IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY

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Department of Management Science

ASPECTS OF LINEAR PROGRAMMING AND ACCOUNTING IN A MANUFACTURING COMPANY

by

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ABSTRACT

This thesis is concerned with the development of tools to aid the management of a general manufacturing company to both plan their activities in the short and medium term, and to evaluate their proposals in terms of balance sheet projections.

The modelling technique used is linear programming and the two models concentrate on the physical activities of production planning (i.e. work-centre and labour force scheduling), purchasing, and cashflow planning in the short term, extended to include capital expenditure under capital rationing in the medium term.

It has been proposed that the dual linear programme may be used for valuation purposes and that such a procedure may overcome some of the problems experienced by currently accepted accountancy procedures.

This thesis investigates the conditions that give rise to (the often experienced) alternate and/or degenerate solutions to corporate models formulated above.

These particular linear programming solutions require a revision to our previous understanding and use of the dual. In particular, the existance of an alternate dual space at degenerate primal solutions presents a considerable obstacle to the proposal to use the dual linear programme for valuation purposes.

ACKNOWLEDGEMENTS

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I am indebted to Hymac Ltd. for the original sponsorship of this project and to Esso Petroleum Ltd. who have helped in the latter stages to produce this thesis.

And last, but in no sense least, I record my lasting debt to my principal sponsor - Louise - without whose untiring vigilance over my flagging efforts this thesis would never have seen light of day.

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ABBREVIATIONS AND NOTATION.

$\underline{x}$	vector x
x_j	the j'th element of vector x
S	either the set or matrix S (differentiation is clear from the text
∅	zero where confusion may occur (otherwise 0)
st	such that
$\sum_i$	sums over i
⩾,⩽	greater than or equal to, less than or equal to

Common notation has been used throughout this report. The following are the most important examples:

$\underline{A}$	the matrix of technological coefficients (dimension m x n)
$\underline{b}$	the right hand side vector (dimension m)
$\underline{c}$	the objective function vector (dimension n)
$\underline{B}$	the square basic matrix
$\underline{\pi}$	the vector of dual variables (dimension m)
$\underline{x}$	the vector of primal variables (dimension n)
*	superscipt denoting the optimal value
α,β,λ	scalar multipliers

References are denoted as follows:

- 9 - 99 -	page 99 of the thesis that forms part of Chapter 9
- X - 999 -	page 999 of the thesis that forms part of Appendix X
(999)	reference 999
Figure 9.99	Figure 99 from Chapter 9
Table 9.99	Table 99 from Chapter 9
Equation 9.99	Equation 99 from Chater 9
Chapter 9.99.i	Subsection i of Section 99 of Chapter 9
Jrn.	journal

INTRODUCTION

The need for planning a company's affairs is now well accepted. We proceed from that basis to propose a particular model that may help management govern a company's activities in the short and medium term and then investigate a method for evaluating the company's performance.

The first part of the thesis is concerned with the development of a Linear Programming (LP) model of a company's activities. In Chapter 1 we confine these activities to the short-term management decisions of production, sales and short-term financing. In Chapter 2 we propose a formulation of longer-term decisions involving capital expenditures, changing the product mix and changing the amount of value added during production, under conditions of capital rationing that overcome problems experienced by previous capital budgeting formulations.

The second part of the thesis is concerned with the valuation process. In reporting to the parties interested in the company's affairs (shareholders, employees, creditors, bankers, government, etc.) the company presents a balance sheet drawn up to give a picture of the company at its year end. In Chapter 3 we inspect the currently accepted accounting procedures and point to a number of problem areas. It has been suggested that the dual LP from a corporate LP model of the type described earlier may provide a means to apply an opportunity valuation to the company's resources in the context of their use in achieving the company's goals.

However, this proposal is itself not problem free and we investigate the meaning of the dual LP under two commonly experienced conditions; namely that the optimal solutions are Primal Alternate and/or Primal Degenerate. In Chapter 4 we discuss the geometry that gives rise to alternate solutions. In Chapter 5 we show that the Kuhn-Tucker Necessary Conditions for optimality need not apply to degenerate solutions. Furthermore, the generally accepted two-sided nature of the dual variable at the degenerate solution is shown to be a gross simplification of the behaviour of the dual LP: we shall show the existence of an alternate dual space at the primal degenerate optimum and the collapse of this space, under simple degeneracy, to the two-sided dual variable under sensitivity analysis.

In Chapter 6 we discuss the implication of these results to the proposal to use the dual LP for valuation. We suggest the introduction of a "Holding Value" to highlight the company's over-availability of some resources (corresponding to the slack of certain constraints). Making the problem totally degenerate appears attractive in that it allows the LP to allocate the objective function value over all the resources (and represents the ideal determinstic economic system). However, the existence of the alternate dual space allows the user to construct an infinity of different balance sheets that all purport to portray the same physical solution. The resulting loss of information leads us to question the use of the dual LP (even for non-degenerate problems) for valuation.

CHAPTER 1

MATHEMATICAL PROGRAMMING & SHORT-TERM CORPORATE PLANNING

1. Introduction

This thesis takes as read the justification of the need for systematic planning (see (4), (47)); the discussion will concentrate on the use and respective merits of various analytical techniques in helping to construct short-term corporate plan (or budgets).

Planning may be identified with many different aspects of the company's activities: it can concern itself with the solution of particular problems, such as the raising of finance (3); the selection of product mix (90); the scheduling problem (52), and many others. Or it may concern itself with establishing the global corporate strategy (see (96), (101)).

This thesis is concerned with the latter aspect of corporate planning, and in this chapter a global corporate model is developed and tested for the particular company described below.

2, The Company

The test company being modelled, is a wholly owned subsidiary of a small conglomorate holding company. It is a medium-engineering company making earth-moving equipment of two distinct types: a 360-degree tracked excavator (termed a HYMAC, since that company is the market leader for this product in the U.K.) manufactured at Rhymney and a 180-degree wheeled excavator loader (termed a JCB) manufactured at Great Yeldham. The company also markets (and manufactures some) spare-parts for its primary products, and in the course of business it also undertakes some part-exchange of customers' old machines which are then resold on the second-hand market.

2.1 Product Range

By inspection of the value and volume of units of separately identifiable products seven different HYMAC and six different JCB models were identified. These 13 models accounted for the major proportion of all sales. (Products excluded consisted of one HYMAC model of which only one or two were sold annually, and some wheeled-shovels which could be similarly ignored). The market for these products was split into two sectors, namely "HOME" and "EXPORT". Further subdivision of these markets could not be identified by specific marketing factors. However, sales of JCB's in Germany required different production specifications, , so three extra 'special' JCB models were included in the product range. These were sold in the specific market sector "GERMANY' only.

Consideration of spare-parts proved to be a more difficult problem: each finished product is made up of some thousands of component items and it would be impractical to try to treat each one individually. Attempts to combine a number of parts to form a pseudo-unit spare-part proved impossible; nor was there any information to relate the sales of spare-parts with the population of machines already sold. Explicit treatment of this facet of the company's activities had therefore to be shelved.

Since the number of products -16- is small, no grouping of products was required and this greatly eased the data collection exercise.

2.2 Manufacturing Facilities

The manufacturing process at both factories is similar: raw materials are processed in the machine and fabrication shops, then assembled with other bought-in components, and finally painted and tested prior to despatch. This is shown schematically in Fig. 1.1 At Great Yeldham the progress of work is more along the line-system than at Rhymney, where scheduling is more batch oriented, but the difference was not so marked that separate treatment was required. At both factories the manning of work-centres can be done by more than one labour group.

The differentiation between labour groups was initially ignored but on subsequent discussions with the management it was felt that distinction between skilled, semi-skilled and unskilled labour would better represent their operations.

When required production exceeds capacity, there exists some facility to subcontract at both locations. At the time of writing, there is no interchange of work between the two factories, but this need not always be the case.

2.3 The Company's Objectives

The difficult task of making an explicit statement of the company's objectives is exacerbated by the fact that the objectives appear differently to the different people in the hierarchy of the company. Discussion with the managing director revealed that he was under pressure from a variety of sources that tried to pull him in a number of different directions: due to the previous disasterous record of the company (requiring them to be bailed out by the parent) he was concerned to report good results. However he was aware that an attractive long term strategy might be to sacrifice present results and aim for a high turnover in order that in later years he might capitalise on the sale of spares to a high 'captive' market. But he was also aware of the fierce competitive nature of the business and forecast that a number of companies might be forced to the wall in the forthcoming recession and was also considering the possibility of growth by acquisition.

Above all, the company is conscious of being a subsidiary: every year it has to present to the parent company a proposed budget plan for the coming year. Since its actual performance will be judged against its proposals, is is important that, while the budget must appear attractive in order that the company receive any finance it requires from the parent in the coming year, the budget be feasible. Shortfalls are regarded disfavourably, even if the actual performance is a 'good' one!

It is for this reason that we searched for a process to model the company encompassing as many of its activities as possible, and model them in sufficient detail to help management in their budget preparation.

3. Modelling Approaches

From the wide variety of modelling techniques our objective was to chose the technique that

(i) allowed us to model the company in the detail required;

(ii) allowed explicit recognition of the stochastic nature of real life, or allowed easy sensitivity testing;

(iii) allowed us to test a number of alternative strategies quickly (in order that management could then impose on the model, and test the effect of, a variety of assumptions about the future);

(iv) allowed us to present the results of the model in recognisable managerial accounting format.

3.1 Simulation

GERSHEFSKI has found (67) that the major proportion of corporate model builders use simulation, as opposed to mathematical programming techniques. BATT & GRINYER tentatively conclude from their survey (9) that the majority of such models are top-down, deterministic simulation finance models (in effect input-output models) and this has since been substantiated by other surveys (74). The reasons for this is that such corporate models are cheap and fast to develop (giving a quick return on the effort invested), easy to understand (an important criterion for acceptance by management) and flexible. Inclusion of statistical relationships is hampered by the lack of data, but when that can be encorporated (e.g. Chapter 4 of (55)) simulation proves to be the only method to tackle the problem of risk evaluation in large systems.

The disadvantage of simulation is that it merely evaluates a particular solution (in light of a number of uncertain variables), and that the evaluation is done according to given precepts. The technique cannot search for a solution that may result in best attainment of the goal(s) of the company. Nor can the technique itself suggest a manner for evaluating the particular solution selected.

3.2 Stochastic Programming

This technique aims to incorporate recognition of random events into the analytical solution of model.

Despite the difficulty in doing so, numerous (small) models have been developed dealing with a variety of problems (see Chapter 4 (131)).

BEALE ((13) and (14)) has noted that the technique places an enormous burden on the management for more data; for example in production planning the assumptions have to be clearly stated about the variation in demand for each product, and the correlations in demand between the different products. This factor alone would present an insuperable obstacle to widespread use of this technique for corporate planning, were it even possible to solve the large, detailed model envisaged above.

3.3 Linear Programming

Linear Programming (LP) is a technique that can handle very large models (of the order of several thousand constraints and hundreds of thousands of variables) with ease and speed. Furthermore it is a procedure that searches among all the points that comprise the feasible set (defined by the constraints) for the 'best' solution to the stated goals of the enterprise. The well developed computer based LPs also allow the user to test, with ease, points close to the chosen solution to gauge the sensitivity of that solution. In standard form the model can be expressed as

$$\begin{aligned} \text{Max} \quad & \underline{c}\,\underline{x} \\ \text{s.t} \quad & \underline{A}\,\underline{x} \leq \underline{b} \\ & \underline{x} \geq \underline{\emptyset} \end{aligned} \qquad (1.1)$$

where $\underline{x}$ is the set of variable (activities); $\underline{c}\,\underline{x}$ is the objective function and $\underline{A}\,\underline{x} \leq \underline{b}$ define the structural constraints on the model.

There are two further attributes of this technique. The first is the well known economic interpretation of the dual LP which attributes an opportunity valuation on

* This arises from the relationship $\frac{\partial z}{\partial b} = \pi$: marginal changes in the availability of a resource are valued by the effect such changes have on the overall achievement of the goal z. This in turn results in variables having an associated "reduced cost" (= $\underline{\pi}.\underline{A}^T - \underline{c}$). The reduced cost derived from the optimal solution can be considered as an opportunity cost in accounting terminology: the cost associated with basic variables is zero, whereas the cost associated with non-basic variables (which are not active - i.e are set to zero) is negative. If some non-basic variables are introduced to the solution (to satisfy some other objective than that expressed in the objective function) the achievement of the stated goal will be reduced in accordance with the associated reduced/opportunity cost.

the assets that comprise the enterprise when these assets are encorporated as constraints on the model.*

The second is the manner in which the simple formulation can be extended to broaden the scope for application of this modelling technique. These are described in the following subsections.

The problems associated with the technique are dealt with in Section 8.

3.3.1 Linear Fractional Programming

Many companies use financial ratios as expressions of goals or for purposes of control. GOLD has shown (69) how the commonly used ratio of 'profit to total investment' can be broken down into a series of simple ratios such as 'profit to output', 'output to capacity', etc. These simple ratios, made up of variables that relate to the activities under the direct control of a manager, can then be used to set targets, compatible with the overall objective, for the various echelons within the company.

The modelling of ratios used for target-setting can be transferred to a standard LP.

Targets, such as $\frac{x_p}{x_q} \geq t$, can be included in the model as

$$x_p - tx_q \geq 0 \; ; \quad x_p, x_q \geq 0 \qquad (1.2)$$

The treatment of ratios used as objectives is more complex: CHARNES AND COOPER (39) transfer such a model -

$$\begin{aligned}\text{Max} \quad & \frac{(\sum_j c_j x_j + \alpha)}{(\sum_j d_j x_j + \beta)} \\ \text{s.t.} \quad & \sum_j a_{ij} x_j \leqslant b_i \\ & x_j \geqslant 0\end{aligned} \qquad (1.3)$$

to the standard form -

$$\begin{aligned}\text{Max} \quad & \sum_j c_j tx_j + \alpha t \\ \text{s.t.} \quad & \sum_j a_{ij} tx_j - tb_i \geqslant 0 \\ & \sum_j d_j tx_j + t\beta = 1 \\ & t, x_j \geqslant 0\end{aligned} \qquad (1.4)$$

Thus by suitable modification the standard LP model formulation (Equation 1.1) can be extended to deal with linear ratios in addition to linear constraints.

3.3.2 Goal Programming

Goal programming, a term first coined by CHARNES & COOPER (41), is a technique that allows the mathematical programme to more closely approximate managerial behaviour in real life.

Goal (or target) setting is widely used as a means of motivating and controlling management (see (80)). The task that confronts the enterprise is then to achieve a plan that meets (or comes as close as possible to) that goal:

$$\begin{aligned}\text{Min} \quad & U + O \\ \text{s.t.} \quad & \underline{A}\,\underline{x} \leqslant \underline{b} \\ & \underline{G}\,\underline{x} + U - O \geqslant \underline{g} \\ & \underline{x}, U, O \geqslant \emptyset\end{aligned} \qquad (1.5)$$

where U is the extent by which the goal $\underline{G}\ \underline{x}$ underachieves the target $\underline{g}$; O the extent by which it is overachieved (termed "underage" and "overage").

IJIRI (88) points to the considerable psychological difference between these variances in target achievement.

> "For managers, there is a qualitive difference between profit and loss, although arithmetically the two are on the same continuous scale. For them the difference between $1000 profit and $1000 loss seems to be greater than the difference between $10000 profit and $8000 profit or the difference between $10000 loss and $8000 loss".

To account for this the model requires that the underage and overage are weighted differently. These weights would need to be non-linear if the model is to take account of the further difference in appreciation of achieving 1, 100, or 1000 over (or under) target.

It has long been recognised that companies pursue a number of goals simultaneously (see (22)), and that these may even be conflicting. Optimizing methods described above, epitomized by LP, consider only one objective and this is a serious limitation on the applicability of such techniques. In attempts to overcome this obtacle modellers resort to

i. Optimizing in tandem. The goals are ranked in order and the programme is then optimized with respect to the first, then the second, etc. It is an extremely lax model that has sufficient degrees of freedom to cater for more than a very small number of goals in this fashion.

ii. Trade-offs of one goal vis-a-vis another. In this way a single objective function is built up of the weighted sums of all the others. This suffers from all the drawbacks of utility functions and presents severe problems for practical applications.

iii. Converting goals to constraints by setting targets. The goal programming formulation appears to be ideally suited since it can cater for any number of goals. It would appear that a definition of terms is required: MAO states (114) that one should be clear about the distinction between "goals - which refer to management desire - and constraints - which refer to environmental conditions under which management makes its decisions" EILONS. disagrees (54) - "Essentially all constraints are goals since they express desirable (or undesirable) modes of operation" KENDALL agrees (95) and has formulated the LP in terms of 'Hard' and 'Soft' constraints to represent 'Musts' and 'Wants' constraint/goals.

Consider the multiple-goal programme:

$$\text{Min} \left(\sum_{k} \sum_{l} D^{-}_{kl} M^{-}_{k} V_{k} + D^{+}_{kl} M^{+}_{k} O_{k} \right)$$

$$\text{s.t.} \quad \underline{A}\,\underline{x} \leq \underline{b}$$

$$\underline{G}_k \underline{x} + U_k - O_k \geq \underline{g}_k \quad \forall k \qquad (1.6)$$

$$\underline{x}, \underline{U}, \underline{O} \geq \emptyset$$

where M^{+}_{k}, M^{-}_{k} are the weights representing the difference between overage and underage; D_{kl} is the weight representing the desirability of attaining goal k in preference to goal l (i.e. an ordering of goals). The objective function is beginning to collect-some of the unfavourable attributes associated with utility functions and trade-offs. Furthermore, the validity of the assumption of additivity becomes questionable - can one add the (weighted) deviation from one goal to that from another? The remarks made by EILON S. highlight this problem and he goes on to advocate the approach of satisficing - any feasible solution satisfying the 'do' and 'don't' requirements is acceptable. (If there is no feasible solution then that itself is information useful for management to set them thinking about their conflicting goals!).

The problems of setting the target (or norm), of dealing with the inevitable internal pressures to achieve always a better performance than previously (ultimately coming up agains the ceiling of the 'optimal' solution) remain. But the attractiveness of the technique has led it to be used widely: starting with breakeven analysis (42) it has encompassed merger modelling (93), production planning (71), and in many areas involving non-financial attributes - e.g. medicare, government, and academic planning (104).

Clearly a modelling technique that allows complex fractional, multi-goal models to be solved with ease goes a long way to meet our objectives and this forms the basis of our selection of this technique.

3.4 Integer Programming

In reality, a number of the variables in any corporate model, require to be integral (e.g. number of products made or sold, investment projects, etc.). Many computer based mathematical programming systems can now cater for large mixed-integer problems (see Chapter 2, (63)), but the solution process still remains cumbersome and lengthy (see Appendix in (147)). Furthermore, the economic interpretation of the dual programme is lost.

4. Corporate Models

Before developing our own model we inspected a number of models described in the literature.

4.1 The KROUSE Model

The global corporate model, developed by KROUSE (101), centres on a "multi-attribute criterion function to measure financial performance, and state-transition equations to impose a variety of behavioural, technical and accounting-identity relationships which set out the firms financial process from one period to the next".

Let the state of the system be defined by a N-vector $\underline{X}(t)$. The state variables X(t), defining the company's profile at a moment in time t, are attributes such as profitability, liquidity, capital structure, etc.

Let $\underline{d}(t)$ be the M-vector of decision variables open to management control, e.g. increase long-term debt pay out dividend, etc. Let $\underline{u}(t)$ be some disturbance vector to reflect uncertainty. Then the model formulation is:-

Optimize $E(\underline{G})$ The expected value of some multi-objective performance index

s.t.

$X(t+1) = f(X(t), d(t), \underline{u}(t), t)$ the financial process

$\underline{X}(0) = x^o$ initial state

$\underline{h}(\underline{X}(t), \underline{d}(t)) \geq 0$ policy/institutional constraints.

(1.7)

The above model is not constrained to be linear, nor deterministic, and KROUSE claims to have established procedures for solving such general models in his thesis.

We have rejected this model as being too abstruse. The decision variables described above are too far removed from the activities over which the management can excercise direct control (eg. a production plan, stock levels, levels of drawdown of finance facilities etc). The behavioural and regulatory constraints suggested (e.g. the share price and stock market valuations, etc) are too ill defined to be expressed explicitly. Furthermore, while the quadratic objective function propounded by KROUSE may have certain appealing attributes for the management scientist - it can encompass diminishing marginal returns and trade-offs between objectives-it was felt that the resulting complexity would further deter management of the company from accepting the model.

4.2 The LAHIRI Model

LAHIRI developed (103) a basic linear production-planning model for a manufacturing company. The large number of products - 80 - made by the company were collected into a smaller number of product groups, and these were processed through their production facilities. The company aimed to maximize the contribution from production, subject to capacity and demand constraints.

This approach appears to be amenable to the application we described above (Section 3) and this model will be compared with our own model formulation described below.

4.3 The BP Model

BP have developed (17) a mixed linear-integer model of their refining and marketing operations. The model incorporates

i) a description of currently available resources
ii) estimates of demand and price for each product
iii) estimates of marginal costs
iv) capacity required by existing processes
v) future investment possibilities.

The model determines the optimal production and marketing plans together with the required investment profile for the entire planning period that will maximise the present value of the net after-tax cashflow (that the account of debt and equity servicing).

We adopt a similar direct approach in modeling the company's activities. The model described below is retained to short-term planning (assuming a given capacity availability) and we extend this to the medium-term in the following Chapter. A point of divergence between the models lies in the definition of the goals pursued by the Company.

5. The Imperial College Model

The corporate model that we have developed (first formulated by KORUBLUTH (98) and since augmented ((70) and (24)) is based on a similar approach to LAHIRI'S.

The model is a multi-period, deterministic, linear model of the activities of the company. These activities centre on the production required to meet some sales demand, and the financial implication of any selected plan. Since the function of this model is to aid budget preparation and control, time periods of one month have been chosen. This is long enough to render unnecessary the need for attention to the finer details and problems of day-to-day scheduling, and yet is short enough to give a detailed plan of the coming year's activity.

The physical activities are shown in Fig. 1.1: raw materials flow through a series of identified work-centres, which are manned by some labour group, end up as finished goods, and are then despatched to the customer.

The financial activities, shown in Fig. 1.2, are modelled in accordance with the marginal costing concept. Financial flows are distinguished between five types:

a. Fixed ("Overhead") costs required to keep the facilities open for business. This includes rent, rates, possibly electricity (though this could be included in type b), and normal time wages if the company does not act on a hire-fire basis. While in the final analysis no cost

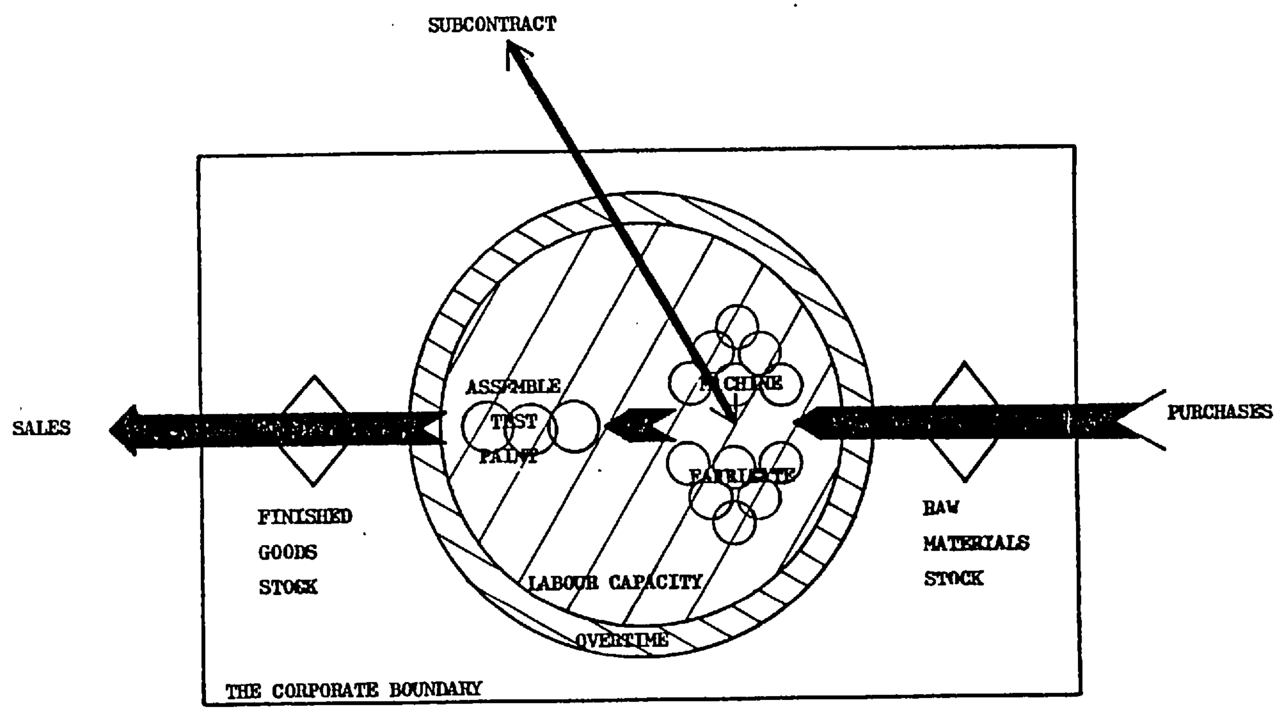


FIGURE 1.1. THE MANUFACTURING PROCESS

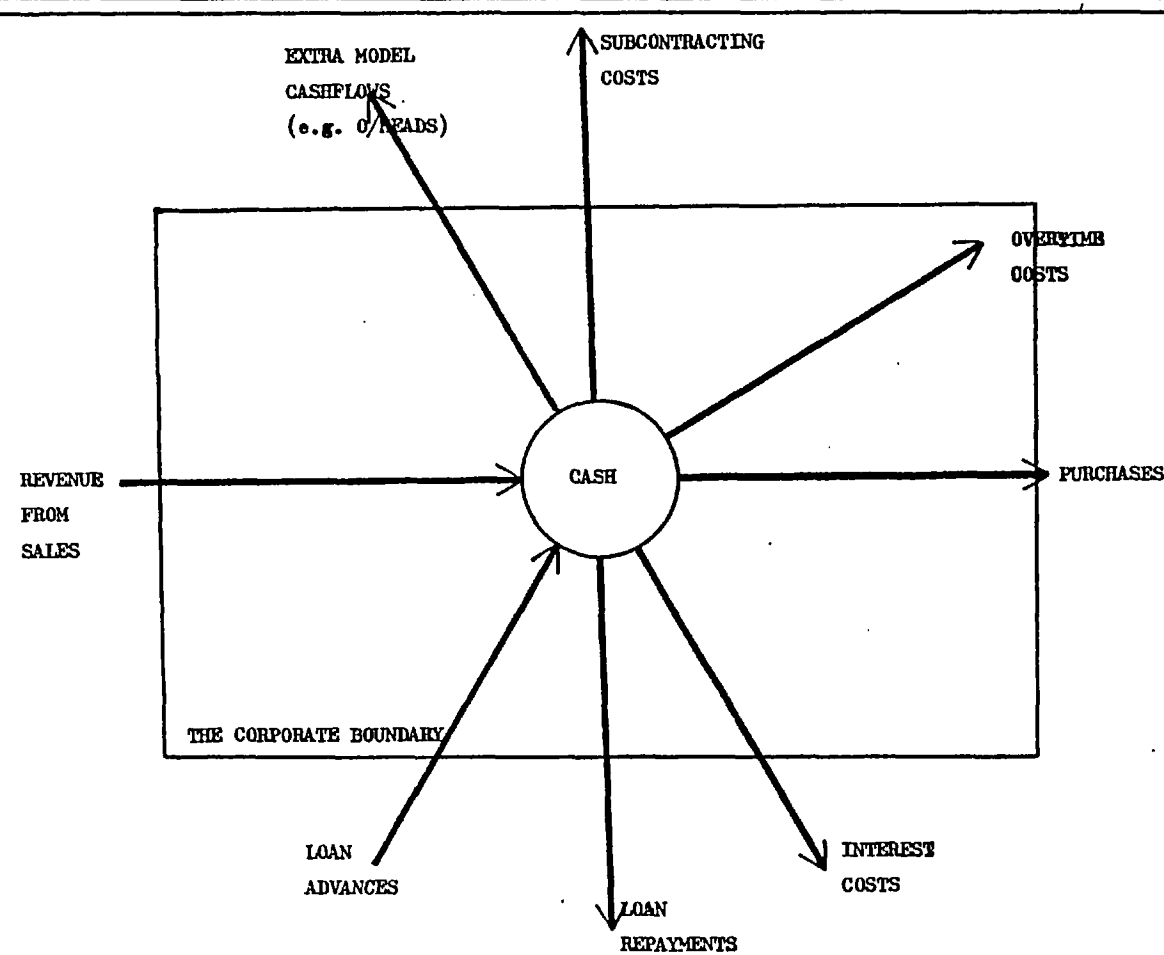


FIGURE 1.2 CORPORATE FINANCIAL FLOWS

can be considered to be irrevocable, in defining a cost to be 'fixed' the assumption is made that the business intends to operate.

b. Operating Costs. These are costs incurred which depend on the volume of production, but which cannot be allocated to the individual finished product - e.g. overtime payments, subcontracting costs, maintenance, etc.

c. Marginal or unit costs. This is made up of material costs, warranty, etc. - all costs that can be identified with each unit of production.

d. Revenues from the sales of finished goods.

e. External financing arrangements. In this test case the company has only one source of external finance (namely its parent) and this can be considered as a (bank) loan facility.

Having defined these costs, one is left with the problem of extracting this marginal costing data from the information available to the firm, usually in standard cost form. This is not an insignificant problem, but it is not one that I wish to persue here (see RAINE (126) for a full treatment of associated problems and solutions).

All of these financial activities are considered explicitly in the model since the timing of these flows influence the optimal plan, (i.e. a plan made when only some of these flows

had been considered, as in the LAHIRI model).
In this way the financial aspects of the company are directly linked to the physical activities undertaken by the company: physical events that the management controls.

6. Model Formulation

The following sections give a detailed explanation of the structure of the model and the modifications required to ensure that it can be implemented. Details concerning the problems of implementation are to be found in Appendix A.1, and a summary of the equations, together with definitions of the variables used and data required by the model, are given in Appendix A.2 and A.3.

6.1 The Production Function

(i) Workcentre Capacity Restrictions

Production required by any plan must comply with capacity restrictions. For each factory a number of distinct work-centres was identified and the time required by a unit of product on that work-centre was established.

This time was established from standard times in hours (used in calculating the standard costs). While it is true at marginal utilisation may result in different times, it was felt that using such 'average' times was reasonable for short medium term planning purposes.

At the time of formulation there was no interchange of production between the similar work-centres at both factories and so they are considered separately: welding at Rhymney and Great Yeldham comprise two work-centres - JCB model production will make use of the latter, while HYMACs will only be produced at the former.

The model will compare the requirement for a particular work-centre against its capacity and ensure that only feasible plans are allowed. This simple statement must be qualified by the following points:

Firstly it was deemed desirable to split the HYMAC products into three constituents sections - Crawler, Platform and Front-end. This was done in an attempt to be able to cater for the number of different options available with each product (e.g, a wide-tracked Crawler with a long-armed Front-end)

Secondly it is necessary to establish the length of the production cycle for each product and then to establish the requirement by each product for each work-centre in each period of its production cycle. Then the total work-centre requirement in any period is calculated from the production planned to be completed in that period multiplied by the requirement for that work-centre in the last period of the production cycle, added to the product of the production planned to be completed in the next period and the requirement in the last but one period of the production cycle etc.

Thirdly, normal work-centre capacity is calculated by multiplying the number of hours in the normal shift time and the number of units that comprise the work-centres. The number of units need not be integral - if there are three good lathes and one poor one, which is seldom used because of breakdown problems, then this latter machine may be included by considering it as a fraction of one of the good machines. The resultant work-centre capacity is finally down-graded to take account of activities that lie outside the scope of the model: emergency breakdown and maintanance; manufacture of spares and of the exceptional product not included within the product range.

The equation then reads:

> Requirement for work-centre M in period I must not exceed the normal-time capacity of that work-centre in that period plus any overtime worked plus any subcontracting done. (1.8)

(ii) <u>Labour Force Allocation.</u>

In order that work-centres be available for production they must be manned. The number of men of any labour-type (i.e. skilled, semi-skilled, or unskilled) required to man each work-centre was found and was found to be independant of the product being worked at the time. It was established that some work-centres could only be manned by a particular labour group, whereas other work-centres could be manned by more than one type. In these circumstances a direct allocation of men to work-centres is required. The equation reads:

> Total in-house production (i.e. total work-centre requirement less any work subcontracted out) on work-centre M in period I must be allocated amongst the labour groups capable of doing the work. (1.9)

6.2 Physical Constraints

(i) Production

Production is also limited by the availability of jigs required to load the job onto the work-centres. (The problem concerned with production cycles that are greater than one period are dealt with in Section 8.3). This may be considered to be a 'soft' constraint since the limit could be increased fairly easily (by the acquisition or manufacture of extra jigs). However, this does not detract from the validity of constraint being imposed in the model - if this constraint were to limit the achievement of the objectives severely then this ought to be reflected in the dual LP. The equation reads:

> Limit production (of division J) of product K in period I by some upper bound. (1.10)

(ii) Labour Force Capacity

Having allocated the in-house labour requirement amongst the labour groups, the model further requires that the Labour Force Allocation does not exceed available capacity. Observation has shown that the company is not a 'hire-fire' type and is thereby committed to a given labour force (which may be augmented in a regular fashion in periods of heavy demand). Overtime variables are explicitly included in both this equation and the Work-Centre Capacity equation to ensure that when either the labour force requirement or the work-centre requirement reach their respective upper bounds then both machines and men switch to working overtime. The equation reads:

> Requirement for labour group L in period I must be less than or equal to the work scheduled to be done in overtime plus that scheduled to be done in normal shift time. (1.11)

(iii) Labour Overtime Capacity

Clearly the labour force will not accept too great an over-time burden. Thus the equation reads:

> Limit the total overtime load on labour-group L in period I by some upper bound. (1.12)

If some minimum overtime scheduling has been promised to the labour force then this requirement can be included as a lower bound.

(iv) Market Constraints

The model tries to construct a plan to meet some demand condition. This is a forecast made by the company of their expectation of their share of the market for the goods they produce. To give the model some freedom these conditions are expressed as upper bounds on the sales variables. (The possibility of dealing with situations where the elasticity of demand differs from one, is discussed in Section 7.4).

The possibility exists that the model will yield a solution that declares a particular product to be undesirable in terms of the stated objectives. In other - words the model suggests that no volume of that product be made nor sold. If this is unacceptable for any reason (for example the company may desire representation in all sectors of some market) then the Market Constraints can be augmented by some lower bound condition to ensure such representation as is deemed necessary. The equation reads:

Limit sales of product K in market O in period I by some upper (and possibly lower) bound. (1.13)

(v) Storage Capacity

Storage of raw materials and finished goods requires space and the available space is limited. If these different items are kept in different storage modes, then a number of different storage areas can be considered separately. A problem might also exist with work-in-progress as it moves through the production process (especially at Rhymney which is batch oriented). No such problem was found to exist: work-in-progress can be stored anywhere on site (even gangways if necessary); finished goods are stored outside as are most raw materials. However, a general company model would include the equation.

Storage space required in period I must be less than or equal to storage space available. (1.14)

6.3 Financial Flows

(i) Cash Position

The model considers that all cash transactions resulting from the physical activities pass through some central cashbook. The cash transactions are of two types:

a. Those that relate directly to activities within the scope of the model e.g. purchase of raw materials resulting in some outflow of money; a bank loan resulting in some inflow etc.

b. Those that relate to activities outside the scope and direct control of the model. These extra-model cashflows comprise all overhead payments (including normal payment of the labour force), net income from sales of spares, trade-ins, etc.

Both the quantity and timing of these flows are important facets of the plan. For this reason time-lags obtained before paying creditors and time-lags to debtors have to be established to the nearest integral number of periods. The equation reads:

> Cash at the close of period I equals cash position at opening of period I plus inflows resulting from sales made in previous periods and loans negotiated, less outflows resulting from loans repayed; overtime payment for work done in that period; subcontracting costs, purchases and bank charges incurred in previous periods and net out-flows from extra-model activities. (1.15)

(ii) Creditors Account

Because of credit facilities that the company has been able to establish, it will owe money for activities previously under-taken but not yet paid for, The credit account is reflected by the equation:

Credit at close of period I equals purchases of raw materials made but not yet paid for, plus subcontracted work carried out but not yet paid for, plus interest charges outstanding, plus any extra-model credit. (1.16)

(iii) Debtors Account

Similar credit facilities are allowed to the company's clients, and so the debt position is modelled by:

Debt at the close of period I equals revenue due, but not yet received, from sales made in prior periods plus any extra-model debt. (1.17)

6.4 Inter-period Continuity Equations

These equations are included to ensure internal consistency of the multi-time period model.

(i) Cash Continuity.

The cash position at the close of one period equals the cash position at the opening of the next. There is no need to explicitly model this equation since suitable substition in the Cash Position Equation above (1.15) will result in the elimination of the variable "cash at opening of period I".

(ii) Finished Goods Stock.

There is a physical requirement that product be in stock in order that it can be sold. In other words, the number of items in stock must never be negative. A further assumption is made that a unit of product completed in any period is eligible for sale in the same period. This is expressed as:-

> Stock of product K at the close of any period equals the stock at close of previous period plus newly completed production less sales made in the period. (1.18)

(iii) Raw Materials Stock

A similar 'restriction' applies to the stocks of raw materials:

> Stock level of raw material R at the close of any period equals the stock level at close of the previous period plus any new purchases less any amount used in production during the period. (1.19)

6.5 Objective Functions

Three different objective functions were proposed and tested:

(i) Profit Earnings.

The concept of profit is one that will evoke considerable discussion in Chapter 3, but since this short-term model assumes a constancy (or some known expansion) of the physical resources endowed to the firm, the amount of cash, or cash equivalent, accumulated during the planning horizon can be attributed as profits earned as a result of the planning decisions accepted at the begining of the year. Thus:

Maximise cash plus debtors less creditors positions at the close of the planning period. (1.20)

(ii) Turnover

Maximise total sales revenue accrued during the planning period. (1.21)

(iii) Sales Penetration

This objective function was proposed with the view of increasing market share in order to market spares in the future:

Maximise the total number of units of primary product sold during the planning period. (1.22)

6.5 Modification Required

The equations of the model just described above relate to a general period within the planning horizon. However, periods at the beginning and the end of the model will have to bear some extra conditions in order that they be a more realistic representation of the activities of the company.

6.5.1 Starting Conditions

In order that the planning process be an on-going system the first (few) periods of the model must reflect conditions as they exist at that time. Certain decisions taken in periods prior to the commencement of the modelled period will control the levels of a number of the variables of the model:

a. Production variables. If the production cycle is greater than one period then work started in previous periods already determines the level of finished product output in the first (few) periods of the model. For example, if the production cycle is 2 periods, then the amount of finished product that can be completed in the first period of the planning horizon will depend entirely on how many units were started in the previous period. Even if the production cycle is only one period long, a lower bound on production is determined by the amount of work in progress at the beginning of the planning horizon.

b. Stock levels. The number of items in stock at the begining of the planning period of both finished goods and raw materials is fixed by previous actions.

c. Purchases. The model takes no account of the lag encountered between ordering and receipt of raw materials - a lapse of time that may in fact be quite substantial. Therefore purchases may well be fixed for a number of periods into the planning horizon.

d. Financial position. The opening debt, credit and cash positions will be fixed as will the extra-model financial flows.

In practice these starting conditions are quite easy to establish.

6.5.2 End Conditions

If the model were left to run in the state described above, the solution would suggest that the company run itself down at the end of the horizon. In other words the stock levels would be reduced to zero and no new production would be undertaken. This problem, relevant to all multi-time period models, is clearly critical.

GRINOLD (72) and HOPKINS (84) (among others) have tried to circumvent this problem by formulating an infinite horizon model. As a prototype GRINOLD uses

$$\begin{aligned} \text{Max} \quad & \sum_{t=o}^{\infty} a_t X_t \\ \text{s.t.} \quad & A_t X_t \leq b_t + \sum_{s=o}^{t-1} K_{ts} x_s \\ & X_t \geq 0 \qquad \forall t \end{aligned} \tag{1.23}$$

where K_{ts} x_s represents some inter-temporal relationship.

The dual programme is

$$\begin{aligned} \text{Min} \quad & \sum_{t=o}^{\infty} \pi_t b_t \\ \text{s.t.} \quad & \pi_t A_t \geq a_t + \sum_{s=t+1}^{\infty} \pi_s K_{st} \\ & \pi_t \geq 0 \qquad \forall t \end{aligned} \tag{1.24}$$

He shows that the dual solution only has meaning when $\sum_{s=t+1}^{\infty} \pi_s K_{st}$ converges - a rather arbitrary restriction on the problem!

HOPKINS attempts to solve the problem of equipment replacement with an infinite horizon approach. His procedure for solution is as follows: having formulated the infinite horizon model, truncate at some finite horizon date T. This requires some statement about the terminal wealth to be included in the objective function which is done according to a specific formula. Then the solution to the finite problem can (under certain sufficient conditions) be used as a solution to the infinite problem. In the model he describes, it becomes clear that GRINOLD's convergence requirement becomes translated as the need for expansion of capacity.

It seems unlikely that our corporate model will satisfy the condition of convergence of the intertemporal relationships. Instead we have tried to overcome the horizon problem by demanding that certain end conditions be met:

a. Stock levels. A minimum level can be imposed on the final stocks of finished goods and raw materials.

b. Production. If the production cycle is longer than one period, then in order that there be continuity, the model is forced to start production within the planning horizon so that the flow of finished goods may continue in post-horizon periods. This is ensured by either of two approaches:-

One is to make some forecast of the production required to be completed in the first (few) post-horizon periods and then reduce the capacity of the production facilities in the last (few) periods of the model by the amount required to have work started on that production.

This would mean that post-horizon production requirements are met in the normal shift time. But owing to the possibility of allocating different labour groups to man the machines it is not possible to reduce the normal-time labour capacity by the required amount. The simple solution to this dilemma is to introduce a dummy product that will only be produced in the last (few) periods of the model, and that will require the use of production facilities to the extent that is determined by the post-horizon production forecasts. In this way the work is allocated among the work-centres and labour groups in the same manner as all other work.

The second approach is to make use of unequal time-periods. This was first suggested by ORGLER (124) in a model formulated to aid cash management decisions. The planning period of his model spanned a year but required day-by-day decisions.

Since the model would be too big to handle on that basis, he suggested that the length of the period should increase as the horizon is approached.

This method could be adopted with our model: periods close to the horizon would be longer than those at the start, with the length of the last period being at least as great as the longest production cycle of the products produced. In this way continuity of post-horizon production is ensured since production can then be completed within a single period.

Clearly these end conditions that are imposed on the model are quite arbitrary in nature and will interface to some degree with the planning freedom of the model. The extent to which the plan proposed by the model differs from the 'optimal' plan due to the imposition cannot be determined and a great deal more theoretical work needs to be done in this area. The manner in which this problem has been met in three-fold:

Firstly extend the planning horizon and hope that the deviations will iron themselves out by the time they work back from the extended horizon to the desired planning period.

Secondly alter the end conditions and use only those plans that can handle the widest variety of situations, i.e. stable solutions are sought.

Thirdly use the plan for the first few periods only. The model has been designed for budget preparation and control and it is envisaged that it will be rolled forward and rerun every period.

A report on the results of using this model in the test-case company is included in Appendix A.

7. Further Sophistication

Section 5 above reflects the current status of the model. The activities of the firm that have been modelled are quite general in character and the model can form the basis of a general manufacturing company model. Topics that have not been expanded, owing to the particular nature of the company use as the case-study, are the following.

7.1 Managerial Constraints

As the case-study company is left very free to make its own decisions, this area has not been developed to the extent that it might. Other companies may be required to report on a number of different criteria and may therefore wish to include certain minimum achievement levels of these key factors. They might include a minimum debt to credit ratio, a minimum turnover level, some maximum interest payment, etc. Other managerial constraints may reflect a desire for a smooth production plan, a growing profit profile, etc. These requirements self-imposed by the management, are quite easy to establish in practice and to include in the model.

7.2 External Financing

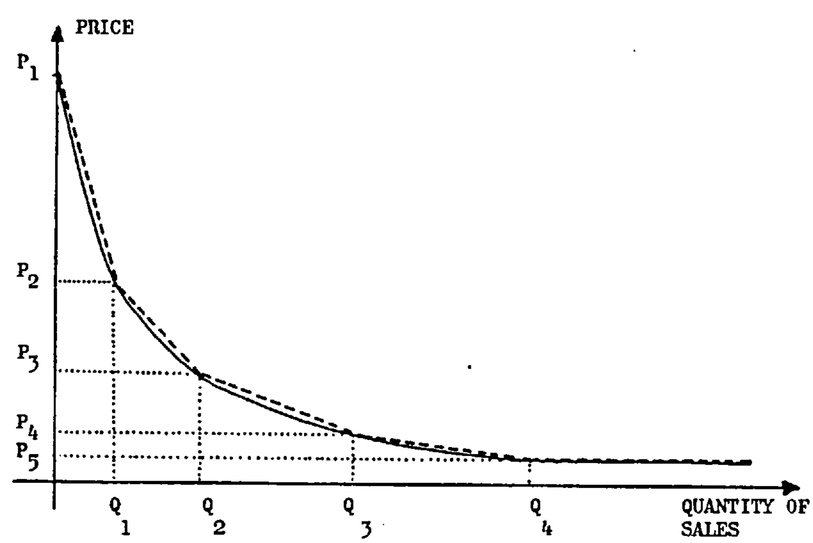
As the company is a wholly owned subsidiary, it has only one source of money. This has been modelled as "Bank Loans". The general company will have a number of sources of raising money (and a number of alternatives of investing any excess cash) and these can be incorporated without too much difficulty (see Chapter 4 (142)). Assessment and payment of taxes (and dividend payment) can also the included.

7.3 Inflation

Prices have been assumed to be constant over the planning horizon but these can be factored by some inflation rate if desired. The rate (either the Retail Price Index or a more specialised sector price index) may be constant, or may itself change over the planning horizon. The prices of one's own products may or may not be allowed to rise depending on Price Commision controls.

7.4 Sales

The model acts on a given, fixed demand and price vector. However, if the demand curve can be ascertained this can be included in the model by piece-wise linearization. The demand curve is approximated to the step-wise function shown in Fig. 1.3. The product is then subdivided into a number of pseudo-products, each having a production and materials requirement identical to the original product.



PSEUDO PRODUCT 1:($Q < Q_1$) :

No of units for sale $< Q_1$ at price $\frac{P_1 + P_2}{2}$

PSEUDO PRODUCT 2: ($Q_1 > Q < Q_2$):

No of units for sale $< Q_2 - Q_1$ at price $\frac{P_2 + P_3}{2}$

etc

FIGURE 1.3 PIECE-WISE LINEARIZATION OF THE DEMAND CURVE

Each pseudo-product will be priced in accordance with the demand curve shown, and their sales will be similarly restricted to the levels. Then provided the demand curve is downward sloping, the hill climbing search process of the simplex algorithm (used for solving the LP) will ensure that the higher priced pseudo-products are sold first.

8. Assumptions of LP in Planning

Three major assumptions have been made in the formulation of our corporate model and these must be borne in mind when interpreting the results of the model.

8.1 Determinism

The model makes no attempt to deal with the stochastic nature of real life: unique values (for sales forecasts, productions times, etc.) are used in place of samples from some probability distribution. It is maintained that post-optimal sensitivity analysis will test stability of the solution to variations in certain elements of the data. However, this is, in general, limited (e.g. (16), (98)) to parametric programming on the right hand side (RHS) vector $\underline{b}$ of the matrix (corresponding in the main to data on capacities).

Due to the very nature of the model being multi-time period, variations in other data will typically change a number of elements of the $\underline{A}$ matrix simultaneously. This makes sensitivity analysis much more complex:

FLAVELL shows (62) how the effect of change in any underlying datum on the objective function realization can be measured. It has been shown that at any basic feasible solution

$$\frac{dZ}{d\alpha} = \frac{d\underline{c}}{d\alpha} \cdot \underline{x} - \underline{\pi} \cdot \frac{dA}{d\alpha} \cdot \underline{x} + \underline{\pi} \cdot \frac{db}{d\alpha} \qquad (1.25)$$

where $Z = \underline{c}.\underline{x}$ subject to $\underline{A}\ \underline{x} \lesseqgtr \underline{b}$, $x \geq o$ with corresponding duals π. α is any parameter.

If α is chosen to be the datum then if $\frac{dZ}{d\alpha} = 0$

small changes in α will not effect the solution. (Similarly, if α is known deterministically then sensitivity analysis is not required). But if α is uncertain (with a standard deviation σ_α about a mean value that has been used to calculate the relevant coefficients in A) then

$\left|\frac{dZ}{d\alpha}\right| \sigma_\alpha$ is a measure of the importance of the datum, and allows datum to be ranked so that management can know on which particular index to focus their attention.

8.2 Linearity

The error produced by linearizing a non-linear environment need not provide a solution better than a randomly selected initial point. This is hardly a surprising result but serves to focus attention on the critical problem of linear approximations.

First the case when approximations are made with too little evidence. Fig. 1.4 shows a common example:

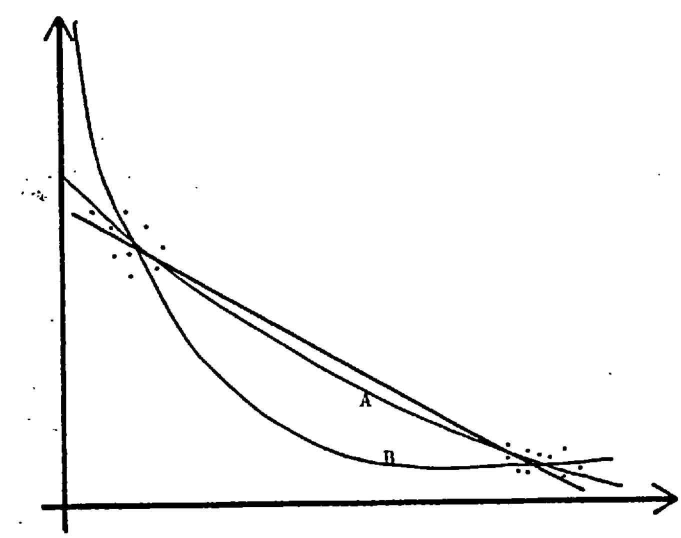
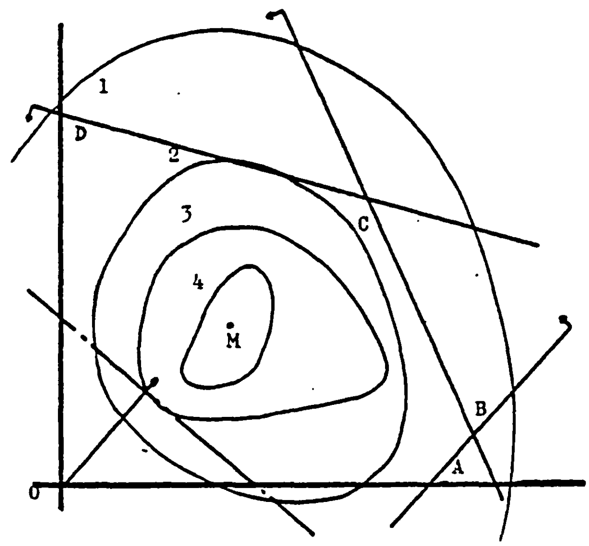


FIGURE 1.4 DANGERS OF LINEAR APPROXIMATIONS

experience offers data about two locations only. If the true demand curve were A then the linear approximation may be acceptable, but if it were B then serious errors would be introduced into the model.

Secondly, BAUMOL & BUSHNELL (10) show (see Fig. 1.5) the result of imposing a linear model on a non-linear programme.

The iso-profit curves rise to a peak at M. A profit maximising linear programme would select point C. Yet any point inside the iso-profit curve 2 would yield a better solution!



1.5 DANGERS OF LINEAR OPTIMIZATION

Much of the structure of the corporate model described in Section 5 is based on input-output relations (which are linear). The number of 'potential' nonlinearities which are constrained into a linear form is small:

(i) Batch production and setup times. This cannot be modelled explicitly without recourse to integer or separable programming. Instead, assumptions about average batch sizes have been made and the setup times, divided amongst the batch, have been included in the work-centre requirement for each product. This approximation is not thought to present too severe a distortion in this company.

(ii) Indivisibility. Linear programmes assume that the problem is continuous over all its variables. Yet in practice the number of items sold or produced must be integral and interpretation of fractional solutions is difficult.

8.3 Passage of time

The passage of time over the planning horizon is assumed to progress in jumps from one period to the next. All the events within a period are considered to occur the split second before the period is over there is no flow of time. In reality, however, production is a continuous processs with new products being started, completed and sold throughout the period.

This incorrect modelling of real life behaviour may result in the number of any product being completed in any period being wrong since the model can only recognise production being started at the beginning of any period. This in turn results in the Work Centre and Labour Force Requirements (Equations 1.8 and 1.9) and the Stock Equations (1.18 and 1.19) being incorrect.

Using a very long time period would reduce the error since the number of products actually in process of manufacture at the close of any period would be small compared to the number of units completed. However, long periods result in a loss of detail (that might hide, for example, a peak in stockholding which would exceed the available storage capacity).

Using a very short time period would also eliminate this problem since the start of production may be accurately pinpointed. But this too has its problems: a short time period involves modelling the day-by-day variabilities (and necessary contingency actions) of the production process. Some compromise must be reached!

8.4 Constant Returns to Scale

The construction of the Objective Function (OF) implies that the problem is concerned with optimizing a function of constant returns to scale. In our particular model this causes no problems since the formulation requires the optimizing of a (number of) variables. Where non-constant returns to scale may apply (e.g. the demand curve), the formulation can be adjusted to include such factors - see Section 7.4 above.

9. Conclusions

Our objective in this chapter was to search for an appropriate formulation of a corporate model that would aid management in their task of budget preparation and control. Such a model has been constructed and despite the presence of some non-linearities, we have opted to use Linear Programming as the model solution technique. This choice is based on the ease with which large models can be solved and the sensititity of the solution tested to a variety of assumptions about the future, and the derivation (from the dual LP) of a means to value the inputs to the model.

CHAPTER 2

LONG-TERM CORPORATE PLANNING

1. Introduction

Long-term planning has been researched by both qualitative and quantitative approaches. The former concentrates on the system of planning (92) the methodology (5) and control (53) aspects of the task of deciding where one wants to go and how, in general, to go about it. It encompasses analysis of business goals, of strategic business policy and of liaison and communication within the organisation.

The latter approach, with few exceptions (e.g. (128)), has concentrated on the tactical decision concerning acceptance or rejection of individual long-term ventures, and it is with this area that this chapter is concerned.

We first inspect investment appraisal methods that are widely used, but that are found deficient in practice. The corporate model described in the previous chapter is then extended to encompass long-term decision-making in the framework of the manufacturing company as a going concern. The detailed equations are contained in Appendix B.

2. Widespread Investment Appraisal Techniques

2.1 Payback

The payback period is the number of time periods that elapse from the initiation of the project till the capital outlay in recouped. Used as a selection procedure it recommends acceptance of projects in order of increasing payback. As a means of evaluating long term investments it has been widely condemmed by academic writers since

- it only considers flows of funds for a portion of the life of the project

- it takes no explicit account of the time-value of money

- it takes no account of the relative sizes of the projects.

Despite this, payback is widely used and WEINGARTNER (146), in an attempt to assess the reasons for this, makes the following points:

i. It is important to note the difference between using a single figure of merit as the <u>sole</u> basis on which the decision is made and using it as a means of communication within the organization. In addition it is important to differentiate between use of a merit figure as the criterion for choice and its use as a constraint (in conjunction with others) on the selection of projects.

ii. LEVY proved (107) that, in the (artificial) case of an infinite project with regular return, the payback period is the reciprocal of the internal rate of return. Thus for long-lived projects having regular income, minimising the payback is approximately equivalent to maximising the internal rate of return. The increasing importance of capital tax-allowances which distort the cash flows in the first (few) periods, has resulted in fewer projects approximating to the infinite, regular project.

iii. Acceptance of projects with short paybacks is a manifestation of the company's desire to maintain liquidity. In minimising his 'lost opportunity risk' the investor attempts to return to his original situation as quickly as possible in order to be in a position to accept other attractive investment possibilities that may arise. This approach, however, takes no account of the (hopefully improved) change in company's circumstances resulting from previous investments.

iv. Payback seems to be used widely as a break-even concept: it identifies that point of indifference beyond which the management expect an accounting profit to be generated. The qualification of 'accounting profit' is used advisedly since break-even analysis implies an (arbitrary) allocation of overhead costs and managerial effort over some portion of the life of the project. However, it must be noted that for projects of a given net present value the shorter the payback period of the selected project, the sooner will profitability be known (enabling the manager to receive the reward of his wise decision!).

v. Payback is a relatively stable measure under random variations of the cashflows. It is arguable whether this represents an advantage - stable measurements, while reassuring, are useless if they are insensitive to changes in the events they attempt to illuminate.

