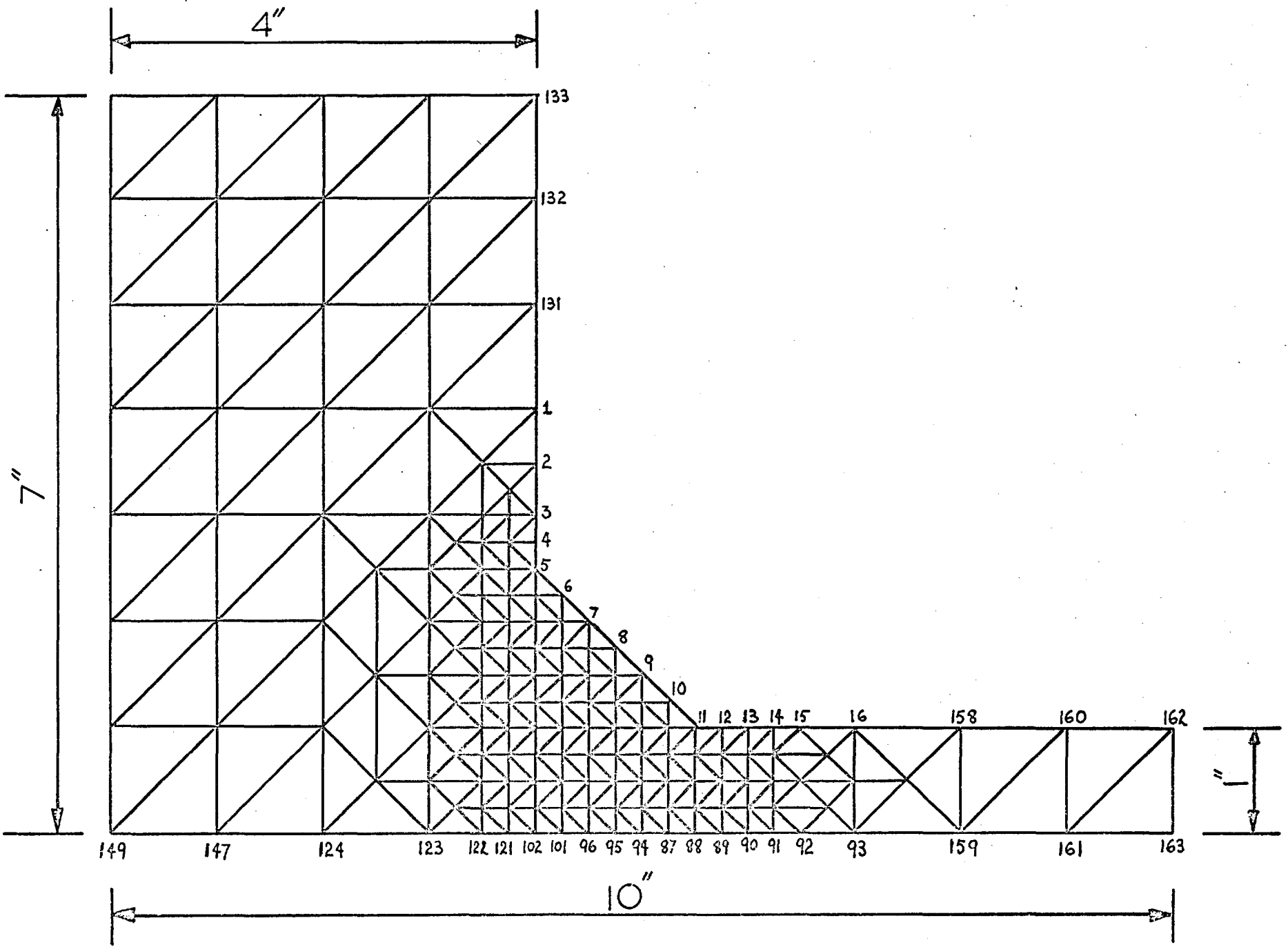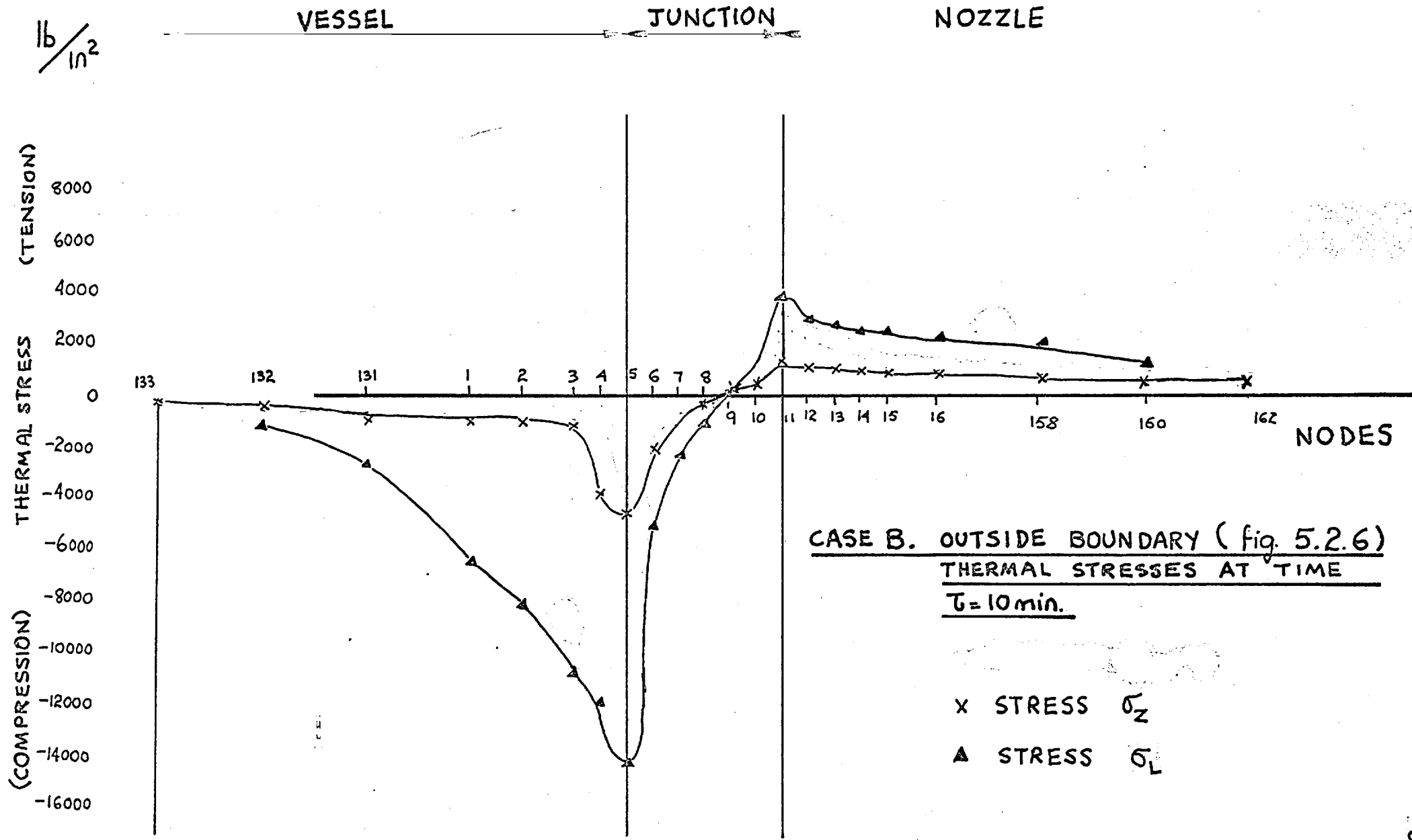
304

FIGURE 5.2.7

VESSEL   JUNCTION   NOZZLE

lb/in^2

(TENSION)

THERMAL STRESS

(COMPRESSION)

NODES

CASE B. OUTSIDE BOUNDARY (fig. 5.2.6)
THERMAL STRESSES AT TIME
$\tau = 10\,min.$

× STRESS σ_z

▲ STRESS σ_L

305

FIGURE 5.2.8

VESSEL   JUNCTION   NOZZLE

CASE B. OUTSIDE BOUNDARY (fig. 5.2.6)

THERMAL STRESSES

AT TIME $\tau = 10$ min.

▲ Maximum principal stress σ_{maxp}

○ Maximum shear stress $\tau_{max(12)}$

306

FIGURE 5.2.9

CASE B.   INSIDE BOUNDARY (fig. 5.2.6)

THERMAL STRESSES AT TIME

$\tau = 10$ min.

$\triangle$ Maximum Principal Stress $(\sigma_{max}$

$\circ$ Maximum Shear Stress $(\tau_{max}$
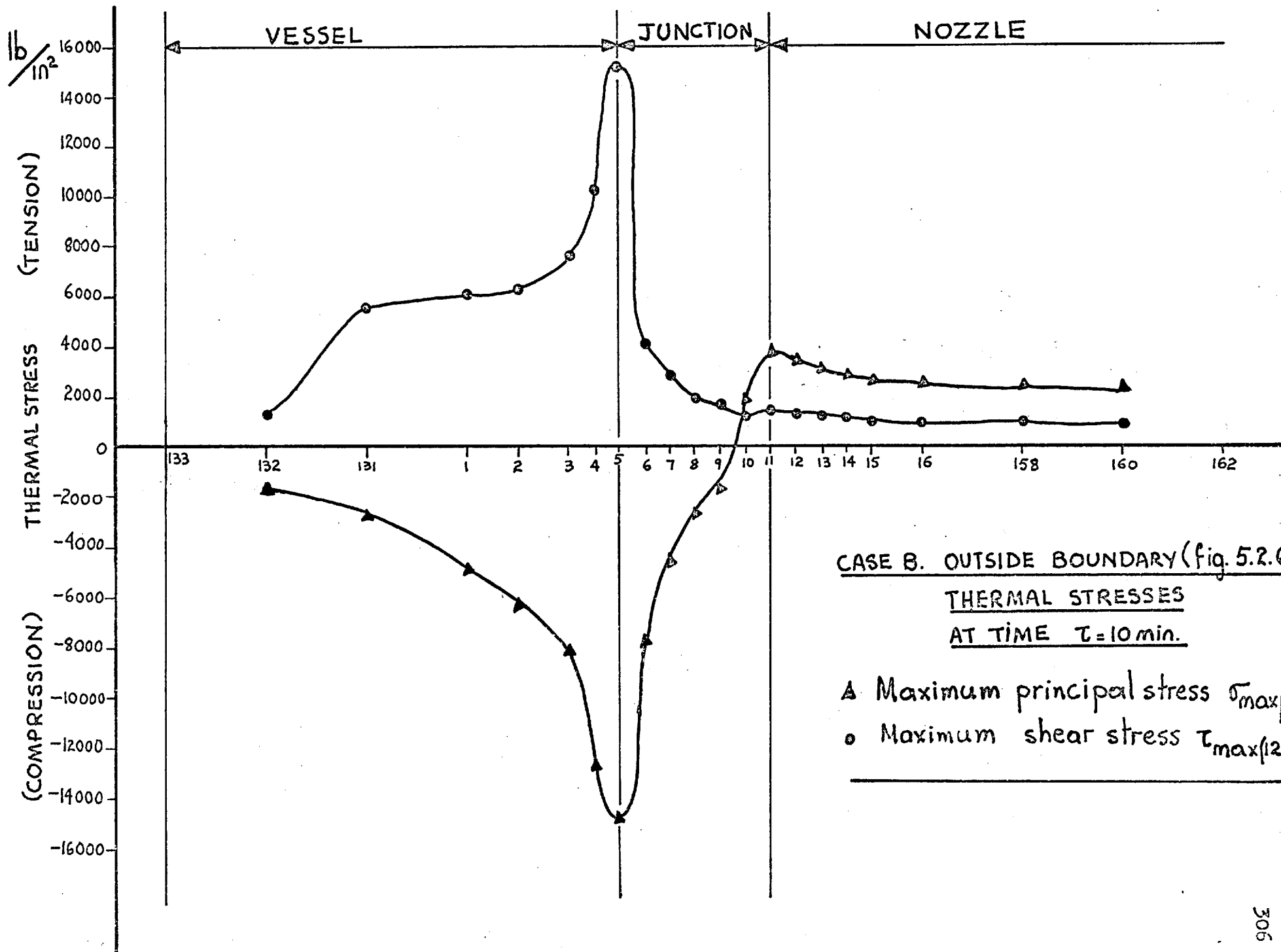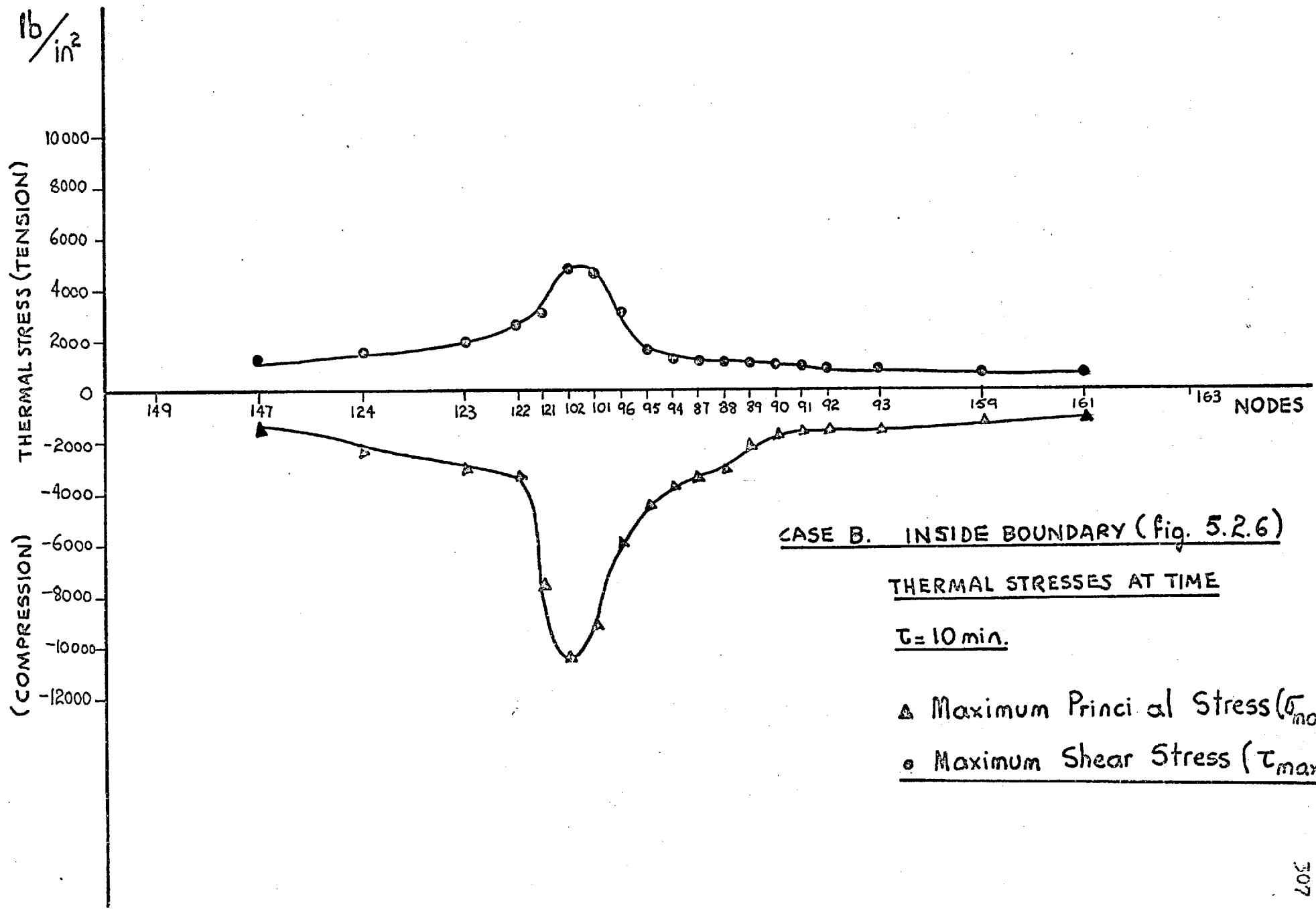
FIGURE 5.3.1 CASE C. FINITE ELEMENT MESH.

