STUDIES IN SUPERSYMMETRY

by

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This thesis is dedicated to

Liz and John

ABSTRACT

The research in this thesis is concerned solely with the properties of supersymmetric theories. First, we examine what supersymmetric models are renormalizable and ghost free. We show that the Wess-Zumino Lagrangian and its gauge extension are the only physically viable supersymmetric Lagrangians which can be constructed from scalar superfields, that has an interaction at most cubic in the scalar superfield.

In the rest of the thesis we examine symmetry breaking in supersymmetric theories. The pattern of spontaneous and explicit symmetry breaking at the tree level is investigated in SU(N) supersymmetric gauge theories for the simplest representations. We calculate the one loop effective potentials for two supergauge models (SU(2) adjoint representation and U(1)) and find that they vanish if supersymmetry is conserved. Generalizing this result we prove that the effective potential vanishes to all orders in perturbation theory for those vacuum expectation values of the fields which conserve supersymmetry. Finally, we calculate the one loop effective potential for a general theory, which does not contain gauge particles, and speculate on the higher order effects.

These calculations show that despite the aesthetic appeal and attractive technical features of supersymmetry there are severe difficulties in trying to construct realistic models where supersymmetry is not explicitly broken. However, further progress may be made by considering, in more detail, models in which super-

symmetry is explicitly broken; in the hope that some unknown mechanism may justify this somewhat ad hoc approach.

PREFACE

The work presented in this thesis was carried out in the Department of Theoretical Physics, Imperial College, between October 1973 and May 1976 under the supervision of Professor Abdus Salam. The material presented is the original work of the author, except where stated in the text, and has not been submitted in this or any other University for any other degrees.

The author wishes to thank Professor Abdus Salam for suggesting many avenues of research and for his constant help and encouragement while the work was in progress. Thanks are also due to Dr. R. Delbourgo for many very helpful discussions, to all members of the Theoretical Physics Department, and to Mrs. V. Maughan who so patiently typed the manuscript.

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CONTENTS

CHAPTER I

INTRODUCTION

1.1 Motivation and general introuction

Supersymmetry arose in the context of dual models in 1+1 dimension. The basic idea was put forward by Ramond(1) and later developed by Gervais and Sakita, Iwasaki and Kikkawa, Neveu and Schwarz(2). However, it was only when Wess and Zumino(3) constructed a supersymmetric Lagrangian in the context of a conventional field theory in 3+1 dimensions that general interest was aroused. This model had the remarkable property that fermions and bosons were transformed into each other under the supersymmetric transformation and therefore belonged to the same multiplet. Hence, it was possible to construct models in which there existed a deep connection between fermions and bosons. Such a connection has been believed by some to be fundamental to an explanation of particle physics.

Since the transformation mixes commuting and anticommuting fields it must contain anticommuting parameters. Salam and Strathdee(4) showed that supersymmetry could be understood in the context of an eight dimensional space (4 dimensions labelled by anticommuting parameters and usual 4 space-time dimensions). There remained the question of what are the properties of supersymmetric models. These properties fall naturally into two categories;

1) The first category being those properties which ensure that the model is physically sensible and tractable; that is, it is renormalizable and has a well behaved energy spectrum.

2) The second category being how far the properties of the physically sensible and tractable models relate to those of the actual world.

Chapter one comprises an introduction to supersymmetry and a derivation of the more important formulae used later in the thesis. Properties of category (I) are the concern of chapter two. As a consequence of the remarkable renormalization properties of the Wess-Zumino Lagrangian[5] it is natural to ask if there exist supersymmetric models that are renormalizable although they would be conventionally regarded as non-renormalizable (meaning they contain interaction terms of mass dimension higher than four). Or, if there exist conventionally renormalizable supersymmetric models other than the Wess-Zumino Lagrangian. Because we can not carry out renormalization programmes on all such models we only consider the most likely class in chapter two. Here we show that the Wess-Zumino Lagrangian is the only viable supersymmetric Lagrangian which can be constructed from scalar superfields and the covariant derivative that has an interaction cubic in the scalar superfield.

Properties in category (II) are discussed in the remaining chapters of the thesis. However, given a Lagrangian there exists no known way in general to calculate, with confidence, the physical scattering amplitudes except for the limit of small coupling. Therefore, we are forced to compare the properties of supersymmetric theories not with those of the real world, but with the most acceptable theoretical models of reality; namely spontaneously broken gauge theories. Salam and Strathdee and Ferrara and Zumino[6] showed that supergauge theories can be constructed. In the conventional gauge theories a realistic mass spectrum is obtained by spontaneously breaking the gauge symmetry via the

Higgs-Kibble mechanism. It is a trivial consequence of the supersymmetry algebra that, if supersymmetry is conserved, all particles in the same multiplet have the same mass. As spontaneous symmetry breaking is required in gauge theories it would be desirable if we could spontaneously break both the gauge and supersymmetry.

However, in contrast to conventional gauge theories, where the choice of tachyonic mass terms in the scalar sector will lead to breaking of the internal symmetry, there exists no such simple mechanism in supergauge theories. In fact, having chosen the fermionic content of the theory, the Higgs potential is completely determined. It is the search for spontaneous symmetry breaking mechanisms which is the central problem in constructing realistic supersymmetric theories.

In chapter three we examine the pattern of symmetry breaking, at the tree level, for SU(N) supergauge theories for the simplest representations. We found that if a singlet was introduced it was possible to spontaneously break the internal symmetry, however, it was not possible to break supersymmetry. Also, we discuss the effect of introducing mass terms which explicitly break supersymmetry.

In the remaining chapter we consider the effect of the quantum corrections to the classical potential. These corrections are particularly important in supersymmetry for two reasons. Firstly, including the quantum corrections may radically enlarge the representations for which it is possible to induce spontaneous symmetry breaking; secondly, they are required to remove the physical ambiguity in the vacuum state which is often encountered at the tree level. In chapter four we calculate the one-loop effective potential for two simple supergauge models (SU(2) adjoint representation and U(1)). It is found that the potential is complex if supersymmetry is violated and zero if supersymmetry is conserved. This result leads us to suspect that the supersymmetric effective potential would vanish to all orders of perturbation theory for fields whose vacuum expectation values conserve supersymmetry. This result is proved for a general theory in chapter five.

This investigation leaves open the question of the general behaviour of the supersymmetric effective potential in the region where the fields acquire vacuum expectation values which break supersymmetry. The result of chapter four would indicate that there might exist large areas in which it is complex in this region. Hence, in chapter six we partially answer this question by calculating the one loop effective potential for a general theory which does not contain gauge particles and speculate on the higher order effects.

To conclude, supersymmetry has been shown to be a much more restrictive symmetry than originally supposed. We found that, at tree level, although it was possible, to break spontaneously the internal symmetry, it was not possible to break supersymmetry for the representations considered. Also, we proved that if supersymmetry is conserved the effective potential vanishes to all orders of perturbation theory and hence the degeneracy often present at the tree level is unresolved and so leads to a physical ambiguity in the vacuum state. (Put another way, if supersymmetry is conserved pseudo-goldstone bosons remain pseudo-goldstone bosons to all orders.) Several one loop calculations indicated that there is unlikely to exist a stable supersymmetry violating vacuum for theories which do not break supersymmetry at the tree level. However, explicit soft breaking (mass terms) of supersymmetry could lead for some representations to asymptotic freedom and spontaneously broken internal symmetry. Therefore, despite the aesthetic appeal and attractive technical features of supersymmetry there are severe difficulties in trying to construct realistic models where the supersymmetry is not explicitly broken. Nevertheless, a future programme for research could be to consider, in more detail, models in which supersymmetry is explicitly broken. It may be hoped that some mechanism will be discovered with improved calculation techniques that will justify this somewhat ad hoc symmetry breaking.

The research contained in chapter two is published in Nuclear Physics B[7] and that contained in chapter five is to be published in Nuclear Physics B[10]. The research in chapter three is to be published in the Journal of Physics. Although the work on the SU(2) adjoint representation in chapter four was not submitted for publication by myself a similar piece of work was published in Physical Review D by G. Woo.

1.2 Introduction to Supersymmetry

This section is devoted to an account of the early development of supersymmetry and an explanation of the notation and techniques which will be used in the rest of the thesis. It is convenient to use the formalism developed by Salam and Strathdee(7).

This supposes the existence of an eight dimensional space. Four dimensions are those of space-time and the other four are labelled by anticommuting Majorana spinors; each point is represented by the pair (x_μ, θ_α) where

$$\theta_\alpha \theta'_\beta + \theta'_\beta \theta_\alpha = 0 \qquad (1)$$

The group action on this space is the Poincare group and supersymmetry. The transformation being

$$\begin{aligned} x_\mu &\longrightarrow x'_\mu = \Lambda_{\mu\nu} x^\nu + b_\mu \\ \theta_\alpha &\longrightarrow \theta'_\alpha = a_\alpha^\beta(\Lambda)\,\theta_\beta \end{aligned} \qquad (2)$$

and

$$\begin{aligned} x_\mu &\longrightarrow x'_\mu = x_\mu + \tfrac{i}{2}\bar{\epsilon}\gamma_\mu\theta \\ \theta_\alpha &\longrightarrow \theta_\alpha + \epsilon_\alpha \end{aligned} \qquad (3)$$

Where $a_\alpha^\beta(\Lambda)$ denotes the Dirac spinor representation of the homogeneous Lorentz transformation Λ and ϵ like θ is a Majorana spinor. The properties of Majorana spinors and the related conjugation matrix are given in appendix A.

It is easy from equation (2) and (3) to calculate the algebra of the group generators:-

$$\{ S_\alpha , S_\beta \} = -(\gamma_\mu C)_{\alpha\beta} P_\mu$$

$$[S_\alpha , P_\mu] = 0$$

$$[S_\alpha , J_{\mu\nu}] = \frac{1}{2} \sigma_{\mu\nu} S_\alpha$$

The generators of the Poincare group are P_μ and $J_{\mu\nu}$ and S_α is the generator of supersymmetry and is a Majorana spinor. It would of course be possible to drop the Majorana constraint throughout. However, this would lead to vastly more complex theories.

Supersymmetry consists in constructing field theories, in the eight dimensional space, which are invariance under the transformations given by equation (2) and (3). A scalar field is defined by

$$\phi(x, \theta) = \phi(x', \theta') \tag{5}$$

Higher spin fields may be defined, but these are not considered here and no realistic model has been constructed from such fields as, in general, they are non-renormalizable. It follows from the anticommuting nature of θ that terms of the form

$$\theta_{\alpha_1} \theta_{\alpha_2} \ldots\ldots \theta_{\alpha_n}$$

vanish for $n > 4$. Therefore, we can expand ϕ in the form

$$\phi(x,\theta) = A(x) + \bar{\theta}\psi(x) + \frac{1}{4}\bar{\theta}\theta F(x) + \frac{1}{4}\bar{\theta}\gamma_5\theta\, G(x) + \frac{1}{4}\bar{\theta}\, i\gamma_\nu\gamma_5\theta\, A_\nu(x) + \frac{1}{4}\bar{\theta}\theta\bar{\theta}\chi(x) + \frac{(\bar{\theta}\theta)^2}{32} D(x)$$

Differentiation with respect to θ can be defined in a straight forward way.

$$f(\theta + \delta\theta) = f(\theta) + \delta\bar{\theta}^\alpha \frac{\partial f}{\partial\bar{\theta}^\alpha}$$

where $\delta\bar{\theta}^\alpha$ is infinitesimal. The above equation defines the right derivative and provided we always consider differentiation with the order of factors above it is unnecesary to consider the left derivative.

It will prove extremely useful to the development of the theory to define a differential operator which is a scalar under supersymmetry and a spinor under the Lorentz group:-

$$D_\alpha = \frac{\partial}{\partial\bar{\theta}^\alpha} - \frac{i}{2}(\gamma_\mu\theta)_\alpha \frac{\partial}{\partial x_\mu} \qquad (7)$$

The important properties of D are given in appendix B.

Although the scalar superfield is invariant under supersymmetry, we now consider its decomposition into irreducible parts. First consider the identity for the operator $\bar{D}D$

$$(\bar{D}D)^3 + 4\,\partial^2(\bar{D}D) = 0 \qquad (8)$$

Since $\bar{D}D$ is a covariant operator, subspaces on which it has the same eigenvalues will form representations of supersymmetry. However, we can from equation (8) deduce these eigenvalues and the projectors onto the representation subspaces[8]. They are

$$E_+ = -\frac{1}{4\partial^2}\bar{D}\left(\frac{1-i\gamma_5}{2}\right)D\,\bar{D}\left(\frac{1+i\gamma_5}{2}\right)D$$

$$E_- = -\frac{1}{4\partial^2}\bar{D}\left(\frac{1+i\gamma_5}{2}\right)D\,\bar{D}\left(\frac{1-i\gamma_5}{2}\right)D$$

$$E_1 = 1 + \frac{1}{4\partial^2}(\bar{D}D)^2 \tag{9}$$

Applying these projectors to the scalar superfield and assuming the expansion given in equation 6 we obtain the following reduction into the subspaces ϕ_+, ϕ_- and ϕ_1

$$\begin{aligned}
A &= A_+ + A_- + A_1 \\
\psi &= \psi_+ + \psi_- + \psi_1 \\
F &= F_+ + F_- \\
G &= iF_+ - iF_- \\
A_\mu &= i\partial_\mu A_+ - i\partial_\mu A_- + A_{1\mu} \\
\chi &= -i\not{\partial}\psi_+ - i\not{\partial}\psi_- + i\not{\partial}\psi_1 \\
D &= -\partial^2 A_+ - \partial^2 A_- + \partial^2 A_1
\end{aligned} \tag{10}$$

$A_{1\mu}$ is transverse ($\partial^\mu A_\mu = 0$) and $\psi_\pm$ are chiral Majorana spinors. ϕ_+, ϕ_- and ϕ_1 are irreducible representations of supersymmetry.

To construct supersymmetric Lagrangians we must state some important properties of $\phi_\pm$ and ϕ_1 which can be derived from equation (10). $\phi_\pm(1)\ \phi_\pm(2)$ is a representation of supersymmetry of $\phi_\pm$ type, but any other product is a general scalar superfield. The scalar superfield transforms under an infinitesimal supersymmetric transformation are

$$\delta A = \bar{\epsilon}\psi$$

$$\delta\psi = \frac{1}{2}\left[F + \gamma_5 G + i\gamma_\mu\gamma_5 A_\mu - i\not{\partial}A\right]\epsilon$$

$$\delta F = \frac{1}{2}\bar{\epsilon}\chi - \frac{i}{2}\bar{\epsilon}\not{\partial}\psi$$

$$\delta G = \frac{1}{2}\bar{\epsilon}\gamma_5\chi - \frac{i}{2}\bar{\epsilon}\gamma_5\not{\partial}\psi$$

$$\delta A_\nu = \frac{1}{2}\bar{\epsilon}i\gamma_\nu\gamma_5\chi + \frac{i}{2}\bar{\epsilon}\gamma_\mu i\gamma_\nu\gamma_5\partial_\mu\psi$$

$$\delta\chi = \frac{1}{2}\left[D - i\not{\partial}F - i\not{\partial}\gamma_5 G - i\gamma_\nu\gamma_5 i\not{\partial}A_\nu\right]\epsilon$$

$$\delta D = -i\bar{\epsilon}\not{\partial}\chi. \tag{11}$$

The Lagrangian must consist of terms which are Lorentz and supersymmetric invariant or at least invariant up to a divergence. The only possibilities are the F and D terms of chiral superfields ($\phi_\pm$) and the D terms of the other types (ϕ, ϕ_1). These components can be selected in an invariant way by the appropriate number of operations with the covariant derivative; twice for an F term and four times for a D term.

The simplest model is in fact the Wess-Zumino Lagrangian

$$\mathcal{L} = \frac{(\bar{D}D)^2}{8}(\phi_+ \phi_-) - \frac{\bar{D}D}{2}\left(V(\phi_+) + V(\phi_-)\right)$$

where

$$V(\phi_\pm) = \frac{m}{2}\phi_\pm + \frac{\lambda}{3!}\phi_\pm^3$$

When calculated in components.

$$\mathcal{L} = \partial_\mu A_+ \partial^\mu A_- + \frac{1}{2}\bar{\Psi} i \not{\partial} \Psi + F_+ F_-$$

$$+ m\left(F_+ A_+ + F_- A_- - \frac{1}{2}\bar{\Psi}_+^c \Psi_+ - \frac{1}{2}\bar{\Psi}_-^c \Psi_-\right)$$

$$+ \lambda\left(F_+ A_+^2 + F_- A_-^2 - \frac{A_-}{2}\bar{\Psi}_-^c \Psi_- - \frac{A_+}{2}\bar{\Psi}_+^c \Psi_+\right) \qquad (12)$$

In appendix C we give some useful formulae for the expansion of superfields into components.

Salam and Strathdee and Ferrara and Zumino[6] demonstrated that it was possible to construct supersymmetric gauge Lagrangians. Since these involve spin 1 particles they must include a full superfield, Ψ. The Lagrangian is

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_m + \mathcal{L}_I$$

$$\mathcal{L}_G = \frac{\bar{D}D}{8}\left[\bar{\Psi}_{\alpha+}\Psi_{\alpha-} + h.c\right]$$

where

$$\Psi_{\alpha\pm} = \frac{-i}{2\sqrt{2}\,g}\bar{D}\frac{(1 \mp i\gamma_5)}{2}D\left\{e^{\mp 2g\Psi}\left(\frac{(1 \pm i\gamma_5)}{2}D\right)_\alpha e^{\pm 2g\Psi}\right\}.$$

and

$$\mathcal{L}_m = \frac{(\bar{D}D)^2}{8}\left(\phi_+^{\dagger} e^{2g\psi} \phi_+ + \phi_-^{\dagger} e^{-2g\psi} \phi_- \right)$$

$\mathcal{L}_I$ are the possible invariant interaction terms. $\mathcal{L}$ is invariant under

$$\phi_\pm \longrightarrow \phi_\pm' = e^{i\Lambda_\pm(x,\theta)} \phi_\pm(x,\theta)$$

$$e^{g\psi} \longrightarrow e^{g\psi'} = e^{i\Lambda_\pm^{\dagger}} e^{g\psi} e^{-i\Lambda_\pm}$$

where $\Lambda_\pm(x,\theta) = \mathbb{F}^r \epsilon_{r\pm}(x,\theta)$ with $\mathbb{F}^r$ the group generator and $\epsilon_{r\pm}(x,\theta)$ an arbitrary chiral field. The component form $\mathcal{L}$ is given in chapter three.

The calculation of Feynman diagrams in supersymmetric theories is considerably simplified if it is performed in a manifestly supersymmetric theory. Here we outline the supersymmetric Feynman rules. The propagator for the Wess-Zumino Lagrangian can be derived using functional techniques.

$$\mathcal{L} = \frac{(\bar{D}D)^2}{8}(\phi_+\phi_-) - \frac{m}{2}\frac{\bar{D}D}{2}(\phi_+^2 + \phi_-^2)$$

$$- \frac{\bar{D}D}{2}(j_+\phi_+ + j_-\phi_-) + \text{interaction}.$$

$j_\pm$ are the chiral superfield sources for the fields. The resulting equation of motion is

$$\bar{D}D\,\phi_\pm - 2m\,\phi_\mp = 2j_\mp$$

The propagator is given by

$$\Delta_{\substack{+-\\-+}}(1,2) = \frac{i}{\hbar} < \phi_{\pm}(x_1, \theta_1)\, \phi_{\mp}(x_2, \theta_2)>$$

$$= \frac{\delta\, \phi_{\mp}(x_1, \theta_1)}{\delta\, J_{\pm}(x_2, \theta_2)}$$

Using supergauge invariance of the vacuum

$$= \frac{i}{\hbar} < \phi_{\pm}(x_1 - x_2 + \tfrac{i}{2}\bar{\theta}_1 \gamma \theta_2, \theta_1 - \theta_2)\, \phi_{\mp}(0, 0)>$$

$$= \frac{i}{\hbar} < \phi_{\pm}(x_1 - x_2 + \tfrac{i}{2}\bar{\theta}_1 \gamma \theta_2, \theta_1 - \theta_2)\, A(0)>$$

$$= \frac{\delta\, \phi_{\mp}(x_1 - x_2 + \tfrac{i}{2}\bar{\theta}_1 \gamma \theta_2, \theta_1 - \theta_2)}{\delta\, J_{A\pm}(0)}$$

From the equation of motion

$$\phi_{\pm} = -\frac{1}{\partial^2 + M^2}\left\{ M J_{\pm} + \frac{\bar{D}D}{2} J_{\mp} \right\}$$

we obtain

$$\Delta_{\pm\mp}(1,2) = -\frac{1}{\partial^2 + M^2} \exp\left\{ \mp \frac{1}{4} \left(\bar{\theta}_1 \not{\partial} \gamma_5 \theta_1 \right.\right.$$

$$\left.\left. + \bar{\theta}_2 \not{\partial} \gamma_5 \theta_2 + \frac{i}{2} \bar{\theta}_1 (1 \pm i\gamma_5) \not{\partial} \theta_2 \right\} \delta(x_1 - x_2)\right)$$

Similarly for $\Delta_{\pm\pm}$

At vertices constructed from superfield of the same chirality we must take $\frac{\bar{D}D}{2}$ and for vertices constructed from superfields of different chirality or non-chiral superfields we must take $\left(\frac{\bar{D}D}{8}\right)^2$

Finally, we note some important results concerning spontaneous symmetry breaking. It is easy to see from the supersymmetric transformations given in equation (11) that Lorentz invariant field expectation values break supersymmetry if and only if the auxiliary fields (fields whose derivatives do not appear in the Lagrangian that is the F and D's) acquire vacuum expectation values. If supersymmetry is broken there appears a Goldstone fermion associated with the broken generators of supersymmetry. The expression for the Goldstone fermion was shown by Salam and Strathdee[9] to be

$$\frac{1}{N}\left\{ \sum_j \langle F_j - \gamma_5 G_j \rangle \psi_j(x) + \right.$$

$$\left. \frac{1}{2} \sum_{jk} \langle -\gamma_5 D_{jk} \rangle \lambda_{jk} \right\} .$$

N is a normalization factor and λ_{jk} and D_{jk} are the λ and D components of the gauge field.