It is clear that payback can yield useful information to management, but when used as a criterion for project selection it represents a simplistic analysis of long-term investments and must remain in the class of rule-of-thumb techniques.

2.2 Discounted Cash Flow (DCF)

In an attempt to take account of the time-value of money arising over the entire lifetime of the project, two measures have been proposed:

Net Present Value (NPV)

$$NPV = \sum_{t=1}^{T} \frac{c_t}{(1+r)^t} \tag{2.1}$$

Internal Rate of Return (IRR)

$$\sum_{t=1}^{T} \frac{c_t}{(1+R)^t} = 0 \tag{2.2}$$

where c_t is the net flow of money in period t; T is the lifetime of the project; r is the discount (interest) rate (kept constant for simplicity), and R is the internal rate of return.

Proponents of the DCF approach argue that the firm should act to maximise its NPV in order to best represent the interest of its shareholders. HIRSHLEIFER, in his classical paper (83) on the theory of optimal investment decisions, emphasises Fisher's conclusion that no search for "a rule or formula which would indicate optimal investment decisions independently of consumption decisions" can succeed. Investment takes place only because one is able to consume more (than one could if no investment occurred) at a later date.

HIRSHLEIFER's approach was to use isoquant analysis:

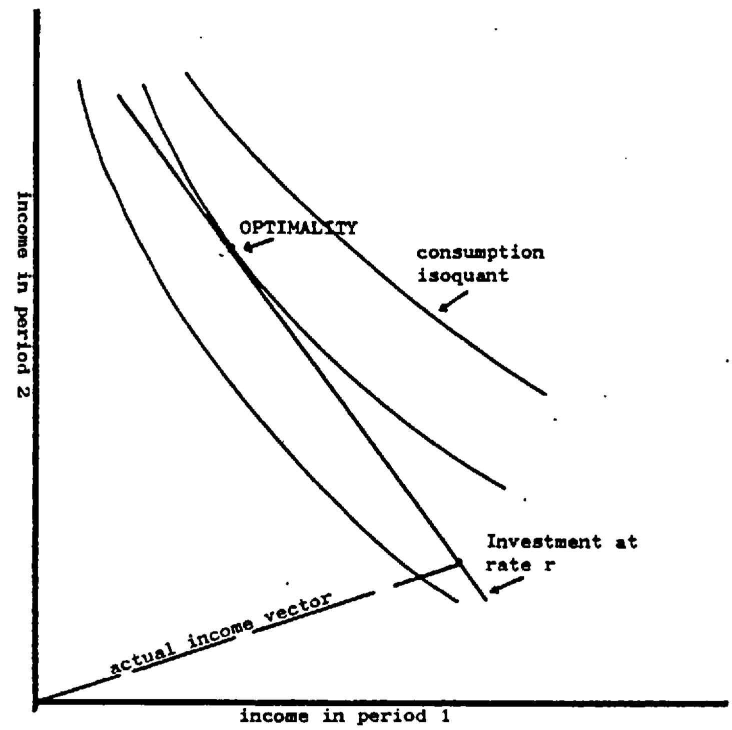


FIGURE 2.1 ISOQUANT ANALYSIS

Fig 2.1 shows 'typical' indifference curves of consumption over two periods. Transference of consumption from one period to the next is at a rate of interest r, and the point of tangency is the optimal pattern of consumption and investment. The analysis is extended for market imperfections of different borrowing and lending rates and HIRSHLEIFER concludes that maximising NPV is correct for <u>independent</u> projects.

With this crucial qualification HIRSHLEIFER excludes the most important category of investment opportunities from his analysis. Furthermore, he gives no hint as to the manner in which the isoquants can be established, and he makes no distinction between the company/entrepreneur and its shareholders - due to market imperfections these people will be treated differently (i.e. different interest rates will apply) resulting in a multiplicity of solutions.

Realisation that market imperfections play a substantial role in investment decision making has led to a great deal of discussion ((58), (30)) about the validity of the concept of the 'cost of capital', and methods of calculating the discount rate.

This in turn led to emphasis being focused on the IRR - it is calculated independently of any 'cost of capital' and is easily understood by management as the 'return' that the project will yield. However the IRR suffers from a number inconsistencies:

i. Calculation of the IRR assumes that excess funds can be re-invested for the life of the project at an interest rate exactly equal to the IRR rate. This is extremely unlikely to occur in practice.

* In two classic papers (119A and 119B) MONTALBANO, ROBICHEK and TEICHROW attempted to answer the problems posed by multiple roots by proposing that such projects were "mixed" projects - projects that were alternately pure financing projects (i.e sources of finance for the company) and pure investment projects (i.e sources of return for the company). They advocated that cash inflows should be treated differently from cash outflows.

Had these arguements been put forward to improve the methodology of calculating NPV's I would wholeheartedly concur but I can see little merit in trying to explain away multiple roots which arise directly from the algebra of Equation 2.2.

ii. Solution of the IRR equation yields T (the number of time periods that make up the project horizon) different values for R (see equation 2.2). Ignoring imaginary solutions one is left with as many real solutions as there are changes in sign in the net-cash flow vector, and the analyst is left in a quandary about which rate to accept, and about the meaning of the other rates.*

In a strong attack on the widespread use of DCF techniques ADELSON (1) emphasises the (often ignored) assumptions that must be shown to hold before DCF techniques can be used.
These are:-

i. <u>that the cashflows can be forecast over the life of the project with certainty</u>. Very little work has been done (see Chapter 1.3.2 and Chapter 8, (114)) dealing with situations where this assumption is recognised not to hold true - thus excluding from analysis the vast majority of all 'real' investment decision-making.

ii. <u>that a perfect capital market exists</u>. The DCF requirement that a firm acts to maximise its NPV on behalf of its shareholders (who for the purpose of arguement arrange their own financing) should result in a certain uniform

Price-Earnings ratio applying to all companies. That this is not so is manifestly obvious. Capital markets are imperfect: they differentiate between lender and borrower and according to the 'quality' of the market entrant; rationing exists; and they take account of the time span and riskiness of the ventures presented.

iii. <u>that the projects are strictly independent</u>. In his paper ADELSON shows how a project reacting with itself, interferes with the DCF calculation. The problem concerns the acquisition of extra capacity to cater for a steadily increasing demand. The 'optimal' size of capacity increases selected will depend on whether each purchase is considered individually or whether the increase is considered to be made of a number of equally sized blocks over time. FLAVELL has shown (61) that the total present value of all the installations will never equal the present value of a single instalation except for a single installation of infinite size! In general projects will interact to an even greater extent.

A further problem encountered when using DCF methods is that the technique takes no account of liquidity requirements. Consider the selection to be made between two projects which are independent, where there is a perfect capital market and where the cashflows are known with certainty. Investment A would yield a steady income over its life, whereas investment B yields no income till the end of its life when it recoups its return in a large lump sum. Even were investment B to come ahead on a DCF basis, many investors would prefer A to B because of liquidity pressures.

2.3 Capital Budgeting

Analytical methods to cater for cases that are known to violate the necessary assumptions for DCF analysis have centred on dealing with the problem of the market imperfection of limited capital resources. The problem was first posed by LORIE & SAVAGE (109): select project j in period t while ensuring that the total expenditures c_{tj} do not exceed the budget C_t for each period t.

In the single period - t=1 - (and ignoring problems of indivisibility) their solution was to rank projects in decreasing order of $\frac{y_j}{c_{1j}}$, where y_j is the Net Present Value of project j, and to accept K project in that order till the budget is exhausted.

So

$$\sum_{j=1}^{K} c_{1j} = C_1 \qquad (2.3)$$

At the marginal project K they propose a value λ such that

$$y_K - \lambda_1 c_{1K} = 0 \qquad (2.4)$$

and that

$$y_j - \lambda_1 c_{1K} \geq 0 \quad \forall \text{ accepted projects}$$

$$< 0 \quad \forall \text{ rejected projects}$$

This procedure is not so simple in the multi-time period case (see example on p59 (100)) and LORIE & SAVAGE have to resort to a process of trial and error to find values $\lambda_1, \lambda_2 \ldots \lambda_T$ to establish the cut off project (as 2.4 above)

$$y_j - \sum_{t=1}^{T} \lambda_t c_{tj} \quad \begin{matrix} > 0 & \forall \text{ accepted projects} \\ < 0 & \forall \text{ rejected projects} \end{matrix} \qquad (2.5)$$

The inadequacy of this selection process provided the impetus for the decisive work in this area by WEINGARTNER (144). He formulated the capital budgeting problem as a linear programme:

$$\text{Max} \quad \sum_{t=1}^{T} \sum_{j=1}^{J} \frac{a_{jt}}{(1+r)^t} \cdot x_j$$

$$\text{s.t} \quad \sum_{j=1}^{J} c_{jt} \cdot x_j \leq C_t \qquad \forall t \qquad (2.6)$$

$$0 \leq x_j \leq 1$$

where a_{jt} is the net income from project j in period t; J is the total number of projects under consideration; r is the discount rate; T is the horizon; and x_j is the fraction of project j undertaken. This is limited to be less than one to prevent the programme selecting only the most favourable project and investing in it till the capital is exhausted.

This formulation, taking an overview of the problem, overcomes the deficiencies of LORIE & SAVAGE's procedure but highlights a number of other problems discussed below:

i. Indivisibilities.

In the strictly linear formulation, the budget will be used to its full extent, resulting in proposals that may include factional projects. (The number of such fractional projects is bounded by the number of constraints in the model). WEINGARTNER examines this in two ways. First he suggests a relaxation on the rigidity of the expenditure constraint. Then "it is possible to regard fractional acceptance as a signal for expansion of the ceiling by an amount sufficient to permit acceptance of the marginal project in toto, or to reject it, leaving some funds unemployed". Secondly he suggests that fractional projects are less disconcerting that it may appear at first glance - "in the case of two basically different bridge designs, for example, the solution may appear to call for, say, two-thirds of a steel bridge and one-third of a wooden one. While construction of such an object is a patent absurdity, this outcome may be interpreted in a way that provides still further information on a possible 'optimal' bridge which may be better than either the steel or wooden one..." Such rationalisations are clearly unsatisfactory, and, as we have seen in Chapter 1.34, the procedure calls for integer programming to cater for the problem of indivisibility.

ii. The discount rate. Consider the dual of the linear programme (2.6):

$$\text{Min} \quad \sum_{t=1}^{T} \pi_t C_t \qquad + \sum_{j=1}^{J} \alpha_j$$

$$\text{s.t} \quad \sum_{t=1}^{T} \pi_t c_{tj} + \alpha_j \geq \sum_{t=1}^{T} \frac{a_{jt}}{(1+r)^t} \qquad \forall_j \qquad (2.7)$$

α_j arises from the limitation on project acceptance (x_j 1) and it can be shown to relate to the cutoff factor ( ) in LORIE & SAVAGE's selecting procedure. π_t is used to 'evaluate' the t'th budgetary constraint - it is the marginal (or opportunity) value of capital in that period. This poses the question of what value to use as the discount rate?

ELTON argues (57) that the budgetary constraint is externally imposed on the company and (following HIRSHLEIFER's isoquant analysis) proceeds to relate the discount rate to the shareholders' indifference curves. BAUMOL & QUANDT (11) take precisely the opposite view: they argue that capital rationing is internally determined and that the discount rate ought to be the true marginal opportunity rate (π_t) which can only be determined after the programme has been run.

To overcome this they propose the following model.

$$\text{Max} \quad \sum_{t=1}^{T} U_t W_t$$

$$\text{s.t} \quad W_t - \sum_{j=1}^{J} a_{jt} x_j \leq C_t \qquad \forall\, t \tag{2.8}$$

where W_t is the withdrawal for owners' consumption (c.f. dividends) and U_t is the marginal utility of consumption in period t. How these utilities are to be established is not explained. Nor, as WEINGARTNER (143) points out, does it cater for different share-holders, nor how they can be expected to determine their utilities for consumption prior to the withdrawal possibilities being known.

CARLETON suggests (29) that distinctions be drawn between the investor's requirements, the requirements of the firms as a going concern, and the characteristics of the specific set of projects being evaluated. Care must be taken to ensure that the capital investment decision fits in with some larger corporate objective (e.g. maximising the present value of ordinary shares). In this event, capital budgets more closely resemble an administrative device (as suggested by WEINGARTNER) and MYERS demonstrates (122) the close connection between the 'utilities of consumption' and the cost of capital. This resolves the dispute between BAUMOL & QUANDT's and WEINGARTNER's models, but is no help in determining the discount rate that will be consistent with the ensuing marginal value for capital.

LUSZTG & SCHWAB propose (110) the use of sentivity analysis to overcome the problem: first (intuitively) select a value for the discount rate; then solve the programme and inspect the rate of return of the most attractive proposal foregone. Using this as a new discount rate recalculate the present values of all the projects (i.e. re-establish the objective function at this new rate) and check to see if the previous proposal is now violated. If it is not, then the solution is the optimal one; if the previous solution does not hold, then start the procedure again. AMEY & WHITMORE point out (3) that despite the arbitrary starting point, a consistent price will be attained, but that it is not unique! Other information is required - resolution of the problem concerning the owners' collective time preferences - before a unique economic discount price can be established.

Despite these problems WEINGARTNER's model provides the basis for all work concerning capital budgeting. The model has been considerably sophisticated ((144) and (19)) to include tax and dividend payments and sources for extra investments and loans. It also provides the means for dealing with some degree of interdependence between projects. In particular, the model can be augmented to include

$$\text{mutually exclusive projects} \quad \sum_{j=1}^{N} x_j \leq 1 \qquad \text{and contingent projects} \quad x_f \leq x_g \tag{2.9}$$

3. Capital Expenditure Formulation

While the previous analysis may be relevant for particular companies (e.g. investment or holding companies; conglomerates, and giant industrial companies) it is, in our view, quite unsuited for the analysis required by an average manufacturing company (such as our case-study company). The long-term proposals that such companies consider are the possibility of increasing production capacity, the possibility of some vertical integration (i.e. manufacturing some of the components presently being bought-in), and the possibility of expanding their product range. Such companies are far more concerned with the manner in which these projects will integrate with their day-to-day operations, and as such with the detailed physical returns of expenditure (e.g. the acquisition of a new lathe for the machine shop; an extension of the storage facility; etc). It is impossible to draw boundaries such that the monetary returns for such projects may be identified.

3.1 Capacity Expansion

Previous work in this area (notably by MANNE (113))inspects the problem of additional capacity size-increments to cater for (linearly or exponentially) increasing demand for that capacity. 'Capacity' is left loosely defined - e.g. pipeline capacity for oil shipments, telephone line capacity or chemical plant capacity. To the average manufacturing company this concept would be too abstruse: the management are concerned about increasing the number of lathes, drills, etc. and they wish to know in what numbers and when these ought to be purchased in order to achieve an increase in the production capabilities of the plant to cater for the expected increase in sales. We have formulated this problem as an extension of the company model described in the previous chapter.

i. <u>The Production Function</u>

If A_{mt} is the technological matrix concerned with the production of the product vector $\underline{x}$ on work-centre type m in period t, then the Work-Centre Constraint restricting that the work done be less than the available capacity C_{mt} (cf. Equation 1.10) is

$$A_{mt}\underline{x} \leq C_{mt} \qquad \forall\, m, \forall\, t \qquad (2.10)$$

Now allow for an increase in the available capacity by an amount I_{mt}. The new equation is

$$A_{mt}\underline{x} - I_{mt} \leq C_{mt} \qquad \forall\, m, \forall\, t \qquad (2.11)$$

Note that while the equation is designed to allow for an increase in capacity, it will also cater for a decrease, and that a similar expression can be constructed for the Labour Force Capacity Constraint to model the hiring (or firing) of new staff.

ii. The Projects.

The increase in capacity occurs as a result of the company having undertaken a project in any previous time period. The resulting addition of capacity may depend on the lapse of time between the initiation of the project and the period t under consideration. This allows the model to take account of the ordering period, learning curves, or projects with limited lifetimes. Let P_j^α be project j initiated in period α, and let Y_{jm}^β be the increase in capacity type m resulting from project j being undertaken β periods prior to the current period. Then the increase in capacity m in the current period is

$$I_{mt} = \sum_{j=1}^{J} \sum_{\alpha=1}^{t} P_{jm}^{\alpha} . Y_{jm}^{t-\alpha} \qquad \forall\ m,t \qquad (2.12)$$

In general there is no need to limit acceptance of any project by any amount (unlike WEINGARTNER's model). However, in practice, it is unlikely that management would wish to undertake more than a certain number of new ventures simultaneously (for administrative reasons). It is clear that the variables representing the projects need to be integral, and that the model can incorporate project dependence expressed in Equations 2.9.

iii. Financial Implications.

The cost of a project may also be dependent on the time lapse between initiation of the project and the current period.This will cover leasing arrangements as well as credit lags. If c_j^{β} is the outlay required to service project j undertaken β periods prior to the current period then

$$COST_t = \sum_{j=1}^{J} \sum_{\alpha=1}^{t} P_j^{\alpha} . c_j^{t-\alpha} \qquad \forall \ t \qquad (2.13)$$

This outflow of money will be included in the Cash Position Equation. Note that sources of cash to fund (capital) expenditure are already included in that equation, that these sources may be bounded by such external budgetary constraints that exist, and that internal restrictions on the amount spent on capital in any period can be easily included.

This formulation allows the company to evaluate their investment proposals with available information (project costs and physical returns) and ensures that expansion (or contraction) is triggered to respond to sales demand.

3.2 Vertical Integration

Whether the company ought to manufacture some of the components it presently purchases is a very complex question. Furthermore, it must be recognized that, the decision to undertake such a venture may be irrevocable - the supplier may not accept intermittent ordering.

i. Manufacture

Consider the particular bought-in component explicitly as a raw material type. Now introduce to the product range a dummy product (having its own raw material and production requirement) that has no sales outlets. Instead, this item is included in the Raw Material Stock Equation as another source for this component: Let S_t be the stock level of this component at the close of period t; let U_t be the useage and R_t be the purchase of the component in the period; and let Pr_t be the amount of the dummy product completed during the period. Then the revised Stock Equation can be written

$$S_t = S_{t-1} - U_t + R_t + Pr_t \quad \forall t \qquad (2.14)$$

ii. Irrevocability

Once the project is undertaken the sole source for this component will be in-house manufacture. (If this is not so then this equation can be ignored). This is modelled by the introduction of a dummy relationship. If P^{α} represents this project undertaken in period , then

$$R_t + M.\sum_{\alpha=1}^{t} P^{\alpha} \leq 0 \quad \forall t$$
$$M >> 0 \qquad (2.15)$$

The decision to extend the in-house manufactured content of the company's products is now taken 'simultaneously' yet 'independently' of capacity considerations, since that is now considered elsewhere in the model.

3.3 Product Range Expansion

Extension of the product range is done by simply including the new proposals in the product vector. With bounds limiting the sales to expectations of the market, the model will include these new products in the final plan only if they make suitable contribution to the achievement of the objectives.

If it envisaged that sales of the new products will correlate (either help or hinder) with sales of other products, this too can be included in the model. Let S_j be the sales of product j, bounded by B_j, and let F_{jk} be the cross-correlation of demand for products j and k. Then

$$S_j + F_{jk} S_k \leq B_j \tag{2.16}$$

The equation can be extended to take account of lags between the sales of a product and its effect being felt by another.

4. Conclusions

Manufacturing companies considering investment are concerned with how such projects will fit in with their current operations. They may not be able to identify the monetary returns of individual projects explicity, thereby nullifying traditional appraisal techniques, even were we happy that the assumptions underlying these techniques were met otherwise.

For these reasons we have extended the corporate model to cover a longer planning horizon and to include investment proposals, whose returns are expressed in physical terms. In this way long-term investments are considered as part of the on-going system, and are accepted only if this helps the company achieve its stated global objective. This also enables investment appraisal in the context of goals other than maximising NPV of the company for its shareholders. A detailed formulation of the model and our experience in its use is contained in Appendix B.

CHAPTER 3

THE ACCOUNTING FUNCTION

1. Introduction

The purpose of the accounting function has become ill defined and there has been little, if any, development in the accounting report to take account of the different needs of the various users. It is widely accepted (119) that users of accounts are interested-in, amongst other matters, the changes in the value of the company's net assets and of its profits and it is on this 'basis' that present-day reports are drawn up. This begs the question - what does 'value' and 'profit' mean? Do universal definitions for these concepts exist?

In this chapter, we begin with a brief history of the development of the accounting function. This will lead to a statement of the principles of presently accepted accounting practice and we shall highlight some major problem areas which arise. This in turn will require that we address ourselves to the question posed above. The chapter will conclude with a discussion of the role that LP may play in accounting.

2. The Development of Accounting Principles

2.1 History

The earliest recorded objectives of the accounting function (dating back to the era of the Babylonian Empire) were to present the owner with a record of the extent of his belongings, in the form of lists of objects. With the advent of trade and business the accounting system developed to enable the owner to keep track of the flow of his belongings. (A detailed description of the history of accounting development may be found in (23) and (27).

The concept of control arose with the appointment by the owner of a steward to take care of the owner's business affairs. In the need for an independent check of the steward's actions the control process reflects a basic trait of human nature. As companies grew in size three processes took place:

i) The double-entry-book-keeping system was invented. It is widely accepted that the double-entry system forms the foundation of modern accounting and that its absence would have severely hindered the growth of business activities. It is as well to bear in mind that the device merely ensures control over the accounting process itself.

(EDWARDS (page 38, (2)) points out that double-entry book-keeping "can never add anything to the original data, though it may well present data in such a way that information becomes available which would not otherwise be disclosed".)

ii) Ownership in the business entity was diluted-among many shareholders resulting in a weakening of control by the owners over the managers. Fear of divulging information to competitors; belief in caveat emptor with respect to shareholders; the heritage of entrepreneurial attitudes and the disavowal of the rights of the public to knowledge about the company's affairs all these led to management refusing to publish (accurate) information about their performance in managing the company (see (81)).

iii) With the associated increase in the complexity of the organisation arose the problem of control by the steward of the activities entrusted to his care. The response to this has been the use of formal budgeting procedures: control is exercised by comparing the predictions with the accounts of actual performance "with the object of defecting to what extent errors have been made, controlling the work of different members of the organisation and providing material for further forecasts about the future" (49).

Pressures from creditors, from the public, and from government (in the form of legal requirements of disclosure) led to an awareness of the professional responsibilities of accounting and resulted in the following concepts, now accepted as fundamental principles of the accounting process (85):

2.2 The Accounting Precepts

1. Boundaries are drawn in order to define the business entity on which the accounting-function focuses,

2. The entity is considered as a going concern.

3. A single (monetary) unit of measurement is applied.

4. Consistency requires that "generally accepted accounting principles be applied on a basis consistent with that of the previous year".

5. Realisation - under historical cost accounting this results in the changing value of an asset being recognised only in the period when revenues from the sale of the asset are deemed to be earned.

6. Objectivity requires that accounting entries be subject to verification.

7. Conservatism requires that measurement be undertaken anticipating losses but not predicting gains.

2.3 Critiques and Comments

One might expect that accountants, guided by the principles outlined above, could report on the economic activities of the company to the satisfaction of all concerned parties. Such is not the case. Some of the problems experienced by users of accounting reports stem from the specific way in which accounts report on the 'value' of the assets of the company - we shall deal with these in more detail in Section 3. Other problems derive from dissatisfaction with the precepts themselves:

1. The Business Entity

The business entity postulate states that a distinction can be drawn between the business and the owner. This is in contrast to a possible view of the corporation as an association of owner shareholders, a social institution, or merely a prescribed set of legal relations (see (133)). The accounting function concentrates on relations of the business with its environment (including the owners amongst other factors). This does not provide sufficient guidance to establish on what basis the accounts should be drawn up: that is left to the discretion of the directors and auditors.

2. The Going Concern

When considering a company it is usual to view it as a going concern rather than as a seriously embarrassed or insolvent enterprise (86). Despite this, the procedure of valuing the assets of the company at their realisable market value has become an accreditted method among the 'generally accepted accounting procedures' (to the delight of the company's creditors, one of whose major concerns is the realisable asset cover of their exposure).

3. A Single Unit of Measurement

It has been advanced that in order that diverse objects (and events) may be classified, some common attribute must be perceived to pertain to them all.

CHAMBERS states (34) that in a 'monetary economy' buyers, holders and sellers of non-monetary assets are concerned with the number of monetary units given or received in exchange for the asset. "The monetary scale is simply a scale of numerosity of monetary units".

For the concept to operate satisfactorily the units of measurement must be stable. TIERNEY identified (137) two factors that may affect this requirement. The first is that in an environment of a stable monetary unit, prices for individual items will still change in response to supply and demand conditions.

The second factor is that in an environment where the 'value' of the monetary unit is itself time dependent, measurement using an 'uncorrected' unit will distort the situation that it aims to portray.

The question of corrected units of measurement was brought up by the Accounting Standards Steering Committee in May 1974 when they attempted to provide the means for accounts to deal with inflationary conditions. Their suggestion (in essence that the units be 'updated' by the Retail Price Index) were wholeheartedly rejected by the Sandilands Committee (129): leaving aside the objection that the RPI is not a good measure of the change in company's (or their shareholders) purchasing power, it is extremely doubtful whether a single index can be used to reflect the changes effected over the spectrum of activities undertaken by a company.

However, the basic premise that man utilises a single dimension for the basis of comparison is itself questionable - six apples are more than five but are they better?. Nor is a single unit of measurement required by the philosophy of double-entry book-keeping; IJIRI proves this as follows (page 83 (88)):

let A be an asset account, with a 'value' $\underline{Y}_A$
let E be an equity account of amount $\underline{Y}_E$
then

$$\sum_A \underline{Y}_A = \sum_E \underline{Y}_E$$

since asset or equity partition is simply a <u>different</u> way of apportioning the same total value of assets. Let a transaction (i.e. an action causing change) to an account be designated by
$\underline{Y}^+_A$ or $\underline{Y}^+_E$ ($\underline{Y}^-_A$ or $\underline{Y}^-_E$)
for the amount by which the account is

increased (decreased).

Then

$$\sum_{A}(\underline{Y}_{A}^{+} - \underline{Y}_{A}^{-}) = \sum_{E}(\underline{Y}_{E}^{+} - \underline{Y}_{E}^{-}) \qquad (3.2)$$

or after rearranging

$$\sum_{A} \underline{Y}_{A}^{+} + \sum_{E} \underline{Y}_{E}^{-} = \sum_{A} \underline{Y}_{A}^{-} + \sum_{E} \underline{Y}_{E}^{+} \qquad (3.3)$$

resulting in the double-entry addage that every transaction is a two-fold aspect i.e. debit and credit flows result from each transaction.

It is clear that the vectors $\underline{Y}_A$ and $\underline{Y}_E$ are not constrained to be uni-dimensioned in equation (3.1).

4. Consistency

Clearly, bad procedures applied consistently would have no inherent quality worthy of upholding. Nevertheless, it is also clearly undesirable to keep changing the basis on which the evaluation is undertaken since this robs the resulting figures of much of their significance. (An example is cited of a major U.S. steel company who changed the basis on which depreciation was calculated, with the resulting lowering of reported profit against a background of general business bouyancy, enabling the company to press for higher prices. Yet later in the decade a further change was made to the assessment of depreciation, and the resulting increased reported profits, enabled the company to stave off a take-over bid. But the overriding result was that no understanding could be derived about the performance of the company!)

5. Realization and Matching

Problems of matching are tied up with the periodicity in accounting reports. This becomes a considerable problem in industries with very long production cycles (such as ship building and contracting) but is less of a problem with manufacturing companies since the component of costs paid in one period but carried over to the next (in the form of w.i.p. or stock is small). The concept of realisation proves to be a problem for investment companies since the 'value' of the investment is only recorded when that investment is realised (135).

In a period of rapid inflation in value of long term assets (e.g. property) the under-valuation of these assets in the accounts leads to companies being vulnerable to asset stripping take-over ventures.

This problem has led to a fundamental re-examination by the Sandilands Committee (129) of the question "what is business income", and how it should be measured - should it include holding gains, or should the principle of realization be upheld? We return to deal with this question more fully in Section 3.2.

7. <u>Objectivity</u>

The concept of objectivity has let to confusion and controversy in the literature. The component of independence (expressed by BIRD (23)) is clear: no reliance may be placed on reports prepared by auditors where colusion with one of the interested parties is suspected. This established the ethical responsibilities of accountants, but in no way indicates how the process of measurement might proceed.

VATTER argues (141) that in preparing accounts one should only recognise those figures that are amenable to verification - he views objectivity as being intrinsically linked with the principle of realisation. This view is of no help, as ARNETT (7) points out, since accountants will accept a wide range of evidence as being objective. While the amount of cash in the bank is easily verifiable, depreciation charges and cost allocations are subjective factors that cannot be treated as 'arms length' transactions without seriously damaging the view of the entity as a going concern (e.g. by including depreciation <u>only</u> when the asset is retired from service).

Furthermore, insistence on linking objectivity to realization would damage the postulate of matching expense to revenues:- views must be taken about the future in allowing for deferred taxation, depreciation and allowances for bad debts. (Acceptance of the accrual approach recognises that some subjective allocation of costs or revenues may be appropriate under some circumstances.) ARNET concludes that the ultimate test for the acceptance (or rejection) of data is whether it helps to reflect the entity events - it is better to be approximately right than precisely wrong.

This brings us to the crux of the question: CARSBERG, HOPE & SCAPNES point out (32) that accounting reports are intended to satisfy some useful purpose and the efficacy of reports can only be judged when their purpose is clearly understood. They suggest that accounting reports attempt to satisfy the following objectives:

(a) provide information to assist shareholders decisions whether to sell, buy or hold shares in the company;

(b) provide information to shareholders about the (legal) use of their funds;

(c) provide information to creditors to decide on future credit allowances;

(d) provide information to employees about future relationships with the company;

(e) provide information to managers to aid management;

(f) provide information to society to check whether activities are consistent with national objectives;

(g) provide information to government for the levy of taxation;

(h) provide information to financial institutions for negotiated financial assistance;

CHAMBERS, postulates (p384 (35)) that since the number of individual actors within the system, enumerated above, have different wants (which themselves change over time) and since the information processor must proceed without full knowledge of these, he does best to process the information neutrally. What does neutrality mean? If it means accounting recognition of economic data only when arms length (verifiable) transactions occur (i.e. realization), then the conclusion does not help the information processor in those areas where strict neutrality leads to clearly unsatisfactory results (as indicated above). Nor is the conclusion derived from the full set of precepts: better results might be obtained with more study about the needs of the classes of users listed above - a study that might indicate that each objective requires, ideally, different accounting procedures for its satisfaction (32)!

8. Conservation

The adage 'anticipate no gain but provide for all possible losses ....if in doubt write it off' conflicts with the principles expressed above (specifically those of realisation and objectivity) to an extent that the theoretical objections are clear.

3. Valuation of Assets

The principle concern of all parties concerned with a company (i.e. workers, managers, shareholders, creditors, debtors, etc.) is the general welfare of the company and the expectation that payments (in the form of monies, goods or services) due will be met.

Since the qualitative nature of the criterion 'stability' and 'welfare' defies quantitative measurement, the accounting function has concentrated on the measurement of income (from which the parties' stake in the business can be serviced).

3.1. Definition of Profit

The transference of Hicks' classical definition of income - that an individual's income may be defined by the maximum amount that he can consume during a week and still expect to be as well off at the end of the week as he was the beginning - to the corporate environment remains the subject of fierce debate amongst economists (Chapter 5 (82)). Measurement of corporate income is attempted by two distinct approaches:

(i) Profit and Loss Accounting

An attempt is made to match revenue in any stated period with the costs of achieving that revenue. 'Profit' pertaining to the period is defined as the difference between revenues and costs. This process is fraught with the problems of identifying the relevant costs (discussed above in Section 2.2.5), and one is left with the problem of deciding what proportion of the profit should be properly allocated to capital maintenance in order that the company be 'as well off at the end of the period as it was at the beginning'.

(ii) Balance Sheet Accounting

In a dynamic environment, where companies may take advantage of changes in the environment and chose to replenish their asset base by technically more advanced plant or by the acquisition of assets that will work in a new area, profit in any period may be defined as the change in net assets that takes place during that period.

This procedure concurs with EDEY's conclusion (50) that no unique or objectivity determined 'true' figure of profit exists, and that profit must emerge from the valuation process rather than contribute to it. (Henceforth, any reference to profit will imply this process of measurement).

3.2 Proposals for Asset Valuation

This brings us to the crux of the problem - namely the process whereby assets may be valued. This is an area rife with proposals and counter proposals where supporters vehemently defend their respective positions. These concepts may be summarized (8) as follows:

1. HISTORIC COST

   The most widely used basis for constructing accounts is to record the cost of the asset at the time of purchase and to write down that value over some pre-specified life.

2. ADJUSTED HISTORIC VALUE

   To account for general inflation, monetary units are factored to take account of the age of the assets to which they are linked.

3. REALISABLE VALUE

It has been widely suggested that the value of an asset can be established by referring to the market place to find the price at which exchange will be effected. However, this value, when it can be ascertained, can only be used as a guide: the purchaser of an asset clearly values the item at more than the price agreed since he would otherwise not involve himself in the exchange. Similarly, the seller must value the item at less than the price agreed. Due to the principle of conservatism market price is often used adjunct to historic cost (e.g. in stock valuation) when market valuation would be less than the depreciated historic value.

4. REPLACEMENT COST

This would value the assets at the current (often (depreciated) purchase price) i.e. the basis for valuation is the cost of replacing the asset with an identical replica in the event that the asset is lost to the business.

5. ECONOMIC VALUE

This is defined as the present value of all associated future cash flows. We find this measure particularly unsatisfactory since:

(a) the discount factor remains undefined (despite continuing ingenius attempts e.g. (28));

(b) forecasts of future cash flows cannot be objectively determined (this factor alone should, but often does not, prevent widespread application of this approach). In particular it is not clear whether the cash flows are to ascertained with the view that the asset will continue to be used in its existing or its alternative use;

(c) it may be very difficult - if not impossible - to allocate cash flows to individual assets (as discussed above in Chapter 2).

The Sandilands Committee, before putting forward their own recommendations, searched to define which basis for valuation is most appropriate for inclusion in balance sheets. From the weight of evidence presented to them, they concluded that the process of valuation should be undertaken from the view of "value to the business as a going concern". From the choice of different methods (summarized above) they decided on three principle contenders: net realizable value (NRV); present (economic) value (PV), and replacement cost (RC). The respective valuations may be ranked in six ways:

1. NRV > PV > RC
2. NRV > RC > PV
3. PV > RC > NRV
4. PV > NRV > RC
5. RC > PV > NRV
6. RC > NRV > PV

In cases 1-4, where either the net realisable value or the present value (or both) are more than the replacement cost, the value of the asset to the business can be recorded as the replacement cost since this is the amount that would be required to replace the asset in order to reap the benefit of either the realisable or present value. Only in cases 5 and 6 will it not be worth replacing the asset, since no benefit to the company will ensue. The Committee took the (intuitive) view that these cases are relatively unlikely to occur in practice and therefore opted for the replacement cost method of valuation. (In theory the value of any asset to the optimally efficient company as a going concern should be limited by the purchase price of the asset at the lower end and by the realizable value at the higher end, since if either limit is violated the asset should not be a member of the set of assets that comprise the company.)

The Committee proceeded to outline detailed proposals for revision to the currently accepted accounting procedures starting from the premise that holding gains be seperated from operating gains. The important features of the Sandilands Committee recommendations are:

i) That land and buildings be valued regularly on an "existing use" basis.

ii) That for other fixed assets, the Government Statistical Service should publish a series of capital goods price indices to provide a standard reference for estimating replacement costs of such assets. Directors may include their own valuations in balance sheets if they deem it desirable, but must show (in a note) how such valuation differs from the standard reference.

iii) That no change is immediately required to the existing bases used to value stock (since FIFO, used in the majority of cases, is a reasonable approximation to current value).

iv) That stock , when consumed, be valued at "value to the business" for inclusion in the Profit and Loss account.

v) That depreciation be provided on all assets except land. The level of depreciation should be calculated on the basis of the value of the asset to the business at the end of the period - the purpose is to charge the Profit and Loss account with the value of the assets consumed during the year, and not to finance replacement of assets. However, for comparative purposes, the historical cost figures for depreciation should be included as a note to the accounts.

3.3 Some Problems with Accounting Valuation Processes

Dissatisfaction with the accounting process of valuation has been brought to a head by the realization that valuation procedures based on historical cost are totally unsatisfactory in inflationary conditions. Such few defenders of historical cost that now remain argue that historical valuation is objective and should be retained since alternative procedures rely on subjective valuation. Further arguement is propounded (e.g. (116)) on the basis that it is incorrect to say that accounts prepared on historical cost basis are of no use since they are widely used by all (ignoring the fact that while no alternative exists one must make do with what is available). The widespread acceptance of the Sandilands Committee recommendations will ensure that this problem will be overcome to some extent.

However, other problems still exist:

1. Multiplicity of reports

    This, problem, highlighted in the main by SPACEK (132), concerns the large variety of accounts that may be drawn up, all of which claim to represent with a 'true and fair view' of the same set of transactions. Practitioners who rely on the fact that in preparing these accounts they do not break the 'accepted' principles outlined above, and that their own brand of calculation is explained in (cursory) footnotes attached are shirking their auditing responsibility. His clear message is that a more unified approach be sought.

2. Additivity

CHAMBERS points (34) to the assumption of additivity and questions whether measures made under different conditions (or according to different procedures) can, in fact, be added. How can one add "the amount of cash held by a company today to the amount of cash paid 20 years ago for a piece of freehold land which the company still holds"? How different is the notion of adding values calculated according to one procedure for one item (e.g. fixed assets by historical, or even replacement cost) and values for another item calculated according to different principles (e.g. debtors by book value less some provision)?

3. Standard Costing

The long run objective of business survival can be translated to the requirement that the company must ensure that costs are met. Absorption costing is the generally accepted manner of accounting for manufacturing costs, and specifically for fixed overhead costs (25). The widespread acceptance of standard costing has been helped by the advent of budgetary control systems. The calculation and the application of standards, corrected by variances, (discussed in Chapter 15, (2)), is a long and complex operation. The commitment involved on the part of the company makes it tempting to overlook some of the disadvantages and inherent assumptions:

i) The level of standard set has an effect on the motivation of workers affected by that standard.

ii) Despite the oft stated principle that underage cannot necessarily be viewed as 'gain' (and overage as 'loss') to the system, this is very often the way in which managers are judged.

iii) No level of intricacy of calculation will negate the fact that allocation of overheads is arbitary in situations where more than one product is produced from common manufacturing facilities. This has been the subject of much research (see Chapter 6 (2) for a review) with results ranging from simple formulas to suggested LP models (94). We show in Chapter 6 that even such sophistication is to no avail.

(iv) Standard costs are calculated on an assumption of a particular level of throughput and resource utilisation. Once calculated the costs are often used out of context in situations where the utilisation and throughput bear no resemblance to the base used.

The case for marginal costing to replace standard costing is now being put forcibly forward (87) with the argument that cost control is best effected if identified with the responsible manager - allocating arbitrary cost will side-track management into arguing amongst themselves how those allocations may be done 'fairly'. Clearly a separate exercise of control needs to be carried out on those costs central to the organisation (such as payments of rates or central management emoluments) and management needs to ensure that prices will be sufficient to cover these additional 'fixed' costs over the period.

Advocates of standard costing argue that the full unit cost needs to be known when determining the unit price to be charged: this is countered by the fact that in many industries the price is determined by considerations external to the firm.

Furthermore, since overhead allocations is subject to a variety of 'acceptable' methods, the price would become a function of the methods, and not of cost. It is further argued by standard costing proponents that it is correct to include overhead recovery in stock valuation. This is a moot point in terms of philosophy - advocates of marginal costing argue (page 15 (87)) that fixed costs may be identified with a period in time and should not, therefore, be carried forward as part of the cost of an asset into subsequent periods unless the unit of asset is very large (such as exists in the ship building and contracting industries). It is also moot whether inclusion of overhead recovery in stock valuation is an efficient system: BULL (page 160 (25)) illustrates this with an example of a company operating over three years with no changes in price, sales volume, marginal or overhead costs. The only change was in volume of production. Standard costing resulted in fluctuating income whereas under marginal costing the income is steady.

4. Modular Valuation

A further implication of the standard costing approach is that accounts approach the task of valuation from a modular standpoint. In other words, having found the value of a unit of asset (stock for example), the value of ownership of that asset is calculated by multiplying the value of each unit by the number of units present. Unless the 'value' of each unit truly represents the average value, such a procedure is patently wrong since it does not reflect the decreasing utility for more of a particular class of item.

This attitude is extended to viewing each class of asset individually, whereas in fact assets need to be brought together to be of use to a company. Plant and machinery are to no avail if there is no factory to house them, and likewise, an empty factory is of limited value to a manufacturing company.

5, Valuation of a Subset of Assets

The accounting process concentrates on those components of the business that are measurable. This ignores vital resources - for example the human asset.

Failure to measure and report on the value of the human resource may lead to decisions being made that may appear beneficial, but that would lead to an unintended depletion of human resources (by way of reduced motivation, higher turnover, etc.) (64).

It may be argued that since the company does not own the individual it cannot lay claim to ownership of the human asset. This is denied by LEV & SCHWARTZ (106) who counter the argument by stating that ownership of the individual is immaterial with respect to the labour force as a whole, The problem of how to attribute a value to the human resource has been the subject of much research (reported by GILES AND ROBINSON (68)) and it is clear that much work in this field remains to be done.

6. Multiple Goals

While there is currently widespread acceptance that a company does not pursue a single goal (and that different companies may pursue different set of possibly conflicting goals) this realisation has not permeated through to the accounting function. BEDFORD & DOPUCH argue (15) that the accounting profession must consider means of broadening its scope to report on the effectiveness with which non-income-oriented goals are attained and that the area to start with is that of the measurement of the nature and extent of benefits that would accrue to the participants where alternative, additional, goals are adopted.

4. Examples of Problem Balance Sheet Entries

Before proceeding to discuss a technique that aims to deal with some of the problems listed above, we shall consider the particular problems concerning the valuation of inventory, depreciation and goodwill.

4.1 Inventory Valuation

This single item has given rise to much, contradictory, literature, with advocates ranging-from those who insist that the LIFO view represents the only realistic approach to the problem (117); those who argue against (120); as well as others who press for a variety of other methods - FIFO or standard (historical) costs (see (105)) replacement at market prices (21) or a multiple combination of these (eg (73), (121)). The practitioner is left with such a bewildering array of tools at his disposal, but with little direction about their use resulting in the ability to value inventory by whichever method is convenient - UNDERWOOD has calculated (137) that over 108 methods are accepted by the Institute of Chartered Accountants.

The overriding emphasis of establishing some idea of the cost of inventory appears to be ignoring the very essence for the presence of stock. Inventory is held in order to cater for the possibility of a variety of outcomes in the future:-

(a) inventory holdings smooth out the random variations in demand;

(b) inventory acts as a substitute for production capacity when demand temporarily outstrips that capacity.

Both of these functions demand that the valuation for inventories be measured by looking forward to the use that such stocks will be put to - a utility that has little, if any, connection with the historical means of producing and holding the inventories.

4.2 <u>Depreciation</u>

Practitioners may approach the problem of calculating depreciation by adopting a variety of rule of thumb measures - straight line; fixed proportion; rule of years etc - on the value of the relevant asset (which may itself be measured according to cost; replacement; economic; opportunity; market prices, etc.) This seeming complexity obscures the fact that the single figure in the accounts serves three distinct functions:

(a) The entry in the profit and loss account represents a "fair" cost to the process of manufacture of goods using the asset. (To some extent this cost will be borne whether or not the assets are used.)

(b) It is a means to accumulate funds necessary to repurchase a replacement at such time as that is required (thereby preventing any temptation to distribute such income as dividend).

(c) It is used in the balance sheet to give a net measure of value of the asset. WRIGHT maintains (148) that to find value one needs, perforce, to reject all accounting conventions since they merely attempt to allocate cost over the life of the asset rather than value its services. He argues for particular valuation based on a minimum cost or loss suffered by the owner of the asset by its absence. There are problems connected with this approach. We show later, in Section 5.2.ii that such a cost allocation procedure results in an overvaluation of the enterprise. A further point to note is that the value placed on the asset by sequencial losses of the asset to the business will depend on the order in which the assets are lost!

From that it becomes clear that the figure need not be the same for each purpose.

(a) It can be argued that the cost of the "physical" depreciation of the asset is linked with the historical cost of acquisition. This still leaves the accountant the task of allocating such cost over the life of the asset - an allocation that will be inherently arbitrary in nature.

(b) The provision of a replacement fund requires to be linked with the costs of replacing the asset. This is extremely difficult to define in view of the changes in price levels and in the technology of production.

(c) We have argued above that value will depend on the utilisation of the asset and is divorced from concepts of costs.

4.3 Goodwill

The concept of goodwill, and the role that that entry plays in the balance sheet, is still the subject of a great deal of discussion. The concept was initially conceived to take account of the good relations a proprietor of a business established with his customers - a relationship that could be expected to endure (to some degree) even after the departure of this entrepreneur. It was at this juncture - the sale of a share of the business - that the valuation of goodwill arose. To this day, a point on which accountants agree is that purchased-goodwill represents the excess of the purchase price paid over the fair value of the net assets when the business is acquired. (Care must be taken in the reading of accounts since the amounts shown for acquired goodwill may sometimes be more as a result of bookkeeping or consolidation techniques than of supportable concepts".)

The existence of goodwill is now recognized to exist at all times and not merely at those moments in time when sale is considered. SPICER & PEGLER consider (132) goodwill to be "that element arising from the reputation connectionor other advantages possessed by a business which enables it to earn profits greater than the return normally to be expected on the capital invested in the tangible assets employed in the business". GYNTHER agrees (75) - "goodwill exists because assets are present, even though they are not listed with the tangible assets. For example, 'special skill and knowledge', 'high managerial ability' ... are assets in this category".

There remains the difficult problem of finding a method of valuing such goodwill - a procedure that will require neither the subjective valuation by the management of their own skill and performance, nor the speculation of a price that will be set by a mythical purchaser of the business. A method often quoted expresses goodwill in terms of excess or super profits. But GYNTHER makes it clear that this will not do:- "goodwill is not 'the discounted value of the estimated excess earning power - the amount of the net income anticipated in excess of income sufficient to clothe the tangible resources involved with a normal rate of return'. This is not what goodwill is. This is merely a rationalization of the method commonly used to calculate the value of Goodwill".

Research undertaken by the American accounting profession, into the nature of goodwill, and the manner in which it ought to be treated in the accounts (a matter that appears to take up an inordinate amount of accountants' attention (e.g. (56)) to the detriment of an understanding of other aspects of goodwill), concluded by characterising goodwill - as distinguished from other elements of value in the business - as follows:

i. The value of goodwill has no reliable or predictable relationship to costs which may have been incurred in its creation.

ii. Individual intangible factors which may contribute to goodwill cannot be valued. This results from the inability of such factors to exist apart from the business as a whole.

iii. Goodwill attaches only to a business as a whole. It forms an inseperable part of the business and cannot be realised seperately.

iv. The value of goodwill may, and does, fluctuate suddenly and widely because of the innumerable factors which influence that value.

5. LP IN ACCOUNTING

LP has been applied to the analysis of numerous accounting and financial problems. The applicability of the technique rests on its ability to overcome the disadvantages of other valuation processes (discussed in section 3.3) in the following ways:

i) LP takes an overview of the company and enables a valuation of all the component assets in their existing usage. This overcomes the problems of additivity of valuations made under different procedures, and the problems arising from valuing the assets individually and attempting to arrive at a value of the whole by merely summing the value of the parts.

ii) LP models, utilising a marginal costing approach, obviate the need for arbitrary allocations of overhead or fixed costs.

iii) LP models can take some account of such assets as the labour force by the inclusion of equations modelling the scarcity of such resources.

iv) LP models can take account of multiple goals.

When LP is applied to the process of valuation two distinct approaches have been adopted.

5.1 Spread Sheet Planning

This approach, initiated by CHARNES, COOPER & IJIRI (42) and developed and extended by the latter author (88), is founded on the double-entry book-keeping system. The mathematical basis for double-entry system, generated from equation (3.1), i.e. $\sum_A \underline{Y}_A = \sum_E \underline{Y}_E$ arises from the fundamental observation that the equation is time independent. Changes that do occur over a time period to the asset accounts are balanced by counter-changes to the equity accounts.

Let the accounts be numbered sequentially (1, 2, ...n, n+1, ... n+m) starting with the asset accounts and followed by the equity accounts. Let w_{ij} represent a transaction debiting account i and crediting account j. Then

$$\sum_{i=1}^{n+m} w_{ij} = \sum_{j=1}^{n+m} w_{ij} \qquad (3.4)$$

mirrors equation 3.2

Since a large number of transactions may involve any single account the net flow for any amount may be found by establishing an incidence matric T: a matrix having only two non-0 entries in each column:-

for the kth element of $\underline{w}$ (w_{ij}) the kth column of T has an entry 1 in row i and -1 in row j; 0 elsewhere

Now $\underline{T}\ \underline{w} = \underline{d}$ the net flow for any account (3.5)

It is simpler to visualise the process of transactions in terms of a network with accounts represented by nodes and transactions by arcs.

IJIRI gives the following example (page 91 (82)) as an aid to understanding.

Let the opening balance of the accounts be as shown below in Table 3.1, and the transactions undertaken in the period shown in Table 3.2

ACCOUNT $\underline{Y}_A$		$\underline{Y}_E$	
1. CASH	30	4. EQUITY	50
2. FINISHED GOODS	10		
3. MATERIALS	10		
	50		50

TABLE 3.1 OPENING BALANCE SHEET

Purchase of Materials in Cash	5
Consumption of Materials	2
Fixed Operating Expenses	2.5
Cost of (Cash) Sales	3
Profit on Sales of Finished Goods	3

TABLE 3.2 TRANSACTIONS

Then $\underline{w} = \begin{bmatrix} w_{31} \\ w_{23} \\ w_{41} \\ w_{12} \\ w_{14} \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 2.5 \\ 3 \\ 3 \end{bmatrix}$

And $\underline{T} = \begin{bmatrix} -1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$

$\underline{T}\,\underline{w} = \begin{bmatrix} -1.5 \\ -1 \\ 3 \\ -.5 \end{bmatrix}$

which can be shown as Fig. 3.1

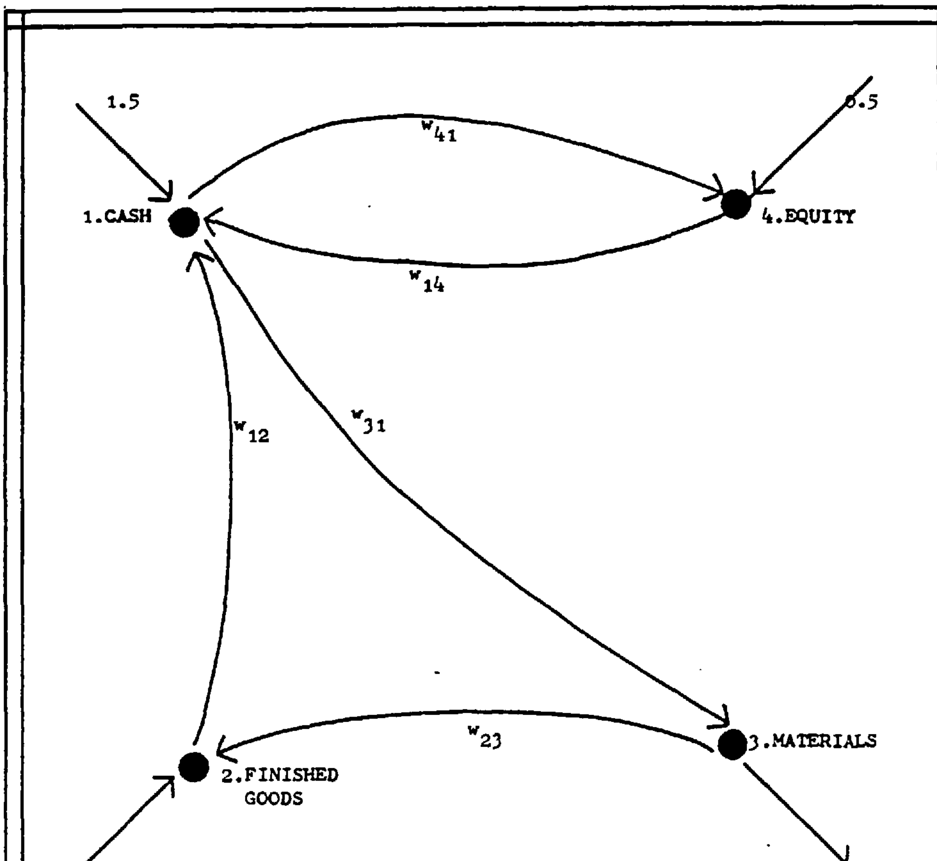


FIGURE 3.1 SPREAD SHEET FLOWS.

Spread sheet models allow the analyst to investigate the effects on the balance sheet of pursuing different, or multiple goals. Furthermore, the skeleton model of the company in terms of its balance sheet components and the accounting flows between them can be further extended to take in multi-time period horizons and other managerial, technological and environmental constraints (89). The apparent versatility and wide ranging scope for the technique and the straightforward manner in which results are presented are its major advantage.

A fundamental criticism of the spread sheet approach is that it provides no new information, neither to management nor to the analyst. Balance sheet entries may be linked (through any particular view-point) to the results of a model of physical activities of the company at a report writer stage: GAMBLING very neatly demonstrates this (66) by taking the same example used by CHARNES, COOPER & IJIRI to exemplify their spread sheet approach, and solves it with a direct LP model of the physical activities and constraints.

Furthermore, claims for the use of the dual interpretation of the model are suspect: in the spread-sheet model many of the accounting flows are linked to the physical flows of goods or materials by some (arbitrary) standard costing system. However, some of the dual variables extracted from the optional solution of the model will also, 'price' the flow of goods and materials. If the dual 'prices' and 'standard costs' are different we are presented with a problem comparable with that of pricing the value of money under capital rationing (see Chapter 2.2.3.ii).

5.2 Valuation by the Dual LP

The interpretation of the dual variables of an LP as an opportunity evaluator is well established (developed by DANTZIG (46) from the concept of the dual as a pricing mechanism for inputs). The basis for this interpretation lies in the Strong Duality Theorem.

$$\underline{c}\,\underline{x}^{*} = \underline{\pi}^{*}\,\underline{b} = Z^{*}$$

(The mathematical foundation for this interpretation is to be found in Chapter 5.2.2.)

This combines the aspect of marginality with average value: each unit of resource input is valued by the amount that the optimal value attributed to the company as a whole would change by the addition (or decrease) of a single unit of that resource, (see HADLEY, page 484,(78)). This marginal valuation is attributed to the entire availability of the asset. The value is thereby determined at the margin of a known and given availability, and in a given combination, of other assets. A necessary result from this is to value at zero resources which have slack (space capacity) since any marginal increase or decrease in availability will not effect the solution.

The general use of this interpretation is to inspect the solution to a (maximising) LP and argue for the increase in availability of resources with high positive duals - with a decrease in requirement for activities with negative duals. (e.g. (48)).

WRIGHT argues (147) that the dual LP is a viable system for imputing values to assets, the sum of which comprises the business as a "going concern". As we have seen previously (Section 4.2.c). he argues that valuation should be on an opportunity value basis but discards the approach of 'discounting the future stream of income', because of the difficulty of allocating income to a part of a large and complex man-machine organisation.

The straightforward acceptance of the dual variable as the opportunity evaluator for the inputs to the company (or as the allocation of the value of the enterprise as a whole over the constraining inputs) is not problem free:

(i) <u>Limited range of validity</u>

BERNHARD, (20) in commenting on SAMUEL's proposal (128A) to bill departments (on the basis of opportunity costs) for deviation in profit achievement due to their variance in planned performance, points to the problems of changes in basis. The example used is shown in figure 3.2.

