RESPONSE OF SOIL DEPOSITS TO EARTHQUAKE DISTURBANCES; A TIME-HISTORY APPROACH

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by

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IN MEMORY OF MY MOTHER

I used to dream and think of you
to keep you close.
So shall I do now,
to keep you alive.
And you shall never die!

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TABLE OF CONTENTS

ABSTRACT

An analytical solution is developed to the problem of the shear response of a homogeneous layer to an arbitrary horizontal disturbance (acceleration, velocity or displacement) which includes the effects of radiation and material damping.

The main characteristics of the solution are:

a) It is exact and can be computed exactly.

b) It has in itself a consistent physical interpretation.

c) The final expressions for the layer response involve only the shifting and scaling of the input disturbance, their computation being, therefore, fast and simple.

d) Time-histories of acceleration, velocity or displacement are readily obtained at any depth in the layer.

e) To obtain the input disturbance, deconvolution of the layer motion may be performed with the same degree of accuracy and simplicity as in the response problem.

The method of solution is based on the fact that the layer motion obeys a phenomenon of propagation, and for the cases studied, the partial differential equation which describes the problem analytically is basically a wave equation, i.e. it has a D'Alambert solution. It is shown that such a solution may be obtained by means of the Laplace Transform with which the response of the layer may be expressed in terms of travelling waves.

The problem of radiation is studied analytically and its effect on the response is considered.

A visco-elastic constitutive law, in which the viscous component is assumed to be a volumetric force takes care of material damping effects.

The formulation of a general case for multi-layered deposits is made and discussed.

NOTATION

$g(t)$	A prescribed displacement time-history
$\dot{g}(t)$	" " velocity " "
$\ddot{g}(t)$	" " acceleration " "
$U(y,t)$	Absolute layer displacement at depth y at time t
$\dot{U}$	" " velocity " " " " "
$\ddot{U}$	" " acceleration " " " " "
$g_s(t) = U(0,t)$	Displacement time-history at the free surface of a deposit
$g_b(t) = U(h,t)$	Displacement time-history at the base of a deposit
$g_{rec}(t)$	Displacement time-history at rock outcrop
$W_g(t)$	Displacement Layer Wave
$W_{\dot{g}}(t) = \dot{W}_g(t)$	Velocity " "
$W_{\ddot{g}}(t) = \ddot{W}_g(t)$	Acceleration " "
G	shear modulus
ρ	mass density
η	specific viscosity
$S = (G/\rho)^{1/2}$	shear wave velocity
$\alpha = G_1S_2/G_2S_1$	Impedance ratio
$\beta = \dfrac{1-\alpha}{1+\alpha}$	Radiation coefficient
$k = \dfrac{\eta}{2\sqrt{G\rho}}$	damping parameter
$\xi = k/\sqrt{1+k^2}$	damping coefficient
$\mu = (1-\xi)/(1+\xi)$	damping parameter
$\gamma = \mu \dfrac{1-\alpha\sqrt{1+k^2}(1+\xi)}{1+\alpha\sqrt{1+k^2}(1-\xi)}$	dissipation coefficient
T_L	undamped layer period
e	the base of natural logarithms

INTRODUCTION

The problem of the response of a soil deposit to a seismic excitation is of particular importance in the engineering study of earthquakes, as it has to be considered both in the process of design of a specific structure, as well as in the general search for earthquake parameters that may, from an engineering point of view, characterise a seismic event.

A soil deposit disturbed by an earthquake motion at its base, will not transmit the motion unchanged, but it will cause it to advance in accordance with its material and geometrical characteristics; consequently, the vibration to which a structure, founded on the deposit, may be subjected as a result of the base movement will largely depend, both in magnitude and in nature, upon the properties of the deposit, The excitation for which the structure ought to be designed is, therefore, the response of the deposit to the base motion, and not the base motion itself.

It is also clear from the previous argument that records obtained from an earthquake at different locations of a particular region will show individual characteristics dictated by the geological conditions existing at each particular site. If those records are to be compared in order to study the general characteristics of the earthquake motion in that area, the effect of local geology has to be filtered from each record. Thus, in studying a natural deposit subjected to an earthquake, it is as important a problem to determine the response of the deposit for a given disturbance as it is to find the input disturbance from the recorded motion of the deposit.

The analytical study of the dynamic response of a deposit fundamentally involves firstly defining a constitutive equation for the material in the deposit, then, establishing the equation of motion for the medium, and finally solving this equation for the conditions imposed at the boundaries and the initial conditions of motion.

The simplest assumption regarding material behaviour is that of linear elasticity, where the stresses developed in the material are proportional to the strains which cause them, no matter how large these are. It may also be assumed that this behaviour is valid up to a certain level of stress and that when it is reached either the proportionality constant between stresses and strains changes (bilinear model), or the material yields, strain increasing while stress remains constant (elasto-plastic model). Another approach would be to consider the material to be visco-elastic, that is to assume that stresses have an elastic component proportional to the strain applied, and a viscous component proportional to the time rate of strain (Voigt or Kelvin model). A discussion on the different considerations made on the nature of the viscous constant is found in Newmark and Rosenblueth (1971). Other rheological models to represent the material behaviour may be found in Jaeger (1969) or Kolsky (1963). Based on experimental evidence the stress-strain curve for a material may be assumed to be a loop. The shape of this loop may be defined by means of three parameters (Ramberg & Osgood 1943), or it may also be supposed that such a shape corresponds to a hyperbolic relationship (Hardin & Drnevich, 1972b).

Hysteretic models may be linearised by considering equivalent properties, such as a constant of proportionality between stress and strain given by the slope of the line joining the extreme points of the actual loop shaped curve, and a damping coefficient proportional to the ratio of the energy loss to the total elastic energy, both in a cycle, as originally suggested by Jacobsen (1930).

Once the material behaviour is defined by a constitutive law, the equation of motion for the medium may be established from the condition of dynamic equilibrium. For deposits formed by homogeneous material, or with properties varying regularly with depth, a second order partial differential equation results which may be solved in a closed form. Ambraseys, for example, presents the shear response of a layer, with homogeneous material and surface loading (1960a), and with varying rigidity (1959). Idriss and Seed (1968) provide a general form of the equation of motion for horizontal deposits subjected to horizontal seismic motion and review the work of others.

For the case of layered deposits, a differential equation of motion is obtained for each layer, and the response of the deposit is found by solving this system of equations simultaneously considering the compatability of stresses and displacements at each interface, and the boundary conditions at the surface and base of the deposit.

Alternatively, the whole deposit may be idealised as a series of discrete masses interconnected, and then the equation of motion of the deposit is given in a matrix form. Idriss and Seed (1967, 1968) present the formulation of this lumped mass analysis, and Papastamatiou (1971) utilises this approach to illustrate the effect of radiation and plastic yielding on the response of foundation materials.

Response of deposits whose geometry cannot be described by a one-dimensional model may be given in a closed form in simple cases; Ambraseys (1960b) is an example. More irregular deposits may be idealised as an assemblage of elements interconnected at a finite number of nodal points in a Finite Element analysis, used, for example, by Idriss and Seed (1974). Also, the medium may be visualised as a lattice work of one-dimensional linear elements (Streeter & Wylie, 1968).

The equation or equations of motion for the deposit may be solved by numerical integration techniques, either directly or, if possible, after the time and space variables in the equations have been separated (Modal Analysis). A review of integration techniques can be found in Ayala-Milian (1973).

A Fourier analysis may also be used in the solution, using for example, the expressions given by Kanai (1951) for a deposit formed by horizontal visco-elastic layers under steady-state harmonic motion. Roesset (1970), Herrera & Rosenblueth (1965), and Schnabel et al (1972) use this kind of analysis in their studies; the last two references consider frequency independent damping coefficients. Streeter et al (1974a) have applied the method of characteristics, originally suggested by Westergaard (1933), to wave propagation problems. Basically, this is a finite difference method of integration suitable for the numerical solution of hyperbolic partial differential equations.

From a physical point of view, the dynamic response of a soil deposit may be seen essentially as a phenomenon of propagation. When this consideration has been brought into the analytical formulation of the problem, it has proved to be of great use simplifying both the formulation itself and the numerical process involved, and obviously providing a general understanding of the problem.

The boundary condition used by Papastamatiou (1971) to deal with radiation, and the basic idea which gives place to the method of characteristics (Westergaard (op. cit.), Newmark and Rosenblueth (1971)), are but two examples illustrating the simplicity which may be attained by considering the propagative nature of the response. We feel, however, that full use has not been made so far of the physical meaning attached to the soil response and its analytical implications.

We may state that if the motion of a deposit is the result of a disturbance being propagated in it, then such a motion may be known at any point in the deposit and at any instant if we know the disturbance and the way in which this travels inside the medium. The analytical solution to the problem must reflect this situation, and therefore it should be possible to find in this solution a clear relationship between excitation and response in the time domain. A relationship which is obviously not in terms of frequencies, modes, amplification factors or spectra, but which indicates a process performed on the disturbance to obtain the response.

The purpose of this thesis is to present this approach - a time-history approach - to the response of soil deposits for the simple case of uni-dimensional propagation. The basic assumptions in the thesis are as follows:-

a) Horizontal shear disturbances propagating vertically

b) Horizontal boundaries

c) Layers formed by homogeneous material with properties constant in time.

This work is divided into five chapters. The first of them considers the problem of a linear elastic layer on a rigid base in order to present, in the simplest of cases, a particular procedure of inversion for the Laplace transform which enables us to obtain the layer response directly as a time-history.

The second chapter is concerned with the problem of radiation, namely, the process of energy loss which takes place when the foundation of the layer is assumed to be a deformable semi-infinite medium. A closed form analytical solution is obtained following a time-history approach.

In chapter three a model of damping compatible with wave propagation is introduced so that the effect of internal dissipation of energy may also be included in our approach.

Chapter four combines the findings of the previous chapters; a complete wave solution is given for an homogeneous layer, and the formulation of a general case is made for multi-layered deposits, discussing both the problems of response and deconvolution.

Finally, in chapter five, a general discussion is made of the advantages and limitations of a time-history approach and conclusions are drawn.

CHAPTER ONE

THE BASIC SOLUTION

Introduction

This chapter considers the problem of the response of an homogeneous linear elastic layer subjected to an arbitrary shear motion at its base. This problem in itself does not offer many possibilities for its application, mainly, as the assumptions made for the behaviour of the constitutive material of the layer are too simple as to idealise adequately the behaviour of an actual soil deposit. It is simplicity, however, which makes this problem appropriate to introduce some basic concepts and discuss the general physical meaning of the analytical expressions for the layer response.

Hence, this chapter is intended to be a basic reference for the whole of this thesis. Here, a particular procedure for inverting Laplace transforms is presented, which will be used throughout this work. Such a procedure, by making use of the operational properties of the transformation, simplifies considerably the inversion problem and enables the layer response to be expressed as a time-history with a consistent physical interpretation. Also, it is in this chapter where concepts such as Disturbance, Layer Wave and Response Motion are defined and given a specific meaning which will be held all along this study.

A brief review of the Modal Analysis solution, which is obtained by using a conventional procedure of inversion is included, so that the advantages of the solution developed in this chapter may be fully appreciated.

The complete mathematical development can be found in Appendix 1.

Formulation of the Problem

Consider an elastic layer of thickness h on a rigid base as shown in Figure 1.1a. The material in the layer is characterized by a mass density ρ and a shear modulus G, both constant with depth throughout. If the base is given a displacement g(t), as the one produced by an SH wave, a differential element dy will be stressed as shown in Figure 1.1b.

$U = U(y,t)$ denotes the absolute horizontal displacement of a point in the layer at a depth y, at a time t.

The equation of motion 1.1 is obtained equating the net force in an elemental mass with the product of that mass and the absolute acceleration (dynamic equilibrium)

$$\ddot{U}\,(y,t) \;=\; S^2\,U''\,(y,t) \tag{1.1}$$

where

$S^2 = G/\rho$, the shear wave velocity of the layer material;

dots and dashes indicate derivations with respect to t and y respectively.

To fully specify the problem it will be considered that throughout the movement the surface of the layer is free of stresses and that there is no sliding between the base and the layer. Hence, the solution sought, i.e. $U(y,t)$, should satisfy the conditions:

$$U(h,t) \;=\; g(t) \tag{1.2}$$

and

$$U'(y,t)\Big]_{y=0} \;=\; 0 \tag{1.3}$$

Furthermore, it will be assumed that the motion starts from rest, i.e.

$$g(0) \;=\; U(y,0) \;=\; 0 \tag{1.4}$$

$$\dot{g}(t)\Big]_{t=0} \;=\; \dot{U}(y,t)\Big]_{t=0} \;=\; 0 \tag{1.5}$$

The problem is thus represented by a second order partial differential equation, and for the conditions given, it may be shown to be soluble.

The Solution Procedure

The use of the Laplace Transformation for finding the response of the layer leads to an expression in the transformed domain which provides the relationship between the transforms of the input displacement and of the layer displacement. Such an expression has the general form:

$$\bar{U}(y,p) = \bar{T}_{og}\,\bar{g}(p) \tag{1.6}$$

where p is the parameter of the transformation, and bars denote transformed functions. The meaning of $\bar{T}_{og}$ will now be discussed.

Equation 1.6 has a mathematical meaning but not a physical one (BRACEWELL, 1965). Therefore it is not proper to associate the parameter p with a frequency, nor to define $\bar{T}_{og}$ as a transfer function. Rather, if $\bar{U}$ and $\bar{g}$ are referred to as transforms only, without attaching any physical meaning to them, then by the same nature of the Laplace Transformation $\bar{T}_{og}$ may be seen to be the transform of an operation, namely, the Transfer Operation, which may be defined as the operational procedure performed upon the input to obtain the response.

Hence, from the previous definition it may be implied that, as equation 1.6 is an operational relation, a convenient way of inversion should involve finding a suitable expression for $\bar{T}_{og}$ so that the elementary properties of the Laplace Transformation may be applied to the right hand side of 1.6. This brings not only simplicity to the whole process of inversion, but in addition, expressions for the layer motion obtained by this procedure may be easily interpreted in physical terms.

Furthermore, the transform of the Transfer Operation, $\bar{T}_{og}$ may be defined uniquely for layer displacement, velocity, and acceleration;

this can be done simply by associating to each one of them the corresponding input, i.e.

$$\bar{\dot{U}}\,(y,p) = \bar{T}_{og}\,\bar{\dot{g}}\,(p) \tag{1.7}$$

$$\bar{\ddot{U}}\,(y,p) = \bar{T}_{og}\,\bar{\ddot{g}}\,(p) \tag{1.8}$$

Then, it can be claimed that no generality whatsoever has been lost by assuming stationary initial conditions (equations 1.4 and 1.5); any non-stationary condition may be introduced in the process of integration.+

Also, with this unique definition of $\bar{T}_{og}$, layer displacements, velocities or accelerations may be obtained with the same operational procedure, and in consequence, with the same degree of accuracy.

To illustrate these points, we may return to the layer problem, where if the Laplace Transformation is applied to equations 1.1, 1.2 and 1.3 with the initial conditions 1.4 and 1.5 the following relation is obtained in the transformed domain:

$$\bar{U}\,(y,p) = \frac{\cosh(p\ y/S)}{\cosh(p\ h/S)}\ \bar{g}\,(p) \tag{1.9}$$

which, after some manipulations may be written

$$\bar{U}(y,p) = \left[e^{-\frac{p}{S}(h-y)} + e^{-\frac{p}{S}(h+y)}\right]\left[\sum_{n=0}^{\infty} (-1)^n\ e^{-2np\frac{h}{S}}\ \bar{g}(p)\right] \tag{1.10}$$

Equation (1.10) is readily suitable for the application of the Shifting Theorem, which gives for the layer motion, as it may be easily shown:

+For engineering applications it is common to have the input as a record of accelerations; hence, the use of equation (1.8) will provide layer accelerations, and then velocities and displacements may be obtained by integration.

$$U(y,t) = \sum_{n=0} (-1)^n \, g(t-t_{yn1}) + \sum_{n=0} (-1)^n \, g(t-t_{yn2}) \tag{1.11}$$

and therefore

$$\dot{U}(y,t) = \sum_{n=0} (-1)^n \, \dot{g}(t-t_{yn1}) + \sum_{n=0} (-1)^n \, \dot{g}(t-t_{yn2}) \tag{1.12}$$

$$\ddot{U}(y,t) = \sum_{n=0} (-1)^n \, \ddot{g}(t-t_{yn1}) + \sum_{n=0} (-1)^n \, \ddot{g}(t-t_{yn2}) \tag{1.13}$$

where

$$t_{yn1} = (2n+1)\,\frac{h}{S} - \frac{y}{S}$$

and

$$t_{yn2} = (2n+1)\,\frac{h}{S} + \frac{y}{S}$$

The Solution and its Interpretation

A few remarks are needed before discussing the meaning of expressions 1.11, 1.12 and 1.13. First, it should be considered that the input disturbance $g(t)$ is ALWAYS defined for positive times only, and usually for a finite duration t_d. Hence, the arguments of g in the previous equations must satisfy the conditions:

$$t - t_{yni} \geqslant 0 \qquad i=1,2 \tag{1.14}$$

and

$$t - t_{yni} \leqslant t_d \qquad i=1,2 \tag{1.15}$$

These two inequalities provide the limits of the summations, and it may be seen that the number of terms in the summations is are FINITE even when the duration t_d is infinite. Furthermore, the number of terms involved in the computation of the motion at any particular time may be seen to depend upon the properties of the layer only. Therefore, the solutions found are not only exact, but may be computed exactly as there is no problem of convergence.

Notice should also be taken that the argument of g in the summations has dimensions of time. Consequently, equation 1.11, 1.12 and 1.13 as they stand, are suitable for the computation of time histories of response at a given depth y. Should the variation of response throughout the layer at a particular time t_o be of interest, such an argument ought to be expressed as $St_o - y_{tni}$, so that it has dimensions of length.

What any of the solution equations shows is that the material of the layer is a perfect transmitter in which the input disturbance travels with a velocity S without being at all modified. When the disturbance reaches the surface of the layer, it is reflected (in consequence its amplitude is doubled). After this reflection, the disturbance travels back towards the base, where a further reflection takes place and a change of sign. As there is no dissipative mechanism, this process continues for ever.

Therefore, the motion of the layer at a specific depth y, at an instant t, is simply the addition of the different points of the disturbance which happen to be passing by y at that instant.

This argument is shown in Figure 1.3. There, a layer of thickness h and shear wave velocity S is shown excited by a triangular pulse, say of acceleration, of duration 3/2 h/s. The appearances of the disturbance at the base, the middle depth and at the surface of the layer are shown and so the resulting accelerations at each one of these levels.

The Layer Wave

Another way of looking at the solution equations may also be noticed in Figure 1.3. If all the disturbances travelling upwards at any level are considered as a whole, it can be seen that they form an wave which has identical shape at all levels but is shifted in time from one depth y_1, to another y_2 by an amount $|y_2-y_1|/S$, giving the appearance of the complete wave travelling towards the surface with a velocity S. Also, the disturbances travelling downwards form an identical wave, now appearing to travel towards the base of the layer with the same velocity S. This wave shall be defined as the <u>Layer Wave.</u>

The analytical expression for the layer wave may be found from equation 1.10 which may be written

$$\bar{U}(y,p) = \left[e^{-\frac{p}{s}(h-y)} + e^{-\frac{p}{s}(h+y)} \right] \bar{W}_g(p) \qquad (1.16)$$

where

$$\bar{W}_g(p) = \sum_{n=0}^{\infty} (-1)^n e^{-2np\frac{h}{s}} \bar{g}(p) . \qquad (1.17)$$

Thus, after inversion:

$$W_g(t) = \sum_{n=0}^{[t/(2n/s)]} (-1)^n g(t-2nh/s) \qquad (1.18)$$

where

$W_g(t)$ is the Displacement Layer Wave, and

$[x]$ the largest integer less or equal to x.

Therefore, the layer displacement is, in terms of the layer wave:

$$U(y,t) = W_g[t-(h-y)/s] + W_g[t-(h+y)/s] \qquad (1.19)$$

and similar expressions are found for velocities and accelerations.

The layer wave thus, characterises the response of the layer as it combines in a unique form the properties of the layer and the input disturbance. Also, it simplifies considerably the numerical solution process as in fact, once this wave is determined, the response time-history at any depth or the configuration of the layer at any time may be easily obtained.

It should be stressed that it is the input disturbance that actually travels inside the layer. The layer wave is only an apparent motion and it is more a convenient mathematical concept than a precise description of the physical phenomenon. For the purposes of this thesis, disturbance shall be defined as a prescribed history of acceleration, velocity or displacement, and layer motion as the corresponding layer response. The Layer Wave is a characteristic of this response, and as such it cannot be prescribed since it is not known in advance. These definitions that may seem trivial, will prove to be of great use in the more complex problem of radiation.

It is to be noticed that the properties of the layer, both material and geometric, are summarised in one parameter only when no dissipative mechanisms are present. This parameter is the travelling time h/s which takes a disturbance to propagate from the base of the layer to its surface. This parameter may be expressed in a more conventional form considering the time between two consecutive appearances of a point of the disturbance, with the same sign and coming from the same direction, at a particular depth. This time is the Layer Period, T_L, and it may be easily shown that

$$T_L = \frac{4h}{s} \tag{1.20}$$

Deconvolution of Recorded Motions

Strong ground motion instruments are usually installed to record surface accelerations. Therefore, in practical terms, records from these instruments are layer accelerations $\ddot{U}(o,t)$ rather than acceleration disturbance $\ddot{g}(t)$. It is then of interest to study the problem of given the properties of a deposit and the recorded motion at its surface, finding the original disturbance; or in other words, deconvolve the effect of the deposit from the recorded motion.

At the surface of the layer, y=0, and equation 1.9 for this particular value of y may be written

$$\overline{U}(o,p) = \frac{1}{\cosh(p\,\frac{h}{s})}\ \bar{g}\ (p) \tag{1.21}$$

The fact that 1.21 is an operational relation justifies writing it as

$$\bar{g}\ (p) \quad = \quad \cosh\ (p\ \frac{h}{s}) \quad \overline{U}\ (o,p) \tag{1.22}$$

A procedure of inversion identical to that previously followed to find the layer motion, may be shown to lead to

$$g(t) = \frac{1}{2}\left[g_s\ (t+h/s) \ + \ g_s\ (t-h/s)\right] \tag{1.23}$$

where

$$g_s\ (t) = U\ (o,t),\ \text{the recorded displacement.}$$

Also,

$$\dot{g}(t) = \frac{1}{2}\left[\dot{g}_s\ (t+h/s) + \dot{g}_s\ (t-h/s)\right] \tag{1.24}$$

and

$$\ddot{g}(t) = \frac{1}{2}\left[\ddot{g}_s\ (t+h/s) + \ddot{g}_s\ (t-h/s)\right] \tag{1.25}$$

If the value of U(o,t) in 1.19 is substituted, then

$$g(t) = W_g(t) + W_g\ (t-2h/s)\ . \tag{1.26}$$

Similarly,

$$\dot{g}(t) = W_{\dot{g}}(t) + W_{\dot{g}}\ (t-2h/s) \tag{1.27}$$

$$\ddot{g}(t) = W_{\ddot{g}}(t) + W_{\ddot{g}}\ (t-2h/s) \tag{1.28}$$

where

$W_{\dot{g}}$ velocity layer wave [+]

$W_{\ddot{g}}$ acceleration layer wave

[+] The notation $W_{\dot{g}}$, $W_{\ddot{g}}$ and not $\dot{W}_g$, $\ddot{W}_g$ stresses the fact that the concept of layer wave is valid to represent the layer response to a disturbance irrespective of whether this disturbance is acceleration, velocity or displacement. However, it may be seen that $W_{\dot{g}}$ and $W_{\ddot{g}}$ are the first and second derivatives of W_g, respectively, and consequently $W_{\dot{g}} = \dot{W}_g$, and $W_{\ddot{g}} = \ddot{W}_g$.

Comparison with Modal Analysis

Modal analysis solution to the layer problem expresses the response of a layer as a combination of responses of one degree of freedom systems. Such a solution has the form

$$U_r(y,t) = \sum_{n=1}^{\infty} \phi_n (y)\, X_{n,g} (t) \tag{1.29}$$

where,

$U_r(y,t)$ layer displacement relative to the base movement

$$X_{n,g} (t) = - \frac{1}{w_n} \int_0^t \ddot{g}(\tau) \sin w_n (t-\tau)\, d\tau \tag{1.30}$$

$X_{n,g}(t)$ is the response displacement of a one degree of freedom system with frequency W_n to a disturbance g(t).

$\phi_n(y)$ is the n-th modal shape of vibration of the layer, which, at a particular y, defines the contribution of the single system with frequency w_n to the total reponse.

The long numerical process necessary to evaluate equation 1.29 has made usual practice to consider the alternative expression

$$U_{r\ max}(y) = \sum_{n=1}^{N} \phi_n (y)\, S_d (w_n) \tag{1.31}$$

where S_d is the displacement response spectrum and is simply the maximum absolute value of 1.30. N is fixed such that $\phi_n(y)$ is less than a certain percentage of $\phi_{n-1}(y)$.

It is evident that, because not all maxima are likely to occur at the same time, expression 1.31 is only an upper boundary to the actual maximum response at the depth y, and so, further assumptions are necessary to be made about the real contribution of each of the modes.

A review of the procedure followed to find 1.29 is necessary to compare this solution with the one introduced in this Chapter. It may be shown that the relationship in the transformed domain between the input disturbance and the layer motion which leads to expression 1.29 is of the form

$$\overline{U}(y,t) = \frac{\cosh(py/s)}{p^2\cosh(ph/s)} \ \ddot{\overline{g}}(p) \qquad (1.32)$$

where transformed layer displacements are related to transformed accelerations disturbance.

Inversion of the fraction in the right hand side of (1.32) following the Standard Bromwich Contour gives the expression

$$L^{-1}\left[\frac{\cosh(ph/s)}{p^2\cosh(ph/s)}\right] = T_o(y,t) = t - \frac{2h}{s\pi^2}\sum_{n=0}(-1)^n\cos\ (n+\tfrac{1}{2})\pi y/h \ \times$$

$$\times\ (n+\tfrac{1}{2})^{-2}\ \sin\ (n+\tfrac{1}{2})\pi st/h \quad (t>0) \quad (1.33)$$

where (see page 111 for definition of n)

$L^{-1}\left[\overline{f}(p)\right] = f(t)$, the Inverse Laplace Transform of $\overline{f}(p)$.

Equation 1.33 is actually the departing point between the two solutions. Modal analysis assumes

$$w_n = (n + \frac{1}{2})\ \pi\ \frac{s}{h} \qquad n=0,1,\ldots \qquad (1.34)$$

then, (1.33) is written

$$T_o(y,t) = t - 2\frac{s}{h}\sum_{n=0}^{\infty}(-1)^n\ \frac{\cos(w_n y/s)\ \sin\ (w_n t)}{w_n^2} \qquad (1.35)$$

and convolution of $T_o(y,t)$ with the acceleration disturbance $\ddot{g}(t)$ gives

$$U(y,t) = g(t) - \int_o^t 2\ \frac{s}{h}\sum_{n=0}(-1)^n\ \frac{\cos(w_n y/s)}{w_n^2}\ \sin w_n(t-\tau)\ddot{g}(\tau)\ d\tau \quad (1.36)$$

Finally, by reversing the order of the summation and integration involved, and considering relative displacements, equation 1.29 is obtained; now it is clear that:

$$\phi_n(y) = \frac{1}{2}(-1)^n\ \frac{\cos(w_n\ y/s)}{w_n\ h/s} \qquad (1.37)$$

Completeness of the modal analysis solution 1.29 is difficult to prove due to the fact that $\phi_n(y)$, though in itself independent of t, its interval of definition does depend on the time variable. Hence, the last step taken to find 1.29 is far from being a rigorous procedure.

Two points which are not considered in Modal Analysis but which are implicit in the time-history solution give to it its simplicity and accuracy. The first point is that the summation in equation 1.33 is in fact the Fourier expansion of the sum of two perfectly defined functions shown in Figure 1.4. (Notice that their interval of definition depends upon y). Therefore $T_o(y,t)$ in the same equation, may be shown to be the sum of the functions $f_{y1}(t)$ and $f_{y2}(t)$, which are illustrated in Figure 1.5a.

The consideration of this fact alone represents a great advantage as, in the first place, all the terms of the summation have been included, and therefore there is no problem of convergence. Also, it is undoubtedly easier to work numerically with the final expressions for $f_{y1}(t)$ and $f_{y2}(t)$ than with the trigonometric functions involved in their Fourier expansion.

The second point, however, has more important implications. This point, which has already been mentioned, is the operational nature of the relationship between the transforms of the input disturbance and the response. In the Modal Analysis solution, it may be seen that, once $T_o(y,t)$ is obtained, the layer response is given by equations

$$U(y,t) = \int_0^t T_o(y,t-\tau)\,\ddot{g}(\tau)\,d\tau \qquad (1.38)$$

$$\dot{U}(y,t) = \int_0^t \dot{T}_o(y,t-\tau)\,\ddot{g}(\tau)\,d\tau \qquad (1.39)$$

$$\ddot{U}(y,t) = \int_0^t \ddot{T}_o(y,t-\tau)\,\ddot{g}(\tau)\,d\tau \qquad (1.40)$$

$\dot{T}_o(y,t)$ and $\ddot{T}_o(y,t)$ are shown in their closed form, in Figures 1.5b and 1.5c respectively. It may be seen that $T_o(y,t)$ consists of two series of shifted ramp functions, while $\dot{T}_o(y,t)$ is given by two series of shifted step functions and $\ddot{T}_o(y,t)$ by two series of pulses. Therefore, the convolution integrals in equations 1.38, 1.39 and 1.40 may be shown to be operations performed upon $\ddot{g}(t)$. Equation 1.38 implies double integration and shifting; 1.39 integration and shifting; and (1.40) shifting only.

Modal analysis disregards these operational considerations, and consequently it complicates unnecessarily the numerical solution process and brings in serious problems of convergence, particularly in the evaluation of accelerations.

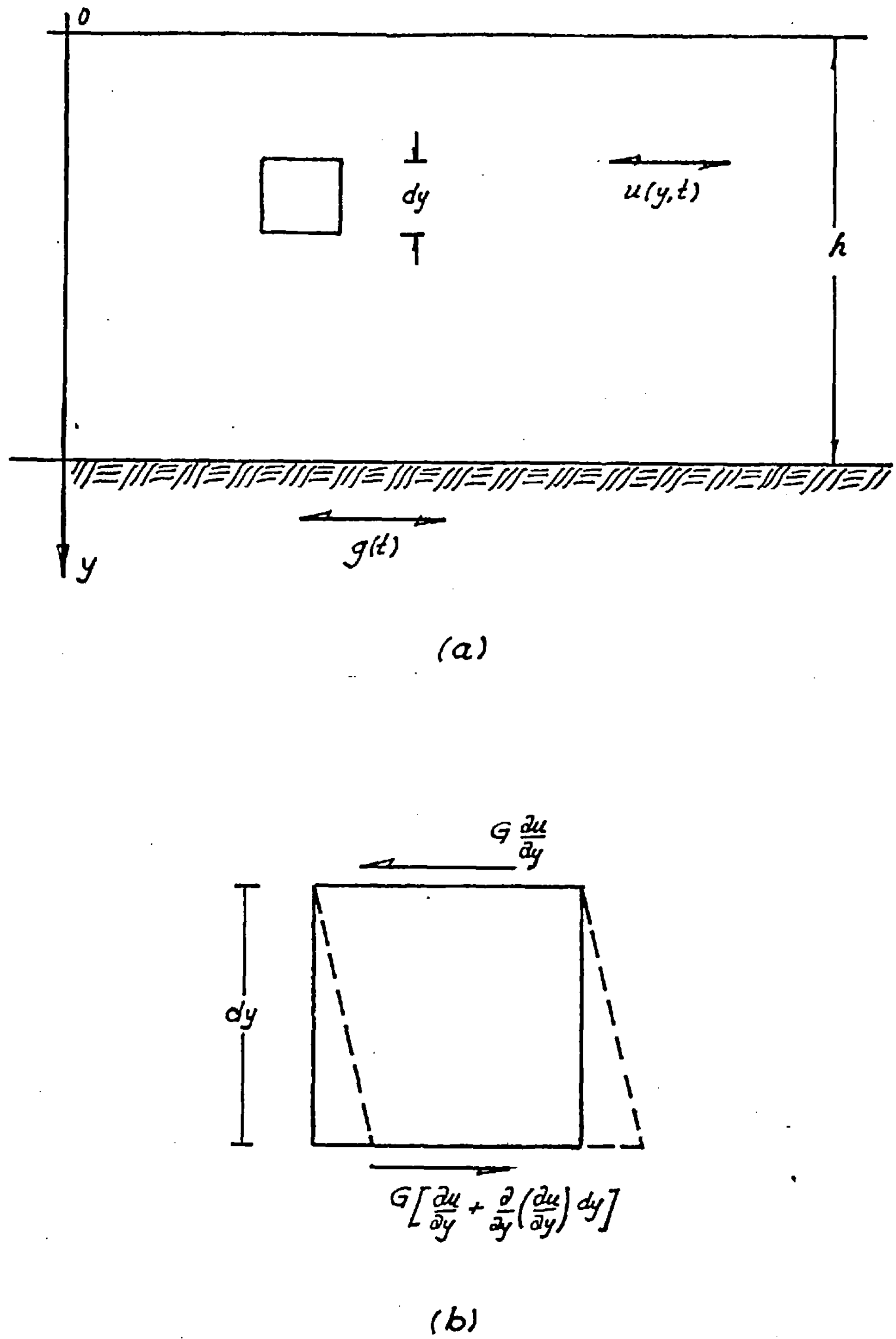

0
dy
u(y,t)
h
g(t)
y
(a)
$G\frac{\partial u}{\partial y}$
dy
$G\left[\frac{\partial u}{\partial y}+\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)dy\right]$
(b)

FIG. 1.1

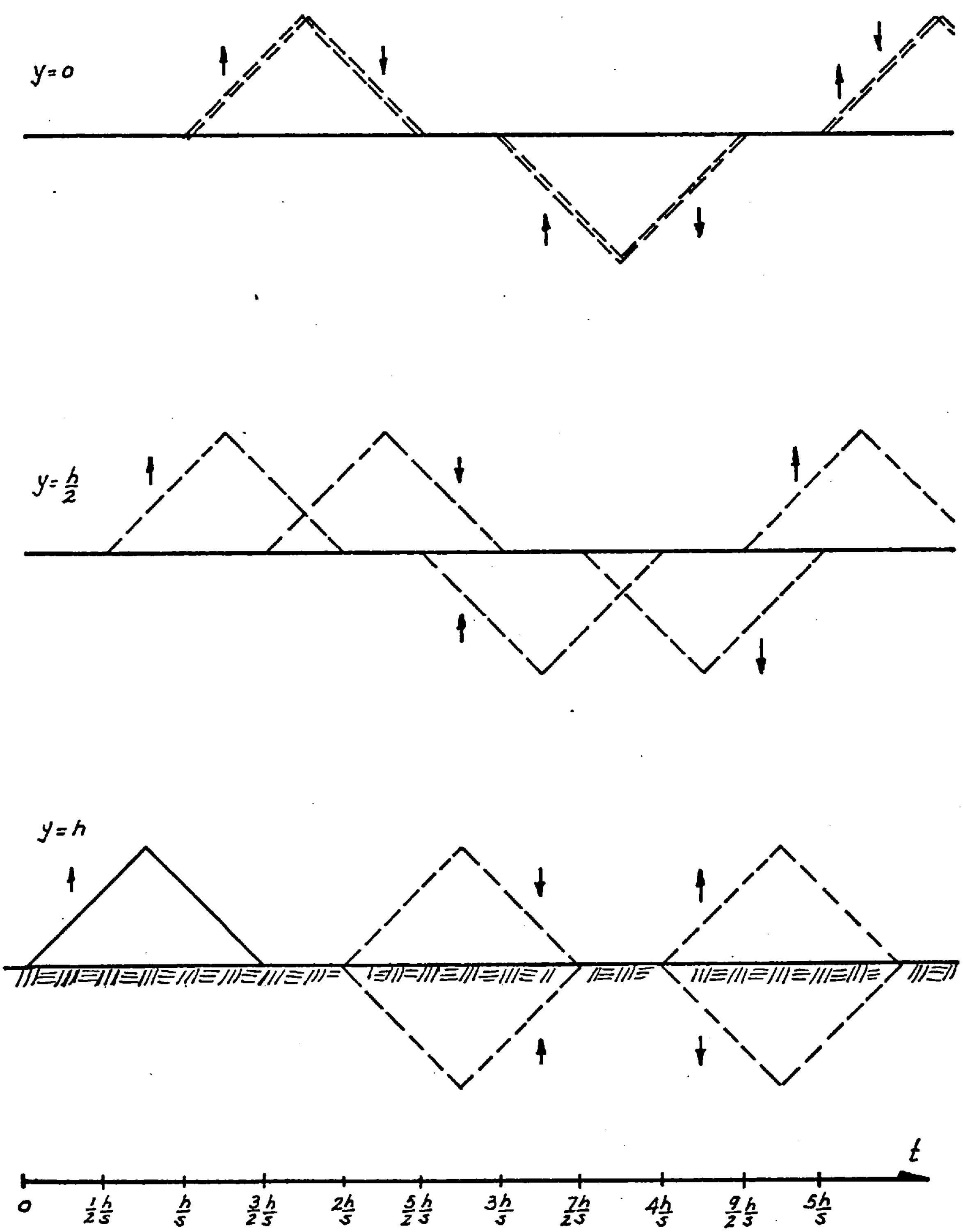
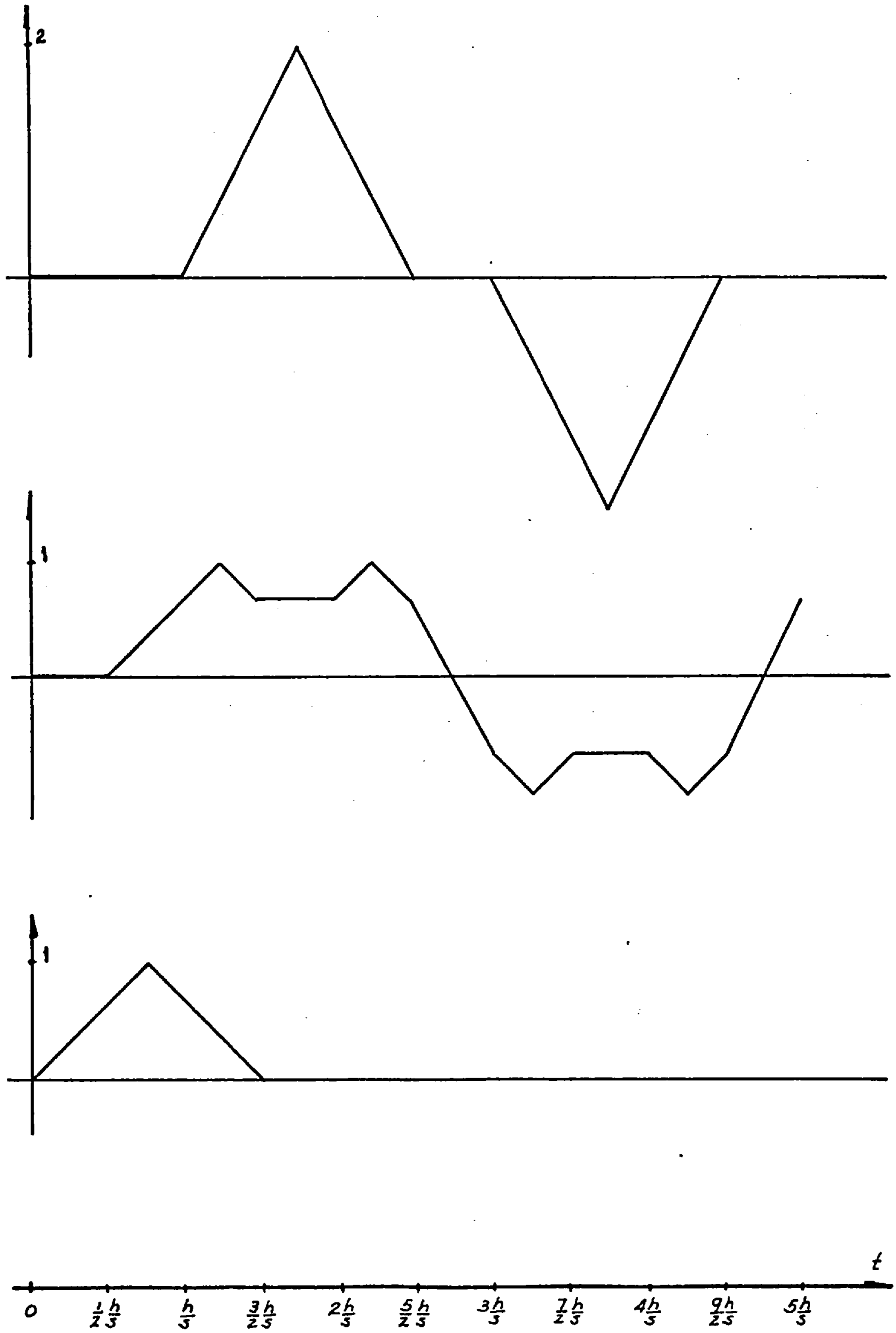


FIG. 1.3


2
1
1
t
0
$\frac{1}{2}\frac{h}{s}$
$\frac{h}{s}$
$\frac{3}{2}\frac{h}{s}$
$2\frac{h}{s}$
$\frac{5}{2}\frac{h}{s}$
$3\frac{h}{s}$
$\frac{7}{2}\frac{h}{s}$
$4\frac{h}{s}$
$\frac{9}{2}\frac{h}{s}$
$5\frac{h}{s}$

