DIGITAL PROCESSING OF SIGNALS IN THE PRESENCE OF INTERSYMBOL INTERFERENCE AND ADDITIVE NOISE

BY

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This Thesis is dedicated to my Parents and to my Wife

ABSTRACT

This thesis describes an investigation into the performance of various digital processing techniques used in high speed modems in the presence of intersymbol interference and additive noise. The limitations of the existing techniques for such applications are stated and new kinds of equalizers are proposed to overcome some of these limitations.

The first part deals with a new kind of non-recursive digital filter equalizer which leads to a significant reduction in the settling time of high speed modems. This is achieved by reducing the effects of intersymbol interference. For deterministic, discrete time channels this equalizer is initialized with a single isolated pulse and then is switched to the decision directed mode for on line adaptive operation to allow for continual tracking of the variations in the channel characteristics. Conditions for the Wiener solution are obtained using mean square error criterion. Since the rate of convergence depends on the eigenvalues of a discrete time channel output correlation matrix, the convergence rate can be improved by modifying the matrix by linear transformation. This results in a new structure, a generalized non recursive digital filter equalizer, which is transformed into a set of non recursive digital filter sections connected in parallel. The weighted outputs of these sections are added to produce the equalizer output. It is shown that by properly selecting the multiplier coefficients of each non recursive digital filter section, the new equalizer structure can be adjusted to yield minimum mean square error in one iteration of the algorithm. This equalizer may be suitable for systems (e.g. distributed processing, digital networks, multiparty polling systems etc.) in which the period of the training sequence may be shorter than the duration

of the impulse response of the equalizer. In this case signals entering the multipliers may not be linearly independent and hence the correlation matrix would be singular. In such a case, therefore, a Moore Penrose Pseudoinverse algorithm is proposed to obtain a better approximation to an orthogonality condition. An orthogonal Hadamard matrix is used and it is shown that the correlation matrix is equal to a projection matrix for which all eigenvalues are either '0' or '1' which is the desired condition for fast initialization (or minimum settling time).

For noisy discrete time channels the proposed design algorithm is modified and the convergence is studied in statistical sense. On-line adaptation is used and the tracking ability is improved for this digital noisy channel by using more than one initializing pulses. A diagnostic is proposed to cater for excess errors. The effects of several key parameters are studied.

Six constrained recursive digital filter equalizers are proposed. The rate of convergence of these equalizers, using mean square error as the performance criterion have been studied by using optimization techniques. Also, the relationship between the probability of error and signal to noise ratio has been investigated using Monte Carlo simulation techniques. The performance of these equalizers has been compared with conventional non recursive digital filter equalizers and the results indicate that proposed methods are superior.

Finally the use of a constrained recursive digital filter equalizer has been made with the maximum likelihood sequence estimator, also known as the Viterbi algorithm. Two receiver structures have been proposed and simulated for truncated discrete time channel impulse response samples. The results show that there is a scope for application for these recursive digital filter equalizers into the Viterbi algorithm.

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CHAPTER 1

INTRODUCTION AND BACKGROUND REVIEW

You have a right to perform prescribed duty, but you are not entitled to the fruits of action. Never consider yourself to be the cause of the results of your activities, and never be attached to not doing your duty.

BHAGAVAD-GĪTĀ (CH. 2, TEXT 47)

1.1 INTRODUCTION:

Man has devised numerous machines to perform tasks that are either too difficult or too tedious for him to do unaided. These devices have grown more complex over the years so that today it is necessary for machines to communicate not only with men but also with other machines. Communications between computers, between business machines and computers, between spacecraft and ground stations are examples of machine-to-machine communication. This type of communication is really important today; its impact on society is felt quite clearly now a days in the form of video telephone, space shuttle, lunar mars landings, etc. This widespread use of machine-to-machine communication has lead to dramatic changes in man's way of life. Therefore, it is felt that today's immense growing communication needs can be met not only with developing new equipment but also, to a certain extent, by increasing the throughput of already existing data channels. A cost effective design will in both cases involve a tradeoff between the total power and bandwidth allocated to the digital signal on one hand, and complexity of the decision device (algorithm) on the other hand. A theoretical limit to what could be achieved in this respect is imposed by the concept of channel capacity.

The maximum rate at which digital signals can be transmitted through a channel corrupted by additive White Gaussian Noise (WGN) is given by Shannon's Capacity formula [1-1].

$$C = BW \log_2 [1 + \frac{P}{N_0 BW}] \quad \text{bits/sec}$$

where BW is the channel bandwidth, P is the maximum average received signal power, and $(N_0/2)$ is the two-sided noise spectral density.

When the available signal-to-noise ratio, SNR $\overset{\Delta}{=} \frac{P}{N_0 BW}$, is small, bandwidth is not the restricting factor and then we say that the capacity of the channel is power-limited. We can see this by using the expansion:

$$\log_2(1+X)=(X - \frac{X^2}{2} + \frac{X^3}{3} - \frac{X^4}{4} + \ldots)/\ln 2 \qquad (|X|<1)$$

If $\frac{P}{N_0 BW} << 1$, the above formula reduces to

$$C=BW \left[\left(\frac{P}{N_0 BW}\right) - \left(\frac{P}{N_0 BW}\right)^2/2 + \ldots\ldots \right]/\ln 2$$

$$\simeq P/(N_0 \ln 2)$$

which is independent of the bandwidth, BW. In other words, we can say that even when the information is to be transmitted at rates close to capacity, we have ample bandwidth available and the simple pulse shaping techniques would suffice to ensure that the pulse overlap (henceforth called the intersymbol interference and abbreviated as ISI) is not a problem.

When the available signal-to-noise ratio, SNR $\overset{\Delta}{=} \frac{P}{N_0 BW}$ is high, that is

$$C << P/(N_0 \ln 2)$$

then the bandwidth limitation becomes the main obstacle to high data transmission rate.

According to the Sampling Theorem for a channel of bandwidth BW, the maximum rate at which these pulses or information symbols can be decoded without ISI is equal to the Nyquist rate (2xBW). When the symbol rate is close to the Nyquist rate, say $S_p = \beta x BW$ (where $\beta \simeq 1$ and is a constant) and the actual information rate R_a is a fraction λ_f of the channel capacity, then the number of bits of information per channel symbol equals

$$\left(\frac{R_a}{S_p}\right) = \lambda_f \left(\frac{C}{\beta x BW}\right)$$

$$= \left(\frac{\lambda f}{\beta}\right) \left(\frac{C}{BW}\right) = \left(\frac{\lambda f}{\beta}\right) \mathrm{Log}_2 \,(1+SNR)$$

This expression tends to infinity as SNR$\rightarrow \infty$, which shows that for high SNR multi-level signalling becomes essential if we wish to achieve transmission rates that are a reasonable fraction of the capacity. However, because of the power constraint any increase in the number of levels forces energy levels of the signals closer together, and therefore causes the symbol error probability to be drastically higher. Therefore, we are forced to look for the smallest number of levels consistent with the information rate $\left(\frac{R_a}{S_p}\right)$. One is therefore tempted to push the pulse rate S_p towards its limit of about 2BW. For most channels the ISI thus introduced does not cause a loss in error performance when compared with the case of no ISI.

We concern ourselves with the area of high SNR typically 30db, i.e., existing telephone channel. In this channel bandwidth, usually 2.8KHz, is the major constraint. The capacity of this channel, assuming White Gaussian Noise (WGN), can be estimated at 2800 $\log_2(1+10^3) \approx 27.892$ Kilobits per second. Therefore in theory, reliable communication should be possible not only at 4,800 bits/sec., 9,600 bits/sec., but even at 19,200 bits/seconds.

Various schemes have been derived to achieve such high rates of transmission of data. Also, it is hoped that this work might provide the means to operate a larger fraction of existing channels including those with undesirable amplitude and phase characteristics at rates up to 9,600 bits/sec.

1.2 INTERSYMBOL INTERFERENCE (ISI):

We have seen that the most important restraint for the channel under consideration is the Bandwidth limitation. The aim is to transmit data at very high rate with the available bandwidth, using the lowest permissible number of levels (i.e., Binary signalling), consistent with the power requirements, and reliably. The reliability is usually specified in terms of the probability of error (P_e). But, when the data is transmitted it suffers time-dispersion. This time-dispersion imparted on the transmitted signal takes the form of symbol overlap in the time between successive symbols known as intersymbol interference (ISI). This thesis studies the ways of reducing the ISI digitally. This is usually accomplished by means of an equalizer and the process is called digital equalization.

All real channels exhibit some form of time-dispersion. We mention the details about two time-dispersive channels. The first is the telephone channel, where the time-dispersion is attributed to imperfect frequency response characteristics of the channels, which are usually expressed in terms of attenuation and envelope delay as functions of frequency [Figure (1.1)]. An ideal channel has a constant attenuation and envelope delay across its frequency band.

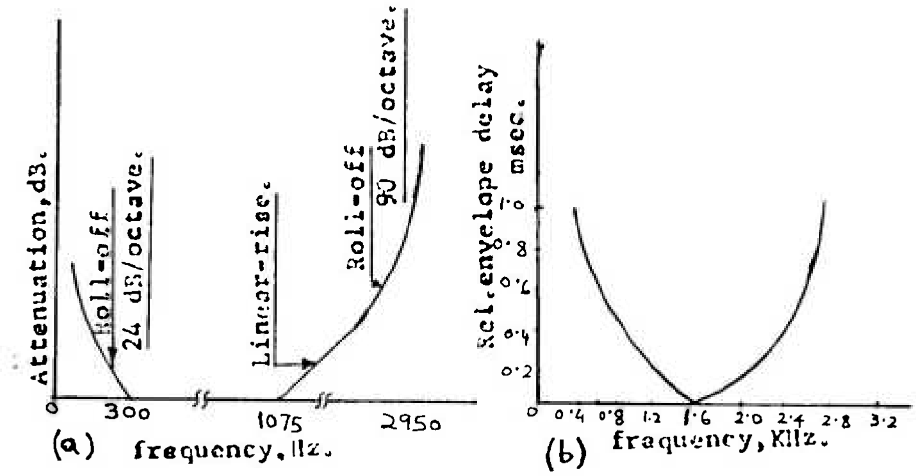


FIG. (1.1) (A) AMPLITUDE CHARACTERISTICS FOR A TELEPHONE CHANNEL

(B) DELAY CHARACTERISTICS FOR A TELEPHONE CHANNEL.

Since neither the envelope delay nor the attenuation delay is generally constant across its frequency band, the input signal suffers amplitude and delay distortion respectively both of which contribute heavily to ISI.

The second channel considered is a high-frequency radio channel using tropospheric scatter, on which the time-dispersion and, hence, ISI is the result of multiple propagation modes or paths with different path delays. The number of paths and the relative time delays among the paths vary with time and, for this reason, these radio channels are usually called time-variant multipath channels. The time-variant multipath conditions give rise to a wide variety of frequency response characteristics. Consequently, the frequency response characterization that is used for telephone channels is unsuitable for time-variant multipath channels. Instead, one should characterize statistically these channels in terms of scattering functions; in brief, this is a 2-dimensional representation of the average received signal power as a function of relative time-delay and frequency.

ISI represents a kind of deterministic channel impairment. If the channel characteristic is known, it is usually feasible, in theory, to remove ISI. As such, it is desirable at this point to emphasize two important aspects in the problem of data transmission through time-dispersive channel:

(I) the channel characteristics (i.e., channel impulse or frequency responses) are never known, a priori,

and, (II) the response characteristic of the channel varies with time.

Telephone Channels exhibit slow and small variations in their transmission characteristic while multipath channels exhibit relatively rapid and large variations in their transmission characteristics.

In any case, the digital signal processing technique that is employed to cope with time-variant ISI conditions must include some means of measuring and tracking the channel response. Such operations should be and are performed at the receiving terminal.

1.3 THE EQUALIZATION CONCEPT OF DISCRETE TIME NON-RECURSIVE MODEL CHANNELS:

As stated earlier, the reduction of ISI by means of digital processing techniques is known as digital equalization and the device

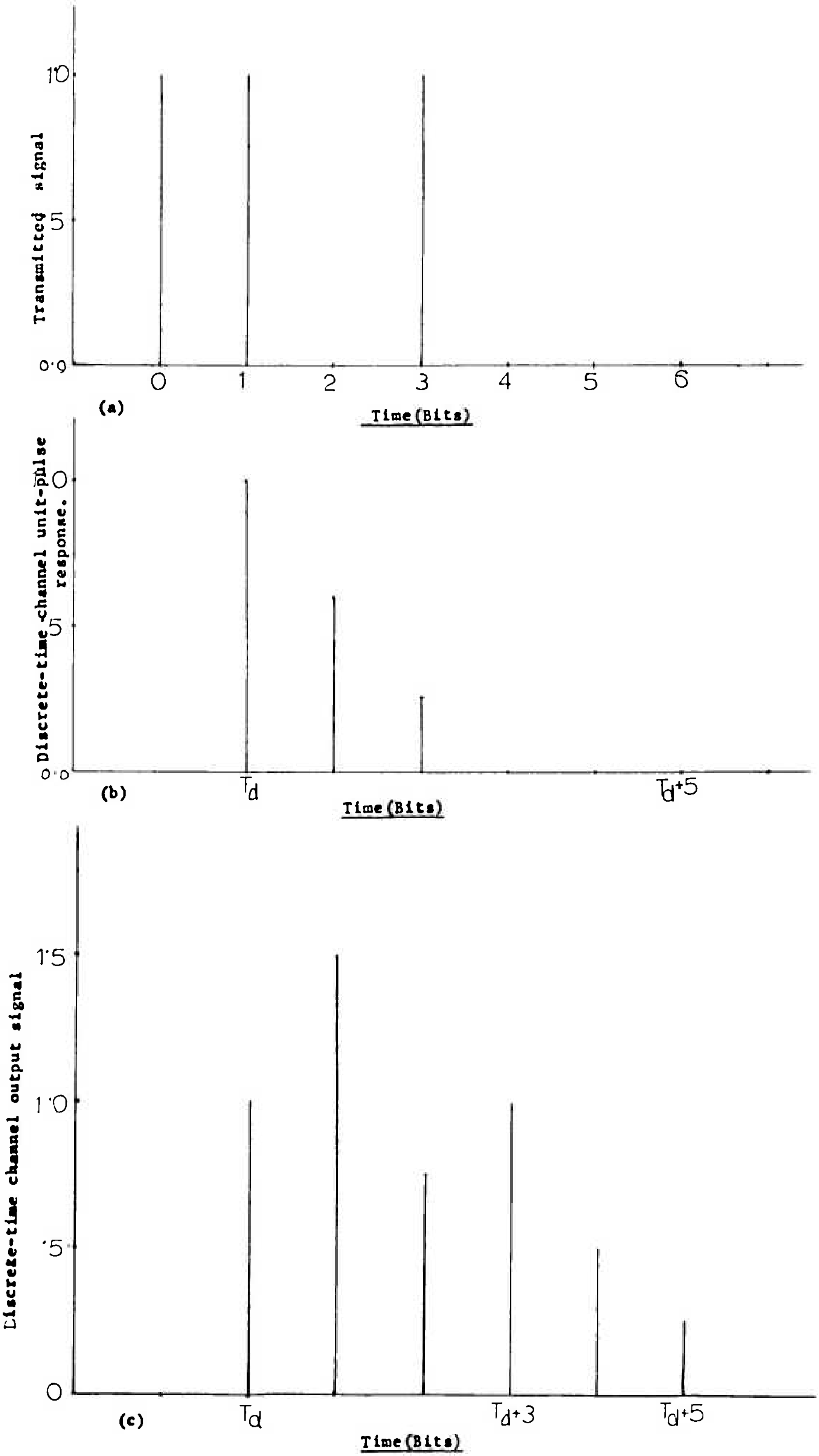


FIG. (1.2) INTERSYMBOL INTERFERENCE (ISI) DUE TO TIME-DISPERSION.

with which this is carried out is called a digital equalizer. ISI is the time-dispersion, deterministic impairment and the way in which this occurs in real time is explained below:

Figure (1.2) depicts the time-dispersion of a pulse by the band-limited discrete-time non-recursive channel. In figure (1.2), time has been normalized and therefore, T_d, represents a normalized time delay of the transmitted pulse. On examining the whole phenomena closely, we see that the channel output at the sampling point (T_d+3) is unity [Figure (1.2(C))], whereas the pulse transmitted corresponding to that point was zero [Figure (1.2(a))]. Thus a message error has occurred. Even if ISI does not cause message errors by itself, it will certainly reduce the immunity of the system to noise. So, this error somehow has to be reduced (preferably eliminated!). Some ways to reduce the error rate include:

(a) raising the signal-to-noise ratio (SNR),

(b) introducing redundancy through coding,

and, (c) the use of pulse-shaping filters to reshape the transmitted or received signals. This latter (c) approach is called discrete-time channel equalization and the filter used for it is termed an equalizer. These equalizers are classified here. A number of classification schemes have been proposed depending upon the type of signals being processed. They are:

BASED ON LINEARITY:

(i) Linear,

and, (ii) Non-Linear.

BASED ON FEEDBACK:

(i) Non-Recursive,

and, (ii) Recursive.

BASED ON SIGNAL-FLOW:

(i) Parallel,

(ii) Series,

and, (iii) Cascade.

BASED ON HISTORICAL NOMENCLATURE:

(i) Transversal,

(ii) Sampled-Data,

and, (iii) Frequency Sampling.

BASED ON STATISTICAL PROPERTIES:

(i) Moving Average (MA),

(ii) Auto-Regressive (AR),

(iii) Moving Average Auto-Regressive (MAAR),

and, (iv) Maximum Likelihood Sequence Estimator (MLSE) or the Viterbi Algorithm Detector (VA).

In this Thesis, we shall concern ourselves with:

(i) Non-recursive digital filter equalizer (henceforth abbreviated as NRDFE),

(ii) Recursive digital filter equalizer (henceforth abbreviated as RDFE) and,

(iii) Maximum-likelihood sequence estimator (henceforth abbreviated as MLSE or the VA).

Next, we discuss the terminology connected with the equalizers.

1.4 FUNDAMENTALS ABOUT EQUALIZERS:

Equalizers fall within three categories:

Fixed (preset),

Automatic,

and, Adaptive.

Fixed Equalizers [1-8] are adjusted to provide the amplitude and phase compensation necessary to correct the average distortion characteristics of dispersive channels, while Automatic and Adaptive Equalizers (also called Self-Adjusting Equalizers), on the other hand, have generally found applications in Synchronous data transmission to equalize the discrete-time channel, i.e., reducing the ISI at the sampling times. The automatic equalizers [1-9] transmit their own pulse waveforms prior to data transmission, and they utilize these in adjusting the multiplier coefficients of equalizers [1-11]. Adaptive equalizers differe from the automatic equalizers in that they adjust their parameters during data transmission, using either a training sequence or their own data in decision-directed mode until the mean-square error becomes acceptable [1-10, 1-12, 13, 14, 15]. It is assumed that a high percentage of the output decisions are correct so that the sequence of output decisions closely approximate the transmitted sequence. The error signal is then the difference between the signals before and after the decisions are made. The limitation of the decision-directed mode is obvious; if the equalizer is adjusted poorly initially, then the output sequence contains so many errors that the use of an equalizer foils its own purpose. Therefore, it is usually necessary to initialize by transmitting an initial known training or test sequence which can be reproduced at the receiver. Comparison with the equalizer output before the decision device then gives the desired error signal. This known test sequence, transmitted during the training or initialization period, is frequently selected to be a string of unit pulses with sufficient spacing between pulses to prevent the received waveforms from overlapping. After each such isolated test pulse, equalizer parameters are incremented according to an iterative procedure called an algorithm. Supposing that the algorithm is Convergent, the training period is continued till the initialization is achieved, at which time the message transmission commences. Once the algorithm converges, with a particular value of convergence increments, there is always some amount of residual error. It is intended that this error should be made as small as possible. To do this a control parameter called convergence factor (Δ) is selected which should be small. Small convergence factor means lengthy test sequences to achieve equalization. Such sequences are highly undesirable in the case of equalizers incorporated in high speed modems intended for networks of sensors or multiparty polling systems (Airline reservation systems, On-line banking systems, etc.). These are generally real-time information retrieval systems where the inquiry and the response times are short (the message lengths are usually less than 1,000 bits).

With a 9,600 bits/sec. modem this message will require 104 msec. to transmit. Consequently, the actual transmission time can be much less than the initialization of the system (which can be 3 sec. or even higher). In order to allow additional stations to be served or to reduce the response time of the system (response time is important in real-time systems), it is necessary to reduce the initialization time (also called start-up time*) of the high speed modems drastically. This means that the settling time of the self-adjusting equalizers must be reduced considerably. So our aim here is the development of an equalizer with a short initializing period that can be operated in an adaptive manner after initialization and be suitable for noisy time-varying networks that include switching.

1.5 LITERATURE SURVEY:

We survey here the relevant literature from 1949 to date. This survey is being carried out under two generalized headings of equalizers, viz, linear and non-linear equalizers. Linear equalizers embrace the conventional non-recursive digital filter equalizers and some of recursive equalizers. From the survey on the linear equalizers during the mid and late 1960's most of the emphasis on receiver design for dispersive channels centred on transversal systems and on their practical realizations in the form of adaptive equalizers. For channels of interest at that time, generally those exhibiting small distortion, linear systems perform almost as well as is theoretically

*Start-up time is the time during which the receiver locks onto the carrier, establishes bit synchronization, and performs automatic equalization.

possible for any receiver. This could be shown by the use of the one-shot receiver bound, i.e., by considering optimum (matched filter) reception of a single transmitted pulse. Since 1970 attention has been focused on the area of speed of convergence and the search is continuing. Non-linear equalizers include decision feedback and Viterbi algorithm equalizers. The interest in non-linear receiver structures was initially academic, but several factors combined to swing work away from linear systems to non-linear. The limitations of the non-recursive model became apparent in the late 1960's so the search started for better results. This aroused interest in the non-linear model. In practice an interest arose in partial response (correlative) signalling schemes, in which a large amount of ISI is intentionally introduced for spectrum control. Furthermore, a trend toward higher symbol repetition rates also led to more severely distorted received pulses. For these badly distorted pulses, the SNR performance of the linear receiver is often several dB poorer than either the one-shot receiver bound or another generally tighter, receiver bound proposed by Forney [1-16] based on minimum distance between received sequences. Thus, non-linear receivers have a definite performance advantage to offer. In order to obtain this advantage, a variety of techniques have been proposed, but aside from certain noteworthy, but isolated proposals [1-17, 20] the most imporatnt contributions concern a particular structure known as the decision-feedback equalizer (DFE).

However, this also proved insufficient, by the beginning of 1970, to cope with the yet greater demand of high speed data transmission (9,600 bits/sec.). A Search started for statistical equalizers based upon coding theory. It became apparent that the algorithm of Viterbi [1-21] is the only suitable alternative. This search is very much alive at the present moment.

1.5.1 CONVENTIONAL NON-RECURSIVE DIGITAL FILTER EQUALIZERS:

The earliest application of the tapped delay line (TDL) or "transversal filter" to pulse shaping for data transmission was made by Boothroyd and Creamer [1-22]. Tufts [1-23], and George [1-24], have shown that under a mean square error criterion the optimal receiver structure includes a TDL with delay between taps equal to the symbol period. Aaron and Tufts [1-25] have shown that the same receiver structure is needed to minimize the average error probability for binary data transmission. The basic approach to adaptive adjustment of a set of weights where a mse criterion is used with gradient search procedure was noticed by Widrow and Hoff [1-26] who observed that no derivatives computation is needed. Narendra and McBride [1-27] proposed a self-optimizing Wiener filter using a continuous time gradient algorithm and a filter structure whose transfer function is a weighted sum of fixed functions. Koford and Groner [1-28] used a mse criterion and a gradient learning algorithm to find an optimum set of weights for pattern classifying. Widrow [1-29] described a general adaptive filtering with the TDL filter. Coll and George [1-30] discussed the performance of George [1-24] optimum equalizer and indicated a possible adaptive adjustment technique. Lucky and Rudin [1-31, 1-11] were the first to apply the mse criterion with the gradient search procedure to the field of adaptive equalization. Before that, Lucky [1-9] made the first noted attempt to make equalizers, using another criterion called peak-distortion error criterion. By defining the peak-distortion error criterion as a measure of ISI he obtained conditions for the optimal parameter settings using a series of isolated initializing pulses prior to message transmission in conjunction with a steepest-descent gradient algorithm. By computing the approximate gradient with only the polarity of the variables involved he was able to realize the equalizer using only digital logic. Lucky subsequently combined the initialization procedure with a decision-directed on-line adaptive scheme for slowly varying channels, and was again able to realize the equalizer with digital circuits [1-10]. Although these equalizers performed well when initialized, the initialization procedure generally required a large number of initializing pulses and did not converge at all when the peak distortion was greater than unity. Such equalizers were easy to implement and were inherently stable. They were named zero-forcing (Z.F.), since the unit pulse response of the equalized channel was made zero at a number of points adjacent to the main pulse (head pulse), with the number of such zeros dependent on the equalizer length. Since 1972 there has been continuing effort to understand thoroughly the choice of error criterion for equalization (MSE, MINIMAX, SNR, PROB. OF ERROR, etc.). This has been successful to a great extent [1-3, 36]. Guida [1-37] re-examined the minimax criterion for non-independent data and the analysis of the mse was extended [1-38]. In 1970, three papers on practical equalizers [1-12, 13, 1-39] appeared. But, after 1970, various implementations of adaptive equalizers for voiceband data modems have appeared [1-40, 49]. Some have discussed specific implementations of equalizers at higher frequencies for television, cable, and microwave applications [1-50, 55].

New algorithms for equalizer adjustment using quadrature tapped delay lines at passband for phase modulated systems have been discovered [1-56, 58] while other authors have described algorithms for zero-forcing [1-59] and for mean-square equalization [1-60] which are useful in special circumstances. An equalizer for analog data has been constructed [1-61] and the theory of channel equalization to an ideal condition using a mean-square fidelity [1-62] has been extended.

An adaptive equalizer interacts in a critical fashion with the timing and phase recovery circuits of a receiver. The problem of optimum timing instant has been studied [1-63, 64, 1-34], the best choice of reference tap has been derived [1-65], and the effect of phase jitter has been analyzed [1-66]. The use of data aided tracking loops for deriving carrier phase in systems using adaptive equalization became important [1-67, 68].

1.5.2 FASTER CONVERGING EQUALIZERS (NRDFE):

Need for faster converging equalizers arose since a decade ago prototype equalizers took about 10 seconds to converge. Settling time seemed unimportant then, but now a considerable effort is being devoted to speeding equalizer convergence. A number of authors have offered various analyses of equalizer convergence [1-69, 73]. A large number of new algorithms have been put forward claiming faster convergence [1-74, 84]. A new algorithm using Kalman filter techniques for speeding convergence of a tapped delay line equalizers has offered especially dramatic improvement at the expense of increased complexity [1-85, 87]. Finally, some new canonic forms for equalizers have been proposed for the purpose of speeding convergence [1-88, 89]. It seems difficult to compare the already numerous schemes touted for fast convergence. No definition of what constitutes convergence has been accepted. Furthermore, the convergence time is a function of the particular channel selected (from mse point of view, reference [1-87] suggests differently). By choice of definition and example channel, different orderings of system speeds can be obtained.

1.5.3 NEW LINEAR STRUCTURES FOR EQUALIZERS (NRDFE TYPE):

Though the problem of optimum linear equalization receives some small perennial attention [1-90, 94], most linear equalizer effort is structurally constrained or based. Van Gerwen, et. al. [1-94] have introduced a digital filter consisting of a feedforward (NRDF) and a simple recursive network. The coefficients of the NRDF part are equal to integer powers of two or zero; thus complicated multipliers are avoided and instead a simple routing circuit is used. Use of several types of the recursive network makes the filter applicable in different frequency ranges. The coefficients of the NRDF can be interpreted as the differences of successive values of the impulse response of the filter. It has been shown that this difference routing digital filter (DRDF) is especially suited for data transmission.

The use of an adaptive recursive filter for equalization, i.e., giving moveable poles rather than zeros is an interesting alternative which has started receiving attention [1-95]. The Kalman filter approach to equalization, which also uses a recursive structure, has been championed by other authors [1-97, 99]. Miscellaneous linear equalizers included bump equalizer [1-100], and adaptive prefiltering [1-101].

1.5.4 DECISION FEEDBACK EQUALIZERS (DFE):

In the latter part of the 1960's the limitations of the linear equalizer for compensating severely distorted pulses without concomitant noise enhancement became apparent. The class of non-linear equalizers offered a distinct advantage for high speed transmission where pulses were necessarily highly distorted. A number of authors resuscitated the 50 year old idea of tail cancellation embodied in decision feedback equalization and showed substantial advantages to this scheme. Austin [1-102] was the first to consider this equalizer. The fundamental optimization of the decision feedback receiver for minimum mean-square error (mmse) was first accomplished by Monsen [1-103]. Subsequently, Price [1-104] examined the receiver under the premise that the forward and feedback equalizers are constrained to eliminate completely ISI - the so called zero - forcing mode of operation. The existence of such an equalizer is of conceptual interest and provides a clue to the performance advantage of decision feedback over conventional non-recursive equalizers. Price also observed that the mse for DFE is never greater than that for the linear system. Furthermore, the mse for DFE remains finite for algebraic zeros (channel nulls) in the folded spectrum; the linear equalizer (NRDFE) does not exist in this event. Messerschmitt [1-105] has explored the various aspects of duality between NRDFE and DFE.

Salz [1-106] has optimized the DFE and solved the joint optimization problem under a mse criterion. He did not force the equalizer to eliminate ISI instead he allowed a trade off some of this effect against noise. Salz's simple expression for the MSE of an optimized receiver is identical to Price's quoted previously, except for a factor of +1 being added to the argument of the algorithm. Similarly, adding 1 to the term in brackets for the linear system converts it to a mean-square solution.

A number of authors [1-103], 113] have calculated or simulated performance of DFE and compared them with NRDFE under specific channel conditions. In general, these results show the superiority of the DFE - slight for good channels, moderate for channels with severe attenuation distortion, and substantial for channels with spectral nulls in the Nyquist band. The problem of error propagation when incorrect decisions are fed back was studied and shown not to be of crucial importance [1-114]. However, an ingenious circumvention of this difficulty has been suggested by Gerrish & Howson [1-115], Tomlinson [1-116], and Harashima and Miyakawa [1-117]. Their suggestion uses generalized precoding at the transmitter rather than decision feedback at the receiver. The precoder is combined with a modulo-A operation which keeps the transmitted signal in the range $\pm A$; the received signal is subsequently reduced modulo-A. Although this system eliminates error propogation, it does result in correlating the transmitted symbols, and the exact spectral density or transmitted power is not known. An application of decision feedback to a passband system (QAM) has been described [1-118, 119]. Aside from the DFE there have been a few isolated proposals involving other suboptimum non-linear equalizer structures [1-120, 123]. Experiments on the performance of DFE on real telephone lines were reported exclusively in Britain [1-124, 125].

1.5.5 MAXIMUM LIKELIHOOD EQUALIZERS:

These equalizers are well known by the name of Viterbi algorithm (VA). In 1967 Viterbi [1-126] published an algorithm which was shown to provide maximum likelihood decoding of convolution codes. In retrospect it seemed obvious that this algorithm can be applied to the ISI problem for PAM systems. However, the realization did not immediately occur to workers in the ISI area.

However, breakthrough came in 1970 in Noordwijk, Holland where three talks relating to the VA were given by three people - Forney, Kobayashi and Omura. The initial papers on the VA for PAM appeared on 1971 and 1972 by Kobayashi [1-127, 128], Omura [1-129], and Forney [1-130]. Kobayashi [1-128] treated in detail the correlative (Partial Response) signalling format in which the channel impulse transforms have simple integer values such as $f(Z^{-1}) = 1+Z^{-1}$ or $f(Z^{-1})=1-Z^{-2}$. In PAM this "Controlled ISI" is sometimes used for spectral shaping whereas in digital recording system [1-127] it arises naturally. Omura [1-129] viewed the ISI problem as a regulator control problem and he used the control theoretic principle and applied dynamic programming for its solution. Forney [1-130] examined the general PAM system, derived the finite state machine model (either Moore or Mealey finite state machine model) shown in Figure (1.6) using a whitened-matched filter (WMF), and described the application of the VA for maximum-likelihood sequence estimation. He showed that the Viterbi decoding algorithm could be used for optimum, i.e., MLSE, detection of symbols in the presence of noise and ISI. Further analysis of maximum likelihood systems was reported by other authors [1-17, 1-131]. Forney [1-130] has derived rather tight bounds on the probability of an error event in the form

$$K_L \, Q\left(\frac{d_{min}}{2\sigma_n}\right) \leq P_e \leq K_U \, Q\left(\frac{d_{min}}{2\sigma_n}\right)$$

K_L is the lower bound and K_U is the upper bound constant

where Q is the customary integral of the Gaussian density and (d_{min}) is the minimum euclidean distance between any two received paths. Magee and Proakis [1-131] have derived a bound on (d_{min}). It has been shown by Forney [1-130] that the error probability differs in most circumstances only by a small factor from the best that can be achieved for the given channel - thus implying that in many cases the VA uses all the energy in pulse detection and in effectively as good as if the ISI were absent [1-132]. Adaptive Viterbi receivers using NRDFE have been described by Magee and Proakis [1-33]. Qureshi and Newhall [1-134] combined the Viterbi algorithm with an adaptive NRDFE which compensates the channel to a predetermined response.

The VA (a dynamic programming application) is in itself an extremely complex system, whose complexity grows exponentially with the channel memory (duration of impulse response). Much effort in this direction is at the present moment under way and some results have appeared where the necessary search in the VA itself for the correct data sequence has been made limiting. Ungerboeck [1-136] has reported a complete Viterbi receiver, including phasing and timing recovery loops. The fact that the Viterbi detector is a computer algorithm also has future significance. At the present moment, handshaking of computers, packet switching, multiparty polling systems all need data at tremendous speed and therefore, for all those applications the future lies with the VA [1-137].

1.6 PROBLEM STATEMENT AND THESIS STRUCTURE:

The problem here is that of transmitting data at a rate considerably higher than those being used at present over channels which are time-dispersive and have severe phase and amplitude distortions. Such high transmission rates are particularly suited to computer networks and multiparty polling systems where the quick set-up and response of communication link has become of paramount importance.

Therefore, this thesis contains the possible answers to the problem outlined above. We have developed a theory for a non-recursive digital filter equalizer which is intended to reduce the settling time drastically. The approach is completely new, as far as we are aware, since this equalizer converges and a solution is obtained even if the channel correlation matrix becomes singular. Under the circumstance a method is proposed for making the selection using the MOORE PENROSE PSEUDOINVERSE (MPPI) of the channel correlation matrix, which gives a best solution to the problem. A second proposed method use a new orthogonal matrix called Hadamard matrix. It has been shown that a Hadamard matrix is similar to a projection matrix. Thus, all eigenvalues are either 1 or 0, which is the desired condition for rapid initialization in one algorithm iteration.

Another equalizer structure proposed is a recursive digital filter equalizer (RDFE) which has been found advantageous over severely amplitude distorted channels. We have explored the possibilities of using six different forms of adaptive RDFE and studied their properties by using optimization and Monte-Carlo simulation techniques.

We have also tried to explore the possibility of using this recursive digital filter equalizer in conjunction with a maximum likelihood sequence estimator (MLSE).

Finally, to facilitate continuity and ready reference in the exposition, certain basic transform, theories, and statistical methods used in communication system are outlined in appendices that are not reviewed here.

This thesis has been divided into seven chapters:

CHAPTER 1:

We discuss some introductory material on digital equalizers and provide an exhaustive literature survey. Also, since the aim is entirely digital processing of signals, we have therefore developed a mathematical model of the channel. This channel model remains the basis for all the chapters and whereever possible is referred to as the DISCRETE TIME CHANNEL (D.T. CHANNEL) or simply the channel.

CHAPTER 2:

We recapitulate the existing design procedures (and their related algorithms) of the NRDFE. Detailed analysis of an iterative procedure has been provided which leads on to the convergence properties of various algorithms.

CHAPTER 3:

We develop the initialization theory for an automatic NRDFE and design an NRDFE under the assumption that the channel is deterministic (i.e., no background noise). The use of iterative techniques and adaptive desired impulse response NRDFE. Computer simulation has been done for channels of length 3, 5, 7.

CHAPTER 7:

This chapter contains the conclusion and the possible problems for future investigations.

Every chapter (except 1) contains its own summary and comments which we have found important and beneficial to grasp the immediate outcome. At the end we provide an exhaustive bibliography.

Finally, an appendix has been given which contains many known theories used in the course of this thesis.

1.7 BASIC ASSUMPTIONS:

Although every chapter contains its own data communication model and assumptions, certain basic assumptions are enumerated here:

(I) Equalization process is located at the receiver.

(II) The Discrete Time Channel of the NRDF type is assumed (Sec. 1.9).

(III) The D.T. channel is either time invariant or else varies slowly with time.

(IV) The equalizer is held correctly matched to the channel.

(V) The signal input to the transmitter is a sequences of binary impulses regularly spaced at intervals of T-secs. They are statistically independent and equally likely to have either binary value.

(VI) The delay in transmission, other than that involved in the time-dispersion of the transmitted signal is neglected for Chapter 5.

(VII) White Gaussian Noise (WGN) with zero mean and a two-sided spectral density of N_0 is added to the data signal at the output of transmission path (see Sec. 1.9).

(VIII) Signal is digitized before entering the equalizer.

(IX) The transfer function of the transmitter filter is such that the energy of an individual transmitted signal element is unity, and the transfer function of the receiver filter is such that the noise samples at its output are statistically independent Gaussian random variables with zero mean and variance σ_N^2.

(X) Sampling instants are correctly synchronized to the received signal.

(XI) No switching action occurs during the initialization (training) period.

1.8 COMMENTS ON NOTATION USED:

The theory of digital filters and equalizers has developed from a number of mathematical and engineering disciplines. Some of the disciplines are:

1. Networks
2. Communications (including Information and Coding theory)

3. Control Systems (including Estimation, Identification)
4. Statistics (including the Probability and Time Series analysis)
5. Optimization, and
6. Mathematical Programming, etc.

As a consequence, the notations and terminology used here vary considerably. On many occasions we have used the same word over and over again and also often different words having the same meanings (e.g., training or initialization, on-line adaptation or tracking).

All vectors are taken to be column vectors (if not exclusively described), and vector and matrix variables are indicated by an underline. Several types of sequence notation are also used. Sequences have also been represented by the corresponding Z-transforms.

1.9 DERIVATION OF A DISCRETE TIME EQUIVALENT CHANNEL MODEL:

In order to deal with the problems encountered during digital processing of signals in the presence of ISI (and additive noise) on a quantitative basis we find it necessary to develop a mathematical model for the data communication systems.

A data communication system, Figure (1.3), consists of three basic blocks:

(a) The Transmitter,

(b) The Channel,

and, (c) The Receiver.

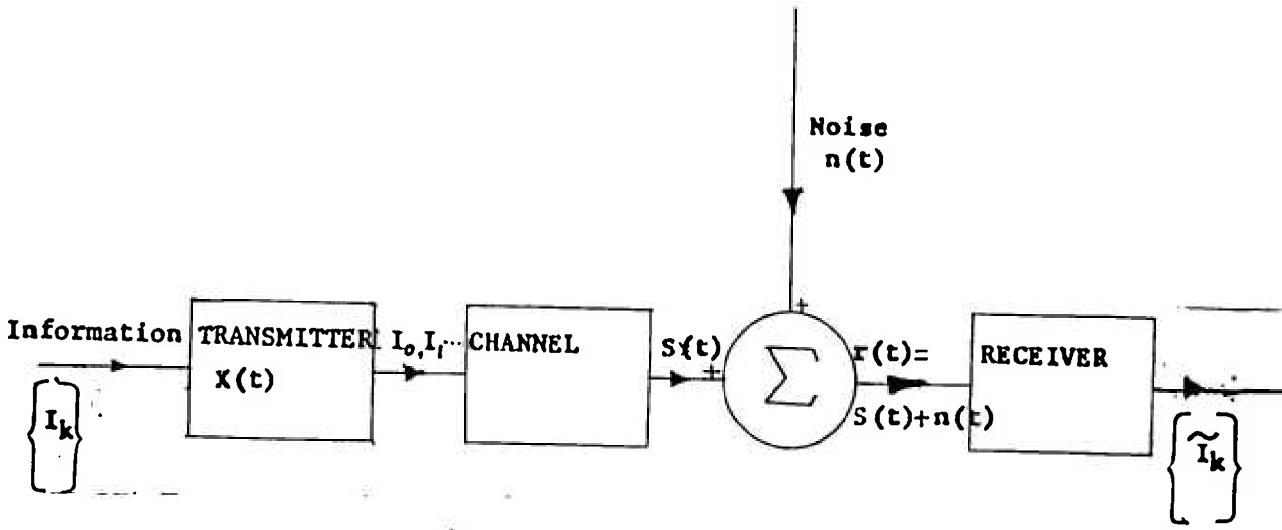


FIG. (1.3) MODEL OF DATA COMMUNICATION SYSTEM

Information is transmitted by some form of carrier modulation through a bandpass channel. It is convenient, however, to represent such a channel by an equivalent lowpass or baseband (i.e., transmits energy to zero frequency [1-1]) channel. Moreover, a bandpass signal which is to be transmitted over a bandpass channel has a lowpass equivalent channel. Thus, all signals and filter response functions can be treated in complex-valued lowpass equivalent form although they are real valued bandpass signals and filters.

In figure (1.3), the data sequence $\{I_K\}$ modulates a basic transmitting filter pulse x(t) at a rate (1/T).

The total transmitted signal is

$$= \sum_{k=0}^{\infty} I_k \, x(t-kT) \tag{1.1}$$

where T is the duration of the signalling interval, and, I_k is the information symbol transmitted in the interval $kT < t < (k+1)T, k=0, 1,2........$

x(t) in (1.1) may be either partial response pulses or pulses having a raised cosine spectrum. When I_k take one of M real values equally spaced about zero, then the signal in (1.1) is a pulse amplitude modulated (PAM) signal. If x(t) is complex-valued, then (1.1) represents a PAM Vestigial Sideband (VSB) signal or PAM single-sideband (SSB) signal [1-1]. However, phase (PM) and QAM (PAM+PM) modulations also fit the mathematical formulation given by (1.1).

The channel is characterized, in general, by a time-variant impulse response. For our purposes, we shall assume that this time variation is much slower than the duration of the signalling interval. In other words, the channel can be considered as being relatively time-invariant over a large number of signalling intervals. This assumption is necessary and realistic to design adaptive equalizers. Therefore, the channel output,

$$s(t) = \sum_k I_k \, h(t-kT) \tag{1.2}$$

$$\text{where } h(t) = \int_{-\infty}^{\infty} g(\tau) \, x(t-\tau) d\tau \tag{1.3}$$

and, g(t) is the channel impulse response

When the noise n(t) is added to the system then the input to the receiver is represented as:

$$r(t) = \sum_k I_k \, h(t-kT)+n(t) \tag{1.4}$$

The noise process, assumed to be stationary, zero mean, white and Gaussian, is called random impairments. The response, h(t) is assumed to be square integrable, and from a practical point of view it is finite in duration, spanning a length of L symbol intervals, i.e., L is the smallest integer such that

$$h(t) = 0 \text{ for } t \geq LT, \text{ and,}$$

$$||h||^2 \triangleq \int_{-\infty}^{\infty} h^2(t)\, dt \quad < \infty$$

The problem at the receiver, then, is to detect the information sequence $\{I_k\}$ from the available observation r(t). From a computation of the likelihood function [Figure (1.4)] it is easily shown that the output of a filter matched to h(t) and sampled periodically every T seconds, once every signalling interval, constitutes a set of sufficient statistics for the detection of the sequence $\{I_k\}$.

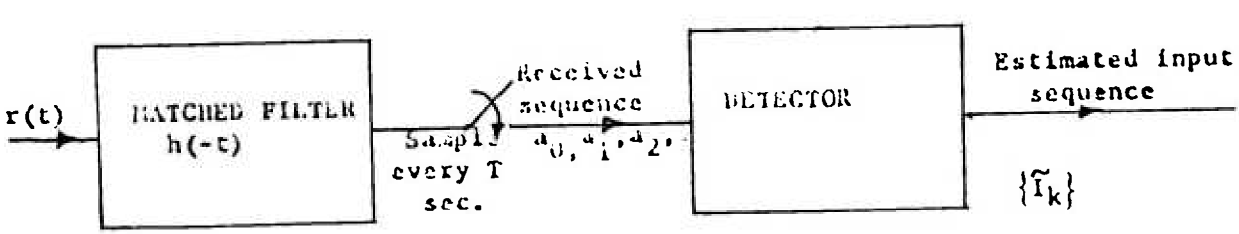


FIG. (1.4) LIKELIHOOD SEQUENCE ESTIMATION

With $h(-t)$ as the filter matched to $h(t)$ and $r(t)$ as its input, the output of the matched filter (MF) is:

$$a(t) = \int_{-\infty}^{\infty} r(t)\ h(\tau - t) d\tau \tag{1.5}$$

The corresponding sampled values

$$a_m \stackrel{\Delta}{=} a(mT) \text{ are}$$

$$a_m = \sum_k I_k R_{m-k} + n_m \tag{1.6}$$

where R_k denotes the sampled autocorrelation function of $h(t)$ i.e.,

$$R_k = \begin{cases} \int_{-\infty}^{\infty} h(t)h(t+kT)dt & 0 \leq k \leq L \\ 0 & \text{otherwise} \end{cases} \tag{1.7}$$

and $\{n_m\}$ denotes the additive noise sequence . at the output of the matched filter

i.e., $$n_m = \int_{-\infty}^{\infty} n(\tau)\ h\ (\tau - mT) d\tau \tag{1.8}$$

Manipulating (1.5), we have

$$a_m = I_m R_0 + \sum_{k \neq m} I_k R_{m-k} + n_m \tag{1.9}$$

where $I_m R_0$ represents the desired signal at the m^{th} sampling instant.

$\sum_{k \neq m} I_k R_{m-k}$ represents the ISI for the L adjacent symbols and n_m additive noise component.

Up to this point, we have seen that the transmitter sends discrete time symbols/second and the sampled output of the MF is also a discrete time signal with samples occurring at a rate (1/T) per T

second, therefore, it will be appropriate to represent the cascade of filters (analogue) x(t), g(t), h(-t) and the sampler by an equivalent discrete time non-recursive filter figure (1.5) spanning a time interval of 2LT seconds and having multiplier coefficients the sampled values R_k, $0 \le k \le L$, of the autocorrelation function of h(t).

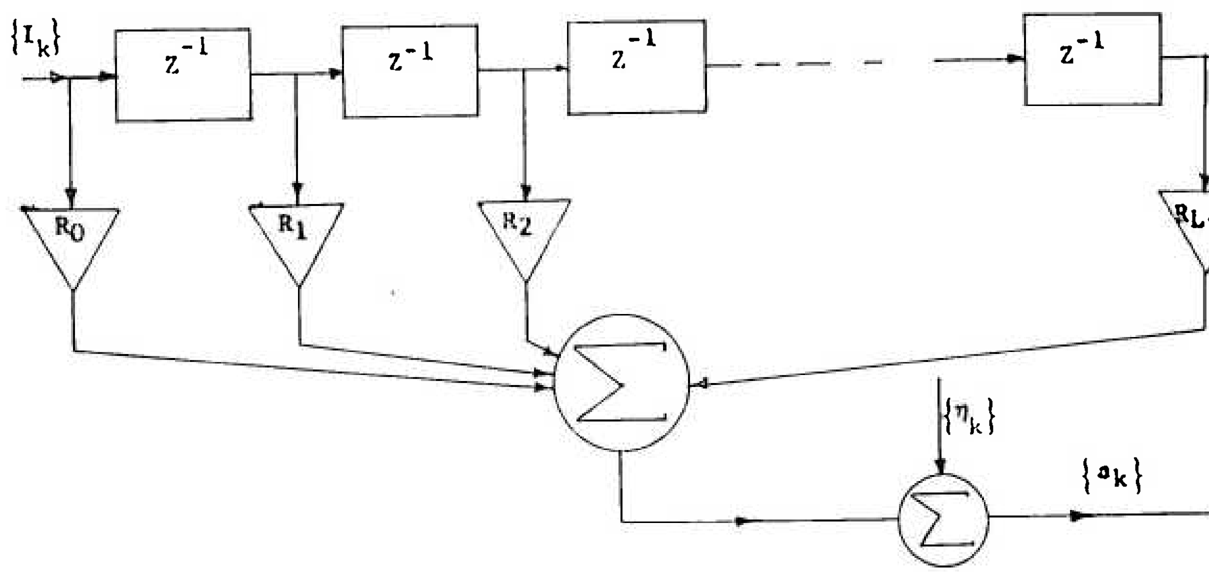


FIG. (1.5) DISCRETE TIME MODEL OF DATA COMMUNICATION SYSTEM

Input to this model [Figure (1.5)] is the sequence of information symbols $\left\{I_k\right\}$. To complete the equivalence between the continuous time and discrete time model it is necessary to add a noise component η_k to the k^{th} output of the NRDF. However, this particular approach is unsuitable to the different filtering techniques to be described in the subsequent part of the thesis since the noise $\left\{\eta_k\right\}$ now becomes coloured but Gaussian with autocorrelation function

$$E[\eta_i \eta_k] = \begin{cases} N_0 R_{i-k}\,, & |i-k| \le L \\ 0 & \text{otherwise} \end{cases} \qquad (1.10)$$

where $(N_0/2)$ is the two-sided spectral density of the noise. So, our immediate concern is to whiten this noise sequence by further filtering the sequence $\{a_m\}$. We shall, therefore, describe Forney's [1-5] discrete time, noise whitening filter.

Let R(Z) denotes the two-sided Z-transform of the sampled autocorrelation function

$$\text{i.e.,} \qquad R(Z) = \sum_{k=-(L-1)}^{L-1} R_k Z^{-k} \tag{1.11}$$

since $R_k = R_{-k}$, it follows that $R(Z)=R(Z^{-1})$ and the L roots of R(Z) have the symmetry that if r is a root, 1/r is also a root. Hence, R(Z) is factored as

$$R(Z)=F(Z)F(Z^{-1}) \tag{1.12}$$

Where F(Z) is a polynomial of degree L having the roots $r_0, r_1 \ldots\ldots\ldots\ldots r_{L-1}$ and $F(Z^{-1})$ is a polynomial of degree L having the roots

$$\frac{1}{r_0}, \frac{1}{r_1}, \ldots \ldots \frac{1}{r_{L-1}}$$

Then an appropriate noise-whitening filter has a Z-transform $[1/F(Z^{-1})]$. As there are 2^L possible choices for the roots of $F(Z^{-1})$, each choice resulting in a filter characteristic that is identical in magnitude but different in phase from other choices of the roots, we propose to choose the unique $F(Z^{-1})$ having minimum phase, i.e., the polynomial having all its roots inside the unit circle. Then $[\frac{1}{F(Z^{-1})}]$ is a discrete time recursive filter which is stable and causal. Forney [1-5], has also considered the cases when $F(Z^{-1})$ has some roots on the unit circle. Consequently, passage of the sequence $\{a_m\}$ through the digital filter $[1/F(Z^{-1})]$ results in an output sequence $\{y_m\}$ which can be expressed as:

$$y_m = \sum_{k=0}^{L-1} \rho_k I_{m-k} + n_m \tag{1.13}$$

where $\{n_m\}$ is a White Gaussian Noise sequence and $\{\rho_k\}$ is a set of multiplier coefficients of an equivalent discrete time NRDF having a transfer function F(Z).

In summary, the cascade of the transmitting filter x(t), the channel g(t), the matched filter h(-t), the sampler, and the discrete time noise whitening filter $[1/F(Z^{-1})]$ can be represented as a FINITE MEMORY EQUIVALENT DISCRETE TIME NRDF having the set $\{\rho_k\}$ as its multiplier coefficients. The additive noise sequence $\{n_k\}$ corrupting the output of this NRDF is a WHITE GAUSSIAN NOISE (WGN) se ıqence having zero mean and variance N_0.

Figure (1.6) illustrates the model. This model has been derived not only from a mathematical point of view but it appears to be reasonably consistent with experience on real channels (Telephone circuits, H.F. Radio links, 600 ohm pair, coaxial cable links, and Fibre optic links.) We shall illustrate the above fact using the following example:

EXAMPLE: Suppose that the transmitter signal pulse x(t) has duration T and unit energy and the received pulse is h(t)=x(t) + a x(t-T). Find the value of multiplier coefficients.

The sampled autocorrelation function is given as

$$R_k = \begin{cases} a & ,k=1 \\ 1+|a|^2 & ,k=0 \\ a & ,k=-1 \end{cases} \tag{1.14}$$

The Z-transform of R_k is

$$\begin{aligned} R(Z) &= \sum_{k=-1}^{1} R_k Z^{-k} \\ &= R_{-1}Z^{1} + R_0 + R_1 Z^{-1} \\ &= aZ+(1+ a^2)+aZ^{-1} \\ &= (a+Z^{-1})\ (a+Z) \end{aligned} \tag{1.15}$$

Under this assumption that $|a|<1$, one chooses

$F(Z)=(a+Z^{-1})$ so that the equivalent NRDF consists of two multipliers $\rho_0=a$, $\rho_1=1$.

When the channel impulse response is varying slowly with time, the MF becomes a TIME VARIABLE FILTER and the time variable filter pair give rise to a discrete time filter with TIME VARIABLE COEFFICIENTS. As the channel impulse response is not known at the receiver, therefore, the realization of time variable MF implies some kind of an ADAPTIVE SYNTHESIS TECHNIQUE. This can be accomplished by approximating the MF arbitrarily close by an NRDF with multiplier coefficients that are adjustable continuously by a cross correlation technique based on a known pseudo-random or customer's data which can be embedded in the information bearing signal and transmitted through the channel. There are, however, several instances of practical communication system when we abandon the idea of a MF in favor of some kind of a simpler fixed filter that may either be matched to the transmitted (known) signal pulse x(t) or more simply one that has a frequency response characteristic appropriate for passing the received signal relatively undistorted and simultaneously limiting the noise at its input. In such cases, sampling the output of the fixed time filter at the symbol rate introduces some loss in detectability which is usually small provided the sampling time within the sampling interval T is chosen judiciously. Time variations in the channel impulse response still give rise to time variable ISI effects which can be modelled now as in figure (1.6) by a discrete time non-recursive filter with time-variant multiplier coefficients.

This model [Figure (1.6)] will be assumed everywhere unless otherwise stated.

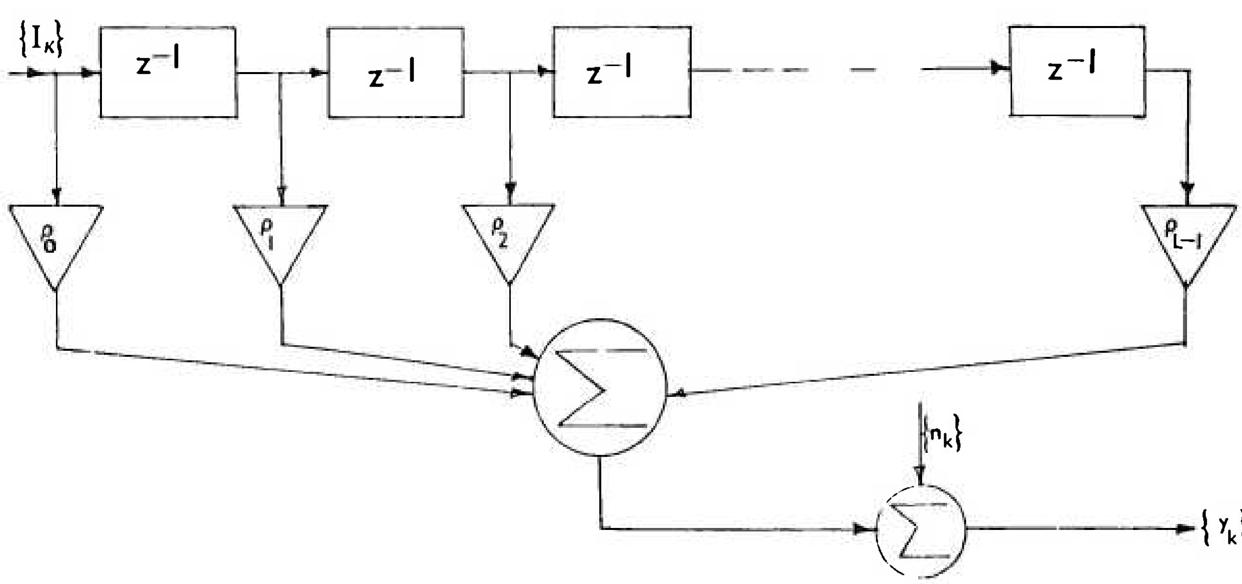


FIG. (1.6) FINITE MEMORY EQUIVALENT DISCRETE TIME CHANNEL MODEL WITH WHITE GAUSSIAN NOISE

CHAPTER 2

DESIGN OF ADAPTIVE NON-RECURSIVE DIGITAL FILTER EQUALIZERS

A man must see, do and think things for himself, in the face of those who are sure that they have already been over all that ground. In science, there is no substitute for independence.

J. BRONOWSKI, "SCIENCE AND HUMAN VALUES"

2.1 INTRODUCTION:

This chapter is a recapitulation of the existing design techniques of adaptive non-recursive digital filter equalizer (NRDFE). We have used the word adaptive (i.e., on-line operation) since almost no a priori information about the channel is assumed. Also at the present moment about 95% of the high speed modems employ either NRDFE or its analogue version called a transversal filter equalizer. The literature survey of ch. 1 also proves the fact that the same amount of work has been reported on NRDFE which adapt (track) to a quasistationary unknown channels [2-3, 16]. The difference between the various approaches is in the performance index used as a goal function for the adapting procedure and in the adaptive algorithm used to optimize the goal function [2-1, 2].

While using conventional NRDFE, there has been a tremendous effort to understand thoroughly the choice of performance (error) criterion for equalization (e.g., Peak-distortion, MSE, Prob. of error, Signal-to-Noise Ratio, . [illegible] etc.) [2-3, 4, 2-18] and the consequences of this choice. Since the most meaningful measure of performance for a data communication system is the probability of error P_e (or Pr(e)), it is desirable to choose the coefficients $\{\alpha_k\}$ to minimize this performance index [2-19]. However, P_e is a highly non-linear function of $\{\alpha_k\}$. In addition, it is difficult to compute such a function (except in a few isolated cases) let alone minimize it. As a consequence, P_e as a performance index for optimizing the coefficients $\{\alpha_k\}$ has been impractical.

Professor R. E. Guilleman, in 1953, posed an intriguing question: "What is Nature's error criterion"? [2-1]. His answer is that nature adopts as a criterion a minimum-mean-square-error [2-2]. While the truth of this question and answer is debatable it is never-the less true that the MSE criterion is the most widely used in mathematics and engineering. It is mathematically easily tractable and leads on to simple minimization algorithms. We shall be using, particularly, MSE criterion for our analysis and occasionally will make deviations to other error criteria, in the course of this thesis in order to establish more meaningful facts.

In brief resume, we shall define the first three performance indices in the next section.

2.2 DESIGN OF AN NRDFE USING PEAK-DISTORTION ERROR CRITERION:

This criterion is also known as Lucky's criteria [2-3, 4]. According to this criterion, the peak-distortion $D(\underline{\alpha})$ is defined

$$D(\underline{\alpha}) = \left|\frac{1}{q_i}\right| \sum_{k\neq i} |q_k| \tag{2.1}$$

Where $\{q_n\}$ represents the impulse response of the cascade of the equalizer and the equivalent discrete-time channel and q_i is the head pulse. The peak eye closure for an m-level system is $(m-1)D(\underline{\alpha})$ and the eye opening is $[1-(m-1)D(\underline{\alpha})]$. The input pulse to the equalizer is assumed to have initial peak distortion $D_0(\underline{\alpha})$. The reference sample q_0 will also be normalized to unity. The problem, here, is to determine the values of multipliers $\{\alpha_k\}_0^{K-1}$ for the K shift-register taps which minimizes the final peak-distortion, e.g., (2.1) where

$$q_n = \sum_{j=0}^{K-1} \alpha_j Y_{n-j} \tag{2.2}$$

There is an arbitrary gain factor involved in the equalizer setting; therefore the central sample (or head pulse) q_i must be constrained to be unity. All the other q_k, $k \neq i$ give rise to residual ISI at the output of the equalizer.

The multiplier coefficients $\{\alpha_k\}$ are chosen to minimize $D(\underline{\alpha})$. Since there are (K) adjustable parameters and K+L-1 outputs $\{q_k\}$, it is generally impossible to eliminate completely the ISI at the output of the equalizer. There is always some residual ISI when the optimum coefficients are used.

It can be easily shown that the functions $D(\underline{\alpha})$ is a convex functions of the coefficients $\{\alpha_k\}$. In general, we can carry out its minimization numerically, using for example, a steepest descent technique. For one special but important case, however, the minimum is easily obtained. This is the case in which the distortion prior to equalization is small (i.e., $D_0(\underline{\alpha})<1.00$).

The equalizers using this criterion are called Zero-Forcing (Z.F.) equalizers.

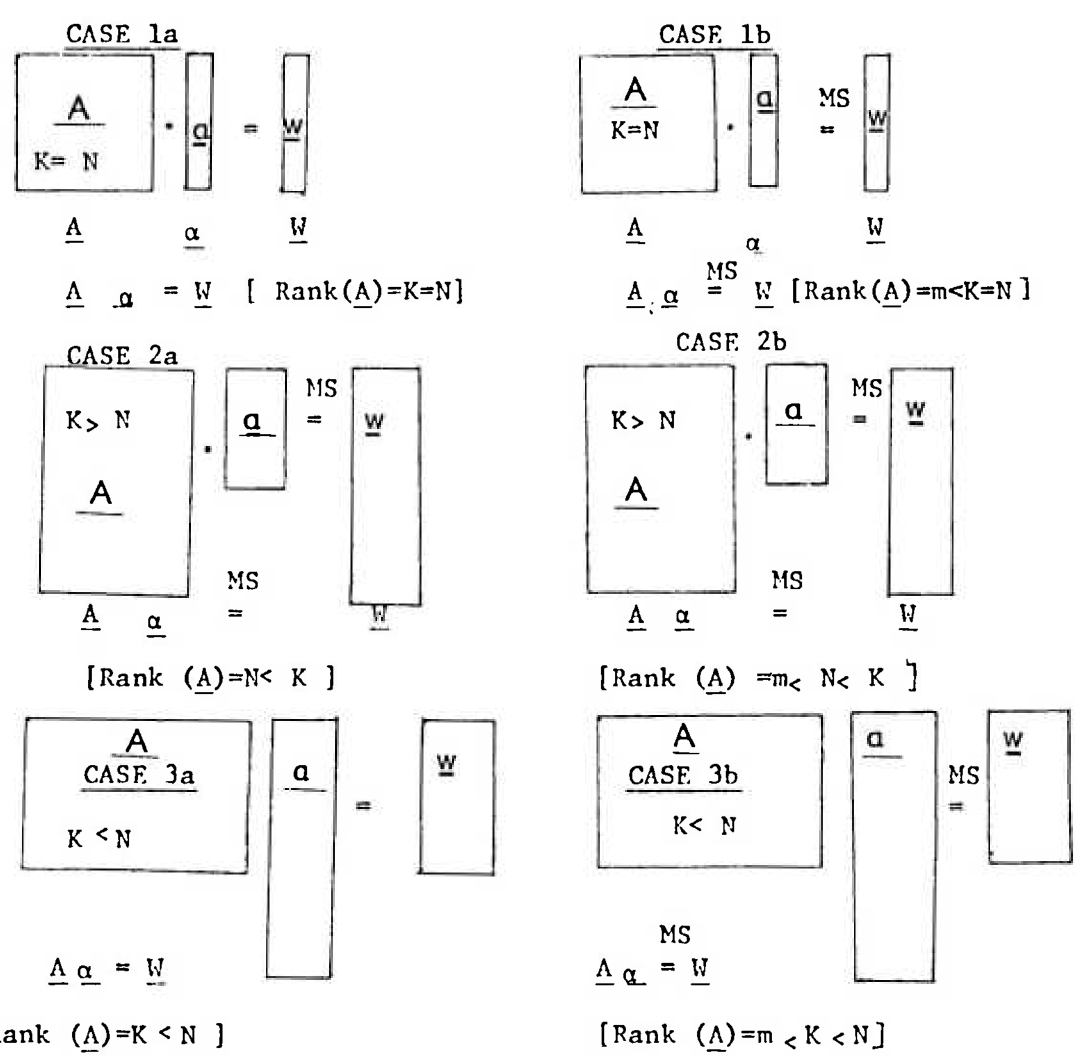
The major advantages of the peak distortion criterion are that the function $D(\underline{\alpha})$ possesses no relative minima but just a global minimum, and the zero-forcing algorithm is easily implemented. On the other hand, there are two disadvantages with the performance index. The first is that this algorithm does not cater for the additive noise, in other words, the additive noise is not included in the performance index. The second disadvantage is when $D_0>1$, the Z. F. algorithm does not necessarily yield the minimum of $D(\underline{\alpha})$ and sometimes may not even converge to any solution. However, on many telephone channels, $D_0 < 1$. therefore the Z. F. algorithm is presently being used on some commercially available adaptive NRDFE.

2.3 DESIGN OF NRDFE BASED ON MEAN SQUARE ERROR CRITERION:

The central problem of mean square error criterion can be stated as follows:

> " Given a real K x N matrix $\underline{A}$ of rank $m \leq \min(K,N)$, and given a real M-vector $\underline{W}$, find a real N-vector $\underline{\alpha}_{opt}$ minimizing the Euclidean length of $\underline{A}\,\underline{\alpha}_{opt} - \underline{W}$".

If we donot specify the size of K & N, then the MS problems can be shown as below (symbol $\underline{\underline{M}}S$ means mean square problem):



We concern ourselves in this thesis, depending upon the situations, with all the cases depicted here. However, we confine ourselves with case(1a) to illustrate the basic mathematics involved in the mean square analysis of a problem.

A performance index that possesses the desired characteristics of being a convex function of the multiplier coefficients $\{\alpha_k\}$, but does not suffer from the limitations inherent in the peak distortion criterion is the mean-square error (MSE) or Widrow criterion [2-20] and will be denoted by $J(\underline{\alpha})$. We shall apply this criterion to design an adaptive NRDFE and study its various properties. An adaptive NRDFE in the decision directed mode is shown in Fig. (2.1). The input signal vector $\underline{Y}_k$ is defined as:

$$\underline{Y}_k \overset{\Delta}{=} \begin{bmatrix} y_{ok} \\ y_{1k} \\ \cdot \\ \cdot \\ y_{(K-1)k} \end{bmatrix} \overset{\Delta}{=} [\, y_{0k} \;\; y_{1k} \;\; \cdots \;\; y_{(K-1)k} \,]^T \qquad (2.3)$$

The input signal components are assumed to appear on all input lines at discrete time indexed by the subscript k. The component y_j is a constant normally set to ± 1. The multiplying coefficients are given by:

$$\underline{\alpha} \overset{\Delta}{=} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \cdot \\ \cdot \\ \alpha_{K-1} \end{bmatrix} \qquad (2.4)$$

Therefore the output $\hat{I}_k$ is equal to the inner product of $\underline{Y}_k'$ and $\underline{\alpha}$

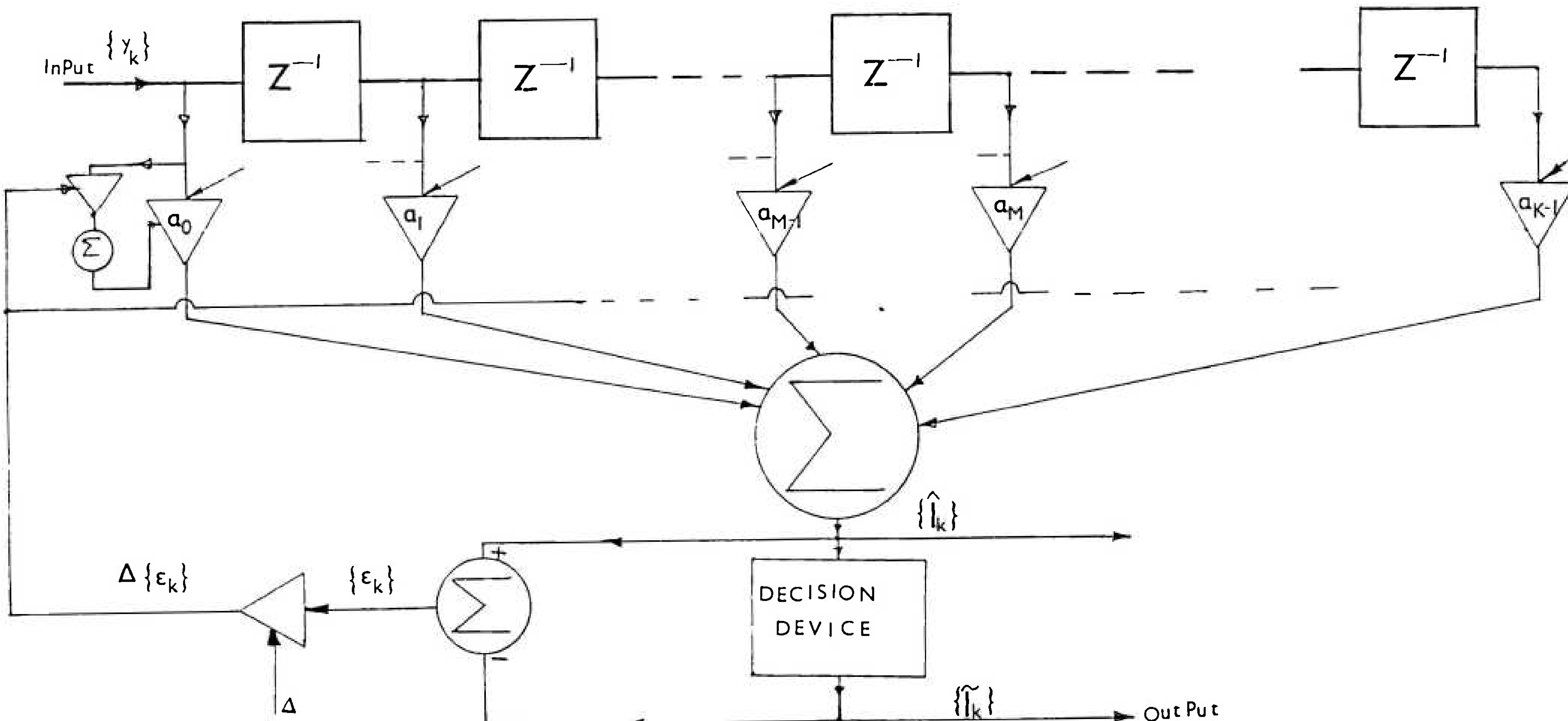


FIG. (2.1) AN ADAPTIVE NON RECURSIVE DIGITAL FILTER EQUALIZER [DECISION DIRECTED MODE]

$$\hat{I}_k = \underline{Y}_k^T \underline{\alpha} = \underline{\alpha}^T \underline{Y}_k \tag{2.5}$$

The error ε_k is defined as the difference between the desired response $\{I_k\}$ and the actual response $\{\hat{I}_k\}$:

$$\begin{aligned}\varepsilon_k &\triangleq I_k - \hat{I}_k \\ &= I_k - \underline{Y}_k^T \underline{\alpha} \\ &= I_k - \underline{\alpha}^T \underline{Y}_k \end{aligned} \tag{2.6}$$

In most applications some ingenuity is required to obtain a suitable input for I_k (therefore we use decision directed mode output $\tilde{I}_k$). After all, if we had already known the actual data at the output, then why bother to have an adaptive processor ?

A general expression for MSE as a function of the multiplier value, assuming that the input data and the desired response are statistically stationary - and that the ε_k are fixed, can be derived in the following manner. Expanding (2.6) and defining the MSE by

$$\begin{aligned} J(\underline{\alpha}) &\triangleq E[\varepsilon_k^2] \\ &= E\left[I_k - \underline{\alpha}^T \underline{Y}_k\right]^2 \\ &= E\left[I_k\right]^2 - 2E\left[I_k \underline{Y}_k^T \right] \underline{\alpha} + \underline{\alpha}^T E\left[\underline{Y}_k \underline{Y}_k^T \right]\underline{\alpha} \end{aligned} \tag{2.7}$$

We shall now define $\underline{W}$ the cross-correlation between the desired response (a scalar) and the $\underline{Y}$ vector; this yields

$$\underline{W} \triangleq E\left[I_k \underline{Y}_k\right] \triangleq E \begin{bmatrix} I_k y_{0k} \\ I_k y_{1k} \\ I_k y_{2k} \\ \cdot \\ I_k y_{(K-1)k} \end{bmatrix} \tag{2.8}$$

The input correlation matrix $\underline{A}$ is defined as:

$$\underline{A} \triangleq E\left[\underline{Y}_k\ \underline{Y}_k^T\right] = E\begin{bmatrix} y_{0k}y_{0k} & y_{0k}y_{1k} & y_{0k}y_{2k} & \cdots & y_{0k}y_{(K-1)k} \\ y_{1k}y_{0k} & y_{1k}y_{1k} & y_{1k}y_{2k} & \cdots & y_{1k}y_{(K-1)k} \\ y_{2k}y_{0k} & y_{2k}y_{1k} & y_{2k}y_{2k} & \cdots & y_{2k}y_{(K-1)k} \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\ y_{(K-1)k}y_{0k} & y_{(K-1)k}y_{1k} & y_{(K-1)k}y_{2k} & \cdots & y_{(K-1)k}y_{(K-1)k} \end{bmatrix}$$

This matrix $\underline{A}$ is real, symmetric, positive definite (or in rare cases positive semi-definite, this is our case in chapter (3))

Therefore we can express the MSE as

$$J(\underline{\alpha}) \triangleq E\left[\varepsilon^2\right]$$

$$= E\left[\ I_k^2\ \right] - 2\ \underline{\alpha}^T\ \underline{W} + \underline{\alpha}^T\ \underline{A}\ \underline{\alpha} \tag{2.10}$$

We shall note here that the error is a quadratic function of the multipliers coefficients, that can be pictured as a concave hyperparaboloidal surface, a function that never goes negative. Adjusting the multiplier coefficients to minimize the error involves discending along this surface with the objective of getting to the bottom of the bowl [2-20]

The gradient $\frac{\partial J(\underline{\alpha})}{\partial \underline{\alpha}}$ (or g) of the error function is obtained by differentiating (2.10) i.e.

$$\underline{g} \triangleq \frac{\partial J(\underline{\alpha})}{\partial \underline{\alpha}} = \begin{bmatrix} \frac{\partial J(\underline{\alpha})}{\partial \alpha_0} \\ \frac{\partial J(\underline{\alpha})}{\partial \alpha_1} \\ \vdots \\ \cdot\cdot \\ \frac{\partial J(\underline{\alpha})}{\partial \alpha_{K-1}} \end{bmatrix} = -2\ \underline{W} + 2\ \underline{A}\ \underline{\alpha} \tag{2.11}$$

The optimal value of $\underline{a}$ (i.e. $\underline{a}_{opt}$) usually called the Wiener multiplier vector, is obtained by setting the gradient of the MSE function to zero, yielding

$$\underline{a}_{opt} = \underline{A}^{-1} \underline{W} \tag{2.12}$$

This equation is a matrix form of the WIENER-HOPF EQUATION.

One way of finding the optimum set of multiplier vectors is to solve (2.12). This solution is generally straight forward, but present serious computational problems when the number of multipliers K is large and when data rates are high. In addition to the necessity of inverting a (KxK) matrix, this may require as many as (K+1)(K+2)/2 autocorrelation and cross-correlation measurements to obtain the elements of $\underline{A}$. Furthermore, this process generally needs to be continually repeated in most practical situations where the input data changes slowly. No perfect solution of (2.12) is possible in practice because of the fact that an infinite statistical sample would be required to estimate perfectly the elements of the correlation matrices.

So, we shall apply an approximation method to get a solution of (2.12). The accuracy of this method is limited by the statistical sample size, since it determines multiplier values based on finite-time measurements of input data signals. These methods do not require explicit measurements of correlation functions or matrix inversion. They are based on gradient-search techniques applied to MSE functions. Minimization is usually accomplished by gradient search techniques. One such method is the LMS algorithm[2-20] which is an implementation of the method of STEEPEST DESCENT. According to this method, the next multiplier vector at any iteration is equal to the present multiplier vector plus a change proportional to the negative gradient. Accordingly,

$$\underline{\alpha}^{(k+1)} = \underline{\alpha}^{(k)} - \tfrac{1}{2}\,\Delta^{(k)}\, g^{(k)}_{(\underline{\alpha}_k)} \qquad \ldots \qquad (2.13)$$

The parameter Δ is the factor that controls stability and the rate of convergence. Each iteration occupies a unit time period. The true gradient at the k^{th} iteration is represented by $g^{(k)}_{(\underline{\alpha}_k)}$ or simply g_k

The algorithm mentioned here estimates an instantaneous gradient in a crude but efficient manner by assuming that ε_j^2 , the square of a single error sample, is an estimate of the MSE and is given by differetiating ε_j^2 with respect to $\underline{\alpha}$. The relationship between true and estimated gradients are given by the following expressions:

$$g_k \triangleq \begin{bmatrix} \frac{\partial J(\underline{\alpha})}{\partial \alpha_0} \\ \frac{\partial J(\underline{\alpha})}{\partial \alpha_1} \\ \vdots \\ \frac{\partial J(\underline{\alpha})}{\partial \alpha_{K-1}} \end{bmatrix}_{\underline{\alpha} = \underline{\alpha}_k}$$

$$\tilde{g}_k = \begin{bmatrix} \partial \varepsilon_k^2 / \partial \alpha_0 \\ \partial \varepsilon_k^2 / \partial \alpha_1 \\ \vdots \\ \partial \varepsilon_k^2 / \partial \alpha_{K-1} \end{bmatrix}_{\underline{\alpha} = \underline{\alpha}_k} = 2\,\varepsilon_k \begin{bmatrix} \partial \varepsilon_k / \partial \alpha_0 \\ \partial \varepsilon_k / \partial \alpha_1 \\ \vdots \\ \partial \varepsilon_k / \partial \alpha_{K-1} \end{bmatrix}_{\underline{\alpha} = \underline{\alpha}_k} \qquad (2.14)$$

The estimated gradient components are related to the partial derivatives of the instantaneous error with respect to the multiplier coefficient, which can be obtained by differentiating ε_k^2 . Thus the expression for the gradient estimates can be simplified to

$$\tilde{\underline{g}} = - 2 \, \varepsilon_k \, \underline{Y}_k \tag{2.15}$$

using this estimate in place of the true gradient in (2.13) yields the Widrow-Hoff LMS algorithm (henceforth we shall refer to it by the name Widrow estimate [2-20] given by:

$$\underline{\alpha}^{(k+1)} = \underline{\alpha}^{(k)} + \Delta \, \varepsilon^{(k)} \, \underline{Y}^{(k)} \tag{2.16}$$

this algorithm is simple and generally easy to implement. Although this makes use of gradients of MSE functions, it does not require squaring, averaging, or differentiation.

The gradient estimate used here is unbiased in that the expected value of the multiplier coefficient vector values converge to the Wiener multiplier coefficient vector (2.12) $\underline{\alpha}_{opt}$, when the input vectors are uncorrelated over time (although they could, of course, be correlated from input components to component). Starting with an arbitrary $\underline{\alpha}$, the algorithm will converge in the mean and will remain stable as long as the parameter Δ is greater than zero but less than the reciprocal of the largest eigenvalue λ_{max} of the matrix $\underline{A}$:

$$2 / \lambda_{max} > \Delta > 0 \tag{2.17}$$

We shall prove (2.17) in the following section. A great deal of information is available on iterative algorithm etc. in reference [2-2] some of which are discussed in section (2.5). The basic algorithm given by (2.16), and some of the possible variations of it have been incorporated in many commercial adaptive equalizers. These variations of the basic algorithm are obtained by using only sign information contained in the error signal $\varepsilon^{(k)}$ and/or in the components of $\underline{Y}^{(k)}$. Hence, we outline the following possible variations:

(a) $$\underline{\alpha}^{(k+1)} = \underline{\alpha}^{(k)} + \Delta \operatorname{Sgn}\{\varepsilon^{(k)}\} \underline{Y}^{(k)} \tag{2.18}$$

$$k = 0, 1, 2, \ldots (K-1)$$

(b) $$\underline{\alpha}^{(k+1)} = \underline{\alpha}^{(k)} + \Delta \varepsilon^{(k)} \operatorname{Sgn}\{ \underline{Y}^{(k)}\} \tag{2.19}$$

$$k = 0, 1, 2, \ldots (K-1)$$

(c) $$\underline{\alpha}^{(k+1)} = \underline{\alpha}^{(k)} + \Delta \operatorname{Sgn}\{\varepsilon^{(k)}\} \operatorname{Sgn}\{ \underline{Y}^{(k)}\} \tag{2.20}$$

$$k = 0, 1, 2, \ldots (K-1)$$

$$\operatorname{Sgn}(x) = \begin{cases} 1 \text{ if } & x \geq 0 \\ -1 \text{ if } & x < 0 \end{cases} = \text{either } 1 \text{ or } -1$$

As we can see algorithm (2.20) is the one which is most easily implemented, but this gives the smallest rate of convergence relative to the others.

In the above discussion, we assumed that the receiver had *a priori* knowledge of the transmitted information sequence in forming the error signal between the desired symbol and its estimate. Such knowledge can be made available during a short initialization (training) period in which a signal with a known information sequence is transmitted to the receiver for initially adjusting the multipliers [2-10]. This is not the practical scheme for continuous adjustments of $\{\alpha_k\}$. In practice $\{\alpha_k\}$ may be adjusted either in a decision directed mode of operation in which the decisions on the information symbols are assumed to be correct and are used in place of I_k in forming the error signal ε_k, or a known pseudo-random probe sequence may be inserted in the information bearing signal either additively mod 2 or by interleaving in time and the multiplier coefficient vectors are adjusted by comparing the received probe symbols with the known transmitted probe symbols. Therefore we can write

$$\left.\begin{array}{ll} \{\varepsilon_k\} = \{\tilde{I}_k\} - \{\hat{I}_k\} & \text{for the decision directed mode} \\ \{\varepsilon_k\} = \{I_k\} - \{\hat{I}_k\} & \text{for the training mode} \end{array}\right\} \quad (2.22)$$

As long as the receiver is operating at low error rates an occasional error will have a negligible effect on the convergence of the algorithm.

If the channel response changes, then the effect of this change will be reflected in the coefficients $\{\rho_k\}$ of the equivalent discrete time channel. Consequently, the error signal will change. Hence, the $\{\alpha_k\}$ will be changed according to (2.16) reflecting the change in the channel. A similar change in $\{\alpha_k\}$ occurs if the statistics of the noise or the information sequence change. Thus the equalizer is said to be adaptive.

2.4 DESIGN OF NRDFE BASED ON THE PROBABILITY OF ERROR (P_e or Pr(e)):

As we have pointed out in section (2.1) the most meaningful criterion for optimality of an equalization technique is usually the probability of error Pr(e) , but, it is also the most cumbersome, nonlinear and difficult to analyze mathematically. However, according to [2-10] , the SNR is related to the minimum MSE and there exists a relationship between the output SNR and the Pr(e) In the absence of ISI, the output is gaussian, i.e. the estimate $\{\hat{I}_k\}$ is a gaussian random variable. For this case, the P_e becomes a function of ρ where

$$\rho\rho = \frac{\text{M S value of the desired signal}}{\text{M S value of additive noise}}$$

However, in the presence of ISI, the estimate $\hat{I}_k$ is no longer gaussian and, as a result, there is no simple correspondence between the output SNR and the Pr(e). When the MS value of the ISI is small relative to the MS value of the additive gaussian noise, a good estimate of the Pr(e) can be obtained by using $\rho\rho$ given by equation (30) of reference [2-10] , and assuming that the total ISI can be characterized as a gaussian variable. However, when the ISI dominates the additive gaussian noise, the quality of the gaussian approximation is poor, especially when estimating low error probabilities. In that case, we resort to a simple computational procedure as described by Proakis and Miller [2-10] for PAM signalling with equally probable, statistically independent information symbols. However, this method is useful for computing small probabilities of error. Since, we shall be dealing with error probabilities of 10^{-4} and larger, a more convenient method for estimating the Pr(e) (of the adaptive receivers operating in the presence of ISI) is by Monte-Carlo simulation. The Monte-Carlo simulation technique is described in the Appendix and in [2-10, 2-21]. Programs are available in the Library subroutine packets.

Aaron and Tufts [2-19] used Pr(e) as a performance index under the assumptions that an error occurs only if the centre symbol of the message is in error. They failed to make it suitable as an adaptive algorithm; this is yet to be found.

Throughout this work our performance index will be MSE except occassional crossings to other criterion for computer simulation purposes.

2.5 ITERATIVE ALGORITHMS FOR MEAN SQUARE ERROR:

An iterative method is a route for operating on previous approximate solution to obtain an improved solution. These methods are preferred for solving large sparse systems because they can take advantage of zeros in the matrix and tend to be self correcting and hence tend to minimize the roundoff errors. Such methods are particularly good for almost diagonal or dominant diagonal systems in which convergence is rapid. Our aim here is to solve the Wiener solution (2.12)

$$\underline{\alpha}_{opt} = \underline{A}^{-1}\,\underline{W} \tag{2.12}$$

iteratively. We shall start with a first degree iteration written as

$$\underline{\alpha}^{(k+1)} = \underline{\alpha}^{(k)} - \Delta^{(k)}\,\underline{p}^{(k)} \tag{2.23}$$

where step-size, $\Delta^{(k)}$, and direction matrix, $\underline{p}^{(k)}$, are functions of k, $\underline{A}$, $\underline{W}$ and $\underline{\alpha}^{(k)}$.

Keeping in mind Fig.(2.1), the above representation must leave the Wiener solution vector $\underline{\alpha}_{opt}=\underline{A}^{-1}\underline{W}$ invariant i.e., if $\underline{\alpha}^{(k)} = \underline{A}^{-1}\,\underline{W}$, then

$$\underline{\alpha}^{(k+1)} = \underline{A}^{-1}\,\underline{W}$$

Hence it is required that $\Delta^{(k)}\,\underline{p}^{(k)} = \underline{0}$ when $\underline{\alpha}^{(k)} = \underline{A}^{-1}\,\underline{W}$

We define, residual vector (at the k-th iteration) as

$$\underline{r}^{(k)} \stackrel{\Delta}{=} \underline{g}^{(k)} = \underline{A}\,\underline{\alpha}^{(k)} - \underline{W} = \partial J(\underline{\alpha})/\partial\underline{\alpha} \tag{2.24}$$

Multiplier error vector is defined as:

$$\underline{e}^{(k)} = \underline{\alpha}^{(k)} - \underline{\alpha}_{opt} \tag{2.25}$$

Since $\underline{\alpha}_{opt}$ is unknown, therefore, it is usually either the length of the residual vector $||\underline{g}^{(k)}||$ or the length $||\underline{\alpha}^{(k)} - \underline{\alpha}^{(k-1)}||$ is used in a convergence test.

For the first representation of the first degree iteration to let $\underline{p}^{(k)} = \underline{g}^{(k)} = \underline{r}^{(k)}$, the iteration may be written as:

$$\underline{\alpha}^{(k+1)} = \underline{\alpha}^{(k)} + \Delta^{(k)} \underline{g}^{(k)}$$

This is satisfied by $\underline{\alpha}^{(k)} = \underline{\alpha}^{(k+1} = \underline{A}^{-1} \underline{W}$. Therefore, we have

$$\underline{A}^{-1} \underline{W} = (\underline{I} - \Delta^{(k)} \underline{A}) \underline{A}^{-1} \underline{W} + \Delta^{(k)} \underline{W}$$

Subtracting the quantity

$$\underline{\alpha}^{(k+1)} = (\underline{I} - \Delta^{(k)} \underline{A}) \underline{\alpha}^{k} + \Delta^{(k)} \underline{W}$$

yields

$$(\underline{A}^{-1} \underline{W} - \underline{\alpha}^{(k+1)}) = (\underline{I} - \Delta^{(k)} \underline{A})(\underline{A}^{-1} \underline{W} - \underline{\Delta}^{(k)})$$

or,
$$\begin{aligned} \underline{e}^{(k+1)} &= (\underline{I} - \Delta^{(k)} \underline{A}) \underline{e}^{(k)} \\ &= \prod_{i=0}^{k} (\underline{I} - \Delta^{(i)} \underline{A}) \underline{e}^{0} \end{aligned} \tag{2.26}$$

The matrix $(\underline{I} - \Delta^{(i)} \underline{A})$ is called error or iteration matrix. This matrix converges if and only if

$$M_{k+1} = \prod_{i=0}^{k} (I - \Delta^{(i)} \underline{A}) \rightarrow \underline{0} \quad \text{as} \quad k \rightarrow \infty \tag{2.27}$$

If all eigenvalues of M_{k+1} are less than 1 in absolute value then the iteration converge. If we suppose $\lambda(M_k)$ be the spectral radius of M_k that is the magnitude of the eigenvalue of M_k of largest magnitude then the average rate of convergence for fixed k is defined by

$$R(M_k) \triangleq - (1/k) \log(\lambda(M_k)) \tag{2.28}$$

and this is also the asymptotic rate of convergence for stationary case

We can put the convergence conditions into the following words with the help of Fig. (2.2)

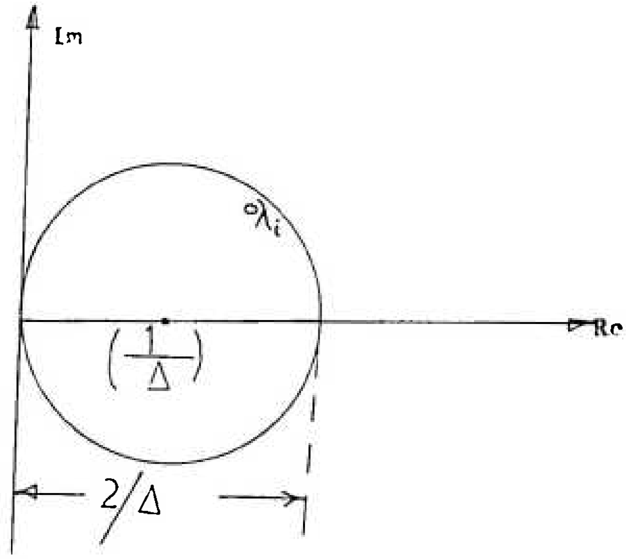


FIG. (2.2) NECESSARY RELATION OF Δ TO LOCATION OF EIGENVALUES.

