STUDY OF GENERALISED STRENGTH CHARACTERISTICS OF GRANULAR SOILS IN A THREE-DIMENSIONAL APPARATUS

by

Mehmet Ufuk Ergun B.Sc.(Eng.), M.Sc.(Soil Mech.).

April, 1976

A thesis submitted for the degree of Doctor
of Philosophy of the University of London.

Civil Engineering Department,
Imperial College of Sci.and Techn.,
London, SW7.

ABSTRACT

An experimental program has been carried out to investigate the strength of granular materials in the generalised stress state. The 3-Dimensional apparatus designed by Green (1969) has been modified to apply the intermediate stress using flexible plattens (reinforced rubber bags and metal encasing plattens) the main idea being to enlighten the variation of strength at high intermediate stress state and near extension ($\sigma_1 = \sigma_2 > \sigma_3$) state, and hence to decide on the failure criterion of granular soils.

Different series of generalised tests with flexible and rigid plattens at high intermediate stress states together with triaxial extension tests on Ham River sand reveal that the strengths are significantly affected by the way of imposing stresses and strains on to the specimen boundaries and the type of boundary conditions in the apparatus. Deformation characteristics are also affected.

A critical examination of the data coming from all other cubical apparatuses has been found to support this finding.

Generalised tests on a high ϕ' material (volcanic sand) was found to give a similar behaviour to Ham River sand data with a major difference that the strength variation was magnified quantitatively throughout the change of intermediate stress, and it has been concluded that qualitatively the strength behaviour must be similar for all sands.

An equation for a failure criterion in the generalised state has been proposed, and reasonable correlation was obtained with the data from the two sands tested in dense and loose states as well as with other materials from several other apparatuses.

A brief discussion of the deformations has also been presented.

ACKNOWLEDGEMENTS

The research work presented in this thesis was carried out in the Civil Engineering Department, at Imperial College of Science and Technology, University of London, under the general supervision of Professor A.W. Bishop. The Writer thanks Professor Bishop for his interest and valuable advice during the course of the study.

The Writer is greatly indebted to Dr.G.E. Green and Dr.A.E. Skinner, his supervisors, for their keen interest and many helpful discussions and suggestions during the course. The project was initially supervised by Dr.Green who was ready to help at all times during the development of the apparatus and the experimental work. When Dr. Green departed halfway through the project, he was replaced as supervisor by Dr.A.E. Skinner. The Writer would like to express his gratitude to Dr. Skinner not only for his continuous encouragement and support during the experimentation period but also for his reading the draft of the manuscript and his constructive suggestions during the preparation of the thesis.

The Writer wishes to thank Dr.J.R.F. Arthur for allowing the Writer to visit his laboratory to study the reinforced rubber bag technique and for his advice on the materials involved in this technique.

Other members of the academic staff, especially Dr.P.R. Vaughan, and research students, contributed through stimulating discussions. The assistance of Mr.J. Apted and Dr. M. Maguire, both research students, assisted in the laboratory during the preparation of loose ISC samples. Other research students also helped in many other ways. The help of all these people is gratefully acknowledged. The Soils Laboratory staff; Mr. David Evans, the Chief Technician; Mr.L. Spall, Mr.F. Evans, Mr.E. Harris and Mrs. E. Gibbs have at all times done their utmost to help. Mr.D. Evans was responsible for the technical development and repair of any malfunction of the electronic apparatus and supply of required equipment from industry. The mechanical parts were constructed by Mr.L. Spall. Their help was greatly appreciated.

The photographic reproduction was carried out by Miss J. Gurr who also took the photographs except for those of of the failed samples, and the Writer thanks her for her assistance.

The Writer would like to thank Miss V. Collins for typing the manuscript neatly and patiently.

The financial support from the Ministry of Education of the Turkish Government (government scholarship under Law No.1416) is gratefully acknowledged as without it this study would never have materialized.

Finally, the Writer wishes to thank his wife, Canan, for her continious support, encouragement and patience throughout the period of study.

CONTENTS

CHAPTER 1

INTRODUCTION

1.1. General.

The triaxial test has been the most widely accepted and standardised laboratory test to determine the stress-strain and strength properties of soils in the last thirty years, and still seems to be a relatively reliable tool to use in the immediate future. As in many other areas of study the triaxial testing technique has been continually improved. On the other hand, the triaxial test is criticised mainly because only a special state of stress, an axisymmetric state, can be represented, wheareas the stress state of most field problems are not axisymmetric. Plain strain type of problems are common relative to the other states. But plain strain testing of soil samples creates additional difficulties in measurements.

Recently, with the aid of high speed computers powerful numerical techniques, like finite element method for example, have been put into use in soils engineering, and such techniques improving so rapidly that standard laboratory and field measurements are getting more and more insufficient to supply the required parameters in analyses. Therefore in the recent years a great interest has been shown by various researchers in the stress-strain and especially strength properties of soils in the generalised state, $\sigma_1 \neq \sigma_2 \neq \sigma_3$ being the principal effective stresses. Various forms of apparatuses have been designed and built, A brief review of various apparatuses, with the emphasis on measurement techniques, is given in Chapter 2 together with short reviewson triaxial testing and plain strain testing.

Most of the work has been done on cohesionless soils mainly because testing of cohesive soils - especially undisturbed - would add extra complexities to analysis of results. Apart from that it would be quite difficult to obtain enough representative material for a series of tests to start with. In the Writer's opinion the behaviour of cohesionless soils is by no means simple, but they offer certain advantages in laboratory testing.

1.2. Scope of the Study.

As mentioned above, with the advance of finite element methods the tendency is becoming to design testing apparatuses in which generalised stresses can be applied, and extract stress-strain-strength properties from the material in question, and then apply these findings to predict the behaviour of field deposits of the same material under various boundary conditions and loading. There are some fundamental questions like; Would the behaviour of this "point-like" element in an apparatus resemble the behaviour of that element in-situ, i.e. if it were not taken out of the ground and regardless of the boundary conditions of the field deposit ? or is it possible that the design of a generalised apparatus itself and testing techniques influence the results obtained from the apparatus ? The latter question is one of the main concerns in this study.

It is strongly felt that it is essential to examine apparatus effects of the three - dimensional apparatuses together with the data coming from them before going further and trying to apply these to the solution of engineering problems. It is regarded that the present level of generalised test research is embriyonic and the data must be examined carefully.

Green (1969), among others, attempted to design an apparatus to load a cuboidal sample with independent stresses (Reported also by Bishop (1967a),Green and Bishop (1969), Green (1971a),Bishop, Green and Skinner (1973). It was called independent stress control apparatus (ISC)- from now on it will be referred as ISC-. As briefly summarised in Chapter 3 it is basically a large triaxial cell to apply major and minor principal stresses exactly in the same way as in a triaxial compression test. The additional feature is a frame hanging from the cell top containing a jack mechanism (ram) and a proving ring at opposite ends, each carrying a stainless steel loading platten to load and measure the sample stressed in one of the two lateral directions in excess of the cell pressure. This frame will be frequently called a "belt". Fig. 3.4.

Green performed a series of ISC tests on dense samples on Ham River sand to cover a range from triaxial compression to extension, namely, from $\sigma_1 > \sigma_2 = \sigma_3$ to $\sigma_1 = \sigma_2 > \sigma_3$. He reported an increase of 5^o in ϕ' ($= \sin^{-1} \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$) from triaxial compression to plain strain and thereafter constant ϕ' up to the extension state.

Then Reades carried out a more extensive series of tests in the ISC apparatus to establish strength and deformation patterns for all porosities, and indicated, Reades (1972), the effect of gaps allowed between the loading plattens, and also disclosed the need for a more realistic and larger mean stress level correction. This required a modification of Green's results, and somewhat increased values of ϕ' were obtained with increasing intermediate stresses, Reades and Green (1974). He found considerably increased ϕ' values for loose samples for $\sigma_2 > \frac{\sigma_1 + \sigma_3}{2}$ or $b = (\frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3})$ value of 0.5, and noticed that the difference in ϕ' between loose ISC samples at b = 1 and triaxial extension tests was 5^o-6^o while the similar difference in dense samples was 1^o-2^o. When the Writer started research at the College he was introduced to this problem. Reades (1972) had not concluded on it clearly. Although he mentioned a possibility of interference in his final conclusion - section 11.4 he discounted this possibility in his conclusions at the end of the chapter on ISC results, for example section 9.6.6. (p.284) "...the observed material behaviour in the region b = 1 was not undesirably influenced by the apparatus ...," and in Section 9.8 (p.304) he explains why any interference did not occur. Then again states (p.305) "...it is not clear why there is an increase in ϕ' for ISC tests on loose samples in the region $0.6 < b < 1.0^{o}$, "whether the increased axial strains and higher ϕ' values are due to one of several possible undesirable factors such as platten interference or whether they reflect real material behaviour cannot be established at this stage".

P.V. Lade working on an almost identical apparatus to ISC concluded independently as follows; Lade (1972), last Chapter; "some serious discrepancies between different investigators at normal confining pressures for values of b greater than one-half still remain to be cleared up". It was therefore decided to put more effort into the generalised states between plain strain and extension.

One of the first questions in mind was whether axial and lateral (belt) steel plattens which are very close at the edges of the sample cause interference and result in higher ϕ' values. Since the stress distribution on them was not known, it was considered that use of flexible plattens or instrumentation of the rigid plattens or elaboration on the modes of testing in the apparatus would help in the explanation of the observed behaviour for $b > 0.5$. The first and third considerations were selected for this research programme.

It was thought that since flexible plattens apply uniform normal pressures they would eliminate any doubts about the normal stress distribution on the plattens. Flexible belt plattens have been designed to apply stresses corresponding to the stress levels in rigid platten tests. These plattens employ reinforced rubber bags, and the preparation of the bags has been along similar lines to those used at University College, London, Arthur and Menzies (1972), Arthur (1973). Design and construction of the flexible plattens are described in Chapter 3. Generalised tests with flexible plattens were carried out on dense and loose samples of Ham River sand. The cell pressure, σ_3, was the same as used in the rigid platten tests. Results and discussion of the findings are presented in Chapter 5.

The orientation of principal stress directions has been noticed to be completely disordered with respect to the orientation of sample dimensions in ISC and triaxial extension tests by Green (1969) and Reades (1972). This can be clearly seen in figure 6.1. Therefore, a series of triaxial extension tests on prismatic shaped samples (mostly loose) has been conducted. In this series the ratio of the dimensions of the samples in the direction of the major principal stress to those of minor principal stress has been adjusted to be the same as in ISC tests by making a short - in axial direction - sample mould with wider lateral dimensions, Chapter 6.

The use of another possible testing mode of the apparatus (other than that in ISC tests) has been investigated on loose samples. This mode which will be referred as "the second mode" was already tried in his three special tests by Reades, not as a study of the mode but to be able to apply a different stress path to the sample. He was interested in the effect of stress path on generalised tests. In other words, to apply different stress paths the direction of the application of the principal stresses had to be changed.* He obtained fundamentally different results in these few tests but the difference he obtained was not of the effect of stress path, as will be shown in Chapter 7. The need for an investigation of this mode was obvious, and as discussed in Chapter 6 it was strongly felt that it would be of help in the discussion of observed ϕ' values near extension, SP9 - 16 series of generalised tests on loose Ham River sand has been conducted for this purpose and is reported in Chapter 6.

Driving axial and belt plattens simultaneously inwards with strain controlled systems - especially near b = 1 where the two rates are about the same - was suspected of creating or actuating a load transfer mechanism between the plattens along the edges. SP1 - 8 series of tests has been planned where belt plattens were stress and not strain controlled. Results are reported in Chapter 5.

Failure characteristics in all generalised tests

---

*With respect to the machine axes.

on Ham River sand have been critically examined in Chapter 7. Data obtained by Green (1969) and especially by Reades (1972) have been included in the discussion. Results from generalised testing programs by other researchers have also been discussed where relevant.

At the suggestion of Prof.Bishop a high strength material - a volcanic sand from Iceland - has been tested in three-dimensional stress field in loose and dense states. This series of tests were for comparison of strength theories with such a high ϕ' material and to see whether the previous results for Ham River sand could be taken as representative for other granular materials. Results and discussion are given in Chapter 8.

Triaxial 'control tests' and a few generalised tests for the effect of mean stress level on strains and strength have been presented in Appendix 6. Review of apparatuses rather than the data is the main issue in Chapter 2.

Chapter 4 outlines the technique used in the preparation of samples, the description of the materials and the testing procedure.

In Chapter 9 the behaviour of the materials has been assessed throughout the intermediate stress space. Also various stress-strain and strength theories have been discussed. It was always kept in mind that without reliable tests and careful inspection of the data it might be a waste of time to try to express the data in various analytical forms or to set theories on the basis of the data. Therefore considerable attention has been given first to the data, and then they have been correlated. A failure criterion has been proposed. It involves all three principal stresses.

In the final Chapter (10) conclusions reached and recommendations for future research are given.

There are very many factors affecting the strength and deformation of granular materials, so the conditions under which the results are obtained must be clearly indicated in a research program.

In this study the effect of the intermediate stress is the main concern, but on the other hand, a few other important factors such as density and mean stress level have been considered when examining the results. The stress level used in the tests is at the level to be encountered under normal foundation loads i.e. excepting high dams etc.

1.3. Notation.

The test program consists of drained tests, and all stresses refer to effective stresses (the usual dash sign above them is left off for convenience). During the discussions and even in the present section the term "degrees" refer to $\phi' = \text{arc sin } (\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3)$ namely

Mohr-Coulomb b angle. Compressive stresses and volume decreases (contractions) have been taken as positive. All three pairs of stresses acting on the six faces of the sample are imagined to be principal stresses, and hence, their directions are constant throughout shearing. Strains are engineering strains, and all test calculations have been performed assumming the cuboidal sample deforms as a right prism. Stresses on the rigid plattens - in rigid platten ISC tests on volcanic sand and special series on Ham River Sand - have been assumed uniform and calculated simply dividing the recorded load by the corresponding sample area.

1.4. List of Symbols.

A	Cross-sectional area of the sample in the axial direction .
B	Length of the sample in the belt direction.
b	$(\sigma_2-\sigma_3)/(\sigma_1-\sigma_3)$ ratio at failure.
C	Length of the sample in the cell direction (width)
c	Subscript referring to the end of consolidation stage.
D	Diameter of cylindrical samples or $1 + (d\varepsilon v/d\varepsilon a)$, "dilatancy factor" in Stress-dilatancy formulation of Manchester University.
E_s	Secant modulus (based on a certain threshold strain or deviator stress.).
e	Void ratio.
G_s	Specific gravity of solids.

H	Height of the sample.
I_1, I_2, I_3	Invariants of stress (see Appendix 4).
i	subscript referring to the initial conditions (i.e. specimen is under suction and before any loading commences).
J_1, J_2, J_3	Invariants of stress deviation (see Appendix 4).
n	Porosity.
p	Mean effective principal stress, $^{1}/_{3}(\sigma_1+\sigma_2+\sigma_3)$
q	The stress difference $(\sigma_1-\sigma_3)$
R	Stress ratio σ_1/σ_3
W	Weight.
V	Volume.
$\sigma_a, \sigma_b, \sigma_c$	Axial, belt and cell effective normal stresses
$\sigma_1, \sigma_2, \sigma_3$	Major, intermediate and minor principal effective stresses.
ϕ'	Maximum angle of shearing resistance, Mohr-Coulomb angle, arc sin $(\sigma_1-\sigma_3)/(\sigma_1+\sigma_3)$
$\varepsilon_a, \varepsilon_b, \varepsilon_c$	Strains in the axial, belt and cell directions calculated on the basis of end of consolidation.
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	Principal strains
εV	Volumetric strain
τ	Shearing stress.
σ	Normal stress.
σ_{oct}	Octahedral normal stress (see App.4).
τ_{oct}	Octahedral normal strain (see App.4).
ε_{oct}	Octahedral normal strain (see App.4).

γ_{oct}	Octahedral shear strain (see App.4).
μ	Lode parameter of stress; $\frac{(\sigma_2-\sigma_1)+(\sigma_2-\sigma_3)}{\sigma_1-\sigma_3}$
β	Parameter defined by Equation 9.9.
α	Parameter used in the extended von Mises and Tresca failure criteria (not generally identical for the intermediate stress change).
Ko	Coefficient of earth pressure at rest.
ϕ_μ	Angle of interparticle friction
ϕ_{cv}	Angle of shearing resistance at constant volume
σ_m	Mean normal effective stress, $1/3\ (\sigma_1+\sigma_2+\sigma_3)$

CHAPTER 2

A SHORT REVIEW OF LABORATORY SHEAR TESTING OF GRANULAR SOILS WITH AN EMPHASIS ON THE GENERALISED STATE.

2.1. Introduction.

After soil mechanics was accepted as a separate discipline, many varieties of laboratory testing machines were developed to measure stress-strain-strength properties of soils. Until the present time testing techniques and overall accuracy in measurements have been progressively improved. In this chapter the main types of tests will be briefly reviewed, and the data from these tests will be dealt with together with triaxial compression tests. The mechanics of a triaxial sample will be considered mainly because the triaxial compression test is taken as a standard for comparison of the other tests.

A review of generalised testing apparatuses will be presented. Direct shear and simple shear tests are excluded because of the uncertainity involved in the principal stress directions. Generalised hollow cylindrical tests are mentioned very briefly in a later section.

2.2. Triaxial Testing.

2.2.1. Triaxial Compression Test.

This has been the most widely accepted test to determine the properties of soils. The soil sample is usually a solid cylinder with a height to diameter ratio of

2-2.5 to 1. It is loaded axially through rigid metal plattens, and this direction is taken as the major principal stress direction. The sample is in a water-pressurised cell so that lateral stresses are applied on the membrane which encloses the sample (i.e. equal minor and intermediate principal stresses).

The mechanics of the triaxial test has been investigated both experimentally and analytically. It sustains an axi-symmetrical stress state, but when the equilibrium equations are written - more conveniently in polar coordinates - with respect to the stresses away from the boundaries, it is seen that it is indeterminate, and therefore the internal variation of stresses and strains from the boundary conditions can not be solved rigourously. The most common treatment up to present has been to take the Haar & Von Karman hypothesis for granted (1909) (Also given in Haythornthwaite (1960b)). According to this hypothesis the cylindrical element deforms uniformly and the radial stresses (σ_r) are equal to tangential stresses (σ_θ) inside the element, and this implies a homogeneous stress state through the element. For example, in a drained test $\sigma_r=\sigma_\theta$ = cell pressure. Although this assumption is taken as a correct one especially in the mid-section of the sample - not necessarily of soils only - in the elastic range, Haythornthwaite (1960b), it is usually assumed true for any state, elastic or plastic. Green (1969) summarized the discussion on this assumption by Broms (1963), Casbarian and Jamal (1963), Jamal (1963), Broms and Jamal (1965), Bemben (1966) and Harr (1966). Except for the latter, these researchers tested hollow cylindrical samples together with solid cylinders and concluded that $\sigma_r > \sigma_\theta$. Green (1969) argued that their testing techniques might well influence their results hence their conclusions.

An important contribution to the subject was made by Kirkpatrick (1967) and Kirpatrick and Belshaw (1968). By placing lead shot in a grid pattern inside the samples and using an X-ray technique it was possible to detect deformations inside the samples at various stages of a test. Using lubricated ends at the top and bottom plattens and for H/D = 1 sand samples -28cm high - it was discovered that tangential and radial strains were equal, until quite large strains probably implying the equality of tangential and radial stresses. In the case of rough ends there were big discrepancies. Perloff and Pombo (1969) gave a finite element solution of a triaxial sample with constrained ends using a non-linear constitutive relation, and concluded that there were smaller tangential stresses than radial stresses in most parts of the sample. Haythornthwaite (1960b) gave a solution based on simple plasticity.

The uniformity of stresses and strains inside a triaxial sample at various stages of a test were considered by many researchers. Sample dimensions and end restraint are directly relevant to this problem. Some of the available analytical solutions for stresses in triaxial compression tests on cylinders were compared by Balla (1960). Perloff and Pombo (1969) in their FEM analysis stressed the importance of stress-strain relations in relation to end constraints and stress distributions in the specimen.

A number of researchers tried to measure the stresses and/or strains inside or at the boundaries of specimens. Shockley and Ahlvin (1960) reported tests on dry sand specimens experiencing volume increase in the middle third and volume decrease near the ends, and large dry sand samples - specially instrumented - showed higher vertical stresses and strains near the axis below mid height of the specimen. Although some aspects of the stress and strain measurements made are questionable, the Authors' contention that stress and strain distribution inside a triaxial sample are non-uniform are supported by their volume change and direct stress-strain measurements. See also Januskevicius and Vey (1965) and Kirkpatrick, Seals and Newman (1974) who instrumented the base platten with pressure cells and showed that in the case of rough plattens, stresses near the centre were at a minimum. They measured uniform stresses for lubricated ends.

Kirkpatrick (1967), Kirkpatrick and Belshaw (1968) Kirkpatrick and Younger (1970) using X-ray techniques showed that classical triaxial specimens with rough ends and H/D = 2 deform non-uniformly with the formation of two rigid conical regions near the plattens with the major deformations in the middle third. Whereas when lubricated ends - see the next paragraph - were used, strains were uniform both axially and laterally. Therefore lubricated ends were highly recommended especially when stress-strain behaviour of the material was required.

The importance of friction between specimen and rigid end plattens as a major cause of non-uniformities in a deforming soil sample has long been recognised. Many researchers tried to find ways of eliminating it. The most convenient and practical way seems to use one or two thin greased rubber sheets between plattens and specimen. The first use of this method is credited to Roscoe (1953) and Blight (1961) then with more comprehensive studies to Rowe and Barden (1964) and Bishop and Green (1965). Usually there is a common belief that once the height to diameter ratio is two or more the effect of end restraint is not important, this belief originated from early tests; Taylor (1941), Waterways Experiment Station (1947), Bishop and Henkel (1962) (and see also Hvorslev (1957)). Although this was confirmed by a more detailed study later on, Bishop and Green (1965), as far as peak strength was concerned, it is by no means true for strains. Roscoe et al. (1963) and Kirkpatrick and Younger (1970,1971) pointed out the non-uniformity of strains in standard triaxial specimens.

Sample dimensions in relation to end restreint and sample strength was considered in detail by Rowe and Barden (1964) and Bishop and Green (1965). The former concluded that short specimens - as short as H/D = 1 - with lubricated rubber sheet ends deform more uniformly than classical H/D = 2 geometry with rough ends. The rigid zones near the plattens were said to be eliminated with a tendency towards multiple failure surfaces. They also found that enlarged plattens were useful in obtaining uniform deformation. Green and Bishop (1965) found that 4" x 4" samples with two lubricated sheets at each ends gave similar strengths compared with 4" x 8" rough end specimens but the former deformed more uniformly with higher axial and volumetric strains. 4" x 4" samples with one lubricated sheet at each end gave 1^{o}-2^{o} higher ϕ' compared with conventional dimensions or 4" x 4" with two lubricated sheets indicating some restraint at the ends.

Triaxial compression tests have been carried out by so many investigators that it is not possible and perhaps not justified to give a review of them because the great majority of them are not related to other stress states, which is the main issue in this study.

2.2.2. Triaxial Extension Test.

While the triaxial compression test has been used as a standard for the measurement of strength of soils, a number of researchers were interested in another form of triaxial test, namely, triaxial extension test which can be also performed in a triaxial cell employing the cell pressure as the major and intermediate principal stresses and the stress on the axial rigid plattens as the minor principal stress, i.e. $\sigma_1 = \sigma_2 > \sigma_3$. Although this state is rare relative to other stress states in field problems, it is important in the study of failure criteria of soils being another limiting case like triaxial compression.

The results of the triaxial extension test are more susceptible to measurement errors, and the failure mechanism is more sensitive than the triaxial compression test. Since the axial stress is the minor principal stress in extension tests, an error in axial load measurements (like bushing friction if external load measurement methods are used) results in an appreciable variation in ϕ'. In the last ten years or so this problem has been overcome using internal load cells as in Bishop and Green (1965), Barden and Khayatt (1966) so that a direct recording of the axial load can be achieved. High quality calibration scheme and sensitive axial load transducer are necessary in a proper axial load measurement that is free from errors.

A major problem of concern are the non-uniform deformations observed in extension tests. Free ends, as mentioned in the previous section, are certainly helpful in obtaining uniform deformations compared to rough ends but cannot stop non-uniform deformations that tend to develop in extension tests. This is presumably due to a certain amount of frictional restraint which still exists and to other effects of introducing a rigid boundary to the relatively highly compressible soil sample. The centre position of the sample usually deforms more and a "neck" forms. Since the axial principal stress (minor) is computed on the basis of the measured axial load per unit axial sample area, appreciable errors can result from the usual assumption of a sample deforming as a right cylinder or prism. A reasonable estimate of the influence of this effect is given in Appendix 2.

Among others Taylor (1941), Eldin (1951), Bishop and Eldin (1953), Cornforth (1961, 1964), Roscoe et al. (1963a) Esrig and Bemben (1965), Barden and Khayatt (1966), Green (1969), Mesdary (1969), Dyson (1970), Barden and Procter (1971) and Reades (1972) performed triaxial extension tests on various granular material and compared the ϕ' values found relative to those in triaxial compression tests. The effect of mean stress level must be considered for any strength or moduli comparison and can be accounted for by a normalisation procedure. Some of the researchers took the initial porosity as a basis of comparison, for example, Green, Cornforth and Reades while some others employed the porosity after consolidation like Mesdary and Dyson. Since the samples are first consolidated to a relatively high pressure, and then the mean stress level is decreased to failure in a standard extension test, after consolidation porosities would differ between two samples, both prepared to the same initial porosity, one tested in standard compression and the other in extension. But this effect is relatively less significant than the effects of non-uniformity and small errors in axial load measurement.

If a survey is done on published results of comparative strengths in triaxial compression and extension it can be noticed that there are conflicting results as to whether the ϕ' values in the two types of tests are the same or one or the other is higher. It was established in more recent studies (e.g. Green (1969), Dyson (1970), Reades (1972)) that the ϕ' values in compression and extension were almost the same for loose samples, and for higher densities ϕ' in extension gets bigger than that in compression, the difference reaching a maximum for very dense samples. Therefore a researcher who performed a series of tests at a selected density, say loose samples, would report similar strengths in the two states while another on dense samples would report a big difference. See Chapter 6.

The ϕ' values obtained in this type of test must be affected by the structural anisotropy with respect to ϕ' values in triaxial compression test. Because usually all sand samples are prepared depositing in the vertical direction. For clean laboratory sand samples this effect seems to be relatively insignificant. For example, Arthur and Menzies (1972) gave a difference of ten percent in major stress ratio through a 90 degree change in the deposition direction. In the case of field deposits it may be significant due to layering and cementation.

The uncertainity over the cylindrical and rectangular prism sample shapes in triaxial extension seems to be cleared up by Reades (1972). Green (1969) and Barden and Procter (1969) reported higher extension ϕ' values for rectangular shaped samples compared with cylindrical samples. The latter attributes this to non-uniformity of strains along the sharp edges of the rectangular sample which was shown not to be true by Kirkpatrick and Younger (1971). Reades (1972) found the opposite; Both shapes gave similar results in medium dense and loose states, but dense cylindrical samples gave higher strengths relative to rectangular samples due to supposedly premature failure of the latter. The difference in ϕ' between triaxial compression and extension tests varied between being about the same to eight degrees difference, depending on the type of material and the porosity in the studies by the aforementioned researchers. As in the triaxial compression sample the stress, distribution inside the extension sample is unknown and needs exploring future research.

2.3. Plain Strain Test.

The plain strain state is most frequently encountered in field problems. The representative test for it was developed after the triaxial testing technique was improved substantially, because it is a more sophisticated test. It requires a frame in the intermediate stress direction in the triaxial cell to take the reaction of intermediate stresses on the lateral faces of the sample or a special design is needed if a conventional triaxial cell is not employed. The shapes of top and bottom plattens are the same as the sample cross-section which has not been standartised. Various dimensions of a plane strain sample were tried on an experimental basis by Marachi et.al. (1969) who tried few different ratios of the dimensions. Height to length ratio was found to be insignificant due to similar behaviour of samples when it was changed from 1:1 to 1: 8.5 A range of height to width ratios were tried by Manchester School, Barden, Ismail and Tong (1969), Barden and Procter (1971) and they found similar results for ratios from 1:1 to 2.2 :1.

Consolidation of plane strain samples led to some difficulties. Because if a fixed rigid frame is used to apply the intermediate stress, consolidation other than K_o consolidation is not possible. For example an isotropic consolidation pressure would compress the dimension in the intermediate stress direction so that there would be a gap between the side platten and the sample before shearing is commenced, and the shearing stage would be without end plate support, thus, deviating from plain strain state. That is why most researchers use anisotropic consolidation in their plain strain tests. Control of the lateral, loading plattens from outside the cell, and provision to measure the intermediate stress is desirable but requires additional design effort.

Plain strain tests on granular soils were performed by Cornforth (1961, 1964), Bjerrum and Kummeneje (1961), Wade (1963), Leussink and Wittke (1963), Sultan and Seed (1967), Barden, Khayatt and Wightman (1969), Barden, Ismail and Tong (1969), Manachi et.al.(1969), Lee (1970) and Dyson (1970). Several others could also have been included but they report a less comprehensive series of tests. The above apparatuses usually have a height to length ratio of more than 1:1. Several researchers conducted plain strain tests as a part of their generalised testing programme with three dimensional apparatuses. These plain strain samples were more cuboidal - height/ length ratio near 1:1-,they are reported in the next section. A survey of the ϕ' values obtained in plain strain tests shows that they are higher than ϕ' values in triaxial compression both for loose and dense samples. Typical increase above triaxial compression values is about 1°-3° for loose samples and about 3°-7° for dense samples depending on the material. On the material and mean stress level,for example, the difference in ϕ' between the two types of tests reduces to almost nil under high pressures.

Major principal strains associated with plain strain and triaxial tests are remarkably different, and hence perhaps more important than the difference in ϕ' value. The small strains observed for relatively low stress levels in plain strain tests must be correctly indicated in any soil model. They imply high moduli.

The intermediate stress in the plain strain condition was found to be sensitive to small intermediate strain variations (Marachi et.al.) but did not affect the value of ϕ' measured. However, total abandonement of side plattens resulted in appreciably lower ϕ' values, Lee (1970), unlike the results of Bjerrum and Kummeneje (1961). The value of the intermediate principal stress was approximated simply with $\sigma_2 = K_o\sigma_1$ which showed reasonable correlation with the tests at Imperial College, Bishop (1966). Green (1969) proposed $\sigma_2 = (\sigma_1\sigma_3)^{\frac{1}{2}}$ for tests on Ham River Sand. These included loose tests at zero rate of volume change.

2.4 Generalised Testing.

2.4.1. Introduction.

In this section the true triaxial apparatuses will be reviewed. The survey is mainly from the point of view of apparatus design and application of loads to the sample. Measured data will be mentioned in some cases. The data from them are discussed in Chapter 7 together with the data from Ham River sand tests. Some of the generalised apparatuses are limited in capability in certain ways and do not produce "good quality" data throughout the intermediate stress range. These will be mentioned very briefly and their data will not be discussed in Chapter 7. Generalised testing techniques which make use of hollow cylindrical samples are mentioned very briefly in a later section.

2.4.2. Three-Dimensional Testing Apparatuses Which Make Use of Six Identical Plattens All Around.

Kjellman (1936) made the first attempt to measure the influence of intermediate stress on the strength of granular soils. His apparatus was completely composed of mechanical systems, and the plattens were identical to load a cubical sample. The plattens consisted of rods so providing a certain degree of flexibility. A triaxial test having a ϕ' of 35^o was reported. The same material was reported to have a ϕ' of 43^o at both $b=0.38$ and 0.50. In these tests the minor principal stress was decreased below the consolidation pressure to failure while the other principal stresses were increased or kept constant.

Jakobson (1957) published some results of the tests conducted in Kjellman's apparatus which was slightly modified. Again the sample was a 62 x 62 x 62mm cube. The materials were two quartz sands of approximately same grain size one being better graded. He summarized the results of the tests which were mainly of two types, loading and unloading, and showed the importance of the effect of shearing stress on poisson ratio and modulus of elasticity, he tried to relate the stresses and corresponding strains by an empirical power law. Although he admitted that the tests could not be carried through to failure, due to limitations of the movement of the plattens, his estimates of peak strengths for these materials are worth mentioning. For both materials tests on the extension side with major and intermediate stresses equal gave more than six degrees higher ϕ' relative to the tests in which minor and intermediate stresses were equal.

Ko(1966) and Ko and Scott (1967a) described a three-dimensional apparatus in the form of a cubical box inside which is contained the soil sample which is itself within six preformed rubber membranes clamped by the box sides. The sample is stressed by pressurising between the box and rubber membranes and to prevent bursting or balooning of the membranes at the edges, where the fluid pressure is differentially applied, a spacing frame is used. The stresses applied to the sample were generated by a stress control device which was a mechanical-hydraulic analogue of an octahedral plane in principal stress space.

Although the system was very clever, several researchers working in the field of three-dimensional testing pointed out the possible interference of the rigid spacing frame with the stressed sand sample. Green (1967), Bell (1968) and Arthur and Menzies (1968) all claimed that the findings were not of the true material behaviour.

The deformation of the sample was limited to 1-1,5 percent strain in every direction in the apparatus so that Ko and Scott (1968) were only able to define failure as a state at which interparticle movements of sand grains distinctly started, and therefore resulting in relatively larger deformations than the initial stages of the test. But this state was appreciably below the peak and cannot be directly related to ϕ'.

Lomize and Kryzhanovsky (1967) and Lomize, Kryzhanovsky and Vorontsov* (1969), gave sketches of three dimensional apparatuses with flexible boundaries on all faces. Their 1969 design differed in some respects from the early design, e.g. design of perforated backing plates behind the rubber membranes. The sample was a 71mm.cube. The stress control system used compressors and air-water interface volume gauges. Deformations were calculated on the basis of water leaving or entering the subcells which were formed by rubber membranes and metal frame block.

Although the Authors did not mention any mechanical difficulties or problems with membranes during the test, the Writer can at least imagine the problem of interference along edges and at corners, because rubber membranes would probably be stretched enough to allow interference between differentially pressurised subcells especially near failure. Results obtained using these aparatuses were presented using stress and strain invariants, and octaletial stresses, and as it will be discussed later on, "simple" and "complex loading" were differentiated clearly. It was stated that

*and Goldin.

for three sands, ϕ' in compression and extension were 36° - 45°, 36°-52°, and 36°-58°. Conventional triaxial compression and extension tests were not reported.

Menzies (1970, 1972) and Arthur and Menzies (1968, 1972) reported the design of a cubical triaxial test machine very similar to that of Ko (1966), and Ko and Scott (1967a), using six rubber bags on all six faces of a 100mm - cube sample. The main differences were the cut-back side vanes to separate the differentially stressed bags, and a rubber sample sheath was used apart from the bags. Bags were fully reinforced, details are given in Menzies (1970).

Similar strengths were reported in triaxial compression in cubical and conventional triaxial apparatuses. A problem with this equipment may well be the movement of bags to cope with the sample deformation at large strains. Although lubrication between the vanes and the bags can ease the movement, in a cubical extension test, for example, top and bottom and one pair of the side bags will be inflated greatly to suit the considerable deformations in those directions. These two pairs of bags may tend to inflate more at the central portion relative to the edges. This and other problems such as adequate contact area of the bags with the sample made it probably difficult to perform reliable tests at intermediate stresses, and in fact no such tests were reported.

Very recently Al-Ani (1975) modified the University College apparatus and was able to conduct generalised tests. The design of the reinforced bags was improved. Starting with Menzies (1970) several types of bags and reinforcements were tried to achieve full coverage of the sample faces, prevention of "lift off" at the corners and to stop balooning between differentially pressurised bags especially at the middle part of the edges of the cubical sample. The design finally used involves partially reinforced bags. The idea of vanes was abandoned (45° solid corners were used instead). Corners of the bags at the rear were not reinforced while mid portions along the edges were heavily reinforced so that the bags were able to follow the deformations of the sample with full coverage on the faces without stretching and balooning between them along the edges. Strain measurements were done using lead shot-x-ray technique. An increase of 11° in ϕ' was observed between triaxial compression and $b = 0.40$. ϕ' values then peaked at about $b = 0.5 - 0.6$ and lowered $3\text{-}4^{\circ}$ until $b = 1.0$ for dense to medium dense samples. The minor principal stress used was low ($42kN/M^2$), and no corrections were applied to the test results.

The apparatus described by Gudehus (1971) and Goldscheider and Gudehus (1973) employed the nested platten concept of Hambly (1969). Six rigid plattens larger than the sample faces were positioned in such a way that they slid and loaded the sample without allowing gaps or causing collision between them. Dry sand was tested in this apparatus whose mechanical design seems to be more complex compared to other apparatuses. As will be seen in Chapter 7 the results were similar to those found in this study.

2.4.3 Apparatuses With Mixed Boundary Conditions.

Proctor and Barden (1969) and Barden and Proctor (1971) reported some test results obtained from a three-dimensional apparatus which is similar to those obtained in the Imperial College ISC apparatus in many respects. They reported that the strength in plain strain and near extension were the same for River Welland sand, and conventional extension tests on cubical samples gave the same strength.

Sutherland and Mesdary (1969) described the design of a three dimensional test apparatus and gave results of series of tests performed. A large triaxial cell was used. The sample was a 4 in. cube. Top and bottom rigid plattens were used. "Free ends" were employed on these. On the sides there were two fixed side panels on which water filled bags were mounted. Edges of the rubber bags were said to be reinforced with a brass mesh. All measurements were of classical type (i.e. triaxial testing instrumentation), and the deformations in the intermediate stress direction were estimated from the amount of water leaving and coming into the bags during test. At the extension side (approximately b = 0.75 - 1.00) tests reported were similar to conventional extension tests, i.e. applying the confining stress then reducing the axial stress. In the remaining intermediate pressure range samples were failed into the cell direction. Drained strengths increased from triaxial compression to a maximum value near b = 0.5, then decreased to approximately triaxial compression strength in extension, (For loose sample 3.3°, dense samples 5.5° increase relative to ϕ' in triaxial compression).

It is not clear to the Writer how brass mesh worked at the edges. If it was free it would not prevent any balooning. If it was attached to the bag and/or the backing frame it would be difficult for this flexible platten to cope with the deformations in the intermediate stress direction, and contact areas would presumably be poor near the edges of the sample. In Chapter 7 results from this apparatus are examined.

Ramamurthy (1970) and Ramamurthy and Rawat (1973) presented a description of a three-dimensional test machine and gave results of a series of generalised tests with different intermediate stresses. The 7.6cm cube specimen was tested in conventional testing frame without a triaxial cell, instead, four sides of the sample were surrounded by four square flexible plattens which were formed by special conical shaped rubber bags. These flexible plattens were inset into backing frames so designed that the rubber bags were supported at the rear. The position of the backing frames could be controlled mechanically.

Lateral deformations of the centers of the lateral surfaces of the sample were determined by rod-dial gauge systems operating through the backing frames. Up to plain strain samples were failed by increasing the axial stress and at the same time increasing one of the lateral stresses to a predetermined value at which it was kept constant. The other group of tests were carried out by applying side pressures and decreasing the axial stress until failure as in classical extension tests. The strength (ϕ') of dense Ottawa sand in triaxial compression was found to be 40°, to increasing to plain strain at 46.4° and then decreasing to approximately extension strength of 40° as in triaxial compression.

Lade (1972) and Lade and Duncan (1973) performed three-dimensional tests on a sand with a generalised testing apparatus which was very similar to the design of Green (1969) which was that used in this study. It consisted of a large triaxial cell to load in the axial direction and a horizontal frame to load in one of the two lateral directions. In the other lateral direction cell pressure acted. The problem of gaps between axial and lateral pairs of rigid plattens was avoided by making the lateral rigid plattens in strips of metal and balsa wood so that the plattens could compress to cope with the axial deformations (compressive) taking place in the sample. An inital size of gap was maintained during the test. The platten surfaces were reported to stay plane throughout a test. A frame above the top cap compressed the lateral pair of rigid plattens on the roller bearing at the sides without influencing the axial load measurement. The jack mechanism in the lateral direction was very similar to the one used in the present study. The generalised tests were conducted at constant b values during shearing. The sample shape was cube. As will be discussed their results were similar to those from ISC apparatus.

The axial load to compress the lateral plattens will be transferred to the horizontal direction through side bearings. No explanation is given whether this lateral force affects the load measurement in the horizontal direction.

Dyson (1970) and Bennett (1969, 1971) described a three dimensional apparatus which employed a triaxial cell. The cell pressure acted as the minor principal stress. Water filled rubber bags encased in subcells applied major and intermediate stresses in the axial and remaining lateral direction. The axial load measurement was external, and the cell pressure was kept low to prevent high differential pressures because the bags were not reinforced. The Authors reported appreciable friction values on the platten at such low stress levels. Tests at stress states higher than plain strain could not be carried out with this apparatus due to the bags balooning.

Lenoe's (1966) apparatus was composed of rigid top and bottom plattens and air inflated rubber side membranes to apply the intermediate stress. Vacuum was applied in the remaining lateral direction to obtain the minor principal stress. Very low pressures (deviatoric stresses of the order of 20kN/M^2) were used and only very low b values were attained (b = 0.0 - 0.1).

Daniel (1954,1957) tested dry sand in (rigid) box type of apparatus which had a height to width ratio of two and had a square cross section. The lateral plattens were loaded using a lever-dead weight system. The data were not comprehensive but the largest difference of about 15° in ϕ' was recorded in the intermediate range.

Details of the apparatus presented by Malyshev and Fradis (1968) were not clear although it was known that rigid plattens were used.

Bell's (1965) design employed rubber bags on the faces of the sample. They were separated by plastic hinges at the corners. The sample height was very low, (5 (height) x 41 x 41 cm. were the dimensions). A major criticism at the time came from Ko (1966) who stated that only 60 percent of the base square area of the sample had contact with the rubber bag so that the tests would be in error. Mean normal stress was 120 kN/M^2 in the tests, thus, corrections for friction or sample sheath rigidity would become noticable. He obtained a 12° difference in ϕ' between plain strain and triaxial compression.

Hansen (1973) reported a generalised apparatus with mixed boundary conditions constructed at the Danish Geotechnical Institute. Major and intermediate stresses were applied by rigid plattens in the axial and one of the two lateral directions, similar to ISC design, vacuum was applied to the sample, and it was used as the minor principal stress. Samples were prepared and tested in the dry state and had the dimensions of 20 x 20 x 20cm. The failure characteristics were "very similar" to those by Lade (1972), except that the failure planes could clearly be observed.

Matsuoka and Hashimoto (1973) reported in Matsuoka (1974) made a design of a "true triaxial apparatus". It made use of three pairs of rigid plattens around a 7cm cube sand sample which was deposited dry. The plattens applying minor principal stresses were able to compress and expand laterally in the plane of the platten (i.e. in the intermediate principal stress direction) so that there were no large gaps between the moving (intermediate stress) plattens, which were larger in size, and the minor principal stress plattens. Compressibility of the (σ_3) plattens was achieved by a spring loading system. The Writer is not aware of any comprehensive generalised test series from this equipment at the present (1975).

2.4.4. Generalised Tests on Cohesive Soils.

The apparatuses in which cohesive soil samples were tested will not be reviewed. Very brief summaries can be found in Green (1969) for the period before 1969 and Reades (1972) for the period 1969-1972. Since 1972 there are a few additional contributions, Wood (1973), Goldstein et.al. (1973), Mitchell (1973) and Mitchell and Wong (1973).

2.4.5. Hollow Cylindrical Tests on Cohesionless Soils.

It was pointed out earlier that hollow cylindrical tests would not be emphasized in this study. Several researchers conducted generalised tests in hollow cylindrical apparatuses. Among others Kirkpatrick (1957), Haythornthwaite (1960a), Whitman and Luscher (1962), Wu, Loh and Malvern (1963), Proctor (1967), Arnold and Mitchell (1973), Frydman et.al. (1973).

This type of test has not been favored by the Writer. The uncertainities involved in it are more than those in cubical tests. The internal stress distribution is the major problem and analyses are usually disputed. Use of bigger wall thicknesses (between bore and cell pressure) may not justify a linear assumption of stress distribution which is generally used. See Harr (1966), for example, for a discussion of this problem.

Due to the uncertainity of principal stresses in the specimen ϕ' values calculated may be in significant error, but at least qualitatively the variation of strength (ϕ') with increasing intermediate stress state can be traced. Provided that the assumptions in calculations are approximately true the ϕ' values found show an increase from triaxial compression to about b = 0.5, then a decrease to extension state is observed.

CHAPTER 3

APPARATUS, INSTRUMENTATION, TEST PROGRAM AND TEST CALCULATIONS.

3.1 Introduction

The apparatus and laboratory instrumentation used will be briefly described in this chapter. The reason for being concise is that the main body of the apparatus has already been described in detail in Green (1969), (1971a). The apparatus was originally designed by Green and has been subjected to a major modification in this study. Details of the new components will be described. Also instrumentation related to the new system of loading will be described. For mechanical and electrical design details the reader should refer to Green (1969). Major components will be summarised below. Types of tests conducted on different materials and test calculations will be presented in later sections.

3.2. Description of the Apparatus

3.2.1. General.

A general sectional elevation and two sections of the apparatus are given in figures 3.1 and 3.2 after Green (1971a). It is basically a large triaxial cell. Axial load application is conventionally strain controlled. A horizontal loading frame which consists of a jack mechanism (hydraulic ram) and a proving ring is suspended from the cell top. The hydraulic ram has previously always been used as a strain controlled mechanism but in this work it has been used as a stress controlled system in certain series of tests as well. A prismatically shaped soil sample can be loaded vertically and laterally in excess of the cell pressure. Cell pressure acts directly on the sample sheath. Directions of load and stress application are imagined to represent the principal directions on the soil element. Of the two lateral directions one is referred as the cell direction along which the cell pressure acts on the sample sheath. The other lateral direction is refferred as the belt direction. Horizontal loading plattens apply loads on the sample faces in this direction -intermediate stresses are applied in this direction in most of the tests-. This is the same terminology as in Green (1969) and Reades (1972) who employed polished stainless steel axial and belt loading plattens in these directions. In the present study apart from rigid plattens flexible plattens have been used in the belt direction.

3.2.2. Cell Body.

The main features of the cell are that the upper and lower plates are held apart by four large diameter rods, which are internal to the cell, these rods are clamped by closed end nuts which are sealed with 'Dowty' bonded seals.

The cell wall is a large thick walled perspex cylinder retained between an upper and lower annular rings by six tie-rods. The lower ring is clamped to the cell base utilising nuts engaging on extensions of the tie rods and is sealed to the base by an O-ring sandwiched between the lower annular ring and the base. The upper annular ring is sealed to the cell top by means of an O-ring squeezed by a clamping system.

A bushing is screwed into the cell top to guide the ram for axial load application. The cell base contains a large klinger valve to empty the cell quickly. Electrical lead-outs to the belt proving ring and all kinds of tubing – to the lateral belt, top and bottom axial plattens, flexible platten, cell water supply etc. - pass through the cell base. The recess underneath the base has the same diameter as the pedestal of loading frame. Through the six holes on the loading frame pedestal and tapped holes in the cell base, the cell can be bolted to the pedestal for extension tests. There are three legs underneath the cell, to support it when it is free standing.

3.2.3. Axial Loading System.

Axial plattens are formed by bolting three separate pieces together; polished stainless steel square top, perspex body and cylindrical brass back plate. The perspex body is a transition zone from square steel platten to a cylindrical cross-section used to seal the standard cylindrical rubber membrane with O-rings.

The bottom platten is clamped on a brass plinth concentric with the cell base. There are recesses at the centre on each platten to insert 12mm diameter porous stones which protrude about a milimetre from the platten surface. The top platten drainage lead (saran tubing) is connected to the cell base and provides no handling difficulties during the preparation of the test set up.

The axial load measuring system utilizes an internal proving ring immersed under oil in a special perspex casing with polythene cover sheet at the back to cope with the changes in cell pressure without developing body stresses of its own, in other words there is no differential pressure across the polythene sheet which is flexible enough to equalise the oil pressure and the cell pressure. Bishop and Green (1965). It has a capacity of 11 kN (2500 lbs) both in tension and compression with a maximum deflection of ±0.6 mm. Deformation of the steel ring is measured by an inductive type of displacement transducer which is located at the centre of the ring. The loading anvil of the proving ring is in direct contact with the circular brass back plate of the top axial platten to achieve a stable, non-tilting loading mechanism. The cell body which is on the ram of the 30 kN (3 ton) capacity loading machine and travels up (in compression) against a fixed frame supported by two identical tie rods. See figure 3.3. For another view of the apparatus see Green (1971a). Strain controlled movement of the loading ram was obtained from (Kopp) variator controlled wormgear system.

3.2.4 Sample Sheath

Conventional cylindrical rubber sample sheath was employed. Triaxial compression, extension and ISC tests all required 88-92 mm diameter sheets. As will be pointed out in Chapter 4 ISC tests required slightly different diameters at different b values due to small changes in dimension. It was established that a membrane with an oversize perimeter of a few millimeters relative to that of the mould gave the best fit. However it is not practical to order membranes which only differ a few millimeters in perimeter. Fortunately, there was a slight variation in the perimeters of membranes supplied by the manufacturer which was very helpful. Use of specially shaped membranes was considered impracticle and expensive. The thickness of the membranes ranged roughly between .25 - .35 mm.

3.2.5. Triaxial Compression, Extension Tests versus ISC Tests:

The same large cell was used for triaxial compression and extension tests. The equipment described is sufficient to perform them. But a few modifications must be done for extension tests. The ram of the loading frame was bolted to the cell base otherwise the cell body would hang freely in air because the anvil of the axial proving ring was attached to the loading frame by two strong bolts and an extension plate - a circlip around the anvil prevented it being disengaged from the plate -. The loading reversed from compression to tension so the proving ring calibrations in tension were necessary, see Appendix 3. Another provision required was the connection of the top loading platten to the proving ring. It was achieved by screwing a boss into the proving anvil. A spigot on the boss was inserted into the recess at the top axial platten and a shear pin was located which went through the brass back plate of the axial platten and the spigot.

3.2.6. Horizontal Loading Frame ("ISC Belt").

Perhaps the most interesting component of the ISC apparatus is the horizontal loading frame which is suspended from the cell top with wires through four pulleys on the tie bars. The belt frame consists of two duralumin crossheads tied together with four stainless steel bars. On one side there is the hydraulic ram mounted to operate in the horizontal direction, on the other side a proving ring with an inductive type of displacement transducer to sense its deformation. The proving ring is encased in an rectangular perspex box in oil, and connected to the crosshead in such a way that the load measurement can be done in the lateral direction. The proving is identical to the axial one, only the casing is different in shape. The polythene sheet at the back again provides flexibility. During the test program the belt load cell had to be dismantled due to oil leaks and because of a small rupture in the polythene sheet it was replaced. The tie bars are screwed into the belt proving ring cross-head permenantly whereas the ram crosshead can be separeted from the frame to enable the belt to be assembled around the sample, the ram cross-head is secured on to the tie bars with four heavy nuts. On the inner side of the crosshead there are nuts whose position are carefully fixed to locate the ram crosshead in vertical plane parallel to the other crosshead. The locations of these marking nuts are selected on the basis of the expected travel in the test series in question.

On the upper tie bars, near the crossheads, there are four pulleys, two on each tie bar. Brass wires which are connected to adjustable studs on the inside surface of the cell top pass around the pulleys and are connected to another group of adjustable studs (four in number) on the belt suspension support plate which is positioned just above the axial load cell. Therefore the belt frame is completely free to move in the cell. (which makes the task of correctly aligning the belt system very simple).

The horizontal load is applied by the hydraulic ram carried in the belt assembly. The ram is pressurised with low viscosity machine oil through flexible 'saran' tubing. The operating pressure can be generated by a strain controlled screw jack,or by using other pressure sources the pressure can be stress controlled. Alternatively a simple manual control can be used. The ram is 51 mm diameter and runs in a brass bushing with a rubber seal and has a stroke of 19mm.

To aid in the assembly of the belt system, temporary support is provided by adjustable screws mounted in plates supported from the cell base. The whole assembly is supported in this way prior to suspending it with the pulley system. Once suspended the temporary support screws are withdrawn. A general view of the belt frame with rigid plattens mounted on is seen in figure 3.4.

3.2.7. Belt Loading Plattens

3.2.7.1. Rigid Plattens.

Green (1969) and Reades (1972) used highly polished stainless steel loading plattens to apply the stresses in the belt direction in their all generalised tests. The Writer also used the same rigid plattens in his series of tests on volcanic sand and for the special series on Ham River sand. See the test program in section 3.4. Two pairs of stainless steel plattens which were machined and polished to a mirror finish were 88.8 and 77.5 mm high. The former pair was used in plain strain tests and the latter for generalised tests by Green. The other dimension (i.e the width) was not important as long as it covered the sample face. These plattens were mounted on ram and belt proving ring anvils. Since loose volcanic sand samples were very compressible, ISC tests at high b values made it necessary to construct shorter belt plattens (67mm high).

Dial gauges were employed to measure the travel of belt plattens towards each other. In some of the series two dial gauges were used, only one was used in some others. They were connected to the platten on the proving ring side and their spring operated arms rested on the L-shaped support plates which were attached to the plattens on the ram side. After the initial series of tests it was noticed that the difference in readings between the two dials was not significant probably due to a later modification of the clearance between the ram and the bushing - it was decreased to 0.012mm -. It was decided to use only one gauge in the later series of tests. There were other reasons for this decision. Since the major principal strain was measured using a single external dial gauge (in majority of the tests axial direction corresponded to major principal stress direction) a greater accuracy was not required in the belt direction, and when using flexible plattens only average strains could be measured. An additional advantage was that it was possible to observe closely one of cell faces of the sample during the test, and so to gauge the uniformity of strains.

The internal dial gauge was made of an ordinary dial gauge encapsulated in a perspex casing filled with oil. Plastic bellows covered both the spring loaded arm and the other end so that oil inside was transferred from one bellows to the other if a deformation was experienced. See Green (1969) for details.

3.2.7.2. Design of Flexible Plattens.

3.2.7.2.1. General.

For the reasons explained briefly in Chapter 1 and 5 a decision was taken to use flexible belt plattens. Initially use of axial flexible plattens was also considered, but it was abandoned later on for various reasons.

First it was considered whether the present belt frame would be made use of or whether another system would be necessary. It was evident that a fixed frame to apply intermediate stresses would not be suitable, because the large deformations expected at high intermediate stress states would require the flexible plattens to be extensible. It was not possible to make a flexible platten which was extensible and resistant to high differential pressures at the same time. The constant cell pressure used in the rigid platten ISC tests was about 207 kN/M^2. Thus the deviator stress $(\sigma_2 - \sigma_3)$ was then of the order of 1000 kN/M^2 for dense Ham River samples near b=1. Such a large differential pressure could not be resisted by any sort of rubber bag. Thus there was a need for very strong and yet flexible material. Attention was drawn to the fabric reinforced rubber bags manufactured at University College, London, which were used in a cubical soil testing apparatus employing six flexible plattens, Menzies (1970), Menzies and

Phillips (1972), Arthur and Menzies (1972). After a close examination of the reinforced fabric it was decided to use a similar method, because it was the right material for the purpose, Arthur (1973).

The decision to use flexible plattens of fabric reinforced rubber caused many problems associated with incorporating these plattens within the existing equipment layout, the solution to these problems culminated in adopting a nested frame system accomodating the reinforced rubber bags.

To avoid problems associated with the expected large deformations for tests at large b values the bags were designed as a closed hydraulic system within its supporting frame and the deformation required was to be provided through the built-in hydraulic jack. This system ensured proper platten contact over the whole sample face while maintaining a stress boundary, this boundary stress was monitored by measuring the pressure within the prefilled and sealed fluid space within the bag.

3.2.7.2.2. Design of Backing Frames for Bags

By trimming the dimensions of existing equipment just enough space was made available for the backing frames which are shown in detail in figure 3.6. The frame was made of brass, and its dimensions were determined by the dimensions of the test sample in the belt direction. The height of the sample could be modified slightly if desired but the dimension in the cell direction (C) was more difficult to change. The inside width of the frame was made slightly wider than the sample, and so it follows that the bags would slightly overlap the sample. Therefore during the initial stages of a test there would be more bag face exposed to the cell pressure at the edges than at failure, and the lower deviator stresses at the early pre-peak strains could be coped with by the membrane across the exposed area of the bags which were open to the cell pressure.

The space provided in the frame had the same shape as the flexible bag except the bag was little deeper, so that the face was not restrained by the side walls of the casing. Two drainage lines were located at the mid height on the sides, one reaching the conical recess at the center, the other joining the recess at its sides. The conical shape was introduced to aid in driving the air bubbles out of the bags. The purpose of these two drainage lines was to provide a circulatory system so that the bag could be filled and de-airedwater at the same time, see section 4.2.4. The line joining the top (centre) of the cone was the outlet.

The major problem was to seal the bag. Two O-rings and a brass plate inside the bag were used for this purpose. The inner plate had a circular opening at the centre. Eight allen screws secured the plate to the back frame and sandwiched the membrane between them. The holes in the inner brass plate were blind tapped holes. The screw heads were in flush with the surface of the recess at the back of the casing frame so that a good fit could be obtained with the ram and proving ring anvil. These cylindrical recesses at the back of both plattens had the same dimensions as the diameter of belt proving ring anvil, and four side screws provided a grip. A circular extension piece which had the same diameter as the recess was screwed on to the ram to fit it into the recess. Since the eight allen screws were lined on a diameter between the two O-rings a complete seal for the system was obtained.

U-shaped brass strips were connected on top and bottom of the plattens to connect the encapsulated dial gauges. Other required pieces to fix the dial gauges to the plattens, and the reaction plates to support the dial gauge arms were designed with the requirements of the limited space, figure 3.7. The nuts which were normally used for fixing the crosshead positions in rigid platten tests were abandoned and circular threaded pieces were used instead because the nuts caused an obstruction to the flexible plattens during the erection of the belt.

A photograph of the belt frame with the flexible plattens and a dial gauge can be seen as placed on the bench in figure 3.8. (Ram crosshead is not fixed properly, and mark rings on the tie bars are missing in the picture).

3.2.7.2.3 Construction of Reinforced Rubber Membranes.

Once the brass backing frame was designed, the exact dimensions of the bags with the exception of the depth was fixed. The depth of the bag was limited due to the area that would be exposed to differential pressure application. The Writer was well aware of the importance of this feature, because as differential pressures were built up during a test, balooning and puncturing could occur. Increasing the depth would result in a bigger unsupported area in the gaps at the sides of the platten despite full coverage of the sample face. An optimum size of gap in the belt direction was determined, thus specifying the bag depth.

A detailed section on the manufacture of the membranes is given in Appendix 1. Only the highlights are given in this section. Reinforcement technique was in line with Menzies (1970), Arthur and Menzies (1972), Phillips (1972), Arthur (1973). First perspex formers were prepared. They were little larger than the actual dimensions of the bag desired to allow for the shrinkage expected during curing. Ordinary rubber bags were prepared according to a standard method of dipping the former into a coagulant, driving off the solvent by drying, dipping it into the liquid rubber (Latex) solution for a specified period of time and then curing the rubber skin formed on the former.

A fine trylene mesh fabric was glued on to the surface of the bag smoothly without getting any accumulated glue spots on the bag, then it was dried and a very fine and smooth final latex finish was made on the surface of the bag. The extent of the reinforcement that covered the back of the bag was important since it was impossible to take the former out of a bag reinforced fully all around. This fact plus the location of O-rings determined the final shape of the reinforcement, see figure 3.9. The circular cut at the centre - back - of the bag had the same diameter as the hole through the inner brass plate.

The design worked well, but as will be described in section 4.2.4, the preparation of flexible plattens to conduct a test was very time consuming. The preparation of eight holes through the membrane and their alignment with the screw holes was tedious. Leaks occured everytime the circular cut at the centre of the membrane was not correct. In such cases a fresh start was made every time. The sealing mechanism was improved by trimming the back surface of inner brass plate as much as the thickness of the membrane, from between the two O-rings to the outer edges of the plate. This made it possible to eliminate the need for piercing holes in the membrane because the central piece of the bag could then be cut to pass the screw holes, see the dotted line in figure 3.9.

A problem with the flexible plattens was the membrane tension (drag) developed due to high differential pressures.

They were especially dangerous when there was a little large exposed area to the cell pressure. They caused the membrane to slip on the O-rings along the narrower dimension of the platten at mid height. This directly resulted in a leak because the reinforcement was not on the O-ring and the tension caused the pure rubber bag to stretch and to get thinner thus resulting in a loose contact between the O-ring and the membrane leading to a leak. To prevent this, the back of the bag was glued on to the inside surface of the backing frame. Glue was also applied along the vertical edges of the frame and the bag. This was necessary for tests near b=1. This solved the problem but the plattens had to be cleaned before a new test which was difficult due to the strong adhesion between the brass base and the bag.

Another problem with the flexible plattens was that they sometimes slipped off the sample faces sideways together with the whole belt frame before peak stress was attained. It was a sudden angular movement around a vertical axis. This was caused by the inflated bag area exposed to the cell pressure along the vertical edges of the plattens. A couple of tests were lost in this way so it was decided to use small supporting guide arms to arrest any tendency for rotation. Guide arms were inserted through the holes which were drilled through four heavy reaction bars in the cell and fixed at the desired position by locking side screws. They were eight in number, two on each bar at the levels of lower and upper tie bars of the belt frame, see figure 3.10. A gap of about a milimeter was allowed at each guide location. They served the intended purpose very well.

It was seen that the reinforced bags were very strong and any difficulty such as the one just mentioned or the leakage through the outer O-ring in the belt platten was not caused by the reinforced membrane itself. The bags coped with relatively large gaps between the sample and the backing frame in some tests without any significant balooning or bursting. Initial trial tests with dummy samples showed that provided the seal system worked properly the bursting pressure was outside the range of the pressure recording system (1200 kN/M^2)

3.3. Instrumentation and General Arrangement.

The same large triaxial cell was used for triaxial compression, extension and generalised tests. The belt frame was omitted for triaxial series of tests and all entry points (tubing, electrical cable etc.) related to 3-dimensional testing were sealed properly. Axial load was provided by 30kN capacity worm gear screw jack testing machine driven by an electric motor and a gear box. A (Kopp) variator which was connected to the gear box provided a variation of speed from 1.25mm/min to 0.000025mm/min. Speed could be changed during a test by varying the gear ratio continuously and smoothly. If a large range of change of speed was required, then the manual change of the gear ratio required a temporary stoppage of one or two seconds.

A frame supported by twin tie rods supplied the reaction for the internal axial proving ring. The triaxial cell of 33 cm internal diameter, axial and belt proving rings have all been described in the previous sections. The inductive displacement transducers in the both proving rings were energised, and read with a (Boulton Paul) C61 null balance transducer meter which had very good sensitivity and long term stability. Matching transformers could be used in the meter to match the type of transducer in use.

The cell was filled with de-aired water and pressure was supplied by self-compensating mercury control system, Bishop and Henkel (1962). Use of de-aired water was essential, mainly because large quantity of tap water from which dissolved air could be released may have required mercury pots of larger capacity than available due to compression of air with increasing pressure. The cell pressure was normally measured using a (Bourdon) pressure gauge which had range of 0-1100 kN/M^2. In quite a large number of tests a (Budenberg) dead weight tester was also connected to the system for greater accuracy, but general use of the tester in the laboratory by other research workers made it impossible to use it permenantly. In some of the test series the cell pressure was varied during the test - for example SP(9-16) or some of EX series -, and a pressure transducer was used to record the cell pressure more accurately. A motor-gear box mechanism was used, to drive the mercury pots in these tests. Both the cell pressure and the oil pressure in the cylinder of the ram were increased by driving the necessary pots in some of the tests, so two separate motor-gear box systems had to drive two different pots. Sometimes the range of a single mercury column was exceeded, in these cases there was a temporary halt to the test to enable a second mercury column to be linked in. (Two separate double mercury lines had to be used in some tests, for example, in SP14, 15, 16).

All tests were conducted as drained tests and volume changes were measured with a 50cc burette open to atmosphere. The oil pressure required for the ram cylinder was normally supplied by a screw jack - piston mechanism (strain controlled) as in all tests by Green (1969) and Reades (1972), but in certain special test series - see the test programme in section 3.4. - the ram pressure was controlled by the mercury pots through an oil-water interface. Oil pressure in the pressure cylinder (ram) could be observed visually during the test using another pressure gauge (4200kN/M^2 capacity) so that any build-up pressure due to any malfunction in the system could be detected immediately before any damage was done to the hydraulic ram. The water pressure in each bag was measured using pressure transducers independently. The pressure transducers used were of the strain gauge resistance type, and had a range of about 1100kN/M^2.

To circulate water, de-air and fill the bags a simple system was designed and constructed. It also had the function of measuring the pressure, Each flexible platten had two fittings at the sides through which (saran) tubing was connected. Deaired water was pumped into each bag through one of these two connections and left the bag through the other. (This is the preparation stage of the flexible plattens which will be described in section 4.2.2). At the start of the test the valves were so adjusted that water pressure in each bag was measured without any water circulation independently. See figures 3.5 and 3.11.

Four (saran) tubing lines coming from the plattens, one (saran) tubing line from the hydraulic ram, another large diameter tubing to fill the cell and electrical cable from the belt all passed through the cell base using proper fittings and seals. Care was taken not to damage the plastic electrical cables inside the cell. Electrical wire connections were sealed in bulk araldite to prevent the pressurised water from penetrating into the electric cables and consequently leaking out. The two pressure transducers were also used in conjunction with the C61 transducer meter. Since the displacement transducers worked with inductive principle and the pressure transducers with resistance principle, each set of readings required use of different transformers in the C61 meter. Additionally, a minimum of four electrical output had to be recorded for each set of readings for a single point on stress-strain curve. This was accomplished by employing a 20- channel switch-box which had two locations for push-in transformers controlled by a switch.

3.4. Test Programme.

3.4.1. General.

In the present study of strength behaviour of granular soils at truely triaxial stress state, all tests were carried out drained, and saturated sand samples were used. Testing of dry sand was not considered due to varying degrees of moisture absorbed by the sand particles that would cause certain changes in the frictional behaviour which could not be isolated from the present study of global strength parameters. See, for example, Skinner (1969),(1975), Horn and Deere (1962).

Not all of the tests performed were planned from the start of the research. Some series were decided on the basis of the results of previous series. All tests were isotropically consolidated prior to shearing. Although anisotropic consolidation could be a closer presentation of field deposits, shearing behaviour of anisotropic and isotropic samples was found to be "essentially the same" by Lee and Seed (1970) in drained tests, and yet isotropic consolidation procedure offers simplicity compared to anisotropic consolidation. Behaviour of undrained tests differed with respect to the type of consolidation but they have no relevance to the present series of tests. Majority of tests were carried out under a constant cell pressure of 207kN/M^2.

No cylindrical samples were tested, the sample shape was a right rectangular prism which is frequently referred to as "cuboidal". The dimensions of the cuboidal samples were rather different in triaxial tests compared with those in generalised tests which also differed slightly among themselves at various intermediate stress states. A high b value test required several millimetres longer dimension in the belt direction so that the belt plattens would not become arrested under the axial plattens at failure. An average size for a generalised sample was 90 x 85 x 53mm and for triaxial compression and extension samples 85 x 76x 58mm. Some short extension samples were also tested and they will be described below. Letters B, C refer to the dimensions in the belt and cell directions. The strain rates during shearing did not present any problem for the free draining materials used, and the value selected was a matter of convenience. Most of the tests fell in the range 0.04-0.10 mm/min. Belt strain rates were selected on the basis of the axial strain rate decided and the specific b value.

In the following survey of the series of tests the main philosophy behind each series of tests is only given very briefly. See the respective sections which are referenced to in each section below for additional information. All dense samples were prepared by tamping and loose samples by deposition under water, see Section 4.1.

3.4.2. Tests on Ham River Sand.

3.4.2.1. Generalised Tests Using Flexible Plattens.ISC(F)1-20.

Reinforced bags were used to apply intermediate stresses in one of the lateral (belt) directions. Average dimensions of the samples were H = 92mm B = 85mm C = 53mm. The samples were first consolidated approximately to 207 kN/M^2. Then the axial rigid plattens and belt flexible plattens were driven inwards both in a strain controlled way to apply major and intermediate principal stresses respectively until failure. The cell pressure was held constant throughout shearing. The sample was deformed into the cell water against a constant pressure. This was the shearing procedure used by Green (1969) and Reades (1972) who used two pairs of rigid plattens in the axial and belt directions.

There were two porosity groups, namely, dense and loose. Dense tests covered a wide range of intermediate stress states wheareas loose tests were confined to b range of roughly 0.65 - 1.00. The latter group were ISC(F) 10, 11, 12, 13, 15, 17, 18 & 19 and the rest were dense tests, see Chapter 5.

The main idea behind the dense series was to key in with the test results of the rigid platten tests conducted by the aforementioned researchers, their testing method was strictly followed in this series. The sharply increasing ϕ' values found after b = .60 until b = 1.0 in rigid platten tests on loose samples by Reades (1972) were suspect, as there was a suspicion as to whether they were affected by the presence of rigid plattens, Therefore the loose, flexible group of generalised tests were confined to this range of b values. See Chapters 5 and 7 for thorough discussion.

3.4.2.2. Triaxial Extension Tests.

3.4.2.2.1. Triaxial Extension Tests on Short Samples EX1-EX12.

These were short cuboidal samples of the dimensions 78mm x 78mm x 4.9mm (height). They were mainly loose to medium loose, EX9, 10 being the only dense samples. They were in two different subgroups. EX1, 2, 3, 4, 6, 8, 9 were consolidated to a cell pressure of 207 kN/M^2, and cell pressure was increased to failure as the major and intermediate principle stresses, while the stress on the axial platten was kept approximately constant as the minor stress. EX5, 7, 10, 11, 12 were first consolidated to 700-1000 kN/M^2 under the cell pressure which was then kept constant while the axial platten was withdrawn to failure (minor), in the same way as in a conventional extension test.

3,4,2,2,2 Triaxial Extension Tests on Long Samples.

These series were for the purpose of investigating the orientation of the sample dimensions with respect to the direction of the principal stresses in ISC and extension tests. If more information is desired at this stage see Chapter 6 and figure 6.1. Stress path effects were also considered because up to the present only mean stress level-decreasing-type of extension tests consist of EXII-1, EXII-3 (failure was not reached in this test), EXII-4 and SP17. SP SP17 was conducted at the end of another series but was essentially the same type as in these groups. Dimensions were 85 x 78 x 58mm, and they were performed in a similar way to the tests in the first subgroup of the short samples mentioned above.

3.4.2.3. Special Series of Generalised Tests SP1-16 (Rigid plattens).

3.4.2.3.1. SP1-8 series.

These were all on loose samples which had dimensions of 90 x 85 x 53mm, i.e. almost the same as those of ISC(F) series. They were all carried out using rigid plattens. Although SP2 and SP5 were described under this group they were the usual type of rigid platten ISC tests. As also explained in Section 5.4, they were intended as control tests because they were at 44 percent porosity and thus could be compared with ISC(F) series directly. It will be seen in Chapter 4 that the Writer's and Reades' loose samples differed by one percent in porosity due to differences in preparation.

They also presented a good chance to compare with Reades' rigid platten tests. The rest of the tests covered a wide range of b values.

Orientation of principal stresses were the same as in ISC(F) series but the testing method and the instrumentation were different. Rigid belt plattens were stress controlled. After the samples were consolidated to about 840 kN/M^2 isotropically, axial plattens were driven at a selected strain rate as usual but intermediate stresses were kept at about consolidation (cell) pressure while the cell pressure was decreased to failure.

3.4.2.3.2. SP 9-16 Series.

In this series the orientation of the principal stresses were different with respect to the axes of the apparatus compared with other generalised tests in the ISC apparatus. The tests were all on loose samples, and had b values higher than 0.50. Sample dimensions were similar to other ISC samples. SP10, 11, 12 were consolidated to a relatively high all-round pressure, then while the cell pressure was kept constant (intermediate stress) and belt rigid plattens were kept stress controlled at a constant stress (major), the axial plattens were withdrawn and the sample was failed into the axial direction ie. mean stress level was decreased, and the stress applied by axial plattens represented the minor principal stress.

SP 14, 15, 16 were similarly tested by applying major principal stresses through rigid belt plattens, but the consolidation pressure was 207 kN/M^2 as in all other usual types of ISC tests. Then the cell pressure (intermediate) and the belt stress (major) were increased while the axial plattens were withdrawn to failure again. Belt stress was stress controlled despite its increasing nature i.e. mercury pots were driven up simultaneously with the pots which were driven to increase the cell pressure. SP9, 13 were abandoned as explained in Chapter 6 where results of this series of tests were discussesd.

3.4.2.4. Control and Mean Stress Level Tests.

At the start of the test programme preliminary triaxial compression tests were performed. These were to provide a comparison with the previous triaxial compression tests performed by Green (1969) and Reades (1972) to check the Writer's sample preparation method. This first group of tests, six in number, was designated as TC1-TC6. TC7 was conducted at the end of tests series on Ham River sand, see Appendix 6.

Mean stress level series were composed of three triaxial compression tests; ASL- TC2, 3, 4 and three generalised tests with rigid plattens; ASL - ISC 1, 2, 3. These are reported in Appendix 6. The idea behind these series was simply to see the effect of mean stress level on strength and deformation and to strengthen the existing limited data on Ham River sand with respect to varying mean stress levels and were used to normalise the results.

3.4.3. Tests on Volcanic Sand.

3.4.3.1. Generalised Tests on Volcanic Sand.

These were rigid platten tests, and were carried out by driving the (strain controlled) axial and belt plattens inwards as being the major and intermediate principal stresses respectively . Consolidation pressure was kept constant up to failure and in a majority of tests, it was 207kN/M^2. Dense tests were designated as ISC D-1, 2, 3, 4, 5, 6, 7, 8 and loose tests as ISC L - 1, 2, 3, 4, 5, 6, 7, 8, 9. Tests covered almost the whole intermediate stress range at high and low porosities. They were intended to investigate the behaviour of a high ϕ' material and to compare it quantitatively and qualitatively with the Ham River sand data and to test the existing failure theories.

3.4.3.2. Triaxial Compression and Extension Tests.

Five triaxial compression tests, TC 1-5, and three triaxial extension tests EX V1-3 were conducted to complete the picture for strength variation. They were at dense and loose porosities. The results are presented in Chapter 8.

3.5. Test Calculations.

Test calculations were done on the assumption that the samples deformed uniformly as right prisms. Another major assumption was the uniformity of stresses on the rigid plattens. The total load on the axial and belt proving rings was directly divided by the sample area in the respective direction to obtain the stresses. The total deformation measurements in the axial and belt directions were again directly divided by the initial sample dimensions to obtain the strains at any stage of the stress-strain curve. All strains were based on the dimensions at the end of consolidation.

Contraction of the sample was taken as leading to positive compressive strains. The cell pressure acted on one pair of lateral faces of the sample in generalised tests and on two pairs in compression and extension tests. It was considered to be absolutely uniform, and regarded as a principal stress in all cases.

Thickness of the sample sheath and axial free ends were subtracted from the measured quantities. Compression of free ends were subtracted from the measured quantities. Compression of free ends and the loading system was assessed by calibration and considered in the calculations. See Appendix 3. In the case of flexible plattens their own compression was measured by calibration and considered in the calculations (Appendix 3). Good quality calibrations were done frequently for the axial proving ring both in compression and tension and for belt proving in compression and for pressure transducers as well, see Appendix 3 for the details.

The load on the belt proving ring was used for calculating belt stresses in flexible platten series rather than the bag pressures. The uniformity of stresses on the sample forces was a good assumption in this case.

The weight of the top cap was subtracted from axial load reading in the extension test and added in the triaxial compression test. These considerations can be formulated. (see the list of symbols at the end of Chapter 1).

$$Hc\ (1-\varepsilon a)\quad Bc\ (1-\varepsilon b)\quad Cc\ (1-\varepsilon c) = Vc\ (1-\varepsilon v) \qquad 3.1.$$

$A = Ac. \frac{1-\varepsilon v}{1-\varepsilon a}$ where A is the average axial cross-sectional area at any strain during the test. Subcript "c" always refers to consolidation. i.e. A_c is the average axial area after consolidation etc.

$$\sigma_{a(c)} - \sigma_{c(a)} = \frac{\text{Load recorded by proving ring} \mp \text{weight of top cap}}{A}$$

Equation 3.1 can be used generally without any reference to any specific test.

For example lateral (major) strain in an extension test;

$$(1-Eb) = (\frac{1-\varepsilon v}{1-\varepsilon a})^{\frac{1}{2}}$$

Various strain rates were calculated simply by the ratios of finite increments, i.e.

$$\frac{dEv}{dEa} = \frac{\varepsilon v_2}{\varepsilon a_2} = \frac{\varepsilon v_2 - \varepsilon v_1}{\varepsilon a_2 - \varepsilon a_1}$$

Generalised tests presented an additional feature of loading in the lateral direction i.e. belt loading which was always in excess of the cell pressure (compressive). The sample area on the lateral (belt) face against belt plattens at any stage of a generalised test which is loaded in the axial, belt and cell directions independently is;

$A_b = \frac{A_{bc}(1-\varepsilon v)}{1-\varepsilon b}$ where A_{bc} is the sample belt area after consolidation.

Deviator stress in the belt direction

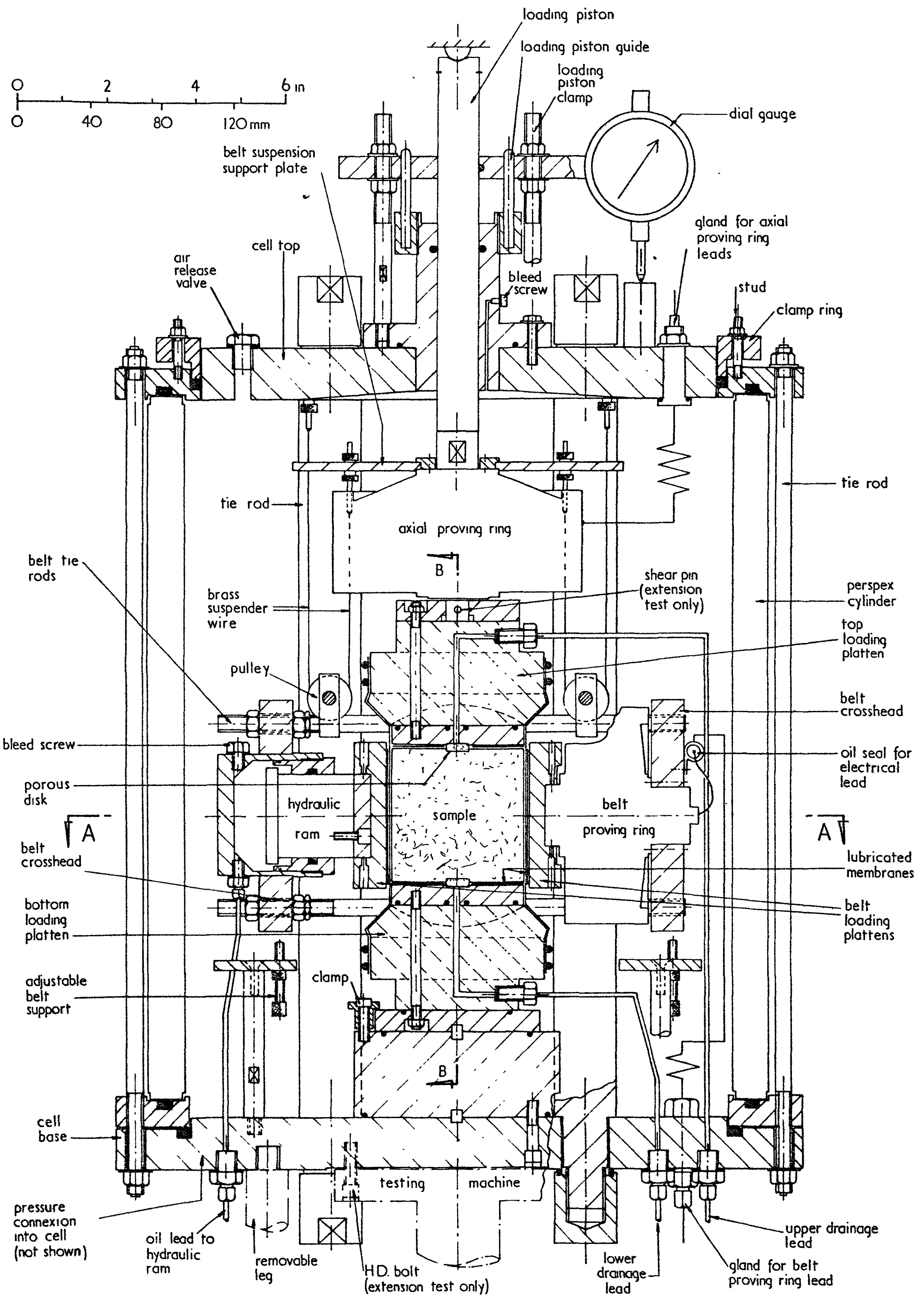
$$\sigma_b - \sigma_c = \frac{\underline{\text{Load recorded by belt proving ring}}}{A_b}$$

An actual example of a test calculation for a generalised test is given in Appendix 7.

Porosity was calculated on the basis of its usual definition as an index propertiy of the material

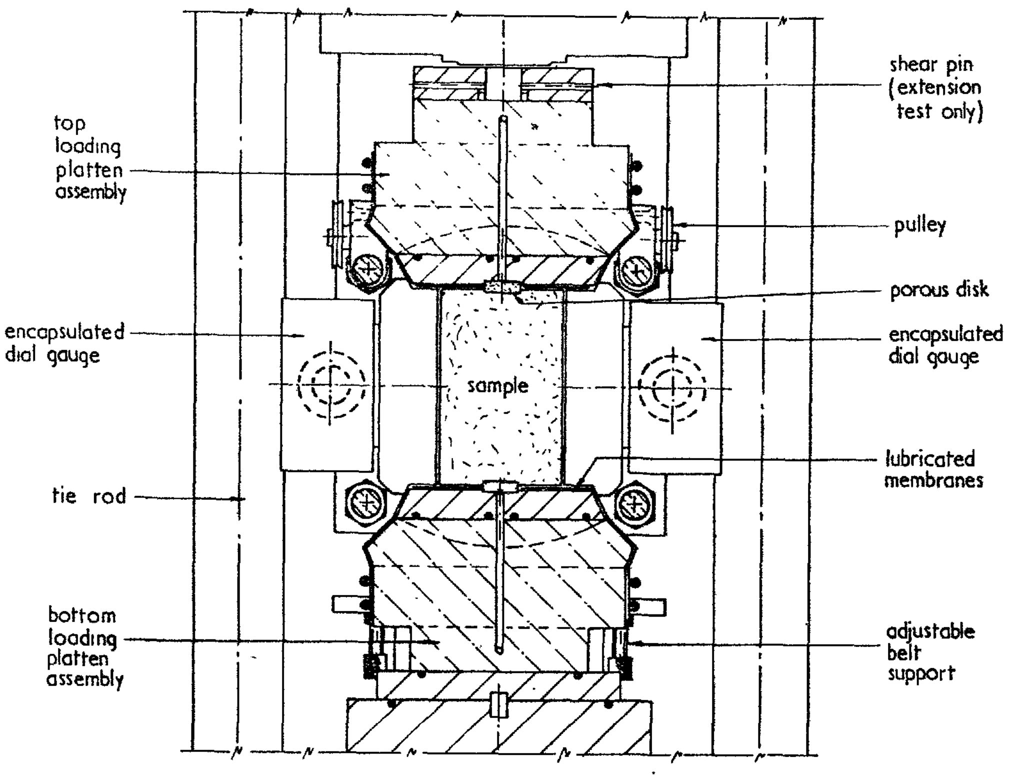
(%) $n=1 - \frac{Ws}{G\gamma_w V}$ (see list of symbols)

And finally Mohr-Coulamb angle ϕ' was calculated as arc sin $\left\{(\sigma_1-\sigma_3)/(\sigma_1+\sigma_3)\right\}$.

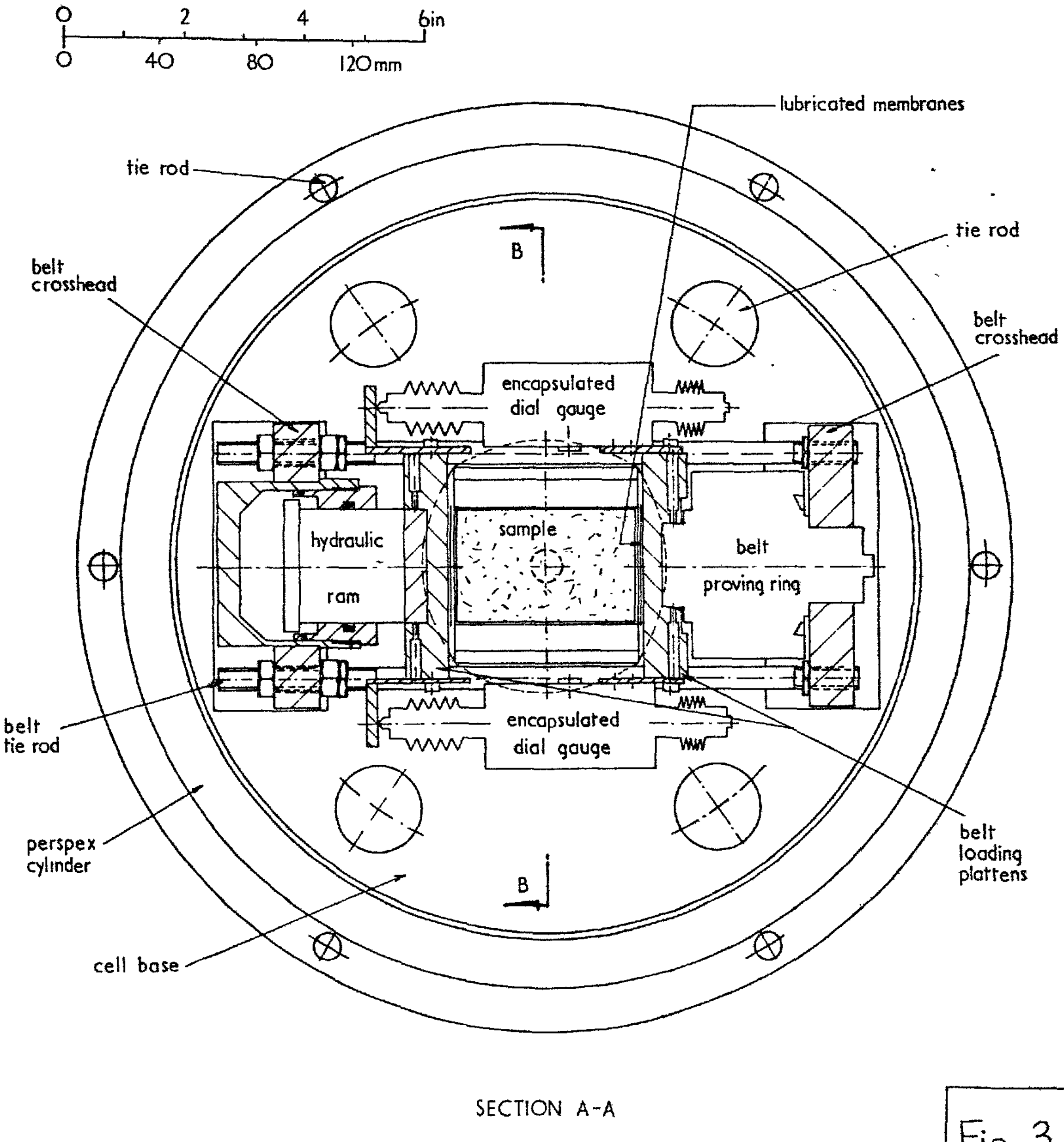


Independent stress control cell, general elevation, After Green (1971a).

Fig. 3.1



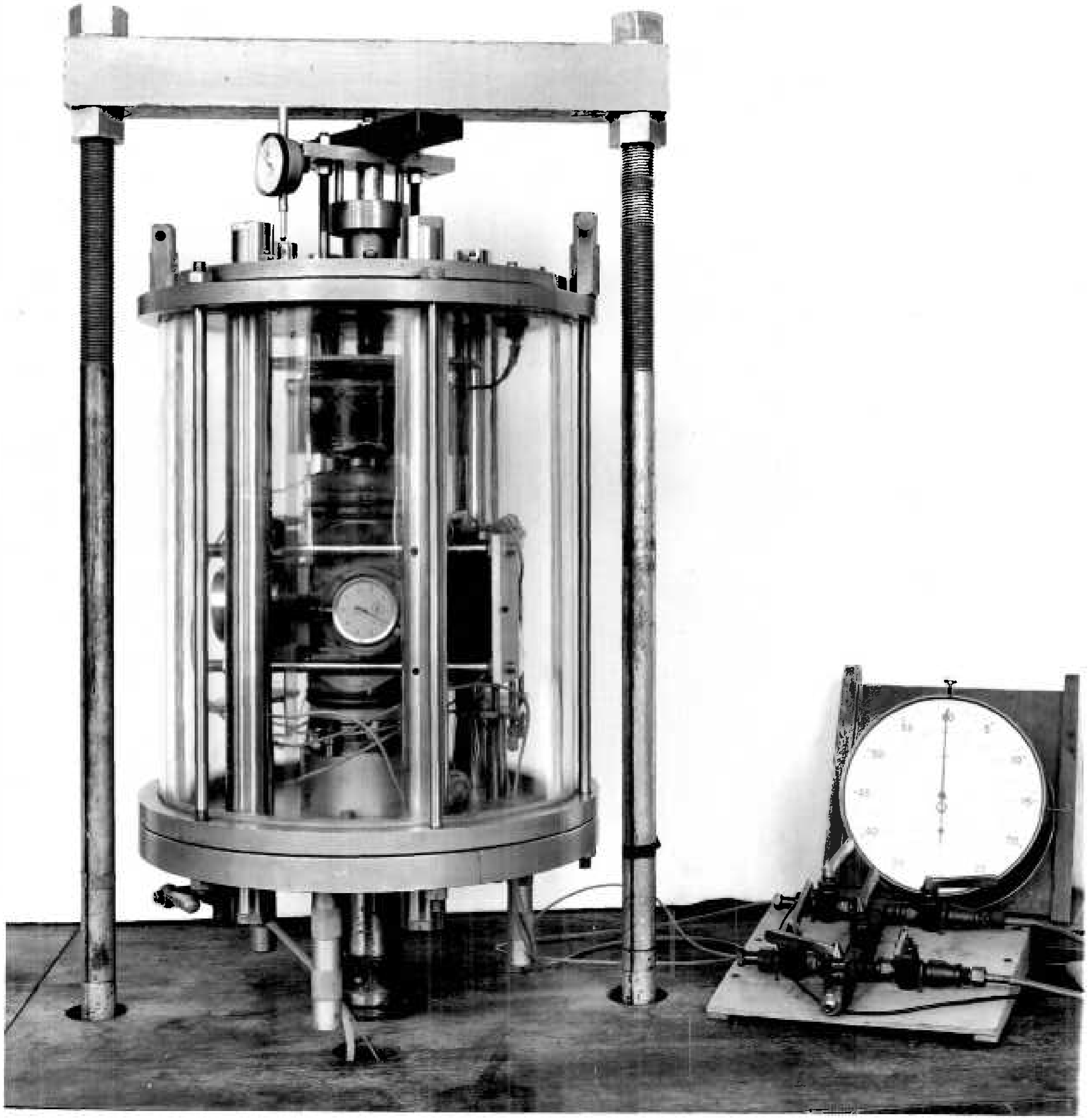
SECTION B-B



SECTION A-A

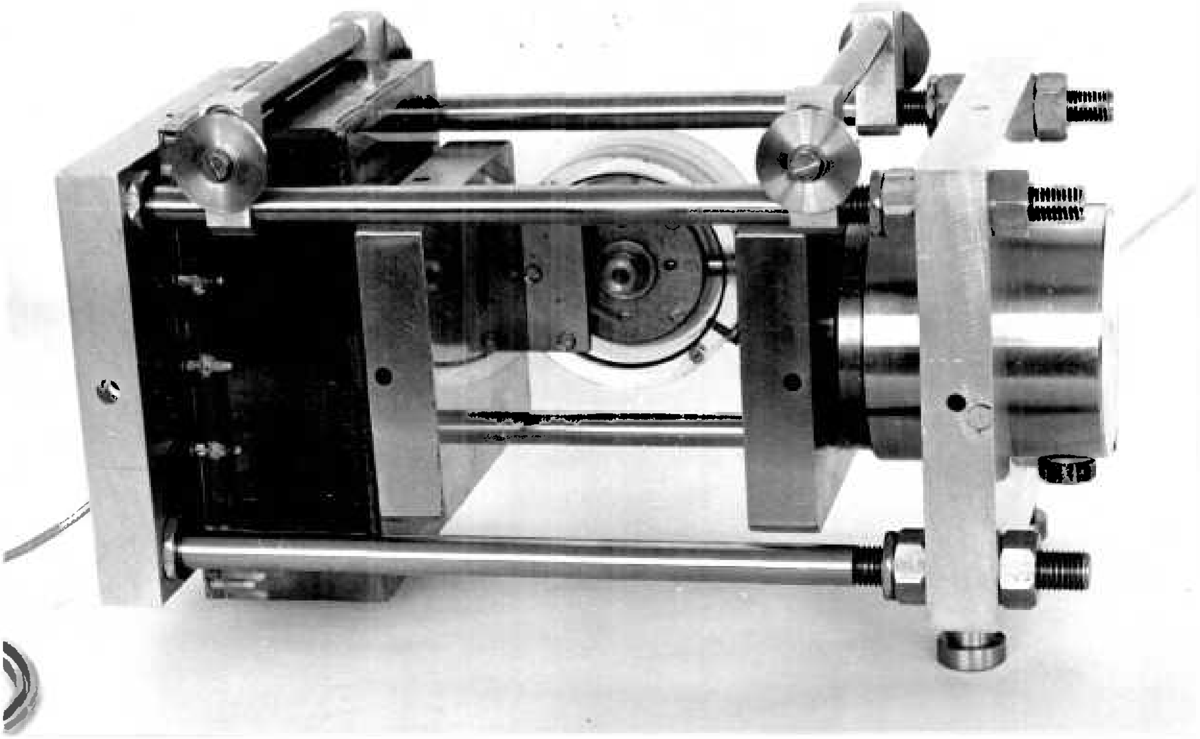
Fig. 3.2

ISC cell, sections A-A and B-B. After Green (1971a)



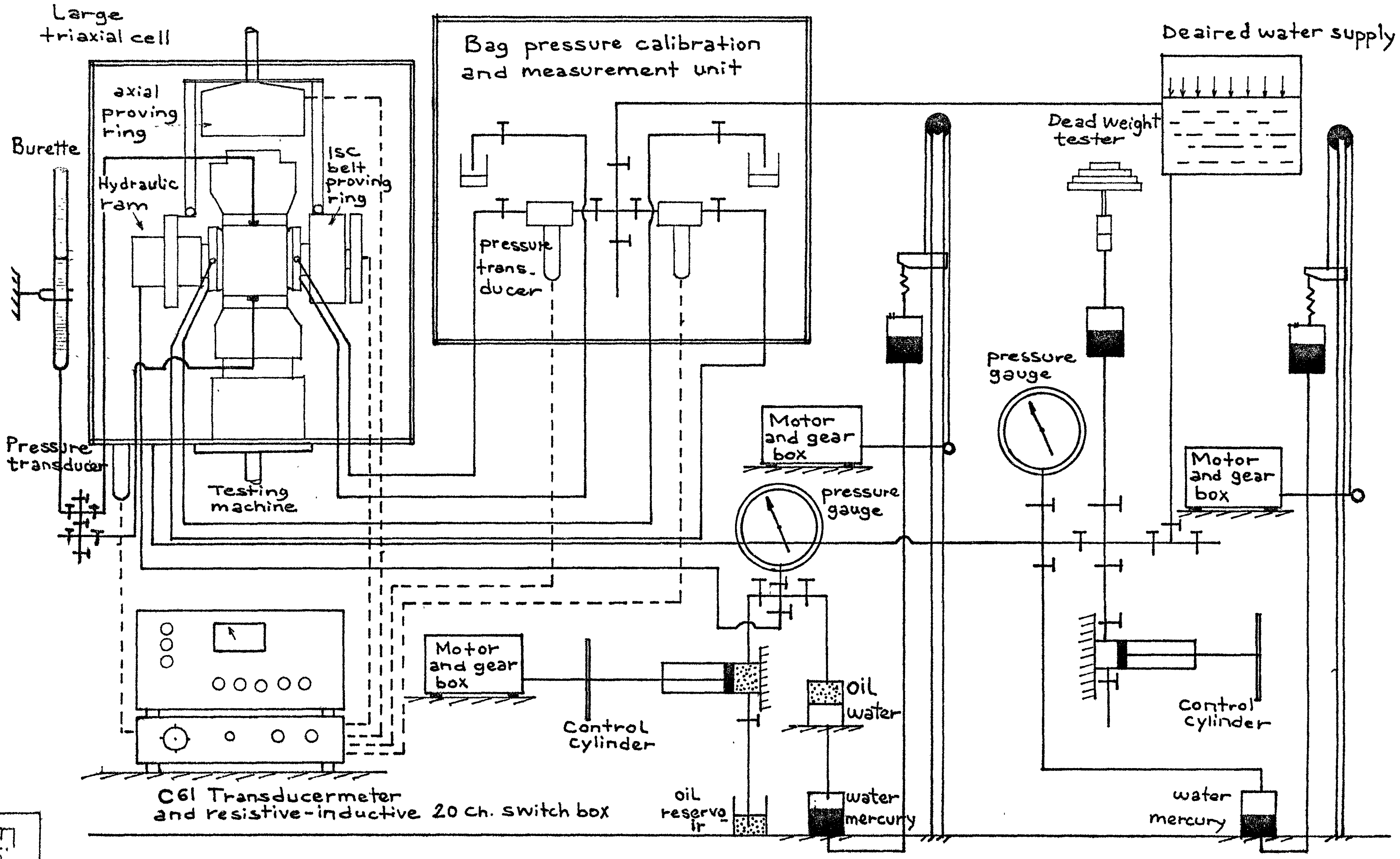
ISC cell

Fig.3.3



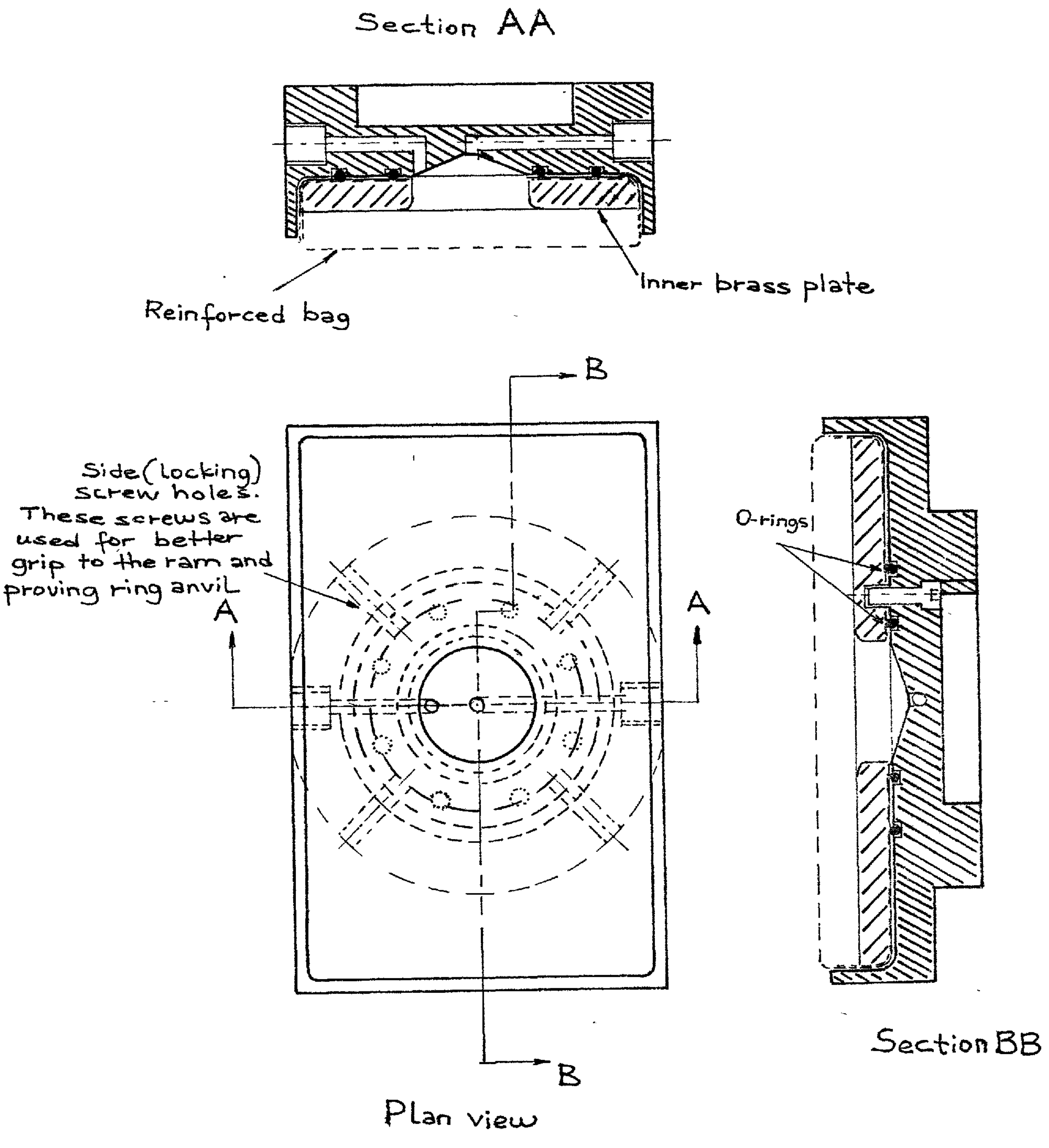
The horizontal loading frame ("belt") with the rigid plattens mounted on

Fig. 3.4



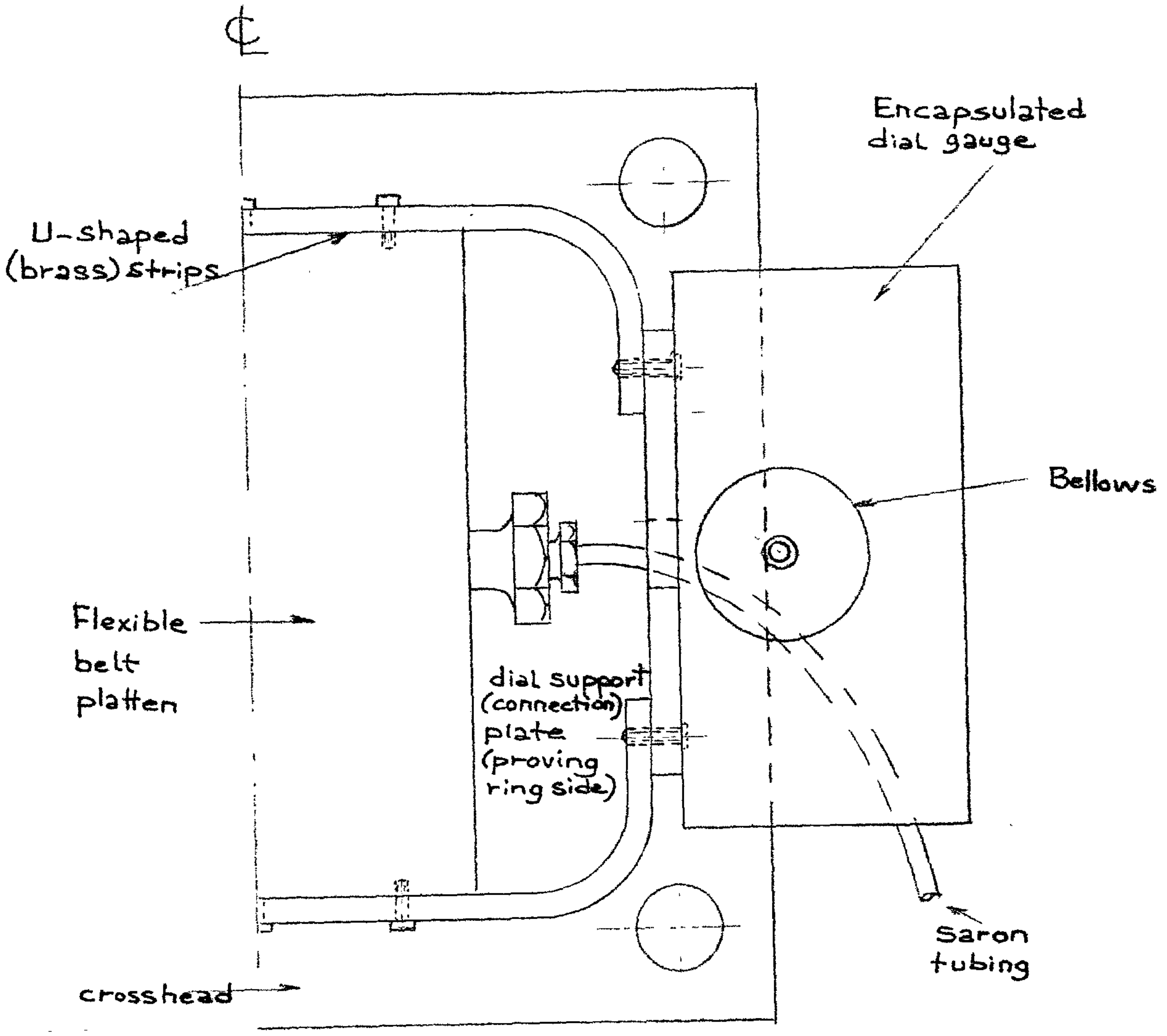
General Layout of the instrumentation both for rigid and flexible platten ISC tests (drained). During rigid platten tests bag pressure unit is disconnected

Fig. 3.5

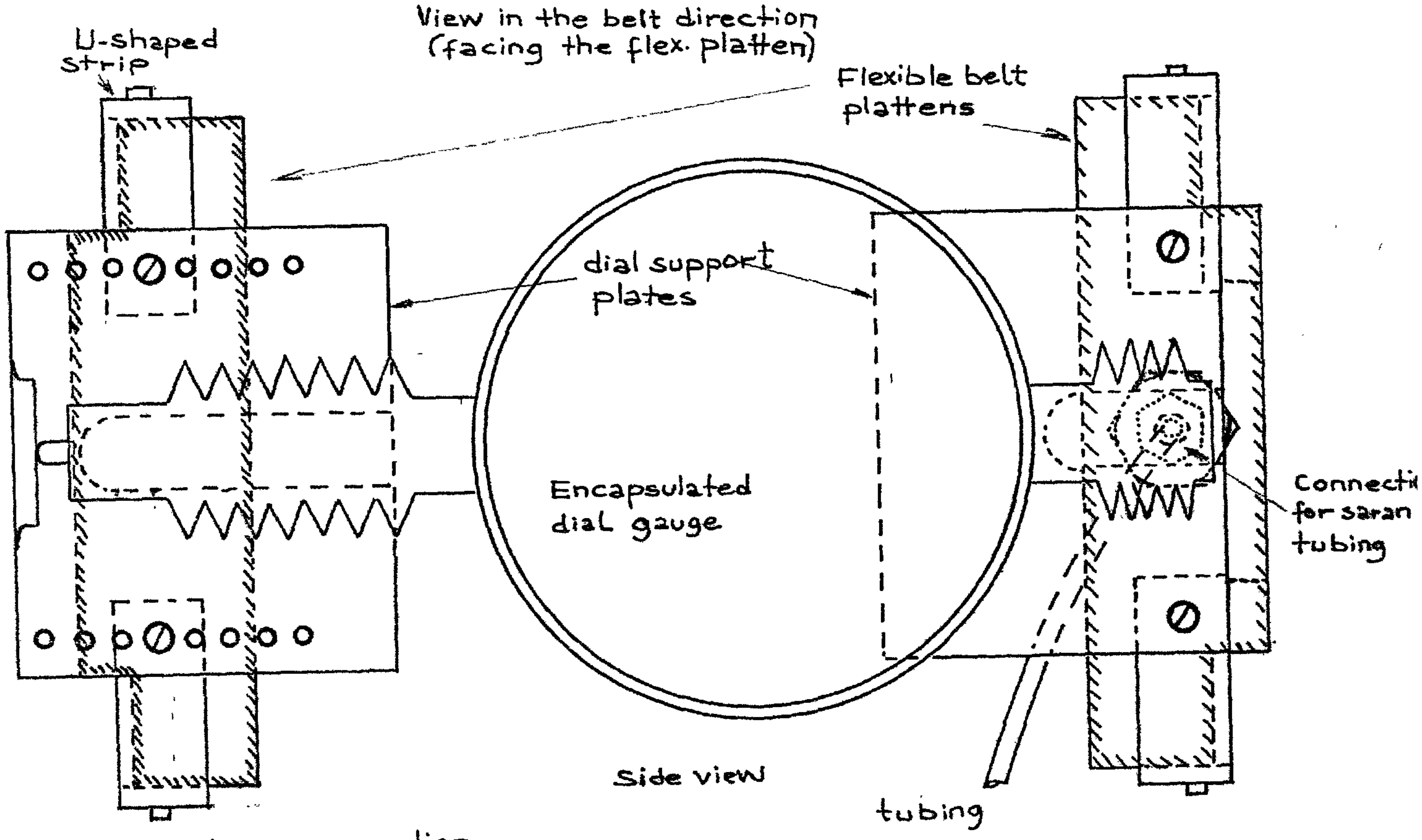


Design of the flexible belt platten (back frame)

Fig. 3.6



View in the belt direction (facing the flex. platten)

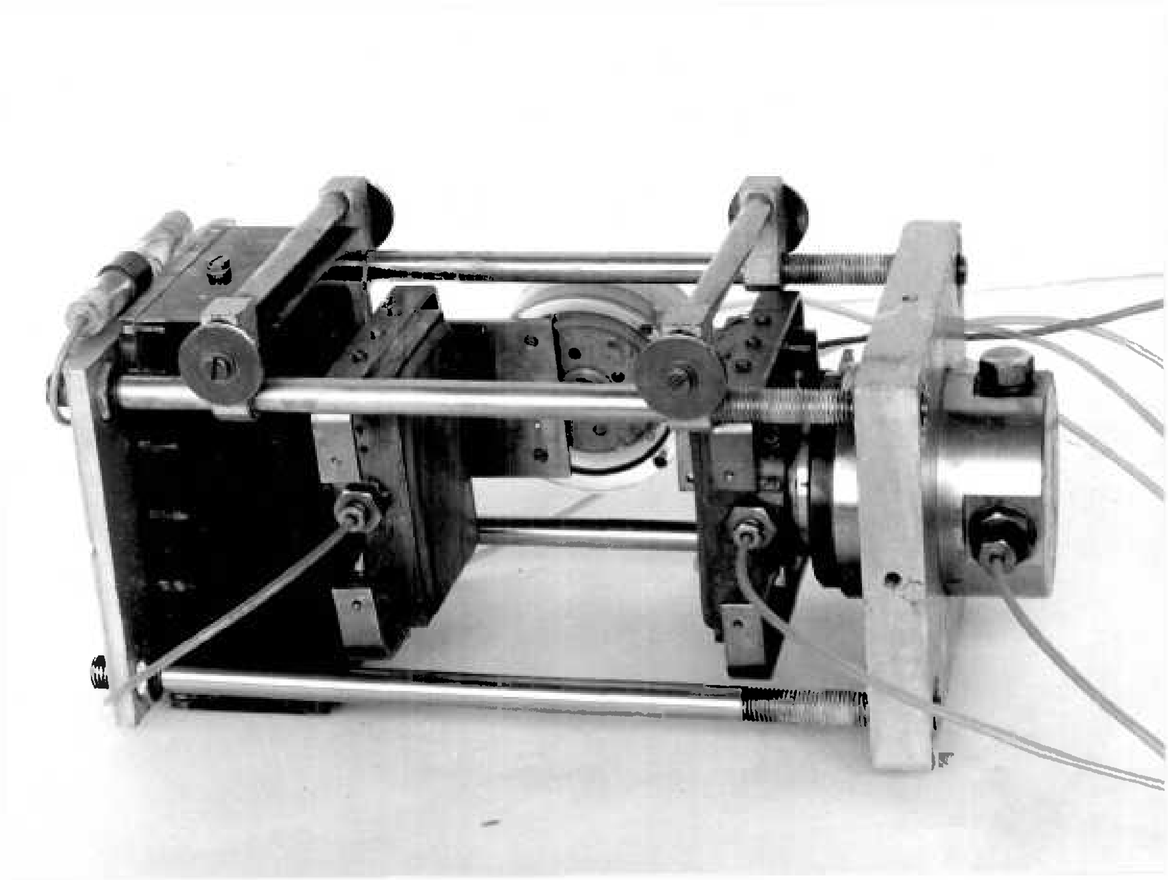


Side view

Saran tubing connection is not shown on this (ram) side.

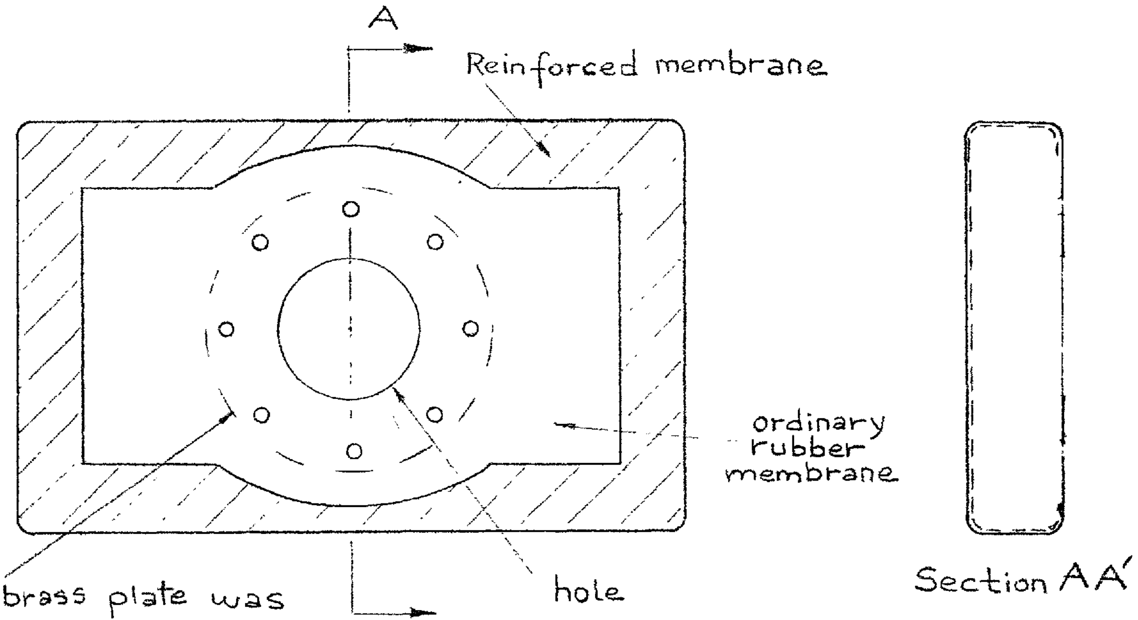
(App.) full size (minor details missing)

Fig. 3.7



The horizontal loading frame ("belt") with the flexible plattens mounted on

Fig. 3.8

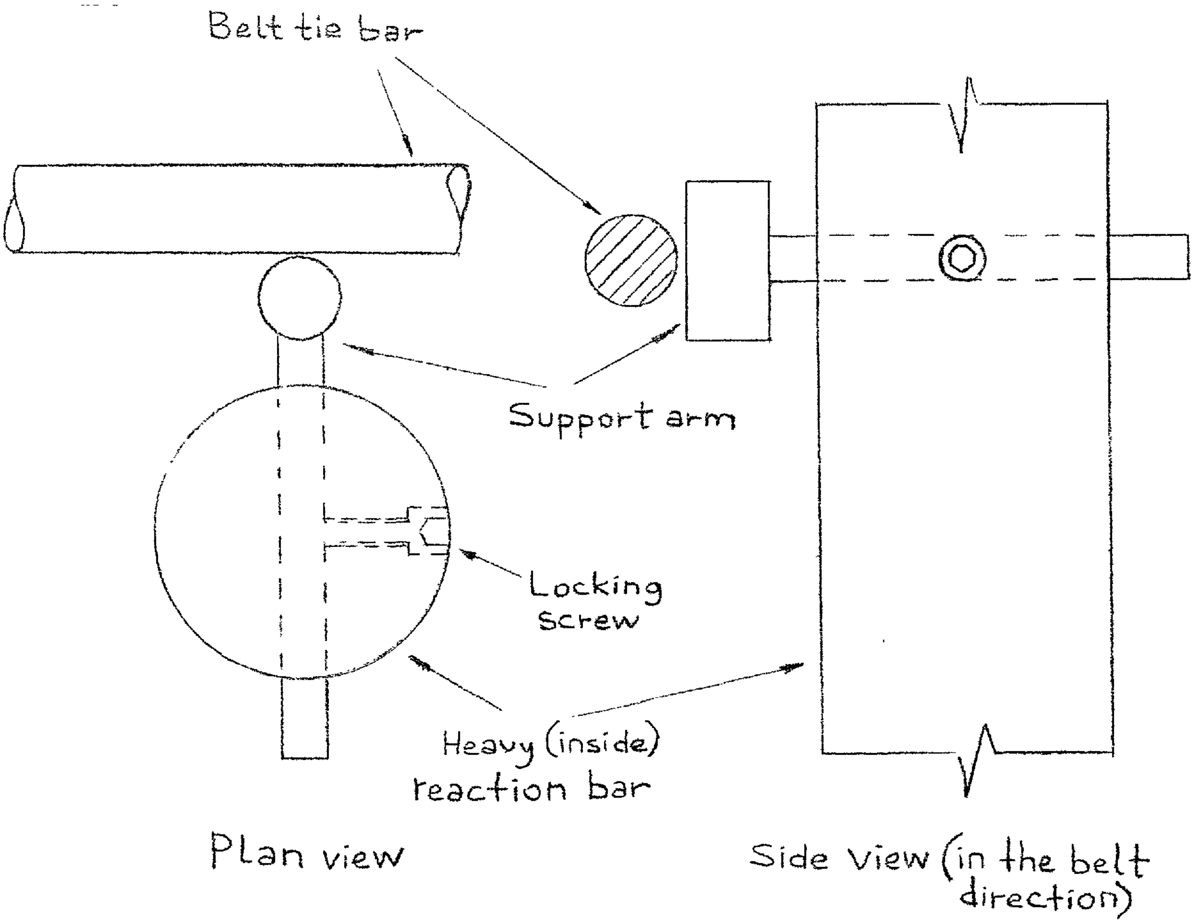


The inner brass plate was trimmed outside this diameter A' as an alternative sealing method. (This dashed circle is not related to the figure)

Back view of a reinforced bag

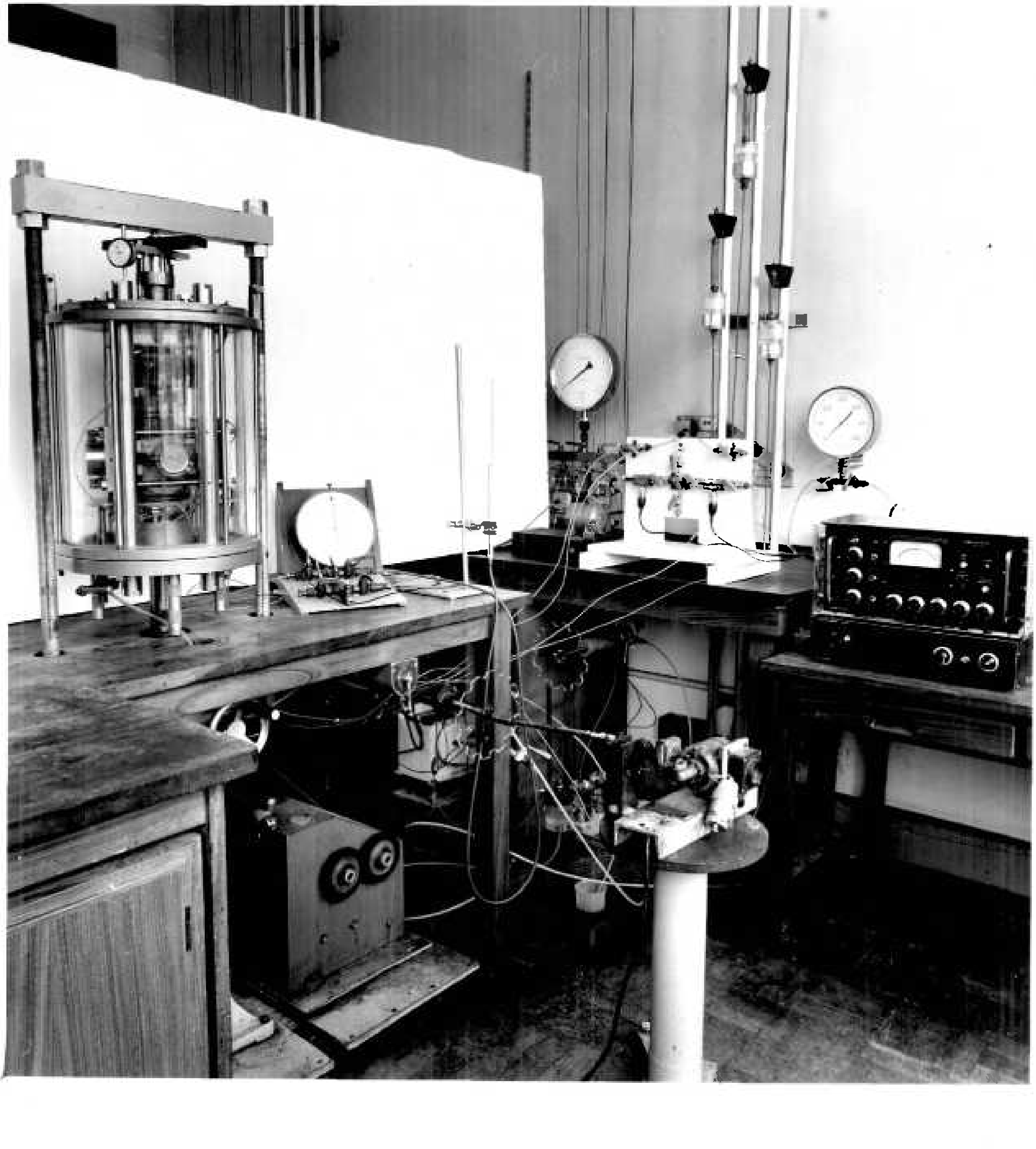
Full scale

Fig. 3.9



Full scale

Fig. 3.10



A general view of the instrumentation

Fig. 3.11

CHAPTER 4

SAMPLE PREPARATION, TEST PROCEDURE, AND MATERIALS TESTED.

4.1. Sample Preparation.

4.1.1. Preparation of the Mould.

The required quantity of sand to be used in each test was estimated and placed in small polythene bags. The bulk material was prepared by washing without losing the fines to clean dirt etc. if there was any and drying and separating homogeneously by a divider. Sufficient amount of sand was always kept in the oven in porcelen dishes. A dish of sand was taken out of the oven on the day of a test and weighed immediately, then immensed in de-aired water.

The rubber membrane to be used in the test was tested for leakage before the test by a set-up constructed for this purpose. All tests in this study were performed on cuboidal samples. Triaxial compression and extension samples were 85 x 78 x 58mm, short extension samples were 78 x 78 x 49mm and generalised test samples were 90 x 85 x 53 mm in dimension. Two 100mm O-rings were placed around the bottom platten. The components of the brass split mould for the sample were placed on the bench and assembled with special care to the exact dimensions depending on the test. The sample mould was originally constructed by Green (1969). A similar mould was made by the Writer for short triaxial extension samples(Ham R.s).It was composed of four brass plates attached together by two tie rods. Each plate had small diameter copper tubing outlet where plastic tubing from the vacuum pump was connected. This system sucked the rubber into the corners by vacuum. Small brass and cardboard pieces were used along four-vertical-edges and inside the mould plates corresponding to belt faces in generalised tests, (original mould). These were required to provide for the exact gaps and dimensions for a specific test at a particular density. See locations x and y in Green's (1971a) figure 3.9, figure 4.1.

Use of flexible plattens made it necessary to employ higher samples, so to raise the mould height, perspex strips were stuck on the rims of the brass mould plates with araldite with the inner surfaces flush with the brass plates. Special care was used to ensure that the mould produced right prisms, by using a dummy prism insert and checking for squareness after assembly.

After placing the rubber membrane and the O-rings around the bottom platten, the sheath was folded down the sides of the bottom platten. Then a"free end" as described in Rowe and Barden (1964) or Bishop and Green (1965) was prepared on the bottom platten. A hole of 12mm in diameter was cut at the centre of the lubricated rubber sheets to accomodate the porous stone. The top platten was also prepared in the same way separately on the bench. The excess silicon grease was taken out using a straightedge by applying gentle pressure on it. The top platten was then placed on the bottom platten with four lubricated sheets in between, and the gap between them measured with a vernier. This measurement was used in the calculation of sample dimensions. The top platten was placed back on to the bench again, and the split mould, which was already assembled on the bench and connected to a vacuum pump through four outlets at the sides was placed and fixed on to the bottom platten by clamping screws. Care was taken to prevent creases especially at the corners. Then the membrane was stretched over the rim and side walls. It proved very useful to employ paper tissue between the rubber sheath and the mould otherwise it stuck to the walls of the mould and made it difficult to work with the membrane. It was noticed that water in the mould helped to supress any small creases on to the walls. Great attention was paid not to crease the free end on the bottom platten while working with the sample sheath.

It was found extremely difficult to form sharp corners especially at the initial stages of the test programme Reades (1972) faced exactly the same problem. This was mainly due to the use of cylindrical sample sheath for prismatical shaped samples. Circumferential difference in length between the sample sheath and the mould directly affected forming good, sharp corners. Since slightly different dimensions were required for ISC tests at different b values and the same size of cylindrical membrane was used, a good fit could not be obtained in every test. In the case where the perimeter of the sample sheath was a few milimetres larger than the (inner) perimeter of the mould, a better fit could be obtained. It was thought that poor corners would not affect the behaviour of samples. Allowance was made for this during calculation of initial porosities, the allowance was not significant.

As it was shown in Section 3.2.7.2. top and bottom plattens were connected to a burette through a system of valves. A flow of water from the burette to the bottom platten was provided to eliminate any air bubbles on the line, and some de-aired water was placed into the mould and the porous stone was inserted into its recess under water. Porous stones were always kept under water after being saturated by boiling.

4.1.2 Deposition of Sand.

The sample was then built in different ways depending on the porosity required. Tests series in this study consisted of dense and loose samples. Dense samples can be formed by tamping or vibrating generally. The former was selected, because first, Reades (1972) concluded that vibrated samples were more anisotropic structurally compared to samples prepared by tamping, and secondly a comparison of the tests on Ham River sand by Reades was intended, and any change in the method of preparation would introduce another variable in the correlations. Cornforth (1961) prepared his dense samples by vibrating the mould initially, but later on he concluded that the samples were rather inhomogeneous.

The mould was filled with water up to about three quarters of the height, and a small spoon was used to put the sand (already saturated under water in the dish) into the mould in approximately 30 gm quantities. Layers of 2.3 cm were tamped at a time with a metal tamper of roughly 1 x 2 cm base dimensions. An effort was made to cover the cross sectional area as evenly as possible for each layer to achieve uniformity on horizontal planes. The same number of tamps were applied to each layer. It can be argued that in this method lower layers will get more tamping energy and will presumably be denser compared to upper layers. Hafiz (1950) suggested an increase in the number of tamps on upper layers. On the other hand the depth below each layer which is being tamped, that is influenced by tamping is not known. If the effect of tamping is to die out after a relatively shallow depth, this effect may not be very significant. Another point is the uniformity of manual tamping load at each tamp. Effort was spent in applying uniform tamping load.

Loose samples were prepared by depositing small quantities of sand (2-10gm) under water. The mould was nearly filled with water and sand was carefully spooned into the mould. Among other methods "dry raining" was considered but it was not practical for the present set-up. Pouring sand in dry may be another practical way, Kirkpatrick and Younger (1971). As it will be mentioned in Chapters 7 and 10 the sample preparation and deposition procedure is significant, and must be clearly indicated in every study.

Initially larger increments were possible due to long falling lengths of the particles before settling at the bottom i.e. the effect of pouring at the surface will not be felt at the bottom, whereas when the level of the loose deposit approached the top of the mould the spooning could directly affect the placement porosity, therefore extreme care was taken in the final quarter of the height. The water in the spoon was a potential disturbing factor if proper attention was not paid.

Besides, the amount of sand placed each time must be decreased in the last stage, otherwise denser lumps of sand will settle down over a short distance. While the time to build a dense sample was about 30-40 minutes (excluding everything; only spooning) loose samples took 1.5-2.0 hours. In the latter case water in the mould had to be emptied 1-2 cm in level several times because continious supply of water and sand with the spoon causes it to overflow. Sucking with a piece of plastic tubing proved efficient for the purpose. For small adjustments of the water level bottom drainage was sufficient.

The axial platten which was already prepared together with the bottom platten was deaired, and the porous stone was inserted. Then it was carefully lowered on top of the sample with the water level slightly below the surface of the sand so that it did not cause overflow due to liquefaction in loose samples. The procedure was pretty straightforward in the case of dense samples. Then the sample sheath which was stretched over the rim of the mould was rolled over the sides of the top platten simultaneously from all sides. This step necessiated another person for loose samples, because quite significant disturbance could result. Formation of sand pockets and any creases around the periphery of the top platten were avoided, but air bubbles always formed initially. They were completely driven out by using a wash bottle whose point was inserted inside the membrane. Finally, with the aid of a two-part O-ring expander two O-rings were placed around the top cap and it was made horizontal using a spirit level.

Excess water in the sample was drained into the burette by placing it about 20-30cm below the mid-height of the sample. After an equilibrium was reached valves were closed, and the burette was lowered 1-1.3 m below the level of the sample to apply suction. After taking an initial reading and starting the chronometer the valves were opened. Very frequent readings were taken initially, then preferably at regular time intervals. Water level in the burette gradually moved upwards until it reached an equilibrium in the case of a sample that was free from any kind of leak. However, if there was a leak the water level would move continiously upwards without equilising at a certain value. This check lasted from ten to twenty minutes in very dense samples to 2-3 hours in loose samples. Any water pockets were checked. Creeping volume changes in loose samples could easily be mixed with the possibility of a tiny leak. Loose volcanic sand samples crept appreciably due to high compressibility of the material.

The pump was stopped after equilibrium was reached, and the mould was dismantled without significantly disturbing the specimen. The top cap was again checked with the spirit level. Distmantling was always associated with some volume change, especially in loose samples. Any trapped water pockets at the transition sections or loss of rigid support on the faces could be reasons for it, and this volume change occurred in a relatively short period of time. At this stage the sample was ready for a test and was standing under a small suction. If everything went normal it took 4-5 hours to attain this stage. The remaining quantity of sand in the dish was placed back into the oven to determine the dry weight of the sand used.

As will be pointed out in later chapters, porosity of loose samples varies depending on the experimenter. Since part of the testing program in this study was aimed at a comparison with the results of Reades (1972) and Green (1969), porosities had to be normalised. Reades prepared 45. - 45.5 percent porous loose samples while Green and the Writer prepared 44. - 44.5 percent. This variation may be a function of the amount of sand spooned into the mould each time.

Since no cylindrical samples were tested, the difference in volume change behaviour between cylindrical and cuboidal samples due to probable trapped water at the corners in the latter is not relevant in the present series of tests.

The more slender the dimensions of the rectangular cross-section of a sample, the harder it is to form a sharp-cornered sample. Since short extension samples had a square cross-sectional area they were the easiest to form, the most difficult being the tests at high b values.

Another point worth mentioning is that there was probably some disturbance to the upper portion of loose samples due to the placement of the heavy top cap. Unfortunately no study was conducted to determine the variation of porosities within the sample.

4.2. Test Procedure

4.2.1. Introduction.

After the sample was constructed - and standing under suction - its dimensions were measured using a vernier calipher without disturbing it. Then, the procedures for triaxial compression, extension tests and various series of generalised tests were somewhat different although there were many similarities. They are summarised in order of complexity below, and are broadly similar to those of Green (1969) and Reades (1972) with the exception of the flexible platten series and certain special series with rigid plattens. Many descriptions and comments of minor importance were excluded for the sake of being concise.

4.2.2. Triaxial Compression Tests.

These were the most straightforward tests. After measuring the sample the four steel tie bars were placed in the cell base, and the cell top was lifted by a hand-operated winch and positioned on the rods by slowly lowering it. Domed nuts on the cell top and under the cell base were tightened. The perspex cell body was lowered on the cell base and tightened through six tie rods (around the perspex chamber) which pass the cell base through six holes. The nuts were tightened evenly to seal the large O-ring properly. De-aired water was then admitted to the cell which took about an hour to fill. During this period the two clamp rings were tightened through the studs on the cell top to seal the O-ring against sides of the cell top, and the triaxial frame was set up around the cell. The volume changes of the sample which was under suction were checked and recorded at each major stage. The axial proving ring anvil was lowered to a point just clear of the top platten.

The cell filling was completed, and then it was isolated, and a pressure (10-20kN/M^2) was applied to the cell water. After equilibration the burette which was on the floor to apply suction to the sample was raised until the water level in the burette was at the same level as the sample mid-height. The cell pressure was raised in increments until the required pressure was attained, and at each stage volume changes were recorded for the equilibrium condition which was reached 5-15 minutes after pressure changes were made, thus depended on the density and the material. The axial dial gauge was installed. Each time the water level in the burette was repositioned.

After allowing sometime at the consolidation pressure to check for leaks, the axial proving ring anvil was lowered down to make slight contact with the top platten. If any leak was observed the test was discarded. A check was made whether there was a sufficient distance of travel between the horizontal guide plate attached to the axial piston and the clamping nuts on the guide screws. (see figure 3.3). If the clamping nuts are not taken down damage would occur. The pedestal of the loading machine was raised until axial loading shaft made a contact with the crosshead of the loading frame through a steel ball. The transducer meter was closely watched not to load the sample significantly. The self-compensating mercury control was already on due to the application of consolidation pressure.

If a budenberg tester was also required, the necessary connection would be prepared at this stage. Initial readings of axial proving ring, burette and axial dial were taken, and a strain rate was selected with the (Kopp) variator. Then the shearing was started and readings of time, axial dial, axial load, burette were taken with continous adjustment of the water level in the burette.

The peak of the stress-strain relation could be readily observed, and the tests were stopped after peak since there was no intended study of residual angles. Axial piston was separated from the crosshead of loading frame, and axial proving anvil was also separated from the top platten, cell pressure still being on. The burette was lowered on to the floor again, and cell pressure was disconnected from the source and released to atmosphere. At both stages, volume changes were recorded. An alternative was to close the drainage system and then release the all pressure. The difference was that swelling under isotropic pressure was prevented in the latter case.

Both methods do not give the sample dimensions at failure because releasing the cell pressure with drainage lines open would result in some amount of water intake. In the latter procedure the sample being unsupported would slump to a certain extent. The cell was emptied and dismantled in the reverse order. Sample dimensions were measured.

The sample material was placed in a dish, dried and weighed to compare with the initial weight to check the amount of dry sand used in the sample. This was expected to agree closely with the value obtained initially. The membrane and the rubber sheets that were used for construction of the free ends were washed, and the sample sheath was rechecked for any leaks with the set up constructed for the purpose, after which it was dried, powdered and stored for another test if it was free from any leak. But no membrane was used more than a few times.

4.2.3. Triaxial Extension Tests.

The cell was constructed around the sample as in a triaxial compression test. One different feature was that an extension boss was screwed into the axial proving anvil. The boss was lowered into the recess on top of the axial platten without disturbing the sample making use of the clamp screws. Any disturbance was monitored with the C61 meter. The boss and the recess were not aligned usually because it was not possible to prepare a sample which had an axial symmetry axis exactly coinciding with the symmetry axis of the apparatus. Top platten was slightly moved in the horizontal plane to fit them in together. The holes through the boss and the circular brass back plate of the top platten were aligned, an extension pin was inserted. The loading frame was constructed, and an extension collar around the axial piston was fixed to the machine cross-head by two strong bolts, and it was used to suspend the axial loading piston around a circlip at its end. Loading pedestal was also screwed to the cell base. Therefore the idea was to fix the axial loading piston to the machine cross-head and then to pull the cell downwards.

Cell filling was started after provision was made to obtain tensile loads with the axial proving ring. When the sample was submerged under water, settlements occured, and these were either compensated for by a little loose contact of the extension pin in its hole or by an adjustment of the clamp nuts for the axial load cell.

The consolidation scheme was similar to compression tests but the consolidation pressure was high in some of the extension tests. Due to contraction of the sample small tensile loads were applied to the proving ring. These were released after each increment of pressure either by slightly lowering the proving ring, or by raising the cell with manual control. In extension tests with low consolidation pressure and increasing mean stress level this problem was not significant. It was easier to locate the peak in constant cell pressure tests than increasing mean stress level tests. That's why the latter group of tests could not be stopped near peak which was desirable to allow proper neck area calculations. (Appendix 2).

Readings of cell pressure (transducer), axial proving ring, burette axial dial were taken during shearing. The large klinger valve under the base of the cell helped in speedy emptying. Extension connections to the machine crosshead were released, and the cell body was lifted off using the winch. Sample dimensions were measured with a smaller vernier calipher before the four large tie bars and the cell top were dismantled. Because taking the shear pin out resulted in the immediate collapse of the sample.

Calibrations of the axial proving ring were carried out at the end of each series of tests. Since changes from compressive to tensile loads (or vice versa) affected its behaviour, frequent changes of the load cycle were avoided. See Appendix 3 for calibrations of the axial proving ring both in compression and extension.

4.2.4. Generalised Tests.

The test procedure for generalised tests were more involved compared to the previous groups of tests as would be expected. The type of the three-dimensional test and the intended value of "b" directly influenced the details like the selection of the diameter of the sample sheath to be used, exact sample dimensions, height of the rigid platten to be used, selection of the right gear for the belt motor, initial location of the ram etc. The test procedure for ISC tests on volcanic sand and to some extent ISC(F) series on Ham River Sand were similar to the procedure followed by Green (1969).

The procedure was the same as for the triaxial compression tests up to the stage at which the sample was ready on the loading pedestal with the axial proving ring anvil just clear of the top platten. Preliminary preparations were done for the belt frame in the mean time.

In section 3.2.7.2.3 and Appendix 1 the manufacturing process of the reinforced rubber bags was explained. They were cut circular at the centre and sealed against the brass frame with two O-rings and an inside plate. The eight clamp screws were tightened evenly pulling the inner brass plate against the O-rings and the brass base clamping the membrane between them.

Each platten was filled through one of the inlets at the side with deaired water, and at the same time the other drainage line which led to the bottom of the conical groove was opened. The platten was placed with the membrane face against the bench. Air bubbles left the bag through the top of the conical groove, this process was continued until all the air bubbles were cleared from the platten, and after a predetermined quantity of water was forced into the bag, the drainage lines were both closed. This quantity was just sufficient for the bag to remain plain during the test, and was determined by tests on dummy samples. The other platten was prepared exactly in the same way. Then both plattens were mounted on the belt frame which stood vertical on the bench and a steel dummy sample with the same dimensions of an ISC sample was loaded.

During loading only a minute volume change occurred in the belt plattens that required to pressurise both the bags and the pressure transducer. The ram was driven manually to pressurise the bags. The pressure was increased in stages, and at each stage checks for leaks were made. Pressure transducers were sensitive leak detectors of any leakage.

The procedure was repeated until both plattens were checked as leak-proof. This took a long time, and was usually done one or two days before an intented test. If the time spent in manufacturing the reinforced bags was also taken into account, the time to conduct a test was considerable. With the tests lost due to several reasons (slip of the belt frame around the sample, puncturing the sample sheath during preparation etc.) the procedure was extremely time consuming.

Sticking the back of the bag to the base and sides of the brass frame (casing) was a technique adapted later, and as explained previously, stretching associated with leakage was observed in high b value tests. The reason for this was the high differential pressure which caused problems with the small unreinforced part of the bag near the mid height of the plattens. The preparation technique then became more cubbersome because if a leak occured during loading on dummy samples the glue had to be cleaned off before another attempt was made. When a pair of plattens were prepared ready, they were fixed to the ram and the belt proving ring anvil using the annular clamp screws of the plattens after a final loading to secure a proper setting. The belt dial gauge and its support arm were then installed. The treaded annular pieces on the belt tie bars were used to mark the location of the ram crosshead. They were adjusted together with the nuts behind the crosshead to a distance measured with a vernier caliper to fix the distance between the two crossheads which had to be parallel to each other and vertical to the horizontal plane. Since the maximum travel of the ram was 19 mm, the ram crosshead had to be positioned with care in tests with high b value.

Initially (first four tests) friction was reduced by the use of a lubricated single rubber sheet placed on the bag face which was lubricated. The sample sheath on the belt faceswas also lubricated. Later the finish on surface of the reinforced rubber bags was improved, and lubrication of the flexible membrane and the belt faces of the sample was considered sufficiently good to ensure that frictional losses were negligible.

Construction of the belt frame around the sample was the main difference from the procedure in triaxial tests.

First, the proving ring crosshead complete with the proving ring, flexible platten and tie bars was passed around the sample and supported by two screws used for the adjustment of the height of the belt frame. These adjustment screws were connected to belt support plates underneath the crossheads on each side, see figure 3.3. During this most delicate stage of the procedure extreme care was taken not to touch the sample. While holding the major part of the belt frame with one hand, the ram crosshead (with the other flexible platten mounted on it) was pushed through the four holes at its corners against the tie bars, and nuts were tightened to fix it in position. The vertical adjustment screws were adjusted at four points to obtain a level belt frame position. A spirit level was used in the two horizontal directions for this purpose. The ram was manually operated, and any extra distance between the sample faces and the flexible plattens was eliminated at this point they were just clear from the faces.

It was also important that the height of the flexible plattens had to centre the belt face area leaving roughly equal gaps at top and bottom relative to the both axial rigid plattens. The size of gaps allowed depended on the state of the intermediate principal stress, porosity and the material. Sample height was specified to obtain the right size of gaps for the current test. In the case of rigid plattens, the height of the loading plattens could be varied whereas the height of the flexible plattens were fixed.

Since axial and especially belt plattens did not cover the whole sample faces in the tests, ratios were employed to indicate the percentages of coverage in these directions. "Axial platten contact ratio" and "belt platten contact ratio" are explained in figure 4.2.

The belt was then suspended with brass wires from the cell top with the system described before. The wires were tensioned to clear the belt from the supporting screws which were then retracted by a few centimetres to permit free movement of the blet. It was then checked that the symmetry axis of the belt frame was perpendicular to the belt faces of the sample. Before lowering the cell body and assembling the cell and the loading frame, a general check was made to ensure everything was alright. Due to many, small but vital details a flow chart was closely followed item by item. The cell body and the frame of the machine were set up in exactly the same way as in a triaxial compression test. Filling the cell and consolidation stage were also similar.

Preloading in the belt direction was essential before shearing commenced otherwise large belt dial gauge readings would be recorded virtually without any loading in the belt direction. It was actually more important for rigid plattens because any deviation of the rigid plattens from being vertical would impose non-uniform stresses.

Flexible plattens were capable of adapting themselves to the surfaces of the sample in these directions should there be any slight irregularity. Observation of volume changes during preloading was a good indication of having a proper contact especially in the case of rigid plattens. If a big volume change was observed for small loading increments it meant plattens were overstressing the sample face locally somewhere. In such a case the test would not be started and the exact cause would be found, and if it could not be found, the belt was dismantled, and dimensions were checked thoroughly again. The amount of volume change observed normally varied between .10 - .20cc depending on the porosity of the sample. Preloading by about 80-120N was usual and slightly more for dense samples.

Axial contact of the top platten with the proving ring was checked again, and initial readings of axial and cell pressure (if varying) were taken. The gear box-motor system was already connected to the screw-piston mechanism during the consolidation stage (Its gear ratio was set before depending on the b value of an intended axial strain rate). Strain rates in axial and belt directions were so chosen that there was sufficient time to watch the sample carefully other than taking the data. Any significant feature observed was noted down. Consolidation period usually lasted 1-2 hours, and so did the shearing period. Total time required for a test excepting the preparation of the plattens was 12-15 hours on average if everything worked properly. So, there were two alternatives; either to perform the test in two working days, say, one day for preparation and consolidation the other for shearing or to do it in a very long day. The former was adopted in several of the tests by Green (1969) whereas Reades (1972) and the Writer preferred the latter which was followed for most tests. Specific loading precedure for each group of tests was different and it was summarised in section 3.4.

After the peak had been reached, motors were stopped in axial and belt directions, and the cell was dismantled in the same way as in triaxial compression tests. It proved helpful to decrease the oil pressure in the hydraulic ram while cell pressure was still on to retract the ram otherwise it would be difficult to do it manually later on. The belt frame was rested on support screws after the cell body was separated from the cell base, and then the suspension wires were released. Nuts on the ram crosshead were taken out, and the ram crosshead was separated from the belt, then the other part was removed without touching the sample. Dimensions and failure plane inclination (if any) were measured, the sheath was washed and sand from the sample was put into the oven.

During the course of describing the procedure for flexible plattens that adapted for rigid plattens was mentioned when relevant. The procedure in their case was entirely similar to flexible plattens, with an exception that they were much easier to use than flexible plattens. Their height could be changed with respect to a type of test. "Free ends" were used on them.

4.3. Materials Tested.

4.3.1. Ham River Sand.

This material was supplied by the BRS on several occasions to the laboratory. Green prepared two different batches, first one was abandoned after a series of tests due to large proportion of fines so that his second batch was washed. Reades' supply was also unused and for exactly the same reason he washed a large quantity of sand including Green's used samples and used this throughout his programme.

Ham river sand used in this study was that final batch (batch 2) by Reades (1972). Since a considerable time elapsed between him leaving and the Writer's initiation of his programme and to make sure that the material was not mixed with other used batches of the sand in the laboratory, a sufficient quantity of the sand was washed, without loosing the fines, dried and filled into small plastic bags in small quantities, (a sufficient amount for a sample) A divider was used for the purpose.

Grain size distribution for this material was almost the same as Reades' batch 2. It was probable that the sand treated by the Writer was infact directly a part of batch 2. This was advantageous because during the comparison of results by both researchers the material factor would not exist. The gradation curve for the material used is plotted in figure 4.3, together with those of other batches of Ham River Sand as reported by Green (1969) and Reades (1972).

It was observed that the specific gravity measurements did not differ significantly even among the previous batches. Since the material used in this study was almost identical to batch 2 by Reades it was decided not to carry out a group of specific gravity tests but to adapt the final concluded value of 2.677 by Reades (1972). Maximum dry porosity by rapid tilt test, Kolbuszewski (1948) was around 48 percent as reported by aforementioned researchers. Since stresses were low enough, no significant particle breakage could be expected. Green (1969) presented photographs of grains before and after a test which showed some increasing portion of fines. Cornforth (1961) used the material (brasted sand) repetatively in several tests but concluded that there was no change in the behaviour of the material in the tests. The material was used at most twice before being prepared for this study and most probably only once.

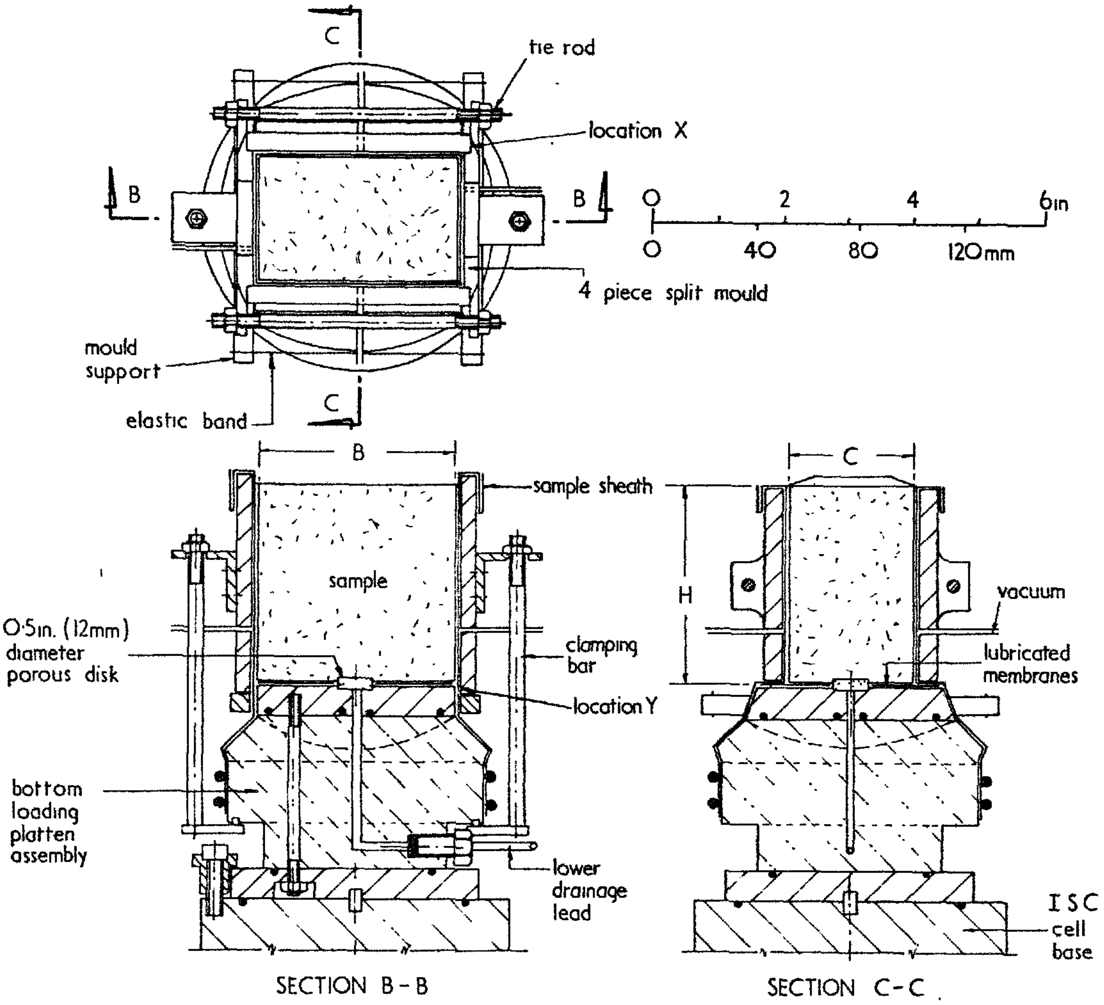
4.3.2. Volcanic Sand.

This material was supplied in a limited quantity to the laboratory from Iceland. It was just enough for the intended serious of triaxial and generalised tests. Only one or two triaxial extension and compression tests had to re-use material at the end of the test programme. The material was like ash, black in colour contained coarse porous particles and very fine dust. The particle size distribution curve is seen in figure 4.3. About twenty percent is finer than 200 mesh with the majority lying between 100 and 200 meshes.

Four specific gravity determinations gave values of 2.795, 2.798, 2.785 and 2.807. A value 2.79 was adapted in the calculations.

