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SOME NOVEL ASPECTS OF FLOW-DRIVEN AERODYNAMIC NOISE

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ABSTRACT

Kinetic energy levels associated with a mean fluid flow are many orders of magnitude larger than those in a sound field. Mechanisms that are capable of converting kinetic energy to sound energy, however inefficient they appear to be, cannot be ignored. That a mean flow driven jet instability leads to an enhanced sound field is already well recognised. The radiation field generated by the interaction of compliant surfaces and a mean flow (with the possibility of a mean flow driven surface instability) is investigated here. We also outline a simple scheme which enables us to estimate the noise field radiated by systems in which changes take place rapidly, such as impulsive motions. These noise fields are quite substantial, a fact that is highlighted by the approximation scheme.

CONTENTS

INTRODUCTION

Our main interest is the investigation of interactions between aerodynamic noise generating mechanisms and subsonic mean fluid flows. Some of the mean flow effects, such as the Doppler effect, are simply small perturbations to the zero flow situations and are well understood. Great care must be taken in these situations, however, since kinetic energy levels in the fluid are many orders of magnitude higher than those in sound fields. Consequently we cannot afford to ignore any process, however inefficient it may appear to be, that is converting mean flow kinetic energy into sound.

A configuration which is of practical concern is that of a fluid flowing over a compliant boundary in the pressence of acoustic sources. In many practical situations, such as in ventilation ducts and aero-engines, surfaces which are effectively rigid are lined with a compliant material with the aim of attenuating the transmitted noise. There is always the possibility of the mean flow inducing an instability wave in a compliant wall, a phenomenon well known as panel flutter.

If this unstable wave is well coupled to a sound wave, as it will be if scatterers are present, then a disastrously large sound field will be generated!

There is always a difficulty in finding a model problem, which, whilst rendering itself susceptible to straightforward analysis, retains the characteristic physical properties we wish to describe. The work of the first three chapters investigates problems of this type. In the first a compliant surface is modelled by an array of vibrating pistons. This idea was first suggested by Lord Raleigh and has been recently extended by Ffowcs-Williams, The changes brought about by mean flow in the characteristic properties of a single piston are easy to catalogue, and use of the work of Ffowcs-Williams clarifies the role of a mean flow in a stable interaction situation. Little insight of the qualitative or quantitative effects of the instability can be gained from this model but the fact that instability must take place at a high enough flow speed emerges. In order to discover further properties of the instability we must consider a continuous compliant wall. In chapter II we discuss the scattering at the interface of a semi-infinite wave bearing wall and a semi-infinite rigid wall in the pressence of stagnant fluid.

Using the Wiener-Hopf technique an exact solution (at all levels of fluid loading) is obtained, relating the scattered intensity to the incident surface wave energy due to an acoustic line source situated far from the interface. In particular, in the heavy fluid loading limit the energy scattered at the interface exceeds the 'direct' energy radiated by the source, even in the case when the driving source is a line source in the wall. This is not surprising since the wave bearing wall transfers near field energy from the source to the interface without attenuation. In practice, with panels which are necessarily finite and scattering must occur, we can already see that surface compliancy may greatly increase the radiation due to sources close to the wall.

It would obviously be convenient if we were able to apply these results to the corresponding problem with flow. However we find that the growth and decay rates of the unstable modes are not equal, and therefore we cannot model a finite panel, even one which is many plate wavelengths long by considering two independant semi-infinite problems. In chapter III there is a broad discussion of the difficulties in selecting an appropriate model for the problem of a finite panel with flow.

We choose a method involving conjugate modes on an infinite wall, whose amplitudes are arranged so that only a finite section of the wall is vibrating.

This highly idealised problem does reveal the important process of the instability removing kinetic energy from the fluid and then being scattered with a resulting substantial sound field.

The final chapter is a self-contained one describing a method of approximating the sound field radiated in situations where rapid accelerations of a compact body take place. Ray theory, a high frequency approximation, is used to estimate the pressure at the surface in terms of the normal velocity there. Curle's theorem is then used to determine the sound field. The accuracy of the scheme is checked in the case of the impulsive motion of a single sphere, a problem which can be solved exactly, and the agreement is found to be good. The application to the impulsive motion of a cylinder is also given and compared to results obtained by crude experiments using model piledrivers. The agreement again appears to be good, but the experimental results indicate that 'ringing', due to internal vibration of the piston, dominates the noise field in this case. Whilst recognising that this impulse does not form the dominant term in the noise field radiated from an impact, we must recognise that it cannot be removed. We expect this type of scheme to give a useful estimate of the scattered potential of many diffraction problems provided events take place on a time scale which is small compared to the time taken for sound to travel a typical length scale of the body.

CHAPTER 1

INTRODUCTION

Compliant surfaces are widely employed as noise attenuators in situations where a mean flow interacts with an acoustic field. Thus whenever there is a flow in a duct, the duct may be lined in an attempt to reduce the noise output as in the case of an aircraft engine or a ventilation duct. A conformal sonar array which consists of a series of vibrating panels in a nominally rigid baffle is another application of a compliant surface that may give rise to an acoustic flow interaction.

At first sight the effect of low Mach number flow is likely to produce only small pertubations to static fluid results. In practice one can find many problems where flow effects are small and are simply accounted for by phenomena such as the Doppler effect.

However, it is becoming increasingly widely recognised that mean flow instability effects can be large in many cases, and in this context we mention the work done on orderly jet structures by Crow and Champagne (1971), Ronneberger (1967) and Dean (1972).With incompressible fluid we find that flow can induce flutter in compliant surfaces - Rayleigh waves and fluttering flags (see Lamb (1932) for example). We consider that it is still an open question whether or not there is an important coupling between sound and flow instabilities. Practical experience seems to suggest that it is safe to ignore instabilities. On the other hand some modes are known to grow in the presence of a flowing fluid (Dean (1972) and Tester (1973) so it is not an entirely academic point.

The instability problems such as panel flutter are straightforward enough when the boundary conditions are uniform, and one usually finds that instability speeds are too slow to be well coupled to sound waves.

Scattering is a feature that is capable of coupling the waves so, if linear theory is relevant, instabilities and sound are always coupled whenever scatterers are present. The compact surface is a most efficient scatterer and this is the problem we treat below.

The flow velocity is considered constant and we assume that the effects of the vibration have no bearing on the overall mean flow. The fluid is inviscid. Boundary layer effects are obviously important in practice but they are too difficult to treat analytically. Our model will consequently have limited application but should be valid whenever the boundary layer thickness is much smaller than the characteristic scale of surface vibration. Since this problem can be solved exactly the model forms a useful base from which we can gain an understanding of the mechanics underlying the more complex situation.

Our problem can be sensibly divided into two sections. There is the purely acoustic effect of a compact source (of constant strength) moving uniformly at low Mach number relative to a fluid. We treat this problem without specifying the details of the source but assuming that the monopole strength is independent of motion. Motion is known to increase the radiated energy by an amount which is proportional to M^2 (Lighthill (1952), Morse and Ingard (1968)). We show in our problem that the increase is $5M^2$ times the energy radiated in static fluid. This energy is provided by two distinct effects. Firstly, there is an increased mechanical damping, and secondly, energy is extracted from the mean flow in overcoming the mean (compressible) drag on the vibrating surface. These increases contribute energy in the ratio 2:1. The flow will also have a non-acoustic effect on the surface vibration (the response amplitude, the resonance frequency and the effective mechanical damping will all be affected, and this in turn alters the acoustic source strength. The effect is usually bigger than the purely acoustic effect described above.

The problem we consider is that of a circular piston which is free to vibrate normally in an otherwise rigid plane wall. This is probably a fairly good but obviously simplified model of one element of a sonar array, but we do not expect even this local problem to be simple. There are easily foreseen difficulties associated with the singularities of two dimensional potential flow at the sharp edges of the piston and baffle. Batchelor (1967) shows that the included suction force is logarithmically singular. He goes on to say "what we learn ... is that the total force depends on the precise shape of the two boundaries close to their intersection". We emphasise this point by considering "pistons" which do not have discontinuities at their edges, and draw conclusions about the effect of the degree of edge curvature on the magnitude of this force. The actual magnitude of the force will be difficult to find, but we expect it to be governed by a length scale on which our potential theory model has failed, possibly a boundary layer scale or the finite curvature of practical 'sharp' edges.

The local effects of motion are obvious and can be classified as:-

1. Increased mass and damping due to the fluid loading.
2. Decreased stiffness due to the flow.
3. Increased radiation damping due to Mach number effects.

These effects are quantified below, and are shown to be quite substantial. The greatest effect occurs when the flow brings the piston close to resonance (or detunes a resonating piston), and there is indeed the possibility that the flow can lead to an instability. We deal only with the stable case although the possibility of the instability leads to a limiting value of the flow velocity for which the work is valid. That limit is reached when the flow induced suction force on the piston exactly balances the stiffness of the restoring spring.

SOUND RADIATED INTO MOVING FLOW BY COMPACT SURFACE VIBRATION

We consider homogeneous inviscid fluid in uniform motion parallel to a plane boundary surface, S. The fluid occupies the upper half-space and moves with velocity cM,c being the speed of sound, A ~~company~~ compact section of the boundary vibrates, radiating sound to infinity. We choose a co-ordinate system $\underline{X}$, fixed in the fluid, and write p($\underline{X}$, t) for the sound pressure far away from the vibrating part of the surface

$$p(\underline{X},t) = \frac{\rho_0}{2\pi|\underline{x}|}\int_S \frac{\partial^2 \xi}{\partial t^2}\left(\underline{Y}, t - \frac{|\underline{X}-\underline{Y}|}{c}\right) d^2\underline{Y} \tag{1.1}$$

$$= \frac{\rho_0}{2\pi|\underline{x}|}\int_S \int_\tau \xi(\underline{Y},\tau)\,\delta''\left(\tau - t + \frac{|\underline{X}-\underline{Y}|}{c}\right) d\tau\, d^2\underline{Y} \tag{1.2}$$

δ is Dirac's delta fundtion, primes denote differentiation with respect to the argument, ρ_0 is the density of the undisturbed fluid and ξ is the small displacement of the surface from its mean position. ξ is finite only within a compact region moving relative to this co-ordinate system with velocity -c$\underline{M}$. We therefore choose a co-ordinate system $\underline{x}$, whose origin moves with the centre of the vibrating region, and write $\eta(\underline{x},\tau)$ for the surface elevation at time τ.

$$\xi(\underline{X},\tau) = \eta(\underline{x},\tau) \tag{1.3}$$

$$\underline{x} = \underline{X} + c\underline{M}\tau \tag{1.4}$$

The Jacobian of this transformation is unity, so that,

$$d^2\underline{Y} = d^2\underline{y} \tag{1.5}$$

and

$$p(\underline{X},t) = \frac{\rho_0}{2\pi|\underline{X}|}\int_S \int_\tau \eta(\underline{y},\tau)\,\delta''\left(\tau - t + \frac{|\underline{X}-\underline{Y}|}{c}\right) d\tau\, d^2\underline{y} \tag{1.6}$$

This equation can now be integrated by parts with respect to τ, and on making use of the fact that,

$$\left.\frac{\partial}{\partial \tau}\right|_{y\,const} \left(\tau - t + \frac{|\underline{X} - \underline{y}|}{c} \right) = 1 + \underline{M} . \frac{(\underline{X} - \underline{y})}{|\underline{X} - \underline{y}|}$$

$$= 1 + \frac{\underline{M} . \underline{X}}{|\underline{X}|}$$

$$= (1 + M\cos\theta) \qquad (1.7)$$

$$p(\underline{X}, t) = \frac{\rho_o}{2\pi |\underline{X}| |1 + M\cos\theta|^3} \int_S \frac{\partial^2 \eta}{\partial \tau^2} \left(\underline{y}, t - \frac{|\underline{X} - \underline{y}|}{c} \right) d^2\underline{y} \qquad (1.8)$$

We now restrict our attention to very compact surface sources, so that the small changes in retarded time are negligible. The wave field is then determined by the instantaneous source strength Q,

$$Q\left(t - \frac{|X|}{c} \right) = \int_S \frac{\partial^2 \eta}{\partial \tau^2} \left(\underline{y}, t - \frac{|\underline{X} - \underline{y}|}{c} \right) d^2\underline{y} \qquad (1.9)$$

The sound radiated to large distances in the moving flow is therefore given by:

$$p(\underline{X}, t) = \frac{\rho_o \, Q\left(t - \frac{|\underline{X}|}{c} \right)}{2\pi |\underline{X}| \, |1 + M\cos\theta|^3} \qquad (1.10)$$

$$\overline{p^2}(\underline{X}, t) = \frac{\rho_o^2 \, \overline{Q^2}\left(t - |\underline{X}|/c \right)}{4\pi^2 |\underline{X}|^2 \, |1 + M\cos\theta|^3} \qquad (1.11)$$

The overbar denotes an average taken over many 'cycles'.

We now examine conditions on the distant hemisphere S_∞, that lies parallel to the phase fronts radiated by the vibrating surface (see figure). The centre of the hemisphere, of radius $|\underline{X}|$, has drifted with the fluid a distance $M|\underline{X}|$ downstream of the vibrating section. On this surface the radiation is statistically stationary in time, because the surface vibration $(\underline{x}, t)$ is supposed stationary.

Sound energy crosses S_∞ at a mean rate,

$$\left\{ \frac{\overline{p^2}}{\rho_o c} + \frac{\underline{M} . \underline{X}}{|\underline{X}|} \frac{\overline{p^2}}{\rho_o c} \right\}$$ per unit area.

The first term represents the rate of working of the sound pressure and the second the rate at which the mean flow convects the energised fluid across S_∞. The mean radiation energy density is $\frac{\overline{p_0^2}}{\rho_0 c^2}$ (the sum of equal kinetic and potential parts) and the volume flux out of S_∞ is $\frac{c\underline{M}\cdot\underline{X}}{|\underline{X}|}$, or $cM\cos\theta$, per unit area.