The necessary and sufficient condition is that the eigenvalues of $(\underline{I} - \Delta\underline{A})$ must lie inside the unit circle or the eigenvalues of $\underline{A}$ must lie in a circle centered at $(1/\Delta)$ with radius $(1/\Delta)$. It is shown that this convergence is monotonic [2-7] . The number of iterations k is given by the following relation:

$$\max_{\underline{e}^{(0)} \neq 0} \frac{|| \underline{e}^{(k)} ||}{|| \underline{e}^{(0)} ||} = \lambda^k < \eta \text{ (say)} \tag{2.28}$$

so that the required k satisfies

$$k > \frac{-\log \eta}{-\log \lambda} \tag{2.29}$$

For the zero-forcing equalizers, if Δ is small in the iteration procedure the eigenvalues of $\underline{A}$ must lie in the right half complex plane. This implies that the phase of the channel must lie within $\pm 90^{\circ}$. Therefore, for finite Δ, absolute convergence cannot be guaranteed unless initial distortion $D_0 < 1$ [2-3] .

For the mean square error equalizer, the coefficient matrix of the simultaneous linear equations is Hermitian and positive definite Therefore the convergence of many iterative algorithms are guaranteed [2-9]. A particular iteration (be it linear or nonlinear, stationary or non stationary) depends on the choice of $\Delta^{(k)}$ and $\underline{p}^{(k)}$. Often we make our choice in such a way as to minimize some measure of error. Three such measures (related to the MSE criterion) are:

(i) $J_1(\underline{\alpha}) = \langle \underline{\alpha}_{opt} - \underline{\alpha},\ \underline{\alpha}_{opt} - \underline{\alpha} \rangle = \langle \underline{e}, \underline{e} \rangle = \langle \underline{A}^{-1}\underline{r}, \underline{A}^{-1}\underline{r} \rangle = \langle \underline{r}, (\underline{A}\underline{A}^T)^{-1}\underline{r} \rangle$

(ii) $J_2(\underline{\alpha}) = \langle \underline{r}, \underline{r} \rangle = \langle \underline{e}, \underline{A}^T\underline{A}\,\underline{e} \rangle$

(iii) $J_3(\underline{\alpha}) = \langle \underline{e}, \underline{A}\,\underline{e} \rangle = \langle \underline{e}, \underline{g} \rangle = \langle \underline{A}^{-1}\underline{g}, \underline{g} \rangle = \langle \underline{g}, (\underline{A}^T)^{-1}\underline{g} \rangle$

$J_1(\underline{\alpha})$ is the square of the length of the error vector and $J_2(\underline{\alpha})$ is the square of the length of the residual vector. $J_3(\underline{\alpha})$ should be used only when $\underline{A}$ is symmetric positive definite, otherwise its minimum might be negative. For every error measure the generalized version of Eq.(2.15) hold good. That is the determination of gradient of $J(\underline{\alpha})$ and the optimum value of Δ . Detailed analysis of this is beyond the scope of this thesis but the useful discussion can be found in [2-17]. However, the pertinent results are given here from Westlake [2-21] . Assume basic iterative algorithm:

$$\underline{\alpha}^{(k+1)} = \underline{\alpha}^{(k)} - \Delta^{(k)}\ \underline{p}^{(k)}$$

and the simultaneous equation $\underline{A}\ \underline{\alpha} = \underline{W}$. Then the gradients for various measure of errors are given by:

$$\text{gradient for } J_1(\underline{\alpha}) = -2\,\underline{A}^{-1}\underline{r}$$

$$\text{gradient for } J_2(\underline{\alpha}) = -2\,\underline{A}^{T}\,\underline{r} \qquad (2.30)$$

$$\text{gradient for } J_3(\underline{\alpha}) = -2\,\underline{r}$$

OPTIMUM VALUES OF Δ FOR ARBITRARY $\underline{\alpha}$ AND $\underline{p}$:

$$\text{for } J_1(\underline{\alpha}), \quad \Delta = \frac{\langle \underline{g},\, (\underline{A}^T)^{-1}\,\underline{p} \rangle}{\langle \underline{A}\,\underline{p},\, (\underline{A}^T)^{-1}\,\underline{p} \rangle} = \frac{\langle \underline{A}^{-1}\,\underline{g},\, \underline{p} \rangle}{\langle \underline{p}, \underline{p} \rangle}$$

$$\text{for } J_2(\underline{\alpha}), \quad \Delta = \frac{\langle \underline{g},\, \underline{A}\,\underline{p} \rangle}{\langle \underline{A}\,\underline{p},\, \underline{A}\,\underline{p} \rangle} \qquad (2.31)$$

$$\text{for } J_3(\underline{\alpha}), \quad \Delta = \frac{\langle \underline{A}^{-1}\,\underline{g},\, \underline{A}\,\underline{p} \rangle}{\langle \underline{p},\, \underline{A}\,\underline{p} \rangle} = \frac{\langle \underline{A}^T\,\underline{A}^{-1}\,\underline{g},\, \underline{p} \rangle}{\langle \underline{p},\, \underline{A}\,\underline{p} \rangle} = \frac{\langle \underline{g},\, \underline{p} \rangle}{\langle \underline{p},\, \underline{A}\,\underline{p} \rangle}$$

There are three standard iterative numerical methods for performing the minimizations of $J(\underline{\alpha})$ used in the equalization techniques. They are

(i) The steepest descent method

(ii) The conjugate gradient method [2-17, 2-21, 2-24]

(iii) The Fletcher-Powell method [2-25]

These methods are discussed next.

2.6 STEEPEST DESCENT GRADIENT METHODS:

To solve the equation $\underline{A}\,\underline{\alpha} = \underline{W}$ we start with a trial point $\underline{\alpha}^{(0)}$ in n-space and move in the direction $\underline{p}^{(k)}$ to the new approximation $\underline{\alpha}^{(1)}$, adding a correction of $\Delta^{(k)}\,\underline{p}^{(k)}$ to $\underline{\alpha}^{(0)}$. Gradient methods are characterized by the fact that $\underline{p}^{(k)}$ is chosen to be the gradient direction. If in addition, $\Delta^{(k)}$ is taken as the optimum Δ required to minimize $J(\underline{\alpha})$ for arbitrary $\underline{\alpha}$ and $\underline{p}$, the steepest descent method are obtained. These methods are stationary, explicit, and nonlinear. For each of the three error measures, the summary below gives $\underline{p}^{(k)}$, $\Delta^{(k)}$ for the iteration

(i) $J_1(\underline{\alpha})$:

$$\underline{p}^{(k)} = \underline{A}^{-1} \underline{r}^{(k)}$$

$$\Delta^{(k)} = \frac{<\underline{A}^{-1} \underline{g}^{(k)}, \underline{p}^{(k)}>}{<\underline{p}^{(k)}, \underline{p}^{(k)}>} = \frac{||\underline{A}^{-1} \underline{g}^{(k)}||^2}{||\underline{p}^{(k)}||^2} = 1$$

This is not suitable since it involves $\underline{A}^{-1}$.

(ii) $J_3(\underline{\alpha})$:

$$\underline{p}^{(k)} = - \underline{g}^{(k)} = \underline{r}^{(k)}$$

$$\Delta^{(k)} = \frac{<\underline{p}^{(k)}, \underline{g}^{(k)}>}{<\underline{p}^{(k)}, \underline{A}\, \underline{p}^{(k)}>} = \frac{<\underline{g}^{(k)}, \underline{g}^{(k)}>}{<\underline{g}^{(k)}, \underline{A}\, \underline{g}^{(k)}>}$$

$$= \frac{||\underline{g}^{(k)}||^2}{<\underline{g}^{(k)}, \underline{A}\, \underline{g}^{(k)}>}$$

This is the familiar form of steepest descent algorithm.

We can therefore write

$$\underline{\alpha}^{(k+1)} = \underline{\alpha}^{(k)} - \Delta^{(k)} \underline{g}^{(k)} = (\underline{I} - \Delta^{(k)} \underline{A}) \underline{\alpha}^{(k)} + \Delta^{(k)} \underline{W} \tag{2.32}$$

with error vector

$$\underline{e}^{(k+1)} = (\underline{I} - \Delta^{(k)}\underline{A}) \underline{e}^{(k)} = \prod_{i=0}^{k} (\underline{I} - \Delta^{(i)}\underline{A}) \underline{e}^{(0)} = M_{k+1} \underline{e}^{(0)} \tag{2.33}$$

Iteration Matrix $(\underline{I} - \Delta^{(k)} \underline{A})$ provides the eigenvalues. The process converges if $M_{k+1} \to 0$ as $k \to \infty$ for every eigenvalues< unity. In the method of steepest descent, the vector $\underline{r}^{(k)}$ is chosen as the negative of the gradient vector $\underline{g}^{(k)}$, i.e. $\underline{r}^{(k)} = - \underline{g}^{(k)}$. Consequently we obtain algorithm for determining Wiener solution $\underline{\alpha}_{opt}$ by (2.32). With this method, $\underline{g}^{(k)} \to \underline{0}$

$$\Delta^{(k)} \to 0 \quad \text{as} \quad k \to \infty$$

The method requires an infinite number of iterations to converge to $\underline{\alpha}_{opt}$ but, practically, the algorithm may be stopped at a point where magnitude of the gradient components fall below some specified limit.

2.7 CONJUGATE-GRADIENT METHOD

In this method [2-24, 2-26] the direction vector $\underline{p}^{(k)}$ is chosen according to the relation

$$\underline{p}^{(k+1)} = \underline{r}^{(k+1)} + \Delta_c^{(k)} \underline{p}^{(k)} \tag{2.34}$$

where the coefficients Δ_c is chosen to satisfy the generalized orthogonality condition

$$<\underline{p}^{(k+1)}, \underline{A}\, \underline{p}^{(k)}> = 0$$

From this condition we can obtain the result in two forms

$$\Delta_c^{(k)} = \frac{||\underline{r}^{(k+1)}||^2}{||\underline{r}^{(k)}||^2} = \frac{<\underline{g}^{(k+1)}, \underline{g}^{(k+1)}>}{<\underline{g}^{(k)}, \underline{g}^{(k)}>} \tag{2.35}$$

$$\Delta_c^{(k)} = \frac{<\underline{r}^{(k+1)}, \underline{A}\, \underline{p}^{(k)}>}{<\underline{p}^{(k)}, \underline{A}\, \underline{p}^{(k)}>} = \frac{<\underline{g}^{(k+1)}, \underline{A}\, \underline{g}^{(k)}>}{<\underline{p}^{(k)}, \underline{A}\, \underline{p}^{(k)}>} \tag{2.36}$$

The value of $\Delta_c^{(k)}$ given by (2.35) is simpler to compute, but the relation (2.36) gives better results according to Hestenes and Stiefel [2-24].

It can be shown that [2-24] the gradients $\{\underline{g}_i\}$ are orthogonal, i.e., $<\underline{g}_i, \underline{g}_j> = 0$ for $i \neq j$ and also $<\underline{p}_i, \underline{A}\, \underline{p}_j> = 0$ for $i \neq j$. The direction vectors, $\underline{p}_0, \underline{p}_1, \ldots \underline{p}_{K-1}$ form a basis in K - dimensional space and, hence, the minimum of $J(\underline{\alpha})$ is obtained in at most K iterations where K represents the number of filter coefficients. This rapid rate of convergence to Wiener solution $\underline{\alpha}_{opt}$ is to be contrasted with the relatively slow rate of convergence given by the steepest descent technique.

However, a word of caution, that the orthogonality methods usually tend to accumulate round off errors and thereby this method may disturb the stability [2-21, p. 48] .

2.8 THE FLETCHER POWELL ALGORITHM:

The Fletcher-Powell method [2-25] generates the direction vectors according to the relation

$$\underline{p}^{(k)} = - \underline{H}^{(k)} \underline{g}^{(k)} \tag{2.37}$$

where $\underline{H}^{(k)}$ is an (KxK) positive definite matrix that converges to the inverse of the Hessian $\underline{A}$ and which initially is set equal to the identity matrix. Then $\underline{H}^{(k)}$ is generated by the recursive relation

$$\underline{H}^{(k+1)} = \left(\underline{H}^{(k)} + \Delta_F^{(k)} \frac{\underline{H}^{(k)} \underline{g}^{(k)} [\underline{g}^{(k)}]^T \underline{H}^{(k)}}{[\underline{g}^{(k)}]^T \underline{H}^{(k)} \underline{\delta}(\underline{g}^{(k)})} \right) - \left(\frac{\underline{H}^{(k)} \underline{\delta}(\underline{g}^{(k)}) [\underline{\delta}(\underline{g}^{(k)})^T] \underline{H}^{(k)}}{[\underline{\delta}(\underline{g}^{(k)})]^T \underline{H}^{(k)} \underline{\delta}(\underline{g}^{(k)})} \right) \tag{2.38}$$

where $\underline{\delta}(\underline{g}^{(k)}) = \underline{g}^{(k+1)} - \underline{g}^{(k)}$

and T denotes the transpose. It can be shown [2-25] that $J(\underline{\alpha})$ attains its minimum value in at most K iterations just as the conjugate gradient method.

All these numerical methods need to know the gradients which are estimated according to the method outlined in the section (2.3). Therefore the multipliers are updated by the algorithm

$$\underset{\sim}{\hat{\alpha}}^{(k+1)} = \underline{\hat{\alpha}}^{(k)} + \Delta_1^{(k)} \varepsilon^{(k)} \underline{Y}^{(k)} \tag{2.39}$$

Eq. (2.39) may be thought upon as the optimization of the averaging interval since the E [...] has to be averaged over, say, LL symbols. Therefore by making LL=1 i.e. corrections are to be made after each symbol and no averaging at all is done. This method is usually called the stochastic approximation because the corrections become stochastic quantities whose means equal the desired gradient [2-22] .

This way the implementation of the algorithms become easier and cheaper.

2.9 SUMMARY AND COMMENTS:

(A) We have recapitulated the existing design techniques for designing NRDFE and discussed the three main error criteria. However, our performance crfiterion is the mean square error.

(B) We have mentioned and detailed the numerical control algorithms for updating the coefficient. There are of course other control algorithms, such as that proposed by Di Toro [2-8] , Gersho [2-9] , as weil. In addition to all these following algorithms are also used.

(a) Fixed Shift Jacobi

(b) Seidel (successive displacements)

(c) Jacobi

with the list order generally reflecting increasing hardware comlexity. It is clear that every iterative method for solving a set of simultaneous equations gives rise to a multiplier coefficient adjustment algorithm and there is no shortage of methods - Forsythe [2-17] mentions over 500 papers on the subject.

(C) Monte carlo simulation technique is the only method to study the most meaningful criterion- the probability of error P_e.

CHAPTER 3

A DETERMINISTIC DESIGN FOR A FAST-INITIALIZING NON-RECURSIVE DIGITAL FILTER EQUALIZER USING HADAMARD MATRIX

"The formulation of a problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skill. To raise new questions, new possibilities, to regard old problems from a new angle, requires creative imagination and marks real advance in science".

ALBERT EINSTEIN, (The evolution of physics (p.95))

3.1 INTRODUCTION:

With the development of computer networks and multiparty polling systems a large demand for very high speed modems became inevitable. Because messages in such a system may consist of only a few hundred bits, it is essential that the start up period of the modem be very short. Preferably the modem start up time should waste fewer bits than are contained in an average message to guarantee a reasonable system throughput. This could suggest that the start up time should be inversely proportional to the data rate; this is a contradictory requirement, since more accurate and complex operations are usually required during start up of a high speed modem than with a low speed version. Timing recovery, carrier recovery (if required), and initialization of the automatic equalizer are the most important start up operations. The time that must be allowed for the equalizer initialization, is usually the major delay in start up; thus, it seems worthwhile to concentrate some of our efforts in the direction of reducing start up time.

Throughout the development of digital NRDFE there have been continuing attempts to reduce the time required for initialization. In the early days an equalizer using the mean square error criterion was reported to require several thousand isolated initializing symbols [3-1] and a considerably larger number of symbols were required when the equalization was done using the more slowly convergent z.f. equalizer. A number of methods have been proposed to speed up this convergence. Schonfeld and Schwartz [3-2] allowed the loop gains to be time varying throughout adaptation, so as to provide optimal convergence at the end of specified duration. In another paper [3-3] the same authors incorporated a second order tracking scheme in order to minimize the norm of the tap gain error at each iteration . Richman and Schwartz [3-4] have used a dynamic programming approach to the adjustment of loop-gains while Walzman and Schwartz [3-5] have developed a discrete frequency domain technique to obtain faster convergence. In this latter technique the use of the fast fourier transform allows dealing with the frequency domain, where optimum gain constants are more readily available than in the time domain where a similar optimization would require determination of the eigenvalues of a matrix related to the system impulse response. Chang [3-6] orthogonalized the signals present at the various tap gains through a fixed weighting matrix to improve convergence. Chang and Ho [3-7] selected maximum length pseudo-random sequences of short periods for training purposes. They showed that the equalizer multiplier convergence rate is independent of the phase characteristic of the communication channel and of the choice of pseudorandom sequences which have the same period.

Kosovy and Pickholtz [3-8] have proposed fast automatic equalization using a successive overrelaxation iterative technique during a training period using isolated symbols for the minimization of the mean square error. Mueller [3-9] has presented a new, generalized mean square algorithm to adjust the taps of an adaptive transversal equalizer. He has taken into account any knowledge of the channel or signalling format to speed up the the convergence process mainly for partial response signalling by eliminating the interaction between the individual tap increments. This is achieved by decorrelating the components of the gradient in a fixed weighting matrix prior to adjustment. In any case he claims this algorithm is extremely fast.

In another paper Mueller and Spaulding [3-10] have described a new rapidly converging equalization technique for synchronous data communication judiciously called Cyclic Equalization. A special training sequence whose period in symbols is equal to the number of equalizer taps is used initially to achieve an open eye pattern, but, the resulting equalizer coefficients may be cyclically displaced from their proper positions. After the eye is opened by this process, the equalizer coefficients are rotated to their proper positions, and decision directed equalization is used with either a longer training sequence or random data to achieve final tap settings. They have shown that the cyclic equalization provides perfect equalization at evenly spaced points in the frequency domain.

Recently Kalman filtering theory [3-11,13] has been proposed for obtaining fast convergence of the tap gains of NRDFE to their optimal settings. Kalman filter applied to channel equalization requires (1) a knowledge of the initial state variable estimate and the initial covariance matrix, and (2) a knowledge of the channel impulse response. So, this equalization approach largely depends upon the correct initial estimate of (1) and (2). Under a known channel condition the Kalman filter, which is dual to the channel model, represents the optimum linear equalizer in that the number of equalizer taps need be only the same as the number of channel taps. A NRDFE performance is a direct function of the degree of freedom associated with it. Therefore a Kalman equalizer may be superior to a conventional NRDFE with the same number of taps

Lawrence and Kaufman [3-14] have used a Kalman filter for channel equalization under a known channel conditions. Mark [3-18] has studied the modified Kalman filter for channel equalization in which the channel tap gains are estimated via decision feedback approach and the initial state variable is estimated by a prediction process.

Godard [3-19] in this paper has shown how a Kalman filter may be applied to the problem of tap setting of transversal equalizers to minimize the mean square distortion. In the presence of noise and without prior knowledge about the channel, the filter algorithm leads to faster convergence than other methods and its speed of convergence depends only on the number of taps.

No doubt, Kalman filter is an optimal equalizer , but, is nonlinear and too complex to be implemented even in the presence of modern LSI technology.

Kobayashi [3-15] used Hestens-Stiefel algorithm [3-16] to channel equalization. This algorithm is an iterative method and possesses the proprty of fast convergence. Nevertheless, it is quite difficult to implement. De and Davis [3-17] have proposed the method of conjugate gradients as the MSE control algorithm. Their method has many desirable properties as a computation tool; study of implementation requirements shows that it is more complex than the more natural automatic Gram-Schmidt (AGS) control algorithm [3-20].

We have surveyed iterative (simple, complex) and finite step algorithms. They all seem justified in their approaches. Moreover, all of them, presumbly, have the same applications in their minds as us. All papers [3-1,20] stress the problem of fast initialization. It is therefore imperative that we talk about the choice of the training sequence. Obviously, a strictly random data pattern would be a bad choice, since transitions occur on a probabilistic basis and cannot be guaranteed. The variability of repeated convergence runs would be large. This can be avoided by transmitting a short period training sequence. Even if the starting point occurred at random, convergence would be more predictable. But, this condition may lead on to a situation where correlation matrix $\underline{A}$ would become singular and a unique solution for the optimum multiplier coefficient vector $\underline{\alpha}$ would not be possible.

However, we intend to study the case when the matrix $\underline{A}$ is singular and find a solution by proposing a new algorithm called MOORE-PENROSE PSEUDOINVERSE (MPPI).

3.2 FUNDAMENTALS:

For convenience we shall draw the following digital communication system for the noise-free channel [Fig. (3.1)].

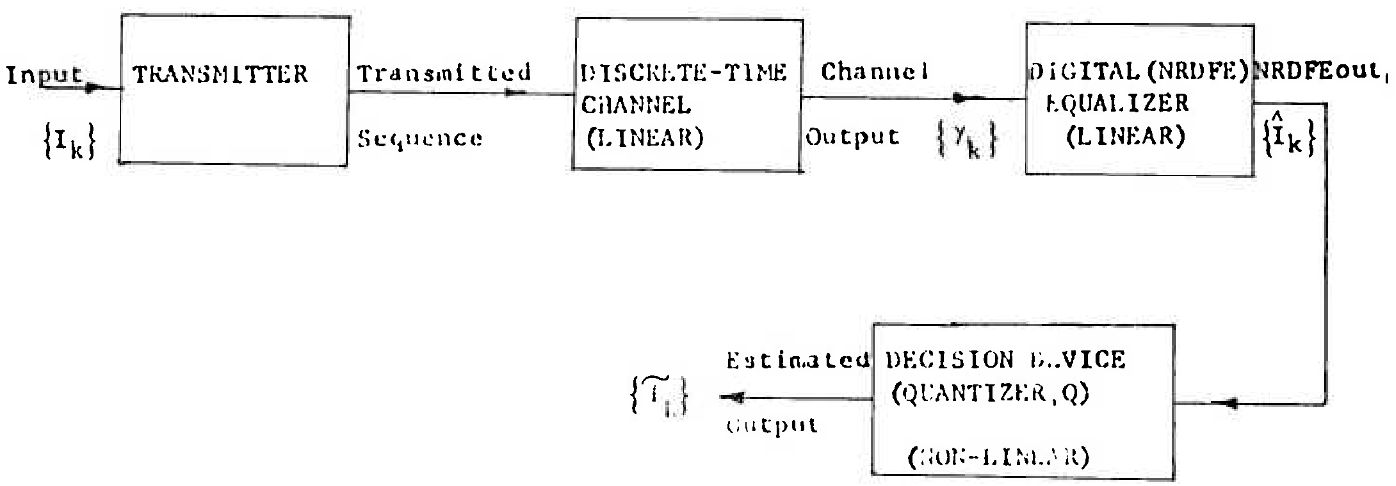


FIG. (3.1) NOISE-FREE DIGITAL COMMUNICATION SYSTEM.

$\{ I_k \}$ is a sequence of binary data. This sequence is sent through a D.T. NRDF channel which distorts the input to produce the channel output sequence $\{ y_k \}$. The impulse response of this channel is given by $\{ g_k \}$. In order to compensate for the effect of the channel imperfections, an equalizer is employed at the D. T. channel output and finally a nonlinear decision device [Quantizer Q] is used to obtain an estimate of the transmitted sequence based on the observation of the equalizer output sequence { $\hat{I}_k$ }. Basic assumptions outlined in chapter 1 are supposed to apply in the foregoing discussions.

The function of an equalizer has been explained in Chapter 2. We shall, however, explain the following results on convergence on the basis of chapter 2, [section 2.3] and relate them with the problem of equalization at hand. From Eq. (2.10), we have the MSE given by

$$J[\underline{\alpha}] = E[I_k^2] - 2\,\underline{\alpha}^T\,\underline{W} + \underline{\alpha}^T\,\underline{A}\,\underline{\alpha} \qquad (2.10)$$

where $\underline{\alpha}$, $\underline{W}$ and $\underline{A}$ are given by Equations (2.4), (2.8) and (2.9) respectively. On the assumptions made in section (2.3) the elements of $\underline{Y}_k$ of $\underline{A}$ are linearly independent. As a consequence $\underline{A}$ is symmetric, positive definite and hence non-singular. However{ Y_k} may not be linearly independent, in that case $\underline{A}$ is positive semidefinite and hence singular [3-28]. (The mean square problem will be, then, like cases 2a to 3b (chapter 2) where we have the rank deficiency.)

The gradient of the error function is given by

$$\underline{g} = \frac{\partial\,[\,J\,(\,\underline{\alpha}\,)\,]}{\partial\,\alpha_j} = -2\underline{W} + 2\,\underline{A}\,\underline{\alpha}$$

$$= 2 < \underline{Y}_k\ ,\ \varepsilon > \qquad (3.1)$$

where ε is the error given by (2.6) and < , > is the inner product notation. The second derivative of $J(\underline{\alpha})$ is given by

$$\frac{\partial^2[\,J\,(\underline{\alpha}\,)\,]}{\partial\,\alpha_j\,\partial\alpha_j} = 2\,\underline{A} = 2 < \underline{Y}_k\ ,\ \underline{Y}_j > \qquad (3.2)$$

This means that the error surface is a concave surface, that never goes negative [3-29] and therefore any standard gradient and related algorithm can be used to find optimum multiplier coefficients. In order to illustrate the convergence properties we shall assume that the signals correlation matrix $\underline{A}$ is positive definite. The Wiener multiplier vector is given by equating $\underline{g} = \underline{0}$ in (3.1). That is,

$$\underline{\alpha}_{opt.} = \underline{A}^{-1}\,\underline{W} \tag{3.3}$$

The multiplier coefficient error vector, $\underline{e}^{(k)}$, after $k^{th.}$ adjustment is given by

$$\begin{aligned}\underline{e}^{(k)} &= \underline{\alpha}^{(k)} - \underline{\alpha}_{opt.} \\ &= \underline{\alpha}^{(k)} - \underline{A}^{-1}\,\underline{W}\end{aligned} \tag{3.4}$$

and the MSE $J(\underline{\alpha})$ becomes

$$J(\underline{\alpha}) = \underbrace{E[I_k^2] - \underline{W}^T\,\underline{A}^{-1}\,\underline{W}}_{J_i(\underline{\alpha})} + \underbrace{[\underline{e}^{(k)}]^T\,\underline{A}\,\underline{e}^{(k)}}_{J_R(\underline{\alpha})} \tag{3.5}$$

This consists of two parts, the first part $J_i(\underline{\alpha})$ is the irreducible part [which is the MMSE, $J_{min}(\underline{\alpha})$] and is given by

$$J_i(\underline{\alpha}) = J_{min}(\underline{\alpha}) = E[I_k^2] - \underline{W}^T\,\underline{A}^{-1}\underline{W} \tag{3.7}$$

The second part $J_R(\underline{\alpha}) = [\underline{e}^{(k)}]^T\,\underline{A}\,\underline{e}^{(k)}$ (3.8)

is, in fact, zero when $\underline{e}^{(k)} = \underline{0}$. i.e. when,

$$\underline{\alpha}^{(k)} = \underline{\alpha}_{opt}. \quad k = 0, 1, 2, \tag{3.9}$$

Using the steepest descent gradient algorithm the $(k+1)^{th.}$ adjustment of multiplier coefficient is done according to the Eq. (2.32), i.e.

$$\underline{\alpha}^{(k+1)} = \underline{\alpha}^{(k)} - 1/2\ \Delta^{(k)}\ \underline{g}^{(k)}$$
$$= [\ \underline{I} - \Delta^{(k)}\ \underline{A}\ ]\ \underline{\alpha}^{(k)} + \Delta^{(k)}\ \underline{W} \tag{3.9}$$

It can be easily shown that the multiplier error vector is [Eq. (2.33)]

$$\underline{e}^{(k+1)} = [\ \underline{I} - \Delta^{(k)}\underline{A}\ ]\ \underline{e}^{(k)}\ ,\ k=0,1,2,3... \tag{3.10}$$

We now proceed to study the convergence properties with the help of Eq. (3.8). Let the eigenvalues of $\underline{A}$ be related to each other as:

$$\lambda_0 \leq \lambda_1 \leq \lambda_2 \leq . \quad ... \quad \leq \lambda_{K-1} \tag{3.11}$$

and let $\underline{U}_i$, $i = 0, 1, 2, \ ...\ (K-1)$ be a set of orthonormal eigenvectors of $\underline{A}$ ($\underline{U}_i$ is the eigenvector of $\underline{A}$ corresponding to eigenvalues λ_i). Since $\underline{A}$ is symmetric, it can easily be proved that it is real (appendix). Since all eigenvalues are real therefore, there exists a unitery transformation $\underline{Q}$ such that

$$\underline{A} = \underline{Q}\ \underline{\Lambda}\ \underline{Q}^T \tag{3.12}$$

where $\underline{\Lambda}$ is a (K x K) diagonal matrix whose i^{th} diagonal element is λ_i and $\underline{Q}$ is a (KxK) matrix whose i^{th} column is the eigenvector $\underline{U}_i$. It is also well known that $\underline{Q}$ is an orthogonal matrix, i.e.

$$[\underline{Q}]^T = [\underline{Q}]^{-1} \tag{3.13}$$

From (3.12) and (3.13)

$$[\ \underline{I} - \Delta^{(k)}\ \underline{A}\ ] = \underline{Q}\ [\ \underline{I} - \Delta^{(k)}\ \underline{\Lambda}\ ]\ \underline{Q}^T \tag{3.14}$$

Repeated application of (3.10), with decreasing index k, results in the telescoping form, [Eq. (2.26)].

$$\underline{e}^{(k)} = [\,\underline{I} - \Delta^{(k-1)}\underline{A}\,]\,[\,\underline{I} - \Delta^{(k-2)}\underline{A}\,] \ldots [\,\underline{I} - \Delta^{(0)}\underline{A}\,]\,\underline{e}^{(0)}$$

$$= \prod_{i=0}^{k-1} [\,\underline{I} - \Delta^{(i)}\,\underline{A}\,]\,\underline{e}^{(0)}$$

$$= [\,\underline{I} - \Delta^{(i)}\,\underline{A}\,]^k\,\underline{e}^{(0)} \qquad (3.15)$$

Substituting (3.14) into (3.15) and noting that

$\underline{Q}^T\,\underline{Q} = \underline{I}$ gives

$$\underline{e}^{(k)} = \underline{Q}\,[\,\underline{I} - \Delta^{(i)}\,\underline{\Lambda}\,]^k\,\underline{Q}^T\,\underline{e}^{(0)} \qquad (3.16)$$

which when substituted in (3.8) yields

$$J_R^{(k)}(\underline{\alpha}\,) = \sum_{i=0}^{K-1} [\{\,\underline{e}^{(0)}\}^T\ \underline{U}_i\,]^2\lambda_i\,(\,1-\Delta^{(i)}\lambda_i)^{2k} \qquad (3.17)$$

From Eq. (3.17) it is evident that two sets of parameters $\Delta^{(0)}$, $\Delta^{(1)}$, $\ldots$ $\Delta^{(K-1)}$ and λ_0, λ_1, $\ldots$ λ_{K-1} control $J_R(\underline{\alpha}\,)$, in effect, the convergence. The first set of parameters corresponds to the magnitudes of the multiplier control adjustments. The second set of parameters λ_0, λ_1, $\ldots$ λ_{K-1} depend on the modulation scheme and channel characterstics. The eigenvalue spread of $\underline{A}$ (i.e. the difference between the largest and the smallest eigenvalues) is an important factor in determining the rate of convergence. If

$$\lambda_0 = \lambda_1 = \ldots = \lambda_{K-1}$$

and $\quad \Delta^{(k)} = 1/\,\lambda_k$, for all k

then $J_R^{(k)}(\underline{\alpha}\,) = 0$

and convergence takes place on first iteration only. Consequently the mean square error $J(\underline{\alpha}\,)$ is reduced to its irreducible value $J_i(\underline{\alpha}\,) \stackrel{\Delta}{=}$ MMSE, after only one adjustment.

This kind of convergence is obtained regardless of the initial equalizer settings, the carrier phase and system timing, the phase characteristic of the transmission medium, and the phase characteristics of the transmitting and the re ceiving filters.

However, the signalling format, the channel attenuation , and the choice of initializing (training) pulse determine the eigenvalues. In practical situations, eigenvalues will not be equal so in order to get fast convergence, the difference between the eigenvalues should be maintained as small as possible.

But, because of the nature of arguments we may ourselves ask a few questions, namely

(i) What causes the eigenvalues to be different?

(ii) Is it possible to reduce the eigenvalue spread by altering the equalizer structure?

Partial answers to these questions have been given; however, we shall elaborate these answers a bit further. From chapter (2), we know that the matrix

$$\underline{A} = [\ a_{ij}\ ] = E\ [\ y_{k-i}y_{k-j}\ ]\quad i,j = 0,1,\ldots (K-1)$$

Since these are the inputs to the i^{th} and the j^{th} multipliers, therefore, a_{ij} is simply the cross-correlation between these inputs and $\underline{A}$ is the correlation matrix. Therefore, we might state that it is the correlations between the inputs to the multiplier coefficients that result in the differences between the eigenvalues. When these inputs are orthonormal, $\underline{A}$ is an identity matrix and the eigenvalues are equal. Therefore, in order to answer our questions we should construct a new equalizer, where the inputs to the multipliers are orthonormal. Such an equalizer is called a Generalized Equalizer Structure and is shown in Fig. (3.2).

3.3 A GENERALIZED EQUALIZER STRUCTURE:

For convenience, we divide Fig. (3.2) into two blocks, block (1) and block (2). Block (1) contains a band of filters, each having transfer functions $P_i(z)$, connected in parallel. The multiplier coefficients of these filters, block (1), once determined, do not change with the change in channel characteristics and are held fixed while the remaining multipliers of block (2) are adaptively adjusted. The input $\{y_k\}$ enters the system through the first member of block (1) and gives output $\{v_k\}$. The output of the i^{th} filter [of block 1] is connected through a variable multiplier $\{\alpha_i\}$ to the summer of block (2). The output from block (2) is the generalized equalizer output which is given by

$$\hat{I}_k = \sum_{i=0}^{K-1} \alpha_i v_i \quad , k = 0,1,\ldots.. \tag{3.18}$$

using z-transform notation, we can write

$$\hat{I}(z) = \underline{\alpha}^T \underline{V}(z) = \underline{\alpha}^T [\underline{P}(z) Y(z)] \tag{3.19}$$

where $\hat{I}(z)$, $\underline{V}(z)$, and $Y(z)$ are the z-transform of $\hat{I}_k$, v_k, and y_k respectively. From Fig. (3.2) we define,

$$\underline{\alpha} \overset{\Delta}{=} [\alpha_0 \; \alpha_1 \; \ldots \; \ldots \; \alpha_{K-1}]^T \tag{3.20}$$

$$\underline{V}(z) \overset{\Delta}{=} [V_0(z) \; V_1(z) \; \ldots \; V_{K-1}]^T \tag{3.21}$$

and $$\underline{P}(z) \overset{\Delta}{=} [P_0(z) \; P_1(z) \; \ldots \; P_{K-1}]^T \tag{3.22}$$

The signals, v_i, i=0, 1, $\cdots$ (K-1) are orthonormal provided

$$< v_i , v_j > = \delta_{ij} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases} \tag{3.23}$$

We may show that it is always possible to choose a fixed set of filters with transfer functions $p_i(z)$ such that the outputs are orthogonal for all inputs.

Consider a set of filters as above for which the set of impulse responses are $p_i(z)$. Then it is possible to choose $p_i(z)$ such that the outputs are orthogonal. One such set would be

$$p_i(z) = \alpha_i z^{-i}$$

Now by superposition , if y(z) is the z-transform of the input sequence, the output sequence will be given by y(z). $p_i(z)$ corresponding to the sequence $v_i = y_k * p_i^{(k-j)}$

Since the p_k are orthogonal then the sequence v_i will also be orthogonal.

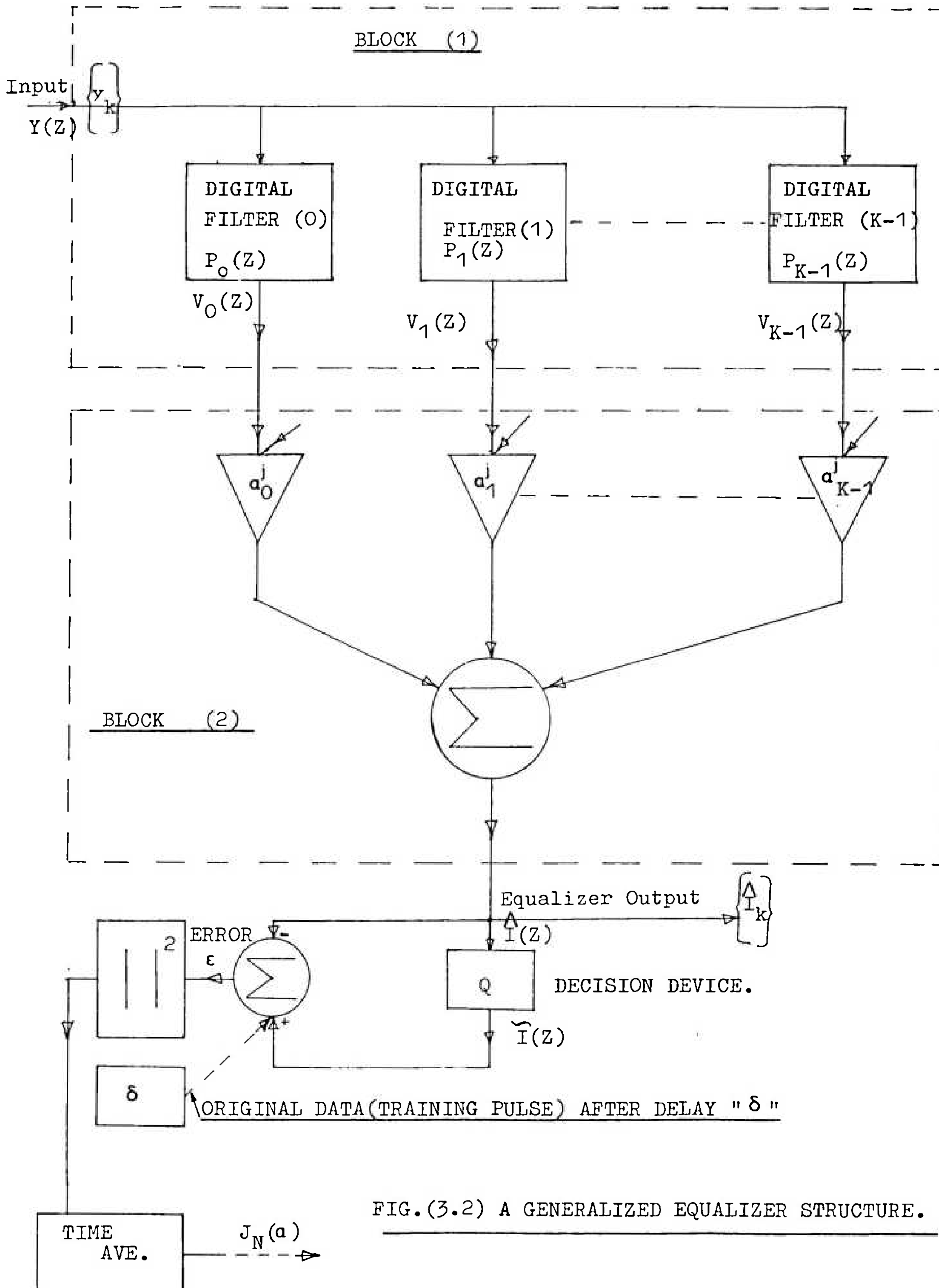


FIG.(3.2) A GENERALIZED EQUALIZER STRUCTURE.

In the following, we show that for any given channel, the filters in Fig. (3.2) can be designed to satisfy Eq. (3.23).

In section (3.2), we illustrated certain convergence properties by assuming the matrix $\underline{A}$ is positive definite implying that the system of signals are linearly independent. However, in high speed data communication systems (sensor networks, polling systems, digital networks etc.) the available signal correlation matrix $\underline{A}$ is positive semi-definite [3-30]. In that case $\underline{A}$ is singular.[It is our aim to consider such a case and look for a possible solution.]

However, the matrix $\underline{A}$ can be made nonsingular and therefore, strictly positive definite, by the proper selection of filter sections. In particular, if

$$P_i(z) = z^{-i} \text{ , for } i=0,1,\dots \quad \dots (K-1) \qquad (3.24)$$

and allow all the filter sections to be connected to a common shift-register, then those filter sections are, in effect, just non-recursive digital filter sections. One such filter section employing an NRDF is shown in Fig. (3.3). From now onwards, singular and non-singular matrices will be denoted by $\underline{\mathring{A}}$ and $\underline{\tilde{A}}$ respectively.

Chang, in a recent paper [3-6] has proposed a generalized equalizer structure having the following shortcomings and limitations:

(i) The channel correlation matrix is strictly non-singular ($\underline{\tilde{A}}$). That is, the structure deals with the linearly independent signals only.

(ii) Number of filter sections (K) equals the length of individual filter section (N).

(iii) He does not consider the telephone channel in particular.

(iv) No averaging over a priori known channel is performed.

(v) He does not mention anything about the kind of orthogonal matrix.

The possible explanations put forward to account for the disadvantages outlined above may be the following:

1. In the case of sensor network and polling systems, very fast initialization is required. Fast initialization inevitably means a short period training sequence. But by making the period of the training sequence shorter than the duration of the impulse response of the equalizer, the input signals to the multipliers remain nolonger linearly independent and the signal correlation matrix becomes singular, $\mathring{\underline{A}}$, and positive semidefinite. There may be other circumstances where the matrix $\underline{A}$ will be singular. A generalized structure must have the capability of tackling such a situation if it exists.
2. The choices of K and N equally affect the singularity conditions. If K = N, then the matrix is $\tilde{\underline{A}}$, otherwise always $\mathring{\underline{A}}$.
3. Averaging takes up considerable time. The first order steepest descent gradient algorithm can be written

$$\underline{\alpha}^{(k+1)} = \underline{\alpha}^{(k)} - \frac{\Delta^{(k)}}{2} \underline{g}^{(k)} \tag{3.9}$$

where

$$\underline{g} = 2< \underline{Y}_k \text{ , } \varepsilon_k> \tag{3.1}$$

From an implementation point of view, this is a convenient quantity because Y_k is readily available, and the error $\varepsilon_k = \hat{I}_k - \tilde{I}_k$ can be estimated. A difficulty still persists in that the expected value is not available in real time and must be estimated by averaging over a finite number of symbols, say, LL. Equation (3.9) can be rewritten using systems in which $\underline{A}$ is non-singular for equalizers where this is appropriate

$$\underline{\alpha}^{(k+1)} = \underline{\alpha}^{(k)} - \Delta^{(k)} [\underline{\tilde{A}}\,\underline{\alpha}^{(k)} - \underline{W}]$$

$$= \underline{\alpha}^{(k)} - \Delta^{(k)} [E\{\underline{Y}_j\underline{Y}_j^T\}\underline{\alpha}^{(k)} - E\{I_{j-\delta}\,\underline{Y}_j\}] \qquad (3.25)$$

$$= \underline{\alpha}^{(k)} - \Delta^{(k)}\frac{1}{LL}\sum_{j=0}^{L-1} \underline{Y}_j[\underline{Y}_j^T\,\underline{\alpha}^{(k)} - I_{j-\delta}]$$

where δ is the delay between the arrival at the equalizer of the first precursor of the channel unit pulse response and the beginning of the locally generated desired pulse. Usually averaging is recommended for the channels possessing severe distortions.

In order to concentrate upon the shortcomings we propose a generalized NRDFE which will be based upon the cases (2a) to (3b) outlined in chapter (2). These cases always give rank deficient matrices (which are, of course, singular). Solutions to such cases are proposed by MOORE PENROSE PSEUDOINVERSE algorithm described in subsquent sections. The question here arises as to whether there exists some N x K matrix $\underline{A1}$, uniquely determined by $\underline{\mathring{A}}$, such that the (unique) minimum length solution of problem MS is given by $\underline{\alpha} = \underline{A1}\underline{W}$. This is indeed the case and this matrix $\underline{A1}$ is called the MPPI pseudoinverse of $\underline{A}$. We provide following theorems which lead to a constructive definition of the pseudoinverse of an K x N matrix $\underline{\mathring{A}}$.

<u>THEOREM 1:</u>

Suppose that $\underline{A}$ is an K x N matrix of rank m and that

$$\underline{\mathring{A}} = \underline{H1}\ \underline{R1}\ \underline{K1}^T$$

Where

(a) $\underline{H1}$ is an K x K orthogonal matrix

(b) $\underline{R1}$ is an K x N matrix of the form

$$\underline{R1} = \begin{bmatrix} \underline{R1}_{11} & 0 \\ 0 & 0 \end{bmatrix}$$

(c) $\underline{R1}_{11}$ is a m x m matrix of rank m

(d) $\underline{K1}$ is a N x N orthogonal matrix

Define the vector

$$\underline{H1}^T \underline{W} = \underline{g1} = \begin{bmatrix} \underline{g1}_1 \\ \underline{g1}_2 \end{bmatrix} \begin{matrix} \} \, m \\ \} \, K-m \end{matrix}$$

and introduce the new variable

$$\underline{K1}^T \underline{\alpha} = \underline{y1} = \begin{bmatrix} \underline{y1}_1 \\ \underline{y1}_2 \end{bmatrix} \begin{matrix} \} \, m \\ \} \, N-m \end{matrix}$$

Define $\tilde{y1}$ to be the unique solution of

$$\underline{R1}_{11} \, \underline{\tilde{y1}}_1 = \underline{g1}_1$$

(1) Then all solutions to the problem of minimizing $||\underline{A} \, \underline{\alpha} - \underline{W}||$ are of the form

$$\underline{\hat{\alpha}} = \underline{K1} \begin{bmatrix} \underline{\hat{y1}} \\ y1_2 \end{bmatrix} \quad \text{where } y1_2 \text{ is arbitrary.}$$

(2) Any such $\underline{\hat{\alpha}}$ gives rise to the same residual vector $\underline{r}$ satisfying

$$\underline{r} = \underline{W} - \underline{\dot{A}} \, \underline{\hat{\alpha}} = \underline{H1} \begin{bmatrix} 0 \\ \underline{g1}_2 \end{bmatrix}$$

(3) The norm of $\underline{r}$ satisfies

$$||\underline{r}|| = ||\underline{W} - \underline{A} \, \underline{\hat{\alpha}}|| = ||\underline{g1}_2||$$

(4) The unique solution of minimum length is

$$\underline{\alpha}_{opt.} = \underline{K1} \begin{bmatrix} y1 \\ 0 \end{bmatrix}$$

THEOREM 2:

Let $\overset{o}{\underline{A}}$ be a K x N matrix of rank m with an orthogonal decomposition

$$\overset{o}{\underline{A}} = \underline{H1}\ \underline{R1}\ \underline{K1}^T$$

as in hypotheses of Theorem (1). Then the unique minimum length solution of problem MS is given by:

$$\underline{\alpha}_{opt.} = \underline{K1} \begin{bmatrix} \underline{R1}_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \underline{H1}^T\ \underline{W}$$

THEOREM 3:

Let $\overset{o}{\underline{A}}_{K \times N} = \underline{H1}\ \underline{R1}\ \underline{K1}^T$ as in Theorem (2) .

Define

$$\underline{A1} = \underline{K1} \begin{bmatrix} \underline{R1}_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \underline{H1}^T$$

Then $\underline{A1}$ is uniquely defined by $\overset{o}{\underline{A}}$; it does not depend on the particular orthogonal decomposition of $\overset{o}{\underline{A}}$

In view of Theorems (2) and (3), we make the following definition

DEFINITION:

For a general K x N matrix $\overset{o}{\underline{A}}$, the MPPI of $\overset{o}{\underline{A}}$, denoted by $\underline{A}^+$, is the N x K matrix whose $j^{th.}$ column z_j is the unique minimum length solution of the mean square problem

$$\underline{A}\ \underline{\alpha}_j = \underline{\theta}_j$$

where θ_j is the $j^{th.}$ column of the identity matrix $\underline{I}_k$.

Next, we consider the structures and its related algorithms.

3.4 A GENERALIZED NON-RECURSIVE DIGITAL FILTER EQUALIZER:

It has been proposed in the previous section that the filter sections are to be replaced by Non-Recursive digital filters and that all the filter sections are connected to the same shift register. A typical generalized NRDFE is shown in Fig. (3.3). This fulfills the conditions that the matrix given by $< v_i, v_j >$ is now strictly positive definite. Secondly, for convenience as well as to fulfil other conditions laid down in section (3.3), we assume

(1) All the filter sections are of equal length N

and (2) N is strictly less than K.

Then the filter section shown in Fig. (3.3) can be defined as

$$P_i(z) = \sum_{j=0}^{N-1} p_i^{(j)} z^{-j} , \; i=0,1,\ldots (K-1) \qquad (3.26)$$

where $p_i^{(j)}$ is the j^{th} multiplier coefficient of i^{th} section.

Expanding Eq. (3.26), we get

$$\begin{bmatrix} P_0(z) \\ P_1(z) \\ \cdot \\ \cdot \\ \cdot \\ P_{K-1}(z) \end{bmatrix} = \begin{bmatrix} p_0^{(0)} & p_0^{(1)} & p_0^{(2)} & \ldots & p_0^{(N-1)} \\ p_1^{(0)} & p_1^{(1)} & p_1^{(2)} & \ldots & p_1^{(N-1)} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ p_{K-1}^{(0)} & p_{K-1}^{(1)} & p_{K-1}^{(2)} & \ldots & p_{K-1}^{(N-1)} \end{bmatrix} \begin{bmatrix} z^{-0} \\ z^{-1} \\ \cdot \\ \cdot \\ \cdot \\ z^{-(N-1)} \end{bmatrix} \qquad (3.27)$$

We define a (K x K) matrix $\underline{P}^T$ by adding (K-N) zeros to each row of (K x N) matrix of Eq. (3.27)

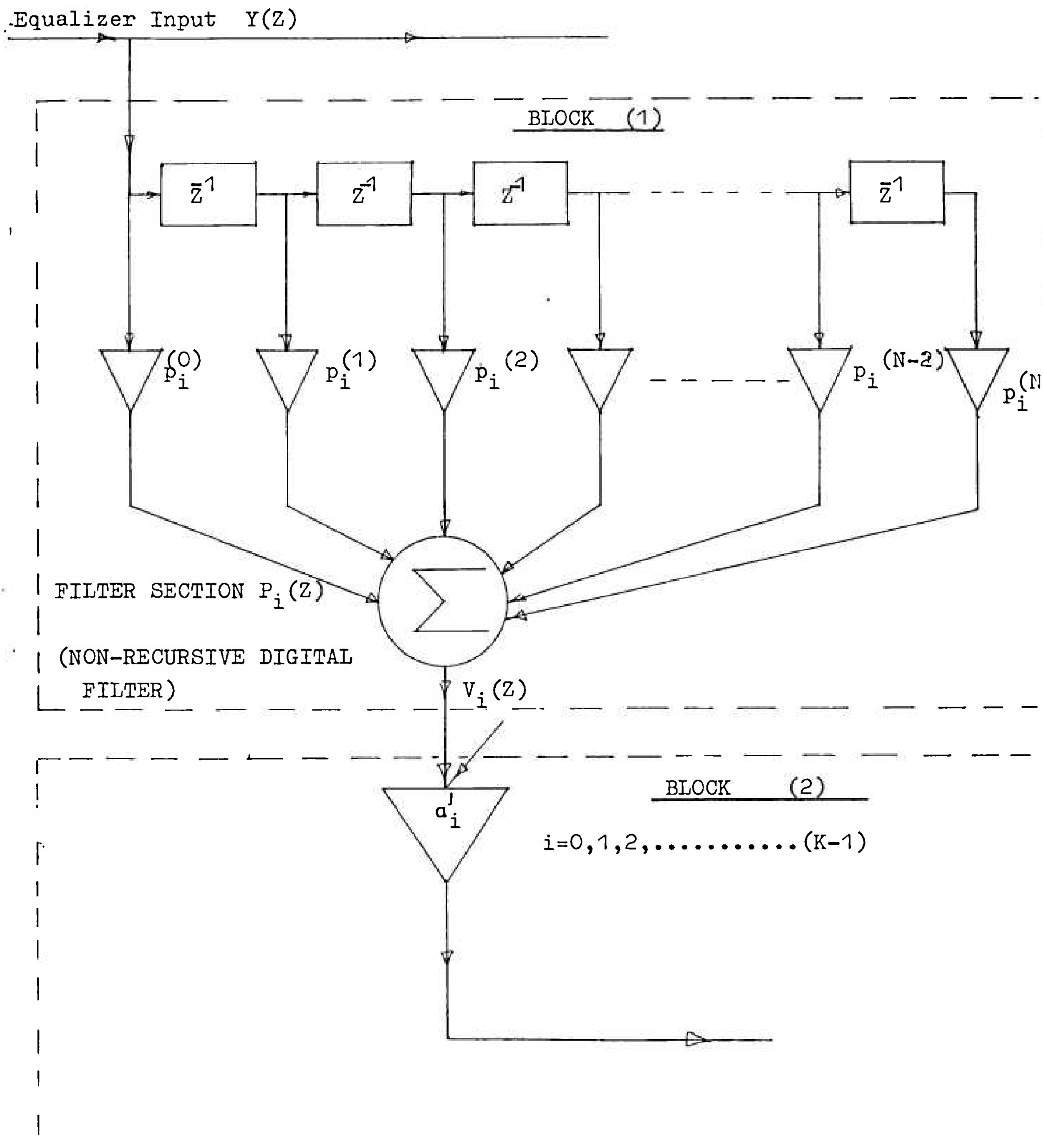


FIG.(3.3) A GENERALIZED NON-RECURSIVE DIGITAL FILTER EQUALIZER SECTION $P_i(Z)$.

$$\underline{P}^T \triangleq \begin{bmatrix} p_0^{(0)} & p_0^{(1)} \cdots & p_0^{(r)} \cdots & p_0^{(N-1)} \cdots & \overbrace{0 \cdots \cdots 0}^{(K-N)\text{zeros}} \\ p_1^{(0)} & p_1^{(1)} \cdots & p_1^{(r)} \cdots & p_1^{(N-1)} \cdots & 0 \cdots \cdots 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \qquad \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \qquad \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \qquad \cdot \\ p_{K-1}^{(0)} & p_{K-1}^{(1)} \cdots & p_{K-1}^{(r)} \cdots & p_{K-1}^{(N-1)} \cdots & 0 \cdots \cdots 0 \end{bmatrix}_{K \times K} \tag{3.28}$$

It can be seen from Eq. (3.28) that the column i of $\underline{P}$ corresponds to the coefficients of $P_i(z)$. This means that we are physically extending the shift register length from N to K and assigning zeros to the new multiplier coefficients. Next, we define a K-vector given by:

$$\underline{F}^T \triangleq [\, z^{-0} \quad z^{-1} \quad z^{-2} \cdots \quad \cdots \quad z^{-K+1} \,]_{K \times 1} \tag{3.29}$$

From (3.27), (3.28) and (3.29), we get

$$\underline{P}(z) = \underline{P}^T \underline{F} \tag{3.30}$$

The output of the filter section is given by:

$$\underline{V}(z) = \underline{P}^T \underline{F}\, Y(z) \tag{3.31}$$

The New MSE is given by:

$$J_{N'}(\underline{\alpha}) = \underline{\alpha}^T \underline{P}^T \underline{\tilde{A}}\, \underline{P} - 2\, \underline{\alpha}^T \underline{P}^T \underline{W} + E\,[\, I^2_{k-\delta} \,] \tag{3.32}$$

where N' in $J_{N'}(\underline{\alpha})$ stands for New.

Differentiating Eq. (3.32) w.r.t. α, we obtain the gradient

$$\underline{g}_{N'} \overset{\Delta}{=} \frac{\partial J_{N'}(\underline{\alpha})}{\partial \underline{\alpha}} = 2\,(\underline{P}^T \underline{\tilde{A}}\, \underline{P} - \underline{P}^T \underline{W}) \tag{3.33}$$

$$\overset{\Delta}{=} 2\,(\underline{\overset{o}{A}}\,\underline{\alpha} - \underline{W}) \tag{3.34}$$

It has been shown that $\underline{\overset{o}{A}}$ [$= \underline{P}^T \underline{\tilde{A}}\, \underline{P}$] is singular, although $\underline{\tilde{A}}$ is non-singular. It is possible to find $\underline{\alpha}_{opt.}$ since $\underline{W}$ is always in the range space of $\underline{\overset{o}{A}}$ [3-29,33]. The optimal value of $\underline{\alpha}$ is given by

$$\underline{\alpha}_{opt.} = \underline{A}^{+} \underline{W} \tag{3.35}$$

where $\underline{A}^{+}$ is the MPPI of $\underline{\overset{o}{A}}$ [Appendix A.4]. The multiplier error ($\underline{e}_m$) is given by

$$\underline{e}_m = \underline{\alpha} - \underline{\alpha}_{opt.} = \underline{\alpha} - \underline{A}^{+} \underline{W} \tag{3.36}$$

and the Eq. (3.32) is given by:

$$J_{N'}(\underline{\alpha}) = \underbrace{E\,[\,I^2_{k-\delta}\,] - \underline{W}^T \underline{A}^{+} \underline{W}}_{J_{N'i}} + \underbrace{\underline{e}_m^T \underline{\overset{o}{A}}\, \underline{e}_m}_{J_{N'R}} \tag{3.37}$$

As before, the irreducible (or minimum) MSE is given by:

$$J_{N'i}(\underline{\alpha}) = E\,[\,I^2_{k-\delta}\,] - \underline{W}^T \underline{A}^{+} \underline{W} \tag{3.38}$$

whereas the reducible MSE

$$J_{N'R}(\underline{\alpha}) = \underline{e}_m^T \underline{\overset{o}{A}}\, \underline{e}_m \tag{3.39}$$

Since $\underline{\overset{o}{A}}$ is real and symmetric, therefore, the matrix $\underline{Q}$ having the normalized eigenvectors { $\underline{U}_i$} of $\underline{\overset{o}{A}}$ as columns, forms a unitary transformation that diagonalizes $\underline{\overset{o}{A}}$ according to

$$\underline{Q}^T \underline{\overset{o}{A}}\, \underline{Q} = \underline{\Lambda} \tag{3.40}$$

where

$$\underline{\Lambda} = \begin{bmatrix} \lambda_0 & & & & & \underline{0} \\ & \lambda_1 & & & & \\ & & \lambda_2 & & & \\ & & & \lambda_3 & & \\ & & & & \ddots & \\ \underline{0} & & & & & \lambda_{K-1} \end{bmatrix} \tag{3.41}$$

and $\{ \lambda_k \}$ are the set of eigenvalues of $\underline{\mathring{A}}$. The first order steepest descent algorithm is given by:

$$\underline{\alpha}^{(k+1)} = [\, \underline{I} - \Delta^{(k)} \underline{\mathring{A}} \,] \underline{\alpha}^{(k)} + \Delta^{(k)} \underline{W} \tag{3.42}$$

and

$$\underline{e}_m^{(k+1)} = [\, \underline{I} - \Delta^{(k)} \underline{\mathring{A}} \,] \underline{e}^{(k)} \quad ,k=0,1, \ldots \tag{3.43}$$

Also, after some manipulations and noting that $\underline{Q}^T = \underline{Q}^{-1}$, we obtain

$$\underline{e}_m^{(k+1)} = \underline{Q} \, [\, \underline{I} - \Delta^{(k)} \underline{\Lambda} \,]^k \, \underline{Q}^T \, \underline{e}_m^{(0)} \tag{3.44}$$

Substituting Eq. (3.44) in Eq. (3.39) yields

$$J_{NR}^{(k)}(\underline{\alpha}) = \sum_{i=0}^{K-1} [\{ \underline{e}_m^{(0)} \}^T \underline{U}_i \,]^2 \lambda_i \, (1 - \Delta^{(k)} \lambda_i)^{2k} \tag{3.45}$$

From (3.45), if

$$\lambda_0 = \lambda_1 = \lambda_2 = \ldots \ldots . = \lambda_{N-1} \, , \quad \lambda_{N+1} = \cdots = \lambda_K = 0$$

and $\quad \Delta^{(k)} = 1/ \lambda_N$

then $\quad J_{NR}^{(k)} (\underline{\alpha}) = 0, \quad \text{for} \quad k \geq 1$

This means the convergence will take place on first iteration only. Since $\underline{P}^T$ and hence $\underline{\mathring{A}}$ is singular, it is possible that one or more of the eigenvalues λ_i will be zero, therefore, the corresponding terms of Eq. (3.45) will become zero as well. Therefore, $\underline{P}^T$ has to be selected in such a way that all non-zero eigenvalues of $\underline{\mathring{A}}$ are equal to $(1/\Delta)$. This, in effect, will ensure rapid initialization to take place in the neighborhood of first iteration only.

Next we consider this rapid initialization by the suitable design of $\underline{P}^T$.

3.5 CONDITIONS FOR RAPID INITIALIZATION:

As said earlier we intend to design a Non-Recursive Digital Filter $\underline{P}^T$. Therefore, we proceed by making certain elementary approaches, that is, suppose an input sequence $\{I_k\}$ of length I is transmitted over the D.T. channel, then, the output of the channel is given by

$$Y(z) = I(z)\,G(z)$$

$$= \sum_{k=0}^{I-1} I_k z^{-k} \sum_{l=0}^{G-1} g_l z^{-l}$$

$$= \sum_{l=0}^{G-1} \sum_{k=0}^{I-1} g_l I_k z^{-(l+k)}$$

Rearranging the summation and putting $i \overset{\Delta}{=} l+k$, we obtain

$$Y(z) = \sum_{i=0}^{I+G-2} \Big[\sum_{l=0}^{i} g_l I_{i-l}\Big] z^{-i} \quad \text{provided, } I_j = 0 \quad \begin{matrix} j \geq I \\ j < 0 \end{matrix}$$

$$\overset{\Delta}{=} \sum_{i=0}^{L-1} y_i z^{-i} \qquad \text{where } L \overset{\Delta}{=} I + G - 1 \text{ is the length of}$$

the D.T. channel output sequence.

As can be seen from above the channel output sequence $\{y_k\}$ is given by the discrete convolution of the channel input $\{I_k\}$ and the channel unit pulse response $\{g_k\}$. On this argument we can, therefore, say that the output of the i^{th} filter section of length N is the discrete convolution as shown below:

$$V_i(z) = \sum_{k=0}^{L+N-2} \Big[\sum_{n=0}^{k} p_i(n)\, y_{k-n}\Big] z^{-k}$$

$$= \sum_{k=0}^{PP-1} v_i(k)\, z^{-k} \qquad (3.46)$$

where $PP \overset{\Delta}{=} L+N-1$ is the length of the sequence $\{v_i(k)\}$

In order to make the discussion more vivid, it is necessary to make some assumptions which will lead to the filter section length N and the number of such sections K . Since our channel is of NRDF type, a finite upper limit of the D.T. channel output sequence, assumed known, does exists. Also, we assume the following constraint on the length of of the filter section output sequence PP, i.e. $K \geq$ PP. This inequality also ensures that the output sequence length can always be extended from PP to K by adding (K-PP) zeros to the transmitted information sequence $\{I_k\}$. Elaborating the discrete convolution relationship [Eq. (3.46)],

$v_i^{(k)} = \sum_{n=0}^{k} p_i^{(n)} y_{k-n}$ with $y_j = 0$ for $j \geq L$, we obtain

$$\begin{bmatrix} v_i^{(0)} \\ v_i^{(1)} \\ v_i^{(2)} \\ \cdot \\ \cdot \\ v_i^{(L-1)} \\ v_i^{L} \\ \cdot \\ v_i^{(PP-1)} \\ \cdot \\ v_i^{(K-1)} \end{bmatrix} = \begin{bmatrix} y_0 & 0 & 0 & 0 & 0 \\ y_1 & y_0 & 0 & 0 & 0 \\ y_2 & y_1 & y_0 & 0 & 0 \\ \cdot & y_2 & y_1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{L-1} & y_{L-2} & \cdot & \cdot & y_{L-N} \\ 0 & y_{L-1} & y_{L-2} & \cdot & y_{L-N+1} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & y_{L-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_i^{(0)} \\ p_i^{(1)} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ p_i^{(N-1)} \end{bmatrix} \tag{3.47}$$

Writing briefly, we have

$$\underline{v}_i = \underline{YY}\,\underline{p}_i \tag{3.48}$$

Where $i=0,1,\ \ldots\ \ \ldots\ ,(K-1)$

$\underline{YY}$ is a $K \times N$ matrix of rank N (since $N \times N$ principal minor is lower triangular with non-zero diagonal elements)

$\underline{v}_i$ is a filter section output vector of length K,

and $\underline{p}_i$ is filter section multiplier vector of length N.

The correlation matrix, from the outputs of an arbitrary pair of filter sections j and k, is given by

$$\overset{o}{\underline{A}} \overset{\Delta}{=} \underline{P}^{T}\,\underline{\tilde{A}}\,\underline{P} = [\,a_{jk}\,] = <\underline{v}_j,\,\underline{v}_k> \tag{3.49}$$

From Eq. (3.49) it is evident that the elements of $\overset{o}{\underline{A}}$ depend on the filter section output vectors $\{v_i\}$, therefore, their relationship to the eigenvalues of $\overset{o}{\underline{A}}$ is very important. For rapid (i.e. one-step) initialization a sufficient condition, $\overset{o}{\underline{A}} = \underline{I}$, and the necessary orthonormality conditions given by Eq.(3.23) must be fullfilled. These two conditions, in this case, allow us to assume a theoretical solution to determine a set of theoretical multiplier vectors $\{\underline{p}_{it}\}$ that would make the corresponding output set $\{\underline{v}_{it}\}$ orthonormal (suffix t stands for theoretical). Equation (3.48) represents K-scalar equations in N unknowns with $N< K$ which can be solved using MPPI algorithm only. Therefore, a best optimum solution

$$\underline{p}_i = \underline{YY}^{+}\,\underline{v}_{it} \tag{3.50}$$

exists.

Where $\underline{YY}^{+}$ is the MPPI of $\underline{YY}$ and is unique [3-31, pp. 26 & 49]

and $\underline{p}_i$ is the actual filter section multiplier vectors.

An approximate relationship between $\underline{v}_i$ and $\underline{v}_{it}$ can be obtained from Eq. (3.48) and Eq. (3.50) as

$$\underline{v}_i = \underline{YY}\,\underline{p}_i = \underline{YY}\,\underline{YY}^{+}\,\underline{v}_{it} \tag{3.51}$$

Now, from Eqs. (3.49) and (3.51), we obtain

$$\underline{\mathring{A}} = [\, a_{jk}\,] = \langle \underline{v}_j \,;\, \underline{v}_k \rangle$$

$$= \langle \underline{YY}\,\underline{YY}^{+}\,\underline{v}_{jt} \,,\, \underline{YY}\,\underline{YY}^{+}\,\underline{v}_{kt} \rangle$$

$$= \langle \underline{v}_{jt} \,,\, \underline{YY}\,\underline{YY}^{+}\,\underline{v}_{kt} \rangle \quad \text{from Appendix} \tag{3.52}$$

Let $\{ \underline{\psi}_i \,,\, i = 0,1, \ldots \ \ldots (K-1)\}$ be the orthonormal set of vectors obtained by Gram Schmidt orthonormalization applied to column of $\underline{YY}$. Then we have [3-31, pp. 210 and from Appendix (A.4) using the spectral representations for $\underline{YY}$ and $\underline{YY}^{+}$]

$$\underline{YY}\,\underline{YY}^{+} = \sum_{i=0}^{N-1} \underline{\psi}_i \, \underline{\psi}_i^{T} \tag{3.53}$$

From Appendix (A.2), since the set $\{\underline{\psi}_i\}$ is orthonormal, they are linearly independent, thus, it forms a basis for the real vector space of dimension K. It should be noted that the vectors $\underline{v}_i$ and $\underline{v}_{it}$ belong to the real vector space of dimension K. As a result, we may represent $\underline{v}_{it}$ in the basis by:

$$\underline{v}_{it} = \sum_{j=0}^{K-1} h_{ij}\, \underline{\psi}_j \tag{3.54}$$

where h_{ij} a scaler , is the coordinate of $\underline{v}_{it}$ in the direction of $\underline{\psi}_j$. It can easily be shown that $\underline{\psi}_j$ is also unitary [3-29]. Then,

$$\langle \underline{v}_{it} \,,\, \underline{v}_{jt} \rangle = \langle \sum_{m=0}^{K-1} h_{im}\, \underline{\psi}_m \,,\, \sum_{n=0}^{K-1} h_{jn}\, \underline{\psi}_n \rangle$$

$$= \sum_{m=0}^{K-1} \sum_{n=0}^{K-1} h_{im}\, h_{jn} \langle \underline{\psi}_m \,,\, \underline{\psi}_n \rangle$$

$$= \sum_{k=0}^{K-1} h_{ik}\, h_{jn} \tag{3.55}$$

We can now obtain $\underline{\mathring{A}}$ from Eqs. (3.52), (3.54), and (3.55), i.e.,

$$\underline{\mathring{A}} = [\, a_{ij}\,] = \sum_{k=0}^{N-1} h_{ik}\, h_{jk} \tag{3.56}$$

Next, suppose that $\underline{H}$, $\underline{V}_t$, and $\underline{\psi}$ be the matrices corresponding to the columns coinciding with the ordered sets $\{h_{ij}\}$, $\{\underline{v}_{it}\}$, and $\{\underline{\psi}_i\}$ respectively, then, from Eq. (3.55) it can be shown that by making $\underline{H}$ orthogonal, inevitably, makes $\underline{V}_t$ orthogonal. Also, $\underline{H}$ must be orthogonal to make the set $\{h_i\}$ orthonormal. Since there are an infinite number of orthogonal matrices, we shall be using one special type called Hadamard matrix, henceforth abbreviated as "$\underline{H}$-matrix or simply $\underline{H}$". Since the design and analysis of $\underline{H}$ is complicated we shall refer to [3-40] for an advanced treatment.

The eigenvalues of $\overset{o}{\underline{A}}$ are most important; therefore, we shall determine them first. By observing Eq. (3.49), we write

$$\overset{o}{\underline{A}} = \underline{V}^T \underline{V} \tag{3.57}$$

where $\underline{V}$ is defined to be a (K x K) matrix having columns coinciding with $\{v_i\}$. Now, using the projection operator ($\underline{\Phi}$) properties,

$$\begin{aligned} \underline{\Phi} &\overset{\Delta}{=} \underline{YY}\,\underline{YY}^{+} \\ &= \underline{\Phi}^T \underline{\Phi} = \underline{\Phi}^2 \end{aligned} \tag{3.58}$$

We can, from Eq. (3.51), write

$$\underline{V} = \underline{\Phi}\, \underline{V}_t \tag{3.59}$$

Putting Eq. (3.54) into Eq. (3.59) yields

$$\underline{V} = \underline{\Phi}\,\underline{\Psi}\,\underline{H} \tag{3.60}$$

From Eqs. (3.57), (3.58) and (3.60), we get

$$\begin{aligned} \overset{o}{\underline{A}} &= [\underline{\Phi}\,\underline{\Psi}\,\underline{H}]^T\, [\underline{\Phi}\,\underline{\Psi}\,\underline{H}] = \underline{H}^T \underline{\Psi}^T \underline{\Phi}^T \underline{\Phi}\,\underline{\Psi}\,\underline{H} \\ &= \underline{H}^T \underline{\Psi}^T \underline{\Phi}\,\underline{\Psi}\,\underline{H} \end{aligned} \tag{3.61}$$

From Appendix (A.6.5), $\underline{\Psi}^T \underline{\Phi}\,\underline{\Psi}$ equals the block diagonal matrix diag. [$\underline{I}_N$ | $\underline{0}_{K-N}$]. Since $\underline{H}$ is orthogonal, Eq. (3.61) implies that $\overset{o}{\underline{A}}$ is similar to diag. [$\underline{I}_N$ | $\underline{0}_{K-N}$] and therefore has the same eigenvalues, N of which are equal to unity and (K-N) equal to zero. This gives the desired condition for rapid (one step) initialization.

Thus by making $\Delta^{(k)} = 1$ in Eq.(3.45), we get

$$J_{NR}^{(k)} (\underline{\alpha}) = 0$$

and the rapid initialization has been achieved regardless of the values of $\underline{e}_m^{(0)}$.

3.6 CALCULATION OF FILTER SECTION MULTIPLIERS:

There are many methods discussed in Ref.[3-32 pp.209-217] Here we describe a method based on the spectral representation definiti -on of the MPPI. This is an indirect method where we donot require the direct computation of $\underline{YY}^{+}$. Since we have the relationships:

$$\underline{YY}\ \underline{YY}^{+} = \sum_{i=0}^{N-1} \underline{\psi}_i\ \underline{\psi}_i^{T} \tag{3.53}$$

$$\underline{v}_{it} = \sum_{j=0}^{K-1} h_{ij}\ \underline{\psi}_j \tag{3.54}$$

and

$$\underline{p}_i = \underline{YY}^{+}\ \underline{v}_{it} \tag{3.50}$$

therefore, from the spectral representation definition of $\underline{YY}^{+}$, we have

$$\begin{aligned} \underline{p}_i &= \left[\sum_{k=0}^{N-1} \xi_k^{-1}\ \underline{\Psi}_k\ \underline{\psi}_k^{T} \right] \left[\sum_{j=0}^{K-1} h_{ij}\ \underline{\psi}_j \right] \\ &= \sum_{k=0}^{N-1} \xi_k^{-1}\ h_{ik}\ \underline{\Psi}_k \end{aligned} \tag{3.62}$$

where $i=0,1,...(K-1)$

This expression requires only the first N rows of $\underline{H}$ and the eigenvalues $\{\xi_K{}^2\}$and eigenvectors $\{\underline{\Psi}_K\}$of the N x N real symmetric matrix $\underline{YY}^{T}\ \underline{YY}$ as discussed in Ref. [3-31 , pp. 224; 3-32]. A number of numerical methods exist for finding eigenvalues and eigenvectors of real symmetric matrices. We mention the names of two such methods used in the computation ,the details of which are available in the Ref. [3-33,34]:

1. The Jacobi Method [3-33]

and 2. The Givens-Householder Method [3-34]

Both methods are iterative and make use of orthogonal transformation, as a basic tool, thereby reducing the given matrix to a simplified form.

A detailed description of the Jacobi method along with Computer program in Fortran IV is given by Greenstadt [3-33] whereas the Givens-Householder method has been explained at length by Ortega [3-34] Library subroutines are also available ao CDC 6600 and IBM 360 computers.

3.7 ORTHOGONAL MATRIX (HADAMARD MATRIX):

In section (3.3) we indicated about the definite choice of an orthogonal matrix which has been lacking in [3-6] . Out of an infinite number of orthogonal matrices we select a special one called Hadamard Matrix ($\underline{H}$) because of its binary nature.

A Hadamard matrix ($\underline{H}$) is a square matrix whose elements are ONES and MINUS ONES and whose row (column) vectors are mutually orthogonal [3-35] . For example (a) and (b)

$$\text{(a)} \quad \underline{H}_0 \overset{\Delta}{=} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}_{2x2}$$

$$\text{(b)} \quad \underline{H}_1 \overset{\Delta}{=} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}_{4x4}$$

are the two Hadamard matrices of (2x2) and (4x4) order respectively. It is clear from the definition of these matrices that one may

(1) interchange rows

(2) interchange columns

(3) change the sign of every element in a row

and (4) change the sign of every element without disturbing the Hadamard property.

If $\underline{H}_1$ and $\underline{H}_2$ are equivalent Hadamard matrices, then $\underline{H}_2 = \underline{P}\ \underline{H}_1\ \underline{Q}$ (*)

---

* All notations, except $\underline{H}$, are independent of the similar notations used previously.

where $\underline{P}$ and $\underline{Q}$ are monomial permutation matrices of +1's and -1's. By this we mean that $\underline{P}$ and $\underline{Q}$ have exactly one non zero entry in every row and in every column, and this non-zero entry is +1 or -1. $\underline{P}$ gives the permutation and change of sign of rows; $\underline{Q}$ of columns. Given an $\underline{H}$ matrix, we can always find an equivalent matrix whose first row and first column consist entirely of +1's. Such a Hadamard matrix is called normalized.

3.7.1 CONSTRUCTION OF HADAMARD MATRICES:

Very little work has been reported [3-35, 40] on Hadamard matrices. Our work, although independent resembles the works of Paley [3-37] and Williamson [3-39] . The order of $\underline{H}$ matrix plays an important role in the mathematical analysis, therefore we shall investigate the conditions for orders under which we may construct $\underline{H}$ with all its elements ±1. Let K be the order of the matrix, then apart from the exceptional cases K = 1, 2, it is necessary that K should be divisible by 4. To prove this let the matrix $\underline{H}$ be represented by

$$\underline{H}_{i,j} \quad (0 \leq i \leq K-1, 0 \leq j \leq K-1, \quad K \geq 3)$$

then
$$\sum_{j=0}^{K-1} [\underline{H}_{1,j} + \underline{H}_{2,j}] [\underline{H}_{1,j} + \underline{H}_{3,j}] = \sum_{j=0}^{K-1} \underline{H}^2_{1,j} = K \quad (3.63)$$

Since
$$[\underline{H}_{1,j} + \underline{H}_{2,j}] [\underline{H}_{1,j} + \underline{H}_{3,j}] = 4 \text{ or } 0$$

We see that it must be 4 or ¼K values of j, and that K must be divisible by 4. It has been shown that, whenever K is divisible by 4, it is possible to construct a $\underline{H}$ matrix of order K($\leq$ 200) composed of ±1 except when K=116, 156, 188 [3-35] . For those orders which are also powers of two, the construction can be done recursively by starting with $\underline{H}_0$ iteratively as

$$\underline{H}_{k+1} \triangleq \begin{vmatrix} \underline{H}_k & \underline{H}_k \\ \underline{H}_k & -\underline{H}_k \end{vmatrix}$$

However, when K is not a power of 2 then the construction of $\underline{H}$ is somewhat more complicated (since it requires the notions of finite field, Legendre symbol, prime, quadratic residue etc.). It is possible to construct Hadamard matrices of the following orders K (in this list, p is an odd prime)

I $K = 2^r$

II $K = p^r + 1 \equiv 0 \pmod 4$

III $K = h(p^r+1)$, $h \geqslant 2$, order of $\underline{H}$-matrix

IV $K = h(h-1)$, h a product of numbers of forms I and II

V $K = h(h+3)$, h and h+4, both products from I and II

VI $K = h_1h_2(p^r+1)p^r$, $h_1 > 1$ and $h_2 > 1$, orders of Hadamard matrices.

The construction of these matrices are described in the appendix.

3.8 COMPUTER SIMULATION:

The NRDFE rapid (one-step) initialization theory developed in this chapter was tested by using a digital computer. A brief description of the simulation is given here. Two discrete time channels, one theoretical and another practical, have been used to verify the technique developed here.

The simulation, written in Fortran IV for CDC 6600 computer consists of a main routine called MAIN, and a number of subroutines which together simulate a transmitter, D.T. channel, and the NRDFE. A GRAPH subroutine has been used for producing graphical output of the equalizer performance. MAIN also uses two other subroutines, namely FLTSET and HADGEN to compute the initial multiplier settings in accordance with Eq. (3.62). Each of these, in turn calls two or more subroutines as shown in Fig.(3.4). Two numerical methods for finding eigenvalues and eigenvectors of real symmetric matrices are described. One is used by the IBM library subroutine EIGEN whereas the other by UNIVAC library subroutine TRIDMX, EIGVAL, and EIGVEC.

3.8.1 DESCRIPTION OF PROGRAM MAIN:

The program MAIN consists of two basic parts:

(a) The first part simulates the transmission of an initializing pulse through the D.T. channel and performs the NRDFE initialization based on the theory developed in this chapter.

(b) The second part generates the transmitter message sequence and uses the initialized equalizer to adapt the channel. This is well illustrated by the flow chart diagram shown in Fig. (3.5).

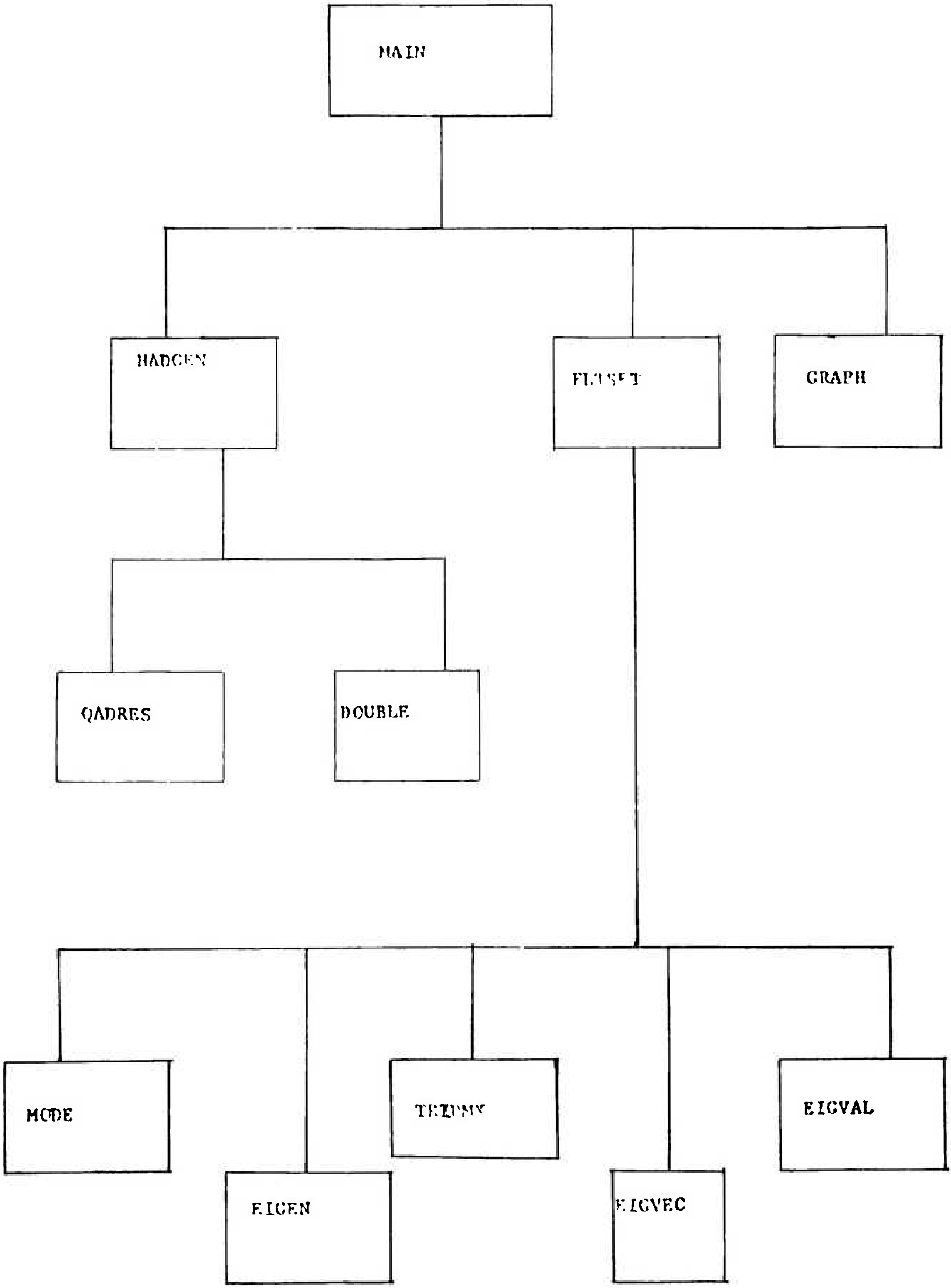


FIG. (3.4) BLOCK DETAILS OF COMPUTER PROGRAMS USED.