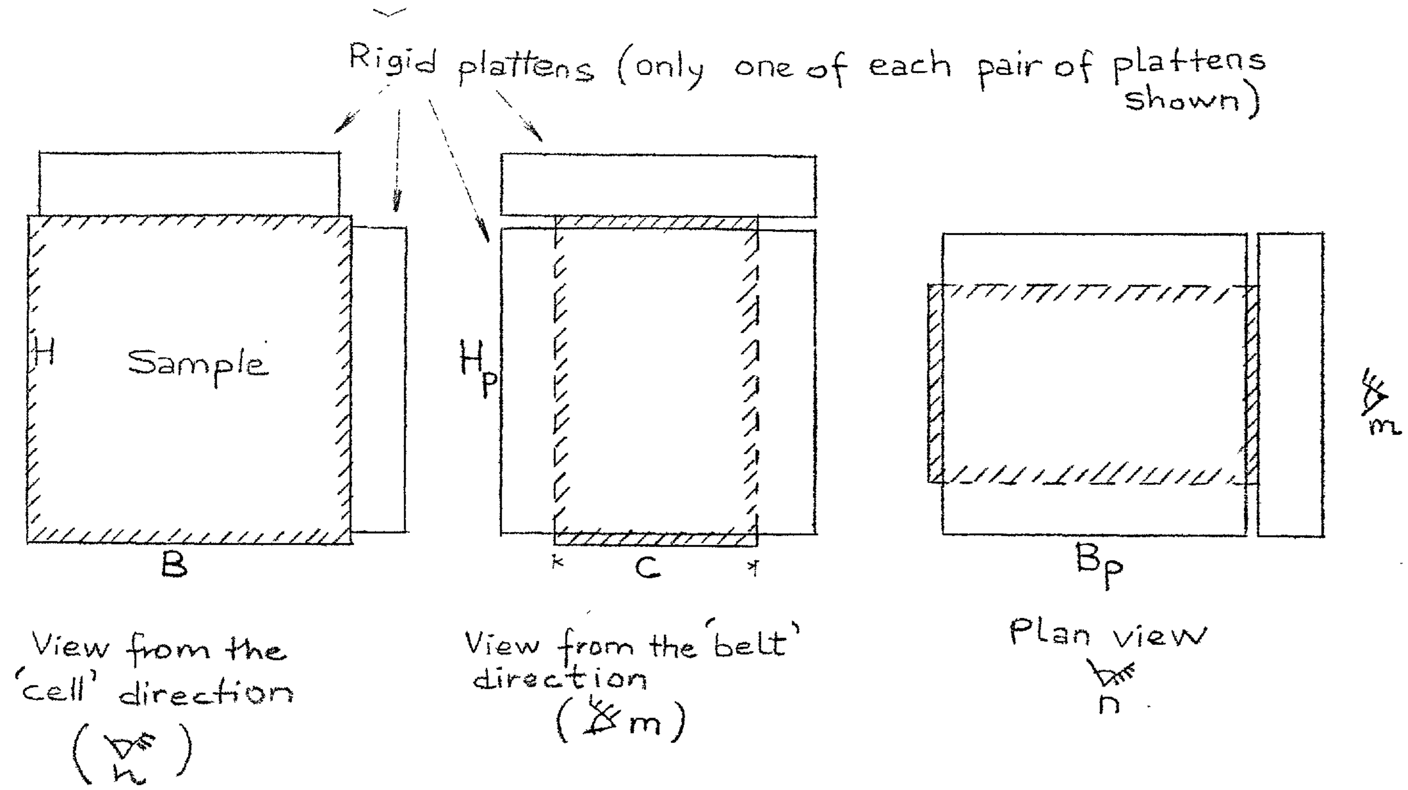
The Writer expects difficulty when performing undrained tests with such a material due to high air absorbtion capacity of the material which was evident in de-airing the specific gravity bottles.

An interesting comparison of the grain structures of the two sands can be seen in figure 4.4. Ham River sand consists of subrounded particles where volcanic sand particles were angular or needle-like.



Sample Mould
After Green (1971a)

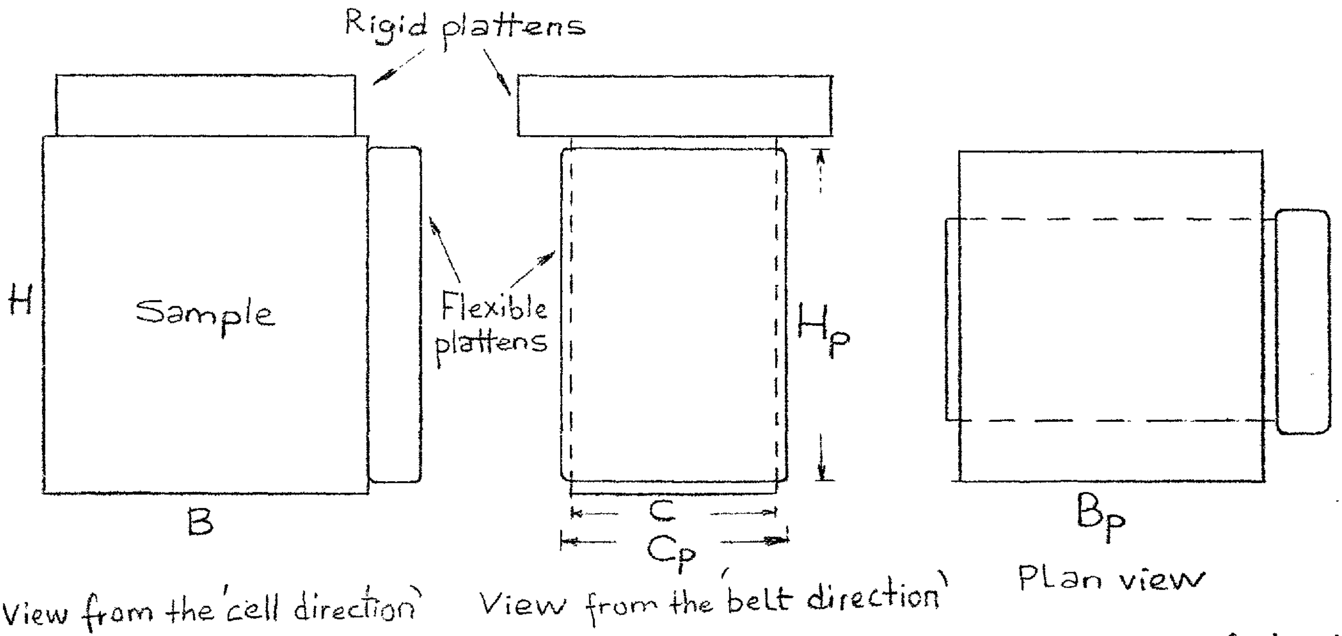
Fig. 4.1



H, B, C; sample dimensions, p stands for 'platten'.

$$\text{Axial platten contact ratio} = \frac{\text{Loading area}}{\text{Sample area}} = \frac{B_p \cdot C}{B \cdot C} = \frac{B_p}{B}$$

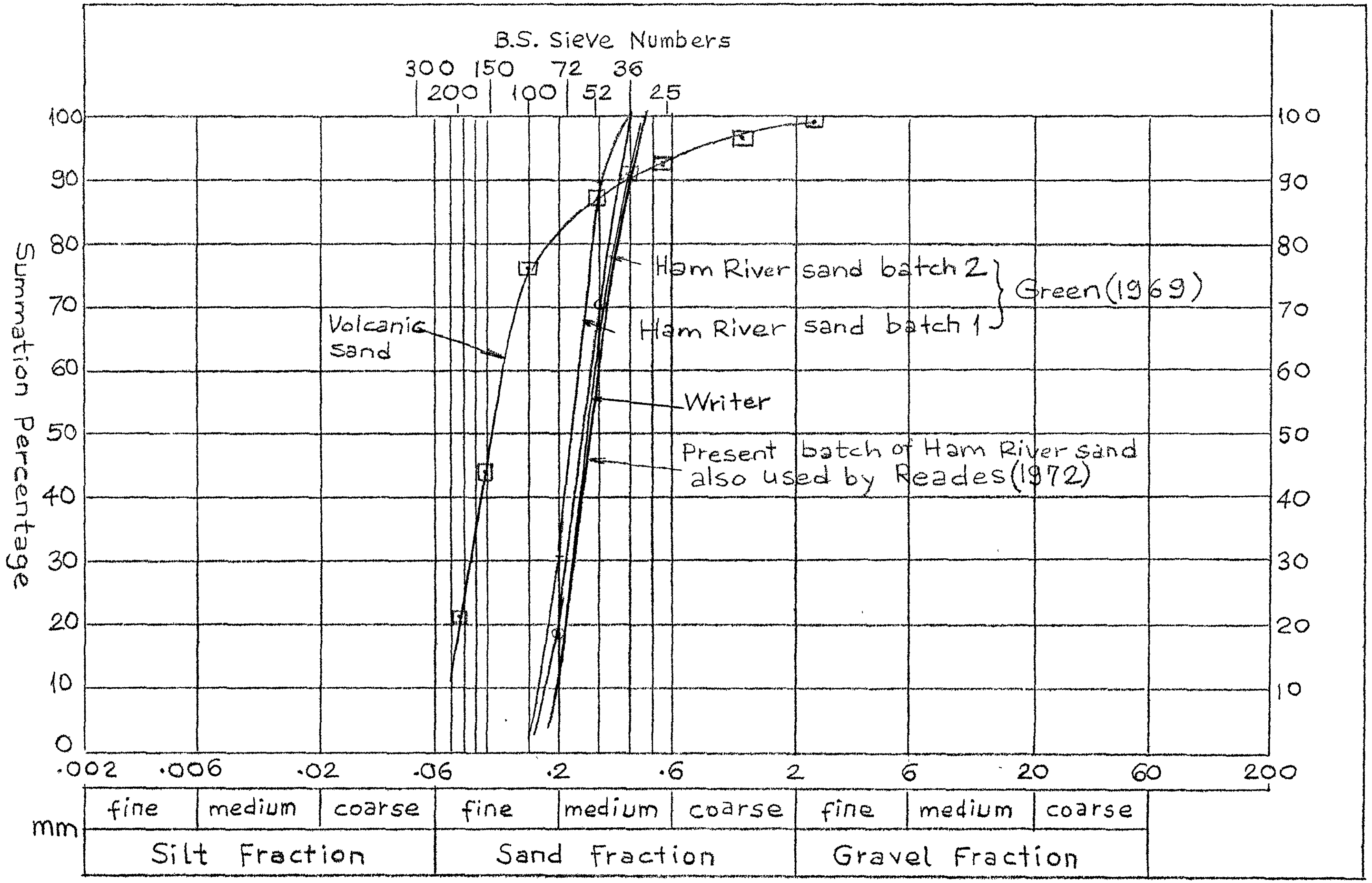
$$\text{Belt platten contact ratio} = \frac{\text{Loading area(belt)}}{\text{Sample area(belt)}} = \frac{H_p \cdot C}{H \cdot C} = \frac{H_p}{H}$$



In the case of flexible platten tests there are two types of belt contact ratios: Belt platten contact ratio (in the axial direction) $= \frac{H_p}{H}$

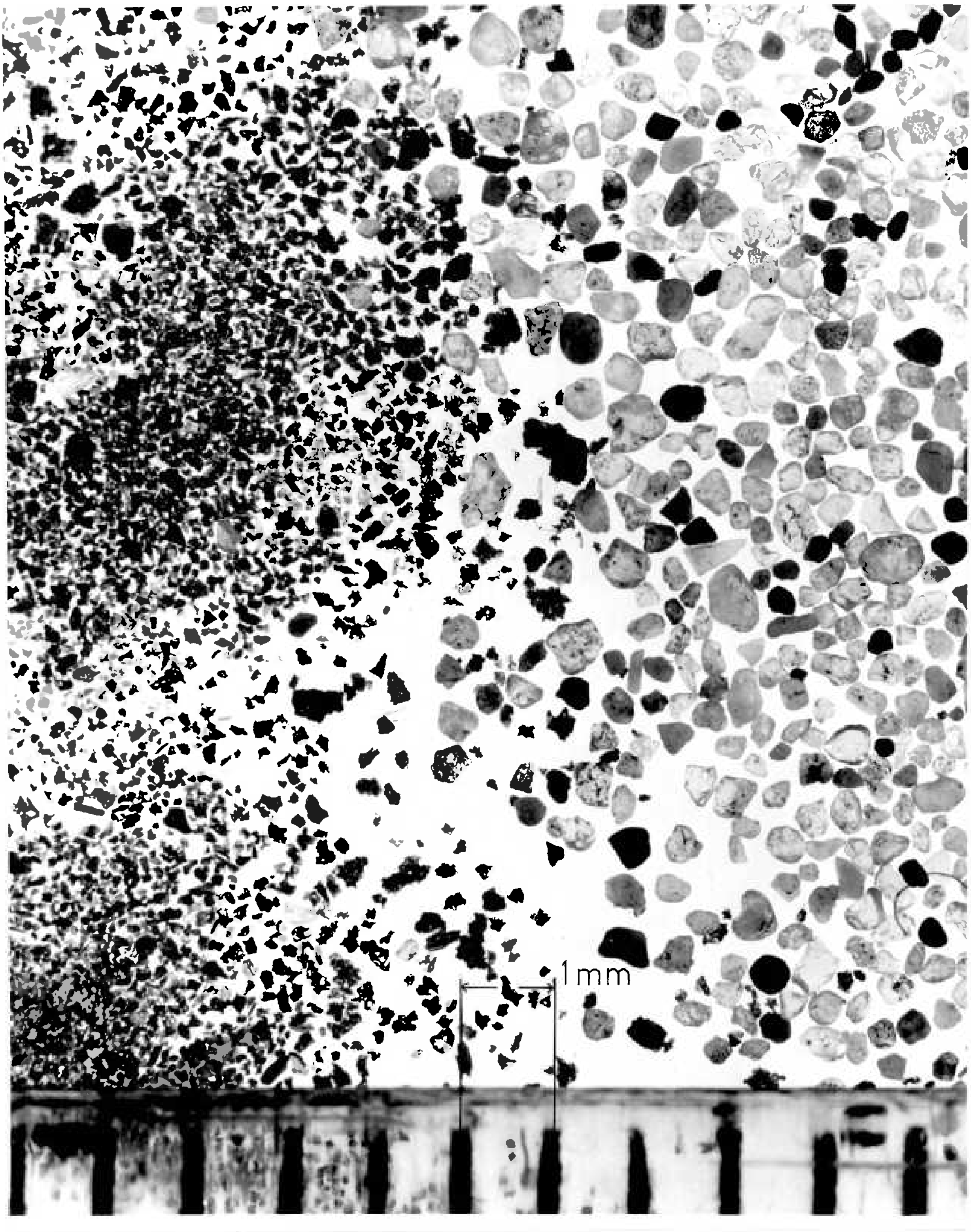
Belt platten contact ratio (in the cell direction) $= \frac{C_p}{C}$

Fig. 4.2



Particle size distribution

Fig.4.3



Comparison of size and shape of the particles in volcanic sand and Ham River sand (on the right)

CHAPTER 5

RESULTS AND DISCUSSION OF GENERALISED TESTS ON HAM RIVER SAND.

5.1. Introduction.

As it has been already explained in Chapter 1, generalised tests with flexible plattens in the intermediate principal stress direction have been carried out. Major and minor principal stresses have been applied by rigid plattens and cell fluid pressure respectively in the same way as tests by Green (1969) and Reades (1972). The first series of flexible platten tests are on dense samples. The main purpose of these is to compare the failure characteristics with those of samples tested with rigid plattens. This series covers most of the intermediate stress range except b=0 -0.15 and .85 - 1.00. The second series of tests are on loose samples and they were mainly performed to study the effect of inward moving plattens at high intermediate stresses and to throw light on the controversial finding of high strengths using rigid plattens at high intermediate stresses relative to conventional extension tests. All of the tests are above b = .70. In this section only test results are presented, for testing techniques see Chapter 4.

5.2. Generalised Tests on Dense Samples with Flexible Plattens and Their Comparison with the Tests Using Rigid Plattens.

This series includes tests ISC (F) 1-9, 14, 16 and 20. Failure characteristics are given in table 5.1 and plotted in figures 5.1, 5.2, 5.3, 5.4 against b together with all other dense tests by Green (1969) and Reades (1972) performed in the ISC apparatus using rigid plattens. The curve proposed by Reades and Green (1974) based on Green's (1969) results has also been plotted. Although measured peak strengths from flexible platten tests in the plain strain region seem to be approximately a degree higher than rigid platten tests, the Writer's dense samples are little denser than others so that the corrected strengths are higher by less than a degree than all other tests in the region. For corrections see Appendix 2. The four tests ISC (F) 1-4 strictly are not plain strain tests, because the belt motor was not driven, and lubricated membranes, the belt itself (hydraulic jack mechanism) and more important the flexible plattens themselves were compressed. Therefore the b values are between .16 -.20. Plain strain state is realised at about b= .30. Vertical belt contact ratios are greater than one - see section 4.2 - and axial platten contact ratios are lower than one, except ISC (F)1, so that the plattens cannot touch each other. Reades (1972) concluded that the effect of belt platten contact ratio on peak strength was negligible if major principal stress was applied by axial plattens,and the platten contact ratio in the direction of application of major principal stress influences the peak strength characteristics only. In his figures 9.20 and 9.21 he plots platten contact ratios in the major principal stress direction versus peak strength, major principal strain and the value of b. There it is seen that reduction of platten contact ratio from 1.00 to .90 results in reduction of peak strengths by 0.4° and 1.4° for dense and loose samples respectively. Thus ISC (F) 2-4 peak strengths can actually be increased by small fractions of a degree (0.2°) - Platten contact ratios of .93 relative to average of .98 in four plain strain tests - It was also found that unlike peak strengths, intermediate stresses were affected by lower belt platten contact ratios.

Peak strengths have been corrected for the effects of sample sheath rigidity, platten friction, mean stress level and initial porosity. Other failure characteristics have also been corrected where necessary. The effects of mean stress level and initial porosity on the failure characteristics of the tests are presented in Appendix 2 in the form of graphs. These graphs will help to explain any discrepancies in the 'as measured' properties. Rate of volume change is affected by porosity changes at all porosities. Axial strains are not sensitive to porosity differences for relatively denser specimens. Volumetric strains again show a variation through the whole range, looser samples being more sensitive. Peak strengths from these four flexible tests are almost the same as all other tests by rigid platten within the b span of 0.15 - 0.26. Ten tests - six rigid platten tests (four by Green, two by Reades) and four flexible - agree within 0.8 degree, and the rates of volume change with respect to the axial strains from Green's tests are little higher than the others. A careful examination of all figures reveals that most of the properties in the test series by Green (1969) differ somewhat from the tests conducted by Reades (1972). This recognisable difference may come from several sources. Most probable ones are differences in material - although both are called Ham River sand, different batches may be in different gradations etc.- and sample preparation techniques. Since the material used in this study is the same as that of Reades' (1972), and the sample preparation techniques are similar in both test programmes - see Section 4.1 - his results are regarded as more comparable to the present series of tests. It should be pointed out that Green prepared his dense samples using vibrations whereas Reades and the Writer used tamping. Axial strains to failure of the Writers' tests are on average about one percent less than the rigid platten tests by Reades. This difference can not be explained by experimental error, since it is derived from a direct measurement by a dial, and slight variations in the calibration curves for the compression of lubricated sheets and the axial proving ring itself - see Appendix 3 - can not account for the difference, and axial strains obtained in dense samples do not change with porosity. The difference is less than half percent when compared with Green's tests. A reasonable explanation for this observational fact will be considered in a later section. Volumetric strains from flexible tests match up with the trend set up by rigid platten tests. In the very sharp-rise section- see figure

5.4 - a small change in b due to lower axial or belt platten contact ratios may in fact result in appreciable volumetric strain differences (remember that b = $\frac{\sigma_2-\sigma_3}{\sigma_1-\sigma_3}$), Volume changes of denser samples at failure are not much affected by initial porosity variations.

Remaining dense sample group consist of ISC(F) 5-9, 14, 16, 20. These cover a large part of the intermediate stress range. Since Reades concentrated more on loose samples, there are no intermediate tests by him except one with b = .72, the other four are near b = 1. The whole range is covered by Green's (1969) tests. Since Green underestimated the mean stress level correction, the test points plotted in figure 5.1 are the strengths which have been corrected on the basis of the Writer's tests and other tests, see Appendix 2. They range from .9° - 1.4°. It is seen in figure 5.1 that Green's plain strain tests are about half a degree higher than those of Reades'. This difference gradually increases to a degree till b = .85. In this interval tests with flexible plattens are higher than both. The scatter is quite large 1.3°max.),but tests with flexible plattens are recognizably above those with rigid plattens on average. The difference starts with about half a degree at b = .3, and at b = .70 it is nearly 1.5 degrees. Calibration errors cannot account for such a difference. Examination of stress-strain curves indicates that all samples have clear peaks with sharp decreases post-failure free from any undesirable restraining effects similar to the initial series ISC(F) 1 to 4.

Sample height was increased by 9-10mm in this series due to the expected inward lateral deformations to prevent any direct contact of the plattens. At failure, except ISC(F) 16 and ISC(F) 20, all samples had a belt contact ratio of at least .91. Belt contact ratios in the cell direction were all about one. In the case of belt contact ratios of one, b values for all tests would increase so that test points in ϕ' - b plot would shift to right. This is especially true for tests ISC(F) 16 and 20 in which the belt contact ratios are .87 and .86 respectively. For example, to bring them up to a ratio of .92 which is the average in other tests, would increase b from .74 to .78 and .82 to .88 respectively. It is interesting to note first in the Green's series that approximately after b = .80, strength values decrease quite sharply. Reades (1972) simply joined the plain strain values to his b = .95 - 1.0 strength data by an approximate straight line. As will be seen in following sections this feature is really worth considering from the point of mechanics of cubical tests in general and also for comparison of data derived from tests in other apparatuses. There are no dense generalised tests with flexible plattens near b = 1.0 but a careful examination of the data after b = .65 indicates decreasing strengths. (ISC(F) 16 and ISC(F) 20 have actually larger b values as explained above). If an approximate curve for ϕ' is to be drawn through the ISC(F) series of tests together with the support of the Green's data; at b = .2 it is 43.6°, then gradually increases until b = .7 -.75 to the value of 46.3°, then a drop to 45.2° at b = 1.0.

But as will be seen in following chapters this may not be regarded a final conclusion.

Failure characteristics other than ϕ' are given in figures 5.2,5.3,5.4. Rate of volume changes with respect to axial strains are little lower (approximately by 0.1) than Green's data except ISC(F) 5 and 14. But Reades' ISC 23(b=.72) is also a little lower than Green's data,and so are the cluster of data near b=.95 by Reades. Corrections have been applied for initial porosity differences. Maximum scatter in $d\varepsilon v/d\varepsilon a$ values is .2 considering all kinds of tests at any specified value of b which can be regarded as within the experimental uncertainity. Unfortunately there are no tests by Reades (1972) in the intermediate range to compare with directly. Axial strains to failure follow the previous trend of being less than the rigid platten tests. At the middle intermediate stress range both type of tests give similar values. After b=.60 it is seen that while the axial strains from flexible platten tests remain constant, those from rigid platten tests increase. With small allowances for the porosity differences volumetric strains follow a similar pattern. But tests ISC(F)16 and especially ISC(F)20 give a tendency like the sharp volumetric strain increase at very high b values would become milder especially if their low belt platten contact ratios are taken into account, figure 5.4.Writer's and Reades" tests show slightly lower volumetric strains than Green's tests. Belt strains seem to agree with rigid platten pattern of results, figure 5.5, but the Writer feels that they may be slight overestimates of the actual strains.

This is because at high intermediate stresses, the bags are balooned slightly due to very high stresses and sometimes due to a larger surface exposed to the cell water pressure, and since the conditions for each test are unique, the calibrations for the compression of bags by dummy samples can not represent the actual test conditions, and dial readings between the jack and the proving ring are in fact over registered. Although a more detailed calibration scheme with various sizes of gaps could have helped representing the actual belt deformations better, the extremely painstaking procedure of carrying out a test resulted in abandoning it.

Mean stress level doubles from triaxial compression tests to extension tests by the nature of the testing technique adapted in ISC tests i.e., a constant cell pressure (σ_3) throughout a test. Its effect on strength has been first correctly considered by Reades (1972). On the other hand, both Green (1969) and Reades (1972) presented their test results -mainly at failure - by curves indicating the change of test characteristics against b value. Except the strength points all other characteristics were plotted as measured by Reades and Green. The Writer has plotted all test characteristics taking initial porosity differences into account. Also, bearing in mind the possible influence of doubling mean stress level over the b range, the solid curve has been plotted showing the corrected $d\varepsilon v/d\varepsilon_1$ for this effect taking the mean stress level in triaxial compression test as the basis at that porosity. figure 5.2. This may be particularly important when comparing tests of different stress paths and stress levels even at the same b values. The mean stress range considered is not a very wide but is practically significant because it is of the order of magnitude usually encountered under structures.

Graphs showing volumetric strain rate and volumetric strain against mean stress level are presented in Appendix 2. It is seen that $d\varepsilon v/_{d\varepsilon a}$ vs. b and εv vs. b graphs - figures 5.2 and 5.4 - should be corrected for this effect. For higher mean stress level differences see, for example, Bishop (1966), Lee and Seed (1967).

5.3 Results of Generalised Tests Using Flexible Plattens on Loose Samples At High b Values.

5.3.1, Introduction.

At the end of his test programme, Reades attempted to investigate the effect of stress path by conducting five tests. Three of these tests (64, 65, 66) required the sample to be reoriented with respect to the normal major, intermediate and minor principal stress directions. Major, intermediate and minor stresses were usually applied by axial plattens, belt plattens - both pairs were driven inwards towards each other - and the cell pressure - constant- respectively. These three tests were performed increasing the belt stresses by driving the belt plattens (major) and

?

keeping the cell pressure constant (intermediate) and decreasing the load on the axial plattens to failure by withdrawing it upwards. He noticed that the results were significantly different from those obtained with usual series of tests. So these tests did not serve their originally intended purpose but they happened to disclose an effect in the functioning mechanism of the apparatus. He concluded that the results from the normal type of ISC tests were the correct representation of three-dimensional behaviour of Ham River sand. However, he was aware, at the same time, of the difference in behaviour between ISC tests and triaxial extension tests together with these three "stress-path" tests[x]. It was strongly felt at the time by the Writer that one of the promising approaches to resolve this inconsistency would be to use flexible-bag type-plattens at least in the belt direction or to instrument the existing rigid plattens along the belt faces of the sample in a sophisticated way. The former was adopted. The present series of tests originated in this way. Flexible plattens have been designed and used in this series which includes tests ISC(F) 10-13, 15, 17, 18, 19. They are all high intermediate stress tests with b greater than .70.

5.3.2. Results.

With the exception of ISC(F) 10 and 15 tests were stopped at failure or after failure. Failure characteristics are given in table 5.2 and plotted in figures 5.6, 5.7, 5.8,

---

[x] Full discussion about this will be given in Chapter 7.

5.9 against b together with the tests with rigid plattens by Reades (1972). It must be noted that the Writer's loose samples are approximately one percent denser than Reades' despite very similar methods of preparation, namely, 44.- 44.5 vs. 45.- 45.5. (Green's loose triaxial samples were also 44.0 - 44.5 percent porosity).

Measured strengths were corrected for the effects of rigidity of sample sheath, platten friction, initial porosity and mean stress level. Remaining failure characteristics were corrected for initial porosity variations. $d\varepsilon v/d\varepsilon a$ values are represented by an average (broken) line in figure 5.7. It has been redrawn (solid line) taking into account the effect of mean stress level. See Appendix 2 for change of several failure characteristics with initial porosity and mean stress level.

Peak strengths from flexible tests, figure 5.6, are extremely interesting. The flexible tests between b = .70 and b = .80, ISC(F) 12 and 13, are comparable to the tests using rigid plattens. They are a degree higher than an average line passing through rigid platten test data. ISC(F) 11 gives almost the same ϕ' value as an average rigid platten test at that specific b value.

On the other hand peak strengths measured in the flexible tests between b = .89 - .95, namely ISC(F) 17, ISC(F) 18 and ISC(F) 19 are strikingly lower than that would be expected from them considering the plot from Reades' tests using rigid plattens. See figure 5.10 where stress-strain curves of all loose flexible tests are plotted. It is also seen that tests ISC(F) 18 and 19 are almost identical. There is an average difference of 3° between them and the rigid platten tests at the same value of b. The Writer regards this difference as an important observational fact that can enlighten a number of points. It can not be explained by any sort of error in testing and instrumentation, in contrast, these series of tests on loose samples being at the end of flexible series of tests, were the result of long accumulated practice, (say, compared to first four tests). Mean stress level corrections for the measured values are 0.5° on average being very similar to those of rigid platten tests. Initial porosity differences have been taken into consideration. Axial platten contact ratios are 0.97 - 0.98 (in rigid platten series; 0.99, 0.99, 1.00, 96, 1.01, 1.02 for Reades' tests ISC 35, 36, 38, 41, 43, 67 respectively). Some little portion of the difference may be explained in this way. See for example Reades' figure 9.9 - σ_1 vs.size of gaps - Belt platten contact ratios in the vertical direction are .95 - .96 in ISC(F) 11, 12, 13 and .89 - .90 in ISC(F) 17, 18, 19 whereas in rigid platten tests .88 -.90 except with .92 in ISC 67. This implies higher b values for all mentioned tests except ISC(F) 11, 12, 13. Belt contact ratios in the cell direction for flexible tests (always greater than one in rigid platten tests, because plattens are much wider than the belt faces of sample) are nearly unity. Therefore it can be said that peak strengths decrease as b value increases (until extension state) after a certain intermediate stress range in flexible platten tests. This region seems to lie in between b = .65 and b = .85. In a way peak strength versus b curve peaks in this sector. Similar behaviour was observed for dense samples in flexible series. And this is in contrast with the behaviour observed from rigid platten tests, although for dense samples the Writer is aware of a small drop. Full discussion on this point will be presented in Chapter 7.

Differences in initial porosities of loose samples, and increasing mean stress level make it imperative to correct the volume change rates as well. In figure 5.7 corrected line is drawn. The broken line represents an average line through initial porosity corrected data whereas solid line is obtained increasing the values for the effect of mean stress level. Measured and corrected values of $d\varepsilon v/d\varepsilon a$ seem to be somewhat different only after b value of 0.5. Rigid platten tests indicate an increase in $d\varepsilon v/d\varepsilon a$ after b = .5. Flexible platten tests ISC(F) 12 and 13, at b = 0.72 and 0.71 respectively, give very close volume change rates to those of rigid platten tests, and the flexible platten tests with b values higher than 0.80 give lower values of $d\varepsilon v/d\varepsilon a$ than those of rigid platten tests on average.

Major principal strain εa is plotted in figure 5.9. Initial density corrections have been considered in the figure. Sharp decrease in εa from triaxial compression to b = 0.50 is noticable for both densities. Three flexible platten tests with b values .71, .72, .79 agree with the data obtained from rigid platten tests. Flexible platten tests after b = .88 consistently give lower values than those of rigid plattens, the difference being about 1.5 percent. Control rigid platten tests ISC-SP2 and ISC-SP5 have also been plotted,see Section 5.4. Following the correction procedure they entirely agree with Reades' tests. Reades' tests are also corrected for the effect of initial density in figure 5.9.

In figure 5.8 volume change characteristics of both rigid and flexible platten tests are given. They are again connected for initial porosity, see Appendix 2. It is clearly seen that after mid range flexible tests show higher compressive volumetric strain at failure than rigid platten tests. The difference gets larger with increasing values of b. ISC-SP2 and ISC-SP5 are also placed. They match with rigid platten data by Reades.

5.4. Special Series of Generalised Tests on Loose Samples.

Eight tests were carried out in this group, namely ISC-SP1-8. They were all loose prepared to an approximate initial porosity of 44 percent. SP2 and SP5 were classical type of ISC tests with b values of .81 and .85 respectively.

They were performed to see whether the Writer's testing technique affected the comparison of tests between in this study and in Reades (1972). Secondly the Writer's 44 percent porous loose samples were compared to those of Reades (1972) at 45 percent using an interpolation precedure. SP2 and SP5 which were at 44 percent could be directly compared with flexible tests.

Others were first consolidated isotropically to about 840 kN/M^2 then while cell pressure was being lowered at a constant rate the axial plattens were driven using the conventional worm gear frame at different strain rates suitable for the specific part of the stress-strain relation. Immediately after consolidation belt faces of the sample was preloaded by screw-piston mechanism, and this pressure was maintained throughout the test by conventional mercury pots. Details were seen in Chapters 3 and 4. Therefore, failure was reached as the cell pressure being minor principal stress, axial pressure major principal stress and the belt stress as intermediate like a conventional generalised test in the ISC apparatus but with a very important difference; without the two pairs of rigid plattens being driven against each other at the same time. In the next section its importance will be explained.

Failure characteristics of all tests in this series are given in Table 5.3. Failure characteristics are also plotted in the previous figures: 5.6, 5.7, 5.8, 5.9 together with all ISC loose tests. Failure characteristics of tests SP1, SP3, SP4 - with b values .21, .39, .39 respectively - were very similar to the conventional ISC tests. Giving allowance to initial porosity differences, peak strengths were about the same as those from other conventional ISC tests.

This is also true for the rate of volume changes, figure 5.7. Major principal strains reached at failure are close to the curve obtained by other ISC tests after corrections, figure 5.9, SP1 and SP4 being somewhat lower. This is interesting because these tests followed a completely different stress path, see figure 5.11, and similarity in deformation properties cannot be expected. Volume changes were more dilatant but SP1, 2, 3 would have comparable values to ISC tests in the corresponding intermediate stress ranges if required corrections are estimated correctly.

ISC-SP2 and SP5 are ISC tests of usual type and their failure characteristics are also plotted in the figures.

It can be said that all characteristics conform to the previous tests performed by Reades (1972) at that intermediatory range initial porosities were different though. Tests ISC-SP6,SP7 and SP8 were at higher intermediate stress range with b values 0.91,0.80,.88 respectively. Peak strength values are very interesting; figure 5.6, SP7 has a value of $39^{o}.7$ which lies almost at the bottom of the peak values obtained in that range by other ISC tests with both flexible and rigid plattens whereas tests SP6 and SP8 gave $37^{o}.6$ and $37^{o}.2$ respectively. The difference of $2-2^{o}.5$ is quite large especially if the very narrow gap of b values are considered, namely from .80 to 0.91. This sharp drop of strength at very high intermediate stress range is similar to the drop - at exactly the same range - in generalised tests using flexible plattens. A full discussion of this point will be given in the next section and Chapter 7.

Since the test procedure was to consolidate to high stresses and than to reduce the stresses to failure, mean normal stress level was half of the other generalised tests in the same range. If (together with the porosity differences) the change of $d\varepsilon v/d\varepsilon_1$ against mean stress level was considered very high. Values of rate of volume change would be decreased to the values obtained from other rigid platten tests. Axial strains were noticably lower (2-3%) than usual generalised tests in spite of the corrections applied which was expected in such an unloading stress path, figure 5.11. Measured volume changes are negative -.85, -.63, -.62 percent for SP6, SP7 and SP8 respectively. But low mean stress level and initial porosity differences could account for this big difference relative to the ISC group of tests. Inspection of plots σ_m vs. εv and n_i vs. εv in Appendix 2, suggested big corrections if a comparision was desired. Corrected values of εv for these tests were close to other rigid platten generalised tests.

5.5. Brief Discussion of the Results and Conclusions:

In the previous section strength characteristics of generalised tests has been presented both with flexible and rigid plattens for dense and loose Ham River sand. Many arguments can be held about differences among platten contact ratios, initial density variations, variations in material characteristics and in preparation of the samples to explain relatively less important differences in strength characteristics, but regarding that the tests altogether were performed in a time span of almost eight years by three different researchers on different batches of basically the same material, the results are quite consistent.

A survey on the strength characteristics of all tests indicates that there is a general consistency of results both for dense and loose samples regardless of whether rigid or flexible plattens are used except at certain state of the intermediate stress. It is a fair assumption that water pressurised bags apply uniform stresses on to the belt faces of the sample. Then a direct deduction will be that the normal stress distribution on the belt rigid plattens is uniform in generalised tests using rigid plattens when applying intermediate stresses up to a certain value of intermediate stress which is seen to be around b = .5 - .6 (see discussion below). Quite noticable differences have been observed between generalised tests with flexible and rigid plattens at the intermediate stresses which give b values higher than o.7, therefore mention will be given first to the group of tests which have b values lower than .7, then the other group will be examined.

The maximum scatter among dense flexible tests is about 1,5^{o} - exactly the same scatter Green (1969) obtained in his tests.- Slightly higher strengths are detectable in ISC (F) 5, 6, 9, 14 relative to rigid platten tests. In figures 5.12,5.13, the stress-strain curves from both groups have been plotted together at similar b values.

It may be argued that this difference can be explained in the physical sense by the strong systone hypothesis by Trollope (1971). He argues that rigid plattens will cause amplification of the stresses in the strongest systones and hence will lead to lower strengths.

Slightly higher flexible platten strengths do not correspond to a higher trend of volume change rates except in ISCF (5), little lower axial strains to failure may be explained as follows; When intermediate stresses applied by water filled bag plattens, if the regions of different density will deform in varying amounts in the sample and the bags will follow them on the belt faces. This will help quicker unification of the density, and this may well lead to a quicker dilation and mobilization of the peak strength. Again slightly low volume changes at failure by flexible plattens may be at least partly associated with this explanation.

Simultaneous changes in εa and εv prevents to detect any changes in $d\varepsilon v/d\varepsilon a$ with respect to rigid plattens tests. The comparison of belt deformations in the ISC tests by flexible and rigid plattens is dubious especially in the case of dense samples where the deformations are small and because the compliance of bags may not be estimated for each test correctly. Minor principal strains may not be specified accuretely as well firstly because they are not measured directly and, uniformity of sample deformation is assumed generally.

The Writer took the curvatures of the sample corners into account when calculating the initial volume of the samples whereas Reades and Green do not report such corrections even though they argue that such somewhat round-cornered sample shape would not affect the sample behaviour. It is practically impossible to obtain very sharp corners with a circular membrane - for example, a very poor cornered dense sample will yield an initial porosity correction of .4 percent which implies approximately 0.4 degrees. This means that Writer's tests must have been higher if compared with those of other researchers by an amount which is differing for each test.

Another point deserves mentioning is friction on the belt faces of the sample. It may be argued that the friction coefficient between rigid plattens and sample faces may be different from that of flexible plattens and the sample - say, a lubricated rubber sheet is placed between in both cases. - Green (1969) and Reades (1972) used coefficients of friction of 0.01 and 0.015 respectively, see Appendix 2. After an initial series of flexible tests it was experienced that intermediate rubber sheet between the flexible platens and the sample was quite awkward to use and it frequently caused creases to form at failure, thus, reducing the sensitivity of the contact between the bags and the sample therefore, use of it was abandoned after first series of ISC tests (1-4). Instead, the bags and the sample face were both greased efficently. Now it may be reasoned that strengths in ISC(F) series were a little higher because coefficient of friction was a little higher. For such an argument to be true the friction coefficent has to be considerably large. For example, for $1-1.5^{o}$ increase in ϕ' to be caused by the frictional forces between the belt plattens and the sample, the friction coefficent has to be about .065 - .10 which corresponds to friction angle of 3.7^{o} and 6^{o} respectively.

The writer did not carry out a frictional testing program with flexible plattens. As can be seen in Appendix 2, most researches get frictional coefficients around 0.010-0.030 for two greased rigid surfaces and a lubricated sheet between them. In the case of two lubricated surfaces there are not many test data. Duncan and Dunlop (1968) give friction values of 0.010 - .040 depending on the thickness of the grease film, between polished lucite - grease - and rubber covered lucite block. Roscoe and Bassett and Cole (1967) report a friction coeffficent of 0.11 between glass and sand without any greasing.

The ISC(F) tests conducted at b values 0.7 or higher on dense and especially loose samples disclose important facts. Strength of dense samples show a peak at about b = .7. ISC(F) 16 and 20 give lower values towards extension. True b values for these tests are .78 and .88 considering the low belt platten contact ratios (indicated by arrows). Very high b values for dense samples (b = .90 -1.0) could not be attained, because for a direct comparison a constant cell pressure of 207 (30 psi) kN/M^2 was adopted for all present and past tests, and an ISC(F) test at b=1 meant bag pressures at the order of 1100-1200 kN/M^2 (160 - 175 psi). As explained in Chapter 3, the bags resisted such pressures but high traction forces resulted and caused sides of the bags to slip between the O - ring and internal supporting plate leading to a leak.

Another alternative might be to perform the tests at 140 kN/M^2 (20 psi) constant cell pressure instead of 207 kN/M^2 (30 psi) so that the bag pressure would create no problem, but this was not tried mainly because all test characteristics would necessiate an elaborate correction scheme to compare with the other tests. The Writer is able to observe a similar decrease in strength near extension in Green's dense ISC rigid platten tests.

Similar decreases in strength at high b values can be seen in the data by other researchers, for example, clearly in Lomize and Kryzhanovsky (1967), Lomize et.al. (1969) Al-Ani (1975), to a lesser extent in Lade (1972). A noticable decrease has also been observed in Writer's loose ISC(F) series. The reason for this behaviour is not clear. As will be stressed in the final discussion, researchers who perform their tests at high intermediate stresses by pulling the top platten up (applying minor principal stresses) while maintaining the major and intermediate stresses also observe this decrease more sharply but at lower values, say from b = 0.5 - 0.6 onwards. Possible explanation for this behaviour will be given in Chapter 7, after presenting the results of the triaxial extension tests and the special series ISC SP9-16.

In a cubical generalised test with rigid plattens and especially in the tests with high intermediate stresses there is an important point to be considered in relation to the correctness of the peak strength values of the samples. Since both pairs of rigid plattens are driven inwards at similar speeds, the question that immediately comes to mind is the possibility of interference of these plattens. This question has been discussed in Roscoe Memorial Symposium (1971b) by Green and Sutherland, and also by Sutherland and Mesdary (1970) before . It seems to the Writer that these researchers do not distinguish the two types of phenomena occuring during a test with two pairs of rigid plattens are made use of. First, as will be examined in Chapter 7 in detail, stress-strain-strength response of cuboidal samples loaded by two pairs of rigid plattens driven inwards applying major and intermediate principal stresses under a constant cell pressure as minor stresses will be different from that of generalised tests in which axial rigid platten is opened up until failure applying the minor principal stresses and belt plattens are driven inside applying major principle stresses and cell pressure being the intermediate stress. This will affect a wide range of three-dimensional tests since except the tests up to plain strain, all tests in the ISC apparatus are carried out by driving the plattens in at various speeds. The other phenomenon is the possible interference of the rigid plattens driven at similar speeds, starting to become effective from b = .5 - .6 onwards. By "interference" it is meant that the plattens do not collide - this can be detected immediately - but presumably some sort of load transfer occurs along the edges where the two pairs of plattens meet each other though not necessarily through trapped sand pockets in between the platens like some researchers claim. Sample faces along the gaps remain quite vertical during the tests, but any dense pocket - relative to the overall sample porosity - along the edges may help such a transfer. The latter case may be particularly true for loose samples at b values of ,80 - 1,00 which means considerable deformations are taking place - at the order of 3-4 mm, both vertically and laterally at the edges, and especially when a very small, insufficient, gap is allowed at the start of a test it will end with a tiny one between the plattens at failure, and will probably amplify such load transfers. Therefore in this category of tests both the peak strengths and the final portion of the stress-strain curves will be higher than they would be otherwise.

In figures 5.14, 5,15, 5.16, 5.17, 5,18, and 5,19 stress strain curves of several loose samples with high b values have been shown. All are rigid platten tests by Reades (1972) except SP5 (figure 5.14). Tests ISC 18 and 42 by Reades have also been included. The latter two tests were stopped because of direct collision between the rigid plattens. A careful examination of these stress-strain curves in comparison with those from other tests like, triaxial compression or ISC tests at lower b values reveals certain facts. Along a stress-strain curve of a soil sample, "the modulus of deformation" that is the ratio of stress increment to the corresponding major strain increment is the highest initially (called tangential Modulus), then it decreases as the total strains get larger until it becomes zero at the peak. In the figures the moduli at the final stages of stress-strain curves (slopes of tangents along the curve) do not decrease, but stay constant or even increase in some cases. It seems that this can only be explained by a kind of effect which has been postulated above. Investigation of the stress-strain curves shows that this additional increase in the stress ratio gives extra few degrees (1^{o} - 3^{o}) increase in ϕ'. This is presumably one of the major reasons why Reades reports sharply increasing ϕ' - b graphs near extension.

It is interesting to note that if the tests ISC 18, 42 had been stopped little before the collision of the plattens they would probably have been reported as "good" tests. Before collision stress-strain curves were almost flattened, and the plattens were very near each other. Final sections of the stress-strain curves are similar to other tests in the figures with almost constant slope of the tangents.

Since the Writer's loose samples were denser than those of Reades', and comparisons between flexible and rigid plattens would be made, ISC SP2 and SP5 were performed. They are rigid platten tests at high b values, and are a perfect match with the other rigid platten tests.

Writer's flexible platten tests support the above argument. Peak strengths in tests ISC(F) 12 and 13 give almost exactly the same value of 40.8° at b values .72 and .71 respectively. At b = .79 ISC(F) 11 gives 39.5°, and tests ISC(F) 17, 18 and 19 give peak strengths of 38.0°, 38.4°, 38.8° at b values of .88, .94, .92 respectively. This decrease in strength from b = .7 till b = .95 is no scatter but an observational fact; material, sample preparation method, testing technique etc. all being identical in every test.

On the other hand it has been shown that rigid platten tests indicate a continious increase in ϕ' from approximately b = 0.6 until extension. At b = .90 as rigid plattens indicate a value of ϕ' 41°, flexible tests show 38.5, at extension (b=1) rigid and flexible tests again suggest ϕ' values of 42° and 37° respectively. This divergence between the two groups of tests at high intermediate stresses can be explained by the same hypothesis in the above paragraphs as follows; As a result of the method of application of the loads in the ISC apparatus, loading plattens affect each other along the edges of sample starting from mid-intermediate stress range onwards. This must be true whether plattens are rigid or flexible. They are expected to be more pronounced in the case when two pairs of rigid plattens are driven in, because flexible plattens would allow more freedom to displacing groups of soil grains relative to rigid plattens between which the soil particles are "locked" in a way, the only alternative left being more decrease in void ratio. This will directly influence the stress conditions and distribution on the plattens. Hence stress distribution on the axial and belt rigid plattens are expected to be higher along the edges than the more central parts especially at larger strains - at failure for example - .

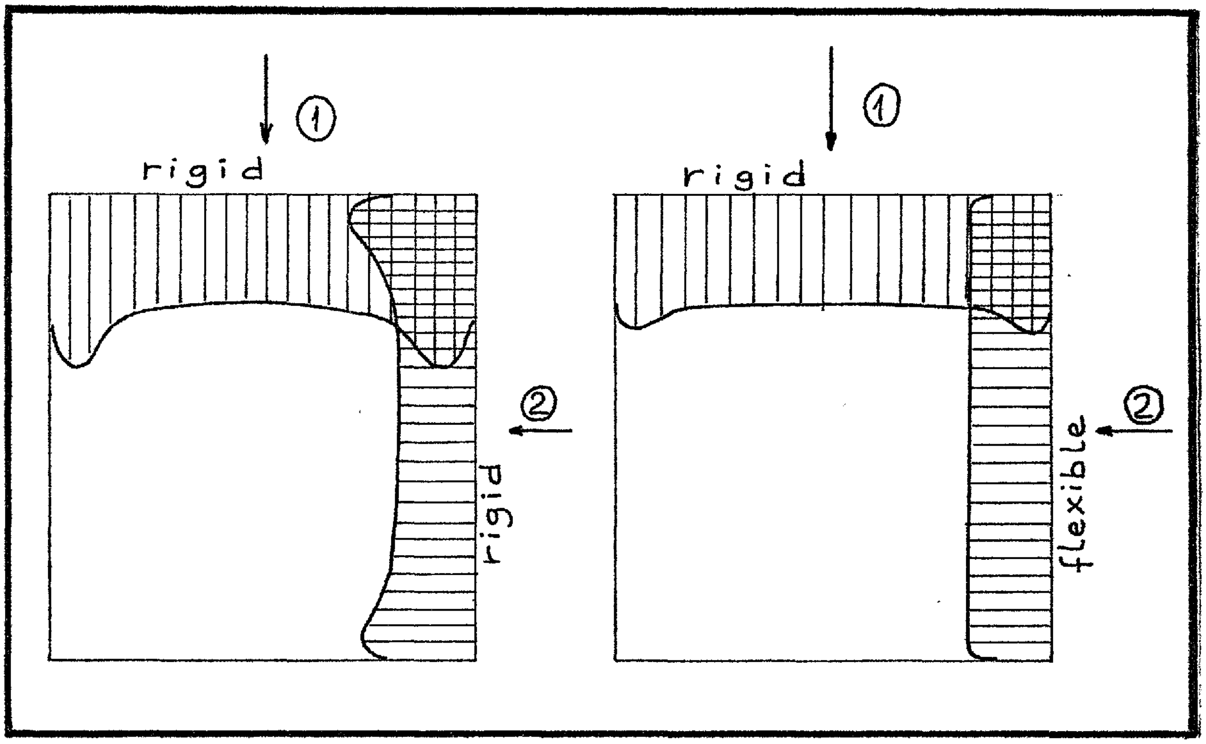


Figure 5.20: A qualitative (exaggerated) sketch of normal stress distributions on the loading plattens in two ISC tests one with two pairs of rigid plattens, the other one rigid, one flexible. (At b= .75-1.00, failure)

In figure 5.20 a sketch has been made to illustrate this possible explanation. Such stress concentrations will be more severe when belt plattens are driven faster to achieve higher intermediate stresses. When bag plattens are used load shedding is still expected to exist but by the nature of their flexibility they can not sustain differential normal pressures, and hence the normal pressure distribution on them can be correctly assumed as uniform. Since the bags have been designed as a closed system - see the discussion in Chapter 7-, the proving ring which is actually measuring total belt load is affected. Therefore, while rigid platten tests give high ϕ' values at very high intermediate stress range due to the expected non-uniformity of pressure distributions, flexible tests will exhibit smaller resistance to failure due to uniform pressures on the belt plattens, and relatively less non-uniform pressure distributions on the axial plattens. In the extension state the difference will be maximum because while this effect is the most severe on the rigid plattens, flexible plattens will still be able to manage to maintain uniform pressures.

An elaborate experimental program may well disclose such stress concentrations. Sketchley (1972) gives normal and shear stresses measured along the rigid plattens of a plain strain apparatus. Normal stresses near the edges are a little higher than the rest of the plattens, but the system that is loading the sample is different from ISC apparatus.

This design by Hambly (1969) makes use of nested plattens. There is no mention whether the normal stress distribution belongs to a sample at failure, and the material is kaolin slurry Top and bottom faces of the apparatus are fixed plate glasses so that nested plattens work easily. It must be also remembered that plain strain tests in the ISC apparatus did not present any difficulties in this respect, so the above measurements can be regarded as "normal". Note the difference between the readings in the transducers at the centre and the edge of the rigid axial platten, 599 kN/M^2 against 634 kN/M^2, 6% difference, even for a plain strain test.

The Writer always keeping in mind the possible interference effects resulting from driving the rigid plattens in, intended to carry out few special tests for the purpose. ISC-SP-6, 7 and 8 were performed by stress controlling the belt plattens while cell pressure was decreased to failure as the minor principal stress. (see also section 3.4) Therefore these series of tests were very similar to main ISC series of tests with respect to the plattens applying the principal stresses, only differences were an approximately constant and lower mean stress level path - against increasing mean stress level in ISC tests - and stress controlled belt plattens rather than strain controlling. Strengths have been corrected for platten friction and sample seath, mean stress level and initial porosity. As it is seen in figure 5.6 results are quite interesting.

Tests SP 6, 7 and 8 have b values .92, .80, and .88 respectively. The peak strength of SP7 is in complete accord with all the other ISC rigid platten data, whereas tests SP6 and 8 - closer to extension - have surprisingly low peak strengths, even about a degree lower than Writer's flexible ISC tests.

This sharp decrease in strength is at high intermediate stresses reminiscent of flexible ISC tests. So, this finding supports the thesis that there are load transfers between pairs of rigid plattens at high intermediate stresses when they are driven towards each other.

It has been shown by several researchers that the effect of stress path on the peak strength characteristics of granular soils is almost nil. Hence it can be deduced that the different results obtained in SP tests compared to ISC tests can only be some sort of apparatus effect no matter which group is closer to the "truth".

Another interesting observation can be made by plotting the principal stress ratios, σ_1/σ_3, at axial strains (major principal strains) corresponding to shear at minimum volume, namely, axial strains at which rearrangement of particles stops and sample prepares itself for dilation to give the final peak resistance, figure 5.21. This stress ratio is little lower than the peak strength ratio, and the sample is yielding continiously. Such a plot is justified, because all tests have been carried out under a constant cell pressure, and major and intermediate stresses have been increased monotonically to failure. It is certain that under various complex stress paths such a comparison might not be much meaningful. The logic behind the preparation of this plot has been the suspected peak strengths due to a possible interference in high b value tests. Once a sample starts yielding especially loose samples at an axial strain+ (or at any strain), the reserve shearing resistance to reach the peak value is small relative to the current shearing stress at that strain, and this final proportion of the shearing resistance before the peak is spanned over rather large amount of yielding deformations. In the case of suspected interference it has been thought that it is more informative to plot a shearing resistance which is marginally lower than peak resistance but having a much smaller strain than failure strain, thus being a record of a deviatoric stress without driving the both pairs of rigid plattens extensively. Therefore, if ISC rigid platten tests are free from any interference effects, such a plot against b value (b value is defined as $b = \frac{R_2-1}{R_1-1}$ in this case where $R_2 = \sigma_2/\sigma_3$ and $R_1 = \sigma_1/\sigma_3$), should follow the same trend of the peak strength versus b value plot. Writer's flexible and rigid platten tests have been plotted together with Reades' rigid platten tests. ISC(F) 10 and 15 which were stopped before

---

\+ There is no such strain that samples start yielding, actually, because they start yielding almost from the beginning, but it is meant the axial strain at which deformations start increasing in a more noticable way.

peak are also included in the graph.

It is seen that stress ratios do not increase after b = .40 until extension like peak strengths, but they increase very little (essentially the same) above plain strain tests in the mid b range until b = .7, then decrease towards extension. In spite of the difference in stress ratios between flexible and rigid platten tests, decreasing trend after b = .7 is clear for both groups of tests, even more noticable for flexible tests. The difference can be explained in the same way as peak strength differences between the two types of tests. It is becoming clearer to the Writer that the increase of peak strengths more and more when approaching b = 1 limit is most probably caused partly by interference of very near rigid plattens along the edges.

In loose ISC tests near extension ($b \simeq .9-1.0$) σ_1/σ_3 and σ_2/σ_3 values follow each other very closely when plotted on a graph, but before failure,belt stress ratio starts falling while the major principal stress is still increasing towards peak strength. This is due to the gaps, and shorter belt plattens in a way punch against the belt faces of the sample. Similar cases have also been observed in Writer's loose ISC tests on volcanic sand. This complicates the reported b values at failure, because, for example, an expected b = .92 test would lower to b = .80 at the final section of the stress-strain curve. Reades' tests ISC 38 and 35 are typical examples of this behaviour.

They are both reported to have a b value of .84 but actually ISC 38 is at least a b = .9-.95 test. The actual test points should have been placed at higher b values, thus they would show milder ϕ' increases at extension. It is seen that plot of stress ratios little lower than peak stress ratio also avoids this effect. But it should be emphasized that the above observation and consideration does not apply to all high b value tests.

The apparatus is restricted in measuring the behaviour of samples at ultimate strength, namely the post-peak properties, and so are almost all generalised apparatuses. The dimensions of sample are so chosen that a clear failure plane forms at failure without intersecting top and bottom plattens. This is clearly seen in stress-strain curves with distinct peaks, and there is always no end restraint for at least another 2 percent axial strain after peak.

5.6. Conclusion

In the light of the foregoing discussion the Writer has been convinced that for considerable part of the intermediate stress range, ISC apparatus with rigid plattens in σ_2 direction works properly, say until mid - b range. Similar behaviour with flexible plattens in the belt direction implies a fairly uniform normal pressure distribution along the rigid plattens. Slightly higher strengths by flexible plattens may originate from a slightly different internal shearing mechanism in the sample or they may result from a rather different friction coefficient between the bags and the sample, which is less probable. In any case the difference is not significant.

But for the whole range of intermediate stresses the Writer does not agree with Reades (1972). He claims that "Material behaviour was correctly measured" (p.279) and "...observed material behaviour in the region b=1 was not undesirably influenced by the apparatus so that the measured strengths were in error, due for instance to sand being trapped between adjacent inward-moving plattens", (p.284). And he reasons that (a) sample photographs indicate sample faces remain plane at failure (b) All tests near b=1 (35, 53, 63, 67 etc) present very similar behaviour.(p.304).

The Writer reckons that the photographs of the failed samples after the cell has been dismantled can not be used as a proof that the samples are free from any restraints etc. Direct contact of rigid plattens or any trapped sand pocket causing direct load transfer between them is out of question because it can be detected immediately. In other words, plane belt sample faces at failure are not necessary and sufficient condition for a test to be free from such effects. The latter argument is not valid because if certain effects exist in ISC tests near b=1 then it will of course exist in all tests in this region.

It is the Writer's contention that after mid-intermediate stress range, interference between axial and lateral loading platens occur due to inward movement of them meeting along the edges of the sample. This results in non-uniform normal pressure distributions on the platens, and cause higher record of ϕ' values. It is true for both rigid and flexible platen tests, being more pronounced in the former because of the two pairs of rigid platens. Several facts supporting this argument are; lower strengths obtained by flexible platten tests near b=1, lower strengths obtained in special tests near b=1, in which rigid belt plattens are stress controlled rather than being driven, plot of pre-peak stress ratios throughout the intermediate stress range and, careful examination of all stress-strain curves of the tests near b=1.

It is concluded that the two effects governing the shearing mechanism in ISC apparatus must be differentiated which are the interference phenomena after b = .5-.6 causing somewhat higher strengths and the way of loading a cuboidal sample by three different normal stresses. The latter will be discussed in Chapter 7 in detail.

For the loading mode adapted for ISC tests, peak strengths have been observed to increase from triaxial compression tests to plain strain tests approximately (5^{o} and 3^{o} for dense and loose samples) then further to increase until b = .7-.9 (2^{o}-3^{o} for dense and loose samples),finally, to decrease at near b=1 (1^{o}-2^{o} for dense samples and 3^{o}-4^{o} for loose samples). In ISC tests with rigid plattens the final drop of strength at the extension side can not be observed except a slight decrease in dense samples. If underestimated b values are taken into account at very high b values due to punching effect, it can be said that a decrease also exists for loose rigid platen ISC tests. Other test characteristics are similar for rigid and flexible platten ISC tests excepting the b = .70-1.00 interval.

TEST NO.	INITIAL POROSITY n_1^o %	b	σ_a kN/M^2	σ kN/M^2	ε_a %	ε_b %	ε_v %	$\frac{d\varepsilon_v}{d\varepsilon_a}$	AXIAL PLATTEN CONTACT RATIO	BELT PLATTEN CONTACT RATIO		AS MEASURED ϕ'^o_o	PLATTEN FRICTION & S.SHEATH ϕ'^o_1	MEAN STRESS LEVEL ϕ'^o_2	NORMALISED 39% ϕ'^o_3
										1	2				
ISC(F)1	37.5	.19	1175.6	367.7	4.1	- .6	- 1.25	- .65	.98	1.06	.98	44.7	44.5	45.2	43.6
ISC(F)2	38.3	.16	1129.0	126.1	4.4	- .3	- 1.45	- .28	.93	1.05	.99	44.0	43.9	44.5	43.7
ISC(F)3	38.4	.20	1120.3	365.2	3.1	- .7	- 0.90	- .65	.93	1.04	1.02	43.6	43.5	44.2	43.5
ISC(F)4	38.5	.19	1155.8	375.8	2.9	- .8	- 0.90	- .70	.93	1.04	1.02	44.2	44.1	44.9	44.3
ISC(F)5	39.4	.37	1162.7	543.7	2.5	.1	- 0.65	- .65	.96	.93	1.02	44.4	44.2	45.1	45.5
ISC(F)6	39.0	.54	1210.2	733.2	2.2	.3	- 0.23	- .70	.97	.91	1.03	45.2	45.1	46.3	46.3
ISC(F)7	38.6	.58	1160.4	685.7	2.0	.6	- 0.30	- .71	.97	.92	1.02	44.4	44.2	45.3	44.9
ISC(F)8	37.9	.46	1195.5	650.9	1.7	.2	- 0.35	- .71	.96	.93	1.03	45.0	44.8	45.9	44.8
ISC(F)9	38.0	.72	1324.8	1003.2	2.0	1.5	- 0.60	- .88	.95	.93	1.01	47.1	46.9	48.4	47.4
ISC(F)14	39.1	.68	1225.0	903.8	1.9	1.7	- 0.65	- .83	.99	.92	1.00	45.5	45.3	46.7	46.8
ISC(F)16	39.6	.74	1155.1	911.8	2.1	1.8	- 0.47	- .77	.95	.87	1.00	44.3	44.1	45.5	46.1
ISC(F)20	38.4	.83	1186.2	1013.3	1.8	1.6	- 0.55	-1.0	.99	.86	1.00	44.9	44.7	46.2	45.6

Failure Characteristics of Dense ISC Tests Using Flexible Plattens

Table 5.1

Test No	Initial Porosity Ni %	b	σ_a	σ_b	ε_a %	ε_b %	ε_v %	$\frac{d\varepsilon v}{d\varepsilon a}$	Axial Platten Contact Ratio	Belt Platten Contact Ratio	As Measured ϕ'^{o}_{0}	Platten Friction & S. Sheath ϕ'^{o}_{1}	Mean Stress Level ϕ'^{o}_{2}	Normalised 44% ϕ'^{o}_{3}
ISC(F)10	44.3		STOPPED		BEFORE	FAILURE								
ISC(F)11	43.6	.79	926.4	777.5	4.1	2.9	.48	-.22	.97	.95.99	39.7	39.4	40.0	39.5
ISC(F)12	44.0	.72	967.7	752.4	4.4	3.1	.55	-.25	.98	.96.99	40.6	40.4	41.0	41.0
ISC(F)13	44.5	.71	968.5	754.6	4.4	2.6	.70	-.19	.97	.96.99	39.8	39.6	40.2	40.7
ISC(F)15	44.5		STOPPED		BEFORE	FAILURE	at	4% ε_a	σ_a/σ_c =	4.5, b=	.86 $\sigma_a-\sigma_c$	= 706	kN/M^2	
ISC(F)17	43.8	.88	860.3	784.0	3.40	3.8	.57	-.29	.98	.90.99	38.0	37.8	38.3	38.0
ISC(F)18	44.4	.94	852.4	815.2	3.4	3.7	.73	-.30	.97	.89.97	37.8	37.6	38.1	38.5
ISC(F)19	43.9	.92	886.0	820.1	3.7	4.1	.38	-.41	.97	.89.97	38.7	38.4	39.0	38.8

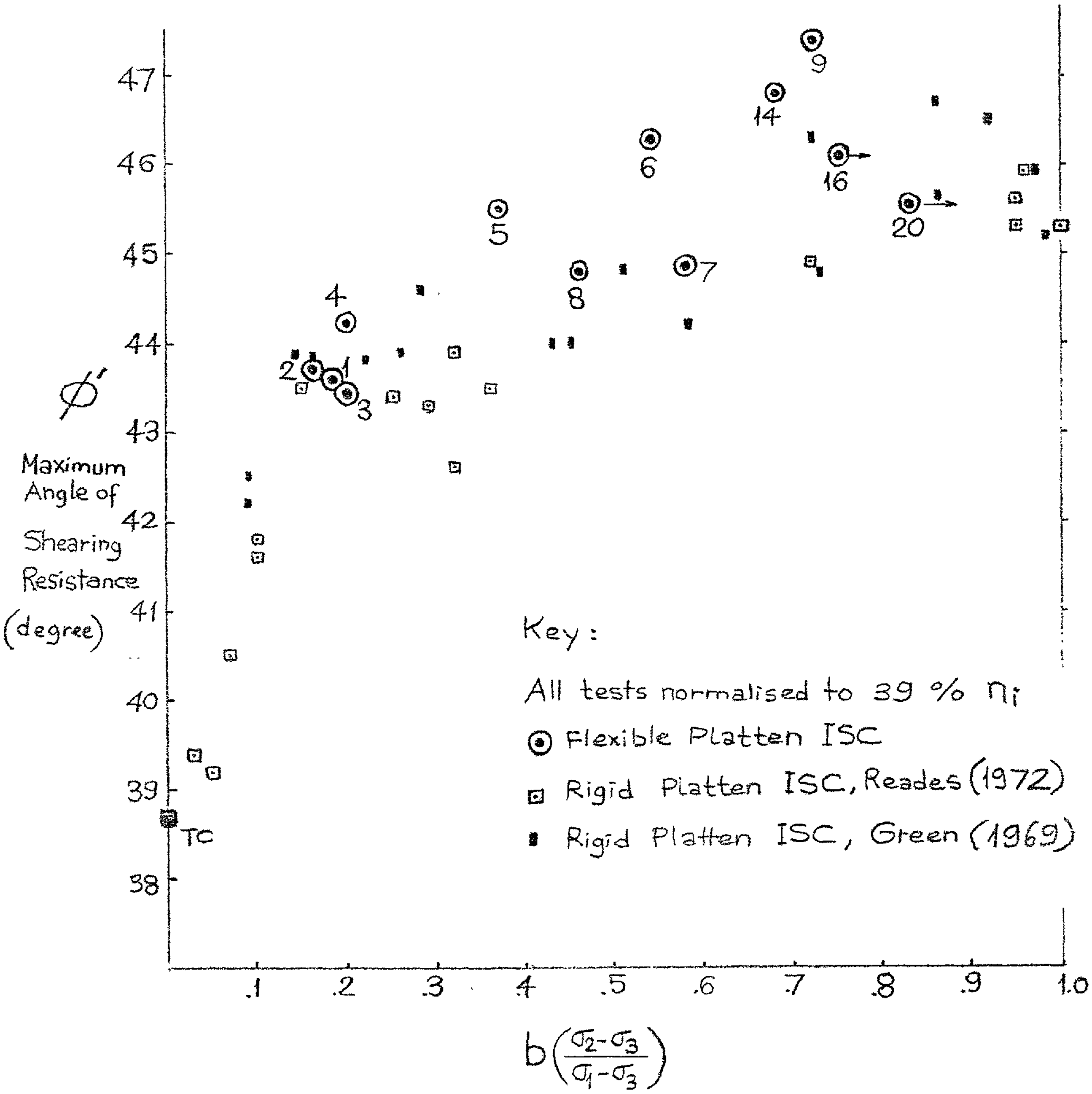
Failure Characteristics of Loose ISC tests with Flexible Plattens.

Table 5.2

Test No	Initial Porosity N_i %	b	kN/ M^2												
			σ_a	σ_b	σ_c	ε_a %	ε_b %	ε_v %	$\frac{d\varepsilon v}{d\varepsilon a}$	Axial Platten Contact Ratip	Belt Platten Contact Ratio	Measured ϕ'	Platten Frict. & S.Seath	Mean Stress Level	Norm. 44% N_i
ISC-SP1	44.2	.23	1117.5	450.2	276.7	5.2	-.23	-.38	-.094	.94	.92	37.2	37.1	37.6	37.7
ISC-SP3	44.2	.41	846.9	458.4	210.0	4.4	.55	.22	-.17	.95	.90	37.2	37.1	37.4	37.6
ISC-SP4	44.0	.41	857.0	460.3	203.3	3.9	1.0	.34	-.18	.96	.90	38.2	38.1	38.4	38.4
ISC-SP2	44.0	.82	921.1	788.	204.9	5.6	5.8	.10	-.35	1.00	.91	39.8	39.5	40.0	39.9
ISC-SP5	43,9	.86	952.4	841.9	207.9	4.8	4.7	.30	-.29	.99	.90	40.2	39.9	40.5	40.4
ISC-SP6	43.7	.92	397.1	371.0	93.8	1.5	1.75	-.85	-.75	.97	.87	38.4	38.2	38.0	37.6
ISC-SP7	44.3	.80	471.6	402.9	103.1	2.5	2,6	-.63	-.75	.99	.89	39.8	39.6	39.5	39.8
ISC-SP8	44.3	.88	421.5	382.6	104.6	1.7	2.0	-.62	-1.13	.97	.88	37.3	37.0	36.9	37.2

Table 5.3

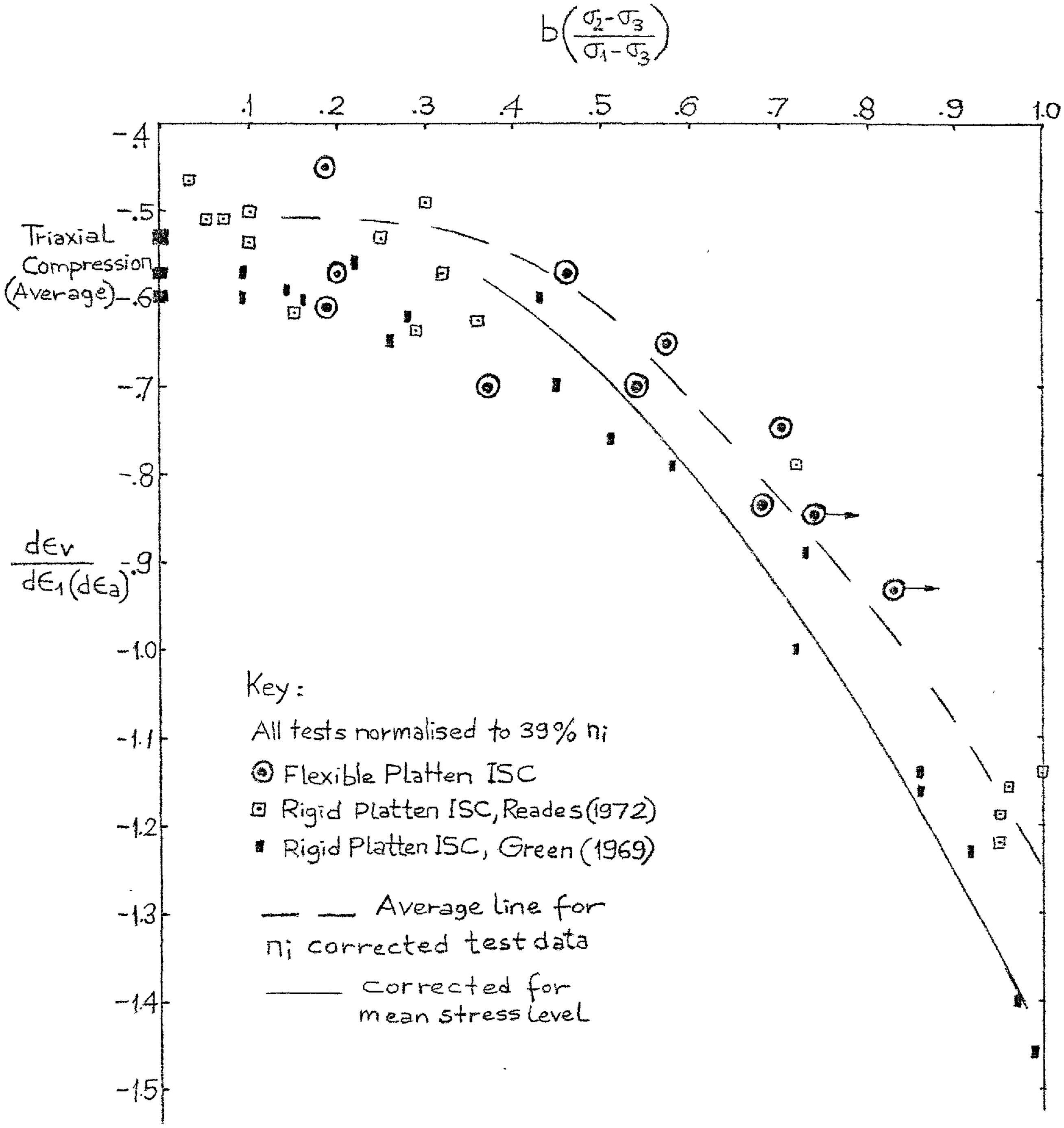
Failure Characteristics of Loose (ISC-SP(1-8) Tests on Ham River Sand.



Corrected peak strengths for dense ISC tests on Ham River sand based Mohr-Coulomb criterion.

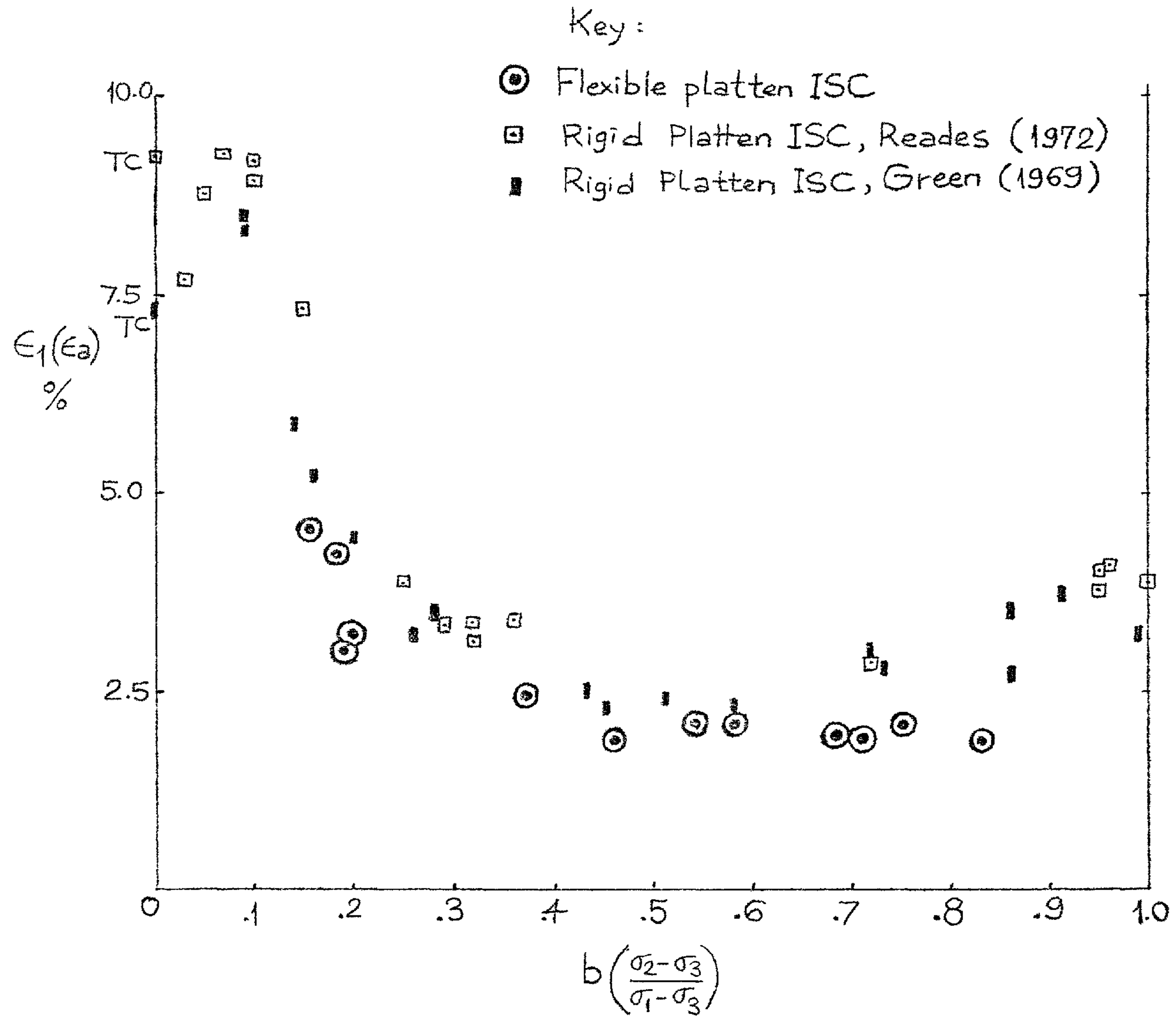
Corrected for a) Rigidity of sample sheath
b) Platten friction
c) Mean stress level

Fig. 5.1



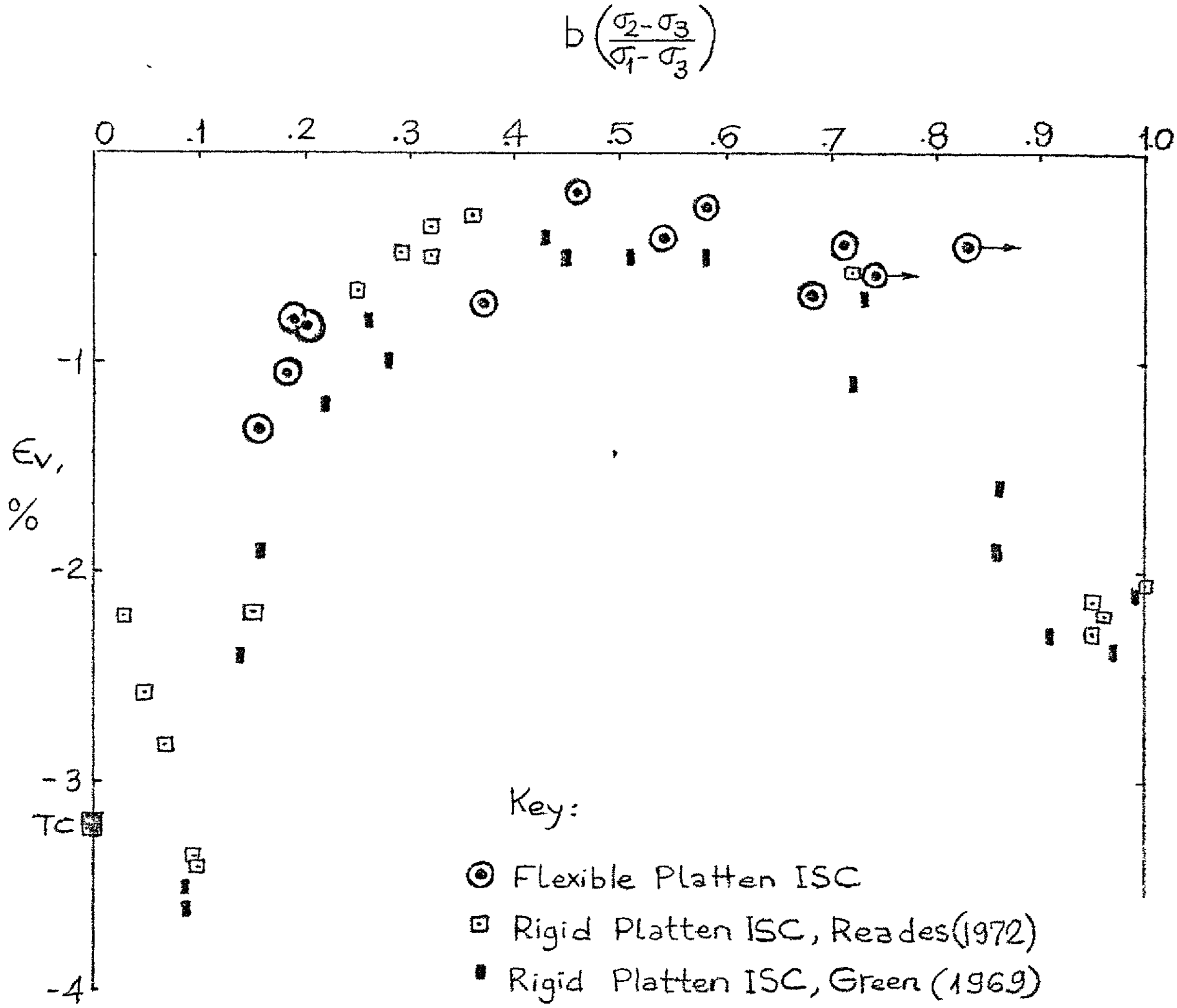
Rate of volume change for dense ISC tests on Ham River sand at failure.

Fig. 5.2



Axial (major principal) strains for dense ISC tests on Ham River sand at failure

Fig. 5.3



Volume change at failure in dense ISC tests on Ham River sand.

Fig. 5.4

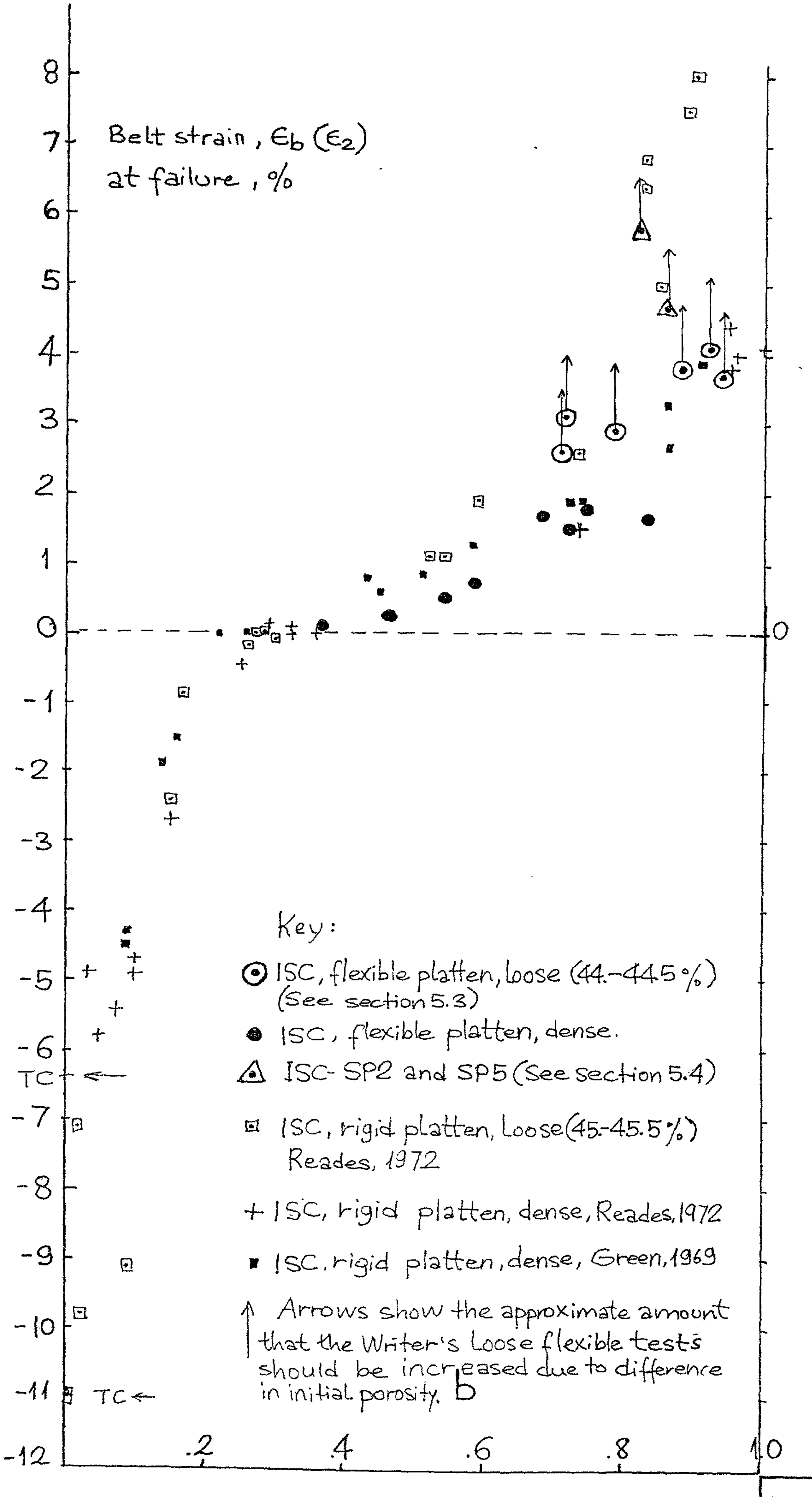
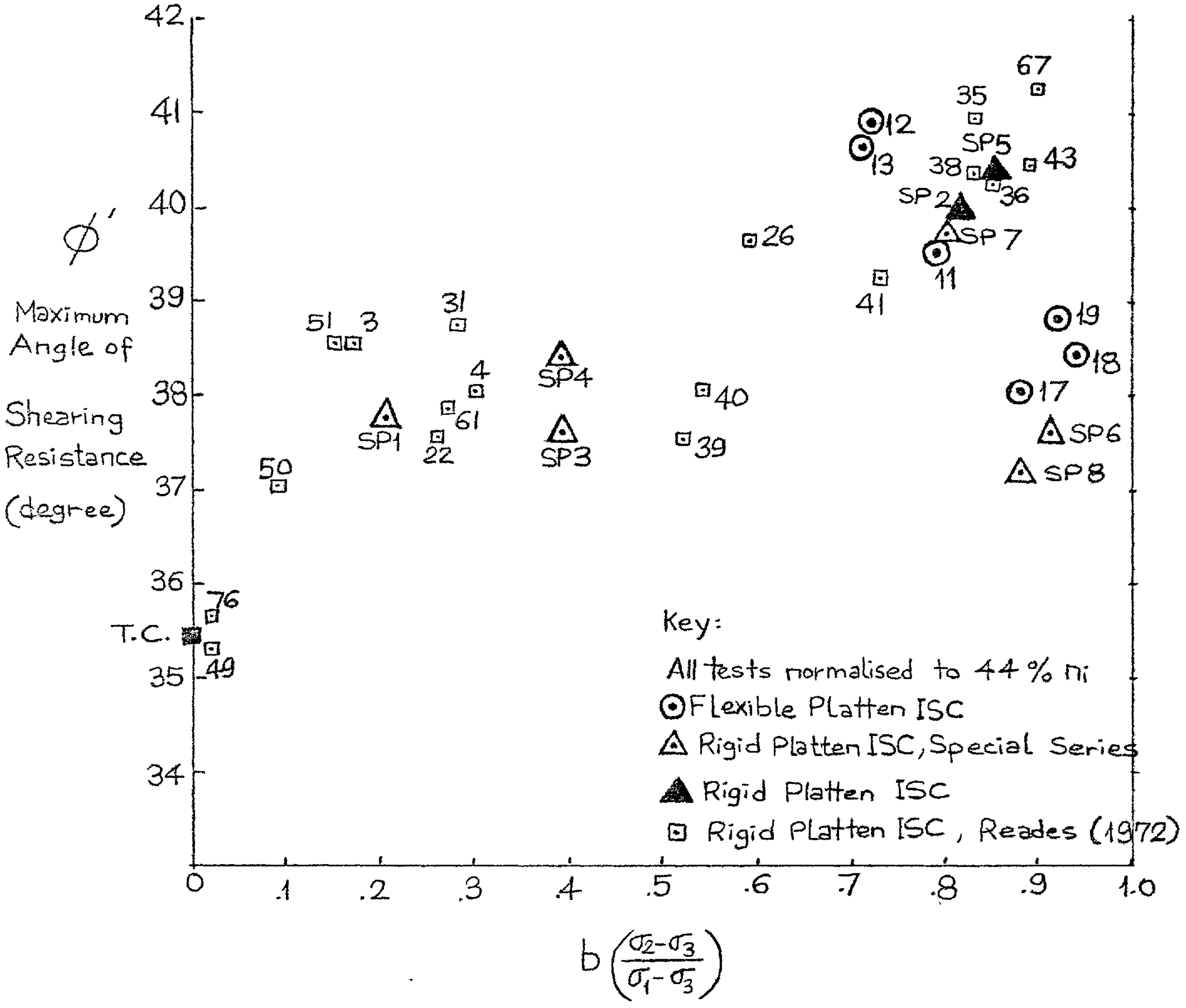
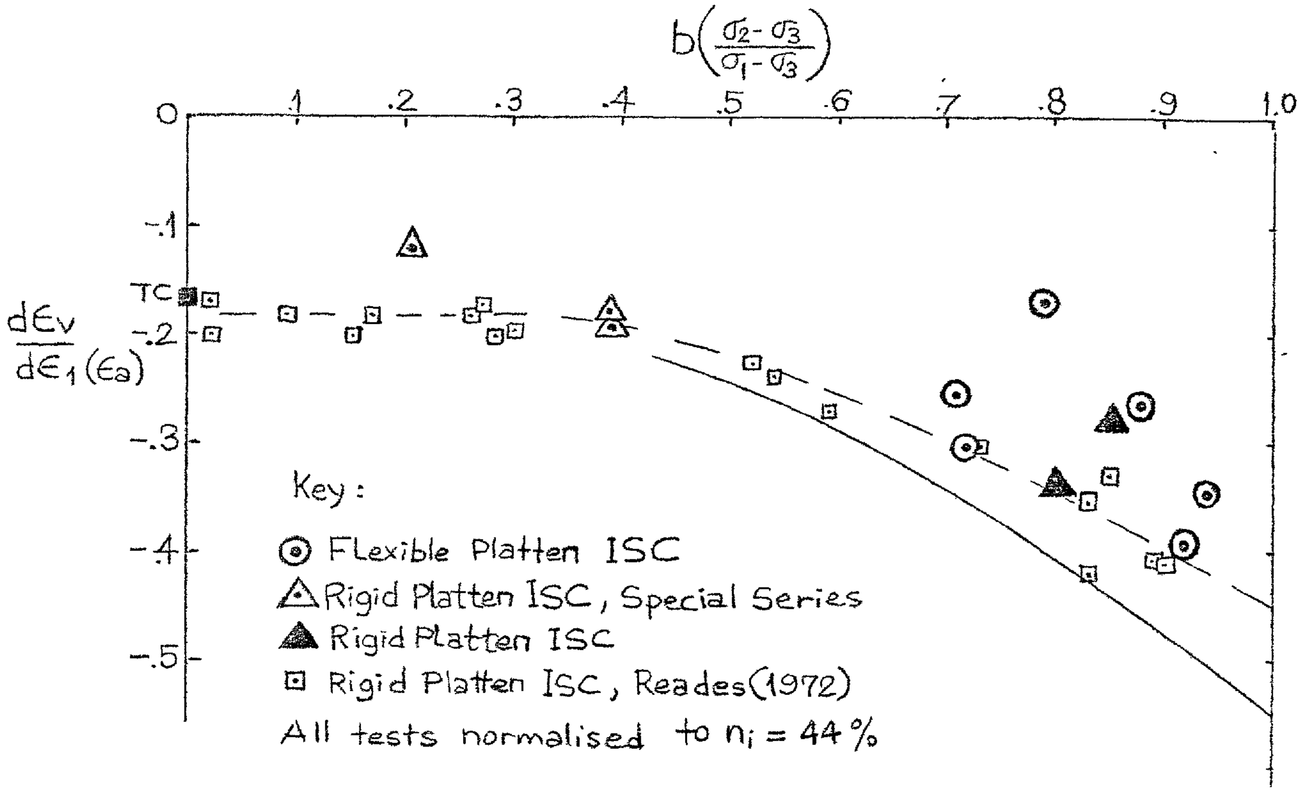

Belt strain, ϵ_b (ϵ_2) at failure, %
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
TC ←
-7
-8
-9
-10
-11
TC ←
-12
.2
.4
.6
.8
1.0
0
Key:
⊙ ISC, flexible platten, loose (44.-44.5%) (See section 5.3)
● ISC, flexible platten, dense.
△ ISC- SP2 and SP5 (See section 5.4)
⊡ ISC, rigid platten, Loose (45.-45.5%) Reades, 1972
+ ISC, rigid platten, dense, Reades, 1972
▪ ISC, rigid platten, dense, Green, 1969
↑ Arrows show the approximate amount that the Writer's Loose flexible tests should be increased due to difference in initial porosity.
b

Fig. 5.5



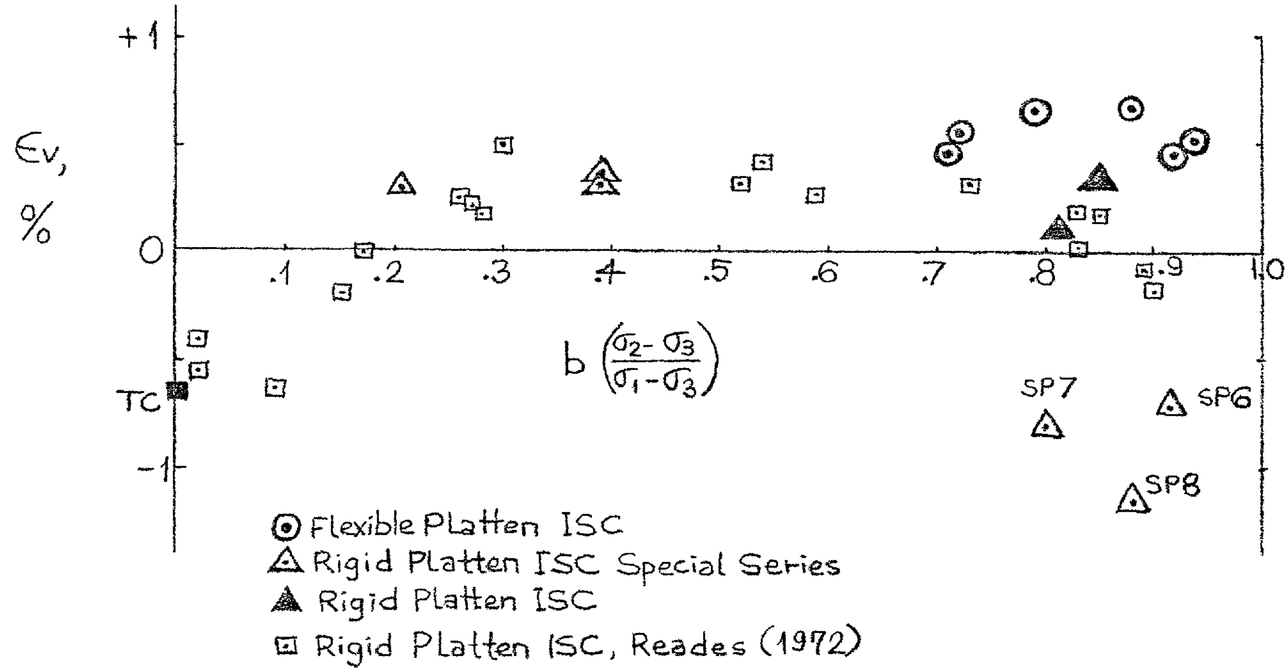
Corrected peak strengths for Loose ISC tests
on Ham River sand based on Mohr-Coulomb criterion.
Corrected for a) Rigidity of sample seath
b) Platten friction
c) Mean stress level

Fig. 5.6



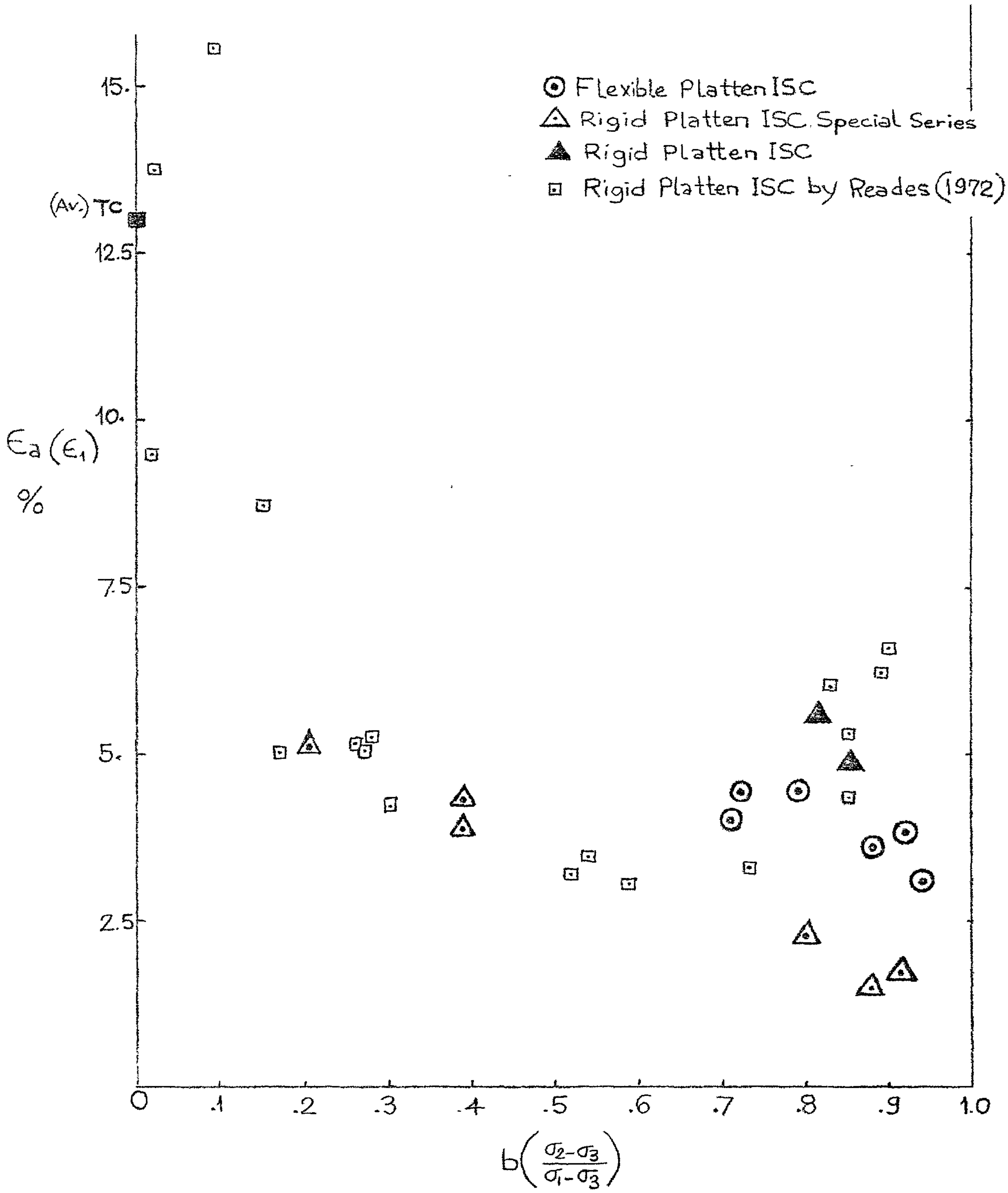
Volume change rates at failure in Loose ISC tests on Ham River sand.

Fig. 5.7



Volume changes at failure in Loose ISC tests on Ham River sand.

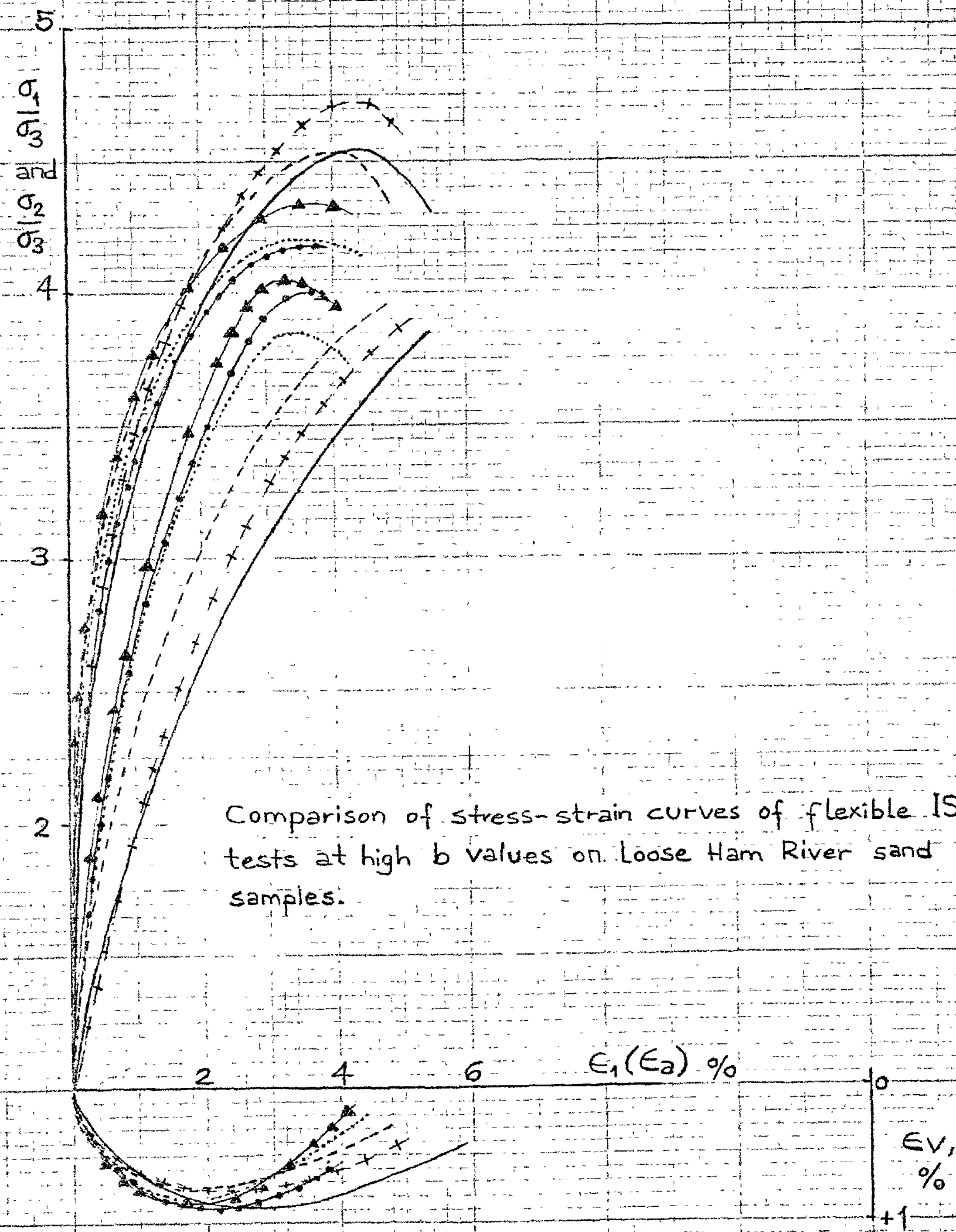
Fig. 5.8



Axial (major principal) strains at failure in loose ISC tests on Ham River sand.

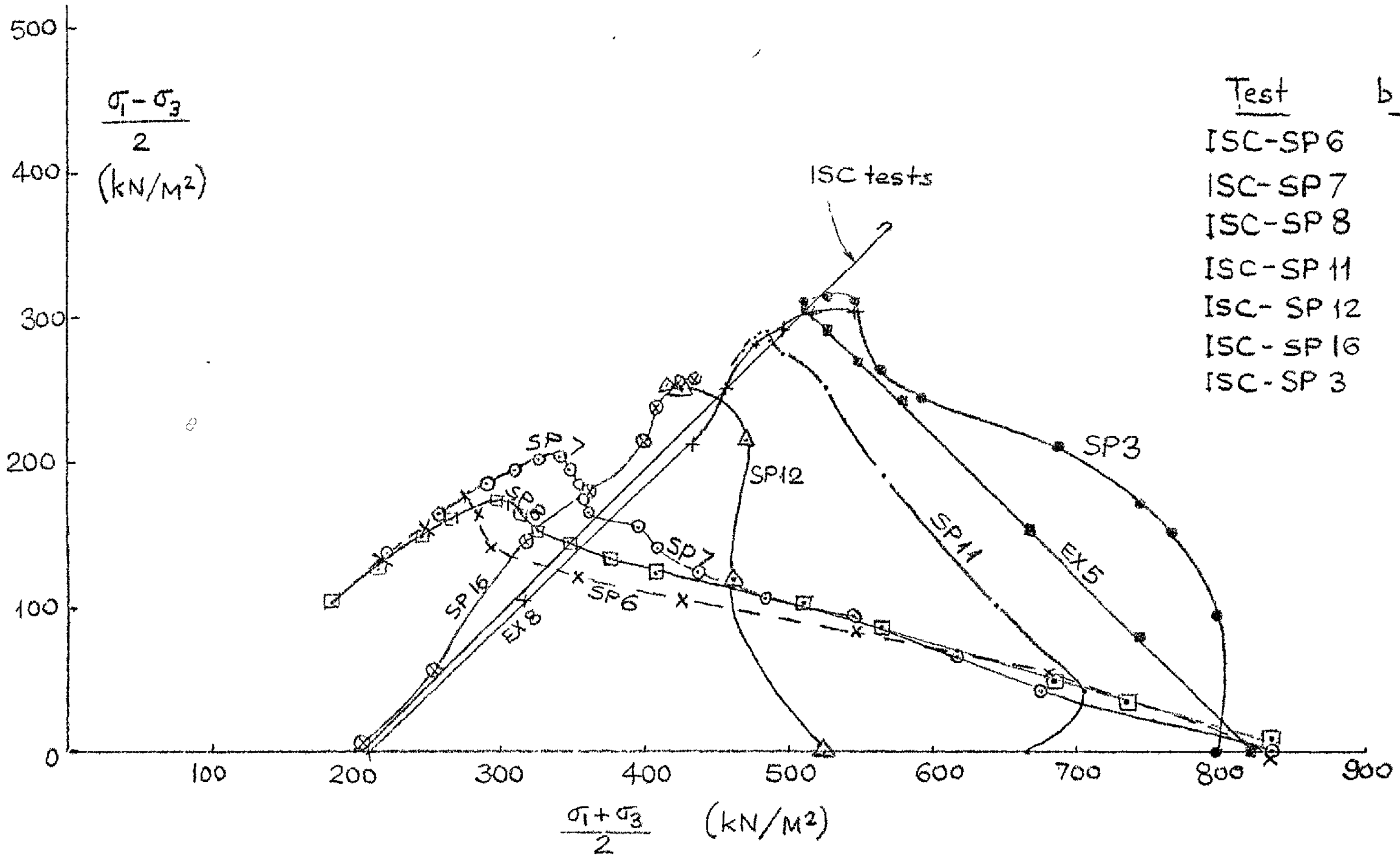
Fig. 5.9

TEST NO.	ISC(F)11	ISC(F)12	ISC(F)13	ISC(F)17	ISC(F)18	ISC(F)19
DESIGNATION	- - - - -	—+—+—+—	———	··········	—•—•—•—•—	—▲—▲—▲—
INITIAL POR. : n_i (%)	43.6	44.0	44.5	43.8	44.4	43.9
b :	.79	.72	.71	.89	.94	.92



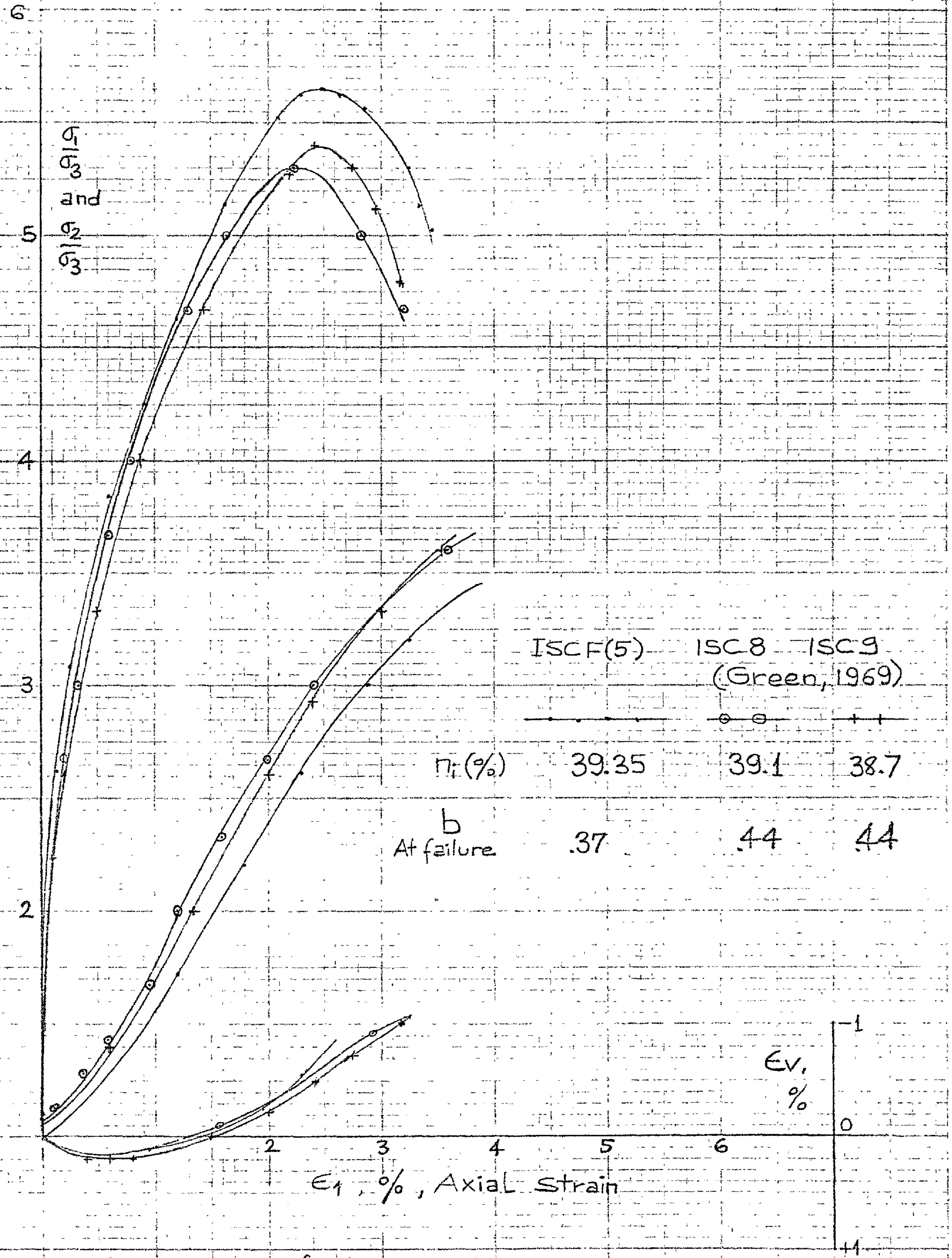
Comparison of stress-strain curves of flexible ISC tests at high b values on loose Ham River sand samples.

Fig. 5.10



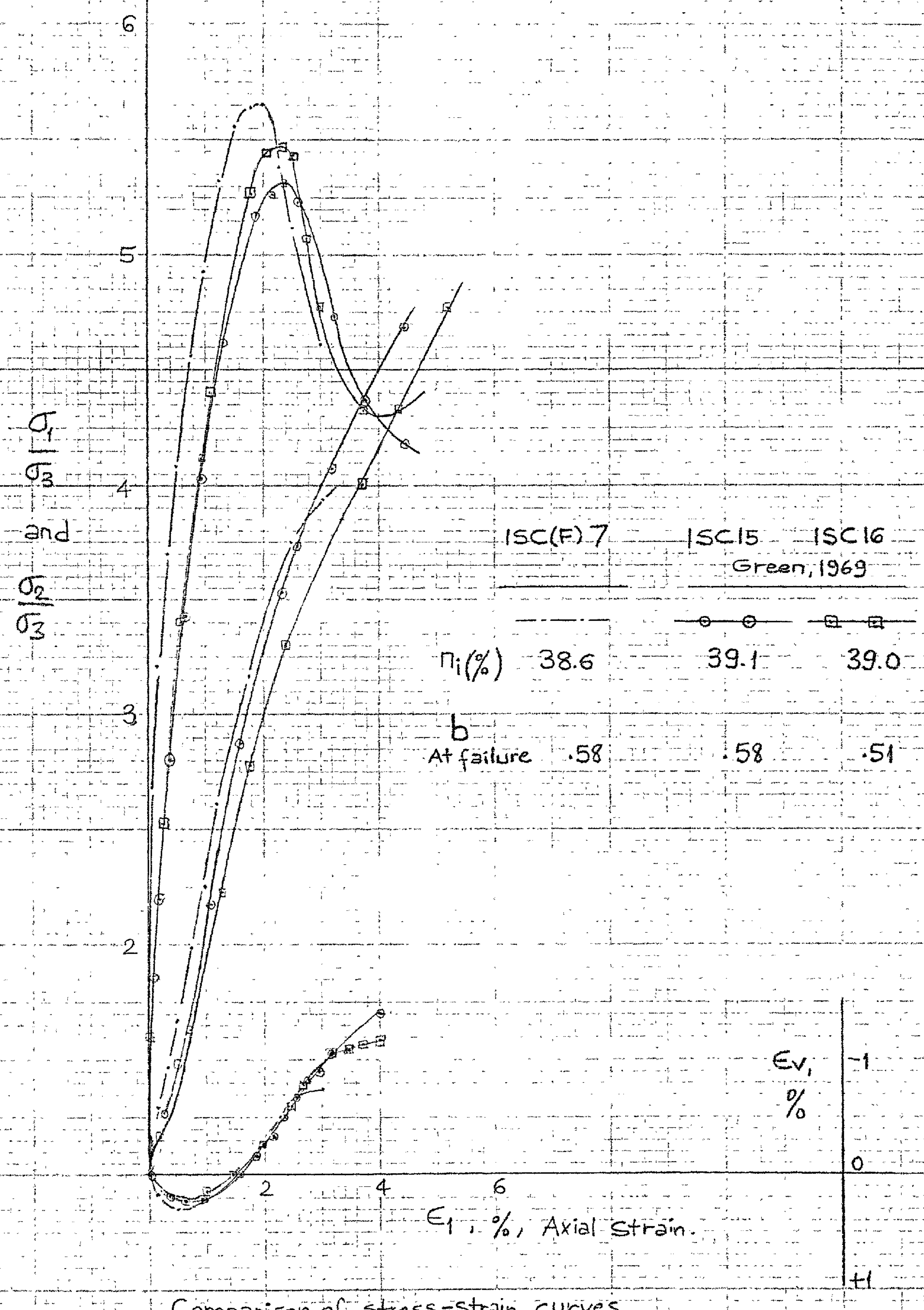
Test	b value
ISC-SP 6	.92
ISC-SP 7	.80
ISC-SP 8	.88
ISC-SP 11	.82
ISC-SP 12	.70
ISC-SP 16	.70
ISC-SP 3	.41

Fig. 5.11



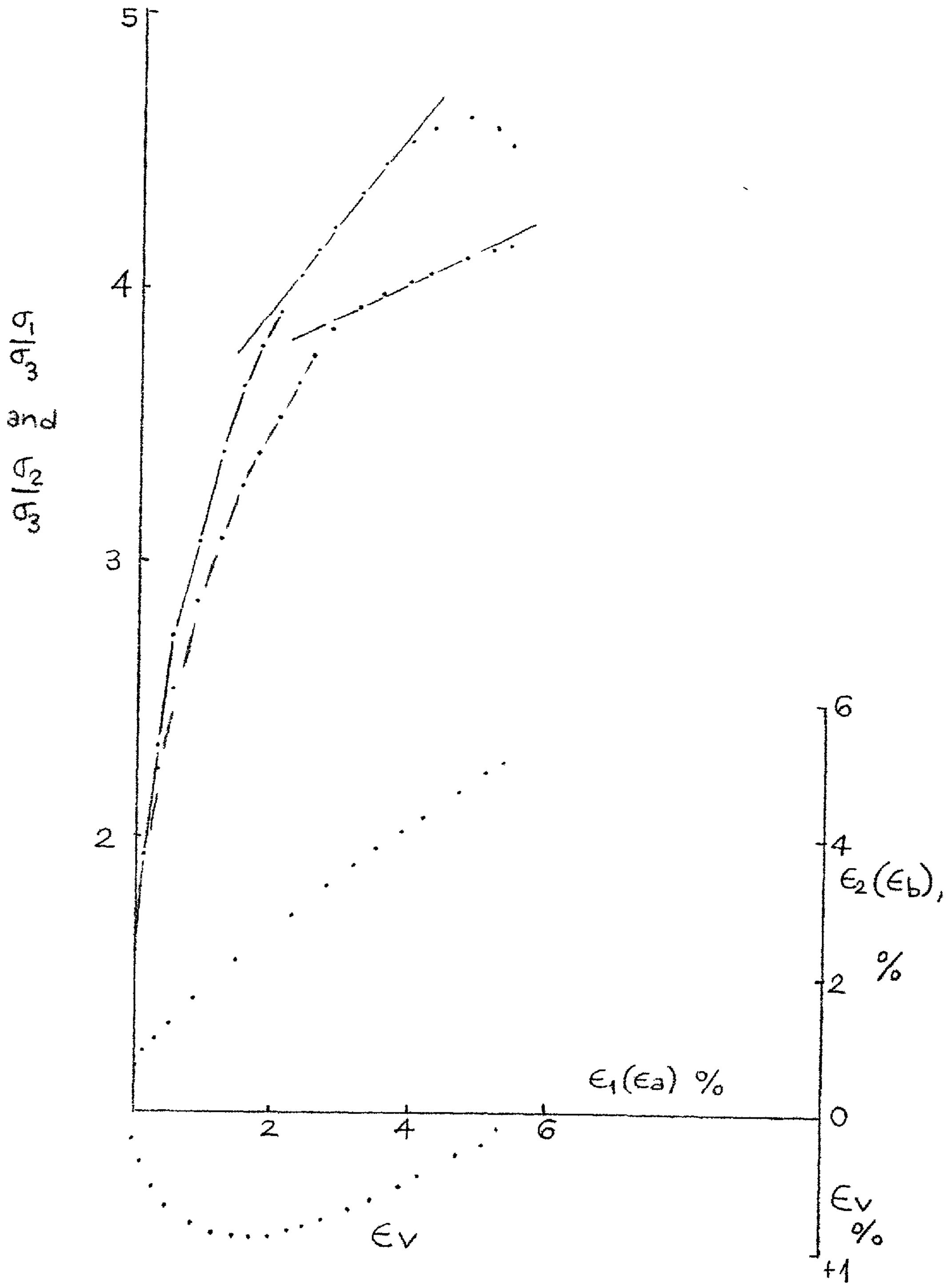
Comparison of stress-strain curves

Fig. 5.12



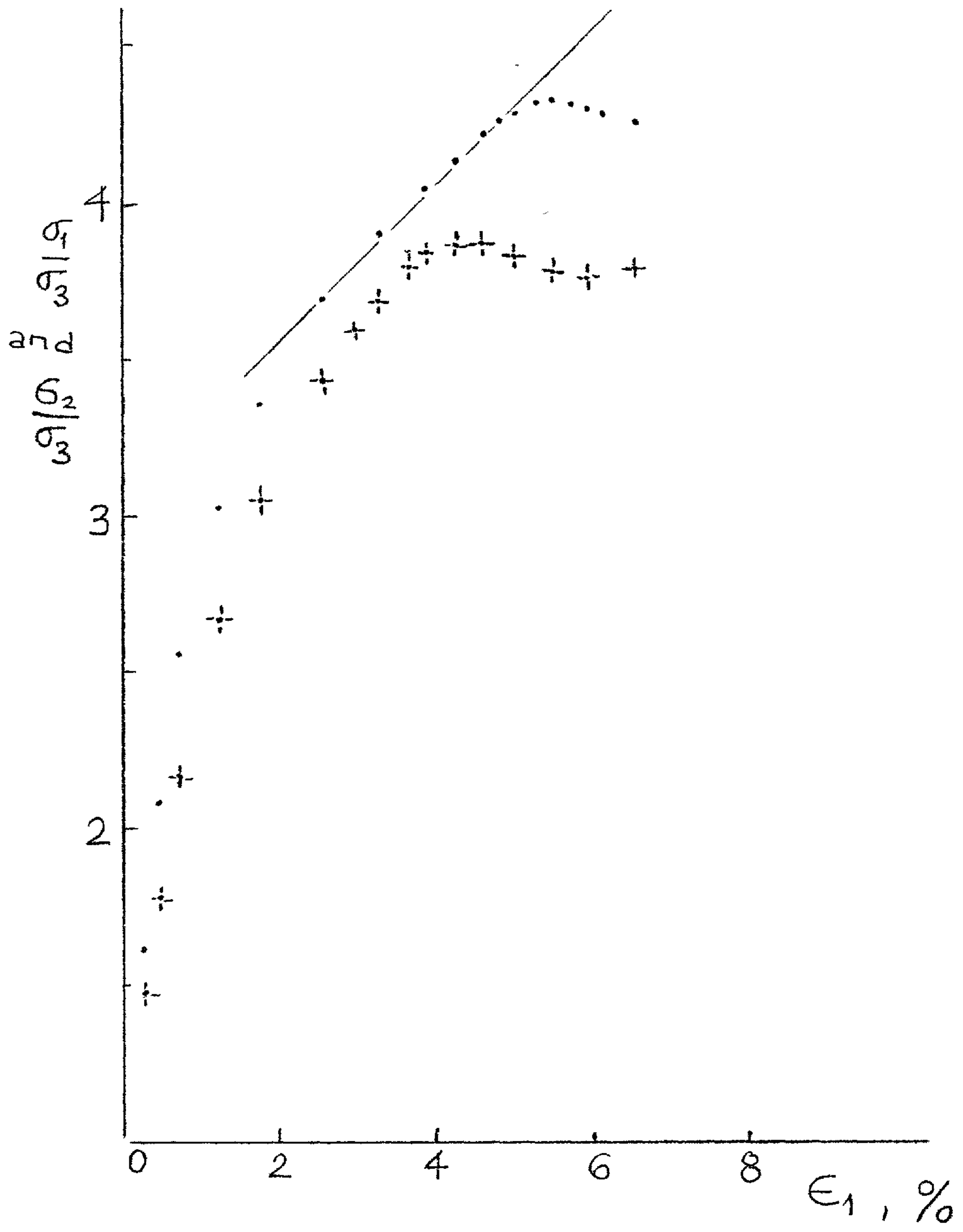
Comparison of stress-strain curves

Fig. 5.13



Stress-strain curve of ISC-SP5, Rigid plattens, Ham River sand, Loose, b=.86.

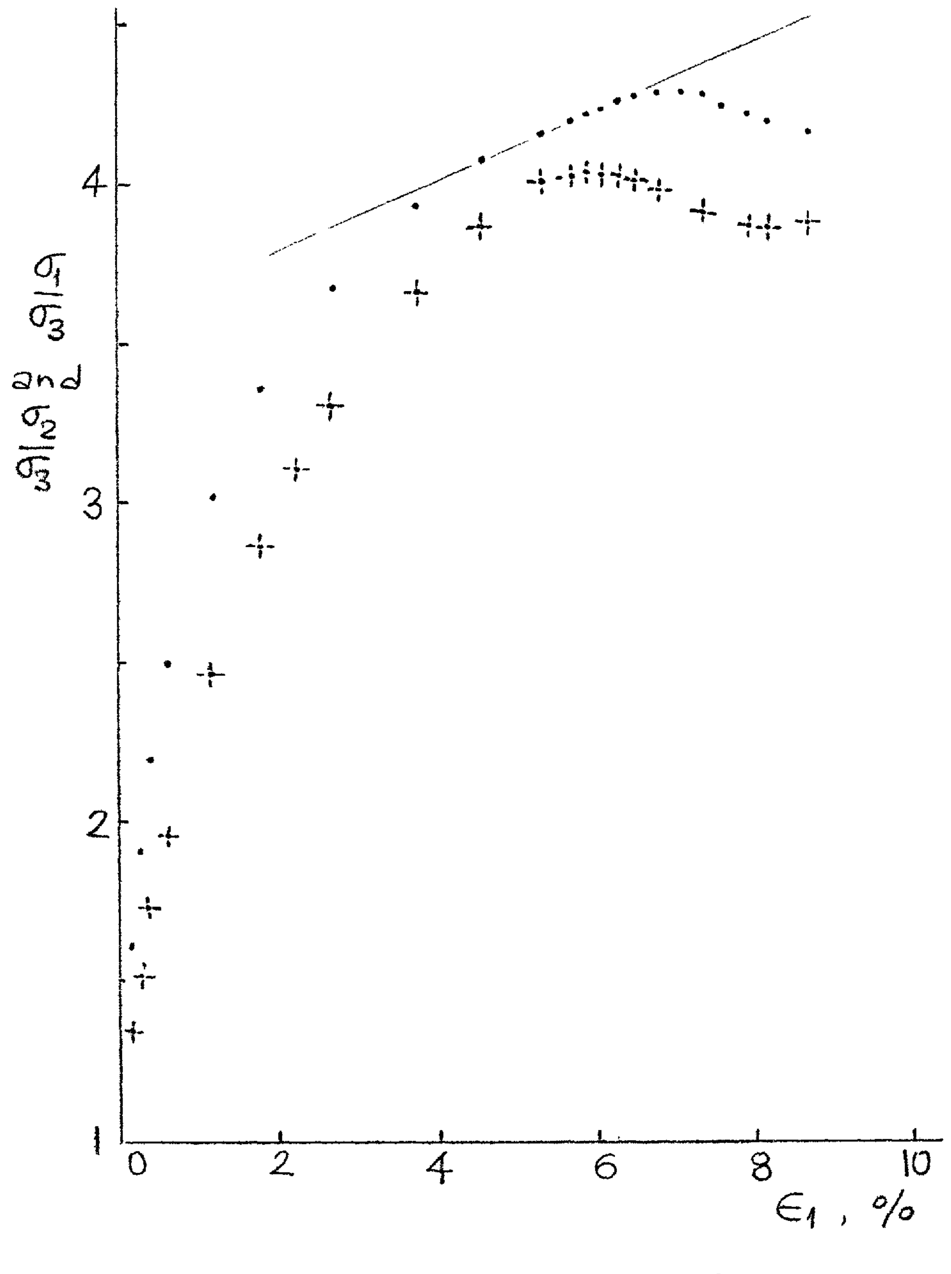
Fig. 5.14



ISC 38 , after Reades (1972)

$n_i = 45.4$ % , $b = .84$

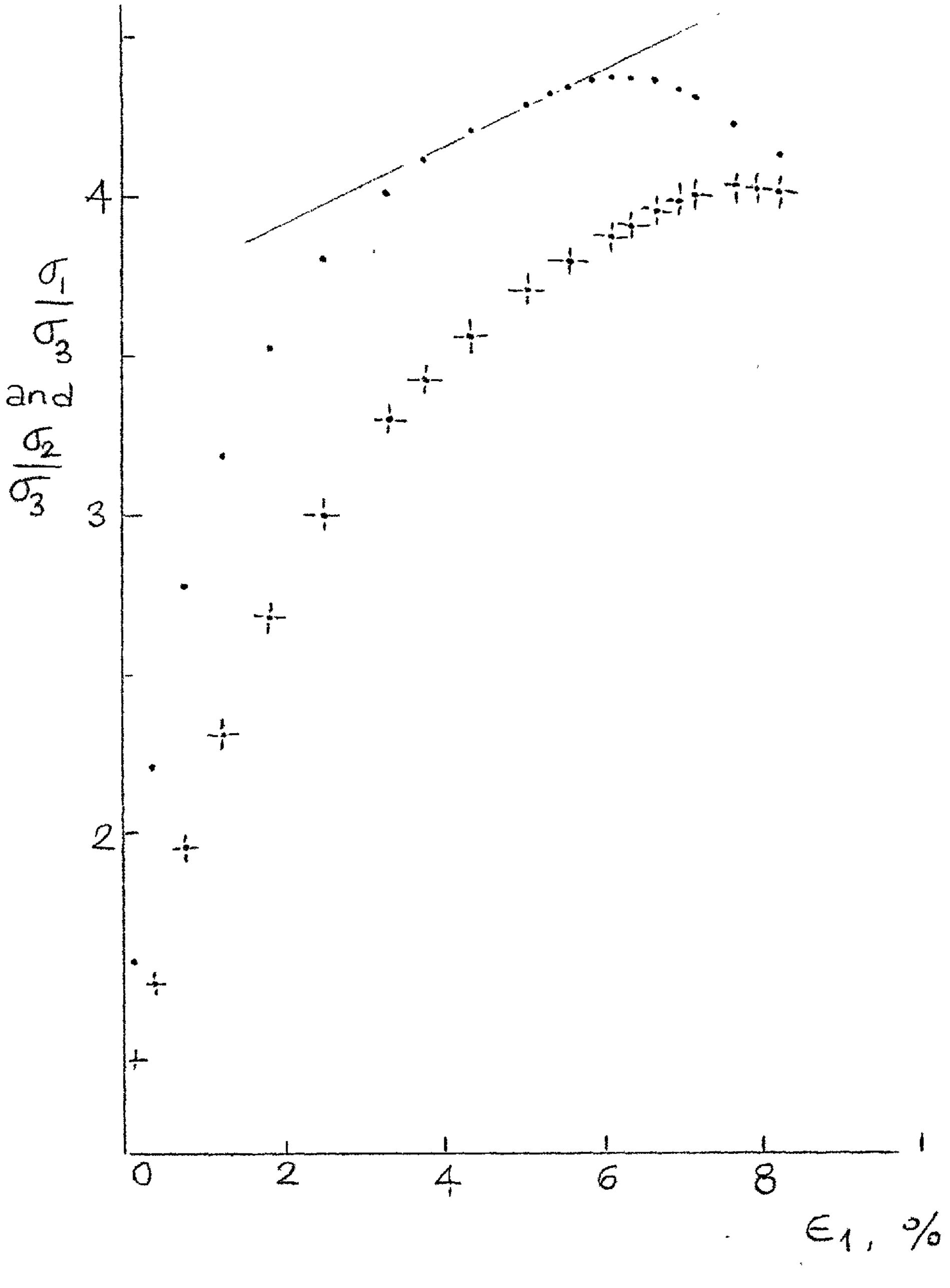
Fig. 5.15



ISC 43 , after Reades (1972)

n_i = 45.6 % b = .90

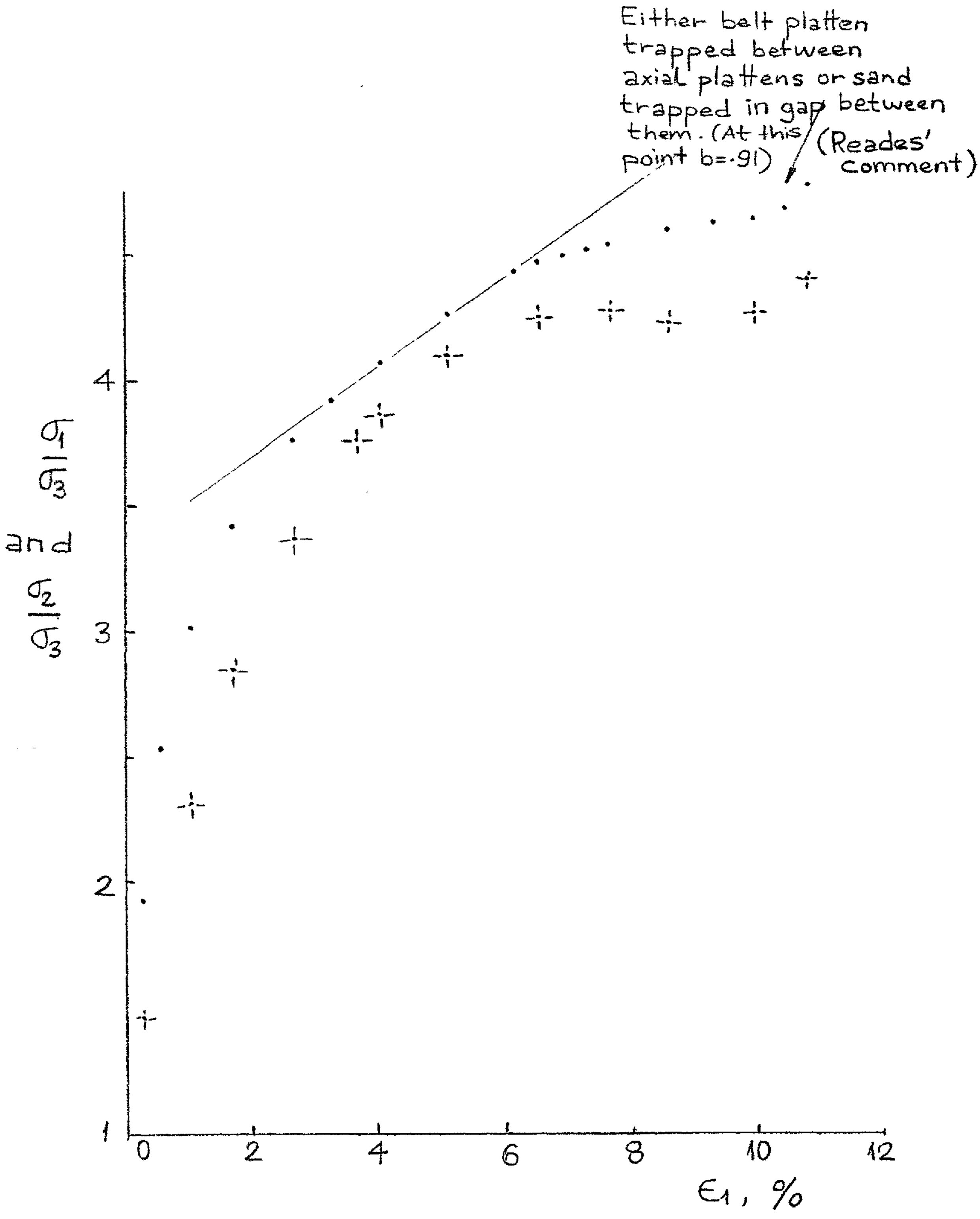
Fig. 5.16



ISC 36 , after Reades (1972)

n_i = 45.1 % b= .86

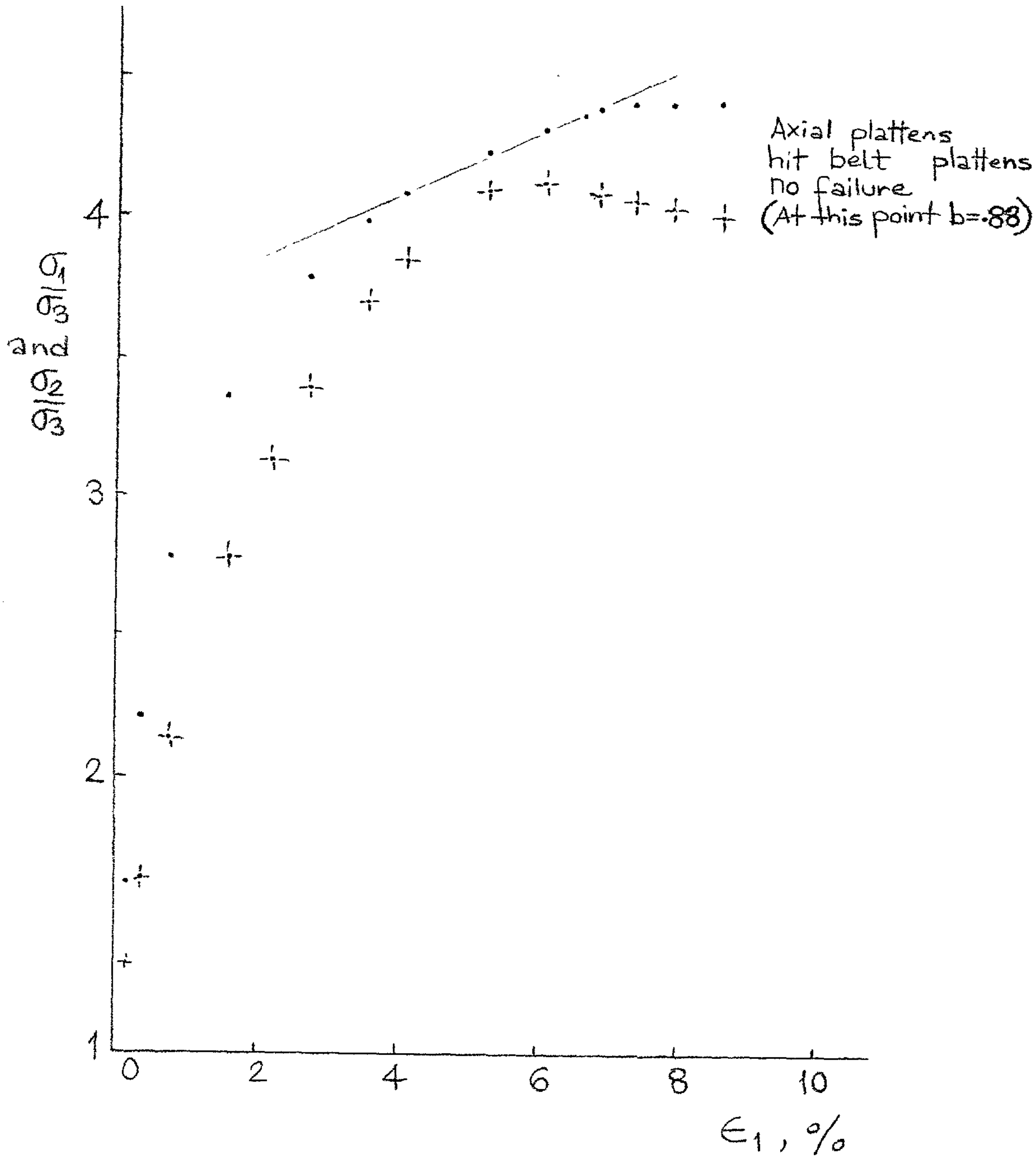
Fig. 5.17



ISC 18, after Reades (1972)

$n_i = 45.2$ %

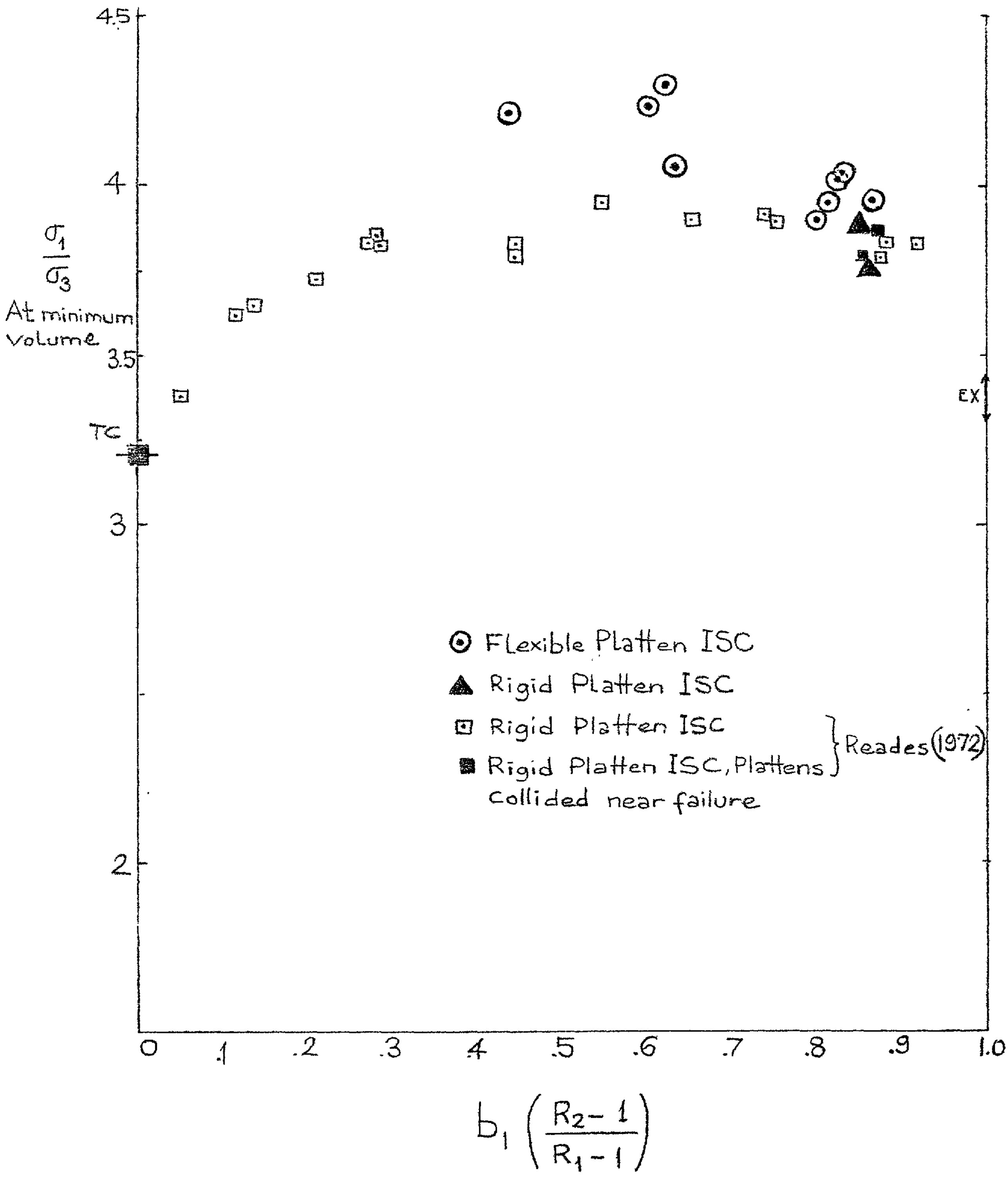
Fig. 5.18



ISC 42 , after Reades (1972)

$n_i = 45.2$ %

Fig. 5.19



Stress ratios at minimum volume in Loose ISC tests on Ham River sand.

Fig. 5.21

CHAPTER 6

RESULTS AND DISCUSSION OF TRIAXIAL EXTENSION AND SP9-16 SERIES OF TESTS ON HAM RIVER SAND

6.1. Introduction

A number of cuboidal triaxial extension tests on Ham River sand were carried out. They are in different series each aiming at the study of one or more specific points. The main series of tests from EX1 to EX12 inclusive are cuboidal very short samples about 7.8 x 7.8 x 4.9cm (height) in dimensions. As mentioned in Chapter 4 a special mould was made for these series. The idea behind this group of tests is as follows: When comparing the results of triaxial extension tests and ISC tests at b = 1.0, it was noticed that the dimensions of the samples in the two groups of tests were not compatible with respect to the orientation of principal stress directions. ISC samples have the dimensions 5.3 x 8.5 x 8.5cm whereas classical extension samples are 5.8 x 7.8 x 8.5cm - the last figures being always the height-. Ratios of the samples dimensions in major vs. minor principal directions are 8.5/5.3 = 1.60 and 5.8/8.5 = 0.68 respectively, see figure (6.1). In the present series this ratio is 7,8/4.9 = 1.59 being very similar to ISC tests.

In addition, for ISC tests, samples are consolidated to 207kN/M^2 and then sheared with mean normal stress increasing with σ_3 constant. Whereas in classical extension tests samples are consolidated to about 700-800kN/M^2 and then sheared with mean normal stress decreasing. Some of the tests in the present series of triaxial extension tests were first consolidated to 207 kN/M^2 then sheared with average stress level increasing. The second group of tests had the same dimensions of samples as used by Green (1969) and Reades (1972), but the mean stress level was increased during shearing. In all tests 2/0.30mm lubricated rubber sheets were used on top and bottom stainless steel polished plattens to reduce the friction.

The Writer also thought that it was imperative to perform extension tests especially on loose samples, because in the final analysis and comparison with the other tests it would not be sound to base the argument on the tests which were carried out by another researcher where differences might arise because of differing technique in sample preparation, calibrations etc.

The presentation and discussion of the tests will be for the Writer's own tests and Reades' tests on extension samples which are more in number than the Writers'. For other tests, comparison and discussion of the findings will be presented when the analysis of each group of tests has been completed. It must be also emphasized that the main interest will be focused on loose samples of Ham River sand.

6.2. Test Program and Presentation of Results.

First series; EX1-EX12: Short - this term has its meaning relative to the vertical - cuboidal samples, 7.8 x 7.8 x 4.9cm, mostly medium dense to loose samples. Type A series consists of tests EX1, 2, 3, 4, 6, 8, 9. They were consolidated to a cell pressure of 207 kN/M^2 the same as the previous ISC samples, then while cell pressure was increased gradually the minor principal stress at the top platten was kept approximately constant resembling the constant minor principal stress-cell pressure-in ISC tests. This was achieved by adjusting the strain rate in the conventional worm gear mechanism. Type B tests include EX5, 7, 10, 11, 12. They were first consolidated to 700-1000 kN/M^2 and sheared at constant major principal stress (cell pressure) by decreasing the minor principal stress on top platten by withdrawing them. Tests EX2, 4 did not reach failure but their stress-strain curves have been included in Appendix 8. The kink in EX1 was due to unreleased clamping screws for the axial displacement dial gauge.

Second series: EX11 1, 4 and SP17. These are loose cuboidal samples of 5.8 x 7.8 x 8.5cm (height). Testing method is the same as type A tests in first series.

In the following paragraphs failure characteristics of samples tested will be presented and discussed together with the results of tests performed by Green (1969) and Reades (1972) on Ham River sand. The emphasis is on the loose and medium loose samples. Much controversy exists in stress-strain-strength properties of extension tests as already stated in Chapter 2. When it comes to the comparison of results for one type of test, such as, plain strain or triaxial compression with an extension test, extra care is needed because of the uncertainities involved in extension tests. This is expecially true when peak strengths are compared.

The most important problem is the non-uniform deformation of the samples. Extension samples do deform non-uniformly no matter whether dense or loose though in different modes. In extension tests neck forms at some point during the tests. In some of the very non-uniformly deformed samples it may even start growing at very small strains, but, near or at failure, it is more definite, and much of the deformation takes place in a limited section of the sample. All test calculations are done assuming a right cylinder or a rectangular prism deforming uniformly. Therefore a correction must be applied to take this necking effect into account. In Appendix 2, details of this non-uniformity correction is given. Briefly it consists of measuring the dimensions of the failed sample and then deducing the effective sample area at failure. Unless continious monotoring and recording of the deformations by transducers and recording equipment or by some other means, it seems logical to deal with non-uniform deformations in this way. Since the deformations are more and more concentrated into the neck area it is important that the test should be stopped at the peak as soon after as possible.

In the calculation of correct ϕ' sample sheath resistance during shearing, average stress level and porosity differences have all been accounted for (see Appendix 2).

Failure characteristics are given in table 1, and all measured strengths have been plotted against initial porosity in figure 2, results of extension tests on rectangular and cylindrical samples by Reades (1972) and the line proposed by Green (1969) are also included. It is clearly seen that ϕ' values obtained from the first series are on average 1^o- 2^o higher than the second series. Although a few tests were obtained for the second series of tests, which had the usual sample shape (H/D=1.5) they give much the same ϕ' values as obtained by previous researchers. No noticable effect of the different stress paths in the results can be discerned of the first series. The 1^o- 2^o difference in ϕ' either originates from the size or shape of a short sample in the vertical direction or from other factors.

One of the possibilities in the former case is the type of necking and non-uniformity of short samples relative to longer ones. Since when calculating uncorrected strengths the area calculations are based on uniformly deforming samples, neck formation directly corresponds to under-estimation of measured strengths. Therefore uniformly deformed samples will result in higher measured strengths. But, it will be seen that short samples do not yield any more uniformity. Long samples usually neck in the middle, short samples near the bottom plattens.

Extremely good quality calibration scheme is necessary in axial load measuring mechanism in extension tests, because any small change in minor principal stress will cause recognisable strength changes. The writer has always been very careful in performing extension calibrations for the axial proving ring, and has adapted a dead load method up to 3600N instead of using the calibrator (see details in Appendix 3). In this respect, care should be taken when quoting peak strengths from tests by other researchers, especially from some of the early studies where external axial load measurements requiring corrections for bush friction may not be accurate enough. The Writer's calibrations with the calibrator - "budenberg system" yielded higher values compared to dead load calibrations. Since Reades (1972), used the calibrator in his laboratory programme, the 1^{o}- 2^{o} difference in ϕ' obtained, must be a minimum. The second series of tests indicate lower values with respect to the tests in first series. Two dense samples in the first series EX9 and 10 also show higher values.

Although the Writer knew the importance of stopping an extension test immediately after failure, he was not able to stop many of his first series of tests near peak values in particular those in EX1-8. The main reason for this hinges on increasing the cell pressure as major principal stress during shearing in several tests in the group whereas the end of test strains are fairly close to peak strains in the constant cell pressure tests (like EX5). In EX3, 6, 8 for example, end of test strains are well after peak with apparent failure planes. The bottom half of the samples underwent higher strains relative to the top half in all three, and failure planes were inclined at 15°- 16° to horizontal intersecting the edge of bottom plattens.

Unfortunately the importance of stopping strain after peak was not realised until the actual non-uniformity corrections were being considered. In tests with increasing cell pressure it is not possible to locate the peak without actually calculating the stress ratio or approximately plotting the volume changes, so such tests were stopped after observing the failure plane. Sometimes even a drop in deviator load reading in constant cell pressure extension tests does not necessarily mean that peak has been passed. It may be logical to think that shorter samples should fail more uniformly but the end of test measurements do not reveal more uniformity. EX11 and 12 were intended to be stopped very close to peak, they were stopped just before peak, and together with their photographs they help in showing what short samples look like at failure strains. Figure 3.

Failure characteristics like $d\varepsilon v/d\varepsilon a$, $d\varepsilon v/d\varepsilon b$, ε_a, ε_c, ε_v have been plotted in figures 4, 5, 6. Again Green and Reades' results have been included for comparison (solid and dashed lines respectively). Volume change rates in the first series with respect to ε_b (major principal strain) are little higher than those of the second series and Reades' except EX1 and 3.

Axial and lateral strains in the first series are appreciably higher than second series and Reades' tests. This is especially clear in the two short dense samples EX9 and EX10.

The agreement between Writer's second series and Reades' - which have the same sample dimensions as the second series - is good, and this is evidence that the differences in behaviour between Writer's first series of tests and Reades' tests do not come from changes in technique calibrations or measurements etc. Volumetric strains are higher in the tests with increasing mean stress level compared to the tests with decreasing stress level. Although the second group used long samples they were not the classical type of tests with a constant cell pressure but with increasing cell pressure. This is reflected in the volume changes measured. EX9 and 10 also give a similar comparative behaviour.

6.3. General Discussion and Corrected ϕ' Values.

The scatter in measured ϕ' values is reasonable which is about a degree in the first series excepting EX1 and 7. The scatter might be functions of the mode of deformation of the sample and non-uniform deformations which either originate from an initially non-uniform sample shape - which is usually more common in dense extension tests - or internal density differences or both. Sample preparation method directly influences internal porosity distribution. As summarised in Chapter 4, the method of depositing in small quantities under water proved to be one of the best methods of loose sample preparation.

The same top and bottom plattens in the ISC apparatus were used for rectangular compression and extension tests. The top platten weighs about 16N which is quite heavy for loose samples. If the placement of it is considered - stretching the membrane around it, placing O-rings and especially inserting the extension pin in earlier tests - it is obvious that the disturbance cannot be avoided completely. This suggests a denser structure near the top platten relative to bottom, and this may well be one of the reasons for samples experiencing larger strains and failure planes at the lower half of the sample most of the time.*

As expected and observed, every test was unique in its deformation mode even in the same group of tests although there were general similarities. In the second series, failing samples formed a neck at the middle of the height

---

* Green (1969) reported similar observations in his series of extension tests.

rather than near the bottom as observed in the first group. By calculating the deformations based on the assumption of a right rectangular prism, Reades (1971) measured and concluded that axial strains were underestimated up to 30 percent in loose samples.

It is interesting to note that the more uniform the sample at failure is the higher the measured ϕ'. EX7 and 12 (see figure 2), are good examples of this where they are 1.5^o and 1^o higher than the average line.

In figure 7, stress-strain graphs of several tests have been plotted for comparison, including the two loose tests, XR5 and XR12 by Reades. Short samples are seen to have, in general, higher strengths and moduli. They are a little denser than the long ones but this would not account for the difference.

Increasing or decreasing the average stress level seems to have no effect on the ϕ' value. SP17, an average stress level increasing test, which has not been included in the graph has an almost identical stress-strain curve to EXII-4 and XR5. This is good because there was a period of half a year between these tests, and more important, loading cylcle of the proving ring was reversed in the intervening period. (The proving ring can be used both in tension and compression). Agreement between the dead load calibrations was extremely good in spite of the long duration between calibrations. (see Appendix 3).

Which void ratio should be used in correlating and comparing test results is a point of controversy among many researchers. Green and Reades always used initial porosities for Ham River sand. It is true that the initial dimensions of a sample are the most reliable, yet it is also true that especially loose samples experience considerable volume changes (contraction) during the consolidation stage of tests. Therefore, two identically prepared samples result in different porosities if the consolidation pressures are different. However their new dimensions (and porosities) can not be measured exactly unless a special instrumentation is used. A plot of measured ϕ' values from the Writer's extension tests and Reades' tests against consolidation porosities is seen in figure 8. It is interesting that the difference between the ϕ' values from short and long samples has been increased as can be seen from the comparison of figures 2 and 8. This may also have significance in comparisons with other type of tests. End of test porosities are also very important, but unfortunately, non-uniform deformations and probable density differences between the actual failing zone with respect to the other parts of the sample (or average density) make it very difficult to estimate the porosity at failure.