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CHAPTER 2

PHYSICAL VIABILITY OF A CLASS OF SUPERSYMMETRIC LAGRANGIANS

2.1 Introduction

Perhaps the most remarkable technical feature of supersymmetric theories is their renormalization properties. In order to make a conventional renormalizable field theory finite one is required to regard all parameters in the bare Lagrangian as infinite (coupling constants, masses and the implicit wavefunction normalization). Hence, a conventional renormalizable theory comprising one scalar, one pseudo scalar and a fermion field would require 13 infinite bare constants to renormalize the theory. The analogous supersymmetric theory is the Wess-Zumino Lagrangian[1] which involves only 3 coupling constants at the tree level. The relations between the couplings are preserved when the theory is renormalized. However, only one infinite (wavefunction) renormalization is required, instead of the three expected, to make the theory finite. Consequently, it is natural to ask if there exist some conventionally non-renormalizable theories which become renormalizable when the coupling constants are constrained to admit supersymmetry. Or, if there exist theories, other than the Wess-Zumino Lagrangian and its gauge extension, which are renormalizable on conventional grounds.

This chapter consists of a search for such theories. Because it is not possible to test all supersymmetric theories we examine only those which are closest to conventionally renormalizable theories (containing no term in the Lagrangian of dimension greater than four). The actual class considered is supersymmetric Lagrangians constructed only from superfields and the covariant derivative that has an interaction at most cubic in the superfields.

Theories which have interactions quartic in the superfield will contain terms with scalar fields to the sixth power. The simplest such theory was examined by Delbourgo[3]

$$\mathcal{L} = \left(\frac{\bar{D}D}{8}\right)^2 \phi_+ \phi_- - \frac{\bar{D}D}{2}\left\{ m\left(\phi_+^2 + \phi_-^2\right) + \lambda\left(\phi_+^3 + \phi_-^3\right) + f\left(\phi_+^4 + \phi_-^4\right)\right\}$$

and was found to be non-renormalizable.

A renormalization programme consists in absorbing all the infinities which arise in perturbation theory into the coupling constants. More precisely, given the bare Lagrangian $\mathcal{L}(\phi, g_0)$ we shift all coupling constants $g_0 = g + \delta g$ such that g is finite and δg may be infinite.

$$\mathcal{L}(\phi, g_0) = \mathcal{L}(\phi, g) + \Delta\mathcal{L}(\phi, \Delta g)$$

Although $\mathcal{L}(\phi, g)$ is finite, infinities occur when its Feynman graphs are calculated. In a renormalizable theory it is possible to choose δg such that the Feynman for $\mathcal{L}(\phi, g_0)$ graphs contributing to a given process are finite at each order of perturbation theory. For example, if the infinities occuring in the first order of pergurbation theory can not be cancelled by the infinites occuring in $\Delta\mathcal{L}$ the theory would be non-renormalizable.

To examine the physical viability of the Lagrangians we must check that the theory has a well behaved energy spectrum as well as being renormalizable. Hence, having expressed the Lagrangian in component form we must check for the presence of ghosts, because these lead to an energy spectrum which is unbounded below. If no ghosts are present we examine the renormalizability of the Lagrangian. This entails calculating the counter-terms required to cancel all infinities occuring in the first order graphs arising from $\mathcal{L}(\phi, g)$. If these counter-terms can not be found in $\Delta\mathcal{L}$ then the theory is non-renormalizable. Since it is hoped that cancellations due to supersymmetry will render the theories renormalizable, the calculations will be more transparent and quicker if performed in supersymmetric formalism. To this end the supersymmetric Feynman rules are derived by the methods set out in chapter one.

The Lagrangians to be considered are

$$\mathcal{L}_1 = (\bar{D}D)^2 \left\{ \tfrac{1}{2}\phi(\bar{D}D - 2m)\phi + g\phi^3 \right\}$$

$$\mathcal{L}_2 = (\bar{D}D)^2 \left\{ \tfrac{1}{2}(\phi_+ + \phi_-)(\bar{D}D - 2m)(\phi_+ + \phi_-) + g(\phi_+ + \phi_-)^3 \right\}$$

$$\mathcal{L}_3 = \frac{(\bar{D}D)^2}{8}(\phi_+ \phi_-) + (\bar{D}D)^2 g(\phi_+ + \phi_-)^3$$

$\mathcal{L}_1$ having been already suggested by Salam and Strathdee[2].

It will be shown that $\mathcal{L}_1$ is non-renormalizable and the source of the non-renormalizability is ϕ_1. This leads us to $\mathcal{L}_2$ which is renormalizable, but contains ghosts. On the other hand $\mathcal{L}_3$ is non-renormalizable. In the class of supergauge lagrangians considered above, the Wess-Zumino lagrangian is unique in that it alone is renormalizable and contains no ghosts.

2.2 Non-Renormalizability of $\mathcal{L}_1$

(A) The action

$$S = \int d^4x \, \mathcal{L}(x, \theta)$$

is rendered supergauge invariant by demanding that $\mathcal{L}$ is transformed into a 4 divergence under variation. This is implemented by the rule already stated in the introduction.

A comparison of the infinities present in the second order diagrams and the normal counter terms generated by $\mathcal{L}_1$ quickly shows that $\mathcal{L}_1$ is non-renormalizable. This calculation is grossly simplified if we work in superfield notation rather than try to treat each component field individually. The superpropagators are calculated using functional methods. To this end we introduce a classical supercurrent j into the lagrangian

$$\mathcal{L}_1' = \mathcal{L}_1 - (\bar{D}D)^2 \, (2 j \phi)$$

can be expanded in a complete set of θ 's

$$\phi(x,\theta) = A(x) + \bar{\theta}\psi(x) + \frac{1}{4}\bar{\theta}\theta F(x) + \frac{1}{4}\bar{\theta}\gamma_5\theta G(x)$$

$$+ \frac{1}{4}\bar{\theta}_i \gamma_\nu \gamma_5 \theta A^\nu(x) + \frac{1}{4}\bar{\theta}\theta\bar{\theta}\chi(x) + \frac{1}{32}(\bar{\theta}\theta)^2 D(x) .$$

In order to get the correct source-field combinations we define J to be

$$2j(x,\theta) = j_D - \frac{1}{2}\bar{\theta} j_\chi + \frac{1}{8}\bar{\theta}\theta j_F + \frac{1}{8}\bar{\theta}\gamma_5\theta j_G$$

$$+\frac{1}{8}\bar{\theta}i\gamma_\nu\gamma_5\theta j_{A_\nu} + \frac{1}{8}\bar{\theta}\theta\bar{\theta}(-j_\psi) + \frac{1}{32}(\bar{\theta}\theta)^2 j_A$$

When evaluating $\mathcal{L}_1'$ in terms of its component fields we need only pick out the coefficient of the θ^4 term because

$$\int dx\, (\bar{D}D)^2 = \int dx \left(\frac{d}{d\theta_\alpha}\frac{d}{d\bar{\theta}^\alpha}\right)^2$$

$$\mathcal{L}_1' = 2\{-2\partial_\nu A\partial^\nu F - 2FD - 2G\partial^\nu A_\nu + 2A_\nu\partial^\nu G$$

$$+\bar{\chi}^c\chi - \bar{\chi}^c i\partial\!\!\!/\psi - \bar{\psi}^c i\partial\!\!\!/\chi + \partial^\nu\bar{\psi}^c\partial_\nu\psi - 2m(AD$$

$$+F^2 + G^2 - \bar{\psi}^c\chi - \bar{\chi}^c\psi) + j_A A + \ldots + \bar{j}_\psi^c\psi\}$$

+ interaction terms

(2.1)

The equations of motion for j = 0 are

$$\partial^2 F = mD, \quad \partial_\nu A^\nu = -mG, \quad \partial^2 A = 2mF + D,$$

$$\partial_\nu G = mA_\nu, \quad F + mA = 0, \quad \partial^2\psi + i\partial\!\!\!/\chi = 2m\chi, \quad i\partial\!\!\!/\psi - \chi = 2m\psi$$

Using the equations of motion F, D, A_ν and χ can be eliminated. The equations of motion can be rewritten in superfield notation:

$$(\bar{D}D - 2m)\,\phi = 2j \tag{2.2}$$

from which it follows that†

$$\phi = -\frac{1}{\partial^2 + m^2}\left\{\frac{\bar{D}D}{2} + \frac{(\bar{D}D)^2}{4m}\right\} j - \frac{j}{m} \tag{2.3}$$

(B) Using supergauge invariance of the vacuum

$$\Delta(1,2) = \frac{i}{\hbar} \langle T\, \phi(x_1, \theta_1)\, \phi(x_2, \theta_2) \rangle$$

$$= \frac{i}{\hbar} \langle T\, \phi(x, \theta_{12})\, \phi(0,0) \rangle = \frac{\delta \phi(x, \theta_{12})}{\delta j_A(0)} \tag{2.4}$$

where

$$x = x_1 - x_2 + \frac{i}{2}\,\bar{\theta}_1 \gamma \theta_2 \quad , \theta_{12} = \theta_1 - \theta_2$$

Using (2.3) and (2.4) and

$$-\frac{1}{2}\bar{D}Dj = \frac{\bar{\theta}\theta j_A}{16} \qquad + \text{ terms not involving } j_A$$

$$-\frac{1}{2}(\bar{D}D)^2 j = \frac{1}{2} j_A - \frac{(\bar{\theta}\theta)^2}{64}\partial^2 j_A \qquad + \text{ terms not involving } j_A$$

---

† We used the fact that $(\bar{D}D)^3 = -4\partial^2 \bar{D}D$.

it follows that

$$\phi = \frac{1}{\partial^2 + m^2}\left\{ -\frac{j_A}{4m} + \frac{(\bar{\theta}\theta)^2}{128}\partial^2 j_A + \frac{\bar{\theta}\theta}{16} j_A \right\} - \frac{(\bar{\theta}\theta)^2}{64m} j_A$$

+ terms not involving j_A.

Hence

$$\Delta(1,2) = -\frac{e^{\frac{i}{2}\bar{\theta}_1 \not{\partial} \theta_2}}{4m(\partial^2 + m^2)}\left\{ e^{-\frac{m}{4}\bar{\theta}_{12}\theta_{12}} + \frac{(\bar{\theta}_{12}\theta_{12})^2(\partial^2 + m^2)}{32} \right\} \delta(x_1 - x_2)$$

(C) Armed with the propagator we can calculate the Feynman graphs. The second order diagrams are produced by

$$-\frac{g^2}{2} T \left\{ \int (\bar{D}D)^2 \phi^3(1)\, dx_1 \int (\bar{D}D)^2 \phi^3(2)\, dx_2 \right\}$$

The second order self energy diagram with only A external lines is

$$\Sigma_{AA} = \left(\frac{d}{d\theta_{1\alpha}} \frac{d}{d\bar{\theta}_1{}^{\alpha}}\right)^2 \left(\frac{d}{d\theta_{2\beta}} \frac{d}{d\bar{\theta}_2{}^{\beta}}\right)^2 \Delta(1,2)^2$$

Now†

$$\Delta^2(1,2) = \frac{e^{\frac{i}{2}\bar{\theta}_1 \not{\partial} \theta_2}}{16m^2}\left\{ \frac{e^{-\frac{m}{4}\bar{\theta}_{12}\theta_{12}}}{\partial^2 + m^2}\delta(x_1 - x_2) + \frac{(\bar{\theta}_{12}\theta_{12})^2}{32}\delta(x_1 - x_2) \right\}^2$$

The only term to survive the θ differentiation is $\theta_1{}^4 \quad \theta_2{}^4$, but

---

† As $e^{\frac{i}{2}\bar{\theta}_1 \not{\partial} \theta_2}$ is just a translation on the δ function.

$$(\bar{\theta}_{12}\theta_{12})^2\,\bar{\theta}_1 \not\partial \theta_2 = (\bar{\theta}_{12}\theta_{12})^2\,\bar{\theta}_{12} \not\partial \theta_2' = 0$$

$$(\bar{\theta}_{12}\theta_{12})\;(\theta_1 \not\partial \theta_2)^3 = 0$$

Then $\Delta\,(1,2)^2 = \dfrac{(\bar{\theta}_1 \not\partial \theta_4)^4\,\Delta^2(x_{12})}{4!\;2^4\;16m^2}$ + terms with less θ's

$= \dfrac{(\bar{\theta}_1\theta_1)^2(\bar{\theta}_2\theta_2)^2(\partial^2)^2\;\Delta^2(x_1-x_2)}{m^2\;8^2\;16^2}$ + terms with less θ's

So

$$\Sigma_{AA} = \frac{g^2}{4m^2}\,(\partial^2)^2\,\Delta^2(x_1-x_2).$$

In momentum space this gives rise to a logarithmic divergence (analogous to ϕ^3 theory) $\propto (q^2)^2 \ln \Lambda$ (q the external momenta).

Similarly we can calculate the second order self energy for other external lines. For example A and F

$$\Sigma_{AF} = g^2\left(\frac{d}{d\theta_{1\alpha}}\,\frac{d}{d\bar{\theta}_1{}^{\alpha}}\right)^2\left(\frac{d}{d\theta_{2\beta}}\,\frac{d}{d\bar{\theta}_2{}^{\beta}}\right)(\bar{\theta}_2\theta_2)$$

$$\times\ \Delta(1,2)^2 \quad \propto \quad g^2\ \partial^2\ \Delta^2$$

(in momentum space $\propto q^2 \ln\Lambda$).

(D) The normal counterterms are generated through the substitutions

$$\phi_0 = \sqrt{Z_1}\ \phi \quad , \quad g_0 = \frac{Z_2}{Z_1^{3/2}}\, g \ ; \ m_0 = m + \delta m$$

in the bare Lagrangian. From the expression for $\mathcal{L}_1'$ written out in terms of component fields, equation (1), it is clear that the above substitutions can not produce a counterterm looking anything like $\Sigma_{AA} \propto (q^2)^2 \ln\Lambda$ (The Σ_{AF} graph, however does have a counterterm from the wave function renormalization to the $\partial_\nu F\, \partial^\nu A$ term). Since there exists no counterterm coming from wave function, coupling constant or mass renormalization to account for Σ_{AA} we could only put in an arbitrary counterterm $(\bar{D}D)^2(\bar{D}D\phi)^2$ to remove the divergence. This counterterm has no immediate physical interpretation and we conclude that $\mathcal{L}_1$ is non-renormalizable in the conventional sense.

If we started with the Lagrangian $\mathcal{L}_1 + (\bar{D}D)^2\{\bar{D}D\phi\}^2$ we could generate the counterterm required to cancel the troublesome divergence. However, this would lead to ghosts in the theory.

2.3 Origin of the Non-Renormalizability of $\mathcal{L}_1$

In order to see which irreducible part of the superfield causes the non-renormalizability and to compare $\mathcal{L}_1$ with $\mathcal{L}_{WZ}$, the renormalization of which was carried out in superfield notation by Delbourgo and Capper[3], we calculate the propagators between the irreducible parts of the superfield. This is achieved by applying the projectors $E_\pm$, E_1 to equation (2.4).

$$E_\pm = -\frac{1}{4\partial^2}\bar{D}\left(\frac{1 \mp i\gamma_5}{2}\right)D\,\bar{D}\left(\frac{1 \pm i\gamma_5}{2}\right)D$$

$$E_1 = 1 + \frac{1}{4\partial^2}(\bar{D}D)^2$$

Then

$$E_\pm \frac{i}{\hbar}\langle T\,\phi(x,\theta_{12})\,\phi(0,0)\rangle = \frac{i}{\hbar}\langle T\,\phi_\pm(x,\theta_{12})\,\phi(0,0)\rangle = \Delta_\pm(1,$$

$$= \frac{m}{8\partial^2(\partial^2+m^2)}\left\{1 + \frac{\bar{\theta}_{12}\theta_{12}\partial^2}{4m} \pm \frac{\bar{\theta}_{12}\gamma_5\theta_{12}i\partial^2}{4m} \pm \frac{\bar{\theta}_{12}i\gamma_\nu\gamma_5\theta_{12}i\partial^\nu}{4} - \frac{(\bar{\theta}_{12}\theta_{12})^2\partial^2}{32}\right\}\delta(\qquad (2.5)$$

Similarly

$$\Delta_1(1,2) = E_1\langle T\,\phi(x,\theta_{12})\,\phi(0,0)\rangle = -\frac{1}{4m}\left\{\frac{1}{\partial^2} + \frac{(\bar{\theta}_{12}\theta_{12})^2}{32}\right\}\delta(x$$

(2.6)

Using the properties of the projectors and propagators we find that

$$E_{\binom{\pm}{1}}(x,\theta_{12}) < T\,\phi(x,\theta_{12})\,\phi(0,0) > \; = \; < T\,\phi_{\binom{\pm}{1}}(x,\theta_{12})\,\phi(0,0) >$$

$$= < T\,\phi(0,0)\,\phi_{\binom{\pm}{1}}(x,\theta_{12}) > \; = \; < T\,\phi(x_2,\theta_2)\,\phi_{\binom{\pm}{1}}(x_1,\theta_1) >$$

Hence

$$< T\,\phi_{\binom{\pm}{1}}(x_1,\theta_1)\,\phi(x_2,\theta_2) > \; = \; < T\,\phi(x_2,\theta_2)\,\phi_{\binom{\pm}{1}}(x_1,\theta_1) >$$

Application of the projectors to the above equation gives the

$$\Delta_{\binom{\pm}{1}\binom{\pm}{1}}(1,2) = < T\,\phi_{\binom{\pm}{1}}(x,\theta_{12})\,\phi_{\binom{\pm}{1}}(0,0) >$$

propagators in terms of $E_{\binom{\pm}{1}}$ acting on the propagators of equations (2.5) and (2.6). Use was made of the result $(\bar{D}D)^2 E_{\pm} = -4\partial^2 E_{\pm}$

$$\Delta_{\pm\pm}(1,2) = \frac{e^{\frac{i}{2}\bar{\theta}_1 \not{\partial} \theta_2}}{32(\partial^2+m^2)}\,\bar{\theta}_{12}(1 \pm i\gamma_5)\theta_{12}\,\delta(x_1-x_2) \tag{2.8}$$

$$\Delta_{\mp\pm}(1,2) = \frac{m\, e^{\mp\frac{i}{4}\bar{\theta}_{12}\, i\not{\partial}\gamma_5\theta_{12}}}{8\partial^2(\partial^2+m^2)}\,\delta(x_1-x_2) \tag{2.9}$$

$$\Delta_{1\pm}(1,2) = 0 \tag{2.10}$$

$$\Delta_{11}(1,2) = -\frac{1}{4m}\left\{\frac{1}{\partial^2} + \frac{(\bar{\theta}_{12}\theta_{12})^2}{32}\right\} e^{\frac{i}{2}\bar{\theta}_1\not{\partial}\theta_2}\,\delta(x_1-x_2) \tag{2.11}$$

$\Delta_{\pm\pm}$ and $\Delta_{\mp\pm}$ are very similar in form to those of $\mathcal{L}_{\omega-z}$ except for an extra $\frac{1}{\partial^2}$ in $\Delta_{\mp\pm}$. That is for(2) $\mathcal{L}_{\omega-z}$

$$\Delta_{\pm\pm} = -\frac{m}{2} \exp\left\{\frac{i}{2}\bar{\theta}_1 \not\partial \theta_2\right\} \bar{\theta}_{12} \frac{(1 \pm i\gamma_5)}{2} \theta_{12} \quad \Delta(x_1 - x_2) \tag{2.12}$$

$$\Delta_{\pm\mp} = \exp\left\{\frac{i}{2}\bar{\theta}_1 \not\partial \theta_2 \mp \frac{1}{4} \bar{\theta}_{12} \not\partial \gamma_5 \theta_{12}\right\} \Delta(x_1 - x_2) \tag{2.13}$$