FIG.(3.5) DIGITAL COMPUTER SIMULATION FLOW-CHART DIAGRAMS

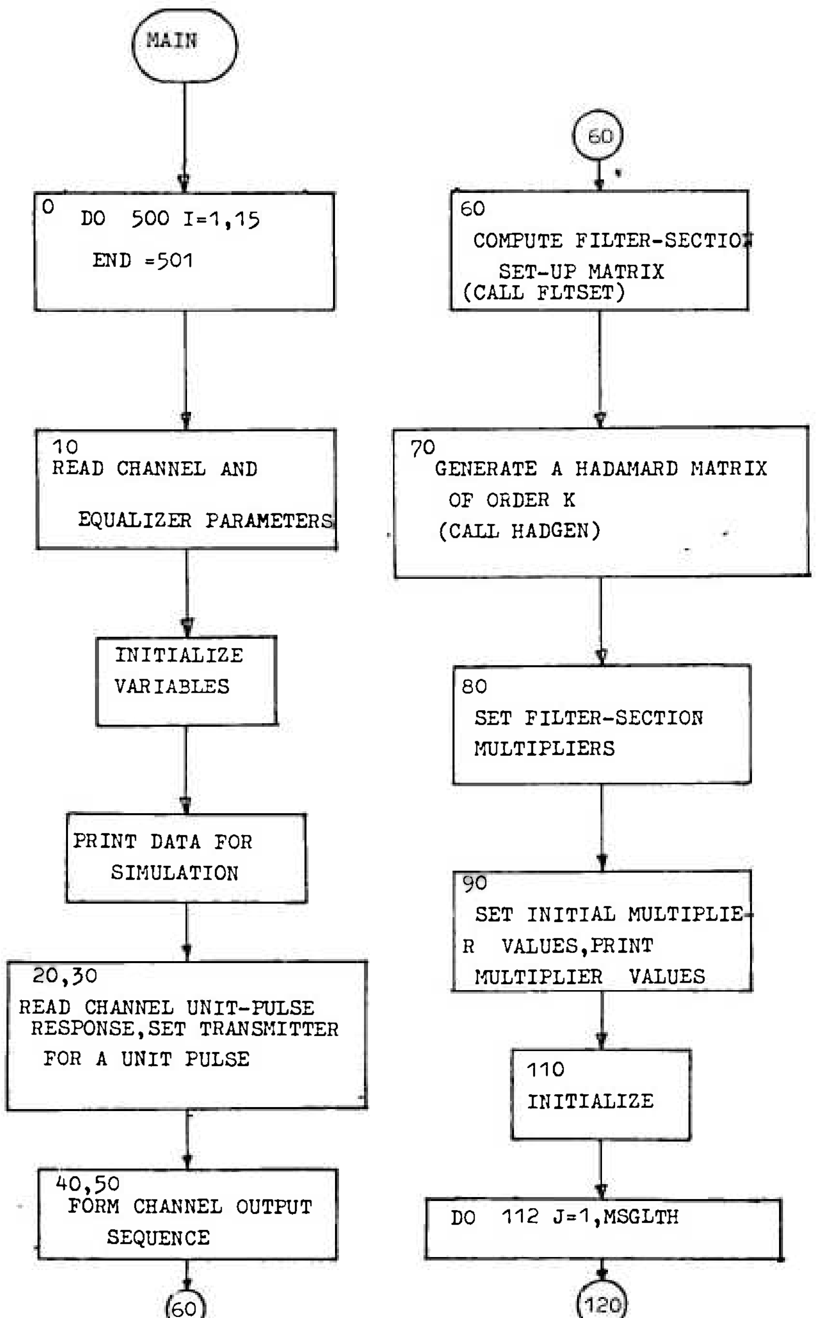
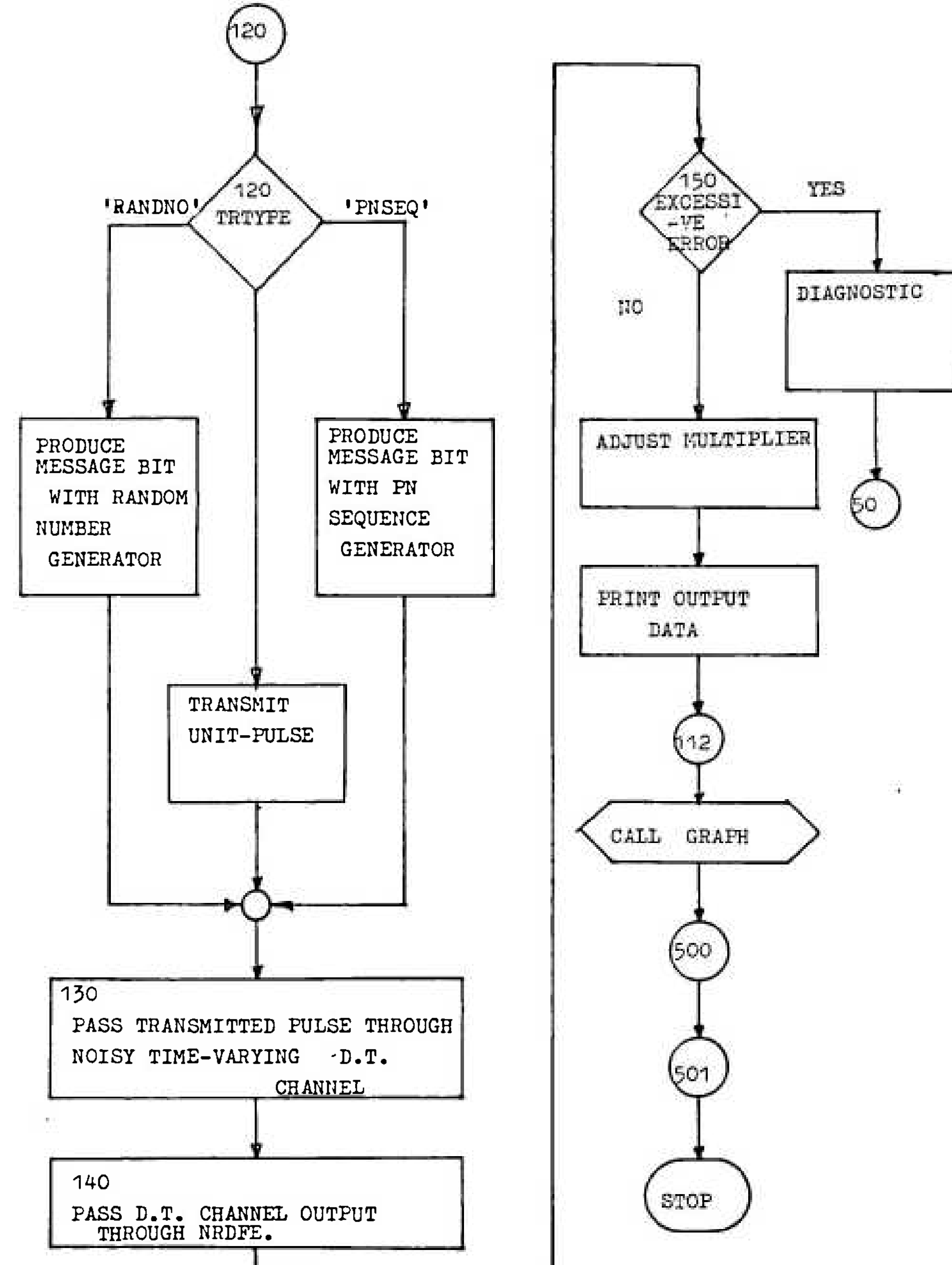
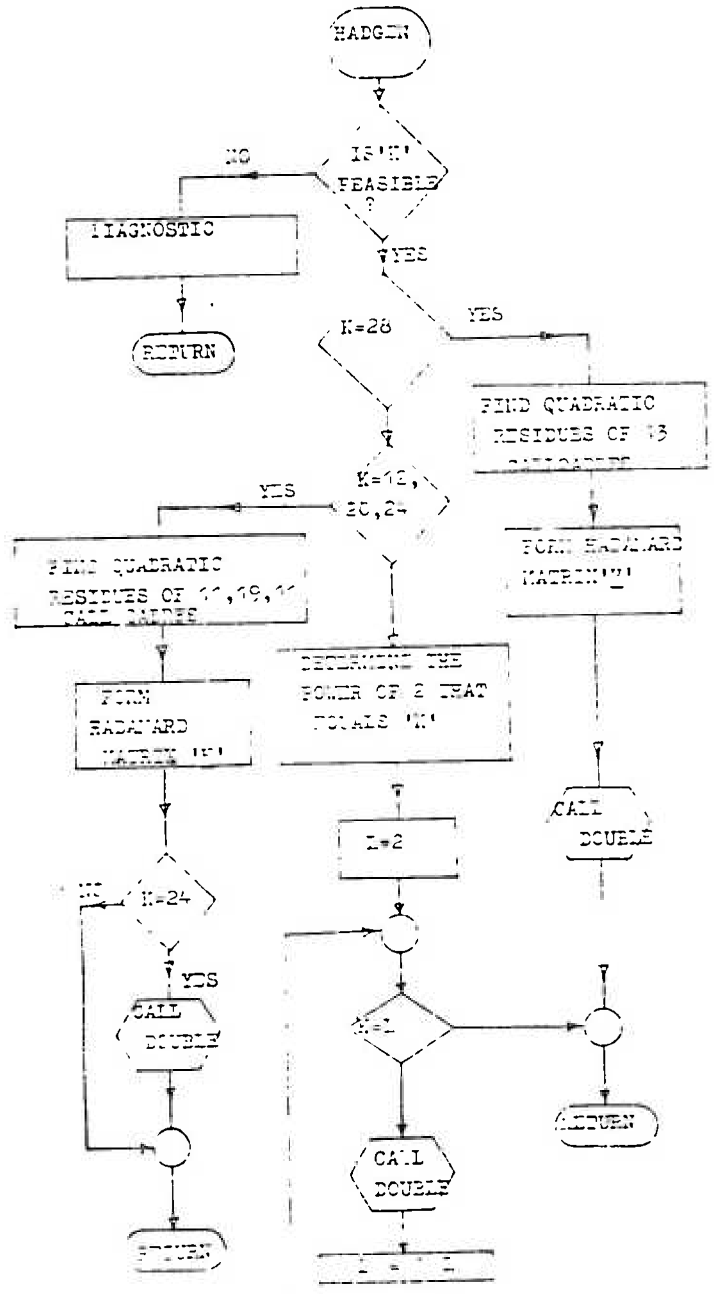


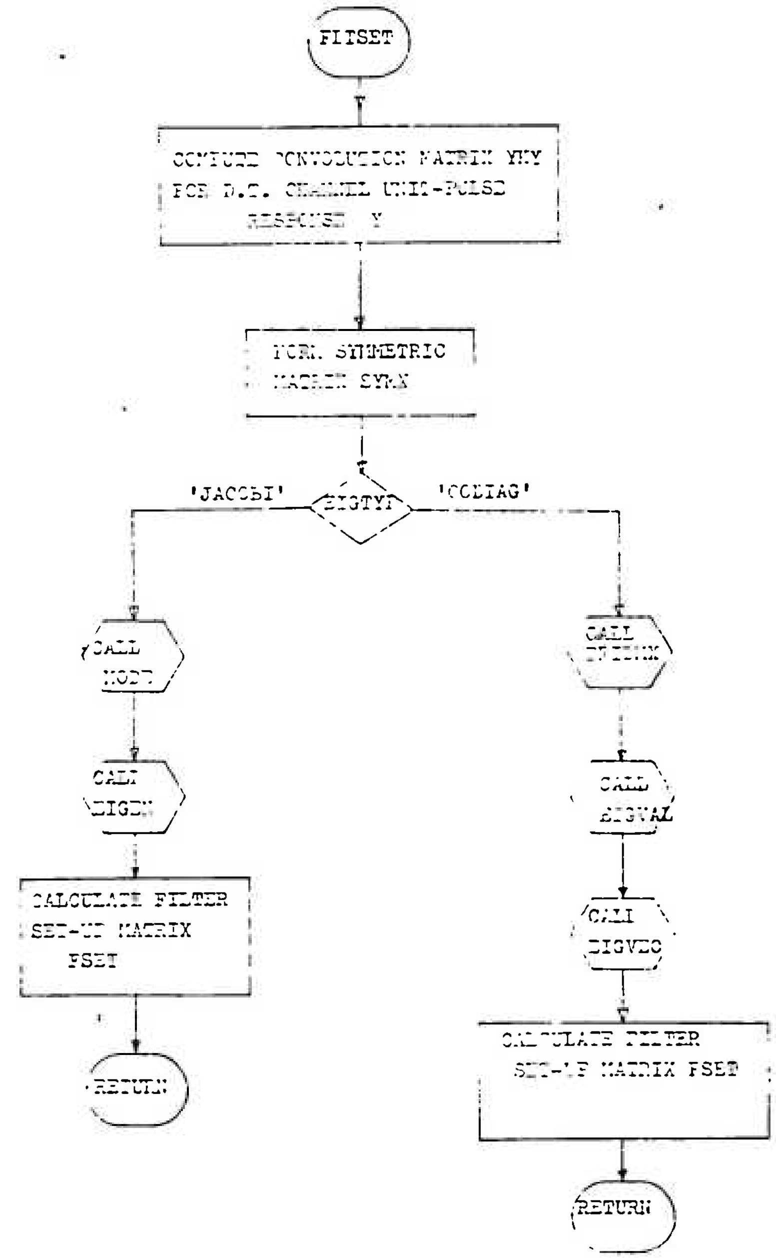
FIG.(3.5) CONTINUED



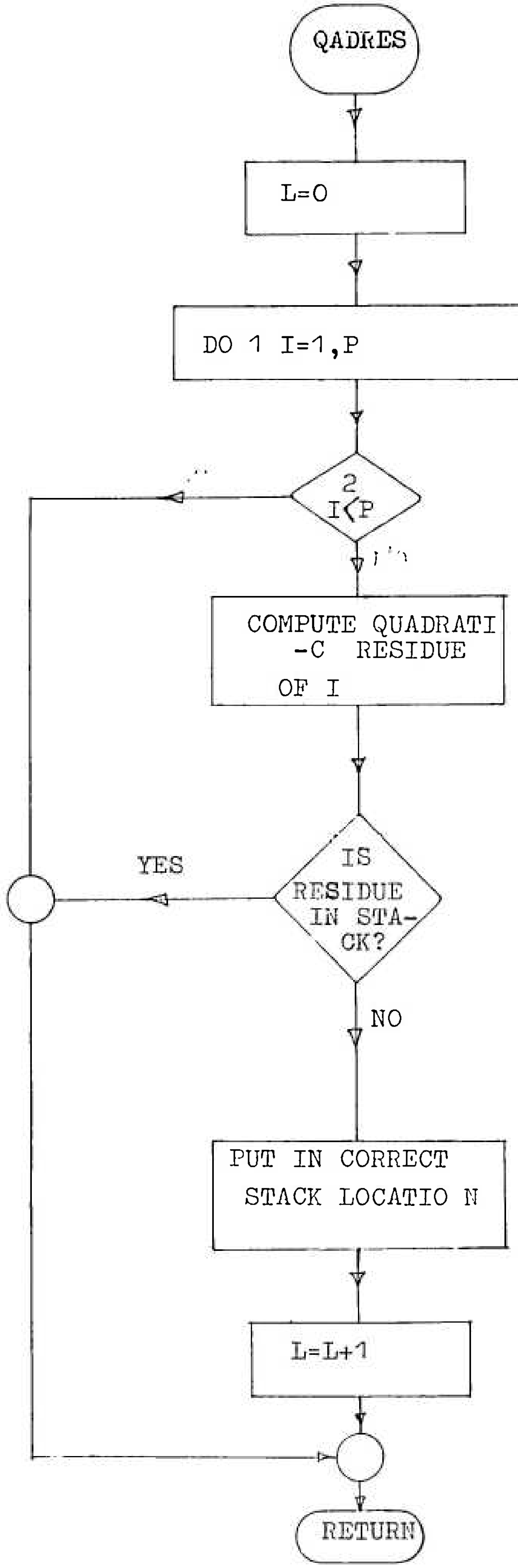
SUBROUTINE HADGEN



SUBROUTINE FLTSET



SUBROUTINE QADRES



For reasons of continuity a few parameters, usually pertaining to small blocks of code, are set locally as they occur rather than at the beginning of the program.

A concise description of the program is found in block 0. The program is designed to repeat until no further data cards are found, at which time it terminates normally. After reading the simulation parameters and initializing variables in blocks 0-30, the channel output sequence $\{Y\}$ is formed in blocks 40-50 by passing an initializing pulse through the noisy channel (CHNL, G, CNOISE). Block 60 calls the subroutine FLSET which returns in common the eigenvalue and eigenvector information (FSET) for computing Eq.(3.62). In block 70, an (KxK) Hadamard matrix $\underline{H}$ is formed by subroutine HADGEN, and the matrix $\underline{P}$, whose columns are the filter-scetion multiplier coefficients is then computed as the product of FSET and $\underline{H}$. The NRDFE is now initialized in block 80 by computing the multiplier corrections (DELALF), then adding the result to then existing multipliers (ALF) in accordance with Eq.(3.9).

The remaining part of MAIN, starting with a description in block 100, is essentially a large loop that iterates once for each transmitted pulse. All coding inside the loop has been done in-line i.e., there are no subroutines, in order to reduce the program execution time. After a number of variables are initialized in block 110, the loop begins. A transmitter pulse (DD) is generated in block 120 using either a pseudo-noise sequence (TRTYPE='PNSEQ') or a random binary number (TRTYPE='RANDNO') generated by quantizing random numbers that are uniformly distributed between 0 and 1. If no transmitter type is specified then a unit pulse is transmitted. In block 130 the pulse is next passed through the channel which is corrupted by gaussian noise. Since time variation in communication channels is rather arbitrary, this effect is simulated by inserting a user supplied description immediately after staement 130. The transmitted pulse (YY) is then filtered by the equalizer in block 140 to produce the NRDFE output (I). The average MSE/Symbol (ASME) and total (cumulative) MSE(TMSE) are next calculated and the symbol error (E) is compared against the maximum acceptable error value (ELIM). If the maximum value exceeded the limit an error message is printed and the NRDFE either goes for reinitialization by returning to block 50, or the run can be terminated by transfer to 500. Otherwise at block 150 the NRDFE multiplier coefficients are adjusted in accordance with Eq. (3.62) and, when indicated by a local parameter (NLINE), a line of data concerning the iteration is printed before repeating the loop for the next transmitter symbol. After the indicated number (MSGLTH) of information symbols has been transmitted, the total number of symbol errors is printed and GRAPH is called to plot the frequency of occurrence of each value of the NRDFE output.

3.8.2 DISCRETE TIME CHANNEL DESCRIPTION:

We simulated a large number of channels. However, for convenience two of the channels are described here.

D. T. CHANNEL 1:

This channel is a low pass filter (LPF) having a normalized cut-off frequency of 1.0 Hz and a 40 dB/decade roll-off . It can be described mathematically by the Fourier Transform pair

$$G_1(f) = (1 + jf)^{-2} \leftrightarrow g_1(t) = 4\pi^2 t\, e^{-2\pi t}, \quad t \geqslant 0$$

Samples of the unit pulse response, taken at 10 msec interval, are given by {2.11, 2.25, 1.80, 1.28, 0.85, 0.55, 0.34, 0.210, 0.120, 0.07, 0.04, 0.02} . Normalizing the main pulse of 2.25, the unit pulse-response {0.938, 1.000, 0.800, 0.569, 0.378, 0.244, 0.151, 0.093, 0.053, 0.031, 0.018, 0.009} has been plotted in Fig. (3.6a). This is a hypothetical D.T. channel for actual data communications, nevertheless, it is useful to demonstrate the equalizer performance with non-negative samples which are relatively large with respect to unity. As can be seen from Fig. (3.6a) the normalized peak distortion Eq. (2.1) is 3.284, implying that the binary eye* is completely closed.

D. T. CHANNEL 2:

This channel model represents the unit-pulse response of a vestigial-sideband (VSB) amplitude-modulated data link composed of two carrier systems and 160 Km of loaded cable. When sampled at 0.4167 msec, corresponding to a transmisiion rate of 2,400 baud , the unit pulse response of length 9 is given by { -0.05, 0.05, -0.20, -0.05, 0.90, 0.12, 0.15, 0.05, 0.03} and is shown in Fig.(3.7a). The magnitude of the distortion depicted in Fig. (3.7a) is such that binary transmission would be marginal, the binary eye being impaired by some 14 dB, and multi-level operation is never possible without adaptive equalization. Unlike D.T. channel (1), this has relatively large number of precursors as well as nonlinear phase characteristics and is an example of typical British Public Switched Telephone Network (BPSTN) channel [3-27] .

---

* The " binary eye" is a convenient way of displaying on an oscilloscope the aggregate ISI over an element period when transmitting a wavetrain [3-26] . Binary eye is given by E = (1 - peak distortion)

Since, NOISE FREE channels are being considered, therefore, we programmed entire digital communication system by assuming SNR=80 dB on a per-bit basis, which means [$E_b/N_0 = 1 / \sigma^2$] .

An isolated unit pulse (1000 ...) was transmitted MSE was computed on the basis of algorithm (3.3). We studied the effects of three parameters involved;

(i) DELAY 'δ' : between the arrival at the equalizer of the first precursor of the channel unit-pulse response and the begining of the locally generated desired pulse.

(ii) N : the number of shift register stages in each filter section.

(iii) K : the number of filter sections. $K > N$

Finally we studied the equalization of the channels under investigation.

3.8.3 EFFECT OF DELAY (δ):

D.T. CHANNEL 1: [Fig. (3.6C)]

For the study of δ we worked through various equalizer orderings and finally settled on two particular orders which in a way exhibited the largest and smsllest equalizers:

Largest Equalizer Structure: K = 32, N = 20

Smallest Equalizer Structure: K = 24, N = 12

The graphs plotted [Fig.(3.6c)] show the effect on MSE of the delay ' δ ' 'δ ' (being varied) while keeping K and N fixed. These graphs show an interesting point that with K = 32, any value of delay less than six essentially reduces the MSE. Because of the large amplitudes of the first two samples of the channel output, either could be considered to correspond to the transmitted unit pulse. Hence the channel delay is small and could easily be zero, depending on which sample the equalizer chooses to correspond to the unity output. Due to the nature of the computed multipliers for this channel the smallest residual MSE values occur when the channel output sequence begins to enter the shift register, rather than after most of the postcursors are included as might be expected intuitively.

D. T. CHANNEL 2: [Fig.(3.7d)]

As before the graphs plotted show the effects of 'δ ' on MSE while maintaining K and N fixed. Equalizers with less than 6 delays seem quite appropriate to reduce the MSE to an acceptable level.

3.8.4 EFFECTS OF SHIFT REGISTER LENGTH (N):

In order to study the effects of shift register length (N) on MSE we assigned K=32 and nominal value to δ . On the arguments provided, the values of $\delta = 0.25$ N and $\delta = 0.50$ N for channel 1 and 2 respectively were found to give the lowest acceptable MSE. A basic question now arises, why should we have to assign a nominal value to 'δ ' at all? We can put forward the following arguments in support of providing the nominal value to 'δ '. We shall describe the filter section in the frequency domain.

Since each filter section is a NRDF with, in general, no symmetry in the set of multipliers $\{ p_i^{(k)} \}$, the phase versus frequency characteristics is nonlinear and the corresponding delay cannot be expected to be constant. And therefore the different frequencies will be delayed by different amounts. This nonlinearity is also referred to as the delay distortion which tends to cause relatively long precursors in addition to an average time delay of the signal maximum. Skewing of the signal peak is another phenomena attributed to this phase nonlinearity. The transmitted signal also undergoes amplitude distortion in the channel. This effect is due to nonuniform amplitude versus frequency characteristics in both the channel passband and the stopband,and it causes dispersion of the transmitted pulse with the resulting precursors. Based on these arguments we assigned $\delta = 0.25\ N$ for D. T. channel 1 since this channel exhibits phase linearity over most of the passband. A nominal value of approximately $\delta = 0.50\ N$ for D. T. channel 2 was assigned in view of the loading and modulation.

Finally, after simulation runs with these values of δ , we found that the larger values of N produce better performance. [Figs. (3.6d) and (3.7e)]

3.8.5 EFFECTS OF FILTER SECTIONS (K):

In order to study the effects of filter sections (K) on the MSE both channels were simulated and the results clearly indicated the fact that the larger values of K is needed to reduce the MSE [Figs. (3.6e) and (3.7f)].

3.8.6 EQUALIZATION:

Finally both the channels were successfully equalized [Figs. (3.6b), (3.7b), (3.7c)]. As could be seen that the channel 1 was easy to equalize and attained convergence quickly ($\delta = 0$) with smaller equalizer structure whereas channel 2 required longer to converge

(δ = 14, 10) and needed longer structures.

This is conceptually satisfying since the channel 2 [Fig. (3.7a)] exhibits both positive and negative sidelobes, thereby introducing a cancellation effect, which is absent in channel 1 [Fig. (3.6a)] where all the sidelobes are positive.

3.9 SUMMARY AND COMMENTS:

(A) We have developed and designed independently a new equalizer for fast data communication which affords considerable reduction of the ISI for the noise free channels [assumed SNR = 80 dB] considered.

(B) All the channels were successfully equalized even when N was less than L, the channel unit pulse response. This is an important observation because the ammount of computation required is N^3. This would have been difficult to achieve with ordinary NRDFE.

(C) The maximum shift register length N is clearly limited by the digital computer used for the initialization computation. The ammount and speed of digital hardware available for the shift register, multipliers, summers, limits N, and K as well.

(D) Good estimates of the delay δ is required. But, this in turn require some a priori knowledge of the channel characteristics or some preliminary experimental data. D. T. channels are random in nature therefore larger values of N provides a good solution in the absence of good estimates of δ . However, it is conjectured that the value of δ between 0.25 N and .85 N provides a good estimate for all kinds of D. T. channels.

(E) We have outlined in this chapter first the importance of Moore Penrose Pseudoinverse matrix and shown that MPPI provides a best estimate multiplier values;second, we have shown that when the signals are linearly dependent, the matrix $\underline{\mathring{A}}$ singular, then the signals can be orthogonalized and very fast convergence (one step initialization) can be achieved by solving an orthogonal matrix ($\underline{H}$) called Hadamard Matrix. The idea that the communication systems resulting in singular correlation matrix ($\overset{\circ}{\underline{A}}$) cannot be solved has been succesfully challenged.

However, it could be said that this equalizer is general in nature and the design of it may be approached from quite different points of view.[1-91]

DISCRETE TIME CHANNEL 1.

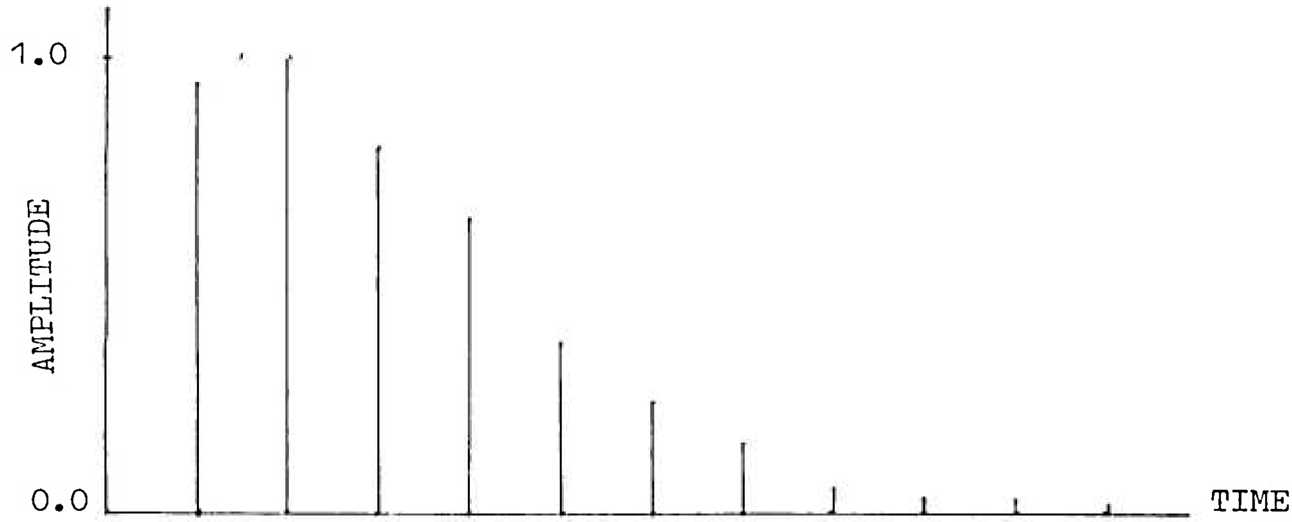


FIG.(3.6a)IMPULSE RESPONSE OF D.T. CHANNEL 1.

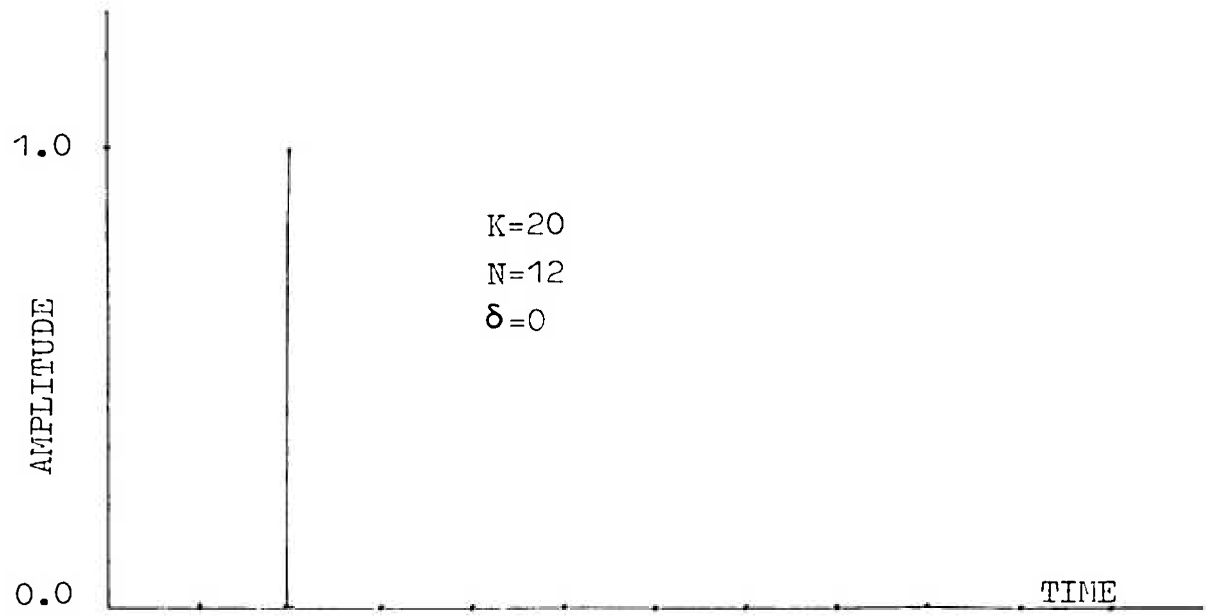


FIG.(3.6b) IMPULSE RESPONSE AFTER EQUALIZATION USING NRDFE(GENERALIZE
(K=20,N=12,δ=0)

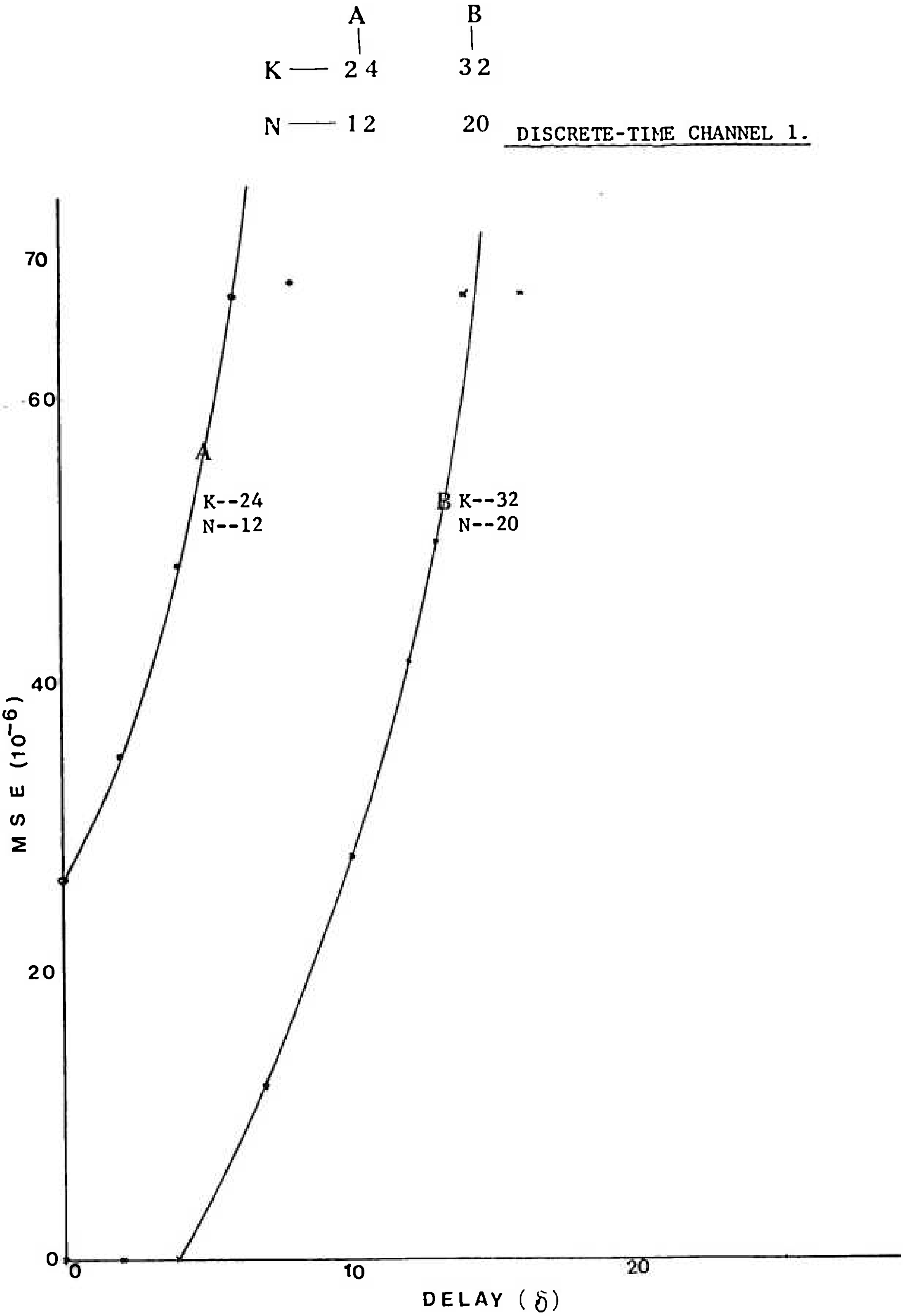


FIG. (3.6c) EFFECTS OF DELAY.

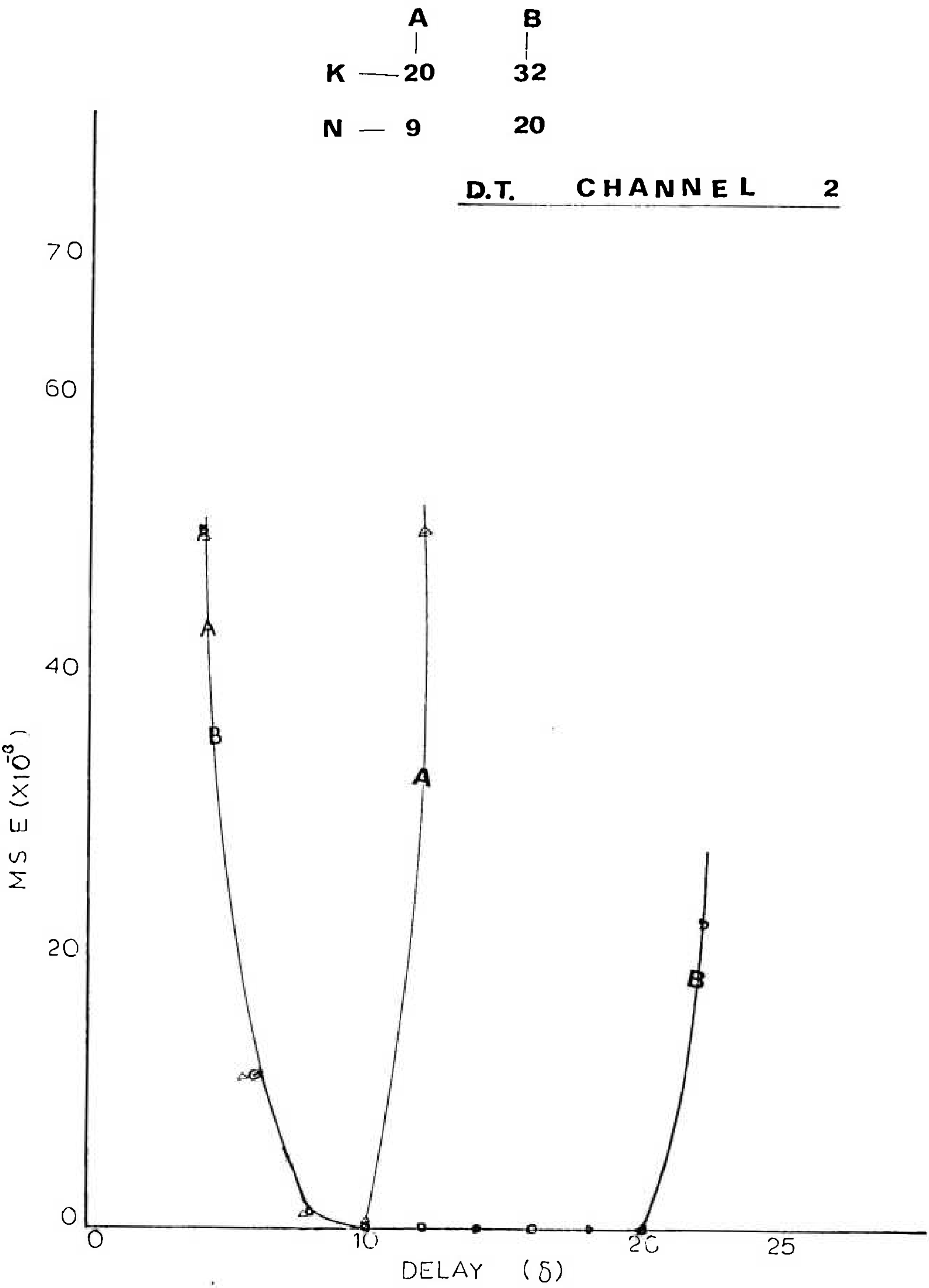


FIG. (3.7d) EFFECTS OF DELAY

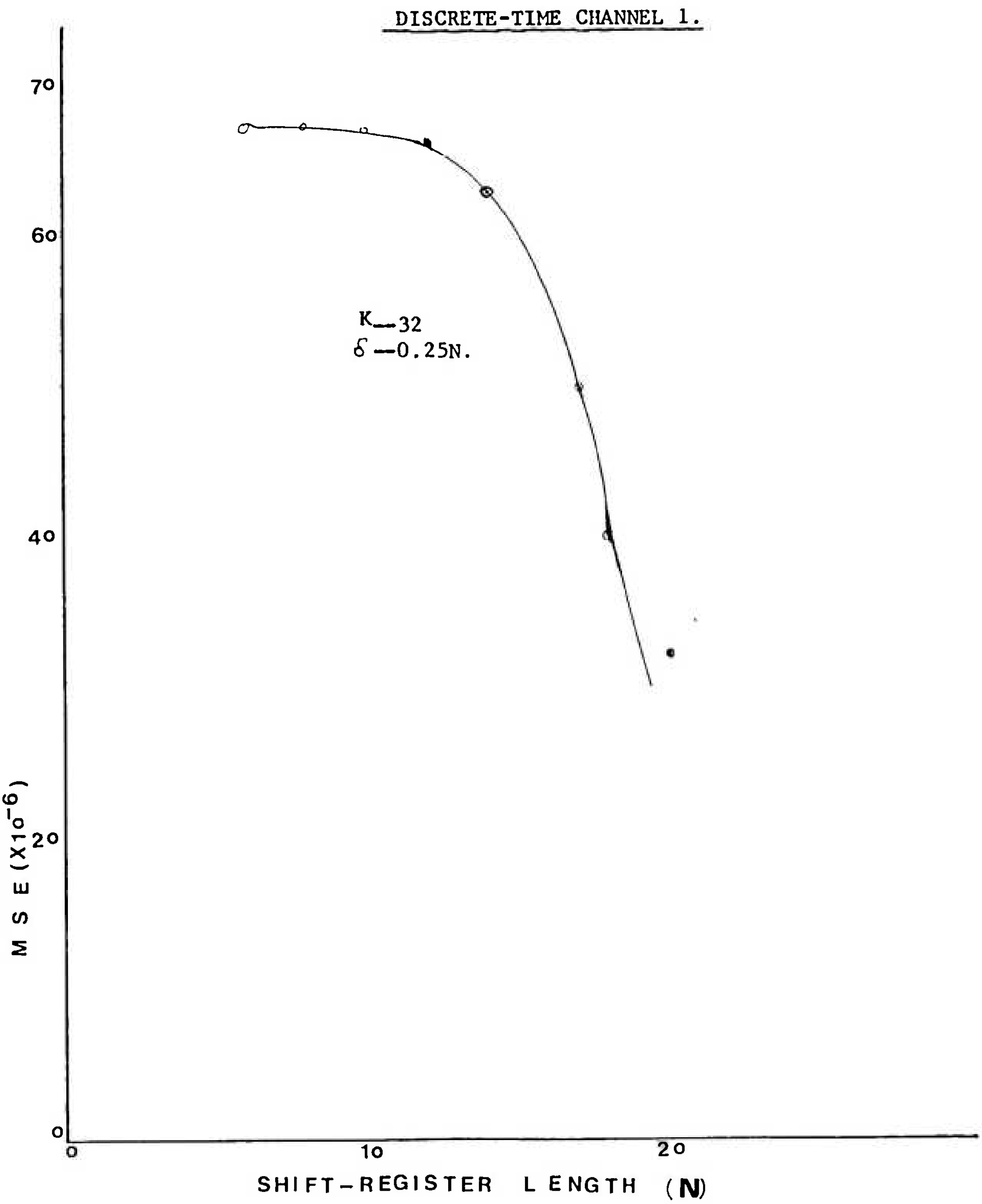


FIG. (3.6d) EFFECTS OF SHIFT-REGISTER LENGTH (N).

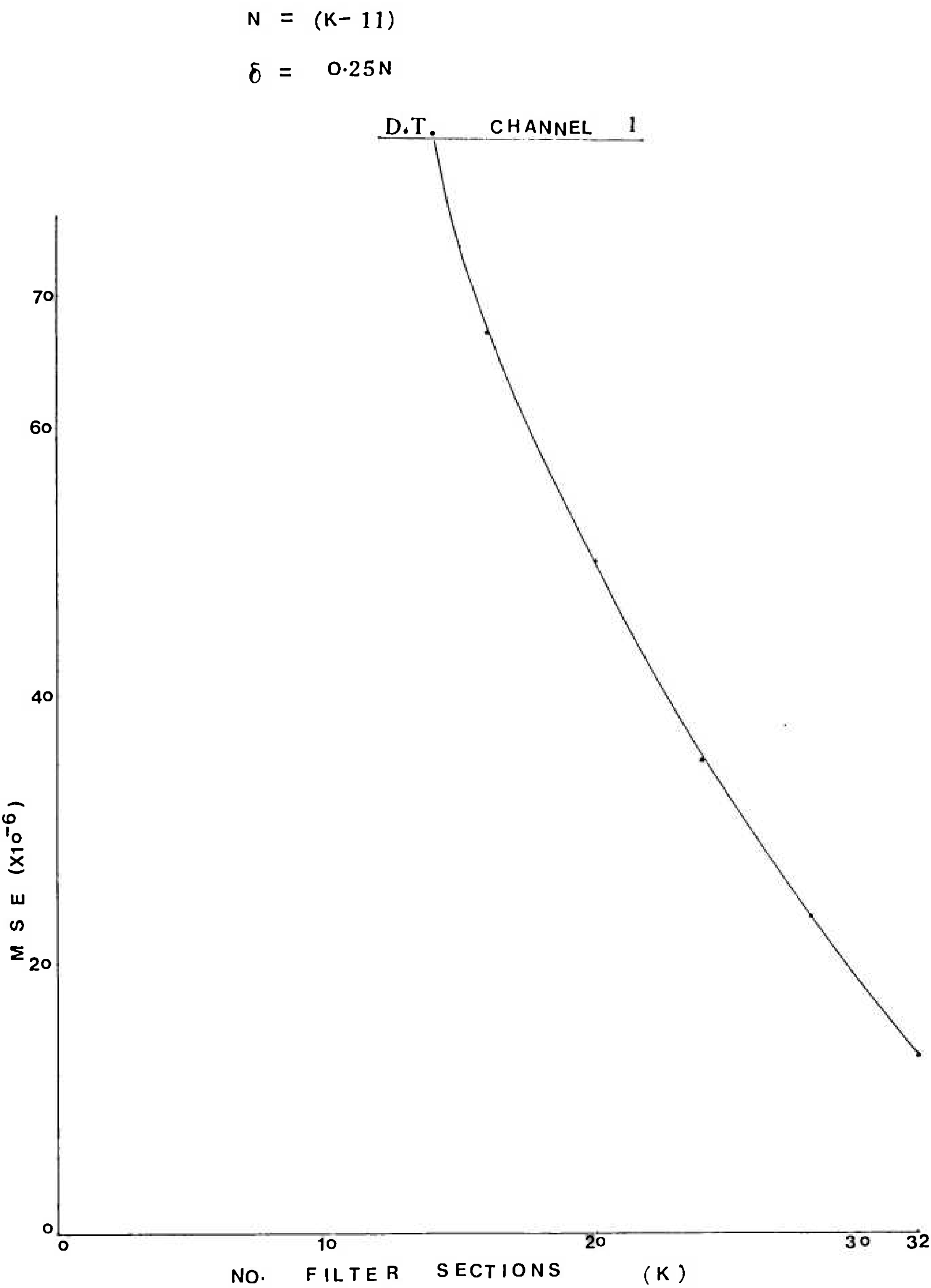


FIG.(3.6 e) EFFECTS OF NUMBER OF FILTER SECTIONS

DISCRETE TIME CHANNEL 2.

VSB 2,400 BAUDS 2-CARRIER SYSTEMS 160 KILOMETRE LOADED CABLE.

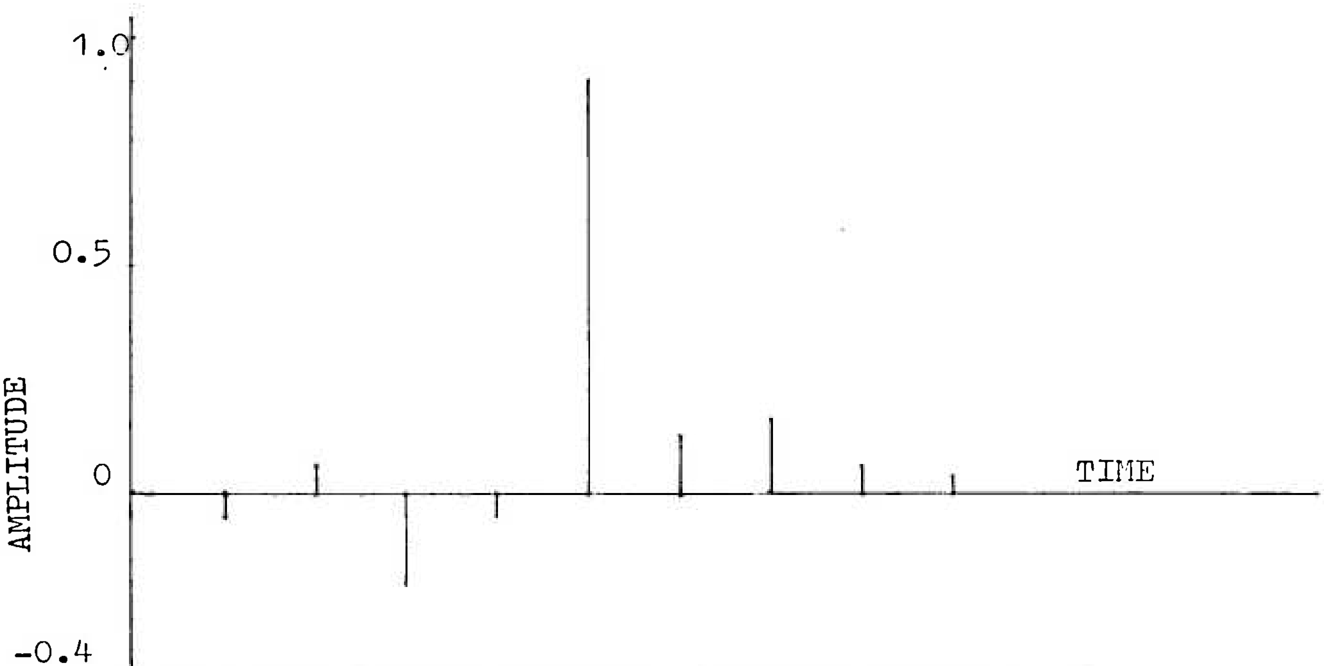


FIG.(3.7a) IMPULSE RESPONSE OF D.T. CHANNEL 2.

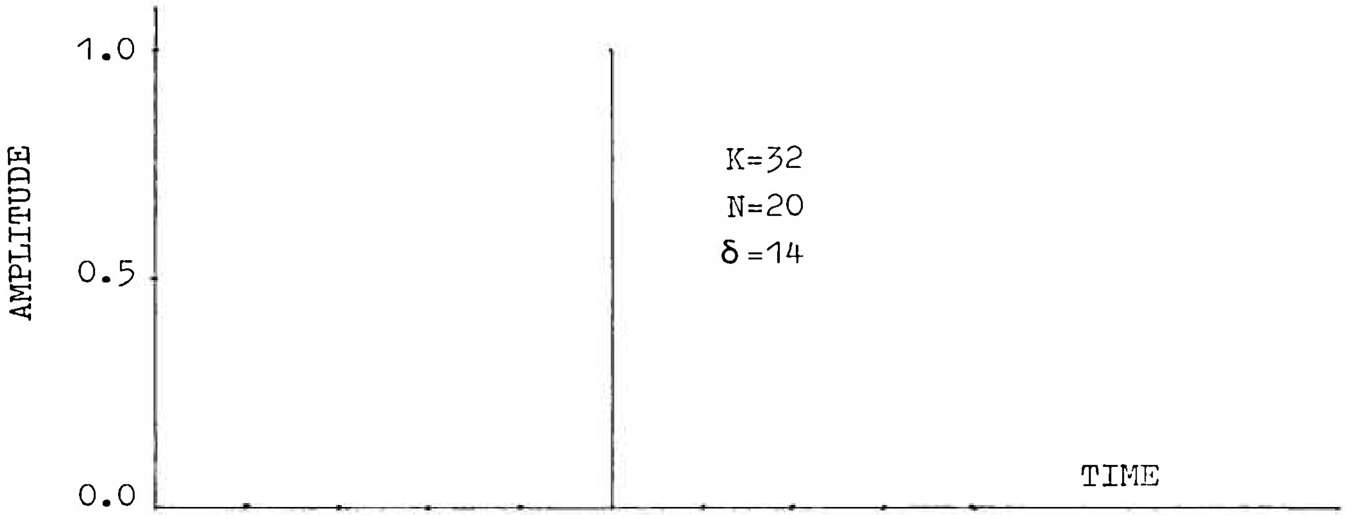


FIG.(3.7b) IMPULSE RESPONSE AFTER EQUALIZATION USING NRDFE (GENERALIZED) (K=32,N=20, δ=14)

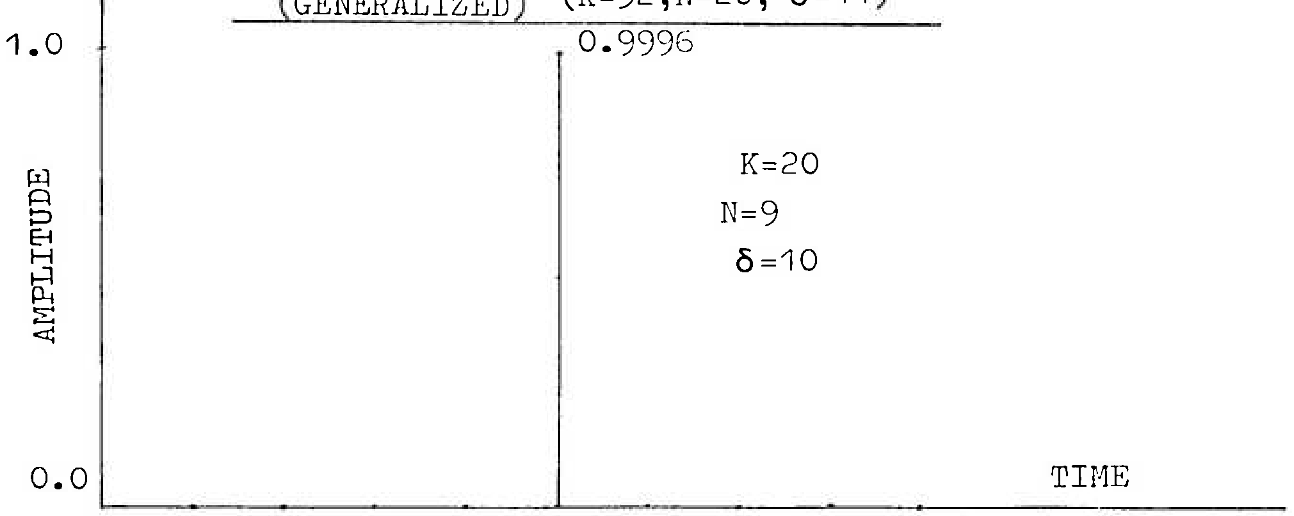


FIG.(3.7c) IMPULSE RESPONSE AFTER EQUALIZATION USING NRDFE(GENERALIZED) (K=20,N=9, δ =10)

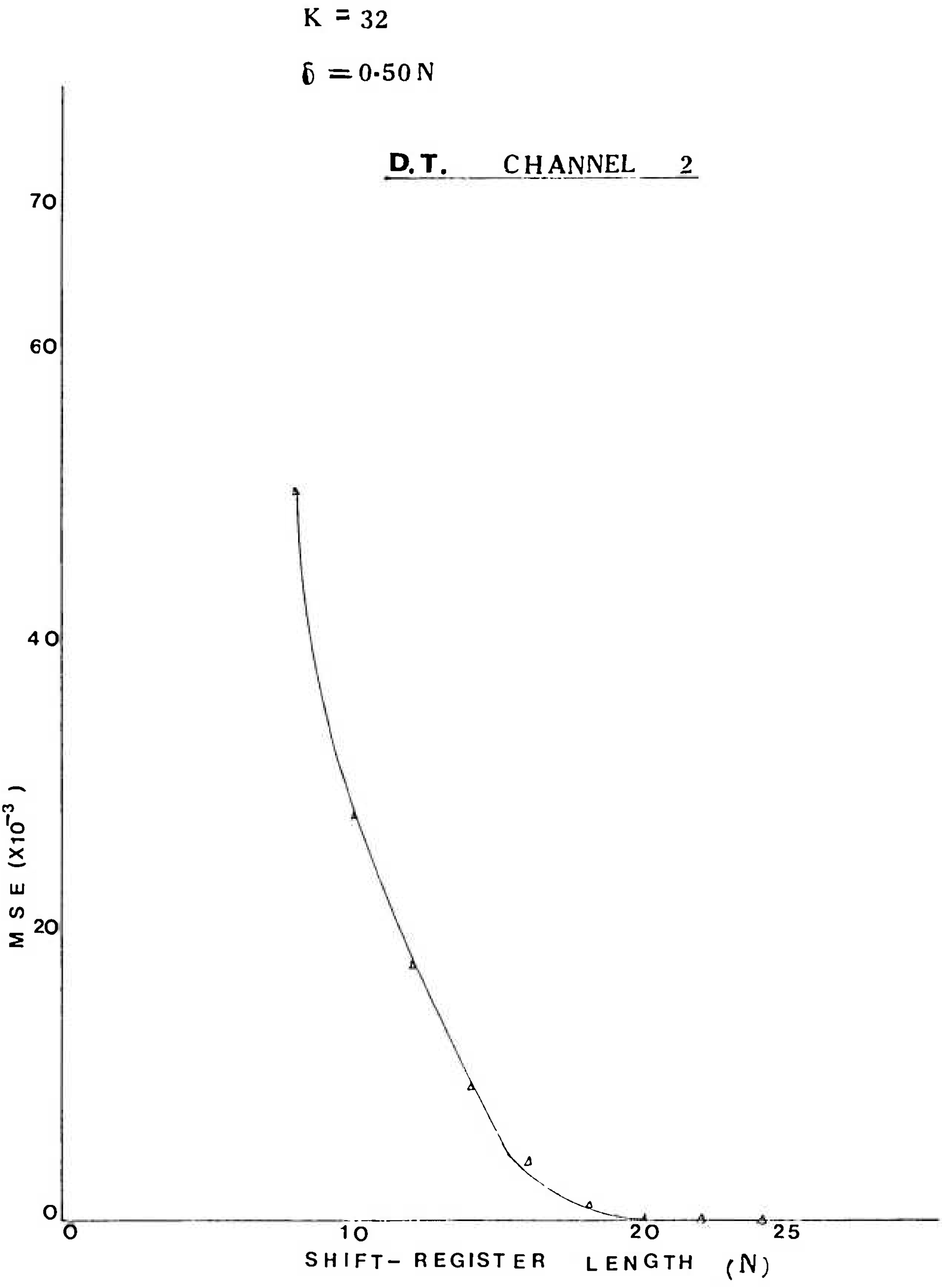


FIG.(3.7 e) EFFECTS OF SHIFT-REGISTER LENGTH

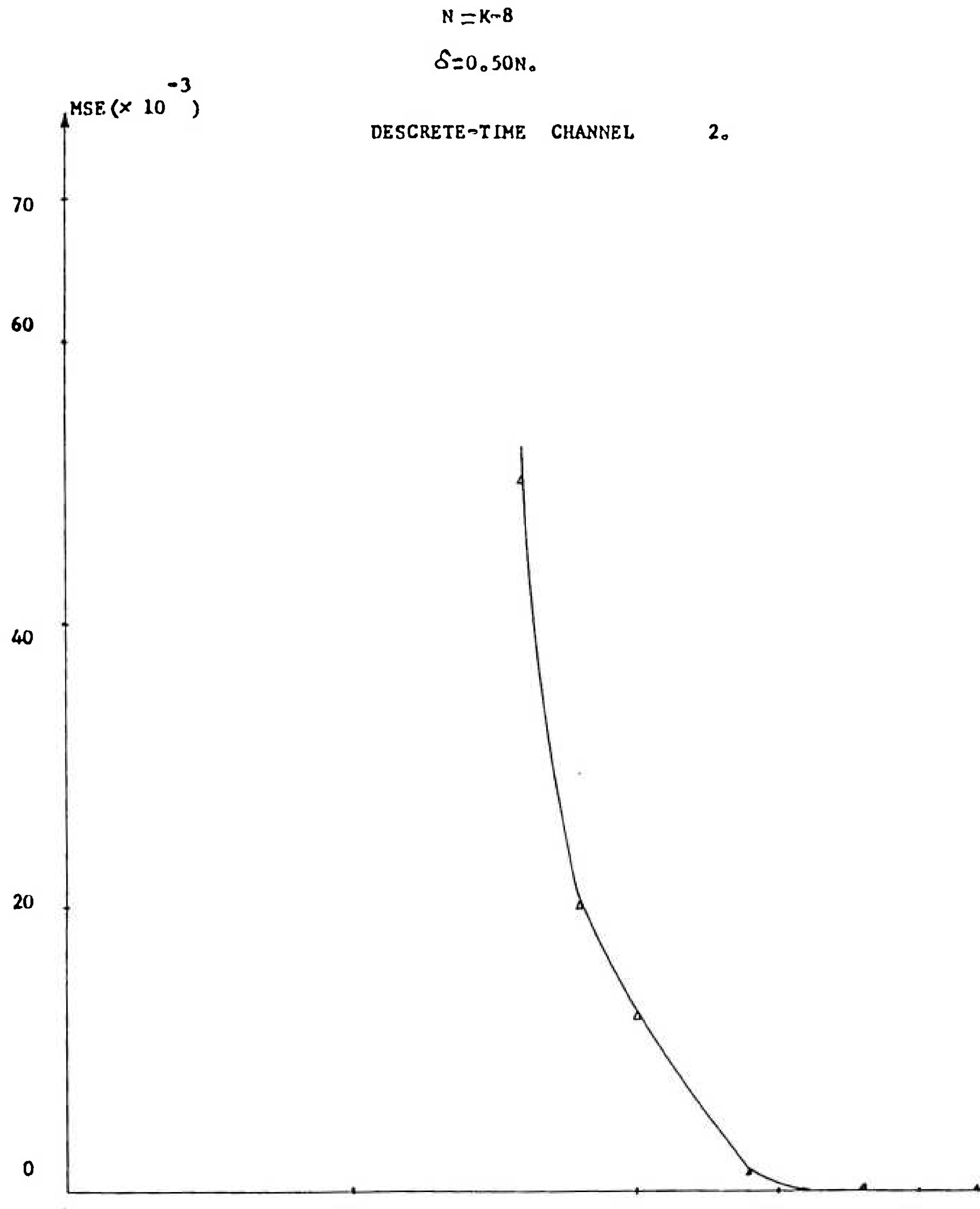


FIG. (3.7f) EFFECTS OF NUMBER OF FILTER SECTIONS.

CHAPTER 4

REAL - TIME ADAPTIVE OPERATION OF THE FAST INITIALIZED GENERALIZED NON-RECURSIVE DIGITAL FILTER EQUALIZER(NRDFE) IN NOISY ENVIRONMENT.

Satyam, Shivam, Sundaram

(The Truth is Beauty.)

A SANSKRIT PROVERB.

Lest men suspect your tale untrue,

Keep probability in view.

JOHN GAY (1727)

4.1 INTRODUCTION:

In chapter (3) we developed and designed a one-step initialized non-recursive digital filter equalizer. In this chapter, the aim is to modify the design to make it applicable for real-time adaptive operation in the presence of additive noise. Chapter (2) contains some of the basic ingredients of this chapter(e.g. Widrow's estimate). The relevant digital communication system can be shown as Fig.(4.1)

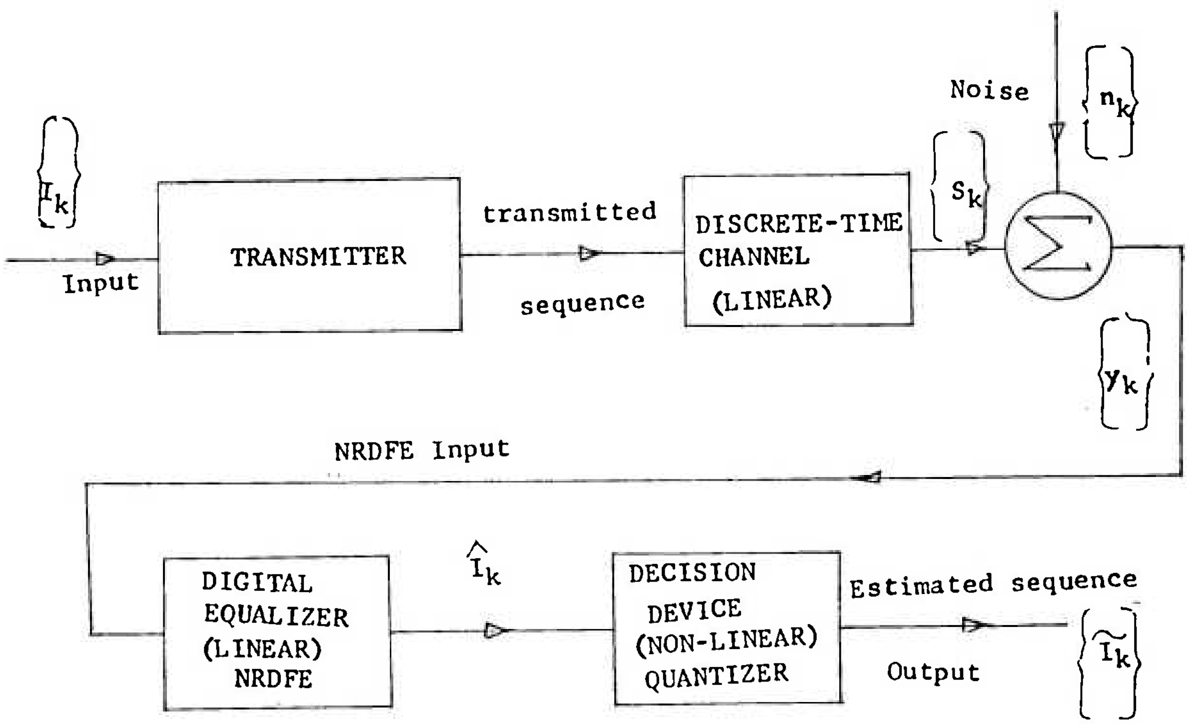


FIG. (4.1) NOISY DIGITAL COMMUNICATION SYSTEM.

Since it is impractical to compute the exact gradient of the system shown in Fig.(4.1), so a simple method of estimating this quantity is presented. Some convergence properties of the resulting algorithm are then developed and an eigenvalue condition is obtained. This eigenvalue condition ensures convergence in the stochastic sense under the stated assumptions.

When these assumptions are violeted the possibility of divergence exists and then a simple diagnostic determines the need for reinitialization. Finally the effect of noise on the initialization procedure and the on-line adaptation is studied.

4.2 FUNDAMENTALS:

In chapter (3),fast initialization was achieved for a noise-free channel in one iteration only using an isolated test pulse.It was, therefore,quite feasible to compute the gradient of $J(\underline{\alpha})$.

However,the present situation is not so straightforward due to the following reasons:

(i) An equalizer is to be used for real-time adaptive signal processing using decision-directed mode in the presence of noise.

(ii)Due to the presence of noise,the equalizer output sample is a discrete random variable,therefore,the need for some kind of statistical averaging arises

and, (iii)Signal pulses are no longer isolated at the channel output,therefore,it is necessary to distinguish between error samples corresponding to each pulse transmitted.

Since in real-time operation we need to adjust the equalizer iteratively for each pulse transmitted,it will be convenient to denote the error ε, at iteration k by $\varepsilon(k)$ rather than $\varepsilon^{(k)}$ used previously.Similar pattern will follow for other variables as well.

From Eq. (2.22)

$$\varepsilon(k)=\hat{I}(k)-\tilde{I}(k) \tag{4.1}$$

using the MSE as the performance criterion we define

$$J(\underline{\alpha}) = E\{\varepsilon^2(k)\} \tag{4.2}$$

The on-line algorithm for adjusting the multiplier coefficients is given by:

$$\underline{\alpha}(k+1) = \underline{\alpha}(k)+ \Delta/2\ \underline{g}(k) \tag{4.3}$$

where $\underline{g}(k) \triangleq$ estimated gradient vector of $J\{\underline{\alpha}(k)\}$ with respect to $\underline{\alpha}$.

$\Delta_s \triangleq$ scaler constant controlling the rate of convergence and stability.

Differentiating Eq.(4.2), we get the gradient

$$\underline{g}(k) = 2\,E\{\varepsilon(k)\nabla\varepsilon(k)\} \tag{4.4}$$

where $\nabla\varepsilon(k) \triangleq \frac{\partial\varepsilon(k)}{\partial\underline{\alpha}}$

We define the filter section output vector $\underline{v}(k)$ as

$$\underline{v}(k) \triangleq \{v_0(k)\ v_1(k)\ \ldots\ldots\ v_k(k)\}^T \tag{4.5}$$

Therefore, using inner-product notation,

$$\hat{I}(k) = \langle\underline{\alpha}(k),\underline{v}(k)\rangle = \underline{\alpha}^T(k)\underline{v}(k) \tag{4.6}$$

and $$\varepsilon(k) = \langle\underline{\alpha}(k),\underline{v}(k)\rangle - \tilde{I}(k) \tag{4.7}$$

From Eq. (4.4) and Eq.(4.7), we obtain

$$\underline{g}(k) = 2E\,\{\varepsilon(k)\ \underline{v}(k)\} \tag{4.8}$$

$$= 2E\,\{\underline{v}(k)\underline{v}^T(k)\ \underline{\alpha}(k)\} - 2E\{\ \tilde{I}(k)\underline{v}(k)\}$$

From an implementation point of view Eq. (4.8) is a convenient quantity. Because the signal vector $\underline{v}(k)$ is readily available and the error is also known, however, a difficulty still remains in that the expected value is not available in real time and, therefore, must be estimated by averaging over a finite number of symbols. To circumvent this difficulty the gradient is estimated using Widrow's noisy estimates method [4-1] given by:

$$\underline{g}(k) \simeq 2\ \varepsilon(k)\ \underline{v}(k) \tag{4.9}$$

which is an unbiased estimate [4-1]. Thus, we may say that for a given multiplier vector, the expected value of the estimate equals the true value. The algorithm given by Eq.(4.3) reduces to

$$\underline{\alpha}(k+1) = \underline{\alpha}(k) - \Delta_s\ \varepsilon(k)\ \underline{v}(k) \tag{4.10}$$

As can be seen here, Eq.(4.10) involves no a priori statistical information, nevertheless, it is based upon the decision directed hypothesis that the output decision of the equalizer $\{\tilde{I}_k\}$ closely approximates the original information transmitted $\{I_k\}$.It should be noted that the initialization procedure requires a priori information in the form of the locally generated desired pulses.In that case,better known algorithms are available [4-2,3].

Using Eq.(4.10),the implementation of the new equalizer structure for real-time on line adaptive operation is shown in Fig.(4.2). All variables shown in Fig.(4.2) correspond to the output $\hat{I}(k)$.Noise $n(k)$ is added to the discrete channel output $s(k)$ to obtain the equalizer input $Y(k)$.As can be observed from Fig.(4.2),that a filter section output $\underline{v}_i(k)$, $i=0,1,\ldots\ldots,(K-1)$,is obtained by multiplying N input samples with the multiplier coefficients p_i^k ,$k=0,1,\ldots\ (N-1)$ of block (1).The filter section multiplier vector is given by (Eq.3.62,chapter 3):

$$\underline{P}_i = \sum_{K=0}^{N-1} \xi^{-1} h_{ik} \underline{\psi}_K \quad i=0,1, \quad \ldots\ (K-1) \tag{4.11}$$

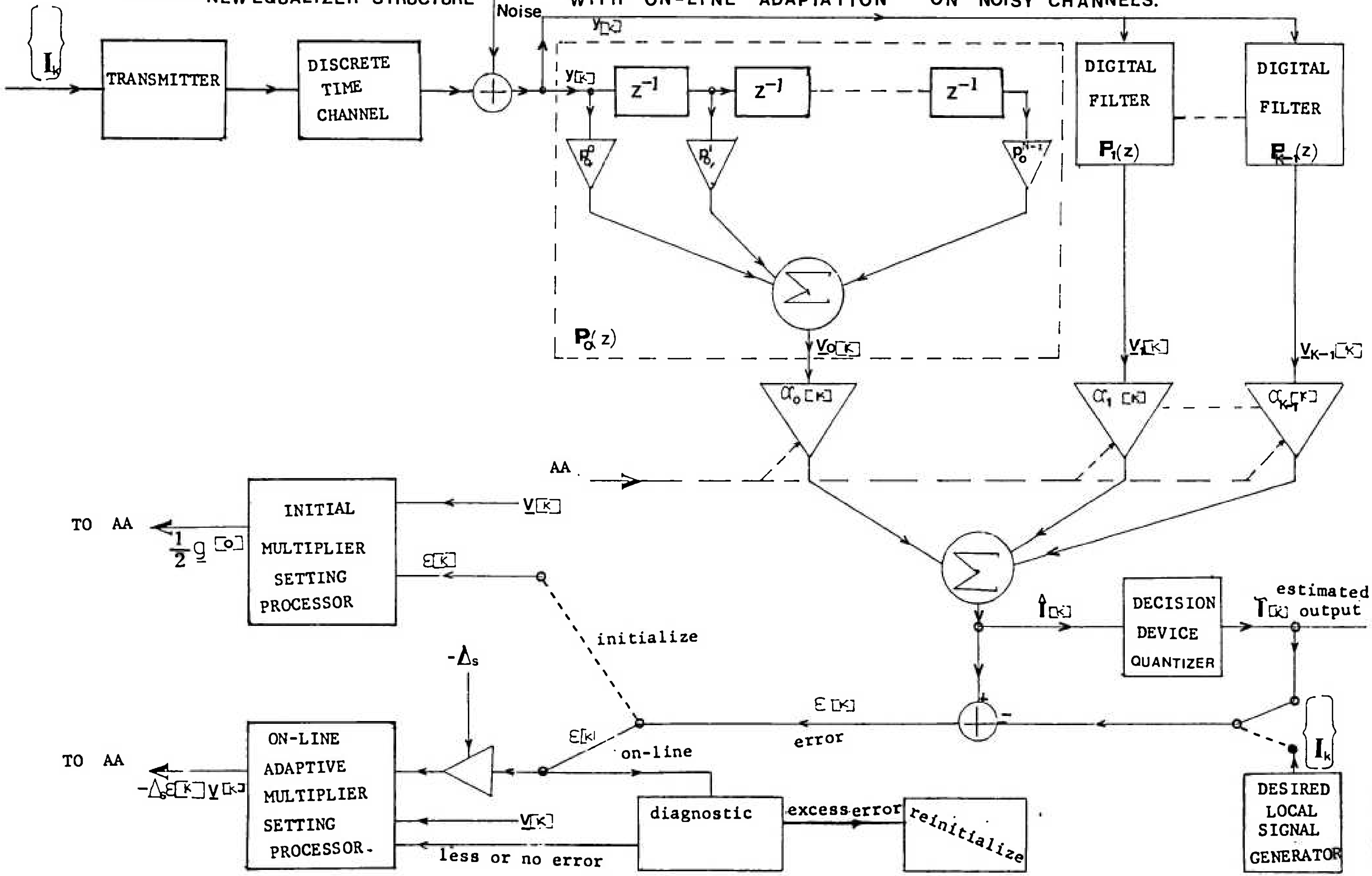
The equalizer output is given by:

$$\hat{I}(k) = \langle \underline{\alpha}\ (k), \underline{v}(k) \rangle \tag{4.12}$$

From Fig.(4.2),it can be seen that the output $\hat{I}(k)$ is fed back,when switched on to the on-line adaptive operation,to form the error signal $\varepsilon(k)$ in a decision-directed fashion.The error signal, $\varepsilon(k)$ after being multiplied by the convergence factor Δ_s is fed into the multiplier setting processor along with $\underline{v}(k)$ to produce $-\Delta_s\ \varepsilon(k)\ \underline{v}(k)$ which is applied to the equalizer multiplier vectors$\underline{\alpha}$ as shown by AA.

Finally,a diagnostic has been provided to take care of the fact that when the error $|\varepsilon\ (k)|$ becomes larger than a predetermined positive threshold value, ε_{max}, then the multiplier setting processor automatically stops and the equalizer goes for reinitialization.However,when the error is less than the predetermined positive threshold value ε_{max} then the adaptation ensues undisturbed.

FIG. 4.2 NEW EQUALIZER STRUCTURE WITH ON-LINE ADAPTATION ON NOISY CHANNELS.



4.3 CONVERGENCE PROPERTIES OF THE ON-LINE ADAPTIVE ALGORITHM:

The multiplier coefficient vector $\underline{\alpha}(k)$ obtained through repeated use of this stochastic iterative Equation (4.10) is a random vector. Therefore, the convergence properties of the algorithm may be conveniently explained in the statistical sense, using the first and second statistical moments of $\underline{\alpha}(k)$. To this effect we restate the assumptions made in chapter (1) that the signal and the interfering noises can be simply modeled as stationary random processes with zero mean and finite variance and that they are uncorrelated. From Fig. (4.2), we define

$$\begin{aligned} E\{\underline{s}(k)\} &= E\{\underline{n}(k)\} = 0 \\ E\{\underline{s}(k)\,\underline{s}^T(k)\} &\triangleq \underline{\Lambda}_{ss} \\ E\{\underline{n}(k)\,\underline{n}^T(k)\} &\triangleq \underline{\Lambda}_{nn} \\ E\{\underline{s}(k)\,\underline{n}^T(k)\} &= 0 \end{aligned} \tag{4.13}$$

Now, taking the first moment of (4.10), we get

$$E\{\underline{\alpha}(k+1)\} = E\{\underline{\alpha}(k)\} - \Delta_s\left(E\{\underline{v}(k)\underline{v}^T(k)\underline{\alpha}(k)\} - E\{\tilde{I}(k)\underline{v}(k)\}\right) \tag{4.14}$$

We define

$$E\{\underline{v}(k)\underline{v}^T(k)\} \triangleq \underline{\Lambda}_{vv}$$

$$= \text{The filter section correlation matrix} \tag{4.15}$$

$$E\{\tilde{I}(k)\underline{v}(k)\} \triangleq \underline{\Lambda}_{Iv}$$

$$= \text{A cross correlation vector} \tag{4.16}$$

Then Equation (4.14) reduces to

$$E\{\underline{\alpha}(k+1)\} = E\{\underline{\alpha}(k)\} - \Delta_s\left(\underline{\Lambda}_{vv}\, E\{\underline{\alpha}(k)\} - \underline{\Lambda}_{Iv}\right) \tag{4.17}$$

A channel output K-vector $\underline{Y}(k)$ at iteration k consists of the current shift register contents, which produce $\hat{I}(k)$ for the first N components and the most recent (K-N) samples to have passed through the shift register as the remaining components. Therefore, we define, the channel auto-correlation matrix,

$$\underline{\Lambda}_{yy} \triangleq E\{\underline{Y}(k)\,\underline{Y}^T(k)\} \tag{4.18}$$

and cross-correlation vector,

$$\underline{A}_{Iv} \triangleq E\{\tilde{I}(k)\underline{Y}(k)\} \qquad (4.19)$$

Also from chapter (3), Fig. (3.3), we write

$$\underline{v}(k) = \underline{P}^T \underline{Y}(k) \qquad (4.20)$$

it follows that

$$\underline{A}_{vv} = \underline{P}^T \underline{A}_{yy} \underline{P} \qquad (4.21)$$

and $$\underline{A}_{Iv} = \underline{P}^T \underline{A}_{Iy} \qquad (4.22)$$

Since we have assumed that the noise $n(k)$ and signal $\underline{s}(k)$ are independent therefore from the relationship

$$\underline{Y}(k) = \underline{s}(k) + \underline{n}(k) \qquad (4.23)$$

and from (4.18)& (4.13), we get

$$\underline{A}_{yy} = \underline{A}_{ss} + \underline{A}_{nn} \qquad (4.24)$$

Since $\underline{A}_{ss}$ is the signal correlation matrix, hence it is positive semi-definite. Next, $\underline{A}_{nn}$ can be diagonalized by a unitary transformation matrix $\underline{Q}$ to yield

$$\underline{Q}^T \underline{A}_{nn} \underline{Q} = \underline{\Lambda}_{nn} \qquad (4.25)$$

However, upon defining, $\underline{m}(k) \triangleq \underline{Q}^T \underline{n}(k)$ a new representation of the noise results with correlation matrix

$$E\{\underline{Q}^T \underline{n}(k) \underline{n}^T(k) \underline{Q}\} = \underline{Q}^T \underline{A}_{nn} \underline{Q} = \underline{\Lambda}_{nn} \qquad (4.26)$$

i.e., the cross-correlation matrix of $m(k)$ is diagonal. The elements are variances, thus are all positive. It follows that $\underline{A}_{nn}$ and $\underline{A}_{yy}$ are positive definite and thus non-singular.

From chapter (3), (Eqs. 3.27&3.28), we note that $N < K$, $\underline{P}^T$ is a rectangular matrix, hence $\underline{A}_{vv}$ of (4.21) is positive semidefinite. From Eq. (4.17), we have

$$E\{\underline{\alpha}(k+1)\} = [\underline{I} - \Delta_s \underline{A}_{vv}] \; E\{\underline{\alpha}(k)\} + \Delta_s \underline{A}_{Iv} \qquad (4.27)$$

where $\underline{I}$ is an identity matrix. With an initial multiplier vector $\underline{\alpha}(0)$, $(k+1)$ iterations of (4.27) become

$$E\{\underline{\alpha}(k+1)\} = [\underline{I} - \Delta_s \underline{A}_{vv}]^{k+1} \underline{\alpha}(0) + \Delta_s \sum_{i=0}^{k} [\underline{I} - \Delta_s \underline{A}_{vv}]^{i} \underline{A}_{Iv} \qquad (4.28)$$

Eq.(4.28) may be put in diagonal form by using the appropriate similarity transformation $\underline{\Omega}$ for the matrix $\underline{\Lambda}_{vv}$, that is,

$$\underline{\Lambda}_{vv} = \underline{\Omega}^{-1}\underline{D}\,\underline{\Omega} \quad \cdots \quad \cdots \tag{4.29}$$

where

$$\underline{D} \triangleq \begin{bmatrix} \lambda_o & o & o & o & o & \cdots & \cdots & o \\ o & \lambda_1 & o & o & o & \cdots & \cdots & o \\ o & o & \lambda_2 & o & o & \cdots & \cdots & o \\ \cdot & \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot\cdot & & & & & \cdot \\ \cdot & \cdot & \cdot & & & & & \cdot \\ o & o & o & o & o & \cdots & \cdots & \lambda_{N-1} \end{bmatrix}$$

is the diagonal matrix of eigenvalues. The non-zero eigenvalues are positive, since $\underline{\Lambda}_{vv}$ is positive semidefinite. Equation (4.28) may be now expressed as

$$E\{\underline{\alpha}(k+1)\} = \left[\underline{I} - \Delta_s\, \underline{\Omega}^{-1}\underline{D}\underline{\Omega}\right]^{(k+1)} \underline{\alpha}(0) + \Delta_s \Sigma_0 \left[\underline{I} - \Delta_s\, \underline{\Omega}^{-1}\underline{D}\,\underline{\Omega}\right]^{i} \underline{A}_{Iv}$$

$$= \underline{\Omega}^{-1}\left[\underline{I} - \Delta_s \underline{D}\right]^{(k+1)} \underline{\Omega}\, \underline{\alpha}(0) + \Delta_s\, \underline{\Omega}^{-1}\, \Sigma_o \left[\underline{I} - \Delta_s \underline{D}\right]^{i} \underline{\Omega}\, \underline{A}_{Iv} \tag{4.30}$$

Consider the diagonal matrix $(\underline{I}-\Delta_s\,\underline{D})$, as long as its diagonal terms are all of magnitude less than unity

$$\lim_{k\to\infty} \left[\underline{I} - \Delta_s\, \underline{D}\right]^{(k+1)} \to \underline{0}$$

and the first term of (4.30) vanishes as the number of iterations increases. The second term in (4.30) generally converges to a nonzero limit. The summation factor $\sum_{i=0}^{k} \left[\underline{I} - \Delta_s\, \underline{D}\right]^{i}$ becomes

$$\lim_{k\to\infty} \sum_{i=0}^{k} \left[\underline{I} - \Delta_s\, \underline{D}\right]^{i} = 1/\Delta_s \underline{D}^{-1} \tag{4.31}$$ *

---

* $\sum_{i=o}^{\infty} (1+\Delta_s\,\lambda_p)^i = 1/\{1-(1+2\,\Delta_s\lambda_p)\} = -1/2\,\Delta_s\,\lambda_p$, Geometric Series property.

Thus, in the limit Eq. (4.30) becomes

$$\lim_{k\to 0} E\{\underline{\alpha}(k+1)\} = \underline{Q}^{-1} \underline{D}^{-1} \underline{Q}$$

$$= \underline{A}_{vv}^{+} \underline{A}_{Iv} \tag{4.32}$$

where $\underline{A}_{vv}^{+}$ is the MPPI of $\underline{A}_{vv}$. But, as the number of iterations increases without limit, the expected value of $\underline{\alpha}$ converges to the Wiener solution. Therefore, we have

$$\underline{\alpha}'_{opt} = \underline{A}_{vv}^{+} \underline{A}_{Iv} \tag{4.33}$$

as the optimal expected solution of the on-line adaptive algorithm in which Widrow's noisy estimate has been used.

Suppose $\underline{A}_{vv}$ has rank $N < K$, then the uniqueness theorem implies the existence of one optimum vector. Therefore, Eq. (4.33) has an infinity of solutions each of which is an optimum multiplier vector that generates the resulting signal $\hat{I}$. Let π denote the null space of $\underline{A}_{vv}$ then

$$\underline{\alpha} \text{ in } \pi \Longrightarrow \underline{A}_{vv} \underline{\alpha} = 0 \tag{4.34}$$

Let $\mathcal{R}$ denote the range space of $\underline{A}_{vv}$ so that

$$\underline{\alpha} \text{ in } \mathcal{R} \Longrightarrow \underline{\alpha} = \underline{A}_{vv} \underline{v} \text{ for some } \underline{v}.$$

Since $\underline{A}_{vv}$ is symmetric, π & $\mathcal{R}$ are orthogonal and any vector $\underline{\alpha}$ has a unique representation as the sum $\underline{\alpha}_{\mathcal{R}} + \underline{\alpha}_{\pi}$ with $\underline{\alpha}_{\mathcal{R}}$ in $\mathcal{R}$ & $\underline{\alpha}_{\pi}$ in π [4-4]. This implies that there exists only one optimum vector $\underline{\alpha}^{P}$ contained in $\mathcal{R}$ that satisfies (4.33). Therefore every optimum vector $\underline{\alpha}_{opt}$ can be expressed in the form

$$\underline{\alpha}_{opt} = \underline{\alpha}^{P} + \underline{\alpha}^{N1} \tag{4.35}$$

for some $\underline{\alpha}^{N}$ in π and $\underline{\alpha}^{P}$ orthogonal to $\underline{\alpha}^{N1}$ (p and N_1 imply positive and null respectively). The orthogonality implies

$$|\underline{\alpha}'_{opt}| > |\underline{\alpha}^{P}| \quad , \text{ unless } \underline{\alpha}_{opt} = \underline{\alpha}^{P} \tag{4.36}$$

so that of all optimum vectors satisfying (4.33), $\underline{\alpha}^{P}$ has the smallest norm or smallest energy.

Next we study the convergence conditions, by defining the multiplier error vector as:

$$\underline{e}(k) \overset{\Delta}{=} \underline{\alpha}(k) - \underline{\alpha}'_{opt} \tag{4.37}$$

and using the relationship derived in Eq. (2.26) as

$$\underline{e}(k) = \left[\underline{I} - \Delta_s \; \underline{A}_{vv}\right]^{(k-1)} \; \underline{e}(1) \tag{4.38}$$

A vector $\underline{e}$ can be decomposed in the form

$$\underline{e}(1) = \underline{e}^{P}(1) + \underline{e}^{N1}(1)$$

$$= \underbrace{\sum_{0}^{N} a_{vvi}\underline{u}_{vvi}}_{\mathcal{R}\text{ - Space}} + \underbrace{\sum_{0}^{K-N} a_{vvj}\underline{u}_{vvj}}_{\pi\text{ - Space}} \tag{4.39}$$

where a_{vvi}, a_{vvj} are the suitable coefficients for the basis $\underline{u}_{vv}$ and $\{\underline{u}_{vvi}\}$, $i = 0,\ldots\ldots\ldots(N-1)$ are the eigenvectors of $\underline{A}_{vv}$ with nonzero eigenvalues and $\{\underline{u}_{vvj}\}$, $j = 0, \ldots\ldots, (K-N)$ are the remaining eigenvectors.

Now from (4.38)

$$\left[\underline{I} - \Delta_s \underline{A}_{vv}\right] \underline{e}^{\mathcal{R}}(1) = \sum_{0}^{N-1} a_{vvi}\lambda_{vvi}\underline{u}_{vvi} \tag{4.40}$$

where λ_{vvi} is the eigenvalue and

$$\underline{e}(k) = \underline{e}^{P}(k) + \underline{e}^{N1}(k)$$

$$= \sum_{0}^{N-1} a_{vvi}(1 - \Delta_s \lambda_{vvi})^{k-1}\underline{u}_{vvi} + \sum_{0}^{K-N} a_{vvj}\underline{u}_{vvj} \tag{4.41}$$

where $(1 - \Delta_s \lambda_{vvi})$ are the diagonal terms of $\left[\underline{I} - \Delta_s \underline{A}_{vv}\right]$.

From (4.41) it is apparent that the last term is simply $\underline{e}^{N1}(1)$ i.e.

$$\underline{e}^{N1}(k) = \underline{e}^{N1}(1) \quad \text{for all} \quad k.$$

The remaining component $\underline{e}^{P}(k)$ will converge provided Δ_s is so chosen as to ensure that

$$\max_{i \epsilon p} \{ | 1 - \Delta_s \lambda_{vvi} | \} < 1 \tag{4.42}$$

since in this case

$$|| \underline{e}^{P}(k) || < \max. \{| 1 - \Delta_s \lambda_{vvi}|\} \, || \sum_{0}^{N-1} a_{vvi} \underline{u}_{vvi} ||$$

$$< || \underline{e}^{P}(1) ||$$

Therefore the condition to ensure convergence of (4.33) is found from (4.42) to be

$$0 < \Delta_s < 2/(\lambda_{vv})_{max}. \tag{4.43}$$

where $(\lambda_{vv})max > (\lambda_{vvi})$ for all i

4.4 COMPUTER SIMULATION:

Basic structures of the computer programs for chapters (3) & (4) remain the same.. In particular we have made provisions for

(i) generating a random binary sequence to simulate the transmitted information,

(ii) adding gaussian noise with specified statistics,

and, (iii) adjusting the multipliers iteratively at each symbol interval.

Also, we have devised a scheme to detect the errors during on-line operation using decision directed mode,

Different equalizer structures have been used to compensate the effects due to ISI and the SNR variation from 80* dB down to 15 dB over the discrete time channels discussed in chapter (3) [section (3.8)] . Choice of K has been made on the basis of Hadamard Matrix ($\underline{H}$) i.e. K will assume values either 2^2, 2^3, 2^4, 2^5 etc. or 12,20,24,28 etc. At 15 dB the noise standard deviation (s.d.), calculated using the formula (SNR/20) = antilog$_{10}$ (1/s.d.), is 0.1778 and the symbol error rate is frequent and significant.

As before we studied the effects of parameters delay (δ), shift register length (N) and the number of filter sections (K). We specified the convergence factor $\Delta_s = 0.025$ which is equivalent to 5% of the decision threshold value. The transmitted message was simulated by repeatedly generating a 255 symbol pseudorandom sequence, and the error criterion was taken to be the average MSE $\varepsilon^2(r)$ after 600 iterations of the on-line adaptation algorithm. For programming convenience, we determined $\varepsilon(r)$ by transmitting the symbol $I(r)$ in place of the output decisions $\tilde{I}(r)$. This assumption has negligible effect when there are no decision errors.

---

* This was our assumption for the noise-free case.

4.4.1 EFFECTS OF DELAY (δ) :

Delay (δ), we studied, is the time difference between the arrival at the equalizer of the first precursor of the channel unit-pulse response and the beginning of the locally generated desired pulse { I_r }.

DISCRETE- TIME CHANNEL 1 [Figs. 4.3(d) or 3.4 (a)]:

The simulation results have been plotted in Fig. 4.3(a), which shows the variation of average MSE as a function of δ for SNR, 40 dB. and 20 dB. (Assuming 30 dB as the working SNR level). This channel has been successfully equalized for SNR of 30 dB or greater [Figs.4.3(a)&4.3(c)]

DISCRETE - TIME CHANNEL 2 [Fig. 3.5(a)] :

The simulation results have been plotted in Fig. 4.4(a), which shows the variation of average MSE as a function of δ for SNR, 40 dB and 20 dB. This channel was successfully equalized for SNR of 30 dB or greater [Curves (b) & (c) , Curve 4.4(a)] . These curves indicate that there is no significant improvement in performance for the larger structures provided δ is properly chosen, in fact, the smaller structure perform slightly better. However, there is a much wider range of feasible values of δ for larger structures and this range is not so much sensitive to SNR. Thus the possibility of bad equalization , due to incorrect selection of δ, with a small structure must be compared with the complexity and cost of the larger structure. Delay δ also depends on the knowledge of channel characteristics which is usually unknown and time varying, in that case , a larger structure will perform better. Therefore, a proper selection of the nominal values of δ will certainly be advantageous. Based on these considerations nominal values of δ for noisy discrete-time channels (1) and (2) were selected to be 0.60 N and 0.85 N respectively.