Corrected Strengths: Results of triaxial extension tests need to be corrected to enable comparisons to be made with other types of tests. Corrections for the rigidity of the sample sheath, mean stress level and non-uniform deformations have to be applied to obtain the corrected strengths. Details of these corrections have been presented in Appendix 2. Sample sheath corrections to ϕ' normally vary between $.3^{o}$-$.5^{o}$ and are subtracted from the measured values. In all tests except EX3 and EX7 (σ_3= 687 and 704 kN/M^2 respectively), the cell pressure was higher than 790 kN/M^2. Mean stress level corrections to ϕ' for loose samples are between .2-.35^{o}, and for the two dense samples, EX9, 10; .90^{o} and .75^{o} respectively. Sample sheath and mean stress level corrections roughly cancel each other so that the non-uniformity correction is the one which dictates the overall correction. In fact, it is much larger than the others. Unfortunately this correction, is the most uncertain. While Roscoe et.al.(1963) exaggrated it, others - see for example Cornforth (1961), Barden and Khayatt (1966), Bishop and Green (1965), Green (1969) - tried to apply corrections by using a smaller average plan area in the axial direction within the zone of the failure plane or minimum neck area rather than using an overall average area. Barden and Khayatt (1966) applied ϕ' corrections of 3.2^{o}, 6.2^{o}, 8.2^{o} to some of their 8"x4" cylindrical dense and medium dense samples on the basis of neck area measurements. Cornforth (1961) applied corrections up to 3^{o} to his samples, and Green (1969) used corrections between 2.7^{o} - 4.2^{o} for his 7" long cylindrical dense samples. He concludes that even in his "best tests" corrections 1^{o} - 3^{o} are necessary, which he describes as "far from satisfactory".

The Writer is no better off since the majority of his extension tests were not stopped at peak, the worst being those in which increasing cell pressure and constant axial stress were aimed at.

The width ratios of the sample at various heights will not be the same at every stage of the test, and changes in these ratios will probably accelerate towards failure. End of test measurements may not represent the true ratio at failure, besides, upon release of the cell pressure, changes in sample dimensions may not be the same. In any case, there is no better way than stopping the test near peak and measuring the dimensions unless a special instrumentation system is used. As outlined in Appendix 2 plot of $(\varepsilon_{a_{stop}} - \varepsilon_{a_{peak}})$ strain differences against area ratios (minimum or average neck) may yield a reasonable relationship for corrections.

It was somewhat unexpected that the first series of tests on shorter samples did not deform more uniformly than long samples. For the first series of tests the non-uniformity corrections for ϕ' varied between 2°- 2.5° in loose samples which is higher than those of Reades' for his rectangular samples - which were 1.5°- 1.8° -. The Writer's few long samples suffered either puncturing - EXII-3,4- or were disturbed before reliable measurements were taken. (EXII.1) Few tests by Reades were used to estimate the corrections for the long samples. But these corrections can never be exact.

EX11 and 12 were stopped at 3.8 and 2.6 percent axial strain(minor) before peak. They must give the most conservative values (i.e. least correction) if the end-of-test rather than the extra polated non-uniformities at the peak are taken into account. They yield ϕ' corrections of .9° and 1.6° respectively. Corrected ϕ' values are plotted against initial porosities in figure 9. Reades' upper and lower bounds for his rectangular extension tests have also been shown for better comparison. If an average line is passed through Writer's tests it is a degree or more higher than Reades' Upper bound, and if an average value is taken for Reades' data there is approximately a difference of 2° between the Writer's short extension tests and Reades' extension tests on average. This is partly due to 1°-2° difference in measured ϕ' and partly the difference in non-uniformity corrections. Dense samples EX9 and EX10 are in agreement with the upper bound where as EXII-1 and EXII-4, long samples, average on the lower bound.

Corrected ϕ' values plotted against the rate of volume change with respect to lateral (major) strain can be seen in figure 10. Here again the Writer's tests are above the line proposed by Reades from his cylindrical and rectangular tests. But, in fact, most of his rectangular samples fall somewhat below this line (i.e. it is an upper bound for them).

6.4. Conclusions

The purpose of the present extension tests must be remembered; They are, first to test a similar sample geometry to ISC samples in triaxial extension as far as the directions of the principal stresses are concerned, secondly, not to base the comparisons between extension tests and ISC test near b=1 on Reades' extension tests only, and finally to observe the differences, if any, between the group of tests with increasing and decreasing mean normal stresses.

There is 1°-2° difference in measured ϕ' between short samples and long samples - for both the Writer's and Reades' tests - the former being higher. The location of the failure planes developed and deformation patterns are different. Short samples experience higher strains in the bottom half, and failure planes join the bottom loading platten, whereas long samples form a neck in the middle of the sample.

Measured ϕ' values have been corrected for sample sheath rigidity, mean stress level and non-uniform deformations. The latter governs the total correction applied. Net corrections applied amounted to 2°.-2.5° almost the same as the non-uniformity correction whose magnitude is rather uncertain. Lower ϕ' measured in Writer's few long extension samples precludes the possibility that the ϕ' values of short samples in the first series are higher than Reades' long extension samples because of the various differences involved in test techniques. The difference is thought to have come partly from dissimilar sample shape and mode of deformations and partly from the difference in non-uniformities which can not be determined exactly.

Examining all the results carefully and bearing in mind the uncertainity in the corrections, it is concluded that 36.5° must be a good estimate for the corrected ϕ' value at 44 percent initial porosity although it is a bit conservative. Long samples vary between $\phi' = 34.5^{\circ}$ and $\phi' = 36.2^{\circ}$ i.e. upper and lower bounds. If such limits are to be imposed on short samples they should be roughly $\phi' = 35.7^{\circ}$ to $\phi' = 38.0^{\circ}$.

Short samples show larger axial and lateral strains than long samples. Volume change rates with respect to lateral and axial strains agree with long samples generally but a more careful inspection reveals slightly larger values for short samples.

Stress path to failure, whether increasing or decreasing mean normal stress seemed to have no observable effect on the ϕ' values. Deformation properties like volume changes are affected, resulting in larger values.* Even a few tests in the second series are sufficient to indicate the trend of higher values relative to Reades' tests under constant cell pressure.

---

*In the case of increasing σ_m

6.5 Special Series of Generalised Tests

6.5.1. Introduction:

Before bringing the test series on Ham River sand to an end, it was thought it would be very useful to perform generalised tests in another mode of the ISC apparatus. Actually these rather restricted number of tests had long been planned but due to several reasons were delayed until later stages of the test programme. They are designated as ISC-SP, or for convenience SP, and there are eight in number, from SP9 to SP16 inclusive. Researchers in 3-Dimensional testing should carefully and clearly explain the mode of testing they adapt when reporting their results. As will be fully discussed in the next chapter much confusion in 3-Dimensional testing literature originates because different modes oftesting are employed. Almost all the ISC tests throughout the research period by Green, Reades and the Writer were carried out by using a constant cell pressure and increasing the axial and lateral stresses by driving the axial and belt plattens inwards and thus failing the sample into the cell pressure-minor principal stress - direction, axial stresses being the major principal stress. Some researchers in generalised testing have used one of the lateral directions to apply major principal stresses and load on the axial plattens to provide minor principal stresses, and in the remaining lateral direction intermediate stresses have been applied. See, for example, Sutherland and

Mesdary (1969), Ramamurthy and Rawat (1973). One may argue that sample response should be the same whether it is sheared vertically or horizontally provided that similar conditions exist in them. This may not be true if a cubical generalised testing apparatus does not have six identical loading plattens. Even with an apparatus like this it may not be true if stress or strain control and loading or unloading modes are not identical in the two cases. Therefore, in the present series of tests samples are consolidated and then sheared by applying major principal stresses laterally through the belt plattens. In some of them consolidation pressures are initially high, and loads on the axial plattens are decreased to failure as in the conventional extension test while constant cell pressure acts as the intermediate stress. In some others the consolidation pressure is 207kN/M^2 (cell pressure) initially, then it is gradually increased together with the belt load while axial loading platten is again withdrawn, providing virtually the same mode as the previous group the only difference being that approximate constant mean stress level is held, instead of a decreasing one. Tests were planned to cover the intermediate stress space for $b > 0.50$ that is, in the range when there is more controversy as will be clearer from the following chapter. Belt loading technique used in these series is different from other ISC tests in that the belt jack system is connected to self compensating mercury pots rather than to a motor-screw piston system, i.e. stress rather than strain control. Decreasing mean stress level tests were performed after the pot height was fixed while in the increasing stress level tests the pots were driven to keep pace with the cell pressure.

6.5.2 Results and Discussion

Failure characteristics ϕ', $^{d\varepsilon v}/_{d\varepsilon b}$, ε_b, ε_v, ε_a are given in table 2, and are plotted against b in figures 6, 11, 12, 13, 14, & 15, and a plot of ϕ' vs. $^{d\varepsilon v}/_{d\varepsilon b}$ is presented in figure 6.16 Most of the measured ϕ' values are between 36^o-37^o. Measured ϕ' values in SP12 and 16 are exactly the same at the same b value. SP10, 11 and 14 agree to within a degree. It must be noticed that SP10,11,12 are constant cell pressure tests (σ_m decreasing) whereas the others, 14, 15, 16 were sheared while the cell pressure was increasing. SP9 and 13 were stopped because well-developed necks were observed.

When plotting the failure characteristics other than ϕ' no corrections were applied. Initial porosities are very close, and average stress levels are not much different with the exception of SP15. When comparing with other types of tests due care will be given for such variables.

Before further examining the test properties an important issue must be emphasized. It is the sample has a tendency to form a neck in these series of tests. Since the top platten is withdrawn as for a classical extension test, lateral deformations can not follow the change of shape uniformly. Since the major principal stresses are applied through a pair of rigid plattens in one of the two lateral directions in excess of the cell pressure, it directly suggests non-uniform normal stress distributions along the belt plattens virtually resulting in no or very little normal stress at central sections of the samples*. This causes the belt stresses to shoot up. This behaviour has been noticed in tests with high b values, i.e. when the belt deviatoric load is relatively low. SP9 and 13 were intended as b> .90 tests, and as it can be seen in the photograph , figure 6.17(a), the non-uniform shape is no better than that of a triaxial extension test. Both tests were stopped and abandoned, and the other tests were watched with utmost care. In SP14 at b= .84 necking was observed again though not to the extent as in SP9 and 13. SP10 and 11 were both at b=.82 and no apparent neck was observed in both tests, and the Writer, aware of this general inclination at high b values, was always suspicious about the uniformity of the stress distribution on belt plattens, because any slight amount of necking which may not be clear to eyes can cause such non-uniform stress distributions. SP12 and 16 (b=.70) seemed to have deformed uniformly and so did SP15 at b = .55. Rate of volume changes are comparable, and SP15 and SP16 gave little lower than the others. Belt-

---

*This should not be mixed with the discussion in Chapter 5.

major-strains agreedwithin 2 per cent. Constant or slightly increasing mean stress level tests SP14, 15, 16 gave agreeably higher volume changes than those in SP10, 11, 12 in which stress level was increasing. This was already expected.

Corrected ϕ' values are given in figure 6.11, with the total correction indicated. Corrections are for the rigidity of sample sheath, average stress level, non-uniform deformations and low belt platten contact ratio. For details see Appendix 2. Non-uniformity corrections are much lower than extension tests, only SP14 has a comperatively high one, 1.7°, it was stopped at peak with an apparent necking though not to the extent observed in triaxial extension tests. SP10, 11 have non-uniformity corrections of 0.55° and 0.65° respectively which are higher than those for SP12, 15 and 16, namely .15°, .25°, .35° respectively. This also supports Writer's suspicions about the distribution of belt stresses in tests SP10 and SP11.

At the start of a test, axial and belt plattens were set as close as possible to each other in these series, because the gaps grow larger during the tests due to the withdrawal of top platten, the gap will be largest at failure (Belt contact ratios were about 0.94 initially, they decreased to 0.82 - 0.87 at failure.) Although the belt stress σ_b, is obtained by adding the deviator stress to cell pressure, σ_c, (the gaps will be under cell pressure) the lack of deviator stress on the gaps must influence the strength because the belt stress is no longer the intermediate but the major principal stress. Reades (1972) concluded after his series of tests on horizontal loose plain strain samples (shearing with belt stresses as major, axial stresses as intermediate principal stresses) that 20 percent undersized platten (on the lateral sample face) in the direction of the major principal stress resulted in the underestimation of measured strength by more than 2.5°. But in that case, the belt deviatoric stress was much in excess of the low cell pressure of 207 kN/M^2, The present estimated corrections are at the order of .5° - .6° except for SP15 with a degree correction.

To be able to see the variation of the strength the tests should be normalised to a common density which was taken to be at 44 percent porosity. The net corrections are between 0.35° - 1.6° except SP14 which has 2.6° net correction. Comparisons between these series and other type of tests will be done in the next chapter.

Two distinct failure planes were observed near top and bottom plattens in SP11 and 15, and a single plane near the top platten in SP12 and 16. SP11 can be seen in figure 6.17 (b), photographed after failure .

Three stress path tests intended to investigate certain deformational properties of Ham River sand were carried out by Reades (ISC 64, 65, 66) at the end of his test program which required a change in the direction of application of the principal stresses in the ISC apparatus - This is the mode in which present series of tests were planned -. The fundamentally different mechanism of failure which will be explained later prevented him assessing the effect of stress path, and Reades, rather bewildered by the results, did not recognise these tests as representing the behaviour of Ham River sand, but he concluded that ISC tests in the normal mode were the true representations of the generalised behaviour. These three tests are in general agreement with the present series of tests.

6.5.3. Conclusion:

Generalised tests on loose sand samples by another mode of the ISC apparatus reveal that deformation pattern of the samples are different tending to form a neck especially at higher intermediate stresses. This mode is characterised by the laterally applied (belt) stresses being the major principal stress while axial plattens are withdrawn in the same way as for a classical extension test. Corrected ϕ' values are between 36^o - $38,5^o$ at 44 percent porosity.

Test No.	Ni %	Nc %	AT FAILURE σ_1 kN/M^2	σ_3 kN/M^2	$\varepsilon_{b=c}$ %	ε_2 %	ε_v %	$\frac{d\varepsilon_v}{d\varepsilon_b}$	$\frac{d\varepsilon_v}{d\varepsilon_a}$	Degrees ϕ' Meau-sed.	$\Delta\phi'$ S. Sheath	$\Delta\phi'$ Mean Stress.	$\Delta\phi'$ Non-Uni.	44% ϕ' Norm.
EX1	42.6	41.8	838.9	195.5	5.8.	- 9.6	2.08	- .24	.12	38.7	-.33	+.35	+2.30	39.4
EX2	41.0		Membrane puncture, no failure characteristics											
EX3	42.8	42.1	687.4	172.2	6.1	-11.7	1.52	- .18	.07	36.8	-.47	+.20	+2.10	37.2
EX4	39.8	39.2	Membrane puncture					-(.70)	(.24)					
EX5	43.3	41.8	821.2	217.4	5.3	-10.9	0.62	- .29	.11	35.5	-.32	+.30	+2.36	37.0
EX6	43.8	43.2	793.7	217.2	6.3	-11.6	1.96	- .23	.09	34.8	-.39	+.30	+2.26	36.7
EX7	43.5	42.2	704.1	170.3	6.5	-14.8	-0.05	- .33	.12	37.6	-.59	+.20	+2.51	39.1
EX8	43.3	42.6	787.9	203.3	5.7	-11.8	.91	- .31	.11	36.2	-.40	.30	+2.41	37.6
EX9	39.5	38.9	1031.9	206.0	4.4	-10.0	- .46	- .97	.27	41.8	-.28	+.90	+ .27	43.7
EX10	38.8	36.7	965.9	183.0	3.3	- 8.3	-1.25	-1.03	.31	43.0	-.28	+.75	+1.36	45.2
EX11	43.9	42.5	833.2	222.8	(6.8)	-13.8	(.90)	-(.29)	.10	35.3	-.31	+.35	+2.53	37.7
EX12	44.2	42.9	790.9	207.6	(6.7)	-13.7	(.70)	-(.34)	.12	35.8	-.36	+.30	+2.30	38.2
EXII-1	44.1	43.5	790.3	246.6	5.4	- 9.5	1.81	- .17	0.07	31.6	-.33	+.30	+1.65	33.4
EXII-2	43.9	43.3	Leakage											
EXII-3	44.2	43.5	Membrane puncture at						3.50					
EXII-4	44.1	43.5	832.1	245.0	5.4	- 9.7	1.87	- .17	0.07	33.0	-.32	+.35	+1.70	34.9
SP17	44.5	43.7	863.7	255.9	(6.5)	- 9.6	2.25	- .11	(.20)	32.9	-.36	+.37	+3.47	37.0

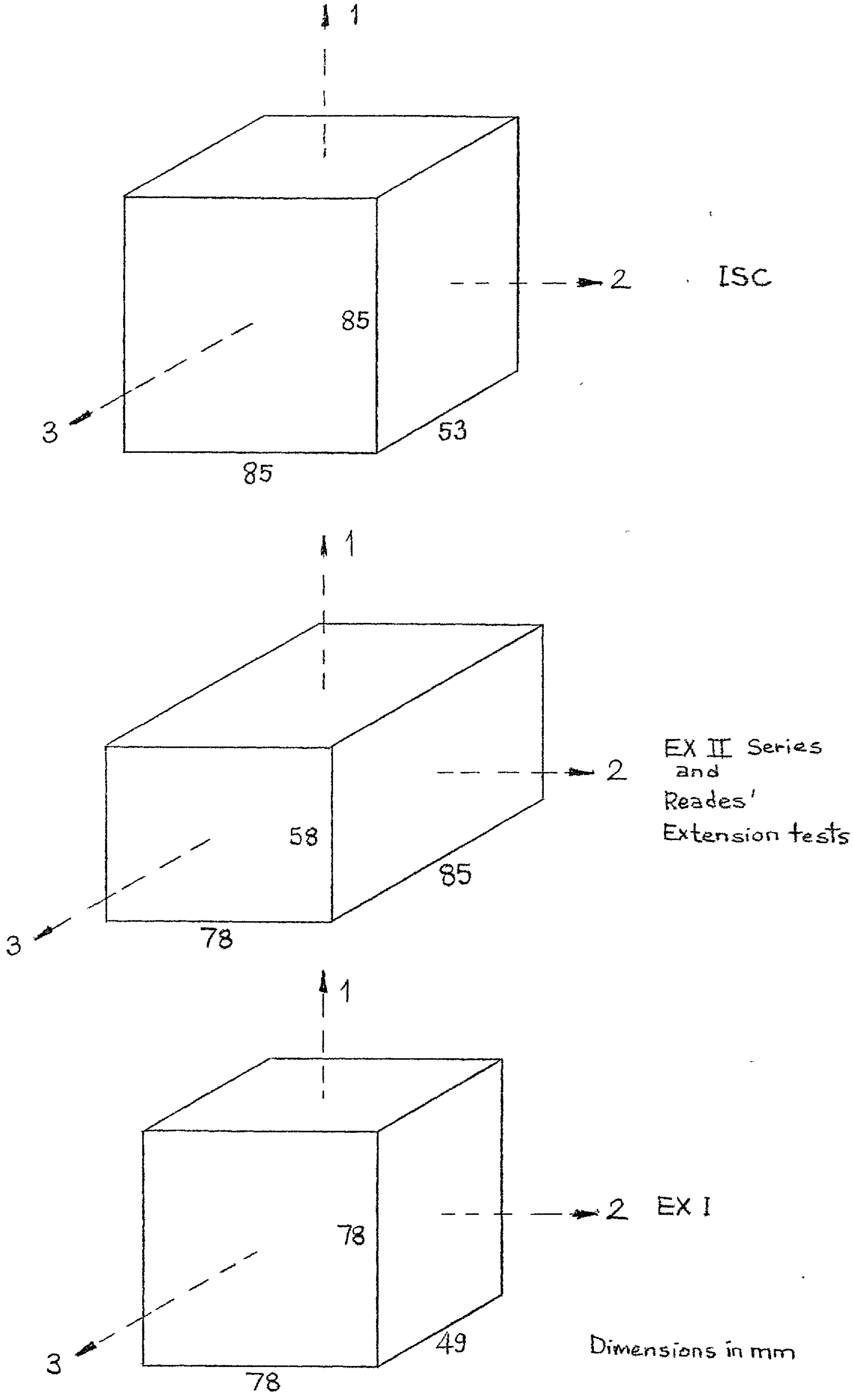
Failure Characteristics of Extension tests on Ham River Sand.

Table 6.1

Test No. ISC			AT FAILURE														
	N_i %	N_c %	b	σ_b kN/M^2	σ_c kN/M^2	σ_a kN/M^2	ε_b %	ε_a %	ε_v %	$\frac{d\varepsilon v}{d\varepsilon b}$	Belt Cont-act Ratio	ϕ' Meas-ured	$\Delta\phi'$ Belt C	$\Delta\phi'$ Non-Uni.	$\Delta\phi'$ S. Sheath	$\Delta\phi'$ Mean S.	Norm. ϕ' 44%
SP9	43.8	42.5	>.9	Stopped due to excessive necking													
SP10	44.4	43.1	.82	764.0	661.1	187.3	7.40	-13.0	1.10	-.19	.82	37.3	+ .55	+ .57	-.58	+.32	38.6
SP11	44.4	43.2	.82	766.0	663.6	194.1	7.70	-12.6	0.80	-.19	.83	36.6	+ .47	+ .65	-.60	+.32	37.9
SP12	43.9	42.7	.70	670.0	522.2	170.9	6.00	-7.6	1.02	-.19	.87	36.4	+ .60	+ .15	-.38	+.10	36.8
SP13	43.7	42.3	>.9	Stopped due to excessive necking													
SP14	44.6	43.9	.84	752.0	662.4	194.4	7.50	-12.8	1.70	-.17	.83	36.1	+ .47	+1.71	-.58	+.32	38.7
SP15	44.6	43.8	.55	606.0	405.5	165.6	7.10	- 8.3	1.70	-.10	.86	34.8	+1.05	+ .26	-.44	+.05	36.4
SP16	44.2	43.3	.70	675.8	523.0	172.3	6.00	- 7.5	1.72	-.13	.87	36.4	+ .68	+ .36	-.36	+.10	37.4

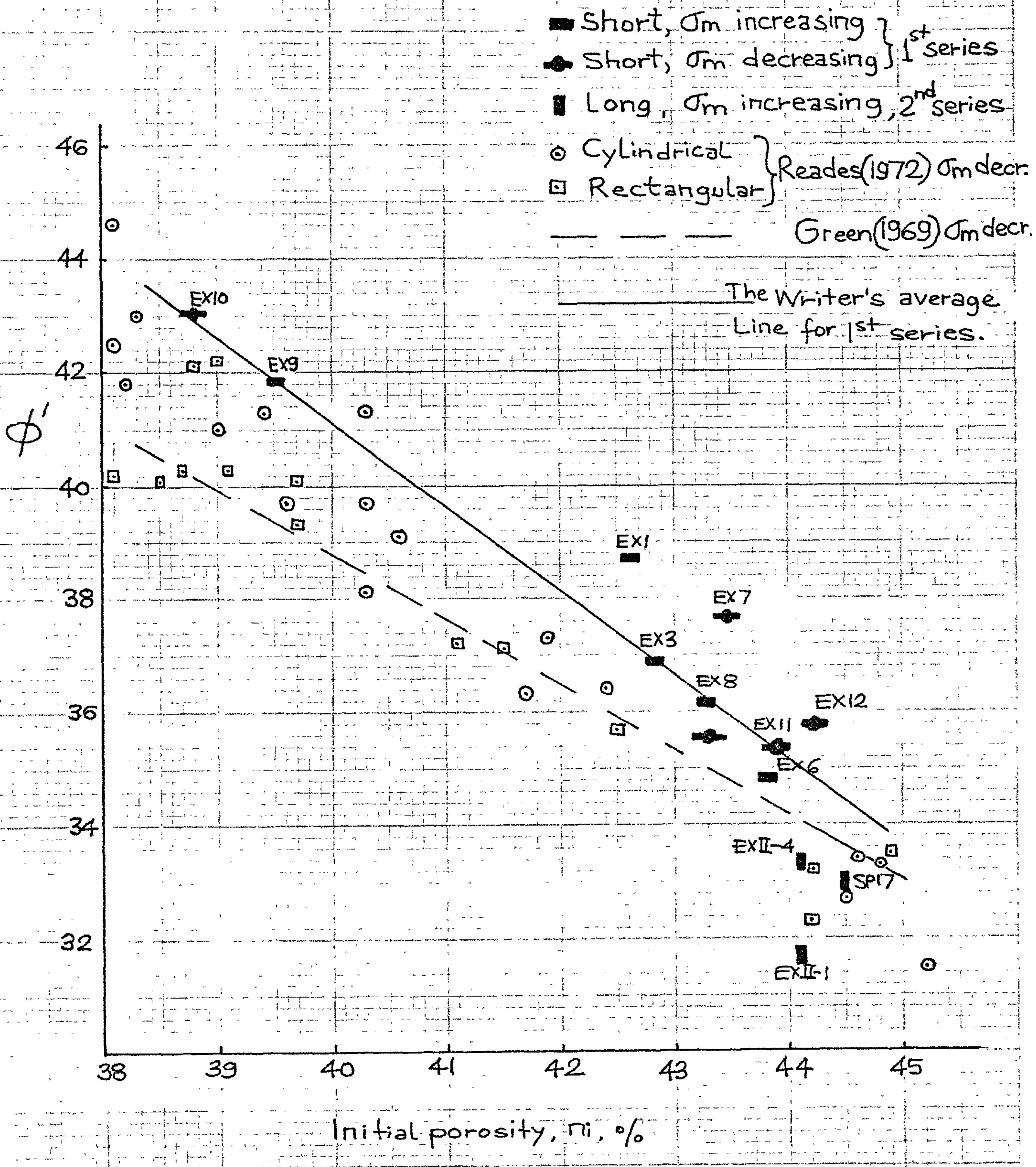
Table 6.2

Failure Characteristics of SP9-16 Series of Tests on Ham River Sand.



Orientation of principal stress directions with respect to sample dimensions in different types of tests.

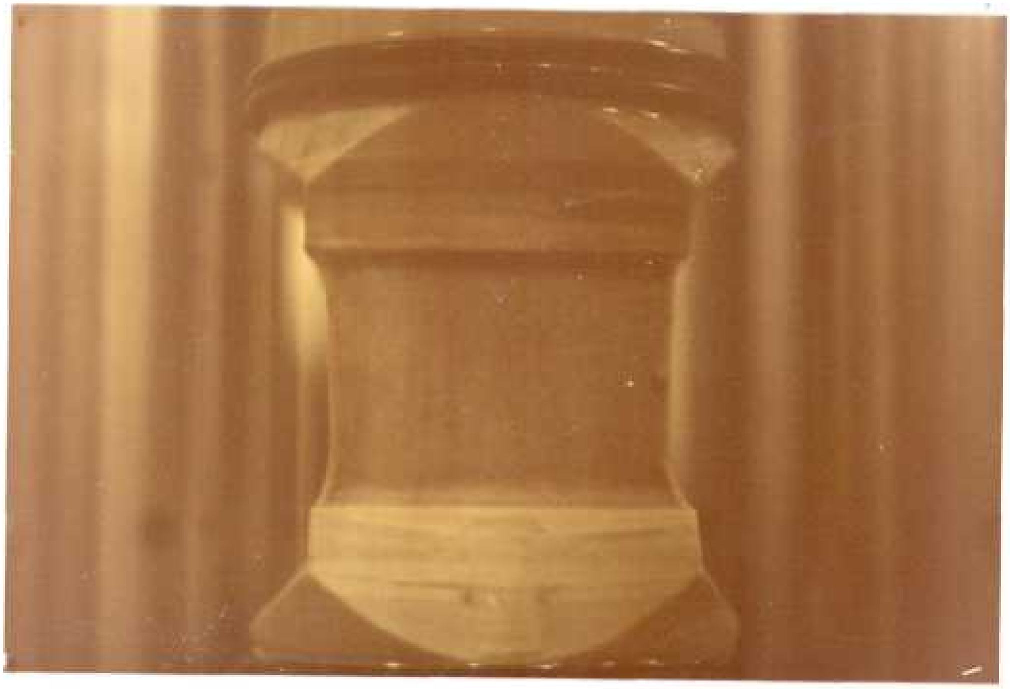
Fig. 6.1



Measured peak strengths in triaxial extension tests on Ham River sand sample

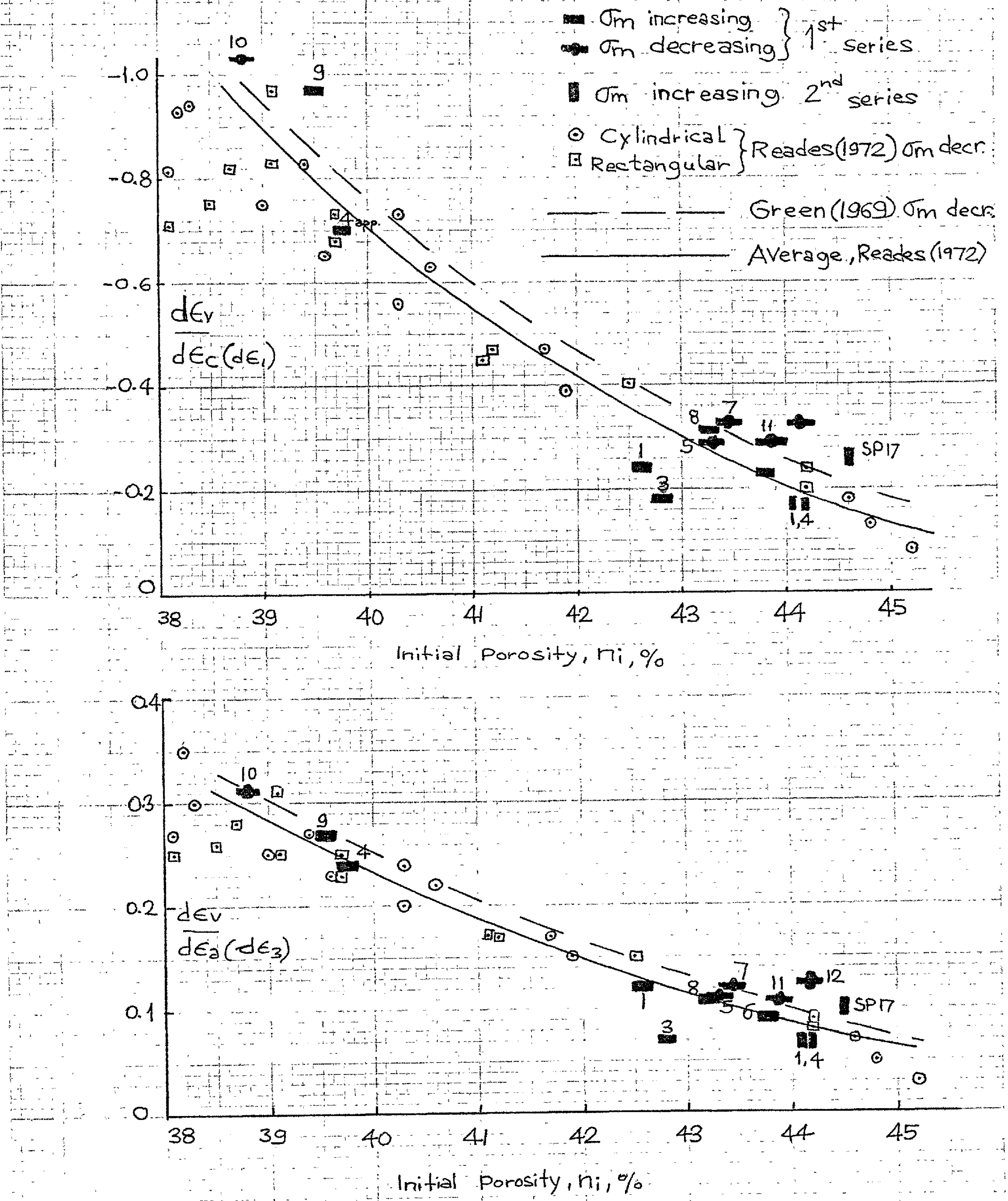
Fig. 6.2

(a) EX 11 ,
at failure



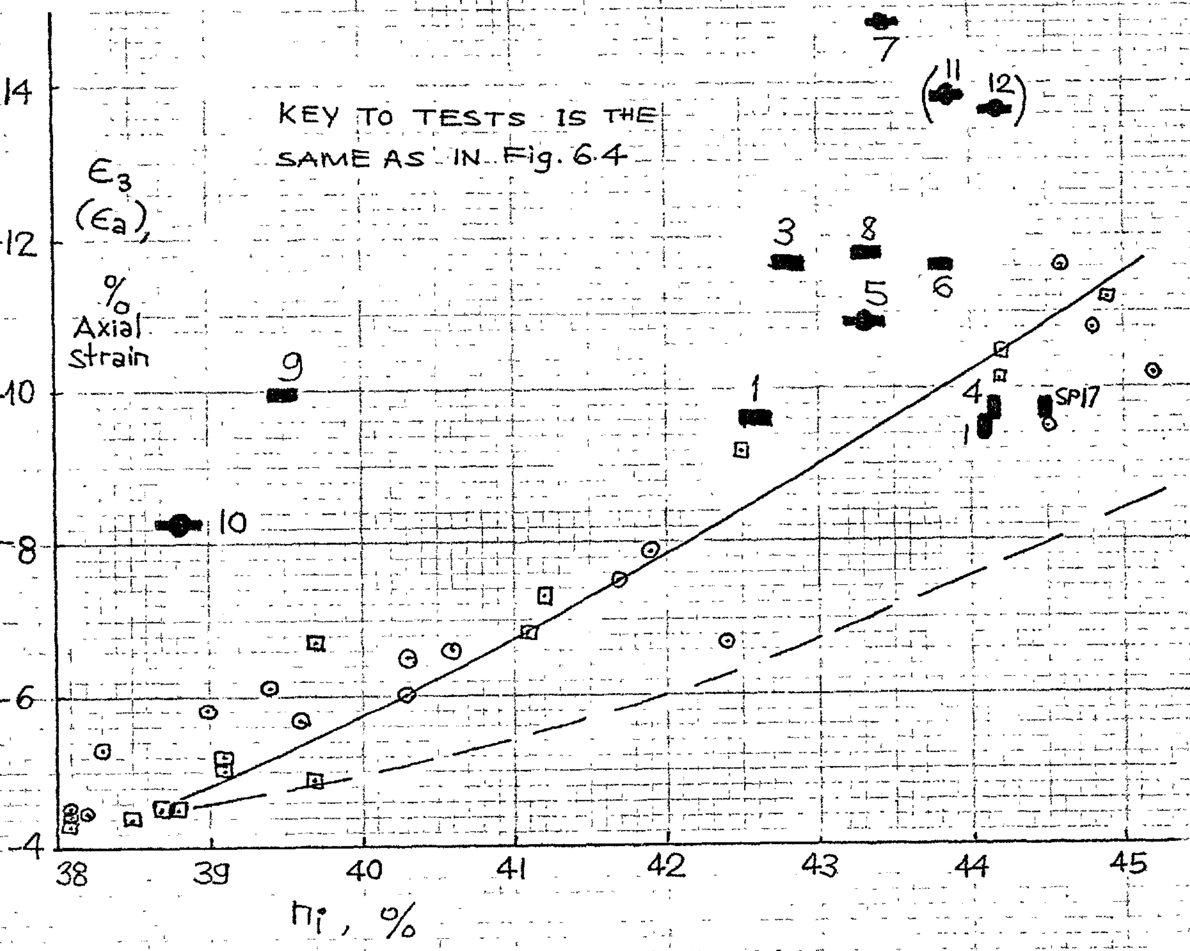
(b) EX 12 , at failure

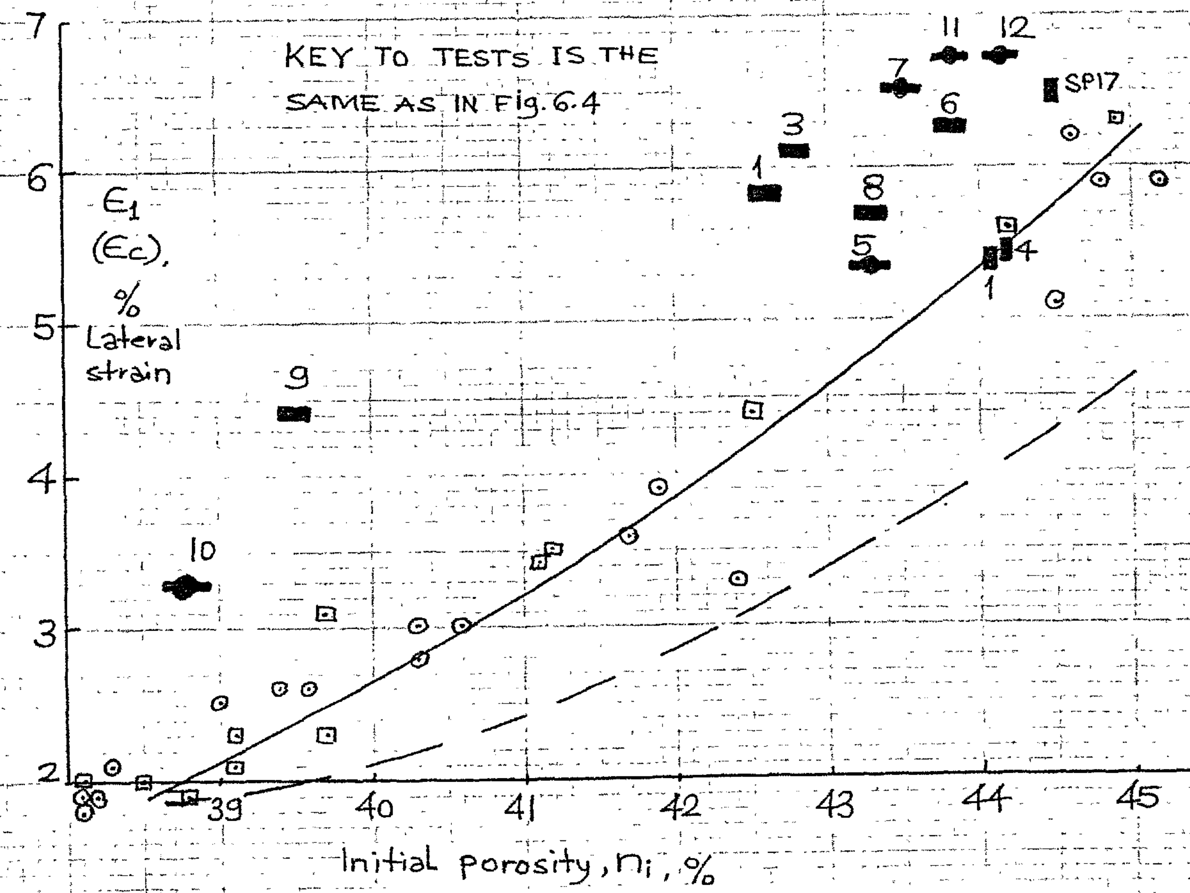
Fig. 6.3



Volumetric strain rates in triaxial extension tests on Ham River sand samples at failure

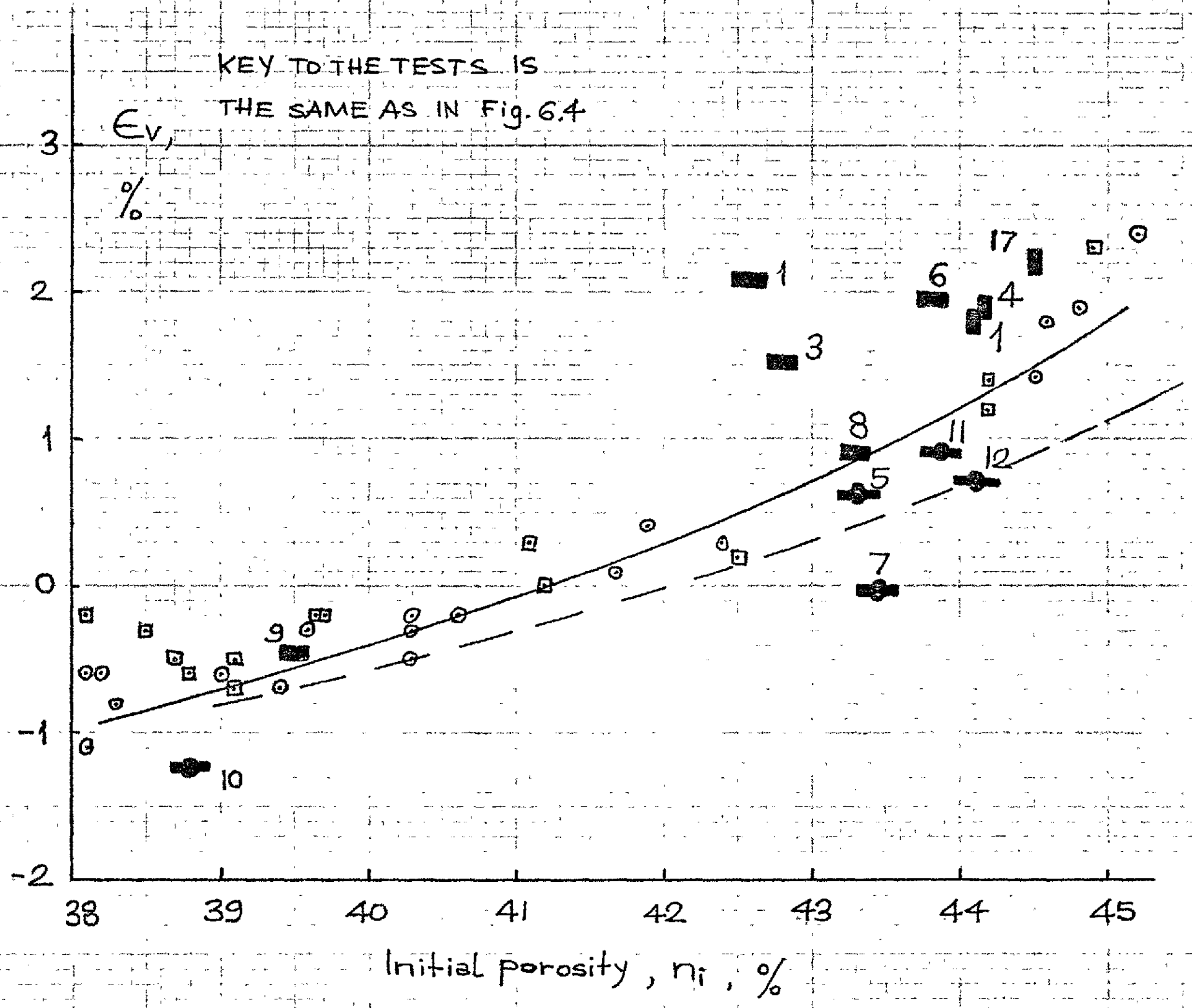
Fig. 6.4





Linear strains in triaxial extension tests on Ham River sand samples at failure.

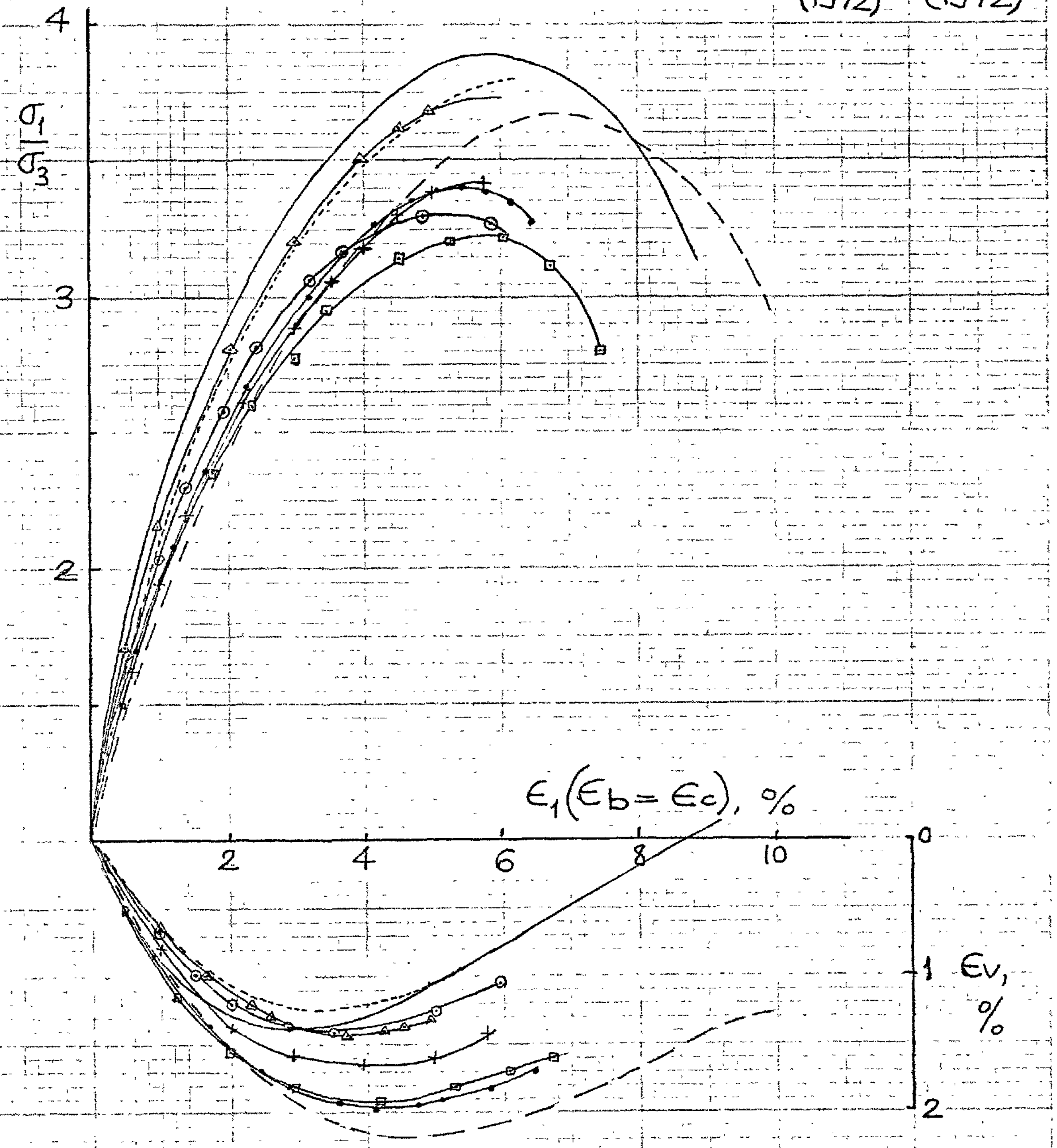
Fig. 6.5



Volumetric strains in triaxial extension tests on Ham River sand samples at failure.

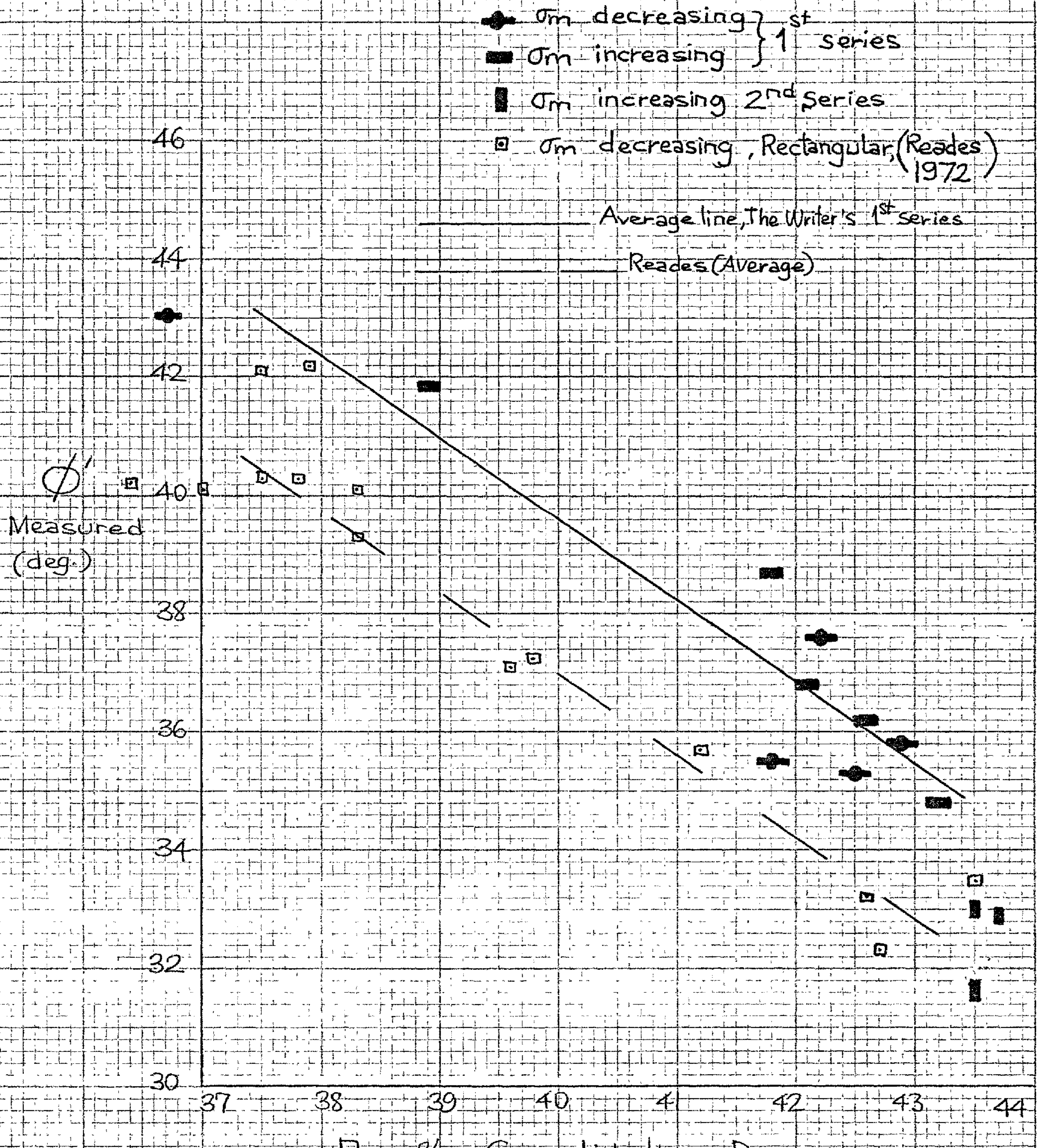
Fig. 6.6

Test No.	EX6	EX8	EX11	EX12	EXII-1	EX II-4	XR5	XR12
Designation	— —	———	—△—△—	-------	—⊡—⊡—	—●—●—	—⊙—⊙—	—+—+—+
n_i-n_c, %	43.8-432	43.3-426	43.9-426	44.2-429	44.1-43.5	44.1-43.5	44.2-42.7	44.2-426
Long vs.short	Short	Short	Short	Short	Long	Long	Long	Long
Cell pres.	Increasing	Increasing	Const.	Const.	Increasing	Increasing	Const. ↓ Reades (1972)	Const. ↓ Reades (1972)



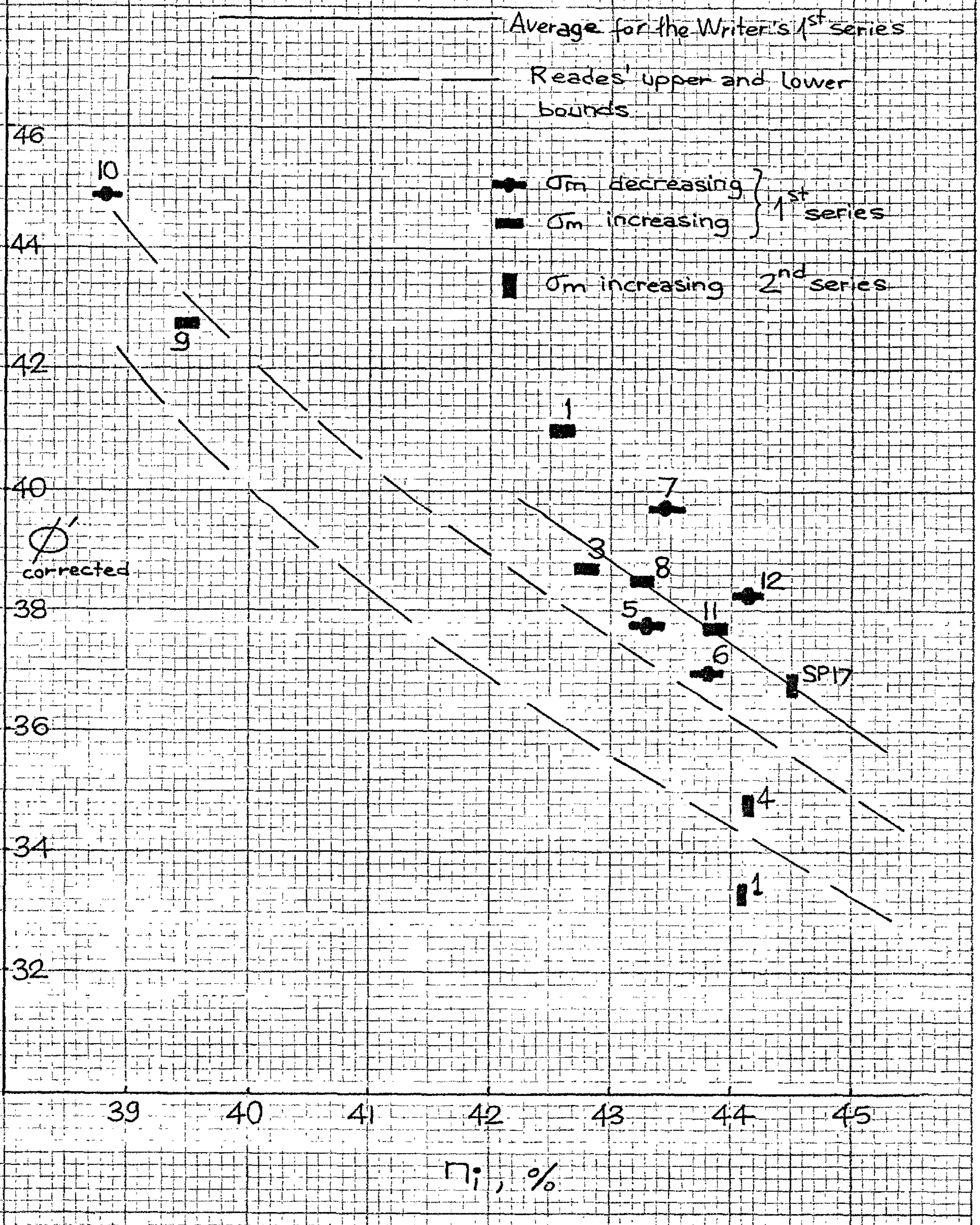
Comparison of triaxial extension tests in different series on Ham River sand samples

Fig. 6.7



Variation of the Maximum angle of Shearing Resistance (Measured) with Consolidation Porosity in Triaxial Extension Tests on Ham River Sand.

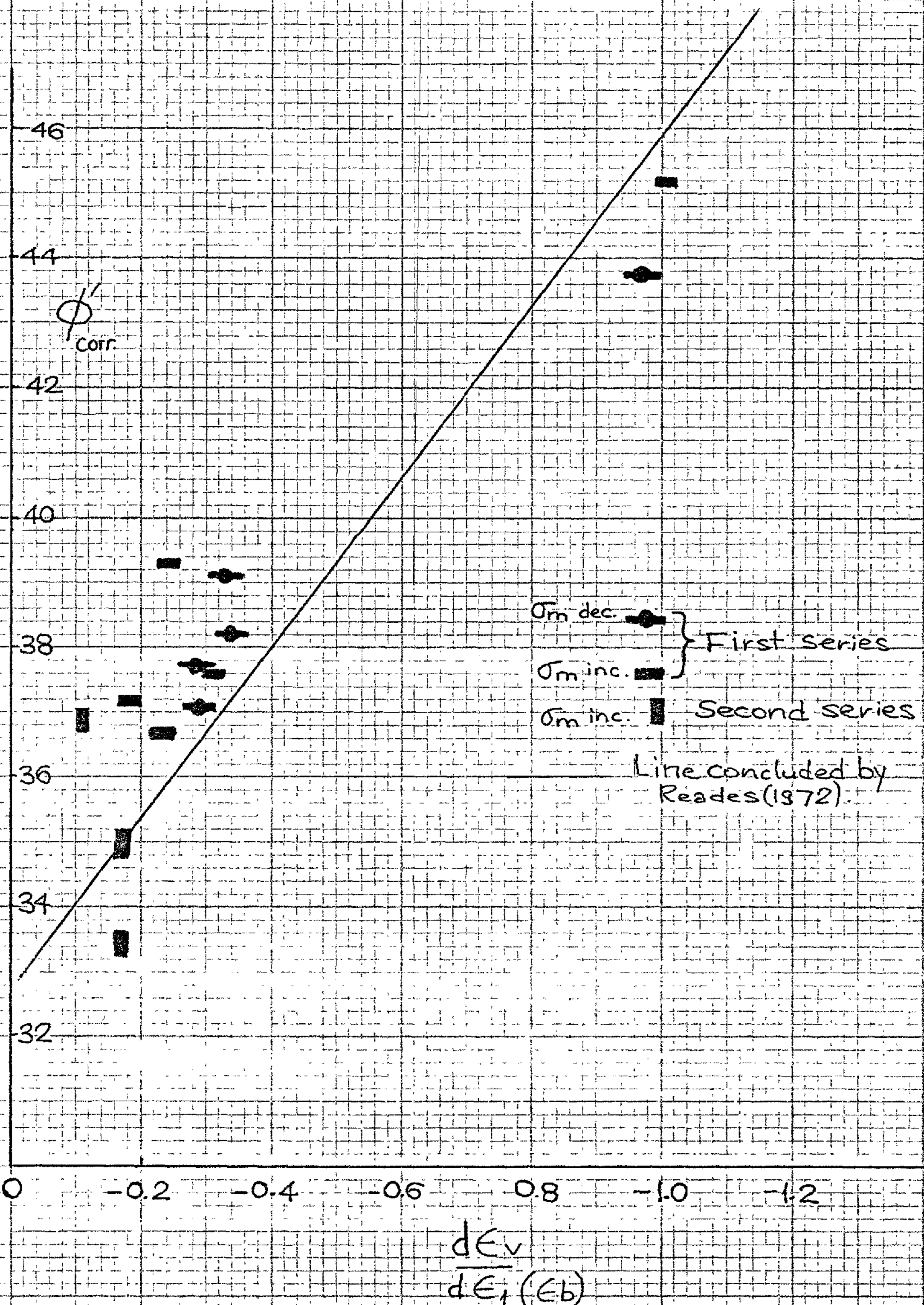
Fig. 6.8



Corrected peak strengths in triaxial extension tests on Ham River sand.

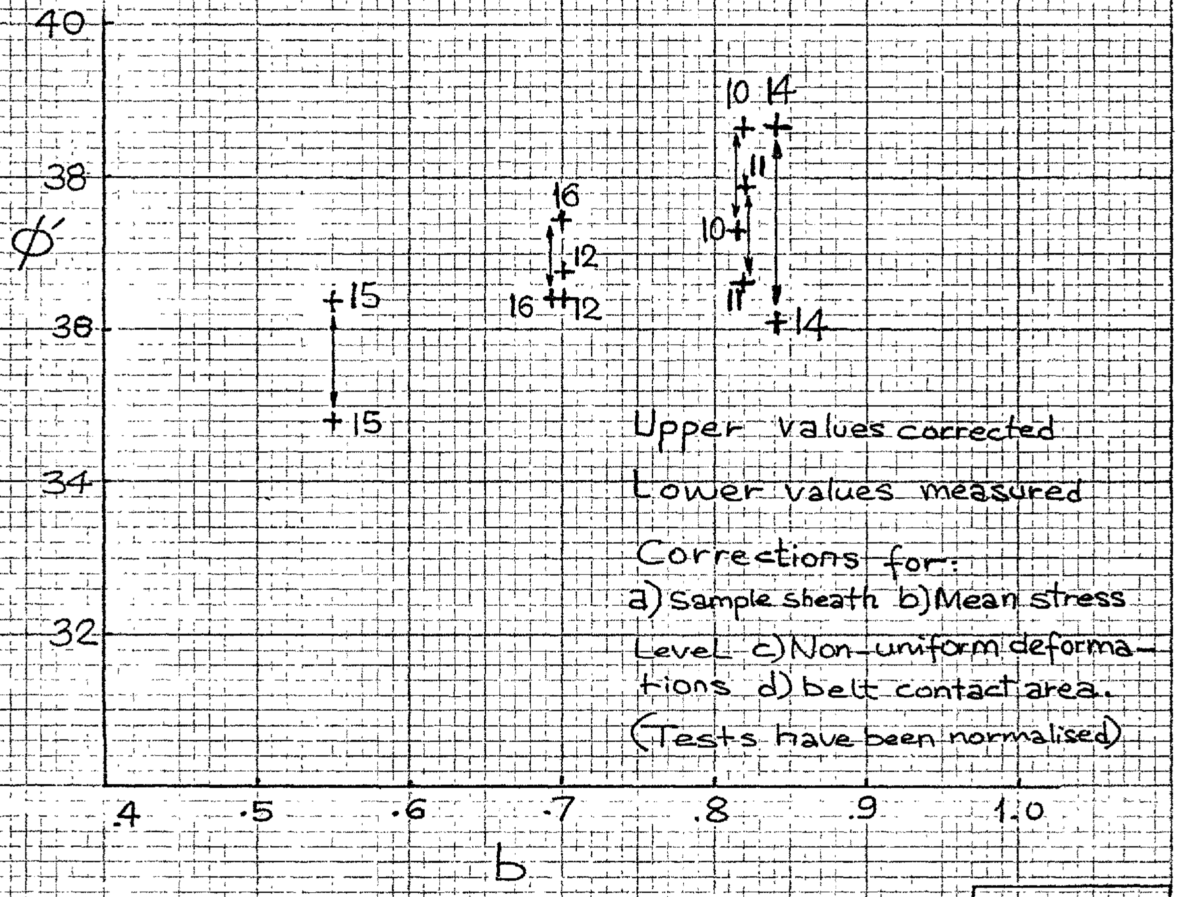
Corrected for: a) Sample sheath rigidity
b) Mean stress level
c) Non-uniform deformation

Fig. 6.9



Corrected peak strengths vs. rate of volume change with respect to major principal strains in triaxial extension tests on Ham River sand samples.

Fig. 6.10



Upper values corrected
Lower values measured
Corrections for:
a) sample sheath b) Mean stress level c) Non-uniform deformations d) belt contact area.
(Tests have been normalised)

Fig. 6.11

SP 9-16 series of generalised tests on Loose Ham River sand samples.
("2nd mode")

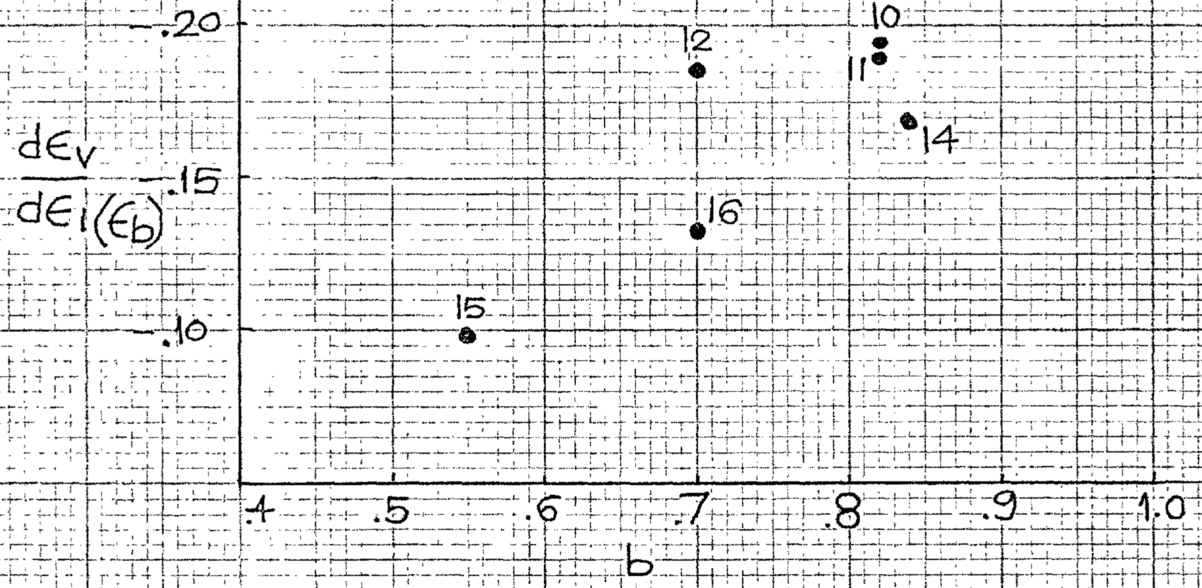


Fig. 6.12

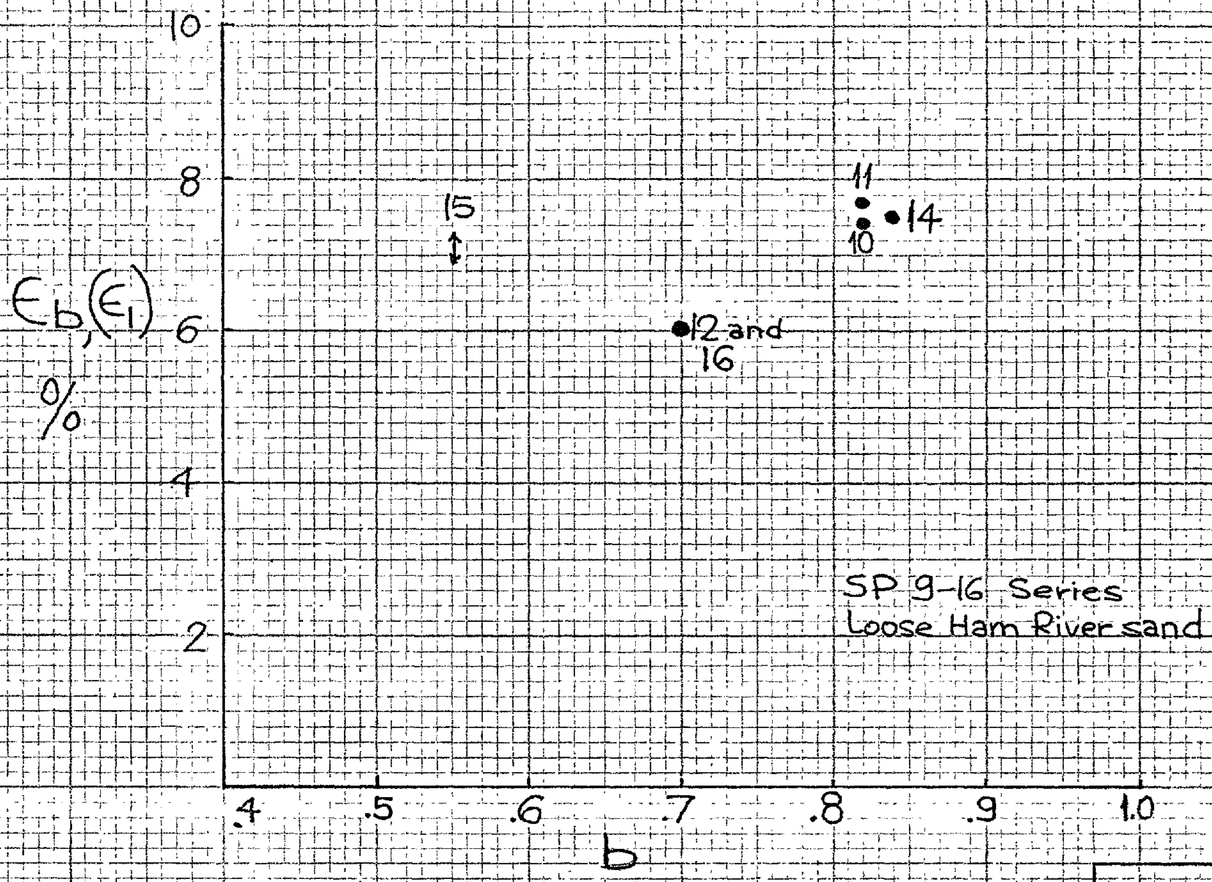


Fig. 6.13

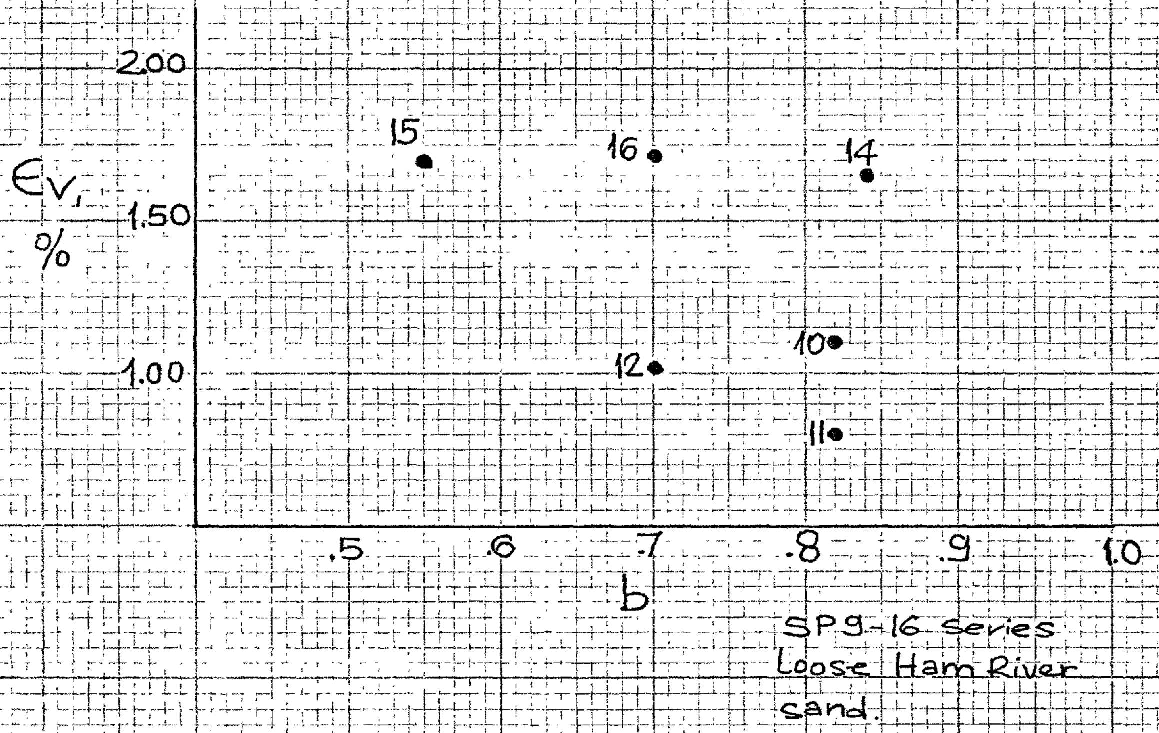
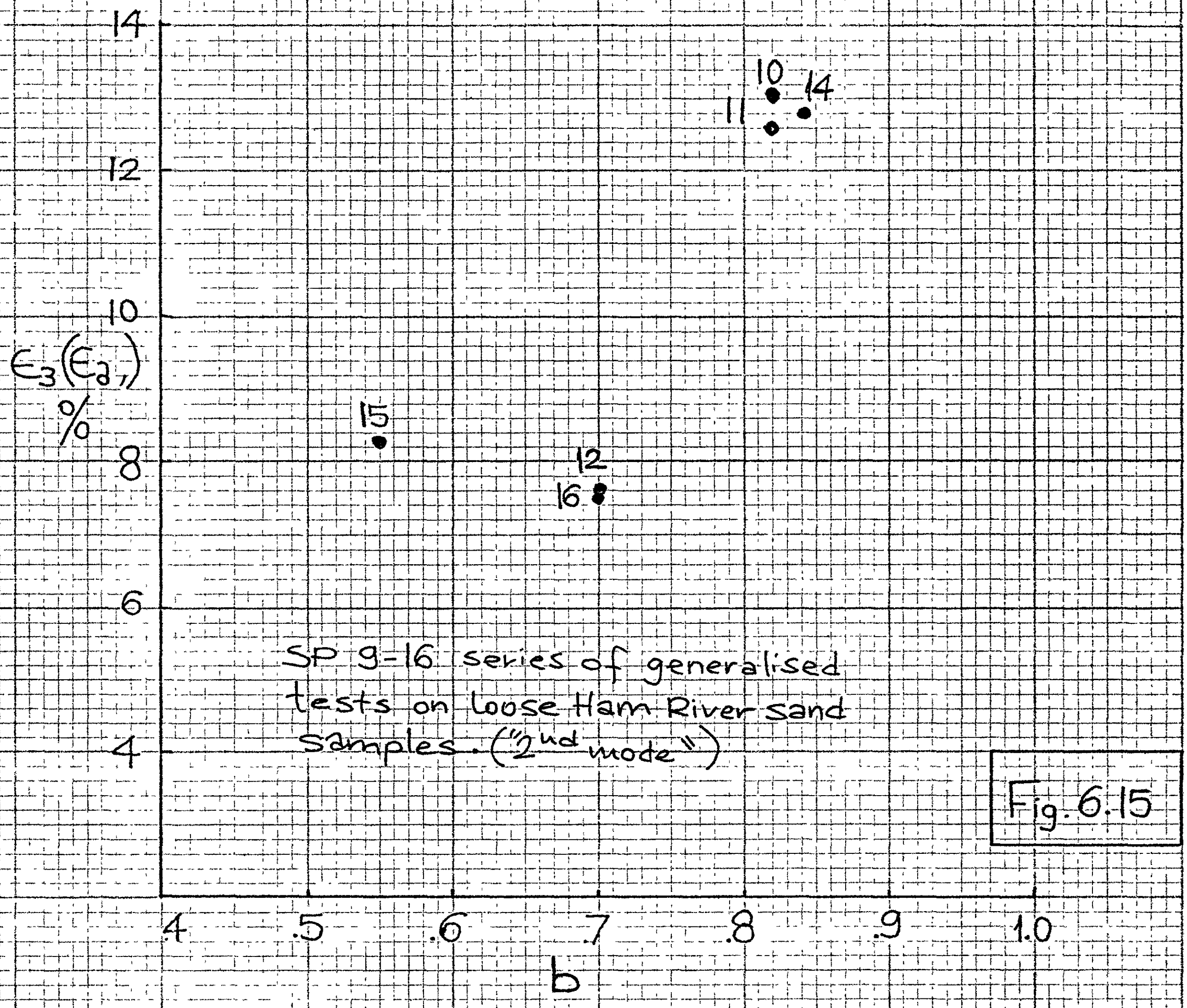
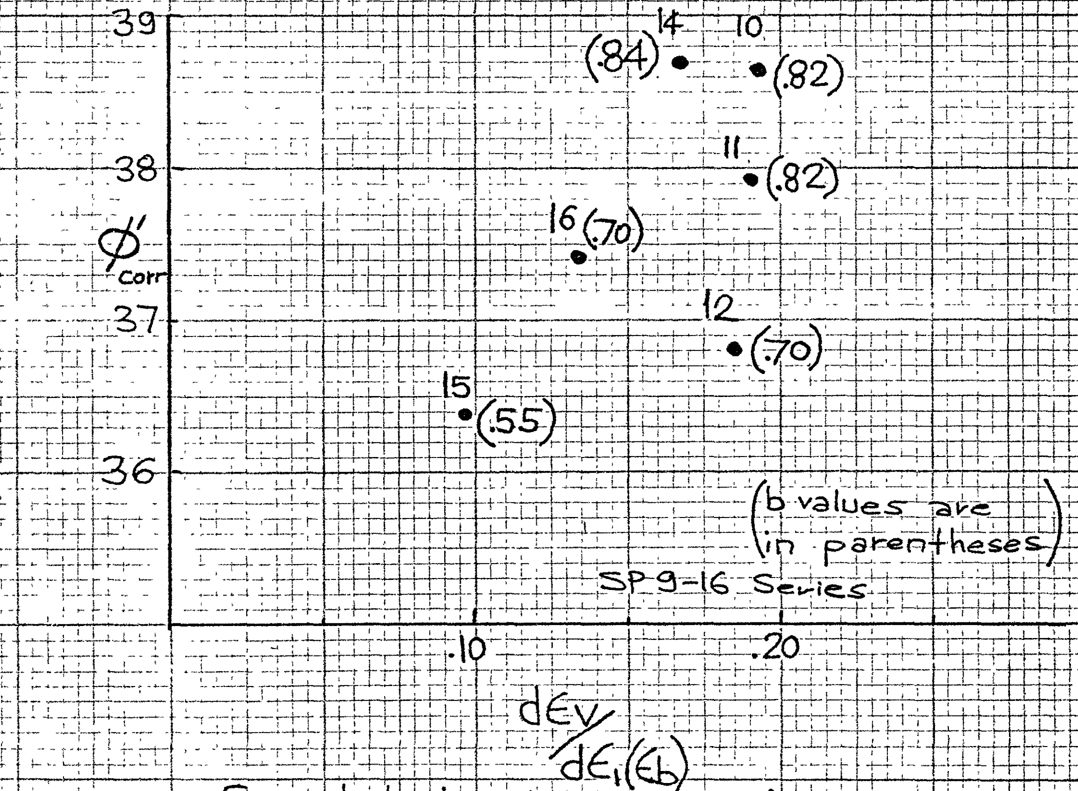


Fig. 6.14



SP 9-16 series of generalised tests on loose Ham River sand samples. ("2nd mode")

Fig. 6.15

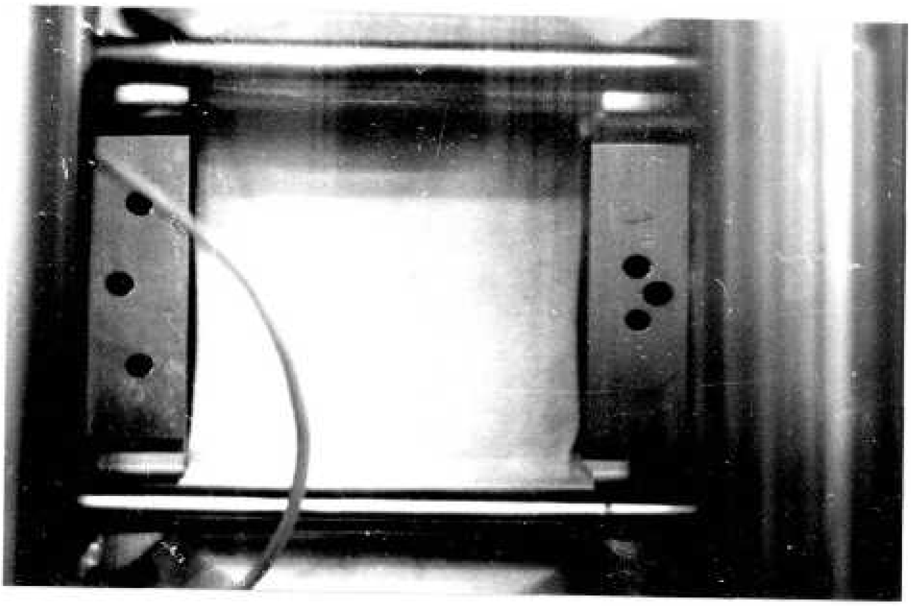


(b values are in parentheses)

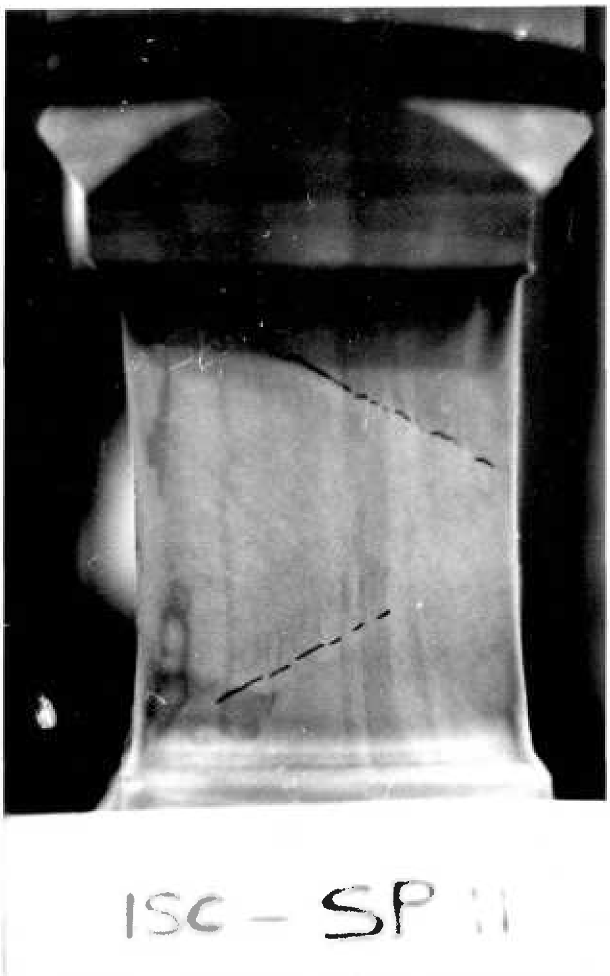
SP 9-16 Series

Corrected strength vs. rate of volume change (with respect to major principal strain)

Fig. 6.16



(a) SP 13, abandoned due to necking



(b) SP 11, at failure

Fig.6.17

CHAPTER 7

GENERAL DISCUSSION OF FAILURE CHARACTERISTICS OF ALL TESTS ON HAM RIVER SAND.

7.1. Introduction and objectives.

Until now different types of tests on Ham River sand have been presented and discussed, the discussions were limited to the tests in each section without cross-reference. Since the main purpose of this study was to find out any variation in behaviour with changing intermediate stress level, this Chapter will be devoted to comparison and analysis of all tests performed on Ham River sand. Tests by Reades (1972) and Green (1969) will be referred to where appropriate. Some observations for Ham River sand will be compared with results for other materials from other apparatuses which apply generalised stress states to samples.

In this discussion, data from apparatuses which make use of hollow cylindrical samples will be excluded because these apparatuses impose a completely different stress system on samples, and they have different mechanics relative to cubical apparatuses. They are outlined in Chapter 2.

The results of using of rigid or flexible boundaries will be discussed, and testing techniques will be critically reviewed with respect to the results obtained in both ISC and several other apparatuses which apply generalised stress-states. Comparisons will be made between the group of ISC tests in Chapter 5 and the extension and SP9-16 series in Chapter 6. From this arises the main theme in the examination of conflicting generalised test data by several researchers.

The whole effort in this Chapter has been aimed at the correct determination of the test characteristics for various intermediate stresses. These characteristics are of the utmost importance when considering failure criteria for soils and for modelling for numerical computations. It must be remembered that the emphasis will be on the stress states other than triaxial compression and plain strain, especially for b > .60.

The most challanging question is whether generalised apparatuses themselves affect the "true" stress-strain-strength characteristics of the material. The Writer has always been at pains both in the laboratory and during the analysis of the data, to eliminate or take into account apparatus effects.

The effect of anisotropy and geometry of failure planes are briefly considered.

Failure characteristics, especially the peak strength, are given priority in the discussions because without a settlement in the controversy surrounding strength it may not be justified to go into a more elaborate discussion of deformation characteristics. This was also the reason for placing emphasis on strength during the test program since it might be a total waste of time and effort to become involved in stress path testing without properly assessing the problems associated with the peak strengths observed in these tests.

7.2. General.

Most soil testing apparatuses attempt to apply only normal stresses to a soil sample, and fail it by increasing one of these principal stresses (major) relative to the other two, that is, applying a deviatoric stress. Unless special designs are involved no gradual rotation of principal stress directions occurs.

One of the most controversial problems of soil mechanics has been whether intermediate stress effects stress-strain-strength behaviour of soils, and if it does, how? As summarized in Chapter 2 several researchers attempted to test soil samples under different intermediate stress states, and the triaxial test has always been taken as the absolute standard against which all other tests have been compared. This may seem logical but the Writer has the opinion that a test which can represent the triaxial compression stress state better rather than the standard triaxial test should be taken as a standard, ideally, at least in the near future. More clearly, instead of failing the sample under fluid pressure in a cell between two rigid platens, it may be more promising to fail a cylindrical or cuboidal sample by applying loads through platens of the same type. Mechanical difficulties involved in such apparatuses relative to a standard triaxial cell has made it impossible to adapt such a test in a general scale. Besides, researchers like Arthur and Menzies (1968), Pearce (1971) obtained similar strengths in their apparatuses to conventional triaxial compression strengths. But the deformation response is not the same, Arthur and Menzies (1968). Also in Chapter 5 an examination of figure 5.3. shows that dense ISC data implies higher axial strain for triaxial compression state compared to the values measured in conventional triaxial compression tests. Actual deformations taking place along height of the samples may be significantly different than average values based on total register of platen travel depending on the uniformity of deformations in conventional tests. Therefore all deformation characteristics will be affected automatically, and due care must be given before accepting the conventional test as an absolute standard.

Conventional triaxial compression tests will be taken as the basis especially when peak strengths are compared in the following paragraphs. The corrected strengths from all tests on Ham River sand have been presented in figure 7.1. Each group of tests is clearly indicated on the plot. The full lines represent the finally concluded variation of strength on Ham River sand as the intermediate stress changes on the basis of discussions given in Chapter 5, and the following paragraphs. The dotted lines are the average lines passing through flexible and rigid platten test data. The latter are mainly given by Reades and Green on the same sand . Location of these curves may be arguable because of the scatter.

It is apparently seen in figure 7.1 that SP9-16 series of generalised tests (section 6.5) give lower strength values than ISC tests (section 5.3.) in the range b = 0.50 - 1.00, and the difference is not small, at least being 3^{o}-4^{o}. More clearly, at any specific b value in this range an ISC test and a SP9-16 series of test yield different ϕ' values. Both ISC and SP9-16 tests have been corrected for possible errors like rigidity of sample sheath, mean stress level, density, and SP9-16 series has also been corrected for non-uniform deformations and for lower platten contact ratios in the direction of the major principal stress (belt). This is a serious setback in the interpretation of generalised tests, because the material is the same, apparatus is the same the only difference being in the mode of testing. It will be reasoned below that it is not only associated with the ISC apparatus but it is believed to be true for many other generalised apparatuses as well. The difference can not be explained by decreasing or increasing mean stress level to failure, because extension tests do not support this argument and, more important, SP14 , 15 and 16 were not carried out by decreasing the mean stress level.

Now, two tests at the same b value are considered; an ISC type and a SP type: In the former axial and belt plattens move towards each other applying major and intermediate stresses whereas in the latter belt plattens move inwards applying major principal stresses, there are no plattens in the intermediate direction to impose loads on the remaining pair of the lateral faces of the sample, but the cell pressure is acting as intermediate stresses, and soil particles have much greater freedom to rearrange themselves on this stress controlled interface when loads (deformations in a strict sense) are being imposed from the belt direction by plattens. In an ISC test there are again plattens in the intermediate direction imposing inward, uniform deformations restricting the particles from moving freely. Note, for example, the larger volume change (decrease) response of the samples in SP9-16 series. As it has been explained in Chapter 5 simultaneous driving of two pairs of plattens towards each other is expected to induce additional restraint along the edges causing load cells to register more than actually required for failure. The strengths from the second mode - SP9-16 - seem to have approximately matched up with SP6 and SP8 (see Section 5.3) and the three flexible platten tests for b > .85. Generalised soil testing apparatuses which use six identical plattens should not produce this difference if the loading procedure is identical but along different axes. It may be very difficult to design an apparatus with 3-fold loading symmetry although the six plattens are identical. In Writer's opinion cubical apparatuses with six identical rigid plattens are not free from edge restraints even if the same strength is obtained in two different modes, because moving rigid plattens near the edges will most probably experience load shedding effects which is quite a different case relative to an ideal, but mechanically very difficult loading by three different fluid pressure acting on the six sides of an cuboid. - Ideal in the loading sense, not the deformations.- Therefore apparatuses which use six identical flexible plattens may be more promising.

The discussion of sample boundary conditions will be given in one of the coming paragraphs in this chapter. It is Writer's contention that this discrepancy observed in the results in the two separate modes of the ISC apparatus is one of most important facts that caused much controversy over the variation of strength with the change of the intermediate principal stress mainly at higher intermediate stress range. Various patterns of maximum angle of sharing resistance, ϕ', as proposed by several researchers, have been plotted against b in Figure 7.2. Some others might have been included in this figure but those who covered the entire range of intermediate stresses were given priority. All investigators agree that strength increases from triaxial compression up to plain strain.

The main controversy lies in the shape of the ϕ' vs. b relationship between plain strain and extension. Similar shapes are reported by Sutherland and Mesdary (1969), Mesdary (1970) and Ramamurthy and Rawat (1973) who find the largest ϕ' values at mid-b range, then they observe decreasing ϕ' values until extension state at $b = 1.0$ at almost the same ϕ' values as in triaxial compression (i.e. ϕ' vs. b relationship peaks at about $b = 0.50$ with no significant strength increase after plain strain). On the other hand, Lade and Duncan (1973), Lade (1972), Reades (1972), Lomize et.al. (1967), (1969), Al-Ani (1975) observe further increase in ϕ' after mid-intermediate stress range as b increases. Lomize et.al. (1967), (1969) observe peaks at $b = .7 - .85$ after continuous increase from triaxial compression onwards. ϕ' decreases few degrees after this peaking ϕ'-b relationship until extension. Although the two apparatuses are similar Al-Ani's (1975) ϕ'-b relationship peaks at $b = 0.5 - 0.6$ compared to $b = .70 - .85$ in Lomize et.al. (1967), (1969).

Reades (1972) observes increases both for loose and dense samples, strengths for the looser samples starting to increase after $b = .4 - .5$ in addition to already high plain strain strengths and higher extension strengths with respect to the triaxial compression strengths. Dense samples have been shown to have smaller increases after plain strain compared with loose ones. Lade and Duncan (1973) conclude at a rather sharp increase in ϕ' from triaxial compression to extension for loose samples similar to Reades. Dense samples did not show any increase after plain strain. - Note that no σm corr. was applied.- Both curves tended to show a very slight drop just before extension. If the test points are examined instead of the line placed by Lade (1972) it can be noticed that loose samples show a similar strength response as Reades' curve for loose samples, that is, no significant increase occurs in strength until mid-intermediate stresses.

The disagreement among several researchers can be explained when their testing methods are investigated. Sutherland and Mesdary (1969), Mesdary (1970) perform tests in their second, third and fourth series - up to b = .50 - similar to Writer's first mode of ISC series, increasing top and lateral stresses failing into the remaining lateral direction (cell pressure). But after b = .50 they adapt another failure mechanism in their apparatus (their series five and six) up to b = 1.00, namely, increasing the cell pressure and the belt stresses to a certain consolidation pressure, then increasing belt stresses further, axial top platten is withdrawn to failure, very similar to Writer's second mode (SP9-16). The abrupt change in the testing method and no mention of any tests with b values larger than b = 0.50 in the first mode suggest that they were presumably not able to increase the lateral bag pressures to high values to perform generalised tests at high b values failing into the cell direction. This is most probably because in this case the major deviatoric stress would have to be operated through the wire mesh-supported side panels into the low cell pressure, and therefore problems along the edges would soon appear especially after b = .6 - .7 due to high differential pressures. As stressed in Chapter 2 the fixed nature of the side panels makes it even worse if the application of large deformations are imagined to be realised by these side panels.

All series of generalised tests, except F series, by Ramamurthy and Rawat (1973) are up to b = .35, and peak strengths increase until that value. As summarised in Chapter 2, this apparatus consists of four side water bags and conventional rigid top and bottom plattens driven by a wormgear system. Series A, B, C, D, E are all failed by increasing axial load on the rigid plattens while one pair of bags provided the minor principal stress, the second pair supplied the intermediate stress, the same as the conventional ISC test technique. After b = .35 the Author conducted four more tests - F series - but again like Mesdary, he abandoned the method he used in all other series. He explains that his lateral platten bags would not stand differential pressures developing between the two pairs after b = 0.35. Even until b = .35 he reports to have used sponge pieces to prevent balooning and the interference of the differentially pressurised bags. The maximum differential pressure between the two pairs of bags was 350 kN/M^2 at b = .35. (This value is 1000 kN/M^2 for the Writer's bags.) Therefore F series of tests - four tests with b values .58, .77, .85, 1.00 - were conducted by decreasing the axial load to failure withdrawing the top rigid platten after consolidation while the two pairs of bags supply major and intermediate stresses constantly roughly similar to Writer's second mode.

Lade's (1972) tests were conducted in exactly the same way as in ISC apparatus. Lomize et al. (1967), (1969) do not clearly state their testing mode, but the Writer expects that plattens were simply driven all together, of course, one pair moving out. It must be also emphasized that Ramamurthy and Rawat (1973) plotted measured ϕ' values, Sutherland and Mesdary (1969) did not apply mean stress level correction to their ϕ' values, and Lade and Duncan (1973) also did not apply any mean stress level correction. The importance of this correction in any comparison has been indicated by Reades and Green (1974) especially when the mean stress level is low. It is interesting that Lade's (1972) proposed curve for dense sand is almost identical in shape with Green's (1969) initial proposal for dense Ham River Sand (i.e. no increase after plain strain). Lade and Duncan (1975) admit that although σ_m correction would not change the general shape of their ϕ'-b relationships it would certainly make them steeper and their curve for dense samples would be very similar to that of Reades and Green (1974).

Dyson (1970) has made an attempt to design a generalised apparatus but he concentrated on triaxial compression and plain strain tests. Triaxial extension tests were also performed. This similar apparatus to Writers' was limited in the pressure capacity of its axial and lateral stress bags (see Chapter 2).

This was the reason, actually, that he only reported one test after plain strain state. He concluded for a dense sand that ϕ' increases by more than 7° from triaxial compression (38.8°) to b = .3 - .35 and placed plain strain tests on the slope of this steep rising increase at b = .15 - .20 which is quite a low b range for plane strain. It is not practical to maintain a strict plain strain condition with flexible plattens, and it may well be that plain strain strength could be on the top of the slope at b = .30. Dyson then joins this highest value with the strengths obtained from triaxial extension tests. The only intermediate test is b = .83 in which axial stress is decreasing failure and the result of which is not clear to Dyson himself.

Goldscheider and Gudehus (1973) have been more interested in the deformations rather than the maximum angle of shearing resistance - see Chapter 2 about the apparatus.- Although they try to find constitutive parameters, they needed to carry out generalised tests at various principal stress combinations to see which failure states were not possible to attain. Their tests are not clearly presented about the failure condition, but they propose envelopes for glass beads and a loose sand. The strength is seen to increase from triaxial compression to plain strain, remain the same throughout the mid-intermediate range and decrease little after about b = 0.6 - 0.7, as will be seen in the following paragraphs, a very similar conclusion to the Writers'.

Ko and Scott (1968), defined failure as a certain point on the stress-strain curve after which deformations increase at a higher rate. This and the criticism over the rigid frame separating the bags by some researchers (see Chapter 2) make it difficult to compare the results with those of other investigators. They admit that the apparatus has been mainly designed for pre-failure strains. Their "failure" can only be defined in a band because the apparatus can give only incremental loads, and the exact points can not be located. They conclude that "failure" envelope rises from triaxial compression until triaxial extension. In their "equivalent coulomb ϕ" it increases 4° - 6° for medium dense samples, 6° - 8° for medium loose samples.

Malyshev and Fradis (1968) plot their findings in the form of Lode parameter, $\mu = \frac{(\sigma_2-\sigma_3)+(\sigma_2-\sigma_1)}{\sigma_1-\sigma_3}$, against ϕ'. Although the details of the apparatus are not given clearly, one can notice that there are almost no generalised tests after plain strain and in some cases after $b = 0.5$ ($\mu=0$) until $b = 1.0$. These relatively higher plain strain or mid-b range strengths are presumably directly joined to lower conventional triaxial extension strengths (ϕ'). Therefore their curves for generalised strengths show big drops in ϕ' at high intermediate stresses.

In the light of the preceeding discussion, it is convincing to the Writer that the main controversy about the variation of maximum angle of shearing resistance at high intermediate stress states is a matter of method of imposing the loads on samples in apparatuses which are capable of applying the principal stresses separately. Other factors such as stress path, platten friction, gaps between the plattens etc. have relatively less significance.

Before saying a final word and concluding on the change of ϕ' with intermediate stress, triaxial extension tests must be compared with other tests. There are quite a number of extension tests conducted on Ham River Sand by Green, Reades and the Writer if considered altogether. It was previously pointed out that the errors involved and basically unstable nature of the extension test have caused conflicting results to be reported in the literature. More recent studies eliminate errors associated with mechanical components such as axial load measuring mechanism. But non-uniform deformations, homogeneity of the initial density etc. are factors which can still affect the reported results. Some researchers do not take all factors into account when plotting their results which makes the comparisons more difficult. Assessment of strength at the extension state is further complicated by differences observed between cylindrical and prismatical shaped samples.

Procter and Barden (1969) report a very close - 1^o lower - triaxial extension ϕ' on a rectangular sample to generalised tests on dense samples (50^o) while their three triaxial extension tests on dense cylindrical samples yield few degrees lower strengths (45^o, 46^o, 47^o). The Authors try to explain it by possible complications during straining at the corners. But Kirkpatrick and Younger (1971) make it clear by X-ray measurements that rectangular shaped triaxial extension samples were no different than the cylindrical extension tests with respect to the uniformity of internal strains. A similar difference of 2.9° has also been reported by Green and Bishop (1969) between the strengths obtained from cylindrical and rectangular dense triaxial extension tests. (Rectangular samples giving higher.)

More extensive tests by Reades (1972) later on have thrown more light on the problem, and showed that denser the sample is more susceptibility to non-uniformities and more scatter in the results, and very small initial irregularities can develop into non-uniform deformations. He obtained a general agreement between cylindrical and rectangular samples in triaxial extension although the former inclined to give values 1° - 2° higher; exactly opposite to the researchers mentioned above. Cylindrical samples have been reported to deform more uniformly in general. In spite of the corrections applied, the scatter was about 4° in the dense state. In Figure 7.1 it is seen as 2° because "better" tests have been given additional weight. Reades' upper bound for dense triaxial extension tests is only 0.5° lower than dense ISC tests at b = 1.0. Writer's two short dense triaxial extension tests almost average on the upper bound. On the looser side the scatter is about 2°. Writer's short samples give higher (loose) ϕ' - both measured and corrected - than Reades'. Details have been given in Chapter 6. It is clear that triaxial compression and extension strengths are not the same for the whole density range. In Figure 7.3 it is seen the difference starts from almost being equal in the case of loose samples, reaches a value of $4^o - 5^o$ on dense samples, triaxial extension strengths being higher. This again explains the conflicting results in the literature, because relative magnitudes of triaxial compression and extension strengths have always been reported without much notice given to the relative densities of the samples. While dense ISC tests near b = 1 more or less compare with the triaxial extension tests, loose ISC strengths near b = 1.0 are considerably higher than loose triaxial extension tests (about 4^o at 44 percent porosity). ISC tests with flexible plattens, special rigid platten series SP 1-8 and 9-16 indicate lower strengths than those from rigid platten tests near b = 1, and they are little higher than triaxial extension tests, see Figure 7.1.

Lade and Duncan (1973) do not report any conventional extension tests, but it is interesting to find out that two such tests - dense and loose - were performed and reported in Lade (1972). They had the same geometry as their cubical tests, and similar cell pressures were used corresponding to platten stresses in cubical tests, namely, 600 and 400 $^{kN}/_{M^2}$ $(\sigma_1 \simeq \sigma_2)$ for dense and loose samples respectively. These conventional tests give much lower strengths than cubical extension tests, approximately equal to triaxial compression tests; 46.5^o and 37.5^o in conventional triaxial extension tests compared to 57.1^o and 45.9^o in cubical tests for dense and loose samples respectively. Calculated corrections for extension tests are 1.4° and 0.7° for dense and loose samples exclusive of the above values. These are due to errors in measurements. It seems that no other corrections such as non-uniform deformation or mean stress level have been applied despite the observed necking that is reported in the dense sample for example. In any case the difference cannot be explained by such errors. Lade (1972) tries to explain it by his zone versus line failure hypothesis which will be explained in one of the following paragraphs.

Before concluding it is essential to differentiate the types of sample boundary conditions. Rigid plattens are usually made of stainless steel and they are the simplest and practical way of applying loads to a sample, and present the least mechanical difficulty to prepare. They are used with the highly polished surfaces and with greased lubricated rubber sheets to reduce the friction to a minimum. Some of the problems associated with them can be seen in detail in Rowe and Barden (1964), Barden and Mc Dermott (1965), Blight (1965), Bishop and Green (1965). They automatically apply uniform deformations at the loading interface but not necessarily uniform stresses. Flexible plattens can apply uniform stresses to a loading boundary. Uniformity of stresses does not imply uniformity of deformations on the surface of application. Reinforced rubber bags encased in backing frames were used in a part of the test programme in this study together with rigid plattens (Chapter 3). The bags were closed systems except the provision for the displacement of a tiny amount of fluid to operate the pressure transducers. Pressure in the bags was increased by pushing it against the sample with a jack mechanism.

7.3 Concluding Comments on the Variation of ϕ'.

It has been generally agreed that peak strength values increase markedly from the axially symmetric compression state to the plain strain and then hardly increase up to b = 0.5 - 0.6 range, with dense samples showing slightly higher strengths. The researchers also agree that the increase is larger when the material is denser. Therefore, it can be said that there is a general agreement as to the variation of strength in the first half of the intermediate stress space.

Most investigators observe strength drops in the second half of the space, different groups of tests in different patterns though. Generally, researchers who test in the first mode observe smaller drops at about b = 0.70 - 1.00. Rigid platten tests give very small or no strength drops very near to extension state, say at b = .9 - 1.0, whereas flexible tests give relatively larger drops starting at about b = 0.70 - 0.80, but these are observed after a record of increasing strengths from b ≈ 0.5 onwards. Researchers who use the second mode after plain strain find bigger strength decreases from b ≈ 0.5 right through to b = 1.00 where strengths approaching values corresponding to triaxial compression tests in looser materials are obtained and a few degrees higher than compression strengths in denser materials. Another interesting observation is that the looser the material the larger the drop.

This is true for the both modes - note especially the data by Lomize et.al. (1967, 1969) and from the ISC for the first mode, and ISCSP9-16 and extension data for the second mode.

Therefore, another general conclusion will be that strengths do decrease before reaching the extension state, but the extent of these drops and the point at which they start decreasing are complicated by the two loading modes.

It must be understood clearly that the strength increase over plain strain values after $b \simeq 0.5$ in the first mode is not entirely an effect coming from non-uniform normal stress distributions on rigid plattens. This noticeable increase is the superposition of two phenomena, and so are the strength drops near extension. These are, first, the effect coming from imposition of loading boundary conditions, and secondly the non-uniform normal pressure distributions on the plattens. It seems to the writer that the latter effect is especially true for final stages of tests (around peak values) in the case of rigid plattens, and it seems to be a larger effect when the extension state is approached (see the discussion in section 5.5). But this cannot amount to more than a few degrees. The imposition of loads on the sample is a more significant effect and is the main cause of higher strengths in the first mode, Figure 7.4.

The discrepancy in ϕ' values observed in generalised hollow cylinder tests is very interesting, and seems to be related to the type of loading constraint on the sample. There are a few ways of failing a hollow cylindrical sample.

It may be failed by increasing the axial stress (major principal) while the bore and outside pressure are kept under a differential fluid pressure. Or, alternatively, the bore pressure may be increased as the major principal stress while the axial rigid plattens apply the intermediate stress. Arnold and Mitchell (1973) used the first method, they reported $18^{\circ} - 19^{\circ}$ increase in $b = 0 - 0.5$ range on medium dense samples.* On the other hand Kirkpatrick (1957) used the latter method and found ϕ' increase of $2 - 3^{\circ}$ in the same range for dense samples.[x] Procter (1967) who made use of the former procedure also found large increases of ϕ' on dense samples[†] (10.5° at $b \approx 0.50$ over triaxial compression value for 1.5" I.D. samples; 9.5° and 7° increase at the plain strain state for 1.5" and 2.5" I.D. samples respectively). The shape and geometry of failure planes were totally different in the two modes.

The degree of freedom is one of the basic concepts behind stress-dilatancy hypotheses (see Chapter 9). As claimed by several researchers in stress-dilatancy field it may be true that the minimum degree of freedom is materialised in plane strain state associated with the maximum angle of shearing resistance. This implies that higher strengths than plain strain values obtained in the tests are apparatus, rather than material effects. In the Writer's opinion the statement that the plain strain state has the maximum shearing resistance

---

[*] Sample dimensions; 142 mm (5.6") height, 152 mm (6") O.D., 102 mm (4") I.D.

[x] Sample dimensions; 6" height, 4" O.D., 2.5" I.D.

[†] Sample dimensions; 6" height, 4" O.D., 2.5" and 1.5" I.D.

with minimum degree of freedom is open to discussion and further experimentation. Also, the argument may take different forms with respect to whether a "field element" or a laboratory sample is considered.

Laboratory tests in soil mechanics are performed to determine the behaviour of an "element" in a stressed soil mass. The "field element" is represented by "the sample" in the apparatus and it is usually loaded by applying three principal stresses in various combinations. There are some fundamental questions here; will this element be represented correctly in the apparatus as far as the stress-strain boundary conditions in the soil mass are concerned (qualitatively)? What is the relevance of so called "degrees of freedom" in the field element? A unique strength behaviour of an element may be expected when it is exposed to a certain stress state, at a certain stress level and for a particular stress (strain) history, excluding any creep effects. Since the discrepancy observed between different modes at high b values in the ISC apparatus - and in several other generalised apparatuses as well - is related to the boundary loading conditions and method of loading of the sample, what is the relevance of the behaviour in these modes to the "true" behaviour? It is usually assumed that the "true" behaviour is associated with uniform stresses and strains occurring simultaneously on the boundaries of the element.

It seems to the Writer that neither ISC-SP9-16 series and extension tests nor ISC series can represent the "true" strength behaviour after b = .50. SP 9-16 series and extension tests are presumably somewhat underestimates while usual ISC tests are overestimates of the "true" values in this range.

Variation of peak strengths from generalised tests at high intermediate stresses can be assessed now in the light of previous discussions. In Figure 7.5 variations of ϕ' have been shown for dense and loose samples respectively. Curves marked (1) and (2) represent the observed shape of ϕ' for tests with two pairs of rigid plattens and one pair rigid, one pair flexible respectively (all in first mode). Curves marked (3) are imagined to represent the "true" behaviour (with respect to field element) that the Writer expects on the basis of discussions and observations presented. The strengths have basically the plain strain values throughout the middle range. The relative amount of the strength drop near extension is governed by the density of the material. For loose samples, conventional extension tests are little lower than the envisaged generalised strength at b = 1.00. They are about the same in dense sand data. The zones between curves (1), (2) and (3) are the total effects coming from more than a single source as explained above.

The Writer believes that lower degrees of freedom may be attained in generalised tests with $b > 0.5$ than those in plain strain tests although this may not be true for the "field element".

The peaks observed after $b \approx 0.5$ (i.e. ϕ'-b relation) in the case of using flexible plattens have already been mentioned. It was explained that strength decrease near extension was a reflection of the true material behaviour. The existence of edge effects when using rigid plattens, especially near $b = 1$, prevented the strengths from decreasing. It was also strongly felt that the peak of the ϕ'-b relation was associated with the failure mechanism in the sample which failed along distinct slip planes until the b values corresponding to the peak, afterwards there were no observable slip planes.

An equally important observation was the larger ϕ' increases over triaxial compression state when using flexible plattens. The Writer's introduction of a pair of flexible plattens to the ISC apparatus resulted in slightly higher strengths. Researchers like Lomize et.al. (1967, 1969) or Al-Ani (1975) using generalised apparatuses with six flexible plattens measured sharp increases of ϕ' at the plain strain state and mid b range. The former researchers reported maximum increases of 12.5°, 18° and 22° for three sands and the latter found 11° increase for a dense sand. The Writer does not expect such increases in the ISC and other generalised apparatuses for the same materials. The only possible explanation at the present can be the different internal shearing mechanism resulting from six uniform boundary stress application.

7.4 Effect of Anisotropy

Structural anisotropy of granular materials, if any, may play a relatively important role in the comparisons made between tests which are failed in different directions. Several researchers (including the Writer) conducted generalised tests failing the samples both in vertical and horizontal directions and placed the results of both groups of tests on the same ϕ'-b plot and compared other failure characteristics directly. Samples are normally prepared by depositing vertically for both cases unless special precautions are taken to take care of possible anisotropic behaviour. If vertically deposited samples are inherently anisotropic in strength and deformation properties, the test results will be affected according to whether they are sheared vertically or horizontally or in any other direction. For example, conventional extension tests are always performed failing the sample in the axial direction and these tests are generally regarded as a check for generalised strength at b = 1.0. A detailed study of the problem has been done by Arthur and Menzies (1972) who constructed a tilting mold so that it was possible to form air deposited samples at various angles of deposition. Their cubical apparatus was used in the study accompanied by X-ray technique. All tests were performed at triaxial compression state presumably to avoid additional complexities. A difference of 2^{o} was found in maximum angle of shearing resistance through 90^{o} change of angle of the tilting mould - vertical to horizontal -. That is, the samples that are sheared in the direction perpendicular to the plane of deposition give lower strengths.+

Reades (1972) attempted to investigate the same problem by shearing a number of vertically deposited samples from vertical and horizontal directions in the ISC apparatus (Both triaxial compression and plain strain), but his results were complicated by different platten contact ratios in the two directions. So, the problem he looked at was the effect of platten contact ratio on strength. After applying contact ratio corrections he concluded that the horizontally sheared samples show slightly higher - less than a degree - strengths than vertically sheared samples, and deduced that air deposition method would result in more anisotropic structure in samples relative to tamping. The Writer assumes that there is no effect due to testing technique.

Green (1969),(1971a) in a rather limited series of tests concluded his dense plain strain tests sheared vertically were little (about a degree) higher than horizontally sheared samples. This is contrary to the previous findings mentioned above, but Green's samples were prepared by vibration, and this may have an effect on the results. Natural clay soils have been investigated for structural anisotropy extensively but for sands reliable data are lacking. If vertical deposition of samples in the laboratories is assumed normally, then horizontally

+Recently Al-Ani (1975) found similar results (see Chapter 10).

sheared samples - major principal stress horizontal - will give a strength value which will also include the effect due to inherent anisotropy. Therefore, all tests in which the major principal stresses are applied laterally should actually be lowered accordingly.

The more interesting findings have been reported in the stress-strain behaviour where compressibilities are grossly affected by inherent anisotropy. Arthur and Menzies (1972) report differences in axial strain up to 200 percent for a certain stress ratio before failure. Reades (1972), and Green and Reades (1975) indicate higher deformations are taking place in samples sheared in the horizontal direction relative to vertical. This good agreement between researchers must draw attention because such a significant deformational anisotropy will grossly affect the stress-strain behaviour of tests in generalised apparatuses. El-Sohby and Andrawes (1973) also showed the anisotropy of total strains in triaxial compression samples under hydrostatic loading. It was more significant in loose samples.

7.5. Failure Planes.

Observation of the shapes of the failed samples and the failure planes may have a certain degree of significance in the interpretation of results. Tests were usually stopped few percent after the peak except in some extension tests during which the cell pressure was increasing while the axial pressure was held closely to a fixed value, and such tests were stopped after observing the failure plane. Tests that are stopped at or very near the peak may not show any slip planes, although they exist at the micro-scale. Different types of tests show different deformation patterns and failure planes. ISC tests-normal mode - usually fail quite uniformly, and slip planes cross the sample diagonally along the full height. Samples deform and fail into the cell pressure side. Slip planes make angles of 60^{o}-65^{o} with the horizontal. This range covers all porosities. ISC tests in the second mode (axial stress is decreased to failure) give either single or double slip planes starting from one or both of the axial plattens and joining approximately the middle height of the sample - if two; with a distance between them at the middle -. These make angles 20^{o}-27^{o} with horizontal, actually three tests average on 19^{o}-22^{o}. Except SP14, others were uniformly deformed. If it is remembered that the belt stresses are the major principal stresses, the failure planes do not conform to those in the first mode.

Short extension samples show failure planes intersecting the bottom platten along the edges making angles of 15^{o}-20^{o} with the horizontal. It is a fair assumption that slip planes do not rotate between the peak strains and the end - of - test strains after peak if the two values are reasonably close. Failure of short extension samples almost always occur near the bottom platten, and it may be partly explained as follows; Lower part of the sample is deposited under a deeper head of water compared to the upper part in spite of care is taken to decrease the amount of sand being spooned into the mold gradually to achieve homogeneity. Besides, more important in fact, the placement of the heavy top cap makes the upper part even denser. Therefore the actual porosity near the bottom platten is presumably higher than the average porosity which is reported on the basis of dimensions of the sample and amount of dry sand placed into the mold. This automatically implies that the strength of loose short extension samples which have been normalised to 44 percent porosity are actually underestimates of the true strengths. However it is impossible to detect such porosity differences in the sample unless special techniques like waxing, freezing etc. are used. The situation for Reades' loose samples must be no different. On the other hand, the fact that the lower axial platten moves downwards to fail the sample while the top platten is fixed to the crosshead of the triaxial frame may also be responsible for this behaviour.

The angle between slip planes and the planes on which the major principal stresses act are about 5° larger in SP series than ISC series, and yet lower strengths are obtained in SP series. But from the geometry of the failure planes it may be inferred that the failure mechanism is not similar in the two series. Short extension samples show even higher angles. The angles in ISC series give comparable values to 45 + $\phi'/2$, others give higher values. As will be repeated on few other occasions in this thesis, it was very interesting that failure planes did not form at very high b values in the ISC tests. Between the plain strain state and about b $\underset{-}{\sim}$ 0.80 they were clearly observed. For example, flexible platten ISC tests ISC(F) 11,12, 13 failed along slip planes (b values 0.79, 0.72, 0.71) whereas no slip surfaces were observed in ISC(F) 17, 18, 19 (b values 0.88, 0.94, 0.92 respectively).

Slip plane angles have been tried to be interpreted with stress-dilatancy approaches by some researchers, See for example King and Dickin (1970), Rowe (1971c), King and Dickin (1971). Rowe (1971c) tries to indicate the dilating feature of the granular materials and to explain why slip planes whould not be expected at 45 + $\phi'/2$ except for very uniformly deformed triaxial compression samples which form slip planes at large strains well after peak - at critical values -. Bransby (1971) tries to correlate slip plane inclinations from King and Dickin's (1970) tests with zero lines of extension - directions in a sample along which no linear strains occur - as he and other researchers observe from retaining wall tests, and concludes that it gives closer values to the observed angles. General agreement among the researchers has not been reached even in the relatively simpler state of plain strain. Variation of H/D ratio also presents further complexity. While King and Dickin (1970, 1971) report varying slip plane inclinations for several H/D ratios, Rowe (1971c) based on Tong's results (Tong, 1970) reports almost constant slip plane inclinations with varying height to breadth ratio.

In this context Lade's (1972) argument about failure planes is interesting. He differentiates samples failed by the formation of slip planes from those without slip planes. He names the former group as "line failures" versus the latter "zone failures". Line failures appear where strains are not uniform, and they occur on the weakest planes but zone failures are the formation of many failure planes at the same time. Then goes on to explain the difference in strength between generalised tests at b=1 and the conventional triaxial extension tests by zone failure vs. line failure concept. He argues that extension samples in the cubical apparatus are failed as "zone failures" whereas triaxial extension samples experience "line failures". Therefore, he concludes, cubical extension tests give higher strengths. The Writer has been rather dissatisfied with this explanation firstly because the generalised tests from ISC apparatus show bare failure planes most of the time especially if test is stopped few percent strain after the peak. For example, a generalised test at $b = 0.75$ would show a slip surface while ϕ' observed is higher than those in other intermediate stress states. The two generalised apparatuses are almost the same except the compressible belt plattens to reduce the gaps on the belt faces in one of them, and the results from both of them are very similar. Secondly, the Author stopped his tests at the peak, and it is possible that he may not observe a macro-slip line which is visible to the eye at that strain although there may be one which is not apparent yet. Unlike the results of ISC tests Lade does not observe failure planes in his samples most of the time (regardless of b). In fact this observation led his line vs. zone failure hypothesis. Although this hypothesis cannot explain the large ϕ' differences between generalised tests at b=1 and conventional extension tests, it is believed that the peaking ϕ'-b relation at about b = 0.50-0.80 in flexible platten ISC tests and the dissappearance of formation of slip planes after this peak range while ϕ' values show decrease seem to be related which is somewhat similar to what Lade claims in a different context, i.e. the cause of this behaviour is not non-uniformities (as Lade put it), but it is fundamentally related to the mechanics of cuboidal generalised tests. (For example, although there is no apparent difference in the uniformity of strains in tests with $b < .75$ and $b = 0.75 \div 1.00$, the former group of tests show slip planes unlike the latter.)

Apart from the discussion given earlier in this Chapter about the strength variation at higher intermediate stress states, it must be also noticed that the shape of the sample used by the Author is exactly a cube, and this geometry prevents a free slip line pass through the sample (in the case of rigid plattens) but a potential line would intersect top and bottom plattens and could cause higher peak strengths to be recorded even if lubrication is provided at the plattens. Bishop and Green (1965) and Green (1969) clearly indicated the extent of the restraint caused by the plattens in the case of various ratios of sample dimensions in triaxial compression. This was especially true when the ratio of H/D (D is diameter or shortest dimension in the case of a prism) was lower than 1.5. This finding, as the result of triaxial compression tests, was actually the reason for Green's selection of 3.3.2 geometry (ratios between dimensions) for the prism shaped ISC sample rather than a cube. On the other hand, Lade (1972) reports results of tests on triaxial compression samples both cylindrical with H/D = 2.5 without any lubrication and cube and cylindrical samples with H/D = 1.0, one lubricated rubber sheet on top and bottom plattens (and in one test two lubricated sheets at the top). Samples with H/D = 2.5 give approximately 3.5° lower ϕ' value compared with samples with H/D = 1.0 both dense and loose, and he again explains the smaller ϕ' values using his "line failure" vs. "zone failure" concept. It is important to note at this point that the generalised soil testing apparatus at the University College which use six flexible plattens gives identical ϕ' value to that in conventional triaxial compression test. For example, Green's lubricated H/D = 1.5 samples give the same ϕ' as H/D = 2 samples with rough ends, but H/D = 1.0 samples give a degree or more higher ϕ' values than these values.

It seems to the Writer that this is both a sample shape and end platten constraint effect, and since Lade considers the strengths from H/D = 1.0 samples are correct ones, his entire strength data (ϕ'-b) in the generalised field may be overestimated in all states by as much as 3.5° due to end constraint.

7.6. Failure Characteristics Other than ϕ'

If a comparison on a deformation parameter is going to be made between any two different kind of tests, care must be given to the stress paths followed during the tests, otherwise a direct comparison may be misleading. Before trying to compare some of the failure characteristics of ISC, ISC-SP and extension tests it may be helpful to see the stress paths followed along representative tests in each group, figure 5.11. The variation of b until failure state must also be considered. Volume change rates may be less affected than principal strains or volumetric strains. In figure 7.6 volume change rates with respect to major principal strains from ISC, SP1-8, SP9-16 and extension tests have all been plotted. The solid line represents the corrected values for initial porosity and the effect of mean stress level. The dashed line shows the values corrected only for the effect of differing initial porosities. Tests points for SP9-16 series have been shown twice, upper values have been modified for porosity lower ones both for porosity and average stress level. Relevant data from triaxial extension tests have been placed in two zones, again the upper, dashed one is for porosity corrected values, the lower one both for porosity and stress level. Stress paths in SP14, 15, 16 conform to those in ISC tests, and so EX1, EX3, EX6, EX8, EXII-1, 2, 4 do.

It can be noticed in figure 7.6 that between triaxial compression (b=0) and b= 0.5 $d\varepsilon v/d\varepsilon_1$ values are constant in ISC tests on loose samples, thereafter they gradually increase until b = 1.0. On the other hand SP 9-16 series and triaxial extension tests indicate very close values to this $d\varepsilon v/d\varepsilon_1$ value in b = 0.50 - 1.00 range, therefore, suggesting a constant $d\varepsilon v/d\varepsilon_1$ value for the whole intermediate stress space. It is noticed that the difference in strength between loose triaxial extension tests and generalised (ISC) tests b = 1.0 is also reflected in volume change rates.

Dense samples also show a difference. It must be borne in mind that the major principal strains are in different directions in ISC and SP1-8 series relative to SP9-16 and extension tests. It was indicated in the previous paragraphs the deformations are affected by structural anisotropy during deposition of the samples. Reades' (1972) horizontal and vertical plain strain tests give very similar volume change rates with respect to the major principal strains. This may be explained by the simultaneous effect of the anisotropy on both volume changes and major principal strains.

Since other deformation characteristics are directly influenced by stress paths, it is more suitable to make comparisons on individually selected tests in which paths are similar rather than comparing them in general. All results are presented in figures 7.7, 7.8 for εv and εa.

Some stress-strain characteristics like deformation moduli etc. have been deferred to later section 9.4 but the following comparative graphs presented here will be referenced back in later sections. Results from other apparatuses that contain generalised tests throughout the whole intermediate stress range will also be compared with the Writers'. In figure 7.9 two tests have been plotted, both at about b = 0.70. ISC(F)12 is a flexible platten test , SP16 is a rigid platten one. SP16 is a test in the second mode while ISC(F)12 is the first mode. And the average stress level increases during shearing in both tests. By coincidence the intermediate stress ratios in ISC(F)12 coincide with the major stress ratios in SP16 throughout shearing. It is apparent that ISC(F)12 yields a higher strength which has already been discussed. Stress paths are alike in both tests, and yet volumetric strains in SP16 are much larger relative to ISC(F)12. The intermediate stress ratios in SP16 are rather unusual with $\frac{\sigma_c(\sigma_2)}{\sigma_a(\sigma_3)}$

increasing at a higher rate near the end of the test. This is because increase in cell pressure was stopped near peak so as to control the axial stresses more effectively. The strains are totally different in two tests. An attempt to explain these observations was made previously when strengths were compared. Higher volume changes and linear strains in SP16 seem to originate from basically different mechanism of shearing the samples.

Five tests have been plotted in figure 7.10 for comparison. SP2 and SP5 are rigid platten tests (first mode). ISC(F) 11 is a flexible test in the first mode and SP10, 11 are tests in the second mode, and their stress paths to failure are different because the mean stress level decreases. b values and initial porosities are similar. SP10 and 11 are seen to be almost identical in stress-strain behaviour, they give lower stress ratios at higher principal stresses at the peaks - about 0.5 lower -. Volumetric strains at failure are larger than those of the ISC group of tests. This very definite difference in behaviour is like the one seen in figure 7.9, and it can not be explained by increasing or decreasing average stress level. It is also seen that no major difference results from whether rigid or flexible plattens are used.

In figure 7.11, results of two triaxial extension and two flexible ISC tests at b values larger than .90 have been displayed. The extension tests have similar stress paths to the ISC tests. The disparity between the two groups of tests is even more pronounced it must be noted EX8 is denser than the others. Mean stress level in the extension tests are close to the ISC tests. A point worth bearing in mind is that because of the non-uniform deformation in extension tests the peak stress ratios may be somewhat higher than that plotted on the graph, because they have been drawn on the basis of calculations that assume uniform deformations.

A number of vertical and horizontal plain strain tests were performed by Reades (1972) on loose Ham River sand in the ISC apparatus which indicated that, at failure, 30 percent higher linear strains resulted from shearing horizontally. The major principal strains in extension tests are 75 percent larger than those in ISC18 and 19 at failure. Therefore, nearly half of the difference may be due to structural anisotropy. Arthur and Menzies' (1972) measured differences of up to 200% must be treated cautiously because it is the difference in linear strains for a certain prepeak stress ratio, not at failure. For example, for stress ratio of 3.00 (3/4 of the peak approximately), the major principal strain (axial) in ISC(F)19 is 0.4%, it is 3.35% (Lateral) in EX6 for the same stress ratio; 800% difference! Therefore the difference is not at the order to be reconciled with the effect of anisotropy. It is directly related to the method of applying loads to fail the sample which has been already discussed early in this Chapter.

Volumetric strains in extension tests are appreciably higher than ISC tests in spite of the average stress level in the latter being a little higher. Three extension tests EX5, 11, 12 have been plotted in Figure 7.12 together with SP11 (b = .82). Mean stress level decreases in all tests. Stress-strain curves for EX11 and 12 are very similar throughout the entire shearing range. EX5 would probably be very close to them if it were somewhat looser. It is interesting to note how the stress-strain curve of SP11 resembles to those of extension tests. It follows them up to $^3/_4$ of the peak ratio. Major principal and volumetric strains are comparable for all tests.

The difference in behaviour in the two testing modes adapted can also be seen in Sutherland and Mesdary's (1969), figures 8 and 9 where the former test is failed as in the Writer's first mode while the latter in the second mode, b values are not much different, .67 and .77 and porosities are 43.3 and 39.5 percent respectively. The former has a major principal strain of 2.5 percent compared with the latter failing at 6.0 percent strain although the latter is much denser. Similarly, it can be seen in figure 7.10 that ISC-SP2 and ISC(F)11 (both tested in the first mode) fail at 5.5. and 4.5 major principal strains while SP10 (second mode) fails at 8 percent strain at almost identical b values.

Ramamurthy and Rawat (1973) did not perform tests on loose samples. The major principal strains in his F series (dense) hardly change from b = .60 to 1.0, a similar observation to the Writers'. Although Lade's (1972) two triaxial extension tests can not be taken as definite (one dense, one loose) loose test gives higher strains at failure compared with the cubical test at b = 1.0, and the dense test gives comparable strains to cubical test at b = 1.0. It may be misleading to compare the strains in ISC tests directly with his results, because his belt loading mechanism was stress-controlled in his cubical series so that b was kept constant throughout shearing whereas b was increased from low to higher stress-ratios during shearing in ISC series. Major principal strains in his tests do not show significant increases after b = 0.50 (in the case of dense samples there is even a little decrease) unlike the major principal strains observed in ISC tests with rigid plattens (first mode). The Writer's flexible platten series do not indicate any change in ε_1 for b > 0.50. It is interesting to note that Al-Ani's (1975) tests do not yield any change for b > 0.50 as well.

7.7. Conclusions

In this chapter results from various series of tests on Ham River sand were discussed. The main issue was the strength behaviour of samples with changing intermediate stress. Results of the tests were examined together with the data from other cohesionless soils in other generalised testing apparatuses. It was found that the design of the apparatus and loading method of the sample are significant factors influencing the observed behaviour especially in the high intermediate stress range. Since the effects due to apparatus and type of loading were already built in the observed ϕ' values, an attempt was made to estimate their extent of influence, both qualitatively and quantitatively. It was concluded that the plain strain strength was more or less the maximum strength value that could be obtained in the intermediate stress space. ϕ' observed in dense samples seemed to show some increase a little more after plain strain state. It was most marked in dense volcanic sand samples. Apparatuses that use six identical flexible loading plattens as in Lomize et. al. (1967), (1969) and Al-Ani (1975) show larger increase over plain strain strength.

In addition to the effect of method of loading of the sample, platten interference effect, was concluded to exist in the mode which employs two rigid pairs of plattens driven toward each other applying major and intermediate stresses. Larger stresses were expected to be mobilised on the rigid plattens near the edges where the two pairs met. A sharp increase in ϕ' values was observed in the tests with very high b values - especially on looser samples - in this mode were linked with both the way of straining the sample and interference effects along the edges. Flexible plattens were observed to decrease the effect of platten interference because they did not allow non-uniform stresses to develop on their faces. Therefore ϕ' values measured in the tests on loose samples with flexible plattens near b = 1 were lower than those using rigid plattens although the testing modes were the same, but ϕ' values (using flexible plattens) were still larger than those in the second mode of generalised tests (SP9-16) and triaxial extension tests due to the relatively more significant effect of method of straining and failing the sample.

It was deduced that ϕ' at b = 1 was not represented realistically by triaxial extension tests nor with ISC tests, and they were imagined to be lower and upper bounds respectively. On the other hand it was concluded that ϕ' decrease near b=1 was related to material behaviour.

Generally, it is concluded that variation of strength with changing intermediate stress as measured from any 3-dimensional testing apparatus should be carefully examined with respect to the apparatus itself, and the results must be treated with caution before they are used in engineering computations.

Failure characteristics other than ϕ' were also affected by the different modes of testing. For example, volume changes and linear strains were larger in the second mode than in the first mode, and volume change rates after b = 0.5 were larger in the first mode.

Corrected strengths for all tests on Ham River sand

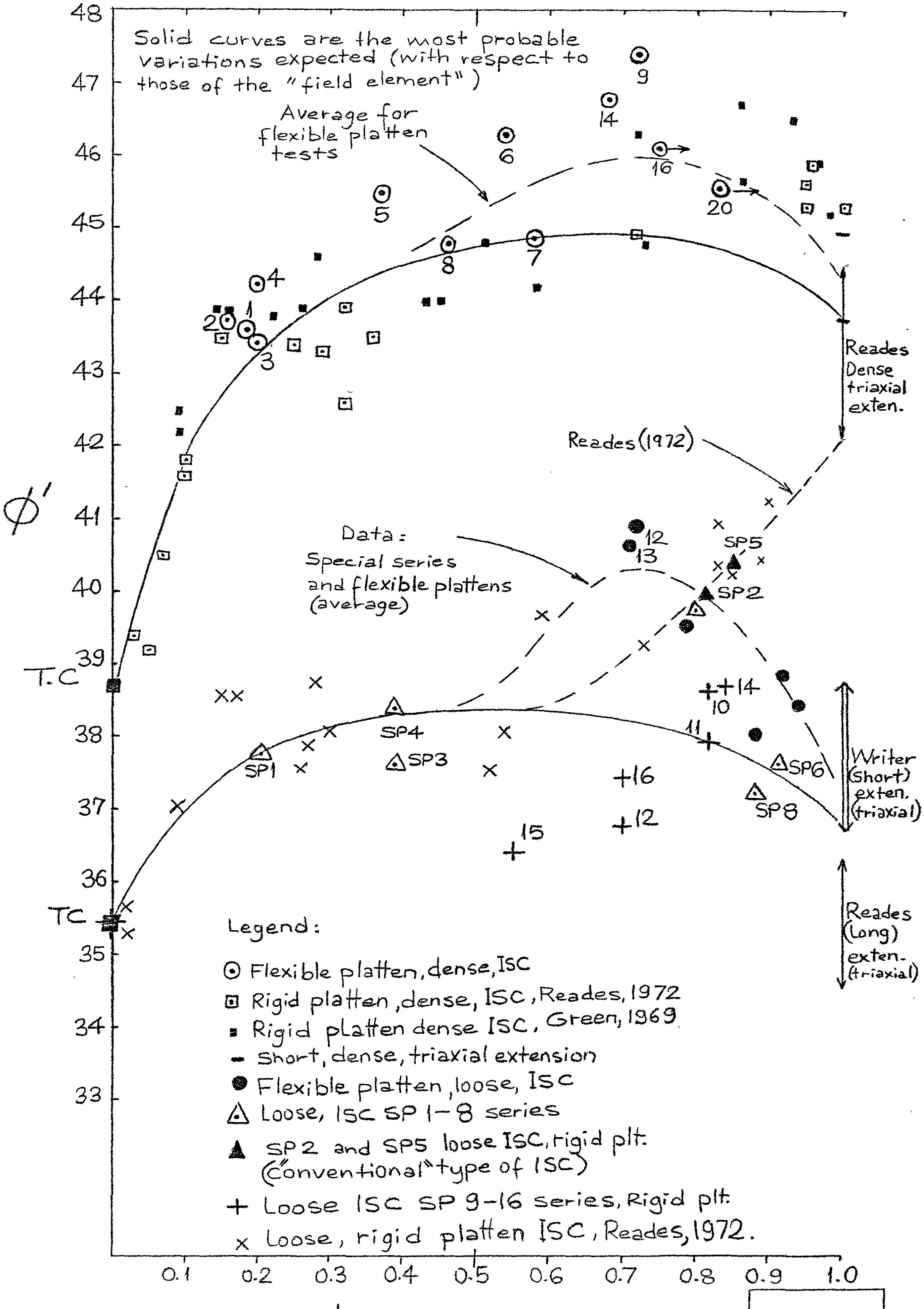
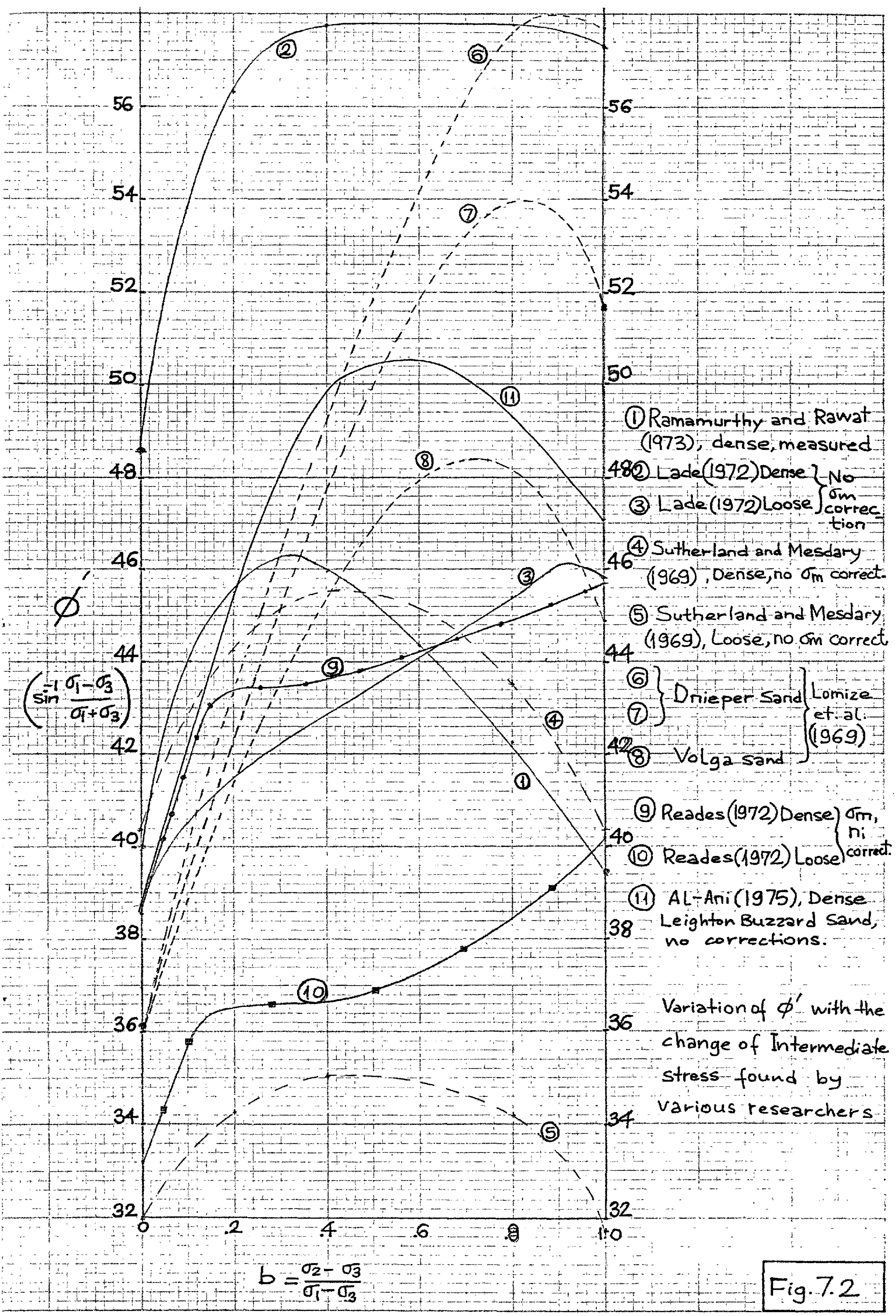
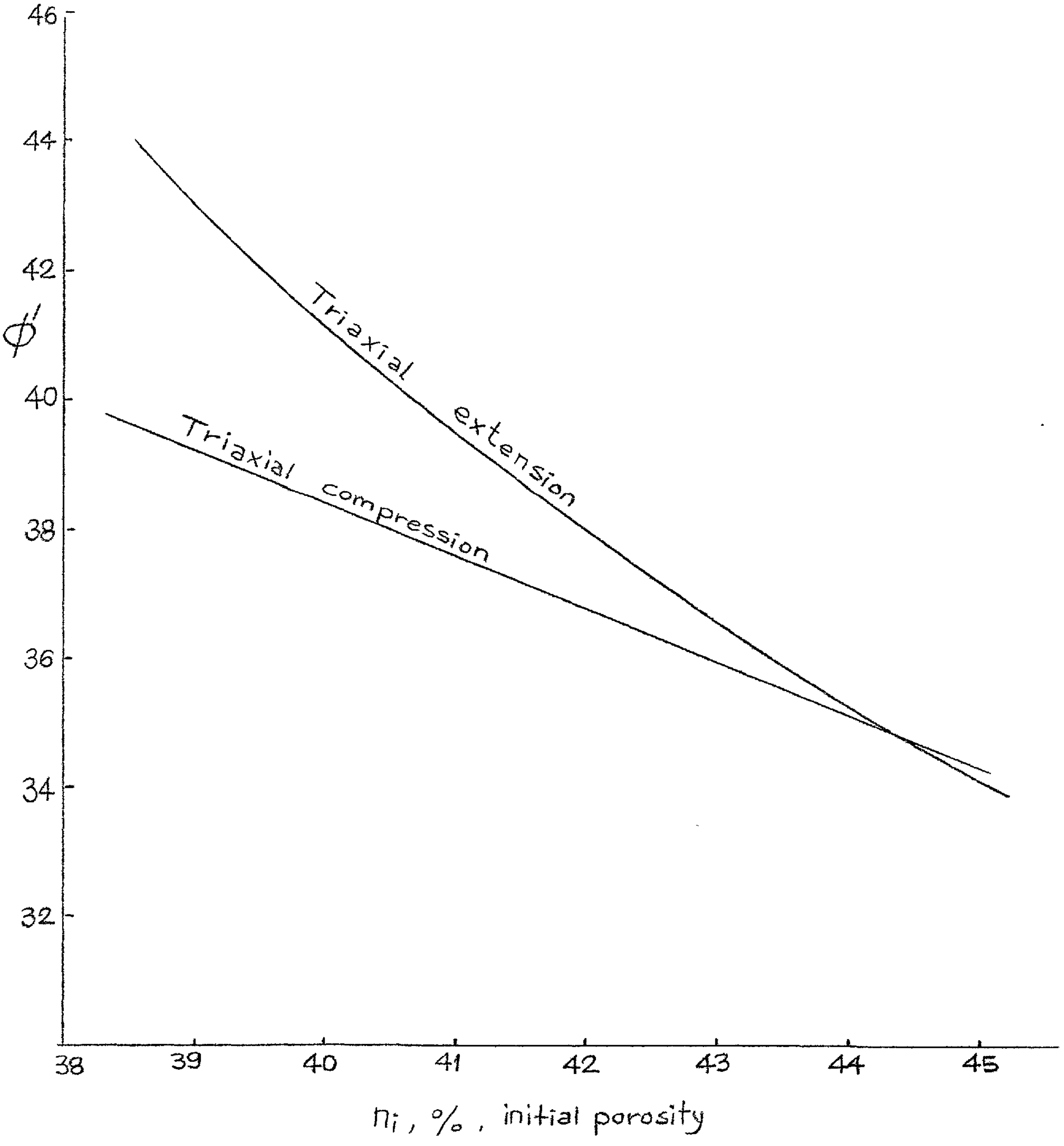


Fig. 7.1



Variation of ϕ' with the change of Intermediate stress found by various researchers

Fig. 7.2



Comparison of peak strengths in triaxial compression and extension tests. (Average values)

Tests corrected for:

a) Non-uniform deformation
b) Sample sheath strength
c) Mean stress level

Data from Reades (1972)

Fig. 7.3

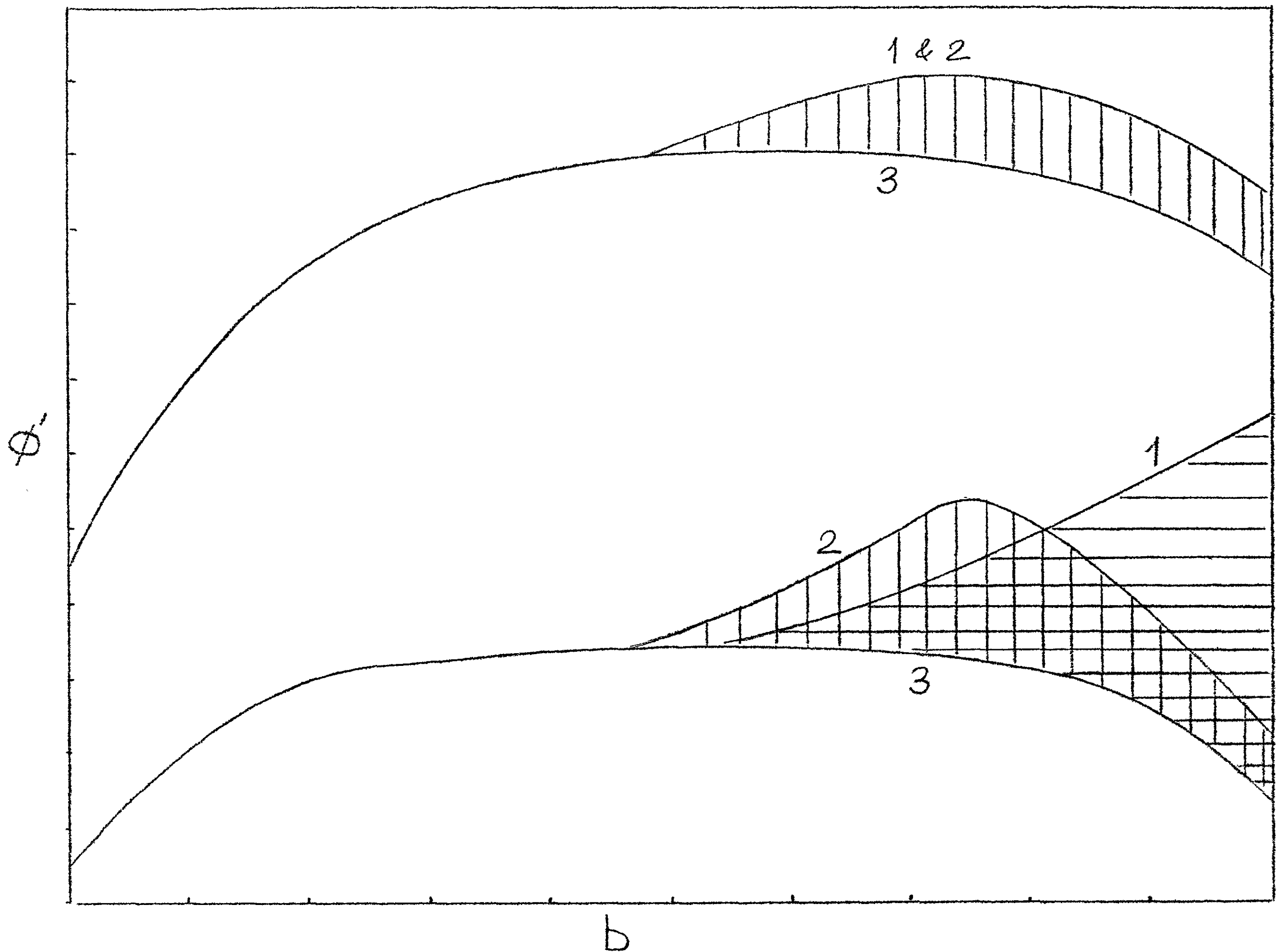
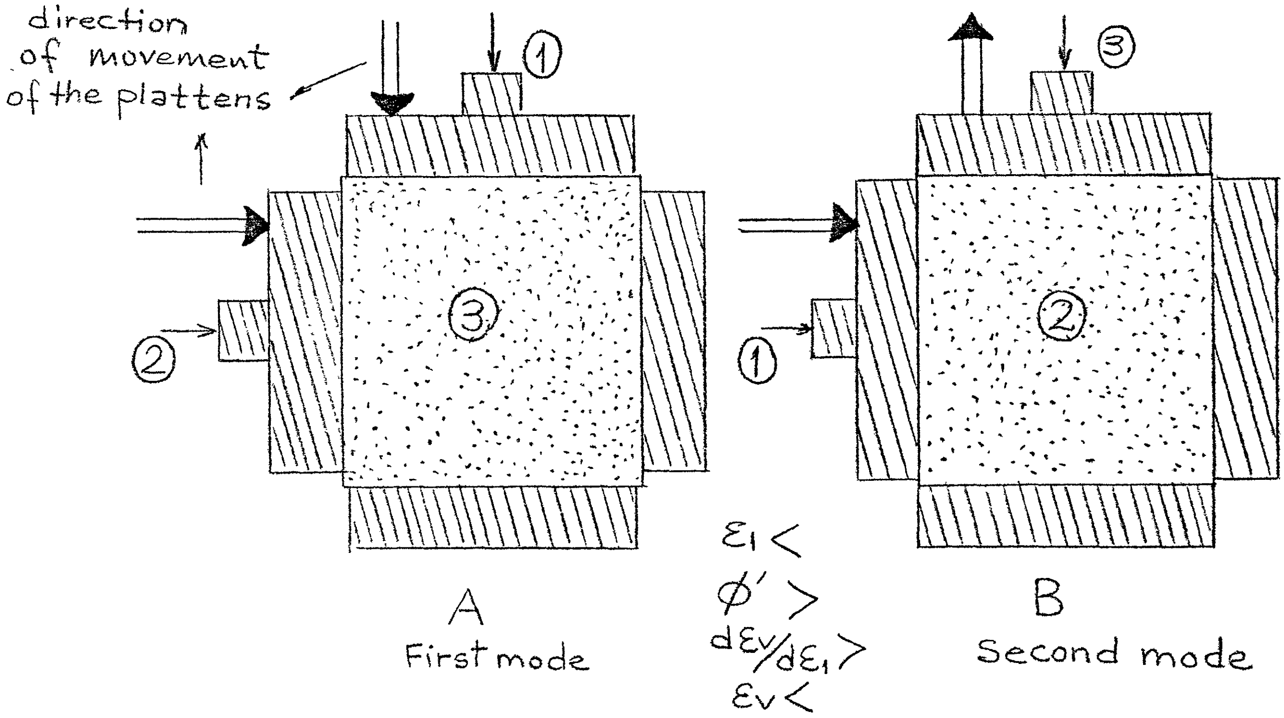


Fig. 7.5

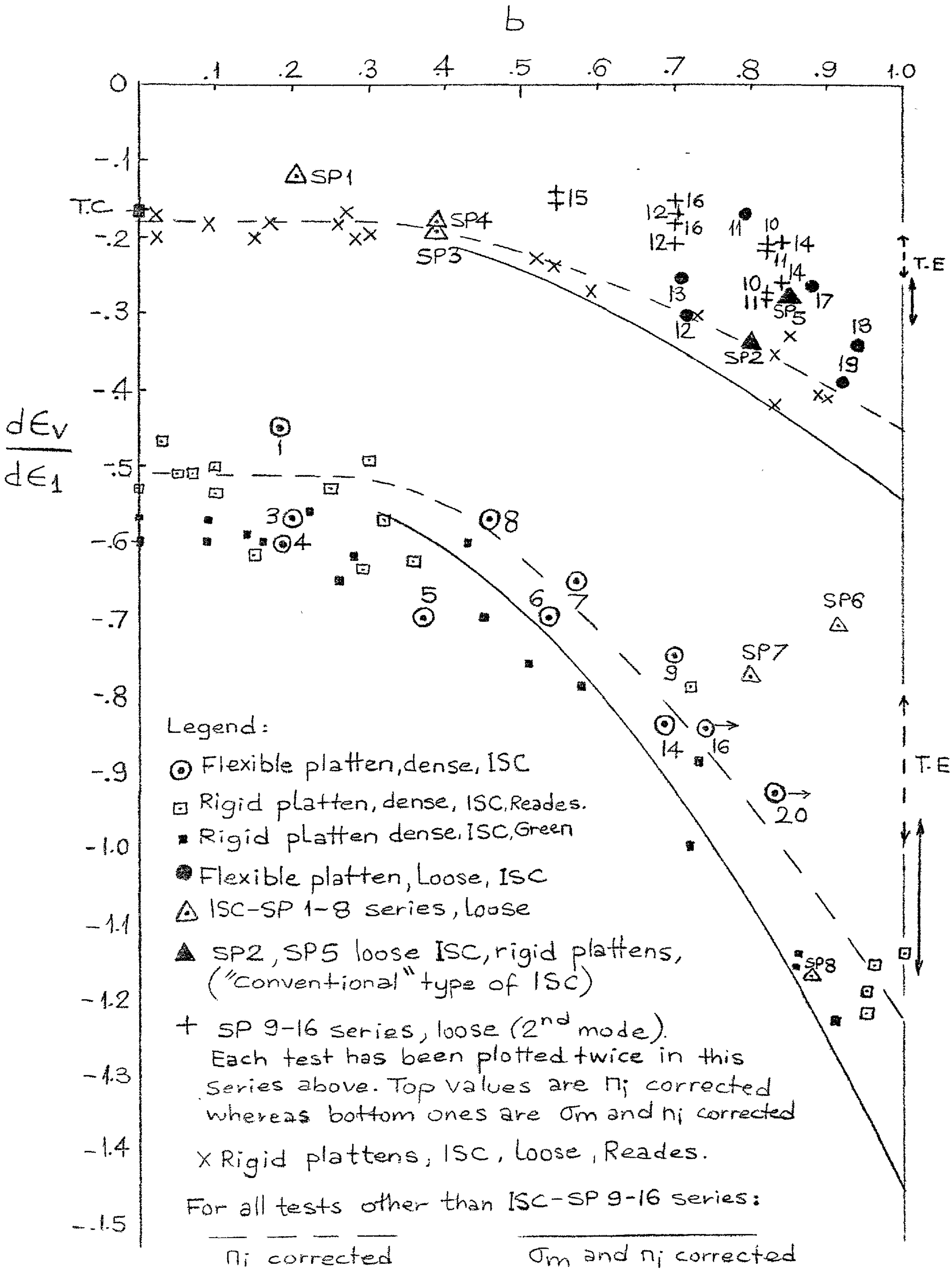


First mode

Second mode

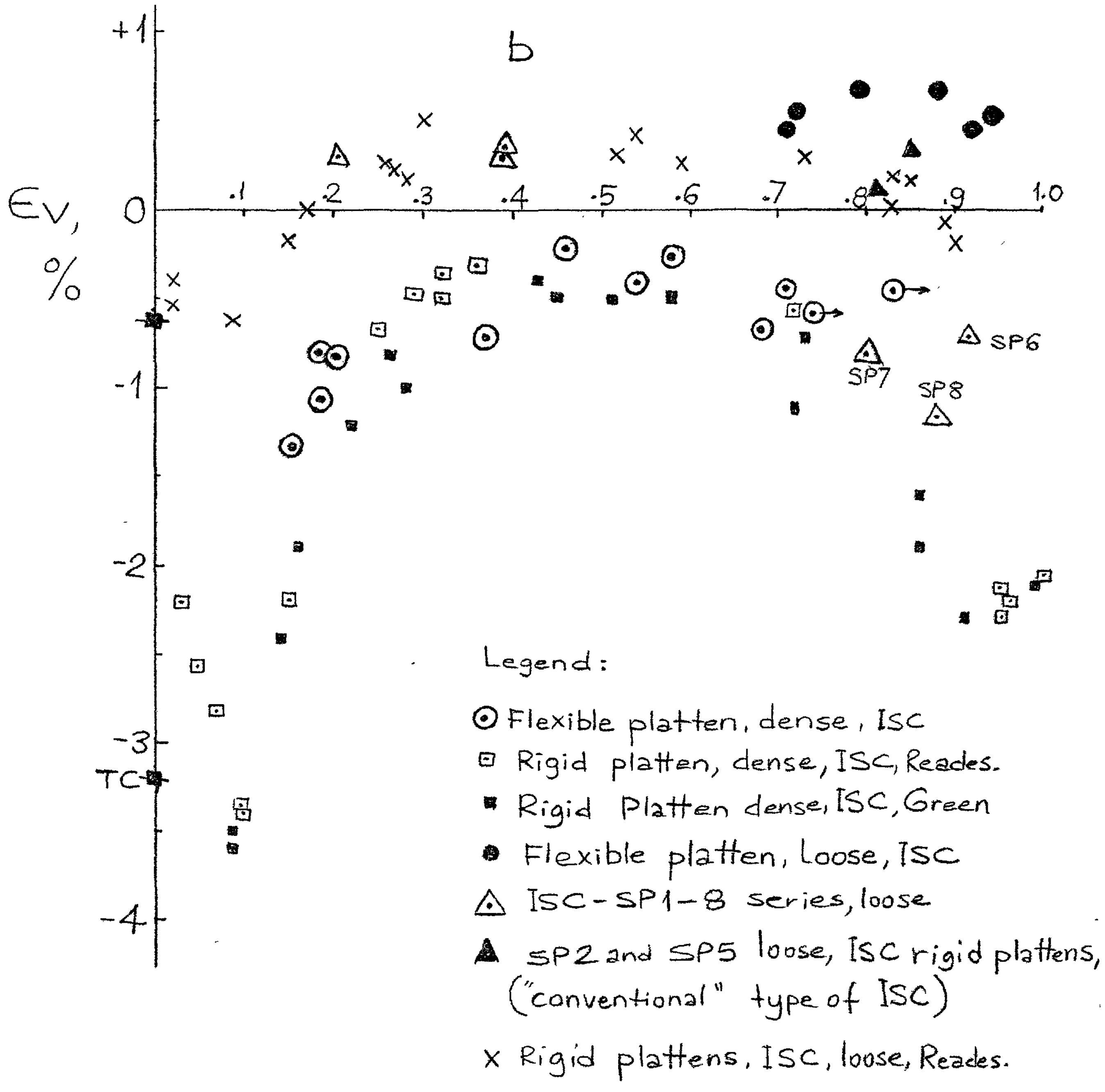
The two testing modes in the ISC apparatus using rigid plattens; ①, ②, ③ principal stresses
Fluid pressure is applied perpendicular to the plane of the figure.