We can now recalculate the self-energy diagrams with only A external lines

$$\Sigma_{\substack{++\\++}} = \Sigma_{\substack{--\\--}} = \Sigma_{\substack{++\\--}} =$$

$$\Sigma_{\substack{++\\+-}} = \Sigma_{\substack{--\\+-}} = 0$$

as the highest possible number of θ 's from the propagators is $(\bar{\theta}_{12}\theta_{12})^2$

$$\Sigma_{\substack{+-\\+-}} \quad \propto (\partial^2)^2 \left\{ \left\{ \frac{1}{\partial^2(\partial^2+m^2)} \right\} \delta(x_1 - x_2) \right\}^2 \text{ which is finite}$$

$$\Sigma_{\substack{+-\\-+}} \quad \propto (\partial^2)^2 \left\{ \frac{1}{\partial^2(\partial^2+m^2)} \delta(x_1 - x_2) \right\}^2 \text{ which is finite}$$

$$\Sigma_{\substack{++\\11}} = \Sigma_{\substack{--\\11}} = 0$$

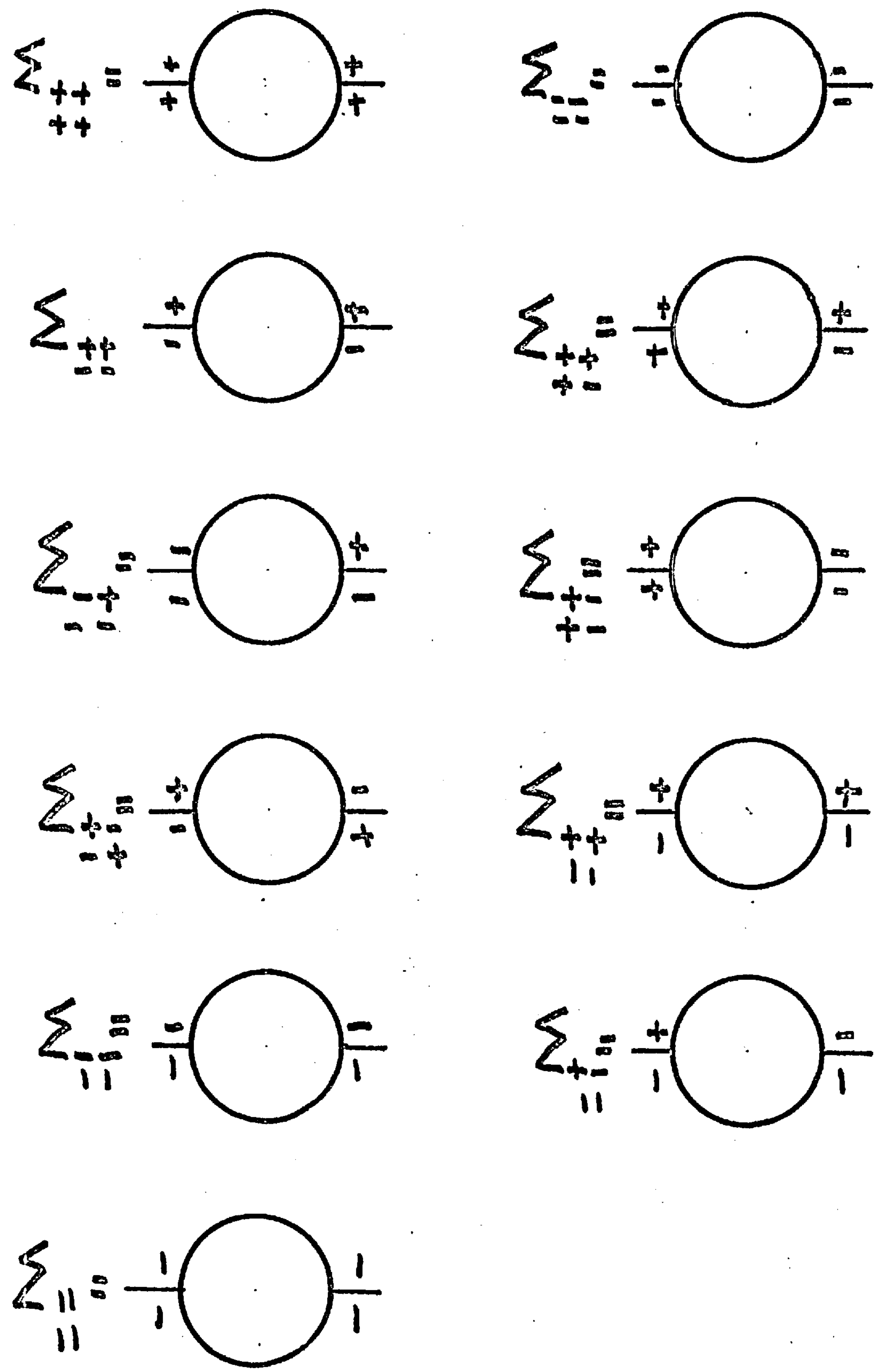

fig 2.1.

$$\sum_{\substack{+-\\11}} \qquad \propto \partial^2 \left\{ \frac{1}{\partial^2(\partial^2+m^2)} \delta(x_1-x_2) \frac{1}{\partial^2} \delta(x_1-x_2) \right\}$$

which is finite.

There remains

$$\sum_{\substack{11\\11}} \qquad \propto (\partial^2)^2 \left\{ \frac{1}{\partial^2} \delta(x_1-x_2) \right\}^2$$

which is the cause of the logarithm divergence. It is clearly ϕ_1 which is responsible for the non-renormalizability of the theory. Further this difficulty is inherent in any theory which contains a self-interacting superfield ϕ, i.e. constructed out of ϕ and D_α, as can be seen by the following: ϕ_1, unlike $\phi_\pm$, has the property that ϕ_1^2 is a general superfield and so that Lagrangian must be of the form

$$\mathcal{L} = (\bar{D}D)^2 \left\{ f(\phi ; D_\alpha \phi) \right\}$$

We now consider the case in which f is allowed to be an arbitrary function of ϕ and $D_\alpha \phi$ consistent with $\mathcal{L}$ being at least superficially a sensible Lagrangian when written in terms of its component fields. The kinetic term can only be constructed from $D_\alpha \phi$, but $(\bar{D}D)^2$ has the property that

$$(\bar{D}D)^2 \left\{ g(\phi) D_\alpha h(\phi) \right\} = -(\bar{D}D)^2 \left\{ (D_\alpha g(\phi)) h(\phi) \right\}$$

and so can be cast in the form $(\bar{D}D)^2\{\phi\,\bar{D}D\phi\}$. The difficulty stems from the fact that $\bar{D}D\,\phi_1 = 0$ so the ϕ_1 part of $\mathcal{L}$ has the form

$$(\bar{D}D)^2\,\{m\,\phi_1^{\,2} + \text{interaction}\}$$

This results in the Δ_{11} propagator of equation (2.11) and so the divergent graph $\propto (q^2)^2 \ln\Lambda$. The counterterm must be generated from $(\bar{D}D)^2\{(\bar{D}D\,\phi)^2\}$ which if added to the original Lagrangian produces ghosts.

If the theory were not self interacting, but described a gauge interaction between a ϕ_1 superfield and two other chiral superfields the gauge symmetry can make it renormalizable. The $(\bar{D}D)^2\,(\bar{D}D\phi)^2$ term is still needed to make the Lagrangian renormalizable but this is the usual gauge fixing term. The ghosts introduced from this term decouple from the physical matric elements as a consequence of gauge Ward identities. We note that this argument does not include theories which make explicit use of ordinary differentiation, in the supersymmetric Lagrangian.

2.4 Renormalizability and Ghosts of $\mathcal{L}_2$

Motivated by the preceding argument we examine $\mathcal{L}_2$. Since

$$(\bar{D}D)^2\{(\phi_+ + \phi_-)\,\phi_1\} = 0 = (\bar{D}D)^2\{\phi_1\,\bar{D}D(\phi_+ + \phi_-)\}$$

the propagators of equations (2.8) and (2.9) are those for $\mathcal{L}_2$. Using these propagators it can be shown that to second order only mass-renormalization is required (see Appendix D). By working with the

$$(\Delta_{++} + \Delta_{--} + \Delta_{+-} + \Delta_{-+})\Big|_{m=0} = \frac{\bar{\theta}_{12}\theta_{12}}{16}\, e^{\frac{1}{2}\bar{\theta}\not{\partial}\theta_2}\,\Delta(x_1 - x_2)$$

propagator it can be shown that $\mathcal{L}_2$ is renormalizable to all orders in the m = 0 case. The essence of the proof is to find the highest number of ∂ 's which can come from the exponentials in the propagators subject to the restrictions that for an n vertex graph any polynomial of θ 's greater than 4n vanishes and $\bar{\theta}_{12}\theta_{12}(\bar{\theta}_1 \not{\partial}\, \theta_2)^3 = 0$. This number enables us to calculate the superficial degree of divergence of the graphs and leaves only graphs for which Vertex + External lines $\leq$ 8 whose finite nature is not accounted for. Of these remaining graphs all are finite except those in Fig. 2.2 which are accounted for by mass-renormalization and adding a harmless $(\bar{D}D)^2\{\phi\}$ to the Lagrangian respectively (see Appendix D). The theory has in the m = 0 case exactly the same renormalization pattern as ordinary ϕ^3 theory.

Examination of the propagator of equation (2.9) leads us to

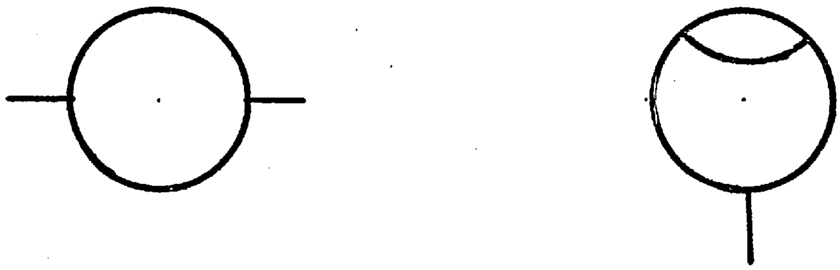

fig 2.2

suspect that the theory has ghosts. Writing $\mathcal{L}_2$ out in terms of its component fields confirms this possibility.

$$\mathcal{L}_2 = -4\partial_\nu A\partial^\nu F - 4\partial_\nu G\,\partial^\nu B + 4\,\partial^\nu\bar{\Psi}^c\partial_\nu\Psi - 2m(\partial_\nu A\partial^\nu A$$

$$+F^2 - G^2 - \partial_\nu B\partial^\nu B + 2\bar{\Psi}^c i\not{\partial}\Psi) + \text{interaction terms}$$

Where the component fields are labelled by

$$(\phi_+ + \phi_-) = A + \bar{\Theta}\Psi + \frac{1}{4}\bar{\Theta}\Theta F + \frac{1}{4}\bar{\Theta}\gamma_5\Theta i\,G$$

$$+\frac{1}{4}\bar{\Theta}i\gamma_\nu\gamma_5\Theta i\,\partial^\nu B + \frac{1}{4}\bar{\Theta}\Theta\,\bar{\Theta}(-i\not{\partial}\Psi) + \frac{(\bar{\Theta}\Theta)^2}{32}(-\partial^2 A)$$

The equations of motion are

$$mF = \partial^2 A \qquad + \text{ interaction terms}$$

$$mG = \partial^2 B \qquad + \text{ interaction terms}$$

After eliminating F and G we obtain terms like $\partial_\nu A\,\partial^\nu\partial^2 A$ i.e. ghosts. If we renormalized so that the physical mass was zero then we could no longer eliminate F and G as above. However, as

$$\partial_\nu A\,\partial^\nu F = \frac{1}{2}\left\{\partial_\nu(A+F)\partial^\nu(A+F) - \partial_\nu(A-F)\partial^\nu(A-F)\right\}$$

where A and F are hermitian fields, we again have ghosts.

Whether the problem concerned with ghosts can be consistently avoided in this theory has not been examined. A possibility for eliminating such ghosts is to consider the larger class of Lagrangians which contain terms of the form $i\partial^{\nu}\phi$ which is a supergauge invariant because S_{α} the supergauge infinitesimal operator commutes with displacements: $[S_{\alpha}, P_{\mu}] = 0$ To use such terms is in some senses a departure from the superfield techniques.

2.5 Non-renormalizability of $\mathcal{L}_3$

It is sufficient to consider only second order diagrams with only A external lines. The propagators for $\mathcal{L}_3$ are the same as for $\mathcal{L}_{W-Z}$ and were given in equations (2.12) and (2.13). The second order self energy graphs for A external lines

$$\Sigma_{\substack{++\\++}} = \Sigma_{\substack{--\\--}} = \Sigma_{\substack{+-\\++}} = \Sigma_{\substack{+-\\--}} = 0$$

$$\Sigma_{\pm\mp} = \Sigma_{\substack{+-\\+-}}$$

$$= \qquad \propto (\partial^2)^2 \left\{ \frac{1}{\partial^2 + m^2} \delta(x_1 - x_2) \right\}^2$$

as the only term to contribute is $(\bar{\theta}_1 \not{\partial}\, \theta_2)^4$. In momentum space these divergent graphs are $\propto (q^2)^2 \ln \Lambda$. For which there is no normal counterterm generated by $\mathcal{L}_3$.

The reader may wonder why the Lagrangian

$$\mathcal{L} = \frac{(\bar{D}D)^2}{8}\left\{ \phi_+ \bar{D}D\phi_+ + \phi_- \bar{D}D\phi_- \right\}$$

$$+\frac{m^2}{2}(\bar{D}D)\left\{ \phi_+ + \phi_- \right\} \qquad + \text{ interaction}$$

is not considered. This is because when $\mathcal{L}$ is written out in terms of components it contains terms like $\partial^\nu \psi \partial_\nu \psi$ and so has ghosts. It is nevertheless of interest to note that provided the interaction does not couple ϕ_+ to ϕ_- then in this theory the only non-zero propagators are

$$\Delta_{\pm\pm}(1,2) = \bar{\theta}_{12} \frac{(1 \pm i\gamma_5)}{\partial^2 + m^2} \theta_2 \exp\left(\frac{i}{2}\bar{\theta}_1 \not{\partial} \theta_2\right) \delta(x_1 - x_2)$$

as a consequence of which only tree graphs are non-zero.

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CHAPTER 3

SYMMETRY BREAKING IN SUPERSYMMETRIC

SU(N) GAUGE LAGRANGIANS AT THE TREE LEVEL

3.1 Introduction

In the previous chapter we saw that there probably exists only one well defined type of supersymmetric theory; namely the Wess-Zumino Lagrangian and its gauge extension. It is the comparison of the properties of these supersymmetric theories with those of experiment that will decide if supersymmetry is a realistic symmetry. The theoretical models which have been most successful are the spontaneously broken gauge theories proposed by Salam and Weinberg. However, their popularity only arose after spontaneously broken gauge theories were shown to be renormalizable by t'Hooft. The reason for this was that one could use the Higgs-Kibble mechanism to spontaneously break the symmetry and give masses to the vector particles. The actual gauge symmetry to be used is not well established and is the subject of considerable debate. The most favoured symmetry groups being SU(N) ($N \geqslant 3$) and SU(N) x SU(M) ($N,M \geqslant 3$). The presence of U(1) subgroups is not desirable as these would destroy the asymptotic freedom properties of the theory. Therefore, it is natural to try to construct realistic supergauge theories. An inherent feature of supersymmetric theories in which supersymmetry is conserved is that all particles in the same supermultiplet have the same mass. Consequently, it is hoped that supersymmetry and the internal symmetry will be broken.

However, in contrast to conventional gauge theories where the choice of tachyonic mass terms in the scalar sector will lead to breaking of the internal symmetry, there exists no such simple mechanism in supergauge theories. Supersymmetry places very strong constraints on the classical potential and we will find that only for certain representations will any symmetry breaking take place.

Despite the initial speculations of Wess and Zumino that supersymmetry was never spontaneously broken, Ilioupolous and Fayet[1] found two supergauge models where supersymmetry was spontaneously broken at the tree level (for a U(1) and SU(2) x U(1) local symmetry).

In this chapter we do not attempt to construct realistic theories, but rather to explore the general difficulties and pattern of spontaneous symmetry breaking in supersymmetric gauge theories. As a suitable class of Lagrangians to study we examine those having SU(N) local symmetry for the simplest representations:- quark and self adjoint. We also consider the $(m,\bar{n})$ representation of SU(M) x SU(N).

Rather than regard supersymmetry as an exact symmetry we could consider it to become exact only in the high energy limit. In this case, as long as the coefficients of the leading terms (terms which have dimensionless coupling constants) in the Lagrangian are constrained by supersymmetry, we are free to choose the coefficients of the non-leading terms. In particular, we can choose the mass terms to conserve the internal symmetry and violate supersymmetry. If we choose them so as to produce tachyonic masses they will inevitably lead to spontaneous symmetry breaking at the tree level. In part two of chapter two we consider this possibility in the context of Lagrangians having an SU(N) local symmetry for the quark and adjoint representation. We also consider the possible implications for asymptotically free and infra red stable theories.

2. SUPERSYMMETRIC GAUGE LAGRANGIANS

Supersymmetric gauge Lagrangians have been developed by Salam and Strathdee[2] and Ferrara and Zumino[3]. We refer the reader to these papers for the techniques and formalism used. These theories are constructed out of ϕ_+ and ϕ_- chiral irreducible superfields which also form the basis vectors for the representation of a group, G, and the gauge fields, $\psi = \psi^r F_r$ in the adjoint representation. F_r are the matrix generators of G in the given representation and must obey

$$[F_r, F_s] = i f_{rst} F_t$$

where f_{rst} are the structure constants of the group, G.

The Lagrangian is generally of the following form

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_M + \mathcal{L}_I .$$

where

$$\mathcal{L}_G = \frac{\overline{D}D}{8} \quad \overline{\psi}_+ \psi_- \quad + \text{h.c.}$$

$$\psi_{\alpha\pm} = -\frac{i}{2\sqrt{2}g} \overline{D} \left(\frac{1\mp i\gamma_5}{2}\right) D \left\{ e^{\mp 2g\psi} \left(\frac{1\pm i\gamma_5}{2} D\right) e^{\pm 2g\psi} \right\}$$

In a special gauge[4] ψ can take the form

$$\psi = \tfrac{1}{4} \overline{\theta}\, i\, \gamma_\nu \gamma_5\, \theta\, W_\nu + \frac{1}{2\sqrt{2}} \overline{\theta}\, \theta\, \overline{\theta}\, \gamma_5\, \lambda + \left(\frac{\overline{\theta}\theta}{16}\right)^2 D$$

Then

$$\mathcal{L}_G = \frac{\text{Tr}}{2} \left\{ -\tfrac{1}{4} V^2_{\mu\nu} + \overline{\lambda}_-\, i \not\nabla \lambda_- + \tfrac{1}{2} D^2 \right\}$$

where

$$V_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + g\, (W_\mu \times W_\nu)$$

$$\nabla_\mu \lambda = \partial_\mu \lambda + g\, (W_\mu \times \lambda)$$

where $(A \times B)_r = f_{rst} \; A_s B_t$.

$$\mathcal{L}_m \doteq \left(\frac{\bar{D}D}{8}\right)^2 \left(\phi_+^+ \, e^{2g\psi} \, \phi_+\right)$$

If ϕ_+ is labelled as

$$\phi_\pm = e^{\mp\frac{1}{4}\bar{\theta}\not{\partial}\gamma_5\theta} \left(A_\pm + \bar{\theta}\,\psi_\pm + \tfrac{1}{4}\,\bar{\theta}(1 \pm i\gamma_5)\theta\, F_\pm\right)$$

Then

$$\mathcal{L}_M = \nabla_\mu A_+^+ \nabla_\mu A_+ + \bar{\psi}_+ \, i \not{\nabla} \psi_+ + F_+^+ F_+$$

$$+ \, ig\sqrt{2}\left(A_+^+ \bar{\lambda}\,\psi_+ - \bar{\psi}_+ \lambda A_+\right) + g\, A_+^+ \, DA_+$$

where the covariant derivatives are given by

$$\nabla_\mu A_+ = \partial_\mu A_+ - ig\, W_\mu A_+ .$$

We can also include a ϕ_- term if required.

The transformation properties which leave $\mathcal{L}$ invariant are

$$\phi_\pm \to e^{i\Lambda_\pm} \phi_\pm$$

$$e^{g\psi} \to e^{i\Lambda_+^+} e^{g\psi} e^{-i\Lambda_+}$$

where $\Lambda_\pm = \varepsilon_{\pm r} \; F^r$ and $\varepsilon_{\pm r}$ are arbitrary functions of x and θ.

$\mathcal{L}_I$ includes all possible interaction terms compatible with the representation used.

3. SPONTANEOUS SYMMETRY BREAKING

The effective potential in the tree approximation is found by eliminating the auxiliary fields and can be written in the form

$$V = \tfrac{1}{2} \sum_{\substack{\text{gauge}\\\text{fields}}} D^2 + \sum_{\substack{\text{matter}\\\text{fields}}} (F_+^+ F_+ + F_-^+ F_-)$$

Supersymmetry places very strong restrictions on the type of potential allowed. V is positive and is a function of the matter fields $A_\pm$. The auxiliary fields are of the form a constant, λ + powers of $A_\pm$, $A_\pm^+$ up to second order.

