CONTRIBUTIONS TO UNIFIED GAUGE THEORIES

OF THE BASIC INTERACTIONS

by

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I know only one thing - that this One is everything.

Confucius

ABSTRACT

In this thesis, we are concerned with certain consequences of the unified gauge models of leptons and hadrons recently proposed by Pati and Salam.

Chapter 1 serves as an introduction to the field by summarising the gauge aspects of the basic interactions and describing the general principles that are employed in the construction of unified gauge theories. This discussion is exemplified in Chapter 2 as we survey some of the more recent unified models.

In Chapter 3, we present the Pati-Salam model based on SU(4) X SU(4) X SU(4) and discuss those aspects most relevant to our own work.

The baryon number violating decays of integrally charged quarks and protons into leptons and the decay of hadronic gluons is analysed in Chapter 4. It is found that, while quarks decay too rapidly to have been observed (with lifetimes $\gtrsim 10^{-10}$ sec.), protons are sufficiently stable ($\tau \gtrsim 10^{32}$ sec.) to agree with contemporary experiments. The vector mesons decay into leptons and mesons with very short lifetimes ($\gtrsim 10^{-17}$ sec. for the SU(3) gluons and $\gtrsim 10^{-25}$ for the exotic SU(4) bosons) and are not likely to have been seen. Finally, the relevance of these decays to anomalous lepton events at SPEAR and the general criteria for colour production is discussed. It is suggested that production of quarks and/or colour gluons could be responsible.

A simplified model (in the sense that there are fewer fundamental particles, called "preons") is described in Chapter 5 and in Chapter 6 we investigate some of its consequences. We are particularly concerned with the possibility of unconfined, unstable preons. We find that, if they exist, free preons will decay into leptons very rapidly ($\tau \gtrsim 10^{-20}$ sec.) and, in some cases, produce new meson states which also decay into leptons. Some of these mesons may live long enough (perhaps as long as 10^{-1} sec.) to be detected. The leptonic decay modes of these mesons suggests that they may be produced in e^+e^- annihilation and this possibility is discussed in the context of anomalous lepton events and direct lepton production in pp collisions. The relation of the charmed mesons to the ψ/J particles is briefly mentioned. The preon hypothesis is not found to adversely alter previous work on proton decay.

PREFACE

The work described in this thesis was carried out under the supervision of Professor Abdus Salam in the Department of Theoretical Physics of the Imperial College of Science and Technology between October, 1973 and June, 1976. Except where explicit reference to others is made, the work is original and has not been submitted for another degree at this or any other university.

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INTRODUCTION

For whatever the reason, the Creator seems to have made considerable use of nonabelian gauge theory in constructing the Universe. It is perhaps because that the minimal couplings required by gauge invariance are the only ones that permit the formulation of renormalisable interacting vector field theories.

Over the past decade, much effort has been devoted to the construction of gauge theories of the fundamental interactions and, at present, it seems both promising and correct to suggest that ordinary weak and electromagnetic interactions are described by SU(2) X U(1) which commutes with the gauge group for strong interactions, believed to be (colour)SU(3).

More recently, attention has been drawn to the esthetically displeasing fact that fundamental matter should belong to two so similar yet distinct group structures. It would be far better to find that strongly and weakly interacting particles were classified by commuting subgroups of a single symmetry group. Needless to say, such a large symmetry would have to be very badly broken to account for huge lepton-hadron mass differences and the highly asymmetric response of matter to strong interactions.

This idea of a universality of the basic interactions has only lately achieved some status. While the actual universality may become visible only at very high energies, the recent suggestion that strong interactions can be described by a gauge theory coupled with the phenomena of scaling and asymptotic freedom have, nevertheless, given it much support. More precisely, scaling suggests that if the strong interactions can be described by a renormalisable field theory, then it should be a nonabelion gauge theory. Further, if we interpret strong interaction phenomena in terms of quarks and gluons, then Bjorken scaling can be understood if one assumes that the quark-gluon coupling constant is small. Indeed, this assumption suggests that quarks behave as free particles approximately in the range 10^{-13} - 10^{-15} cm.. So, the smallness of the strong coupling constant at short distances give credence to our speculation that all the basic interactions are described by a single gauge group that is more than just the direct product of the groups for the individual interactions.

In this work, we will be concerned with certain consequences of one of the more recent and most ambitious attempts at constructing a universal gauge theory, that of Pati and Salam(1,2). But first, in Chapter 1, we set the stage by briefly discussing those aspects of the weak, strong and electromagnetic interactions that are relevant to the construction of a gauge theory. We then mention some very general principles that one uses in the formulation of such theories giving particular attention to our original hypothesis of a single unifying symmetry group and the idea of "maximal" symmetry.

These ideas are exemplified in Chapter 2 as we turn to consider a number of recent and representative candidates for this unifying description. We discuss, in turn, the

SU(3) X SU(3) model of Weinberg[3], the U(3) X U(3) X SU(2) X U(1) model of Bars, Halpern and Yoshimura[4] and the model of Georgi and Glashow[5] based on SU(5).

In Chapter 3, we introduce the Pati-Salam model[2] based on SU(4) X SU(4) X SU(4) and give a fairly complete account of the fundamental features of the so-called "basic" model and a brief summary of the several variants that one may consider. We pay little attention to aspects of the model that do not directly relate to our own work and reference to the original paper will compensate for any deficiencies from which our review may suffer.

One of the most interesting predictions of the Pati-Salam model is that integrally charged quarks will have baryon number violating decays into leptons and, in Chapter 4, we demonstrate this and calculate dominant decay modes and lifetimes for the twelve hadronic quarks. It is found that they all decay rapidly enough to have escaped detection by contemporary means with lifetimes $\gtrsim 10^{-10}$ sec.. The dominating leptonic decay modes of quarks suggests that they are likely candidates for explaining anomalous lepton production at SLAC and this possibility is discussed.

The instability of quarks implies that protons, too, must decay and we compute leading decay modes and lifetimes. Because protons decay only in third order in the effective strength for baryon number violation, it is found that the proton is as stable as the most recent experiments suggest with $\tau \gtrsim 10^{32}$ sec.. In connection with conservation of fermion number in these reactions, it is remarked that past searches may have missed a proton decay event in view of the wrong modes being sought.

Finally, we discuss the decay of the colour vector gluons to leptons (finding $\tau \gtrsim 10^{-17}$ sec. for the SU(3) octet and $\tau \gtrsim 10^{-25}$ sec. for the exotic superheavy SU(4) bosons) and make some comments about similarities to J/ψ particles and the relevance of these leptonic decays to the production and detection of the colour gluons and the question of signals for colour production in general.

Still accepting an SU(4) X SU(4) X SU(4) universal gauge symmetry, we discuss, in Chapter 5, how this symmetry seems to require a disturbingly large number of fundamental fermions. We then describe a new model of Pati and Salam[(6)] that drastically reduces the number of basic particles to the minimum number permitted by the group structure of the gauge theory. It turns out that only eight particles are required, one for each of the fundamental attributes believed present in nature (four colours and four valencies). These particles are called "pre-quarks" or "preons" and all known matter, including quarks and leptons, is presumed to be constructed out of them. We discuss the several alternative formulations of such a model and outline the structure of a gauge theory for preons.

In Chapter 6, we advance the conjecture that they exist as free particles and examine the consequences of unconfined preons. We find several mechanisms that permit preons to decay rapidly ($\tau \gtrsim 10^{-20}$ sec.) thus accounting for their absence from present particle data tables. Many of these decays are to known particles (principally leptons and mesons) but a number involve new meson states (which may be spin 0 or 1) that are di-preon composites. Most of these decay into leptons with very short lifetimes ($\tau \gtrsim 10^{-9}$) but some seem fairly long lived (perhaps as long as 10^{-1} sec.) and, with masses probably in the range of 3 - 5 GeV., they ought to be discovered.

The leptonic decay modes of the preons and their bound states leads us to suggest that the anomalous lepton production at SLAC might be attributed to the production of some of these hypothetical particles. We offer what we hope are convincing reasons why di muon events, $\bar{\mu}e$ production and direct lepton production in pp collisions might be due to preons.

Many of the new meson states we predict carry charm and are also colour non-singlets. Since they may be produced in e^+e^- annihilation (as well as decay into e^+e^-), we tentatively identify certain of these mesons with the recently discovered ψ/J particles (or at least their close relatives). As the quantitative aspects of this suggestion may be subject to attack, it is the general features of the proposed mechanism that we choose to emphasize, namely that the ψ/J particles need not be charm-anti-charm composites but may themselves carry charm, their decay into leptons resulting from a spontaneous breakdown of charm symmetry.

The case for proton decay (which is now very much more complicated than before) is also treated and while precise estimates are not obtained, it is ascertained that the preon hypothesis does not adversely affect our original results and that the same parameters govern the effective strength of the interaction.

In Appendix A, we summarize the basic decay modes, approximate matrix elements and lifetimes for quarks, protons and vector gluons. In Appendix B, we indicate all the relevant diagrams for our work.

CHAPTER 1

UNIFIED GAUGE THEORIES OF THE BASIC INTERACTIONS

(1.1) GAUGE ASPECTS OF THE BASIC INTERACTIONS

(a) The Strong Interactions

Whatever symmetry group we choose for our universal theory, it is evident that the subgroups responsible for the strong gauges on one hand and the weak and electromagnetic gauges on the other must commute. If this were not the case, we would have no explanation for the existence of symmetries respected by the strong interactions but not by the weak and electromagnetic ones.

As is well known, the necessity to have low-lying baryon states of zero triality and the quark model calculation of $\pi^0 \to 2\gamma$ suggests that strong interactions require an additional SU(3) reflecting the three basic hadron colours[7].

It is usually accepted that only colour singlets are seen in nature which implies that colour must be an exact symmetry. To construct a gauge theory of strong interactions based on exact SU(3) colour symmetry, one would start with the Lagrangian[8]:

$$\mathcal{L} = -\tfrac{1}{4} \sum_{i=1}^{8} G^{i}_{\mu\nu} G^{\mu\nu}_{i} + \bar{q}\left(i\gamma^{\mu}(\partial_{\mu} + if\,\theta^{i} A_{i\mu}) - M\right)q \qquad (1)$$

where θ_i (i = 1, 2, ..., 8) are the SU(3) generators satisfying:

$$(\theta_i, \theta_j) = if_{ijk}\, \theta_k$$

$$tr(\theta_i, \theta_j) = 2\delta_{ij}$$

and

$$G^i_{\mu\nu} = \partial_\nu A^i_\mu - \partial_\mu A^i_\nu - ff^{ijk} A^j_\mu A^k_\nu \ .$$

The quark mass matrix has been denoted by M.

Such a theory is asymptotically free provided there are not more than sixteen quark triplets present[8]. This is a rather important point and we wish to discuss it in the context of scaling.

As is well known from the work of Symanzik[9] on ϕ^4 theory, any change in an amplitude brought about by a scaling of the external momenta can be compensated by a change in the coupling constant and the mass parameter of the ϕ field. Further, the sign of the change in these quantities is determined by the one loop diagrams.

Now, it is known that in ϕ^4 theory increasing the external momenta is equivalent to increasing the magnitude of the coupling constant provided that the sign of the coupling constant is taken such that the energy in the corresponding classical theory is bounded from below[9]. But the opposite happens if the sign of the coupling constant is reversed. As Symanzik has suggested that negative energies in a classical theory do not necessarily imply the same in a quantum theory, this indicates that a ϕ^4 theory with the "wrong" sign for the coupling constant is asymptotically free for large external momenta[9]. Symanzik also demonstrated that in the limit of large momenta amplitudes can be computed with arbitrary accuracy and obtained a logarithmic behaviour for them. Parisi[10] showed that this asymptotic behaviour implies Bjorken scaling. Then, 't Hooft[11] showed that for Yang-Mills theories the sign of the change in the coupling constant is such as to provide asymptotic freedom at large momenta. More recently, Gross and Wilczek[12] and Politzer[13] have also shown that Yang-Mills theories are asymptotically free. Finally, Coleman and Gross[14] discovered that any renormalisable field theory that is asymptotically free must contain nonabelian gauge fields.

Thus, we are led to the conclusion stated in the Introduction, namely that if we are to describe the strong interactions by a renormalisable field theory then this theory must be a nonabelian gauge theory.

The theory described by the Lagrangian of eq. (1) is infrared unstable due to infrared divergences caused by the massless gluons. This instability is responsible for the essential dynamics of the quark-gluon theory in that it may produce the strong force between colour singlet hadrons even if the quark-gluon coupling constant is small[8] as is suggested by asymptotic freedom. It has recently been pointed out that a small quark-gluon coupling constant could be responsible for Bjorken scaling[15, 16].

It has also been suggested that this infrared instability is responsible for shielding colour from the physical world[8]. We will have little to say about the explicit dynamics of colour production but it is worthwhile to point out that if contemporary hypotheses about the perfect confinement of colour are not true, then it is a common feature of unified gauge models that colour should be very difficult to see at present energies[17].

We close this section by remarking that gauge models incorporating hadron symmetries may be classified according to whether these symmetries are built in "naturally" or "artificially"[18]. This division arises from the theorem that a spontaneously broken gauge theory is renormalisable in the strict sense if it contains all possible terms of dimension 4 or less which are gauge invariant[18, 19]. This condition being satisfied, the Lagrangian will contain all the counter terms necessary for renormalisation. (This is what is meant by renormalisability in the strict sense[20, 21].) In an "artificial" model, the hadronic symmetry is exhibited in lowest order only if we consider a subset of these terms or if the coefficients of gauge invariant terms are subject to some specified constraints. In such a model, the symmetry is not preserved in higher orders because the terms excluded in lowest order must be supplied as renormalisation counter terms in higher order[18]. A "natural" model is one in which the hadronic symmetry is preserved in the presence of all possible gauge invariant terms.

Models which are artificial in this sense are, for example, the SU(2) X U(1) theory of Salam[22] and Weinberg[23] extended to hadrons by means of the GIM scheme[24] and the O(3) model of Georgi and Glashow[25]. The model of Bars, Halpern and Yoshimura[4], reviewed in the next chapter, is natural in the above sense.

(b) Leptonic Weak and Electromagnetic Interactions

Although the weak and electromagnetic interactions are adequately described by the gauge group SU(2) X U(1), this can hardly be regarded as a unification (in the sense of the Introduction) as it still requires two fundamental coupling constants arising from the two independent gauge groups. To obtain a proper unification, this group should be nontrivially embedded in a single gauge group that permits only one coupling constant.

Such a model has been proposed by Weinberg[3] and we shall discuss it in the next chapter. We mention that two of the most important consequences of this unification are that it fixes the ratio of the coupling constants of the SU(2) X U(1) theory and it permits the electron and muon to belong to the same representation of the gauge group which is not the case in the Salam-Weinberg model. It is therefore possible, at least in principle, to calculate the mass of one in terms of the other. If one of the leptons should acquire mass through coupling to the Higgs scalars, the mass for the other could be generated by emission and absorption of the gauge mesons coupled to the e-μ system. Then, the e-μ mass ratio could be calculated in terms of the basic coupling constant and the mass of the appropriate gauge particles.

(c) Weak Interactions of Hadrons

The most obvious requirement of a gauge theory incorporating weak interactions of hadrons is that the W-mesons responsible must couple to hadrons. This coupling must be arranged so that the gauge particles mediating weak hadronic interactions couple to vector or axial vector currents whose charges generate the symmetry group of the quark fields.

The W's mediating semi-leptonic interactions should couple only to left handed hadronic currents. This is best established for $\Delta S = 0$ currents.

There must also be intermediate vector bosons that couple to both $\Delta S = 0$ and $\Delta S = 1$ currents to account for strangeness violating non-leptonic decays.

Finally, there is the well known universality principle[26, 27]:

$$\frac{G_{\Lambda}^{0}}{G_{\mu}^{0}} = \sin\theta_C, \quad \frac{G_{\beta}^{0}}{G_{\mu}^{0}} = \cos\theta_C$$

where G_{μ}^{0} and G_{β}^{0} are the bare constants for μ and β decay, respectively, G_{Λ}^{0} is the bare constant for $\Delta S = 1$ decay and θ_C

is the Cabibbo angle.

(1.2) UNIFICATION OF THE BASIC INTERACTIONS[28]

In this section, we briefly discuss, in a very general fashion, the basic principles that one might use in constructing a unified gauge theory and which have been employed in formulating the models on which our work is based.

We have, first, what one may call the "Principle of Maximal Local Symmetry"[29] which requires that our Lagrangian be invariant under all possible symmetries or, equivalently, that we gauge the maximal symmetry group of the kinetic term for the fundamental fields.

This leads to the second principle, that of "Elementarity" which requires that the fundamental fields are fermions and all other particles are bound states of these fields. In the usual formulations, leptons are elementary and the basic degrees of freedom for hadrons are quarks. This is the approach we follow in most of this work. In Chapters 5 and 6, however, the existence of a new class of fundamental fermions (preons) for both leptons and hadrons will be discussed.

Finally, we rely on the "Gauge Principle" which requires that all interactions arise within a nonabelian gauge theory. For a genuinely unified theory, the symmetry group must be simple or semi-simple such that the factor groups are related to each other by a discrete transformation. This assumes that there will be only one fundamental coupling constant.

The maximal gauge group required by the first principle, is, as we have stated, the symmetry group of the kinetic term for the elementary fermions. Parity invariance requires that the kinetic bilinear $\psi \, i \, \not\partial \, \psi$ consist of equal numbers of right and left handed fields so, if ψ is an n-component spinor (i.e., if there are n fundamental fermions) the maximal symmetry group in $U(2n) \approx U(1) \times SU(2n)$. The generator of the factored U(1) group corresponds to a γ_5 transformation.

It is possible (and even natural) to identify one of the generators of SU(2n) as the fermion number operator[8, 29]. When the SU(2n) symmetry is broken, the gauge boson associated with fermion number will acquire a mass and we expect this boson will participate in fermion number violating transitions[8, 29]. Naturally, such a boson will have to be superheavy.

The first stage of symmetry breaking, then, is from SU(2n) to the $\Delta F = 0$ subgroup which is $SU(n)_L \times SU(n)_R$, the chiral symmetry group of n massless fermions. This is the maximal subgroup of SU(2n) which commutes with fermion number but does not contain it. The basic Lagrangian for this group is[8, 30]:

$$\mathcal{L} = -\tfrac{1}{4}\, \mathrm{tr}\, G^L_{\mu\nu} G^{\mu\nu}_L - \tfrac{1}{4}\, \mathrm{tr}\, G^R_{\mu\nu} G^{\mu\nu}_R$$

$$+ \bar{f}\left[\gamma^\mu \frac{1+\gamma_5}{2}(i\partial_\mu - gB^L_\mu) + \gamma^\mu \frac{1-\gamma_5}{2}(i\partial_\mu - gB^R_\mu)\right] f$$

where L and R refer, respectively, to gauge bosons coupled to the left and right handed fermions and the fermion spinor f transforms like (n,1) + (1,n) under $SU(n)_L \times SU(n)_R$. The field tensor is:

$$G^{L,R}_{\mu\nu} = \partial_\nu A^{L,R}_\mu - \partial_\mu A^{L,R}_\nu + ig\left[A^{L,R}_\nu , A^{L,R}_\mu\right] .$$

The adjoint representation of the group U(n) is described by the E^n_m - matrices of Weyl[8, 31] which are nxn matrices that satisfy:

$$\left(E^n_m , E^p_q\right) = \delta^n_q E^p_m - \delta^p_m E^n_q .$$

The adjoint representation of SU(n) is then given by the matrices

$$M^n_m = E^n_m - \frac{1}{n} \delta^n_m \cdot 1$$

which satisfy the same algebra as the E^n_m. The U(1) operator of $U(n)_L \times U(n)_R$ which commutes with $SU(n)_L \times SU(n)_R$ then defines the fermion number operator. All the generators of this subgroup of SU(2n) carry a well defined fermion number and the irreducible representations can be classified according to this fermion number content[29].

CHAPTER 2

A SURVEY OF SOME POPULAR MODELS

(2.1) THE WEINBERG MODEL[3]

As we remarked earlier, the Salam-Weinberg model based on SU(2) X U(1) does not represent a true unification of weak and electromagnetic interactions because it requires two fundamental coupling constants. This is the reason why it is necessary to introduce the "mixing" angle

$$\tan\theta = \frac{g_{U(1)}}{g_{SU(2)}}$$

in terms of which all physical predictions are made. Furthermore, such a model is displeasing because it fails to provide a rationale for the existence of two different types of neutrinos and because it deals with the left and right-handed parts of the leptons field in qualitatively different ways.

To do away with these problems, Weinberg proposed that the SU(2) X U(1) group be embedded in an $SU(3)_L \times SU(3)_R$ group and that there are three fundamental four component leptons which transform as the triplet[32]:

$$\ell = \begin{pmatrix} \mu^+ \\ \nu \\ e^- \end{pmatrix}$$

where the four component neutrino is formed out of the electron and muon type neutrinos:

$$\nu_L \equiv \nu_e \quad , \quad \nu_R \equiv \nu_\mu^c$$

To permit only one fundamental coupling constant, it is further assumed that the Lagrangian conserves parity.

The model contains 16 gauge fields A_M (the index M running over L_α and R_α) whose interaction with leptons is given by:

$$\mathcal{L} = if \sum_M (\bar{\ell} \gamma_\mu t_M \ell) A_M^\mu$$

where f is a single dimensionless coupling constant and,

$$t_{\alpha L} = \tfrac{1}{2}(1 + \gamma_5) t_\alpha$$

$$t_{\alpha R} = \tfrac{1}{2}(1 - \gamma_5) t_\alpha$$

with t_α the usual Hermitians SJ(3) matrices.

Unlike the treatment of SU(3) X SU(3) by Salam and Ward(33), Weinberg assumes gauge invariance under the whole group so the model will include ten additional charged vector bosons as well as three massive meutral ones. Many of these gauge particles give rise to unobserved interactions such as those which violate the conservation of muon number.

To account for the suppression of these exotic interactions, it is necessary that the gauge bosons mediating them acquire very large masses $\gtrsim$ 200 GeV.. This is much greater than the masses of the particles which mediate the known interactions so there must exist two scales of spontaneous symmetry breaking represented by the diagram

$$
\begin{array}{ll}
\text{I} & SU(3)_L \times SU(3)_R \\
 & \downarrow \\
\text{II} & SU(2) \times U(1) \\
 & \downarrow \\
 & U(1)
\end{array}
$$

and the breaking in the first stage must be stronger than in the record. To the extent that effects due to exchange of the superheavy gauge bosons are ignored, the known weak interactions will appear as if the fundamental gauge group were still SU(2) X U(1) instead of SU(3) X SU(3).

However, even neglecting the effects of the superheavy gauge mesons, one new feature of the SU(3) X SU(3) model remains and that is the ratio of the coupling constants which is now fixed and

$$\tan\theta = \frac{1}{\sqrt{3}}$$

In effect, the SU(3) X SU(3) model is exactly like the SU(2) X U(1) theory with $|\theta| = 30^o$ apart from terms of order $^{m_e}/_{m_\mu}$, the effects of the superheavy bosons and a richer spectrum of scalar bosons.

(2.2) THE BARS, HALPERN, HOSHIMURA MODEL[4]

This model represents a unification of the strong interactions theory of Bardakci and Halpern[34] and the weak-electromagnetic theory of Salam[22] and Weinberg[23]. The gauge group is $U(3)_L \times U(3)_R \times SU(2) \times U(1)$ though difficulties with strangeness changing processes require the $SU(2) \times U(1)$ to be embedded in a global $U(4)_L \times U(4)_R$.

If we denote the strong vector and axial vector fields by V^μ_α and A^μ_α and the weak gauge fields by W^μ_k and B^μ and define:

$$V^\mu_{L,R} \equiv \sum_0^8 (V^\mu_\alpha \mp A^\mu_\alpha)\tfrac{1}{2}\,\lambda^\alpha$$

$$\tilde{W} \equiv \sum_1^3 \tau_k\, W_k$$

where λ^α and τ_k are the generators of the strong and weak groups, respectively, the transformation properties of all the gauge bosons may be specified in terms of a diagonal matrix with entries:

$$V^\mu : \left\{ f\,V^\mu_L,\ f\,V^\mu_R,\ g'(\tau_3 + \tfrac{1}{3}\,\tau_0)B^\mu,\ g\,\tilde{W}^\mu + \tfrac{1}{3}\,g'\,\tau^\mu_0\,B \right\}$$

The leptons of Weinberg and Salam can be specified (with $\nu_L = \nu_e$, $\nu_R = \nu^c_\mu$) by:

$$\psi_D = \begin{pmatrix} \nu_L & \mu^+_R & 0 \\ e^-_L & -\nu_R & \\ 0 & & ? \end{pmatrix}$$

$$\psi_S = \begin{pmatrix} 0 & \mu_L^+ & 0 \\ e_R^- & 0 & \\ 0 & & ? \end{pmatrix}$$

$$\ell = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{\psi}_S & \frac{1}{2}\sqrt{2\tilde{\psi}_D^c} \\ 0 & 0 & \frac{1}{2}\sqrt{2\tilde{\psi}_D} & 0 \end{pmatrix}$$

where D and S mean doublet and singlet, respectively, and the question marks stand for heavy leptons that will have to be added to deal with anomalies. Only three fractionally charged quarks are needed

$$q^T = (q_L, q_R, 0, 0)$$

and the scalar mesons are denoted by:

$$M = \begin{pmatrix} 0 & \Sigma & 0 & M_L \\ \Sigma^+ & 0 & M_R & 0 \\ 0 & M_R^+ & 0 & \tilde{\phi}^+ \\ M_L^+ & 0 & \tilde{\phi} & 0 \end{pmatrix}$$

where $\Sigma = \sigma + i\pi$ is the usual $(3, \bar{3}) + (\bar{3}, 3)$ multiplet of scalars and pseudoscalars and $M_{L,R}$ are three by four complex matrices. The field ϕ is as in the Salam-Weinberg model and

$$\phi = \phi_o \tau_o + i\phi.\tau$$

These scalars are the only connections of the weak and electromagnetic interactions with the strong and will give mass to the strong vector mesons.

The completion of spontaneous symmetry breaking is very complicated and we will only briefly mention the important features. Twenty-one degrees of gauge freedom are required to eliminate the 3 x 3 submatrices of $M_L - M_L^+$ and $M_R - M_R^+$ and all the components of ϕ except ϕ_o. To give masses to all gauge fields except the photon, the following vacuum expectation values are assigned:

$$\langle\phi\rangle \equiv \lambda \tau_o$$

$$\langle M_L\rangle = \langle M_R\rangle \equiv \kappa \ .$$

These, in turn, generate a linear term in Σ in the Lagrangian and Σ acquires an expectation value:

$$\langle\Sigma\rangle \equiv V$$

which is the $(3, \bar{3}) + (\bar{3}, 3)$ hadronic symmetry breaking of Gell-Mann, Oakes and Renner[35]. The following complexions are allowed.

$$\kappa = \begin{pmatrix} \kappa_1 & 0 & 0 & 0 \\ 0 & \kappa_1 & 0 & 0 \\ 0 & 0 & \kappa_2 & 0 \end{pmatrix}$$

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} f_\pi & 0 & 0 \\ 0 & f_\pi & 0 \\ 0 & 0 & 2f_k - f_\pi \end{pmatrix}$$

There are no Goldstone bosons.

The spontaneous breakdown is such that only the photon remains massless. The diagonalisation of the photon is such that electromagnetic mixing is induced between bare ρ^0, ω and ϕ such that the physical particles have order e^2/f couplings directly to leptonic electromagnetic currents. This simulates vector-dominated electromagnetic form factors in lowest order.

Because the Cabibbo rotations affect only $W^\pm$, there are no $\Delta S = 1$ neutral currents. The particles which make this suppression possible are the (unobservable) $M_{L,R}$ and not additional quarks.

As indicated earlier, the model has anomalies and it is not possible to cancel hadronic against leptonic ones. So, a "doubling" is proposed where (heavy) q', $\psi'_{S,D}$ are introduced that couple to the same gauge bosons as q, $\psi_{S,D}$ but with the opposite chirality. Thus, the complete 4 x 4 lepton matrices are:

$$\psi_D = \begin{pmatrix} \nu_{eL} & \mu_R^+ & 0 & 0 \\ e_L^- & -(\nu_\mu^c)_R & 0 & 0 \\ 0 & 0 & -\nu'_{L\mu} & e_R'^- \\ 0 & 0 & \mu_L'^+ & (\nu_e'^c)_R \end{pmatrix}$$

$$\psi_S = \begin{pmatrix} 0 & \mu_L^+ & 0 & 0 \\ e_R^- & 0 & 0 & 0 \\ 0 & 0 & \beta'\nu'_{\mu R} & e_R'^- \\ 0 & 0 & \mu_R'^+ & \alpha'\nu_{eR}' \end{pmatrix}$$

In the leptonic system, anomalies are cancelled without difficulties. To avoid suppressing $\pi^0 \to 2\gamma$, one has to introduce a "heavy pion" field $\Sigma' = \sigma' + i\pi'$ which couples only to q'. It can then be arranged that $\pi^0 \to 2\gamma$ proceeds only through q.