Equalizer structures for this case

	DISCRETE TIME CHANNEL 1 N	K	DISCRETE TIME CHANNEL 2 N	K
Equalizer # 1	20	32	17	32
Equalizer # 2	16	28	13	28
Equalizer # 3	12	24	9	20

4.4.2 EFFECTS OF SHIFT REGISTER LENGTH (N) :

Both channels were simulated by using $K= 32$ and $\delta=0.6N$ (LPF) and $\delta=0.85N$ (VSB). SNR was varied from 80 dB to 20 dB (It was not possible to simulate for 15 dB SNR).

For both channels, high SNR values , performance either improved or remained constant. As can be seen from Figs. 4.3 (b) & 4.4 (b) that the performance degraded slightly for N greater than 10. This degradation in performance can surely be attributed to the accumulated round-off noise. Also, for SNR greater than and equal to 30 dB , there is a lower value of N above which there is very little change in the performance, in other words, there is very little improvement in performance by using shift-register length (N) greater than the channel impulse response. Smaller N makes computation easy but makes the selection of δ difficult. It is interesting to note that , in LPF channel, equalization was achieved for SNR = 20 dB with N=5 [Fig. 4.3 (b)]. For VSB channel [Fig. 4.4 (b)], at 20 dB, the performance was poor and errors were detected which eventually improved.

4.4.3 EFFECTS OF SNR AND MULTIPLE INITIALIZING PULSES :

DISCRETE TIME CHANNEL 1 [Fig. 4.3 (c)]

We selected two structures:

K = 32		K = 24
N = 20	and	N = 12
δ = 12		δ = 8

Simulations were carried out with these two equalizer structures. Fig. 4.3 (c) shows the effect of SNR on equalizer performance when both a single initializing pulse and an average of two initializing pulses were used by the equalizer to better identify the channel. It is clear from the curves (c) & (d) that the use of multiple initializing pulses is quite benificial when the SNR is low and the noise represents a significant part of the unit pulse amplitude. However, this is not the case at SNR > 30 db, because multiple initializing pulses offer very little advantage.

DISCRETE TIME CHANNEL 2 [Fig. 4.4 (c)]

We selected two structures:

K = 32		K = 20
N = 17	and	N = 9
δ = 15		δ = 8

Simulations were carried out with these two equalizers. As we can see the similarity in curves between Figs. 4.3 (c) & 4.4 (c) , therefore,similar interpretations are applicable in this channel as well.

4.4.4 EQUALIZATION:

Channels (1) & (2) were successfully equalized with a 20 dB SNR [Figs. 4.3 (d) & 4.4 (d)] ,however, it was not sufficient for very many other channels which required 30 dB to be equalized without errors.

As said earlier, the noise has been superimposed on the ISI. This has not been shown before equalization part of Figs. 4.3 (d) & 4.4 (d). However,the after equalization part of Figs. 4.3 (d) & 4.4 (d) have been drawn only when the channel impulse response with added noise has been equalized.

4.5 SUMMARY AND COMMENTS:

(A) We have designed, developed and extended the design of the new initialized equalizer structure of chapter (3) to make it suitable for real-time on line adaptive operation on noisy channels.

(B) It has been shown that the channels (1) and (2) could be equalized successfully even at 20 dB noise. However, certain other channels with more overshoots required 30 dB at least to get equalized but it has not been considered here .

(C) We have used the stochastic approximation for estimating the noisy gradients.

(D) We found that no value of N was reached above which the equalizer performance remained fairly constant. It suggested that the better equalization is attainable for larger equalizer parameters than the maximum of (30, 20, δ) and (32, 17, δ).

(E) From the foregoing considerations we have found that there are a number of tradeoffs in selecting the equalizer parameters:

By selecting large values of N & K gives a larger range of values from which to select the nominal delay δ. This is, particularly important if little is known about the channel a priori. On the other hand, small value of shift register length (N) greatly reduce the amount and time of computation required for initialization, while smaller values of K reduce the number of on-line adaptive multiplications required at each iteration. We can think of these considerations as tradeoffs between the accuracy desired from the equalizer and the cost that must be paid to achieve it.

(F) If the nominal value of δ is uncertain or if considerable variation is expected in the channel unit-pulse response then a large value of N will make the equalizer perfopmance less sensitive to δ .

(G) Since single symbol errors are highly probable when the SNR is low or the channel badly equalized, some form of averaging must be used to avoid an excessive number of reinitialization diagnostic calls. This could be made possible by keeping a running count of the number of times $|\varepsilon(k)|$ exceeds a predetermined limit ε_{max} over the last , say, k iterations and reinitializing only if it is exceeded $n < k$ times, where n and k are known integers.

(H) The scheme outlined in (G) would also prevent shorst bursts of impulse or switching noise from causing needless reinitialization.

	NOISE	K	N
A.	40 db;	32	20
B.	20 db;	32	20
C.	40 db;	24	12
D.	20 db;	24	12
E.	40 db;	28	16
F.	20 db;	28	16

DISCRETE-TIME CHANNEL 1.

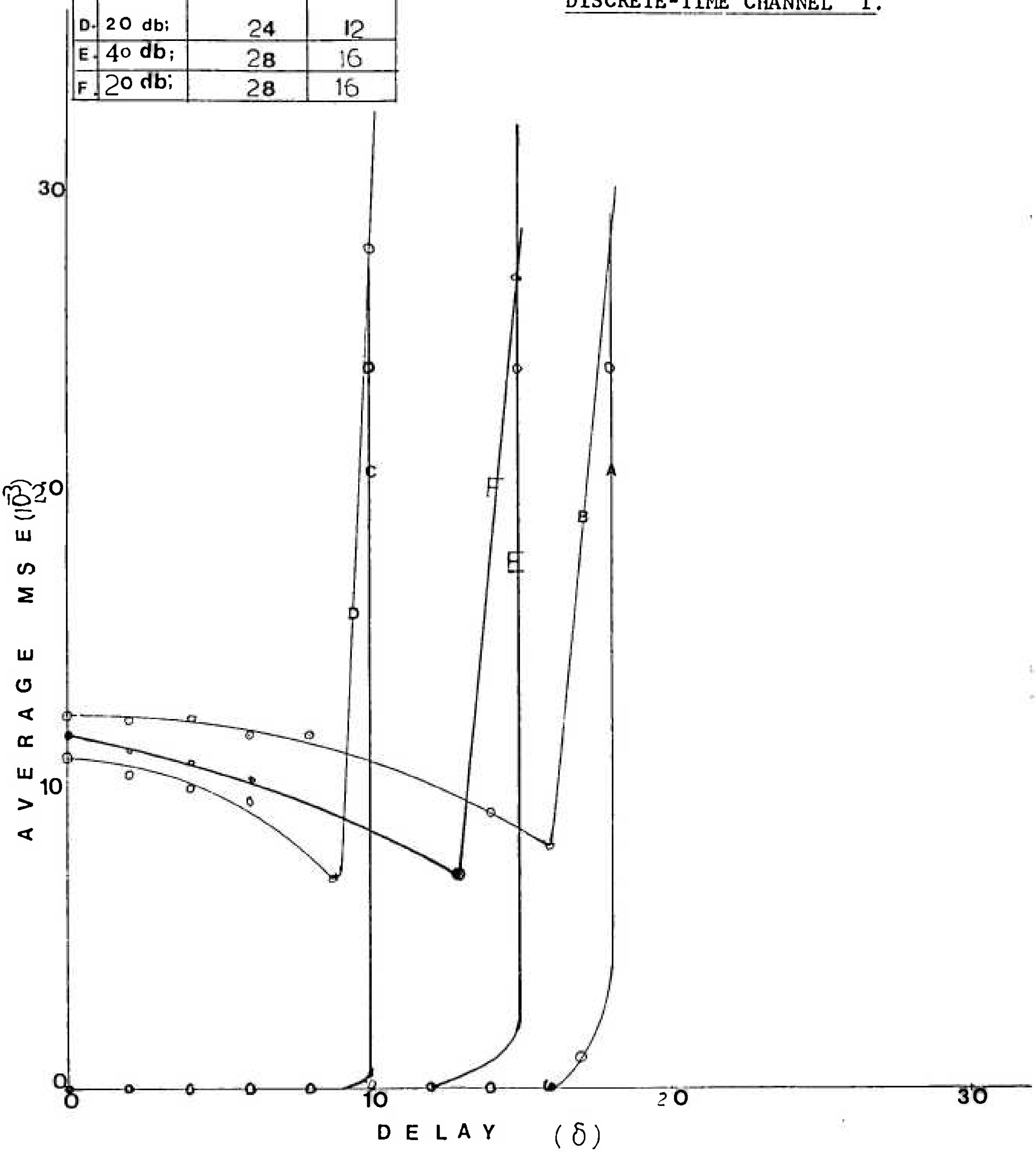


FIG. (4.3a) EFFEECT OF DELAY (δ).

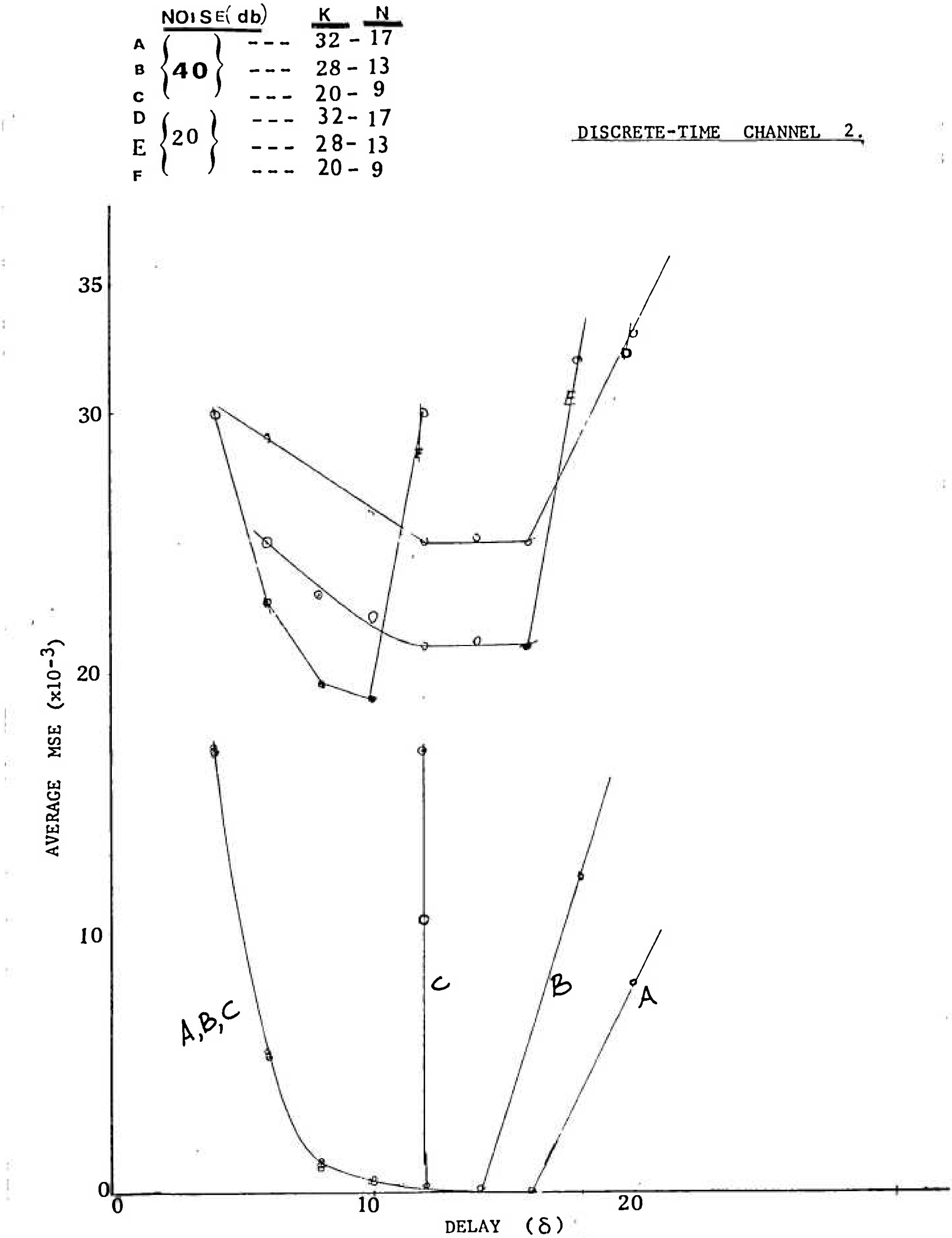


FIG. (4.4a) EFFECT OF DELAY (δ)

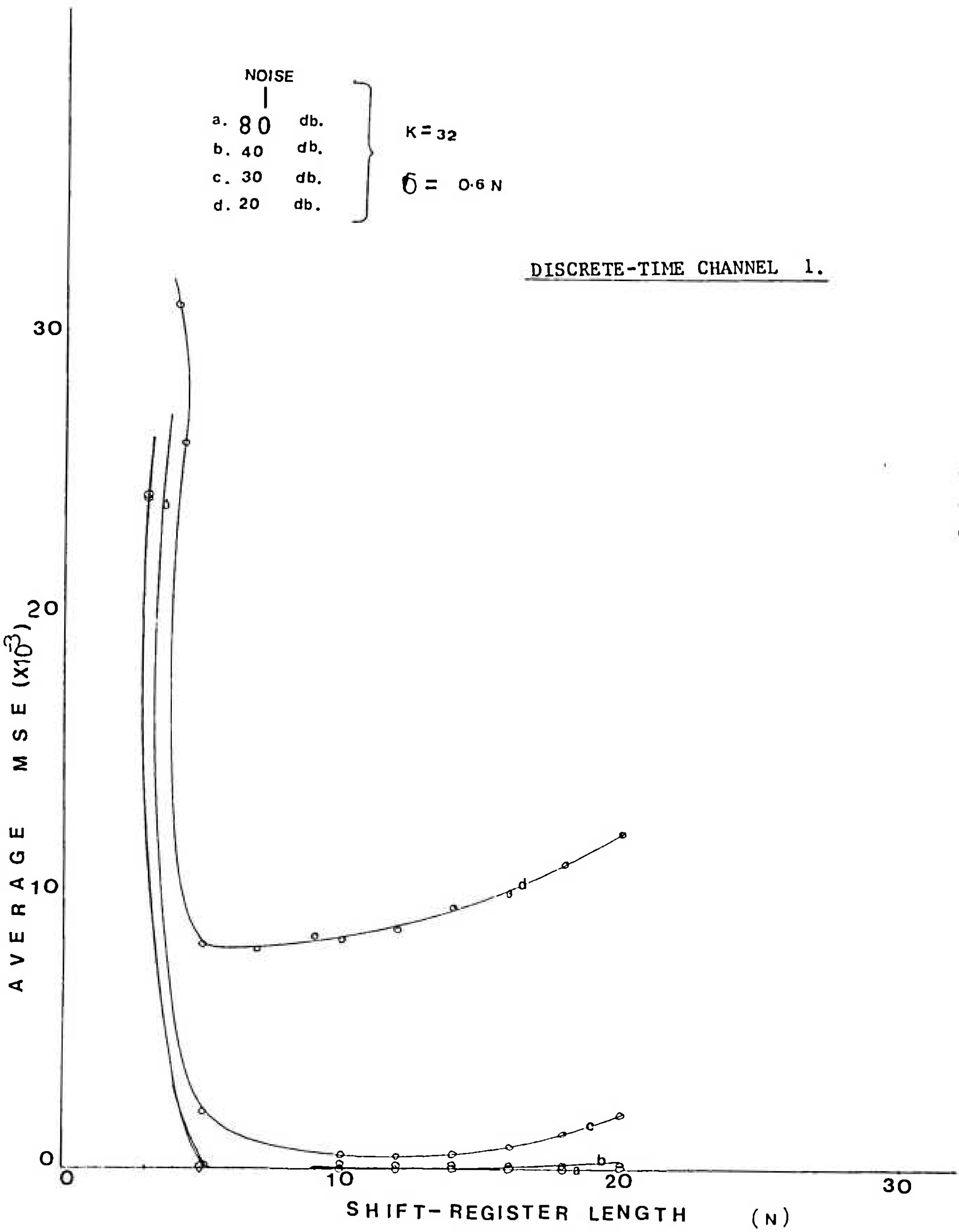


FIG. (4.3b) EFFECT OF SHIFT REGISTER LENGTH (N)

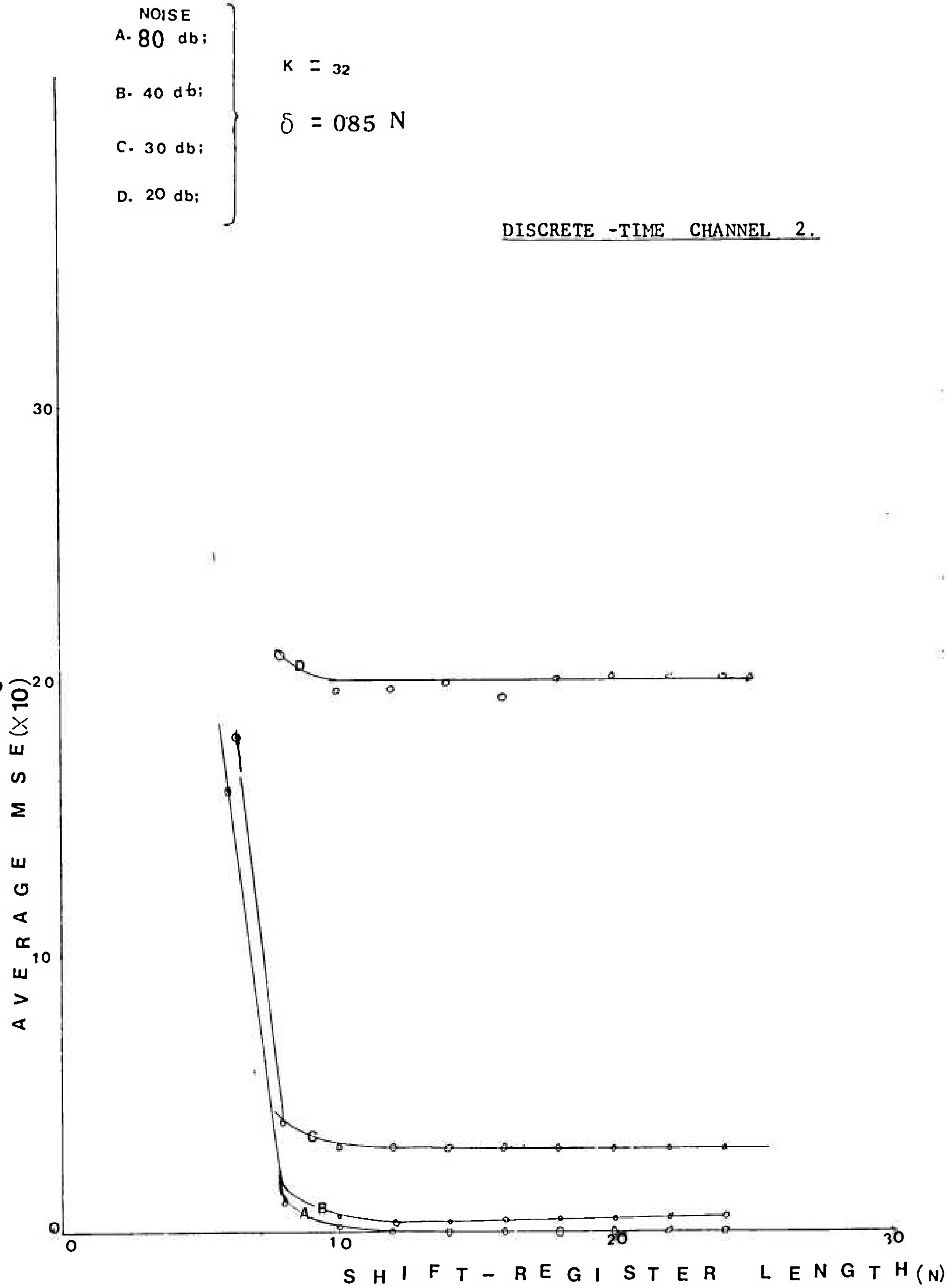
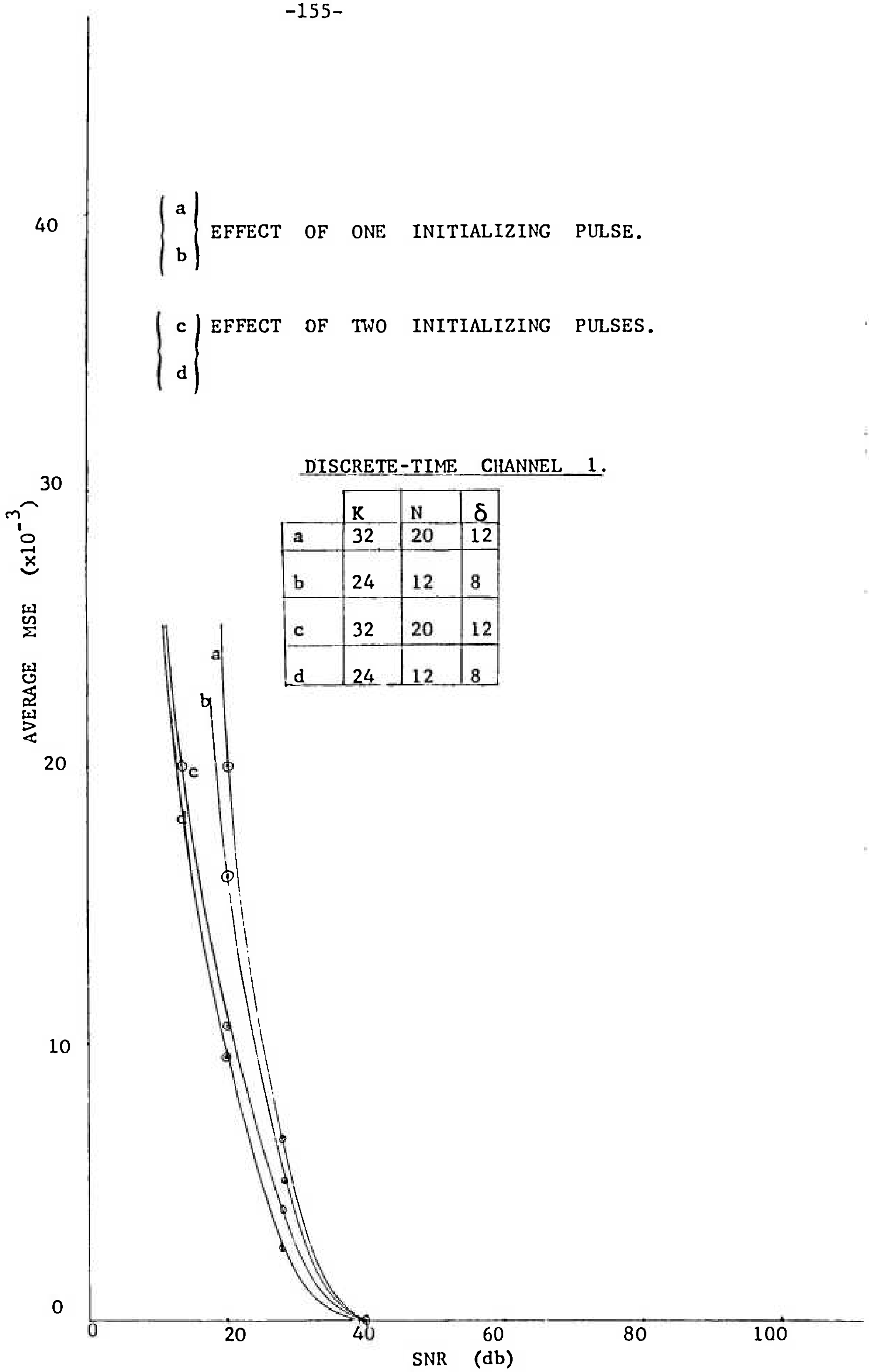


FIG. (4.4b) EFFECT OF SHIFT REGISTER LENGTH (N)



	K	N	δ
a	32	20	12
b	24	12	8
c	32	20	12
d	24	12	8

FIG. (4.3c) EFFECT OF SNR AND INITIALIZING PULSES ON AVERAGE MSE OF A GENERALIZED NRDFE.

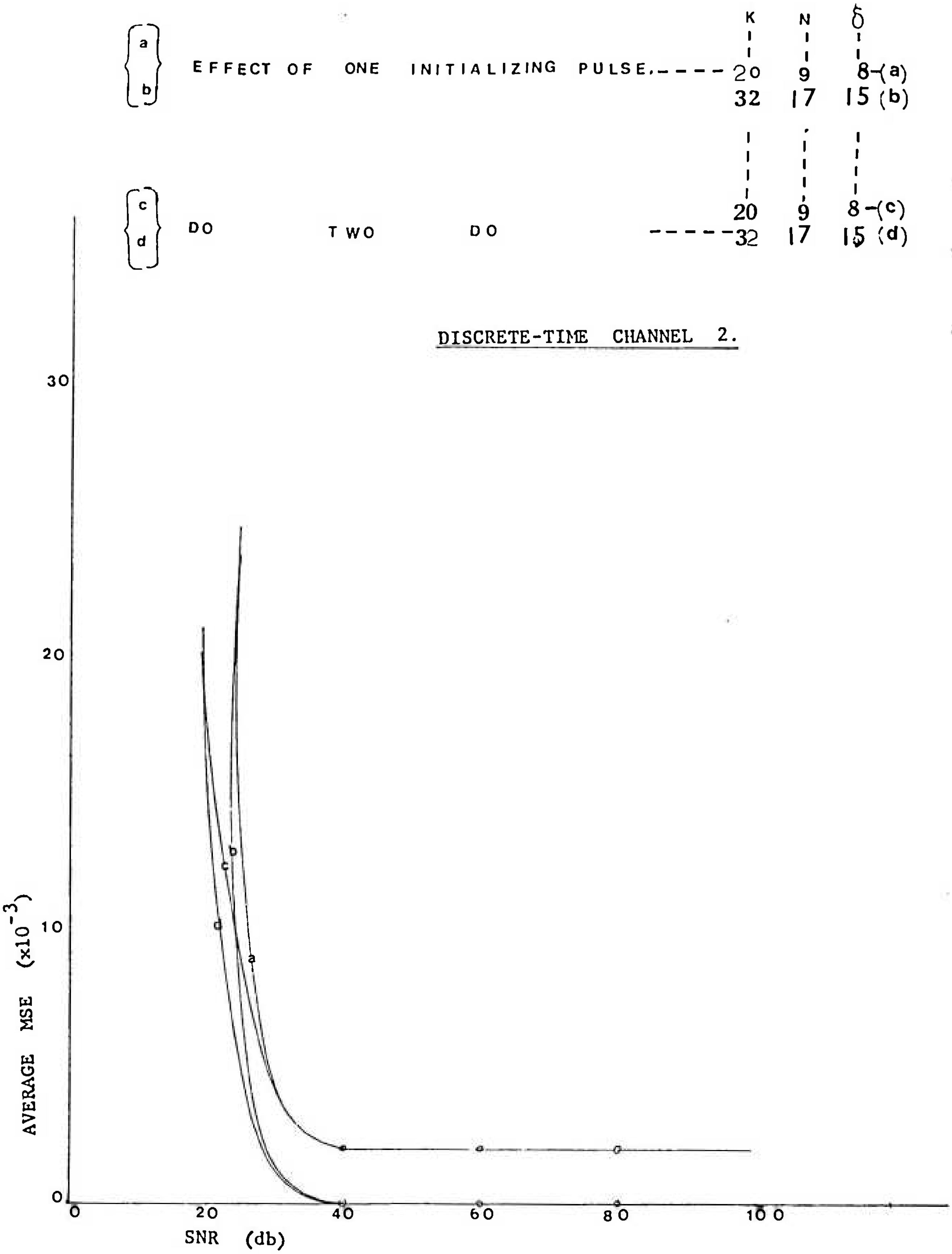


FIG. (4.4c) EFFECT OF SNR AND INITIALIZING PULSES ON AVERAGE MSE OF A GENERALIZED NRDFE.

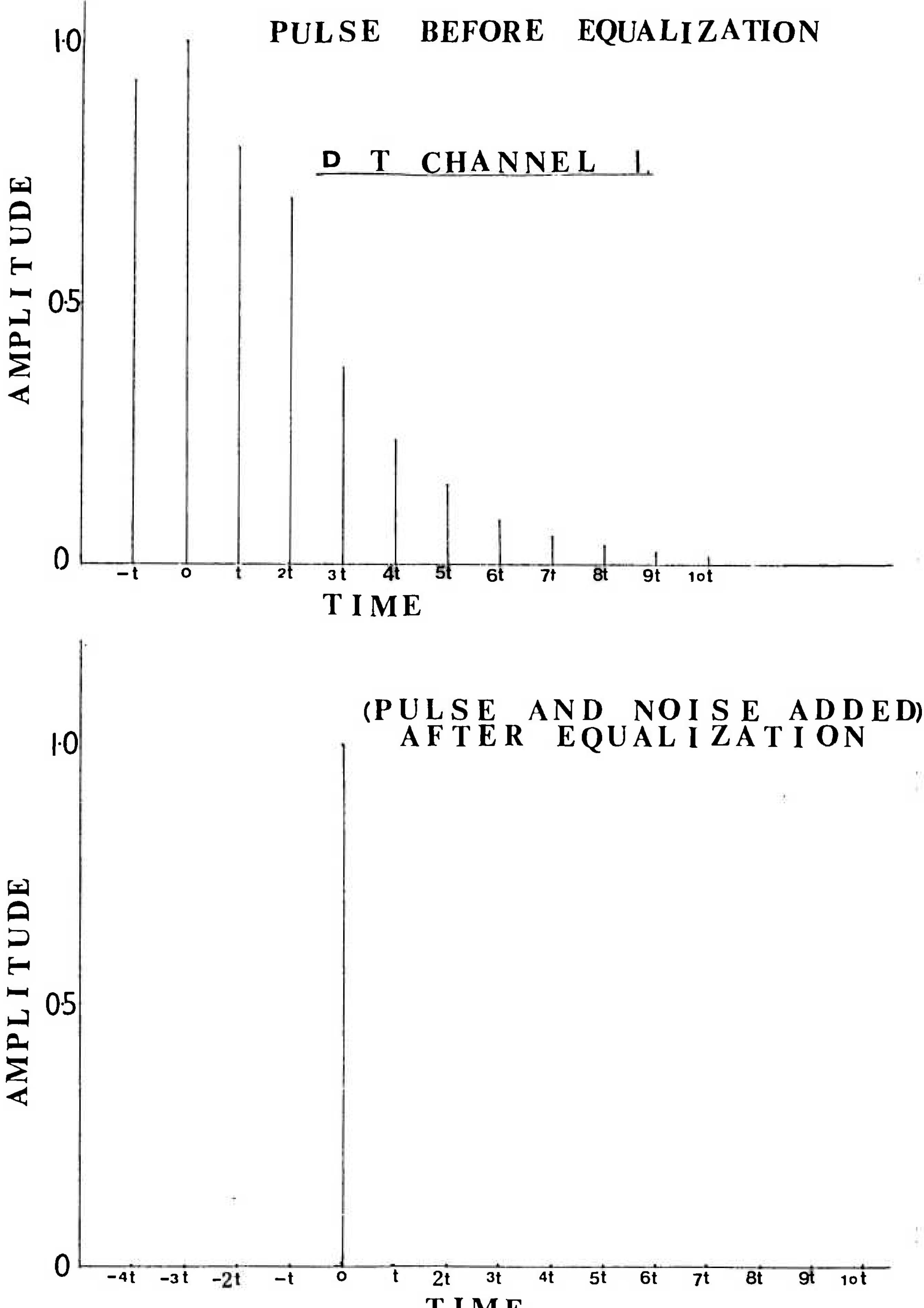
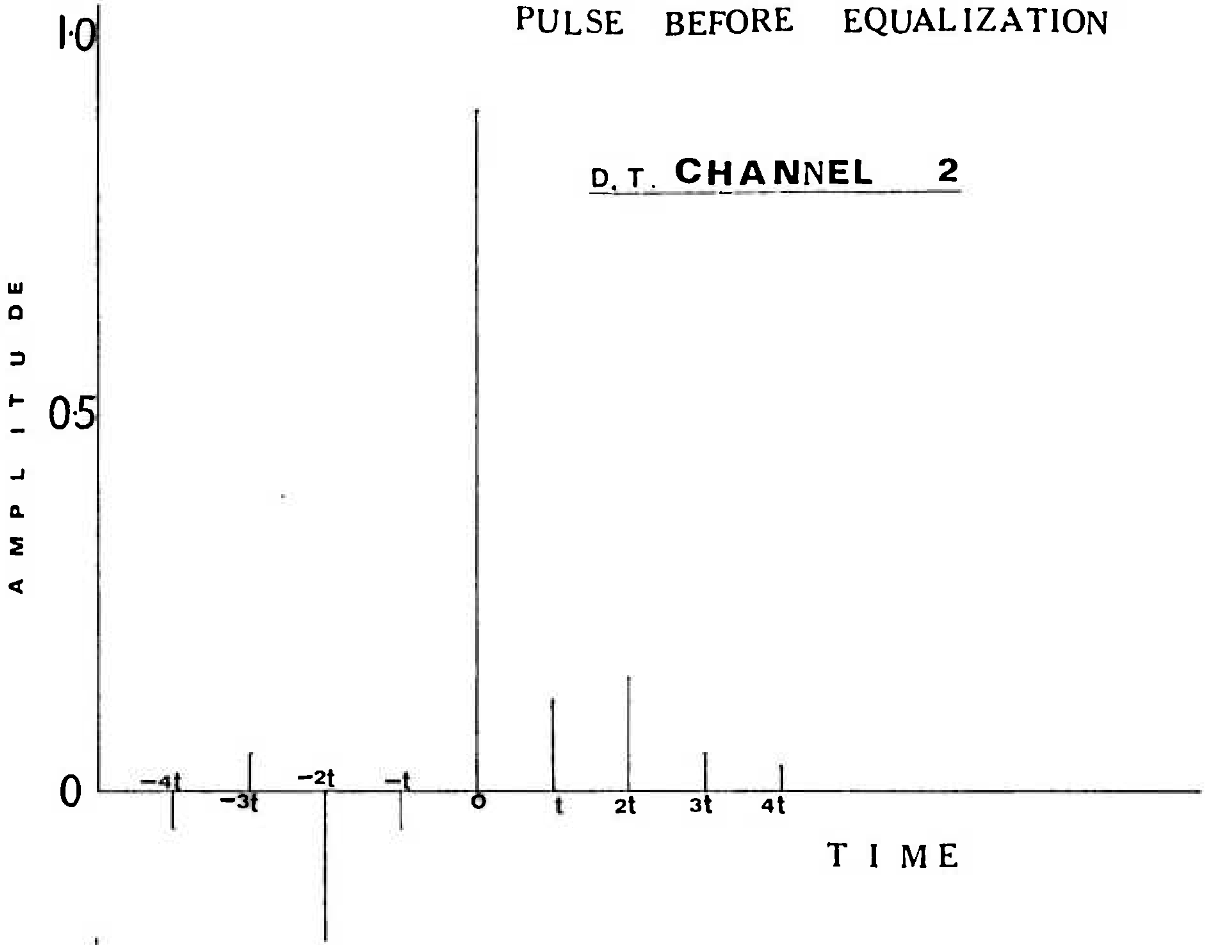


FIG.(4·3d) IMPULSE RESPONSE



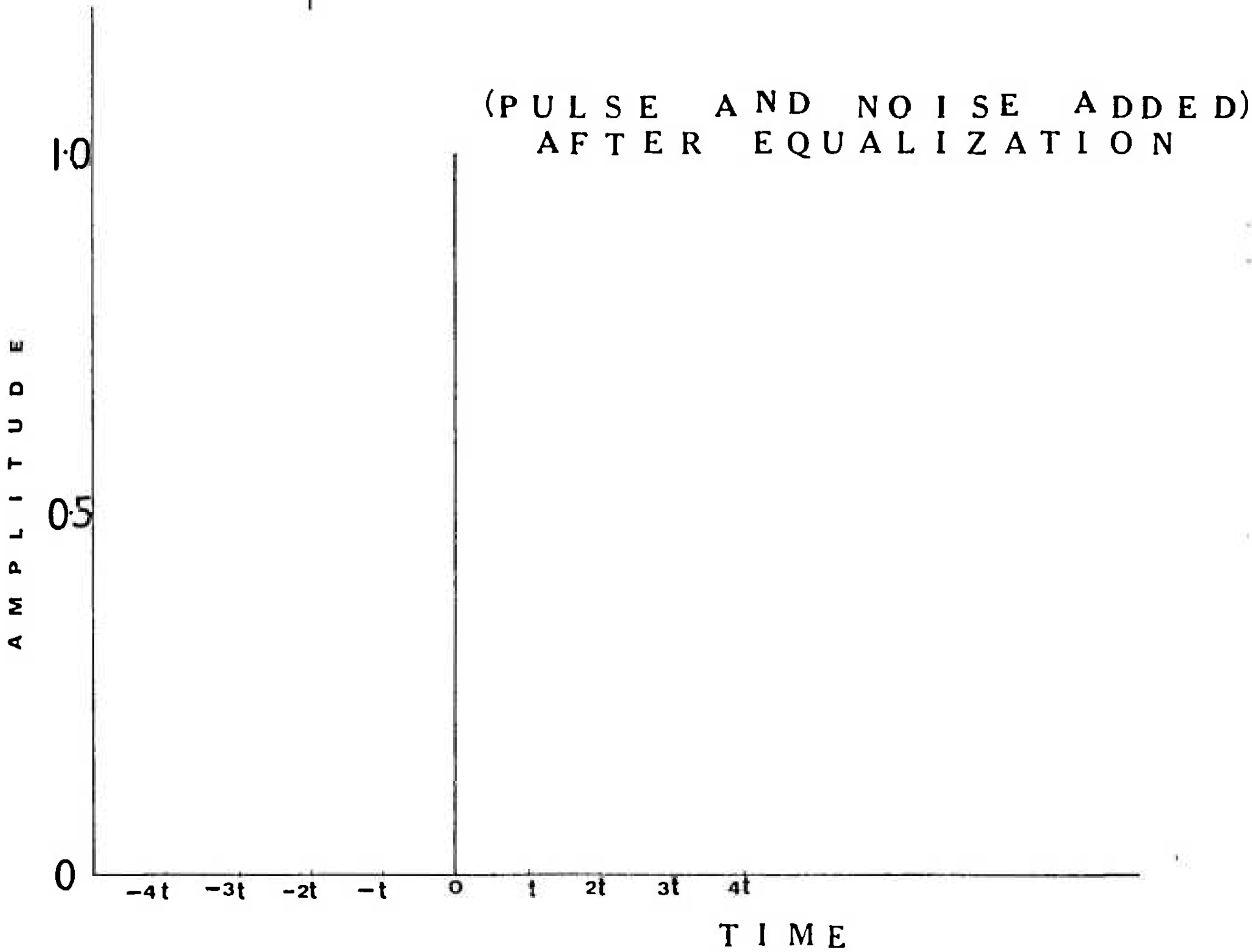


FIG. (4·4d) IMPULSE RESPONSE

CHAPTER 5

DESIGN OF RECURSIVE DIGITAL FILTER EQUALIZERS (RDFE)

There is nothing more difficult to take in hand, more perilous to conduct, or more uncertain in its success, than to take the lead in the introduction of a new order of things.

-NICCOLO' MACHIAVELLI (The Prince, 1513)

5.1 INTRODUCTION:

In this chapter we consider a recursive digital filter as an equalizer (initialized and on-line adaptation) as a means to equalize discrete time channels (noise free and noisy) by reducing ISI.

Fitch and Kurtz [5-1] have proposed a design procedure for the receiver of a PAM system in the presence of ISI and noise. In the receiver an estimate of each source symbol is made based on the Weiner-Kolmogorov theory of minimum variance estimation for stationary time series. The resulting structure has been called a recursive equalizer and its taps have been obtained by taking the Canonical factorization of the spectral density function followed by the operations of polynomial multiplication, and division. A comparison has been made with conventional NRDFE. Some improvements have been reported, but the basic question of on-line adaptation has not been touched upon.

Mantey [5-2], proposed a method for adapting a recursive network which will converge on a unique minimum. His method involved breaking the feedback link during the on-line adaptive operation and exciting the feedback branch with the desired response. He has shown that the adaptation using modified mean-square error algorithm yeilds weights which give the best estimates of the desired response based upon weighting the past inputs and the desired responses. If the minimum mean-square error of this minimization is small, then the system when returned to the operate mode yields very satisfactory performance. His modified MSE is an approximate criterion {eq.(29) P.22, [5-2]} and fails to operate in noisy conditions as an on-line adaptive equalizer.

So, from the survey of literature it appears that upto now very little has been known about recursive digital filter equalizer (RDFE). The basic question of its suitability has vaguely been proposed but its implementation with detailed analyses are yet to come. It is our aim, here, to add some useful investigations in the form of important communication properties e.g. convergence, on-line adaptation using decision directed mode, probability of error using Monte-Carlo simulations etc. However, it must be emphasised that the general approach is the digital filtering where in itself, a vast amount of literature is available but none on its implementation as an equalizer.

We have made an independent investigation on the various forms of RDFE and compared their suitability with NRDFE by means of computer simulation for severe phase and amplitude distortion channels. The most meaningful error, P_e, has been obtained using Monte-Carlo simulation technique. Also, the pole-zero constellation equalization has been explained.

5.2 FUNDAMENTALS:

A digital communication system using a recursive digital filter equalizer (RDFE), is shown in Fig. (5.1).

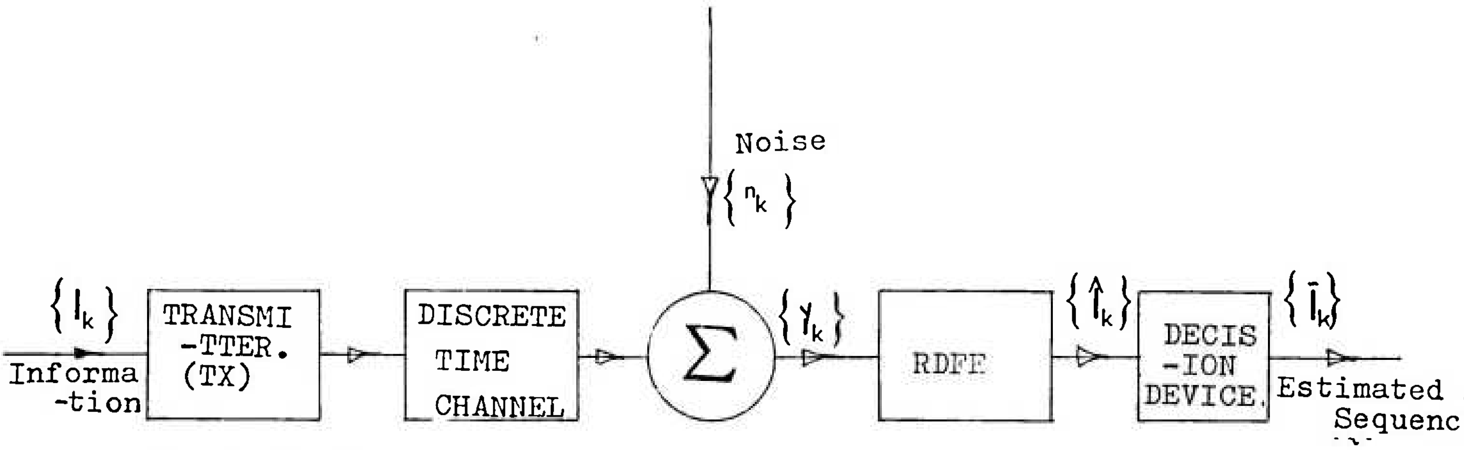


FIG.(5.1) DIGITAL COMMUNICATION SYSTEM USING RDFE.

The basic element of an adaptive RDFE ia a recursive digital filter (RDF) which is described in terms of z- transform as a rational polynomial in z^{-1} and is given by,

$$H(z) = \frac{\hat{I}(z)}{Y(z)}$$

$$= \frac{\sum_{i=0}^{M} \alpha_i \; z^{-i}}{1+\sum_{i=1}^{N} \beta_i \; z^{-i}} \quad , \; M > N \tag{5.1}$$

where at least one of the β_i is nonzero and all the roots of the denominator are not cancelled exactly by the roots of the numerator. The filter of eq. (5.1) has, in general, M finite zeros and N finite poles. The zeros of H(z) can be anywhere in the z-plane but the poles must lie inside the unit circle for stability i.e. poles must satisfy the following sufficient inequality condition

$$|\beta_i| < 1 \tag{5.2}$$

The input-output relationship from Eq. (5.1) is given by the difference equation

$$\hat{I}_k = \sum_{i=1}^{M} \alpha_i\, y_{k-i} - \sum_{i=1}^{N} \beta_i\, \hat{I}_{k-i} \tag{5.3}$$

where $\{y_k\}$ and $\{\hat{I}_k\}$ are the input and output sequence, and $\{\alpha_k\}$ and $\{\beta_k\}$ are the filter adjustable coefficients. In practical circumstances the characteristic of H(z) should be such that the output $\{\hat{I}_k\}$ approximates the input $\{I_k\}$ by fulfilling the performance,MSE, criterion, which is defined as

$$J(\underline{\alpha},\underline{\beta}) \triangleq \sum_{k=-\infty}^{\infty} \varepsilon_k^2 \triangleq E[\varepsilon_k^2] \tag{5.4}$$

where the error, $\varepsilon_k \triangleq \hat{I}_k - I_k$ (5.5)

in the initializing mode ,

and $\varepsilon_k \triangleq \hat{I}_k - \tilde{I}_k$ (5.6)

when using on-line adaptation using decision-directed mode. Next it remains to be seen that the minimum of the performance criterion yields a quadratic surface. From (5.3) to (5.6), we have

$$J(\underline{\alpha},\underline{\beta}) = E[\varepsilon_n]^2 = E[\hat{I}_n - I_n]^2$$

$$= E[\sum_{i=0}^{M} \alpha_i\, y_{n-i} - \sum_{i=1}^{N} \beta_i\, \hat{I}_{n-i} - I_n]^2 \tag{5.7}$$

Differentiating (5.7) w.r.t. $\underline{\alpha}_k$ we get

$$\frac{\partial J(\underline{\alpha},\underline{\beta})}{\partial \underline{\alpha}_k} = 2\sum_{n=0}^{\infty} \frac{\partial \varepsilon_n}{\partial \underline{\alpha}_k}\varepsilon_n \qquad k=0,1,\ldots..M$$

$$= 2 \sum_{n=0}^{\infty} \frac{\partial}{\partial \alpha_k} \left[\sum_{i=0}^{M} \alpha_i y_{n-i} - \sum_{i=1}^{N} \beta_i \hat{I}_{n-i} \right] \varepsilon_n$$

$$= 2 \sum_{n=0}^{\infty} y_{n-k} \left[\sum_{i=0}^{M} \alpha_i y_{n-i} - \sum_{i=1}^{N} \beta_n \hat{I}_{n-i} \right] \qquad (5.8)$$

Differentiating (5.8) w.r.t. $\underline{\alpha}_k$, it yields

$$\frac{\partial^2 J(\underline{\alpha},\underline{\beta})}{\partial \alpha_k^2} = 2 \sum_{n=0}^{\infty} y_{n-k}^2$$

$$= 2 \underline{\Lambda}_{yy} \geq 0 \qquad (5.9)$$

where $\underline{\Lambda}_{yy}$ is the autocorrelation function of the input signal and this is always positive for all values of k's and is constant. Similarly differentiating (5.7) w.r.t. $\underline{\beta}_k$'s, we get

$$\frac{\partial J(\underline{\alpha},\underline{\beta})}{\partial \underline{\beta}_k} = 2 \sum_{n=0}^{\infty} \hat{I}_{n-k} \varepsilon_n \quad [k = 1,2,\ldots\ldots N] \qquad (5.10)$$

$$\frac{\partial^2 J(\underline{\alpha},\underline{\beta})}{\partial \underline{\beta}_k^2} = 2 \sum_{n=0}^{\infty} \hat{I}_{n-k} \hat{I}_{n-k}$$

$$= 2 \underline{\Lambda}_{II} \geq 0 \qquad (5.11)$$

and remains the same for all values of β_j.

Since both sets of second partial derivatives are everywhere positive and constant, therefore, $J(\underline{\alpha},\beta)$ has a quadratic surface and this assures single minimum convergence and hence any standard iterative search procedure can be applied to it.

As the direct solution to Eq.(5.7) involves complicated circuitry and also because of the quadratic nature of MSE surface it is customary to adopt some form of iterative procedure. Therefore, the coefficients adjustments are made using the following iterative algorithms:

$$\underline{\alpha}_k^{(j+1)} = \underline{\alpha}_k^{(j)} - \frac{1}{2}\Delta_2^{(j)} \underline{g}_{\underline{\alpha}_k}^{(j)} \tag{5.12}$$

and
$$\underline{\beta}_k^{(j+1)} = \underline{\beta}_k^{(j)} - \frac{1}{2}\Delta_3^{(j)} \underline{g}_{\underline{\beta}_k}^{(j)} \tag{5.13}$$

where the gradient vectors after jth iteration are given by:

$$\underline{g}_{\underline{\alpha}_k}^{(j)} = \left\{\frac{\partial J(\underline{\alpha},\underline{\beta})}{\partial \underline{\alpha}_k}\right\}^{(j)} = E\left[2\,\varepsilon_n \frac{\partial \hat{I}_n}{\partial \underline{\alpha}_k}\right] \tag{5.14}$$

and
$$\underline{g}_{\underline{\beta}_k}^{(j)} = \left\{\frac{\partial J(\underline{\alpha},\underline{\beta})}{\partial \underline{\beta}_k}\right\}^{(j)} = E\left[2\,\varepsilon_n \frac{\partial \hat{I}_n}{\partial \underline{\beta}_k}\right] \tag{5.15}$$

and $\Delta_2^{(j)}$, $\Delta_3^{(j)}$ are the convergence constants. However, a difficulty always remains in that the expected value is not available in real time and has to be estimated by averaging over a finite number of symbols, say LL, or by using the stochastic approximation techniques for noisy environment as suggested by Widrow (Ch.2). In the former case, equations (5.12) and (5.13) are given by

$$\underline{\alpha}_k^{(j+1)} = \underline{\alpha}_k^{(j)} - \frac{\Delta_2^{(j)}}{LL} \sum_{n=0}^{LL-1} \varepsilon_n^{(j)} \frac{\partial \hat{I}_n}{\partial \underline{\alpha}} \tag{5.16}$$

and
$$\underline{\beta}_k^{(j+1)} = \underline{\beta}_k^{(j)} - \frac{\Delta_3}{LL} \sum_{n=0}^{LL-1} \varepsilon^{(j)} \frac{\partial \hat{I}_n}{\partial \underline{\beta}_k} \tag{5.17}$$

In the stochastic approximations, coefficients are adjusted on a sample by sample basis. Therefore, Eqs. (5.12) & (5.13) are written as:

$$\underline{\alpha}(j+1) = \underline{\alpha}\ (j) - \Delta_4\ \varepsilon(j) \frac{\partial I(j)}{\partial \underline{\alpha}} \tag{5.18}$$

and

$$\underline{\beta}(j+1) = \underline{\beta}\ (j) - \Delta_5\ \varepsilon(j) \frac{\partial\ I(j)}{\partial \underline{\beta}} \tag{5.19}$$

Next we consider the convergence conditions of the algorithm.

5.2.1 CONVERGENCE CONDITIONS:

Following the results of chapter (2), the iterative algorithms of Equations (5.12) and (5.13) will converge to optimum solutions provided that:

$$|\underline{I} - \Delta_2^{(j)}\ \underline{A}\ | < 1 \tag{5.20}$$

and,
$$|\underline{I} - \Delta_3^{(j)}\ \underline{A}\ | < 1 \tag{5.21}$$

where $\underline{A}$ is a correlation matrix with jk^{th} element $E[y_{n-j}\ y_{n-k}]$ and $\underline{I}$ is an identity matrix.

That is to say, in order to ensure convergence the step sizes Δ_2 and Δ_3 must be restricted to the same range of values as shown by

$$0 < \Delta_2^{(j)} < \frac{2}{\lambda_{max}} \tag{5.22}$$

and $$0 < \Delta_3^{(j)} < \frac{2}{\lambda_{max}} \tag{5.23}$$

where λ_{max} is the maximum eigenvalue of $\underline{A}$.

5.2.2 STABILITY:

Stability is the most important problem in the case of a RDF(E). We examine the stability criterion for an individual second order system. [5-5,6].

Let us consider an N^{th} order recursive digital filter with the transfer function given by, say

$$H_1(z) = \frac{1}{1 + \sum_{k=1}^{N} \beta_k z^{-k}} = \frac{\hat{I}(z)}{Y(z)} \tag{5.24}$$

The input output relationship is given by

$$\hat{I}_n = - \sum_{k=1}^{N} \beta_k \hat{I}_{n-k} + y_n \tag{5.25}$$

where $\{y_n\}$, $\{I_n\}$ are input and output sequences, respectively. Defining the state variables as

$$x_n^k \triangleq \hat{I}_{n-k} \qquad k=1,2 \quad \ldots \quad N \tag{5.26}$$

Then the state variable description of RDF(E) becomes

$$\left.\begin{aligned} \underline{x}_{n+1} &= \underline{A}\ \underline{x}_n + \underline{B}\ \underline{y}_n \\ \hat{\underline{I}}_n &= \underline{C}\ \underline{x}_n + \underline{D}\ \underline{y}_n \end{aligned}\right] \tag{5.27}$$

where $$\underline{x}_n = \left[x_n^1 , x_n^2 , \ldots \quad x_n^N \right]^T \tag{5.28}$$

is the N-state vector, and

$$\underline{A} = \begin{bmatrix} -\beta_1 & -\beta_2 & \cdots & -\beta_{N-1} & \beta_N \\ 1 & 0 & \cdots & 0 & 0 \\ & & \cdots & & \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}_{N\times N} \tag{5.29}$$

$$\underline{B} = \begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}_{N\times 1} \tag{5.30}$$

$$\underline{C} = [\,-\beta_1, \quad -\beta_2, \quad \cdots \quad -\beta_N\,] \tag{5.31}$$

$$\underline{D} = [\,1\,] \tag{5.32}$$

and y_n is a scalar.

We shall use Jury's stability test [5-7,8] and define

$$\left.\begin{aligned} a_{1,k} &\triangleq \beta_{N-k} \qquad k=0,\ 1,\ \ldots.\ (N-1) \\ a_{1,N} &\triangleq 1 \end{aligned}\right] \tag{5.33}$$

where, as usual, β's are the multiplier coefficients as in (5.24).

Therefore, the conditions for stability are

$$\left.\begin{array}{ll} \text{(i)} & a_{1,N} + a_{1,N-1} + \cdots + a_{1,0} > 0 \\ \text{(ii)} & a_{1,N} - a_{1,N-1} + \cdots + (-1)^N a_{1,0} > 0 \\ \text{(iii)} & |a_{1,0}| < 1 \\ & |a_{2,0}| > |a_{2,N-1}| \\ & |a_{N-1,0}| > |a_{N-1,2}| \end{array}\right] \tag{5.34}$$

where

$$a_{n,r} = \begin{vmatrix} a_{n-1,0} & \cdots & a_{n-1,N-n+2-r} \\ & \cdots & \\ a_{n-1,N-n+2} & \cdots & a_{n-1,r} \end{vmatrix}$$

Thus the system (5.27) is stable in the sense that its poles are located inside the unit circle of the z-plane if and only if (5.34) is true.

Examples:

(A) When N=2 (second order filter) the conditions for stability are

$$\left.\begin{array}{l} |\beta_1| < (1+\beta_2) \\ |\beta_2| < 1 \end{array}\right] \qquad (5.35)$$

These conditions can be shown by the stability triangle in the (β_1, β_2)-plane Fig.(5.2)

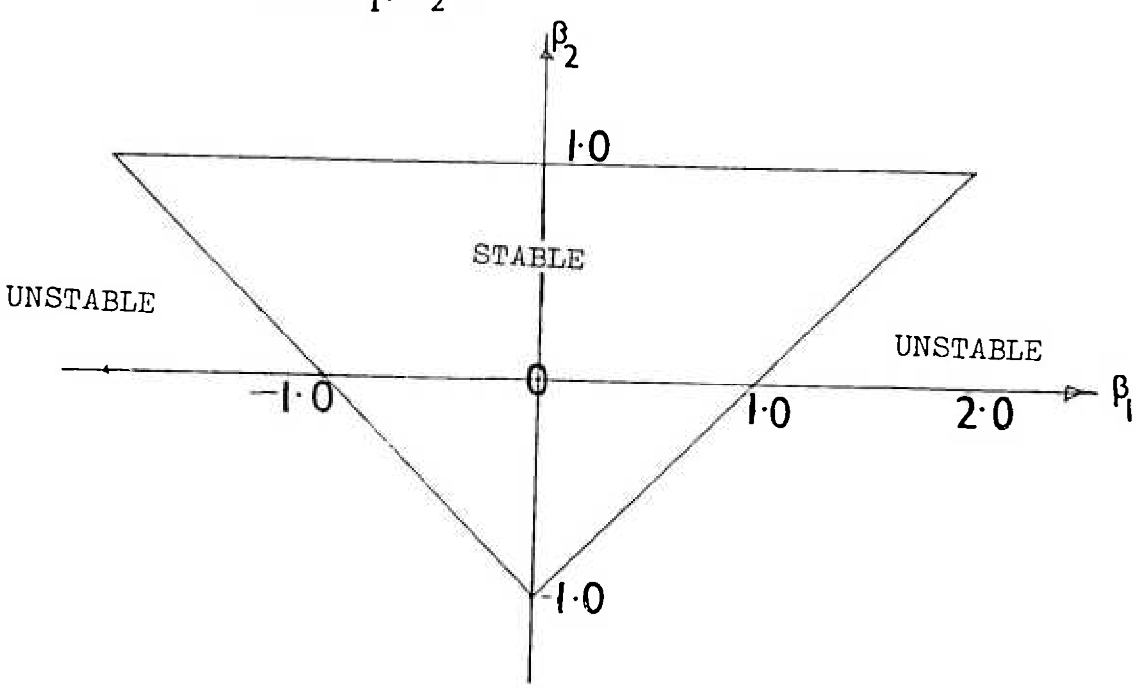


FIG.(5.2) STABILITY DOMAIN FOR A SECOND_ORDER RDFE.

(B) When N=3 (Third-order filter)

The conditions for stability are

$$
\begin{aligned}
|\beta_1 + \beta_3| &< |1+\beta_2| \\
|\beta_3| &< 1 \\
|\beta_1 \beta_3 - \beta_2| &< |\beta_3^2 - 1|
\end{aligned}
\qquad (5.36)
$$

These inequalities will be applied in the subsequent sections where we consider the implementation of RDFE.

5.2.3 BLOCK DIAGRAM REPRESENTATION OF RDFE:

Basic structure of an RDFE can be shown as Fig. (5.3)

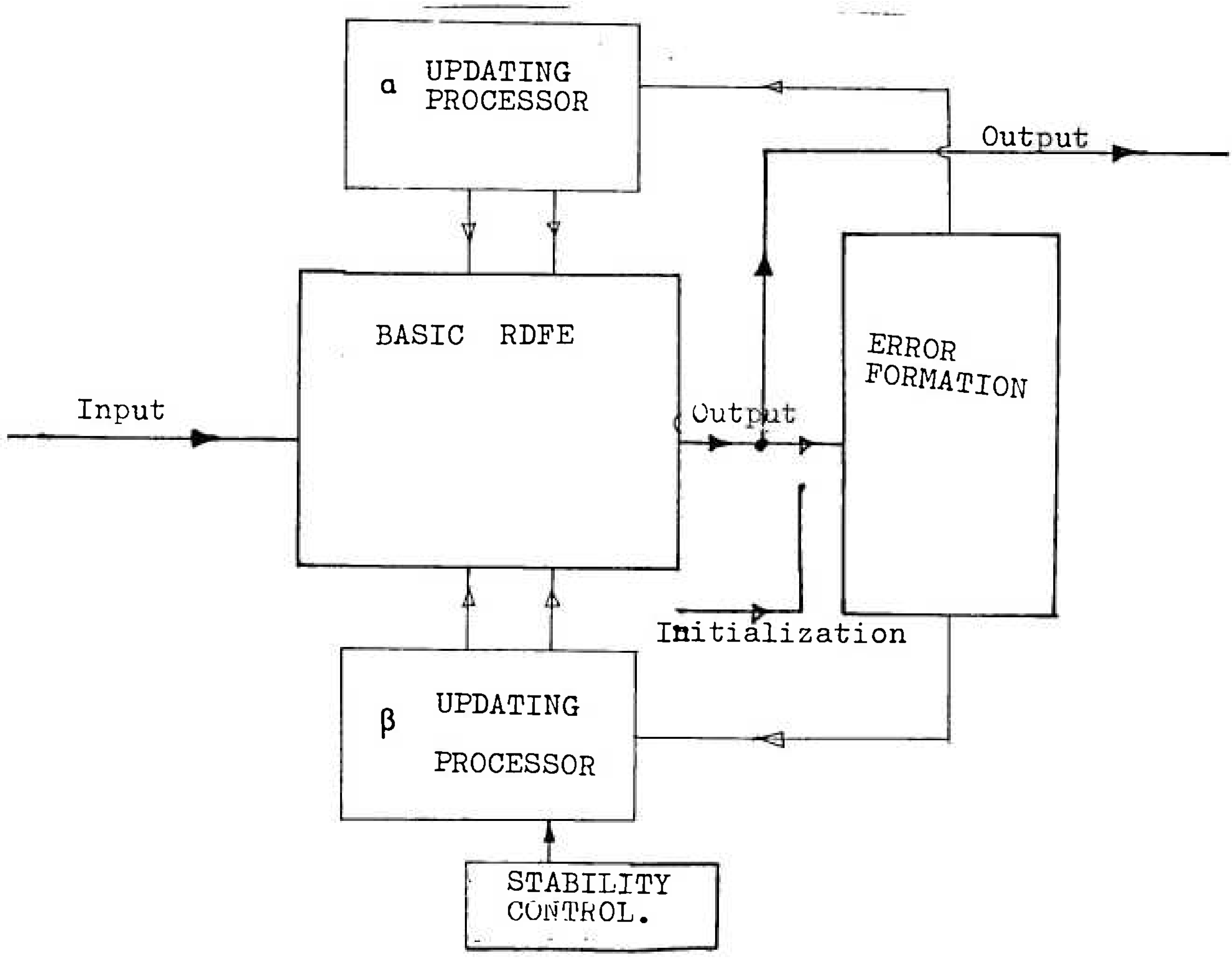


FIG. (5.3) BLOCK DIAGRAM REPRESENTATION OF RECURSIVE DIGITAL FILTER EQUALIZER.

With the help of this block diagram we shall try to study the various forms of RDFE.

5.3 IMPLEMENTATION OF DIRECT FORM (I) ADAPTIVE RDFE:

This is the simplest form of RDFE, and its basic structure shown in Fig. (5.4), is the network corresponding to equation (5.3).

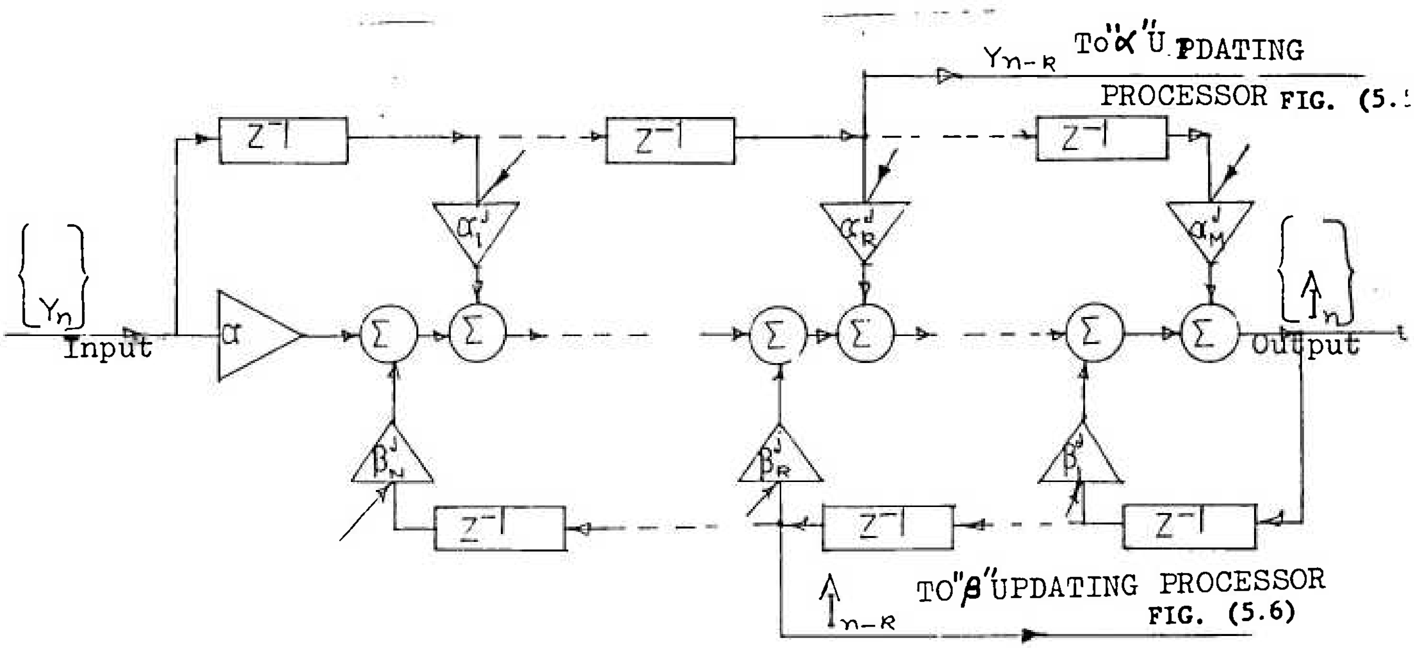


FIG. (5.4) BASIC DIRECT FORM(I) RDFE (M=N)

Error is formed according to equation (5.5) or (5.6) and the MSE is calculated from (5.7). The multiplier coefficient 'α' is updated using Eq. (5.16) where, from Eq. (5.3), on differentiation,

$$\frac{\partial \hat{I}_n}{\partial \alpha_k} = y_{n-k} - \sum_{i=0}^{N} \beta_i \frac{\partial \hat{I}_{n-i}}{\partial \alpha_k} \quad \ldots \tag{5.37}$$

where $k=0,1, \ldots\ldots M$

An 'α' updating processor is shown in Fig. (5.5).

Similarly, the multiplier coefficient 'β' is updated using Eq. (5.17) where from Eq. (5.3), on differentiation,

$$\frac{\partial \hat{I}_n}{\partial \beta_k} = - \hat{I}_{n-k} - \sum_{i=0}^{N} \beta_i \frac{\partial \hat{I}_{n-i}}{\partial \beta_k} \tag{5.38}$$

where $k=1,2, \ldots\ldots N$

A 'β' updating processor including stability control given by Eq. (5.39) is shown in Fig. (5.6).

$$| \beta_i^{(j)} | < 1 \tag{5.39}$$

Fig. (5.8) shows the full implementation of a direct form (I) RDFE, (M=N) which can be obtained by combining Figures (5.4) to (5.7).

As can be seen from the structure Fig. (5.8) the number of multiplications required for the basic direct form (I) RDFE and for the coefficient adjustment algorithms are (M+N+1) and $(1+M+2N+MN+N^2)$ respectively.

5.4 IMPLEMENTATION OF DIRECT FORM (II) ADAPTIVE RDFE:

Since the set of coefficients $\{\alpha_k\}$, $\{\beta_k\}$ correspond to the numerator and denominator polynomials of H(z), we can interpret the basic direct form (I) RDFE structure as consisting of a cascade of two networks, the first realizing the zeros and the second realizing the poles. In other words, rewriting Eq. (5.1), we get

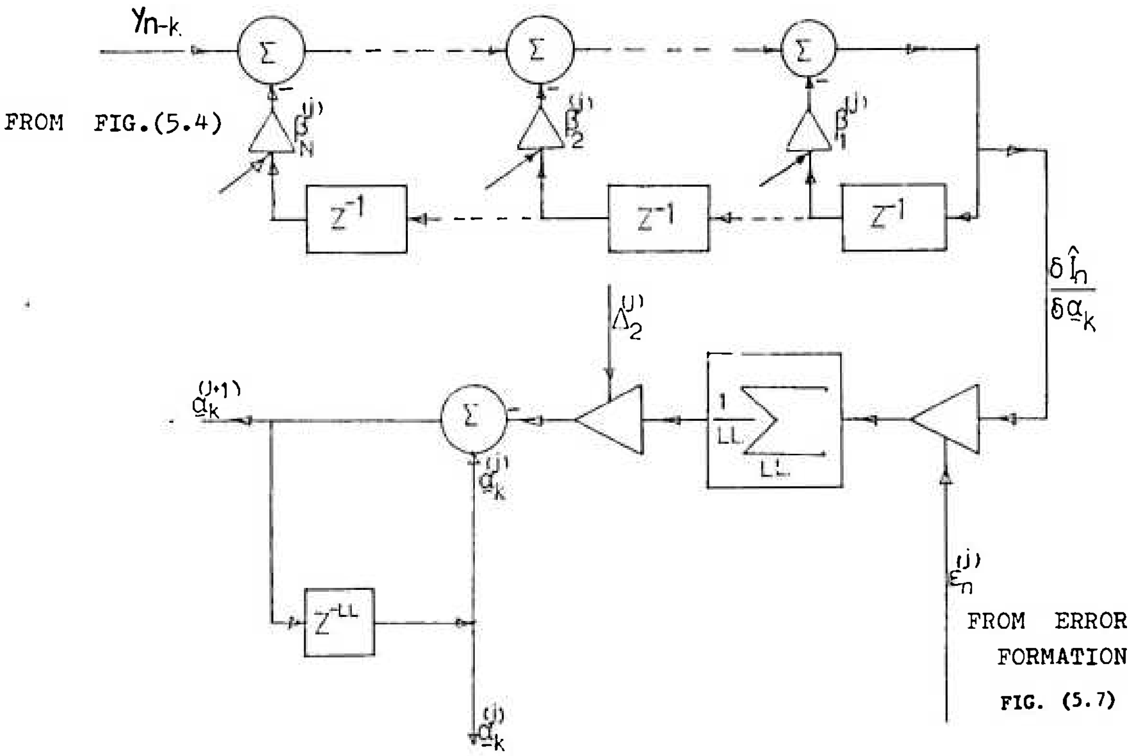


FIG.(5.5) A "a" UPDATING PROCESSOR.

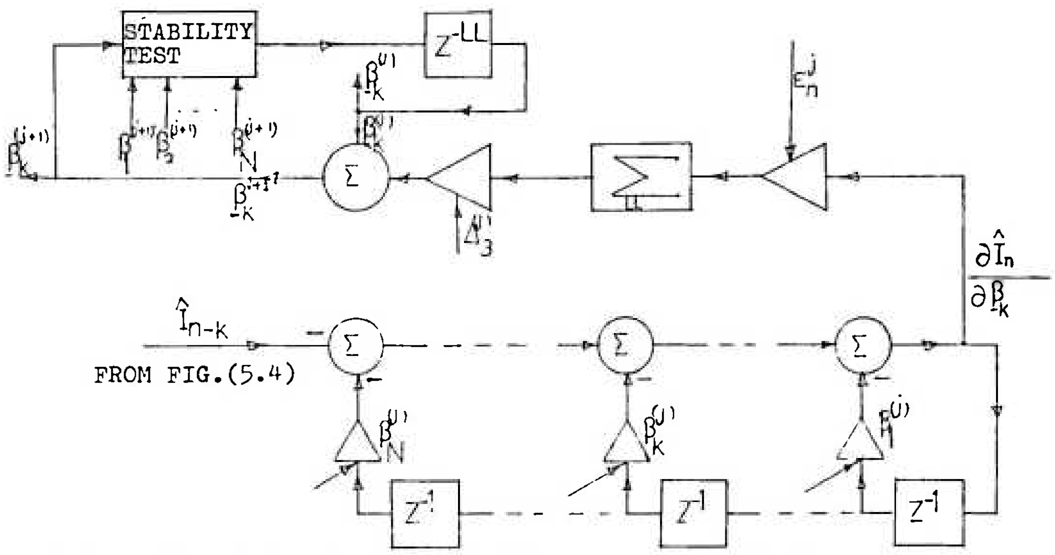


FIG.(5.6) A 'β' UPDATING PROCESSOR WITH STABILITY CONTROL.

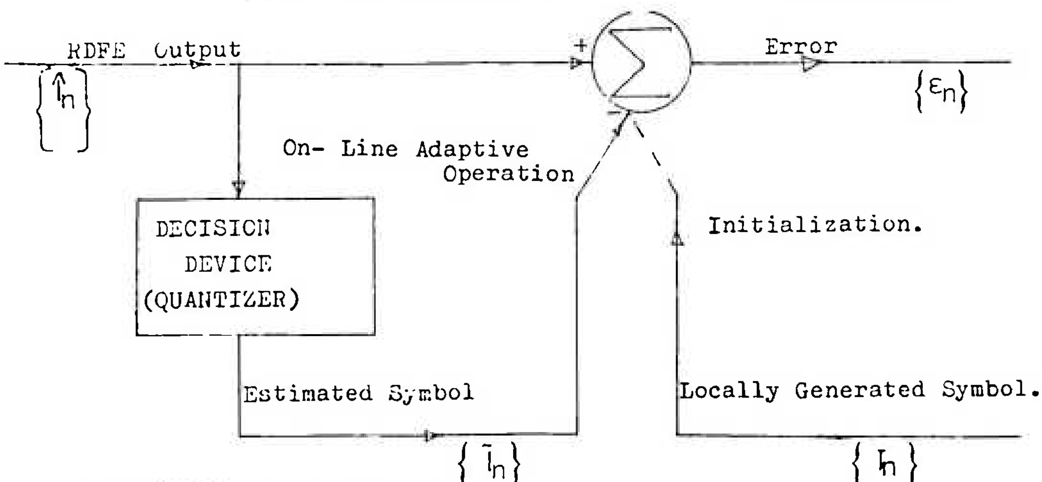


FIG.(5.7) ERROR FORMATION.

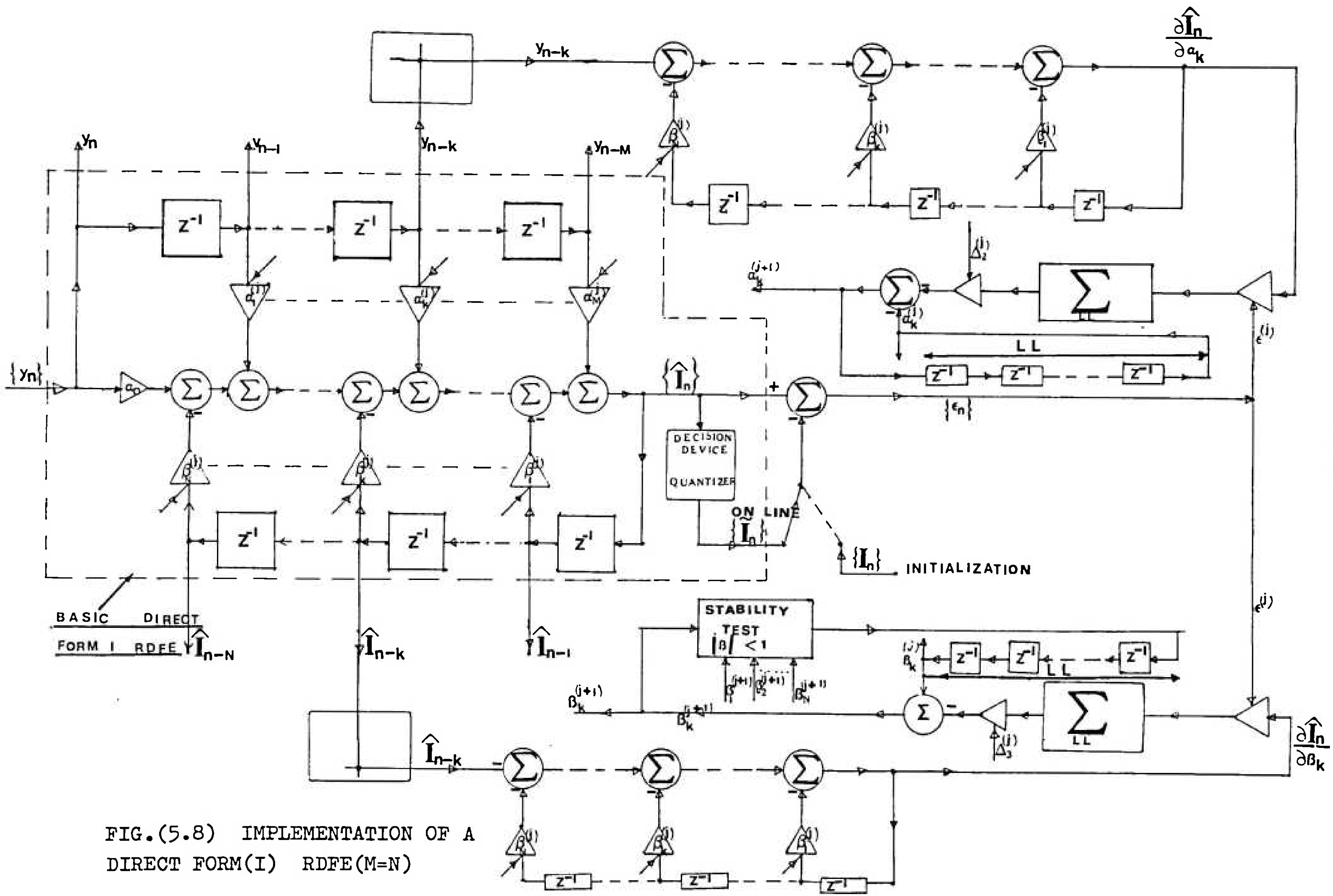


FIG.(5.8) IMPLEMENTATION OF A DIRECT FORM(I) RDFE(M=N)

$$H(z) = \frac{I(z)}{Y(z)} = \left(\sum_{i=0}^{M} \alpha_i z^{-i} \right) \left(\frac{1}{1+ \sum_{i=1}^{N} \beta_i z^{-i}} \right) \tag{5.40}$$

Equation (5.40) can be described by the following pair of difference equations

$$w_n = y_n - \sum_{i=1}^{N} \beta_i w_{n-i} \tag{5.41}$$

and
$$\hat{I}_n = \sum_{i=0}^{M} \alpha_i w_{n-i} \tag{5.42}$$

where $\{w_n\}$ is the state (intermediate) variable.Eqs. (5.41) and (5.42) can be implemented in either canonic or non-canonic forms. We shall consider the canonic form only since one set of delays is sufficient for the entire RDFE. Fig. (5.9) shows the basic direct form (II) RDFE and it is canonic in the sense that it has minimum number of multiplier, adder, and delay elements. However, we shall drop the word canonic and will use simply direct form (II) RDFE.

The coefficients α, $\underline{\beta}$ are adjusted according to the iterative relationships given by Equations (5.16) and (5.17) respectively. However,

$$\frac{\partial \hat{I}_n}{\partial \underline{\alpha}_k} = w_{n-k} \tag{5.43}$$

$$\frac{\partial \hat{I}_n}{\partial \underline{\beta}_k} = \sum_{i=0}^{M} \alpha_i \frac{\partial w_{n-i}}{\partial \underline{\beta}_k} \tag{5.44}$$

and
$$\frac{\partial w_n}{\partial \underline{\beta}} = -\sum_{i=1}^{N} \beta_i \frac{\partial w_{n-i}}{\partial \underline{\beta}_k} - w_{n-k} \tag{5.45}$$

An 'α' updating processor implemented by using Equations (5.16) and (5.43) is shown in Fig. (5.10).

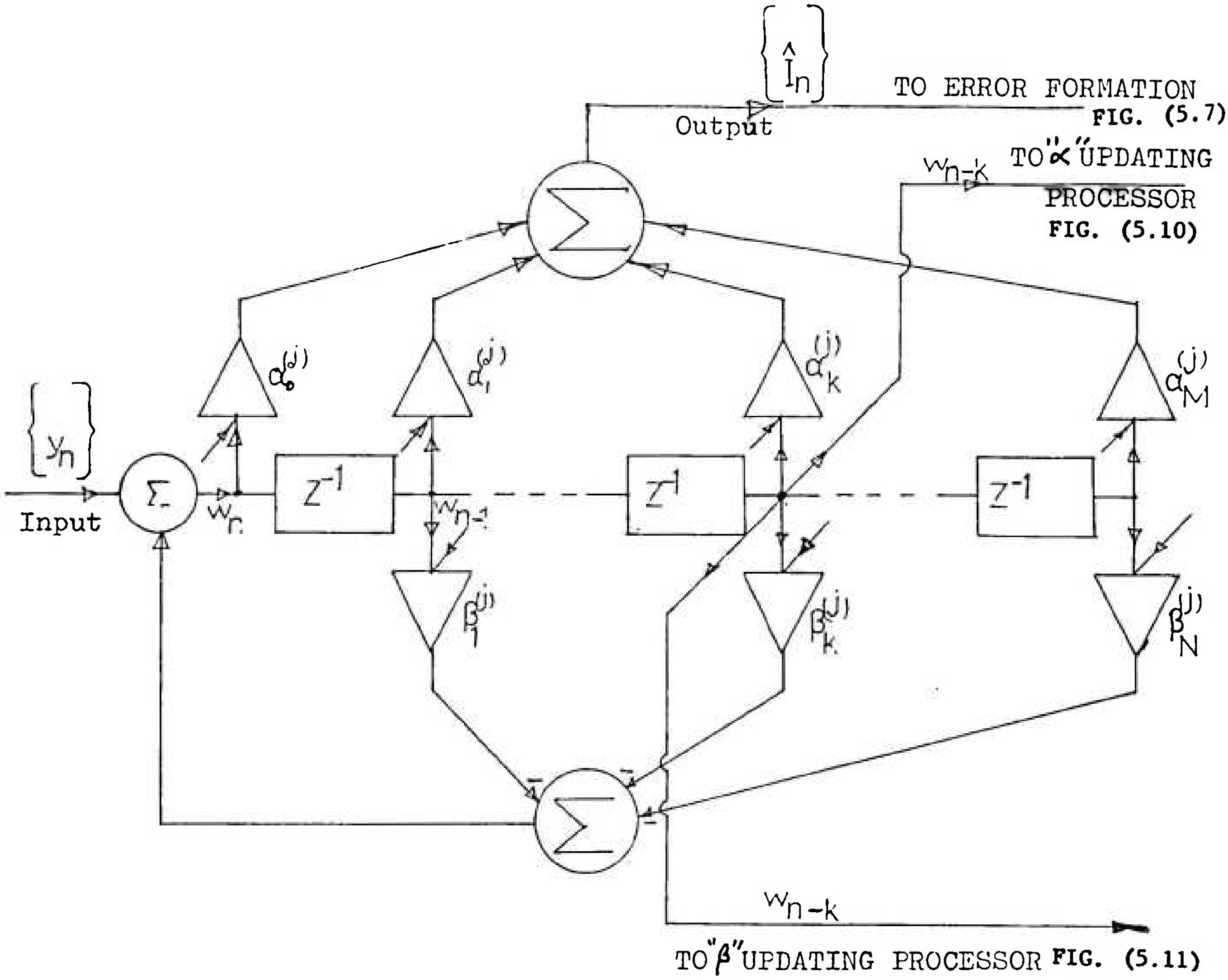


FIG. (5.9) BASIC DIRECT FORM (II) RDFE (M=N).

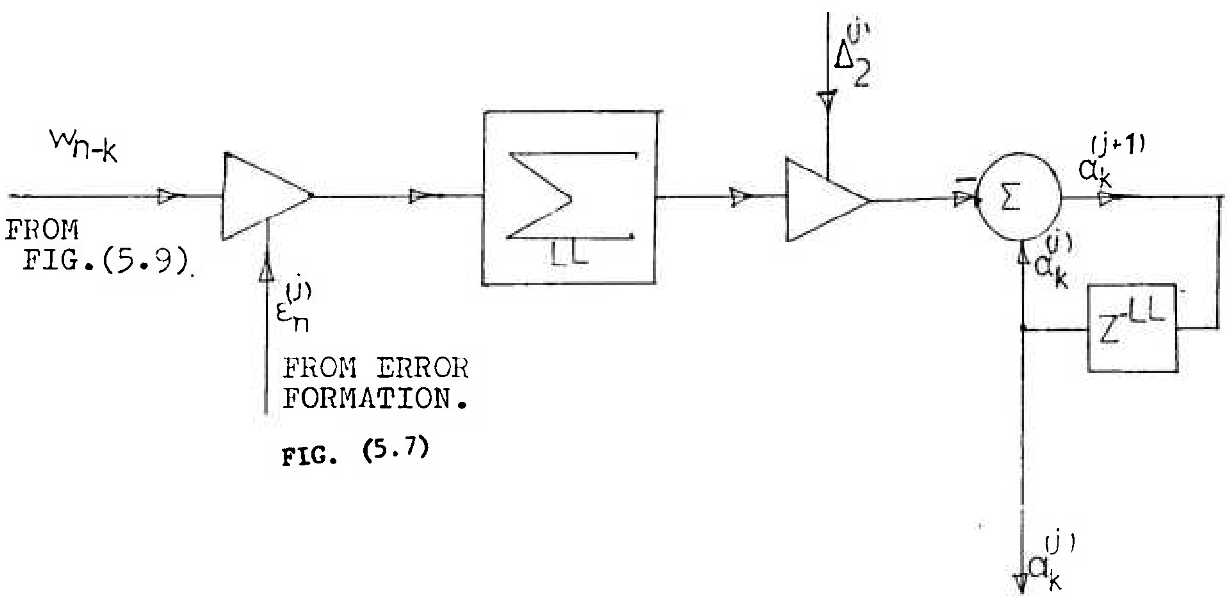


FIG. (5.10) A "α" UPDATING PROCESSOR.