If $\lambda = 0$ then the choice $A_\pm = 0$ implies $V = 0$ which also minimizes V. Therefore, if spontaneous symmetry breaking is to occur a term linear in the auxiliary fields must be present in the Lagrangian ($\bar{D}D\lambda\phi_+$). The requirement that such a term be compatible with the symmetry of the Lagrangian places strong conditions on the fields needed. For there to be a term linear in a gauge auxiliary field a $U(1)$ symmetry must be present, since under a gauge symmetry

$$\delta\psi = i\left(\Lambda_+^+ - \Lambda_+\right) + \frac{i}{2}\left(\Lambda_+^+ + \Lambda_+, \psi\right)$$

to first order in the group parameter. A term linear in a matter auxiliary field can only arise from the F-term of a singlet under the group, because under a gauge transformation $\delta\phi_+ = i\,\Lambda_+\,\phi_+$.

The signal for spontaneous symmetry breakdown is when one of the auxiliary fields acquires a vacuum expectation value at the minimum of $V^{(5)}$. This is equivalent to saying $V > 0$ in the ground state.

Since in this paper we only consider the group $SU(N)$ we can only trigger a spontaneous symmetry breakdown by the addition of a singlet.

4. QUARK REPRESENTATION

Invariants in $SU(N)$ are formed from the fields and the two invariant tensors: - the Kronecker delta(δ_a^b) and the alternating symbol $\varepsilon_{a_1 \ldots\ldots a_n}$.

Since

$$(e^{i\Lambda_+})^+ e^{i\Lambda_+} = e^{-i\epsilon^+_{+r} F^r} e^{i\epsilon_{+r} F^r} \neq 1$$

many terms which are invariant with respect to ordinary symmetries are not invariants in supersymmetric theories. The possible interactions divide into two separate classes depending whether ϵ_+ is related to ϵ_- or not.

If $\epsilon^+_+ \neq \epsilon_-$ the only possible interaction terms are for SU(2) and SU(3). For SU(2)

$$\bar{D}D\ f\ \epsilon_{ab}\ \phi^i_{+a}\ \phi^j_{+b}\ \epsilon_{ij}$$

A global SU(2) symmetry has been introduced as the antisymmetry of the ϵ tensor would make this term vanish if more than one field was not present.

For SU(3)

$$\bar{D}D\ f\ \epsilon_{abc}\ \phi^i_{+a}\ \phi^j_{+b}\ \phi^k_{+c}\ \epsilon_{ijk}$$

The global SU(3) symmetry has been introduced for the same reason.

The other case is for $\epsilon^+_+ = \epsilon_-$. We can construct a mass term for all SU(N): -

$$f\ \bar{D}D\ \{\ \phi^+_-\ \phi_+ + \phi^+_+\ \phi_-\}$$

Terms cubic in the superfields are not invariant.

Apart from the kinetic terms the only terms compatible with renormalization are the F-components of terms at most cubic in the superfields. A singlet, $S_\pm$ is required to break the symmetry. However, given an invariant mass term we can always couple in $S_\pm$: -

$$f\ \bar{D}D\{\phi^+_-\ \phi_+\ S_+ + \phi^+_+\ \phi_-\ S_-\}$$

We now consider the above possibilities.

(a) The most general Lagrangian with $\varepsilon_+^+ = \varepsilon_-$ apart from the direct mass terms and the terms cubic in $S_\pm$ (see last section)

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_m + \mathcal{L}_I$$

Where

$$\mathcal{L}_M = \left(\frac{\overline{D}D}{8}\right)^2 \left(\phi_+^+ e^{g\psi} \phi_+ + \phi_-^+ e^{-g\psi} \phi_-\right)$$

$$\mathcal{L}_I = \mathcal{L}_{W-Z}(S_+, S_-) + \frac{\overline{D}D}{2} \{f \phi_+^+ \phi_- S_- + f^1 \phi_-^+ \phi_+ S_+ + \lambda^1 S_+ + \lambda S_-\} + h.c.$$

$$\mathcal{L}_{W-Z} = \left(\frac{\overline{D}D}{8}\right)^2 \left(S_+^+ S_+ + S_-^+ S_-\right)$$

The inclusion of the direct mass terms and the cubic scalar term results in no spontaneous breakdown of supersymmetry or internal symmetry because we can make $V = 0$ by setting $A_\pm = 0$ and choose the value of $A_\pm^1$ so as to eliminate the one mass parameter λ.

The above Lagrangian can be rewritten in a more illuminating way. Define

$$\chi_- = \frac{(fS_- + f^{1*} S_+^+)}{\kappa}; \quad \chi_+ = \frac{f^* S_-^+ + f^1 S_+)}{\kappa}$$

$$\chi_-^1 = \frac{f^1 S_- - f^* S_+^+}{\kappa} \quad ; \quad \chi_+^1 = f^{1*} \frac{S_-^+ - f\, S_+}{\kappa}$$

$$\kappa = \sqrt{|f|^2 + |f^1|^2}$$

Then
$$\mathcal{L}_I = \frac{(\overline{D}D)^2}{8} \left(\chi_+ \chi_- + \chi_-^1 \chi_+^1\right)$$

$$+ \frac{\overline{D}D}{2} \kappa \{\chi_- \phi_+^+ \phi_- + \chi_+ \phi_-^+ \phi_+\}$$

$$+ (\lambda^1 f^{1*} + \lambda^* f)\frac{1}{\kappa} \chi_+ + (\lambda^{1*} f^1 + \lambda f^*)\frac{1}{\kappa} \chi_-$$

$$+(-\lambda^1 f^* + \lambda^* f^1)\frac{1}{\kappa} \chi_+^1 + (-\lambda^{1*} f + \lambda f^{1*})\frac{1}{\kappa} \chi_-^1 .$$

From which we see that χ^1_- and $\chi^1_+ = \chi^{1+}$ decouple from the rest of the Lagrangian.

$$V = g^2(1-\tfrac{1}{N})\ (A_+^+ A_+ - A_-^+ A_-)^2$$

$$+ 2g^2(A_+^+ A_+ A_-^+ A_- - |A_+^+ A_-|^2)$$

$$+ |\kappa A_+^+ A_- + (\lambda^1 f^{1*} + \lambda^* f)\tfrac{1}{\kappa}|^2 + |(-\lambda^1 f^* + \lambda^* f^1)\tfrac{1}{\kappa}|^2$$

We can rotate A_+ to the form $\begin{pmatrix} 0 \\ \vdots \\ 0 \\ v_+ \end{pmatrix}$ V assumes a minimum when A_- is of the same form as A_+ with

$$v_+^+ \ v_+ = v_-^+ \ v_-$$

$$v_+^+ v_- = \frac{1}{\kappa^2}\ (\lambda^1 f^{1*} + \lambda^* f)$$

The SU(N) symmetry is broken to SU(N-1). However, supersymmetry is broken in the free piece which decouples from the rest and to that extent is irrelevant to the quark fields. The SU(N) symmetry can be broken to any SU(m) (m<N) by the addition of more quark representations and singlets.

(b) If $\epsilon_+^+ \neq \epsilon_-$ the only non-trivial Lagrangians are for SU(2) and SU(3). Although for SU(3) we have an interaction term cubic in the superfields we cannot couple this to a singlet (as we are liable to destroy renormalisability) so no symmetry breakdown can occur.

However, for SU(2) we can couple in a singlet

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_M + \mathcal{L}_I$$

$$\mathcal{L}_M = \left(\frac{\bar{D}D}{8}\right)^2 T_r\left(\phi_+^+ e^{2g\psi} \phi_+\right)$$

$$\mathcal{L}_I = \frac{\bar{D}D}{2} \{\varepsilon_{ab}\ \phi_{+a\alpha}\ \phi_{+b\beta}\ \varepsilon_{\alpha\beta} S_+ + \lambda S_+\} + \mathcal{L}_{W-Z}(S_+)$$

We do not include a ϕ_- field as ϕ_+ and ϕ_- do not mix. ϕ_+ is a 2 x 2 matrix transforming according to SU(2) local ⊗ SU(2) global.

$$V = \tfrac{1}{2} \sum_k D_k^2 + F_+^{1+} F_+^1$$

$$= g^2 \{\tfrac{1}{2}(A^+_{11} A_{11} + A^+_{21} A_{21} - A^+_{12} A_{12} - A^+_{22} A_{22})^2$$

$$+ 2|A^+_{11} A_{12} - A^+_{21} A_{22}|^2\} + |2f \det A + \lambda|^2$$

We can rotate A_+ such that it is of the form $\begin{pmatrix} \kappa & A_{21} \\ 0 & A_{22} \end{pmatrix}$; κ real.

The choice $A_{21} = 0$; $A_{22} = \kappa . e^{i\delta}$ where $e^{i\delta} k^2 = -\frac{\lambda}{2f}$ minimizes V. The local symmetry is completely broken but supersymmetry is not.

5. SELF ADJOINT

By definition the self adjoint representation has the structure constants of the group as its generators

$$(F^k)_{\ell m} = i\ f_{\ell k m},$$

$f_{\ell k m}$ are antisymmetric for SU(N).

Consider for simplicity that $\varepsilon_+^+ \neq \varepsilon_-$ then the most general Lagrangian is

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_M + \mathcal{L}_I$$

$$\mathcal{L}_M = \left(\frac{\bar{D}D}{16}\right)^2 \mathrm{Tr}\,\left(\phi_+^+ e^{g\psi}\, \phi_+\, e^{-g\psi}\right)$$

$$\mathcal{L}_I = \frac{\bar{D}D}{2}\,\{\frac{M}{2}\,\mathrm{Tr}\,\phi_+^2 + \frac{g'}{2}\,\mathrm{Tr}\,\phi_+^3 + \tfrac{1}{2}f\,\mathrm{Tr}\,\phi_+^2\, S_+$$

$$+ \mu\, S_+^2 + \kappa\, S_+^3 + \lambda\, S_+ + \mathrm{h.c.}\} + \mathcal{L}_{W.Z}(S_+).$$

We emphasize again that ϕ_- is omitted since it cannot couple to ϕ_+.

The Euler-Lagrange equations for the auxiliary fields are

$$D^k + g A_+^+ \times A_+ = 0$$

$$F_{+i}^+ + m\, A_{+i} + 2f\, A_{+i}\, A_+^1 + g' d_{ijk}\, A_{+j}\, A_{+k} = 0$$

$$F_+^{1+} + \mu A_+^1 + \mathrm{Tr}\left(f\, A_+\, A_+\right) + kA_+^{12} + \lambda = 0.$$

The d_{ijk} are defined by

$$\{\lambda_i, \lambda_j\} = \frac{4}{3}\,\delta_{ij} + d_{ijk}\,\lambda_k$$

We choose $\mu = k = 0$ otherwise the solution becomes trivial, viz. the choice $A_+ = 0$ and $kA_+^{12} + \mu A_+^1 + \lambda = 0$ minimizes V and leads to no symmetry breaking. We now show that we can simultaneously choose all auxiliary fields to vanish.

First, we define $A_+ = \lambda^a\, A_{+a} = \lambda^a(A_a + iB_a) = (a + ib)$. where a and b are hermitian traceless matrices.

$D_k = 0$ implies $(a, b) = 0$

Since a and b commute we can utilize the one unitary transformation to diagonalize them both.

$$F^+_+ = 0 \text{ implies } sA_+ + A^2_+ - \frac{\text{Tr } A^2_+}{N} = 0$$

where $s = \dfrac{m + 2f\, A^1_+}{g}$ and we use the relation

$$d_{abc}\, \lambda_c = \{\lambda_a, \lambda_b\} - \frac{\delta_{ab}}{N} \text{Tr } \{\lambda_a, \lambda_b\}$$

If we label the elements of A as $\text{diag}(\lambda_1, \ldots, \lambda_N)$ the equation $F^+_+ = 0$ becomes

$$s\, \lambda_i + \lambda_i^2 = \sum_i \frac{\lambda_i^2}{N} \tag{1}$$

$F^{1+}_+ = 0$ implies

$$\frac{1}{N} \text{Tr } A_+ A_+ = \sum_i \frac{\lambda_i^2}{N} = - \frac{2\lambda}{Nf} = \lambda^1 \tag{2}$$

As λ_a is traceless so is A_+ i.e. $\sum_i \lambda_i = 0$

Equations (1) and (2) are equivalent to

$$s\, \lambda_i + \lambda_i^2 = \lambda^1 \tag{3}$$

$$\sum_i \lambda_i = 0$$

The solutions to equation (3) are

$$\lambda_i = \frac{-s \pm \sqrt{s^2 + 4\lambda^1}}{2}$$

Let us suppose that in n cases we take the positive square root and in t cases we take the negative square root (n+t = N).

Then

$$\sum_i \lambda_i = 0 = \frac{-N\,s + (n-t)\,\sqrt{s^2 + 4\lambda'}}{2}$$

$$\rightarrow \qquad s^2 = \frac{(n-t)^2\, 4\lambda'}{N^2 - (n-t)^2}$$

Hence

$$\lambda_i = \pm\sqrt{\lambda'}\,\sqrt{\frac{N-n}{n}} \quad \text{if +ve square root}$$

$$\mp\sqrt{\lambda'}\,\sqrt{\frac{n}{N-n}} \quad \text{if -ve square root}$$

The pattern of symmetry breaking is arbitrary up to the choice in n($n < N$ or $\lambda' = 0$)

$$SU(N) \rightarrow SU(n) \otimes SU(N-n)$$

For SU(3) n can only equal 1 or 2 in either case

$$A = \pm \frac{\sqrt{\lambda}}{2}\,\mathrm{diag}\,\{1, 1, -2\} \quad \alpha\,\lambda_8\,.$$

So $SU(3) \rightarrow SU(2) \otimes U(1)$.

6. $(\bar{m}, n)$ REPRESENTATION OF SU(m) x SU(n)

$\phi_\pm$ is an n x m matrix which transforms as

$$\phi_\pm \rightarrow e^{i\Lambda_{\pm 1}} \phi_\pm e^{-i\Lambda_{\pm 2}}$$

$$\Lambda_{\pm 1} = F_1^r \, \varepsilon_{\pm r} \; ; \quad \Lambda_{\pm 2} = F_2^r \, \varepsilon_{\pm r}$$

F_1^r and F_2^r are the n x n and m x m matrix representation of the group generaters respectively. To introduce an interaction we assume $\Lambda_{+i}^+ = \Lambda_{-i}$.

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_M + \mathcal{L}_I$$

$$\mathcal{L}_M = \left(\frac{\bar{D}D}{16}\right)^2 \mathrm{Tr}\,(\phi_+^+ e^{2g_1\psi_1} \phi_+ e^{-2g_2\psi_2}$$

$$+ \phi_-^+ e^{-2g_1\psi_1} \phi_- e^{2g_2\psi_2})$$

where $\psi_i = F_i^r \psi_{ir}$

$$\mathcal{L}_I = \frac{\bar{D}D}{2} \{f\mathrm{Tr}\, \phi_+^+ \phi_- S_- + \lambda S_- + f^1 \mathrm{Tr}\, \phi_-^+ \phi_+ S_+ + \lambda^1 S_+\} + \mathcal{L}_{W-Z}(S_+, S_-).$$

The Euler Lagrange equations for the auxiliary fields are

$$D_{1k} + g_1 \mathrm{Tr}(A_+^+ \lambda_{1k} A_+ - A_-^+ \lambda_{1k} A_-) = 0$$

$$D_{2k} + g_2 \mathrm{Tr}(A_+ \lambda_{2k} A_+^+ - A_- \lambda_{2k} A_-^+) = 0$$

$$F_+^{1+} + f^1 \mathrm{Tr}\, A_+^+ A_- + \lambda^1 = 0$$

$$F_-^{1+} + f\, \mathrm{Tr}\, A_+^+ A_- + \lambda = 0$$

Then

$$V = {}_{k}(D^2_{1k} + D^2_{2k}) + F^{1+}_{+} F^{1}_{+} + F^{1+}_{-} F^{1}_{-}$$

The analysis to find the minimum of V is very similar to that for the simple quark case.

$$D_{2k} = D_{1k} = 0 \qquad\qquad F^{1}_{+} \neq 0 \text{ and } F^{1+}_{-} \neq 0$$

with

$$\mathrm{Tr}\ A^{+} A_{-} \ = \ - \frac{f^{1}\ \lambda^{1} + f^{*}\ \lambda^{x}}{|f|^{2} + |f^{1}|^{2}}$$

However, there are more variables than equations and the exact pattern of internal symmetry breaking is not determined at this level.

7. FERMION NUMBER

The apparent arbitrariness with which certain terms were disguarded from the Lagrangians so that spontaneous symmetry breaking would occur can be made compatible with a symmetry - fermion number(6) (2)

$$S \to e^{-\alpha\,\gamma_5}\, S$$

S is the supergauge generator.

In the quark case the following fermion number assignments to the fields will prohibit the terms $\phi_+^\dagger\,\phi_-$, $S_\pm^2$, $S_\pm^3$, $S_+^\dagger\,S_-^2$, $S_-^\dagger\,S_+^2$ and allow the terms $\phi_+^\dagger\,\phi_-\,S_-$, $\phi_-^\dagger\,\phi_+\,S_+$; $\lambda\,S_\pm$.

$$\begin{bmatrix} A_+ \\ \psi_+ \\ F_+ \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} A_- \\ \psi_- \\ F_- \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} A'_+ \\ \psi'_+ \\ F'_+ \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} A'_- \\ \psi'_- \\ F'_- \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

or $\phi_\pm(x,\theta) \to \phi_\pm(x,\, e^{-\alpha\,\gamma_5}\,\theta)$

$$S_\pm(x,\theta) \to e^{\mp 2i\,\alpha}\; S_\pm(x,\, e^{-\alpha\,\gamma_5}\theta)$$

Similar assignments exist for the adjoint representation and quark representation of SU(2) (involving only the ϕ_+ field) such that the direct mass terms and terms cubic in the singlet are excluded.

3.8 Explicit soft breaking of supersymmetry

In the introduction it was noted that it is not possible to obtain a realistic model which has supersymmetry as an exact symmetry. The natural way to overcome this difficulty is for supersymmetry to be spontaneously broken. However, at the tree level, this possibility is only realized in a few exceptional cases. Perhaps a more tractable approach is to explicitly break supersymmetry in a soft way. That is we would allow the mass terms to break supersymmetry, although still conserve the gauge symmetry.

This approach could be viewed in two not unconnected ways. Firstly, we could regard supersymmetry as becoming manifest only at high energies and so not relevant to the mass terms which do not affect the high energy behaviour; secondly we could regard this ad hoc choice of mass terms as a prescription which might be justified by some as yet undiscovered underlying mechanism. (Similar to the breaking of gauge symmetries by hand before the advent of spontaneously broken symmetries.) This is not altogether implausible when one considers our inability to calculate Feynman diagrams, in general, to more than a few loops.

Here, we make a preliminary study of this procedure in context of SU(N) in order to contrast the results with those of spontaneous symmetry breaking. At first sight there would appear to be no problem, we could make the mass terms tachyonic and so induce the internal gauge symmetry to be spontaneously broken. However, the quartic terms in the classical potential are restricted to be supersymmetric invariant and there may exist directions in the field space for which they vanish. It is possible that in these directions the now tachyonic mass terms do no vanish and result in the classical potential being unbounded below and so the theory is unstable and hence rejected.

(i) Adjoint representation of SU(N)

The quartic terms of the potential are given in the previous section and are

$$|A_+^{\dagger} \times A_+|^2 \qquad , \quad |d_{ijk} A_{+j} A_{+k}|^2$$

For these to vanish implies that

$$A_+^{\dagger} \times A_+ = 0 \tag{1}$$

$$d_{ijk} A_{+j} A_{+k} = 0 \tag{2}$$

Equation one implies that we can use the group symmetry to diagonalize both degrees of freedom in A_+. Let diag $A_+ = \mathrm{diag}\{\lambda_1 \ldots \lambda_i \ldots \lambda_n\}$.
Equation two then implies

$$\lambda_i^2 = \sum_i \frac{\lambda_i^2}{N} \overset{\mathrm{df}}{=} \lambda^2$$

$$\therefore \quad \lambda_i = \pm \lambda$$

However, $\quad \mathrm{Tr}\, A_+ = 0 = \sum_i \lambda_i = 0$

For N odd, $\lambda_i = 0$ and there exist no directions in which the quartic terms vanish. Consequently, the potential for N odd is bounded below regardless of the mass terms present and so spontaneous symmetry breaking can always be induced. For N even the quartic terms vanish if A_+ has an equal number of $+\lambda$ and $-\lambda$ down its diagonal.