(2.3) THE GEORGI-GLASHOW MODEL[5]

This approach is interesting because of the originality and simplicity of the scheme. We begin by listing some of the assumptions it requires.

The minimal number of fundamental fermions is desired and, in particular, no unobserved leptons are postulated. In this case, the Salam-Weinberg model is unique up to an extension of the gauge group. The known leptons may be represented by six left-handed fields (e_L^-, μ_L^-, ν_L, ν_L', e_L^+, μ_L^+) and their charge conjugates. These fields are to transform as a representation of the gauge group which can be any one of the 23 subgroups of U(6) which contains SU(2) X U(1).

To include hadrons in the theory, the GIM mechanism is employed and three quartets of fractionally charged quarks are introduced. Strong interactions are mediated by an SU(3) colour octet of gluons and it is assumed that there are no fundamental strongly interacting scalar mesons. This assures that parity and hypercharge are conserved to order α and maintains renormalisability.

A simple unification based on $\mathfrak{J}$ = SU(3) X SU(2) X U(1) is rejected on the grounds that several coupling constants are required so a larger group, containing $\mathfrak{J}$, is sought. In searching for a group of the form SU(3) X W where W contains SU(2) X U(1), it is remarked that, since leptons are singlets under colour SU(3), leptons and quarks must lie in separate representations of W. If only the six known leptons are involved, W must be one of the 23 afore mentioned subgroups of U(6). The only possibilities involving a single coupling constant are SU(3), SU(3) X SU(3) and SU(6). But none of these models can describe hadrons (so it is claimed) because the electric charge operator does not admit fractional charges and, because it is traceless, it does not explain why the sum of the quark charges is not zero. Thus, groups of the form SU(3) X W are rejected.

The remaining alternative is that the gauge group is simple (or a direct product of isomorphic simple groups) and contains $\mathfrak{J}$ as a subgroup. The unique aspects of this assertion are that quarks and leptons must lie in the same irreducible representation of the gauge group and that the same coupling constant (the fine structure constant) characterises all basic interactions. The latter idea has been given support in Chapter 1 where the concept of "infrared slavery" suggesting that the infrared divergences of Yang-Mills theory lead to phenomonological strong interactions was introduced. The former concept will reappear in Chapter 3.

The goal, then, is to find a gauge group whose only coupling constant is the unit of electric charge and which contains $\mathscr{G}$ in the appropriate fashion. This symmetry group will be spontaneously broken leaving only the direct product of colour SU(3) (which causes infrared slavery and strong interactions) and electromagnetic U(1) as exact local symmetries. All gauge fields except the photon will acquire mass and those associated with SU(3) X U(1) will mediate know interactions. The rest will be responsible for exotic interactions and must be heavier.

The unifying group must be at least of rank 4. The nine rank 4 lie groups which can involve only one coupling strength are: $(SU(2))^4$, $(O(5))^2$, $(SU(3))^2$, $(G_2)^2$, O(8), O(9), Sp(8), F_4 and SU(5). The first two can be discarded as they do not contain SU(3). To distinguish between the remaining cases, it is necessary to examine the behaviour of quarks and leptons under $\mathscr{G}$.

All fermion fields are represented by left-handed two-component spinors of which there are thirty: four leptons $(\mu^-, \nu', e^-, \nu)_L$, two antileptons $(\mu^+, e^+)_L$, twelve quarks

$(p_i', p_i, n_i, \lambda_i)_L$ and twelve antiquarks $(\bar{p}_i', p_i, \bar{n}_i, \bar{\lambda}_i)_L$ where $i = 1, 2, 3$ is the colour index. Under SU(3) X SU(2), the leptons are SU(3) singlets and SU(2) doublets whereas the antileptons are singlets under both groups. The quarks are SU(3) triplets and SU(2) doublets while the antiquarks are SU(3) antitriplets and SU(2) singlets. Thus, the SU(3) X SU(2) content of the fundamental fermion can be represented as:

$$2(\mathbf{1}, 2) + 2(\mathbf{1}, 1) + 2(\mathbf{3}, 2) + 4(\bar{\mathbf{3}}, 1).$$

This representation is complex and not equivalent to its complex conjugate so the same must be true for the fundamental group. Of the groups just mentioned, only $(SU(3))^2$ and SU(5) admit complex representations. The first choice has already been ruled out even for a unification of weak and electromagnetic interactions which leaves just SU(5).

Under SU(3) X SU(2), the first fundamental (five dimensional) representation of SU(5) transforms like $(\mathbf{1}, 2) + (\mathbf{3}, 1)$ and the complex conjugate $\bar{\mathbf{5}}$ like $(\mathbf{1}, 2) + (\bar{\mathbf{3}}, 1)$. The irreducible ten dimensional representation given by the antisymmetrised tensor product of two $\mathbf{5}$'s transforms like $(\mathbf{1}, 1) + (\bar{\mathbf{3}}, 1) + (\mathbf{3}, 2)$. So, if the thirty fundamental left-handed fermions transform like two $\mathbf{10}$'s and two $\bar{\mathbf{5}}$'s, the $\mathcal{G}$ content will be correct. Replacing the two $\bar{\mathbf{5}}$'s of left-handed fields by two $\bar{\mathbf{5}}$'s of their right-handed charge conjugates, the representations containing electrons are then a $\underset{\sim}{5}$ and a $\underset{\sim}{10}$ and can be displayed as follows:

$$\begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ e^+ \\ \nu \end{pmatrix}_R , \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{p}_3 & -\bar{p}_2 & -p_1(\theta) & -n_1 \\ -\bar{p}_3 & 0 & \bar{p}_1 & -p_2(\theta) & -n_2 \\ \bar{p}_2 & -p_1 & 0 & -p_3(\theta) & -n_3 \\ p_1(\theta) & p_2(\theta) & p_3(\theta) & 0 & -e^+ \\ n_1 & n_2 & n_3 & e^+ & 0 \end{pmatrix}_L$$

where $p(\theta) = p \cos \theta - p' \sin \theta$. The $\underset{\sim}{5}$ and $\underset{\sim}{10}$ containing muons are obtained from these by the replacements:

$$e^+ \to \mu^+, \ \nu \to \nu', \ n \to \lambda, \ p \to p'$$

and

$$p(\theta) \to p'(\theta) = p' \cos \theta + p \sin \theta .$$

The $\mathcal{G}$ subgroup is known to be free of anomalies (for the representation chosen) and, remarkably enough, so is SU(5) for the representations given. The $\underset{\sim}{\bar{5}}$ and $\underset{\sim}{10}$ anomalies cancel. It seems that SU(5) is the only group of any rank with a thirty-dimensional, anomaly free representation with the correct $\mathcal{G}$ content.

To produce the required pattern of spontaneous symmetry breaking, two irreducible representations of Higgs mesons are introduced. The strongest breaking, from SU(5) to SU(3) X SU(2) X U(1) is accomplished with 24 real scalars that transform like the adjoint representation and acquire very large vacuum expectation values. All the gauge mesons except the twelve associated with $\mathcal{G}$ will attain superheavy masses and their effect will be small. The Higgs mesons which give masses to the fermions and the weak bosons are taken to be five complex scalars transforming like the fundamental representation and forty-five complex scalars transforming like the $\underset{\sim}{45}$ contained in $\underset{\sim}{\bar{5}}$ X $\underset{\sim}{10}$. This provides the most general zeroth order mass matrix consistent with exact SU(3) colour symmetry. If only the $\underset{\sim}{5}$ were present, the p and p' masses and the Cabibbo angle are arbitrary.

We should mention that the superheavy gauge particles that arise in the first stage of symmetry breaking mediate exotic interactions like $K^0 \rightarrow \mu^+ e^-$ and proton decay, similar to the Pati-Salam model to be discussed in Chapter 3 though there are fewer restrictions in this scheme on the allowed rates.

The only easily tested prediction of the model is the ratio of the SU(2) X U(1) coupling constants given as:

$$\sin^2 \theta = \frac{3}{8}$$

CHAPTER 3

LEPTON NUMBER AS A FOURTH COLOUR

(3.1) THE "BASIC" MODEL AND ITS VARIANTS

The assumption underlying all of the models we will discuss is that quarks possess four colours. Three of these correspond to the generally accepted hadron colours and the fourth corresponds to lepton number. The unification of hadronic and leptonic matter is formally implemented by extending the ordinary colour SU(3') for hadrons to colour SU(4').

Assuming that quarks also possess four valencies representing an $SU(4)_L \times SU(4)_R$ group structure, the global symmetry that suggests itself is

$$G = SU(4)_L \times SU(4)_R \times SU(4')$$

This symmetry is realized by the following set of fermions:

$$\psi_{L,R} = \begin{pmatrix} p_a & p_b & p_c & p_d & = & \nu \\ n_a & n_b & n_c & n_d & = & e^- \\ \lambda_a & \lambda_b & \lambda_c & \lambda_d & = & \mu^- \\ \chi_a & \chi_b & \chi_c & \chi_d & = & \nu' \end{pmatrix}_{L,R} \qquad (2)$$

which transform as:

$$\psi_L = (4,\ 1,\ \bar{4})_G$$
$$\psi_R = (1,\ 4,\ \bar{4})_G.$$

The three hadronic quartets have been identified with the twelve "accepted" quarks and the leptonic quartet with the four known leptons[36].

The maximal anomaly free subgroup of the valency group is $Sp(4)_L \times Sp(4)_R$ and the maximal anomaly free coloured group is $SU(4')_{L+R}$. Thus, we could gauge

$$Sp(4)_L \times Sp(4)_R \times SU(4')_{L+R}.$$

However, no essential features of the model are lost if we gauge the smaller group

$$\mathcal{G} = \left(SU(2)_L^{I+II}\right) \times \left(SU(2)_R^{I+II}\right) \times SU(4')_{L+R}$$

under which the fermions transform as:

$$\psi_L = (2+2,\ 1,\ \bar{4})_{\mathcal{G}}$$
$$\psi_R = (1,\ 2+2,\ \bar{4})_{\mathcal{G}}$$

The groups $SU(2)_{L,R}^{I+II}$ represent the diagonal sum of groups acting on the Cabibbo rotated $(p,\ n)_{L,R}$ and $(\lambda,\ \chi)_{L,R}$ indices, respectively.

The gauge particles associated with the weak groups are:

$$W_L = (3,\ 1,\ 1) = \left(\begin{array}{c|c} \underset{\sim}{\tau}\cdot\underset{\sim}{W}_L & 0 \\ \hline 0 & \tau_1(\underset{\sim}{\tau}\cdot\underset{\sim}{W}_L)\tau_1 \end{array}\right)$$

$$W_R = (1,\ 3,\ 1) = \left(\begin{array}{c|c} \underset{\sim}{\tau}\cdot\underset{\sim}{W}_R & 0 \\ \hline 0 & \tau_1(\underset{\sim}{\tau}\cdot\underset{\sim}{W}_R)\tau_1 \end{array}\right)$$

which couple with strengths $g_L^2/4\pi \simeq \alpha$, $g_R^2/4\pi \simeq \alpha$, respectively and the strong gauge particles are:

$V(1,\ 1,\ 15)_{gs}$ =

$$\begin{pmatrix} V_3 + \sqrt{\frac{1}{3}}\,V_8 - \frac{1}{2}\sqrt{\frac{1}{6}}\,S^o & V_\rho^- & V_K^{-}* & \bar{X}^o \\ V_\rho^+ & \sqrt{\frac{1}{3}}\,V_8 - V_3 - \frac{1}{2}\sqrt{\frac{1}{6}}\,S^o & \bar{V}_K^o* & X^+ \\ V_K^+* & V_K^o* & -2\sqrt{\frac{1}{3}}\,V_8 - \frac{1}{2}\sqrt{\frac{1}{6}}\,S^o & X'^+ \\ X^o & X^- & X'^- & \sqrt{\frac{3}{8}}\,S^o \end{pmatrix}$$

which couple with strength $f^2/4\pi \simeq 1\text{-}10$. The usual SU(3') octet of gluons has been augmented by S^o which is an SU(3') singlet and the exotic X's which carry B = +1, L = -1 and are an SU(3') triplet.

The Lagrangian describing the interaction of these gluons with the fermions is:

$$\mathcal{L}_{int.} = g_L \sum_{\alpha=a,b,c,d} (\bar{p}_\alpha \; \bar{n}_\alpha \; \bar{\lambda}_\alpha \; \bar{\chi}_\alpha)(W_L)_\mu \, \gamma_\mu \frac{1+\gamma_5}{2} \begin{pmatrix} p_\alpha \\ n_\alpha \\ \lambda_\alpha \\ \chi_\alpha \end{pmatrix}_L$$

$$+ g_R \sum_{\alpha=a,b,c,d} (\bar{p}_\alpha \; \bar{n}_\alpha \; \bar{\lambda}_\alpha \; \bar{\chi}_\alpha)_R \, (W_R)_\mu \, \gamma_\mu \frac{1-\gamma_5}{2} \begin{pmatrix} p_\alpha \\ n_\alpha \\ \lambda_\alpha \\ \chi_\alpha \end{pmatrix}_R$$

$$+ f \sum_{i=p,n,\lambda,\chi} (\bar{\psi}^i_a \; \bar{\psi}^i_b \; \bar{\psi}_c \; \bar{\psi}^i_d)_{L+R} \, V_\mu \, \gamma_\mu \begin{pmatrix} \psi^i_a \\ \psi^i_b \\ \psi^i_c \\ \psi^i_d \end{pmatrix}_{L+R}$$

When we discuss the spontaneous symmetry breaking mechanism, we will display the complete Lagrangian after the appropriate Cabibbo rotations.

The electric charge operator is given by[37]:

$$Q = I_{3L}^{I+II} + I_{3R}^{I+II} + \left(\alpha F_3' + \frac{\beta}{\sqrt{3}} F_8' - \sqrt{\tfrac{2}{3}}\, f_{15}'\right)$$

where $I_{3L,R}^{I+II}$ represent the diagonal generators of $SU(2)_{L,R}^{I+II}$ and F_3', F_8' and F_{15}' are the diagonal generators of $SU(4')_{L+R}$. The coefficients α and β are arbitrary. While this operator can provide for both integrally ($\alpha = \beta = 1$) and fractionally ($\alpha = \beta = 0$) charged quarks, the charge assignments for the lepton quartet (0, -1, -1, 0) are unique.

We close this section by listing the possible alternatives to the "basic" model just described. In each case we distinguish between electron colour and muon colour.

(a) <u>The "Economical" Model</u>

The basic fermions are:

$$(\psi_e)_{L,R} = \begin{pmatrix} p_a & p_b & p_c & \nu \\ n_a & n_b & n_c & e^- \end{pmatrix}_{L,R}$$

$$(\psi_\mu)_{L,R} = \begin{pmatrix} \lambda_a & \lambda_b & \lambda_c & \mu^- \\ \chi_a & \chi_b & \chi_c & \nu' \end{pmatrix}_{L,R}$$

with symmetry group:

$$SU(2)_L \times SU(2)_R \times SU(4')_e \times SU(4')_\mu .$$

The physical SU(3') must now be regarded as a diagonal sum of $SU(3')_e$ and $SU(3')_\mu$.

(b) The "Prodigal" Model

The underlying group structure is the same as the "basic" model but there are more fundamental fermions. Here,

$$(\psi_e)_{L,R} = \begin{pmatrix} p_a & p_b & p_c & E^o \\ n_a & n_b & n_c & E^- \\ \lambda_a & \lambda_b & \lambda_c & e^- \\ \chi_a & \chi_b & \chi_c & \nu \end{pmatrix}_{L,R}$$

$$(\psi_\mu)_{L,R} = \begin{pmatrix} p'_a & p'_b & p'_c & M^o \\ n'_a & n'_b & n'_c & M^- \\ \lambda'_a & \lambda'_b & \lambda'_c & \mu^- \\ \chi'_a & \chi'_b & \chi'_c & \nu' \end{pmatrix}_{L,R}$$

In addition to new (primed) quarks, we have additional heavy leptons. Note that both neutrinos may carry charm. Apart from the "basic" $SU(4)_L \times SU(4)_R \times SU(4')$, the group structure may be taken to be $SU(4)_L \times SU(4)_R \times SU(4')_e \times SU(4')_\mu$ making the distinction between e and μ even sharper.

(c) The "Five-Colour" Model

Finally, the electron, muon and the new, heavy leptons may be put in a single 20-fold multiplet with an $SU(4)_L$ X $SU(4)_R$ X SU(5) group structure. Here,

$$\psi_{L,R} = \begin{pmatrix} p_a & p_b & p_c & E^o & M^o \\ n_a & n_b & n_c & E^- & M^- \\ \lambda_a & \lambda_b & \lambda_c & e^- & \mu^- \\ \chi_a & \chi_b & \chi_c & \nu & \nu' \end{pmatrix}_{L,R}$$

While there is little that distinguishes one variant from the others, all are distinguished from the "basic" model by the forbidden transition $K^o \rightarrow e^- + \mu^+$. In the "basic" model, this reaction is mediated by the X mesons which must, therefore, be superheavy ($m_X \gtrsim 10^4 - 10^5$ GeV) while in the variants the X's may be as light as $10^2 - 10^3$ GeV. Another consequence related to X mass is quark and proton decay discussed in sections (4.1) and (4.2).

(3.2) RESTRICTIONS ON GLUON MASSES

As a result of gauging the full SU(4) colour symmetry and insisting that the Lagrangian exhibit complete symmetry between left and right helicities ($g_L = g_R$), all the models discussed in the last section predict exotic interactions in both the weak and strong sector. The absence of these interactions at presently observed energies imposes restrictions on the masses of the gauge mesons and we now come to consider these.

The X triplet, whose interactions in the "basic" model are:

$$f\big(X^0(\bar{\nu}p_a + \bar{e}\, n_a + \bar{\mu}\,\lambda_a + \bar{\nu}'\,\chi_a)$$

$$+ X^-(\bar{\nu}p_b + \bar{e}\, n_b + \bar{\mu}\,\lambda_b + \bar{\nu}'\,\chi_b)$$

$$+ X^{-'}(\bar{\nu}p_c + \bar{e}\, n_c + \bar{\mu}\,\lambda_c + \bar{\nu}'\,\chi_c) + h.c.\big),$$

contribute to η^0, $\pi^0 \to e^+e^-$, $\mu^+\mu^-$ and, in the "basic" model only, to $K^0 \to e^- + \mu^+$, $\bar{K}^0 \to e^+ + \mu^-$. As the observed amplitude for $K_L \to \mu^+ + \mu^-$ is of order $G_F\,\alpha^2$ and no events of the type $K_L \to \mu^\pm + e^\mp$ have ever been observed, we must have $f^2/m_X^2 \lesssim G_F\,\alpha^2$. For $f^2/4\pi \approx 1$, $m_X > 10^4$ GeV. In the variants, $K^0 \to e^- + \mu^+$ is forbidden and the restrictions on X mass come chiefly from nuclear β decay and hadronic interactions of ν_e (in the "economical" model). As known hadrons do not carry charm, the hadronic interactions of ν_μ (the charmed neutrino) which are mediated by X are naturally suppressed. In the "prodigal" model, where both neutrinos are charmed, the restrictions coming from their hadronic interactions is even less severe. In fact, an X mass on the order of 100 GeV may be tolerated[38].

The S^0 meson, whose interaction is

$$f\Big(\sum_{\alpha=a,b,c} \{\bar{p}_\alpha\, p_\alpha + \bar{n}_\alpha\, n_\alpha + \bar{\lambda}_\alpha\,\lambda_\alpha + \bar{\chi}_\alpha\,\chi_\alpha\}$$

$$- 3(\bar{\nu}\nu + \bar{e}e + \bar{\mu}\mu + \bar{\nu}'\,\nu')\Big)\, S^0,$$

leads to order f^2 interaction of neutrinos with hadrons and leptons. So that the effective strength of these interactions at low energies is of order G_F, we must have $f^2/m^2_{S^0} \lesssim G_F$.

Weak interaction data indicates that V + A amplitude are no more than 10% of V - A amplitudes. Since we are assuming $g_L = g_R$, we expect $m_{W_R^\pm} \gtrsim 3m_{W_L^\pm}$.

Finally, the masses of $W_L^\pm$ and the SU(3') octet are restricted by the fact that they mediate the known V - A weak and effective strong interactions. Thus, we expect $(m_{W_L^\pm})^2 \gtrsim G_F^{-1}\alpha$ and $m(V(8)) \approx$ (3-10) GeV.

(3.3) SPONTANEOUS SYMMETRY BREAKING IN THE "BASIC" MODEL

As always, spontaneous symmetry breaking is affected by choosing a set of Higgs scalars that acquires non-vanishing vacuum expectation values that minimize the scalar potential. In the "basic" model, a satisfactory set is the three 16-fold complex multiplets

$$A = (4, \bar{4}, 1)_G$$
$$B = (1, 4, \bar{4})_G$$
$$C = (\bar{4}, 1, 4)_G$$

where G is the global group $U(4)_L \times U(4)_R \times U(4')$.

The most general renormalizable potential, V(A, B, C), for these multiplets, which contains twelve parameters (or fifteen if we restrict the global group to $SU(4)_L \times SU(4)_R$

X $SU(4')_{L+R}$), can be shown to possess a minimum[39] if the vacuum expectation values are of the form

$$<A> = R(\theta_L)R(\phi_L)<A_D>R^{-1}(\theta_R)R^{-1}(\phi_R)$$

$$<B> = R(\theta_R)R(\phi_R)<B_D>$$

$$<C> = <C_D>R^{-1}(\theta_L)R^{-1}(\phi_L).$$

where $R(\theta)$ and $R(\phi)$ represent Cabibbo type rotations[40] in the (n, λ) and (p, χ) spaces, respectively. For example,

$$R(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The matrices $<A_D>$, $<B_D>$ and $<C_D>$ are diagonal and of the form:

$$<A_D> = \begin{pmatrix} a_1 & & & \\ & a_1 & & \\ & & a_1 & \\ & & & a_4 \end{pmatrix}$$

$$<B_D> = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & b_4 \end{pmatrix}$$

$$\langle C_D \rangle = \begin{pmatrix} c_1 & & & \\ & c_1 & & \\ & & c_1 & \\ & & & c_4 \end{pmatrix}$$

We may now exhibit the complete Lagrangian for the "basic" model which is:

$$\mathcal{L} = \mathrm{tr}\Big[\sum_{L,R} (\bar{\psi} \not{\nabla} \psi) + \sum_{A,B,C} |\nabla A|^2 + \sum_{V, W_L, W_R} |\nabla V|^2$$

$$+ V(A, B, C) + f\, \bar{\psi}_L A \psi_R + \text{h.c.}\Big]$$

where

$$\not{\nabla} \psi_{L,R} = \gamma_\mu (\partial_\mu \psi + ig\, W_\mu\, \psi - if\, \psi\, V_\mu)_{L,R}$$

$$\nabla A = \partial A + ig_L\, W_L A - ig_R A W_R$$

$$\nabla B = \partial B + ig_R\, W_R B - ifBV$$

$$\nabla C = \partial C + ifVC - ig_L C W_L .$$

In view of the Cabibbo rotations in the SU(2) spaces, we should write $W_{L,R}$ as[41]:

$$W_{L,R}(\theta_{L,R}, \phi_{L,R}) = R^{-1}(\theta_{L,R}) R^{-1}(\phi_{L,R}) W_{L,R} R(\theta_{L,R}) R(\phi_{L,R})$$

$$
= \begin{pmatrix}
W^o_{L,R} & W^-_{L,R}\cos\omega_{L,R} & -W^-_{L,R}\sin\omega_{L,R} & 0 \\
W^+_{L,R}\cos\omega_{L,R} & -W^o_{L,R} & 0 & W^+_{L,R}\sin\omega_{L,R} \\
-W^+_{L,R}\sin\omega_{L,R} & 0 & -W^o_{L,R} & W^+_{L,R}\cos\omega_{L,R} \\
0 & W^-_{L,R}\sin\omega_{L,R} & W^-_{L,R}\cos\omega_{L,R} & W^o_{L,R}
\end{pmatrix}
$$

where

$$\omega_{L,R} \equiv \theta_{L,R} + \phi_{L,R}.$$

For the considerations in the following chapters, it will be necessary to examine the gauge meson sector of the mass matrix in detail. Making the appropriate Cabibbo rotations on the relevant part of the Lagrangian and replacing the Higgs fields by their vacuum expectation values, we find(42):

$$\mathcal{L}_{\text{mass}} = \sum_{A,B,C} |\nabla A|^2$$

$$= g_L^2\{(a^2+c^2)((W_L^o)^2+2W_L^+ W_L^-)\}$$

$$- 2\, g_L\, g_R\{a^2\, W_L^o\, W_R^o+(a_1^2+a_1 a_4)(\cos\,\omega_L\,\cos\,\omega_R$$

$$+ \sin\,\omega_L\,\sin\,\omega_R)(W_L^-\, W_R^+ + W_L^+\, W_R^-)\}$$

$$+ g_R^2\{(a^2+a_4^2+b_4^2)((W_R^o)^2+2W_R^+\, W_R^-)\}$$

$$- 2\, g_R f b_4^2\, \sqrt{\tfrac{3}{8}}\, W_R^o\, S^o - 2\, g_L f\{c_1^2((2\, V_3+2\sqrt{\tfrac{1}{3}}\, V_8+\tfrac{1}{2}\sqrt{\tfrac{1}{6}}\, S^o)W_L^o\, +$$

$$+ (W_L^- V_\rho^+ + W_L^+ V_\rho^-)\cos\omega_L - (W_L^- V_{K^*}^+ + W_L^+ V_{K^*}^-)\sin\omega_L)$$

$$+ c_1 c_4((W_L^+ X^- + W_L^- X^+)\sin\omega_L + (W_L^+ X'^- + W_L^- X'^+)\cos\omega_L)$$

$$+ c_4^2 \sqrt{\tfrac{3}{8}}\, W_L^0 S^0\} + f^2\{2c_1^2(V_3^2 + V_8^2 + V_\rho^- V_\rho^+ + V_{K^*}^- V_{K^*}^+$$

$$+ \bar{V}_{K^*}^0 V_{K^*}^0)\} + (c_1^2 + c_4^2 + b_4^2)(X^0 \bar{X}^0 + X^- X^+ + X'^- X'^+)$$

$$+ (c_1^2 + c_4^2 + b_4^2)\tfrac{1}{8}(S^0)^2\} \tag{3}$$

where $a^2 = 3\,a_1^2 + a_4^2$, $c^2 = 3\,c_1^2 + c_4^2$ and $\omega_{L,R} = \theta_{L,R} + \phi_{L,R}$.