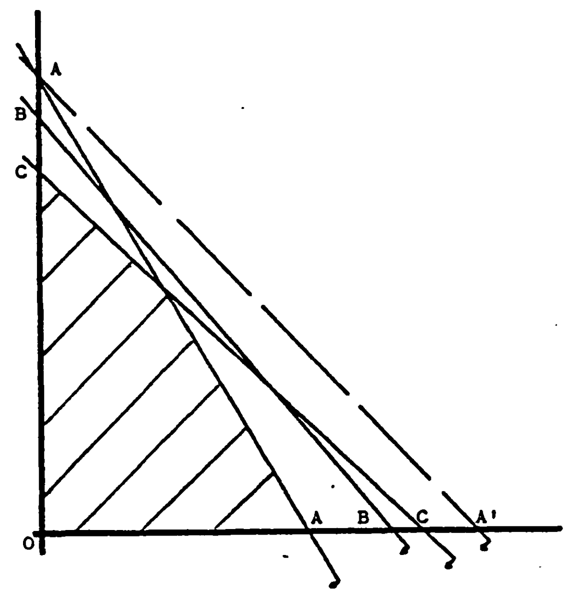


FIGURE 3.2 BOTTLENECK TRANSFERENCE

The model is defined by 3 constraints, constraint BB being redundant. Interpreting the solution of the dual programme would lead to efforts being made to improve the performance of constraint AA. If this were successful and resulted in the constraint being AA' Samuels would value the improvement of performance by the entire change multiplied by π_A (the old dual value). However, we can see from Figure 3.2 that AA' is now redundant and some of the apparent improvement is to waste since overall performance is now constrained by BB.

(ii) Degeneracy

STRUM notes (135) that in degenerate solutions the duals are asymmetric and must be interpreted according to whether the change in asset availability is an increase or decrease.

It has been argued that valuation of inputs may be accomplished by denying the organisation each asset in turn and equating the value of that asset with the change in attainment of the goal(s).

i.e. Value of asset $_i$ = $Z^* - \hat{Z}$ Where $\hat{Z}$ is the optimal solution of the programme missing resource i.

It can be shown (91) that such an attempt results in the sum of the values of the inputs greatly exceeding the value of the company as a whole. CARSBERG demonstrates this phenomenon (31) by the use of a simple example and concludes that the cause is due to jointness in the production process. The particular example he uses is degenerate - "unfortunately this problem with dual prices may occur frequently in practice since comprehensive formulations of a firm's operations will often be degenerate".

6. Conclusions

One of the principal concerns of accounts is the ability to report to the participants on the value of business, and from this valuation to derive the profit earned by the business during the accounting period. We have shown that despite the development of guiding principles and postulates to accounting procedures over a long period of time, the reporter of the company's affairs is left with a bewildering array of tools at his disposal, with little guidance concerning their appropriate use. Acceptance of the Sandilands Committee report goes a long way to providing a unifying approach to the task of drawing up balance sheets, but a number of problems remain. These centre on the modular approach to valuation, the lack of recognition of the different goals pursued by the company and the valuation of only some of the assets that comprise the business.

We have seen that LP models offer the possibility of allocating the overall value of the business as a whole (in terms of the stated objectives of the company) over the constituent assets. An average value for the unit of asset can be determined at the margin of a given availability of that asset, and in a given combination with other assets. However, such a system is not devoid of problems (namely those arising from degeneracy and from the limit of validity of the dual to the optimal basic solution). We shall return to deal with these problems in greater depth in Chapter 6.

CHAPTER 4

ALTERNATE SOLUTIONS

1. Introduction

The solutions to multi-period LP planning models (of the type formulated in Chapters 1 and 2) have two prevailing characteristics:

(i) the solutions are often primal-alternate (i.e. there exist alternate primal solutions).

(ii) the solutions are often primal-degenerate.

We shall discuss the derivation and definitions of primal-degenerate solutions in the following chapter. The implication of interpretation of the dual LP's will be dealt with in Chapter 6.

In discussing the conditions pertaining to primal-alternate solutions we first deal with the geometry of primal-feasible solutions and then with primal-optimal solutions. This results in an explanation of how alternate solutions arise. (The basic mathematical foundation for the analysis is to be found in Appendix C).

2. Geometry of Feasibility

Consider the LP.

$$\max \quad z = \sum_j c_j x_j$$

(i) $$\text{s.t.} \quad \sum_j a_{ij} x_j + S_i = b_i \qquad i = 1, \ldots\ldots, m \qquad (4.1)$$

(ii) $$S_i,\ x_j \quad 0 \qquad j = 1, \ldots\ldots, n$$

$$M = m + n$$

The set of equations (i) are termed "structural" equations, equations (ii) are the "non-negativity" constraints on the variables; x_j are the "structural" variables and S_i the "slack" variables.

In matrix form this is summarized as

$$\begin{aligned} \text{Max } z &= \underline{c}\,\underline{x} \\ \text{s.t. } \underline{A}\,\underline{x} + \underline{I}\,\underline{S} &= b \\ \underline{x}, \underline{S} &\geq 0 \end{aligned} \tag{4.2}$$

where $\underline{A}$ is the matrix of structural equations and I is the mxm unit matrix

or

$$\begin{aligned} \text{Max } z &= \underline{c}\,\underline{X} \\ \text{s.t. } \underline{A}\,\underline{X} &\leq \underline{b} \\ \underline{X} &\geq 0 \end{aligned} \tag{4.3}$$

where $\underline{A} = (A, \underline{I})$; $\underline{X} = \begin{matrix} \underline{x} \\ \underline{S} \end{matrix}$

These equations define a feasible convex region F in E^M

i.e. $F = [\, X / \underline{AX} = \underline{b};\ \underline{X} \geq 0 \,]$

Let S be the set of extreme points of F. Then F, if bounded, is the convex hull of S (see Appendix C.1.8).

The question of feasibility is concerned with whether F is empty or not. This can be determined geometrically by considering the column vectors of $\underline{A}$ (let these be $\underline{\alpha}_j$). A solution to Equation 4.3 exists if the vector $\underline{b}$ can be expressed as a convex combination of these column vectors (thereby taking account of the non-negativity requirements).

$$\underline{\lambda} \in F \quad \text{if} \quad \sum_j \lambda_j \underline{\alpha}_j = \underline{b} \; ; \; \lambda_j > 0$$

This condition defines an interior ray to a convex cone formed by the vectors $\underline{\alpha}_j$ (Theorem C.5) and leads us to state the following Theorem on feasibility.

THEOREM 4.1: The problem is feasible if the vector $\underline{b}$ is contained by the m dimensional cone formed by the column vectors of the matrix of equations.

Proof: $\underline{b}$ is m dimensional and there exist in the set of column vectors ($\underline{\alpha}_j$) m unit vectors spanning E^m (from the slack variable). $\underline{b}$ is therefore dependent on this set and we have shown above that if this dependency is convex then $\underline{b}$ lies interior to the cone formed by the column vectors.

Since $\underline{b}$ is m dimensional only m vectors will, in general, be required to describe it. All the other multipliers (λ_j) will be zero. This represents a basis, and HADLEY proves (78) that each basic feasible solution corresponds to an element of S.

3. Geometry of Optimality

LEMMA 1: If the feasible region is not empty, and is closed in the direction of optimization, then the programme will yield a finite optimal solution.

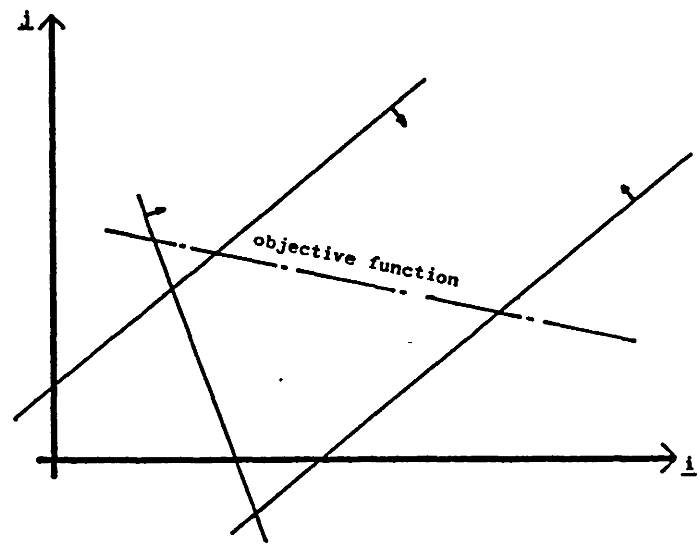


FIGURE 4.1 BOUNDEDNESS OF THE PRIMAL REGION IN DIRECTION OF OPTIMIZATION.

In the simple example shown in Fig. 4.1, maximising the objective function entails moving that line as far away as possible from the origin; minimising requires that the objective function be brought as close to the origin as possible. Clearly the latter problem does yield an optimal solution for the example shown, whereas the problem is unbounded for the maximising case.

<u>LEMMA 2:</u> The optimal solution exists at the extreme point where the objective function forms a supporting hyperplane to the feasible region.

This follows directly from the definition of a supporting hyperplane (Theorem C.2.)

Consider the feasible region F defined in E^n (eliminating the slack variables and converting the structural equations to inequalities) by the intersection of M feasible half-spaces.

<u>LEMMA 3:</u> An extreme point to the feasible region F in E^n is defined by the intersection of n linearly independent hyperplanes from the M hyperplanes defining the region.

<u>Proof:</u> This follows from the requirement that the rank of a matrix must be equal to the dimension of the vector $\underline{x}$ for a single solution to exist for the equations $\underline{A}\,\underline{x} = \underline{b}$.

<u>COROLLARY:</u> More than n hyperplanes passing through the same point will be linearly dependent.

Since we have shown that the optimal objective function hyperplane forms a supporting hyperplane then it follows that the objective function is linearly dependent on the hyperplanes intersecting to form the optimal vertex. i.e.

$$\underline{c} = \sum_{i=1}^{R} \pi_i \underline{a}_i \qquad R > n \qquad (4.3)$$

<u>THEOREM 4.2:</u> Optimality exists when the inward normal to the objective function is contained by the cone formed by the inward normals of the intersecting hyperplanes at a point.

Proof: Consider the objective function hyperplane at the optimal point $\underline{x}^*$ $\underline{c}\ \underline{x} = z^*$

we have shown that the objective function forms supporting hyperplane at the optimal vertex

i.e. there exists no point y such that

$$\underline{c}\ \underline{y} = z^* \qquad (4.4)$$

$$a_i\ y < b_i \qquad \forall_i$$

(i.e. no point on the objective function hyperplane is an interior point of the Primal feasible space F.)
Note that we can restrict the inequalities in Equation 4.4 to those satisfied at equaltiy at the optimum $(\underline{a}_i\ \underline{x}^* = b_i)$ since for any $\underline{y}$ in the neighbourhood of $\underline{x}^*$ the other constraints will still be satisfied.

Assuming, on the contrary, that some $\underline{y}$ exists satisfying Equation 4.4, and substituting for $\underline{c} = \sum_i \pi_i\ \underline{a}_i$ and for $z^* = \underline{\pi}\ \underline{b}$ (from the Strong Duality Theorem)

$$\sum_i \pi_i^*\ \underline{a}_i\ \underline{y} = \sum_i \pi_i^* b_i$$

$$(4.5)$$

$$\underline{a}_i \underline{y} < b_i \quad \forall_i$$

Now substitute for $b_i = \underline{a}_i \underline{y} + \lambda_i$; $\lambda_i > 0$ $\forall_i$

$$\sum_i \pi_i^*\ \underline{a}_i \underline{y}_i = \sum_i \pi_i^* \cdot (\underline{a}_i \underline{y} + \lambda_i) \qquad (4.6)$$

Simplifying yields

$$\sum_i \pi_i^* \cdot \lambda_i = 0 \qquad (4.7)$$

Since $\lambda_i > 0$ a solution to Equation 4.7 may exist if some $\pi_i^* < 0$.

We have shown above that no solution to Equation 4.4 may exist. Therefore we conclude that

$$\pi_i^* > 0 \quad \forall_i$$

and so the vector $\underline{c}$ is contained by the convex cone of vectors $\underline{a}_i$.

A further discussion on the conditions of optimality is to be found in Chapter 5.

4. Alternate Solution to the Primal Programme

Let a supporting hyperplane that touches the feasibile region in more than one point be termed an "extremal supporting hyperplane".

LEMMA 4: Alternate solutions occur when the optimal objective function forms an extremal supporting hyperplane to the feasible region.

In other words alternate solutions occur when more than one point on the optimal objective function hyperplane is a member of F.

THEOREM 4.3: Alternate solutions occur when the inward normal to the objective function is containyd by some sub-cone of the inward normals of the intersecting hyperplanes.

Let there be R hyperplanes intersecting at the optimal vertex, then

$$\underline{c} = \sum_{i}^{r} \pi_i \underline{a}_i \quad ; \quad \pi_i > 0 \quad i=1,\ldots,r$$

$$\pi_i = 0 \quad i=r+1,\ldots,R \qquad (4.8)$$

Proof: Since the objective function is an extremal supporting hyperplane it must be coincident with a face of a feasible region. All faces are formed by the intersection of r (< n) supporting hyperplanes.

A simple example is shown in Figure 4.2.

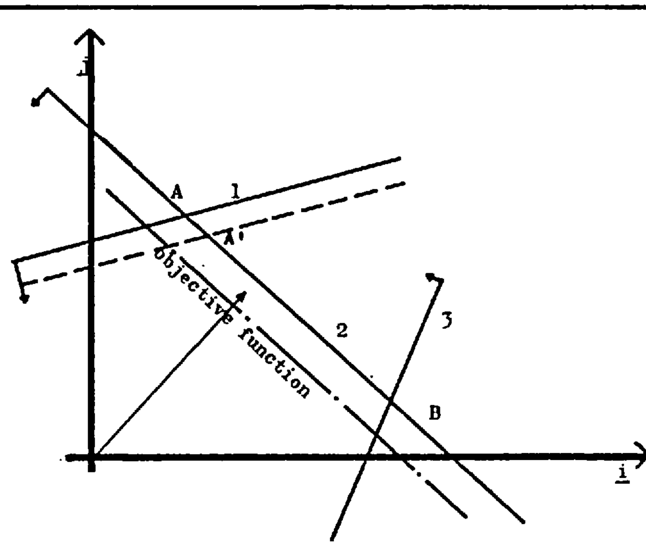


FIGURE 4.2 EXAMPLE OF ALTERNATIVE SOLUTION

In this two dimensional problem the objective function is parallel to constraint 2. Therefore the objective function is described as a convex combination of this line alone. Basic solutions to the problem exist at vertices A and B, but the other lines (constraints) intersecting at these points play no part in determining the optimality of the vertex.

LEMMA 5: If $\underline{x}_i$ ($\forall$ i ϵ I) is an optimal vertex, then any point contained by the convex hull of the points of the set I is also optimal.

Proof: Let $\underline{X} = \sum_{i \in I} \lambda \underline{x}_{i\,i}$; $\lambda_i > 0$; $\sum_{i \in I} \lambda_i = 1$

Since $\underline{Ax}_i = \underline{b}$ then so does $\underline{AX} = \underline{b}$ i.e. $\underline{X}$ is a feasible point. And since $\underline{cx}_i = z^*$ then so does $\underline{cX}$ yield the optimal value z^*.

In terms of the Fig 4.2. any point between A and B is also optimal.

5. Conclusions

The geometry of the primal LP solutions reveals that each basis represents an extremal point of the feasible region.
Expressed in the geometry of cones, the requisite that the solution be feasible may be expressed by the statement that the r.h.s. vector $\underline{b}$ must be internal to the cone formed by the column vectors $\underline{a}_j$ of the matrix of equations in Equation 4.1
Since $\underline{b}$ is m-dimensional, different m-cones containing $\underline{b}$ will represent different feasible (basic) solutions.

From the dual LP we determine that an optimal solution exists where the vector $\underline{c}$ is contained by the cone formed by the internal normal $\underline{a}_i$ to the hyperplanes forming the particular (feasible) vertex. (With the problem expressed in E^n, at least n hyperplanes (of the M that define the problem) will intersect to form the vertex; i.e. the cone formed is an n-cone).

An alternate primal solution exists where the objective function is itself an extremal supporting hyperplane to the feasible region. In that event, the vector $\underline{c}$ may be expressed by a sub-cone of the n-cone defining the 'optimal' basic vertex.

Other cones may be formed with the same subcone representing other feasible vertices) resulting in alternate optimal solutions.

From HADLEY'S definition (78) of a degenerate solution: "A basic solution to $\underline{A}\ \underline{x} = \underline{b}$ is degenerate if one or more of the basic variables vanish" we may infer the analogy that

A Primal Alternate Solution is Dual-Degenerate.

We shall investigate this more closely in the following Chapter.

CHAPTER 5

DEGENERATE SOLUTIONS

1. Introduction

In this chapter we start by deriving a simple definition of degeneracy in terms of the geometry of the primal solution. Recognition of the existance of degeneracy has in the past been mainly focused on theoretical problems that may arise in the procedural search for the optimum. We show that optimal degenerate solutions result in non-unique dual optimal solutions*. The results require that we inspect the behaviour of the dual LP around the optimum with care and we show how the non-unique duals conform to the well known "two-sided dual variable". The economic interpretation of these results is left to Chapter 6.

2. DESCRIPTION AND DEFINITION

We have already shown (by Theorem 4.1) that primal feasibility requires that the resource vector $\underline{b}$ be contained by the m-dimensional cone formed by the column vectors of the matrix of equations. CHARNES & COOPER state (page 415 (38)) that

> "Degeneracy is present whenever, at some stage of a simplex iteration, the right-hand-side vector can be expressed as a (positive) linear combination involving fewer than the m matrix column vectors which are needed for a basis spanning the dual space."

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* Some of the results presented in this chapter have been published (51)

$$\underline{b} = \sum_{i=1}^{R} \lambda_i \underline{a}_i \quad ; \quad \lambda_i \geq 0 \quad ; \quad 0 \leq R < m \tag{5.1}$$

In this Chapter we shall be using the formulation of the LP defined by Equation 4.1 with the added conditions that the equations i = 1,...m refer only to the equations that are tight (i.e. the slack $S_i = 0$) at the optimum $\underline{x}^*$. This is not a restrictive condition since our investigations of the behaviour of the problem will be confined to points in the neighbourhood of the optimum.

The solution is defined by m (tight) equations and n non-negativity constraints. The number of the non-negativity conditions satisfied as equalities is n-R (corresponding to that number of structural variables being zero - there being no positive slack variables!). Therefore the total number of tight constraints is

$$m + (n-R)$$

In terms of the geometry of the primal space defined in E^n, the vertex that is described by the basic solution reached at this stage of the simplex iterations has m+n-R hyperplanes satisfied at equality - i.e. the vertex occurs at the point of intersection of these m+n-R hyperplanes. Since m-R > 0 we derive the following simple definition of a degenerate vertex:

DEFINITION: A vertex in E^n is degenerate when it is formed by the intersection of more than n hyperplanes.

2.1 DEGENERACY DURING THE SEARCH FOR OPTIMALITY

DANTZIG (46) became aware of degeneracy, when in terms of his simplex algorithm, the search procedure is presented with a choice of pivot elements. This results in a halt to the progressive change in the value of the objective function, and in such circumstances a particular basis may be repeated. This raises the spectre of an infinite cycle preventing the algorithm from completing the search for optimality.

HOFFMAN created such an example (of 3 equations and 11 variables) where resolving the ambiguity of choice by selection of the first potential pivot element results in cycling. BEALE constructed a second example (believed to be the simplest possible with only 3 equations and 7 variables) where the same phenomenon can be observed. Yet despite the "common experience, based on the solution of thousands of practical linear programming problems by the simplex method, that nearly every problem at some stage is degenerate" DANTZIG observed no examples of cycling other than the two specially constructed examples.

However, for the sake of mathematical completeness, two methods were developed, independantly, to resolve the degeneracy problem.

i. <u>DANTZIG, ORDEN & WOLFE'S Lexicographic Rule (p183(78))</u>

A vector $\underline{x}$ is lexicographically positive if its first non-zero component is positive. To resolve degeneracy the procedure constructs a generalized L.P. as follows:

The variables x_j are replaced by a m+1 component row vector $\underline{X}_j$ having x_j as the first component.

The resource vector $\underline{b}$ is replaced by a m-by-(m+1) matrix $\underline{P}$ where

$$\underline{P} = (\underline{b}, \underline{I}_m)$$

$\underline{I}_m$ is the m-dimensional unit matrix, and $\underline{P}$ is lexico-positive. The set of constraints $\underline{A}\ \underline{x} = \underline{b}$ becomes $\underline{A}\ \underline{X} = \underline{P}$.

The objective funtion value is replaced by the (m+1) vector

$$\underline{z} = \sum_{j=1}^{n} c_j \underline{X}_j$$

The generalized LP problem is now defined as:
Find a set of lexicographically non-negative variables $\underline{X}_j$ which maximise the objective vector $\underline{z}$ in a lexicographic sense (i.e. $\underline{z}^*-\underline{z}$ is lexico-positive for all other feasible solutions) and satisfy the constraints $\underline{A}\ \underline{X}=\underline{P}$.

It can be shown that the selection of the pivot element is unique in the lexicographic sense, and that degeneracy never arises, thus allowing the generalized simplex algorithm to reach an optimal solution in a finite number of steps.

ii. <u>Charnes' Perturbation Method. (37)</u>

Figure 5.1 shows a 3-dimensional example of the geometry of equation 5.1.

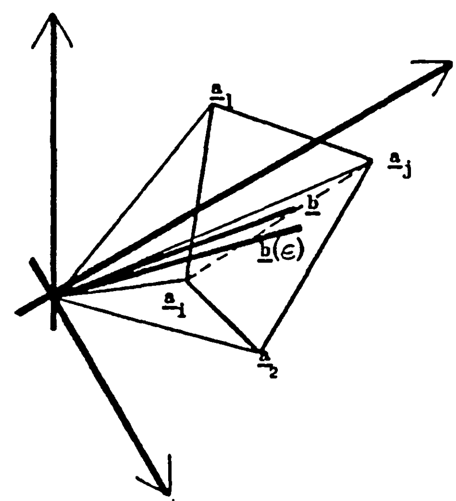

FIGURE 5.1 CHARNES' PERTURBATION

In this example the vector $\underline{b}$ is expressed as a positive linear combination of $\underline{a}_j$ and $\underline{a}_i$ alone. The perturbation method shifts the vector $\underline{b}$ as shown by creating

$$\underline{b}(\epsilon) = \underline{b} + \sum_{j=1}^{n} \epsilon^j \underline{a}_j$$

Care is taken to ensure that this perturbed vector remains enclosed by the cone of column vectors. CHARNES then shows that the polynomial in e requires that $\underline{b}(\epsilon)$ be expressed by a positive linear combination of a full set of basic vectors, and that the perturbed problem is never degenerate.

2.2 DEGENERACY AT THE OPTIMUM

With the resolution of primal degeneracy in the mathematical sense, the behaviour of LPs that are degenerate at the optimal solution has received little attention. This is a reflection of the fact that degeneracy is considered as an obstacle in the path of solution, and has resulted in important properties of degenerate optima being overlooked.

i. <u>The Primal Solution</u>

In one of the few papers concerned with this topic, GAL & HABR (65) attempt to differentiate between programmes that are degenerate (i.e. have zero basic variables) in slack or structural variables. However, the very nature of degeneracy results in the existance of a number of bases. Each basis can be formed from the previous basis by the interchange of a zero basic variable by <u>any</u> non-basic variable leaving the numerical values to <u>all</u> the variables unchanged. This renders the distinction proposed above void. Problems may be distinguished as follows:

Class 1 - Those with no structural variables at zero level

Class 2 - Those with some zero value structural variables.

(In GAL & HABR'S terms Class 1 problems conform to their catagory of degeneracy of slack variables; while Class 2 problems comprise of both their catagories).

This differentiation of problems will be of only minor use in understanding the phenomena of degeneracy.

Problems that fall into the catagory of Class 1 are optimal at some interior point of the positive orthant and sensitivity analysis can be applied to all the constaints active at the optimum. Class 2 type problems are optimal at some boundary point of the positive orthant, thereby limiting sensitivity analysis to some subset of the active equations. In the mathematical formulation there is no difference between structural constraints and the non-negativity conditions, but since the latter have been introduced to represent the behaviour of the variables in the real world this condition must be recognised by any post-optimal sensitivity analysis that is undertaken.

ii. The Dual Solution

The most interesting aspects of degenerate primal LPs comes to light by inspecting the associated dual programme. Our interest in the dual programme stems from the proposal to value the inputs to the company by their respective dual variable (see Chapter 3.5.2). Now consider the sensitivity of the solution value to changes in the availability of a particular resource - i.e. consider the effect on the objective function value when a particular constraint is shifted. CHARNES & COOPER prove (39) that for the programme

$$z^* = (\text{Max } z=\underline{cx})$$
$$\text{s.t} \quad \underline{A}\,\underline{x} \leq \underline{b} \qquad (5.2)$$
$$\underline{x} \geq 0$$

z* (<u>b</u>) is a convex function of <u>b</u>; is (finitely) piecewise linear in <u>b</u>, and that the partial derivatives (where they exist) are equal to the dual variables. This result is shown in Figure 5.2 (the points of discontinuity representing changes in the basis).

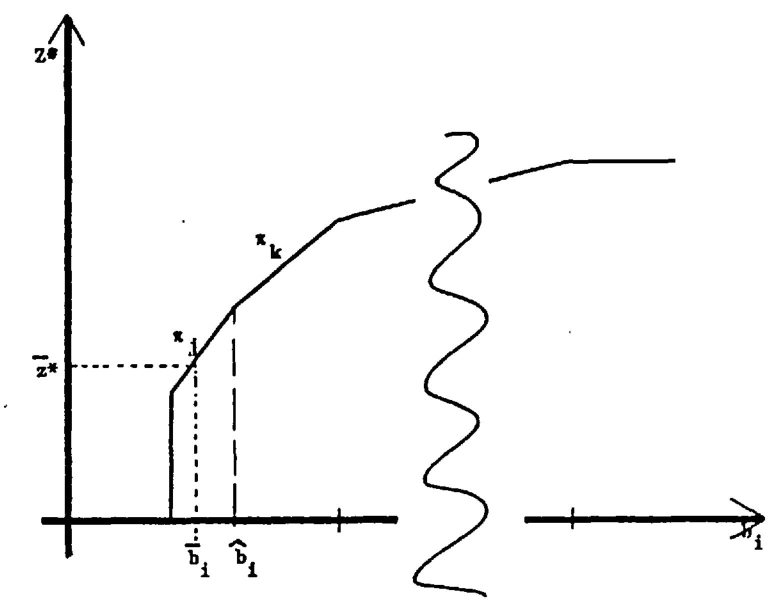

FIGURE 5.2 PIECE-WISE LINEAR FUNCTION OF Z^* (b_i)

An increase in the availability of a resource results in an increase of the objective function value but at a progressively diminishing rate till no further increase in value can be obtained. This occurs when the amount of resource is increased (i.e. the hyperplane is moved in the direction of the outward normal) to the extent that the constraint is no longer binding!

Let the actual availability of resource i in the programme be at $\bar{b}_i$. The value of the objective function (from Fig 5.2) is $\bar{z}^*$ and the sensitivity of that value to incrementa changes in the level of b_i is π_j. This leads to the well known interpretation of the dual variable as the marginal evaluator of this resource at this level.

However, with availability of the resource at the level $\hat{b}_i$ the programme is degenerate and the sensitivity of the objective function value is direction dependent.

$$\left.\frac{\partial z^*}{\partial b_i}\right|_{b_i^+} = \pi_k \qquad\qquad (5.3)$$

$$\left.\frac{\partial z^*}{\partial b_i}\right|_{b_i^-} = \pi_j$$

This two-sided nature of the marginal evaluator was brought to the attention of accountants by STRUM (135) who stressed the difference between the "gain in having one more unit of resource" as opposed to the "loss in having one less unit".

EILON A. & FLAVELL have shown ((51)-see Appendix E.2) that this statement is a gross simplification of the true behaviour of the dual linear programme when the problem is primal degenerate. Using the same example as STRUM (shown in Figure 5.3) they pointed to the existance of a third, hitherto unconsidered, basis (shown in Table 5.1).

Basis	(x, y, s_1)	(x, y, s_2)	(x, y, s_3)
Marginal value	$(0, c_1 - \frac{1}{2}c_2, -2c_1 + \frac{3}{2}c_2)$	$(\frac{2}{5}c_1 - \frac{1}{5}c_2, 0, -\frac{1}{5}c_1 + \frac{3}{5}c_2)$	$(\frac{4}{9}c_1 - \frac{1}{3}c_2, -\frac{1}{9}c_1 + \frac{1}{3}c_2, 0)$
Marginal value ($c_1 = 2$, $c_2 = 3$)	$(0, \frac{1}{2}, \frac{1}{2})$	$(\frac{1}{5}, 0, \frac{7}{5})$	$(-\frac{1}{9}, \frac{7}{9}, 0)$
Limitation	$\frac{1}{2} \leq \frac{c_1}{c_2} \leq \frac{3}{4}$	$\frac{1}{2} \leq \frac{c_1}{c_2} \leq 3$	$\frac{3}{4} \leq \frac{c_1}{c_2} \leq 3$

Table 5.1 THE MARGINAL VALUES OF THE ALTERNATE OPTIMAL BASES

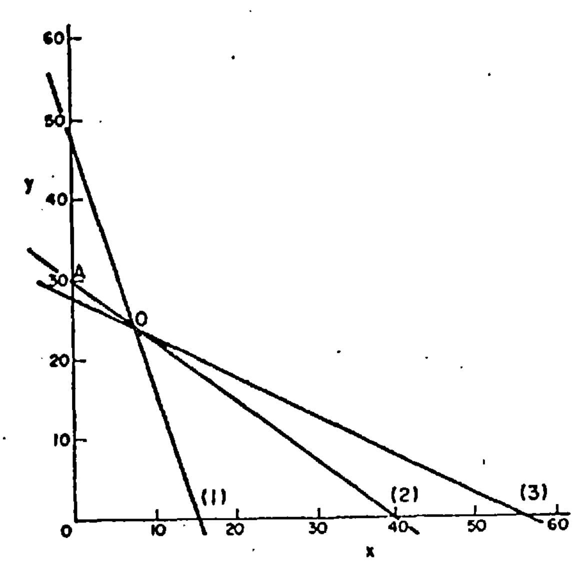

Figure 5.3. STRUM's Example

Each basis representsa particular set of independant hyperplanes that form the vertex by their intersection. In the example in E^2 shown in Figure 5.3, the number of ways of selecting two independent lines from the set of three that passthrough the optimal vertex A is three. Each basis results in a new set of dual values giving rise to the non-symetric nature of the dual values that does not, however, conform to the generally accepted non-negativity requirement on the dual variables.

Negative dual valuescan arise in an optimal solution in the following manner: consider the cone C formed by the inward normals of the hyperplanes (lines in Fig 5.3) that form the optimal vertex (A) . Since the vertex is the optimal point of the problem, the inward normal to the objective function is included by the cone C (by Theorem 4.2). This is shown in Figure 5.4.

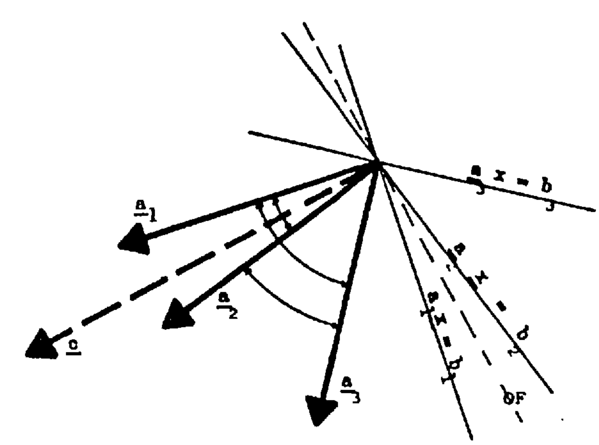

FIGURE 5.4 GEOMETRY OF OPTIMALITY

A <u>basic</u> solution corresponds to the description of the vertex by a <u>n-ray subcone</u> from C. In this example (from Figure 5.4) the subcones formed by rays (1&2), (&3), and (2&3) correspond to the three bases given in Table 5.1.

The first two subcones (corresponding to the bases produced by STRUM) themselves contain the inward normal to the objective function.

$$\underline{c} = \pi_1 \underline{a}_1 + \pi_2 \underline{a}_2 \quad ; \quad \pi_3 = 0$$

$$\underline{c} = \pi'_1 \underline{a}_1 + \pi'_3 \underline{a}_3 \quad ; \quad \pi'_2 = 0 \tag{5.4}$$

Equations 5.4 result in <u>different</u> sets of <u>non-negative</u> dual values.

The third subcone does not itself contain the objective function vector $\underline{c}$ (although it adequately represents the vertex A) and the relationship

$$\underline{c} = \hat{\pi}_2 \underline{a}_2 + \hat{\pi}_3 a_3 \, ; \; \hat{\pi}_1 = 0 \tag{5.5}$$

results in a <u>new</u> set of dual values that are <u>not non-negative</u>.

This startling result requires a revision of previous classical tests for optimality.

3. CLASSICAL TESTS FOR OPTIMALITY

3.1 The Simplex Algorithm

Acting on the programme (Equation 5.2) the simplex algorithm tests for the need for further iterations by checking the revised objective function for the existance of negative components. HADLEY (78) gives the following definition for the completion of the simplex search:

> "Given a basic feasible solution $\underline{x}_B = B^{-1}\underline{b}$ with $z_o = \underline{c}_B\underline{x}_B$ to the linear programming problem $A\underline{x} = \underline{b}$, $\underline{x} \geqslant 0$, max $z=\underline{cx}$ such that $z_j - c_j \geqslant 0$ for every column $\underline{a}_j$ in A. Then z_o is the maximum value of z subject to the constraints, and the basic feasible solution is an optimal basic feasible solution".

Clearly this statement holds true but results in all final optimal bases being those that correspond to subcones that themselves contain the objective function normal, and neglects the other optimal bases. It follows that, this condition may require a number of iterations to be undertaken that do not move the final location of the primal solution - the algorithm continues its search merely because it cannot recognise that the optimum has been reached.

3.2 'Classical' Derivation of Necessary & Sufficient Conditions for Optimal L.PS

CURRY & TAHA (44) transformed the standard LP (Equation 5.2) by expressing the (non-negative) variables as perfect squares, and applying the Jacobian method to the resulting non-linear programme. Their analysis led to the construction of the following conditions.

"The necessary condition shows that a candidate for the optimal solution is a basic feasible solution. The sufficiency condition on the other hand, stipulates that for every nonbasic variable x_j the optimality indicator (c_j-z_j) must be non-positive (non-negative) in order for a maximization (minimization) linear programme to be optimal".

Clearly these results do not conform to our findings above. RAIKE & TAYLOR point out (127) that optimal solutions need not be basic solutions (in the case of alternate optima) and that degenerate problems result in the disappearance of the Jacobian matrix, thereby taking our problem outside the scope of CARRY & TAHA's assumptions.

(In a rejoinder (45) CURRY & TAHA state that the aim of their paper was to show that 'classical' optimization techniques may be used when investigating the behaviour of LPs - their aim was not to develop rigorous optimality conditions since that would "only be a repetition of results well known in the literature").

3.3 The KUHN-TUCKER Conditions

KUHN & TUCKER, in their classical paper (102), generalise the optimality conditions from the LP to conditions for local optimality for mathematical programmes that are differentiable continuous, and that satisfy a Constraint Qualification by adapting classical calculus methods that are normally applied to constrained equations. The (linear) problem

$$\text{Max} \quad g(x) = c_j x_j$$

$$\text{s.t.} \quad f_i(x) = b_i - a_{ij} x_j \geq 0 \qquad i=1,..,m \qquad (5.6)$$

$$x_j \geq 0 \qquad j=1,,..,n$$

is transformed into an equivalent saddle value (minimax) problem by constructing the Lagrangian function

$$\emptyset(x,u) = g(x) + \sum_i u_i f_i(x).$$

KUHN & TUCKER state that, subject to a constraint qualification,

> "a particular vector x^o maximises g(x) subject to the m+n constraints if, and onlyif, there is some vector u^o with nonnegative components such that
>
> $$\emptyset(x,u^o) \leq \emptyset(x^o,u^o) \leq \emptyset(x^o,u) \qquad (5.7)$$
>
> for all nonnegative x,u".

Clearly the necessary condition has been shown not to hold for degenerate LP's fall outside the constraint qualification:

<u>THEOREM 5.1</u> All linear programmes (including degenerate systems) comply with the KUHN-TUCKER Constraint Qualification.

<u>Proof:</u> The KUHN-TUCKER constraint qualification has been defined as follows (79):

> For each line originating at the optimum $\underline{x}^*$ and lying in the convex set (the feasible region) there is an ϵ-neighbourhood of $\underline{x}^*$ a continuous differentiable curve also in the convex set and tangent to the line of $\underline{x}^*$.

For the general system $\quad \min \ f(\underline{x})$

$\quad\quad\quad\quad\quad\quad\quad\quad \text{st} \quad \underline{g}(\underline{x}) \geq 0$

then at the optimal solution $\underline{x}^*$ to the system define the following sets.

$\underline{I}$ = (active equations at $\underline{x}^*$)

$\underline{J}$ = (zero variables of $\underline{x}^*$)

The Constraint Qualification may be translated to mathematical requirements as follows:

(i) There must be more than one feasible solution;

(ii) The neighbourhood surrounding $\underline{x}^*$ will satisfy the constraints imposed by the inactive equations, and the non-negativity requirements of the non-zero variables at $\underline{x}^*$;

(iii) Consider any feasible point $\underline{y}$ ($\neq \underline{x}^*$)

then $g_i(\underline{y}) \geq 0 \quad \forall\, i \epsilon \underline{I}$

Now define $\underline{ho} = \underline{y} - \underline{x}^*$

and $\underline{h} = \lambda . \underline{ho}$; $\quad \lambda \geq 0$ (5.8)

where λ=0 represents $\underline{h}$=$\underline{x}^*$

Now the Constraint Qualification can be written as

$$\nabla f(\underline{x}^*).\underline{h} \geq 0 \ \ \forall \underline{h} \quad \text{s.t} \quad g_i(\underline{x}^*).\underline{h} \geq 0 \ \ \forall\ i \epsilon \underline{I} \qquad (5.9)$$

$$h_j \geq 0 \quad \forall\ j \epsilon J$$

Our system is a degenerate linear programme. Substituting for

$$f(\underline{x}) = \sum_{j=1}^{n} c_j \, x_j$$

$$g_i(\underline{x}) = a_i(\underline{x}) - b_i \geq 0 \quad \forall\ i(=1,\ldots m)\ \epsilon\ I$$

Equation 5.9 becomes:

$$\underline{c}\,\underline{h} \geq 0 \quad \forall \underline{h} \text{ s.t. } \underline{a}_i \underline{h} \geq 0 \quad \forall i \in \underline{I} \qquad (5.10)$$
$$h_j \geq 0 \quad \forall j \in \underline{J}$$

For $h_j = 0$ $\underline{h} = \underline{x}^*$

and $\underline{a}_i\,\underline{h} = 0$

$\underline{c}\,\underline{h} = 0$

For $h_j > 0$

$a_i h > 0$ since $\underline{h}$ is feasible (from 5.8)

and $\underline{c}\,\underline{h} > 0$ since $\underline{c}\,\underline{x}^* = 0$ is the minimum value!

There is no restriction on the number of elements in I.

Therefore all LPs satisfy the Constraint Qualification.

MANGASARIAN makes the point (112) that the multipliers are non-negative due to the problem (5.6) being expressed in terms of inequalities. Problems with equality constraints impose no restriction on the sign of their respective Lagrangian multipliers. (This follows from the treatment of equality constraints as two opposite inequalities.) We have shown that the non-negativity condition can be excluded for inequality problems that are degenerate.

3.4 Theorems of Alternatives

Farkas' Theorem states that for any fixed mxn matrix A and some fixed vector $\underline{c}$ in R^n then

either

I	or	II	
$\underline{A}\,\underline{x} \leq 0$		$\underline{A}'\underline{y} = \underline{c}$	(5.11)
$\underline{c}\,\underline{x} > 0$		$\underline{y} \geq 0$	
has solution $\underline{x}$ in R^n		has solution $\underline{y}$ in R^m	

but never both.

MANGASARIAN uses this theorem (112) to derive the necessary optimality conditions for the LP

$$\underline{c}\,\underline{x}^* = \text{Max } \underline{c}\,\underline{x}$$

$$x \in X$$

$$X = (x/x \in R^n \; ; \; \underline{A}\,\underline{x} \leq b)$$

These are:

i.	$\underline{A}\,\underline{x}^* \leq \underline{b}$	(primal feasibility)	
ii.	$\underline{A}'\underline{u}^* = \underline{c}$)	(dual feasibility)	
iii.	$\underline{u}^* \geq 0$)		(5.12)
iv.	$\underline{c}\,\underline{x}^* = \underline{b}\,\underline{u}^*$	(strong duality)	

Derivation: Define the index sets $\underline{I}$, $\underline{J}$, and $\underline{M}$ as

$$\underline{I} \cup \underline{J} = \underline{M} = (1,\ldots,m)$$

$$\underline{I} = (i/\underline{a}_i\underline{x}^* = b_i)$$

$$\underline{J} = (j/\underline{a}_j\underline{x}^* < b_j)$$

so that $\underline{A}_I\underline{x}^* = \underline{b}_I$ the active eqns. (5.13)

$\underline{A}_J\underline{x}^* < \underline{b}_j$ the inactive eqns. (5.14)

If $\underline{I}$ is empty analysis shows that $\underline{c} = 0$

If $\underline{I} \neq \emptyset$ then analysis asserts that

$$A_I\underline{x} \leq 0 \;;\; \underline{cx} > 0 \text{ has no solution } x \in R^n \qquad (5.15)$$

If there were a solution, say $\hat{x}$, then $\lambda\hat{\underline{x}}$; $\lambda > 0$ would also be a solution. In that case the point $\underline{x}^*+\lambda\hat{\underline{x}}$ would yield

$$\underline{c}(\underline{x}^*+\lambda\hat{\underline{x}}) > \underline{cx}^* \quad \text{from 5.15} \qquad (5.16)$$

$$A_I(\underline{x}^*+\lambda\hat{\underline{x}}) - \underline{b}_I \leq 0 \quad \text{from 5.15 \& 5.13} \qquad (5.17)$$

$$A_J(\underline{x}^*+\lambda\hat{\underline{x}}) - \underline{b}_J \leq -\alpha\underline{e} + \lambda A_J\hat{\underline{x}} \leq 0 \qquad (5.18)$$

from 5.15 & 5.14.

where $\underline{e}$ is the unit vector and α is defined as

$$-\alpha = \underset{j \in J}{\text{Max}}\ (a_j\underline{x}^* - b_j) < 0 \;;\; \alpha > 0 \qquad (5.19)$$

Equations 5.17 and 5.18 imply that $\underline{x}^* + \lambda\hat{\underline{x}} \in X$ but Equations 5.16 denies that $\underline{x}^*$ is the optimal solution in X. Hence the system 5.15 has no solution.

By Farkas' Theorem the systems

$$A_I'\underline{y} = \underline{c} \;;\; y \geq 0 \qquad (5.20)$$

must have a solution $\underline{y}^* \in R^I$.

If $\underline{\emptyset} \in R^J$ then

$$\underline{b}_I\underline{y}^* + \underline{b}_J\underline{\emptyset} = \underline{y}^*A_J\underline{x}^* = \underline{cx}^* \qquad (5.21)$$

from 5.20 & 5.13

becomes condition 5.12-iv, and

$$A_I'\underline{y}^* + A_J'\underline{\emptyset} = \underline{c} \;;\; \underline{y}^* \geq 0 \qquad (5.22)$$

yields conditions 5.12-ii and 5.9-iii.

This last statement takes Farkas' Theorem too far. While it is true that some $\underline{y}^* \geqslant 0$ does satisfy Equation 5.22 there is no exclusion of some $\underline{y}^* < 0$ also satisfying the equation.

A more careful interpretation of Farkas' Theorem leads VAJDA (110) to state the following Necessity Theorem for the existance of a LP solution:

"If x_o minimizes $c'x$ subject to $b-Ax \leqslant 0$, then there exists a vector $y \geqslant 0$ such that $c-A'y_o=0$ and $(b-Ax_o)'y_o=0$".

As we have seen above (page 134) this statement is correct as far as it goes, but does not bring to light the existence of other solutions where the dual multiplier need not be constrained to be non-negative.

4. MATHEMATICAL DESCRIPTION OF THE BEHAVIOUR OF DEGENERATE LPs

It is clear that the characteristics of degenerate - primal solutions do not confirm to previously accepted precepts. In this section we investigate the behaviour of such optimal solutions, by inspecting the structure of the solution at points close to the optimal vertex.

For problems of the type:

Find $\underline{x}^*$, if it exists, s.t $\theta(\underline{x}^*) = \min_{x \in X} \theta(\underline{x})$; $X = (\underline{x}/g(\underline{x}) \leqslant 0)$

where $\theta(x)$ and $g(x)$ are differentiable continuous functions, the Kuhn-Tucker conditions for optimality can be expressed as follows (p94,(112)):

From the Lagrangian $\quad \phi(\underline{x},\underline{\pi}) = \theta(\underline{x}) + \underline{\pi}g(\underline{x})$

$$\nabla_x \phi(\underline{x}^*,\underline{\pi}^*) = 0 \qquad \nabla\theta(\underline{x}^*) + \underline{\pi}^*\nabla g(\underline{x}^*) = 0 \tag{5.23}$$

$$\nabla_\pi \phi(\underline{x}^*, \underline{\pi}^*) \leq 0 \qquad g(\underline{x}^*) \leq 0 \tag{5.24}$$

$$\underline{\pi}^*\nabla_\pi \phi(\underline{x}^*,\underline{\pi}^*) = 0 \qquad \underline{\pi}^* g(\underline{x}^*) = 0 \tag{5.25}$$

$$\underline{\pi}^* \geq 0 \qquad \underline{\pi}^* \geq 0 \tag{5.26}$$

For the Linear Programme the functions $\theta(\underline{x}) = -\underline{c}\ \underline{x}$

$g(\underline{x}) = \underline{A}\ \underline{x} - \underline{b}$

are clearly differentiable. Substitution yields the following conditions:

$$\left.\begin{array}{ll} -c_j + \sum_i \pi_i^* a_{ij} = 0 & j=1,..,n \\ (\underline{a}_i\underline{x}^* - b_i) \leq 0 & i=1,..,M \\ \pi_i^*(\underline{a}_i\underline{x}^* - b_i) = 0 & i=1,..,M \\ \pi_i^* \geq 0 & i=1,..,M \end{array}\right) \tag{5.27}$$

For degenerate problems (dropping the non-negativity condition and) considering only the m (m>n) active equations, the system

$$\left.\begin{array}{l} \underline{c} = \underline{\pi}\ \underline{A} \\ 0 = \underline{\pi}\ (\underline{A}\ \underline{x}^*-\underline{b}) \\ \underline{c}\ \underline{x}^* = \underline{\pi}\ \underline{b} \end{array}\right) \tag{5.28}$$

has no unique solution $\underline{\pi}^*$. Equations 5.28 define the alternate dual space overwhich the dual variables $\underline{\pi}$ are free-ranging.

The discussion of the economic implications of the existence of this dual space is left till the next chapter. To understand more about the behaviour of the dual variables we shall demonstrate the behaviour of degenerate problems under small perturbations of the resource vector. It will be helpful to make the following distinction between 'simple' and 'non-simple' degeneracy:

DEFINITION: Simple degeneracy occurs when all sets of n hyperplanes of those that intersect at the degenerate vertex in E^n are linearly independent.

This can be expressed in the form of two assertions.

ASSERTION 1. Any set of n hyperplanes of the active equations is linearly independent.

This ensures that the vertex may be defined uniquely by any sufficient set of hyperplanesand precludes the type of problem shown in Fig. 5.5, where the three planesin E^3 are dependant and do not define a unique point.

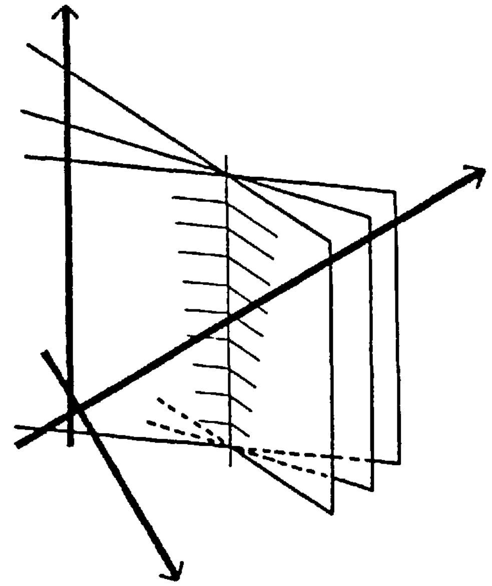

FIGURE 5.5 NON-SIMPLE DEGENERACY

ASSERTION 2. Any set of n-1 hyperplanes of the active equations and the objective function hyperplane is linearly independent.

This precludes the existance of alternate primal solutions (for the sake of simplicity).

Unless otherwise stated all further analysis will assume that these assertions hold.

It is convenient to restate at this point terminology that will be used in the following analysis.

$\underline{J}$ is the index set of inactive eqns. $\underline{a}_j\underline{x}^* < b_j$

$\underline{I}$ is the index set of active eqns $\underline{a}_i\underline{x}^* = b_i$

P is the set of extremal supporting hyperplanes

Q is the set of nonextremal supporting hyperplanes

Let $\underline{a}_p \in P$, $\underline{a}_q \in Q$; then $I = U\ (p,q.)$

R_K is an index set such that the n-ray cone formed by the inward normals of the indexed (active) hyperplanes contains the objective function normal: $\Sigma\ \pi_{ik}\underline{a}_i = \underline{c}\ ;\ \pi_{ik} > 0$

R is the family of such sets. $R = U_k(R_k)$

T is the set of indices common throughout R

$T = \cap_k R_k$

4.1 BEHAVIOUR OF THE PRIMAL PROBLEM UNDER SMALL PERTURBATION OF b.

THEOREM 5.2 Perturbation of the r.h.s. vector $\underline{b}$ at a degenerate optimum will resolve the degeneracy, under 'simple' degeneracy.

i. <u>Change in the Availability of Resource s in the Direction of the Inward Normal.</u>

$$\hat{b}_s = b_s - \partial_s$$

now $$\underline{a}_s\underline{x}^* = b_s > \hat{b}_s$$

so $\underline{x}^*$ is now infeasible. Therefore remove from consideration all non-extremal hyperplanes (other than s iff $\underline{a}_s \epsilon Q$) since they intersect the feasible region at $\underline{x}^*$ only.

Degeneracy is resolved by such perturbation since either:

a. $\underline{a}_s \epsilon$ P and /P/=n. i.e. there are only n hyperplanes now active. This condition follows from the definition of degeneracy.

or

b. The system $\underline{a}_i\underline{x} = b_i \qquad \forall \underline{a}_i \epsilon P \; ; \; i \neq s$ (5.30)

$$\underline{a}_s\underline{x} = \hat{b}_s$$

where /P/ $\geq$ n or $\underline{a}_s \epsilon$ Q, has no feasible solution.

This can be shown to hold by application of Farkas' Theorem. There will be no solution to the set of Equations 5.30 if there exists a non-zero solution to the system

$$\left.\begin{aligned} &\sum_{\underline{a}_i \epsilon P} y_i \underline{a}_i + y_s \underline{a}_s = 0 \\ &\sum_{\underline{a}_i \epsilon P} y_i b_i + y_s \hat{b}_s = \alpha \quad ; \alpha \neq 0 \end{aligned}\right) \qquad (5.31)$$

now $$\underline{a}_s = \sum_{\underline{a}_i \epsilon P} \lambda_{si} \underline{a}_i \; ; \; b_s = \sum_{\underline{a}_i \epsilon P} \lambda_{si} b_i \; ; \; \underline{\lambda}_s \neq 0 \qquad (5.32)$$

Hence $$\sum_{\underline{a}_i \epsilon P} (y_i + y_s \; _{si}) \underline{a}_i = 0 \qquad (5.33)$$

$$\sum_{\underline{a}_i \epsilon P} (y_i + y_s \lambda_{si}) b_i - \partial_s y_s = \alpha$$

A possible solution to Equations 5.33 is

$$y_s = -\overset{\alpha}{\partial}_s \neq 0 \; ;$$

$$y_i + y_s \lambda_{si} = 0 \qquad \forall \; \underline{a}_i \in P$$

Since a solution $\underline{y} \neq 0$ exists for the system 5.31 there can be no solution to Equations 5.30.

Solutions do exist for sets of equations comprising of equation s and any n-1 equations from the set P. By definition, these solutions are non-degenerate.

ii. <u>Change in Availability of Resource s in the Direction of the Outward Normal.</u>

$$\hat{b}_s = b_s + \partial_s$$

Now $\underline{a}_s \underline{x}^* = b_s < \hat{b}_s$ - i.e. $\underline{x}^*$ remains feasible.

While $\underline{x}^*$ remains feasible under such perturbation it need not remain optimal.

Iff $s \in T$ then x^* will not remain optimum.

The objective function normal is contained by subcones indexed by the sets R_k. Furthermore T is contained byall these sets, so the exclusion from the vertex $\underline{x}^*$ of a hyperplane in T results in no cone containing $\underline{c}$. The resulting dual infeasibility is interpreated as primal non-optimality.

For pertubation of any $s \in T$ it remains for us to show that new non-degenerate vertices are formed, enabling a new optimum to be established. Consider the set $\mathcal{Z}$ formed by the n vectors

$$\underline{a}_s \text{ ; p elements from P; and n-p-1 elements of Q}$$

The intersection of the hyperplanes of this set will yield a feasible point if that point is contained in the feasible half-space of all the other hyperplanes. Furthermore, this point will be non-degenerate if it is contained by the open half-space of the other hyperplanes.

$$\left.\begin{array}{lll} \text{Let } \underline{z} \text{ be defined by } \underline{a}_s\underline{z} = b_s + \partial_s & \text{one } s \in T \\ \underline{a}_i\underline{z} = b_i & \underline{a}_i \in P \\ \underline{a}_j\underline{z} = b_j & \underline{a}_j \in Q \end{array}\right\} \text{ the set } \mathcal{Z} \qquad (5.34)$$

For $\mathcal{Z}$ to be non degenerate we need to show that such $\underline{z}$ results in $\underline{a}_k\underline{z} < b_k \quad \forall \quad \underline{a}_k \notin \mathcal{Z}$:

Let equations 5.34 be summarised as $\underline{B}\ \underline{z} = \begin{bmatrix} b_s + \partial_s \\ \underline{b}_i \\ \underline{b}_j \end{bmatrix}$

$$\text{Now} \qquad \underline{z} = B^{-1}\begin{bmatrix} b_s + \partial_s \\ \underline{b}_i \\ \underline{b}_j \end{bmatrix} \text{ and } \underline{x}^* = B^{-1}\begin{bmatrix} b_s \\ \underline{b}_i \\ \underline{b}_j \end{bmatrix} \qquad (5.35)$$

$$\text{From } \underline{a}_k\underline{x}^* = b_k \text{ substitution yields } \quad b_k = \underline{a}_k B^{-1}\begin{bmatrix} b_s \\ \underline{b}_i \\ \underline{b}_j \end{bmatrix} \qquad (5.36)$$

$$\text{Thus } \underline{a}_k\underline{z} < b_k \text{ reduces to } \underline{a}_k B^{-1}\begin{bmatrix} b_s + \partial_s \\ \underline{b}_i \\ \underline{b}_j \end{bmatrix} < \underline{a}_k B^{-1}\begin{bmatrix} b_s \\ \underline{b}_i \\ \underline{b}_j \end{bmatrix}$$

Since $\underline{a}_k = \underline{\lambda}_k B \; ; \; \underline{\lambda}_k \neq 0$ equation 5.36 reduces to

$$\underline{\lambda}_k I \begin{bmatrix} b_s + \partial_s \\ \underline{b}_i \\ \underline{b}_j \end{bmatrix} < -\underline{\lambda}_k I \begin{bmatrix} b_s \\ \underline{b}_i \\ \underline{b}_j \end{bmatrix}; \text{ I is the unit matrix} \qquad (5.37)$$

This results hold only if $\lambda_{ks} < 0$.

Thus the set of hyperplanes Z will form a feasible non-degenerate vertex if and only if $\lambda_{ks} < 0$ for all $\underline{a}_k \notin Z$.

iii. Summary

The previous analysis, using the geometry of the cones formed by the inward normals of the hyperplanes (intersecting to form the degenerate vertex) can be summarised by the matrix algebra of the LP.

Consider the simplex tableau (shown in Fig. 5.6) describing the optimum $\underline{x}^*$.