The total acoustic power, P, radiated by the surface vibration into the moving stream is therefore:

$$P = \int_{S_\infty} \frac{\overline{p^2}}{\rho_0 c}\,(1 + M\cos\theta)\,dS_\infty \qquad (1.12)$$

and because

$$\overline{p^2} = \overline{p^2}(\underline{x} - c\underline{M}t, t) = \overline{p^2}(\underline{x}, 0) = \overline{p^2}(\underline{X}, 0). \qquad (1.13)$$

$$P = \int_0^\pi \frac{\rho_0^2\,\overline{Q^2}}{4\pi^2|\underline{X}|^2}\,\frac{(1 + M\cos\theta)^{-5}}{\rho_0 c}\,\pi|\underline{X}|^2\sin\theta\,d\theta \qquad (1.14)$$

$$= \frac{\rho_0\,\overline{Q^2}}{2\pi c}\left\{1 + 5M^2 + O(M^4)\right\} \qquad (1.15)$$

Low Mach number flow evidently increases the sound energy radiated from a vibrating surface, the increase being $5M^2$ times the power radiated into fluid at rest. Where does this extra energy come from? There are two potential sources as can be seen by considering the rate at which energy crosses the boundary surface S.

$$P = \int_S p(\underline{X}, t)\,\frac{\partial\xi}{\partial t}(\underline{X}, t)\,d^2\underline{X} \qquad (1.16)$$

But,

$$\frac{\partial\xi}{\partial t} = \frac{\partial\eta}{\partial t} + \frac{\partial\eta}{\partial x_i}\left.\frac{\partial x_i}{\partial t}\right|_{\underline{X}} \qquad (1.17)$$

$$\frac{\partial\eta}{\partial t} + cM_i\frac{\partial\eta}{\partial x_i}$$

so that,

$$P = \int_S \overline{p\frac{\partial\eta}{\partial t}}\,d^2\underline{x} + cM_i\int_S \overline{p\frac{\partial\eta}{\partial x_i}}\,d^2\underline{x} \qquad (1.18)$$

The first of these is the definition of the power extracted from the surface vibration, P_v say, and the second is the power extracted from the mean flow to overcome the steady drag force on the boundary surface, for $p \frac{\partial \eta}{\partial x_i}$ is the resolved component of the surface stress in the i direction. We will denote this drag induced power by P_d, and evaluate it by determining the steady drag on the surface S_∞, which must equal the boundary drag on S. The distant pressure, with its zero mean value, cannot contribute to the drag on S_∞, which is therefore comprised entirely of terms arising from the mass flux times the unsteady part of the momentum per unit mass, (the steady part amounting to zero by mass conservation). The drag on S_∞ is therefore,

$$D = -\int_{S_\infty} \ell_j \rho u_j \frac{p}{\rho_o c} \cos\theta \, d^2\underline{x} \tag{1.19}$$

$$= -\int_{S_\infty} \overline{\left\{\rho_o \frac{p}{\rho_o c} + \frac{p}{c^2} \cdot cM\cos\theta\right\} \frac{p}{\rho_o c}} \cos\theta \, d^2\underline{x}$$

$$= -\int_{S_\infty} \frac{\overline{p^2}}{\rho_o c^2} (1 + M\cos\theta) \cos\theta \, d^2\underline{x}$$

$$= -\int_0^\pi \frac{\rho_o \overline{Q^2}}{4\pi^2 c^2} (1 + M\cos\theta)^{-5} \cos\theta \, \pi \sin\theta \, d\theta \tag{1.20}$$

$$D = \frac{5}{6} M \frac{\rho_o \overline{Q^2}}{\pi c^2} \left(1 + O(M^2)\right) \tag{1.21}$$

The power absorbed from the mean flow to overcome this drag is therefore,

$$P_d = cMD = \frac{5}{6} M^2 \frac{\rho_o \overline{Q^2}}{\pi c} \tag{1.22}$$

By difference, since $P = P_v + P_d = (1+5M^2) \frac{\rho_o \overline{Q^2}}{2\pi c}$ (1.23)

$$P_v = \left(1 + \frac{10M^2}{3}\right) \frac{\rho_o \overline{Q^2}}{2\pi c} \tag{1.24}$$

Evidently then, the increased radiation induced by the mean flow draws its energy from both the surface and fromthe mean flow, the two parts being in the ratio 2:1. The radiation damping of the surface motion will thus be increased by the flow to a value $(1 + \frac{10}{3} M^2)$ times its value in stagnant fluid.

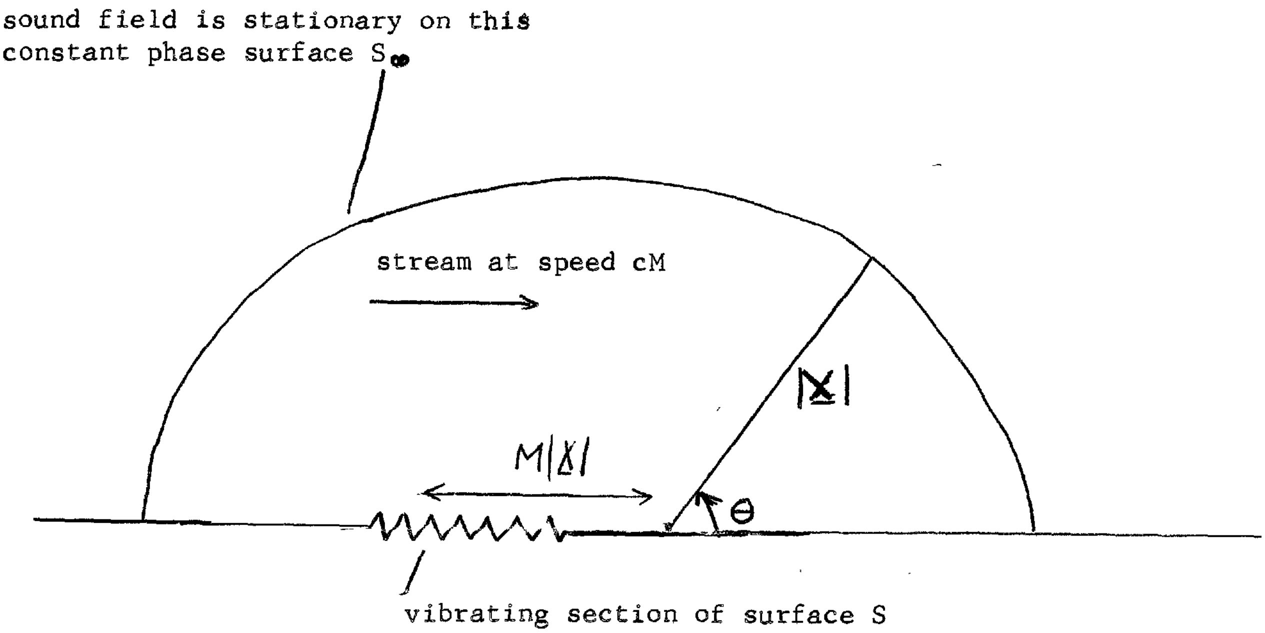

Figure 1 Diagram of the co-ordinate system.

THE LOCAL INTERACTION BETWEEN A FLOW AND A COMPACT VIBRATING SURFACE

We continue to consider homogeneous inviscid fluid in uniform motion parallel to a plane boundary surface. The fluid occupies the upper half-space, and moves with velocity $c\underline{M}$, c being the speed of sound. A compact circular section of the boundary vibrates, and we refer to this section as the piston. The remainder of the boundary, which is rigid, is described as the baffle. The piston is held in place by a spring, whose undisturbed length keeps the piston face flush with the baffle.

To investigate the local effect of the flow on the piston, we first calculate the force exerted on the piston, due to its own small oscillations. We choose a co-ordinate system $\underline{X}$, fixed in the fluid, and write the pressure at $\underline{X}$ as:

$$p(\underline{X},t) = \frac{\rho_0}{2\pi}\int_{S(\underline{Y},t)} \frac{\partial^2 \xi}{\partial t^2}\left(\underline{Y}\ ,\ t - \frac{|\underline{X}-\underline{Y}|}{c}\right)\frac{d^2\underline{Y}}{|\underline{X}-\underline{Y}|} \tag{2.1}$$

Here ρ_0 is the density of the undisturbed fluid and ξ is the (small) displacement of the surface from its mean position ξ is non-zero only on the piston face, which is moving relative to this co-ordinate system with velocity $-c\underline{M}$. The force on the piston, due to its motion is thus given by

$$F(t) = \int_{S(\underline{X},t)} p(\underline{X},t)\, d^2\underline{X} \tag{2.2}$$

We evaluate the integrals required by expanding the retarded time in a Taylor series about the point $\underline{X} = \underline{Y}$, and this procedure is valid provided

$$\left|\frac{|\underline{X}-\underline{Y}|}{c}\frac{\partial}{\partial t}(\ln \xi)\right| << 1 \tag{2.3}$$

We now transfer the integrals to the piston-fixed co-ordinate system $\underline{x}$, where

$$x_i = X_i + cMt\delta_{1i} \tag{2.4}$$

and we write the surface displacement

$$\xi(\underline{Y},t) = \eta(\underline{y},t) \tag{2.5}$$

then we formally write down the result as

$$F(t) = \frac{\rho_0}{2\pi}\int_{S(\underline{x})}\int_{S(\underline{y})}\frac{1}{|\underline{x}-\underline{y}|}\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial y_1}\right)^2 \eta(\underline{y}\ t)\, d^2\underline{y}\, d^2\underline{x} \tag{2.6}$$

$$+ \frac{\rho_0}{2\pi}\int_{S(\underline{x})}\int_{S(\underline{y})}\sum_{j=1}^{\infty}\frac{(-1)^j|\underline{x}-\underline{y}|^{j-1}}{c^j\, j!}\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial y_1}\right)^{j+2}\eta(\underline{y},t)\, d^2\underline{y}\, d^2\underline{x}$$

where the conditions for the validity of this expression are now

$$\left|\frac{|\underline{x}-\underline{y}|}{c}\frac{\partial}{\partial t}\left(\ln \eta\right)\right| << 1 \tag{2.7}$$

and

$$\left||\underline{x}-\underline{y}|M\frac{\partial}{\partial t}\left(\ln \eta\right)\right| << 1 \tag{2.8}$$

The first of these tells us that the piston must be compact with respect to a typical sound wavelength, the second that the flow Mach number be small or that the surface remain nearly plane. Both of these limits are satisfied for sufficiently large c, so it is sensible to consider F(t) as an expression in this form.

The terms which are independent of c (ie the terms remaining non-zero on letting $c\to\infty$) are the "incompressible" flow terms, and they are

$$\frac{\rho_0}{2\pi}\int_{S(\underline{x})}\int_{S(\underline{y})}\frac{1}{|\underline{x}-\underline{y}|}\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial y_1}\right)^2\eta\left(\underline{y},t\right)d^2\underline{y}\,d^2\underline{x} \tag{2.9}$$

The $\underline{x}$ integration can be performed immediately, and the result is

$$\int_{S(\underline{x})}\frac{d^2\underline{x}}{|\underline{x}-\underline{y}|} = 4a\,E\left(\frac{|\underline{y}|}{a}\right)$$

where a is the radius of the piston, E(k) is the elliptic integral of the second kind. [This result is obtained by the use of Copson's formula, (1947) which states that

$$\int_0^{2\pi}\frac{d\theta}{(\rho^2+r^2-2\rho r\cos\theta)^{1/2}} = 4\int_0^{\min(\rho,r)}\frac{dt}{(\rho^2-t^2)^{1/2}(r^2-t^2)^{1/2}}\,.\;]$$

As we have already indicated, we do not expect to be able to evaluate the exact value of the force in the case of a piston whose angle of contact is $\frac{\pi}{2}$. In this case we can write

$$\eta(\underline{y},t) = H(a-|\underline{y}|)\,\eta(t) \tag{2.10}$$

and we indeed find that the expression for the incompressible force has a singularity arising from the integral

$$\frac{2\rho_0 a U^2\eta(t)}{\pi}\int_{S(\underline{y})}\left\{\frac{\partial^2}{\partial y_1^2}H(a-|\underline{y}|)\right\}E\left(\frac{|\underline{y}|}{a}\right)d^2\underline{y}\,. \tag{2.11}$$

We can see that if we integrate by parts once we obtain

$$\int\frac{y_1^2}{|\underline{y}|^2}\,\delta(a-|\underline{y}|)\,\frac{\partial}{\partial|\underline{y}|}E\left(\frac{\underline{y}}{a}\right)dy_1\,dy_2$$

leading to the term E'(k) as $k\to1^-$. The singularity is negative and logarithmic.

As we indicated in the introduction, the singularity has arisen through the inability of our potential theory model to deal with the discontinuity at the piston edge. This point can be demonstrated by the following examples. Suppose there is a flexible piston which is flush with the baffle at its edge, rising to a peak at its centre. During an oscillation the peak varies from $+|\eta|$ to $-|\eta|$, but the edge remains flush. The elevation of the 'piston' can be represented, for example, by the functions

$$\left(1-\frac{|y|^2}{a^2}\right)H(a-|y|)\eta(t) \; ; \; \left(1-\frac{|y|^2}{a^2}\right)^2 H(a-|y|)\eta(t)$$

and these 'pistons' lead to values

$$-\frac{4}{3}\rho a U^2\eta(t) \; ; \; -\frac{32}{45}\rho a U^2\eta(t)$$

respectively, for the mean flow force. It is evident that this term is a mean flow effect. It represents a suction (when the piston is proud of the boundary). The magnitude of the force is highly sensitive to any variation in the piston geometry.

One way to allow for the non-linearity of the problem would be to perform one of the integrations required along the actual surface of the piston, i.e. at a height η above the undisturbed height. The integral required is then

$$\frac{\rho U^2\eta}{2\pi}\int_{S(x)}\int_{S(y)}\left\{\frac{\partial^2}{\partial y_1^2}H(a-|y|)\right\}\frac{d^2y\; d^2x}{\left\{|x-y|^2+\eta^2\right\}^{3/2}} \tag{2.12}$$

and this limits the previous singularity to a value

$$\rho a U^2\eta \ln\left\{\frac{|\eta|}{a}\right\} + O(\eta), \tag{2.13}$$

provided

$$|\eta| << a.$$

In truth we cannot hope to get a true value for this mean flow suction, as we would have to attempt to solve the full viscous problem. The actual magnitude of this force is of little concern, especially as we can write it in the form

$$\rho a U^2\eta \ln\left(\frac{\varepsilon}{a}\right) \tag{2.14}$$

Where ε is the length scale on which our potential modelling has broken down. The presence of the logarithmic function ensures that although our estimate of $\frac{\varepsilon}{a}$ may not be very accurate, the error will have little effect on the value of the force. We write the suction force as

$$-\rho a U^2\eta(t) A_0 \tag{2.15}$$

where A is a constant.

The remainder of the terms required in (2.6) can be evaluated, to give us the expression as accurately as we require. The majority of these terms do not pose any difficulty of the type discussed, indeed it is only terms of the form

$$M^{2n}\,|\underline{x}-\underline{y}|^{2n-1}\,\frac{\partial^{2n}}{\partial y_1^{2n}}H(a-|\underline{y}|)$$

which are singular. These terms all have singularities of the same type as that discussed, and must be treated in the same way. Of course each such term is at least of order M^2 smaller than the first.

We finally obtain

$$F(t)=\frac{8}{3}\rho a^3\eta''(t)-\rho_0 a U^2\eta(t)A_0-\frac{\rho_0 a^4\pi}{2c}\eta'''(t)$$
$$+\frac{32}{45}\frac{\rho_0 a^5}{c^2}\eta^{IV}(t)+4\rho_0 a^3\eta''(t)M^2+\rho_0 a U^2\eta A_1$$
$$-\frac{\rho_0 a^6\pi\eta^{V}(t)}{12c^3}-\frac{5\rho_0 a^4\pi\eta'''(t)M^2}{3c}+O(c^{-4}) \qquad (2.16)$$

where

$$A_1\sim\frac{4\pi}{a}\int_{S(\underline{x})}\int_{S(\underline{y})}|\underline{x}-\underline{y}|\left\{\frac{\partial^4}{\partial y_1^4}H(a-|\underline{y}|)\right\}d^2\underline{x}\;d^2\underline{y}$$

this integral has a singularity, and must be regarded as a constant whose magnitude is not determined by our potential modelling.

We shall now examine the significance of these terms.

THE EFFECT OF THE FLOW ON THE MECHANICAL PROPERTIES OF THE PISTON

The piston has mass m, and we imagine that it is retrained by a spring whose stiffiness is K. The damping in the system is β. More generally we can define the impedance of the piston to be

$$Z(\omega) = -\beta + \frac{i}{\omega}\left(m\omega^2 - K\right) \tag{3.1}$$

where ω is the frequency under consideration (time dependence $\exp(-i\omega t)$.

The piston is excited by some external forcing, which in the absence of any piston response, would induce a force $F_s(t)$ on the piston. This force could be exerted mechanically, via the support system, or it could be produced by an acoustic source in the fluid.

Then the equation of motion for the piston is

$$F(t) + F_S(t) + m\eta''(t) + \beta\eta'(t) + K\eta(t) = 0 \tag{3.2}$$

and bearing in mind the nature of F(t) as given in (2.16) this is the required equation, telling us the effect of flow upon the piston response as represented by the amplitude $\eta(t)$.

Defining the Fourier transform $\bar{\eta}(\omega)$ of $\eta(t)$ in the usual manner, where

$$\bar{\eta}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\eta(t)\exp(-i\omega t)\,dt \tag{3.3}$$

then the transform of equation (3.2) is

$$\left\{m_u\omega^2 - K_u + \beta_u i\omega\right\}\bar{\eta}(\omega) = \bar{F}_S(\omega) \tag{3.4}$$

We have rearranged the terms to show the way in which the flow has affected the piston constants, with the motion being forced externally.