From eq. (3), we find the following masses for the gauge mesons.

$$m_{W_L^\pm} \approx g_L\,(a^2 + c^2)^{\frac{1}{2}}$$

$$m_{W_R^\pm} \approx g_R\,(a^2 + b_4^2)^{\frac{1}{2}}$$

$$m(V(8)) \approx \sqrt{2}\, f\, c_1$$

$$m(X^0, X^-, X^{-\prime}) \approx \sqrt{2}\, f\,(c_1^2 + c_4^2 + b_4^2)^{\frac{1}{2}}$$

$$m_{S^0} \approx \frac{f}{2\sqrt{2}}\,(c_1^2 + c_4^2 + b_4^2)^{\frac{1}{2}}$$

The restrictions on the masses of these particles outlined in Section (3.2) can be satisfied if we take[43]:

$$(c_1, c_4) \approx 1 \text{ GeV}$$

$$(a_1, a_4) \approx 300 \text{ GeV}$$

$$b_4 \approx 10^4 - 10^5 \text{ GeV}$$

The neutral eigenstates of the mass matrix are[44]:

$$\frac{A}{e} = \left(\frac{W_L^3}{g_L} + \frac{W_R^3}{g_R}\right) + \frac{(V_3 + V_8/\sqrt{3} - \frac{2}{\sqrt{3}} S^o)}{f}, \quad m_A = 0$$

which is the photon,

$$Z^o = \frac{f(g_R\ W_R^3 - g_L\ W_L^3) - \frac{2}{3}\ g_R^2\ S^o + 0(\frac{g^2}{f^2}, \frac{c^2}{a^2})}{\left(f^2(g_R^2 + g_L^2) + \frac{2}{3}\ g_R^4\right)^{\frac{1}{2}}}, \quad m_{Z^o} \approx \frac{(g_L^2+g_R^2)^{\frac{1}{2}} a}{2},$$

$$\tilde{S}^o = \frac{fS^o + \sqrt{\frac{2}{3}}\ g_R\ W_R^3 + 0(\frac{g^2}{f^2}, \frac{c^2}{a^2})}{\left(f^2 + \frac{2}{3}\ g_R^2\right)^{\frac{1}{2}}}, \quad m_{\tilde{S}^o} \approx \sqrt{\frac{3}{8}}\ f\ b_4$$

$$\tilde{U}_o = \frac{g_R\ W_L^3 + g_L(1+\Delta)W_R^3 - \sqrt{3}\ \bar{f}\ U^o - \sqrt{\frac{2}{3}}\ \frac{g_L\ g_R}{f}(1+\Delta+\frac{4}{3}\ \frac{c_1^2}{b_4^2})S^o}{\left(g_R^2 + g_L^2 + 3\bar{f}^2 + \frac{2}{3}(g_L\ g_R/_{f^2})\right)^{\frac{1}{2}}}$$

$$m_{\tilde{U}_o} \approx \frac{fc_1}{\sqrt{2}}$$

and

$$V^o = \frac{V_3 - \sqrt{3}\ V_8}{2}, \quad m_{V^o} \approx \frac{fc_1}{\sqrt{2}}$$

where

$$\frac{1}{e^2} = \frac{1}{g_L{}^2} + \frac{1}{g_R{}^2} + \frac{2}{f^2}$$

$$U^{o} = \tfrac{1}{2}\left(\sqrt{3}\, V_3 + V_8\right)$$

$$\Delta = 2({}^{f^2}/{}_{g^2})({}^{c_1^2}/{}_{a^2})$$

$$\bar{f} = \frac{f(g_R^{\,2} + g_L^{\,2})}{2g_R\, g_L^{\,2}}$$

The charged eigenstates are[45]:

$$\tilde{V}_\rho^\pm \approx \cos\beta\, V_\rho^\pm + \sin\beta\, W_L^\pm$$

$$\tilde{V}_{K^*}^\pm \approx \cos\alpha\, V_{K^*}^\pm + \sin\alpha\, W_L^\pm$$

$$\tilde{W}_L^\pm \approx W_L^\pm - V_\rho^\pm \sin\beta - V_{K^*}^\pm \sin\alpha$$

where

$$\sin\alpha = -\sin\omega_L\, ({}^{m_V}/{}_{m_{W_L}})^2\, ({}^{g}/{}_{f})$$

$$\sin\beta = -\cos\omega_L\, ({}^{m_V}/{}_{m_{W_L}})^2\, ({}^{g}/{}_{f})$$

The W_L - X mixing, to which we have previously drawn attention, is responsible for the violation of baryon-lepton number and gives rise to an effective propagator

$$\langle W_L\, X\rangle = \frac{f\, g\, c_1\, c_4}{(k^2 + m_X^{\,2})(k^2 + m_W^{\,2})}$$

which is manifestly convergent so no infinities are encountered in closed loop calculations. In fact, the effective coupling for baryon-lepton number violation is $f\ g\ c_1\ c_4/m_X^2$ by standard arguments.

(3.4) FERMION MASSES

The only surprise in the fermion sector[46] is that the neutrinos ν and ν' are four component objects that can acquire mass through both spontaneous symmetry breaking and radiative corrections. The situation is remedied in the following way[2].

Secure in the knowledge that a physical spin-½ particle is massless only if it can be described by a two component spinor, one introduces two left handed two component spinors ξ_L^e and ξ_L^μ, which are singlets under the gauge group and whose interactions with the unphysical ν and ν' will produce massless two component objects. In fact, the only renormalizable interaction the ξ's possess is

$$h\ \bar{\xi}_L^\mu\ \mathrm{tr}\ \bar{B}\ \psi_R + h'\ \bar{\xi}_L^e\ \mathrm{tr}\ \bar{B}\ \Gamma\ \psi_R + \mathrm{h.c.}$$

where

$$\Gamma = \left(\begin{array}{c|c} 0 & \tau_1 \\ \hline \tau_1 & 0 \end{array}\right)$$

A consideration of the mass matrix for the complex $(\xi_L^\mu, \nu_L', \nu_R')$ reveals there is precisely one massless two component left handed particle which should be identified with the physical ν^{μ}. It is

$$\nu^{\mu} = \frac{f\ a_4\ \xi_L^{\mu} - h\ b_4\ \nu_L'}{(f^2\ a_4^2 + b_4^2\ h^2)^{\frac{1}{2}}}$$

An identical situation obtains for the complex (ν_L, ν_R, ξ_L^e).

(3.5) SOME EXPERIMENTAL CONSEQUENCES

Apart from the spontaneous breakdown of baryon and lepton number conservation which leads to leptonic decay of quarks and protons as discussed in the next chapter, there are a number of other distinguishing predictions of the model some of which we outline below.

(a) The Colour Octet of Gluons

At sufficiently high energies, these could be produced in pairs in hadron collisions with observable cross sections. The question of whether these particles have already been seen is taken up in Chapter 4.

(b) The Colour Component of the Photon

If the photon contains a colour octet component U^0 (assuming quarks carry integral charge), then the U^0 can be produced in photon reactions such as

$$\gamma + P \rightarrow P + U^0$$

$$e^+ + e^- \rightarrow U^0$$

with production cross sections comparable to that of the ρ^0 at appropriate energies[2]. If there exist colour octet states lighter than U^0, then it would decay strongly into these plus hadrons. It may also decay to $\pi^0 + \gamma$ with secondary decays to e^+e^- and $\mu^+\mu^-$. So a search for U^0 by missing mass measurements, $\mu^+\mu^-$ production in $\gamma + P$ reactions and e^+e^- annihilation may offer a way of establishing whether the photon has colour octet pieces. If it does, this would favour the integer charge quark model.

(c) <u>Neutral Neutrino-Current Processes</u>

Through exchange of Z^0, S^0 and U^0, processes of the following kind can occur.

$$\nu_\mu + e \to \nu_\mu + e$$
$$\nu_e + e \to \nu_e + e$$
$$\nu_\mu + p \to \nu + \text{hadrons}$$

All three neutral bosons couple to both leptonic and semi-leptonic currents though with different strengths. The lightest member is the U^0 and will have a net effect towards leptonic amplitudes only of the order of 10^{-6} X (strong amplitude).

As these interactions arise from new neutral bosons not predicted by former schemes, a study of the cross sections of neutral neutrino-current leptonic and semileptonic processes could determine what departures from the simple SU(2) X U(1) theory are needed[2]. The current from the S^0 (whose existence is a consequence of gauging the full SU(4') symmetry) is purely isoscalar so a study of the isospin structure of the hadron current for semileptonic processes could determine if there is an S^0 contribution[2].

(d) RIGHT-HANDED CURRENTS

The left-right symmetry imposed on the theory implies the existence of V+A currents. If the W_R's are not super-heavy, one should observe V+A amplitudes at about 10% of the level of V-A amplitudes.

CHAPTER 4

SELECTION RULES IN THE "BASIC" MODEL

(4.1) DECAYS OF INTEGRALLY CHARGED QUARKS

Before spontaneous symmetry breaking, the gauge invariant Lagrangian for the "basic" model conserves baryon number (B), lepton number (L) and, of course, their sum fermion number (F). As we have seen, spontaneous symmetry breaking induces a mixing between the valency gauge mesons W and the exotic triplet X that will lead to decay of integrally charged quarks into leptons with the rule $\Delta B = -\Delta L = -1$, $\Delta F = 0$. Because W bosons will couple only to colour singlet currents (e.g., purely leptonic ones) whereas the X mesons will couple to colour non-singlet currents (e.g., quark-lepton ones), b and c-type quarks may decay as $q_{b,c} \to \ell + \ell + \bar{\ell}$ via the simple tree diagram of fig. 1. Diagrams with X will describe decays of b-type quarks and those with X' c-type.

Some typical modes are:

$$(p^+_{b,c},\ n^o_{b,c},\ \lambda^o_{b,c},\ \chi^+_{b,c}) \to (\nu_e,\ e^-,\ \mu^-,\ \nu_\mu) + (e^+ + \nu_e).$$

Of course, we may replace the electron current coupled to the W by a muon current in each case.

Approximate matrix elements and phase space factors for these modes are listed in Table 1 where G_F is the usual

Fermi constant (about 10^{-5} GeV^{-2}) and $G_B = f^2/m_X^2 \sim 10^{-8}$ GeV^{-2}. Our phase space factors include factors of $(2\pi)^{-3}$ assigned to each final state particle and factors of π arising from angular integrations. A_n is a numerical factor arising from n-particle phase space integrations and is usually >>1. (For example, A_5 = 2304.)[47]

With $c_1 \sim$ 1-3 GeV, $c_4 \sim$ 100 GeV and $m_q \sim$ 5 GeV, we find $\tau_q \gtrsim 10^{-3}$ sec.[48].

We may replace the $\ell\bar{\ell}$ current by a colour singlet $q\bar{q}$ current as shown in fig. 2 leading to decays of the sort $q \to \ell+\pi$. Some typical modes are:

$$(p^+_{b,c},\ n^o_{b,c},\ \lambda^o_{b,c},\ \chi^+_{b,c}) \to (\nu_e,\ e^-,\ \mu^-,\ \nu_\mu) + \pi^+.$$

Here, we find $\tau_q \gtrsim 10^{-2}$ sec., somewhat longer than the three body process in spite of what appears to be greater available phase space. This has resulted from the insertion of a form factor proportional to pion mass and the corresponding reduction of phase space by dimensional arguments.

These modes, however, are not dominant and our surprise at finding such large lifetimes for quarks was exceeded only by that in observing that it is not the Born terms that contribute most heavily to quark decay but that higher order terms may play an equal and often greater rôle[49].

In fact, the dominant decay mechanisms all arise from the basic one loop diagram of fig. 3 from which we extract the selection rules: $\Delta S = \Delta C = 0$ with $\Delta I_3 = 1$ and $\Delta S = -\Delta C = -1$ with $\Delta I_3 = 0$.

A physical example of this process which is of roughly the same order as the tree diagram is given in fig. 4 and the matrix element is:

$$\frac{g^4 \; G_B}{196 \; \pi^2} \; \frac{c_1 \; c_4}{(t - m^2_{\tilde{U}_o})} \; \ell n \; \frac{m_X^2}{m_W^2} \; <q|\gamma^\mu \; \tfrac{1}{2}(1-\gamma_5)|\ell><\ell|\gamma^\mu \; \tfrac{1}{2}(1-\gamma_5)|\ell>$$

The complexion of the vector $\tilde{U}^o$ is given in Section (3.3). There are similar diagrams with other neutral eigenstates in place of $\tilde{U}^o$ but they will be suppressed owing to the greater mass of the other mesons. Because $\tilde{U}^o$ contains a part proportional to $\sqrt{3}\; V_3 + V_8$, it will couple to hadrons and quarks with strength f. It will couple to leptons with strength $g^2/_f$ owing to the part proportional to S^o. We note that as $\tilde{U}^o$ contains W^o and S^o only in the combination $W^o - S^o$ and as W^o and S^o couple to the charged leptons with opposite sign and to the neutral leptons with the same sign (S^o is proportional to the 15-generator of SU(4)), $\tilde{U}^o$ will couple only to e^+e^- (and, of course, $\mu^+\mu^-$).

Some typical modes here are:

$$(p^+_{b,c}, \; n^o_{b,c}, \; \lambda^o_{b,c}, \; \chi^+_{b,c}) \rightarrow (e^+, \; \nu_e, \; \nu_\mu, \; \mu^+) + (e^+ + e^-)$$

with lifetimes about the same as the tree diagram modes. We observe, in passing, that this loop cannot lead to decays of the type $q_{b,c} \rightarrow \ell + \pi$ as the pion will not couple to $\tilde{U}^{o}$ which is an SU(2) singlet.

The dominant modes for decays of b and c-type quarks arise from the diagram of fig. 5 where the process $q_{b,c} \rightarrow \ell + \pi$ is pictured[50]. The $qq\pi$ coupling constant can be estimated from Goldberger-Trieman and is found to be about 65. The matrix element for this process is:

$$\frac{f^2\ g^2\ c_1\ c_4}{m_X{}^2}\ \frac{1}{32\pi^2}\ \{3(\not p - \not p') - 4m_q\}\ (1-\gamma_5)\ell n\ \frac{m_X^2}{m_W^2}\ \frac{1}{\not p - \not p' - m_q}\ g_{qq\pi}$$

leading to a decay width:

$$\Gamma = \frac{9}{4065\pi^5}\ \frac{g_{qq\pi}^2 g^4\ G_B^2 (c_1\ c_4)^2\ (\ell n\ {}^{m_X^2}/m_W^2)^2}{m_\ell}$$

and we find $\tau_q \gtrsim 10^{-13}$ sec.. Some typical processes are:

$$p^+_{b,c} \rightarrow \pi^+ + \nu_e,\ K^+ + \nu_\mu$$

$$n^o_{b,c} \rightarrow \pi^o + \nu_e,\ K^o + \nu_\mu$$

$$\lambda^o_{b,c} \rightarrow \eta^o + \nu_\mu$$

$$\chi^+_{b,c} \rightarrow \nu_e + D^+,\ \nu_\mu + F^+$$

where D^+ and F^+ are new charmed meson states[51]. If the D, F mass is large, however, these χ decays are likely to be suppressed and replaced by the ordinary weak decays:

$$\chi^+_{b,c} \rightarrow \pi^o + p^+_{b,c},\ \pi^+ + \lambda^o_{b,c}$$

Owing to the very large $qq\pi$ coupling, it would seem that we could achieve an even shorter lifetime by adding a greater number of pion lines. Indeed, we find that a considerable number of pions can be added before phase space becomes the critical factor.

A very rough guide to the behaviour of the lifetime can be obtained by multiplying the above expression for Γ by an "enhancement factor" for decay into a lepton and n pions which is:

$$\frac{1}{(2\pi)^{3n}} \left(\frac{\pi}{2}\right)^{n-1} \frac{n-1}{((n-1)!)^2} \left(\frac{m_q(m_q-m_\pi)\ldots(m_q-nm_\pi)}{m_\pi^{n+1}}\right)^2$$

This expression has a maximum at n = 7-8 and is about 10^3. Thus, we expect b and c-type quarks to decay into a lepton and seven or eight pions(52) with $\tau_q \gtrsim 10^{-16}$ sec..

We wish to emphasize that the dominant semi-leptonic decay modes just listed all involve exclusively neutral leptons. The restrictions of charge and fermion number conservation forbid the diagram of fig. 5 from leading to production of charged leptons unless the pion is emitted from inside the loop in which case, we have, for example:

$$p^+_{b,c} \rightarrow e^- + \pi^+ + \pi^+$$

$$n^o_{b,c} \rightarrow e^- + \pi^+.$$

However, in view of the two large masses (m_W and m_X) running around the loop, these modes will be strongly suppressed and are, in fact, of about the same order as the tree diagram modes of fig. 1 which are the only other case where b and c-type quarks have charged semi-leptonic decay modes. This means, in particular, that decays of b and c-type quarks are not likely to be responsible for the $\bar{\mu}e$ events seen at SPEAR[53].

The a-type quarks do not couple directly to X so these cannot decay directly into leptons. They must first change to a b or c-type quark by coupling to a member of the SU(3) colour octet. Thus, the only sort of tree diagram decays for a-type into leptons are those of the form $q_a \to 3\ell + 2\bar{\ell}$ illustrated in fig. 6. Some typical processes are:

$$(p_a^o,\ n_a^-,\ \lambda_a^-,\ \chi_a^o) \to (\nu_e,\ e^-,\ \mu^-,\ \nu_\mu) + (\nu_e + \bar{\nu}_e + e^- + e^+).$$

From Table 1, we find $\tau_{q_a} \gtrsim 10^{-2}$ sec.. Making the usual replacement of electron (or muon) current by a colour singlet valency non-singlet $q\bar{q}$ current, we arrive at the following new decays:

$$(p_a^o,\ n_a^-,\ \lambda_a^-,\ \chi_a^o) \to (\nu_e,\ e^-,\ \mu^-,\ \nu_\mu) + (\pi^+ + \pi^-)$$

with $q_a \gtrsim 10^{-1}$ sec. where again we find that the decay into a larger number of particles is preferred despite what appears to be less available phase space.

As before, the dominant contributions to q_a decay come not from trees but rather from a class of one loop diagrams the first of which is pictured in fig. 7 leading to decays like $q_a \rightarrow \ell + \ell\bar{\ell}$ some examples of which are:

$$(p_a^o,\ n_a^-,\ \lambda_a^-,\ \chi_a^o) \rightarrow (\nu_e,\ e^-,\ \mu^-,\ \nu_\mu) + (e^+ + e^-).$$

Of course, we may couple any $\ell\bar{\ell}$ current to this diagram. The approximate matrix element is found in Table 1 and we estimate $\tau_{q_a} \gtrsim 10^{-2}$ sec.. By coupling colour singlet quark currents to the right hand side of the diagram, one may achieve a variety of mesons in the final state with lifetimes of roughly the same order.

The a-type quarks may change their colour by producing a physical V_ρ as shown in fig. 8. The matrix element for this diagram is the same as that for fig. 5 save that f replaces $g_{qq\pi}$ and we get an extra factor of three from summing over the polarizations of the V_ρ. The typical modes here are:

$$p_a^o \rightarrow V_\rho^+ + e^-$$

$$n_a^- \rightarrow V_\rho^- + \nu_e$$

$$\lambda_a^- \rightarrow V_\rho^- + \nu_\mu$$

$$\chi_a^o \rightarrow V_\rho^+ + \mu^-$$

with $\tau_q \gtrsim 10^{-11}$ sec.. The previous arguments concerning the size of $g_{qq\pi}$ still apply and we expect an enhancement of about 10^3 for decay of a-type quarks into a V_ρ, a lepton and as many as seven or eight pions. As we will see in Section

(4.3), the V_ρ decays into two leptons very much more rapidly than the q_a decays into a V_ρ, a lepton and several pions. Thus, the dominant process for the decay of an a-type quark into leptons and mesons will be sequential and the evidence for the decay of such a quark should be the presence of three leptons and seven or eight pions.

There is also an important class of loop diagrams where no physical V_ρ is produced and the basic structure is exhibited in fig. 9 from which we conclude that a very simple selection rule is obtained for a-type quark decay, namely that I_3, charm and strangeness are conserved separately.

To obtain physical decays, we may attach meson lines as displayed in fig. 10 leading to the following typical processes:

$$p_a^o \to \pi^+ + e^-,\ \nu_e + \pi^o$$
$$n_a^- \to \pi^o + e^-,\ \nu_e + \pi^-$$
$$\lambda_a^- \to \nu_e + K^-,\ e^- + K^o,\ \mu^- + \eta$$
$$\chi_a^o \to \nu_e + D^o,\ e^- + D^+$$

where, from Table 1, we estimate $\tau_{q_a} \gtrsim 10^{-10}$ sec..

Similar to the case of one loop $q_{b,c}$ decay, we may also attach the SU(2) singlet $\tilde{U}^o$ leading to processes like:

$$(p_a^o,\ n_a^-,\ \lambda_a^-,\ \chi_a^o) \to (\nu_e,\ e^-,\ \mu^-,\ \nu_\mu) + e^+ + e^-)$$

though here we find only $\tau_{q_a} \gtrsim 10^{-3}$ sec..

The double mixing V_ρ - W - X manifests itself also in two loop processes an example of which is given in fig. 11 where a meson line is attached. The allowed modes here are identical to those admitted by the one loop double mixing diagram though, as we find from the table, $\tau_{q_a} \gtrsim 10^{-5}$ sec.. New exclusive leptonic decay modes do not result from attaching a $\tilde{U}^0$ to the diagram of fig. 11 but, here $\tau_{q_a} \gtrsim 10^2$ sec., the process being suppressed by a factor $\approx \frac{\alpha}{m^2_{\tilde{U}^0}}$.

When a-type quarks decay inside protons, the dominant sequential process involving the V_ρ will not be realized as the V_ρ, with mass of 3-10 GeV, cannot be on-shell. Thus, decays of a-type quarks within low lying baryons must be via any of the sub-dominant modes just discussed or the diagram of fig. 12 where $q_a \to 2\ell+\bar{\ell}$, some typical processes being:

$$p^o_a \to e^- + e^+ + \nu_e$$

$$n^-_a \to \nu_e + e^- + \bar{\nu}_e$$

$$\lambda^-_a \to \nu_\mu + e^- + \bar{\nu}_e$$

$$x^o_a \to \mu^- + e^+ + \nu_e$$

The appropriate matrix element is found in Table 1 leading to a lifetime $\tau_{q_a} \gtrsim 10^{-2}$ sec.. As this is not a dominant mode and, therefore, will apply only to quarks inside baryons, this number has little observational significance and we mention it only by way of comparison.

We wish to call special attention to the fact that while the suppppression of charged semi-leptonic decay modes of b and c-type quarks rendered it unlikely that they could account for SPEAR $\bar{\mu}e$ events, the opposite situation has been obtained for a-type quarks and it is possible to explain the $\bar{\mu}e$ events in terms of pair production of a-type quarks if they are heavier than the SU(3) colour gluons(53). A typical process would be:

$$e^+ + e^- \rightarrow n_a^- + n_a^+$$
$$\downarrow \qquad\qquad \downarrow$$
$$V_\rho^- + \nu_e \qquad V_\rho^+ + \bar{\nu}_e$$
$$\lfloor \rightarrow e^- + \bar{\nu}_e \qquad \lfloor \rightarrow \mu^+ + \nu_\mu$$

The $\bar{\mu}e$ pairs would appear, within the present statistics, like three body leptonic decays of the parent quarks. Further, the observed contribution of the $\bar{\mu}e$ signals to

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

at SPEAR could be attributed to quark decay provided the square of the quark-electromagnetic form factor is of order unity(53).

(4.2) PROTON DECAY

Given that integrally charged quarks undergo baryon-lepton number violating transitions to leptons and mesons, it is a virtually inescapable conclusion that the proton must similarly decay[54]. This, however, is not in conflict with the observed stability of the proton since said decay will occur only in order G_B^3 (or higher) and while G_B is large enough, as we have seen, to guarantee that free quarks would decay too rapidly to have yet been observed, in third order it is sufficiently small to accommodate the extraordinary stability of the proton.

In giving estimates for proton lifetime, we will be concerned with lower bounds and will therefore assume that the effective strength for proton decay is the product of the strengths for the decays of the constituent quarks. We will refrain from offering exotic speculations as to the details of quark binding and other unknown aspects of the strong dynamics or the question of how SU(3) colour symmetry is broken and the presence of additional selection rules which would conspire to suppress proton decay even further.

Combining a two loop decay of the a-type quark with the one loop decay of the b and c-type quarks, we arrive at a proton decay of the form $p \rightarrow 3\ell + \pi$ (e.g., $p \rightarrow 3\nu_e + \pi^+$) pictured schematically in fig. 13. The factor M that appears in the matrix elements of Table 2 is an unknown quantity inserted in place of a precise form factor where the proton disintegrates into three quarks. Making the not unjustified assumption that $M \approx m_p$, we find that $\tau_p \gtrsim 10^{32}$ sec..

If the a-type quark decays via the diagram of fig. 12, we find decays of the sort $p \rightarrow 3\nu_e + \pi^+ + \pi^0$ displayed in fig. 14. From Table 2, we estimate $\tau_p \gtrsim 10^{34}$ sec..

By combining sub-dominant loop and/or tree decays of quarks into leptons, we can obtain decays of protons exclusively into leptons (e.g. $p \to 4\nu_e + e^+$) as in fig. 15. This is not a dominant mode and, from the table, $\tau_p \gtrsim 10^{54}$ sec..

We note that, unlike the case of quark decay, we do not shorten the proton's lifetime by adding π lines. Indeed, we find the "enhancement factor" for proton decay (which is the same as that given for quarks save that m_p replaces m_q) decreases rapidly as n increases.

Regarding possible searches for proton decay, we wish to emphasize that multi-particle decays of the type just described are the only sort allowed. The structure of the model and fermion number conservation forbids the decay of the proton into fewer than three neutrinos and a pion. In the classic experiments on proton decay(55), the dominant modes were assumed to be two body so that the presence of high energy charged secondaries received the most attention. This leaves open the possibility that some of the unidentified low energy charged secondaries could have come from the multi-particle decays we have described, a fact which should be borne in mind for future experiments.

(4.3) THE GAUGE MESONS

From the Lagrangian given in eq. (3), we see that the charged members of the SU(3) colour octet of gluons ($V_\rho^\pm$, $V_{K^*}^\pm$)

may decay only through their mixing with the charged W's via the tree diagrams of fig. 16. This will lead to decays of the sort $(V_\rho^+, V_{K^*}^+) \to (\mu^+ + \nu_\mu, e^+ + \nu_e)$. The matrix elements are given in Table 3 and, assuming the gluon mass to be about 3 GeV, we find $\tau_{V_\rho^\pm, V_{K^*}^\pm} \gtrsim 10^{-17}$ sec.. The mixing with the weak gluons will also give rise to decays of the sort(56):

$$(V_\rho^+, V_{K^*}^+) \to \pi\pi,\ 3\pi,\ 4\pi,\ \bar{K},\ \text{etc.}$$

$$\to \pi\pi e\nu,\ K\bar{K}e\nu,\ \eta\eta e\nu$$

where, in the case, of the last three modes, the hadrons must be in an I = 0 state. A very important remark is that decays into a single π, K or η are forbidden to $O(\alpha)$ in contrast to other mesons with new quantum numbers such as the charmed D and F, and, therefore, if one finds charged short lived particles that exhibit semi-leptonic decay modes involving only two π's (or K's, η's, etc.) then, unquestionably, they must be either colour gluons or similar colour octet states lighter than the gluons(56).

The seven exotic members of the SU(4) 15-plet have two principle decay modes. As the $X^\pm$ (and $X'^\pm$) and S^0 couple directly to charged and neutral W's respectively, they will also decay into $\ell\ell$ or $\ell\bar{\ell}$ via the diagram of fig. 16. (The previous statements for π's, K's, η's, etc. in the final state still apply.) The enormous mass of these particles (10^4 - 10^5 GeV) leaves them a great deal of phase space and we predict $\tau_{X}^{\pm}{}_{;X'^{\pm}} \gtrsim 10^{-25}$ sec. and $\tau_{S^0} \gtrsim 10^{-28}$ sec. the difference arising from the different mixings between the strong and weak gauge mesons which is proportional to $c_1\ c_4$ for X and c_4^2 for S^0. The neutral X does not mix with the W even after spontaneous symmetry breaking and can decay into leptons (or mesons) only via the three point interaction displayed in fig. 17 arising from the term $\mathrm{tr}\ f_{abc}\ V_\mu^b\ V_\nu^c\ (\partial_\mu\ V_\nu^a - \partial_\nu\ V_\mu^a)$ in the kinetic part of the Lagrangian and leading to decays like $X^0 \rightarrow 2\ell + 2\bar{\ell}$ where, from Table 3, we estimate $\tau_{X^0} \gtrsim 10^{-34}$ sec.. Again, decays of the form $X^0 \rightarrow \pi^+\pi^-$, K^+K^-, 2η, etc. are possible. The final member of the 15-plet of gauge mesons is the V_K^{0*}. It, too, has a decay mode arising from the three point interaction leading to modes like $V_K^{0*} \rightarrow 2\ell + 2\ell$ displayed in fig. 18. From the table, we find $\tau_{V_K^{*}{}^0} \gtrsim 10^{-7}$ sec.. (The contributions from W-X mixing can be ignored as they are strongly suppressed by the X mass.) From this diagram, we may also achieve decays like $V_K^{0*} \rightarrow \pi^+\pi^-$, etc. by the usual procedure of attaching the appropriate colour singlet $q\bar{q}$ current to the W's. Making the standard modifications in the matrix element, we find, for the two pion mode, $\tau_{V_K^{0}{}^*} \gtrsim 10^{-8}$ sec..

Concerning this diagram, it is interesting to observe that, unlike the case of $V_\rho^\pm$, $V_K^{\pm *}$, decays involving only a single π (or K, η, etc.), for example, $V_K^{0*} \rightarrow \pi^+e^-\ \bar{\nu}_e$, are not strongly suppressed relative to those modes involving many mesons and are, in fact, of about the same order.

Thus, colour octet gluons may be present even without the previously mentioned signal for semi-leptonic decays.

The importance of this observation should not be over emphasized because, for the $V^o_{K^*}$, the dominant decay modes, which arise from the convergent loop diagrams of fig. 19, are pure leptonic. The first loop arises from the previously mentioned three point interaction and the second from the four point interaction $|f_{abc}\, V^b_\mu\, V^c_\nu|^2$. From Table 3, we deduce $\tau_{V^o_{k^*}} \gtrsim 10^{-17}$ sec..