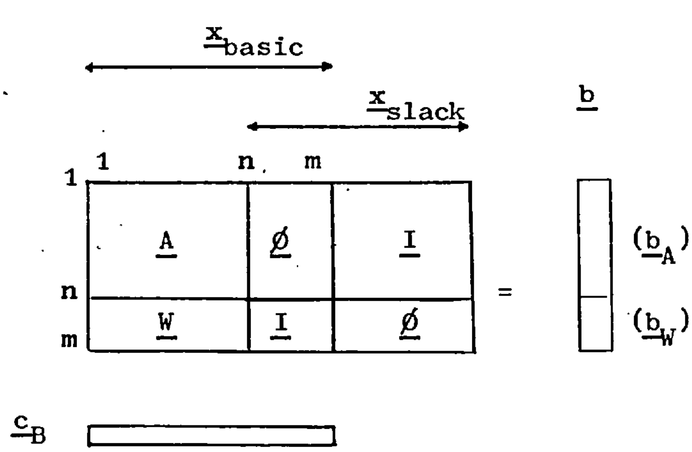

FIGURE 5.6 OPTIMAL DEGENERATE TABLEAU.

The basis B is constructed of all the structural variables and m-n of the slack variables: in effect the vertex is described by the first n hyperplanes. (This basis is of quite general construction since all the non-negativity constraints are included explicitly).

$$\underline{x}^* = \underline{B}^{-1}\,\underline{b}$$

Where the inverse of the basis takes the form $\begin{bmatrix} \underline{A}^{-1} & \underline{\emptyset} \\ -\underline{W}\,\underline{A} & \underline{I} \end{bmatrix}$

Under Assertion 1 $\quad W = \underline{\lambda} A \; ; \; \underline{\lambda} \neq 0$

so that $$\underline{x}^* = (A^{-1}\underline{b}_A , \quad -WA^{-1}\,\underline{b}_A + \underline{b}_W) \tag{5.38}$$

becomes $$\underline{x}^* = (A^{-1}\underline{b}_A , \quad \underline{b}_W - \underline{\lambda}.\underline{b}_A)$$

and since the slack variables are zero (- all the equations are active)

$$-\underline{\lambda}\,\underline{b}_A + \underline{b}_W = 0 \tag{5.39}$$

a. Perturbation of $b_s \in \underline{b}_W$

$$\hat{\underline{b}}_W = \underline{b}_W + \underline{\partial} \quad \text{where } \partial_i = 0 \;\forall\; i \neq s$$
$$(\partial_s \text{ unrestricted in sign}) \tag{5.40}$$

Clearly from 5.40 if $\partial_s > 0$ then the slack of hyperplane s becomes positive while all the others remain unchanged (i.e. the basis B is still feasible but degenerate). If $\partial_s < 0$ then the slack of hyperplane s is negative and the basis can no longer remain feasible.

b. Perturbation of $b_s \in \underline{b}_A$

$$\hat{\underline{b}}_A = \underline{b}_A + \underline{\partial} \quad ; \quad \partial_i = 0 \; \forall \; i \neq s \tag{5.41}$$

Such perturbation will not affect the feasibility conditions of the structural variables: from Equation 5.38 $A^{-1}\underline{b}_A > 0$. Segmenting the matrix A so that all the non-negativity constraints come together:

$$A = \begin{bmatrix} A_a & A_w \\ \emptyset & -I \end{bmatrix} \; ; \quad A^{-1} = \begin{bmatrix} A_a^{-1} & A_w A_a^{-1} \\ \emptyset & -I \end{bmatrix} \; ; \quad \underline{b}_A = \begin{bmatrix} \underline{b}_a \\ \emptyset \end{bmatrix} \tag{5.42}$$

If $b_s \in \underline{b}_a$ then $\underline{x} = A_a^{-1}\hat{\underline{b}}_a$ remains strictly positive for small ∂_s while all the zero variables remain unaltered. If $b_s = 0$ then only that variable constrained by that equation is altered by such perturbation. (In the mathematical sense the perturbation may be in either direction; physical interpretation of perturbation in the direction of the outward normal for the non-negativity constrainst may be difficult!)

The basis B remains feasible (and becomes non-degenerate if

$$\underline{-\lambda\partial} > 0 \qquad \text{(from 5.39)} \tag{5.43}$$

4.2 BEHAVIOUR OF THE DUAL PROGRAMME UNDER SMALL PERTURBATION OF b.

Similar analysis applied to the feasibility of the dual programme leads to the following theorem:

THEOREM 5.3. Under simple degeneracy perturbation of a hyperplane in the direction of the inward normal results in the associated dual variable taking its maximal value defined by the alternate dual space; perturbation in the opposite direction results in the dual variable taking its minimum, non-negative value.

Proof: The values of the dual variables may be extracted from the simplex tableau shown in Figure 5.6

$$\underline{\pi}_B = \underline{c}_B \underline{B}^{-1} \qquad (5.44)$$

Previous theory relating to the simplex algorithm has proved that the dual variables associated with equations having their slack variables in the basis take a zero value.

Therefore $\underline{\pi}_B = (\underline{\pi}, \emptyset)$; $\underline{\pi} = \underline{c}\,\underline{A}^{-1}$

Now select the set of n hyperplanes $\bar{A}$ (represented by the matrix A) such that $\bar{A} \in R$, whence $\underline{\pi} \geq 0$.

<u>i. Let $s \in \bar{A}$</u> and $-\underline{\lambda}_s . \pi_s > 0$

Now perturbation of hyperplanes will result in the selected basis remaining optimal and becoming non-degenerate. Furthermore the vector $\underline{\pi}$ remains unchanged, and now becomes the unique set of feasible dual variables.

Instead perturb hyperplane $\underline{r}$.

a. $r \in \bar{A}$ and $-\underline{\lambda}_r \pi_r > 0$. In this event the selected basis still remains optimal, leaving π_s unchanged.

b. $r \notin \bar{A}$ <u>or</u> $-\underline{\lambda}_r \pi_r < 0$. Now one (or more) hyperplane(s) from the set $\bar{A}$ must be replaced in order that the condition for primal feasibility be attained - i.e. pivoting must occur.

If in the course of pivoting hyperplanes is removed from the set $\bar{A}$ its dual variable will take a value of zero (since the slack variable will enter the basis). In that event it is clear that the Theorem holds.

In the general case consider the change (pivot) of hyperplane t for hyperplane 1 with hyperplane s remaining in the basis. (This will be extended to the case when more than one row change is required in A.)

Now $\hat{A} = (A + E)$ where $\left.\begin{array}{l} e_{ij} = 0 \;\forall\; i \neq 1 \\ e_{1j} = a_{tj} - a_{1j} \end{array}\right) \in E$

$$a_{ij} \in A \;\forall\; i \leq n \qquad a_{ij} \in W \;\forall\; i > n \qquad (5.42)$$

$$\hat{A}^{-1} = (I + EA^{-1})\,A^{-1} \quad = A^{-1}\,(I + EA^{-1})^{-1} \qquad (5.46)$$

Letting $\alpha_{ij} \in A^{-1}$, and $F = EA^{-1}$

$$\left.\begin{array}{l} f_{ij} = 0 \;\forall\; i \neq 1 \\ f_{1j} = \sum_{k=1}^{n} (a_{tk} - a_{1k})\alpha_{kj} \quad \in F \\ \quad = \sum_{k=1}^{n} a_{tk}\alpha_{kj} - \beta_{ij} \text{ where } \beta_{ij} = 0 \;\forall\; i \neq j,\; \beta_{ii} = 1 \end{array}\right) \qquad (5.47)$$

Under Assertion 1 $\quad a_{t1} = \sum_{i \in \bar{A}} \lambda_{ti} a_{i1}; \quad \lambda_{ti} \neq 0$

Substituting into 5.47

$$f_{ij} = \sum_i \lambda_{ti} \cdot \sum_k a_{1k}\alpha_{kj} - \beta_{ij} \qquad (5.48)$$
$$= \lambda_{ti} - \beta_{ij}$$

Letting $G = (I + EA^{-1})$ $\quad \left.\begin{array}{l} g_{ij} = \beta_{ij} \;\forall\; i \neq 1 \\ g_{ij} = \lambda_{tj} \end{array}\right) \in G \qquad (5.49)$

In general Equation 5.49 can be extended to include more than one (say q) change(s). G can then be partitioned

$$G = \left[\begin{array}{c|c} & \lambda \\ \hline \emptyset & I \end{array}\right]$$ where I is the n-q unit matrix

To ease invertion of G further partion the matrix $\underline{\lambda}$ into a qxq matrix $\underline{\lambda}_Q$ and a qx(n-q) matrix $\underline{\lambda}_R$. Then

$$G^{-1} = \begin{bmatrix} \underline{\lambda}_Q^{-1} & -\underline{\lambda}_R \underline{\lambda}_Q^{-1} \\ \underline{\emptyset} & I \end{bmatrix}$$

Returning to Equation 5.43 perturbation will result in the dual variables taking new values:

$$\hat{\underline{\pi}} = \underline{c}A^{-1} = \underline{c}A^{-1}G^{-1} = \underline{\pi}G^{-1} \tag{5.50}$$

$\hat{\underline{\pi}}_Q = \underline{\pi}_Q \underline{\lambda}_Q^{-1}$ are the 'values' of the newly introduced hyperplanes.

($\underline{\pi}_Q$ refer to the old, now removed, variables).

Note that $\hat{\underline{\pi}}_Q \geqslant 0$ ensures dual feasibility.

$$\hat{\underline{\pi}}_R = -\underline{\pi}_Q \underline{\lambda}_R \underline{\lambda}_Q^{-1} + \underline{\pi}_R$$

$$\hat{\underline{\pi}}_R = -\underline{\pi}_Q \underline{\lambda}_R + \underline{\pi}_R \tag{5.51}$$

are the values of the dual variables of the hyperplanes, remaining in the basis. Since hyperplane s remains in the basis

$$\hat{\pi}_s = \pi_s - \sum_{i \in Q} \pi_{Qi} \lambda_{is}$$

The assumption made at the beginning of this section (4.2.i) of the proof (on page) needs amplification:

a. Hyperplane s is perturbed in the direction of the inward normal i.e. $\partial_s < 0$ requiring $\underline{\lambda}_s > 0$.

In that event $\hat{\underline{\pi}}_s < \underline{\pi}_s$ when any other resource r is perturbed.

b. Hyperplane s is perturbed in the direction of the outward normal i.e. $\partial_s > 0$ requiring $\underline{\lambda}_s < 0$.

In that event $\hat{\pi}_s > \pi_s$ when any other resource is perturbed.

<u>ii. s ∉ A.</u>

This can only apply for s ∉ T (since $\overline{A} \in R$) and is deemed to apply for all such hyperplanes. Furthermore such a basis can only remain feasible under perturbation in the direction of the outward normal (see Section 4.1.iii.a on page). In that event resource s becomes non-binding with the result that the associated dual remains at zero.

5. CONCLUSION

We have shown that degeneracy can be defined by over definition of the primal vertex. This results in the dual programme having alternate solutions (while maintaining the strong duality condition) that are not confined by non-negativity constraints.

Under conditions pertaining to 'simple' degeneracy small perterbation of any of the defining hyperplanes in the direction of the inward normal will always resolve the degeneracy by defining a new optimal vertex. Perturbation in the opposite direction leaves the originally optimal vertex feasible and, unless the hyperplane moved belongs to a special identifiable set, optimal. Perturbation of hyperplanes that are members of this special set results in a new, non-degenerate optimum.

The dual 'space' (over which the dual vector is free ranging) collapses to a single point when degeneracy is resolved. For perturbation of hyperplane s in the direction of its inward normal the dual solution point-vector is characterised by the s'th dual variable having its maximum value (over the 'space'); for perturbation in the direction of the outward normal the s'th variable takes its minimal, non-zero, value.

CHAPTER 6

INTERPRETATION OF LP SOLUTIONS

1. Introduction

Having shown the underlying geometry of alternative and degenerate solutions, and their behaviour under small pertubations, we now discuss the economic interpretation that can be read into such solutions.*

Dealing first with alternate solutions we show how these may be resolved, and how such resolution can introduce new and important measurement to the valuation process.

Degenerate solutions are shown to be economically desirable, and would appear to have an attraction for valuation procedures. However the existence of the alternate dual space causes insurmountable problems by denying the valuation process the ability to divulge information to the user of the accounts.

2. Alternate Solutions

We have shown in Chapter 4 that alternate solutions arise when the objective function can be expressed by a positive linear combination of less than n intersecting hyperplanes that form '<u>the</u>' optimal vertex in E^n. In fact, any point on the face of the primal polytope formed by these intersecting hyperplanes will be optimal and these hyperplanes represent the sole constraints that restrict the programme.

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* Some of the material in this chapter was first presented to the Operations Research Society Annual Conference in October 1974. (See Appendix E.1).

The simplex search procedure always moves from vertex to vertex of the primal feasible space in search of the optimum. The only way to determine whether the optimal solution is a member of the set of alternate solutions is by inspection of the dual. If the optimal vertex is non-degenerate then those tight constraints that do not in actual fact restrict the solution will have duals of zero (since marginal alteration in the availability of that resource will not affect the objective function achievement). In this manner a number of basic dual variables are zero resulting in alternate primal problems being dual degenerate (118). (If the optimal solution selected by the simplex algorithm is primal-alternate-degenerate, then simple inspection of the dual programme may not identify the alternate-hyperplanes because the dual solution is itself alternate - we describe a procedure for overcoming such problems in Appendix D.2).

2.1 Physical Interpreation of the Primal-Alternate Solution

The existence of alternate optima presents the decision maker with an infinity of possible optimal solutions. Given this choice, at which point should he choose to operate? No definative answer can be given to this question, but in general it is clear that some other guidelines must be used to assist the decision.

(i) Secondary Objectives

The manager might be in a position to define a secondary objective that he would wish to pursue, given that he achieved a maximal performance of his primary objective. While it is theoretically possible that this second optimization procedure would also result in alternate solutions, allowing the pursuit of tertiary objectives (and more), it is unlikely that a large number of objectives can be dealt with in practice in this manner. The procedure of ranking objectives and optimizing the programme with respect to each in turn is termed Optimization in Tandem (see Chapter 1.3.3.2.i.).

(ii) Utility Functions

The manager may chose to impose some utility function of his own. These, in general, will attempt to reflect 'qualitative' aspects relating to the variables under his control For example, he may prefer a solution with a smooth cashflow profile; with a growing profit profile; with even shop-loading; etc.

(iii) Stability

A possible requirement may be that the desired solution be stable. In other words, the search is for a solution that can allow for some (maximal) degree of alteration in the availability of the resources without the necessity of changing one's operating plan, thereby avoiding the commotion invariably linked with such a change. Refering back to Fig 4.3 we see that change in availability of resource 1 will necessitate a change in the operating plan from A to A', and that the new plan still yields the same objective function value. No account is taken of the disruption caused by the move from A to A'. Only changes in availability of resource 2 will actually affect the attainment of the objective.

One can extend the desire for stability to include the restrictive resource too if the manager can stipulate some percentage of attainment that he would be prepared to sacrifice to this end - i.e. revert from an optimiser to a satisficer.

The stability requirement is encompassed into the programme as follows. Consider the normalised hyperplane

$$\frac{\underline{a}}{|a|}\,\underline{x} = \frac{b}{|a|} \qquad (6.1)$$

$\frac{b}{a}$ is the distance of the hyperplane from the origin, and

$\frac{\underline{a}}{a}$ is the unit normal to the hyperplane. The distance of any point $\underline{y}$ to this hyperplane is found by substituting the point into the normalised equation:

$$\text{distance} \quad \lambda = \left| \frac{b}{|a|} - \frac{\underline{a}}{|a|}\underline{y} \right| \qquad (6.2)$$

In a linear programme each constraint divides the linear space into a feasible closed half-space and an infeasible half-space. With the inward normal pointing into the feasible half-space, and if λ is constrained to be positive then Equation 6.2 can be written omitting the modulus sign. This enablesus to write the problem of maximising stability as a new linear programme.

To maximise stability, while maintaining optimal objective function attainment, maximise the minimum distance from all non-vital constraints of the original programme.

$$\begin{aligned} &\max \quad \lambda \\ &\text{s.t.} \quad \frac{b_i}{\underline{a}_i} - \frac{\underline{a}}{\underline{a}} \geq \lambda \qquad \forall\, i \text{ "non-vital"} \qquad (6.3) \\ &\qquad\qquad\qquad\qquad \text{i.e. } \pi_i^* = 0 \\ &\qquad\quad \lambda \geq 0 \\ &\qquad\quad \underline{c}\,\underline{x} = z^* \end{aligned}$$

A more general formulation would extend the desire of stability to all constraints, while ensuring that some given proportion of the optimal objective function value is achieved. Furthermore, stability should be measured in terms of some percentage of the resource availability allowing all constraints to be treated in a comparable manner. This is done by weighting the distance by the amount of resource $\frac{b_i}{\underline{a}_i}$. The resulting programme is:

$$\begin{aligned} &\text{Max} \quad \lambda \\ &\text{s.t.} \quad b_i - \underline{a_i x} \geq \lambda b_i \qquad \forall\, i \qquad (6.4) \\ &\qquad\quad \underline{x}, \lambda \geq 0 \\ &\qquad\quad \underline{c}\,\underline{x} = z^* \end{aligned}$$

Equation 6.4 places equal weight on the stability on each constraint. It may be possible for the various parties concerned with the company's affairs to negotiate an uneven weighting that would place a greater requirement for a stable solution on the availability of some resources (e.g. availability of capital) then others (e.g. availability of man-power).

The examples (based on a simplified version of the Corporate Mode described in Chapter 1) used hereafter will have selected amongst the alternate solutions in order to ensure a maximally stable solution.

2.2 Implications of a Selected Solution on the Valuation of the Assets

Clearly, whichever solution is finally accepted to be implemented, certain resources will not be used to their full capacity. In effect the company is overstocked with certain resources, and this it is prepared to accept in order to be able to operate the chosen plan - a hedge against uncertainty.*

Whether this is an explicit acceptance or not - even when the situation is unavoidable (due to the necessity of purchasing assets in block amounts) - is irrelevant. The fact remains that the company has an overabundance of assets paid for but not being used.

We propose that this situation ought to be reflected in the accounts by reference to the difference between the value (to the company pursuing its chosen plan) of that portion of the assets that are fully utilised, and some external reference value of the stock of assets available to the company.

It would appear reasonable to use Realisable Value (since the excess resource availability could always be sold!) as a measure of valuation that can be ascertained by reference to the environment in which the company operates. Reference to such an external, objective basis for valuation would surely find

*The introduction of such economic inefficiencies has a price and we propose a method to value this 'overcapacity'.

favour with the accountants! To find the value of the assets in the context in which they will be used, our proposal is that the programme be rerun with just the required availability of assets present, and the value of the selected plan attributed via the dual LP. The programme so constructed is totally degenerate, and the interpreation of such a programme is discussed in the following section.

The difference between the value of the assets in use, and the assets as stock could be reflected as "Holding Value" of the assets, and form the type of entry as Goodwill in the Balance Sheet. This value clearlycannot constitute a part of the Goodwill of the business since that is attributable to the company as a whole (see Chapter 3.4.3) whereas the Holding Value is identifiable with each asset. We feel that this information will be useful to readers of the accounts.

3. Degenerate Solutions

We have mentioned previously that the solutions to Corporate LP models (as described in Chapter 1 and 2) are almost invariably degenerate. In Chapter 5 we have shown that degenerate primal solutions result in alternate dual solutions. This would appear to add to the problems of using the dual LP as a means to value the assets of the company that were discussed in Chapter 3.5.2. Before proceeding with a discussion of the economic interpretation of the optimal alternate dual soltuion, we shall inspect the desirability of degenerate primal solutions.

3.1. Desirability of a Degenerate System

In two papers (65) and (76) GAL & HABR brought the desirable attributes of degenerate solutions to the attentions of LP modellers who were accustomed to viewing degeneracy disfavourably. The desirable attribute of a system that results in degenerate solutions is that it represents a more perfect economic system. in that there is less wastage than would be encountered in a non-degenerate system.
The production process is built up by the joint acquisition of resources of different types, each of which plays a vital role: vital in the sense that the absence of any particular resource would bring a halt to the process of production. In general, however, acquisition is achieved by purchasing the resources in different block sizes with the result that capacity is not equal throughout the production process. Rather, particular 'bottlenecks' can be identified. Furthermore, the easing of that bottleneck by the acquisition of more of the offending asset results in the bottleneck moving to some different function (and in general, only some portion of the newly acquired asset is used - the remaining capacity is un-utilised owing to the existance of this new bottleneck).

Degenerate solutions are tight in more constraints than are required by the geometry of the problem. The resulting system is at capacity equilibrium in more than the 'normal' number of resources with the corollary that there is less slack capacity being under-utilised. The perfect economic system is the wholly degenerate system where assets are present just to the extent that is required - a system with no waste!

It must be accepted that the price paid for such perfection is that the system is more vulnerable to changes in availability of resource. With over-availability (i.e. the presence of more resource capacity than originally conceived) the solution will remain acceptable (with the possibility that a better goal attainment might be available if a change in solution were undertaken). On the other hand, under-availability of any resource (rather than the fewer tight constraints found in non-degenerate solutions) render the solution infeasible and require a change in the planned activity. This 'perfect' solution is totally inflexible: it applies to a single product mix and a given set of costs and prices.

3.2 Desirability of Degeneracy for Valuation Using the Dual.

The application of the information that can be derived from the dual LP for the purposes of valuation has been described in Chapter 3.5.2. A major obstacle to the widespread application of this method of valuation arises from the inherent foundation of marginal costing on which LP is based.

A marginal evaluation scheme views the contribution that resources advance to the attainment of any stated goal (or goals) for the enterprise as whole. This contribution is valued at the margin - ie. the valuation is based on the amount by which the last (or next) unit of resource has increased the achievement of that goal. As a result resources with spare capacity have a marginal value of zero.

The problem of convincing management (and accountants) that the ownership of fully paid up assets (even though they may not be fully utilised!) have a zero value is extremely difficult! Indeed there is a paradox contained by this view (first propounded by WRIGHT (149): consider a particular asset - say stock - which is available in greater quantity than required. Accordingly its value is zero. But if the excess amount alone were to be lost then the remaining quantity (of stock) would have a value since the programme is now restricted by this condition!

Since a primal-degenerate solution is tighter in more than the 'normally required' number of constraints, the value of the company's goal attainment can be spread over a larger set of the the company's constituent resources.

3.3 Proposed Valuation Procedure

Our intention was to provide a facilityfor allocating the value of the firm over all the constituent resources by means of the dual LP.

Such a procedure would be to solve the corporate model (described in Chapters 1 and 2); to include any other subjective criteria to resolve the alternate solutions (we have used the desire for maximum stability); to restate the problem with resources allocated such that the solution would be attained without slacks, and to re-solve the resulting totally degenerate problem. In this way, the value of the firm can be allocated over all the constituent inputs (with the Holding Value ascribed to the difference between this value and some measure of value made with reference to the external environment, such as the Realisable Value if that is applicable.).

Such a valuation process would overcome many of the problems associated with other techniques (discussed in Chapter 3.3.3):

- Consistent valuation is achieved over all the modelled resources (e.g. including labour) thereby resolving the problems of additivity.

- Rather than build up the value of the company as a whole by the sum of values of the individual resources, this procedure allocates the value of the firm over the constituent resources. The value so attributed is the average value for the given amount of resource available in the programme and calculated in a given combination with the other resources. This is particularly relevant for such resources as stock.

- By choosing an interior point of the alternate dual space non-zero value may be attributed to all the resources.

- Valuation of assets in the context of their use to the company obviates the need for depreciation for any purpose other than as a 'charge' to profit made in order to build up a fund with which to replace assets worked out in service.

- The valuation is undertaken recognising the particular goals pursued by the company.

However the existence of the alternate dual-space results in two problems that must be resolved before the proposed procedure can be considered viable:

(i) Dual Variables Exist as a Vector

We have shown that the alternate dual space collapses to a point on the resolution of primal-degeneracy. We have linked the previouslynoted two-sided nature of the dual variable to changes in availability of that particular resource. However, the analysis makes no statement about the behaviour of other dual variables under such perturbation. Clearly, reduction in availability in resource s (resulting in π_s taking its maximum value) need not require that π_r (for any resource r) be at any particular value.

This results in the requirement that the dual variables always be treated as a cohesive set. It is wrong to use a value of π_s relating to the last unit of resource s availability, together with a similar value for π_r if both these values do not appear together as elements in a dual vector!

(ii) Explanation of the Difference between Alternate Dual Vectors

An interpretation of the difference between alternate dual vector remains an unsolved problem. We do not understand the basis for the difference between dual vectors, nor the meaning of negative variables.

It may be thought that resolution of the choice of a dual vector may be effected in a similar manner to the resolution of the alternate primal solutions. In other words, select a dual vector that

- allocates positive values to all the resources (if possible)

- results in the balance sheet showing a steadily increasing profit
- debt/equity ratios shown in the accounts remain with certain bounds etc.

It has proved possible to draw up a large number of balance sheets by selecting different dual vectors, and this is shown in Appendix D. Thisresults in the possibility that the company can construct different balance sheets for presentation to different interested parties. In constructing a balance sheet for presentation to shareholders the directors may feel that they would like to show large investment in resources, and steadily increasing profits with no running down of the assets. For presentation to the labour force, management may wish to show a declining profit with an increasing committment to the labour resource. Government may be shown a steadily increasing working capital requirement with increasing depreciation against a background of declining profits.

All of these different presentations of the companies affairs may be drawn by appropriate selection of dual vectors from the alternate space. Yet <u>all</u> the different representations are derived from the <u>same</u> underlying physical solution.

It may at first appear that the different balance sheets take account of and reflect the different views adopted by the interested parties in the company's affairs.

This is not so. Each valuation is made on the basis of <u>value to the company</u> (as a going concern): management now has the opportunity to present its affairs to outsiders in numerous different ways.

This brings up a basic question: does the use of the dual vector to 'price' the inputs actually impute value to the assets under conditions of degeneracy? Our previous understanding of the dual was that it represents the marginal value to the programme as a whole, of the last (or next) unit of the associated resource. Given a linear system , we can attribute this value to the entire availability of the resource, although we know that the range over which the dual is valid is more limited.

However, the dual in a degenerate problem occurs at a point of discontinuity. In effect $\frac{\partial z^*}{\partial b_i}$ does not exist. What we have is an average value that may not be linked in any way to the effect on the programme of change in availability of the resource in question. The dual vector represents an allocation of value of the whole over the constituent inputs. But it is not unique. By analogy given a value V to be allocated over inputs a and b then

$$V = V_a + V_b \qquad (6.5)$$

does not define unique values to V_a or V_b.

Selection of dual vectors from the alternate dual space implies a particular allocation of the objective function value over the resources. Different vectors result in different allocations, each of which is consistent. However, attempting to derive information from such an allocation is futile. In our example (Equation 6.5) there would be no basis for any interpretation of what to do about resource a were a particular allocation to give a high value V_a!

This result, deriving from the fundemental existence of jointness, should also make us look more closely at our interpretation of the dual for non-degenerate problems. The allocation of value by the dual has been shown to have drawbacks since the value is valid over a limited range and it invariably allocates zero value to resources with slack. The fact that this allocation is based on a valid marginal valuation approach should not prevent realisation that other allocations of value may be as valid, and that allocation by the dual LP may not be useful for application to balance sheet construction.

4. Conclusion

We have shown that alternate-primal solutions may be resolved by imposing other goals on the problem. These may be sejondary objectives; subjective selection based on 'quality' factors (such as even work-loading; smooth buildup of certain financial measures, etc) or on a desire for a solution that will remain feasible under variation in availability of the resources.

Realisation that the company holds an overabundance of assets in order that it may operate at its close solution has led us to suggest that the difference between some value ascribed by reference to the external environment, such as Realisable Value, and the value to the company of that amount of asset in actual use, should be included in the balance sheet as a Holding Value.

We have suggested that in order to find the value to the company of assets in use in the business, the problem be restated as a totally degenerate problem and that the dual LP be used to give an allocation of value of the overall objective function attainment to the constituent inputs. A totally degenerate system has the attractive attribute that firstly it represents a economically desirable system since there are no slacks (no wastage), that secondly by appropriate selection of a dual vector from the alternate dual space, non-zero value may be attributed to all the inputs.

However, we have found that the different valuations resulting from selection of different dual vectors, while being equally valid, are arbitrary allocations and do not allow the user the ability to put an interpretation on the results and thereby gain an understanding of the underlying factors affecting the results. The different valuations that may be constructed <u>all</u> represent the <u>same</u> physical solution. This result has led us to question the validity of using the dual LP (for both degenerate and non-degenerate problems) for the valuation process.

CHAPTER 7

CONCLUSIONS

Our objectives in this thesis were two-fold. First we set out to test a model formulation representing a company's short-term activities. Our intention was then to extend this formulation to the medium-term to encompass the capital investment under capital rationing problem. Our second objective was to investigate the application of the dual LP for valuation in constructing balance sheets for the company.

In Chapters 1 and 2 we have put forward a LP formulation that models the company's affairs in sufficient detail to be of use to management in budget preperation and planning. We have maintained this approach (of directly modelling the flows of cash and physical goods) in extending the model to the medium term. Encorporating capital expenditure in terms of projects that result in changes to the physical structure of the company overcomes two problems: the problem of linking financial returns to the project and the problem of relating the discount rate to the opportunity value of money (given by the dual LP) encountered by classical capital budgeting formulations.

We managed to test the short-term model in a 'field' application to a certain degree but data collection difficulties resulted in the final test on the entire company, having to be cancelled. There was no opportunity to test the medium term model, nor to consider the effect on the model of different objective functions.

Use of the dual LP for valuation has been proposed as a means of overcoming some of the problems recognised to pertain to currently accepted accounting conventions. The attributes of the LP approach are that

- it overcomes the problem of modular valuation. Since the LP takes an overview of the company as a whole, the value of the company is not built up as the sum of values attributed to the comparent units in isolation; rather the value to each unit arises from the contribution that the resources plays in attaining the value of the whole company.
- units of resource are valued as part of a company as a going concern. This is a view recognised by accountants being the ideal but which has proved immensely difficult to compute.

However, use of the dual LP has been limited since the dual 'evaluator' is recognised to hold only over the range of the associated basis. While this in no way detracts from the information about the marginal value of increasing (or decreasing) the availability of a particular resource, it has proved to severely limit the application of the dual for valuation purposes.

It has been argued that since the system is linear, (resulting in marginal values being equal to average values) one could use the dual variable as an average evaluator - the value of resource i being $\pi_i^* b_i$. Unfortunately, a marginal valuation process results in resources not being used to their full availability being valued at zero and this has been a difficult concept to include in balance sheet preparation!

To take account of the limited range of validity of the dual variable, and to overcome the problem of zero valuation for some resources it has been suggested that the resource availability be segmented, and each portion be valued at its corresponding marginal value. It has been shown that such a process is unsatisfactory since it results in the sum of such values being much greater that the value of the company as a whole (except for a company that has only one resource). We have shown, further, that such a procedure is likely to be incorrect.

In investigating the possible application of the dual LP for valuation purposes we have noted that corporate models (of the type formulated earlier) often yield alternate and/or degenerate solutions. We have shown in Chapters 4 and 5 how these solutions arise in terms of the geometry of the problem. This has led us to conclude that the KUHN-TUCKER necessary condition for optimality - that the dual variables be non-negative - does not apply to degenerate solutions. Furthermore we have shown that primal degenerate solutions are dual alternate (and vice versa).

Our proposed valuation procedure was to run the corporate model, to resolve the existance of alternate solutions (if they arise) by resorting to some secondary objective and to rerun the model with revised resource availability so that the new "optimal" solution can be attained but without any slack. The revised, totally-degenerate, problem appears to have a number of advantages:

(i) it allows the slack of the original ('real') problem to be valued, and this value - termed Holding Value - to be included in the balance sheet. This will show the extent to which the company is prepared to overstock itself with certain assets in order that it can operate at its chosen solution.

(ii) the totally degenerate system is the ideal deterministic economic system since there are no unused resources. (The price paid is that it is an inflexible system!)

(iii) since there are no slacks, the dual LP can attribute an average value to all the resources.

However the implications of the alternate dual space that exists at primal degenerate solutions are:

1. The dual variable must be considered as an element of a set of values. It is <u>incorrect</u> to pick a dual variable to value a certain resource (say π_i) with another dual variable for a different resource (π_j) unless they are both elements of the same set ($\pi_i, \pi_j \in \underline{\pi}$).

2. The infinity of different sets of dual variables enables us to construct different balance sheets that <u>all</u> purport to reflect the <u>same</u> physical solution. The resulting loss of information to the user of the accounts renders our proposed valuation process invalid.

Our conclusion is that the dual LP cannot provide a tool to break down the problem of valuing joint components - any consistent allocation of values is valid, but arbitrary.

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APPENDIX A

THE SHORT-TERM CORPORATE MODEL

This appendix discusses the use of the general manufacturing company planning model described in Chapter 1. The discussion is divided into five parts:

i. Use of the planning model.
ii. Definition of the variables and data used in the model formulation.
iii. A detailed construction of the equations forming the model.
iv. A listing of the Matrix Generator Program (with examples of the data requried).
v. A listing of the Report Writer Program (with examples of output).

1. The System in Use.

The planning model described in Chapter 1 (and detailed below) portrays a typical manufacturing company: the flow of raw materials through the manufacturing process is activated in order to satiate (as far as is possible) the expected demand for the finished goods produced, while taking account of a number of environmental, financial and managerial constraints.

In order to aid the corporate budget preparation exercise the model has been designed to reflect these activities in some detail. In the text case company we are modelling a firm over 16 periods, making 16 products, each sold in two outlets, on some

30 work-centres (with half of these having some sub-contracting facility) with 6 different labour groups from some 20 raw materials. This would result in a model of some 2500 rows by 8000 variables. In order to handle a LP model of this size, we used the MPSX system on an IBM 360/70.

However it is clearly unrealistic to expect management to have the detailed and technical knowledge required to manipulate the matrix of equations. It is envisaged that this job will be undertaken by a suite of computer programmes to the extent that requests made by the user will be translated into answers presented in intelligible user-oriented reports. In this way information requested by the factory manager will be reported back in terms of shop-loading; the finance director would receive cash-flow or projected profit/loss reports, etc. Examples of the type of reports envisaged are included in Section 5 of this Appendix.

The <u>fundemental</u> component of such a system is the data base and the adage "Garbage in ... Garbage out" must be borne in mind. The model requires (as will be shown in Sections 2 and 4) a substantial amount of detailed information: some say a requirement that places an intollerable burden on management. This is a weak critisism - the information, although voluminous, is the same as that used for decision making in the various departments of the company (accounts, sales, production, etc.) This model merely requires that this information be brought together and maintained,* while noting that any data that is suspected by the local users remains marked as suspect. In this

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* It must be reported that this requirement proved too onerous a task for the test-case company.

respect the importance of ranking the data sensitivities described in Chapter 1 becomes clear: local data problems may be shown to be globally unimportant (and vice versa).

It follows that the data bank maintenance systems will be company-dependant and will be structured round the company's other control systems.

Modification to the data bank by the user will be either permanent (due to updated knowledge) or temporary 'what-if' information (i.e. the user wants to run the planning model on the basis of some speculative data). The data-bank/user interface will be able to differentiate between these distinct operations and will then generate a complete re-run of the model, or merely a revision of the existing model, depending on the extent of the new information. The new (or revised) matrix is fed to the MPSX Linear Programming Package to be solved and the solution is interpretted for the user by a Report Writer. It is anticipated that the planning model will be run as shown in Figure A.1.

The state of application at the moment is limited to a Matrix Generator Program (MGP) (reading data in arbitrarily chosen format from card) which generates the entire matrix for the LP Package, and a Report Writer (RP) which generates sets of specialist reports. The programs are listed in Sections 4 and 5. For reasons of confidentiality the examples of data input and of output reports attached are from a specially constructed (simple) test problem.

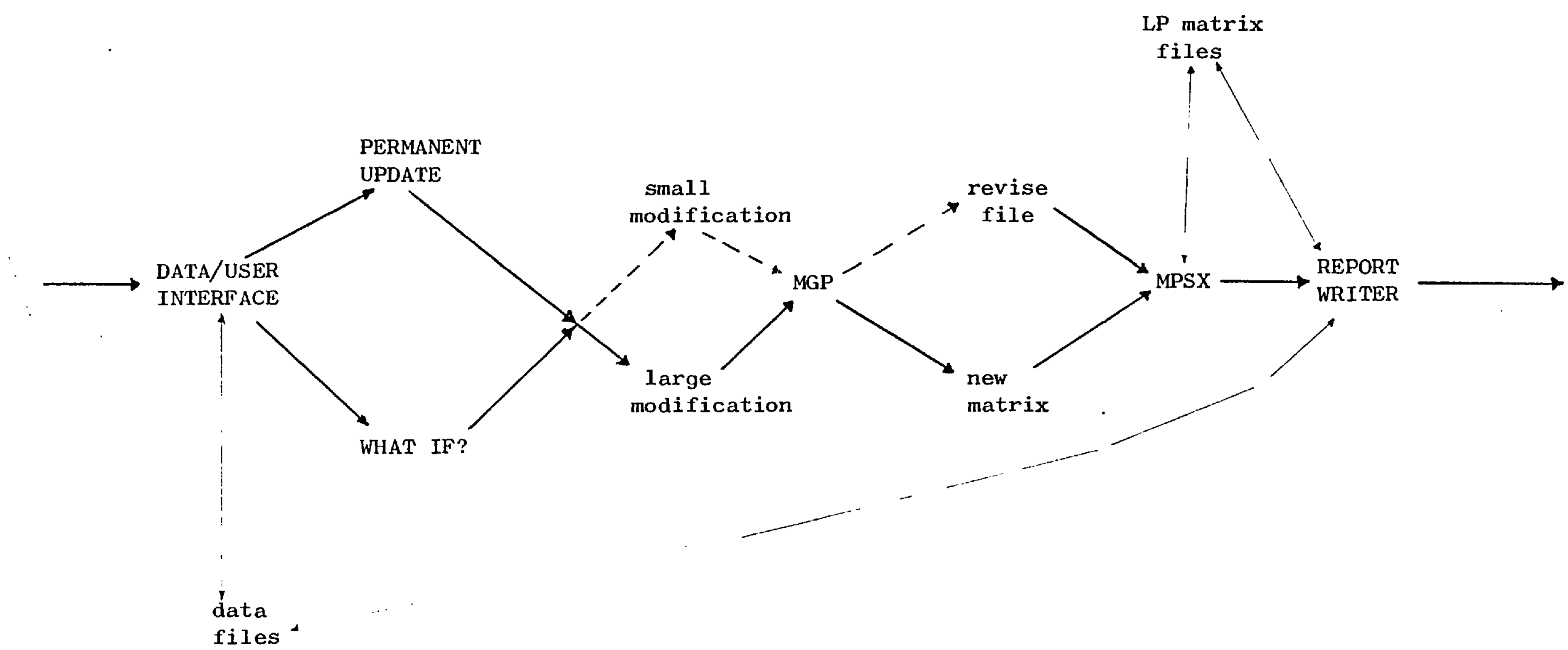

FIGURE A.1 THE LP MODEL IN USE

The model was tested by collecting real data from the test-case company for the year just ended, and comparing the actual results with the results of the model in a number of dimensions.

The test was protracted owing to difficulties in convincing the management that the required data were available (e.g. the length of the production cycle, the usage of raw materials through that period, etc), and other internal political activities that diverted management attention. Tests were conducted in each factory location individually and the resulting raw material usage; stock levels; subcontracting and overtime schedules were shown to compare with the actual results. Financial indicators such as profit and cashflow, could only be judged to be correct since the company had combined the accounting function for both locations and could not break the consolidated accounts down. No test of the entire company was undertaken before the project terminated.

2.1 Data Definition

alphlg	No. of periods lag obtained before payment of interest charge required.
alpha	Percentage interest charge (per period).
caplf(I,L)	Normal-time capacity (in hours) of labour type L in period I. This is calculated from the number of men in group L and the number of hours available for work in normal shift time in the period, and should take account of reductions to capacity due to holidays, absenteeism, etc.
caplo(I,L)	Overtime capacity (in hours) of labour type L in period I.
capst(I)	Storage capacity available in period I. The defined "unit of storage" (e.g. sq feet) is also used in vol(R) and space (K,J,I). We have assumed that only one distinct type of storage facility exists.
capwc(I,M)	Normal-time capacity (in hours) of workcentre M in period I. This is calculated from the number of machines in group M and the number of hours available in normal shift working time (factored by expected holidays, shutdowns, etc). The number of machines in the group need not be integral: machines known to be used in emergency only can be included as fractions of a whole machine. Workcentres such as PAINT need careful examination to ensure inclusion of preparatory work being done outside the paint booths. ASSEMBLY and WELD are also very difficult to ascertain correctly: the equivalent "number of machines in the

group" has been defined to be the number of products on which work can in general be carried out simultaneously.

Finally, capacity is factored by an amount to take account of any extra-model production activities (- in the case of the test-case company to take account of the production of spares and of products being phased out by the company with expected actual production of only one or two units).

cashi	Value of the bank balance at the beginning of the modelling period.
crti	Value of the creditors account at the beginning of the model.
dbti	Value of the debtors account at the beginning of the model.
excash(I)	Extra-model cash flow in period I. This total can be broken down into any number of items that are normally detailed in the accounts e.g. wages; management levies; tax bills; receipts from sales of spares, etc.
excrt(I)	Extra-model credit in period I.
exdbt(I)	Extra-model debt in period I.
ifbl	This takes a value of 1 if the model is allowed to accept overdraft facilities.
kdiv(K,1)	No. of sections which make up product K. (Section 1 refers to the complete product.)
kdiv(K,J,1)	Two letter name identification for section J of product K.

knout(K)	No. of sales outlets for product K. Each outlet is identified as an area in which either the product specification or the net price differs from elsewhere.
lagexcrt	No. of periods lag before extra-model credit is paid.
lagexbdt	No. of periods lag before extra-model debt is received.
list(K,J)	Gross price for product K in sales outlet J.
market (I,K,J)	Market forecasts for sales of product K in outlet J in period I. These act as upper bounds on demand. Lower bounds can also be included in the model to cater for a management constraint of representation in certain (if not all) sectors of the market.
mcreq(I,K,J,M)	Machine time required on workcentre M by section J of product K in period I of the production cycle. This time (in hours) is derived from the standard times, available from the production control department, to which is added a component representing the set-up times (making some assumptions about batch sizes). In practice, work is done at a faster rate. The required factor by which the 'standard' time is multiplied is found from the wages department.
month(I)	Name of period I.
nlf	No. of labour force types considered by the model.
nm	No. of periods in the planning horizon.
nout	Total number of sales outlets encompassed by the model.
nprod	No. of products in range.
nrm	No. of different raw materials.

nsub	No. of workcentres that can be subcontracted.
nwc	No. of distinct workcentres.
owage(L)	Overtime rate for labour type L.
prod(K,J)	Limit on production of section J of product K in any period. This limit is the smallest bottleneck created by the scarcity of jigs required by certain jobs in the production process.
prodi(I,K,J)	Amount of production of section J of product K completed in period I. This data is determined by the work in progress underway at the beginning of the planning horizon. If no limit exists the entry is infinity.
rmb(R)	Net price per unit of raw material R.
rmlag(R)	No. of periods lag obtained before payment for supply of raw material R is required.
rmp(I,R)	Fixed amount of raw material R due for delivery in period I. This is determined by existing activities at the beginning of the planning horizon. If no limit exists the entry is infinity.
rmreq(I,K,J,R)	Units of raw material R required for the production of section J of product K in period I of the production cycle.
sblag(M)	No. of periods credit obtained before payment required on work subcontracted to subcontractor M.
slag(K,J)	No. of periods credit given on the sale of product K in outlet J before payment collected.
space(K,J,I)	Storage space required by section J of product K in period I of its production cycle.

spred(K,J)	Length of production cycle of section J of product K.
stockcf(K)	Desired minimum stock levels at horizon of finished goods product K.
stockcr(R)	Desired minimum stock levels at horizon of raw material type R.
stockif(K)	No. of units of finished goods product K in stock at the beginning of the planning horizon.
stockir(R)	No. of units of raw material R in stock at beginning of model horizon.
subcd(K,J)	Two letter name identification of sales outlet J for product K.
subp(M)	Net price (per hour) charged by subcontractor M.
subwc(M)	List of workcentres whose capacity may be enhanced by subcontracting.
wclf(L,M)	No. of men of type L required to man workcentre M. A value of infinity indicates that labour of this category cannot man a particular workcentre.
wcload(M)	Scheduling efficiency in loading workcentre M. This is calculated as the ratio of busy time (i.e. total less idle time) to the total time available, and is used to factor the times given in mcreq(I,K,J,M) to obtain a realistic production time.
wcnm(M)	Two letter name identification of workcentre M.
vol(R)	Storage space required by a unit of raw material R.

2.2 Definition of Variables

BANKL(I)	Bank loan taken in period I.
BANKR(I)	Bank repayment made in period I.
CASH(I)	Cash position at the close of period I.
CRT(I)	Creditors position at the close of period I.
DBT(I)	Debitors position at the close of period I.
LABOT(I,L,M)	Overtime scheduled in period I for labour type L manning workcentre M.
LABREQ(I,L,M)	Total required labour time for labour type L manning workcentre M in period I.
PROD(I,J,K)	No. of units of section J of product K completed in period I.
RMB(I,R)	Units of raw material R purchased in period I. (Assume no lag between purchase and delivery.)
SALES(I,K,J)	No. of units of product K sold (and delivered) in market J in period I.
STOCK(I,K)	No. of completed units of product K in stock at the close of period I.
STOCKR(I,R)	No. of units of raw material R in stock at the close of period I.
SUB(I,M)	Amount of work subcontracted out for workcentre M in period I.

3. The Model Equations

A. Production Function

i. Workcentre Capacity (cf. Eqn.1.10)

Requirement for workcentre M in period I must be less than or equal to the normal time capacity of that workcentre in that period plus any overtime worked plus any subcontracting done.

$$\sum_{K=1}^{nprod} \sum_{J=1}^{kdiv(K,1)} \sum_{\hat{I}=0}^{spred(K,J)-1} PROD(I-\hat{I},J,K).mcreq(\hat{I}+1,K,J,M) \qquad (A.1)$$

$$- SUB(I,M) - \sum_{L=1}^{nlf} \frac{LABOT\ (I,L,M)}{wclf(L,M)}$$

$$\leq capwc(I,M) \qquad \forall\ I,M$$

Initial condition:

$$PROD\ (I,J,K) = prodi\ (I,K,J) \quad \forall\ prodi\ (I,K,J) \neq \infty$$

ii. Labour Force Allocation (cf. Eqn. 1.11)

Total in-house production (i.e. total workcentre requirement less any work subcontracted out) on workcentre M in period I must be allocated amongst the labour groups capable of doing the work.

$$\sum_{K=1}^{nprod} \sum_{J=1}^{kdiv(K,1)} \sum_{\hat{I}=0}^{spred(K,J)-1} PROD(I-\hat{I},J,K).mcreq(\hat{I}+1,K,J,M)$$

$$- SUB(I,M) \qquad (A.2)$$

$$= \sum_{L=1}^{nlf} \frac{LABREQ(I,L,M)}{wclf(L,M)} \qquad \forall\ I,M$$

B. Physical Constraints.

i. Production (cf. Eqn. 1.12)

Limit production (of division J) of product K in period I by some upper bound.

$$PROD(I,J,K) \leq prod(K,J) \qquad \forall\ I,K,(J) \qquad (A.3)$$

ii. Labour Force Capacity (cf. Eqn. 1.13)

Requirement for labour group L in period I must be less than or equal to the work scheduled to be done in overtime plus that scheduled to be done in normal shift time.

$$\sum_{M=1}^{nwc} LABREQ(I,L,M) - LABOT(I,L,M) \leq caplf(I,L) \qquad (A.4)$$

$$\forall\ I,L$$

iii. Labour Overtime Capacity (cf. Eqn. 1.14)

Limit the total overtime load on labour group L in period I by some upper bound.

$$\sum_{M=1}^{nwc} \text{LABOT}(I,L,M) \leq \text{caplo}(I,L) \qquad \forall\ I,L \qquad (A.5)$$

iv. Market Constraints (cf. Eqn. 1.15)

Limit sales of product K in market J in period I by some upper (and/or lower) bound.

$$\text{SALES}(I,K,J) \leq \text{market}(I,K,J) \qquad \forall\ I,K,J \qquad (A.6)$$

v. Storage Capacity (cf. Eqn. 1.16)

Storage space required in period I must be less than or equal to the storage space available.

$$\sum_{K=1}^{nprod} \sum_{J=1}^{kdiv(K,1)} \sum_{\hat{I}=0}^{spred(K,J)-1} \text{PROD}(I-\hat{I},J,K).\text{space}(K,J,\hat{I}) \qquad (A.7)$$

$$\sum_{R=1}^{nrm} \text{STOCKR}(I,R).\text{vol}(R) \leq \text{capst}(I) \qquad \forall\ 1$$

C. Financial Flows

i. Cash Position (cf. Eqn. 1.17)

Cash at the close of period I equals cash position at the opening of period I plus inflows resulting from sales made in previous periods and loans negotiated, less outflows resulting from loans repayed; payment for overtime worked in that period; subcontracting costs; purchases, and bank charges incurred in previous periods and net outflows from extra-model activities.

$$CASH(I) = CASH(I-1) + BANKL(I) + exdbt\ (I-lagexdbt)$$

$$+ \sum_{K=1}^{nprod} \sum_{J=1}^{knout\ (K)} SALES\ (I-slag(K,J),K,J).list(K,J).discp(K,J)$$

$$- BANKR(I) - \sum_{\hat{I}=0}^{alphlg-1} BANKL(I-\hat{I})-BANKR(I-\hat{I})\ .alpha$$

$$- excash(I) - excrt(I-lagexcrt) \qquad (A.8)$$

$$- \sum_{L=1}^{nlf} \sum_{M=1}^{nwc} LABOT(I,L,M).owage(L)$$

$$- \sum_{R=1}^{nrm} RMB\ (I-rmlag(R),(R).rmb(R)$$

$$- \sum_{M=1}^{nwc} SUB(I-sblag(M),(M).subp(M) \qquad \forall I$$

Initial Condition:

$$CASH(0) = cashi$$

ii. Creditors Position (cf. Eqn. 1.18)

Credit at close of period I equals purchases of raw materials made by not yet paid for, plus subcontracted work carried out but not yet paid for plus interest charges outstanding plus any extra-model credit.

$$CRT(I) = \sum_{I=0}^{lagexcrt-1} excrt\ (I-I)$$

$$+ \sum_{R=1}^{nrm} \sum_{\hat{I}=0}^{rmlag\ (R)-1} RMB\ (I-\hat{I},R).rmb(R) \qquad (A.9)$$

$$+ \sum_{\hat{I}=0}^{alphlg} (BANKL(I-\hat{I}) - (BANKR(I-\hat{I})).alpha \qquad \forall I$$

Note - the credit account does not include the outstanding overdraft.

Initial Condition:

$$excrt\ (0) = crti.$$

iii. Debtors Position (cf. Eqn. 1.19)

Debt at the close of period I equals reserves due, but not yet received, from sales made in prior periods plus any extra model debt.

$$DBT\ (I) = \sum_{I=0}^{lagerdbt-1} exdbt(I-I) \qquad (A.10)$$

$$+ \sum_{K=1}^{npod} \sum_{J=1}^{knout(K)} \sum_{\hat{I}=0}^{slag(K,J)-1} SALES\,(I-\hat{I},K,J).list(K,J).discp(K,J) \qquad \forall I$$

Initial Condition:

$$exdbt(0) = dbti$$

D. Inter-Period Continuity

(i) Finished Goods Stock (cf. Eqn. 1.20)

Stocks of product K at the close of any period equals the stock at the close of the previous period plus newly completely production less sales made in the period.

$$STOCKF(I,K) = STOCK\ F(I-1,K) + PROD\ (K,1,I) - \sum_{0=1}^{knout(K)} SALES\ (K.O.) \qquad (A.11)$$

$\forall$ K,I

Initial Condition:

STOCKF(O,K) = Stockif(K)

Closing Condition:

STOCKF(nm,K) $\geq$ Stockcf(K)

(ii) Raw Material Stock (cf. Eqn. 1.21)

Stock level of raw material R at the close of any period equals the stock level at the close of the previous period plus any new purchases less the amount used in production during the period.

$$STOCKR(I,R) = STOCKR(I-1,R) + RMB(I,R) \qquad (A.12)$$

$$- \sum_{K=1}^{nprod} \sum_{J=1}^{kdis(K,)} \sum_{\hat{I}=0}^{spred\,(K,J)-1} PROD(I-\hat{I},J,K).rmreq(I+1,K,J,R)$$

$$\forall I,R$$

Opening Conditions:

$$STOCKR(0,R) = Stockir(R)$$

$$RMB(I,R) = rmp\,(I,R) \quad \forall\; rmp(I,R) \neq \infty$$

Closing Condition:

$$STOCKR(nm,R) \geq Stockfr(R)$$

E. Alternative Objective Functions

(i) Profit Earning (cf. Eqn. 1.22)

Max cash plus debtors less creditors positions of the close of the planning period.

$$\text{Max} \quad CASH(nm) + DBT(nm) - CRT(nm) \qquad (A.13)$$

We have used this objective function in all the examples that follow.