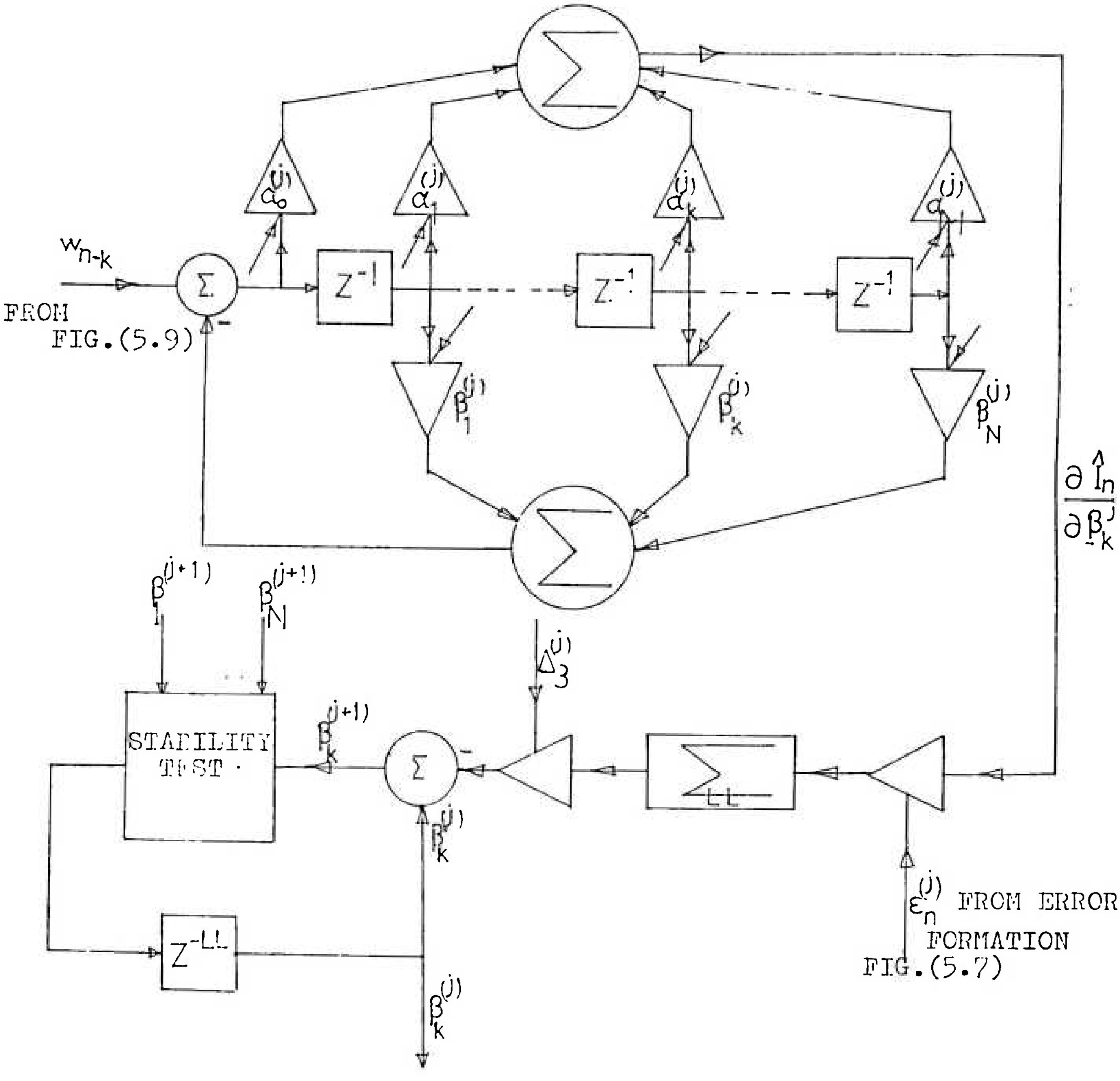


FIG. (5.11) A " β " UPDATING PROCESSOR WITH STABILITY CONTROL

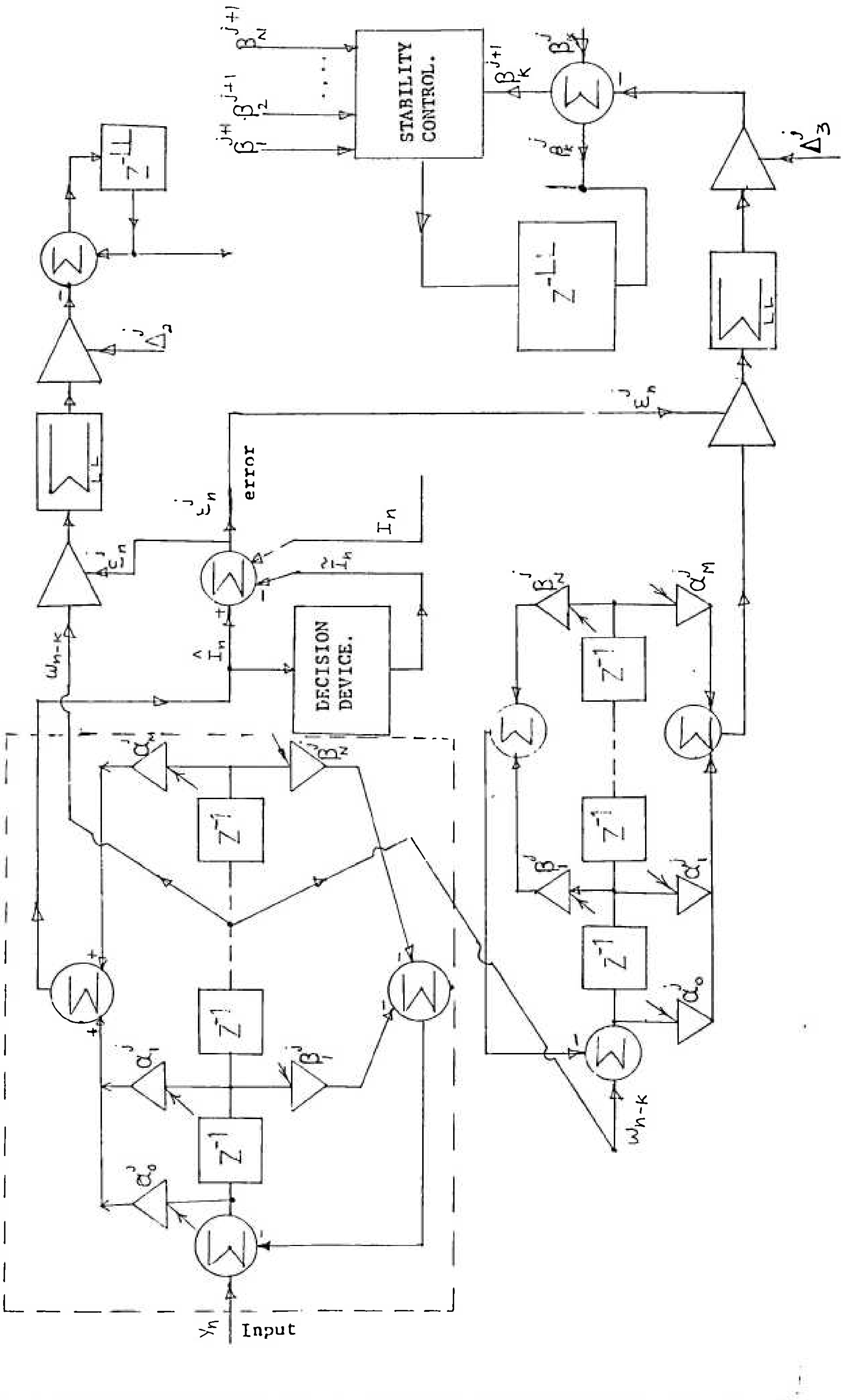


FIG..(5.12) IMPLEMENTATION OF A COMPLETE DIRECT-FORM(II) RDFE.(M= N)

A 'β' updating processor along with stability control, implemented by using Equations (5.17), (5.45) and (5.39), is shown in Fig. (5.11). Error formation is similar to Fig. (5.7).

Fig. (5.12) shows the full implementation of a direct form (II) RDFE, (M=N) which can be obtained by combining Figures (5.9) to (5.11). As can be seen from the structure [Fig. (5.12)] the number of multiplications required for the basic direct form (II) RDFE and for the coefficient updating algorithms are given by $(M+N+1)$ and $(1+M+2N+MN+N^2)$ respectively.

Since these two forms RDF are susceptible to instability therefore, the need for more stable structures are sought. Next, we consider such structures.

5.5 IMPLEMENTATION OF CASCADE FORM ADAPTIVE RDFE:

The realization of RDFE by cascading second order sections has many desirable features, such as better noise performance than the direct form realization, and permitting a modular realization of high-order digital filter equalizer in a flexible manner. The realization is not as straight forward as it appears to be. It is because of the fact that when a fixed point digital filter (equalizer) is realized by cascading its second order sections under dynamic range constraints, the resulting round-off error due to the use of finite word length, convergence, and MMSE are highly dependent upon the pole-zero pairings, initial conditions and specific oredrings. We shall follow closely with Jackson [5-10] for these requirements. Briefly stated, the procedure we propose calls for choosing a random assignment and performing a " local optimization ". This is repeated a number of times and the best of these local optima is taken as the " near optimal " solution (assignment).

When a RDF is realized in the cascade form, its transfer function H(z) is factorized into ratios of second order polynomials as follows:

$$H(z) = \alpha_0 \frac{\prod_{k=1}^{K} (\alpha_{k1} + \alpha_{k2} z^{-1} + \alpha_{k3} z^{-2})}{\prod_{k=1}^{K} (1 + \beta_{k2} z^{-1} + \beta_{k3} z^{-2})} = \frac{I(z)}{Y(z)} \qquad (5.46)$$

where K is the number of second order sections to be cascaded and is an integer. Let the k^{th} quadratic factor of the denominator be paired with the n^{th} quadratic factor of the numerator to form the i^{th} second order section. That is

$$H(z) = \alpha_0 \prod_{i=1}^{K} H_i(z)$$

where (5.47)

$$H_i(z) = \frac{1 + \alpha_{n2} z^{-1} + \alpha_{n3} z^{-2}}{1 + \beta_{k2} z^{-1} + \beta_{k3} z^{-2}}$$

The total number of such assignments is $(K!)^2$. Again taking into account the scaling coefficients $\{S_i\}_{i=0}^{K}$, So as to avoid overflow at certain branch nodes, we can express Eq. (5.47) as

$$H(z) = S_0 \prod_{i=1}^{K} S_i H_i(z)$$

(5.47a)

with $S_0 . S_1 . S_2 .... S_N = \alpha_0$

Assuming $S_0 = S_1 = S_2 = ... S_N = 1$, the realization of (5.47a) can be shown in Fig. (5.13). (page 191)

The individual second-order section of Fig. (5.5) may be realized in either direct form (I) or (II). The reasons for choosing a cascade of second order sections is that a second order section is required to realize a complex pole, or zero, with real multiplier coefficients.

It is extremely difficult to describe the time domain relationship for the entire cascade structure because it depends on the realization of the subsystems $H_i(z)$. For mathematical convenience, we suppose that the subsystems are of direct form (I), and describe the outputs in the following difference equations form:

$$\left.\begin{aligned}
\hat{I}_{1,n} &= \sum_{k=0}^{2} \alpha_{k,1}\, \hat{I}_{0,n-k} - \sum_{k=1}^{2} \beta_{k,1}\, \hat{I}_{1,n-k} \\
\hat{I}_{2,n} &= \sum_{k=0}^{2} \alpha_{k,2}\, \hat{I}_{1,n-k} - \sum_{k=1}^{2} \beta_{k,2}\, \hat{I}_{2,n-k} \\
&\;\;\vdots \\
\hat{I}_{K,n} &= \sum_{k=0}^{2} \alpha_{k,K}\, \hat{I}_{(K-1),(n-k)} - \sum_{k=1}^{2} \beta_{k,K}\, \hat{I}_{K,n-k} \\
\text{where}\quad \hat{I}_{0,n} &= \alpha_0\, y_n
\end{aligned}\right] \tag{5.48}$$

Coefficients are updated according to equations (5.16) and (5.17), i.e. using

$$\left.\begin{aligned}
\underline{\alpha}_{k,i}^{(j+1)} &= \underline{\alpha}_{k,i}^{(j)} - \tfrac{1}{2}\, \Delta_{2i}^{(j)}\, \underline{g}_{\alpha\ k,i}^{(j)} \\
\underline{g}_{\alpha_{k,i}}^{(j)} &= \left\{\frac{1}{LL} \sum_{LL} \varepsilon_n \frac{\partial \hat{I}_n}{\partial \underline{\alpha}_{k,i}}\right\}^{(j)} \\
\text{where}\quad k&=0,\ 1\ ; \qquad \text{and}\ i=1,\ 2,\ \ldots\ K
\end{aligned}\right] \tag{5.49}$$

and

$$\left.\begin{aligned}
\underline{\beta}_{k,i}^{(j+1)} &= \underline{\beta}_{k,i}^{(j)} - \tfrac{1}{2}\, \Delta_{3i}^{(j)}\, g_{\beta\ k,i}^{(j)} \\
\underline{g}_{\beta_{k,i}}^{(j)} &= \left\{\frac{1}{LL} \sum_{LL} \varepsilon_n \frac{\partial \hat{I}_n}{\partial \underline{\beta}_{k,i}}\right\}^{(j)} \\
\text{where}\quad k&=1,\ 2\ ; \qquad \text{and}\ i=1,\ 2,\ \ldots\ K
\end{aligned}\right] \tag{5.50}$$

As before, Δ's denote the step sizes and $\underline{g}$'s denote the gradient vectors. The partial derivatives of $\hat{I}_n$ with respect to the RDFE adjustable parameters is very complicated. However, considering equation (5.48),

$$\frac{\partial \hat{I}_n}{\partial \underline{\alpha}_{k,i}} = \frac{\partial \hat{I}_{K,n}}{\partial \underline{\alpha}_{k,i}} \quad \text{and} \quad \frac{\partial \hat{I}_n}{\partial \underline{\beta}_{k,i}} = \frac{\partial \hat{I}_{K,n}}{\partial \underline{\beta}_{k,i}}$$

are given by the following (K-i+1) difference equations,

$$\left.\begin{aligned}
\frac{\partial \hat{I}_{i,n}}{\partial \underline{\alpha}_{k,i}} &= \hat{I}_{(i-1),(n-k)} - \sum_{r=1}^{2} \beta_{y,i} \frac{\partial \hat{I}_{i,(n-r)}}{\partial \underline{\alpha}_{k,i}} \\
\frac{\partial \hat{I}_{(i+1),n}}{\partial \underline{\alpha}_{k,i}} &= \sum_{r=0}^{2} \alpha_{r,(i+1)} \frac{\partial \hat{I}_{i,(n-r)}}{\partial \underline{\alpha}_{k,i}} - \sum_{r=1}^{2} \beta_{r,(i+1)} \frac{\partial \hat{I}_{(i+1),(n-r)}}{\partial \underline{\alpha}_{k,i}} \\
&\vdots \\
\frac{\partial \hat{I}_{K,n}}{\partial \underline{\alpha}_{k,i}} &= \sum_{r=0}^{2} \alpha_{r,K} \frac{\partial \hat{I}_{(K-1),(n-r)}}{\partial \underline{\alpha}_{k,i}} - \sum_{r=1}^{2} \beta_{r,K} \frac{\partial \hat{I}_{K,(n-r)}}{\partial \underline{\alpha}_{k,i}}
\end{aligned}\right] \quad (5.51)$$

Similarly,

$$\frac{\partial \hat{I}_{K,n}}{\partial \underline{\beta}_{k,i}} = - \hat{I}_{K,(n-k)} - \sum_{r=1}^{2} \beta_{r,i} \frac{\partial \hat{I}_{K,(n-k)}}{\partial \underline{\beta}_{k,i}} \quad (5.52)$$

Basic cascade form adaptive RDFE structure, "α" and "β" updating processors are shown in Figures (5.14) to (5.16) respectively. Error formation is made according to Fig. (5.7). Finally, Fig. (5.17) shows the implementation of a cascade form adaptive RDFE. The number of

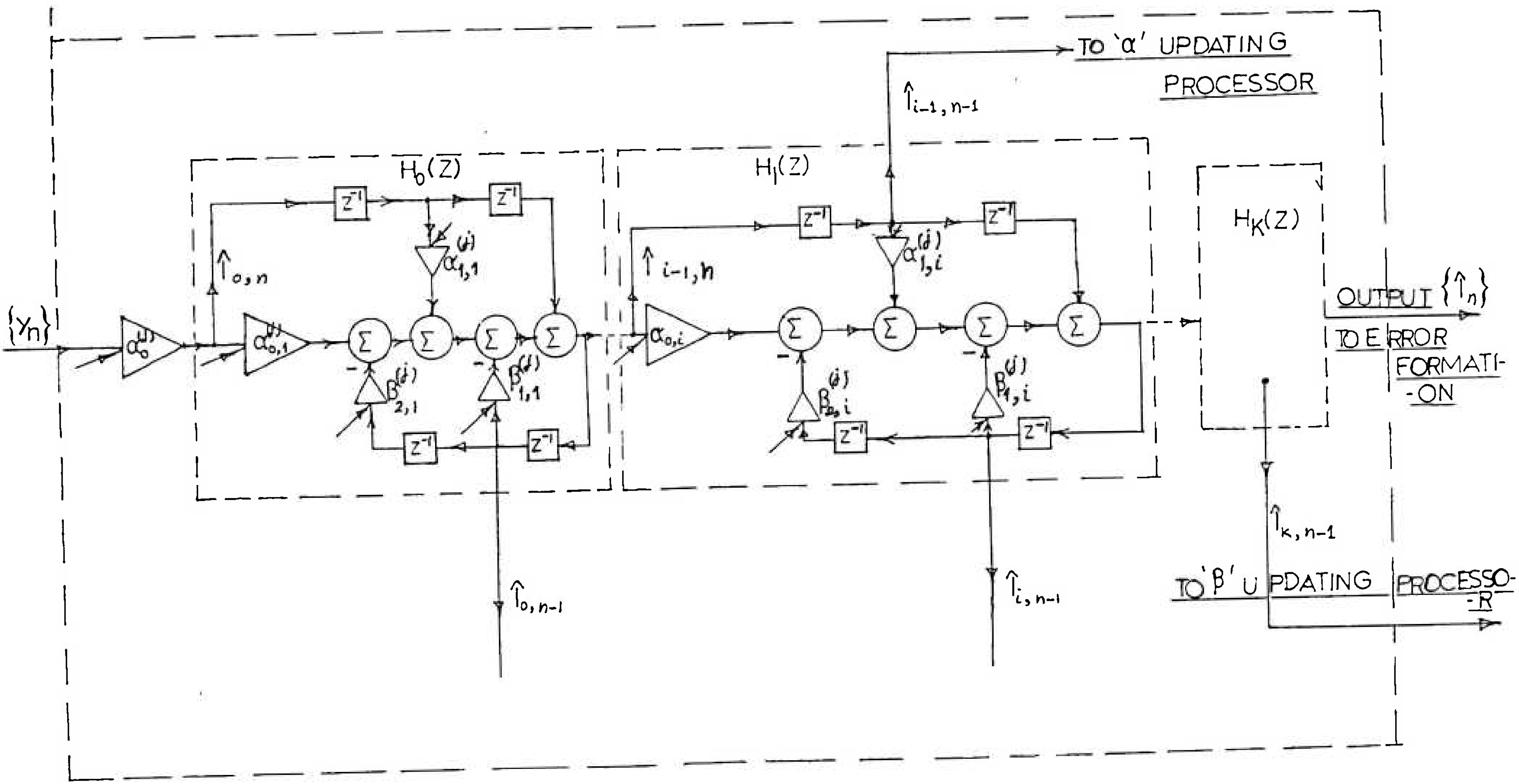


FIG.(5.14) BASIC CASCADE RDFE STRUCTURE(M=N)

FROM EQ. (5.46,5.47,5.48)

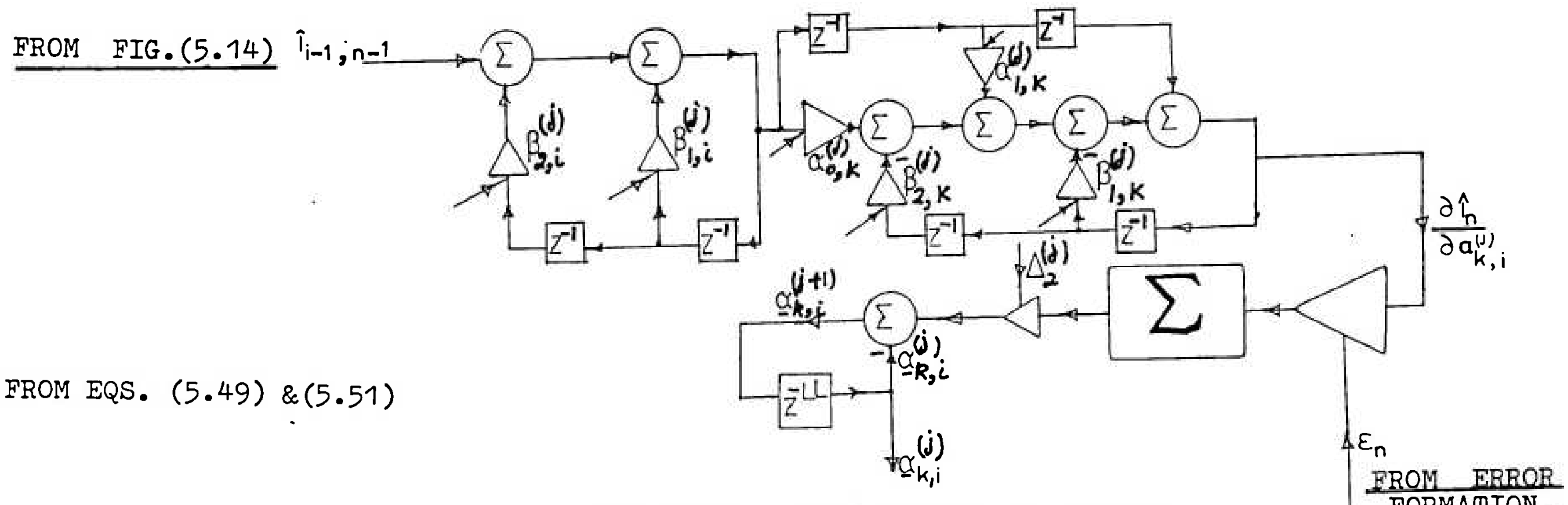


FIG.(5.15) A "α" UPDATING PROCESSOR.

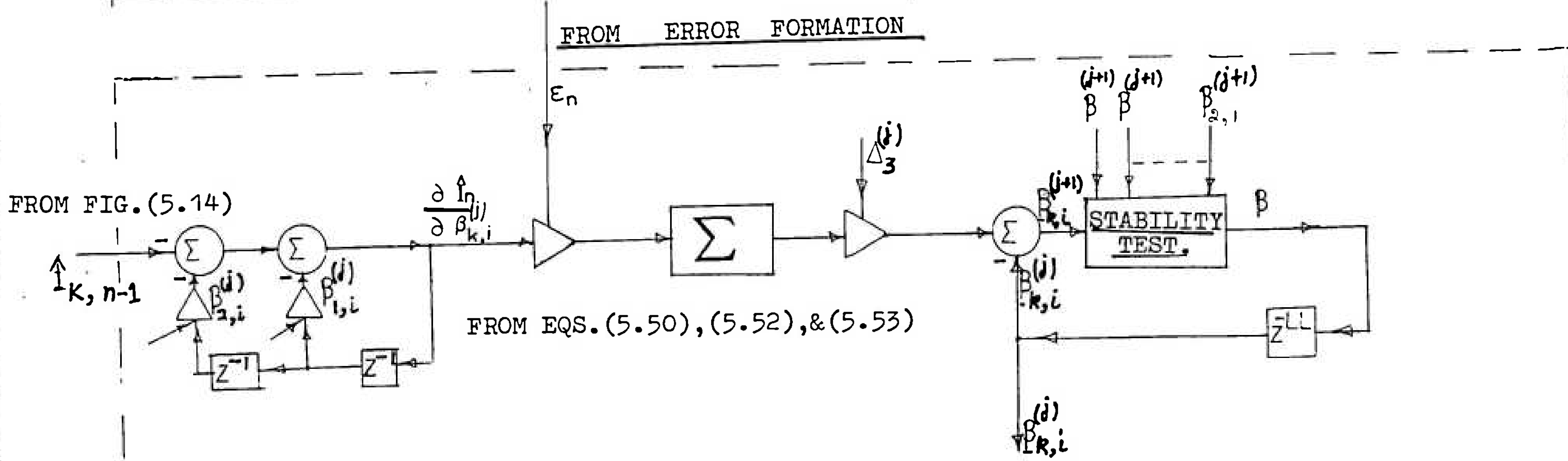


FIG.(5.16) A "β" UPDATING PROCESSOR WITH STABILITY CONTROL.

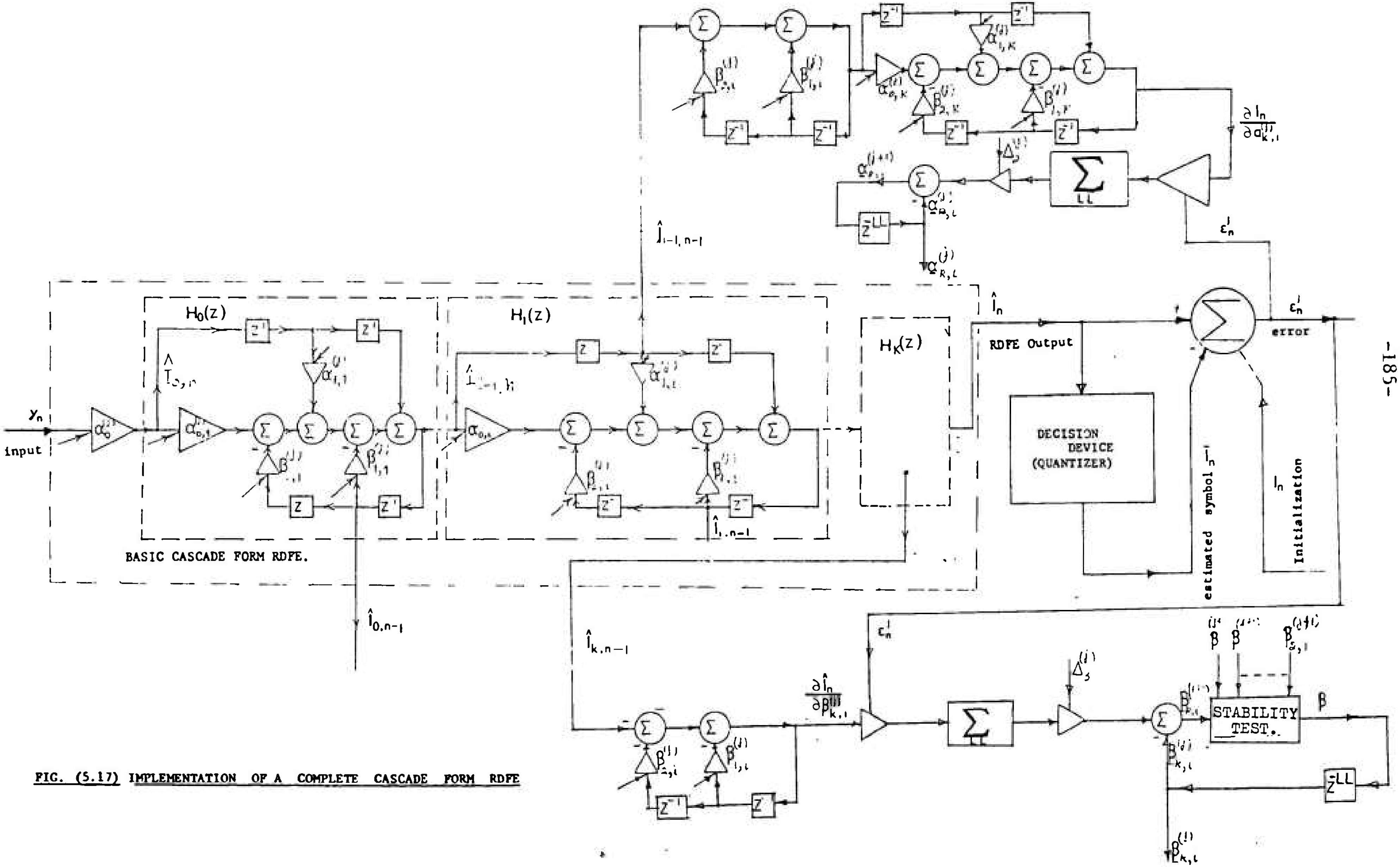
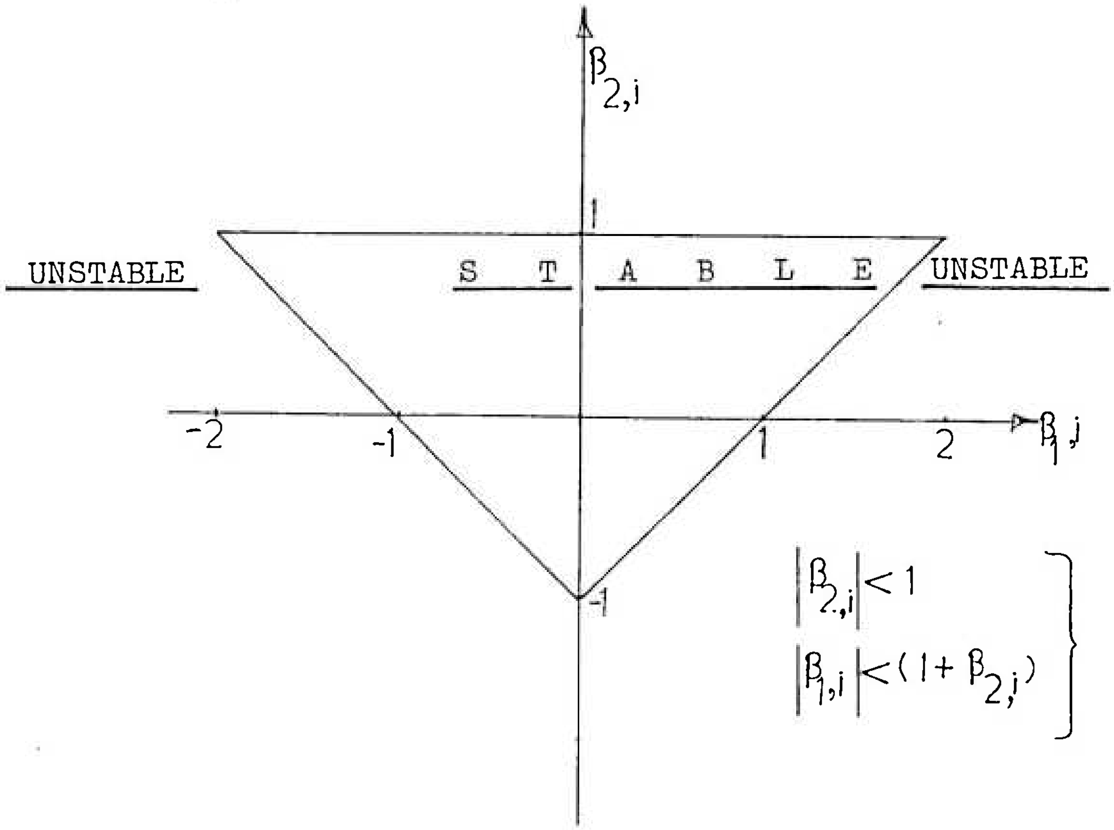


FIG. (5.17) IMPLEMENTATION OF A COMPLETE CASCADE FORM RDFE

multiplications required for the basic cascade form RDFE and for the coefficient adjustment algorithms are (1 + 2N) and (2 + 4N + N^2) respectively (assuming M=N). Stability for individual section is tested according to Fig. (5.18).



$$\left.\begin{array}{l} |\beta_{2,i}| < 1 \\ |\beta_{1,i}| < (1+\beta_{2,i}) \end{array}\right\} \qquad (5.53)$$

FIG.(5.18) STABILITY DOMAIN FOR CASCADE STRUCTURE.

5.5.1 INITIAL CONDITIONS:

In cascade form RDFE, there are two important factors relating to convergence and MMSE.

(a) The pairing of poles and zeros and

(b) Orderings of the K-sections

There is considerable flexibility in the manner in which the poles and zeros are paired together and in the order in which the resulting second order subsystems are cascaded. It appears that all such pairing and orderings are equivalent for infinite precision arithmetic, though they may differ considerably in practice, owing to finite word-length effects. A detailed analysis is complicated. However, the above mentioned factors are easily accomplished by optimizing the filter design in the direction of minimizing either peak or total round off noise energy. It has been found that peak energy can be minimized by arranging sections approximately in order of decreasing Q[5-10]. In this case the high Q-section which has the largest peak response and hence will tend to amplify the round off noise most, is placed early in the basic structure and encounters the maximum subsequent filtering action. Also, this arrangement requires the largest amount of pre-scaling before entering the filter to prevent overflow. In the somewhat more common case of desiring to minimize total noise energy, the sections are arranged roughly in the order of increasing Q [5-10] . However, this is the case usually used in practice.

The pole-zero pairing and the difficulty encountered can be overcome by the proper selection of initial conditions which are given by (5.54) [5-6, Page 313]

$$\left.\begin{array}{ll} \alpha_{0,i} = \dfrac{1}{R^2} & \beta_{1,i} = -2\ r\ \mathrm{Cos}\left(\dfrac{2\pi i}{K}\right) \\ \alpha_{1,i} = -\dfrac{2}{R}\ \mathrm{Cos}\left(\dfrac{2\pi i}{K}\right) & \beta_{2,i} = r^2 \\ \alpha_{2,i} = 1 & \end{array}\right] \qquad (5.54)$$

where $i = 1, 2, \ldots\ldots K$

with $r = 1/R$ and $0.2 \leq r \leq 0.5$

5.6 IMPLEMENTATION OF PARALLEL-FORM ADAPTIVE RDFE:

An alternate approach is to write equation(5.1) in the partial fraction expansion form.

$$H(z) = C(z) + \sum_{i=1}^{K} H_i(z) \tag{5.55}$$

where $H_i(z)$ is the second order section of the form

$$H_i(z) = \frac{\alpha_{0,i} + \alpha_{1,i}\, z^{-1}}{1 + \beta_{1,i}\, z^{-1} + \beta_{2,i}\, z^{-2}} \tag{5.56}$$

K is the integer part of $(N + 1)/2$ and $C = (\alpha_N / \beta_N)$ (5.57)

The realization of Equation (5.55) ia called the parallel form and is shown in Fig. (5.19).

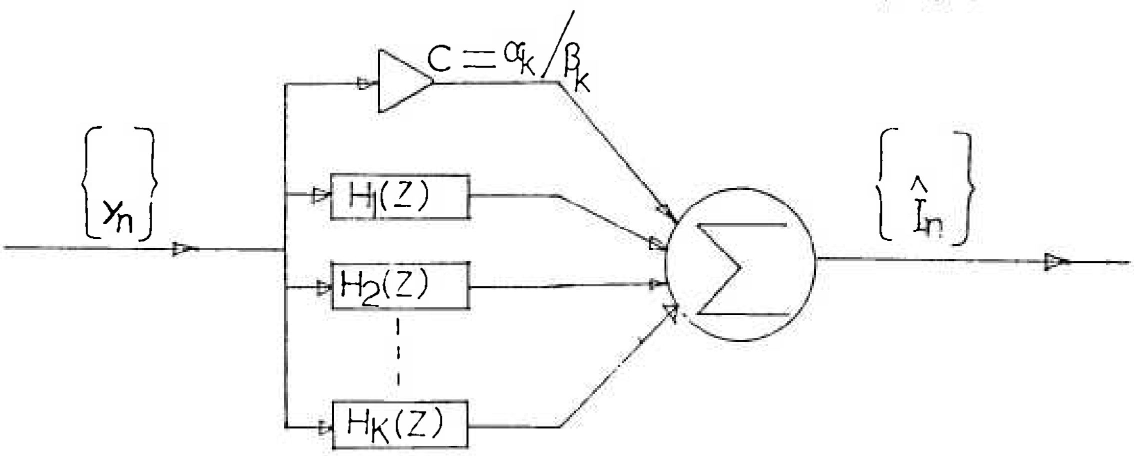


FIG.(5.19) PARALLEL FORM.

The input-output relationship is given by the following difference equations. For mathematical convenience, considering only the direct form (I) second-order systems we have

$$\left.\begin{aligned}
\hat{I}_{0,n} &= \sum_{k} \left(\alpha_k / \beta_k \right) y_{n-k} \\
\hat{I}_{1,n} &= \sum_{k=0}^{2} \alpha_{k,1}\, y_{n-k} - \sum_{k=1}^{2} \beta_{k,1}\, \hat{I}_{1,n-k} \\
&\;\vdots \\
\hat{I}_{K,n} &= \sum_{k=0}^{2} \alpha_{k,K}\, y_{n-k} - \sum_{k=1}^{2} \beta_{k,K}\, \hat{I}_{K,n-k} \\
\hat{I}_{n} &= \sum_{i=0}^{K} \hat{I}_{i,n}
\end{aligned}\right] \qquad (5.58)$$

Coefficient adjustments are done according to Equations (5.49) and (5.50) i.e.

$$\frac{\partial \hat{I}_n}{\partial \underline{\alpha}_{k,i}} = \frac{\partial \hat{I}_{i,n}}{\partial \underline{\alpha}_{k,i}} = y_{n-k} - \sum_{r=1}^{2} \beta_{r,i} \frac{\partial \hat{I}_{i,n-r}}{\partial \underline{\alpha}_{k,i}} \qquad (5.59)$$

and $$\frac{\partial \hat{I}_n}{\partial \underline{\beta}_{k}} = \frac{\partial \hat{I}_{i,n}}{\partial \underline{\beta}_{k}} = - \hat{I}_{i,n-k} - \sum_{r=1}^{2} \beta_{r,i} \frac{\partial \hat{I}_{i,n-r}}{\partial \underline{\beta}_{k,i}} \qquad (5.60)$$

The coefficient of (α_k / β_k) is adjusted using

$$\frac{\partial \hat{I}_n}{\partial (\alpha_k/\beta_k)} = \frac{\partial \hat{I}_{0,n}}{\partial (\alpha_k/\beta_k)} = y_{n-k} \qquad (5.61)$$

in

$$\left[\frac{\alpha_k}{\beta_k}\right]^{(j+1)} = \left[\frac{\alpha_k}{\beta_k}\right]^{(j)} - \Delta_6^{(j)}\, g^{(j)}_{(\alpha_k/\beta_k)} \qquad (5.62)$$

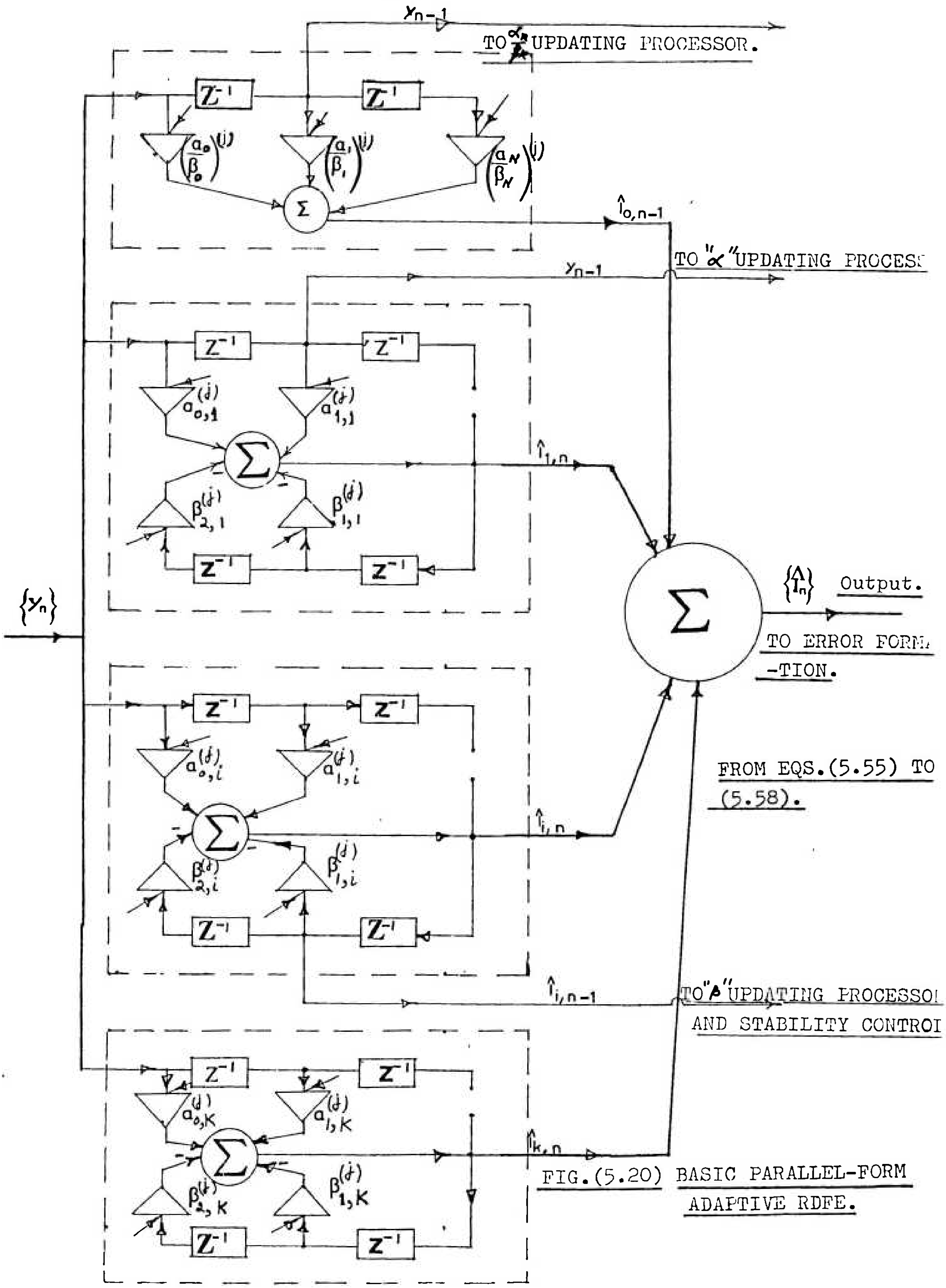


FIG.(5.20) BASIC PARALLEL-FORM ADAPTIVE RDFE.

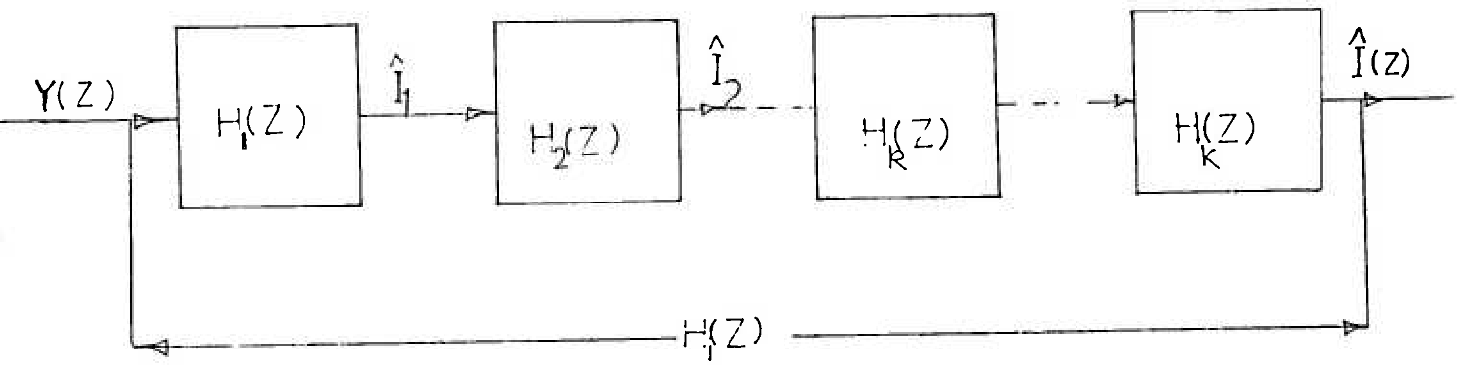


FIG.(5.13) CASCADE FORM.

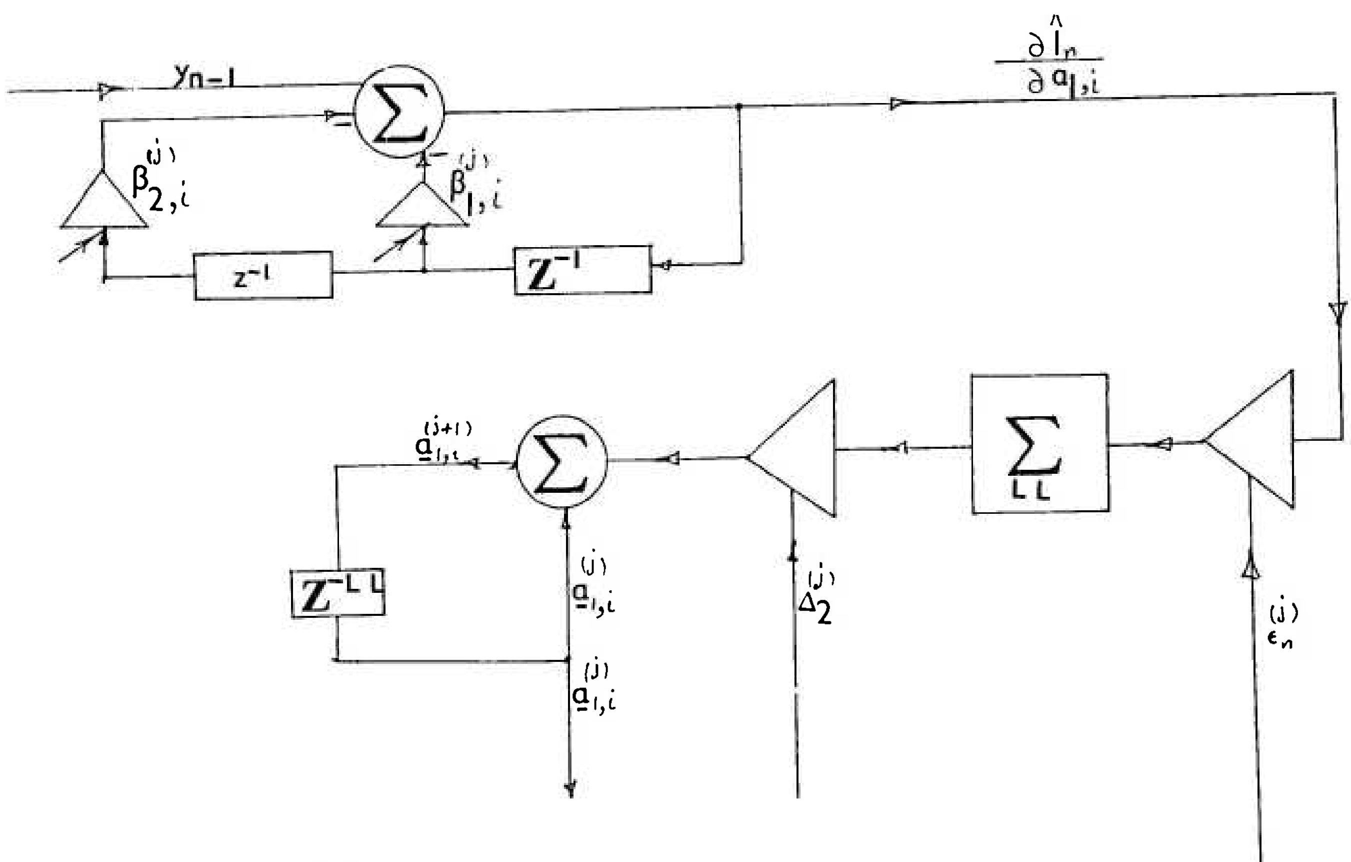
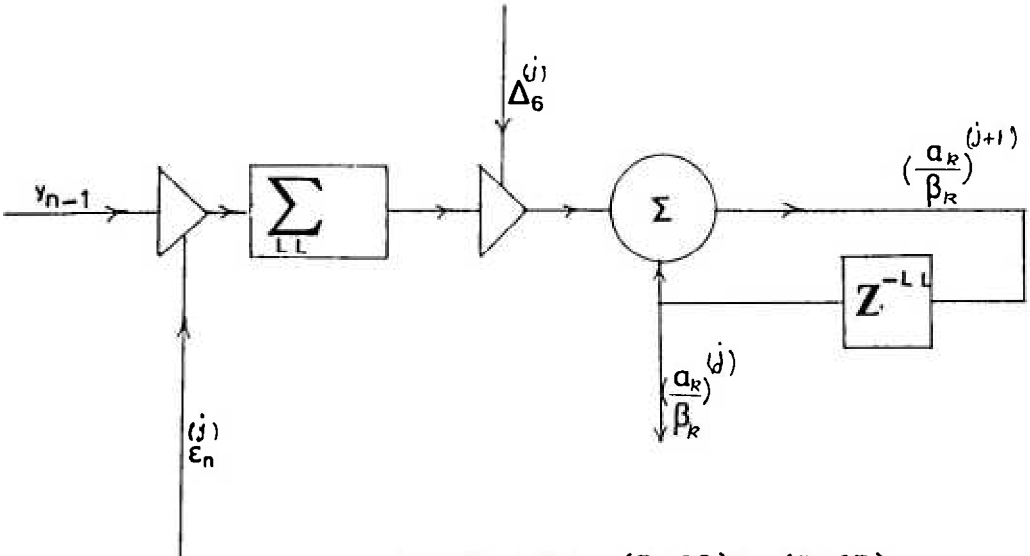


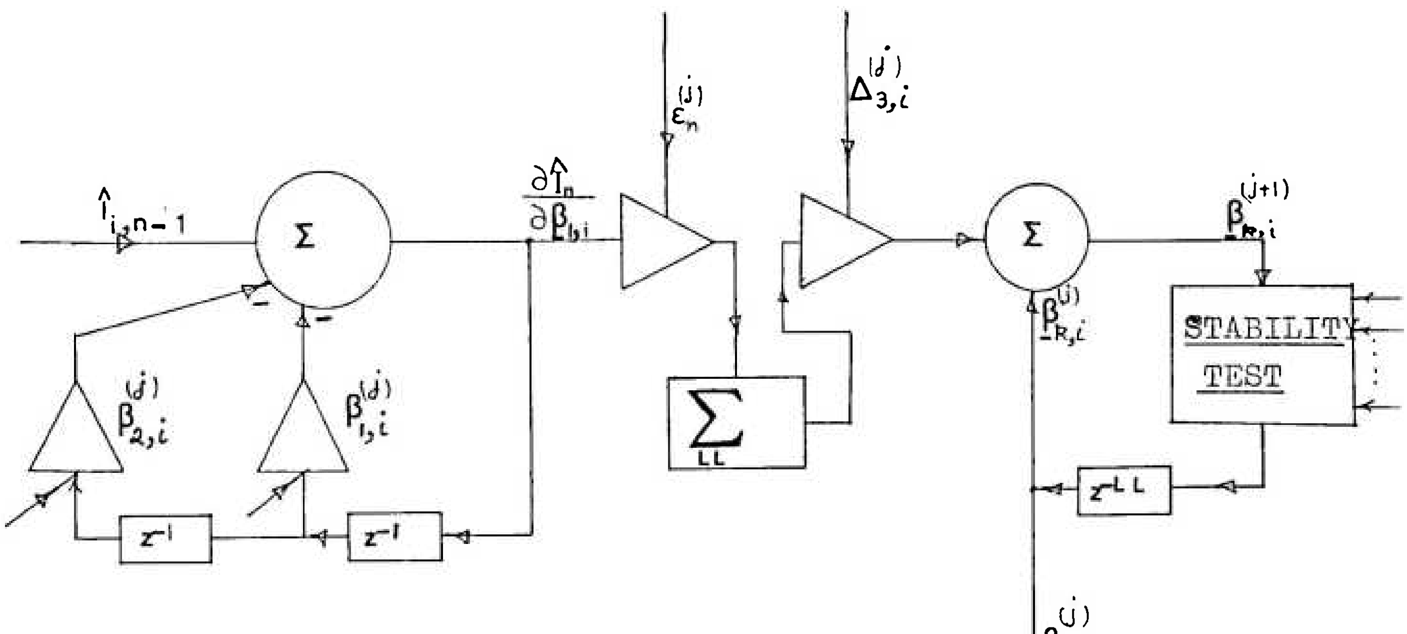
FIG.(5.21) A "a" UPDATING PROCESSOR.

FROM EQS.([illegible]),([illegible])



FROM EQS. (5.62)& (5.63)

FIG.(5.22) A $\left[\frac{\alpha}{\beta}\right]$ UPDATING PROCESSOR.



FROM EQS. (5.50) &(5.60)

FIG.(5.23) A β UPDATING PROCESSOR WITH STABILITY CONTROL.

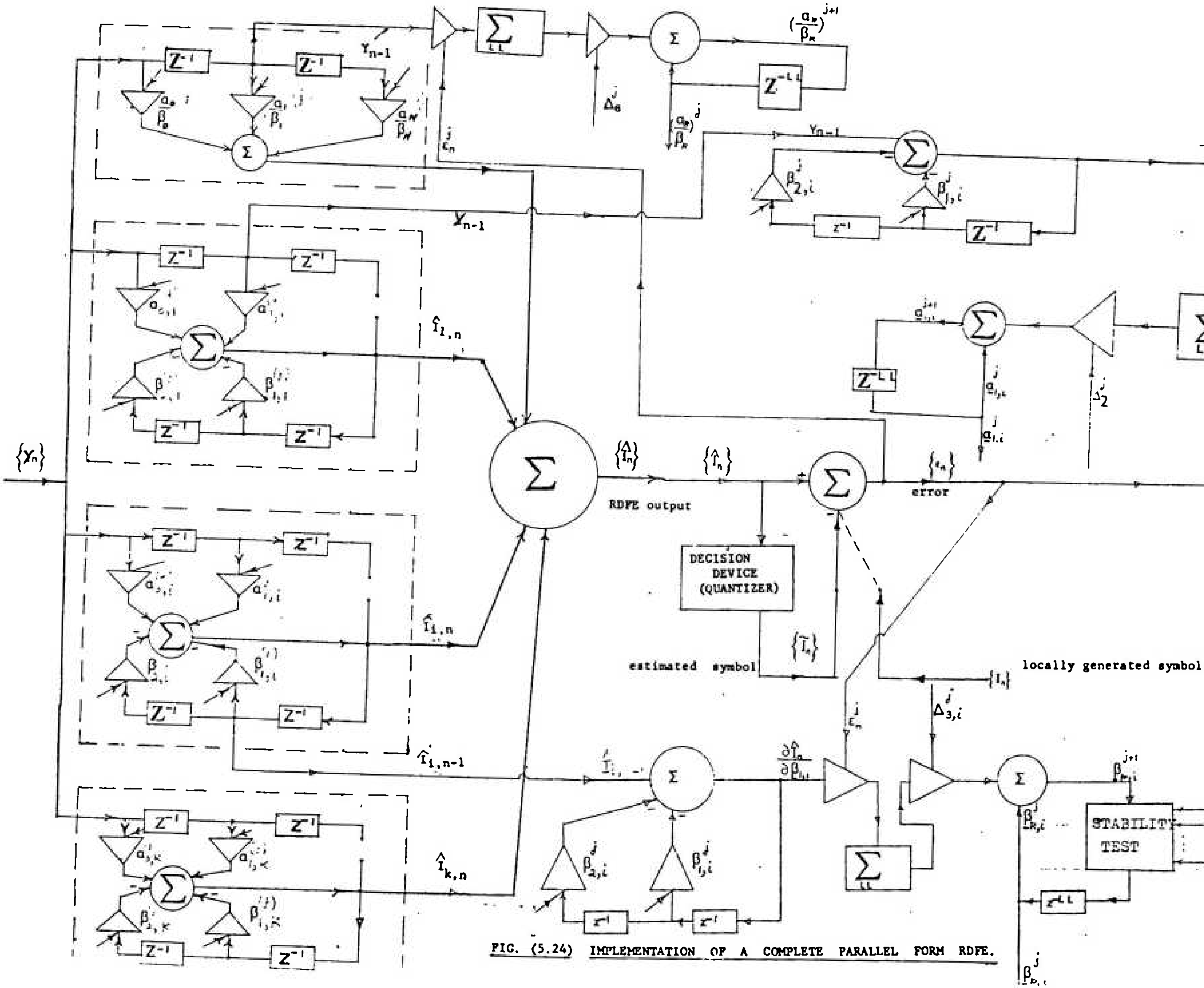


FIG. (5.24) IMPLEMENTATION OF A COMPLETE PARALLEL FORM RDFE.

where

$$g^{(j)}_{(\alpha_k/\beta_k)} = 1/LL \sum_{n=0}^{LL-1} \varepsilon_n \frac{\partial \hat{I}_n}{\partial(\alpha_k/\beta_k)} \qquad (5.63)$$

The implementation of basic parallel form adaptive RDFE structure "α_k", " α_k/β_k ", "β_k" along with stability control, updating processors are shown in Figs. (5.20) to (5.23) respectively. The formation of error is made according to Fig. (5.7) and Fig. (5.24) shows the implementation of complete parallel form adaptive RDFE.

The number of multiplications required for the basic parallel form RDFE and for the coefficients adjustment algorithms are (1+2N) and (2+6N) respectively (assuming M=N)

5.6.1 STABILITY OF PARALLEL-FORM ADAPTIVE RDFE:

From Eq. (5.55) it can be seen that H(z) consists of a non-recursive part C(z) in parallel with a recursive part $\sum_{i=1}^{K} H_i(z)$ made up of second order elements as in Eq. (5.56). The problem of stability in the case of NRDF does not arise, whereas for the recursive individual section the question of stability has already been dealt with in section (5.2.2) and the same conditions apply here as well.

5.6.2 INITIAL CONDITIONS:

Approximate initial conditions have been discussed in section (5.5.1) and the same apply here as well.

5.7 IMPLEMENTATION OF LATTICE-FORM ADAPTIVE RDFE:

There is evidence that in addition to standard digital filter forms such as the direct (I and II), cascade, and parallel forms, digital lattice and ladder filters may play an important role in finite word-length implementation of equalizers.

We consider here lattice form digital filter proposed by Gray and Markel [5-11] as an adaptive RDFE. This structure is based on the properties of orthogonal polynomials. It has the advantage that its stability is easy to test and its stability conditions are given by its feedback multiplier coefficients $\{k_i\}_{i=1}^{N}$

$$\left| k_i \right| < 1 \qquad i = 1, 2, \dots N \tag{5.64}$$

Gray and Markel [5-11] have considered three models of this particular form i.e. (a) Two-multipliers, (b) one-multiplier and (c) Three multipliers. We have selected "two-multiplier model" since each section contains two-multipliers and that the other models can be easily deduced from it. A general implementation of lattice (and ladder) form and the prototype filter for the two multiplier model are shown in Figs. (5.25) and (5.26) respectively.

The input-output relationship of this model is given by the following state-equations in the discrete form

$$\left.\begin{aligned} \underline{W}_{n+1} &= \hat{\underline{F}}\, \underline{W}_n + \hat{\underline{G}}\, Y_n \\ \hat{I}_n &= \hat{\underline{H}}\, \underline{W}_{n+1} \end{aligned}\right] \tag{5.66}$$

where

$$\underline{W}_n^T = \left[w_{1,n}, \;\; w_{2,n} \;\; \dots \;\; w_{N,n} \right] \tag{5.66}$$

is the vector of state variables

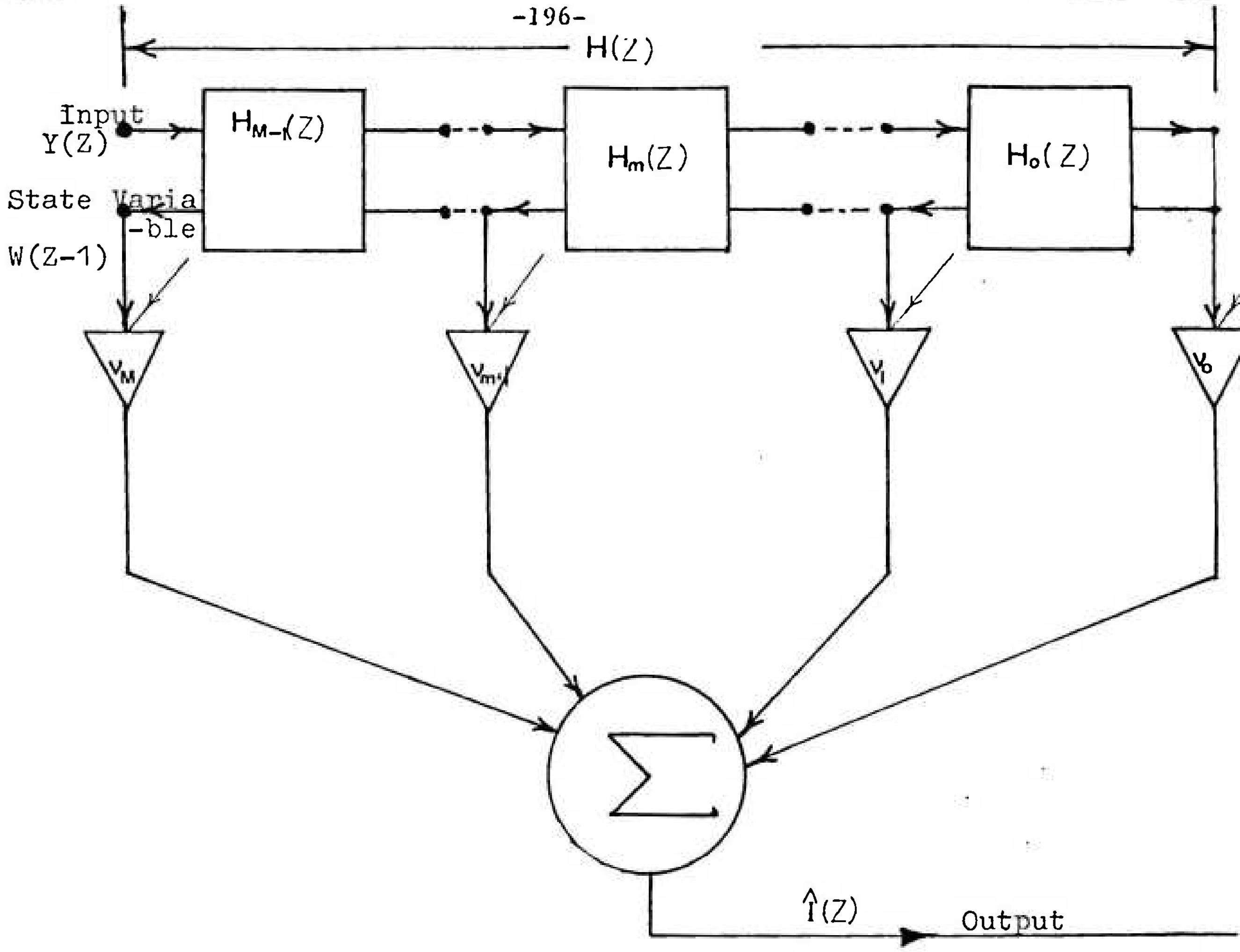


FIG.(5.25) A LATTICE FORM IMPLEMENTATION.

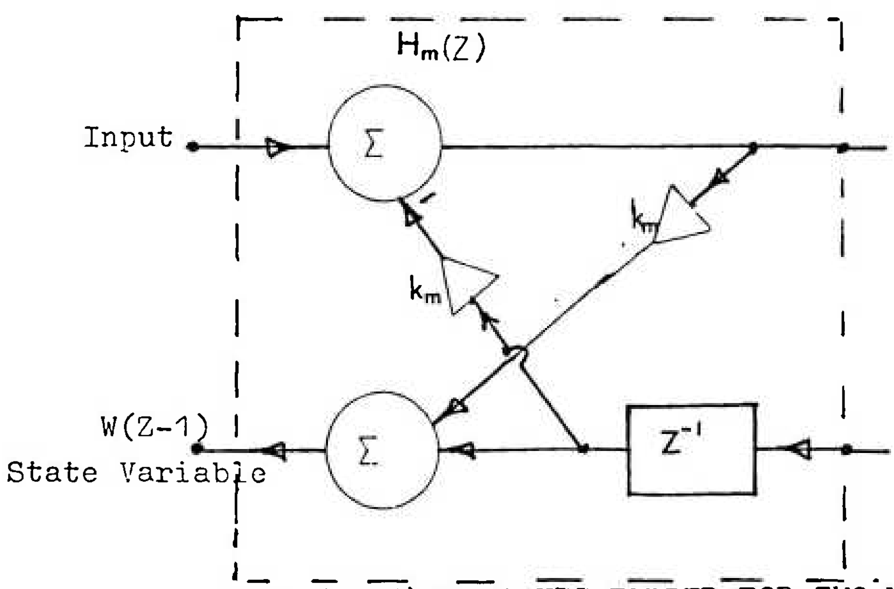


FIG.(5.26)PROTOTYPE FILTER FOR TWO MULTIPLIER LATTICE MODEL.

$$\underline{\hat{F}} = \begin{bmatrix} -k_1 & -k_2 & \cdots\cdots & -k_{N-1} & -k_N \\ 1-k_1^2 & -k_1k_2 & \cdots\cdots & -k_1k_{N-1} & -k_1k_N \\ 0 & 1-k_2^2 & \cdots\cdots & -k_2k_{N-1} & -k_2k_N \\ 0 & 0 & \cdots\cdots & \cdots & \cdots \\ 0 & 0 & & & \\ 0 & 0 & 0 & -k_{N-2}k_{N-1} & -k_{N-2}k_N \\ 0 & 0 & 0 & 1-k_{N-1}^2 & -k_{N-1}k_N \end{bmatrix} \tag{5.67}$$

$$\underline{\hat{G}}^T = [1,\ k_1,\ k_2, \quad \cdots \quad k_{N-2},\ k_{N-1}] \tag{5.68}$$

and $$\underline{\hat{H}} = [\nu_0,\ \nu_1,\ \nu_2, \qquad \nu_{N-1}] \tag{5.69}$$

The adjustments of the coefficients of the lattice-form adaptive RDFE are given by the following iterative relationships.

$$\underline{k}_m^{(j+1)} = \underline{k}_m^{(j)} - \Delta_7^{(j)} \ \underline{g}_{k_m}^{(j)} \tag{5.70}$$

and $$\underline{\nu}_m^{(j+1)} = \underline{\nu}_m^{(j)} - \Delta_8^{(j)} \ \underline{g}_{\nu_m}^{(j)} \tag{5.71}$$

where gradient vectors are given by

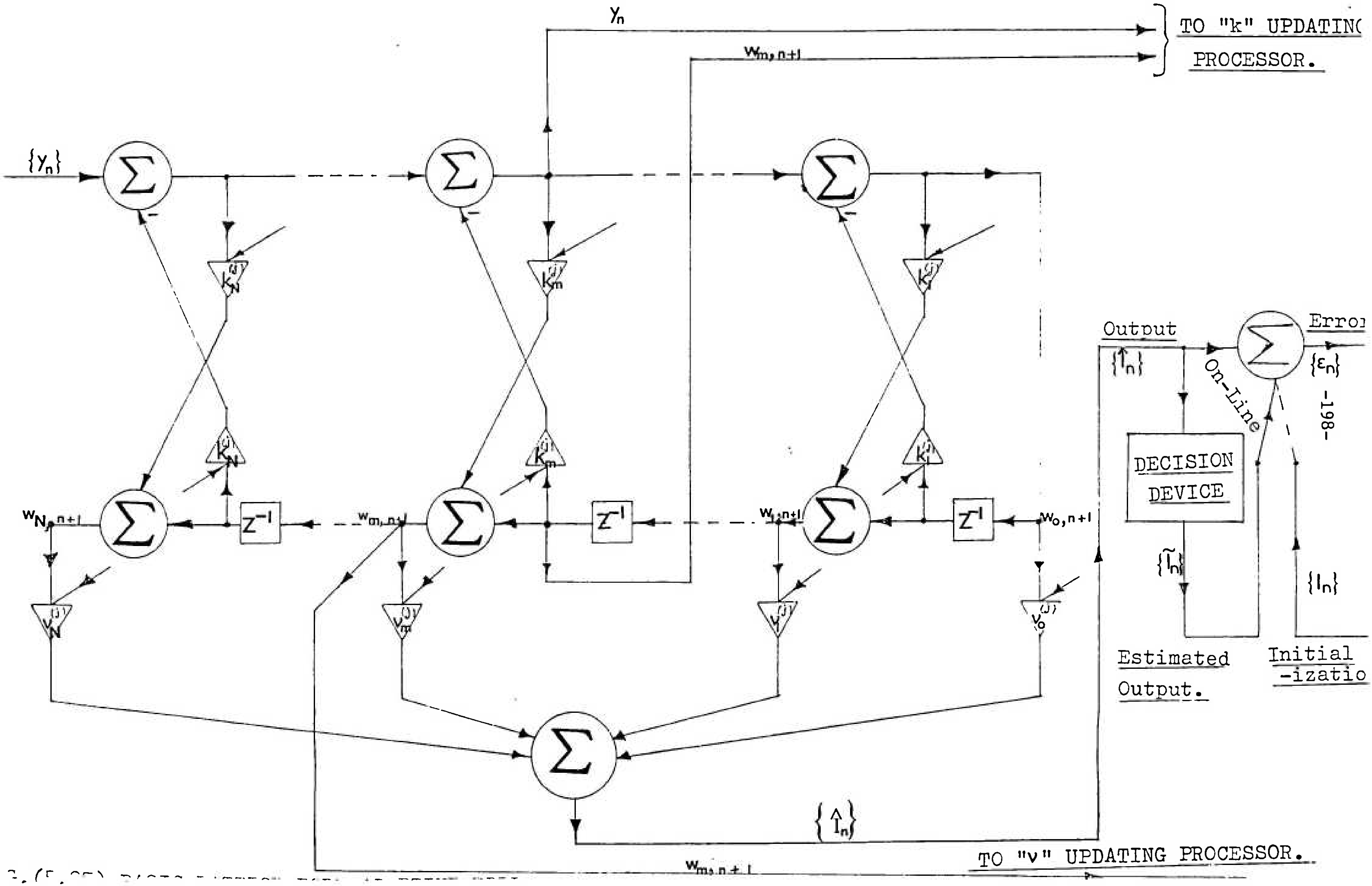
$$\underline{g}_{k_m}^{(j)} = 1/LL \sum_{n=0}^{LL-1} \varepsilon_n^{(j)} \left\{ \frac{\partial \hat{I}_n}{\partial k_m} \right\}^{(j)} \quad \text{for } m=1,2,\ldots N \tag{5.72}$$

and $$\underline{g}_{\nu_m}^{(j)} = 1/LL \sum_{n=0}^{LL-1} \varepsilon_n^{(j)} \left\{ \frac{\partial \hat{I}_n}{\partial \nu_m} \right\}^{(j)} \quad \text{for } m=0,1,\ldots M \tag{5.73}$$

for which

$$\frac{\partial \hat{I}_n}{\partial \nu_m} = w_{m,n} \tag{5.74}$$

and $\dfrac{\partial \hat{I}_n}{\partial k_m}$ is calculated using method described in Eq.(5.48).



y_n
$w_{m,n+1}$
TO "k" UPDATIN(
PROCESSOR.
$\{y_n\}$
$k_N^{(j)}$
$k_m^{(j)}$
$k_1^{(j)}$
Z^{-1}
$w_{N,n+1}$
$w_{m,n+1}$
$w_{1,n+1}$
$w_{0,n+1}$
$v_N^{(j)}$
$v_m^{(j)}$
$v_1^{(j)}$
$v_0^{(j)}$
Output
$\{\hat{I}_n\}$
Erroı
$\{\varepsilon_n\}$
On-Line
DECISION DEVICE
$\{\tilde{I}_n\}$
$\{I_n\}$
Estimated Output.
Initial -izatio
$\{\hat{I}_n\}$
TO "ν" UPDATING PROCESSOR.
$w_{m,n+1}$

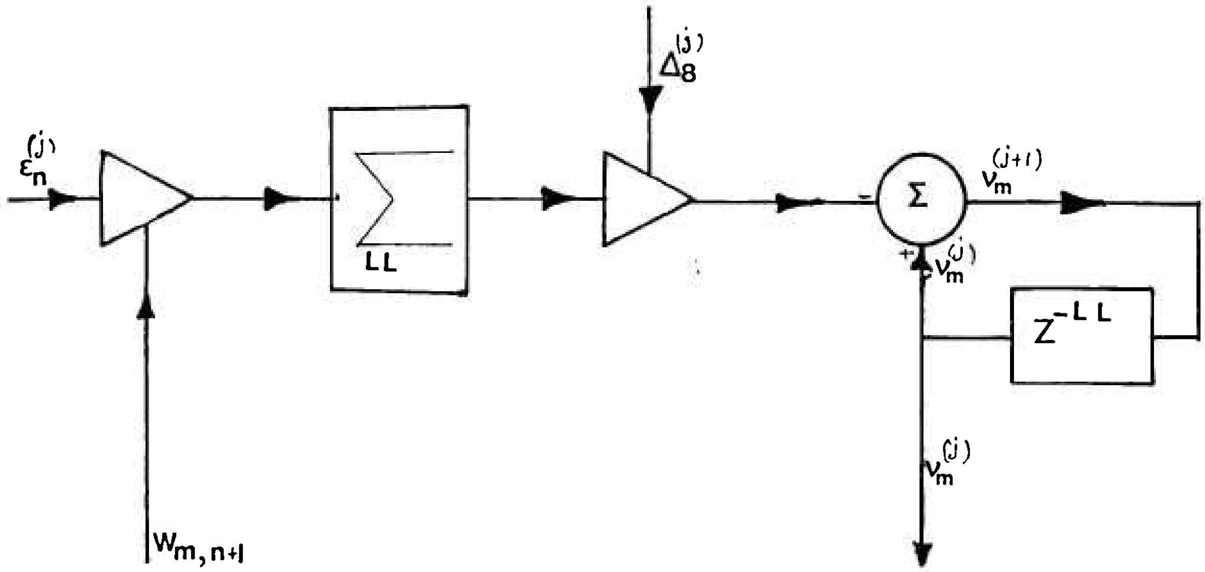
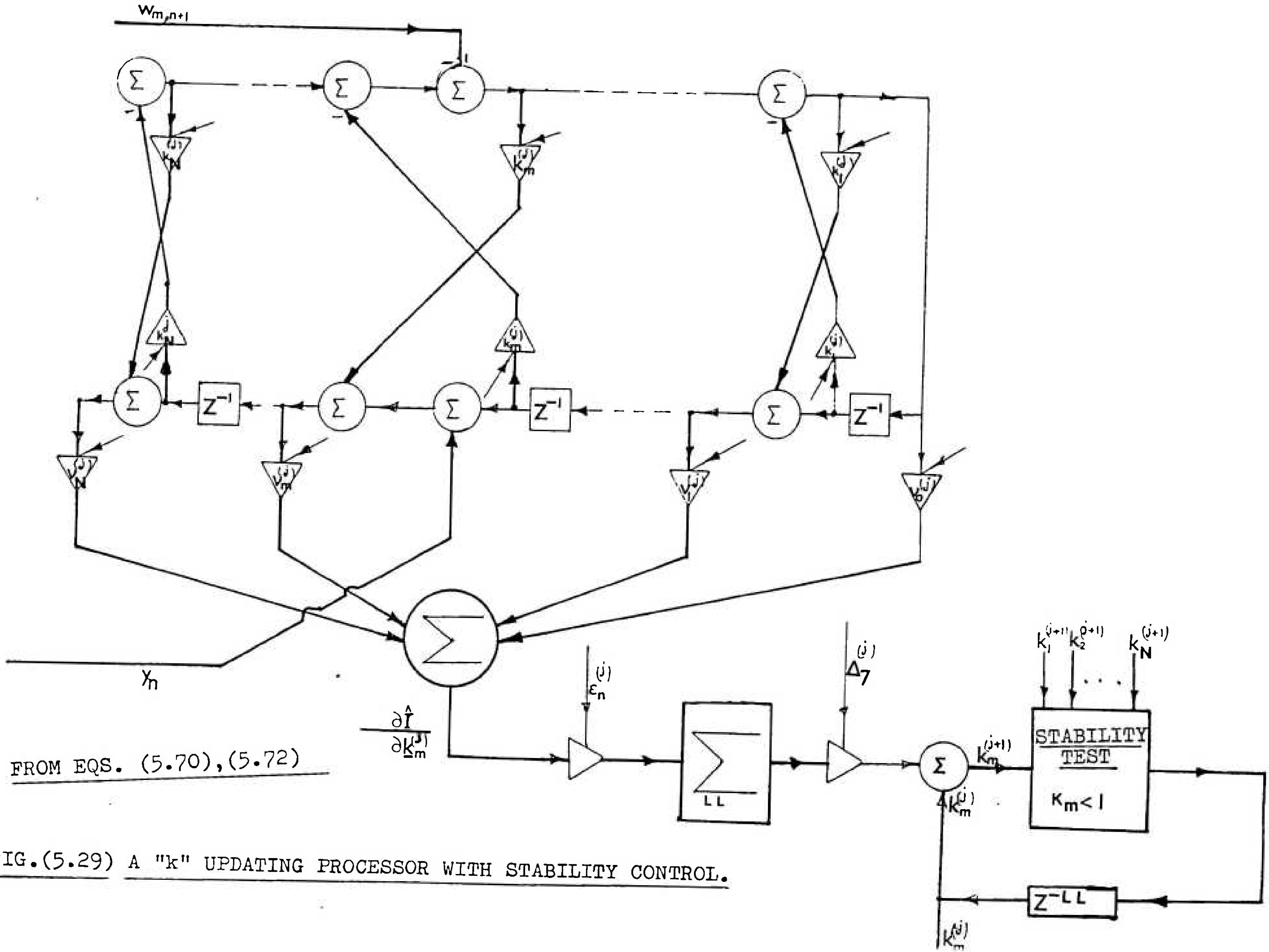
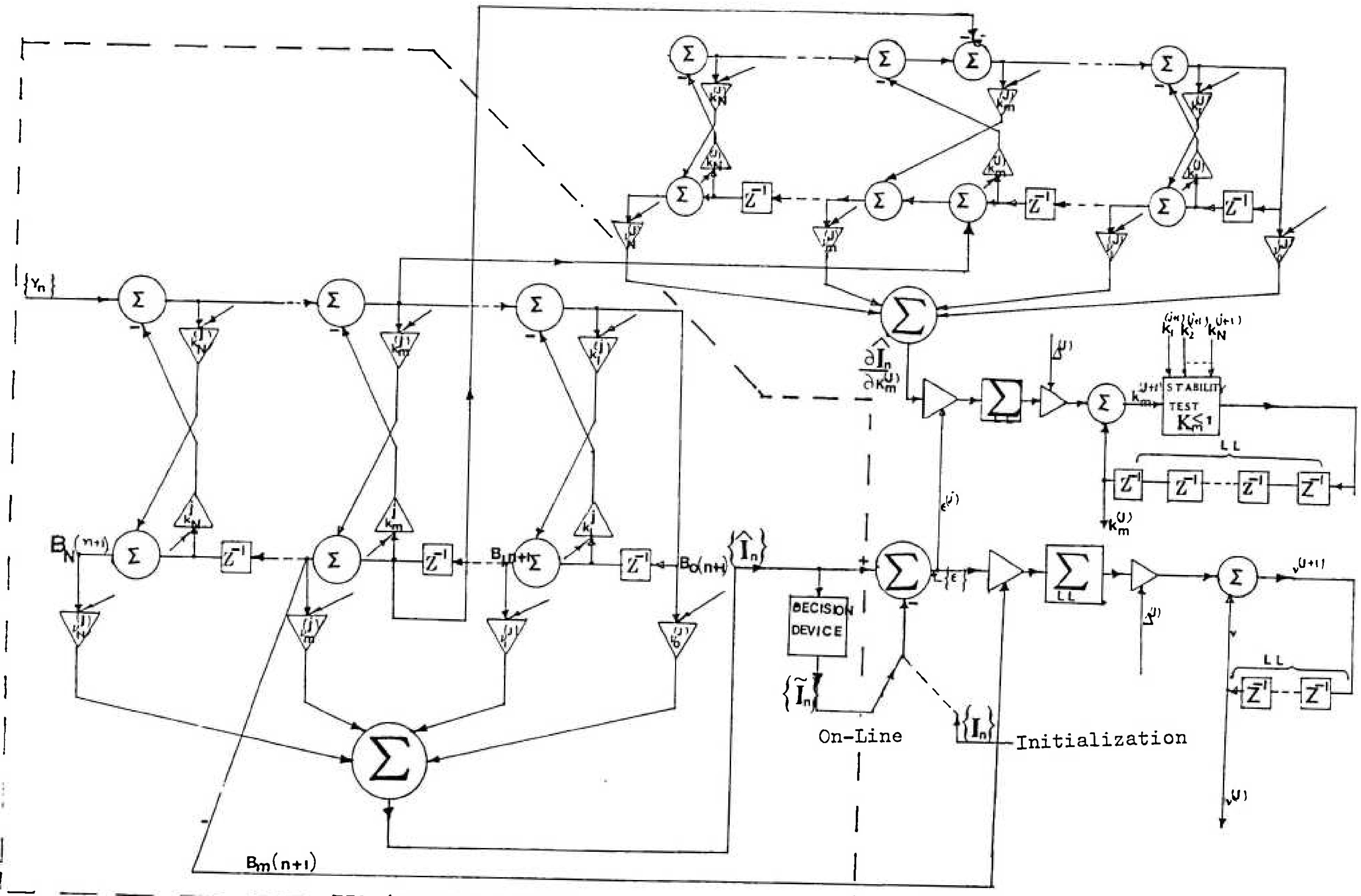


FIG.(5.28) A "ν " UPDATING PROCESSOR.



FROM EQS. (5.70),(5.72)

FIG.(5.29) A "k" UPDATING PROCESSOR WITH STABILITY CONTROL.



DECISION DEVICE
STABILITY TEST
On-Line
Initialization

The implementation of basic lattice form adaptive RDFE; "v", "k" along with stability control updating processors are shown in Figures (5.27), (5.28) and (5.29) respectively. The formation of error is made according to Fig. (5.7) and Fig. (5.30) shows the implementation of complete lattice-form adaptive RDFE.

The number of multiplications required for the basic lattice form RDFE and for the parameters adjustment algorithms are (1+M+2N) and ($1+M+2N+MN+2N^2$) respectively.

5.7.1 STABILITY CONDITIONS:

A built-in stability test exists within the synthesis process which is given by the inequality (5.64). If any k-parameter magnitude exceeds or equals unity, the equalizer is unstable, otherwise stable.

5.8 IMPLEMENTATION OF A DECISION-FEEDBACK RDFE:

If the discrete-time channel to be equalized has a zero ON the unit circle, the recursive digital filter equalizers discussed earlier will become unstable. In order to avoid this situation we propose, that quantized versions of output $Q\left[I_{n-k}\right]$, are fed back leading to the modified difference equation:

$$\hat{I}_n = \sum_{k=0}^{M} \alpha_k y_{n-k} - \sum_{k=1}^{N} \beta_k Q\left[\hat{I}_{n-k}\right] \qquad (5.75)$$

Implementation of Eq. (5.75) shown in Fig. (5.31) is called a decision feedback RDFE. The partial derivatives of Eq. (5.75) are given by

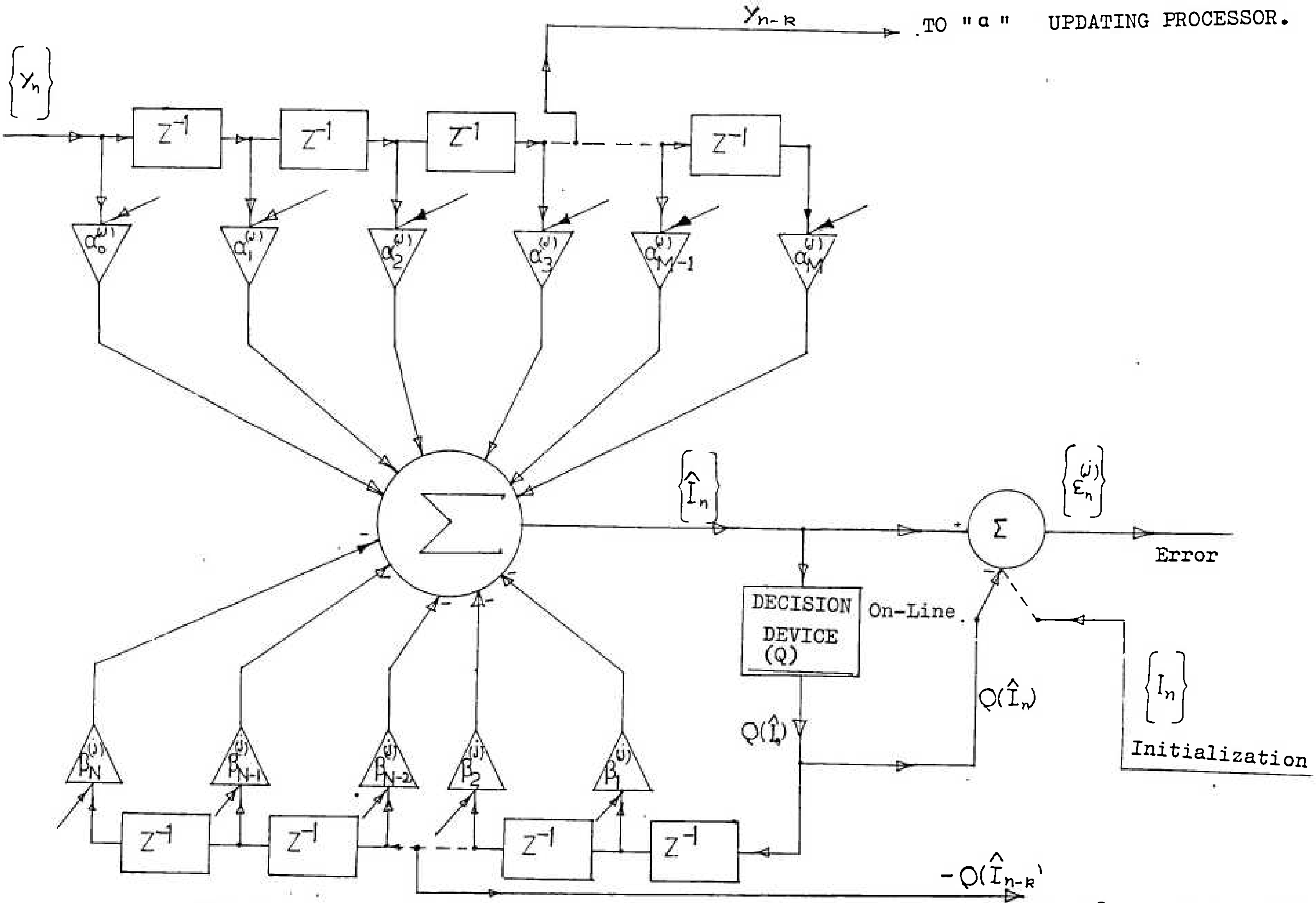


FIG.(5.31)BASIC DECISION-FEEDBACK RDFE WITH ERROR FORMATION.

$$\frac{\partial \hat{I}_n}{\partial \alpha_i} = y_{n-i} - \sum_{k=1}^{N} \beta_k \frac{\partial Q[\hat{I}_{n-k}]}{\partial \alpha_i} \qquad (5.76)$$

where $i=0, 1, \ldots M$

$$\frac{\partial \hat{I}_n}{\partial \beta_i} = - Q[\hat{I}_{n-i}] - \sum_{k=1}^{N} \beta_k \frac{\partial Q[\hat{I}_{n-k}]}{\partial \beta_i} \qquad (5.77)$$

where $i=1, \ldots N$

Now if we assume the changes in α_k ($\delta\alpha_k$) and β_k ($\delta\beta_k$) to be very small then the equalizer is not disturbed materially and, therefore, we can write

$$Q[\hat{I}_{n-k}] = I_{n-k} = \text{A desired information symbol} \qquad (5.78)$$

(Assuming no decision errors by the decision device)

$$\therefore \quad \frac{\partial Q[\hat{I}_{n-k}]}{\partial \alpha_i} = 0 \quad [\, i= 0, \ldots M \,] \qquad (5.79)$$

and

$$\frac{\partial Q[\hat{I}_{n-k}]}{\partial \beta_i} = 0 \quad [\, i=1 \ldots N \,] \qquad (5.80)$$

From (5.76) and (5.79) we obtain

$$\frac{\partial \hat{I}_n}{\partial \alpha_i} = y_{n-i} \quad [\, i=0, 1 \ldots M \,] \qquad (5.81)$$

and from (5.77) and (5.80), we get

$$\frac{\partial \hat{I}_n}{\partial \beta_i} = - Q[\hat{I}_{n-i}] \quad [\, i=1, \ldots N \,] \qquad (5.82)$$

The multiplier coefficients are updated using the following iterative algorithms

$$\underline{\alpha}_k^{(j+1)} = \underline{\alpha}_k^{(j)} - \tfrac{1}{2}\,\Delta_9^{(j)}\; g_{\alpha_{k(Q)}}^{(j)} \tag{5.83}$$

and

$$\underline{\beta}_k^{(j+1)} = \underline{\beta}_k^{(j)} - \tfrac{1}{2}\,\Delta_{10}^{(j)}\; g_{\beta_{k(Q)}}^{(j)} \tag{5.84}$$

where

$$g_{\alpha_{k(Q)}}^{(j)} = 2/LL \sum_{n=0}^{LL-1} \varepsilon_n^{(j)} \left\{\frac{\partial \hat{I}_n}{\partial \alpha_k}\right\}^{(j)} = 2/LL \sum_{n=0}^{LL-1} \varepsilon_n^{(j)}\; y_{n-k} \tag{5.85}$$

and

$$g_{\beta_{k(Q)}}^{(j)} = 2/LL \sum_{n=0}^{LL-1} \varepsilon_n^{(j)} \left\{\frac{\partial \hat{I}_n}{\partial \beta_k}\right\}^{(j)} = 2/LL \sum_{n=0}^{LL-1} \varepsilon_n^{(j)}\; -\{Q(\hat{I}_{n-k})\} \tag{5.86}$$

Finally multiplier coefficients updating algorithms are given by

$$\underline{\alpha}_k^{(j+1)} = \underline{\alpha}_k^{(j)} - \frac{\Delta_9^{(j)}}{LL} \sum_{n=0}^{LL-1} \varepsilon_n^{(j)}\; y_{n-k} = \underline{\alpha}_k^{(j)} - \delta\underline{\alpha}_k^{(j)} \qquad k=0,\ 1,\ \ldots\ M \tag{5.87}$$

and

$$\underline{\beta}_k^{(j+1)} = \underline{\beta}_k^{(j)} - \frac{\Delta_{10}^{(j)}}{LL} \sum_{n=0}^{LL-1} \varepsilon_n^{(j)}\; \{-\,Q\,[\hat{I}_{n-k}]\,\} = \underline{\beta}_k^{(j)} - \delta\underline{\beta}_k^{(j)} \qquad k=1,\ \ldots\ N \tag{5.88}$$

A very interesting observation from these two equations, that is the change in multiplier coefficients, are obtained by the cross correlations between the error and the input or the quantized decision feedback. Since the quantized decisions are fed back, therefore the name decision-feedback RDFE seems justified.