In equation (3.4) the constants M_u, K_u and β_u are defined to be

$$\begin{aligned} m_u &= m + \frac{8}{3}\rho_0 a^3 - \frac{32}{45}\rho_0 a^3\left(\frac{\omega a}{c}\right)^2 + 4\rho_0 a^3 M^2 \\ K_u &= K - \rho_0 a U^2 A_0 + \rho_0 a U^2 A_1 M^2 \\ \beta_u &= \beta + \frac{\rho_0 a^4\omega^2\pi}{2c} + \frac{5\rho_0 a^4\omega^2\pi M^2}{3c} - \frac{\rho_0 a^4\omega^2\pi}{12c}\left(\frac{\omega a}{c}\right)^2 \end{aligned} \tag{3.5}$$

correct to the order M^2 and $(ka)^2$

Now the meaning of the terms becomes clear. $\frac{8}{3}\rho_0 a^3$ is the traditional virtual mass of a baffled piston oscillating into a fluid of density ρ_0.

The terms

$$-\frac{32}{45}\rho a^3\left(\frac{\omega a}{c}\right)^2 ; \qquad 4\rho a^3 M^2$$

are respectively the lowest order compressible fluid correction to the virtual mass and the Mach number correction because of the Doppler effect.

Similarly in the second of expressions (3.5) the terms represent a mean flow suction of the lowest order Doppler correction to it. The additional terms for the damping are all compressible effects. The term

$$\frac{\rho a^4 \omega^2 \pi}{2c}$$

is the radiation damping, the others again being small corrections to it.

At this stage we can see that the most important effect of the flowing fluid has been the reduction of the effective stiffness of the spring. Another effect which we note at this stage is that the radiation damping is increased by a factor

$$\left(1 + \frac{10M^2}{3}\right)$$

over the static fluid value, confirming our general result of the first section.

Equation (3.4) tells us that the system is stable to small oscillations provided that

$$K_u > 0 .$$

When this condition fails, the piston-spring system becomes unstable and the equations break down. This condition then defines a critical velocity, $\hat{U}$ say, and we are restricted to cases where

$$U < \hat{U} = \left(\frac{K}{\rho a A}\right)^{1/2} \qquad (3.6)$$

Equation (3.4) also tells us that, provided $U < \hat{U}$, the resonance frequency of the piston has been reduced from its zero velocity value

$$\Omega_0 = \left(\frac{K}{m + \frac{8}{3}\rho a^3}\right)^{1/2} \qquad (3.7)$$

to a value at velocity U of

$$\Omega_u = \left(\frac{K - \rho a U^2 A}{m + \frac{8}{3}\rho a^3}\right)^{1/2} = \Omega_0\left(1 - \frac{U^2}{\hat{U}^2}\right)^{1/2} \qquad (3.8)$$

and we see that one effect of flow is that it will always detune a vibrating piston.

In the case of the sonar array, the surface impedance will have been arranged to resonate at a given frequency, in the presence of still water. The questions we must answer quantitatively are therefore:-

1. Assuming a value for the mass of the resonator and an operating frequency, what can we deduce about the magnitude of the critical velocity?

2. Assuming that the operating velocity is significantly below critical, what effect does the flow have on the resonance of the piston?

The mass of the piston is given approximately as

$$m \sim O(\rho' a^3)$$

where ρ' is the piston density, and this leads to a value of critical velocity

$$\hat{U} = \left(\frac{\rho+\rho'}{A\rho}\right)^{1/2} a\Omega_0$$

at the expected frequency Ω_0.

The quantity within the square root is O(1), and so if we assume values of $a \sim 10\,\text{cms}$, $\Omega_0 \sim 10^3\,\text{c/s}$.

this leads to a critical velocity of $\hat{U} > 10^4\,\text{cms/sec} \sim 200$ knots

Thus we should not expect any instability to occur in most nautical situations. A typical value of $\frac{U}{\hat{U}}$ is thus around 0.1, implying that the maximum change in resonance frequency is likely to be around 1%.

These points are discussed further in the conclusion.

Turning now to the acoustic source strength, this is given by equation (1.9) as

$$Q\left(t-\frac{|x|}{c}\right) = \int_S \frac{\partial^2\eta}{\partial t^2}\left(y, t-\frac{|x-y|}{c}\right) d^2y$$

and as before provided the source is compact we can neglect variations in retarded time. Thus the mean square acoustic source strength is

$$\overline{Q^2} = \frac{(\pi a^2)^2 |F_s(\omega)|^2\ \omega^4}{(m_u\omega^2 - K_u)^2 + \beta_u^2\omega^2} \tag{3.9}$$

$$= \left\{\frac{\pi a^2\ |F_s(\omega)|}{m+\frac{8}{3}\rho a^3}\right\}^2 \frac{\alpha^4}{\left[\alpha^2 - \left(1-\frac{U^2}{\hat{U}^2}\right)\right]^2 + \gamma_u^2\alpha^2}.$$

In this second expression α is a non-dimensional frequency defined by

$$\alpha = \frac{\omega}{\Omega_0} \tag{3.10}$$

and γ_u is a non-dimensional damping parameter, defined by

$$\gamma_u^2 = \frac{\beta_u^2}{\Omega_0^2 \left(m + \frac{8}{3}\rho a^3\right)^2} \tag{3.11}$$

If the piston is being irradiated above its no-flow resonance frequency ($\alpha \gg 1$), the source strength is dominated by the mass term, and the flow will have little effect on the acoustic energy output. However if the forcing is below resonance ($\alpha < 1$) there exists a flow velocity which brings the piston to resonance. At this point the radiation energy of course goes up enormously as it is controlled only by the damping. The flow velocity will have to be very near to the critical velocity before any appreciable shift in frequency, and consequent energy gain is obtained.

Perhaps the most significant change occurs when the piston is initially set to resonate in the no-flow situation. Now any flow immediately detunes the piston, with a consequent reduction in radiated energy.

CONCLUSIONS

We have seen that the flow has two distinct effects on sound radiation from a compact vibrating surface. Firstly, whenever linearised boundary conditions are appropriate, a finite and constant mass displacement by the source gives rise to pressure increase proportional to $(1-M\cos\theta)^{-3}$. This implies an energy increase of $5M^2$ times the energy radiated into static fluid. The increased energy is drawn in the ratio 2:1 from the surface and mean flow respectively. However this is only one application of a general result for a vibrating surface characterised by $\frac{\partial^n f}{\partial t^n}$. Then the increase in radiated pressure is $(1-M\cos\theta)^{-(n+1)}$, the increased energy being drawn in the ratio n:1.

Thus the effect of flow on the acoustics of vibrating surfaces is often substantial, even when the amplitude of vibration is not altered by the flow.

This brings us to the second flow effect. The amplitude is altered, and there is a critical flow velocity, $\hat{U}$, ($\hat{U} = \left(\frac{K}{\rho a A}\right)^{1/2}$, see equation (3.6)), above which local instability sets in. The flow also changes the effective stiffness and alters the resonance frequency and these effects can be substantial.

Although the problem treated is only an idealised model it contains the basis for anticipating definite trends in flow-acoustic vibration problems. Ffowcs Williams (1972) models a compliant surface by several isolated pistons, and following his method we may anticipate:-

a) reduced absorption

b) a change in tuning

c) instabilities

This model is thought to represent practical acoustic liners. Therefore the effect of flow is far from a small perturbation and is inevitably so when surface motions can be unstable and are coupled to sound by a scattering mechanisms.

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CHAPTER 2

INTRODUCTION

The first person to discuss the effects of boundaries upon Lighthill's (1952) theory of aerodynamic noise was Curle (1955). He showed that the action of a rigid boundary was equivalent to that of a distribution of surface dipoles. His work was not deductive since it required independent knowledge of the quadrupole and dipole strengths. Theories linking quadrupole and dipole strengths have perforce been developed to avoid paradoxes arising through mis-application of Curle's results. Considerable progress has been made; thus in the theory of rigid surface scattering we have the work of Powell (1960, the infinite rigid plate), Ffowcs-Williams and Hall (1970, the rigid semi-infinite plate) and Leppington (1970, the semi-infinite rigid circular duct).

In practice, especially in underwater applications, solid surfaces cannot truly be considered rigid and research must develop theories allowing for wall compliancy. Such work is indeed being performed and we mention papers by Ffowcs-Williams (1965, infinite flexible wall and 1966, infinite flexible wall with simple (rigid) supports). A paper by Crighton and Leppington (1970, semi-infinite compliant plate) introduces the Wiener-Hopf technique to scattering problems. This technique is a powerful tool in dealing with two-part boundary conditions.

When a compliant wall is capable of sustaining travelling waves an interesting possibility of long-range interaction between an acoustic source and the edge is opened up. In the case of two dimensional fluid motion the energy arriving at the edge via surface waves is independent of the separation distance, h, of the source and the edge, whereas the direct energy (via the fluid) is decreasing at least as quickly as $(kh)^{-1}$, where k is the acoustic wavenumber. Heckl (1967, 1969) has shown that the power generated in a surface by a point source close to the surface may greatly exceed that radiated into space. Even when the edge scattering is relatively inefficient the edge can thus provide a large increase in sound power. Both these points have been raised by Crighton (1972) who discussed the scattering problems by ignoring the presence of the fluid except in the determination of the radiation.

It is the aim of the paper to give an exact treatment of this mechanism, using the simplest model situation which contains the essential features and yet which is susceptible to exact and complete analysis.

The scheme adopted in this paper is simple. Far from the interface between a rigid wall and a wave-bearing wall we have a mechanism generating both surface waves and radiated sound (we consider both a line force situated in the wall and a line quadrupole adjacent to the wall in the fluid). Both mechanisms excite surface waves as if the wall were infinite rather than semi-infinite. The scattering process at the interface is treated by the Wiener-Hopf technique.

It is perhaps worth emphasising the type of problem we expect to be able to solve by this method. We are specifically excluding problems with direct interaction between the forcing mechanism and the edge. Problems involving sources close to the edge may be solved, employing the reciprocal theorem. The interested reader is referred to Crighton and Leppington (1971) for a full discussion of this point. Wherever the source is situated there is, of course, the possibility of interaction of the source with the wave reflected from the interface. This is also specifically excluded from the problem, although we could regard our problem as the steady-state outcome of such an interaction.

Wiener-Hopf problems are notoriously liable to extreme algebraic complexity. In the interests of clarity we will specify particular physical conditions describing the compliant surface. Crighton and Leppington (1970) considered the "locally reacting" mass-loaded plate and they noted the absence of surface waves. Provided that we can find a physical interpretation for it, by the simple mathematical step of reversing the sign of "m" (mass/unit area) surface waves become an important feature. The model thus obtained may be interpreted physically as involving a locally reacting plate (locally reacting in a vacuum, that is) with each element restrained by a spring, of stiffness K, say. For excitation at frequency w, the mass term will be replaced by the term

$$-\left(\frac{K - m\omega^2}{\omega^2}\right)$$

with m again representing the areal density. Provided excitation is below the in vacuo resonance frequency of the wall, then the presence of the fluid ensures the existence of surface waves.

We first perform the full Wiener-Hopf calculation for the case of the line force. Details of the edge scattering and the reflection coefficient of the surface waves are obtained. The problem with the point quadrupole can then be treated fairly easily, since we will only require comparision of the incident surface wave amplitudes, and these are the same as if the flexible wall were infinite. An appendix shows that we can derive simple expressions for both the scattered energy and the reflection coefficient at all levels of fluid loading.

THE BASIC EQUATIONS AND THEIR SOLUTION

An inviscid fluid occupies the half space $y \geq 0$. The two-part boundary consists of a rigid wall occupying $x > 0, y = 0$ and a compliant wall occupying $x < 0, y = 0$. A harmonic line force, of strength F_0, is situated in the wall at $x = -h$. The force excites small disturbances in the wall and the fluid and we describe the motion by the velocity potential $\phi(x, y, t)$. Suppressing the time factor $\exp(-i\omega t)$ throughout, then the equations to be satisfied by ϕ are

$$\nabla^2 \phi + k^2 \phi = 0 \tag{1.1}$$

$$\frac{\partial \phi}{\partial y} = 0 \tag{1.2}$$

$$F_0 \delta(x+h) - \rho i \omega \phi = z \frac{\partial \phi}{\partial y} \tag{1.3}$$

Here ρ is the fluid density and z is the wall impedance, and in fact

$$z = \frac{K - m\omega^2}{-i\omega} \tag{1.4}$$

where the significance of K and m has been explained in the introduction.

The equations (1.1)-(1.3) are solved in the standard D.S. Jones method (Noble (1958)). We follow the notation of Noble as far as is possible. Introducing half range transforms

$$\Phi_+ = \frac{1}{(2\pi)^{1/2}} \int_0^\infty \phi(x, y) \exp(i\alpha x)\, dx$$

$$\Phi_- = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^0 \phi(x, y) \exp(i\alpha x)\, dx \tag{1.5}$$

and

$$\Phi = \Phi_+ + \Phi_-$$

the original equations become

$$\frac{\partial^2 \Phi}{\partial y^2} = (\alpha^2 - k^2)\, \Phi(\alpha, y) \tag{1.6}$$

$$\frac{\partial \Phi_+}{\partial y} = 0 \tag{1.7}$$

$$\Phi_- = -\mu^{-1} \frac{\partial \Phi_-}{\partial y}(\alpha, 0) + F_0 (\rho i \omega)^{-1} (2\pi)^{-1/2} \exp(-i\alpha h) \tag{1.8}$$

with the notation

$$\mu = \frac{\rho i \omega}{z} \tag{1.9}$$

Provided that the excitation is below the in vacuo resonance frequency for the wall, we see that $\mu > 0$.

The equation (1.6) has the solution

$$\Phi(\alpha, y) = A(\alpha) \exp(-\gamma(\alpha) y) \tag{1.10}$$

where $\gamma(\alpha)$ is the branch of $(\alpha^2 - k^2)^{1/2}$ chosen to ensure a sensible radiation condition away from the wall. Thus if α and k are real then $(\alpha^2 - k^2)^{1/2}$ denotes the positive root when $|\alpha| > k$, while $(\alpha^2 - k^2)^{1/2} = -i(k^2 - \alpha^2)^{1/2}$ when $|\alpha| < k$. The boundary conditions alone with the solution (1.10) can be rewritten in the form,

$$\frac{\partial \Phi_-}{\partial y}(\alpha, 0) \left\{ \frac{1}{\gamma(\alpha)} - \frac{1}{\mu} \right\} + \Phi_+(\alpha, 0) = -F_0 (\rho i \omega)^{-1} (2\pi)^{-1/2} \exp(-i\alpha h) \tag{1.11}$$

This equation is in the standard form for solution by the Wiener-Hopf procedure (Noble pp 36 - 38). The kernel of the equation $K(\alpha)$ is given by

$$K(\alpha) = \frac{1}{\gamma(\alpha)} - \frac{1}{\mu} \tag{1.12}$$

and it is now obvious that the kernel contains zeros within the α plane. We assert that these zeros correspond to free surface waves, a fact which we justify later. Essentially the zeros of $K(\alpha)$ correspond to simple poles in the α plane when we perform the inverse transform, giving rise to outgoing waves in the normal way. The zeros present little analytic difficulty to the Wiener-Hopf calculation and the required split is obtained by first isolating the zeros, writing

$$K(\alpha) = \frac{-(\alpha-p)(\alpha+p)}{\mu(\gamma(\alpha))} \left\{ \frac{1}{\mu + \gamma(\alpha)} \right\} \tag{1.13}$$

with $p = (\mu^2 + k^2)^{1/2}$

The multiplicative split for the function $\mu + \gamma$ has been given by Crighton and Leppington (1970) and if

$$\{\mu + \gamma(\alpha)\}^{-1} = K_{1+}(\alpha)\, K_{1-}(\alpha) \tag{1.14}$$

we can write

$$K(\alpha) = K_+(\alpha)\, K_-(\alpha)$$

with

and

$$K_+(\alpha) = i(\alpha+p)(\alpha+k)^{-1/2}\mu^{-1/2} K_{1+}(\alpha)$$
$$K_-(\alpha) = i(\alpha-p)(\alpha-k)^{-1/2}\mu^{-1/2} K_{1-}(\alpha) \tag{1.15}$$

Further details of the $K_{1+}(\alpha)$, $K_{1-}(\alpha)$ functions are given in the appendix. We see that the strip in which both the plus and minus functions are regular is limited by the position of the poles $\alpha = \pm p$.