(ii) Turnover (cf. Eqn. 1.23)

Maximise total sales revenue acquired during the planning period.

$$\text{Max} \sum_{K=1}^{nprod} \sum_{J=1}^{knout(K)} \sum_{I=1}^{nm} SALES(I,K,J).list(K,J).discp(K,J) \quad (A.14)$$

(iii) Sales Penetration (cf. Eqn. 1.24)

Maximise the total number of primary products sold during the planing horizon.

$$\text{Max} \sum_{K=1}^{prod} \sum_{J=1}^{knout(K)} \sum_{I=1}^{nm} SALES(I,K,J) \quad (A.15.)$$

4.1 The Matrix Generator Programme.

```
C
C                 .....................................................
C                 .  THIS PROGRAMME IS THE MATRIX GENERATOR             .
C                 .....................................................
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
COPYRIGHT    ---    AMIR ELLOW    IMPERIAL COLLEGE (MANAGEMENT SCIENCE)
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C
C                 .....................................................
C                 .  THE INPUT DATA IS READ ON 2                        .
C                 .  THE DATA RECORDS ARE WRITTEN ON LINEPRINTER 3      .
C                 .  THE GRAPHS ARE PLOTTED ON LINEPRINTER 5            .
C                 .  THE OUTPUT FILE IS 8                               .
C                 .....................................................
      COMMON MONTH(13),SPRED(20,4),N1,K,KDIV(20,5),CPCTY(15,20)
      REAL SALESS(2,2,16)
      INTEGER IGRPH(16,2)
      INTEGER BTYPE,TURN,TYPE,SPRED,HTIME,TIME,STYPE,NUMBR(20)
      INTEGER ALFLG,STURN(20),FCAST(4,20),STOCKI(20)
      INTEGER COLMN(5),KCODE(20),SLAG(20,2),RMLAG(9),SBLAG(17),
     1SUBNC(17),SUBCD(20,2),KNOUT(20),SUBCD,DENM, TYPE1,CCODE(15)
      REAL CHAGE(5),ANAME(2),CAPHC(15,17),WCLOAD(20)
      REAL RMB(9),VOL(9),SUBP(17),SPACE(20,4),SUBL(20),FGP(20),DUMMY(20)
      REAL WCLF(6,20),LIST(20,2),DISCP(20,2)
      REAL MCREQ(4,9,1,20),RMREQ(4,9,1,10)
C
C                 .....................................................
C                 .  INITIALISE                                         .
C                 .....................................................
      DATA CCODE/'PD','SL','SG','RD','KA','CH','BL','BR','BC','CR','DE',
     1 'LB','LD','SR','SF'/
      DATA SUBCD/'  '/
      DATA WCLF/120*0.0/,FCAST/80*0/
      DATA RMREQ/360*0.0/,MCREQ/720*0.0/
      DATA ALFLG/2/,ALPHA/9.5/
      WCLEFT=0.0
      DATA NUMBR/'01','02','03','04','05','06','07','08','09','10',
     1           '11','12','13','14','15','16','17','18','19','20'/
      ISCALE=100
      ALPHAC=ALPHA/1200.0
C
C                 .....................................................
C                 .  READ IN THE BASE DATA                              .
C                 .....................................................
      READ(2,4001) NPROD,NOUT,NWC,NLF,NSUB,NRM,NM,  IFBL
      WRITE(3,5001)NPROD,NOUT,NWC, NLF,NSUB,NRM,NM
      READ(2,4020)(RMB(J),RMLAG(J),VOL(J),J=1,NRM)
      WRITE(3,5020)
      WRITE(3,5030)(RMB(J),RMLAG(J),VOL(J),J=1,NRM)
      READ(2,4030)((WCLF(M,N),M=1,NLF),N=1,NWC)
      READ(2,4035)  (WCLF(NLF+1,N),N=1,NWC)
      WRITE(3,5040)
      DO 8010 N=1,NWC
 8010 WRITE(3,5050)WCLF(NLF+1,N)   ,(WCLF(M,N),M=1,NLF)
      READ(2,4190)  (WCLOAD(J),J=1,NWC)
      READ(2,4030)  (CHAGE(L),L=1,NLF)
      WRITE(3,5052)  (CHAGE(L),L=1,NLF)
      IF(NSUB.EQ.0)  GO TO 8021
      READ(2,4040)(SUBNC(KA),SUBP(KA),SBLAG(KA),KA=1,NSUB)
      WRITE(3,5060)
      WRITE(3,5070)  (SUBNC(KA),SUBP(KA),SBLAG(KA),KA=1,NSUB)
 8021 CONTINUE
C
C                 .....................................................
C                 .          - FOR EACH PRODUCT                         .
C                 .....................................................
      DO 8020  K=1,NPROD
      READ(2,4050)ISK ,KDIV(K,1),KNOUT(K)
      WRITE(3,5080) K,KDIV(K,1),KNOUT(K)
      KNO=KNOUT(K)
      WRITE(3,5130)
C
C                 .....................................................
C                 .     -    SALES INFORMATION                          .
C                 .....................................................
      DO 8050J=1,KNO
      READ(2,4080)ISK,SUBCD(K,J),LIST(K,J),DISCP(K,J),SLAG(K,J)
 8050 WRITE(3,5140)SUBCD(K,J),LIST(K,J),DISCP(K,J),SLAG(K,J)
      KDV=KDIV(K,1)
      DO 8020 KDV1=1,KDV
      WRITE(3,5090) KDV1,K
C
C                 .....................................................
C                 .     -    MC REQUIREMENT                             .
C                 .....................................................
      READ(2,4070) ISK,KDIV(K,KDV1+1),KT,SPRED(K,KDV1),SPACE(K,KDV1)
      NSPR=SPRED(K,KDV1)
      NSPR=NSPR-1
      READ(2,4071)(I,J,MCREQ(I,K,KDV1,J),IJK=1,KT)
      WRITE(3,5100)
      DO 8030 J=1,NWC
```

```
8050   WRITE(3,5050)KDEF(NEF+1,J),(HOREQ(I,K,KDV1,J),I=1,NSPR)
C                ....................................................................
C                .    - -    RAW MATERIAL REQUIREMENT                               .
C                ....................................................................
       READ(2,4070) IOK,KP,KT
       IF(KT.EQ.0) GO TO 8070
       READ(2,4071) (I,J,RMREQ(I,K,KDV1,J),IOK=1,KT)
       WRITE(3,5120)
       DO 8040 J=1,NRM
8040   WRITE(3,5050) J,(RMREQ(I,K,KDV1,J),I=1,NSPR)
8070   CONTINUE
8020   CONTINUE
C                ....................................................................
C                .        - POST HORIZON FORCAST                                    .
C                ....................................................................
       DO 8100 K=1,NPROD
       KP=KDIV(K,1)
       III=0
       II=SPRED(K,1)-1
       IF(KP.LE.1) GO TO 8080
       DO 8090 KP1=2,KP
       IF(III.GT.SPRED(K,KP1))GO TO 8090
       III=SPRED(K,KP1)
8090   CONTINUE
       II=II+III
8080   IF(II.EQ.0) GO TO 8105
       WRITE(3,5150)
       READ(2,4090) (FCAST(I,K),I=1,II)
       WRITE(3,5160) K,(FCAST(I,K),I=1,II)
       GO TO 8100
8105   READ(2,4100) SKPS
8100   CONTINUE
C                ....................................................................
C                .  SET UP THE ROWS                                                 .
C                ....................................................................
        READ(2,4100)(MONTH(I),I=1,NM)
       READ(2,4110)NROW,XNAME
       WRITE(8,5170)XNAME
       WRITE (8,5180)
       DO 8111 N=1,NROW
       READ(2,4120)ICODE,RCODE(N),NDIM,IFLAG
C                ....................................................................
C                . IFLAG = 0   -   GENERAL                                           .
C                .       = 1   -   LAST PERIOD ONLY                                  .
C                .       = 2   -   FIRST PERIOD ONLY                                 .
C                .       = 3   -   FOR THE PRODUCT DIVISIONS                         .
C                .       = X0Y-    ALLOCATION OF TYPE Y TO TYPE X                    .
C                ....................................................................
       IFLAG=IFLAG+1
8120   IF(NDIM.EQ.0) GO TO 8111
       DO 8115 J=1,NDIM
       DO 8110 I=1,NM
       STYPE=SU500
       KP1=0
       GO TO(8140,8160,8170,8175,8177),IFLAG
8160   IF(I-NM)8110,8140,8110
8170   IF(I-1) 8110,8140,8110
8175   KP=KDIV(J,1)
       IF(KP.EQ.1) GO TO 8115
       DO 8112 KP1=2,KP
       STYPE=NDIV(J,KP1+1)
8140   IF(J-10)8180,8190,8190
8180   WRITE(8,5190)ICODE,MONTH(I),RCODE(N),J,STYPE
       IF(KP1.EQ.0) GO TO 8110
       GO TO 8112
8190   WRITE (8,5200) ICODE,MONTH(I),RCODE(N),J,STYPE
       IF(KP1.EQ.0) GO TO 8110
8112   CONTINUE
8110   CONTINUE
8115   CONTINUE
       GO TO 8111
8177   NN=NDIM/100
       NDIM=NDIM-100*NN
       DO 8200 J=1,NDIM
       DO 8200 I=1,NM
       DO 8200 J2=1,NN
       J3=J+100*J2
       IF(J3.GT.1000) GO TO 8210
       WRITE(8,5203)ICODE,MONTH(I),RCODE(N),J3,STYPE
       GO TO 8200
8210   WRITE(8,5204)ICODE,MONTH(I),RCODE(N),J3,STYPE
8200   CONTINUE
8111   CONTINUE
       WRITE(8,5210)
       GO TO 100
```

```
8130   WRITE(6,8220)
       GO TO 1
100    CONTINUE
C                ...........................................................
C                .  SET UP THE VARIABLES                                   .
C                ...........................................................
C
C                ...........................................................
C                .      *****      PRODUCTION VARIABLES          *******    .
C                ...........................................................
       NAME=CCODE(1)
       STYPE=SUBDD
       DO 101 K=1,NPROD
       KP=KDIV(K,1)
       DO 101 KP1=1,KP
       KK=100 + KP1
102    DO 101 I=1,NM
       CALL CCLUM(COLNM,I,NAME,KK,STYPE)
C                ...........................................................
C                .  WORK CENTRE CAPACITY                                   .
C                ...........................................................
       ROWNM=RCODE(1)
       CALL P3(COLNM,ROWNM,I,NWC,WCREQ,KP1)
C                ...........................................................
C                .  STORAGE CAPACITY                                       .
C                ...........................................................
       IF(KP1.EQ.1) GO TO 103
       COEFF=SPACE(K,KP1)
       ROWNM=RCODE(3)
       TYPE=1
       LTIME=1
       CALL P2(COLNM,ROWNM,LTIME,NM,TYPE,COEFF)
103    CONTINUE
C                ...........................................................
C                .  RAW MATERIALS STOCKS                                   .
C                ...........................................................
       ROWNM=RCODE(4)
       CALL P3(COLNM,ROWNM,I,NRM,RMREQ,KP1)
C                ...........................................................
C                .  FINISHED GOODS STOCKS                                  .
C                ...........................................................
       IF(KP1.GT.1) GO TO 106
       IF(I.GT.NM) GO TO 106
105    LTIME=I
       COEFF=-1
       ROWNM=RCODE(5)
       CALL P1(COLNM,ROWNM,LTIME,    K,COEFF)
106    CONTINUE
C                ...........................................................
C                .  NUMBER OF PARTS PER F/GOOD                             .
C                ...........................................................
       IF(KP.EQ.1) GO TO 140
       ROWNM=RCODE(15)
       TYPE=100 + KP1
       IF(KP1.GT.1) GO TO 135
       IF((I-SPRED(K,1)+1).LE.0) GO TO 140
       II=I
       COEFF=1
       GO TO 139
135    COEFF=-1
       II=I+SPRED(K,1)-1
       IF(II.GT.NM) GO TO 140
139    CALL P1(COLNM,ROWNM,II,TYPE,COEFF)
140    CONTINUE
C                ...........................................................
C                .  TOTAL LABOUR TIME-ALLOCATION                           .
C                ...........................................................
       NTYPE=NWC
       ROWNM=RCODE(17)
       CALL P3(COLNM,ROWNM,I,NTYPE,WCREQ,KP1)
C
   101 CONTINUE
C
C                ...........................................................
C                .      *****      SALES VARIABLES               *******    .
C                ...........................................................
       NAME=CCODE(2)
       DO 201 K=1,NPROD
       KNO=KNOUT(K)
       DO 201 J=1,KNO
       STYPE=SUBCD(K,J)
       DO 201 I=1,NM
       CALL CCLUM(COLNM,I,NAME,K,STYPE)
C                ...........................................................
C                .  FINISHED GOODS STOCK                                   .
```

```
C     ..................................................................
      ROWNM=RCODE(5)
      TYPE=K
      COEFF=1.0
      LTIME=I
      CALL P1(COLNM,ROWNM,LTIME,    TYPE,COEFF)
C     ..................................................................
C     .  DEBT                                                          .
C     ..................................................................
      ROWNM=RCODE(6)
      TYPE=1
      LTIME=I
      II=I+SLAG(K,J)
      HTIME=II-1
      IF(SLAG(K,J))208,208,205
205   IF(HTIME-NM)207,207,206
206   HTIME=NM
207   CALL P2(COLNM,ROWNM,LTIME,HTIME,TYPE,VALUE)
208   CONTINUE
C     ..................................................................
C     .  CASH CONTINUITY                                               .
C     ..................................................................
      ROWNM=RCODE(8)
      II=I+SLAG(K,J)
      TYPE=1
      VALUE=(LIST(K,J)*(100-DISCP(K,J)))/100
      IF(II-NM)203,203,204
203   CALL P1(COLNM,ROWNM,II,TYPE,VALUE)
204   CONTINUE
C     ..................................................................
C     .  TOTAL SALES VALUE OBJECTIVE                                   .
C     ..................................................................
      ROWNM=RCODE(13)
      TYPE=1
      CALL P1(COLNM,ROWNM,NM,TYPE,VALUE)
C     ..................................................................
C     .  TOTAL SALES PENETRATION OBJECTIVE                             .
C     ..................................................................
      TYPE=2
      ROWNM=RCODE(13)
      COEFF=1
      CALL P1(COLNM,ROWNM,NM,TYPE,COEFF)
C
201   CONTINUE
C     ..................................................................
C     .        *****      SUBCONTRACTING ACTIVITIES        *******     .
C     ..................................................................
      IF(NSUB.EQ.0) GO TO 399
C
      NAME=CCODE(3)
      STYPE=SUBOD
      DO 301 I=1,NM
      DO 301 M=1,NSUB
      TYPE=SUBNC(M)
      CALL COLUM(CCLNM,I,NAME,TYPE,STYPE)
C     ..................................................................
C     .  WORK CENTRE CAPACITY                                          .
C     ..................................................................
      ROWNM=RCODE(1)
      COEFF=-1.
      CALL P1(COLNM,ROWNM,I,TYPE,COEFF)
C     ..................................................................
C     .  CREDIT                                                        .
C     ..................................................................
      ROWNM=RCODE(7)
      TYPE=1
      COEFF=SUBP(M)
      II=SBLAG(M)+I
      HTIME=II-1
      IF(HTIME-I)308,305,305
305   IF(HTIME-NM)307,307,306
306   HTIME=NM
307   LTIME=I
      CALL P2(COLNM,ROWNM,LTIME,HTIME,TYPE,COEFF)
308   CONTINUE
C     ..................................................................
C     .  CASH CONTINUITY                                               .
C     ..................................................................
      ROWNM=RCODE(8)
      TYPE=1
      II=I+SBLAG(M)
      IF(II-NM)303,303,304
303   COEFF=-SUBP(M)
      CALL P1(COLNM,ROWNM,II,TYPE,COEFF)
304   CONTINUE
```

```
C           ..............................................................
C           .   TOTAL LABOUR TIME-ALLOCATION                             .
C           ..............................................................
      ROWNM=RCODE(17)
      TYPE=SUBMC(M)
      COEFF=-1.
      CALL P1(COLNM,ROWNM,1,TYPE,COEFF)
C
301   CONTINUE
399   CONTINUE
C           ..............................................................
C           .      *****          ALLOCATED LABOUR             *******   .
C           ..............................................................
      NAME=CCODE(12)
      DO1301 I=1,NM
      DO1301 L=1,NLF
      DO1301 K=1,NWC
      IF(WCLF(L,K).EQ.0.0) GO TO1350
      STYPE=NUMBR(K)
      CALL  CCOLUM(COLNM,I,NAME,L,STYPE)
C           ..............................................................
C           .   LABOUR CAPACITY                                          .
C           ..............................................................
      TYPE=L
      COEFF=1.0
      ROWNM=RCODE(2)
      CALL P1(COLNM,ROWNM,I,TYPE,COEFF)
C           ..............................................................
C           .   TOTAL LABOUR TIME-ALLOCATION                             .
C           ..............................................................
      ROWNM=RCODE(17)
      COEFF=-1/WCLF(L,K)
      TYPE=K
      CALL P1(COLNM,ROWNM,I,TYPE,COEFF)
C
1350  CONTINUE
1301  CONTINUE
C           ..............................................................
C           .      *****          RAW MATERIALS PURCHASED      *******   .
C           ..............................................................
      STYPE=SUBGD
      NAME=CCODE(5)
      DO 501 L=1,NRM
      DO 501 I=1,NM
      CALL CCLUM(COLNM,I,NAME,L,STYPE)
C           ..............................................................
C           .   RAW MATERIALS STOCKS                                     .
C           ..............................................................
502   ROWNM=RCODE(4)
      TYPE=L
      COEFF=-1
      CALL P1(COLNM,ROWNM,1,     TYPE,COEFF)
503   CONTINUE
C           ..............................................................
C           .   CREDIT                                                   .
C           ..............................................................
      IF(RMLAG(L))508,508,507
507    ROWNM=RCODE(7)
      TYPE=1
      COEFF=RMS(L)
      II=I+RMLAG(L)-1
      IF(II.LE.NM) GO TO 509
      II=NM
509   CALL P2(COLNM,ROWNM,I,II,TYPE,COEFF)
508   CONTINUE
C           ..............................................................
C           .   CASH CONTINUITY                                          .
C           ..............................................................
      II=I+RMLAG(L)
      IF(II-NM)504,504,505
504    ROWNM=RCODE(8)
      TYPE=1
      COEFF=-RMS(L)
      CALL P1(COLNM,ROWNM,II,TYPE,COEFF)
505   CONTINUE
C
501   CONTINUE
C           ..............................................................
C           .      *****          RAW MATERIALS STOCKS         *******   .
C           ..............................................................
      NAME=CCODE(14)
      STYPE=SUBGD
      DO 1201 L=1,NRM
      DO 1201 I=1,NM
      CALL CCLUM(CCLNM,I,NAME,L,STYPE)
```

```
C                   ....................................................
C                   .   RAW MATERIALS STOCKS                           .
C                   ....................................................
      ROWNM=RCODE(4)
      TYPE=L
      COEFF=1
      CALL P1(COLNM,ROWNM,I,TYPE,COEFF)
      IF(I.EQ.NM) GO TO 1202
      II=I+1
      COEFF=-1
      CALL P1(COLNM,ROWNM,II,TYPE,COEFF)
 1202 CONTINUE
C                   ....................................................
C                   .   STORAGE CAPACITY                               .
C                   ....................................................
      ROWNM=RCODE(3)
      TYPE=1
      COEFF=VOL(L)
      CALL P1(COLNM,ROWNM,I,TYPE,COEFF)
C
 1201 CONTINUE
C                   ....................................................
C                   .     *****     FINISHED GOODS STOCKS     *******  .
C                   ....................................................
      NAME=CCODE(15)
      STYPE=JOBOD
      DO 1401 K=1,NPROD
      DO 1401 I=1,NM
      CALL COLUM(COLNM,I,NAME,K,STYPE)
C                   ....................................................
C                   .   FINISHED GOODS STOCKS                          .
C                   ....................................................
      ROWNM=RCODE(5)
      TYPE=K
      COEFF=1
      CALL P1(COLNM,ROWNM,I,TYPE,COEFF)
      IF(I.EQ.NM) GO TO 1402
      II=I+1
      COEFF=-1
      CALL P1(COLNM,ROWNM,II,TYPE,COEFF)
 1402 CONTINUE
C                   ....................................................
C                   .   STORAGE CAPACITY                               .
C                   ....................................................
      ROWNM=RCODE(3)
      TYPE=1
      COEFF=SPACE(K,1)
      CALL P1(COLNM,ROWNM,I,TYPE,COEFF)
C
 1401 CONTINUE
C                   ....................................................
C                   .     *****     LABOUR OVERTIME           *******  .
C                   ....................................................
      DO1701 I=1,NM
      DO1701 L=1,NLF
      DO1701 K=1,NWC
      IF(WCLF(L,K).EQ.0.0) GO TO1750
      NAME=CCODE(13)
      STYPE=NUMBR(K)
      CALL COLUM(COLNM,I,NAME,L,STYPE)
C                   ....................................................
C                   .   WORK CENTRE CAPACITY                           .
C                   ....................................................
      ROWNM=RCODE(1)
      COEFF=-1/WCLF(L,K)
      TYPE=K
      CALL P1(COLNM,ROWNM,I,TYPE,COEFF)
C                   ....................................................
C                   .   LABOUR CAPACITY                                .
C                   ....................................................
      ROWNM=RCODE(2)
      TYPE=L
      COEFF=-1
      CALL P1(COLNM,ROWNM,I,TYPE,COEFF)
C                   ....................................................
C                   .   CASH CONTINUITY                                .
C                   ....................................................
      ROWNM=RCODE(8)
      COEFF=-OWAGE(L)
      TYPE=1
      CALL P1(COLNM,ROWNM,I,TYPE,COEFF)
C                   ....................................................
C                   .   LABOUR OVERTIME CAPACITY                       .
C                   ....................................................
      ROWNM=RCODE(10)
```

```
      COEFF=1
      TYPE=1
      CALL P1(COLNM,ROWNM,1,TYPE,COEFF)
C
1750  CONTINUE
1701  CONTINUE
C
C              ..........................................................
C              .      *****      CASH ON HAND              *******      .
C              ..........................................................
      TYPE=1
      STYPE=SUBOD
      NAME=CCODE(6)
      DO 601 I=1,NM
      CALL COLUM(COLNM,I,NAME,TYPE,STYPE)
C              ..........................................................
C              .  CASH CONTINUITY                                       .
C              ..........................................................
      COEFF=-1
       ROWNM=RCODE(6)
      CALL P1(COLNM,ROWNM,1,TYPE,COEFF)
      IF(I-NM)602,603,603
602   II=I+1
      COEFF=1
      CALL P1(COLNM,ROWNM,II,TYPE,COEFF)
603   CONTINUE
C              ..........................................................
C              .  EARNINGS OBJECTIVE                                    .
C              ..........................................................
      IF(I.LT.NM) GO TO 601
      COEFF=1
       ROWNM=RCODE(12)
      CALL P1(COLNM,ROWNM,1,TYPE,COEFF)
601   CONTINUE
C
      IF(IFBL.EQ.0) GO TO 999
C
C              ..........................................................
C              .      *****      BANK LOANS                *******      .
C              ..........................................................
      NAME=CCODE(7)
      STYPE=SUBOD
      TYPE=1
      DO 701 I=1,NM
      CALL COLUM(COLNM,I,NAME,TYPE,STYPE)
C              ..........................................................
C              .  CREDIT                                                .
C              ..........................................................
      TYPE=1
      ROWNM=RCODE(7)
      COEFF=1
      LTIME=I
      CALL P2(COLNM,ROWNM,LTIME,NM,TYPE,COEFF)
C              ..........................................................
C              .  CASH CONTINUITY                                       .
C              ..........................................................
      TYPE=1
       ROWNM=RCODE(8)
      COEFF=1
      CALL P1(COLNM,ROWNM,I,TYPE,COEFF)
C              ..........................................................
C              .  LOAN CHARGES                                          .
C              ..........................................................
       ROWNM=RCODE(9)
      TYPE=1
      COEFF=-ALPHAC
      LTIME=1
      CALL P2(COLNM,ROWNM,LTIME,NM,TYPE,COEFF)
C
701   CONTINUE
C              ..........................................................
C              .      *****      BANK REPAYMENTS           *******      .
C              ..........................................................
      NAME=CCODE(8)
      STYPE=SUBOD
      TYPE=1
      DO 801 I=1,NM
      CALL COLUM(COLNM,I,NAME,TYPE,STYPE)
C              ..........................................................
C              .  CREDIT                                                .
C              ..........................................................
      TYPE=1
       ROWNM=RCODE(7)
      COEFF=-1
      LTIME=I
      CALL P2(COLNM,ROWNM,LTIME,NM,TYPE,COEFF)
```

```
C
C               .............................................................
C               .  CASH CONTINUITY                                          .
C               .............................................................
      TYPE=1
      COEFF=-1
      ROWNM=RCODE(8)
      CALL P1(COLNM,ROWNM,I,TYPE,COEFF)
C               .............................................................
C               .  LOAN CHARGES                                             .
C               .............................................................
      TYPE=1
      ROWNM=RCODE(9)
      COEFF=ALPHAC
      LTIME=I
      CALL F2(COLNM,ROWNM,LTIME,NM,TYPE,COEFF)
C
801   CONTINUE
C               .............................................................
C               .       *****      BANK CHARGES                 *******     .
C               .............................................................
      NAME=CCODE(9)
      TYPE=1
      STYPE=SUBDD
      DO 901 I=1,NM
      CALL COLUM(COLNM,I,NAME,TYPE,STYPE)
C               .............................................................
C               .  CREDIT                                                   .
C               .............................................................
      IF(ALFLG)905,905,902
902   HTIME=I+ALFLG-1
      IF(HTIME-NM)904,904,903
903   HTIME=NM
904   ROWNM=RCODE(7)
      TYPE=1
      COEFF=1
      LTIME=I
      CALL F2(COLNM,ROWNM,LTIME,HTIME,TYPE,COEFF)
905   CONTINUE
C               .............................................................
C               .  CASH CONTINUITY                                          .
C               .............................................................
      II=I+ALFLG
      IF(II-NM)906,906,907
906   ROWNM=RCODE(8)
      TYPE=1
      COEFF=-1
      CALL P1(COLNM,ROWNM,II,TYPE,COEFF)
907   CONTINUE
C               .............................................................
C               .  LOAN CHARGES                                             .
C               .............................................................
      TYPE=1
      COEFF=1
      ROWNM=RCODE(9)
      CALL P1(COLNM,ROWNM,I,TYPE,COEFF)
C
901   CONTINUE
C
999   CONTINUE
C
C               .............................................................
C               .       *****      CREDIT                       *******     .
C               .............................................................
      TYPE=1
      STYPE=SUBDD
      NAME=CCODE(10)
      DO 1001 I=1,NM
      CALL COLUM(COLNM,I,NAME,TYPE,STYPE)
C               .............................................................
C               .  CREDIT                                                   .
C               .............................................................
      ROWNM=RCODE(7)
      COEFF=-1
      CALL P1(COLNM,ROWNM,I      ,TYPE,COEFF)
C               .............................................................
C               .  EARNINGS OBJECTIVE                                       .
C               .............................................................
      IF(I.LT.NM) GO TO 1001
      ROWNM=RCODE(12)
      CALL P1(COLNM,ROWNM,I,TYPE,COEFF)
C
1001  CONTINUE
C               .............................................................
C               .       *****      DEBT                         *******     .
C               .............................................................
```

```
      TYPE=1
      STYPE=SUBDO
      NAME=CCODE(11)
      DO 1101 I=1,NM
      CALL COLUM(COLNM,1,NAME,TYPE,STYPE)
C               ..........................................................
C               .  DEBT                                                  .
C               ..........................................................
      ROWNM=RCODE(6)
      COEFF=-1
      CALL F1(COLNM,ROWNM,1,TYPE,COEFF)
C               ..........................................................
C               .  EARNINGS OBJECTIVE                                    .
C               ..........................................................
      IF(I.LT.NM) GO TO 1101
      COEFF=1
       ROWNM=RCODE(12)
      CALL F1(COLNM,ROWNM,1,TYPE,COEFF)
C
 1101 CONTINUE
C               ..........................................................
C               .       *****       THE RHS VECTOR           *******     .
C               ..........................................................
      WRITE(6,5240)
C               ..........................................................
C               .  WORK CENTRE CAPACITIES                                .
C               ..........................................................
      DO 3011 L=1,NWC
 3011 READ(2,4130) (CPCTY(I,L),I=1,NM)
      WRITE(3,5255)ISCALE
      WRITE(3,5250)
      WRITE(3,5257) WCLEFT
      WRITE(3,5260)
      WRITE(3,5270)MONTH
      WRITE(3,5260)
      WRITE(3,5280)
C               ..........................................................
C               .  FACTOR FOR EXTRA-MODEL PRODUCTION AND BREAKDOWN       .
C               ..........................................................
      DO 3006 I=1,NM
      DO 3006 L=1,NWC
 3006 CPCTY(I,L)=((100.0-WCLEFT)*CPCTY(I,L))/(100.*ISCALE)
      DO 3001 L=1,NWC
 3001 WRITE(3,5290)WCLF(NLF+1,L), (CPCTY(I,L),I=1,NM)
      WRITE(3,5260)
      WRITE(3,5300)
C               ..........................................................
C               .   -  ADJUST FOR POST HORIZON PRODUCTION REQUIREMENTS .
C               ..........................................................
      DO 3002 J=1,NWC
      DO 3003 I=1,NM
 3003 DUMMY(I)=0
      DO 3004 K=1,NPROD
      KP=KDIV(K,1)
      DO 3004 KP1=1,KP
      II=SPRED(K,KP1)-1
      IF(II.EQ.0) GO TO 3004
      DO 3008 I=1,II
      HTIME=SPRED(K,KP1)-I
      DO 3008 LL=1,HTIME
      TIME=NM+I+LL-SPRED(K,KP1)
      DUMMY(TIME)=DUMMY(TIME)+FCAST(I,K)*WCREQ(LL,J,K,KP1)
 3008 CONTINUE
 3004 CONTINUE
 3007 DO 3005 I=1,NM
      CPCTY(I,J)=ISCALE*CPCTY(I,J)-DUMMY(I)
 3005 DUMMY(I)=DUMMY(I)/ISCALE
      WRITE(3,5290)WCLF(NLF+1,J), (DUMMY(I),I=1,NM)
      DO 3009 I=1,NM
      II=NM+1-I
      IF(CPCTY(II,J).GE.0.0) GO TO 3009
      I1=II-1
      IF(I1.LE.0) GO TO 2
      CPCTY(I1,J)=CPCTY(II,J)+CPCTY(I1+1,J)
      CPCTY(II+1,J)=0.0
 3009 CONTINUE
      DO 3010 I=1,NM
 3010 CAPWC(I,J)=CPCTY(I,J)
 3002 CONTINUE
      WRITE(3,5310)
      LTIME=1
      ROWNM=RCODE(1)
      CALL F4(ROWNM,LTIME,NM,NWC,CPCTY)
C               ..........................................................
C               .  LABOUR FORCE CAPACITIES                               .
```

```
C                 ...................................................
      DO 3025 L=1,NLF
3025  READ(2,4130)  (CPCTY(I,L),I=1,NM)
      WRITE(3,5320)
C                 ...................................................
C                 .  FACTOR FOR EXTRA-MODEL PRODUCTION AND BREAKDOWN  .
C                 ...................................................
      DO 3026 I=1,NM
      DO 3026 L=1,NLF
3026  CPCTY(I,L)=((100.0-NOLEFI)*CPCTY(I,L))/100.
      KORNM=KCODE(2)
      CALL P4(KORNM,LTIME,NM,NLF,CPCTY)
      DO 3027 I=1,NM
      DO 3027 L=1,NLF
3027  CPCTY(I,L)=CPCTY(I,L)/ISCALE
      DO 3021 L=1,NLF
3021  WRITE(3,5291)L,(CPCTY(I,L),I=1,NM)
      WRITE(3,5260)
C                 ...................................................
C                 .  LABOUR OVERTIME CAPACITIES                       .
C                 ...................................................
      DO 3035 M=1,NLF
3035  READ(2,4130)  (CPCTY(I,M),I=1,NM)
      WRITE(3,5340)
       KORNM=KCODE(10)
      CALL P4(KORNM,LTIME,NM,NLF,CPCTY)
      DO 3032 I=1,NM
      DO 3032 M=1,NLF
3032  CPCTY(I,M)=CPCTY(I,M)/ISCALE
      DO 3031 M=1,NLF
3031  WRITE(3,5290) M,(CPCTY(I,M),I=1,NM)
      WRITE(3,5260)
C                 ...................................................
C                 .  STORAGE CAPACITIES                               .
C                 ...................................................
      READ(2,4130)(CPCTY(I,1),I=1,NM)
      KORNM=KCODE(3)
      CALL P4(KORNM,LTIME,NM,LTIME,CPCTY)
       DO 3041 I=1,NM
3041  CPCTY(I,1)=CPCTY(I,1)/ISCALE
      WRITE(3,5350)(CPCTY(I,1),I=1,NM)
      WRITE(3,5310)
C                 ...................................................
C                 .  INITIAL RAW MATERIALS STOCKS                     .
C                 ...................................................
      READ(2,4130)(CPCTY(1,M),M=1,NRM)
      WRITE(3,5370)
      WRITE(3,5360)
      DO 3052 M=1,NRM
3052  WRITE(3,5390)M,CPCTY(1,M)
      WRITE(3,5400)
      WRITE(3,5410)
      WRITE(3,5255)ISCALE
      WRITE(3,5260)
      WRITE(3,5270) MONTH
      WRITE(3,5260)
C                 ...................................................
C                 .    -  ADJUST FOR POST HORIZON PRODUCTION REQUIREMENTS .
C                 ...................................................
      DO 3056 J=1,NRM
      DUMMY(1)=0.0
      DO 3053 I=2,NM
      CPCTY(I,J)=0.0
3053  DUMMY(I)=0.0
      DO 3059 K=1,NPROD
      KP=KDIV(K,1)
      DO 3054 KP1=1,KP
      II=SPRED(K,KP1)-1
      IF(II.EQ.0) GO TO 3054
      DO 3058 I=1,II
      HTIME=SPRED(K,KP1)-I
      DO 3058 LL=1,HTIME
      TIME=NM+1+LL-SPRED(K,KP1)
      DUMMY(TIME)=DUMMY(TIME)+FCAST(I,K)*PMREQ(LL,K,KP1,J)
3058  CONTINUE
3054  CONTINUE
3059  CONTINUE
3057  DO 3055 I=1,NM
      DO 30571 II=1,NM
30571 CPCTY(II,J)=CPCTY(II,J)-DUMMY(I)
      SUREX(I,J,1,1)=DUMMY(I)
3055  DUMMY(I)=DUMMY(I)/ISCALE
      WRITE(3,5291)J,(DUMMY(I),I=1,NM)
3056  CONTINUE
      KORNM=KCODE(4)
```

```
      CALL P4 (KOWNM,LTIME,NM,NM1,CPCTY)
      WRITE(3,5510)
C                 ..............................................................
C                 .  SKIP WIP FIGURE                                           .
C                 ..............................................................
      READ(2,4100) XSKP
C                 ..............................................................
C                 .  INITIAL FINISHED GOODS STOCKS                             .
C                 ..............................................................
      READ(2,4130)(CPCTY(1,K),K=1,NPROD)
      DO 3061  K=1,NPROD
3061  STOCKI(K)=CPCTY(1,K)
      WRITE(3,5420)
      WRITE(3,5380)
      DO 3062K=1,NPROD
3062  WRITE(3,5390)K,CPCTY(1,K)
      WRITE(3,5400)
      KOWNM=KCODE(5)
      CALL P4(KOWNM,LTIME,LTIME,NPROD,CPCTY)
C                 ..............................................................
C                 .  EXTRA-MODEL DEBT ACCOUNT                                  .
C                 .       -  ADJUST CASHFLOW FOR INITIAL DEBT POSITION         .
C                 ..............................................................
      READ(2,4130) DUMMY(3),DUMMY(4)
      WRITE(3,5530) DUMMY(3),DUMMY(4)
      READ(2,4130)(CPCTY(I,1),I=1,NM)
      WRITE(3,5540)
      WRITE(3,5255)ISCALE
      WRITE(3,5470)
      WRITE(3,5460)MONTH
      WRITE(3,5470)
      DO 3112 I=1,NM
3112  CPCTY(I,1)=CPCTY(I,1)/ISCALE
      WRITE(3,5490)(CPCTY(I,1),I=1,NM)
      DO 3111 I=1,NM
3111  CPCTY(I,1)=CPCTY(I,1)*(-ISCALE)
      KOWNM=KCODE(6)
      CALL P4(KOWNM,LTIME,NM,LTIME,CPCTY)
      WRITE(3,5500)
C                 ..............................................................
C                 .  EXTRA-MODEL CREDIT ACCOUNT                                .
C                 .       -  ADJUST CASHFLOW FOR INITIAL CREDIT POSITION       .
C                 ..............................................................
      READ(2,4130) DUMMY(5),DUMMY(6)
      WRITE(3,5550) DUMMY(5),DUMMY(6)
      READ(2,4130)(CPCTY(I,1),I=1,NM)
      WRITE(3,5560)
      WRITE(3,5255)ISCALE
      WRITE(3,5470)
      WRITE(3,5480)MONTH
      WRITE(3,5470)
      DO 3122 I=1,NM
3122  CPCTY(I,1)=CPCTY(I,1)/ISCALE
      WRITE(3,5490)(CPCTY(I,1),I=1,NM)
      WRITE(3,5500)
      DO 3121 I=1,NM
3121  CPCTY(I,1)=CPCTY(I,1)*(-ISCALE)
      KOWNM=KCODE(7)
      CALL P4(KOWNM,LTIME,NM,LTIME,CPCTY)
C                 ..............................................................
C                 .  EXTRA-MODEL CASH-FLOWS                                    .
C                 .       -  INITIAL CASH HOLDING                              .
C                 ..............................................................
      READ(2,4130) DUMMY(1)
      WRITE(3,5450) DUMMY(1)
C                 ..............................................................
C                 .       -  DIFFERENTIATE BETWEEN INFLOWS AND OUTFLOWS        .
C                 ..............................................................
      READ(2,4180) KEXC,(CPCTY(I,2),CPCTY(I,3),I=1,KEXC)
      KEXC=KEXC+3
      DO 3082 J=4,KEXC
      READ(2,4130) (CPCTY(I,J),I=1,NM)
3082  CONTINUE
      KEXC=KEXC-3
      WRITE(3,5460)
      WRITE(3,5255)ISCALE
      WRITE(3,5470)
      WRITE(3,5480)MONTH
      WRITE(3,5470)
      DO 3084 I=1,NM
      CPCTY(I,1)=0.0
      DO 3083 J=1,KEXC
3083  CPCTY(I,1)=CPCTY(I,1)+(CPCTY(J,3)*CPCTY(I,J+3))
3084  CONTINUE
      CPCTY(1,1)=CPCTY(1,1)-DUMMY(1)
```

```
      JJ=DUMMY(4)
      DO 3007 J=1,JJ
3007  CPCTY(J,1)=CPCTY(J,1)-DUMMY(3)/JJ
      JJ=DUMMY(6)
      DO 3009 J=1,JJ
3009  CPCTY(J,1)=CPCTY(J,1)+DUMMY(5)/JJ
       KOWNM=KODE(8)
      LTIME=1
      CALL P4(KOWNM,LTIME,NM,LTIME,CPCTY)
      KEXC=KEXC+3
      DO 3081 I=1,NM
      CPCTY(I,1)=CPCTY(I,1)/ISCALE
      DO 3081 J=4,KEXC
      CPCTY(I,J)=CPCTY(I,J)/ISCALE
3081  CONTINUE
      KEXC=KEXC-3
      DO 3086 J=1,KEXC
      IF(CPCTY(J,3).GT.0.0) GO TO 3086
      WRITE(3,5495) CPCTY(J,2),(CPCTY(I,J+3),I=1,NM)
      L=J
3086  CONTINUE
      IF(J.EQ.KEXC) L=KEXC+1
      JJ=DUMMY(4)
      DO 3088 I=1,NM
      CPCTY(I,L+3)=0.0
      IF(I.LE.JJ) CPCTY(I,L+3)=DUMMY(3)/(JJ*ISCALE)
3088  CONTINUE
      WRITE(3,5496) (CPCTY(I,L+3),I=1,NM)
      WRITE(3,5498)
      DO 3085 J=1,KEXC
      IF(CPCTY(J,3).LT.0.0) GO TO 3085
      WRITE(3,5495) CPCTY(J,2),(CPCTY(I,J+3),I=1,NM)
      L=J
3085  CONTINUE
      JJ=DUMMY(6)
      DO 3090 I=1,NM
      CPCTY(I,L+3)=0.0
      IF(I.LE.JJ) CPCTY(I,L+3)=DUMMY(5)/(JJ*ISCALE)
3090  CONTINUE
      WRITE(3,5497) (CPCTY(I,L+3),I=1,NM)
      WRITE(3,5498)
      WRITE(3,5499) (CPCTY(I,1),I=1,NM)
      WRITE(3,5500)
C
      IF(IFBL.EQ.0) GO TO 3099
C               ..............................................................
C               .  LOAN CHARGES                                              .
C               ..............................................................
      READ(2,4100)CPCTY(1,1)
      WRITE(3,5510)CPCTY(1,1)
      WRITE(3,5520) ALPHA
      IF(CPCTY(1,1).EQ.0) GO TO 3092
      DO 3091 I=2,NM
3091  CPCTY(I,1)=CPCTY(1,1)
       KOWNM=PCODE(9)
      LTIME=1
      CALL P4(KOWNM,LTIME, NM  ,LTIME,CPCTY)
3092  CONTIN UE
C
3099  CONTINUE
C               ..............................................................
C               .       *****      THE RANGES SECTION              *******   .
C               ..............................................................
      WRITE(8,5720)
C               ..............................................................
C               .  WORK CENTRE LOWER BOUNDS                                  .
C               ..............................................................
      KOWNM=KODE(1)
      DO 3901 L=1 ,NWC
      DO 3901 I=1,NM
      IF(L.GE.10) GO TO 3902
      WRITE(8,5730) MONTH(I),KOWNM,L,CAPWC(I,L)
      GO TO 3901
3902  WRITE(8,5740) MONTH(I),KOWNM,L,CAPWC(I,L)
3901  CONTINUE
C               ..............................................................
C               .       *****      THE BOUNDS SECTION              *******   .
C               .    -  THERE ARE 3 DATA TYPES                               .
C               .    -   1  SALES BOUNDS                                     .
C               .    -   2  DATA FOR ALL NM PERIODS                          .
C               .    -   3  DATA FOR A SPECIFIC PERIOD                       .
C               ..............................................................
      WRITE(8,5800)
C               ..............................................................
C               .  SALES BOUNDS                                              .
```

```
C                ..........................................................
C                .  THE PLOTTING ROUTINE                                  .
C                .         THIS PLOTS TWO GRAPHS PER PAGE OF LINED O/P    .
C                ..........................................................
      TYPE1=0
      J=0
 3801 CONTINUE
      READ(2,4140) CLNM,TYPE,STYPE
      IF(TYPE1.NE.TYPE) GO TO 3850
 3810 TYPE1=TYPE
      IF(TYPE.EQ.0) GO TO 3803
C                ..........................................................
C                .           OUTPUT THE BOUNDS                            .
C                ..........................................................
      KT=KNOUT(TYPE)
      J=J+1
      DO 3802 K=1,KT
      READ(2,4150) (SALESD(J,K,I),I=1,NM)
      STYPE=SUBCD(TYPE,K)
      DO 3802 I=1,NM
      CALL COLUM(COLNM,I,CLNM,TYPE,STYPE)
      WRITE(3,5640) STYPE,COLNM,SALESD(J,K,I)
 3802 CONTINUE
      GO TO 3801
C                ..........................................................
C                .           DETERMINE THE SCALE                          .
C                ..........................................................
 3850  IF(TYPE1.EQ.0) GO TO 3810
      ISCALE=0
      DO 3851 K=1,KT
      DO 3851 J1=1,J
      DO 3851 I=1,NM
      IF(SALESD(J1,K,I).LE.ISCALE) GO TO 3851
      ISCALE=SALESD(J1,K,I)
 3851 CONTINUE
      IF(ISCALE.LE.25)  ISCALE= 25
      IF((ISCALE.GT.25).AND.(ISCALE.LE.50))  ISCALE= 50
      IF(ISCALE.LE.50) GO TO 3860
      DO 3852 IC=100,10000,100
      IF(ISCALE.GT.IC) GO TO 3852
      ISCALE=IC
      GO TO 3860
 3852 CONTINUE
 3860 CONTINUE
C                ..........................................................
C                .           PLOT THE GRAPHS LINE BY LINE                 .
C                ..........................................................
      IKT=(KT+1)/2
      IZ=KT-2*(KT/2)
      IF(IZ.EQ.0) IZ=2
      DO 3870 IKT1=1,IKT
      IST=(2*IKT1)-1
      IFIN =(2*IKT1)-2+IZ
      WRITE(5,5710) TYPE1
      DO 3861 IC=1,26
      ISMAX=(27-IC)*ISCALE/25
      ISMIN=(26-IC)*ISCALE/25
      DO 3862 I=1,NM
      DO 3862 K=IST,IFIN
      IGRPH(I,K+1-IST)=SUBDD
      DO 3862 J1=1,J
      IF((SALESD(J1,K,I).LT.ISMAX).AND.(SALESD(J1,K,I).GE.ISMIN))
     1     IGRPH(I,K+1-IST)=SUBCD(TYPE1,K)
 3862 CONTINUE
      IF((((IC-1)    )-5.0*((IC-1)/5)).EQ.0.0) GO TO 3863
      WRITE(5,5700) (SUBDD,(IGRPH(I,K),I=1,NM),K=1,IZ)
      GO TO 3861
 3863 ISCL=(ISCALE*(26-IC))/25
      WRITE(5,5705) (SUBDD,ISCL, (IGRPH(I,K),I=1,NM),K=1,IZ)
 3861 CONTINUE
      WRITE(5,5707) (SUBDD,K=1,IZ)
      WRITE(5,5703) ((MONTH(I),I=1,NM),K=1,IZ)
 3870 CONTINUE
      J=0
      GO TO 3810
C                ..........................................................
C                .  BOUNDS FOR NM PERIODS                                 .
C                ..........................................................
 3803 STYPE=SUBDD
 3804 READ(2,4160) CLNM,STYPE,TYPE,STYPE,(DUMMY(I),I=1,NM)
      IF(TYPE.EQ.0) GO TO 3806
      DO 3805 I=1,NM
      CALL COLUM(COLNM,I,CLNM,TYPE,STYPE)
      WRITE(3,5640) STYPE,COLNM,DUMMY(I)
```

```
3805  CONTINUE
      GO TO 3804
C                 ..................................................................
C                 .  SPECIFIC BOUNDS                                               .
C                 ..................................................................
3806  READ(2,4170) CLNM,TYPE,STYPE,I,VALUE
      IF(TYPE.EQ.0) GO TO 3807
C  IF THE COL IS PO THEN BOUNDS MUST WHOLD FOR DIVISIONS AS WELL AS **
      IF(CLNM.NE.CCODE(1)) GO TO 3808
      KP=KDIV(TYPE,1)
      K=TYPE
      DO 3809 KP1=1,KP
      TYPE=100*KP1
      CALL COLUM(COLNM,I,CLNM,TYPE,STYPE)
3809  WRITE(3,5640) STYPE,COLNM,VALUE
      GO TO 3806
3806  STYPE=SCODE
      CALL COLUM(COLNM,I,CLNM,TYPE,STYPE)
      WRITE(3,5640) STYPE,COLNM,VALUE
      GO TO 3806
C                 ..................................................................
C                 .  CLOSE THE OUTPUT FILE   .                                      .
C                 ..................................................................
3807  WRITE(8,5650)
      WRITE(3,5660)
      END FILE 8
      REWIND 8
1     STOP
2     WRITE(3,5670)
      GO TO 1
C                 ..................................................................
C                 .  THE FORMAT SECTION                                            .
C                 .    -    NUMBERS 4XXX REFER TO READ STATEMENTS                  .
C                 .    -    NUMBERS 5XXX REFER TO WRITE STATEMENTS                 .
C                 ..................................................................
4001  FORMAT(20(1X,I2))
4011  FORMAT(24I2)
4020  FORMAT(2(F8.3,I4,F8.3))
4030  FORMAT(9F8.3)
4035  FORMAT(15A4)
4037  FORMAT(10F6.4)
4040  FORMAT(I3,F8.3,I5)
4050  FORMAT(3I2)
4060  FORMAT(6F10.4)
4070  FORMAT(I2,A2,2I2,F8.3)
4071  FORMAT(5(2I2,F8.3,1X))
4080  FORMAT(I2,A2,2F8.3,I4)
4090  FORMAT(12I6)
4100  FORMAT(20A2)
4110  FORMAT(I3,2A4)
4120  FORMAT(A2,1X,A2,I5,I3)
4130  FORMAT(6F10.0)
4140  FORMAT(A2,I2,A2,2A4)
4150  FORMAT(10F6.0)
4160  FORMAT(2A2,I2,A2,12F6.0)
4170  FORMAT(A2,1X,I4,1X,A2,1X,I3,1X,F10.0)
4180  FORMAT(I2,20(A4,F2.0))
4190  FORMAT(10F6.3)
C
5001  FORMAT(1H1,'NO. OF PRODUCTS ',I2,10X,'NO. OF OUTLETS ',I2//
     1' NO. OF WORK CENTRES ',I2, 5X,'NO. OF TYPES OF LABOUR ',I2//
     2' NO. OF SUBCONTRACTED ',I2, 5X,'NO. OF RAW MATERIALS ',I2//
     4' NO. OF PERIODS ',I2)
5020  FORMAT(//,' DATA ON RAW MATERIALS',///2X, ' PRICE/UNIT   LAG ON PA
     1YMENT     VOL/UNIT',//)
5030  FORMAT(4X,F8.3,6X,I6,6X,F8.3)
5040  FORMAT(/////' WORK CENTRE LABOUR FORCE REQUIREMENTS HRS/HR',//)
5050  FORMAT(2X,A4,10(4X,F8.3))
5052  FORMAT(1H0,'OVERTIME RATES'/6X,10(4X,F8.3))
5054  FORMAT(1H0,' WORK CENTRE LOADING EFFICIENCY'/1X,28('-'))
5055  FORMAT(1H0,20(A4,2X))
5056  FORMAT(1H ,20(F6.3))
5060  FORMAT(/////' SUBCONTRACTING ARRANGEMENTS'//'    WORK CENTRE
     1ICE/HR      LAG ON PAYMENT        ')
5070  FORMAT(/5X,I5,3X,F8.3,3X,I5)
5080  FORMAT(1H1,'  PRODUCT DETAILS'//'  PRODUCT',I4,'  IS MADE OF',
     1I4,'  SECTIONS'//'   THE NUMBER OF OUTLETS FOR THIS PRODUCT IS',
     2I3)
5090  FORMAT(///////////////'  DIVISION',I6,'   OF PRODUCT',I5)
5100  FORMAT(/////' MACHINE TIME REQUIREMENTS MINS/UNIT')
5120  FORMAT(/////' RAW MATERIAL REQUIREMENTS UNITS/UNIT')
5130  FORMAT(/////' SALES DATA BY MARKET OUTLET'//5X,'CODE',8X,'LIST PRI
     1CE      DISCOUNT     LAG ON PAYMENT')
5140  FORMAT(/7X,A2,6X,F12.3,7X,F8.3,8X,I5)
5150  FORMAT(///,'  FORECAST FOR POST HORIZON PRODUCTION'///,' PRODUCT',
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     1'   ARGUMT'//)
5160  FORMAT(/,I6,12I10)
5170  FORMAT ('NAME',10X,2A4)
5180  FORMAT ('ROWS')
5190  FORMAT(    A2,2X,2A2,I1,A2)
5200  FORMAT(    A2,2X,2A2,I2,A2)
5203  FORMAT(    A2,2X,2A2,I3,A2)
5204  FORMAT(    A2,2X,2A2,I4,A2)
5210  FORMAT( 'COLUMNS')
5220  FORMAT(' ROW DIMENSION TOO LARGE')
5230  FORMAT( ' THERE IS AN INCOMPATIBILITY BETWEEN THE TURNROUND
     0  REQUIRED AND THE LAGS IN PRODUCT ',I5)
5240  FORMAT( 'RHS')
5250  FORMAT(1H1,//,20X,'CAPACITY OF PRODUCTION FACILITIES'//19X,35('*')
     1//)
5255  FORMAT(1H ,10X,32('*')/11X,'*  THE UNITS ARE IN',  I9 ,' S *'/,
     111X,32('*')//)
5257  FORMAT( '  NOTE---',F6.1,'   PERCENT OF CAPACITY IS LEFT OUT OF
     1 MODEL FOR EXTRA MODEL ACTIVITIES')
5260  FORMAT(1X,113('-')/1X,'I',7X,17('I',6X))
5270  FORMAT(' I',7X,'I',16(2X,A2,2X,'I'))
5280  FORMAT(1X,'I',1X,'WORK  I',16(6X,'I')/1X,'I CENTREI',
     116(6X,'I'))
5290  FORMAT(1X,'I',2X,A4,1X,'I',16(F6.1,'I'))
5291  FORMAT(1X,'I',3X,I2,2X,'I',16(F6.1,'I'))
5300  FORMAT(' REDUCTION IN WORK CENTRE CAPACITY FOR POST HORIZON PRODUC
     1TION')
5310  FORMAT(1X,119('-'))
5320  FORMAT(1X,'I',1X,'LABOUR','I',16(6X,'I')/1X,'I',1X,'FORCE',1X,'I',
     016(6X,'I'))
5330  FORMAT(' REDUCTION IN LABOUR FORCE CAPACITY FOR POST HORIZON PRODU
     1CTION')
5340  FORMAT(1X,'I',1X,'LABOUR','I',16(6X,'I')/1X,'I',1X,'FORCE',1X,'I',
     016(6X,'I')/1X,'I',1X,'O/TIMEI', 16(6X,'I'))
5350  FORMAT(1X,'I','STORAGE','I',16(6X,'I')/1X,'I','CAPACTY','I',
     116(F6.1,'I'))
5360  FORMAT(1H0//1X,120('*')////)
5370  FORMAT(20X,'INPUT OF RAW MATERIALS'/19X,24('*')//)
5380  FORMAT(21X,20('-')/21X,'I',8X,'I',9X,'I'/21X,'I',' TYPE',3X,'I',
     1'   INPUT',2X,'I'/21X,20('-'))
5390  FORMAT(21X,'I',2X,I2,4X,'I',F9.1,'I')
5400  FORMAT(     21X,20('-'))
5410  FORMAT(' REDUCTION IN RAW MATERIAL CAPACITY FOR POST HORIZON PRODU
     1CTION'///)
5420  FORMAT(1H1,20X,'INPUT OF FINISHED PRODUCT'/19X,27('*')//)
5430  FORMAT(///20X,'INITIAL VALUE OF FINISHED GOODS = £',F11.1/19X,
     050('*'))
5440  FORMAT(//' EXTRA W.I.P. FOR POST HORIZON PRODUCTION',/1X,40('*')
     1//,1X,10F8.1)
5450  FORMAT(///,' INITIAL INPUT OF CASH',F10.1,/,1X,22('*'))
5460  FORMAT(1H0//20X,'OUTFLOWS OF CASH'/19X,18('*')//)
5470  FORMAT(8X,111('-')/8X,'I',6X,16('I',6X))
5480  FORMAT(8X,'I',16(2X,A2,2X,'I'))
5490  FORMAT( '              I',16(F6.1,'I'))
5495  FORMAT( ' I',A4,'   I',16(F6.1,'I'))
5496  FORMAT( ' IIN.DBTI',16(F6.1,'I'))
5497  FORMAT( ' IIN.CRTI',16(F6.1,'I'))
5498  FORMAT( ' I       I',16(6X,'I'))
5499  FORMAT( ' I TOTALI',16(F6.1,'I'))
5500  FORMAT(12X,112('-'))
5510  FORMAT(///20X,'INTEREST ON INITIAL BANK LOAN= £',F8.1/19X,30('*'))
5520  FORMAT(///,20X,'RATE OF INTEREST',F10.3,/,19X,16('*'))
5530  FORMAT(//' INITIAL DEBT OWED TO COMPANY  ',F10.1,'   PAYED OVER',
     1F3.0,'  PERIODS'//)
5540  FORMAT(1H0//20X,'   EXTRA MODEL DEBT    '/19X,23('*')//)
5550  FORMAT(//' INITIAL CREDIT OWED BY COMPANY  ',F10.1,'  PAYED OVER',
     1F3.0,'  PERIOUS'//)
5560  FORMAT(1H0//20X,'   EXTRA MODEL CREDIT   '/19X,23('*')//)
5570  FORMAT(//20X,'GROWTH IN STOCK OF FINISHED PRODUCTS '/19X,40('*')/)
5571  FORMAT(//20X,'GROWTH IN STOCK OF RAW MATERIALS      '/19X,40('*')/)
5575  FORMAT(20X,' THE GROWTH IS',F8.1,'   PERCENT',20('-'),'WHICH IMPLI
     1ES -'//)
5580  FORMAT(21X,20('-')/21X,'I',8X,'I',1X,'STOCK',3X,'I'/21X,'I',
     1'  TYPE',2X,'I',1X,'GROWTH',2X,'I'/21X,20('-'))
5590  FORMAT(4X,'WOC',7X,2A2,I1,5X,F12.4)
5591  FORMAT(4X,'WOC',7X,2A2,I2,4X,F12.4)
5600  FORMAT(4X,'LFC',7X,2A2,I1,5X,F12.4)
5610  FORMAT(4X,'IRM',7X,2A2,I1,5X,F12.4)
5630  FORMAT( 'BOUNDS')
5640  FORMAT(1X,A2,1X,'POLICY',4X,2A2,2I1,A2,2X,F12.4)
5650  FORMAT( 'ENDATA')
5660  FORMAT(///'  BOUNDS DATA HAS BEEN READ IN')
5670  FORMAT (' THERE IS TOO MUCH POST HORIZON FCAST')
5700  FORMAT(1H ,2(A2,6X,'I',16(A2,1X), 8X))
5705  FORMAT(1H ,2(A2,I5,1X,'I',16(A2,1X), 8X))
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5707  FORMAT(1H ,2(A2,7X,40('-'),11X))
5708  FORMAT(1H ,2(10X,10(A2,1X),6X))
5710  FORMAT(1H1,///////,25X, ' UPPER AND LOWER SALES BOUNDS FOR PRODUCT
     1 ',I6,//////////)
5720  FORMAT( 'RANGES')
5730  FORMAT(4X,'RANGES',4X,2A2,I1,5X,F12.4)
5740  FORMAT(4X,'RANGES',4X,2A2,I2,4X,F12.4)
55100 FORMAT(1H ,10A2)
C
C
      END
C                .................................................
C                . ALL VARIABLE NAMES ARE MADE UP AS FOLLOWS -    .
C                . 'PERIOD,GENERIC NAME, TYPE, (EXTRA)IDENTIFIER' .
C                .................................................
      SUBROUTINE COLUM(COLNM,I,NAME,TYPE,STYPE)
C                .................................................
C                . THIS ROUTINE CONSTRUCTS THE COLUMN VARIABLE NAME .
C                .................................................
      COMMON MONTH(16),SPRED(20,4),NM,K,KDIV(20,5),CPCTY(16,20)
      INTEGER COLNM(5),TYPE,STYPE,SPRED
      COLNM(1)=MONTH(I)
      COLNM(2)=NAME
      IF(TYPE.LT.100) GO TO 10
      TYPE=TYPE-100
      STYPE=KDIV(K,TYPE+1)
      TYPE=K
10    IF(TYPE-10)1,2,2
    1 COLNM(4)=TYPE
      COLNM(3)=0
      GO TO 3
    2 COLNM(3)=TYPE/10
      COLNM(4)=TYPE-10*COLNM(3)
    3 COLNM(5)=STYPE
      RETURN
      END
      SUBROUTINE P1(COLNM,ROWNM,TIME,TYPE,COEFF)
C                .................................................
C                . THIS ROUTINE WRITES ONE MATRIX ELEMENT          .
C                .................................................
      COMMON MONTH(16),SPRED(20,4),NM,K,KDIV(20,5),CPCTY(16,20)
      INTEGER TYPE,TIME,COLNM(5),SPRED,ROWNM
      IF(TYPE.GE.1000) GO TO 5
      IF (TYPE.GT.100) GO TO 10
      IF(TYPE-10)1,2,2
    1 WRITE(8,100)COLNM,MONTH(TIME),ROWNM,TYPE,COEFF
      RETURN
    2 WRITE(8,200)COLNM,MONTH(TIME),ROWNM,TYPE,COEFF
      RETURN
5     TYPE=TYPE/10+K
      IF(TYPE.GT.1000) GO TO 6
      WRITE(8,310)COLNM,MONTH(TIME),ROWNM,TYPE,COEFF
      RETURN
6     WRITE(8,400)COLNM,MONTH(TIME),ROWNM,TYPE,COEFF
      RETURN
C                .................................................
C                .     -  THIS SECTION FOR THE PRODUCT DIVISIONS  .
C                .................................................
10    TYPE=TYPE-100
      IF( TYPE.GT.1) GO TO 20
      KP=KDIV(K,1)
      DO 30 KP1=2,KP
30    WRITE(8,300)COLNM,MONTH(TIME),ROWNM,K,KDIV(K,KP1 +1),COEFF
      RETURN
20    WRITE(8,300)COLNM,MONTH(TIME),ROWNM,K,KDIV(K,TYPE+1),COEFF
      RETURN
C                .................................................
C                . THE FORMAT SECTION                              .
C                .................................................
  100 FORMAT(4X,2A2,2I1,A2,2X,2A2,I1,5X,F12.4)
  200 FORMAT(4X,2A2,2I1,A2,2X,2A2,I2,4X,F12.4)
300   FORMAT(4X,2A2,2I1,A2,2X,2A2,I1,A2,4X,F12.4)
C  NOTE--LESS THAN 10 PRODUCTS ONLY CAN BE MADE OF SECTIONS
310   FORMAT(4X,2A2,2I1,A2,2X,2A2,I3,3X,F12.4)
400   FORMAT(4X,2A2,2I1,A2,2X,2A2,I4,2X,F12.4)
      END
      SUBROUTINE P2(COLNM,ROWNM,LTIME,HTIME,TYPE,COEFF)
C                .................................................
C                . THIS ROUTINE IS USED FOR TRIANGULAR MATRIX ENTRIES .
C                .     -   WRITES IN ONE COLUMN THE SAME ENTRY BETWEEN .
C                .             TWO ROW IDENTIFIERS                 .
C                .................................................
      COMMON MONTH(16),SPRED(20,4),NM,K,KDIV(20,5),CPCTY(16,20)
      INTEGER COLNM(5),TYPE,HTIME,SPRED,ROWNM
      IF(TYPE-10)1,2,2
```

```
    1 DO 3 I=LTIME,HTIME
    3 WRITE (8,100) COLNM,MONTH(I),ROWNM,TYPE,COEFF
      RETURN
    2 DO 4 I=LTIME,HTIME
    4 WRITE(8,200)COLNM,MONTH(I),ROWNM,TYPE,COEFF
      RETURN
C                .....................................................
C                . THE FORMAT SECTION                                .
C                .....................................................
  100 FORMAT(4X,2A2,2I1,A2,2X,2A2,I1,5X,F12.4)
  200 FORMAT(4X,2A2,2I1,A2,2X,2A2,I2,4X,F12.4)
      END
      SUBROUTINE P3(COLNM,ROWNM,TIME,NTYPE,ARRAY,KP1)
C                .....................................................
C                . THIS ROUTINE WRITES MATRIX ELEMENTS FROM AN ARRAY .
C                .....................................................
      COMMON MONTH(16),SPRED(20,4),NM,K,KDIV(20,5),CPCTY(16,20)
      DIMENSION ARRAY(4,9,1,20)
      REAL DUMMY(16)
      INTEGER COLNM(5),TIME,SPRED,ROWNM
      N=0
      IF(KP1.GT.1000) GO TO 12
      IF(KP1.GT.100) GO TO 10
      GO TO 9
12    N=KP1/1000
      KP1=KP1-1000*N
9     NDIM= SPRED(K,KP1)
      DO 201 L=1,NTYPE
      DO 1 JJ=1,NDIM
      IKK=TIME-NDIM+JJ
      IF(IKK)1,1,4
4     IF(IKK.GT.NM) GO TO 201
      IF(N.GT.0) GO TO 6
    5 IF(L-10)2,3,3
    2 WRITE(8,200)COLNM,MONTH(IKK),ROWNM,L,ARRAY(JJ,K,KP1,L)
      GO TO 1
    3   WRITE(8,300)COLNM,MONTH(IKK),ROWNM,L,ARRAY(JJ,K,KP1,L)
    1 CONTINUE
201   CONTINUE
3     RETURN
6     LL=L+N*100
      IF(LL.GT.1000) GO TO 7
        WRITE(8,400)COLNM,MONTH(IKK),ROWNM,LL,ARRAY(JJ,K,KP1,L)
      GO TO 1
7       WRITE(8,500)COLNM,MONTH(IKK),ROWNM,LL,ARRAY(JJ,K,KP1,L)
      GO TO 1
C                .....................................................
C                .     - FOR RAW MATERIALS SUMS OVER THE PRODUCTION CYCLE.
C                .....................................................
10    KP1=KP1-100
      NDIM=SPRED(K,KP1)
      DO 20 L=1,NTYPE
      DO 11 I=1,NM
11    DUMMY(I)=0.0
      DO 21 JJ=1,NDIM
      IKK=TIME-NDIM+JJ
      IF(IKK) 21,21,22
22    IF(IKK.GT.NM) GO TO 21
      DO 23 IKK1=IKK,NM
23    DUMMY(IKK1)=DUMMY(IKK1)+ARRAY(JJ,K,KP1,L)
21    CONTINUE
      DO 25 I=1,NM
      IF(DUMMY(I).EQ.0.0) GO TO 25
      IF(L.GE.10) GO TO 26
      WRITE(8,200) COLNM,MONTH(I),ROWNM,L,DUMMY(I)
      GO TO 25
26    WRITE(8,300) COLNM,MONTH(I),ROWNM,L,DUMMY(I)
25    CONTINUE
      CPCTY(1,L)=DUMMY(NM)
20    CONTINUE
      RETURN
C                .....................................................
C                . THE FORMAT SECTION                                .
C                .....................................................
  200 FORMAT(4X,2A2,2I1,A2,2X,2A2,I1,5X,F12.4)
  300 FORMAT(4X,2A2,2I1,A2,2X,2A2,I2,4X,F12.4)
400   FORMAT(4X,2A2,2I1,A2,2X,2A2,I3,3X,F12.4)
500   FORMAT(4X,2A2,2I1,A2,2X,2A2,I4,2X,F12.4)
      END
      SUBROUTINE P4(ROWNM,LTIME,KTIME,KTYPE,ARRAY)
C                .....................................................
C                . THIS ROUTINE WRITES THE RHS VECTOR ELEMENTS FROM ARRAY.
C                .....................................................
      COMMON MONTH(16),SPRED(20,4),NM,K,KDIV(20,5),CPCTY(16,20)
      DIMENSION ARRAY(16,20)
```

```
      INTEGER ROWNM,SPRED                                             MGP1544
      DO 3 L=1,NTYPE                                                  MGP1545
      DO 3 I=ITIME,KTIME                                              MGP1546
      IF(ARRAY(I,L).EQ.0.0) GO TO 3                                   MGP1547
      IF(L     -10)1,2,2                                              MGP1548
    1 WRITE(3,100)MONTH(I),ROWNM,L,ARRAY(I,L)                         MGP1549
      GO TO 3                                                         MGP1550
    2 WRITE(3,200)MONTH(I),ROWNM,L,ARRAY(I,L)                         MGP1551
    3 CONTINUE                                                        MGP1552
    5 RETURN                                                          MGP1553
C                 ...................................................  MGP1554
C                 . THE FORMAT SECTION                               .  MGP1555
C                 ...................................................  MGP1556
  100 FORMAT(4X,'RMS',7X,2A2,I1,5X,F12.4)                             MGP1557
  200 FORMAT(4X,'RMS',7X,2A2,I2,4X,F12.4)                             MGP1558
      END                                                             MGP1559
```