Fig. 7.4



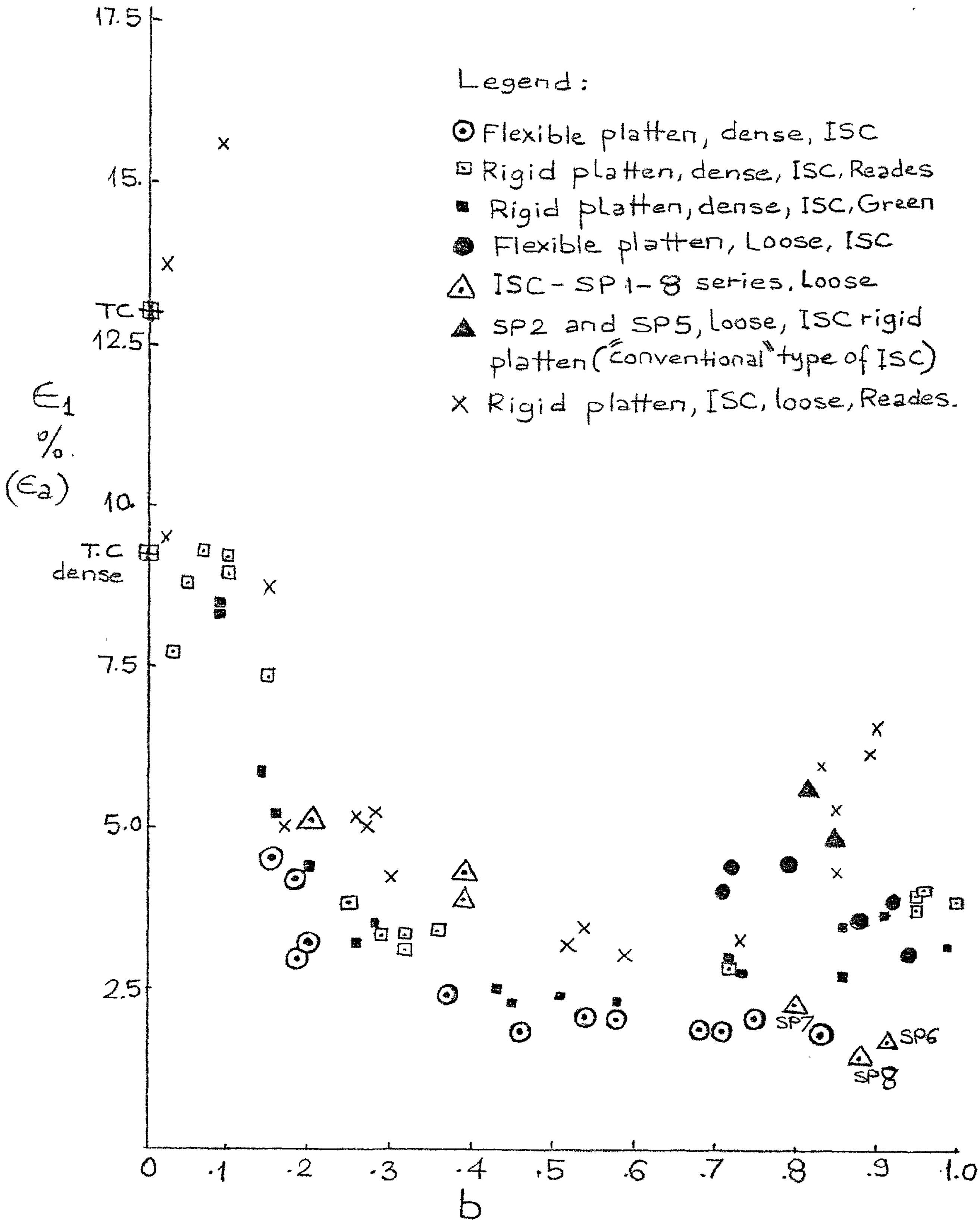
Rate of volume change with respect to the major principal strain for all tests on Ham River sand

Fig. 7.6



Volume change in tests on Ham River sand.
(1st mode only)

Fig. 7.7



Major principal strains in generalised tests on Ham River sand (1st mode only)

Fig. 7.8

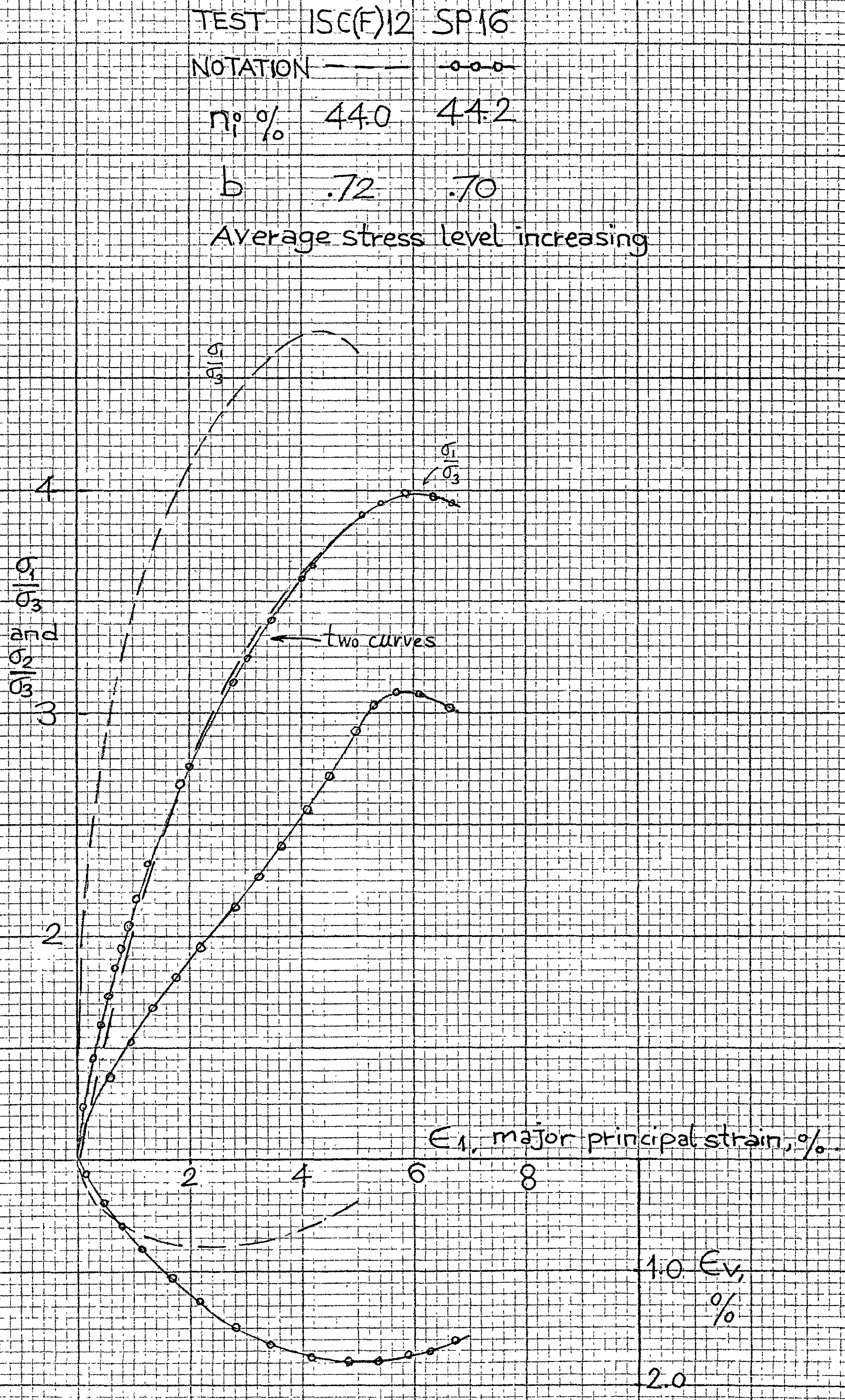

TEST ISC(F)12 SP16
NOTATION —— —— -o-o-o-
n_i % 44.0 44.2
b .72 .70
Average stress level increasing
$\frac{\sigma_1}{\sigma_3}$
$\frac{\sigma_1}{\sigma_3}$
two curves
$\frac{\sigma_1}{\sigma_3}$ and $\frac{\sigma_2}{\sigma_3}$
4
3
2
ϵ_1, major principal strain, %
2
4
6
8
1.0 ϵ_v, %
2.0

Fig. 7.9

TEST	SP2	SP5	SP10	SP11	ISC(F)11
NOTATION	— — —	———	- - - - -	—■—■—	—•—•—•—
n_i %	44.0	43.9	44.4	44.4	43.6
b	.82	.86	.82	.82	.79
	σ_m increasing		σ_m decreasing		σ_m increasing

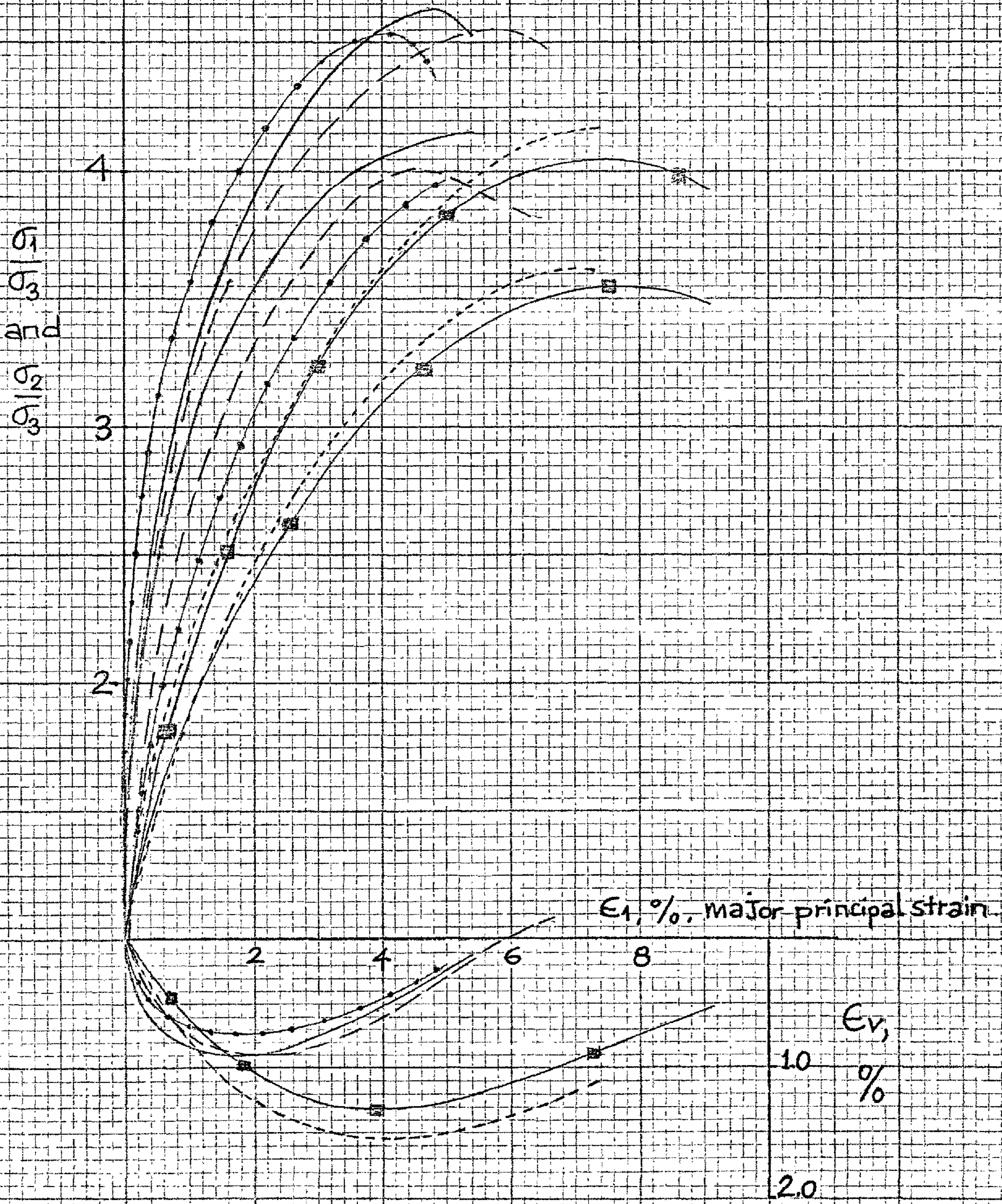


Fig. 7.10

TEST	EX6	EX8	ISC(F)18	ISC(F)19
NOTATION	—■—■—	— — —	- - - -	———
n_i %	43.8	43.3	44.4	43.9
b	1.0	1.0	.94	.92

Average stress level increasing

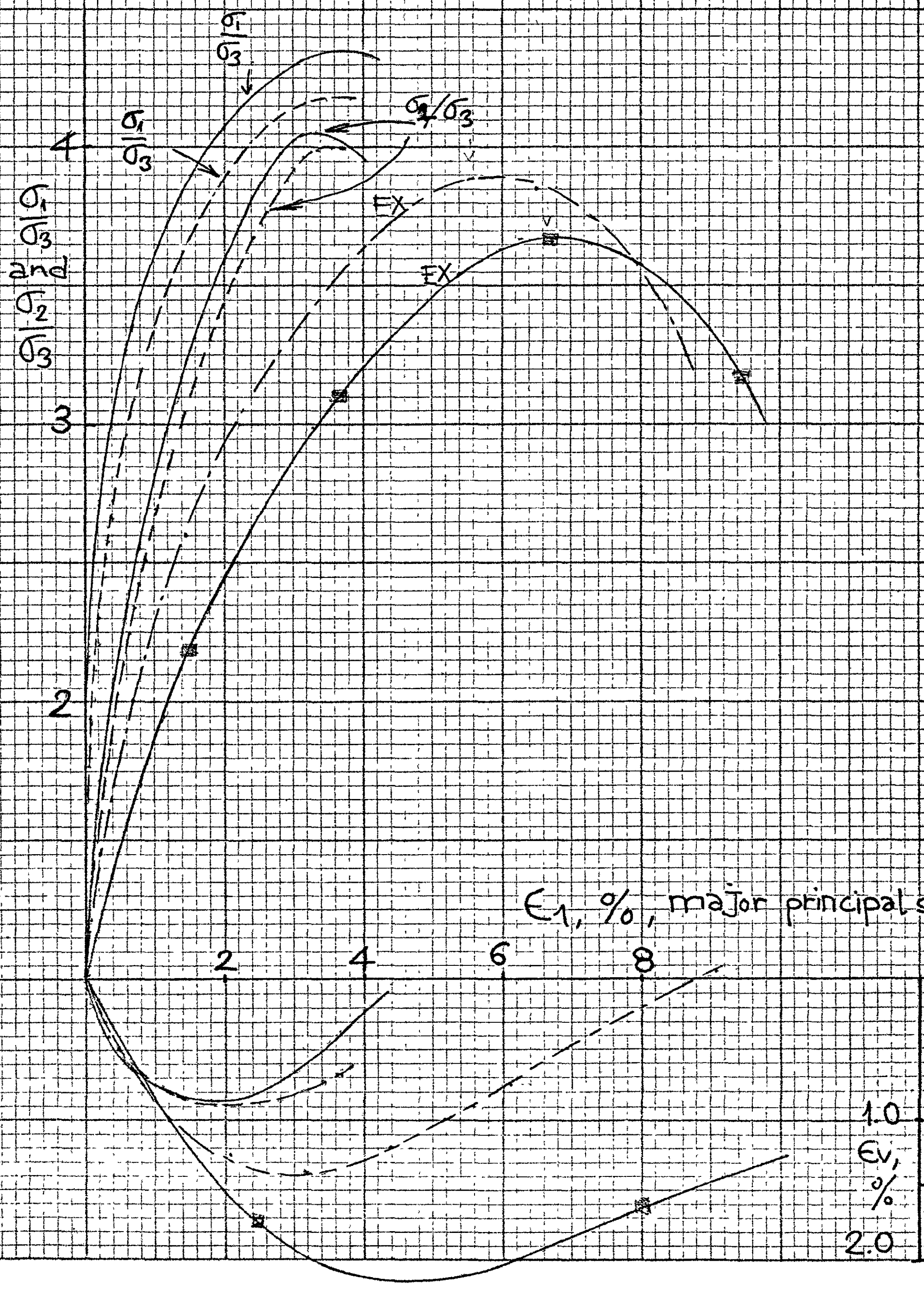


Fig. 7.11

TEST	EX5	EX11	EX12	SP11
NOTATION	———	— — —	- - - - -	—— ——
n_i %	43.3	43.9	44.2	44.5
b	1.0	1.0	1.0	0.82

Average stress level decreasing

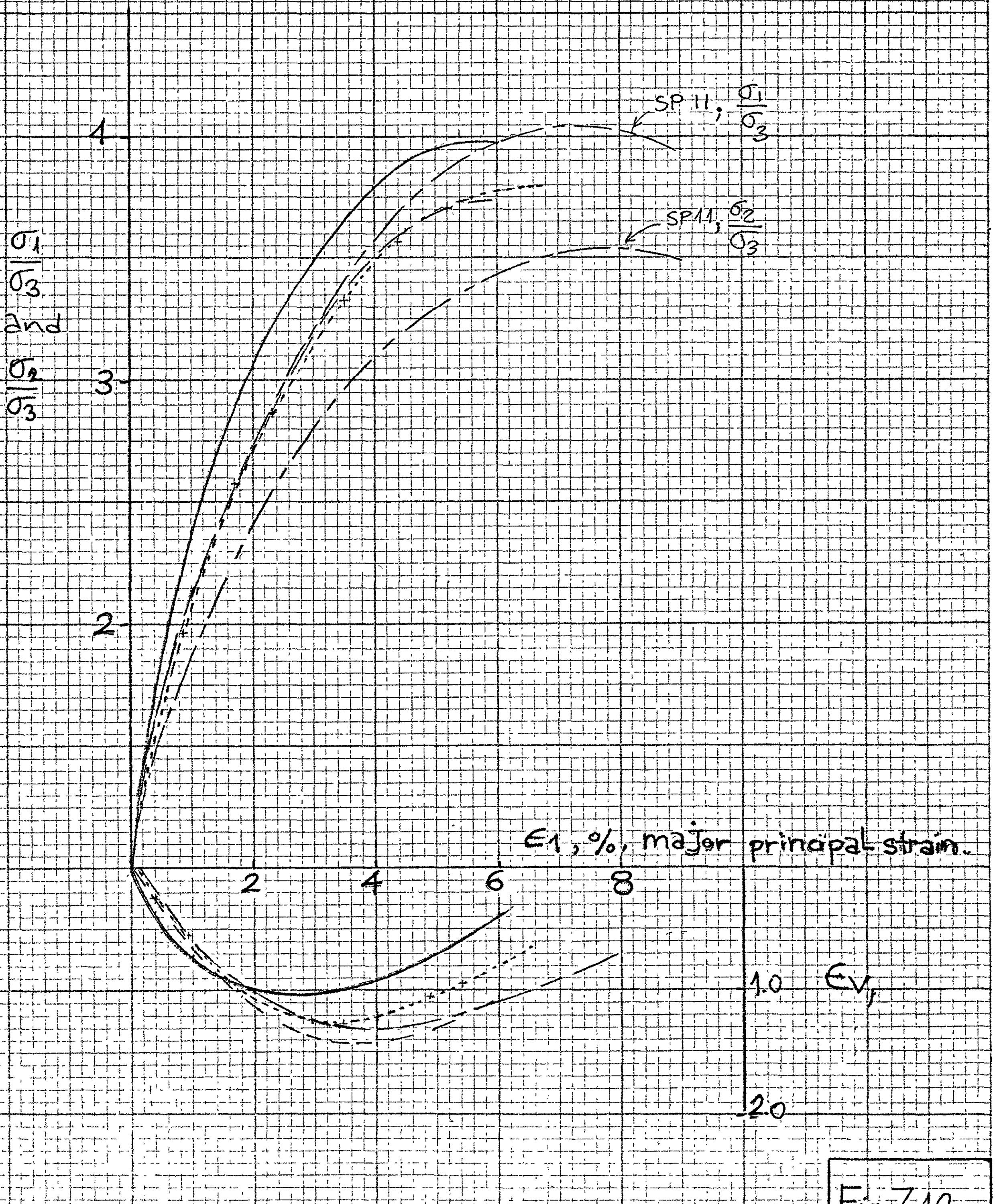


Fig. 7.12

CHAPTER 8

FAILURE CHARACTERISTICS OF VOLCANIC SAND SAMPLES IN GENERALISED TESTS

8.1. Introduction.

Granular material with differing ϕ' values, must be tested in order to generalise the findings. On the suggestion of Prof.Bishop a volcanic sand was chosen. The particular characteristics of the volcanic sand chosen is that it has different index properties than Ham River sand- see Chapter 4,- and more important, it yields appreciably higher strength than Ham River sand in triaxial compression at similar relative densities and mean normal stresses.

An adequate number of tests were planned to cover the intermediate stress space and to disclose the effects of mean stress level and porosity changes. Seventeen generalised tests were performed in the ISC apparatus with rigid plattens and five triaxial compression and three triaxial extension tests were also performed. Roughly half of the samples were loose and the remaining half were dense. Presentation and discussion of the results are given together to start with and comparison of the behaviour with Ham River sand will be given in a final section.

8.2. Results of Tests on Volcanic Sand.

8.2.1. Dense Samples.

Failure characteristics for dense samples are presented in table 8.1. ϕ' values are plotted against b in figure 8.1. They increase from triaxial compression at about 45° to 56° at b = .75, an eleven degree increase. After b = .75 a decrease is apparent at b = 1.0 to 52°.

The material has a suprisingly high compressibility as well as strength, and special attention had to be paid to the size of the gaps initially allowed between the plattens. The height of the mould was raised. Brass plattens were used in tests on loose samples at high intermediate stresses. See Chapter 4 for details. ISC L2, a high b value test (loose), had to be stopped before reaching failure due to insufficient gaps. During the test ISCD7 major and intermediate stress ratios were almost equal until near failure. Belt stress ratio then became larger and reached peak before the axial stress ratio. If belt stresses are taken as the major, b can be said to be equal to .96, but then it must be somewhat increased because the platten contact ratio was .89 (Such a correction would approximately amount to 1.9°) Belt contact ratios lower than one imply that b values are underestimated. This is more true for the tests that have high b values at failure. So, test points should actually have been shifted somewhat to the right in figure 8.1. This would not change the general shape of the ϕ' - b curve but the peak would be shifted closer to extension from b = 0.75. The amount of correction can be calculated approximately as in SP9-16 series on Ham River where the belt stresses were major principal stresses.

Triaxial extension test EXV2 (it was explained in Chapter 3 that V stands for volcanic sand) gave a value of 47.4^{o}which was lower than the value implied by ISC tests. Medium dense test EXV2, if interpolated using ϕ' - porosity relation would give 48.1^{o}. These two values seem to agree but its not known where they would have their exact location if extensive series of extension tests had been carried out and a range of ϕ' values were obtained.

All measured ϕ' values in ISC tests have been corrected for the effects of sample sheath rigidity, platten friction and mean stress level. Non-uniformity correction has been additionally applied to triaxial extension tests. The strength of the material was significantly affected with the level of mean stress level. In figure A.2.3 (in Appendix 2) it is indicated on the basis of Writer's tests and five triaxial compression tests on 38mm (1.5in.) diameter samples by Walbancke(1974). This effect is mainly responsible for increasing ϕ' values after plain strain state in figure 8.1. Sample sheath and platten friction correction is almost the same for both Ham River and volcanic sand. Mean stress level correction for volcanic sand is at least three times larger than that of Ham River sand both for loose and dense states. Typical values for the b range are 0.7^{o} - 1.5^{o} for dense Ham River samples. This is 2^{o} - 5^{o} in the case of dense volcanic sand samples. Values of 0.6^{o} and 1^{o}- 3^{o} can be given for loose samples of Ham River and volcanic sand respectively.

Tests ISC D1, D2, D3, D4, D8 gave close measured strengths with a small increase throughout the change of intermediate stress as can be seen in table 8.1. Since the material was relatively compressible and some of the samples were consolidated to different isotropic pressures the ϕ' - b plot in figure 8.1 is based on consolidation porosity rather than initial porosity, and 58 and 64 percent were selected as normalising consolidation porosities for dense and loose samples respectively.

8.2.2. Loose Samples.

ϕ' values for loose samples are also plotted in figure 8.1, and the properties of tests are given in table 8.2. Loose samples deformed more non-uniformly compared to dense samples and some of them expanded at the base. ISCL1 and ISCL9 at $b = .23$ and $.41$ gave almost the same ϕ' about four degrees higher than that in triaxial compression.

After $b = 0.50$ another four degrees increase is observed until $b = .70 - .85$ where there is a peak after which ϕ' decreases till $b = 1.0$ limit. The dashed curve passes through the data points after $b = 0.50$ in figure 8.1, and the full line represents a possible variation without any apparatus effect for the reasons given in Chapter 7. In the case of dense samples such a line has not been shown, and it is expected that little error exists in the observed data certainly not as large a difference as suggested for the loose tests.

ISCL8 is seen to indicate a very sharp drop after the peak in ϕ' - b plot but the b value for this test is actually higher than reported. It was pointed out in Chapter 5 that incomplete coverage of the sample faces by belt plattens caused punching into the larger sample face if the sample was loose, and high intermediate stress states were involved. This in turn caused belt stress ratio to drop sharply before major principal stress ratio attained its peak, therefore peak stress ratio was accompanied by a lower b value which is misleading. - See Appendix 8 for stress-strain curves - The same phenomenon was observed in ISCL7 to a lesser extent where the b value of .78 should be around .83. b value in ISCL8 was probably near extension.

Tests ISCL3, 5, 6, 7 gave exactly the same ϕ' around b = .80. Two triaxial extension tests EXV1 and 2 (medium dense sample which was also interpolated to dense porosities) yielded similar strengths, and were more than two degrees higher than triaxial compression values.

Volume change rates and other deformation properties were observed to be influenced by average stress level and porosity variations. It may be misleading to plot them as measured. Corrected $d\varepsilon v/d\varepsilon a$ values are plotted in figure 8.2. Due to limited number of tests corrections may be regarded as approximate but they are believed to be good estimates. Their magnitude increased from triaxial compression until extension in dense samples. Plain strain values were slightly larger than those in triaxial compression. Much of the increase was in between b = 0 - .50. Volume change rates showed no increase at all up to b = .5 - .6 in loose samples thereafter higher values were observed.

Axial strains at failure whether measured or corrected decreased sharply from triaxial compression to values of about b = 0.60 thereafter corrected values showed little change. Loose samples if plotted as measured showed a certain amount of increase until extension after that range, figure 8.3.

Volume changes at failure gave a different picture if corrected values rather than measured values were considered. For example, dense samples were observed to have larger volume changes than triaxial compression, the higher the b values, however, measured values slightly decreased from triaxial compression to approximately b = .80, figure 8.4.

Eb-b plot, figures 8.5, indicates the intermediate stress level corresponding to zero lateral deformation (K_o) condition between b = 0.25 - .30.

8.3 Comparison of the Results with Ham River Sand and General Discussion.

If results of tests on volcanic sand are compared with those of Ham River Sand, the similarity is apparent.

Variation of ϕ' against b followed exactly the same patterns. Loose volcanic sand samples ISC L1,9 at b = .23 and .41 gave the same strength. This was exactly the same observation as on loose Ham River sand samples in that range. Then a sharp increase up to b = .7 - .8 followed for both materials. ISCL8 which was actually near extension gave a lower ϕ' than the tests around b = .80 in parallel with the observations on dense Ham River sand and Volcanic sand samples in that region, which is unlike rigid platten, loose, Ham River sand samples. It was explained in Chapter 5 that if local punching of the belt plattens was taken into account, ISC tests (Ham R.) having b values larger than .80- .85 should be closer to extension. Since due to the nature of the material ϕ' variation was magnified in tests on volcanic sand, strength reduction near extension was magnified too.

Two loose triaxial extension tests gave ϕ' values two degrees higher than triaxial compression compared to $1 - 1.5^{o}$ for Ham River sand.

Variation of strength - at constant mean normal stress - of dense samples was again very similar in both materials. Despite the lack of a test between plain strain and b = .56, the peak around b = .80 and the decrease afterwards was apparent.

A significant observation, as mentioned above, was that although the pattern of ϕ' variations was similar, the quantities were not. ϕ' differences between various states was appreciably higher compared to Ham River strength data. Plain strain versus triaxial compression ϕ' differences were 5° and 7° for loose and dense volcanic sand samples compared with 2.5° - 3° and 5° for Ham River sand respectively. The total ϕ' increase from triaxial compression to $b = 0.80$ was 11° for dense volcanic sand samples compared to a corresponding figure of 7.5°. Strength (ϕ') decrease near $b = 1$ was of the order of 3° - 4° which can be compared with a degree observed by Green (1969).

On the other hand, ϕ' increases after mid - b values were different in both materials when dense and loose samples were examined comparatively. It was generally observed that denser assemblies gave higher ϕ' increases from triaxial compression to plain strain state, but the increases from plain strain - more correctly from $b = 0.5$ - to higher b values was the same for both densities. (Approximately 4° in tests on volcanic sand, 2.5° on Ham River sand). It brings into mind the question whether this increase of ϕ' in loose samples and at high b values (which was not in match with the whole ϕ' variation) was purely of the material behaviour.

Volume change rate with respect to the major principal strain varied in agreement with Ham River sand data. The $d\varepsilon_v/d\varepsilon_a$ values stayed constant from triaxial compression to $b = .5$ - $.6$ range in loose samples, and they indicated a certain increase afterwards. That constant value also agreed with the values from triaxial extension tests in loose state for both materials. Dense samples of both materials yielded a continious increase from b = 0 - 0.25 where as two dense volcanic samples at b = .21 and .22 gave little higher values relative to those in triaxial compression.

A possible explanation for the above observations can be as the following; Loose samples between plain strain and relatively high intermediate stress levels (about b=0.6) virtually showed no increase in volume change rate, with respect to the major principal strain although the intermediate principal strains and stresses continiously increased. This was because particles had enough void space to rearrange themselves - possibly both rotating and displacing - without resorting to dilate. With further imposition of belt strains (tests with higher b values) the void ratios reached a certain limit so that there might not be any more interparticle rearrangements readily feasible (of course with respect to a certain stress level), and samples started to show a tendency to dilate to shear off. On the other hand, expressing various quantities in soil tests with respect to major principal strains has no fundamental basis. (e.g. plotting stress-strain curves in $\sigma_1-\sigma_3$ vs. ε_1 or σ_1/σ_3 vs. ε_1). This is, in fact, more true in the case of generalised tests. The term "dilation" for example, used in this paragraph would be more correct if it were expressed by a more generalised parameter rather than $d\varepsilon v/d\varepsilon 1$, although the latter gives an idea about volume change. A dilation angle $\nu = -\sin^{-1} \frac{d\varepsilon_1 + d\varepsilon_3}{d\varepsilon_1 - d\varepsilon_3}$, for example, could be used but there is no independent measurement of displacements in the direction of cell pressure application in this study, and they are inferred from volume change and other principal strains.

In dense samples the rearrangement of particles was more restricted due to already existing very close contacts among them, and the samples started dilating with the application of strains and stresses. More strain imposition in the belt direction was associated with more dilation with respect to (compressive) axial strains. But it must be borne in mind that this behaviour and its possible explanation was associated with the first mode. In Chapters 6 and 7 it was disclosed through Ham River sand data that loose generalised tests in the second mode did not yield an increasing $d\varepsilon v/d\varepsilon 1$ at high intermediate stress states, and it was explained that the greater freedom allowed on the boundaries in that mode at high b values prevented samples from dilating which is in agreement with the explanation above.

A parallel may be drawn between volume change rates at failure and observed peak strengths. Like volume change rates ϕ' values stayed constant from plain strain until about b = 0.6 in loose samples and thereafter an increase in both quantities was noticed. Continious increase both in observed strengths and volume change rates (after plain strain state) in dense tests could also be seen clearly. The behaviour of Ham River sand was similar in this respect. These observations suggests that any further strength increase after plain strain could be associated with a more dilatant tendency whose sources could originate from stress state, packing density and boundary conditions imposed on the sample. This entirely supports the discussions in the previous chapters.

The comparison of rate of volume change with respect to major principal strain between plain strain and triaxial test data was interesting. They were almost the same excepting a little higher values in plain strain in dense state for both materials. This observation is not only clear in the tests in the ISC apparatus but in other plain strain apparatuses and data. See, for example, Cornforth (1964) where volume change rates with respect to major principal strains were measured to be nearly identical for triaxial and plain strain data throughout the porosity range. Behaviour of dense samples are in a way in contrast with the explanation in the previous paragraphs, because particles were more restricted to move in plain strain than in triaxial compression, and also there were a considerable ϕ' difference between the two stress states.

It was pointed out earlier that the average porosity at failure was the least certain. In fact, the porosity in the shearing band in the sample must be the most important parameter with regard to the description of packing state of the material but unfortunately it is very difficult or impossible to measure directly at present, and therefore plots of average (bulk) porosity against ϕ' and all other characteristics have their limitations in this respect. How the average porosity in the upper and lower wedges of a failing sample in the vicinity of the shear band would affect the actual porosity in the shearing band is not known either.

Relative changes of major and intermediate stress ratios with respect to the increasing b values were very similar to the behaviour observed in Ham River Sand data, that is, while the major stress ratio peaked, the intermediate stress ratio was in continious increase in the tests between plain strain and $b \simeq 0.80$, but after $b = .80$ and especially in ISC tests near $b = 1.0$, major and intermediate stress ratios both peaked simultaneously. In Chapter 7 it was explained that the sample was, infact, failing in both directions. Single failure planes formed approximately up to $b = 0.80$ at failure, and no distinct failure planes could be observed near $b = 1.00$. In the former case the failure planes were at right angles to the belt plattens, (i.e. shearing displacements were parallel to the belt plattens).

It is interesting to note that the observed ϕ' values attain a maximum in the region where formation of single failure planes ceases to appear. Peaking ϕ' - b relation seems to have something to do with failure plane formation. In b = 0.75 - 1.00 region although ϕ' values show a decrease, $d\varepsilon v/d\varepsilon 1$ values still increase which is in contrast with the explanation which associated high ϕ' values with high $d\varepsilon v/d\varepsilon 1$ values between b = 0.20 - 0.80, but it must be clear that $d\varepsilon v/d\varepsilon 1$ values cannot be regarded as governing factors that determine values of ϕ'; in other words dilation cannot be regarded as the sole cause of ϕ' variation. The failure mechanism near b = 1.00 which is very different affects the failure load.

As will be seen in Chapter 9 plain strain tests on volcanic sand roughly yielded 44.5^{o} at zero rate of volume change. Tests ISCD 2 and ISCD 3 (b = .76 and .56 respectively) were the only tests in which large strains were purposely imposed in an attempt to determine the residual values. It was observed that the post-peak stresses dropped rapidly and stabilised rather quickly after the peak stresses had beeen passed. It was pointed out earlier that residual angles could not be attained in the ISC apparatus because rupture planes met the axial plattens or they were very near to them. The residual σ_1/σ_3 ratio of 3.80 in the above tests agrees with the ϕ' at zero rate of volume change. Since the post-peak shearing displacements were very small, this correlation may not be fortuitous. Residual values may be attained before the effect of axial platten restraint becomes effective.

The brittle post-peak behaviour in volcanic sand samples is reflection of the material behaviour. A quantitative measurement of brittleness like "brittleness index", Bishop (1967b),would be useful in the case of a comparison with Ham River sand but the significant effect of the mean stress level on brittleness index and small number of tests in which post-peak strains were attained made such a comparison impossible.

It is speculative at this stage to discuss about the effect of generalised state on residual angles which may not be insignificant. Study of the effect in cubical apparatuses is not easy.

In figure 8.6 a plot of major stress ratios at minimum volume is given (A similar plot for Ham River sand samples was given in figure 5.21). This ratio, at minimum volume, being somewhat lower than the peak ratio, was expected to be free from the effect due to platten interference etc. b* values on the abcissa were not the usual b values at failure but at minimum volume, i.e. $b^* = (R_2-1)/R_1-1)$ wher R_1 and R_2 were major and intermediate stress ratios at minimum volume. The plot is for dense samples. It was observed that the minimum volume was attained at the peak stress ratio in loose samples, so making it inconvenient to use such a graph for its purpose, another reference strain could have been specified for the same purpose though.

The decrease of stress ratios in the second half of the b span was very clear. It amounted to a difference in stress ratio of about one. The decrease in ϕ' along the same span is close to the above value suggesting that the peak values for dense samples were not significantly influenced by apparatus effects which is in total agreement with previous agruments in Chapter 5,7 and in this chapter.

Loose samples seemed to be more influenced by such phenomena. This can be noticed by examining the stres-strain graphs of tests ISCL 3, 4, 5 with b values .72, .56, .79 respectively. Although there was an increase of almost 3^{o} from b = .56 (ISCL 4) to b = .72 (ISCL 3, 5) in ϕ', the stress-strain graphs of these three tests were very close to each other until the final stages. Stress-strain curves for these tests can be seen in Appendix 8.

8.4. Conclusion.

A fine volcanic sand was tested in the ISC apparatus to investigate the failure characteristics in a truely three dimensional stress field. This material was the second tested in the apparatus after Ham River sand and constituted a chance to compare the findings. The failure characteristics were similar to those of Ham River sand generally. The material has a high compressibility and strength. ϕ' values yielded a maximum increase of 11° in the intermediate state in the case of dense samples which had also high ϕ' values in triaxial compression relative to Ham River sand. The variation of ϕ' in dense samples was composed of a sharp increase to plain strain, a gradual increase to about $b = .70$ and then a decrease to $b = 1.00$. In the case of loose samples, ϕ' values again increaed rather sharply to plain strain, then until mid b values there was no appreciable increase, and higher ϕ' values were observed at about $b = .80$ with a tendency to decrease until extension. It may be deduced that behaviour of other granular materials would be similar with quantitative differences.

														CORRECTED STRENGTHS		
Test No.	Initial Porosity. N_i %	Conslolidation Porosity N_c %	b	σ_a kN/M^2	σ_b kN/M^2	σ_c kN/M^2	ε_a %	ε_b %	ε_v %	$\frac{d\varepsilon V}{d\varepsilon a}$	Axial Platten Cont. R.	Belt Platten Contact Ratio	φ' Measured	φ' S.Sheath, Pl.friction	φ' Mean Stress level	φ' Normalised
ISCD1	59.4.	58.7	.21	1453.4	482,2	206.85	6.3	-0.4	0.9	- .23	.97	.90	48.66	48.56	50.46	51.3
ISCD2	59.7	59.0	.76	1551.3	1231.8	206.5	5.1	3.3	1.8	- .29	1.00	.90	49.88	49.67	54.42	55.6
ISCD3	59.1	58.5	.56	1589.1	990.3	206.8	4.9	1.9	1.6	- .37	.99	.89	50.33	50.16	54.36	54.9
ISCD4	57.8	57.1	.87	1603.5	1430.3	206.0	4.6	5.2	0.9	- .65	.99	.89	50.56	50.33	55.43	54.3
ISCD5	58.7	58.3	.91	1078.5	990.5	124,5	4.2	3.9	0.1	- .87	.97	.88	52.46	52.23	54.23	54.5
ISCD6	59.6	59.4	.88	589.9	528.7	50.7	3.3	3.1	-1.2	-1.58	.97	.87	57.33	57.1	53.2	54.8
ISCD7	58.2	57.5	.96 (1.04)	1341.	1389.4	205.8	3.1	4.1	1.8	-0.49	.99	.89	47.85	47.65	52.35	51.8
ISCD8	58.2	57.4	.22	1515.5	521.2	207.0	5.8	-0.6	1.3	- .35	1.00	.91	49.43	49.35	51.75	51.0
TC 2	57.0	56.4	0	1322.9	213.4	213.4	8.4	-	-2.2	-0.55	-	-	46.25	46.17	46.17	44.3
TC 4	58.1	57.4	0	1375.8	220.3	220.3	9.8	-	-2.0	-0.51	-	-	46.38	46.31	46.31	45.6
TC 5	57.4	56.8	0	1301.7	206.85	206.85	9.4	-	-1.6	- .47	-	-	46.53	46.4	46.4	45.0
EXV 3	57.2	55.6	1	161.4	1036.7	1036.7	-4.0	2.0	.2	- .49	-	-	46.93	46.76	49.16	47.4

Failure Characteristics of Dense Volcanic Sand Samples.

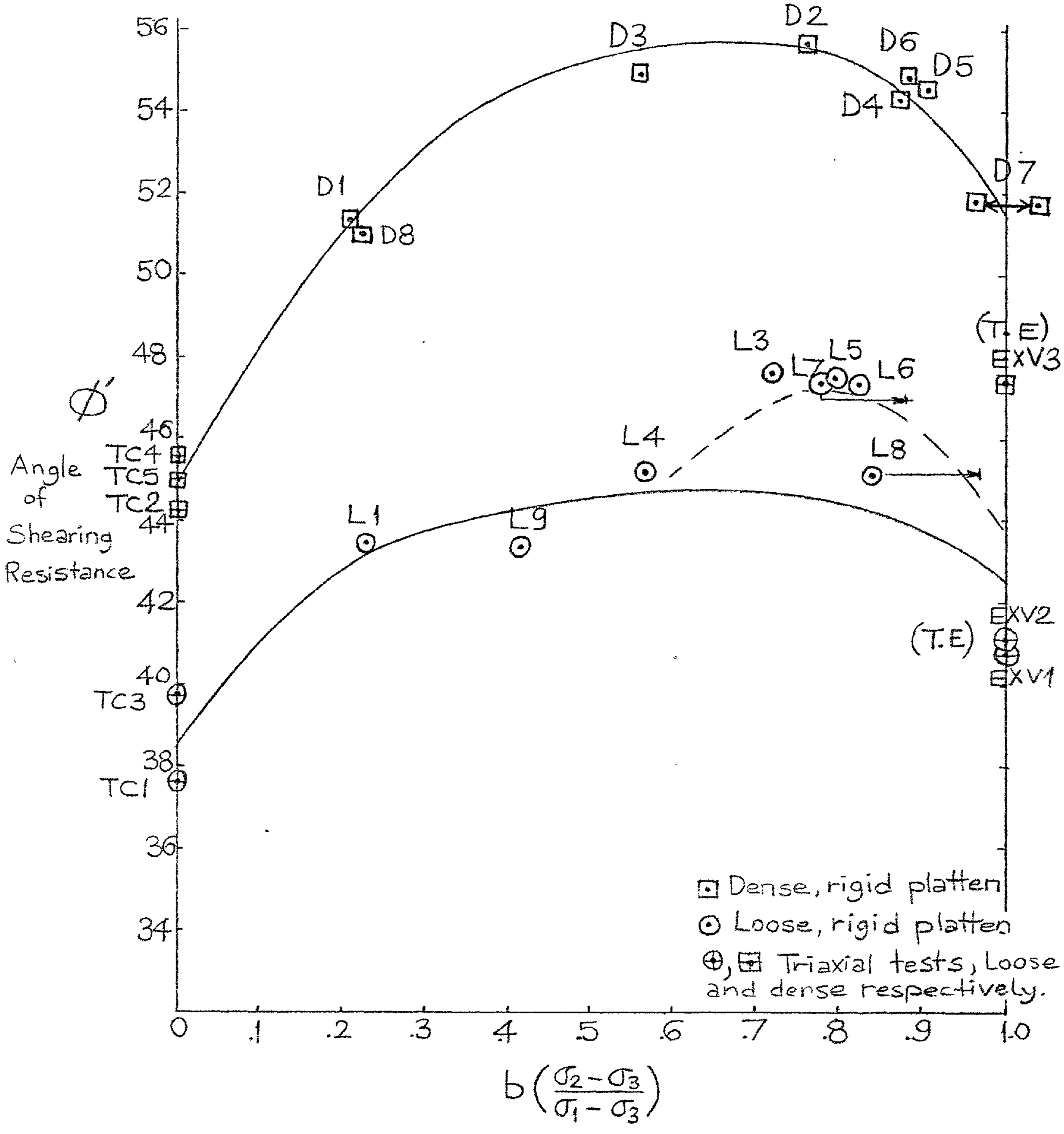
Table 8.1

TEST No.	INITIAL POROSITY η_1^o %	CONSOLIDATION POROSITY η_c %	b	σ_a kN/M^2	σ_b kN/M^2	σ_c kN/M^2	ε_a %	ε_b %	ε_v %	$\frac{d\varepsilon_v}{d\varepsilon_a}$	AXIAL PLATTEN CONTACT RATIO	BELT PLATTEN CONTACT RATIO	ϕ' MEASURED	CORRECTED STRENGTHS		
														ϕ' S.SHEATH PLATTEN FRIC.	ϕ' MEAN STRESS	ϕ' NORMAL
ISCL1	64.1	63.1	.23	1118.2	426.4	206.2	11.0	-0.4	4.7	+.05	.98	.96	43.50	43.36	44.6	43.5
ISCL3	64.9	64.1	.72	856.9	646.3	134.1	9.0	4.4	4.2	+.03	.99	.81	46.83	46.56	47.51	47.6
ISCL4	65.2	64.4	.56	766.3	494.3	131.3	8.0	2.8	4.0	+.05	.98	.81	45.00	44.79	44.79	45.2
ISCL5	64.8	64.1	.79	758.1	627.0	115.5	9.8	6.9	3.9	-.06	1.02	.82	47.35	47.05	47.45	47.5
ISCL6	64.4	63.3	.82	1231.0	1052.4	206.8	11.0	7.8	5.8	0	1.04	.84	45.42	45.13	48.23	47.4
ISCL7	65.5	65.0	.78	565.8	462.1	85.5	8.3	8.5	2.7	-.19	1.04	.81	47.50	47.2	46.3	47.4
ISCL8	64.5	63.9	.84	543.0	472.0	86.5	7.0	8.7	2.6	-.10	1.03	.79	46.50	46.15	45.25	45.1
ISCL9	62.3	61.2	.41	1192.6	628.9	206.85	8.4	0.9	3.4	0	1.00	.88	44.78	44.62	46.62	43.3
TC1	62.7	61.7	0	972.5	206.5	206.5	15.5	-	2.2	-.11	-	-	40.52	40.37	40.37	37.6
TC3	65.5	64.0	0	958.6	208.6	208.6	21.8	-	5.0	0	-	-	40.0	39.79	40.37	39.8
EXV1	63.6	60.7	1.0	208.6	990.0	990.0	-7.2	4.3	1.96	0	-	-	40.7	40.44	44.74*	40.9
EXV2	62.5	59.9	1.0	198.8	1006.0	1006.0	-8.7	(5.1)	1.60	-.08	-	-	42.06	41.76	45.86*	41.1

*Non-uniform deformation correction included.

Table 8.2

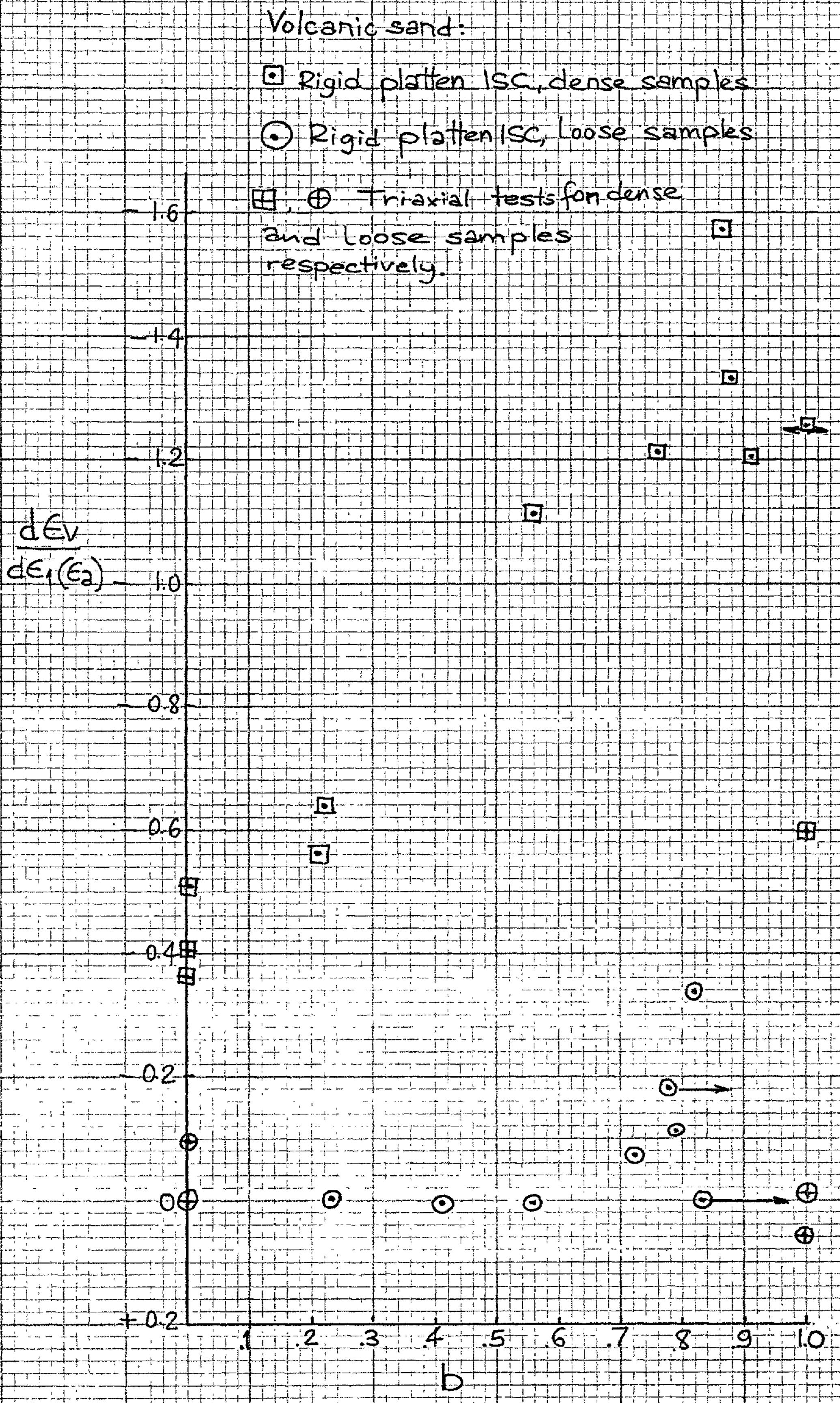
Failure Characteristics of Loose Volcanic Sand Samples



Corrected peak strengths of dense and Loose ISC tests on volcanic sand samples based on Mohr-Coulomb criterion. Tests are corrected for:

a) Rigidity of sample sheath
b) Platten friction
c) Mean stress Level

Fig. 8.1



Rate of volume change with respect to major principal strain at failure

Fig. 8.2

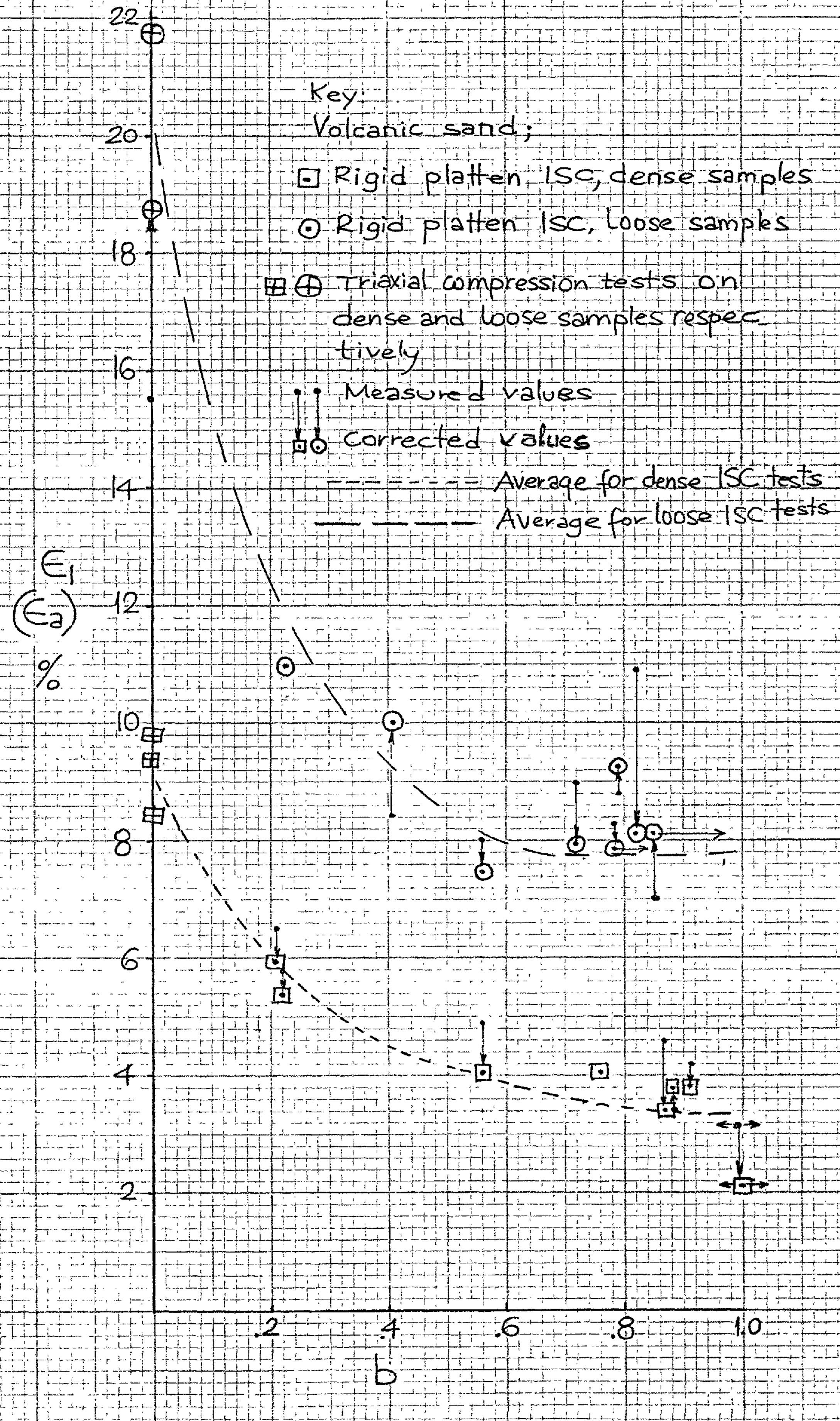
22
20
18
16
14
12
10
8
6
4
2
ϵ_1 (ϵ_a) %
.2
.4
.6
.8
1.0
b
Key:
Volcanic sand;
⊡ Rigid platten ISC, dense samples
⊙ Rigid platten ISC, loose samples
⊞ ⊕ Triaxial compression tests on dense and loose samples respectively
Measured values
Corrected values
Average for dense ISC tests
Average for loose ISC tests

Fig. 8.3

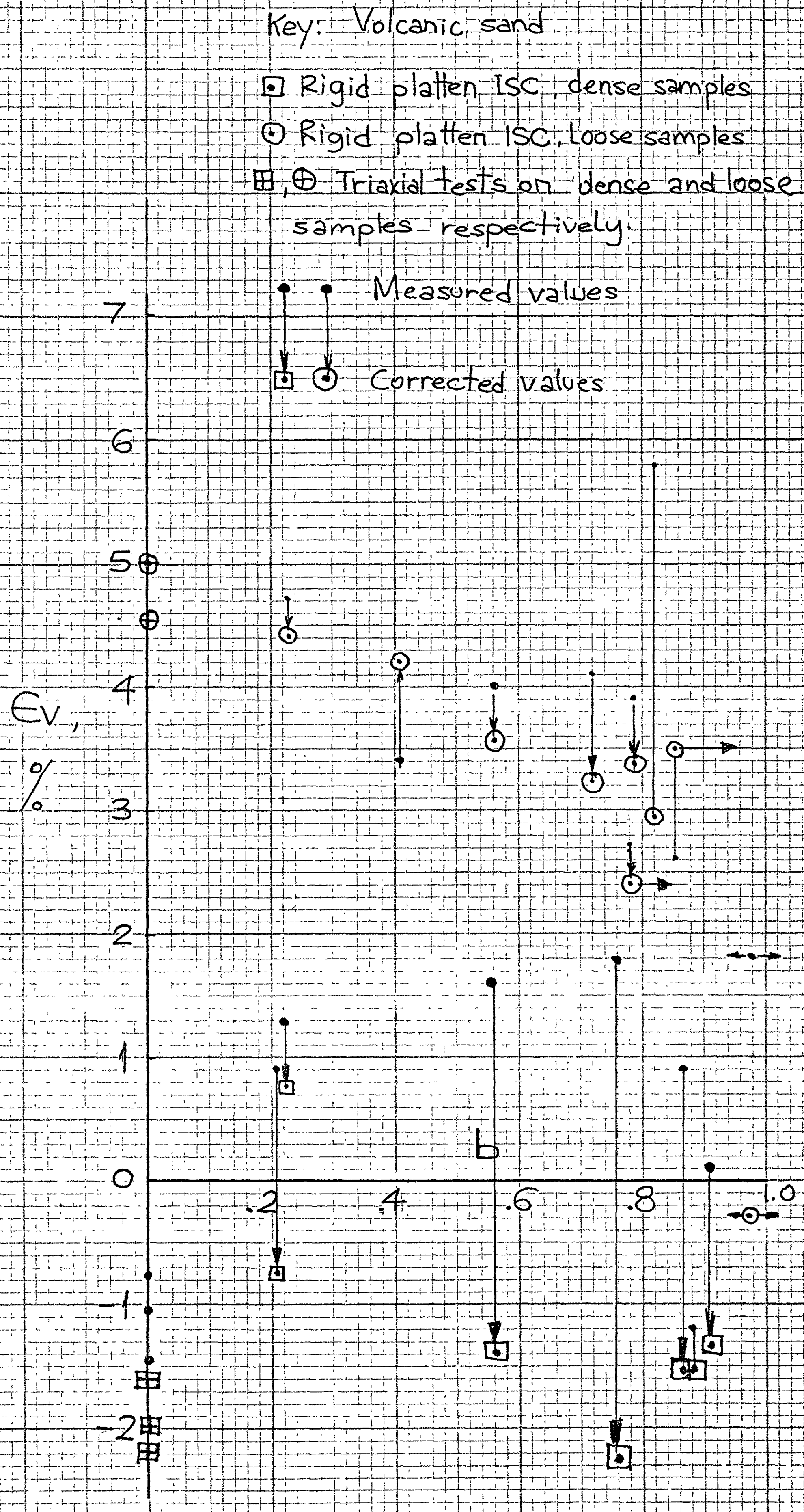

Key: Volcanic sand
⊡ Rigid platten ISC, dense samples
⊙ Rigid platten ISC, Loose samples
⊞, ⊕ Triaxial tests on dense and loose samples respectively.
Measured values
Corrected values
Ev, %
7
6
5
4
3
2
1
0
-1
-2
b
.2
.4
.6
.8
1.0

Fig. 8.4

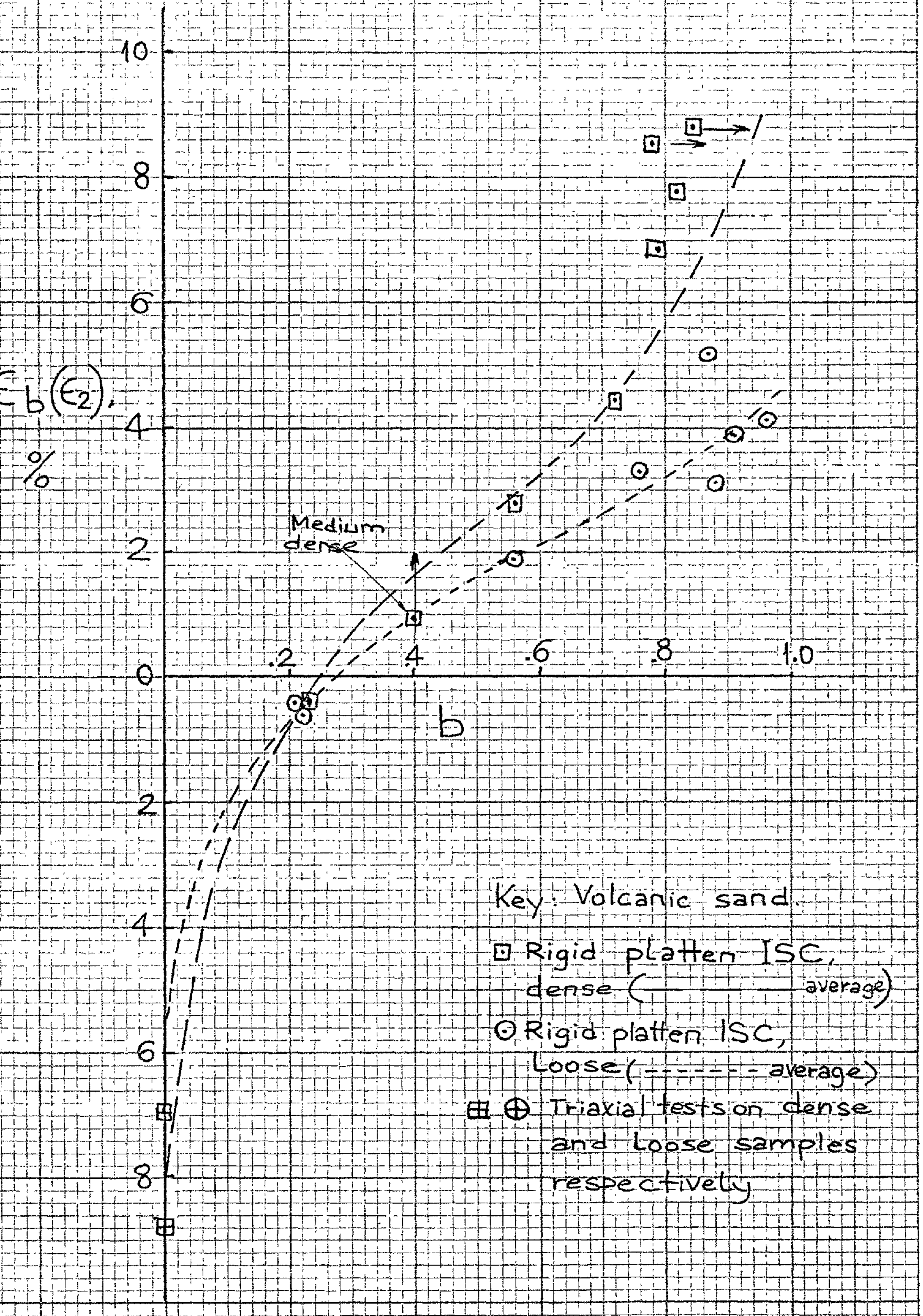

$\epsilon_b(\epsilon_2)$, %
10
8
6
4
2
0
2
4
6
8
.2
.4
.6
.8
1.0
b
Medium dense
Key: Volcanic sand
⊡ Rigid platten ISC, dense (——— average)
⊙ Rigid platten ISC, Loose (- - - - - - average)
⊞ ⊕ Triaxial tests on dense and Loose samples respectively

Fig. 8.5

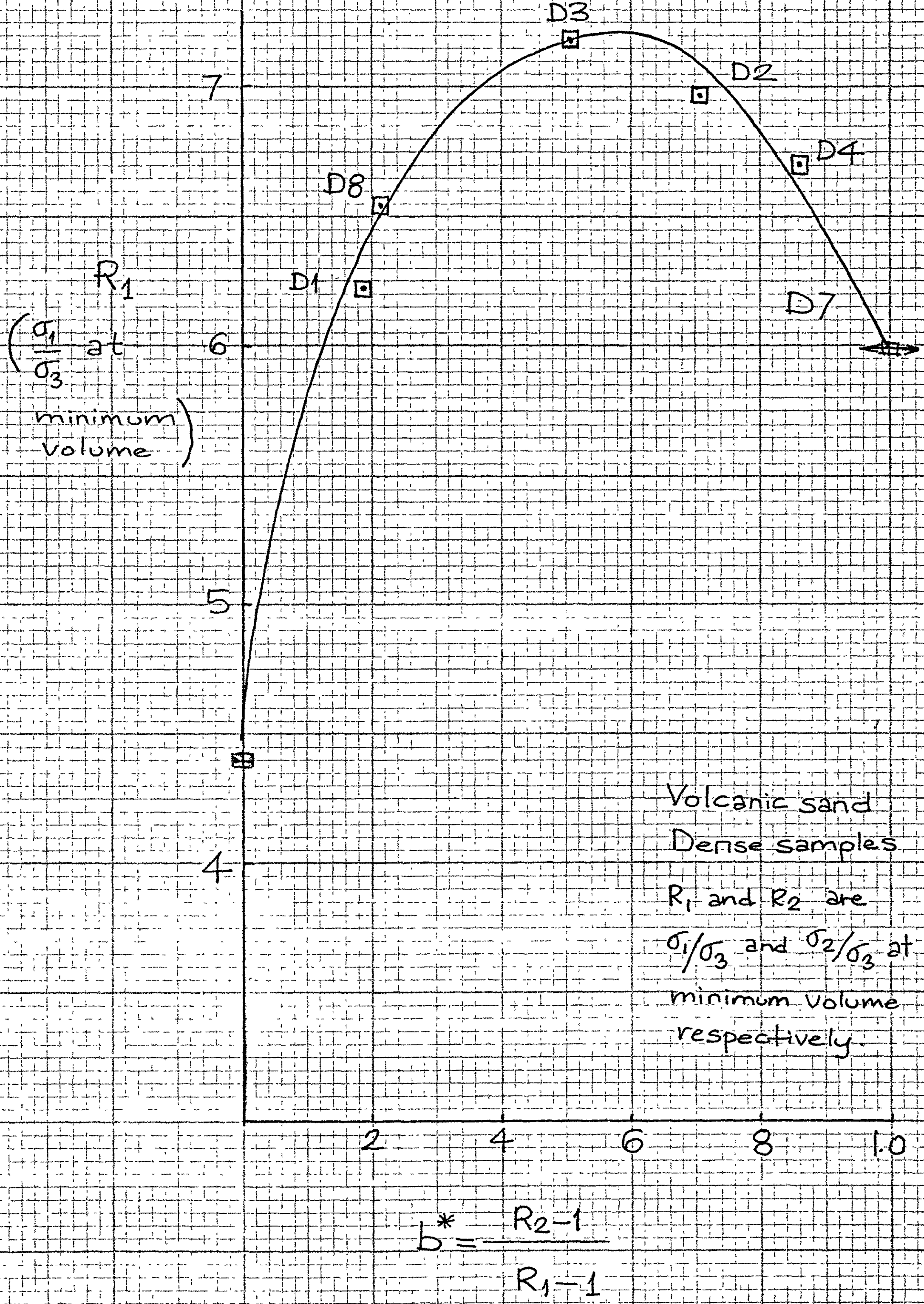

D3
D2
D4
D8
D1
D7
7
6
5
4
R_1
$\left(\frac{\sigma_1}{\sigma_3}\right.$ at
minimum
volume$\left.\right)$
2
4
6
8
1.0
$b^* = \frac{R_2 - 1}{R_1 - 1}$
Volcanic sand
Dense samples
R_1 and R_2 are
σ_1/σ_3 and σ_2/σ_3 at
minimum volume
respectively.

Fig. 8.6

CHAPTER 9

ANALYSIS OF RESULTS IN THE LIGHT OF STRESS-STRAIN - STRENGTH THEORIES

9.1. Introduction.

In this chapter, first, the conditions which lead to failure of granular soils will be examined. Various proposed criteria will be briefly outlined, and in the light of tests conducted in this study and elsewhere which have already been discussed in the previous chapters, a criterion for failure will be proposed which is capable of predicting peak strengths throughout the intermediate stress space. Secondly, studies on pre-peak behaviour will be surveyed. A section on the components of shearing strength of granular materials is presented. In a final section, attempts to explain the behaviour of granular material using plasticity concepts will be shortly examined.

9.2. Failure in Cohesionless Soils.

It has been always believed that there must be a relation between principal stresses at the limit equilibrium of a soil element subjected to stresses. Various attempts to quantify this relation were made on metals, concrete and other materials by several researchers before soils were investigated in this way. See, for a general review, for example, Nadai (1950) and Prager and Hodge (1951). Several criteria for the failure condition of mostly solid materials were proposed to fit the observations. Among others, Mohr-coulomb (generally for brittle materials), constant octahedral shearing stress (for metals) proposed by Huber, Hencky, von Mises etc. and Tresca's hypothesis which claim yielding occurs when maximum shearing stress reaches a constant value (mostly originates from observations on soft, flowing, metals) drew the attention of investigators in soil mechanics as possible failure criteria for soils. But it was soon realised that soils behaviour differed from other solids in certain ways. An important difference was the strong influence of the level of normal stresses on failure state in soils. The last two criteria mentioned above were adapted to soil mechanics with the inclusion of normal stresses, and they are usually referred as extended von Mises and Tresca criteria*. Mohr-Coulomb criterion was directly considered without any change since it already included some sort of stress level effect.

Several soil testing apparatuses were designed mainly to investigate the failure state. A brief review of them was given in Chapter 2. Attempts were made to correlate results of these investigations with the failure criteria mentioned above. Failure criteria for soils have become one of the most controversial topics in soil mechanics. See, for example, Kirkpatrick (1957), Hvorslev (1960), Scott (1963), Roscoe, Schofield and Thurairajah (1963), Bishop (1966, 1971).

*Extended Tresca criterion was attributed to Sandel by Johansen (1958), and Exten. von Mises to Schleicher (1925, 1926).

The failure criteria which are mostly discussed in relation to soils, namely, Mohr-Coulomb, extended von Mises and extended Tresca are concisely considered below in principal effective stress space, and cohesive soils in which interparticle chemical forces etc. govern the behaviour are excluded. $\sigma_1, \sigma_2, \sigma_3$ are effective principal stresses ($\sigma_1 > \sigma_2 > \sigma_3$). An important assumption here is the isotropy of the properties of the soil element. This assumption may lead in fact, to significant errors in engineering computations for certain field deposits and where significant rotation of principal stresses occurs. For laboratory prepared clean sand samples it is a reasonable assumption at failure.

The three criteria are written in equation form as follows:

Mohr-Coulomb; $$\sigma_1 - \sigma_3 = \sin\phi' \, (\sigma_1 + \sigma_3) \qquad 9.1$$

Extended Tresca; $$\sigma_1 - \sigma_3 = \alpha \left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{3}\right) \qquad 9.2$$

Extended von Mises; $$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\alpha^2 \left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{3}\right)^2 \qquad 9.3$$

It is apparent that they only contain the principal stresses and certain parameters, and the effect of normal stresses are built into the expressions in different ways. The relative magnitude of the intermediate stress affects the failure state in extended Tresca and von Mises criteria whereas Mohr-Coulomb criterion implies that the intermediate stress has no effect on the failure state. Considerable research efforts in soil mechanics were directed to this point, because only generalised testing could resolve it.

Bishop (1966, 1971) critically examined these three criteria. He took $\sin\phi'$ value of Mohr-Coulomb as the basis, and expressed the other two equations in terms of $\sin\phi'$ and $b = \frac{\sigma_2-\sigma_3}{\sigma_1-\sigma_3}$, α's are equal in the two expressions for the state of triaxial compression. Fixing the ϕ' to a constant value in the Mohr-Coulomb expression - and so α automatically in triaxial compression state - he was able to plot the equivalent ϕ' values corresponding to extended von Mises and Tresca criteria against varying b, i.e. the intermediate principal stress. He noted that of the three criteria, after all starting from the same triaxial compression value, extended Tresca and von Mises gave unreasonably high ϕ' values for b values larger than for the plain strain state. The Mohr-Coulomb criterion, shows no change throughout the change of the intermediate state. For a triaxial compression ϕ' value of 39°-40° at the dense state von Mises and Tresca hypotheses indicate ϕ' value of 90° at $b = .64$ and $b = .87$ respectively. For example for a material which has a ϕ' of 50° in triaxial compression von Mises criterion would give 90° at $b = .46$. For b values higher than indicated, ϕ' is indeterminate.

The equations for the criteria if plotted in the effective principal stress space correspond to failure surfaces which are prymidal shapes whose apex are at the origin of the cartesian coordinate system. They have different shapes. Shapes of the failure surfaces are demonstrated in figure 9.1 where the section in which $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$ and equations 9.1, 9.2, 9.3 hold is indicated. Bishop (1966, 1971) also explained why such an abnormality was faced in Tresca and von Mises critria. As can be seen in figure 9.1 if the failure surfaces are cut by an octahedral plane the shapes obtained are a regular hexagon and a circle respectively. For $\alpha = 1.50$ or $\phi' = 36.9^{\circ}$ (triaxial compression) the failure surfaces touch the coordinate planes at $b = 1$ (that is for $\alpha = 1.5, \sigma_3 = 0$ for $b=1$). For ϕ' values higher than 36.9° the surfaces will pass into σ_3 negative stress space which cannot be possible for cohesionless materials. Malyshev (1970) also concluded that von Mises criterion "Leads to absurdity", and practical application of it was not recommended.

On the other hand, Roscoe, Schofield and Thurairajah (1963) reasoned that data based non-uniformly deformed laboratory samples and definition of "failure" were not reliable and that until more reliable results were obtained extended von Mises criterion should be preferred. As it was made clear in the previous chapters, the Writer's tests and other good quality data indicated that extension strengths were not underestimated to the extent claimed by Roscoe et.al.

(1963). If failure is defined on the basis of collapse of the sample after showing a maximum possible resistance to shearing the underestimation on Roscoe's argument would amount to 40^o-50^o. However it is calculated to be only of the order of a few degrees. Definition of failure may however be argued. If a parallel is drawn to the criteria in certain full scale earth structures like a deformational limit or a deformational constraint imposed by the superstructure in the case of soil-structure work together etc., certain stages of the stress-strain relation may be taken as failure or more truely excessive yielding, but it is again improbable that von Mises criterion will be satisfied. It must be borne in mind that Roscoe et.al. (1963) defined α at ultimate strains in their subsequent studies. The Writer reckons that one of the main reasons for their selection of von Mises criterion was the suitability of this expression to plastic analysis of deformations in general. (See section 9.4)

Several investigators tried to find failure surfaces to best represent the material behaviour. Although not entirely involved in soils it is interesting to note Johansen's (1958) modified extended von Mises and Tresca expressions. He gave more weights to certain terms as shown below:*

---

* λ is used in place of α which was originally used in the equations. This is not to mix the α values in von Mises and Tresca equations with the α values in these equations.

$$\tau_\lambda = \frac{\sqrt{\lambda^2(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + \lambda^2(\sigma_2-\sigma_3)^2}}{2+\lambda^2} \qquad \sigma_\lambda = \frac{\sigma_1+\lambda^2\sigma_2+\sigma_3}{2+\lambda^2}$$

$$F(\sigma_\lambda, \tau_\lambda) = K \qquad 9.4$$

$$\tau = \frac{\sigma_1-\sigma_3}{2} \qquad \sigma = \frac{\sigma_1+\lambda^2\sigma_2+\sigma_3}{2}$$

$$F(\sigma, \tau) = K \qquad 9.5$$

He was able to fit curves for different materials. λ is a parameter in the equations above between 0 and 1. It can be noted that if $\lambda=0$ both equations reduce to Mohr-Coulomb expression while $\lambda=1$ corresponds to von Mises and Tresca criteria in equations 9.4 and 9.5 respectively.

Coleman (1960) assuming equal triaxial compression and extension strengths developed an expression as a function of first stress invariant and second and third invariants for stress deviation.

Lomize and Kryzhanovsky (1967) proposed an equation for the failure state as shown below:

$$\left(\frac{I_1^{\,3}}{I_3}\right)^{\alpha} \frac{\sqrt{J_2}}{I_1} = \text{Constant.} \qquad 9.6$$

Where α is an experimental constant related to the properties of the material and its initial state, see Appendix 4 for invariants. It was also claimed to give a unique relation for volumetric and linear strains with respect to various generalised states in the case of "simple loading" which is loading at constant stress ratios, and implies a constant b value (or lode parameter) during shearing. Equation 9.6 was fitted on the basis of generalised tests in their apparatus which yielded large increases of ϕ' from triaxial compression to extension.

Malysev (1963) fitted analytical equations between equal compression and extension strengths in ϕ'-μ (see notation) plot for certain soils he tested. He also attempted to form a modified Mohr-Coulomb relation employing many emprical constants. (1967a,b). Compression and extension strengths were set equal for all cases in his equation, and the curves peaked at $\sigma_2=\frac{1}{2}(\sigma_1+\sigma_3)$, i.e. b = 0.5.

Nagaraj and Someshekar (1974) assumed again equal triaxial compression and extension values, and suggested varying α values in extended von Mises and Tresca criteria (instead of using the α value in triaxal compression). α was expressed in terms of b and emprical coefficients m and n determined from experimental results by Green (1971), Ramamurthy (1973), Kirkpatrick (1957) etc. for Tresca and Mises Criteria respectively.

Lade and Duncan (1975) adapted a relation to define the failure surface in the effective stress space. It was:

$$\frac{I_1^{\,3}}{I_3} = K \qquad 9.7$$

They observed approximately the invariance of this ratio with changing b values. This is a simpler form of Lomize and Kryzhanovsky's (1967) equation (equation 9.6) which was claimed to be promising in the analysis of deformations as well. Lade and Duncan (1975) used the same ratio in describing the shape of the plastic potential surface. (see section 9.4).

Goldscheider and Gudehus (1973), based on their results, either recommended the use of a Mohr-Coulomb angle or the following relation;

$$C_1 \frac{J_2}{I_1^{\,2}} + C_2 \frac{J_3}{J_2^{\,3/2}} = 0 \qquad 9.8$$

The latter recommendation was made in the case of a fully developed plastic flow.

Parkin (1964, 1965) specified two separate regions in the intermediate principal stress space using a model developed from analysis of packed spheres. Major stress ratio (σ_1/σ_3) increased from triaxial compression until plain strain constituting the first part. After a discontinuity, in the remaining sector, stress ratio was constant till extension. The magnitude of the predicted increase of strength from triaxial to plain strain state seems to be larger than actually observed in tests later on. - It was only a theoretical analysis there was no attempt at experimental correlations.- Green (1969) was interested in his theory, and compared his (Green) results with theoretical findings.

It is noticable in the brief review above that the general tendency is to try to find out equations which involve invariants to define the generalised failure surface. In the Writer's opinion this may be advantageous if a deformation model is going to be developed in conjuction with the failure criterion. Mathematically speaking, derivations and transformations will presumably be more handy to work out.

It is apparent that analytical efforts are made to develop failure criteria mainly on the basis of experimental data. It must be emphasized that good quality generalised testing data and a thorough examination of the data are extremely significant. Early generalised tests were on hollow cylindrical samples the analysis of which raises uncertainity greater than that of cubical tests. All tests at b=1 were conventional triaxial extension tests. It seems that this accounts for most of the analytical attempts assuming equal compression and extension strengths.

Examining the experimental findings in this and other studies it was strongly felt that the intermediate stress must have a bearing on failure in the generalised state. The extent of its influence is the main issue. The failure state, at the same time, must be controlled both by normal and deviatoric stresses which are not independent of each other. Investigation of the extended von Mises criterion shows that stress deviators should not have an equal effect on the generalised shearing behaviour of soils and other granular materials. Johansen (1958) early recognised this fact, and tried to give weights to the deviator components.

After a detailed examination of deviatoric stress components a potentially useful failure criterion has emerged it is given below as;

$$(\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2 = \beta^2 \left(\frac{\sigma_1+\sigma_2+\sigma_3}{3}\right)^2 \qquad 9.9$$

where $\sigma_1 \geq \sigma_2 \geq 3$. In more exact form in the principal effective stress space it can be written as the following;

$$\begin{array}{lll}
(\sigma_1-\sigma_3)^2+ (\sigma_2-\sigma_3)^2- \beta^2\ \sigma_{oct}^2 & = 0 & \sigma_1 \geq \sigma_2 \geq \sigma_3 \\
(\sigma_1-\sigma_2)^2+ (\sigma_3-\sigma_2)^2- \beta^2\ \sigma_{oct}^2 & = 0 & \sigma_1 \geq \sigma_3 \geq \sigma_2 \\
(\sigma_2-\sigma_3)^2+ (\sigma_1-\sigma_3)^2- \beta^2\ \sigma_{oct}^2 & = 0 & \sigma_2 \geq \sigma_1 \geq \sigma_3 \\
(\sigma_2-\sigma_1)^2+ (\sigma_3-\sigma_1)^2- \beta^2\ \sigma_{oct}^2 & = 0 & \sigma_2 \geq \sigma_3 \geq \sigma_1 \\
(\sigma_3-\sigma_1)^2+ (\sigma_2-\sigma_1)^2- \beta^2\ \sigma_{oct}^2 & = 0 & \sigma_3 \geq \sigma_2 \geq \sigma_1 \\
(\sigma_3-\sigma_2)^2+ (\sigma_1-\sigma_2)^2- \beta^2\ \sigma_{oct}^2 & = 0 & \sigma_3 \geq \sigma_1 \geq \sigma_2
\end{array} \qquad 9.10$$

The only parameter is β in the equation. Before an analytical study of the criterion is done, the actual β values from the tests have been investigated. In figures 9.2 and 9.3 β values are plotted against b values from the

Writer's and other researchers' generalised tests. There is a fairly good uniformity of β. Loose and medium dense samples give a better constant fit, Dense volcanic sand shows a relatively larger variance than others, but all values are plotted as measured. For example, mean stresses are indicated in figure 9.2 for dense volcanic sand samples at the same b value. This material, as already pointed out in Chapter 8, is very stress level dependent especially in the dense state, and is a high-strength material compared to other sands at the same mean normal stress level. Ramamurthy and Rawat's (1973) data are from two groups of 3- dimensional tests. Each group has a constant β value. They are in different modes, and it is expected that if the modes were identical the resulting β values would not differ.

The invariance of β values throughout the intermediate stress state (for a specific mode) has a significant convenience. Since generalised testing cannot be conducted in laboratories, a β value obtained in conventional triaxial tests for the appropriate density will apply to all intermediate stress states, and the variation of strength can be predicted fairly accurately without actually performing generalised tests. The failure surface can be plotted on an octahedral plane in the principal effective stress space for a constant value of β as in figure 9.1. See Appendix 5 for more information on this failure surface.

To study the implications of the above failure condition a similar procedure to that used by Bishop (1966) in examining the extended von Mises and Tresca criteria will be followed. It consists of expressing the equation in terms of ϕ' so that any change in the intermediate stress will correspond to an equivalent ϕ'. So, equation 9.9 becomes:

$$\sin\phi' = \frac{1}{\frac{1}{3} + \frac{2}{\beta}\sqrt{1+b^2} - \frac{2}{3}b} \qquad 9.11$$

For the state of triaxial compression ($\sigma_2=\sigma_3$);

$$\sin\phi' = \frac{1}{\frac{2}{\beta} + \frac{1}{3}} \qquad 9.12$$

which is an identical expression to the von Mises and Tresca equations.

Therefore β and α are identical in triaxial compression.

In the case of extension state ($\sigma_1=\sigma_2$);

$$\sin\phi' = \frac{1}{\frac{2\sqrt{2}}{\beta} - \frac{1}{3}} \qquad 9.13$$

which is not the same as those in others. If ϕ' against β value is plotted it can be seen that for $\phi' = 90^\circ$ $\beta = 2.12$ while ϕ' in triaxial compression is 51.6° at this β value.

In figure 9.4 ϕ'-b plots are shown in which equation 9.9 is drawn for ϕ' (or β) values in triaxial compression of 35.5°, 39°, and 45° (β; 1.44, 1.59, 1.85 respectively). Peak strength variations concluded in tests on loose and dense Ham River and Volcanic sand samples in generalised state are placed for comparison. Predictions by Tresca and Mises criteria are also plotted. Usually parameters in equations defining failure states are keyed to values obtained for triaxial compression, then they are treated as constants to search for the other states. This method is just a matter of convenience to compare intermediate states with triaxial compression state, and generally there is no physical reason behind it. But in this study this method is followed not for the sake of convenience but due to the fundamental reason that β values are actually invariant along the variation of b.

Correlation of equation 9.9 with experimental results is reasonably good. For very high strength materials in triaxial compression it will predict very high strengths in the intermediate state. A 3^{o} difference is noticable after b=.50 for dense volcanic sand samples ($\phi'_{tc} = 45^{o}$).

Like α in von Mises and Tresca equations, β in equation 9.9 once specified, predicts the variation of ϕ' for intermediate stress changes, more clearly, if two cohesionless materials give the same ϕ' in triaxial compression, predicted variation of ϕ' at various intermediate states will be identical. The porosity of the material should not influence the results. Although this statement cannot be considered conclusive a good indication as to its validity can be obtained from the peak strength variation along b for dense Ham River sand (39 percent initial porosity) and loose volcanic sand (64.5 percent initial porosity) which have the same ϕ' in triaxial compression and are very close to each other, see figure 9.4.

Equation 9.9, like Tresca and Mohr-Coulomb criteria, is not completely composed of invariants, but as pointed out before they are desirable but not essential. A certain amount of care needs to be exercised to insure the correct ordering of the principal stresses when using the postulated criterion for the solution of field problems.

The criteria mentioned in the previous paragraphs are plotted in figure 9.5 on a 60^o - sector (one sixth) of an octahedral plane together with equation 9.9 for a material which has a triaxial compression ϕ' of 39^o (since dense Ham River sand and loose volcanic sand have both compression ϕ' 's of 39^o). Some experimental points are also plotted individually for comparison.

An assumption usually made is the linear effect of mean stress level on the strength. It is known that the failure envelope is not a straight line but has steeper slope at lower stress levels. Since it is on the safe side it is usually neglected. But for certain problems in which the stress level is very low it may be more realistic to take the curved shape of the envelope into account.

The failure criterion which is widely accepted and used in soil mechanics is Mohr-Coulomb, equation 9.1.

This semi-empirical relation agrees quite well with many failure cases in the field. In the Writer's opinion it is mainly because this criteria is the lower limit for the failure state and does not change with increasing intermediate stresses provided that the material does not exhibit an excessive degree of anisotropy. In engineering analyses for bearing capacity, slope stability etc. ϕ' in triaxial compression is used throughout the failure surfaces, and therefore computations are conservative due to higher strengths actually available at intermediate stresses.

9.2.1 Summary and Conclusion

In this section attempts to formulate the failure condition for granular soils have been briefly reviewed and discussed. The majority of propositions do not have any connection with deformational behaviour which has been the target point for critical attacks - see, for example, Roscoe (1970)-. Stress-strain behaviour is certainly important, however there has been a lack of success in formulating a model for the behaviour of granular material. The models which have been proposed are not suitable for the anisotropic states of field deposits. Peak strength seems to be less affected by anisotropy. It is not meant that analysis of strains cannot be put in use in engineering problems, but it seems that there is a long way before any link between the failure state and pre-failure deformation behaviour can be established. A review and discussion which are relevant to this significant subject are presented in sections 9.3 and 9.4.

A failure criterion has been found equation 9.9, it expresses the peak strength variation with increasing intermediate stress states quite well since it can predict the higher increases in peak strength for stronger - with respect to the same mean normal stress level - soils, a fact that is observed in laboratory tests. A very useful nature of the proposed relation is the predictability of β values from standard triaxial compression tests.

9.3 A Review on Physical Components of Shearing Strength.

9.3.1. Introduction.

In this section shear strength of granular materials will be briefly reviewed from the point of view of physical components of the observed behaviour. Certain hypotheses which have been claimed to explain the shearing behaviour of granular material have been assessed. Studies which seek to separate the components of shearing strength have been reviewed. Although the observed strength is a complex combination of certain components which can hardly be mobilised individually, it is justified to investigate and explain these components to enable a better understanding to be developed.

9.3.2. General.

The dilatant behaviour of granular materials has long been recognised, Reynolds (1886), but research into the components of shear strength started with the first international conference on soil mechanics (1936). When any type of test is performed to determine the shearing strength of a mass of granular material the quantity measured is a combined effect of contributing factors. The most dominant are friction between the particles and dilatancy. It was noticed that stress-strain curves of dense samples passed through a maximum stress point while loose samples did not, and post peak shearing stresses showed a decrease in the former case which almost reached the level of those of loose samples at very large strains - In more recent terminology "the residual" state -. From such observations the concept of critical void ratio was postulated. Hvorslev (1937), obviously referring to the residual state, remarked that a state could be reached where shearing resistance and void ratio do not change while deformations still took place regardless of the initial packing of the mass. He referred to it as "the critical void ratio". Bishop (1971) attributes it to Casagrande originally. Casagrande (1940), Taylor (1948) also implied the same concept while using the term. But some others used other critical void ratio definitions, see for example, Lee and Seed (1967). Rowe and Barden (1969) try to make the original definition clear. It was also noticed that an increase in the mean stress level caused a decrease in the critical void ratio.

The shearing strength at critical void ratio has practical implications in the analysis of field problems but direct measurement of it is not straightforward. Direct shear tests have always been criticised because of the indeterminate orientation of the principal stresses, and triaxial test value has been considered approximate due to large displacements required and inaccurate effective shearing area determinations. Bishop (1966) proposed a procedure to determine the shearing strength at critical void ratio. He plotted ϕ' against rate of volume change with respect to major principal strain and noted that different tests at various densities and mean stress levels fell approximately on a single line. The intersection of this line with the zero rate of volume change axis was thought to give ϕ'_{cv} Rowe (1971b) critisized it stating that it had no derivation and since it covered all densities of a granular material its use was limited. The Writer believes that this plot is significant because it is only at the peak state at which volume changes take place almost wholly in the shearing zone. In other words, at pre-failure strains the total volume change observed is the result of general shearing throughout the sample. In Chapter 2 it was pointed out how a triaxial sample responded to the shearing load imposed on it, and that there were virtually two semi rigid wedges which moved relative to one another. Inside the wedges particles presumably compress throughout shearing until failure. The particles within the shearing zone may compress and arrange themselves to a certain degree initially - which must depend on the initial packing - then they start dilating when the failure state is approached. Therefore, analysis of pre-failure strains are very complex in a triaxial sample, and the rate of volume change at peak is an important parameter.

Two ϕ' - $d\varepsilon v/d\varepsilon l$ plots are given in figures 9.6, and 9.7 for Ham River and volcanic sands respectively. In each of them triaxial compression, plain strain, triaxial extension and generalised tests have been plotted. Bishop (1966) suggested that if both variables were to be plotted as measured any error would be avoided because simultaneous variation of them with respect to stress level and density would roughly cancel out.

In figure 9.6, the line for triaxial tests is the result of triaxial tests by Reades (1972). The Writer's seven triaxial compression control tests also agree with this line. The problem with the extension tests is that the non-uniformity correction has nothing to do with the fundamental variables such as stress level or density which affect the shearing strength but it is directly related to the use of the incorrect area in the minor principal stress direction due to necking, so this correction actually must be included in the ϕ'. In Chapter 6 and Appendix 2, it was explained how uncertain the non-uniformity correction was. It was thought it would be suitable to plot the extension strengths as measured in figure 9.6 and then consider the correction when comparing all the tests. Reades' rectangular extension tests have also been shown. It is seen that the measured ϕ' - $d\varepsilon v/d\varepsilon_1$ line given by triaxial extension tests lies about a degree lower than the curve representing triaxial compression tests. Non-uniformity corrections are a maximum for loose samples. Reades applied 1.7° for his loose samples while the Writer's short loose samples required an average correction of 2.4°, that yields a line which is a degree or more higher than the triaxial compression values.

Plain strain tests (both flexible and rigid) give a line which is above both curves and has a little steeper slope. ϕ' values at zero volume change rate for compression, extension and plain strain states are 33.5°, 34.3° and 35.6° respectively. The definitions are ambiguous most refer to no change in shearing resistance without specifying the type of test or of any constraint, they therefore, do not imply that it is unique.

Roughly speaking generalised test points follow the plain strain line except for dense flexible platten tests at b = .88, .92, .94 which are near (non-uniformity corrected) triaxial extension values. Note the flexible platten tests at b = .71, .72, .79 which agree very well with the plain strain line. Generalised tests in the second mode are considerably below the plain strain line.

In a similar plot for volcanic sand, figure 9.7, three extension tests fall on a line-sample sheath rigidity corrected only -. The Writer's triaxial compression tests on cuboidal samples and five tests on 38mm diameter samples by Walbancke (1974) again give a straight line. The difference from Ham River sand data is that measured triaxial extension strengths are higher than in compression tests at the same rate of volume change. Applying non-uniformity corrections they will be appreciably above the triaxial compression line - by approximately 3° -. The plain strain line is little steeper, and lies above the compression and extension states. ϕ' values at zero rate of volume change for triaxial compression, extension and plain strain are 39.4°, 42.5° and 44.5° respectively. Very high b value, dense tests present similar results to Ham River sand data in that they depart from the plain strain line indicating higher volume change rates without corresponding to very high ϕ' values, although there is a certain increase in ϕ'. This finding is not as clear cut as the big strength increase from compression to plain strain tests at the same rate of volume change. One probable deduction may be that since loose samples results are near or on the plain strain line generally and the higher the b value the higher the ϕ' value from b = .50 onwards up the curve, the observed increasing ϕ's from loose tests at high b values may be associated with increasing dilatancy rates which are believed to be the result rather than the cause of constraint from larger inward movement of the two pairs of rigid plattens in parallel with the discussions in Chapters 5, and 7.

Higher strengths in extension tests relative to compression tests for volcanic sand can easily be seen from the $\phi' - (d\varepsilon v/d\varepsilon_1)$ plot, and due to the nature of the material this amplified difference is a good example for comparing relative strengths in the two states.

9.3.3. Components of the Strength of Granular Materials.

The two fundamental components of resistance to shearing in a granular mass, namely, friction between the grains and the dilation or interlocking have been said to have been recognised for a long time, but it has not been and will not be easy to quantify this behaviour, and possibly additional phenomena are involved in the shearing mechanism. Most attempts to obtain a reasonable quantitative correlation are based on energy principles. Taylor (1948) and Bishop (1950) worked out, independently, an energy calculation to explain the results of drained shear box tests on loose and dense samples and to separate the two components. The angle of friction was calculated after subtracting the dilatancy effect from the total (observed) angle of friction. The equivalent angle of shearing resistance corresponding to the dilation component was obtained by equating the total work done to that done against the normal stress and the shear stress associated with friction. If τ_f is the shear stress associated with friction, then;

$$\tau = \tau_f + \sigma_n \frac{\delta V}{\delta H} \quad \text{hence } \tau_f = \tau - \sigma_n \frac{\delta V}{\delta H}$$

So $\tau_d = \sigma_n \frac{\delta V}{\delta H}$ where τ_d is shear stress required to overcome dilation, and δV and δH are vertical and horizontal displacements. The conclusion was that the work done in friction was the same for all densities, and observed different shearing angles were due to varying degrees of dilation.

Following similar lines, Bishop and Eldin (1953) and Bishop (1954) derived a relation for conventional triaxial test - i.e. constant cell pressure -. Deviatoric stress required for dilation was determined to be $(\sigma_1-\sigma_3)_d = \frac{d\varepsilon V}{d\varepsilon_1}\sigma_3$, and again it could be subracted from the observed deviatoric stress to obtain the frictional component.

Newland and Allely (1957) made an analysis based on the principles of equilibrium and continuity. They used "Θ" as the average angle between the potential shearing plane and movement of particles at contact points, and it was imagined to have a maximum value when sliding begins, as was the rate of volume change, the average angle then diminishes to zero at the residual value. For any micro-slip surface; $\tan(\phi_f+\Theta) = \frac{\tau_{max}}{n}$ and $\left(\frac{dV}{d\varepsilon}\right)_{max} = \; = \tan\Theta$

or for triaxial sample;

$$(\sigma_1/\sigma_3)_{max} = \tan^2(45 + \phi f/2 + \frac{\theta}{2}), \quad \tan\theta = f((\sigma_1/\sigma_3)_{max}, (d\varepsilon V/d\varepsilon_1)_{max})$$

The assumption here was that the potential slip plane made an angle of $(45+\phi'/2)$ with the major principal plane. Therefore, ϕ_{cv} was implied by ϕ_f due to the aforementioned definition, because θ would be zero at the residual state. Clearly it is a different ϕ_f obtained from the Bishop (1954) analysis. But ϕ_f values from Newland and Allely's analysis were not constant throughout the porosity range, dense assemblies giving more dilation correction and lower ϕ_f. ϕ_f itself is not the same at various stages of the shearing.

As will be repeated at times in this Chapter and Chapter 10, the concept of friction used in the above analysis must be clearly understood. It is believed that the friction value between the particles is the same regardless of the stress state or density with exceptions at high stresses or if very weak particles are tested so that plastic contacts or crushing are present. The ϕ_f values obtained above are not pure friction coefficients but parameters which involve factors like dilation. Although ϕ_f is defined as an angle obtained after dilation is subtracted and another kind of friction, namely, "friction in dilation" is put forward, the Writer believes that quantitatively it is very difficult to separate a dilation angle and to say the remaining portion is friction.

The ϕ_f angles proposed by Bishop (1954) are higher because after the dilatancy correction is subtracted, the remaining energy is imagined to be spent on friction. Newland and Allely (1957) also recognised the rearrangement of particles as an important phenomenon in samples other than very dense ones, thus they restricted the use of ϕ_μ.

Ladanyi (1960) as reported by Koerner (1970) also attempted to separate the frictional and dilational components of shearing strength of granular materials in drained tests. His analysis yielded the following equation;

$$\frac{\mathrm{Sin}\phi_f}{\cos^2\phi_f} = \frac{\sin\phi'}{\cos^2\phi'} + \frac{(d\varepsilon v/d\varepsilon_1)_{\text{failure}}}{3-(d\varepsilon v/d\varepsilon_1)_{\text{failure}}} \cdot \frac{3-\sin\phi'}{\cos^2\phi'} \qquad 9.14$$

Poorooshasb and Roscoe (1961) stated that energy transmitted across the boundaries of a sample was not identical to the work done overcoming friction except in special circumstances and they proposed that this transmitted energy must have two components, namely, energy absorbed in consolidation and in shearing and arrived at the following dilation correction;

$$(\sigma_1-\sigma_3)_d = p\,\frac{dV}{d\varepsilon_1} \Big/ \left(1 - \frac{1}{3}\,\frac{dV}{d\varepsilon_1}\right)$$

Rowe (1962, 1963), originally on the basis of tests on samples which were composed of metal rods and balls developed certain rules through his observations. His axisymmetric formulation was developed first by making use of equilibrium and continuity then defining an energy ratio and minimizing it. The energy ratio was described as:

$$E = \tan(\phi_\mu + \beta)/\tan\beta$$

where β was the angle between the micro-slip directions and the direction of major principal stress. Applying the principle of least work ($dE/d\beta = 0$) β was found to be $45-(\phi_\mu/2)$ and he ended up with the equation below.

$$\sigma_1/\sigma_3 = (1 + d\varepsilon v/d\varepsilon_1)\ \tan^2(45 + \phi_\mu/2) \qquad 9.15$$

In words, it was imagined that shearing takes place with the greatest degree of freedom and minimum internal energy absorbtion. Test results showed that only dense samples followed the theoretical relation up to peak.

Several researchers examined and criticized this analysis. See, among others, Gibson and Morgenstern (1963), Trollope and Parkin (1963), Scott (1963) and Roscoe and Schofield (1963). See also Rowe (1964). One of the points which was most severely criticized was the energy manipulation (especially the differentiation) in obtaining equation 9.15, and the validity of principle of least work in the case of granular assemblies of earthy materials. Medium dense and looser granular materials do not correlate with it. This can be explained mainly by the additional energy required for the rearrangement of particles, and hence shearing does not occur at the minimum energy. Skinner (1975) clarifying the assumptions involved in stress-dilatancy hypothesis showed that straight R vs. D lines were not possible at high stress levels where plastic junctions between the particles and crushing existed, and higher R values were found at the same D values compared with those at lower stress levels.

An angle ϕ_f was introduced in equation 9.15 instead of ϕ_μ. It was assigned different values at different densities and states as will be seen subsequently. The Writer reckons that this is the step where more empricism comes into scene. ϕ_f here is not the one defined by Newland and Allely (1957) nor by Bishop (1954), and it is not a constant but a parameter to match the data in equation 9.15.

It was hypothesized that the more the interparticle slip directions (β) deviated from a mean value, the more frictional energy was spent hence higher ϕ_f values. Thus, with increasing void ratio β values increased from low to high deviations from the mean, and loose samples were prescribed to have maximum ϕ_f values which were later claimed to be ϕ_{cv} values. It was claimed that the straining condition directly affected ϕ_f, and that the axisymmetric state offered maximum freedom while the plain strain state was thought to give the minimum, therefore, highest ϕ_f values were attained in plain strain, namely, ϕ_{cv}. As will be recalled ϕ_{cv} is the angle of shearing resistance at constant volume, at a constant stress level and with zero rate of volume change. It is associated with large strains in drained tests.

It can be noted that if Rowe's (1962) dilatancy hypothesis in triaxial compression is taken for granted, observed high ϕ' values in plain strain and almost identical rate of volume changes in both states make it imperative that a different interparticle shearing mechanism occurs, which results in higher strengths.

Horne (1965) attempted to show theoretically that the maximum ϕ_f that could be attained was ϕ_{cv}. More truely, he set upper and lower limits for the dilatancy component $(1 + d\varepsilon v/d\varepsilon_1)$ in triaxial compression which was generally in between one and two.

Therefore, ϕ_f was assumed to have the following values: Dense triaxial samples $\phi_f = \phi_\mu$ up to peak, and in all loose triaxial tests $\phi_f = \phi_{cv}$ at the peak. In plain strain, regardless of the density $\phi_f = \phi_{cv}$ up to the peak stress ratio, and finally in all other densities in the axisymmetric state $\phi_f \leq \phi_{cv}$, and ϕ_f increased to ϕ_{cv} after peak to the residual state.

Procter (1974) recently sought a possible maximum value for ϕ_f and concluded that maximum ϕ_f was in between ϕ_{cvt} and ϕ_{cvps}, which are the ultimate values measured in triaxial tests and plain strain tests which were not the same.

The sketch in figure 9.8 can be regarded as a summary which indicates the generally accepted view by research workers who support stress-dilatancy hypothesis with reference to a specific stress state (mainly triaxial compression).

Detailed extension of the so called "stress-dilatancy" approach can be seen interalia Rowe (1969, 1971a), Barden, Ismail and Tong (1969) - especially with regard to high pressures, particle crushing, supressed dilatancy, plain strain state etc. -, Barden and Proctor (1971) and in a general summary form in Barden (1969). King and Dickin (1970) made a general comparison of the two approaches by Newland and Allely (1957) and Rowe (1962). They demonstrated that the major part of the derivations by employing equilibrium conditions are identical. (Then Rowe started an energy treatment). Actually ϕ_f values are close to each other in both treatments throughout the porosity variation. They also proposed an extension of the equations by Newland and Allely (1957) employing them for all stress ratios, because originally this approach did not consider the pre-failure state whereas Rowe had considered it. Rowe's main objection to the extended model was (1971c) that while dilatancy was taken into account, the angle between the slip plane (macro) and the minor principal stress direction - α in Rowe's terminology - was always taken constant equal to $45 + \phi'/2$. As it has been pointed out before, this is only kinematically possible for two rigid blocks, one is sliding on top of the other, without any expansion. The inclination of the rupture plane has been briefly considered in Chapter 7.

An important point to emphasize is that the dilatancy approach by Rowe has been claimed as a global theory for calculating stresses and strains, in other words, a soil sample has been taken as a representative volume element of soil under yielding. The laboratory data to support the hypothesis are from standard triaxial and plain strain tests in which the failure is known to occur along diagonal shearing bands. It was pointed out earlier that the linear and volumetric strain behaviour of this zone is quite different from the overall measured quantities on the sample. The Writer feels that the field application of the theory in predicting strains in soil masses may not be on a sound basis in this respect.

Rowe, Barden and Lee (1964) in their energy treatment of the triaxial test emphasized the internal energy component of friction associated with dilation compared with the external energy calculation for dilation, Bishop (1954). In fact they were in error but Bishop (1964) cleared it up. They also extended the dilatancy approach to cover the triaxial extension state - i.e., $\sigma_1/\sigma_3 = \tan^2(45 + \phi_f/2)/(1+ d\varepsilon v/d\varepsilon_1)$ - . Barden and Khayatt (1966) also treated the extension state.

There are several other works involving energy calculations during tests but a discussion of them is outside the scope of this study. The same can be said about the research effort devoted to the concept of angle of interparticle friction.

A generally accepted hypothesis is that there is a fundamental friction coefficient between the grains (denoted by tan ϕ_μ), and it is constant for a certain granular material, and it is also believed that the angle of shearing resistance at constant volume for an assembly of grains is a direct reflection of the interparticle friction between grains. Caquot (1934), Bishop (1954) - Based on the work of Hafiz (1950) - expressed ultimate strength (constant volume) in triaxial and plain strain as a near linear function of the interparticle friction. Recently (1969) Horne both theoretically and experimentally showed for triaxial compression that there was an almost directly proportionate relationship between ϕ_{cv} and ϕ_μ. On the other hand Skinner's (1969) results - see also Bishop (1969) - did not agree with this trend. His tests on ballotini, steel balls and lead shot demonstrated that there was no one-to-one relationship between the strength of a particulate mass and the friction between the individual grains. While flooded ballotini grains showed higher friction values between them relative to when they were dry, ϕ_{cv} values did not show any difference. Skinner (1969) concluded that high interparticle friction must be associated with higher degree of particle rolling to release more energy to give the same bulk shearing resistance as the dry assembly.

The rolling mechanism was assumed not to exist by Horne (1969) in his theoretical analysis, and it was ignored most of the time by several others. More recently Sharma (1976) with the aid of an x-ray measurement technique was able to detect significant body rotation of particles in his plane strain tests on ballotini and natural gravel. In this study individual particles were marked by inserting wires in predrilled holes. These particles were then easily distinguished by x-ray equipment.

It seems to the Writer that the way of measuring of interparticle friction plays an important role in the results. The techniques of measuring interparticle friction differs a lot, While Skinner (1969, 1975) considers two individual particles, the Manchester School generally uses a shear box test with particles and a parent block of material shearing one on another, See Proctor and Barton (1974) for a discussion of the methods.

Interpretation of generalised tests are more complex with respect to components of the measured strength values. The stress dilataney equation has no relevance in this case because the differentiation of the energy ratio is not valid, in other words, shearing does not take place with maximum degree of freedom and with minimum internal energy absorption. Different modes of testing which were discussed in Chapter 7 also presented further complications.

Barden and Proctor (1971) after conducting various types of tests including generalised tests concluded that D_{max} in the equation $R = D.K$ is a function of density, and K is function of degree of freedom (boundary conditions etc.) and of the straining system. It is clear to the Writer that D_{max} will be grossly affected by the boundary conditions and method of testing and will not only be determined by density. This implies that the D and K factors are not independent of each other; thus, the equation will yield much less meaningful results than those obtained from the axisymmetric tests. It must be emphasized that separation of "K" as"the friction component" is misleading because it contains significant factors other than friction.

Frydman et,al.(1973) based on pure deviatoric tests on hollow cylindrical samples hypothesized that slip would occur at a point in the soil mass when shear stress "on average" + was equal to $\sigma_{oct} \tan \phi_\mu$ and using an energy balance arrived at;

$$\tau_{oct}/\sigma_{oct} = \tan \phi_\mu - 2dv/3d\gamma_{oct}$$

Definition of the terms can be found in Appendix 4. They defined yielding at a stress-strain level (i.e. corresponding to a certain deviatoric stress level) after which significant shearing strains (and volume changes) occured.

9.3.4. Undrained Tests.

An interesting subject is the significance or the meaning of observed strengths in undrained tests. Early

---

+ Authors' italics.

reliable data was given in Bishop and Eldin (1950) who stressed the importance of the test conditions like degree of saturation, existance of negative pore pressures etc., when conducting undrained tests, the results of these tests can be appreciably affected by these factors which cause deviations from the expected apperant angle of shearing resistance $\phi_u = 0$. They also presented a theoretical discussion on the physical behaviour granular assemblies to explain their findings. Factors affecting the $\phi_u = 0$ condition were also discussed by Newland and Allely (1959).

Bishop and Eldin (1953) obtained a good agreement between ϕ' in their undrained triaxial compression tests, and ϕ' obtained after subtracting the deviator load associated with external work done against dilation of the sample in drained tests, Bishop (1954),and emphasized the usefulness of the Cu/p ratio. Seed and Lee (1967b) made similar comparisons between drained and undrained strengths but they again made use of so called "critical σ_3" which has no fundamental basis.

Another interesting feature of the behaviour of loose samples in undrained shear was pointed out by Bjerrum (1961), Bishop, Webb and Skinner (1965), Castro (1969) and Bishop (1971). This was the very low mobilised Coulomb angles at maximum $(\sigma_1 - \sigma_3)$ values before higher strains were reached. This feature was more pronounced in the case of relatively high consolidation pressures.

Drained ϕ' values corrected for the external work due to dilation using Bishop (1954) approach , rather than Rowe's,give ϕ' values which agree with the ϕ'actually measured in undrained tests. Koerner (1970) reports a ϕ_f value by Ladanyi correlates with ϕ' from an undrained test, ϕ_f being obtained from equation 9.14.

9.4. A Brief Review of Stress-Strain Behaviour of Granular Soils.

9.4.1. Introduction.

In this brief section stresses and strains before the failure state is attained will be reviewed. Although the immediate aim is the determination of the generalised failure state, pre-failure stress-strain behaviour cannot be overlooked. Since all structures transmit loads to their foundations, and induce stresses at pre-peak strains, the deformation calculations must clearly be based on a continuum type of formulation because it is not possible to conduct tests at all stress levels and states because there are infinite in number. This boils down to the determination of the properties of the materials considering all factors affecting it and at all stress and strain levels. The practice in foundation engineering has been to use very simple principles to estimate roughly the expected vertical deformation. In recent years more sophisticated deformational approaches have been proposed. Theories from linear-elasticity to various forms of plasticity have been tried to suit the observed behaviour of granular materials. The following account is intended as a short survey rather than a complete review of the subject, since the amount of work being carried on the subject is so enormous that it might be a general reporter's task to give a full account of the work. Studies which have a more generalised approach will be of interest.

9.4.2. Elasticity Applied As a Model.

Succesful use of the theory of linear elasticity in problems concerning metals tempted researchers to use it, at least partly, as a tool in soil deformation calculations, see, for example, the early studies, among others, Chen (1948), Jakobson (1957). It is known that the behaviour of granular assemblies is not ideally elastic. This is more marked at higher deviatoric stress levels where major particle slippages occur. Even during the initial portions of the stress-strain relation the use of a simple modulus of elasticity and poisson ratio cannot be justified, first, because these values change along the curve significantly, secondly granular assemblies are not isotropic.

Holubec (1968) defines elastic constants for a soil element at a given initial void ratio under a stress increment. Elastic strain increments are expressed in terms of elastic constants and stress increments both for isotropic and cross-anisotropic medium - see Barden (1963), Pickering (1970) and Jaeger (1969) for a complete treatment of the equations in cross-anisotropy - . Axial modulus of elasticity ratio of different moduli and poisson ratios are shown by Holubec in a p-q plot and indicated that they are a function of p,q and e (see notation).

Merkle and Merkle (1969) notice the incosistancy in Holubec's cross-anistropic equations, and more important they recommend a through investigation before a statement - as has been made by Holubec - that elastic moduli are defined on the basis of tests of different types of stress paths uniquely in p.q, e space.

Linear elasticity model seems to be too simplified to reflect the pre-peak stress-strain behavour especially in granular assemblies other than for a very dense packing state. Elastic constants must be assumed to hold for large stress and strain increments and to be applicable along the entire path. Holubec (1968) also proposes an incremental computation for elastic strains which are imagined to be composed of hydrostatic and deviatoric parts, and isotropic consolidation tests and triaxial compression tests at constant mean stress level are used to obtain those components respectively.

Coon and Evans (1969, 1971), seek a constitutive law for recoverable deformations both in isotropic and anisotropic form but they emphasize the fact that recoverability does not directly imply elasticty and in fact they claim the path dependency of elastic strains so called (hypoelasticity) in disagreement with Holubec (1968).

Although realised by most researchers, it must be clearly pointed out at this stage that elastic strains associated with experiments on granular material must be differentiated from the overall strains in the pre-peak stress-strain behaviour. Medium dense to loose assemblies will yield substantial interparticle slippages or rearrangements so that the deformations other than elastic occur and have to be modelled adequately. Elastic-rigid plastic idealisations which are widely used in the applications of the finite element method may be justified in problems which involve no unloading, e.g. monotonical loading. Strain hardening plasticity, the approach by the Cambridge group, also treats the material as elastic at stresses below the yield value. So in a granular soil element under generalised stresses and at a certain pre-peak stress level, observed principal strains ε_1, ε_2, ε_3 are the total-built-in-register of reversible spherical and shearing strains and irreversible spherical and shearing strains the relative quantities of which are functions of level of deviatoric stresses, density, mean normal stress, the stress path, structural anisotropy etc. Since the material is not linear, simple superposition of components may not be valid. This complex behaviour explains why there has not been an integrated deformation model for granular material taking all variables properly into account.

Frydman and Zeitlen (1969) performing triaxial compression tests (standard, σ_m constant and isotropic consolidation), on dense sand and ballotini conclude that at a certain deviatoric stress level, a so called "yield point" corresponding to no volume change has been observed in tests with constant mean stress level, and this stage approximately corresponds to the end of linear-like portion of the stress-strain curves, and they observe a "reasonable" possibility of superposition of the spherical and deviatoric strain components to give the combined effect in a stress path which is the resultant of corresponding spherical and deviatoric stress components similar to isotropic elastic materials. No mention is given to the recoverable deviatoric strains, and presumably they are thought to be negligible. It has been pointed out above that at relatively higher stress ratios all assemblies other than very dense should give a certain amount of permenant deformation on unloading. The set back behind these Authors' model seems to lie in that only dense assemblies are treated.

El-Sohby (1969a,b) conducting constant stress ratio loading and unloading tests draws attention to almost totally elastic deformations during unloading and shows the relatively insensitive effect of stress ratio on elastic volumetric strains - dense samples even show a slight increase in elastic volume change in contrast to his analytical prediction of decreasing volume change with increasing stress ratio - and suggests an estimation of spherical elastic volume changes which will have sufficient accuracy to be used in other tests.

Domaschuk and Wade (1969) also carried out isotropic triaxial consolidation tests and triaxial compression tests at constant mean stress level for various porosities of a sand and expressed volumetric strains as a function of mean stress, and deviatoric stresses in a hyperbolic form of equation as originally proposed by Kondner and Zelasko (1963). Separation of these two components has been reasoned on the basis of isotropic, elastic behaviour. It seems to the Writer that these Author's do not make any differentiation between elastic and permenant strains although they consider the whole porosity range. Calculations of strains along different stress paths especially those which involve unloading can not be predicted by their model.

Makhlouf and Steward (1965) summarize the factors which influence the moduli calculations. For a more practical use of moduli in engineering calculations the emprical method by Janbu (1963, 1965) may be used. GiriJavallabhan and Reese (1968) use varying tangents moduli and possion ratios with changing stress levels.

One of the detailed experimental studies on deformation of sands has been done by Ko and Scott (1967b,1968) Pure deviatoric and hydrostatic stresses have been applied to the samples apart from a wide variety of stress paths in an apparatus which is capable of testing cubical samples using six rubber bags on the faces, Ko and Scott (1967). Although this apparatus has been severely criticised for the peak strength measurement (see Chapter 2) pre-failure strains are possibly less affected by the inbuilt restraint. They have differentiated between elastic and plastic strains and have found significant elastic strains associated with unloading in agreement with El-Sohby (1969). Their pure deviatoric tests give a similar behaviour to the tests by Frydman and Zeitlen (1969) in that after a certain shearing stress level strains suddenly start increasing and a yield point is defined at this stage before the conventional peak value. Ko and Scott have pointed out that a complete separation of hydrostatic and deviatoric components has not been observed. Volume changes that occur during pure deviatoric tests are partly the reason for this coupling. Investigation of states other than pure shear and hydrostatic loading and stress path behaviour of strains have been left open.

El-Ruwayih (1975) has obtained nearly elastic behaviour in unloading and reloading cycles completely in agreement with most of the aforementioned researchers, and also noticed appreciable degree of induced anisotropy on the elastic behaviour. Stress path dependency of recoverable strains has also been observed.

Since this study was not planned as a deformation study there are no unloading-reloading tests nor any tests at constant mean stress level nor any other type of test aimed at a particular feature of the deformation behaviour. The majority of tests used a cell pressure of 207 kN/M^2. If only major to minor principal stresses are considered the stress paths were similar to those in triaxial compression tests.

It is felt that direct use of elastic parameters in the theory of linear, isotropic elasticity applied to granular materials is not satisfactory. The concept of poisson ratio has a different status. When there are any interparticle slips poisson ratio starts changing continiously. It is not a constant but a function of density, mean stress level, shearing stress level, anisotropy (both structural and induced) for a specific stress path. Tangent modulus is similarly affected by all these factors. These parameters will be different in loading and unloading. A general theory taking all these factor into account may be difficult to obtain. Elasticity of pre-failure strains is extensively discussed for granular materials in El-Ruwayih (1975).

9.4.2.1. Secant Moduli in ISC Tests.

Generalised data can be examined with respect to the moduli. Secant modulus has been thought to represent the moduli better than tangent modulus mainly because the latter is more suceptible to errors like the graphical adjustment of initial tangent and bedding errors etc. A certain value of strain on the stress-strain curve or a certain proportion of the failure load is usually selected as basis for secant modulus determination. The former is possible in this case because stress paths are very similar (σ_1 vs. σ_3) otherwise a certain amount of strain would not mean anything.

The loading path in ISC tests is always "complex". "Simple" or proportional loading might actually be of help in a generalised deformation analysis. In a test with constant cell pressure simple loading can be achieved by keeping a proportional increase of the intermediate stress compared to major stress (i.e. σ_2/σ_1 = const.), namely, a constant b value during the test. If b values are observed in ISC tests it can be noticed that they vary, always starting with a lower value and ending with a higher one. Change of b values during ISC tests are presented in figure 9.9 for some of the typical tests. In words, it means that deformations take place under lower intermediate stresses compared to the failure state where the b value is reported, thus the deformations at failure are due to the combined accumulation of a range of intermediate stresses along the stress-strain curve which are lower than the one at failure. Only in SP5 was a relatively simple loading path followed. Therefore, secant moduli investigation has this reservation which would be more important in an attempt to investigate strains in the ISC apparatus.

Secant moduli versus b values are plotted for ISC tests on Ham River Sand in figures 9.10, 9.11, 9.12, 9.13. 0.5, 1, 2, 3 percent major principal strains and 60 percent of shearing stress at failure have been selected as a basis for the comparison. Since b values vary during the tests, moduli have also been plotted against b values at the corresponding strains in figures 9.11 and 9.13, whereas in figures 9.10 and 9.12 b values are at failure. Most of the dense flexible tests, all loose flexible tests and the special SP9-16 series (figure 9.13) have been graphed. It is seen that dense samples show an increase with increasing b value at failue while a plot against b values at specific strain values seems not to yield any increase after mid b values. Figures 9.10, 9.11. Moduli based on 60 percent of the failure load give a steeper curve. Loose flexible tests after b = .60 give constant moduli until extension at all strains. The scatter for 0.5 percent strain is probably largely due to the initial setting of the sample. A steep curve for moduli based on 60 percent failure load is interesting. Moduli plotted against b values corresponding to the specific strains at which Es are defined also show constant values. Moduli from SP9-16 series (figure 9.13) cannot be directly compared with ISC series, but an idea about the moduli can be obtained in the series itself which again gives constant values. Apart from differences in failure mode and stress path, the effect of anisotropy can be very significant. Similar to the strength variation the

SP series yield lower moduli.

Comparing the moduli obtained from flexible tests against rigid platten tests it can be said that the flexible tests give higher moduli. See Bishop, Green and Skinner (1973), who give secant moduli for rigid platten tests on Ham River Sand. Two rigid platten loose tests SP2 & SP5, are also seen to be noticably lower than the average line from loose flexible tests for 0.5 and 1.0 percent axial strain in figure 9.12.

But it should be borne in mind that the mean stress level almost doubles due to the constant cell pressure used throughout the tests. The Writer's average stress level tests (ASL) conducted on generalised and triaxial samples - which are reported in Appendix 6 - give lower values at low stresses. Tests by Makhlouf and Steward (1965) and Lee (1970) indicate the significant change in E (initial modulus) with the mean stress level. Therefore the observed plots in actual fact have to be corrected with respect to the stress level in triaxial compression tests, and so higher moduli would be obtained. Similar plots can be prepared for volcanic sand.

9.4.3. Granular Material as Viewed from Plasticity Concepts.

The following is a brief review and commentary on the behaviour of granular materials in relation to plasticity. Use of plasticity to describe soil behaviour is increasingly gaining momentum in soil mechanics and the number of attempts to date are so enormous that the considerations given below are far from being complete or comprehensive.

The stress-strain behaviour before failure was not given much attention during the test program in this study which was aimed mainly at the failure state. As pointed out before the loading path in ISC tests is not "simple" and this is a very significant restriction in any sort of analysis of the deformations in the tests. On the other hand, pre-peak stress-strain behaviour of granular soils has still not been clearly defined in relation to various forms of plasticity even in the triaxial state. Pre-failure behaviour, therefore, will be briefly reviewed in relation to triaxial data.

Early attempts to relate plastic behaviour to soils - with the exception of limit analysis approach in slope or bearing capacity failures etc. which was in use much long ago - were done by Drucker and his co-workers in 1950's at Brown University. Drucker and Prager (1952) suggested the use of a generalised Mohr-Coulomb failure envelope* as a yield curve and studied implications of assuming the soil as a perfectly plastic material. Predicted volumetric strains using these assumptions are highly dilatational due to the normality rule. (The yield surface will also become the plastic potential). Experimental findings - mainly triaxial compression tests - indicate that less dilation than predicted occurs. Moreover depending on the test conditions there may not necessarily be dilation

---

*By this expression they actually meant the extended von Mises expression.

but compression or no volume change.

Drucker, Gibson and Henkel (1957) proposed a more realistic yield surface. It was composed of the Mohr-Coulomb surface capped by a hemisphere in effective principal stress space, figure 9.14. This surface would embrace the possibility of yielding of a soil element under spherical stresses or a combination of normal stresses and deviatoric stresses. It was also possible to predict volume decrease as well as increase depending on the location on the yield surface. Equally important; the consolidation curves obtained in standard tests were likened to strain-hardening behaviour; increasing volume decrease implied increasing stresses. The transition region from the Mohr-Coulomb line to the cap is the only section that obeys normality. They speculated that a perfectly plastic state might be attained in this small section where the yield surface was locally parallel to the hydrostatic axis, and there was no further hardening.

Roscoe, Schofield and Wroth (1958), Wroth and Bassett (1965), Schofield and Wroth (1968) tried to set up a theory to explain the behaviour of granular materials in parallel with the ideas summarized above. Volume change of the material was represented by void ratio changes and the behaviour was visualized in a three dimensional coordinate system of p, q and e. ($p = (\sigma_1+2\sigma_3)/3$, $q = \sigma_1-\sigma_3$ due to consideration of triaxial state only). Projection of the so called "critical curve" in this space on p, q and e, p coordinate planes give critical state line and a consolidation curve respectively, equations of which are:

$$q = Mp \qquad 9.16(a)$$

$$e = e_o - \lambda \ln p \qquad 9.16(b)$$

Equation 9.16(a) corresponds to extended von Mises failure criterion and the yield surface is hypothesized as being similar to that of Drucker et.al. (1957) with the difference that the cap is not circular and the tangent at the transition point is parallel to the hydrostatic axis, figure 9.15. The three dimensional form of equation 16(a), is given in Appendix, Schofield and Wroth (1968), and is exactly the extended von Mises criterion. Although M corresponds to a frictional ratio at constant volume and not to a value when there is volume change taking place (like ϕ', for example, in Mohr-Coulomb equation), it was emphasized in section 9.2 that the extended von Mises criterion could not represent the failure of granular materials in the generalised state even in loose materials which fail almost without volume change.

A plastic analysis requires certain elements. A yield surface must be defined crossing which initiates plastic deformations and inside the surface an elastic behaviour is assumed to exist. There are supposed to be infinite number of successive yield surfaces. They are assumed to be similar in shape and expand until a failure surface is reached which is the limiting yield surface, and collapse occurs for the stress combinations on it. The stiffness of the material qualitatively changes with expanding yield surfaces. Tatsuoka and Ishihara (1974) determined yield loci for sand in p.q space and indicated that they approximately agreed with the equation proposed by Poorooshasb (1971). It is $f = q/p + m\ln p$ where m is constant for a sand irrespective of density and stress path and it is also shown in figure 9.15.

Secondly, a "flow rule" is needed. It relates the plastic strain increments to current stresses. It is usually expressed as a function which is referred to as a "plastic potential". The normality of the plastic strain increment vector to the potential surface forms the basis of a flow rule. For perfectly plastic material the yield surface and plastic potential are identical. Another concept is "strain hardening" which is the property of increasing shear stresses required, in plastic flow state, with increasing plastic shearing strains. This concept originates from metal plasticity like the others (plastic potential etc.).

Almost zero volume change during plastic flow and insignificance of mean stress level are basic differences between metals and granular soils. While a strain hardening function can be written in terms of shearing stresses and strains in the case of metals, granular materials require additional parameters like porosity and mean stress level.

Thus a strain hardening fuction for granular soils could be of the form;

$$F = F(\tau_*, \gamma_*, \sigma_m, e)$$

where τ_* and γ_* are overall shearing stress and strains indicating the state of distortion. They may be octahedral values or the followingexpressions;

$$\tau_* = \left\{(\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2\right\}^{\frac{1}{2}}$$

$$\gamma_* = \left\{(\varepsilon_1-\varepsilon_3)^2 + (\varepsilon_2-\varepsilon_3)^2\right\}^{\frac{1}{2}}$$

It may be noticed that the Cambridge group tries to simplify the picture by introducing a surface ('state boundary surface') in p.q,e space for Granta-Gravel, and varitions in the quantities are projected on to (p,q), (e,p),(e.q) planes, and problems are usually considered in the triaxial compression state. Volume changes due to shearing stresses are not included in the graphical set-up.

An approach may be obtaining semi-emprical relations for a specific density group.

A usual assumption is that isotropic hardening of the material occurs. Otherwise analytical models get complicated. Granular soils have been proved to show strong anisotropic hardening behaviour which is the basis of stress induced anisotropy, Barden (1969), El-Ruwayih (1975). It is believed that this is the most significant limitation of all types of plastic approaches in the case of a granular assembly.

A subject which is under dispute is the shape of the plastic potential surface. A closed surface symmetrical around the hydrostatic line is usually assumed. It first expands from the origin of the stress and strain axes, then closes on the $\sigma_1 = \sigma_2 = \sigma_3$ axis. Drucker et.al. (1957), Schofield and Wroth (1958,* 1968), Christian (1966), Tang and Hoeg (1968), Di Maggio and Sandler (1971) proposed different curved surfaces as being the closing caps while Jenike and Shield (1959) assumed an octahedral plane to be the end cap of the closed pyramidal prism shape. If non-association is involved in a model it is customary to assume a shape for plastic potential surface which is entirely similar to yield surface, for example, Barden and Khayatt (1966), Lade and Duncan (1975).

Based on tests on granite rockfill, marble chippings and Ham River sand at stress ratios below failure El-Ruwayih (1975) concluded that the plastic strain increment vectors had different directions at a point in p,q space, depending on the stress path followed, in the triaxial compression state. In other words the stress increment direction affected the strain increment direction. This was also noted by Lewin and Burland (1970). Strong stress path dependancy of plastic strain increment vector is another important drawback for the concept of 'plastic potential'. This indicates the impossibility of an unique plastic potential surface. El-Ruwayih also found that the

*Roscoe, Schofield and Wroth.

directions of strain increment vectors along a constant stress ratio path were not constant but were changing with increasing stress level in contrast with findings of Holubec (1966) and Pooroohasb et.al. (1966,1967). Strain increment vectors rotated anti-clockwise with increasing mean stress level.

It can be said that the normality rule of plasticity does not hold for granular materials. Although various forms of yield surfaces are assumed to obtain a normality condition at failure by manipulating the tangent of yield surface parallel to the hydrostatic axis, it seems that this is a hypothetical condition rather than real. For example, for a yield curve OABCE in figure 9.16, which is typical of the type assumed by several researchers - Schofield Wroth (1968), DiMaggio and Sandler (1971) etc.- point B represents failure state. Point B' is another point on yield surface which is close to B, and a drastic change in the direction of plastic strain increment vector may not be expected from B' to B.

Definition of the term "yielding" is quite ambigious and is responsible for certain complexities. If interparticle structural changes are considered as yielding, then this definition applies to a wide variety of spherical and deviatoric stress states, and presumably holds true from low percentages of the peak stress ratio (σ_1/σ_3) until failure during shearing. Another criterion may be taken as a deviatoric stress level after which rate of increase of deformations becomes appreciable (for example Frydman et.al. (1973) or Ko and Scott (1968)). Although it may not be very accurate to define this point under different stress path and densities it is believed that this definition is more reasonable in the engineering sense.

In figure 9.16 when it is hypothesized that the material will yield along O A B C D E it is not clear, for example, why the material should yield at D and not D' or D'' nor the criterion for yield at D. If we take another point C, accepting the second definition above it is expected that the yielding will occur at any point like C' in the crosshatched zone.

In the case when the normality rule does not apply limit theorems of plasticity cannot be valid, and a unique solution can not be guaranteed. On the other hand there is no reason why granular materials should follow the normality rule. It is only a facility in predicting the deformations in the plastic state. This can be also achieved if the position of resultant strain increment vectors can be formulated with respect to tangents of plastic potential curves at any stress level.

A very interesting observation is the projection of strain increment vectors on the octahedral planes in the generalised state. In figure 9.17, these projections are shown for dense Ham River sand samples, and they are plotted in figure 9.18, for dense and loose volcanic sand samples. It can be noticed easily that increment vectors are almost normal to the failure surface in $\sigma_1, \sigma_2, \sigma_3$ effective principal stress space if principal strain increments are also plotted in the same coordinate system. It is true regardless of the initial density and for both materials. There is a slight deviation from the normality in the mid intermediate state on the deviatoric planes - say,between b = .3-.6 -. This interesting observation was also made by Goldscheider and Gudehus (1973) and Lade and Duncan (1973), for other sands. The meaning of this normality on π - planes must be made clear, and possible ways of making use of it must then be sought.

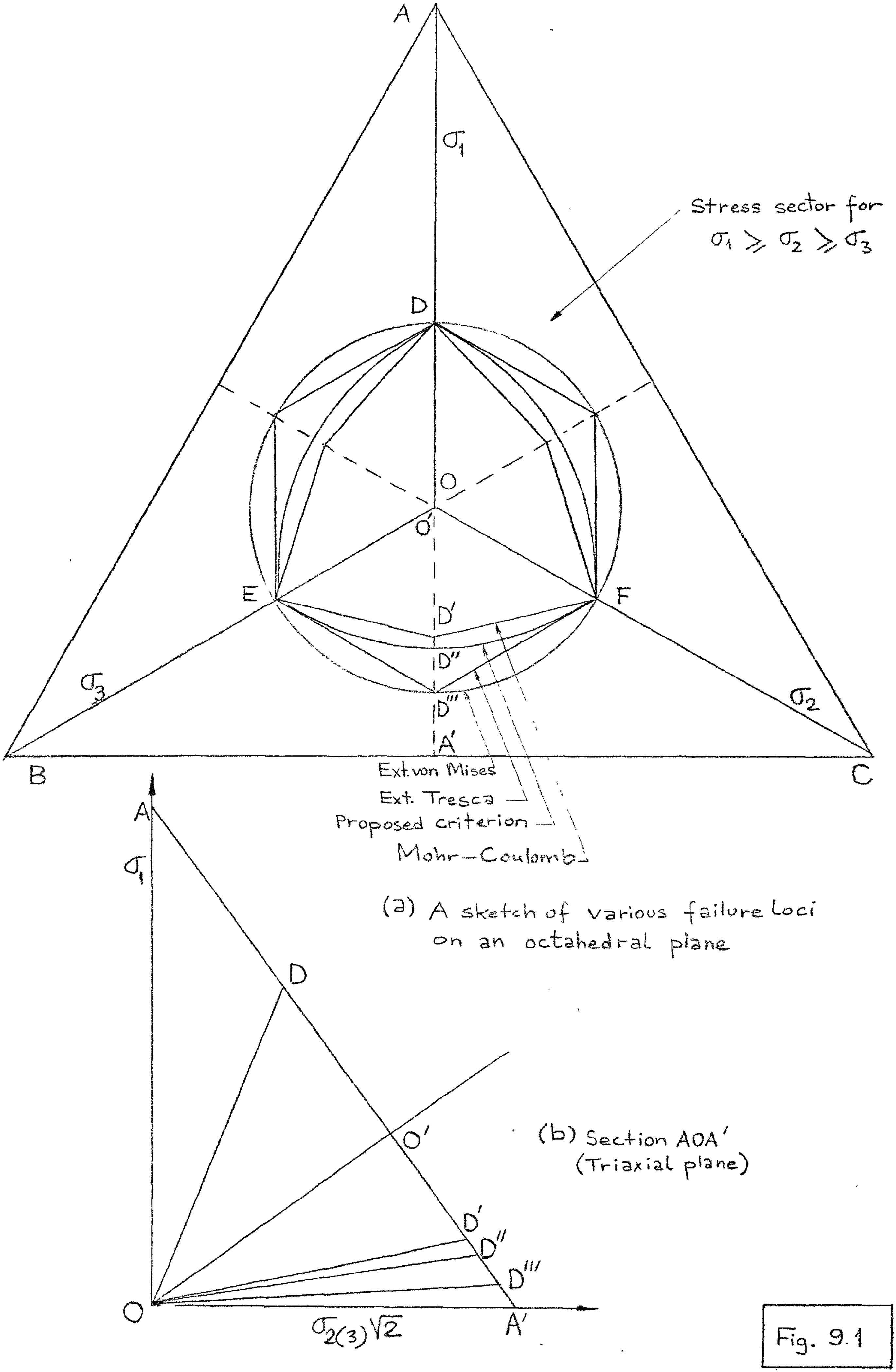
It is shown by triaxial compression tests that plastic strain increment vectors are not normal to the failure envelope in the triaxial plane. This is true for all other planes; see for example figure 9.19, where strain increment vectors are plotted for two tests in a triaxial and a generalised plane.

The normality observed on octahedral planes, as the Writer sees it, is due to the similarity of stress and strain rate fields at failure. For example, if a generalised stress state is imagined with the smaller deviatoric stress half the major one (b = 0.50) it is not expected that the minor and the intermediate strains or strain increments will be equal at failure as in the triaxial compression state.

Otherwise strain increment vectors would lie in the triaxial plane and the failure locus on π - plane would make an acute angle with the vector. If a parameter "t" is defined as $(d\varepsilon_2-d\varepsilon_3)/(d\varepsilon_1-d\varepsilon_3)$ relative magnitudes of strain increments can be compared with the stress state at failure. A plot of t versus b is given in figure 9.20, for many generalised tests for both materials. It can be seen that there is almost a linear relationship between the parameters, and more interesting, the deviation from 45° - line increases towards the middle range b values. It was mentioned that at mid range intermediate stresses the normality was followed more approximately.

It is seen generally that although application of plasticity concepts to soil mechanics have been increasing recently, a synthesis is needed. On one side there are data from various types of laboratory shear tests, on the other, laboratory model tests give valuable data (e.g. Arthur, James, Roscoe (1964), Rowe and Peaker (1969), James and Bransby (1970)). Another group of researchers, work on more mathematical side on modelling, e.g. de Josselin De Jong (1959, 1971 & 1973), Mandl and Fernandez Luque(1970).

It is believed that concluding on a generalised model to describe the granular material at <u>all</u> spherical and deviatoric stress strain levels is a difficult task awaiting further research which must be directed towards combining the behaviour observed in all kinds of tests (including model experiments) into unified principles as well as developing new techniques to enable measurements of the required parameters in analyses.