In closing this section, we remark that the remaining neutral eigenstates of the diagonalised mass matrix, $\tilde{U}^o$, V^o and Z^o, whose complexions were exhibited in Section (3.3), will all decay into leptons (and mesons) through their mixing with the W's (except for the $\tilde{U}^o$ which couples directly to leptons and is an SU(2) singlet). The lifetimes vary slightly owing to the various effective strengths of the mixings but are all in the range 10^{-17} - 10^{-14} sec..

(4.4) AN ALTERNATE SOLUTION

In the foregoing, we have been discussing the effects of the solution given in Section (3.3) for the vacuum expectation values of the Higgs fields which leads, as seen in the Lagrangian of eq. (3), to mixing between the X's and the left-handed W's from which arises the baryon-lepton number violating transitions we have examined. However, this is not the only solution for the vacuum expectation values that yields the required minimum of the potential. Indeed, one would expect from the left-right symmetry imposed on the theory that we should be able to obtain a viable solution exactly interchanging the roles of the left and right-handed weak bosons which would amount simply to interchanging their coefficients in (3). This is equivalent to taking a solution where:

$$\langle A\rangle = \begin{pmatrix} a_1 & & & \\ & a_1 & & \\ & & a_1 & \\ & & & a_4 \end{pmatrix},$$

$$\langle B\rangle = \begin{pmatrix} b_1 & & & \\ & b_1 & & \\ & & b_1 & \\ & & & b_4 \end{pmatrix}$$

$$\langle C\rangle = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & c_4 \end{pmatrix}$$

with a_1, a_4, b_4 and c_4 as before and $b_1 \sim 1 - 3$ GeV. Now, the effective propagator for baryon number violation is:

$$\langle W_R X \rangle = \frac{f g b_1 b_4}{(k^2 + m_{W_R}^2)(k^2 + m_X^2}$$

which, being proportional to b_4, appears very much larger than before.

However, the size of m_{W_R} must be taken into account. While the suppression of V+A weak currents imposes a lower limit on the W_R mass ($\gtrsim 3m_{W_L}$), it may be considerably larger than this. Therefore, processes whose matrix elements contain sufficient powers of $1/m_{W_R}$ would be unchanged with respect to this new solution. Thus, we find that decays of b and c-type quarks would be greatly enhanced owing to the participation of the W_R's but decays of a-type quarks would be unaffected is we take $m_{W_R} \sim m_X$[57]. Admittedly, these results are of limited relevance as neither the lifetime nor the existence of free quarks has been established. The case for proton decay is quite different, however, and if we make the replacements $c_4 \rightarrow b_4$, $m_{W_L} \rightarrow m_{W_R}$ in the matrix elements in Table 2, we find that proton lifetime could be shortened by a factor as great as 10^6, bringing the predicted decay time well below the experimentally established value. We are thus motivated to reject this alternate solution on the grounds that a universe described by it would have long ago decayed into leptons[58].

(4.5) SUMMARY AND CONCLUSIONS

We have seen that, as a result of gauging the unifying symmetry between hadrons and leptons, there has arisen exotic interactions some consequences of which we have explored in this chapter. All of these consequences have in common the spontaneous breakdown of baryon and lepton number conservation resulting in the decays of quarks, protons and colour gluons into leptons and mesons. By way of summary, we will restate some of our results emphasizing their significance in the light of present and future experiments.

An underlying assumption throughout our work is that quarks may exist as free particles and carry integral charge. This is, admittedly, a prejudice. Yet is no more severe, and certainly no less justifiable, than the experimentalists' prejudice that quarks are fractionally charged or the theorists' bias that quarks are permanently confined inside hadrons. Indeed, the result that we have quite easily obtained that a-type quarks decay predominantly into only leptons provides a perfectly viable explanation for the anomalous SLAC $\bar{\mu}e$ events. Further, the preferred decay of b and c-type quarks into quite a large number of pions is a unique signal that future quark hunters may well remember. We cannot overemphasize that while quarks may not exist as free, integrally charged particles, there is little evidence that suggests this and the concept of an unconfined, unstable quark should be at least as acceptable as the contrary notions.

The instability of the quark has led, unavoidably, to an unstable proton. Yet even the most naive estimates of the degree of this instability, which have all but totally ignored the internal dynamics of the proton, have shown that the proton is sufficiently stable to be in perfect accord with the most recent determinations of its lifetime. Of course, we would urge a continuation of the search for baryon number violation and note that there is some reason to believe it has already been observed(59).

The discovery of the J/ψ particles has created an understandable enthusiasm for searching for new particles and quantum numbers and, in this context, it is appropriate to recall that we have shown that the decays of colour gluons are into leptons and mesons. This leaves open the possibility of pair-producing these particles in hadronic collisions through strong interactions or in e^+e^- annihilation(56):

$$p + p \rightarrow V_\rho^+ + V_\rho^- + \text{hadrons}$$

$$e^+e^- \rightarrow V_\rho^+ + V_\rho^- + \text{hadrons}.$$

Also, the recently observed dimuon events can be attributed to the production of charged colour gluons through the reaction:

$$\nu_\mu + N \rightarrow \mu^- + V_\rho^+ + \text{hadrons}.$$

Finally(60), the decay modes we have described make it possible to distinguish between quarks and new heavy leptons as explanations for $\bar{\mu}e$ events. Toward this end, one should search for pair-production of these objects in hadron collisions. For while neutral and charged quarks pairs are expected to be produced through strong interactions with cross sections $\approx 10^{-32}$ cm^2 at Fermilab energies[56], neutral and charged heavy lepton pairs should be produced through weak and electromagnetic interactions respectively with cross sections $\approx 10^{-38}$ cm^2 for the neutral pair and $\approx 10^{-34}$ cm^2 for the charged pair at Fermilab energies (assuming quark mass to be $\approx$ 2 - 3 GeV and heavy lepton mass $\approx$ 2 GeV). It therefore seems promising to search for anomalous dilepton production (involving e^+e^-, $\mu^\pm e^\mp$ and $\mu^+\mu^-$ pairs) in hadron collisions.

Further, quark pairs may be produced in e^+e^- annihilation via:

$$e^+e^- \rightarrow q + \bar{q} + \text{hadrons}$$

which is not possible for heavy leptons[56]. Also, the semileptonic decay modes of a-type quarks, for example,

$$n_a^- \rightarrow V_\rho^- + \pi^+ + \pi^- + \nu_e$$

or colour gluons,

$$V_\rho^- \rightarrow \pi^+ + \pi^- + e^- + \bar{\nu}_e$$

can give rise to anomalous semileptonic signals such as

$$e^+ + e^- \rightarrow \bar{\mu}e + \pi^+\pi^- + \text{missing momentum}$$

which could be as small as one tenths of the leptonic $\bar{\mu}e$ signal[56]. If electron and muon number are separately conserved, such semileptonic signals would not be produced by the decays of heavy leptons. Thus, a search for such semi-leptonic $\bar{\mu}e$ signals is advised.

Another distinction of the quark hypothesis for the $\bar{\mu}e$ events is that a significant fraction of the total hadronic cross section must involve real quark-anti-quark pair production[56] (again assuming that the square of the quark-electromagnetic form factor is of order unity). Since all b and c-type quarks (and x_a^o and p_a^o) decay into neutrinos and mesons, this may explain the energy crisis and jet structure observed at SPEAR.

CHAPTER 5

FUNDAMENTAL ATTRIBUTES AND PRE-QUARKS

(5.1) HOW MANY FUNDAMENTAL PARTICLES?

The introduction of quarks by Gell-Mann and Zweig provided a remarkably elegant and simple foundation for hadron physics and physicists of the last decade could have enjoyed an understandable sense of well being, secure in the belief that there were only three fundamental particles for strong interactions. Ensuing developments, however, have considerably enlarged the basic SU(3) symmetry leading to an alarming increase in the number of apparent fundamental particles.

The introduction of charm, required to rid gauge theories of $\Delta S = 1$ neutral weak currents, has increased the original three valencies to four. The introduction of three colours, required to resolve the problem of quark statistics, leaves us with twelve fundamental fermions and if, as in the last two chapters, one regards lepton number as a colour, we will have sixteen. Finally, the extensions to higher symmetries[61] and the hypothesis of gauging the maximal symmetry[8,29] requires the introduction of "mirror" fermions that increases the number of basic entities to thirty two. It thus appears prudent to consider more economical ways of accomodating all these elementary attributes and, toward this end, it may be necessary to regard the quarks themselves as being composed of even more basic particles[62].

The ideas that we will discuss here are based on the simplest mathematical realisation of the structure of the basic fermions set down in Section (3.1). The basic 16-fold multiplet of eq. (2) may obviously be written:

$$\psi_{L,R} = \begin{pmatrix} p \\ n \\ \lambda \\ \chi \end{pmatrix}_{L,R} \quad X \quad (a, b, c, d) \qquad (4)$$

which emphasizes the simple fact that the basic $(4, \bar{4})$ representation of SU(4) X SU(4') can be constructed from two quartet spinors. The logical next step is to regard the light elements of these spinors as the fundamental fermions - one particle for each elementary attribute believed present in nature. These particles will be called "pre-quarks" or "preons" and, out of them, the thirty two quarks will be constructed. In the following, we will denote the valency spinor by $\mathcal{Q}$ and the colour spinor by $\mathcal{C}$ and refer to them frequently as "valons" and "colourons", respectively.

(5.2) <u>THE POSSIBLE STRUCTURES FOR PRE-QUARK GAUGE MODELS</u>

If we are to construct quarks only out of $\mathcal{Q}$ and $\mathcal{C}$, (i.e. $\psi_{L,R} = \mathcal{Q}_{L,R}\,\bar{\mathcal{C}}$), there are two possible variants:

(A) $\mathcal{Q}$ is a fermion and $\mathcal{C}$ a boson,

(B) $\mathcal{Q}$ is a boson and $\mathcal{C}$ a fermion.

We mention that neither of these readily admit an SU(6) type classification of preons (perhaps not a significant disadvantage) but scheme B cannot be used for a gauge theory of weak interactions and we shall have no more to say about it.

If we introduce a neutral singlet pre-fermion, there are two more cases in which the quarks appear as three body composites, :

(C) $\mathcal{Q}$ and $\mathcal{C}$ are bosons

(D) $\mathcal{Q}$ and $\mathcal{C}$ are fermions.

Scheme C has the same problem as B and we reject it. Thus, only A and D seem to be candidates for a viable gauge theory.

In the case of model D, however, a further simplification can be achieved by identifying the four components of the valency spinor with the known leptons which, of course, are no longer regarded as composite objects. The colour spinor then requires only three degrees of freedom and the twelve hadronic quarks appear as $L\bar{\mathcal{C}}\mathcal{S}$ composites.

We will, in the following, concern ourselves exclusively with model D and outline below the general features the resulting gauge theory. We will indicate, also, the effects of the modification just mentioned.

(5.3) A GAUGE MODEL FOR PREONS

The underlying global symmetry group is

$$G = \left(SU(4)_L \times SU(4)_R \times U(1)\right)_{\text{valency}} \times \left(SU(4') \times U(1')\right)_{\text{colour}} .$$

As in Chapter 3, the presence of anomalies compels one to gauge only the subgroup $SU(2)_L \times SU(2)_R \times U(1)$ in the valency sector. The U(1) groups, which have been introduced ad hoc, serve two ends. First, their generators appear in the expression for the electric charge operator and are responsible for the preons acquiring integer charge. Secondly, the gauge fields associated with the U(1)'s permit the singlet $\mathcal{S}$ to couple with the $\mathcal{Q}$'s and $\mathcal{C}$'s so that $\mathcal{Q}\bar{\mathcal{C}}\mathcal{S}$ bound states can arise. The transformation properties of the fundamental fermions under the gauge group are:

$$\mathcal{Q}_L = (2+2,\ 1,1)_{1,0}, \quad \mathcal{Q}_R = (1,\ 2+2,\ 1)_{1,0},$$
$$\mathcal{C} = (1,1,4)_{0,1}, \qquad \mathcal{S} = (1,1,1)_{-1,1}$$

where the $U(1) \times U(1')$ quantum numbers are given as subscripts. The electric charge operator is:

$$Q = I_{3L} + I_{3R} + F_3' + \frac{1}{\sqrt{3}} F_{15}' - \tfrac{1}{2} I_0 - \tfrac{1}{2} I_0'$$

which takes the form (0, -1, -1, 0) on the quartets and is zero for the singlet. Note that it yields fractional charges if we discard the U(1) generators. The $U(1) \times U(1')$ assignments for $\mathcal{S}$ are opposite to those for $\mathcal{D}$ and $\bar{\mathcal{C}}$ so as to produce attractive gluon forces in the $\mathcal{D}\bar{\mathcal{C}}\mathcal{S}$ composite.

The gauge invariant Lagrangian for the theory is:

$$\mathcal{L} = -\mathrm{tr}\Big[\sum_{L,\,R} \bar{\mathcal{D}}\not{\nabla}\mathcal{D} + \bar{\mathcal{C}}\not{\nabla}\mathcal{C} + \bar{\mathcal{S}}\not{\nabla}\mathcal{S} + \sum_{A,B,C,D} |\nabla A|^2$$

$$+ \sum_{\substack{V,\, W_L,\, W_R \\ T,\, T'}} |\nabla V|^2 + V(A,B,C,D) + m_{\mathcal{C}}\bar{\mathcal{C}}\mathcal{C} + m_{\mathcal{S}}\bar{\mathcal{S}}\mathcal{S}$$

$$+ k\bar{\mathcal{D}}_L A^+ \mathcal{D}_R + h.c.\Big]^{(63)} \qquad (5)$$

where

$$\nabla\mathcal{D}_{L,R} = \partial\mathcal{D}_{L,R} + ig_{L,R} W_{L,R}\mathcal{D}_{L,R}$$

$$\nabla\mathcal{C} = \partial\mathcal{C} + ifV\mathcal{C}$$

$$\nabla\mathcal{S} = \partial\mathcal{S} + i(hT\mathcal{S} - h'T'\mathcal{S})$$

$$\nabla A = \partial A + ig_L W_L A - ig_R AW_R$$

$$\nabla B = \partial B + ig_R W_R B - ifBV + i(hBT - h'BT')$$

$$\nabla C = \partial C + ifVC - ig_L CW_L + i(h'CT' - hCT)$$

$$\nabla D = \partial D + ifVD + ih'DT'.$$

The valency gauge particles form two chiral SU(2) triplets, W_L and W_R as before, and a singlet T while the coloured gauge mesons constitute the, by now, familiar 15-plet V and another singlet T'.

The Higgs scalars required to generate mass for the gauge mesons is[64]:

$$A = (4,\bar{4},1)_{0,0}$$
$$B = (1,4,\bar{4})_{1,-1}$$
$$C = (\bar{4},1,4)_{-1,1}$$
$$D = (1,1,4)_{0,1} \quad .$$

The first three multiplets are the same as those introduced in Chapter 3. The new multiplet is required by the presence of the new gauge particles T and T'.

The vacuum expectation values for A, B and C which minimized the scalar potential in Chapter 3 and gave acceptable values for the masses of the twenty-one gauge particles still apply and we still expect $b \sim 10^4$-10^5 GeV, $a \sim 300$ GeV and $c \sim 1$ GeV. This same situation obtains if we take:

$$\langle D \rangle = (0,0,0,d)^{(65)}$$

For our purposes, as before, the most important part of the Lagrangian is the gauge meson sector of the mass matrix which determines the nature of the gauge interactions. Apart from the addition of the singlets T and T', this matrix is the same as that described in Section (3.3). Except for slightly modifying the masses of the charged vectors, the main contribution of the D matrix will be to induce new mixing among the neutral components. The new terms, which must be added to those of eq. (3), are:

$$\begin{aligned}\mathcal{L}'_{mass} = -\{ & g_R\, h\, b_4^2\, W_R^o\, T - g_R\, h'\, b_4^2\, W_R^o\, T' \\ & + g_L\, h'(c_1^2 - c_4^2)\, W_L^o\, T' - g_L\, h(c_1^2 - c_4^2)\, W_L^o\, T \\ & - fh\left[\sqrt{\tfrac{3}{8}}\,(c_4^2 - b_4^2)\, S^o T + c_1^2(V_a T + V_b T + V_c T)\right] \\ & + fh'\left[\sqrt{\tfrac{3}{8}}(b_4^2 + c_4^2 + d_4^2)\, S^o T' + c_1^2(V_a T' + V_b T' + V_c T')\right] \\ & + f^2\left[d_4^2(X^o\, \bar{X}^o + X^+\, X^- + X'^+\, X'^- + \tfrac{3}{8}(S^o)^2)\right] \\ & + h^2\left[(b_4^2 + 3c_1^2 + c_4^2)T^2\right] \\ & + h'^2\left[(b_4^2 + 3c_1^2 + c_4^2 + d_4^2)T'^2\right]\} \end{aligned} \tag{6}$$

where:

$$V_a = V_3 + \frac{V_8}{\sqrt{3}} - \frac{S^o}{2\sqrt{6}}$$

$$V_b = \frac{V_8}{\sqrt{3}} - V_3 - \frac{S^o}{2\sqrt{6}}$$

$$V_c = \frac{-2}{\sqrt{3}}\, V_8 - \frac{S^o}{2\sqrt{6}}$$

We wish to call particular attention to the mixing induced between valency, represented by the W bosons, and the new U(1) quantum numbers, represented by T and T'.

The neutral eigenstate which is least affected by the new mixing is the photon which is now given by:

$$\frac{A}{e} = \frac{1}{g}(W_L^0 + W_R^0) + \frac{1}{f}\left(V_3 + \frac{1}{\sqrt{3}} V_8 - \sqrt{\tfrac{2}{3}}\, V_{15}\right) + \frac{T}{h} + \frac{T'}{h'}$$

The remaining states are more drastically mixed. An approximate diagonalisation is given in ref. (6).

We close this section by indicating the slight modifications required for the alternative to model D. As we remarked in Section (5.2), since leptons are no longer composite, we require only three colourons and the twelve hadronic quarks will be $L\mathcal{S}\bar{\mathcal{C}}$ composites, provided that the lepton and baryon-number assignments of $\mathcal{S}$ and $\mathcal{C}$ conform to

	L	B
$\mathcal{S}$	0	1
$\mathcal{C}$	1	0

or

	L	B
$\mathcal{S}$	-1	0
$\mathcal{C}$	0	-1

with electric charges

$$Q_{\mathcal{S}} = 0,\ Q_{\mathcal{C}} = (0, -1, -1)$$

or

$$Q_{\mathcal{S}} = -1, \quad Q_{\mathcal{C}} = (1, 0, 0),$$

respectively.

The gauge group now is:

$$= \left(SU(2)_L \times SU(2)_R \times U(1)\right) \times \left(SU(3') \times U(1')\right)$$

with the following transformation properties for the fermions.

$$L_L = (2+2,\ 1,\ 1)_{1,0}$$
$$L_R = (1,\ 2+2,\ 1)_{1,0}$$
$$\mathcal{C} = (1,\ 1,\ 3)_{0,1}$$
$$\mathcal{S} = (1,\ 1,\ 1)_{-4,3}$$

The electric charge operator is now:

$$Q = I_{3L} + I_{3R} + F_3' + \frac{1}{\sqrt{3}} F_8' - \frac{1}{2} I_o - \frac{2}{3} I_o'$$

which corresponds to the assignments $Q_{\mathcal{C}} = (0, -1, -1)$ and $Q_{\mathcal{S}} = 0$. The photon becomes:

$$\frac{1}{e} A = \frac{1}{g}(W_L^o + W_R^o) + \frac{1}{f}(V_3 + \frac{1}{\sqrt{3}} V_8) + \frac{1}{2h} T + \frac{1}{2h'} T'.$$

Spontaneous symmetry breaking can be induced by the abbreviated set of Higgs' scalars:

$$A = (4,\ \bar{4},\ 1)_{0,0}$$
$$C = (\bar{4},\ 1,\ 3)_{-1,1}$$
$$D = (1,\ 1,\ 3)_{0,1}$$

that acquire vacuum expectation values:

$$\langle A \rangle = \begin{pmatrix} a_1 & & & \\ & a_1 & & \\ & & a_1 & \\ & & & a_4 \end{pmatrix}$$

$$\langle C\rangle = \begin{pmatrix} c_1 & & \\ & c_1 & \\ & & c_1 \end{pmatrix}$$

$$\langle D\rangle = (d,\ 0,\ 0).$$

A Lagrangian model can now be completed "mutatis mutandi" as for the original scheme D.

(5.4) THE STRUCTURE OF MESONS AND BARYONS

At present energies, where quarks are not likely to be produced much less disassociate, the preon theory in no way contradicts or supercedes the quark model and we are equally free to regard mesons and baryons as composed of two or six or three or nine fundamental entities, respectively. However, we are also free to imagine that, for example, known mesons are $\bar{\mathcal{C}}\,\mathcal{C}$ or $\bar{\partial}\,\partial\,\bar{\mathcal{C}}\,\mathcal{C}$ composites or, indeed, any combination of preons with the right quantum numbers. Further, the possibility that some of the new meson states (such as the J/ψ particles) are similar preon bound states may be considered.

Assuming that baryons are three quark composites allows us to present an amusing (and, of necessity, qualitative) picture[66] of the internal dynamics of hadrons. To account for the absence of colour, we imagine that the colourons are localized in a very tightly bound core at the centre of hadrons. This superstrong binding is a result of the U(1') forces introduced in Section (5.3). Since these forces must be at least as strong as ordinary strong interactions, we must have $h'^2/4\pi \gtrsim 1$-10. Less tightly bound around this core are the valons, held in place by the weaker U(1) gauge interaction. Since even the weak interactions respond to valency, we must have $h^2/4\pi \gtrsim \alpha$.

The picture that emerges is that hadrons are like tiny atoms, their coloured "nuclei" being held together by a super-strong force and their valency "electrons" orbiting around the nucleus but held in place by an effective weak interaction. The $\mathcal{S}$ fermions, which are the only particles which respond to both the U(1) and U(1') forces, lie between the "nucleus" and the valon "cloud" yielding an even more effective screen for colour. Thus, hadron physics in the twentieth century is analogous to the "high energy" physics of the last century when only enough energy was available to excite electrons and the nucleus played no rôle in the interactions of atoms. We prefer to regard the hadron physics of today as the low energy hadron chemistry of tomorrow when sufficiently high energies are reached to split the colour nucleus and smash the hadronic atom. Naive though this model may be, it does provide a clear physical basis for the absence of colour at present energies.

Despite the highly qualitative and speculative nature of this section, we may at least derive one more useful number, the masses of the mesons T and T' carrying the U(1) forces holding our hadronic atom (or atomic hadron) together. One could suggest that quarks and leptons should have a size on the order of $^1/_{m_T}$ (not $^1/_{m_T'}$ since quarks and leptons do not "readily" exhibit the T' interaction). Since we have assumed that the strength of T force $^{h^2}/_{4\pi} \gtrsim \alpha$, the known size of leptons < 10^{-15} cm permits us to estimate m_T > 25 GeV. The mass of the T' must be at least this but, since it leads to strong interactions of leptons, it could be ~ 100 GeV or larger(6).

(5.5) WHY ARE LEPTONS LIGHTER THAN QUARKS?

If both leptons and quarks are composites of the same kind and number of particles, there is no evident reason for the enormous mass difference. An obvious, though unsatisfactory, suggestion is that the lepton colouron ($\mathcal{C}_d$) is sufficiently lighter than the hadronic ones ($\mathcal{C}_{abc}$). However, the presence of a light object inside leptons is bad for g-2 of the muon and we immediately reject this idea(67).

The solution we propose requires a new symmetry, expected by leptons, that will enable $\mathcal{D}$ to bind more strongly to lepton colour than hadron colour. To implement this new symmetry, we imagine that $\mathcal{C}$ and $\mathcal{D}$ are part of an SU(5) symmetry(68) and write:

$$\mathcal{F} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

Now, the Higgs scalar D which was required to give mass to T and T' transforms like $\mathcal{L}\overline{\mathcal{C}}$ and acquires a vacuum expectation value:

$$\langle D\rangle = (0,\ 0,\ 0,\ d_4)$$

which suggests that the vacuum expectation value of $\mathcal{F}$ be:

$$\langle \mathcal{F}\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ d_1 \\ d_2 \end{pmatrix}, \qquad d_1 d_2 = d_4 .$$

We thus identify, at this stage, a residual SU(3') X SU(2) symmetry(69). The SU(3') corresponds to ordinary hadron colour and the SU(2) is responsible for the additional binding force between $\mathcal{L}$ and $\mathcal{C}_d$. Clearly, this SU(2) symmetry has to be badly broken by hadrons to account for the presence of $\mathcal{L}$ (without $\mathcal{C}_d$) inside quarks.

It is clear that this SU(2) force saturates(70) and will not lead to exotic bound states of leptons. More precisely, if we write the force between $\mathcal{L}$ and $\mathcal{C}_d$ as

$$v_{ij} = V\, \underset{\sim}{I}_i \cdot \underset{\sim}{I}_j$$

where V represents all other possible degrees of freedom besides SU(2) (which we call ℓ-spin) and $\underset{\sim}{I}$ is the ℓ-spin generator. Then, for an n particle system consisting of

$\mathcal{L}$'s and $\mathcal{C}_d$'s, the interaction is:

$$V(n) = \tfrac{1}{2} \sum_{i \neq j} v_{ij} = \tfrac{1}{2}\left[\sum_{i,j} V\, T_i . T_j - \sum_{i} V\, T_i \cdot T_j\right]$$

$$= \frac{V}{2}\left[L(L+1) - n\ell(\ell+1)\right]$$

where L is the total ℓ-spin and ℓ is the ℓ-spin eigenvalue for a single $\mathcal{C}_d$ or $\mathcal{L}$ (= ½). The force is attractive only for L = 0 in which case a lepton would appear neutral with respect to ℓ-spin forces and not attract other leptons.

To obtain a guide to preon mass, we first make the obvious assumption that preons must be at least as strongly bound inside quarks as quarks are bound inside low lying baryons. If quarks have mass on the order of 3-5 GeV (itself an assumption), then in low lying hadrons (mass about 1 GeV) something like 90% of the constituent quark mass is taken up by binding energy. Assuming the same proportion of preon mass is taken up inside quarks, we have $m_{preon} \sim$ 10 GeV.

In the following chapter, we will assume that the magnitude of the mass splitting between hadronic and leptonic colourons is determined by the magnitude of the breaking of SU(4) symmetry down to SU(3) and that lepton colourons are lighter than the hadronic ones. It has, however, been suggested that, to account for the point - like behaviour of leptons, they may contain particles of very great mass, perhaps as large as 10^5 GeV. and the lepton colouron could conceivably be this massive. It is superfluous to add that such particles would have to be very strongly bound.

The only effect of this assumption of ultra-heavy preons on our work is to eliminate decay modes involving a free d-colouron. As most of the decays we discuss involve only $\mathcal{C}_d$ composites and as the super strong binding is already accommodated by the work of this section, we need pay little attention to this possibility in what follows.

CHAPTER 6

UNCONFINED, UNSTABLE PREONS

(6.1) TRANSITIONS TO KNOWN PARTICLES

Encouraged by the predictions of Chapter 4, we are prepared to speculate on the possibility that preons also exist as free particles. In order that they would have escaped detection, we must assume, as for quarks, that their decays are sufficiently rapid. With a natural reluctance to predict a plethora of new particles, we will first examine the possibility that preons decay into known particles, especially leptons and mesons.

Because all known particles are supposed to be preon composites, a single preon must decay into a bound state of itself and at least two others and this essentially requires the creation of a pair in the vicinity of the original preon. A mechanism for spontaneously creating a virtual preon pair out of the vacuum is furnished by a set of scalars which transform under the gauge group like the particle pair we wish to create and acquire nonvanishing vacuum expectation values. Although it is not necessary to identify these fields with the Higgs scalars already introduced, for reasons of economy, we will choose to do so.

Fig. 20 depicts a valon-lepton transition that is possible because the field D (which transforms like $\mathcal{S}\overline{\mathcal{C}}$) has an expectation value. In effect, this transition occurs because the vacuum spontaneously breaks colour symmetry. We note that if <D> had the form (d_1, 0, 0, d_4) then transitions of the sort $\mathcal{Q} \rightarrow q_a$ would also be possible.