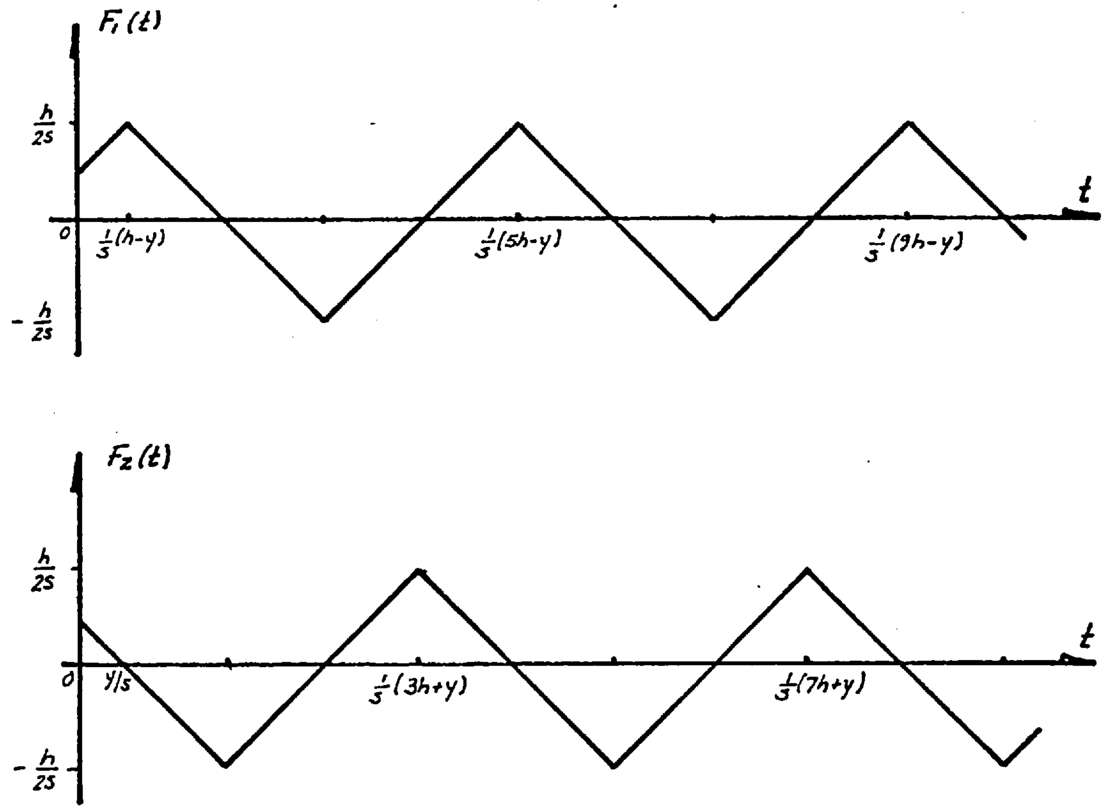
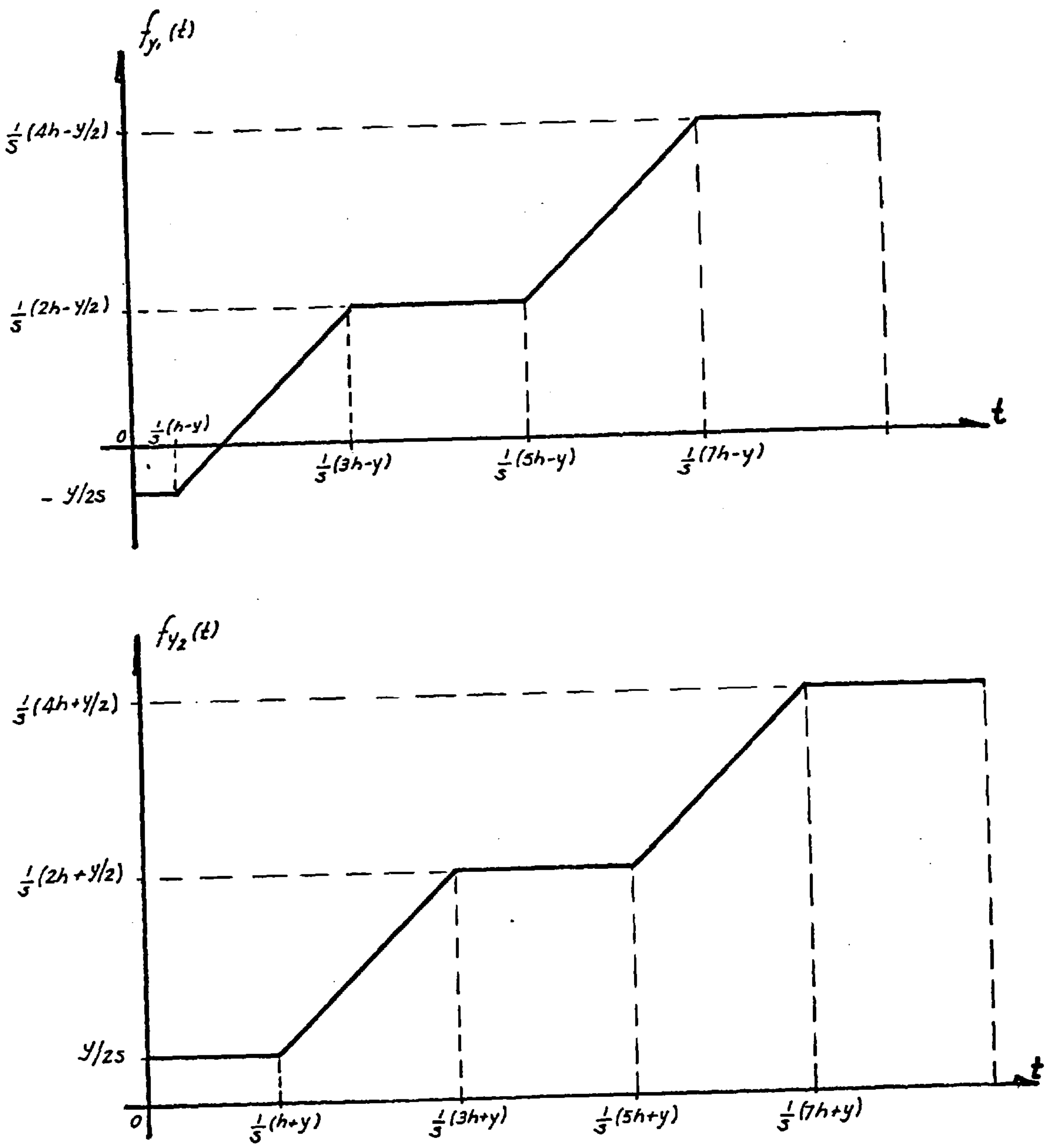
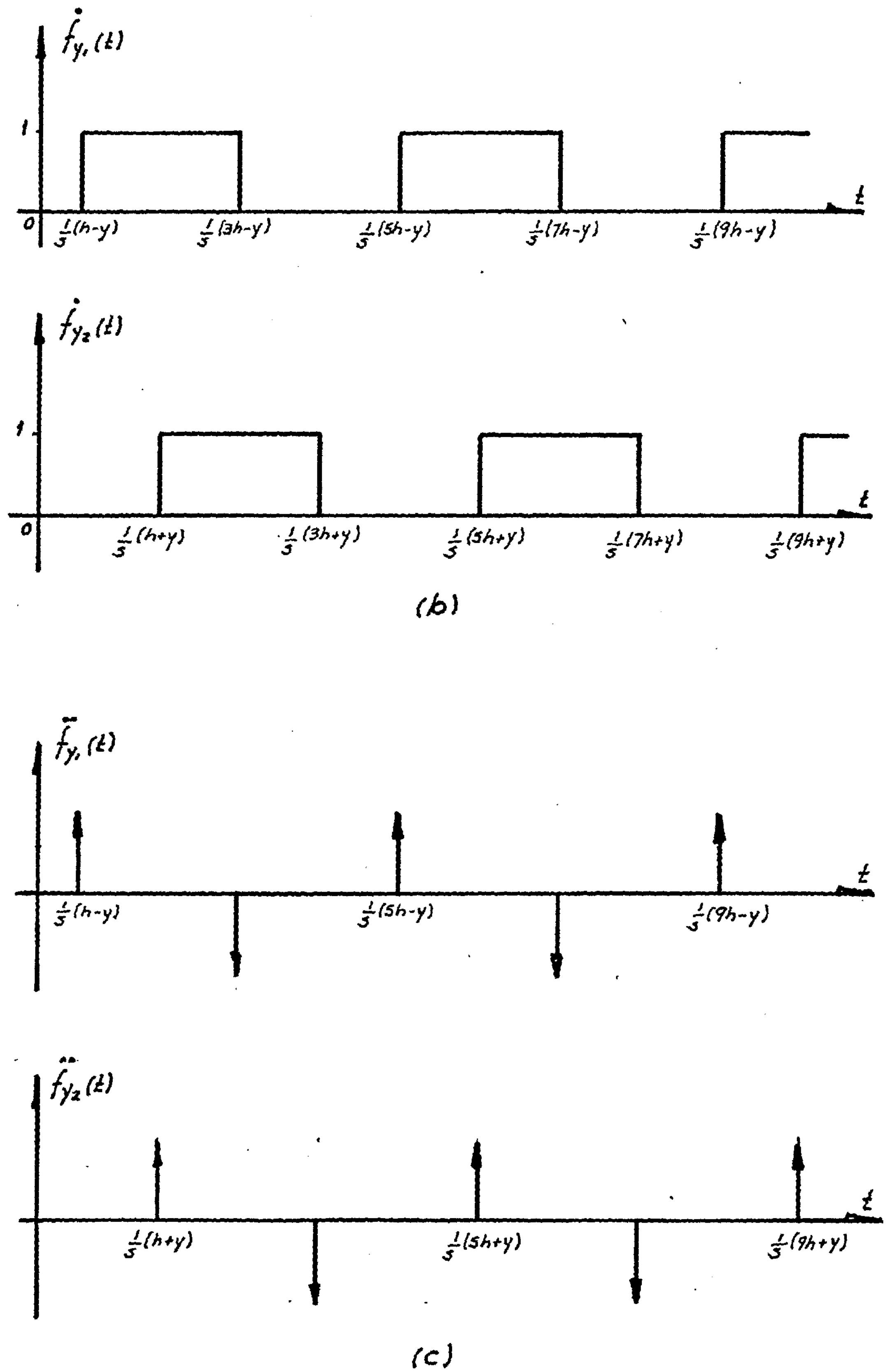

$F_1(t)$
$\frac{h}{2s}$
$-\frac{h}{2s}$
0
$\frac{1}{s}(h-y)$
$\frac{1}{s}(5h-y)$
$\frac{1}{s}(9h-y)$
t
$F_2(t)$
$\frac{h}{2s}$
$-\frac{h}{2s}$
0
y/s
$\frac{1}{s}(3h+y)$
$\frac{1}{s}(7h+y)$
t

FIG. 1.4



$f_{y_1}(t)$
$\frac{1}{s}(4h - y/2)$
$\frac{1}{s}(2h - y/2)$
0
$\frac{1}{s}(h-y)$
$-y/2s$
$\frac{1}{s}(3h-y)$
$\frac{1}{s}(5h-y)$
$\frac{1}{s}(7h-y)$
t
$f_{y_2}(t)$
$\frac{1}{s}(4h + y/2)$
$\frac{1}{s}(2h + y/2)$
$y/2s$
0
$\frac{1}{s}(h+y)$
$\frac{1}{s}(3h+y)$
$\frac{1}{s}(5h+y)$
$\frac{1}{s}(7h+y)$
t

FIG. 1.5 a



$\dot{f}_{y_1}(t)$
1
0
t
$\frac{1}{3}(h-y)$
$\frac{1}{3}(3h-y)$
$\frac{1}{3}(5h-y)$
$\frac{1}{3}(7h-y)$
$\frac{1}{3}(9h-y)$
$\dot{f}_{y_2}(t)$
1
0
t
$\frac{1}{3}(h+y)$
$\frac{1}{3}(3h+y)$
$\frac{1}{3}(5h+y)$
$\frac{1}{3}(7h+y)$
$\frac{1}{3}(9h+y)$
(b)
$\ddot{f}_{y_1}(t)$
t
$\frac{1}{3}(h-y)$
$\frac{1}{3}(5h-y)$
$\frac{1}{3}(9h-y)$
$\ddot{f}_{y_2}(t)$
t
$\frac{1}{3}(h+y)$
$\frac{1}{3}(5h+y)$
$\frac{1}{3}(9h+y)$
(c)

FIG. 1.5 (cont.)

CHAPTER TWO

EXTERNAL MECHANISM OF DISSIPATION - RADIATION

Introduction

It was mentioned in the first chapter that the model of an elastic layer on a rigid base was not realistic enough to represent the response of actual soil deposits subjected to earthquake disturbances. The main criticism which may be made of the solution obtained in Chapter one is that the layer response, though finite in magnitude, continues indefinitely, whatever the duration of the input disturbance, as the model does not take into consideration any dissipative mechanism.

The incorporation of a dissipative mechanism in the response of the layer, without changing the one-dimensional nature of the problem, may be achieved either by proposing a more complex constitutive relation for the material in the layer, or by considering the base of the soil deposit to be deformable and not rigid. The first of these alternatives provides an internal mechanism of dissipation, usually termed as Damping, while the second defines an external dissipative process, known generally as Radiation.

A mathematical formulation considering internal damping involves the solution of an equation different to the one seen in the previous chapter. On the other hand, the formulation of the radiation problem, though it involves a more complex solution, implies simply a modification of one of the boundary conditions. Therefore, it may be said that, while consideration of internal damping constitutes a different problem, the inclusion of radiation is merely an extension to the model previously studied; and as such, it is of convenience to deal with it first.

This chapter, therefore, considers the problem of the response of an elastic layer on a semi-infinite elastic base which is disturbed by an arbitrary shear motion. A definition of the mechanism of radiation for a disturbance is given, which helps to formulate the problem and solve it analytically without additional assumptions on the nature of the disturbance at infinity.

The solution is followed by an interpretation in physical terms and a discussion.

Definition of the Problem

The concept of radiation is difficult to work with if a precise definition of its meaning is not provided.

It is clear that such a definition is merely a matter of convenience for the purposes of this thesis as radiation effects are nowadays generally considered in the dynamic response of soil deposits (PAPASTAMATIOU, 1971; ROESSET, 1970; SCHNABEL et al., 1972; etc.). There is, however, no analytical model available which can be explained unrestrictedly in physical terms and, at the same time, provide an exact solution to the radiation problem. Hence, the search for a specific conception is justifiable.

Before attempting any definition (which will determine the mathematical formulation of the problem and its capability to be interpreted), consider the two elastic layers shown in Figure (2.1), founded on a rigid base. The material in the upper deposit has a mass density ρ, a shear modulus G, and therefore a shear wave velocity $S = (G/\rho)^{1/2}$. The layer of thickness H is characterised by a mass density ρ_s, rigidity modulus G_s, and shear wave velocity $S_s = (G_s/\rho_s)^{1/2}$.

Let the upper deposit be referred to as the OBJECT MEDIUM and the subjacent layer as the BASE MEDIUM.

A disturbance g(t) applied to the base of the system at a time t=0, will travel upwards and reach the interface of the two media after an interval H/S_s. There, part of the disturbance is transmitted to the object medium while another part is reflected back to the base medium. The conditions which should satisfy the response motions of the base and object media at the interface, define alone the amounts of the disturbance which are either transmitted or reflected.

Let $C_{To}\, g(t)$ be the disturbance transmitted to the object medium and $C_{Ro}\, g(t)$ the one reflected, and consider $C_{To}\, g(t)$ only. Once this disturbance is inside the object medium, it travels unaltered for an interval 2h/s after which reaches again the boundary between the two layers. There, it is split once more, $C_{T1}\, g(t)$ remaining in the object medium, and $C_{R1}\, g(t)$ sent back to the base medium. Hence, considering only disturbances which remain in the object medium, it may be seen that $C_{T1}\, g(t)$ splits into C_{T2} and C_{R2}; C_{T2} into C_{T3} and C_{R3}, and so forth, until after sufficient reflections there is no disturbance present in the object medium.

Therefore, it may be said that, by the generation at the interface of disturbances which are radiated to the base medium, the presence of the original disturbance g(t) in the object medium has been gradually dissipated. This phenomenon is what is understood in this thesis as RADIATION.

Four points are to be stressed from the previous statement:

1) The radiation phenomenon is associated to a disturbance arriving to the object medium from the base medium.
2) The phenomenon takes place only at the interface.
3) Radiation is related to disturbances remaining in the object medium.
4) What occurs to the disturbances which are radiated away into the base medium is of no concern to the problem of radiation.

From these remarks it may be said that the occurrence of the radiation phenomenon is independent of whether the base medium is finite or infinite. An infinite base medium presents the problem of radiation once, but a finite medium does not rule out radiation; in this case, the phenomenon simply occurs as many times as disturbances arrive from the base medium to the interface.

A proper mathematical formulation for the response of the object medium considering the base medium semi-infinite is not possible, unless an assumption (difficult to justify in physical terms) is made on what occurs at infinity. On the other hand, the case of a finite base medium offers no difficulty at all to be formulated, and if a proper method of solution is used, it is possible to isolate the radiation problem, it is to determine the response of the object medium to the disturbance which arrives first at the interface, without including the effect of further disturbances arriving from the base to the object medium.

Such an approach to the radiation problem has the advantage that, for an earthquake disturbance, the problem of propagation is separated completely from those of attenuation and source mechanism, as the original disturbance considered is prescribed on its first arrival at the interface of the elastic media.

Mathematical Formulation and Solution Procedure

Consider the elastic deposit on a semi-infinite elastic medium shown in Figure (2.2). Material properties for the deposit are those of the object medium previously described, while the elastic semi-space is characterised by the properties of the base medium.

A shear displacement disturbance $g(t)$ travels upwards in the base medium producing absolute displacements $U_s(y,t)$ in the base medium and $U(y,t)$ in the upper deposit. The motion $U(y,t)$ is required.

The equation governing the motion of the deposit may be shown to be

$$\ddot{U}(y,t) = S^2\, U''(y,t) \qquad 0 \leq y \leq h \tag{2.1}$$

The surface of the deposit is free of stresses. Therefore

$$U'(y,t)\Big]_{y=0} = 0 \tag{2.2}$$

It is assumed that there is no sliding at the interface of the two media; i.e.

$$U(h,t) = U_s(h,t) \tag{2.3}$$

and

$$G\, U'(y,t)\Big]_{y=h} = G_s\, U'_s(y,t)\Big]_{y=h} \tag{2.4}$$

Where $U_s(y,t)$ is the absolute horizontal displacement of a point in the elastic semi-space at a depth y, at a time t. Hence,

$$\ddot{U}_s(y,t) = S_s^2\, U_s''(y,t) \qquad y \geq h \tag{2.5}$$

Furthermore, it will be assumed that the motion starts from rest, i.e.

$$g(0) = U(y,0) = U_s(y,0) = 0 \tag{2.6}$$

$$\dot{g}(0) = \dot{U}(y,t)\Big]_{t=0} = \dot{U}_s(y,t)\Big]_{t=0} = 0 \tag{2.7}$$

Equations (2.1) to (2.7) constitute the formulation of the radiation problem, but for the disturbance g(t) which remains to be defined. The condition

$$U_s(h+H,t) = g(t) \tag{2.8}$$

defines the disturbance arriving to the interface for the first time as g(t) and makes it possible to solve the system of two partial differential equations implied in the formulation.

On the other hand, the introduction of such a condition makes the base medium finite and hence, further disturbances from the base are to appear at the interface. The solution procedure should then be able to disregard the effect of such disturbances.

The use of the Laplace Transformation for the solution of the problem formulated above leads to a system of two ordinary linear differential equations in y, being the transforms of U and U_s the unknowns of the system. These equations are easily integrable and their solution may be expressed in terms of two constants of integration only if the conditions at the surface of the deposit (y=0) and at the imposed base of the semi-space (y=h+H) are applied. The value of such constants is then found by the application of the conditions at the interface of the two media.

It may be shown that the expression for the transform of the displacement at any point of the deposit obtained from this procedure is

$$U\,(y,p) = \frac{\cosh \frac{p}{S} y}{\alpha_{o/m} \sinh \frac{p}{S}h \quad \sinh \frac{pH}{S_s} + \cosh \frac{p}{S}h \quad \cosh \frac{pH}{S_s}} \quad \bar{g}(p) \qquad (2.9)$$

$$0 \leq y \leq h$$

where $\overline{U}(y,p)$ is the transform of the absolute horizontal displacement at any depth in the elastic deposit, and

$$\alpha_{o/m} = \frac{G\,S_s}{G_s S} = \sqrt{\frac{G\rho}{G_s \rho_s}} \qquad (2.10)$$

is the impedance ratio of the deposit to the semi-space.

The expression for $\overline{U}_s$, the transform of the displacement response of the semi-space, has been of use to find $\overline{U}$; in what follows, it is irrelevant.

Equation (2.9) may be written

$$\overline{U}(y,p) = \frac{2\, e^{-\frac{p}{S}(h-y)} \, (1+e^{-2\frac{p}{S}y}) \quad e^{-\frac{p}{S_s}H} \, \overline{g}(p)}{\alpha_{o/m}(1-e^{-2\frac{p}{S}h}) \, (1-e^{-2\frac{p}{S_s}H}) + (1+e^{-2\frac{p}{S}h}) \, (1+e^{-2\frac{p}{S_s}H})} \qquad (2.11)$$

According to the concept of transfer operation, the factor $e^{-\frac{p}{S_S}H}$ in the numerator of the right hand side of (2.11) indicates that the disturbance g(t) appears in the solution shifted an interval H/S_s, that is, the time taken by the disturbance to arrive from the depth y=h+H to the base of the elastic deposit. Therefore, to omit this factor is equivalent to considering that the time origin is the time of the first arrival of the disturbance to the base of the elastic deposit. And, once made this consideration, the thickness H of the base medium may be made infinite so that no other disturbances from the base are considered in the solution.

Hence, if the right hand side of (2.11) is multiplied by $e^{+\frac{p}{S_S}H}$ and then limits are taken when H tends to infinite, the expression

$$U(y,p) = \frac{2\,e^{-\frac{p}{S}(h-y)}\left(1+e^{-2\frac{p}{S}y}\right)}{\alpha_{o/m}\left(1-e^{-2\frac{p}{S}h}\right)+\left(1+e^{-2\frac{p}{S}h}\right)}\,\bar{g}(p) \qquad (2.12)$$

is obtained, which shows the relationship between the transforms of the input disturbance and the displacement of the deposit IN THE RADIATION PROBLEM.

The inversion procedure indicated in Chapter ONE may be shown to lead to the expressions:

$$U(y,t) = \frac{2}{1+\alpha_{o/m}} \sum_{n=0} (-1)^n\,\beta^n_{o/m}\,g(t-t_{yn1}) + \sum_{n=0} (-1)^n\beta^n_{o/m}\,g(t-t_{yn2}) \qquad (2.13)$$

$$\dot{U}(y,t) = \frac{2}{1+\alpha_{o/m}} \sum_{n=0} (-1)^n\,\beta^n_{o/m}\,\dot{g}(t-t_{yn1}) + \sum_{n=0} (-1)^n\beta^n_{o/m}\,\dot{g}(t-t_{yn2}) \qquad (2.14)$$

$$\ddot{U}(y,t) = \frac{2}{1+\alpha_{o/m}} \sum_{n=0} (-1)^n\,\beta^n_{o/m}\,\ddot{g}(t-t_{yn1}) + \sum_{n=0} (-1)^n\,\beta^n_{o/m}\,\ddot{g}(t-t_{yn2}) \qquad (2.15)$$

where

$$\beta_{o/m} = \frac{1 - \alpha_{o/m}}{1 + \alpha_{o/m}} \tag{2.16}$$

is the RADIATION coefficient, and

$$t_{yn1} = (2n+1)\,\frac{h}{s} - \frac{y}{s}$$

$$t_{yn2} = (2n+1)\,\frac{h}{s} + \frac{y}{s}$$

as previously defined.

The Solution - Interpretation and Discussion

It may be seen from equations (2.13) to (2.15) that the general form of response of the elastic layer has not been altered by considering the effect of radiation. Each one of these equations shows that a disturbance in the base medium is scaled by a factor $2/(1+\alpha_{o/m})$ when it arrives at the elastic interface. Then, this modified disturbance travels with a velocity S toward the surface of the stratum, where it is reflected backwards, being therefore doubled at that particular level. The disturbance continues to travel towards the base where it is again reflected, but now with opposite sign and multiplied by a factor $\beta_{o/m}$. Hence, unlike the case of a rigid substratum, the disturbance is not cancelled out at the base when reflected; which explains why the response motion at this depth cannot be prescribed, as it is neither equal to g(t) nor to $2/(1+\alpha_{o/m})$ g(t).

After the reflection at the base, the amount of disturbance left in the upper stratum, travels again towards the surface and the whole process of reflection and scaling down of the amplitude is repeated time after time. In consequence, as the solution shows, the response motion at any depth of the upper stratum at a time t, may still be visualised as the superimposition of the different points of the disturbance, now with different amplitudes, passing by that depth at that time.

It may be noticed that the velocity of propagation in the upper stratum is S as before. Thus, it will still take a time h/s for a disturbance to travel from base to surface of the upper stratum; and consequently it may be concluded that radiation DOES NOT change the layer period, which remains

$$T_L = 4\,\frac{h}{S} \tag{2.17}$$

Notice also, that the initial disturbance in the upper stratum is $2/(1+\alpha_{o/m})\; g(t)$, a fact which indicates that the amplitude of the original disturbance $g(t)$ is magnified when passing from a harder medium to a softer one ($\alpha_{o/m} < 1$) or diminished in the opposite case, $\alpha_{o/m} > 1$. Dissipation of the disturbance, however, does always take place as the modulus of radiation coefficient $\beta_{o/m}$ is for both cases less than unity. Furthermore, it may be seen that $\alpha_{m/o}$ the impedance ratio of the base medium to the object medium is

$$\alpha_{m/o} = \frac{1}{\alpha_{o/m}} \tag{2.18}$$

hence, from (2.16) the corresponding radiation coefficient is

$$\beta_{m/o} = \frac{1 - \dfrac{1}{\alpha_{o/m}}}{1 + \dfrac{1}{\alpha_{o/m}}} = -\,\beta_{o/m} \tag{2.19}$$

which implies an equal rate of dissipation for both impedance ratios $\alpha_{o/m}$ and $\alpha_{m/o}$. This, together with the fact that

$$\frac{2}{1+\alpha_{o/m}} = 1 + \beta_{o/m} \tag{2.20}$$

indicates that the solution to the radiation problem when the material properties of the object medium are those of the base medium and vice versa, is obtained by substituting $-\beta_{o/m}$ for $\beta_{o/m}$ in the solution equations (2.13) to (2.15).

A more general definition of impedance ratio and radiation coefficient may now be given as follows:

$$\alpha = \begin{cases} \alpha_{o/m} & \alpha_{o/m} < 1 \\ \dfrac{1}{\alpha_{o/m}} = \alpha_{m/o} & \alpha_{o/m} > 1 \end{cases} \tag{2.21}$$

$$\beta = \begin{cases} \beta_{o/m} & \alpha_{o/m} < 1 \\ -\beta_{o/m} = \beta_{m/o} & \alpha_{o/m} > 1 \end{cases} \tag{2.22}$$

Figure (2.3) shows graphically the relation between α and β.

The similarity in form and interpretation between the solution equation (2.13) to (2.15) and the corresponding equations in Chapter One suggests the consideration of a Layer Wave in the radiation problem. It may be shown that when radiation is considered, the displacement Layer Wave is given by

$$W_g\,(t) = (1+\beta) \sum_{n=0}^{[t/(2h/s)]} (-1)^n\, \beta^n\, g(t-2nh/s) \tag{2.33}$$

with similar expressions for velocities and accelerations. Hence, equation (2.13) may be written

$$U(y,t) = W_g\,[t-(h-y)/s] + W_g\,[t-(h+y)/s] \tag{2.24}$$

an expression which is identical to (1.19) and indicates that using the concept of Layer Wave, the radiation problem may be reduced to one which is equivalent to that of an elastic layer on a rigid base.

Deconvolution of Recorded Motions

It may be seen from equation (2.12) that the relation between the transforms of the displacement at the surface of the deposit and of the disturbance g(t) is

$$\overline{U}(0,p) = \frac{4}{1+\alpha} \; \frac{e^{-\frac{p}{s}h}}{1+\beta\, e^{-2\frac{p}{s}h}} \; \bar{g}(p)$$

hence,
$$\bar{g}(p) = \frac{1+\alpha}{4}\left[e^{\frac{p}{s}h} + \beta\, e^{-\frac{p}{s}h} \right] \overline{U}\,(0,p) \tag{2.25}$$

Inversion of (2.24) is shown to give the relation

$$g(t) = \frac{1}{4}\,(1+\alpha)\left[g_s\,(t+h/s) + \beta\, g_s\,(t-h/s) \right] \tag{2.26}$$

where $g_s(t) = U(0,t)$.

Similar expressions are found for velocities and accelerations.

In a similar fashion, the expression for the response at the interface in terms of the motion at the surface may be proved to be

$$g_b(t) = \frac{1}{2}\left[g_s\,(t+h/s) + g_s\,(t-h/s) \right] \tag{2.27}$$

where $g_b(t) = U\,(h,t)$.

Therefore, considering the definition of Layer Wave given in equation (2.23), it is possible to write

$$g_s(t) = 2\,W_g\,(t-h/s) \tag{2.28}$$

$$g(t) = \frac{1}{2}\,(1+\alpha)\left[W_g(t) + \beta\, W_g(t-2h/s) \right] \tag{2.29}$$

and
$$g_b(t) = W_g(t) + W_g(t-2h/s) \tag{2.30}$$

with similar expressions for velocities and accelerations.

Reference Motion

The solution found in this chapter does not include the case of a rigid base, as this is not a particular case of the formulation which has been made. The response which is obtained by considering in equation (2.13) an impedance ratio equal to zero, corresponds to the problem in which there is no upper layer, ($G=\rho=0$); and hence, under these conditions, such an equation provides the motion at the surface of a semi-infinite elastic medium, a motion which is twice the input disturbance.

Comparing the response of a layer for the conditions of an elastic and a rigid base, the fact that for an elastic base the initial disturbance in the layer differs in amplitude from the original disturbance in the base, may lead to confusion, as depending on which of these disturbances is considered to be the reference motion for the comparison, the response of the layer on a rigid base may or may not be the largest.

It is possible to include the condition of a rigid base in the solution found and to avoid all ambiguity if the reference motion for all cases is considered to be the motion $g_{rec}(t)$ which would be recorded at the free-surface of the base medium. It is clear that, $g(t)$ being the original disturbance in the base,

$$g_{rec}(t) = 2\, g(t) \tag{2.31}$$

for an elastic base; or

$$g_{rec}(t) = g(t) \tag{2.32}$$

for a rigid one.

The layer response given in equations (2.13) to (2.15) may thus be expressed as

$$U(y,t) = \frac{1}{1+\alpha} \sum_{n=0} (-1)^n \beta^n g_{rec}(t-t_{yn1}) + \sum_{n=0} (-1)^n \beta^n g_{rec}(t-t_{yn2}) \tag{2.33}$$

$$\dot{U}(y,t) = \frac{1}{1+\alpha} \sum_{n=0} (-1)^n \beta^n \dot{g}_{rec}(t-t_{yn1}) + \sum_{n=0} (-1)^n \beta^n \dot{g}_{rec}(t-t_{yn2}) \tag{2.34}$$

$$\ddot{U}(y,t) = \frac{1}{1+\alpha} \sum_{n=0} (-1)^n \beta^n \ddot{g}_{rec}(t-t_{yn1}) + \sum_{n=0} (-1)^n \beta^n g_{rec}(t-t_{yn2}) \quad (2.35)$$

and the expression for the displacement layer wave in terms of $g_{rec}(t)$ is thus

$$W_g(t) = \frac{1}{1+\alpha} \sum_{n=0}^{[t/(2h/s)]} (-1)^n \beta^n g_{rec}(t-2n\ h/s) \quad (2.36)$$

with similar expressions for velocities and accelerations.

The previous equations may be used for the evaluation of the response of a layer on either an elastic or a rigid base.

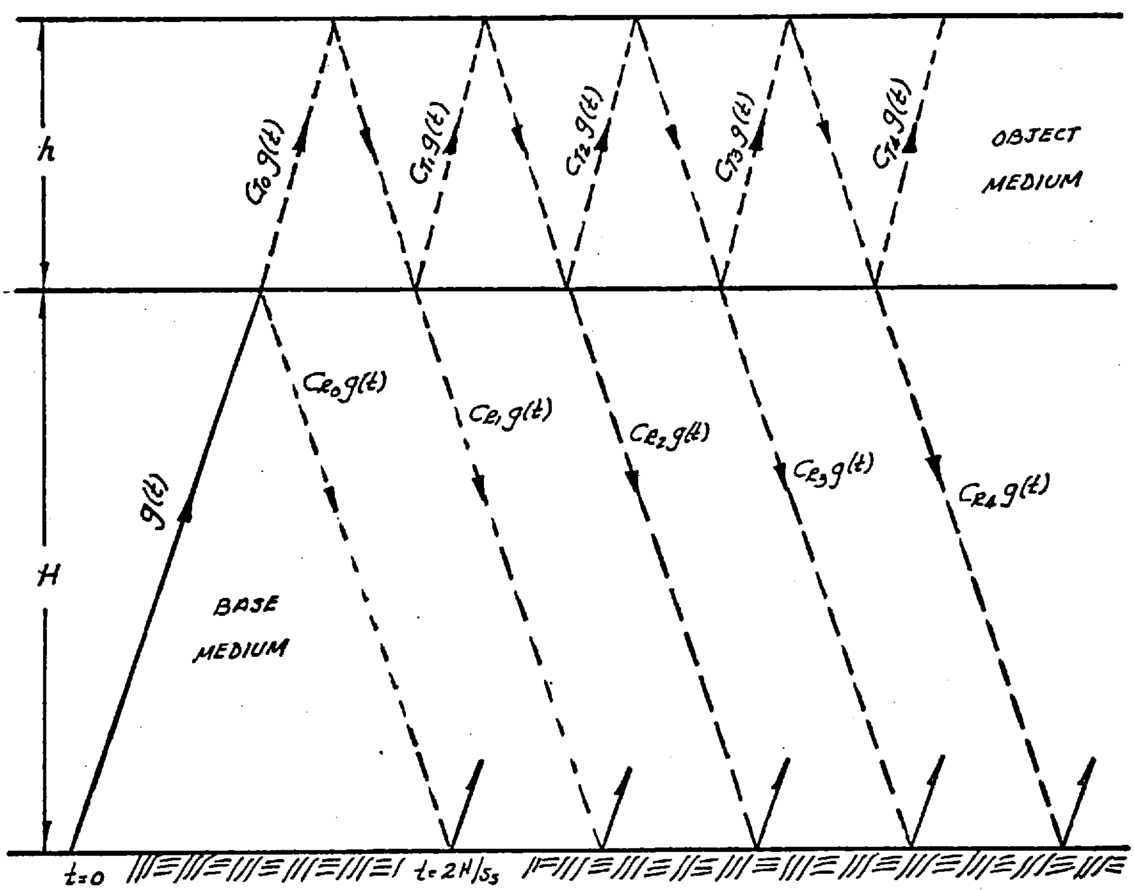

h
H
OBJECT
MEDIUM
BASE
MEDIUM
$C_{T0}\,g(t)$
$C_{T1}\,g(t)$
$C_{T2}\,g(t)$
$C_{T3}\,g(t)$
$C_{T4}\,g(t)$
$g(t)$
$C_{R0}\,g(t)$
$C_{R1}\,g(t)$
$C_{R2}\,g(t)$
$C_{R3}\,g(t)$
$C_{R4}\,g(t)$
$t=0$
$t=2H/S_s$

FIG. 2.1

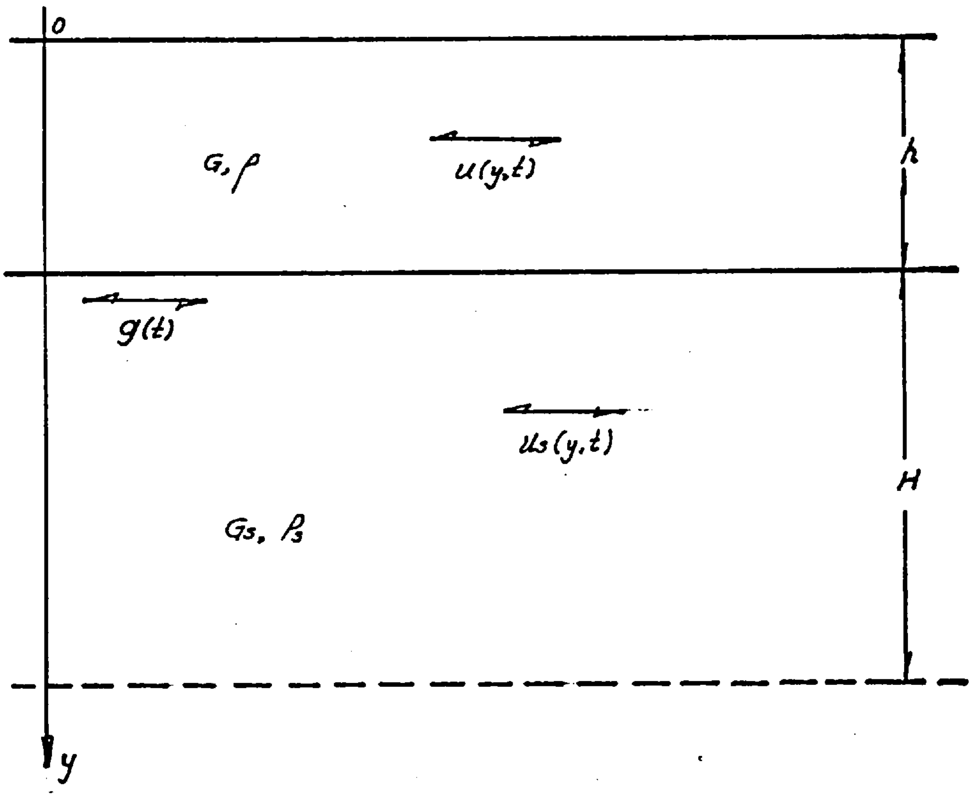
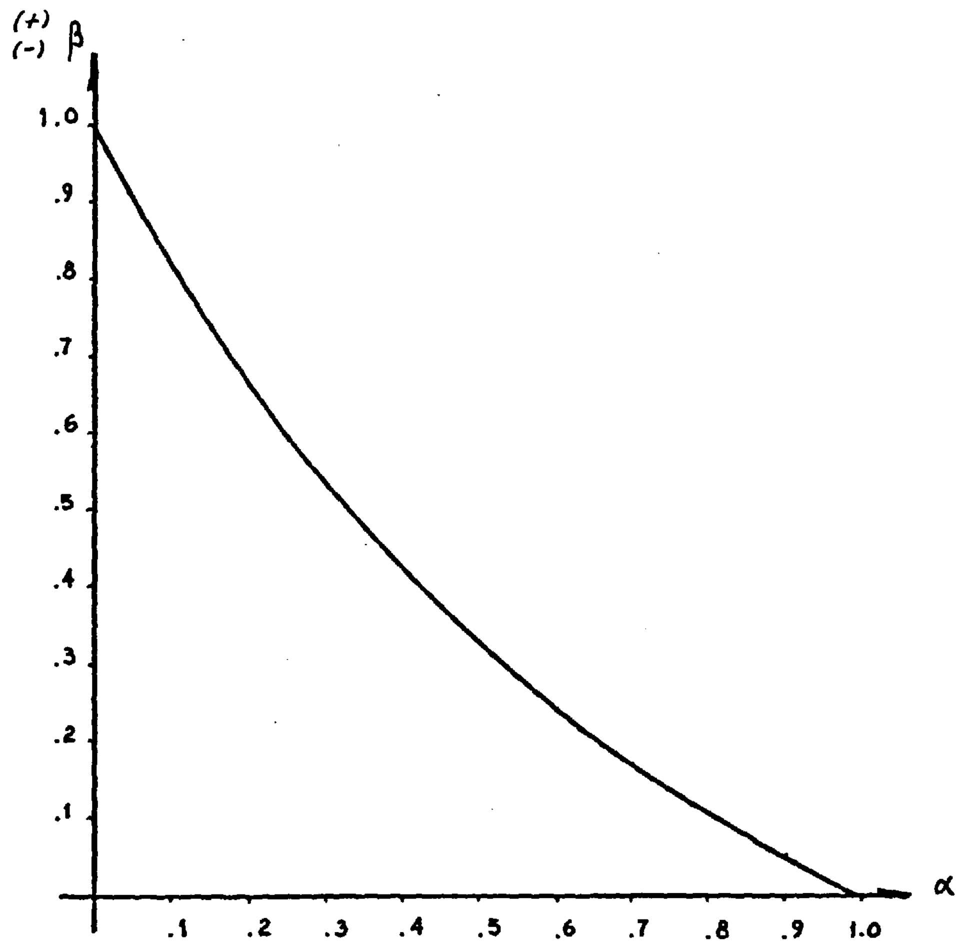


FIG. 2.2



$$\alpha = \begin{cases} \alpha_{o/m} & (\alpha_{o/m} < 1) \\ \dfrac{1}{\alpha_{o/m}} = \alpha_{m/o} & (\alpha_{o/m} > 1) \end{cases} \qquad \beta = \begin{cases} \beta_{o/m} & (\alpha_{o/m} < 1) \\ -\beta_{o/m} = \beta_{m/o} & (\alpha_{o/m} > 1) \end{cases}$$

FIG. 2.3

CHAPTER THREE

INTERNAL MECHANISM OF DISSIPATION - DAMPING

Introduction

During the motion of a soil deposit, friction among particles, material heterogeneity and other factors contribute to dissipate into heat the energy supplied to the deposit by a disturbance. The combined action of these factors is usually termed as damping or internal friction.

Experimental research on the dynamic behaviour of actual soils (HARDIN & DRNEVICH, 1972 a and b) shows that under cyclic loading the stress-strain relationship may be considered to be a loop, which makes the existence of damping evident. The nature of damping, however, is not yet well understood as to attribute to it a specific material property, nor to propose a satisfactory theory for its explanation. Therefore, in general, the inclusion of damping in the analytical model of a vibrating system is made in a way "which is most expedient in the mathematical solution of the problem rather than on purely physical considerations." (VAISH & CHOPRA, 1973).

Some models, briefly reviewed in Appendix 3, have been proposed to consider the effect of damping in the response of a layer. However, these models, which are suitable either for a Fourier analysis or a modal type of solution, cannot be used to obtain a *wave-form* solution such as those given in the previous chapters.

This chapter includes the effect of damping in the layer response by adding to the equation of motion that has been used until now, a viscous force, proportional to the time rate of deformation in an elementary volume. Damping here, is thus related to a volumetric force and not directly to the constitutive equation of the material.

This assumption, which is compatible with findings from experiments about the dependance of damping on effective mean principal stress, void ratio, degree of saturation, etc. (HARDIN & DRNEVICH opp.cit), enables the layer response to be expressed in a time-history form.

Equations of Motion

Consider the layer of thickness h shown in Figure (3.1) on a rigid foundation. The material in the layer has as properties a mass density ρ, a shear modulus G, and a specific viscosity η. A horizontal displacement g(t) of the base produces, at a depth y, an absolute displacement U(y,t).

Assuming that the net force acting in an elementary volume of material is equal to the elastic force G U''(y,t) dV plus a viscous force proportional to the time rate of deformation in that volume dV, the equation of motion of the layer is given by

$$\ddot{U}(y,t) = S^2\, U''(y,t) + \lambda\, \dot{U}''(y,t) \qquad (3.1)^{*}$$

where,

$$S^2 = \frac{G}{\rho} \quad \text{and} \quad \lambda = \frac{\eta}{\rho} \qquad (3.2)$$

The surface of the layer, y=0, is free of stresses; therefore

$$U'(y,t)\Big]_{y=0} = 0 \qquad (3.3)$$

and there is no sliding between the base and the layer; i.e.

$$U(h,t) = g(t) \qquad (3.4)$$

Furthermore, the motion is considered to start from rest, i.e.

$$U(y,0) = \dot{U}(y,t)\Big]_{t=0} = 0 \qquad (3.5)$$

and

$$g(0) = \dot{g}(t)\Big]_{t=0} = 0 \qquad (3.6)$$

* For a clarification of the validity and consequences of the last term in this equation see discussion on page 49.

Solution

The application of the Laplace Transform to equation (3.1) and its boundary conditions gives a relationship between the transforms of the absolute displacement of the layer and of the disturbance displacement g(t), which may be shown to be

$$U(y,p) = \frac{e^{(k-\sqrt{1+k^2})\frac{p}{s}h}\left[e^{(-k+\sqrt{1+k^2})\frac{p}{s}y} + \mu\, e^{-(k+\sqrt{1+k^2})\frac{p}{s}y}\right]}{1 + \mu\, e^{-2\sqrt{1+k^2}\,\frac{p}{s}h}}\,\bar{g}(p) \qquad (3.7)$$

where

$$\mu = \frac{1 - k/\sqrt{1+k^2}}{1 + k/\sqrt{1+k^2}} \qquad (3.8)$$

$$k = \frac{\eta}{2\sqrt{G\rho}} = \frac{\lambda}{2s} \qquad (3.9)$$

The denominator in (3.7) may be written as

$$\left[1 + \mu\, e^{-2\sqrt{1+k^2}\,\frac{p}{s}h}\right]^{-1} = \sum_{n=0}^{\infty} (-1)^n\, \mu^n\, e^{-2n\sqrt{1+k^2}\,\frac{p}{s}h} \qquad (3.10)$$

and then, it is possible, by the sole use of the shifting theorem of the transformation, to find the expression for U(y,t), which may be shown to be

$$U(y,t) = \sum_{n=0} (-1)^n\, \mu^n\, g(t_y - t_{yn1}) + \mu \sum_{n=0} (-1)^n\, \mu^n\, g(t_y - t_{yn2}) \qquad (3.11)$$

where

$$t_y = t + \frac{k}{s}(h-y) \qquad (3.12)$$

$$t_{yn1} = (2n+1)\sqrt{1+k^2}\,\frac{h}{s} - \sqrt{1+k^2}\,\frac{y}{s} \qquad (3.13)$$

and

$$t_{yn2} = (2n+1)\sqrt{1+k^2}\,\frac{h}{s} + \sqrt{1+k^2}\,\frac{y}{s} \qquad (3.14)$$

both $(t_y - t_{yn1})$ and $(t_y - t_{yn2})$, should satisfy the inequalities

$$0 \leq (t_y - t_{yni}) \leq T_d \qquad i=1,2 \qquad (3.15)$$

where T_d is the duration of the input motion g(t).

Expressions similar to (3.11) relate the layer velocity and acceleration to the corresponding base disturbance.

Properties of the Solution

Equation (3.11) shows that the inclusion of an internal mechanism of dissipation brings both a decrease in the amplitude of the layer response, and an increase in the layer period.

It may be noticed from the argument of g(t) in the previous equation, that the actual velocity with which a disturbance is propagated inside the layer is

$$S_d = \frac{S}{\sqrt{1+k^2}} \tag{3.16}$$

and consequently, the layer period will now be

$$T_{Ld} = 4\,\frac{h}{S_d} = \sqrt{1+k^2}\;T_L \tag{3.17}$$

where,

$T_L = 4\,\dfrac{h}{S}$ is the undamped layer period, and

T_{Ld} damped layer period.

From the relation between these two periods, it is possible to define a damping coefficient for the layer. If it is considered that

$$\frac{2\pi}{T_{Ld}} = \sqrt{1-\xi^2}\;\frac{2\pi}{T_L} \tag{3.18}$$

where ξ is the layer damping coefficient, then it is found from (3.17) that

$$\xi = \frac{k}{\sqrt{1+k^2}} \tag{3.19}$$

which may be seen to be independent of frequency.