The possible mass terms for SU(N) are

$$\frac{1}{2} \mathrm{Tr} \left\{ \mu_1 A_+^2 + \mu_1 A_+^{\dagger 2} + \mu_{12} A_+^\dagger A_+ \right.$$

$$\left. + i\mu_2 \left(A_+^{\dagger 2} - A_+^2 \right) \right.$$

Substituting the directions above for N even and defining $\lambda = a + i\, b$ the mass term becomes

$$= \frac{N}{2} \left\{ (\mu_{12} + 2\mu_1)\, a^2 + (\mu_{12} + 2\mu_1)\, b^2 + 4\mu_2 ab \right\}$$

$$= \frac{N}{2} \left\{ \left(\sqrt{\mu_{12} + 2\mu_1}\; a + \sqrt{\mu_{12} - 2\mu_1}\; b \right)^2 \right.$$

$$\left. + \; 2ab \left(2\mu_2 - \sqrt{\mu_{12}^2 - 4\mu_1^2} \right) \right\}.$$

Hence, the potential is bounded below for N even if and only if

$$\mu_{12} \pm 2\mu_1 \geq 0$$

$$2\mu_2 \leq \sqrt{\mu_{12}^2 - 4\mu_1^2} \qquad (3)$$

The question to be answered is do the above constraints prevent spontaneous symmetry breaking for N even. Because it is difficult to calculate the minimum of the potentials resulting from even quite small N we perform the calculation for SU(2). (In particular, we have in mind the 'clever model', the effective potential of which is calculated in the next chapter.)

$$V = c_1 \underline{A}^2 + c_{12} \underline{A}.\underline{B} + c_2 \underline{B}^2 + \frac{g^2}{2}(\underline{A} \times \underline{B})^2$$

Where A and B are real fields.

The constraints for stability given by equation (3)

$$c_1 > 0, \quad c_2 > 0, \quad c_{12}^2 \leq 4 c_1 c_2$$

The minimum of V is given by

$$\frac{dV}{d\underline{A}} = 2 c_1 \underline{A} + c_{12} \underline{B} + g^2 \underline{B} \times (\underline{A} \times \underline{B}) = 0$$

$$\frac{dV}{d\underline{B}} = 2 c_2 \underline{B} + c_{12} \underline{A} + g^2 \underline{A} \times (\underline{B} \times \underline{A}) = 0$$

This implies that

$$(\underline{A} \pm \underline{B})^2 + 2 \underline{A}.\underline{B} \left(\frac{c_1 + c_2}{c_{12}} \mp 1 \right) = 0$$

Using the constraints on c_1, c_2 and c_{12} implies that the minimum is at

$$\underline{A} = \underline{B} = 0$$

Hence, for SU(2) adjoint representation no spontaneous symmetry breaking is possible even if supersymmetry is softly broken.

(ii) The quark representation of SU(N)

Since the mass terms need no longer be invariant under supersymmetry, we do not need both ϕ_+ and ϕ_- to form a mass term. Retaining only $\phi_+^\dagger$ and ϕ_+ the classical potential is

$$V = +\frac{\mu}{2} A_+^\dagger . A_+ + \frac{g'^2}{4} (A_+^\dagger . A_+)^2$$

This theory is stable for all μ and for $\mu < 0$ spontaneous symmetry breaking takes place. SU(N) $\rightarrow$ SU(N-1). In particular, SU(2) $\rightarrow$ no symmetry.

To summarize, provided we are prepared to introduce mass terms which break supersymmetry the gauge symmetry can be spontaneously broken for the quark representation of SU(N) and the adjoint representation of SU(N) for N odd. For the adjoint representation of SU(2) it is not possible to induce spontaneous symmetry breaking of the guage symmetry and it is plausible that this is the case for higher even N.

There is an important point to be made concerning the asymptotic freedom properties of these models. Firstly, consider the supersymmetric (without the mass breaking terms) quark representation. The only coupling to enter into the theory is the gauge coupling. The formula for the β function of the gague coupling, g was given by Gross and Wilezeck[7] to be

$$\beta(g) = -\frac{g^3}{16\pi^2}\left\{\frac{11}{3}\, C_2(G) - \sum_{\text{fermions}} \frac{4}{3}\, T(R) - \sum_{\text{scalars}} \frac{1}{6}\, T(R)\right\}.$$

Where

$$C_2(G) = N$$

$$T(N) = \begin{cases} \frac{1}{2} & \text{quark} \\ N & \text{adjoint} \end{cases}$$

The SU(N) quark representation has one adjoint fermion, one quark fermion and one quark scalar and so

$$\beta(g) = -\frac{g^3}{16\pi^2}\left\{\frac{11}{3}\cdot N - \frac{1}{2}\cdot\frac{4}{3}\left(\frac{1}{2} + N\right) - \frac{1}{6}\cdot\frac{1}{2}\right\}$$

$$= -\frac{g^3}{16\pi^2}\left\{3N - \frac{5}{12}\right\}.$$

(Majorana fermions have a factor $\frac{1}{2}$).

Hence the theory is asymptotically free for all N.

We now present a heuristic argument to demonstrate that for N = 2 the theory is asymptotically free and infra-red stable. If we introduce the supersymmetric breaking masses, as shown above, we can cause the SU(2) symmetry to be completely spontaneously broken. Explicit calculations confirm the fact that all vector mesons acquire mass and there is only one massive scalar in accordance with the Goldstone theorem. Hence, with the mass terms the theory is infra-red stable. However, we have destroyed the supersymmetry and the dimensionless coupling constants are no longer related to one another. To examine the asymptotic freedom properties of the theory we would have to calculate a β function for each coupling constant. The β function is independent of the masses in the theory and if we adjust the dimensionless coupling constants to satisfy the supersymmetric constraints when renormalized we must recover the β function of equation (4) (for consistency). Therefore, we have an infra-red stable asymptotically free theory.

It was shown by Cheng, Eichten and Ling-Fong Li[8] that SU(2) with one quark representation is not asymptotically free. Consequently, the ultra-violet fixed point we have found is unstable in the group space and we have really found a solution in the same class as those found by Chang.[9] Even the slightest deviation of the coupling constant renormalization conditions from the supersymmetric values would not result in an asymptotically free theory.

A similar argument can be applied to the adjoint case. O'Raifeartaigh and T. Sherry[10] showed that the adjoint representation is asymptotically free. Once the mass terms are included it is likely that the SU(3) will break to U(1) or U(1) x U(1) and hence will be infra-red stable and asymptotically free according to the above argument.

Conclusion

We have seen that spontaneous symmetry breaking can only be forced to occur in SU(N) gauge theories if a singlet is introduced. The internal symmetry is broken in a way similar to that in a normal gauge theory(11). In the representations considered, however, supersymmetry does not break. A fermion number conservation is required in order to exclude certain terms which would prevent symmetry breaking.

Nevertheless, if we are prepared to introduce supersymmetry breaking mass terms we can in many theories spontaneously break the gauge symmetry. This is the case for SU(N) adjoint representation for N odd and for SU(N) quark representations for all N. However, we showed that this is not always the case; it is not possible to induce spontaneous symmetry breaking of SU(2) adjoint representation without the theory becoming unstable. These theories are likely to be asymptotically free and those in which the residual symmetry is no symmetry at all or a U(1) gauge symmetry will be infra-red stable and asymptotically free.

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CHAPTER 4

TWO ONE LOOP SUPERSYMMETRIC

EFFECTIVE POTENTIALS

4.1 Introduction to the Effective Potential

In the remaining chapters we consider the quantum corrections to the classical potential of supersymmetric theories. These corrections were first calculated by Coleman and Weinberg[1] and here we outline a derivation of the equations used to calculate the effective potential later in the thesis.

In the presence of an external source j(x) the vacuum to vacuum amplitude is given by

$$\exp\left\{\frac{i}{\hbar} W[j]\right\} = \langle + \mid - \rangle_j$$

$$= \int [d\phi] \exp\left\{\frac{i}{\hbar}\left\{ I[\phi_i] + j_i \phi_i \right\}\right\} \qquad (1)$$

Where $I[\phi_i]$ is the action of the theory. Repeated indices are to be taken to imply summation over group, Lorentz and space-time indices. We define the function φ_{ci} by

$$\varphi_{ci} = \frac{\delta W}{\delta j_i} = \frac{\langle + \mid \varphi_i \mid - \rangle_j}{\langle + \mid - \rangle_j} \qquad (2)$$

The effective action, $\Gamma[\varphi_{ci}]$ is now defined by a functional Legendre transform

$$\Gamma[\varphi_{ci}] = W[j] - j_i \varphi_i \qquad (3)$$

Γ can be expanded in a Taylor series,

$$\Gamma[\varphi_{ci}] = \sum_n \frac{1}{n!} \Gamma^{(n)}_{i_1 \dots i_n} \varphi_{ci_1} \cdots \varphi_{ci_n}$$

It is possible to show that the coefficients, $\Gamma^{(n)}_{i_1 \ldots i_n}$ in this series are the one particle irreducible Greens functions[2]. However, we can also expand Γ in powers of momentum,

$$\Gamma[\varphi_c] = \int d^4x \left\{ -V(\varphi_c) + \frac{1}{2}(\partial_\mu \varphi_c)^2 Z(\varphi_c) + \cdots \right\} \tag{5}$$

The quantity of interest is $V(\varphi_c)$, and is the effective potential. As must be the case, it is an ordinary function of φ_c and reduces, at the tree level, to the classical potential. It is not hard to show[3] that $V(\varphi_c)$ is the energy density of the theory when the fields of the theory acquire vacuum expectation values φ_c Comparing the expansions of Γ given by equations (4) and (5) we note that V is just the sum of all one particle irreducible graphs with all possible field insertions at zero momentum.

However, apart from exceptional cases, one of which is considered in chapter 5, we can not compute an infinite sum of Feynman diagrams. As such, we need an approximation which is invariant under the field translations required when theories are spontaneously broken. The crudest approximation which fulfills this requirement is an expansion in Planks constant, $\hbar$, or equivalently a loop expansion.[4]

The expression for the one loop approximation can easily be derived from the functional formalism given above. Let us denote φ_0 to be the solution to the classical field equations i.e.

$$\left. \frac{\delta I[\varphi]}{\delta \varphi_i} \right|_{\varphi = \varphi_0} = -j_i \tag{6}$$

Consider expanding $I[\varphi] + j_i \varphi_i$ in a Taylor series about φ_{oi},

$$I[\varphi] + j_i \varphi_i = I[\varphi_o] + j_i \varphi_{io}$$

$$+ \frac{1}{2!} \mathcal{D}_{ij}(\varphi_o) \, (\phi_i - \phi_{oi})(\phi_j - \phi_{oj}) + \cdots$$

Where

$$\mathcal{D}_{ij}(\varphi_o) = \left. \frac{\delta^2 I}{\delta \varphi_i \delta \varphi_j} \right|_{\varphi = \varphi_o}$$

Utilizing this expansion in the vacuum to vacuum amplitude of equation (1) gives

$$\exp\left\{\frac{i}{\hbar} W[J]\right\} = \exp \frac{i}{\hbar} \left\{ I[\varphi_o] + j_i \varphi_{io} \right\} \times$$

$$\int [d\phi] \exp \frac{i}{2!} \left\{ \mathcal{D}_{ij}(\varphi_o)(\phi_i - \varphi_{oi})(\phi_j - \varphi_{oj}) + \cdots \right\}.$$

Carrying out the Gaussian integration

$$\exp\left\{\frac{i}{\hbar} W[J]\right\} = \exp \frac{i}{\hbar} \left\{ I[\varphi_o] + j_i \varphi_{oi} \right\}$$

$$\mathcal{D}et^{-\frac{1}{2}} \left\{ \mathcal{D}_{ij}(\varphi_o) \right\}. \quad + \text{ higher order corrections}$$

Implying

$$W[J] = I[\varphi_0] + \varphi_{i0} J_i + \frac{1}{2} i\hbar \ln \mathrm{Det}\,(\mathcal{D}_{ij}(\varphi_0))$$

$$+ \quad O(\hbar^2).$$

Equation (2) implies that

$$\varphi = \varphi_0 + O(\hbar^2).$$

and hence

$$\Gamma[\varphi] = I[\varphi] + \frac{i\hbar}{2} \ln \mathrm{Det}\,\{\mathcal{D}_{ij}(\varphi)\} + O(\hbar^2)$$

To obtain V we set φ to be a constant

$$V(\varphi) = V_{tree}(\varphi) - \frac{1}{2} i\hbar \int d^4k \ln \mathrm{Det}\,(\mathcal{D}(k,\varphi)) + O(\hbar^2). \qquad (7)$$

In equation (7) the determinat is to be taken to operate only on internal indices and not those of space-time. Also $\mathcal{D}(k,\varphi)$ is the Fourier transform of $\mathcal{D}(x\varphi)$

Finally we can carry out the integration in equation (7) to obtain

$$V(\varphi) = V_{tree}(\varphi) + \frac{\hbar}{64\pi^2} \sum_j (2j+1)(-1)^{2j} \times$$

$$(M_j^2)^2 \ln M_j^2 \qquad (8)$$

Where $m_j(ce) = \mathrm{diag}\, \mathcal{D}_{ij}(k^2 = 0)$ and the sum of j is a sum of overspins for all the particles.

4.2 Two One Loop Supersymmetric Effective Potentials

The results of the previous chapter demonstrate that spontaneous symmetry breaking is much more difficult to induce in supersymmetric gauge theories than in the corresponding conventional gauge theories. In fact, demanding spontaneous symmetry breaking at the tree level requires the presence of certain representations of specific symmetries (global and/or gauge). However, once the symmetry is broken the pattern of spontaneous symmetry breaking is analogous to that in conventional gauge theories.

Although Fayet and Ilioupoulos[5] demonstrated in the context of a U(1) gauge symmetry that it is possible to break supersymmetry spontaneously, this is not the case for most symmetry groups and their representations. We saw that it was often possible, under certain conditions, to break the internal symmetry at the tree level. Nevertheless, it is often the case that the pattern of breaking of the internal symmetry is not completely resolved at the tree level. This results in a physical ambiguity in the vacuum state.

It might be hoped that the higher order contributions to the effective potential may change this situation. In particular, they may lead either to supersymmetry being broken or, if supersymmetry is conserved, to a resolution of the physical degeneracy in the vacuum state often occuring at the tree level.

We saw in the introduction to this chapter that to calculate even the modest one loop correction requires us to diagonalize an arbitrary mass matrix. Further, to find the vacuum state in this approximation we must minimize the effective potential which is a function of as many variables as there are fields which acquire vacuum expectation values. Such a calculation is only possible for theories with a small number of fields. Hence, a suitable candidate is the so called 'clever model' of Salam and Strathdee[6] which is an SU(2) gauge model.

$$\mathcal{L} = \frac{(\bar{D}D)^2}{8} \left\{ \phi_- e^{g\psi} \phi_+ \right\}$$

$$= -\frac{1}{4} \underset{\sim}{F}_{\mu\nu}^2 + \bar{\chi}\, i \not{\nabla} \chi + \frac{1}{2}(\nabla_\mu A)^2$$

$$+ \frac{1}{2}(\nabla_\mu B)^2 - i g\, \underline{\bar{\chi}} \times (\underline{A} + \gamma_5 \underline{B}) . \underline{\chi}$$

$$- \frac{g^2}{2} (\underline{A} \times \underline{B})^2 .$$

Where

$$\underset{\sim}{F}_{\mu\nu} = \partial_\mu \underline{V}_\nu - \partial_\nu \underline{V}_\mu + g\, \underline{V}_\mu \times \underline{V}_\nu$$

$$\nabla_\mu \underline{\chi} = \partial_\mu \underline{\chi} + g\, \underline{V}_\mu \times \underline{\chi}$$

$$\nabla_\mu \underline{A} = \partial_\mu \underline{A} + g\, \underline{V}_\mu \times \underline{A} \qquad (4.9)$$

The Majorana fermions from the matter and gauge fields combine to produce the Dirac spinor, $\underline{\chi}$. This model admits the fermion number symmetry which excludes the possible mass term. We consider the two cases (i) $\langle \underline{D} \rangle = \langle \underline{A} \times \underline{B} \rangle = 0$ and (ii) $\langle \underline{D} \rangle \neq 0$ corresponding to supersymmetry conserving and breaking solutions respectively.

As shown in the previous part, the one loop effective potential is most easily calculated by shifting the fields by arbitrary amounts and then calculating the resulting mass matrix. In order to avoid mixing terms of the form $\partial_\mu \underline{A} \cdot \underline{V}_\mu \times \langle \underline{A} \rangle$ we must choose the Unitary[7] or the Landau gauge. The choice only affects the scalar mass matrix and we choose the Unitary gauge, which for this model is

$$\underline{A} \times \langle \underline{A} \rangle + \underline{B} \times \langle \underline{B} \rangle = 0 \qquad (4.10)$$

(i) $\underline{D} = 0$. We choose the field expectation values

$$A = (0,0,a)$$
$$B = (0,0,b)$$

The unitary gauge condition becomes

$$b B_2 + a A_2 = 0$$
$$b B_1 + a A_1 = 0 \qquad (4.11)$$

We now calculate the mass matrix when the fields are shifted for each sector of the model.

The vectors acquire mass from

$$\frac{1}{2} g^2 \{ (\underline{V}_\mu \times \langle \underline{A} \rangle)^2 + (\underline{V}_\mu \times \underline{B})^2 \}$$

$$= \frac{1}{2} g^2 (a^2 + b^2) \{ V_{\mu 1}^2 + V_{\mu 2}^2 \}$$

The fermions acquire mass from

$$-ig \, \overline{\underline{\chi}} \times \langle \underline{A} + \gamma_5 \underline{B} \rangle . \underline{\chi}$$

$$= (-iga) \{ \overline{\chi}_2 \chi_1 - \overline{\chi}_1 \chi_2 \}$$
$$-(igb) \{ \overline{\chi}_2 \gamma_5 \chi_1 - \overline{\chi}_1 \gamma_5 \chi_2 \}$$

$$= \overline{\chi}^+ (a + \gamma_5 b) \chi^+ - \overline{\chi}^- (a + \gamma_5 b) \chi^-$$

where $$\chi^\pm = (\chi_1 \pm i \chi_2) \frac{1}{\sqrt{2}} .$$

The scalars acquire mass from

$$-\frac{g^2}{2} \{ (\underline{A} + \langle \underline{A} \rangle) \times (\underline{B} + \langle \underline{B} \rangle) \}^2$$

Imposing the Unitary gauge and keeping terms only bilinear in the fields this term becomes

$$-\frac{g^2}{2} \left\{ \frac{(a^2 + b^2)^2}{b^2} \right\} \{ A_2^2 + A_1^2 \} .$$

However, imposing the Unitary gauge condition on the kinetic terms in the Lagrangian forces us to renormalize the fields, i.e.

$$A_1 = \sqrt{\frac{b^2}{a^2+b^2}}\, A_1 \,, \qquad A_2 = \sqrt{\frac{b^2}{a^2+b^2}}\, A_2'$$

The free part of the Lagrangian becomes

$$\tfrac{1}{2}(\partial_\mu A_1')^2 + \tfrac{1}{2}(\partial_\mu A_2')^2 + \tfrac{1}{2}(\partial_\mu A_3)^2 + \tfrac{1}{2}(\partial_\mu B_3)^2$$

$$- \frac{g^2}{2}(a^2+b^2)\left\{ A_1'^2 + A_2'^2 \right\}.$$

To summarise, the mass spectrum resulting from shifting the fields is given in Figure 1.

Upon substituting these values into the equation for the one loop effective potential (equation 4.8) we obtain

$$V_1 = \frac{\hbar}{64\pi^2}\left\{ (g^2(a^2+b^2))^2 \ln(g^2(a^2+b^2)) \right\}$$

$$\times \left\{ 2 - 2\times 2\times 2 + 3\times 2 \right\} \qquad = 0$$

(ii) $\underline{D} \neq 0$. We choose the field expectation values to be

$\underline{A} = (0,0,a)$; $B = (0,b,0)$.

The unitary gauge conditions becomes

$$\begin{aligned} -A_2 a + bB_3 &= 0 \\ A_1 &= 0 \\ B_1 &= 0 \end{aligned} \qquad (4.12)$$

FIGURE 4.1

Particle	Type	Mass
$V_{\mu 1}$	vector	$g^2(a^2+b^2)$
$V_{\mu 2}$	"	"
$V_{\mu 3}$	"	0
χ_3	Dirac fermion	0
χ^+	"	$g^2(a^2+b^2)$
χ^-	"	$g^2(a^2+b^2)$
A_1'	real scalar	$g^2(a^2+b^2)$
A_2'	"	"
A_3	"	0
B_3	"	0

The vectors acquire mass from

$$\frac{1}{2}\, g^2 \left\{ \left(\underline{V}_\mu \times \langle \underline{A} \rangle \right)^2 + \left(\underline{V}_\mu \times \langle \underline{B} \rangle \right)^2 \right\}$$

$$= \frac{1}{2} g^2 \left\{ a^2\, V_{\mu 2}^2 + b^2\, V_{\mu 3}^2 + (a^2 + b^2)\, V_{\mu 1}^2 \right\}$$

The fermions acquire mass from

$$-ig\, \overline{\underline{\chi}} \times \langle \underline{A} + \gamma_5 \underline{B} \rangle \cdot \underline{\chi}$$

$$= (-iga) \left\{ \bar{\chi}_2 \chi_1 - \bar{\chi}_1 \chi_2 \right\}$$

$$+ (-igb) \left\{ \bar{\chi}_3 \gamma_5 \chi_1 - \chi_1 \gamma_5 \chi_3 \right\}$$

$$= -\sqrt{2}\, g \sqrt{a^2 + b^2} \left(\bar{\chi}^+ \chi^+ - \bar{\chi}^- \chi^- \right)$$

where

$$\chi^{\pm} = \chi_1 \pm \frac{i}{\sqrt{2}} \frac{(b\chi_3 - a\chi_2)}{\sqrt{a^2+b^2}}$$

The remaining combination

$$z = \frac{1}{\sqrt{2}} \frac{(a\chi_3 + b\chi_2)}{\sqrt{a^2+b^2}}$$ is

massless.