[In using the Wiener-Hopf technique we introduce a small damping factor k_2, writing the wavenumber as $k = k_1 + ik_2$. Then the poles corresponding to $p = (\mu^2 + k^2)^{1/2}$ can be written as $p = p_1 + ip_2$, and for $k_2/k_1 \ll 1$ we see that $p_2 < k_2$, and then the poles lie within the strip $-k_2 < \operatorname{Im}\alpha < k_2$.]

It is the step of obtaining the multiplicative split $K(\alpha) = K_+(\alpha)\, K_-(\alpha)$ which presents algebraic difficulty for more "practical" surfaces, although it can be shown formally that the required split functions can be obtained in the form of the integrals similar to those in the appendix.

Use of the functions $K_+(\alpha)$, $K_-(\alpha)$ enables us to rewrite (1.10) as

$$\frac{\partial \Phi}{\partial y} K_- + \frac{\Phi_+}{K_+} = L_+(\alpha) + L_-(\alpha) \tag{1.16}$$

where $L_+(\alpha)$ and $L_-(\alpha)$ are the two functions forming the additive split of the function

$$\frac{-F_0 \exp(-i\alpha h)}{\rho i\omega (2\pi)^{1/2}\, K_+(\alpha)}$$

Alternatively, we separate the plus and the minus functions and write

$$\frac{\partial \Phi_-(\alpha)}{\partial y} K_-(\alpha) - L_-(\alpha) = -\frac{\Phi_+(\alpha)}{K_+(\alpha)} + L_+(\alpha) \equiv J(\alpha).$$

$J(\alpha)$, the entire function, is the common representation of both the plus and the minus functions within the strip where both are regular.

From the appendix we see that

$$L_\pm(\alpha) = O(\alpha^{-1}) \quad \text{as} \quad |\alpha| \to \infty,$$

and that

$$K_\pm(\alpha) = O(1) \quad \text{as} \quad |\alpha| \to \infty.$$

A simple application of Liouville's theorem shows that, provided $\phi = O(1)$, and $\nabla\phi = O(|x|^{-\alpha})$, $(\alpha < 1)$ $\quad |x| = 0$,

then the entire function is identically zero. Thus the boundary conditions yield the relationships

$$\frac{\partial \Phi_-}{\partial y}(\alpha, 0) = L_-(\alpha)\{K_-(\alpha)\}^{-1}; \tag{1.17}$$

$$\Phi_+(\alpha, 0) = L_+(\alpha)\ K_+(\alpha). \tag{1.18}$$

The final expression for $\phi(x,y)$ is obtained as the inverse Fourier transform of (1.10). $A(\alpha)$ is obtained from the boundary conditions, (1.17) and (1.18), enabling us to write

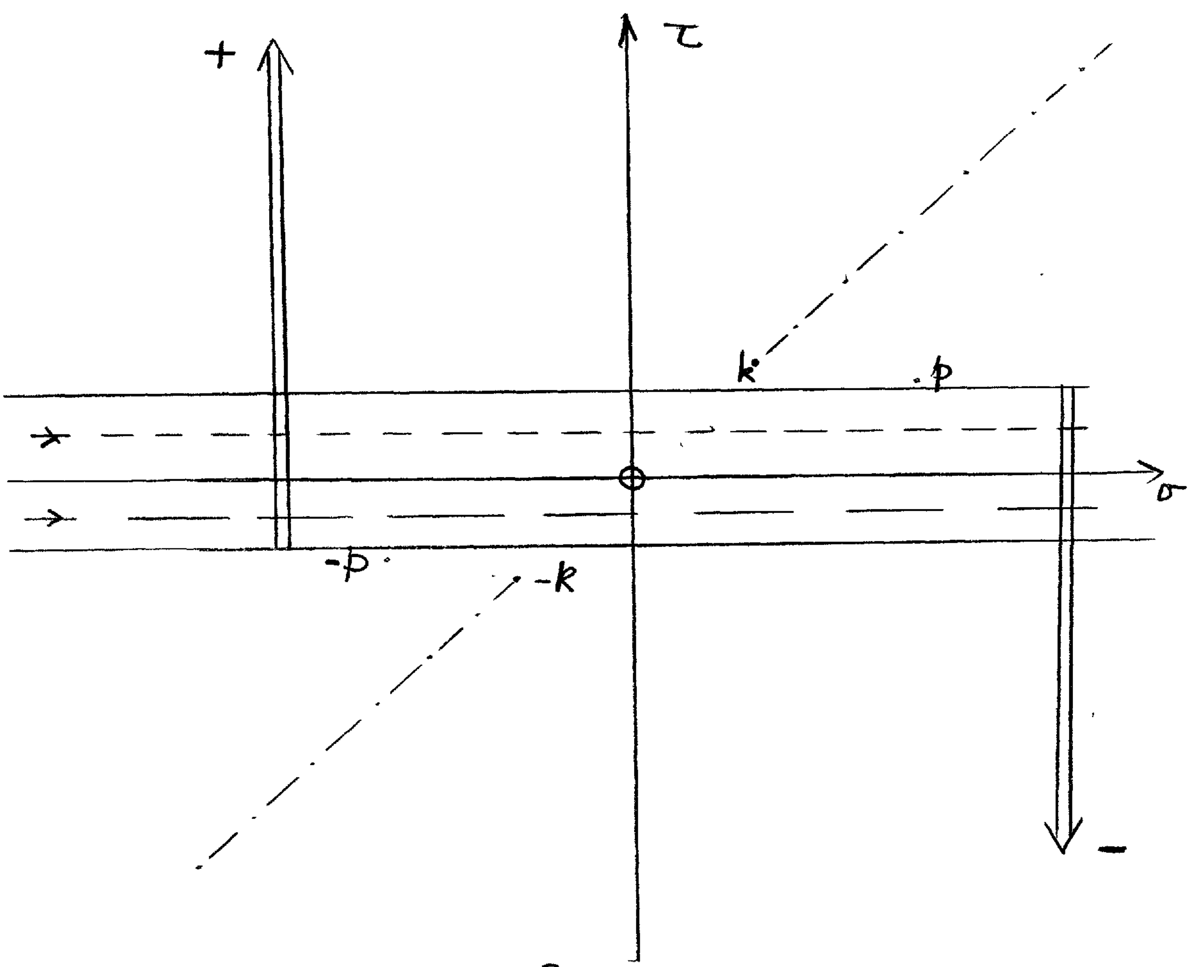
$$\phi(x,y) = -\frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty} \frac{L_-(\alpha)\exp(-i\alpha x)\exp(-\gamma y)}{K_-(\alpha)\ \gamma(\alpha)}\, d\alpha \tag{1.19}$$

EVALUATION OF THE INTEGRALS

The equation (1.19) contains all the essential information which we expect to obtain from the investigation. A stationary phase integration tells us the far field radiation, whilst differentiation with respect to y leads to an expression for the surface velocity. We must first obtain a useful expression for the function $L_-(\alpha)$. Noble (p13) gives the expression for $L_-(\alpha)$ as

$$L_-(\alpha) = \int_{-\infty+id}^{\infty+id} \frac{\exp(-i\beta h)\, d\beta}{K_+(\beta)\,(\beta-\alpha)} \cdot \frac{-F_0}{\rho i \omega (2\pi)^{1/2}} \qquad (2.1)$$

where d is a constant chosen such that the path of integration lies within the overlapping strips of regularity of the plus and the minus functions, and also such that the path of the β integration lies above the path for the α integration. The plane of the β and α integration is shown in fig. 1. Our goal is to evaluate the integral by stationary phase.



– – – – – – – – β contour
—— —— —— α. contour
. —— . —— . —— . —— cuts

Figure (1)

The integral is the limiting form of the integral

$$\int_{-\infty}^{\infty} f(\beta)\exp(-i\beta h\cos\theta)\exp(-\delta(\beta)\,h\sin\theta\;\;d\beta \tag{2.2}$$

as $\theta\to 0$. Following the argument of Noble (pp 31-35) we see that the integral should be deformed on to a (degenerate) hyperbola around -k. Before we can do this, the path of integration must cross two poles, one at $\beta=\alpha$ and the other at $\beta=-p$. Including the contribution from these poles, we obtain the result

$$L_-(\alpha) = \frac{-F_0}{\rho i\omega(2\pi)^{1/2}}\left\{\frac{\exp(-i\alpha h)}{K_+(\alpha)} - \frac{\mu^3\exp(iph)K_-(-p)}{p(p+\alpha)}\right\}$$

$$+\frac{F_0}{2\pi i\rho i\omega(2\pi)^{1/2}}\int\frac{\exp(-i\beta h)\;K_-(\beta)}{K(\beta)\;(\beta-\alpha)}\,d\beta$$

Where the path of the β integration may now be deformed around the cut as required.

The leading term of the stationary phase integration is zero since, in general, the integral (2.2) when evaluated is

$$A\sin\theta\; f(-k\cos\theta)\exp(ikh).(kh)^{-1/2}$$

with A independent of θ. The next term in the stationary phase estimate of the integral is proportional to $(kh)^{-3/2}$ and is negligible compared with the polar contributions.

Consideration of this zero indicates that it will occur whenever a compliant surface is present. Thus the direct field at the edge will in fact decay in proportion to $(kh)^{-3}$ and not $(kh)^{-1}$ as it would for a rigid wall.

The outcome is that we have

$$L_-(\alpha) = \frac{-F_0}{\rho i\omega(2\pi)^{1/2}}\left\{\frac{\exp(-i\alpha h)}{K_+(\alpha)} - \frac{\mu^3\exp(iph)K_-(-p)}{p(p+\alpha)}\right\}$$

and $$\phi(x,y) = \frac{F_0}{2\pi\rho i\omega}\int_{\alpha=-\infty}^{\infty}\frac{e^{-i\alpha(h+x)}e^{-\gamma y}}{K(\alpha)\gamma(\alpha)}\,d\alpha - \frac{F_0\mu^3 e^{iph}K_-(-p)}{2\pi\rho i\omega p}\int_{\alpha=-\infty}^{\infty}\frac{e^{-i\alpha x}e^{-\gamma y}}{K_-(\alpha)\gamma(\alpha)(p+\alpha)}\,d\alpha \tag{2.3}$$

Expression (2.3) has been written deliberately to show the two distinct effects. The first integral is exactly that which would occur if the compliant wall were infinite, whereas the second contains all the information about the edge scattering and reflection processes.

The surface velocity is given by $\partial\phi/\partial y = 0$. The integrals are evaluated by closing contours in the appropriate half planes and, apart from a near field correction term (decaying in proportion to $x^{-1/2}$ or $(x+h)^{-1/2}$) we obtain

$$v(x) = \frac{F_0\mu^3}{p\omega\rho}\left\{\exp\left(ip(x+h)\right)\left\{H(x+h) - H(x)\right\} + \exp\left(-ip(x+h)\right)H(-(x+h)) \right. \tag{2.4}$$

$$\left. - \mu^3/2p^2 \exp(ip(h-x))K_+(p)\,K_-(p)H(-x)\right\}$$

Here H is the unit Heaviside function. As required the surface velocity is zero for $x>0$. On the flexible boundary we can recognise the waves generated as if on an infinite flexible wall, plus the wave reflected from the interface at $x=0$. The reflection coefficient is given by

$$R = -\frac{\mu^3\{K_+(p)\}^2}{2p^2} \tag{2.5}$$

where we have used the result from the appendix that $K_+(\alpha) \equiv K_-(-\alpha)$

Equation (2.4) has confirmed our assertion that the wavenumber is given by the zeros, $\pm p$, of the equation $K(\alpha)=0$. The waves are always subsonic. This is a useful restriction (it may not be the case on a bending plate) because supersonic waves are capable of radiating sound without the aid of a scattering process.

Returning to the expression (2.3) we evaluate the radiation by the stationary phase method. Writing

$$x+h = r'\cos\theta' \qquad x = r\cos\theta$$
$$y = r'\sin\theta' \qquad y = r\sin\theta$$

the stationary phase integral gives, with no difficulty,

$$\phi(x,y) = \begin{array}{l} -\dfrac{F_0\exp(i\pi/4)\mu k\sin\theta'\exp(ikr')}{(2\pi)^{1/2}\rho\omega(\mu+ik\sin\theta')(kr')^{1/2}} \\ -\dfrac{F_0\exp(-i\pi/4)\exp(iph)\mu^3K_-(-p)\exp(ikr)}{(2\pi)^{1/2}\rho\omega p(p-k\cos\theta)(p-k\cos\theta)K_-(-k\cos\theta)(kr)^{1/2}} \end{array} \tag{2.6}$$

Again, we recognise this as being due to two fundamentally different sources. The first term, ϕ_d say, represents radiation as if from a line force in an infinite compliant wall. In the light fluid loading limit $\mu k \ll 1$ we see that $\phi_d \sim O(1)$ (except near $\theta'=0$), whereas in the heavy fluid loading case we have

$$\phi_d \sim O(\sin\theta')$$

Thus the fluid loading is inducing a dipole in place of the in vacuo monopole.

In fact the energy radiated directly as sound by the force is

$$\frac{F_0^2 k \mu^2}{2\rho c\, p(p+\mu)} \tag{2.7}$$

As with most problems with non-compact scatterers, the edge radiation, given by the second term in (2.6) cannot be pictured as a simple combination of compact multipole sources. We do see, however, that the directivity of the "edge" scattering is independent of the forcing term.

The form of the expressions in (2.5) and (2.6) may appear alarming, but we show in the appendix that simple exact expressions may be obtained for both $|R|^2$ and for $|\phi_c|^2$. [ϕ_c is the scattered potential represented by the second term of (2.6)].

These quantities are of prime importance as far as energy transfer is concerned. In fact we obtain

$$|R|^2 = \frac{\mu^4}{p^2(p+k)^2} \tag{2.8}$$

and

$$\int_{\text{space}} |\phi_c|^2 \frac{\rho\omega^2}{c} = \frac{F_0^2 \mu^3 (p^2+\mu^2+pk)}{2\rho c\, p^3 (p+k)} \tag{2.9}$$

The reflection coefficient varies from zero to one as the fluid loading parameter μ/k varies from zero to infinity.

Comparing the energy scattered from the edge with the energy radiated from the force itself we see that the ratio of the energy output is given by a factor which is close to μ/k in both the light and heavy fluid loading limits.

EXCITATION OF SURFACE WAVES BY A POINT MULTIPOLE

The equivalent problem for excitation by a point quadrupole situated in the fluid far from the interface follows fairly easily. The results of the previous section indicate that the details of the edge scattering process depend only on the amplitude of the incident wave, that is, the incident wave that would be induced were the flexible wall infinite.

We consider a problem where a line monopole lies just above an infinite flexible wall. Results for a line quadrupole can be obtained by differentiation with respect to the source position. Once again we formulate the problem in terms of the velocity potential ϕ_t which consists of a direct and scattered potential ϕ_d and ϕ_s respectively. ϕ_d is the field radiated if the wall was not present. Suppressing the harmonic time dependence the scattered potential satisfies

$$\nabla^2\phi_s + k^2\phi_s = 0 \tag{3.1}$$

$$\phi_s + \frac{Q}{4i} H_0\left\{k\left(\{x+h\}^2+y_0^2\right)^{\frac{1}{2}}\right\} = -\frac{z}{\rho i\omega}\frac{\partial\phi_s}{\partial y} \tag{3.2}$$

where the source is situated at the point $(-h, y_0)$. The impedance z is identical to that used in the previous section.