The decrease in amplitude of response is given by the parameter μ, which is related to the damping coefficient by the equation

$$\mu = \frac{1-\xi}{1+\xi} \tag{3.20}$$

The parameter μ appears in equation (3.11) multiplying the disturbance g(t) every time it reaches the surface of the layer. Thus, it may be seen that during an interval T_{Ld} the amplitude of the disturbance at any depth is reduced by a factor d, such that

$$d = \mu^2 \tag{3.21}$$

Hence, d may be used to evaluate the damping coefficient from a vibrational test. It may be shown that

$$\xi = \frac{1-\sqrt{d}}{1+\sqrt{d}} = \frac{1-\mu}{1+\mu} \tag{3.22}$$

From equation (3.21) and the definition of logarithmic decrement for a viscous damping, one may see that

$$\partial = 2\pi \frac{\xi}{\sqrt{1-\xi^2}} = \log_e \frac{1}{d} \tag{3.23}$$

which gives place to the following relation

$$\mu = e^{-\pi k} \tag{3.24}$$

A layer wave may also be considered to express the layer response given in (3.11). It may be shown that for a damped motion, the displacement layer wave is given by

$$W_g(t) = \sum_{n=0}^{[t/2h/S_d]} (-1)^n \mu^n g\left(t - 2n \sqrt{1+k^2}\, \frac{h}{s}\right). \tag{3.25}$$

The layer displacement may be written

$$U(y,t) = W_g\left[t_y-(h-y)/S_d\right] + \mu\, W_g\left[t_y-(h+y)/S_d\right] \tag{3.26}$$

with similar expressions for layer velocities and accelerations.

Finally, as in the radiation problem, the previous solution may be deconvolved easily to find the input motion. An identical procedure to the one presented in the previous chapters leads to an expression for the original disturbance g(t) in terms of the motion recorded at the surface of the layer. Such an expression may be proved to be

$$g(t) = \frac{1}{1+\mu} \left\{ g_s\left[t-kh/s + \sqrt{1+k^2}h/s\right] + \mu\, g_s\left[t-kh/s-\sqrt{1+k^2}h/s\right]\right\} \quad (3.27)$$

where

$$g_s(t) = U(0,t) .$$

Similar expressions relate input velocities and accelerations to the corresponding responses at the surface.

As is the case with the other analytical models for damping, the solution in this chapter accounts for the energy loss inside the layer, but it does not provide a precise description of the damping phenomenon in physical terms.

One may see from equation (3.11) that the amplitude of the input disturbance is not decreased continuously as it travels inside the layer but only when it reaches the layer surface. The application of (3.11) is thus confined to bounded media where continuous reflections of the disturbance propagated in the medium ensures the action of the internal dissipative mechanism.

In practical terms, this presents no serious restrictions to the use of this model of damping for the study of actual soil deposits of limited thickness; and advantages such as an explicit time-history form of solution for any arbitrary input, expedience in computation, and an easily measurable damping coefficient, together with the fact that there is no satisfactory theory of damping (EWING et al 1957), fully justify the use of the model proposed.

It may be thought that the apparent discontinuous decrease in amplitude of the disturbance which occurs every time the wave reaches the surface is due to the initial assumption of considering the damping term in the equation of motion to be a volumetric force. However, this is not the case. Had it been assumed, for example, that the damping term were derived from a constitutive equation such as,

$$\tau = G \frac{\partial u}{\partial y} \pm \eta \frac{\partial u}{\partial t} \tag{3.28}$$

a solution identical in form to (3.11) would have been obtained. The solution corresponding to the positive sign in (3.28) is unacceptable as in this case the response increases with time; and for the negative sign, the final equation is identical to (3.11) but for the time t_y which in this case is defined as

$$t_y = t - \frac{k}{s}(h-y) \tag{3.29}$$

Also, if one considers the equation of motion

$$\ddot{U}(y,t) + 2k_1 S \dot{U}(y,t) + S^2 k_1^2 U(y,t) = S^2 U''(y,t) \tag{3.30}$$

with the boundary conditions

$$U'(y,t)\Big]_{y=0} = 0 \tag{3.31}$$

and $$U(h,t) = g(t) \tag{3.32}$$

the layer displacement $U(y,t)$ may be shown to be given by

$$U(y,t) = e^{-k_1(h-y)} \sum_{n=0} (-1)^n \mu_1^{2n} g\left[t - \frac{h-y}{s} + 2n\frac{h}{s}\right] +$$

$$+ e^{-k_1(h+y)} \sum_{n=0} (-1)^n \mu_1^{2n} g\left[t - \frac{h+y}{s} + 2n\frac{h}{s}\right] \tag{3.33}$$

where

$$\mu_1 = e^{-k_1 h} \tag{3.34}$$

It may be seen that in equation (3.30), the terms $2k_1 S \dot{U}$ and $k_1^2 U$, which account for the internal dissipation of energy, have been considered in the equilibrium of an elementary volume without being directly related to the constitutive equation of the material; thus acting as volumetric forces. The solution (3.33) however, shows a continuous decrease in amplitude of response, though not an increase in the layer period.

It may thus be suggested that a discontinuous decrease may be due to the assumption that damping is viscous in nature only. However, an irrefutable proof to substantiate this argument cannot be provided; mainly because the other analytical models which consider a viscous damping (see Appendix 3) do not have a solution which may be easily expressed as a time-history, and it is only in this type of solution where the complete form of the response motion may be visualised.

If, for example, a modal solution corresponding to the equation of motion (3.1) is considered the poles of (3.7) have to be found. By making the denominator of (3.7) equal to zero, it may be seen that the poles of $\overline{U}(y,p)$ are given by

$$p = \frac{1}{2\sqrt{1+k^2}} \, \frac{s}{h}\left[\pm (2n-1)\,\pi\, i + \log_e \mu \right] \qquad n=1,2,\ldots \qquad (3.35)$$

If the undamped frequency of the n-th mode of vibration is defined as

$$W_n = (2n-1)\,\frac{\pi}{2}\,\frac{s}{h} \qquad n=1,2,\ldots \qquad (3.36)$$

then, from (3.22), (3.24) and after some manipulations, (3.35) may be written

$$p = -\xi W_n \pm \sqrt{1-\xi^2}\; W_n\, i \qquad n=1,2,\ldots \qquad (3.37)$$

Hence, (3.37) ensures that in the subsequent convolution integral the term

$$e^{-\xi W_n (t-\tau)}$$

will appear, giving the impression that the amplitude of response decreases continuously in time.

Relation between Dissipative Mechanisms

Although different in nature, radiation and damping produce similar effects on the response at the surface of the layer. The layer displacement, for example, at the surface y=0, when damping is considered is given, from (3.11), by

$$U(0,t)_D = (1+\mu) \sum_{n=0} (-1)^n \mu^n g\left[t + k \frac{h}{s} - (2n+1) \sqrt{1+k^2} \frac{h}{s}\right] \quad (3.38)$$

While the displacement at the same surface for the case of radiation, considering the same reference recorded motion, is

$$U(0,t)_R = \frac{2}{1+\alpha} \sum_{n=0} (-1)^n \beta^n \; g\left[t - (2n+1) \frac{h}{s} \right] \quad (3.39)$$

where α is the impedance ratio and β is the radiation coefficient.

For purposes of comparison, the term k h/s in (3.38) is irrelevant, as it is independent of n and indicates simply a different time origin for the responses.

If the value of μ is made equal to the radiation coefficient β, or what is the same, the damping coefficient ξ to the impedance ratio α, then, it is found that, as

$$\frac{2}{1+\alpha} = 1 + \beta \quad (3.40)$$

expressions (3.38) and (3.39) are identical but for the different velocities of propagation s and $S/\sqrt{1+k^2}$. The value of $\sqrt{1+k^2}$ does not differ from unity more than 10%, even for values of damping coefficient as high as 0.4. Thus, for most practical cases,

$$\sqrt{1+k^2} \doteq 1$$

and hence, it may be said that

$$U(0,t)_D \doteq U(0,t)_R \quad (3.41)$$

and similarly for velocities and accelerations.

Figures (3.3) to (3.10) show the validity of this simplificatory consideration. Each of these Figures presents in the left hand side the acceleration at the surface of a layer on a rigid base (no radiation) with a given undamped period T_L, for a given value of ξ. The surface acceleration for the layer with the same period T_L, now with an impedance ratio α, equal to the value of ξ, but without damping, appears on the right hand side of the Figure.

Responses have been computed for the same reference motion at the base of the layer, which is the North-South component of the earthquake recorded at Port Hueneme on 18/3/1957, whch is shown in Figure (3.2).

Notice that differences between the responses are more pronounced as either the dissipation coefficient (α or ξ) or the layer period increases.

A point worth mentioning is that in both dissipative mechanisms the radical $\sqrt{G\rho}$ appears as an important parameter. Both k, from which the damping coefficient is obtained, and α, the impedance ratio, are dependent on the value of this radical; which suggest that $\sqrt{G\rho}$ may be a measure of the dissipative capacity of the material in the layer. However, the inability to produce a model for damping based only on physical considerations, makes any conclusion on the influence of this radical on the material behaviour, merely speculative.

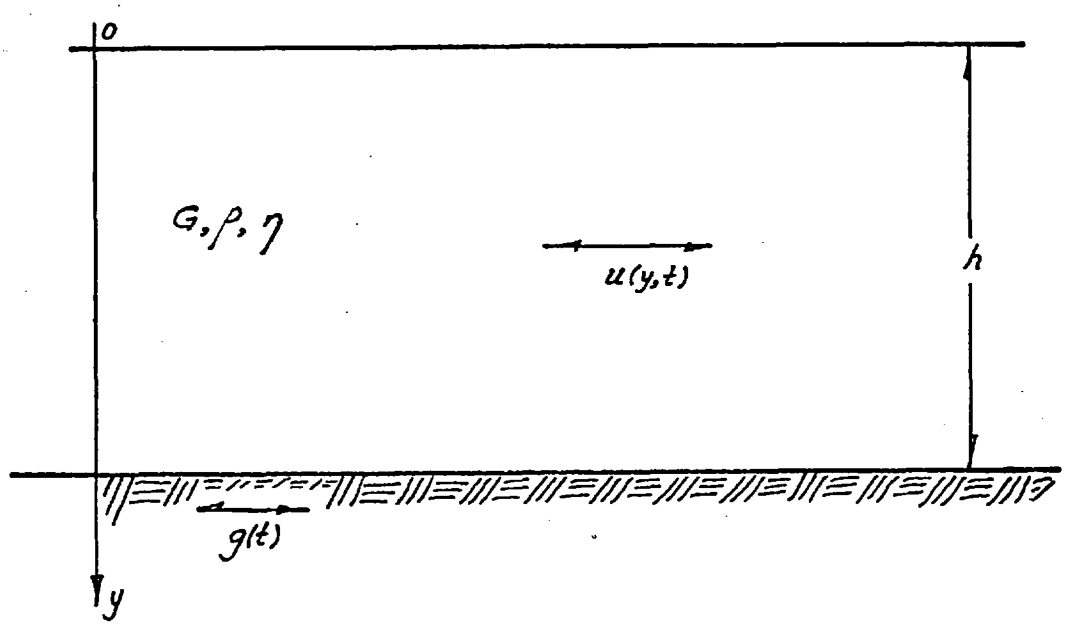


FIG. 3.1

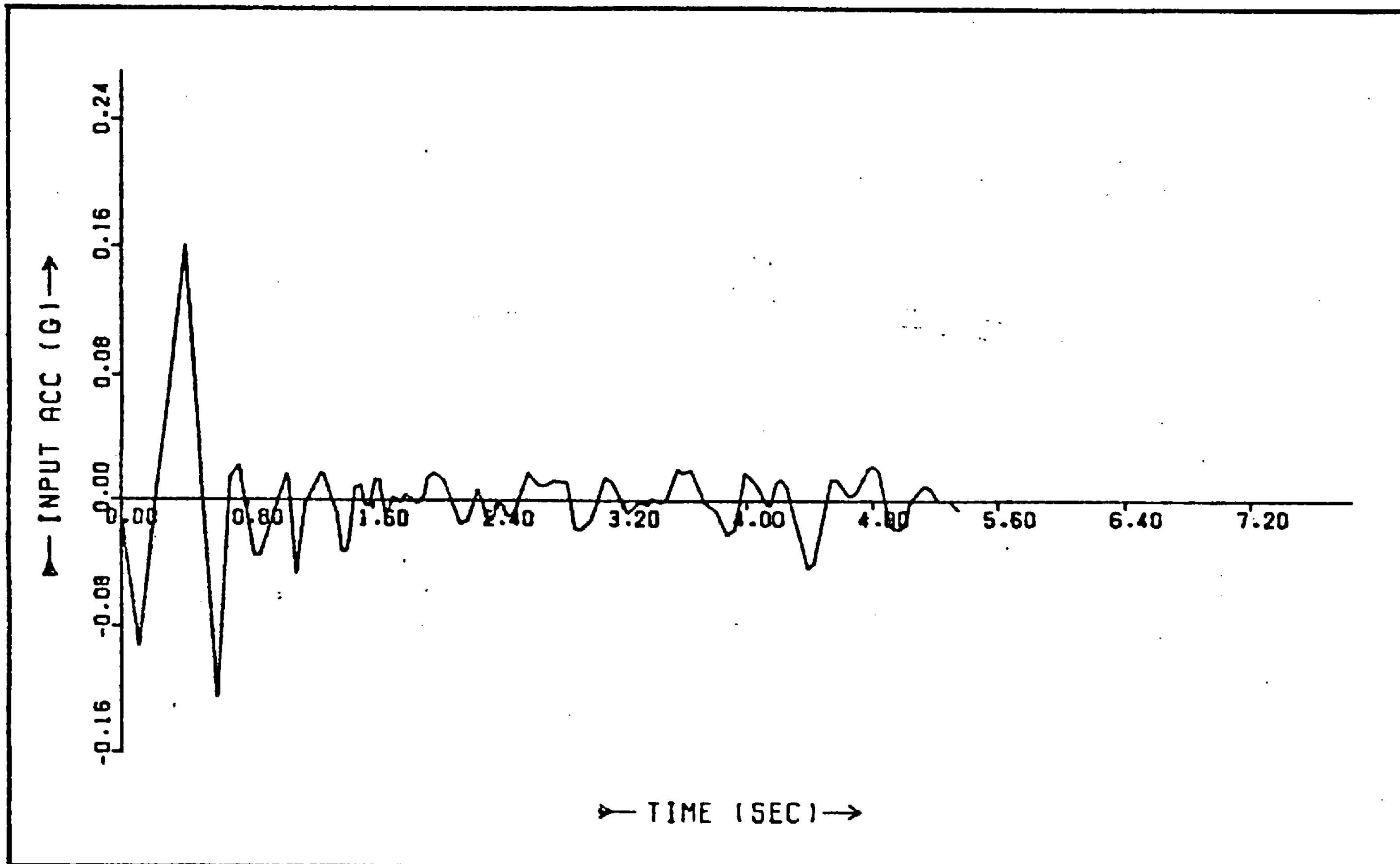
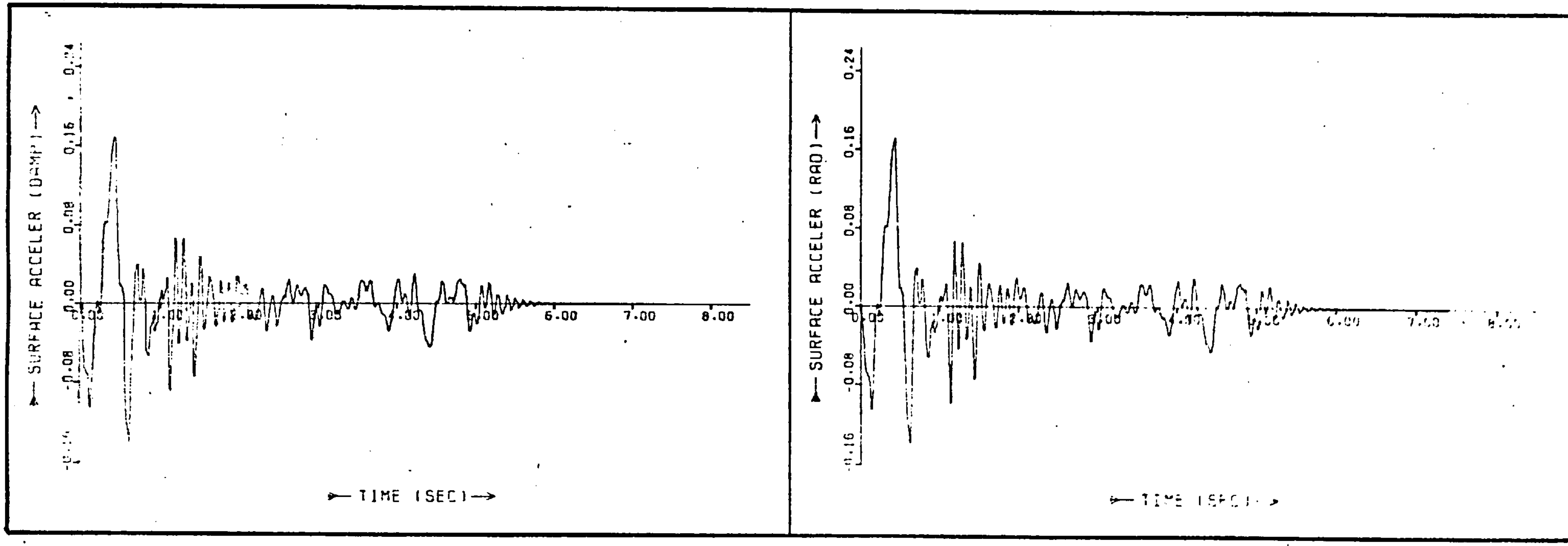

INPUT ACC (G)
0.24
0.16
0.08
0.00
-0.08
-0.16
0.00
0.80
1.60
2.40
3.20
4.00
4.80
5.60
6.40
7.20
TIME (SEC)

FIG. 3.2



$T_L = 0.1$ sec.

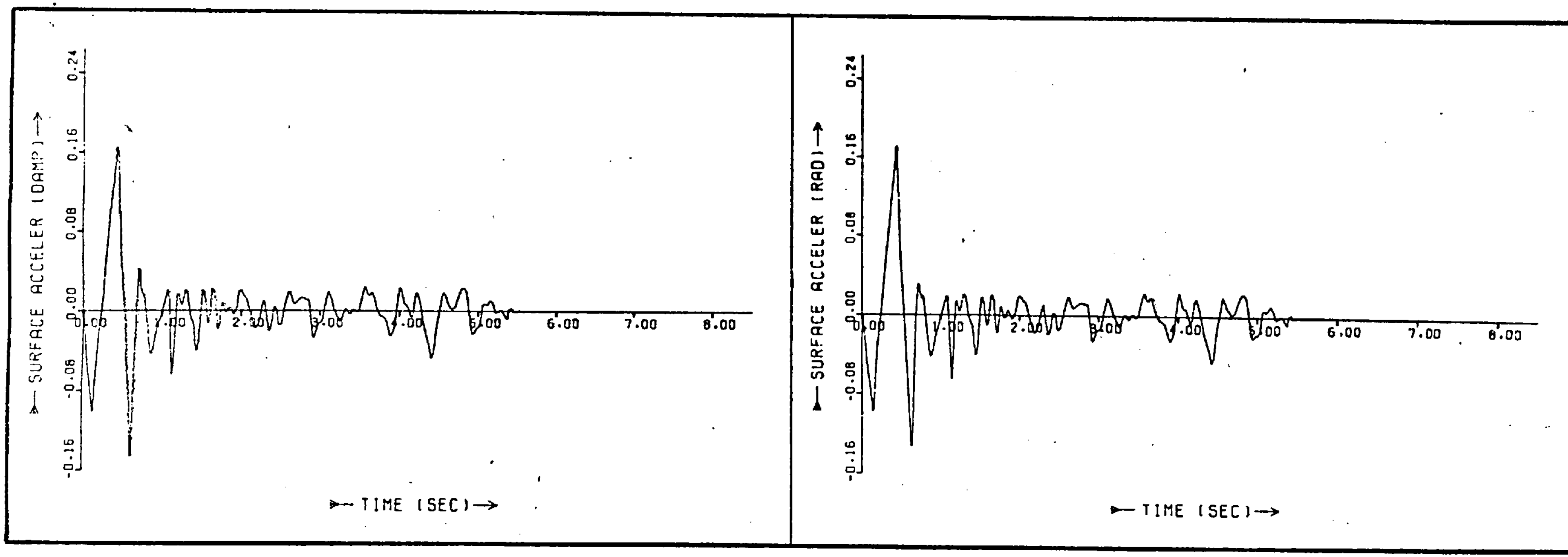
$\xi = 0.1$

$\alpha = 0$

$\alpha = 0.1$

$\xi = 0$

FIG. 3.3

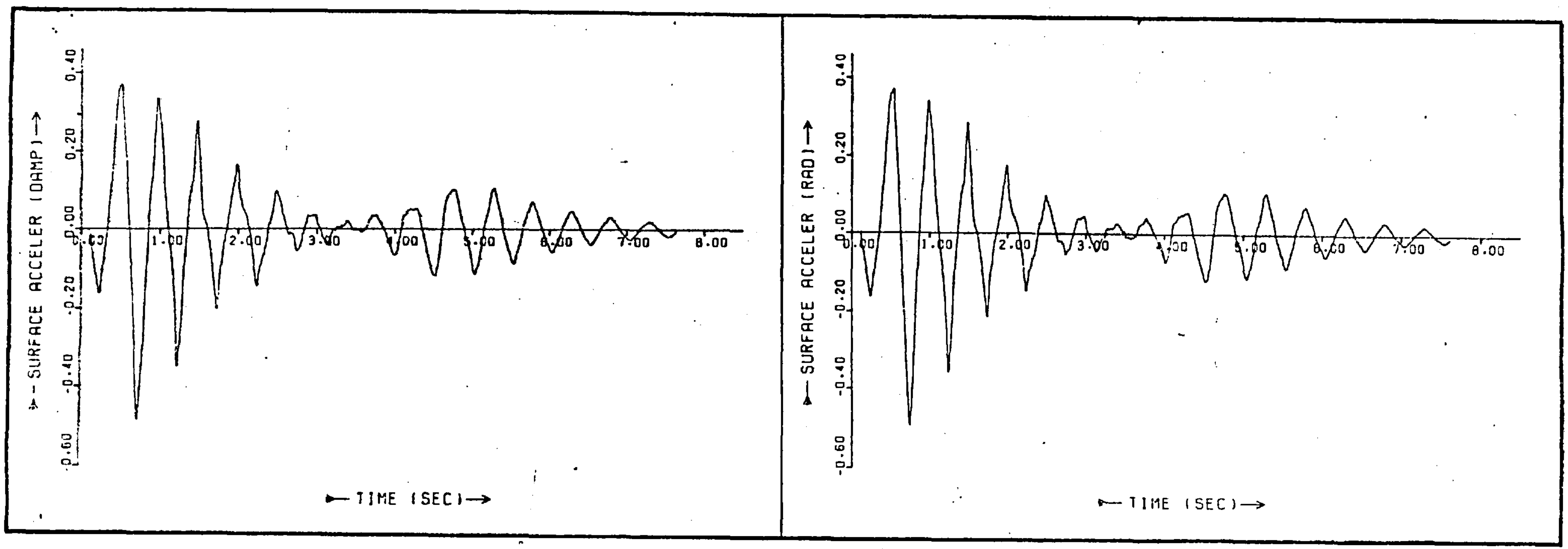


$T_L = 0.1$ sec.

$\xi = 0.5$
$\alpha = 0$

$\alpha = 0.5$
$\xi = 0$

FIG. 3.4

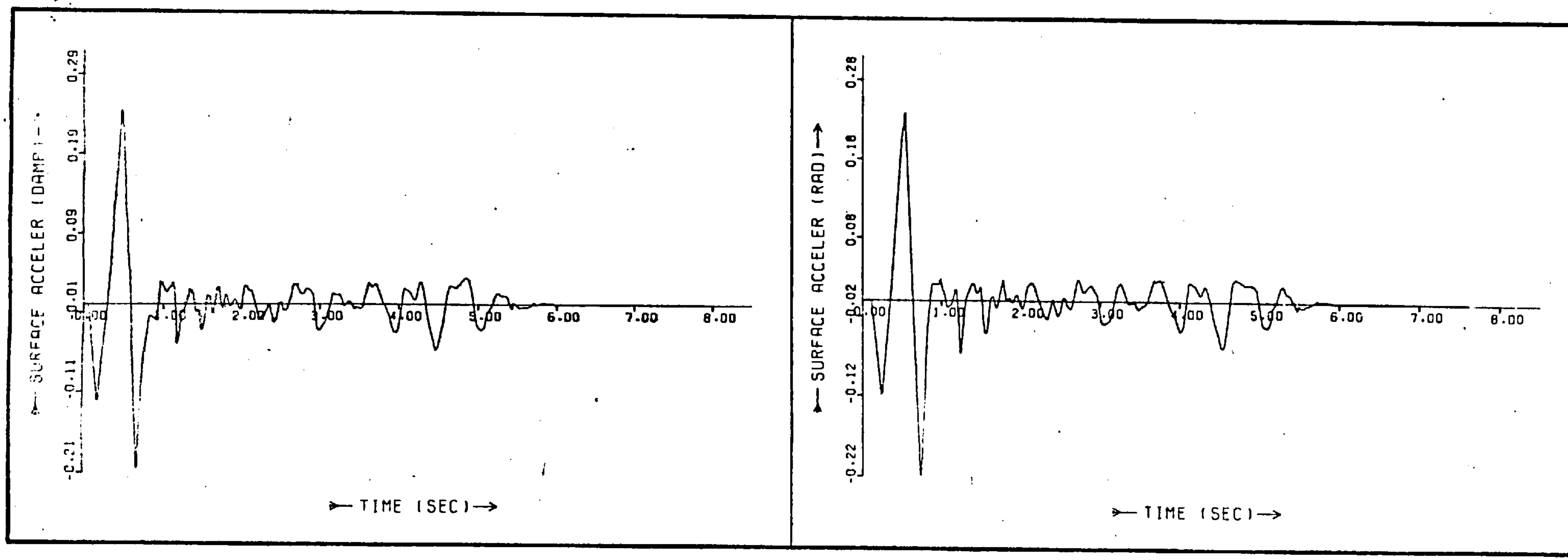


$T_L = 0.5$ sec.

$\xi = 0.1$
$\alpha = 0$

$\alpha = 0.1$
$\xi = 0$

FIG. 3.5

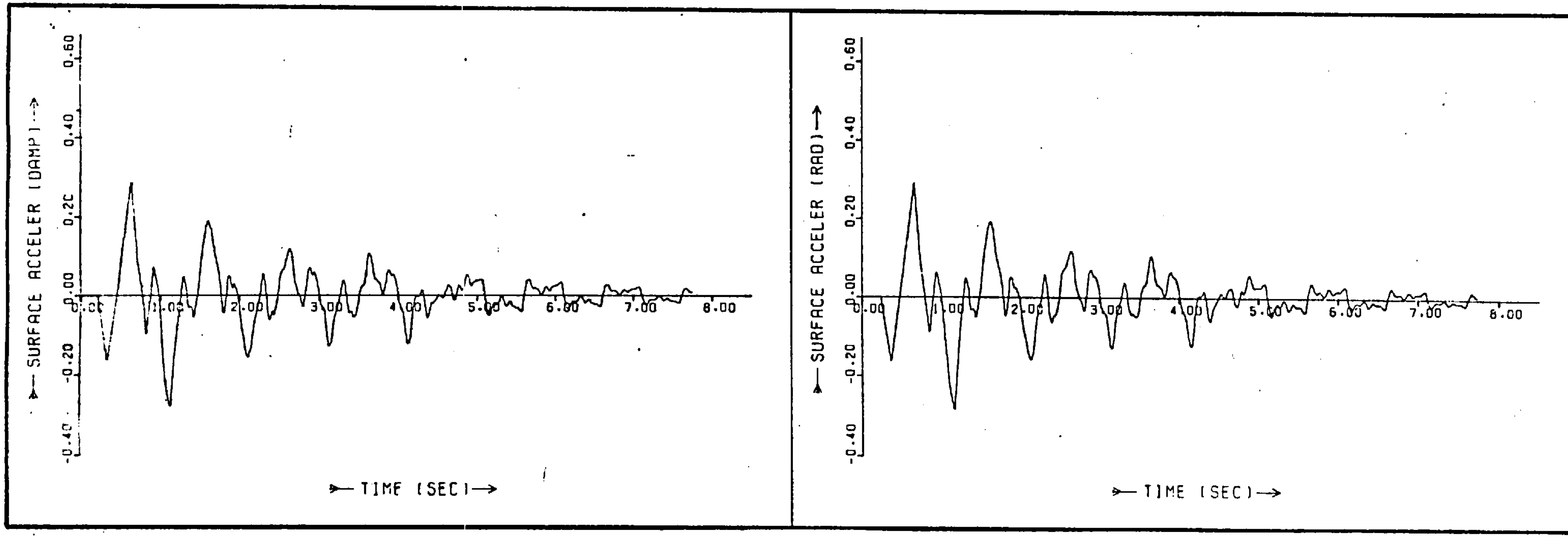


$T_L = 0.5$ sec.

$\xi = 0.5$
$\alpha = 0$

$\alpha = 0.5$
$\xi = 0$

FIG. 3.6

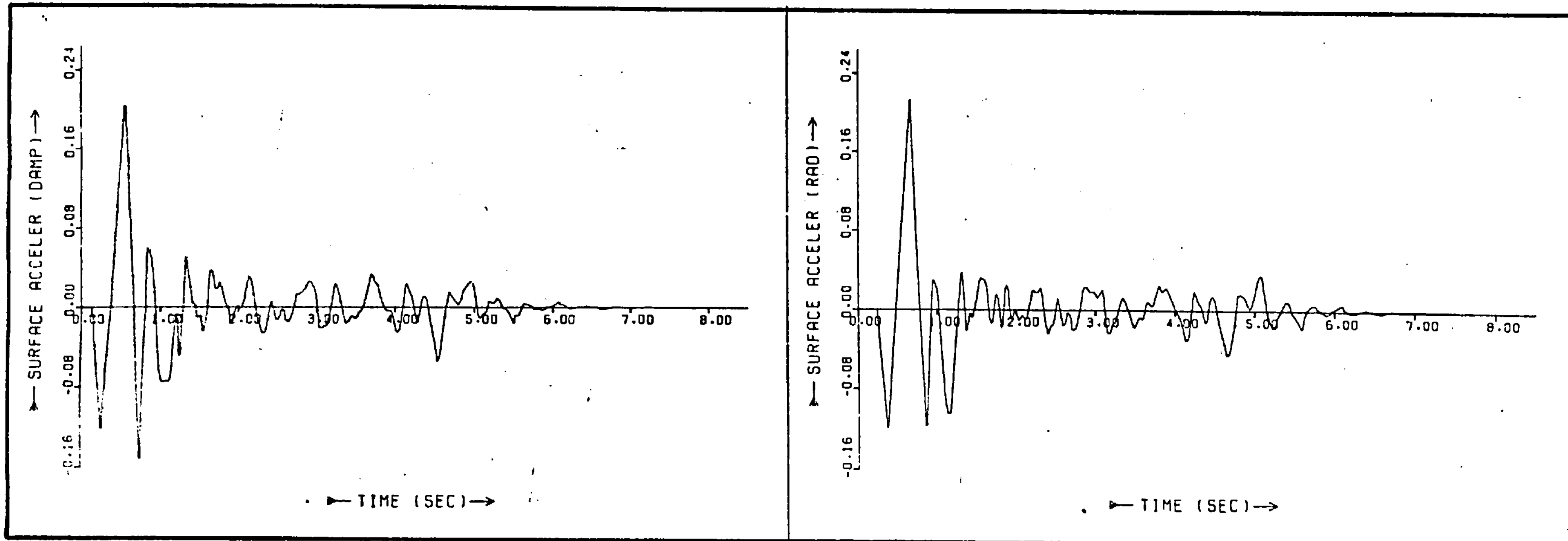


$T_L = 1.0$ sec.

$\xi = 0.1$
$\alpha = 0$

$\alpha = 0.1$
$\xi = 0$

FIG. 3.7

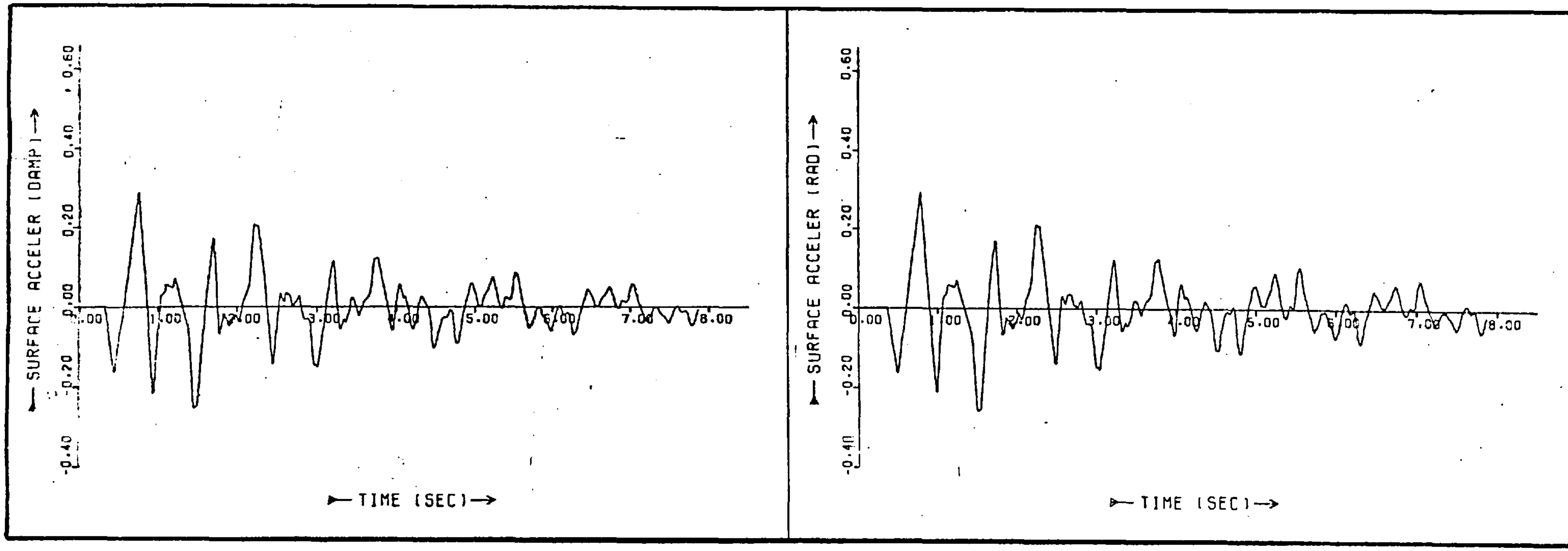


$T_L = 1.0$ sec.

$\xi = 0.5$
$\alpha = 0$

$\alpha = 0.5$
$\xi = 0$

FIG. 3.8

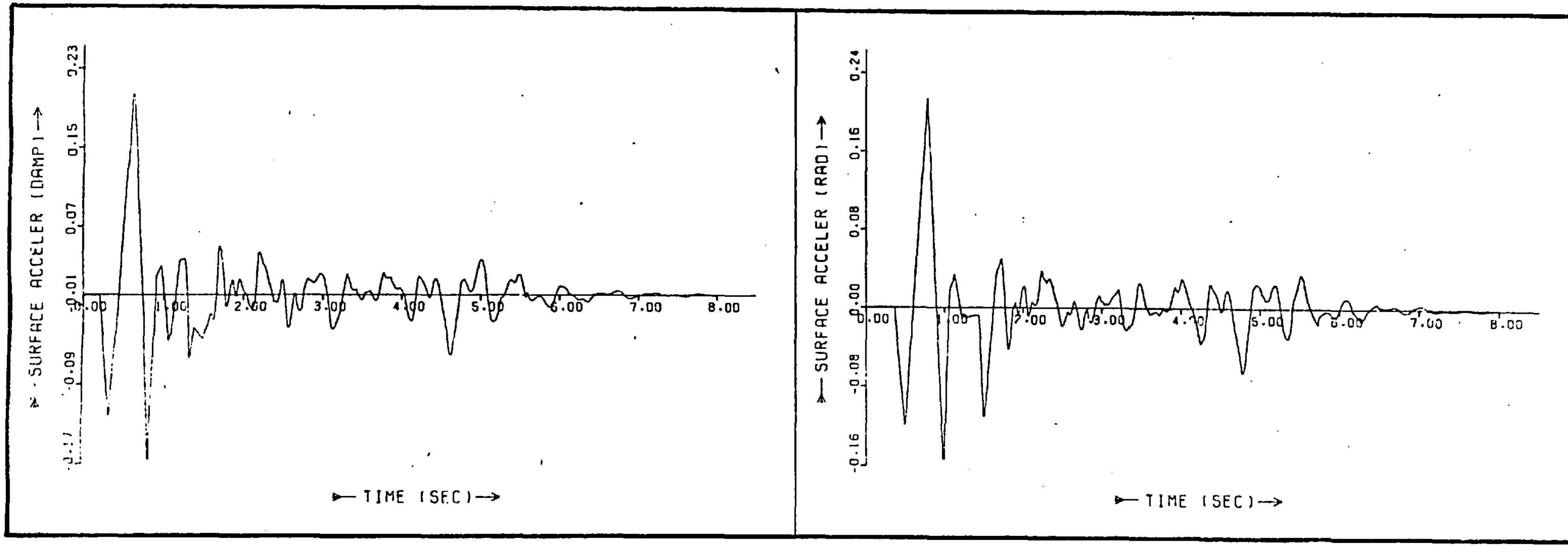


$T_L = 1.5$ sec.

$\xi = 0.1$
$\alpha = 0$

$\alpha = 0.1$
$\xi = 0$

FIG. 3.9



$T_L = 1.5$ sec.

$\xi = 0.5$
$\alpha = 0$

$\alpha = 0.5$
$\xi = 0$

FIG. 3.10

CHAPTER FOUR

GENERAL WAVE SOLUTION

Introduction

The main elements involved in the response of an elastic layer have been discussed in the previous chapters, and to provide a general equation for the layer motion; it is, considering both radiation and material damping, requires no further assumptions than those which have already been made. The analytical process to arrive to such a general solution, though it deals with more complex expressions, does not present any problem which had not been previously discussed.

There is, however, a special interest for dealing with the general case in a chapter of its own, and it is to show that a solution in an wave form is one of the main characteristics of the layer response, rather than a mere convenient way of expressing it.

General Formulation

The homogeneous elastic layer of thickness h shown in Figure (4.1) has material properties G, ρ and η and is placed on a semi-infinite elastic base of shear modulus G_s and mass density ρ_s. A horizontal displacement disturbance g(t) travelling upwards in the base, produces absolute horizontal displacements $U_s(y,t)$ at a depth y inside the base ($y \geq h$) and U(y,t) at a depth y inside the layer ($y \leq h$). The displacement U(y,t) is required.

The equation of motion for the layer is (Chapter Three):

$$\ddot{U}(y,t) = S^2 U''(y,t) + \lambda \dot{U}'(y,t) \tag{4.1}$$

where

$$S^2 = \frac{G}{\rho}$$

and
$$\lambda = \frac{\eta}{\rho}$$

At the surface of the layer, y=0

$$U'(y,t)\Big]_{y=0} = 0 \tag{4.2}$$

while at the interface y=h, (Chapter Two)

$$U(h,t) = U_s(h,t) \tag{4.3}$$

and

$$G\,U'(y,t)\Big]_{y=h} = G_s\,U'_s(y,t)\Big]_{y=h} \tag{4.4}$$

U_s, satisfies the equation

$$\ddot{U}_s(y,t) = S_s^{\,2}\,U''(y,t) \tag{4.5}$$

where

$$S_s^{\,2} = \frac{G_s}{\rho_s}$$

The disturbance is defined (Chapter Two) as:

$$U_s(h+H,t) = g(t) \tag{4.6}$$

which enables us to solve the problem uniquely. Limits, however, should have to be taken once the disturbance g(t) arrives for the first time to the interface, as it was pointed out in Chapter Two.

As before, it will be assumed that all motion starts from rest, i.e.

$$U(y,0) = U_s(y,0) = g(0) = 0 \tag{4.7}$$

and

$$\dot{U}(y,t)\Big]_{t=0} = \dot{U}_s(y,t)\Big]_{t=0} = \dot{g}(t)\Big]_{t=0} = 0 \tag{4.8}$$

The Wave Solution

The Laplace Transformation shall be used to solve the problem described by equations (4.1) to (4.8).

If equation (4.1) is transformed, it may be shown that an ordinary differential equation in y is obtained, and that the general solution of such an equation is:

$$\overline{U}(y,p) = A\, e^{(\sqrt{1+k^2}-k)\frac{p}{s}y} + B\, e^{-(\sqrt{1+k^2}+k)\frac{p}{s}y} \qquad (4.9)$$

where

$$k = \frac{\lambda}{2s}$$

and $A = A(p)$ and $B=B(p)$ are to be found from the conditions at the boundaries.

Before entering into the process of determining A and B, it may be seen that inverting (4.9) in the form described in Chapter One, a general expression for the layer displacement is found of the form:

$$U(y,t) = W_{Ag}\left[t+(\sqrt{1+k^2}-k)y/s\right] + W_{Bg}\left[t-(\sqrt{1+k^2}+k)y/s\right] \qquad (4.10)$$

where, clearly

$$W_{Ag}(t) = L^{-1}\left[A(p)\right] \qquad (4.11)$$

$$W_{Bg}(t) = L^{-1}\left[B(p)\right] \qquad (4.12)$$

are the inverse Laplace transforms of A and B respectively.

Therefore, the layer displacement may be interpreted as being the superimposition of the two travelling waves W_{Ag} and W_{Bg}.

Notice that this conclusion has been reached by considering only the general solution to the equation of motion, consequently the previous interpretation is generally valid, <u>irrespective</u> of the conditions at the boundaries. Notice also, that for layer velocities and accelerations as

$$\overline{\dot{U}}(y,p) = p\,\overline{U}(y,p) \qquad (4.13)$$

and

$$\overline{\ddot{U}}(y,p) = p^2\,\overline{U}(y,p) \qquad (4.14)$$

an identical interpretation can be made.

Once *the general wave nature of the response has been established*, what remains of the solution process is to find the actual expressions for the waves appearing in equation (4.10). Such expressions are obtained from the consideration of the boundary conditions.

Free Surface

From equation (4.2), back to the transformed domain, the following condition at the free boundary y=0 is obtained:

$$\bar{U}'(y,p)\Big]_{y=0} = 0$$

Hence, from the first derivative of $\bar{U}(y,p)$ in (4.9) it may be shown that

$$B = \mu A \tag{4.15}$$

where

$$\mu = \frac{\sqrt{1+k^2} - k}{\sqrt{1+k^2} + k} \tag{4.16}$$

A and B being the transforms of the waves W_{Ag} and W_{Bg}, equation (4.16) implies that these waves have identical form and differ in magnitude only by the constant μ. Therefore, it may be concluded that for a layer with a free surface, there is only one wave characteristic of its displacement response, and (4.10) may be written

$$U(y,t) = W_g\left[t+(\sqrt{1+k^2}-k)y/s\right] + \mu W_g\left[t-(k+\sqrt{1+k^2})y/s\right] \tag{4.17}$$

where,

$$W_g(t) = W_{Ag}(t) \tag{4.18}$$

the displacement layer wave.

Elastic interface

From equations (4.3) and (4.4) the conditions which are to be satisfied at the elastic interface, y=h, in the transformed domain are

$$\bar{U}(h,p) = \bar{U}_s(h,p) \tag{4.19}$$

and $$G\,\bar{U}'(y,p)\Big]_{y=h} = G_s\,\bar{U}'_s(y,p)\Big]_{y=h} \tag{4.20}$$

Following an identical procedure to the one used to obtain equation (4.9), $\overline{U}_s(y,p)$ may be shown to be given by

$$U_s(y,p) = C\, e^{\frac{p}{S_s}y} + D\, e^{-\frac{p}{S_s}y} \qquad y \geqslant h \tag{4.21}$$

Therefore, (4.19) and (4.20) may be be written

$$A\left[e^{(\sqrt{1+k^2}-k)\frac{p}{s}h} + \mu\, e^{-(k+\sqrt{1+k^2})\frac{p}{s}h}\right] = C\, e^{\frac{p}{S_s}h} + D\, e^{-\frac{p}{S_s}h} \tag{4.22}$$

$$\alpha\, A \quad (\sqrt{1+k^2}-k)\, e^{(\sqrt{1+k^2}-k)\frac{p}{s}h} - \mu(\sqrt{1+k^2}+k)\, e^{(\sqrt{1+k^2}+k)\frac{p}{s}h} = C\, e^{\frac{p}{S_s}h} - D\, e^{-\frac{p}{S_s}h} \tag{4.23}$$

where $$\alpha = \frac{G\, S_s}{S\, G_s} \tag{4.24}$$

The expression for $A = A(p)$ may now be determined from equations (4.22), (4.23) and the resultant equation from transforming (4.6). Once this expression for A is found, it has to be multiplied by $e^{p\frac{H}{S_s}}$ and then its limit taken for H tending to infinity. (Chapter Two).

All this somewhat tedious algebraic process may be avoided if consideration is given to the wave form of equation (4.21). It may be seen that what was said of the expression for $\overline{U}(y,p)$ is also valid for $\overline{U}_s(y,p)$. Thus, C and D are the transforms of the two travelling waves which characterise the response displacement of the elastic base.

The exponentials which multiply C and D in (4.21) indicate that C corresponds to an wave propagating upwards in the base medium, while D is related to an wave travelling downwards.

The reason for defining g(t) as in (4.6) was to ensure that the first disturbance arriving to the elastic interface from the base medium were g(t). This, may also be ensured if it is stated that

$$C\, e^{+\frac{p}{S_s}h} = \overline{g}(p) \tag{4.25}$$

with the advantage that (4.25) also ensures that no other disturbance will arrive to the layer from the base medium.