The scalars acquire mass from

$$-\frac{g^2}{2} \left\{ (\underline{A} + \langle \underline{A} \rangle) \times (\underline{B} + \langle \underline{B} \rangle)^2 \right\}^2$$

Upon imposing the Unitary gauge condition and only keeping terms bilinear in the fields this term becomes

$$-\frac{g^2}{2}\left\{-2a^2A_2^2 + 2ab\,B_2A_3 + (aB_2 + bA_3)^2\right\}.$$

Imposing the Unitary gauge condition on the kinetic part of the Lagrangian requires us to renormalize the field; $A_2' = \sqrt{\frac{a^2+b^2}{b^2}}\,A_2$

The scalar masses are

$$\frac{1}{2}g^2 \left\{ (a^2+b^2) \pm \sqrt{(a^2+b^2)+12a^2b^2} \right.$$

and

$$-\frac{2g^2a^2b^2}{(a^2+b^2)}.$$

To summarise, the mass spectrum is given in Figure 2.

In accordance with Goldstones theorem. Substituting these masses into the equation for the one loop effective potential gives a complex effective potential.

To conclude, the potential vanishes if supersymmetry is conserved and is complex is supersymmetry is violated. A complex effective potential signals an instability in the theory and so no supersymmetry violating vacuum state is stable. On the other hand, if supersymmetry is conserved we gain no further information about the true nature of the minimum than we learnt from the tree potential i.e. $\underline{D} = \underline{A} \times \underline{B} = 0$. This leaves us with a physical degeneracy in the vacuum state. Or stated another way pseudo-Goldstone bosons remain massless at the one loop level.

Performing the calculation in the Landau gauge does not affect

FIGURE 4.2

Particle	Type	Mass
$V_{\mu 1}$	vector	$g^2(a^2+b^2)$
$V_{\mu 2}$	"	$g^2 a^2$
$V_{\mu 3}$.	"	$g^2 b^2$
χ^+	Dirac fermion	$2g^2(a^2+b^2)$
χ^-	" "	"
Z	" "	0
A_2'	real scalar	$-\frac{2a^2b^2}{a^2+b^2}$
Two	" "	$\frac{g^2}{2}\{ (a^2+b^2) \pm \sqrt{(a^2+b^2)+12a^2}$

any of the conclusions, although the actual form of the potential is changed. The recent work of Iliopoulos and Papanicolaou [8] on the gauge invariance of physical quantities associated with the effective potential leads us to expect the conclusion to be valid in all gauges.

Finally, we note that had the one loop effective potential been non-zero in the supersymmetry conserving region we would have obtained an infra-red stable asymptotically free theory. The usual problems of the region of validity of the one loop potential would not be encountered as the tree potential vanishes in this region.

As a second example, we calculate the one loop effective potential for the U(1) supergauge model of Fayet and Iliopoulos:[9]

$$
\begin{aligned}
\mathcal{L} = {} & \tfrac{1}{2}(\partial_\mu A_1)^2 + \tfrac{1}{2}(\partial_\mu A_2)^2 + \tfrac{1}{2}(\partial_\mu B_1)^2 \\
& + \tfrac{1}{2}(\partial_\mu B_2)^2 - \tfrac{1}{4} V_{\mu\nu}^2 + \tfrac{1}{2}\bar{\lambda}\not{\partial}\lambda + \tfrac{1}{2}\bar{\Psi}_1\not{\partial}\Psi_1 \\
& + \tfrac{1}{2}\bar{\Psi}_2\not{\partial}\Psi_2 - \tfrac{M}{2}(\bar{\Psi}_1\Psi_1 + \bar{\Psi}_2\Psi_2) \\
& + V_\mu g(B_2\partial^\mu A_2 + A_1\partial^\mu B_1 - B_1\partial^\mu A_1 - A_2\partial^\mu B_2 \\
& - \bar{\Psi}_1\gamma_\mu\Psi_2) - \frac{i\bar{\lambda}}{\sqrt{2}}\Big[(A_1 - B_2 + \gamma_5(A_2+B_1))\Psi_2 - ((A_2-B_1)-\gamma_5(A_1+B_2)) \\
& - V
\end{aligned}
$$

where

$$
\begin{aligned}
V = {} & \frac{m^2}{2}(A_1^2 + B_1^2 + B_2^2 + A_2^2) \\
& + \frac{1}{2}(g(A_1^2 + B_1^2 - A_2^2 - B_2^2) + 2\xi)^2
\end{aligned}
$$

The calculation is analogous to that for the 'clever model'. For simplicity we only shift the A_2 field, $\langle A_2 \rangle = v$

The result is

$$V_1 = 2\left(m^2 + \xi g - \frac{v^2}{2}g^2\right)^2 \ln\left(m^2 + \xi g - \frac{v^2 g^2}{2}\right)$$

$$+ \left(m^2 - \xi g + \frac{v^2}{2}g^2\right)^2 \ln\left(m^2 - \xi g + \frac{v^2}{2}g^2\right)$$

$$+ \left(m^2 - \xi g + 3\frac{v^2}{2}g^2\right)^2 \ln\left(m^2 - \xi g + 3\frac{v^2}{2}g^2\right)^2$$

$$+ 3\left(g^2 v^2\right)^2 \ln\left(g^2 v^2\right)$$

$$- 4\left(m^2 + v^2 g^2\right)^2 \ln\left(m^2 + v^2 g^2\right)$$

The classical potential is minimized by $\frac{v^2}{2}g^2 = \xi g - m$ and we chose ξg to be positive and also $(m^2 - \xi g) < 0$

This result is not very illuminating but two useful features can be noticed. V_1 is complex for $\frac{1}{2}v^2 g^2 - (\xi g - m^2) < 0$ reminiscent of a $\lambda\phi^3$ potential.

By insisting that the model admit a slightly different fermion number conservation we can force m = 0. In this case the model does not break supersymmetry at the tree level because the classical potential is

$$\frac{1}{2}\left(g\left(A_1^2 + B_1^2 - A_2^2 - B_2^2\right) + 2\xi\right)^2$$

Setting m = 0 in the above formula for the one loop potential gives

$$V_1 = 2\left(\xi g - \frac{v^2}{2}g^2\right)^2 \ln\left(\xi g - \frac{v^2}{2}g^2\right)$$

$$+ \left(-\xi g + \frac{v^2}{2}g^2\right)^2 \ln\left(-\xi g + \frac{v^2}{2}g^2\right)$$

$$+ \left(-\xi g + 3\frac{v^2}{2}g^2\right)^2 \ln\left(-\xi g + 3\frac{v^2}{2}g^2\right)$$

$$+ 3\,(g^2 v^2)^2 \ln g^2 v^2$$

$$- 4\,(v^2 g^2)^2 \ln(g^2 v^2)$$

Hence, V_1 is complex unless $\xi g = \frac{v^2}{2}g^2$ (supersymmetry is conserved) and in this case vanishes, similar to the clever model.

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CHAPTER 5

GENERAL SUPERSYMMETRIC EFFECTIVE POTENTIAL

WHEN SUPERSYMMETRY IS CONSERVED

5.1 Introduction

Spontaneous symmetry breaking has played an important role in many attempts to construct realistic supersymmetric theories. However, it is very often the case that the classical potential has a larger symmetry than the supersymmetric Lagrangian. Therefore, a knowledge of the higher order contributions to the effective potential is required to resolve the resulting degeneracy in the physical vacuum state.

Now, in a general supersymmetric theory the tree potential can be written as the modulus squared of auxiliary fields (fields whose derivatives do not appear in the Lagrangian). If supersymmetry is conserved these auxiliary fields have zero expectation values. Consequently, at the tree level conservation of supersymmetry implies the effective potential vanishes. Several authors have noted that this result persists at the one loop level in some models, an example being the supergauge theory based on the SU(2) adjoint representation (Salam, Strathdee and Duff, myself (unpublished) and Woo[1]). We prove that this result is true to all orders in perturbation theory and therefore the vacuum state degeneracy when it exists, remains unresolved if supersymmetry is conserved.

5.2 Lagrangian and Propagators

A sufficiently general renormalizable supersymmetric Lagrangian is

$$\mathcal{L} = \frac{(\bar{D}D)^2}{8}\left(\phi_+^\dagger e^{g\psi}\phi_+ + \phi_-^\dagger e^{-g\psi'}\phi_-\right)$$

$$-\frac{(\bar{D}D)}{2}\left(\phi_+^2 + \phi_-^2 + \phi_+^\dagger\phi_- + \phi_+^3 + \phi_-^3 + \phi_+^2\phi_-^\dagger\right.$$

$$\left. + \phi_+\phi_-^{\dagger 2}{}_+ \; h.c\right) + \mathcal{L}_G(\psi) + \mathcal{L}_G(\psi')$$

Where ϕ_+ and ϕ_- are the matter chiral superfields (ϕ_- may equal $\phi_+^\dagger$) and ψ is the gauge superfield. $\mathcal{L}_G(\psi)$ is a complicated function of ψ which is no more than the usual kinetic term for the gauge fields. The group indices and matrices are not indicated, but are understood to be present. Since the actual group structure is irrelevant to the proof indices will be assumed to be present, but not actually indicated from now on.

The superpropagators for the matter fields were derived by Salam and Strathdee(2) and are

$$\Delta_{\substack{+\,+\\-\,-}}(1,2) = -\frac{m}{2}\,\bar{\theta}_{12}\,\theta_{12} \pm \exp\left\{\pm\frac{i}{4}\left(\bar{\theta}_1 \not{\partial}\gamma_5\theta_1\right.\right.$$

$$\left.\left. - \bar{\theta}_2\not{\partial}\gamma_5\theta_2\right)\right)\;\Delta$$

$$\Delta_{\pm\mp}(1,2) = \exp\{\bar{\theta}_1 \not{p}\, \theta_{2\mp} \pm \frac{i}{4}(\bar{\theta}_1 \not{p}\gamma_5\theta_1 + \bar{\theta}_2 \not{p}\gamma_5\theta_2)\}\Delta \tag{2}$$

where

$$\theta_{12} = \theta_1 - \theta_2 \quad , \quad \theta_\pm = \frac{(1 \pm i\gamma_5)}{2}\theta \quad , \quad \Delta = \frac{1}{p^2 - m^2}$$

Ferrara and Piguet(3) showed that in the Feynman gauge the vector propagator is

$$\Delta(1,2) = (\bar{\theta}_{12}\theta_{12})^2 \frac{\Delta}{16} \tag{3}$$

All gauge calculations will be carried out in this supergauge.

The supersymmetric Feynman rules are given in the introduction. They are those for an ordinary theory except that at every matter vertex we must take two derivatives with respect to θ :

i.e. $$\nabla(\theta) = \left(\frac{d}{d\theta_\alpha} \frac{d}{d\bar{\theta}^\alpha}\right)$$

and at every gauge vertex four derivatives with respect to :

i.e. $$(\nabla(\theta))^2 = \frac{1}{2}\nabla(\theta_+)\nabla(\theta_-). \tag{4}$$

5.3 Effective Potential in the absence of Gauge Particles

The quantum corrections to the effective potential were shown by Coleman and Weinberg[4] to be all one particle irreducible graphs with all possible insertions of the field at its vacuum expectation value at zero momentum. We will show that all such super-graphs vanish. Since the proof will only depend on the structure of the anticommuting objects in the graph, in what follows the phrase two graphs are equivalent will mean they have the same θ structure up to multiplicative c numbers (mathematically we will denote this by the symbol $\cong$). In this new notation the propagators are:

$$\Delta_{\pm\pm}(1,2) \cong \bar{\theta}_{12}\,\theta_{12}\,\exp\left\{\pm\frac{i}{4}\left(\bar{\theta}_1 \not{p}\,\gamma_5\theta_1 - \bar{\theta}_2\not{p}\,\gamma_5\theta_2\right)\right\}$$

$$\Delta_{\pm\mp}(1,2) \cong \exp\left\{\bar{\theta}_1 \not{p}\,\theta_{2\mp} \pm \frac{i}{4}\left(\bar{\theta}_1\not{p}\,\gamma_5\theta_1 + \bar{\theta}_2\not{p}\,\gamma_5\theta_2\right)\right\}.$$

We note that the factors like $\exp\left(\frac{i}{4}\theta\gamma_5\not{p}\theta\right)$ always cancel at any internal vertex (in the absence of gauge particles) and so we may just work with the propagators

$$\Delta_{\pm\pm}(1,2) \cong \bar{\theta}_{12}\,\theta_{12\pm}$$

$$\Delta_{\pm\mp}(1,2) \cong \exp\left(\bar{\theta}_1\not{p}\,\theta_{2\mp}\right)$$

We are considering the effective potential only in the region where the vacuum expectation values of the fields, $\langle\ |\phi_\pm|\ \rangle$

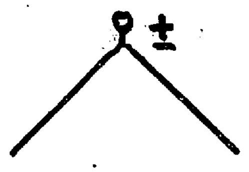

fig 5.1

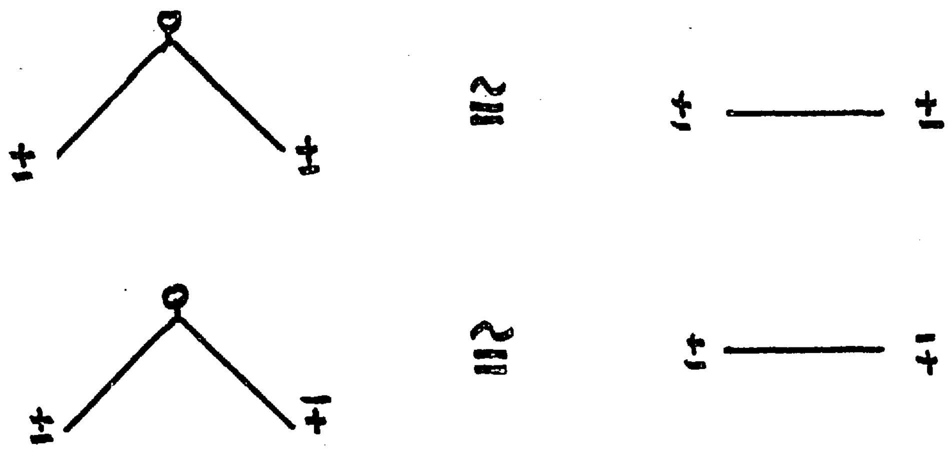

fig 5.2

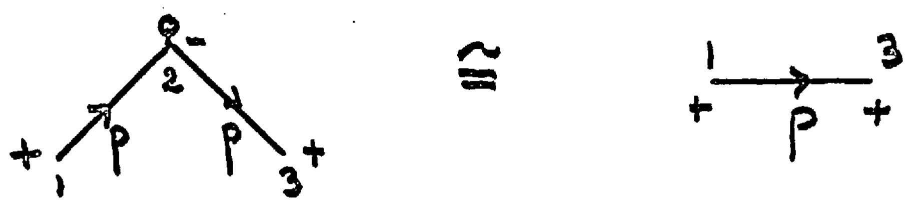

fig 5.3

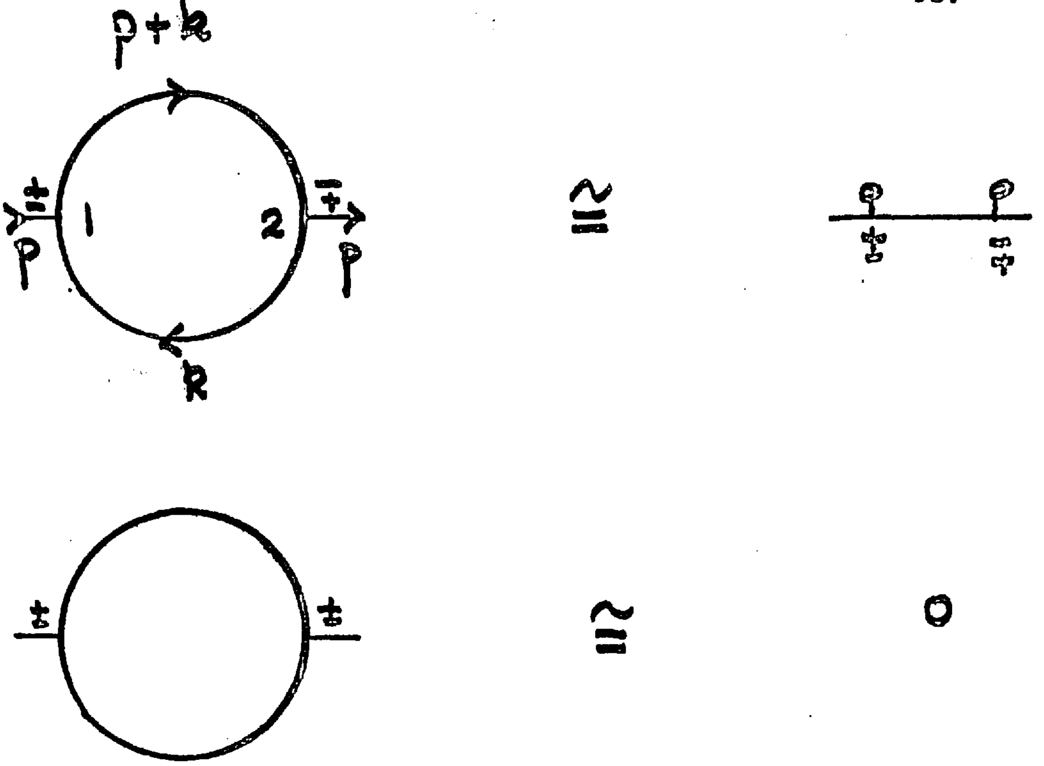

fig 5.4

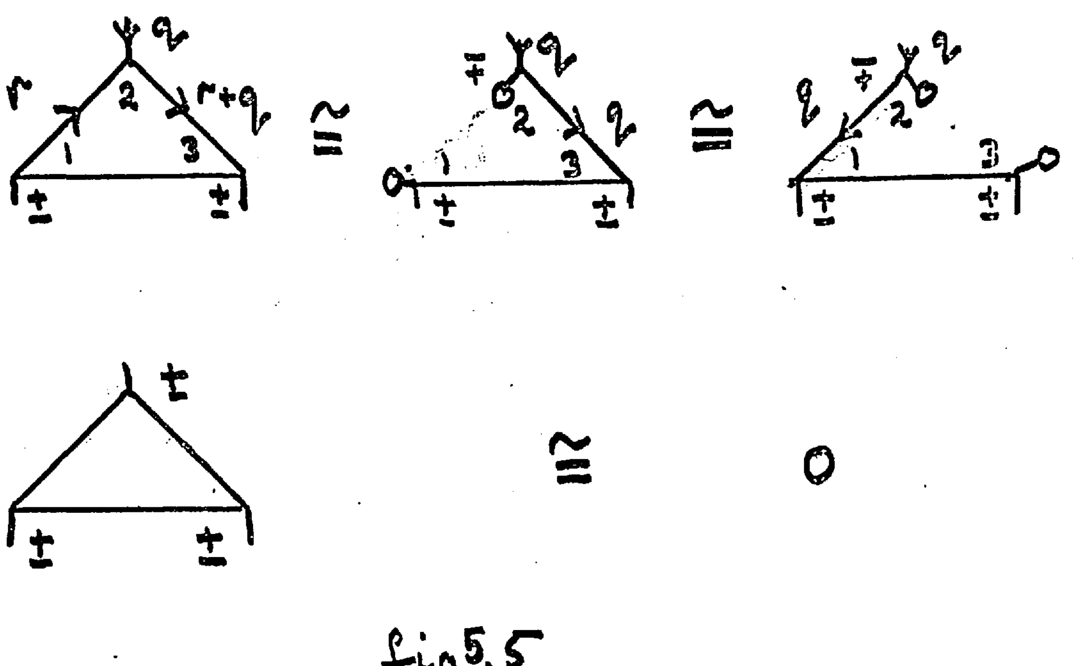

fig 5.5

conserve supersymmetry. This implies that $< | \phi_\pm | >$ is independent of θ, i.e. only the $< A >$ term is non-zero. There is only one kind of field insertion corresponding to a $\phi_\pm^3$ vertex which we depict in Figure 5.1.