308
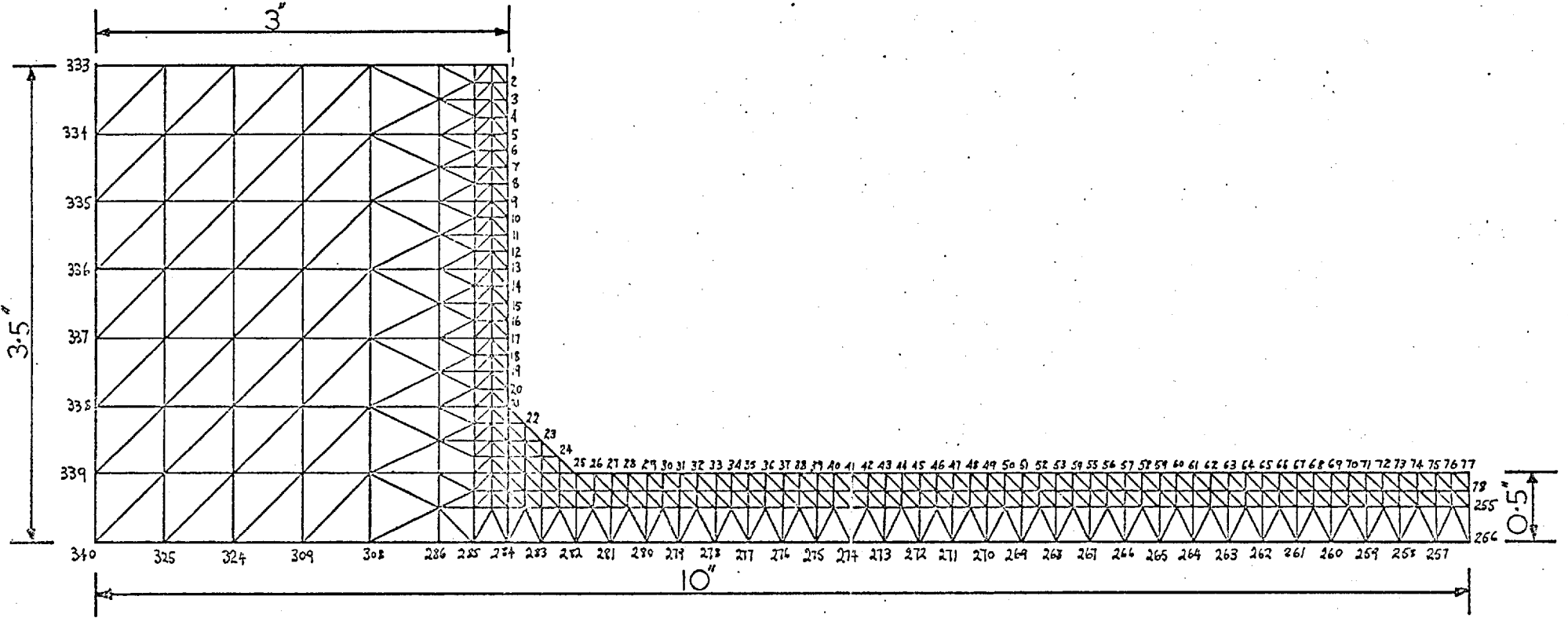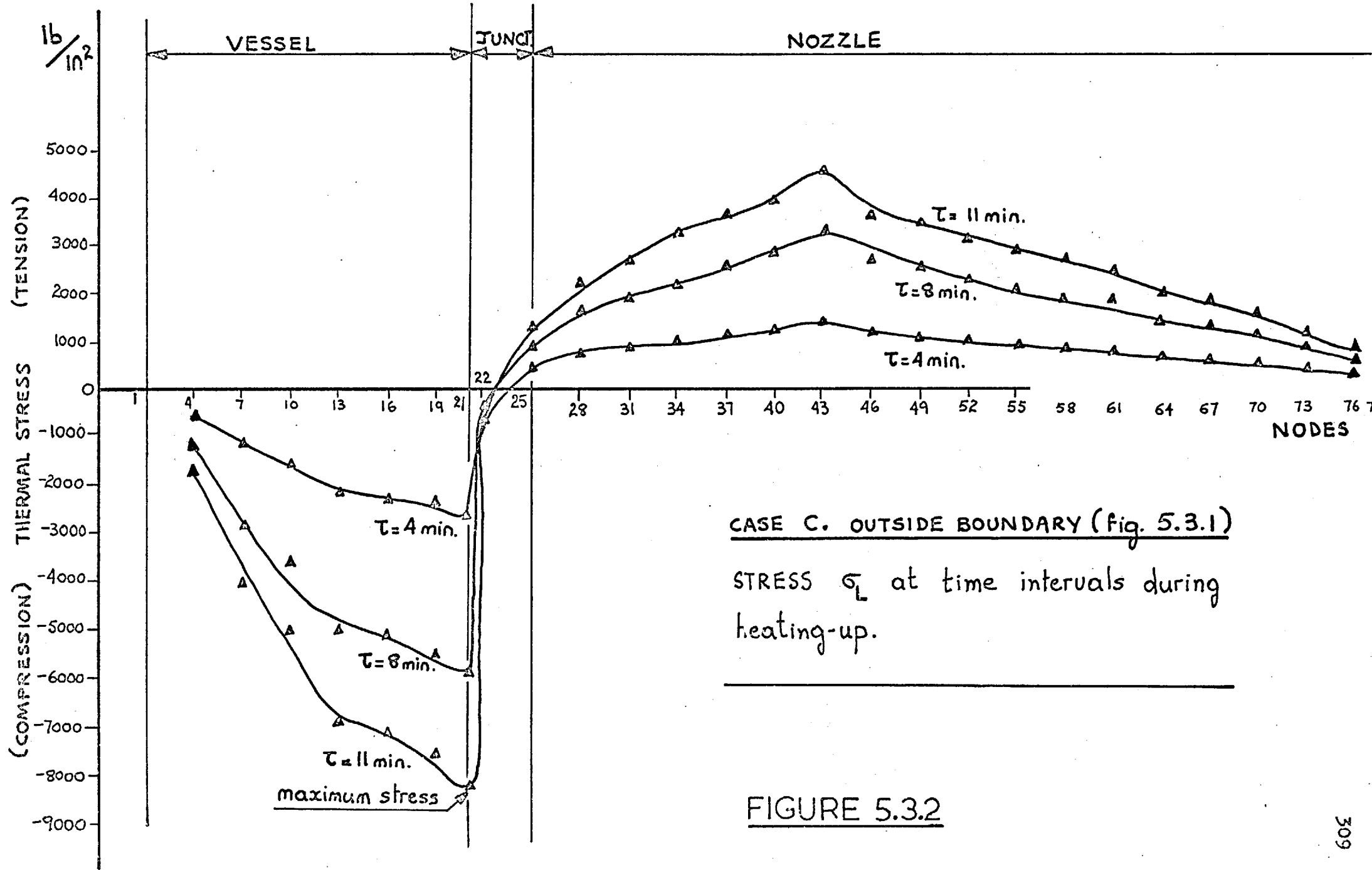
CASE C. OUTSIDE BOUNDARY (fig. 5.3.1)

STRESS σ_L at time intervals during heating-up.

FIGURE 5.3.2
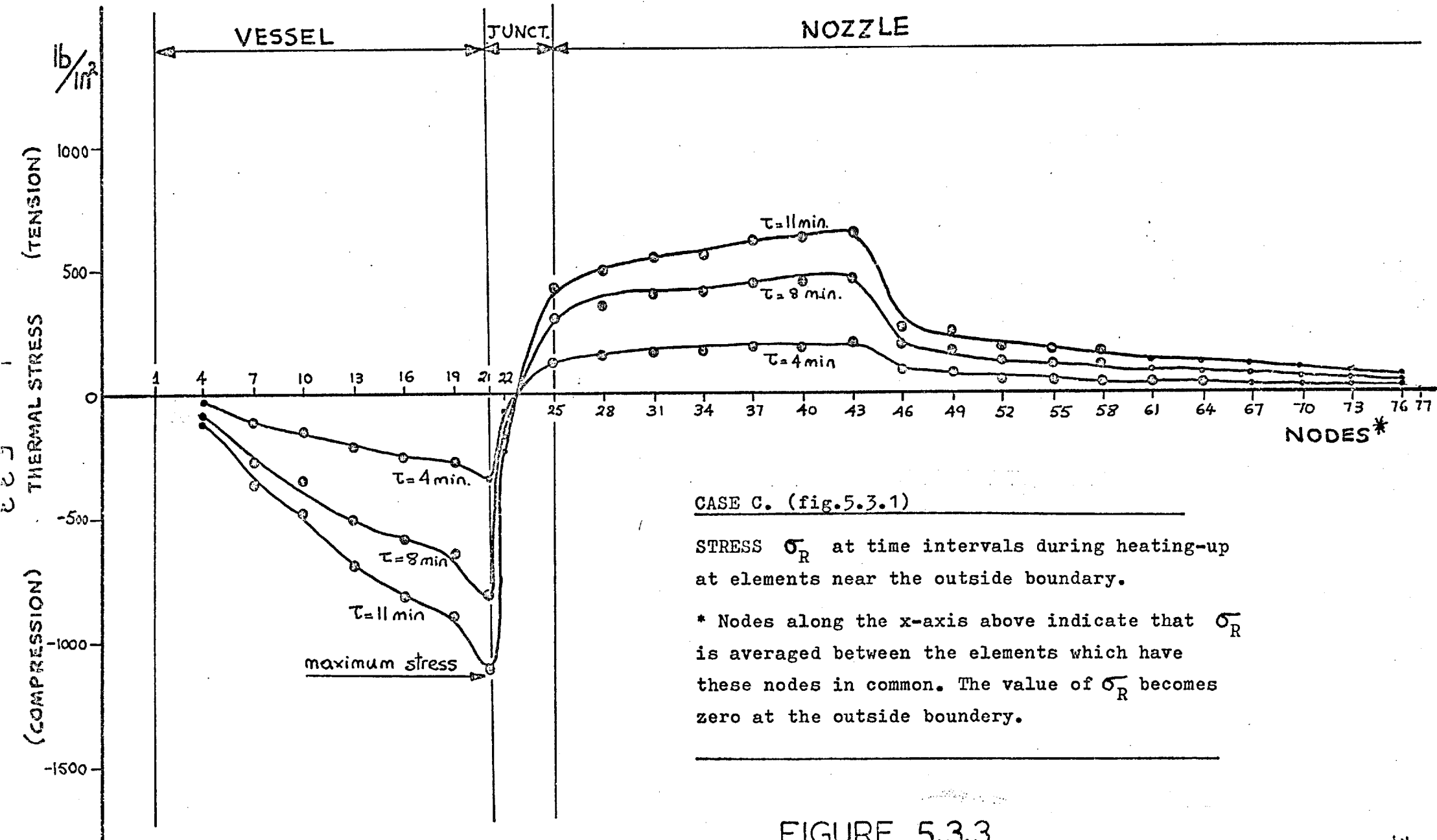
CASE C. (fig.5.3.1)

STRESS σ_R at time intervals during heating-up at elements near the outside boundary.