The previous statement thus implies a semi-infinite base, but it does not imply that either the response motion of the base or the value of D may be known. However, as the base response is not required and as D may be easily eliminated (not made zero, which would imply a rigid base!) from equations (4.22) and (4.23) a solution for the layer motion may be obtained. Therefore, adding (4.22) and (4.23) term by term, and considering (4.25) it may be shown that for a semi-infinite base

$$A(p) = \frac{2\,\bar{g}\,(p)}{\left[1+\alpha(\sqrt{1+k^2}-k)\right]e^{(\sqrt{1+k^2}-k)\,\frac{p}{s}h} + \mu\left[1-\alpha(\sqrt{1+k^2}+k)\right]\,e^{-(\sqrt{1+k^2}+k)\,\frac{p}{s}h}} \tag{4.26}$$

Equation (4.26) may be inverted following the procedure described in Chapter One, and it is then found that

$$W_g(t) = \frac{2}{1+\alpha(\sqrt{1+k^2}-k)}\sum(-1)^n\gamma^n g\left[t-(2n+1)\sqrt{1+k^2}h/s+k\ h/s\right] \tag{4.27}$$

where

$$\gamma = \mu\ \frac{1-\alpha(\sqrt{1+k^2}+k)}{1+\alpha(\sqrt{1+k^2}-k)} \tag{4.28}$$

And as,

$$W_{\dot{g}}(t) = L^{-1}\left[p\ A(p)\right]$$

and

$$W_{\ddot{g}}(t) = L^{-1}\left[p^2A(p)\right]$$

performing the derivative process implied by p and p^2, on the input disturbance, it is obtained that

$$W_{\dot{g}}(t) = \frac{2}{1+\alpha(\sqrt{1+k^2}-k)} \sum (-1)^n \gamma^n \dot{g}\left[t+k\ h/s-(2n+1)\sqrt{1+k^2}\ h/s\right] \tag{4.29}$$

and

$$W_{\ddot{g}}(t) = \frac{2}{1+\alpha(\sqrt{1+k^2}-k)} \sum (-1)^n \gamma^n \ddot{g}\left[t+k\ h/s-(2n+1)\sqrt{1+k^2}\ h/s\right] \tag{4.30}$$

In order to include in the solution the case of a rigid base, (4.27), (4.29) and (4.30) may be expressed in terms of the motion recorded at the free surface of the base medium, as it was seen in Chapter Two.

Thus, following general expressions may be given:

$$W_{Grec}(t) = \frac{1}{1+\alpha(\sqrt{1+k^2}-k)} \sum (-1)^n \gamma^n G_{rec}\left[t+k\ h/s-(2n+1)\sqrt{1+k^2}\ h/s\right] \tag{4.31}$$

where

$G_{rec}(t)$ is either $g_{rec}(t)$, $\dot{g}_{rec}(t)$, or $\ddot{g}_{rec}(t)$; a motion recorded at the free surface of the semi-infinite base.

$W_{Grec}(t)$, depending of the meaning of $G_{rec}(t)$, may stand for the layer wave of either displacements, velocities or accelerations, and

$$\underset{\sim}{U}(y,t) = W_{Grec}\left[t+(\sqrt{1+k^2}-k)y/s\right] + \mu\ W_{Grec}\left[t-(\sqrt{1+k^2}+k)y/s\right] \tag{4.32}$$

where

$\underset{\sim}{U}(y,t)$ is $U(y,t)$, $\dot{U}(y,t)$ or $\ddot{U}(y,t)$ according to the layer wave considered.

Deconvolution of Recorded Motions

It has been shown that in order to evaluate the response motion of a layer, it is required to obtain the layer wave from the input disturbance. It is now of interest to consider the problem of finding the input disturbance from the layer wave, so that a wave approach may be used to deal with the deconvolution of response motions.

From equation (4.26) it is possible to express the transform of the input disturbance in terms of the transform of the layer wave, i.e.

$$\bar{g}(p) = \frac{1+\alpha(\sqrt{1+k^2}-k)}{2}\left[e^{(\sqrt{1+k^2}-k)\,\frac{p}{s}h} + \gamma\, e^{-(\sqrt{1+k^2}+k)\,\frac{p}{s}h} \right] A(p) \qquad (4.33)$$

and, after inversion

$$g(t) = \frac{1+\alpha(\sqrt{1+k^2}-k)}{2}\left\{ W_g\left[t+(\sqrt{1+k^2}-k)h/s\right] + \gamma\, W_g\left[t-(\sqrt{1+k^2}+k)h/s\right]\right\} \qquad (4.34)$$

with similar expressions for velocity and acceleration disturbances.

In the deconvolution problem, it is assumed that the response motion at the surface of the layer is known and the disturbance $g(t)$ is required to be found. Thus, if (4.34) is used, the only further step involved in the deconvolution process is to determine the Layer Wave in terms of the surface motion; a simple task, as from (4.32) it may be seen that

$$W_g(t) = \frac{1}{1+\mu}\, g_s(t) \qquad (4.35)$$

where $\quad g_s(t) = U(0,t) \qquad (4.36)$

It may now be concluded that in an wave formulation both the response and the deconvolution problems involve an identical procedure of solution, which consists of first determining the Layer Wave and then by shifting and scaling such a wave, obtaining either the response motion or the input disturbance. However, as the expression for the Layer Wave is much simpler in terms of the surface motion than in those of the input disturbance, the problem of deconvolution is always simpler than that of finding the response.

Wave Formulation for Multi-layered Deposits

Considering still the assumption that the motion of a deposit is primarily the result of upward propagation of shear waves, and thus keeping the response problem one-dimensional, a more realistic representation of an actual soil deposit is achieved if it is considered to be formed by a series of horizontal strata, with different material properties, but each stratum homogeneous.

For deposits idealised in such a manner, it is easy to extend the theory presented in this chapter in view of the fact that the wave nature of the response only depends on the type of the differential equation of motion, as has been shown.

Consider the deposit shown in Figure (4.2) which is assumed to be formed by a series of n homogeneous horizontal strata. The material in the i-th layer, of thickness h_i, is characterised by a shear modulus G_i, a mass density ρ_i and a specific viscosity η_i.

The deposit is founded on a rigid base which is given an arbitrary horizontal displacement g(t). Such a disturbance produces a motion at any depth inside the i-th layer, which, accordingly to what has been shown in previous chapters, is described analytically by the equation

$$\ddot{U}_i(y,t) = S_i^{\,2}\, U''_i(y,t) + \lambda_i\, \dot{U}'(y,t) \qquad (4.37)$$

$$H_{i-1} \leq y \leq H_i$$

where $S_i^{\,2} = \dfrac{G_i}{\rho_i}$,

$\lambda_i = \dfrac{\eta_i}{\rho_i}$,

and $H_i = \sum_{j=1}^{i} h_j$

Transforming equation (4.37) it is found that for each of the strata

$$U_i(y,p) = A_i\, e^{(\sqrt{1+k_i^2}-k_i)\frac{p}{S_i}y} + B_i\, e^{-(\sqrt{1+k_i^2}+k_i)\frac{p}{S_i}y} \tag{4.38}$$

where $k_i = \dfrac{\lambda_i}{2S_i}$

Therefore, the response displacement at any depth inside the i-th layer may be considered to be the superimposition of two waves travelling with a velocity $S_i/\sqrt{1+k_i^2}$, one upwards and downwards the other, i.e.

$$U_i(y,t) = W_{Agi}\left[t_y + \sqrt{1+k_i^2}\,y/S_i\right] + W_{Bgi}\left[t_y - \sqrt{1+k_i^2}\,y/S_i\right] \tag{4.39}$$

where,

$$W_{Agi}(t) = L^{-1}\left[A_i(p)\right] \tag{4.40}$$

$$W_{Bgi}(t) = L^{-1}\left[B_i(p)\right] \tag{4.41}$$

$$t_y = t - k_i\, y/S_i$$

As the deposit is formed by n strata, it is thus required to find the expression for 2n wave transforms in order to determine the motion at any depth in the deposit. The boundary conditions imposed at the interface between layers, and at the surface and base of the deposit, provide 2n-1 homogeneous and one non-homogeneous equations in the unknowns A_i and B_i. Therefore, it is always possible to determine these unknowns uniquely.

The conditions imposed at the boundaries are given by:

at the interface $y = H_i$

$$U_i(H_i,t) = U_{i+1}(H_i,t) \tag{4.42}$$

and

$$G_i\, U'_i(y,t)\Big]_{y=H_i} = G_{i+1}\, U'_{i+1}(y,t)\Big]_{y=H_i} \tag{4.43}$$

at the free surface of the deposit, $y = 0$

$$U'_1(y,t)\Big]_{y=0} = 0 \tag{4.44}$$

and at the base of the deposit

$$U_n\ (H_n, t) = g(t) \tag{4.45}$$

It may be seen from equations (4.42) and (4.43) that expressions may be found which relate the wave transforms A_{i+1} and B_{i+1} of the i+1-th layer, with those of the immediate upper layer i. It follows then that it is possible for the wave transforms of any stratum to be expressed in terms of those of the surface layer, namely, A_1 and B_1.

Also, from condition (4.44) the relationship between B_i and A_i is easily found; and in so doing the wave transforms of any layer i may be given as the product of A_1 and a certain coefficient; i.e.

$$A_i = C_A^{(i)}\ A_1 \qquad i=1,2,3,\ldots,n \tag{4.46}$$

$$B_i = C_B^{(i)}\ A_1 \qquad i=1,2,3,\ldots,n \tag{4.47}$$

Finally, from the condition at $y=H_n$, the base of the deposit, it is found that A_1 and the transform of the input disturbance are related by the expression

$$A_1 = \frac{\bar{g}(p)}{C_A^{(n)}\, e^{p(1-\zeta n)H_n/S_n} + C_B^{(n)}\, e^{-p(1+\zeta n)H_n/S_n}} \tag{4.48}$$

The effect of radiation may be considered by assuming the n-th layer to be an elastic semi-space. The time origin would, in this case, be set at the time of the first arrival of g(t) to the depth H_{n-1}, and then limits would be taken for h_n tending to infinity, as discussed in Chapter Two. Proceeding in the way described above, it is found that the expression relating A_1 and $\bar{g}(p)$ for the case of radiation is:

$$A_1 = \frac{\bar{g}(p)}{C_A^{(n)}\ e^{pH_n/S_n}} \tag{4.49}$$

It may be shown that the coefficients $C_A^{(i)}$ and $C_B^{(i)}$ are given by the recurrent relations

$$C_A^{(1)} = 1 \tag{4.50}$$

$$C_B^{(1)} = \mu_1 \tag{4.51}$$

$$C_A^{(i+1)} = \frac{e^{pt_{i+1}}}{(1+\mu_{i+1})} \left[(1+\mu_i\alpha_{di})C_A^{(i)} + (1-\alpha_{di})e^{-2pH_i/S_{di}} C_B^{(i)} \right] \tag{4.52}$$

$$C_B^{(i+1)} e^{-2pH_{i+1}/S_{di+1}} = \frac{e^{p(t_{i+1}-2H_{i+1}/S_{di+1})}}{(1+\mu_{i+1})} \left[(\mu_{i+1}-\mu_i\alpha_{di})C_A^{(i)} + (\mu_{i+1}+\alpha_{di})\, e^{-2pH_i/S_{di}} C_B^{(i)} \right] \tag{4.53}$$

where,

$$\xi_i = \frac{k_i}{\sqrt{1+k_i^2}} \; ; \qquad \mu_i = \frac{1-\xi_i}{1+\xi_i} \; ;$$

$$\alpha_{di} = \frac{(\sqrt{1+k_i^2}+k_i)G_i\, S_{i+1}}{(\sqrt{1+k_{i+1}^2}+k_{i+1})G_{i+1}S_i} \; ;$$

$$S_{di} = \frac{S_i}{\sqrt{1+k_i^2}} \; ; \qquad t_i = (1-\xi_i)\frac{H_i}{S_{di}} - (1-\xi_{i+1})\frac{H_i}{S_{di+1}}$$

It is clear from the previous discussion, that the fundamental step involved in obtaining the motion of the deposit is finding the wave $W_{A1}(t)$, whose transform is A_1. If such a wave is known, the motion at any depth may be easily obtained by a process which involves only shifting and scaling, as will be seen later. One may say then that $W_{A1}(t)$ is a characteristic of the motion of the deposit, and despite the obvious complications brought forth by different material properties, the claim that a wave form of solution is feasible for a multi-layered deposit is thus justified.

As it is the case for a single layer, the degree of simplicity in the process of evaluating the motion at any depth inside a multi-layered deposit, very much depends on whether the surface or the base motion of the deposit is known; or in other words, on whether one is dealing with a problem of deconvolution or response.

When the surface motion of the deposit is known, $W_{A1}(t)$ is readily found as

$$\bar{g}_s(p) = \bar{U}_1(o,p) = (1+\mu_1)A_1$$

then,

$$W_{A1}(t) = \frac{1}{(1+\mu_1)} \; g_s(t) \tag{4.54}$$

$W_{B1}(t)$, as indicated by equation (4.51) is obtained simply by multiplying the previous expression by μ_1. The waves in the subsequent layers are then obtained progressively following the operational procedure described by the inverses of equations (4.52) and (4.53); i.e.

$$W_{A_{i+1}}(t_o) = \frac{1}{(1+\mu_{i+1})}\left[(1+\mu_i\alpha_{di})\, W_{A_i}(t) + (1-\alpha_{di})\, W_{B_i}(t_i') \right] \tag{4.55}$$

and

$$W_{B_{i+1}}(t_o'{}_{i+1}) = \frac{1}{(1+\mu_{i+1})}\Big[(\mu_{i+1}-\mu_i\alpha_{di})W_{A_i}(t-2h_{i+1}/S_{di+1}) + $$

$$+ (\mu_{i+1}+\alpha_{di})W_{B_i}(t_i'-2h_{i+1}/S_{di+1}) \Big] \tag{4.56}$$

where,

$$t_i' = t - 2H_i/S_{di} \qquad i=1,2,\dots n$$

$$t_{oi}' = t_o - 2H_i/S_{di} \qquad i=1,2,\dots n$$

One may notice that by adding (4.55) and (4.56) the motion at the base of the i+1-th layer is obtained; the time origin, $t_o = 0$, being the time of the beginning of the motion at that particular level. Also, if h_{i+1}, where it appears in (4.56) is replaced by $y-H_i$ and the resulting wave is added to the one obtained from (4.55), as it stands, then the motion at the depth y (y inside the layer i+1) is found with a time origin set now at the start of the motion at that depth. The relationship between t and t_o is thus not required, as equations (4.55) and (4.56) are related to local times.

In the response problem, with or without radiation, despite the fact that the waves in the base layer are related to a known disturbance, each one of these waves is not known, and one may see that they cannot be determined but after the surface waves are known. Hence, in this kind of problem, one has to work out either equation (4.48) or (4.49) by progressively using (4.52) and (4.53). Once the expression for A_1 is found, it has to be inverted and a general expression for the wave $W_{A_1}(t)$ is obtained. From then on, motion at any depth is evaluated by the process of deconvolution defined by (4.55) and (4.56).

Derivation of a general expression for the characteristic wave $W_{A_i}(t)$ is beyond the scope of this thesis. However, it may be proved that such an expression for a deposit of N layers will be given by a multiple series with 2N-1 indices, which, although it appears to be appallingly complicated, may be evaluated merely by progressive shiftings and scalings of the input motion.

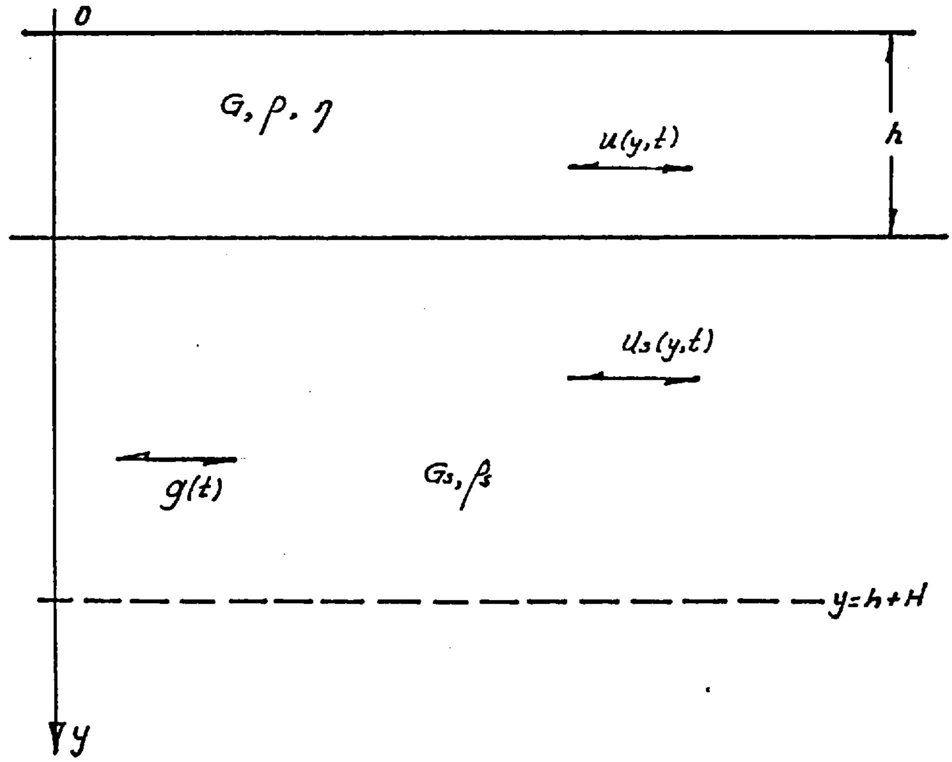


FIG. 4.1

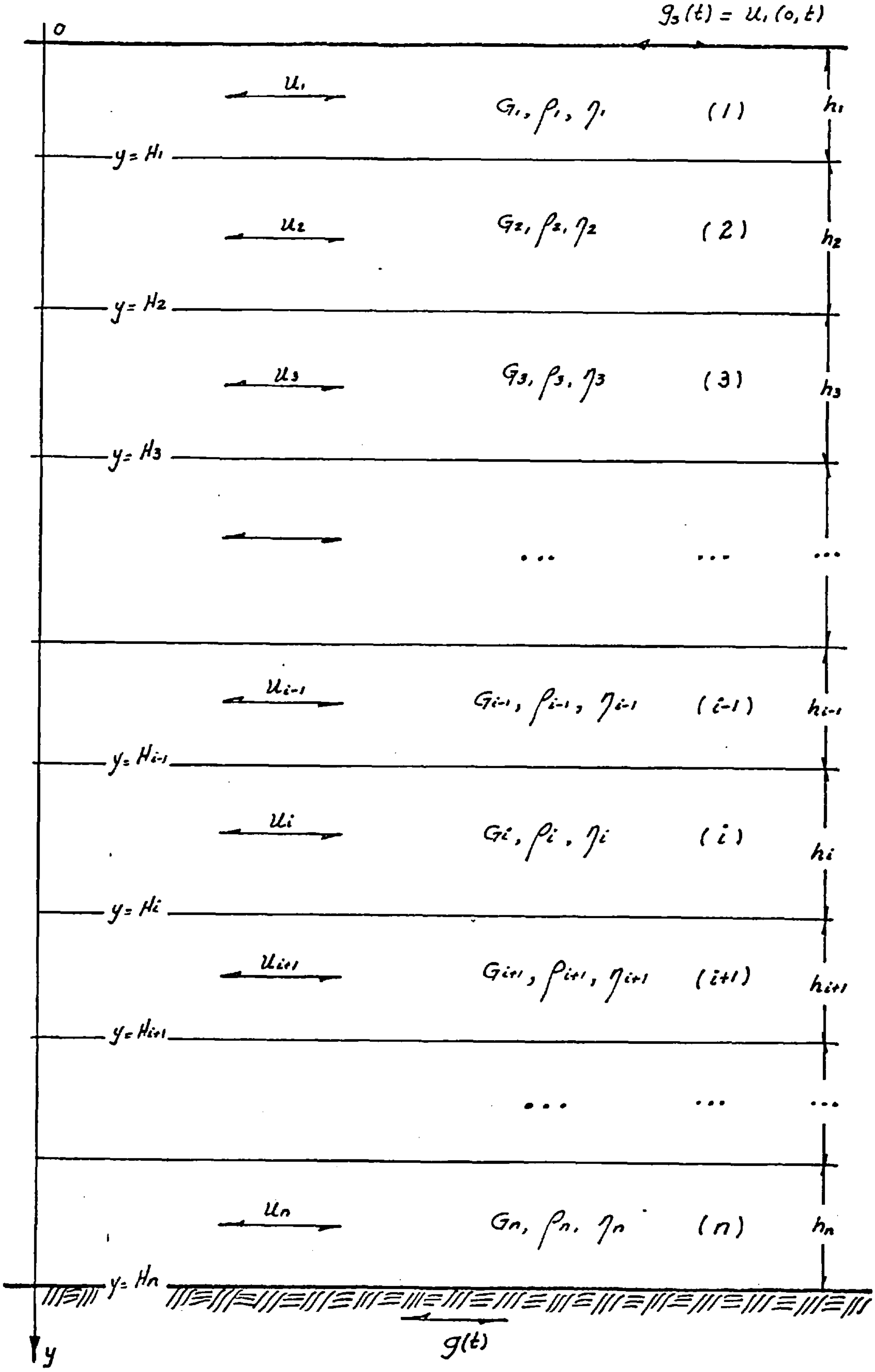

$g_3(t) = u_1(o,t)$
o
u_1
G_1, ρ_1, η_1
(1)
h_1
$y = H_1$
u_2
G_2, ρ_2, η_2
(2)
h_2
$y = H_2$
u_3
G_3, ρ_3, η_3
(3)
h_3
$y = H_3$
...
u_{i-1}
$G_{i-1}, \rho_{i-1}, \eta_{i-1}$
(i-1)
h_{i-1}
$y = H_{i-1}$
u_i
G_i, ρ_i, η_i
(i)
h_i
$y = H_i$
u_{i+1}
$G_{i+1}, \rho_{i+1}, \eta_{i+1}$
(i+1)
h_{i+1}
$y = H_{i+1}$
u_n
G_n, ρ_n, η_n
(n)
h_n
$y = H_n$
$g(t)$
y

FIG. 4.2

CHAPTER FIVE

GENERAL DISCUSSION AND CONCLUSIONS

Before entering into a discussion on the advantages and limitations of the method of solution proposed, it would be desirable, as an introduction to such a discussion, to verify the results which are obtained for simple cases.

To this effect, consider the profile of a certain site shown schematically in Figure (5.1). There, two horizontal soil deposits of thickness h1 and h2 are shown on a rock formation. The deposits may be assumed to be homogeneous, and their relevant material properties are found to be G_1, ρ_1, η_1, and G_2, ρ_2 and η_2, respectively. The bedrock material has a shear modulus G_B, a mass density ρ_B, and it is assumed that the rock formation is semi-infinite.

All these properties are summarised in an undamped layer period T_L, a damping coefficient ξ, and an impedance ratio α for each one of the deposits.

In the event of an earthquake, consider the problem of determining the acceleration time-histories at the surface of both deposits and at the rock outcrop, assuming that there is only one strong-motion instrument, installed at the surface of layer 1, from which a record of acceleration has been obtained. In other words, the problem of obtaining $\ddot{g}_{rec}(t)$ and $\ddot{g}_{s2}(t)$, knowing $\ddot{g}_{s1}(t)$.

The problem involves, first to deconvolve the given record $\ddot{g}_{s1}(t)$ to find the reference motion $\ddot{g}_{rec}(t)$, and then, evaluate the response at the surface of layer 2 for such a motion.

Some numerical examples have been worked out. The results obtained are shown in Figures (5.2) to (5.12). Each of them shows the datum acceleration, $\ddot{g}_{s1}(t)$ in the lower left hand side corner of the figure; the response acceleration, $\ddot{g}_{s2}(t)$, in the upper left; and in the lower right hand side corner, the reference motion $\ddot{g}_{rec}(t)$. The properties of both layers are also shown in each figure. A simple input motion, the N-S component of Port Hueneme Earthquake 18/3/57, has been chosen for all the examples, to facilitate a visual comparison.

The first point to verify is whether departing from a known motion at the surface of a layer with known properties, we arrive at the very same motion after first deconvolving it, and then with the base motion obtained evaluating the response at the surface of the layer. Cases 1 to 4, shown in Figures (5.2) to (5.5), where both deposits have been given identical properties, show that $\ddot{g}_{s1}(t)$, the input motion, and $\ddot{g}_{s2}(t)$, obtained after both deconvolution and response, are identical.

Case 5 illustrates the situation when both deposits and the bedrock are formed of the same material; thus, the impedance ratio for both deposits is equal to one, and the three motions, $\ddot{g}_{s1}(t)$, $\ddot{g}_{s2}(t)$, and $\ddot{g}_{rec}(t)$ are the same, as shown in Figure (5.6)

Cases 6 and 7 consider the first deposit to be of the same material as the rock formation. Hence, the datum acceleration $\ddot{g}_{s1}$ is equal to the reference motion $\ddot{g}_{rec}$; and only $\ddot{g}_{s2}$ is different, as it is shown in Figures (5.7) and (5.8).

For the remaining cases arbitrary layer properties were given. In Case 8, layer 1 is a strong deposit, as indicated by a high impedance ratio and a short layer period. Also, this period is not near to the predominant one of the datum motion, which is around 0.5 sec. Then, as one may see in Figure (5.9), the motion at the rock outcrop does not differ much from $\ddot{g}_{s1}$. Layer 2 is a weaker layer whose period coincides with the predominant of the original record. The maximum acceleration at the surface of this layer is twice as much as the maximum of the reference motion.

Figure (5.10) illustrates the case of two deposits with identical material properties, but one, layer 2, twice as thick as the other. One may see that the maximum accelerations at the surface of both deposits are almost the same, but the motions are rather different. Also from this Figure, we may see the reference motion corresponding to a surface record which has been deconvolved for a layer period which is equal to the predominant period of the surface motion.

It is interesting to point out that the only operations involved in the computation of time-histories, by means of the wave solution, are additions and multiplications and that the layer response is evaluated point by point (see Appendix 2). The numerical process does not involve the computation of any series at all, as it is the case of a modal analysis, nor is the validity of the solution restricted to the interval of definition of the input disturbance, as in a Fourier analysis. The layer response may be obtained for durations as short or as long as one wishes, and in all cases with identical accuracy.

Another advantage is that with the wave solution we may obtain the layer response at certain specific times without the need of computing the complete time-history.

Therefore, from a numerical point of view, the solution proposed presents a considerable advantage over other available methods both in accuracy as well as in efficiency and simplicity in the computational algorithm. These considerations alone, fully justify the use of this type of solution. It must be said, on the other hand, that a great deal of time and effort has been spent, rewardingly, on the numerical solution of wave propagation problems. Nowadays, standard works are available (Idriss et al., 1973; Schnabel et al., 1972; Streeter et al., 1974b; Chen, 1975 etc.) which provide the response of soil deposits for less restricted conditions than those we have considered, regarding both the material behaviour as well as the geometry of the deposit. Hence, an extension to the model which we have used, with more relaxed assumptions, would be necessary in order to make the applicability of the wave solution comparable to that of others found in the literature.

This extension should not be difficult, at least on what is concerned with the material behaviour. An equivalent linear analysis such as that suggested by Idriss and Seed (1968) may be easily implemented to the wave solution. In this case, the fast computation of our proposed solution should prove to be invaluable in the iterative process implied in finding soil properties compatible with the effective strains, and it also offers the possibility of considering different values of these properties at different stages of the motion.

It is difficult to assess at this stage the practicality of a wave solution in a closed form for two and three dimensional problems. However, it should be possible to obtain such a solution for simple boundary conditions.

The major advantage of the method of solution used in this thesis is that it relates directly the time-history of the response with that of the input disturbance, namely, the relationship between excitation and response for a deposit in the time domain is found. This gives to the solution a close mathematical form and makes it capable of being interpreted in simple physical terms; thus providing a description rather than a mere numerical evaluation of the response phenomenon. In this sense, the application of the solution obtained goes far beyond the limitations imposed by simplistic assumptions as it enables us actually to visualise the physical problem, to have a better understanding of it, and hence, to establish judgements of general nature which may prove to be of great value.

For example, using the time-history approach we were able to give a definite explanation, and under the most simple considerations (Chapter One), to the slow convergence, using a modal solution, in the evaluation of responses from corresponding inputs (accelerations from accelerations, etc.) It was shown that it obeys the condition that the sum of participation coefficients is at its limit a series of pulses. We mention that this explanation is definite, as this is the case for all problems of wave propagation, irrespective of whether or not energy dissipation is considered. On this evidence, it may be suggested to improve convergence in modal analysis, to relate layer displacements and velocities to acceleration disturbances.

The simple formulation of the radiation problem made in Chapter Two may also be seen as an application of a time-history approach. There, as the meaning of the expression for the layer response in the transformed domain was known in terms of an operation upon the input disturbance in the time domain, it was then possible to make the necessary modifications to a model of two finite layers in order to consider the foundation layer infinite and the disturbance acting at the base of the upper layer.

After this formulation, the solution given in that chapter was also obtained following an approach suitable to express the layer motion directly as a time-history. This solution analytically verifies the expression for the displacement at the surface of the layer given by Newmark and Rosenblueth (1971), which, as they report, has been confirmed experimentally in the case of a shallow alluvial formation resting on rock by Takahasi and Hirano (1941). Also, it came to be clear, from the physical interpretation of the solution, that the period of the motion of the layer is not altered by including the effect of radiation. This conclusion which was arrived at so easily in the time domain, is very difficult to visualise using other types of solution where the layer frequencies are related to the roots of a transcendental equation or the layer response is given an interpretation in the frequency domain.

When an internal mechanism of dissipation, or damping, is considered in the layer response, it is perhaps the case in which the simplicity of a time-history approach can be more appreciated. From the solution that was found for a model of damping compatible with wave propagation, we may conclude that the effect of damping may be taken into account in a time-history approach, simply by increasing the layer period and gradually decreasing the amplitude of the travelling disturbance.

Reflecting on this conclusion we may see that the model used represents, in essence, the case of material damping. Firstly, the damping coefficient is independent of the frequency content of the disturbance; whatever it is, both the layer period and the decrease in amplitude are given by the properties of the layer only. Secondly, the increase in layer period, as it is closely related to the travelling time h/s, may also be seen as a decrease in the value of shear modulus.

Considering that both shear modulus and damping coefficient are defined in terms of a loop-shaped stress-strain relationship (Hardin & Drnevich, 1972a,b) the decrease in modulus for increasing damping holds true for actual cases. The model proposed, thus, may be considered as a linear approximation to material damping. This model is suitable to be refined in order to relate more realistically both material properties, using, for example, the design equations proposed in the previous references.

Other models which have been proposed to consider the effect of damping (Appendix 3), as we mentioned in the pertinent chapter, cannot be used in an wave solution, simply because the equations of motion which they give place to are not, in a strict sense, wave equations. However, in the case of a modified Voigt model, such as the one used by Schnabel et al. (1972), and which also resembles actual material damping, an approximate wave solution may be obtained by considering an equivalent increase in layer period and decrease in amplitude (Appendix 3).

Figure (5.13) shows the response of a layer of period 0.1 sec. for the well-known Port Hueneme record. The time history in the right hand side was computed for 10% (Schnabel) damping using a frequency analysis. In the left hand side of the Figure, it is shown the time history obtained from an wave solution in which only the equivalent increase in layer period has been considered with no decrease in amplitude whatsoever. A better approximation, certainly, is achieved when such a decrease is taken into consideration; however, we wanted to stress the effect of the change in period which might be thought to be irrelevant.

The capability of the wave solution to be interpreted in physical terms indeed gives to it an advantage over solutions obtained either by modal or frequency analysis, as the meaning attached to these solutions can only be given in terms of mathematical entities and not physical ones.

This is well understood in the case of the former, of which it may be said that it basically intends to relate the response of a continuous medium to that of a series of single degree of freedom systems. But, on the other hand, frequency analysis is thought to have a more direct physical interpretation, and this may lead into an erroneous conception of the response phenomenon.

We must stress that it is not intended to challenge the validity of such an analysis. In fact, the layer responses obtained from it are identical, in most practical cases, to those computed using a wave solution, as it may be seen in Figures (5.14) to (5.16), where the acceleration time-histories evaluated using both approaches are shown for three simple cases. (At this point, it is pertinent to remark that while the frequency or Fourier solution required in each case, for a record of N points a number of operations proportional to N $\log_2 N$ to obtain the frequency image of the input (Cooley & Tukey, 1965) then N/2 complex products and finally another N $\log_2 N$ operations to retrieve the response in the time domain, the wave solution needed only N real additions and multiplications for the whole process).

The point that we like to emphasise is that frequency (or Fourier) analysis does not provide a clear description of the physical problem in the time domain, and that even from a frequency point of view, a better understanding is achieved by using a wave solution.

A Fourier analysis based on the sum of harmonics (see Appendix 4) indicates that an harmonic input of frequency ω produces a layer response of the same frequency, but dephased and with different amplitude. Physically, this is only true, and partially, in the case of sustained vibrations, and when the frequency of the input is lower than that of the deposit; conditions both which are implied in this kind of analysis. If the input is of finite duration, though harmonic and of only one frequency, its representation is no longer a point in the frequency domain, but a continuous function. Hence, to obtain the response of the layer for such an input using a Fourier analysis, it would require that for each point of such a function, we compute the change in phase and amplitude.

It is clear from the above that the frequencies to which the interpretation of the Fourier solution refers are those of the steady-state motions which added up represent the input motion in an interval, and not the frequency of this motion. Such an interpretation is therefore concerned with mathematical entities, and it has no sense if it is taken out of its context of a relationship between ordinates of Fourier spectra.

Fourier analysis must be seen, in a rigorous treatment, as the application of the Fourier transform to solve the equation of motion of the deposit (Appendix 4), and if the relationships between Fourier and Laplace transforms are considered (see Papoulis, 1972 for example), one certainly arrives to a solution identical to the one proposed in this thesis.

The wave solution then, if we are interested in a frequency analysis, may also be seen as the convolution of the input motion with a series of pulses whose amplitudes and the separation between them depend exclusively on the properties of the layer. This series, that for the case of no dissipative mechanisms present in the response is illustrated in Figure (1.5c), is the response of the layer to an impulse disturbance, and therefore, its Fourier transform is the frequency transfer function of the layer (Appendix 4). From regarding the transfer function in this way an important practical point is evident, and this is that although such a function is independent of the input motion, it is not so of the duration of motion considered. For a transient motion of duration Td, the actual transfer function of the layer will be the convolution of the function $\overline{H}(f)$, the Fourier transform of the series of pulses mentioned above, with the function $\overline{S}(f)$

$$\overline{S}(f) = Td \; \frac{\sin(\pi \; Td \; f)}{\pi \; Td \; f}$$

which is the transform of a time window of width Td and unit height, as is shown in Figure (5.17).

We may then see that the problems that may be faced with a frequency analysis are not in the computation of responses for a given input, where both the transfer function and the frequency image are evaluated for the same time interval; but in the use of the transfer function separately.

In the wave solution, rather than considering the response of a layer to an impulse as a separated function, the whole process of convolution with the series of pulses which represent this response has been taken as an operation on the input motion. This, which makes no difference as far as the results obtained are concerned, indeed, it helps to visualise the layer response as a phenomenon of propagation, as convolution of any function with a pulse simply shifts the function to the position of the pulse.

We may therefore conclude, to end this discussion, that a time-history approach to the motion of a deposit leads to a simpler and more efficient numerical algorithm for the computation of this motion; but far more important, it provides both a clear understanding of the process involved in the analytical solution to the problem, and a consistent physical interpretation to this solution. In this way, such an approach enables us to assess the validity and limitations of our analytical model, and also to have a general notion, as good as our model, of the physical phenomenon.

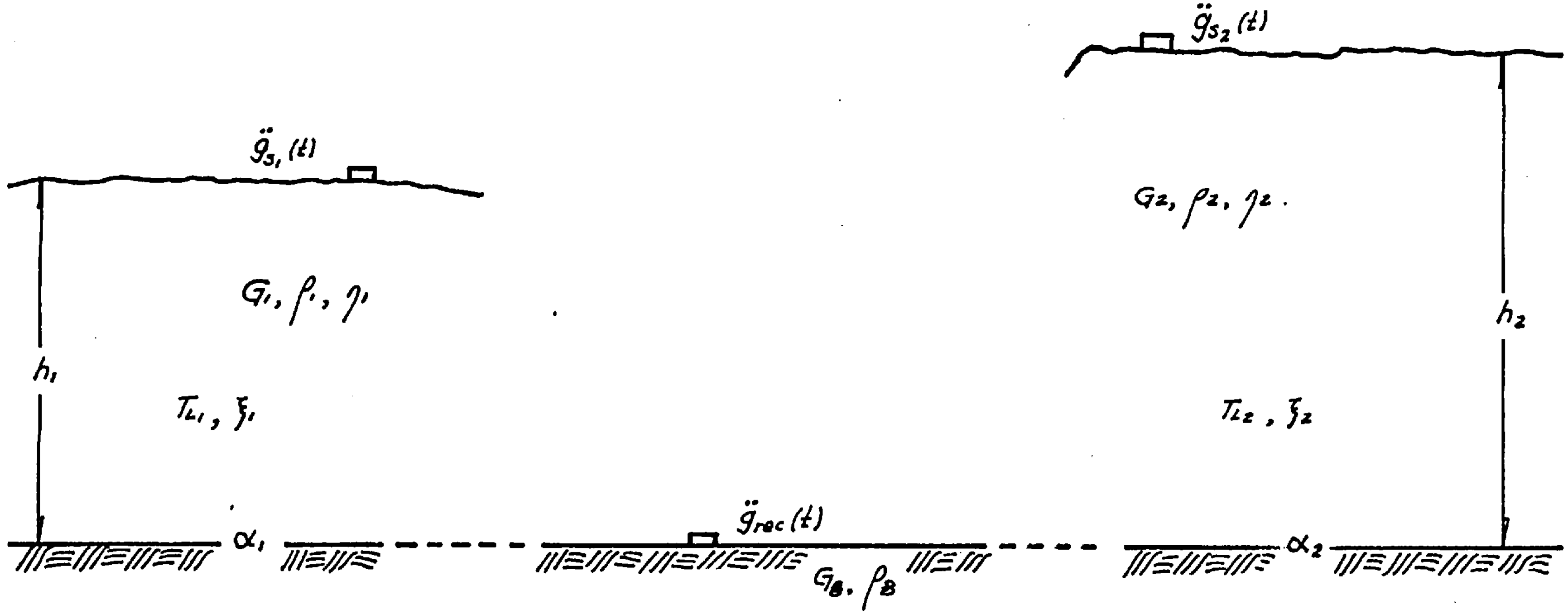

$\ddot{g}_{s_1}(t)$
G_1, ρ_1, η_1
h_1
T_{L1}, ξ_1
α_1
$\ddot{g}_{rec}(t)$
G_B, ρ_B
$\ddot{g}_{s_2}(t)$
G_2, ρ_2, η_2
h_2
T_{L2}, ξ_2
α_2

FIG. 5.1

PHN/N /SS/C N S PORTHUENEME 18/3/57 PAR.CORR. IC 23-

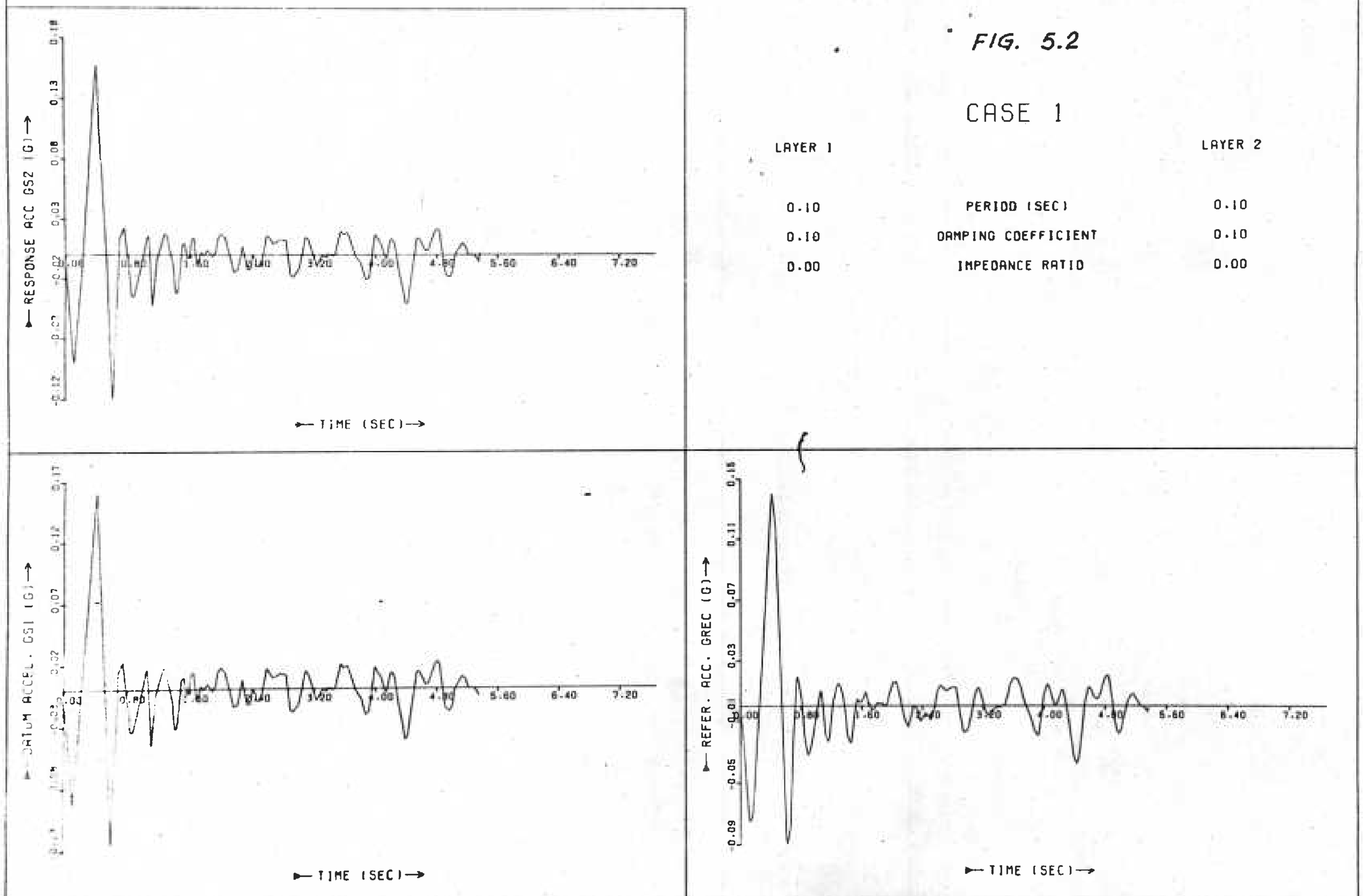


FIG. 5.2

CASE 1

LAYER 1		LAYER 2
0.10	PERIOD (SEC)	0.10
0.10	DAMPING COEFFICIENT	0.10
0.00	IMPEDANCE RATIO	0.00

PHN/N /SS/C N S PORTHUENEME 18/3/57 PAR.CORR. IC 23-

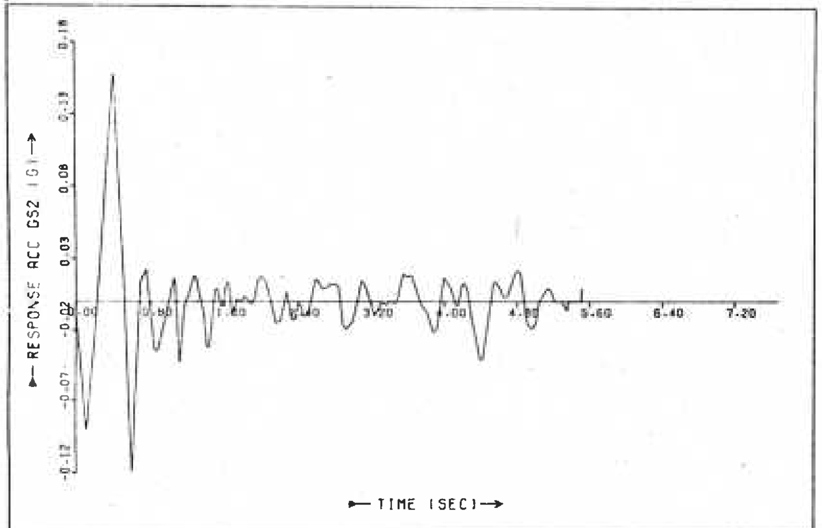
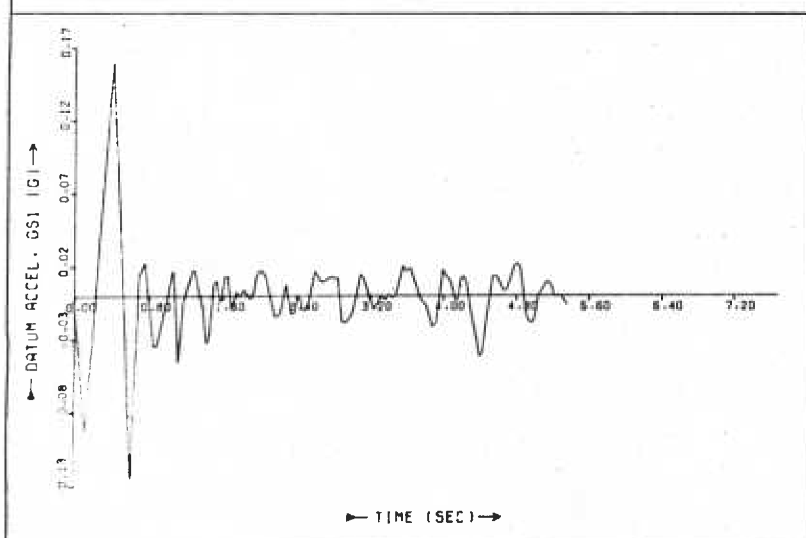
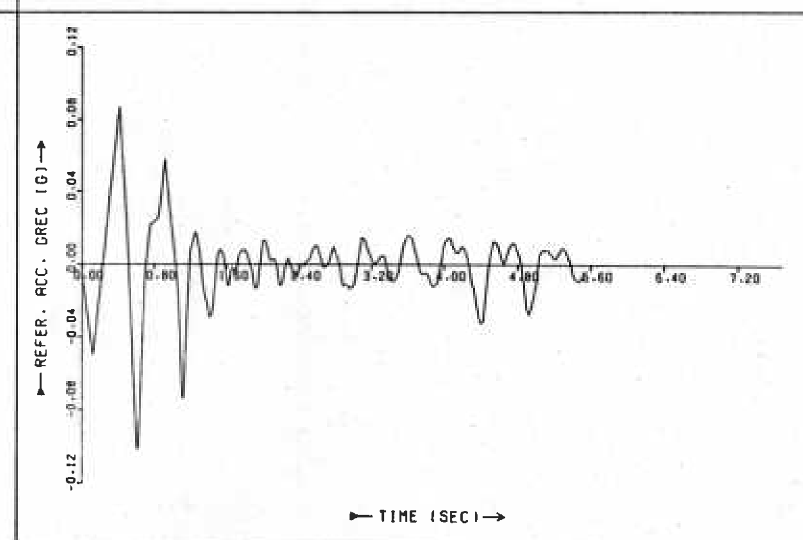