For the singlet preon $\mathcal{S}$ to undergo similar transitions, it is enough to have fields transforming as $\mathcal{Q}_{L,R}\bar{\mathcal{C}}$ acquire a nonvanishing vacuum expectation value and we identify these with $\bar{B}$ and $\bar{C}$, respectively. For the colourons, objects transforming as $\bar{\mathcal{Q}}_{L,R}\bar{\mathcal{S}}$ are needed which are not included in the set of Higgs scalars we have already introduced. Thus, we add ifelds E and F which transform under our local group as $(2+2, 1, 1)_{1,0}$ and $(1, 2+2, 1)_{1,0}$, respectively such that:

$$\langle E\rangle = \begin{pmatrix} e_1 \\ 0 \\ 0 \\ e_4 \end{pmatrix}$$

$$\langle F\rangle = \begin{pmatrix} f_1 \\ 0 \\ 0 \\ f_4 \end{pmatrix}$$

It is easily seen that these fields do not lead to any new mixings in the gauge meson sector and their only effect is to make corrections to the mass of the W and T mesons. Since we expect the vacuum expectation values to be small, these corrections will be ignorable.

To obtain a rough guide to the behaviour of diagrams like fig. 20, we insert a term $\frac{1}{(p'^2 - m_{\mathcal{Q}}^2)^2}$ (needed for convergence) as a form factor where the three preons combine to form a lepton. The Greens function is then:

$$\frac{\pi^2 f}{(2\pi)^4}\, \bar{\ell}(p)\ \frac{<\mathcal{S}\bar{\mathcal{C}}>}{m_{\mathcal{Q}}^2}\, \mathcal{Q}(p).$$

Of course, this does not yet represent a physical amplitude since neither the preon nor lepton can be on-shell. So, we turn now to an examination of diagrams for preon decay where the Greens function of fig. 20 can contribute.

Because of the form chosen for <D>, only valon-lepton transitions are allowed and the most elementary process for valon decay is that shown in fig. 21 where some modes are:

$$(p,\ n,\ \lambda,\ \chi) \rightarrow (\nu_e, e^-,\ \mu^-,\ \nu_\mu) + \ell\bar{\ell}.$$

Using the value of $<D> = <\mathcal{S}\bar{\mathcal{C}}> \sim 10^2$ GeV (which is fixed by the lower energy limit at which leptons undergo strong interactions[6]) we estimate $\tau_{\mathcal{Q}} \gtrsim 10^{-14}$ sec.. In the case of the charged valons n and λ, we may attach photon lines:

$$(n,\ \lambda) \rightarrow (e^-,\ \mu^-) + \gamma$$

and, on the basis of larger coupling and greater available phase space alone, we may expect an enhancement of about 10^3 for the lifetimes.

For the colourons, we observe that the a, b and c-types will undergo transitions to a quark of the same colour and d-type will change only to a lepton. Thus, for d-type colourons, only the diagram of fig. 21 applies as does the analysis following it, the only difference being the field which acquires a nonvanishing vacuum expectation value which, here, is $<\bar{\mathcal{Q}}_{L,R}\bar{\mathcal{L}}>$. Since it contributes to the mass of the T and W mesons, it should not be larger than about 1-10 GeV. With values in this range, we predict $\tau_{\mathcal{C}_d} \gtrsim 10^{-12}$ sec. Modes of the form:

$$\mathcal{C}_d \rightarrow (\bar{\nu}_e, \bar{\nu}_\mu) + \ell\ell$$

as expected. After the strong colourons have changed to a quark, the results of Chapter 4 may be applied to find the resulting decays into leptons and mesons. Transitions of the form:

$$(a,\ b,\ c) \rightarrow (\bar{p},\ \bar{x})_{a,b,c}$$

are possible and the diagram of fig. 22 represents one type of decay for b and c-colourons into a lepton and pion. Decays of the form:

$$\mathcal{C}_a \rightarrow \pi^0 + \nu_e,\ \pi^+ + e^-$$

$$\mathcal{C}_{b,c} \rightarrow \pi^- + \nu_e$$

are allowed and, for the case of a single lepton and meson, we estimate $\tau_{\mathcal{C}_{a,b,c}} \gtrsim 10^{-10}$ sec.. Arguments indicating that charged semi-leptonic decays of b and c-type colourons are suppressed with respect to the neutral semi-leptonic modes can be advanced in the manner of Chapter 4.

Finally, the $\mathcal{S}$ preon may undergo transitions of the form:

$$\mathcal{S} \to (p_a, n_b, \lambda_c, \nu_\mu)$$

if the field $\mathcal{Q}_L \bar{\mathcal{C}} = c$ acquires a vacuum expectation value and, therefore, decays of the sort:

$$\begin{aligned} \mathcal{S} &\to \nu_e + \pi^o,\ e^- + \pi^+ \\ &\to \nu_\mu + K^o,\ \nu_e + \pi^o \\ &\to \eta^o + \nu_\mu \\ &\to \nu_\mu + e^+ e^-,\ \nu_\mu + \mu^+ \mu^- \end{aligned}$$

are allowed. The diagrams of fig. 21 and 22 apply to the transitions to leptons and quarks, respectively. Since $c_1 \sim 1\text{-}3$ GeV and $c_4 \sim 10^2$ GeV, we estimate $\tau_{\mathcal{S}} \gtrsim 10^{-9}$ sec. for decay into a single lepton and meson and $\tau_{\mathcal{S}} \gtrsim 10^{-12}$ sec. for decay into three leptons.

(6.2) PREON DECAYS AND NEW MESON STATES

Interactions mediated by the mesons T and T' give rise to a new class of diagrams that lead to the decays of preons into leptons and new mesons. (Their decays will be discussed in Section 6.4). As with the case for quark decay, we find the dominant contributions come from tree diagrams and convergent loop diagrams.

An example of a tree diagram is given in fig. 23 which represents decays of the form:

$$\mathcal{Q} \rightarrow \ell + \bar{\mathcal{S}}\, d\,.$$

(The same diagram will, of course, permit decays like $\mathcal{Q} \rightarrow \bar{q}_{abc} + \bar{\mathcal{S}}\, \mathcal{C}_{abc}$.) This process occurs to order $h^2/m_T^2 \times h'^2/m_{T'}^2$. The state $\bar{\mathcal{S}}d$ is expected to be a strongly bound spin zero object with a mass of about 3-5 GeV (by arguments similar to those of Section 5.5).

The same decays will arise from loop diagrams owing to the W^o-T' mixing which appears after spontaneous symmetry breaking. Because T' couples to currents of the form $\bar{\mathcal{C}}\gamma_\mu \mathcal{C} + \bar{\mathcal{S}}\gamma_\mu \mathcal{S}$, we may imagine processes like those pictured schematically in fig. 24a and represented more transparently in fig. 24b. The admitted decays are:

$$(p,\ n,\ \lambda,\ \chi) \rightarrow (\nu_e,\ e^-,\ \mu^-,\ \nu_\mu) + \bar{\mathcal{S}}\, d$$

or

$$p \rightarrow p_a + \bar{\mathcal{S}}\, a, \text{ etc..}$$

The matrix element for these decays is:

$$(g^2\, h'^2\, c_4^2)\ \frac{\not{p}}{2}\ \frac{\pi^2}{(2\pi)^4}\ \frac{1}{m_W^2\, m_{T'}^2} \left((m_{T'}^2 + \frac{m_T{}'^4}{m_W^2})\, \ell n(1 + \frac{m_W^2}{m_{T'}^2}) + \right.$$

$$+ \left(m_W^2 + \frac{m_W^4}{m_{T'}^2}\right) \ln\left(1 + \frac{m_{T'}^2}{m_W^2}\right) + 2\left(m_W^2 - m_{T'}^2\right)\Bigg] (1 - \gamma_5)$$

leading to a decay width:

$$\Gamma = \frac{\pi^5}{(2\pi)^{10}} \frac{m_{\mathcal{Q}}\, m_\ell}{m_{\bar{\mathcal{S}}\mathcal{C}}} \frac{(g^2 h'^2 c_4^2)^2}{m_W^4\, m_{T'}^4} \Bigg\{ \left(m_{T'}^2 + \frac{m_{T'}^4}{m_W^2}\right) \times \ln\left(1 + \frac{m_W^2}{m_{T'}^2}\right)$$

$$+ \left(m_W^2 + \frac{m_W^4}{m_{T'}^2}\right) \ln\left(1 + \frac{m_{T'}^2}{m_W^2}\right) + 2\left(m_W^2 - m_{T'}^2\right)\Bigg\}^2 .$$

With $m_{\mathcal{Q}} \approx 10$ GeV, $m_{\bar{\mathcal{S}}\mathcal{C}} \approx 2$ GeV and $m_W \approx m_{T'} \approx 30$ GeV, we predict (for the decay into $\ell + \bar{\mathcal{S}}d$) $\tau_{\mathcal{Q}} \gtrsim 10^{-20}$ sec.. We expect this mode to dominate over the decay into $\bar{q}_{abc} + \bar{\mathcal{S}}$ $\mathcal{C}_{abc}$ from phase space considerations as $m_\ell << m_q$ and $m_{\bar{\mathcal{S}}d} << m_{\bar{\mathcal{S}}\mathcal{C}_{abc}}$, the latter in view of the stronger binding between $\mathcal{S}$ and lepton colourons.

The one-loop diagram for the decay of the strong colourons arises from mixing between $V_{a,b,c}$ and T and the fact that T will couple to currents of the form $\bar{\mathcal{Q}}\gamma_\mu\mathcal{Q} - \bar{\mathcal{S}}\gamma_\mu\mathcal{S}$. ($V_{a,b,c}$ are the diagonal components of the SU(3') octet exhibited explicitly after eq. (6) for example.) The admitted decays are of the form:

$$\mathcal{C}_{abc} \to \bar{q}_{a,b,c} + \mathcal{S}\mathcal{Q}$$

and, since the diagram is identical to fig. 24, we can obtain the matrix element and width from the appropriate expressions for valon decay by making the substitutions $m_W \rightarrow m_V$, $m_{T'} \rightarrow m_T$ (about the same), $m_\ell \rightarrow m_q$ and observing that the magnitude of the V-T mixing is fhc_1^2 (instead of ghc_4^2 for W-T' mixing). With these replacements, we estimate $\tau_{\mathcal{C}abc} \gtrsim 10^{-21}$ sec.. The quarks may now decay as described in Chapter 4.

The process responsible for the one loop decay of the weak colouron arises because of S^0-T mixing which is proportional to fhb_4^2. Making the appropriate alterations in the basic matrix element, we find, for modes of the sort:

$$\mathcal{C}_d \rightarrow \bar{\ell} + \mathcal{Q}\mathcal{L},$$

$\tau_{\mathcal{C}d} \gtrsim 10^{-20}$ sec..

The colour-valency mixing discussed in Chapters 3 and 5 also lead to decays of the strong colourons by permitting them to change to lepton colour. The first of these processes, shown in fig. 25, is analogous to the lepton transitions of a-type quarks and results from V-W-X mixing. The effective amplitude for this process is:

$$\frac{1}{8\pi^2}\left(\frac{f^4\ g^2\ c_1^3\ c_4}{m_X^2}\right)\frac{m_{\mathcal{C}b}}{m_W^2}\ \ln\ \frac{m_X^2}{m_W^2}$$

and, since $m_{\mathcal{C}a} \simeq m_{\mathcal{C}d}$, this process should be physical[71] and we estimate $\tau \gtrsim 10^{-11}$ sec. It thus appears that the direct decay (into $q_a + \mathcal{L}\mathcal{Q}$) is preferred.

The a-type colourons may also change their colour by producing a physical or virtual V_ρ, the latter through V-W mixing, and diagrams for these processes are given in fig. 26 and fig. 27 leading to decays of the sort:

$$\mathcal{C}_a \to V_\rho^+ + \mathcal{C}_b$$

and

$$\mathcal{C}_a \to e^+ \nu_e + \mathcal{C}_b, \quad \pi^+ + \mathcal{C}_b,$$

respectively. Since we expect the strong colourons to be nearly of the same mass (to the extent that SU(3') symmetry is unbroken) the first mode is probably not possible by phase space considerations. The second modes are rather more likely and, with an effective amplitude of $\frac{f^2 \; G_F \; c_1^2}{m_V^2}$ we estimate, for the leptonic mode and assuming a $\mathcal{C}_a$-$\mathcal{C}_b$ mass difference of a few hundred MeV, that $\tau_{\mathcal{C}_a} \gtrsim 10^{-8}$ sec.. However, after changing to b-colour by producing a V_ρ^+, the a-colouron may become a b-type quark which can undergo direct transition to a lepton via W-X mixing as shown in fig. 28 leading to the sequential process:

$$\mathcal{C}_a \to V_\rho^+ + e^- \qquad V_\rho^+ \to e + \nu_e$$

The first stage has an effective amplitude of:

$$M \sim f\left(\frac{\pi^2 f}{(2\pi)^4} \frac{\langle \bar{\delta}\bar{q} \rangle}{m_q^2}\right) \frac{f^2 \; g^2 \; c_1 \; c_4}{m_X^2}$$

leading to a lifetime $\tau \gtrsim 10^{-8}$. As derived in Chapter 4, the second stage occurs very rapidly with $\tau \gtrsim 10^{-14}$ sec.. Since the V's are not expected to be much heavier than 3 GeV, there is a fair amount of phase space available for these decays as evinced by rather large widths.

The b and c-type colourons may also decay into leptons and mesons through W-X mixing as shown in the tree diagram of fig. 29. This process, with an effective amplitude of $G_F\ G_B\ c_1\ c_4$, should be possible as a small mass difference between strong and weak colourons should be expected and, assuming a mass difference as great as 1 GeV, we find, for decays of the sort:

$$\mathcal{C}_{b,c} \rightarrow e^- \bar{\nu}_e + \mathcal{C}_d$$
$$\rightarrow \pi^- + \mathcal{C}_d ,$$

$\tau_{\mathcal{C}_{b,c}} \gtrsim 10^2$ and 10^4 sec., respectively. These modes, therefore, have negligible branching ratios.

In fig. 30, we have indicated a decay mode for strong colourons leading to the production of a new meson state $M_{a,b,c} \equiv \mathcal{C}_{a,b,c} \oplus \mathcal{C}_d$. Since, by the arguments of Section 5.5, we expect bound states of this sort to have a mass of about 3-5 GeV, the process

$$\mathcal{C}_{a,b,c} \rightarrow M_{a,b,c} \oplus \mathcal{C}_d$$

would be forbidden unless there were an appreciable mass difference between the strong and weak colourons[72]. If we permit ourselves the luxury of imagining a mass difference of a few hundred MeV between the final and initial states, this process, of order $h'^2/m_{T'}^2$, could take place with $\tau \gtrsim 10^{-12}$ sec..

The $\mathcal{S}$ singlet will decay through a one-loop mechanism provided by T-W^0-T' mixing and the diagram for the processes

$$\begin{aligned}\mathcal{S} &\to q_{a,b,c} + \mathcal{C}_{a,b,c}\,\bar{\mathcal{Q}} \\ &\to \ell + \mathcal{C}_d\,\bar{\mathcal{Q}}\end{aligned}$$

is given in fig. 3. The effective amplitude is

$$\frac{1}{8\pi^2}\,\frac{(ghh'\,c_4^2)^2}{m_W^2\,m_{\mathcal{S}}}$$

and we estimate $\tau \gtrsim 10^{-28}$ sec.. This is rather more rapid than the other preons and the enhancement appears because of stronger mixing between the participating gauge mesons.

We close this section by restating the dominant decay modes for free preons.

$$\begin{aligned}\mathcal{Q} &\to \ell + \bar{\mathcal{S}}d, \quad \tau \gtrsim 10^{-20} \text{ sec.} \\ \mathcal{C}_{a,b,c} &\to \bar{q}_{a,b,c} + \mathcal{Q}\mathcal{S}, \quad \tau \gtrsim 10^{-21} \text{ sec.} \\ \mathcal{C}_d &\to \ell + \mathcal{Q}\mathcal{S}, \quad \tau \gtrsim 10^{-20} \text{ sec.} \\ \mathcal{S} &\to \ell + \bar{\mathcal{Q}}d, \quad \tau \gtrsim 10^{-28} \text{ sec..}\end{aligned}$$

(6.3) QUARK DECAY

We intend, first, to examine quark decay as resulting from the simultaneous decays of the constituent preons via the dominant processes just described. As the dominant decay of hadronic colourons leads to a quark in the final state, we will have to employ a sequential mechanism where the strong colours first change to lepton colour. In the case of a-colour, this takes place via the double mixing one-loop process of fig. 25 and for b and c-colour through the tree diagram of fig. 29. If quarks have mass as great as 5 GeV, a-colourons could change their colour more efficiently by producing a physical V_ρ which would then decay rapidly to leptons and mesons. The remaining new mesons (such as $\bar{\mathcal{S}}d$ and $\bar{\mathcal{Q}}d$) can then combine to form leptons. The various possibilities can be represented schematically as follows. For a-type quarks, either:

$$\text{(i)} \quad \bar{\mathcal{C}}_a \to \bar{\mathcal{C}}_d \to \ell + \bar{\mathcal{Q}}\bar{\mathcal{S}}$$

$$\mathcal{Q} \to \ell + \bar{\mathcal{S}}d$$

$$\mathcal{S} \to \ell + \bar{\mathcal{Q}}d$$

or

$$\text{(ii)} \quad \bar{\mathcal{C}}_a \to V_\rho + \bar{\mathcal{C}}_b, \qquad V_\rho \to \ell\bar{\ell}, \qquad \bar{\mathcal{C}}_b \to \mathcal{C}_d + \ell\bar{\ell}, \qquad \mathcal{C}_d \to \bar{\ell} + \mathcal{Q}\mathcal{S}$$

$$\mathcal{Q} \to \ell + \bar{\mathcal{S}}d$$

$$\mathcal{S} \to \ell + \bar{\mathcal{Q}}d \, .$$

For b and c-type quarks, we have:

$$\bar{\mathcal{C}}_{b,c} \to \bar{\mathcal{C}}_d + \ell\bar{\ell}, \qquad \bar{\mathcal{C}}_d \to \ell + \bar{\mathcal{Q}}\bar{\mathcal{S}}$$

$$\mathcal{Q} \to \ell + \bar{\mathcal{S}}d$$

$$\mathcal{S} \to \ell + \bar{\mathcal{Q}}d$$

In each of the above cases, the mesons $\bar{\mathcal{Q}}\bar{\mathcal{S}}$, $\bar{\mathcal{Q}}d$ and $\bar{\mathcal{S}}d$ may either combine to form two anti-leptons or decay into leptons by the mechanisms given in the next section. In the case where the new mesons combine to form leptons, a quark could decay into as many as seven leptons and anti-leptons. If for no reason other than phase space considerations, we would expect such modes to be suppressed and will, therefore, assume that not all these leptons appear as physical states[73]. Thus, some allowed decays are, for a-type quarks:

$$(p_a, n_a, \lambda_a, \chi_a) \rightarrow (\nu_e, e^-, \mu^-, \nu_\mu) + (e^+ e^-)$$

after the conversion of a-colour to lepton colour and the rapid decay of the weak preon. Because a-colour changes to d-colour very much more slowly than the preons decay, only the processes just listed will be observed. For the case where a virtual V_ρ is produced (a very rapid decay being of order f^2) we may expect, for example,

$$(p_a, n_a, \lambda_a, \chi_a) \rightarrow (\nu_e, e^-, \mu^-, \nu_\mu) + (\nu_e \bar{\nu}_e).$$

All the virtual processes occur at least as rapidly as the preons decay and the decays listed are what will be observed. Admittedly, these modes do not seem very likely as they presume a great many things happening at the same time not to mention the circumstance that all the exotic mesons are sufficiently localized in space to combine to form leptons.

Thus, our estimates will provide only a rough guide.

A conservative estimate for the first set of q_a decay can be obtained by looking at the amplitude for the $\mathcal{S}$ and $\mathcal{Q}$ preons to decay via their dominant mode simultaneously with the change of the a-colouron to the d-colouron. This is, very approximately,

$$\frac{1}{64(1024)\pi^9} f^2 h'^2 h^4 c_1^3 c_4^7 \frac{m_{\mathcal{P}} m_{\mathcal{Q}}}{m_{\mathcal{S}}} G_F^3 G_B \ln \frac{m_X{}^2}{m_W{}^2} ,$$

leading to a lifetime $\tau_{q_a} \gtrsim 10^{1}$ sec. which suggests that quarks are uncomfortably stable. The situation is not much improved for the alternative decay mechanism of a-quarks nor of b and c-quarks whose decays are built out of the dominant decays of the constituent preons. We are faced with the same situation as in Chapter 4 where free quarks decayed very rapidly yet the proton, decaying only in third order, was perfectly stable.

Fortunately, other mechanisms for quark decay present themselves and the relevant diagram, common to all hadronic quarks is given in fig. 32 where we find decays of the form:

$$q_{a,b,c} \rightarrow \bar{M}_{a,b,c} + (\nu_e, e^-, \mu^-, \nu_\mu) .$$

The states $\bar{M}_{a,b,c}$ are the new mesons introduced in the last section. This reaction is of order $h'^2/m_{T'}^2 \approx 10^{-2}$ GeV^{-2} and, with a q-M mass difference on the order of 3 GeV, we predict $\tau_{q_{a,b,c}} \gtrsim 10^{-23}$ sec., rather less than the vastly more complicated processes which began this discussion.

(6.4) DECAYS OF NEW MESON STATES

The decays of quarks and preons dealt with in the preceeding sections have led to the production of twelve mesons which are bound states of two preons and have been denoted by $\bar{\mathcal{S}}d$, $\bar{\mathcal{D}}d$, $\mathcal{D}\mathcal{S}$ and $M_{a,b,c}$. We now consider the decays of these mesons into leptons.

The nine mesons produced in preon decay can decay into leptons only via the combined transition of the component preons into leptons by means of the mechanisms illustrated in figs. 20 and 21. The simultaneous cónservation of charge and fermion number requires that at least one of the processes be accompanied by a four fermion weak interaction as described in fig. 33 for the decay

$$\bar{\mathcal{S}}d \rightarrow \nu_\mu + \bar{\nu}_\mu$$

With a meson mass of 3-5 GeV, we estimate $\tau \gtrsim 10^{-11}$ sec., having simply multiplied the effective strengths for the individual interactions. We note that an additional four fermion weak interaction can be present in the case of the $\bar{\mathcal{S}} - \bar{\nu}_\mu$ transition leading, for example, to

$$\bar{\mathcal{S}}d \rightarrow e^+ e^- + \nu_\mu \bar{\nu}_\mu$$

However, this process, being of order G_F^2 is highly suppressed and we estimate $\tau \gtrsim 10^{-1}$ sec.. Treating the four $\bar{\mathcal{D}}d$ states similarly, we find the following typical decays:

$$\bar{p}d \rightarrow \nu_e \, \bar{\nu}_e$$

$$\bar{n}d \rightarrow e^+ \, \nu_e$$

$$\bar{\lambda}d \rightarrow \mu^+ \, \nu_\mu$$

$$\bar{\chi}d \rightarrow \nu_\mu \, \bar{\nu}_\mu$$

with lifetimes $\tau \gtrsim 10^{-11}$ sec.. Finally, for the $\mathcal{S}\,\mathcal{Q}$ composites, some typical modes are:

$$\mathcal{S}\,p \rightarrow \bar{\nu}_e + \nu_\mu$$

$$\mathcal{S}\,n \rightarrow e^- + \nu_e$$

$$\mathcal{S}\,\lambda \rightarrow \mu^- + \bar{\nu}_\mu$$

$$\mathcal{S}\,\chi \rightarrow \nu_\mu + \bar{\nu}_\mu$$

with $\tau \gtrsim 10^{-13}$ sec., the difference arising from the different magnitudes of the fields which acquire a vacuum expectation value.

The decays of the negatively charged mesons $M_{b,c}$ is schematically represented in fig. 34 which gives rise to decays of the sort:

$$M_{b,c} \rightarrow e^- \, \nu_e, \; \mu^- \, \nu_\mu .$$

The width for these processes is largely determined by the strength of W-X mixing and, if these mesons have mass in the range of 3-5 GeV, we estimate $\tau_{M_{b,c}} \gtrsim 10^{-8}$ sec. The decay of the neutral M_a requires the change of a-colour to lepton colour through the double V_ρ-W-X mixing as displayed in fig. 35. Decays of the sort:

$$M_a \rightarrow e^+ e^-, \mu^+ \mu^-$$

are expected and we predict $\tau_{M_a} \gtrsim 10^{-9}$ sec.

Having considered the decays of the new mesons that have arisen naturally from the decays of single preons into leptons, it is now appropriate to examine the decay modes of the fifteen remaining two preon bound states, $\mathcal{S}\bar{\mathcal{C}}_{abc}$ and $\bar{\mathcal{Q}}\mathcal{C}_{abc}$ that are the partners of the ones already described.

The states $\mathcal{S}\bar{b}$ and $\mathcal{S}\bar{c}$ may decay through a combination of the processes of fig. 20 (for $\mathcal{S}$) and 22. Thus, we have:

$$\mathcal{S}\bar{b}, \mathcal{S}\bar{c} \rightarrow e^+ \nu_\mu ,$$

for example, and we find $\tau \gtrsim 10^{-5}$ sec., the long lifetime obtaining as this process is at least of order G_B. However, by attaching π lines to the quark propagator in fig. 22, we can achieve decays of the sort:

$$\mathcal{S}(\bar{b}, \bar{c}) \rightarrow \pi^+ + \nu_e + \bar{\nu}_\mu, \; K^+ + \nu_\mu + \bar{\nu}_\mu$$

and there will be an enhancement of about 10^3 for decay into two leptons and several pions as described in Chapter 4.

The situation for $\mathcal{S}\bar{a}$ is rather different (as most previous cases involving a-colour have been) and, here, we must employ the double mixing mechanism of fig. 25, required to change a-colour to lepton colour) combined with the basic mechanism, of fig. 20, for example, that permits $\mathcal{C}_d$-lepton transitions. This process is pictured in fig. 36. Further, in order to conserve fermion number, we must adjoin an additional four fermion interaction to the $\mathcal{S}$ decay diagram. This badly suppresses the process and, for decays of the sort:

$$\mathcal{S}\,\bar{a} \to \nu_\mu\,\bar{\nu}_\mu$$

we estimate $\tau \gtrsim 10^{-1}$ sec.. There appears to be no other mechanism for the decay of these three mesons and new stable particles of mass 3-5 GeV seems to be an unavoidable consequence of the model.

Finally, there are the twelve states $\bar{\mathcal{D}}\mathcal{C}_{a,b,c}$. For the states composed of b and c colourons, decay into leptons is permitted by the simultaneous transition of the valon into a lepton via the mechanism of fig. 20 and the transition of the colouron to a quark followed by transition to a lepton achieved through W-X mixing as in fig. 22. Some allowed decays are:

$$\bar{p}(b,c) \to e^-\,\bar{\nu}_e$$

$$\bar{n}(b,c) \to e^-\,e^+$$

$$\bar{\lambda}(b,c) \to e^-\,\mu^+$$

$$\chi(b,c) \to e^-\,\bar{\nu}_\mu$$

and we predict $\tau \gtrsim 10^{-8}$ sec.. Again, we may attach meson lines to the quark propagator of fig. 22 leading to decays of the kind:

$$\bar{p}(b,c) \to \pi^- + \nu_e + \bar{\nu}_e$$

$$\bar{n}(b,c) \to \pi^- + \nu_e + e^+$$

$$\bar{\lambda}(b,c) \to \pi^- + \nu_e + \mu^+$$

$$\bar{\chi}(b,c) \to \pi^- + \nu_e + \bar{\nu}_\mu \ .$$

For several mesons, there will be the usual enhancement due to the large $qq\pi$ coupling.

(6.5) SUMMARY AND CONCLUSIONS

Because preons decay rapidly into leptons and mesons which in turn produce leptons, our original hope, that preons may exist as free particles which have escaped detection, has been vindicated. Further, although the preon hypothesis has admitted new quark decay modes (via the mesons $M_{a,b,c}$), one still finds quarks decaying sufficiently rapidly into leptons so that they need not have been observed. However, there are many important differences in the present case, as opposed to the situation where quarks were regarded as fundamental, that we now emphasize.

The most obvious difference in the preon theory is that the dominant mode for quark decay does not involve pions as was the case in Chapter 4. Here, we find that quarks decay exclusively via leptonic modes and the arguments concerning the suppression of the charged semi-leptonic decays of b and c-type quarks are no longer relevant. Indeed, all quarks of strong colour may decay into charged leptons with similar widths. For example, we may have:

$$p_a \rightarrow \bar{M}_a + \nu_e \qquad \tau \gtrsim 10^{-23} \text{ sec.}$$
$$\quad \lfloor\rightarrow e^+ e^- \qquad \tau \gtrsim 10^{-8} \text{ sec.}$$

$$p_{b,c} \rightarrow \bar{M}_{b,c} + \nu_e$$
$$\quad \lfloor\rightarrow e^+ \nu_e$$

$$n_a \rightarrow \bar{M}_a + e^-$$
$$\quad \lfloor\rightarrow e^+ e^-$$

$$\lambda_{b,c} \rightarrow \bar{M}_{b,c} + \mu^-$$
$$\quad \lfloor\rightarrow e^+ \nu_e$$

Thus, all hadronic quarks may be the source of SPEAR $\bar{\mu}$ e events (and may contribute to $e^+ e^-$ cross sections) instead of only those of a-colour as was originally the case. We emphasize that these new results do not invalidate those of Chapter 4. Indeed, if mesons are $q\bar{q}$ composites (which may still be the case in the present scheme) then all the semi-leptonic modes and the associated discussion of Chapter 4 remain valid but are only relevant in a higher order. Thus, these competing predictions furnish a possible test of preon versus quark theory.