* Nodes along the x-axis above indicate that σ_R is averaged between the elements which have these nodes in common. The value of σ_R becomes zero at the outside boundery.
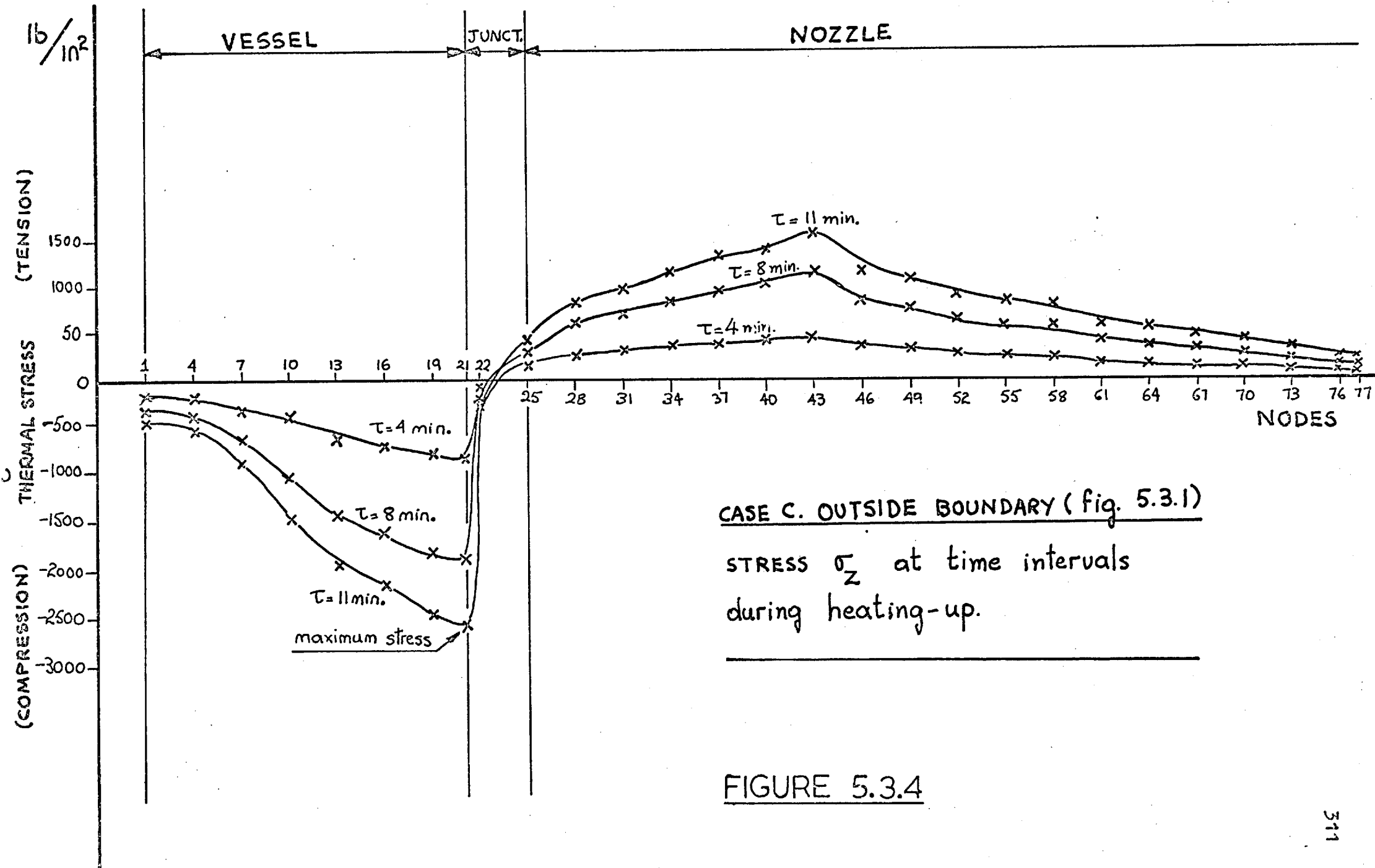
FIGURE 5.3.3

CASE C. OUTSIDE BOUNDARY (fig. 5.3.1)

STRESS σ_z at time intervals during heating-up.

FIGURE 5.3.4

FIGURE 5.3.5

CASE C. OUTSIDE BOUNDARY (fig. 5.3.1)

△ Maximum Principal Stress (σ_{maxp}) at time intervals during heating-up.
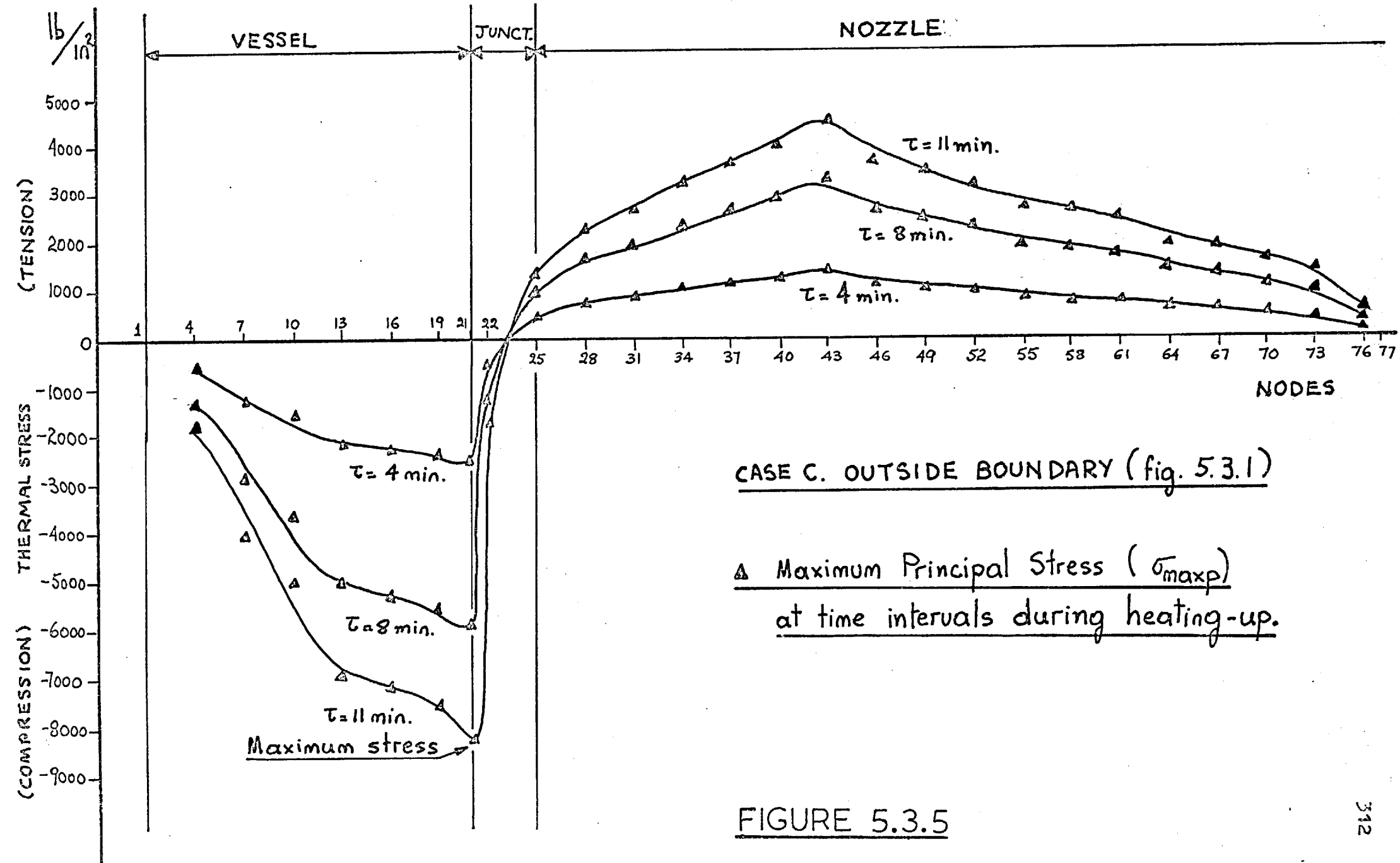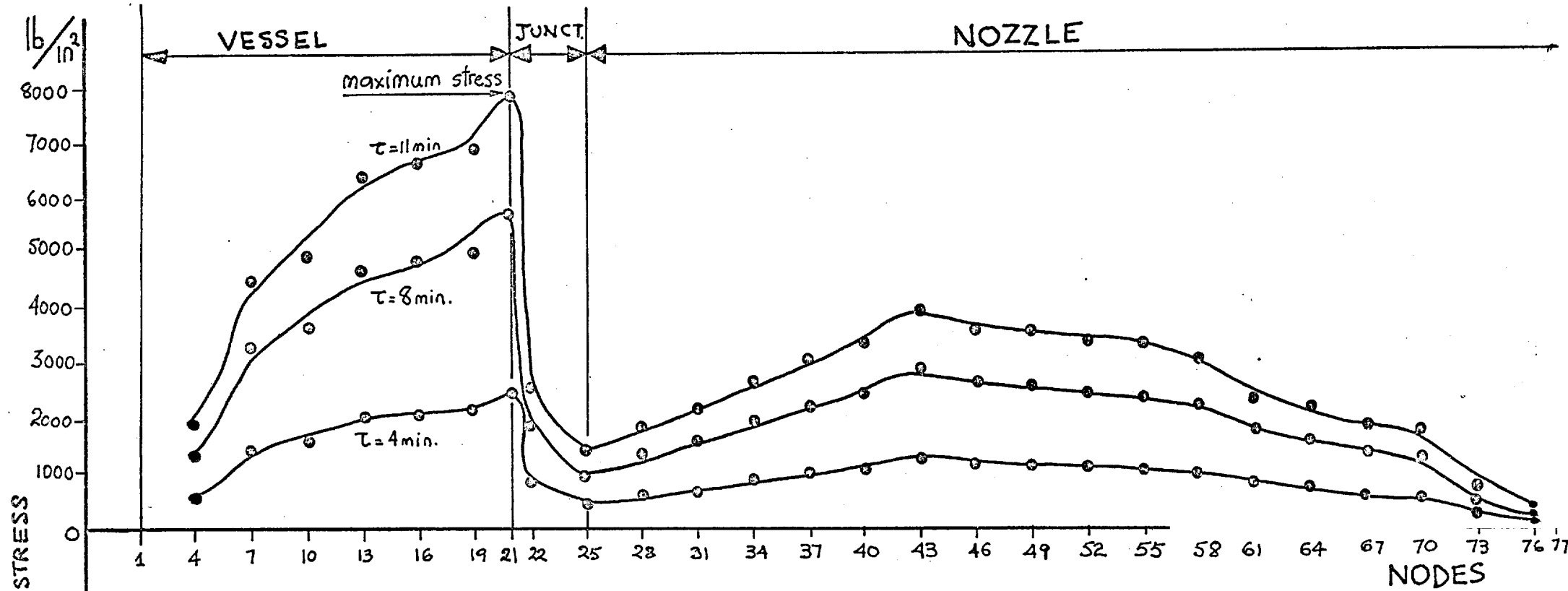
CASE C. OUTSIDE BOUNDARY (fig. 5.3.1)

⊙ Maximum shear stress $\tau_{max(12)}$ during heating-up.

FIGURE 5.3.6

8"

3"

8"

10"

1" I.D
2" O.D
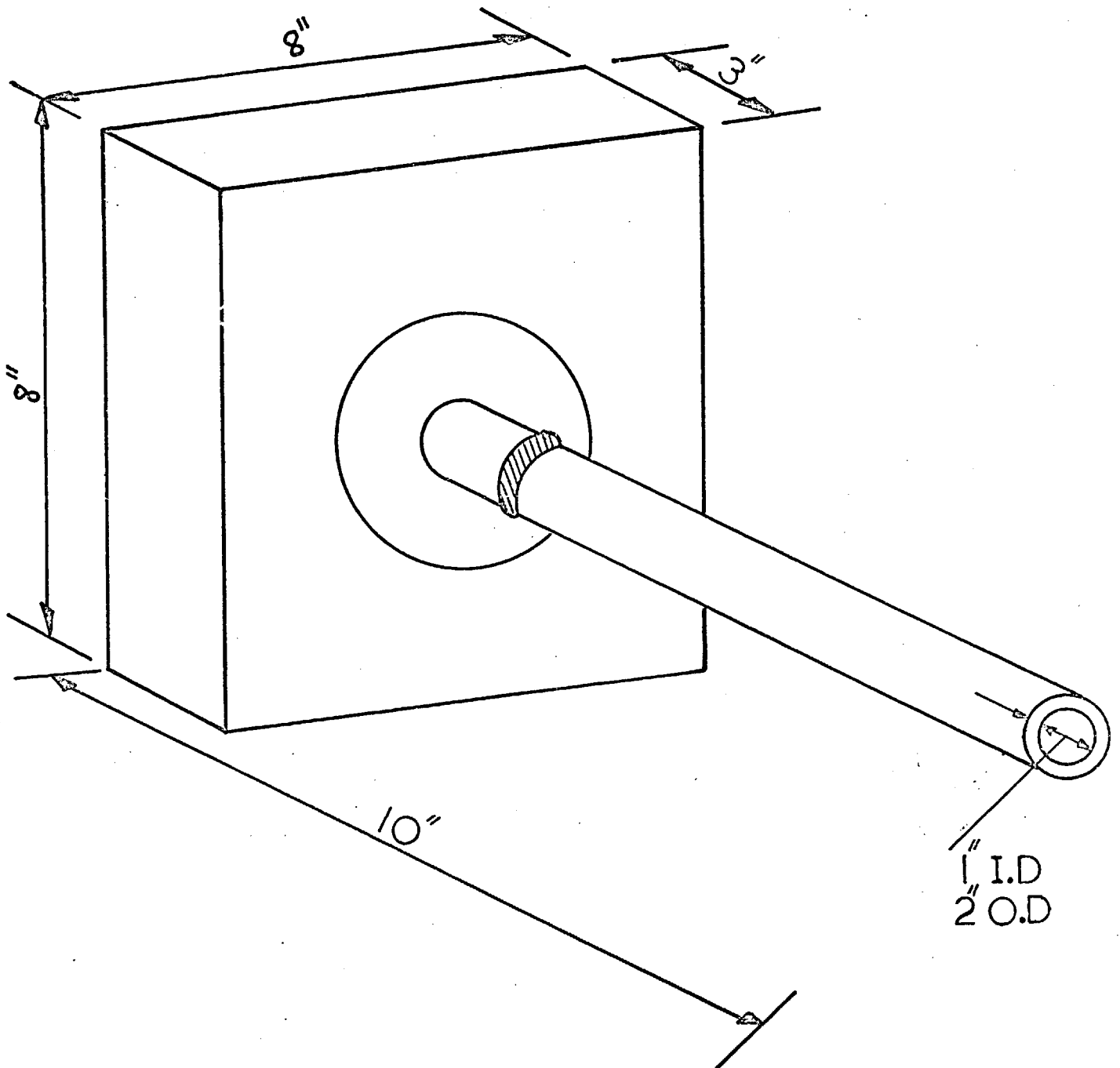
FIGURE 6.1.1. EXPERIMENTAL SPECIMEN.
FULL DIMENSIONS SHOWN IN FIGURE 6.1.2