FIG. 5.3

CASE 2

LAYER 1		LAYER 2
1.00	PERIOD (SEC)	1.00
0.10	DAMPING COEFFICIENT	0.10
0.00	IMPEDANCE RATIO	0.00





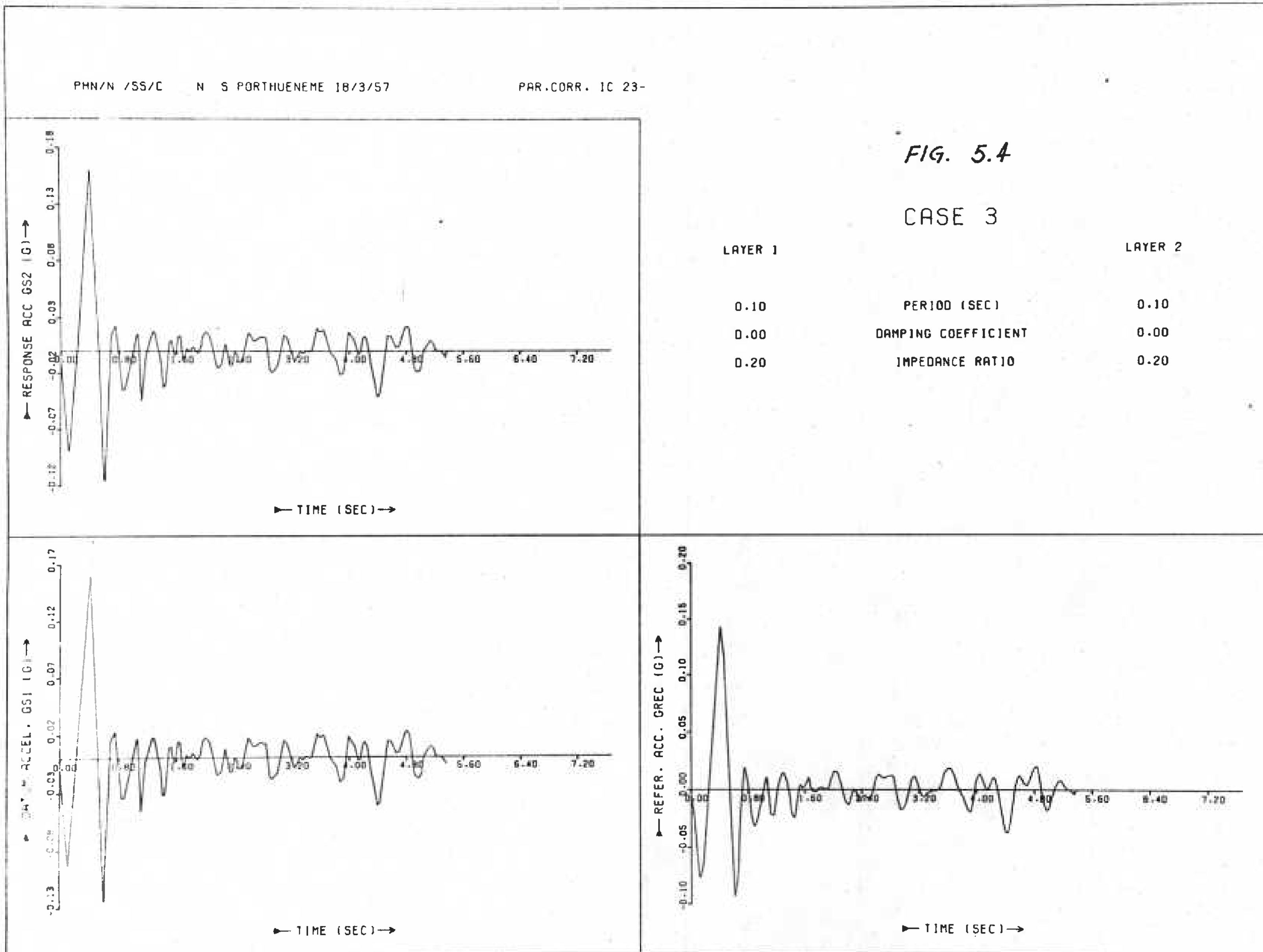

PHN/N /SS/C N S PORTHUENEME 18/3/57 PAR.CORR. IC 23-
CASE 3
LAYER 1
LAYER 2
0.10 PERIOD (SEC) 0.10
0.00 DAMPING COEFFICIENT 0.00
0.20 IMPEDANCE RATIO 0.20
RESPONSE ACC GS2 (G)
TIME (SEC)
TIME (SEC)
REFER. ACC. GREC (G)
TIME (SEC)

FIG. 5.4

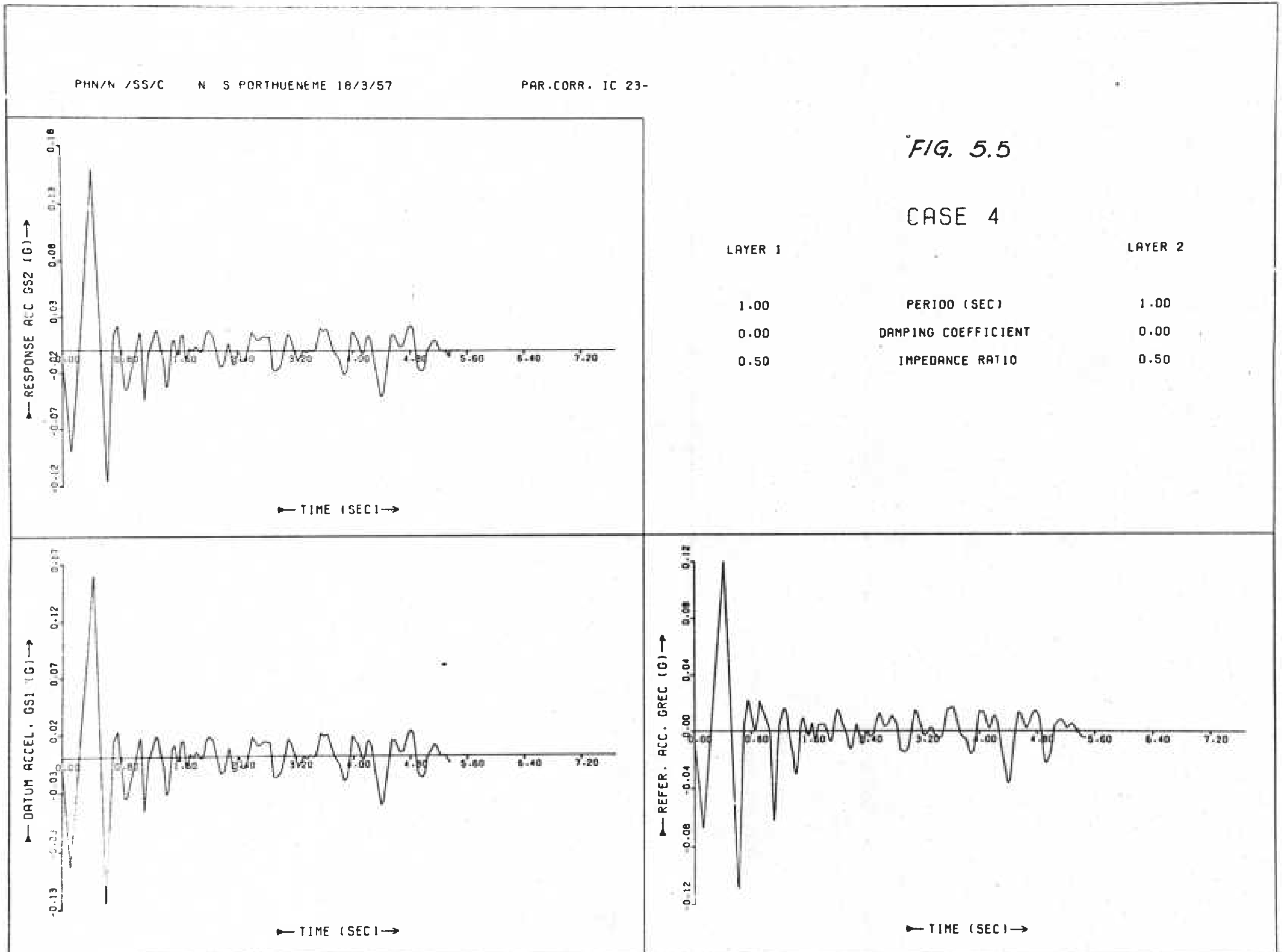


FIG. 5.5

CASE 4

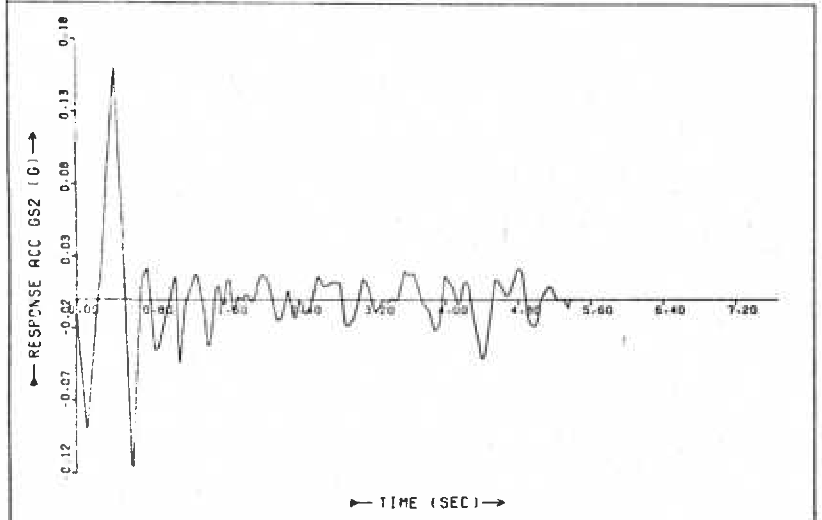
LAYER 1		LAYER 2
1.00	PERIOD (SEC)	1.00
0.00	DAMPING COEFFICIENT	0.00
0.50	IMPEDANCE RATIO	0.50

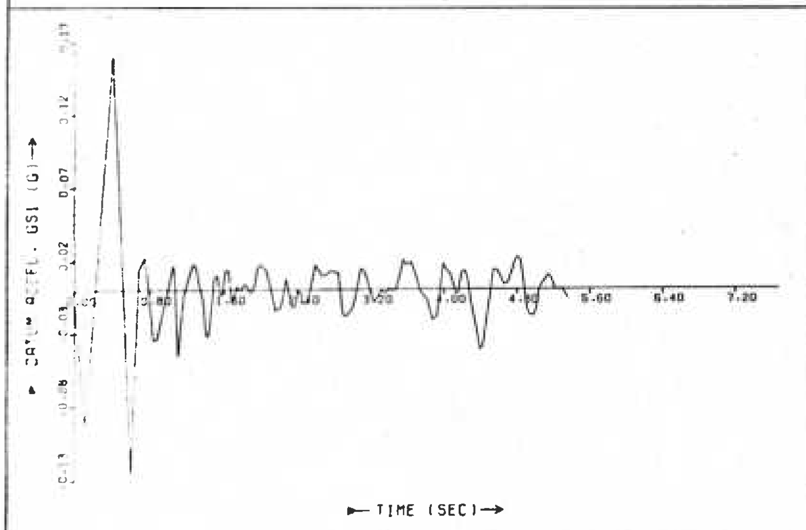
PHN/N /SS/C N S PORTHUENEME 18/3/57 PAR.CORR. IC 23-

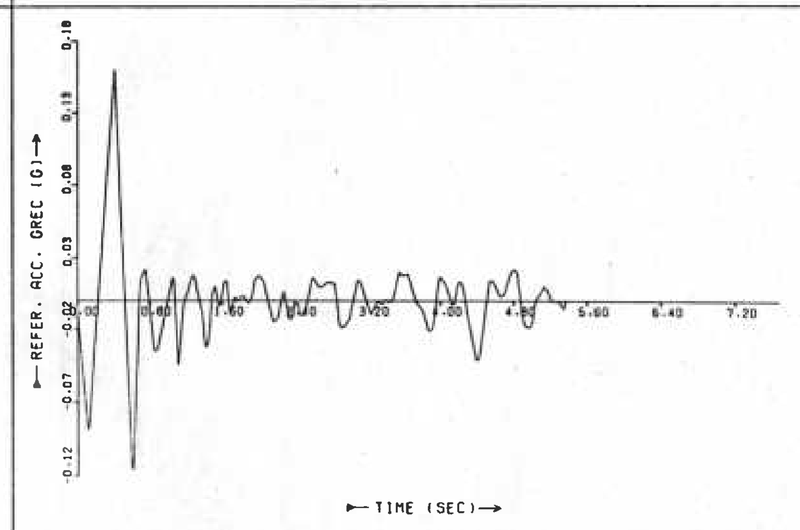
FIG. 5.6

CASE 5

LAYER 1		LAYER 2
0.20	PERIOD (SEC)	0.20
0.00	DAMPING COEFFICIENT	0.00
1.00	IMPEDANCE RATIO	1.00







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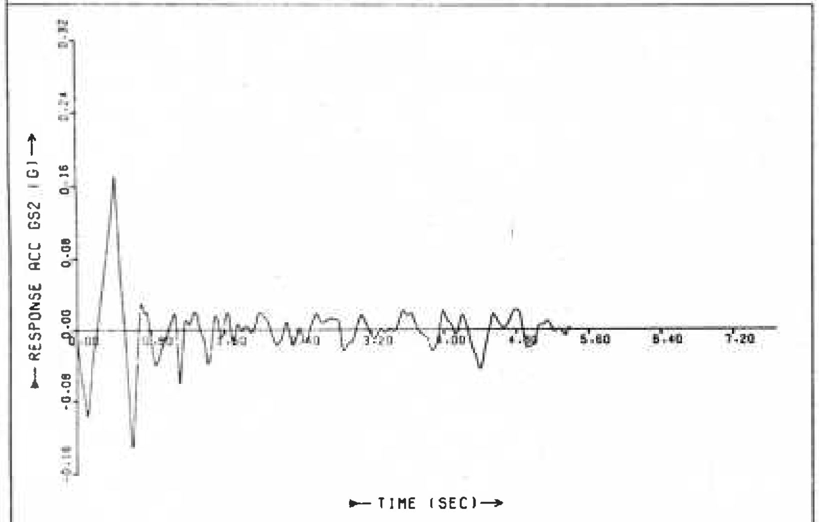
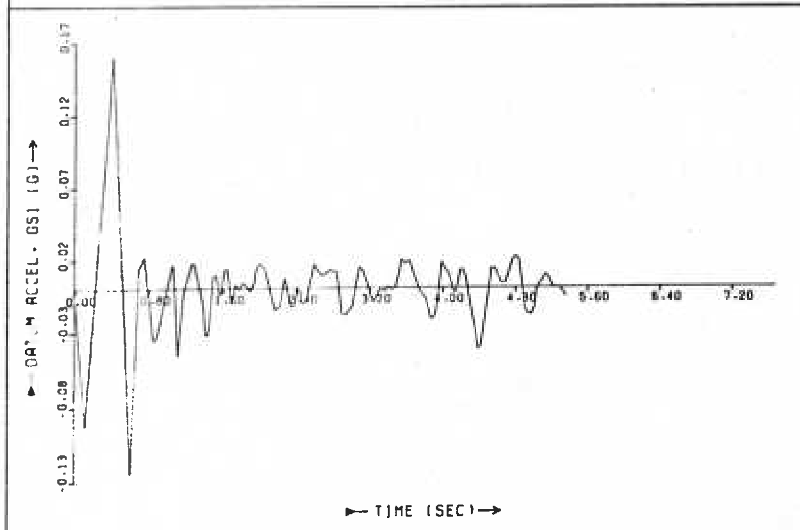
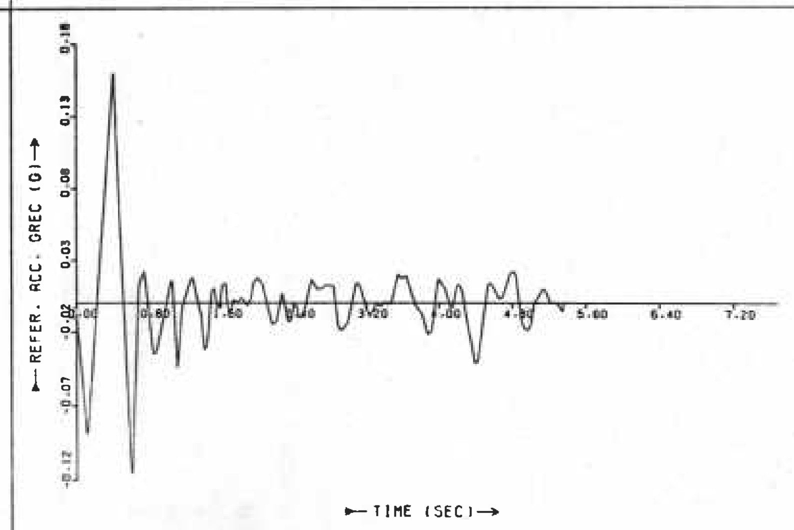


FIG. 5.7

CASE 6

LAYER 1		LAYER 2
0.10	PERIOD (SEC)	0.10
0.00	DAMPING COEFFICIENT	0.05
1.00	IMPEDANCE RATIO	0.50



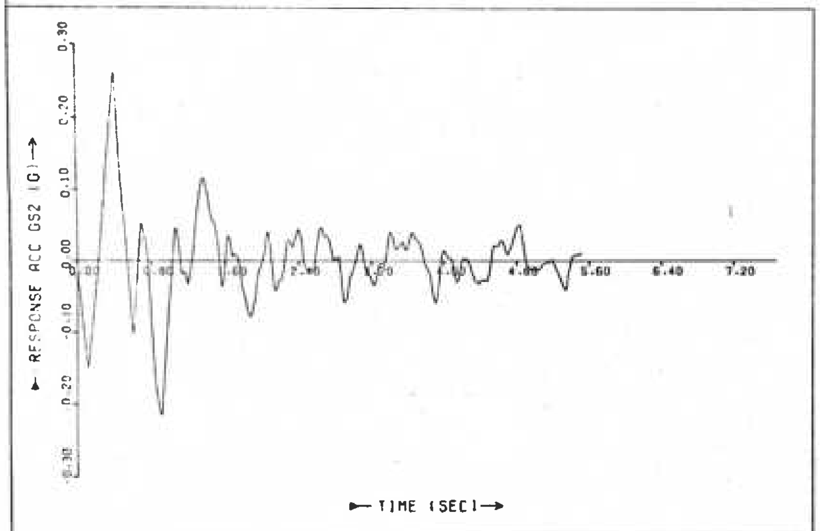


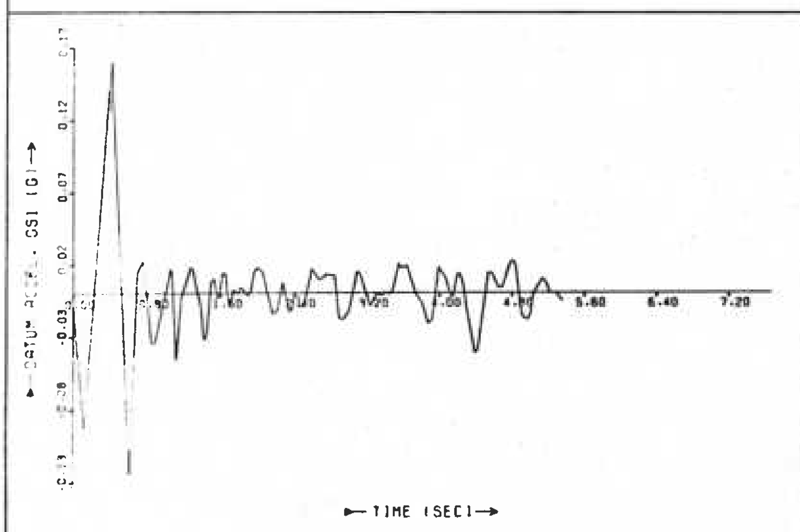
PHN/N /SS/C   N S PORTHUENEME 18/3/57   PAR.CORR. IC 23-

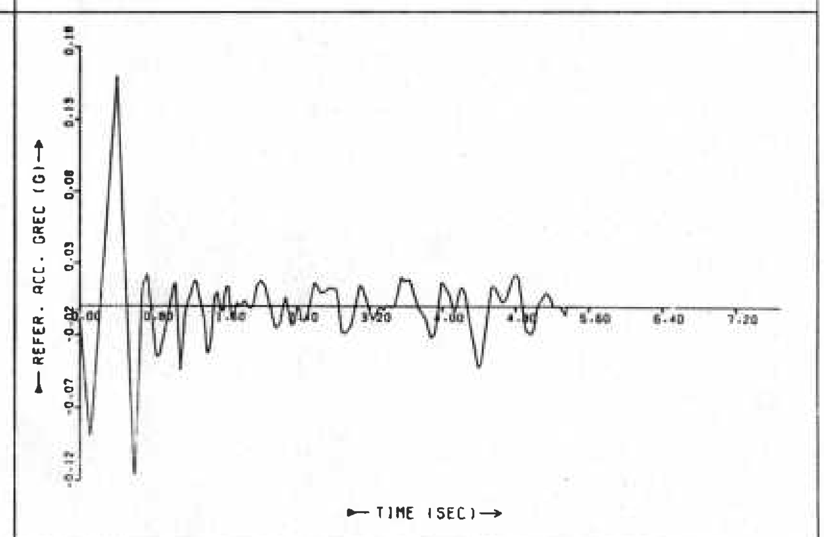
FIG. 5.8

CASE 7

LAYER 1		LAYER 2
0.50	PERIOD (SEC)	1.00
0.00	DAMPING COEFFICIENT	0.10
1.00	IMPEDANCE RATIO	0.10







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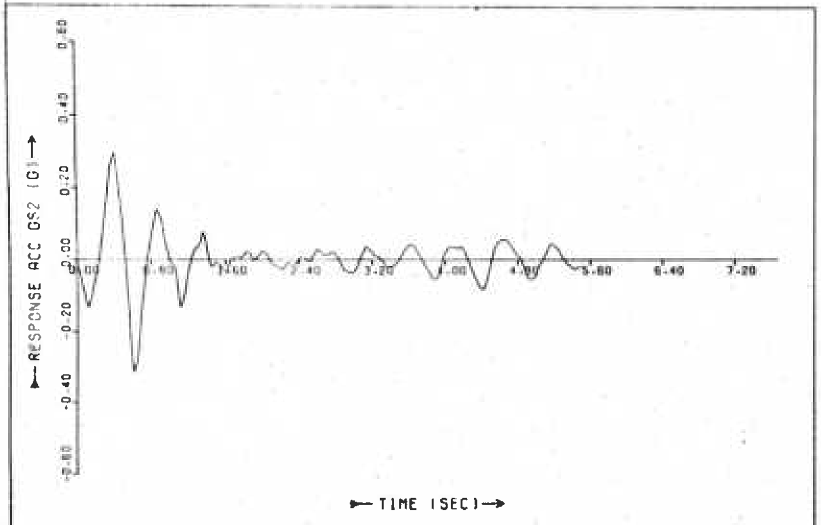
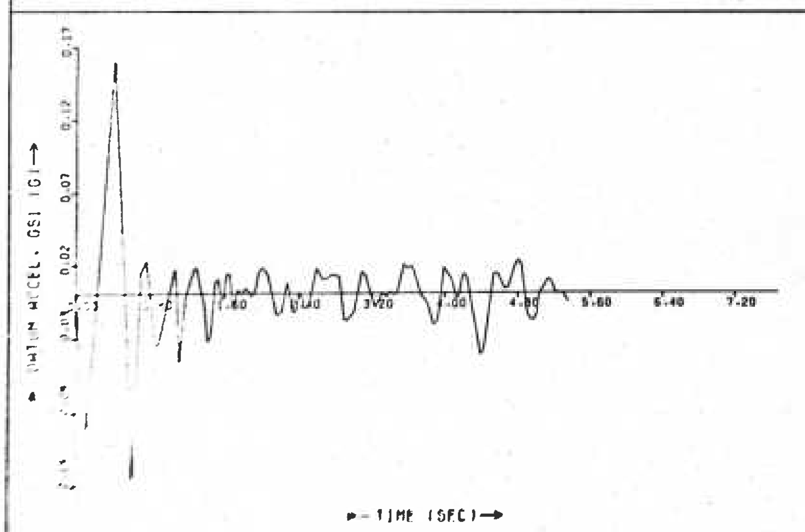
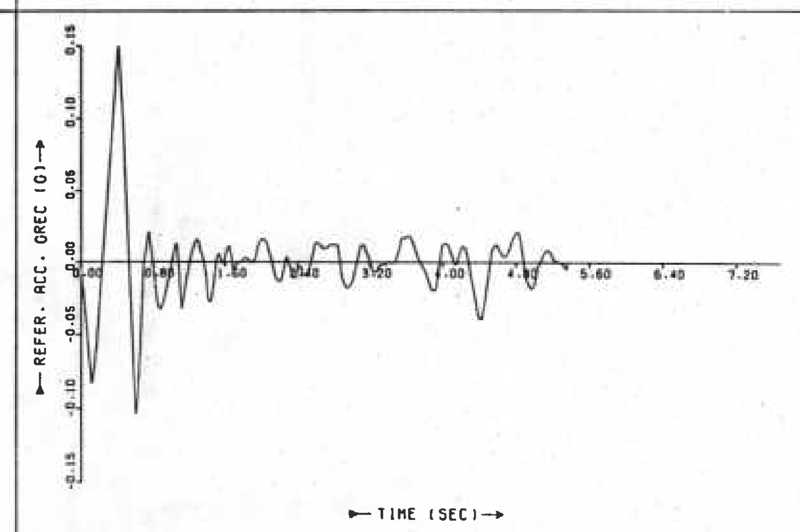


FIG. 5.9

CASE 8

LAYER 1		LAYER 2
0.10	PERIOD (SEC)	0.50
0.10	DAMPING COEFFICIENT	0.05
0.50	IMPEDANCE RATIO	0.20





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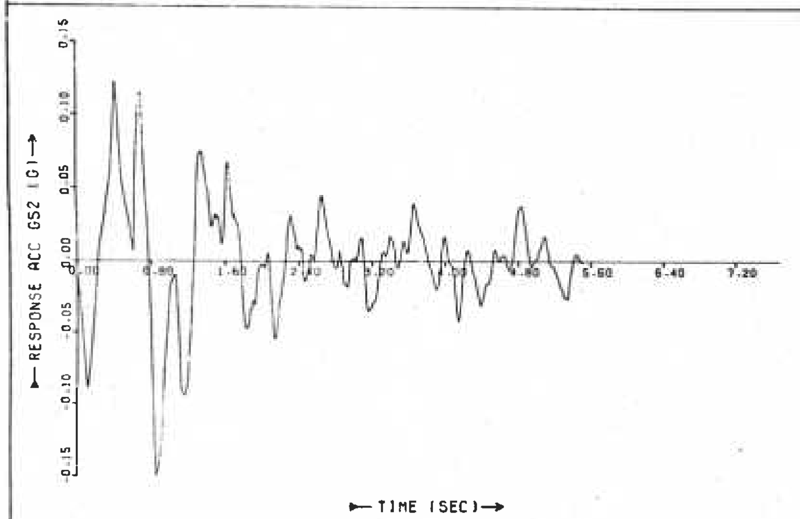
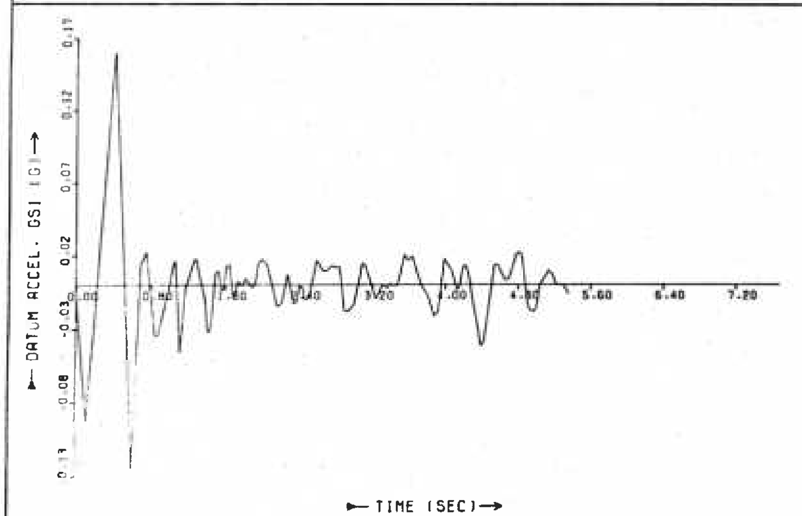
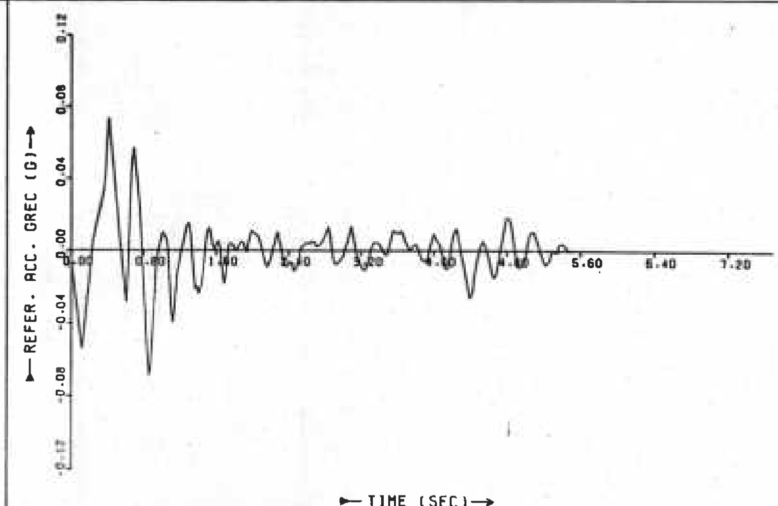


FIG. 5.10

CASE 9

LAYER 1		LAYER 2
0.50	PERIOD (SEC)	1.00
0.10	DAMPING COEFFICIENT	0.10
0.10	IMPEDANCE RATIO	0.10



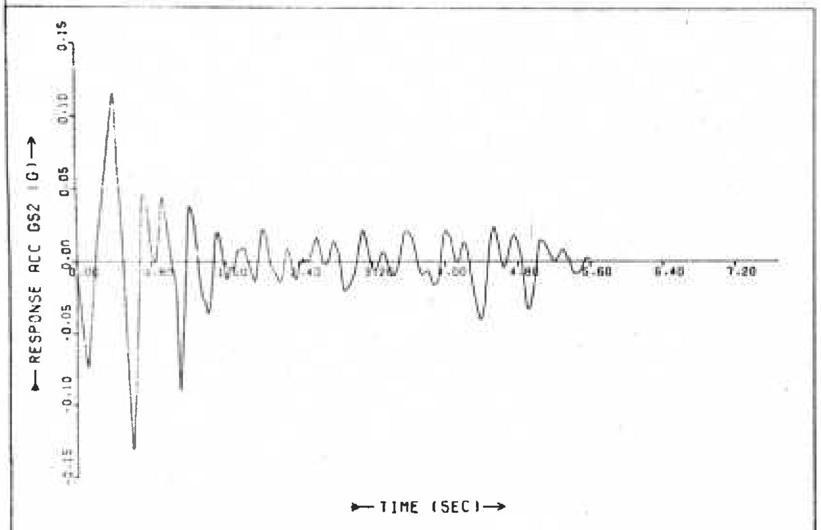


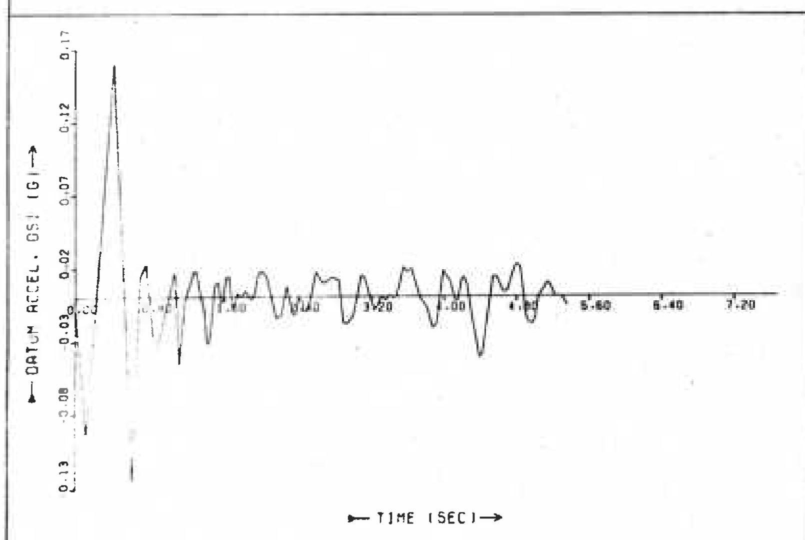
PHN/N /55/C N S PORTHUENEME 18/3/57 PAR.CORR. IC 23-

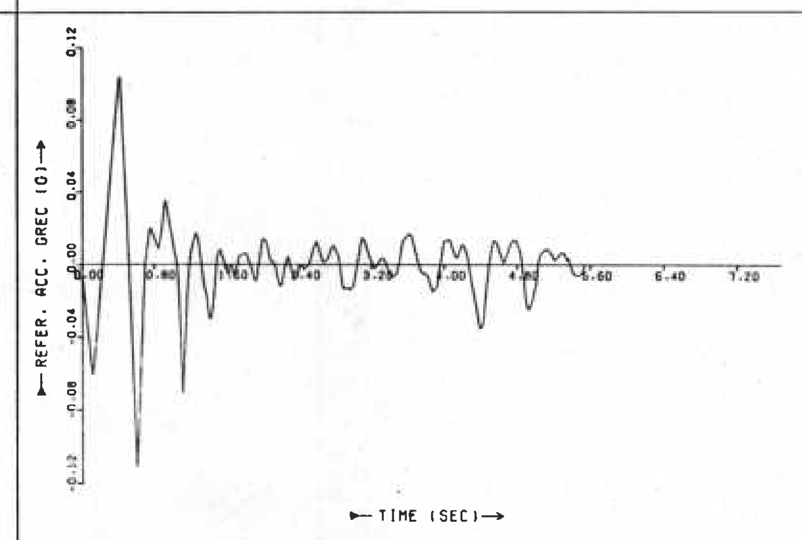
FIG. 5.11

CASE 10

LAYER 1		LAYER 2
1.00	PERIOD (SEC)	0.20
0.15	DAMPING COEFFICIENT	0.05
0.20	IMPEDANCE RATIO	0.50







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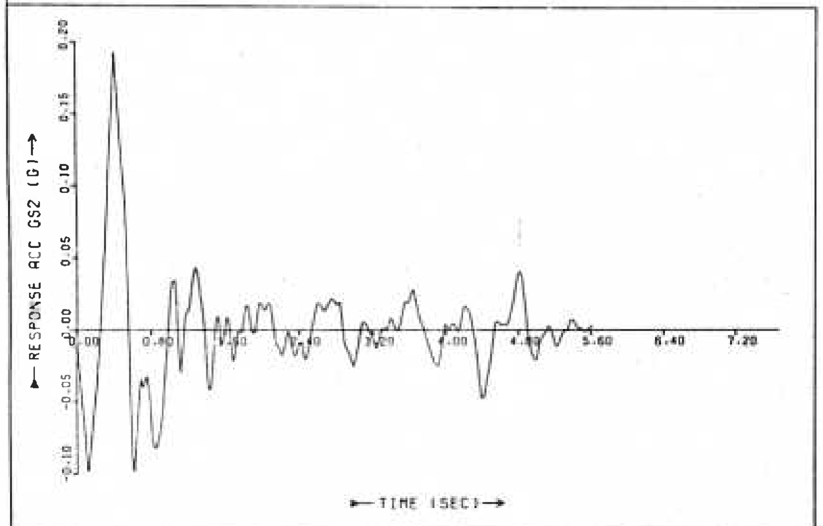
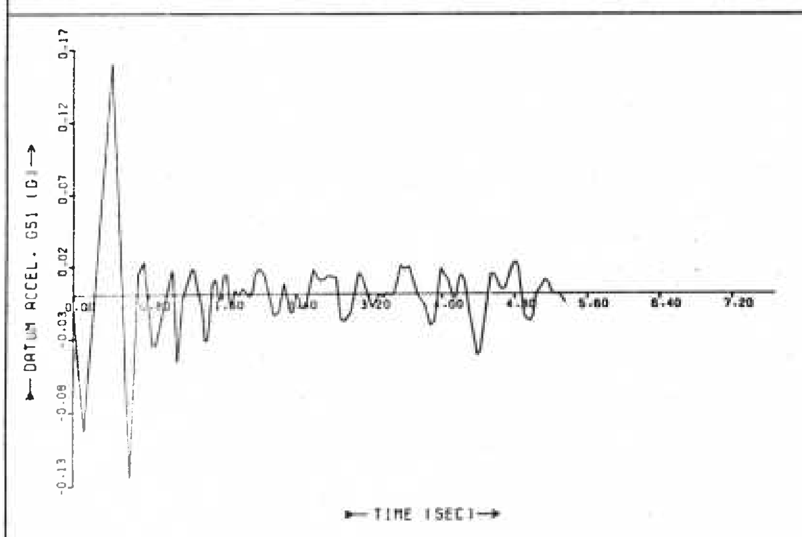
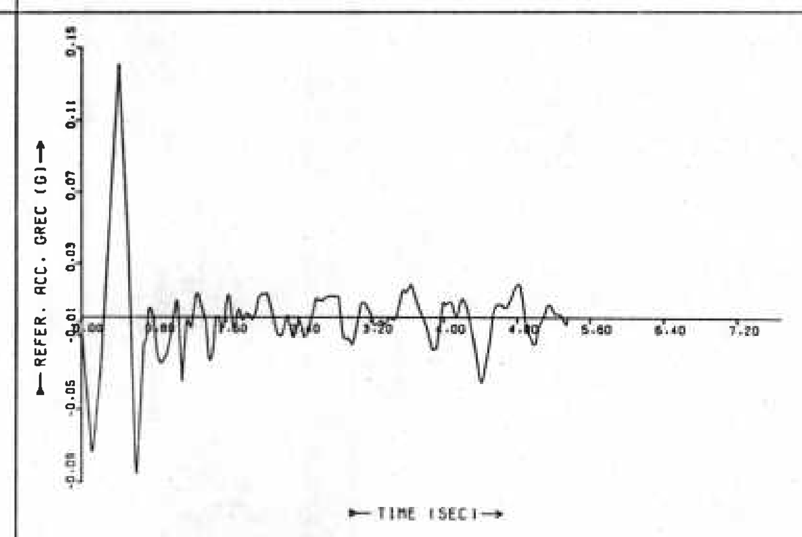


FIG. 5.12

CASE 11

LAYER 1		LAYER 2
0.20	PERIOD (SEC)	0.80
0.05	DAMPING COEFFICIENT	0.10
0.50	IMPEDANCE RATIO	0.40





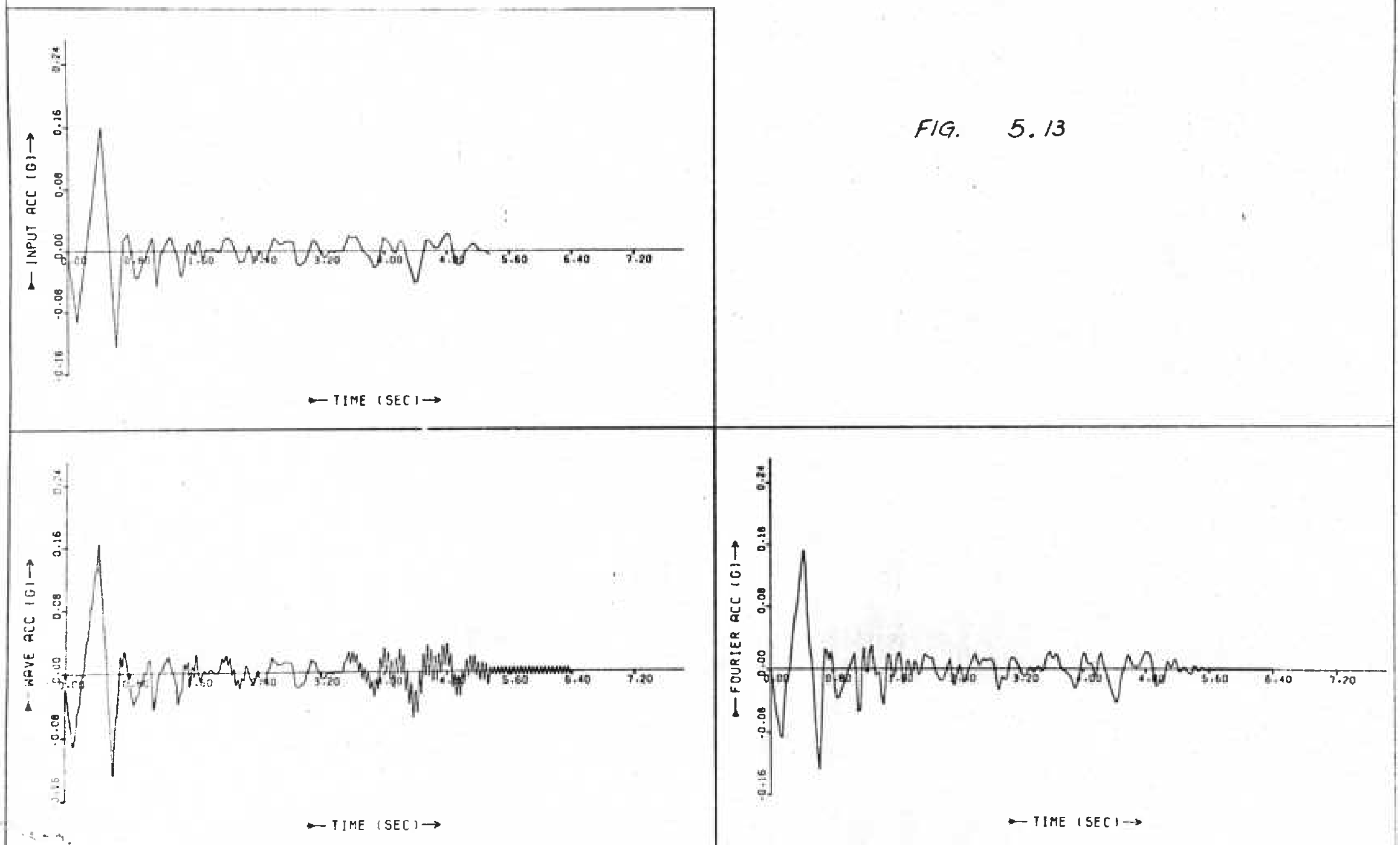

INPUT ACC (G)
TIME (SEC)
WAVE ACC (G)
TIME (SEC)
FOURIER ACC (G)
TIME (SEC)
0.00
0.80
1.60
2.40
3.20
4.00
4.80
5.60
6.40
7.20
-0.16
-0.08
0.08
0.16
0.24