(a) A sketch of various failure loci on an octahedral plane

(b) Section AOA′ (Triaxial plane)

Fig. 9.1

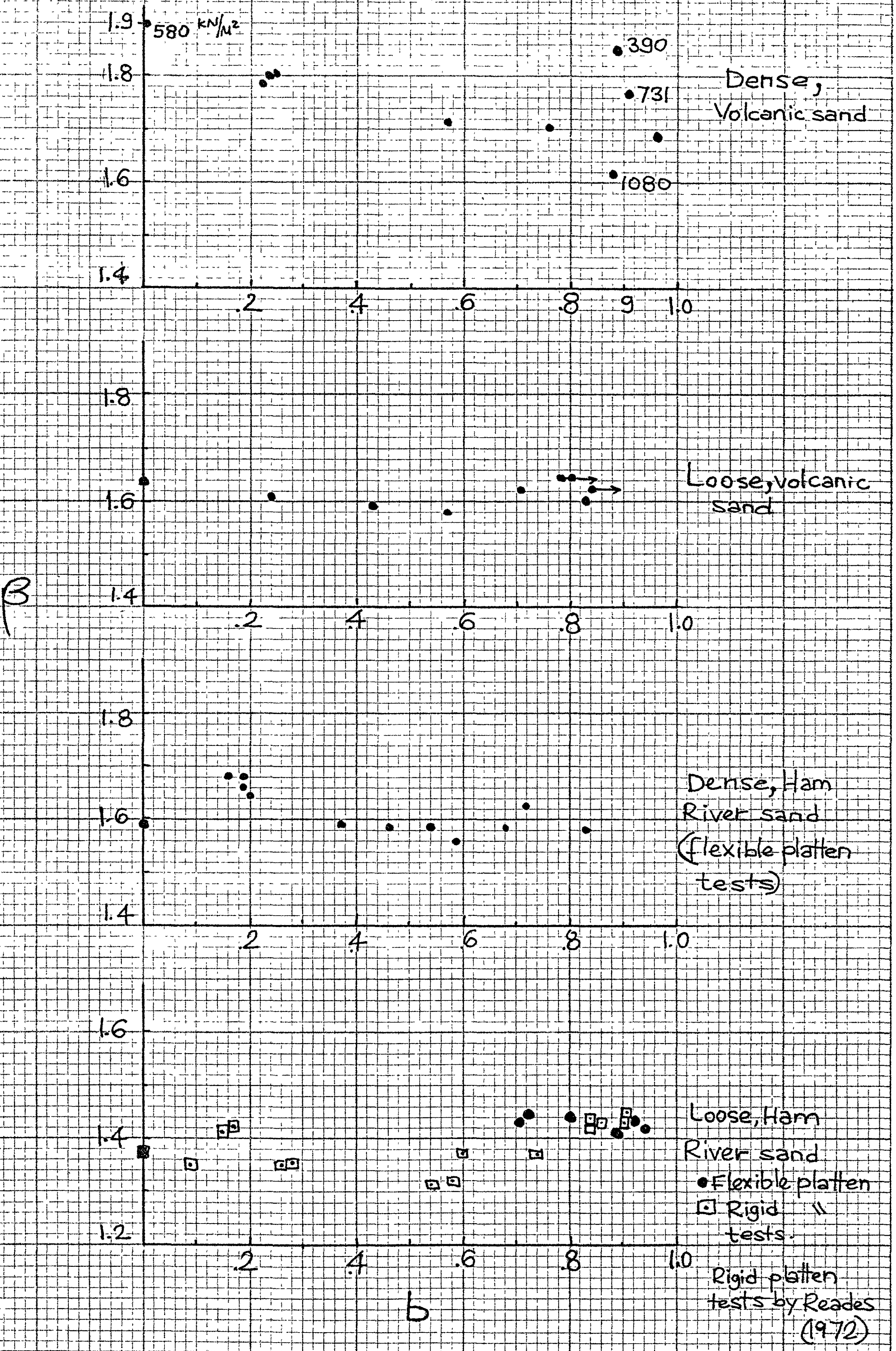

1.9
580 kN/M²
1.8
1.6
1.4
390
731
1080
Dense, Volcanic sand
.2
.4
.6
.8
9
1.0
1.8
1.6
1.4
Loose, volcanic sand
.2
.4
.6
.8
1.0
β
1.8
1.6
1.4
Dense, Ham River sand (flexible platten tests)
.2
.4
.6
.8
1.0
1.6
1.4
1.2
Loose, Ham River sand
● Flexible platten
⊡ Rigid "
tests.
.2
.4
.6
.8
1.0
b
Rigid platten tests by Reades (1972)

Fig. 9.2

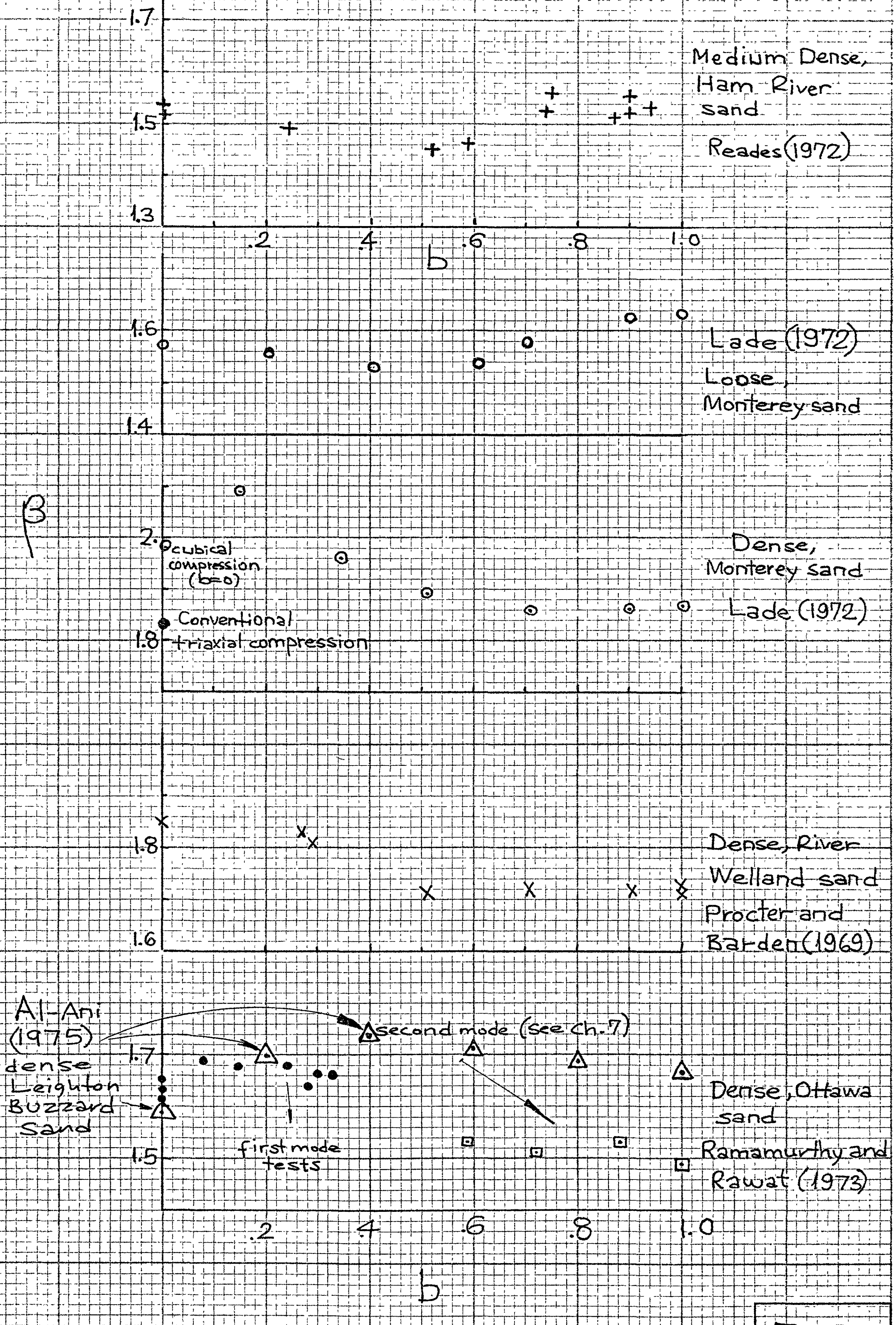
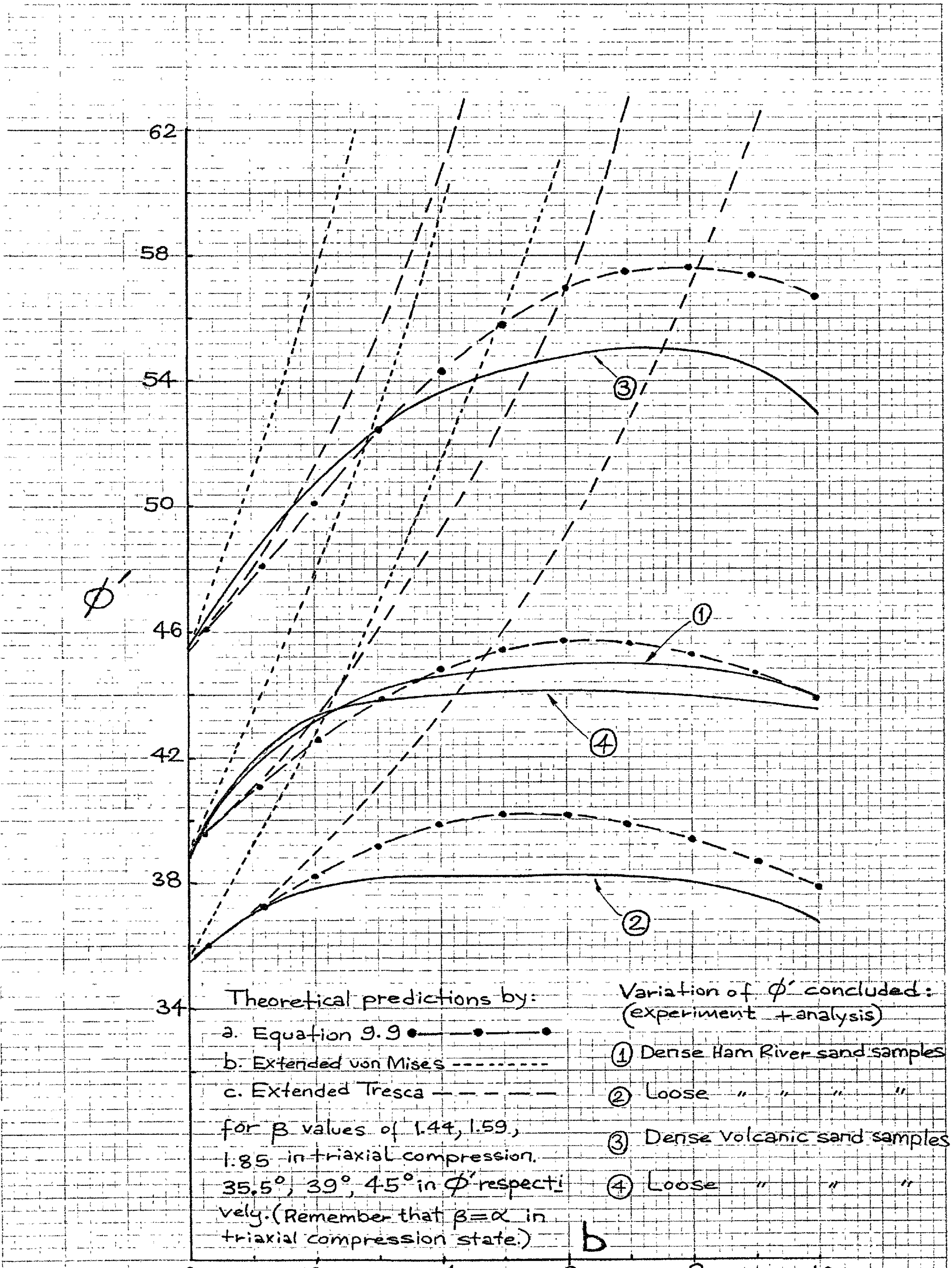
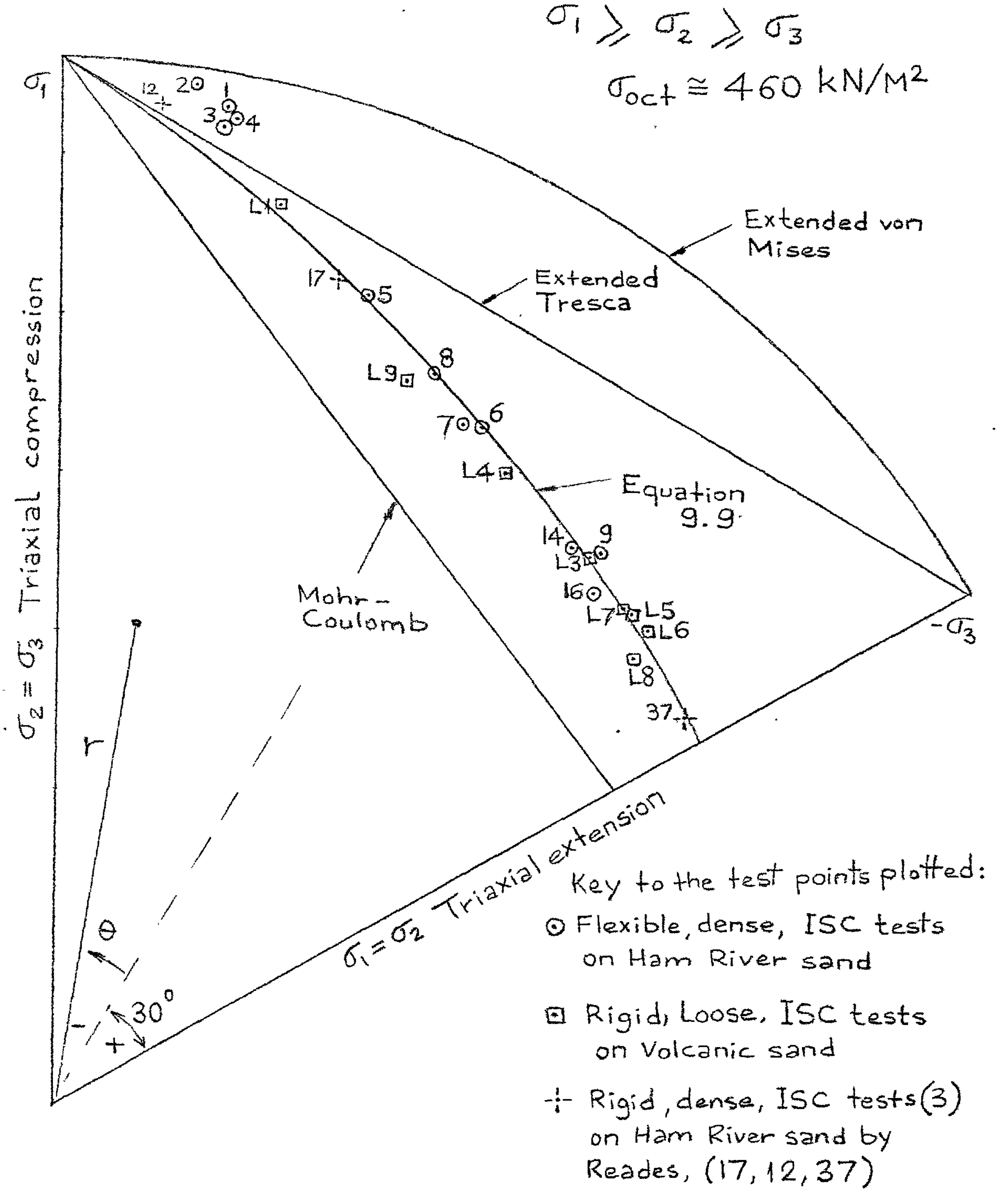

β
b
1.7
1.5
1.3
.2
.4
.6
.8
1.0
Medium Dense, Ham River sand
Reades (1972)
1.6
1.4
Lade (1972)
Loose, Monterey sand
2.0
1.8
cubical compression (b=0)
Conventional triaxial compression
Dense, Monterey sand
Lade (1972)
1.8
1.6
Dense, River Welland sand
Procter and Barden (1969)
Al-Ani (1975)
dense Leighton Buzzard Sand
second mode (See Ch. 7)
first mode tests
1.7
1.5
Dense, Ottawa sand
Ramamurthy and Rawat (1973)
.2
.4
.6
.8
1.0
b

Fig. 9.3



Comparison of strengths from Equation 9.9, von Mises, Tresca criteria (extended) and concluded on the basis various generalised tests

Fig. 9.4



Key to the test points plotted:

⊙ Flexible, dense, ISC tests on Ham River sand

⊡ Rigid, Loose, ISC tests on Volcanic sand

-¦- Rigid, dense, ISC tests (3) on Ham River sand by Reades, (17, 12, 37)

Plot is for $\alpha_0 (= \beta_0) = 1.593$ in triaxial compression ($\phi' = 39^\circ$)

Failure Loci shown in a 60°-sector of an octahedral plane for several failure criteria

$$r = \tan\left[\cos^{-1}\left\{\frac{1}{\sqrt{3}}\frac{(\sigma_1+\sigma_2+\sigma_3)}{(\sigma_1^2+\sigma_2^2+\sigma_3^2)^{1/2}}\right\}\right],\quad \theta = -\tan^{-1}\left\{\frac{1}{\sqrt{3}}\frac{(2\sigma_2-\sigma_3-\sigma_1)}{\sigma_3-\sigma_1}\right\}$$

Fig. 9.5

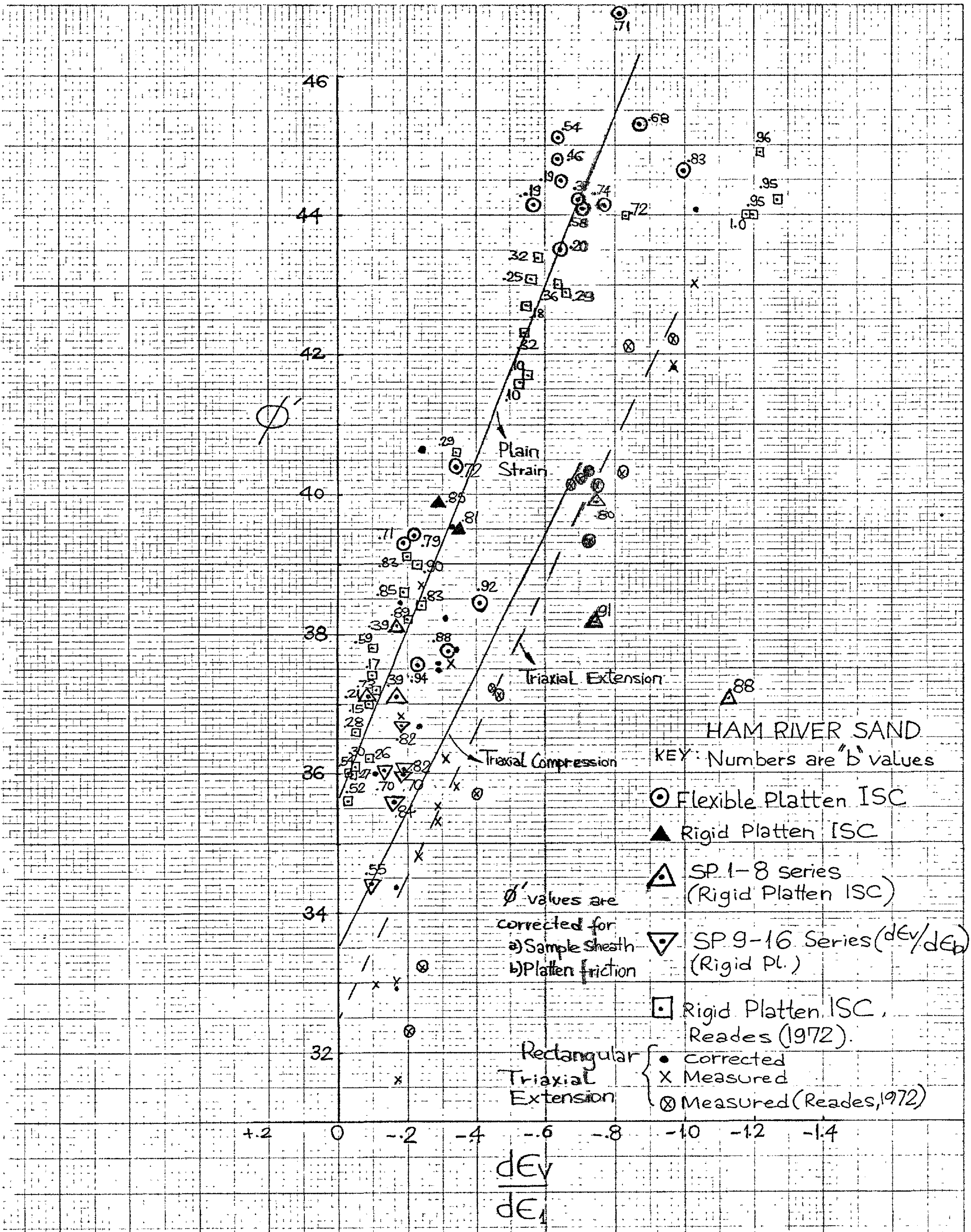

HAM RIVER SAND
KEY: Numbers are "b" values
Flexible Platten ISC
Rigid Platten ISC
SP 1-8 series (Rigid Platten ISC)
SP 9-16 Series (dεv/dεb) (Rigid Pl.)
Rigid Platten ISC, Reades (1972).
Rectangular Triaxial Extension
corrected
Measured
Measured (Reades, 1972)
φ' values are corrected for
a) Sample Sheath
b) Platten friction
Plain Strain
Triaxial Extension
Triaxial Compression
φ'
46
44
42
40
38
36
34
32
+.2
0
-.2
-.4
-.6
-.8
-1.0
-1.2
-1.4
dεv/dε1

Fig. 9.6

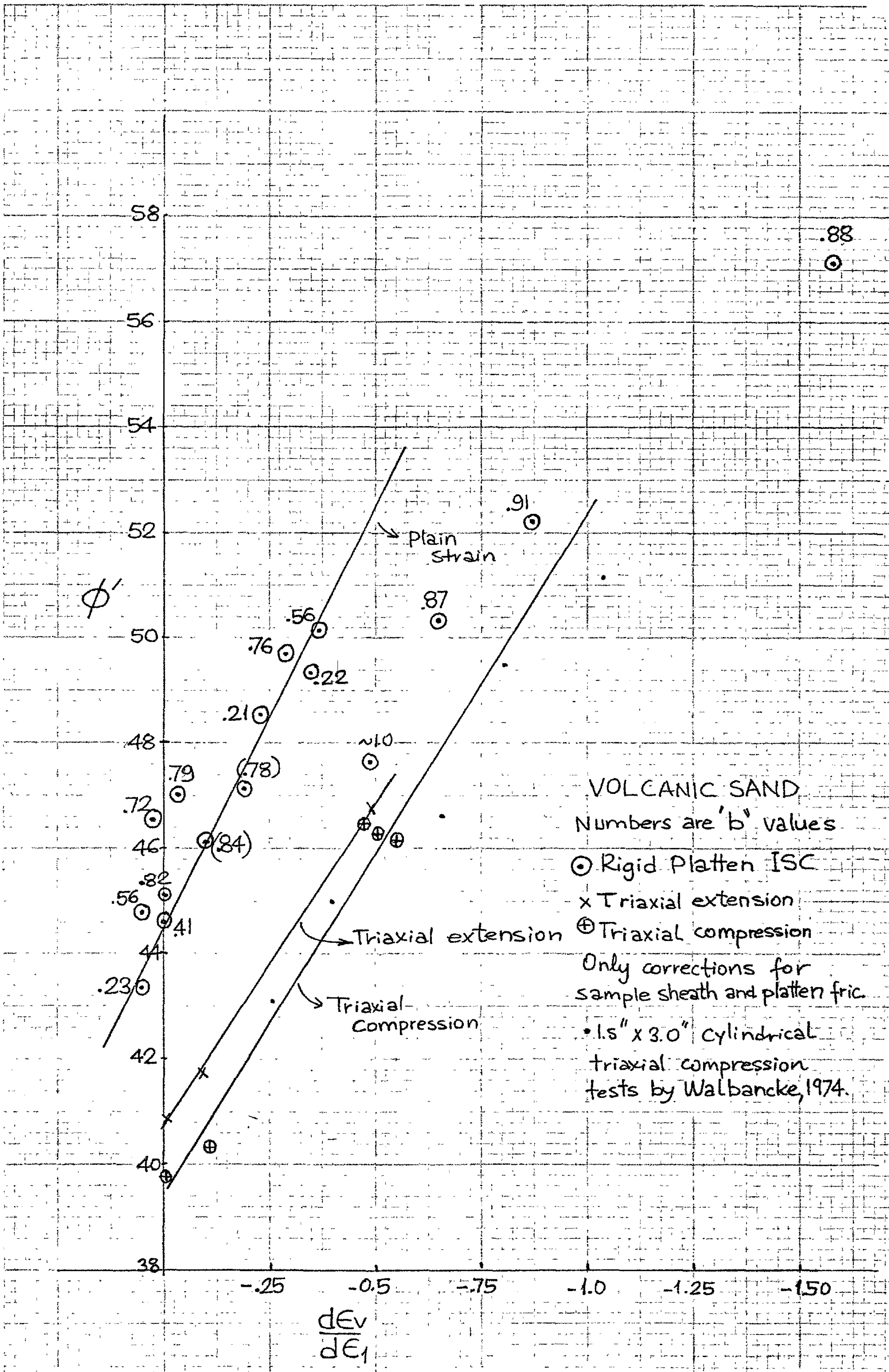
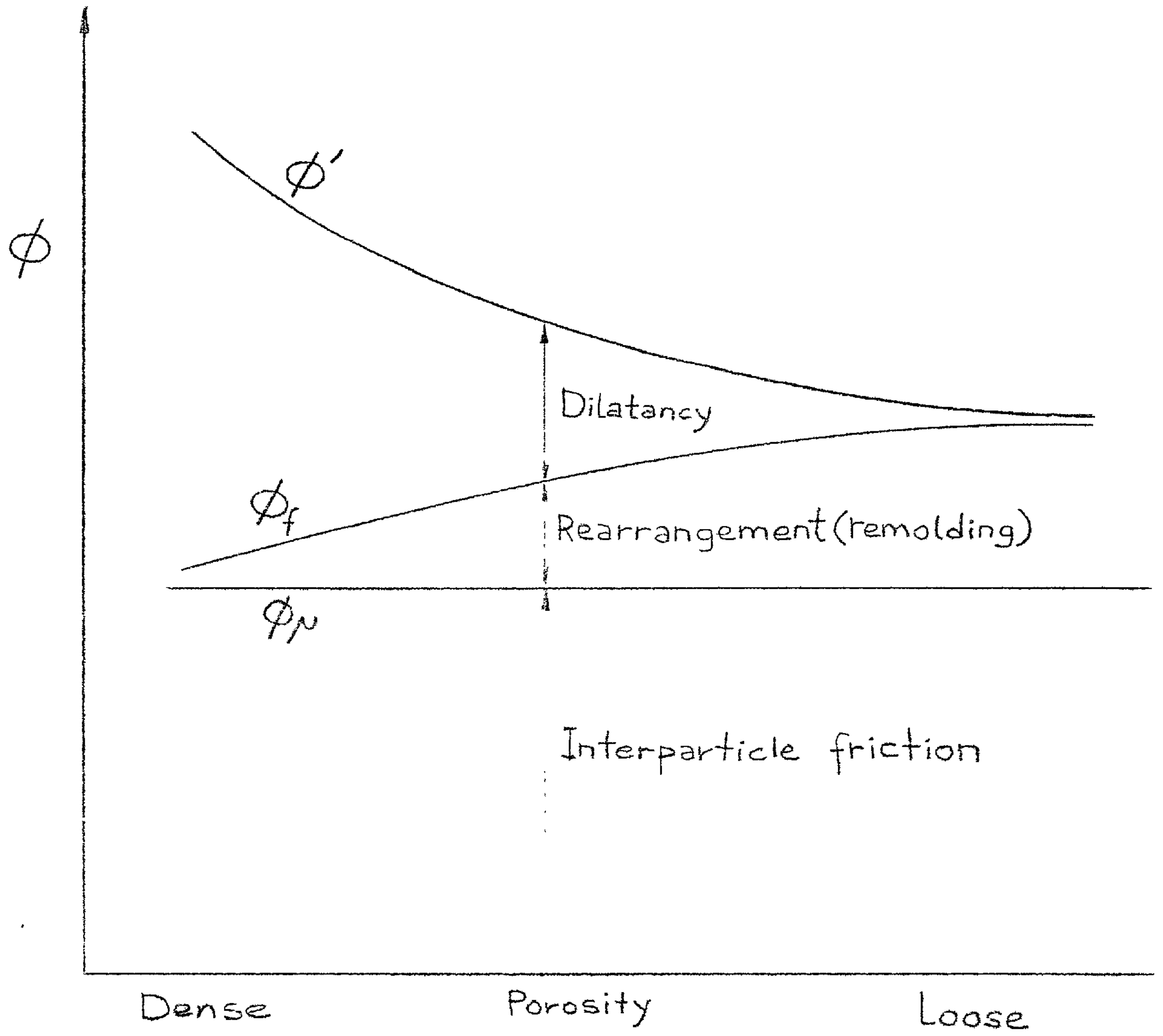
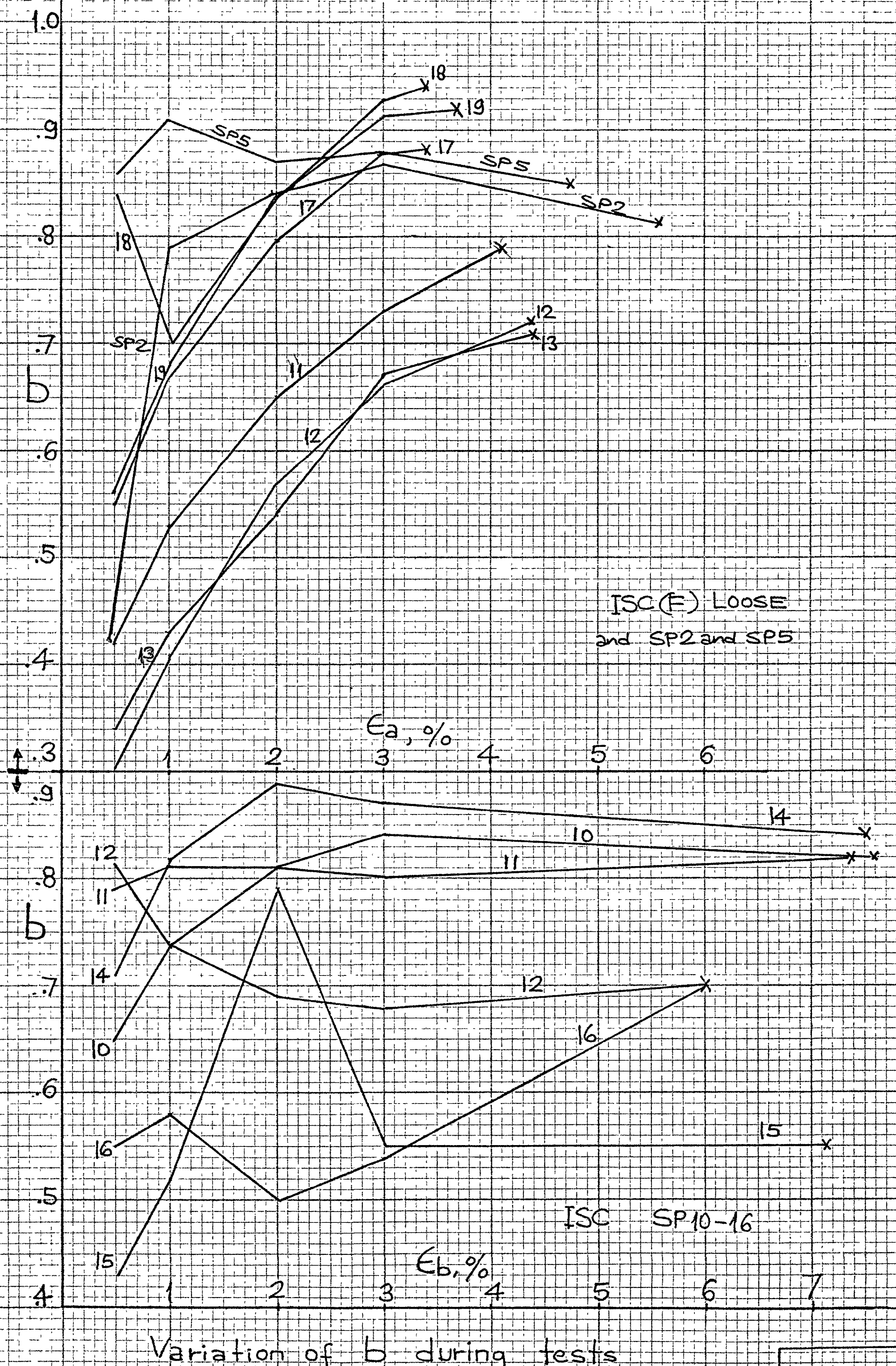

58
56
54
52
50
48
46
44
42
40
38
ϕ′
-.25
-0.5
-.75
-1.0
-1.25
-1.50
$\frac{d\epsilon_v}{d\epsilon_1}$
.88
.91
.87
.56
.76
.22
.21
~1.0
.79
(.78)
.72
(.84)
.82
.56
.41
.23
Plain strain
Triaxial extension
Triaxial compression
VOLCANIC SAND
Numbers are 'b' values
⊙ Rigid Platten ISC
× Triaxial extension
⊕ Triaxial compression
Only corrections for sample sheath and platten fric.
• 1.5" x 3.0" cylindrical triaxial compression tests by Walbancke, 1974.

Fig. 9.7



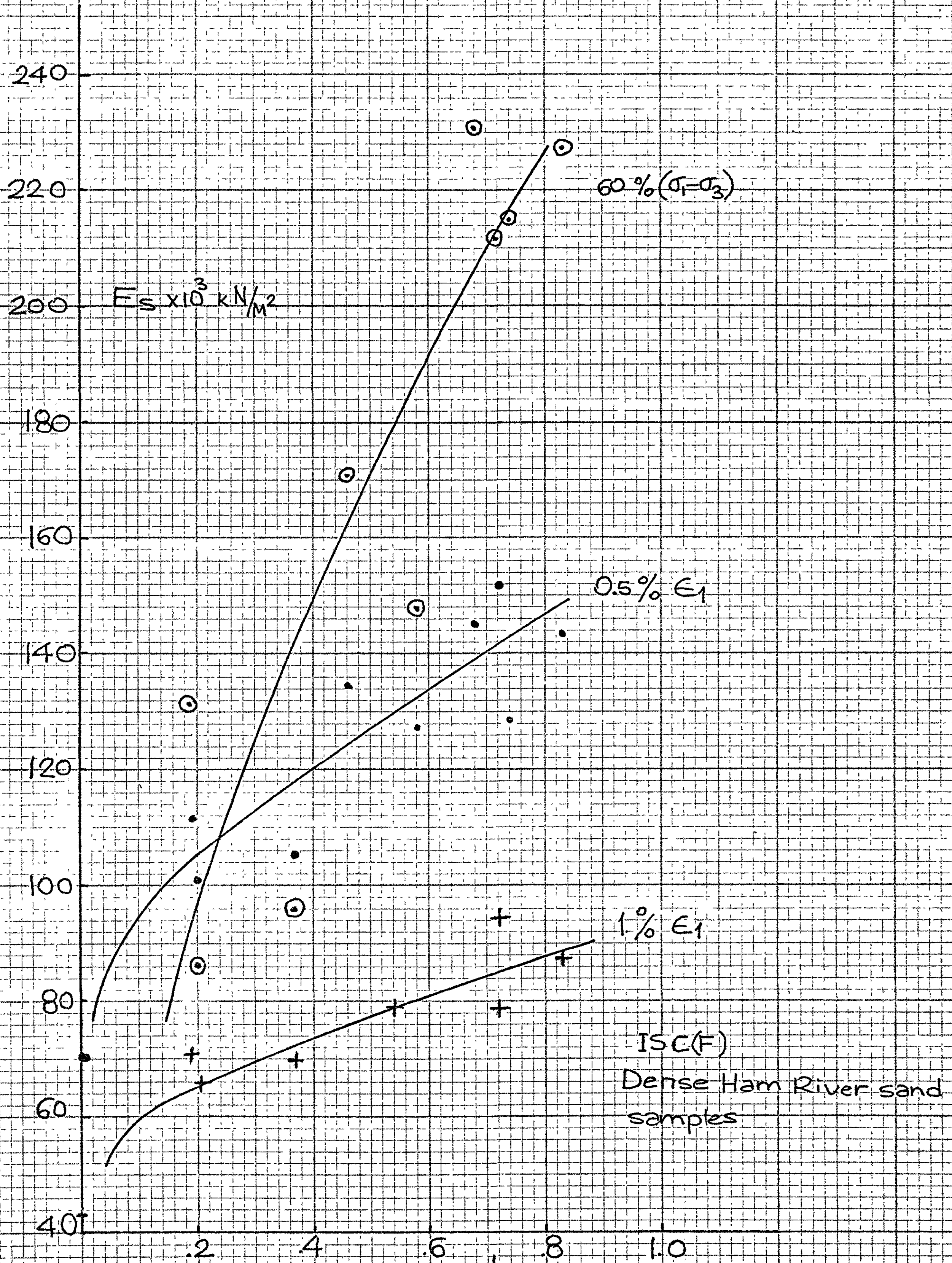
ϕ
ϕ'
Dilatancy
ϕ_f
Rearrangement (remolding)
ϕ_μ
Interparticle friction
Dense
Porosity
Loose

Fig. 9.8



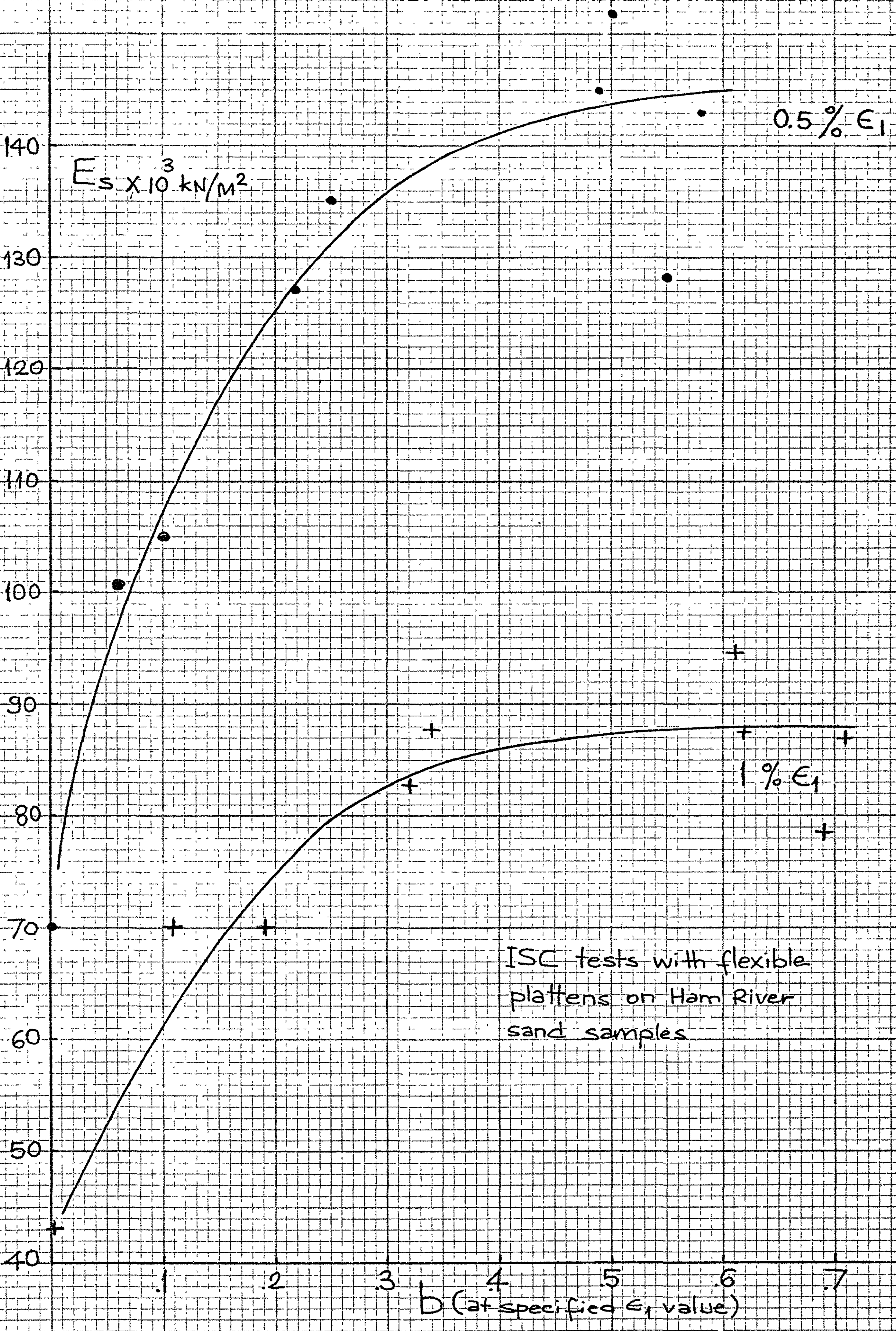
Variation of b during tests

Fig. 9.9



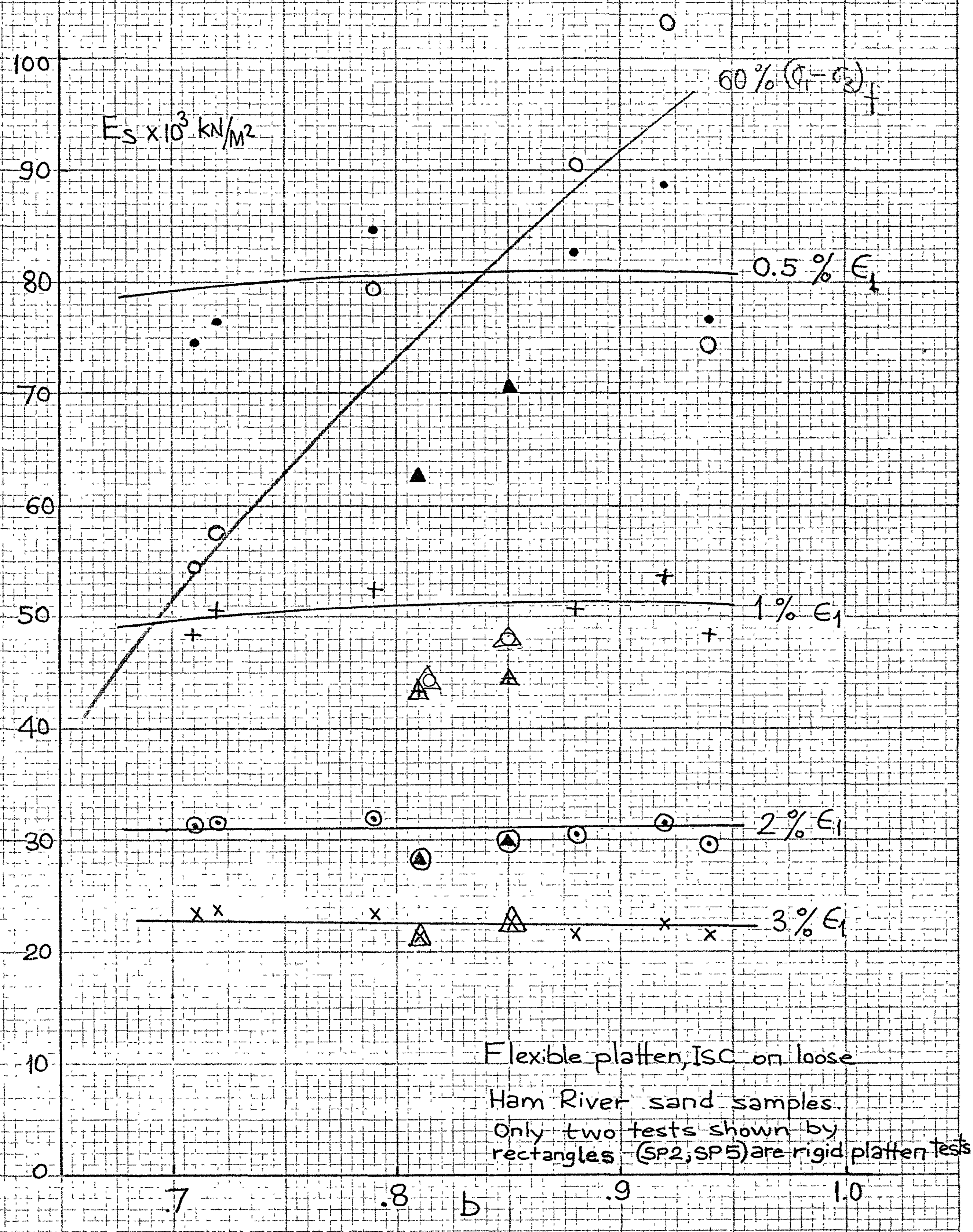
Secant moduli vs. b

Fig. 9.10



Secant moduli vs b (at corresponding ϵ_1)

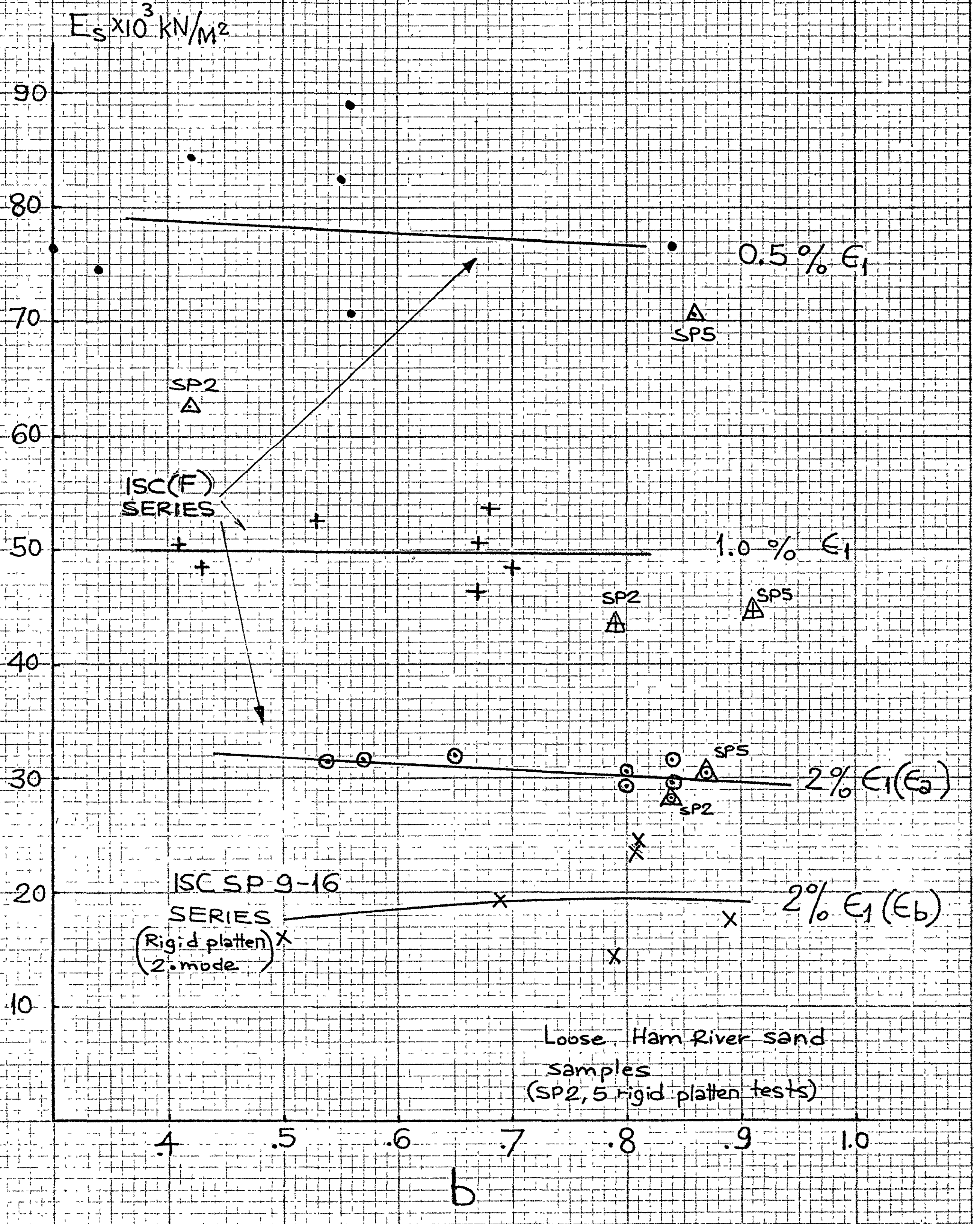
Fig. 9.11



Flexible platten, ISC on loose Ham River sand samples. Only two tests shown by rectangles (SP2, SP5) are rigid platten tests

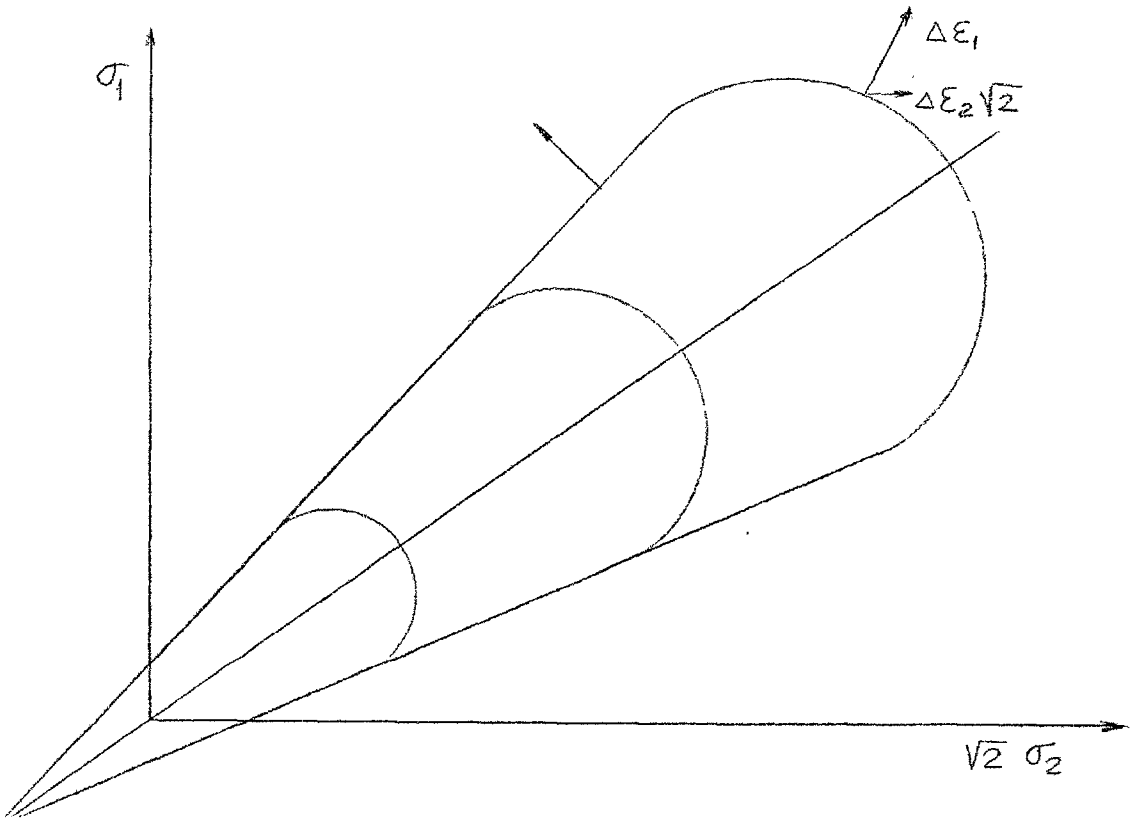
Secant moduli vs. b (failure)

Fig. 9.12



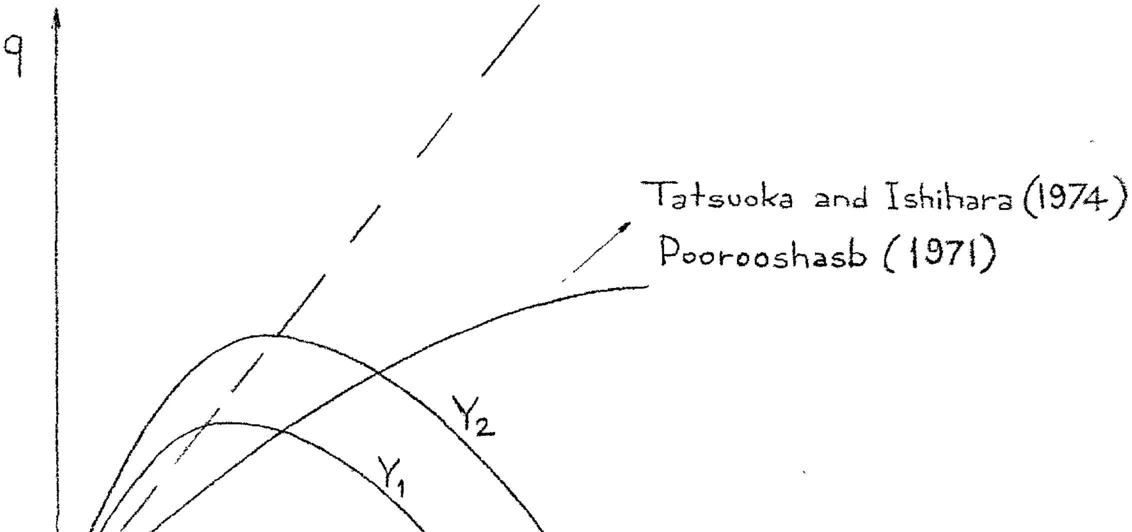
Secant moduli vs. b (corresponding to ϵ_1 values)

Fig. 9.13



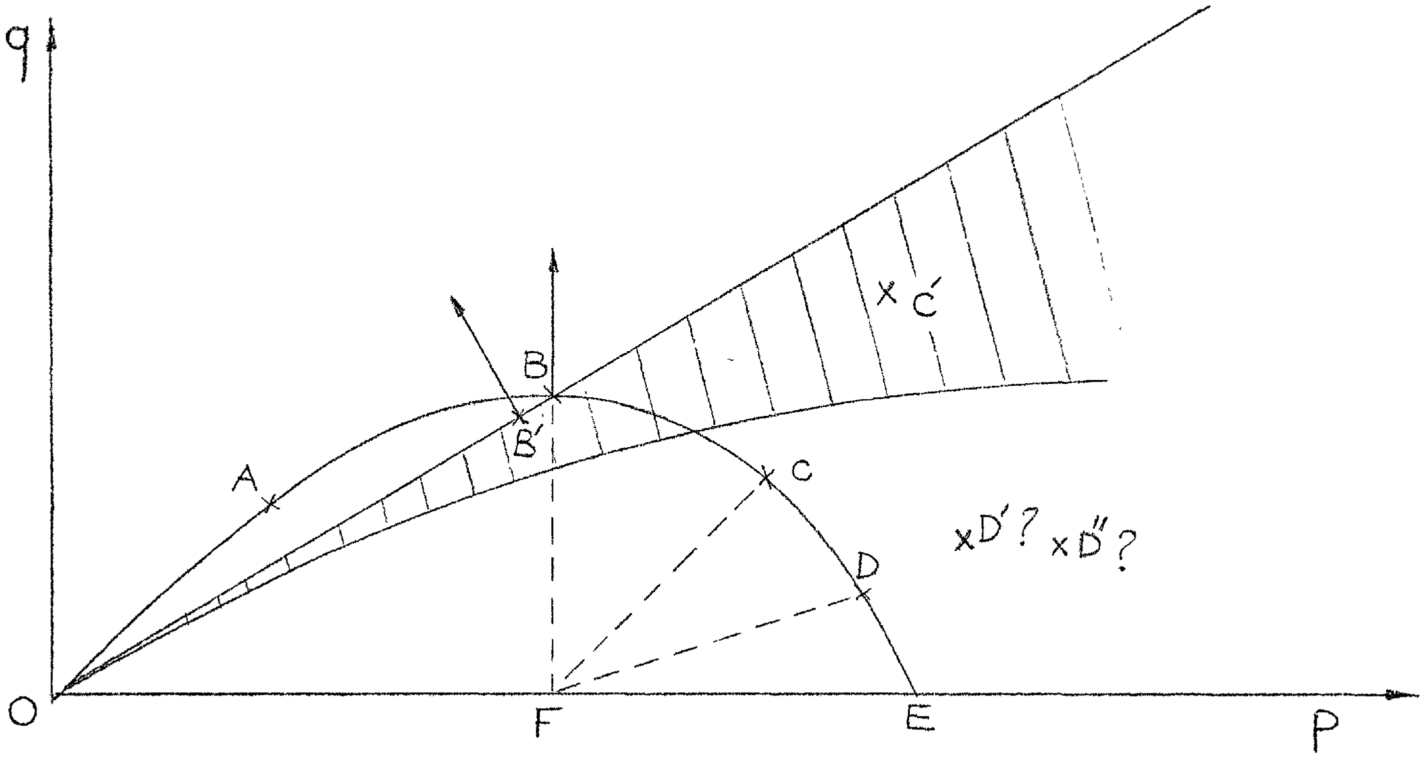
Possible yield surfaces, after Drucker et. al. (1957)

Fig. 9.14



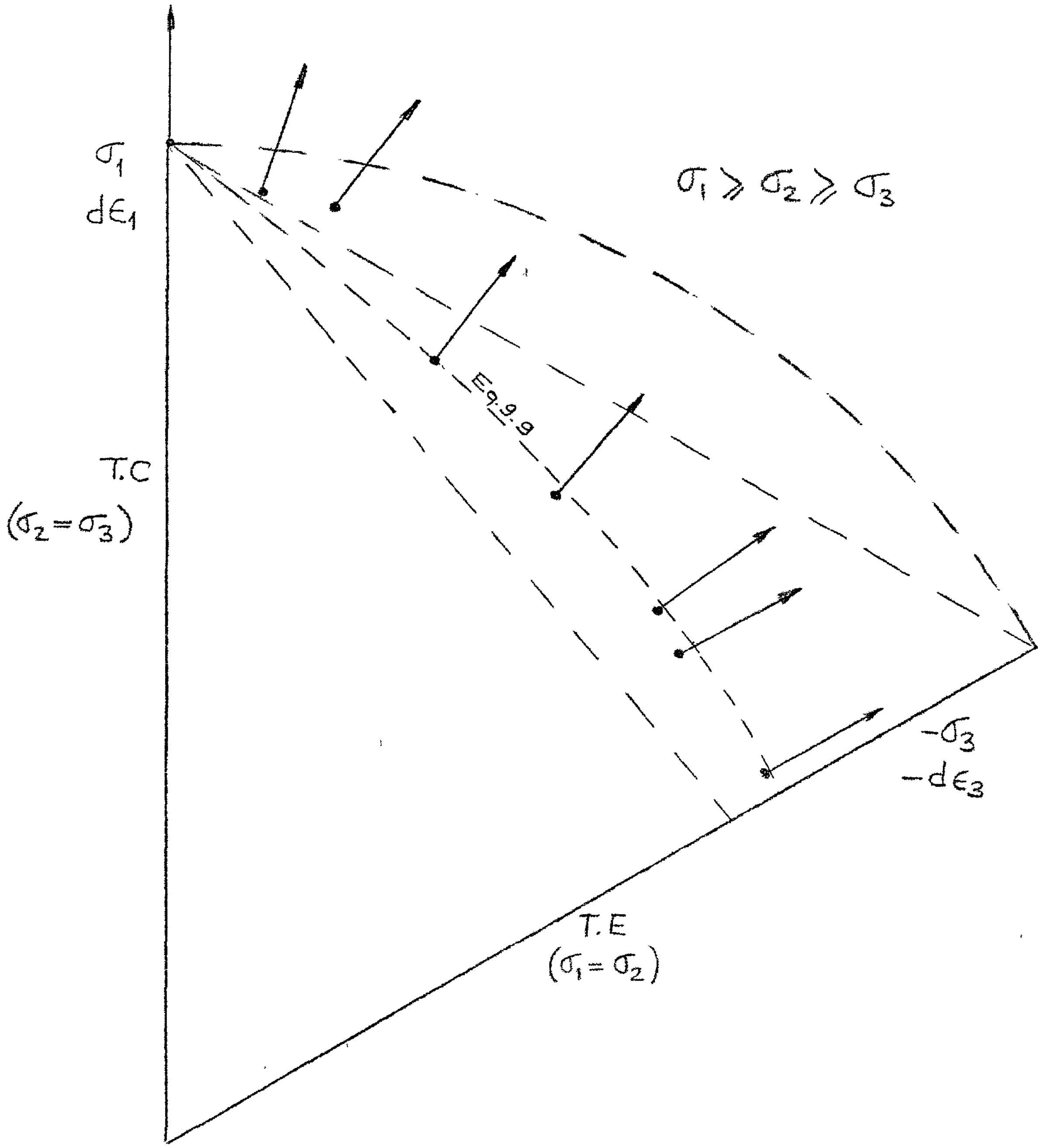
Yield surfaces perceived by Schofield and Wroth (1968), (Y_1, Y_2).

Fig. 9.15



On the definition of "yielding"

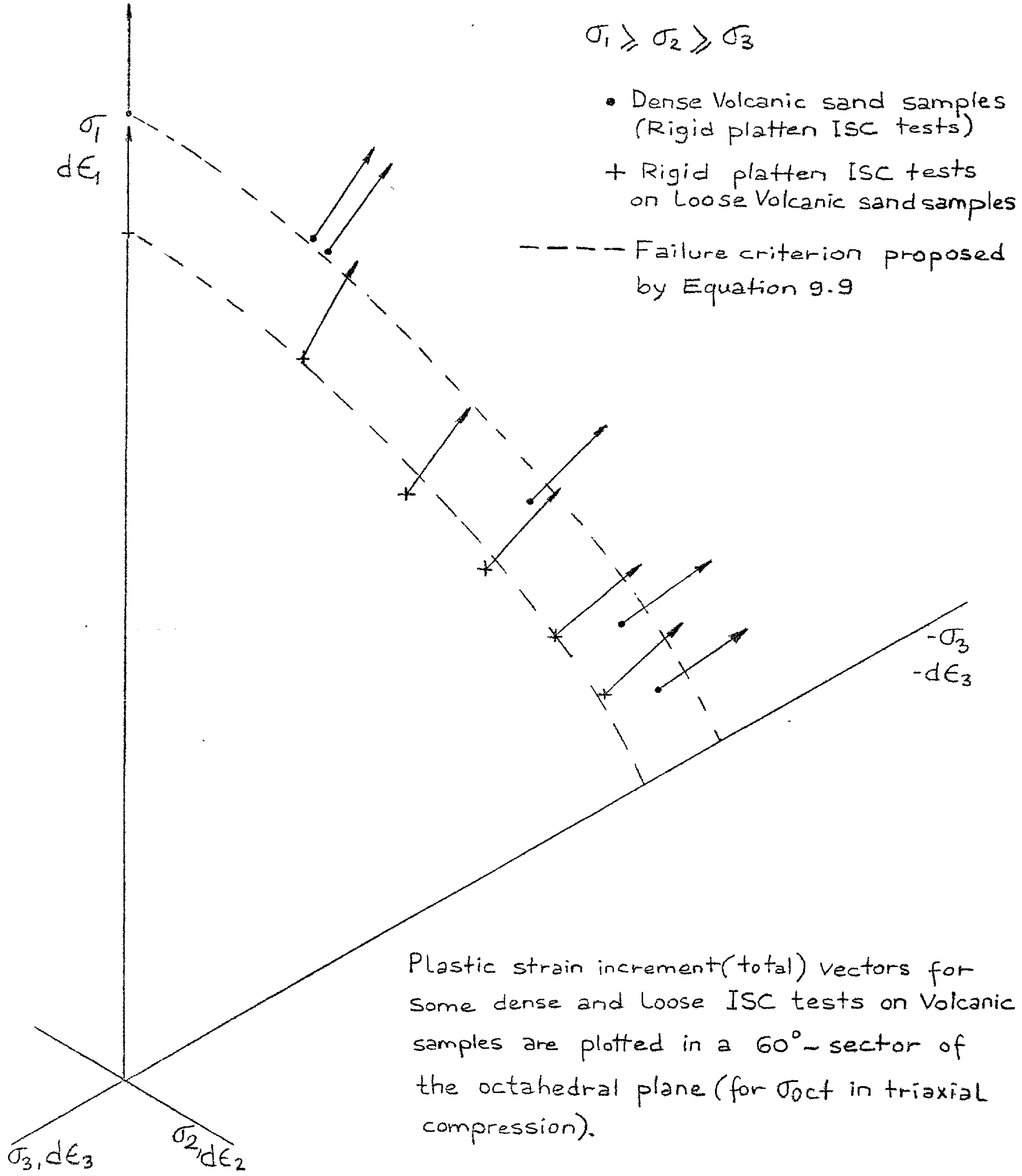
Fig. 9.16



Plastic (total) strain increment vectors for some ISC tests – dense, Ham R. sand – are plotted in a 60^0-sector of the octahedral plane (for σ_{oct} in triaxial compression) together with the failure points.

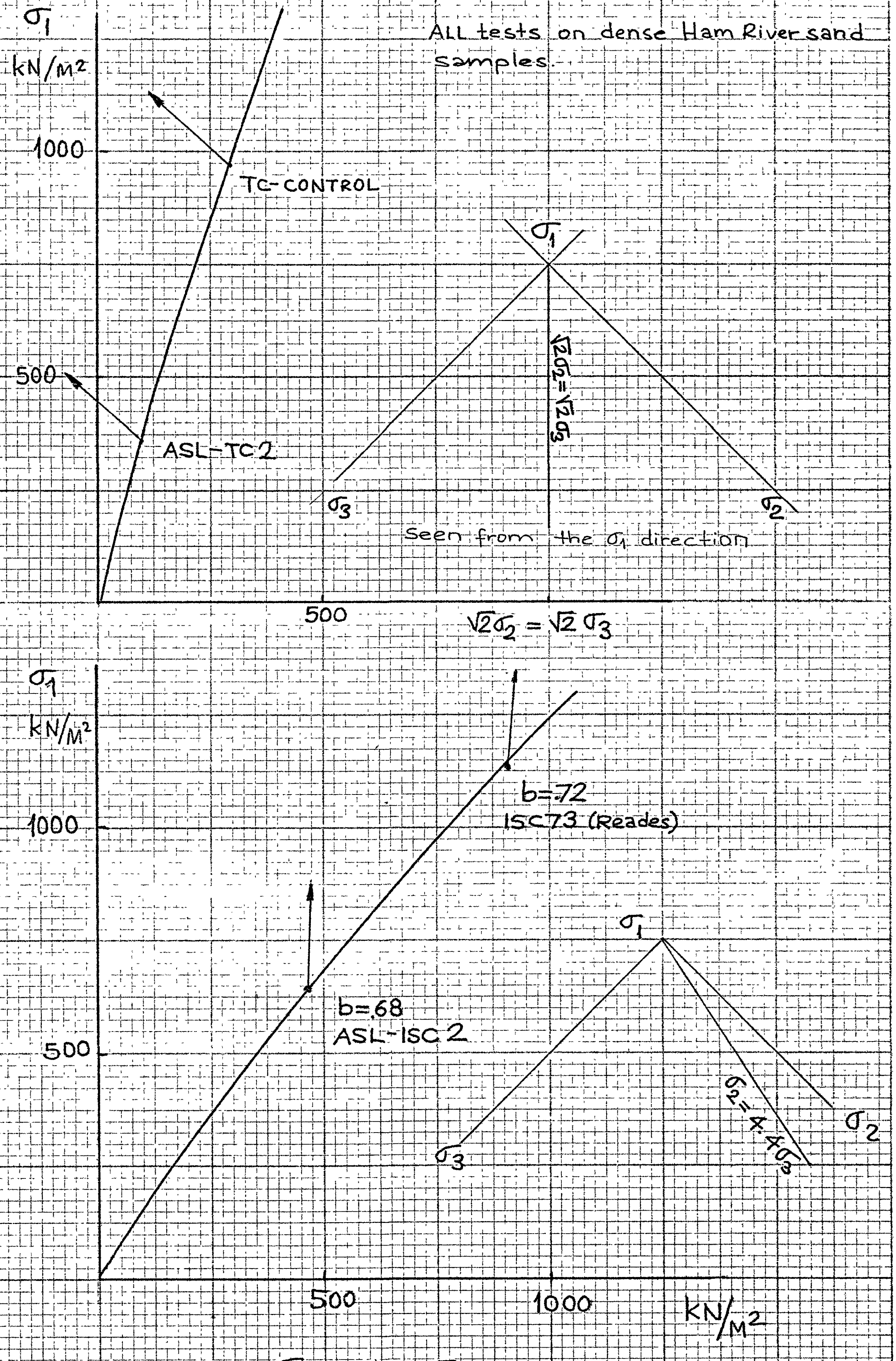
Note that the experimental failure locus (which is very close to the Locus obtained by Eq. 9.9) will be nearly perpendicular to the vectors at the tangents.

Fig. 9.17



Plastic strain increment (total) vectors for some dense and loose ISC tests on Volcanic samples are plotted in a 60°– sector of the octahedral plane (for σ_{oct} in triaxial compression).

Fig. 9.18



ALL tests on dense Ham River sand samples.

$\sigma_2 = 4.4\,\sigma_3$

Plastic strain increment vectors compared with the failure Loci in the triaxial and a generalised plane

Fig. 9.19

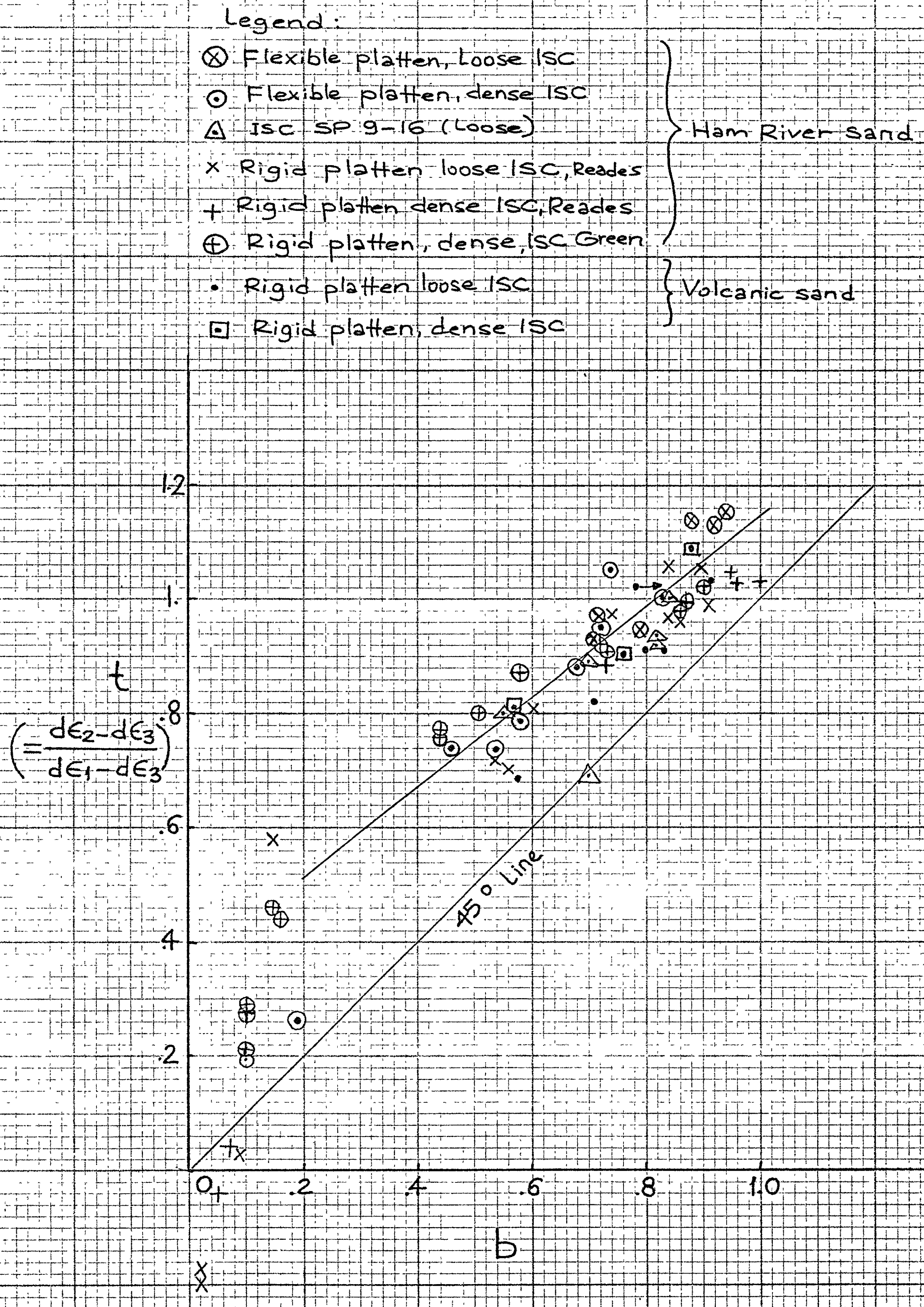

Legend:
⊗ Flexible platten, Loose ISC
⊙ Flexible platten, dense ISC
△ ISC SP 9-16 (Loose)
× Rigid platten loose ISC, Reades
+ Rigid platten dense ISC, Reades
⊕ Rigid platten, dense, ISC Green
Ham River sand
• Rigid platten loose ISC
⊡ Rigid platten, dense ISC
Volcanic sand
t
$\left(= \frac{d\epsilon_2 - d\epsilon_3}{d\epsilon_1 - d\epsilon_3}\right)$
1.2
1.
.8
.6
.4
.2
0
.2
.4
.6
.8
1.0
b
45° Line

Fig. 9.20

CHAPTER 10

CONCLUSION

10.1. Summary and Conclusions.

The major question in this study was the variation of strength in the three dimensional stress field, or more specifically, the effect of intermediate principal stress on the strength behaviour granular soils. It was a continuation of the research efforts by Green (1969) and Reades (1972) using the ISC apparatus. Together with rigid loading plattens in the intermediate stress direction, flexible - bag type - plattens were designed and employed. Tests on Ham River sand and a volcanic sand were performed, and the results were analysed. Results from other generalised soil testing apparatuses around the world were also examined and compared. The major conclusions reached are summarized briefly below.

Results of tests on Ham River sand using flexible plattens, applying constant boundary stress, gave similar ϕ' values to those obtained by Green and Reades with rigid plattens which apply uniform strain to the boundaries, this implies that the stress applied by the rigid plattens are relatively uniform at least between b = 0.0 - 0.80 because there were no tests conducted after that b value, using flexible plattens. But near extension (b=1) a decreasing ϕ' can be detected from the flexible tests near b = 0.70-0.80. The ϕ' values from flexible tests were a degree higher than rigid platten tests, on average, between b = 0.20 - 0.80. It may be inferred that flexible boundary loading leads to mobilization of higher ϕ' values presumably due to a somewhat different internal shearing mechanism. It must be remembered that so called flexible series of generalised tests in this study still used a pair of rigid (axial) plattens to apply the major principal stresses. Other researchers such as Lomize and Kryhazhanousky(1967), Lomize et.al. (1969) or Al-Ani (1975) obtained steep ϕ' increases over the b span in their apparatuses which used six flexible plattens all around in tests on dense sand samples. Sands used were usually of a standard type and were composed of rounded to sub-rounded particles. The first two and the latter obtained ϕ'-b relations which showed maxima at b=0.75 and b = 0.60 respectively.

Triaxial extension tests on dense samples yielded ϕ' values roughly in accord with the decreasing trend of ϕ' values near b=1.

Flexible platten tests on loose Ham River sand samples were at high b values, they were aimed at comparison with rigid platten tests in the same b range. While the ϕ' values in flexible platten tests were a degree higher at b = 0.70 they showed a sharp decrease (almost 4°) up to b = 1.0 in contrast to those from rigid platten tests which indicated a continious increase right up to b = 1.0.

Two series of special generalised tests on loose

Ham River sand samples, namely, SP1-8 and SP9-16 were aimed at clarification of the strength variation at high b values. Two series of triaxial extension tests were also performed.

It was concluded that the procedure for the application of loads to the sample affected the strength (ϕ') response. In other words, the shearing resistance obtained from the sample was directly related to the boundary constraints through which the loads were imposed. This effect should not be related to the type of plattens used (flexible vs. rigid). Note for example, the ϕ' values obtained in ISC(F)12, 13 and ISC-SP12, 16 all around b = 0.70 (figure 7.1). It was believed that this effect started at mid b values (0.50) and it is not only observed in ϕ' values but the failure planes, the strains, hence the stress-strain curves of the two groups of tests were significantly different i.e.(SP9-16 series versus all ISC series). This fact must have directly affected the results and hence the conclusions with regard to the shape of failure surfaces in the generalised stress field by researchers like Sutherland and Mesdary (1969) or Ramamurthy and Rawat (1973) who performed generalised tests in which these two loading methods were indiscriminantly used.

Another phenomenon was the interference of rigid plattens at very high b values. Flexible platten ISC tests, SP (1-8) series and other results (see Chapter 7) supported the view that the axial and belt pairs of rigid plattens which were driven towards each other at similar speeds at high b values interfered along the meeting edges, and presumably caused stress concentrations thus the applied stresses deviated from a uniform pressure distribution - usually assumed-on them. This restraint led to the registration of higher loads on the proving rings which amounted to 1^o-3^o in ϕ' between b = 0.75 and b = 1.0, and it was the cause of steep ϕ' increases on loose samples reported in Reades (1972).

Triaxial extension tests on loose samples which had approximately similar dimensions in shape (but not as far as the orientation of principal stresses were concerned) as ISC samples resulted in lower ϕ' values compared to ISC rigid platten tests in the main- first-mode, but flexible platten tests and SP1-8 series implied close ϕ' values at b = 1 to those in triaxial extension tests. SP(9-16) series of tests turn out to be triaxial extension tests at the limit of b= 1.0 so they naturally implied triaxial extension strengths at b = 1.0.

Correlation was better in short loose extension tests which were the main series. The sample dimensions were such that they were at similar ratios in the particular direction of principal stresses when compared with principal stress directions vs. sample dimensions in ISC samples. They yielded 1-2^o higher ϕ' values than longer samples (longer in the axial direction).

ISC tests with rigid plattens were performed on dense and loose volcanic sand samples covering almost the whole intermediate stress space. This material having high ϕ' value in triaxial compression and high compressibility was a very suitable material to test in the ISC apparatus after Ham River sand because it offered almost the maximum material variation as far as sands are concerned. A high ϕ' value in triaxial compression would also constitute a good chance to test the failure theories generally under discussion.

Variation of ϕ' with increasing intermediate principal stress was generally similar to that of Ham River sand. The increase over triaxial compression was bigger, reaching a maximum difference at about $b = 0.75$ (11° compared to 7°-8° in Ham River sand) in dense samples, figure 8.1. The variation in the case of loose samples was very similar in both materials; ϕ' values increased from triaxial compression to plain strain and then stayed essentially constant until $b = 0.50 - .60$ thereafter they showed an recognizable increase till $b = .80$. Due to bigger differences involved in variations in the case of volcanic sand, ϕ' decrease near $b=1$ was more pronounced for both densities.

An interesting observation was that while failure planes were always observed throughout the intermediate stress region they were not in the tests near the extension stage (say after b = .80- .85) nor the triaxial compression state.

Failure characteristics other than ϕ' generally showed similar variations in both materials. Rate of volume change with respect to the major principal strain was increased continuously from triaxial compression to b=1 in dense samples with the exception that no increase from triaxial compression to plain strain was observed in dense Ham River sand samples (this difference was insignificant in dense volcanic sand samples, too, relative to the amount of variation throughout the range).Loose samples showed an increase in $d\varepsilon v/d\varepsilon_1$ only after b = 0.5-0.60 in both materials.

Major principal strains were decreased from triaxial compression with increasing values of intermediate stress. Flexible platten tests indicated that the decrease was until about b = 0.50 thereafter it stayed constant until extension. Reades (1972) rigid platten tests, however showed noticable increase from mid-b values until b = 1.0. (This contrasts with Lade's (1972) rigid platten tests that did not show this later increase).

A failure criterion was developed (equation 9.9) involving all three principal stresses. It was able to predict, roughly, the variation of strength through increasing intermediate principal stress for granular materials (or more specifically sands because behaviour of other granular materials like rockfill still awaits investigation in the generalised field, but it is broadly expected to be similar). The parameter β can be taken as the value in triaxial compression and can be used for the whole intermediate range due to the approximately invariant values of β at a specified density. The criterion predicts a bigger increase of strength for a stronger material which was observed with the volcanic sand data.

As it has been pointed out earlier, this study was not oriented to the analysis of deformations and consequently no special tests were planned and directed towards the study of deformation characteristics. Besides b values during the tests were not constant hence the proportional loading was not realised. (This could be in itself a major complexity in the interpretation of strains).

Certain studies concerned with the pre-failure state were reviewed and it was seen that there was no universally acceptable, reasonably accurate, model yet to express and predict the material behaviour before failure presumably due to complex behaviour of granular material tested with different stress paths and the possibly coupling of spherical and deviatoric responses.

Plastic concepts can represent certain aspects of the behaviour, but not all. Non-associated flow rules must be employed when these are used. One interesting observation that can be made use of is that the projection of the total plastic strain increment vectors (at failure) showed normality to the failure loci on octahedral planes regardless of the density of granular material.

10.2. Further Comments.

It is believed that the different ϕ' variations with the change of intermediate principal stress reported by several researchers are partly explained by the procedure of imposing stresses and/or strains to the specimen. This is the dominating factor for the variation obtained at high intermediate stress state.

The type of the apparatus and type of boundary conditions are other important factors influencing the results. Therefore, given the identical material (say, a clean, standard sand) the various types of three dimensional apparatuses designed and being used in generalised stress-strain-strength research will yield different results depending on the conditions mentioned above. So, it is not believed that there is a unique behaviour of the material under generalised testing conditions in the laboratory and that a'good' generalised apparatus will disclose this behaviour. 'Good'here basically means that there are no direct errors coming from unsuccesful application of principal stresses and/or strains to the specimen nor from measurements.

Although it is felt that the use of rigid plattens in generalised apparatuses up to the mid-intermediate stress range is perfectly all right, general use of them is not favoured unless they are completely instrumented. The main issue in employing flexible plattens is that their manufacture and operation are difficult, and they are much less robust than rigid plattens. Preparation of soil samples require more elaboration.

Although the results coming from various apparatuses are not the same there are common points to most of them. First, the increase of ϕ' from triaxial compression to about the plain strain state is observed in all of them and in all states of density. But this increase seems to be a little larger in apparatuses which make use of six flexible plattens all around. Secondly, decreasing ϕ' values with increasing intermediate stress state at high intermediate stress states must be associated with the material behaviour. Because it is observed in most of the apparatuses and under various boundary conditions and loading methods. For cases in which this was not observed the reasons were explained in this study. The ϕ' increase after plain strain state is more noticable in dense specimens. Such an increase is associated with testing in the first (main) mode - not only in this study but in general-,and ϕ'-b relation attains a peak at about $b = 0.60 - 0.80$ before ϕ' values decrease until $b = 1.00$. This observation, in the Writer's opinion, is linked with the failure mechanism of the samples and the formation of failure planes which appear only until this peak. This suggests that between plain strain and $b = 0.60 - 0.80$ the failure mechanism in the specimens are similar despite increasing intermediate principal stresses.

When approaching the extension state the deviatoric stresses $(\sigma_1-\sigma_3)$ and $(\sigma_2-\sigma_3)$ are comparable to each other, and the former can not cause any single failure formation in I-III deviatoric direction anymore, and the sample fails in both directions without distinct failure planes.

Lower ϕ' obtained in generalised tests in the second mode and in triaxial extension tests compared to those in the first mode (not only ISC data but in general) lead to the question which is more relevant to the behaviour of a field element? Although it may be speculated about, it seems reasonable to favour a ϕ' variation between these two laboratory constraint limits.

Finally, it must be emphasized that Mohr-Coulomb failure criterion which is being used for most types of earthy materials is conservative for granular soils and that the influence of intermediate principal stress should be taken into account in calculations.

10.3. Recommendations for Future Research.

Considerable effort has already been spent on strength of the granular soil in generalised stress field through design of several three dimensional loading apparatuses, and a pretty clear idea about the strength behaviour of the material has been obtained. It is difficult to perform three dimensional model tests which would also be useful.

A type of test which is difficult to perform from the instrumentation point of view and has potentially a fundamental character is the following; (figure 10.1).

A large sand body is loaded in the three dimensional principal stress space. In the mass a cubical shape is specified initially by placing lead shots. Outside this zone, very small total stress cells are instrumented. (Experimentally speaking problems like the behaviour of embedded cells in the sand mass, large capacity loading scheme, x-ray equipment etc. make it unreasonable)

A relatively easier attempt (relative to the above consideration) would be to place a few tiny total pressure cells in the test samples which usually have the side dimensions 7-12 cm. Electrical wiring (reach) of the cells will present problems in this case. A very thin loose, wiring may have a negligible effect on the behaviour.

The above considerations boil down to the identical uncertainity of the present; stress distribution in the cubical sample. Does uniform strain distribution observed by x-ray-lead shot technique automatically imply that the stresses are uniform? The Writer believes that the stress distribution in cubical test samples must be studied experimentally. The discontinuity of stress distribution along the edges will be spread within the sample, but how? figure 10.2. Three dimensional non-linear finite element analyses may also be potentially useful in the study of this problem.

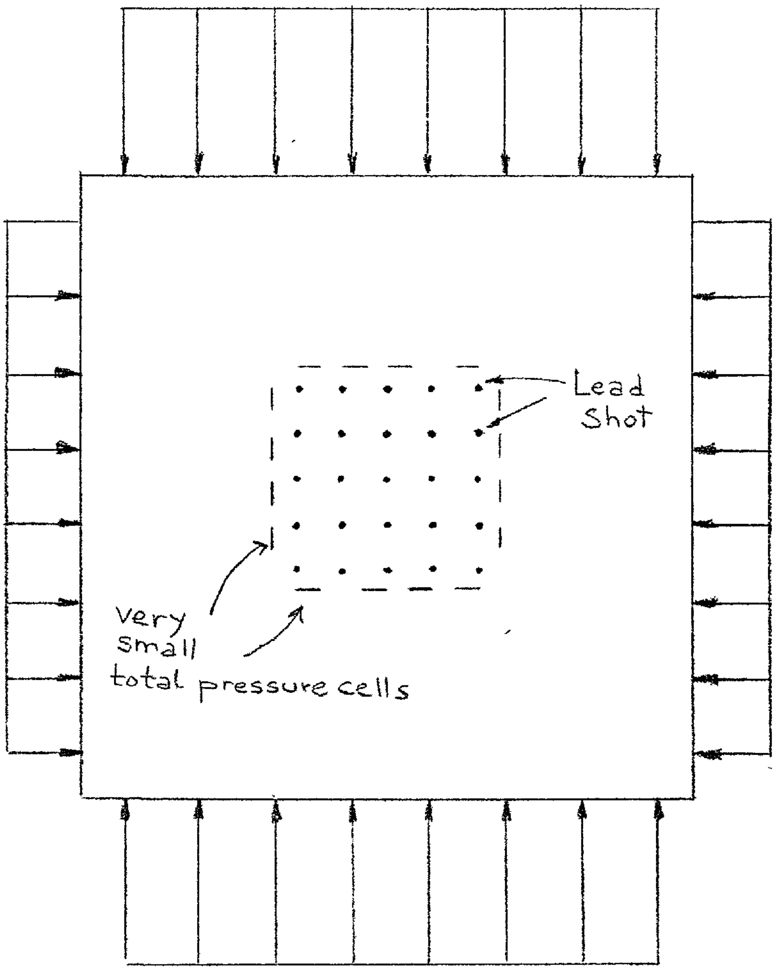
Before switching over to generalised stress-strain behaviour, effects of anisotropy and orientation of principal stresses on strength behaviour must be correctly assessed. These problems presumably are more significant in stress-strain behaviour.

The failure criteria must be in fact, modified due to structural anisotropy. This implies non-symmetrical failure surfaces in effective principal stress space. Al-Ani's (1975) finding of 2°-3° lower ϕ' values for samples sheared with major principal stress direction 90° with the direction of deposition - compared to 0° - shows the importance of the problem, because cohesionless field deposits will be more severely stratified, and if any cementation exists, the structural effect will be much more significant than that of deposition of clean sand.

It is felt that principal stress rotation is closely associated with the problem of induced anisotropy and study of it in three-dimensional testing may be experimentally difficult (especially cubical samples).

Instrumented field cases of cohesionless soils must be examined and compared with the predictions based on laboratory tests, but unfortunately the majority of instrumented field cases involve soft clays.

The effect of the intermediate stress in cohesive soils has been less subjected to research. Both at the pre-peak and failure levels it awaits research. Pore pressure response in the undrained case for various cohesive materials is an interesting subject to explore. "Samples" to be used in this case need more elaboration before shearing. Due to the limitations of time and financial support, an attempt on a three-dimensional testing programme of a cohesive material was abandoned in this study. (Special top and bottom plattens, pore pressure measuring system-probes etc. - were designed and prepared). It is hoped that this project will be undertaken by a future research student.



Plan or side view

Fig. 10.1

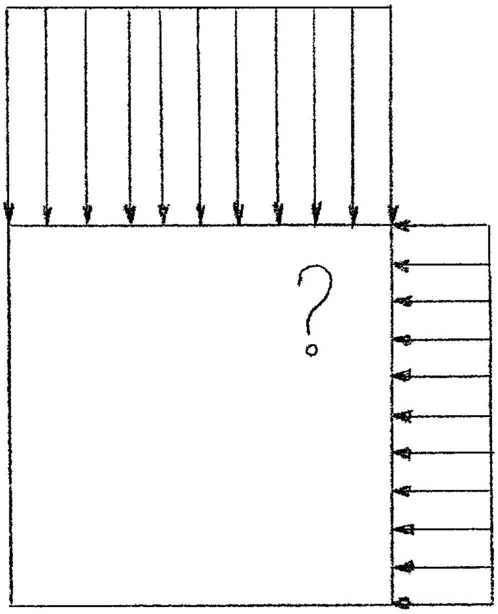


Fig. 10.2

APPENDIX 1

PRODUCTION OF REINFORCED RUBBER BAGS

A.1.1 General.

After the brass backing frame for flexible plattens was designed the shape of the required rubber bags was specified. They would be of rectangular shape and quite flat, Figure 3.6. Since the cell pressure was to be 207 kN/M^2 in the tests, and in the case of extension tests (especially for dense samples) the intermediate principal stresses could reach values of up to 1200 kN/M^2, the intermediate deviatoric stress could be up to 1000 kN/M^2. Naturally, ordinary rubber bags would not stand such high differential pressures. The Writer considered the reinforced rubber bag research at University College, London, and adapted the latest rubber bag construction technique used there. Menzies (1970), Menzies and Phillips (1972), Arthur (1973).

Menzies tried various forms of reinforced bags for his cubical triaxial machine and constructed a simulator to obsreve the behaviour of bags. Details of this bag study can be found in Menzies (1970) in the Appendix. He came out with a final technique which was as follows; first the shape of the required bag was cast using a rubber latex solution, this process will be summarized subsequently. After curing and obtaining the ordinary rubber bag, a reinforcing fabric (fine terylene mesh) was cast on the bag, and it was stuck to it using a special, easily workable, rubber adhesive. This was applied quickly with a fine glass rod. To obtain a well reinforced rubber bag considerable skill was required, especially when preparing the corner folds of the fabric. It was then left to dry for some time, and was dipped into the latex solution again to obtain a smoother surface. Menzies (1970) capped a very thin (half bag-shaped) membrane on top of the reinforced bag and inserted a free lubricated rubber sheet between this "slip" bag and the reinforced bag, and stuck the slip bag on to the reinforced bag along the sides. In the present study proper lubrication of both the sample sheath and well-finished very smooth bags was considered adequate. In the initial tests with flexible bags free ends were tried but they were abandoned later, because they creased and decreased the sensitivity of stress application on the surface of the bags.

A.1.2 Preparation of Rubber Bags.

Prevulcanised latex is commercially available. Latex solution was poured from the large commercial container into a container of suitable size which was able to accomodate the moulds when dipped to a reasonable depth and was left for one or two days covered and airtight. During this period any small air bubbles come to the surface. If this was not done these bubbles create holes in the bag.

Formers of the required shape were prepared allowing 6 - 7 percent for shrinkage of the bag upon drying. Formers may be made of PVC, perspex, dural, araldite, polished metals (except copper alloys) etc. Perspex formers were used in this study. Sharp, convex corners should be avoided, because these form local thin spots or lines (1 - 1.5 mm radii for rounded edges should be the minimum). On the other hand sharp internal corners will produce thick zones due to accumulation of latex. Formers must contain a handle for dipping.

Before starting the process, cleaning of the formers is very important. The writer was very strict about this point, and was successful in preparing membranes without defects. Formers were first treated with household cleaning powder, then soap and with industrial cleaning fluid "Decon", and cleaned surfaces should never be touched. After they were cleaned and dried the formers were coated with coagulant by dipping into a coagulant bath. The coagulant used was calcium nitrate dissolved in methanol. This is a chemical solution which, upon drying, leaves a sticky skin on the former so that when it is dipped into latex solution a rubber coat of certain thickness forms on it. The strength of the coagulant used is directly related to membrane thickness produced. After dipping and removing the former from the coagulant the excess amount was allowed to drain off, having a thin uniform surface. This may be assisted by rotating it around with the handle then it was left for drying.

It may be put into an oven at 60 - 70^{o}C for about 10 to 30 minutes to speed up the process. Excessive heating for a long time causes formation of some crystallized spots on the surface which may later cause weak spots in the rubber membrane. During this drying period methanol is driven off. The former was cooled and then dipped into the latex solution gently at a suitable angle, this was quite critical as it avoided the formation of air holes and resultant poor membranes.

The strength of the coagulant and the viscosity of the latex solution usually determine the required thickness produced for a certain dipping time. This was easily found by practicing the process a few times. This period (which is usually called as dwell time) was 30 seconds to 3 minutes. During this period latex reacted with $Ca(NO_3)_2$ film on the surface of the former. Then the former was removed slowly from the latex bath keeping a similar angle, and the excess latex was drained off. To prevent a local accumulation of latex at any point on the surface, the former was rotated gently with the handle.

It was then left drying for curing. This may be in air, at room temperature for a period, usually overnight or in an oven at 60 - 70^{o}C for 3 or more hours. After the membrane became transparent it was left to cool naturally, then immersed in water overnight or in warm water for 3 - 4 hours to leach out the soluble materials like ammonia. It was then taken out from the water bath, dried, and treated with talcum powder and stored away from light. For the latex supplied by the manufacturer (pH9), and coagulant available in the laboratory one minute dwell time corresponded

.53 - .56 mm (0.020" - 0.022") thickness. A few examples are given in table A.1.1.

A.1.3 Reinforcement of the Bags

If rubber bags formed by the dipping process described above had been used as σ_2 loading plattens in the triaxial cell, the maximum differential pressure that could be resisted across the water pressure in the bag and the cell pressure would be about 50 - 60 kN/M^2 and extensive ballooning problems would be faced as, for example, experienced by Dyson (1970).

As mentioned earlier the belt plattens are designed to travel in the ISC cell so for this study, there is no requirement for the bags themselves to cope with the deformations of the sample, therefore an intextensible bag would perform quite satisfactorily. As already pointed out in the introduction to this appendix the reinforcement method used at the University College, London, was followed. This consisted of applying a fine trylene mesh cloth fabric, rectangular in shape, to the clean surface of the rubber bag which was already cured and was on the mould. It was stuck on the rubber bag with fluid (rubber based) adhesive using a thin cylindrical glass rod. Special attention was paid to form perfect folds at the back corners of the bag and to obtain a very smooth surface without any dirt or accumulation spots. After the fabric was cast smoothly on the bag it was left to dry in an oven for 2 - 3 hours.

Because of the design of the brass backing plattens the geometry of the reinforcement was quite important. For example if the reinforcement had covered the whole back area of the bag then it would not be possible to take the mould off the reinforced bag through the small hole for the handle. It can be noticed that the outer O-ring had to press on a smooth bag surface so the shape finally used is seen in Figure 3.9.

Trial tests showed later on that such a bag was extremely strong against bursting. Within the range of the measurement system - 1100 kN/M^2 (155 psi) - no bursting was experienced the only problems were related to the leakage in the seal system rather than any problems with the bags. So the exact bursting pressure was not determined. Photographs of the moulds and the membranes can be seen (reinforced and unreinforced) in figures A.1.1 and A.1.2.

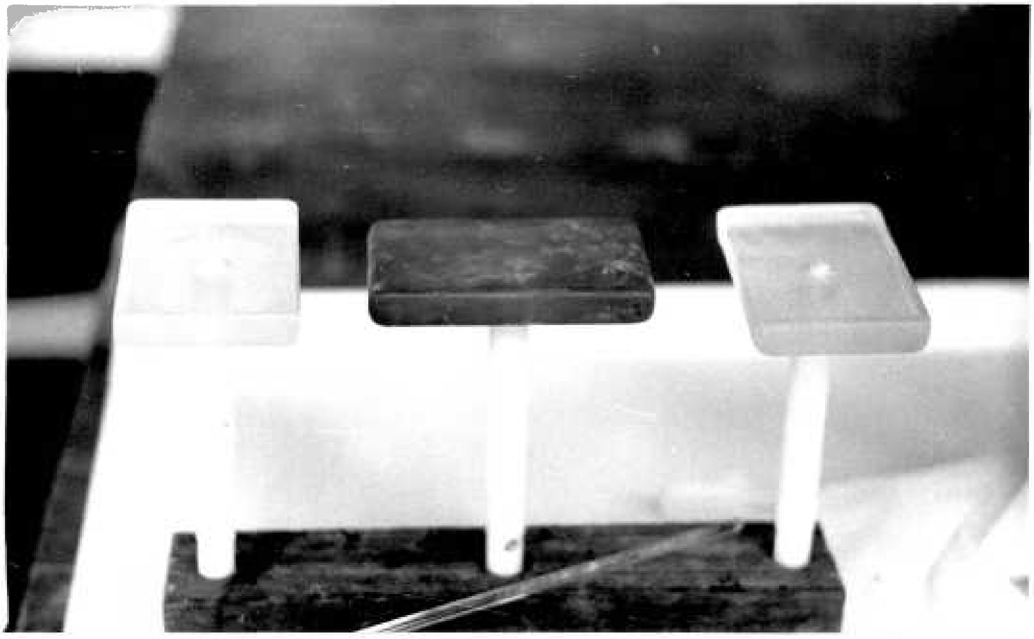
	MOULD 1	MOULD 2	MOULD 3
Dwell time (minutes)	2.0	1.0	1.0
Thickness mm (inches)	0.60 (.023)	.55/.56 (.021/.022)	.545 (.021)
* Shrinkage percent		7.52 – 5.88	7.09 – 6.37
Dwell time (minutes)	1.0	2.0	1.0
Thickness mm (in.)	.54/.55 (.021/.022)	.68 (.026/.027)	.53 (.020/.021)
Shrinkage percent	7.4 – 6.5		
Dwell time (minutes)	1.0	1.0	
Thickness mm (in.)	.50/.53 (.019/.020)	.54 (.020/.021)	
Shrinkage percent	6.55 – 5.73	7.84 – 7.05	

* The two figures given as shrinkage percentages correspond to different directions. (Note the "anisotropy" of the shrinkage). Mould dimensions were altered a few times due to this phenomenon.

Table A.1.1

A general view of some of the materials and equipment used to make reinforced rubber bags

Fig. A.1.1



A close-up view of the moulds

Fig. A.1.2

APPENDIX 2

CORRECTIONS FOR THE TESTS

A.2.1 General.

The main discussion is on the values of ϕ' in this study. There are certain factors which influence the ϕ' values and they must be taken into account. The rigidity of the sample sheath and friction between the belt plattens and the sample faces are two such factors which must be allowed for by corrections in the calculations. The strength behaviour of granular material, more specifically ϕ', is partly governed by the stress level. Since a constant cell pressure was used during the shear stage with increasing axial and belt stresses, tests with higher b values implied higher stress levels. If a comparison of the failure points on an octahedral plane in principal stress space is to be done, the mean stress level must be normalised in all tests. Correction may not be the right word for such a normalisation.

Another factor influencing the behaviour is the porosity. Various comparisons in analysing the results are only meaningful if the tests under discussion are at the same porosity therefore, porosities whether initial or consolidation must also be normalised.

Extension samples tend to neck near failure, this affects the shear test calculations because the usual assumption of right deforming prism is made. Each correction will be briefly described and examples will be given from the test data.

A.2.2 Sample Sheath Rigidity.

The force to stretch the rubber sample sheath is included in deviator load measured by the proving ring and it must be deducted. This force is a function of extension modulus of the rubber and amount of strain applied, Bishop and Henkel (1962). The amount of stress to be deducted from deviator stress is written as:

$$\sigma = \frac{L.M.\varepsilon}{A}$$

Where M is the extension modulus of the membrane in kN/M, L is the initial perimeter of the sample, ε is the axial strain at failure and A is the cross-sectional area of the sample at failure. Green's suggestion of M = 175t relation (M in lb/in, t, thickness of membrane in inches is useful if membranes of different thicknesses are to be used. It is written as M = 1206t in S.I units with M is in kN/M and t in meters. In table A.2.1 examples have been given from different kinds of tests.

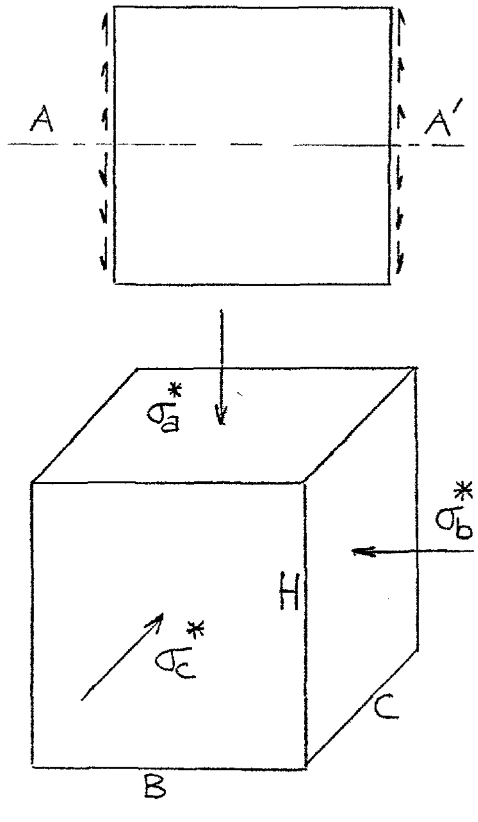
A.2.3 Platten Friction

Any friction that develops between the belt plattens and belt faces of the sample will directly cause an extra load registration in the axial proving ring (or in the SP series the friction will be generated on the axial plattens, and overregistration will occur in the belt platten loads.) In each case there will be an overregistration of ϕ'. The friction coefficient between the sample and the plattens controls the amount of correction to be applied. Green (1969), Reades (1972) give a list of investigators who studied the friction coefficient when using free ends and they also conducted some friction tests to determine it for their own apparatus. See Table A.2.2 for a brief summary. Green used a coefficient of 0.01 while Reades selected 0.015. Since the comparison of results were done mostly with the latter's data the value of 0.015 was also used in corrections for platten friction in this study. The same type of a calculation was followed for the same reason although other forms of calculation of the friction force could be done. The axial stress at central line (AA') was considered and the amount of axial stress due to friction to be subtracted from the axial stress was calculated as:

$$\mu \, \sigma_b^* \, \frac{H}{B}$$

assuming the shear stress distribution on the belt faces as shown in the figure. σ_a^* and σ_b^* are the effective stresses in the axial and belt directions. Since H/B ratio is approximately unity for ISC geometry, the correction is simply $\mu \, \sigma_b^*$.

(Note: $\sigma_b^* = \sigma_b - \sigma_c$, $\sigma_a^* = \sigma_a$)



On the other hand, if full mobilization of the frictional forces on the both plattens is assumed, the correction to be subtracted from the measured axial stress is:

$$\frac{2\mu\ \sigma_b^{*}.H.C}{B.C} = 2\mu\ \sigma_b^{*}\ \frac{H}{B} \simeq 2\mu\ \sigma_b^{*}$$

It is seen that this is twice the former correction which was used in the calculations. But as can be seen in Table A.2.1 the amount of correction applied is not significant.

One lubricated sheet was used on lubricated flexible bags for the initial series of flexible tests, then it was abandoned, and well finished bag surface and sample sheath were lubricated efficiently. See Chapter 5. Tests at relatively low b values showed that, in fact, using one lubricated sheet did not affect the results. But there was the question that this might not be the case at higher b values. A discussion in Chapter 5 indicate that the platten friction correction is not expected to be significantly different in rigid and flexible platten series. Although a check at this point should have been done by performing friction tests on flexible plattens, this would have been an extremely time consuming and elaborate procedure so the idea was abandoned.

A.2.4 Mean Stress Level Correction.

It has been long established that increasing stress levels cause ϕ' to decrease. This is mainly due to suppression of dilatancy and particle crushing at high stress levels. It was necessary to bring all tests to the same mean stress level for more correct comparisons since mean stress levels were not the same in the tests, it increased from triaxial compression up to generalised tests at $b = 1.00$ when it was almost doubled. Green (1969) underestimated this correction in his series of tests on dense samples. Reades (1972) introduced a more realistic correction which was appreciably higher than those by Green. They indicated the significance of this correction in Green and Reades (1974). A logical procedure was to take the mean stress level for the triaxial compression test as the reference stress level at each porosity and to modify the other tests accordingly. Since ϕ' values decreased with increasing stress level, the correction was an addition to the ϕ' obtained in generalised tests. Tests with higher b values required more correction.

A group of tests were conducted aimed at a better evaluation mean stress level effect both in triaxial compression and the generalised state. They are labelled the ASL series. They are given in Appendix 6. Plots of ϕ' versus σ_m are shown in figures A.2.1 and A.2.3 for Ham River and volcanic sand. It includes Writer's tests and tests by other researchers as indicated. It must be noted how important the mean stress level correction is when lower mean stress levels are considered, thus, for any generalised test under low stress levels the correction will be very large. For a better view of the effect higher pressure tests (figure A.2.2) by Skinner -

reported in Bishop (1966) - were plotted against low pressure tests. At 12000 kN/M^2 there is hardly any ϕ' difference between loose and dense samples. It is interesting to note that the effect of the stress level is much more pronounced for volcanic sand than for Ham River Sand.

It was assumed that the effect of mean stress level on the plain strain tests would be same for other generalised tests at the same porosity. This assumption was examined later on volcanic sand by performing ISC tests at different stress levels at around b = 0.87 and was seen to hold true. The need for such an assumption is apparent because it is not practically possible to carry out tests at different stress levels and at all b values. The effect was assumed similar for triaxial compression and extension tests and considered accordingly. The correction for loose and dense samples at low and high b values are given in the respective chapters.

In sP 9-16 series where the axial plattens were withdrawn to failure the corrections were similar to those in extension tests due to the similarities in ϕ'. For Dense Ham River samples at high b values the correction was around 1 - 1.5° and for loose ISC samples it was about 0.6°. Extension samples required smaller corrections, dense and loose samples about .80° and .3° respectively. Volcanic sand samples required corrections of 1°-5°. For a few extension tests corrections around 2° were applied.

A.2.4.1 Effect of Mean Stress Level on Other Failure Characteristics.

Stress level as could be expected not only influences ϕ' but most of the other failure characteristics as well. In figures A.2.4, 5, 6, 7, 8 some of the failure characteristics were plotted against mean normal stress for both types of material tested.

It showed the importance of keeping mean stress level constant if certain deformation properties were to be investigated on octahedral planes. The significance of such tests were in fact clear to the Writer at the beginning of the test program but for the compelling reason that the significant comparisons were to be made with Reades' (1972) data the present programme was adapted. Volcanic sand data indicates very significant changes with stress level. Effort was made in chapter 8 to take such changes into account when plotting the variables against b values.

A.2.5 Effect of Porosity on Failure Characteristics.

Like mean stress level porosity variations must be taken into account when comparing the tests. This also should not be taken as a correction but a normalisation. Some of the failure characteristics are plotted against the initial porosities at several b values. Figures A.2.9, 10, 11, 12, 13, 14, 15, 16. It is not only interesting to see the change of behaviour at various b values and porosities it is also of direct use for normalising the tests around a certain porosity. It is not possible to form samples at an exactly desired porosity.

A.2.6 Non-Uniformity Correction of Triaxial Extension Tests.

Many of the conflicting data concerning the triaxial extension test in soil testing literature stem from the uncertainties involved in this type of test. One of the most important points to be considered is the non-uniformity of the failing sample. Extension samples do deform non-uniformly, and looser samples suffer the most. The cross-sectional area of the samples vary considerably along the height near failure, and test calculations are based on the assumption that the samples deform uniformly as a right cylinder or prism. Actually the failing zone is the one which experiences larger strains and tries to form a neck and the strains concentrate in this part more and more as they increase. Therefore some kind of correction must be applied to the results unless special instrumentation is made to enable measurements of the deformations throughout the test along the sample height, - either surface or internal measurements - . This necessity was felt by many research workers in the past.

Roscoe, Schofield and Thurairajah (1963) drew attention to the non-uniformity of triaxial compression and especially extension tests of 1.5" x 3" diameter samples with rough ends. By measuring the boundary deformations they concluded that the classical area correction in triaxial extension test would lead to large underestimates of peak strengths. For example they increased $\frac{3(\sigma_1-\sigma_3)}{\sigma_1+\sigma_2+\sigma_3}$ ratio from 1.06 on the basis of conventional area correction method to 1.20 by measuring the neck area (12% increase). They reported that at 5% axial strain conventional area correction was 3.9% relative to the observed value of 10.9%, at 10% axial strain they were 7.6% and 28.9% respectively. In the Writer's opinion estimation of cross-sectional area well after peak is much more uncertain compared with that at small strains up to peak. Roscoe et al. (1963) in this respect seem to overestimate the correction by measuring the dimensions of a well-developed neck.

Barden and Khayatt (1966) recognised non-uniform deformation as the main source of error in triaxial extension test calculations. They presented two tests, a 4" x 4" inch sample with free ends and the other 4 x 8" rough ends, both dense. Area corrections based on conventional method and minimum diameter were 4.0%, 4.8% in the former and 3%, 7.5% in the latter respectively. These authors do not present any data on loose samples which are more severely affected. Attention must be paid to the direct comparisons of areas calculated on the assumption of uniformity (or any kind of non-uniformity) and areas obtained by measuring the sample directly. Unless measurements are taken by external methods (i.e. optical, electronic monitoring etc.), or if the sample can be reached directly (vacuum test etc.), the experimenter will normally take the deviator load off and release the cell pressure and even sometimes take the shear pin off, and only then will measure the sample. Sample dimensions will change under such an unloading cycle and so will not be the same as those of the deformed sample in the cell. Cornforth (1961), Reades (1972) being well aware of this fact presented few test data giving the changes of areas at each step. The latter especially drew attention to the difference in the expansion of samples between cylindrical and rectangular shapes (rectangular samples 60% more, double this amount if low cell pressure is used) after the release of the cell pressure.

Following Green (1969) and Reades (1972) the Writer has adapted a similar way to take account of the non-uniformity observed in his extension tests. The reason for that is two-fold. First, it seems to be the most reasonable way if there is no special instrumentation to measure the deformations. Secondly, the present work is an extension of the research efforts by the above mentioned authors. In many tests the material is the same, and therefore there is a good chance of comparing the findings in different tests. For the same reason platten friction correction was applied in the same way.

The method is basically to stop the test immediately after failure, to take the deviator load off, and keeping the required suction in the sample to release the cell pressure and measure the samples. Green (1969) and more comprehensively

Reades (1972) calculated a ϕ' difference based on average area within the neck and overall average area respectively and plot lines of this $\Delta\phi'$ versus the axial strain difference between points at the end of the test and at failure ($\varepsilon_{a_{stop}} - \varepsilon_{af}$). After detecting a sloping trend of points on this plot, a line is passed through any test point parallel to this slope to intersect the ordinate, (i.e. ($\varepsilon_{a_{stop}} - \varepsilon_{af}$)=0). The $\Delta\phi'$ reading on the ordinate will correspond to required correction at failure. Data are plotted in certain porosity groups. The main assumption is that the ratio of the areas of overall average to average neck will be the same at the peak and the strain at which the test is stopped. This seems to be a fair assumption especially if the test is stopped just after failure.

The Writer finds it more convenient and better in general to deal with the area ratios rather than ϕ' differences. The first series of Writer's short extension tests are not standard triaxial extension tests where the cell pressure (major p.p.) is increased at a constant rate after consolidation, the axial pressure is crudely constant. This implies that strain at which peak stress ratio is reached cannot be detected because of the continuous increase in the deviatoric load. So these tests were not stopped until a clear failure plane was observed. Therefore these large strains made it impossible to work out the corrections. The tests which were stopped reasonably close to peak have been measured and the ratio of overall average area to average area within the neck has been plotted against the difference ($\varepsilon a_{stop} - \varepsilon a_{failure}$), Figure A.2.17. Some of Reades' tests have also been included. Test points roughly show the increase in the ratio upon further straining after peak. Short samples are not more uniform than longer samples. Connections for loose samples are the order of 2^{o} - 2.5^{o} degrees which are 0.6^{o} - 0.7^{o} higher than the connections applied by Reades.

A.2.7. Bag Pressure Measurements

It can be noticed that measuring both the bag pressures and the load on the belt proving ring is a double check, but the load measurement of the proving ring was used in the calculations. Measurement of bag pressures was very useful during the preparation stage of the flexible plattens. Leaks could be easily detected.

Bag pressure measurements were compared with the belt proving ring load readings. They were slightly higher than the stress which was calculated dividing the load on the proving ring by the whole area of the belt face. This is expected because the bags, like the rigid plattens, do not cover the whole belt face area of the sample. If belt platten contact ratios in the axial and belt directions are taken into account, the two measurements closely agree.

Test No.	Initial Porosity n_i, %	b	$\sigma_a (\sigma_1)$ Measured kN/M^2	sheath correction kN/M^2	Platten friction kN/M^2	Correct. $\sigma_a(\sigma_1)$ kN/M^2	Measured ϕ'	Corr. ϕ'	$\sigma_b (\sigma_2)$ Measured kN/M^2	Sheath correct. kN/M^2	Platten friction kN/M^2	Correct. $\sigma_b(\sigma_2)$
ISC(F)2	38.3	.16	1133.0	1.9	2.1	1129.0	44.0	43.9	143.1	—	17.0	126.1
ISC(F)13	44.5	.71	978.7	1.9	8.3	968.5	39.8	39.6	770.0	0.7	14.7	754.6
ISC(F)20	38.4	.83	1202.4	0.7	15.5	1186.2	44.93	44.66	1031.9	0.6	18.0	1013.3
ISC 2	59.4	.21	1453.4	2.8	4.2	1446.4	48.66	48.56	482.2	—	18.7	463.5
ISC 5	57.8	.87	1603.5	2.1	18.4	1583.0	50.56	50.33	1430.3	1.3	21.0	1408.0
ISC 13	64.4	.82	1231.0	5.0	12.7	1213.3	45.4	45.1	1052.4	2.0	15.4	1035.0

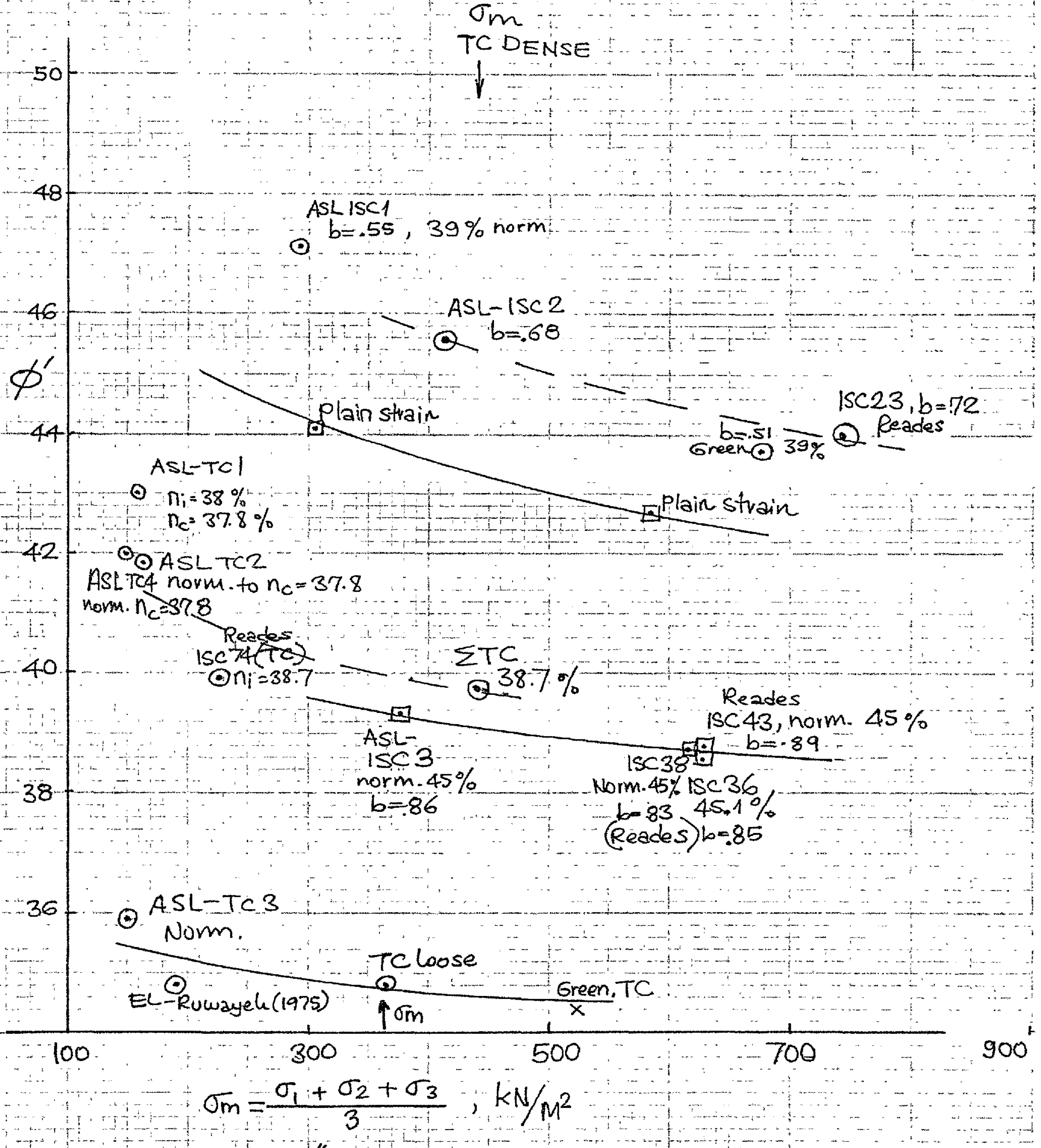
Test No.	Initial porosity n_i, %	$\sigma_1 - \sigma_3$ kN/M^2	S.sheath correct. kN/M^2	$\sigma_1 - \sigma_3$ correct. kN/M^2	σ_{1f}	σ_{3f} Corrected	ϕ' Measured	ϕ' correct.
Ex10	38.3	782.9	2.33	780.57	965.93	185.35	42.97	42.69
Ex11	43.86	610.4	3.0	607.4	833.2	225.8	35.3	35.0
SP10	44.4	473.8	4.94	468.9	764.0	192.2	37.3	36.7

Table A.2.1

Wood (1958) Shear Box was used. Silicone grease. 0.25-0.40 mm thick single membrane. Sandy-gravel material. Coefficient of friction "μ" was found 0.018.
Rowe and Barden (1964). Shear box. Silicone grease. Sand. μ values were reported to be less than one degree. (one degree corresponds to 0.017)
Green (1969). Friction slider technique. Silicone grease. Perspex and Ham rivers was used against a rubber sheet in the two different types of tests. While μ values were 0.006 and 0.007 at low pressures (200 kN/m^2) they were 0.009 and 0.018 at about 900 kN/m^2 respectively.
Dyson (1970). Shear box and friction slider methods were used. Two Lubricated (silicone) sheets. The former method (with metal backing) yielded $\mu = 0.010 - 0.030$ at 250 kN/m^2. The Latter (with sand) gave $\mu \cong 0.080 - 0.100$ at 80 kN/m^2

Values of "coefficient of friction" between two surfaces with lubricated sheets in between as measured by some researchers.

Table A.2.2



TC stands for "triaxial compression"

HAM RIVER SAND

Fig. A.2.1

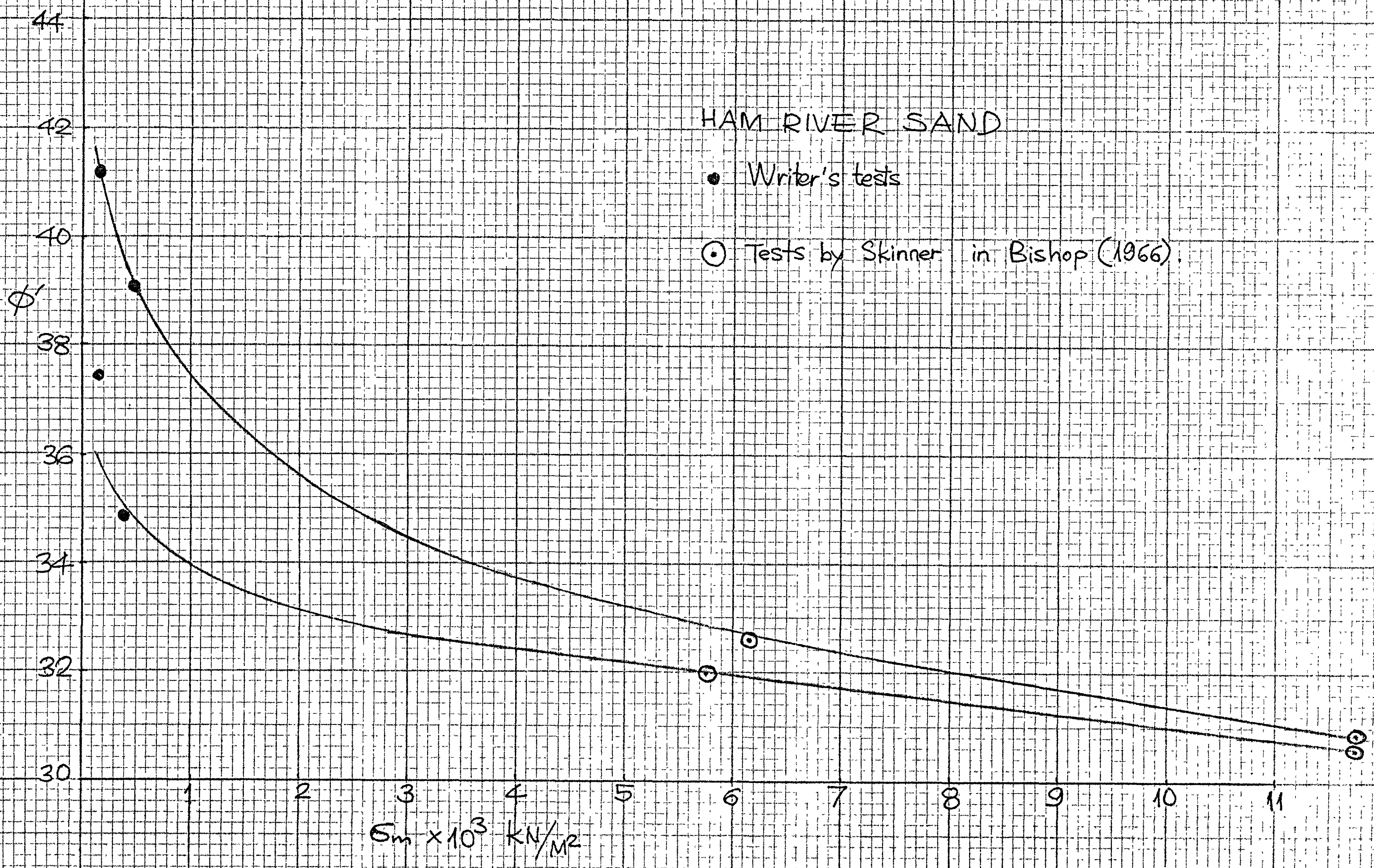
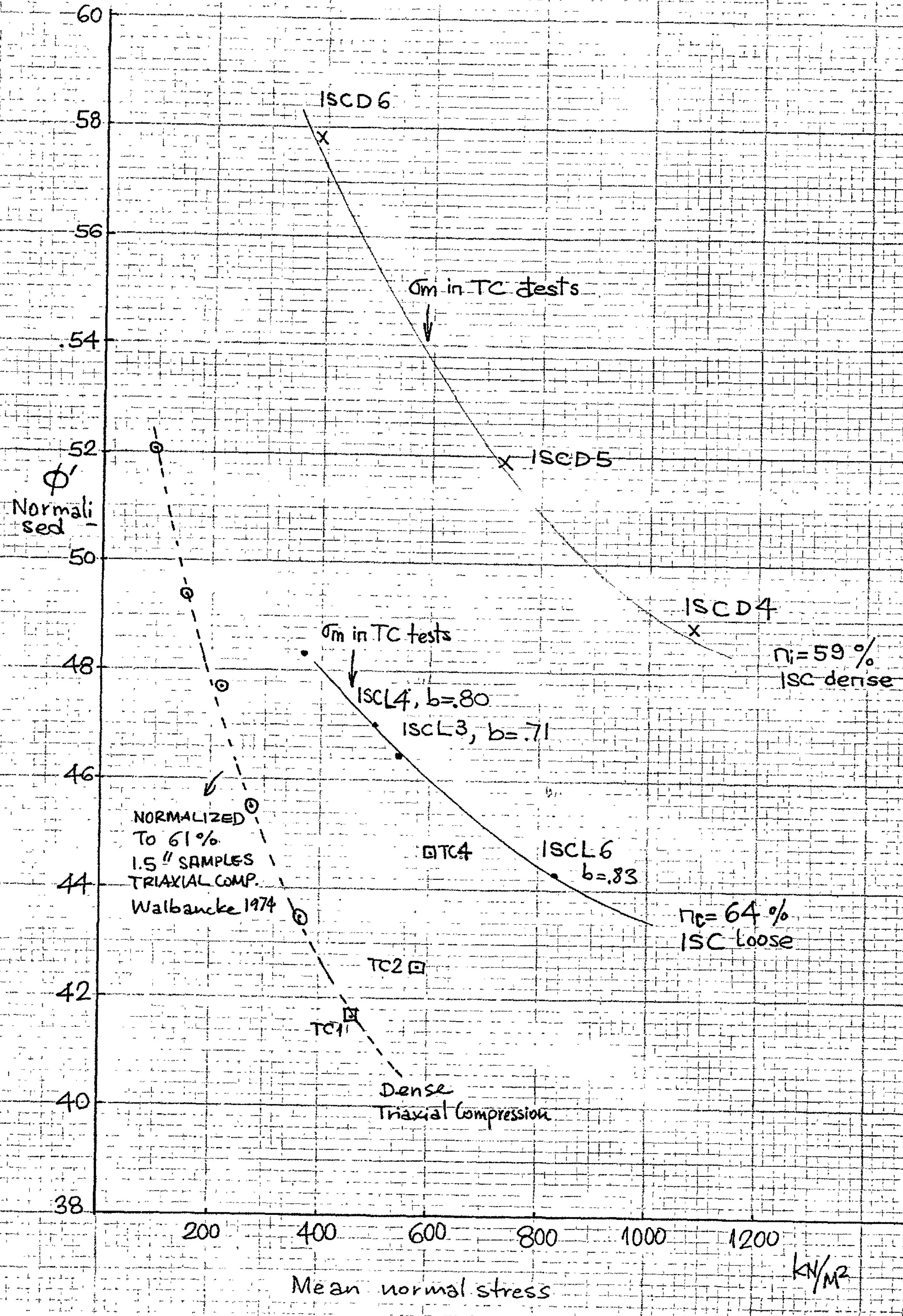
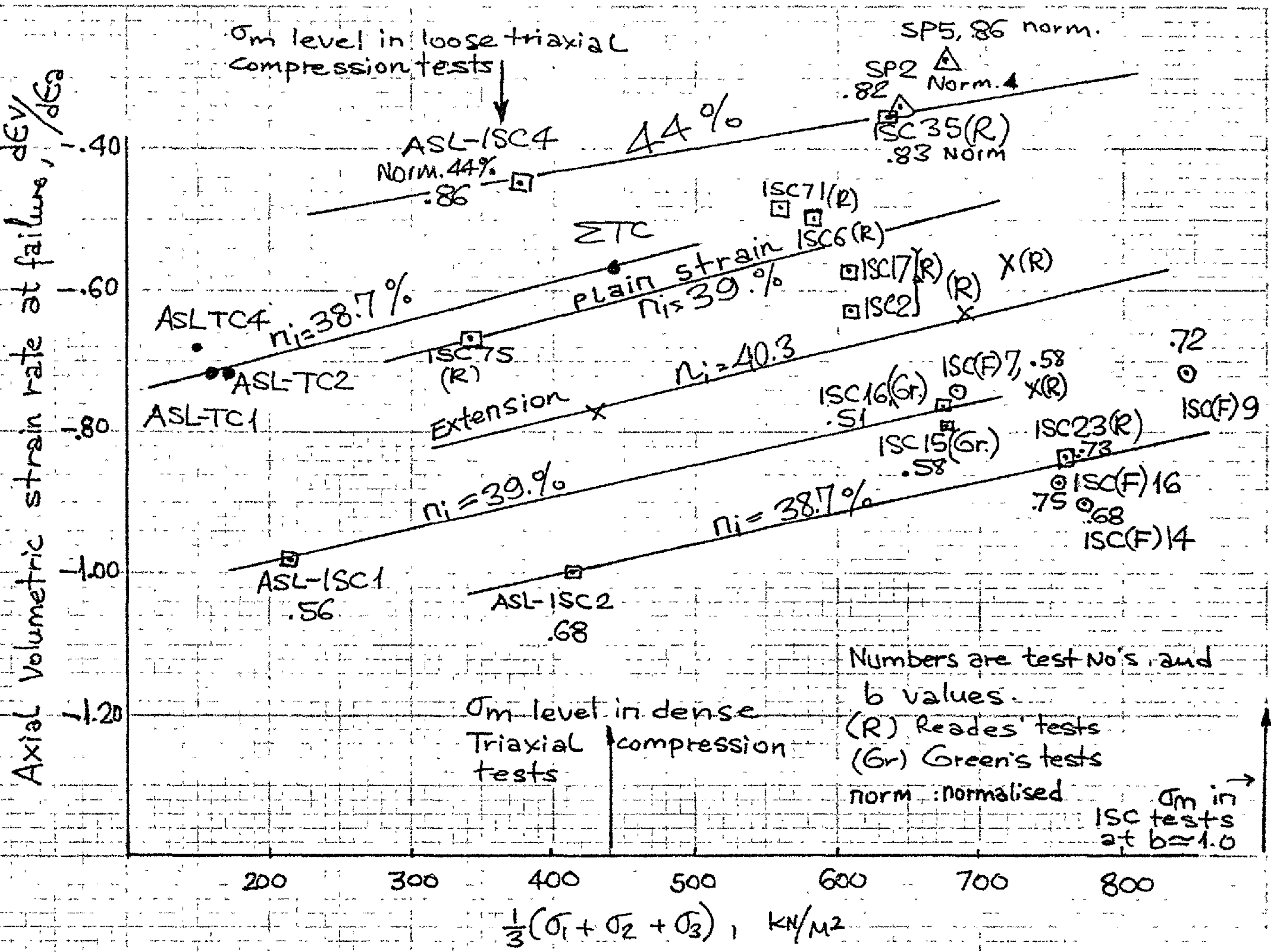

HAM RIVER SAND
● Writer's tests
⊙ Tests by Skinner in Bishop (1966).
Φ'
44
42
40
38
36
34
32
30
1
2
3
4
5
6
7
8
9
10
11
σm ×10³ KN/M²

Fig. A.2.2



VOLCANIC SAND

Fig.A.2.3



HAM RIVER SAND

Fig. A.2.4

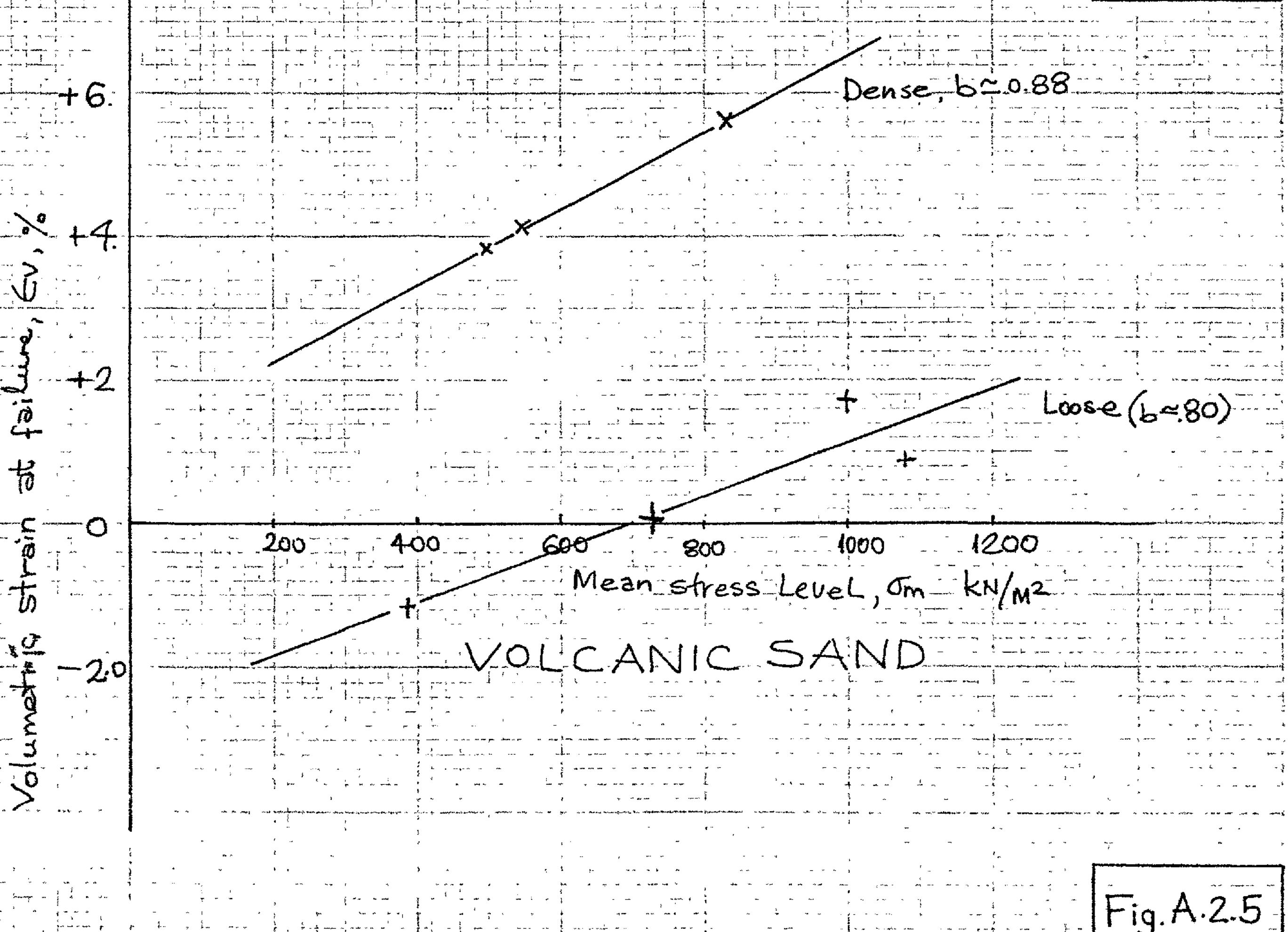
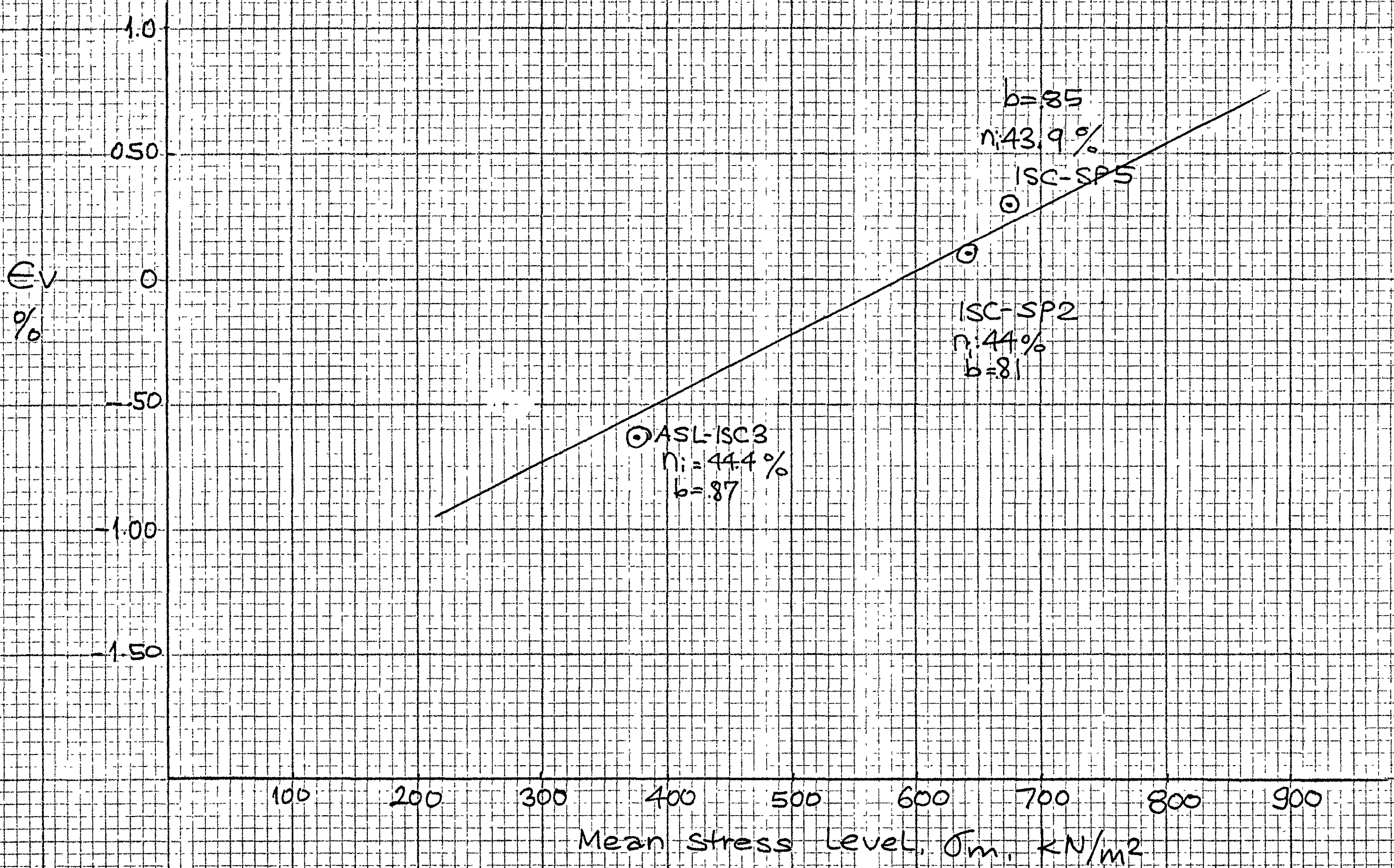
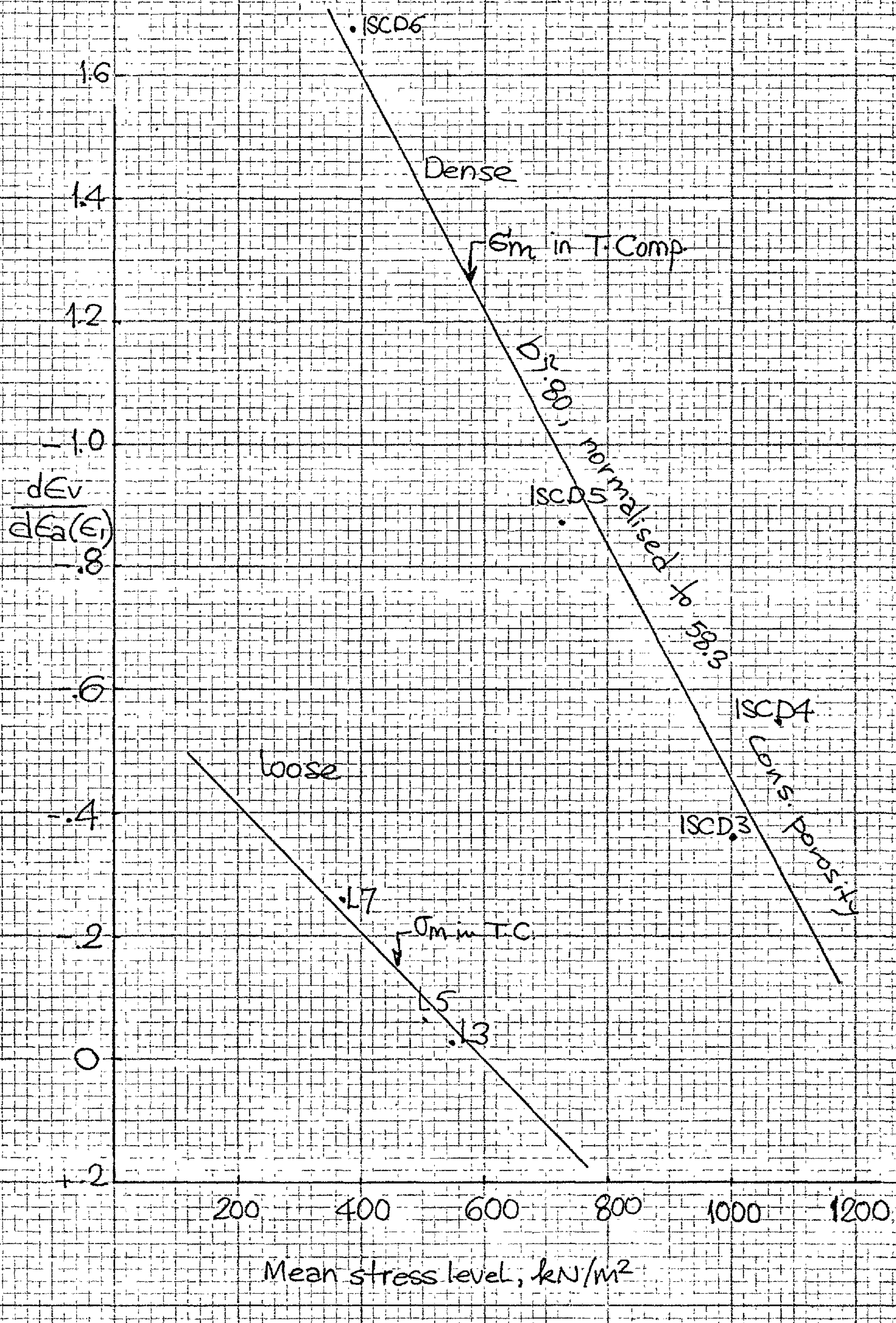


Fig. A.2.5



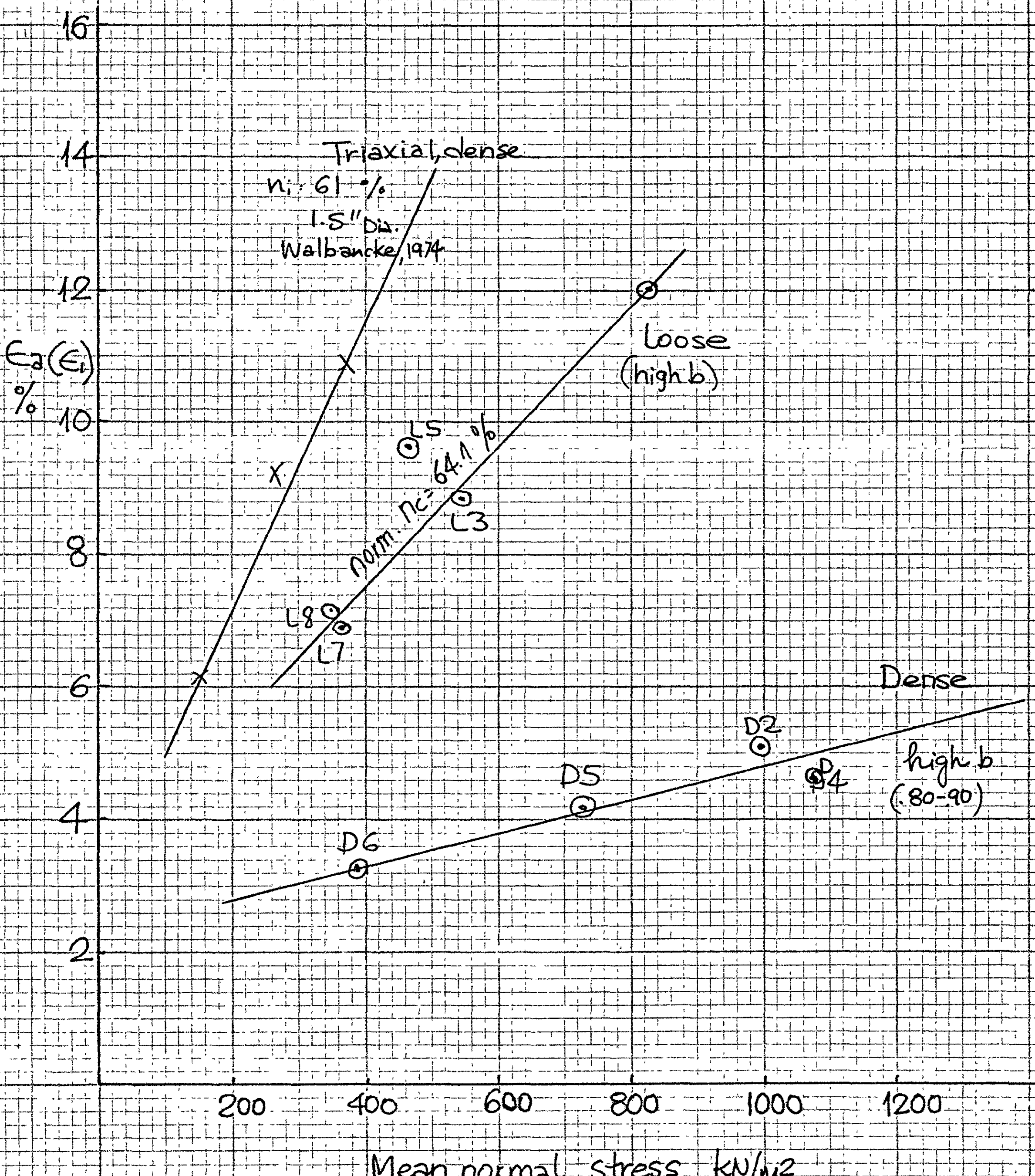
HAM RIVER SAND

Fig.A.2.6



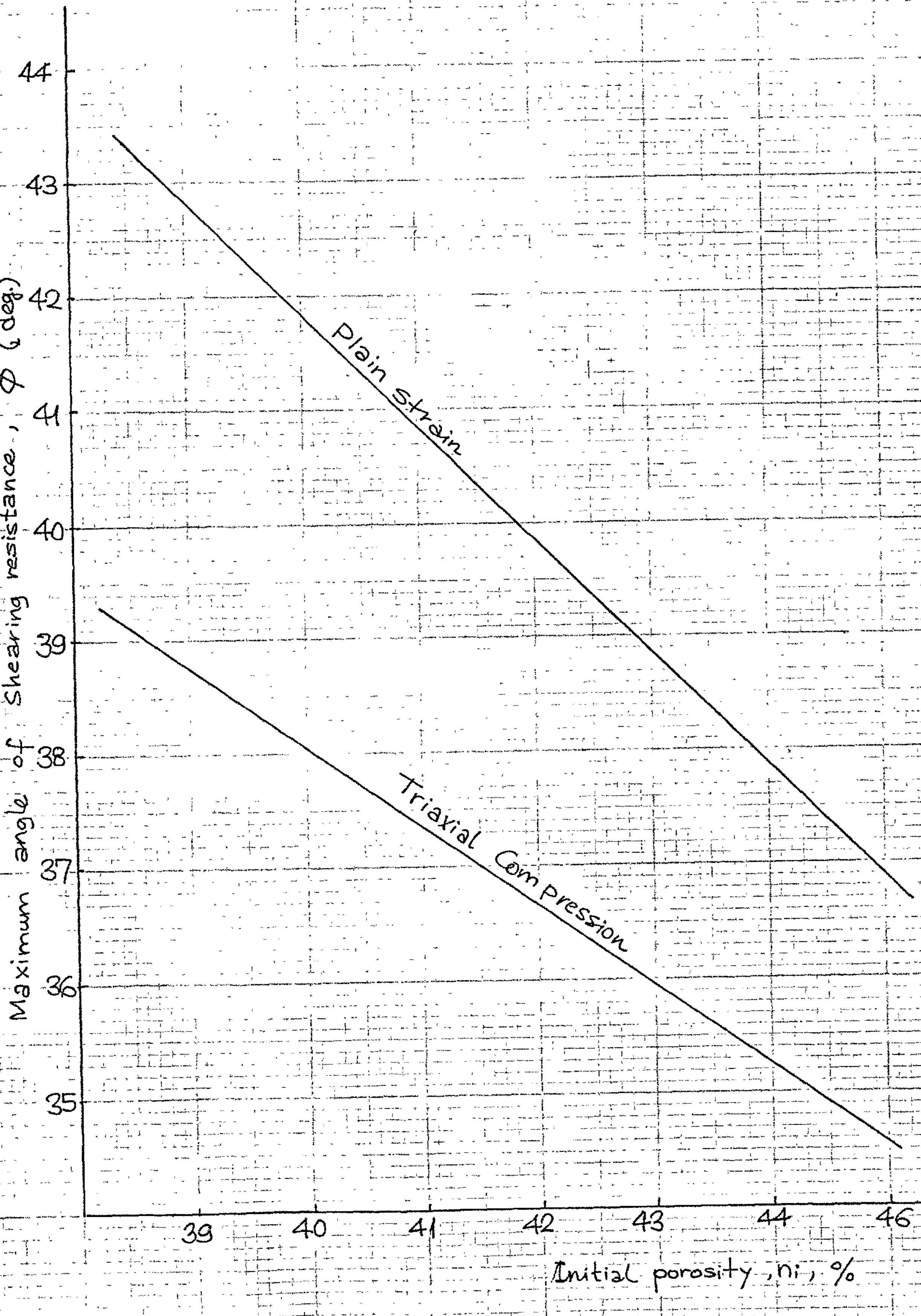
VOLCANIC SAND

Fig. A 27



VOLCANIC SAND

Fig. A.2.8



Peak strength – initial porosity relationships

HAM RIVER SAND

Fig. A.2.9

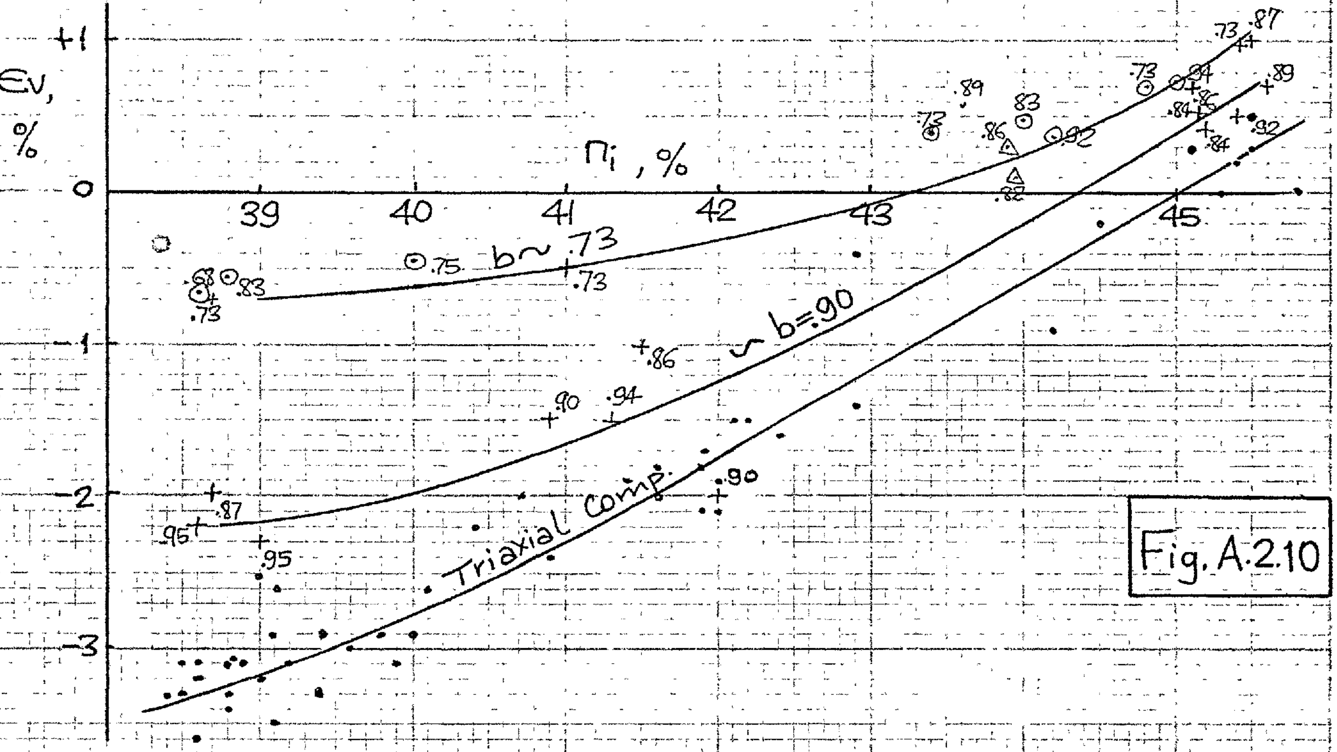
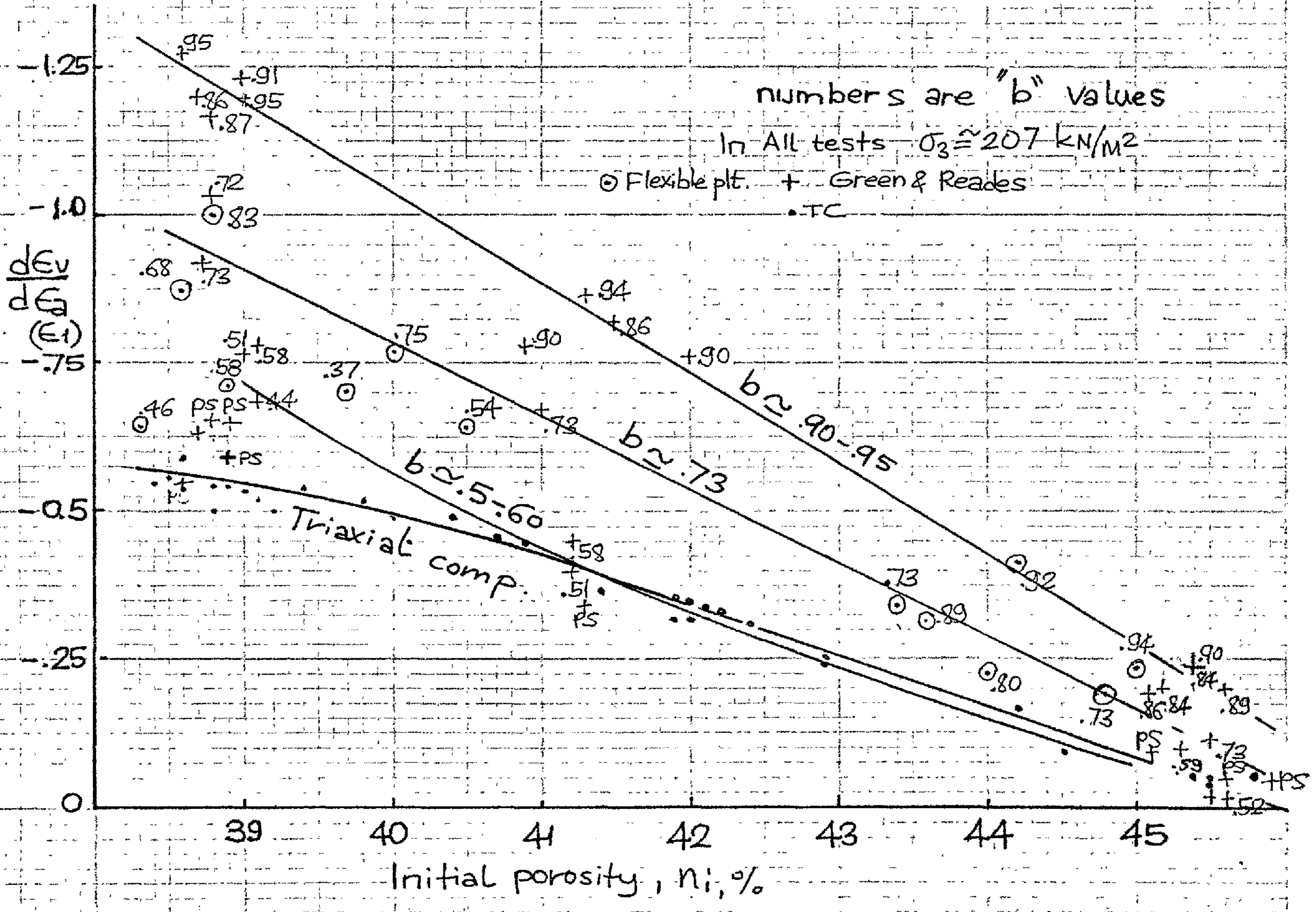
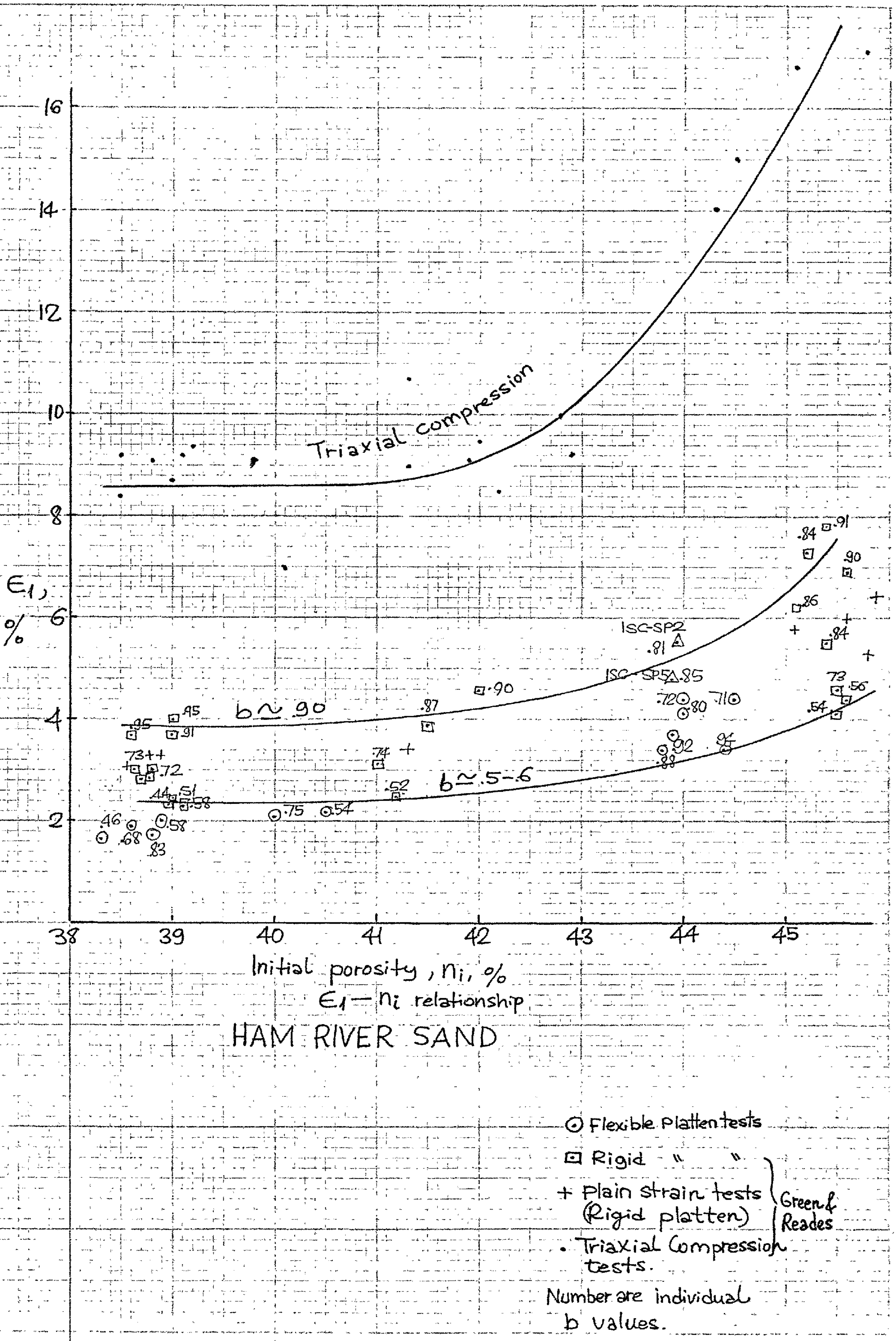


Fig. A.2.10



HAM RIVER SAND

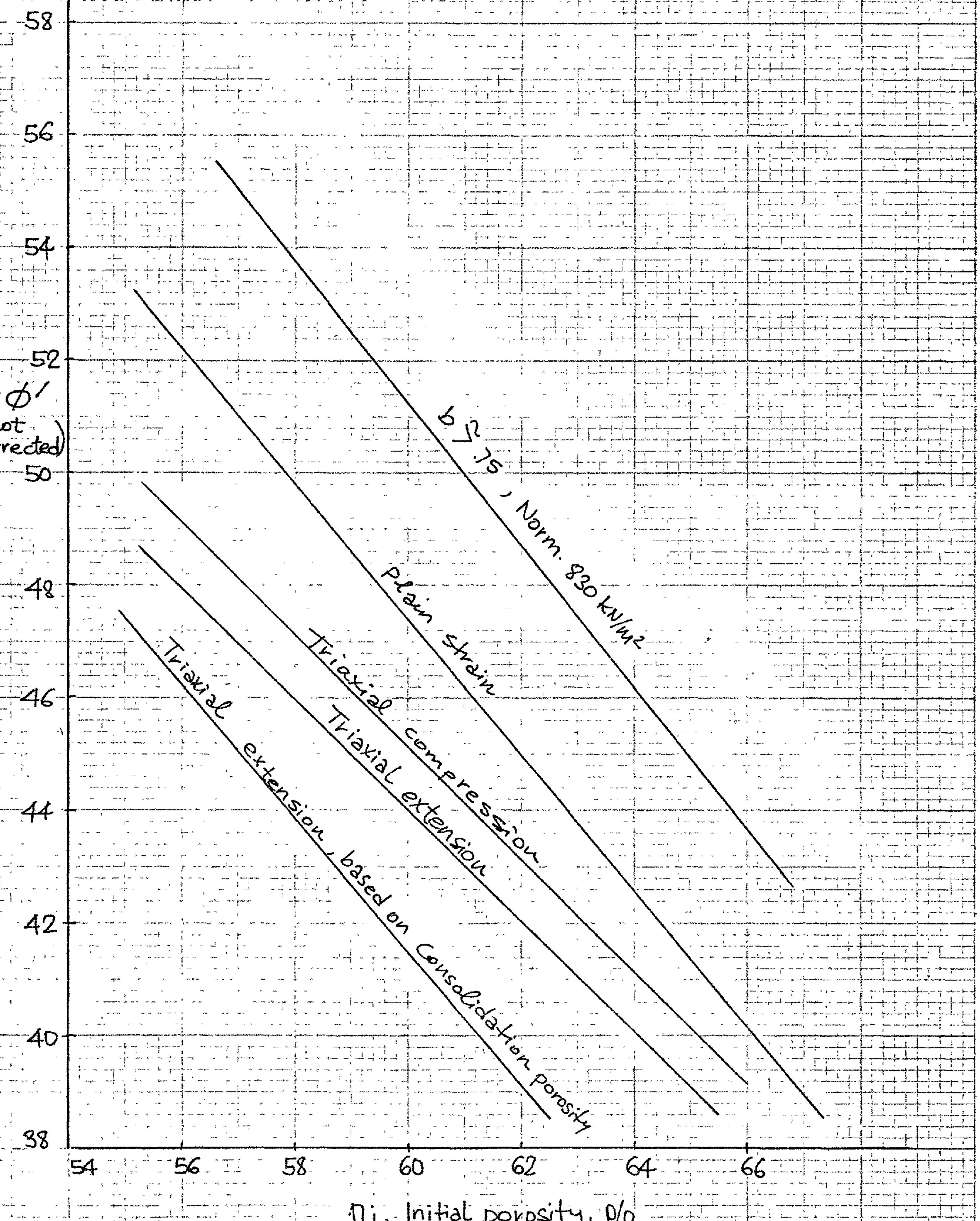
Fig. A.2.11



$\epsilon_1 - n_i$ relationship

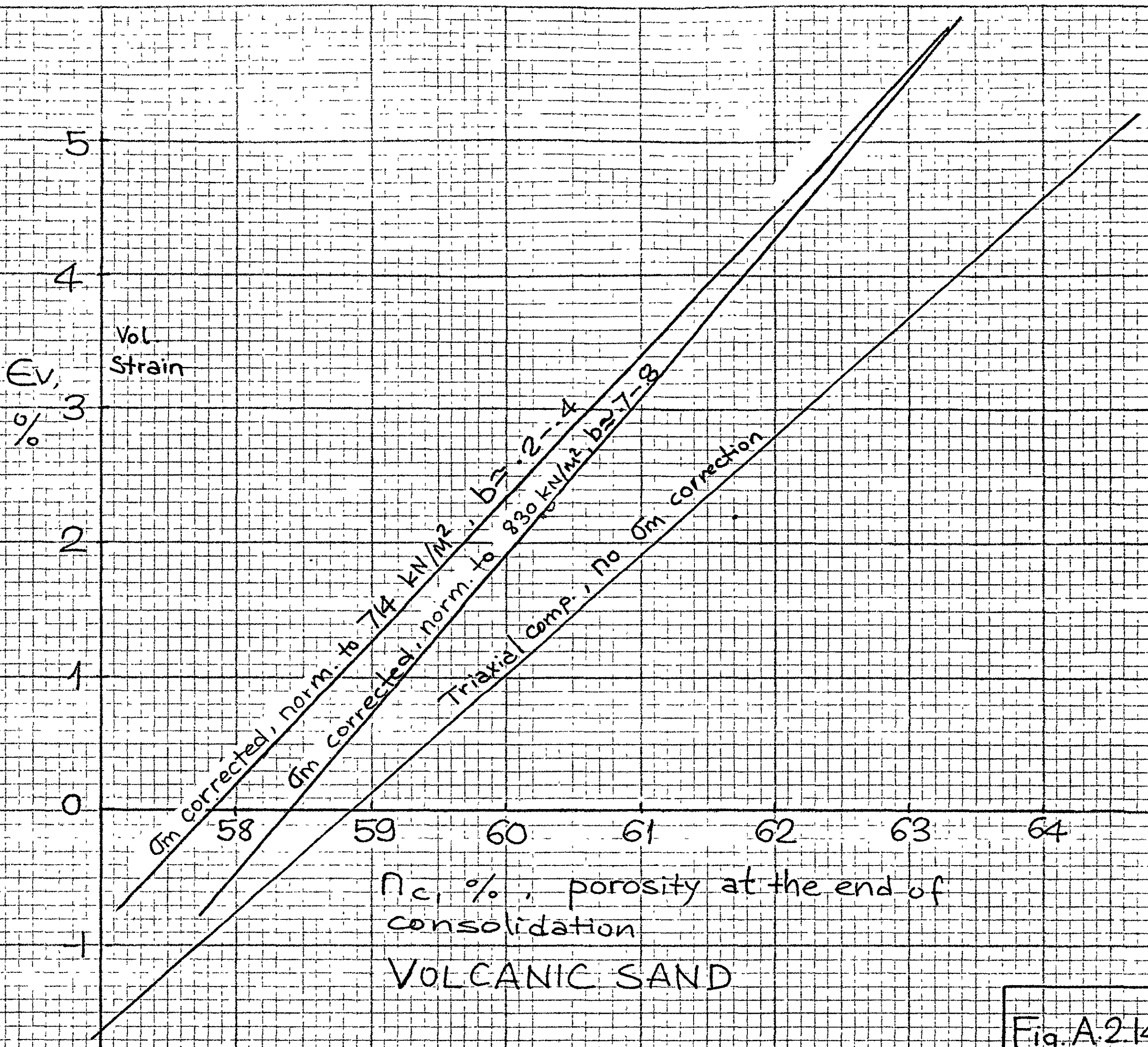
HAM RIVER SAND

Fig. A.2.12



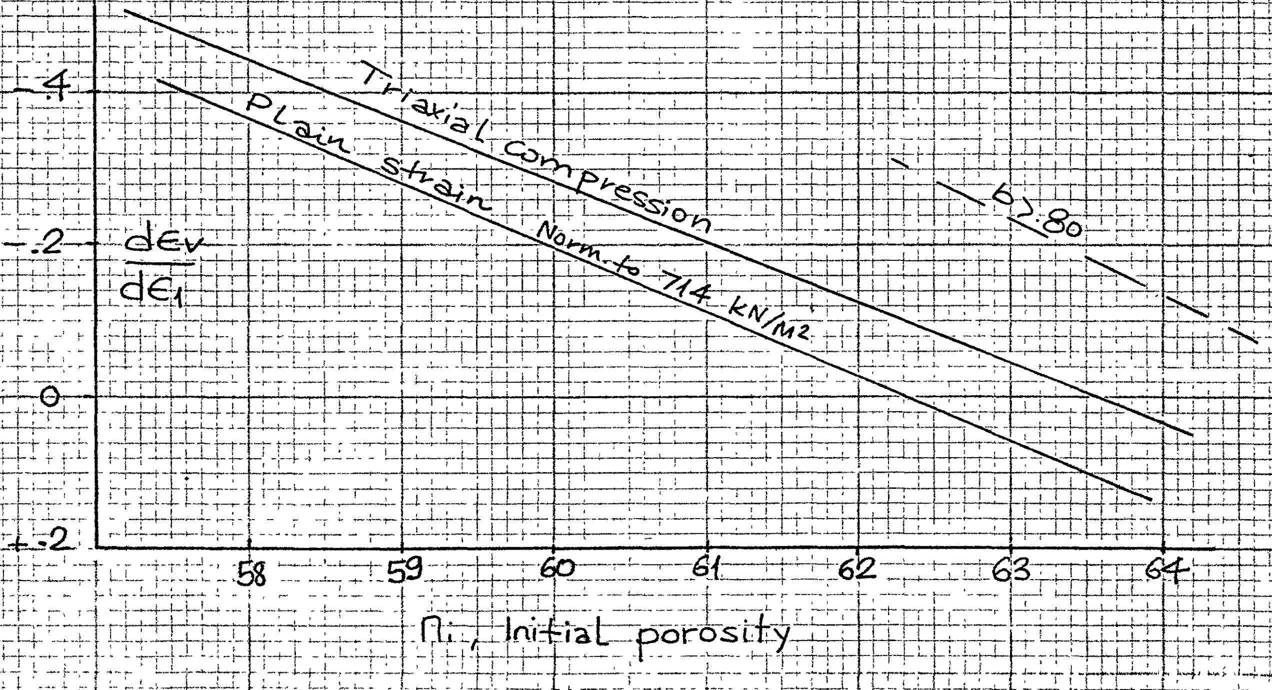
VOLCANIC SAND

Fig. A.2.13



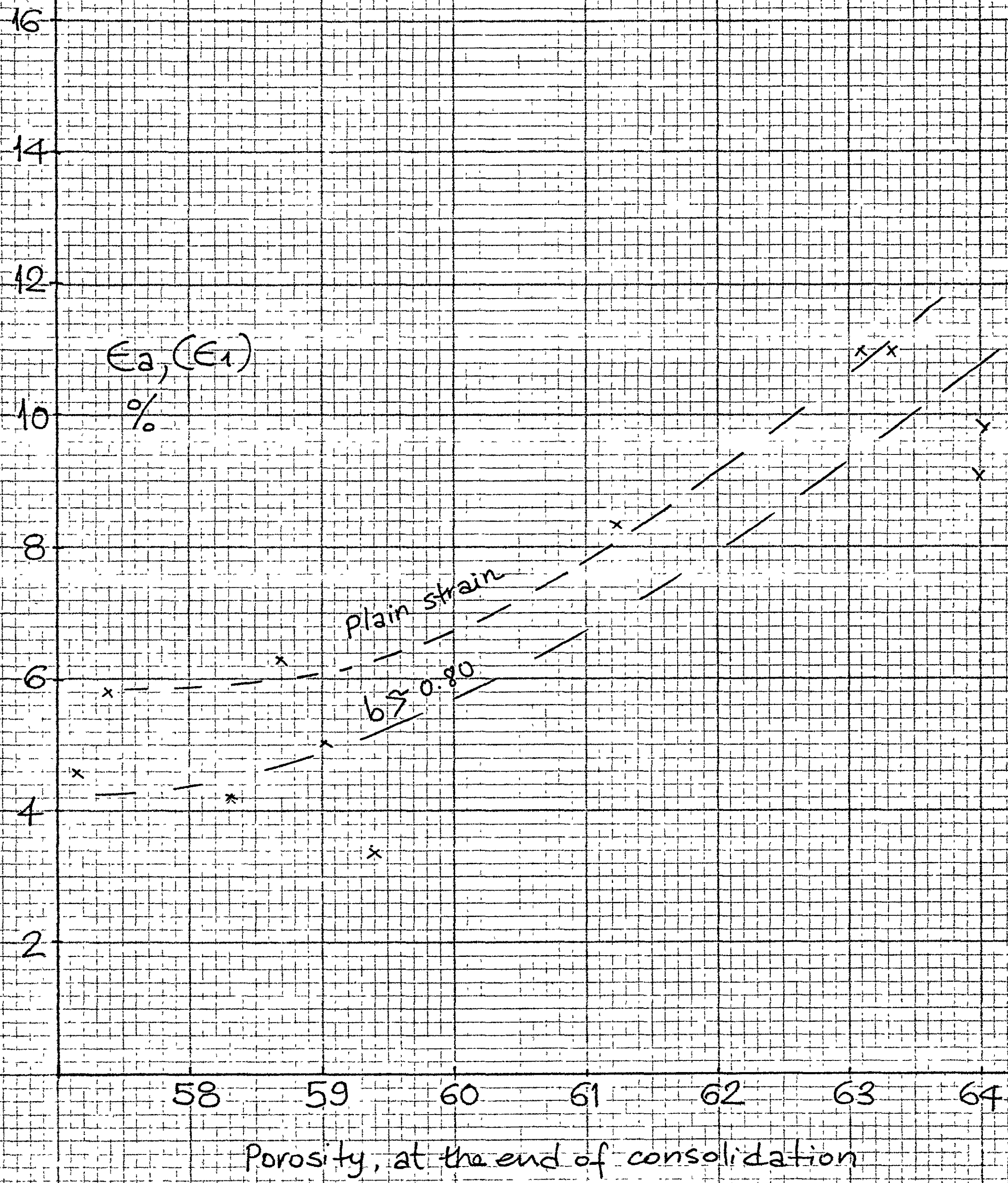
VOLCANIC SAND

Fig. A.2.14



VOLCANIC SAND

Fig. A.2.15



VOLCANIC SAND

Fig. A.2.16

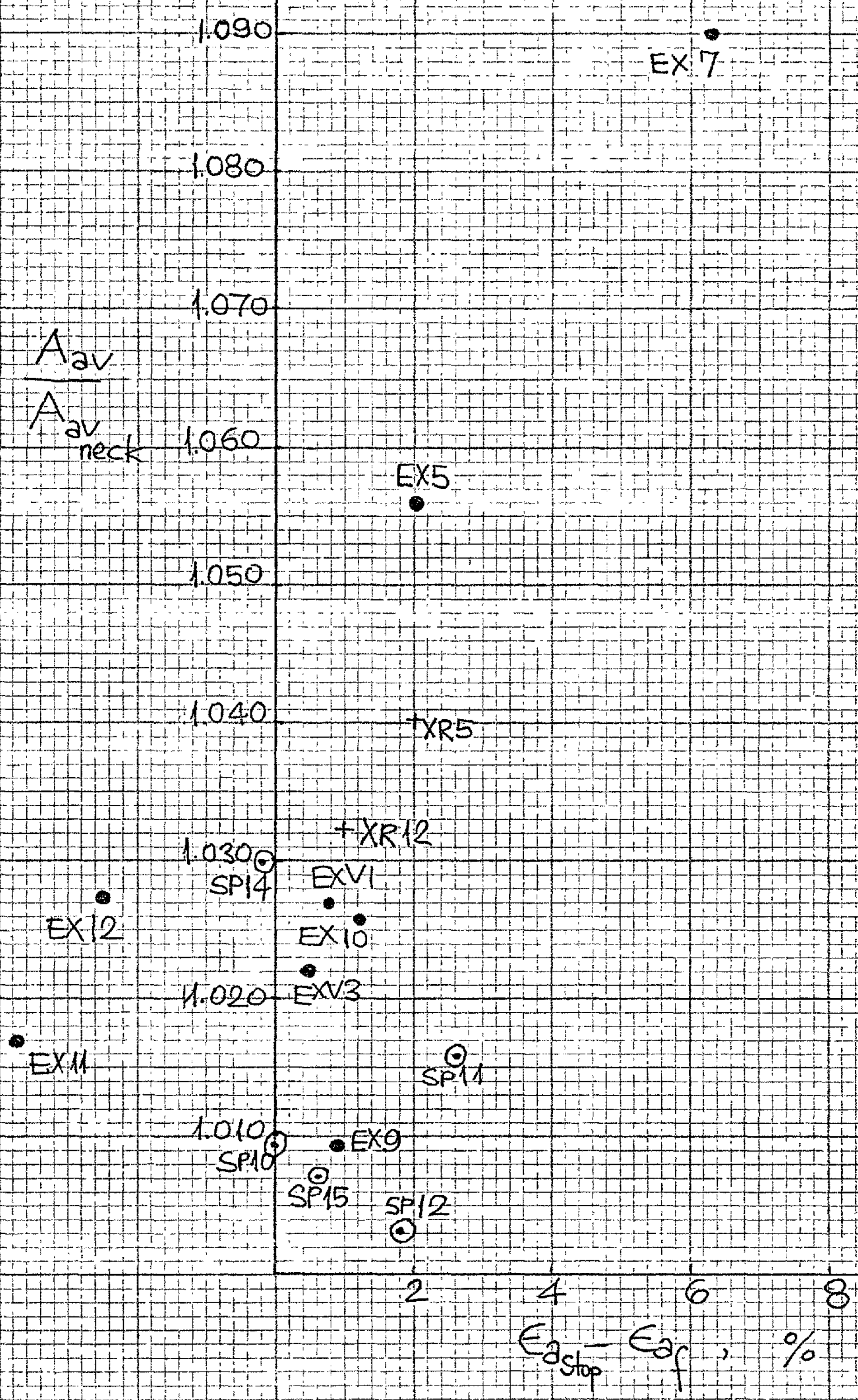
1.090
1.080
1.070
1.060
1.050
1.040
1.030
1.020
1.010
$\frac{A_{av}}{A_{av_{neck}}}$
EX 7
EX5
+XR5
+XR12
SP14
EXV1
EX12
EX10
EXV3
EX11
SP11
EX9
SP10
SP15
SP12
2
4
6
8
$\epsilon_{a_{stop}} - \epsilon_{a_f}$, %

Fig. A.2.17

APPENDIX 3

CALIBRATIONS

A.3.1 General.