FIGURE 6.1.2 SPECIMEN

8"

8"

3"

A'

A

B

B'

2½"

¾

½

5¼"

3"

½"

1"

½"

Cartridge Heater

B

B'

Thermocouple

315

(a)

(b)

(a)

(b)

(a)

(b)

ZPR

MVC

(d)

STANDARD TYPE

'THERMIC'

(c)

SECONDS

'VARISET'
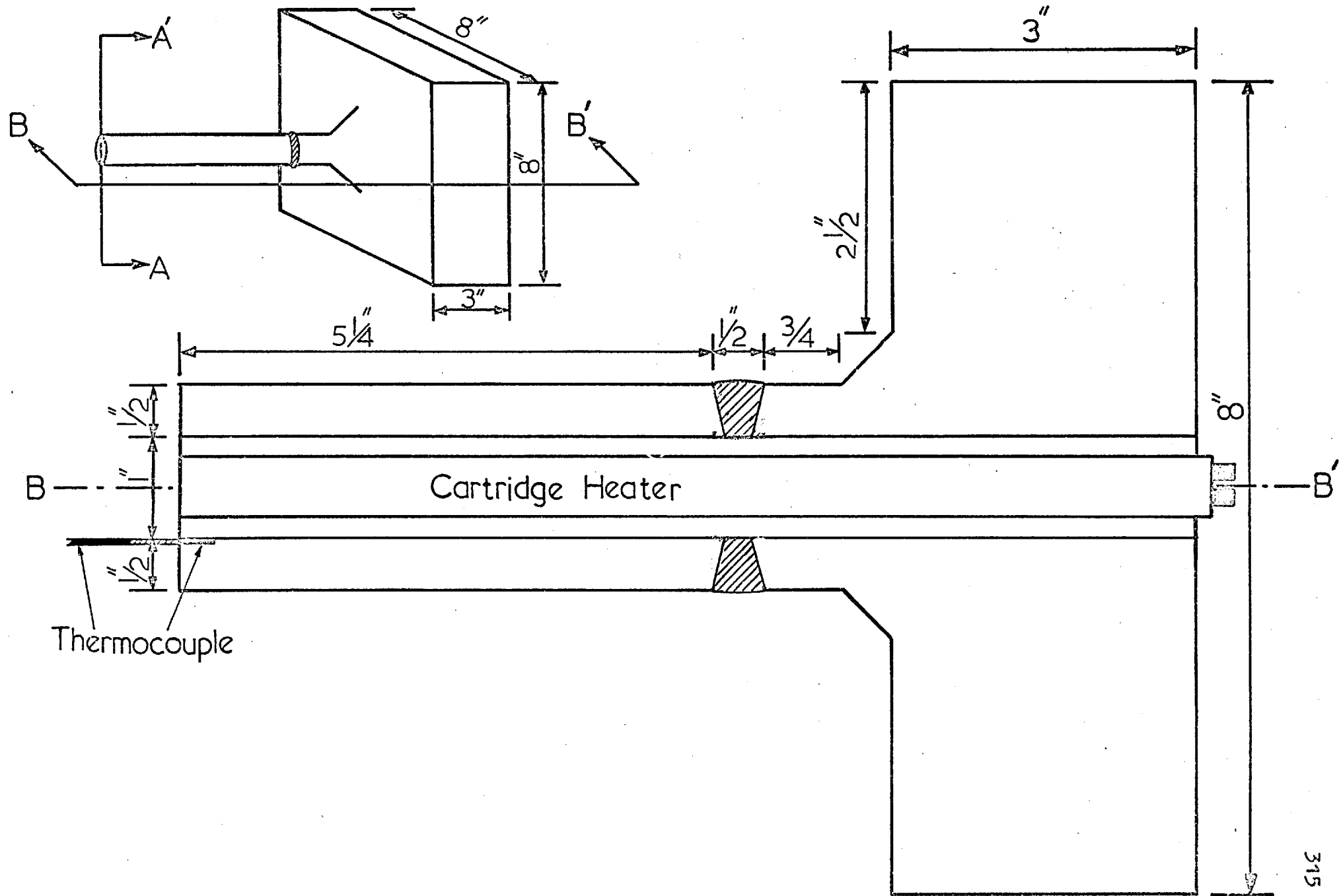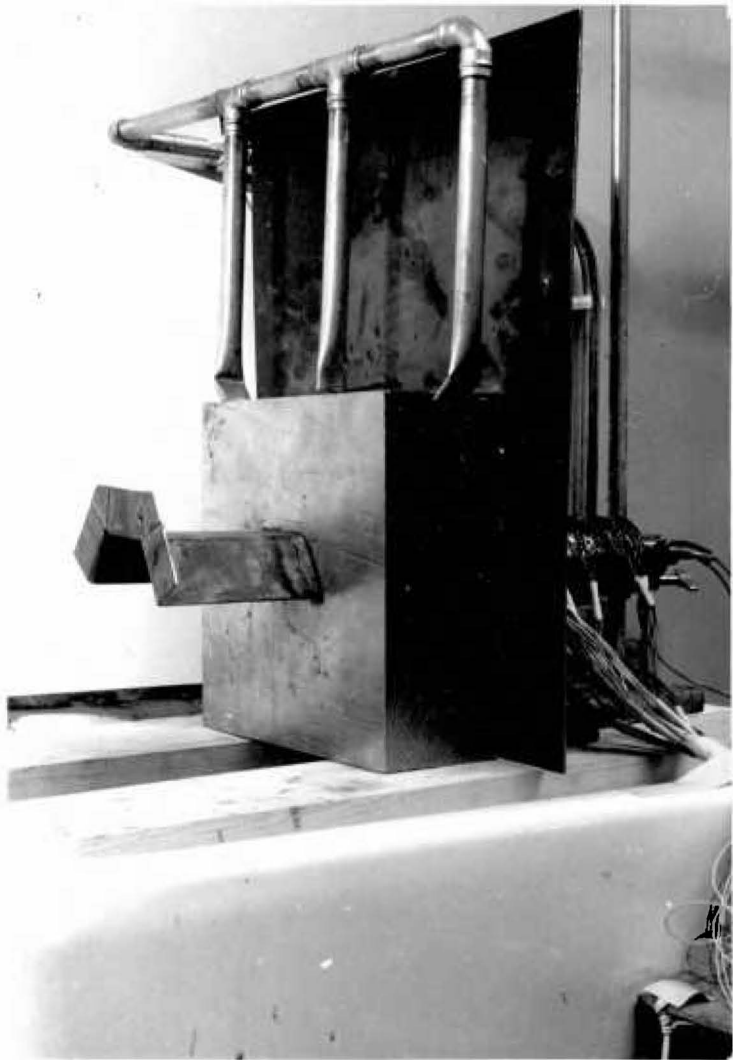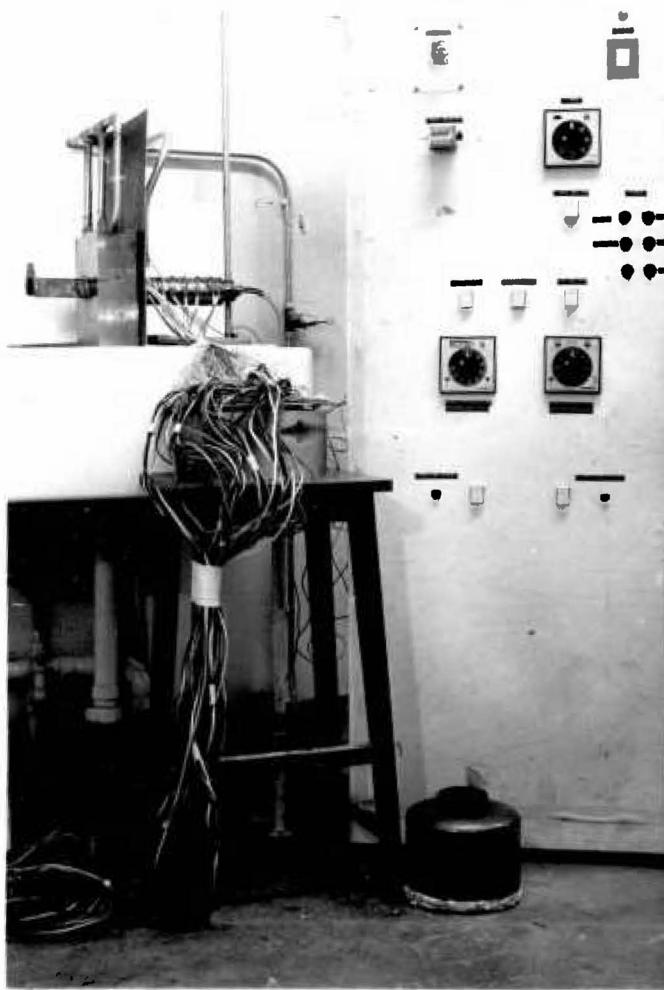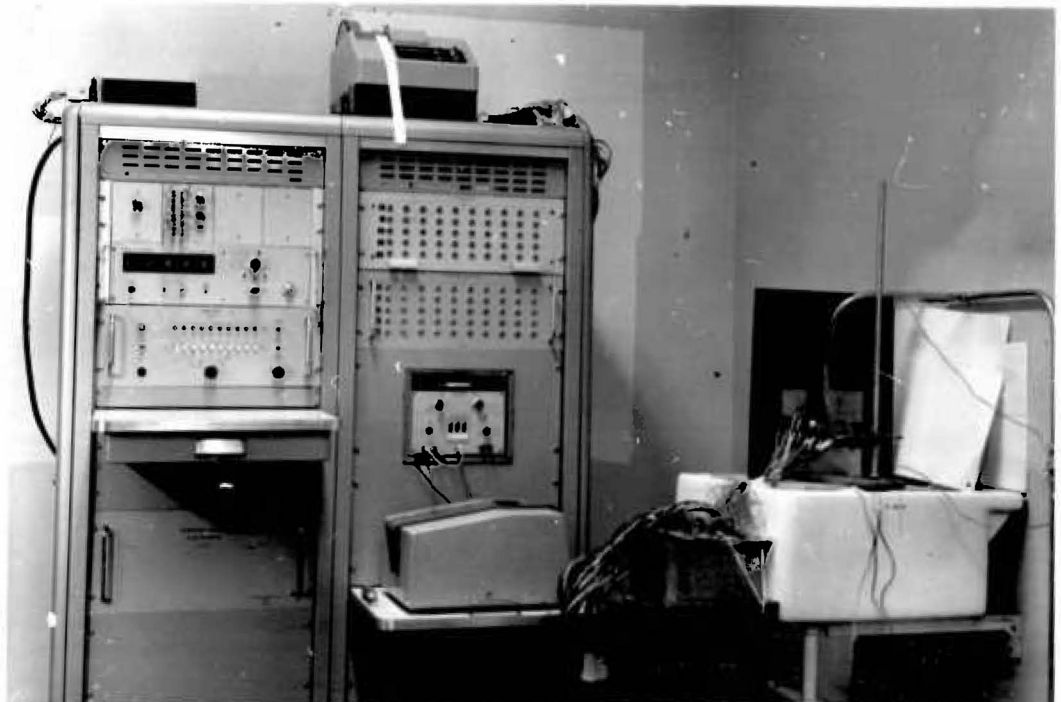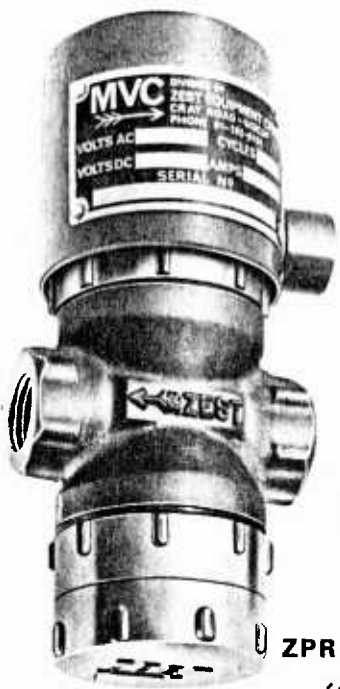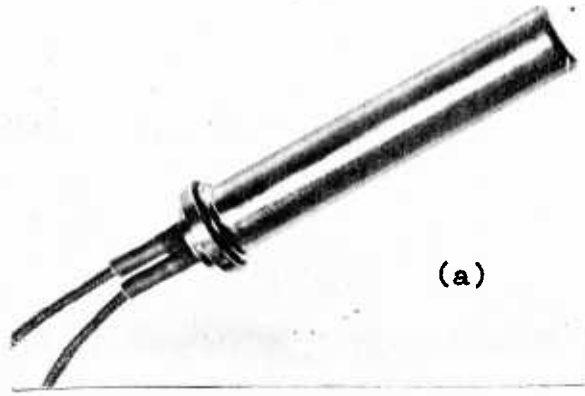
(e)