The dynamics of proton decay is, of course, very much more complicated since it is now a three body composite. For the decay of the proton into leptons, which requires the simultaneous transition of the three internal quarks to leptons, we require the simultaneous conversion of all three colourons to lepton colour and the (virtual) transition of the remaining preons to mesons and then into leptons. Since the effective strength for these interactions is still $\frac{fgc_1c_4}{m_X^2}$, we are at least assured that protons will not decay more rapidly than we have previously suggested. Indeed, the small probability that all these interactions take place simultaneously (which is required as all intermediate states are more massive than the proton) will act to suppress the decay even further.

The existence of leptonic decay modes for preons suggest that they may be produced in lepton collisions and account for direct lepton production in hadron collisions. For example, valons could be produced in $e^+ e^-$ annihilation:

$$e^+ e^- \rightarrow n\bar{n},\ \lambda\bar{\lambda}$$

and then decay rapidly into leptons, the n-valons producing electrons and the λ muons. Thus, a signal for valon-antivalon production should be the anomalous production of $e^+ e^-$ and $\mu^+ \mu^-$ pairs in $e^+ e^-$ annihilation at centre of mass energy greater than 10 GeV. Valons may also be produced in pp collisions via production of a virtual pair of T mesons as shown schematically in fig. 37. Thus, valons may be responsible for direct lepton production in pp collisions(74) via:

$$\begin{array}{l} p + p \to n + \bar{n} + \text{hadrons} \\ \qquad\quad\;\, | \qquad\! |\!\to e^+ + \bar{d}\,\ell \\ \qquad\quad\;\, |\!\to e^- + d\,\bar{\ell} \end{array}$$

$$\begin{array}{l} p + p \to \lambda \qquad \lambda \\ \qquad\quad\;\, | \qquad\! |\!\to \mu^+ + \bar{d}\,\ell \\ \qquad\quad\;\, |\!\to \mu^- + d\,\bar{\ell} \end{array}$$

The discussion of Section 4.5 that suggested that hadronic quarks might be responsible for SLAC $\bar{\mu}$ e and $\mu^+ \mu^-$ events coupled with the demonstrated tendency for colourons to convert themselves to quarks offers the possibility that SLAC is producing them as well. The arguments of Section 4.5 may be employed to assert that production of strong colourons is responsible for anomalous lepton production as they all may be produced in $e^+ e^-$ annihilation. The rapid decay of the colouron pairs to leptons provides a possible explanation of $\bar{\mu}$ e events seen at SPEAR in a manner analogous to the valon contribution to $e^+ e^-$ and $\mu^+ \mu^-$ cross sections.

Colourons may also be pair produced in pp collisions via the virtual production of either T' mesons or the V(8) gluons as represented in fig. 38. As the mass of the V(8) mesons is much less than that of the T' and T mesons, we expect the process involving the V(8)'s to dominate and lead to a larger effective strength for colouron production than valon production. Again, because of the possibility of quark-colouron transitions, we expect that both should be seen in direct lepton production in pp collisions at relatively high energies $\gtrsim$ 20 GeV.

We recall that the preon bound states all have leptonic decays and this suggests that at least some of them may have already been produced. The M_a, for example, may decay into $e^+ e^-$ and $\mu^+ \mu^-$ and is thus a possible explanation for anomalous lepton production at SLAC. The anomalous production of $\bar{\mu}$ e is thought to arise from the pair production of bosons that decay into electrons or muons and their neutrinos[75] and the mesons we have denoted by $M_{b,c}$, $\mathcal{S}(\bar{b}, \bar{c})$, $\bar{p}(b,c)$ and $\bar{\chi}(b,c)$ are possible candidates[76]. The coloured mesons $\bar{n}(b,c)$ and $\bar{\lambda}(b,c)$ decay, respectively, into $e^+ e^-$ and $\mu^+ e^-$ and we draw special attention to the latter mode whose anomalous production may arise from two-body decays of a scalar or pseudo-scalar meson[75]. Of course, we have not suggested what the spin of our new mesons might be and the possibility that theyare vectors in equally intriguing. In fig. 39, we indicate how a spin one $\bar{n}$ b could contribute to $\sigma(e^+ e^- \to e^+ e^-)$. Similarly, a vector $\bar{\lambda}$ b would contribute to $(e^+ e^- \to \mu^+ e^-)$. The temptation to identify these coloured vectors with the ψ/J particles is somewhat reduced by the very small widths we have found for them ($\sim 10^{-16}$ GeV) which is certainly inadequate for ψ/J and is, in fact, so small that it is likely to have escaped observation. Thus, we could suggest that these new particles be considered relatives ψ/J having similar quantum numbers and decay modes and differing only in lifetime and that a very careful search for resonances in $\sigma(e^+ e^+ \to e^+ e^-, \mu^+ \mu^-, \mu^\pm e^\mp)$ in the region of centre of mass energy 3-5 GeV would reveal them. On the other hand, we should recall the assumptions that were introduced to derive these lifetimes. The chief mechanism for the decay of these bound states (as for the decay of the constituent preons) which is shown in fig. 20 requires the creation of a preon pair out of the vacuum. We have seen that this can be achieved by allowing a scalar field which transforms under the gauge group like the required preon pair to acquire a non-vanishing vacuum expectation value. We have chosen to identify these scalar fields with those already introduced to implement the Higgs-Kibble mechanism and the vacuum expectation values these fields acquire are rather tightly constrained by restrictions on gauge meson mass and experimental limits on anomalous interactions. It is these fixed values that we have been obliged to use in our estimates for preon and preon bound state decay widths. However, we emphasize that the importance of diagrams like fig. 20 is not so much for the numerical estimates it produces but rather for the mechanism it suggests, namely that preons and their bound states decay only if the vacuum breaks a symmetry (colour or charm, for example). There is no compelling reason to identify this symmetry breaking with that which gives the gauge particles mass and, in fact, it could be much larger. Thus, it is entirely possible that the widths of our new meson states could be comparable with the ψ/J's and we emphasize that the numbers we have given should be taken only as a guide.

FOOTNOTES AND REFERENCES

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(26) M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960).

(27) N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).

(28) This principles are stated and discussed in ref. (8).

(29) See, also, J.C. Pati, Abdus Salam and J. Strathdee, Nuovo Cimento 26A, 72 (1975).

(30) In ref. (29), a specific example of the breaking $SU(6) \to SU(3)_L \times SU(3)_R \times U(1)_F$ (F = fermion number) is considered.

(31) H. Weyl, The Classical Groups, their Invariants and Representations, Princeton 1946.

(32) E.S. Konopinski and H.M. Mahmoud, Phys. Rev. 92, 1045 (1953).

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(36) None of these fermions are, at this point, physical and later it will be necessary to introduce Cabibbo rotations. Further, left on their own, the indicated neutrinos are four component objects that will acquire mass through spontaneous symmetry breaking.

(37) Note that it is not necessary to introduce a new U(1) gauge for electromagnetism as was the case in ref. (1).

(38) An advantage in having lighter X's would lie in the support of the proposed explanation of the behaviour of $\sigma(e^+ e^- \to \text{hadrons})$ by J.C. Pati and A. Salam, Phys. Rev. Letters 32 (1974) 1083. Here, the X mass is taken as low as 15-50 GeV.

(39) This is discussed more fully in ref. (2). A general discussion of these ideas is provided by Ling Fong Li, Phys. Rev. D9, 1723 (1974).

(40) We note that we do not require $\theta_L = \theta_R$ or $\phi_L = \phi_R$. The

Cabibbo angle is $\theta_L - \phi_L$. However, the requirement of neutrino universality imposes separate restrictions on θ and ϕ as remarked in ref. (2).

(41) It is also appropriate to define the physical Fermi fields via the relation $\psi = R(\theta)R(\phi)\psi_D$. One can then write the Lagrangian for the theory in terms of the fields A_D, B_D, C_D, ψ_D and $W(\theta,\phi)$ as in ref. (2).

(42) In reference (2), $\theta_L = \theta_R = \phi_L = \phi_R = 0$ is taken for convenience and we see here the greater variety of colour-valency mixing which is required for all quarks to decay and for all leptons to possess anomalous strong interactions.

(43) Apart from baryon-lepton number violation, the rôles of c_4 and b_4 are largely interchangeable and c_4 could be very much larger without seriously affecting observable consequences. In the "prodigal" model, b_4 can be as small as $10^2 - 10^3$ GeV and $c_4 \approx c_1 \alpha^2$.

(44) Terms of order (c^2/a^2), (c^2/b^2) and $(g^2_{L,R}/f^2)$ have been neglected when they have no physical consequence. Some terms of this order have been retained in the expression for $\tilde{U}^0$ as the terms proportional to Δ contribute to parity violation in nuclear transitions. See ref. (2) and J.C. Pati and A. Salam, Phys. Rev. D11, 1137 (1975).

(45) Small corrections and W_L-X mixing are ignored for this purpose. Also, although it does not occur in eq. (2), $V_\rho^\pm$ and $V_{K}^{\pm *}$ may mix with $W_R^\pm$ as well.

(46) We mention, in passing, that the Yukawa term $f\ tr(\bar{\psi}_L \langle A \rangle\ \psi_R)$ + h.c. does not provide a distinction between fermions of different colour. Ways of dealing with this problem are discussed in ref. (2).

(47) A simple way to estimate these factors is given by B. Almgren, Ark. Phys. 38 (1961) 161.

(48) The numerical estimates differ slightly from those given by W.R. Franklin, Nuc. Phys. B91, 160 (1975) because we are using more reasonable values for quark mass. There are no differences in the qualitative features and, in particular, results regarding proton decay are unaffected by quark mass.

(49) A discussion of higher order effects in this model has been given by D.A. Ross, Phys. Rev. D11, 911 (1975).

(50) We may also attach K, η, D^+, F^+, etc. lines depending on the quantum numbers of the decaying quark.

(51) This notation is standard. See, for example, M.K. Gallard, B.W. Lee and J.L. Rohner, Fermilab 74/86 THY.

(52) A K^0 may also be produced in $p_{b,c}$, $n_{b,c}$, $\lambda_{b,c}$ decay leading to the analogous processes:

$p^{+}_{b,c} \to \nu_\mu + K^o$ + (several pions)

$n^{o}_{b,c} \to \nu_\mu + K^o$ + (several pions)

$\lambda^{o}_{b,c} \to \nu_e + \bar{K}^o$ + (several pions)

See also J.C. Pati, University of Maryland Technical Report No. 76-071 and J.C. Pati, S. Sakakibara and A. Salam, Trieste Preprint IC/75/93.

(53) See, J.C. Pati, A. Salam and S. Sakakibara, University of Maryland Technical Report No. 76-084.

(54) This is, of course, assuming that both the quark and diquark system are heavies than the proton or that perhaps some confinement mechanism prohibits the appearance of free quarks.

(55) H.S. Gun, W.R. Kropp, F. Reines and B.S. Meyer, Phys. Rev. 158, 1321 (1967).

(56) See the first paper of ref. (52) and ref. (53).

(57) This is certainly the largest value we could assume for M_{W_R}.

(58) Of course, the model itself may have to undergo some modification if experiment offers more precise data on V+A weak currents.

(59) F. Reines and M.F. Crouch, University of California preprint UCI-10-P-19-84, have reported 5 μ events which could have resulted from proton decay.

(60) The implications of our results to the "new physics" has been given only a cursory glance. A more complete treatment will be found in the papers of references (52) and (53).

(61) The largest such symmetry group would be SU(32) or any of its "natural" subgroups such as $SU(16)_L$ X $SU(16)_R$ or $SU(4)_L$ X $SU(4)_R$ X $SU(4')_L$ X $SU(4')_R$. Gauging any of these groups would lead to anomalies unless one postulates the existence of a "mirror" set of fermions coupled with opposite chiral projections to the same set of gauge mesons. See, for example, references (8) and (29) and J.C. Pati and Abdus Salam, ICTP preprint IC/75/73.

(62) The idea of composite quarks is not a new one. See, e.g. O.W. Greenberg, University of Maryland Technical Report 76-012 where several earlier references are given. The ideas that we develop here follow the formulation, of ref. (6).

(63) Couplings of the form $\bar{\mathcal{Q}}_R B \mathcal{C}$ and $\bar{\mathcal{Q}}_L C^+ \mathcal{C}$ can be excluded by imposing the discrete symmetry $\mathcal{Q} \to -\mathcal{Q}$, $\mathcal{C} \to \mathcal{C}$.

(64) From the transformation properties of the scalars, it is evident that $A = \bar{\mathcal{Q}}\mathcal{Q}$, $B = \bar{\mathcal{Q}}_L \mathcal{C}$, $C = \bar{\mathcal{Q}}_R \mathcal{C}$ and $D = \mathcal{S}\mathcal{C}$. To include all possible combinations, we could also introduce additional multiplets $E = \mathcal{S}\mathcal{Q}_L$ and $F = \mathcal{S}\mathcal{Q}_R$. At this point, this would serve no purpose but, later, it will be seen to play a role in preon decay into leptons.

(65) Since the first and fourth element of the quartet carry zero charge, we could give <D> the form (d_1, 0, 0, d_4). However, this gives rise to mixing between V(8) and X gluons (proportional to $d_1 d_4$) which

would contribute to the effective strength for baryon number violation and make corrections to the mass of the SU(3') octet (proportional to d_1^2). If $d_1 \approx d_4$, this would imply $m_{V(8)} \approx m_{T,T'} \approx 25$ GeV.

(66) These ideas were formulated in discussions with Abdus Salam.

(67) It is not clear that this problem does not remain to plague us regardless of the mass of the preons. Of course, this difficulty is avoided in the alternative to model D where leptons are without internal structure.

(68) The Lagrangian of eq. (5) does not exhibit a global SU(5) symmetry and, while it would be possible to make appropriate modifications, it is not necessary as the Lagrangian need be invariant only under the gauge group which is not changed.

We mention, in passing, that this suggests putting $\mathcal{Q}$, $\mathcal{S}$ and $\mathcal{C}$ into a single spinor with SU(9) as the global symmetry. This certainly contains the minimal gauge group we have been using.

(69) The residual symmetry may, in fact, be SU(3') X U(1) X U(1) where the generators for the two U(1)'s are found in $^{SU(5)}/_{SU(3)}$ and are explicitly:

$$I_0 = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & -4 \end{pmatrix}$$

$$I_1 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -3 & \\ & & & & 0 \end{pmatrix}$$

We then have U(1) forces that distinguish lepton colour from hadron colour. This is not fundamentally different and has precisely the same effect as our only goal way to treat the $\ell - \rho_d$ sector preferentially. Similar remarks apply to the situation where SU(5) → SU(4') X U(1) where it is still possible to regard $\ell - \rho_d$ as an SU(2) submultiplet.

(70) The arguments of the latter part of this section are a specialization to SU(2) of a general discussion by H.J. Lipken, Phys. Lett. 45B, 267 (1973) who demonstrates that quark binding forces saturate for $q\bar{q}$ and qqq states.

(71) Perhaps we are relying somewhat too heavily on the extent to which SU(4') is a good symmetry particularly when we later assume a significant mass difference between strong and weak colourons. It might be safer to regard this process as neutral and only taking place inside hadrons as in, for example, quark decay into leptons.

(72) J.C. Pati (unpublished) has considered the case of $m_{\rho_d} \approx 100$ MeV in a development of scheme A. This would render inapplicable our decay mechanism

for $\mathcal{G}_d$ which involves a final state particle of mass ≈ 2-3 GeV. In Pati's work, the $\mathcal{G}_d$ is considered stable.

(73) The same reasoning was applied in the case of proton decay and reference to figs. 14 and 15 will make this clear.

(74) As first suggested by J.C. Pati, unpublished.

(75) M.L. Perl et al., Phys. Rev. Lett. 35, 1489 (1975).

(76) Except, that is, for the too small widths. We discuss this point later.

APPENDIX A

Table 1

Typical Quark Decay Modes

Mode	M	ρ	Γ	τ
$q_{b,c} \to 2\ell + \bar{\ell}$ (tree)	$G_F\, G_B\, c_1\, c_4$	$\frac{1}{(2\pi)^9}\frac{\pi^2}{A_3} m_q^5$	$(G_F\, G_B\, c_1\, c_4)^2 \frac{1}{(2\pi)^5}\frac{\pi^2}{A_3} m_q^5$	$\gtrsim 10^{-3}$ sec.
$q_a \to 3\ell + 2\bar{\ell}$ (tree)	$\frac{f^2 G_F^2 G_B\, c_1^3 c_4}{m_V^2\, m_q}$	$\frac{1}{(2\pi)^{15}}\frac{\pi^4}{A_5} m_q^{11}$	$\frac{f^4 G_F^4 G_B^2\, c_1^6 c_4^2}{m_V^4\, m_q^2}\frac{1}{(2\pi)^{11}}\frac{\pi^4}{A_5} m_q^{11}$	$\gtrsim 10^{-2}$ sec.
$q_a \to \ell+\pi$ (two loop)	$\frac{\alpha f^2}{16\pi^3} G_F\, G_B\, c_1^3\, c_4\, g_{qq\pi}$ $\times \ln\frac{m_X^2}{m_W^2}\ln\frac{m_W^2}{m_V^2}$	$\frac{1}{(2\pi)^6}\frac{\pi}{2} m_q$	$\left(\frac{\alpha f^2}{16\pi^3} G_F G_B c_1^3 c_4 g_{qq\pi} \ln\frac{m_X^2}{m_W^2}\ln\frac{m_W^2}{m_V^2}\right)^2$ $\times \frac{1}{(2\pi)^2}\frac{\pi}{2} m_q$	$\gtrsim 10^{-5}$ sec.
$q_a \to 2\ell + \bar{\ell}$ (loop + tree)	$\frac{\alpha f^2}{8\pi}\frac{G_F\, G_B}{m_V^2} c_1^3\, c_4 \ln\frac{m_X^2}{m_W^2}$	$\frac{1}{(2\pi)^9}\frac{\pi^2}{A_3} m_q^5$	$\left(\frac{\alpha f^2}{8\pi}\frac{G_F\, G_B}{m_V^2} c_1^3\, c_4 \ln\frac{m_X^2}{m_W^2}\right)^2 \frac{1}{(2\pi)^5}\frac{\pi^2}{A_3} m_q^5$	$\gtrsim 10^{-2}$ sec.

Table 1 (continued)

Mode	M	ρ	Γ	τ
$q_a \to 2\ell + \bar{\ell}$ (loop)	$\frac{\alpha f^2}{\pi} G_F\ G_B\ c_1^3\ c_4\ \ln \frac{m_X^2}{m_W^2}$	$\frac{1}{(2\pi)^9} \frac{\pi^2}{A_3} m_q$	$\left(\frac{\alpha f^2}{\pi} G_F\ G_B\ c_1^3\ c_4\ \ln \frac{m_X^2}{m_W^2}\right)^2 \times \frac{1}{(2\pi)^5} \frac{\pi^2}{A_3} m_q$	$\gtrsim 10^{-2}$ sec.
$q_a \to \ell + \pi$ (loop)	$\frac{f^2}{8\pi^2} G_F\ G_B\ c_1^3\ c_4\ \ln \frac{m_X^2}{m_W^2} \times g_{qq\pi}$	$\frac{1}{(2\pi)^6} \frac{\pi}{A_2} m_q$	$\left(\frac{f^2}{8\pi^2} G_F\ G_B\ c_1^3\ c_4\ \ln \frac{m_X^2}{m_W^2}\ g_{qq\pi}\right)^2 \times \frac{1}{(2\pi)^2} \frac{\pi}{A_2} m_q$	$\gtrsim 10^{-10}$ sec.

Table II

Typical Proton Decay Modes

Mode	M	ρ	Γ	τ
$p \to 3\nu_e + \pi$	$\frac{\alpha^3 f^2}{64\pi^5} G_F\ G_B^3\ c_1^5\ c_4^3\ g_{qq\pi}^2 (\ln \frac{m_X{}^2}{m_W{}^2})^3$ $\times \ln \frac{m_W{}^2}{m_V{}^2} \frac{1}{M^3}$	$\frac{1}{(2\pi)^{12}} \frac{\pi^3}{A_4} m_p{}^7$	$\left(\frac{\alpha^3 f^2}{64\pi^5} G_F\ G_B^3\ c_1^5\ c_4^3\ g_{qq\pi}^2 (\ln \frac{m_X{}^2}{m_W{}^2})^3 \times \ln \frac{m_W{}^2}{m_V{}^2}\right)^2 \frac{1}{(2\pi)^8} \frac{\pi^3}{A_4} m_p{}^7$	$\gtrsim 10^{32}$ sec.
$p \to 3\nu_e + \pi^+ + \pi^o$	$\frac{\alpha^3 f^2}{8\pi^3} \frac{G_F\ G_B^3}{m_V{}^2} c_1^5\ c_4^3\ (\ln \frac{m_X{}^2}{m_W{}^2})^3$ $\times g_{qq\pi}^2 \frac{1}{M^3}$	$\frac{1}{(2\pi)^{15}} \frac{\pi^4}{A_4} m_p^{11}$	$\left(\frac{\alpha^3 f^2}{8\pi^3} \frac{G_F\ G_B^3}{m_V{}^2} c_1^5\ c_4^3\ (\ln \frac{m_X{}^2}{m_W{}^2})^3 g_{qq\pi}^2 \times \frac{1}{M^3}\right)^2 \frac{1}{(2\pi)^{11}} \frac{\pi^4}{A_4} m_p^{11}$	$\gtrsim 10^{34}$ sec.
$p \to 4\nu_e + e^+$	$\frac{f^2}{2\pi} \frac{\alpha(G_F\ G_B)^3}{m_V{}^2\ M^3} c_1^5\ c_4^3\ \ln \frac{m_X{}^2}{m_W{}^2}$	"	$\left(\frac{f^2}{2\pi} \frac{\alpha(G_F\ G_B)^3}{m_V{}^2\ M^3} c_1^5\ c_4^3\ \ln \frac{m_X{}^2}{m_W{}^2}\right)^2$ $\times \frac{1}{(2\pi)^{11}} \frac{\pi^4}{A_4} m_p^{11}$	$\gtrsim 10^{54}$

Table III

Gauge Meson Decay Modes

Mode	M	ρ	Γ	τ
$V(8) \to \ell + \ell$	$f\, G_F\, c_1^2$	$\frac{1}{(2\pi)^6} \frac{\pi}{A_2} m_V$	$f^2\, G_F^{\,2}\, c_1^4 \frac{1}{(2\pi)^2} \frac{\pi}{A_2} m_V$	$\gtrsim 10^{-17}$ sec.
$X^o \to 2\ell + 2\bar{\ell}$	$\frac{f\, G_B\, G_F^{\,2}\, c_1^3\, c_4}{m_V^{\,2}} m_X$	$\frac{1}{(2\pi)^{12}} \frac{\pi}{A_4} m_X^{\,7}$	$\left(\frac{f\, G_B\, G_F^{\,2}\, c_1^3\, c_4}{m_V^{\,2}} m_X \right)^2 \frac{1}{(2\pi)^8} \frac{\pi^3}{A_4} m_X^{\,7}$	$\gtrsim 10^{-34}$ sec.
$V_K^o{*} \to \ell + \ell$	$\frac{f^3}{2\pi} \frac{\alpha\, G_F\, c_1^4}{m_V^2} \ln \frac{m_W^{\,2}}{m_V^{\,2}}$	$\frac{1}{(2\pi)^6} \frac{\pi}{A_2} m_V$	$\left(\frac{f^3}{2\pi} \frac{\alpha\, G_F\, c_1^4}{m_V^{\,2}} \ln \frac{m_W^{\,2}}{m_V^{\,2}} \right)^2 \frac{1}{(2\pi)^2} \frac{\pi}{A_2} m_V$	$\gtrsim 10^{-17}$ sec.
$V_K^o{*} \to 2\ell + 2\bar{\ell}$	$\frac{f^3}{m_V^{\,4}} G_F^{\,2}\, c_1^4$	$\frac{1}{(2\pi)^{12}} \frac{\pi^3}{A_4} m_V^{\,9}$	$\left(\frac{f^3}{m_V^{\,4}} G_F^{\,2}\, c_1^4 \right)^2 \frac{1}{(2\pi)^8} \frac{\pi^3}{A_4} m_V^{\,9}$	$\gtrsim 10^{-7}$ sec.