Lemma 1

Two propagators connected by an insertions vertex are equivalent to a single propagator (see Figure 5.2). As an example we prove it for the vertex of Figure 5.3. The relevant part of the graph is

$$\nabla(\theta_2)\{\exp\{\bar{\theta}_1 \not{p}\, \theta_{2-} + \bar{\theta}_2 \not{p}\, \theta_{3+}\}\}$$

$$= \nabla(\theta_2)\{\exp(\bar{\theta}_{13} \not{p}\, \theta_{2-})\}$$

$$\widehat{=} \quad \bar{\theta}_{13}\theta_{13+} \quad \widehat{\cong} \quad \Delta_{++}(1,3)$$

Theorem 1

Any series of propagators connected only by field insertions vertices is equivalent to a single propagator (using Lemma 1). Hence every supersymmetric diagram contributing to the effective potential is equivalent to a supersymmetric vacuum diagram.

Lemma 2

A one loop self energy is equivalent to a single propagator or vanishes (see Figure 5.4). The self energy part is equivalent to

$$\exp\{\bar{\theta}_1(\not{p}+\not{k})\theta_{2\mp} + \bar{\theta}_2 \not{k}\, \theta_{1\pm}\}$$

$$= \exp\{\bar{\theta}_1 \not{p}\, \theta_{2\mp}\} \qquad \text{Q.E.D.}$$

Lemma 3

The one loop vertex correction either vanishes or is equivalent to the same graph with one chirality changing line deleted from the loop (see Figure 5.5). Proof: the triangle part of the diagram is equivalent to

$$\bar{\theta}_{13}\theta_{13\pm} \exp\{\bar{\theta}_1 \not{p}\, \theta_{2\mp} + \bar{\theta}_2(\not{p}+\not{q})\theta_{3\pm}\}$$

$$= \bar{\theta}_{13}\theta_{13\pm} \exp\{\bar{\theta}_2 \not{q}\, \theta_{1\pm}\}$$

$$\cong \Delta_{\underline{\pm\pm}}(1,3)\ \Delta_{\substack{-\,+\\+\,-}}(2,1)$$

Theorem 2

All diagrams up to 5 loops contributing to the effective potential vanish. Proof: since every such diagram is equivalent to a vacuum diagram with the same number of loops (Theorem 1) which

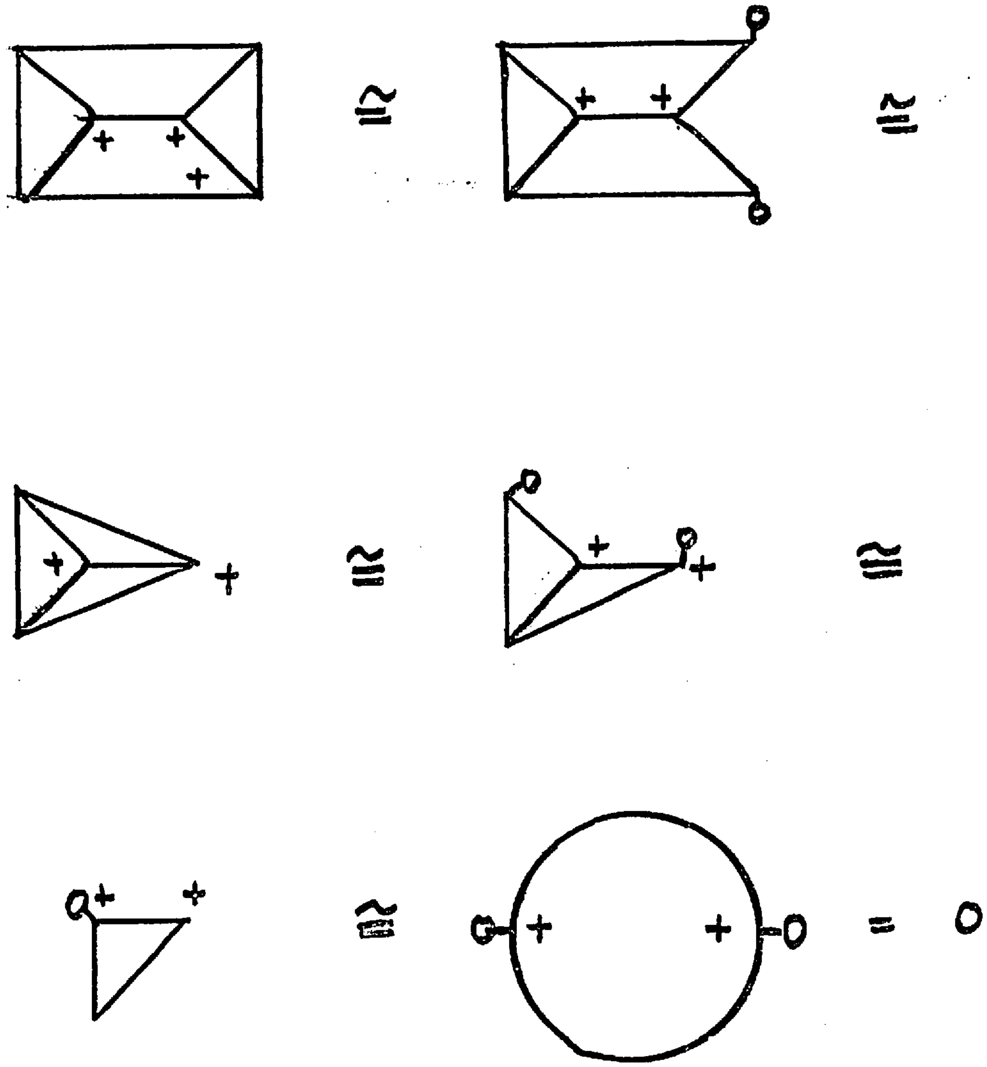

fig 5.6

must contain a one loop self energy or vertex correction graph. The previous lemmas can be used to simplify such loops further and finally to show that the graph vanishes. The most complicated example is drawn in Figure 5.6.

We now prove that for $\langle \, | \phi_\pm | \, \rangle$ independent of θ (i.e. supersymmetry conserving) the effective potential vanishes to all orders. A general diagram will contain $\Delta_{\underline{\pm}\underline{\pm}}$ propagators. However, we note that $\Delta_{\underline{\pm}\underline{\pm}} \cong \bar{\theta}_\pm \theta_\pm$ acts as a delta function in space and therefore carrying out the θ differentiative at one end of the $\Delta_{\underline{\pm}\underline{\pm}}$ propagator is equivalent to contracting the propagator to a point (see Figure 5.7) (momentum still being conserved). The vertices resulting from such a process are equivalent to those arising from terms like $\phi_\pm^n$ (n arbitrary) in the Lagrangian, see Figure 5.8). The vertex in Figure 8 is equivalent to

$$\nabla(\theta_k) \prod_i \exp \{ \bar{\theta}_{i\, i+1} \not{p}_i \, \dot{\theta}_k + \}$$

$$\cong \sum_i \sum_j (p_i \cdot p_j) \; \bar{\theta}_{i\, i+1} \; \theta_{j\, j+1} -$$

Similarly for a - vertex. Using lemma 1 and the above observation we can make any graph equivalent to a vacuum graph constructed from only $\Delta_{\pm\mp}$ propagator. Suppose that in this resulting graph $n_- > n_+$. Carrying out the θ differentiations at every + vertex (as in equation) results in a sum of terms containing the product of $2n_+$ $\theta_{-s\, s+1}$ differences. Now, if $n_- > n_+$.

fig 5.7

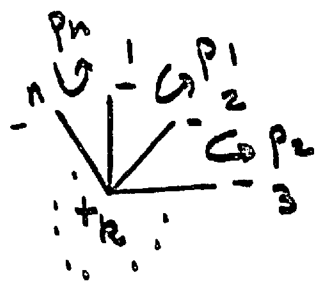

fig 5.8

fig 5.9.

the graph must vanish as there exist more θ derivatives than θ's. If $n_+ = n_-$ there are the same number of θ's as derivatives; however since all θ's occur in the form of differences any θ in a loop can be eliminated in favour of the other θ's in the same loop: i.e. $-\theta_{12} = \theta_{23} + \theta_{34} + \dots + \theta_{n1}$ We can repeat this until there is only one loop left. Since the remaining loop vanishes unless there are twice the number of θ's as there are vertices it must of the form

$$\bar{\theta}_{12}\,\theta_{12-}\,\dots\dots \qquad \dots\,\bar{\theta}_{n1}\,\theta_{n1-}$$

However, this also vanishes by eliminating one of the θ's and using the identity above

$$\dot{\theta}_{-\alpha}\,\theta_{-\beta}\,\theta_{-\gamma} = 0$$

Q.E.D.

5.4 Effective Potential in the presence of the Gauge Particles

It follows from supersymmetry conservation that the vacuum expectation value of the gauge field, $\langle \quad | \psi | \rangle$ must be independent of θ . We now show that graphs in which gauge propagators appear are equivalent to graphs constructed only of matter fields. The gauge propagator can appear in essentially only 3 ways.

(i) Two gauge propagators in succession. This is equivalent to a single gauge propagator (see Figure 5.9). Proof: the vertex is equivalent to

$$\nabla^2(\theta_2)\,(\bar{\theta}_{12}\,\theta_{12})^2(\bar{\theta}_{23}\,\theta_{23})^2 \cong (\bar{\theta}_{13}\theta_{13})^2$$

$$\cong \quad \Delta\,(1,3)$$

(ii) A gauge propagator between two matter propagators. This is equivalent to a single matter propagator (see Figure 5.10). Proof: here we must be careful when gauge lines are present as the factors like exp $(\frac{i}{4}\bar{\theta}_1 \not{\partial} \gamma_5 \theta_1)$ in the propagators no longer cancel at the vertices. Taking this into account the contribution from the gauge line is

$$(\nabla(\theta_2))^2(\nabla(\theta_3))^2 \quad (\bar{\theta}_{23}\;\theta_{23})^2 \times$$

$$\exp\left\{ -\frac{i}{4}\,\bar{\theta}_2 \not{\partial} \gamma_5\,\theta_2 - \frac{i}{4}\;\bar{\theta}_3 \not{\partial} \gamma_5\,\theta_3 \right\}$$

fig 5.10

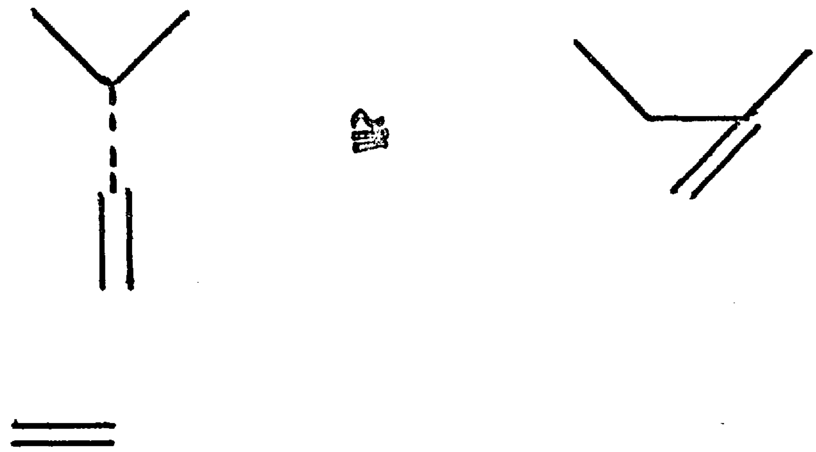

fig 5.11

fig 5.12.

$$= (\nabla(\theta_3))^2 \exp\{\bar{\theta}_3 \not{p}\, \theta_{3-}\}$$

$$\cong \nabla(\theta_{3+}) \nabla(\theta_{3-}) \exp\{\bar{\theta}_{3+} \not{p}\, \theta_{3-}\}$$

$$= \nabla(\theta_+) \nabla(\theta_-) \quad \Delta_{+-}$$

(iii) A gauge propagator leading into a vertex, for example see Figure 5.11.

The factors from the gauge line in Figure 5.11 are

$$(\nabla(\theta_1))^2 (\nabla(\theta_2))^2 (\bar{\theta}_{12}\theta_{12})^2 \exp\{\bar{\theta}_2 \not{q}\, \theta_{2-} + \bar{\theta}_1 \not{p}\, \theta_{2-}\}$$

$$\cong (\nabla(\theta_2))^2 \exp\{\bar{\theta}_2(\not{p} + \not{q})\, \theta_{2-}\}$$

$$\cong \nabla(\theta_+) \nabla(\theta_-) \exp\{\theta_+ (\not{p} + \not{q})\, \theta_-\}$$

$$\cong \nabla(\theta_+) \nabla(\theta_-) \quad \Delta_{+-}$$

(see Figure 5.11). Similarly if the gauge line is between two vertices (see Figure 5.12). Any other possibility trivially reduces to one of the three cases considered above.

Having shown that any gauge line can be removed in favour of matter lines, the super-graph vanishes by the previous theorems.

Conclusion

It has been shown that for field expectation values which conserve supersymmetry the effective potential vanishes to all orders in perturbation theory. Therefore, if supersymmetry is conserved we gain no more information about the vacuum state by computing higher order corrections to the effective potential than was given by the classical potential. Hence, any degeneracy is not removed. It remains an open question as to whether there exists a stable minimum of the effective potential which breaks supersymmetry only when Quantum corrections are included. If such minimi do not exist then apart from a few exceptional models for which there is no vacuum degeneracy the only ambiguity free models are those which break supersymmetry at the tree level.

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CHAPTER 6

THE ONE LOOP EFFECTIVE POTENTIAL

FOR CHIRAL SUPERFIELDS

6.1 Introduction

Although we have proved that the effective potential vanishes for field expectation values which conserve supersymmetry, it is much more difficult to establish a corresponding result for field expectation values which break supersymmetry. In chapter four we showed that for the two models which conserve supersymmetry at the tree level the one loop effective potential was complex for field expectation values which break supersymmetry. This might lead one to speculate that in this region the effective potential has quite large areas in which it is complex and hence unstable. This, coupled with the fact that at the tree level supersymmetry tends to be conserved, leads one to suspect that there does not exist a supersymmetric breaking absolute minimum induced by radiative corrections alone.

The difficulty with such calculations is that the tree potential does not vanish if supersymmetry is conserved and hence when calculating the minimum of a tree and one loop correction to the effective potential one often establishes a minimum only to find it is outside the range of validity of the approximation. (This is certainly the case when only one coupling constant is present.) The complexity of the calculations increases very rapidly as one attempts to go further than one loop.

In this chapter we make a first attempt to establish some of the general features of the supersymmetric effective potential by calculating the one loop effective potential for a Lagrangian containing supersymmetric chiral fields.

6.2 The one-loop effective potential

A sufficiently general renormalizable supersymmetric Lagrangian which does not contain gauge particles is given by

$$\mathcal{L} = \frac{1}{8}(\bar{D}D)^2\{\phi_+\phi_-\} - \frac{1}{2}\bar{D}D\Big\{\frac{1}{2}\phi_{+\alpha}m_{\alpha\beta}\phi_{+\beta}$$

$$= \frac{1}{3!}g_{\alpha\beta\gamma}\phi_{+\alpha}\phi_{+\beta}\phi_{+\gamma} + \frac{1}{2}\phi_{-\alpha}m_{\alpha\beta}\phi_{-\beta}$$

$$+\frac{1}{3!}g_{\alpha\beta\gamma}\phi_{-\alpha}\phi_{-\beta}\phi_{-\gamma}\Big\} \tag{6.1}$$

where $\phi_+^\dagger$ is equal to ϕ_-. We will calculate the one-loop effective potential for this Lagrangian by explicitly calculating all the contributing Feynman diagrams. As explained in the introduction to chapter four these are all one loop, one particle irreducible, connected diagrams with all possible field insertions at zero momentum. The previous calculations demonstrate that the graphs are best evaluated in a manifestly supersymmetric manner. Also the combinatoricswill be considerably simplified if we treat the mass terms as part of the interaction. This is because the $\Delta_{\pm\mp}$ propagators are proportional to the masses and so vanish if the masses are to form part of the interaction. In general, one is not allowed to juggle with parts of the Lagrangian without destroying the approximation being used, however as we are performing an expansion in $\hbar$ it is permissible. Therefore, the propagators for the above Lagrangian are

$$\Delta_{\pm\pm}(1,2)_{\alpha\beta} = 0$$

$$\Delta_{\pm\mp}(1,2)_{\alpha\beta} = \delta_{\alpha\beta} \exp\{\bar{\theta}_1 \not{p} \theta_2 \pm \frac{i}{4}\{\bar{\theta}_1 \not{p} \gamma_5 \theta_1 + \bar{\theta}_2 \not{p} \gamma_5 \theta_2\}\} \quad \Delta$$

where $\Delta = \dfrac{1}{p^2}$

As noticed in the previous chapter factors like $\exp\{\pm\frac{i}{4}\bar{\theta}\not{p}\gamma_5\theta\}$ always cancel at the type of vertex arising in the graphs being considered. Consequently, the propagator may effectively be written as

$$\Delta_{\pm\pm\,\alpha\beta}(1,2) = 0$$

$$\Delta_{\pm\mp\,\alpha\beta}(1,2) = \delta_{\alpha\beta} \exp\{\bar{\theta}_1 \not{p} \theta_2 \mp\} \,\Delta \qquad (6.2)$$

The vertices arising from the above Lagrangian are of either mass or field vertex type. The mass type have a factor $m_{\alpha\beta}$ The field type vertex has a factor

$$g_{\alpha\beta\gamma} < \phi_\pm >_\alpha$$

$$= g_{\alpha\beta\gamma}\left(A_{\pm\alpha} + \frac{1}{2}\bar{\theta}\theta_{\pm} F_{\pm\alpha}\right)$$

For brevity we define the matrices

$$A_{\pm\alpha\beta} = g_{\alpha\beta\gamma} A_{\pm\gamma}$$

$$F_{\pm\alpha\beta} = g_{\alpha\beta\gamma} F_{\pm\gamma} \tag{6.3}$$

The calculation is performed in two summations. First we sum all possible insertions (excluding F type) between two F vertices obtaining an effective F-F propagator; second we then sum the remaining F's. A typical diagram is shown in Figure 6.1.

Two F's linked by propagators and insertions are either of the same chirality or they are not, see Figure 6.2a and 6.2b.

Consider the case given in Figure 6.2a. Using Lemma 1 of chapter five we find that the graph in Figure 6.2a is proportional to that in Figure 6.3. This graph vanishes because the θ structure is of the form

$$\bar{\theta}_1 \theta_{1\pm} \; \bar{\theta}_{12} \theta_{12\pm} \; \bar{\theta}_2 \theta_{2\pm} = 0 \tag{6.4}$$

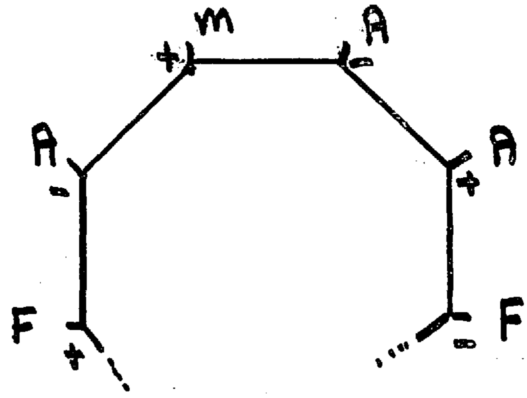

fig 6.1

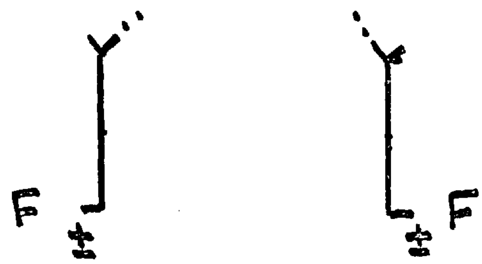

fig 6.2 a

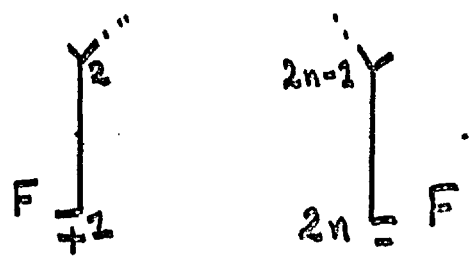

fig 6.2 b

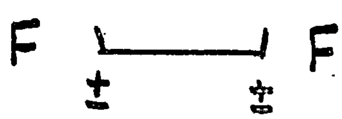

fig 6.3

Now consider the graph in Figure 6.2b. It contains n - 1 (+) vertices and (n - 1)(-) vertices (excluding the two outer $F_\pm$ vertices) which may either be mass type vertices or field type vertices. The mathematical expression for the graph is

$$\left\{ \underset{2}{\nabla}(\theta_1) \ldots\ldots\ldots \underset{2}{\nabla}(\theta_{2n}) \right\} F_+ \,\bar{\theta}_{\underline{1}}\theta_{1+}$$

$$\exp(\bar{\theta}_1 \not{p}\, \theta_{2-})\, A_- \exp(\bar{\theta}_2 \not{p}\, \theta_{3+}) A_+ \ldots$$

$$\ldots\ldots \exp(\bar{\theta}_{2n-1} \not{p}\, \theta_{2n-})\, \bar{\theta}_{\underline{2n}}\theta_{2n-}\, F_+$$

$$\Delta^{2n-1}$$

+ all possible terms with m's in place of $A_\pm$'s

(6.5)

Carrying out the θ differentiations

$$= \left\{ (F_+ A_- A_+ \ldots A_+ F_-) \right.$$

+ all possible terms

with m's in place of A's $\left.\right\} \dfrac{(p^2)^{n-1}}{(p^2)^{2n-1}}$

$$= F_+ \left\{ (m + A_+)(m + A_-) \right\}^{n-1} F_- \left(\frac{1}{p^2}\right)^n$$

To calculate the effect of including all possible number of vertices we must sum over n; resulting in

$$F_+ \left\{ p^2 - (m + A_-)(m + A_+) \right\}^{-1} F_-$$

$$= F_+ \left\{ p^2 - \mathcal{M}^2 \right\} F_-$$

where

$$\mathcal{M}^2 = (m + A_-)(m + A_+) \tag{6.6}$$

The second stage of the calculation is to sum over the remaining $F_\pm$ vertices. A typical graph is shown in Figure 6.4 where the wiggly line represents the effect of the first summation given in equation (6.6).