Taking Fourier transforms as previously defined the equations (3.1),(3.2) yield

$$\frac{\partial^2\Phi_s}{\partial y^2} = (\alpha^2-k^2)\,\Phi_s \tag{3.3}$$

$$\Phi_s + \frac{1}{\mu}\,\partial\Phi_s/\partial y = -\frac{Q\exp(-i\alpha h)\exp(-\gamma y_0)}{(8\pi)^{\frac{1}{2}}\gamma} \tag{3.4}$$

Again choosing the solution of (3.3) which gives a sensible radiation condition away from the wall, the inverse Fourier transform yields

$$\phi_s(x,y) = \frac{Q\mu}{4\pi}\int_{\alpha=-\infty}^{\infty}\frac{\exp(-i\alpha(x+h))\exp(-(y+y_0))\gamma}{\gamma(\alpha)\{\mu-\gamma(\alpha)\}}\,d\alpha \tag{3.5}$$

The form of (3.5) suggests that ϕ_s is centred on the image point $(-h,-y_0)$ of the point source.

The field due to a point quadrupole is obtained by differentiation with respect to either h or y_0, depending on the orientation of the quadrupole. The procedure in each case is the same and the working is not difficult. We deal with the case of a yy oriented quadrupole. The scattered potential is

$$\phi_s^{yy}(x,y) = \frac{Q\mu}{4\pi}\int_{\alpha=-\infty}^{\infty}\frac{\gamma.\exp(-i\alpha(x+h))\exp(-(y+y_0)\gamma)}{\mu-\gamma}\,d\alpha \tag{3.6}$$

leading to a surface disturbance

$$\frac{\partial \phi_s}{\partial y}(y=0) = v(x) = \frac{Qi\mu^4 e^{-\mu y_0}}{2p}\left\{ e^{ip(x+h)} H(x+h) + e^{-ip(x+h)} H(-(x+h)) \right\} \tag{3.7}$$

plus, once again, near field corrections decaying as $(x+h)^{-1/2}$

Comparison with (2.4) indicates that the action of the quadrupole is equivalent to a line force the strength of which is

$$\frac{Q\, i\, \mu\, e^{-\mu y_0} \rho\omega}{2} \tag{3.8}$$

The radiation from the edge, by comparison with (2.9) is

$$\frac{Q^2 \rho^2 \omega^2 e^{-2\mu y_0}}{8\rho c} \cdot \frac{\mu^5 (p^2 + \mu^2 + pk)}{p^3 (p+k)} \tag{3.9}$$

The factor $e^{-\mu y_0}$ indicates that when the quadrupole is far away from the wall virtually no surface wave is excited. This is what we would expect. The near field energy of a quadrupole is much greater than the far field energy, and it is the near field energy which we expect to drive the surface waves.

We compare the edge scattering with the radiation which is produced by a point quadrupole above an infinite flexible wall. This comprises the direct field and the scattered field, the latter being given by the stationary phase estimate of the integral (3.6). This estimate is

$$\phi(\bar{R}, \bar{\theta}) = \frac{Q\mu \exp(ik\bar{R})\exp(-i\pi/4)}{2(2\pi)^{1/2}\,(k\bar{R})^{1/2}} \cdot \frac{-ik^2 \sin^2\bar{\theta}}{\mu + ik\sin\bar{\theta}}$$

and the direct field is

$$\phi(R,\theta) = \frac{Q \exp(ikR)\exp(-i\pi/4)}{2(2\pi)^{1/2}\,(kR)^{1/2}} \; -ik^2 \sin^2\theta$$

where the overbar means that the position is measured relative to the image point. Provided the quadrupole is fairly close to the wall we may safely ignore the differences between source and image point, and the total direct energy is thus

$$\frac{3Q^2 \rho\omega^2 k^3}{8c}\left\{ \frac{1}{8} + \frac{\mu^2}{2k^2} - \frac{\mu^4}{k^4} + \frac{\mu^5}{pk^4} \right\}$$

Thus in the high fluid loading limit the edge radiation is greater by a factor $(\mu/k)^3$ than the direct field from the quadrupole alone. We also see from (3.9) that the energy scattered from the edge tends to zero as μ approaches zero. This is a necessity since $\mu = 0$ implies the absence of surface waves.

CONSERVATION OF ENERGY

Obviously the principle of conservation of energy must hold within a large control surface surrounding the interface. We consider the limit when the driving force lies well outside the control surface, and then we see that the difference between the incident and reflected energy associated with the plate waves must be equal to the scattered energy. In general, the energy associated with the plate waves is carried in both the plate and in the fluid. If we call the incident energy I, then the energy in the reflected wave is

$$|R|^2 I \tag{4.1}$$

Thus, bearing in mind the results (2.8) and (2.9), the conservation principle implies that

$$I = \frac{F_0^2 \mu^3}{2\rho \omega p} \tag{4.2}$$

In the particular case of the "locally reacting" wall which we are describing, no plate waves are possible in vacuo. In this case all the incident energy is contained in the fluid. The incident energy crossing the control surface is thus given by

$$I = \mathrm{Re} \int_{y=0}^{\infty} \left(-\rho \frac{\partial \phi_i}{\partial t} \right) \left(\frac{\partial \phi_i}{\partial x} \right)^* dy \tag{4.3}$$

where ϕ_i is the potential corresponding to the incident wave. This part of the potential can be isolated from expression (2.3); in fact

$$\phi_i = -\frac{F_0 \mu^2 e^{-\mu y}}{p \rho \omega} \cdot e^{i[px - \omega t]} \tag{4.4}$$

Inserting this expression for in equation (4.3) yields, as expected,

$$I = \frac{F_0^2 \mu^3}{2 \rho \omega p}$$

The significance of the principle of conservation of energy would be seen in problems of greater analytic complexity. In general the scattered intensity will depend upon a function equivalent to the $K_-(\alpha)$ function defined in (A.6). Rather than evaluating both $|K_-(-k\cos\theta)|^2$ and $|K_-(-p)|^2$ in order to find the scattered field and the edge reflection coefficient, it may be simpler to evaluate $|K_-(-k\cos\theta)|^2$, perform the integration to find the total scattered energy and use the conservation principle to eliminate the unknown $|K_-(-p)|^2$.

CONCLUSION

As would be expected, we have seen the possibility of enormous increases in radiated sound levels due to this edge effect. Although we have a very simple wall impedance condition we might expect that the characteristics of the results would hold for problems with more realistic impedance condition. This seems likely to be the case at least in the high fluid loading limit. At low fluid loading the surface waves vanish in our problem, which is certainly not the case in a practical situation. However, the approach of Crighton (1972) appears to deal with problems in this limit very successfully.

One related problem which is of great interest is that of a wave bearing wall adjacent to a fluid with a mean flow. Under certain conditions wall waves will grow (especially when the wall has damping) and during this growth the wall is drawing energy from the fluid. When the surface wave is scattered into sound (for example by the type of interface described above) a very substantial noise field can result. Great care is required to formulate a useful mathematical model to deal with problems of this type.

If the surface were slightly damped (as in practice it might well be) the method we have used would not be significantly affected. Of course, the zeros of the kernel are no longer on the real axis, but in fact this is merely the situation we seek to attain by the introduction of the complex wavenumber $k_1 + ik_2$.

We again emphasise that in the problem we have described, we are insisting that the separation of the driving quadrupoles and the edge must be several wavelengths It is at this stage an open question whether a point quadrupole near the interface would radiate more or less efficiently than the quadrupole we have described. We cannot merely compare the results of Crighton and Leppington (1970) because they did not have a wave bearing surface. It may well be the case that with a wave bearing surface all the near field energy is scattered into outgoing surface waves and not into increased radiation. It is clear, however, that there is only a finite volume of fluid available in which to place quadrupoles near the edge (within a fluid or plate wavelength), whereas there is an unlimited volume available for quadrupoles whose near field energy can be scattered by the process we have described. It is also the case that any multipole source placed close to a wave bearing surface cannot be prevented from radiating noise in this fashion. The introduction of any rigid strut simply acts as a scatterer (Ffowcs-Williams 1966). The only apparent way of preventing noise of this type (assuming that the quadrupoles cannot be removed from the proximity of the surface) may be the introduction of large (but continuous) damping mechanisms into wave bearing walls.

APPENDIX

We require the multiplicative split of the function $\{\mu + \gamma(\alpha)\}^{-1}$.

This was obtained by Crighton & Leppington (1970), who actually treated $\mu + \gamma(\alpha)$. The result in our case is

$$K_{1-}(\alpha) = (\alpha - p)^{-\frac{1}{2}} \left(\frac{\alpha+p}{\alpha-p}\right)^{-\frac{1}{2}\mu P_+(p)} \exp\left\{-\mu\int_\alpha^{-i\infty} \frac{P_-(t)\, t\, dt}{t^2-p^2}\right\} \tag{A1}$$

$$K_{1+}(\alpha) = (\alpha + p)^{-\frac{1}{2}} \left(\frac{\alpha+p}{\alpha-p}\right)^{\frac{1}{2}\mu P_+(p)} \exp\left\{-\mu\int_\alpha^{i\infty} \frac{P_+(t)\, t\, dt}{t^2-p^2}\right\} \tag{A2}$$

where $P_+(\eta) = \frac{1}{\pi}\cos^{-1}(\eta/k)(\eta^2-k^2)^{-\frac{1}{2}}$ (A3)

and $P_-(-\eta) = P_+(\eta)$ (A4)

The definitions (1.14)

$$K_+(\alpha) = \frac{i(\alpha+p)}{\mu^{\frac{1}{2}}(\alpha+k)^{\frac{1}{2}}} K_{1+}(\alpha) \tag{A5}$$

$$K_-(\alpha) = \frac{i(\alpha-p)}{\mu^{\frac{1}{2}}(\alpha-k)^{\frac{1}{2}}} K_{1-}(\alpha) \tag{A6}$$

imply the important relation that

$$K_+(\alpha) = K_-(\alpha) \tag{A7}$$

The reflection coefficient, R, is given as

$$R = \frac{\mu^3\{K_+(p)\}^2}{2p^2}$$

$$= \frac{\mu^2(2p)^{\mu P_+(p)}}{p(p+k)} \lim_{\alpha\to p} \frac{\exp\left\{-2\mu\int_\alpha^{i\infty}\frac{P_+(t)\, t}{t^2-p^2}\, dt\right\}}{(\alpha-p)^{\mu P_+(p)}} \tag{A8}$$

The limit $\alpha \to p$ is obtained by setting $\alpha = p + \varepsilon\exp(i\theta_0)$ and taking the limit at $\varepsilon \to 0$. The integral required in (A.6) is performed by contour integration and we close the contour as shown in Fig. 2

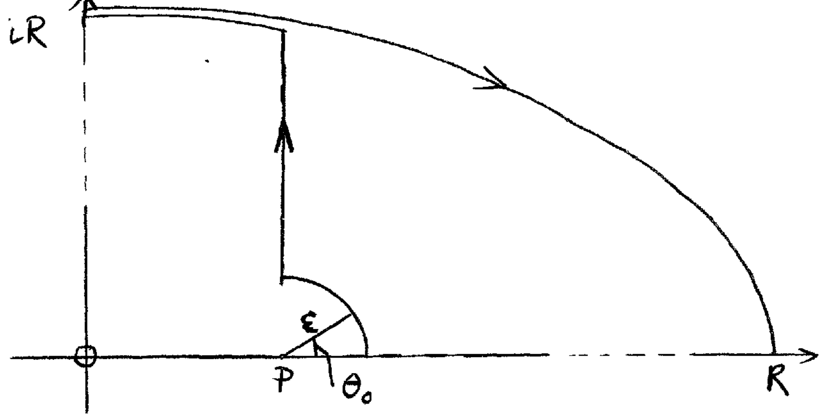


FIG. 2

The only pole affecting the integration is at t = p. The total integral around the chosen contour in zero. Around the large quadrant, when $t = Re^{i\phi}$ the integrand is of order $(\ln R)/R^2$ and the contribution tends to zero as $R \to \infty$. The integral can therefore be rewritten as

$$\int_{p+\varepsilon}^{\infty} \frac{\eta\, P_+(\eta)}{\eta^2 - p^2}\, d\eta \quad - \int_{\theta=0}^{\theta_0} \frac{i\, P_+(p+\varepsilon e^{i\theta})(p+\varepsilon e^{i\theta})\, \varepsilon e^{i\theta}}{2p\, \varepsilon e^{i\theta}}\, d\theta$$

The θ integration can be performed immediately in the limit $\varepsilon \to 0$ and the result is simply

$$\frac{-i\, P_+(\eta)\, \theta_0}{2}.$$

Thus the expression for R becomes

$$R = \frac{\mu^2}{p(p+k)} (2p)^{\mu P_+(p)} \left[\lim_{\varepsilon \to 0} \frac{\exp\left\{ -2\mu \int_{p+\varepsilon}^{\infty} \frac{P_+(\eta)\,\eta}{\eta^2 - p^2}\, d\eta \right\}}{\varepsilon^{\mu P_+(p)}} \right] \tag{A9}$$

and R is independent of θ_0 as expected. The remaining integral in (A9) will not in general be simple to evaluate. We note, however, that throughout the (real) range of integration $P_+(\eta)$ has the form

$$P_+(\eta) = \frac{1}{\pi} \frac{\cos^{-1}(\eta/k)}{(\eta^2 - k^2)^{1/2}} = \frac{i}{\pi} \frac{\cosh^{-1}(\eta/k)}{(\eta^2 - k^2)^{1/2}} \tag{A10}$$

The advantage of the second expression is that it is now obvious that throughout the range of integration (in fact for all $\eta \geq k$) $\cosh^{-1}(\eta/k)$ is a real function. Consequently the contents of the square brackets in equation (A.9) can be expressed in the form $\exp i\lambda$, where λ is some undertermined real constant. We also write

$$(2p)^{\mu P_+(p)} = \exp\left\{ \frac{i}{\pi} \ln(2p) \cosh^{-1}\left(\frac{p}{k}\right) \right\} \tag{A11}$$

and then it is apparent that

$$R = \frac{\mu^2 \exp(i\delta)}{p(p+k)} \tag{A12}$$

or

$$|R|^2 = \frac{\mu^4}{p^2(p+k)^2}. \tag{A13}$$

The integral required in (A.9) may be usefully approximated in the high fluid loading limit and in this limit we find that

$$R \sim \exp(-i\pi/4) \tag{A14}$$

The expression (2.6) which we obtained for the scattered field demands knowledge of $K_-(-k\cos\theta)$. We have

$$K_-(-k\cos\theta) = \frac{(p + k\cos\theta)(2k\mu)^{-1/2}}{\cos\theta/2}\, K_{1-}(-k\cos\theta)$$

and from A.1

$$K_{1-}(-k\cos\theta) = (-k\cos\theta - p)^{-1} \left(\frac{-k\cos\theta + p}{-k\cos\theta - p}\right)^{\frac{1}{2}\mu P_+(p)} \exp\left\{ -\mu \int_{-k\cos\theta}^{-i\infty} \frac{x\, P_-(x)}{x^2 - p^2}\, dx \right\}$$

$$= i(p + k\cos\theta)^{-\frac{1}{2}}\left\{\frac{p - k\cos\theta}{p + k\cos\theta}\right\}^{-\frac{1}{2}\mu P_+(p)} \cdot (-1)^{\frac{1}{2}\mu P_+(p)} \cdot \exp\left\{-\mu\int_{k\cos\theta}^{i\infty}\frac{t\,P_+(t)}{t^2 - p^2}\,dt\right\} \quad \text{(A15)}$$