APPENDIX B

Figure Captions

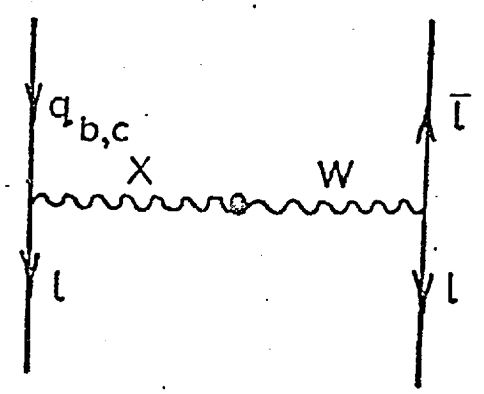


Figure 1

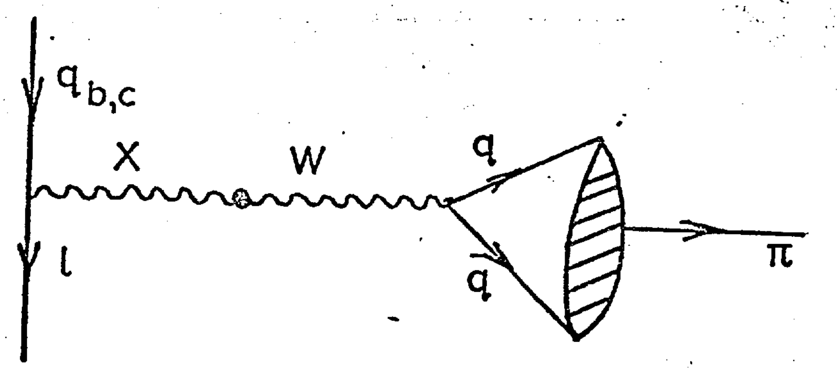


Figure 2

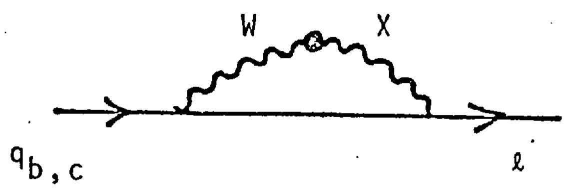


Figure 3

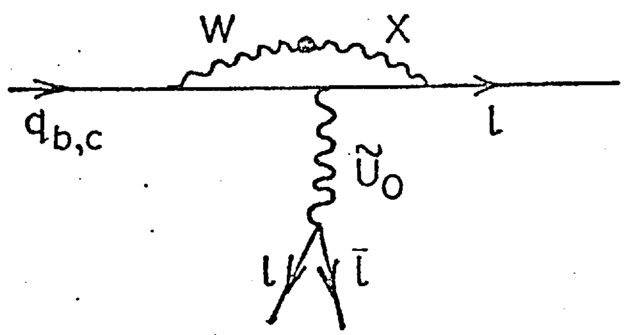


Figure 4

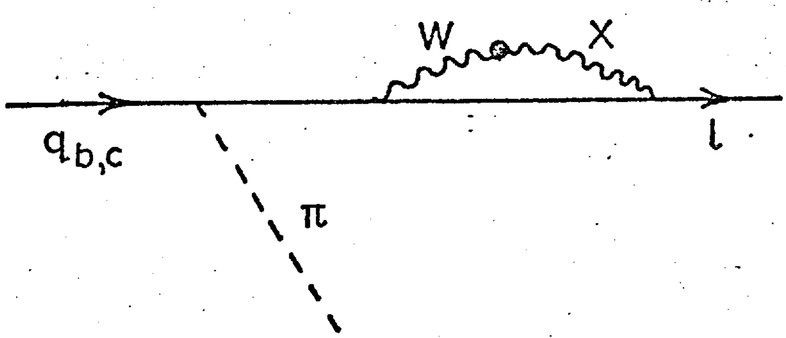
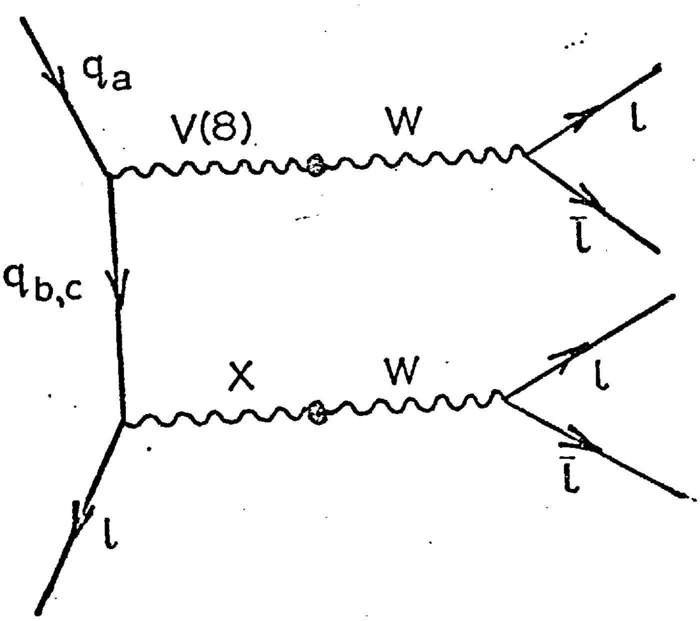


Figure 5



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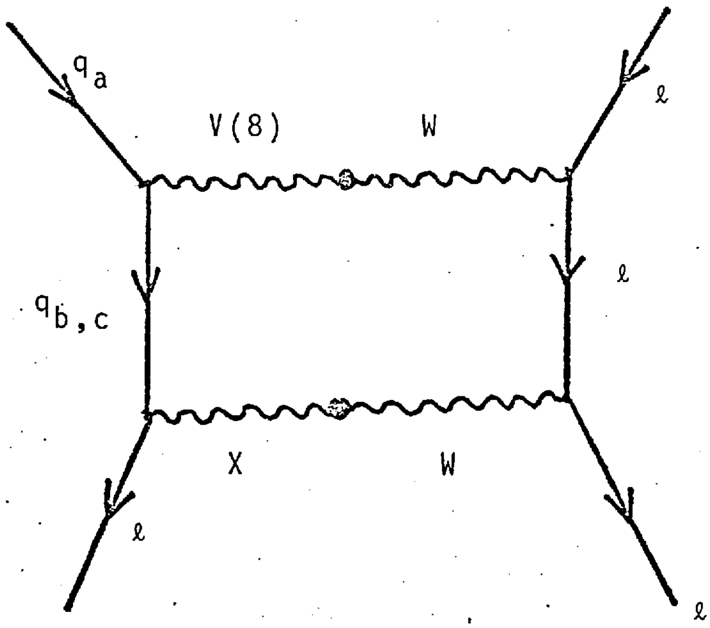

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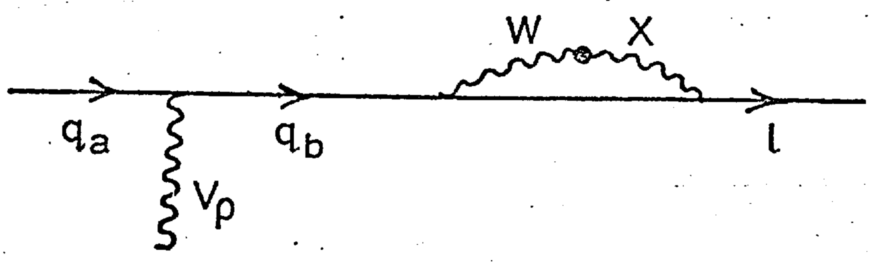

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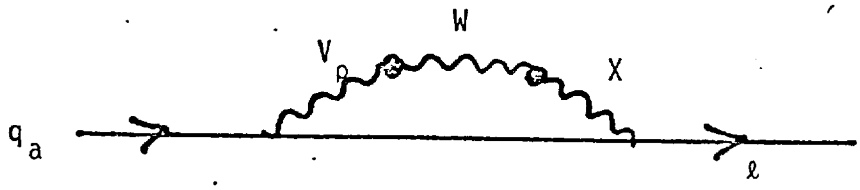

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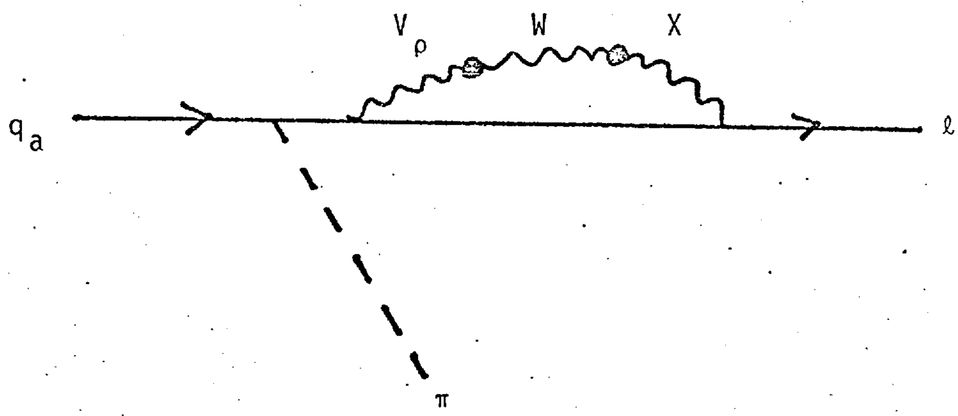

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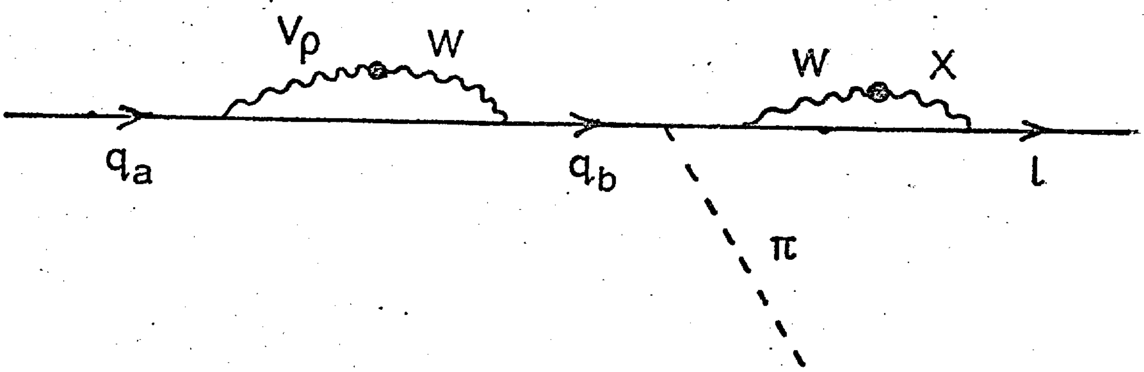

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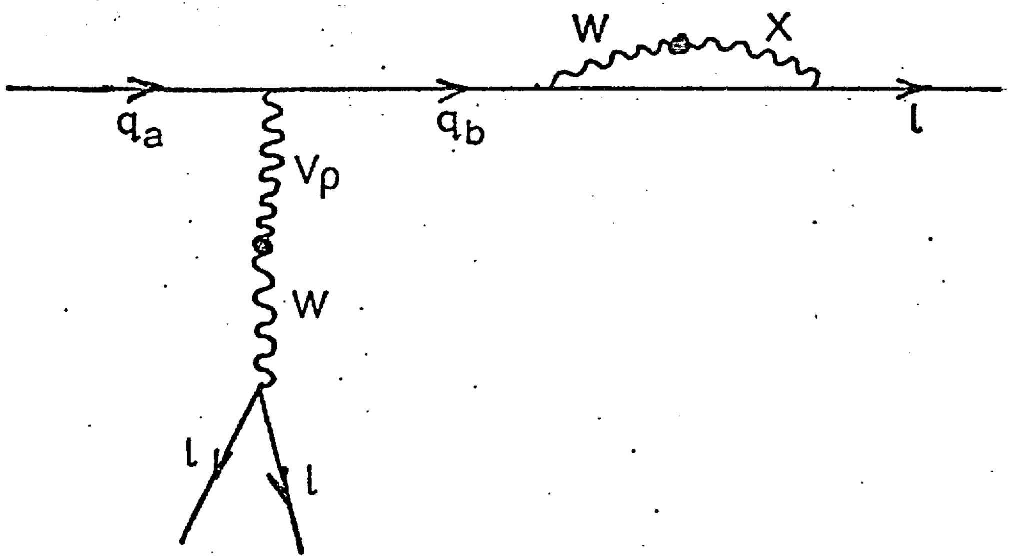

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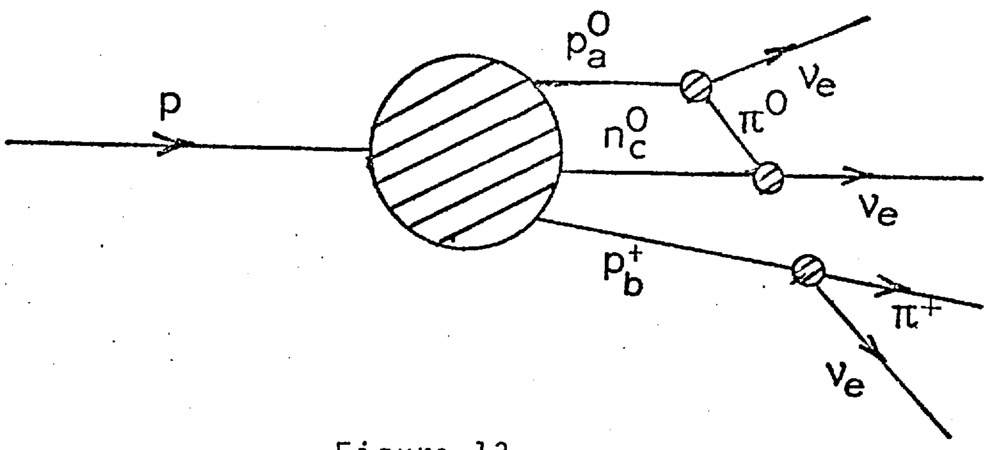

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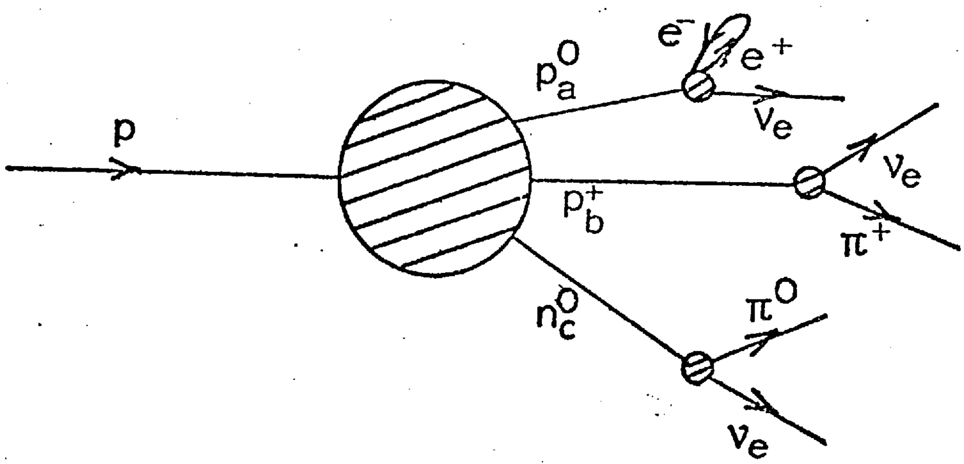

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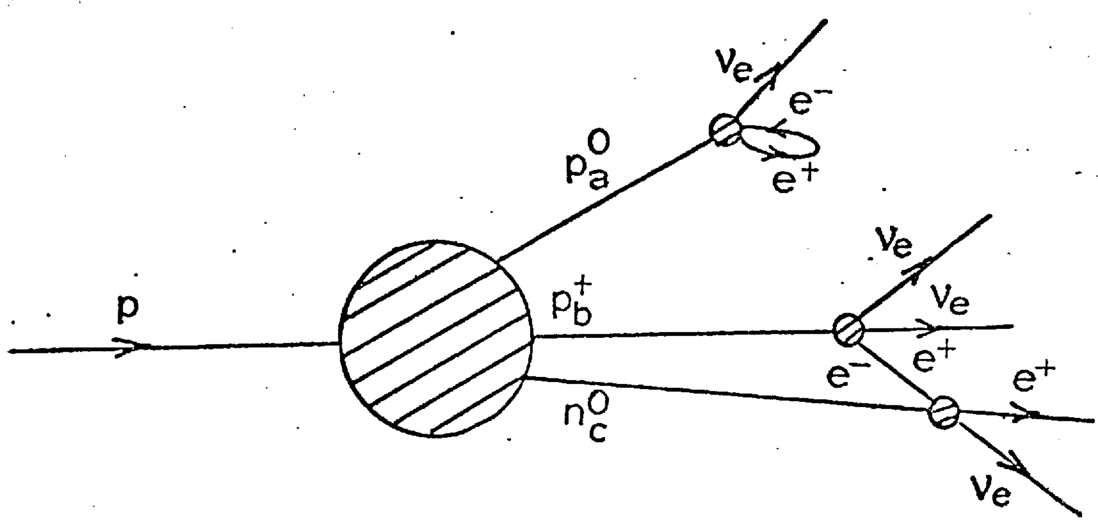

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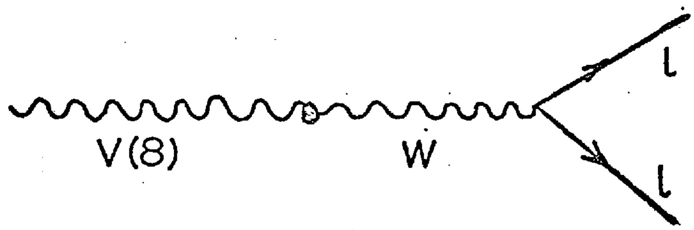

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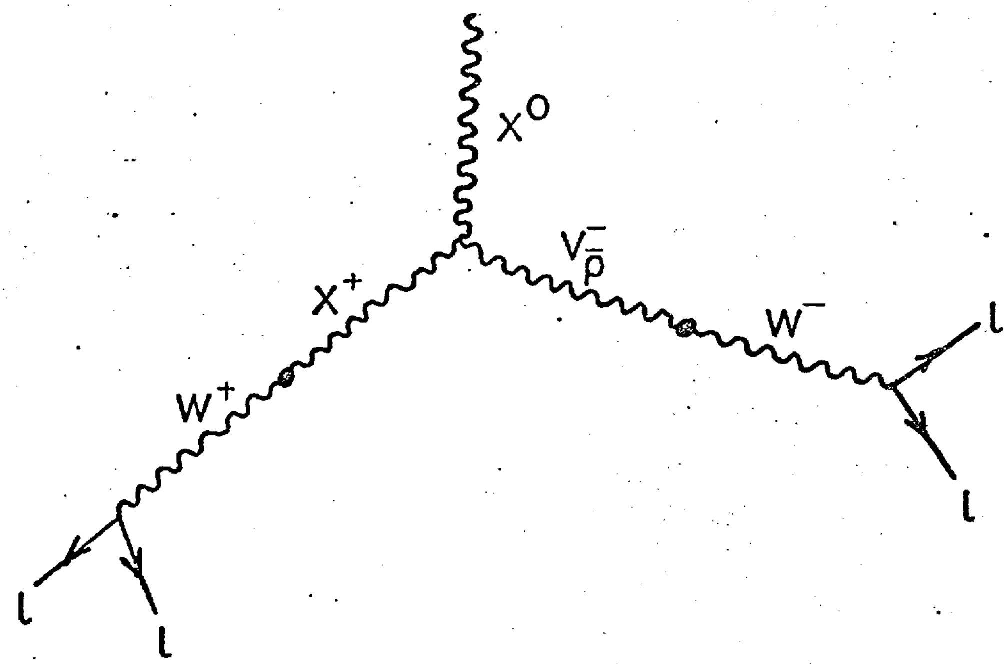

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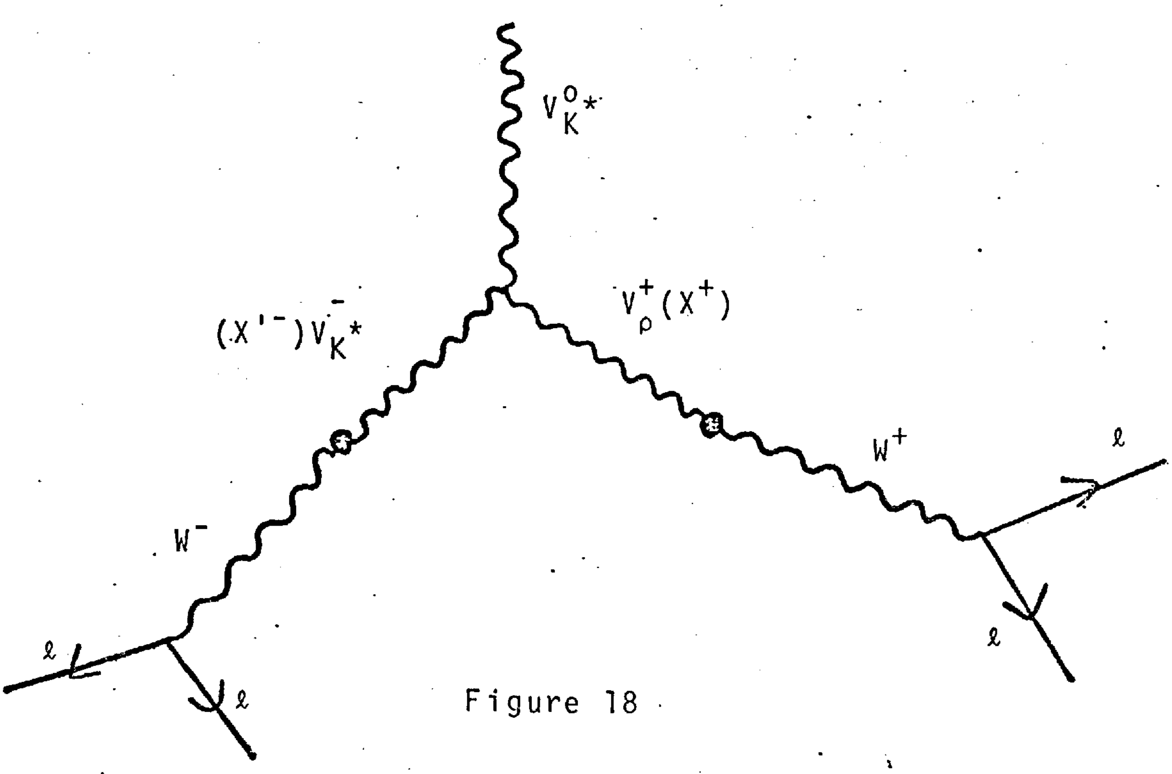

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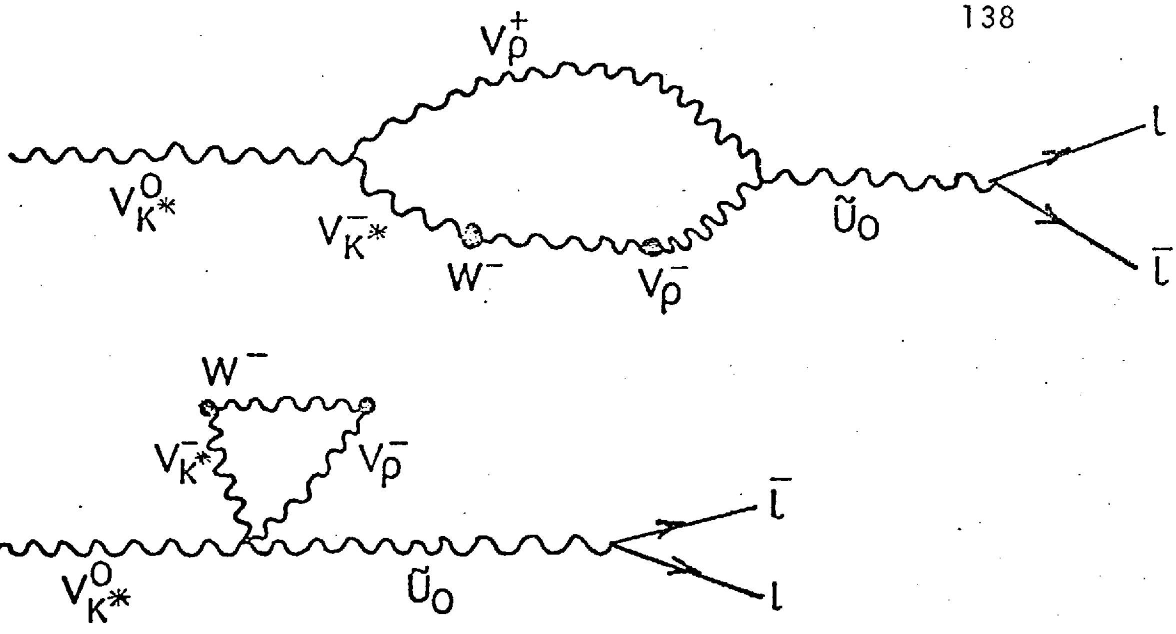

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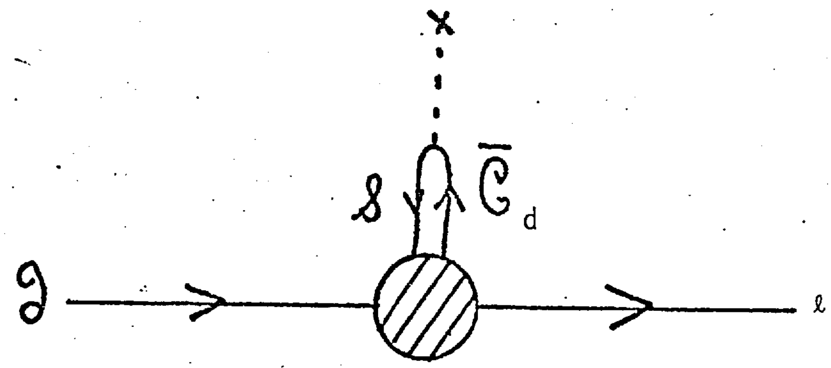

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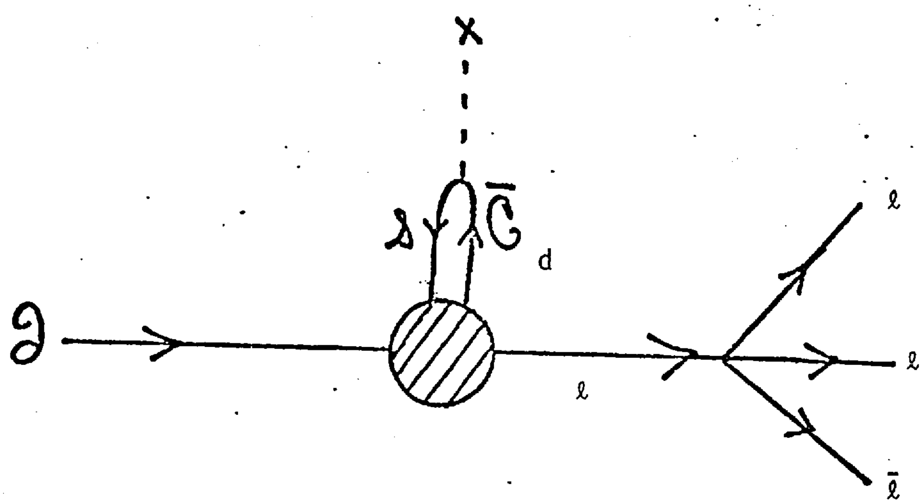

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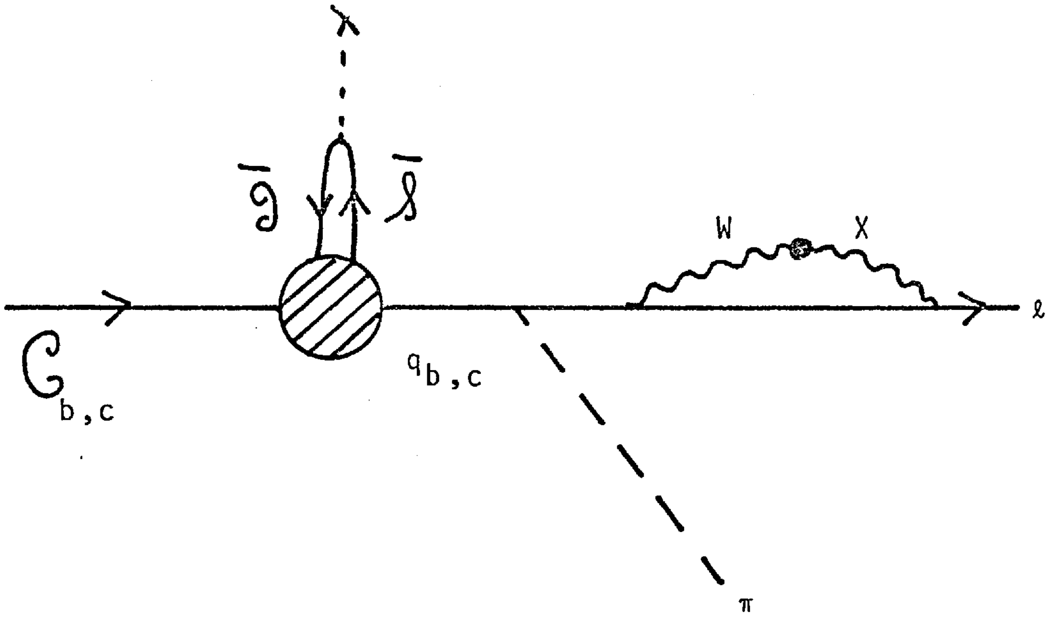

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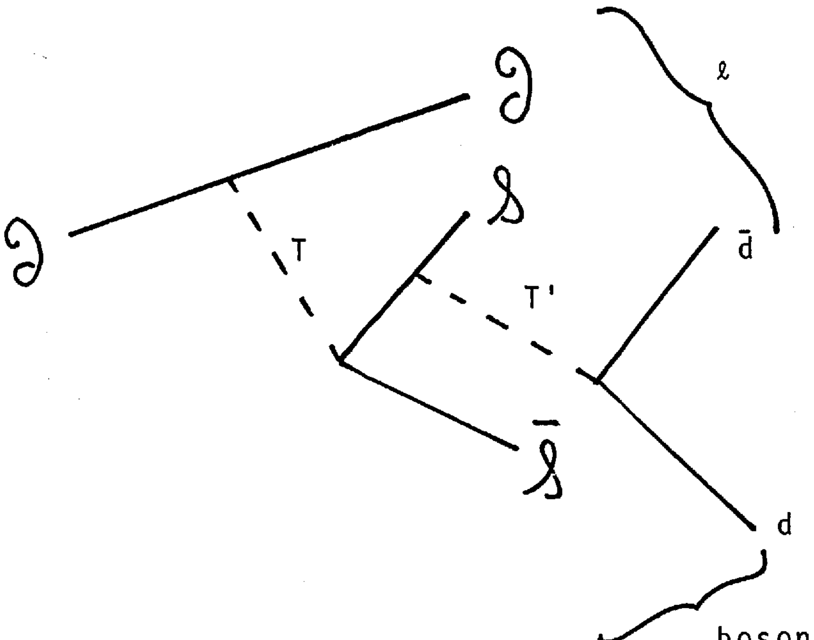

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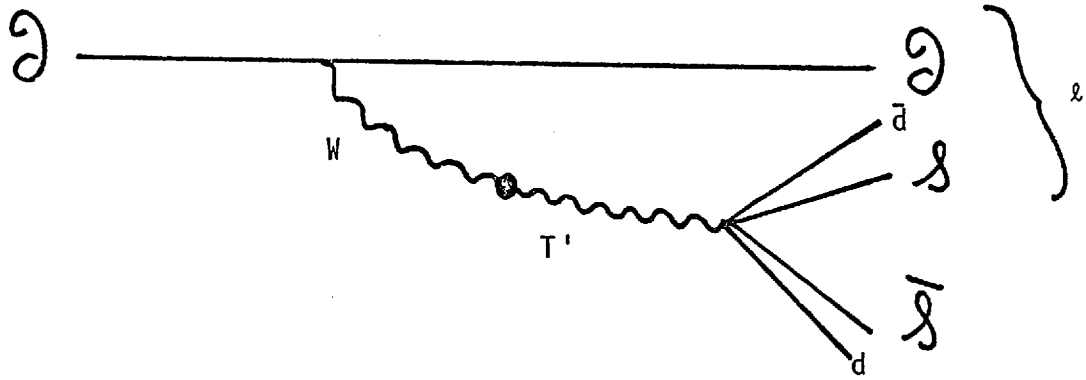