FIG. 5.13

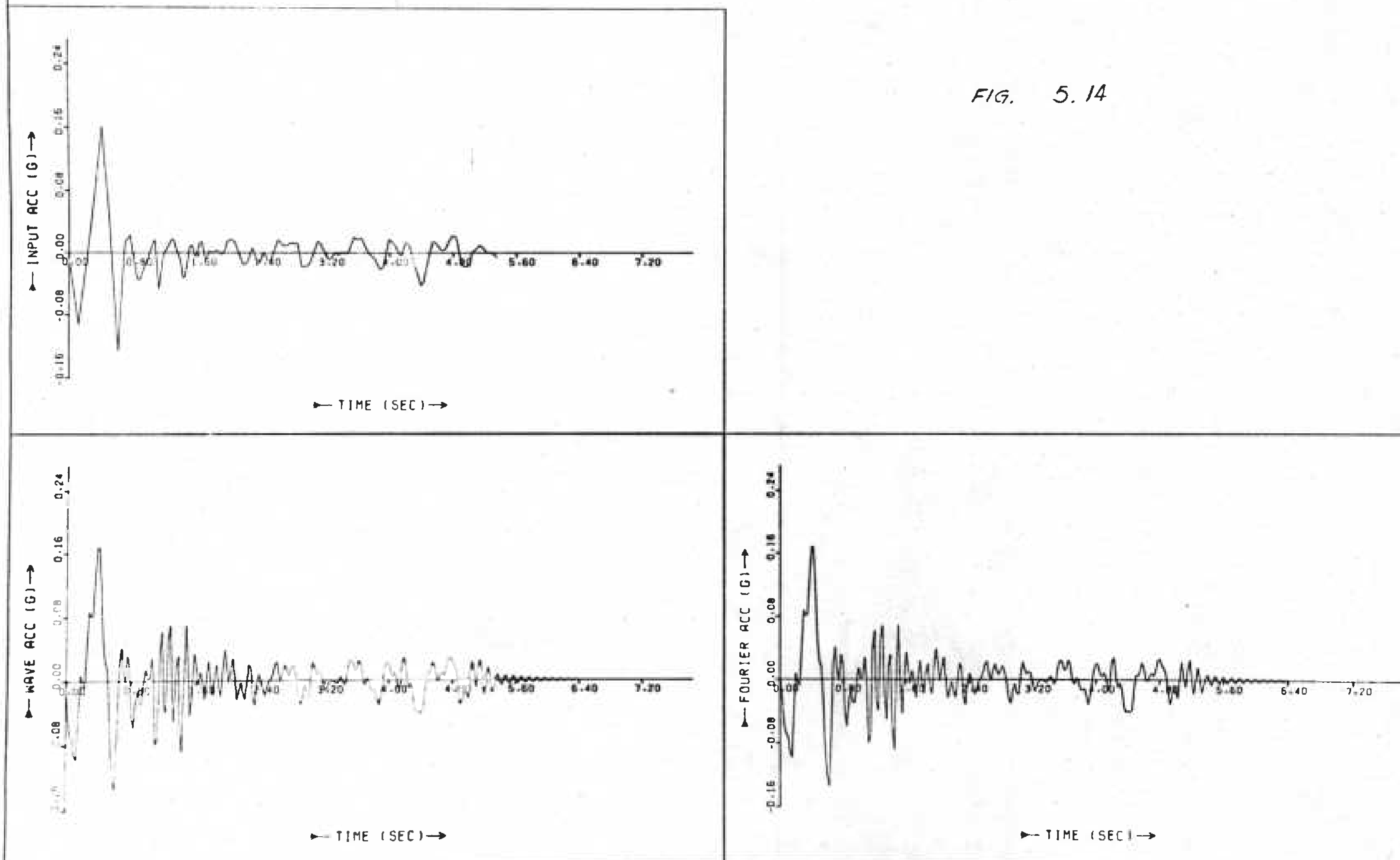
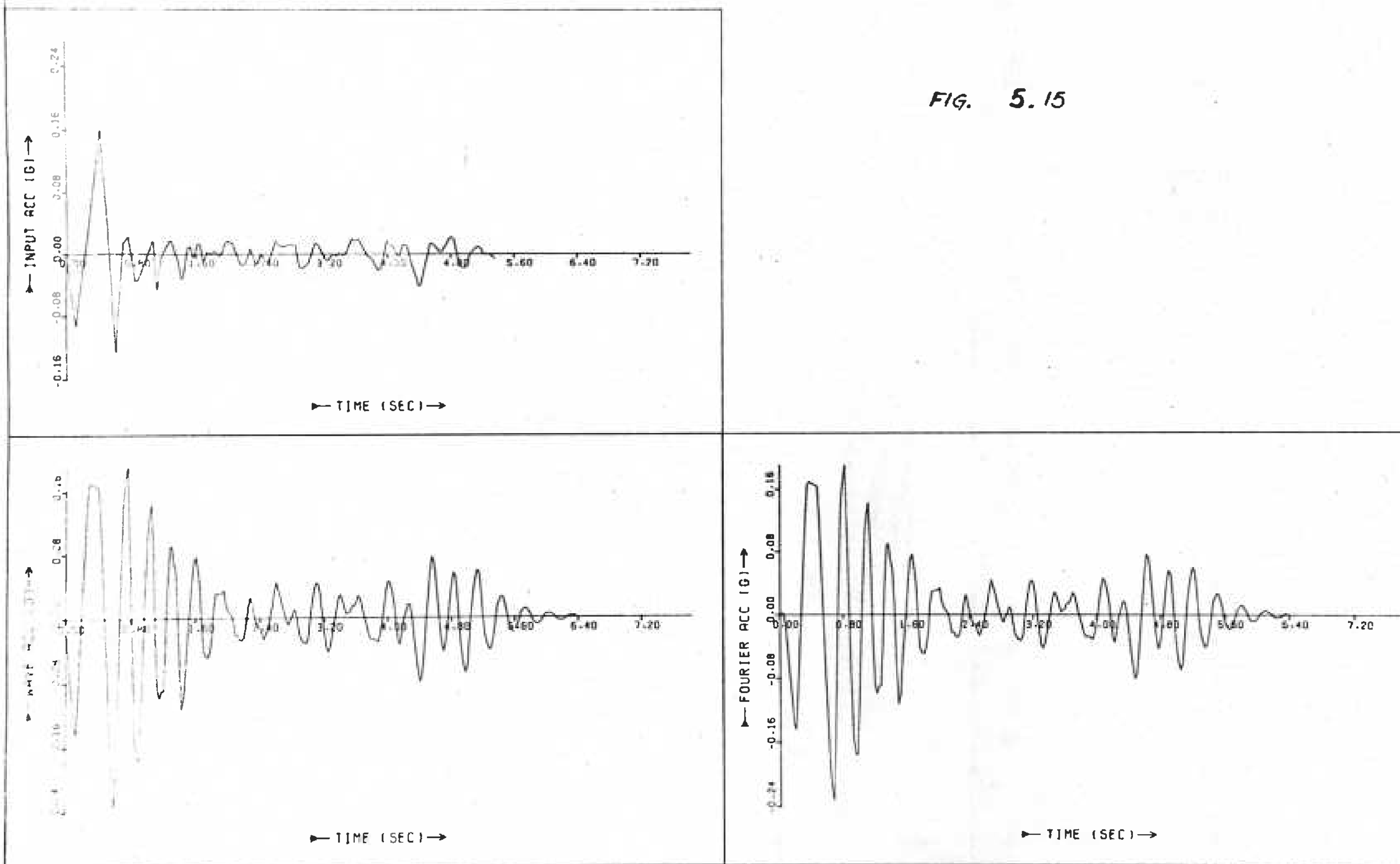

INPUT ACC (G)
TIME (SEC)
WAVE ACC (G)
TIME (SEC)
FOURIER ACC (G)
TIME (SEC)
0.24
0.16
0.08
0.00
-0.08
-0.16
5.60
6.40
7.20

FIG. 5.14



INPUT ACC (G)
0.24
0.16
0.08
0.00
-0.08
-0.16
TIME (SEC)
5.60
6.40
7.20
FOURIER ACC (G)
0.16
0.08
0.00
-0.08
-0.16
-0.24
0.00
0.80
1.60
2.40
3.20
4.00
4.80
5.60
6.40
7.20
TIME (SEC)
TIME (SEC)

FIG. 5.15

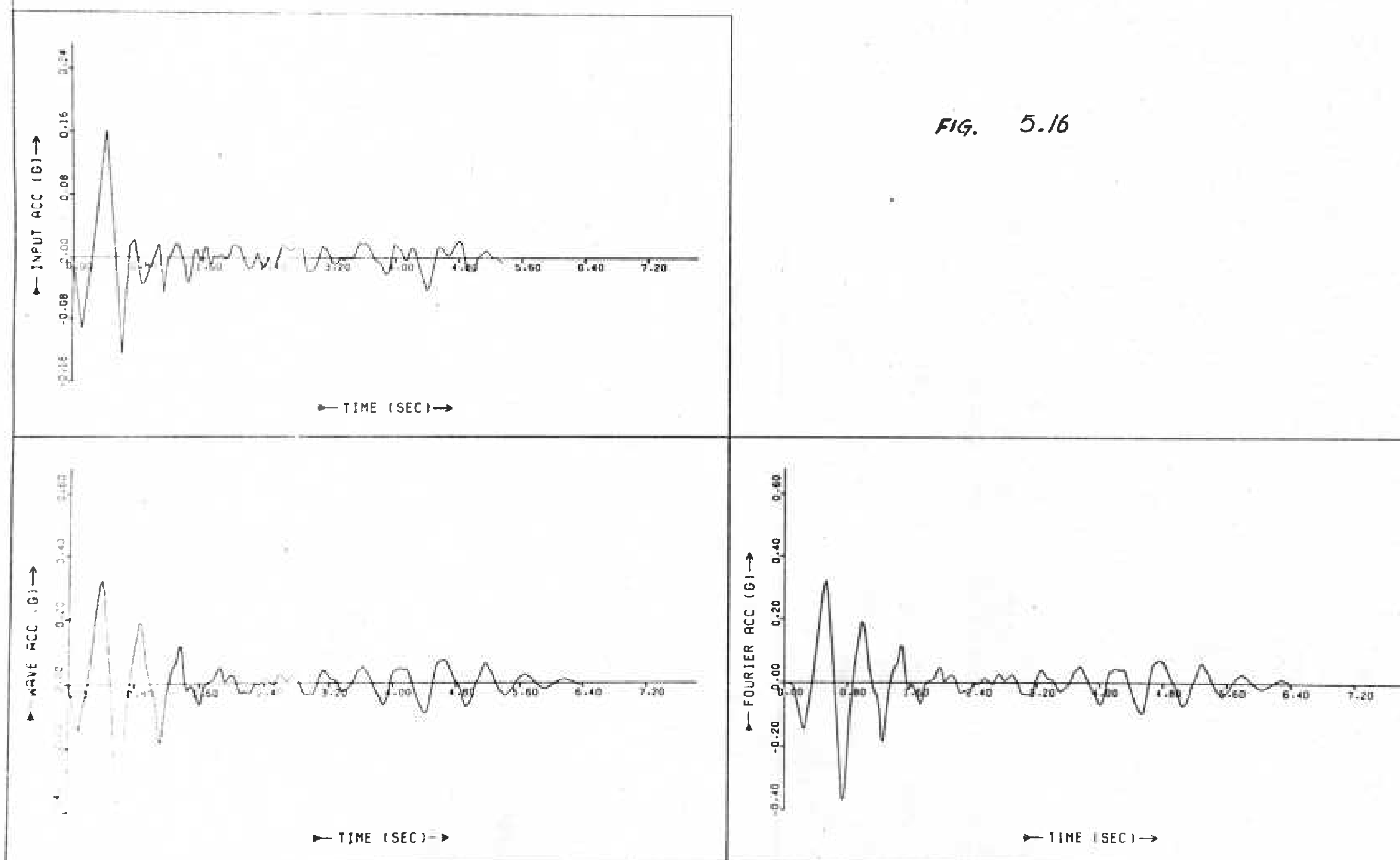


FIG. 5.16

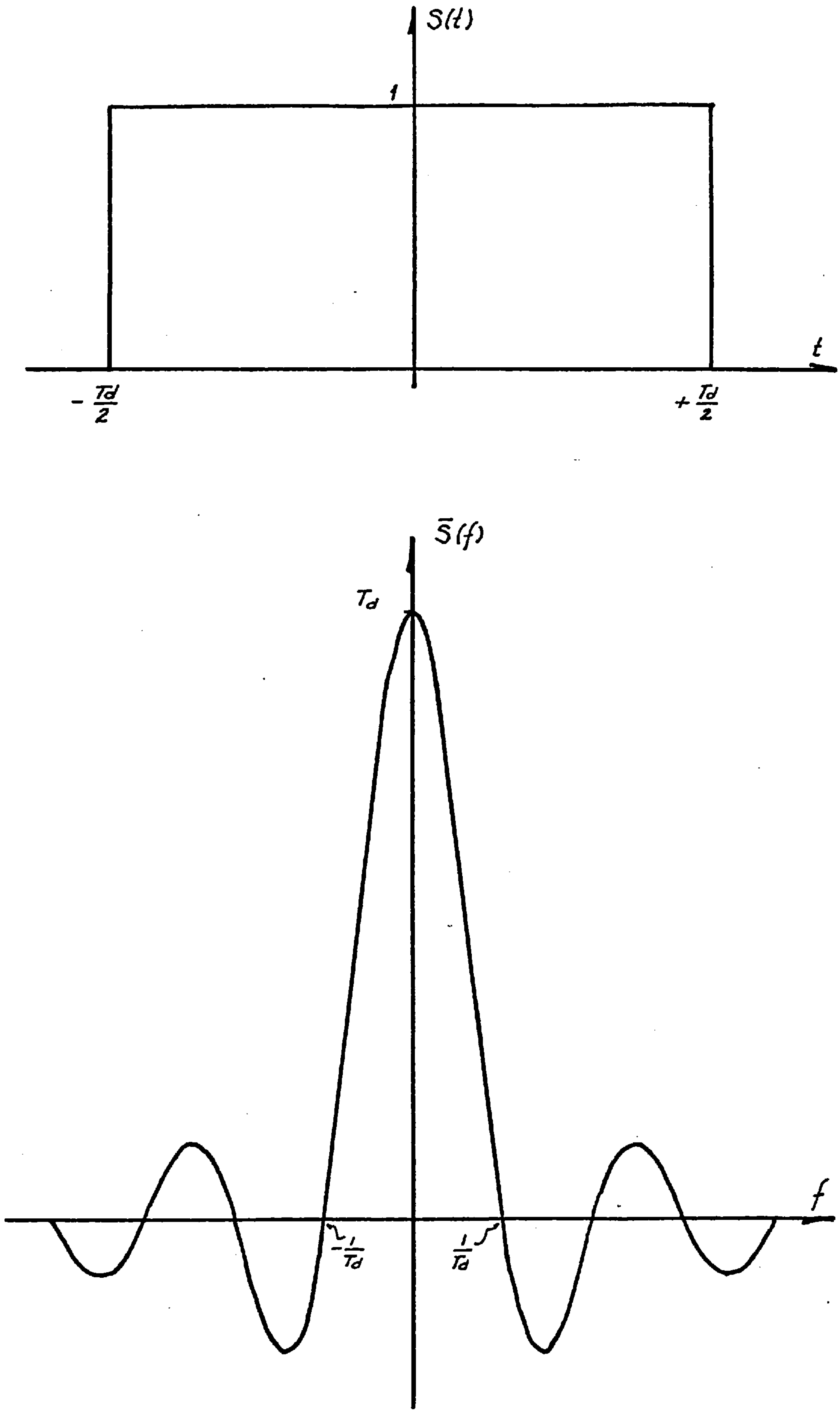


FIG. 5.17

APPENDIX 1

SOME MATHEMATICAL NOTES ON THE SOLUTIONS

The function $\overline{f}(p)$ is the Laplace transform of $f(t)$ if

$$\overline{f}(p) = \int_0^{\infty} e^{-pt} f(t)dt \tag{A1.1}$$

Among the properties of the transformation defined above which are constantly used in this thesis are:

(a) $$L\left[f^{(n)}(t)\right] = p^n\overline{f}(p)-p^{n-1}f(0)-p^{n-2}f^{(1)}(0)- \ldots f^{(n-1)}(0) \tag{A1.2}$$

where

$$f^{(n)}(t) = \frac{d^n}{dt^n} f(t)$$

and the symbol $L[\ \ ]$ denotes the Laplace Transform of the function inside the square brackets.

(b) $$L\left[f(t-b) \right] = e^{-bp} f(p) \tag{A1.3}$$

By the use of (A1.1) the partial differential equation of motion is transformed into an ordinary one, while (A1.3) enables us to avoid, for the case of wave propagation, the complicated process involved in the inversion formula of the transformation, i.e.

$$f(t) = \frac{1}{2\pi i} \int_{c-\infty i}^{c+\infty i} \overline{f}(p)\, e^{pt}\, dp \tag{A1.4}$$

With reference to the equations presented in Chapter One, applying (A1.2) the equation of motion (A-1) is transformed into

$$\frac{d^2\overline{U}(y,p)}{dy^2} - \frac{p^2}{S^2}\, \overline{U}(y,p) = 0 \tag{A1.5}$$

which has a general solution

$$\bar{U}(y,p) = A\, e^{+\frac{p}{s}y} + B\, e^{-\frac{p}{s}y} \qquad (A1.6)$$

After using the transforms of the boundary conditions it is found that

$$A = B = \frac{1}{e^{\frac{p}{s}h} + e^{-\frac{p}{s}h}}\, \bar{g}(p) \qquad (A1.7)$$

and hence,

$$\bar{U}(y,p) = \frac{e^{+\frac{p}{s}y} + e^{-\frac{p}{s}y}}{e^{+\frac{p}{s}h} + e^{-\frac{p}{s}h}}\, \bar{g}(p) \qquad (A1.8)$$

INVERSION

For an wave solution, (A1.8) may be written

$$\bar{U}(y,p) = \frac{\bar{g}(p)}{1+e^{-2\frac{p}{s}h}} \left[e^{-\frac{p}{s}(h-y)} + e^{-\frac{p}{s}(h+y)} \right] \qquad (A1.9)$$

and as

$$(1+x)^{-1} = \sum_{n=0} (-1)^n x^n$$

$$\bar{U}(y,p) = \left[e^{-\frac{p}{s}(h-y)} + e^{-\frac{p}{s}(h+y)} \right] \sum_{n=0} (-1)^n e^{-2n\frac{p}{s}h}\, \bar{g}(p) \qquad (A1.10)$$

which, using (A1.3) alone, gives in the time domain

$$U(y,t) = W\left[t-(h-y)/S\right] + W\left[t-(h+y)/S\right] \qquad (A1.11)$$

where

$$W(t) = \sum_{n=0} (-1)^n g(t-2n\frac{h}{s}) \qquad (A1.12)$$

For a modal solution, the inversion formula (A1.4) is used to obtain U(y,t). The right hand side of that equation equals the sum of residues at the poles of the expression to be inverted multiplied by e^{pt}.

Equation (A1.8) may be written, in terms of an acceleration disturbance, as

$$\overline{U}(y,p) = \left[\frac{1}{p^2} \frac{\cosh(py/s)}{\cosh(ph/s)} \right] \overline{\ddot{g}}(p) \qquad \text{(A1.13)}$$

which may be seen as the product of a function $\overline{U}_1(y,p)$ enclosed in the square bracket, and the transform of the acceleration disturbance. The displacement U(y,t) may thus be obtained by convolution of $\ddot{g}(t)$ and the inverse of $\overline{U}_1(y,p)$ which is found using the inversion formula.

Hence, for $\overline{U}_1(y,p)$ its poles are found to be at

$$p = 0 \qquad \text{(A1.14)}$$

and $\cosh \frac{p}{s} h = 0$; i.e.

$$p = \pm (2n+1) \frac{\pi}{2} \frac{s}{h} i \qquad n=0,1,2,\ldots \qquad \text{(A1.15)}$$

The residue at p=0, a double pole, is found from the expression

$$\text{Res}_{(p=0)} = \frac{1}{1!} \lim_{p \to 0} \frac{d}{dp} \overline{U}_1(y,p)\, e^{pt}$$

and it may be shown that

$$\text{Res}_{(p=0)} = t \qquad t > 0 \qquad \text{(A1.16)}$$

The residues at the single poles in (A1.15) may be evaluated with the formula

$$\text{Res}_{(p=p_o)} = \frac{P(p_o)\, e^{p_o t}}{\left. \frac{d}{dp} Q(p) \right]_{p=p_o}} \qquad \text{(A1.17)}$$

where P(p) and Q(p) are the numerator and denominator of $\overline{U}_1(y,p)$ respectively. Adding the residues found from (A1.17) to (A1.16), it may be shown that

$$U_1(y,t) = t - \frac{2s}{h} \sum_{n=0} (-1)^n \frac{\cos(w_n y/s)}{w_n^2} \sin w_n t \qquad \text{(A1.18)}$$

where $\quad W_n = (2n+1)\,\frac{\pi}{2}\,\frac{s}{h}$ (A1.19)

$n=0,1,2,\ldots$

Finally, convolution of $U_1(y,t)$ with $\ddot{g}(t)$ gives the displacement $U_1(y,t)$, i.e.

$$U(y,t) = g(t) - \frac{2s}{h}\sum_{n=0}^{\infty}(-1)^n\,\frac{\cos(W_n\,y/s)}{W_n^2}\int_0^t \sin W_n(t-\tau)\,g(\tau)\,d\tau \qquad \text{(A1.20)}$$

The series

$$S_1(y,t) = \frac{2h}{s\pi^2}\sum_{n=0}^{\infty}(-1)^n\,\frac{\cos\left[(2n+1)\frac{\pi}{2}\,y/h\right]\,\sin\left[(2n+1)\pi/2\,\frac{s}{h}\,t\right]}{\frac{1}{4}(2n+1)^2}$$

may be written

$$S_1(y,t) = \frac{4h}{s\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^n\sin\left[(2n+1)\frac{\pi}{h}(y+st)\right]}{(2n+1)^2} - \frac{4h}{s\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^n\sin\left[(2n+1)\frac{\pi}{h}(y-st)\right]}{(2n+1)^2}$$

(A1.21)

The two series in (A1.21) are the Fourier series representation of the functions $F_1(t)$ and $F_2(t)$ shown in Figure (1.4). To prove it, consider first the fact that both F_1 and F_2 are periodic functions, and that in a cycle they are defined as

$$F_{1,2}(\tau) = \begin{cases} \frac{1}{2}\tau & -\frac{\pi}{2} < \tau < \frac{\pi}{2} \\ -\frac{1}{2}(\tau-\pi) & \frac{\pi}{2} < \tau < \frac{3\pi}{2} \end{cases} \qquad \text{(A1.22)}$$

where $\quad \tau = \frac{y}{s} \pm t$

and $\quad T = 2\pi = \frac{4h}{s}$, the period of the functions.

Consider now the Fourier series of $F_{1,2}(\tau)$; i.e.

$$F_{1,2}(\tau) = a_o + \sum_{m=1}^{\infty}\left[\,a_m\cos W_m\tau + b_m\sin W_m\tau\,\right] \qquad \text{(A1.23)}$$

where $\quad W_m = mW = m\,\frac{2\pi}{T}$

As the function in (A1.22) is an odd function; i.e.

$$F_{1,2}(-\tau) = -F_{1,2}(\tau)$$

it follows that for all m:

$$a_m = 0$$

Also, $F_{1,2}(\tau)$ has half-wave symmetry; hence, for all even m:

$$b_m = 0$$

For an odd m:

$$b_m = \frac{1}{\pi}\int_T F_{1,2}(\tau) \sin W_m\tau \, d\tau = \frac{2}{\pi m^2} \sin m\frac{\pi}{2}$$

$$m=1,3,5,\ldots.$$

or, considering the value of sin $m\pi/2$

$$b_m = \frac{2}{\pi m^2} (-1)^{(m-1)/2} \tag{A1.24}$$

Hence,

$$F_{1,2}(\tau) = \frac{2}{\pi} \sum_{m=1}^{\infty} (-1)^{(m-1)/2} \frac{\sin W_m \tau}{m^2} \tag{A1.25}$$

$$m=1,3,5,7,\ldots.$$

Finally, if the substitution $n = \frac{m-1}{2}$ is made, and considering the relationship between π, W, and $\frac{4h}{s}$, and after some manipulations, it is found that

$$F_{1,2}(\tau) = \frac{4h}{s\pi^2} \sum_{n=0} \frac{(-1)^n \sin\left[(2n+1)\frac{\pi\tau}{h}\right]}{(2n+1)^2} \tag{A1.26}$$

which proves our initial statement regarding the series in (A1.21).

Now, as the Laplace transform is defined for positive t only, it should be understood that the series in (A1.21) are each multiplied by a unit step function H(t), defined as

$$H(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

and hence, for a particular y, the functions $F_1(t)$ and $F_2(t)$ should be considered with the time origin as it is indicated in Figure (1.4)

The ramp function R(t) is defined

$$R(t) = \begin{cases} 0 & t < 0 \\ t & t \geqslant 0 \end{cases} \qquad \text{(A1.27)}$$

and it may be shown that

$$\int_0^t R(t-\tau)f(\tau)\, d = \int_0^t \int_0^\lambda f(\tau)d\tau\, d\lambda \qquad \text{(A1.28)}$$

also,

$$\frac{d}{dt} R(t) = H(t)$$

hence, $\int_0^t H(t-\tau)f(\tau)\, d\tau = \int_0^t f(\tau)\, d\tau$ (A1.29)

Furthermore,

$$\frac{d}{dt} H(t) = \delta(t)$$

and $\int_0^t \delta(t-\tau)f(\tau)\, d\tau = f(\tau)$ (A1.30)

APPENDIX 2

NUMERICAL COMPUTATION OF THE LAYER WAVE

The computation of the layer wave, and hence of the layer response can be carried out in a fast and simple way when the corresponding disturbance is given as a set of values equally spaced in time.

In general, the expression for the layer wave, for a disturbance g(t), is of the form:

$$W_g(t_o) = k_1 \sum_{n=0} (-1)^n k_2^n \; g(t-2nk_3) \qquad \text{(A2.1)}$$

where k_1, k_2 and k_3 are constants.

Let g(t) be a set of N points g_i, such that

$$g_i = g\left[(i-1) \times \Delta t\right] \qquad i=1,2,\ldots N \qquad \text{(A2.2)}$$

where Δt is the time interval between two of those points.

The disturbance g(t) may be divided into NS segments of NPS points each, as it is shown schematically in Figure (A2.1). NS and NPS are such that

$$NPS = \frac{2k_3}{\Delta t} \qquad \text{(A2.3)}$$

and $$NS \times NPS = N \qquad \text{(A2.4)}$$

The layer wave may now be computed, also in segments of NPS points as it is shown in Figure (A2.2). The first of these segments, SW_1, is obtained by multiplying the first segment of g(t), S_1, by the constant k_1. SW_2 is then computed as

$$SW_2 = k_1 \, S_2 - k_2 \, SW_1 \qquad \text{(A2.5)}$$

and in general,

$$SW_j = k_1 \, S_j - k_2 \, SW_{j-1} \qquad \text{(A2.6)}$$

For j > NS; i.e., once the disturbance has ceased,

$$SW_j = - k_2 \, SW_{j-1} \qquad \text{(A2.7)}$$

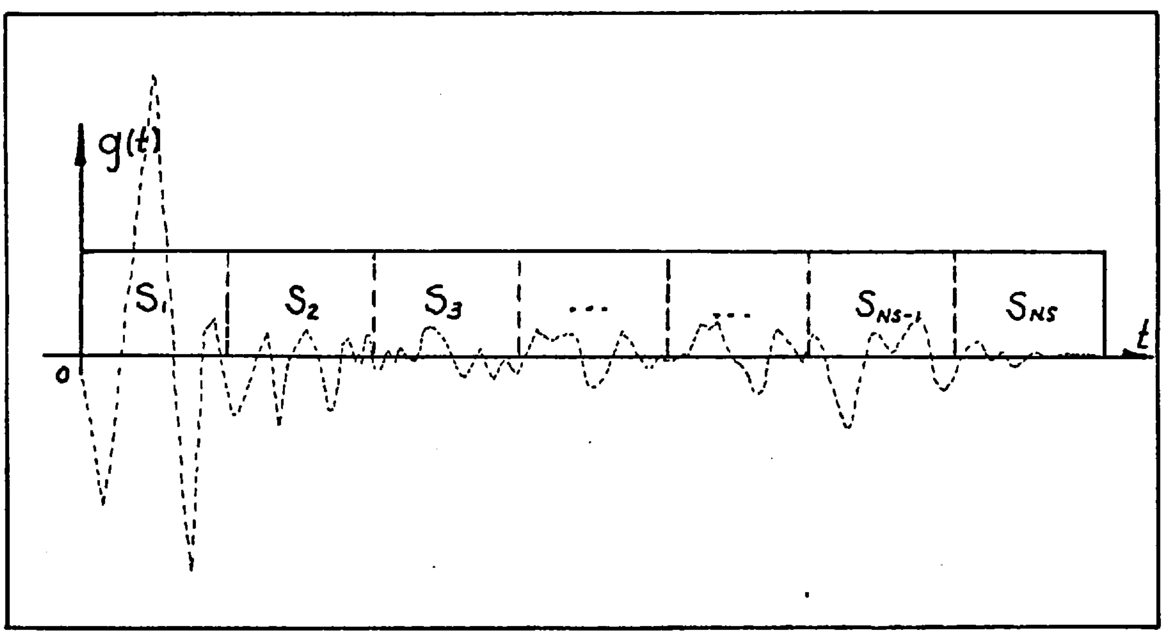


FIG. A2.1

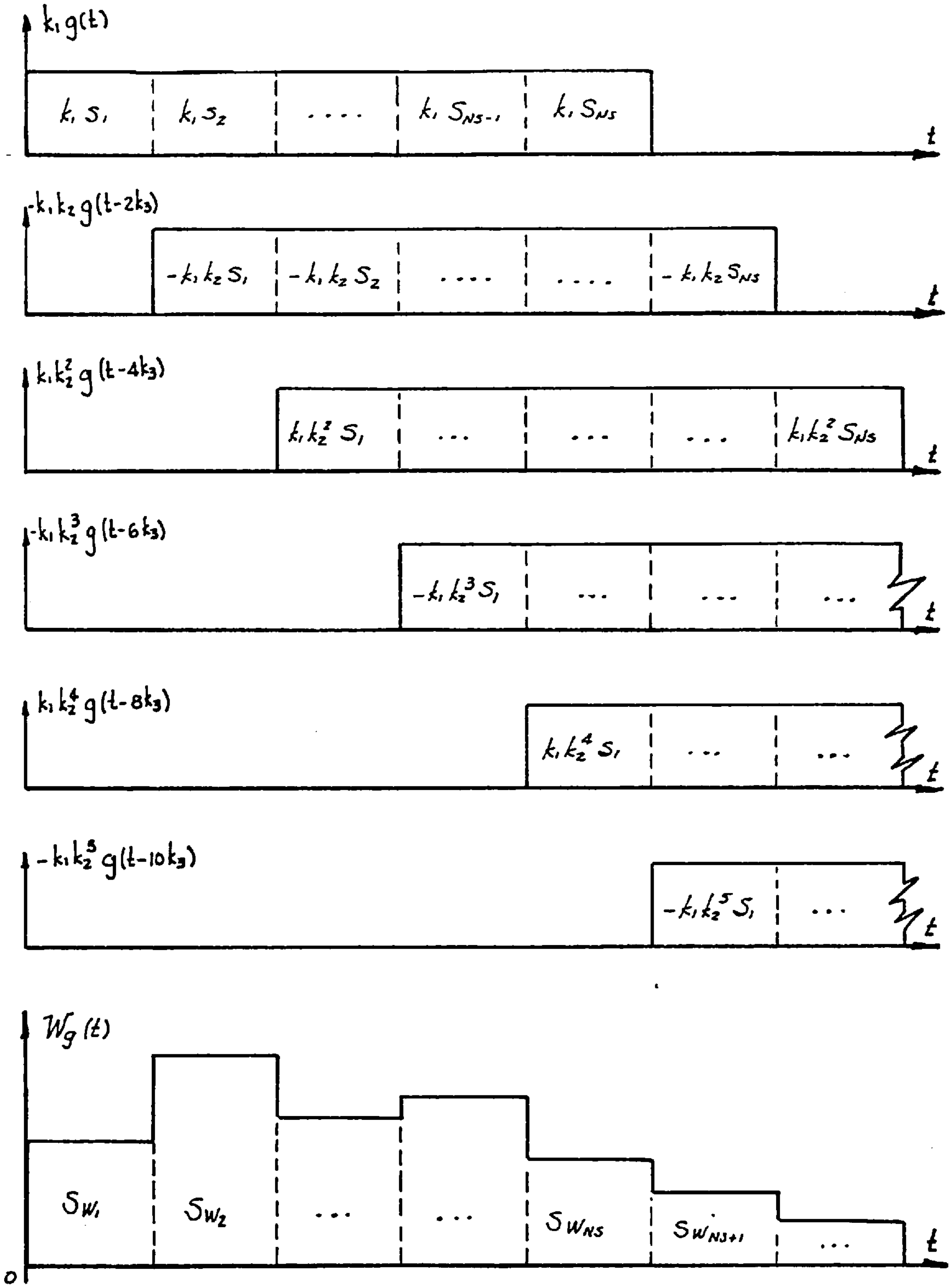

$k_1 g(t)$
$k_1 S_1$
$k_1 S_2$
....
$k_1 S_{NS-1}$
$k_1 S_{NS}$
t
$-k_1 k_2 g(t-2k_3)$
$-k_1 k_2 S_1$
$-k_1 k_2 S_2$
$-k_1 k_2 S_{NS}$
$k_1 k_2^2 g(t-4k_3)$
$k_1 k_2^2 S_1$
$k_1 k_2^2 S_{NS}$
$-k_1 k_2^3 g(t-6k_3)$
$-k_1 k_2^3 S_1$
$k_1 k_2^4 g(t-8k_3)$
$k_1 k_2^4 S_1$
$-k_1 k_2^5 g(t-10k_3)$
$-k_1 k_2^5 S_1$
$W_g(t)$
S_{W_1}
S_{W_2}
$S_{W_{NS}}$
$S_{W_{NS+1}}$
0

FIG. (A2.2)

APPENDIX 3

SOME ANALYTICAL MODELS CONSIDERING DAMPING

Voigt's Solid

The equation of motion

$$\rho \ddot{U}(y,t) = G\, U''(y,t) + \bar{\eta}\, \dot{U}''(y,t) \qquad \text{(A3.1)}$$

is derived from considering a constitutive equation for the material in the layer given by

$$\tau = \left[G + \bar{\eta} \frac{\partial}{\partial t} \right] U'(y,t) \qquad \text{(A3.2)}$$

which corresponds to a Voigt's solid (Kolsky, 1963) where the increase in strength during dynamic loading is assumed to be proportional to the time rate of deformation.

Considering the same reference system which has been used throughout the thesis, the general expression for the layer displacement in the domain of the Laplace Transform may be shown to be

$$\bar{U}(y,p) = A\, e^{+\frac{py}{s}\left(\frac{1}{\sqrt{1+\bar{\lambda}p}}\right)} + B\, e^{-\frac{py}{s}\left(\frac{1}{\sqrt{1+\bar{\lambda}p}}\right)} \qquad \text{(A3.3)}$$

where $s^2 = \dfrac{G}{\rho}$, and $\bar{\lambda} = \dfrac{\bar{\eta}}{G}$

A and B are functions of p alone.

If no further assumptions are made on the nature of the material constants, the general solution of (A3.3) in the time domain does not have a D'Alambert form, as the presence of p in the radicals prevents the use of the Shifting Theorem. The equation of motion (A3.1) is thus not a wave equation.

Papastamatiou (1971) gives a modal solution for equation (A3.1) and Kanai (1951) provides the solution of (A3.1) for steady state motion. Schnabel et al. (1972) present a Fourier analysis to evaluate the layer response to an arbitrary motion, based on Kanai's solution and assuming a complex shear modulus independent of frequency.

For this additional consideration, an equivalent wave solution may be obtained and therefore the complexity of a Fourier analysis may be avoided. Substituting iw for p in (A3.3), and then making $\bar{\lambda}w$ equal to 2β, it is obtained that

$$U(y,iw) = A\, e^{+\frac{iwy}{s}\left(\frac{1}{\sqrt{1+2i\beta}}\right)} + B\, e^{-\frac{iwy}{s}\left(\frac{1}{\sqrt{1+2i\beta}}\right)} \qquad (A3.4)$$

where β is the critical damping ratio considered in the previous reference. For the conditions that y=0 is a free surface and that at y=h the motion is $\bar{g}(iw)$, we arrive at

$$\bar{U}(y,iw) = \frac{e^{-\frac{iw}{s}(a+bi)(h-y)} + e^{-\frac{iw}{s}(a+bi)(h+y)}}{1 + e^{-\frac{2w}{s}hb}\, e^{-\frac{2iwh}{s}a}}\; \bar{g}(iw) \qquad (A3.5)$$

where a and b are the real and imaginary parts of $(1+2i\beta)^{-1/2}$, respectively.

The expression for the layer wave in the time domain is related to the inverse transform of the denominator in (A3.5). This denominator may be written

$$D(iw) = 1 + e^{-\pi b}\; e^{-\frac{2iwh}{s}a} \qquad (A3.6)$$

in which the relationship

$$w = \frac{2\pi}{T} = \frac{2\pi}{4h}\, s$$

was considered to obtain the exponent $-\pi b$.

Substituting now p for iw in (A3.6), we get

$$D(p) = 1 + e^{-\pi b}\; e^{-\frac{2ph}{s}a} \qquad (A3.7)$$

A comparison of the previous expression with equation (3.7) in the main body of the thesis, once the relation given in equation (3.24) is considered, indicates that an equivalent wave solution for Schnabel's model may be obtained if the damping coefficient ξ is made equal to

$$\xi = \frac{k}{\sqrt{1+k^2}} = \frac{a}{b} \tag{A3.8}$$

VISCOUS DAMPING PROPORTIONAL EITHER TO THE ABSOLUTE OR TO THE RELATIVE PARTICLE VELOCITY.

The equations of motion:

$$\ddot{U}(y,t) + k\,\dot{U}(y,t) = S^2\,U''(y,t) \tag{A3.9}$$

and

$$\ddot{U}(y,t) + k_1\left[\,\dot{U}(y,t) - \dot{g}(t)\,\right] = S^2\,U''(y,t) \tag{A3.10}$$

Consider the effect of damping to be proportional either to the absolute particle velocity, (A3.9), or to the velocity relative to the base motion g(t), (A3.10). Modal solutions may be obtained for both equations, but none of them is a wave equation. This may be seen from the general expressions for the layer displacement in the transformed domain, which may be shown to be

$$\bar{U}(y,p) = A\,e^{+\frac{y}{s}\sqrt{p^2+kp}} + B\,e^{-\frac{y}{s}\sqrt{p^2+kp}} \tag{A3.11}$$

corresponding to (A3.9), and

$$\bar{U}(y,p) = A\,e^{+\frac{y}{s}\sqrt{p^2+k_1p}} + B\,e^{-\frac{y}{s}\sqrt{p^2+k_1p}} + \frac{k_1S^2}{p+k_1}\,\bar{g}(p) \tag{A3.12}$$

for equation (A3.10).

The radicals in the previous expressions are functions of the parameter p of the transform, and thus, the shifting theorem is not applicable.

APPENDIX 4.

Frequency Analysis Solution to the Wave Equation

In a linear system such as that of a layer whose equation of motion is an wave equation, the total response of the system to a series of excitations is equal to the sum of responses to each particular excitation acting independently. This property of a linear system is used in frequency analysis.

Consider a disturbance g(t), which in an interval $0 \leq t \leq T_d$, as it is well known, may be represented as a series of harmonics, i.e.

$$g(t) = \sum_{n=1} a_n e^{+iw_n t} + b_n e^{-iw_n t} \qquad 0 \leq t \leq T_d \tag{A4.1}$$

where $$w_n = 2n \frac{\pi}{T_d} \tag{A4.2}$$

The response of a layer to an harmonic motion of frequency w_n may be shown to be

$$U(y,w_n t) = \phi(y,w_n) \left[a_n e^{+iw_n t} + b_n e^{-iw_n t} \right] \tag{A4.3}$$

where $$\phi(y,w_n) = A e^{+iw_n k y} + B e^{-iw_n k y} \tag{A4.4}$$

where k depends upon the material properties of the layer, and A and B upon the boundary conditions.

It follows, from the previous equation that the total response of the layer to the disturbance g(t), within the same interval in which it is defined, is given by

$$U(y,w_t) = \sum_{n=1} \phi(y,w_n) \left[a_n e^{+iw_n t} + B_n e^{-iw_n t} \right] \qquad 0 \leq t \leq T_d \tag{A4.5}$$

In a more rigorous formulation a frequency analysis solution should be seen as the result of using the Fourier transform to solve the differential equation of motion as follows:

The relationship between the input disturbance g(t) and the layer response may be expressed as

$$U(y,t) = \int_{-\infty}^{+\infty} h(\tau)\ g(t-\tau)\,d\tau \tag{A4.6}$$

where h(t) is the response of the layer to an impulse function $\delta(t)$.

The frequency response of the layer is given by the Fourier transform of h(t) (Trifunac & Udwa'dia, 1972), that is

$$\overline{h}(f) = \int_{-\infty}^{+\infty} h(t)\ e^{-2\pi ift}\ dt$$

Hence, if the Fourier transform of (A4.6) is taken, it follows that

$$\overline{U}(y,f) = \overline{h}\ (f)\ \overline{g}\ (f) \tag{A4.7}$$

where

$$\overline{g}(f) = \int_{-\infty}^{+\infty} g(t)\ e^{-2\pi ift}\ dt$$

APPENDIX 5

EARTHQUAKE LAYER SPECTRA

The use of maximum ground acceleration or standard response spectra for design purposes is not always recommended as, according to field evidence, both are far from being a good measure of the damaging potential of an earthquake (Ambraseys 1975). A more sensible approach to the design of important structures, as it is suggested in the previous reference, would be to consider a suitable number of real or artificial time-histories selected in accordance with the source parameters of the design earthquake, and then make a full dynamic analysis.

We may think, however, of response spectra as indexes which provide a general idea and a rough estimate (usually over-conservative in the case of accelerations) of the response of actual structures. For the case of a soil deposit, this estimate is too scant, in view that even for the simplest possible situation, namely, an homogeneous layer, its motion may be seen as the response of various simple oscillators combined rather than of only one of them.

We may thus take the response of this simplest case as a more indicative index of the behaviour of a deposit. Hence, instead of considering a standard response spectrum, we may think of a layer response spectrum or simply layer spectrum, which, say for acceleration, may be defined as

$$S_{AL}(T_L)_{\lambda,\alpha,\xi} = \max \left| \ddot{U}(y,t)_{T_L,\alpha,\xi} \right|$$

where

$\ddot{U}(y,t)_{T_L,\alpha,\xi}$: Acceleration time-history at depth y of an homogeneous layer of period T_L, impedance ratio α, and damping coefficient ξ.

$S_{AL}(T_L)_{\lambda,\alpha,\xi}$	the corresponding layer spectrum
$\lambda = \frac{4y}{ST_L} = \frac{y}{H}$	relative depth
S	shear wave velocity
H	layer thickness

The expedience in the computation of the layer response using a time-history approach makes the evaluation of layer spectra feasible, and even more important, the values which are obtained using this approach are the actual maxima of the response for the corresponding layer characteristics. Hence, an immediate application for these spectra would be to check the accuracy of maximum values obtained from a simplified modal analysis (equation 1.31).

Also, in the case of an earthquake recorded at the outcrop of a rock formation, as this record, under the assumptions of this thesis, represents the input earthquake disturbance for any deposit founded on that formation, layer spectra computed for $\lambda = 0$ would give us the maximum ground response for that particular record. An average of the values obtained for different events recorded at the same location would provide the expected maximum ground response.

Again we insist on the index nature that should be given to these values, both layer spectra and expected maxima, and on their limitations for design.

Acceleration layer spectra have been computed for six actual earthquake records, which are illustrated in Figures (A5.1) to (A5.12) followed by their standard response spectra. Layer spectra are shown in Figures (A5.13) to (A5.24). The name of the record taken as reference motion and the depth at which maxima were computed are indicated in the upper part of the figures.

Four graphs, each one corresponding to one damping coefficient are shown in each figure. The six curves in each graph are the acceleration layer spectra (in percent of g) computed for values of impedance ratio α, from 0.0 to 0.5 in steps of 0.1. The upper curve in each graph obviously corresponds to zero impedance.