WITH DEVIATION
METER

'THERMIC'
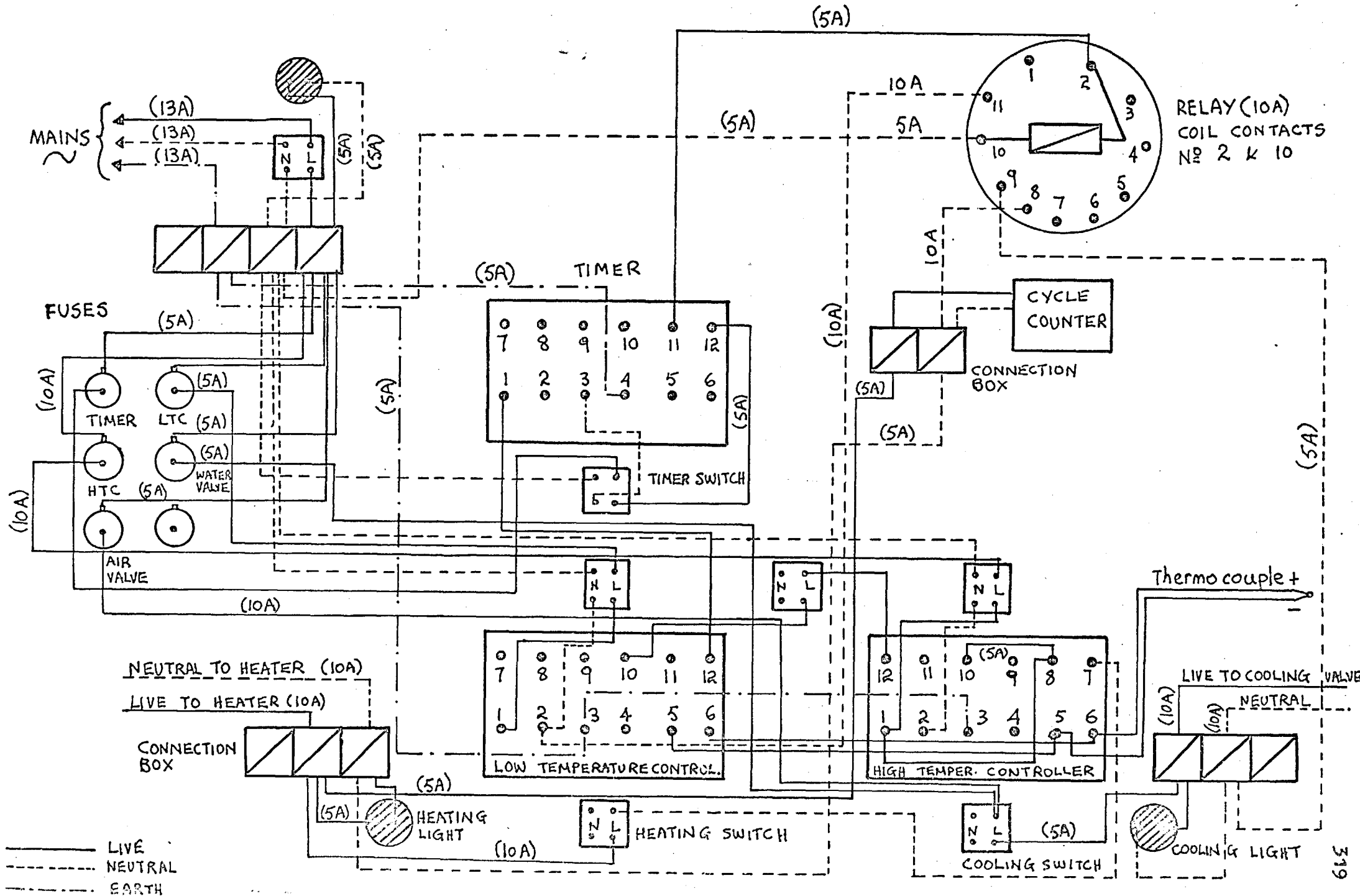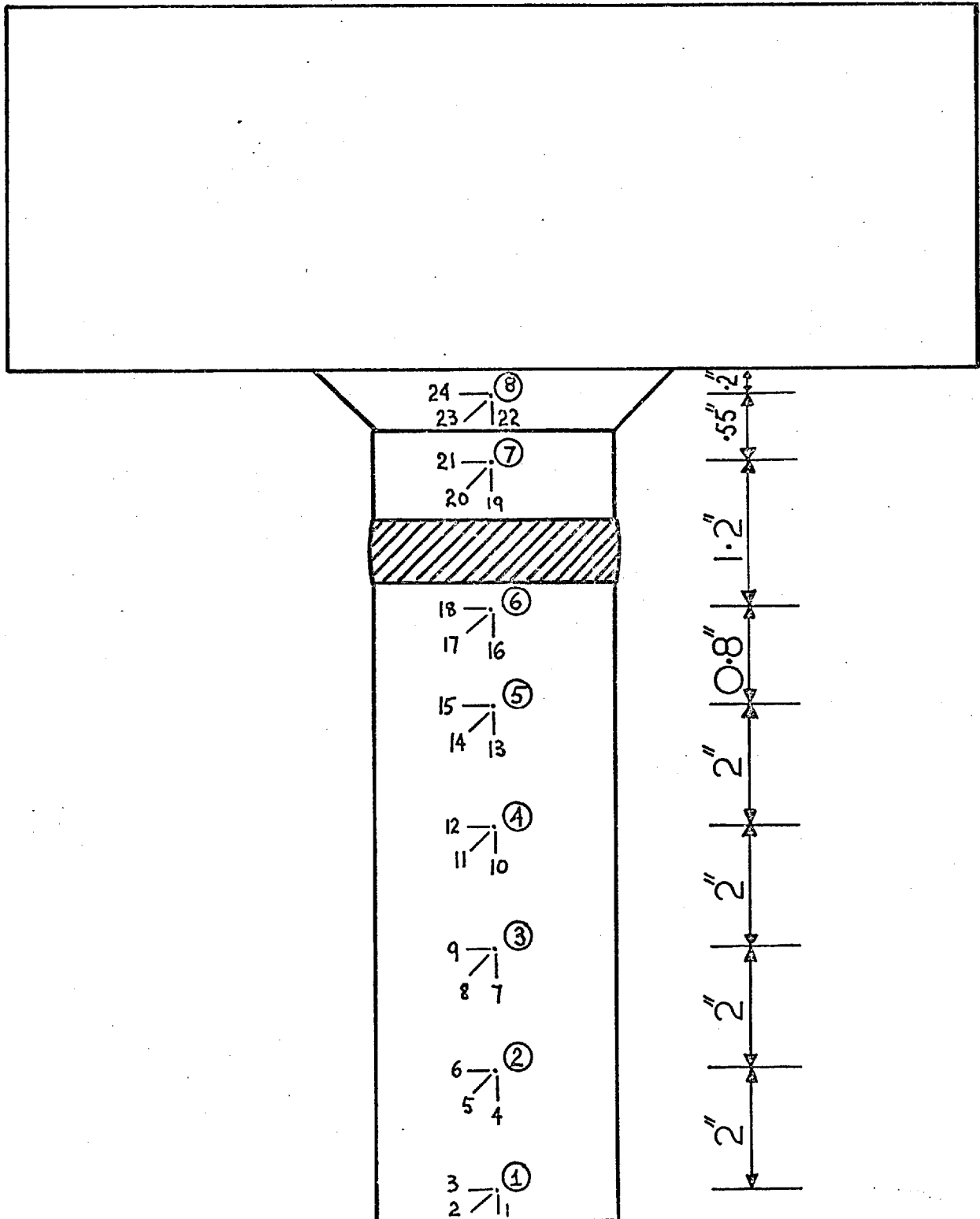
FIGURE 6.2.1

FIGURE 6.3.1(a) POSITIONS OF STRAIN GAUGE
ROSETTES ON THE NOZZLE AND JUNCTION.
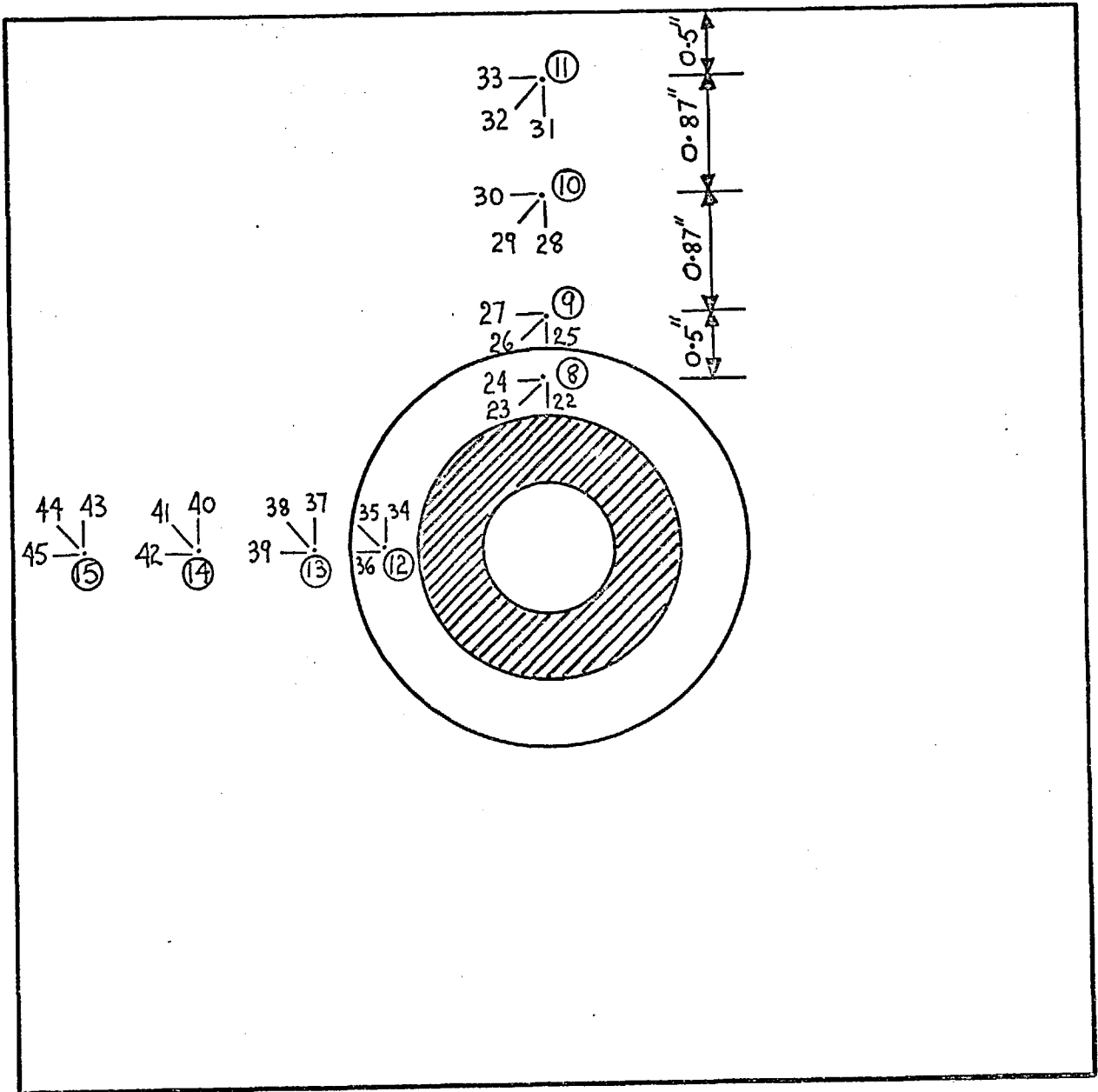
FIGURE 6.3.1(b) POSITIONS OF STRAIN GAUGE ROSETTES ON THE VESSEL.

FIGURE 6.4.1

STRESS σ_z at time = 12 minutes during heating-up.