4.2 Input Data.

i. Base data - line MGP 140

```
NO. OF PRODUCTS  2            NO. OF OUTLETS  1
NO. OF WORK CENTRES  3        NO. OF TYPES OF LABOUR  2
NO. OF SUBCONTRACTED  0       NO. OF RAW MATERIALS  2
NO. OF PERIODS  6
```

ii. Raw material data - line MGP 142

DATA ON RAW MATERIALS

PRICE/UNIT	LAG ON PAYMENT	VOL/UNIT
1.000	1	0.0
3.000	1	0.0

iii. Labour force data - line MGP 145

WORK CENTRE LABOUR FORCE REQUIPEMENTS HRS/HR

M/C1	1.000	0.0
M/C2	1.000	2.00C
M/C3	1.000	1.500
OVERTIME RATES		
	3.000	1.000

iv. product data - line MGP 161

PROOUCT 2 IS MAOE OF 1 SECTICNS
THE NUMBER OF OUTLETS FOR THIS PROCUCT IS 1

SALES OATA BY MARKET OUTLET

COOE	LIST PRICE	CISCCUNT	LAG ON PAYMENT
MK	50.000	0.C	2

OIVISION 1 CF PRODUCT 2

MACHINE TIME REQUIREMENTS MINS/UNIT

M/C1	6.000
M/C2	10.000
M/C3	0.0

RAW MATERIAL REQUIREMENTS UNITS/UNIT

A	20.000
B	2.000

v. Production capacity data - line MGP 802

```
* THE UNITS ARE IN 100 S *
```

CAPACITY OF PRODUCTION FACILITIES

NOTE--- 0.0 PERCENT OF CAPACITY IS LEFT OUT OF MODEL FOR EXTRA MODEL ACTIVITIE

	P1	P2	P3	P4	P5	P6
WORK CENTRE						
M/C1	0.8	0.8	0.8	0.7	0.7	0.7
M/C2	0.8	0.8	0.7	0.7	0.7	0.6
M/C3	0.4	0.4	0.4	0.3	0.3	0.3
REDUCTION IN WORK CENTRE CAPACITY FOR POST HORIZON PRODUCTION						
M/C1	0.0	0.0	0.0	0.0	0.0	0.0
M/C2	0.0	0.0	0.0	0.0	0.0	0.0
M/C3	0.0	0.0	0.0	0.0	0.0	0.0
LABOUR FORCE						
1	1.5	1.5	1.5	1.5	1.5	1.5
2	1.5	1.5	1.5	1.5	1.5	1.5
LABOUR FORCE O/TIME						
A	0.1	0.1	0.1	0.1	0.1	0.1
B	0.1	0.1	0.1	0.1	0.1	0.1
STORAGE CAPACTY	0.5	0.5	0.5	0.5	0.5	0.5

vi. Raw material stock data - line MGP 909

INPUT OF RAW MATERIALS

TYPE	INPUT
1	200.0
2	30.0

REDUCTION IN RAW MATERIAL CAPACITY FOR POST HORIZON PRODUCTION

```
* THE UNITS ARE IN 100 S *
```

	P1	P2	P3	P4	P5	P6
A	0.0	0.0	0.0	0.0	0.0	0.0
B	0.0	0.0	0.0	0.0	0.0	0.0

vii. Finished goods stock data - line MGP 958

INPUT OF FINISHED PRODUCT
****

TYPE	INPUT
1	5.0
2	5.0

viii. Debt data - line MGP 972

INITIAL DEBT OWED TO COMPANY 500.0 PAYED OVER 1. PERIODS

EXTRA MODEL DEBT
****

****
* THE UNITS ARE IN 100 S *
****

P1	P2	P3	P4	P5	P6
0.0	0.0	0.0	0.0	0.0	0.0

ix. Credit data - line MGP 992

INITIAL CREDIT OWED BY COMPANY 400.0 PAYED OVER 1. PERIODS

EXTRA MODEL CREDIT
****

****
* THE UNITS ARE IN 100 S *
****

P1	P2	P3	P4	P5	P6
0.0	0.0	0.0	0.0	0.0	0.0

x. Cashflow data - line MGP 1012

```
INITIAL INPUT OF CASH      100.0
*********************

                    OUTFLOWS OF CASH
                    ****************

              ********************************
              *  THE UNITS ARE IN      100 S *
              ********************************
```

	P1	P2	P3	P4	P5	P6
IN.DBT	5.0	0.0	0.0	0.0	0.0	0.0
EXPN	1.0	1.0	1.0	1.0	1.0	1.0
IN.CRT	4.0	0.0	0.0	0.0	0.0	0.0
TOTAL	-1.0	1.0	1.0	1.0	1.0	1.0

xi. Sales data - line MGP 1118

```
                    UPPER AND LOWER SALES BOUNDS FOR PRODUCT      1

 50 I
    I
    I
    I
 40 I
    I         MK
    I      MK
 30 I
    I            MK
    I   MK          MK
    I
 20 IMK
    I
    I
 10 I
    I
    I
  0 I
    ----------------------------------------------
    P1 P2 P3 P4 P5 P6
```

5. <u>The Report Writer Programme with Sample Outputs.</u>

```
C              .........................................................
C              .  THE REPORT GENERATOR CONVERTS THE LP SOLUTION INTO   .
C              .  LEGIBLE SPECIALIST REPORTS AND USES THE SAME DATA    .
C              .  BASE THAT IS PROCESSED BY THE MATRIX GENERATOR       .
C              .........................................................
      COMMON ALPHA,ALFLG,BANKL(16),BANKC(16),BANKR(16),
     1       CSH(16),CNT(10),CSHI,CTTI,     DSTORE(16),
     2       DISCP(16,2),DUMY(16,20),DMC(16,16),DLF(16,3),DCASH(16),
     3       DLO(16,3),DSN(16,10),DUMMY(20),DTTI,DST(16),DPM(16,9),
     4       DSF(16,10),EXCUT(16),EXCRT(16),EXPSM(11,17),
     5       FUAST(1,18),GROSS(5,16),INT,KDIV(18),KNOUT(18),
     6       LIST(16,2),LF(16,3),LO(16,3),LOCAP(16,3),LFCAP(16,3),
     7       LFREQ(16,3),LABAL(16,3,18), LABOT(16,3,16),MONTH(16),
     8       MO(16,13),MCREQ(1,18,10),NPROD,NOUT,NMC,NLF,NSUB,NRM,NM,
     9       CVT(16),UMAGE(3),PG(10,16)
      COMMON RAML(16,5),RMIN(5),RMLAG(5),RMB(5),
     1       SRM(16,5),SFG(16,10),STORE(15),SUBP(15),SRLAG(16),SUP(16),
     2       SLAG(16,2),SUBRC(15),SOUT(5),SPRED(20,4),SUBN(16,15),
     3       SUBOUT(16,2),SALE(10,16,2),STOCKI(16),   VOL(3),
     4       WCLF(2,18),WCCAP(16,18),WC(16,18),WCREQ(16,18),WIP ,
     5       WCLOAD(13)
      INTEGER ACTIV,LLIM,ULIM,DUAL,VNAME
      REAL LABAL,LABOT,LFREQ
      REAL     LIST,MCREQ,INIT,LFCAP,LOCAP,CSHFL(16),SUBP    ,RMB ,MO
      DOUBLE PRECISION NAME,COLUMN(30),VALUES(7),ENDSEC
      INTEGER SKIP,OVT    ,RR(16,18),SUDES(15),SSLAG,SLAG,RMLAG
      INTEGER           SOUT,DUMMY,NC,SUBRC,SPRED,DES,BLANK,SUBCD,SLOUT,
     1       SCSS,GROSS,BANKR,BANKC,EXP,WIP,FG
      INTEGER  FILE,INDIC,TYPE(30)
      INTEGER Z
      INTEGER ROWS(20,2),RCODE(20)
      DATA ENDSEC/'ENDSEDE'/
      DATA RCODE/'RC','LF','SI','SR','SF','OT','CT','CC','LC','LO',
     1           'CS','EA','SV','MS','PP','CR','TL'/
      ACTIV=1
      LLIM=3
      ULIM=4
      DUAL=5
      VNAME=8
C              .........................................................
C              .  THE LP SOLUTION IS STORED ON FILE                    .
C              .........................................................
      FILE=9
      REWIND 9
C              .........................................................
C              .  INITIALISE                                           .
C              .........................................................
      DO 1 J=1,20
      DUMMY(J)=0
      DO 1 I=1,16
      GROSS(1,I)=0
      GROSS(2,I)=0
      DUMY(I,J)=0
    1 CONTINUE
      DO 2 I1=1,18
      DO 2 I2=1,10
    2 MCREQ(1,I1,I2)=0.0
      DO 3 K=1,10
      DO 3 I=1,16
      DO 3 J=1,2
    3 SALE(K,I,J)=0.0
      DO 4 I=1,16
      DO 4 J=1,3
      DO 4 K=1,18
      LABAL(I,J,K)=0.0
    4 LABOT(I,J,K)=0.0
      ICOUNT=0
      NOUT1=0
      ALFLG=2
      ALPHA=9.5
C              .........................................................
C              .  THE FORMAT SECTION                                   .
C              .........................................................
 5900 FORMAT (1H1,///////////////,15X,'PRINTOUT OF THE FILED ',A8)
 5910 FORMAT (/,50X,A8)
 5920 FORMAT(1H1,'ROWS DONT TALLY',30('.'),A5)
 5950 FORMAT(1H ,A12,'  HAS NOW BEEN READ')
 4000 FORMAT(2513)
 4010 FORMAT(2(F8.3,I4,F6.3))
 4020 FORMAT(5F8.3)
 4030 FORMAT(15A4)
 4035 FORMAT(10F6.4)
 4040 FORMAT(I3,F8.3,I5)
 4050 FORMAT(312)
```

```
4060  FORMAT(I2,A2,2F8.3,I4)
4070  FORMAT(I2,A2,2I2,F8.3)
4080  FORMAT(A2)
4081  FORMAT(15A2)
4090  FORMAT(6F10.0)
4091  FORMAT(6F6.0)
4092  FORMAT(6I10)
4095  FORMAT(I2,16(A2,I2))
4100  FORMAT(5(2I2,F8.3,1X))
4110  FORMAT(3X,A2,I5)
9991  FORMAT(I5,10F10.2)
9999  FORMAT( ' VAL         ',6F15.4)
C              ......................................................
C              .  READ IN BASE DATA                                 .
C              ......................................................
      READ(2,4000) NPROD,NOUT,NRG,NLF,NSUB,NRM, NM,INT
      READ(2,4010) (RMS(J),RMLAG(J),VOL(J),J=1,NRM)
      READ(2,4020) ((WCLF(L,N),N=1,NLF),N=1,NWC)
      READ(2,4030) (WCLF(NLF+1,N),N=1,NWC)
      READ(2,4035) (WCLOAD(L),L=1,NWC)
      READ(2,4020) (DWAGE(L),L=1,NLF)
      IF(NSUB.EQ.0) GO TO 1001
      READ(2,4040) (SUBNO(J),SUBP(J),SBLAG(J),J=1,NSUB)
1001  CONTINUE
C              ......................................................
C              .       - THE DATA FOR EACH PRODUCT                  .
C              ......................................................
      DO 1000 K=1,NPROD
      READ(2,4050) ISK,KDIV(K),KNOUT(K)
      KP=KDIV(K)
C              ......................................................
C              .       - PRODUCT SALES DATA FOR EACH OUTLET         .
C              ......................................................
      KNO=KNOUT(K)
      DO 1010 J=1,KNO
      READ(2,4060)ISK,SLOUT      , LIST(K,J),DISCP(K,J),SLAG(K,J)
      IF(NOUT1.EQ.0) GO TO 1014
      DO 1012 JJ=1,NOUT1
      IF(SLOUT.EQ.SOUT(JJ)) GO TO 1013
1012  CONTINUE
1014  NOUT1=NOUT1+1
      SOUT(NOUT1)=SLOUT
      JJ=NOUT1
1013  JOBCD(K,J)=JJ
1010  CONTINUE
1011  CONTINUE
C              ......................................................
C              .       - M/C REQUIREMENTS  (**REVERSED TO MGP)      .
C              ......................................................
      DO 1000 KP1=1,KP
      READ(2,4070) ISK,ISKF,KT,SPRED(K,KP1)
      IF(KT.EQ.0) GO TO 1020
      NSPR=SPRED(K,KP1)
      READ(2,4100) (IX,IY,DUMY(IX,IY),KT1=1,KT)
      DO 1025 IXX=1,NSPR
      IX=NSPR+1-IXX
      DO 1025 IY=1,NMC
      MCREQ(IX,IY,K)=MCREQ(IX,IY,K)+DUMY(IXX,IY)
1025  DUMY(IXX,IY)=0.0
1020  CONTINUE
C              ......................................................
C              .       - SKIP MATERIALS REQUIREMENTS                .
C              ......................................................
      READ(2,4070) ISK,ISKF,KT
      IF(KT.EQ.0) GO TO 1030
      KT= (KT-1)/5+1
      IF(KT.LE.0) GO TO 1030
      READ(2,4080) (SKIP,J=1,KT)
1030  CONTINUE
1000  CONTINUE
C              ......................................................
C              .       - POST-HORIZON FORECASTS                     .
C              ......................................................
      DO 1040 K=1,NPROD
      KP=KDIV(K)
      II=0
      II=SPRED(K,1)-1
      IF(KP.LE.1) GO TO 1050
      DO 1060 KP1=2,KP
      IF(III.GT.SPRED(K,KP1))GO TO 1060
      III=SPRED(K,KP1)
1060  CONTINUE
      II=II+III
1050  IF(II.EQ.0) GO TO 1070
      READ(2,4091) (FCAST(I,K),I=1,II)
```

```
      GO TO 1040
 1070 READ(2,4080) SKIP
 1040 CONTINUE
C     ...................................................................
C     .          - GENERIC PERIOD AND ROW NAMES                         .
C     ...................................................................
      READ(2,4081) (MONTH(I),I=1,N1)
      READ(2,4080) NROW
      READ(2,4110) ((ROWS(J,JJ),JJ=1,2),J=1,NROW)
C     ...................................................................
C     .          - CAPACITY, INITIAL AND EX-MODEL CONDITIONS            .
C     ...................................................................
      READ(2,4090) ((WCCAP(I,L),I=1,NM),L=1,NWC)
      READ(2,4090) ((LFCAP(I,L),I=1,N1),L=1,NLF)
      READ(2,4090) ((CUCAP(I,L) ,I=1,NM),L=1,NLF)
      READ(2,4090) (STORE(I),I=1,N1)
      READ(2,4093) (RMIN(J),J=1,NRM)
      READ(2,4092) WIP
      READ(2,4090) (STOCKI(K),K=1,NPROD)
      READ(2,4090) USTI
      READ(2,4090) (EXUBT(I),I=1,N1)
      READ(2,4090) CRTI
      READ(2,4090) (EXUKT(I),I=1,N1)
      READ(2,4090) CSHI
      READ(2,4095) KEXC,(EXCSH(1,I+1),EXCSH(I+1,1),I=1,KEXC)
      EXCSH(1,1)=KEXC
      NM=NM+1
      DO 1090 IK=1,KEXC
 1090 READ(2,4090) (EXCSH(IK+1,I),I=2,NM)
      NM=NM-1
      READ(2,4080) SKIP
C     ...................................................................
C     .    PREPARE TO READ FROM THE ROWS SECTION                        .
C     ...................................................................
      READ(FILE) NAME
   10 READ(FILE) NAME
      IF(NAME.NE.ENDSEC) GO TO 10
      READ(FILE) NAME,NOCL
      READ(FILE) (COLUMN(N),N=1,NOCL)
      READ(FILE)
C     ...................................................................
C     .          - WORKCENTRE SCHEDULE                                  .
C     ...................................................................
      ICOUNT=ICOUNT+1
      IF(RCODE(ICOUNT).NE.ROWS(ICOUNT,1)) GO TO 5000
      IF(ROWS(ICOUNT,2).EQ.0) GO TO 5010
      DO 2010 J=1,NWC
      DO 2010 I=1,NM
      READ(FILE)  (VALUES(N),N=1,NOCL)
      WCRED(I,J)=VALUES(ULIM)
      DWC(I,J)=-VALUES(DUAL)
 2010 WC(I,J)=VALUES(ACTIV)
 5010 CONTINUE
C     ...................................................................
C     .          - LABOUR FORCE SCHEDULE                                .
C     ...................................................................
      ICOUNT=ICOUNT+1
      IF(RCODE(ICOUNT).NE.ROWS(ICOUNT,1)) GO TO 5000
      IF(ROWS(ICOUNT,2).EQ.0) GO TO 5020
      DO 2020 L=1,NLF
      DO 2020 I=1,NM
      READ(FILE)  (VALUES(N),N=1,NOCL)
      DLF(I,L)=-VALUES(DUAL)
      LFRED(I,L)=VALUES(ULIM)
 2020 LF(I,L)=VALUES(ACTIV)
 5020 CONTINUE
C     ...................................................................
C     .          - STORAGE                                              .
C     ...................................................................
      ICOUNT=ICOUNT+1
      IF(RCODE(ICOUNT).NE.ROWS(ICOUNT,1)) GO TO 5000
      IF(ROWS(ICOUNT,2).EQ.0) GO TO 5030
      DO 2030 I=1,NM
      READ(FILE)  (VALUES(N),N=1,NOCL)
      USTORE(I)=VALUES(DUAL)
 2030 CONTINUE
 5030 CONTINUE
C     ...................................................................
C     .          - RAW MATERIALS STOCKS                                 .
C     ...................................................................
      ICOUNT=ICOUNT+1
      IF(RCODE(ICOUNT).NE.ROWS(ICOUNT,1)) GO TO 5000
      IF(ROWS(ICOUNT,2).EQ.0) GO TO 5040
      DO 2040 J=1,NRM
      DO 2040 I=1,NM
```

```
      READ(FILE)   (VALUES(N),N=1,NUCL)
2040  DNM(I,J)=-VALUES(DUAL)
5040  CONTINUE
C                 ..........................................................
C                 .        - FINISHED GOODS STOCKS                        .
C                 ..........................................................
      ICOUNT=ICOUNT+1
      IF(RCODE(ICOUNT).NE.ROWS(ICOUNT,1)) GO TO 5000
      IF(ROWS(ICOUNT,2).EQ.0) GO TO 5050
      DO 2050 K=1,NPROD
      DO 2050 I=1,NM
      READ(FILE)   (VALUES(N),N=1,NUCL)
2050  DSF(I,K)=-VALUES(DUAL)
5050  CONTINUE
C                 ..........................................................
C                 .        - SKIP DEBT AND CREDIT EQUATIONS               .
C                 ..........................................................
      DO 2060 J1=1,2
      ICOUNT=ICOUNT+1
      IF(RCODE(ICOUNT).NE.ROWS(ICOUNT,1)) GO TO 5000
      DO 2060 J=1,NM
      READ(FILE)   (VALUES(N),N=1,NUCL)
2060  CONTINUE
C                 ..........................................................
C                 .        - CASH CONTINUITY                              .
C                 ..........................................................
      ICOUNT=ICOUNT+1
      IF(RCODE(ICOUNT).NE.ROWS(ICOUNT,1)) GO TO 5000
      DO 2070 I=1,NM
      READ(FILE)   (VALUES(N),N=1,NUCL)
2070  DCASH(I)=-VALUES(DUAL)
C                 ..........................................................
C                 .        - LOAN CHARGES                                 .
C                 ..........................................................
      ICOUNT=ICOUNT+1
      IF(RCODE(ICOUNT).NE.ROWS(ICOUNT,1)) GO TO 5000
      IF(ROWS(ICOUNT,2).EQ.0) GO TO 5060
      DO 2080 J=1,NM
      READ(FILE)   (VALUES(N),N=1,NUCL)
2080  CONTINUE
5060  CONTINUE
C                 ..........................................................
C                 .        - LABOUR OVERTIME                              .
C                 ..........................................................
      ICOUNT=ICOUNT+1
      IF(RCODE(ICOUNT).NE.ROWS(ICOUNT,1)) GO TO 5000
      IF(ROWS(ICOUNT,2).EQ.0) GO TO 5070
      DO 2090 I=1,NM
2090  DUMMY(I)=0.
      DO 2100 M=1,NLF
      DO 2100 I=1,NM
      READ(FILE)   (VALUES(N),N=1,NUCL)
      DUMMY(I)= DUMMY(I) +(VALUES(ACTIV)*OWAGE(M))
      DLO(I,M)=-VALUES(DUAL)
      LO(I,M)=VALUES(ACTIV)
2100  CONTINUE
      DO 2110 I=1,NM
      OVT(I)=DUMMY(I)
2110  DUMMY(I)=0.
5070  CONTINUE
C                 ..........................................................
C                 .   PREPARE TO READ FROM THE COLUMNS SECTION            .
C                 ..........................................................
2190  READ(FILE) (VALUES(N),N=1,NUCL)
      IF(VALUES(1).NE.ENDSEC) GO TO 2190
      READ(FILE) NAME,NUCL
      READ(FILE) (COLUMN(N),N=1,NUCL)
      READ(FILE)
C                 ..........................................................
C                 .        - PRODUCTION PROFILE                           .
C                 ..........................................................
      DO 3000 K=1,NPROD
      KP=KDIV(K)
      DO 3000 KP1=1,KP
      DO 3000 I=1,NM
      READ(FILE)   (VALUES(N),N=1,NUCL)
      IF(KP1.GT.1) GO TO 3000
      PD(K,I)=VALUES(ACTIV)
3000  CONTINUE
C                 ..........................................................
C                 .        - SALES PROFILE                                .
C                 ..........................................................
      DO 3030 K=1,NPROD
      KNO = KNOUT(K)
      DO 3030 J=1,KNO
```

```
      DO 3030 I=1,NM
      READ(FILE)   (VALUES(N),N=1,NOCL)
      SALE(K,I,J)=VALUES(ACTIV)
 3030 GROSS(J,I)=GROSS(J,I)+VALUES(ACTIV)*LIST(K,J)
     1              *((100-DISCP(K,J))/100.)
C               ..........................................................
C               .      - SUBCONTRACTING (COSTS) INCURRED                  .
C               ..........................................................
      IF(NSUB.EQ.0) GO TO 3062
      DO 3050 J=1,NM
      DUMMY(J)=0.0
      DO 3060 N=1,NSUB
 3060 SUBK(J,N)= 0.
      DO 3070 KA=1,NSUB
      READ(FILE)   (VALUES(N),N=1,NOCL)
      SUBK(J,KA)=VALUES(ACTIV)
 3070 DUMMY(J)=DUMMY(J)+ (VALUES(ACTIV)*SUBP(KA))
      SUL(J)= DUMMY(J)
 3050 CONTINUE
 3062 CONTINUE
C               ..........................................................
C               .      - ALLOCATED LABOUR ACTIVITIES                      .
C               ..........................................................
      DO 3190 I=1,NM
      DO 3190 L=1,NLF
      DO 3190 M=1,NWC
      IF(WOLF(L,M).EQ.0.0) GO TO 3191
      READ(FILE)( VALUES(N),N=1,NOCL)
      LABAL(I,L,M)=VALUES(ACTIV)
 3191 CONTINUE
 3190 CONTINUE
C               ..........................................................
C               .     - RAW MATERIAL PURCHASES                            .
C               ..........................................................
      DO 3080 J=1,NRM
      DO 3080 I=1,NM
      READ(FILE) (VALUES(N),N=1,NOCL)
      RAWM(I,J)=VALUES(ACTIV)
 3080 CONTINUE
C               ..........................................................
C               .      - RAW MATERIAL STOCK LEVELS                        .
C               ..........................................................
      DO 3200 J=1,NRM
      DO 3200 I=1,NM
      READ(FILE) (VALUES(N),N=1,NOCL)
 3200 SRM(I,J)=VALUES(ACTIV)
C               ..........................................................
C               .      - FININSHED GOODS STOCK LEVELS                     .
C               ..........................................................
      DO 3210 K=1,NPROD
      DO 3210 I=1,NM
      READ(FILE)( VALUES(N),N=1,NOCL)
 3210 SFG(I,K)=VALUES(ACTIV)
C               ..........................................................
C               .      - ALLOCATED LABOUR OVERTIME                        .
C               ..........................................................
      DO 3095 I=1,NM
      DO 3095 L=1,NLF
      DO 3095 M=1,NWC
      IF(WOLF(L,M).EQ.0.0) GO TO 3096
      READ(FILE) (VALUES(N),N=1,NOCL)
      LABOT(I,L,M)=VALUES(ACTIV)
 3096 CONTINUE
 3095 CONTINUE
C               ..........................................................
C               .      - CASH BALANCE                                     .
C               ..........................................................
      DO 3100 J=1,NM
      READ(FILE)   (VALUES(N),N=1,NOCL)
      CSH(J)=VALUES(ACTIV)
 3100 CONTINUE
C               ..........................................................
      IF(INT.EQ.0) GO TO 3145
C               ..........................................................
C               .      - BANK LOANS                                       .
C               ..........................................................
      DO 3110 J=1,NM
      READ(FILE)   (VALUES(N),N=1,NOCL)
      BANKL(J) = VALUES(ACTIV)
 3110 CONTINUE
C               ..........................................................
C               .      - BANK REPAYMENTS                                  .
C               ..........................................................
      DO 3120 J=1,NM
      READ(FILE)   (VALUES(N),N=1,NOCL)
```

```
      BANKR(J) = VALUES(ACTIV)
3120  CONTINUE
C
C               ..........................................................
C               .          - BANK CHARGES                                .
C               ..........................................................
      DO 3130 J=1,NM
      READ(FILE)  (VALUES(N),N=1,NOCL)
      BANKC(J) = VALUES(ACTIV)
3130  CONTINUE
C
3145  CONTINUE
C
C               ..........................................................
C               .        - CREDIT                                        .
C               ..........................................................
      DO 3140 J=1,NM
      READ(FILE)  (VALUES(N),N=1,NOCL)
      CRT(J)=VALUES(ACTIV)
3140  CONTINUE
C               ..........................................................
C               .          - DEBT                                        .
C               ..........................................................
      DO 3150 J=1,NM
      READ(FILE)  (VALUES(N),N=1,NOCL)
      DBT(J)=VALUES(ACTIV)
3150  CONTINUE
C               ..........................................................
C               .  PRINT THE REPORTS                                     .
C               ..........................................................
      CALL REPRTA
      CALL REPRTB
      CALL REPRTC
      CALL REPRTG
      CALL REPRTI
      REWIND 9
19    STOP
5000  WRITE(6,5930) RCODE(ICOUNT)
      GO TO 19
      END
```

i. Report A

```
      SUBROUTINE REPRTA
C                 .........................................................
C                 . THIS REPORT DETAILS THE PLANNED PRODUCT-MIX           .
C                 .     -  PER PERIOD                                     .
C                 .........................................................
      COMMON ALPHA,ALFLG,BANKL(16),BANKC(16),BANKR(16),
     1       CSH(16),CRT(16),CSHI,CRTI,     DSTORE(16),
     2       DISCP(10,2),DUMY(16,20),DWC(16,18),DLF(16,3),DCASH(16),
     3       DLO(16,3),DSR(16,10),DUMMY(20),DSTI,DST(16),DRM(16,9),
     4       DSF(16,10),EXDBT(16),EXCRT(16),EXCSH(11,17),
     5       FCAST(1,10),GROSS(5,15),INT,KDIV(10),KNOUT(10),
     6       LIST(10,2),LF(16,3),LO(16,3),LOCAP(16,3),LFCAP(16,3),
     7       LFRED(16,3),LABAL(16,3,18), LABOT(16,3,18),MONTH(16),
     8       MO(16,18),MOREQ(1,18,10),NPROD,NOUT,NWC,NLF,NSUB,NRM,NM,
     9       OVT(16),OWAGE(3),PD(16,16)
      COMMON RAWM(16,5),RMIN(5),RMLAG(5),RMO(5),
     1       SRM(16,5),SFG(16,10),STORE(16),SUBP(15),SBLAG(15),SUB(16),
     2       SLAG(16,2),SUBWC(15),SOUT(5),SPRED(10,4),SUBN(16,15),
     3       SUBCD(10,2),SALE(10,15,2),STOCKI(10),   VOL(9),
     4       WCLF(2,18),WCCAP(16,18),WC(16,18),WCRED(16,16),WIP ,
     5       WCLOAD(18)
      REAL LABAL,LABOT,LFRED
      REAL     LIST,MOREQ,INIT,LFCAP,LOCAP,CSHFL(16),SUBP    ,RMB ,MO
      INTEGER SKIP,OVT      ,MR(16,18),SBDES(15),SBLAG,SLAG,RMLAG
      INTEGER             SOUT,DUMMY,WC,SUBWC,SPRED,DES,BLANK,SUBCD,SLOUT,
     1        SCSD,GROSS,BANKR,BANKC,EXP,WIP,FG
      DO 10 K=1,NPROD
10    DUMMY(K)=STOCKI(K)
C                 .........................................................
C                 . HEADINGS                                              .
C                 .........................................................
      DO 50 J=1,NM
      ISP=0
      DO 11 K=1,NPROD
      KD=KDIV(K)
      DO 11 KD1=1,KD
      IF(ISP.GE.SPRED(K,KD1)) GO TO 11
      ISP=SPRED(K,KD1)
11    CONTINUE
      IF(ISP.EQ.1) WRITE(3,1000) MONTH(J)
      IF(ISP.EQ.1) GO TO 12
      ISP=ISP-1
      WRITE(3,1000) MONTH(J),(ISP1,ISP1=1,ISP)
C                 .........................................................
C                 . PLANNED PRODUCTION (+ STOCK) LEVELS PER PRODUCT       .
C                 .........................................................

12    DO 20 K=1,NPROD
      ISP=0
      KD=KDIV(K)
      DO 9 KD1=1,KD
      IF(ISP.GE.SPRED(K,KD1)) GO TO 9
      ISP=SPRED(K,KD1)
9     CONTINUE
      ISP=ISP-1
      IF(J.EQ.1) OPEN=STOCKI(K)
      IF(J.GT.1) OPEN=SFG(J-1,K)
      WORK1 = 0
      NT=KNOUT(K)
      DO 15 L=1,NT
   15 WORK1 = WORK1 + SALE(K,J,L)
      CLOSE=SFG(J,K)
      IF(ISP.EQ.0) GO TO 17
      DO 16 ISP2=1,ISP
      ISP1=ISP-ISP2+1
      DUMY(ISP1,K)=PD(K,J+ISP2)
      IF((J+ISP2).GT.NM)  DUMY(ISP1,K)=0.0
16    CONTINUE
      WRITE(3,1010) K,OPEN,PD(K,J),WORK1,CLOSE,(DUMY(ISP1,K),ISP1=1,ISP)
      GO TO 20
17    WRITE(3,1010) K,OPEN,PD(K,J),WORK1,CLOSE
   20 CONTINUE
C
   50 CONTINUE
C                 .........................................................
C                 . THE FORMAT SECTION                                    .
C                 .........................................................
 1000 FORMAT (1H1,////,1X,
     1'REPORTA                      PRODUCTION SCHEDULE FOR PERIOD ',A2,///,1X,
     2' PRODUCT          OPENING     PRODUCTION            SALES          CLOSING'
     3,10X,4(I1,' MONTH WIP',5X)/,1X,
     4'                  STOCK                                             STOCK'
     5,/)
1010  FORMAT(/,I8,4(5X,F9.2),10X,4(F10.2,5X))
      RETURN
      END
```

REPORTA PRODUCTION SCHEDULE FOR PERIOD P1

PRODUCT	OPENING	PRODUCTION	SALES	CLOSING
1	5.00	20.56	20.00	5.56
2	5.00	3.89	0.0	8.89

REPORTA PRODUCTION SCHEDULE FOR PERIOD P2

PRODUCT	OPENING	PRODUCTION	SALES	CLOSING
1	5.56	19.67	0.31	24.92
2	8.89	3.67	2.52	10.03

ii. Report B

```
      SUBROUTINE REPRTB
C                 .......................................................
C                 . THIS REPORT DETAILS THE PLANNED PRODUCT-MIX         .
C                 .    -   PER PRODUCT                                  .
C                 .......................................................
      COMMON ALPHA,ALFLG,BANKL(16),BANKC(16),BANKR(16),
     1       CSH(16),CRT(16),CSHI,CRTI,      DSTORE(16),
     2       DISCP(10,2),DUMY(16,20),ONC(16,18),DLF(16,3),DCASH(16),
     3       DLO(16,3),DSR(16,10),DUMMY(20),DUTI,DJT(16),DRM(16,9),
     4       DSF(16,10),EXDBT(16),EXCRT(16),EXCSH(11,17),
     5       FLAST(1,10),GROSS(5,15),INT,KDIV(10),KNOUT(10),
     6       LIST(10,2),LF(16,3),LO(16,3),LOCAP(16,3),LFCAP(16,3),
     7       LFRED(16,3),LABAL(16,3,18), LABDT(15,3,18),MONTH(16),
     8       MO(16,18),MOREQ(1,18,10),NPROD,NOUT,NWC,NLF,NSUB,NRM,NM,
     9       OVT(16),OWAGE(3),PD(10,16)
      COMMON RAWM(16,5),RMIN(5),RMLAG(5),RMB(5),
     1       SRM(16,5),SFG(16,10),STYPE(16),SUBP(15),SBLAG(15),SUB(16),
     2       SLAG(10,2),SUBWC(15),SOUT(5),SPRED(10,4),SUBN(16,15),
     3       SUBCD(10,2),SALE(10,18,2),STOCKI(10),   VOL(9),
     4       WCLF(2,18),WCCAP(16,18),WC(16,18),WCRED(16,18),WIP ,
     5       WCLOAD(18)
      REAL LABAL,LABDT,LFRED
      REAL    LIST,MOREQ,INIT,LFCAP,LOCAP,CSHFL(16),SUBP   ,RMB ,MO
      INTEGER SKIP,OVT     ,WC(16,18),SBUES(15),SBLAG,SLAG,RMLAG
      INTEGER              SOUT,DUMMY,WC,SUBWC,SPRED,DES,BLANK,SUBCD,SLOUT,
     1        SCSS,GROSS,BANKR,BANKC,FXP,WIP,FG
C                 .......................................................
C                 . HEADINGS                                            .
C                 .......................................................
      WRITE (3,1000)
      NN=NM-1
      I = 0
C                 .......................................................
C                 . PLANNED PRODUCTION (+STOCK) LEVELS PER PRODUCT      .
C                 .......................................................
      DO 50 K=1,NPROD
      I = I+1
      KNO=KNOUT(K)
      DO 5 J=1,NM
      DUMY(J,1)=0
      DO 5 L=1,KNO
    5 DUMY(J,1)=DUMY(J,1) + SALE(K,J,L)
      WRITE(3,1010) K
      WRITE(3,1020) (MONTH(J),J=1,NM)
       WRITE(3,1030) STOCKI(K),(SFG(I,K),I=1,NN)
      WRITE (3,1040) (PD(K,J),J=1,NM)

      WRITE(3,1050) (DUMY(J,1),J=1,NM)
      WRITE(3,1060) (SFG(I,K),I=1,NM)
C                 .......................................................
C                 .......................................................
C                 . SKIP TO NEW PAGE FOR EVERY FIVE PRODUCTS
      IF (I-5)  50,20,20
   20 I = 0
      WRITE (3,1000)
   50 CONTINUE
      RETURN
C                 .......................................................
C                 . THE FORMAT SECTION                                  .
C                 .......................................................
 1000 FORMAT (1H1,/)
 1010 FORMAT(///,1X,'REPORT B',24X,'PRODUCTION SCHEDULE FOR PRODUCT ',
     1I6/)
 1020 FORMAT (1X,'     PERIOD      ',16(5X,A2))
 1030 FORMAT(/,1X,'   OPENING STOCK',16F7.2)
 1040 FORMAT (1X,'   PRODUCTION    ',16F7.2)
 1050 FORMAT (1X,'   SALES         ',16F7.2)
 1060 FORMAT (1X,'   CLOSING STOCK',16F7.2)
      END
```

REPORT B   PRODUCTION SCHEDULE FOR PRODUCT   1

PERIOD	P1	P2	P3	P4	P5	P6
OPENING STOCK	5.00	5.56	24.92	40.89	22.44	11.33
PRODUCTION	20.56	19.67	18.44	17.56	16.89	15.67
SALES	20.00	0.31	2.47	36.00	28.00	27.00
CLOSING STOCK	5.56	24.92	40.89	22.44	11.33	0.0

iii. Report C & D

```
      SUBROUTINE REPRTC
C           ..............................................................
C           .  THIS REPORT DETAILS THE SALES PLAN                        .
C           ..............................................................
      COMMON ALPHA,ALFLG,BANKC(16),BANKO(16),BANKR(16),
     1       CSH(16),CRT(15),CSHI,CRTI,      DSTORE(16),
     2       DISCP(10,2),DUMY(16,20),DWC(15,18),DLF(15,3),DCASH(16),
     3       DLO(16,3),DSR(16,10),DUMMY(20),DSTI,DBT(16),DRM(16,9),
     4       DSF(15,10),EXOUT(16),EXCRT(15),EXCSH(11,17),
     5       FCAST(1,10),GROSS(5,16),INT,KDIV(10),KNOUT(10),
     6       LIST(10,2),LF(16,3),LO(15,3),LOCAP(16,3),LFCAP(16,3),
     7       LFRED(15,3),LABAL(15,3,18), LABOT(15,3,13),MONTH(16),
     8       MO(16,18),MOREQ(1,13,10),NPROD,NOUT,NWC,NLF,NSUB,NRM,NM,
     9       OVT(16),OHAGE(3),PD(10,16)
      COMMON RAWM(16,5),RMIN(5),RMLAG(5),RMB(5),
     1       SRM(16,5),SFG(16,10),STORE(16),SUBP(15),SBLAG(15),SUB(16),
     2       SLAG(10,2),SUBWC(15),SOUT(5),SPRED(10,4),SUBW(16,15),
     3       SUBCD(10,2),SALE(10,15,2),STOCKI(10),    VOL(9),
     4       WCLF(2,16),WCCAP(16,18),WC(15,18),WCRED(16,16),WIP ,
     5       WCLOAD(18)
      REAL LABAL,LABOT,LFRED
      REAL     LIST,MCREQ,INIT,LFCAP,LOCAP,CSHFL(16),SUBP    ,RMP ,MO
      INTEGER SKIP,OVT      ,MR(16,13),SBDES(15),SBLAG,SLAG,RMLAG
      INTEGER             SOUT,DUMMY,WC,SUBWC,SPRED,DES,BLANK,SUBCD,SLOUT,
     1       SCSS,GROSS,BANKR,BANKC,FXP,WIP,FG
C           ..............................................................
C           .  REPORTC SHOWS THE SALES FIGURES PER OUTLET (IN UNITS)     .
C           ..............................................................
      DO 150 JA=1,NOUT
      DO 124 K=1,NPROD
      DO 140 I=1,NM
  140 DUMY(I,K)=0
      KNO=KNOUT(K)
      DO 131 J=1,KNO
      IF(SUBCD(K,J).NE.JA) GO TO 131
      DO 130 I=1,NM
  130 DUMY(I,K)=SALE(K,1,J)
131   CONTINUE
  124 CONTINUE
C
      WRITE(3,1000)
      WRITE(3,400)  SOUT(JA)
      WRITE(3,1020)  (MONTH(J),J=1,NM)
      WRITE(3,1050)
      DO 110 K=1,NPROD
      WRITE(3,1030) K,(DUMY(I,K),I=1,NM)
  110 CONTINUE
  150 CONTINUE
C           ..............................................................
C           .  REPORTD SHOWS THE SALES FIGURES PER OUTLET (BY VALUE)     .
C           ..............................................................
      DO 50 JA=1,NOUT
      DO 24 K=1,NPROD
      DO 40 I=1,NM
   40 DUMY(I,K)=0
      KNO=KNOUT(K)
      DO 31 J=1,KNO
      IF(SUBCD(K,J).NE.JA) GO TO 31
      DO 30 I=1,NM
      DUMY(I,K)=SALE (K,1,J)*LIST(K,J)*(100-DISCP(K,J))/100
30    CONTINUE
31    CONTINUE
   24 CONTINUE
C
      WRITE(3,1000)
      WRITE(3,1010)  SOUT(JA)
      WRITE(3,1020)  (MONTH(J),J=1,NM)
      WRITE(3,1050)
      DO 10 K=1,NPROD
      WRITE(3,1030) K,(DUMY(I,K),I=1,NM)
   10 CONTINUE
      WRITE(3,1040)  (GROSS(JA,J),J=1,NM)
   50 CONTINUE
      RETURN
C           ..............................................................
C           . THE FORMAT SECTION                                         .
C           ..............................................................
 1000 FORMAT (1H1,/)
  400 FORMAT (1X,'REPORT C',26X,' SALES PROGRAMME FOR OUTLET ',A2,
     1' IN UNITS',/,37X,40('*'),/)
 1010 FORMAT (1X,'REPORTD',25X,' NET SALES PROGRAMME FOR OUTLET ',A2,
     1' IN POUNDS',/,37X,41('*'),/)
 1020 FORMAT (1X,'       PERIOD   ',15(5X,A2))
 1030 FORMAT(/,5X,I8,3X,16F7.0)
 1040 FORMAT (1X,///,' GROSS SALES',4X,16I7)
 1050 FORMAT(/,5X,' PRODUCT',/)
C
      END
```

REPORT C

SALES PROGRAMME FOR OUTLET MK IN UNITS
****

PERIOD	P1	P2	P3	P4	P5	P6
PRODUCT						
1	20.	0.	2.	36.	28.	27.
2	0.	3.	10.	C.	3.	10.