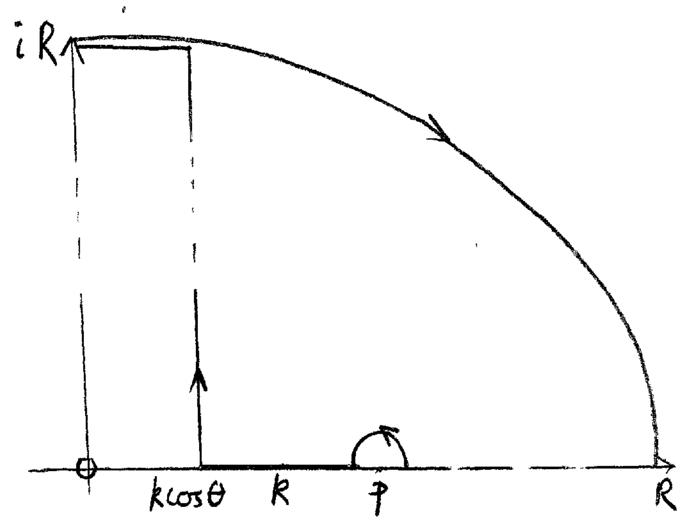


Fig. 3

Figure 3 shows the path of integration and the method of closing the contour. Once again the only pole which concerns us is at p and we avoid this pole as shown. Contributions from the large arc are again of the order $(\ln R)/R^2$, and vanish as $R \to \infty$

The contribution fromthe integral along the real axis is once again of the form $\exp(i\gamma)$, where γ is a real constant. On the real axis, when $\eta > k$ we write

$$P_+(\eta) = \frac{i}{\pi}\cosh^{-1}(\eta/k)(\eta^2 - k^2)^{-\frac{1}{2}}$$

whereas for $\eta < k$ we write

$$P_+(\eta) = \frac{i}{\pi}\cos^{-1}(\eta/k)(k^2 - \eta^2)^{-\frac{1}{2}}$$

The remaining contribution to the integral is from the small semi-circle around the pole. In the limit it is not difficult to show that this contribution is

$$\frac{-i\pi P_+(p)}{2}$$

Writing $(-1)^{\frac{1}{2}\mu P_+(p)} = \exp\left\{-\frac{i\pi\mu P_+(p)}{2}\right\}$

and $$\left(\frac{p-k\cos\theta}{p+k\cos\theta}\right)^{-\frac{1}{2}\mu\, P_+(p)} = \exp\left\{-i\,\ln\left(\frac{p-k\cos\theta}{p+k\cos\theta}\right)\cdot\frac{\cosh^{-1}(p/k)}{2\pi}\right\}$$

we can write
$$K_{1-}(-k\cos\theta) = (p+k\cos\theta)^{-1/2}\exp(i\,\tau_1)$$

and
$$K_-(-k\cos\theta) = \frac{(p+k\cos\theta)^{1/2}\exp(i\tau_2)}{(2k\mu)^{1/2}\cos\theta/2} \qquad \text{(A16)}$$

Equation (2.6) thus implies that the edge scattering has an intensity given by
$$|\phi|^2\frac{\rho\omega^2}{c} = \frac{2F_0^2\mu^6\cos^2\theta/2}{\pi(\rho c r)\,p\,(p+k\cos\theta)(p-k\cos\theta)^2}$$

with the total power
$$\frac{F_0^2\mu^3(p^2+\mu^2+pk)}{2\rho c\;p^3(p+k)}.$$

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CHAPTER 3

INTRODUCTION

There has been a great deal of recent research to study interactions between turbulence generated noise fields and various solid surfaces (Ffowcs-Williams (1965) and (1966), Crighton and Leppington (1970) and Leppington (1972) etc). That work is motivated by aircraft noise problems, although it sometimes finds an underwater application. The theoretical framework is usually founded upon Lighthill's (1952) description of aerodynamic noise generation. It is usual to seek an exact Green's function for the particular geometries rather than use Curle's (1955) general equations that describe the effects of rigid surfaces. That technique was first used by Powell (1960) and later applied by Ffowcs-Williams and Hall (1970) and Leppington (1972) to more complex geometries, all involving simple impedance conditions. In practice, and especially in underwater applications, practical surfaces cannot be regarded as rigid and relevant theories must allow for wall compliancy. In this context we mention the papers by Ffowcs-Williams (1965) and (1966) and Crighton and Leppington (1970). When the compliant wall is capable of sustaining travelling waves the possibility of long range interaction arises. Crighton (1972) has discussed one such case and an exact problem is presented by the author in Chapter 2 of this thesis.

The majority of the work in this field is concerned with situations where the fluid containing the turbulence has no mean flow relative to the boundary surface. Low Mach number mean flow effects have been treated in some cases as small corrections to the no-flow results.

Such a view is probably valid in a large number of applications, but workers are becoming increasingly aware of situations in which the effects of mean flow cannot be represented in this way. (See, for example Orszag and Crow (1970)).

One mean flow effect which cannot be described as a no-flow perturbation is the onset of a flow-driven instability. Recently we have seen experimental work investigating instabilities of shear layers, both in the case of the jet exhaust (Ronneberger (1967) and Crow and Champagne (1971)) and in the flow through a duct (Dean (1972)). In Chapter 1 of this thesis, the author has indicated that instability might always be an important feature whenever a compliant wall interacts with a mean flow. In that work the compliant wall is modelled by an array of vibrating pistons in a rigid baffle, in principle an idea due to Raleigh and recently applied by Ffowcs-Williams (1972).

The analysis is easily extended to include mean flow effects and the possibility of an instability of this type is always present and would give rise to flutter. Great care must therefore be exercised in describing situations with mean flow. Kinetic energy levels in a mean flow can be many orders of magnitude larger than energy levels associated with sound fields and we cannot afford to dismiss any process which is converting that energy to sound however inefficient it may appear to be.

The possible modes of instability are many and various. In a set of papers by Brooke Benjamin (1959, 1960 and 1963) and a companion paper by Landahl (1962), for example, the characteristics of temporally unstable modes on infinite flexible walls have been comprehensively investigated. In particular they isolated a class of wave (class 'A') which exhibits temporal instability due to the action of positive damping. However in practice the relevance of instability problems is not well understood. Investigation of modes does not reveal a sufficiently reliable indication of whether or not a particular mode will be excited in a particular situation. Neither is there a compelling reason for choosing between spatially or temporally growing modes, (thus we may compare the results of Howe (1970) and Jones and Morgan (1972)). Except in a few cases, such as convective flow due to temperature gradients in an otherwise stagnant fluid it is probably more realistic to talk of excitation at a given frequency and to consider spatial growth. The work of Landahl gives a particularly instructive discussion of the energy transfer processes involved and we shall be forced to rely heavily on the results obtained there.

The situation that we wish to describe is relatively straightforward. A harmonic line force, situated in a compliant wall will excite time harmonic surface waves. In certain circumstances, which include the action of positive damping in the wall, a downstream travelling spatially growing wave is generated. In practically interesting situations we may not know the complete details of the disturbances which excite the waves. The practical significance of this model is therefore that it forms a canonical problem in which the interaction of unstable waves with discrete scattering centres produces a secondary sound field.

It is at present an open question whether the potential associated with a spatially growing wave represents a true sound field or not.

At large distances downstream the growth of the waves must be limited by some process. Whether or not a particular limiting process is noisy forms an interesting and novel question. Two contrasting examples may help to clarify this point.

We first consider a rigid sphere moving with a uniform subsonic velocity in an inviscid fluid. This problem which has been treated by Longhorn (1952) is also discussed in Chapter 4. There is a near-field energy associated with the passage of the sphere and we ask; as the sphere is brought to rest does the near field energy represent a sound field or not? It seems that if the sphere is stopped abruptly, all the near field energy is eventually radiated as sound, whereas if the sphere is slowed sufficiently gradually, no sound is radiated.

On the other hand, Crow (1972) has presented a simple model problem based on the experimental results of Crow and Champagne (1971) in which he included a factor exp $(-X^2/b^2)$ to allow for the amplification, levelling off and subsequent decay of the 'preferred' mode (the mode undergoing the greatest amplification). A large sound field is generated although the growth limiting process is exponentially smooth.

The situation is obviously much simpler if scatterers are present. In practice all surfaces are necessarily finite and their edges will act as scattering devices. That is a feature we include in our model.

Despite the simplicity of the physical picture it is still no easy matter to produce a useful mathematical model which, whilst retaining enough physical information, lends itself to straightforward analysis. In discussing a finite panel which is many plate wavelength long an obvious approach is to treat the scattering problem at either end as though the flexible wall were semi infinite. Each of the problems would then involve a two-part Wiener-Hopf calculation, although Crighton and Leppington (1974) have shown that the extension to include flow in general is not trivial and consequently the results of Chapter 2 will not have great relevance here.

A step which brings about a great simplification is the neglect of viscous effects and the assumption that the motion is a linear perturbation about the steady state. Of course unstable modes arise, and according to linear theory grow indefinitely downstream (as long as the flow is parallel)which leads to an inevitable breakdown of the linear model. Since the governing equations are elliptic, departure from linearity may have a significant effect everywhere. Orszag and Crow (1970) and Crighton (1972) have shown however that neglect of this difficulty leads to results which agree with experimental data. There are also several current papers dealing with modes in slowly diverging jets which show growth before they level off and decay, these do not necessarily violate the linearity principle (see e.g. Bouthier (1972) and (1973) and Gaster and Crighton (1974)).

One way of avoiding any such problem is to deal with two conjugate modes. Their relative strengths are set to ensure cancellation of the growing downstream mode by the action of a second surface disturbance of a suitable phase and amplitude placed downstream of the first.

This model will also produce a significant analytic simplification. We would like to solve this forcing problem by Fourier transforms. That technique is satisfactory for problems involving neutrally stable or decaying modes, as the necessary integrals converge at least in the generalised function sense. In most situations involving growing modes the technique breaks down. Jones and Morgan (1972) have solved a causal problem on an infinite vortex sheet using ultradistributions (delta functions of complex argument, these being defined formally by

$$\delta(x+iy) = \sum_{0}^{\infty} \frac{(iy)^n}{n!} \delta^{(n)}(x)$$

See Jones (1966)). Crighton and Leppington (1974) discussed the forcing of growing modes upon a semi-infinite vortex sheet. They employed analytic continuation to solve a harmonic problem, and again found that in real time a solution could only be obtained within the framework of ultradistributions.

Of course the procedure using conjugate modes is very specifically a mathematical tool used in order to manufacture a tractable problem. One would have great difficulty in visualising from it a physical situation of the kind we describe. The essential feature which we are incorporating in our model is the finite length of surface that supports a growing wave. The remainder of the plane surface carries only residual exponentially decaying modes and algebraically decaying near fields. From the model we believe we can isolate the essential properties to be expected in the physical problem where flow and sound interact with a definite bounded section of an otherwise rigid boundary. The model is significant, we think, because it transpires that the surface instabilities that are in general weakly coupled to sound are extremely well coupled by surface discontinuities. The physical process of flow driven instabilities formed on compliant surfaces with exponentially growing energy being coupled effectively to the sound heralds the possibility that sound absorbent walls of flow carrying ducts can lead to strong noise fields that are not present in the rigid wall case.

THE BASIC EQUATIONS AND A DISCUSSION ON THE NATURE OF THEIR SOLUTION

We consider an inviscid fluid occupying the half space $y > o$. The fluid flows parallel to the x-axis, at a subsonic speed U. and is bounded by a locally reacting compliant wall whose impedance is given by

$$Z(\omega) = \frac{K - m\omega^2}{-i\omega} + \beta \tag{1.1}$$

at the frequency ω.

[This simple choice of impedance condition may appear to be unrealistic. It is necessary because even in the problem without flow, determination of the free wave numbers for a "practical" surface such as a fluid loaded plate or membrane is no easy matter. One impedance condition which does lend itself to easy manipulation is that of a locally-reacting compliant wall whose impedance is given by (1.1). Physically we interpret m as a mass/unit area, whilst K represents a restoring force/unit area. β is a small, strictly positive, term which represents damping. In the absence of fluid, of course, such a wall could not support a wave, but provided that fluid is present it is fairly easy to show that waves exist whenever

$$K > m\omega^2 \tag{1.2}$$

ie. provided the excitation is being carried out below the in vacuo local resonance frequency.]

Two harmonic line forces, of frequency exp $(-i\omega t)$ are situated in the wall at $x=-h$ and $x=o$ and they excite disturbances in both the wall and the fluid. Assuming that linearisation is valid everywhere, the equations to be satisfied by the potential $\phi(x,y,t)$ and the surface displacement $\eta(x,t)$ are the convected wave equation within the bulk of the fluid

$$\nabla^2\phi = \frac{1}{c^2}\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)^2\phi \tag{1.3}$$

plus two boundary conditions, to be applied at the mean position of the wall $y=o$. They are the continuity of normal displacement

$$\frac{\partial\phi}{\partial y} = \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\eta$$

and the pressure condition

$$\rho\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) + F_o\,\delta(x+h)\exp(-i\omega t) + Q_o\,\delta(x)\exp(-i\omega t) = Z\frac{\partial\eta}{\partial t} \qquad (1.4)$$

In these equations ρ represents undisturbed fluid density, c is the speed of sound (relative to the static fluid) and Fo is the strength of the line force at $X=-h$. Q_o, the strength of the other line force, is to be determined as a function of F_o.

We assume a time dependence exp(-iωt) throughout. The equations are formally solved by taking Fourier transforms in the X-direction. use of the inverse Fourier transform leads without any algebraic difficulty to the solution

$$\phi(x,y,t) = \frac{\exp(-i\omega t)}{2\pi\rho i}\int_{\alpha=-\infty}^{\infty}\frac{(F_o e^{-i\alpha h}+Q_o)\,e^{-i\alpha x}e^{-\bar{\omega}(\alpha)y}(\omega+U\alpha)}{(\omega+U\alpha)^2-\frac{\omega^2\bar{\omega}(\alpha)}{\mu}}\,d\alpha \tag{1.5}$$

and

$$v(x,t) = \frac{\partial\eta}{\partial t} = \frac{\omega i\exp(-i\omega t)}{\rho 2\pi}\int_{\alpha=-\infty}^{\infty}\frac{(F_o e^{-i\alpha h}+Q_o)\,e^{-i\alpha x}\,\bar{\omega}(\alpha)}{(\omega+U\alpha)^2-\frac{\omega^2\bar{\omega}(\alpha)}{\mu}}\,d\alpha \tag{1.6}$$

Here we have introduced the notation

$$\mu = \frac{\rho i\omega}{z} \tag{1.7}$$

and

$$\bar{\omega}^2(\alpha) = \alpha^2 - \frac{(\omega+U\alpha)^2}{c^2}. \tag{1.8}$$

The convergence of the integral at large distances downstream is ensured by the choice of Q_o, a point which we discuss later. We first examine the nature of the possible modes by considering the zeros of the kernel of the integrands in (1.5) and (1.6)

The branch of $\bar{\omega}(\alpha)$ is chosen to ensure that the potential obeys a radiation condition away from the wall. Formally this branch can be chosen by the following procedure. If we make the Lorentz transformation defined by

$$\begin{pmatrix}\alpha'\\ \Omega\end{pmatrix} = \begin{pmatrix}\Delta & -M/c\Delta\\ 0 & \Delta^{-1}\end{pmatrix}\begin{pmatrix}\alpha\\ \omega\end{pmatrix} \tag{1.9}$$

where $\Delta^2 \equiv 1 - U^2/c^2$, $\Delta > 0$,

then the expression for $\bar{\omega}^2(\alpha)$, (1.8), is transformed into

$$\bar{\omega}^2(\alpha) = \alpha'^2 - \Omega^2/c^2$$

Following the notation of Noble (1958), this can be written as $\gamma^2(\alpha')$ and Noble discusses the selection of the branch of $\gamma(\alpha')$ in order to satisfy a radiation condition at infinity. In future the symbol $\bar{\omega}(\alpha)$ implies that branch of $\bar{\omega}(\alpha)$ chosen according to this procedure.

An estimate of the distant plate disturbance (outgoing travelling waves) depends upon the zeros of the denominator of the integrand in (1.6). Defining

$$K(\alpha) = (\omega + U\alpha)^2 - \frac{\omega^2 \bar{\omega}(\alpha)}{\mu} \tag{1.10}$$

we require the zeros of $K(\alpha)$. Despite the simplifying choice of the wall impedance an exact solution for the zeros of $K(\alpha)$ will be far from trivial.