S 16 E COMP OF SAN FERNANDO EARTHQUAKE 9/2/71 SS IC

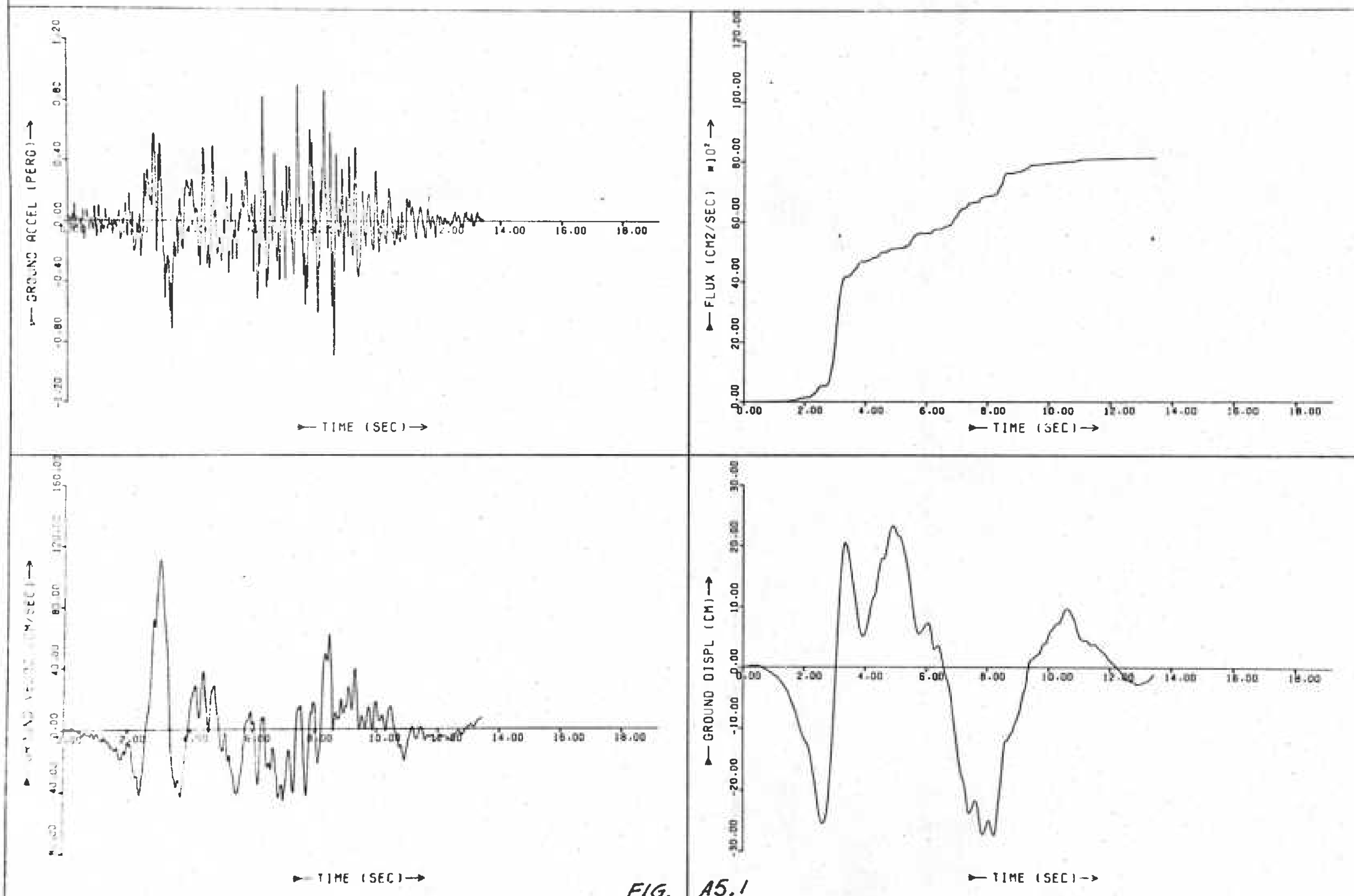


FIG. A5.1

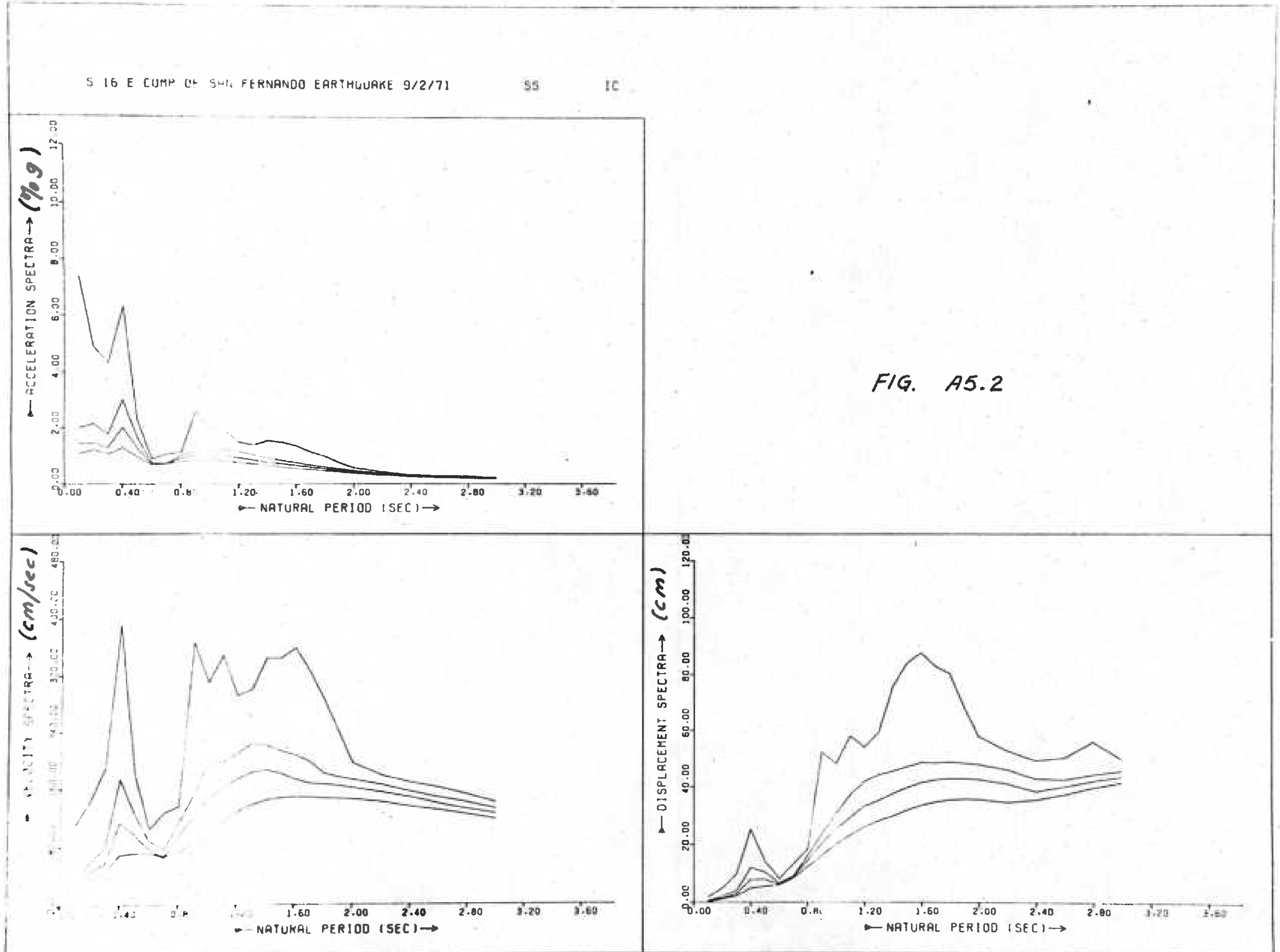

S 16 E COMP OF SAN FERNANDO EARTHQUAKE 9/2/71
SS
IC
ACCELERATION SPECTRA (%g)
NATURAL PERIOD (SEC)
VELOCITY SPECTRA (cm/sec)
NATURAL PERIOD (SEC)
DISPLACEMENT SPECTRA (cm)
NATURAL PERIOD (SEC)

FIG. A5.2

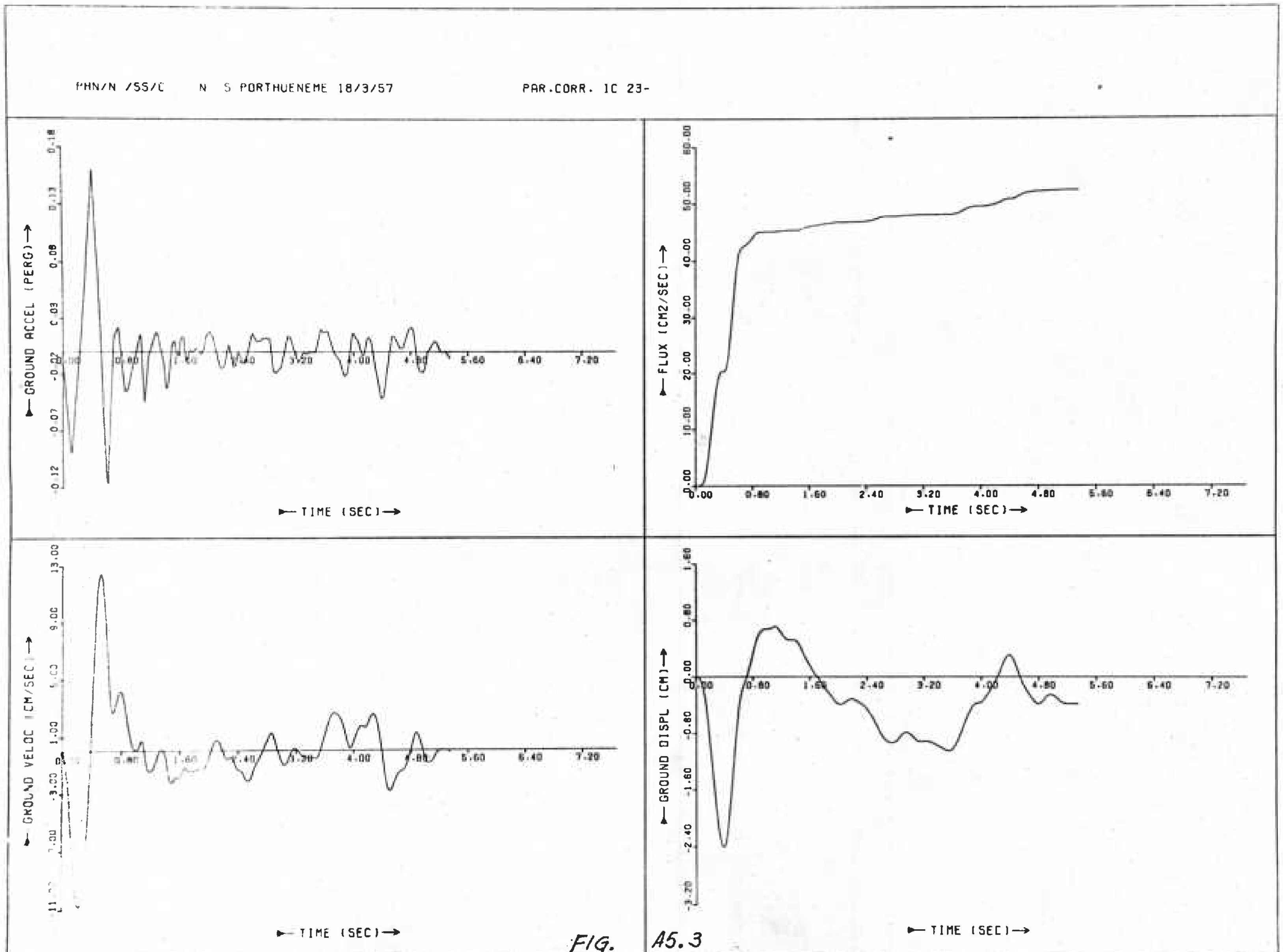

PHN/N /SS/C N S PORTHUENEME 18/3/57 PAR.CORR. IC 23-
GROUND ACCEL (PERG)
TIME (SEC)
FLUX (CM2/SEC)
TIME (SEC)
GROUND VELOC (CM/SEC)
TIME (SEC)
GROUND DISPL (CM)
TIME (SEC)

FIG. A5.3

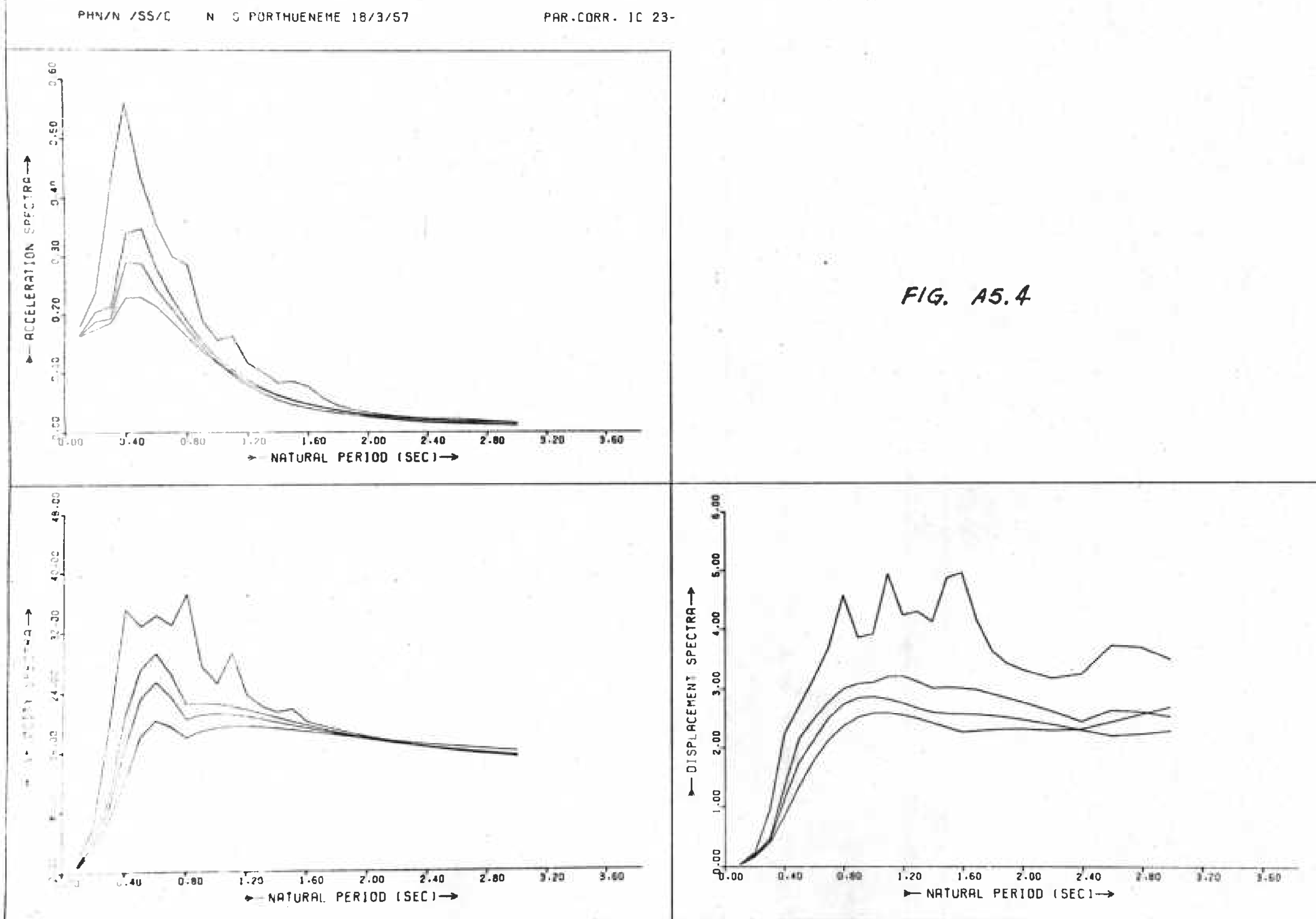

PHN/N /SS/C N S PORTHUENEME 18/3/57
PAR.CORR. IC 23-
ACCELERATION SPECTRA
NATURAL PERIOD (SEC)
NATURAL PERIOD (SEC)
DISPLACEMENT SPECTRA
NATURAL PERIOD (SEC)

FIG. A5.4

LEUKAS EARTHQUAKE (GREECE), 04 NOVEMBER 1973, COMP LONGITUDINAL

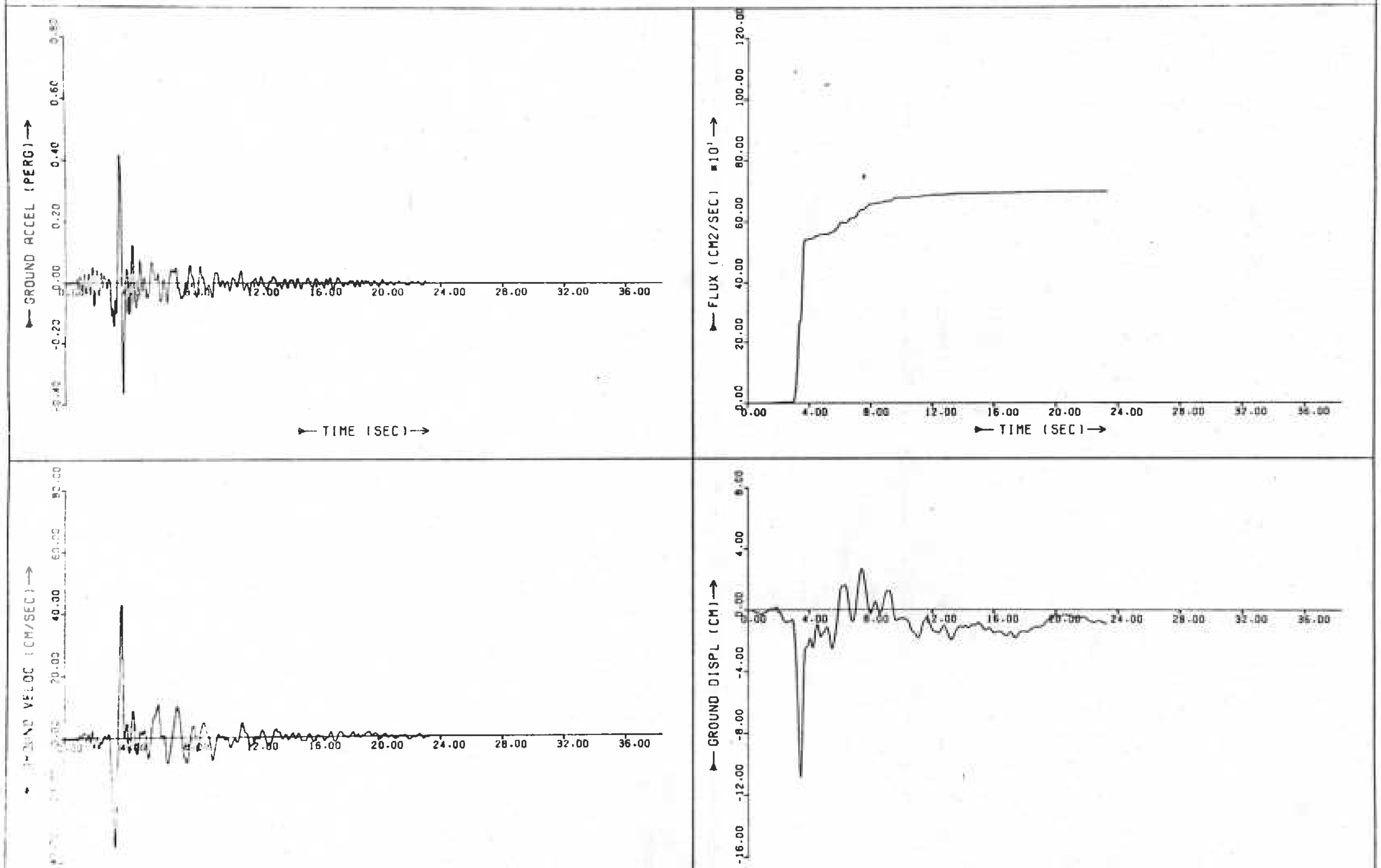


FIG. A5.5

LEUKAS EARTHQUAKE (GREECE), 04 NOVEMBER 1973, COMP-LONGITUDINAL

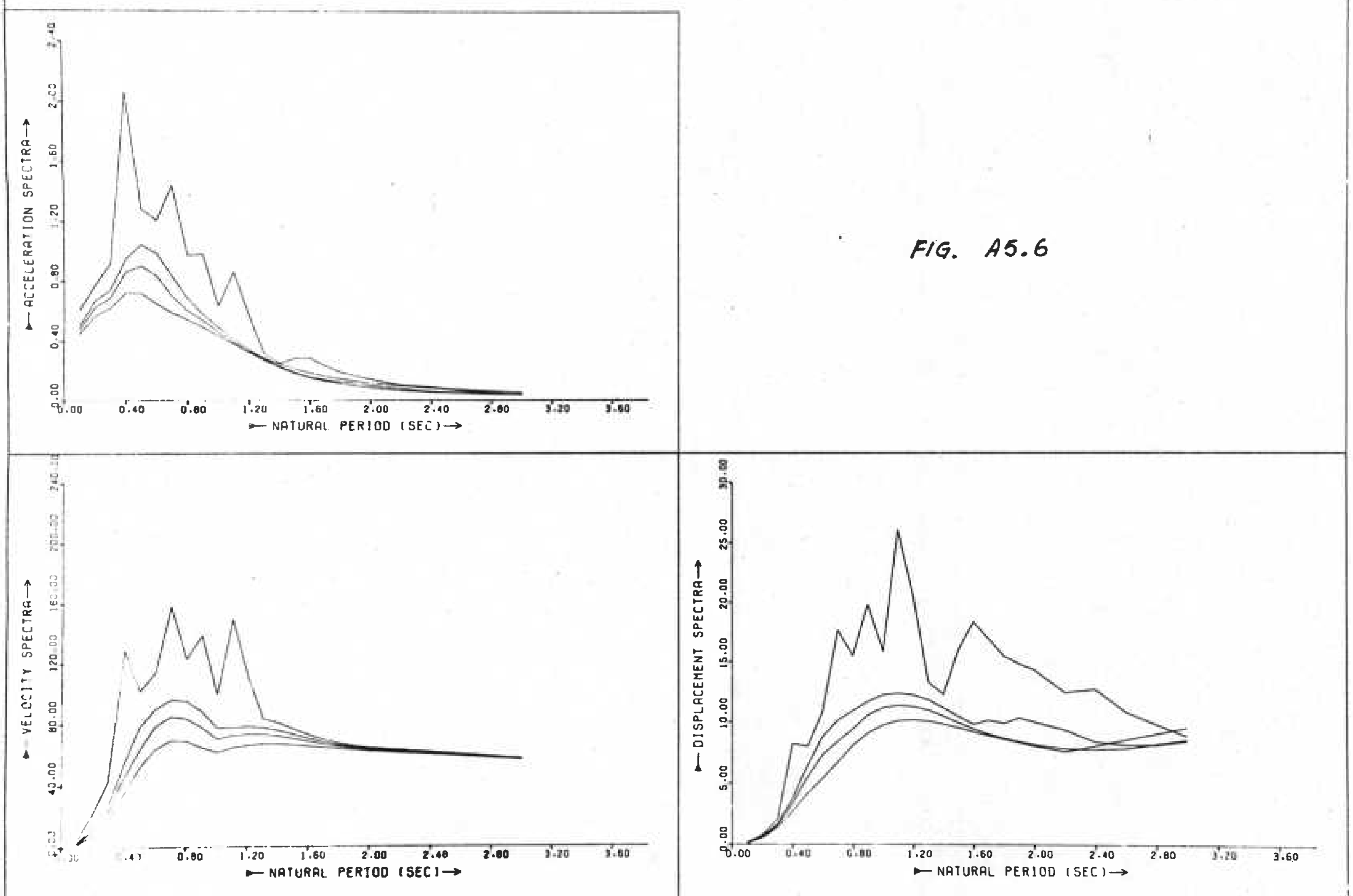


FIG. A5.6

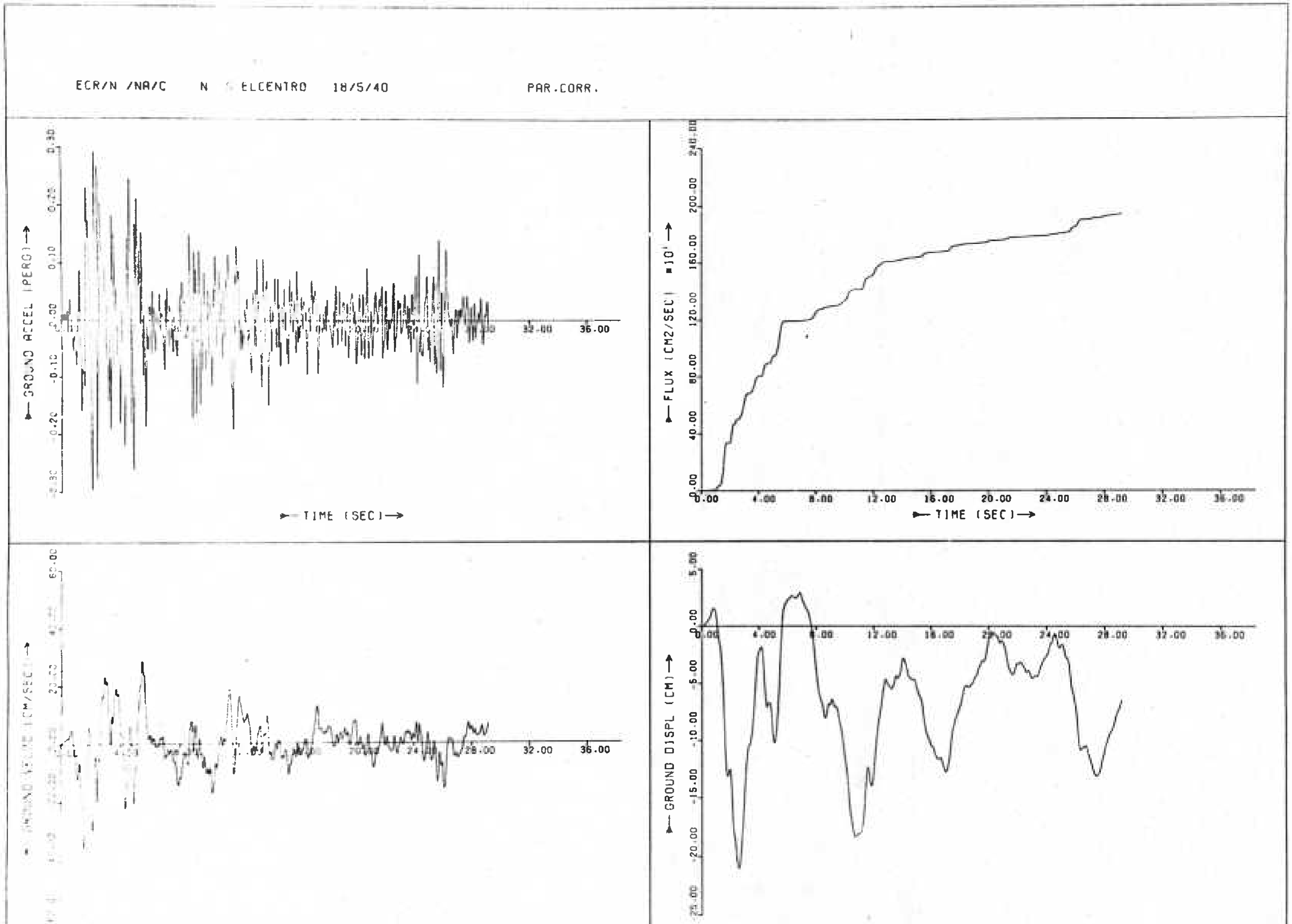
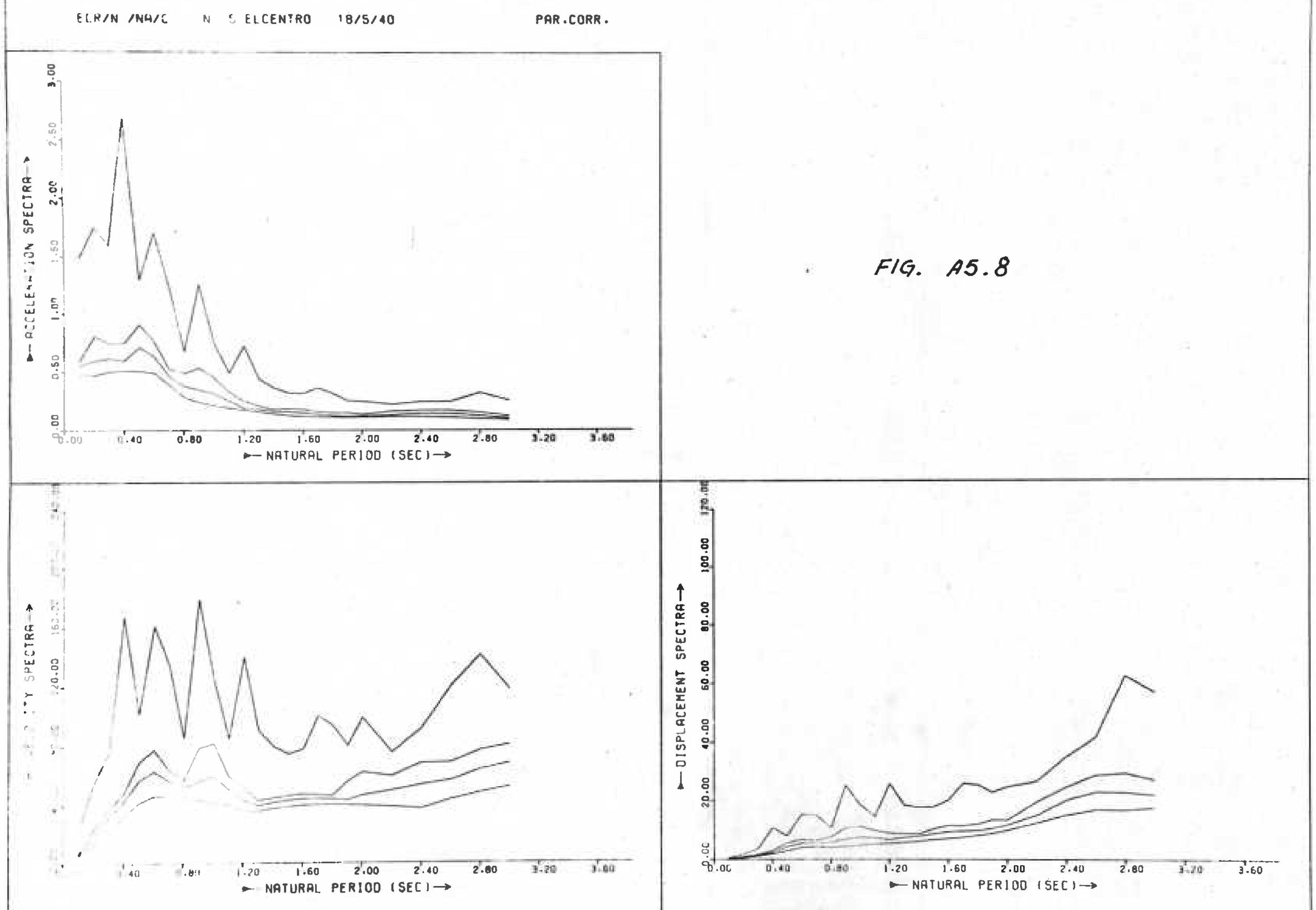
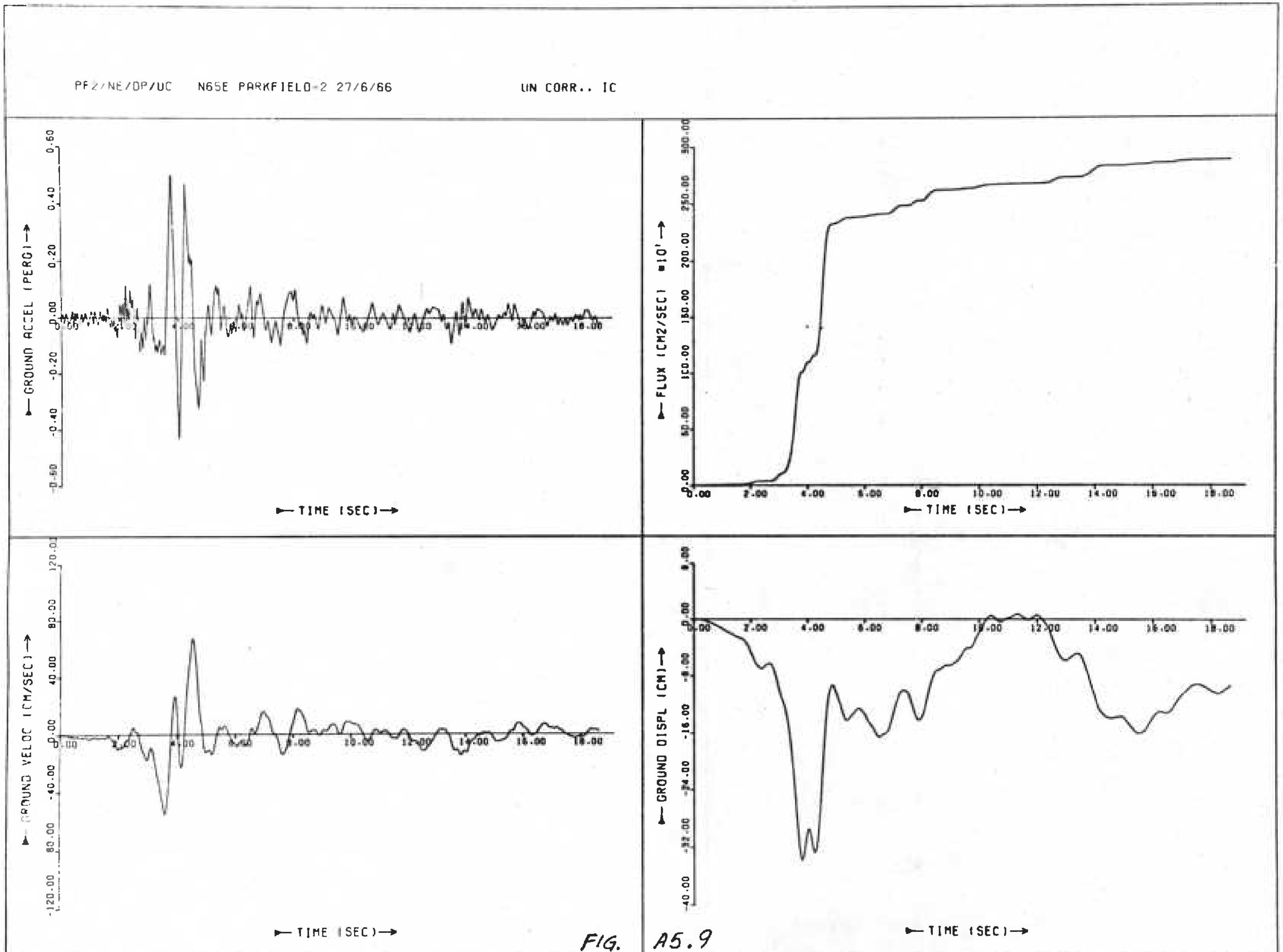

ECR/N /NA/C N ELCENTRO 18/5/40
PAR.CORR.
GROUND ACCEL (PERG)
TIME (SEC)
FLUX (CM2/SEC) *10'
TIME (SEC)
TIME (SEC)
GROUND DISPL (CM)
TIME (SEC)

FIG. A5.7



ELR/N /NA/C N S ELCENTRO 18/5/40
PAR.CORR.
ACCELERATION SPECTRA
NATURAL PERIOD (SEC)
SPECTRA
NATURAL PERIOD (SEC)
DISPLACEMENT SPECTRA
NATURAL PERIOD (SEC)

FIG. A5.8



PF2/NE/DP/UC N65E PARKFIELD-2 27/6/66 UN CORR.. IC
GROUND ACCEL (PERG)
TIME (SEC)
FLUX (CM2/SEC) *10'
TIME (SEC)
GROUND VELOC (CM/SEC)
TIME (SEC)
GROUND DISPL (CM)
TIME (SEC)

FIG. A5.9

PF2/NE/DP/UC N65E PARKFIELD-2 27/6/66 UN CORR.. IC

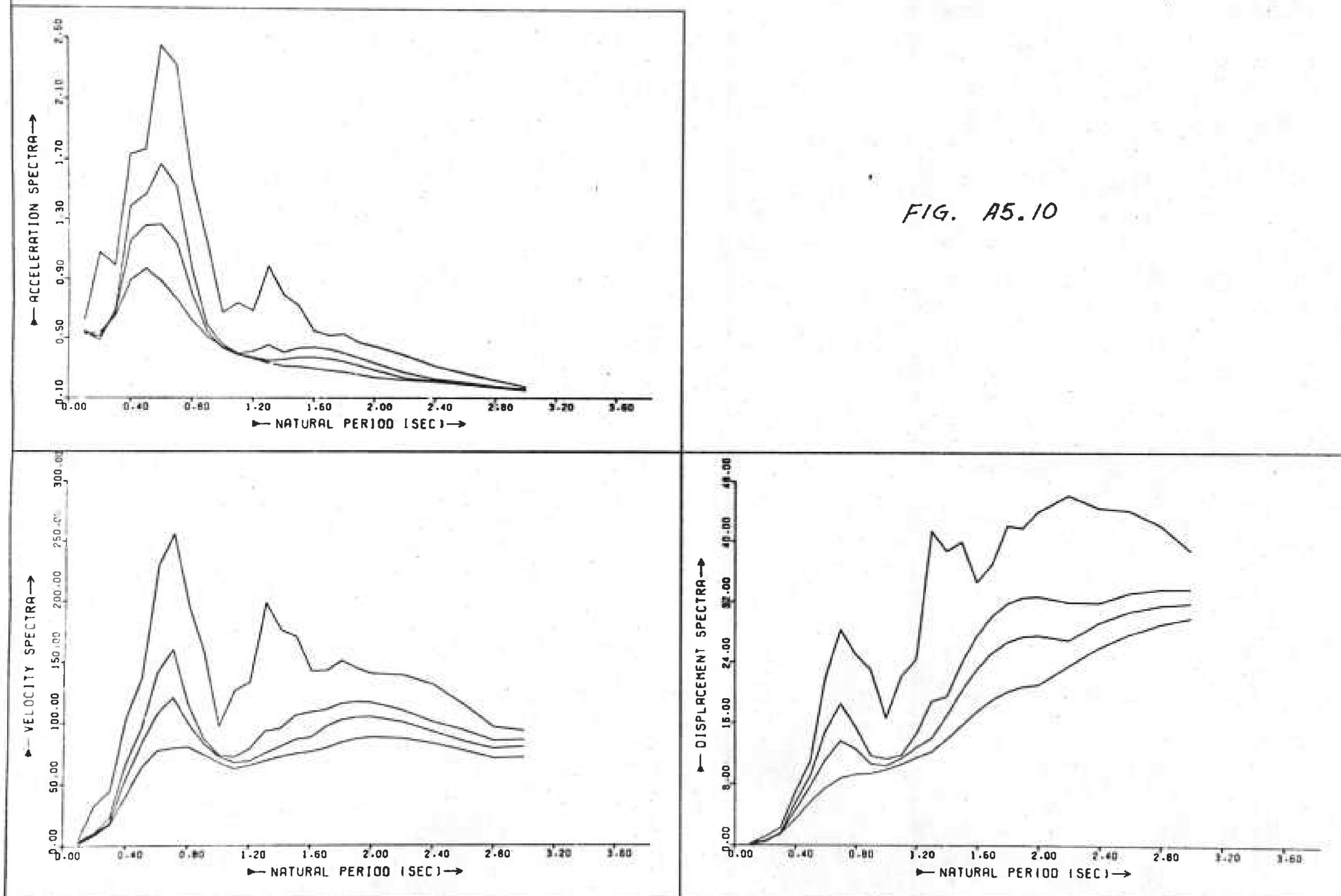


FIG. A5.10

47/KNA-S/L/SS/C LONG KOYNA-SAINI 10/12/67 PAR.CORR. IC

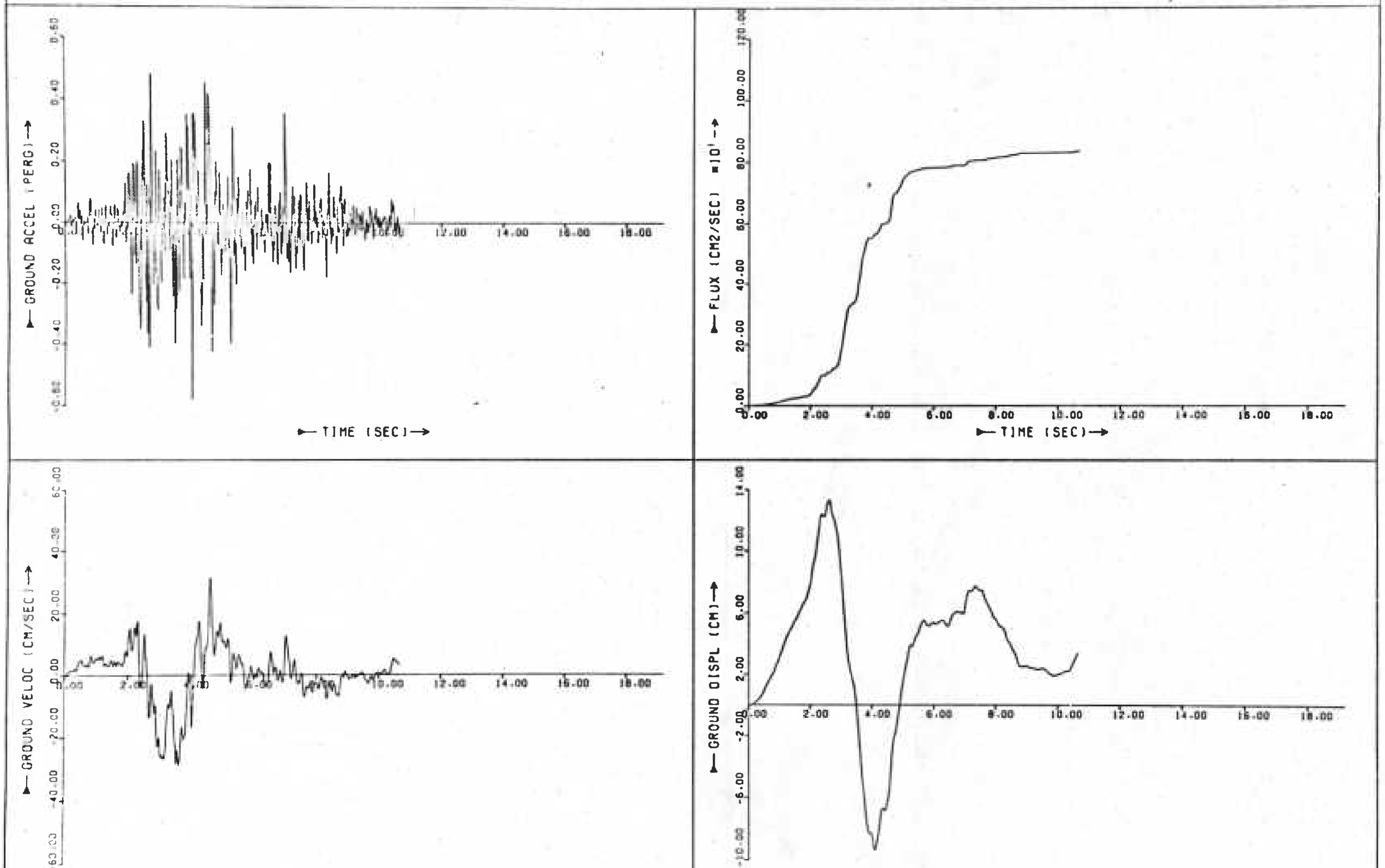


FIG. A5.11

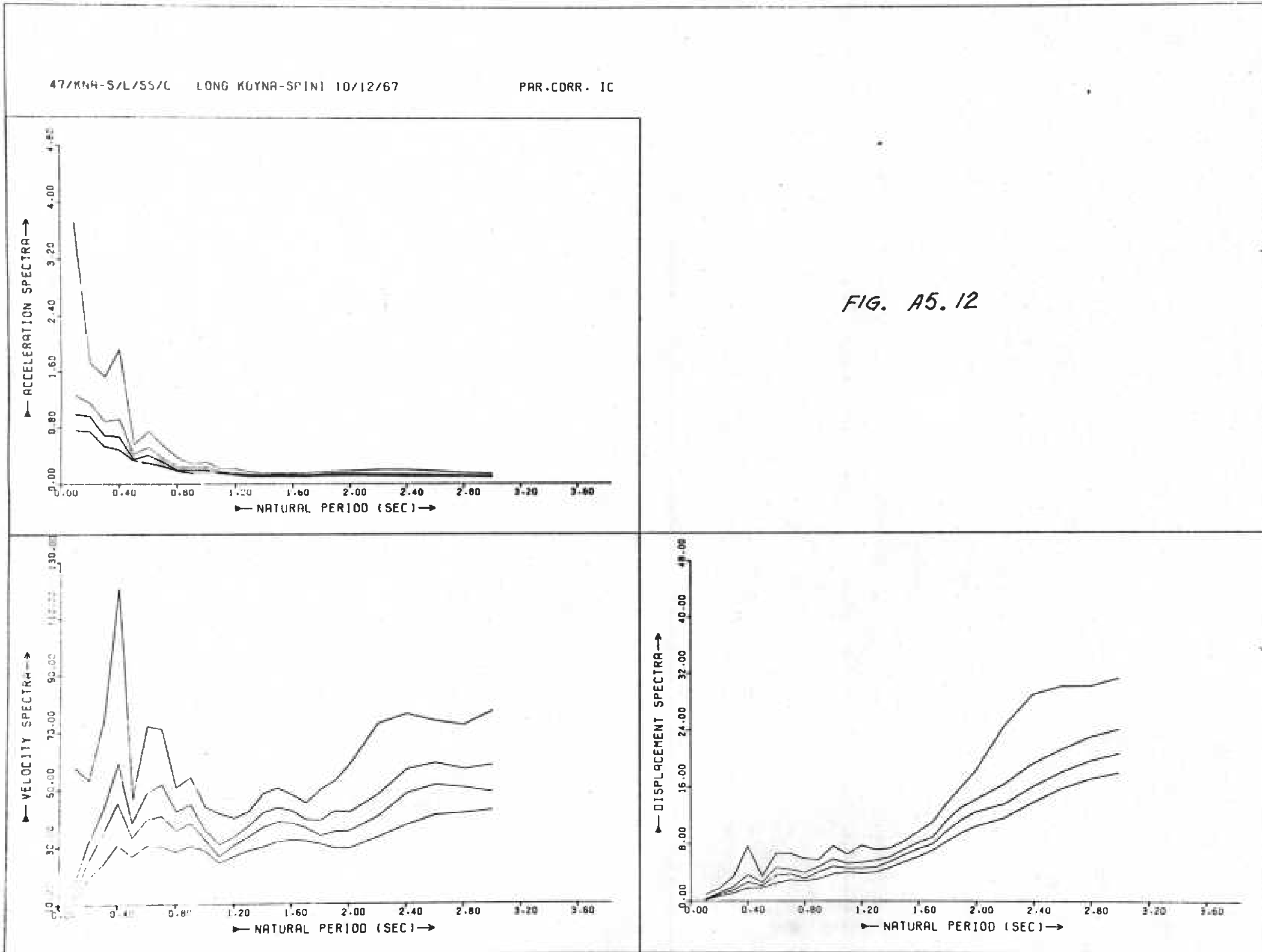

47/KNA-S/L/SS/C LONG KOYNA-SPINI 10/12/67
PAR.CORR. IC
ACCELERATION SPECTRA
0.00
0.80
1.60
2.40
3.20
4.00
VELOCITY SPECTRA
50.00
70.00
90.00
130.00
DISPLACEMENT SPECTRA
0.00
8.00
16.00
24.00
32.00
40.00
0.00
0.40
0.80
1.20
1.60
2.00
2.40
2.80
3.20
3.60
NATURAL PERIOD (SEC)