Calibrations were always thought to be an essential part of the measurements. Therefore each series of tests contain calibrations for the transducers used in that series. In this section calibration methods will be described for different types of measurements. Description below has been kept short. It must be pointed out, however, the time spent on calibrations was enormous.

A.3.2 Calibration of Proving Rings.

The axial proving ring was calibrated by the calibrator which was designed at Imperial College Soil Mechanics Laboratory (see Harris (1966) and Green (1969) - details). It is simply a piston, machined very accurately to fit into a bronze bushing. The oil pressure under the piston is supplied by budenberg dead weight tester. The proving ring was placed between the crosshead of a triaxial frame and the top of the piston using steel balls.[†] The bronze bushing is rotated to prevent friction, and additionally, oil is circulated in the system.

Due to the weight of the piston a chart was prepared and the total load obtained on top of the piston was calculated

---

[†] A special frame has been designed for calibrations recently (Figure A.3.2).

using the piston area. (There was a modification in calibrator in Spring 1976.) Calibration graphs were plotted in the form of force per unit of electrical output versus electrical output for more flexible use and accuracy.

It is seen in Figure A.3.1 that there is a very good agreement among the calibrations throughout a long period of time. Since the calibrator was not designed for very low load levels, a dead load calibration was used for the series of tests in which the axial proving was loaded to a small proportion of its capacity. The dead weight loading procedure was more cumbersome and was up to 1500 - 1600 N. Dead load calibration curve is below the curve obtained by the calibrator. To give an idea of the accuracy an example can be given as follows with the exception of the 26.11.1973 curve (which was not used anyway). The variation in the readings at the usual loads for failure which were at 7000N - 9000N decade readings was 0.003N/div. so, 8000 Divs x 0.003N/Div = 24N. This corresponds to a total variation in ϕ' of 0.1° at 207 kN/M^2 cell pressure. Usually the accuracy is better than this value because it covers several calibrations carried out at different occasions.

In extension tests or generalised tests that required lower axial pressures than cell pressure the axial proving had to be loaded in tension. These series fell in between compressive loading series so that the care had to be taken. The following procedure was adapted. A week or two before the extension test series the axial proving ring was loaded in tension at specified time intervals (one or two days) to the expected level of tensile load or higher in the series and released. After performing the extension series of tests at similar time intervals calibration was done. Change to a compressive cycle was done in the same way. Good agreement of each calibration curve implies that the error due to a change in the calibrations is not significant. Reades (1972) adapted another method of changing the loading cycle from compressive to tensile. He kept the proving ring in tension at some specified load and then released it before an actual test. Whenever the axial proving ring was used in tension it measured the minor principal load in that specific test, and it was clear that ϕ' was influenced by the changes in the minor principal stress. The level of tensile axial load at failure was usually low compared to the compressive loading in generalised tests it was therefore decided to adapt dead load calibration. The load cell was required to be loaded up to 3800N (850 lb). The following system of calibration was used. The load cell was suspended from the large loading crane in the laboratory by a U-bar. A long, two storey hanger was screwed into the tapping in the load cell and loading increments of 450N (100 lb) were applied. Very good agreement between calibrations carried out at different times is obtained, Figure A.3.4. The calibrator extension calibration was appreciably higher than the dead load calibration. At a load of 2700N (600 lb) a difference of 45N (10 lb) exists between the calibration curves. ϕ' values could have been overestimated by up to a degree if the calibrator curve had been used.

Belt load cell was calibrated in exactly the same way as the axial load cell. Four tie bars and the load cell together were placed on top of the ram of the calibrator, Figure A.3.3. Again a very good agreement is obtained for the span of time involved, Figure A.3.5. Dead load calibration was also performed because there were certain tests in which the load level was low. Dead load compression calibrations were done using a frame and a hanger attached to it. The load cell was placed on the frame and the crosshead of the hanger was located on top of the load cell through a steel ball and loads were placed on the hanger. Usually 1400 - 1600N (300 - 350 lb) was the maximum permissible load.

A.3.3 Calibration of the Pressure Transducers and Pressure Gauges.

Pressure transducers - by Bell and Howell Ltd. 1100 kN/M^2 (150 psi) capacity - were calibrated with the budengberg dead weight tester. The oil lead from the tester was connected to the top of an oil-water interface. Water pressure at the bottom was fed to the transducers. First an atmospheric initial reading was taken, and all air bubbles in the lines were avoided. Pressure per unit electrical output versus electrical output was again the convenient way to plot the calibration curves, Figure A.3.6.

The two Bourdon gauges were calibrated using the budenberg dead weight tester and an oil-water interface.

The lower capacity gauge - 1100 kN/M^2 (160 psi) - was employed in conjunction with the cell water pressure. The difference between the budenberg and bourdon gauge pressures varied along with the level of the pressure. The maximum difference was about 11 kN/M^2 (1.6 psi). It was already known during the shearing state in constant cell pressure tests. In the tests with varying cell pressure, the pressure measurement was not done with the bourdon gauge but a pressure transducer at the cell base.

A.3.4 Compression of Free Ends.

Rubber sheets and the grease between them compress under loads and their deformation must be included in the calculation of axial deformation of samples. This is most important in the case of dense camples. One of the most reasonable and practical ways was to place the free ends on top and bottom of a dummy sample (preferably steel) and to load them when recording the deformations (using of course the same axial plattens as in the tests). The axial dial gauge readings include the compressed amount of free ends. Several tests were done to estimate the possible amount of compression of the sheets. They are plotted in Figures A.3.7 and A.3.8. The compression of lubricated sheets - one sheet on each platten - on the belt plattens were also simulated using dummy samples. They were done without using a cell pressure although they were compressed under a cell pressure during the actual tests. Reades (1972) did several calibrationsunder cell pressure and without cell pressure and found that the difference due to 207 kN/M^2 (30 psi) consolidation pressure was about .13 mm (.005 in). He also lightly greased the surface of the dummy sample and dipped it into sand forming a sand skin on the face between lubricated rubber sheets (free ends) and the dummy to be more representative. Due to the variation of the thickness of the membranes and amount of grease between the layers the compression curves were not identical, but the variation was reasonable. In conventional extension tests or generalised tests with axial load decreasing to failure the compression of the free ends would be somewhat different. Although the axial direction in this case would be the minor, and the minor principal strain was not as significant as the major, in principle they were approximately determined loading the greased rubber sheets on the dummy and then unloading (without using a cell pressure). Reades used the pressurised cell to simulate the tests. Both methods resulted in roughly the same relation. Similar considerations apply to the belt direction. One lubricated sheet was used on each rigid belt platten. Compression curves for them are seen in Figure A.3.9.

Flexible plattens were also tested for their compression characteristics. In the majority of the actual tests lubricated sheets were not used on them. Since the reinforced bags were very strong and inextensible the compliance of them was lower than expected. See Figure A.3.10 in which a steel dummy sample was used against the bags. The important point is that there is a difference between the test and the calibration. Due to somewhat rounded corners of the sample and self-positioning of the flexible plattens during the test, the gaps in some of the tests - the fraction of the surface of the bags directly in contact with cell pressure - were larger than those in the calibration tests with the dummy samples. Hence slight inflation of the bags around sample faces would cause a larger compression of the flexible plattens. In other words, calibration for the compliance of flexible plattens was an underestimate in some of the tests.

In conclusion the small variation of the measured quantities with time during calibrations indicates very good stability and the accuracy in load and pressure measurements in this study can be claimed to be high. External measurement of axial deformation has certain shortcomings like, for example, the true strain distribution along the sample height can not be detected. This requires techniques such as X-ray or roughly marking the sample surface at certain heights and observing them at set points of the test.

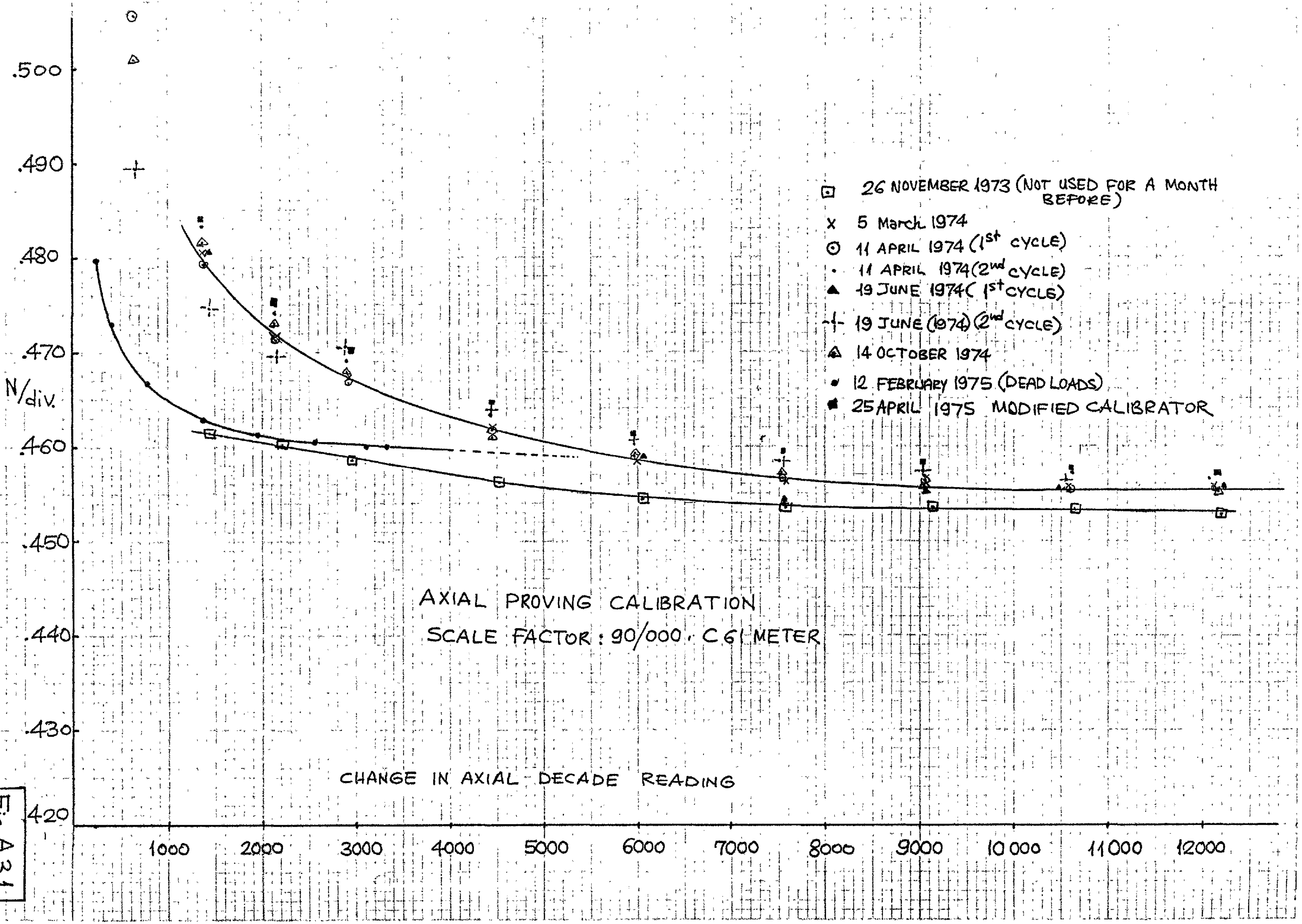

.500
.490
.480
.470
N/div.
.460
.450
.440
.430
.420
26 NOVEMBER 1973 (NOT USED FOR A MONTH BEFORE)
X 5 March 1974
⊙ 11 APRIL 1974 (1st CYCLE)
· 11 APRIL 1974 (2nd CYCLE)
▲ 19 JUNE 1974 (1st CYCLE)
-+- 19 JUNE (1974) (2nd CYCLE)
△ 14 OCTOBER 1974
• 12 FEBRUARY 1975 (DEAD LOADS)
■ 25 APRIL 1975 MODIFIED CALIBRATOR
AXIAL PROVING CALIBRATION
SCALE FACTOR: 90/000 . C61 METER
CHANGE IN AXIAL DECADE READING
1000
2000
3000
4000
5000
6000
7000
8000
9000
10 000
11000
12000

Fig. A.3.1

Calibration of the axial proving ring with the calibrator

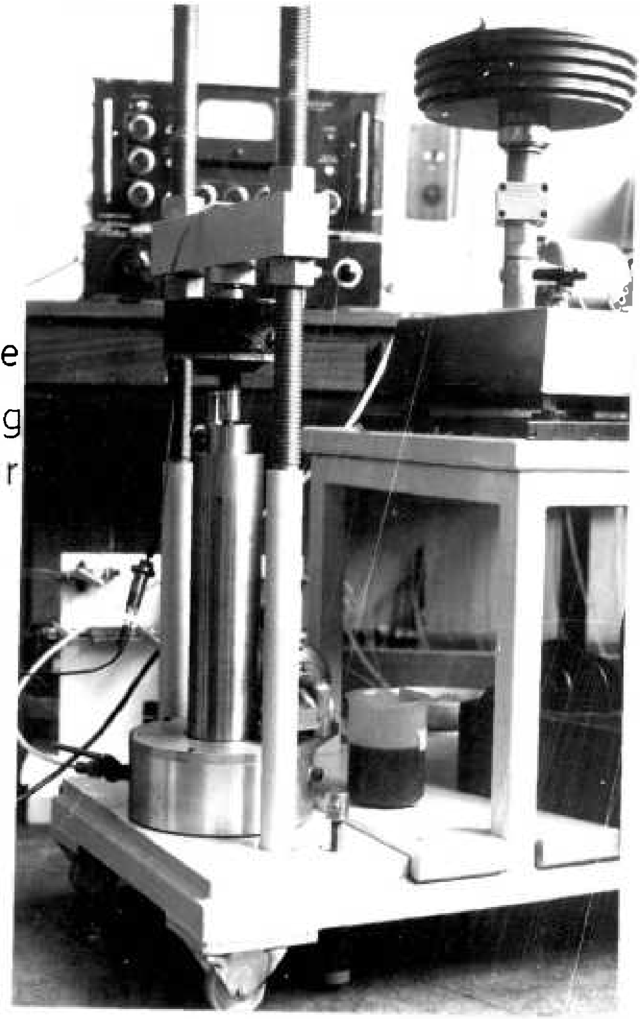


Fig. A.3.2

Calibration of the belt proving ring with the calibrator

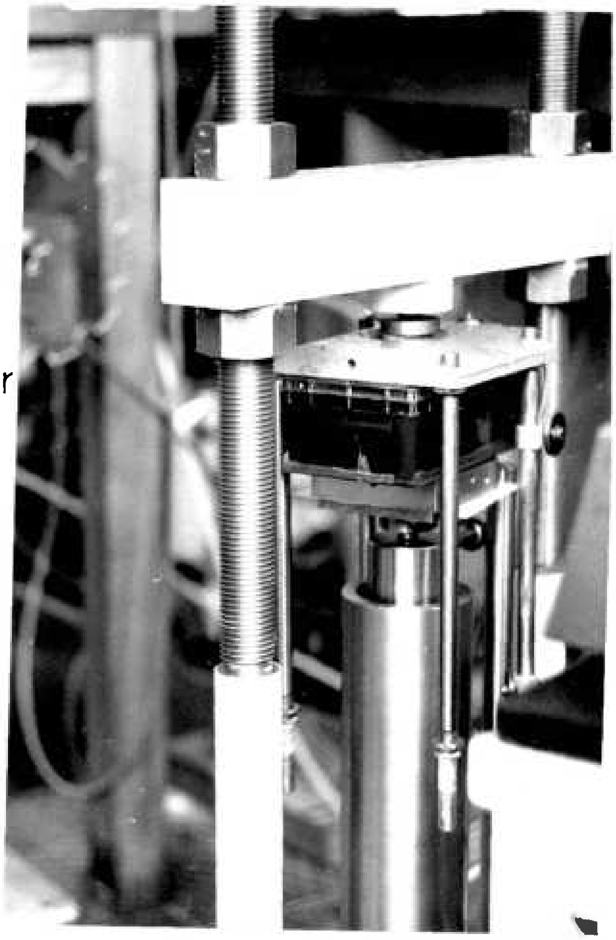
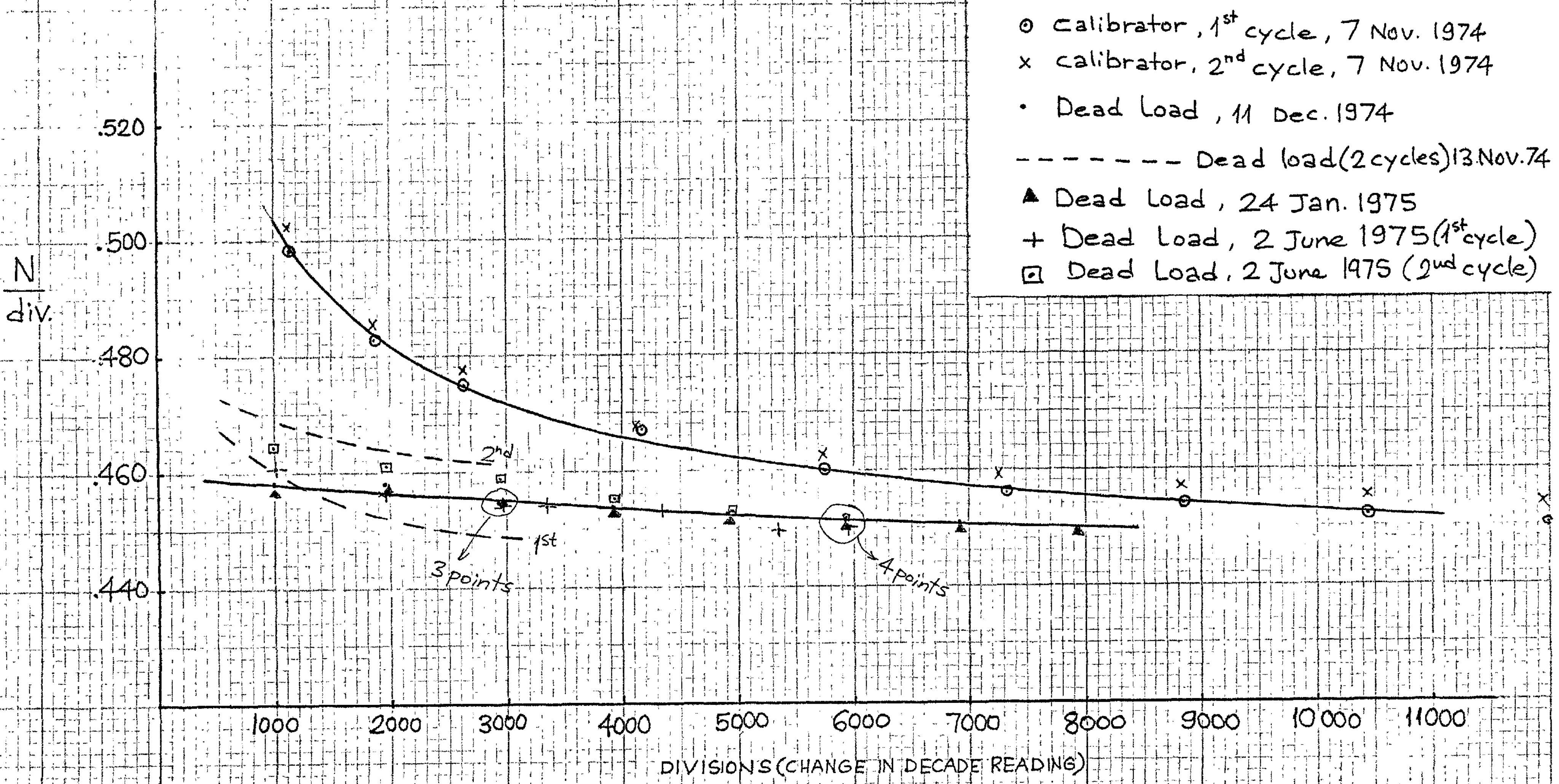


Fig. A.3.3



Extension cabibration of axial proving ring

Fig. A.3.4

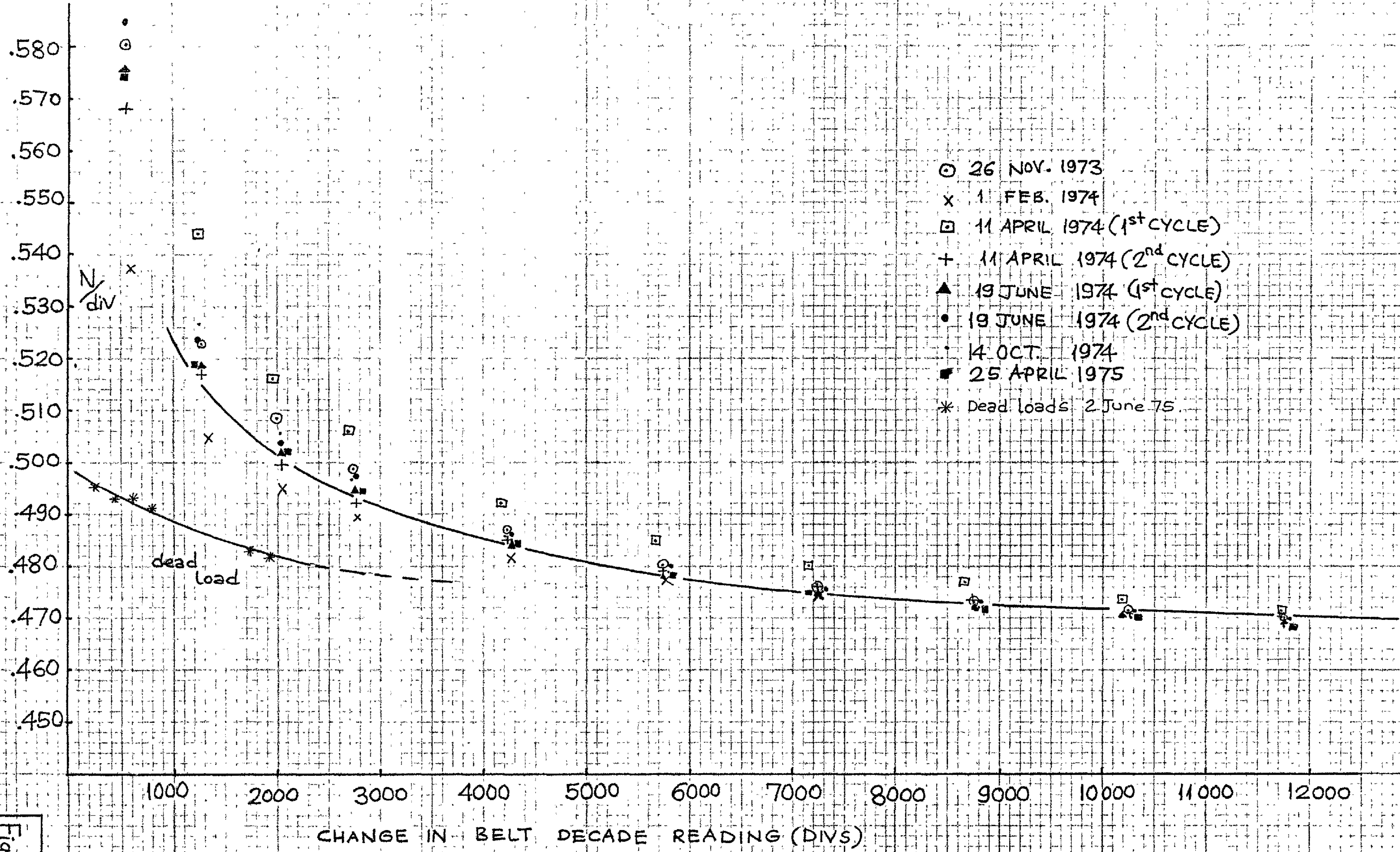
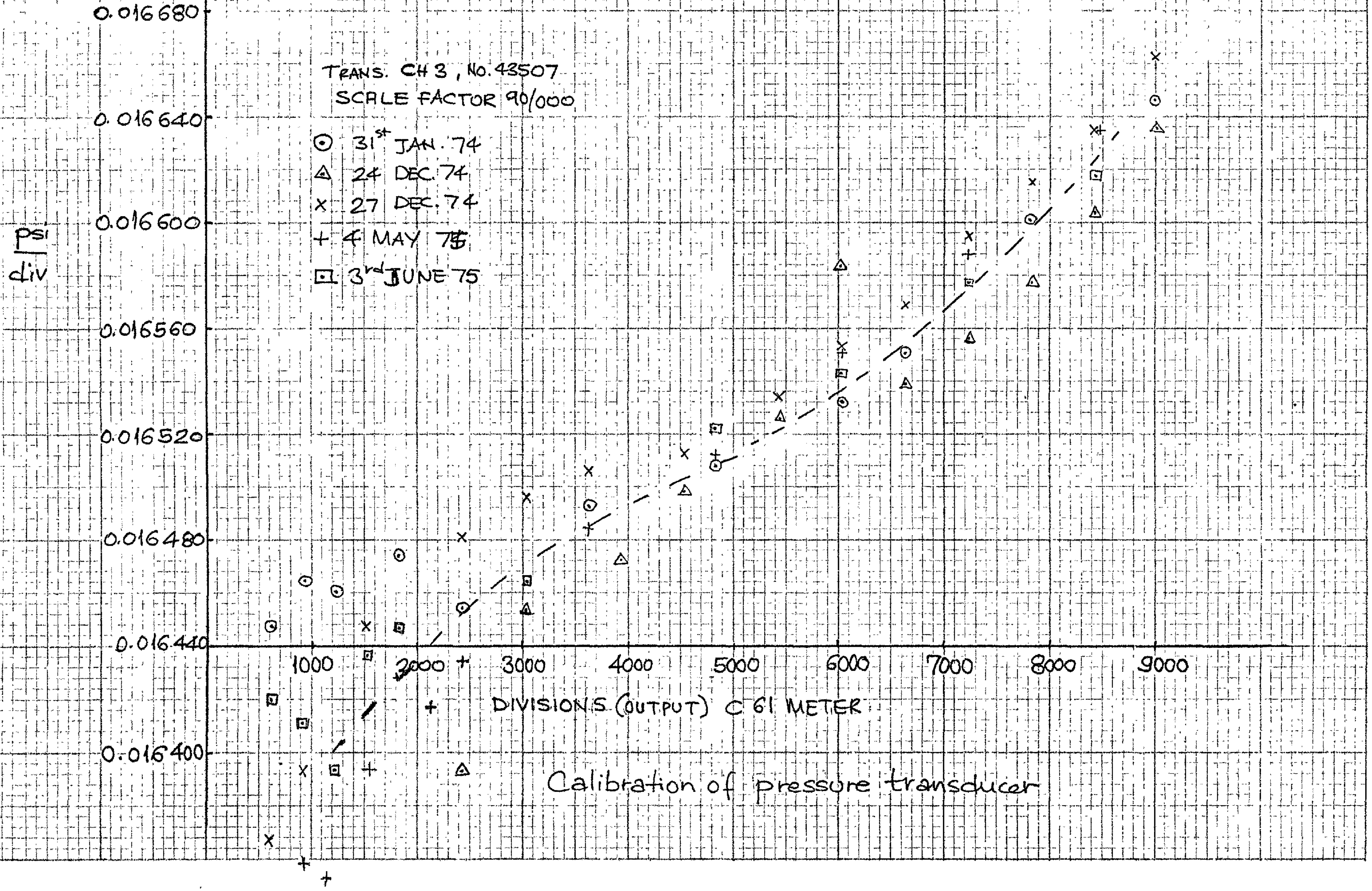


Fig. A.3.5

Calibration of belt proving ring



Calibration of pressure transducer

Fig. A.3.6

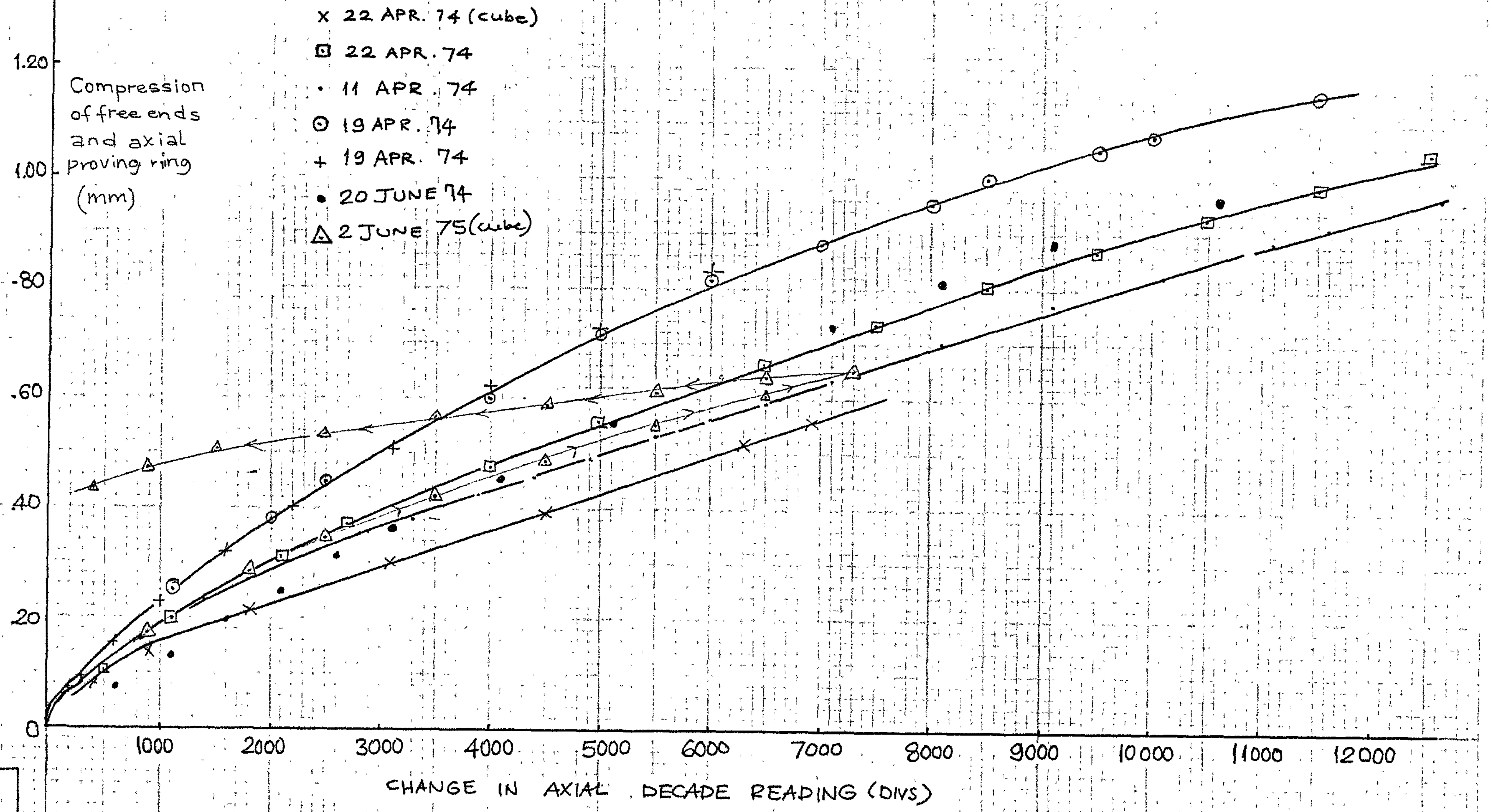


Fig. A.37

Compliance of axial proving ring and Lubricated membranes for rectangular compression samples.

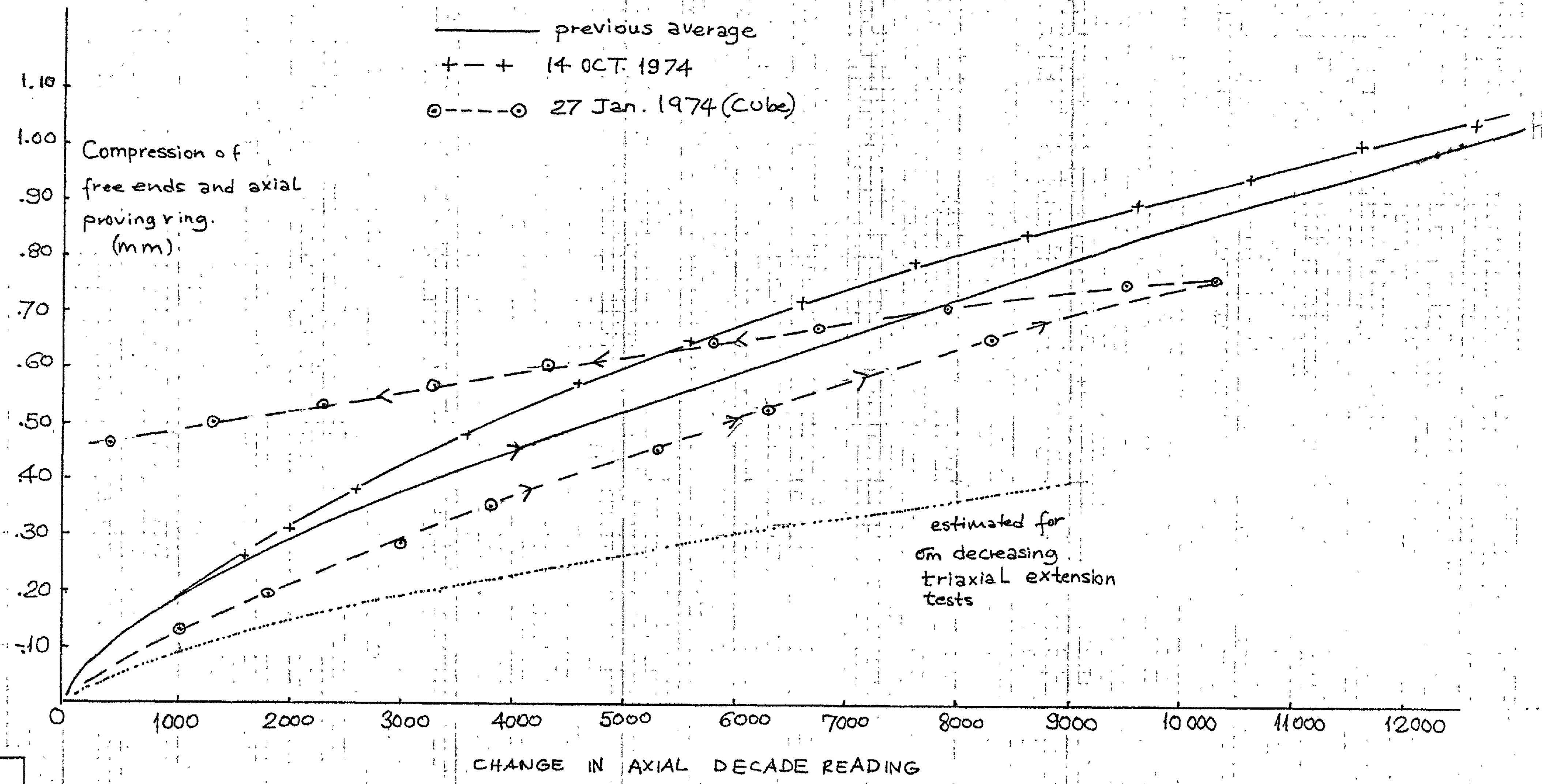

previous average
14 OCT. 1974
27 Jan. 1974 (Cube)
Compression of free ends and axial proving ring. (mm)
1.10
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10 000
11000
12000
CHANGE IN AXIAL DECADE READING
estimated for
σm decreasing
triaxial extension
tests

Fig. A.38

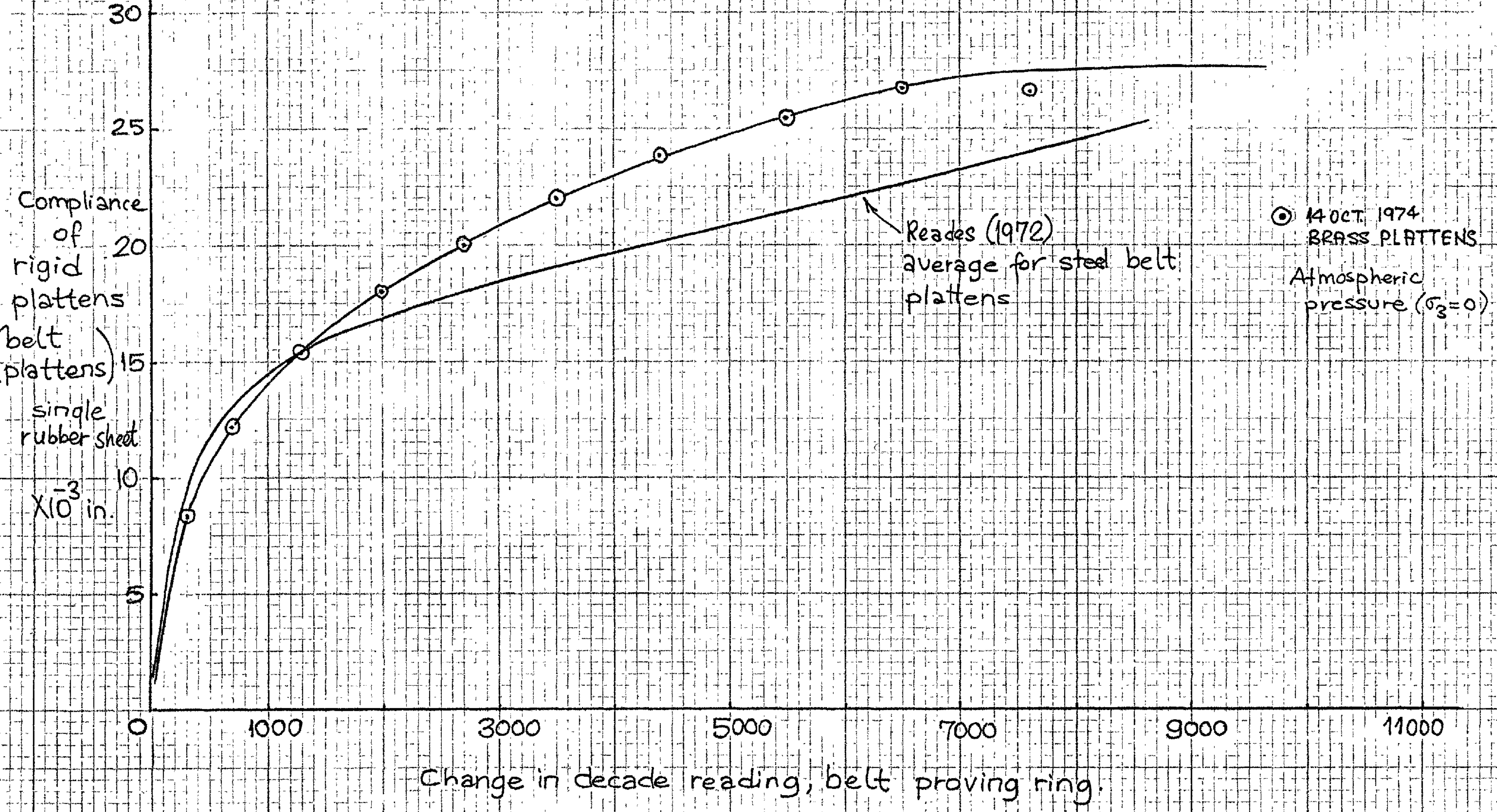


Fig. A.3.9

Compliance of rigid plattens (belt plattens) single rubber sheet $\times 10^{-3}$ in.
30
25
20
15
10
5
0
1000
3000
5000
7000
9000
11000
Change in decade reading, belt proving ring.
Reades (1972) average for steel belt plattens
⊙ 14 OCT. 1974 BRASS PLATTENS
Atmospheric pressure ($\sigma_3 = 0$)

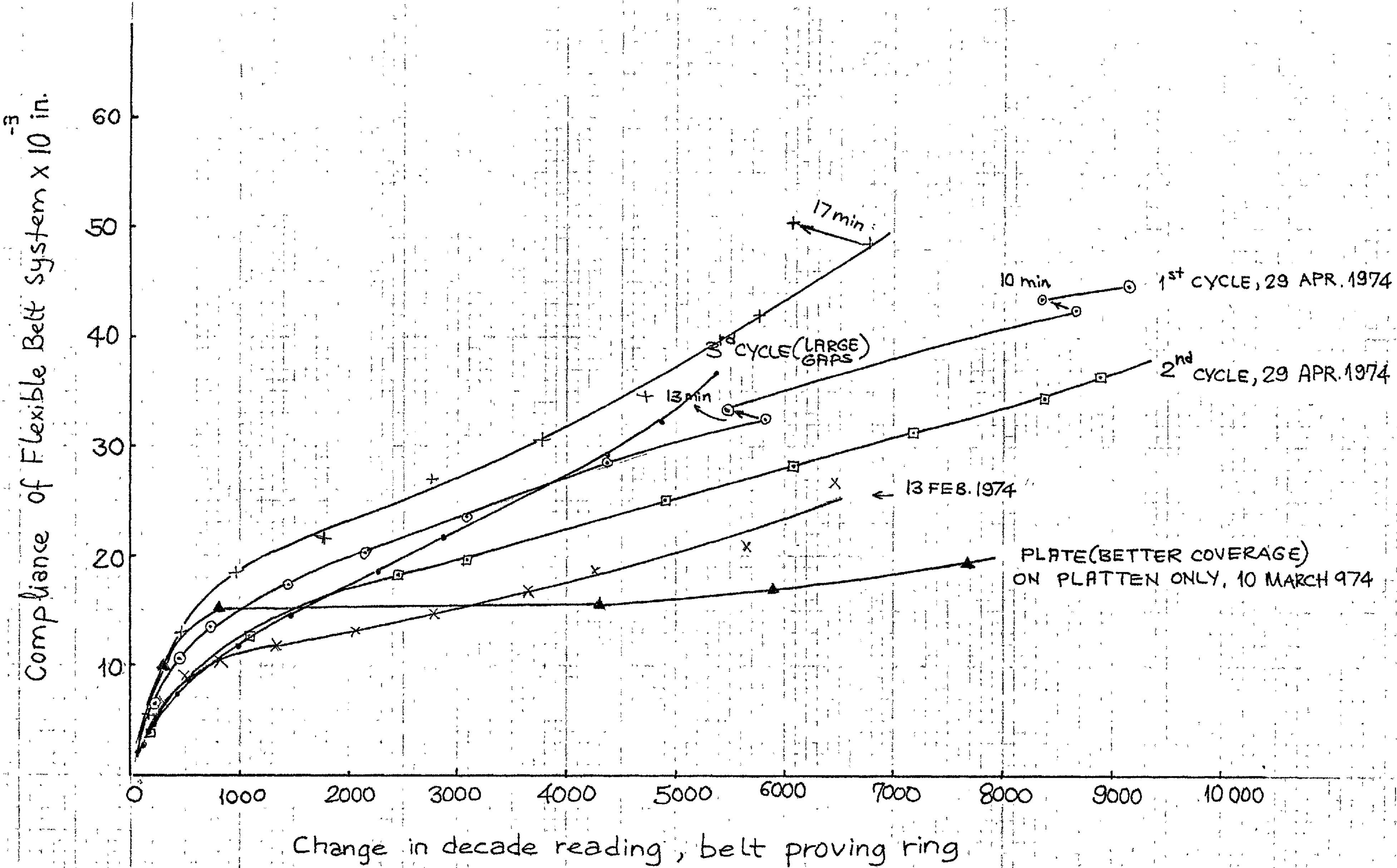

Compliance of Flexible Belt System x 10^{-3} in.
60
50
40
30
20
10
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10 000
Change in decade reading, belt proving ring
17 min
10 min
1st CYCLE, 29 APR. 1974
3rd CYCLE (LARGE GAPS)
13 min
2nd CYCLE, 29 APR. 1974
13 FEB. 1974
PLATE (BETTER COVERAGE)
ON PLATTEN ONLY, 10 MARCH 974

Fig. A.3.10

APPENDIX 4

DEFINITION OF TERMS

In the thesis many terms, invariants etc. have been used. They are given below in explicit form. Together with the list of symbols it is expected that there should not be any confusion. Notation in using invariants is identical to Jaeger (1969) which is quite common in many other studies.

Three invariants of stress tensor are I_1, I_2 and I_3;

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2 = -(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$$

$$I_3 = \sigma_1\sigma_2\sigma_2$$

where $\sigma_1, \sigma_2, \sigma_3$ are the principal stresses. They can also be expressed in terms of normal and shearing stresses (I_1 only in normal stresses) but they were not used in that form in the thesis.

Stress deviation is defined as;

$$S_{ij} = \sigma_{ij} - S \quad \text{where} \quad S = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{I_1}{3} \; (= \sigma_{oct})$$

Principal stress deviations are; $S_1 = \sigma_1 - S, S_2 = \sigma_2 - S$ and $S_3 = \sigma_3 - S$. Invariants of stress deviation (or of deviatoric stress tensor) are J_1, J_2 and J_3 and given in terms of principal normal stresses as;

$$J_1 = 0$$

$$J_2 = \frac{1}{6}\left\{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right\}$$

or $$= -\left\{(\sigma_1-\frac{I_1}{3})(\sigma_2-\frac{I_1}{3}) + (\sigma_2-\frac{I_1}{3})(\sigma_3-\frac{I_1}{3} + (\sigma_3-\frac{I_1}{3})(\sigma_1-\frac{I_1}{3})\right\}$$

or $$= \frac{1}{2}\left\{(\sigma_1-\frac{I_1}{3})^2 + (\sigma_2-\frac{I_1}{3})^2 + (\sigma_3-\frac{I_1}{3})^2\right\}$$

$$J_3 = (\sigma_1-\frac{I_1}{3})(\sigma_2-\frac{I_1}{3})(\sigma_3-\frac{I_1}{3})$$

or $$= \frac{1}{3}\left\{(\sigma_1-\frac{I_1}{3})^3 + (\sigma_2-\frac{I_1}{3})^3 + (\sigma_3-\frac{I_1}{3})^3\right\}$$

Octahedral normal and shearing stresses are defined as ;

$$\sigma_{oct} = \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3) = \frac{I_1}{3}$$

$$\tau_{oct} = \frac{1}{3}\left\{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right\}^{\frac{1}{2}}$$

or $$= \left\{\frac{2}{3} J_2\right\}^{\frac{1}{2}}$$

or $$= \left\{\frac{2}{9}(I_1^2 - I_2)\right\}^{\frac{1}{2}}$$

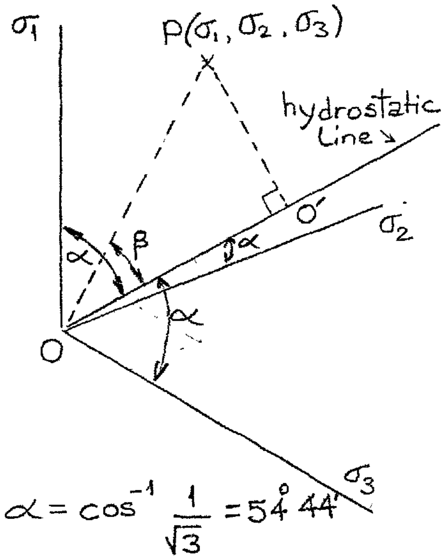
Principal strains and strain invariants are in similar forms. Since they are not referred in the text they are not given here. Octahedral normal and shearing strains are defined as.

$$\varepsilon_{oct} = \frac{1}{3}(\varepsilon_1+\varepsilon_2+\varepsilon_3)$$

$$\gamma_{oct} = \left\{\frac{2}{3}(\varepsilon_1-\varepsilon_2)^2 + (\varepsilon_2-\varepsilon_3)^2 + (\varepsilon_3-\varepsilon_1)^2\right\}^{\frac{1}{2}}$$

where ε_1, ε_2, ε_3 principal normal strains.

Some Geometrical Relations in $\sigma_1, \sigma_2, \sigma_3$ space:



$$\cos\beta = \frac{\sigma_1+\sigma_2+\sigma_3}{\sqrt{3}\,(\sigma_1^2+\sigma_2^2+\sigma_3^2)^{\frac{1}{2}}}$$

$$OP = (\sigma_1^2+\sigma_2^2+\sigma_3^2)^{\frac{1}{2}}$$

$$OO' = \frac{\sigma_1+\sigma_2+\sigma_3}{\sqrt{3}}$$

APPENDIX 5

ON THE FAILURE CRITERION GIVEN IN EQUATION 9.9.

Analytically it may be of interest to examine the proposed failure surface (equation 9.9). It is a right prymidal shape, apex at the coordinate center in $\sigma_1,\sigma_2,\sigma_3$ cartesian coordinates. A right section through this prymid is an octahedral plane in σ_1, σ_2, σ_3. coordinates. The shape of the surface can be best investigated on the right section. In figure A.5.2 equation 9.9 is plotted on a octahedral plane.

If a specific value of mean stress level and β are taken;

$$(\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 = K^2 \qquad \text{A.5.1}$$

A cartesian coordinate system (x,y,z) having the same origin as $\sigma_1,\sigma_2,\sigma_3$ system can be suitably selected. In figure A.5.1 the directions of the axes of the new system are also shown on a octahedral plane (x',y',z') for the sake of clearity - i.e. octahedral planes are parallel to xy plane -. z axis coincides with the hydrostatic axis. If the coordinates are transformed;

$$\sigma_1 = \sqrt{\frac{2}{3}}\, y + \frac{1}{\sqrt{3}}\, z$$

$$\sigma_2 = -\frac{1}{\sqrt{6}}\, y + \frac{1}{\sqrt{2}}\, x + \frac{1}{\sqrt{3}}\, z \qquad \text{A.5.2}$$

$$\sigma_3 = -\frac{1}{\sqrt{6}}\, y - \frac{1}{\sqrt{2}}\, x + \frac{1}{\sqrt{3}}\, z$$

Expressing Equation A.5.1 in x,y,z system;

$$\frac{3}{2} y^2 + \frac{5}{2} x^2 + \sqrt{3}\, xy = K^2 \qquad A.5.3$$

It is significant that the proper octant of the π - plane is chosen. Shape of the surface on the octahedral plane is clearly defined in Equation A.5.3. Slopes of the yield locus at A and B are of interest. So differentiating with respect of x in xy system;

$$3yy' + 5x + \sqrt{3}y + \sqrt{3}y'\, x = 0$$

$$y' = -\frac{5\,x + \sqrt{3}y}{\sqrt{3}\,x + 3y} \qquad A.5.4.$$

at $x = 0$ $y' = -\frac{1}{\sqrt{3}}$, therefore B'A 0 angle is 60°.
It is tangent to the Tresca line at A.

The equation of the line OB' is;

$$y = x \tan 30 = \frac{1}{\sqrt{3}} x \qquad A.5.5.$$

The slope of the tangent at B can be found inserting Equation A.5.5 in Equation A.5.4.

in Equation A.5.4.

$$y_B' = -\sqrt{3}$$

Product of slope of line OB and tangent to failure locus at B is(-1). (i.e. $-\sqrt{3} \times \frac{1}{\sqrt{3}} = -1$).

That is, they are vertical to each other, this implies that the surface is continious at B and corresponding points in other sections.

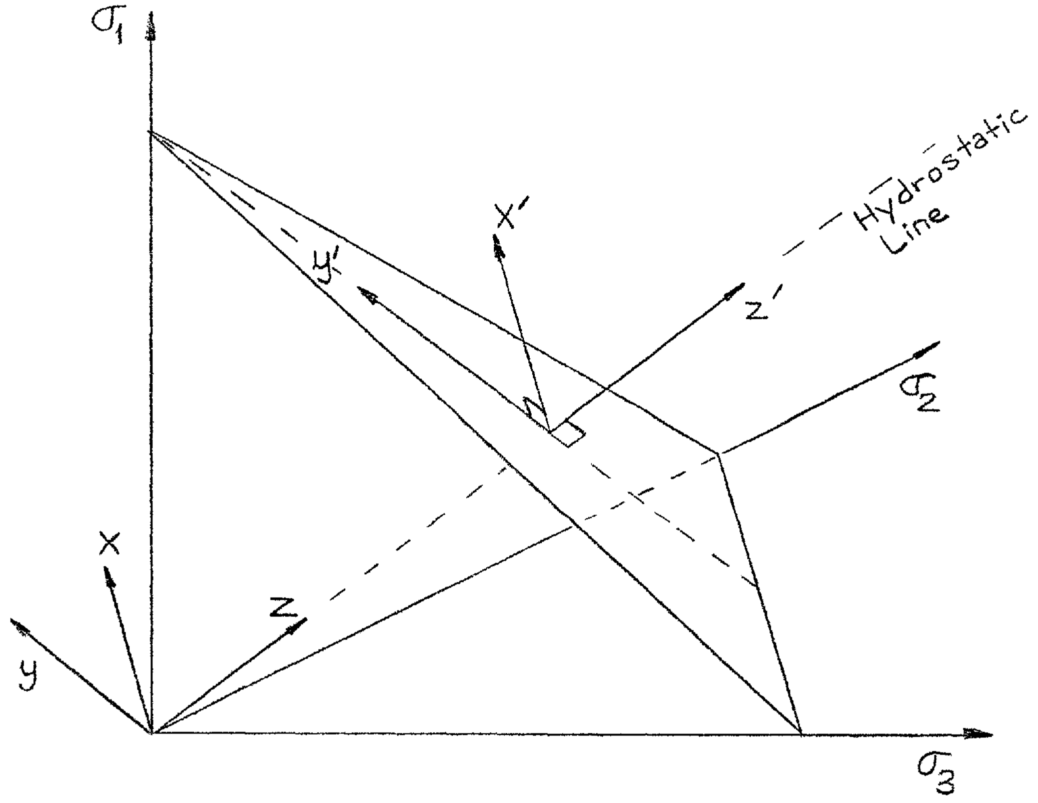


Fig. A.5.1

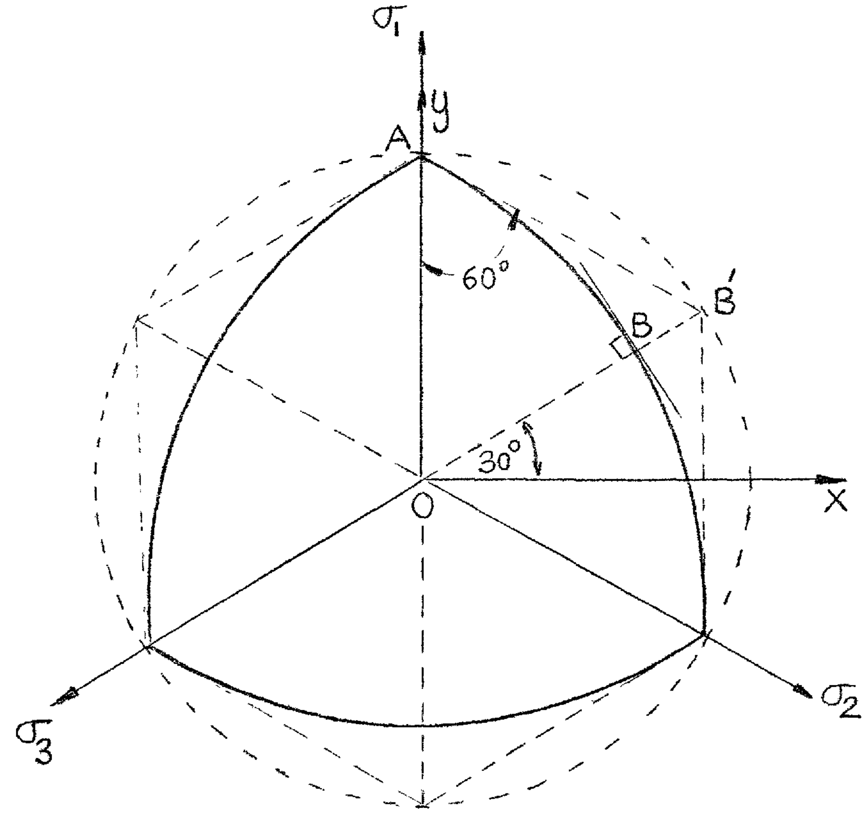


Fig. A.5.2

APPENDIX 6

AVERAGE STRESS LEVEL SERIES (ASL)

TRIAXIAL COMPRESSION TESTS ON HAM RIVER SAND

Hole in the membrane *

			AT FAILURE									
Test No	n_i %	n_c %	$\sigma_1 - \sigma_3$ measured kN/m²	σ_1/σ_3	σ_1 kN/m²	σ_3 kN/m²	ϵ_1 %	ϵ_v %	$\frac{d\epsilon_v}{d\epsilon_1}$	$\frac{\sigma_1+\sigma_2+\sigma_3}{3}$ kN/m²	ϕ' measured	ϕ' Platten fric & S.S
ASL-TC1	38.0	37.75	284.47	5.33	350.1	65.64	6.8	-2.0	-0.75	160.5	43.18	43.05
ASL-TC2	37.2	36.87	290.27	5.21	359.27	69.0	6.3	-4.6	-.78	165.76	42.67	42.56
ASL-TC3	43.55	43.16	232.3	4.15	306.1	73.8	13.5	-1.75	-0.18	151.23	37.7	37.38
ASL-TC4	39.35	39.1	253.65	4.89	318.81	65.16	7.0	-3.8	-.64	149.7	41.35	41.2

INDEPENDENT STRESS CONTROL TESTS ON HAM RIVER SAND

				AT FAILURE													
Test No	n_i %	n_c %	b	σ_1 σ_3 measured kN/m²	σ_2 σ_3 kN/m²	σ_1/σ_3	σ_1 kN/m²	σ_2 kN/m²	σ_3 kN/m²	ϵ_1 %	ϵ_2 %	ϵ_v %	$\frac{d\epsilon_v}{d\epsilon_1}$	$\frac{d\epsilon_v}{d\epsilon_2}$	$\frac{\sigma_1+\sigma_2+\sigma_3}{3}$ kN/m²	ϕ' measured	ϕ' Platten fric & S.S
ASL-ISC 1	38.03	37.77	.56	427.5	240.2	6.79	501.3	314.	73.8	2.1	.45	-1.3	-1.10	-3.07	216.3	48.02	47.85
ASL-ISC 2	38.0	37.64	.68	551.8	377.7	6.29	656.1	482.0	104.3	1.6	.85	-0.83	-1.08	-1.46	414.1	46.52	46.33
ASL-ISC 3	44.4	43.9	.866	422.5	366.2	4.65	538.3	482.0	115.8	5.6	5.26	-0.64	-0.4	-.41	378.7	40.23	39.92

Table A.6.1

PRELIMINARY AND CONTROL TRIAXIAL COMPRESSION TESTS ON HAM RIVER SAND

		AT FAILURE					
Test no:	n_i %	$\sigma_1 - \sigma_3$ kN/m²	σ_3 kN/m²	ϵ_1 %	ϵ_v %	$\frac{d\epsilon_v}{d\epsilon_1}$	ϕ' Deg.
TC1	38.9	608.1	201.	6.0	−1.60	−.47	37.0
TC2	36.7	628.5	201.6	2.8	−2.7	−1.2	37.5
TC3	38.0	635.7	201.2	7.1	−2.0	−.48	37.75
TC4	40.1	574.4	201.3	6.9	−1.8	−.45	36.0
TC5	42.8	539.9	206.8	11.0	+0.15	−.15	34.5
TC6	44.3	562.8	207.	15.0	−.10	−.11	35.2
TC7 CONT.	38.7	762.8	207.5	7.0	−2.9	−.57	40.35

Table A.6.2

APPENDIX 7

A TYPICAL TEST CALCULATION

Test calculations for a flexible platten generalised test on a loose sample, ISC(F)13, are shown below:

Thickness of the sample sheath: .35 mm (t)

Half perimeter of the sample sheath: 141 mm

Weight of dish and dry sand before building the sample: 1615.0 gm

Weight of dish and dry sand after building the sample: 992.0

Weight of dry sand used: 623.0

Weight of dish and dry sand recovered after test: 1069.0

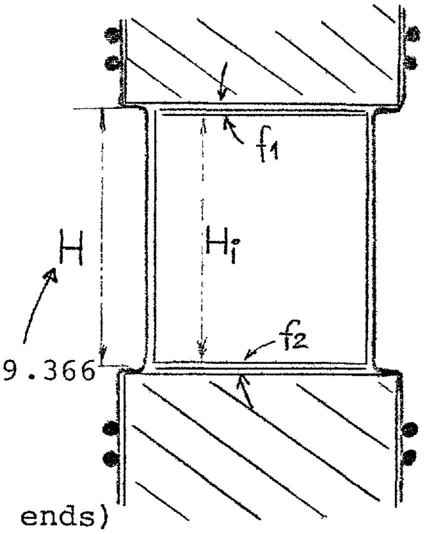
Weight of dish: 446.5

Weight of dry sand (after test): 622.5

Average weight of dry sand in the sample, W_s = 622.75

Initial sample dimensions (measured under suction):

	C_i	B_i	H
Top	5.250	8.735	8.626
Middle	5.275	8.780	8.587
Bottom	5.350	8.795	8.612
			8.590
Mean	5.291	8.770	8.604+.762 = 9.366
-2xthickness	5.221	8.700	



$f = f_1+f_2$ = 1.5 mm (Total thickness of free ends)

$H_i = H+2t-f = 9.366+.035\times2-0.15 = 9.286$ cm

$A_i = B_i \times C_i = 45.4227$ cm^2

Porous disk volume correction : 0.5 Other corrections (disturbance, curved edges) = 2.5 cc

$V_i = A_i.H_i$-corrections = 418.79 cc

$n_i = 1-(W_s/G.V_i) = 44.5\%$

<u>End of consolidation</u>: ΔV during consolidation: 5.65 cc

Reduction in height assuming isotropic compression

$\Delta H = \Delta V.H_i/3V_i = 0.0415$ cm

$n_c = 1-(W_s/GV_c) = 43.7\%$, $H_c = H_i-\Delta H = 9.244$ cm

$C_c = C_i-\Delta V.C_i/3V_c = 5.197$ cm, $V_c = V_i-\Delta V = 413.138$ cc

$B_c = B_i - \frac{\Delta V.B_i}{3\ V_c} = 8.660$, $A_c = V_c/H_c = 44.963\ \text{cm}^2$

Belt cross-sectional area = $A_c^* = H_c.C_c = 48.0442\ \text{cm}^2$

<u>Test Details</u>:

Axial proving ring:

Scale factor (C61 meter) = 90/000

Initial reading in air: -627

Initial reading in water: -626

Axial strain rate and gear ratio: 0.12 mm/min. 2/84

Belt proving ring:

Scale factor (C61 meter) 90/000

Initial reading in air: -4030

Initial reading in water: -4097

Belt strain rate: 0.097 mm/min

Initial reading for pressure transducer (ram side), in air and water: -18, -29

Initial reading for pressure transducer (proving ring) in air and water: +129, +148

Cell pressure: 214.5 kN/m^2

At Failure

Axial strain, $\varepsilon_a = \Delta H/H_c = 4.4\%$

Belt strain, $\varepsilon_b = \Delta B/H_c = 2.6\%$

Volumetric strain, $\varepsilon_v = \Delta V/V_c = .70\%$

Volumetric strain rate, $d\varepsilon_v/d\varepsilon_a = -.19$

Volumetric strain rate, $d\varepsilon_v/d\varepsilon_b = -.24$

Axial deviator stress, $(\sigma_1-\sigma_3)$, $= 763.6$ kN/M^2

Belt deviator stress, $(\sigma_2-\sigma_3) = 555.0$ kN/M^2

Maximum angle of shearing resistance, $\phi' = \arc \sin (\sigma_1-\sigma_3)/(\sigma_1+\sigma_3)$, $= 39.8^\circ$, $b = (\sigma_2-\sigma_3)/(\sigma_1-\sigma_3) = .727$

Dimensions measured after the test:

C	B	H
5.464	8.628	8.040
5.730	8.580	8.010
5.714	8.630	8.06
		8.03

See Table A.7.1.

Table A.7.1

kN/M² cell pres-sure	Axial Dial R. x 10^{-2} mm	Axial Decade R.	Belt Decade R.	Pressure Trands. Ram side	Pressure Trands. P.Ring side	Burette cc	Belt Dial R. x 10^{-3} in.	Corrected Belt def. x 10^{-3} in	Change in Belt Decade Reading	Change in Axial Decade Reading	Corrected Axial Def. mm	$\in_a$ %	ΔV cc	$\in_v$ %	Axial Area $A_c \cdot \frac{1-\in_v}{1-\in_a}$ cm²	$\in_b$ %	$\Delta V/B_c$ cm²	cm² Belt Area $\frac{A_{bc}-\Delta V/B_c}{1-\in_b}$	$\sigma_a-\sigma_c$ kN/M²	$\sigma_b-\sigma_c$ kN/M²	σ_a/σ_c	σ_b/σ_c	$b=\frac{\sigma_b-\sigma_c}{\sigma_a-\sigma_c}$
0(Atm)		floor				16.10																	
15.9		floor				15.4																	
15.9		Bench				16.55																	
44.8						15.2																	
97.9						13.85																	
151.7						12.95																	
214.5						12.1																	
	0	-618	-4046	-1880	+1961	12.0	262	0	0	0	0	0	0	0	44.9630	0	0	48.0442	0	1	0	1	—
	26	+682	-3782	2162	+2250	11.35	276	8	264	1300	.04	.043	0.65	0.16	44.9104	.23	.0750	48.0798	139.23	1.67	29.65	1.14	.21
	44	1482	-3590	-2370	+2460	10.90	279	8.5	456	2100	.14	0.15	1.10	0.26	44.9135	.25	.1270	48.0373	220.69	2.10	50.7	1.24	.23
Cons.	65	2182	-3322	-2642	+2742	10.4	282	9	724	2800	.29	0.314	1.60	0.38	44.9332	0.26	.1847	47.9842	291.63	2.41	79.5	1.38	.27
	85	2782	-3017	-2980	+3080	10.0	285.8	10.6	1029	3400	.446	0.483	2.00	0.48	44.9643	.31	.2309	47.9620	351.61	2.7	111.35	1.54	.32
	111.5	3382	-2646	3336	3426	9.7	289.2	11.2	1400	4000	.665	0.72	2.3	0.55	45.0399	.33	.2656	47.9368	411.19	2.99	148.94	1.71	.36
	145.5	3982	-2192	-3760	+3870	9.2	295.8	16.3	1854	4600	.965	1.04	2.8	0.67	45.1311	.48	.3233	47.9510	470.38	3.27	194.1	1.92	.41
	173	4482	-1752	-4132	+4222	8.9	302.0	21.4	2294	5100	1.205	1.30	3.1	0.75	45.21355	.627	.3579	47.9872	518.27	3.50	237.1	2.13	.46
	193	4782	-1482	-4392	+4492	8.7	307.0	25.3	2564	5400	1.385	1.50	3.3	0.79	45.2271	.74	.3811	48.0124	547.90	3.60	266.86	2.27	.49
	216.5	5082	-1172	-4672	+4762	8.6	311.8	29.6	2874	5700	1.600	1.73	3.4	0.82	45.3794	.87	.3926	48.0698	576.53	3.75	294.16	2.4	.51
	241	5382	-872	-4932	5052	8.45	318.2	35.2	3174	6000	1.825	1.97	3.55	0.85	45.4767	1.03	.4099	48.1300	604.92	3.88	323.14	2.53	.53
	273	5782	-484	-5274	+5364	8.40	326.0	42.0	3562	6368	2.120	2.29	3.60	0.866	45.6183	1.23	.4157	48.2216	639.33	4.04	359.7	2.71	.56
	316.5	6127	-67	-5657	5747	8.30	336.8	51.6	3979	6745	2.570	2.78	3.70	0.89	45.8371	1.51	.4277	48.3465	672.42	4.20	400.0	2.9	.59
	352	6440	+350	-5990	+6080	8.35	347.5	62	4396	7058	2.860	3.09	3.65	0.82	45.9884	1.82	.4215	48.5055	701.37	4.33	438.2	3.02	.62
	379	6650	+600	-6180	+6270	8.45	353.8	67.6	4646	7268	3.140	3.397	3.55	0.85	46.1485	1.98	.4099	48.5965	719.58	4.42	460.8	3.19	.64
	403	6840	+837	-6400	+6480	8.55	360.8	73.6	4823	7458	3.345	3.62	3.45	0.83	46.2646	2.16	.3984	48.6977	736.21	4.50	482.3	3.29	.66
	437	7060	+1130	-6620	+6710	8.7	369.5	81	5176	7678	3.670	3.97	3.3	0.79	46.4519	2.37	.3811	48.8201	754.38	4.51	508.9	3.40	.67
	458	7140	-1338	6788	6898	8.85	376.3	87.6	5384	7758	3.875	4.19	3.15	0.76	46.5727	2.57	.3637	48.9322	760.1	4.54	527.53	3.45	.69
	488	7160	1640	7040	7130	9.1	384.2	94.4	5686	7778	4.173	4.51	2.9	.70	46.7570	2.77	.3349	49.0685	758.22	4.53	554.5	3.59	.73
	507	7130	1800	7150	7250	9.2	389	98	5846	7748	4.365	4.72	2.8	0.67	46.8742	2.87	.3233	49.1309	754.23	4.51	568.8	3.64	.75
	524	7100	1970	7270	7370	9.35	394	103	6016	7718	4.54	4.91	2.65	0.64	46.9820	3.02	.3060	49.2248	749.58	4.49	583.2	3.71	.72
	549	6910	2160	7420	—	9.55	400	108	6206	7528	4.80	5.19	2.45	0.59	47.1445	3.167	.2829	49.3234	728.77	4.39	600.2	3.79	.82

APPENDIX 8

STRESS-STRAIN CURVES FOR GENERALISED TESTS ON HAM RIVER SAND USING FLEXIBLE BELT PLATTENS

ISC(F)1-20

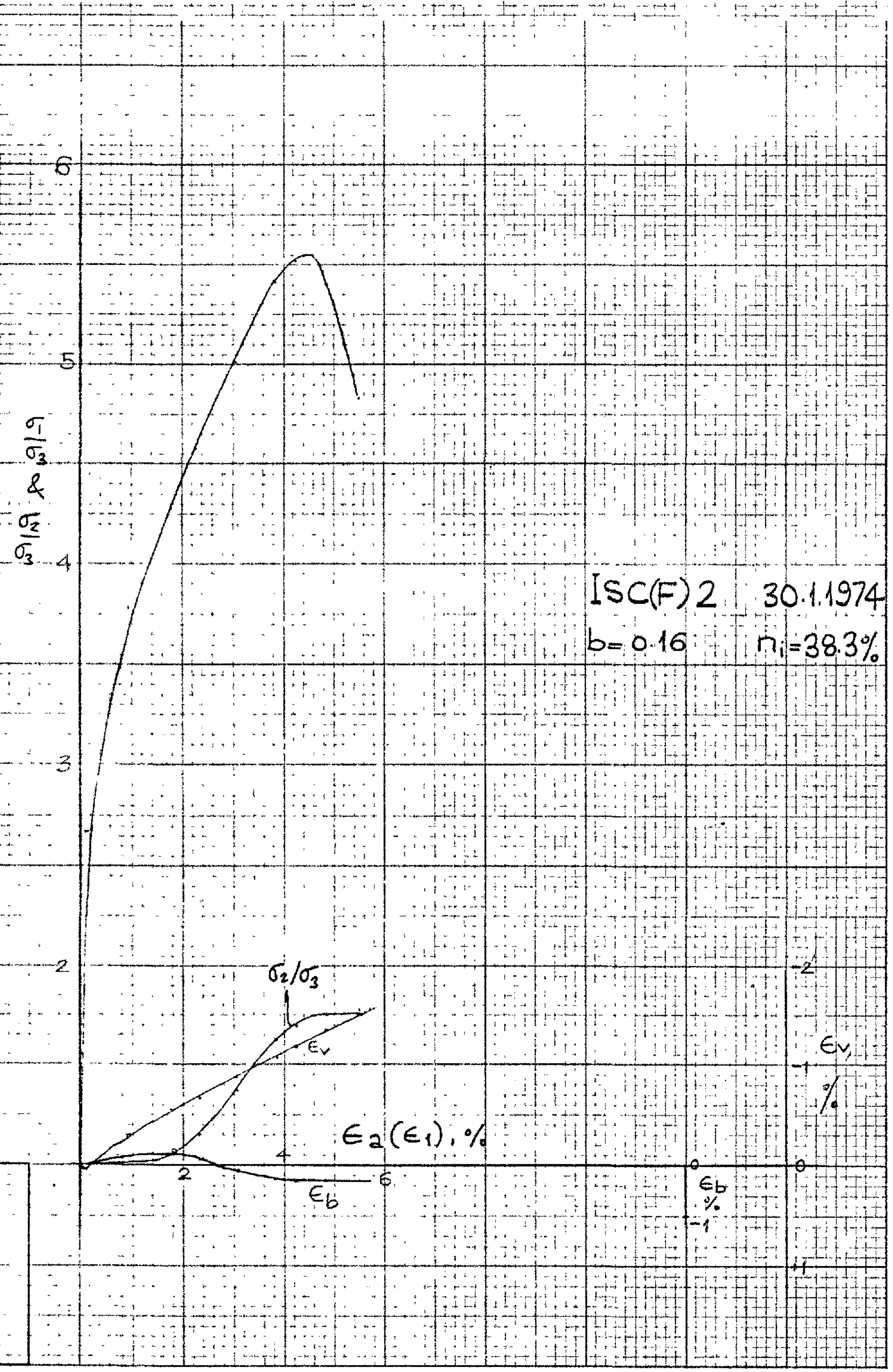


Fig. A 8.2

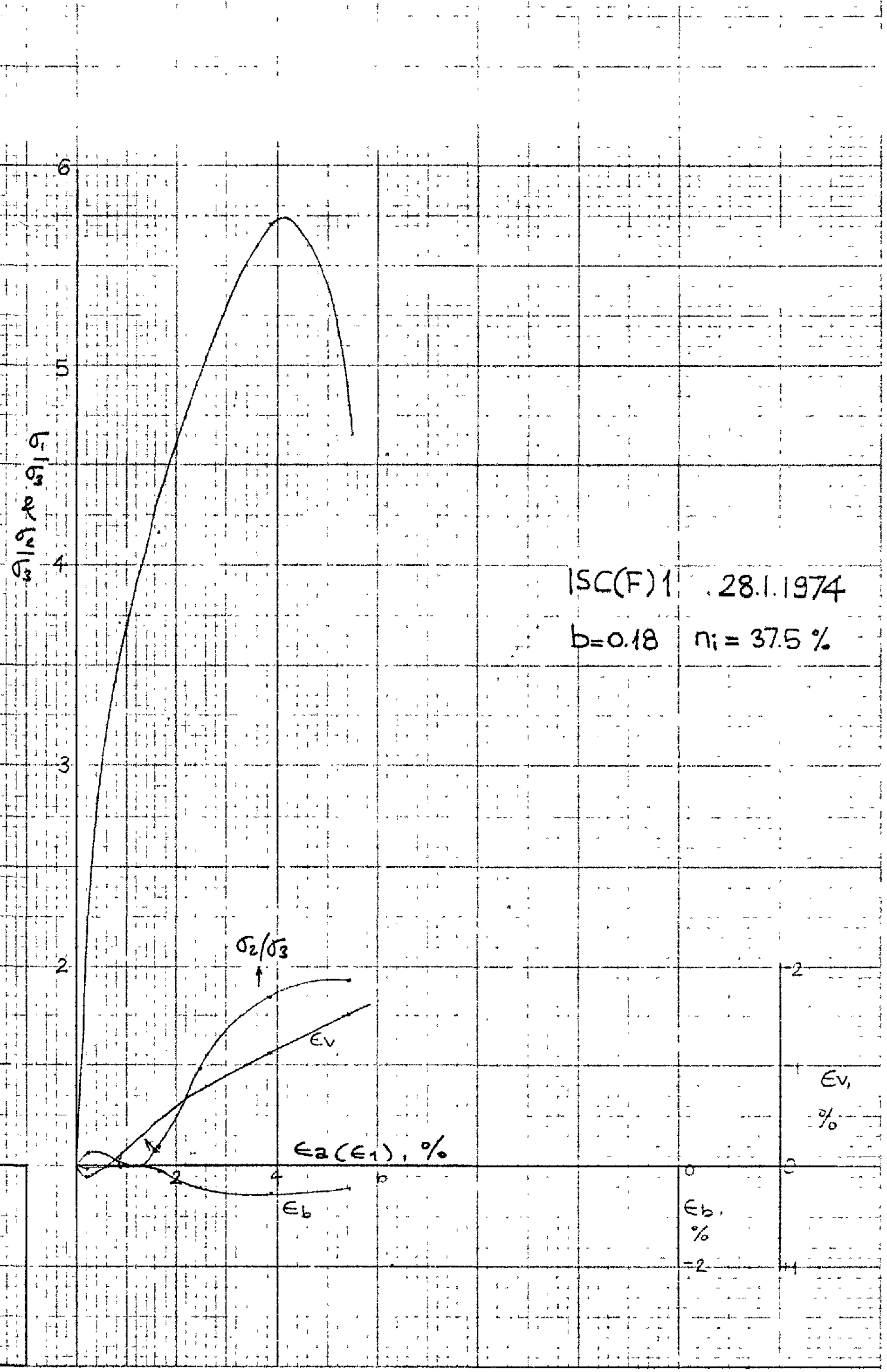
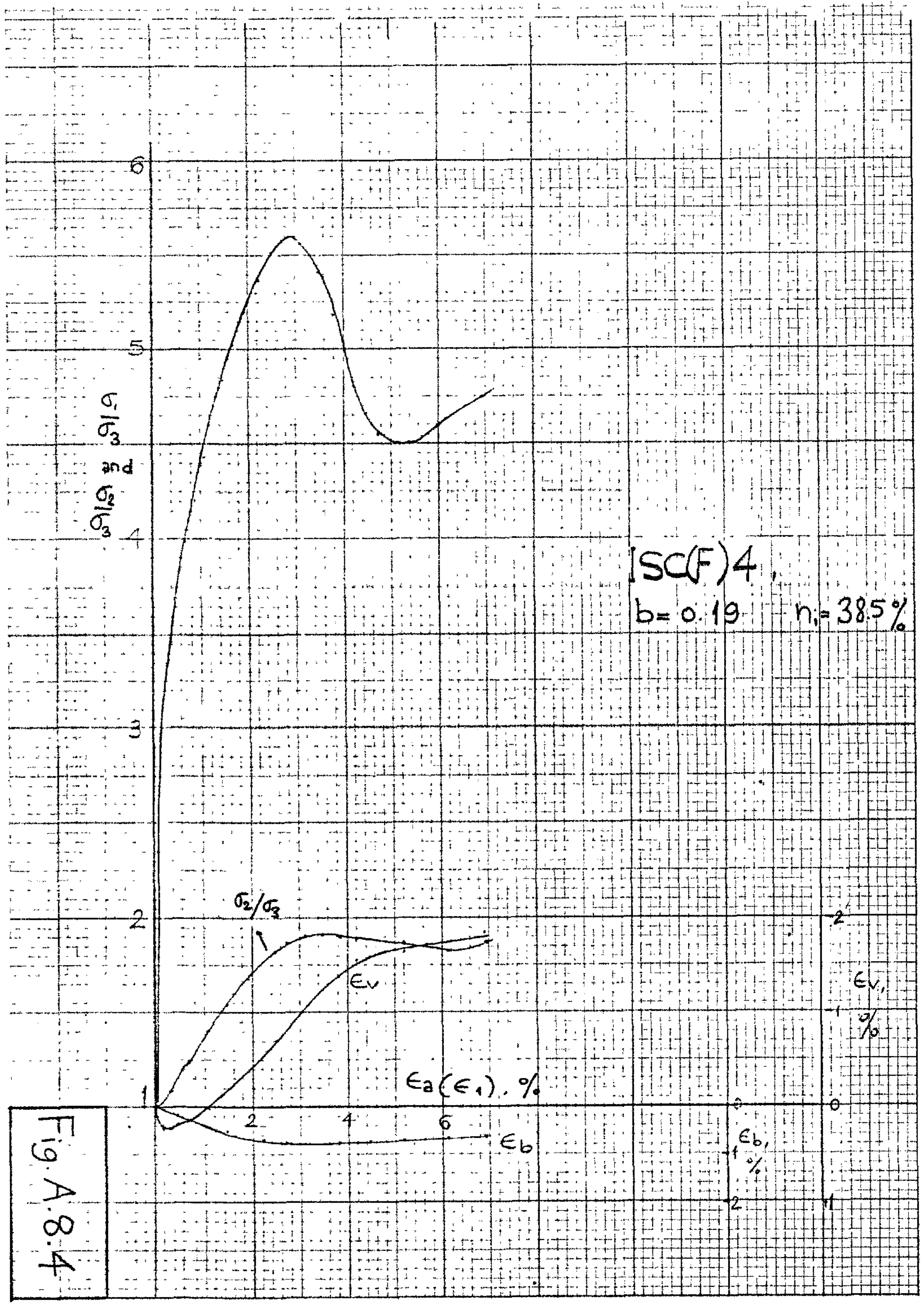
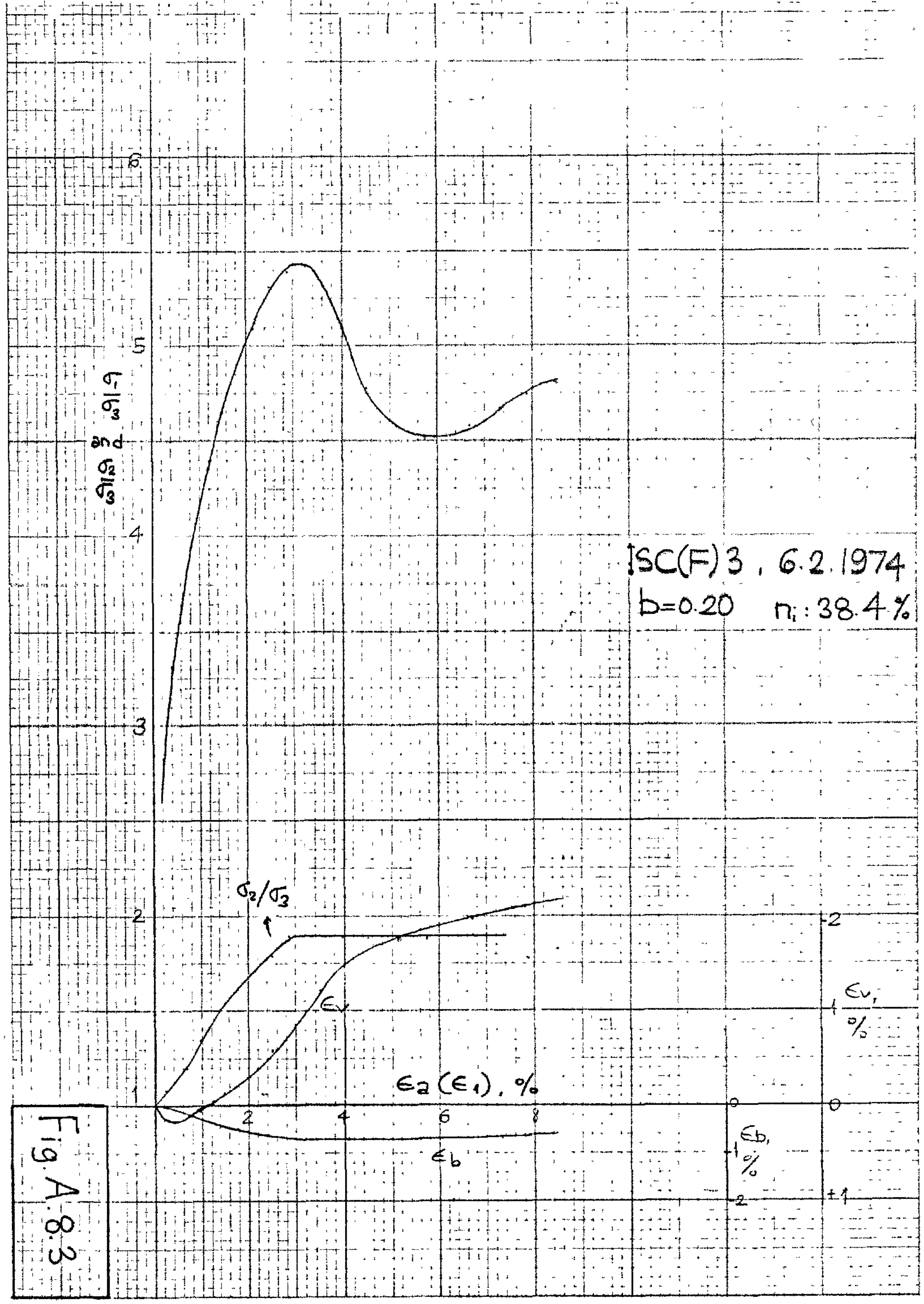


Fig. A 8.1



ISC(F)4,
b= 0.19 n_i: 38.5%

Fig. A.8.4



ISC(F)3, 6.2.1974
b=0.20 n_i: 38.4%

Fig. A.8.3

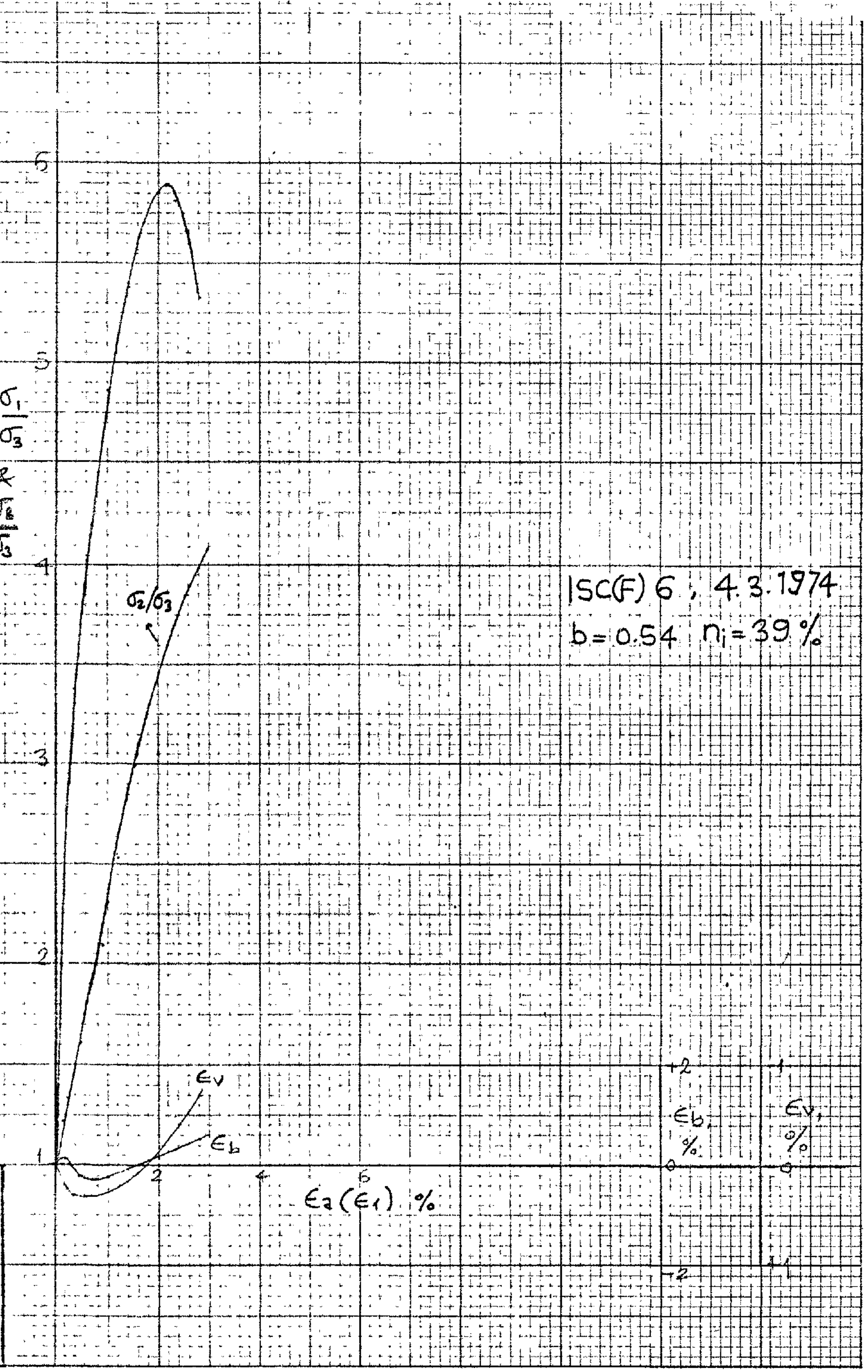


Fig. A.8.6

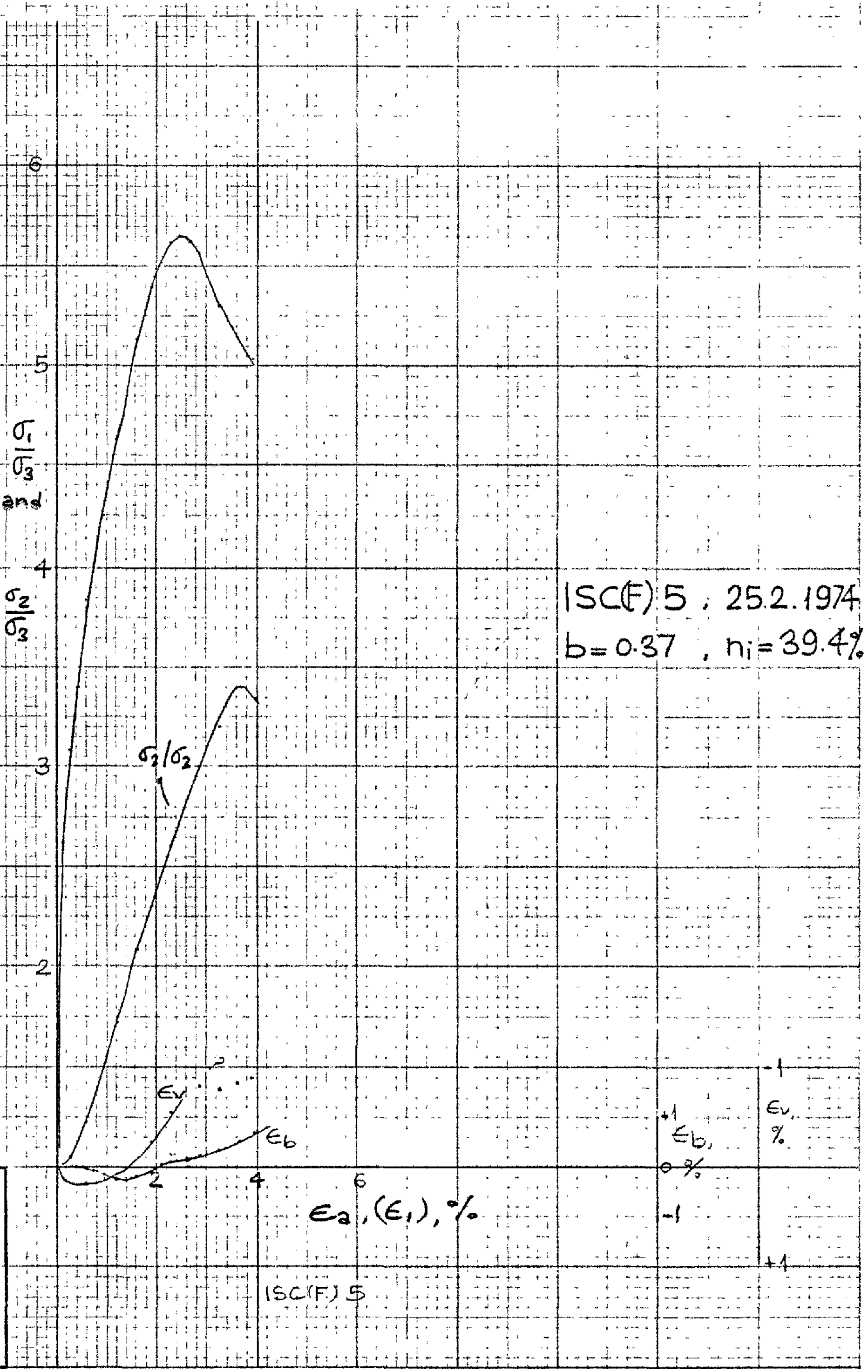


Fig. A.8.5

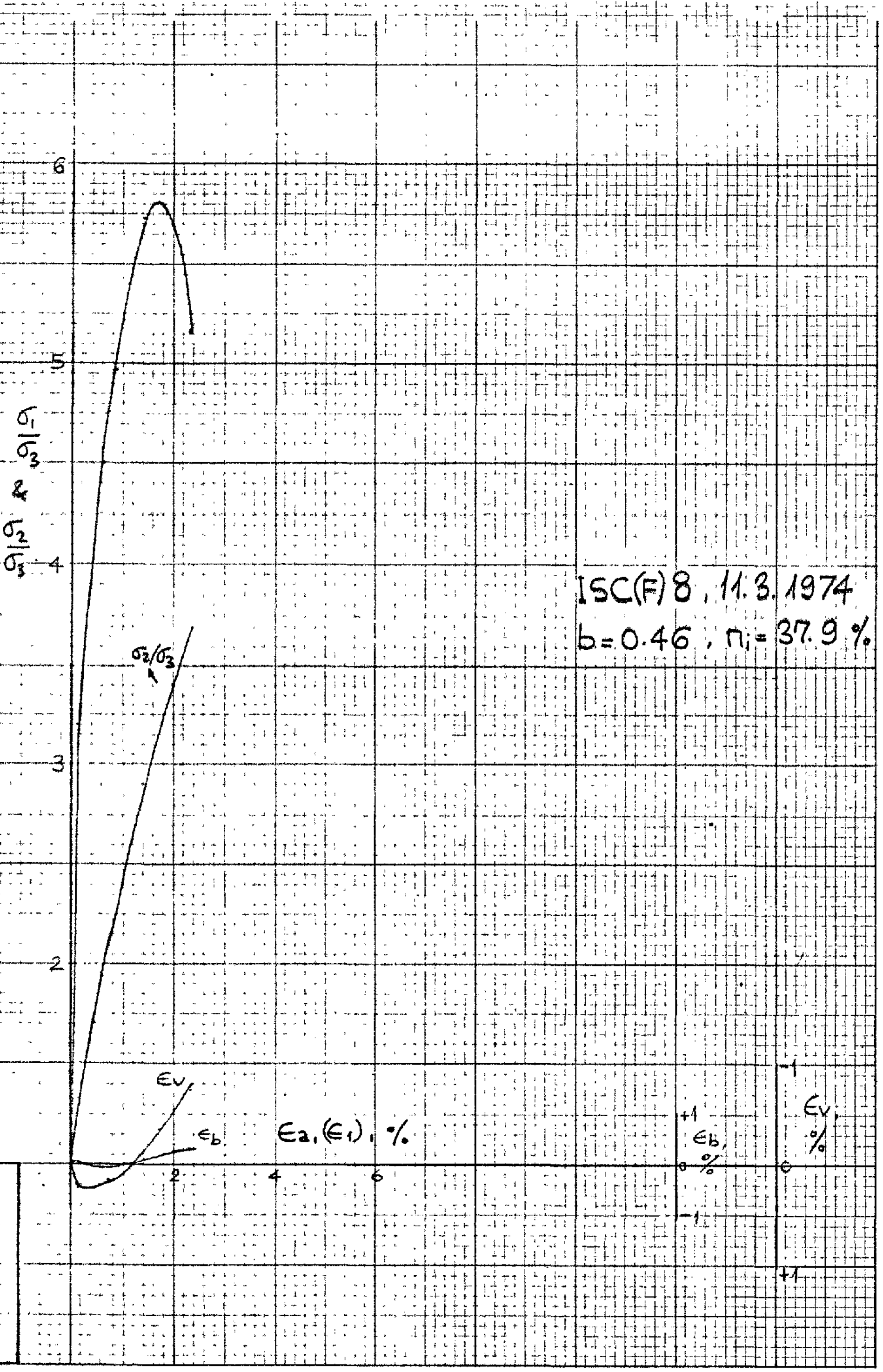


Fig. A.8.8

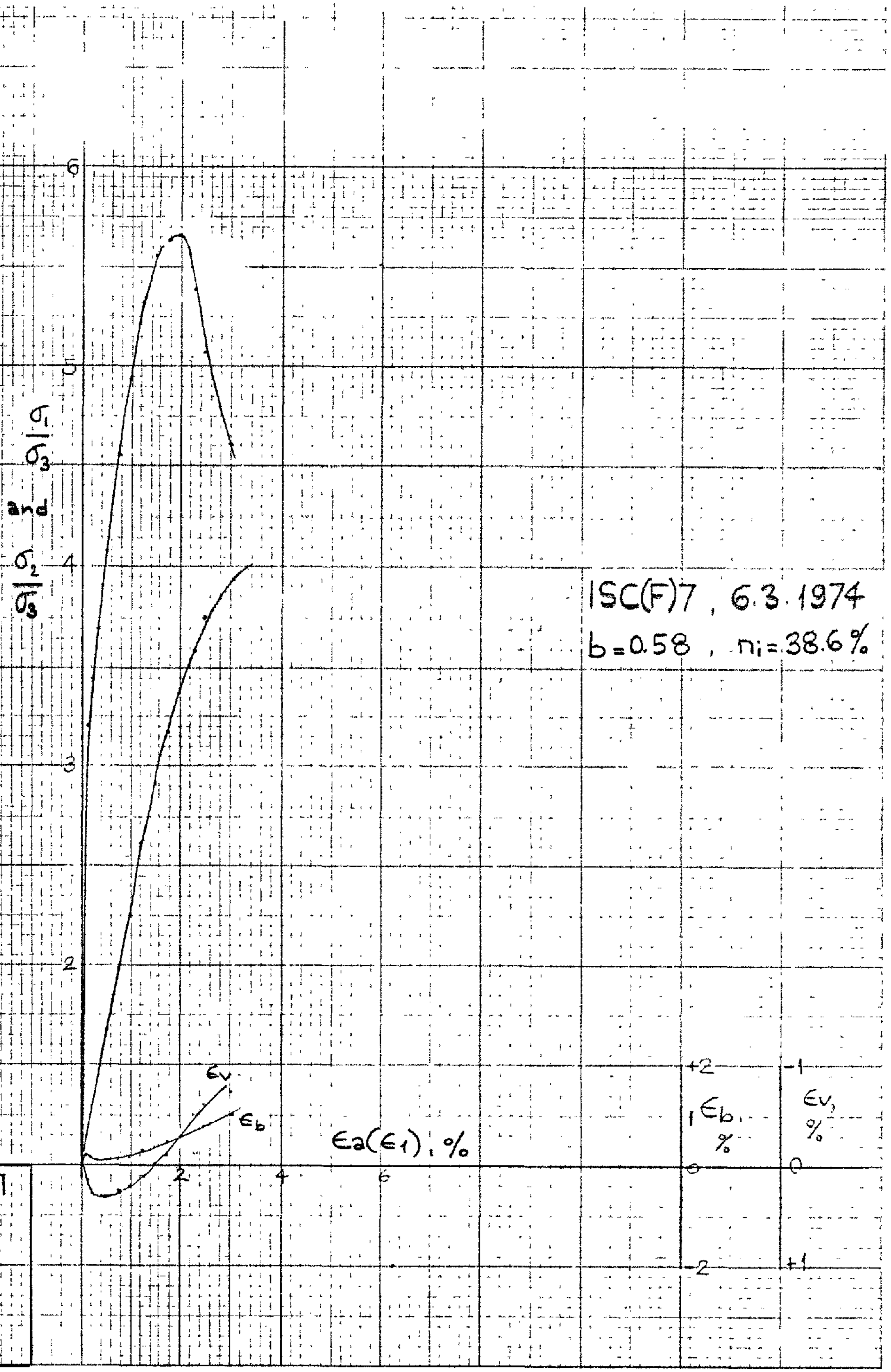


Fig. A.8.7

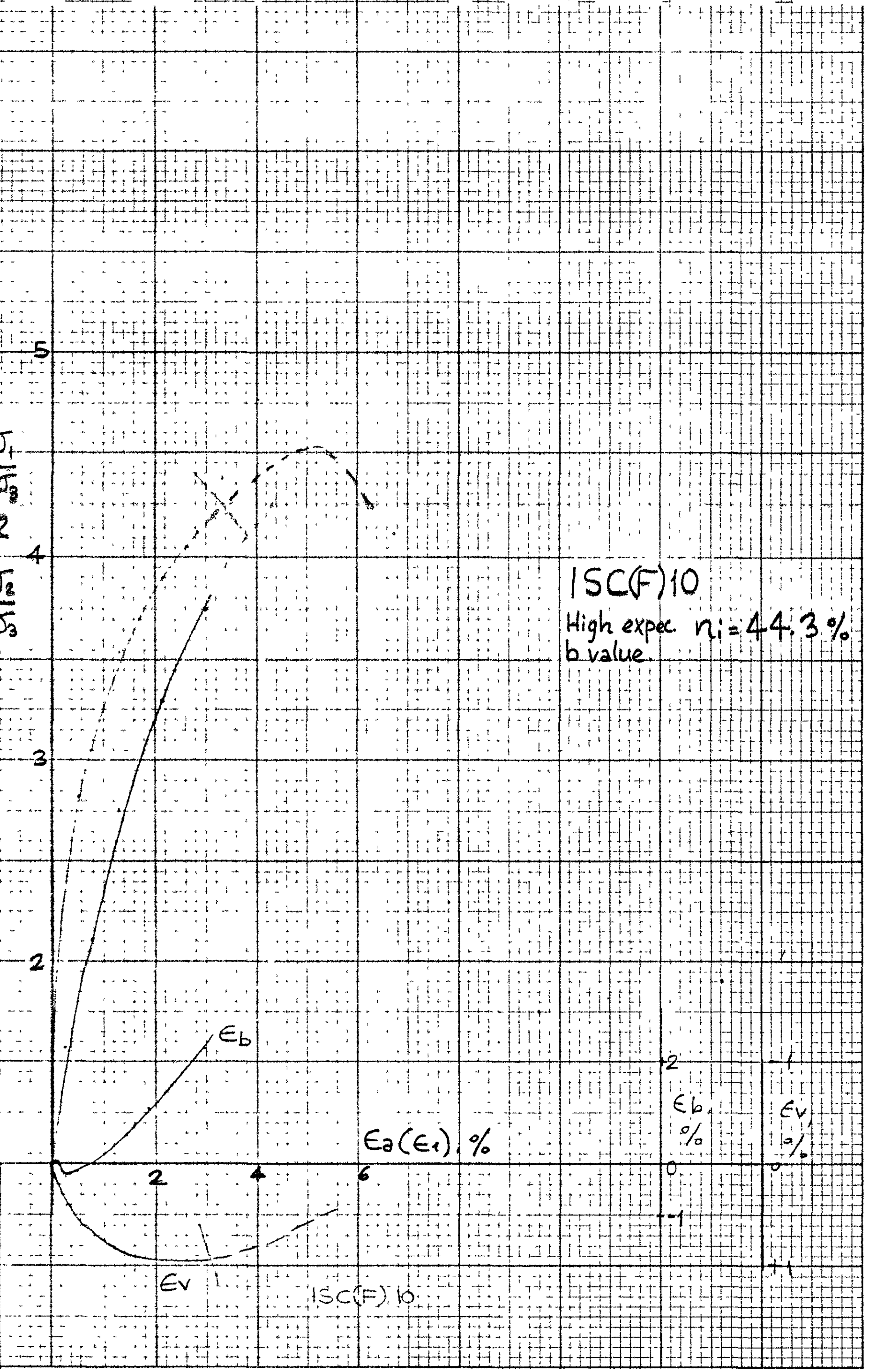


Fig. A.8.10

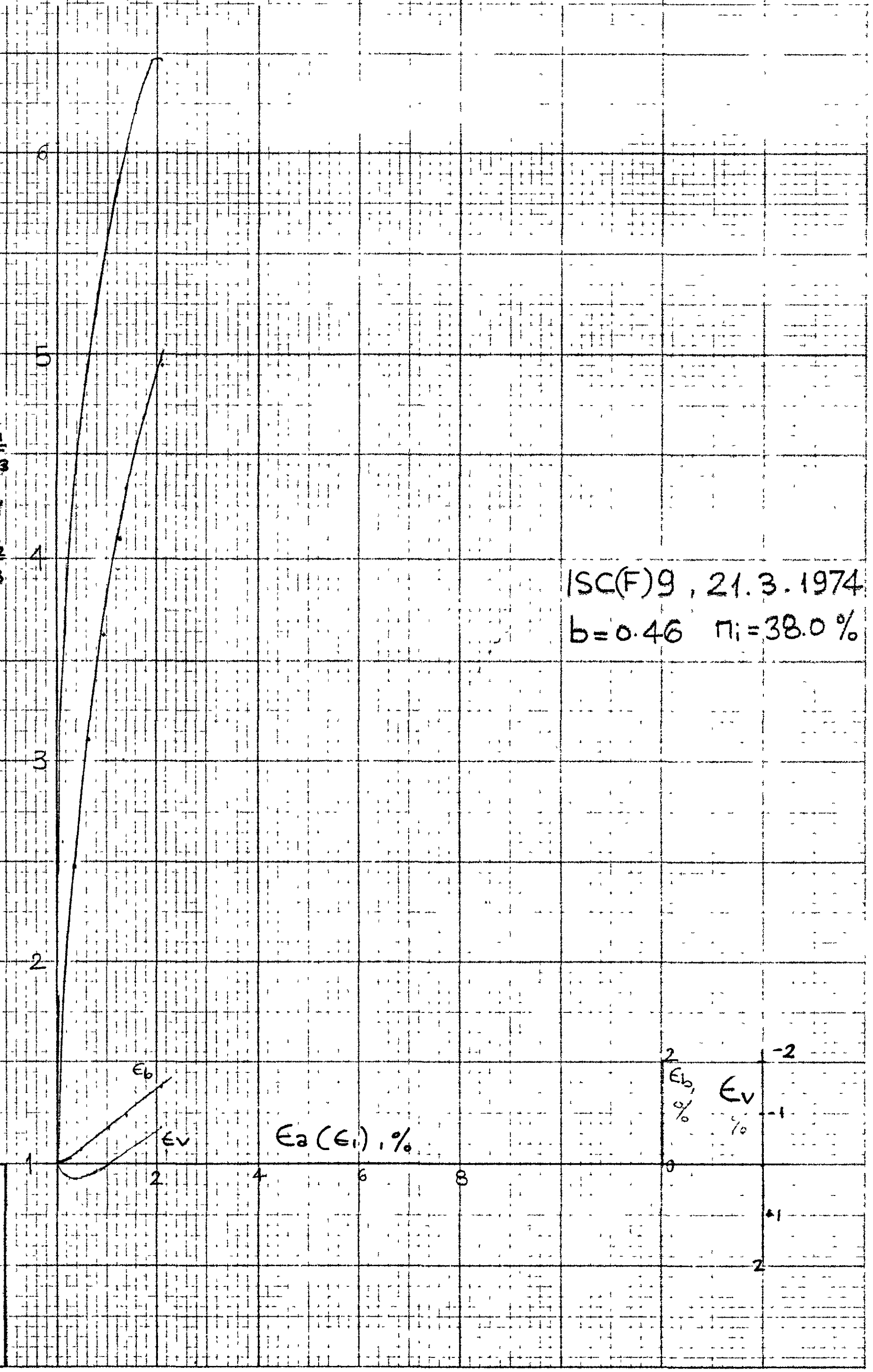


Fig. A.8.9

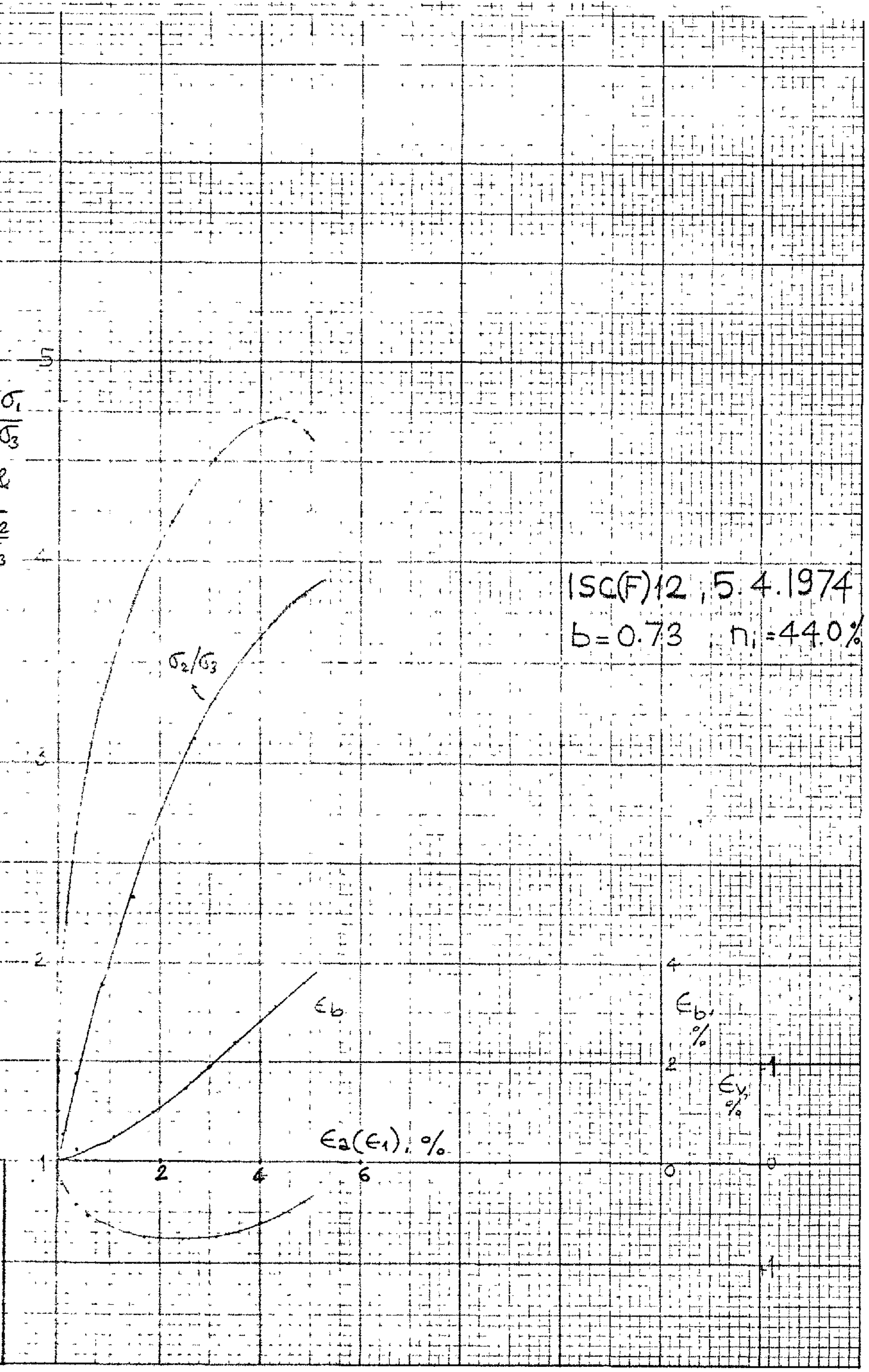


Fig. A.8.12

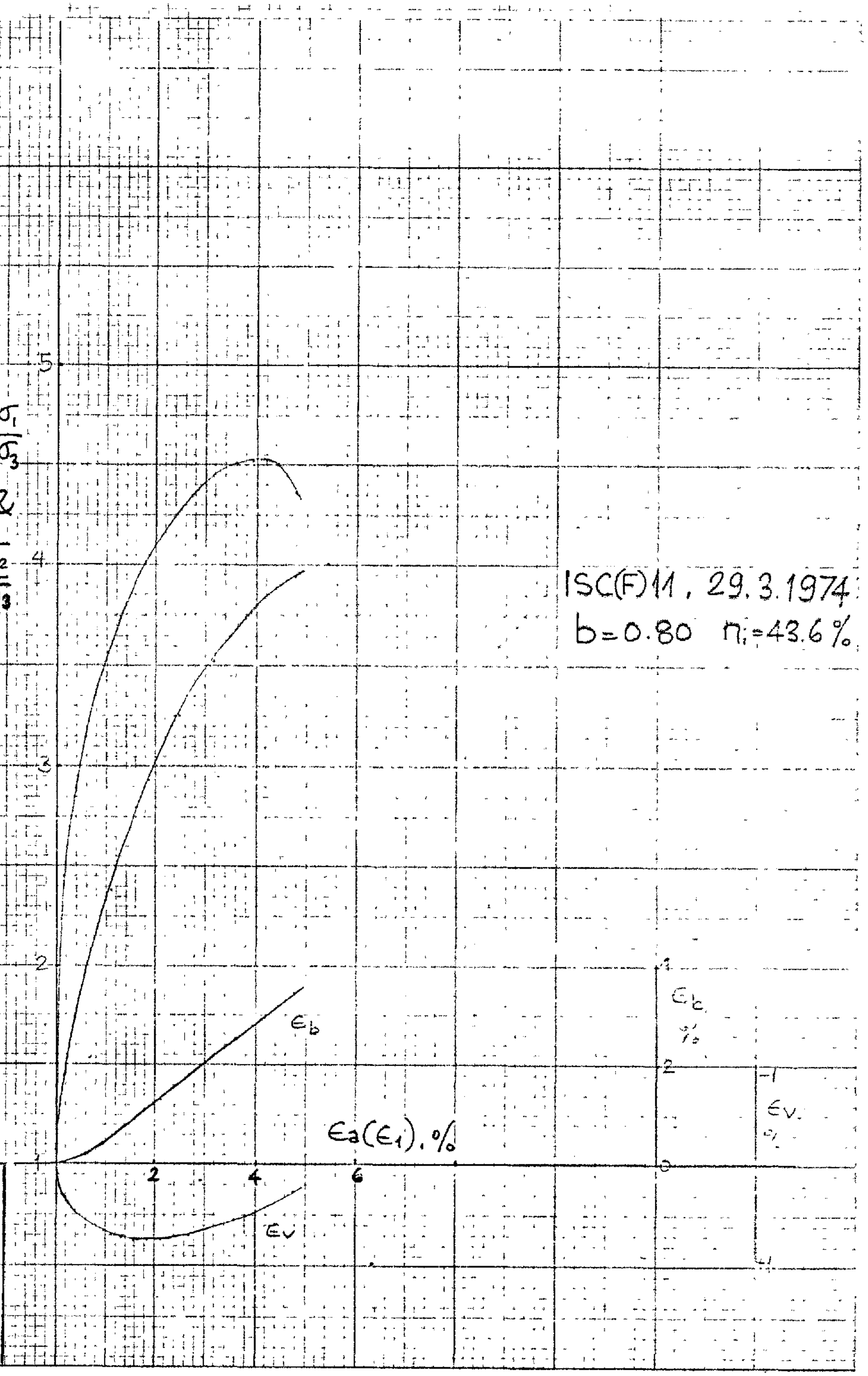
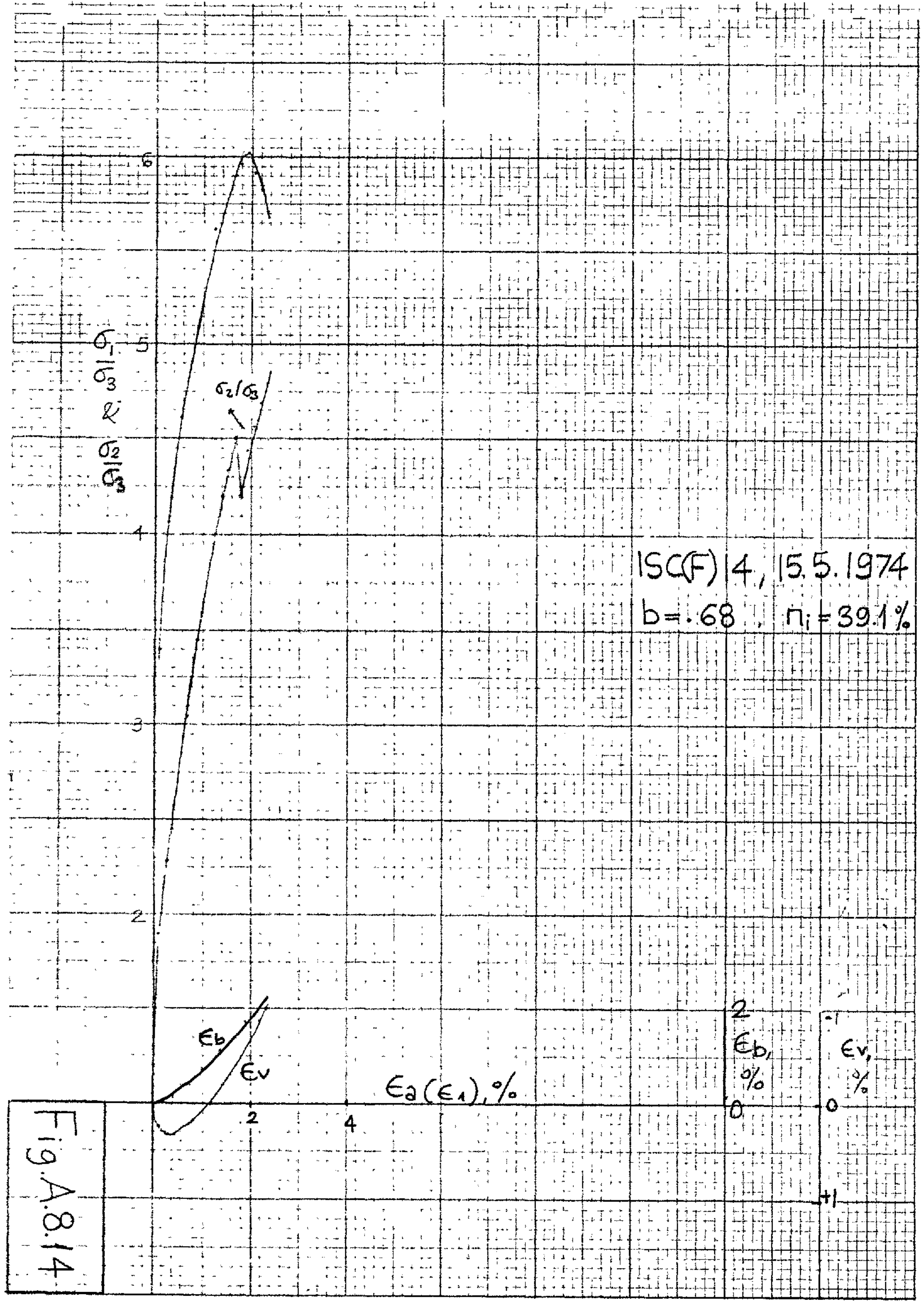
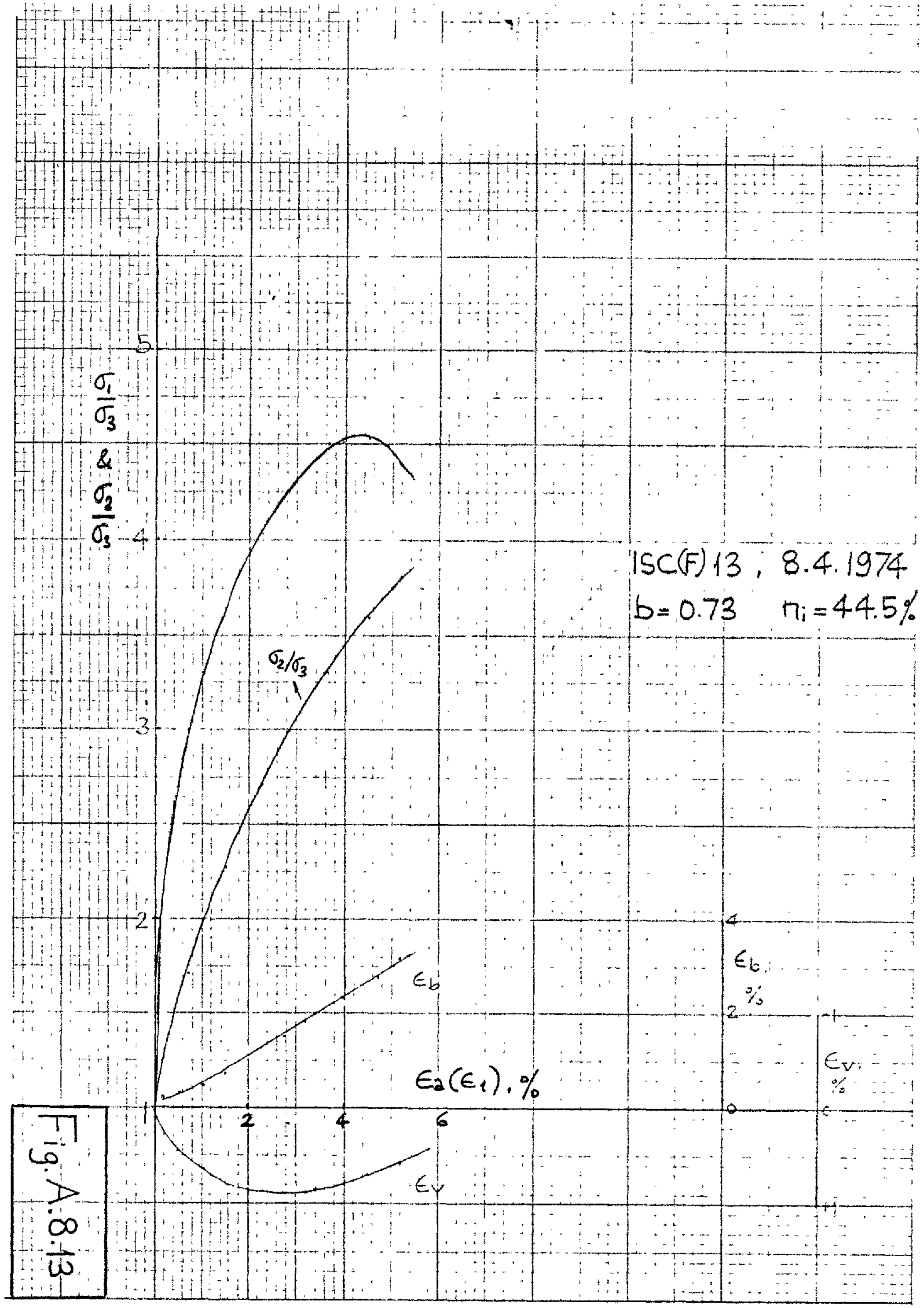


Fig. A.8.11



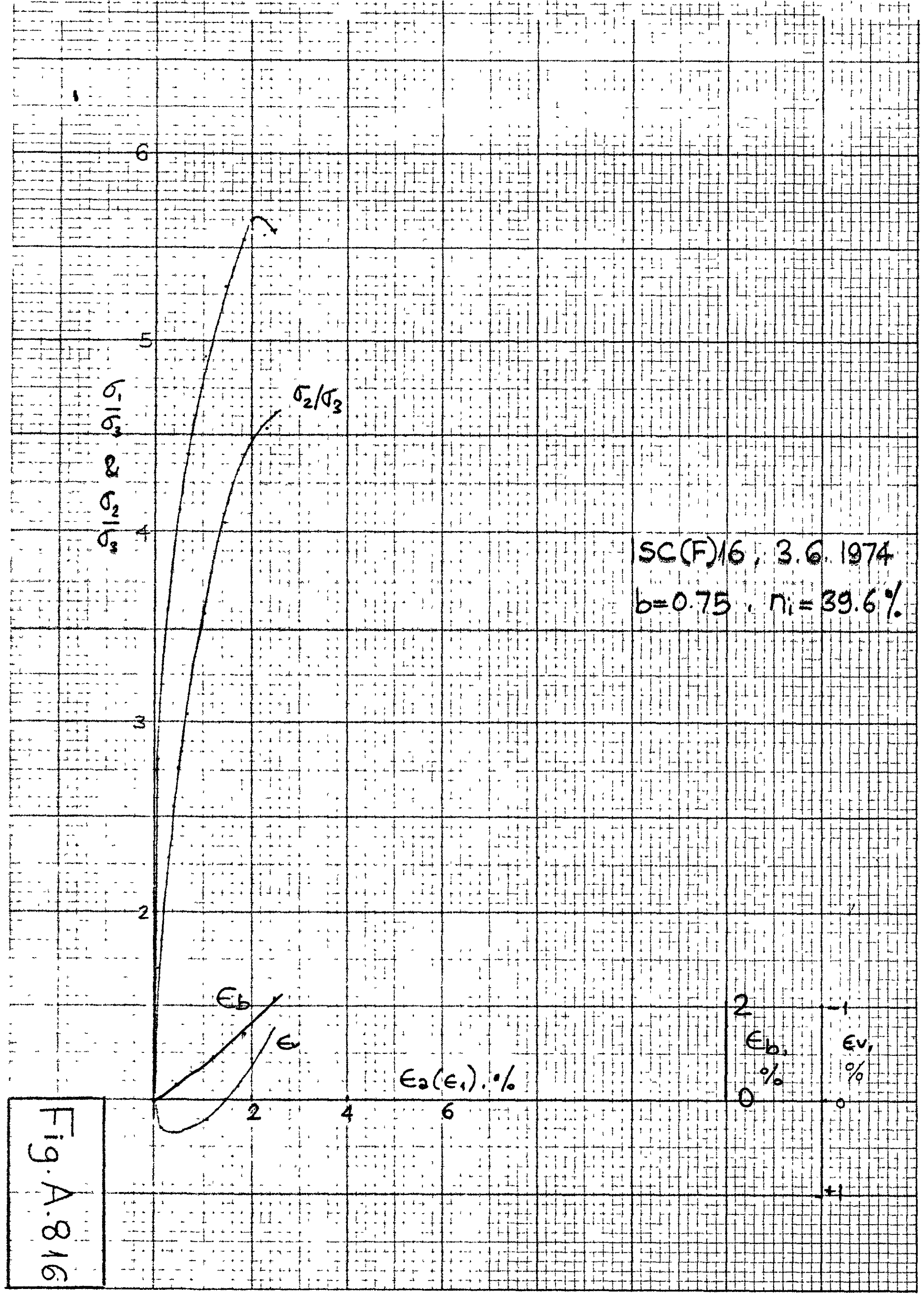
ISC(F) 4, 15.5.1974
b = .68, n_i = 39.1%

Fig. A.8.14



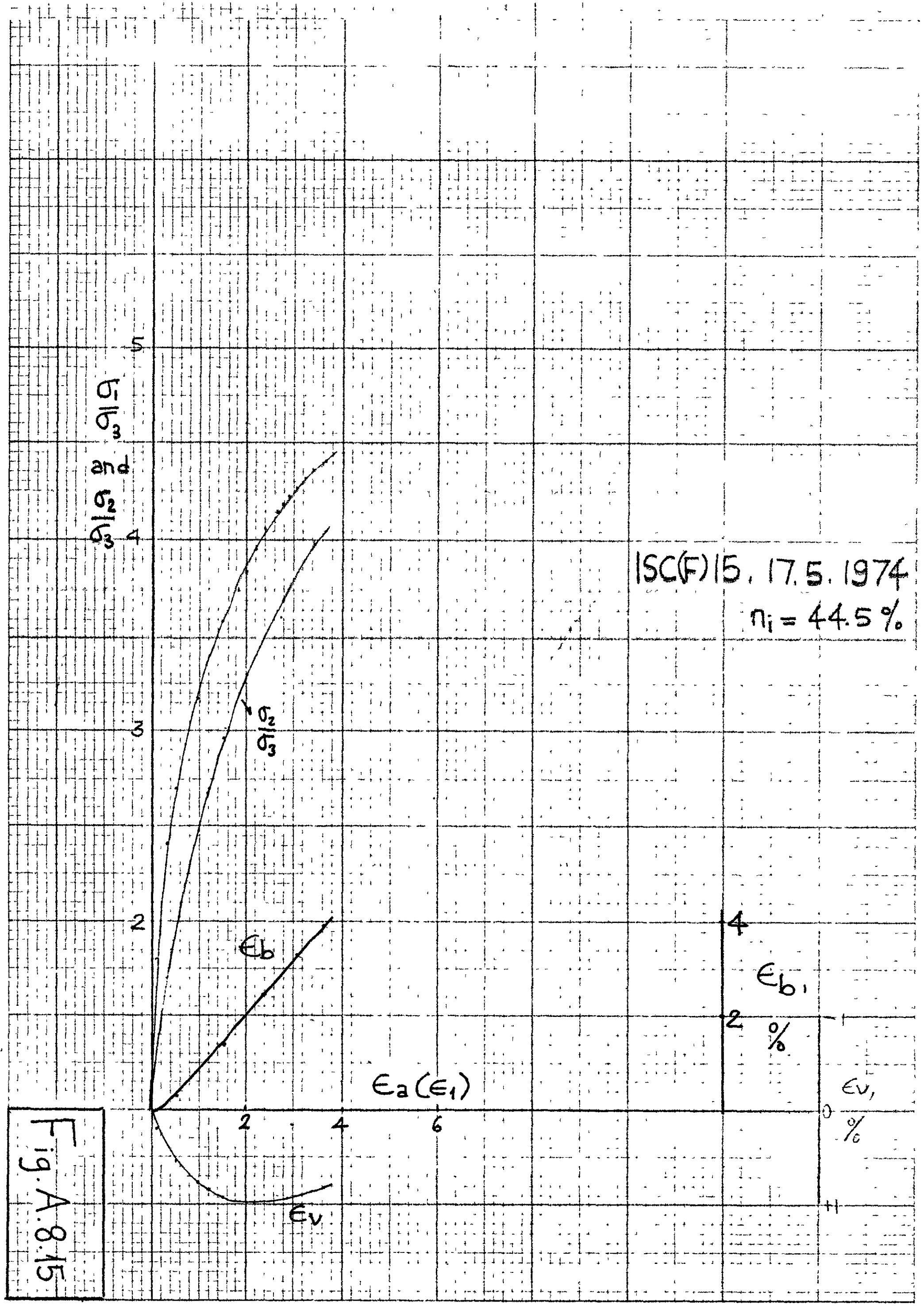
ISC(F) 13, 8.4.1974
b = 0.73 n_i = 44.5%

Fig. A.8.13



ISC(F)16, 3.6.1974
$b = 0.75$, $n_i = 39.6\%$

Fig. A.8.16



ISC(F)15, 17.5.1974
$n_i = 44.5\%$

Fig. A.8.15

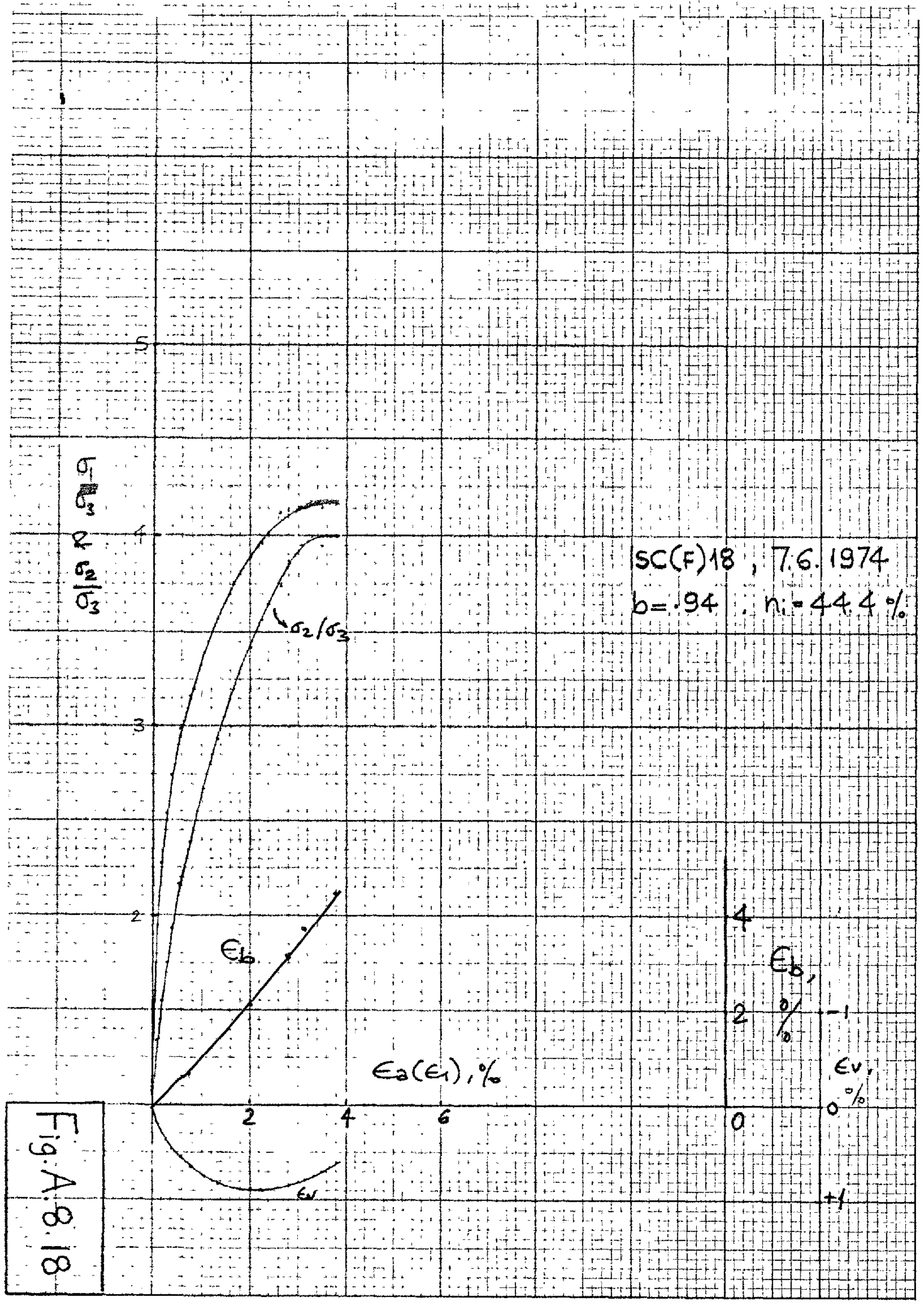


Fig.A.8.18

ISC(F)18, 7.6.1974

b=.94, n_i=44.4%

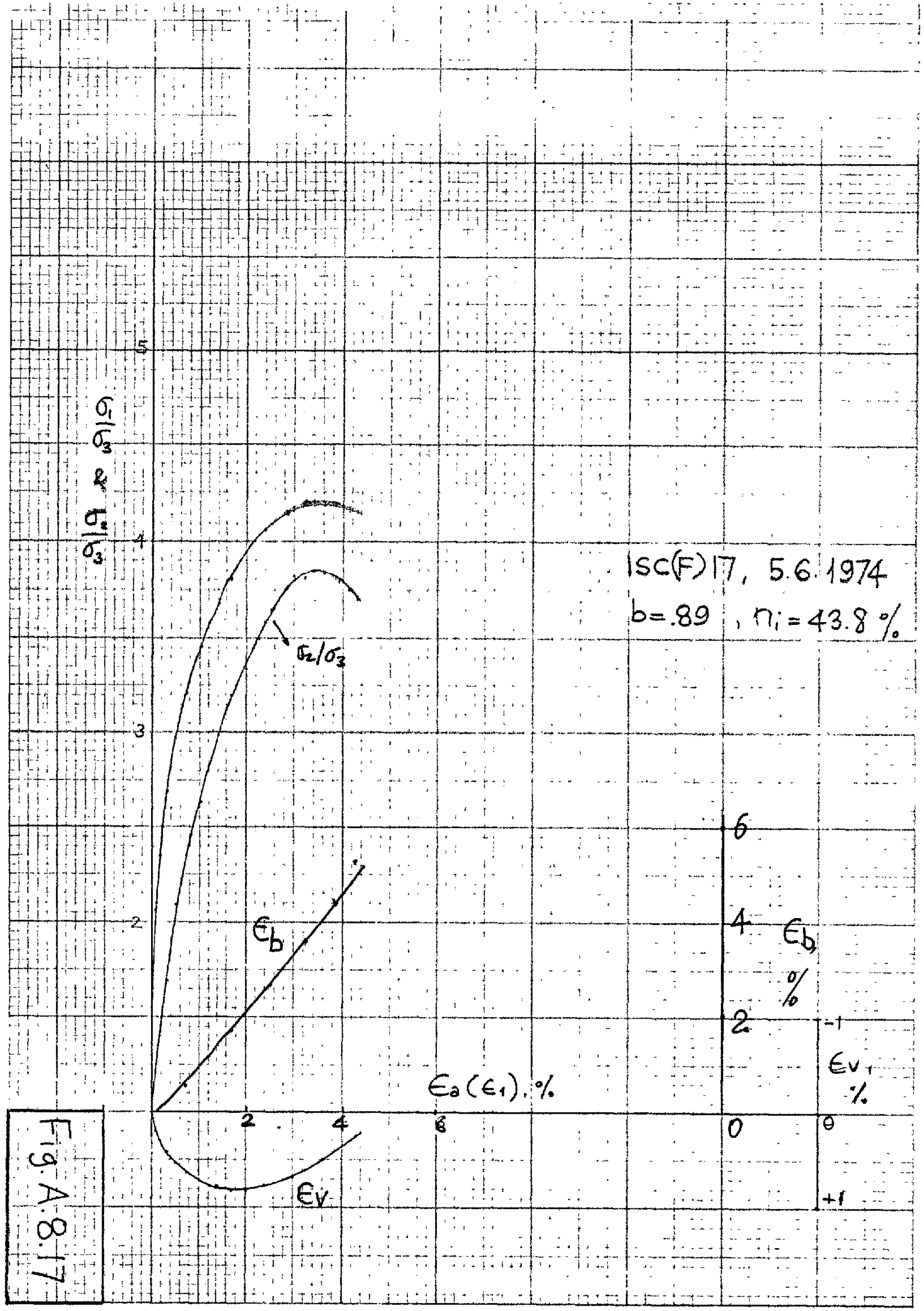
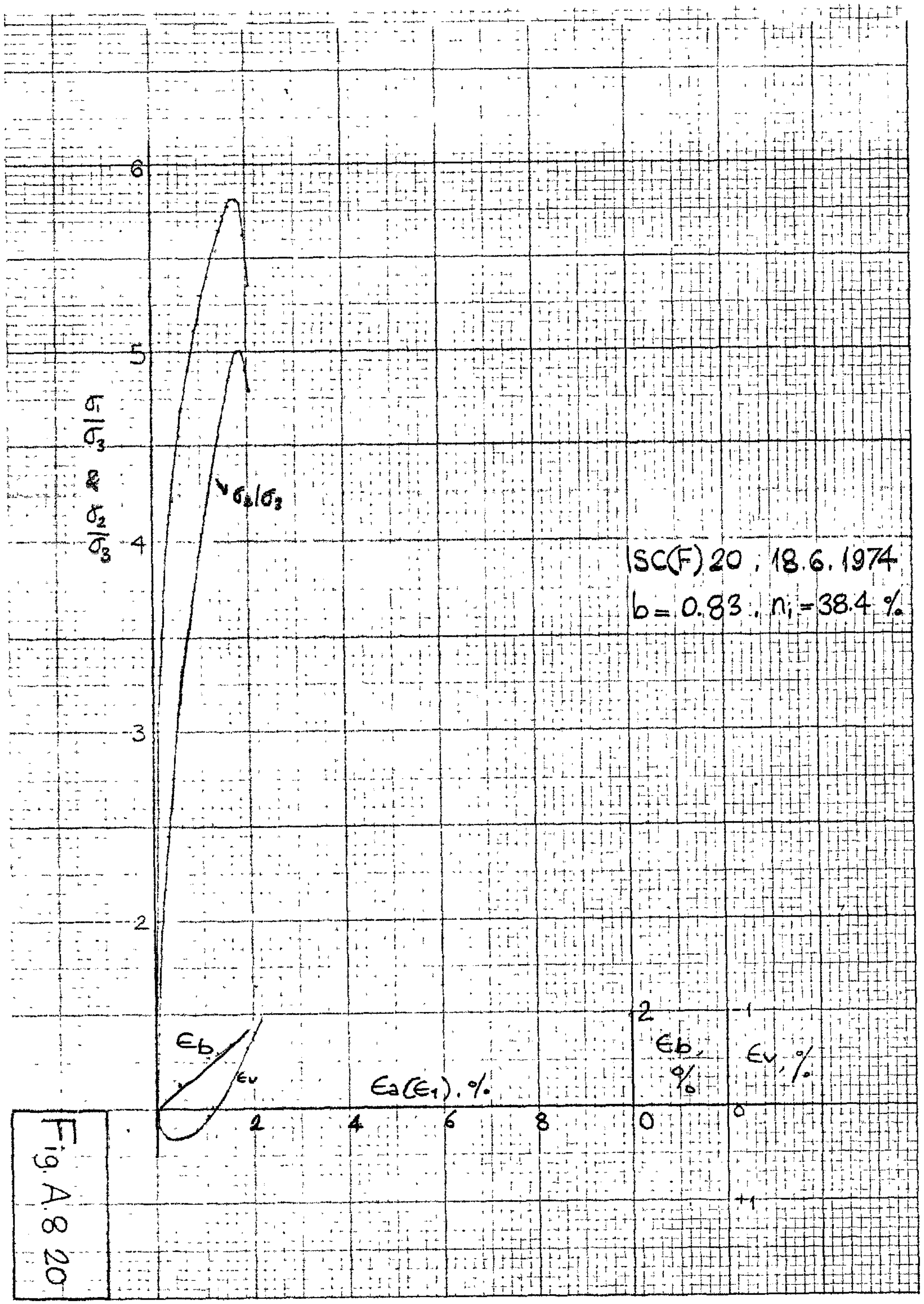


Fig.A.8.17

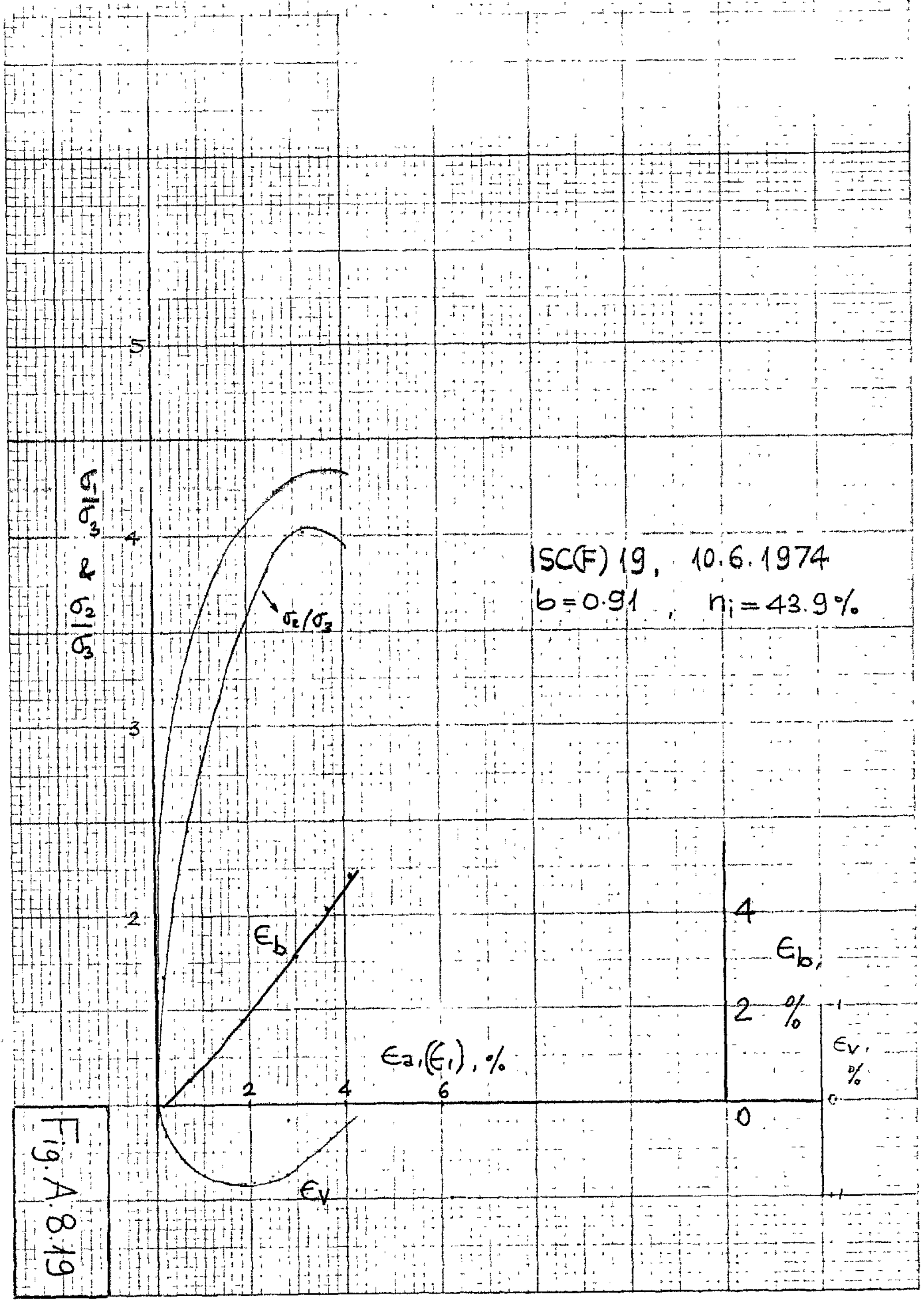
ISC(F)17, 5.6.1974

b=.89, n_i=43.8%



ISC(F) 20, 18.6.1974
b = 0.83, n_i = 38.4 %

Fig. A.8.20



ISC(F) 19, 10.6.1974
b = 0.91, n_i = 43.9 %

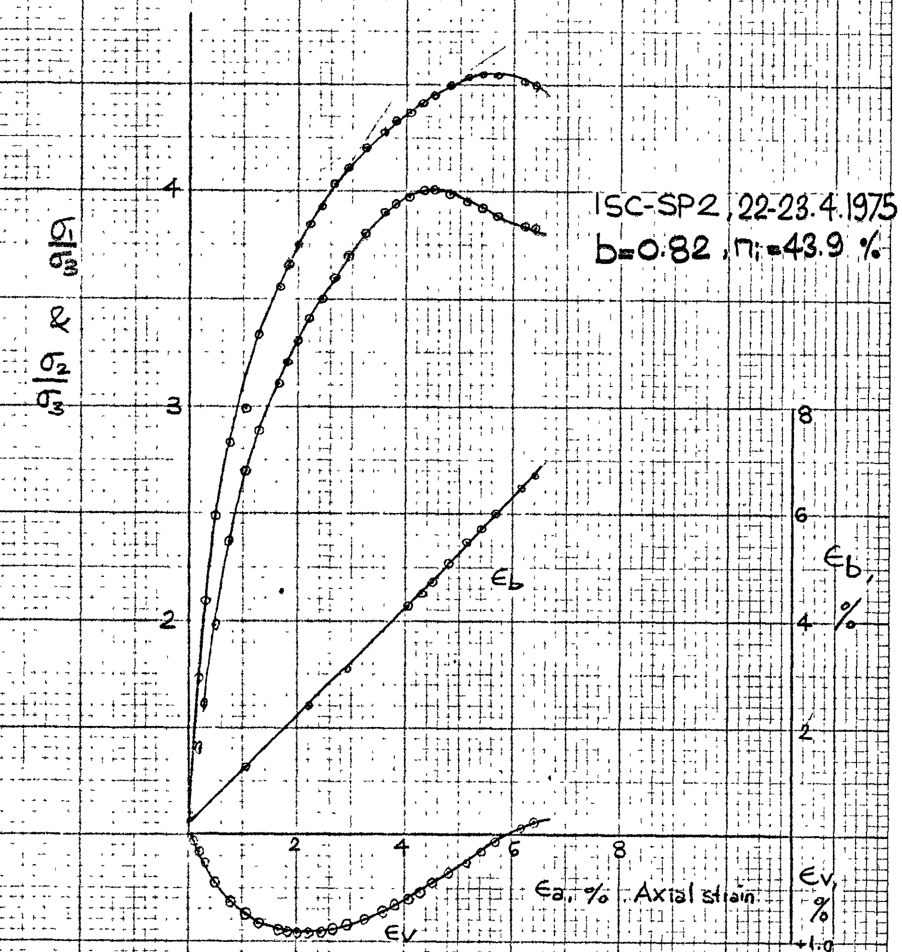
Fig. A.8.19

APPENDIX 9

STRESS-STRAIN CURVES FOR GENERALISED TESTS ON LOOSE HAM RIVER SAND SAMPLES. SPECIAL SERIES.