——— Theoretical

⊚ Strain gauge readings (CORRECTED)

○ Strain gauge readings (OBSERVED)

(See also Table 6.4.2)

FIGURE 6.4.2
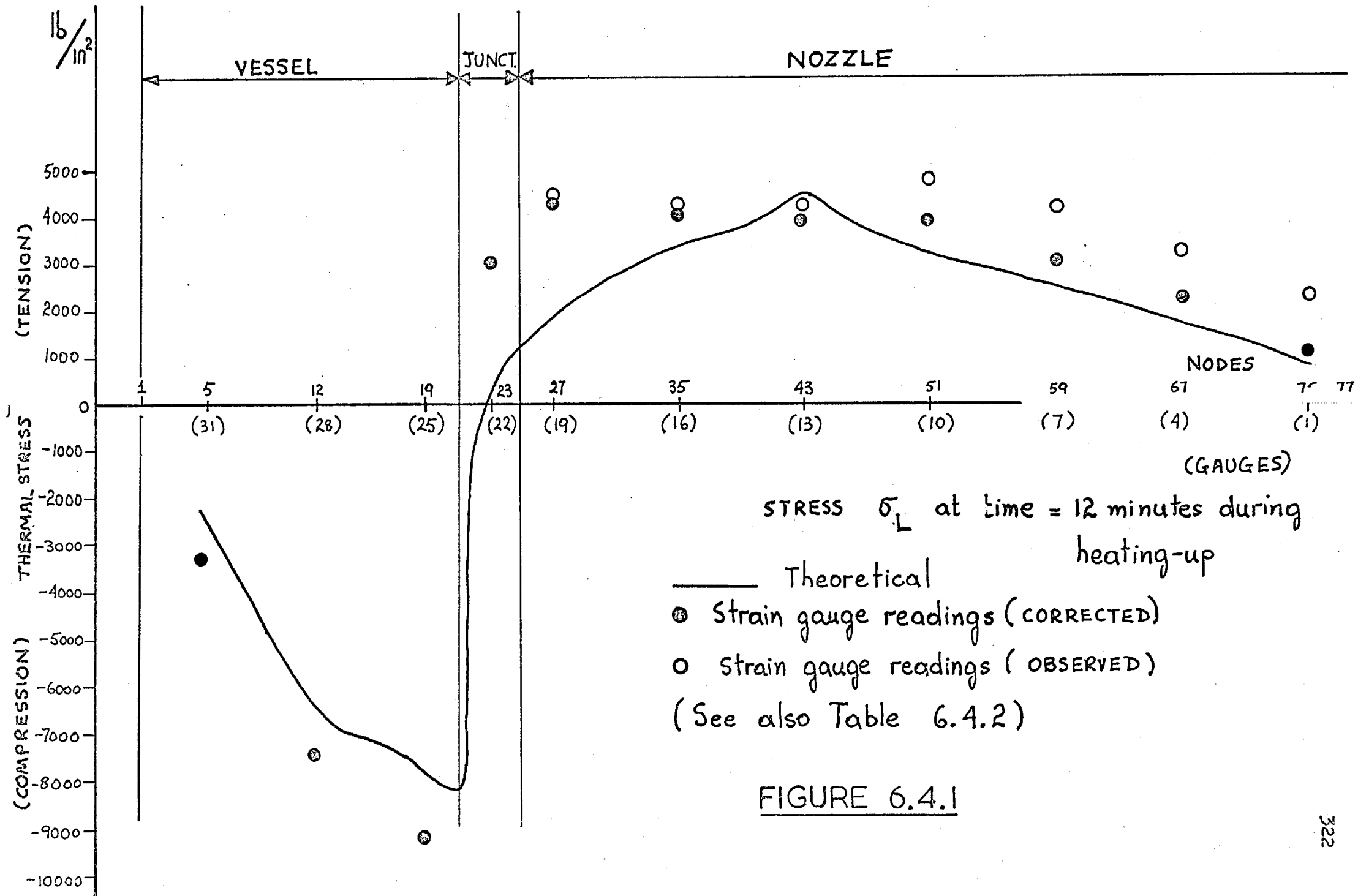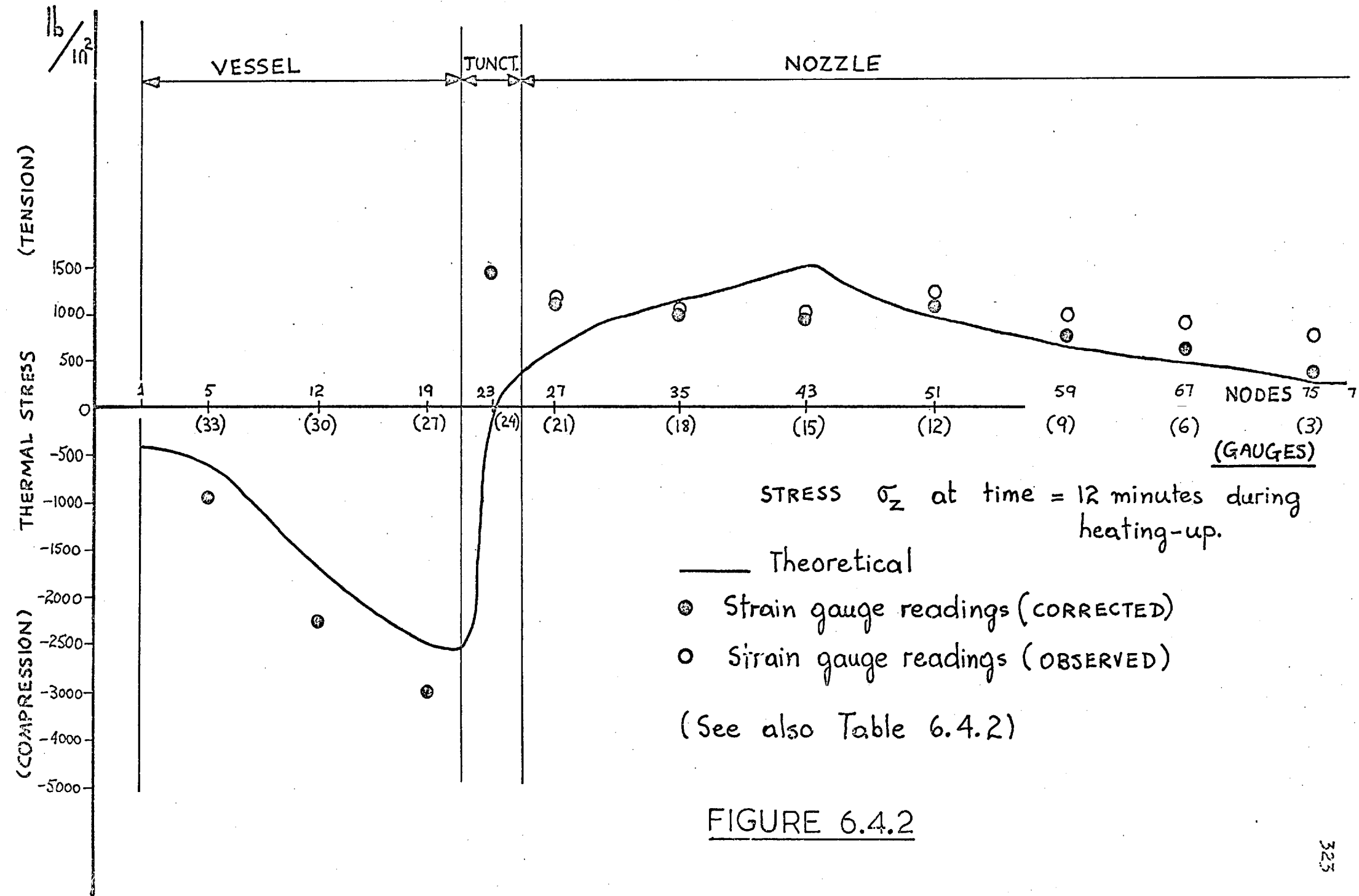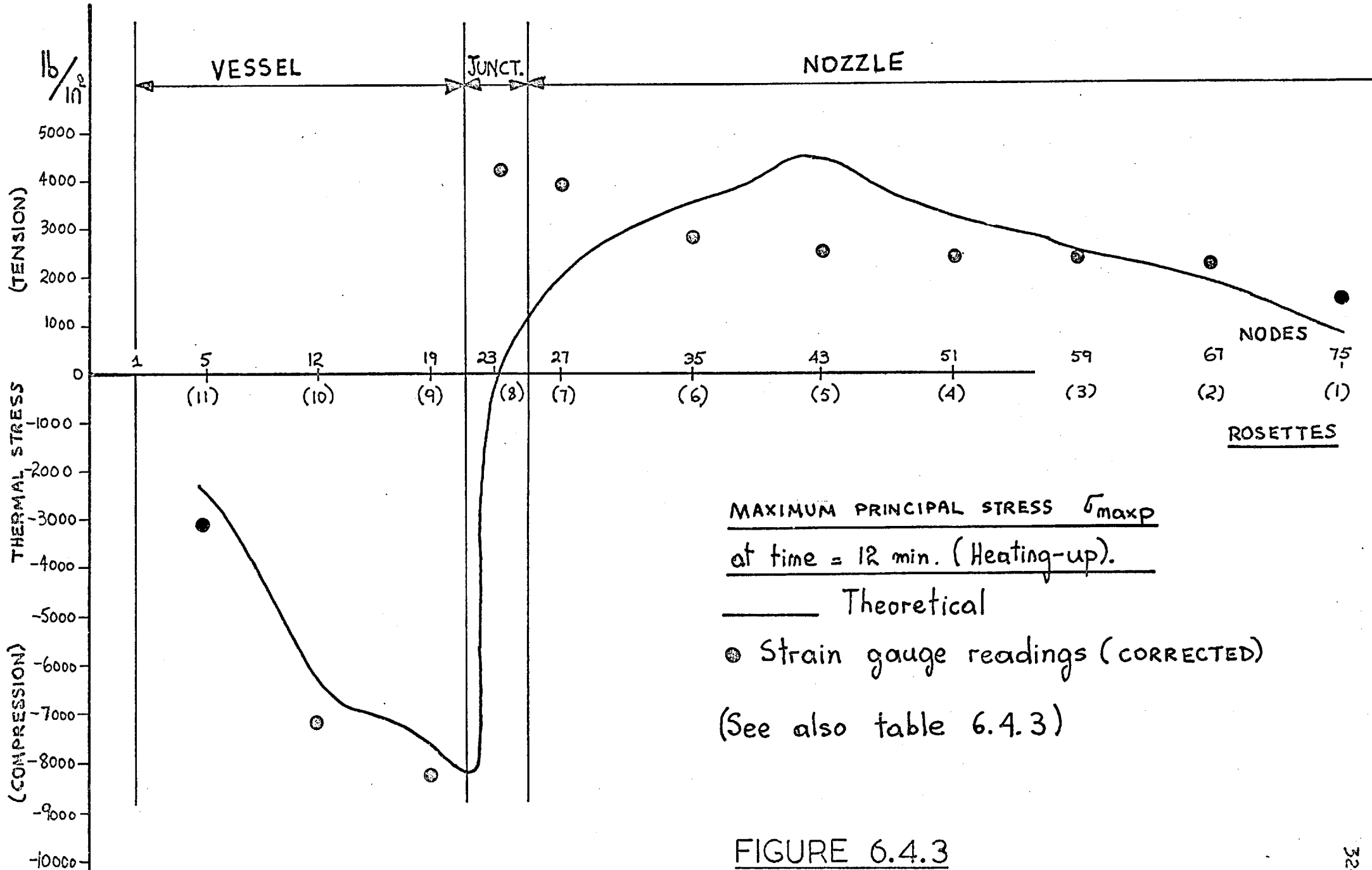
FIGURE 6.4.3

FIGURE 6.4.4

Maximum Shear Stress $\tau_{max(12)}$
at time = 12 min. (heating-up)

—— Theoretical
⊙ Strain gauge reading (CORRECTED)
(See also Table 6.4.3)

FIGURE 6.4.5

STRESS AT VESSEL (NODE 12)

STRESS σ_y

——— Theoretical

⊚ Strain gauge reading (Gauge 28)
(CORRECTED)

(See also Table 6.4.4)

FIGURE 6.4.6

STRESS AT VESSEL (NODE 12)

STRESS σ_z

——— Theoretical

⊙ Strain gauge reading (Gauge²) (CORRECTED)

(See also table 6.4.4)

FIGURE 6.4.6

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FIGURE 6.4.7

STRESS AT VESSEL (NODE 12)

MAXIMUM PRINCIPAL STRESS
(σ_{maxp})

—— Theoretical

⊙ Strain gauge reading (Rosette 1
(CORRECTED)
(See also Table 6.4.4)

STRESS AT NOZZLE (NODE 43)

STRESS σ_x

—— Theoretical

⊙ Strain gauge reading (Gauge 13) (CORRECTED)

○ Strain gauge reading (OBSERVED)

(See also Table 6.4.5)

Cooling-down →

Heating-up →

FIGURE 6.4.8

FIGURE 6.4.9

FIGURE 6.4.10

(a)