We see that zeros again occur for $\mu > 0$, corresponding once again to the condition (1.2) that we found for the existance of 'no-flow' waves. In discussing the modes of the surface disturbance, a useful approximation is given by formally allowing the sound speed to become infinite, since the outgoing travelling waves will be only slightly affected by compressibility.

The zeros of $K(\alpha)$ are then α_i $(i=1,..4)$ where,

$$\left.\begin{aligned} \alpha_1 &= \frac{\omega}{U}\left\{-(1+\varepsilon) - (\varepsilon^2+2\varepsilon)^{1/2}\right\} \\ \alpha_2 &= \frac{\omega}{U}\left\{-(1+\varepsilon) + (\varepsilon^2+2\varepsilon)^{1/2}\right\} \end{aligned}\right\} \tag{1.11}$$

and

$$\left.\begin{aligned} \alpha_3 &= \frac{\omega}{U}\left\{-(1-\varepsilon) - (\varepsilon^2-2\varepsilon)^{1/2}\right\} \\ \alpha_4 &= \frac{\omega}{U}\left\{-(1-\varepsilon) + (\varepsilon^2-2\varepsilon)^{1/2}\right\} \end{aligned}\right\} \tag{1.12}$$

with the notation $\varepsilon = \dfrac{\omega}{2\mu U}$. (1.13)

The modes given by (1.12) correspond to upstream travelling waves and are only valid when

$$\varepsilon > 2.$$

Physically this condition indicates that waves cannot travel against the flow whenever its velocity is greater than a maximum critical velocity, Umax, defined by

$$U < U_{max} = \frac{\omega}{4\mu}. \tag{1.14}$$

The modes given in (1.11) correspond to downstream travelling modes and are valid for all flow speeds. In the absence of damping all these waves are neutrally stable and consideration of the integral (1.6) indicates that all modes are excited by a purely harmonic source.

Inclusion of the damping, which means that we must now write

$$\varepsilon = \varepsilon_R + i\varepsilon_I , \quad \varepsilon_I < 0 \tag{1.15}$$

shows that the waves represented by the wavenumber α_1, and α_4 grow in their respective directions of travel, whereas the other two modes will decay. Our physical insight of the problem indicates that although we might expect a growing wave to travel downstream, we would not expect a growing wave to travel against the flow, as this would contradict energy principles. It would be extremely convenient if we were able to solve a causal problem exactly but the formal analysis required is extremely complicated. We can however give a strong indication that causality will lead to the exclusion of the upstream travelling mode by reconsidering the problem with a loss-less wall.

Lighthill (1964) has shown that wave energy will be found along the line ℓ if an only if the component of the group velocity along ℓ is positive. [The result depends on the direction in which a pole, originally lying on the real axis, moves when we introduce the complex wavenumber, $k=k_1+ik_2$, $k_2/k_1 \ll 1$]

The group velocity for the α_4 mode is given by

$$\frac{\partial \omega}{\partial \alpha_4} = \left(\frac{\partial \alpha_4}{\partial \omega}\right)^{-1} \tag{1.16}$$

and bearing in mind that $\varepsilon = \varepsilon(\omega)$, so that

$$\frac{\partial \varepsilon}{\partial \omega} = -\frac{K+m\omega^2}{2\rho U\omega^2},$$

we obtain

$$\frac{\partial \alpha_4}{\partial \omega} = \frac{\alpha_4}{\omega}\left\{1 - \frac{K+m\omega^2}{2\rho U\omega(\varepsilon^2-2\varepsilon)^{1/2}}\right\} \tag{1.17}$$

now

$$\frac{K+m\omega^2}{2\rho U\omega(\varepsilon^2-2\varepsilon)^{1/2}} > \frac{K+m\omega^2}{2\rho U\omega\varepsilon} = \frac{K+m\omega^2}{K-m\omega^2} > 1$$

so that

$$\frac{\partial \alpha_4}{\partial \omega} = \frac{\alpha_4}{\omega} \text{(negative quantity)}$$

Thus we have given some indication that the α_4 mode will not be excited, although, of course, Lighthill's argument only applies to neutrally stable or decaying modes.

[We must, of course, also ensure that this causality argument does not exclude any of the other modes. It is easily seen that the decaying modes must always be included in a causal problem. The α_1 mode, however, will only obey causality if

$$U > U_{min} = \frac{K m \mu}{\omega \rho^2} \tag{1.18}$$

It may be of interest to ask whether the conditions (1.14) and (1.18) can be satisfied simultaneously. If we define the in vacuo resonance frequency ω_o such that

$$K = m\omega_o^2$$

and then define the non-dimensionalised radiation frequency by

$$\omega = \lambda\omega_o \qquad (0 < \lambda < 1)$$

then the conditions become

$$\frac{1-\lambda^2}{\lambda} > \frac{m\omega_o}{\rho U}$$

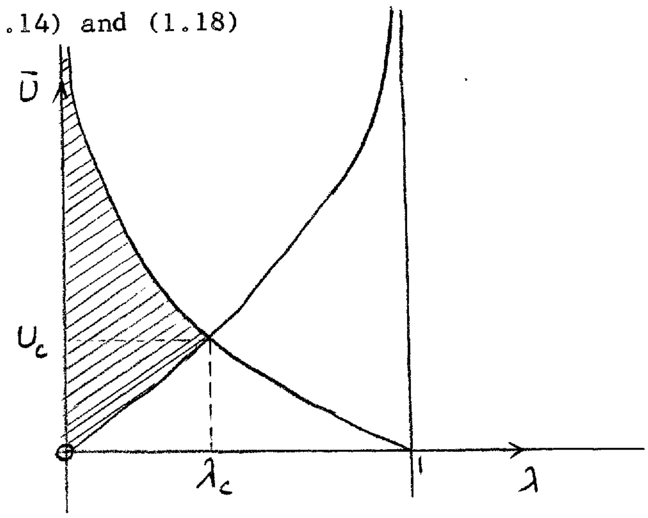
and

$$\frac{m\omega_o}{\rho U} > \frac{4\lambda}{1-\lambda^2}$$

We must have, therefore,

$$\frac{1-\lambda^2}{\lambda} > \frac{4\lambda}{1-\lambda^2} \Longrightarrow \lambda < \lambda_c = \sqrt{2} - 1 \; ; \; \bar{U}_c = \tfrac{1}{2}.$$

Figure 1 shows the possible values of $\bar{U} = \frac{\rho U}{m\omega_o}$, corresponding to both the conditions (1.14) and (1.18)



It is perhaps worth pointing out that the nature of the surface disturbances, as indicated by the modes α_1, α_2, α_3 are qualitatively consistent with the class 'A' and Kelvin-Helmholtz waves of Brooke-Benjamin and Landahl.]

In order to make progress then we must assume that the contribution from the α_4 root can be safely ignored. The choice of the line force strength Q_0 is such as to cancel the contribution from the α_1 mode at large x. It is easy to see that this is achieved by setting

$$Q_0 = -F_0 e^{-i\alpha_1 h} \tag{1.19}$$

The integrals (1.5) and (1.6) can now be regarded as convergent. Estimating the surface velocity from expression (1.6) in the normal way leads to

$$v(x, t) = \frac{F_0 \omega}{\rho U^2} e^{-i\alpha_1(x+h)} \frac{\alpha_1}{\alpha_1 - \alpha_2} e^{-i\omega t}\left[H(x+h) - H(x)\right]. \tag{1.20}$$

plus exponentially decaying waves corresponding to the α_2 and α_3 modes and near field algebraically decaying fields. In order to emphasise the growing nature of the wave we write

$$\alpha_I = -\alpha_{1R} + i\alpha_{1I} \; ; \; \alpha_{1R} > 0 \; ; \; \alpha_{1I} > 0. \tag{1.21}$$

and

$$v(x,t) = \frac{F_0 \omega}{\rho U^2} \cdot \frac{\alpha_1}{\alpha_1 - \alpha_2} e^{\alpha_{1I}(x+h)} \cdot e^{i(\alpha_{1R}(x+h) - \omega t)} \cdot \left[H(x+h) - H(x)\right] \tag{1.22}$$

THE RADIATION FROM THE LINE FORCES, AND AN OBSERVATION ON THE DIRECTION OF ENERGY TRANSFER

We first consider the sound field being radiated by the single force at That field is given by an estimate of the expression (1.5) for large distances from the force. We retain the flow effects, and by employing the transformation indicated in (1.9) find that the expression for the potential becomes

$$\phi(x,y,t) = \frac{F_0 e^{-i\omega t} e^{iMk(x+h)/\Delta^2}}{2\pi\rho i} \int \frac{e^{-i\alpha'(x+h)/\Delta}\, e^{-\gamma(\alpha')y}\,(\Omega' + U\alpha')}{(\Omega + U\alpha')^2 - \frac{\Omega^2 \Delta^4 \gamma(\alpha')}{\mu}}\, d\alpha'. \quad (2.1)$$

We evaluate this integral by stationary phase in the manner indicated by Noble. In deforming the contour we will cross a pole corresponding to the (growing) downstream mode. Taking due account of this contribution, the sound field is given by

$$\phi_{rad} = \frac{F_0 e^{iMk\frac{r'\cos\theta'}{\Delta}} (2k'\pi)^{1/2} e^{ik'r'} e^{-i\pi/4} (\Omega' - Uk'\cos\theta')\sin\theta'}{2\pi\rho i \; r'^{1/2} \left\{(\Omega' - Uk'\cos\theta')^2 + \Omega^2\Delta^4 ik'\sin\theta'/\mu\right\}} e^{-i\omega t}. \quad (2.2)$$

The polar co-ordinates (r', θ') are related to a stretched x-co-ordinate given by

$$\frac{x+h}{\Delta} = r'\cos\theta' \quad ; \quad y = r'\sin\theta' \quad (2.3)$$

Rewriting (2.2) in terms of the conventional polar co-ordinates (r,θ) based on the position of the force, and retaining terms correct to order M, we obtain

$$\phi_{rad} = \frac{\sin\theta\, F_0 \mu\, e^{ikr(1+M\cos\theta)}\, e^{-i\omega t - i\pi/4}(1 - M\cos\theta)}{(2\pi)^{1/2}\rho i c\,(kr)^{1/2}\left\{\mu(1-M\cos\theta)^2 + ik\sin\theta\right\}} \quad (2.4)$$

Examination of the result shows that it is simply that radiation which would have been obtained without flow, modified by the usual flow convection factors.

The radiation from the line force at the origin is given in an identical fashion and is

$$\phi_{rad} = \frac{-F_0 e^{-i\alpha_{1R}h} e^{\alpha_{1I}h} \mu\, e^{ikr_0(1+M\cos\theta_0)} \sin\theta_0 (1 - M\cos\theta_0) e^{-i\frac{\pi}{4}} e^{-i\omega t}}{(2\pi)^{1/2}\rho i c (kr_0)^{1/2}\left\{\mu(1-M\cos\theta_0)^2 + ik\sin\theta_0\right\}} \quad (2.5)$$

where (r_0, θ_0) are polar co-ordinates based on the origin.

The result (2.5) is essentially that which we have been striving to achieve.

The radiation from the downstream end of a growing plate wave is greater, by a factor exp(growth rate x length of plate), than the radiation from the upstream end. We visualise that as the wave grows it is trapping energy from the mean flow, the increase being dependent on the growth rate of the waves. The force at the downstream end merely ensures that this energy does not continue to travel indefinitely in the wall and its adjacent fluid layer. It remains for us to show that the energy in the radiation field is supplied by the fluid, rather than by the support. At the downstream support, the energy which is being lost by the plate wave could be simply transferred back into the mean flow, with the force actually supplying energy to support this process and also to drive the radiation field.

To clarify this important question of the transfer of energy we note that energy crossing <u>into</u> the fluid from the wall is given by

$$Re \int_{S(x)} (\text{pressure}) \left(\frac{\partial \eta}{\partial t}\right)^* dx . \tag{2.6}$$

where S(x) represents the whole of the wall.

The pressure near the downstream force is due to two independent effects, the pressure from the incident travelling wave p_{inc} and that pressure which is induced locally by the force itself, p_{ind}. Similarly the velocity near the force comprises two terms the incident velocity due to the travelling wave v_{inc} and the velocity induced by the force itself, v_{ind}.

We have the relationships

$$\left.\begin{aligned} -Z v_{inc} &= p_{inc} \\ -F_0 e^{-i\alpha_1 h} \delta(x) e^{-i\omega t} - Z v_{ind} &= p_{ind} \end{aligned}\right\} \tag{2.7}$$

and thus the energy transfer into the fluid is given by

$$Re \int \left\{ -Z(v_{inc} + v_{ind}) - F_0 e^{-i\alpha_1 h} \delta(x) e^{-i\omega t} \right\} \left\{ v_{inc} + v_{ind} \right\}^* dx$$

$$= -\beta \int_S |v_{inc} + v_{ind}|^2 ds - F_0 Re\, e^{-i(\alpha_1 h + \omega t)} \int_{x=-\infty}^{\infty} \delta(x) \left\{ v_{inc} + v_{ind} \right\}^* dx . \tag{2.8}$$

The first term represents the energy loss by the fluid in overcoming the mechanical damping in the wall. The remaining term represents the energy transfer locally. The contribution to the energy transfer from the term v_{inc}^{*} is

$$\frac{-F_o^2\omega}{\rho U^2} e^{2\alpha_{1I}h} Re\left\{\frac{\alpha_1}{\alpha_1-\alpha_2}\right\}. \qquad (2.9)$$

We may as well ignore the effect of the damping, except in the exponential growth rate factor, and write this as

$$-\frac{F_o^2\omega}{\rho U^2} e^{2\alpha_{1I}h} \frac{\alpha_{1R}}{\alpha_{1R}-\alpha_{2R}} . \qquad (2.10)$$

The contribution to the energy transfer from the term v_{ind}^{*} is somewhat more difficult to evaluate. The expression for v_{ind} is

$$v_{ind} = \frac{-F_o e^{-i\alpha_1 h} \omega i e^{-i\omega t}}{2\pi\rho}\int \frac{e^{-i\alpha x}\,\bar{\omega}(\alpha)}{(\omega+U\alpha)^2 - \frac{\omega^2\bar{\omega}(\alpha)}{\mu}}\,d\alpha \qquad (2.11)$$

and we require the behaviour of this integral expression for small x. The method we use to evaluate this integral for small u was first suggested by Crighton (1972(3)). The integrand in (2.11) is split, in a manner familiar in the Wiener-Hopf technique, into a sum of functions analytic in upper and lower half-planes, and the response near the site of the force is then readily obtained by the behaviour of the split functions in their respective domains of analyticity.

One fundamental difference is that the presence of the flow removes the δ function singularity found by Crighton (see equ.(7) p.211) at the site of the force. This is due to the replacement of a factor ω^2 by a factor $(\omega+U\alpha)^2$ in the denominator of our integrand. Thus the addition of flow to a response problem of the kind is a singular perturbation, since in the absence of flow there is an apparent singular energy transfer at the site of the force.

Another feature of the flow is that we now no longer expect the response to be symmetric about the force.

As in the argument leading up to the neglect of the upstream travelling wave, the analysis here will be somewhat suspect, as it is relying heavily upon the use of generalised function theory in an application for which it was not intended. The analysis is valid in the limit of zero damping and we must assume that once again we may use results obtained in this limit.

The procedure then, following Crighton, is mechanical and leads to the result

$$\left.\begin{aligned} \underset{x \to 0^+}{\mathrm{lt}}\, v_{ind}(x,t) &= -\frac{F_0 e^{-i(\alpha_1 h + \omega t)}}{\rho U^2\,\omega i}\left\{A \ln x + B\right\}. \\ \underset{x \to 0^-}{\mathrm{lt}}\, v_{ind}(x,t) &= \frac{-F_0 e^{-i(\alpha_1 h + \omega t)}}{\rho U^2\,\omega i}\left\{A \ln(-x) + C\right\} \end{aligned}\right\} \quad (2.12)$$

and the point to note here is that A is purely real and so plays no part in the energy transfer computation. Similarly it is only the imaginary part of B, C that is of interest and close inspection reveals that $B = i + B_R$; $C = i + C_R$, leading to an expression for the energy transfer of

$$\frac{-F_0^2\,\omega}{\rho U^2}\, e^{2\alpha_{1I} h} \qquad (2.13)$$

Thus we have indicated that at the downstream end of the wall energy is actually being transferred into the support from the fluid, which indicates in particular that it is the mean flow which is driving the radiation field.