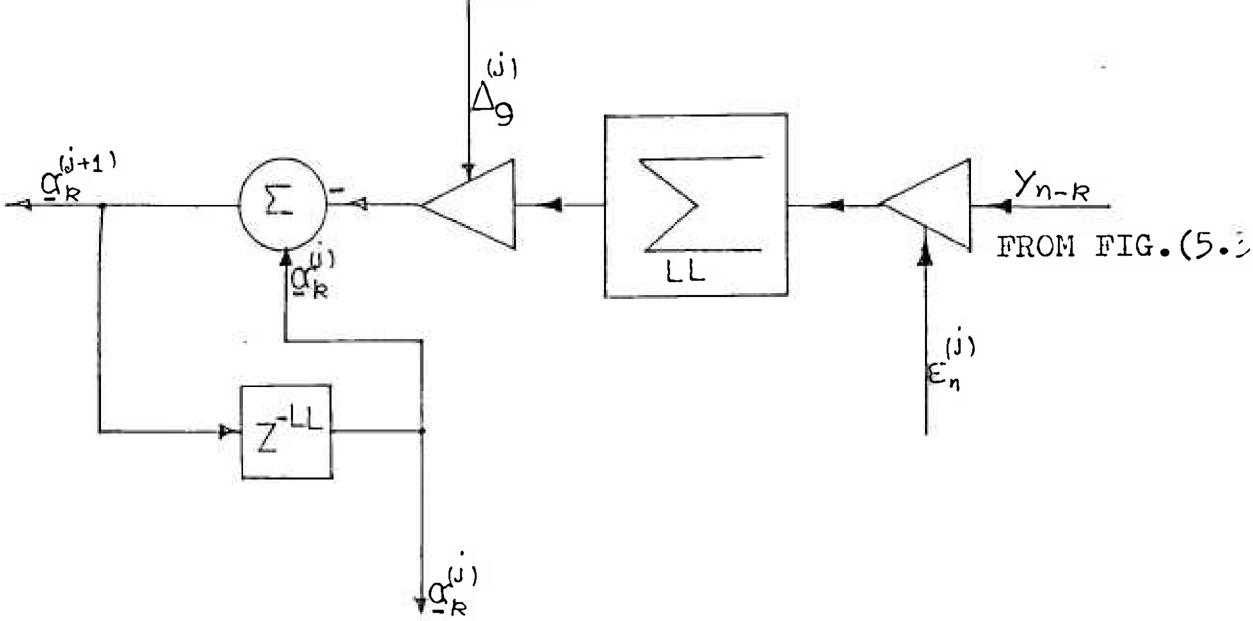


FIG.(5.32) A "α" UPDATING PROCESSOR.

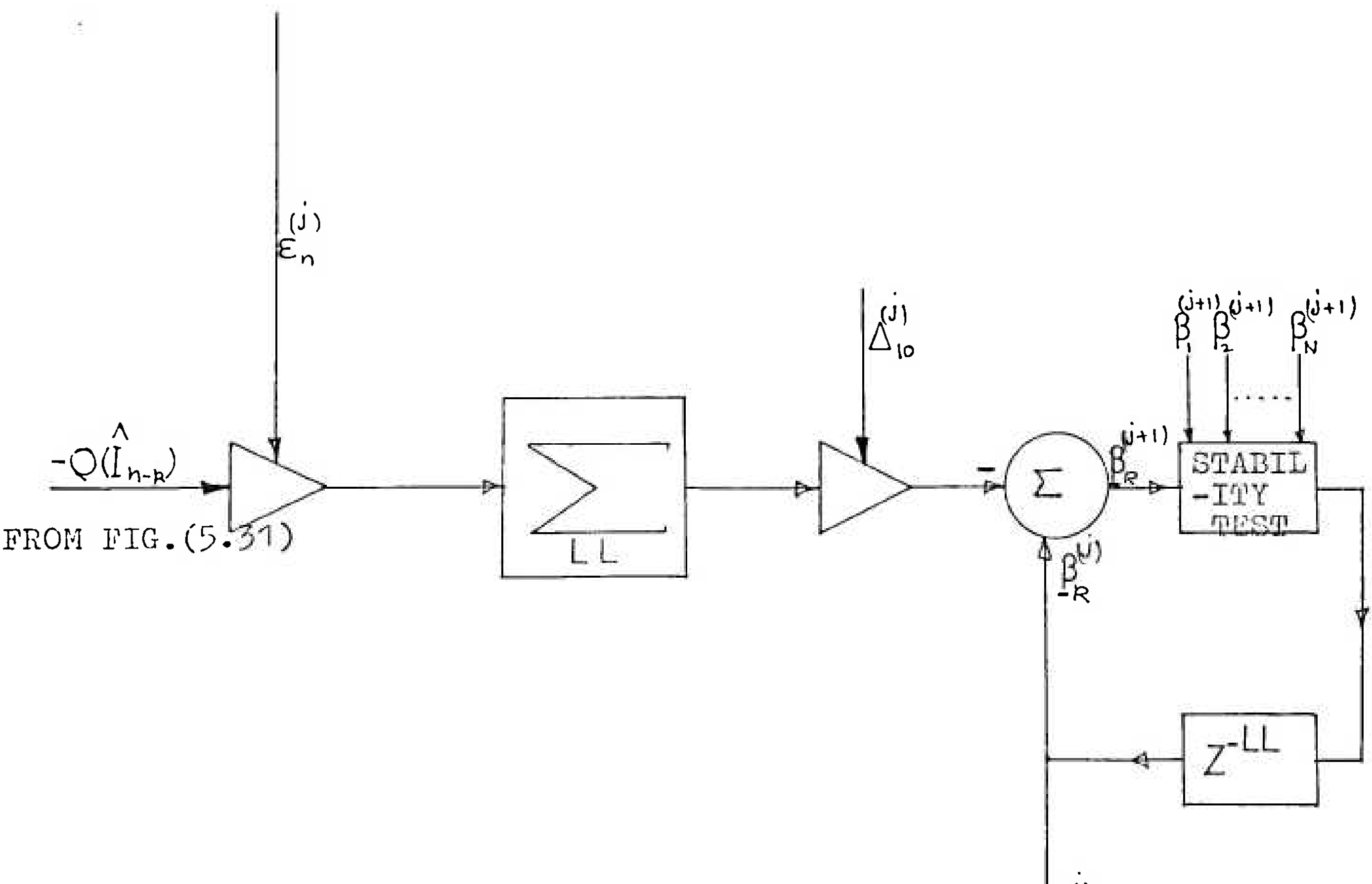


FIG.(5.33) A "β" UPDATING PROCESSOR WITH STABILITY CONTROL.

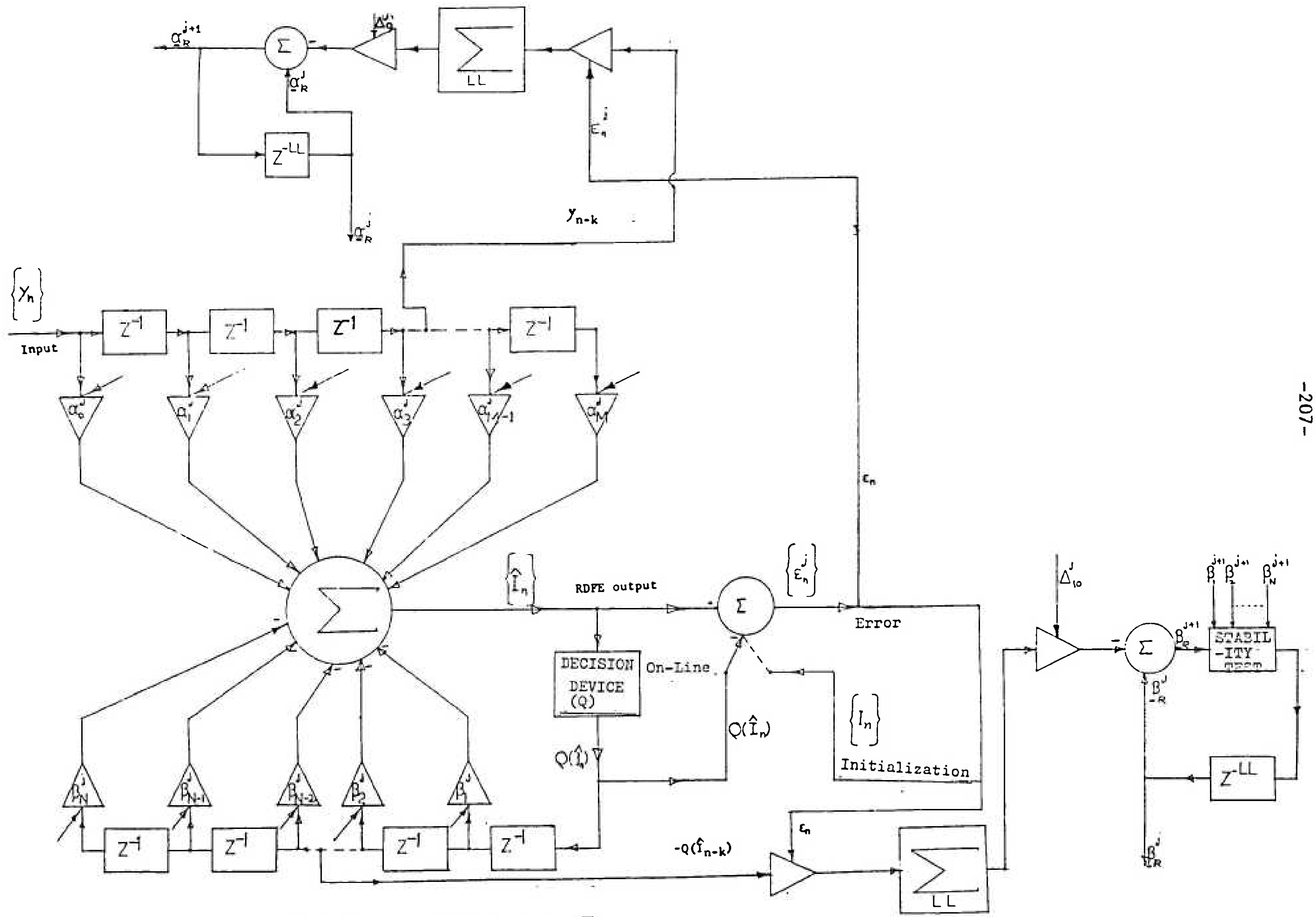


FIG. (5.??) IMPLEMENTATION OF A COMPLETE DECISION FEEDBACK RDFE.

Basic decision feedback RDFE structure, "α" and "β" updating processors, are shown in Figures (5.31), (5.32) and (5.33) respectively.

Fig. (5.34) shows the implementation of a decision feedback RDFE.

5.9 COMPUTER SIMULATION:

Tests were carried out to study

(i) the properties of adaptive recursive digital filter equalizers,

(ii) the comparison of various forms of adaptive RDFE developed in previous sections,

(iii) whether RDFE can adapt better than NRDFE to an unknown or varying channel,

(iv) whether the results are valid for a variety of channels,

(v) whether use of a learning sequence (initialization) is necessary or even advantageous,

(vi) whether or not use of an estimate of $E\left[\varepsilon_j \tilde{I}_{(j+k)}\right]$ results in better adaptation than an estimate of $E\left[\varepsilon_j y_{(j+k)}\right]$ for the NRDF section,

(vii) Probability of error performance of RDFE (P_e) as a function of SNR (dB) using Monte Carlo simulation technique.

We simulated six discrete-time equivalent channels (3,4,5,6,7 and 8). The first three (3,4,5) are the telephone and cable channels where the phase distortion is severe and the amplitude distortion is moderate. The remaining three (6,7,8) are, usually, the multipath radio channels where amplitude distortion is severe and which frequently possess nulls in their time variant spectral characteristics. Therefore, we have embraced the most important channel characteristics apparently met in practice. It can also be observed that channels (3) and (5) exhibit short-time dispersion impulse response while channel (4) exhibits long-time dispersion impulse response. Since the simulation runs were carried out for on-line adaptive equalizers, therefore, we decided first to initialize using a training sequence and then to switch on to the on-line-operation in a decision-directed mode.

5.9.1 DISCRETE TIME CHANNEL (3):

A series of tests were made to explain the concept of pole zero constellation equalization as well as the effects of training sequence and decision directed modes on equalization when the channel characteristics are unknown and quasistationary.

5.9.1.1 EQUALIZATION USING INITIALIZING (TRAINING) SEQUENCE:

The input sequence to the equalizer given by $\{0.17, 1.00, 0.75, -0.95, 0.85, -0.56, 0.28, -0.10, -0.029, -0.0063, 0.00, 0.00\}$ is shown in Fig. (5.35 (a)). Zeros of the z-transform Y(z) of the input signal are given in table (5.1) and are plotted in Fig. (5.35(c)). The effects of these zeros are cancelled by the poles/ zeros of three equalizer structures shown in Figures [(5.35 (d)) to (5.35 (f)] and discussed in the next section. The locations of the z-plane zeros of the input pulse are also useful in determining the rate of convergence of the equalizer time-series. If the input pulse has a zero on the unit circle, the equalizer response cannot converge for some real frequency. If the input pulse does not have a zero on the unit circle the pulse can be equalized and the rate of convergence of the equalizer is determined by the distances from the unit circle of the zeros.

TABLE (5.1) ZEROS OF THE z-TRANSFORM Y(z) OF THE INPUT SIGNAL

ZEROS	Re(z)	Im(z)
0^1	-4.410	0.000
0^2	-2.340	0.000
0^3	-0.099	0.460
0^4	-0.099	-0.460
0^5	0.440	0.000
0^6	0.040	0.430
0^7	0.040	-0.430
0^8	0.330	0.290
0^9	0.330	-0.290

5.9.1.2 POLES OF RDFE [POLE ZERO CONSTELLATION EQUALIZATION]:

In order to study the equalization based on this method we studied the effects of the poles of three RDFE structures viz,

(i) Direct form (I) RDFE having M=N=5.

Since there are seven zeros inside the unit circle to be equalized and this structure can provide maximum of five poles only, therefore such deficient structures will be known as TOO-LOW AN ORDER RDFE.

The poles of this equalizer are given in table (5.2) and are plotted in Fig. (5.35(d)). This equalizer was able to equalize the channel and always had a tendency to use all its poles to realize the best approximation in the mean square sense.

(ii) Direct form (I) RDFE having M=N=7

On the basis of the above argument, this structure will be known as an OPTIMUM ORDER RDFE.

The poles of this equalizer are given in table (5.3) and are plotted in Fig. (5.35(e)). As can be expected, this equalizer used all its poles to cancel the effects of the channel zeros inside the unit circle.

(iii) Direct form (I) RDFE having M=N=9

Since this structure has two more poles than the number of zeros inside the unit circle it will be known as TOO HIGH AN ORDER RDFE.

The poles of this equalizer are given in table (5.4) and are plotted in Fig. (5.35(f)). The two superfluous poles of this equalizer, as can be seen, tend to converge towards the orgin and hence tend to help in achieving convergence with comparatively less MMSE.

TABLE(5.2) POLES OF TOO LOW AN ORDER RDFE (M=N=5) [Fig. (5.35(d))]

POLES	Re(z)	Im(z)
p_3	-0.131	0.570
p_4	-0.131	-0.570
p_5	0.581	0.000
p_6	0.332	0.481
p_7	0.330	-0.481

TABLE (5.3) POLES OF AN OPTIMUM ORDER RDFE M=N=7 [Fig. (5.35(e))]

POLES	Re(z)	Im(z)
P_3	-0.081	0.450
P_4	-0.081	-0.450
P_5	0.470	0.000
P_6	-0.022	0.440
P_7	-0.022	-0.440
P_8	0.342	0.321
P_9	0.342	-0.321

TABLE (5.4) POLES OF TWO HIGH AN ORDER RDFE N=M=9 [Fig.(5.35(f))]

POLES	Re(z)	Im(z)	
P_3	-0.067	0.410	
P_4	-0.067	-0.410	
P_5	0.440	0.000	
P_6	-0.015	0.390	
P_7	-0.015	-0.390	
P_8	0.328	0.290	
P_9	0.328	-0.290	
P_{10}	0.150	0.130	EXCESS POLES
P_{10}	0.150	-0.130	

Let us compare the poles of the three equalizers [Tables 5.2 to 5.4] with the zeros of the input signal [Table 5.1] we note that there is very good correlation between the poles of RDFE (M=N=7) with the zeros of the input signal, but in the other two cases the excess or deficiency of poles in the equalizer causes a compromise to be made which is unlikely to give such a satisfactory impulse response or MSE. This method is called the Pole Zero Constellation Equalization.

5.9.1.3 CONVERGENCE PROPERTIES:

Simulation runs were made using the following initial conditions:

$$\left.\begin{array}{l} \underline{\alpha}^T = [0, 0, \ldots \quad 1] \\ \underline{\beta} = \underline{0} \end{array}\right] \text{Multiplier coefficients}$$

$$\left.\begin{array}{l} -0.004 \le \Delta_1 \le -0.002 \\ \text{and} \quad -0.038 \le \Delta_2 \le -0.015 \end{array}\right] \text{Convergence constants}$$

using an initializing mode, we found all the equalizers converged to some minimum MSE values [Fig. 5.35 (g)]. It was noted that RDFE (N=M=5) was quick to converge with large residual MSE whereas optimum (M=N=7) and larger (M=N=9) structures converged rather slowly but apparently with significantly less MMSE. The final multiplier values (i.e. $\underline{\alpha}$ and $\underline{\beta}$) are given in tables (5.5) and (5.6) and the corresponding RDFE outputs are given in table (5.7)

TABLE (5.5) VALUES OF α AND β AFTER CONVERGENCE (M=N=7)

i	0	1	2	3	4	5	6	7
α_i	-0.C1	0.01	-0.0295	0.071	-0.159	0.350	-0.661	1.00
β_i	1.00	-0.985	0.768	-0.443	0.224	-0.081	0.020	-0.005

TABLE (5.6) VALUES OF α AND β AFTER CONVERGENCE M=N=5

i	0	1	2	3	4	5
α_i	-0.032	0.073	-0.162	0.350	-0.67	1.00
β_i	1.000	-0.985	0.760	-0.443	0.224	-0.081

5.9.1.4 EQUALIZATION USING DECISION DIRECTED MODE:

A test was carried out to compare between the advantages offered by using initializing pulses and those using decision directed mode by adding to each sample of the channel a zero mean gaussian random number with standard deviation 1.17×10^{-3} after each 100 symbols had been passed through the discrete time channel.

Convergence curves, thus obtained [Fig. 5.35(h)] were compared with the curves of [Fig. 5.35(g)]. It was noted that the use of initializing pulse sequence improved the speed of convergence of the adaptive algorithm by a factor of three. Furthermore, the same values of minimum MSE were obtained in both the decision directed and the initializ ed (trained) modes of operation.

TABLE (5.7) EQUALIZER OUTPUTS $\{\tilde{I}_k\}$ AFTER CONVERGENCE

	k	I_k (M=N=7)	I_k (M=N=5)	
	0	-0.56×10^{-3}	-3.20×10^{-3}	
	1	-2.54×10^{-3}	-1.31×10^{-3}	
MAIN PULSE-M	2	0.09×10^{-3}	-0.410×10^{-3}	MAIN PULSE-M
	3	-0.04×10^{-3}	-0.841×10^{-3}	
	4	-0.84×10^{-3}	0.132×10^{-3}	
	5	-0.04×10^{-3}	0.598×10^{-3}	
	6	-0.14×10^{-3}	-0.471×10^{-3}	
	7	0.62×10^{-3}	1.000	MAIN PULSE
	8	-0.42×10^{-3}	-0.342×10^{-3}	
MAIN PULSE	9	1.00	0.672×10^{-3}	
	10	-0.13×10^{-3}	-0.672×10^{-3}	
	11	0.74×10^{-3}	1.00×10^{-3}	
	12	-0.74×10^{-3}	0.03×10^{-3}	MAIN PULSE+N
	13	1.11×10^{-3}	171×10^{-3}	
	14	0.03×10^{-3}	141×10^{-3}	
	15	-0.26×10^{-3}	0.67×10^{-3}	
MAIN PULSE+N	16	0.67×10^{-3}	-2.25×10^{-3}	
	17	-0.73×10^{-3}	0.13×10^{-3}	
	18	-0.07×10^{-3}	0.45×10^{-3}	
	19	0.07×10^{-3}	0.21×10^{-3}	
	20	0.010×10^{-3}	0.43×10^{-3}	
	21	0.000	0.29×10^{-3}	
	22	0.000	0.04×10^{-3}	
		RES. MSE=0.01	RES. MSE=0.03	

5.9.2 DISCRETE TIME CHANNEL (4):

The impulse response of this channel given by

[0.05, 0.2, -0.9, 0.1, 1.0, 0.63, 0.4, -0.4, 0.5, -0.2, 0.3, -0.2, 0.2, -0.1, 0.15, 0.1, -0.10, 0.10, -0.08, 0.06, -0.05, 0.04, -0.02]

is plotted in Fig.[5.36(a)] .

This channel exhibits long dispersion and requires 23 zeros to be equalized. We attempted to equalize this channel with three RDFE structures having M=N=5, 7, and 9 to compensate for 10, 14, and 18 zeros of the input pulse to the equalizer. These structures were selected taking into consideration the quantization errors accumulated by the larger strucrures than 9. We studied the following properties:

5.9.2.1 PERFORMANCE OF THREE DIRECT FORM RDFE:

The simulated results were plotted in Fig. 5.36(c). Equalizer M=N=5 converged [Curve D] but left behind a large MMSE (0.04). Equalizer M=N=7 converged [Curve E] with an acceptable MMSE (0.024) whereas equalizer M=N=9 converged [Curve F] with an MMSE (0.020) which proved to be an optimum case here. Furthermore increase in RDFE structure provided almost no reduction in MMSE since the performance was impaired by the accumulated quantization error. A study to this effect has been widely reported in the annals of digital filters.

For a given channel characteristic, the output SNR is monotonically related to the probability of error and it is much easier to compute even in the case where the residual ISI exists over an infinite number of symbols. A relationship between the output SNR and MMSE exists [5-12, 5-5, 6].

5.9.2.2 COMPARISON OF PERFORMANCES OF VARIOUS RDFE STRUCTURES:

Simulations were made to compare the convergence properties of various derived RDFE. Convergence constants (Δ) for the various structures were judiciously selected from the values outlined in section [5.9.1.3]

We experienced difficulties with the parallel form [Curve 4] structure. Literally it did not converge within 36 iteration steps. However, the convergence took place in the cases of

(a)	Direct form I and II RDFE	[Curves (1) and (2)]	Fig. 5.36(d)
(b)	Lattice form RDFE	[Curve (5)]	
(c)	Decision feedback RDFE	[Curve (6)]	
(d)	Cascade form RDFE	[Curve (3)]	

Curve (6) was different because of the fact that the quantized output symbols have been fedback.

Finally a rough comparison of derived RDFE structures on the basis of hardware and/or software complexities and on the stability can be shown as in table (5.8).

5.9.2.3 COMPARISON OF PERFORMANCES BETWEEN ADAPTIVE RDFE AND NRDFE:

A comparison of performances between adaptive versions of RDFE and NRDFE, over discrete time channel 4 was made. Fig. [5.36 (e)] shows the curve obtained by using different structures. It can be seen that the performance of NRDFE converges slowly towards that of the RDFE. The reasons can be explained easily on the pole-zero constellation equalization technique that during equalization, the zeros of the transfer function of the NRDFE outside the unit circle in the z-plane converge towards the zeros (outside the unit circle) of the RDFE.

TABLE (5.8) COMPARISON ON THE BASIS OF MULTIPLICATIONS AND STABILITY OF DERIVED RDFE STRUCTURES:

Legend: ① ≡M=N=5 ② ≡M=N=7 ③ ≡M=N=9

NO. OF MULTIPLICATIONS REQD.

EQUALIZER TYPE	BASIC RDFE STRUCTURE			COEFF. ADJUSTMENT WITHOUT STEP SIZE MULTIPLICATION			STABILITY TESTING
	1	2	3	1	2	3	
DIRECT FORM (I)	11	15	19	66	91	190	EASY
DIRECT FORM (II)	11	15	19	66	91	190	EASY
CASCADE FORM	11	15	19	47	79	119	EASY
PARALLEL FORM	assuming 11	15	M=N 19	32	(apprx.) 44	56	EASY
LATTICE FORM	assuming 16	22	M=N 28	91	169	271	VERY EASY

5.9.3 DISCRETE TIME CHANNEL (5):

The spectral characteristic for this data quality telephone discrete-time channel possess a depression of about -12 dB. The channel impulse response given by [0.07, -0.08, 0.1, -0.3, -0.62, 1.0, 0.5, 0.0, 0.3, 0.05, 0.1] is shown in Fig. (5.37). The initial distortion is 2.80 and the binary eye is completely closed.

The decision-directed on-line adaptation convergence curves are shown in Fig. [5.37(a)] . As can be seen from Curve (1), that the MMSE equals 0.033 whereas from Curve (2), the MSE equals 0.01. RDFE structure number 1 was unable to compensate for the entire ISI whereas number 2 having superfluous poles was able to reduce the MSE to the absolutely minimum level. RDFE structure number 3 exhibited very little superiority over number 2 and reduced MMSE level to 0.0098. This, once again, proves the fact that the superfluous poles provided by structures

(2) and (3) helped in the convergence of the adaptive algorithm and influenced immensely in reducing the MSE.

5.9.4 DISCRETE TIME CHANNEL (6):

This channel impulse response is symmetrical about the main pulse (1.00), possesses severe amplitude distortion effects and is a difficult channel to equalize [Fig. (5.38)].

This channel was equalized using equalizers in decision-directed mode. On close examination of Fig. [5.38(a)] , we notice that these curves exhibited an overshoot. That is, MSE decreased to some minimum value and then increased slightly after which it tended to decrease again. This effect occured on any channel where very rapid initial convergence was obtained and appears to be an overshoot effect due to the transient properties of the adaptive algorithm.

Also, convergence was the fastest; this could be attributed to the impulse response symmetry about the main pulse. As a matter of fact, the persymmetry properties correspond to the case of a matched-filter preceding the RDFE.

The minimum MSE attained by RDFE was 0.024 whereas by NRDFE was 0.037. The bottoming effect was more pronounced with NRDFE (M=35). The conclusion reached here is that a NRDFE cannot compensate very well for channel characteristics that exhibit large depressions or nulls in their spectral characteristics, whereas, RDFE performs considerably better.

5.9.5 DISCRETE-TIME CHANNELS (7) AND (8):

The impulse responses for these channels are shown in Figures (5.39) and (5.40) respectively. These channels were equalized with structures similar to those used in Fig.[(5.38 (a)], and exhibited the same pattern of curves as shown in Fig.[(5.38(a)] . However, the residual MSE was considerably more in the case of RDFE structure M=N=9.

5.9.6 PERFORMANCE OF RDFE AS A FUNCTION OF THE SIGNAL TO ADDITIVE NOISE RATIO:

As the additive noise level is increased decision errors at the RDFE output, even after convergence has occured, will become more frequent. The most significant measure of equalizer performance in this case is the output probability of error, P_e , as a function of the input SNR, $(SNR)_{IN}$, which in this case is defined as

$$\left[\mathrm{SNR}\right]_{IN} = \left[\frac{\text{Main Pulse}}{N_0}\right]^2 \tag{5.89}$$

where N_0 is the additive noise variance [5-12] . Because of the complexity and the absence of any analytical procedure, we estimated the P_e by means of Monte-Carlo simulation technique [5-2, 5-18] .

Extensive tests were conducted using the same equalizers and channel impulse responses as in the convergence tests. Our main interest here was in the steady-state error rate, namely the value of P_e obtained after the equalizer has converged to within a small neighbourhood of its optimum operating point. We therefore used a training sequence of 510 symbols so as to obtain rapid initial convergence and then waited a further 2000 symbols durations in the decision directed mode to allow ample time for the equalizer to reach its steady state. The error rate was then measured by counting the number of errors that occured. This count was continued until errors had been counted, and the procedure was repeated over a range of SNR's.

The results of these simulations for D.T. channels (5), (6), (7), are shown in Figures [5.41] , [5.42] , and [5.43] respectively where we have plotted the estimated P_e or error rate as a function of $[SNR]_{IN}$ for the channel responses shown. Also, we have plotted in these figures the following curves[5-14, 15]

(1) The ideal case P_e as a function of $[SNR]_{IN}$. This is a lower bound on the attainable P_e for a non-dispersive channel.

(2) The P_e obtained when no equalization is used. This has been obtained by using Hill's method [5-13] .

(3) The corresponding P_e obtained by simulating the RDFE.

We observe:

(i) From Fig. (5.41) that, for phase distorted channels, the performances of RDFE and NRDFE remain within 3 dB of the performance achieved with no interference, however, the gain is achieved in the number of shift register stages.

(ii) The performance of RDFE was remarkably superior on the severe amplitude distorted channels, Figures[5.42 and 5.43]. The performance of NRDFE was the least satisfactory. It could be attributed to the fact the channels, (6), (7), and (8) exhibit spectral nulls and on such channels an adaptive NRDFE cannot compensate for ISI very well.

(iii) During the tests it was observed that by increasing the number of taps of RDFE the performance was improved on amplitude distorted channels, Fig. (5.43). Whereas NRDFE taps did not improve the performance at all.

(iv) The probability of error P_e obtained by using an NRDFE of length 35 on D.T. channels 7 is large. Also, NRDFE appears to encounter the convergence difficulties.

(v) Although sufficient improvement in performance over NRDFE has been noted, still there is considerable degradation in performance of the RDFE due to residual ISI, especially on channels with severe distortion.

5.10 SUMMARY AND COMMENTS:

(A) We have discussed the design procedures of the following forms of adaptive RDFE

(i) Direct form (I)

(ii) Direct form (II)

(iii) Cascade form

(iv) Parallel form

(v) Lattice form

(vi) Decision-feedback

(B) The concept of pole-zero constellation equalization has been explained [Fig. 5.35 (a) to (f)] .

(C) The ratio between the two convergence rates, one using the training sequence and another using the decision directed mode, has been found to be a factor of three [Fig. 5.35 (g) and (h)] and weighs heavily in favour of the training sequence.

(D) Convergence qualities of these forms have been studied for various different conditions and have been found to converge locally except the parallel form adaptive RDFE.

Figures [5.36 (a) to (e)] , [5.37 (a)] and [5.38 (a)]

(E) Because of the digital nature, all the RDFE structures can be easily implemented and time-multiplexed.

(F) Decision feedback RDFE offers very little advantage. Nevertheless, it shows that an ordinary RDFE can be made a decision feedback equalizer easily, and therefore is more versatile than any other forms of equalizers [Fig. 5.36 (d)] .

(G) By Monte-Carlo simulations, we have estimated the probability of error, P_e , and found that

(i) For severe phase distortion channel, both equalizers (NRDFE, RDFE) performances are well within 3-dB of the ideal (No ISI) conditions. Slightly better performance behaviour of RDFE can easily be seen (Fig. 5.41)

(ii) For severe-amplitude distortion channels, the superb performances of RDFE can be compared with the worst performances of NRDFE [Fig. 5.42] and[4.43] .

(H) We would, therefore, expect that an RDFE should provide an attractive replacement for existing equalizers on phase (telephone and cable) and amplitude (radio multipath) distorted channels.

(I) The speed of convergence is dependent on the channel characteristics.

(J) In order not to exceed an acceptable amount of computer time we found that it is impossible to estimate error probability of an order of magnitude less than 10^{-3} by using Monte-Carlo simulation technique (see appendix).

(K) The convergence properties of the derived adaptive RDFE strongly depend on the chosen convergence, constants for their coefficient adjustments. Therefore, a comparison of their performances is difficult and the results thus far obtained must be considered with caution. However, the selection is recommended on the basis of table (5.8).

(L) We have been made aware of the facts that an adaptive RDFE is being considered at various establishments [5-16, 17] .

(M) A detailed study of the quantization errors (Signal, Multiplier coefficients quantization etc.) in adaptive RDFE should be made.

(N) An effort should be made to consider the possible use of the following RDF schemes

(i) Multirate recursive digital filter (propo sed by Wong and King of Imperial College,London)

(ii) Ladder form recursive digital filter (proposed by Mitra and Sherwood of Univ. of California).

(iii) Charge coupled recursive digital filter (proposed by Gersho and Gopinath of Bell Labs. Murray Hill).

All of the above RDF schemes are equally serious contenders.

(O) Implementations of RDFE using currently available microprocessors (INTEL 3000, MOTOROLLA 6800) appear to provide very good oppurtunity to emulate the entire system.

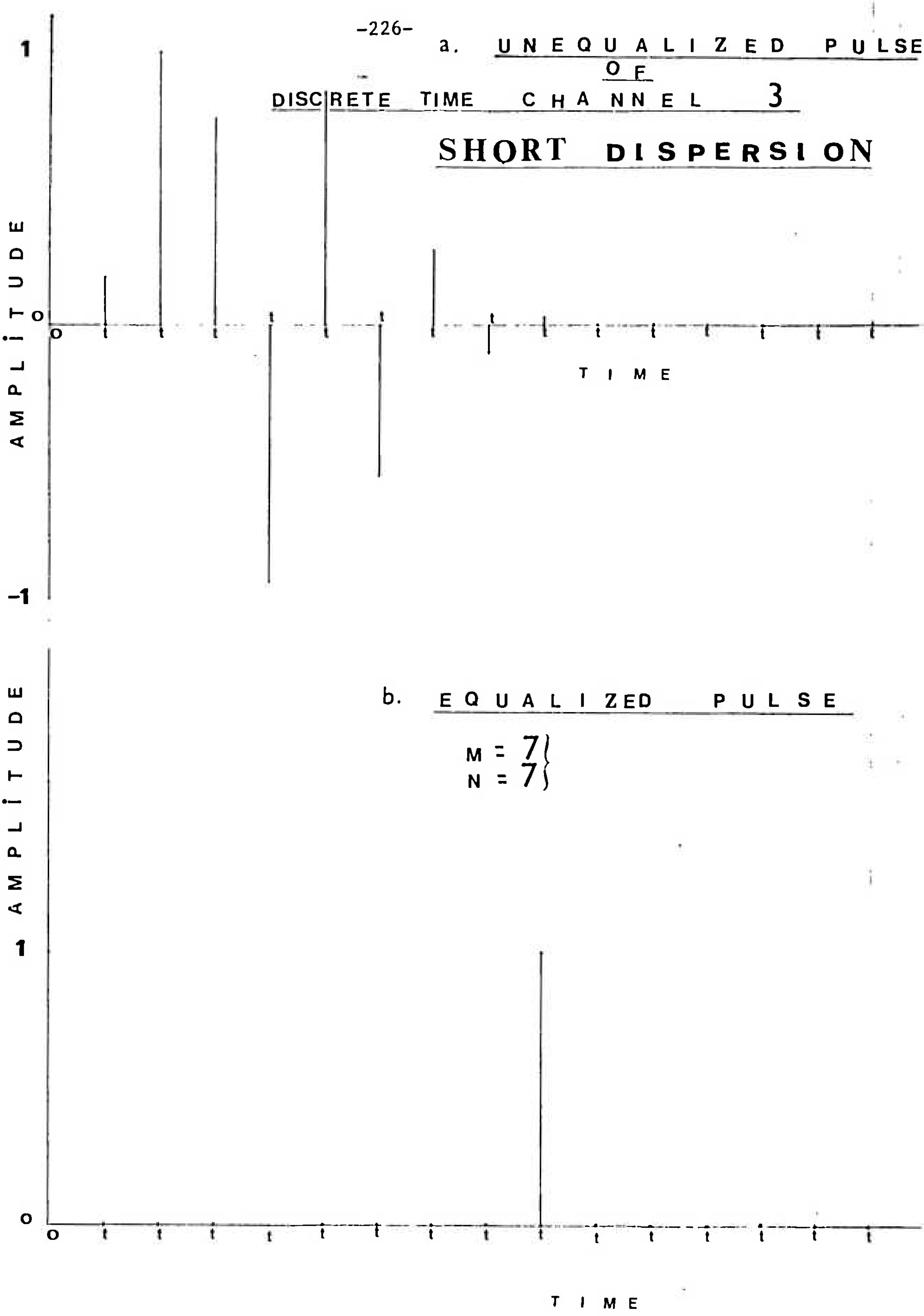
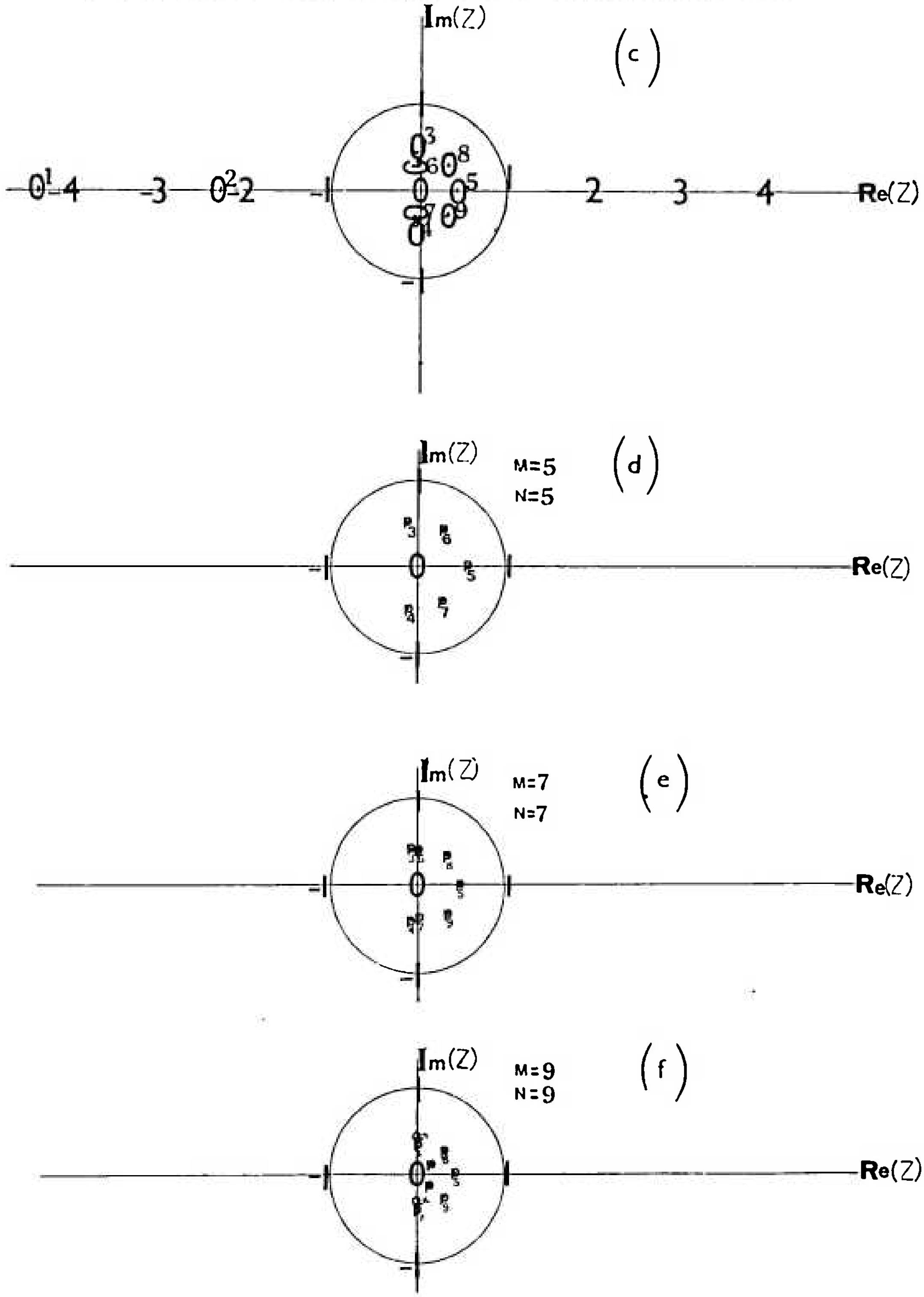


FIG 5·35 IMPULSE RESPONSE OF CHANNEL 3

a. BEFORE EQUALIZATION
b. AFTER DO DO

DISCRETE TIME CHANNEL 3



POLE ZERO CONSTELLATION

FIG. 5.35 (c) ZEROS OF CHANNEL OUTPUT (RDFE INPUT)
(d) POLES OF RDFE (TOO LOW AN ORDER)
(e) DO DO (OPTIMAL ORDER)
(f) DO DO (TOO HIGH AN ORDER)

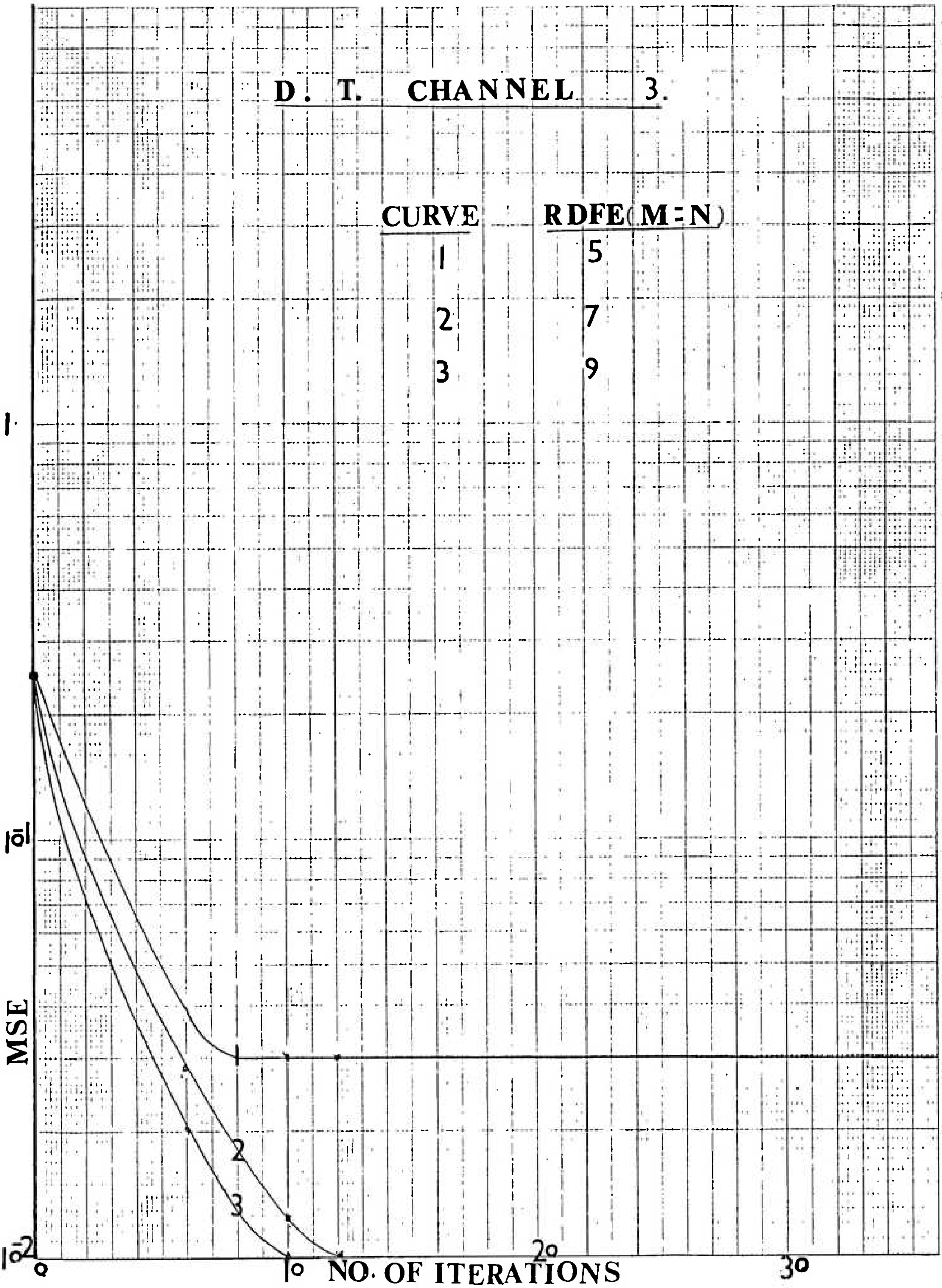


FIG.(5.35g) PERFORMANCE OF THREE RDFE USING TRAINING SEQUENCE.

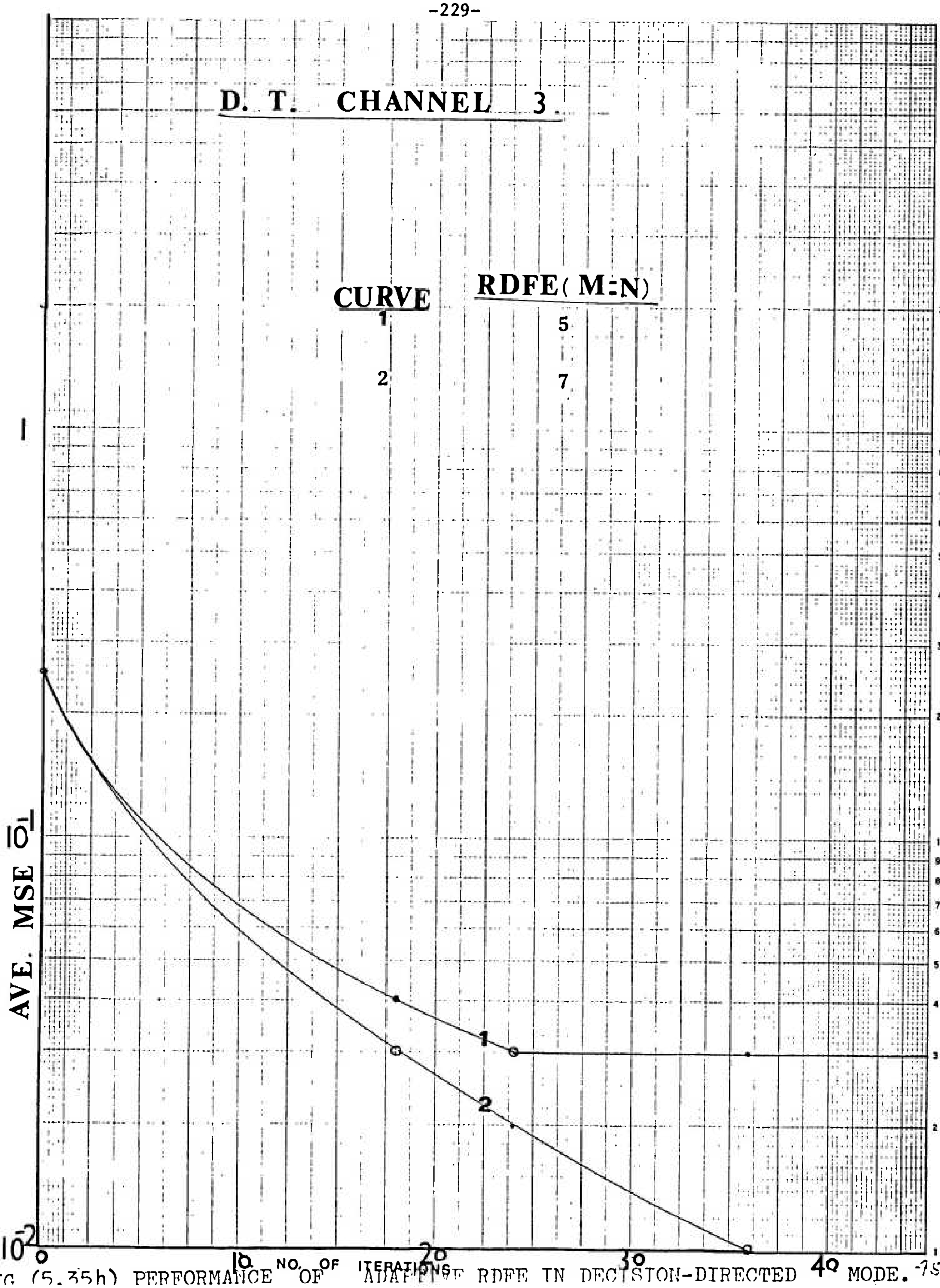


FIG (5.35h) PERFORMANCE OF ADAPTIVE RDFE IN DECISION-DIRECTED MODE.

DISCRETE TIME CHANNEL 4

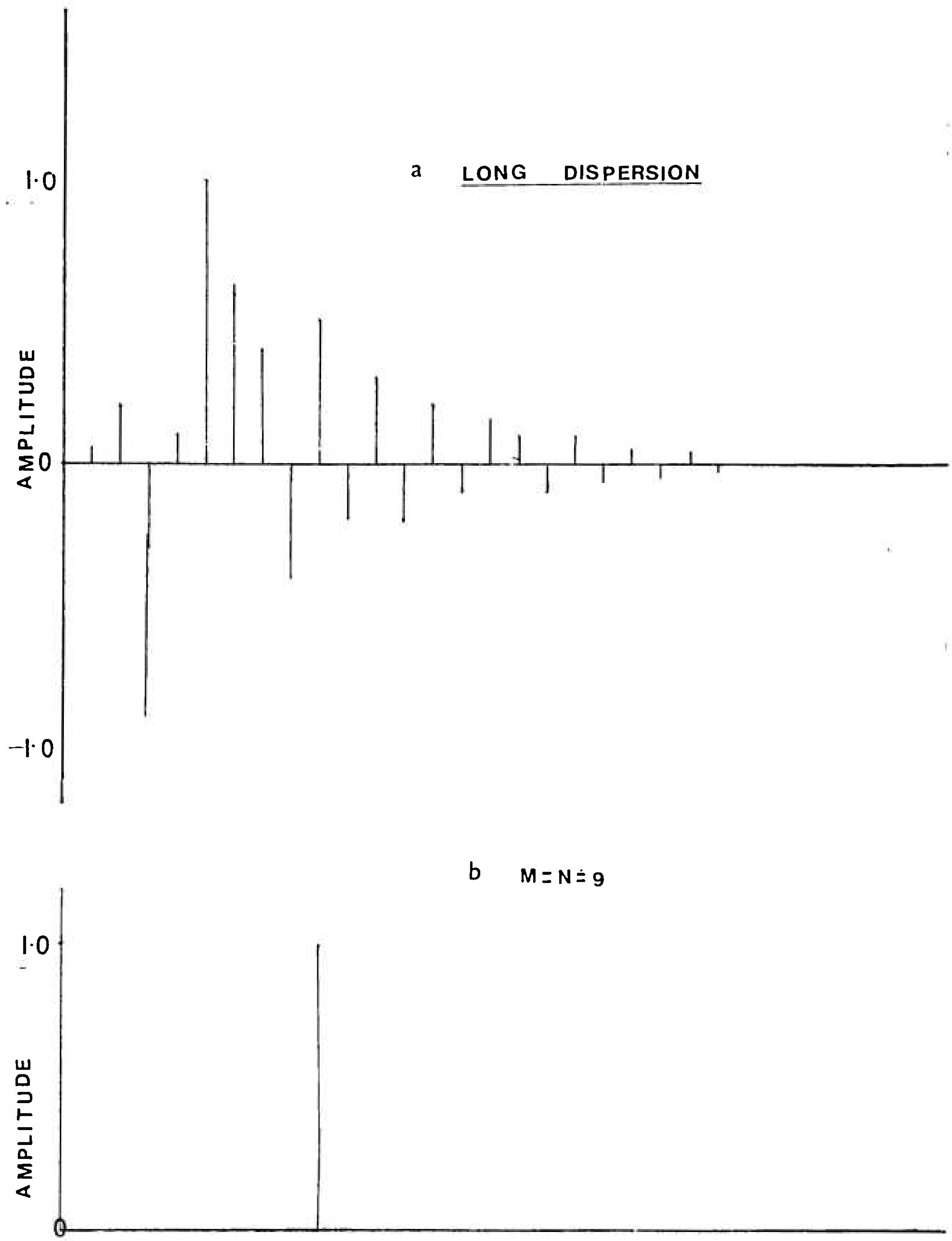


FIG.(5.36) IMPULSE RESPONSE OF CHANNEL 4.

a. BEFORE EQUALIZATION

b. AFTER EQUALIZATION.

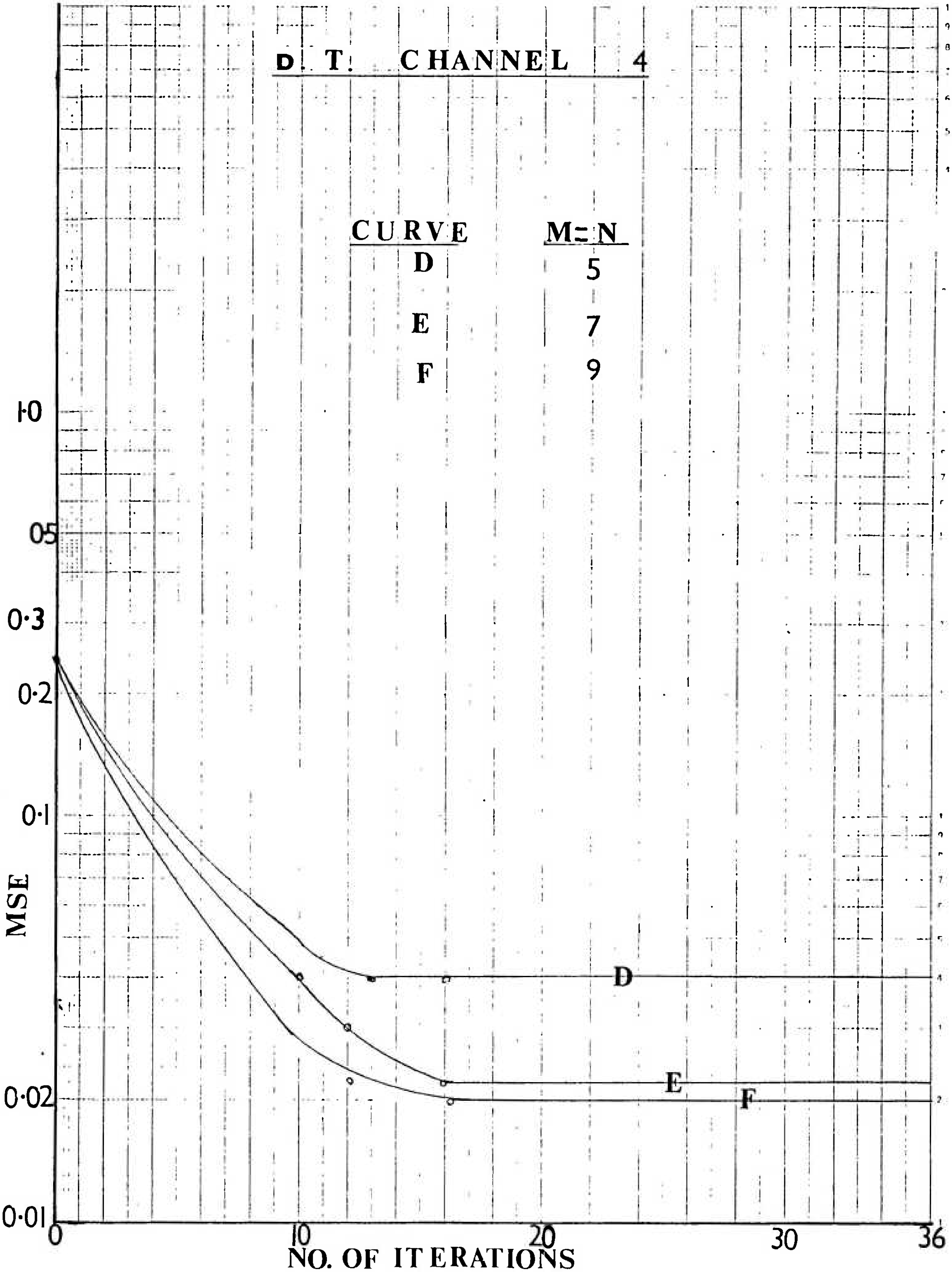


FIG.5.36(C) PERFORMANCE OF THREE DIRECT FORM RDFE.

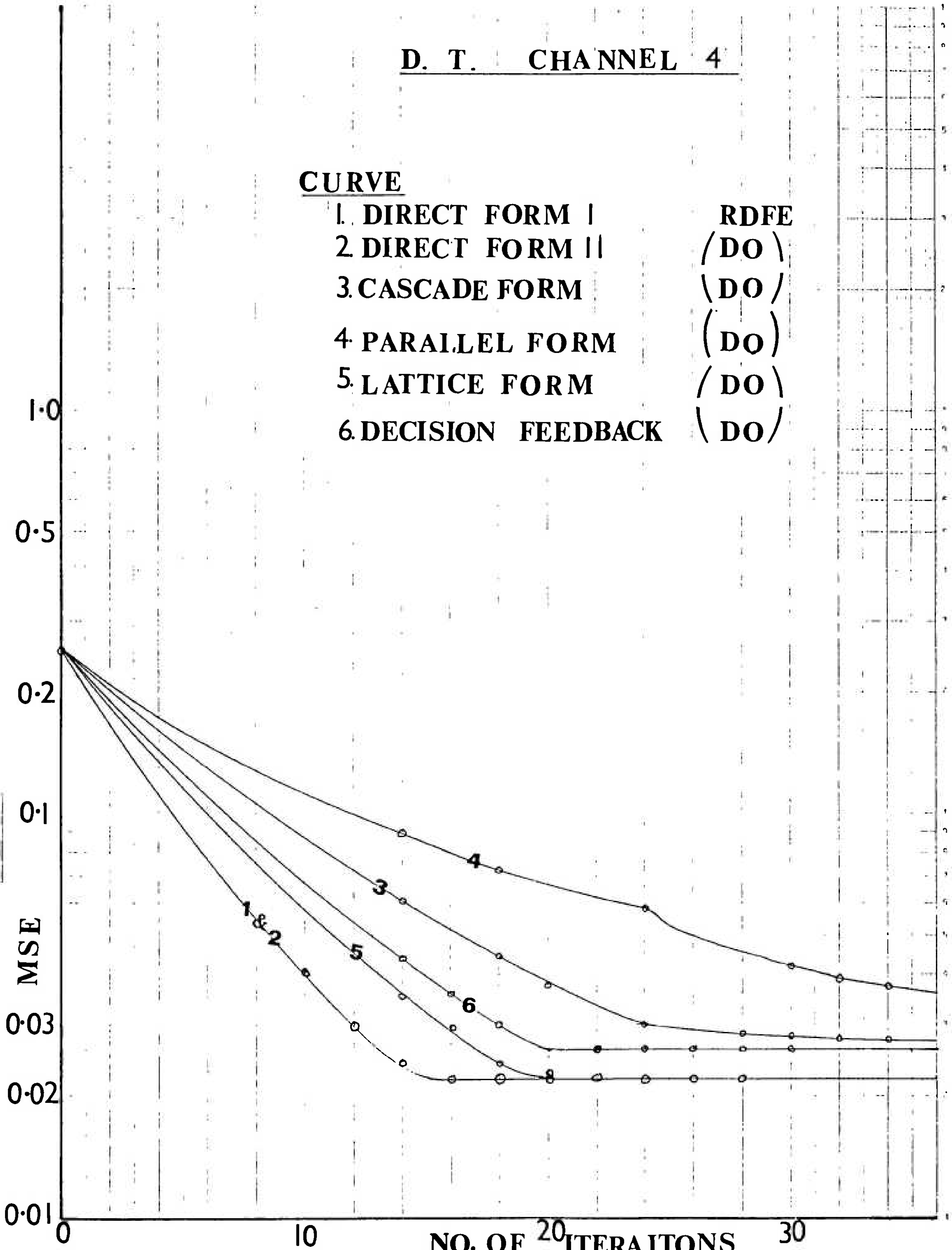


FIG.536(d) COMPARISON OF PERFORMANCES OF VARIOUS RDFE

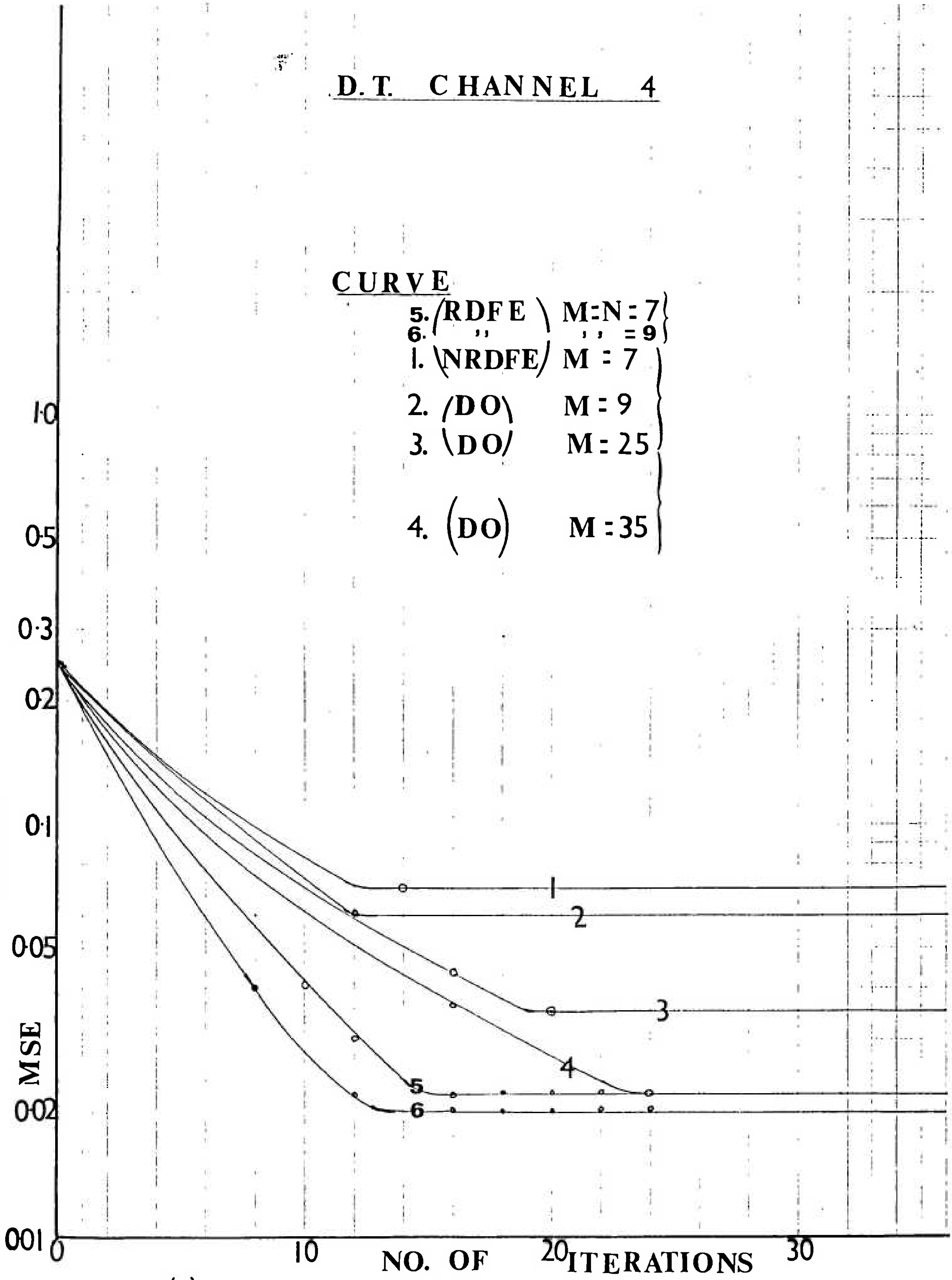


FIG 5.36(e) COMPARISON OF PERFORMANCES BETWEEN RDFE AND NRDFE

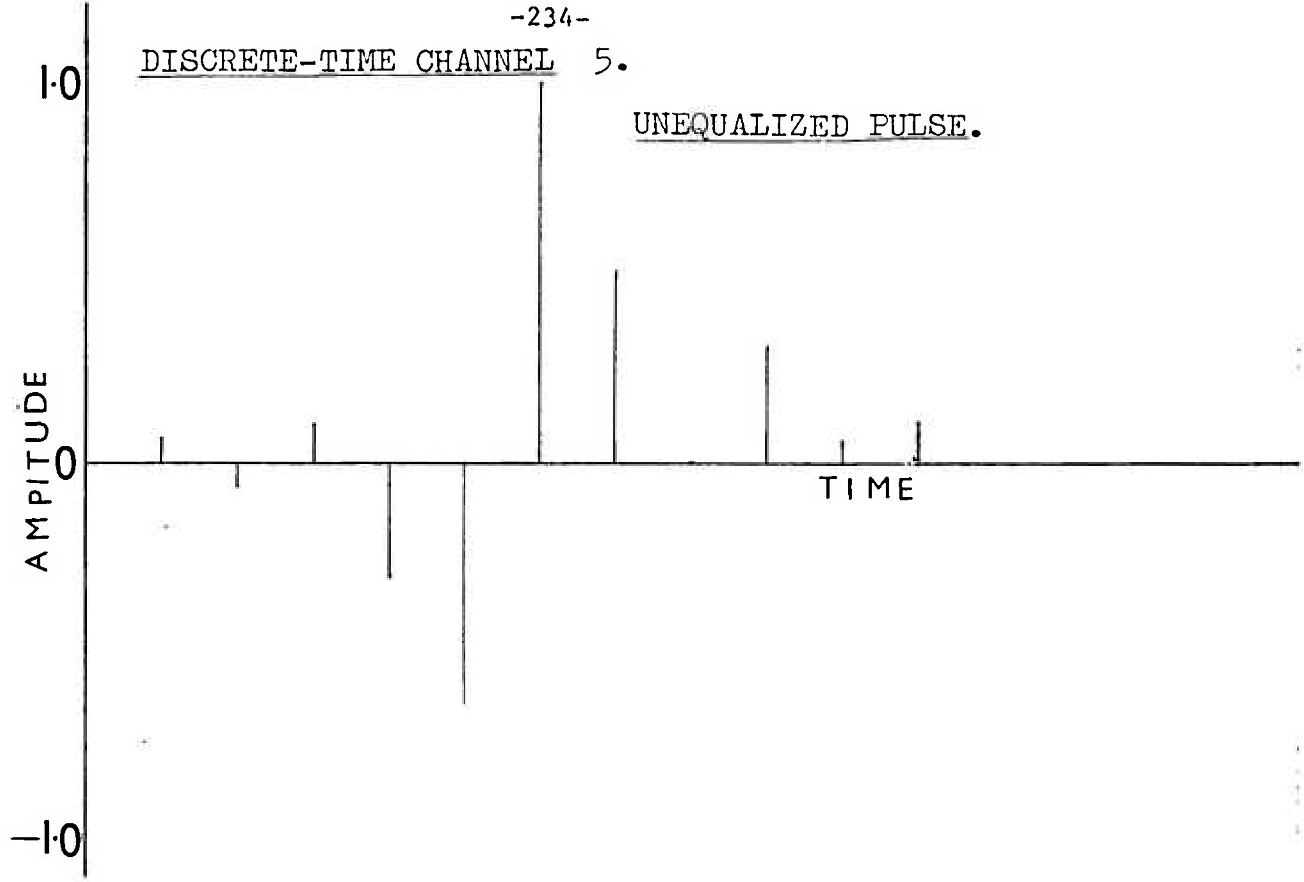


FIG.(5.37) IMPULSE RESPONSE OF CHANNEL 5.

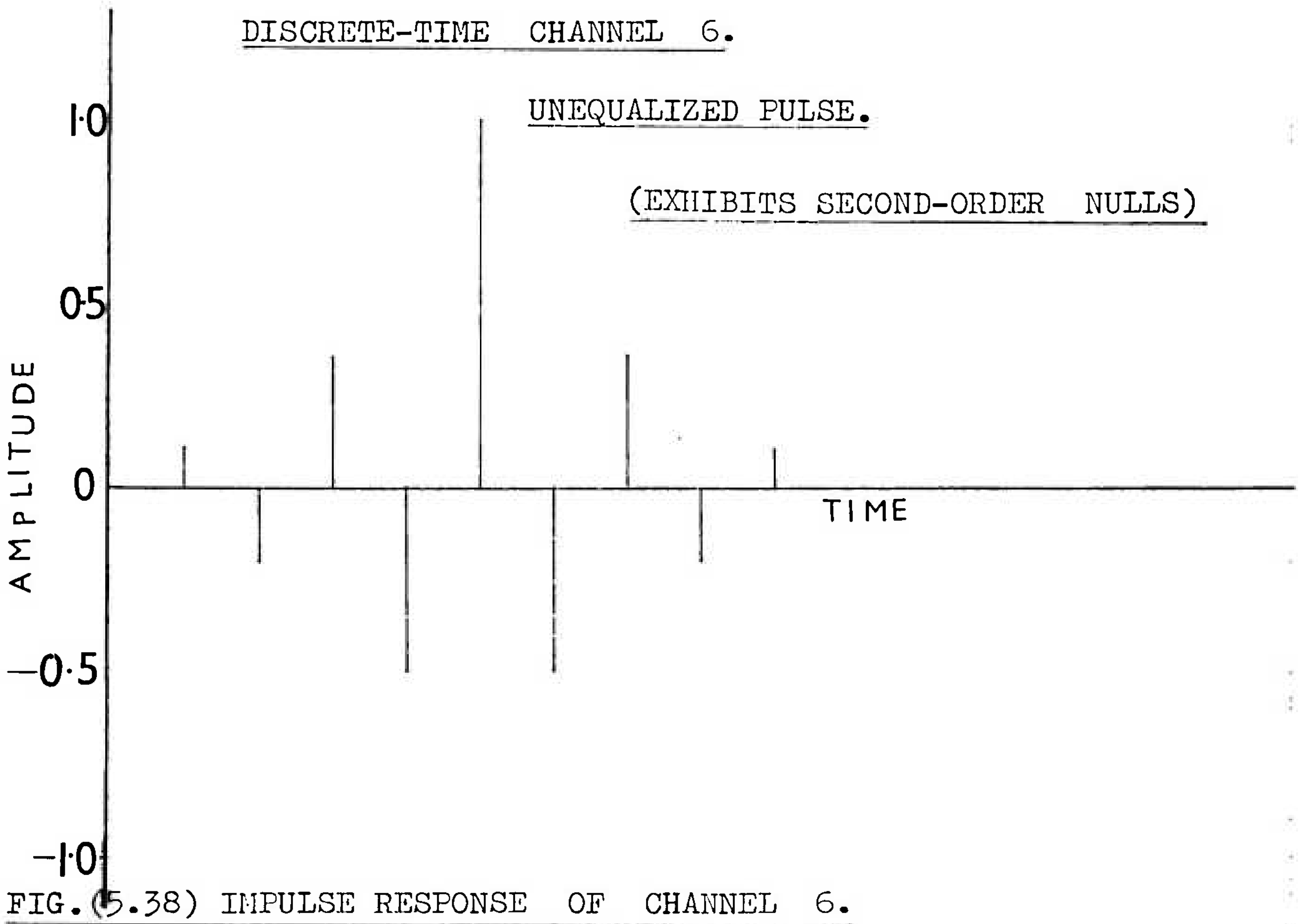


FIG.(5.38) IMPULSE RESPONSE OF CHANNEL 6.

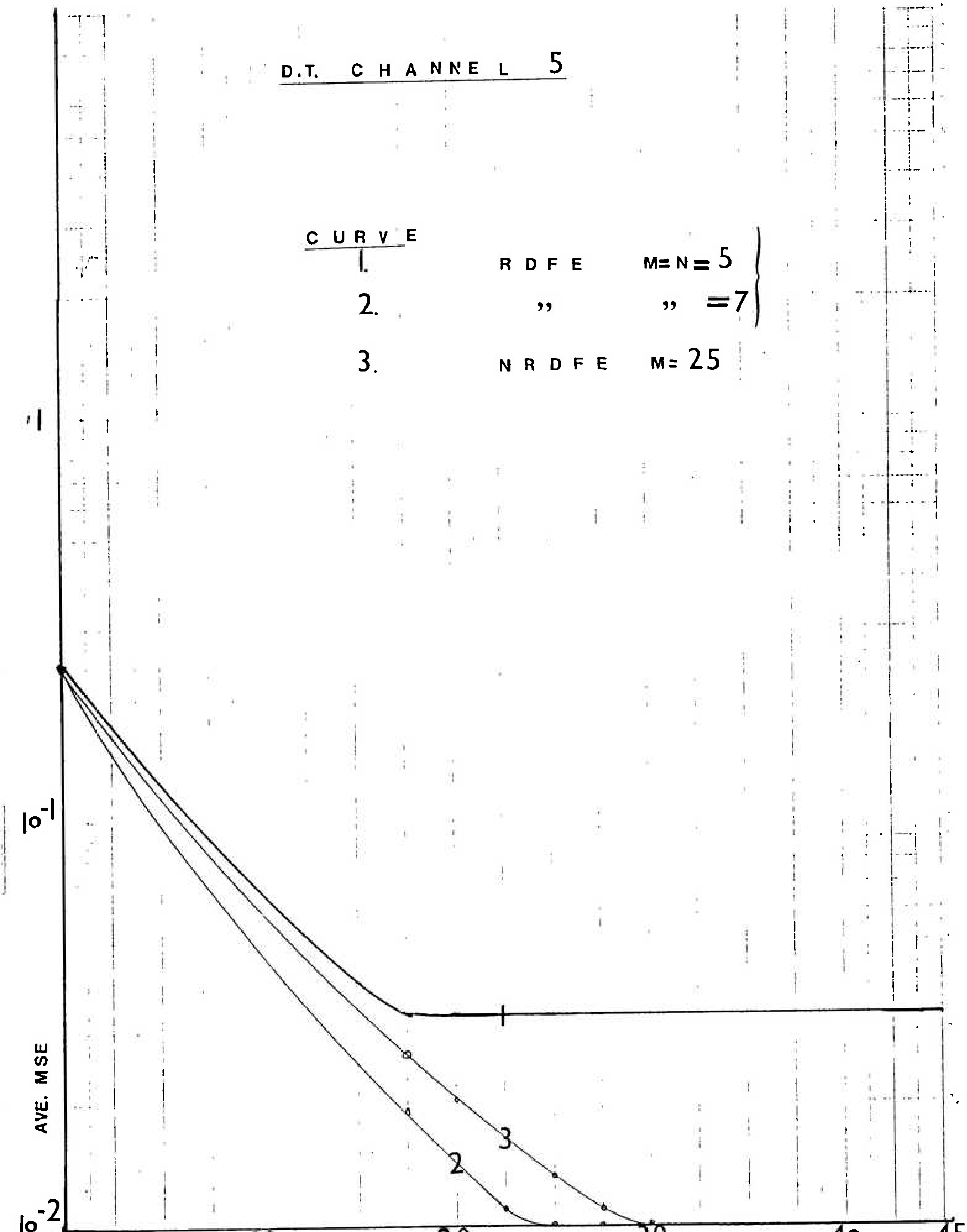


FIG.(5.37a) DECISION-DIRECTED ON-LINE ADAPTATION CONVERGENCE CURVES FOR D.T. CHANNEL 5.

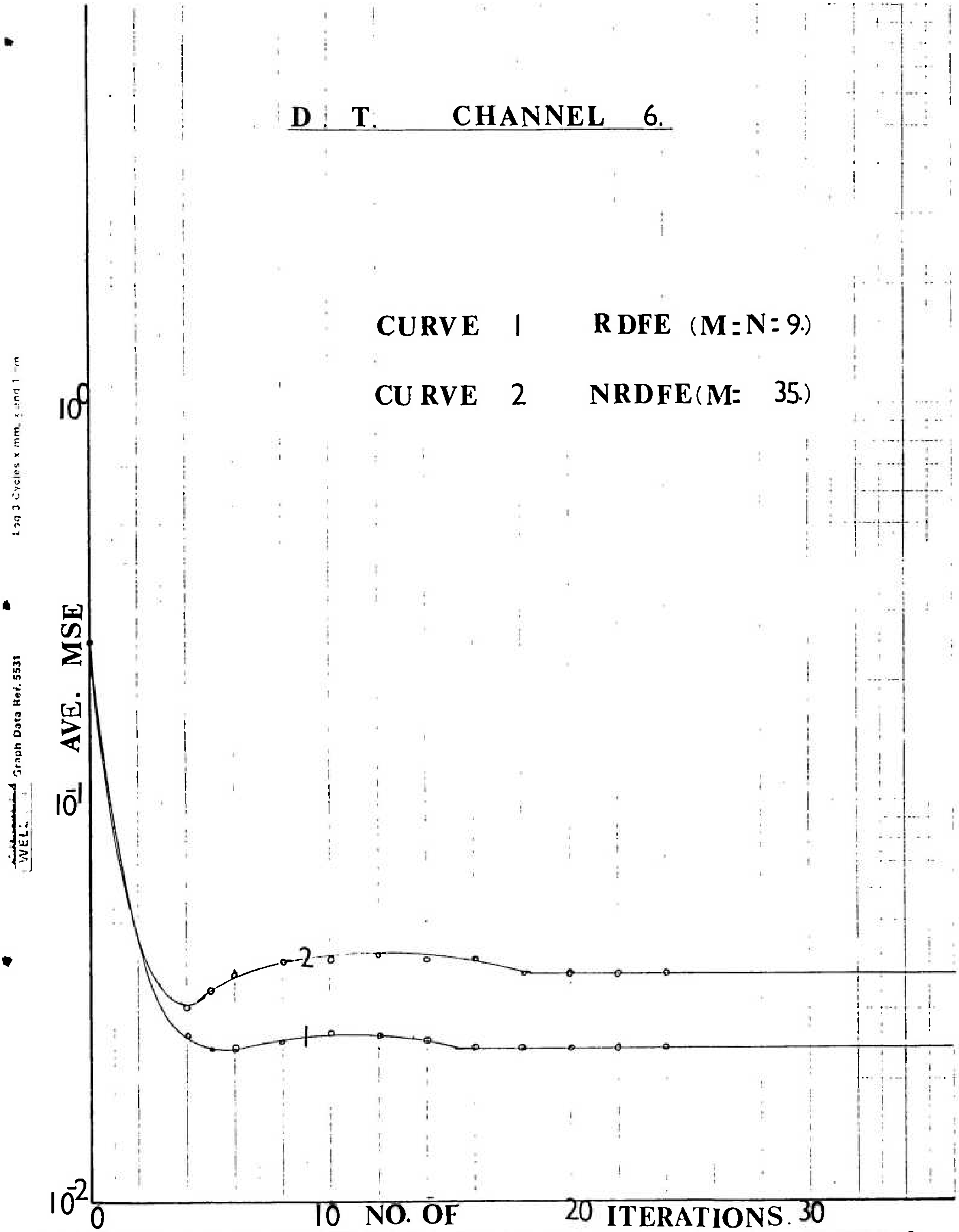


FIG.(5.38 a) DECISION-DIRECTED CONVERGENCE CURVES FOR D.T. CHANNEL 6.

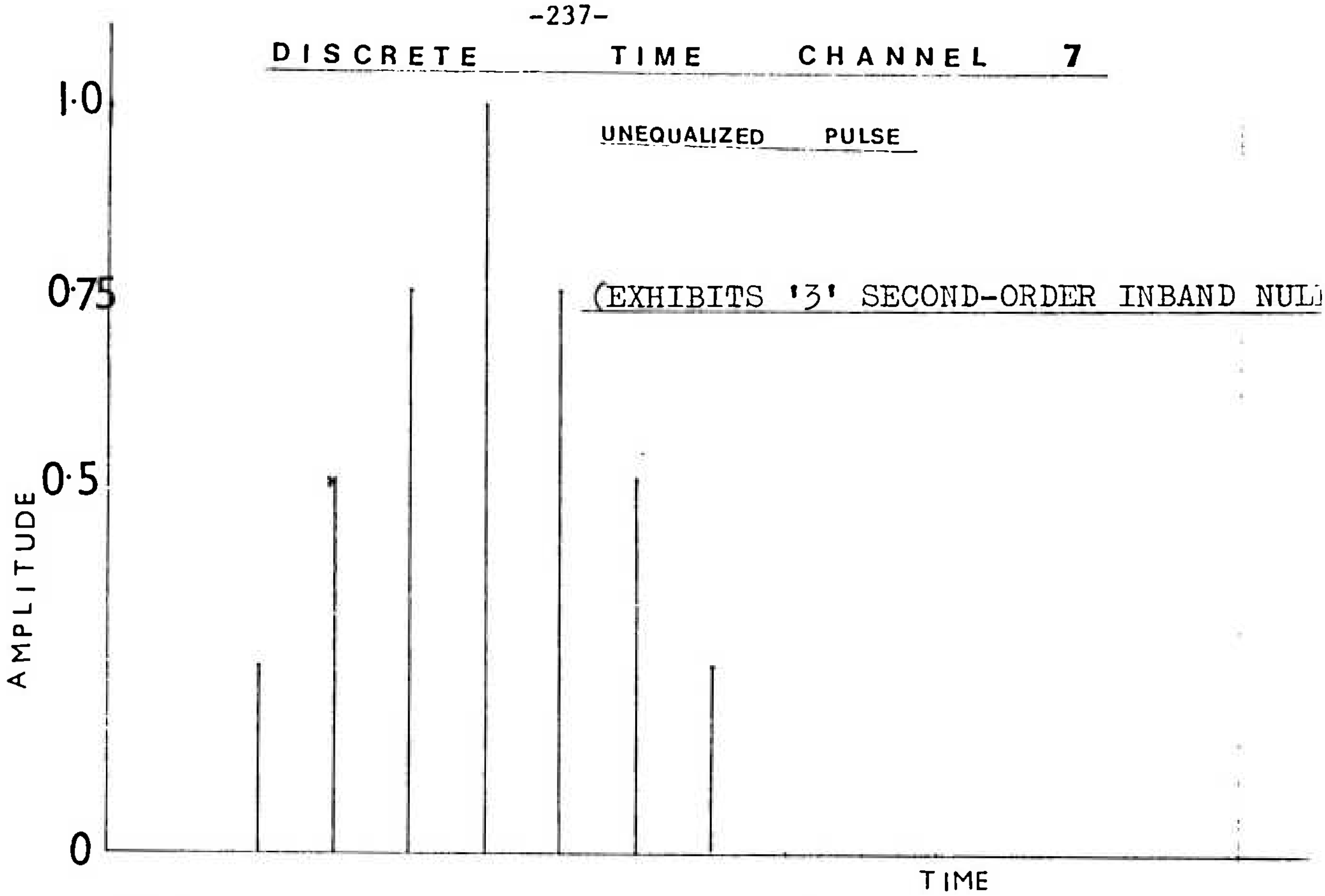


FIG.(5.39) IMPULSE RESPONSE OF CHANNEL 7.

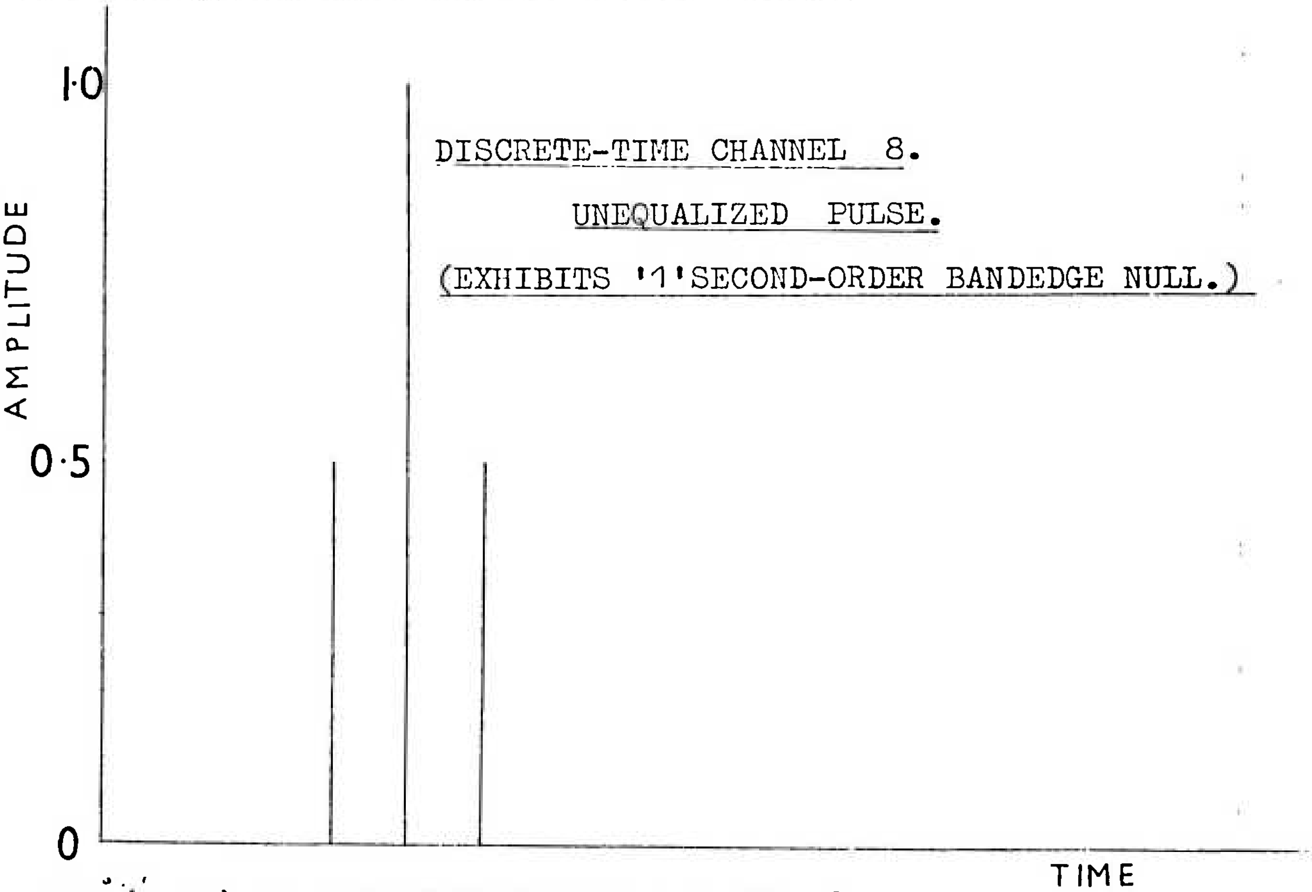


FIG.(5.40) IMPULSE RESPONSE OF CHANNEL 8.

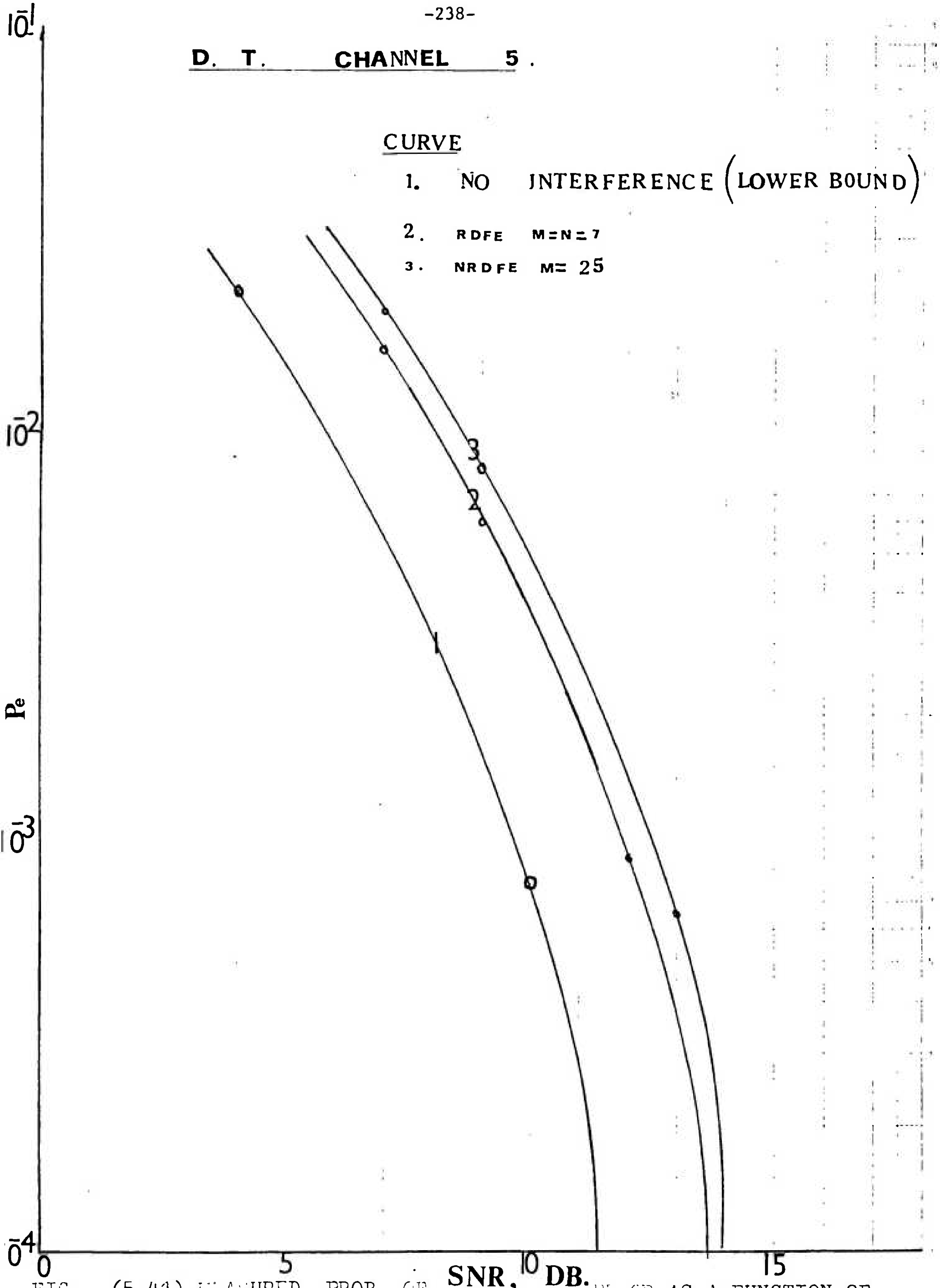


FIG. (5.41) MEASURED PROB. OF ERROR AS A FUNCTION OF SNR FOR AMPLITUDE DISTORTED CHANNEL 5.