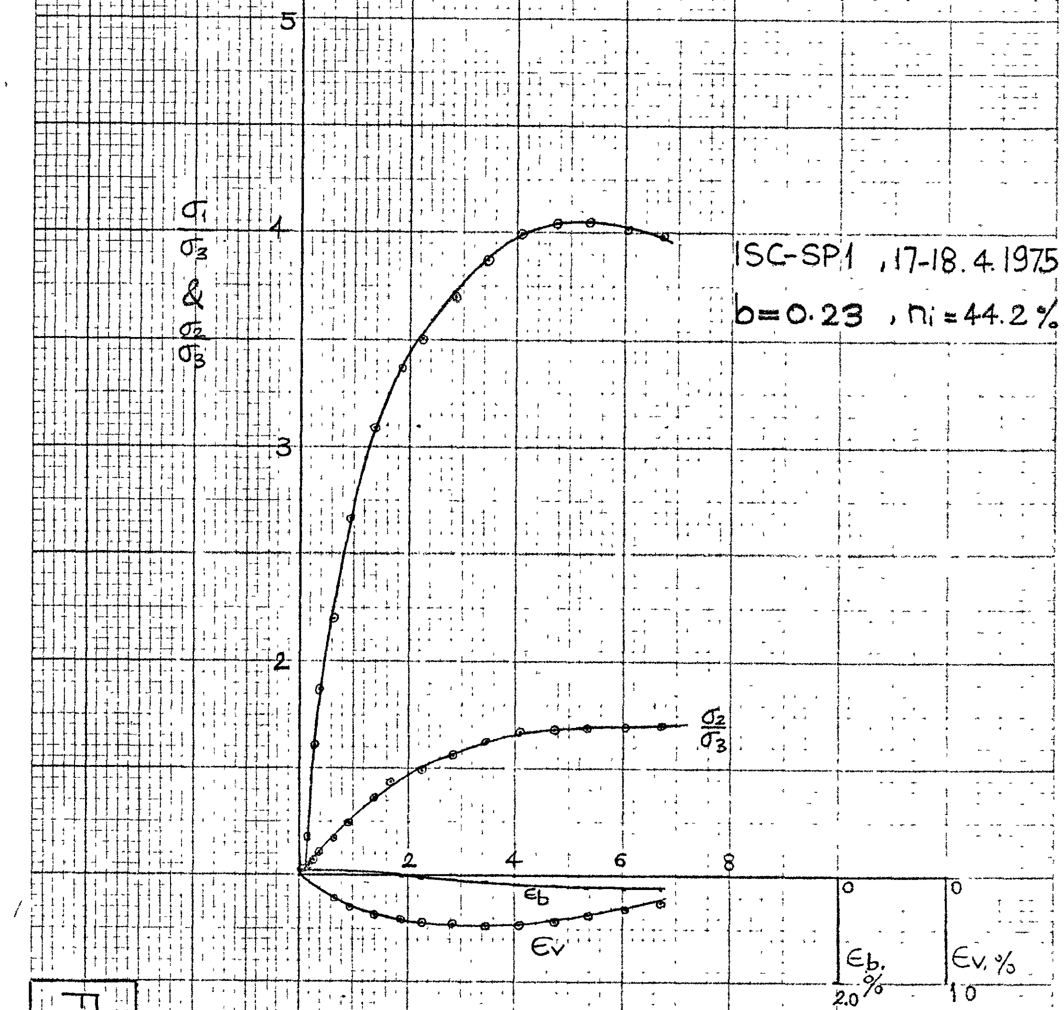
ISC SP1-16

(SP9 and SP13 are at the end)



ISC-SP2, 22-23.4.1975
$b=0.82$, $n_i=43.9$ %

Fig. A.9.2



ISC-SP1, 17-18.4.1975
$b=0.23$, $n_i=44.2$ %

Fig. A.9.1

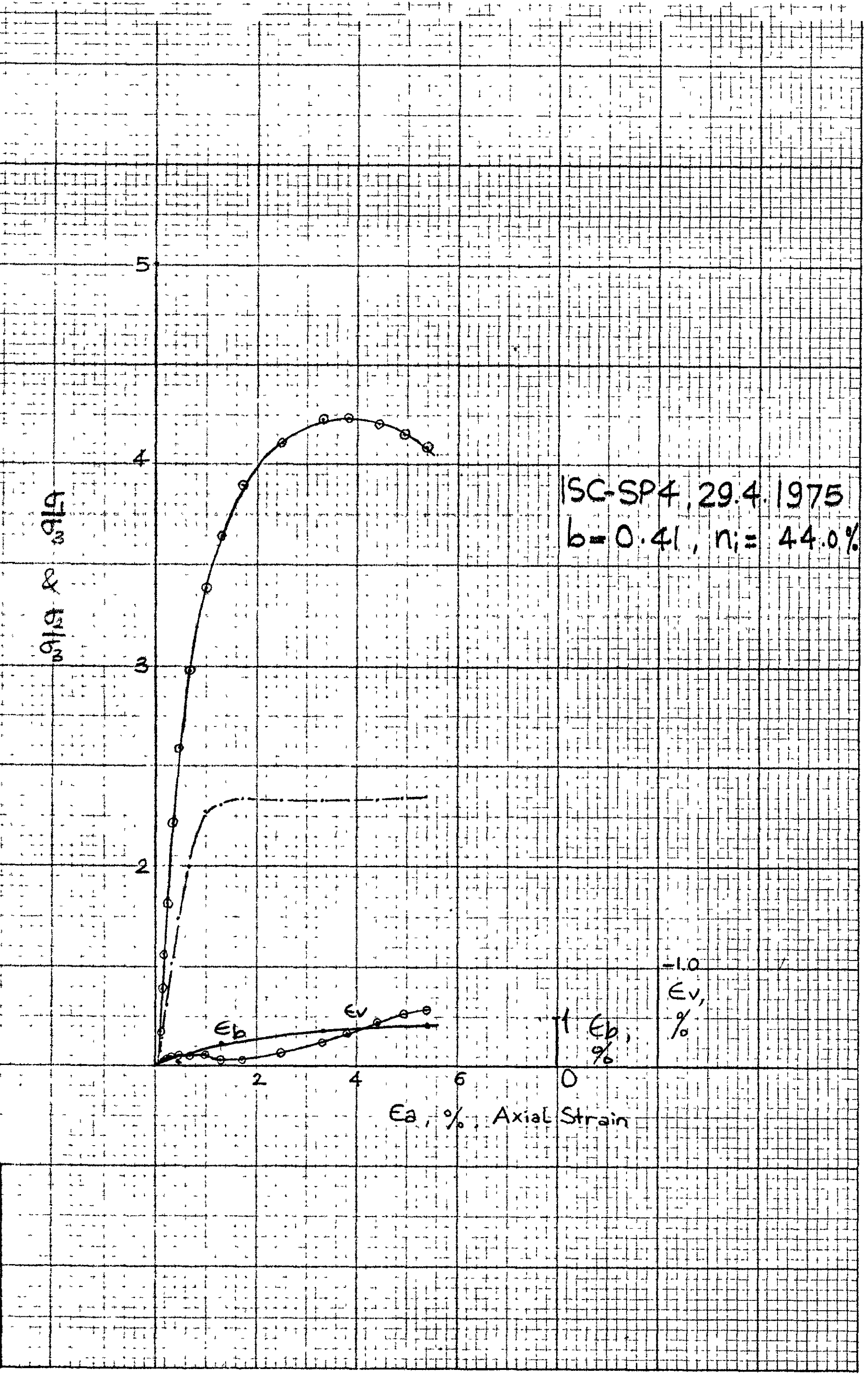


Fig. A.9.4

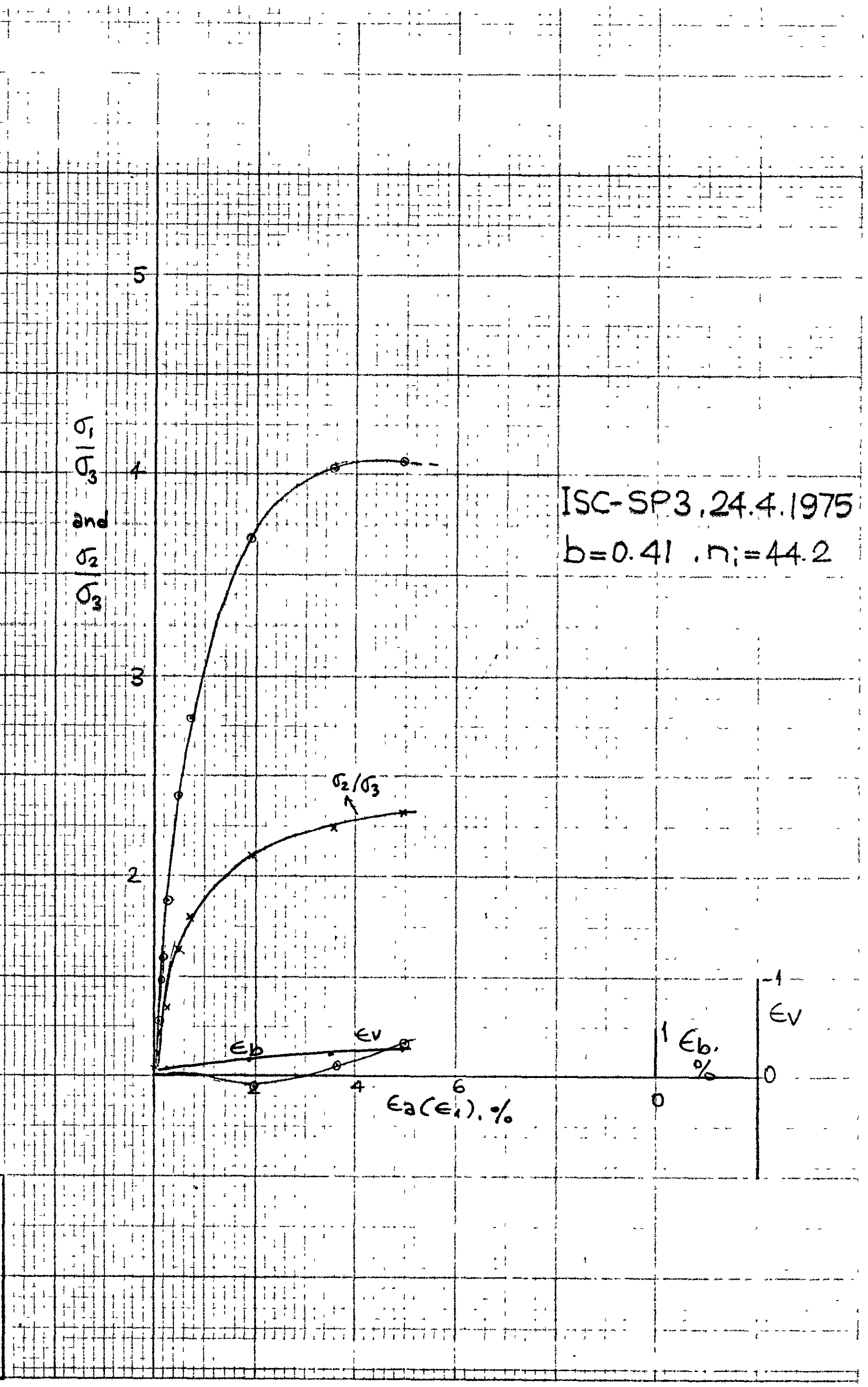


Fig. A.9.3

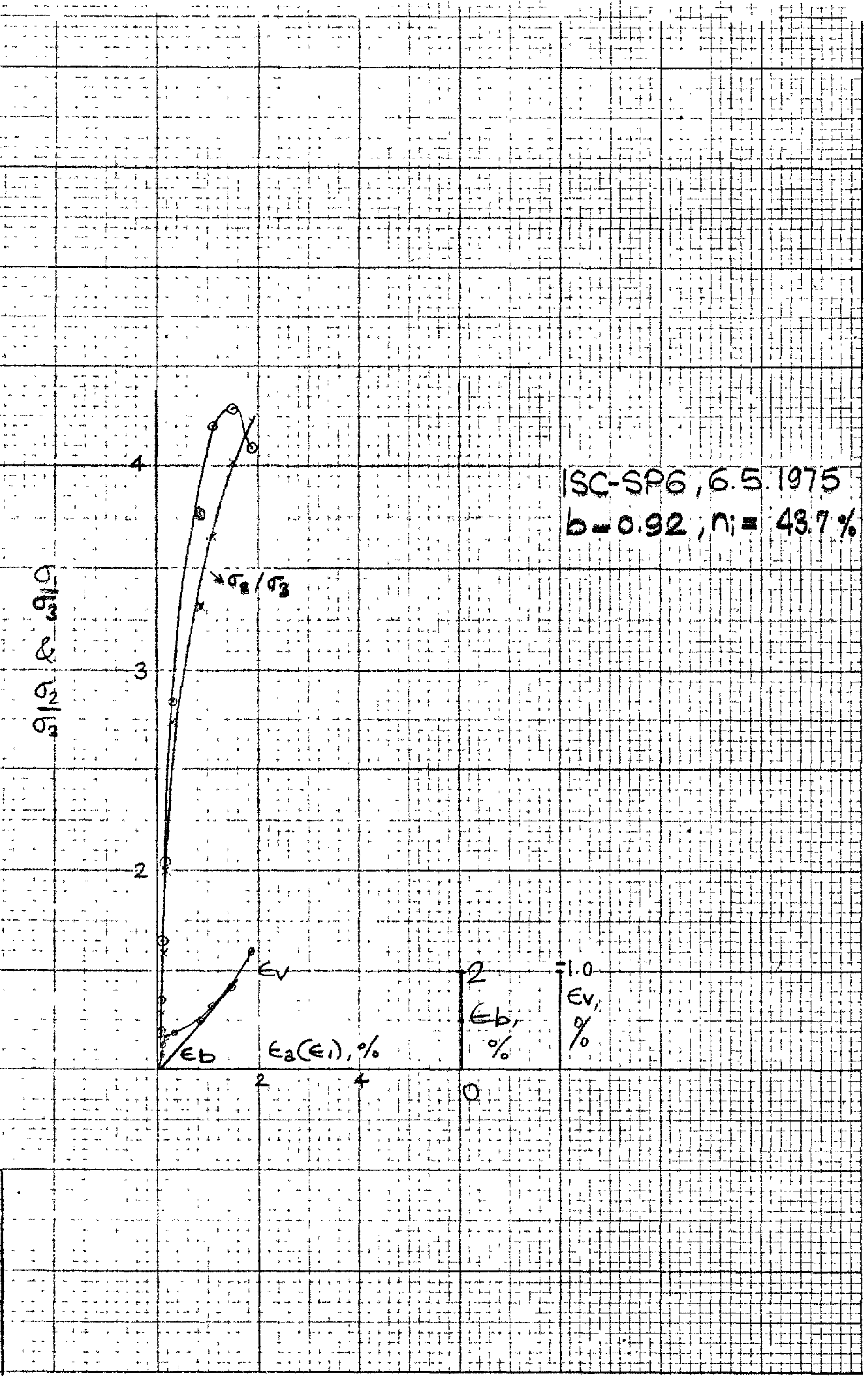


Fig. A.9.6

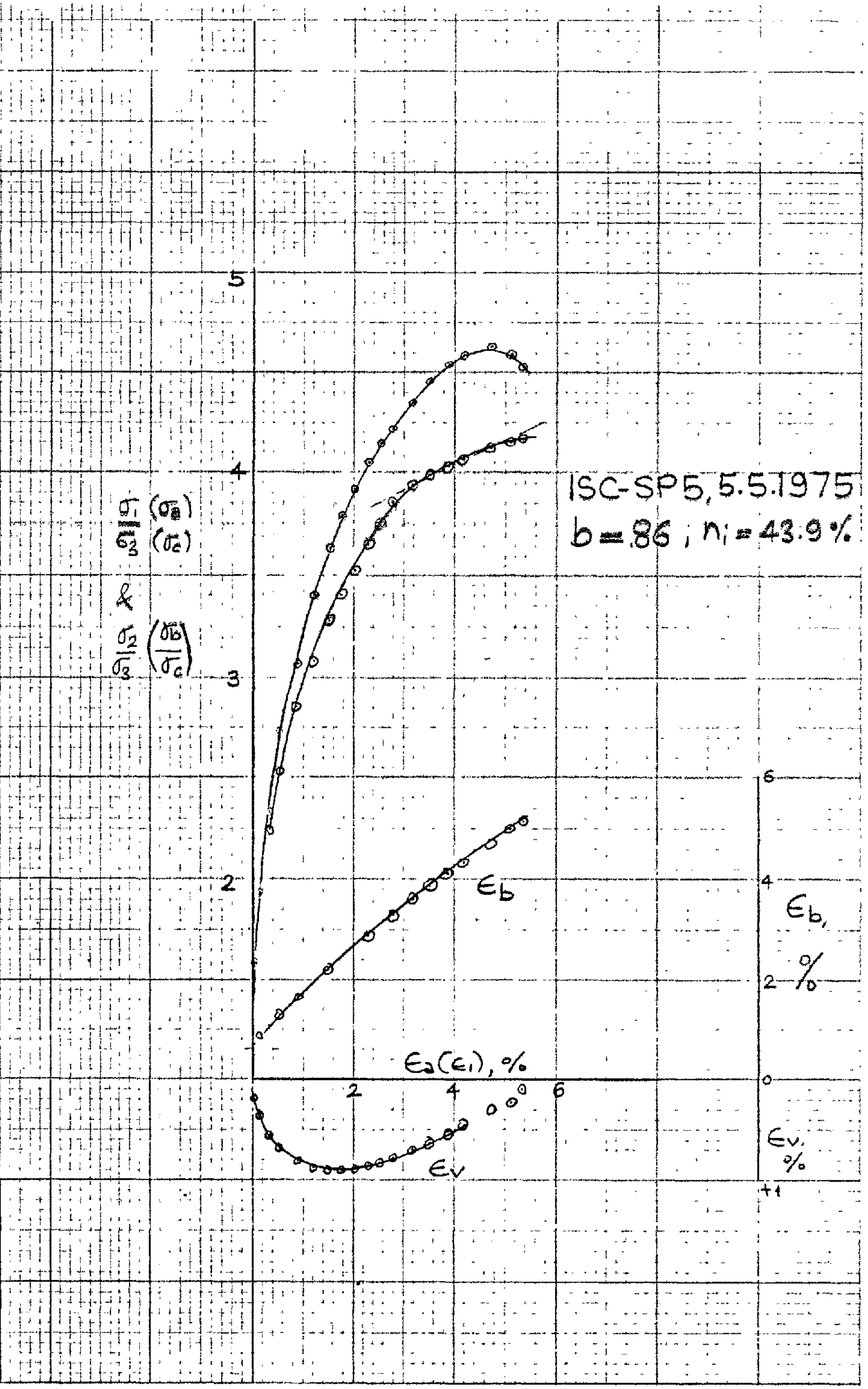


Fig. A.9.5

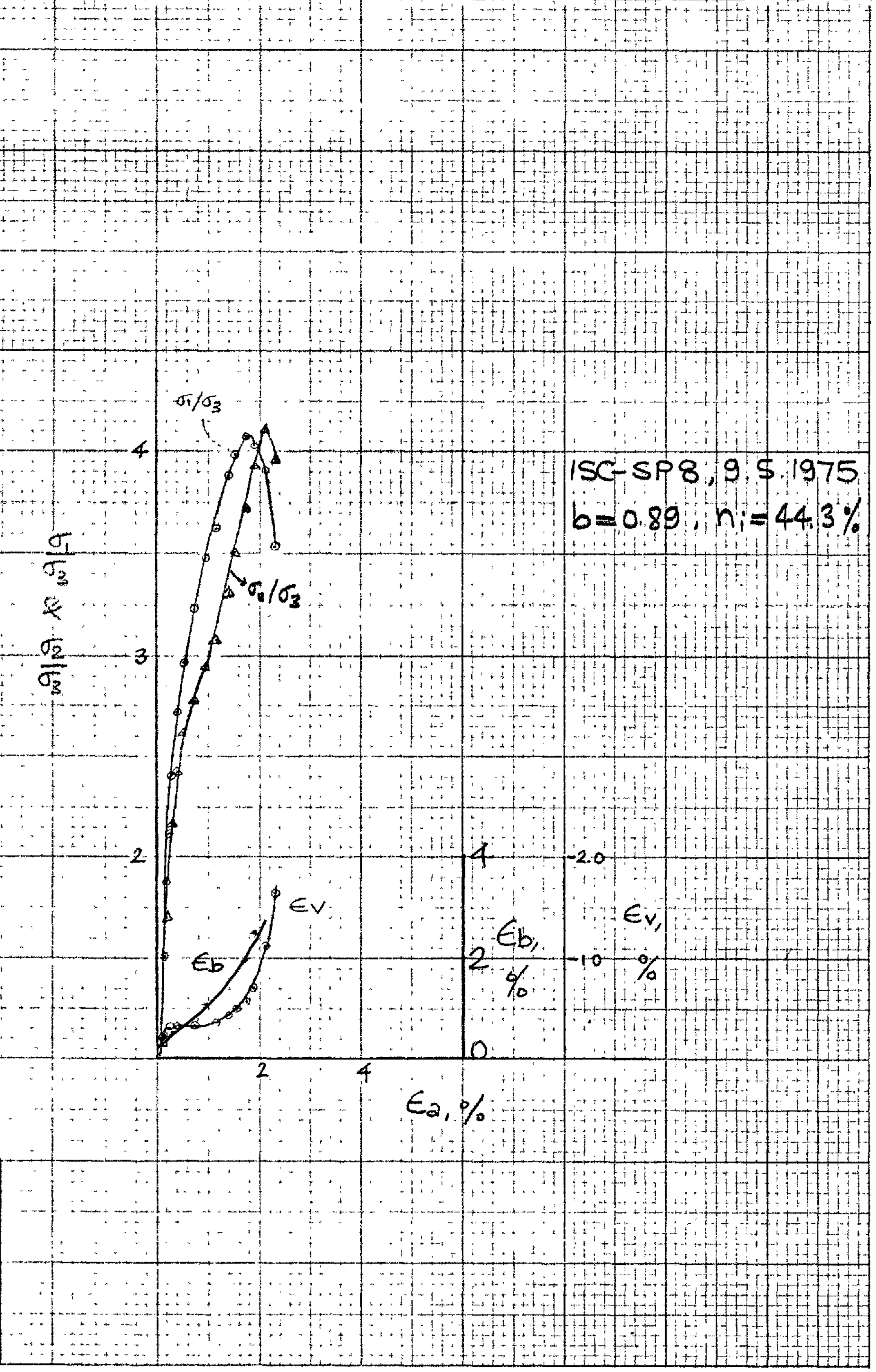


Fig. A.9.8

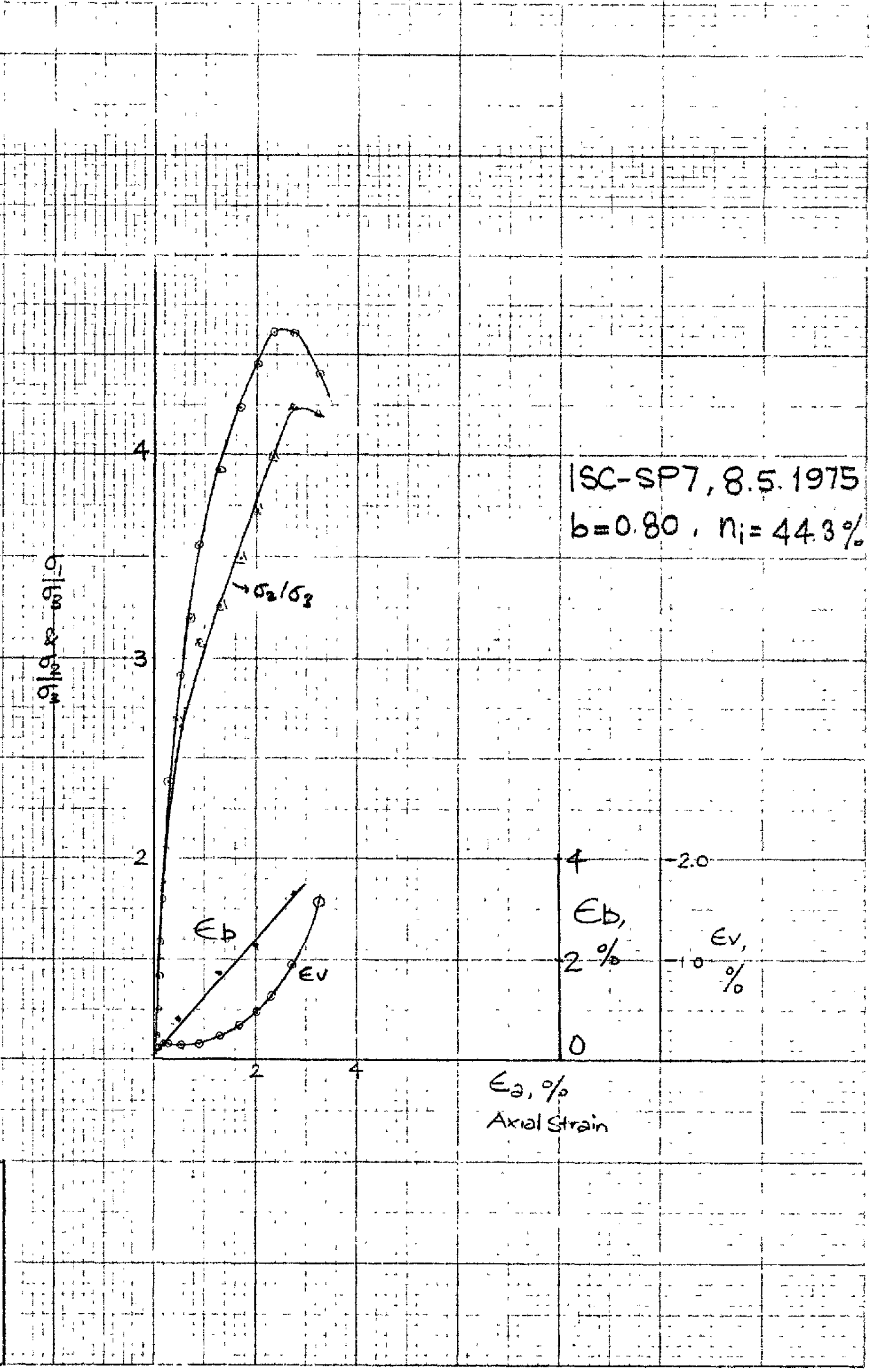


Fig. A.9.7

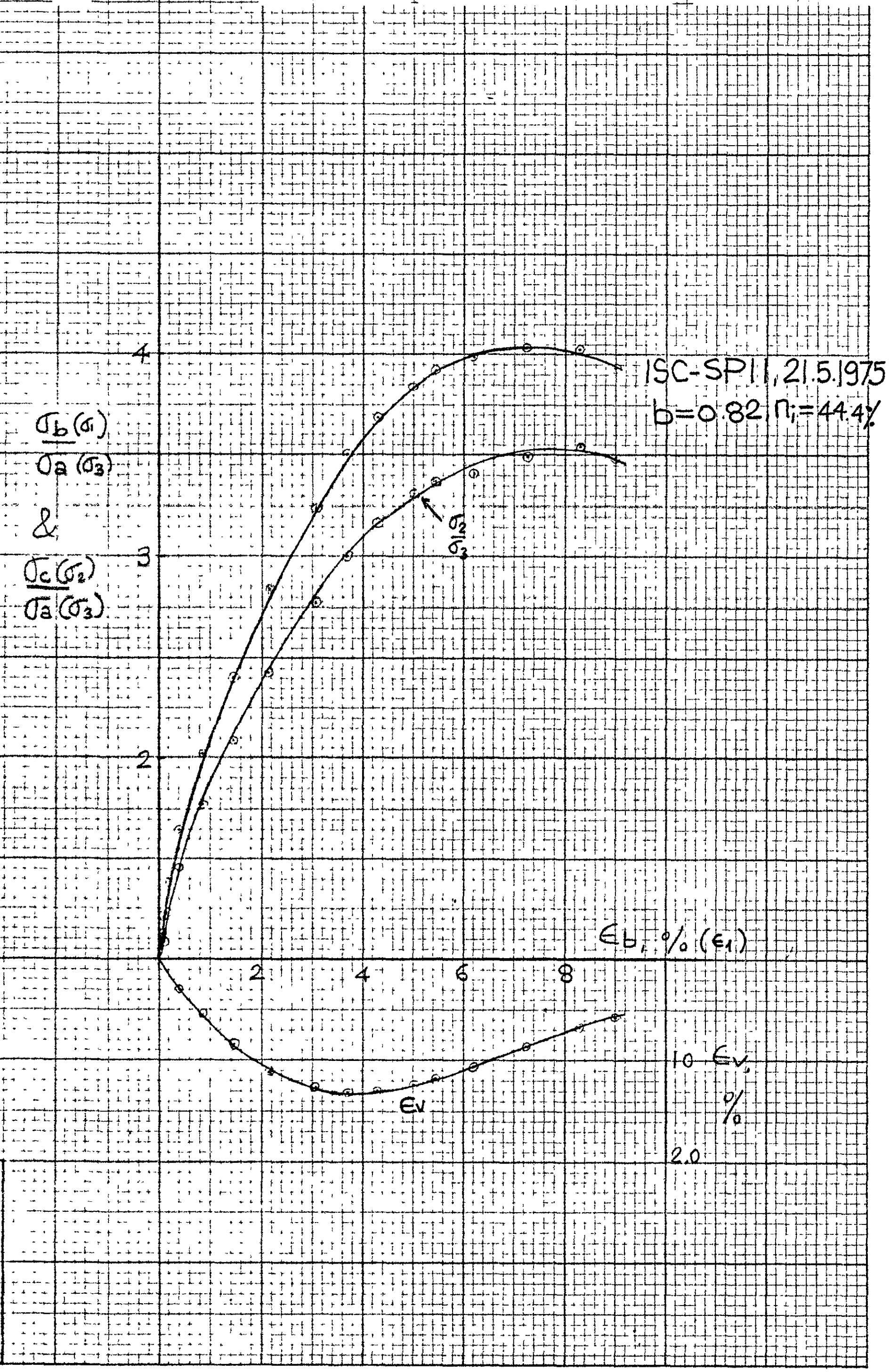


Fig. A.9.10

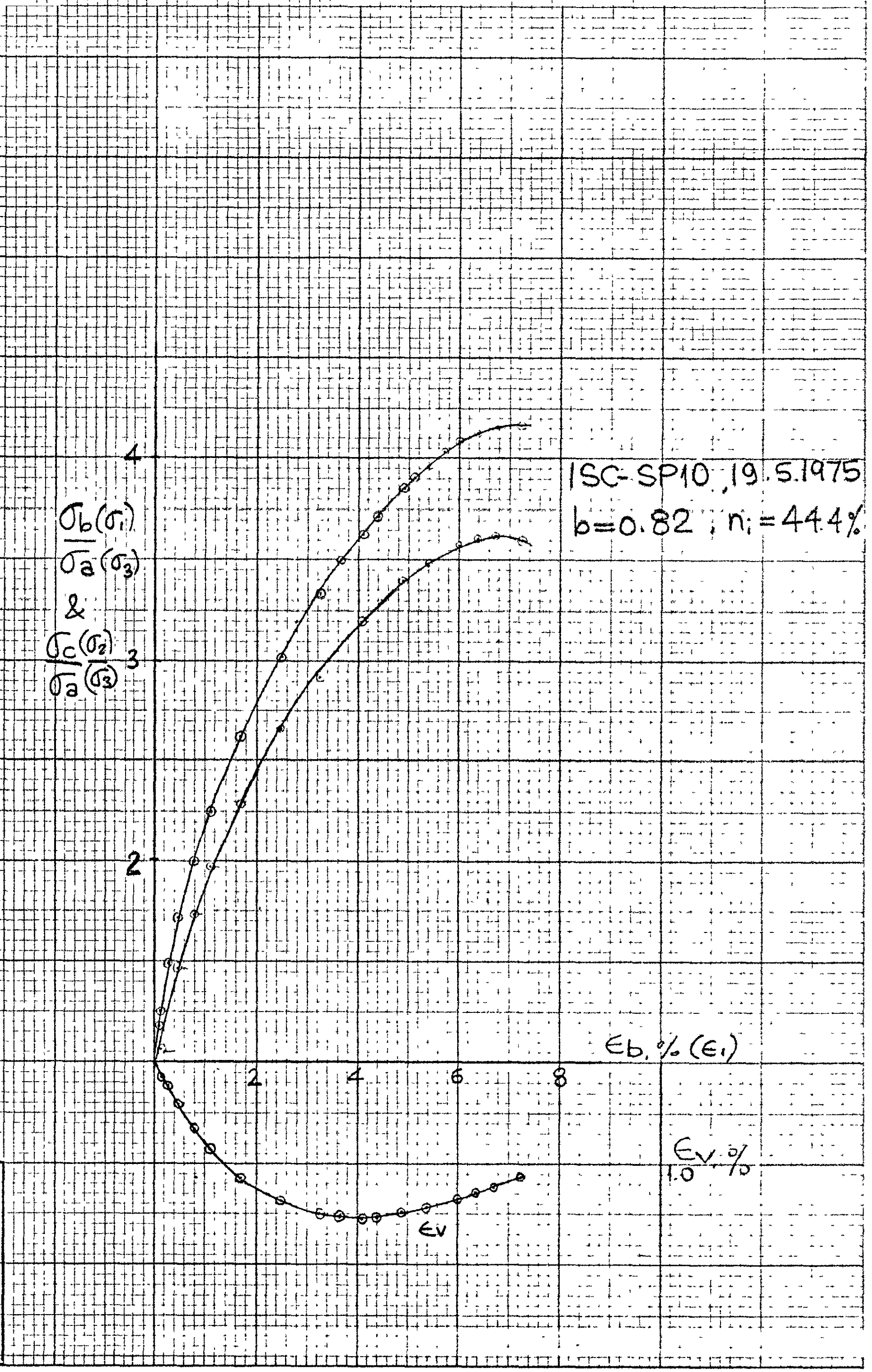


Fig. A.9.9

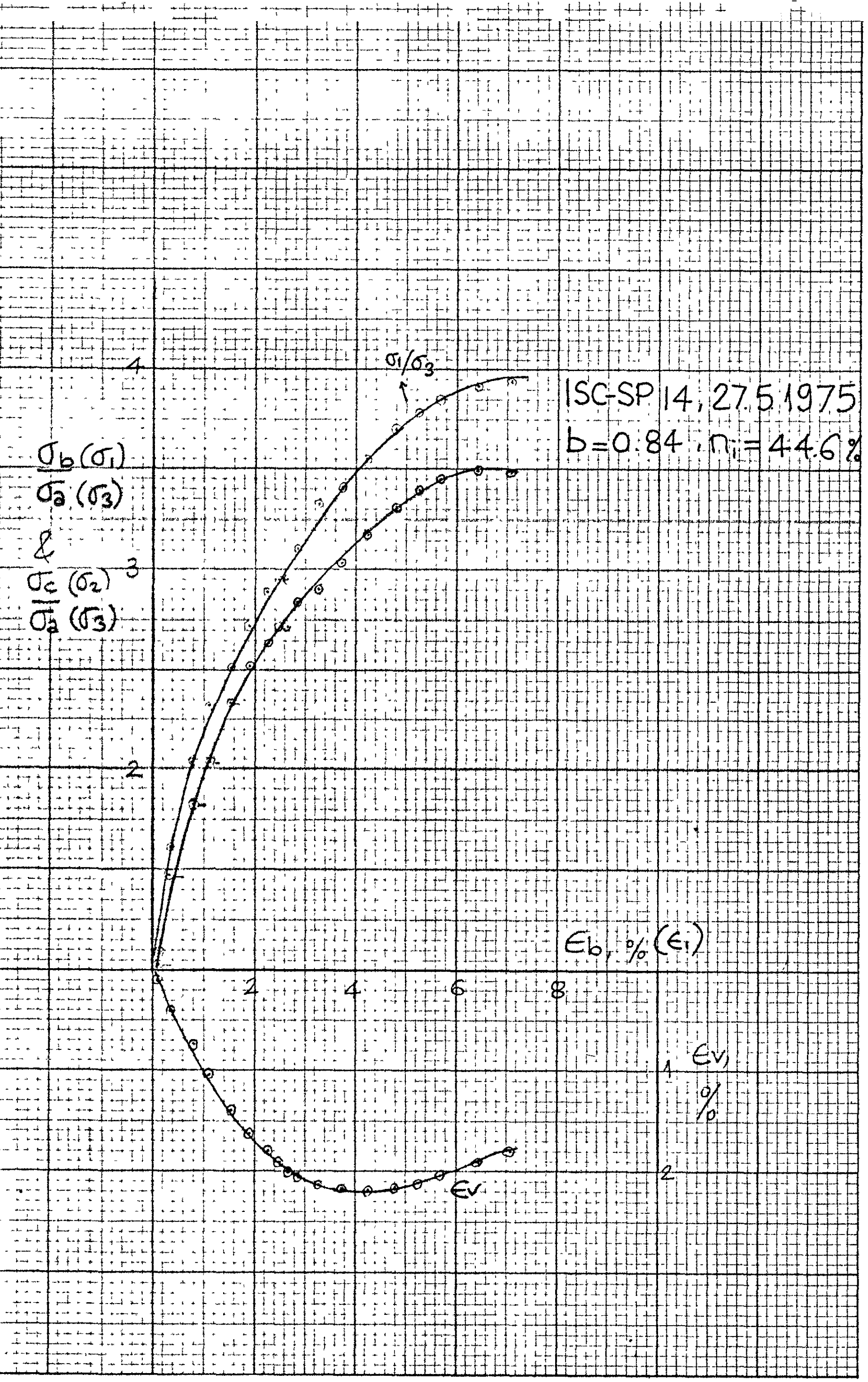


Fig. A.9.12

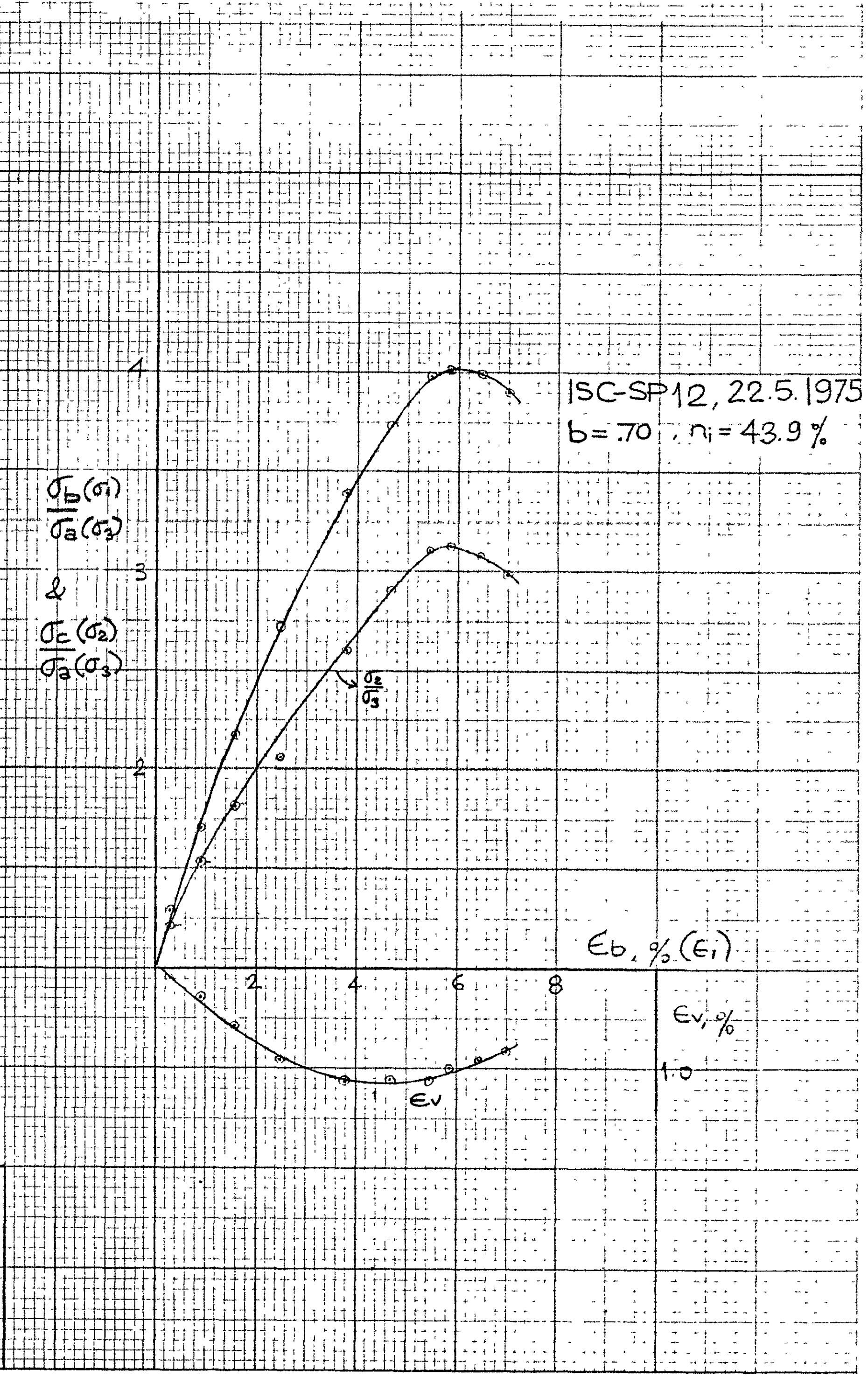


Fig. A.9.11

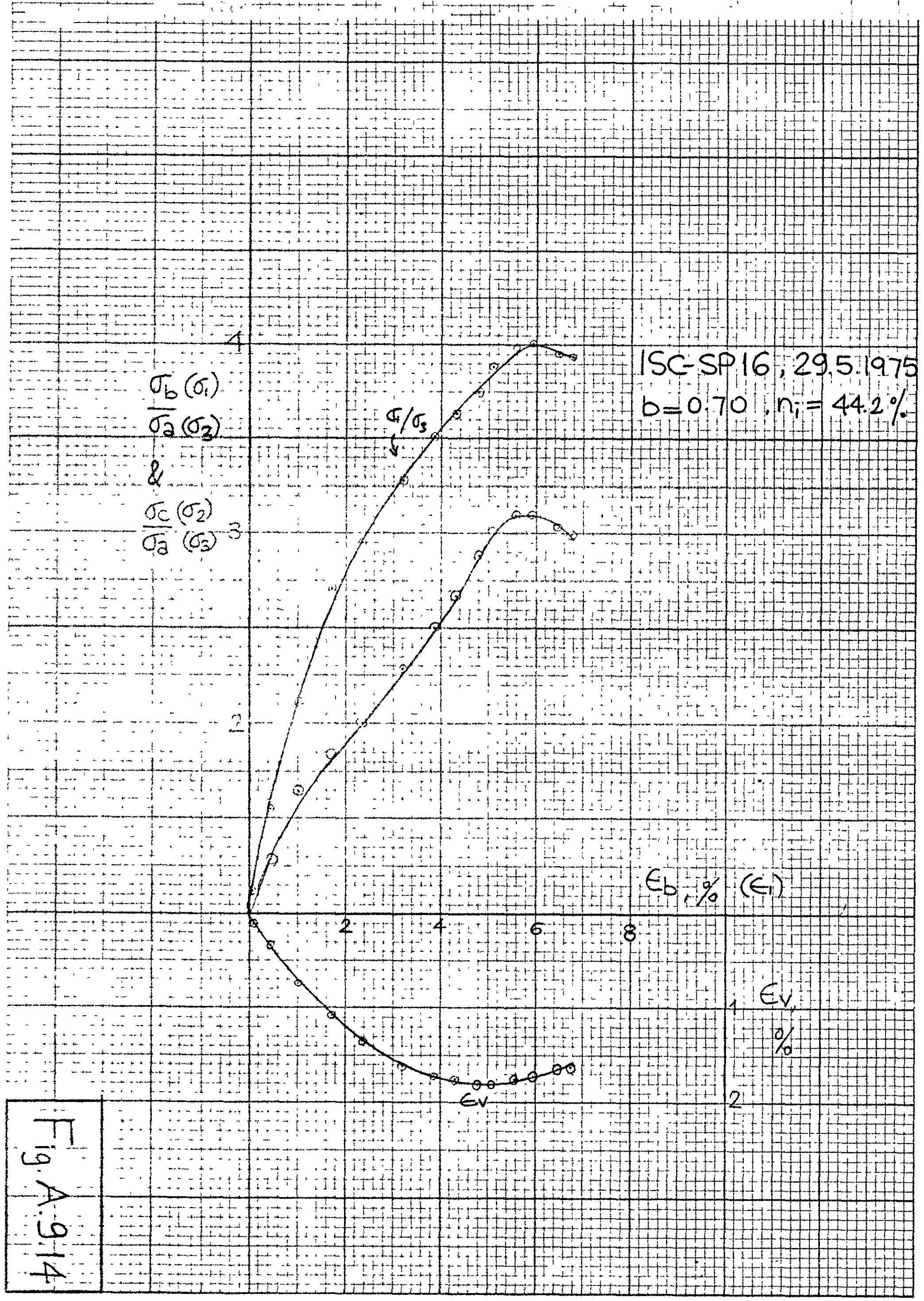


Fig. A.9.14

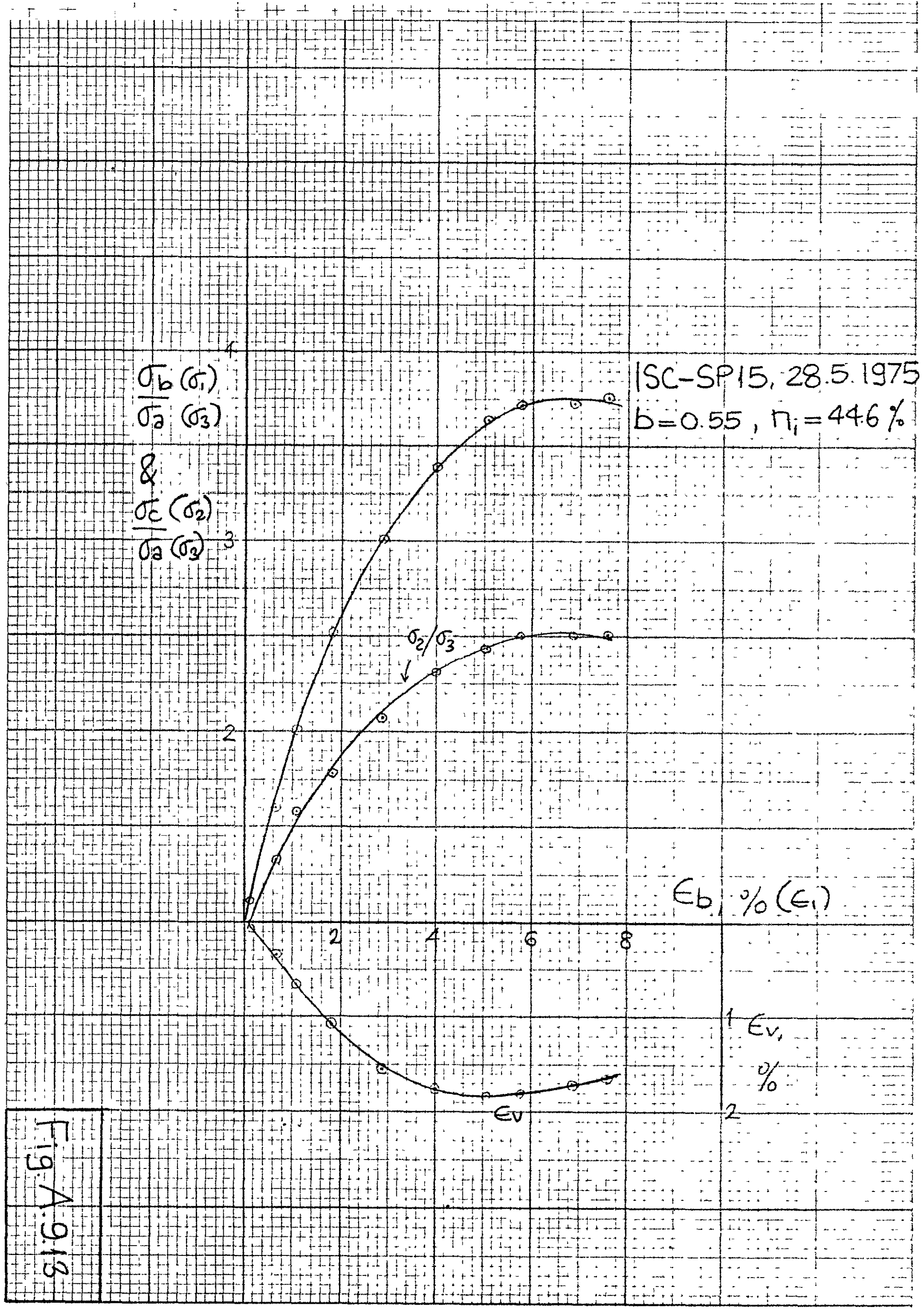
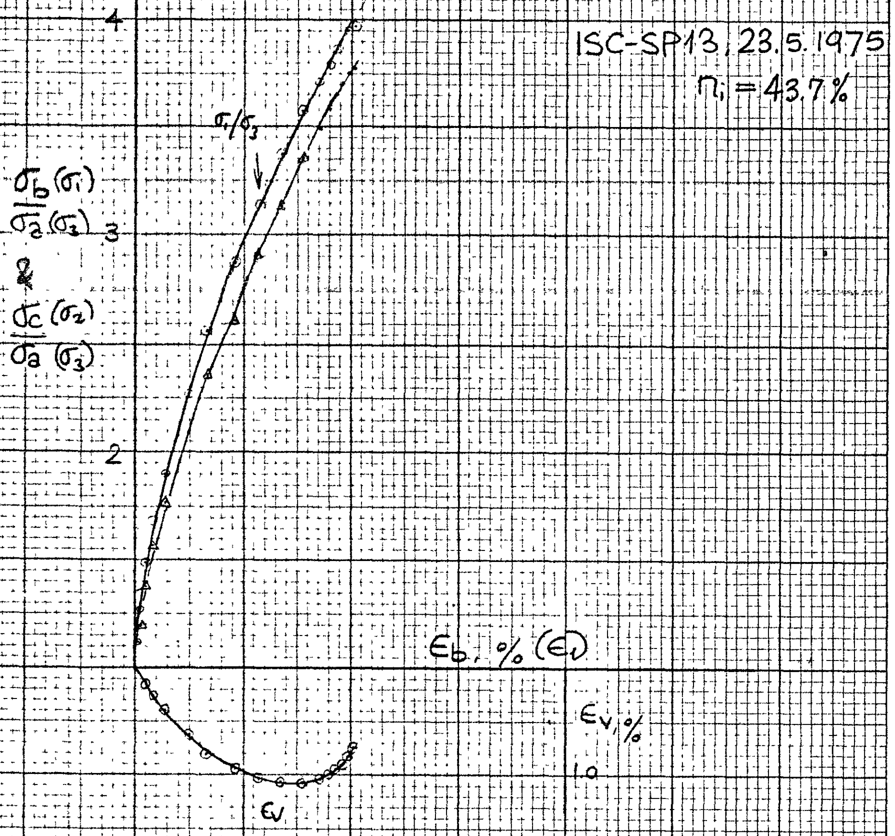
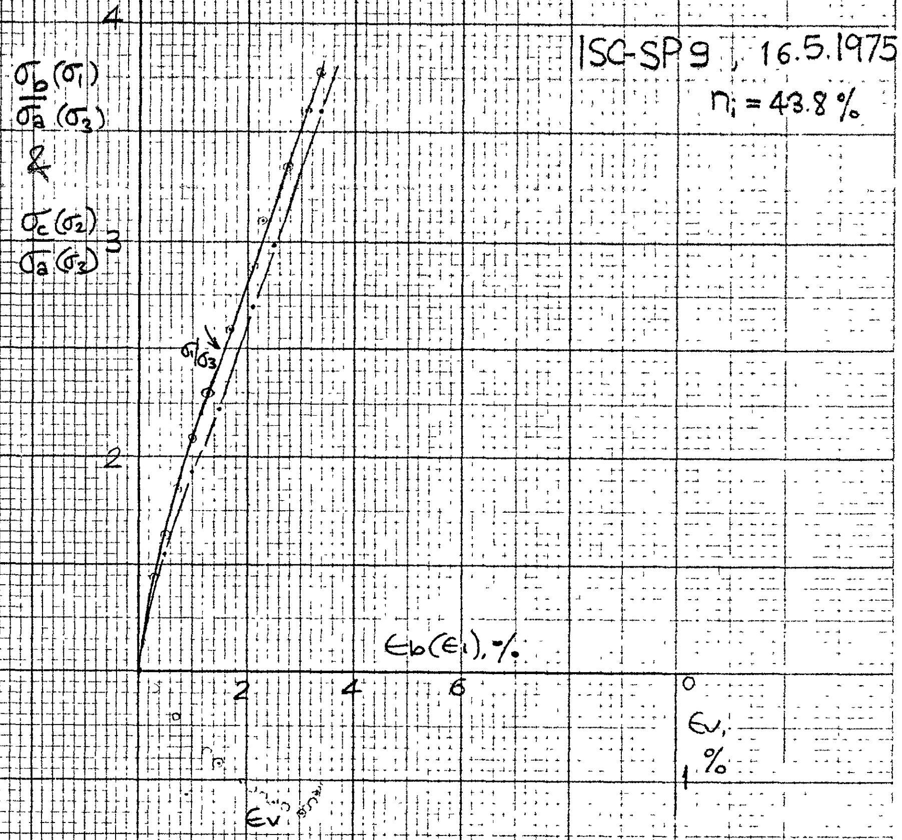


Fig. A.9.13



ISC-SP13, 23.5.1975
$n_i = 43.7\%$

Fig. A.9.16



ISC-SP9, 16.5.1975
$n_i = 43.8\%$

Fig. A.9.15

APPENDIX 10

STRESS-STRAIN CURVES FOR TRIAXIAL EXTENSION TESTS ON HAM RIVER SAND.

EX1-12 FIRST SERIES

EXII-1,3,4 SECOND SERIES

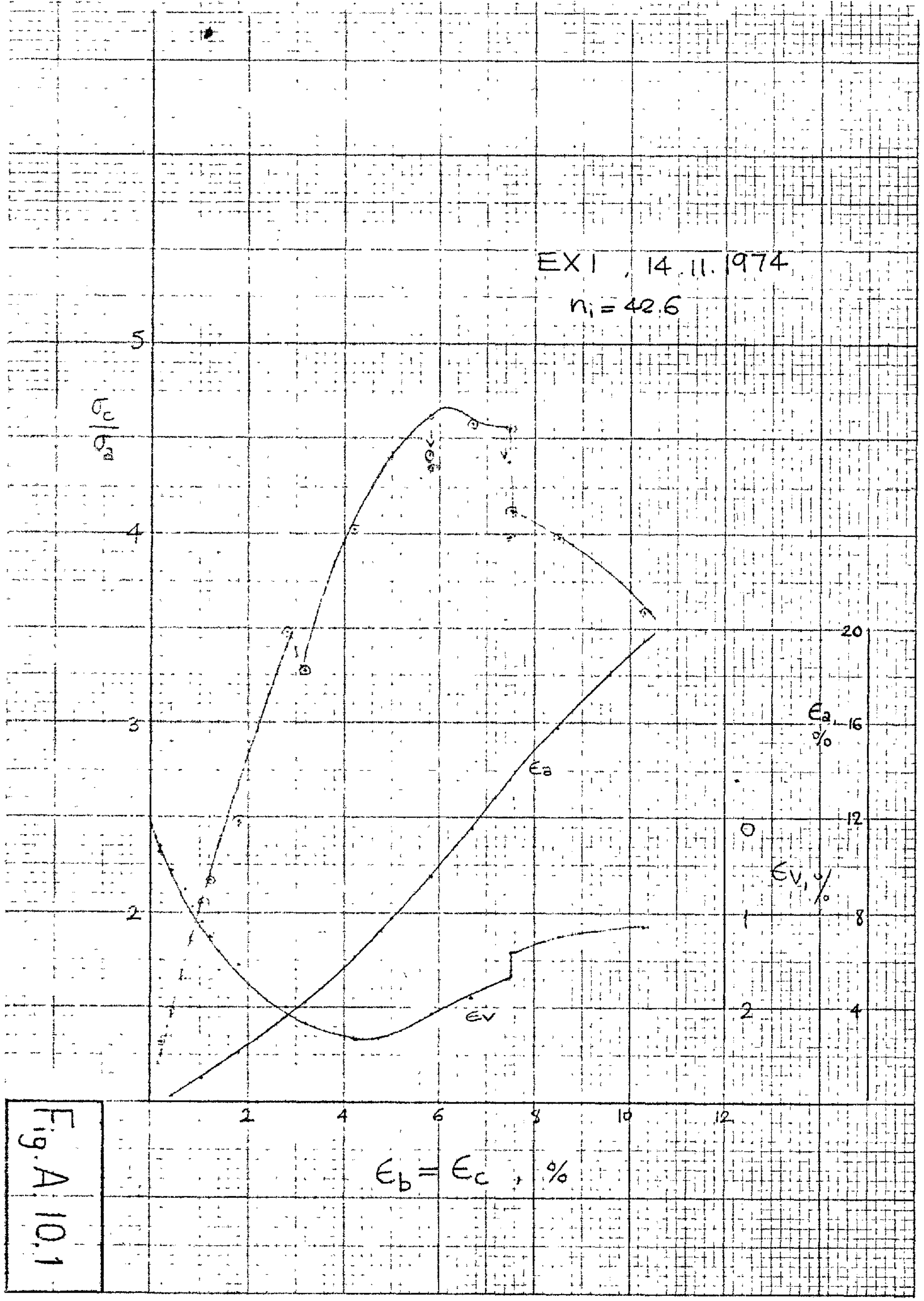


Fig. A 10.1

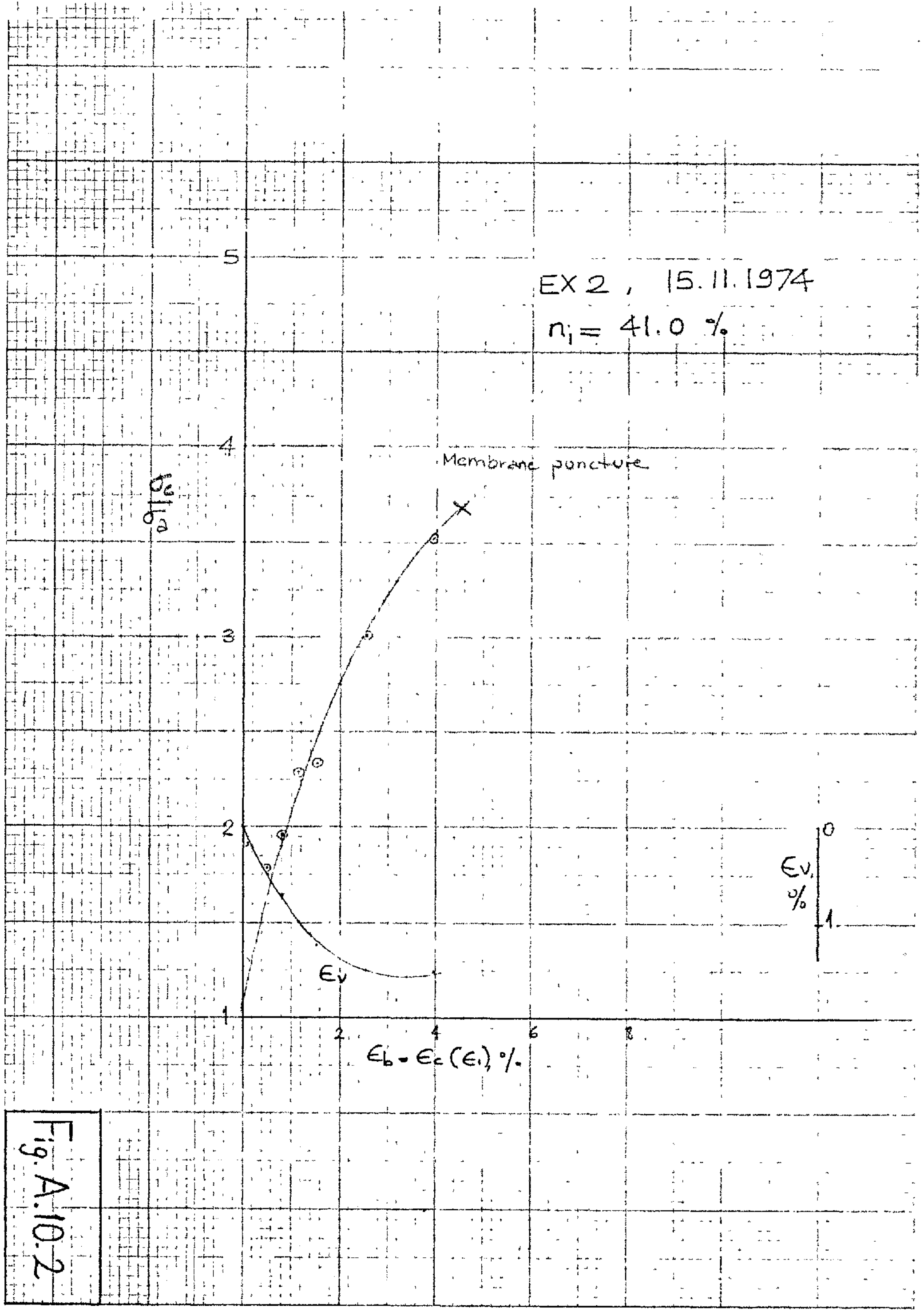


Fig. A.10.2

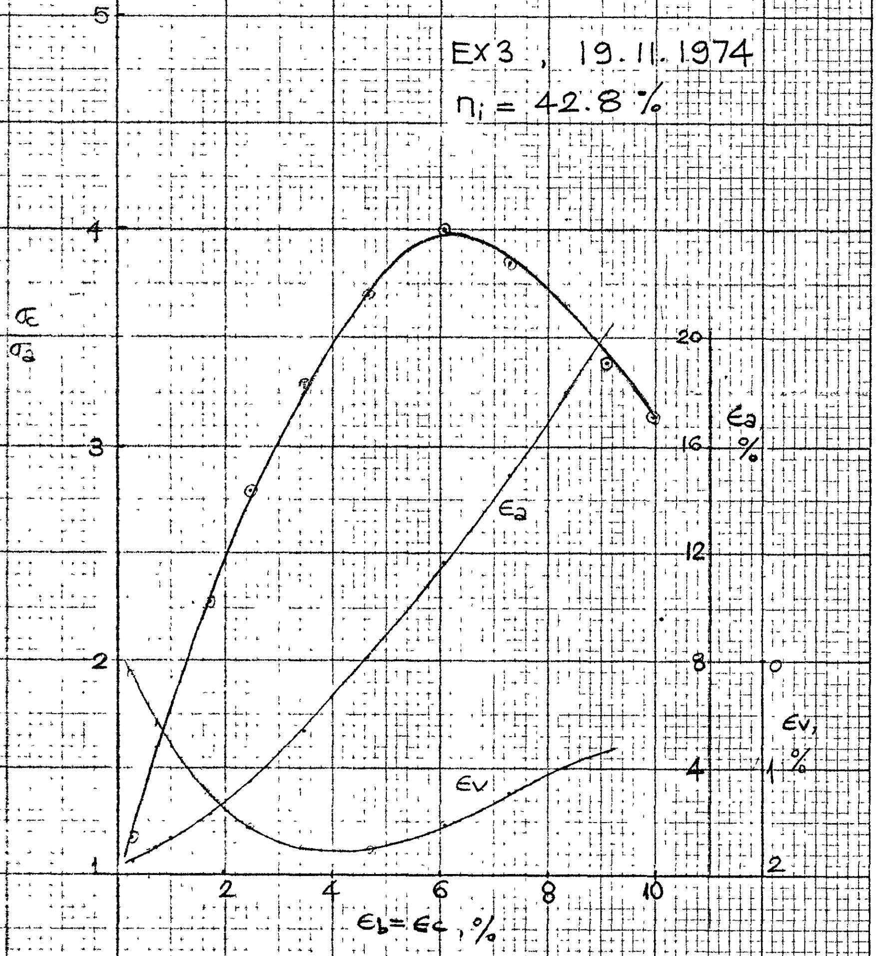


Fig. A.10.3

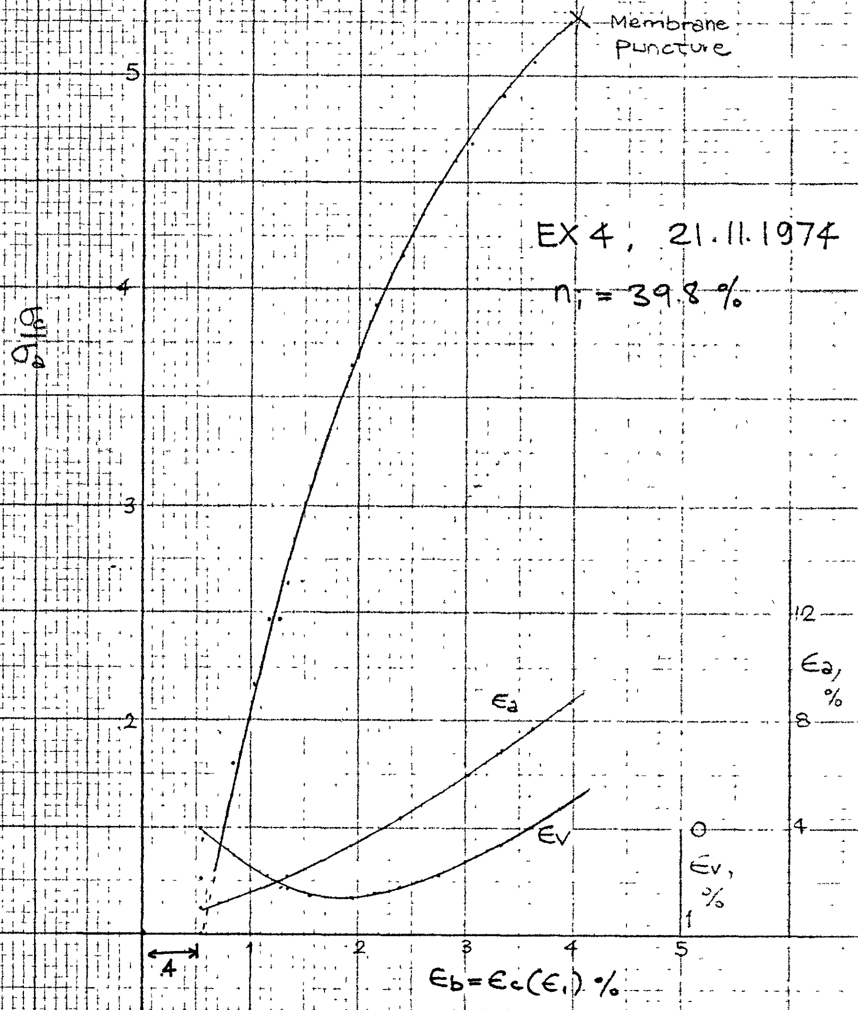


Fig. A.10.4

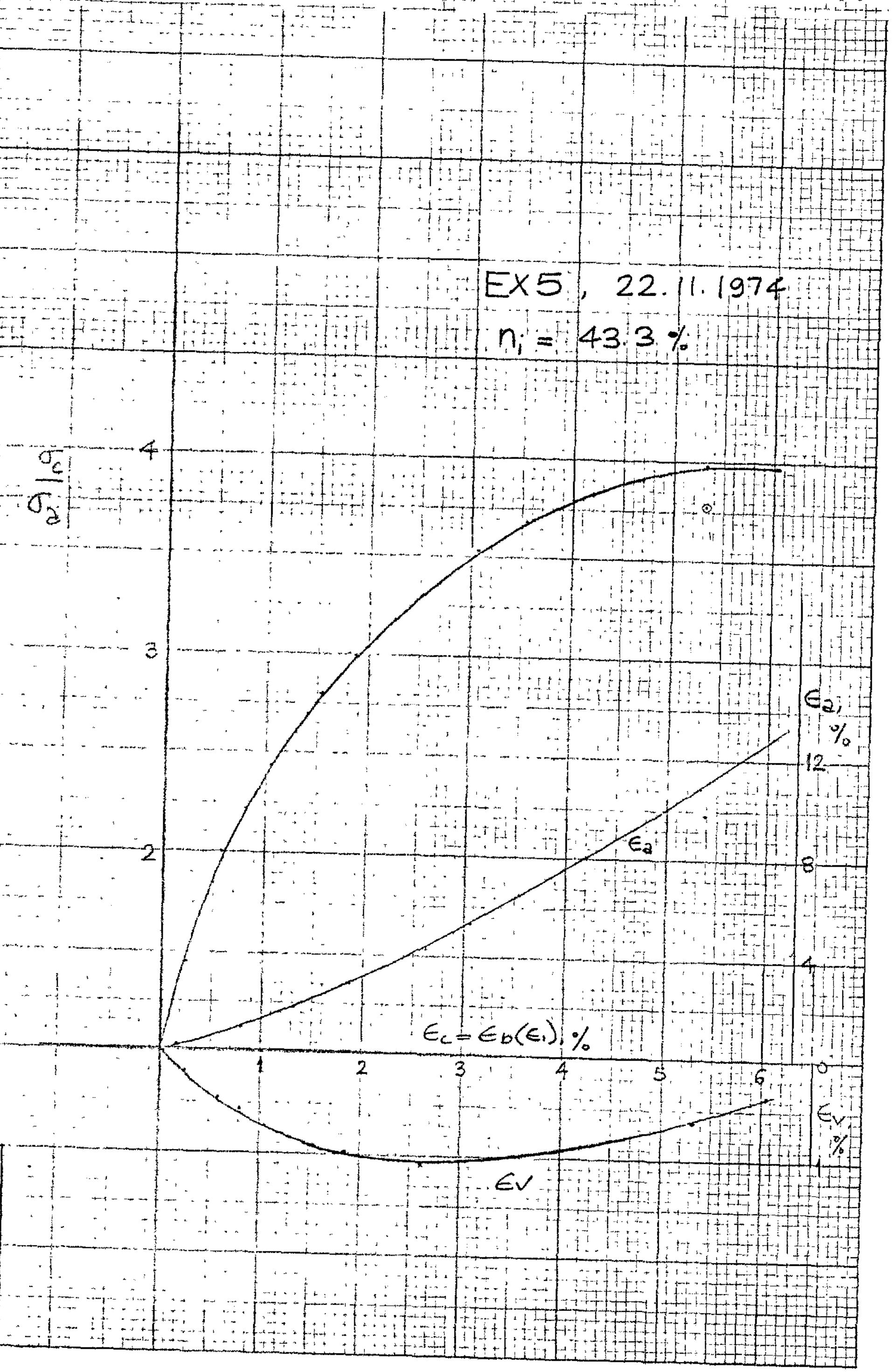


Fig. A. 10.5

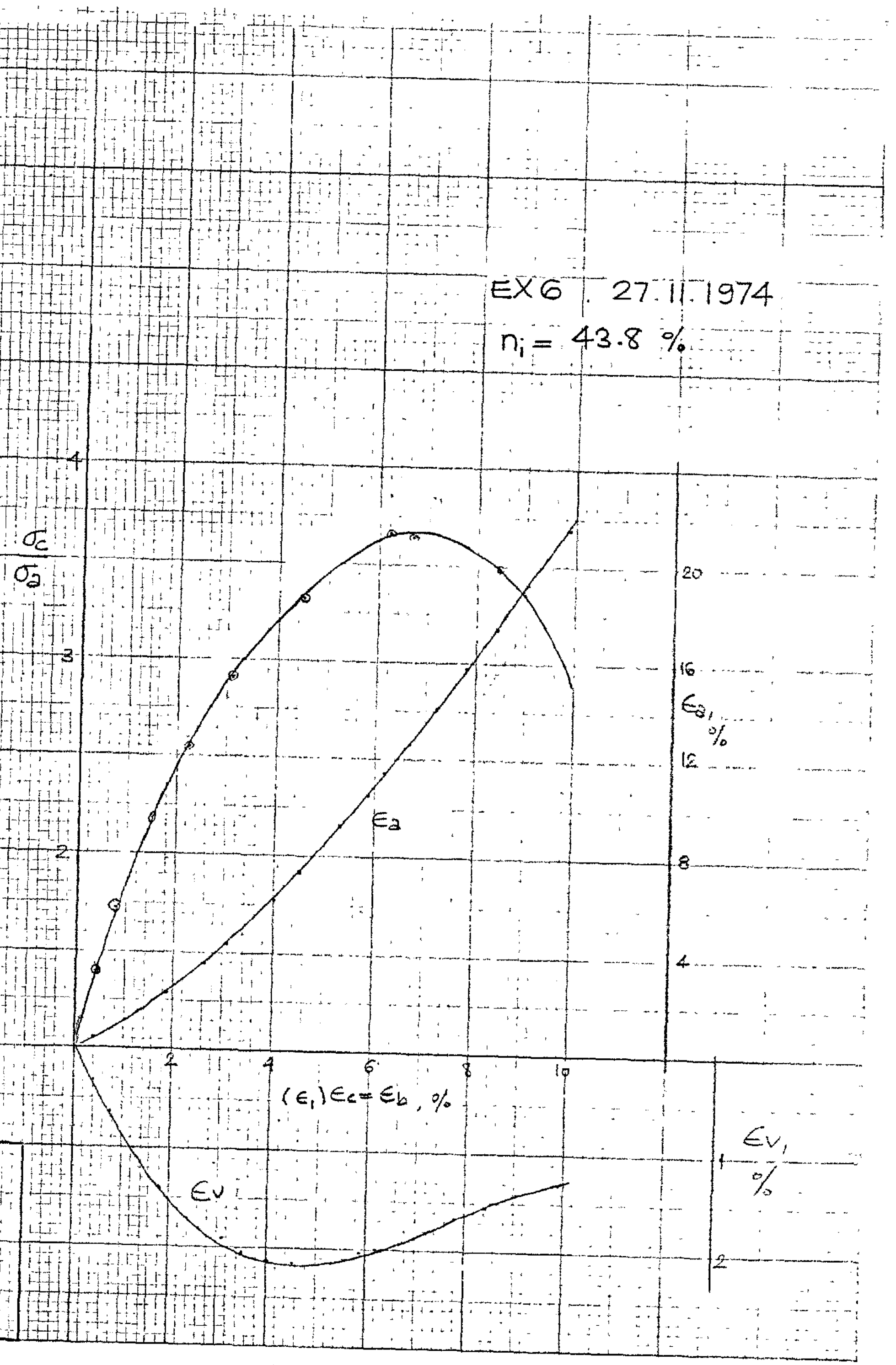


Fig. A. 10.6

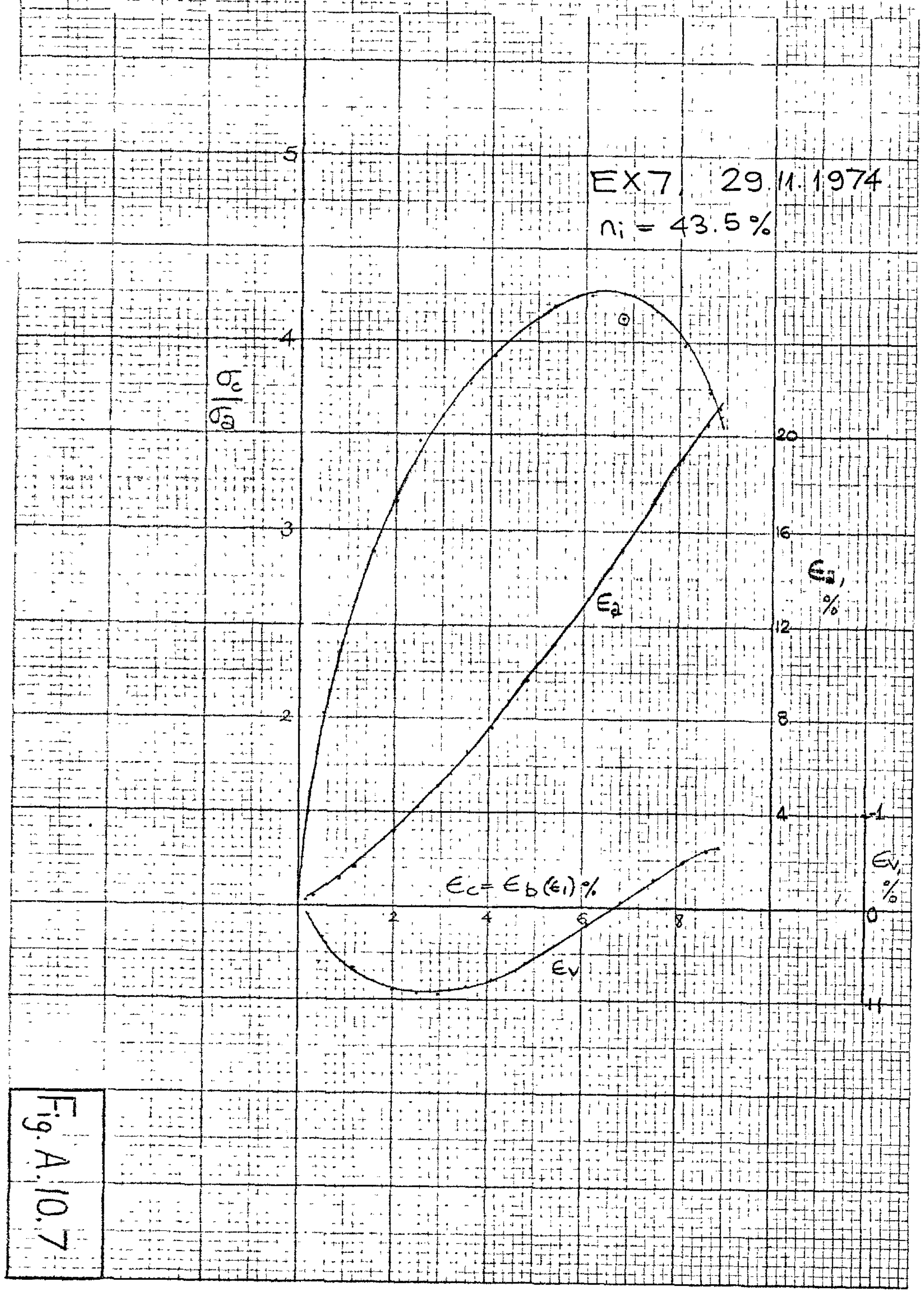


Fig. A.10.7

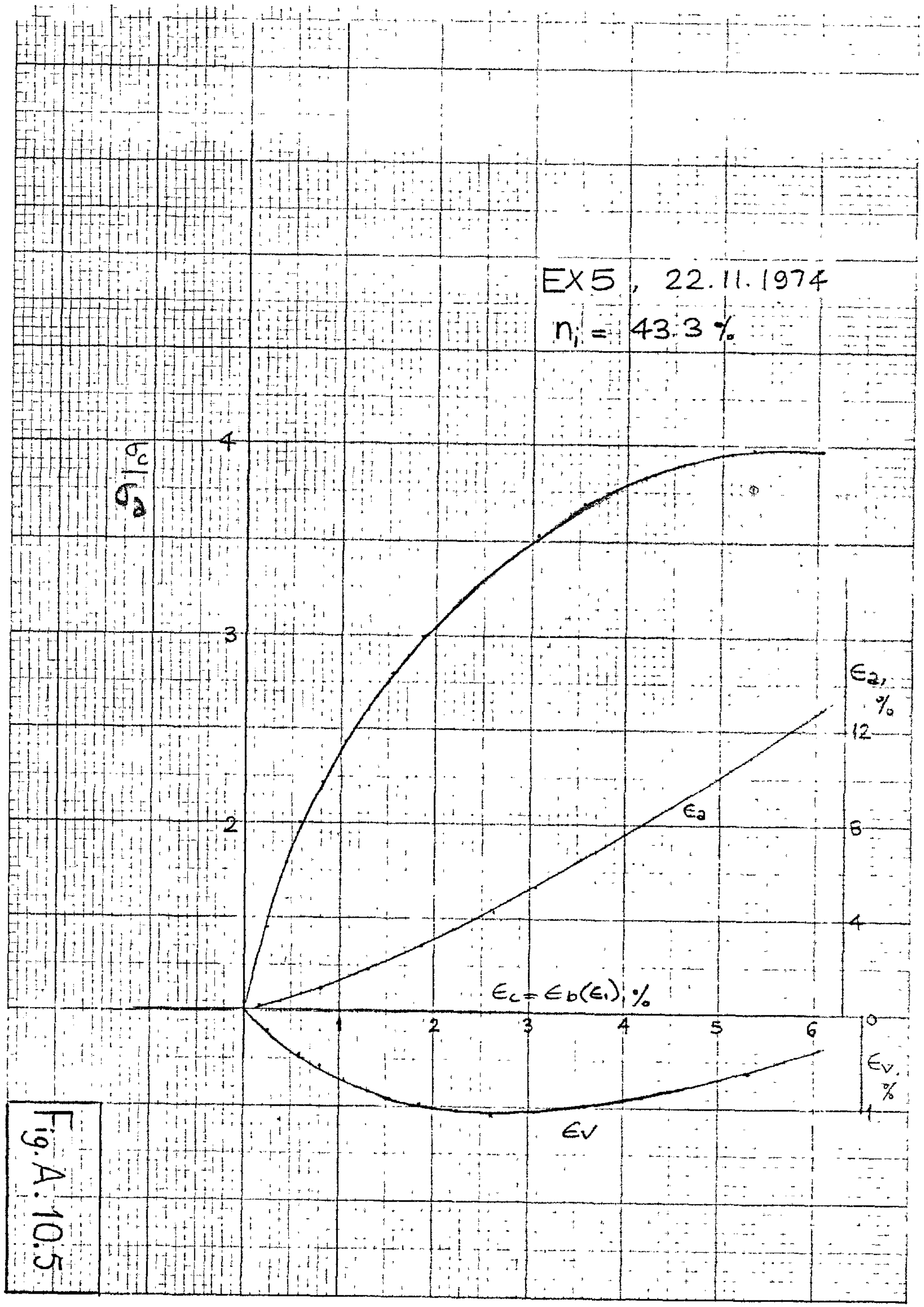


Fig. A.10.5

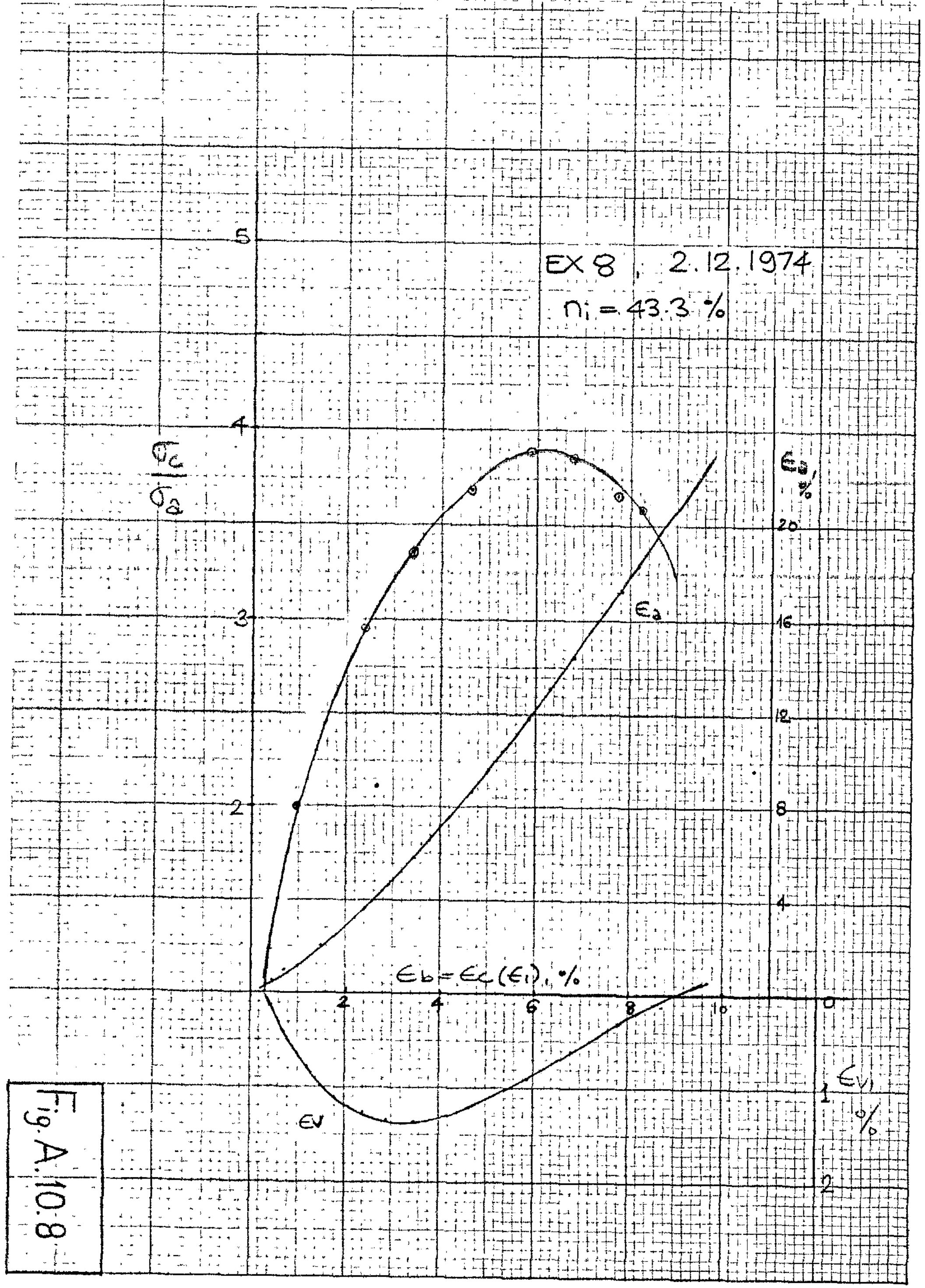


Fig. A.10.8

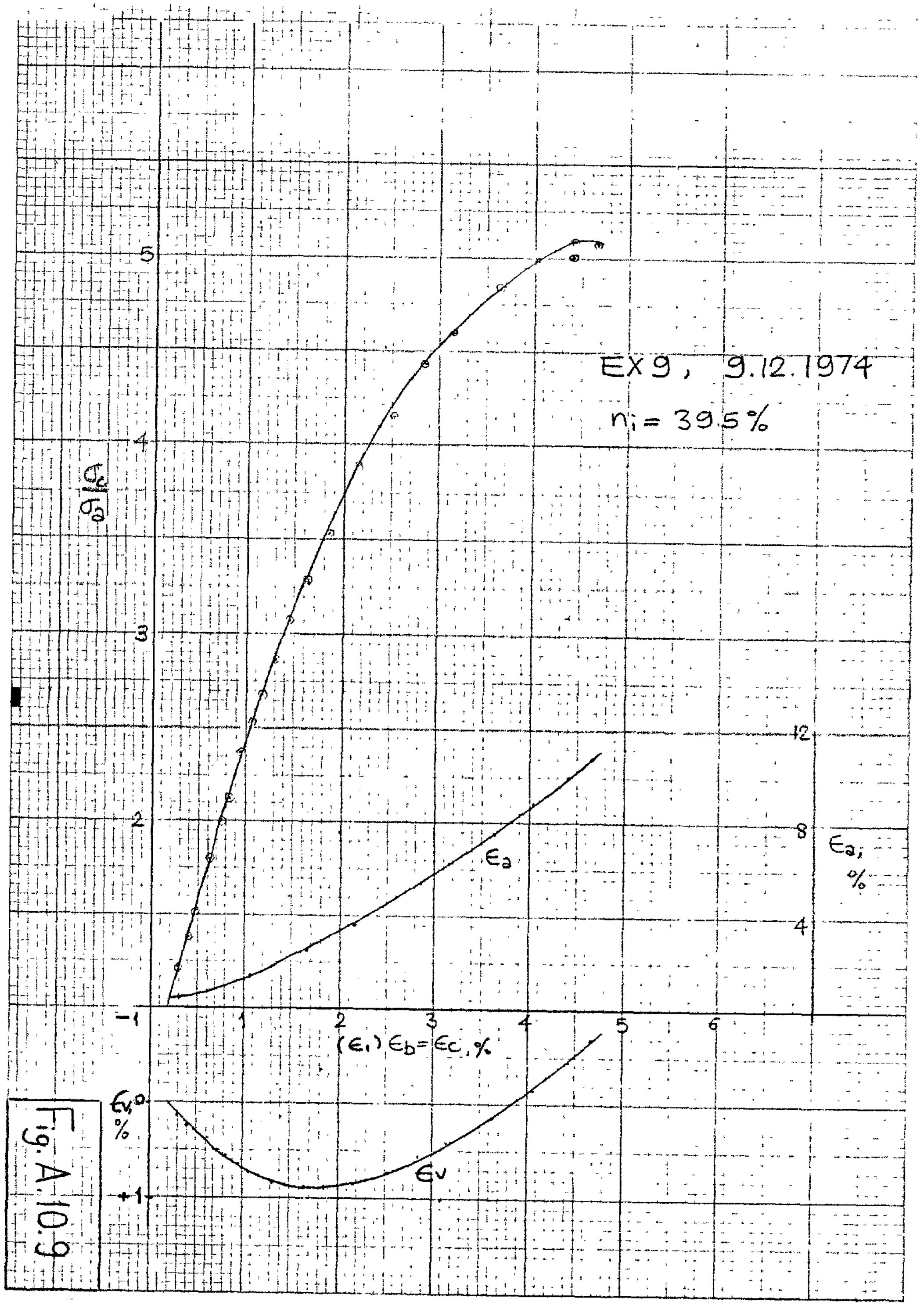


Fig. A.10.9

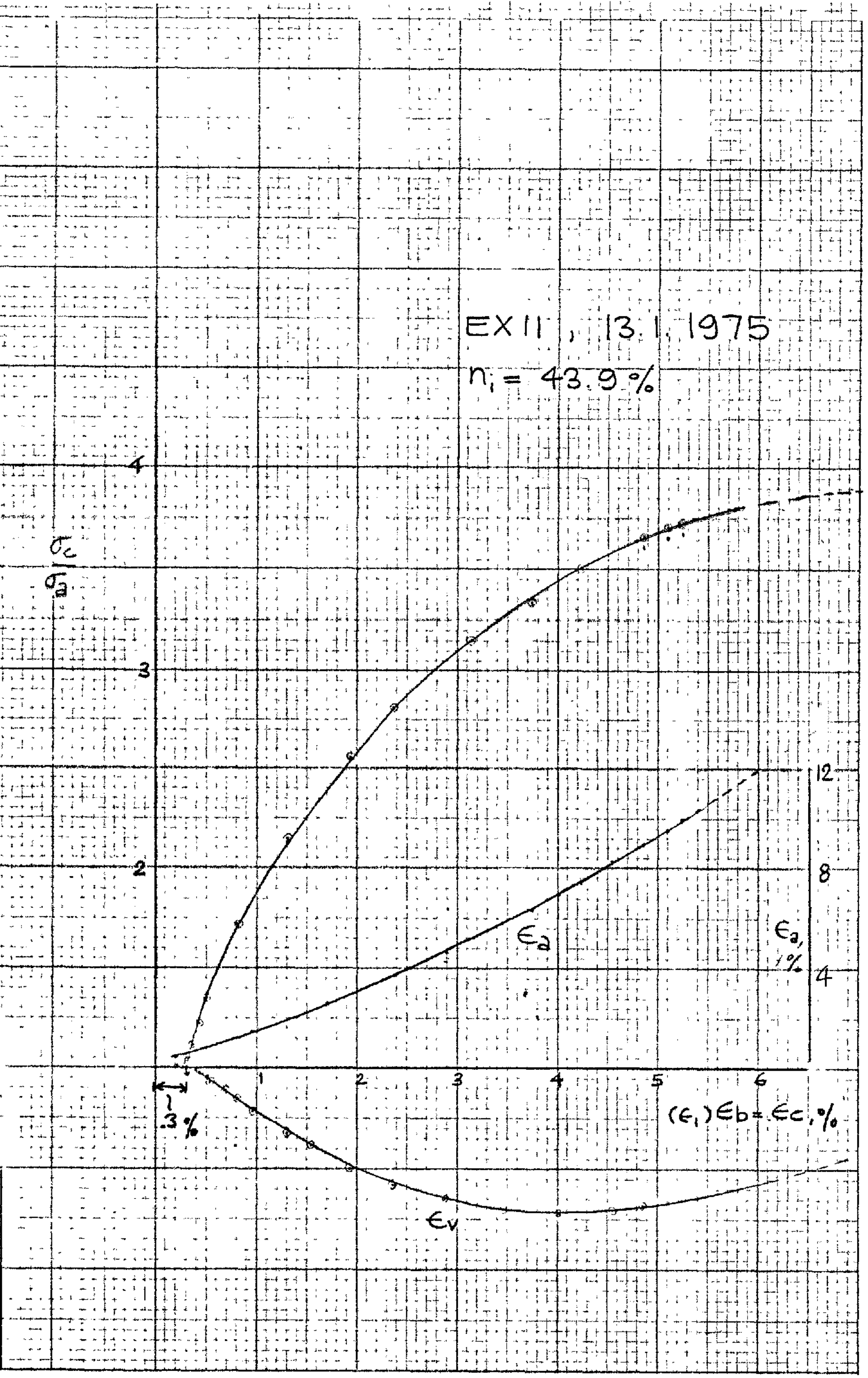


Fig. A.10.11

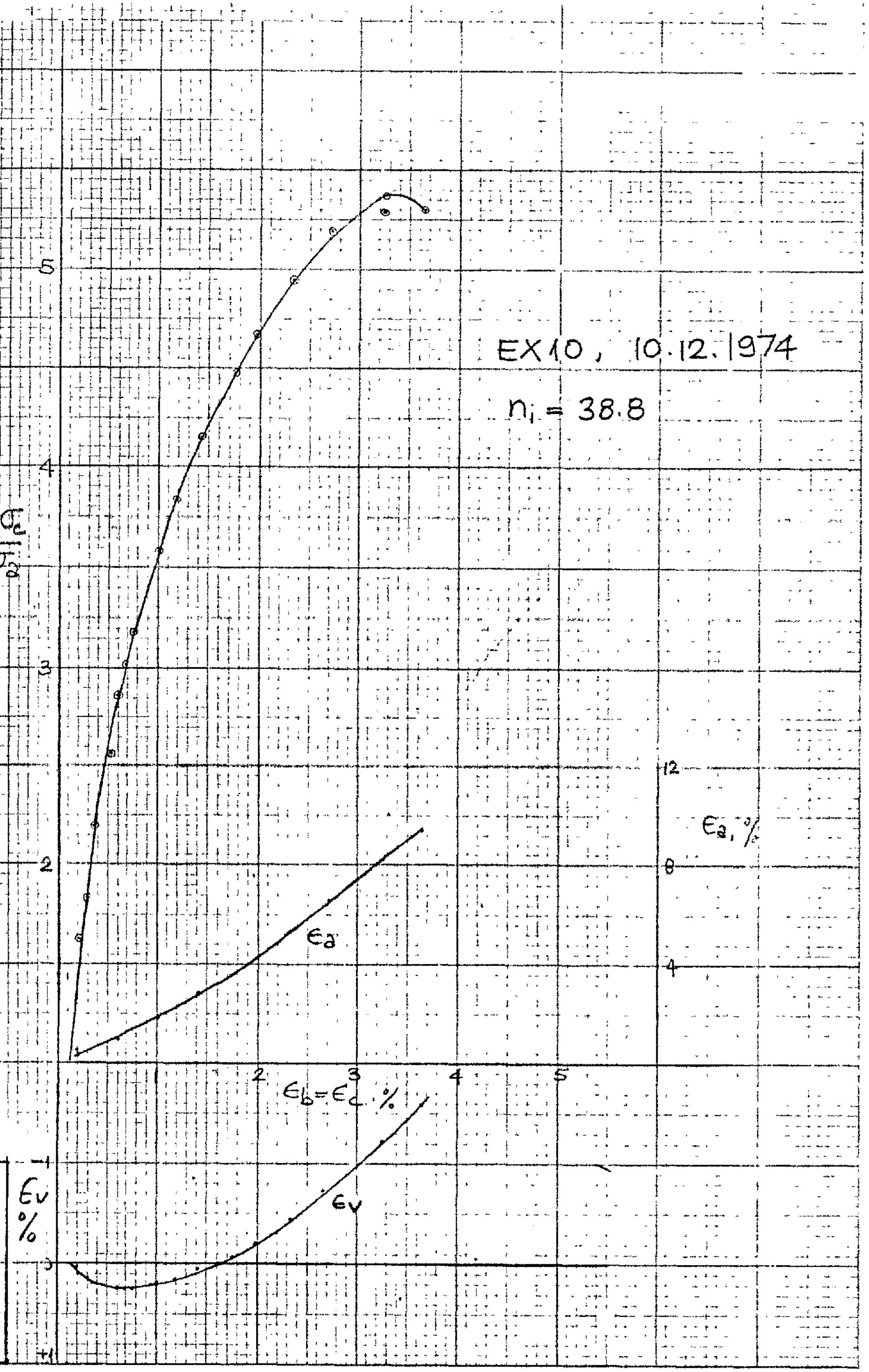


Fig. A.10.10

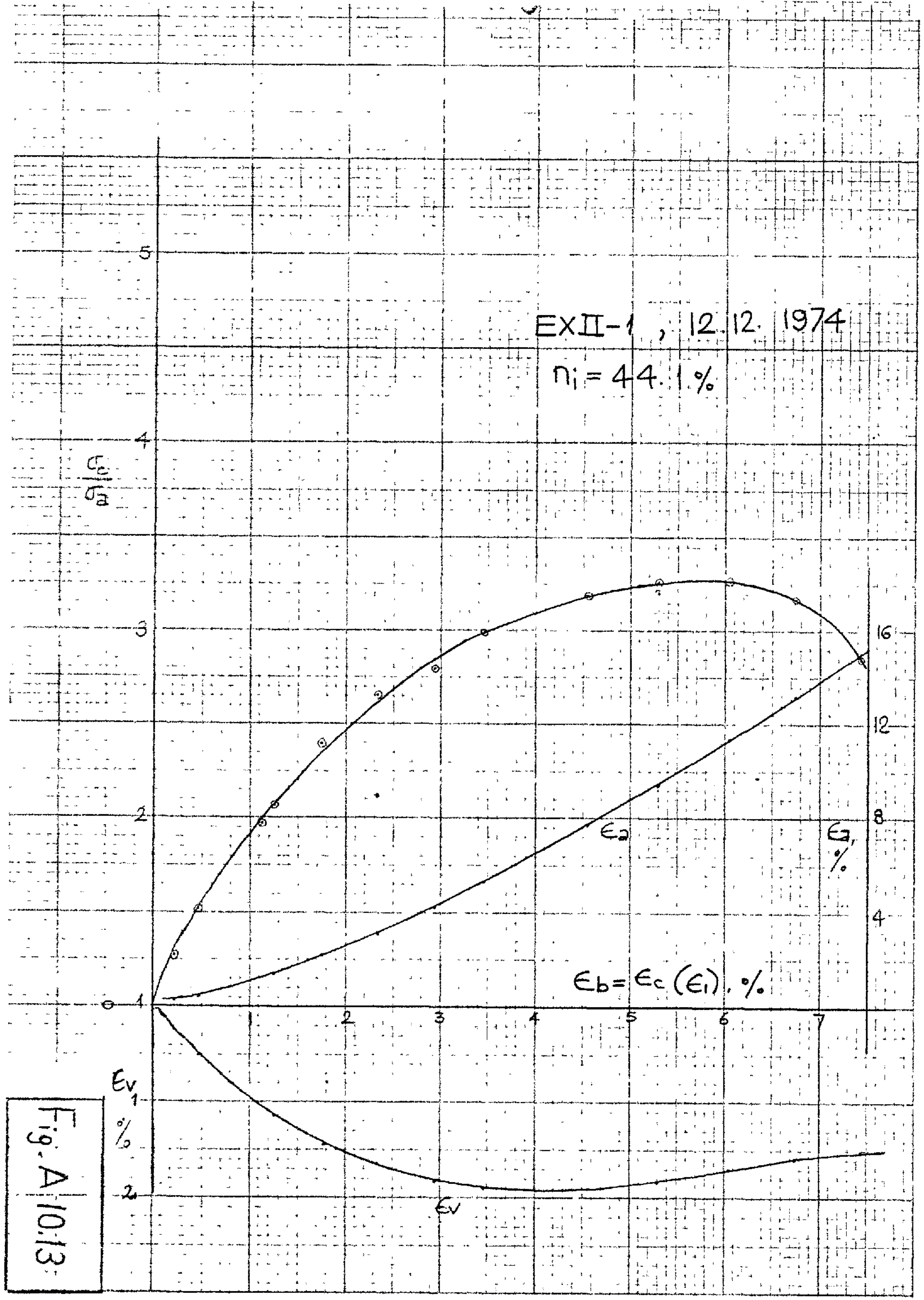


Fig. A 10:13

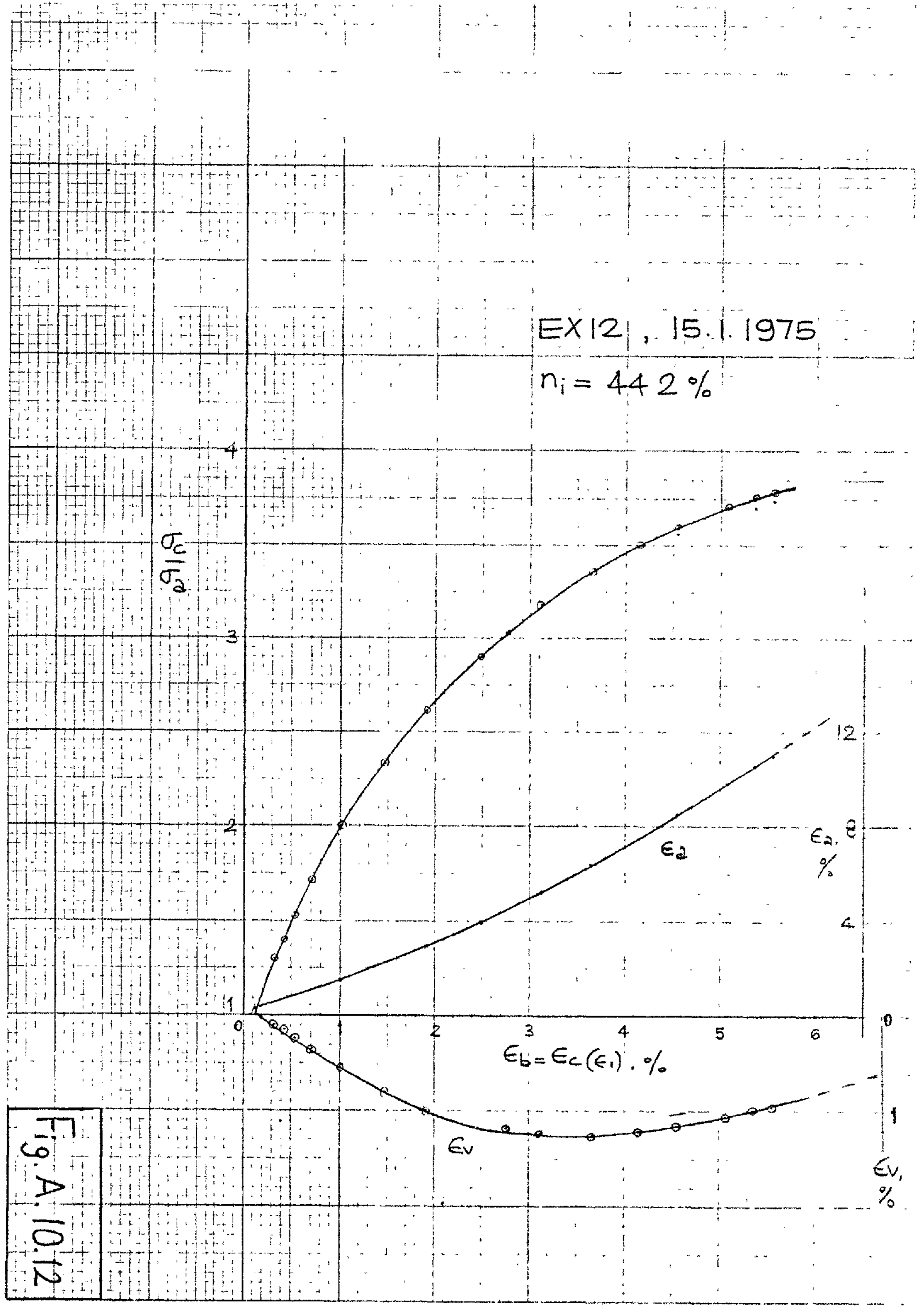


Fig. A. 10.12

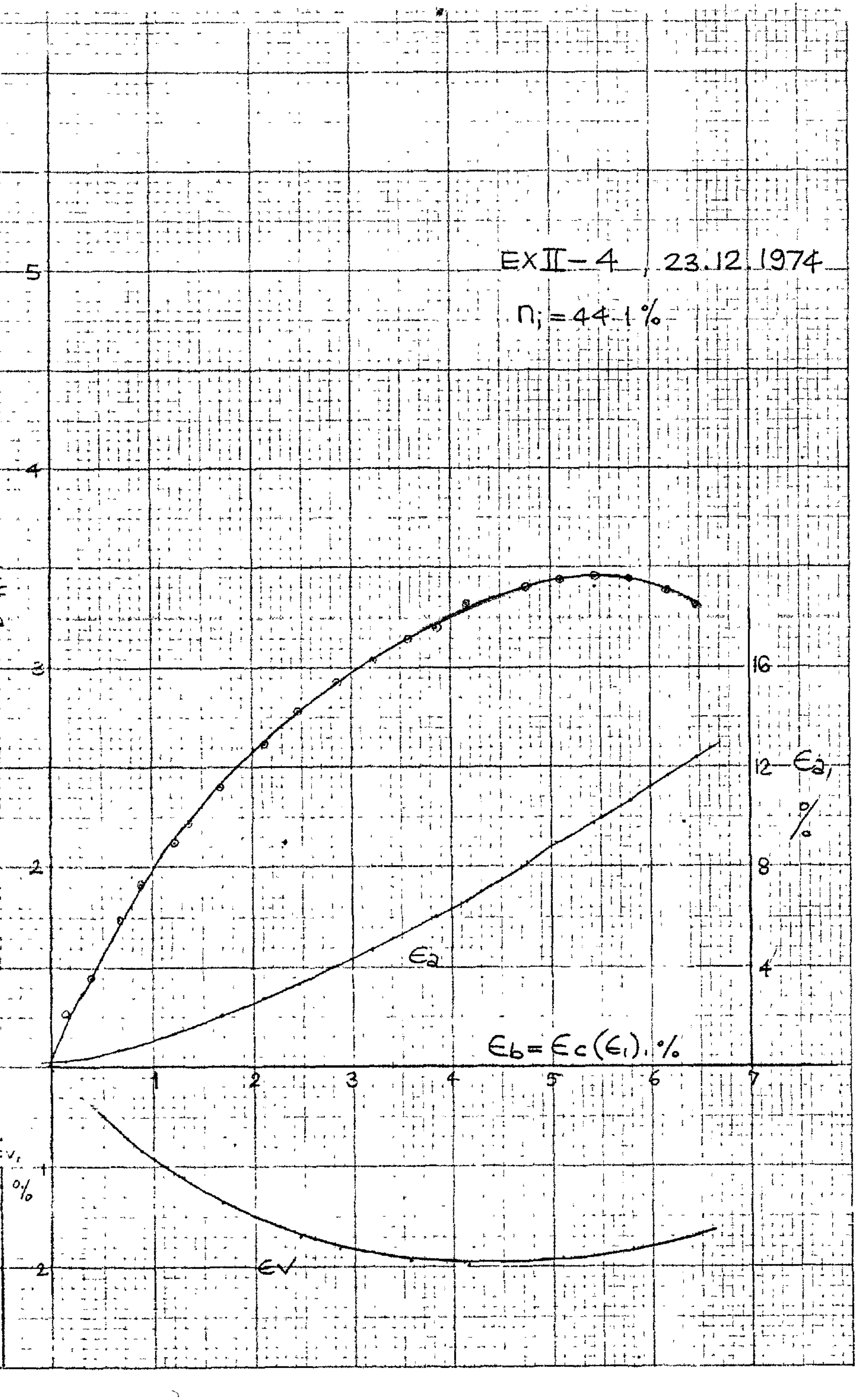


Fig. A.10.15

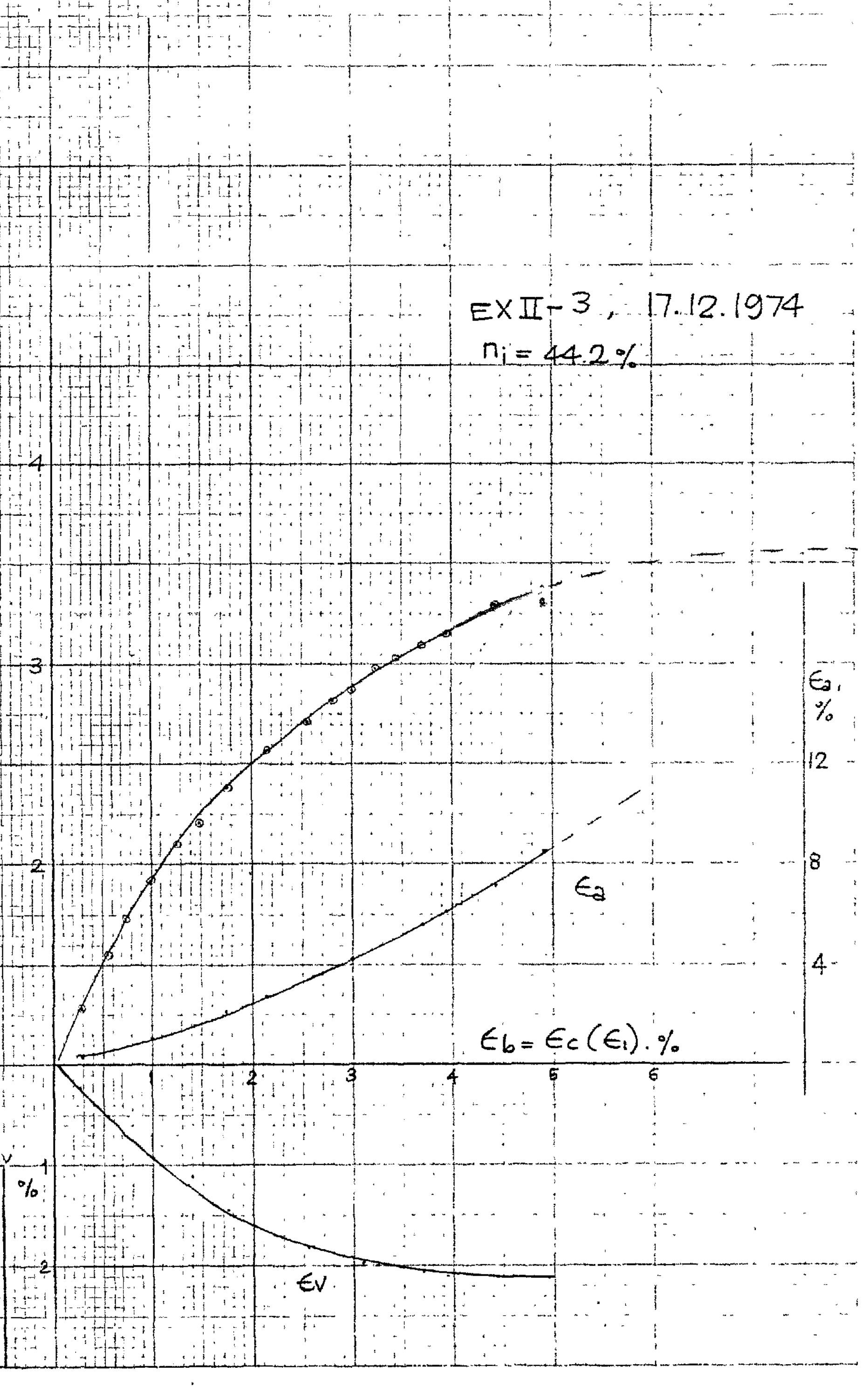


Fig. A.10.14

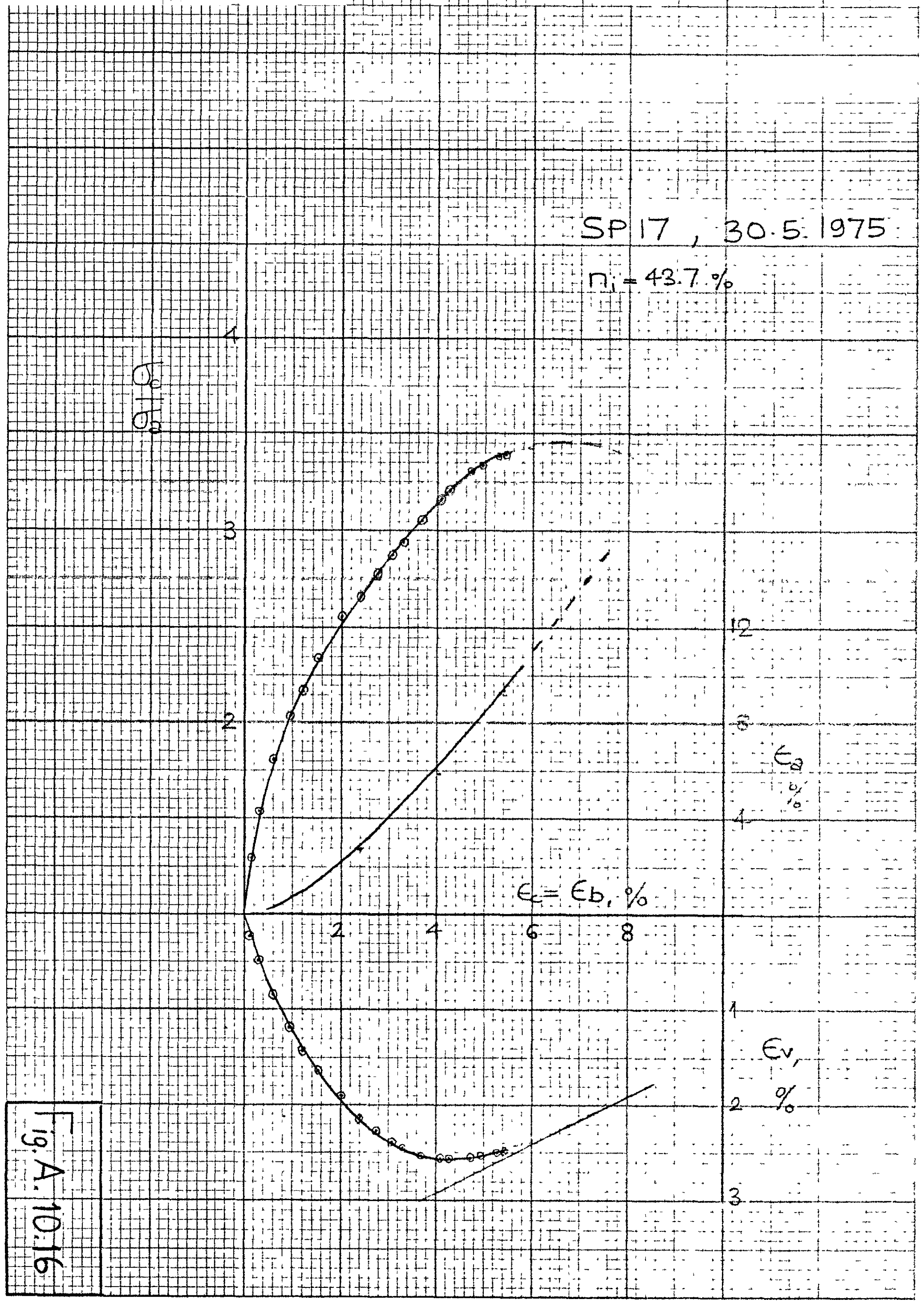

SP 17 , 30.5.1975
n_i = 43.7 %
σ_c/σ_b
4
3
2
12
8
ϵ_a %
4
$\epsilon_c = \epsilon_b$, %
2
4
6
8
1
ϵ_v, %
2
3

Fig. A.10.16

APPENDIX 11

STRESS-STRAIN CURVES FOR GENERALISED, TRIAXIAL COMPRESSION AND EXTENSION TESTS ON VOLCANIC SAND.

ISCD1-D8	DENSE GENERALISED SERIES
ISCL1-L9	LOOSE GENERALISED SERIES
TC1-5	TRIAXIAL COMPRESSION TESTS
EXV1-3	TRIAXIAL EXTENSION TESTS

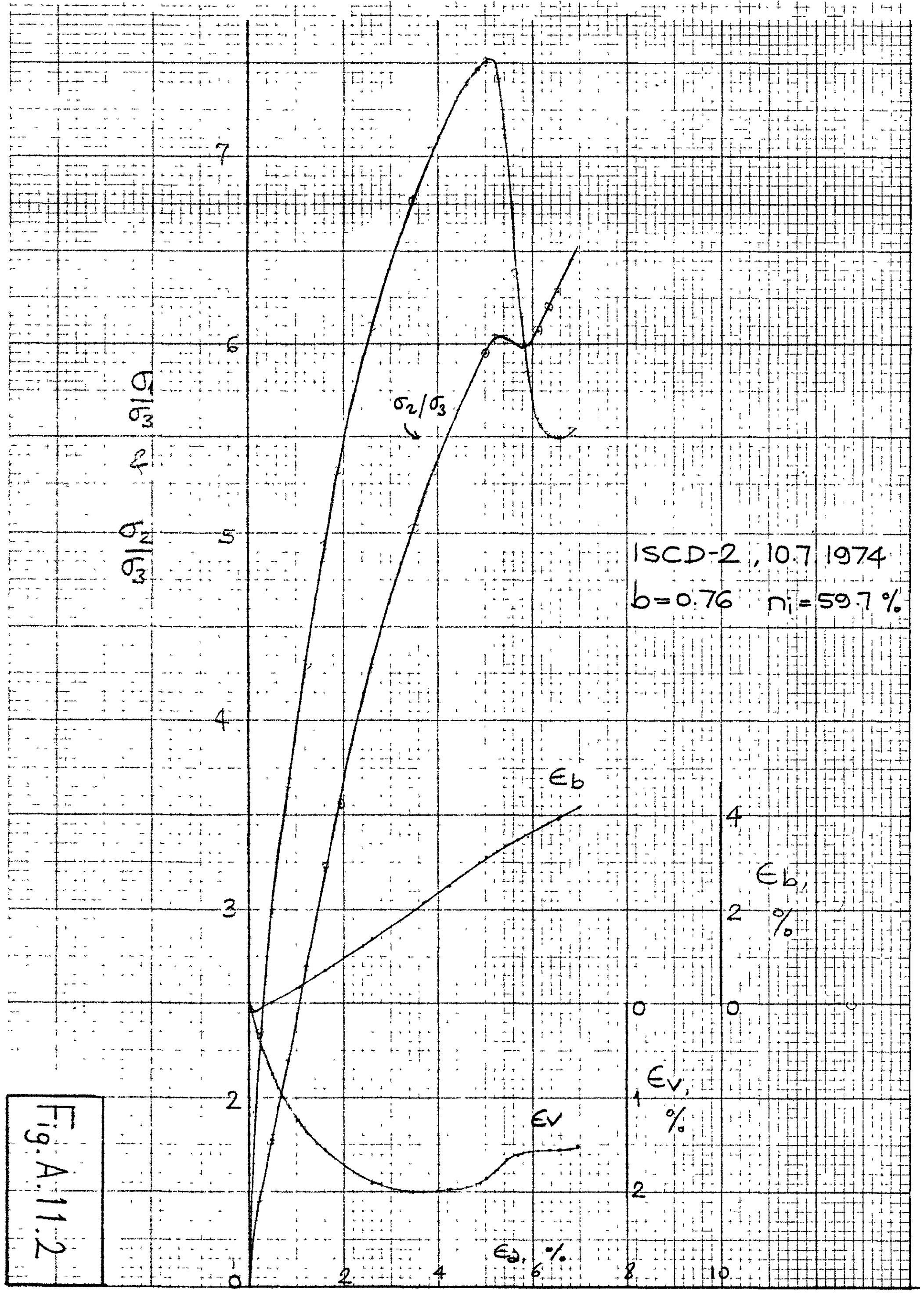


Fig. A.11.2

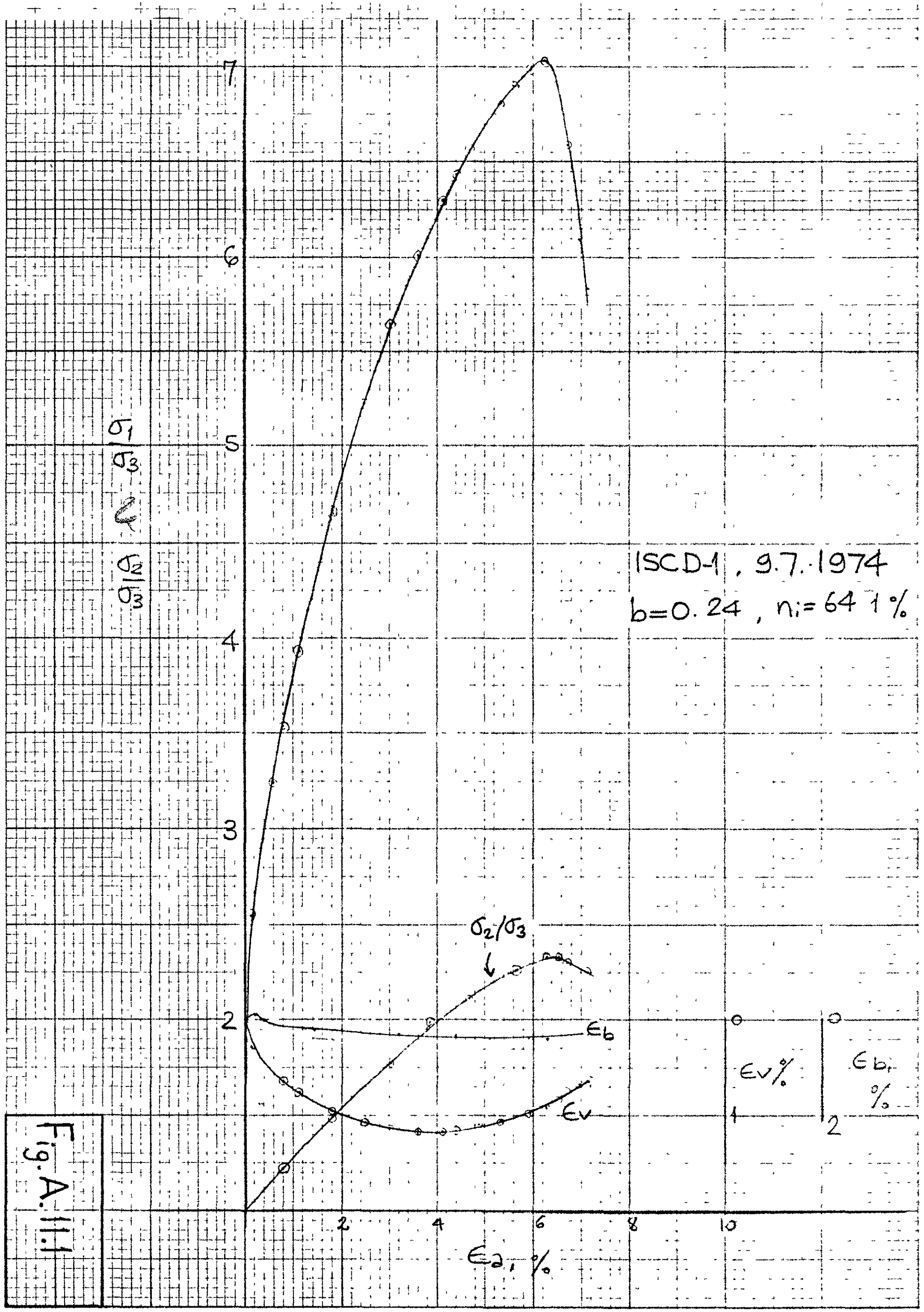
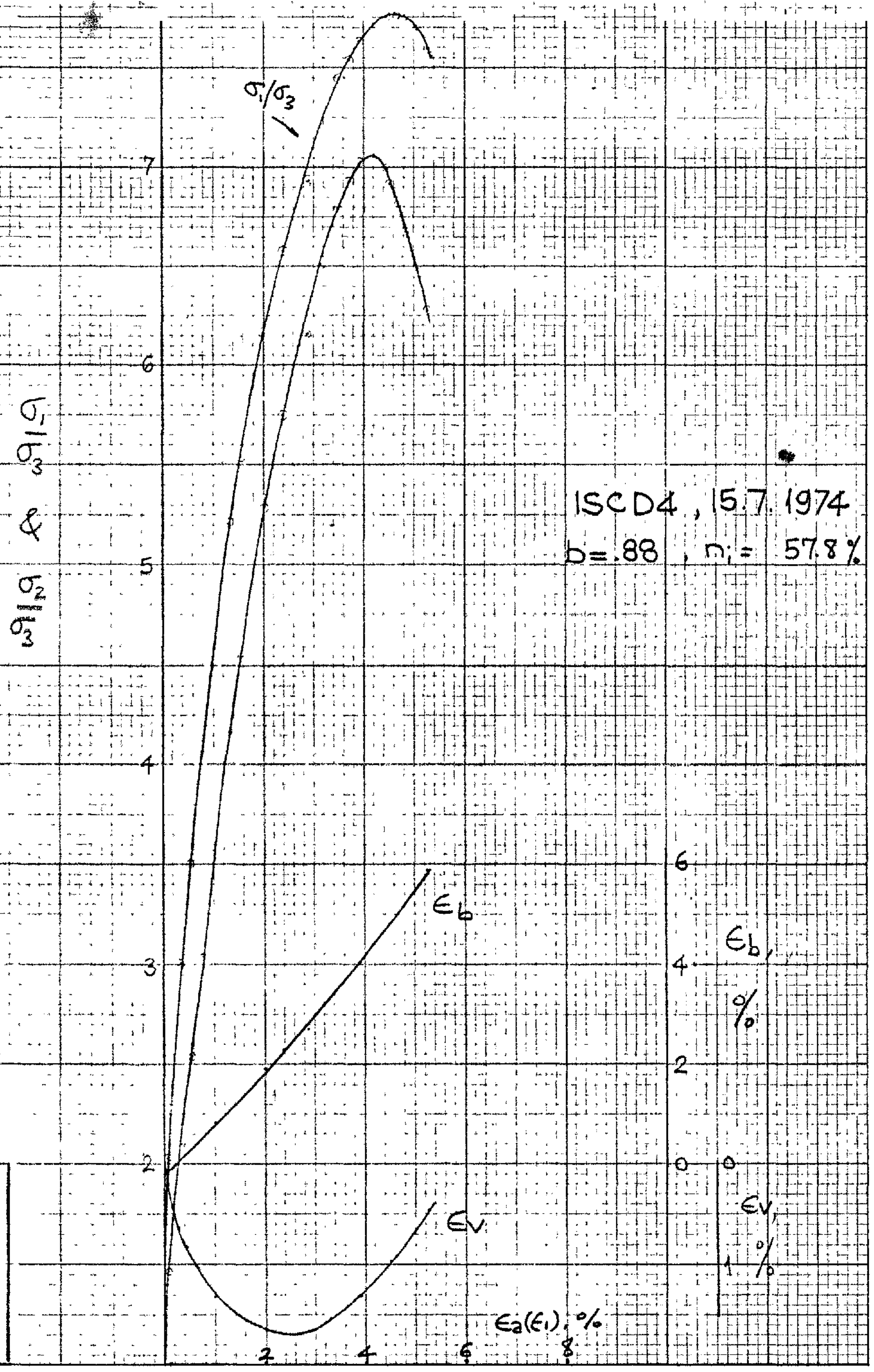