Carrying out the final summation over the $F_\pm$ type vertices the one loop effective potential equals

$$\mathrm{Tr} \left\{ i \int d^4p \sum_{n=1}^{\infty} \frac{1}{n} \left\{ F_+ \frac{1}{p^2 - \mathcal{M}^2} F_- \;\cdots\; F_- \frac{1}{p^2 - \mathcal{M}^2} \right\}_n \right.$$

The bracket $\{ \;\}_n$ is to be taken to contain n F_+'s and n F_-'s. The $\frac{1}{n}$ factor arises from symmetry considerations.

$$= i \int d^4p \, \mathrm{Tr} \ln \left\{ 1 - F_+ \frac{1}{p^2 - \mathcal{M}^2} F_- \frac{1}{p^2 - \mathcal{M}^2} \right\} \tag{6.7}$$

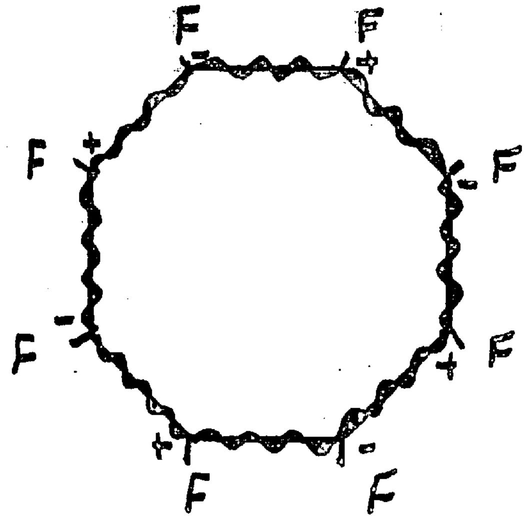

fig 6.4

It is hoped that this compact formula for the effective potential will prove useful for evaluating the one-loop effective potentials and that it will give a clue to the general features of the effective potential for chiral superfields.

We notice that it vanishes for $F_{\pm} = 0$. This must be the case as a consequence of the conclusion of chapter five. Also

$$- F_{\pm\alpha} = m_{\alpha\beta} A_{\mp\beta} + \frac{1}{2} g_{\alpha\beta\gamma} A_{\mp\beta} A_{\mp\gamma}$$

and hence

$$- \frac{d F_{\pm\alpha}}{d A_{\mp\sigma}} = m + A_{\mp}$$

From equation (6.6) we can express $\mathcal{M}^2$ as

$$\mathcal{M}^2 = \frac{dF_+}{dA_-} \frac{dF_-}{dA_+}$$

Consequently, the one loop effective potential is a function of two variables F_+ and $\frac{dF_+}{dA_-}$ (and there conjugates F_- and $\frac{dF_-}{dA_+}$). We expect this to be true to all orders.

Assuming that $[F_+, \mathcal{M}^2] = 0$, as would be the case if only one ϕ_+ and one ϕ_- were present equation (6.7) can be rewritten

$$= i \int d^4p \, \mathrm{Tr} \{ \ln ((p^2 - \mathcal{M}^2)^2 - F_+ F_-)$$

$$- 2 \ln (p^2 - \mathcal{M}^2) \}$$

$$= i \int d^4p \, \mathrm{Tr} \left\{ \ln\left(p^2 - \mathcal{M}^2 + \sqrt{F_+ F_-}\right) \right.$$

$$\left. + \ln\left(p^2 - \mathcal{M}^2 - \sqrt{F_+ F_-}\right) - 2 \ln\left(p^2 - \mathcal{M}^2\right) \right\}$$

If we diagonalized the mass matrices they would be in 3 classes

$$\mathcal{M}^2 + \sqrt{F_+ F_-} \quad , \quad \mathcal{M}^2 - \sqrt{F_+ F_-}$$

$$\mathcal{M}^2$$

A complex effective potential arises from tachyonic masses and these can only arise from $\mathcal{M}^2 - F_+ F_-$

For the case of a single ϕ_z field complexity arises if

$$\left| \left(1 + \frac{A_-}{m}\right)^2 + \frac{1}{3} \right| \leq \sqrt{\frac{2}{9}}$$

For large field expectation values it must be real.

Appendix A

Here we discuss the properties of Majorana fields and the related conjugation matrix. The conjugation matrix C preserves the relation for Dirac matrices. That is

$$\{ \gamma_\mu , \gamma_\nu \} = 2 g_{\mu\nu}$$

and satisfies

$$C^{-1} \gamma_\mu C = - \tilde{\gamma}_\mu$$

It can be chosen to satisfy

$$\tilde{C} = - C$$

It is useful to remember that $\gamma_\mu C$ and $\sigma_{\mu\nu} C$ are symmetric, while $\gamma_5 C$ and $i \gamma_\mu \gamma_5 C$ are antisymmetric.

A Majorana field is defined by

$$\bar{\Psi}^\alpha = (\psi^+ \gamma_0)_\alpha = (C^{-1})^{\alpha\beta} \psi_\beta$$

Given two anticommuting Majorana fields ψ and χ it is easy to deduce

$$\bar{\Psi} \chi = \bar{\chi} \Psi , \quad \bar{\Psi} \gamma_\mu \chi = - \bar{\chi} \gamma_\mu \Psi ,$$

$$\bar{\Psi} \sigma_{\mu\nu} \chi = - \bar{\chi} \sigma_{\mu\nu} \Psi , \quad \bar{\Psi} \gamma_5 \chi = \bar{\chi} \gamma_5 \Psi$$

$$\bar{\Psi} i \gamma_\mu \gamma_5 \chi = \bar{\chi} i \gamma_\mu \gamma_5 \Psi$$

Using the Fiery reshuffling properties we can deduce that

$$\psi \bar{\Psi} \gamma_5 \Psi = - \gamma_5 \Psi \bar{\Psi} \Psi ,$$

$$\psi \bar{\psi} i \gamma_\mu \gamma_5 \psi = -i \gamma_\mu \gamma_5 \psi \bar{\psi} \psi$$

and then prove that

$$(\bar{\psi} \gamma_5 \psi)^2 = (\bar{\psi} \psi)^2$$

$$\bar{\psi} \gamma_5 \psi \bar{\psi} \psi = \bar{\psi} i \gamma_\mu \gamma_5 \psi \bar{\psi} \psi = \bar{\psi} i \gamma_\mu \gamma_5 \psi \bar{\psi} \gamma_5 \psi = 0$$

$$\bar{\psi} i \gamma_\mu \gamma_5 \psi \bar{\psi} i \gamma_\nu \gamma_5 \psi = g_{\mu\nu} (\bar{\psi} \psi)^2$$

These properties are very useful in calculating supersymmetric Feynman diagrams. An example being

$$\bar{\theta}_{12} \theta_{12} (\bar{\theta}_1 \not{\partial} \theta_2)^3$$

$$= \bar{\theta}_{12} \theta_{12} (\theta_{12} \not{\partial} \theta_2)^3 = 0$$

Also frequently used is the fact that

$$\bar{\theta}_{12} \theta_{12\pm} \theta_{12\pm} = 0$$

Appendix B

The properties of the covariant derivative

$$D_\alpha = \frac{\partial}{\partial \bar{\theta}^\alpha} - \frac{1}{2}(\not{p}\theta)_\alpha$$

are given in this appendix. They generate a Clifford algebra,

$$\{ D_\alpha , D_\beta \} = -(\not{p}C)_{\alpha\beta}$$

As such there are 16 independent basic elements

$$\mathbb{1},\ D_\alpha\ ,\ \bar{D}D\ ,\ \bar{D}\gamma_5 D\ ,\ \bar{D}\,i\,\gamma_\mu\gamma_5 D,$$

$$\bar{D}DD_\alpha\ ,\ (\bar{D}D)^2$$

Any product of D's can be reduced to a linear combination of these, for example

$$D_\alpha D_\beta = -\frac{1}{2}(\not{p}C)_{\alpha\beta} + \frac{1}{4}C_{\alpha\beta}\bar{D}D$$

$$-\frac{1}{4}(\gamma_5 C)_{\alpha\beta}\bar{D}\gamma_5 D - \frac{1}{4}(i\gamma_\nu\gamma_5 C)_{\alpha\beta}\bar{D}\,i\,\gamma_\nu\gamma_5 D$$

From this expansion we can prove

$$\bar{D}\gamma_\mu D = 2p_\mu \ ;\ \bar{D}\sigma_{\mu\nu}D = 0$$

The following is a table with which it is possible to simplify a product of four D's.

	$\bar{D}D$	$\bar{D}\gamma_5 D$	$\bar{D}\, i\gamma_\nu\gamma_5 D$
$\bar{D}D$	$(\bar{D}D)^2$	$-2ip^\nu\, \bar{D}\, i\gamma_\nu\gamma_5 D$	$2ip_\nu\, \bar{D}\gamma_5 D$
$\bar{D}\gamma_5 D$	$2ip^\nu\, \bar{D}\, i\gamma_\nu\gamma_5 D$	$(\bar{D}D)^2$	$-2ip_\nu\, \bar{D}D$
$\bar{D}\, i\gamma_\mu\gamma_5 D$	$-2ip_\mu\, \bar{D}\gamma_5 D$	$2ip_\mu\, \bar{D}D$	$\eta_{\mu\nu}(\bar{D}D)^2 - 2ip_\lambda\, \epsilon_{\lambda\mu\nu\rho}\, \bar{D}\, i\gamma_\rho\gamma_5 D - 4(\eta_{\mu\nu}p^2 - p_\mu p_\nu)$

where left (right) factors are listed in the rows (columns).

Frequently useful are the following formulae which give explicitly the action of the more important combination of the covariant derivative on the general superfield.

$$
\begin{aligned}
-\bar{D}\left(\frac{1 \pm i\gamma_5}{2}\right) D\,\phi = {} & F \mp iG \\
& + \bar{\theta}\left(\frac{1 \mp i\gamma_5}{2}\right)(\chi - i\partial\!\!\!/\,\psi) \\
& + \frac{1}{4}\bar{\theta}\left(\frac{1 \mp i\gamma_5}{2}\right)\theta\,(D - \partial^2 A \pm 2i\partial^\mu A_\mu) \\
& + \frac{1}{4}\bar{\theta}\, i\gamma_\nu\gamma_5\,\theta\,(\mp i\partial_\nu)(F \mp iG)
\end{aligned}
$$

$$+\frac{1}{4}\bar{\theta}\theta\,\bar{\theta}\left(\frac{1\pm i\gamma_5}{2}\right)(-i\partial\!\!\!/)(\chi - i\partial\!\!\!/\psi)$$

$$+\frac{1}{32}(\bar{\theta}\theta)^2(-\partial^2)\quad(F\mp iG).$$

$$\frac{1}{2}(\bar{D}D)^2\,\phi = D-\partial^2 A - \bar{\theta}\{\partial^2\psi + i\partial\!\!\!/\chi\}$$

$$-\frac{1}{4}\bar{\theta}\theta(2\partial^2 F) - \frac{1}{4}\bar{\theta}\gamma_5\theta(2\partial^2 G)$$

$$-\frac{1}{4}\bar{\theta}i\gamma_\nu\gamma_5\theta\{2\partial_\nu\partial_\mu A^\mu\}$$

$$+\frac{1}{4}\bar{\theta}\theta\,\bar{\theta}(i\partial\!\!\!/)\{\partial^2\psi + i\partial\!\!\!/\chi\}$$

$$-\frac{1}{32}(\bar{\theta}\theta)^2\partial^2\{D-\partial^2 A\}$$

Appendix C

The expansion into components of the product of two superfields are given below

$$\phi(1,\theta)\,\phi(2,\theta) = A(1)\,A(2)$$

$$+\ \bar\theta\left(A(1)\psi(2) + \psi(1)A(2)\right)$$

$$+\ \frac{1}{4}\bar\theta\theta\left(A(1)F(2) - \bar\psi^c(1)\psi(2) + F(1)A(2)\right)$$

$$+\ \frac{1}{4}\bar\theta\gamma_5\theta\left(A(1)G(2) + \bar\psi^c(1)\gamma_5\psi(2) + G(1)A(2)\right)$$

$$+\ \frac{1}{4}\bar\theta i\gamma_\nu\gamma_5\theta\left(A(1)A_\nu(2) + \bar\psi^c(1)i\gamma_\nu\gamma_5\psi(2) + A_\nu(1)A(2)\right)$$

$$+\ \frac{1}{4}\bar\theta\theta\,\bar\theta\Big(A(1)\chi(2) + \psi(1)F(2) - \gamma_5\psi(1)G(2) + \chi(1)A$$

$$+\ i\gamma_\nu\gamma_5\psi(1)A_\nu(2) + F(1)\psi(2) - G(1)\gamma_5\psi(2) - A_\nu(1)i\gamma_\nu\gamma_5$$

$$+\ \frac{1}{32}(\bar\theta\theta)^2\Big(A(1)D(2) + 2F(1)F(2) + 2G(1)G(2) + 2A_\nu(1)A_\nu(2)$$

$$+\ D(1)A(2) - 2\bar\psi^c(1)\chi(2) - 2\bar\chi^c(1)\psi(2)\Big)$$

where

$$(\overline{\psi}^{c})^{\alpha} = (C^{-1})^{\alpha\beta}\ \psi_{\beta}$$

From this we can derive the expansion for $\phi_{\pm}(1)\phi_{\pm}(2) = \phi_{\pm}(3)$

$$A_{+}(3) = A_{+}(1)\, A_{+}(2).$$

$$\psi_{+}(3) = A_{+}(1)\,\psi_{+}(2) + \psi_{+}(1)\, A_{+}(2)$$

$$F_{+}(3) = A_{+}(1)\, F_{+}(2) - \overline{\psi}^{c}_{+}(1)\psi_{+}(2) + F_{+}(1)\, A_{+}(2).$$

$$\vdots \qquad\qquad \vdots$$

Repeated application of these rules enables one to quickly obtain the component expansion of $\phi_{\pm}{}^{n}$

APPENDIX D

It was stated in chapter two that to second order $\mathcal{L}_2$ required only mass renormalization. The only diagrams at this order which can diverge are of $\Sigma_{\pm\pm}$ type. Of these diagrams only those which pick up θ^4 from external lines will be non-zero because the propagators contribute only a ($\bar{\theta}_{12}$ θ_{12} $)^2$ factor (e.g. $A_\pm$ and $D_\pm$ external lines). On calculating the S-matrix element which gives rise to these divergent diagrams we note that it can be cancelled by the mass renormalization term of $\delta m \int (\bar{D}D)^2 \phi^6 dx$ The relevant S-matrix element being:

$$\int (\bar{D}D(1))^2 dx_1 \int (\bar{D}D(2))^2 dx_2 \; \Delta_{++}(1,2)\, \Delta_{--}(1,2)\, \phi(1)\phi$$

$$= \int dx_1\, dx_2 \frac{1}{8} \Delta^2(x_{12}) \Big\{ A_\pm(1) D_\pm(2) - 2\bar{\psi}_\pm(1)\psi_\pm(2)$$

$$- 2\bar{\chi}_\pm^c(1)\psi_\pm(2) + 2F_\pm(1)F_\pm(2) + 2G_\pm(1)G_\pm(2)$$

$$+ 2A_{\pm\nu}(1)A_\pm^\nu(2) + D_\pm(1)A_\pm(2)\Big\}$$

We noted that for $\mathcal{L}_2$ in the m = 0 limit all graphs for which the number of external lines + the number of vertices are greater than 8 are convergent. In this class, with the exception of the two graphs in Figure 2.2 they all have divergences which are at most logarithmic. Therefore, it suffices to see if they are finite when the momenta on each external line are zero. This proof is considerably simplified by the two lemmas.

Lemma 1

For subgraphs of the form in Figure A.1(a) no derivatives from the exponentials of the propagators can contribute to the divergence because

$$\bar\theta_{12}\theta_{12}\bar\theta_{23}\theta_{23}\bar\theta_{31}\theta_{31}\exp\left\{\frac{i}{2}\left(\bar\theta_{1}\not p\theta_{2}+\bar\theta_{2}\not p\theta_{3}+\bar\theta_{3}\not p\theta_{1}\right)\right\} = \bar\theta_{12}\theta_{12}\bar\theta_{23}\theta_{23}\bar\theta_{31}\theta_{31}\exp\left\{\frac{i}{2}\left(\bar\theta_{12}\not p\theta_{23}\right)\right\} = \bar\theta_{12}\theta_{12}\bar\theta_{23}\theta_{23}\bar\theta_{31}\theta_{31}$$

as $\bar\theta_{12}\theta_{12}\bar\theta_{23}\theta_{23}\bar\theta_{31}\theta_{31}\bar\theta_{12}\not p\theta_{23} = 0$

Lemma 2

Consider graphs which contain a subgraph of the form in Figure A.1(b). If p is the loop momentum then no more than 3p factors can contribute from the exponentials in the propagators.

$$\Delta(1,2)\,\Delta(2,3)\,\Delta(3,4)\,\Delta(4,1) =$$

$$= \bar\theta_{12}\theta_{12}\bar\theta_{23}\theta_{23}\bar\theta_{34}\theta_{34}\bar\theta_{41}\theta_{41}\exp\left\{\frac{i}{2}\left(\bar\theta_{1}\not p\theta_{2}+\bar\theta_{2}\not p\theta_{3}+\bar\theta_{3}\not p\theta_{4}+\bar\theta_{4}\not p\theta_{1}\right)\right\}\left(\frac{1}{p^{2}}\right)^{4}.$$

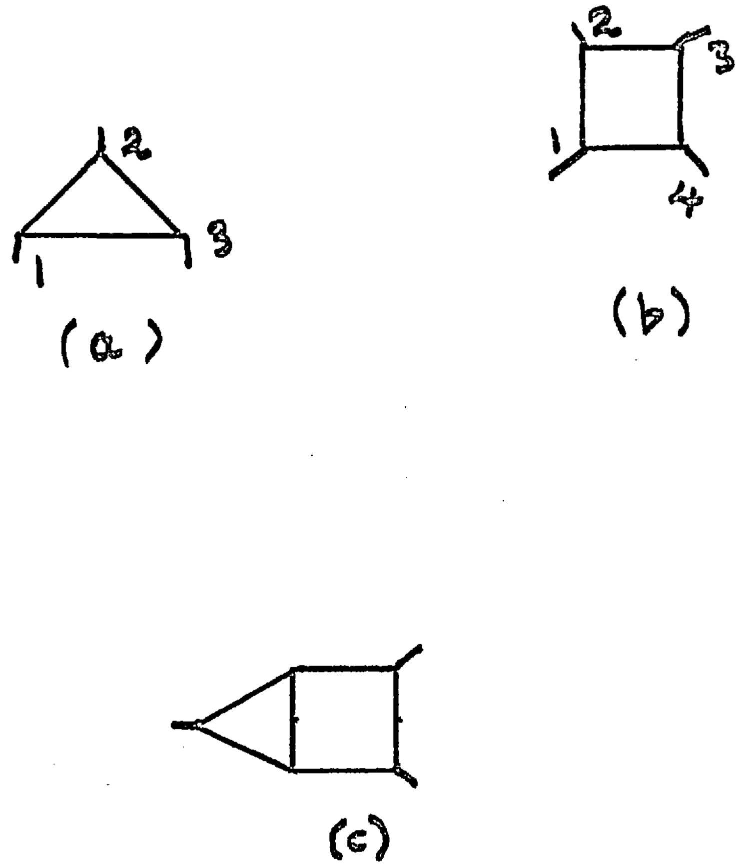

fig A.1.

$$= \bar{\theta}_{12}\theta_{12}\,\bar{\theta}_{23}\theta_{23}\,\bar{\theta}_{34}\theta_{34}\,\bar{\theta}_{41}\theta_{41}\exp\left\{\frac{i}{2}\left(\bar{\theta}_{12}\not{p}\,\theta_{23}+\bar{\theta}_{23}\not{p}\,\theta_{34}+\bar{\theta}_{12}\not{p}\,\theta_{34}\right)\right\}\left(\frac{1}{p^2}\right)^4.$$

Clearly it is not possible to bring down 4p factors.

For example the graph in Figure A.2(c) requires 4 internal momenta factors to diverge and so is finite.