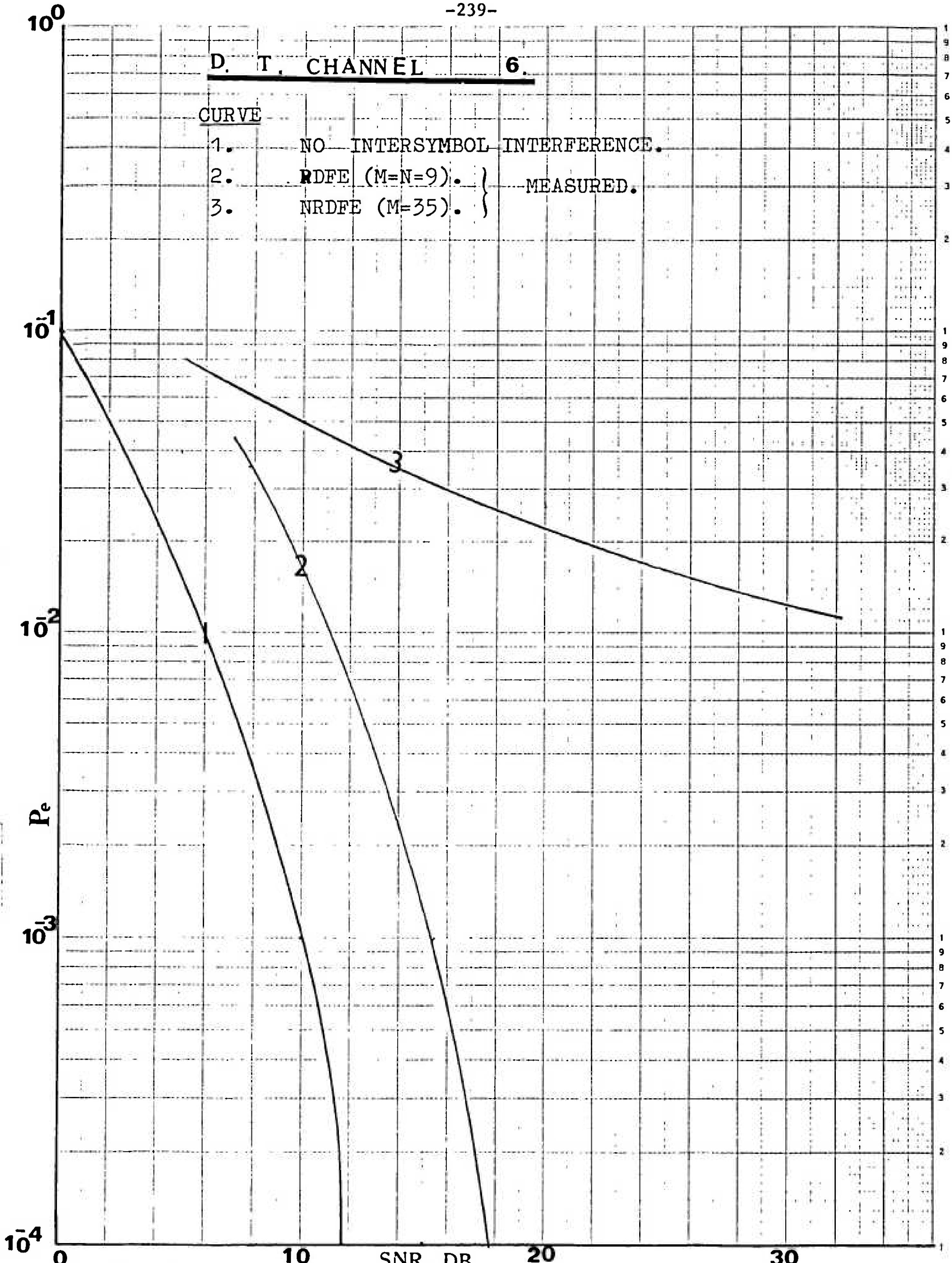


FIG.(5.42) MEASURED PROB. OF ERROR AS A FUNCTION OF SNR FOR AMPLITUDE DISTORTED CHANNEL 6.

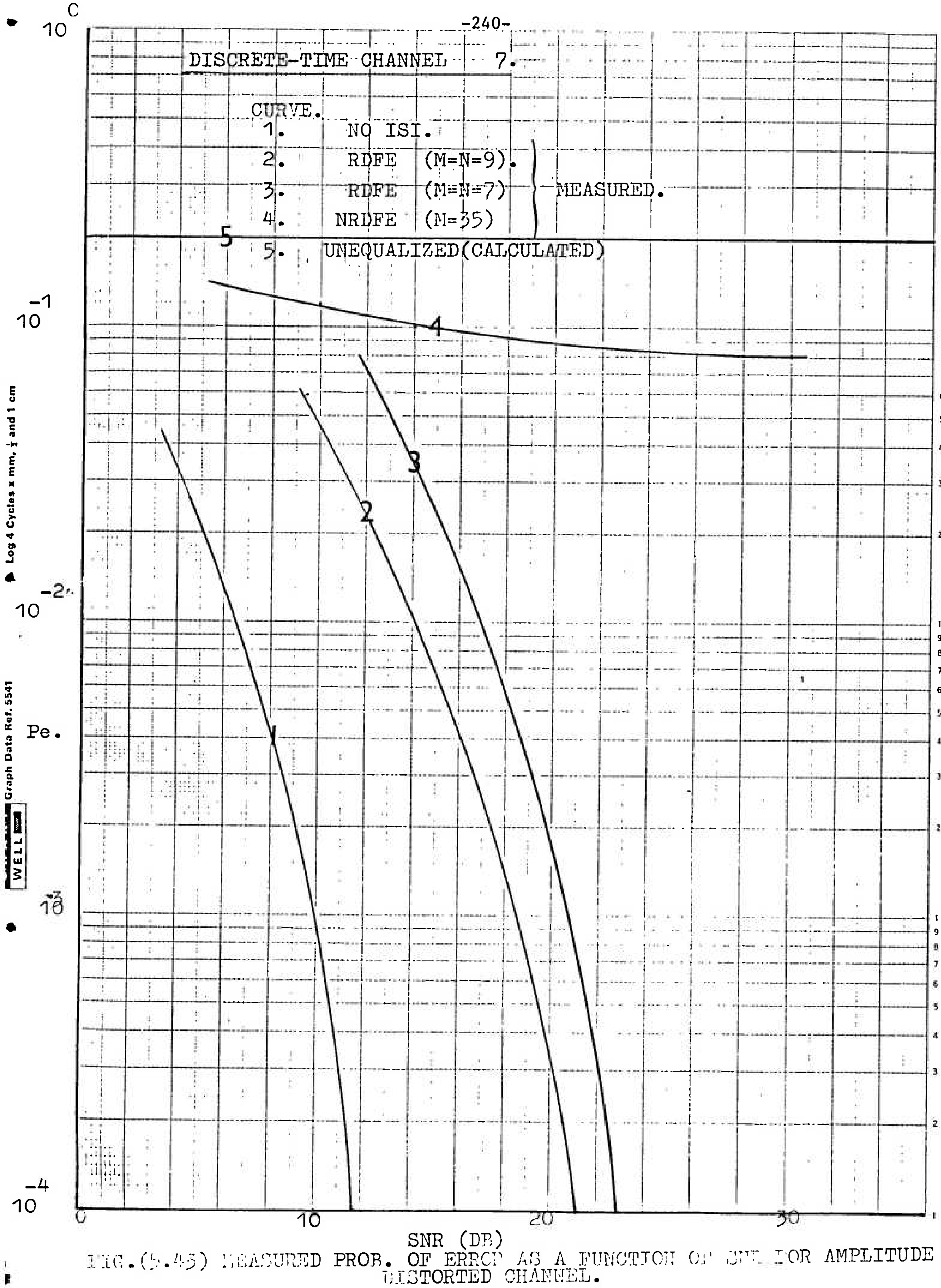


FIG.(5.45) MEASURED PROB. OF ERROR AS A FUNCTION OF SNR FOR AMPLITUDE DISTORTED CHANNEL.

CHAPTER 6

ADAPTIVE RECURSIVE DIGITAL FILTER EQUALIZER RECEIVERS USING MAXIMUM-LIKELIHOOD SEQUENCE ESTIMATION (THE VITERBI ALGORITHM)

"Explain all that", said the MOCK Turtle

"No, No! The adventures first," said the Gryphon in an impatient tone:

"Explanations take such a dreadful time ".

-LEWIS CARROLL (Alice's Adventures in Wonderland)

6.1 INTRODUCTION:

In chapter (1), a discrete time channel model has been derived. The output of this discrete time channel model is, a finite state Markov process corrupted by additive white gaussian noise, a property which is important in our discussion of an optimum detection algorithm for the sequence of symbols $\{I_k\}$.

From chapters (2) to (4), we have discussed NRDFE. These equalizers always entails so called " noise enhancement ", i.e., the noise variance at the output is bigger than at its input [6-1, p. 142]. Also, if the channel possesses nulls in the folded power spectrum of the pulse, then it is even more difficult to find a reasonable size NRDFE. In order to cope with the spectrum nulls in digital channels reasonably well, the existence of an equalizer, viz RDFE has been proposed and evaluated in ch. (5). However, the superiority of RDFE over NRDFE can be impaired badly if the no decision error propagation and the stability conditions are not observed properly. Because of certain superior features of RDFE we look upon , here, its suitability as a prefilter in conjuction with the VITERBI algorithm or maximum likelihood sequence estimator (henceforth to be abbreviated as the VA or MLSE).

The classical theory of ML detection of the transmitted symbols provides a conceptually simple solution to the problem of ISI: for an information of N symbols from an alphabet of size M, M^N matched filters, each matched to one possible information waveform, are used at the receiver. The information sequence with the largest output sample value is the most likely transmitted one and hence is chosen as the receiver's guess of the transmitted sequence [6-2]. However,this approach is impractical since the number of matched filters (M.F.) grows exponentially with the information length.

In 1967, Viterbi [6-3] devised an algorithm for decoding convolution codes, thereafter commonly referred to as the VA. Forney [6-4] pointed out that this algorithm is indeed a maximum likelihood rule, and therefore always optimum.Later it was identified by Omura [6-5] as a version of forward dynamic programming. Recognizing the fact that convolution codes and the ISI process are both shift register processes [6-6] , Omura [6-7] , Forney [6-8] , Kobayashi [6-9] showed that the VA is equally applicable to decoding ISI. As we shall see later the performance of the MLSE is so superb while its complexity grows very fast with the number of ISI terms in the received signal, it is very worthwhile to make an attempt to limit its complexity while retaining its good performance. One immediate approach is to use an equalizer to shape the channel into some desited one whose impulse response is short and then employ the VA to this equalized channel characteristic. This was the scheme put forward by Forney in his original work [6-8]. This resulted in much attention being given to practical methods of applying his results and suggestions. Qureshi and Newhall [6-10] examined this receiver for slowly time-varying channels and analyzed the effects of coloured noise and residual ISI on the performance. It is quite successful, especially when the shapes of unconditioned channel spectrum and the desired one are similar. A slightly better scheme than [6-10] has been proposed by Falconer and Magee [6-11] . They adaptively optimized both the desired impulse response (DIR) itself and the NRDFE parameters in order to minimize the MSE between the output of the NRDFE and the DIR while constraining the energy in the DIR to be fixed. Cantoni and Kwong [6-12] have shown that minimum MSE achievable for this structure is a monotonically decreasing function of both the length of the desired impulse response and the delay involved in updating it. A similar scheme studied by Messerschmitt [6-13] also uses a

NRDF to design a finite impulse response for the VA. The impulse response of the filter is determined to minimize the noise variance while holding the first nonzero sample fixed. A numerical example for coaxial cable channel shows that the performance of the infinite impulse response can be approached by a short impulse response in this way. Recently Fredricsson [6-14] studied the optimization of the transmitting filter, as well as the optimization of the transmitting and receiving filters [6-15]. In both cases he has put a constraint on the receiver complexity by limiting the length of the system impulse response. The results over the coaxial cable channel show that the VA is preferable even when the length of the system impulse response is quite short.

Recently novel approaches have been made to limit the complexity of the MLSE and thereby simplifying the VA itself. One method was discussed by Forney [6-8], which is suitable to the class of partial response system and the system does not incur much loss in performance compared to the original VA. Fredricsson [6-16], Vermeullen and Hellman [6-17] independently derived a two state receiver which keeps only two survivors. This reduced-state receiver was shown to be asymptotically optimal for high SNR under the sufficient condition of $(d_{min}/2) < 0.625$. Kazakos [6-18] saved computations, without affecting the optimality of the algorithm, by observing that certain transitions cannot occur simultaneously in the VA for a given impulse response. Magee and Proakis [6-19] developed an adaptive MLSE and estimated its performance when only the pulse response energy and duration are known. Ungerboeck [6-20] also examined an adaptive receiver in which he used a modified version of the VA that operates directly on the output of the matched filter thus eliminating the need for a noise whitening filter.

Except for the schemes to simplify the algorithm itself, all of the work so far, used a NRDF to shape the impulse into some desired one.

In this chapter, we will study the use of RDFE in place of NRDFE, which is our first and independent effort. We propose and then analyze by computer simulation two receivers consisting of an adaptive RDFE (direct form) with the VA as the decision device. A method of estimating the performance of the MLSE is presented for the case when the delay in the MLSE is limited.

In the proposed receiver structure (1) tentative decisions of the VA are used to cancel the effect of the tail of the channel impulse response. Desired impulse response filter is fixed and the basic RDFE is an adaptive filter.

However, we propose a more practical receiver structure (2), in which a seperate NRDF is used as a DIR which has been considered to be an adaptive filter to cater for the slowly varying channels. Because of the nonlinear nature of the optimization problem main emphasis has been laid on the computer simulation.

6.2 FUNDAMENTALS:

In chapter (1) [section (1.9)], a discrete time channel model has been developed. We have stated that the cascade of the transmitting filter x(t), the channel g(t), the matched filter h(-t), the sampler and the discrete time noise whitening filter $[1/F(z^{-1})]$ can be represented as a finite memory equivalent discrete time NRDF having the set $\{\rho_k\}$ as its multipliers. The output of this model is a finite state Markov process corrupted by white gaussian noise of zero mean and variance N_0. This is illustrated by Figs. (6.1) and (6.2).

We describe receivers which are optimum in the sense that the entire received sequence is correct rather than minimizing the average number of errors in the received sequence. These receivers are in fact asymptotically optimum in terms of the average number of errors as will be shown in subsquent sections .

In section (6.2.1) the method of maximum likelihood is described. In section (6.2.2) the VA, a simplified algorithm for sequence estimation is discussed. In section (6.2.3) the performance of the VA is discussed and an upper bound on performance is developed. In section (6.3) we describe the proposed adaptive versions of VA receivers in conjunction with a recursive digital filter equalizer . Finally we analyze by simulation the receiver structures in section (6.3.1). In section (6.4) a more practical receiver has been proposed in which DIR is an adaptive filter. Section (6.5) deals with the computer simulation.

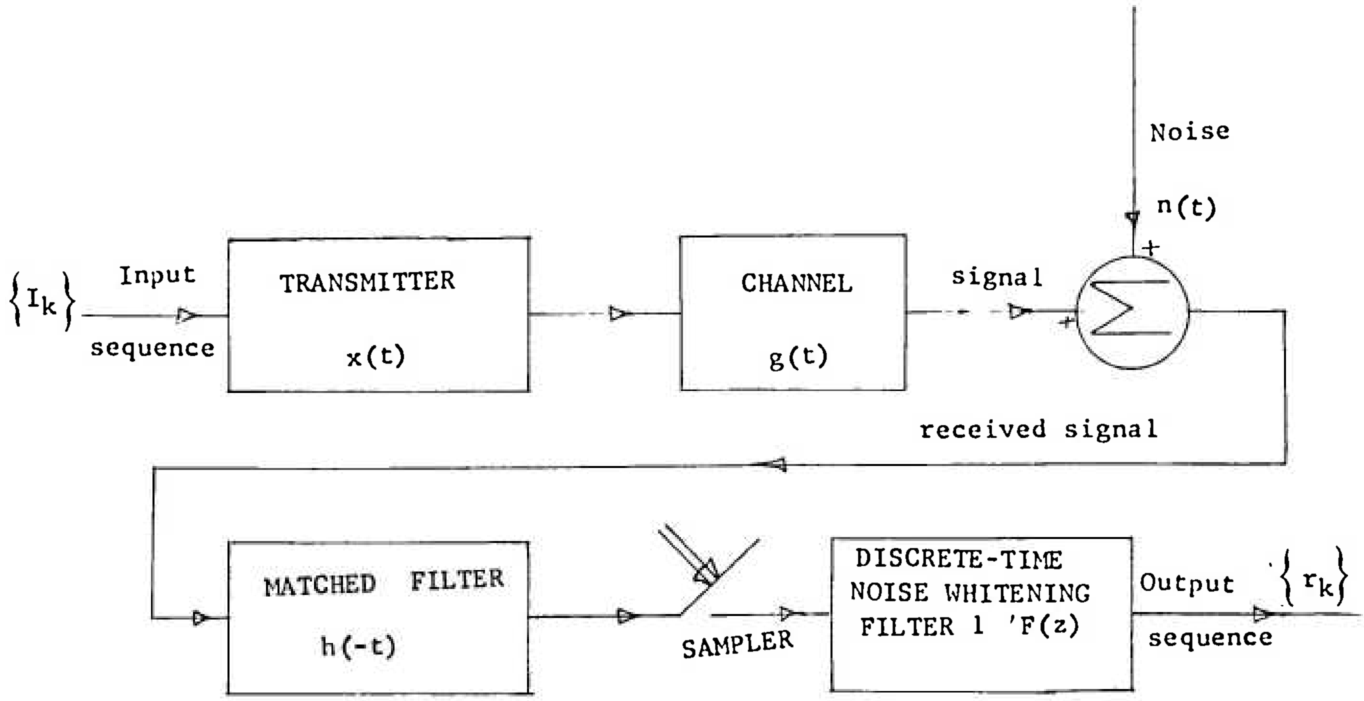


FIG. (6.1) BUILDING BLOCKS OF A DISCRETE-TIME CHANNEL

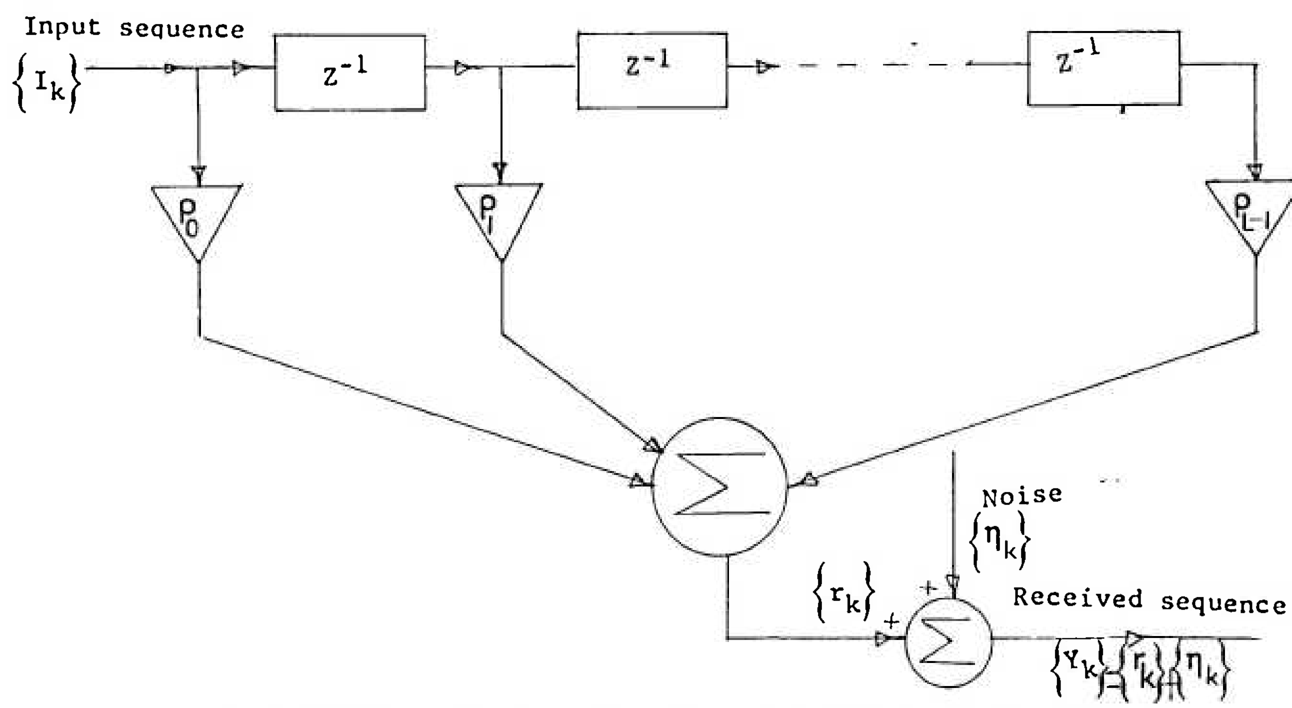


FIG. (6.2) A DISCRETE-TIME CHANNEL MODEL (NRDF TYPE).

6.2.1 MAXIMUM LIKELIHOOD SEQUENCE ESTIMATION:

The classical theory of maximum likelihood detection of the transmitted symbols provides conceptually simple solution to the problem of ISI:

For a message of N symbols from an alphabet of size M, M^N matched filters, each matched to one possible message waveform, are used at the receiver. The message sequence with the largest output sample value is the most likely transmitted one and hence is chosen as the receiver's guess of the transmitted sequence. This approach, which minimizes the average number of errors in the received sequence, is of no practical use, however,since the number of matched filters grows exponentially with the message length and, in any practical situation, is much too large to be implemented.

Thus, having failed to minimize the average number of errors simply, one can investigate maximizing the probability that the entire received sequence is correct. This type of decision making is simply asymptotically optimum, which means that at moderate SNR it is effectively as good as the optimum detector. The VA is a simple efficient algorithm which accomplishes this problem [6-8] .

From Fig. (6.2), the output of the discrete time channel finite state machine is written:

$$r(z^{-1}) = I(z^{-1})\ \rho(z^{-1})$$

This output is then added to a white gaussian noise (WGN) sequence $n(z^{-1})$ with autocorrelation function $\sigma^2\delta_{ij}$ to form the received sequence $Y(z^{-1})$. The channel consists of a shift register of L memory elements, each having m- states(i.e. the number of signal levels). These memory elements contain the L-most recent inputs. The machine has m^L possible states given by the L most recent inputs:

$$s_k \triangleq (\ I_{k-1},\ I_{k-2},\ \ldots\ ,\ I_{k-L}) \tag{6.1}$$

where $I_k = 0$ for $k<o$

Since there are m^L states, therefore, we can set a one-to-one correspondence between the states and the integers ranging from 1 to m^L. The state sequence $s(z^{-1})$ is given by:

$$s(z^{-1}) \triangleq s_0+s_1z^{-1}+s_2z^{-2}+\ldots \quad \ldots \tag{6.2}$$

and two successive states determine an output

$$r_k = r\ (s_k\ ,\ s_{k+1}) \tag{6.3}$$

Since it is the entire received sequence which is to be estimsted, the maximum likelihood sequence estimation rule is defined as selecting that $I(z^{-1})$ for which the likelihood function $p\left[Y(z^{-1})|I(z^{-1})\right]$ is maximum over all allowable source sequences $I(z^{-1})$; that is, the receiver chooses as its estimate the one which gives the largest value of conditional probability density. Since for a given discrete time channel, the mappings from $I(Z^{-1})$ to $s(z^{-1})$ and to $r(z^{-1})$ are one-to-one, therefore, the choice of the most likely $I(z^{-1})$ is equivalent to the choice of most probable $s(z^{-1})$ or $r(z^{-1})$ from the noisy observation $Y(z^{-1})$. The VA is a simple method of performing the computations of MLSE.

6.2.2 THE VITERBI ALGORITHM (VA):

To construct the recursive estimation algorithm known as the VA, we first use the fact that the noise terms n_k are independent. Then the log likelihood function

$$\ln p\ \left[Y(z^{-1})|s(z^{-1})\right]$$

breaks up into a sum of two independent increments:

$$\ln p\left[Y(z^{-1})|s(z^{-1})\right] = \Sigma_k \ln p_n\ \left[Y_k - r(s_k,s_{k+1})\right] \tag{6.4}$$

where

$$p_n(\alpha) = \frac{1}{\sqrt{2\pi}\ \sigma_n} \exp\left(-\frac{\alpha^2}{2\sigma^2_n}\right) \tag{6.5}$$

is the probability density function of the gaussian noise sample n_k with variance $\sigma_n^2 = \frac{N_0}{2}$, and r_k is given by Eq. (6.3) [6-8] . For notational convenience, we denote the partial sum in the log likelihood function from k_1 to (k_2-1) as:

$\Gamma \left[s(z^{-1}) \right]_{k1}^{k2}$ and define it as

$$\Gamma \left[s(z^{-1}) \right]_{k_1}^{k_2} \triangleq \sum_{k=k_1}^{k_2-1} \ln p_n \left[Y_k - r(s_k , s_{k+1})\right] \qquad (6.6)$$

$$0 \leqslant k_1 < k_2$$

then the log likelihood function from, say, k=0 to k=K would be

$$\Gamma \left[s(z^{-1}) \right]_0^K = \Gamma \left[s(z^{-1}) \right]_0^k + \Gamma \left[s(z^{-1}) \right]_k^K \qquad (6.7)$$

for any k, $0 \leqslant k \leqslant K$

For each state j, $j \in \{1, 2, \ldots\ldots\ldots m^L\}$ we call the state sequence having maximum log likelihood $\left[s(z^{-1}) \right]_0^k$ among all allowable sequences evolving at state j at time k the survivor for state j , and denote it by $\tilde{S}_j(z^{-1})$. For any time k, there are m^L survivors in all , one for each state. Principle of optimality dictates that with the final survivor at time K must begin with one of these survivors [6-21] . The function of the VA is simply to compute the log likelihood $\left[s(z^{-1}) \right]_0^k$, select the largest one $\left[\tilde{s}(z^{-1}) \right]_0^k$, one for each state $S_k = j$ and store them with their corresponding survivor sequences $\tilde{S}_j(z^{-1})$. Because it computes the likelihoods recursively therefore it saves computations considerably. Forney [6-8] has shown that the VA can be used to detect on m-level sequence passing through a discrete time channel with known ISI. The complexity of the VA detector or MLSE depends on the shift register length L of Fig.(6.2). The storage required is at least $2(D-L+1)m^{L-1}$ bits, where D is the maximum allowable delay in the detector and m is the size of the input alphabet. The detector must be capable of performing m^L subtractions , m^L squaring operations, m^L additions and $(m-1)m^{L-1}$ binary comparisons every symbol period.

6.2.3 PERFORMANCE OF THE MLSE:

The performance of the receiver based upon the principle of the VA can be greatly superior to other receivers. It has been shown [6-8] that its performance is as good as could be attained by any receiver structure and in many cases as good as if ISI absent. The concept of Euclidean distance in n-dimensional signal space between any two signals, $r_1(z^{-1})$ and $r_2(z^{-1})$ is used. This is defined as the energy in the associated difference square. Let

$$\xi \triangleq r_1(z^{-1}) - r_2(z^{-1}) \tag{6.8}$$

and its square root called the distance between $r_1(z^{-1})$ and $r_2(z^{-1})$ is used. Here

$$\begin{aligned} d^2 &= ||r_1(z^{-1}) - r_2(z^{-1})||^2 \\ &= ||\xi_r(z^{-1})||^2 \\ &= \sum_{i=0}^{n} \xi_{ri}^{\;2} \end{aligned} \tag{6.9}$$

where $\xi_{ri} = (r_{1i}-r_{2i})$ is the difference between two received signal values r_{1i} and r_{2i} at time i. Using this notion Forney [6-8] has given a tight lower bound as well as an approximation to an upper bound on error probability

$$\begin{aligned} K_L\, Q\left[\frac{d_{min}}{2\sigma_n}\right] &\leq P_r(e) \\ &\leq K_u Q\left[\frac{d_{min}}{2\sigma_n}\right] \end{aligned} \tag{6.10}$$

where $Q[\,.]$ is given by

$$Q[x] = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-y^2/2}\, dy \tag{6.11}$$

K_L and K_u are constants which will be described below, d_{min} is the minimum distance between any two distinct allowable received signal sequence. Bounds given by Eq. (6.10) are essential in the study of our receivers, therefore, derivation of these bounds , need the notion of error event 'Ξ '.

An error event 'Ξ' is said to extend from time k_1 to k_2, when the estimated state sequence $\tilde{s}(z^{-1})$ is equal to the correct state sequence $s(z^{-1})$ at time k_1 and k_2, but nowhere in between. The length of the error event is defined as $n \triangleq k_2-k_1-1$. Clearly $n \geq L$, with no upper bound; however, we can prove that n is finite with probability one.

The probability of an error event 'Ξ' is calculated as follows:

Ξ_1 : The input error sequence is such that $I(z^{-1}) + z^{-k_1} [illegible])\xi_I(z^{-1})$ is an allowable sequence $\tilde{I}(z^{-1})$

Ξ_2 : The noise terms n_k, $k_1 < k < k_1+n$, must be such that over this segment $\tilde{I}(z^{-1})$ has greater likelihood than $I(z^{-1})$ or in terms of partial sum notations:

$$\Gamma\left[\tilde{s}(z^{-1})\right]_{k_1}^{k_1+n+1} \geq \Gamma\left[s(z^{-1})\right]_{k_1}^{k_1+n+1}$$

In addition it is useful to define the subevent:

Ξ_2': The noise terms are such that $\tilde{I}(z^{-1})$ has greater likelihood than the true $I(z^{-1})$, but not necessarily the greatest. Clearly Ξ_2 is contained in Ξ_2'. Then we have

$$\begin{aligned} Pr(\Xi) &= Pr(\Xi_1)\, Pr(\Xi_2|\Xi_1) \\ &\leq Pr(\Xi_1)\, Pr(\Xi_2'|\Xi_1) \end{aligned} \tag{6.12}$$

Assuming that the input symbols are independent and equiprobable, it is easy to show that [6-8]

$$Pr(\Xi_1) = \prod_{i=0}^{n-L} \frac{m-|\xi_{Ii}|}{m} \tag{6.13}$$

In terms of partial sum, $(\Xi_2' | \Xi_1)$ is

$$\left\{ \Gamma\left[\tilde{s}(z^{-1})\right]_{k_1}^{k_1+n+1} \geq \Gamma\left[s(z^{-1})\right]_{k_1}^{k_1+n+1} \middle| \Xi_1 \right\} \tag{6.14}$$

Since the noise is white and gaussian with variance, σ_n^2, then from (6.4) and (6.5) we get

$$\ln p_n\,(Y_k - r_k) = -\frac{1}{2}\ln 2\pi\sigma_n^2 - (Y_k - r_k)^2/2\,\sigma_n^2 \qquad (6.15)$$

so that

$$\Gamma\left[\tilde{s}(z^{-1})\right]_{k_1}^{k_1+n+1} \triangleq \left[\tilde{\Gamma}\right]_{k_1}^{k_1+n+1}$$

$$= \sum_{k=k_1}^{(k_1+n+1)-1} \ln p_n\,[Y_k - \tilde{r}_k]$$

$$= \sum_{k=k_1}^{k_1+n} \ln\left[\frac{1}{\sqrt{2\pi}\sigma_n}\exp.\left(-\frac{(Y_k-\tilde{r}_k)^2}{2\,\sigma_n^2}\right)\right]$$

and, similarly,

$$\Gamma\left[s(z^{-1})\right]_{k_1}^{k_1+n+1} \triangleq \left[\Gamma\right]_{k_1}^{k_1+n+1}$$

$$= \sum_{k_1}^{k_1+n} \ln\left[\frac{1}{\sqrt{2\pi}\sigma_n}\exp.\left(-\frac{(Y_k - r_k)^2}{2\,\sigma_n^2}\right)\right]$$

Subtracting, we get

$$\left[\tilde{\Gamma} - \Gamma\right]_{k_1}^{k_1+n+1} = \frac{1}{2\sigma^2{}_n}\sum_{k=k_1}^{k_1+n}\left[(Y_k-r_k)^2 - (Y_k-\tilde{r}_k)^2\right] \qquad (6.16)$$

$$= \frac{1}{2\,\sigma_n^2}\left[\|Y(z^{-1})-r(z^{-1})\|^2 - \|Y(z^{-1})-\tilde{r}(z^{-1})\|^2\right]_{k_1}^{k_1+r}$$

in an obvious notation.

In words, $\tilde{r}(z^{-1})$ is more likely than $r(z^{-1})$ if $\tilde{r}(z-1)$ is closer to $Y(z^{-1})$ than is $r(z^{-1})$ in the (n+1)- dimensional space corresponding to times k_1 to (k_1+n). Since the distance squared between $r(z^{-1})$ and $\tilde{r}(z^{-1})$ is just the Euclidean weight $d^2(\Xi)$ of $\xi_r(z^{-1})$, the probability of Ξ_2' is given by

$$Pr(\Xi_2' \mid \Xi_1) = Q[d(\Xi)/2\sigma_n] \tag{6.17}$$

where $Q(.)$ is defined in Eq. (6.11). Therefore the probability of a particular event Ξ is

$$Pr(\Xi) \leq Q\left[d(\Xi)/2\sigma_n\right] \left[\prod_{i=0}^{n-L} \frac{m-|\xi_{Ii}|}{m}\right] \tag{6.18}$$

The symbol error probability associated with Ξ is computed by weighting the error event Ξ by the number of decision errors $w_H(\Xi)$ it entails:

$$Pr(\Xi) \leq Q\left[\frac{d(\Xi)}{2\sigma_n}\right] \left[w_H(\Xi) \prod_{i=0}^{n-L} \frac{m-|\xi_{Ii}|}{m}\right] \tag{6.19}$$

From the probabilities of individual error events we obtain a bound through the union bound. Let E be the set of all possible error events starting at time k_1 and D be the set of all possible distances $d(\Xi)$, then the probability of symbol error is bounded and

$$Pr(E) \leq \sum_{\Xi \in E} w_H(\Xi) \sum_{\xi \in D} Pr(\Xi) \tag{6.20}$$

Because of the exponential decrease of the gaussian distribution function $Q(.)$ with its arguments, the relationship given by Eq. (6.20) will be dominated at high SNR, i.e. small σ_n, by the term involving the minimum value d_{min} of $d(\Xi)$

$$Pr(E) \leq K_u\, Q\left[\frac{d_{min}}{2\sigma_n}\right] \tag{6.21}$$

where

$$K_u = \sum_{\Xi \in E_{d_{min}}} w_H(\Xi) \prod_{i=0}^{n-L} \frac{m-|\xi_{Ii}|}{m} \tag{6.22}$$

with $E_{d_{min}}$ denoting the set of error events Ξ having the minimum distance. Forney [6-8, pp 371] has proposed Eq. (6.21) to be a bound, which, in fact, is only an approximation to a true upper bound; the accuracy of this bound depends on the assumption that all distances $d(\Xi)$ in D which are not identical to d_{min} and constitute extra terms in Eq. (6.20) are to be deleted. However, the estimate (6.21) is an asymptotic bound [6-22] and will become a true upper bound as the noise variance will be vanishing. In the high SNR cases, Eq. (6.21) gives a good approximation to the actual performance in many cases [6-8] . (It is possible to obtain this upper bound by generating function method which is more involved and therefore not described here [6-8]).

Next we describe the derivation of the true lower bound and its estimates. Let K_L be the probability that the input sequence $I(z^{-1})$ will be such that $\tilde{I}(z) = I(z^{-1}) + z^{-k}\xi_I(z^{-1})$ is an allowable input sequence for at least one $\xi_I(z^{-1}) \in E_{d_{min}}$ and for some k. The prob. that such an $\tilde{I}(z^{-1})$ will be closer than $I(z^{-1})$ to the received sequence $z(z^{-1})$ is exactly $Q[d_{min}/2\sigma_n]$. Hence, with prob. K_L, the prob. of an error event starting at time k for any k is at least $Q[d_{min}/2\sigma_n]$, so

$$\Pr(e) \geq K_L\, Q\left[\frac{d_{min}}{2\sigma_n}\right] \tag{6.23}$$

For comparison purposes, the prob. of error for a matched filter receiver in the absence of ISI is :

$$\Pr(e) = Q\left[\frac{\sqrt{R_0}}{2\sigma_n}\right] \tag{6.23a}$$

where R_0 is the energy of an isolated pulse [(6.23) reduces to (6.23a) in this case]. Forney asserts that the lower bound of Eq. (6.23) is also a lower bound on the error prob. of any receiver [6-23] . Thus, the MLSE achieves within the multiplicative constant (K_u/K_L), the minimum prob. of error attainable by any receiver at high SNR, and, in a very fundamental sense, the quantity (d^2_{min}/R_0) is a measure of the effective decrease in the SNR (relative to the detection of an isolated pulse) resulting from ISI.

The determination of the quantity d_{min}^2 (known as the minimum distance problem) is therefore a very important one for, even if the implementation of the MLSE is not contemplated for a particular channel, d_{min}^2 is a measure of the potential performance which can be obtained using receivers of arbitrary complexity. However, on channels with severe ISI, the exact analytical determination of d_{min}^2 does not appear feasible because of the nonlinear nature of the problem. We therefore determine the minimum distance under a fixed energy and pulse response length constraints.

Since the channel is linear, the input error sequence $\xi_I(z^{-1})$ maps simply to the output, $\xi_r(z^{-1})$, by

$$\xi_r(z^{-1}) = \xi_I(z^{-1})\ g(z^{-1}) \tag{6.24}$$

From Eqs. (6.9) & (6.24), we write d_{min}^2 as:

$$d_{min}^2 = ||\xi_I(z^{-1})\ g(z^{-1})||^2 \tag{6.25}$$

and note that the distance is dependent only upon the relative energies involved. We shall, now, consider specific cases for a few discrete time channels:

<u>CASE 1:</u> A DISCRETE TIME CHANNEL WHOSE IMPULSE RESPONSE CONSISTS OF UNIT PULSE (LENGTH ONE).

It is obvious that in this case the minimum distance is always equal to the energy in the pulse. Hence

$$d_{min}^2 = ||g(z^{-1})||^2 = g_0^2 \tag{6.26}$$

<u>CASE 2:</u> A DISCRETE TIME CHANNEL WHOSE IMPULSE RESPONSE CONSISTS OF TWO PULSES (LENGTH TWO).

This channel, say, is given by $g_0+g_1z^{-1}$.

It is clear that for any input error sequence $\{\xi_I\}$ there must always be $\pm g_0$ at the beginning and $\pm g_1$ at the end of the output error sequence$\{\xi_I(z^{-1})\}\{g(z^{-1})\}$ Thus even if the terms in the middle of the output error sequence cancel, the error distance, d^2_{min} , is at least ($g_0^2 + g_1^2$). We know that ($g_0^2 + g_1^2$) is also the distance caused by the single error, any pulse response length of two will have a minimum distance equal to the pulse response energy. Thus there is no performance loss for a length TWO channel over a length ONE channel except for the constant multiplier K_u.

CASE 3: A DISCRETE TIME CHANNEL WHOSE IMPULSE RESPONSE CONSISTS OF THREE PULSES (LENGTH THREE).

We can express this channel, say, as ($g_0 + g_1 z^{-1} + g_2 z^{-2}$). Then the expression for the error distance for a multiple input error of the form $(1-z^{-1})$ is

$$d^2 = g_0^2 + (g_1-g_0)^2 + (g_2-g_1)^2 + g_2^2 \qquad (6.27)$$

Note that, g_0, g_1, and g_2 must all be of the same sign for the terms in brackets to be minimum. We can express Eq. (6.27), quite conveniently, in the matrix form (since it is in quadratic form). The minimum of Eq. (6.27) under a unit energy constraint on the pulse response, is the minimum eigenvalue of the matrix of the quadratic form. The values of g_0, g_1, and g_2 can be found easily as below:

Since $$||g||^2 = g_0^2 + g_1^2 + g_2^2 \qquad (6.28)$$

Therefore, by

$$\frac{\partial(d^2/\ ||g||^2)}{\partial g_0} = 0 \tag{6.29}$$

$$\frac{\partial(d^2/||g||^2)}{\partial g_1} = 0 \tag{6.30}$$

and

$$\frac{\partial(d^2/||g||^2)}{\partial g_2} = 0 \tag{6.31}$$

yield

$$\left.\begin{array}{l} g_0 = g_2 = 0.5 \\ g_1 = 1/\sqrt{2} \\ \text{and} \\ d_{min} = (\ 2-\sqrt{2}) = 0.586 \end{array}\right] \tag{6.32}$$

We have demonstrated these facts by expressing the error sequence pattern as ($1 - z^{-1}$). However, this error sequence pattern can also be expressed as ($1 + z^{-1}$), ($1 \pm z^{-1} \pm z^{-2}$), ($1 \pm z^{-2}$).

<u>DISCUSSION ABOUT DIFFERENT FORMS OF ERROR SEQUENCE PATTERN :</u>

<u>FORM ($1 + z^{-1}$)</u>:

This form will yield

$$\left.\begin{array}{ll} & g_0 = 0.5 \\ & g_1 = -\ 1/\sqrt{2} \\ & g_2 = 1/2 \\ \text{and} & d^2_{min} = 0.586 \end{array}\right] \tag{6.33}$$

Thus the error sequence form ($1 + z^{-1}$) only result in alternating signs for g_0 , g_1 , and g_2.

FORM $(1 \pm z^{-1} \pm z^{-2})$:

This longer sequence of errors can only result in a larger or equal distance; the same terms which are in Eq. (6.27) will remain, but additional positive definite terms are added.

FORM $(1 \pm z^{-2})$:

We can show, in this case, that this pattern of error results in a larger or equal error distance and does not offer many cancellations in the quadratic form and thus cannot have a smaller error distance.

Therefore, as suggested in [6-8] and confirmed by us , we conclude that the error sequence for the minimum distance channels, should always be expressed in the form $(1 \pm z^{-1})$. Thus for the discrete time channel of length 3, we have

$d^2_{min} = 0.586$ and

therefore , the corresponding upper bound on the prob. of error is

$$Pr(e) \leq K_u Q \left[\frac{0.586}{2\sigma_n}\right] \tag{6.34}$$

Table (6.1) shows the maximum SNR loss and corresponding channel characteristics using the error pattern $(1 - z^{-1})$ only.

TABLE (6.1) PERFORMANCE LOSS AND CORRESPONDING NON-UNIQUE MINIMUM DISTANCE PULSE RESPONSE

D.T. Channel Length	Performance Loss dB $-10 \log_{10}(d^2_{min})$	D.T. Channel Characteristics $\{g_k\}$, k = 0,1, (L-1)
1	0	1
2	0	(a,b) where $a^2 + b^2 = 1$
3	2.3	(0.50 , 0.71 , 0.50)
4	4.2	(0.38,0.60,0.60,0.38)
5	5.7	(0.29,0.50,0.58,0.50,0.29)
6	7.0	(0.23,0.42,0.52,0.52,0.42,0.23)
7	8.2	(0.19,0.35,0.46,0.50,0.46,0.35,0.19)
8	9.2	(0.16,0.30,0.41,0.46,0.46,0.40,0.30, 0.16)
9	10.1	(0.14,0.26,0.36,0.43,0.45,0.43,0.36, 0.26,0.14)
10	10.9	(0.12,0.23,0.32,0.39,0.42,0.42,0.39, 0.32,0.23,0.12)

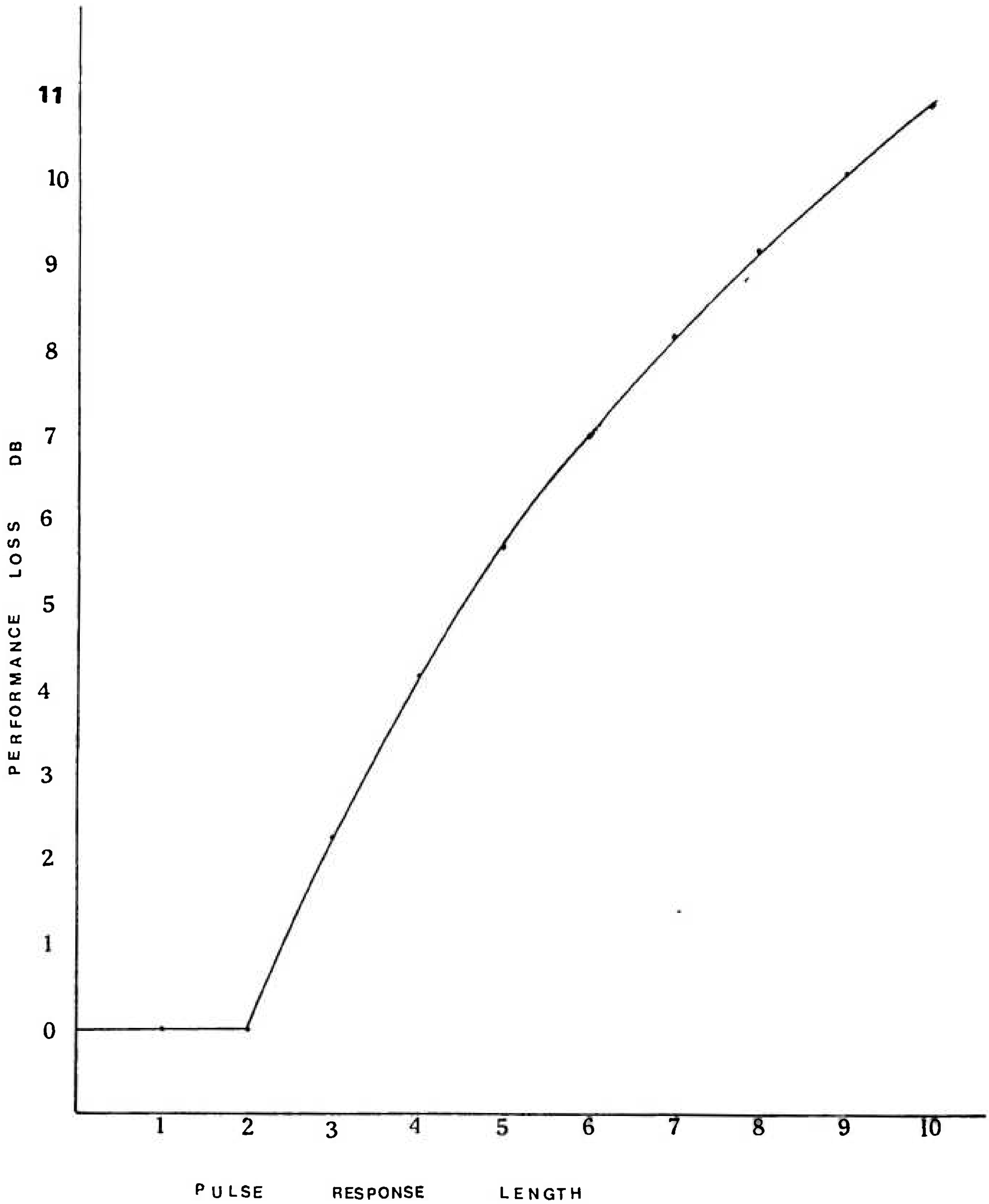


FIG. (6.3) PERFORMANCE 'PERFORMANCE LOSS VRS. CHANNEL PULSE RESPONSE LENGTH.

Fig. (6.3) shows a plot of $-\log_{10}(d^2_{min})$ versus discrete time channel pulse response length. Since the D.T. channels are normalized to unit energy, we find the maximum loss in performance (dB) at high SNR, compared to the best case D.T. channel which has d^2_{min} equal to energy (one) in the channel pulse response.

Finally, K_u of Eq. (6.34) can be calculated by actually computing the channels (the worst possible channel which can have a continuous string of errors without increasing the minimum distance) and then using the following results [6-8] :

$$\left.\begin{aligned} K_u &\le \sum_{n=1}^{\infty} 2n\left[\frac{n-1}{m}\right]^n \\ &= 2\,m\,(m-1) \end{aligned}\right] \qquad (6.35)$$

where m is the number of levels.

Forney's excellent results, thus far, obtained are intended for known discrete time channels. However, the results of this thesis are intended to apply to unknown channels. In order to cope with the unknown channels Forney [6-8] suggested the use of an adaptive receiver with the help of a linear equalizer to track the channel. Recently some works [6-10, 6-11, 6-19, 20] have been reported in this direction, all of them, have made an exclusive use of a nonrecursive digital filter equalizer (NRDFE).

We attempt independently, for the first time, the use of an adaptive recursive digital filter equalizer (RDFE) in place of an NRDFE. The gist of the proposed approach is that the reception of digital signals in the presence of ISI is shared by an RDFE and a VA detector. This allows the VA to be made of a practical size with a small degradation in performance.

6.3 AN ADAPTIVE RECURSIVE DIGITAL FILTER RECEIVER (# 1)USING THE VA:

The block diagram representation of the proposed receiver structure # 1 is shown in Fig. (6.4).

An adaptive RDFE (direct form) , and desired impulse response filter (NRDF type) are shown in Figs. (6.5) & (6.6) respectively. Basic description of RDFE and its coefficient updating processors are described in chapter (5). The received sequence $\{Y_k\}$ is first passed through an adaptive RDFE which limits the ISI by the D.T. channel. The equalized sequence $\{\hat{I}_k\}$ is fed to the VA which produces an estimate $\{\tilde{I}_k\}$ of the information sequence $\{I_k\}$ with a delay " δ" symbol periods. This estimate is the recovered data.

A sequence $\{\bar{I}_k\}$ of tentative decisions chosen from the surviving path with the maximum log-likelihood is used for equalizer adjustment in the feedback circuit. This sequence, which has suffered a delay of at least δ_1 units ($\delta_1 > \delta$) in the VA detector, is passed through a fixed NRDF Fig. (6.6), the coefficient of which represent the desired pulse response $\{g_k\}$ of the channel and the equalizer in cascade. The output of NRDF is then the desired output of the equalizer. The actual RDFE output $\{\hat{I}\}$being delayed by δ symbols, is subtracted from the desired output to produce the error sequence $\{\varepsilon_k\}$. The adaptive RDFE multiplier coefficients $\{\alpha_k\}$ and$\{\beta_k\}$ are adjusted to minimize the MSE $E[\varepsilon_k^2]$ iteratively. The survivors in the VA must be long enough to allow adjustment of the $\{\beta_k\}$. The VA detector assumes a fixed channel with pulse response $\{\lambda_k\}$ and additive WGN. The first assumption is justified provided the RDFE is capable of tracking the slow variations in the channel response. Neglecting the decision errors, we can write:

$$\{\hat{I}_k\} = \{Y_k\} * \{\alpha_k\} - \{\bar{I}_k\} * \{\beta_k\}$$

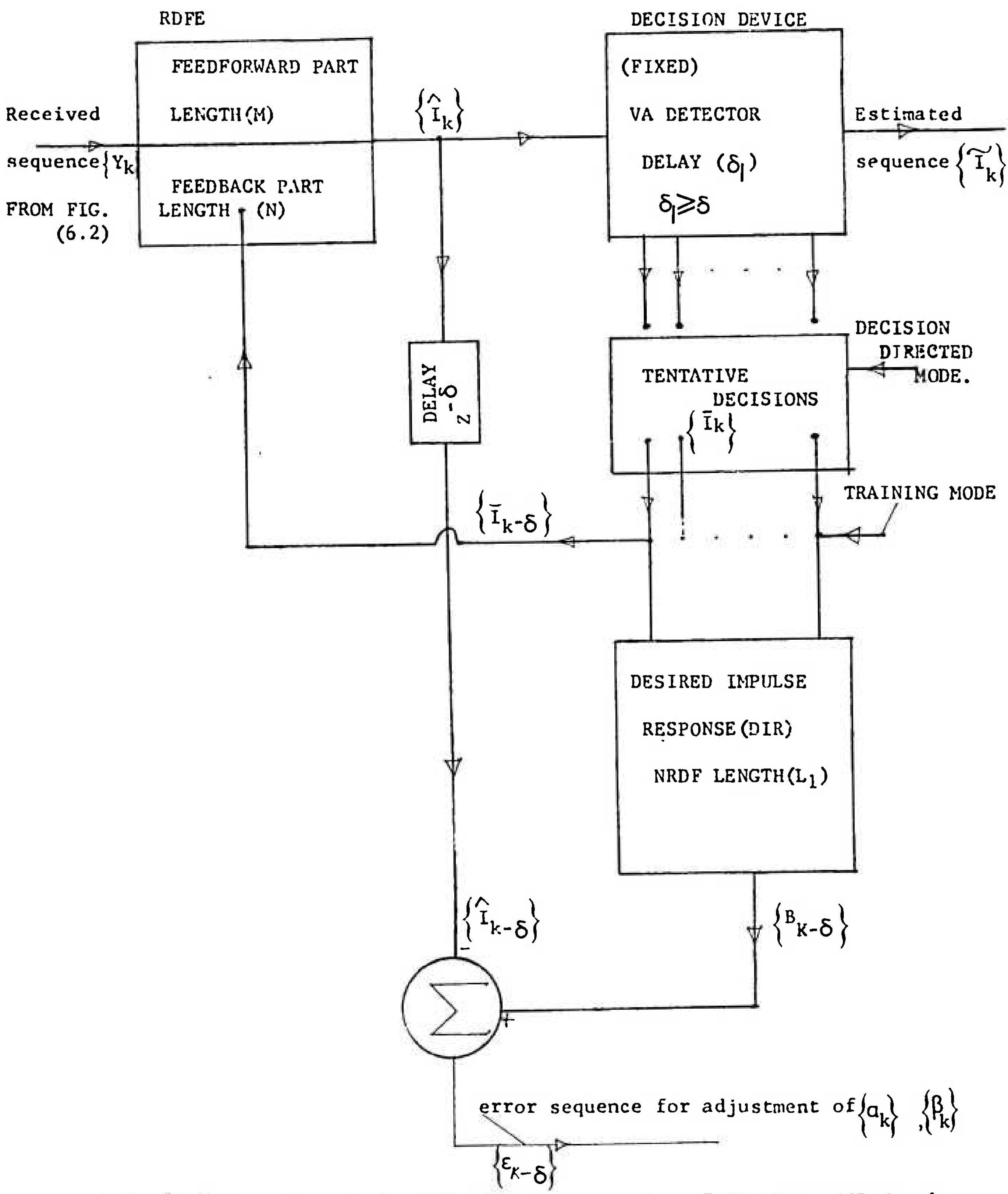


FIG. (6.4) BLOCK DIAGRAM REPRESENTATION OF ADAPTIVE RDF RECEIVER NO. 1.

$$\{Y_k\} = \{I_k\} * \{\rho_k\} + \{n_k\}$$

$$\{B_k\} = \{\bar{I}_k\} * \{g_k\}$$

and $$\{\varepsilon_k\} = \{\hat{I}_k\} - \{B_k\}$$

6.3.1 RECEIVER # 1. PERFORMANCE :

The output (equalized) symbol from a RDFE is given by Fig. (6.5)

$$\hat{I}_k = \sum_{i=0}^{M} \alpha_i Y_{k-i} - \sum_{l=1}^{N} \beta_l \bar{I}_{k-\delta-l} \tag{6.36}$$

From Fig. (6.2)

$$Y_k = \sum_{i_1=0}^{L-1} \rho_{i_1} I_{k-i_1} + n_k \tag{6.37}$$

From Fig. (6.6) the desired truncated received sequence is given by,

$$B_{k-\delta} = \sum_{i_2=0}^{L_1-1} g_{i_2} \bar{I}_{k-\delta-i_2} \tag{6.38}$$

The error in the equalized symbol Fig. (6.7)

$$\varepsilon_{k-\delta} = (\hat{I}_{k-\delta} - B_{k-\delta}) \tag{6.38a}$$

Assuming that the error prob. of the receiver is low enough so that

$$\hat{I}_k = \bar{I}_k$$

Therefore, the MSE is given by

$$J(\underline{\alpha}, \underline{\beta}, \underline{g}) \triangleq E[\varepsilon_{k-\delta}]^2$$

$$= E\left[\hat{I}_{k-\delta} - \sum_{i_2=0}^{L_1-1} g_{i_2} \bar{I}_{k-\delta-i_2} \right]^2 \tag{6.39}$$

The MSE $J(\underline{\alpha}, \underline{\beta}, \underline{g})$ is minimized when

$$g_{\alpha_k} = \frac{\partial J(\underline{\alpha}, \underline{\beta}, \underline{g})}{\partial \underline{\alpha}_k} = 0 \quad , \quad k=0,1, \cdots M$$

$$=2\left[\varepsilon_{k-\delta}\left[Y_{k-i-\delta} - \sum_{l=1}^{N} \beta_l \frac{\partial \bar{I}_{k-l-\delta}}{\partial \underline{\alpha}_k}\right]\right. \tag{6.40}$$

and

$$g_{\beta_k} = \frac{\partial J(\underline{\alpha}, \underline{\beta}, \underline{g})}{\partial \underline{\beta}_k}$$

$$= 2\left[\varepsilon_{k-\delta}\left[-\bar{I}_{k-\delta-1} - \sum_{l=1}^{N} \frac{\partial \bar{I}_{k-\delta-1}}{\partial \underline{\beta}_k}\right]\right] \qquad (6.41)$$

$$= 0, \quad k = 1, 2, \cdots N$$

Solving the last two sets of Equations simultaneously, we obtain

$$\sum_{i=0}^{M} \alpha_i \left(R_{in} - \sigma_I^2 \sum_{j=\delta}^{N} \rho_{j-i}\, \rho_{j-n}\right)$$

$$= \sigma_I^2 \sum_{i=0}^{L-1} \left[g_i\, \rho_{i-n} - \sum_{j=\delta}^{N} g_i\, \rho_{j-n}\right] \qquad (6.42)$$

$$n = 0, 1, \ldots M$$

where

$$R_{in} = E\left[Y_i Y_n\right] \qquad (6.43)$$

The minimum mean square error achievable is given by;

$$J_{min}(\underline{\alpha}, \underline{\beta}, \underline{g}) = \sigma_I^2 \sum_k \left[\sum_{i=0}^{M} \alpha_{0i}\, \rho_{k-i} - \beta_{0k} - g_k\right] + \sigma_n^2 \sum_{i=0}^{M} \alpha_{0i}^2 \qquad (6.44)$$

where α_{0i} and β_{0i} are the optimum values of the i^{th} feedforward and i^{th} feedback multipliers. Multiplier coefficients are adjusted using the iterative algorithms given by Eqs. (6.45) & (6.46) respectively:

$$\underline{\alpha}_k^{(j+1)} = \underline{\alpha}_k^{(j)} - \frac{\Delta_{1i}^{(j)}}{2}\, g_{\underline{\alpha}_k}^{(j)}$$

$$= \underline{\alpha}_k^{(j)} - \Delta_{11}^{(j)}\, \varepsilon_{k-\delta}^{(j)}\left[Y_{k-\delta-i} - \sum_{l=1}^{N} \beta_l \frac{\partial I_{k-1-\delta}}{\partial \underline{\alpha}_k}\right] \qquad (6.45)$$

$$k = 0, 1, \ldots\ldots M$$

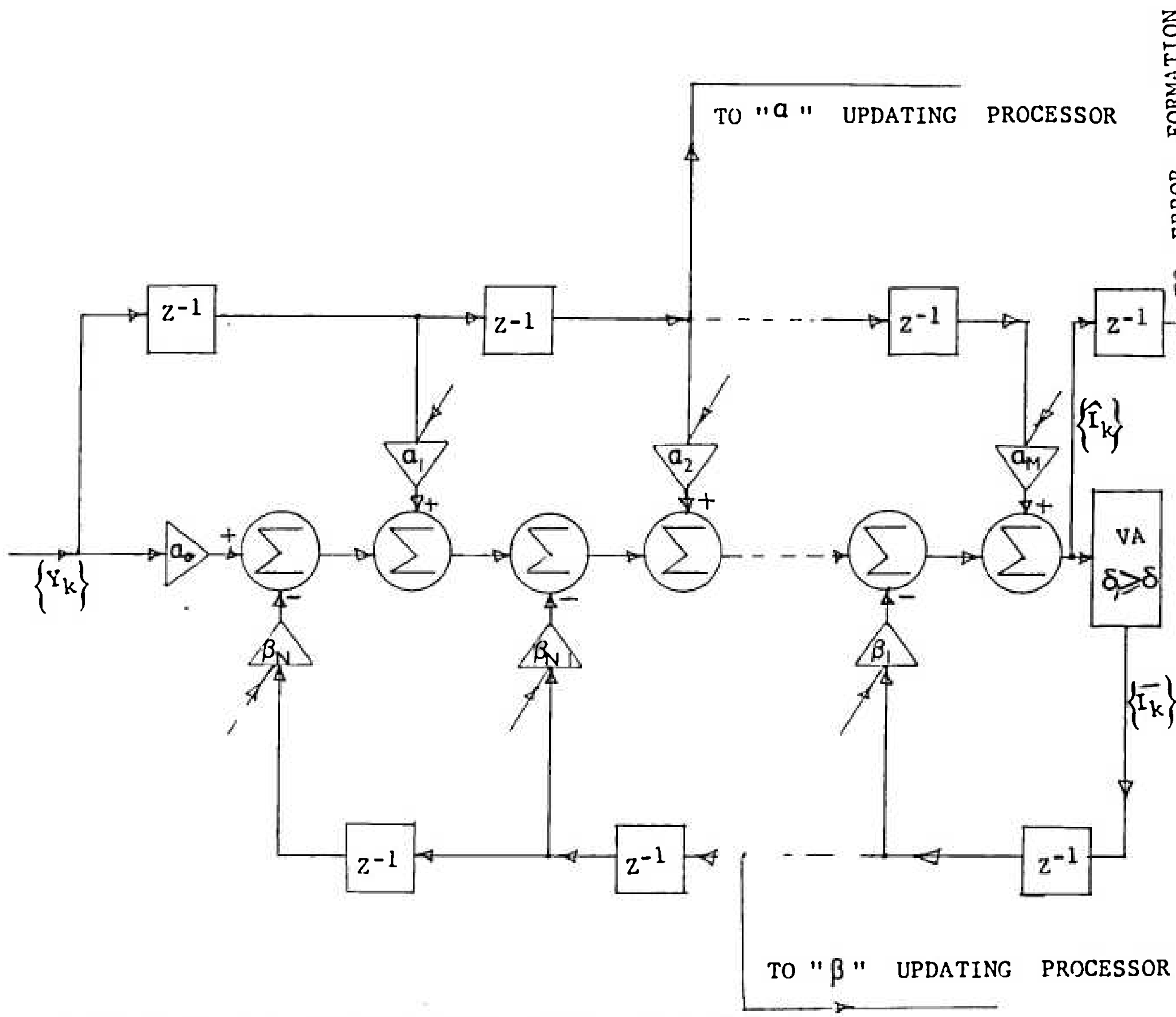


FIG. (6.5) AN ADAPTIVE RDFE USING VA TENTATIVE DECISIONS AS FEEDBACK INPUT.

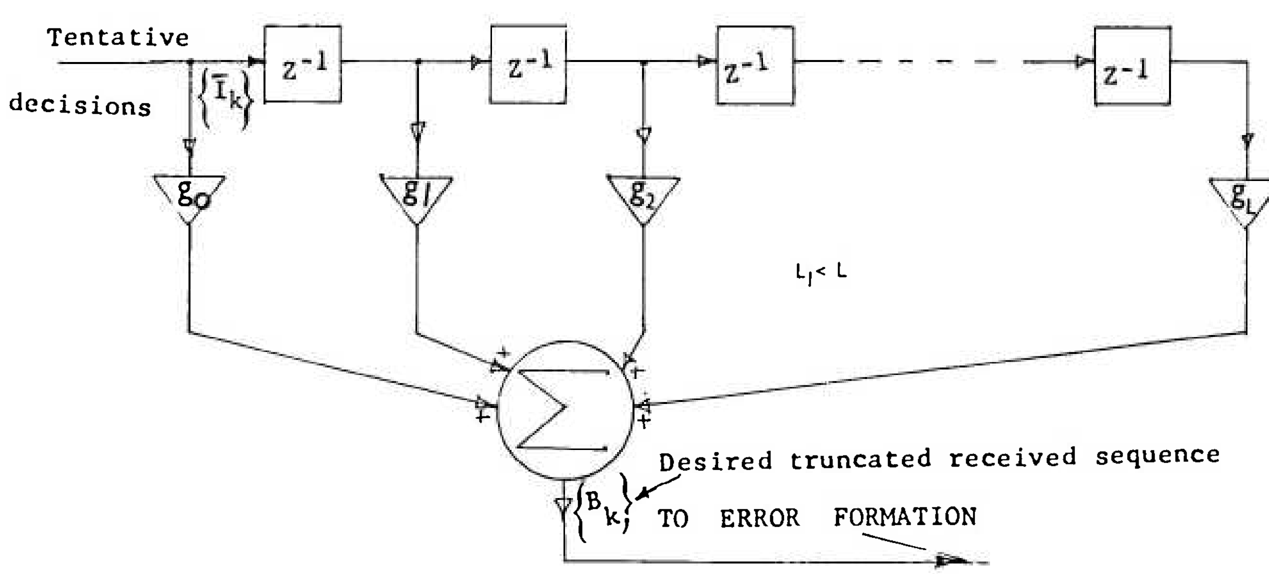


FIG. (6.6) DESIRED IMPULSE RESPONSE, NRDF TYPE, FILTER.

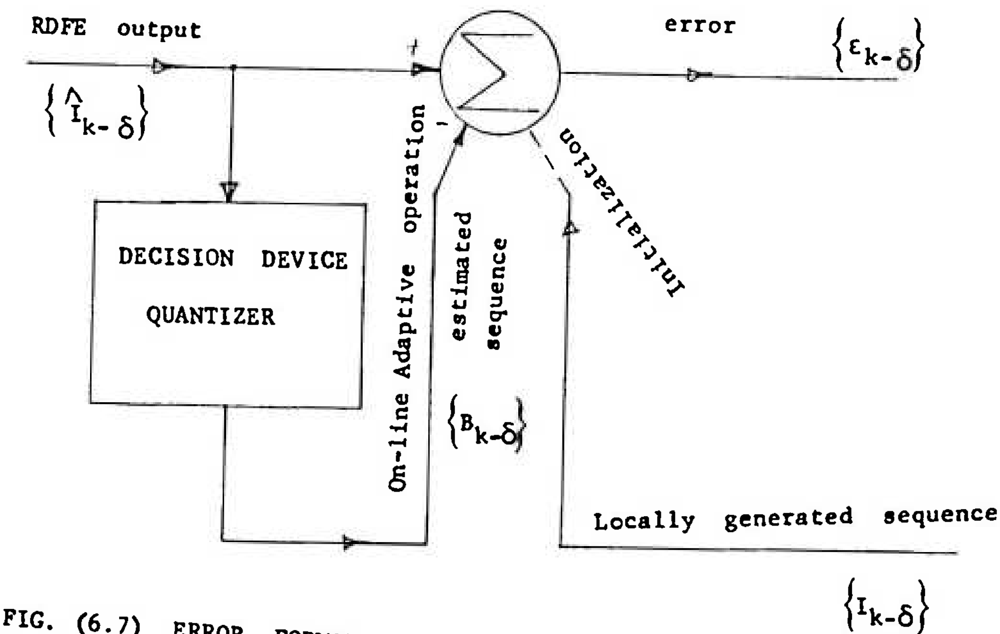


FIG. (6.7) ERROR FORMATION.

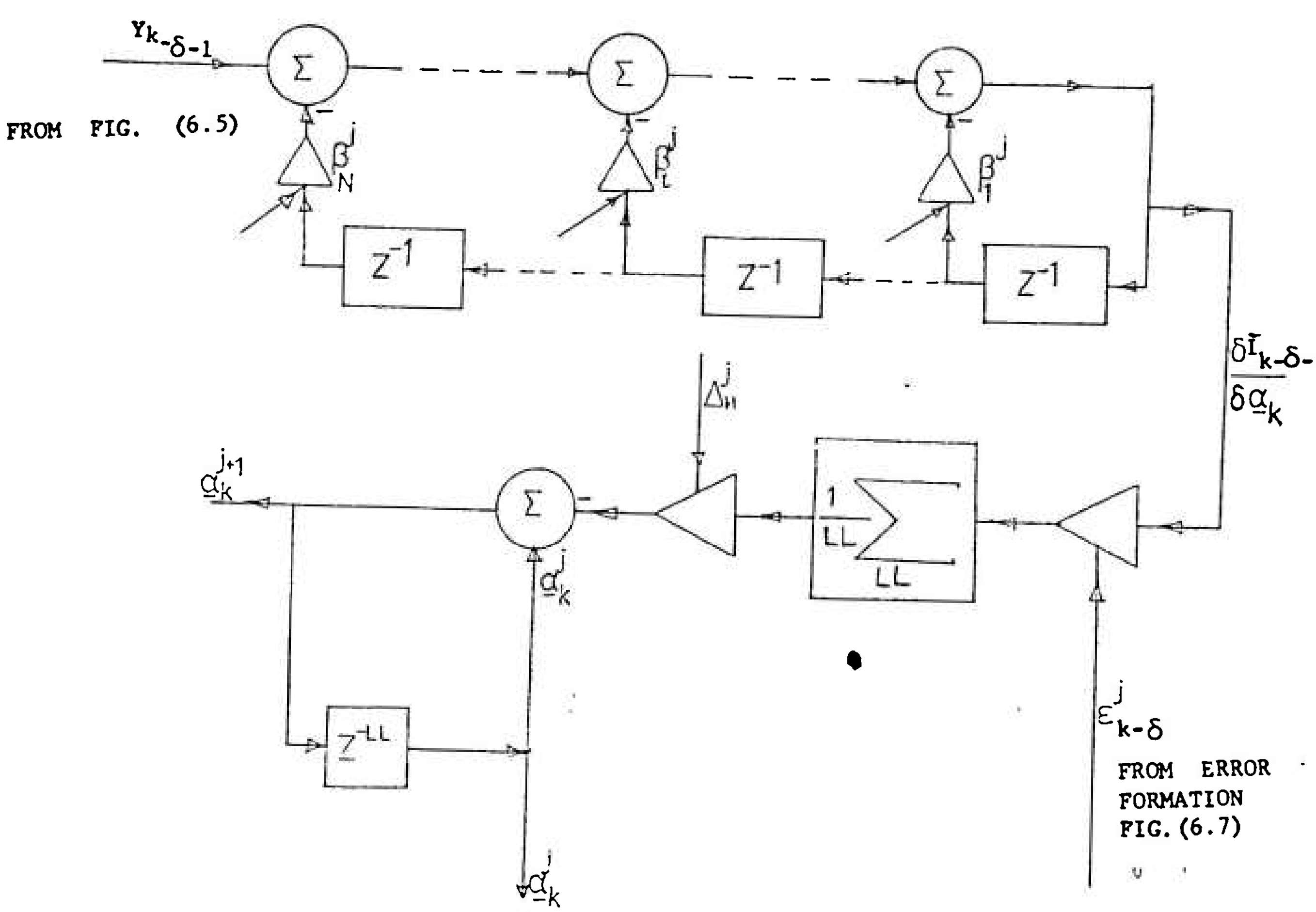


FIG. (6.8) A "α" UPDATING PROCESSOR

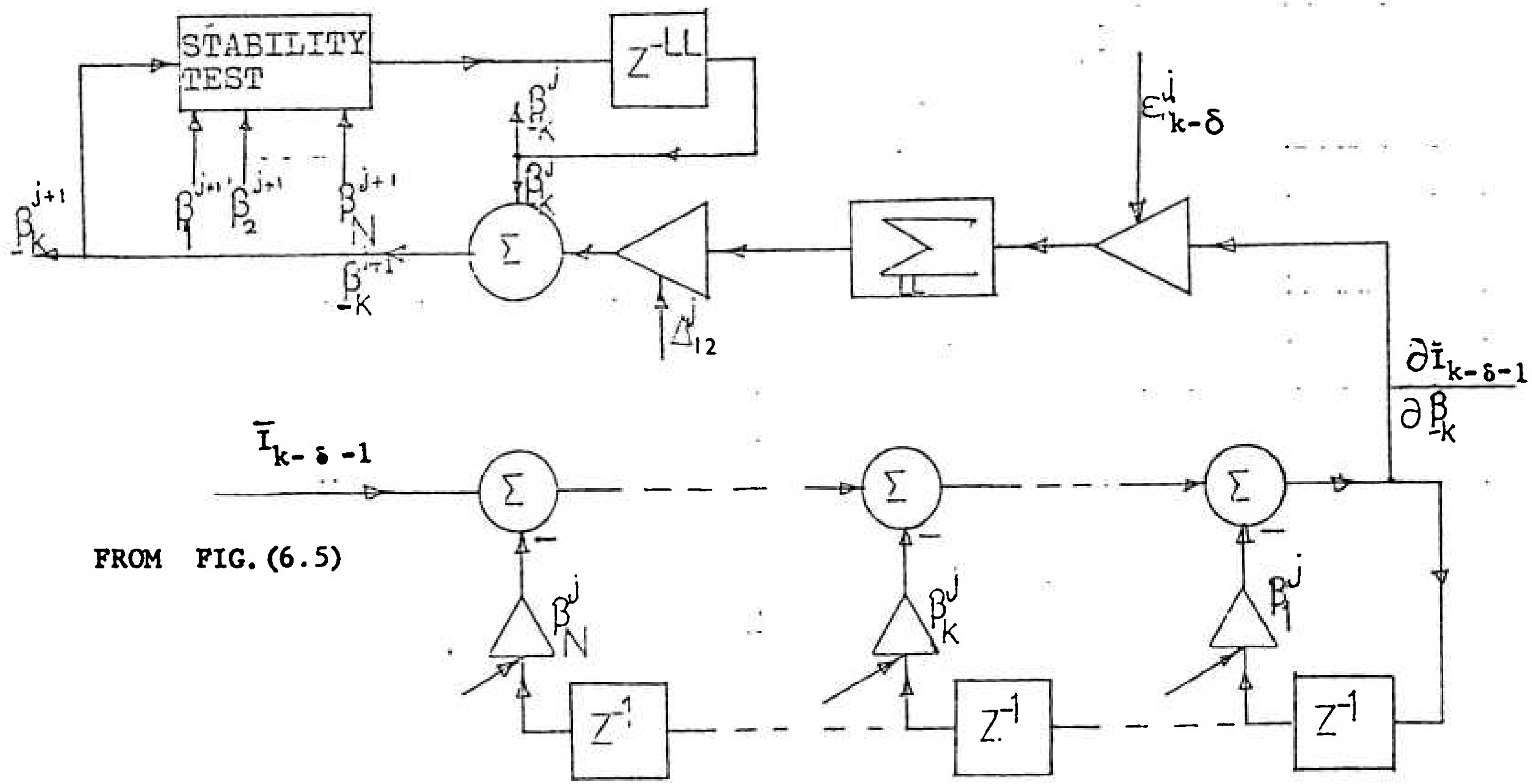


FIG. (6.9) A "β" UPDATING PROCESSOR WITH STABILITY CONTROL.

$$\underline{\beta}_k^{(j+1)} = \underline{\beta}_k^{(j)} - \frac{\Delta_{12}^{(j)}}{2} \underline{g}_{\beta_k}^{(j)}$$

$$= \underline{\beta}_k^{(j)} - \Delta_{12}^{(j)} \varepsilon_{k-\delta}^{(j)} \left[- \bar{I}_{k-\delta-1} - \sum_{l=1}^{N} \beta_l \frac{\partial \bar{I}_{k-\delta-1}}{\partial \beta_k} \right] \qquad (6.46)$$

$k=1,2, \ldots \ldots N.$

Feedback multiplier coefficients β are constrained according to

$$\sum_{i=1}^{N} | \beta_i^{(j)} | < 1 \qquad (6.47)$$

Block diagram description of an error formation, an " α " updating processor, and a " β " updating processor incorporating stability test instrumentation, are shown in Figs. (6.7), (6.8), and (6.9) respectively.

However, in practice, these complicated algorithms (6.45), and (6.46) are simplified by using unbiased but Widrow's noisy estimates of the gradients. Hence they are:

$$\underline{\alpha}_k^{(j+1)} = \underline{\alpha}_k^{(j)} - \Delta_{11}^{(j)} \varepsilon_{k-\delta}^{(j)} Y_{k-\delta-1} \qquad k=0,1,\ldots M \qquad (6.48)$$

and,
$$\underline{\beta}_k^{(j+1)} = \underline{\beta}_k^{(j)} - \Delta_{12}^{(j)} \varepsilon_{k-\delta-1}^{(j)} (- \bar{I}_{k-\delta-1}) \quad k=1,2,\ldots N \qquad (6.49)$$

In the above expressions we have taken into account the delay in the $\underline{\alpha}, \underline{\beta}$ adjustment processes. Δ_{12} and Δ_{11} are very small constants, which determine the rate of convergence of the multipliers and the excess noise due to misadjustment. The " δ " units of delay are caused by the VA which is usually assumed to be between 8 and 18 symbol intervals.

The noise sequence $\{ n_k \}$ at the input to the VA detector consisys of:

(i) WGN passing through the RDFE,

(ii) Excess noise due to random fluctuations in the multiplier coefficients after steady state,

and, (iii) any ISI other than the desired controlled value.

Usually the excess noise due to random fluctuations of the multipliers is negligible compared to the value of minimum MSE, $J(\underline{\alpha},\underline{\beta},\underline{\lambda})$. Therefore, we can split the noise at the RDFE output into two components, that is,

$$\varepsilon_k = v_k + n_k \tag{6.50}$$

where n_k is the zero mean WGN with covariance function [6-10]

$$E\left[n_p n_q\right] = \sigma_n^2 \sum_{i=0}^{M} \alpha_{0i}\ \alpha_{0i+q+p} \tag{6.51}$$

(σ_n^2 =0.001 corresponding to 34 dB SNR)

and zero mean variable v_k is the residual ISI, the probability distribution function of which is unknown. However,

$$|v_k| \leq A|u_k| \tag{6.52}$$

where $$u_k = \sum_{i=0}^{M} \alpha_{0i}\ \rho_{k-i} - \beta_{0k} - g_k \tag{6.53}$$

and A is the peak value taken by $\{I_k\}$. Also, from [6-10]

$$E\left[v_p\ v_q\right] = \sigma_I^2 \sum_k u_k\ u_{k+q-p} \tag{6.54}$$

(σ^2_I = 1/4 for binary 0 or 1 transmission)

The upper bound on the prob. of symbol error of the VA can be evaluated in accordance with Eqs. (6.21), (6.22), (6.35), (6.51-54). However, in the present case, the bound will only be an estimate since the effects of error due to incorrect decision in the feedback path have been assumed negligible.

The stability and convergence to optimum multiplier coefficients of the iterative process defined by Eq. (6.48) and (6.49) is:

$$\left.\begin{array}{l} 0 < \Delta_{11} < 2/(2\delta+1)\lambda_{max} \\ 0 < \Delta_{12} < 2/(2\delta+1)\lambda_{max} \end{array}\right]$$

δ =0,1,....20.
where λ_{max} is the max. eigenvalue of (6.43)

6.4 AN ADAPTIVE RECURSIVE DIGITAL FILTER RECEIVER (# 2) USING THE VA AND ADAPTIVE DESIRED IMPULSE RESPONSE FILTER:

In the receiver # 1,we made no effort to optimize the desired truncated response and thereby failed to make adjustment of DIR filter adaptive. Since the channel pulse response is not usually known prior to the start of transmission , therefore, we consider a more practical receiver in which $\{\alpha\}$,$\{\beta\}$, and $\{g\}$ are adjusted iteratively.

An adaptive RDFE receiver (# 2) using the VA is shown in Fig. (6.10). The received sequence $\{Y_k\}$ feeds a RDF whose function is to shorten the overall impulse response length. The output of this filter feeds the VA which detects the information sequence. The VA makes decisions on the assumptions that the DIR $\{g_i\}$ is the actual overall channel response. The value of L_1 (the length of the DIR NRDF) is much less than L. (Which we have maintained very close to the actual channel response.) We have selected L_1 to make acceptable the complexity of the VA while taking a small penalty in the preprocessing.

An adaptive RDF is shown in Fig. (6.11). The error signal is formed by feeding the estimated information sequence sequence $\{\tilde{I}_{k-\delta}\}$ through the NRDF representing the desired channel impulse response [Fig. (6-10)] . This forms the desired truncated channel received sequence $\{B_{k-\delta}\}$ which is then compared with a delayed version of the actŭal prefilter (RDF) output to form an error sequence. It is this error which we intend to minimize since it is a sum of the additive noise, and the difference between the desired and actual overall impulse response.

The output (equalizer) symbol from RDFE Fig. (6.5) is given by

$$\hat{I}_k = \sum_{i=0}^{M} \alpha_i Y_{k-i} - \sum_{l=1}^{N} \beta_l \hat{I}_{k-l} \qquad (6.55)$$

From Fig. (6.12), the desired truncated received sequence is given by

$$B_{k-\delta} = \sum_{i_1=0}^{L_1-1} g_{i_1} \tilde{I}_{k-\delta-i_1} \qquad (6.56)$$

The error in the equalized symbol, Fig. (6.10)

$$\varepsilon_{k-\delta} = (\hat{I}_{k-\delta} - B_{k-\delta}) \qquad (6.57)$$

Therefore the MSE is defined by

$$J (\underline{\alpha}, \underline{\beta}, \underline{g}) \overset{\Delta}{=} E \left[\varepsilon^2_{k-\delta}\right]$$

$$= E \left[\hat{I}_{k-\delta} - \sum_{i_1=0}^{L_1} g_{i_1} \tilde{I}_{k-\delta-i_1}\right]^2 \qquad (6.58)$$

The mean-square error $J (\underline{\alpha}, \underline{\beta}, \underline{g})$ is minimized when

$$\frac{\partial (\underline{\alpha}, \underline{\beta}, \underline{g})}{\partial \underline{\alpha}_k} = 0$$

$$\frac{\partial (\underline{\alpha}, \underline{\beta}, \underline{g})}{\partial \underline{\beta}_k} = 0 \qquad (6.59)$$

and, $$\frac{\partial (\underline{\alpha}, \underline{\beta}, \underline{g})}{\partial \underline{g}_k} = 0$$

Adaptive RDFE multiplier coefficients are adjusted iteratively by using the following relationships:

$$\underline{\alpha}_k^{(j+1)} = \underline{\alpha}_k^{(J)} - \Delta_{13}\, \varepsilon_{k-\delta} \left[Y_{k-\delta-i} - \sum_{l=1}^{N} \beta_l \frac{\partial \hat{I}_{k-\delta-l}}{\partial \alpha_k}\right] \qquad (6.60)$$

$$k=0,1, \ldots\ldots M$$

$$\underline{\beta}_k^{(j+1)} = \beta_k^{(j)} - \Delta_{14}\, \varepsilon_{k-\delta} \left[-\hat{I}_{k-\delta-1} - \sum_{l=1}^{N} \beta_l \frac{\partial \hat{I}_{k-\delta-l}}{\partial \beta_k}\right] \qquad (6.61)$$

$$k=1,2,\ldots.N$$

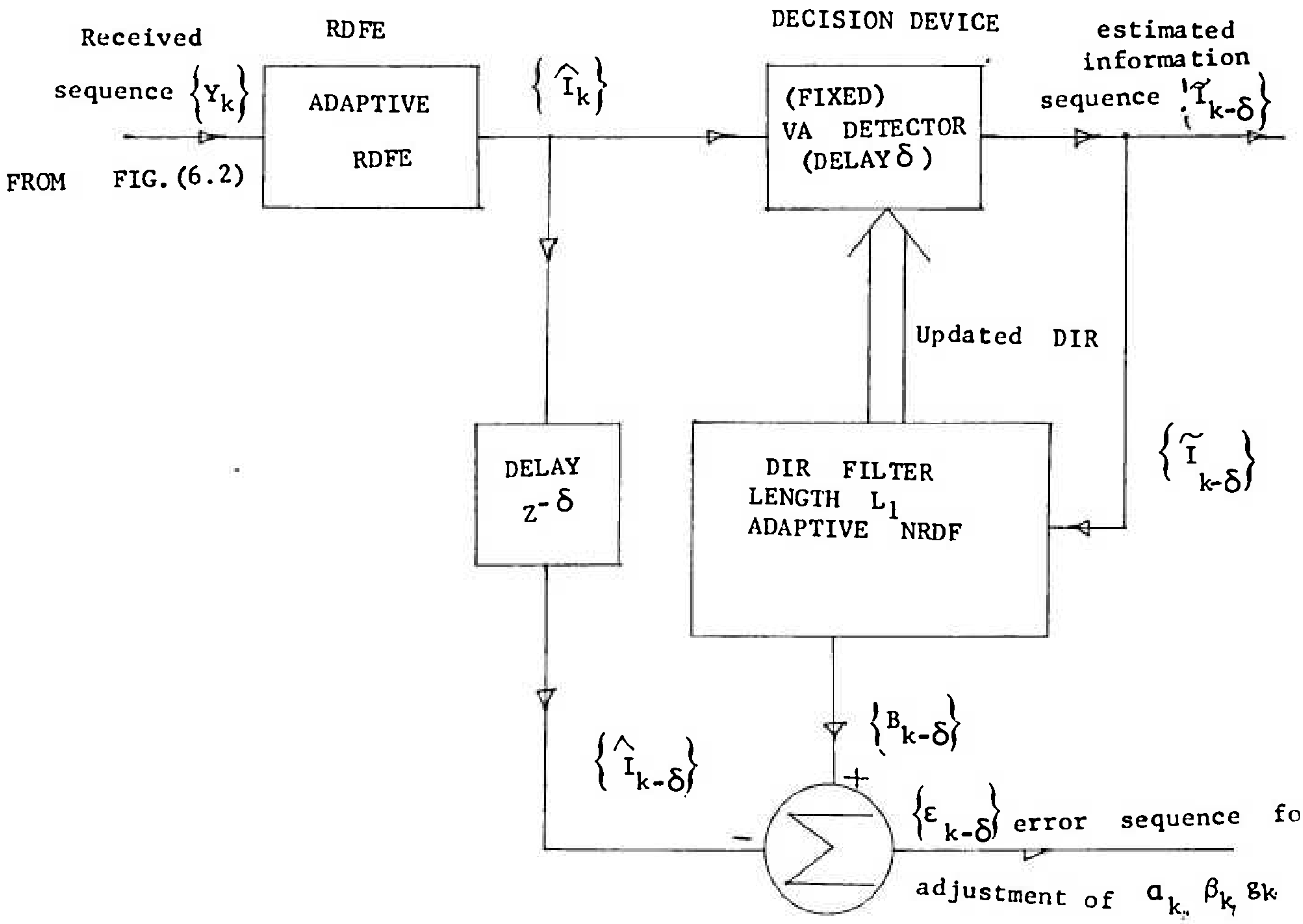
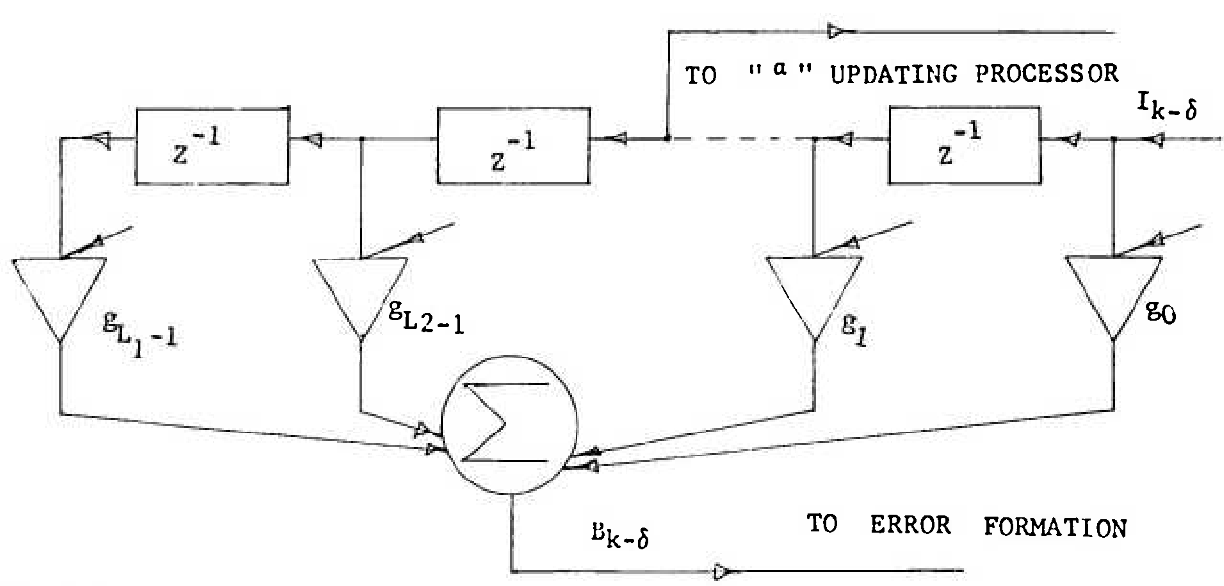


FIG. (6.10) BLOCK DIAGRAM REPRESENTATION OF ADAPTIVE RDF RECEIVER NO.2.