REPORTD

NET SALES PROGRAMME FOR OUTLET MK IN POUNDS
****

PERIOD	P1	P2	P3	P4	P5	P6
PRODUCT						
1	400.	6.	49.	720.	560.	540.
2	0.	126.	500.	C.	171.	500.
GROSS SALES	399	132	548	719	730	1038

iv. Report G & H

```
      SUBROUTINE REPRTG
C             ...........................................................
C             .  THIS REPORT DETAILS THE PRODUCTION SCHEDULE             .
C             ...........................................................
      COMMON ALFHA,ALFLG,BANKL(16),BANKC(16),BANKR(16),
     1       CSH(16),CRI(16),CSHI,CRT1,     DSTOPF(16),
     2       DISCP(16,2),DUMY(16,20),DWC(16,16),DLF(16,3),DCASH(16),
     3       DLO(16,3),DSR(16,10),DUMMY(20),DSTI,DST(16),DRM(16,9),
     4       DSF(16,10),EXDET(16),EXCRT(16),EXCSH(11,17),
     5       FCAST(1,10),GROSS(5,15),INT,KDIV(10),KNOUT(10),
     6       LIST(10,2),LF(16,3),LO(16,3),LOCAP(16,3),LFCAP(16,3),
     7       LFRED(16,3),LABAL(16,3,18), LABOT(15,3,13),MONTH(16),
     8       MO(16,18),MOREQ(1,18,10),NPROD,NOUT,NWC,NLF,NSUB,NRM,NM,
     9       OVT(16),OWAGE(3),PD(16,16)
      COMMON RAWM(16,5),RMIN(5),RMLAG(5),RMB(5),
     1       SRM(16,5),SFG(16,16),STORE(16),SUPP(15),SPLAG(15),SUR(16),
     2       SLAG(10,2),SUBWC(15),SOUT(5),SPRED(10,4),SUBH(15,15),
     3       SUBCD(10,2),SALE(10,16,2),STOCKI(10),   VOL(5),
     4       WCLF(2,16),WCCAP(16,13),WC(16,18),WCRED(16,18),WIP ,
     5       WCLOAD(18)
      REAL LABAL,LABOT,LFRED
      REAL     LIST,MOREQ,INIT,LFCAP,LOCAF,CSHFL(16),SUPP    ,RMB ,MO
      INTEGER SKIP,OVT      ,HR(16,14),SBDES(15),SBLAG,SLAG,RMLAG
      INTEGER           SOUT,DUMMY,WC,SUBWC,SPRED,DES,BLANK,SUBCD,SLOUT,
     1        SCSS,GROSS,BANKR,BANKC,FXP,WIP,FG
      DATA LINE /'--'/,BLANK/'  '/
C             ...........................................................
C             .  THE FORMAT SECTION                                      .
C             ...........................................................
390   FORMAT(1H ,'     IFACTORYI',20(A2,4X,'I'))
400   FORMAT('     ICAPCTY I',20(F5.0,'I'))
405   FORMAT(1H ,'      IWC LOADI',20(F6.3,'I'))
410   FORMAT('     IAVAILBLEI',20(F5.0,'I'))
430   FORMAT('     IREQYRD I',20(F5.0,'I'))
440   FORMAT('     ISUBCNT I',20(F6.0,'I'))
450   FORMAT('     IO/TIME I',20(F6.1,'I'))
460   FORMAT('     IO/T CP I',20(F6.0,'I'))
499    FORMAT(///' REPORTH',7X,'LABOUR FORCE',I3,'  IN-HOUSE MANNING SCH
     1EDULE'//)
500   FORMAT(1H1,1X,' REPORTG', 7X,'WORK CENTRE SCHEDULE FOR PERIOD  ',
     1 A2,//16X,55('*')//)
501   FORMAT('     ---------',20(A2,'-----'))
502   FORMAT(1H ,'      I WORK  I',20(   A4,2X,'I'))
503   FORMAT(1H ,'      I CENTREI',20(A2,4X,'I'))
514   FORMAT('     IPRODUCTI',20(A2,4X,'I'))
520   FORMAT(1H ,'      I',I4,3X,'I',20(F6.1,'I'))
526   FORMAT(1H ,'      I TOTAL I',20(A2,4X,'I'))
525   FORMAT(1H ,'      I       I',20(   A4,2X,'I'))
527   FORMAT(' SHULI',F6.0,' I',20(F6.1,'I'))
528   FORMAT(' O/TMI',F6.0,' I',20(F6.1,'I'))
530   FORMAT(' CHUTI',F6.0,' I',20(A2,4X,'I'))
700   FORMAT('     I',7X,'I',20(A2,4X,'I'))
C             ...........................................................
C             .  INITIALISE                                              .
C             ...........................................................
      DO 10 I=1,NM
      DO 620 J=1,NWC
      DUMMY(J)=0
      DUMY(1,J)=0.0
620   DUMY(2,J)=0.0

      KT=NWC
C             ...........................................................
C             .  THE WORK-CENTRE SCHEDULE                                .
C             ...........................................................
C             ...........................................................
C             ...........................................................
C             .    -    HEADINGS                                         .
      WRITE(3,500) MONTH(I)
      WRITE(3,501) (LINE,J=1,KT)
      WRITE(3,502) (WCLF(NLF+1,J),J=1,NWC)
      WRITE(3,503) (BLANK,J=1,NWC)
      WRITE(3,700) (BLANK,J=1,NWC)
      WRITE(3,501) (LINE,J=1,KT)
      WRITE(3,514) (BLANK,J=1,NWC)
```

```
C                 ..................................................................
C                 .    -    W/C ALLOCATION                                          .
C                 ..................................................................
      DO 610 K=1,NPROJ
      DO 602 J=1,NWC
602   DUMY(1,J)=0.0
      NSPR=0
      KP=NDIV(K)
      DO 601 KP1=1,KP
      IF(SPRED(K,KP1).GT.NSPR) NSPR=SPRED(K,KP1)
601   CONTINUE
      DO 600 J=1,NWC
      DO 590 N=1,NSPR
      IF(I+NSPR-N-NM) 585,585,590
585   DUMY(1,J)=DUMY(1,J)+FD(K,I+NSPR-N)*MCREQ(NSPR+1-N,J,K)
  590 CONTINUE
600   DUMY(2,J)=DUMY(2,J)+DUMY(1,J)
      WRITE(3,520) K,(DUMY(1,J),J=1,NWC)
  610 CONTINUE
C                 ..................................................................
C                 .    -    AVAILABLE CAPACITY                                      .
C                 ..................................................................
      WRITE(3,501)  (LINE,J=1,KT)
      WRITE(3,700)  (BLANK,KT1=1,KT)
      WRITE(3,390)  (BLANK,J=1,KT)
      WRITE(3, 400)  (WCCAP(I,J),J=1,NWC)
      WRITE(3,700)   (BLANK,KT1=1,KT)
      WRITE(3,405)  (WCLOAD(L),L=1,NWC)
      WRITE(3,501)  (LINE,J=1,KT)
      WRITE(3,410)  (WCPED(I,J),J=1,NWC)
      WRITE(3,501)  (LINE,J=1,KT)
      WRITE(3,700)   (BLANK,KT1=1,KT)
      WRITE(3,430)  (DUMY(2,J),J=1,NWC)
      WRITE(3,700)   (BLANK,KT1=1,KT)
C                 ..................................................................
C                 .    -    SUBCONTRACTING                                          .
C                 ..................................................................
      IF(NSUB.EQ.0) GO TO 624
      DO 614 L=1,NWC
614   DUMY(4,L)=0.0
      DO 613 NSUB1=1,NSUB
      L=SUBWC(NSUB1)
613   DUMY(4,L)=SUBW(I,NSUB1)
      WRITE(3,440)  (DUMY(4,L),L=1,NWC)
      WRITE(3,700)   (BLANK,KT1=1,KT)
624   CONTINUE
C
C                 ..................................................................
C                 .    -    OVERTIME                                                .
C                 ..................................................................
      DO 615 J=1,NWC
      DUMY(5,J)=0.0
      DO 616 L=1,NLF
      IF(WCLF(L,J).EQ.0.0) GO TO 616
      DUMY(5,J)=DUMY(5,J)+LABOT(I,L,J)/WCLF(L,J)
616   CONTINUE
615   CONTINUE
      WRITE(3,450)  (DUMY(5,J),J=1,NWC)
      WRITE(3,501)  (LINE,KT1=1,KT)
C                 ..................................................................
C                 .  THE LABOUR FORCE SCHEDULE                                      .
C                 ..................................................................
C                 ..................................................................
C                 .    -    HEADINGS                                                .
C                 ..................................................................
      DO 612 L=1,NLF
      WRITE(3,499)  L
      WRITE(3,501) (LINE,J=1,KT)
      WRITE(3,525)  (WCLF(NLF+1,J),J=1,NWC)
      WRITE(3,526)  (BLANK,J=1,NWC)
      WRITE(3,501)  (LINE,KT1=1,KT)
C                 ..................................................................
C                 .    -    CAPACITY                                                .
C                 ..................................................................
      WRITE(3,530) LFRED(I,L),(BLANK,KT1=1,KT)
C                 ..................................................................
C                 .    -    ALLOCATION                                              .
C                 ..................................................................
      DUMY(1,1)=0.0
      DUMY(1,2)=0.0
      DO 630 J=1,NWC
      DUMY(1,1)=DUMY(1,1)+LABAL(I,L,J)
630   DUMY(1,2)=DUMY(1,2)+LABOT(I,L,J)
      WRITE(3,527) DUMY(1,1),(LABAL(I,L,J),J=1,NWC)
      WRITE(3,700)   (BLANK,KT1=1,KT)
      WRITE(3,528) DUMY(1,2),(LABOT(I,L,J),J=1,NWC)
      WRITE(3,501) (LINE,J=1,KT)
612   CONTINUE
   10 CONTINUE
      RETURN
      END
```

REPORTG WORK CENTRE SCHEDULE FOR PERIOD P4

****

WORK CENTRE	M/C1	M/C2	M/C3
PRODUCT			
1	52.7	35.1	35.1
2	20.3	33.9	0.0
FACTORY CAPCTY	73.	69.	34.
WC LOAD	1.000	1.000	1.000
AVAILBL	73.	69.	34.
REQYRD	73.	69.	35.
O/TIME	0.0	0.0	1.1

REPORTH LABOUR FORCE 1 IN-HOUSE MANNING SCHEDULE

	TOTAL	M/C1	M/C2	M/C3
CPCT	110.			
SHDL	110.	77.0	0.0	33.3
O/TM	0.	0.0	0.0	0.0

REPORTH LABOUR FORCE 2 IN-HOUSE MANNING SCHEDULE

	TOTAL	M/C1	M/C2	M/C3
CPCT	150.			
SHDL	151.	0.0	146.0	5.3
O/TM	1.	0.0	0.0	1.3

v. Report I & J

```
      SUBROUTINE REPRTI
C           ..............................................................
C           .  REPORT I DETAILS THE FLOW OF FUNDS                       .
C           ..............................................................
      COMMON ALPHA,ALFLG,BANKL(16),BANKC(16),BANKR(16),
     1       CSH(16),CRT(16),CSHI,CRTI,     DSTORE(16),
     2       DISCP(16,2),DUMY(16,20),DNC(16,18),DLF(16,3),DCASH(16),
     3       DLO(16,3),DSR(16,10),DUMMY(20),DBTI,DBT(16),DRM(16,9),
     4       DSF(16,10),EXDBT(16),EXCRT(16),EXCSH(11,17),
     5       FCAST(1,10),GROSS(5,16),INT,KDIV(10),KNOUT(10),
     6       LIST(16,2),LF(16,3),LO(16,3),LOCAP(16,3),LFCAP(16,3),
     7       LFRED(16,3),LABAL(16,3,10), LABOT(16,3,18),MONTH(16),
     8       MO(16,16),MCRED(1,18,16),NPROD,NOUT,NHC,NLF,NSUB,NRM,NM,
     9       OVT(16),OWAGE(3),PD(10,16)
      COMMON RAWM(16,5),RMIN(5),RMLAG(5),RMB(5),
     1       SRM(16,5),SFG(16,10),STORE(16),SUBP(15),SBLAG(15),SUB(16),
     2       SLAG(10,2),SUBNO(15),SOUT(5),SPRED(16,4),SUBR(16,15),
     3       SUBCO(10,2),SALE(10,16,2),STOCKI(10),    VOL(9),
     4       WOLF(2,18),WOCAP(16,13),WC(16,10),WCRED(16,18),WIP ,
     5       WCLOAD(18)
      REAL LABAL,LABOT,LFRED
      REAL     LIST,MOREQ,INIT,LFCAP,LOCAP,CSHFL(16),SUBP    ,RMB ,MC
      INTEGER SKIP,OVT      ,MR(16,18),SUDES(15),SBLAG,SLAG,RMLAG
      INTEGER          SOUT,DUMMY,NC,SUBNC,SPRED,DES,BLANK,SUBCO,SLOUT,
     1       SCSS,GROSS,BANKR,BANKC,EXP,WIP,FG
C           ..............................................................
C           .  INITIALISE                                               .
C           ..............................................................
      DO 9 I=1,16
      DO 9 L=1,20
      DUMY(I,L)=0.0
9     DUMMY(L)=0
C           ..............................................................
C           .  THE FORMAT SECTION                                       .
C           ..............................................................
99    FORMAT(1H ,84X,'*')
100   FORMAT(1H1,'REPORTI',26X,'FLOW OF FUNDS STATEMENT FOR THE MONTH '
     $,A2,/34X,42('*')
     1 // 52X,'THIS PERIOD',22X,'*',10X,'YEAR TO DATE'/5X,80('-'),'*',
     225('-'))
101   FORMAT(1H ,60X,24('-'),'*')
102   FORMAT(1H ,13X,'OPENING DEBT',30X,'I',F11.2,11X,'I*')
103   FORMAT(1H ,13X,'OPENING CASH',30X,'I',F11.2,11X,'I*',/
     161X,'I',22X,'I*')
104   FORMAT(1H ,16X,'OPENING CREDIT',30X,'I',11X,F10.2,' I*')
105   FORMAT(1H ,15X,9('***      ***'))
107   FORMAT(1H ,21X,'BANK LOAN',20X,I10,24X,'*',I20)
108   FORMAT(1H ,8X,'RAW MATERIALS PAID FOR ',54X,'*')
109   FORMAT(1H ,I20,F40.2,24X,'*',I20)
110   FORMAT(1H ,13X,'OVERTIME PAID FOR',20X,F10.2,24X,'*',I20)
111   FORMAT(1H ,7X,'SUBCONTRACTING PAID FOR',20X,I10,24X,'*',I20)
113   FORMAT(1H ,10X,'EXTRA MODEL CASHFLOW',54X,'*')
114   FORMAT(1H ,20X,A4,I20,40X,'*')
115   FORMAT(1H ,5X,'OUT',76X,'*')
116   FORMAT(1H ,6X,'IN',76X,'*')
117   FORMAT(1H ,16X,'BANK REPAYMENT',I30,24X,'*',I20)
118   FORMAT(1H ,9X,'BANK CHARGES PAID FOR',I30,24X,'*',I20)
120   FORMAT(1H ,55X,25('-'),24X,'*  ',18('-'))
121   FORMAT(1H ,25X,'TOTAL',I30,24X,'*',I20)
122   FORMAT(1H ,14X,'EXTRA MODEL DEBT',F30.2,24X,'*')
123   FORMAT(1H ,12X,'EXTRA MODEL CREDIT',F30.2,24X,'*')
127   FORMAT(1H ,  17X,'SALE RECEIPTS',54X,'*')
128   FORMAT(1H ,28X,A4,I14,40X,'*',I10)
131   FORMAT(1H ,18X,'CLOSING DEBT',30X,'I',F11.2,11X,'I*')
132   FORMAT(1H ,16X,'CLOSING CASH',30X,'I',F11.2,11X,'I*',/
     161X,'I',22X,'I*')
```

```
130   FORMAT(1H ,18X,'CLOSING CREDIT',30X,'I',11X,F10.2,' I*')
135   FORMAT(1H ,60X,24('-'),'+', 2X,22('-'))
136   FORMAT(1H ,60X,'I',22X,'I+', 2X,'I',20X,'I')
137   FORMAT(1H ,14X,'DEBT+CASH-CREDIT',30X,'I',I15,6X,'I*', 2X,'I',I14,
     16X,'I')
200   FORMAT(1H1,'REPORTJ',40X,'CASHFLOW SUMMARY'////)
205   FORMAT(1H ,5X,121('-')/6X,'I',43X,'II',65X,'I',/
     16X,'I',15X,'RECEIPTS',23X,'II',29X,'PAYMENTS',32X,'I')
210   FORMAT(1H ,'-----',12('*'),99('-'),12('*'))
220   FORMAT(1H ,'IMNTH',
     0                 '* OPN.CASH * BNK LOAN I   SALES  I EX.MODEL II EX.M
     1ODEL I BNK REPY I BNK CHRG I  O/TIME  I  SUBCONT I  RAW MAT * CL.
     2CASH *')
230   FORMAT(1H ,'I ',A2,' *',F9.0,' *',3(F9.0,' I'),
     16('I',F9.0,1X),'*',F9.0,' *')
231   FORMAT(1H ,'I     *',9X,' *',3(9X,' I'),
     15('I',9X,1X),'*',9X,' *')
C
      DO 10 I=1,NM
C               ..........................................................
C               .  HEADING                                                .
C               ..........................................................
      WRITE(3,100) MONTH(I)
      DUMMY(2)=0
      WRITE(3,101)
C               ..........................................................
C               .  OPENING POSITION                                       .
C               ..........................................................
      IF(I.GT.1) GO TO 11
C               ..........................................................
C               .  -  FOR THE FIRST PERIOD                                .
C               ..........................................................
      WRITE(3,102) DBTI
      DUMY(I,1)=CSHI
      WRITE(3,103) CSHI
      WRITE(3,104) CRTI
      WRITE(3,101)
      DUMMY(5)=-(DBTI+CSHI-CRTI)
      GO TO 13
C               ..........................................................
C               .  -  FOR A NORMAL PERIOD                                 .
C               ..........................................................
11    CONTINUE
C      OPENING DEBT
      WRITE(3,102) DBT(I-1)
      DUMY(I,1)=CSH(I-1)
      WRITE(3,103) CSH(I-1)
      WRITE(3,104) CRT(I-1)
      WRITE(3,101)
C
13    CONTINUE
      WRITE(3,106)
C               ..........................................................
C               .  BANK LOANS                                             .
C               ..........................................................
      DUMY(I,2)=0.0
      IF(INT.EQ.0) GO TO 49
      DUMMY(1)=BANKL(I)
      DUMY(I,2)=BANKL(I)
      DUMMY(3)=DUMMY(3)+DUMMY(1)
      WRITE(3,107) DUMMY(1),DUMMY(3)
C               ..........................................................
C               .  SALES REVENUE                                          .
C               ..........................................................
49    WRITE(3,99)
      WRITE(3,127)
      DUMMY(4)=0
      DO 40 K=1,NOUT
      DUMMY(1)=0
      DO 41 J=1,NPROD
      KNO=KNOUT(J)
      DO 42 KNO1=1,KNO
      IF(SOUCO(J,KNO1).EQ.K) GO TO 43
42    CONTINUE
43    IJK=I-SEAS(J,KNO1)
      IF(IJK.LE.0) GO TO 44
      DUMMY(1)=SALE(J,IJK,KNO1)*LIST(J,KNO1)*(100-DISCP(J,KNO1))/100
     1 +DUMMY(1)
44    CONTINUE
41    CONTINUE
      DUMMY(4)=DUMMY(4)+DUMMY(1)
      DUMMY(10+KNO1)= DUMMY(10+KNO1)+DUMMY(1)
40    WRITE(3,125) SOUT(K),DUMMY(1),DUMMY(10+KNO1)
      DUMY(I,3)=DUMMY(4)
      DUMMY(13)=DUMMY(13)+DUMMY(4)
```

```
      WRITE(3,120)
      WRITE(3,121) DUMMY(4),DUMMY(13)
C           .....................................................................
C           .   EXTRA-MODEL CASHFLOW                                            .
C           .....................................................................
      WRITE(3,113)
      KEXC=EXCSH(1,1)
      WRITE(3,116)
      DO 46 IK=1,KEXC
      IF(EXCSH(IK+1,1).GT.0.0) GO TO 46
      DUMMY(1)=-EXCSH(IK+1,I+1)
      DUMMY(2)=DUMMY(2)+DUMMY(1)
      DUMY(I,4)=DUMY(I,4)-DUMMY(1)
      WRITE(3,114) EXCSH(1,IK+1),DUMMY(1)
46    CONTINUE
      WRITE(3,115)
      DO 45 IK=1,KEXC
      IF(EXCSH(IK+1,1).LT.0.0) GO TO 45
      DUMMY(1)=EXCSH(IK+1,I+1)
      DUMMY(2)=DUMMY(2)+DUMMY(1)
      DUMY(I,5)=DUMY(I,5)+DUMMY(1)
      WRITE(3,114) EXCSH(1,IK+1),DUMMY(1)
45    CONTINUE
      WRITE(3,120)
      DUMMY(10)=DUMMY(10)+DUMMY(2)
      WRITE(3,121) DUMMY(2),DUMMY(10)
      WRITE(3,99)
C           .....................................................................
C           .   BANK REPAYMENTS                                                 .
C           .....................................................................
      DUMY(I,6)=0.0
      DUMY(I,7)=0.0
      IF(INT.EQ.0) GO TO 29
      DUMMY(1)=BANKR(I)
      DUMY(I,6)=BANKR(I)
      WRITE(3,117) DUMMY(1)
      WRITE(3,99)
C           .....................................................................
C           .   BANK CHARGES                                                    .
C           .....................................................................
      DUMMY(1)=0
      II=I-ALFLG
      IF(II.LE.0) GO TO 30
      DUMMY(1)=BANKC(II)
      DUMY(I,7)=BANKC(II)
30    CONTINUE
      WRITE(3,118) DUMMY(1)
C           .....................................................................
C           .   OVERTIME WAGES                                                  .
C           .....................................................................
29    WRITE(3,99)
      DUMY(I,8)=OVT(I)
      DUMMY(8)=DUMMY(8)+OVT(I)
      WRITE(3,110) OVT(I),DUMMY(8)
      WRITE(3,99)
C           .....................................................................
C           .   SUBCONTRACTING PAYMENTS                                         .
C           .....................................................................
      DUMMY(4)=0
      IF(NSUB.EQ.0) GO TO 19
      DO 33 NSUB1=1,NSUB
      IJK=I-SLAG(NSUB1)
      IF(IJK.GT.0) GO TO 34
      DUMMY(1)=0
      GO TO 35
34    CONTINUE
      DUMMY(1)=SUBW(IJK,NSUB1)*SUBP(NSUB1)
35    CONTINUE
33    DUMMY(4)=DUMMY(4)+DUMMY(1)
      DUMMY(9)=DUMMY(9)+DUMMY(4)
      WRITE(3,111) DUMMY(4),DUMMY(9)
19    DUMY(I,9)=DUMMY(4)
C           .....................................................................
C           .   PAYMENT FOR RAW MATERIAL PURCHASES                              .
C           .....................................................................
      WRITE(3,108)
      DO 21 NRM1=1,NRM
      DMY=0.0
      IJK=I-RMLAG(NRM1)
      IF(IJK.GT.0) DMY=RAWM(IJK,NRM1)*RMB(NRM1)
      DUMY(I,10)=DUMY(I,10)+ DMY
      DUMMY(5+NRM1)=DUMMY(5+NRM1)+ DMY
21    WRITE(3,109) NRM1,DMY,DUMMY(5+NRM1)
C           .....................................................................
C           .   EXTRA-MODEL CREDIT                                              .
```

```
C     ...........................................................
      WRITE(3,99)
      WRITE(3,123) EXCRT(I)
C     ...........................................................
C     .    EXTRA-MODEL DEBT                                      .
C     ...........................................................
      WRITE(3,99)
      WRITE(3,122) EXDBT(I)
C
      WRITE(3,106)
      WRITE(3,101)
C     ...........................................................
C     .    CLOSING POSITION                                      .
C     ...........................................................
      WRITE(3,131) DBT(I)
      WRITE(3,132) CSH(I)
      WRITE(3,133) CRT(I)
C
      WRITE(3,101)
10    CONTINUE
C     ...........................................................
C     .    REPORT J GIVES A SUMMARY OF THE CASHFLOWS             .
C     ...........................................................
      WRITE(3,200)
      WRITE(3,205)
      WRITE(3,210)
      WRITE(3,220)
      WRITE(3,210)
      DO 300 I=1,NM
      WRITE(3,231)
      DUMY(I,11)=DUMY(I,1)+DUMY(I,2)+DUMY(I,3)+DUMY(I,4)
     1-DUMY(I,5)-DUMY(I,6)-DUMY(I,7)-DUMY(I,8)-DUMY(I,9)-DUMY(I,10)
      WRITE(3,230) MONTH(I),(DUMY(I,J),J=1,11)
      WRITE(3,210)
300   CONTINUE
C     ...........................................................
C     .    REPORT L IS CALLED TO GIVE A PICTORIAL SUMMARY        .
C     ...........................................................
      CALL REPRTL
      RETURN
      END
```

```
REPORTI                          FLOW OF FUNDS STATEMENT FOR THE MONTH  P6
                                 *****************************************

                                                 THIS PERIOD
-----------------------------------------------------------------------------
                                                      -----------------------
            OPENING DEBT                              I     171.11
            OPENING CASH                              I     927.56
                                                      I
            OPENING CREDIT                            I                  0.0
                                                      -----------------------
     ***     ******    ******    ******    ******    ******    ******

            SALE RECEIPTS
                       MK          539
                              -------------------------
                 TOTAL                            539
       EXTRA MODEL CASHFLOW
  IN
  OUT
                EXPN               100
                              -------------------------
                 TOTAL                            100

        OVERTIME PAID FOR                         0.00

     RAW MATERIALS PAID FOR
                1                                 0.0
                2                                 0.0

        EXTRA MODEL CREDIT                        0.0

         EXTRA MODEL DEBT                         0.0
     ***     ******    ******    ******    ******    ******    ******
                                                      -----------------------
            CLOSING DEBT                              I     671.11
            CLOSING CASH                              I    1365.56
                                                      I
            CLOSING CREDIT                            I                  0.0
                                                      -----------------------
```

REPORTJ

CASHFLOW SUMMARY

MNTH	OPN.CASH	RECEIPTS: BNK LOAN	RECEIPTS: SALES	RECEIPTS: EX.MODEL	PAYMENTS: EX.MODEL	PAYMENTS: BNK REPY	PAYMENTS: BNK CHRG	PAYMENTS: O/TIME	PAYMENTS: SUBCONT	PAYMENTS: RAW MAT	CL. CASH
P1	100.	0.	399.	0.	100.	0.	0.	1.	0.	-3.	401.
P2	498.	0.	6.	0.	100.	0.	0.	1.	0.	0.	403.
P3	402.	0.	49.	0.	100.	0.	0.	1.	0.	351.	-0.
P4	0.	0.	845.	0.	100.	0.	0.	1.	0.	119.	625.
P5	625.	0.	1058.	0.	100.	0.	0.	2.	0.	655.	926.
P6	928.	0.	539.	0.	100.	0.	0.	1.	0.	0.	1366.

vi. Report L

```
      SUBROUTINE REPRTL
C
C         THIS REPORT PRINTS OUT THE CASH PROFIE
C
      COMMON ALPHA,ALPLG,BANKL(16),BANKC(16),BANKR(16),
     1       CSH(16),CRT(16),CSHI,CRIT,     DSTORE(16),
     2       DISCP(10,2),DUMY(16,20),DWC(16,18),DLF(16,3),DCASH(16),
     3       DLO(16,3),DSR(16,10),DUMMY(20),DBTI,DBT(16),DRM(16,9),
     4       DSF(16,10),EXDBT(16),EXCRT(16),EXCSH(11,17),
     5       FCAST(1,10),GROSS(5,15),INT,KDIV(18),KNOUT(10),
     6       LIST(10,2),LF(16,3),LO(16,3),LOCAP(16,3),LFCAP(16,3),
     7       LFREQ(16,3),LABAL(16,3,18), LABOT(16,3,18),MONTH(16),
     8       MO(16,18),MOREQ(1,15,10),NPROD,NOUT,NWC,NLF,NSUB,NRM,NM,
     9       OVT(16),OWAGE(3),PD(10,16)
      COMMON RAWM(16,5),RMIN(5),RMLAG(5),RMB(5),
     1       SRM(16,5),SFG(16,10),STORE(16),SUBP(15),SBLAG(15),SUB(16),
     2       SLAG(10,2),SUBWC(15),SOUT(5),SFREQ(10,4),SUBW(16,15),
     3       SUBCD(10,2),SALE(10,15,2),STOCKI(10),   VOL(9),
     4       WCLF(2,18),WCCAP(16,18),WC(16,16),WCREQ(16,18),WIP ,
     5       WCLOAD(18)
      REAL LABAL,LABOT,LFREQ
      REAL     LIST,MOREQ,INIT,LFCAP,LOCAP,CSHFL(16),SUBP    ,RMB ,MO
      INTEGER SKIP,OVT      ,RR(16,13),SBDES(15),SBLAG,SLAG,RMLAG
      INTEGER              SOUT,DUMMY,WC,SUBWC,SFREQ,DES,BLANK,SUBCD,SLOUT,
     1        SUSS,GROSS,BANKR,BANKC,FXP,WIP,FG
      REAL ISMAX,ISMIN
      INTEGER BLNK1,BLNK4,MID(20),CHANGE(20)
      DATA MOTTO/'CASH'/,BLNK4/'    '/,OPN/'OP'/,BLNK1/' '/,IYE/'I'/
C                 ..............................................................
C                 .  INITIALISE                                                .
C                 ..............................................................
      II=NM+1
      XMAX=0.0
      DO 5 I=1,II
      CHANGE(I)=0
      DUMMY(I)=BLNK4
5     MID(I)=BLNK1
C                 ..............................................................
C                 .  CASHFLOW                                                  .
C                 ..............................................................
         DUMY(1,1)=CSHI/1000
      DO 10 I=1,NM
10    DUMY(1,I+1)=CSH(I)/1000
      WRITE(3,99)
C                 ..............................................................
C                 .  DETERMINE THE SCALE                                       .
C                 ..............................................................
      DO 20 I=1,II
      IF(DUMY(1,I).LE.XMAX) GO TO 20
      XMAX=DUMY(1,I)
20    CONTINUE
      MAX=XMAX+0.5
      ISCALE=1
      IF(ISCALE.GE.MAX) GO TO 31
      DO 32 ISCALE=5,100,5
      IF(ISCALE.GE.MAX) GO TO 31
32    CONTINUE
      DO 30 ISCALE=250,2000,250
      IF(ISCALE.GE.MAX) GO TO 31
30    CONTINUE
31    CONTINUE
C                 ..............................................................
C                 .  PLOT THE GRAPH LINE BY LINE                               .
C                 ..............................................................
      DO 50 IC=1,51
      ISMAX=(52-IC)*ISCALE/50
      ISMIN=(51-IC)*ISCALE/50
      DO 51 I=1,II
      IF((DUMY(1,I).GE.ISMAX).OR.(DUMY(1,I).LT.ISMIN)) GO TO 51
      DUMMY(I)=MOTTO
      IF(MID(I).EQ.IYE)   CHANGE(I)=1
      MID(I)=IYE
      IF(I.EQ.1) GO TO 51
      IF(MID(I-1).EQ.IYE)   CHANGE(I-1)=1
      MID(I-1)=IYE
51    CONTINUE
      IF(((IC-1)-5.*((IC-1)/5)).EQ.0.0) GO TO 55
      WRITE(3,100) (DUMMY(I),MID(I),I=1,II)
      GO TO 60
55    ISCL=(ISCALE*(51-IC))*20
      WRITE(3,101)ISCL,(DUMMY(I),MID(I),I=1,II)
60    DO 61 I=1,II
      IF(CHANGE(I).NE.1) GO TO 61
      MID(I)=BLNK1
      CHANGE(I)=0
61    CONTINUE
50    CONTINUE
C                 ..............................................................
C                 .  MARK THE TIME SCALE                                       .
C                 ..............................................................
      WRITE(3,110) OPN,(MONTH(I),I=1,NM)
C                 ..............................................................
C                 .  THE FORMAT SECTION                                        .
C                 ..............................................................
C
4091  FORMAT(1H , 6F15.3)
99    FORMAT(1H1,'REPORT L',30X,'CASH PROFILE'/39X,12('-')////)
100   FORMAT(1H ,20X,'I',20(A4,A1))
101   FORMAT(1H ,I19,' I',20(A4,A1))
110   FORMAT(1H ,16X,85('-')/22X,20(1X,A2,' ;'))
4091  FORMAT(15A2)
4096  FORMAT(6F10.6)
      RETURN
      END
```

REPORT L

CASH PROFILE

```
5000 I
     I
     I
     I
     I
4500 I
     I
     I
     I
     I
4000 I
     I
     I
     I
     I
3500 I
     I
     I
     I
     I
3000 I
     I
     I
     I
     I
2500 I
     I
     I
     I
     I
2000 I
     I
     I
     I
     I
1500 I
     I
     I                             ICASHI
     I                             ICASHI
     I                             ICASHI
1000 I                             ICASHI
     I                        ICASHICASHI
     I                        ICASH CASHI
     I                        ICASH CASHI
     I                   ICASHICASH CASHI
 500 I                   ICASH CASH CASHI
     I    ICASHICASHI    ICASH CASH CASHI
     I    ICASH CASHI    ICASH CASH CASHI
     I    ICASH CASHI    ICASH CASH CASHI
     ICASHICASH CASHI    ICASH CASH CASHI
   0 ICASH CASH CASHICASHICASH CASH CASHI
----------------------------------------------------
       OP \ P1 \ P2 \ P3 \ P4 \ P5 \ P6 \
```

APPENDIX B

THE LONG-TERM CORPORATE MODEL

The model described in Appendix A assumes that the company will continue to operate within the framework laid down by capacity constraints, product mix, and current purchasing patterns. In the longer term all these factors may themselves be changed by management. In this Appendix we introduce modifications to the previous model to incorporate these activities.

1.1 Data Definition

budget(I)	Limitation on amount of capital expenditure in period I.
cost (K,I)	The capital outlay required to service project K in period I after initiation of the project.
excapm(K,I,M)	The amount of additional capacity of work centre M that results from project K, I periods after project initiation. This allows a finite lifespan to be accomodated. Positive entries refer to additions in capacity; negative entries for reductions in capacity.
excapf(K,I,L)	Additional capacity for labour group L resulting from project K, I periods after project initiation.
nproj	No. of different projects considered

1.2 Variable Definition

COST(I)	The capital spent on all projects in period I of the planning period.
EXCAPM(I,M)	The total addition to normal-time capacity of workcentre M in period I of the planning period.
EXCAPF(I,L)	The total addition to normal time capacity of labour group L in period I.
PROJ(K,I)	Project K initiated in period I of the planning period. The life span of the project is defined from the point of initiation in order to take account of possible ordering and planning delays before the first physical or financial activity is effected. This variable is integral.

2. The Model Equations

Three different additions to the basic corporate model may be adopted to model extension of the product range; extension of existing facilities and an extension of 'vertical integration' (namely in-house manufacture of some raw materials currently bought-in). Each of these facets may be considered in isolation - encorporation in the model may allow the relevant decisions to be made in the framework of all the other activities that are simultaneously selected.

2.1 Extension of the Product Range

This is done simply by extending nprod and the SALES (I,K,J) vector to take account of the potential new products that may be manufactured by the company. (This will have to be accompanied by the appropriate 'mcreq' and 'rmreq' data).

2.2 Extension of Production Facilities

(i) Work-Centre Capacity (cf Eqn. 211)

Let the left hand side of Eqn. A.1 (the production requirement to be worked in normal time) be represented by MCAP(I,M). Then Equation A.1. would read

$$MCAP(I,M) \leq capwc(I,M)$$

The revised equation is

$$MCAP(I,M) - EXCAPM(I,M) \leq capwc(I,M) \qquad (B.1)$$

(ii) Labour Force Allocation

It is conceivable that management may wish to increase (or decrease) the labour force. This decrease shall be taken in recognition of the fact that the current labour force availability (caplf(I,L)) is not constant - due to retirements and staff turnover - and against a requirement for manpower to accomplish a production programme in order to satisfy the sales demand.

Let the left-hand side of Equation A.4 (the labour force capacity constraint) be LCAP(I,L). The equation reads

$$LCAP(I,L) \leq caplf(I,L)$$

The revised equation is

$$LCAP(I,L) - EXCAPF(I,L) \leq capf(I,L) \qquad (B.2)$$

Note that the overtime capacity may be linked to the normal time labour capacity. If the overtime is some fraction of such normal time capacity (i.e. is linked to the number of men working) then the Labour Overtime Capacity equation (A.5) would also require reformulation.

(iii) Capacity Expansion (cf. Eqn. 2.12)

An increase to capacity of (for example) work-centre M in period I can come about through the possible adoption of a number of projects in previous periods. This equation sums the cumulative effect of undertaking these projects.

$$\text{EXCAPM}(I,M) = \sum_{K=1}^{nproj} \sum_{\hat{I}=0}^{I-1} \text{PROJ}(K,I-\hat{I}).\text{excapm}(K,\hat{I},M) \qquad (B.3.)$$

To take account of the increase in labour capacity substitute EXCAPL(I,L) and excapf(K,I,L) appropriately in the above equation.

(iv) Cost (cf. Eqn. 2.13)

Projects undertaken in any period may require financial servicing in suceeding periods

$$\text{COST}(I) = \sum_{K=1}^{nproj} \sum_{\hat{I}=0}^{I-1} \text{PROJ}(K,I-\hat{I}).\text{cost}(K,\hat{I}) \qquad (B.4)$$

COST(I) is now included as an outflow of cash in the Cash Position Equation (Eqn. 1.17, A.8)

(v) Budget Constraint

Many companies operate in an environment of capital budgeting. This may be a total constraint on spending in any period.

i.e. COST (I) $\leq$ budget (I) (B.5)

or it may be a constraint imposed by the company on spending on new projects

$$\text{i.e.} \quad \sum_{K=1}^{nproj} PROJ(K,I).cost(K,1) \leq budget(I) \qquad (B.6)$$

2.3 Vertical Integration

(i) Substitute Production (cf. Eqn. 2.14)

In order to consider the possibility of making in-house components presently bought-in, these components must be considered as separate, identifiable, raw materials. Extend the vector of products to include 'pseudo' products corresponding to these raw materials. The only difference between these 'psuedo' products and the others is that no sales outlets will exist for them: in all other respects similar data definition (e.g. mcreq and rmreq) will be required. There is no need to restrict the work-centre requirement to those work-centres available: if the manufacture of a certain 'pseudo' product requires work done on a work-centre not currently owned, then the programme will have to consider, simultaneously, the asquisition of such work-centre capacity!)

The new in-house production is now considered as an extra source of the particular raw material. Let Equation A.12 be expressed as

STOCKR(I,R) = STOCKR(I-1,R) +RMB(I,R) - RUSE (I,R)

where RUSE (I,R) is the amount of raw material R used in production in period I.

The revised quotation is

STOCKR(I,R) = STOCKR(I-1,R) +RMB(I,R) +PROD(R,I)-RUSE(I,R) (B.7)

where PROD(R,I) is the amount production of 'pseudo' product corresponding to material R completed in peirod I.

(ii) Cost

Clearly the cost (Eqn. B.4) and budgeting (Eqn.B.5 or B.6) constraints still apply.

(iii) Irrevocability (cf. Eqn. 2.15)

In the event that a supplier will not accept intermittent ordering or management impose the constraint that the decision to extend the amount of in-house production be made once and for all (to take account of the once off (labour) cost of getting such a project under way) the irrevocability constraint may be modelled as follows:

$$RMB(I,R) + M \sum_{K=1}^{nproj} \sum_{I=1}^{I} PROJ\ (K_R, I) \leq M \quad \forall R \qquad (B.8)$$

Where M is a large multiplier, and PROJ (K_R, I) is a project to produce raw material R in-house undertaken in period I.

3. The Model in Use

This model formulation has been transfered to a computer model by WILSON (147) who experienced a number of difficulties in getting the model to run. One of the problems he encountered was simply that large mixed integer programmes (the number of projects is clearly an integral variable!) are difficult to solve.

Clearly more work needs to be done on the model formulation to resolve a number of remaining problems. These centre on firstly how to deal with projects that extend beyond the planning horizon - will the introduction of unequal time periods be sufficient to cater for the problem or will some post horizon terminal value ascribed to each project have to be included in the objective function - and secondly on the definition of a long term profit maximisation objective function. The definition of profit in the short term model was the difference in the sum of cash plus debtors less creditors positions at the horizon from the position at the beginning of the planning period. This will not do for the long term case since the physical environment is not held constant! Following our definition of profit in Chapter 3.1.ii we will define a profit maximisation objective as the change in the value of the "Shareholders Account" over the planning period. But this in turn requires that we ascribe prices to the assets and liabilities in the balance sheets, which to be consistent with the methodology, should be linked to the dual variables associated with the respective constraints. However, in Chapter 6.3.3.ii, we show that the dual variables themselves form an arbitrary pricing mechanism. This dilema will require resolution before the model can become a useful tool for long-term strategic planning.

APPENDIX C

MATHEMATICAL FOUNDATIONS

This Appendix may be split into 2 sections. The first is concerned with definitions of terms and the second with establishing some basic theorems in Linear Algebra and the geometry of cones.

1. Definitions

1.1 Vector Space. (108)

Let K be some given field of scalars, and V be a non-empty set such that for any $\underline{u},\underline{v}$ elements of V the sum $\underline{u} + \underline{v}$ is also an element of V, and for any $\underline{u} \in V$, $\lambda \in K$ the product $\lambda\underline{u} \in V$.

V is termed the vector space over K. The following axioms hold:

i. $\underline{u} + \underline{v} = \underline{v} + \underline{u}$

ii. $(\underline{u} + \underline{v}) + \underline{w} = \underline{u} + (\underline{v} + \underline{w})$

iii. $\underline{u} + 0 = \underline{u}$

iv. $\alpha(\underline{u} + \underline{v}) = \alpha\underline{u} + \alpha\underline{v}$

v. $(\alpha\beta)\underline{u} = \alpha(\beta\underline{u})$

vi. $(\alpha + \beta)\underline{u} = \alpha\underline{u} + \beta\underline{u}$

vii. $1.\underline{u} = \underline{u}$

1.2 Subspace of a Vector Space. (123)

A subset W of a vector space V is a subspace of V if W is itself a vector space with the binary operations defined in V. i.e.

if $\underline{x},\underline{y} \in W$ then $\underline{x} + \underline{y} \in W$

$\underline{x} \in W$ then $\alpha\underline{x} \in W$, and $\underline{\emptyset} \in W$

1.3 Coset (123)

If W is a subspace of V and $\underline{X}$ is a vector in V then the sum W + [$\underline{x}$] is termed a coset of W.

A coset is not a subspace of V unless $\underline{x}$ is an element of W.

1.4 Dependancy (115)

The vector $\underline{y}$ is linearly dependant on a subset X of V if there are points $\underline{x}_1$, $\underline{x}_2$,.........$\underline{x}_r$ of X and (real) scalars $\lambda_1,\ldots,\lambda_r$ such that

$$\underline{y} = \lambda_1\underline{x}_1 + \ldots\ldots\ldots + \lambda_r\underline{x}_r$$

$\underline{y}$ is affinely dependant on X if in addition

$$\lambda_1 + \ldots\ldots + \lambda_r = 1$$

$\underline{y}$ is convexly dependant if

$$\lambda_i \geq 0 \quad \forall\, i \quad \text{also holds.}$$

1.5 Convex Sets. (60)

A set M is termed convex if M contains every segment joining a pair of points from M.

$$\text{if } \underline{x},\underline{y} \in M \text{ then } \lambda\underline{x} + (1-\lambda)\underline{y} \ ; \ 0 \leq \lambda \leq 1 \quad \in M$$

1.6 Cone (60)

A set C is termed a cone if $\underline{\emptyset}$ is in C and for all $\underline{x} \in C$ $\underline{x} \in C$ for all non-negative scalars .

1.7 Convex Cone (115)

A subset of V, closed under the operations of taking non-negative linear combinations is called a convex cone.

For $\underline{x}_1, \ldots\ldots, \underline{x}_r \in C$

$$\underline{x} = \lambda_1 \underline{x}_1 + \ldots. + \lambda_r \underline{x}_r \; ; \quad \lambda_i \geq 0 \quad \forall\, i$$

If $\underline{x} \in C$ then C is a convex cone.

1.8 Convex Hull. (77)

The 'smallest' convex set which contains A is called the convex hull of A.

A more precise definition in (115) is that the convex hull of a set A is the set of all convex combinations of (finite) subsets of A.

1.9 Convex Polytope (115)

The convex hull of a finite set of points is termed a convex polytope.

1.10 Basis (123)

A basis B of a vector space V is a linearly spanning set for V.

$$\forall\, \underline{x} \text{ in } V \qquad \underline{x} = \sum_{e_i \in B} \lambda_i e_i$$

1.11 Dimension (123)

A vector space is said to be 'finite-dimensional' if it has a finite spanning set.

2. Basic Results

2.1 Convex Sets.

THEOREM C.1. If L is a family of convex sets in a real vector space E^n then the intersection of the sets is also convex.

Proof: Suppose $x_i, x_2 \in \cap(L)$ so that $x_1, x_2 \in C$ for all $C \in L$. Since C is a convex set $\lambda x_1 + (1-\lambda)x_2 \in C$ (for $0 \leq \lambda \leq 1$) for all C in L. Therefore $\lambda x_1 + (1-\lambda)x_2 \in \cap(L)$.

Interior and Boundary Points. (77)

An e neighbourhood about a point a is defined as the set of points inside a hypersphere with centre at a and radius $\epsilon > 0$.

$$N = (\underline{x} / |\underline{x}-\underline{a}| < \epsilon ; \quad \epsilon > 0)$$

A point a is an interior point of a set A if there exists an e neighbourhood about the point which contains only points of the set A.

A point a is a boundary points of a set A if every e neighbourhood about that point (regardless of how small $\epsilon > 0$ maybe) contains points which are in the set and points which are not in the set.

A set is closed if it contains all its boundary points; open if it contains only interior points.

Support Properties. (115)

A hyperplane divides a real vector space E^n into two half-spaces:

Let the hyperplane be $H = (\underline{x} \in E^n / \underline{a}.\underline{x} = \alpha)$ $(\underline{a} \neq 0)$

The set of points lying on, or to one side of, the hyperplane is called a closed half-space.

$$H' = (\underline{x} \in E^n / \underline{a}.\underline{x} \lesseqgtr \alpha)$$

If the inequalities are replaced by strict inequalities, (i.e. removing the boundary points from the set), the set is termed an open half-space.

The vectors $\underline{a}$ and $-\underline{a}$ are the normals to the hyperplane, pointing in either direction. Conventionally, with α positive and the half-space defined as

$\underline{a}.\underline{x} \geqslant \alpha$ the $\underline{a}$ is the inward normal

i.e. it points into the defined half-space.

Consider a closed, bounded convex set K in E^n . A hyperplane H is said to support K if $H \cap K \neq \emptyset$ and K is contained in one of the closed half-spaces defined by H. H is termed a supporting hyperplane of K.

THEOREM C.2. Given any closed convex set K, a point $\underline{y}$ either belongs to the set or there exists a hyperplane H which contains $\underline{y}$ such that all of K is contained in one open half-space formed by H.

Proof: (77) Assuming that $\underline{y}$ is not a member of K find the point $\underline{w}$ in K that is closest to $\underline{y}$ i.e. $|\underline{w}-\underline{y}| = \min\ ]\underline{u}-\underline{y}[\quad \forall \underline{u} \in K$ (note that only one such point exists, since the line joining two points is included in K, and by the triangle inequality the midpoint would be closer to y than either of the two.) Constructing a hyperplane passing through y with the vector joining w to y as the normal, it can be shown that the entire set K lies in one open halfspace.

THEOREM C.3. (115) Every closed bounded convex set K in E^n is the intersection of all its supporting half-space.

THEOREM C.4. (115) Through every point x on the boundary of a closed convex set K passes a supporting hyperplane H.

2.2 Convex Cones. (60)

The dimension of a convex cone is defined as follows:

The smallest subspace S(C) of the vector space E^n which contains the cone C (i.e. the intersection of all subspaces containing C) has dimension d(C). This is termed the linear dimension of the cone C.

A ray (or vector) x of a convex cone C is an extreme ray of C if x is not a positive linear combination of any two linearly independant rays of C.

Clearly any ray which is the only ray in the intersection of a supporting hyperplane and a convex closed cone is necessarily an extreme ray.

THEOREM C.5. If $\underline{x}$ is an interior ray of a convex cone C and $\underline{y}$ is a boundary of interior ray of C then every ray ($\alpha\underline{x} + \beta\underline{y}$) with α, β positive scalars, is an interior ray of the cone.

A cone is termed polyhedral if it is the convex hull of a finite number of rays. The polar of a polyhedral cone is the intersection of a finite number of half-spaces.

2.3 Convex Polytopes. (115)

If H is a supporting hyperplane of a closed bounded (n dimenstional) convex set K then $H \cap K$ is called a face of K.

Every face F of K is convex and $0 \leq \dim F \leq n-1$.
The 0-faces of K are called vertices, the 1-faces are edges, and the n-1 faces are termed facets.

THEOREM C.6. A polytope has only a finite number of distinct faces, and each face is a convex polytope.

THEOREM C.7. A convex polytope is the convex hull of its set of vertices.

It follows that the vertex vector of a convex polytope P is a vector of P which is not a convex combination of the other vectors in P. (123)

THEOREM C.8. If a n-polytope in E^n has d facets, then it can be written as the intersection of d closed half-spaces.

This follows from the fact that a convex polytope is a bounded polyhedral set.

THEOREM C.9. Let F_1 be a face of a polytope P and F_2 be a face of the polytope F_1. Then F_2 is a face of P.

THEOREM C.10. Let $F_1, F_2, \ldots F_r$ be a family of faces of a closed bounded convex set K. Then $\bigcap_{i=1}^{r} F_i$ is also a face of K. (For this purpose the null set is considered a face, but is clearly of little interest.)

Special Polytopes:

Simplicies. If V is any set of n+1 affinely independant points in E^n, then S^n = conv V is called an n-simplex.

THEOREM C.11. Every k-face ($0 \leq k \leq n-1$) of a n-simplex S^n is a k-simplex, and every k+1 vertices of S^n are the vertices of a k-face of S^n.

Pyramids. A n-pyramid P is the convex hull of a (n-1)-polytope Q (termed the basis of P) and a point $\underline{x}$ which is not a member of the affine hull of Q. ($\underline{x}$ is termed the apex of P)

2.4 Basis (77)

THEOREM C.12. The unit ($\underline{e}$) vectors form a basis for E^n.

Proof: The set of unit vectors is linearly independant since

$$\lambda_1\underline{e}_1 + \ldots\ldots\lambda_n\underline{e}_n = (\lambda_1,\ldots\ldots\lambda_n) = \underline{\emptyset}$$

implies that $\lambda_1 = 0,\ldots,\lambda_n = 0$.

Any vector $\underline{x}$ can be written in terms of the unit vectors as

$$\underline{x} = x_1\underline{e}_1 + x_2\underline{e}_1 + x_2\underline{e}_2 + \ldots\ldots\ldots x_n\underline{e}_n$$

with the scalars x_i equal to the components of $\underline{x}$ in each dimension. Hence the set of unit vectors form a basis.

THEOREM C.13. The representation of any vector in terms of a set of basic vectors is unique.

Proof: Assume the contrary. Then

$$\underline{x} = \lambda_1\underline{a}_1 + \ldots. + \lambda_n\underline{a}_n$$

and $\underline{x} = \pi_1\underline{a}_1 + \ldots. + \pi_n\underline{a}_n$

Then $(\lambda_1 - \pi_1)\underline{a}_1 + \ldots\ldots + (\lambda_n - \pi_n)\underline{a}_n = \emptyset$

Since the basic vectors are independant $(\lambda_i - \pi_i)$ must be zero for all i.

THEOREM C.14. Any two bases for E^n have the same number of basic vectors.

It follows that a basis has precisely n members (from Theorem C.12), and that any set of n linearly independant vectors from E^n will form a basis. Furthermore, any n+1 or more vectors are linearly dependant.

APPENDIX D

EXAMPLES OF RESULTS

1. Introduction

In this Appendix we present some results from test runs of the corporate model formulated in Chapter 1. To preserve the anonimity of the test-case company the results shown here arise from a constructed example where a mythical company sells two products through a single marketing outlet. The production process coverts two raw materials through three work-centres, that have no sub-contracting facilities, manned by two different labour groups. The company has no facility to borrow or to lend funds, and the planning horizon, over which the model is run, is six periods.

Section 2 of this Appendix details the procedures and programmes used to construct the alternate dual space from the optimal solution of the model. In Section 3 we discuss the attempts made to resolve this space.

2. Generation of the Alternate Dual Space.

```
            PROGRAM('ND')
  *             ..................................................
  *             .  THIS PROGRAMME CONVERTS THE LP MODEL INTO A   .
  *             .  TOTALLY DEGENERATE SYSTEM                     .
  *             ..................................................
            INITIALZ
  *             ..................................................
  *             .  RUN THE MATRIX GENERATOR                      .
  *             ..................................................
            FREECORE
            MGP
  *             ..................................................
  *             .  CONVERT THE DATA AND RUN THE LP               .
  *             ..................................................
            MOVE(XDATA,'PHDEXMPL')
            MOVE(XPBNAME,'EXMPL')
            CONVERT('FILE','FT08F001')
            SETUP('MAX','RANGES','RANGES','BOUNDS','POLICY','SCALE')
            MOVE(XOBJ,'P6LA1')
            MOVE(XRHS,'RHS')
            CRASH
            PRIMAL
  *             ..................................................
  *             .  STORE ALL THE INFORMATION                     .
  *             ..................................................
            SAVE('NAME','ORIGIN')
            BCDOUT  ('PUNCH','FILE','FT07F001','ONE','NOLIST','BCD')
            SOLUTION('FILE','FT09F001',                              X
               'RSECTION','1/2/3/4D/5D/6D/7C/8D',                    X
               'CSECTION','1/2/3/4D/5D/6D/7C/8D')
  *             ..................................................
  *             .  THE SOLUTION IS LIKELY TO BE PRIMAL ALTERNATE, .
  *             .  DEGENERATE                                    .
  *             .  FIRST - IDENTIFY THE ALTERNATE HYPERPLANE BY FIXING .
  *             .  THE SOLUTION, THEN TRYING TO MOVE AWAY        .
  *             ..................................................
            FREECORE
            ALT
            MOVE(XDATA,'OHYES')
            MOVE(XOBJ,'JECTIVE')
            MOVE(XRHS,'RHS')
            MOVE(XOLDNAME,'EXMPL')
            MOVE(XPBNAME,'EXMPL2')
            REVISE('FILE','FT11F001')
            SETUP('MAX','RANGES','RANGES','BOUNDS','POLICY','SCALE')
            RESTORE
            PRIMAL
            SOLUTION('FILE','FT09F001',                              X
               'RSECTION','1/2/3/4D/5D/6D/7C/8D',                    X
               'CSECTION','1/2/3/4D/5D/6D/7C/8D')
  *             ..................................................
  *             .  THE ALTERNATE HYPERPLANE HAS NON-ZERO DUALS   .
  *             .  SECOND - CHOOSE THE CENTRE POINT TO MAXIMISE THE .
  *             .  INSENSITIVITY OF SOLUTION TO VARIATIONS IN THE RHS .
```

```
      *                 ..........................................................
                     FREECORE
                     CENTALTN
                     MOVE(XDATA,'CENTREAL')
                     MOVE(XOLDNAME,'EXMPL')
                     MOVE(XPBNAME,'EXMPL2')
                     MOVE(XOBJ,'JECTIVE')
                     MOVE(XRHS,'RHS')
                     REVISE('FILE','FT11F001')
                     SETUP('MAX','RANGES','RANGES','BOUNDS','POLICY','SCALE')
                     RESTORE('NAME','ORIGIN')
                     PRIMAL
                     SOLUTION('FILE','FT09F001',                                X
                      'RSECTION','1/2/3/4D/5D/6D/7D/8D',                        X
                        'CSECTION','1/2/3/4D/5D/6D/7D/8D')
      *                 ..........................................................
      *                 .  THIRD - IMPOSE THIS SOLUTION ON THE ORIGINAL MODEL    .
      *                 ..........................................................
                     FREECORE
                     FIXIT
                     MOVE(XOBJ,'EARNINGS')
                     MOVE(XDATA,'NOWDEGEN')
                     MOVE(XRHS,'RHS')
                     MOVE(XOLDNAME,'EXMPL2')
                     MOVE(XPBNAME,'EXMPL3')
                     REVISE('FILE','FT11F001')
                     SETUP('MAX','RANGES','RANGES','BOUNDS','POLICY','SCALE')
                     RESTORE('NAME','ORIGIN')
                     PRIMAL
                     SAVE('NAME','FIRST')
                     SOLUTION('FILE','FT09F001',                                X
                      'RSECTION','1/2/3/4D/5D/6D/7D/8D',                        X
                        'CSECTION','1/2/3/4D/5D/6D/7D/8D')
      *                 ..........................................................
      *                 .  FOURTH - MAKE THE SYSTEM TOTALLY DEGENERATE BY        .
      *                 .  COLLAPSING THE MODEL ON THIS NEW SOLUTION             .
      *                 ..........................................................
                     FREECORE
                     DEGENTOT
                     MOVE(XOBJ,'P6EA1')
                     MOVE(XDATA,'NOWDEGEN')
                     MOVE(XRHS,'RHS')
                     MOVE(XOLDNAME,'EXMPL')
                     MOVE(XPBNAME,'EXMPL3')
                     REVISE('FILE','FT11F001')
                     SETUP('MAX','RANGES','RANGES','BOUNDS','POLICY','SCALE')
                     RESTORE('NAME','ORIGIN')
                     PRIMAL
                     SOLUTION('FILE','FT10F001')
                     SOLUTION
                     SOLUTION('FILE','FT09F001',                                X
                      'RSECTION','1/2/3/4D/5D/6D/7D/8D',                        X
                        'CSECTION','1/2/3/4D/5D/6D/7D/8D')
                     PICTURE
```

```
*          .............................................................
*          .  TO AID SELECTION IN THE ALTERNATE DUAL SPACE             .
*          .  CONSTRUCT THE DUAL PROBLEM EXPLICITLY                    .
*          .............................................................
       FREECORE
       TRANS
       MOVE(XDATA,'NEWMATRX')
       MOVE(XPBNAME,'EXMPL4')
       CONVERT('FILE','FT08F001')
       MOVE(XRHS,'NEWRHS')
       SETUP('MAX','BOUNDS','POLICY','SCALE')
       MOVE(XDATA,'PROFIT')
       MOVE(XOBJ,'NGOALS')
       MOVE(XPBNAME,'EXMPL4')
       MOVE(XOLDNAME,'EXMPL4')
       REVISE
       SETUP('MAX','BOUNDS','POLICY','SCALE')
       PICTURE
       MOVE(XDATA,'REDUCE')
       MOVE(XPBNAME,'EXMPL4')
       MOVE(XOLDNAME,'EXMPL4')
       REVISE
       SETUP('MIN','BOUNDS','POLICY','SCALE')
       MOVE(XDATA,'NEWPRB')
       MOVE(XPBNAME,'EXMPL4')
       MOVE(XOLDNAME,'EXMPL4')
       MOVE(XOBJ,'UNDER')
       REVISE
       SETUP('MIN','BOUNDS','POLICY','SCALE')
       PICTURE('RLIMIT','NEWOBJ',' ')
       PRIMAL
       SOLUTION
       EXIT
       PEND
```

i. ALT - see line 129.

```
//UMTS118H JOB %0120,PMT5,0.59,19CC,: EILON-COURIER:
//  EXEC FORTMOD,PARM.C!:BCD,NOSOURCE:,PARM.L!:LIST,XREF:
//C.SYSIN DD *
C
C                  ..........................................................
C                  .   ALT IDENTIFIES THE ALTERNATE HYPERPLANE              .
C                  .         - FIX THE OLD SOLUTION, THEN TRY TO MOVE AWAY  .
C                  ..........................................................
      INTEGER FILE1,FILE2,ROWS(200,4)
      REAL*8 COLNM(8),VALUES(8),ENDSEC,NAME,COLUM,COLUMN,RHS
      REAL*8 OBJ,XVAL
      INTEGER DUAL,ULIM,LLIM,SLACK,ACTIV,VNAME
      REAL UL,LL,MINUS
      INTEGER FILE
      DATA OBJ/'P6EA1    '/
      DATA ENDSEC/'$ENDSEC$'/,COLUMN/'        '/
      ONE=1.0
      MINUS=-1.0
      XVAL=9999.999
      UL=100000.0
      LL=-100000.0
      ACTIV=1
      LLIM=3
      ULIM=4
      DUAL=5
      VNAME=8
C
C                  ..........................................................
C                  .   THE SOLUTION IS STORED ON FILE2                      .
C                  .   THE NEW REVISE FILE IS WRITTEN ON LIST               .
C                  ..........................................................
      FILE2=9
      LIST=11
C
C                  ..........................................................
C                  .   PREPARE THE FILES                                    .
C                  ..........................................................
      REWIND LIST

      REWIND FILE2
      READ(FILE2) NAME
10    READ(FILE2) NAME
      IF(NAME.NE.ENDSEC) GO TO 10
      READ(FILE2) NAME,NOCOL
      READ(FILE2) (COLNM(IN),IN=1,NOCOL)
      READ(FILE2)
C
C                  ..........................................................
C                  .   NEW OBJECTIVE FUNCTION WILL MAXIMISE LAMDA           .
C                  ..........................................................
      WRITE(LIST,200) OBJ
      WRITE(LIST,205)
      WRITE(LIST,210) ONE
C
C                  ..........................................................
C                  .   ADD SLACK/SURPLUS VARIABLE LAMDA TO EACH CONSTRAINT  .
C                  ..........................................................
50    READ(FILE2) (VALUES(IN),IN=1,NOCOL)
      IF(VALUES(1).EQ.ENDSEC) GO TO 100
      IF(VALUES(LLIM).EQ.VALUES(ULIM))GO TO 50
      IF((VALUES(LLIM).LT.LL).AND.(VALUES(ULIM).GT.UL))  GO TO 60
      IF(VALUES(ULIM).LT.UL)  WRITE(LIST,215) VALUES(VNAME),ONE
      IF((VALUES(LLIM).GT.LL).AND.(VALUES(ULIM).GT.UL))
     1          WRITE(LIST,215) VALUES(VNAME),MINUS
      GO TO 50
60    IF(VALUES(VNAME).NE.OBJ)  GO TO 50
      XVAL=VALUES(ACTIV)
      GO TO 50
C
C                  ..........................................................
C                  .   ENSURE THAT PROGRAMME YIELDS THE SAME VALUE FOR THE  .
C                  .   OLD OBJECTIVE FUNCTION                               .
C                  ..........................................................
100   CONTINUE
      WRITE(LIST,220) OBJ,XVAL
      WRITE(LIST,290)
      END FILE LIST
      REWIND LIST
      REWIND FILE2
      STOP
C
C                      THE FORMAT SECTION                                   .
C                  ..........................................................
200   FORMAT('NAME',10X,'ONYES    '/'ROWS'/'  MODIFY'/' E   ',A8)
205   FORMAT('  AFTER'/' N  JECTIVE ')
210   FORMAT('COLUMNS'/'  AFTER'/4X,'LAMDA     JECTIVE ',F14.4)
215   FORMAT(4X,'LAMDA    ',2X,A8,F14.4)
220   FORMAT('RHS'/'  MODIFY'/4X,'RHS      ',2X,A8,F14.4)
290   FORMAT('ENDATA')
      END
/*
//*
//*              ALT          IS STORED ON AREA UMTMI18A ON DISC 4C002
//*
//L.SYSIN DD *
 ENTRY MAIN
 NAME ALT
```