a = crack length

ρ = tip radius

$2h$ = crack height

(b)
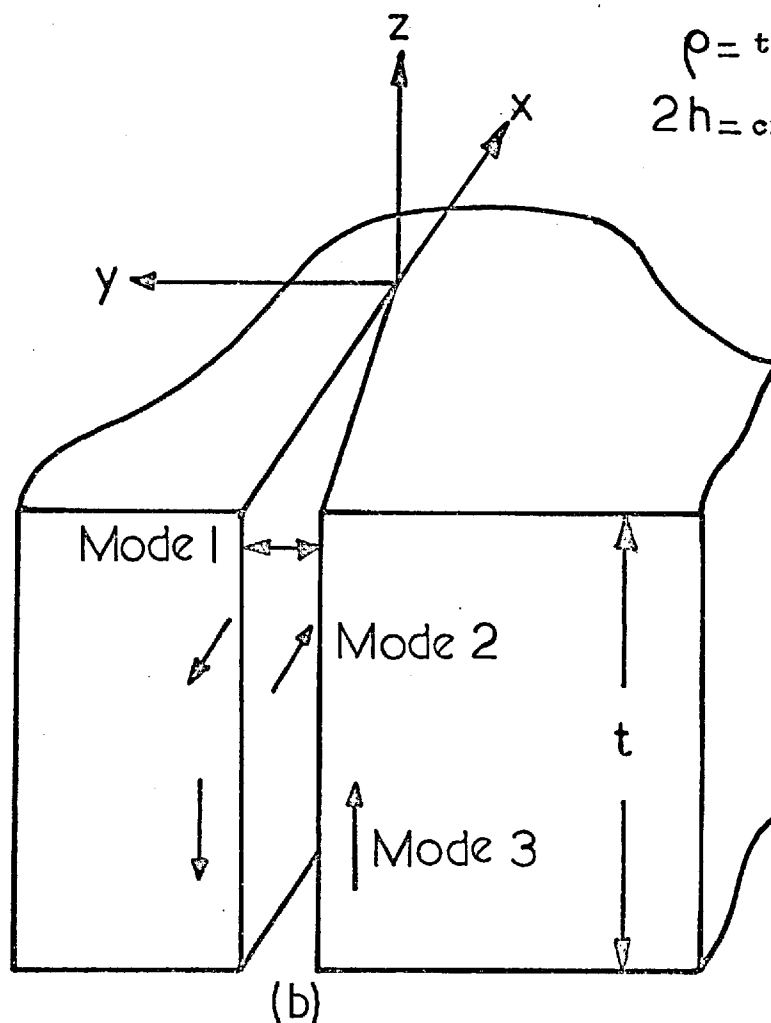
FIGURE 7.2.I CO-ORDINATE SYSTEM FOR CRACK
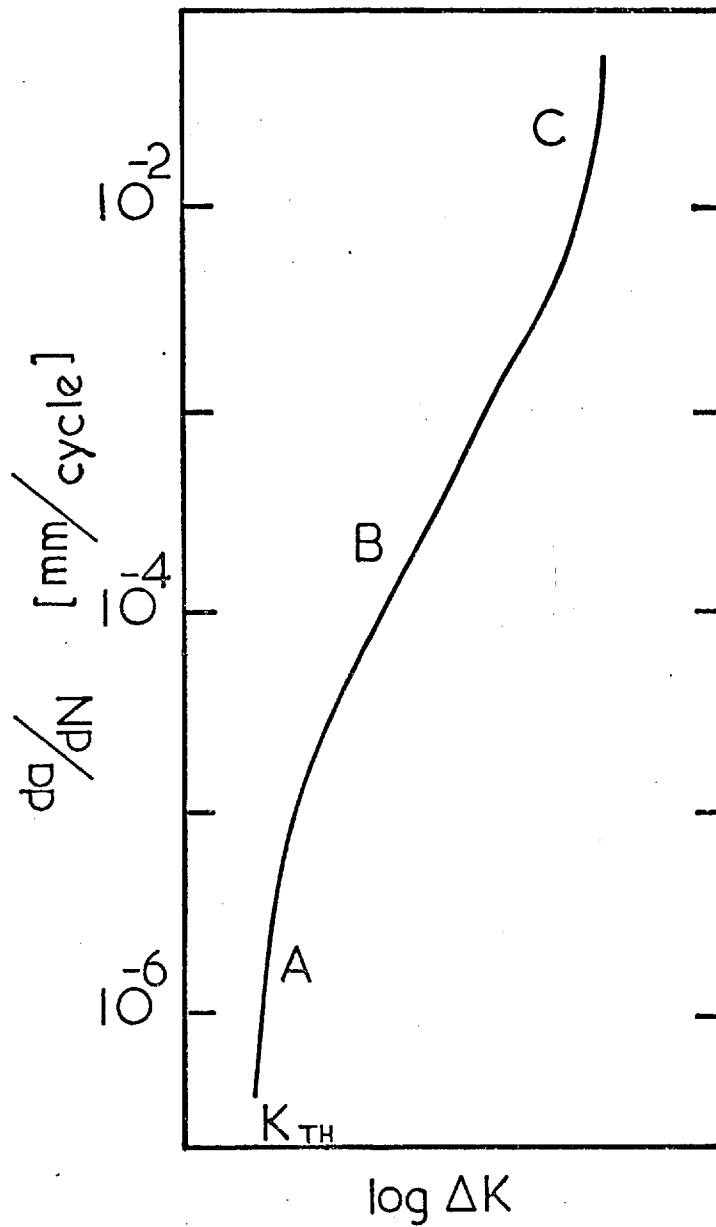
FIGURE 7.2.2 IDEALISED VARIATION OF FATIGUE CRACK GROWTH RATE(da/dN) WITH ALTERNATING STRESS INTENSITY $\triangle$K (R.7-11).
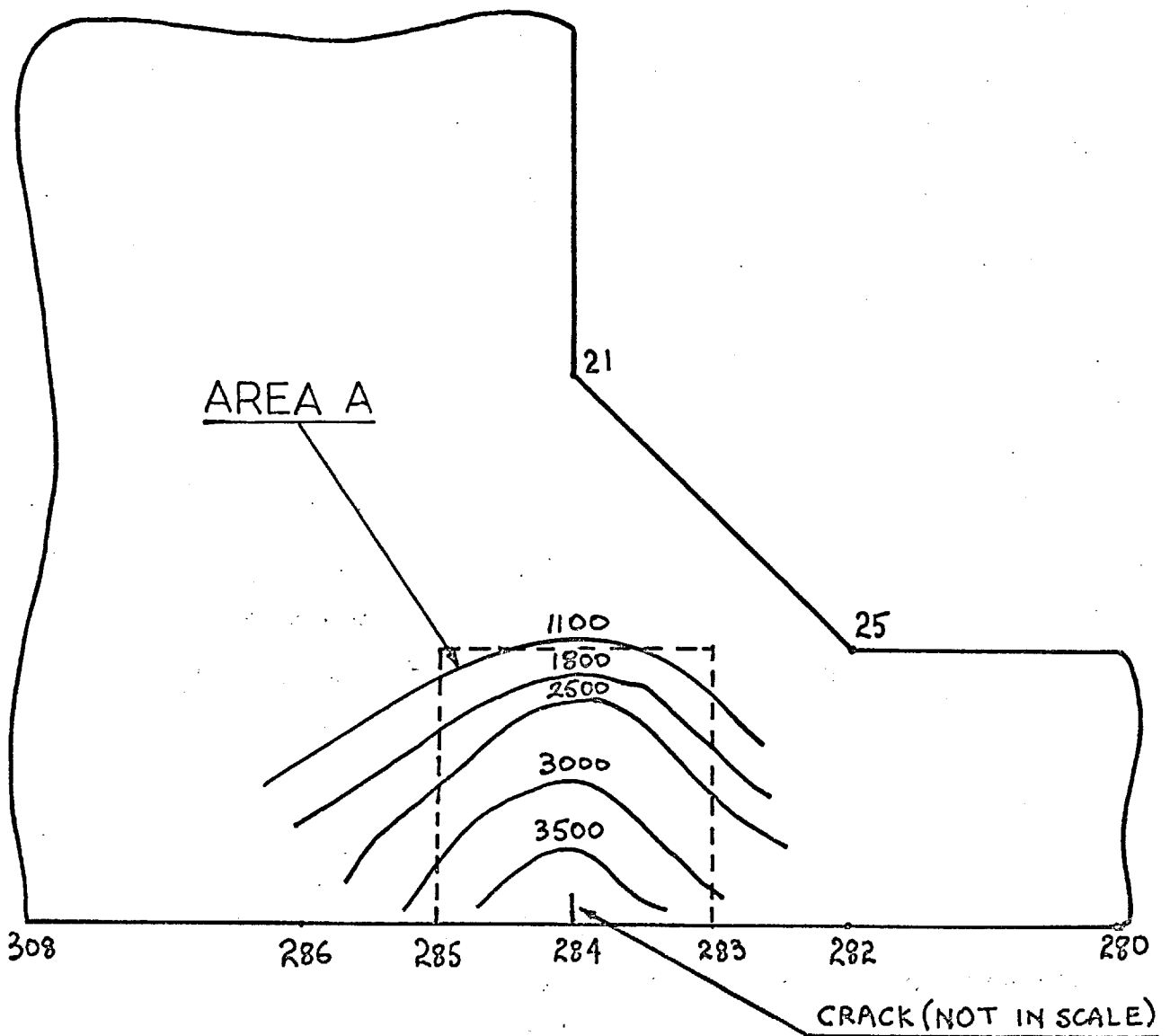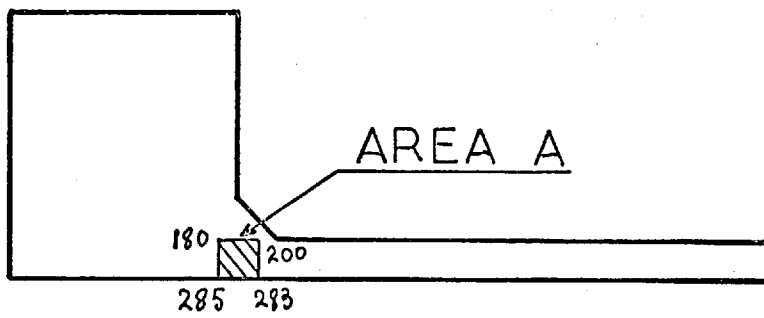
FIGURE 7.3.1.1. MAXIMUM PRINCIPAL STRESSES IN AREA A OF CASE C.
(NUMBERS ALONG THE BOUNDARIES INDICATE NODES)
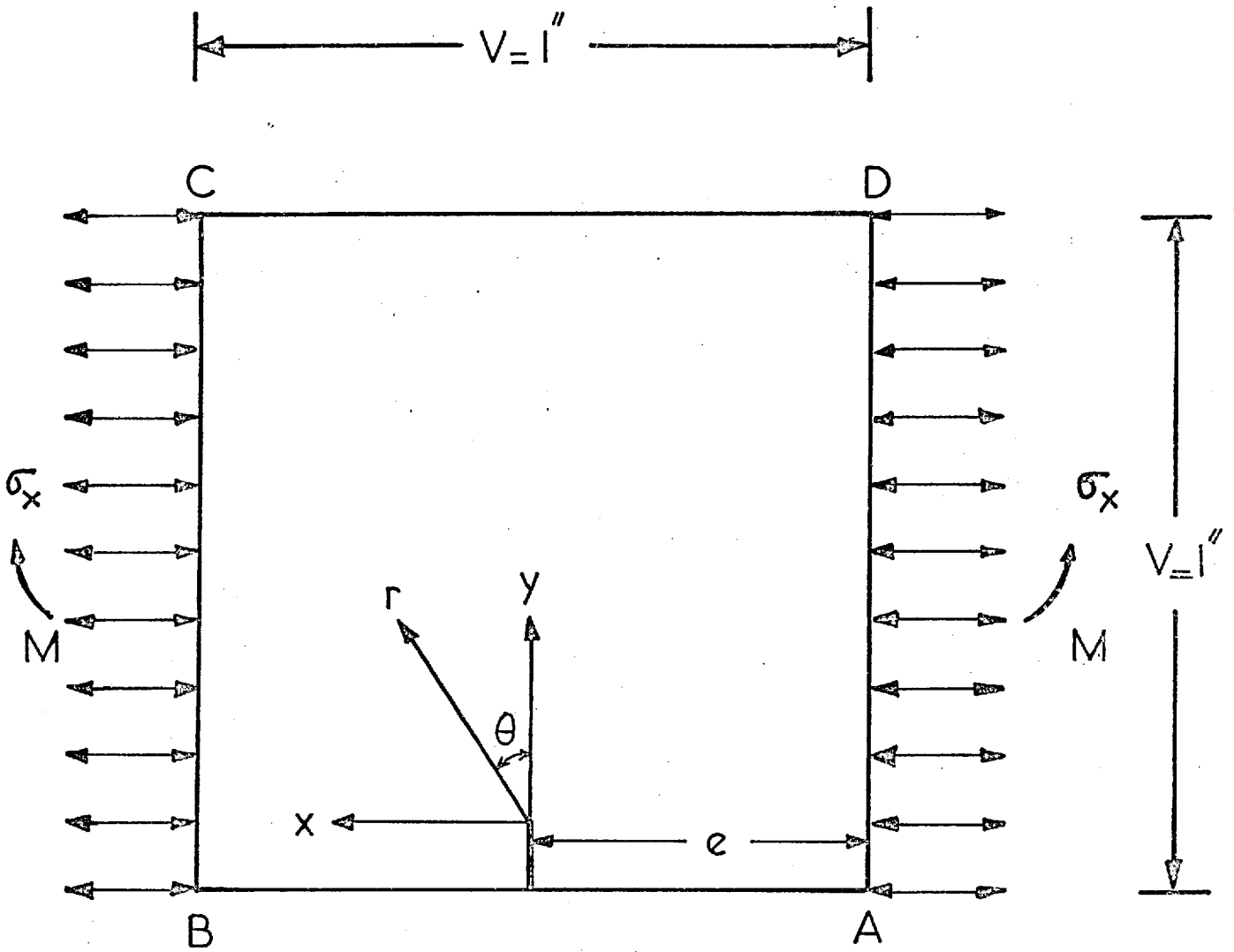
a = CRACK LENGTH (NOT IN SCALE) = 0.04 in.