$L_1 \leqslant L$ (The d.t. channel response length.)

FIG. (6.12) AN ADAPTIVE DIR (NRDF TYPE)

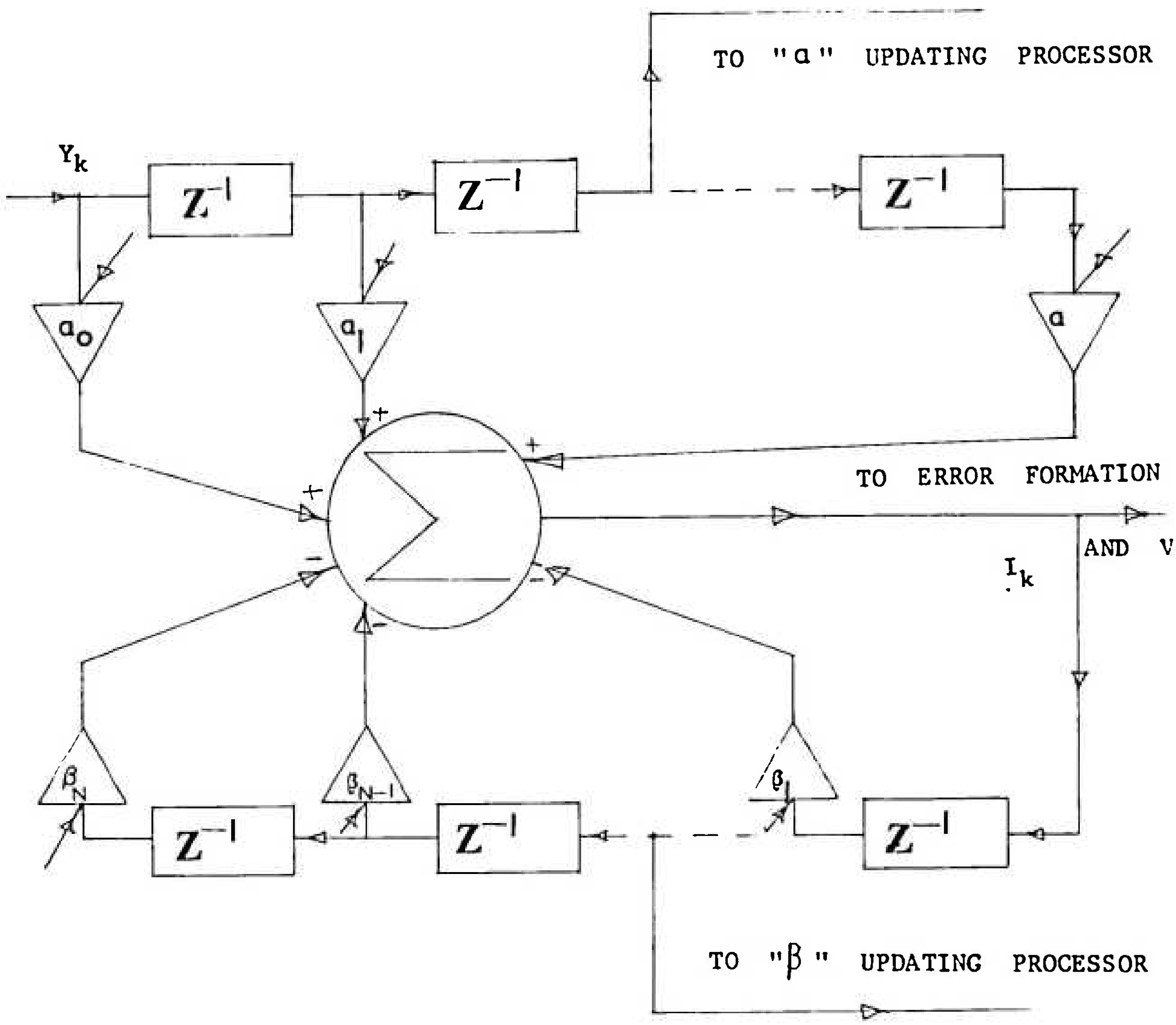


FIG. (6.11) A BASIC RDF AS A PRE-FILTER.

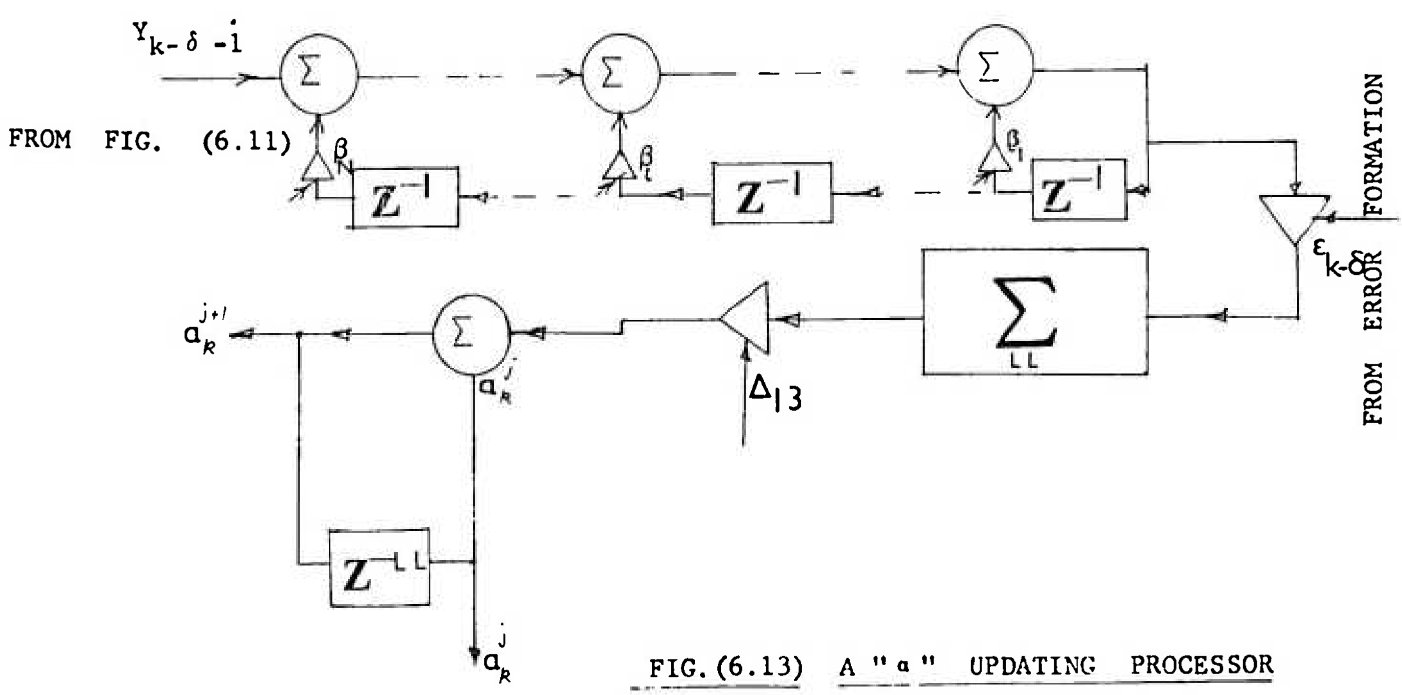


FIG. (6.13) A "a" UPDATING PROCESSOR

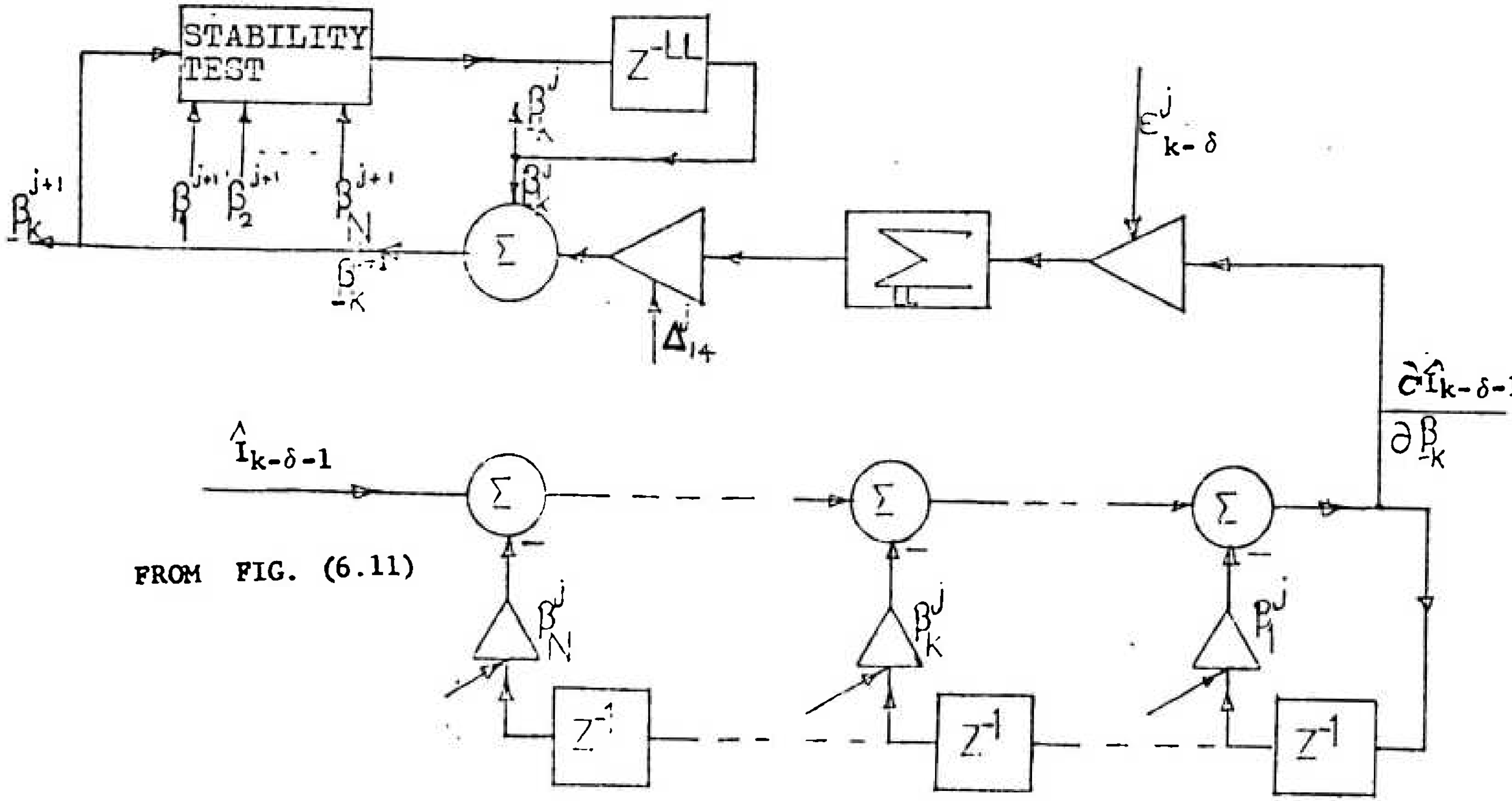


FIG. (6.14) A "β" UPDATING PROCESSOR WITH STABILITY CONTROL.

Feedback multipliers $\{\beta_k\}$ are subject to the constraint

$$|\beta^{(j)}| < 1 \tag{6.62}$$

An "α " updating processor and a "β " updating processor with stability control are shown in Figs. (6.13) & (6.14) respectively. Δ_{13} and Δ_{14} are the adjustment parameters which control the accuracy and the speed of convergence. The algorithm to obtain an iterative solution for $\underline{g}$ is complicated since it has to work under a fixed energy constraint. However, for most practical purpose the unconstrained algorithm:

$$g_k^{(j+1)} = g_k^{(j)} - \Delta_{15}^{(j)} \; \varepsilon_{k-\delta}^{(j)} \; (-\tilde{I}_{k-\delta-i_1}) \tag{6.63}$$

is sufficient. This is a noisy estimate of the required cross-correlation. Δ_{15}, is convergence factor,should never be equal to 1 simply because of the fact that when the noisy estimates are used the algorithm will amplify the noise and therefore will tend to diverge. So Δ_{15} has to be much smaller than 1 .

The lower and upper bound details are somewhat similar to the receiver structure #1.

An exact analysis of the receiver's performance would be very involved, since two nonlinear devices are combined in one framework. Therefore, no effort has been made to take up, analytically, a very difficult task. However, we have developed method to predict the performance numerically. Using several approximations and bounding procedures, we have been able to obtain a lower and upper estimate on the error prob. of the receiver. They are well suited for computer calculations.

6.5 COMPUTER SIMULATION:

Two most difficult discrete time channels (7) and (8) exhibiting in band second order nulls were simulated . The proposed receivers were implemented in the main program for binary signalling system. It simulates an entire communication system consisting of the source, discrete time channel, RDFE loop, the VA and the DIR (NRDF) . The input data, we assumed, consisted of equally likely pseudorandom binary symbols which were transmitted to the unknown (supposed) D.T. channels. The gaussian noise terms were generated using the polar method [6-24]. The channel simply convolves the source sequence with the given discrete time channel characteristic. The VA detector used in the program had a maximum delay of 18 symbol intervals.

6.5.1 RECEIVER STRUCTURE # 1:

The RDFE assumed had M=N=9. Algorithms (6.48) & (6.49) were implemented. The feedback coefficients$\{\beta_k\}$ were constrained to the con-tions of the stability triangle triangle developed in chapter (5). Forney's lower and upper bounds were calculated according to (6.23) & (6.21) respectively. The first RDFE routine makes a tentative decisions on a transmitted symbol. These decisions are fedback to the feedback part of RDFE and to the fixed DIR. The VA performs MLS estimation without any quantization of received signal. In order to avoid degradation due to premature decisions resulting from insufficient storage spaces, each state was allowed to store upto 50 symbols (i.e. a survivor of length 50). The final decisions were made by examining the merging of states. If there was no merge before the allowed 50 symbol storage was exhausted then the decisions were made only on the first half of the stored symbols by choosing the sequence with largest log-likelihood as the detected sequence.

The receiver was switched to the decision-directed mode after the first 400 symbols. The error counts are based on single runs sample sizes of 20,000 for SNR 8 and 9 dB, 100,000 for SNR 10 to 12 dB, and 10^6 for SNR greater than 12 dB respectively. The desired channel equalizer pulse response length in channels (7) and (8) were 7 and 5 respectively. The performances of 35 stage NRDFE; M=N=9 RDFE, were evaluated for channels under consideration. The results are plotted in Figs. (6.15) and (6.16) for channels (7) and (8) repectively.

Although the experience with this simulation is limited, we can draw some important conclusions:

1. First of all,the proposed receiver performs well not only in analysis but also in practice, fulfilling our objective of a compromise between complexity and performance. Over the D.T. channel (7), the receiver suffers a loss of 5 dB relative to the case of no ISI. RDFE suffers even a greater loss. Over the D.T. channel (8) , the receiver # 1 shows about 1.5 dB degradation relative to the case of no ISI.
2. Equally importantly, the analysis agrees very well with the experiment. The performance analysis we developed is remarkably accurate over a wide range of SNR's.

Next, in order to study the effects of tentative decision errors on the VA performance, the simulation program was run with the correct transmitted symbols being fed back into the feedback path. The results are shown by (x) in Figs. (6.15) and (6.16). Also drawn are the performance curves (based on simulation) of the pure RDFE with and without error in the feedback path (*), for the same channels and noise conditions.

3. The errors in the tentative decisions do degrade the final performance more or less, therefore the effects of these errors cannot be ignored in the system design.

6.5.2 RECEIVER STRUCTURE # 2:

In this case we simulated a telephone channel whose impulse response is given in Fig. (6.17). We selected g_0 , g_1 , and g_2 to be the desired impulse response and therefore selected 3 stage adaptive NRDFE in conjunction with the RDFE [M = N =9] and the VA having maximum delay of 18 symbols. The algorithms developed in (6.60) to (6.63) were used for updating $\underline{\alpha}$, $\underline{\beta}$, and the NRDFE multiplier coefficients.

We found that the performance degradation at $Pr(e) = 10^{-3}$ was about 1.5 dB. [Fig. (6.18)]

6.5.3 THE COMPLEXITY OF THE VA DETECTOR:

The complexity of the VA is calculated for D.T. channels (7), (8) and (9) when m = 2 (binary) and m = 4 (for very high speed transmission) for per symbol processing

TABLE (6.2):

For m = 2 or (m = 4)   Delay (D) = 18

	Channel (7)	Channel (8)	Channel (9)
SUBTRACTIONS: (m^{L_1})	128(16,384)	32(1,024)	8(64)
ADDITIONS: (m^{L_1})	128(16,384)	32(1,024)	8(64)
SQUARING: (m^{L_1})	128(16,384)	32(1,024)	8(64)
COMPARISONS: $(m-1)m^{L_1-1}$	64 (12,288)	16(768)	4(32)
STORAGE REQD.: $2(D-L_1+1)m^{L_1-1}$ Bits.	1,536(98,304)	448(7,168)	128(512)

6.6 SUMMARY AND COMMENTS:

(A) In this chapter, we have, for the first time proposed,analysed and simulated a RDFE in conjunction with the VA as a detector and fixed and variable desired impulse response NRDF.

(B) The proposed receiver structures outperformed the NRDFE and the RDFE and can be conveniently implemented digitally.

(C) We have not made use of a Matched filter, as such, some loss in performance is expected. However, it is believed,in practice, they are ordinarily not used, also they offer little gain in performance. But, it is hoped that a Matched filter may be desirable at high data rates because it would eliminate the great sensitivity at these rates to timing and carrier phase.

(D) Theoretically we have shown a relationship between pulse response length and the performance. One may use graph [Fig. (6.3)] as an approximate guideline to compare the performances of various length channels.

(E) We have assumed throughout that the noise is Not Correlated. A more realistic approach, in our opinion, will be to consider the effects of noise correlation. However, some studies, to this effect , have been reported [6-10, 6-14,15]. It has been found that the noise correlation does not affect the performance more than a few dB and it clearly does not affect at the place in the performance curve at which the performance begins to degrade seriously.

(F) The optimization criterion we used , i.e. the minimization of the MSE with respect to the parameters $\underline{\alpha}$, $\underline{\beta}$, and $\underline{g}$; duration, relative delay of the DIR being fixed- is admittedly somewhat ad hoc. Nevertheless, it could safely be said that in view of the performance estimates for the sample channel the use of the VA in conjunction with RDFE (as suggested) appear attractive for high speed data transmission relative to other schemes.

(G) The MLSE technique using the VA suffers a disadvantage that the computational burden on the VA grows exponentially with the number L of the interfering paths and therefore the storage required becomes prohibitive. However, in our case with 3, 5, and 7 samples the VA processor was less burdened.

Also, the VA has been incorporated only in modems designed for the case when L is relatively small, or for example, in the detection of Partial response signals where ISI is purposely introduced to obtain a desired signal spectrum. It is hoped that the Bell Laboratories, Holmdel,New Jersey is engaged in hardware designs of modems incorporating the VA for telephone channels [6-25].

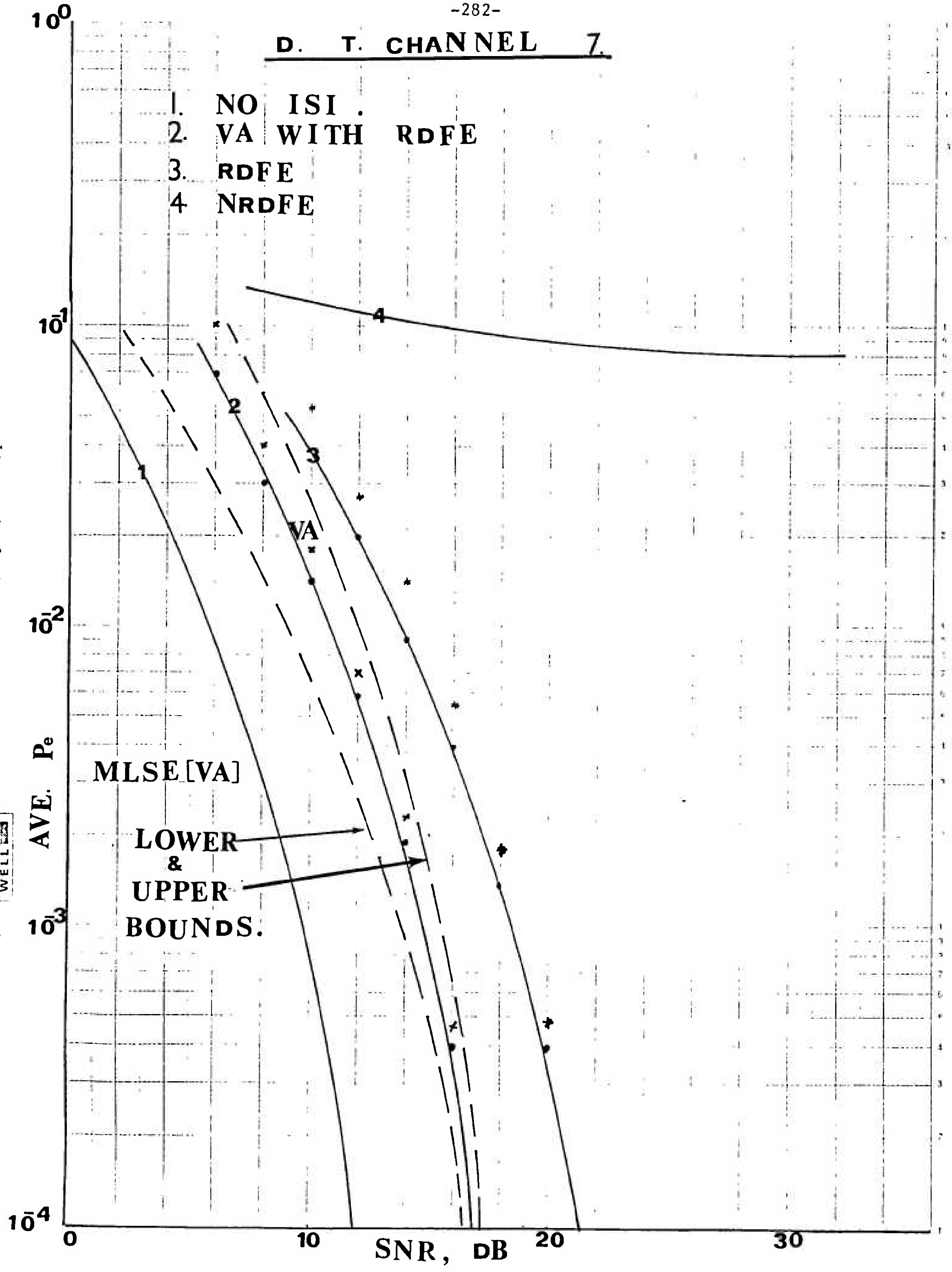


FIG. (6.15) PERFORMANCE OF ADAPTIVE RDFE RECEIVER NO.1 WITH VA AND

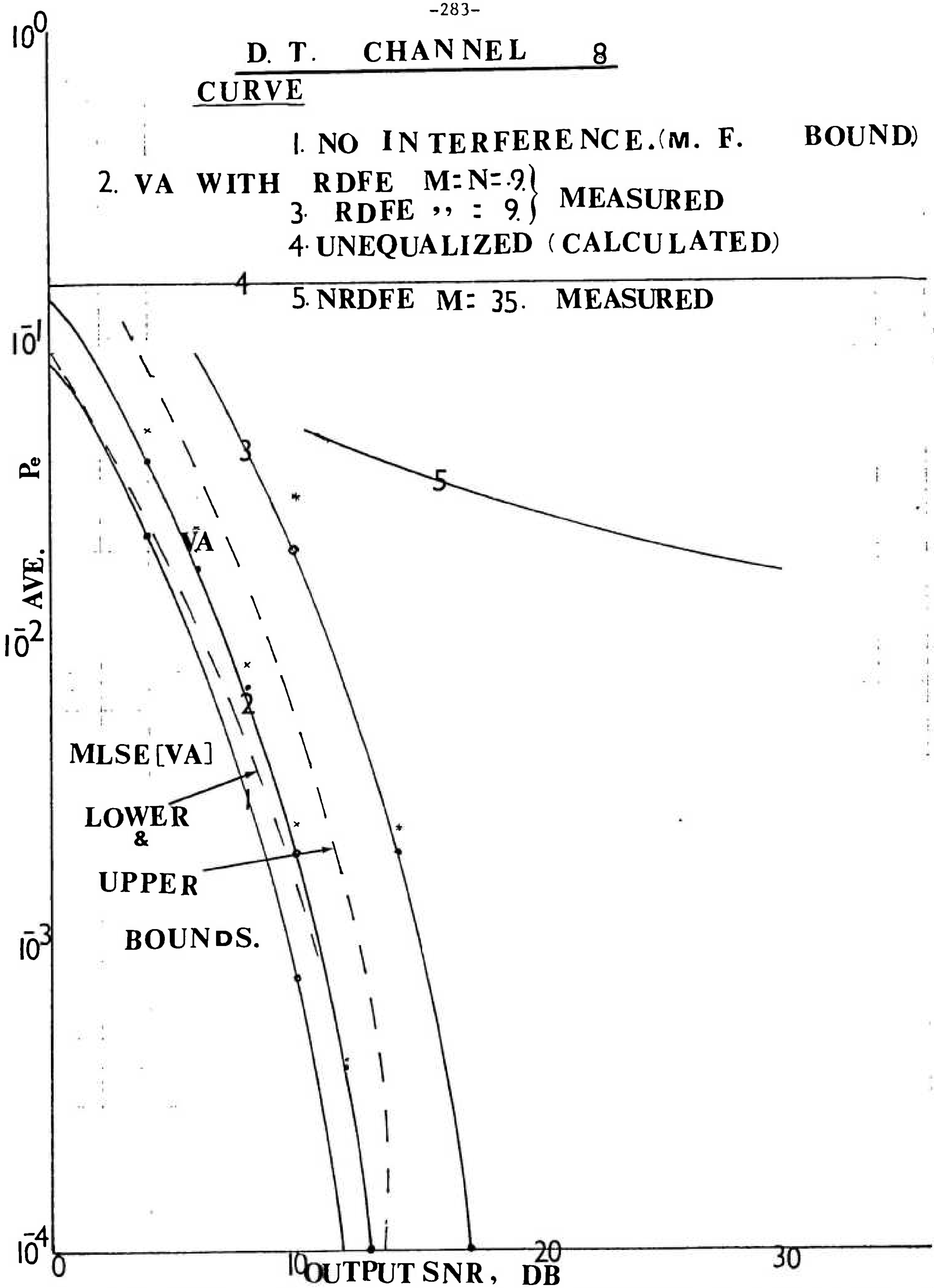


FIG. (6.16) PERFORMANCE OF ADAPTIVE RDFE RECEIVER NO. 2 WITH VA AND FIXED DIR (BINARY SIGNALLING).

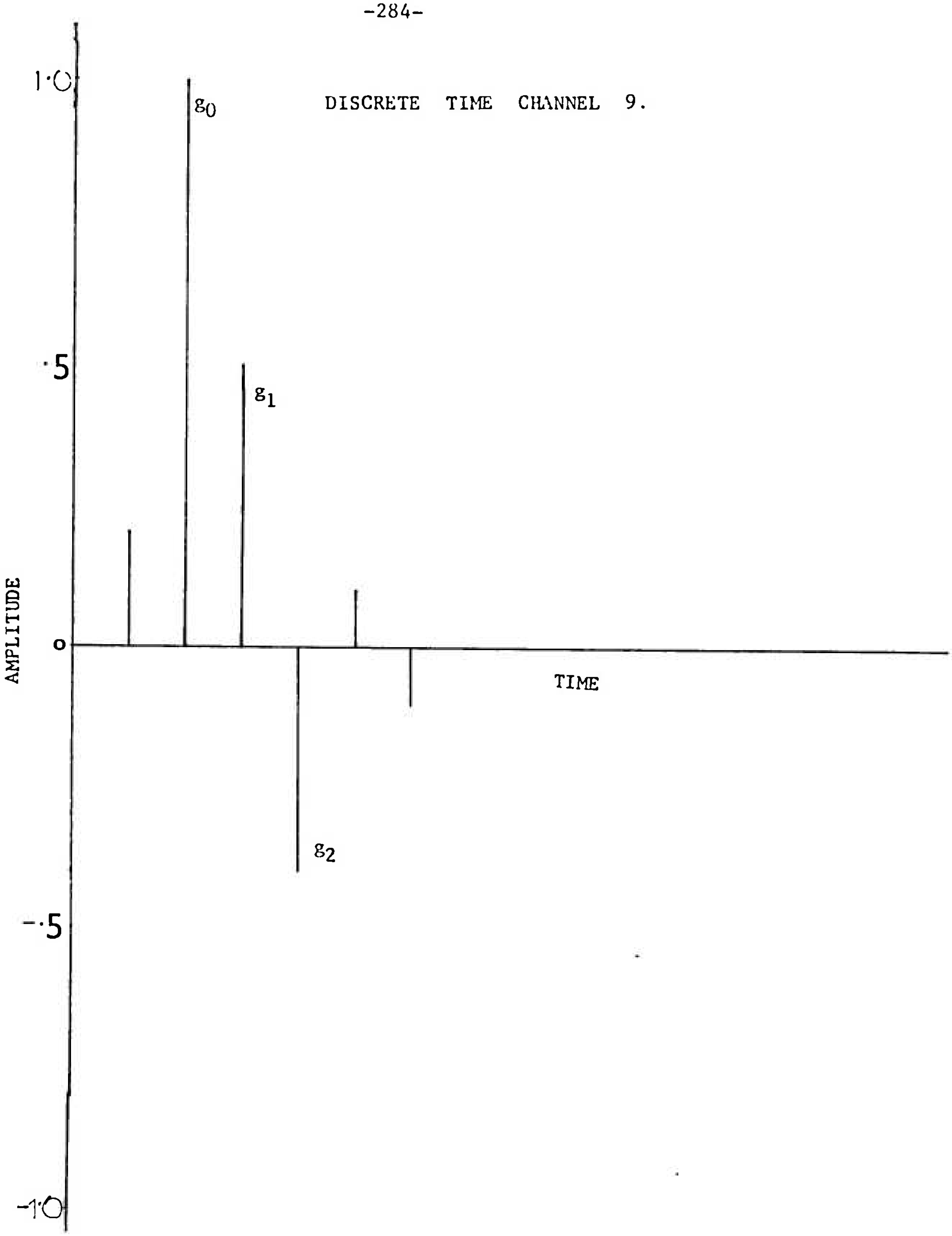


FIG. (6.17) IMPULSE RESPONSE OF A TELEPHONE CHANNEL.

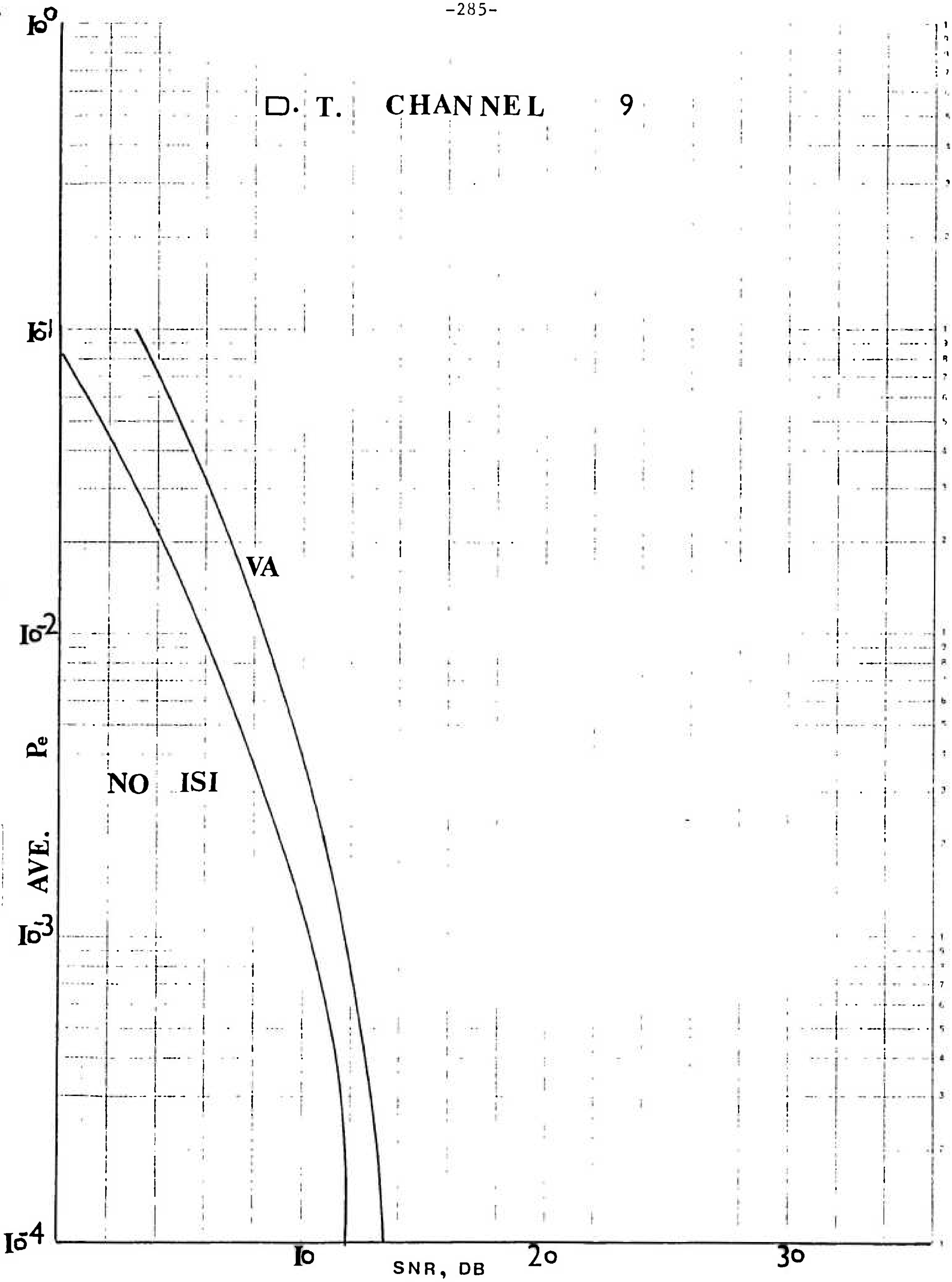


FIG. (6.18) PERFORMANCE OF PROPOSED RECEIVER STRUCTURE NO. 2.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

" And suppose we solve all the problems it presents. What happens ? We end up with more problems than we started with. Because that's the way problems propagate their species. A problem left to itself dries up or goes rotten. But fertilize a problem with a solution - you'll hatch out dozens".

N. F. SIMPSON, (A Resounding Tinkle)

7.1 CONCLUSIONS:

This research led to the designs, analysis and estimation of

(i) fast generalized non recursive digital filter equalizers [Chapters (3) and (4)]

(ii) recursive digital filter equalizers [Chapter (5)], and

(iii) use of direct form RDFE in conjunction with the VA [Chapter (6)]

In Chapter (1) we have developed the discrete time channel model. Chapter (2) deals with the well established design techniques for NRDFE to-date. Also, it contains the summary of mathematics involved in the design of equalizers using the mean square error criterion. The works outlined in this chapter are meant for the conventional NRDFE only.

Chapter (3), we have designed and developed a fast initialization algorithm employing a digital computer. This equalizer is initialized with an isolated pulse provided the SNR is not too low. We have equalized the British Public Switched Telephone Network (PSTN) channels successfully. We have equalized two channels, D. T. channel # 1 (a theoretical channel) and D. T. channel # 2 (a practical channel) with the help of the algorithm developed here. In addition, we studied the effects of three parameters involved; delay, shift register length, number of filter sections, the summary of which is outlined in section (3.9). The use of Moore Penrose Pseudoinverse (MPPI) and an orthogonal Hadamard matrix have been exploited. Convergence properties have been outlined.

In Chapter (4), we have extended the design developed in Ch. (3) to decision directed, on line adaptive mode of operation.

We studied the effects of three parameters involved, the summary and comments of which are outlined in section (4.5). Convergence properties have been outlined. Since the equalizer is working in decision directed mode,threfore, the use of diagnostic ensures reinitialization in case of excess error.

Chapter (5), we have designed, evaluated independently, six different recursive digital filter equalizers on channels (3), (4), (5), (6), (7) and (8). D.T. channels (3), (4) and (5) are severely phase distorted channels whereas the remaining channels possess severe amplitude distortion. We have studied the pole-zero constellation equalization and then the optimization techniques based upon the individual coefficient adjustment algorithms. A comparison based upon the number of multiplications, stability and the coefficient quantization have been suggested with caution! By Monte - Carlo simulation we estimated the probability of error P_e at various SNR. We found little improvement over NRDFE in case of severe phase distorted channels (3), (4), and (5), whereas significant improvement was obtained for severe amplitude distorted channels (6), (7) and (8). Summary of the findings is given in section (5.11). Results dictate in the favour of RDFE over certain channels.

In Chapter (6), we have used RDFE in conjunction with the VA and a desired inpulse response NRDFE. Two receiver structures have been analysed on ideal and most practical (unknown) discrete time channels These designs are based upon the availability of three channel samples, 3, 5 and 7,from channels (9), (8) and (7) respectively.
Summary of the results is given in section (6.6). The complexity of the VA processor / symbol

processing is shown in table (6.2) Section (6.5.3) . We have developed an adaptive asymptotically optimum, Viterbi Algorithm receiver structure that performs effectively as well over an unknown channel as any receiver structure can perform even over a known channel. Since these receiver structures are realizable, we hope it should provide a practical solution to the ISI problem.

7.2 RECOMMENDATIONS FOR FUTURE RESEARCH:

There are a number of interesting areas which could provide motivation for further investigation:

(1) Particularly germane area seems the equalizers (NRDFE, RDFE, MLSE) on time varying channels and the effect of the convergence factor Δ 's on this tracking.

(2) A theoretical study should be made for finding efficient algorithms for finding eigenvalues and eigenvectors and therefore to reduce the amount of computation in order to achieve faster initialization. (This could be done by finding an exact (or near exact) guess for the coefficients in every equalizer structures).

(3) Most important area is to investigate the effects of short word lengths, truncation, and round-off errors. Some work for NRDFE has been reported [7-1, 2] . However, theoretical [7-1] and simulation works [7-2, 3] must be done using MINI-COMPUTERS (16 bits), MICRO-COMPUTERS and MICRO-PROCESSORS (8 bits) and all these works should be extended to the six RDFE structures. Direct inversion of the channel covariance matrix should be done using microprocessors available. Stability of RDFE and the MLSE should be considered.

(4) There should be some study in the possibility of further simplifying the online adaptive algorithm (NRDFE, RDFE, MLSE) by using only polarity information of some of the variables.

(5) The theory which we have described concerns the transmission of a single data sequence. There are many situations in which several data sequences are transmitted in parallel either over a single channel as with multiplexed systems, or over

separate but interacting channels such as multipair cable a study is recommended for this as well.

(6) A study should be made for some modern communication media for instance, satellite links, where channels are non-linear. Can these equalizers cope with the non-linear channels?

(7) The most practical use of an RDFE seems as an adaptive noise canceller for various forms of periodic interference in electrocardiography, in speech signals, tape, turn-table, and in antenna array. A study with a RDFE and an MLSE would be most useful.

(8) Implementation of an RDFE using modulo-arithmetic is recommended. Ref. [7-4] can provide possible guidelines.

(9) Development of simplified or approximate form for the Viterbi Algorithm to enhance VA receiver structure economic feasibility. We have made an adhoc attempt. Low complexity decoder for channels with ISI has been reported [7-5] .

(10) Development of faster convergence algorithms (e.g. Southwell relaxation: Richardson-overrelaxation; second-order gradient, conjugate-gradient etc.) methods for RDFE/ NRDFE should be studied to cope with the problem of more quickly varying channels.

(11) A study should be made at d^2_{min} instead of just MSE since d^2_{min} really controls the P_e for the VA.

(12) The choice of decoding delay of VA in the presence of ISI and the dynamics of the adjustment algorithm should be considered in detail.

(13) The specific implementation of the VA receivers using RDFE will be another interesting topic.

(14) Making the reduced state algorithm adaptive in an efficient manner, especially for large channel constraint length, will be another worthwhile topic to investigate.

APPENDIX - A

MATHEMATICAL PRELIMINARIES

A.1 INTRODUCTION:

This appendix scattered in various subsections is going to provide a brief outline of the topics from linear algebra and from operator theory. A detailed description of the matrices used in this work is also provided to explain the newly developed subject of Moore-Penrose pseudoinverse (MPPI). This is followed by the separate sections on the z-transform and statistics including the Monte-Carlo simulation technique. Finally, certain supplementary results are derived. However, for detailed discussions on these topics a list of references have been provided.

A.2 LINEAR VECTOR SPACES AND OPERATORS:

A mathematical field F consists of a set of elements called scalars which satisfy a particular set of axioms[A-1, (pp. 2-3).] The only fields used herein are the sets of all real and complex numbers. A linear vector space over a field F is denoted (U,F) or just U and F The elements of U, called vectors, also satisfy a certain set of axioms [A-1 , p.5] and are related to each other and to the sc lar by the operations of vector addition and scalar multiplication. Linear vector spaces are also known as linear spaces or simply vector spaces. Vector spaces are called real or complex, according to whether the field is real or complex, respectively. A set of vectors ($\underline{x}_1$, $\underline{x}_2$, ... $\underline{x}_k$) in U span U if every vector $\underline{y}$ in U can be expressed as a linear combination of the set i.e. if there exists a set of scalars (α_1, α_2, ...α_k) such that

$$\underline{y} = \sum_{k=1}^{K} \alpha_k \underline{x}_k \tag{1}$$

A set { $\underline{x}_m$ } is linearly independent if the only way that their linear combination can be made to equal zero is for all of the scalar multiplier to be zero. Any linearly independent set that spans Uis called a basis of U , and the number of elements in any basis is called the dimension of U. Only vector spaces of finite dimension will be considered by us.

Let (U,F) and (V,F) be vector spaces with α,β being arbitrary scalars in F and $\underline{W}$, $\underline{X}$, $\underline{y}$ arbitrary vectors in U. Suppose that to each element of U there is assigned a unique element of V, then the collection of these assignments is called a map T from U into V. Maps are also known as functions , transformations or operators, and we shall use these terms interchangeably.

The identity operator I is defined by $I\underline{x} = \underline{x}$, and a map T^{-1} satisfying the relation $T^{-1}\,T = T\,T^{-1} = I$ is called the inverse of T. The inverse operator may not exist; however, when it does then T is said to be invertible. The set of scalars λ for which the map $(T - \lambda\, I)$ is not invertible is called the spectrum of T and will be denoted $\sigma(T)$. When T satisfies the relation

$$T(\alpha\underline{x} + \beta\underline{y}) = \alpha T(\underline{x}) + \beta T(\underline{y}) \qquad (2)$$

it is known as a linear operator. It is interesting to note that a field is a vector space, and also the set of all linear operators from U into V forms a vector space in which each linear operator is itself a vector space [A-2, p.85] .

There are many important maps between vector spaces. An inner of scalar product is a scalar-valued function of two vectors denoted $<\underline{x}, \underline{y}>$, which is defined by the following four axioms:

(1) $<\underline{y}, \underline{x}> = <\underline{x}, \underline{y}>^*$

(2) $<\alpha\underline{x} + \beta\underline{y}, \underline{W}> = \alpha<\underline{x}, \underline{W}> + \beta<\underline{y}, \underline{W}>$

(3) $<\underline{x}, \underline{x}> \geq 0$ with equality only when $\underline{x} = \underline{0}$

(4) $<\alpha\underline{x}, \underline{y}> = \alpha<\underline{x}, \underline{y}>$

A vector space with an inner- product defined on it is called an inner product space. The inner product determines the relative directions of vectors, and two vectors are said to be orthogonal if $<\underline{x}, \underline{y}> = 0$. The notion of distance in vector spaces is indicated by a non-negative scalar valued map called a norm, $\|\underline{x}\|$, which has the following proper ties.

$$\|\alpha\underline{x}\| = |\alpha|\;\|\underline{x}\|$$

$\|\underline{x} + \underline{y}\| \leq \|\underline{x}\| \cdot \|\underline{y}\|$ (the triangle inequality)

$\|\underline{x}\| \geq 0$ with equality only when $\underline{x} = \underline{0}$

$|<\underline{x}, \underline{y}>| \leq \|\underline{x}\|\;\|\underline{y}\|$ (Schwartz's inequality)

A set of vectors $\{\underline{x}_m\}$ is an orthonormal set if $||\underline{x}_m|| = 1$ for every m and it is an orthogonal set. A vector space with a norm defined on it is a normed vector space. For the finite dimensional vector spaces considered herein it will be convenient to define the inner product of two vectors, say $\underline{x} = (x_1, x_2, \dots x_k)^T$ and $\underline{y} = (y_1, y_2 \dots y_k)^T$ by

$$<\underline{x}, \underline{y}> \triangleq \sum_{k=0}^{K-1} x_k y_k^* = \underline{x}^T \underline{y}^*$$

which then defines a norm of $\underline{x}$ by

$$||\underline{x}|| \triangleq (<\underline{x}, \underline{x}>)^{1/2}$$

A map T, not necessarily linear, is called convex if for every value of α satisfying $0 \leq \alpha \leq 1$, it has the following property:

$$T([1-\alpha]\underline{x} + \alpha\underline{y}) \leq (1-\alpha) T(\underline{x}) + \alpha T(\underline{y})$$

For the strict inequality case, map T is strictly convex.

For every linear operator T on a space U there exists another operator T^A called the adjoint of T defined by

$$< T(\underline{x}), \underline{y} > = < \underline{x}, T^A(\underline{y}) >$$

If $T = T^A$ then T is called a self-adjoint operator, and if $TT^A = T^A T$ then T is a normal operator. A projection operator is one which is both self adjoint and idempotent, i.e. TT=T. Any map for which the adjoint equals the inverse is a unitary operator.

There is an important notion, which deals with the geometric interpretation of a map T form U to V. The set of elements $\underline{x}$ in U such that $T\underline{x} = \underline{0}$ is called the null space, or sometimes the kernel of T, and is denoted by N(T). The set of elements $\underline{y}$ in V which can be written $\underline{y} = T\,\underline{x}$ for some $\underline{x}$ in U is called the range space, or image, of T and is denoted by R(T). The null space of T is related to the range space of its adjoint T^A in the following way: every element in N(T) is orthogonal to every element of $R(T^A)$. We can show this by noting that $\underline{x}$ belongs to N(T) if and only if $< \underline{y}, T\,\underline{x} > = \underline{0}$ for any $\underline{y}$ in U, and since $< \underline{y}, T\,\underline{x} > = < T^A\,\underline{x}, \underline{y} >$ then $\underline{x}$ is orthogonal to every $\underline{W}$ in U such that $\underline{W} = T^A\,\underline{y}$ for some $\underline{y}$. But this set of $\underline{W}$'s is just the range space of T^A hence the desired result is proven.

We can also prove the following important consequence that $R(T^A)$ and $R(T^AT)$ are identical. We know that if $\underline{x}$ is in $N(T^A)$ then $(TT^A)\underline{x} = T(T^A\underline{x}) = T(\underline{0}) = \underline{0}$ so $\underline{x}$ is also in $N(T^AT)$. Coversely, if $\underline{x}$ is in $N(T^AT)$ then

$$||T\underline{x}||^2 = < T\underline{x}, T\underline{x} > = < \underline{x}, T^AT\underline{x} > = < \underline{x}, \underline{0} > = \underline{0}$$

so that $T\underline{x} = \underline{0}$ and $\underline{x}$ is in N(T). We complete this proof by noting that every element of $R(T^A)$ and $R(T^AT)$ is orthogonal to every element of N(T) and $N(T^AT)$, respectively. Hence $R(T^A)$ and $R(T^AT)$ must be identical.

A.3 MATRICES:

An MxN matrix $\underline{A}$ is a rectangular array of elements having M-rows and N-columns; the element in row m and column n is denoted by a_{mn} and the entire matrix $\underline{A}$ is sometimes denoted by $\lfloor a_{mn} \rfloor$. By defining the sum of two matrices and the multiplication of a matrix by a scalar in the usual way, it can be shown that the set of all MxN matrices, with elements in the same field F, forms a vector space [A-3, p.167]. The way that matrices are used to represent linear operators is made precise by the following representation theorem [A-1, p.15].

Let (U,F) and (V,F) be real or complex vector spaces with bases $\{\underline{u}_1, \underline{u}_2, \ldots \underline{u}_N\}$ and $\{\underline{v}_1, \underline{v}_2, \ldots \underline{v}_M\}$ respectively, and let T be a linear map from $U \to V$. Then, with respect to these bases, T is represented by the MxN matrix $\underline{A} = [a_{mn}]$ where the elements of column i of $\underline{A}$ are the components of $T\,U_i$ with respect to the basis $\{\underline{v}_m\}$. When M=N, then $\underline{A}$ is a square matrix and U is identical to V, i.e., T maps U into itself. If M=N and $\{\tilde{\underline{u}}_1, \tilde{\underline{u}}_2, \ldots\ldots \tilde{\underline{u}}_N\}$ is another basis for U in which $\tilde{\underline{A}}$ represents T, then there exists a nonsingular square matrix $\underline{P}$ that relates these representations according to $\tilde{\underline{A}} = \underline{P}^{-1}\underline{A}\,\underline{P}$. Such a mapping is called a similarity transformation, and the matrices $\underline{A}$ and $\tilde{\underline{A}}$ are said to be similar. We therefore say that the similar matrices represent the same operator in different bases.

For a square matrix $\underline{A}$ of order N the spectrum $\sigma[\underline{A}]$ is that set of scalars λ for which $(\underline{A} - \lambda\,\underline{I})$ fails to be invertible, or equivalently for which the determinant $\det(\underline{A} - \lambda\,\underline{I}) = 0$. This equation is a polynomial in λ of order N, so that $\sigma[\underline{A}]$ contains exactly N elements which may not be distinct. These elements are called the eigenvalues of $\underline{A}$, or sometimes called proper or characteristic values. The spectrum is a property of the abstract operator T and does not depend on the particular representation, e.g., $\underline{A}$ or $\tilde{\underline{A}}$. This can be seen by noting that

$$\tilde{\underline{A}} = \underline{P}^{-1}\,\underline{A}\;\underline{P} \tag{3}$$

therefore
$$\begin{aligned}\det(\tilde{\underline{A}} - \lambda\,\underline{I}) &= \det(\underline{P}^{-1}\underline{A}\,\underline{P} - \lambda\,\underline{P}^{-1}\underline{P}) \\ &= \det(\underline{P}^{-1})\det(\underline{P})\det(\underline{A} - \lambda\,\underline{I}) \\ &= \det(\underline{A} - \lambda\,\underline{I})\end{aligned}$$

Any nonzero vector $\underline{b}$ such that $(\underline{A} - \lambda\,\underline{I})\,\underline{b} = \underline{0}$ with λ in $\sigma[A]$, is called an eigenvector of $\underline{A}$ corresponding to the eigenvalue λ.

We shall now elaborate upon some special matrices on the basis of various operators defined in (A.2). The square matrix representing the adjoint T^A of an abstract linear transformation T on vector space U is the complex conjugate transpose of $\underline{A}$ denoted by $\underline{A}^H$ as can be seen from the following

$$< \underline{x}, \underline{A}\,\underline{y} > = \underline{x}^T (\underline{A}\,\underline{y})^* = \underline{x}^T \underline{A}^* \underline{y}^* = < \underline{A}^H \underline{x}, \underline{y}>$$

(note T means transpose)

when T is self adjoint, $\underline{A} = \underline{A}^H$, $< \underline{x}, \underline{A}\,\underline{y} > = < \underline{A}\,\underline{x}, \underline{y}>$ and $\underline{A}$ is called a Hermitian matrix. If the vector space U is real rather than complex then $\underline{A}^H = \underline{A}^T$ and T being self-adjoint implies $\underline{A} = \underline{A}^T$. Such a matrix is called real symmetric.

The eigenvalues of an NxN hermetian matrix $\underline{A}$ are always real, and there always exists a set $\{\underline{p}_m\}$ of N linearly independent eigenvectors which are orthogonal [A-4 , (p 321, theorem 10.10 (ii))] . In fact, since when $\underline{p}$ is an eigenvector then $\underline{p}/||\,\underline{p}\,||$ is too, then the set $\{\underline{p}_m\}$ can always be taken orthonormal. Writing the eigenvector equation $\underline{A}\,\underline{p}_m = \lambda_m \underline{p}_m$ for m=o, 2, ... N-1in matrix forms gives A

$\underline{A}\,[\,\underline{p}_1, \underline{p}_2, \;..\; \underline{p}_N] = [\,\underline{p}_1, \underline{p}_2, \ldots \underline{p}_N\,]\ \mathrm{diag}(\lambda_1, \lambda_2, \ldots \lambda_N)$ or, more concisely in terms of the matrices $\underline{P}$ and $\underline{\Lambda}$,

$$\underline{A}\,\underline{P} = \underline{P}\,\underline{\Lambda}$$

Furthermore, since $<\underline{p}_m, \underline{p}_n> = \delta_{mn}$ and a similar relation exists for the rows of $\underline{P}$ [A-4, (p 333)] , then $\underline{P}^{-1} = \underline{P}^H$ which implies $\underline{P}$ is a unitary matrix. Cosequently the similarity transformation $\underline{\Lambda} = \underline{P}^{-1}\,\underline{A}\,\underline{P}$ that diagonises $\underline{A}$ becomes

$$\underline{\Lambda} = \underline{P}^{T}\,\underline{A}\,\underline{P} \qquad (4)$$

which is called a unitary transformation.

Hermetian matrices can be easily decomposed into their spectral representations by using unitary transformations in the following way,

$$\underline{A} = \underline{P}\,\underline{\Lambda}\,\underline{P}^H = \sum_{m=0}^{N-1} \lambda_m \,\underline{p}_m\, \underline{p}_m^H \qquad (5)$$

When the vector space is real and therefore $\underline{A}$ is real symmetric then the complex conjugate transpose is simply the transpose.

A projection operator is represented by a projection matrix $\underline{A}$ only when $\underline{A}$ is hermitian and possesses idempotent property $\underline{A}^2 = \underline{A}$. Berberian [A-2, p.(163)] has shown that all eigenvalues of a projection matrix are either zero or unity, and that the eigenvectors of unity eigenvalues span the subspace into which the operator projects. Thus any projection operator is similar to the diagonal matrix diag($\underline{I}$ / $\underline{0}$) = diag(1, 1, ... 1, 0, ... 0) where the square matrices $\underline{I}$ and $\underline{0}$ are of orders equal to the number of unity and zero eigenvalues respectively. A quadratic form in real variables x_m associated with a real matrix $\underline{A}$, is a scalar quantity consisting of a sum of multiples of products and squares of the variables:

$$F(x_1, x_2, \dots x_N) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} a_{ij}\, x_i\, x_j \,.$$

$$= a_{11}x_1^2 + (a_{12} + a_{21})\, x_1x_2 + a_{22}\, x_2^2 + \dots$$

Now, if we introduce the real symmetric matrices

$$\underline{x} = \left[x_i \right] \qquad \underline{A} = \left[a_{ij} \right]$$

the quadratic form can be written as

$$F(\underline{x}) = \langle \underline{x}, \underline{A}\,\underline{x} \rangle = \underline{x}^T \underline{A}\, \underline{x}$$

In the more general complex case when $\underline{A}$ is hermitian the function is still a quadratic form [A-4. (p.386)] .

A real symmetric matrix $\underline{A}$ is positive semidefinite if, for any choice of $\underline{x}$, the quadratic form $F(\underline{x}) = \langle \underline{x}, \underline{Ax} \rangle$ is nonnegative.

If $F(\underline{x}) = 0$ only when $\underline{x} = \underline{0}$ then $\underline{A}$ is positive definite.

If $\underline{A}$ is positive semidefinite all eigenvalues are nonnegative while all eigenvalues are strictly positive when $\underline{A}$ is positive definite. If $\underline{A}$ is is positive definite it is invertible too.

A Gramian matrix, $\underline{G}$, for a set of N-real vectors $\{x_m\}$ is defined as

$$\underline{G} = \begin{bmatrix} x_1 \cdot x_1 & \cdots\cdots\cdots & & x_1 \cdot x_M \\ x_2 . x_1 & x_2 \cdot x_2 & \cdots\cdots & \\ \cdots\cdots & & & \\ x_M \cdot x_1 & \cdots\cdots\cdots & & x_M \cdot x_M \end{bmatrix}$$

clearly, the vectors are mutually orthogonal if and if $\underline{G}$ is orthogonal. For a set of real N-vectors $\{x_m\}$ $\underline{G} \geq \underline{0}$. This equality holds if and only if the vectors are linearly dependent [A-5, (p.103)] .

A gramian matrix is always positive definite or positive semidefinite.

An Hadamard matrix [A-6, 17] is a square matrix whose elements are ONES and MINUS ONES and whose row vectors are mutually orthogonal (equivalently, whose column vectors are mutually orthogonal). Examples are shown below

(a) $\begin{bmatrix} 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

(c) $\left[\begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ \hline 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{array}\right]$

(d)
$$\left[\begin{array}{cc|cc|cccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ \hline 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ \hline 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{array}\right]$$

It is clear from the definition of these matrices that one may

(1) interchange rows

(2) interchange columns

(3) change the sign of every element in a row

(4) change the sign of every element in a column, without disturbing the Hadamard property. Using these operations it is possible to establish a normal form for Hadamard matrices by insisting that the first row and first column contain only ones. All examples given above are in Normal form.

Hadamard matrices may exist only for orders which are integer multiple of four, and have been constructed for all such orders upto 200. Assuming for 2x2 case

$$\underline{H}_0 \triangleq \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

We can recursively find the higher order matrices as below:

$$\underline{H}_{k+1} \triangleq \left[\begin{array}{c|c} \underline{H}_k & \underline{H}_k \\ \hline \underline{H}_k & -\underline{H}_k \end{array}\right]$$

The construction of other orders involves the determination of quadratic residues [A-7], [A-17] and is thus more involved but in spite of this fact it is more conveniently implemented using a digital computer. There are a number of methods for constructing Hadamard matrices. [A-17 p.(207)]. Let M be the order of the matrix. In order to construct a Hadamard matrix H, apart from the exceptional cases M=1, 2 it is necessary that M should be divisible by four. If M is a power of 2, i.e. 2^2, 2^3, 2^4, 2^5, then H should be formed in the manner discussed above and doubled (for computational purposes). We shall now consider the case of generating H when M is divisible by 4. This is a bit more complicated and the use of Legendre symbol is quite useful. The Legendre symbol (n/p) is defined for a prime p to be 1 when n is a quadratic residue of p, and -1 when n is a quadratic non-residue. If the congruence relation $r^2 = n(\bmod p)$ has a solution r(mod p) for integers n, p, and r, then n is called a quadratic residue otherwise it is a non-quadratic residue. The remaining results we shall provide in the form of Lemmas.

LEMMA 1: If we have an H matrix of the order M_1 and an H matrix of order M_2, then we may construct an H matrix of order $M_1 M_2$.

LEMMA 2: Let M be of the form (p+1), where p, $\equiv 3(\bmod 4)$, is prime. Then we can construct an H matrix of order M.

LEMMA 3: Let M be divisible by 4 and be of the form $2^K(p+1)$ where p is prime. Then we can construct an H matrix of order M.

In this case p=3(mod 4) i.e. M = 12, 20, 40, then the (p+1)x(p+1) array H' is first formed, with each element computed according to

$$h_{ij} = \begin{cases} 1 & \text{if } i=1 \text{ or } J=1 \\ -1 & \text{if } i=J \text{ and } i > 1 \\ (j-i/p) & \text{otherwise} \end{cases}$$

where the quadratic residues are determined by subroutine.

LEMMA 4: Repeating Lemma 3 for the case $p \equiv 1 \pmod 4$, $K=1$ i.e. $M=28$ then $\underline{H}'$ is computed according to

$$h_{ij} = \begin{cases} 1 \longrightarrow \text{if } i = 1 \text{ or } J = 1 \text{ but } i \neq J \\ 0 \longrightarrow \text{if } i = J \\ (j - i/p) \rightarrow \text{otherwise} \end{cases}$$

A.4 MOORE-PENROSE PSEUDO-INVERSE MATRIX (MPPI) [A-18, 23]

Matrix division is not defined Per se. The concept of matrix inverse effectively allows us to "divide" however. In this section we develope the concept of pseudoinverses and illustrate the computation of the inverse when it exists. In the past it has been conjectured that the inverse of a singular matrix, which is analogous to a scalar, does not exist. The pseudo inverse was introduced for the first time by Moore [A-25] and later rediscovered by Penrose [A-21, 27] whose papers initiated a vogue in the subject. The nascent concept is being tried for the first time for digital processing of PAM data signals. Before defining the MPPI matrix we ought to know a number of preliminary notions. Let the MxN matrix $\underline{A}$ represent a linear map from a vector space U to a normed vector space V of dimension N and M respectively. Assuming that the field is a set of real numbers and that $M \geqslant N$, then all eigenvalues of the real symmetric matrices $\underline{A}^T\underline{A}$ and $\underline{A}\,\underline{A}^T$ are real and nonnegative. The real property follows from symmetry but to show that the matrix $\underline{A}^T\underline{A}$ is nonnegative we let $\underline{V} = \underline{A}\,\underline{U}$ for some arbitrary vector $\underline{U}$ in U. Then

$$0 \leq \|\underline{V}\|_2^2 = \underline{V}^T\underline{V} = (\underline{A}\,\underline{U})^T(\underline{A}\,\underline{U}) = \underline{U}^T\underline{A}^T\underline{A}\,\underline{U}$$

So $\underline{A}^T\underline{A}$ is positive semidefinite from which the desired result follows. The result for $\underline{A}\,\underline{A}^T$ follows analogously. If the rank of $\underline{A}$ is R and is less than N, then it can be shown[A-23, p.84] that both $\underline{A}^T\underline{A}$ and $\underline{A}\,\underline{A}^T$ have exactly R identical nonzero eigenvalues, say $\{\xi_1^2, \xi_2^2, \dots \xi_R^2\}$ with all other eigenvalues zero. Furthermore, from symmetry it follows that $\underline{A}^T\underline{A}$ and $\underline{AA}^T$ each have a set of orthonormal eigenvectors which we shall denote by the sets$\{\underline{\chi}_1, \underline{\chi}_2, \underline{\chi}_3, \dots \underline{\chi}_N\}$ and $\{\underline{\psi}_1, \underline{\psi}_2, \underline{\psi}_3, \dots \underline{\psi}_M\}$ respectively. These sets are ordered so that $\underline{\chi}_m$ and $\underline{\psi}_m$ correspond to ξ_m^2 for m = 1, 2, 3, ... R. The rectangular matrix $\underline{A}$ can now be expressed [2-23, p.85] in the generalized spectral representation

$$\underline{A} = \sum_{k=1}^{R} \xi_k \underline{\psi}_k \underline{\chi}_k^T \tag{6}$$

when $\underline{A}$ is real symmetric, hence square, then (6) reduces to (5). This can be shown by noting that

$$\underline{A}\,\underline{A}^T = \underline{A}^T\underline{A} = \underline{A}^2$$

and from the Frobenius Theorem [A-39, p.244] the nonzero eigenvalues of $\underline{A}$ are $\{\xi_1, \xi_2, \xi_3, \dots \xi R\}$ Let us suppose that $\underline{p}$ is a normalized eigenvector of $\underline{A}$ corresponding to ξ, then it is also an eigenvector of $\underline{A}^2$ corresponding to ξ^2. In order to show this we note that

$$(\underline{A} - \xi\underline{I})\underline{p} = \underline{0}$$

implies that $(\underline{A}^2 - \xi^2 \underline{I})\underline{p} = \underline{A}(\underline{Ap}) - \xi \underline{A}\,\underline{p}$

$$= \underline{A} \quad (\underline{A} - \xi \underline{I})\,\underline{p} \quad = \underline{0}$$

consequently, for this case $\{\underline{p}_m\}$, $\{\underline{\chi}_m\}$, ard$\{\underline{\psi}_m\}$ are all equal and the desired result follows.

We shall now propose a formal definition of Moore-Penrose pseudo-inverse matrix (MPPI) $\underline{A}^{+}$ which is given by

$$\underline{A}^{+} = \sum_{k=1}^{R} \xi_k^{-1} \underline{\chi}_k \underline{\psi}_k^T \tag{7}$$

The most important application of this definition can be seen by considering the matrix equation

$$\underline{A}\ \underline{U} = \underline{V} \tag{8}$$

If $\underline{V}$ belongs to the range space of $\underline{A}$ then an exact solution exists, but is unique only when $\underline{A}$ is non singular, in which case it is given by the usual inverse $\underline{A}^{-1}$ as

$$\underline{U} = \underline{A}^{-1}\ \underline{V} \tag{9}$$

But, when the matrix $\underline{A}$ is singular then there are more linearly independent equations than unknowns and an exact solution does not exist in either case, the MPPI solution [A-23, p.86]

$$\hat{\underline{U}} = \underline{A}^{+}\ \underline{V}$$

is the unique best solution in the sense that, for any other possible solution $\underline{U}'$, the relationship is given by

$$||\underline{A}\ \hat{\underline{U}} - \underline{V}|| \leq ||\underline{A}\ \underline{U}' - \underline{V}||$$

<u>Properties of MPPI</u> [A-4, p.144]

The MPPI matrix $\underline{A}^{+}$ of $\underline{A}$ has the following properties:

(i) $(\underline{A}\ \underline{A}^{+})^T = \underline{A}\ \underline{A}^{+}$

(ii) $\underline{A}\ \underline{A}^{+}\ \underline{A} = \underline{A}$

(iii) $(\underline{A}\ \underline{A}^{+})^T(\underline{A}\ \underline{A}^{+}) = \underline{A}\ \underline{A}^{+}$

An example below shows how to compute MPPI

EXAMPLE

Find the MPPI of

$$\underline{A} = \begin{bmatrix} -1 & 0 & 1 & 2 \\ -1 & 1 & 0 & -1 \\ 0 & -1 & 1 & 3 \\ 0 & 1 & -1 & -3 \\ 1 & -1 & 0 & 1 \\ 1 & 0 & -1 & -2 \end{bmatrix}$$

Solution: Before solving this example we shall state the following theorems.

Theorem 1: If $\underline{A}$ is $m \times n$ and of rank k, and we can partition $\underline{A}$ in the form

$$\underline{A} = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{bmatrix}$$

where $\underline{A}_{11}$ is a nonsingular matrix of rank k, then

$$\underline{A} = \begin{bmatrix} \underline{I} \\ \underline{P} \end{bmatrix} \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} \end{bmatrix} = \begin{bmatrix} \underline{I} \\ \underline{P} \end{bmatrix} \underline{A}_{11} \begin{bmatrix} \underline{I} & \underline{Q} \end{bmatrix} = \begin{bmatrix} \underline{A}_{11} \\ \underline{A}_{21} \end{bmatrix} \begin{bmatrix} \underline{I} & \underline{Q} \end{bmatrix} \tag{10}$$

where $\underline{P} = \underline{A}_{21}\underline{A}_{11}^{-1}$, $\underline{Q} = \underline{A}_{11}^{-1}\, \underline{A}_{12}$

Theorem 2: If $\underline{A} = \underline{B}\ \underline{C}$ where $\underline{A}$, $\underline{B}$, $\underline{C}$ are respectively, MxN, MxK, and KxN, and all three matrices are of rank K, then the solution $\underline{A}\ \underline{x} = \underline{b}$ which minimizes

(a) the sum of the square of the residuals $\underline{r}^T\ \underline{r}$, where $\underline{r} = \underline{b} - \underline{A}\ \underline{x}$

(b) the sum of the squares of the uknowns $\underline{x}^T\ \underline{x}$ is given by

$$\underline{x} = \underline{A}^{+}\ \underline{b} \quad \text{where} \quad \underline{A}^{+} = \underline{C}^T(\underline{C}\ \underline{C}^T)^{-1}\ (\underline{B}^T\ \underline{B})^{-1}\ \underline{B}^T \tag{11}$$

Reducing the matrix $\underline{A}$ to Row-Echelon form, the rank is found to be 2. The 2x2 matrix in the upper left of $\underline{A}$ is nonsingular so that we can choose

$$\underline{A}_{11} = \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix} \qquad \underline{A}_{11}^{-1} = \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix}$$

working in terms of the third formula in (10), we compute

$$\underline{Q} = \underline{A}_{11}^{-1}\,\underline{A}_{12} = \begin{bmatrix} -1 & 2 \\ -1 & -3 \end{bmatrix}$$

In the terminology of theorem 2 we now set

$$\underline{B} = \begin{bmatrix} \underline{A}_{11} \\ \underline{A}_{21} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -1 & 1 \\ 0 & -1 \\ 0 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\underline{C} = \begin{bmatrix} \underline{I} & \underline{Q} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & -3 \end{bmatrix}$$

substituting in (11) we get MPPI

$$\underline{A}^{+} = \frac{1}{102} \begin{bmatrix} -15 & -18 & 3 & -3 & 18 & 15 \\ 8 & 13 & -5 & 5 & -13 & -8 \\ 7 & 5 & 2 & -2 & -5 & -7 \\ 6 & -3 & 9 & -9 & 3 & -6 \end{bmatrix}$$

A.5 STATISTICS:

The essence of communication is randomness. Digital communication involve digital processing of random signals. These random signals usually possess infinite sequence [illegible] energy. Thus, the mathematical representation of such signals lies in their description in terms of averages. We shall develop many (but not all) of the properties of such signals in terms of a finite energy sequence called the Autocorrelation or Autocovariance sequence, for which the z-transform or the Fourier transform often exists. It is impossible to go into all the details of statistics involved, however, a few necessary requirements will be catered for and the rest will be referenced to the standard texts [A-27,36]

The expected value or mean of a random variable $\underline{x}$ is the integral

$$E\,[\underline{x}] = \int_{-\infty}^{\infty} x\, f(x)\, dx \tag{20}$$

where f(x) is the density of $\underline{x}$. If $\underline{x}$ is of discrete type, taking the values x_n with probability p_n, then

$$E\,[\underline{x}] = \sum_n x_n\, P\,(\underline{x}=x_n) = \sum_n x_n p_n \tag{20a}$$

This will be denoted by η . Another important parameter is the Variance or Dispersion which is defined as:

$$\sigma^2 = E[(\underline{x}-\eta)^2] = \int_{-\infty}^{+\infty} (\underline{x}-\eta)^2 f(x)\, dx \tag{21}$$

The square root σ is called the Standard Deviation . If $\underline{x}$ is of discrete type, then

$$\sigma^2 = \sum_n (x_n-\eta)^2\, P\,(\underline{x}=x_n) \tag{21a}$$

$$= E\,[\underline{x}^2] - [E(\underline{x})]^2 \tag{21b}$$

Two random variable $\underline{x}$ and $\underline{y}$ are called Uncorrelated if

$E[\underline{xy}] = E\,[\underline{x}]E[\underline{y}]$. They are orthogonal if $E[\underline{x}\ \underline{y}] = \underline{0}$ and independent if $f(x,y)=f_x(x)\ f_y(y)$. The autocorrelation $R(t_1,t_2)$ of a process x(t) is the joint moment of the random variable $x(t_1)$ and

$x(t_2)$ and is given by:

$$R(t_1,t_2) = E[x(t_1)x(t_2)] \tag{22}$$

The cross correlation of two processes, real or complex, is defined by

$$R_{xy}(t_1,t_2) = E[x(t_1)\, y^*(t_2)] \tag{23}$$

where * denotes the complex conjugate. A stochastic process x(t) is stationary (in the strict sence) if its statistics are not affected by a shift in the time origin. This means that the two processes x(t) and x(t+e) have the same statistics for any e. A process is called stationary in the wide sense (or weakly stationary), if its expected value is a constant and its autocorrelation depends only on (t_1-t_2):

$$\begin{aligned} E[x(t)] &= \eta = \text{a constant} \\ E[x(t+\tau)x(t)] &= R(\tau) \end{aligned} \tag{24}$$

A process x(t) is normal and stationary in the wide sense, then it is stationary also in the strict sense.

So far, we have mentioned about the ensemble averages i.e. E [.], but in a practical sense, we would prefer to deal with a single sequence rather than an infinite ensemble of sequences. To formalize this intuitive notion, we define the time averages <. > of a random process as

$$< x_n > = \lim_{N\to\infty} 1/2N+1 \sum_{n=-N}^{N} x_n \tag{25}$$

Similarly, the time autocorrelation sequence is defined as:

$$< x_n, x_{n+m} > = \lim_{N\to\infty} 1/2N+1 \sum_{n=-N}^{N} x_n x^*_{n+m} \tag{26}$$

It can be shown that the above limits exist if $\{x_n\}$ is a stationary process with finite mean. When the time averages of a process equal ensemble averages then it is called an ergodic process [A_27].Properties of correlation and covariance sequences are tabulated in [A-25,pp.388].

MONTE CARLO SIMULATION TECHNIQUE: [5-3]

In chapter 5, we came across a problem of evaluating the most meaningful criterion, the probability of error Pr(e) or Pe. Since an analytical expressionis not available, we resort to an approximate technique the so called Monte Carlo techniques. In this case the solution is attained in four steps:

(a) Generation of the transmitted data sequence;

(b) Generation of the noise, Which is then combined linearly with the transmitted signal to obtain the received signal;

(c) Application of the equalization operation to the received signal so as to obtain the received data sequence;

(d) Comparison of this sequence with the transmitted one to count the number of errors that have occurred, from which the required estimate of the error prob. is then readily derived.

It is assumed throughout this chapter that the transmitted data sequence is binary and thus step (a) above requires in general the use of a binary pseudo-random generator. However, in all cases studied in chapter (5) the prob. of detecting a digit in error is the same whether it is a 'zero' or a 'one' and therefore it is possible and convenient to asume that the transmitter sends a sequence of identical digits (say, zeros). It is also assumed in this chapter that the receiver processes samples of the received signal. Therefore, step (b) of the simulation procedure can be accomplished by using a pseudo-random number generator to generate the noise samples, which are then combined linearly with the corresponding samples of the transmitted signals.

The generation of the noise samples usually takes most of the computer time in the simulation procedure. Since in order to obtain a sufficiently accurate estimate of the error prob. it is usually necessary to use long transmitted sequence, the noise structure must be simple enough to permit its generation in a reasonable amount of time.

Before the simulation experiment starts it is very desirable to have an estimate of the No. of transmitted digits required to estimate the error prob. with the prescribed accuracy. Assuming for simplicity that the errors are statistically independent events, the No. of errors in n digits obeys a binomial distribution of mean nPe and variance nPe (1-Pe), where Pe is the prob. of error. For sufficiently large n this distribution becomes approximately gaussian and thus the prob. of a given No. of errors can easily be determined. The relative error in estimating Pe is given by (x-nPe)/(nPe), where x is a No. of observed errors. This relative error has an approximately gaussian distribution with zero mean and standatd deviation $\sqrt{(1-Pe)/(nPe)}$. Thus, to find the prob. P of maintaining a given level of accuracy λ it is necessary to determine n from the Equation

$$P = \text{prob.} \left| \frac{x-nPe}{nPe} \right| \leq \lambda$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\lambda_K}^{\lambda K} \varepsilon^{-t^2/2} \, dt = \Phi\ (K\lambda\ )$$

where $$K = \left[\frac{nPe}{1-Pe} \right]^{\frac{1}{2}}$$

This can be written in the form

$$K = \frac{1}{\lambda} \left[\Phi^{-1}\ (P) \right] \quad \text{or} \quad n = K^2 \left[\frac{1-Pe}{Pe} \right]$$

Therefore, if $P = 90\%$ and $\lambda = 0.1$ then

$$n = \frac{300}{Pe}$$

A.6 PROOF OF SUPPLEMENT RESULTS:

A.6.1 Prove that the matrix $\underline{A}$ defined by:

$$a_{pq} = \frac{1}{2\pi j}\oint_c Y(z)\, F_p(z)\, F_q(z^{-1})\, Y(z^{-1})\, z^{-1}\, dz$$

is hermitian and positive semidefinite.

PROOF:

Letting z= exp.(jω), it follows that

$$a_{pq} = \frac{1}{2\pi j}\int_{-\pi}^{\pi} Y(j\omega)\, Y(-j\omega)\, F_p(j\omega)\, F_q(-j\omega)\, j\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} |Y(j\omega)|^2 F_p(j\omega)\, F_q^*(j\omega)\, d\omega$$

$$= a_{qp}^*$$

and this means that $\underline{A}$ is hermitian. To prove that $\underline{A}$ is also positive semidefnite let us suppose that is an arbitrary vector of size M. Then

$$\underline{\alpha}^T \underline{A}\, \underline{\alpha} = \frac{1}{2\pi j}\oint_c \underline{\alpha}^T\, \underline{V}(z)\, \underline{V}^T(z^{-1})\, \underline{\alpha}\, z^{-1}\, dz$$

Since $\hat{I}(z) = \underline{\alpha}^T\, \underline{V}(z)$, it follows that

$$\underline{\alpha}^T\, \underline{A}\, \underline{\alpha} = \frac{1}{2\pi j}\oint_c \hat{I}(z)\, \hat{I}(z^{-1})\, z^{-1}\, dz$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} |\hat{I}(j\omega)|^2\, d\omega \geq 0$$

This proves the positive definiteness [A-42].

A.6.2 Prove that the matrix $\underline{A}$ defined by $P_i(z) = z^{-i}$ in (3.24) is positive definite.

PROOF:

With respect to Fig. (3.2), we have

$$\hat{I}(z) = \sum_{i=0}^{K-1} \alpha_i\, P_i(z)\, Y_i(z)$$

$$= \left[\sum_{i=0}^{K-1} \alpha_i z^{-i} \right] Y(z)$$

$$= C(z)\, Y(z)$$

where $C(z) = \sum_{i=0}^{K-1} \alpha_i z^{-1}$

The quadratic form $\underline{\alpha}^T \underline{A}\, \underline{\alpha}$ can be written as

$$\underline{\alpha}^T \underline{\tilde{A}}\, \underline{\alpha} = \frac{1}{2\pi j} \oint_c \hat{I}(z)\, \hat{I}(z^{-1})\, dz/z$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} |C(j\omega)|^2 |Y(j\omega)|^2\, d\omega$$

Now $C(j\omega)$ is a linear combination of K complex exponentials, and hence must have always nonzero values on any finite interval unless $\underline{\alpha} = \underline{0}$. Also, the input signal is made digital by means of an ADC operating at a sampling rate which is at least twice the highest signal frequency in accordance with the uniform sampling theorem. Consequently, the spectrum $Y(j\omega)$ must have nonzero values on some interval of the primary region $[-\pi, \pi]$, and therefore, $\underline{\alpha}^T \underline{A}\, \underline{\alpha} > 0$ for $\underline{\alpha} \neq \underline{0}$ and $\underline{A}$ is positive definite.

A.6.3 Prove that the vector $\underline{W}$ in Eq. (3.31) belongs to the range space $R(\underline{A})$ of $\underline{A}$.

PROOF: [A-1, 38] Suppose $\underline{\tilde{W}}$ is an arbitrary vector in space $R(\underline{A})$. We have

$$\underline{W} = \underline{P}^T \underline{\tilde{W}}$$

$$\underline{A} = \underline{P}^T \underline{\tilde{A}}\, \underline{P}$$

Therefore, it will suffice to show that $R(\underline{A}) = R(\underline{P}^T)$.

Here, $\underline{\tilde{A}}$ is nonsingular and real symmetric . It has a square root $\underline{\tilde{A}}^{1/2}$ which is also nonsingular and real symmetric: consequently, $\underline{A}$ can be written as

$$\underline{A} = (\underline{\tilde{A}}^{1/2} \underline{P})^T (\underline{\tilde{A}}^{1/2} \underline{P}) = \underline{\hat{X}}^T \underline{X}$$

Also, $\underline{W} = \underline{P}^T \, (\tilde{\underline{A}}^{\frac{1}{2}} \; \tilde{\underline{A}}^{-\frac{1}{2}}) \; \tilde{\underline{W}}$

$= \underline{X}^T \, (\tilde{\underline{A}}^{\frac{1}{2}} \; \tilde{\underline{W}} \,)$. We also observe that

$\underline{X}^T$ is the adjoint of $\underline{X}$ therefore $R(\underline{X}^T) = R(\underline{X}^T\underline{X}) = R(\underline{A})$

However, $\underline{W}$ is obtained by applying

$\underline{X}^T = \underline{P}^T \tilde{\underline{A}}^{\frac{1}{2}}$ to the arbitrary vector $\tilde{\underline{A}}^{\frac{1}{2}} \tilde{\underline{W}}$ and since $\tilde{\underline{A}}^{\frac{1}{2}}$ is

one-to-one, then

$R(\underline{X}^T) = R(\underline{P}^T)$

Hence, $R(\underline{P}^T) = R(\underline{X}^T) = R(\underline{A})$.

This proves the proposition.

A.6.4 Prove that the map

$$\underline{YY}^+ \, \underline{YY} = \sum_{i=0}^{N-1} \underline{\psi}_i \; \underline{\psi}_i^T = \underline{\phi}$$

given by Eq. (3.53) is a projection matrix, i.e., it is both self adjoint and idempotent.

PROOF:

$\underline{\phi}$ is hermitian since it is real

Therefore, $\underline{\phi}^T = [\sum_{i=0}^{N-1} \underline{\psi}_i \; \underline{\psi}_i^T]^T$

$$= \sum_{i=0}^{N-1} \underline{\psi}_i \; \underline{\psi}_i^T = \underline{\phi}$$

Furthermore, since

$$\underline{\phi}^2 = [\sum_{i=0}^{N-1} \psi_i \;\; \psi_i^T] \; [\sum_{j=0}^{N-1} \psi_j \;\; \psi_j^T]$$

$$= \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \underline{\psi}_i (\underline{\psi}_i^T \underline{\psi}_j) \underline{\psi}_j$$

$$= \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \underline{\psi}_i \; \delta_{ij} \; \underline{\psi}_j^T = \underline{\phi}$$

Then $\underline{\phi}$ is also idempotent. Therefore $\underline{\phi}$ represents a projection matrix and the proposition is proved.

A.6.5 Prove that the matrix

$$\underline{\psi}^T \underline{\phi}\, \underline{\psi} = \text{diag.}\ [\ \underline{I}_N \mid \underline{0}_{K-N}\ ]$$

PROOF:

$\{\psi_m\}$ is orthonormal set, therefore,

$$\underline{\psi}^T \underline{\phi}\, \underline{\psi} = \begin{bmatrix} \psi_0^T \\ \psi_1^T \\ \psi_2^T \\ \vdots \\ \psi_{K-1}^T \end{bmatrix} \sum_{i=0}^{N-1} \underline{\psi}_i\, \underline{\psi}_i^T\ [\ \psi_0\ \psi_1\ \psi_2\ \cdots\ \psi_{K-1}\ ]$$

$$= \begin{bmatrix} \underline{I}_N & \underline{0} \\ \underline{0} & \underline{0}_{K-N} \end{bmatrix}$$

$$= \text{diag.}\ [\ \underline{I}_N \mid \underline{0}_{K-N}\ ]$$

GLOSSARY OF SYMBOLS AND TERMS

$\underline{a}_i$	Vector
$\{a_m\}$	Matched filter output in sampled form
$\underline{A}$	Matrix A
$\overset{o}{\underline{A}}$	Singular matrix
$\underline{\tilde{A}}$	Stictly non singular matrix
$\underline{A}^+$	Moore Penrose Pseudoinverse matrix of $\underline{A}$
$\{B_{k-\delta}\}$	Desired truncated received sequence
$\{\underline{\alpha}\}$	Feedforward multiplier coefficient of non recursive digital filter equalizer and recursive digital filter equalizer
$\{\underline{\beta}\}$	Feedback multiplier coeeficient of recursive digital filter equalizer (RDFE)
δ	Delay between the arrival at the equalizer of the first precursor of the channel unit pulse response and the occurrence of the locally generated reference pulse
δ_{ij}	Kronecker delta (if i=j then $\delta_{ij} = 1$; if. $i \neq j$ then $\delta_{ij} = 0$)
ε_k	k th error = ($I_k - \hat{I}_{k-\delta}$) or ($I_k - \tilde{I}_{k-\delta}$) sample
$\underline{\Phi}$	Projection matrix
Δ	Iterative convergence factor
∇	Gradient operator
Ξ	The probability of an error event
Σ	Summation notation
Γ	Partial sum of the log likelihood function
λ_i	i^{th} eigenvalue of matrix $\underline{A}$

$\{\underline{\psi}\}$	Orthonormal eigenvector set of $\underline{YY}^T\,\underline{YY}$
$\{\underline{\Psi}\}$	Orthonormal eigenvector set of $\underline{YY}\,\underline{YY}^T$
$\{\mu^2\}$	Eigenvalues of the N x N real symmetric matrix $\underline{YY}^T\,\underline{YY}$
$\{\eta\}$	Additive coloured gaussian noise component sequence
$\underline{\lambda}$	[K x K] diagonal matrix
σ_n	Standard deviation
$\{\rho\}$	Multiplier coefficients of finite memory discrete time NRDF
$\{\xi\}$	Input error sequence (Chapter 6)
$D(\underline{\alpha})$	Peak distortion criterion
d_{ij}	Coordinate in the direction of $\underline{\psi}_j$ and $\underline{\psi}_i$
d_{min}	Minimum distance weight (the VA algorithm)
$\underline{D}$	Diagonal matrix
$\underline{e}_k$	k^{th} multiplier error vector
$E[x]$	Expected value of the random variable x
$\{f_k\}$	Set of multiplier coefficients of a NRDF model of discrete time channel with finite memory
G	Length of channel unit pulse response
$\{g_k\}$	Channel impulse response
$\underline{g}_k$	Estimated gradient vector of $J[\underline{\alpha}_k]$ w.r.t. $\underline{\alpha}_k$, also residual vector
$\underline{H}$	Hadamard matrix
h	Combined impulse response of channel and transmitter
$\underline{I}$	Diagonal matrix
$\bar{I}$	Length of the input sequence $\{I_k\}$
$\{I_k\}$	Data (Symbol) sequence
$\{\hat{I}_k\}$	Equalizer output before decisions

$\{\Upsilon_k\}$	Equalizer output after decisions
$\underline{I}_N$	N x N identity matrix
$J(\underline{\alpha})$	Mean square error criterion used for NRDFE cases only
$\mathcal{J}(\alpha)$	Noisy performance criterion
$J(\underline{\alpha}, \underline{\beta})$	Mean square error criterion used for RDFE cases only
K	Number of filter sections
K_L	Lower bound multiplier factor
K_U	Upper bound multiplier factor
L	Length of the channel output sequence
LL	Total number of samples
Log (l_n)	Natural logarithm
Log_b	Logarithm to the base b
M	Size of the alphabet ,also used for the NO. Of α's in RDFE
m	Number of levels
N	Number of shift register stages in each filter section (NRDFE), also used for the number of feedback multiplier coefficients β's in RDFE
n	An index integer
n(t)	White gaussian noise component of the channel output
N_0	Two sided noise spectral density
P	Average power
$P_r(e)$, P_e	Probability of error (both notations have been used)
p(i)	Probability of sending i^{th} message
p(E|i)	Conditional probability
$P_i(z)$	Transfer function of
PP	Length of the filter section output
Q	Gain of a digital filter

$\underline{Q}$	A matrix having normalized eigenvector of $\underline{A}$
$Q(\cdot)$	Error function (erf)
$Q[\cdot]$	Quantized output
q_i	Main or Head pulse
$\{r_k\}$	Input to the receiver, Fig. (2.1)
$\underline{r}$	Residual vector
$\underline{R}_{vv}$	Filter section correlation matrix
$\underline{R}_{Iv}$	Cross correlation matrix
$S(t)$	Transmitted signal through the channel
s_k	State sequence
$[\underline{S}]^T$	Transpose of the matrix $\underline{S}$
T	Duration of the signalling interval
$\underline{U}_i$	Eigenvectors of the matrix $\underline{A}$
v_k	Output of the k^{th} filter section [Chs. 3 & 4]
$< u , v >$	Inner product of u and v
$\underline{W}$	Cross correlation vector
$\underline{Y}$	Input vector to the equalizer
$\{y_i\}$	Sequence of input vector(symbols,bits,digits,pulses)

* (on-line)	Convolution
* (superscript)	Complex conjugate of a number (i. e. z^*)
$\approx$	Approximately equal
$\|Z\|$	Modulus of the random number Z
$\geq$	Greater than or equal to
$\leq$	Less than or equal to
$\|\|Z\|\|$	Norm of Z
$\overset{\Delta}{=}$	Is defined as
MLSE	Maximum likelihood sequence estimation or estimator
VA	The Viterbi algorithm
MPPI	Moore Penrose Pseudoinverse
NRDFE	Non recursive digital filter equalizer
NRDF	Non recursive digital filter
RDFE	Recursive digital filter equalizer
RDF	Recursive digital filter
WGN	White gaussian noise
BW	Bandwidth
#	Number
Prob.	Probability

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