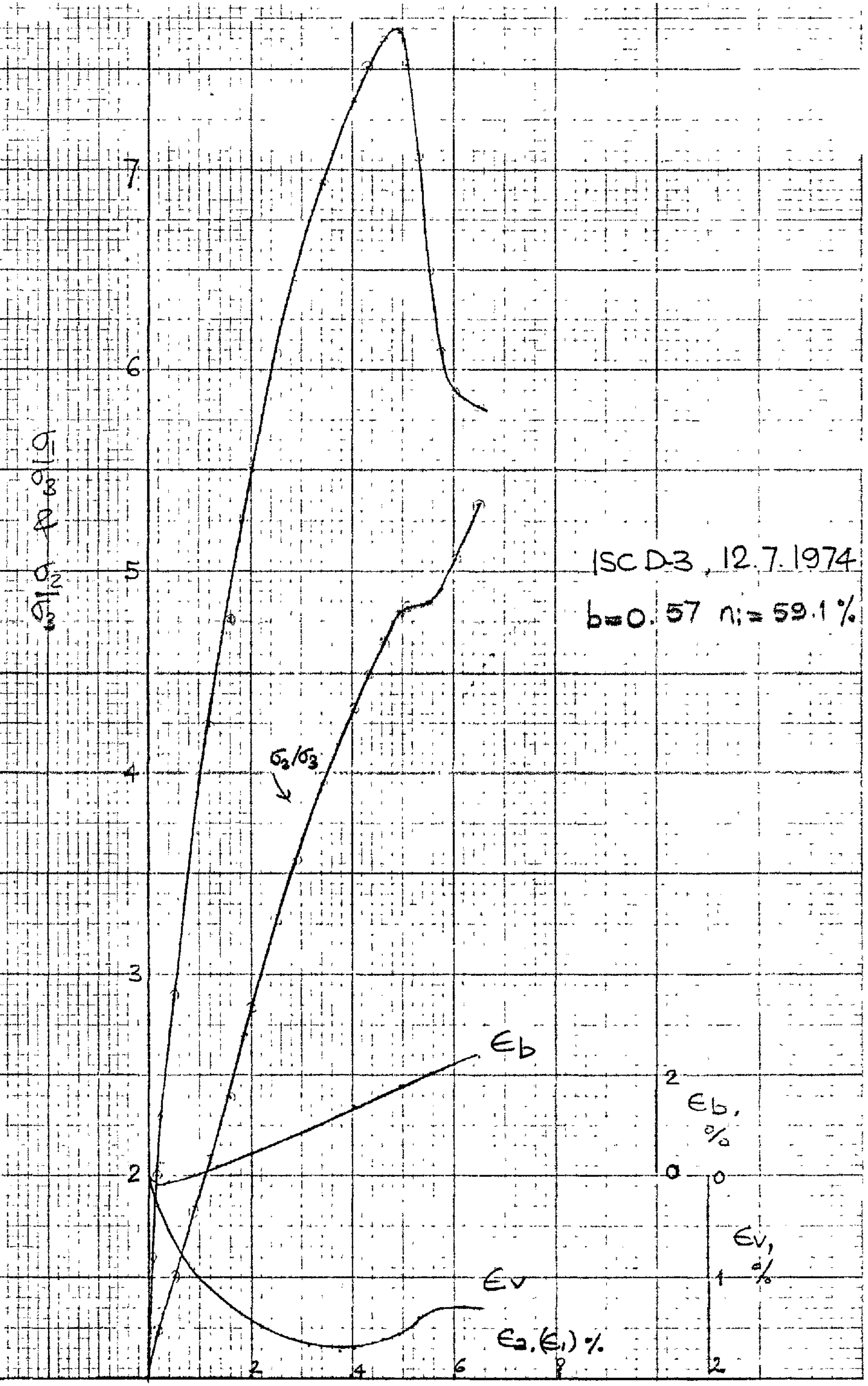
Fig. A.11.1



ISCD4, 15.7.1974

b=.88, n_i= 57.8%

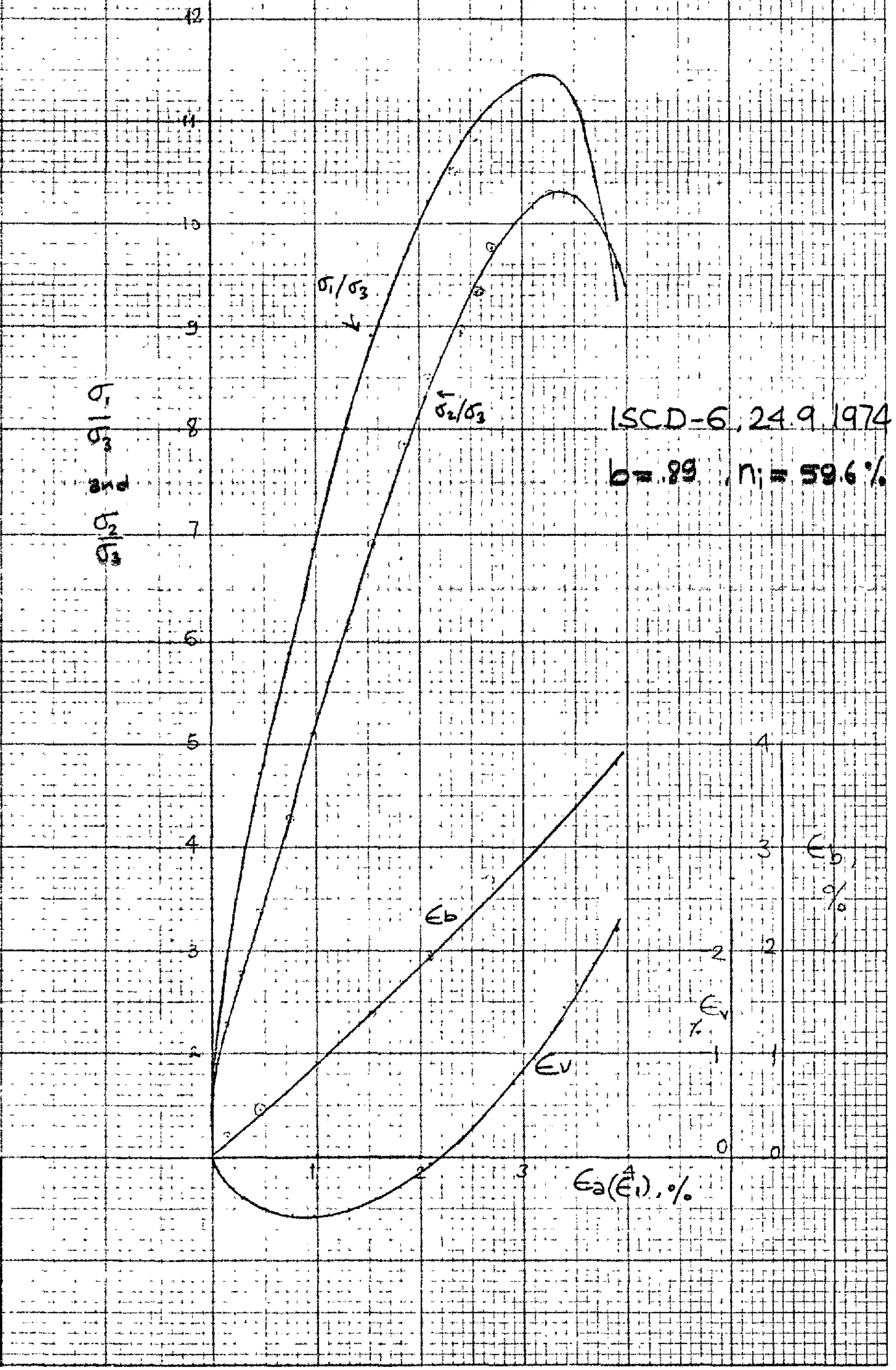
Fig. A.11.4



ISC D-3, 12.7.1974

b=0.57 n_i= 59.1%

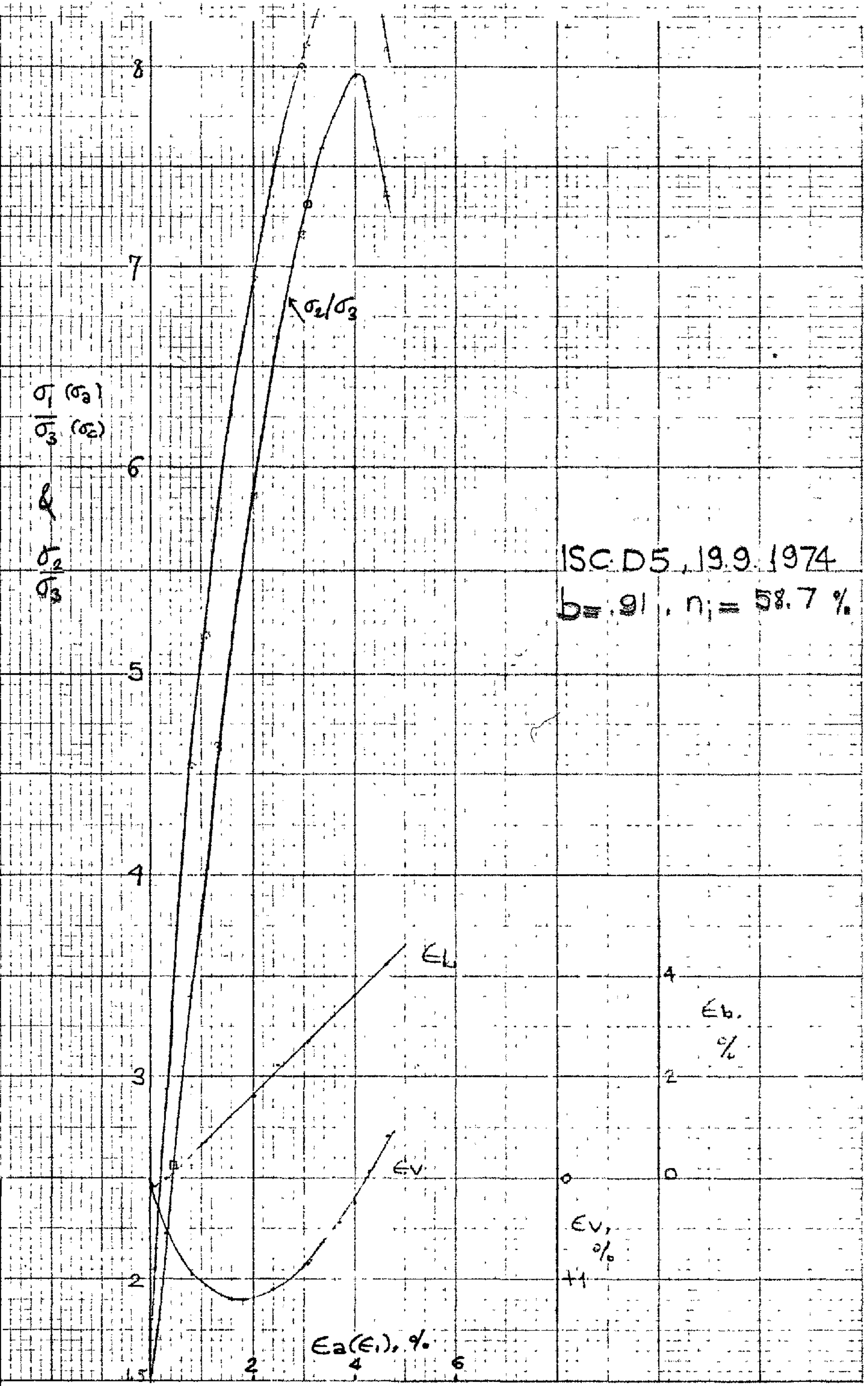
Fig. A.11.3



ISCD-6, 24.9.1974

b = .89, n_i = 59.6 %

Fig. A.11.6



ISC D5, 19.9.1974

b = .91, n_i = 58.7 %

Fig. A.11.5

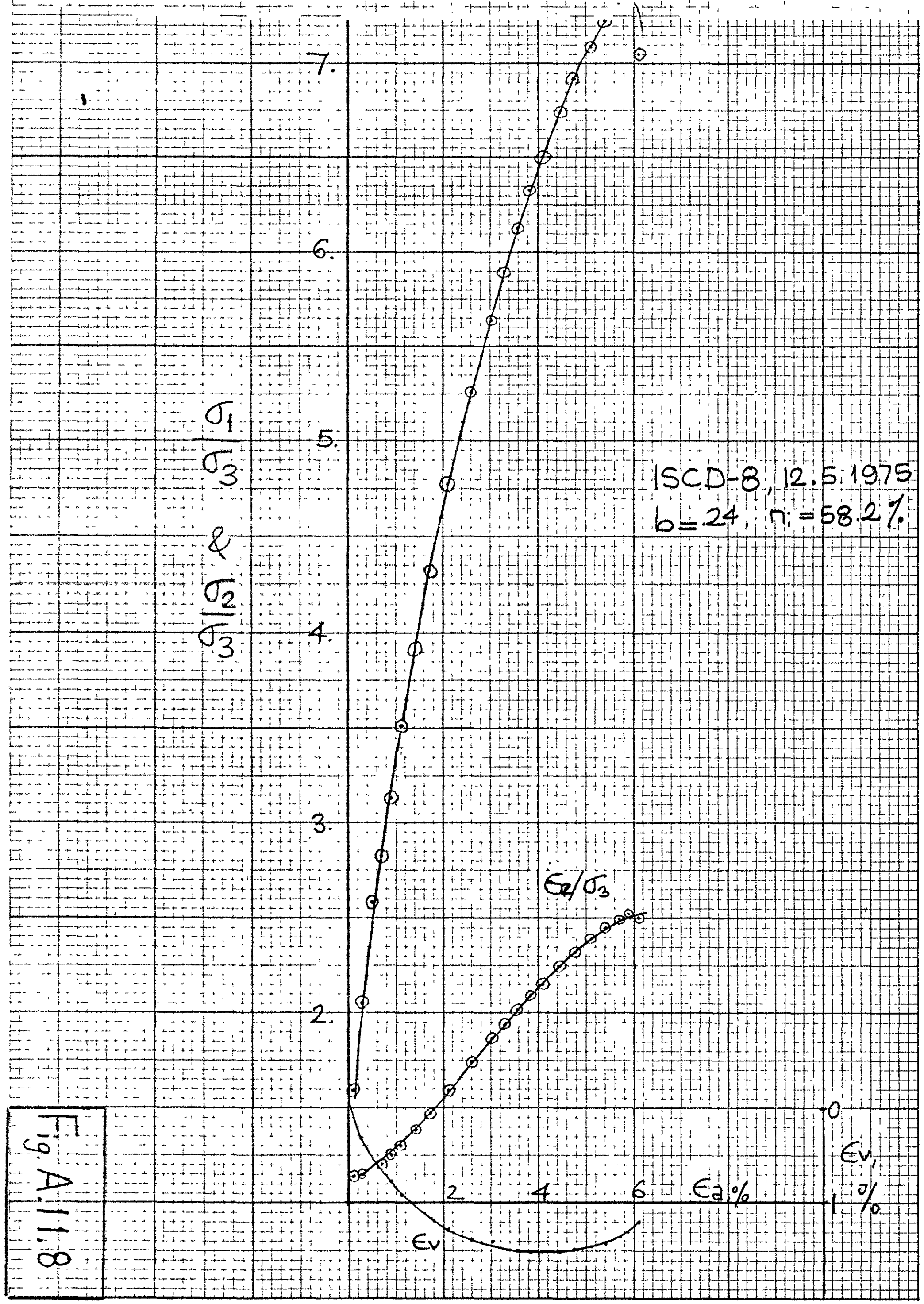


Fig. A.11.8

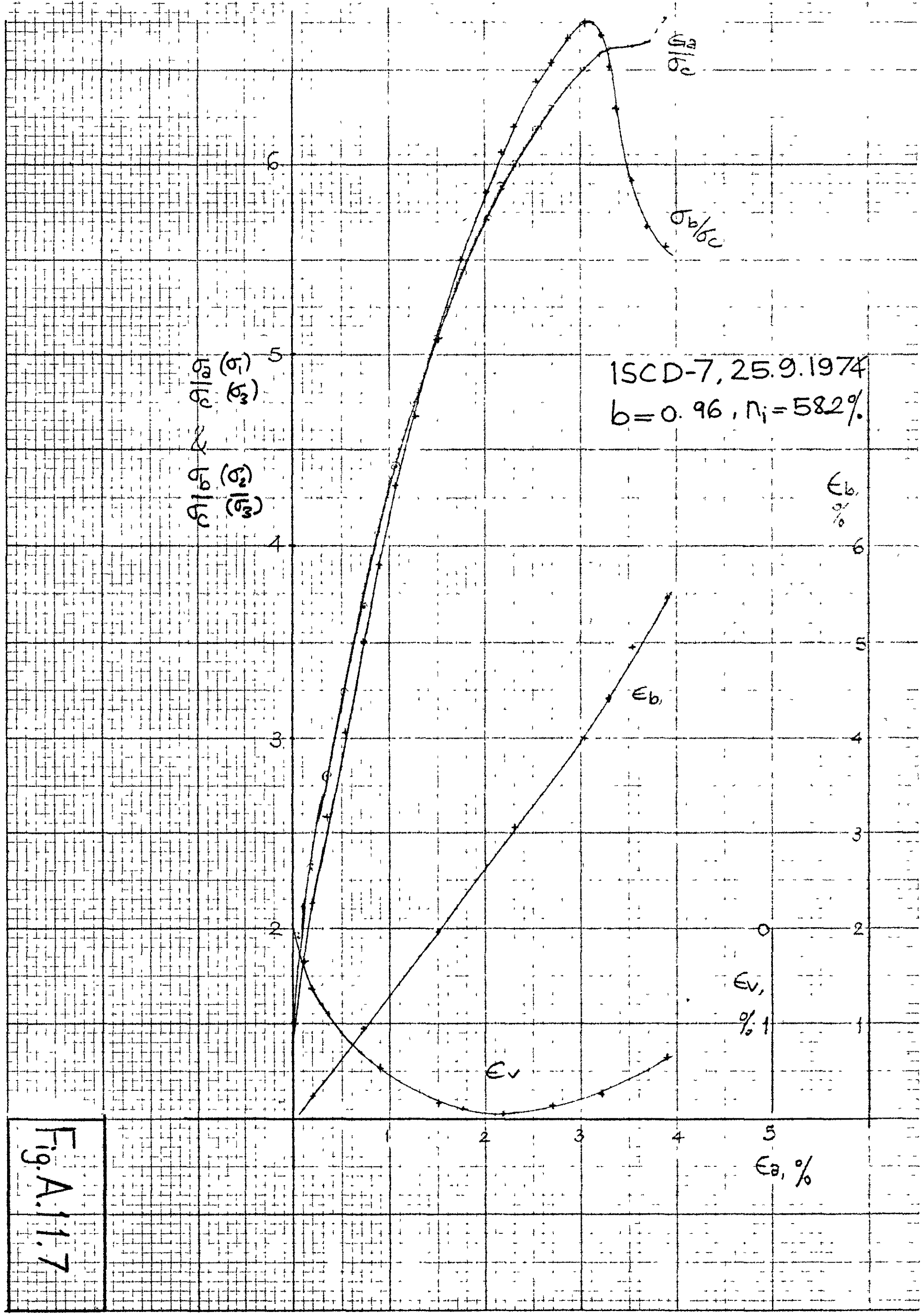


Fig. A.11.7

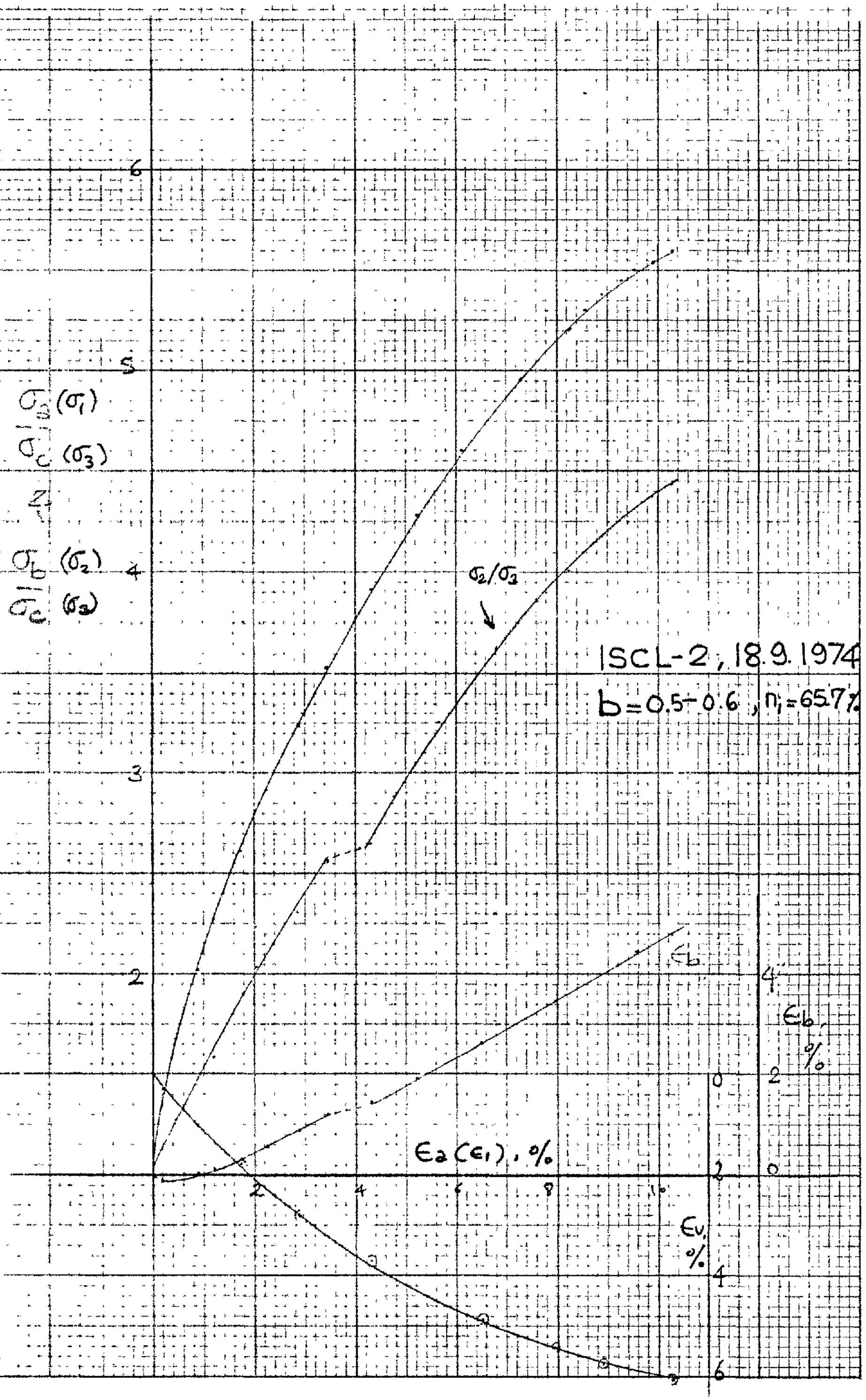


Fig. A.11.10

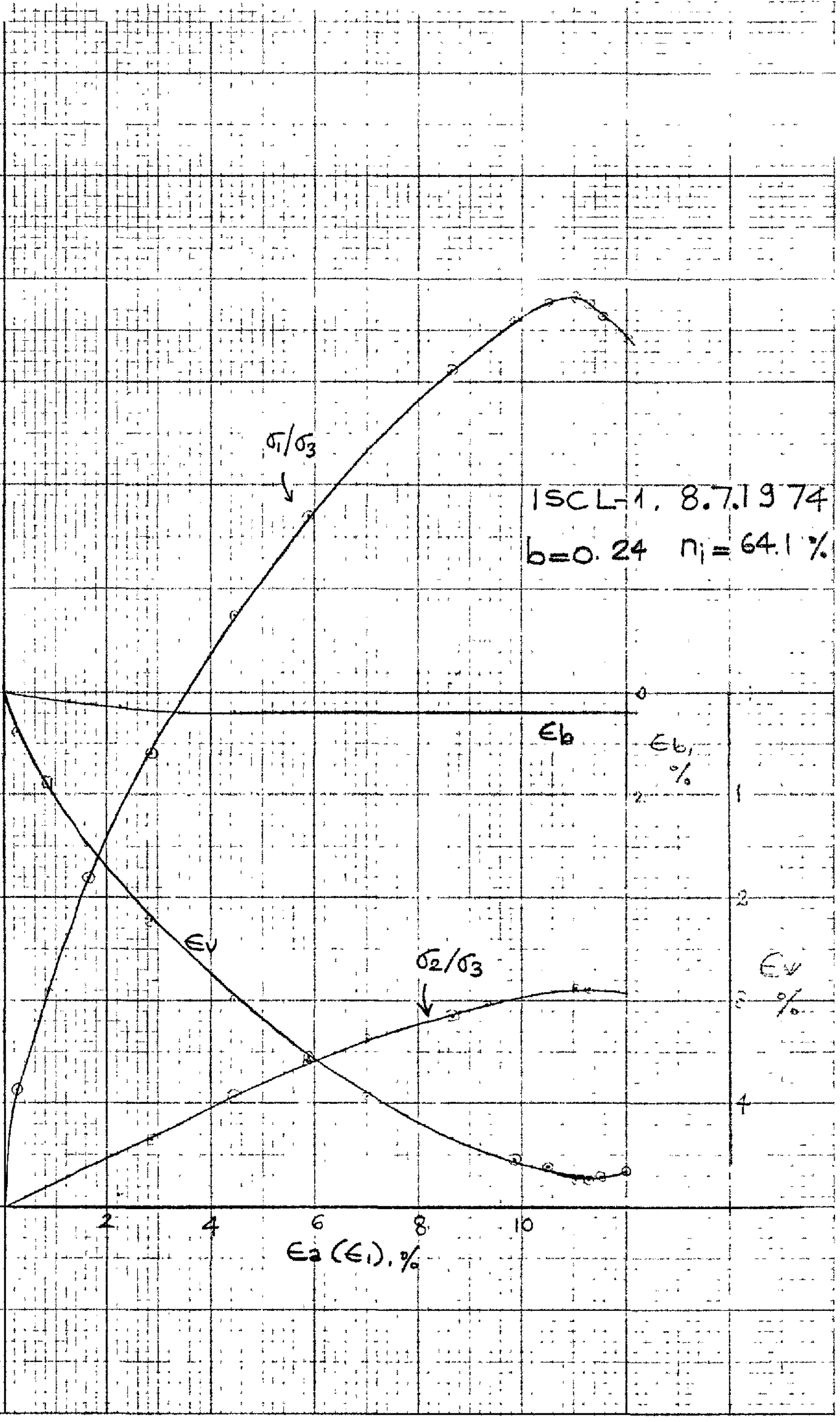


Fig. A.11.9

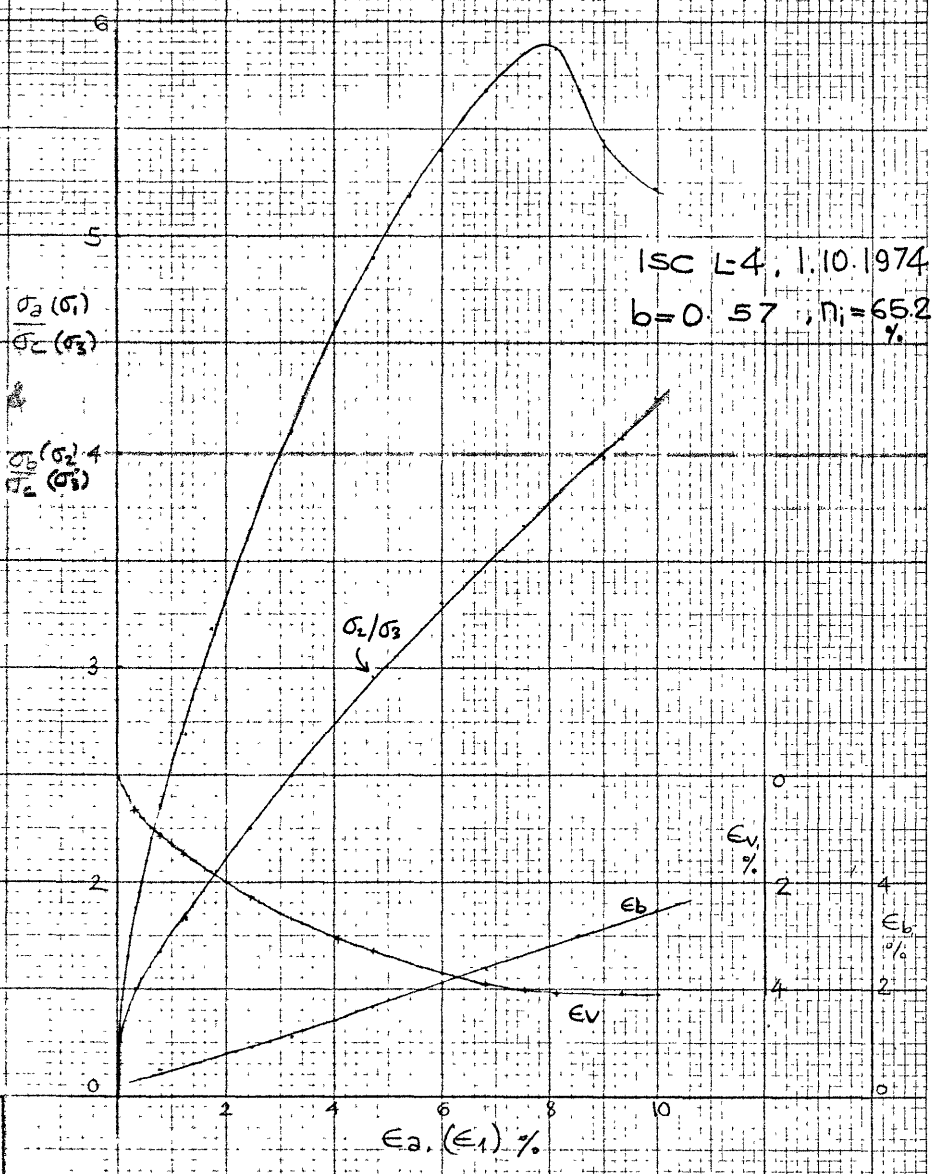


Fig.A.11.12

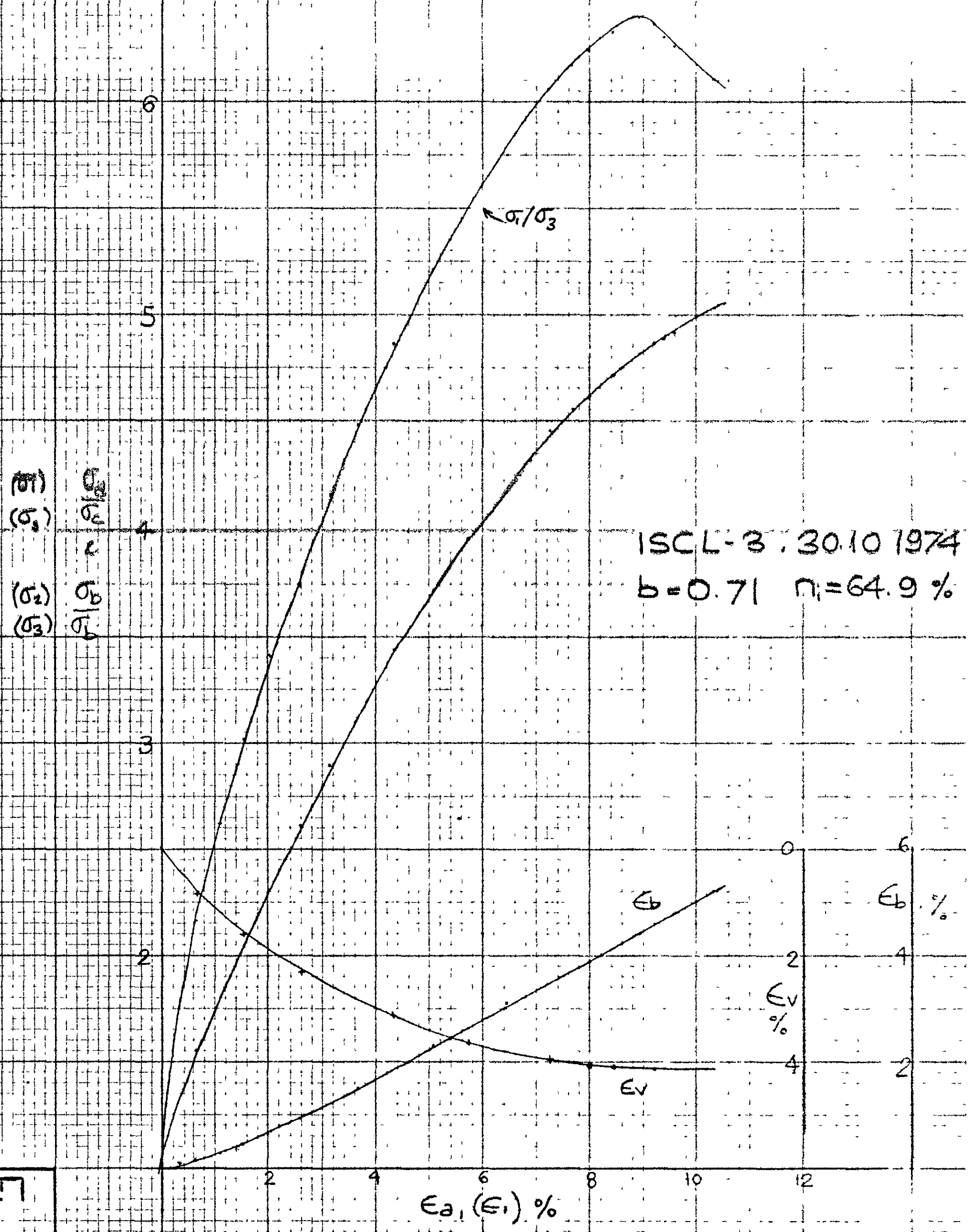


Fig.A.11.11

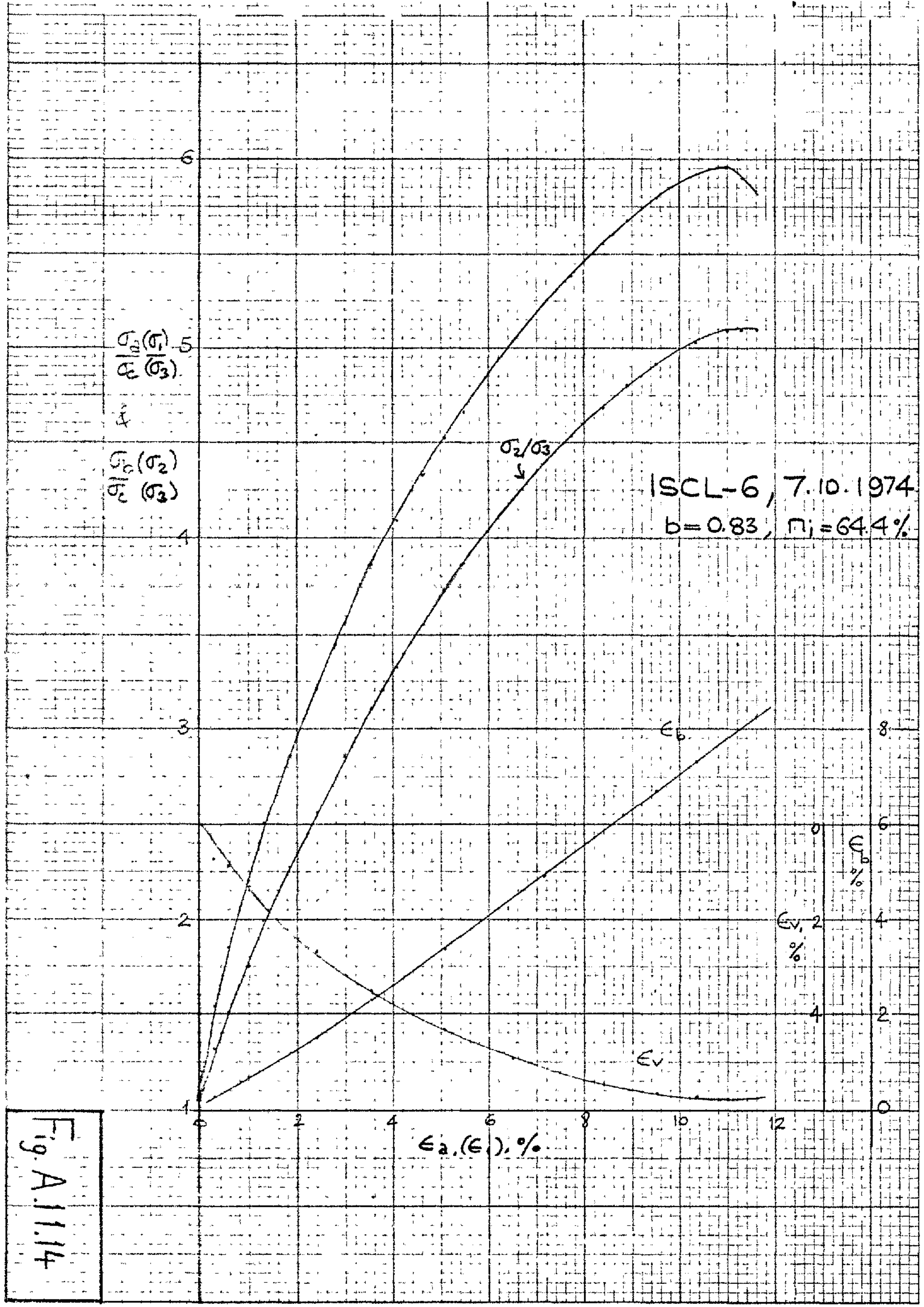


Fig. A.11.14

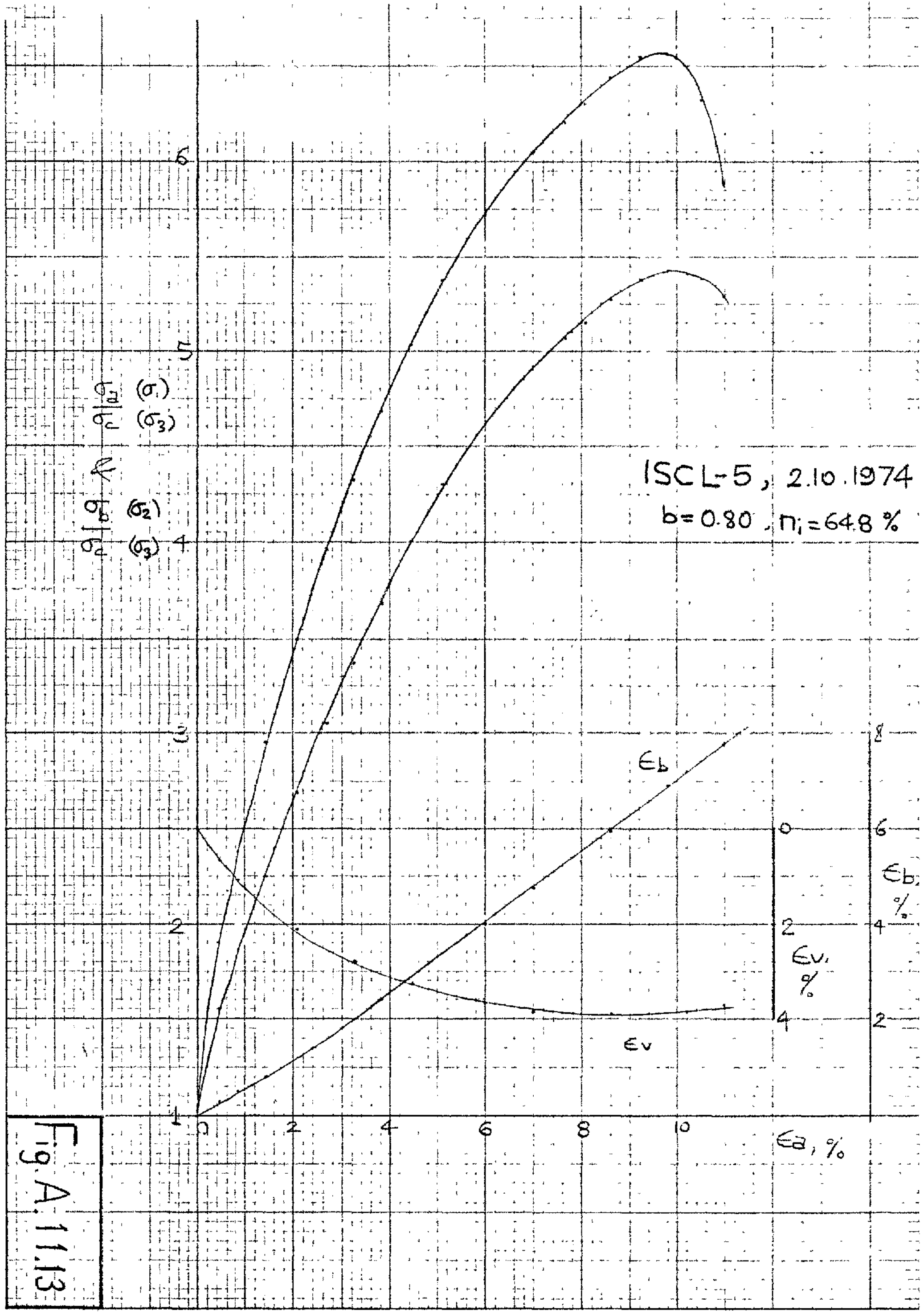


Fig. A.11.13

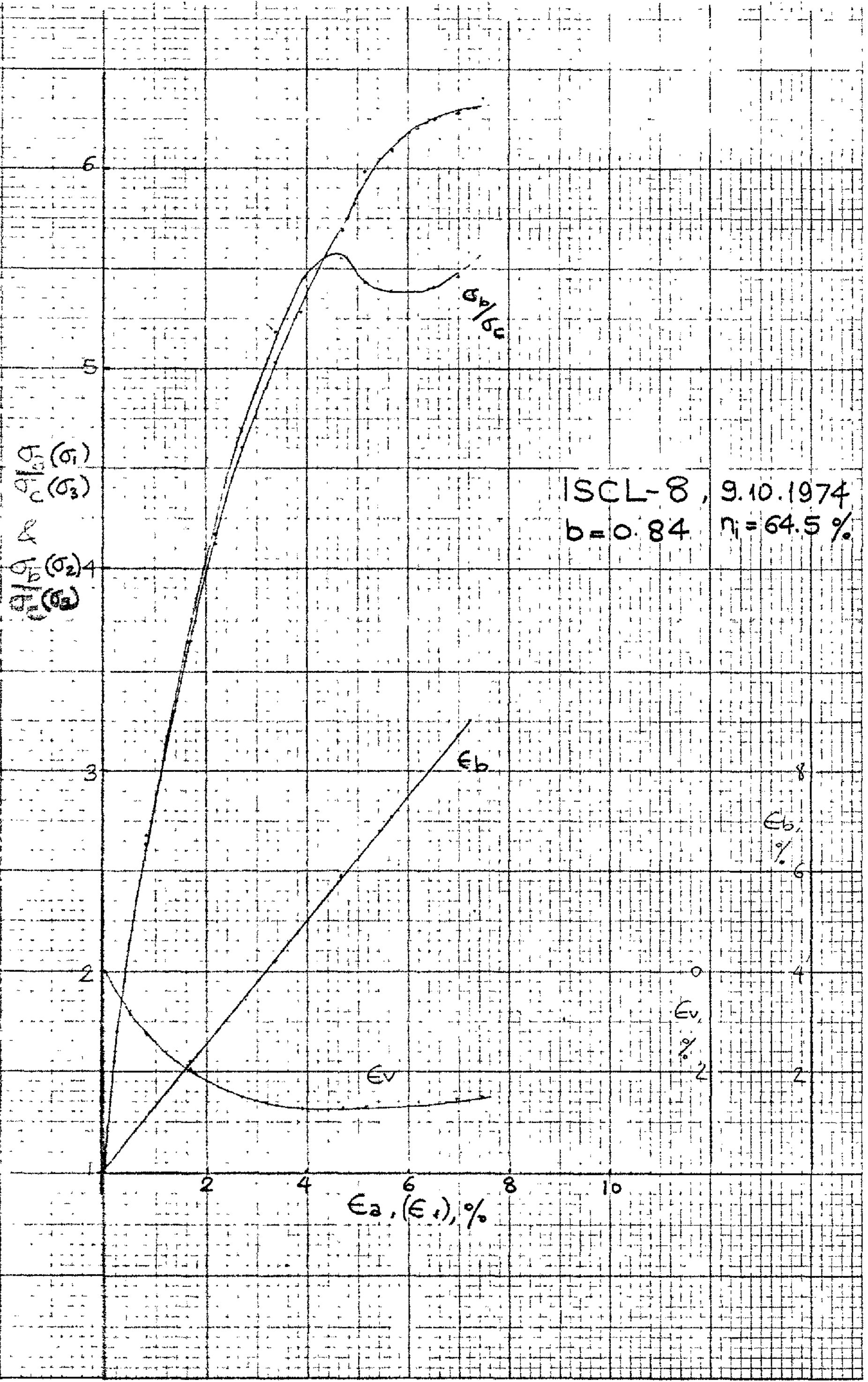


Fig. A.11.16

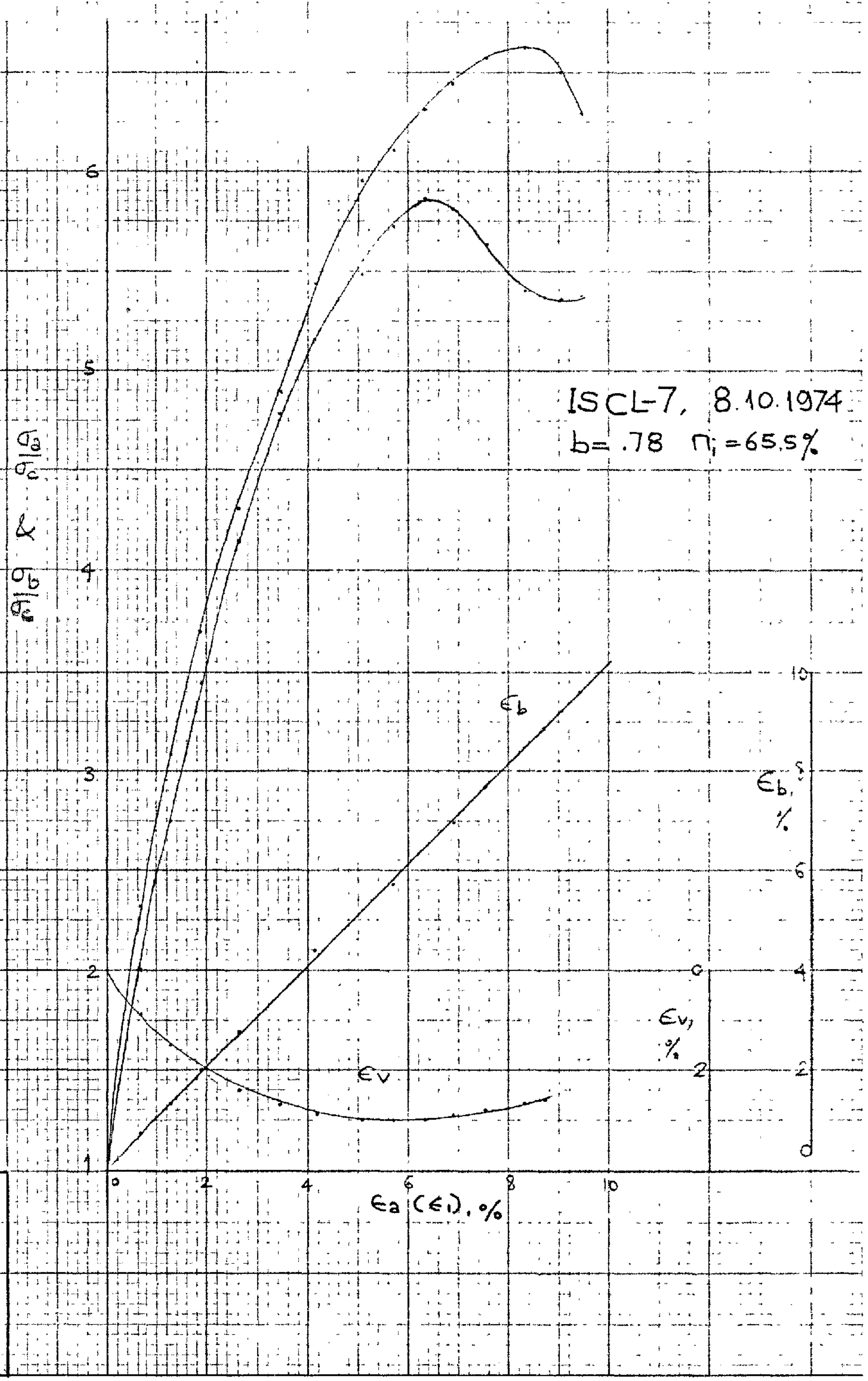
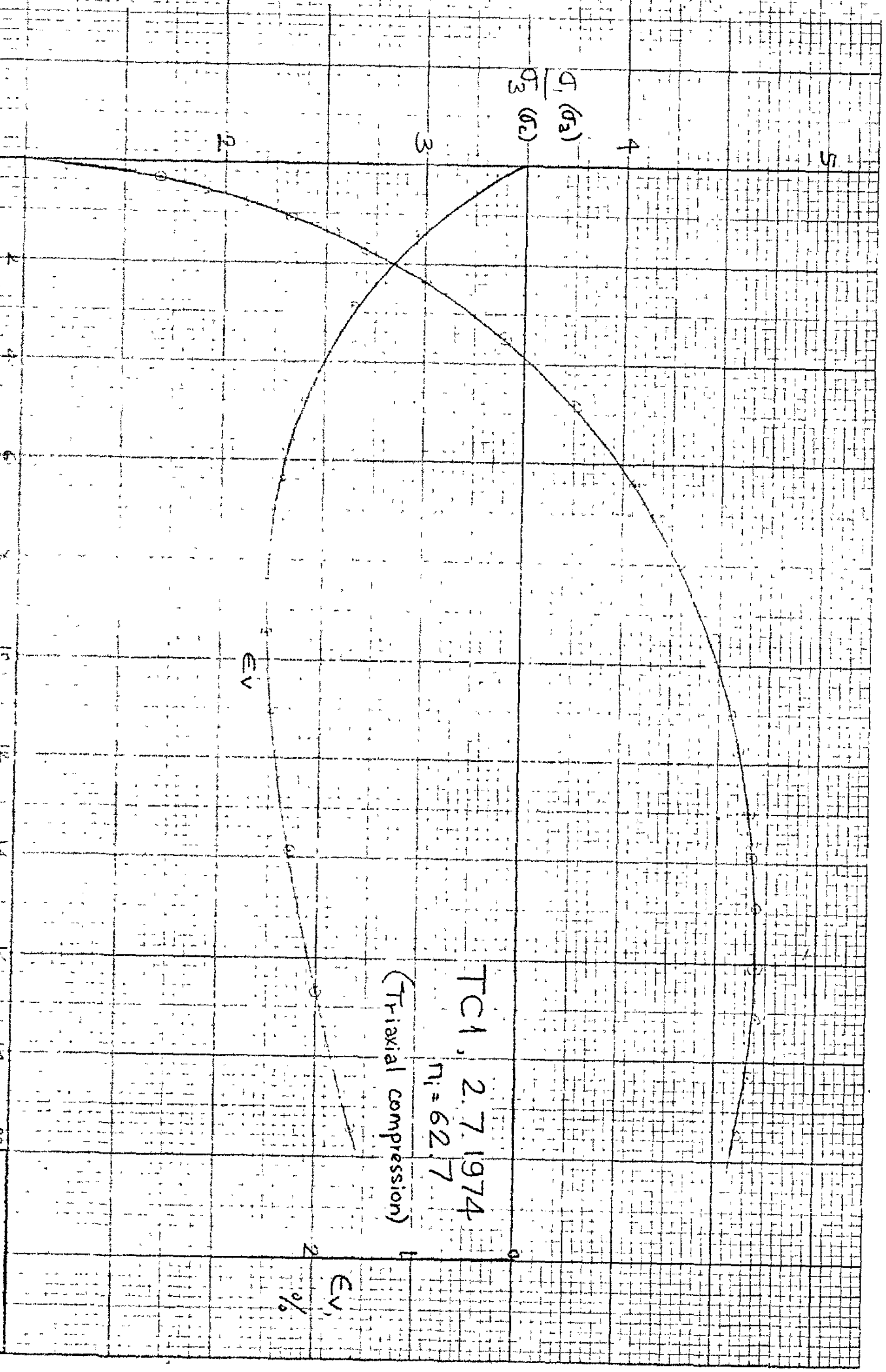
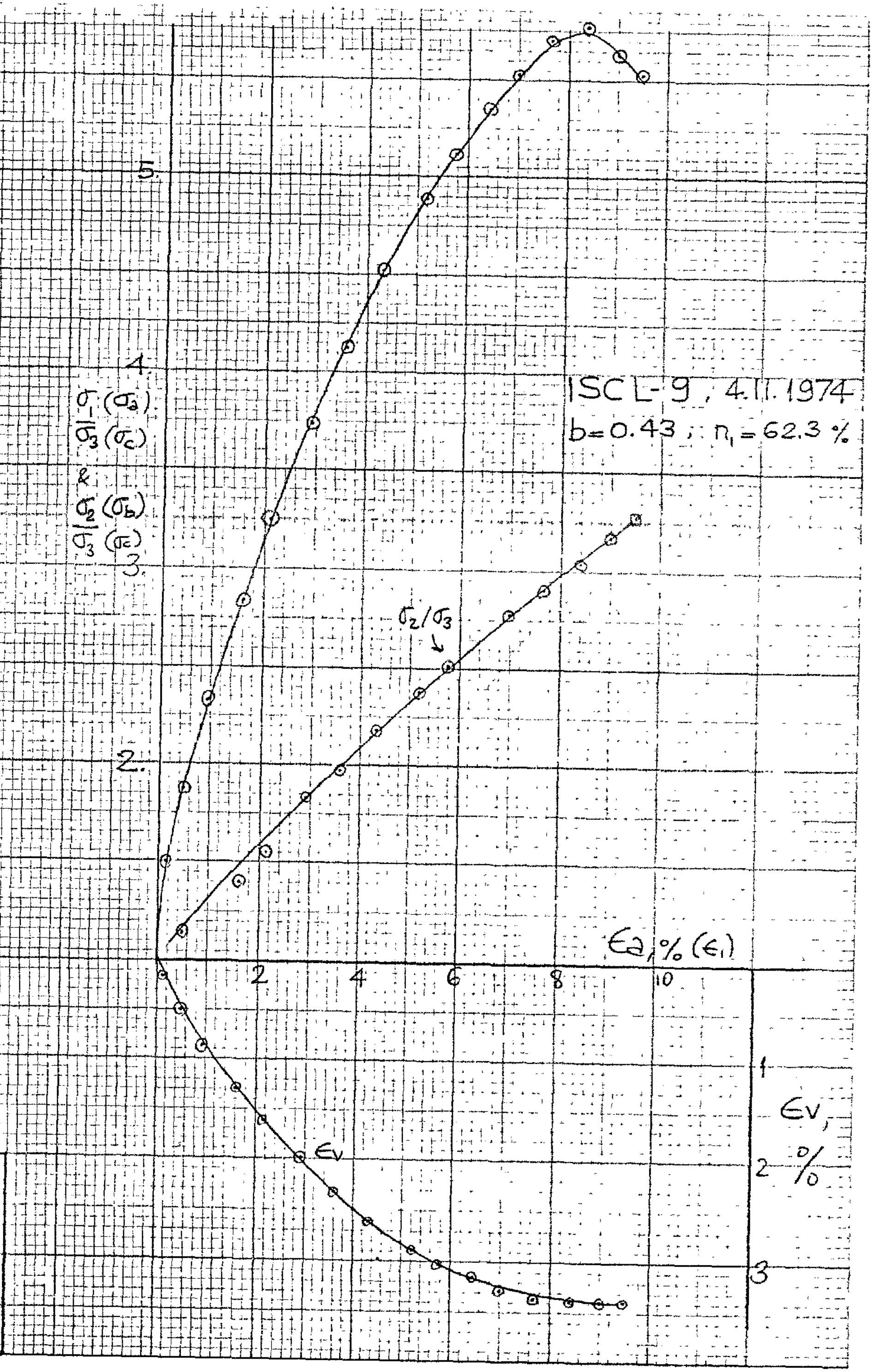


Fig. A.11.15



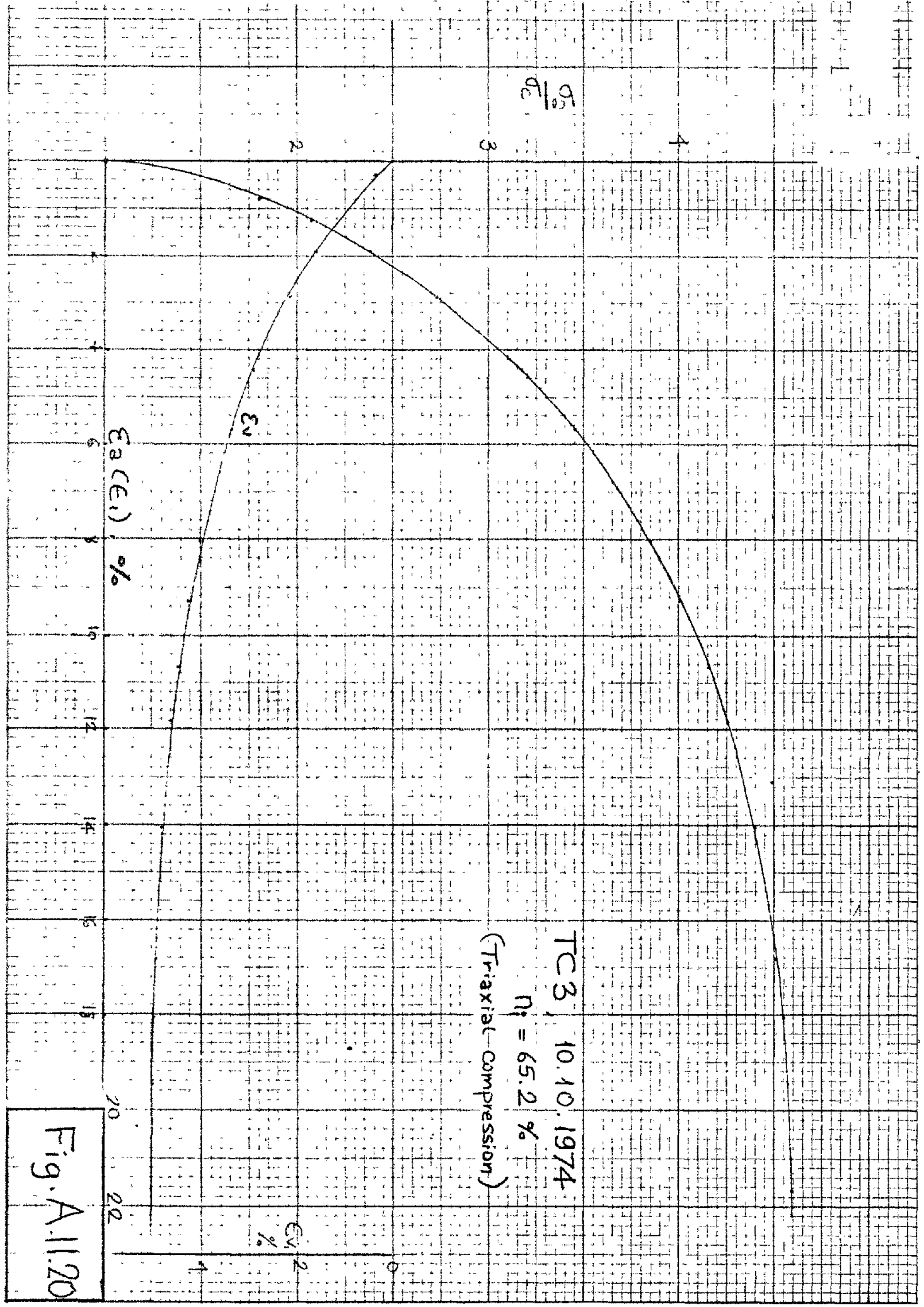
TC1, 2.7.1974
n1 = 62.7
(Triaxial compression)

Fig. A.11.18



ISCL-9, 4.11.1974
b=0.43 ; n1=62.3 %

Fig. A.11.17

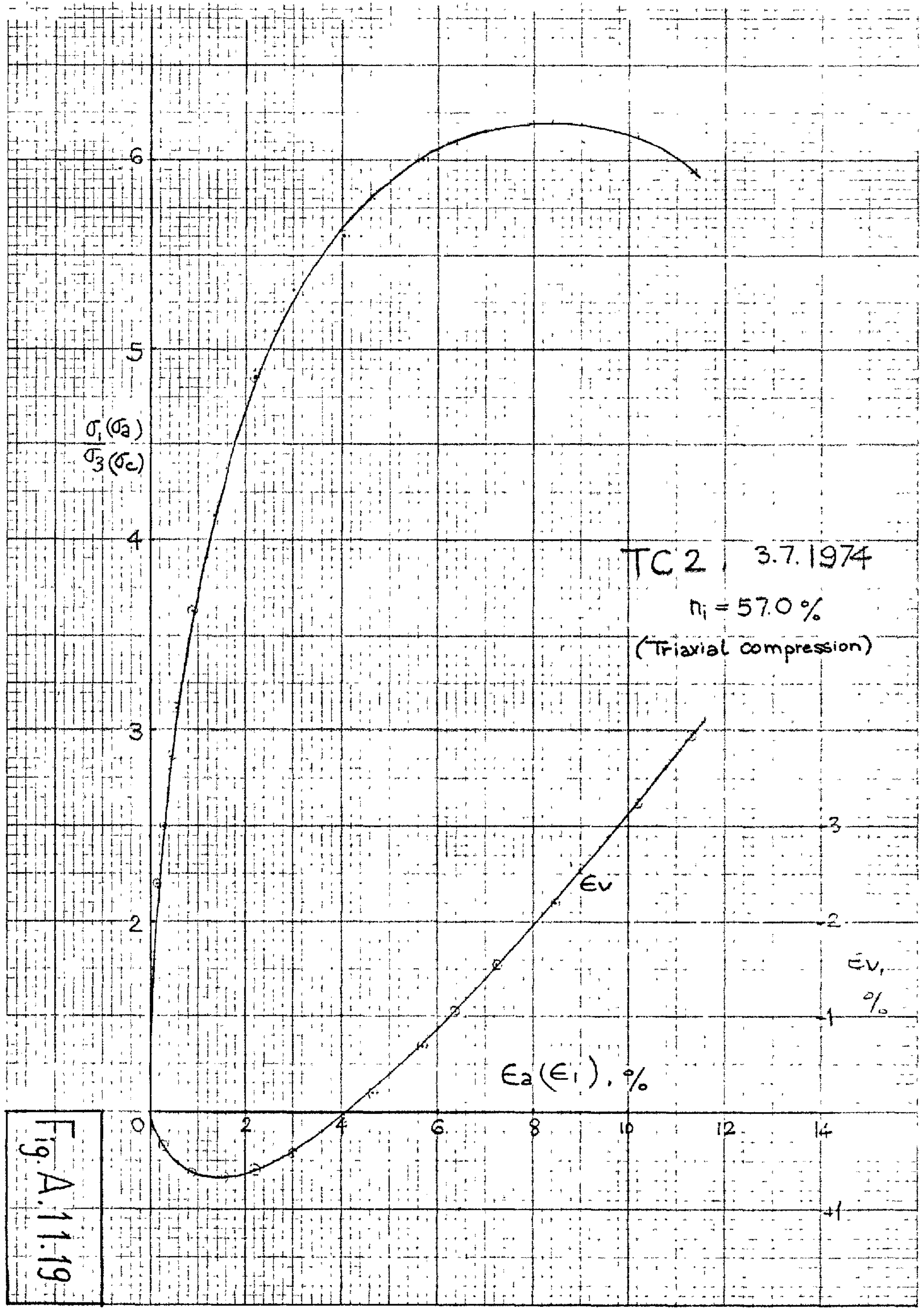


TC3, 10.10.1974

n_i = 65.2 %

(Triaxial compression)

Fig. A.11.20



TC 2, 3.7.1974

n_i = 57.0 %

(Triaxial compression)

Fig. A.11.19

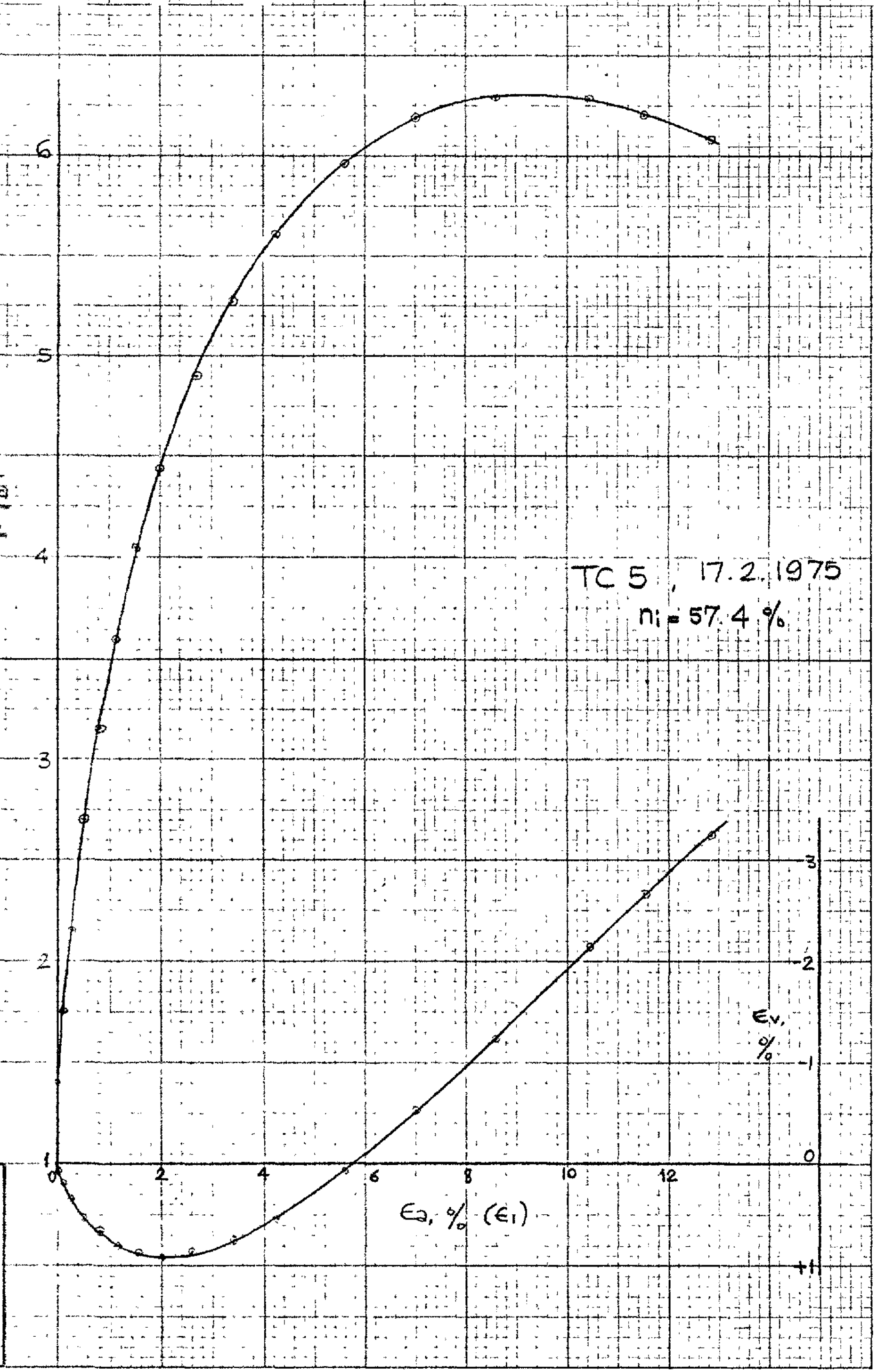


Fig. A.11.22

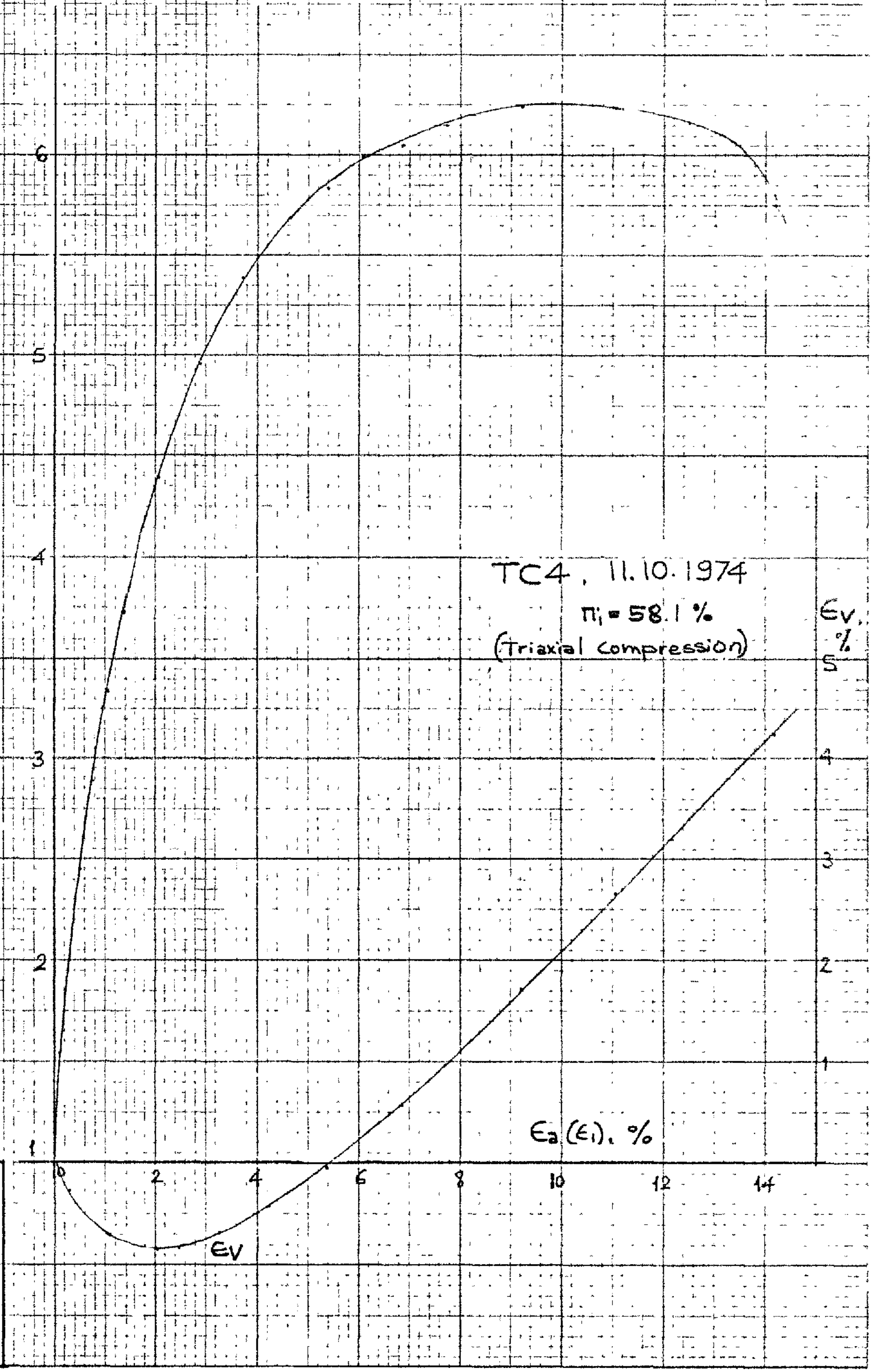
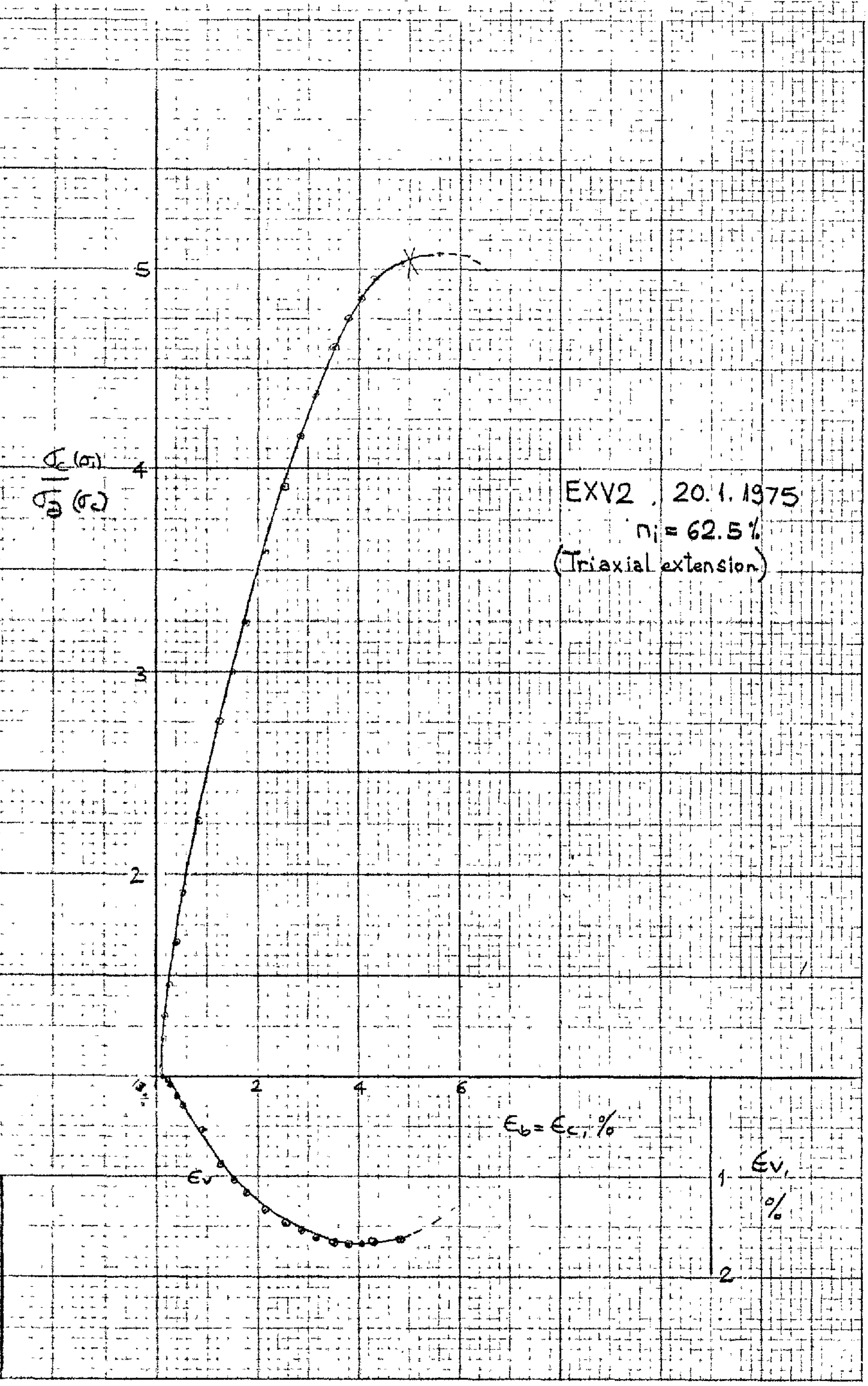
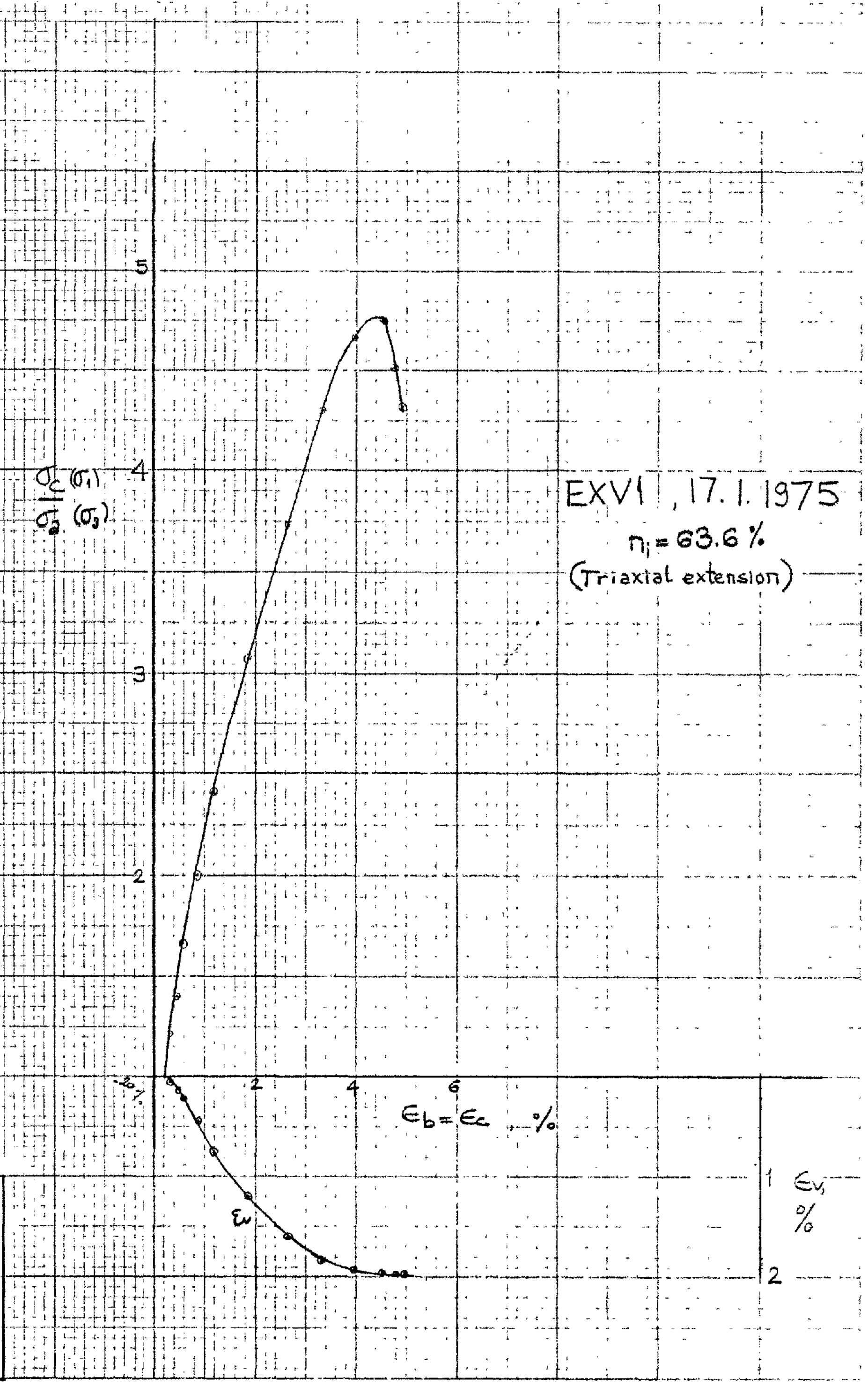


Fig. A.11.21



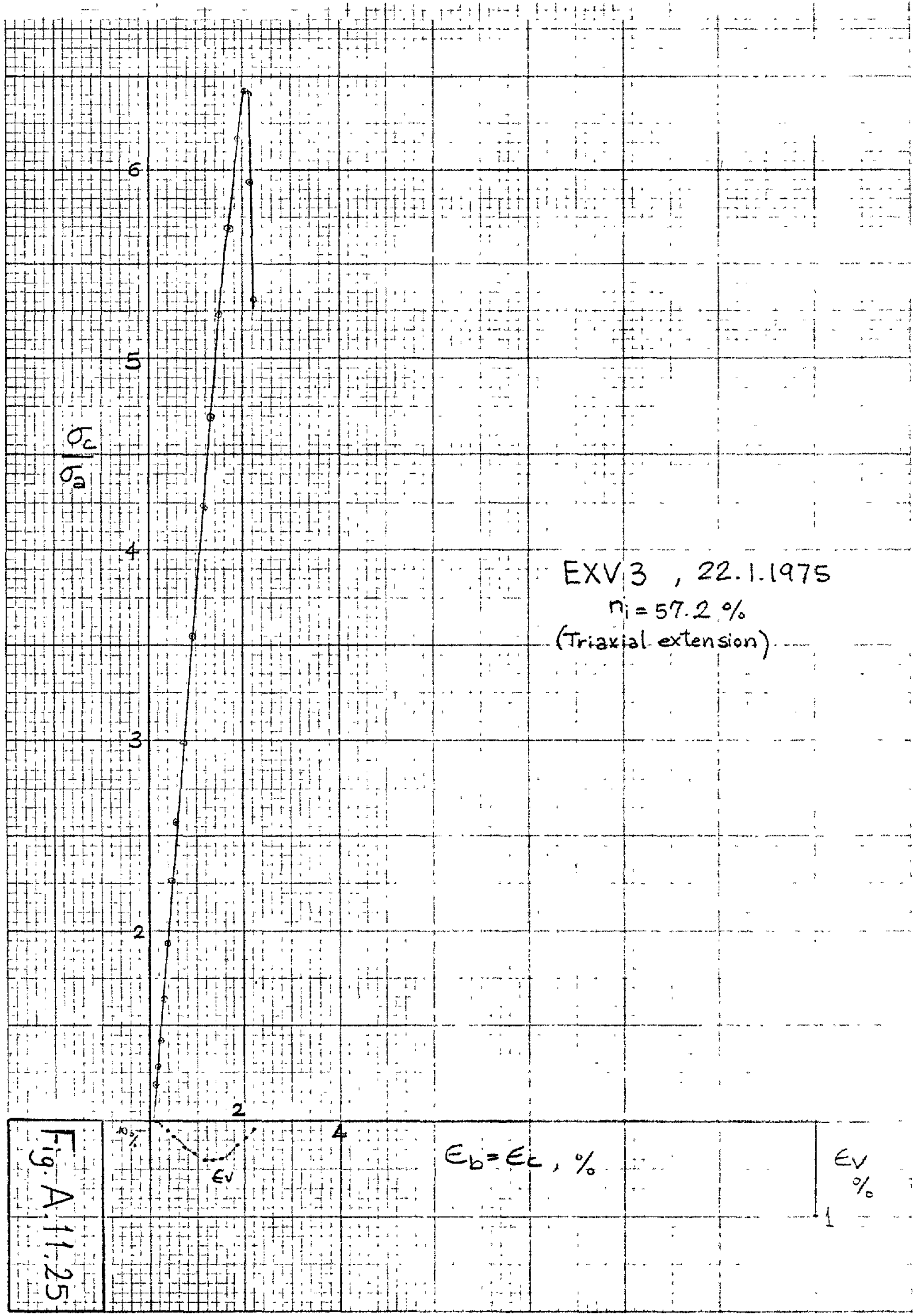
EXV2, 20.1.1975
$n_i = 62.5\%$
(Triaxial extension)

Fig. A.11.24



EXV1, 17.1.1975
$n_i = 63.6\%$
(Triaxial extension)

Fig. A.11.23



EXV 3 , 22.1.1975

$n_i = 57.2$ %

(Triaxial extension)

Fig. A.11.25

APPENDIX 12

STRESS-STRAIN CURVES FOR AVERAGE STRESS LEVEL SERIES AND TRIAXIAL COMPRESSION TESTS ON HAM RIVER SAND.

ASL - ISC1-3

ASL - TC1-4

TC1 - 7

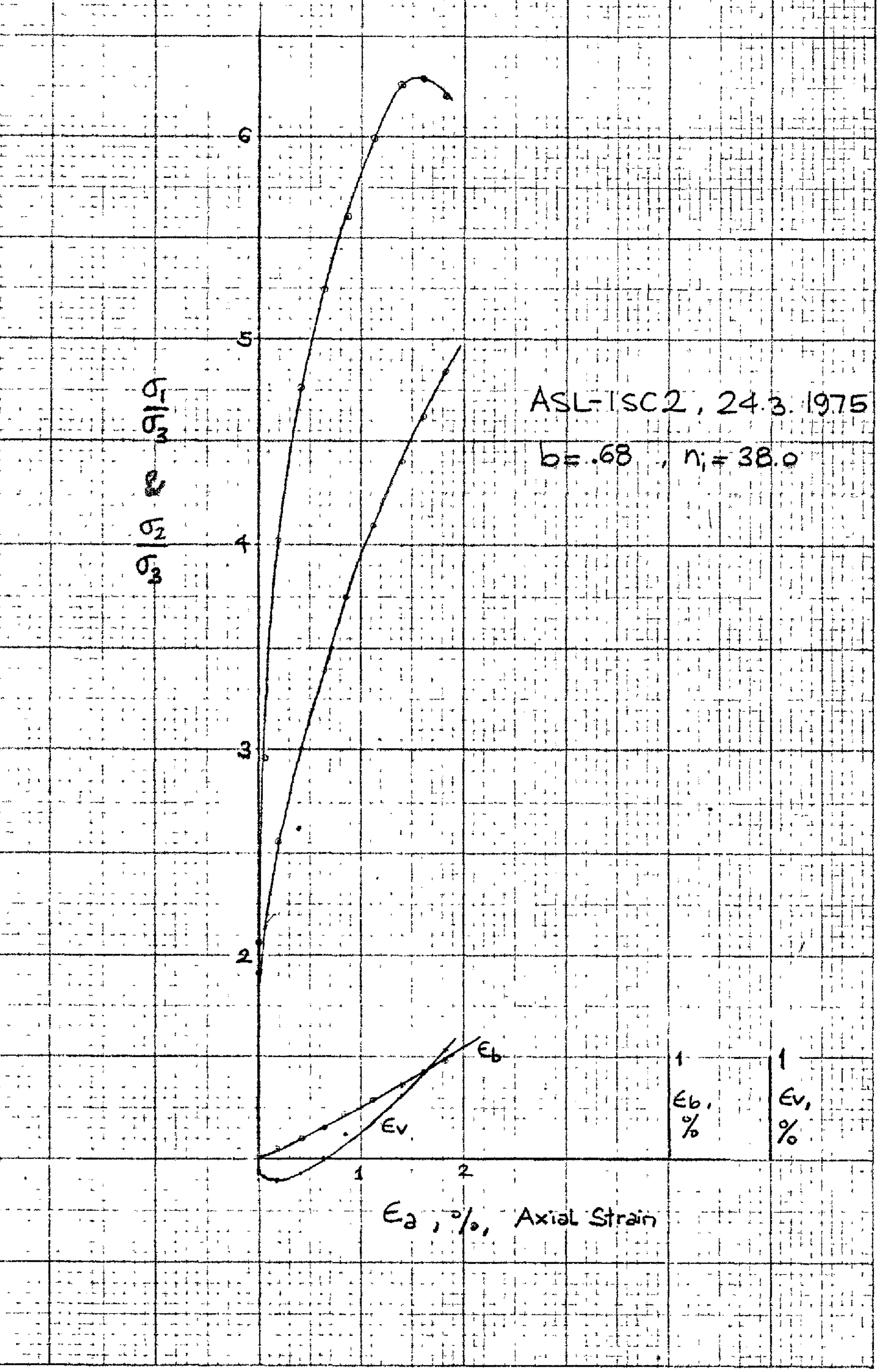


Fig. A.12.2

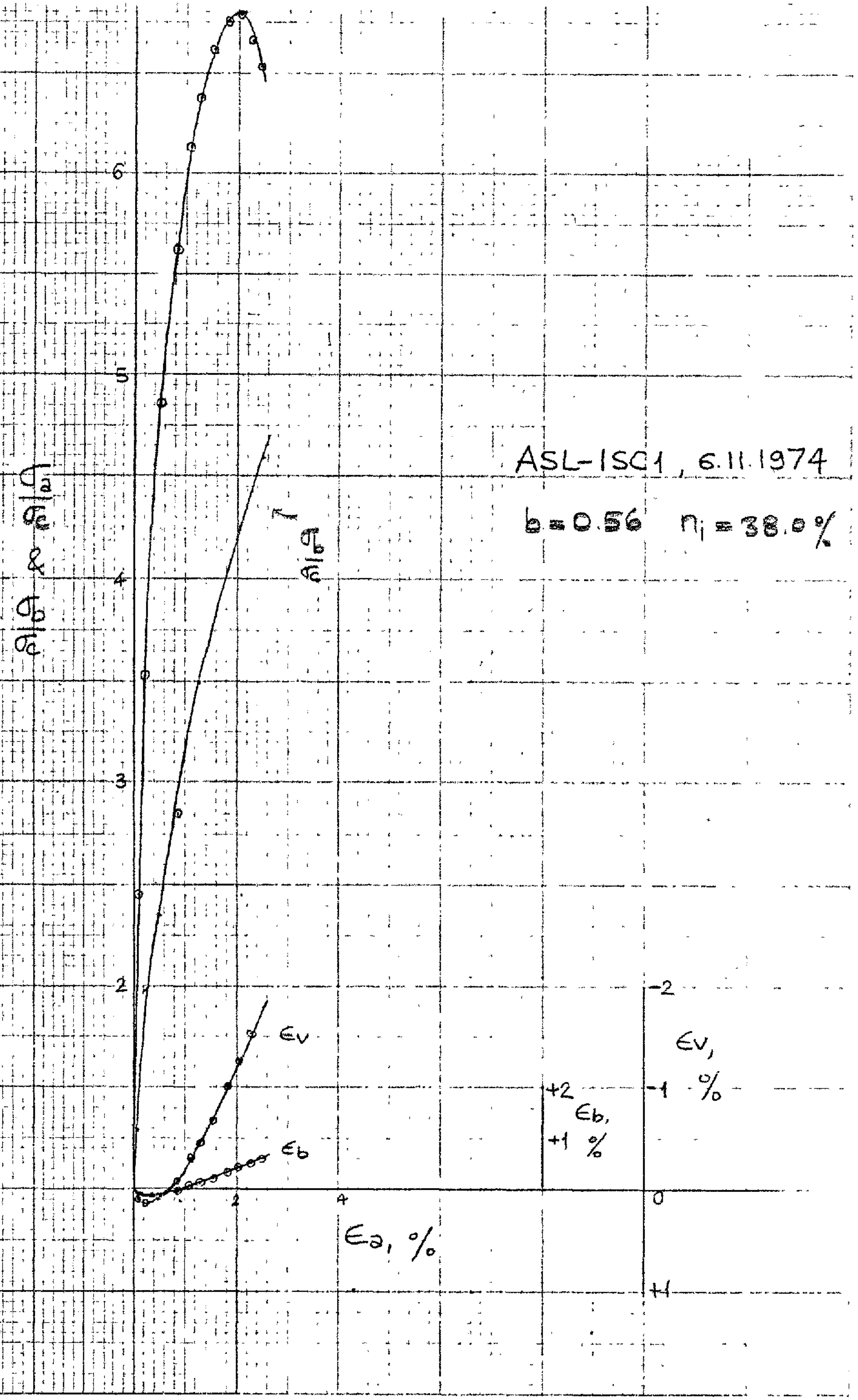


Fig. A.12.1

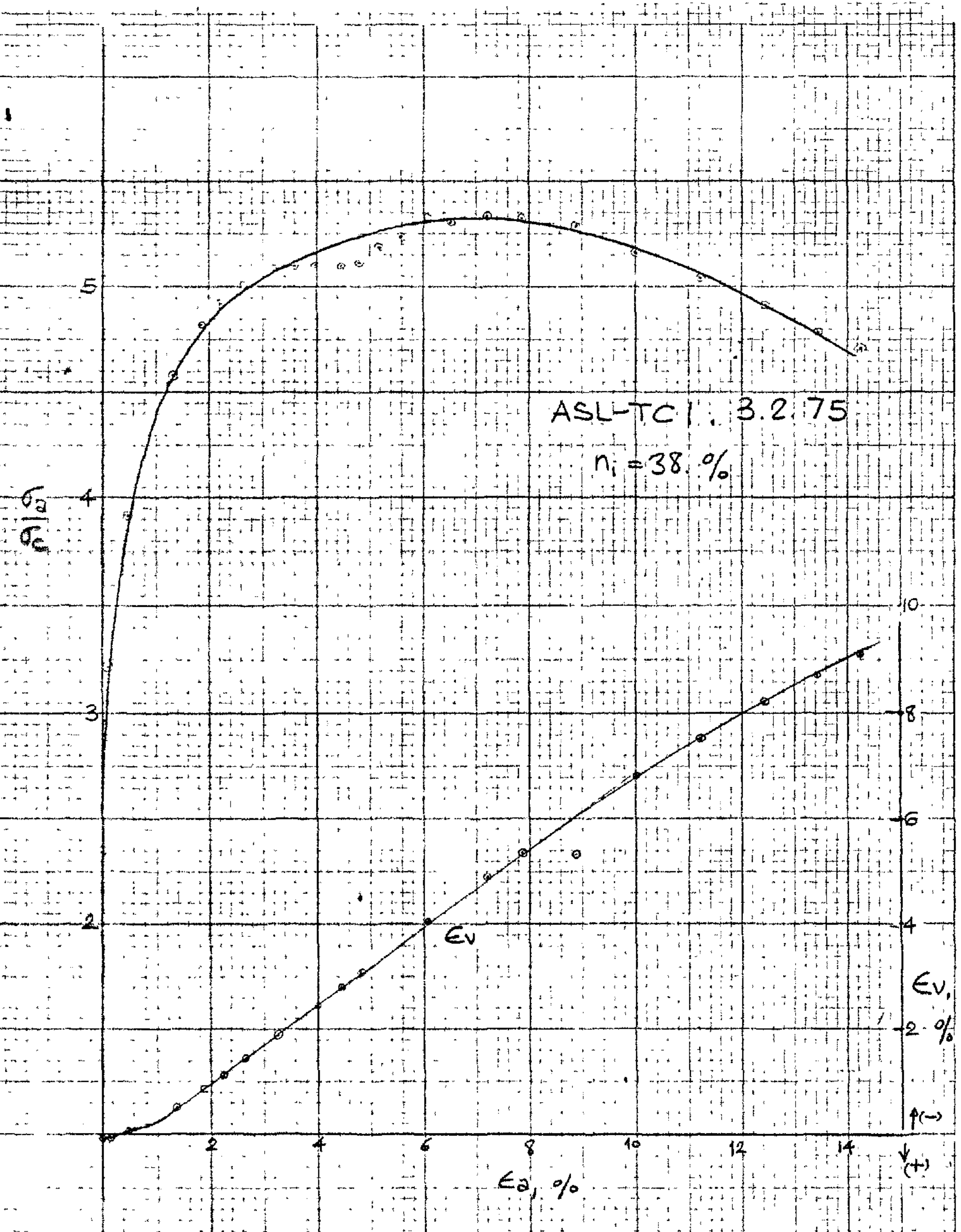


Fig. A.12.4

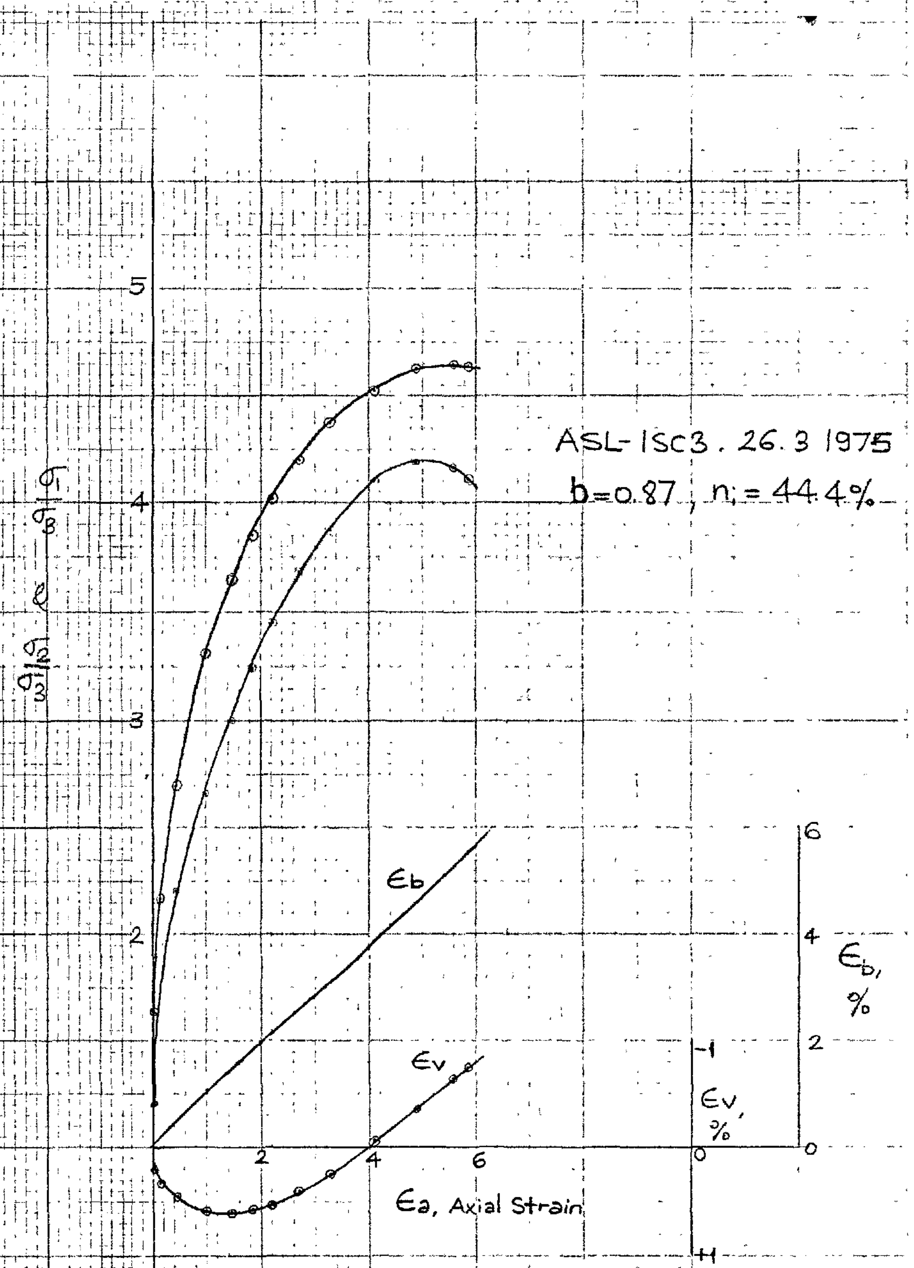


Fig. A.12.3

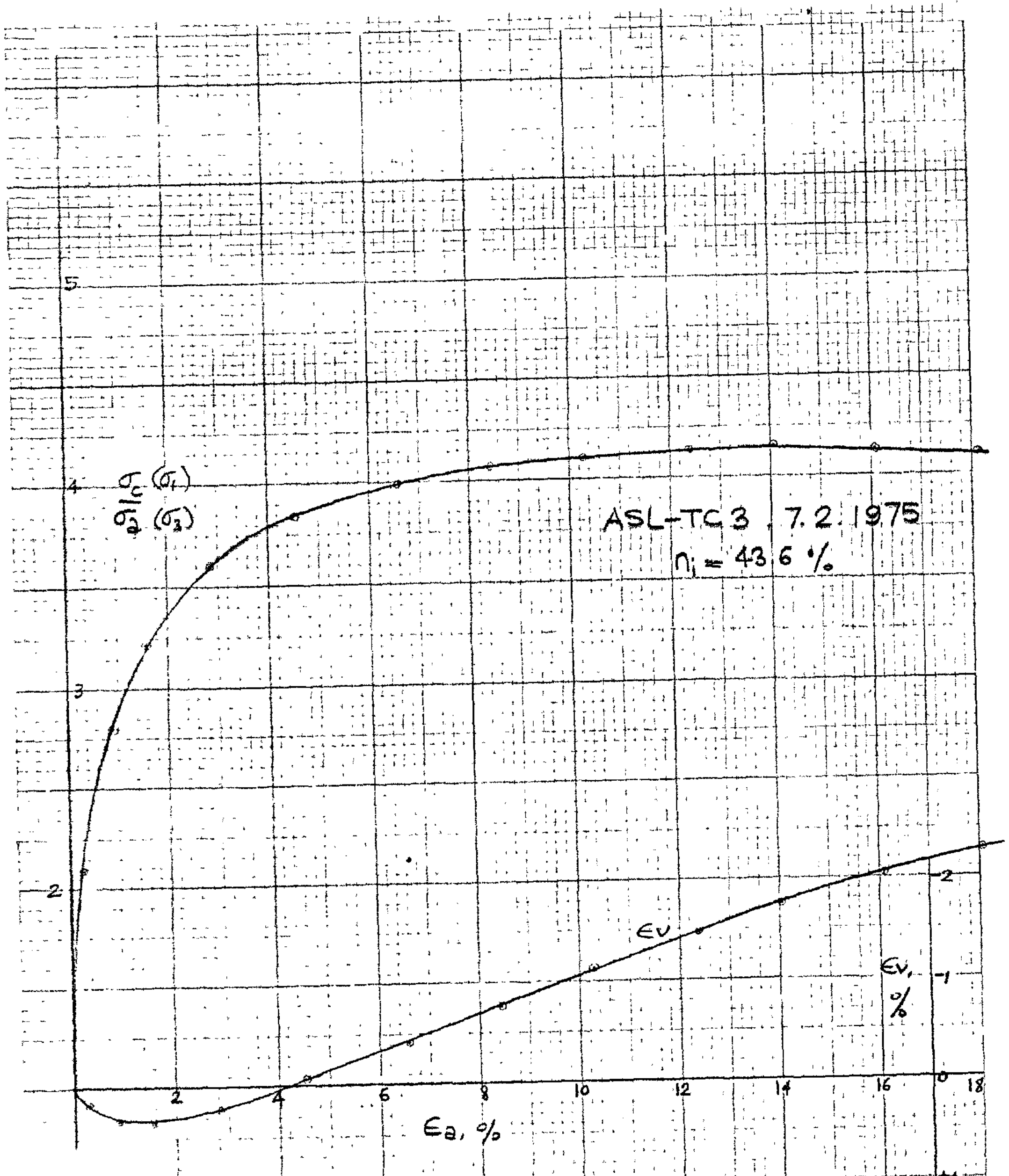


Fig. A.12.6

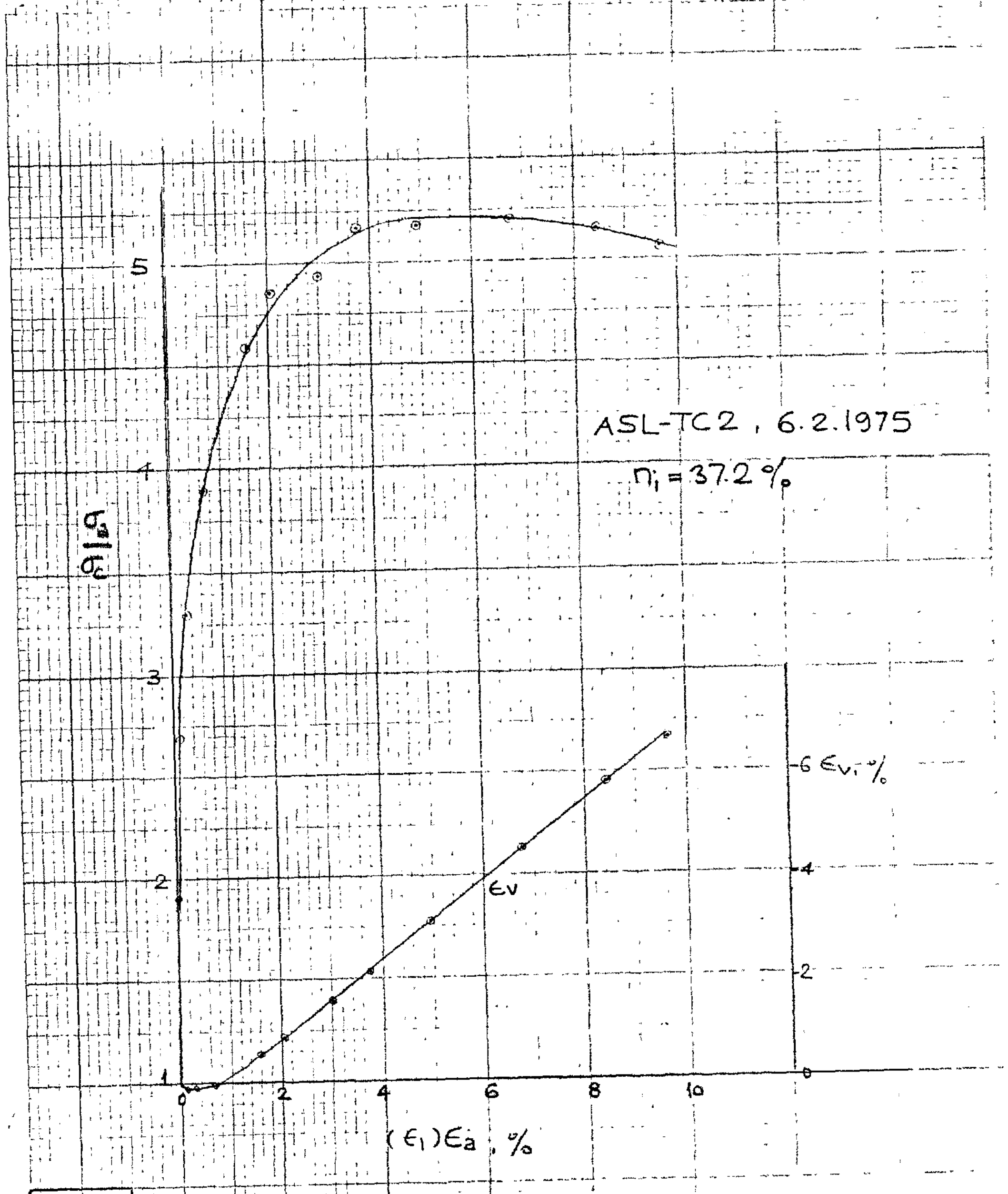
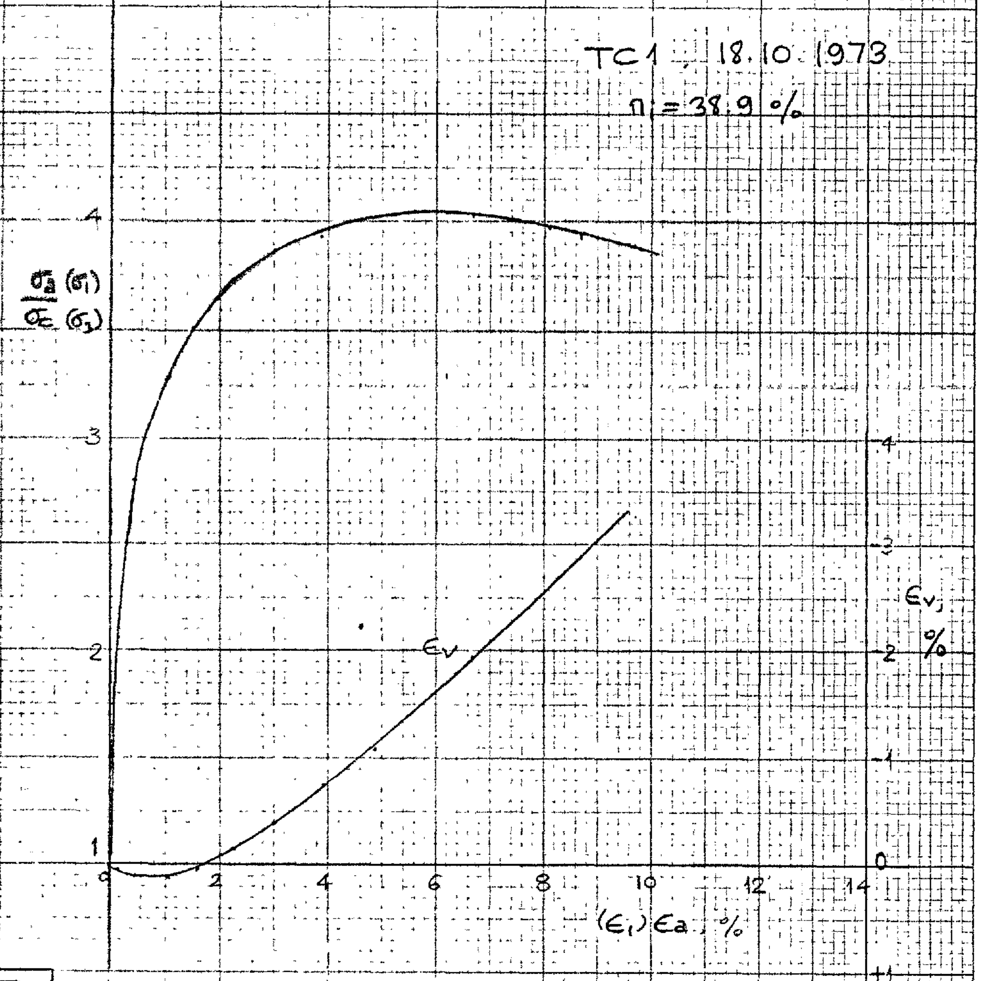


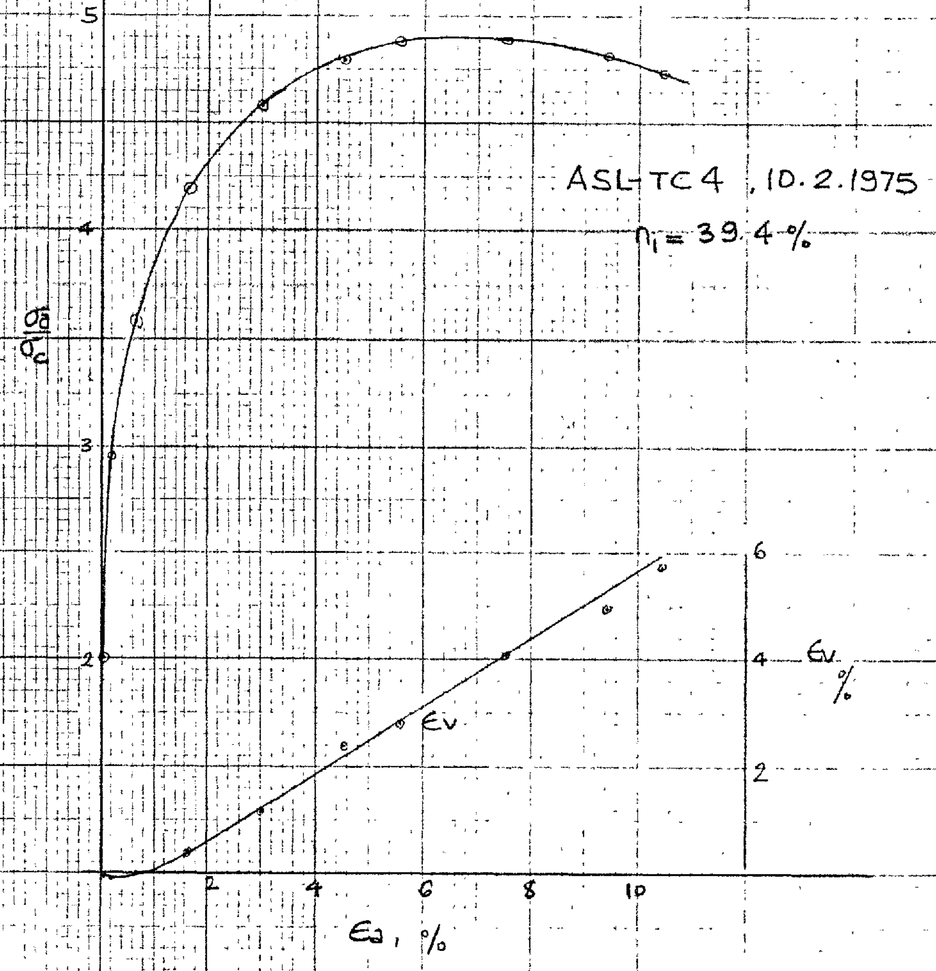
Fig. A.12.5



TC1 , 18.10.1973

n_i = 38.9 %

Fig.A.12.8



ASL-TC4 , 10.2.1975

n_i = 39.4 %

Fig.A.12.7

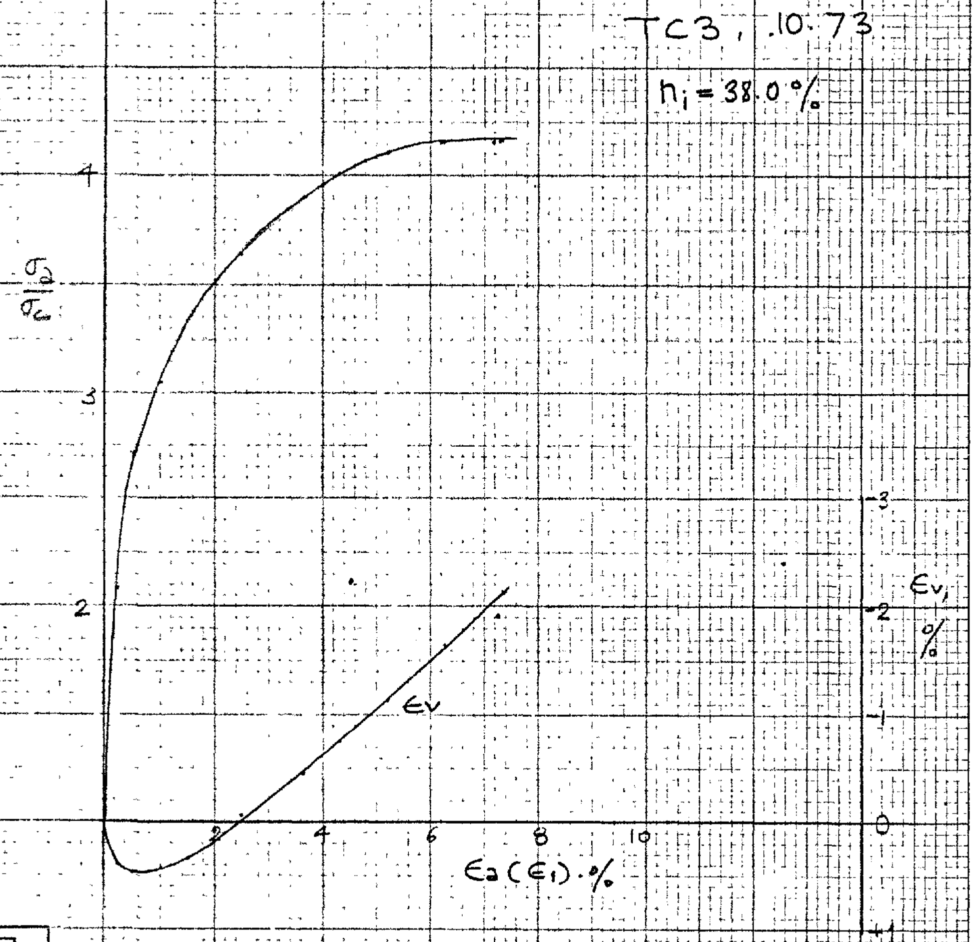


Fig. A.12.10

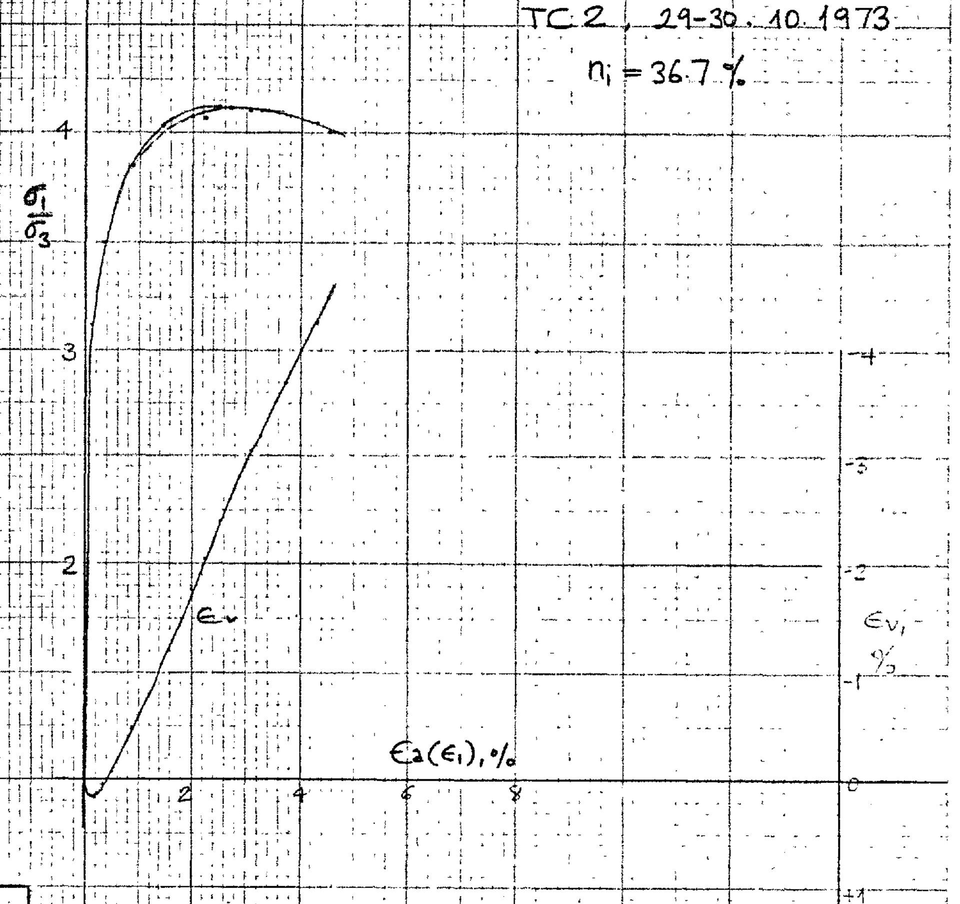


Fig. A.12.9

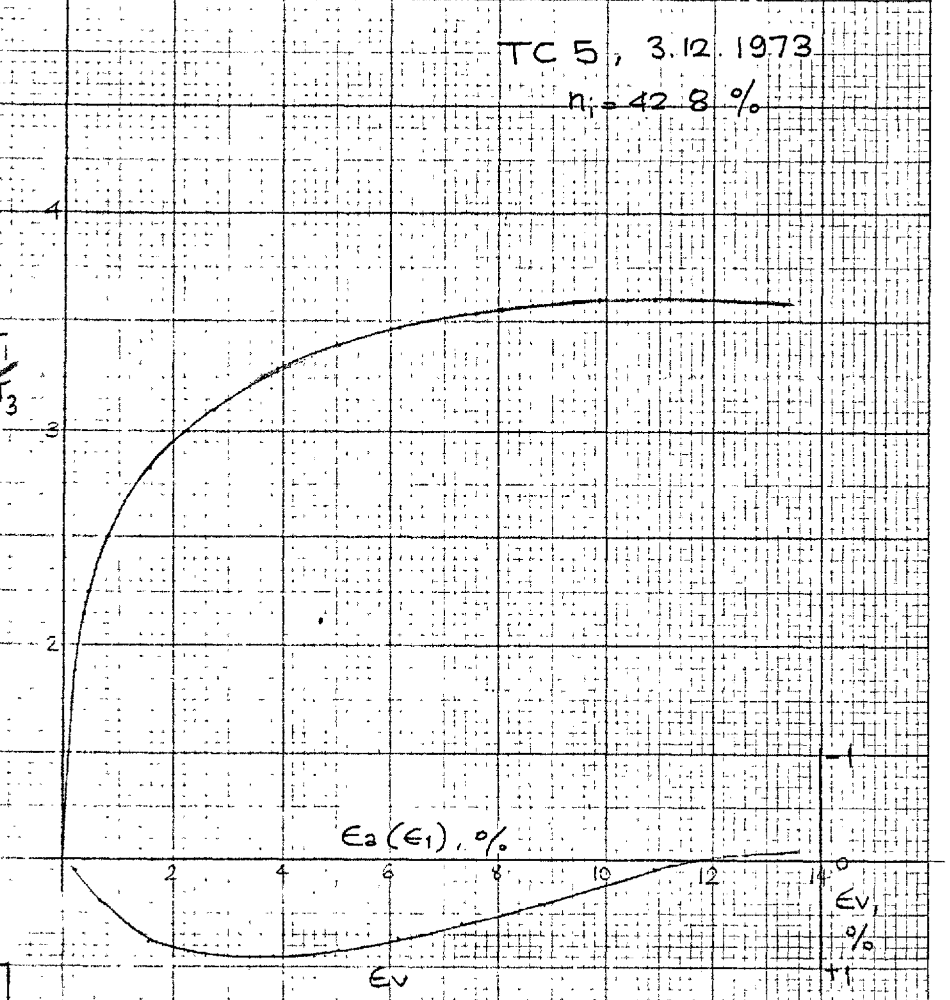


Fig. A.12.12

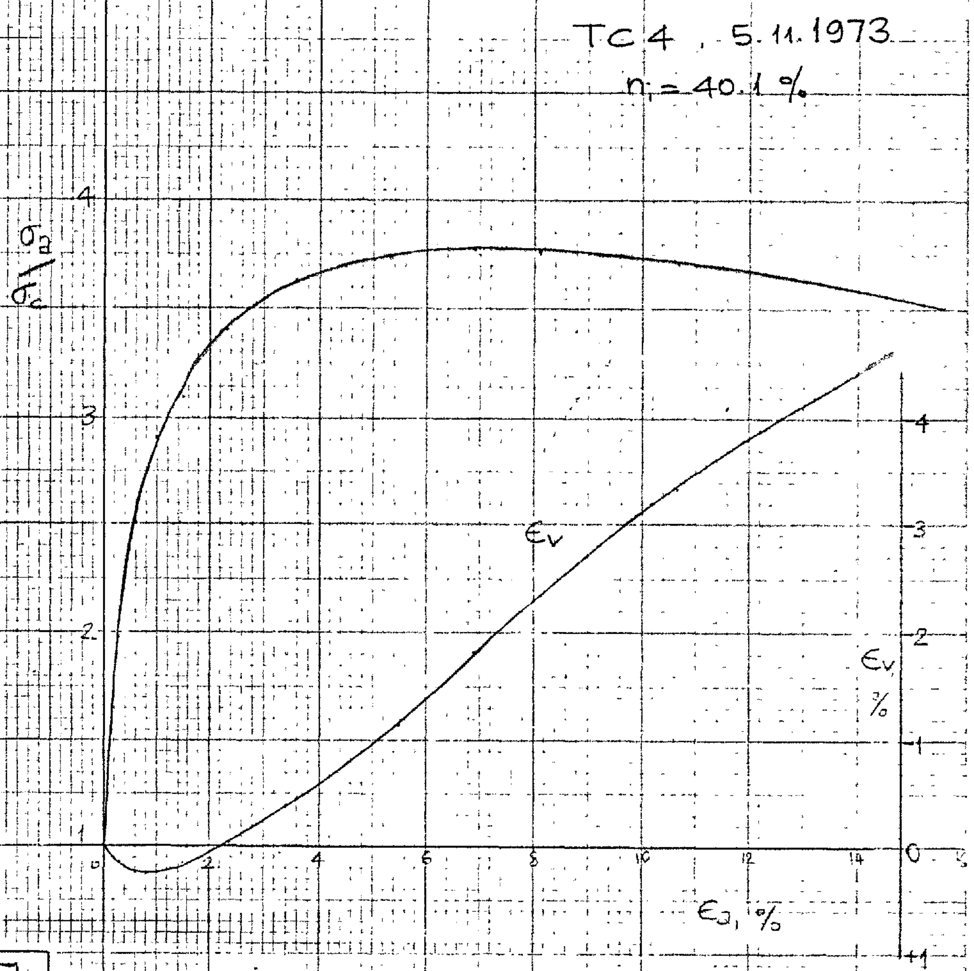


Fig. A.12.11

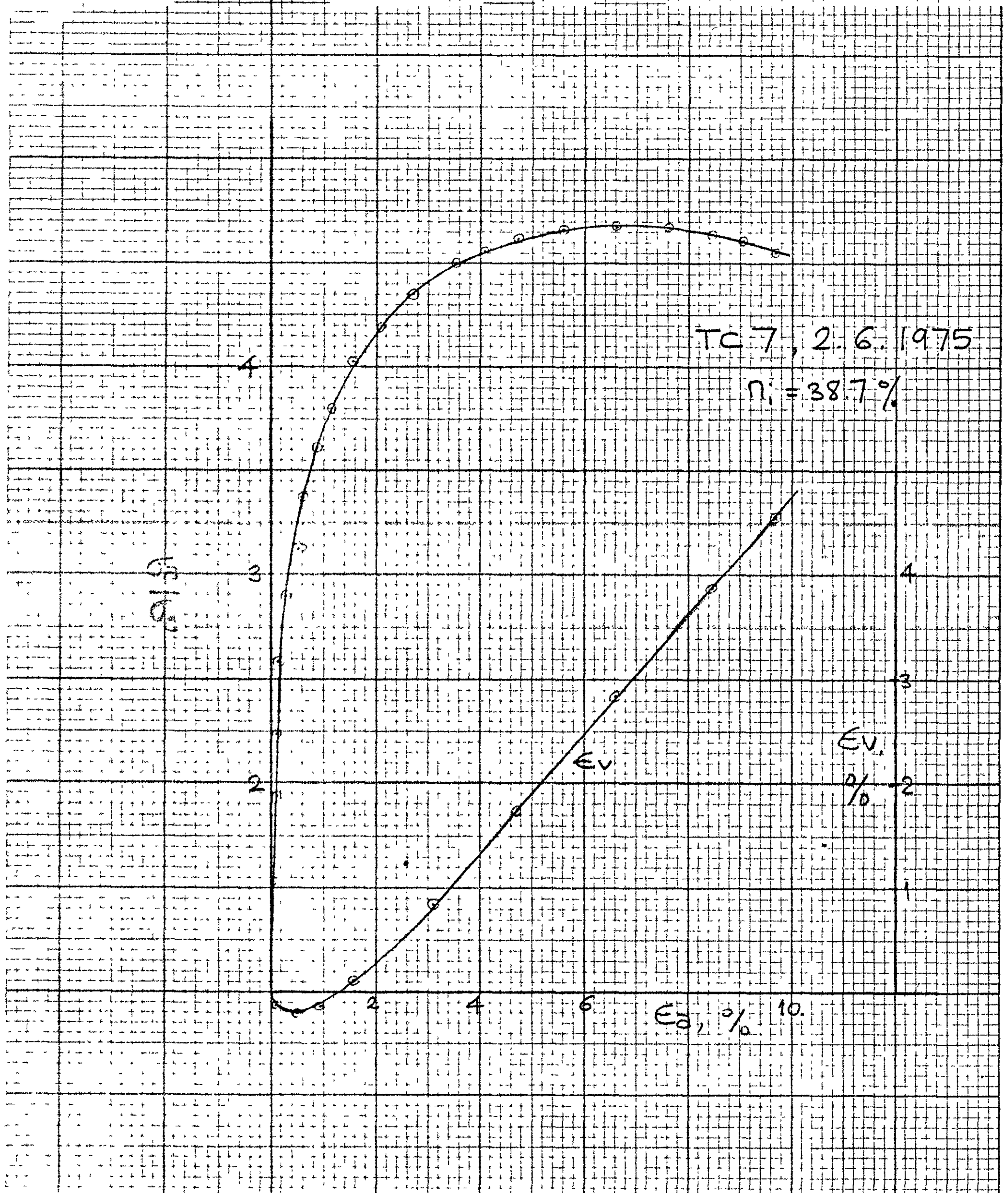


Fig. A.12.14

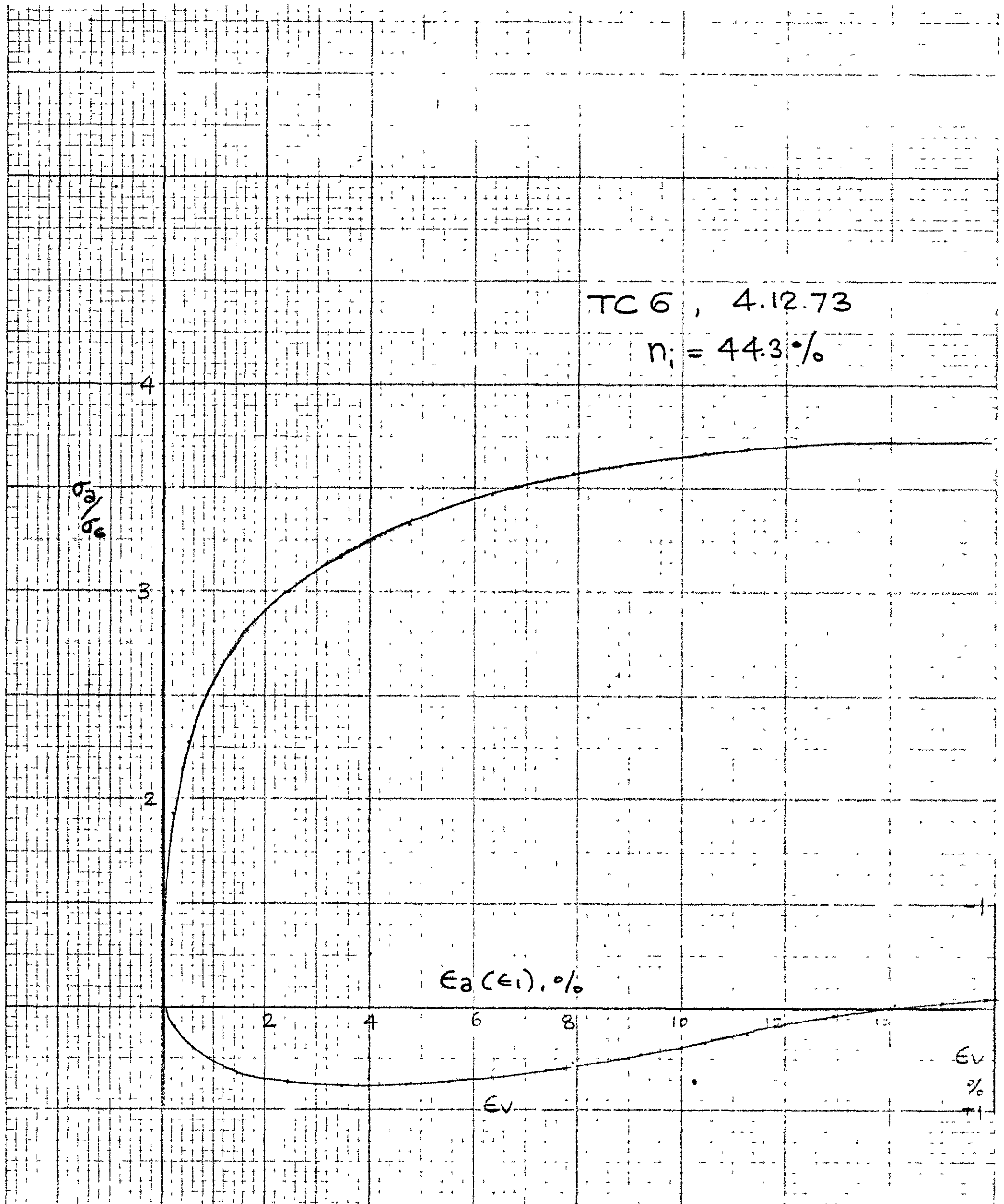


Fig. A.12.13

REFERENCES

AL-ANI, Quasi A.L.J. (1975). "Triaxial Stress and Strain in a sand encompassed by flexible boundaries Ph.D. Thesis. University of London.

ARNOLD, M. and MITCHELL, P.W. (1973). "Sand deformation in three dimensional state". Proc.8th ICSMFE Moscow. Vol.1.1.pp.11-18.

ARTHUR, J.R.F. (1973). Private Communication.

ARTHUR, J.R.F., JAMES, R.G. and ROSCOE, K.H. (1964). "The determination of stress fields during plain strain of a sand mass". Geotechnique Vol.14,pp.283-308.

ARTHUR, J.R.F., and MENZIES, B.K. (1968). Correspondence on "A new soil testing apparatus", by H.Y.Ko and R.F.Scott Geotechnique, Vol.18. No.2 pp. 271-272.

ARTHUR, J.R.F. and MENZIES, B.K. (1972). "Inherent anisotropy a sand". Geotechnique, Vol22, No.1 pp.115-128

BALLA, A. (1960) "Stress conditions in triaxial compression". J.S.M.F.D. ASCE. Vol.88, SM6 pp.57-84.

BARDEN, L. (1963) "Stresses and displacements in a cross-anistropic soil, Geotechnique, Vol.13, No.3, pp pp.198-210.

BARDEN, L. (1969) "A quantitative treatment of the deformation behaviour of granular material in terms of particulate mechanics". Int.Conf.on Structure, Solid Mech. and Eng.Des. Southampton University, paper 56.

BARDEN, L. and ISMAIL, H. and TONG, P. (1969) "Plane strain deformation of granular material at low and high pressures". Geotechnique, Vol.19, No.4, pp.441-452.

BARDEN, L. and KHAYATT, A.J. (1966) "Incremental strain rate ratios and strength of sand in the triaxial test". Geotechnique, Vol,16, No,4, pp.338-357.

BARDEN, L. and KHAYATT, A.J. and WIGHTMAN, A. (1969) "Elastic and slip components of the deformation of sand". Canadian Geol. Jour. Vol.6, No.3, pp.227-240.

BARDEN, L. and MCDERMOTT, R.J.W. (1965) "Use of free ends in triaxial testing of clays". J.S.M.F.D, ASCE, Vol.91, SM6 pp.2-23.

BARDEN, L. and Procter, D.C. (1971) "The drained strength of granular material". Can.Geot.Jn. Vol.8, No.3 pp. 272-383.

BELL, J.M. (1965). "Stress-strain characteristics of cohesionless granular materials subject to statically applied homogeneous loads in an open system". Ph.D.Thesis, California Ins. of Tech., Pasadena, Calif.,

BELL, J.M. (1968). Correspondence on "A new soil testing apparatus" by H.V.Ko and R.F.Scott. Geotechnique, Vol.18, No.2. pp.267-271.

BEMBEN, S.M. (1966). "The influence of controlled strain restraints on the strength and behaviour during shear of a sand tested with a constant volume". Ph.D Thesis, Cornell University.

BENNET, D.H. (1969) "Discussion, Three dimensional stress test", in 'New laboratory methods of investigating soil behaviour, Specialty session 16. Proc.7th.Int.Conf. SMFE, Vol.III, pp.519-520.

BENNET, D.H.(1971) Discussion to session 3, "The meaning measurement of basic soil parameters". Proc. Roscoe Memorial Symposium, Cambridge, pp.369-373.

BISHOP, A.W. (1954) Correspondence on "Shear characteristics of a saturated silt, measured in triaxial compression, by A.D.M. Penman" Geotechnique, Vol.4, No.1, pp.43-45.

BISHOP, A.W. (1966) "The strength of soils ans engineering materials". 6th.Rankine lecture, Geotechnique, Vol.16, No.2, pp.91-128.

BISHOP, A.W. (1967a) "Discussion to session 3, Shear strength of soil other than clay". Proc.Geot.Conf. Oslo. Vol.2, pp.201-204.

BISHOP, A.W. (1967b) "Progressive failure with special reference to the mechanism causing it". Proc.Geot. Conf., Oslo, Vol.2, pp.142-150.

BISHOP, A.W. (1971) Shear strength parameters for undisturbed and remoulded soil specimens. Proc.Roscoe Memorial Symp. Cambridge, pp.3-58.

BISHOP, A.W. (1950) Discussion, Proc.Conf.Measurement of shear strength of soils, Geotechnique, Vol.2, pp.113-116.

BISHOP, A.W. (1964) "Correspondence on energy components during the triaxial cell and direct shear tests". by Rowe, Barden and Lee, _Geotechnique_, Vol.14, No.4, pp.370-371.

BISHOP, A.W. and ELDIN, A.K.G. (1953). The effect of stress history on the relation between ϕ and porosity in sand. _Proc. 3rd Int.Conf. SMFE_, Vol.1, pp.100-105.

BISHOP, A.W. and GREEN, G.E. (1965). The influence of end restraint on the compression strength of a cohesionless soil. _Geotechnique_, Vol.15, No.3, pp.243-266.

BISHOP, A.W. and GREEN, G.E. and SKINNER, E.A. (1973). Recent developments at Imperial College in the measurement of strength and deformation characteristics. _Proc. 8th Int.Conf. SMFE_, Moscow.

BISHOP, A.W. and HENKEL, D.J. (1962). _The measurement of soil properties in the triaxial test_. 2nd.ed., Edward Arnold, Lon.Jou. 1962.

BISHOP, A.W. and GREEN, G.E. (1969). A note on the drained strength of sand under generalised strain conditions. _Geotechnique,_ Vol.19, No.1, pp.144-149.

BISHOP, A.W. and WEBB, D.L. and SKINNER, A.E. (1965). "Triaxial tests on soil at elevated cell pressures". _Proc. 6th. Int.Conf. Soil Mech. and Found Eng._, Vol. 1, pp.170-174.

BJERRUM, L. (1961). Discussion to session 5, Earth Pressures. Proc. 5th Int.Conf. SMFE. Vol.III, pp.317-318.

BJERRUM, L. and KUMMENEJE, O. (1961). "Shearing resistance of sand samples with circular and rectangular cross sections". N.G.I. Publication No.44.

BLIGTH, G.E. (1961). Strength and consolidation characteristics of compacted soils. Ph.D. Thesis, London University.

BLIGTH, G.E. (1965) "Shear stress and pore pressures in triaxial testing". J.S.M.F.D. ASCE. Vol.91, SM6, pp.25-39.

BRANSBY, P.L. (1971), Discussion to session 2, "The meaning and measurement of basic soil parameters", Proc. Roscoe Memorial Symp. Cambridge pp.255-258.

BROMS, B.B. (1963). Discussion on "Basic experiment into soil-structure interection", J.S.M.F.D. ASCE, Vol.89, SM4, pp223-227.

BROMS, B.B. and JAMAL, A.K. (1965). Analysis of the triaxial test cohesionless soils. Proc.6th.Int.Conf. SMFE Vol.1, pp.184-188.

CAQUOT, A. (1934). "Equilibre des massifs á frottement interne stabilité des terres pulverents et cohérentes". Paris. Gauthier Villars.

CASAGRANDE, A. (1940). "Characteristics of cohesionless soils affecting the stability of slopes and earth fills". Contributions to soil mechanics 1925-1940. Boston Society of Civil Engineers. Oct., CASBARIAN, A.O. and JAMAL, A.K. (1963). Discussion on 'study of failure envelope of soils, by T.H.Wu, A.K.Loh and L.E. Malvern, JSMFD ASCE. Vol.89, SM4, pp.243.

CASTRO, G. (1969). Liquefaction of sands. Ph.D. Thesis, Harvard University.

CHEN, L.S. (1948). "An investigation of stress-strain and strength characteristics of cohesionless soils". Proc.2nd Int.Conf.Soil Mechs. and Found.Eng. Vol.5, p.35.

COLEMAN, J.D. (1960). "Suction and the yield and failure surface for soil in the principal effective stress space". Geotechnique, Vol.10, No.4, pp.181-183.

COON, M.D. and EVANS, R.J. (1969). Discussion on, Elastic behaviour of cohesionless soil, by I. Holubec JSMFD, ASCE Vol. 95, SM5, pp.1281-1283.

COON, M.D. and EVANS, R.J. (1971). Recoverable deformation of cohesionless soils. JSMFD, ASLE. Vol.97, SM2, pp.375-391.

CORNFORTH, D.H. (1961). Plane strain failure characteristics of a saturated sand. Ph.D. Thesis, London University.

CORNFORTH, D.H. (1964). Some experiments on the influence of strain conditions on the strength of sand. Geotechnique, Vol.14, No.2, pp.143-167.

DANIEL, A.W.T. (1954). A study of the stress-strain characteristics of granular materials. Ph.D. Thesis, London University.

DANIEL, A.W.T. (1957). "Stress-strain characteristics of a granular material". Engineering, July 12, 1957.

DEJOSSELIN, DE JONG,G. (1959). "Statics and kinematics in the failable zone of a granular material". Uitgeverij Waltman, Delft.

DEJOSSELIN, DE JONG, G. (1971). "The double sliding, free rotating model for granular assemblies". Geotechnique, June pp. 155-163.

DEJOSSELIN, DE JONG, G. (1973). "A limit theorem for material with internal friction" in Symposium on Plasticity and Soil Mechanics, Cambridge, pp. 12-21.

DI MAGGIO, F.L. and SANDLER, I.S. (1971). "Material model for granular soils". J.S.M.F.D. ASCE, E.M.3, pp. 935-948.

DOMASCHUK, L. and WADE, N.H. (1969). "A study of bulk and shear moduli of a sand". J.S.M.F.D. ASCE, Vol. 95, SM.2, pp. 561-579.

DRUCKER, D.C. and PRAGER, W. (1952). Soil mechanics and plastic analysis or limit design. Quar. Applied Math., Vol. 10, No. 2, pp. 157-165.

DRUCKER, D.C., GIBSON, R.E. and HENKEL, D.J. (1957). "Soil mechanics and work-hardening theories of plasticity". Transactions ASCE, Vol. 122, pp. 338-346.

DUNCAN,J.M. and DUNLOP, P. (1968), "The significance of cap and base restraint". JSMFE, ASCE, Vol.94,SM1,pp271-290.

DYSON, S. (1970). The strength and deformation behaviour of a cohesionless soil under generalised stress conditions. Ph.D. Thesis, University of Aston in Birmingham.

ELDIN, A.K.G. (1951). Some fundamental properties controlling the shear strength of sands. Ph.D. Thesis (2 vols.) London University.

EL-SOHBY, M.A. (1969a). "Elastic behaviour of sand". J.S.M.F.D. ASCE, Vol. 95, SM6, pp. 1393-1409.

EL-SOHBY, M.A. (1969b). "Deformation of sands under constant stress ratios". Proc. 7th Int. Conf. Soil Mechanics 1, Mexico, pp. 111-120.

EL-SOHBY, M.A. and ANDRAWES, K.Z. (1973). "Experimental examination of sand anisotropy", 8th I.C.S.M.F.E., Moscow, Vol. 1.1, pp. 103-109.

EL-RUVAYEH, A.A. (1975). Stress-strain characteristics of rockfill, and of clays under high pore water tension. Ph.D. Thesis, London University.

ESRIG, M.I. and BEMBEN, S.M. (1965). "Discussion on soil properties shear strength and consolidation". Proc. 6th Int. Conf. SMFE, Vol. III, pp. 321-323.

FRYDMAN, S., ZEITLEN, J.G. and ALPAN, I. (1973), "The yielding behaviour of particulate media", Canadian Geot. Jr. Vol. 10, No. 3, pp. 341-362.

FRYDMAN, S. and ZEITLEN, J.G. (1969), "Some pseudo-elastic properties of granular media"., proc.7th Int.Cong. S.M.F.E., Vol.1, pp.135-141.

GIBSON, R.E. and MORGENSTERN, N.R. (1963). "Discussion on stress dilatancy earth pressures and slopes", by P.W. Rowe, J.S.M.F.D. ASCE, Vol. 89, SM6, pp. 127-129.

GIRIJAVALLABHAN, C.V. and REESE, L.C. (1968). "Finite element method for problems in soil mechanics". J.S.M.F.D. ASCE, Vol. 94, SM2, pp. 473-496.

GOLDSTEIN, S., BABITSKAYA, M., LOMIZE, G. and KRYZHANOVSKY, M. (1973). "Deformation and strength of soils". Proc. 8th I.C.S.M.F.E., Moscow, Vol. 1.1, pp. 151-158.

GOLDSCHEIDER, M. and GUDEHUS, G. (1973). "Rectilinear extension of dry sand, testing apparatus and experimental results". Proc. 8th I.C.S.M.F.E., Moscow, Vol. 1.1, pp. 143-149.

GREEN, G.E. (1969). Strength and compressibility of granular materials under generalised strain conditions. Ph.D. Thesis (2 vols.), London University.

GREEN, G.E. (1971a). "Strength and deformation of sand measured in an independent stress, control cell". Proc. Roscoe Memorial Symp., Cambridge, pp. 285-323.

GREEN, G.E. (1971b). Discussion leader's closing comments, Session 3, The meaning and measurement of basic soil parameters. Proc. Roscoe Memorial Symposium, Cambridge, pp. 401-409.

GREEN, G.E. (1967). Correspondence on "A new soil testing device by H.Y. Ko and R.F. Scott, Geotechnique, Vol. 17, No. 3, p. 295.

GREEN, G.E. and READES, D.W. (1974). Discussion on"Cubical triaxial tests on cohesionless soil", by Lade and Duncan. Journal of the Geotechnical Eng. Div. ASCE, Vol. 100, GT9, pp. 1065-1068.

GREEN, G.E. and READES, D.W. (1975). "Boundary conditions, anisotropy and sample shape effects on the stress-strain behaviour of sand in triaxial compression and plain strain". Geotechnique, Vol. 25, No. 2, pp. 333-356.

GUDEHUS, G. (1971). Discussion to session 3, The meaning measurement of basic soil parameters. Proc. Roscoe Memorial Symp., Cambridge, pp. 373-375.

HAAR, A. and VON KARMAN, T. (1909). Zur theorie der spannungszustände in plastichen und sandertigen medien. Nachr. Ges. Wiss. Göttingen, Math. Phys. Kl., pp. 204-218.

HAFIZ, M.A. (1950). Strength characteristics of sands and gravels in direct shear. Ph.D. Thesis, London University.

HAMBLY, E.C. (1969). A new true triaxial apparatus. Geotechnique, Vol. 19, No. 2, pp. 307-309.

HANSEN, C.E. Bent (1958). Line ruptures regarded as narrow rupture zones. Basic equations based on kinematic considerations. Proc. Conf. Earth Pressure Problems.

Brussels, Vol. 1, pp. 39-48.

HANSEN, B. (1973). "Contribution to the written discussion by Lade, P.V. in Symposium on Plasticity and Soil Mechanics, Cambridge, p. 136.

HARR, M.E. (1966). "Foundations of theoretical soil mechanics" Mc- Graw-Hill, New York.

HAYTHORNTWAITE, R.M. (1960a). Mechanics of the triaxial test for soils. J.S.M.F.D. ASCE, Vol. 86, SM5, pp. 35-62.

HAYTHORNTWAITE, R.M. (1960b). Discussion in session 1, Failure hypotheses for soils. ASCE Research Conf. Shear Strength Soils, Boulder, pp. 989-995.

HOLUBEC, I. (1966). The yielding of coheionless soils. Ph.D. Thesis, University of Waterloo.

HOLUBEC, I. (1968). Elastic behaviour of cohesionless soil. J.S.M.F.D. ASCE, Vol. 94, SM6, pp. 1215-1231.

HORN, H.M. and DEERE, D.U. (1962). Frictional characteristics of minerals. Geotechnique, Vol. 12, No. 4, pp. 319-335.

HORNE, M.R. (1965). The behaviour of an assembly of rotund rigid, cohesionless particles, Part I and II. Proc. Roy. Soc. London, Series A, Vol. 286, pp. 62-97.

HORNE, M.R. (1969). The behaviour of an assembly of rotund rigid, cohesionless particles, Part III. Proc. Roy. Soc. London, Series A, Vol. 310, pp. 21-34.

HVORSLEV, M.J. (1937). Uber die Festigkeitseigenschaften gestorter bindiger Boden ingeniorvidenskabelige, Skrifter A45, Copenhagen.

HVORSLEV, M.J. (1957). Discussion on irregularities in distribution of stress, strains and volume changes in triaxial test specimens. Proc. 4th I.C.S.M.F.E., Vol. 3, pp. 105-107.

HVORSLEV, M.J. (1960). Physical components of the shear strength of saturated clays. ASCE Research Conf. Shear Strength Soils, Boulder, pp. 169-273.

JAEGER, J.C. (1969). Elasticity, Fracture and Flow. Methuen (3rd edition).

JAKOBSON, B. (1957). Some fundamental properties of sand. Proc. 4th I.C.S.M.F.E., Vol. 1, pp. 167-171.

JAMAL, A.K. (1963). An investigation into the distribution of stresses in a cylindrical specimen loaded to failure in axial compression triaxial tests. M.S. thesis, Cornell University.

JAMES, R.G. and BRANSBY, P.L. (1970). Experimental and theoretical investigations of a passive earth pressure problem. Geotechnique, Vol. 20, No. 1, pp. 17-37.

JANBU, N. (1963). Soil compressibility as determined by oedometer and triaxial tests. Proc. European Conf. Soil Mech. Foundation Eng. I (Wiesbaden).

JANBU, N. (1965). Principal stress ratios and their influence on the compressibility of soils. Proc. 6th I.C.S.M.F.E., Vol. 1, pp. 249-253.

JANUSKEVICIUS, C.K. and VEY, E. (1965). Stresses and strains in triaxial specimens. ASTM STP 392 Instruments and Apparatus for soil and rock mechanics, pp. 37-53.

JENIKE, A.W. and SHIELD, R.T. (1959). "On the plastic flow of Coulomb solids beyond original failure." Journal of Applied Mechanics, Dec. pp. 599-602.

JOHANSEN, K.W. (1958). "Brudbetingelser for sten og. beton". (Failure conditions for rocks and concrete). Bygningsstatiske Meddelelser, Vol. 19, pp. 25-44. Teknisk. Forlag, Copenhagen.

KING, G.J.W. and DICKIN, E.A. (1970). Comparison of stress-dilatancy theories. J.S.M.F.D. ASCE, Vol. 96, SM5, pp. 1697-1714.

KING, G.J.W. and DICKIN, E.A. (1971). Closure to discussion on comparison of stress-dilatancy theories. J.S.M.F.D. ASCE, Vol. 97, SM11, pp. 1598-1601.

KIRKPATRICK, W.M. (1957). The condition of failure for sands. Proc. 4th I.C.S.M.F.E., Vol.1, pp. 172-178.

KIRKPATRICK, W.M. (1967). Influence of end restraint on strain distributions in the triaxial compression test. Discussion to session 3, Proc. Geot. Conf., Oslo, Vol.

KIRKPATRICK, W.M. and BELSHAW, D.J. (1968). On the interpretation of the triaxial test. Geotechnique, Vol. 18, No. 3, pp. 336-350.

KIRKPATRICK, W.M. and YOUNGER, J.S. (1970). Strain conditions in compression cylinder. J.S.M.F.D. ASCE, Vol. 96, SM 5, pp. 1683-1695.

KIRKPATRICK, W.M. and YOUNGER, J.S. (1971). Patterns of strain in strength test samples. Proc. 1st Aust-New Zealand Geomechanics Conf.

KIRKPATRICK, W.M. and SEALS, R.K. and NEWMAN, F.B. (1974). "Stress distributions in triaxial compression samples". Journal of the Geotechnical Eng. Div. ASCE, Vol. 100, No. GT 2, pp. 190-195.

KJELLMAN, W. (1936). Report on an apparatus for consummate investigation of the mechanical properties of soils. Proc. 1st I.C.S.M.F.E., Vol. II, paper A-4, pp. 16-20.

KO, H.Y. (1966). Static stress-deformation characteristics of sand. Ph.D. Thesis, California Inst. of Tech., Pasadena Calif.

KO, H.Y. and SCOTT, R.F. (1967a). A new soil testing apparatus. Geotechnique, Vol. 17, No. 1, pp. 40-57.

KO, H.Y. and SCOTT, R.F. (1967b). Deformation of sand in shear. J.S.M.F.D., ASCE, Vol. 93, SM5, pp. 283-310.

KO, H.Y. and SCOTT, R.F. (1968). "Deformation of sand at failure". J.S.M.F.D. ASCE, Vol. 94, SM4, pp. 883-898.

KOERNER, R.M. (1970). "Effect of particle characteristics on soil strength". J.S.M.F.D. Proc. of the ASCE, Vol. 96, SM4, pp. 1221-1234.

KOLBUSZEWSKI, J.J. (1948). "An experimental study of the maximum and minimum porosities of sands". Proc. 2nd I.C.S.M.F.E., Vol. I, pp. 158-156.

KONDNER, R.L. and ZELASKO, J.S. (1963). "A hyperbolic stress-strain formulation for sands". Proc. 2nd Pan Am. Conf. on Soil Mechanics, Vol. 1, pp. 289-334.

LADANYI, B. (1960). Etude des relations entre les contraintes et les deformations lors du cisaillement des sols pulverulents. Extrait des Annales des Travaux Publics de Belgique, No. 3.

LADE, P.V. (1972). "The stress-strain and strength characteristics of cohesionless soils". Ph.D. Thesis, University of California, Berkeley, Calif.

LADE, P.V. and DUNCAN, J.M. (1973). "Cubical triaxial tests on cohesionless soil". J.S.M.F.D. ASCE, Vol. 99, No. SM 10, Proc. Paper 10057, pp. 793-812.

LADE, P.V. and DUNCAN, J.M. (1975). "Elastoplastic stress-strain theory for cohesionless soil". Journal of the Geot. Eng. Div. ASCE, Vol. 101, No. GT 10, Proc. Paper 11670, pp. 1037-1053.

LEE, K.L. (1970). Comparison of plane strain and triaxial tests on sand. J.S.M.F.D. ASCE, Vol. 96, SM3, pp. 901-923.

LEE, K.L. and SEED, H.B. (1967a). Drained strength characteristics of sand. J.S.M.F.D. ASCE, Vol. 93, SM6, pp. 117-140.

LEE, K.L. and SEED, H.B. (1967b). "Undrained strength characteristics of cohesionless soils". J.S.M.F.D. ASCE, Vol. 93, SM6, pp. 333-360.

LEE, K.L. and SEE, H.B. (1970). Undrained strength of anistropically consolidated sand. J.S.M.F.D. ASCE Vol. 96, SM2, pp. 411-428.

LENOE, E.M. (1966). Deformation and failure of granular media under three-dimensional stress. Experimental Mechanics, Feb. 1966, pp. 99-104.

LEUSSINK, H. and WITTKE, W. (1963). Difference in triaxial and plane strain shear strength. ASTM STP 361, Laboratory Shear Testing of Soils, pp. 77-89.

LEWIN, P.I. and BURLAND, J.B. (1970). Stress-probe experiments on satured normally consolidated clay. Geotechnique, Vol. 20, No. 1, pp. 38-56.

LOMIZE, G.M. and KRYZHANOVSKII, A.L. (1967). On the strength of the sand. Proc. Geot. Conf., Oslo, Vol. I, pp. 215-219.

LOMIZE, G.M. and KRYZHANOVSKII, A.L. and VORONTSOV, E.I., and GOLDIN, A.L. (1969). Study on deformation and strength of soils under under three dimensional state of stress. Proc. 7th I.C.S.M.F.E., Vol. 1, pp. 257-265.

MALYSHEV, M.V. (1963). The effect of the average principal stresses on the strength of soil, and slip planes (in Russian). Osnovoniya fundamenty imekhanika gruntov, No. 1, Moscow, pp. 7-11.

MALYSHEV, M.V. (1970). "Application of the Hubert-Misses-Botkin strength criterion to unconsolidated sands". Soil Mech. and Found. Eng., (Translation from Os-Osnovaniya Fundamenty i Mekhanika Gruntov, No. 5, pp. 3-5, 1969), pp. 302-306.

MALYSHEV, M.V. and FRADIS, A.D. (1968). The strength of sandy soils (in Russian). Proc. 3rd Budapest Conf. SMEE. Section I, pp. 166-174.

MAKHLAUF, H.M. and STEWART, J.J. (1965). "Factors influencing the modulus of elasticity of dry sand". Proc. 6th I.C.S.M. Montreal, Vol. I, pp. 298-302.

MANDL, G. and FERNANDEZ, L.R. (1970). Fully developed plastic shear flow of granular materials. Geotechnique, Vol. 20, No. 3, pp. 227-307.

MARACHI, N., CHAN, C.K., SEED, H.B. and DUNCAN, J.M. (1969). Strength and deformation characteristics of rockfill materials. Report TE-69-5 to State of Calif. Dept. of Water Resources, University of Calif., Berkeley.

MATSUOKA, H. (1974). "Stress-strain relationships of sands based on the mobilised plane". Soils and Foundations Vol. 14, No. 2, pp. 47-61.

MATSUOKA, H. and HASHIMOTO, T. (1973). "A newly manufactured true triaxial apparatus and the analysis of its measured data". Proc. Annual Meeting of J.S.S.M.F.E. (8th), pp. 231-234, (in Japanese).

MENZIES, B.K. (1970). Stress strain anisotropy in sands. Ph.D. Thesis, London University.

MENZIES, B.K. (1972). Discussion to session 3, Roscoe Memorial Symposium, p. 368 in Stress strain behaviour of soils. Faulis and Co. Ltd.

MENZIES, B.K. and PHILLIPS, A.B. (1972). "On the making of rubber membranes" (Technical note). Geotechnique, Vol. 22, No. 1, pp. 153-155.

MERKLE, J.G. and MERKLE, D.H. (1969). Discussion on 'Elastic behaviour of cohesionless soil, by I. Holubec, J.S.M.F.D. ASCE, Vol. 95, SM5, pp. 1276-1280.

MESDARY, M.S. (1969). The shearing behaviour of granular material under a general stress system. Ph.D. Thesis, Glasgow University.

MITCHELL, R.J. (1973). "An apparatus for plane strain and true triaxial testing of undisturbed soil samples". Canadian Geot. Jour., Vol. 10, No. 3, pp. 520-528.

MITCHELL, R.J. and WONG, P.K.K. (1973). "The generalized failure of an Ottawa Valley Champlain Sea Clay". Canadian Geot. Jour. Vol. 10, No. 4, pp. 607-617.

NADAI, A. (1950). Theory of flow and fracture of solids. Vol. 1, Mc Graw-Hill, New York.

NAGARAJ, T.S. and SOMESHEKAR, B.V. (1974). "The failure criteria of soils". (Technical note), Canadian Geotechnical Jour. Vol. 11, No.4, p. 628.

NEWLAND, P.L. and ALLELY, B.H. (1959). Volume changes during undrained triaxial tests on saturated dilatant granular materials. Geotechnique, Vol. 9, pp. 174-182.

NEWLAND, P.L. and ALLELY, B.H. (1957). Volume changes in drained triaxial tests on granular materials. Geotechnique, Vol. 7, No. 1, pp. 17-34.

PARKIN, A.K. (1965). On the strength of packed spheres. Journal Aust. Math. Soc., Vol. 5, Part 4, pp. 443-452.

PARKIN, A.K. (1964). "The application of discrete unit models to studies of the shear strength of granular materials" Ph.D. Thesis, University of Melbourne.

PERLOFF, W.H. and POMBO, L.E. (1969), End restraint effect in the triaxial test. Proc. 7th I.C.S.M.F.E., Vol. 1 pp. 327-333.

PHILLIPS, A.B. (1972). Strength and deformation of layered sand. Ph.D. Thesis, University of London.

PICKERING, D.J. (1970). Anisotropic elastic parameters for soil. Geotechnique, Vol. 20, No. 3, pp. 271-276.

POOROOSHASB, H.B. (1971), "Deformation of sand in triaxial compression", Proc. 4th Asian Regional Conf. on SMFE, Bangkok, Vol. 1, pp. 63-66.

POOROOSHASB, H.B. and ROSCOE, K.H. (1961). The correlation of the results of shear tests with varying degrees of dilatation. Proc. 5th, I.C.S.M.F.E., Vol. I, pp. 297-304.

POOROOSHASB, H.B., HOLUBEC, I. and SHERBOURNE, A.N. (1967). Yielding and flow of sand in triaxial compression, Parts II and III. Canadian Geot. Jour. Vol. 4, No. 4, pp. 376-397.

POOROOSHASB, H.B., HOLUBEC, I. and SHERBOURNE, A.N. (1966). Yielding and flow of sand in triaxial compression, Parts I, Canadian Geot. Jour. Vol. 3, No.4, pp. 179-190.

PRAGER, W. and HODGE, P.G. (1951). Theory of perfectly plastic solids. Wiley, London.

PROCTER, D.C. (1967). Stress dilatancy behaviour of dense sand in the hollow cylinder test. M.Sc. Thesis, Manchester University.

PROCTER, D.C. and R.R. BARTON (1974). "Measurements of the angle of interparticle friction", Geotechnique, Vol. 24, No. 4, pp. 581-605.

PROCTER, D.C. and BARDEN, L. (1969). Correspondence on a note on the drained strength of sand under generalised strain conditions by G.E. Green and A.W. Bishop, *Geotechnique*, Vol. 19, No. 3, pp. 424-426.

PROCTER, D.C. (1974). "An upper bound value for ϕ_f in the stress-dilatancy equation". *Geotechnique*, Vol. 24, No. 3, pp. 269-288.

RAMAMURTY, T. (1970). "A universal triaxial apparatus", *J.S.M.F.E*., India, Vol. 9, No. 3, pp. 251-269.

RAMAMURTY, T. and RAWAT, P.C. (1973). "Shear strength of sand under general stress system". *Proc. 8th I.C.S.M.F.E*. Moscow, Vol. 1.2, pp. 339-342.

READES, D.W. (1972). "*Stress-strain characteristics of a sand under three-dimensional loading*" Ph.D. Thesis, University of London.

REYNOLDS, O. (1886). "Experiments showing dilatancy, a property of granular material, possibly connected with gravitation". *Proc. Roy. Inst*. Vol. 11, pp. 354-363.

ROSCOE, K.H. (1953). An apparatus for the application of simple shear to soil samples. *Proc. 3rd I.C.S.M.F.E.* Vol. I, pp. 186-191.

ROSCOE, K.H., SCHOFIELD, A.N. and WRITH, C.P. (1958). "On the yielding of soils". *Geotechnique*, Vol. 8, pp. 22-53.

ROSCOE, K.H. and SCHOFIELD, A.N. (1963). "A mechanical behaviour of an idealised wet-clay". *Proc. 2nd, European Conf. Soil Mech*. Weisbaden, Vol. 1, pp. 47-54.

ROSCOE, K.H., SCHOFIELD, A.N. and THURAIRAJAH, A. (1963). "An evaluation of test data for selecting a yield criterion for soils". Symp. Lab. Shear Testing of Soils, Ottowa A.S.T.M., STP 361, pp. 111-128.

ROSCOE, K.H. (1970). "The influence of strain in soil mechanics". (10th, Rankine Lecture) Geotechnique, Vol. 20, No. 2, pp. 129-170.

ROSCOE, K.H., BASSETT, R.H. and COLE, E.R.L. (1967). "Principal axes observed during simple shear of a sand". Proc. Geotechnical Conf. Oslo, Vol. 1, pp. 231-237.

ROWE, P.W. (1962). The stress-dilatancy relation for static equilibrium of an assembly of particles in contact. Proc. Roy. Soc. London Series A, Vol. 269, pp. 500-527.

ROWE, P.W. (1963). Stress-dilatancy earth pressures and slopes. J.S.M.F.D. ASCE, Vol. 89, SM 3, pp. 37-61.

ROWE, P.W. (1964). Closure to stress-dilatancy, earth pressures and slopes, J.S.M.F.D. ASCE, Vol. 90, SM4, pp. 145-180.

ROWE, P.W. (1971a). Theoretical meaning and observed values of deformation parameters for soil. Proc. Roscoe Memorial Symposium, Cambridge, pp. 143-194.

ROWE, P.W. (1971b). Discussion to session 1, The meaning and measurement of basic soil parameters. Proc. Roscoe Memorial Symposium, Cambridge, pp. 99-103.

ROWE, P.W. (1971c). Discussion on comparison of stress-dilatancy theories by B.I.W. King and E.A. Dickin, J.S.M.F.D. ASCE, Vol. 97, SM4, pp. 704-709.

ROWE, P.W. and BARDEN, L. (1964). Importance of free ends in triaxial testing. J.S.M.F.D. ASCE, Vol. 90, SM1 pp. 1-27.

ROWE, P.W. and BARDEN, L. (1969). Discussion on drained strength characteristics of sand. J.S.M.F.D. ASCE Vol. 95, SM1 pp. 353-359, Disc. to Lee and Seed 1967.

ROWE, P.W., VARDEN, L. and Lee, I.K. (1964). Energy components during the triaxial cell and direct shear tests. Geotechnique, Vol. 14, No. 3, pp. 247-261.

ROWE, P.W. and PEAKER, K. (1965). "Passive earth pressure movements". Geotechnique, Vol. 15, No. 1, pp. 57-78.

SANDEL, (1925). Über die Festigkeitsbedingungen, Leipzig.

SCHOFIELD, A.N. and WROTH, C.P. (1968). Critical state soil mechanics, p. 130. Mc Graw-Hill, New York.

SCHLEICHER, F. (1925). Zeits ang. Math. Mech. 5 (1925), p. 478.

SCHLEICHER, F. (1926). Der spannungszustand an der Fliessgrenze Z. angew. Math. Mech. Bd. 6, Heft 3, pp. 199.

SCOTT, R.F. (1963). "Principles of soil mechanics", Addison Wesley, London.

SHARMA, Y.K. (1976). Ph.D. Thesis, University of London, in preparation.

SHOCKLEY, W.G. and AHLVIN, R.G. (1960), "Nonuniform conditions in triaxial test specimens". Proc. Research Conf. on shear strength of cohesive soils, Boulder, ASCE, pp. 341-357.

SKETCHLEY, C.J. (1972). Written discussion to Session 3 in Stress-strain Behaviour of Soils, (Roscoe Mem. Symp.) pp. 391-396, G.T. Foulis and Co. Ltd.

SKINNER, A.E. (1969). A note on the influence of interparticle friction on the shearing strength of a random assembly of spherical particles. Geotechnique Vol. 19, pp. 285-290.

SKINNER, A.E. (1975). The effect of high pore water pressures on the mechanical behaviour of sediments, Ph.D. Thesis, University of London.

SULTAN, H.A. and SEED, H.B. (1967). "Stability of sloping earth dams". J.S.M.F.D. ASCE, Vol. 93, SM4, pp. 45-67.

SUTHERLAND, H.B. (1971). Discussion to session 3, "The meaning and measurement of basic soil parameters". Proc. Roscoe Memorial Symposium, Cambridge, pp. 376-377.

SUTHERLAND, H.B. and MESDARY, M.S. (1969). The influence of the intermediate principal stress on the strength of sand. Proc. 7th Int. Conf. SMFE, Vol. 1, pp. 391-399.

SUTHERLAND, H.B. and MESDARY, M.S. (1970). Correspondence on: A note on the drained strength of sand under generalized strain conditions. Geotechnique, Vol. 20, No. 2, pp. 210-212.

TANG, W.H. and HOEG, K. (1968). "Plain strain loading of a strain-hardening soil". Contract Report No. 3-129, Report 5, Contract DA-22-079-eng-471, Dept. of Civ. Eng. Mass., Inst. of Technology.

TATSUOKA, F. and ISHIHARA, K. (1974). "Yielding of sand in triaxial compression". Soils and Foundations. J.S.S.M.F.E., Vol. 14, No. 2, pp. 63-76.

TAYLOR, D.W. (1941). Cylindrical compression research program on stress-deformation and strength characteristics of soils. M.I.T. Soil Mech. Lab. Report, Shear Research, 7th Progress Report to U.S. Eng. Dept.

TAYLOR, D.W. (1948). Fundamentals of soil mechanics. John Wiley and Sons, Book Publishing Co., New York.

TONG, P.Y.L. (1970). Plane strain deformation of sands. Ph.D. Thesis, University of Manchester.

TROLLOPE, D.H. (1971). "The collapse of elastic aggregates". Proc. of 1st Australia-N. Zealand Geomechanics Conf., Vol. 1, pp. 204-210.

TROLLOPE, D.H. and PARKIN, A.K. (1963). Discussion on 'Stress-dilatancy, earth pressures and slopes, by P.W. Rowe, J.S.M.F.D., ASCE, Vol. 89, SM6, pp. 129-133.

WADE, N.H. (1963). Plane strain failure characteristics of a saturated clay. Ph.D. Thesis, London University.

WALBANCKE, J.H. (1974). unpublished test results

WATERWAYS EXPERIMENT STATION (1947). Review of the cooperative triaxial shear research program of the Corps of Engineers, W.E.S. Vicksburg, Miss.

WHITMAN, R.V. and LUSCHER, U. (1962). Basic experiment into soil-structure interaction. J.S.M.F.D. ASCE, Vol. 88, SM6, pp. 135-167.

WOOD, C.C. (1958). Shear strength volume change characteristics of compacted soil under conditions of plain strain. Ph.D. Thesis, University of London.

WOOD, D.M. (1973). "Truly triaxial stress-strain behaviour of kaolin", in Symposium on Plasticity and Soil Mechanics, Cambridge, pp. 67-94.

WROTH, C.P. and BASSET, R.E. (1965). "A stress-strain relationship for the shearing behaviour of a sand". Geotechnique, Vol. XV, No. 1.

WU, T.H., LOH, A.K. and MALVERN, L.E. (1963). Study of failure envelope of soils. J.S.M.F.D. ASCE, Vol. 89, SM1, pp. 145-181.