r,θ = CO-ORDINATES FOR THE TIP OF THE CRACK

FIGURE 7.3.2.1 AREA A WHERE CRACK IS CONSIDERED FOR CASE C.
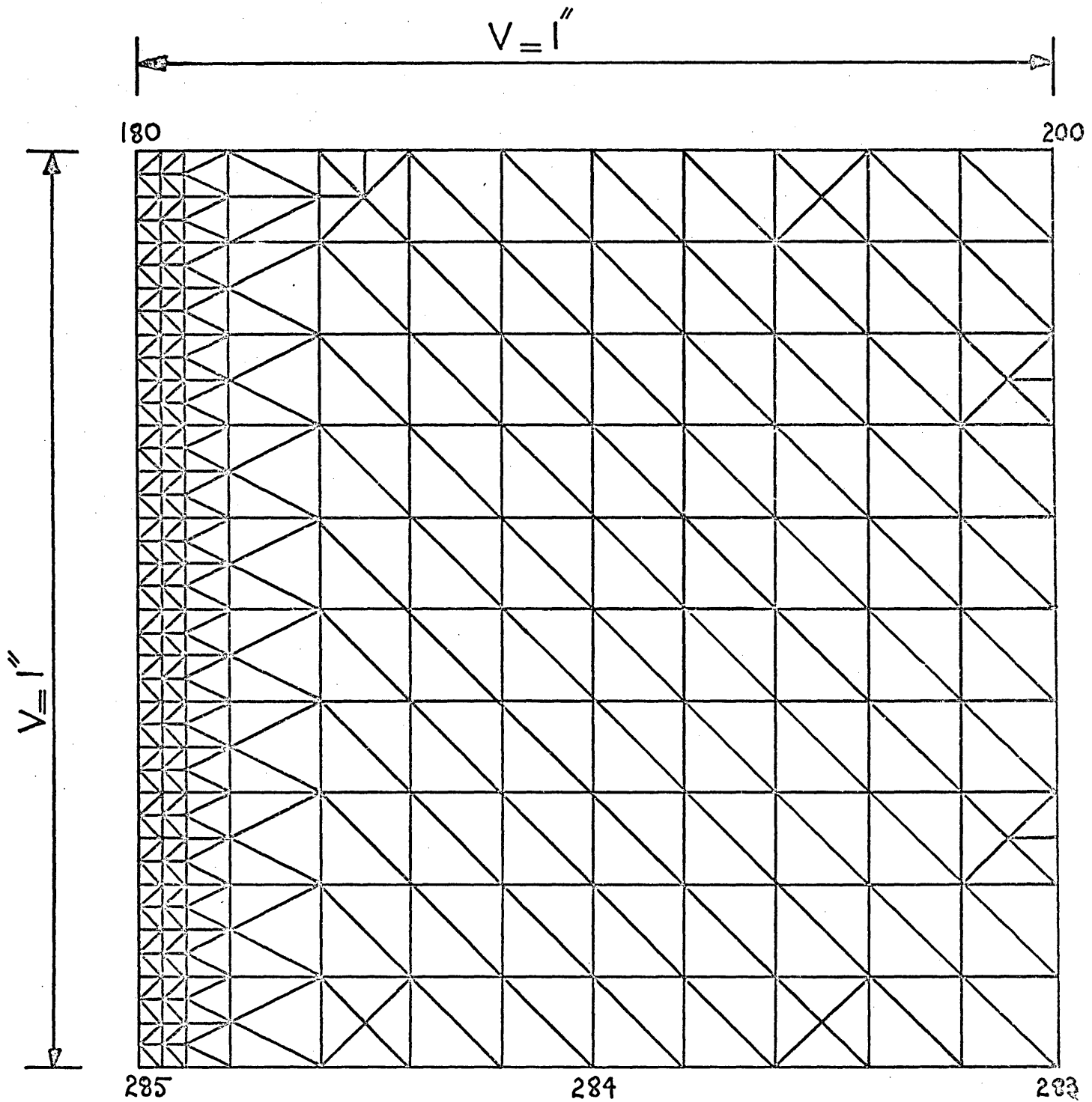
FIGURE 7.3.2.2 FINITE ELEMENT MESH FOR AREA A OF CASE C.
NUMBER OF ELEMENTS = 430
NUMBER OF NODES = 256
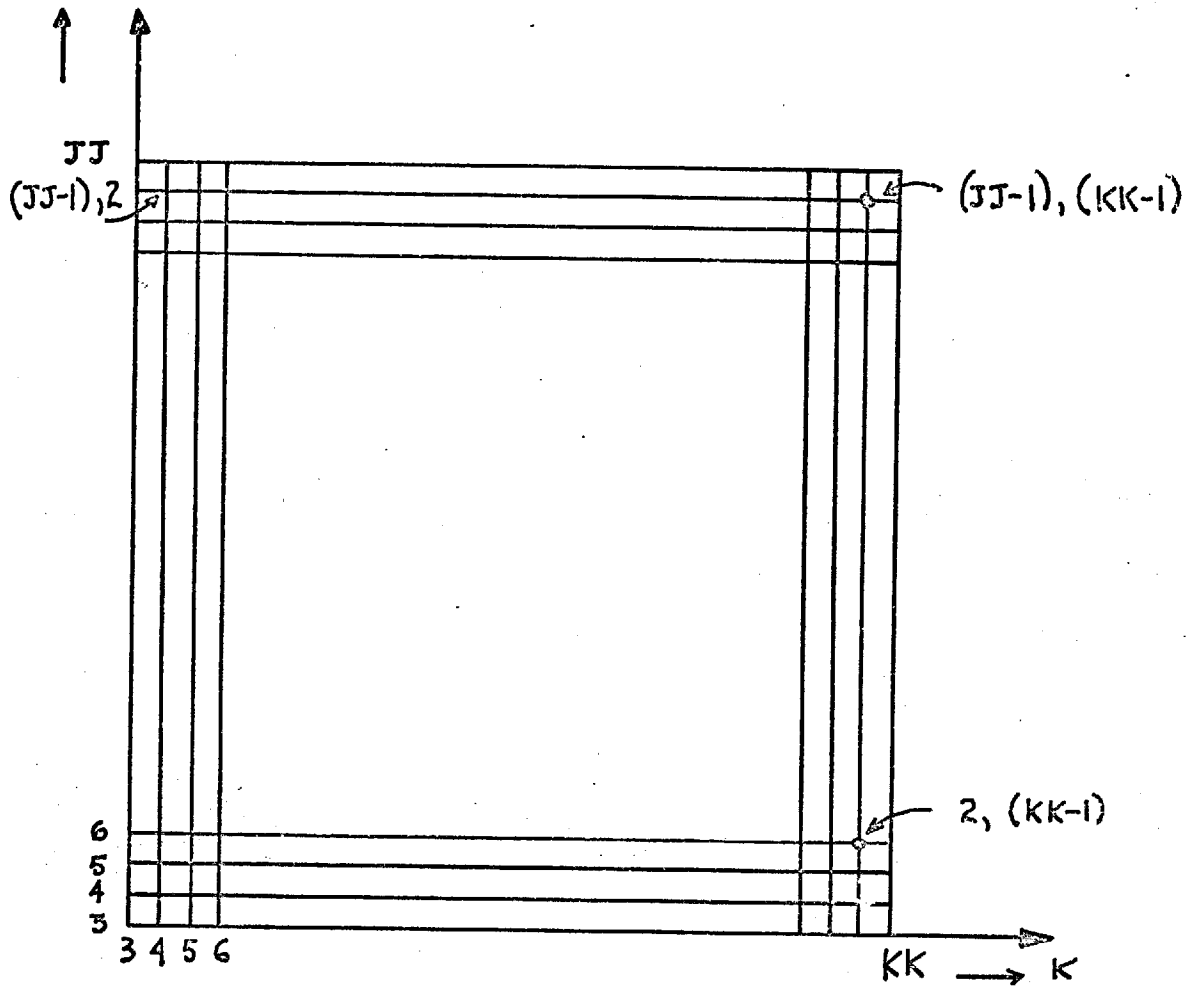
Grid for the determination of stress function χ
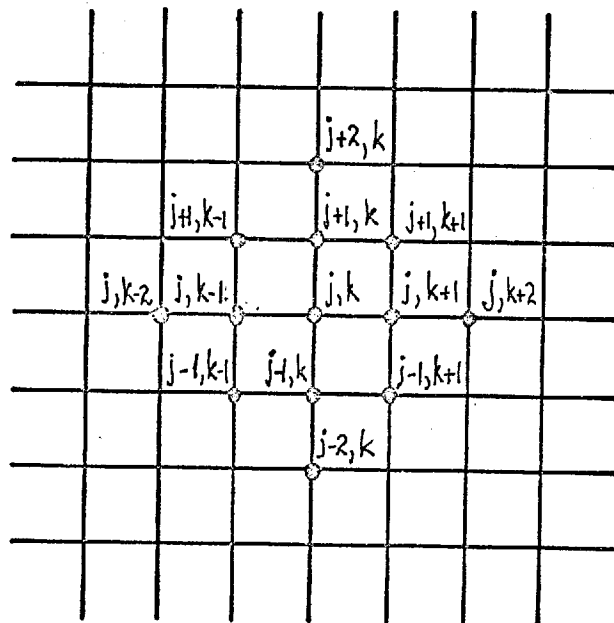
Cell for the finite difference solution of $\nabla^4 \chi = 0$
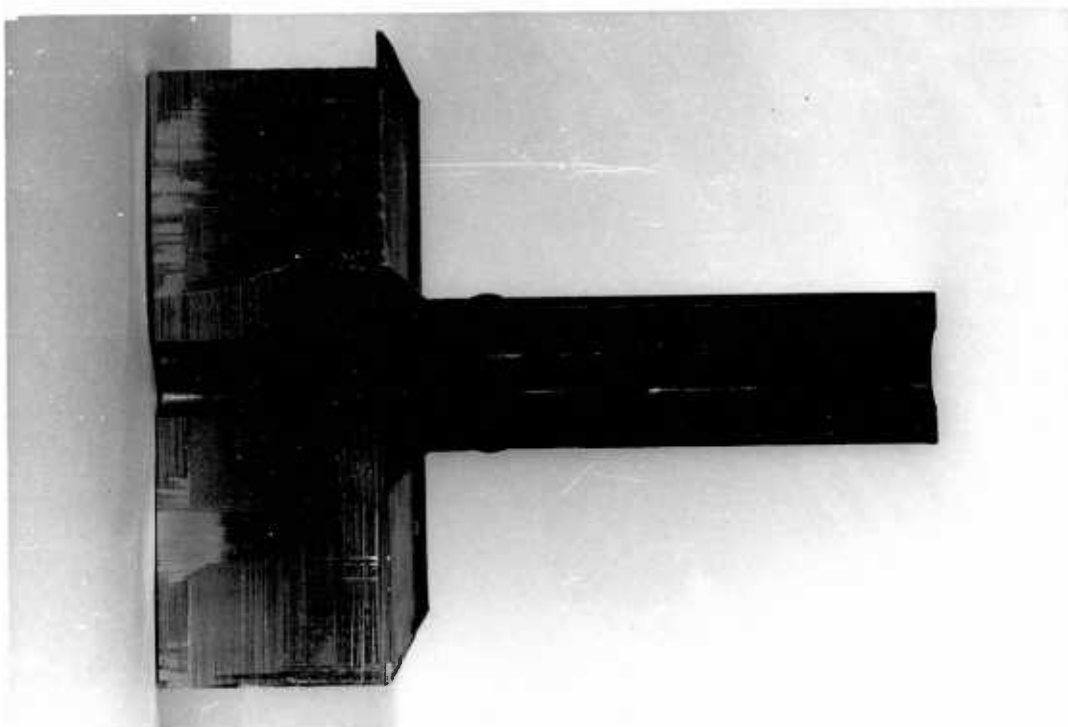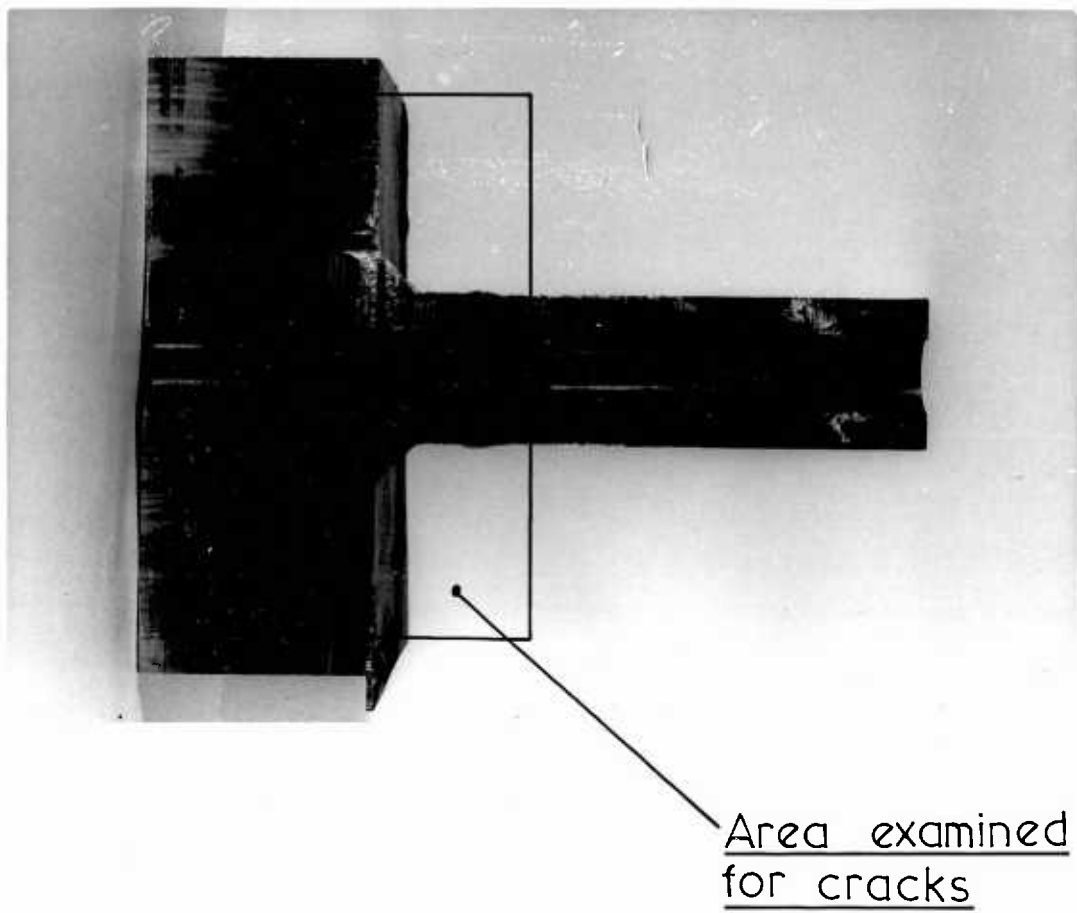
FIGURE 7.3.2.3

Area examined for cracks

FIGURE 7.1.4