ii. CENTALTN - see line 146.

```
//UMTS1180 JOB %0120,RMT9,0.30,9CC,: EILON-COURIER:
//  EXEC FORTMOD,PARM.C!:BCD,NOSOURCE:,PARM.L!:LIST,XREF:
//C.SYSIN DD *
C               ...........................................................
C               .  CENTALTN CHOOSES THAT POINT IN THE ALTERNATE PRIMAL    .
C               .  SPACE THAT MINIMISES THE SENSITIVITY OF THE SOLUTION   .
C               .  TO VARIABILITY IN RESOURCE AVAILABILITY                .
C               .      - THE CHOSEN POINT WILL MAXIMISE THE MINIMUM       .
C               .  DISTANCE FROM ALL NON-ALTERNATE HYPERPLANES            .
C               .  (DISTANCE EXPRESSED AS PRCNT OF RHS FOR COMPRABILITY)  .
C               ...........................................................
      INTEGER ROW,ENDATA
      REAL COEFF(200,2)
      INTEGER GT,LT,STORE,N,E,REVROW(200,3),ROW1,ROW2
      INTEGER FILE1,FILE2,ROWS(200,4),FILE3
      REAL*8 COLNM(8),VALUES(9),ENDSEC,NAME,COLUM,COLUMN,RHS
      REAL*8 XCOL,COL,OVALU,XVALU,OBJ,VALUE
      INTEGER DUAL,ULIM,LLIM,SLACK,ACTIV,VNAME
      REAL UL,LL
      DATA COEFF/400*0.0/
      DATA GT/' G'/,LT/' L'/,E/' E'/,N/' N'/
      DATA XCOL/'********'/
      DATA BLK/'        '/
      DATA ENDSEC/'$ENDSEC$'/,COLUMN/'        '/
      ONE=1.0
      ZERO=0.0
      UL=100000.0
      LL=-100000.0
      I=0
      J=0
      L=1
      ACTIV=1
      LLIM=3
      ULIM=4
      DUAL=5
      VNAME=8
C               ...........................................................
C               .  MATRIX COEFFS STORED ON FILE1                          .
C               .  SOLUTION STORED ON FILE2                               .
C               .  INTER-ROUTINE RECORDS KEPT ON FILE3                    .
C               .  THE NEW REVISE FILE IS WRITTEN ON LIST                 .
C               ...........................................................
      FILE1=7
      FILE2=9
      FILE3=13
      LIST =11
C               ...........................................................
C               .  PREPARE THE FILES                                      .
C               ...........................................................
      REWIND LIST

      REWIND FILE3

      REWIND FILE2
      READ(FILE2) NAME
10    READ(FILE2) NAME
      IF(NAME.NE.ENDSEC) GO TO 10
      READ(FILE2) NAME,NOCOL
      READ(FILE2) (COLNM(IN),IN=1,NOCOL)
```

```
      REWIND FILE2
      REWIND FILE1
      READ(FILE1,200) INAME
      READ(FILE1,200) INAME
C
C                ...........................................................
C                .   IDENTIFY THE ROWS WITH NON-ZERO DUALS                 .
C                .          - THESE ARE THE ALTERNATE HYPERPLANES          .
C                ...........................................................
50    READ(FILE2) (VALUES(IN),IN=1,NOCOL)
      IF(VALUES(1).EQ.ENDSEC) GO TO 100
      I=I+1
      READ(FILE1,210) ROWS(I,1),ROWS(I,2),ROWS(I,3)
      ROWS(I,4)=0
      IF(ROWS(I,1).EQ.GT) ROWS(I,4)=-1
      IF(ROWS(I,1).EQ.LT) ROWS(I,4)= 1
      IF(VALUES(DUAL).EQ.0.0) GO TO 50
      ROWS(I,4) =0
      IF(ROWS(I,1).EQ.E) GO TO 50
      IF(ROWS(I,1).EQ.N) GO TO 50
      J=J+1
      REVROW(J,3)=0
      REVROW(J,1)=ROWS(I,2)
      REVROW(J,2)=ROWS(I,3)
      IF((VALUES(LLIM).GT.LL).AND.(VALUES(ULIM).LT.UL))  REVROW(J,3)=1
      GO TO 50
100   CONTINUE
      LIMR=I
      LIMREV=J
      READ(FILE1,200) INAME
C
C                ...........................................................
C                .   READ THE MATRIX COEFFS, SQUARE, AND STORE             .
C                ...........................................................
630     READ(FILE1,230) BLANK,COL,ROW1,ROW2,VALUE
        IF(BLK.NE.BLANK) GO TO 650
        IF(XCOL.EQ.COL) GO TO 600
        XCOL=COL
        JI=1
600   DO 610 J=I,LIMR
        IF(ROW1.NE.ROWS(J,2)) GO TO 610
        IF(ROW2.EQ.ROWS(J,3)) GO TO 620
610     CONTINUE
620     JI=J+1
        COEFF(J,1)=COEFF(J,1)+VALUE*VALUE
        GO TO 630
C
C                ...........................................................
C                .   FIND THE RHS VALUES                                   .
C                ...........................................................
650   CONTINUE
      I=0
400   READ(FILE1,230) BLANK,RHS,ROW1,ROW2,VALUE
      WRITE(4,300) ROW1,VALUE
      IF(BLANK.NE.BLK) GO TO 500
      I=I+1
      IF(ROW1.NE.ROWS(I,2)) GO TO 420
      IF(ROW2.NE.PONS(I,3)) GO TO 420
410   COEFF(I,2)=VALUE
      GO TO 400
420   DO 430 I=1,LIMR
      IF(ROW1.NE.ROWS(I,2)) GO TO 430
      IF(ROW2.EQ.ROWS(I,3)) GO TO 410
430   CONTINUE
      WRITE(3,310)
500   CONTINUE
C                ...........................................................
C                .   OUTPUT                                                .
C                ...........................................................
      WRITE(LIST,270)
      WRITE(LIST,275)
      IF(LIMREV.EQ.0) GO TO 520
C
C                ...........................................................
C                .   KEEP THE ALTERNATE HYPERPLANES TIGHT                  .
C                ...........................................................
      WRITE(LIST,280) (REVROW(K,1),REVPOW(K,2),K=1,LIMREV)
520   CONTINUE
      WRITE(LIST,284)
      WRITE(LIST,290)
      WRITE(LIST,293) ONE
C     ...........................................................
```

```
C              .  MAXIMISE THE DISTANCE (PRCNT OF NORMALISED RHS) FROM  .
C              .  ALL THE OTHER HYPERPLANES                              .
C              ..........................................................

      WRITE(LIST,260) ONE
      DO 510 I=1,LIMR
         XVALU =COEFF(I,2)*ROWS(I,4)*(SQRT(COEFF(I,1)))
      COEFF(I,2)=XVALU
      IF(COEFF(I,2).EQ.0.0) GO TO 510
      WRITE(LIST,250) ROWS(I,2),ROWS(I,3),XVALU
      WRITE(FILE3,251) ROWS(I,2),ROWS(I,3),XVALU
510   CONTINUE
      WRITE(FILE3,252)
      WRITE(LIST,305) UL
      DO 530 K=1,LIMREV
      IF(REVROW(K,3).GT.0) GO TO 540
530   CONTINUE
590   WRITE(LIST,295)
      END FILE LIST
      REWIND LIST
      REWIND FILE2
      STOP
C              ..........................................................
C              .  IF THE ORIGINAL EQUATION IS RANGED AND NOW REQUIRED   .
C              .  TO BE TIGHT, THE RANGES ENTRY NEEDS MODIFICATION      .
C              ..........................................................
540   WRITE(LIST,310)
      WRITE(LIST,315) UL
      DO 550 KK=K,LIMREV
      IF(REVROW(KK,3).GT.0)  WRITE(LIST,320) REVROW(KK,1),REVROW(KK,2),
     1                               ZERO
550   CONTINUE
      GO TO 590
C              ..........................................................
C              .  THE FORMAT SECTION                                    .
C              ..........................................................
200   FORMAT(A4)
210   FORMAT(A2,2X,2A4)
220   FORMAT(4X,A8,2X,A8,2X,F12.3)
230   FORMAT(A4,A8,2X,2A4,2X,F12.4)
250   FORMAT(4X,'LANDA    ',2X,2A4,2X,F12.8)
251   FORMAT(2A4,F12.8)
252   FORMAT('$ENDSEC$')
260   FORMAT(4X,'LANDA    ',2X,'JECTIVE ',F14.3)
270   FORMAT('NAME',10X,'CENTREAL'/'ROWS')
275   FORMAT('  MODIFY')
276   FORMAT(' E  P6EA1')
280   FORMAT(' E  ',2A4)
284   FORMAT('  AFTER'/' L  DUMMY    '/' N  JECTIVE')
290   FORMAT('COLUMNS'/'  AFTER')
293   FORMAT(4X,'DUMMY    ',2X,'DUMMY    ',F14.3)
295   FORMAT('ENDATA')
300   FORMAT(1H ,'THE LAST ITEM TO BE READ WAS      ',A12,F15.4)
301   FORMAT('  DELETE'/4X,'LANDA    ')
305   FORMAT('RHS'/'  MODIFY'/4X,'RHS      ',2X,'DUMMY    ',F14.3)
310   FORMAT('RANGES'/'  MODIFY'/)
315   FORMAT(4X,'RANGES   ',2X,'DUMMY    ',F14.3)
320   FORMAT(4X,'RANGES   ',2X,2A4,F14.2)
      END
/*
//*
//*            CENTALTH   IS STORED ON AREA UNTM118R ON DISC 4C002
//*
//L.SYSIN DD *
 ENTRY MAIN
 NAME CENTALTH
```

iii. FIXIT - see line 161.

```
//
//UMTS1180 JOB %0120,RMT5,0.30,9CL,: EILON-COURIER:
//  EXEC FORTMOD,PARM.C!:BCD,NOSOURCE:,PARM.L!:LIST,XREF:
//C.SYSIN DD *
C                ..................................................................
C                .   FIXIT READS THE SOLUTION OF THE PREVIOUS RUN AND             .
C                .   FIXES THE VARIABLE BOUNDS AT THEIR ACTIVITY LEVELS           .
C                ..................................................................
      INTEGER FILE2,FILE3
      REAL UL,LL,MINUS
      REAL*8 COLNM(8),VALUES(9),ENDSEC, NAME,COLUM,COLUNN,RHS,DUMMY
      INTEGER DUAL,ULIM,LLIM,SLACK,ACTIV,VNAME
      DATA DUMMY/'DUMMY'/
      DATA ENDSEC/'$ENDSEC$'/
      UL=100000.0
      LL=-100000.0
      ACTIV=1
      SLACK=2
      LLIM=3
      ULIM=4
      DUAL=5
      VNAME=9
C                ..................................................................
C                .   THE SOLUTION IS STORED ON FILE2                              .
C                .   THE NEW REVISE FILE IS WRITTEN ON LIST                       .
C                ..................................................................
      LIST =11
      FILE2=9
C                ..................................................................
C                .   PREPARE THE FILES                                            .
C                ..................................................................
      REWIND LIST

      REWIND FILE2
      READ(FILE2) NAME
10    READ(FILE2) NAME
      IF(NAME.NE.ENDSEC) GO TO 10
      READ(FILE2) NAME,NOCOL
      READ(FILE2) (COLNM(IN),IN=1,NOCOL)
      READ(FILE2)
11    READ(FILE2) NAME
      IF(NAME.NE.ENDSEC) GO TO 11
      READ(FILE2) NAME,NOCOL
      READ(FILE2) (COLNM(IN),IN=1,NOCOL)
      READ(FILE2)
C                ..................................................................
C                .   WRITE REVISE HEADINGS AND KNOWN CHANGES                      .
C                ..................................................................
      WRITE(LIST,200)
      WRITE(LIST,205)
      WRITE(LIST,206)
      WRITE(LIST,210)
      WRITE(LIST,215)
      WRITE(LIST,220)
C                ..................................................................
C                .   FIX THE VARIABLES                                            .
C                ..................................................................
50    READ(FILE2) (VALUES(IN),IN=1,NOCOL)
      IF(VALUES(1).EQ.ENDSEC) GO TO 100
      IF(VALUES(VNAME).EQ DUMMY) GO TO 100
      WRITE(LIST,230) VALUES(VNAME),VALUES(ACTIV)
      GO TO 50
100   WRITE(LIST,290)
      END FILE LIST
      REWIND LIST
      REWIND FILE2
      STOP
C                ..................................................................
C                .   FORMAT SECTION                                               .
C                ..................................................................
200   FORMAT('NAME',10X,'NOVDEGEN')
205   FORMAT('ROWS'/'  DELETE'/'      P6EA1')
206   FORMAT('   BEFORE'/' N  EARNINGS')
210   FORMAT('COLUMNS'/'  DELETE'/'      DUMMY'/'      LANDA')
215   FORMAT('  MODIFY'/
     1                      /'      P6CH01     EARNINGS     1.0'/
     2                      /'      P6CR01     EARNINGS    -1.0'/
     3                      '      P6DE01    EARNINGS     1.0')
220   FORMAT('BOUNDS'/'  MODIFY')
250   FORMAT(' FX POLICY      ',A8,F14.7)
290   FORMAT('ENDATA')
      END
/*
//*
//*           FIXIT       IS STORED ON AREA UNTM1ISM ON DISC 40002
//*
//L.SYSIN DD *
 ENTRY MAIN
 NAME FIXIT
//
```

iv. DEGENTOT - see line 178.

```
//
//UMTS118P JOB %0120,RMT5,0 30,9CI,: EILON-COURIER:
//  EXEC FORTMOD,PARM.C!:BCD,NOSOURCE:,PARM.L!:LIST,XREF:
//C.SYSIN DD *
C                ..............................................................
C                .  DEGENTOT MAKES THE SYSTEM TOTALLY DEGENERATE BY           .
C                .  REDUCING THE RHS VALUE TO THE ACTIVITY LEVEL              .
C                ..............................................................
      INTEGER FILE2,FILE3
      REAL UL,LL,MINUS
      REAL*8 COLNM(S),VALUES(S),ENDSEC, NAME,COLUM,COLUMN,RHS,DUMMY
      INTEGER DUAL,ULIM,LLIM,SLACK,ACTIV,VNAME
      DATA ENDSEC/'$ENDSEC$'/
      UL=100000.0
      LL=-100000.0
      ACTIV=1
      SLACK=2
      LLIM=3
      ULIM=4
      DUAL=5
      VNAME=9
C                ..............................................................
C                .  THE SOLUTION IS STORED ON FILE2                           .
C                .  THE NEW REVISE FILE IS WRITTEN ON LIST                    .
C                ..............................................................
      LIST =11
      FILE2=9
C                ..............................................................
C                .  PREPARE THE FILES                                         .
C                ..............................................................
      REWIND LIST

      REWIND FILE2
      READ(FILE2) NAME
10    READ(FILE2) NAME
      IF(NAME.NE.ENDSEC) GO TO 10
      READ(FILE2) NAME,NOCOL
      READ(FILE2) (COLNM(IN),IN=1,NOCOL)
      READ(FILE2)

      WRITE(LIST,200)
C                ..............................................................
C                .  IF A CONSTRAINT HAS NON-ZERO SLACK PEVISE THE RHS         .
C                ..............................................................
50    READ(FILE2) (VALUES(IN),IN=1,NOCOL)
      IF(VALUES(1).EQ.ENDSEC) GO TO 100
      IF((VALUES(LLIM).LT.LL).AND.(VALUES(ULIM).GT.UL))  GO TO 50
      IF(VALUES(SLACK).EQ.0.0) GO TO 50
      WRITE(LIST,210) VALUES(VNAME),VALUES(ACTIV)
      GO TO 50
100   WRITE(LIST,290)
      END FILE LIST
      REWIND LIST
      STOP
C                ..............................................................
C                .  THE FOFMAT SECTION                                        .
C                ..............................................................
200   FORMAT('NAME',10X,'NONDEGEN'/'RHS'/'  MODIFY')
210   FORMAT(4X,'RHS      ',2X,A8,F14.7)
290   FORMAT('EHDATA')
300   FORMAT('NAME',10X,'NONDEGEN'/'ROWS'/'  MODIFY')
310   FORMAT(' E  ',A8)
      END
//*
//*            DEGENTOT   IS STORED ON AREA UMTM118N ON DISC 4C002




//*
//L.SYSIN DD *
 ENTRY MAIN
 NAME DEGENTOT
/*
```

v.   TRANS - see line 197.

```
//
//UMTS118H JOB %0120,RMT5,0.30,9CC,: EILON-COURIER:
//  EXEC FORTMOD,PARM.C!:BCD,NOSOURCE:,PARM.L!:LIST,XREF:
//C.SYSIN DD *
C
C              ..........................................................
C              .  TRANS CONSTRUCTS THE DUAL PROGRAMME                   .
C              .        - TRANSPOSE THE MATRIX COEFFICIENTS,            .
C              .        - FREE THE DUAL VARIABLES,                      .
C              .        - CONSTRUCT THE NEW RHS AND OBJECTIVE FUNCTION  .
C              ..........................................................
      REAL*8 NAME,ENDSEC,COLNM(8),VALUES(8),XCOL,OBJ,OLDJ
      INTEGER FILE1,FILE2,NEW,ACTIV,VNAME,ULIM,LLIM
      REAL LL,UL,MINUS
      DATA XCOL/'XXXXXXX'/,RHS/'RHS '/,COLUM/'COLU'/
      DATA ENDSEC/'*ENDSEC$'/,BLANK/' .  '/
      DATA OBJ/'P$EA1'/
      ACTIV=I
      ULIM=4
      LLIM=3
      VNAME=8
      UL=10000 0
      LL=-10000.0
      ONE=1.0
      MINUS=-1.
C
C              ..........................................................
C              .  THE COEFFICIENTS ARE STORED ON FILE1                  .
C              .  THE SOLUTION IS STORED ON FILE2                       .
C              .  THE TRANSPOSED MATRIX COEFFS ARE WRITTEN ON FILE NEW  .
C              ..........................................................
      FILE1=7
      FILE2=9
      NEW=8
C
C              ..........................................................
C              .  PREPARE THE FILES                                     .
C              ..........................................................
      REWIND NEW

      REWIND FILE1
10    READ(FILE1,999) XNAME
      IF(XNAME NE COLUM) GO TO 10

      REWIND FILE2
      READ(FILE2) NAME
15    READ(FILE2) NAME
      IF(NAME.NE.ENDSEC) GO TO 15
      READ(FILE2) NAME.NOCL
      READ(FILE2) (COLNM(N),N=1,NOCL)
      READ(FILE2)
C
C              ..........................................................
C              .  NAME THE NEW ROWS TO CORRESPOND TO THE OLD COLUMNS    .
C              ..........................................................
      WRITE(NEW,1000)
20    READ(FILE1,999) XNAME.NAME
      IF(XNAME EQ.RHS) GO TO 100
      IF(NAME.EQ.XCOL) GO TO 20
      WRITE(NEW,1010) NAME
      XCOL=NAME
      GO TO 20
100   WRITE(NEW,1020)
110   READ(FILE2) (VALUES(N),N=1,NOCL)
      IF(VALUES(1).EQ.ENDSEC) GO TO 200
      IF(VALUES(VNAME).EQ.OBJ) OLDJ=VALUES(ACTIV)
C
C              ..........................................................
C                 WRITE THE NEW OBJECTIVE FUNCTION                      .
C              ..........................................................
```

```
      IF((VALUES(ULIM).GT.UL).AND.(VALUES(LLIM).LT.LL)) GO TO 110
      IF(VALUES(ULIM).LT.UL)  WRITE(NEW,1040) VALUES(VNAME),VALUES(ULIM)
      IF(VALUES(ULIM).GT.UL) WRITE(NEW,1040) VALUES(VNAME),VALUES(LLIM)
C                 ...................................................
C                 .  TRANSPOSE THE MATRIX ELEMENTS                  .
C                 ...................................................
      REWIND FILE1
120   READ(FILE1,999) XNAME
      IF(XNAME.NE.COLUM) GO TO 120
130   READ(FILE1,999) XNAME,XCOL,NAME,VALU
      IF(XNAME.NE.BLANK) GO TO 110
      IF(NAME.NE.VALUES(VNAME)) GO TO 130
      WRITE(NEW,1030) NAME,XCOL,VALU
      GO TO 130
200   CONTINUE
C                 ...................................................
C                 .  ENTER THE ORIGINAL BOUNDS EQUATIONS            .
C                 ...................................................
      READ(FILE2) NAME,NOCL
      READ(FILE2) (COLNM(N),N=1,NOCL)
      READ(FILE2)
210   READ(FILE2) (VALUES(N),N=1,NOCL)
      IF(VALUES(1).EQ.ENDSEC) GO TO 300
      I=0
      IF(VALUES(ACTIV).EQ.VALUES(ULIM)) I=ULIM
      IF(VALUES(ACTIV).EQ.VALUES(LLIM)) I=LLIM
      IF(I.EQ.0) GO TO 210
      IF(I.EQ.LLIM) WRITE(NEW,1050) VALUES(VNAME),VALUES(VNAME),MINUS
      IF(I.EQ.ULIM) WRITE(NEW,1050) VALUES(VNAME),VALUES(VNAME),ONE
      WRITE(NEW,1060) VALUES(VNAME),VALUES(ACTIV)
      GO TO 210
C                 ...................................................
C                 .  CONSTRUCT THE NEW RHS                          .
C                 ...................................................
300   WRITE(NEW,1070)
      WRITE(NEW,1075) OLDJ
      WRITE(3,1070)
      REWIND FILE1
310   READ(FILE1,999) XNAME
      IF(XNAME.NE.COLUM) GO TO 310
320   READ(FILE1,999) XNAME,XCOL,NAME,VALU
      IF(XNAME.NE.BLANK) GO TO 400
      IF(NAME.EQ.OBJ) WRITE(NEW,1030) XCOL,VALU
      GO TO 320
C                 ...................................................
C                 .  ALLOW THE DUAL VARIABLES TO BE UNRESTRICTED (I E +OR-).
C                 ...................................................
400   WRITE(NEW,1090)
      REWIND FILE2
415   READ(FILE2) NAME
      IF(NAME.NE.ENDSEC) GO TO 415
      READ(FILE2) NAME,NOCL
      READ(FILE2) (COLNM(N),N=1,NOCL)
      READ(FILE2)
420   READ(FILE2) (VALUES(N),N=1,NOCL)
      IF(VALUES(1).EQ.ENDSEC) GO TO 430
      IF((VALUES(ULIM).GT.UL).AND.(VALUES(LLIM).LT.LL))  GO TO 420
      WRITE(NEW,1100) VALUES(VNAME)
      GO TO 420
430   READ(FILE2) NAME,NOCL
      READ(FILE2) (COLNM(N),N=1,NOCL)
      READ(FILE2)
440   READ(FILE2) (VALUES(N),N=1,NOCL)
      IF(VALUES(1).EQ.ENDSEC) GO TO 450
      IF(VALUES(ACTIV).EQ.VALUES(ULIM))   WRITE(NEW,1110) VALUES(VNAME)
      IF(VALUES(ACTIV).EQ.VALUES(LLIM))   WRITE(NEW,1110) VALUES(VNAME)
      GO TO 440
450   WRITE(NEW,1200)
      STOP
C                 ...................................................
C                 .  THE FORMAT SECTION
C                 ...................................................
999   FORMAT(A4,A8,2X,A8,F14.5)
1000  FORMAT('NAME',10X,'NEWMATRX'/'ROWS')
1010  FORMAT(' E  ',A8)
1020  FORMAT(' F  NEWOBJ'/'COLUMNS')
1030  FORMAT('    P',A8,1X,A8,F14.5)
1040  FORMAT('    P',A8,' NEWOBJ  ',F14.5)
1050  FORMAT('     ',A8,2X,A8,F14.5)
1060  FORMAT('     ',A8,'  NEWOBJ  ',F14.5)
1070  FORMAT('RHS')
1075  FORMAT('    NEWRHS    NEWOBJ  ',F14.5)
1080  FORMAT('    NEWRHS    ',A8,F14.5)
1090  FORMAT('BOUNDS')
1100  FORMAT(' FR POLICY    P',A8)
1110  FORMAT(' FR POLICY     ',A8)
1200  FORMAT('ENDATA')
      END
/*
//*
//*           TRANS        IS STORED ON AREA UMTM118T ON DISC 40002
//*
//L.SYSIN DD *
 ENTRY MAIN
 NAME TRANS
//
```

3. Multiple Balance Sheets

As discussed in Chapter 6.3.3.ii, arbitrary selection of dual variable vectors from the alternate dual space allow us to construct different balance sheets from the same optimal period solution. These results have been published and are included in Appendix E.1: Figure E(.1).3 shows three different balance sheets constructed for the same period, and Figures E(.1).4 and 5 show different profiles over the planning horizon for Profit and Asset Value. It has been this multiplicity of results attempting to portray the same 'physical' solution that has led us to draw our conclusions in Chapter 7.

We attempted to impose secondary conditions on the alternate dual space in order to resolve the alternate dual solutions to select particular dual vectors that would reflect certain desirable attributes. For example, we imposed a variety and combination of conditions that

- the shareholder account (in the balance sheet) grow by at least same specified amount;
- the value of stocks be positive;
- the acid test yield a value greater than one;
- the fixed asset valuation be non-negative;
- the return on assets yield at least some specified level;

etc

The results of this experimentation were disappointing:

We found that the dual space generated was unbounded. The imposition of a variety of the conditions mentioned above resulted in the solution to the 'revised' dual programme switching unstability from an infeasible to an unbounded solution. Furthermore selection of computer solutions (making an unbounded solution) often resulted in many assets taking a zero value, thereby negating one of the prime objectives of this work.

To overcome this problem, one would have to resort to goal programming. By including a number of secondary conditions (that would otherwise result in infeasibility) one can aim to select a point that satisfies these conditions "as closely as possible".

A formulation that may overcome the zero value attributed to assets is shown below:

$$\text{Min } z$$

$$\text{s.t.} \quad \underline{A}\,\underline{x} \leqslant \underline{b}$$

$$\underline{Y} = f\,\underline{x}$$

$$g_j\underline{Y} + u_j \geqslant G_j$$

$$u_j - z \leqslant 0$$

$$Y_j \gg z$$

where Y is same variable (e.g. profit, stock valuation) linked to the physical variables of the model ($\underline{X}$) by same function f_j, to which same special performance G_j is sought, and where some under achievement (u_j) of the goal is permissible.

However, selection of a particular dual vector, resulting in a particular balance sheet will not give a third party reader of the accounts any particular 'true' view of underlying physical activities of the firm.

APPENDIX E.1

VALUATION OF RESOURCES

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ABSTRACT

This paper presents an overview of results of research into problems of financial reporting faced by an average manufacturing company. The paper demonstrates how Linear Programming can be used to overcome some of the shortcomings encountered by standard accounting procedures. However, the widespread occurrence of degenerate solutions to such LP corporate models requires a fundamental revision to our interpretation of the economic meaning of the dual variables: a revision that may be extended to the dual of non-degenerate optimal solutions.

1. INTRODUCTION

The purposes of this paper is to present an overview of some results of research that has been undertaken in the area of financial reporting in a typical manufacturing company.

Some problems of present-day accounting procedures are demonstrated first. It is then shown how Linear Programming (LP) can be used to overcome some of these shortcomings, drawing to the attention of the reader the problems of interpreting degenerate solutions. The problems of interpretation will be demonstrated with the aid of a simple model of a manufacturing company.

It is currently accepted that using different, but equally acceptable, accounting assumptions very different balance sheets can be produced from the same basic economic data. This has been clearly established in the current debate over inflation and how it should be recognised. A number of the assumptions and problems associated with producing accounts are enumerated in Section 2 below.

2. ACCOUNTING PROBLEMS

The currently accepted accounting procedures can be shown to possess a number of shortcomings, viz

(a) The allocation of revenues or, more usually, overheads amongst joint production facilities is arbitrary. Yet the procedures play a central role in standard accounting methods.

(b) Generally accepted accounting procedures normally approach the task of valuation in a modular fashion - each class of asset is valued individually. The value of the entire enterprise is then assumed to be the sum of the values of the component parts. Yet in actual fact, the usefulness of any particular asset is often affected by the holding of others.

(c) There is continuing disagreement about which of the plethora of techniques for valuing stock - LIFO, FIFO, and inflation accounting techniques - is the most appropriate, or presents the 'truest' picture. Yet all of these methods are based on the central concept of historical or replacement cost: a concept at variance with the purpose of stockholding. Companies generally hold stock because of their expectation of demand and variation of demand in the future - demands that they may not be able to meet with their present or expected manufacturing capacity. This idea, that 'future usage is the relevance of stock' is totally divorced from that of cost.

(d) Depreciation hides two very different functions when presented as a single figure in the balance sheet: firstly it represents the means for retention of profits to be used for the replacement of the assets worn out in service, and secondly it gives a value to the remaining assets. (Depreciation serves a further function where it is used in the Profit and Loss accounts as a charge against the revenue produced by the assets). In practice depreciation is calculated according to some arbitrary function related to cost. But valuation ought to reflect the usefulness of the assets to the business as an on-going concern. Once again, such a valuation would be totally divorced from cost.

(e) Traditional balance sheets do not value all the assets that comprise the company - for example labour (an essential input to the company's activities) does not appear on the balance sheet. Furthermore it is not yet widely accepted that the labour force may have a value over and above its wage bill.

(f) In traditional accounting practice the process of valuation is undertaken without reference to the objectives of the organisation. Two similar businesses pursuing different goals would yield similar valuations - a result that is unsatisfactory.

We have searched for an alternative valuation technique in an attempt to overcome these shortcomings.

3. LP MODELLING

In this section we have assumed that the activities of a firm may be accurately represented by a linear deterministic model, so that LP may be used as the model solving technique. (This assumption is primarily for discussion purposes, the conclusions are equally valid for any mathematical programming model.)

The advantages are:

(a) LP is an optimizing technique producing a solution that maximises an objective, or satisfies a set of goals, pursued by the company. If the objective function represents profit over the time period being considered then, through the dual LP, it is possible to allocate such profit to the various inputs used to generate the profit. These allocations can be considered to be valuations of the inputs relative to the objective of the company and to the interactions in their use. From such valuations balance sheets can be constructed with the profit attributed to each period being the difference in the Shareholders' Account between any two consecutive periods.

(b) Financial flows are modelled adopting the marginal costing approach, thereby obviating the need for an allocation of joint costs and revenues. It has also been argued (4) that the dual LP gives a 'realistic' allocation of overheads if that be required (e.g. for pricing purposes).

(c) Despite the assumption of a linear system the average value chosen (the dual variable) for valuation purposes does reflect the phenomenon of decreasing utility in that the value is calculated at the margin of a fixed and known combination of the entire set of assets and this marginal value has been shown to decrease with increasing availability of resource (1).

(d) LP takes an overview of the company as a whole, thereby allowing valuation of component inputs to be made in accordance with the contribution they make in reaching the stated objectives of the company. In this manner valuation is related to the opportunity value and not to the concept of historical cost!

(e) The model can recognise the role played by inputs not normally acknowledged by generally accepted accounting procedures (e.g. the labour force) by the inclusion of extra constraints portraying the scarcity of such resources.

With these principals in mind we have modelled a manufacturing company as shown in Fig. 1. This is a multi-time period model which concentrates on the physical flows (from purchases through to sales). Financial flows are linked through the marginal costing approach so that overheads (including rent, rates, wages etc.) are paid centrally, and only direct costs (such as purchases and overtime payments) and revenues from sales are directly linked to the finally selected plan. A more detailed description of the model is to be found in the Appendix.

4. DEGENERACY

The optimal solution of practical models of this type are almost invariably degenerate, and this renders the normally accepted economic interpretation invalid.

Degeneracy occurs when the solution vertex is overspecified e.g. vertex A in Fig. 2. This could occur, for example, when a product is being manufactured on two machines, one of which is being used to capacity and thereby limiting production, and when the storage capacity is simultaneously being fully utilised: thus the facter that is uniquely limiting the plan cannot be identified.

Degeneracy, although often regarded an an undesirable feature of LP, has a number of desirable attributes. Consider the optimal solution of the LP model. In general there will be a certain number of resources being used to full capacity, with others being under-utilised. The dual LP will only attribute value to those resources without slack: in other words any resource not being used to full capacity is valued at zero. This is consistent with the marginal costing concept, but is obviously unacceptable when trying to construct a balance sheet: no management will accept that some of its resources are <u>totally</u> valueless. Indeed, if the slack of a particular resource were to be removed, then the resource would be used to its full capacity, and should therefore have a non-zero value. By eliminating all the slacks the problem is made totally degenerate, thus providing the opportunity to value <u>all</u> the resources. Furthermore, the totally degenerate system represents the ideally desirable (deterministic) economic system (3), since there are no wasted inputs!

Now let us turn to the problem of interpreting the dual programme. We know that some of the dual variables are asymmetric at a degenerate point (1) but deeper investigation shows that the problem is more complex (2). In fact, it may be shown that, if the primal optimal solution is degenerate, there exists an infinite number of sets of dual variables that satisfy the Strong Duality Theorem (that the optimal value of the dual programme equals the optimal value of the primal). These sets form a closed polytope which may extend beyond the positive orthant.

The implications of this are:

1. We can only consider the dual variables as elements of sets. It is incorrect to use one dual from one set and another dual from a different set since this would lead to non-comparable valuation.

   (In non-degenerate solutions the set of duals in unique so that this condition is met.)

2. Using different sets of duals to construct balance sheets (as suggested in Section 3 (a) above) will result in different valuations of the same physical (primal) solution: Fig. 3 shows three different balance sheets constructed to portray events in the same period; Figs 4 and 5 show how different profiles of profit and asset value emerge over the time horizon. Taking advantage of this, different balance sheets can be drawn up that will show the company

in entirely different lights.

The different combinations (all of which are equally acceptable) of values of inputs appear to rule out any unique or meaningful balance sheet for a company. If, however, the idea of a unique balance sheet at one moment in time is rejected in favour of the view that there are many participants in the subjective interpretation of the economic effects e.g. creditors, bankers, workers shareholders as well as management, then it may be possible to impose constraints on the space of dual variables. Further constraints may also be desired, for example the very practical ones of ensuring that the valuation given to real resources are never negative. These constraints will reduce the dual space, but the multiplicity of choice within the resultant subset may still be too large for the selection of a valuation vector which may be used to construct a presently acceptable balance sheet. Furthermore, a third party could retrieve information from a balance sheet constructed in this manner only if he knew the subjective basis on which it is based.

The attempt to allocate a given (and known) value of the enterprise as a whole over its component, joint facilities is inevitably arbitrary (whether the model solution is degenerate or not!). The view that valuable economic information can be gained by inspecting such an allocation of value is clearly in need of revision.

5. CONCLUSIONS

The shortcomings of standard accounting procedures, when applied to valuation, have been discussed; particularly in light of their cost and modular foundation.

We have shown how a linear deterministic model of the manufacturing company can illuminate some of these problems.

It has been argued that a totally degenerate system is (ideally) economically desirable, and enables us to value all the resources. However, we have shown that at degenerate solutions the duals are not unique; that they need not all be non-negative (even at the optimum); that they must be considered in terms of coherent sets and not as individual elements. These sets result in different allocations of the value of the company over its component assets, and all such allocations are equally valid.

Allocations may be selected so that the resulting balance sheets will satisfy the various interest parties' anticipated requirements.

This leads us to question the economic significance of the dual LP since any allocation of value over joint inputs is necessarily arbitrary, as in reality it must always be.

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APPENDIX

The structure of the multi-time period corporate model is based on the following relationships:

1. The Production Function

(i) Production Capacity

The requirement for each work-centre in each period must not exceed the normal-time capacity of that work centre in that period, plus any overtime worked plus any subcontracting done (if applicable).

(ii) Labour Force Allocation

Total in-house production (i.e. total work-centre requirement less any work sub-contracted out) for each work-centre for each period must be allocated amongst the labour groups capable of doing the work.

2. Physical Constraints

(i) Production Limitations

Limit the production of each product in each period by some upper bound (e.g. availability of jigs).

(ii) Labour Force Capacity

The requirement for each labour group in each period must not exceed the work scheduled to be done in overtime plus that scheduled to be done in normal shift time.

(iii) Labour Overtime Capacity

Limit the total overtime load on each labour group in each period by some upper bound.

(iv) Market Constraints

Limit the sales of each product in each market outlet in each period by some upper (and lower) bound.

(v) Storage Capacity

Storage space required in each period must not exceed the space available.

3. Financial Flows

(i) Cash Position

The cash balance at the close of each period must equal the balance at the opening of the period plus inflows resulting from sales made in previous periods and loans taken up, less outflows resulting from loans repaid; overtime payment for work done in that period; sub-contracting costs, purchases and bank charges incurred in previous periods and net outflows resulting from extra model activities. (This latter factor includes payment of overheads, and net flows from sections of the company not modelled e.g. sale of goods taken in part exchange etc.).

(ii) Creditors Account

Credit at the close of each period must equal purchases of raw materials made plus sub-contracting work carried out but not yet paid for, plus interest charges outstanding.

(iii) Debtors Account

Debt at the close of each period must equal revenues due (from sales made in previous periods) but not yet received.

4. Inter-Period Continuity Equations

(i) Cash Continuity

Cash balance at close of each period must equal the balance at the opening of the next period.

(ii) Finished Good Stocks

The stock of each product of the close of any period must equal the stock at the close of the previous period plus newly completed production less sales made in the period.

(iii) Raw Material Stock

The stock for each type of raw material at the close of each period must equal the stock at the close of the previous period plus any new purchases less the amount used in production during the period.

5. Objective Functions

A number of objective functions have been used e.g.

(i) 'Profit' earnings

Max cash plus debt less credit positions at the close of the flowing period.

(ii) Turnover

Max Turnover.

(iii) Penetration

Max number of units of product sold.

This simple structure can be considerably extended to include inflation, investment of cash surplus, dividend payment, investments in production facilities, as well as taking account of any managerial constraint, such as ensuring that certain financial ratios are adhered to.

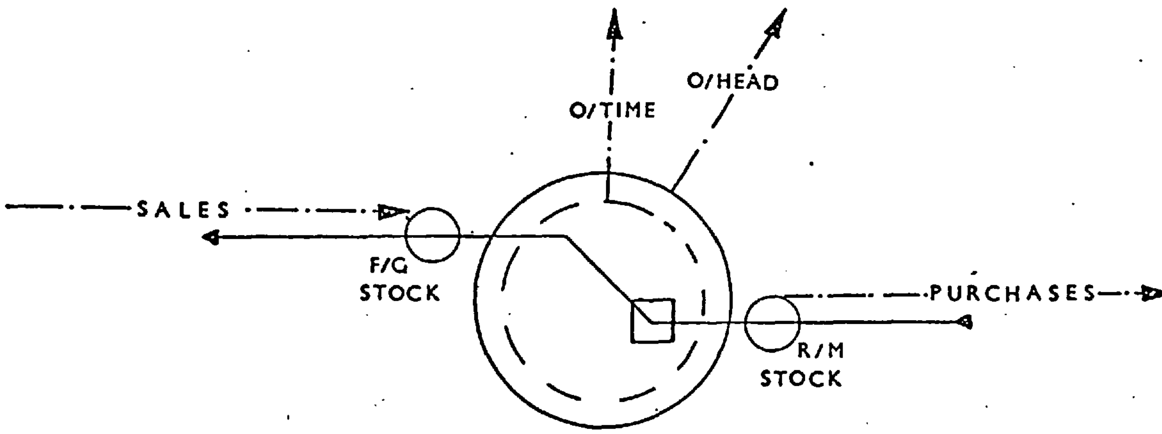

FIGURE 1 THE LP MODEL

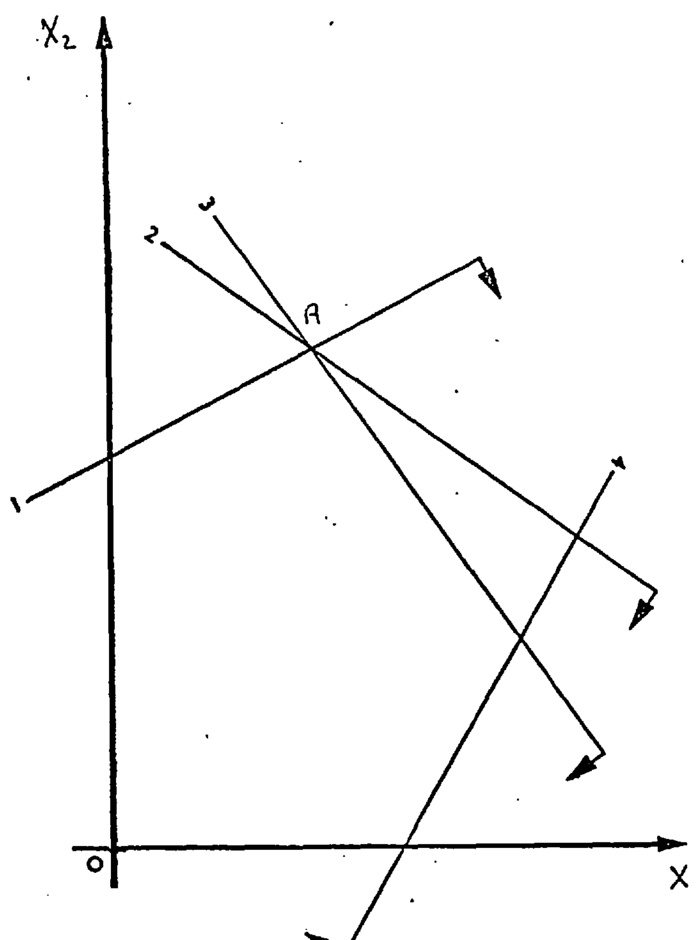

FIGURE 2 EXAMPLE OF A DEGENERATE VERTEX

A

CREDITORS	687.56	DEBTORS			638.89
		CASH			122.33
		STOCK R/M	UNITS	VALUE	
		1	446.1	446.11	
		2	0.0	0.0	
		STOCK F/G			
		1	40.9	817.78	
		2	3.4	169.44	
					1433.33
		WORK CENTRE CAPACITY			308.00
		LABOUR FORCE CAPACITY			0.00
SHAREHOLDERS A/C	2414.97	STORAGE CAPACITY			599.97
	3102.52				3102.52
•PROFIT THIS PERIOD•	1405.19				

B

CREDITORS	687.56	DEBTORS			638.89
		CASH			122.33
		STOCK R/M	UNITS	VALUE	
		1	446.1	446.11	
		2	0.0	0.0	
		STOCK F/G			
		1	40.9	817.78	
		2	3.4	169.44	
					1433.33
		WORK CENTRE CAPACITY			148.93
		LABOUR FORCE CAPACITY			158.94
SHAREHOLDERS A/C	2190.69	STORAGE CAPACITY			387.77
	2888.24				2888.24
•PROFIT THIS PERIOD•	2183.55				

C

CREDITORS	687.56	DEBTORS			638.89
		CASH			122.33
		STOCK R/M	UNITS	VALUE	
		1	446.1	446.11	
		2	0.0	0.0	
		STOCK F/G			
		1	40.9	817.78	
		2	3.4	169.44	
					1433.33
		WORK CENTRE CAPACITY			306.57
		LABOUR FORCE CAPACITY			0.00
SHAREHOLDERS A/C	1963.69	STORAGE CAPACITY			150.93
	2651.25				2651.25
•PROFIT THIS PERIOD•	772.93				

FIGURE 3 DIFFERENT BALANCE SHEETS FOR THE SAME PERIOD

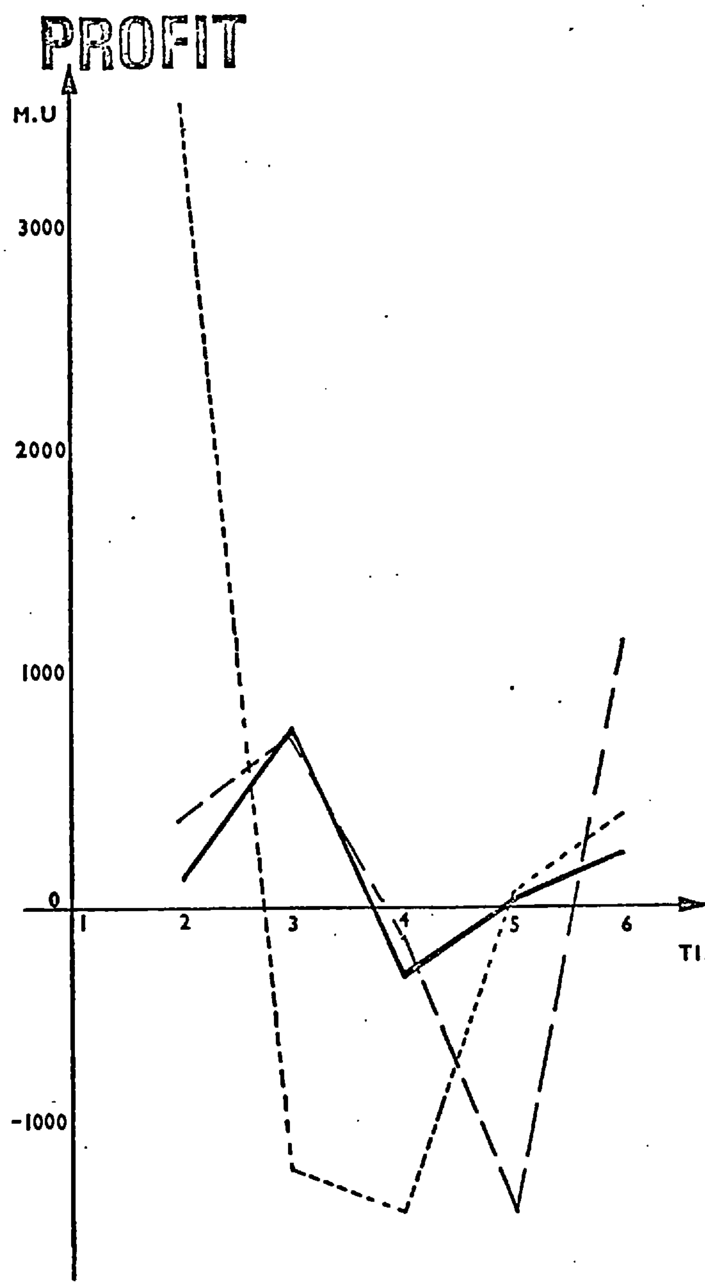

FIGURE 4 DIFFERENT PROFIT PROFILES

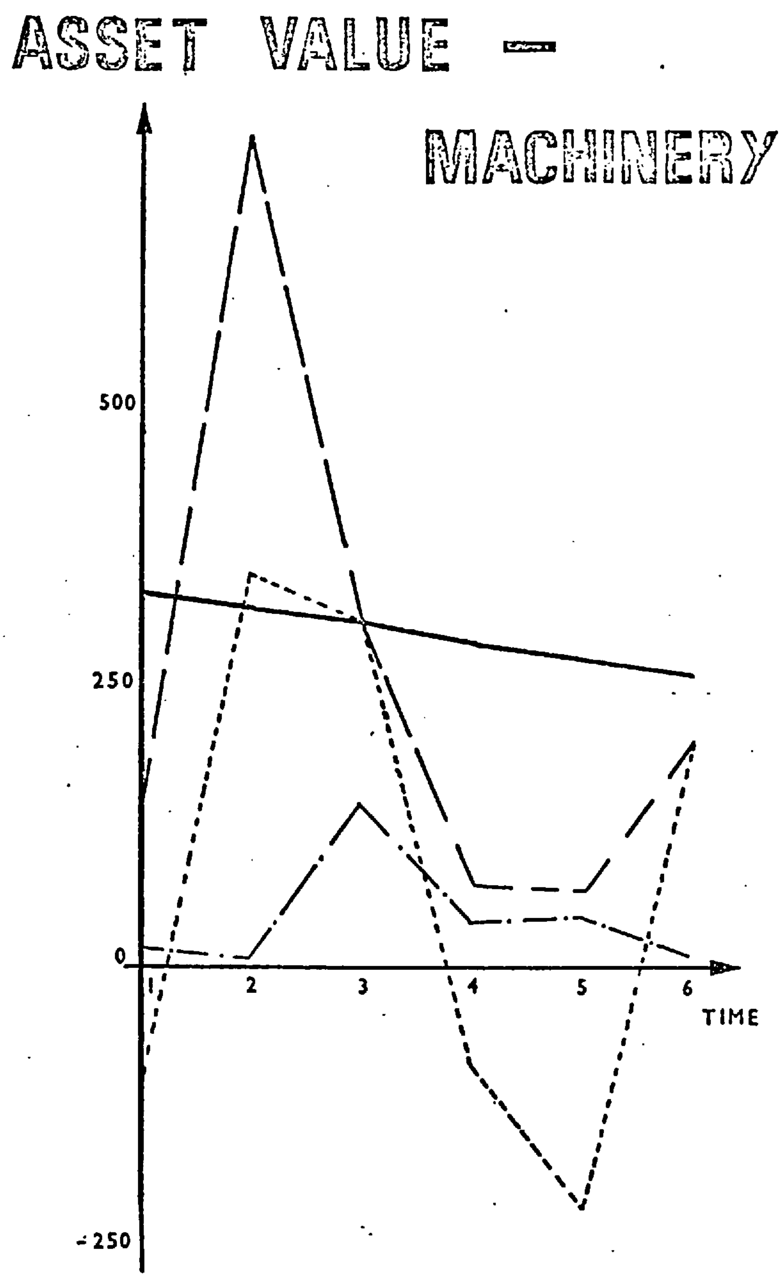

FIGURE 5 DIFFERENT ASSET VALUE PROFILES

APPENDIX E.2

OMEGA, The Int. Jl of Mgmt Sci., Vol. 2, No. 6, 1974

Note on "Many-sided Shadow Prices"

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It has been previously shown, with the aid of a simple example, that linear programmes may have two different non-negative marginal values at the optimum if the solution is degenerate. It is demonstrated in this note that other marginal values exist which may take negative values.

STRUM [1], in his paper entitled 'Note on two-sided shadow prices', brings to the attention of linear programming (LP) users, and particularly accountants, the fact that, at the optimum, a resource may have two marginal values. These may be shown to relate to the change on the objective function (OF) due 'to increasing a resource by one unit' and also 'to decreasing the resource by one unit'. If the optimal vertex is degenerate, then these changes are asymmetric; if it is non-degenerate, the changes are of course symmetrical.

This view is however an oversimplification of the problems that a degenerate LP can cause. As a vehicle for our discussion, let us consider Strum's example; namely

$$\begin{array}{llrcrl} \text{Maximise } P = & 2x + 3y & & & & \\ \text{s.t.} & 3x + y & \leqslant & 48 & (1) \\ & 3x + 4y & \leqslant & 120 & (2) \\ & x + 2y & \leqslant & 56 & (3) \\ & x, \quad y & \geqslant & 0 & \end{array}$$

Basic optimal solutions to this problem yield $P = 88$ with $x = 8$, $y = 24$ (point 0 in Fig. 1) and with the three constraints all being exactly satisfied; hence the problem is degenerate at this point. The alternate optimal bases are (x, y, s_1), (x, y, s_2) and (x, y, s_3) where s_1, s_2 and s_3 are the slack variables in the three constraints respectively. The question is: what are the marginal values of these three bases?

Let us consider this question indirectly by examining the more general problem of maximising $P = c_1x + c_2y$, subject to the same set of constraints and subject also to the proviso that $c_1, c_2 > 0$. At the optimal vertex, we may construct Table 1. The limitation row is calculated from the usual condition that, at the optimum, all the marginal values should be non-negative.

TABLE 1. THE MARGINAL VALUES OF THE ALTERNATE OPTIMAL BASES

Basis	(x, y, s_1)	(x, y, s_2)	(x, y, s_3)
Marginal value	$(0, c_1 - \frac{1}{2}c_2, -2c_1 + \frac{3}{2}c_2)$	$(\frac{2}{5}c_1 - \frac{1}{3}c_2, 0, -\frac{1}{5}c_1 + \frac{3}{5}c_2)$	$(\frac{4}{9}c_1 - \frac{1}{3}c_2, -\frac{1}{9}c_1 + \frac{1}{3}c_2, 0)$
Marginal value ($c_1=2, c_2=3$)	$(0, \frac{1}{2}, \frac{1}{2})$	$(\frac{1}{5}, 0, \frac{7}{3})$	$(-\frac{1}{9}, \frac{7}{9}, 0)$
Limitation	$\frac{1}{2} < \frac{c_1}{c_2} < \frac{3}{4}$	$\frac{1}{2} < \frac{c_1}{c_2} < 3$	$\frac{3}{4} < \frac{c_1}{c_2} < 3$

The first two sets of marginal values in Table 1 are those obtained by Strum; the last is the set generated by the third, hitherto unconsidered, basis. This last set contains a *negative* marginal value, and yet it is derived from a feasible basis describing the optimal vertex. The result would appear to be a direct violation of the Necessity Theorem for defining an optimal point in a LP; for example, Vajda [2] proves the following theorem, 'If x_o minimise $c'x$ subject to $Ax \geqslant b$, then there exist a vector $y_o \geqslant 0$ such that $c = A'y_o$ and $(b - Ax_o)'y_o = 0$'. Clearly, in a degenerate LP where there exists more than one x_o (i.e. alternate bases), it is not necessary for $y_o \geqslant 0$ for all the alternate x_o. Briefly the condition for an optimal vertex is that the inward normal to the OF is internal to the cone of inward normals to the planes defining the vertex. If the vertex is degenerate, then there exist a number of combinations of planes that define the vertex, and it cannot be guaranteed that each cone generated by a combination will contain the normal to the OF; as demonstrated in the above example. Work on this problem, and on similar problems arising from degeneracy in LP's, is continuing and will form part of a forthcoming PhD thesis by one of the authors.

Our concern at this point is the interpretation of the three marginal sets. and in particular the final set. Strum is correct in his interpretation of the marginal values he derives; they do reflect the resultant change in the OF were an individual resource increased or decreased by a small amount. They may however be interpreted somewhat differently by regarding each set as representing the marginal values that would result were that constraint (whose slack is in the particular basis), completely omitted from the problem. As an example, consider the third constraint (3). Strum pointed out that this constraint has two non-zero marginal values associated with it. This implies that the optimal vertex always lies on this constraint, for any small perturbation. Hence it follows that the omission of this constraint entirely would result in a different optimal vertex. This is of course precisely what the negative marginal value reflects; omission of the third constraint implies that s_1 should enter the basis and a higher optimal value of 90 (at $x = o$, $y = 30$) will result (i.e. the optimum will move from 0 to A in Fig. 1) Omission of either of the other two constraints still results in non-negative marginal values and hence the optimal vertex will remain stationary.

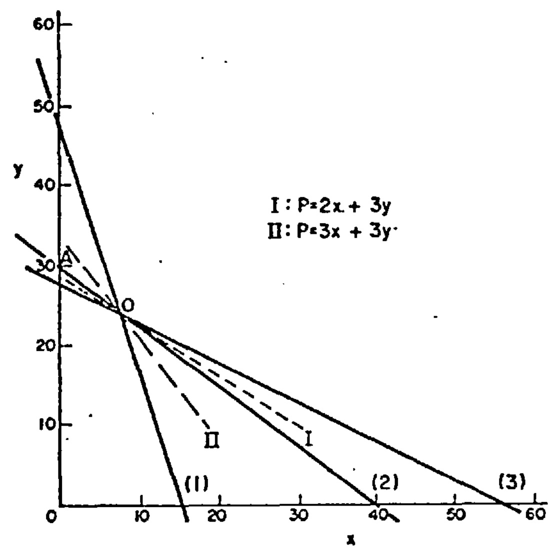

FIG. 1. *Geometry of the problem.*

If the value of c_1 is increased to 3, the optimal vertex remains the same but the set of marginal values relating to the first basis now contains a negative value and the other sets contain only non-negative values. This result is of course expected from the limitations calculated earlier and demonstrates one method of determining all the feasible bases that describe the optimal vertex through the use of the Simplex algorithm.

This note highlights some of the problems that arise in degenerate LP's. Firstly, there exists a number of alternate bases that define the optimal vertex. Secondly, each basis has a different set of marginal variables associated with it and these may not be all non-negative. Hence, at any degenerate vertex, it is possible for any one marginal variable to take a number of different values. Consequently, the use of a LP to value resources at the margin is not as straightforward as Strum indicated because the shadow prices (marginal variables) are many sided.

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