FIG. A5.12

FIG. A5.13

ACCELERATION LAYER SPECTRA (G)

PACOIMA S16E 09/02/71 Y= SURFACE

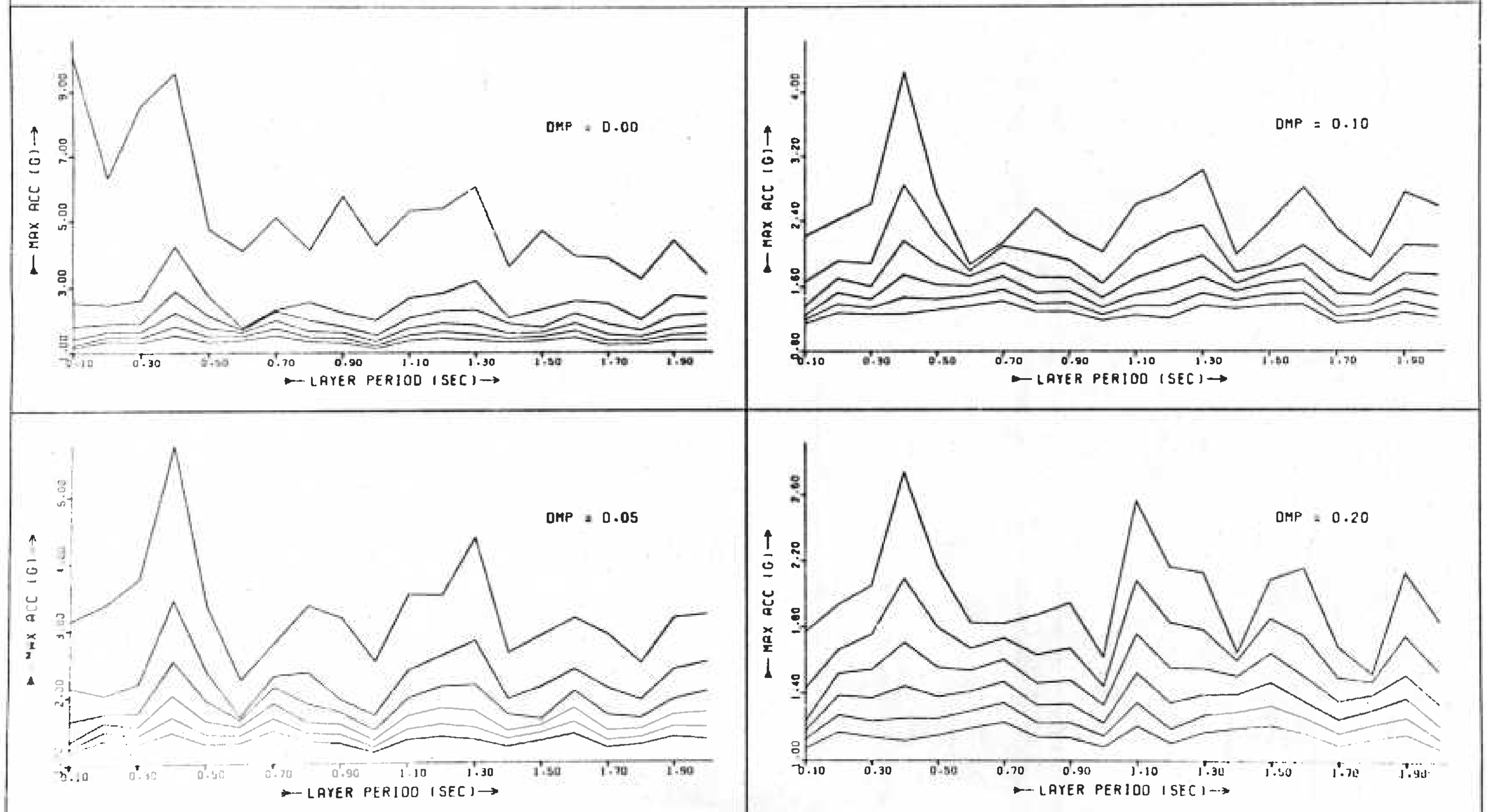


FIG. A5.14

ACCELERATION LAYER SPECTRA (G)

PACOIMA S16E 09/02/71 Y = 0.50 H

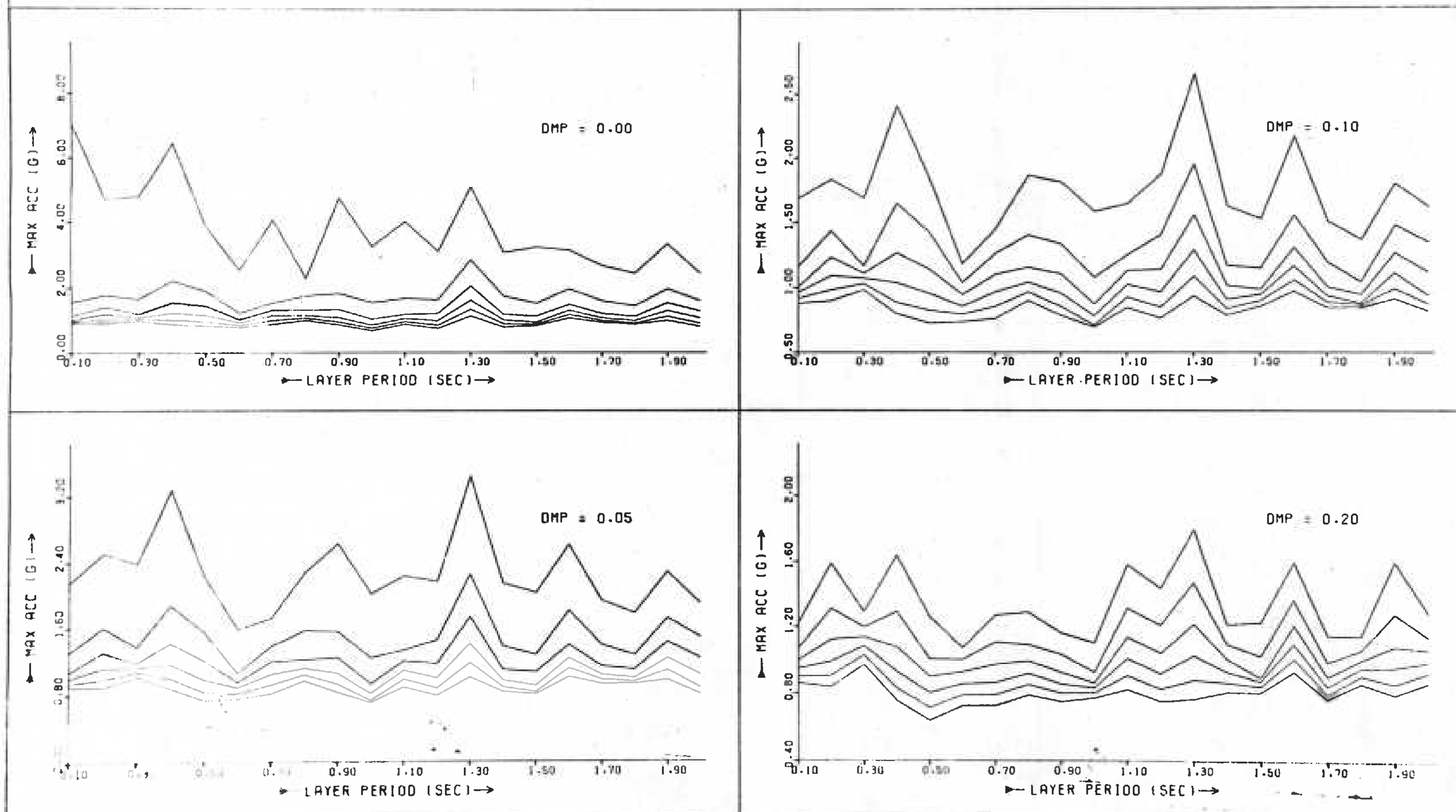
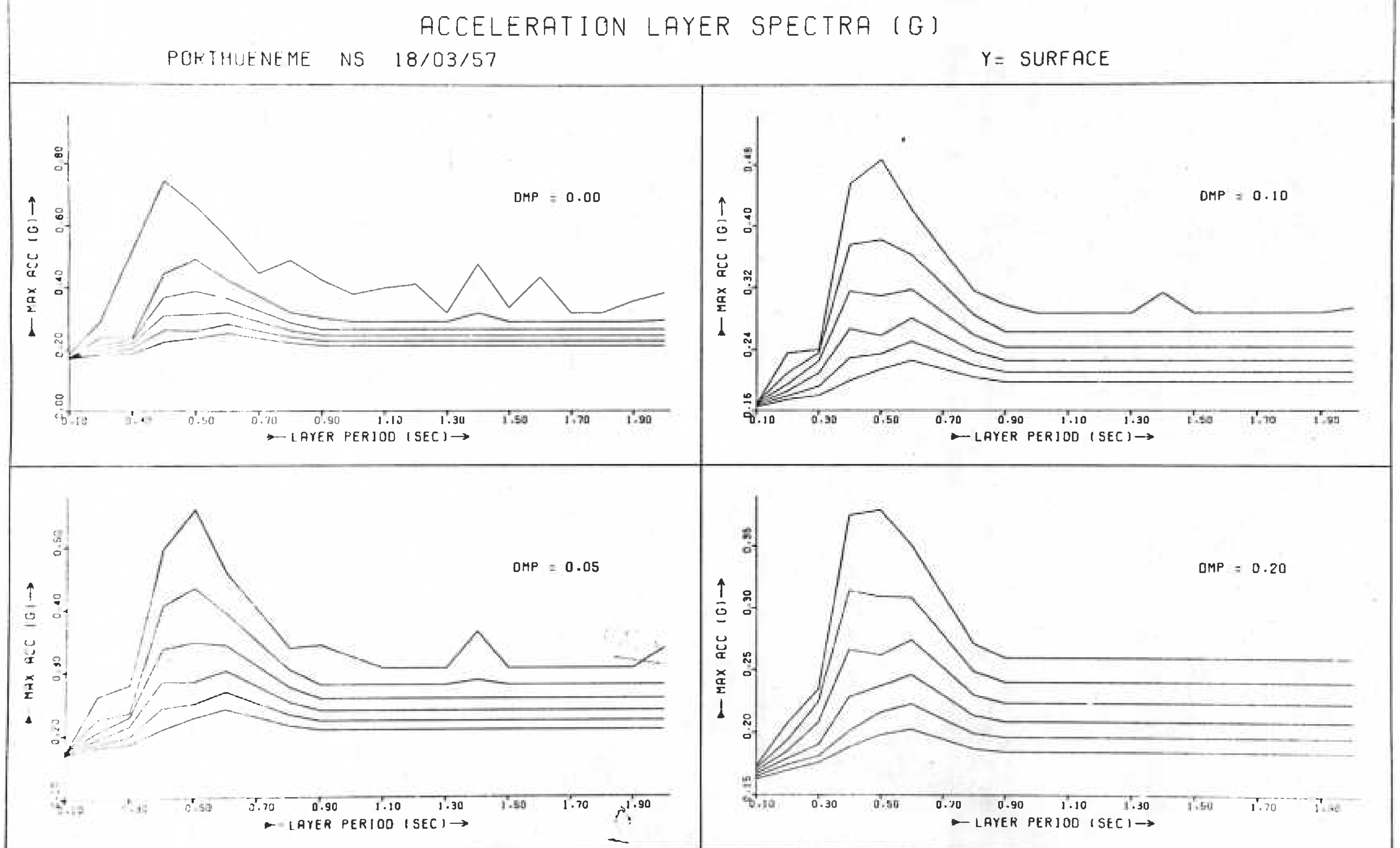


FIG. A5.15


ACCELERATION LAYER SPECTRA (G)
PORTHUENEME NS 18/03/57
Y= SURFACE
DMP = 0.00
DMP = 0.10
DMP = 0.05
DMP = 0.20
MAX ACC (G)
LAYER PERIOD (SEC)

138

FIG. A5.16

ACCELERATION LAYER SPECTRA (G)

PORTHUENEME NS 18/03/57 Y = 0.50 H

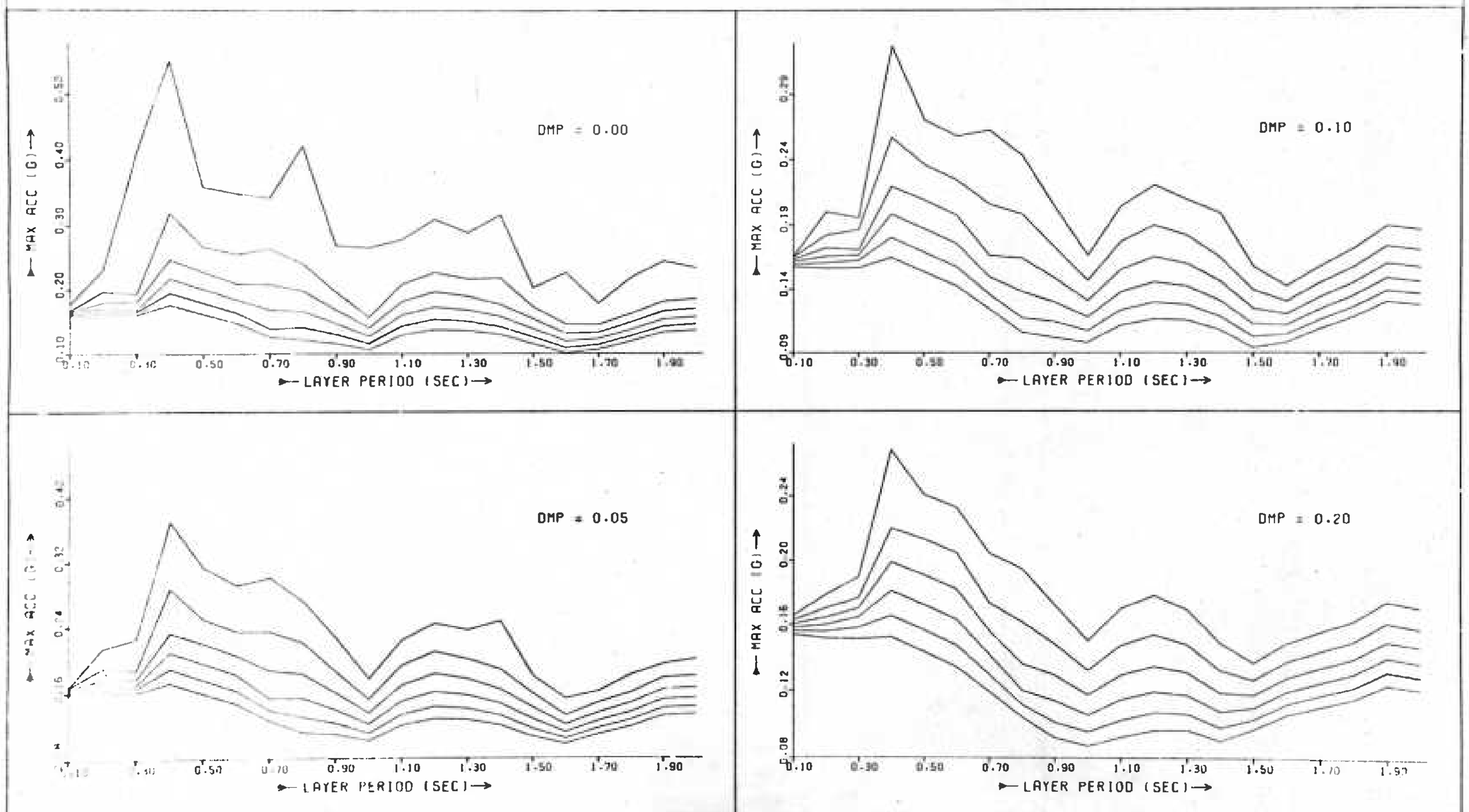


FIG. A5.17

ACCELERATION LAYER SPECTRA (G)

LEUKAS (GR) LON 04/11/73 Y= SURFACE

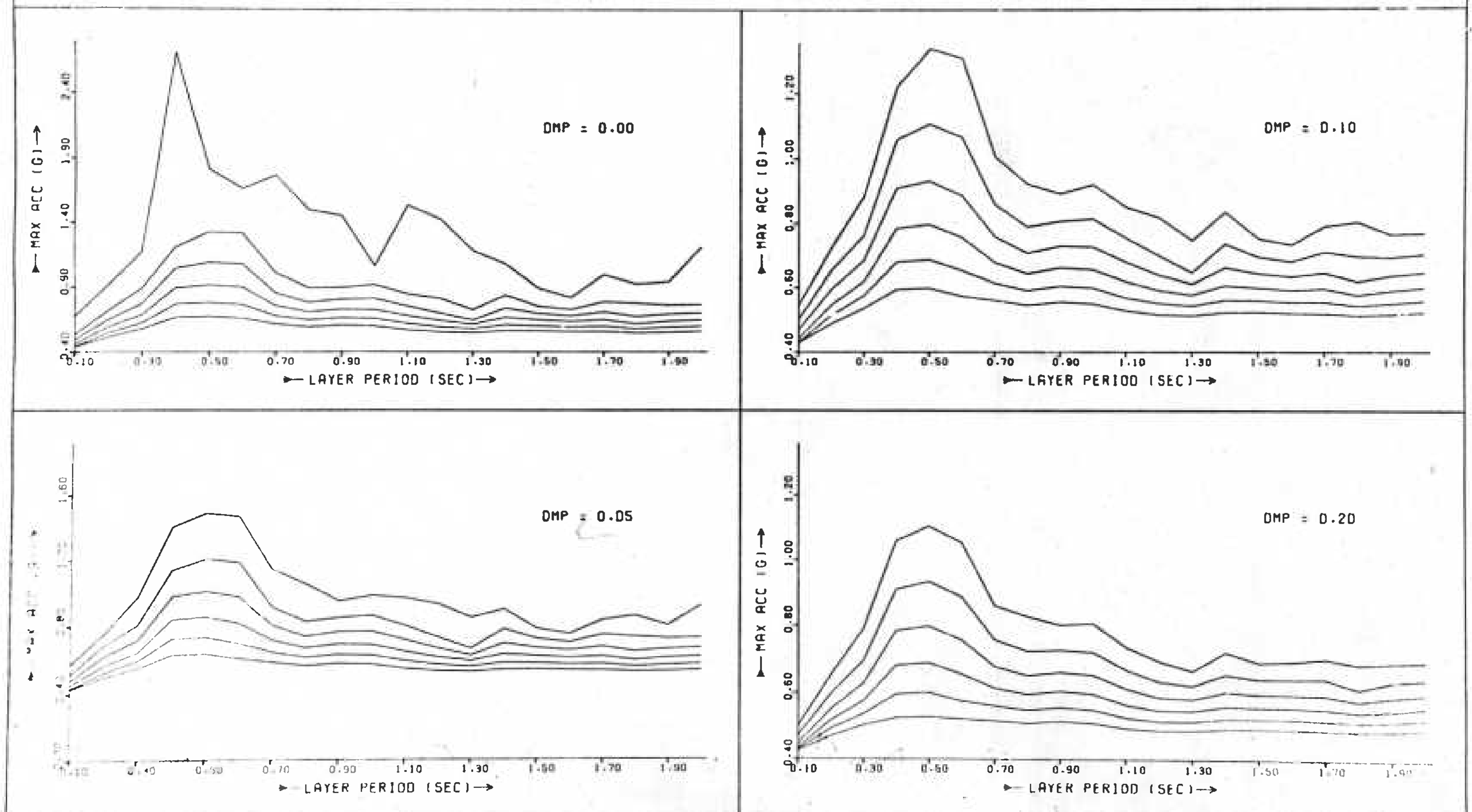


FIG. A5.18

ACCELERATION LAYER SPECTRA (G)

LEUKAS (GR) LON 04/11/73 Y = 0.50 H

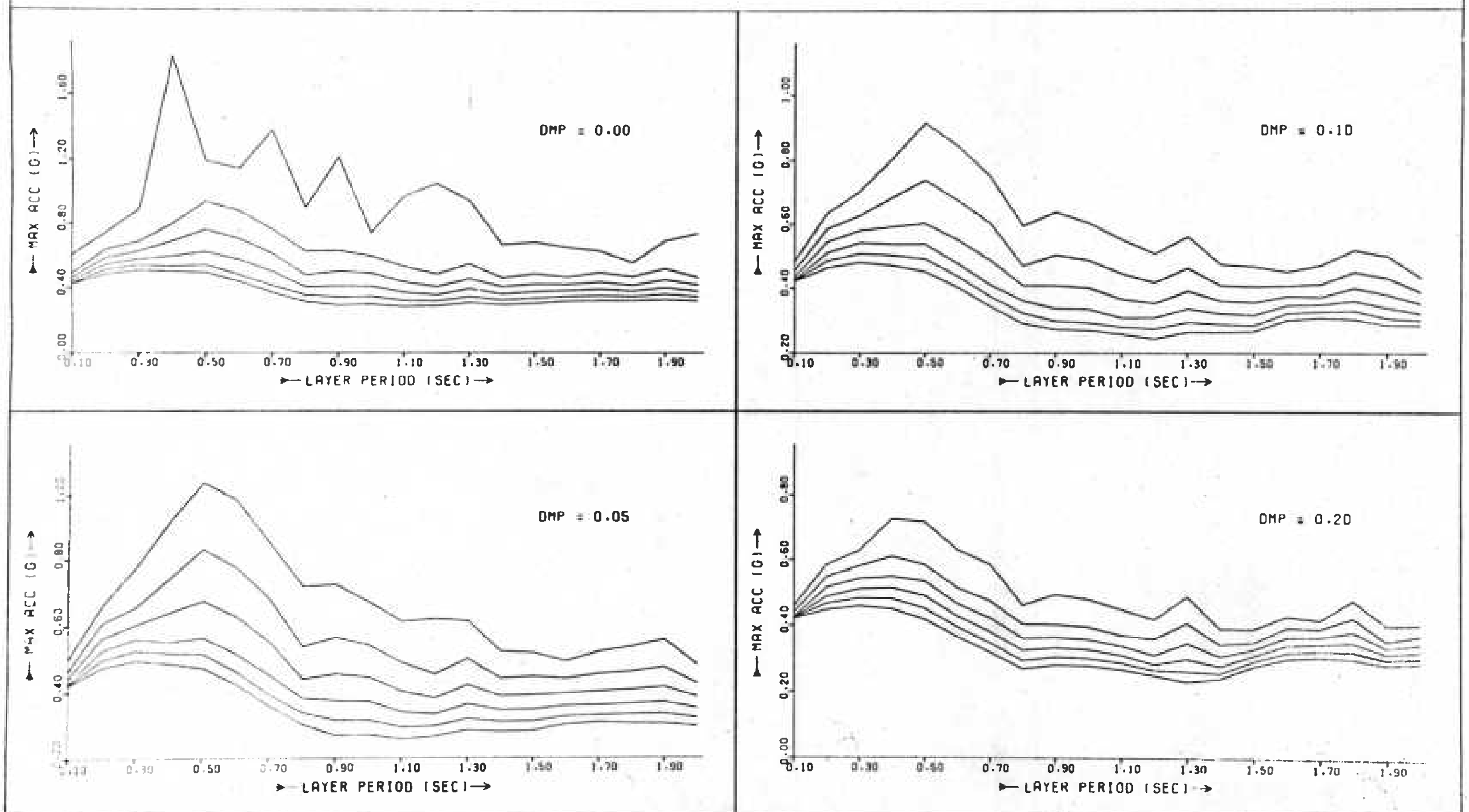


FIG. A5.19

ACCELERATION LAYER SPECTRA (G)

EL CENTRO NS 18/05/40 Y= SURFACE

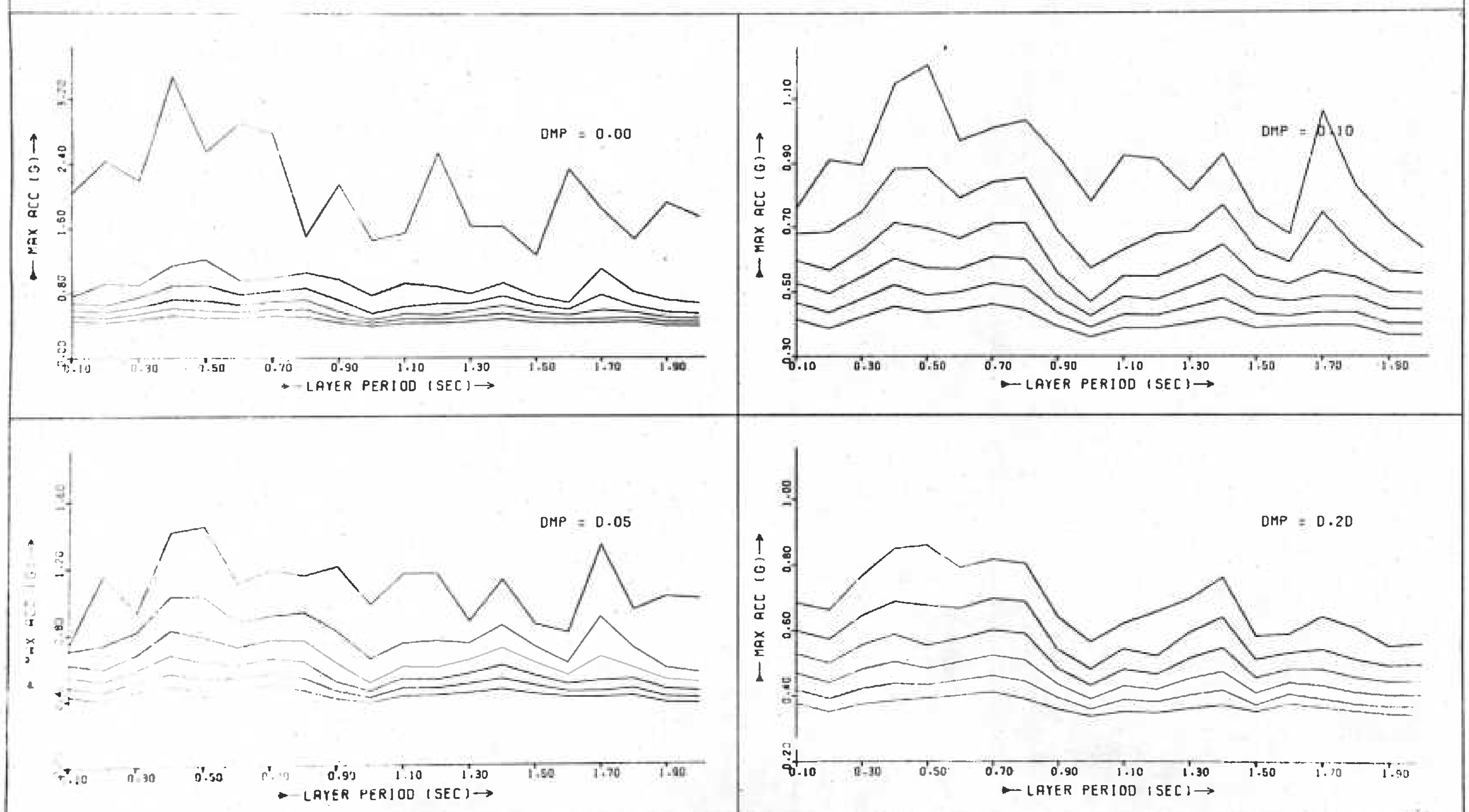


FIG. A5.20

ACCELERATION LAYER SPECTRA (G)

EL CENTRO NS 18/05/40 Y = 0.50 H

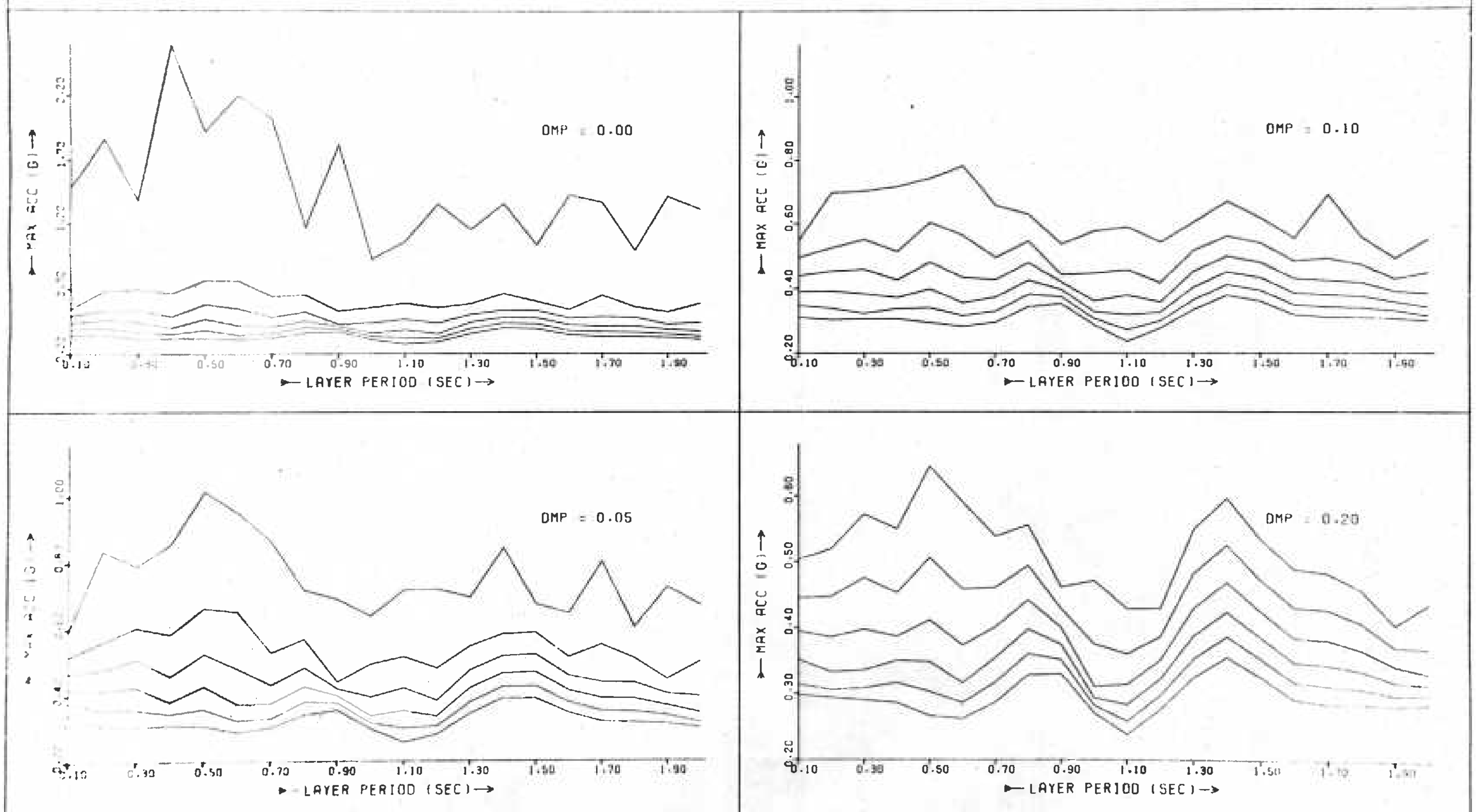


FIG. A5.22

ACCELERATION LAYER SPECTRA (G)

PARKFIELD N65E 27/06/66 Y = 0.50 H

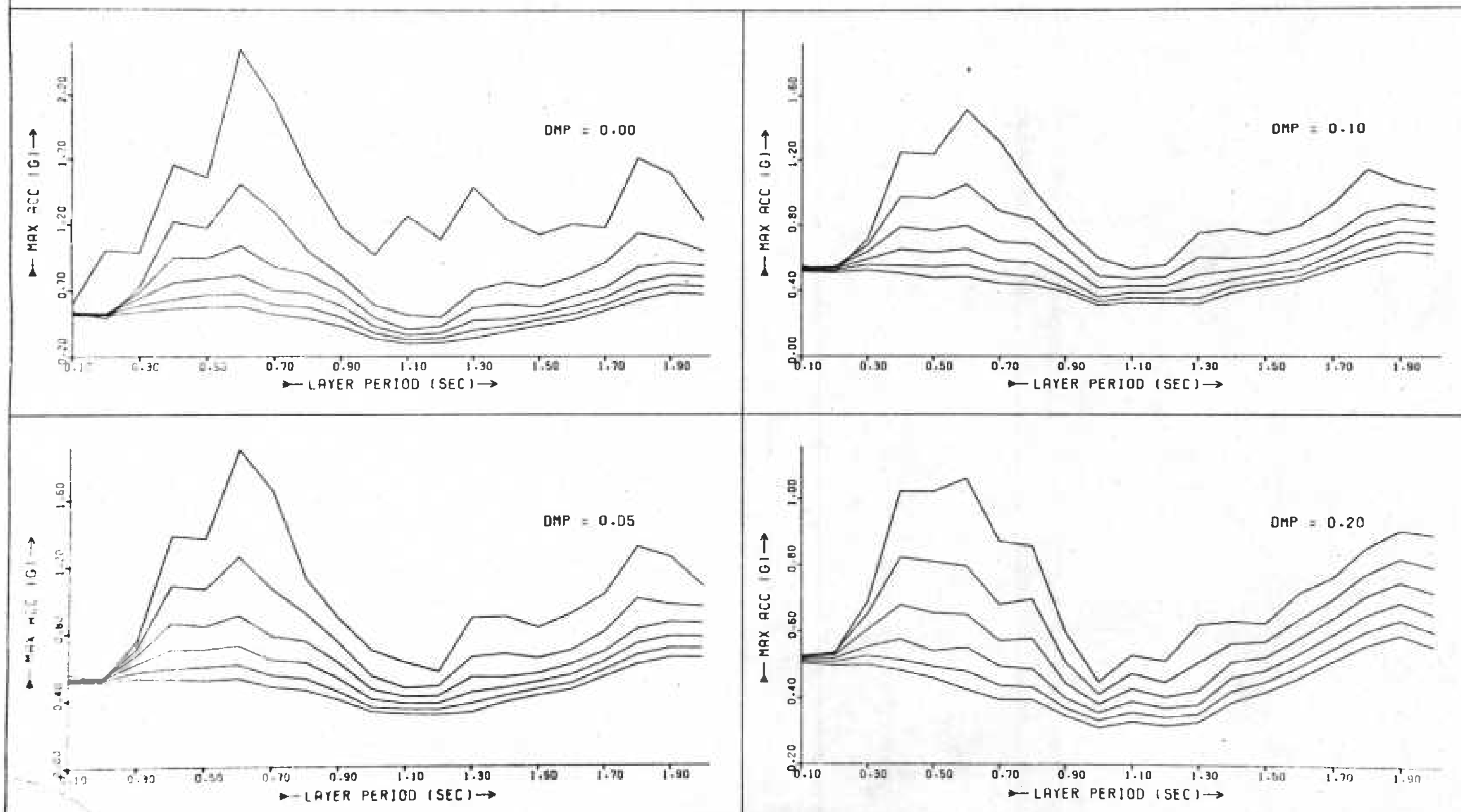


FIG. A5.23

ACCELERATION LAYER SPECTRA (G)

KOYNA-S LONG 10/12/67 Y= SURFACE

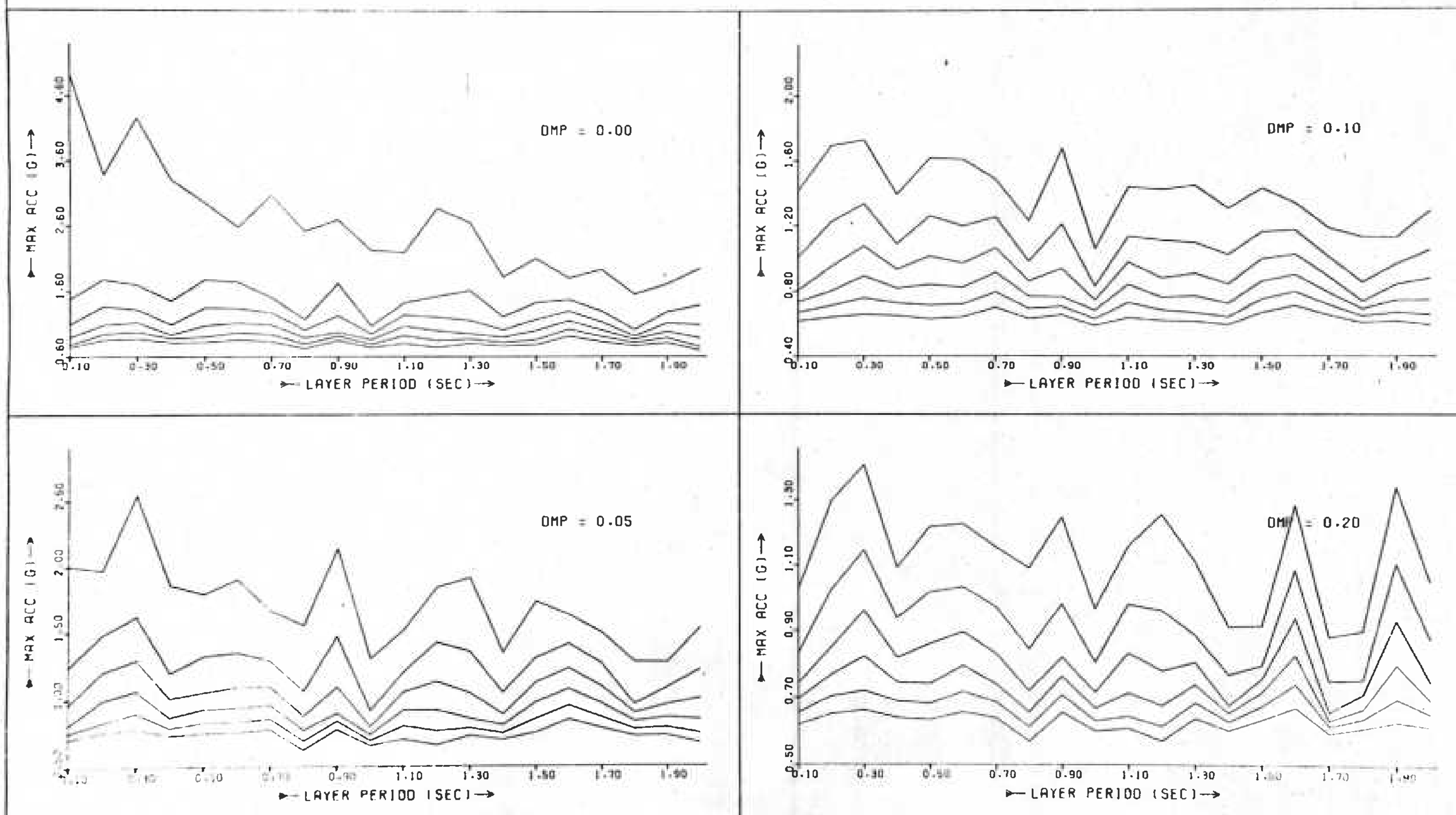
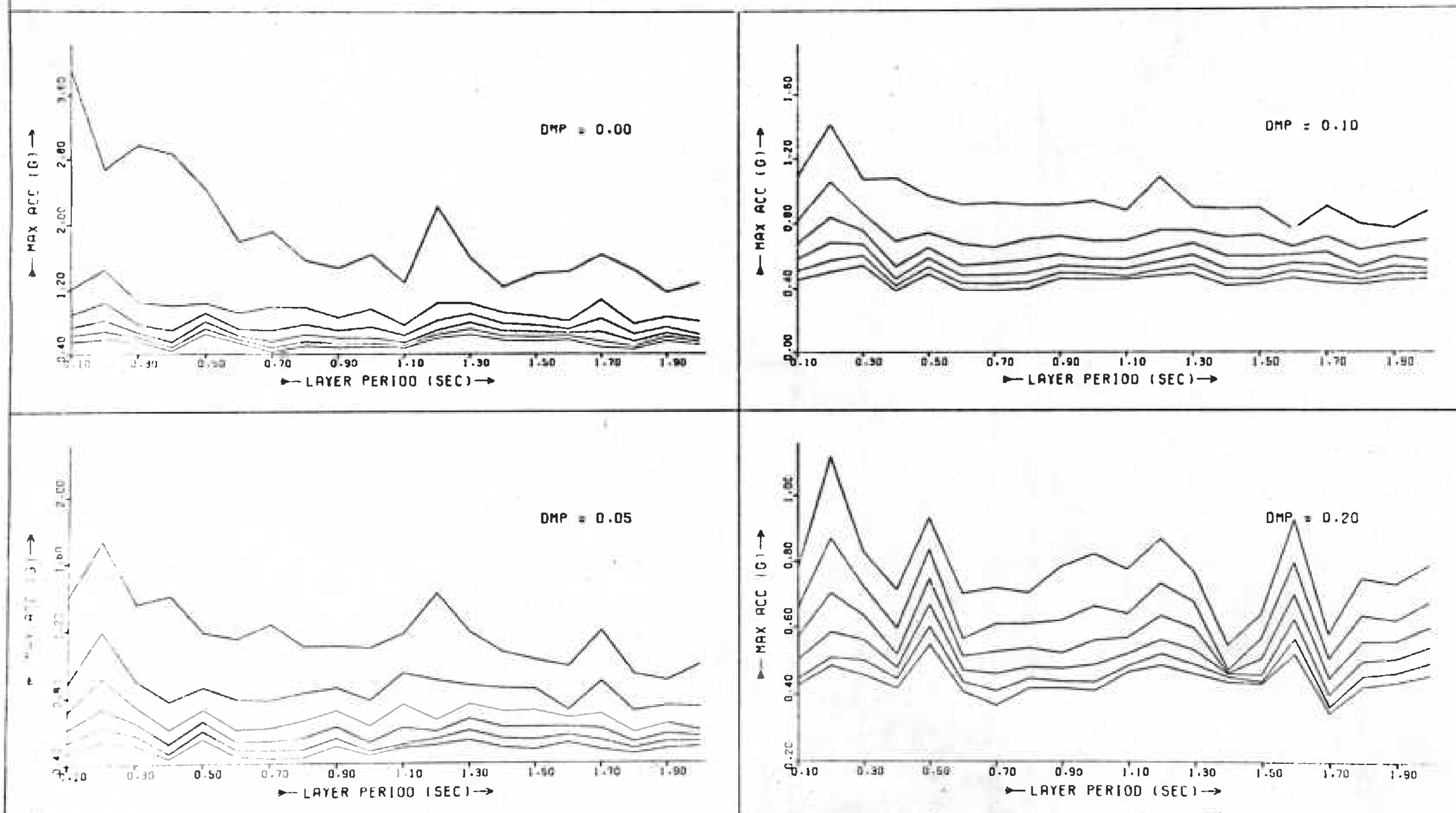


FIG. A5.24

ACCELERATION LAYER SPECTRA (G)

KOYNA-S LONG 10/12/67 Y = 0.50 H



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