Figure 24a

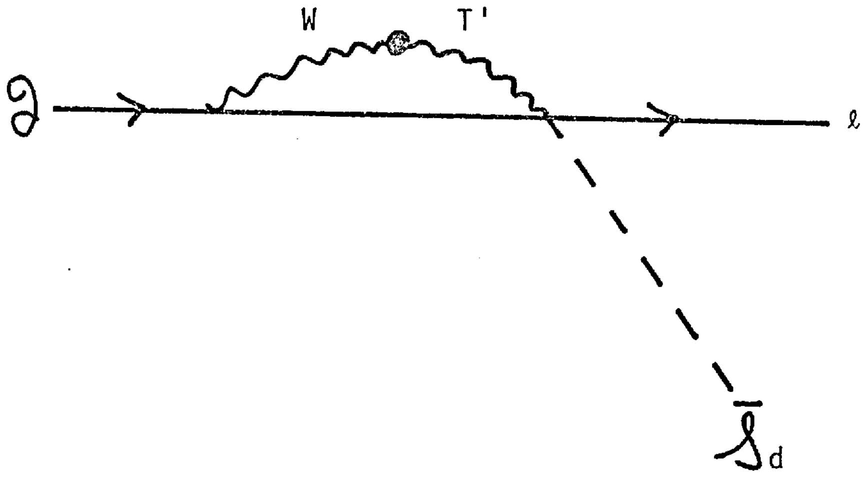
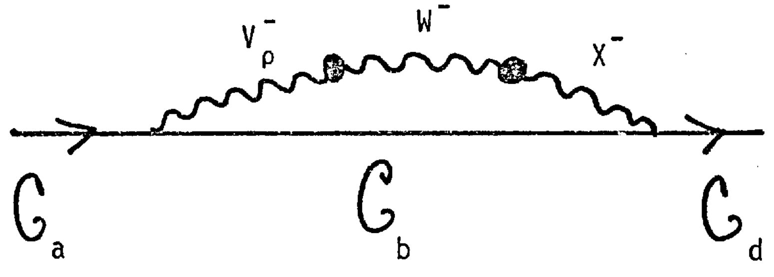


Figure 24b



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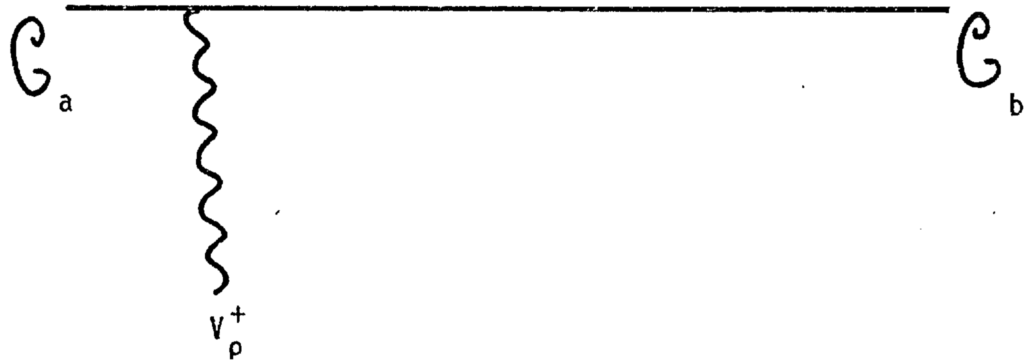

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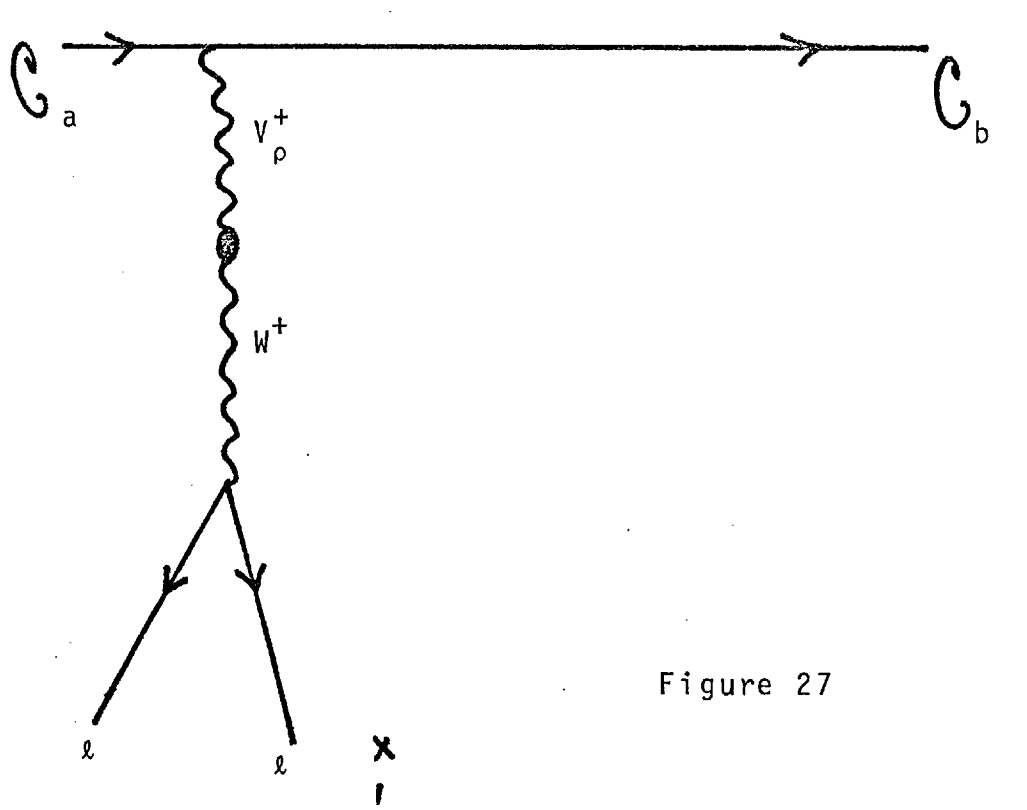

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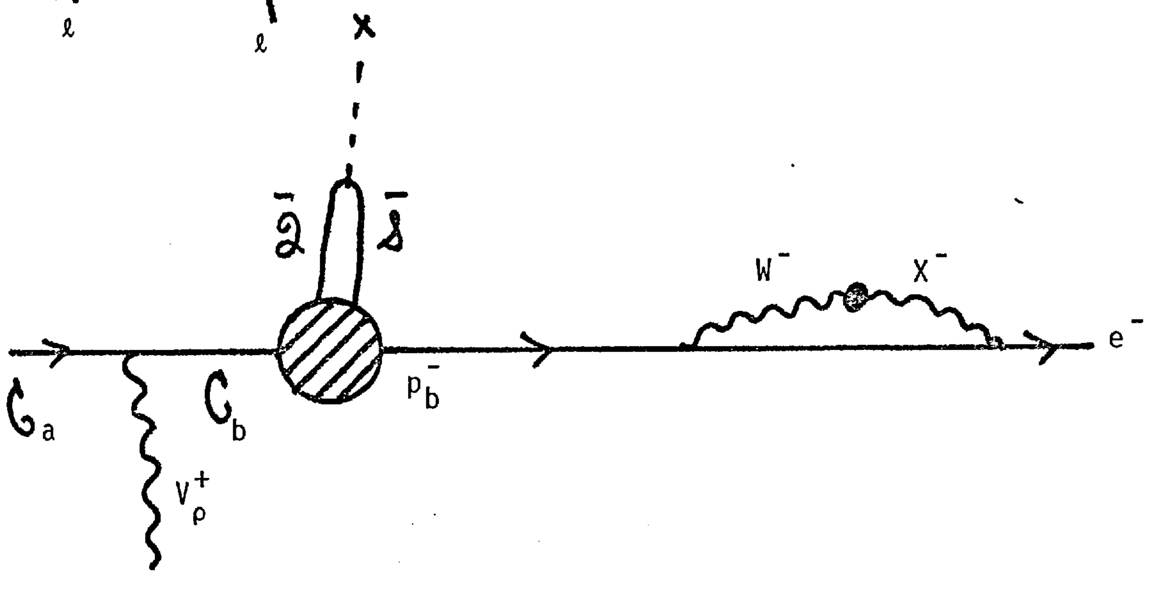

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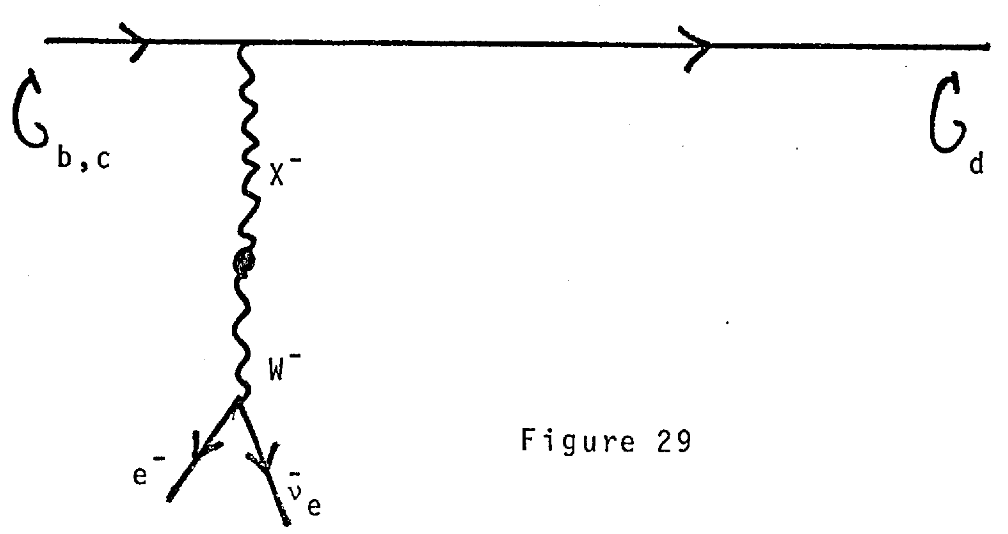

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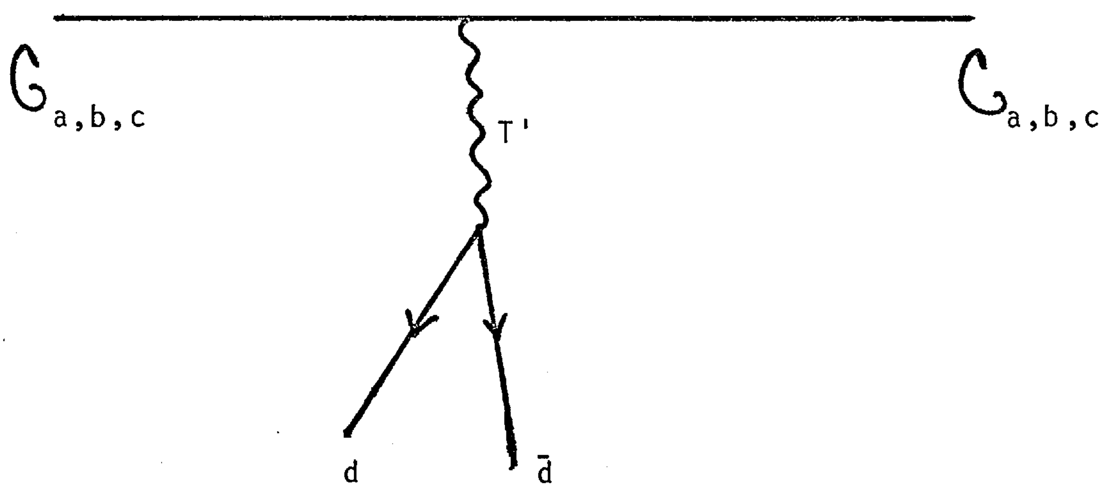

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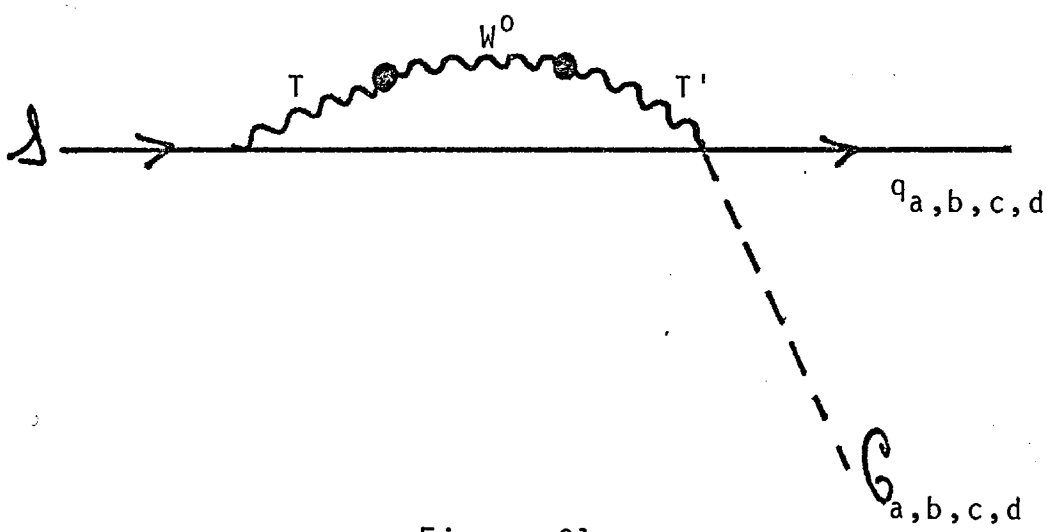

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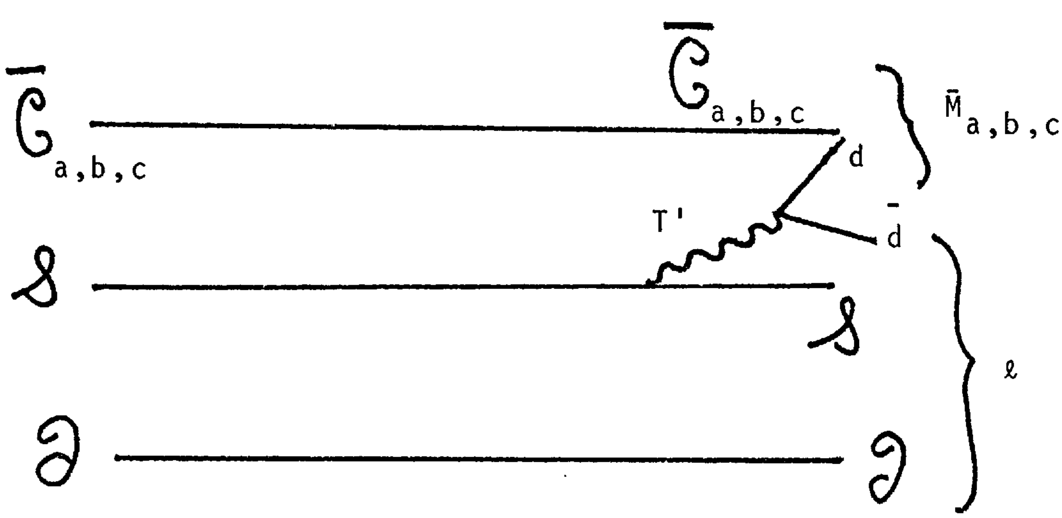

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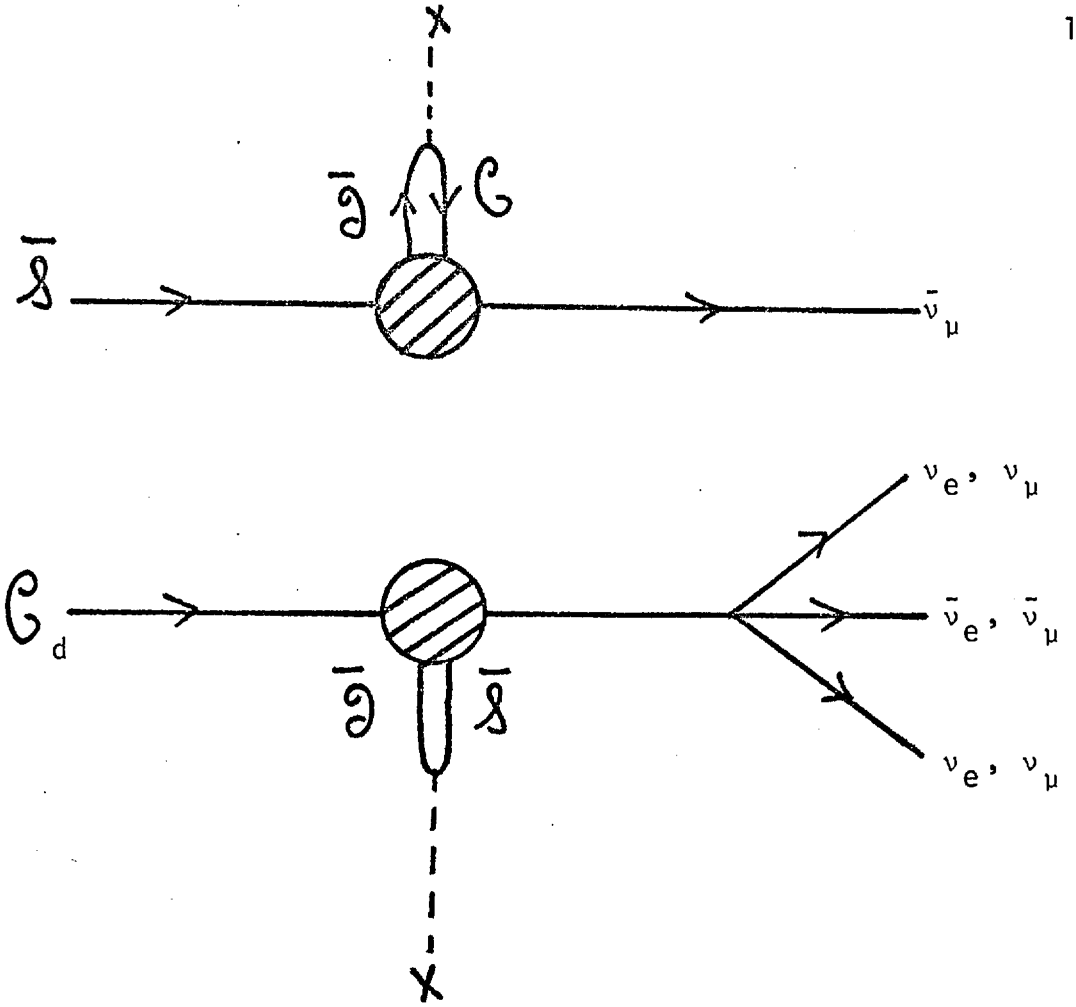

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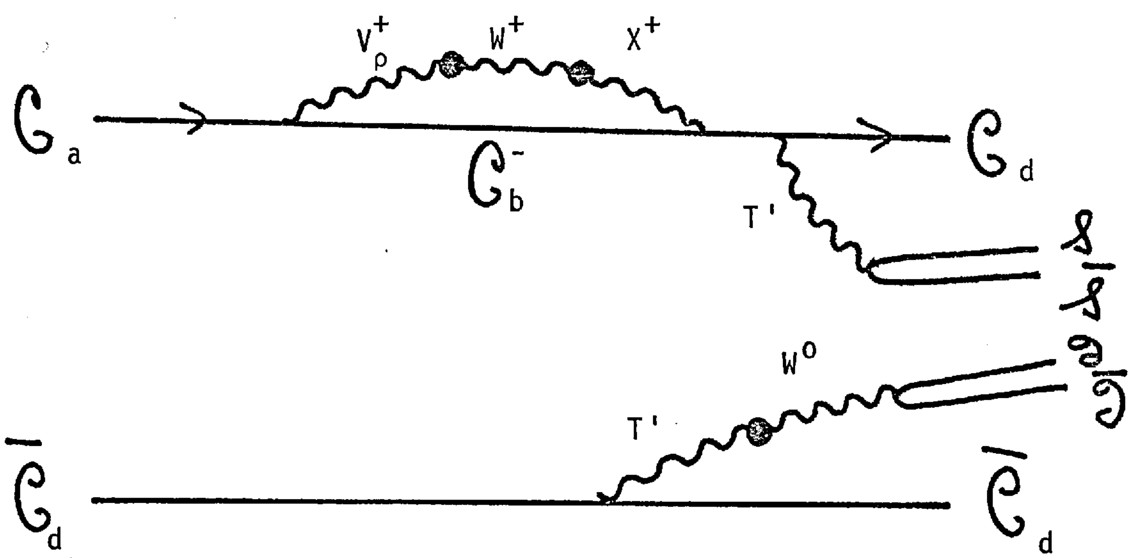

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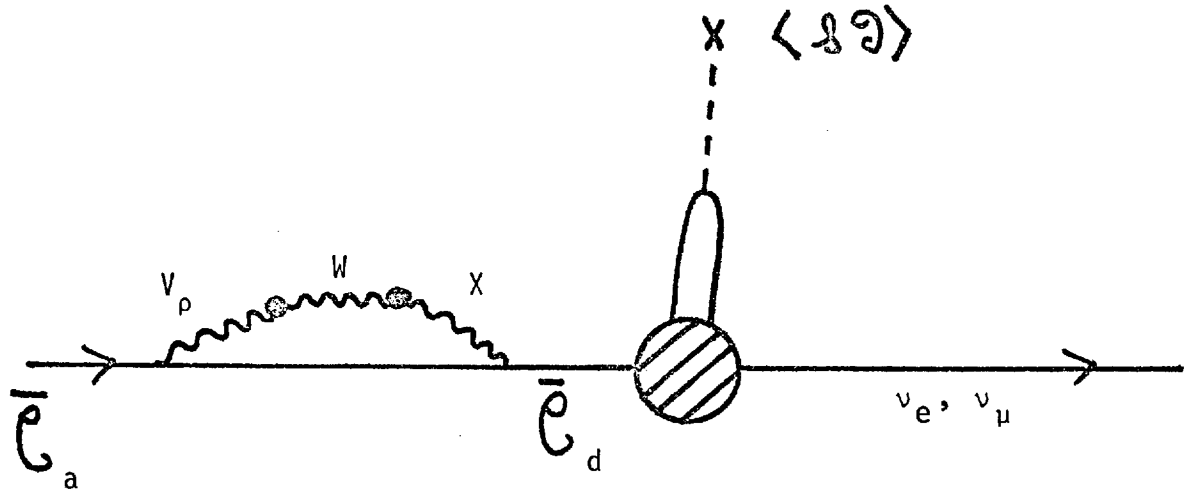

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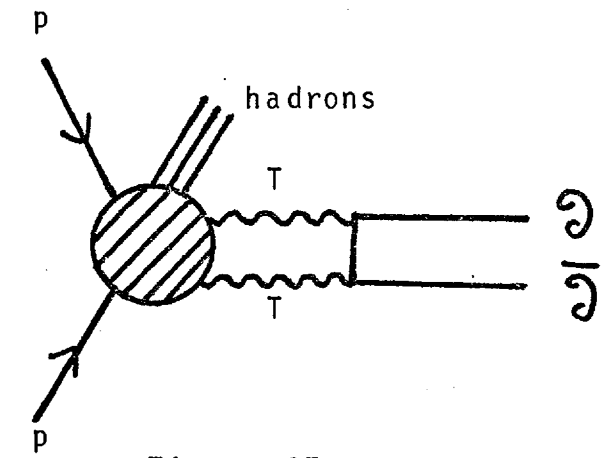

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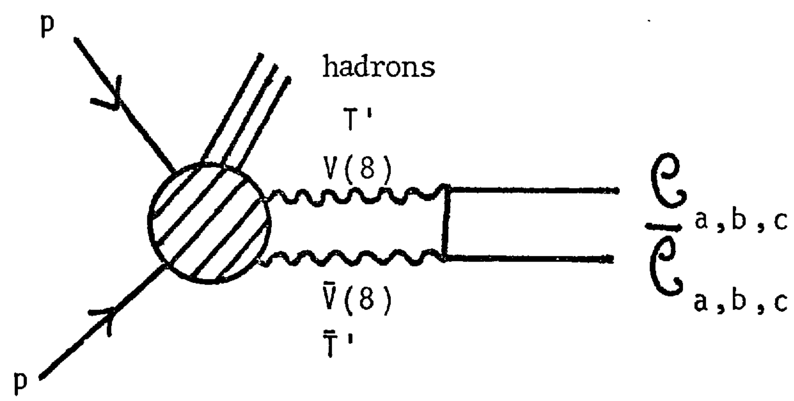

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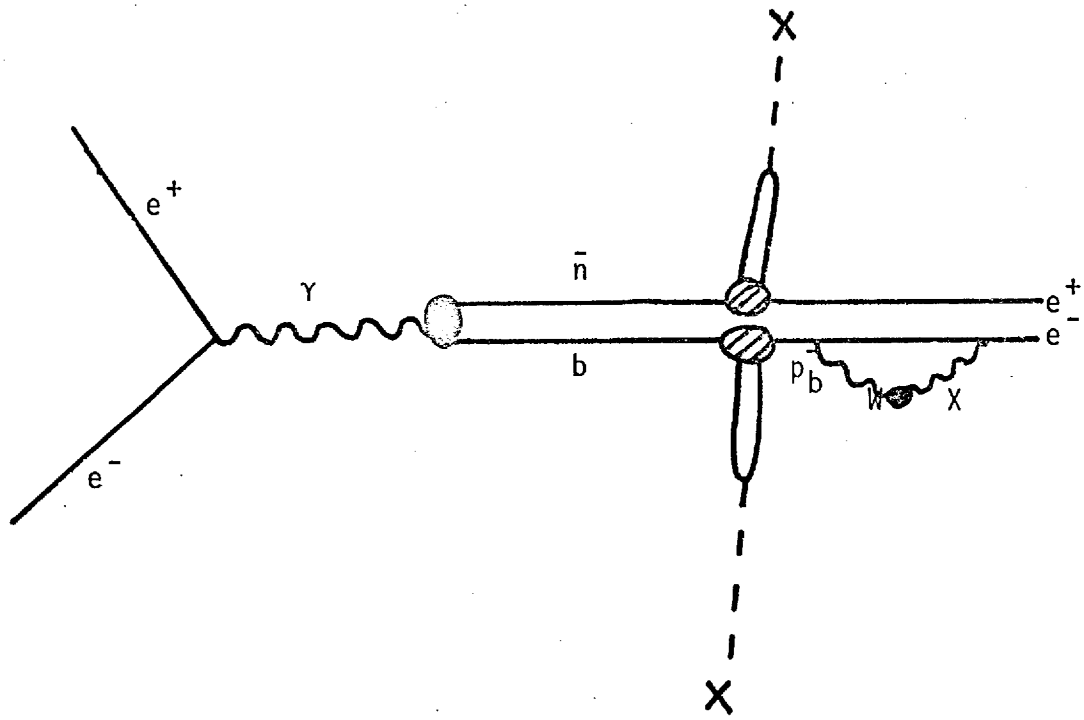

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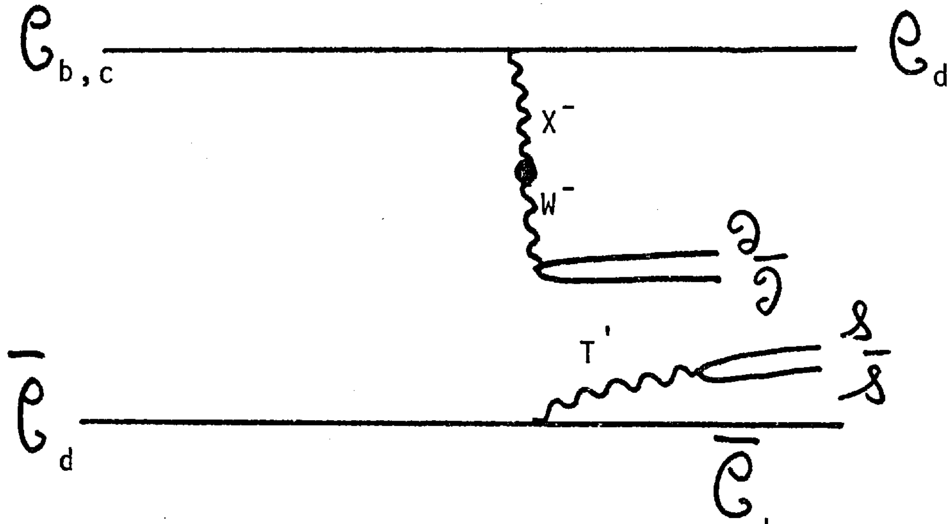

Figure 34