CONCLUSIONS

The aims of this paper have not been to attempt to describe a particular physical situation. They have been directed towards emphasising that a mean flow induced instability coupled with a wave scattering process represents a fundamental conversion of flow energy into sound energy. Even in this, a most idealised problem, the mathematical analysis is far from straightforward.

The firmest conclusion that one can draw from this work along with papers concerning instabilities on jets is that one cannot afford to ignore instability problems. There is now, for example, enough clear theoretical and experimental evidence to indicate that at frequencies and angles characteristic of 'jet mixing noise' the jet instability can act as a 30dB amplifier of tailpipe disturbances (Crow (1972)). We can therefore see the importance of developing the theory of ultradistributions which will allow us to treat problems with unstable modes in the same way that generalised functions allow us to treat neutrally stable modes.

In particular in this problem it is apparent that the instability is dependent upon the damping in the wall, and indeed the larger the damping, the larger the growth rate of the waves and the radiated field. Unfortunately large damping in compliant walls is exactly the requirement for acoustic energy absorption as used, for example, in the acoustic lining of ducts. Obviously the significance of these two contrasting trends will be difficult to quantify. As an example though we may compare the radiated power from the downstream edge of the plate with the energy lost in overcoming the damping in the equivalent problem without flow.

The power radiated from the downstream edge in the high fluid loading limit can be obtained from the expression for the scattered potential (equation (2.5)) in the appropriate limit, and is

$$P = \frac{F_0^2 \omega}{\rho c^2} \exp\left\{2\alpha_I h\right\}$$

The energy lost through internal damping can be evaluated from the equation (4.6) of chapter II. We find that the energy loss for a plate length h is

$$\frac{F_0^2 \omega}{\rho c^2} \exp\left\{-2\mu_I h\right\} \frac{\mu^2}{k^2}$$

Thus there is a parameter

$$\frac{k^2}{\mu^2} \exp\left\{ 2(\alpha_{1I} + \mu_I) \right\} h \}$$

which, if > 1 means that more energy is generated than absorbed by a compliant plate of this type.

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CHAPTER 4

INTRODUCTION

There has been suprisingly little attention paid to the details of the source process responsible for impact noise although the topic is obviously one of great practical importance. No doubt this is due in part to the extreme complexity of most impulsive problems, and also that the impulse itself is often not the immediate source of acoustic waves. They are usually generated as a result of impulsively excited structural vibration. There are at least three distinct sources contributing to the total noise field. The first is the ringing due to resonant vibration of the impacting bodies. Immediately prior to impact, the action of viscous forces cannot be neglected, and we recognise that this leads to a second source concentrated on the body surfaces. The final term is that due to the discontinuous change during impact in both the surface velocity and pressure. It is this final term, which we shall call impulse noise, that is investigated here. This term is not necessarily the dominant one in the total noise field. We concentrate on it because this impulse noise forms an irremovable contribution to the total field and as such forms a lower bound on the noise of impulsive body motions. Furthermore our approximation technique by which this noise can be determined may well have significance in other problems. For example we would expect it to hold in situations where the body dimension is large on the important acoustic scales, as is inevitable in high speed aerodynamic flows.

Curle (1955) showed how the pressence of solid surfaces modified the sources identified in Lighthill's (1952) general theory of aerodynamic noise generation. Application of Curle's work, however, is dependent on a knowledge of the (local) surface pressure and velocity fields and this, in turn, is dependent on a solution being available to the equations of fluid motion, or that the surface terms be known from experiment. Alternatively to Curle's technique we require knowledge of the Green's function for the particular geometry. With that, Lighthill's theory can be used to write down the field exactly. Neither of these schemes is susceptible to simple analysis, except for the very simplest of geometries.

We propose here an approximation scheme which enables us to find this local pressure in the grossly non compact limit, concentrating on the impulsive case. Since the changes which then take place do so instantaneously, ray theory, a high frequency approximation, will be relevant during the period immediately following impact. Since it is events at this time which control the sound field we expect the approximation to be an accurate one. Two schemes are considered: one assumes that the surface is locally plane and the other that the surface is locally curved. In this second example the surface pressure is approximated to by that pressure which would have been induced had the surface instantaneously been part of a radially growing sphere of the same radius of curvature.

The impulsive motion of a sphere is one of the few problems which can be solved exactly (Taylor (1971) and Longhorn (1952)). Comparison of these exact results with those obtained through our approximation schemes provided a useful check on their accuracy. The schemes are shown to be as accurate as is likely to be needed in most practical applications of impulsive sound sources.

In the final chapter a brief description is given of a crude experiment which was performed using 'model' piledrivers. The results appear to verify the linear, plane ray theory estimate of the pressure pulse.

When a body moves unsteadily through an inviscid but compressible medium it is subject to a drag force opposing the motion. In discussing the field induced by an impulsively accelerated sphere we recognise two distinct model situations; in the first an additional external force is supplied to balance the drag exactly so that the subsequent motion of the sphere is steady. In the second there are no external forces so that the subsequent motion is to be determined allowing for the retarding effects of drag.

Longhorn (1952) discussed a problem of the first kind and for easy accessibility we reproduce some of his results. He gave the velocity potential, ϕ, measured in spherical polar co-ordinates (r, θ, κ) fixed in the centre of the sphere as:

$$\phi(r,\theta,\kappa,t) = \frac{Ua^3\cos\theta}{2r^2}\left\{\exp\left(-\frac{c\tau}{a}\right)\left(\cos\frac{c\tau}{a} + \frac{2r-a}{a}\sin\frac{c\tau}{a}\right) + 1\right\} H(\tau). \tag{1.1}$$

U is the steady velocity of the sphere, a, its radius and c is the speed of sound in the stagnant fluid. τ is the retarded time;

$$\tau = t - \frac{r-a}{c} \tag{1.2}$$

Differentiation of (1.1) with respect to r confirms that the potential satisfies the condition

$$\frac{\partial\phi}{\partial r} = U\cos\theta\, H(t)$$

at the spherical boundary, r=a.

The pressure in the fluid is given by the linear term in Bernouilli's equation as,

$$p = -\rho\frac{\partial\phi}{\partial t}, \tag{1.3}$$

where ρ is the mean fluid density. The radiating part of the sound field is given by that part of (1.3) which decays as r^{-1} as $r \to \infty$, i.e.

$$p_{sound} = \frac{\rho U a c \cos\theta}{r}\left\{\cos\frac{c\tau}{a} - \sin\frac{c\tau}{a}\right\}\exp\left(-\frac{c\tau}{a}\right)H(\tau). \tag{1.4}$$

The total radiated energy is found by integrating the acoustic intensity over a distant sphere and all time, and those integrals can be evaluated to give

$$E = \frac{\rho U^2 \pi a^3}{3} \quad . \tag{1.5}$$

It is already apparent that this impulsive source of sound is highly efficient, because the sound energy is not dependent upon the flow Mach number. In fact all the change in the virtual energy in the flow around the body is radiated as sound.

The problem in which the drag force is allowed to retard the sphere after its initial impulsive acceleration has been solved by Taylor (1971) in the particular case of equal fluid and sphere densities. The velocity potential for the general problem of a solid sphere moving with varying speed U(t) is

$$\phi = \frac{-ac\cos\theta}{r^2} \int_0^t U(t-s) \exp\left\{\frac{-c}{a}\left(s - \frac{r-a}{c}\right)\right\} \cdot \left\{ r\cos\frac{c}{a}\left(s - \frac{r-a}{c}\right) + (a-r)\sin\frac{c}{a}\left(s - \frac{r-a}{c}\right)\right\}. \tag{1.6}$$

as a result which was given by Longhorn. Once again the pressure is obtained through Bernouilli's equation and is then used to determine the subsequent motion of the sphere. In fact the equation of motion is

$$\frac{\partial U}{\partial t} H(t) = -\frac{\rho c}{\rho_0 a}\left\{ U_0 \exp\left(-\frac{ct}{a}\right)\cos\left(\frac{ct}{a}\right) H(t) + \int_0^t \frac{\partial U}{\partial s} \exp\left\{-\frac{c}{a}(t-s)\right\}\cos\frac{c}{a}(t-s)\, ds\right\}. \tag{1.7}$$

where ρ_0 is the density of the sphere and U_0 its initial speed. The equation yields the solution

$$U(t) = \frac{U_0}{1+\beta}\left[1 + \left\{\frac{2}{1-\beta}\right\}^{\frac{1}{2}} \beta \exp\left\{-\frac{ct}{a}(1+\beta)\cos\left\{\frac{\lambda ct}{a} + \delta\right\}\right\}\right], \tag{1.8}$$

where $\beta = \rho/2\rho_0 \quad ; \quad \lambda = +(1-\beta^2)^{\frac{1}{2}}$

and $\cos\delta = \left(\frac{1-\beta}{2}\right)^{\frac{1}{2}} \quad ; \quad \sin\delta = \left(\frac{1+\beta}{2}\right)^{\frac{1}{2}}$ (1.9)

The sphere's speed actually decays to a value $\frac{U_0}{1+\beta}$, and does not as Longhorn erroneously stated come to rest.

Once we have the equation of motion (1.8), the sound pressure and total radiated energy follow simply. In fact we find that

$$E = \frac{\rho a^3 U_0^2 \pi}{3} \cdot \frac{1}{1+\beta} . \tag{1.10}$$

The limit $\beta \to \infty$ corresponds to the limit of an infinitely heavy sphere. In that case we would expect the (finite) drag to have no effect on its motion and that it simply continues at its starting velocity (see equ. (2.6)). In fact the limit $\beta \to \infty$ completely reproduces Longhorn's steady case and this is a feature which will be exhibited by each of our future approximation schemes.

We note also that the final kinetic energy of the sphere is

$$\frac{2\pi}{3} \rho_0 a^3 U_0^2 (1+\beta)^{-2}$$

compared to the initial value of

$$\frac{2\pi}{3} \rho_0 a^3 U_0^2$$

The remaining energy has been partitioned in amounts

$$\frac{\rho a^3 \pi U_0^2}{3(1+\beta)} \qquad \frac{\rho a^3 \pi U_0^2}{3(1+\beta)^2}$$

between the sound field and the energy stored as local or virtual kinetic energy. This local energy is the kinetic energy associated with the 'virtual mass' of the sphere. If, once the sphere has settled down to its final speed, $U_0(1+\beta)^{-1}$, it is impulsively stopped then all this local energy will be radiated as sound. The ratio of the radiated energy to the near field energy is unity only in the limit $\beta \to \infty$, which is the special case when the sphere continues to travel at its initial speed.

APPROXIMATION SCHEMES: THE IMPULSIVE MOTION OF A SPHERE

We will first discuss the accuracy of two approximation schemes which we test by application to the known problem of radiation from a single moving sphere.

Lighthill (1952) rewrote the equations of fluid motion in a form which isolated the sources of aerodynamic sound. Curle (1955) and Powell (1960) have extended this theory to include the effects of solid boundaries, work which has been generalised by Ffowcs-Williams and Hawkings (1969) to surfaces moving arbitrarily. In particular Curle's result for the perturbation pressure, when only the linear boundary terms are retained is

$$(p-p_0)(\underline{x},t) = -\frac{\rho}{4\pi}\int_S l_i \left[\frac{\partial v_i}{\partial t}\right]\frac{dS(\underline{y})}{|\underline{x}-\underline{y}|} + \frac{1}{4\pi}\frac{\partial}{\partial x_i}\int_S \left[p_i\right]\frac{dS(\underline{y})}{|\underline{x}-\underline{y}|} \qquad (2.1)$$

The familiar Lighthill quadrupole term has also been disregarded as being negligible. p_i is the force per unit area exerted on the fluid by the surface in the i direction.

The square brackets imply that the contents are to be evaluated at the retarded time $t - \frac{r}{c}$.

In the problem concerning the motion of a sphere, both the surface velocity and pressure are known exactly and their use in equation (2.1) leads to the exact expression for the pressure field.

We propose now, as an approximation scheme, that the surface pressure be estimated according to linear ray theory. There, the velocity normal to the surface, which alone determines the wave field, is taken as part of a high frequency acoustic motion and set equal to $p/\rho c$. This is a scheme that will be precisely correct for sufficiently rapid accelerations and we wish to determine its usefulness in cases where the boundary motion is started impulsively.

Accordingly we set

$$p = \rho c v_n \qquad (2.2)$$

and this leads to an expression for the radiated pressure

$$p_{sound} = \frac{\rho}{4\pi}\frac{\partial}{\partial t}\int_S l_i [v_i]\frac{dS}{|\underline{x}-\underline{y}|} - \frac{\rho c}{4\pi}\frac{\partial}{\partial x_i}\int_S l_i [l_j v_j]\frac{dS}{|\underline{x}-\underline{y}|} \qquad (2.3)$$

Green's theorem allows us to rewrite this integral as a volume integral throughout the interior of the sphere V.

We can fix the axes so that the motion of the sphere is in the 1 direction in which case throughout the sphere we have

$$v_i = \delta_{1i}\, U\, H(t) \tag{2.4}$$

where δ_{ij} is the Kronecker symbol. Thus the expression for the radiated pressure becomes

$$p_{sound} = \frac{\rho U}{4\pi} \frac{\partial}{\partial t} \int_V \frac{\partial}{\partial y_1} \left[H(t) \right] \frac{dV(y)}{|x-y|}$$

$$- \frac{\rho c U}{4\pi} \frac{\partial}{\partial x_i} \int_V \frac{\partial}{\partial y_i} \left[\frac{y_1}{|y|} H(t) \right] \frac{dV(y)}{|x-y|}$$

The sound field is obtained by taking the far-field limit as $|x| \to \infty$ when variations of $|x-y|^{-1}$ are negligible. Similarly we can replace the derivative

$$\frac{\partial}{\partial x_i}$$

by

$$- \frac{x_i}{|x|} \frac{1}{c} \frac{\partial}{\partial t}.$$

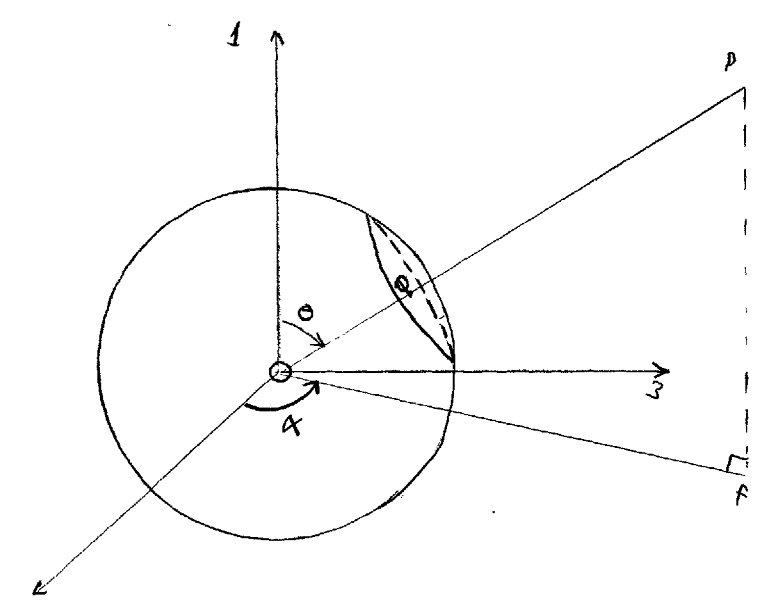

FIG. 1

The integration over the sphere is carried out as shown in Figure 1. We consider a disc whose centre is Q lying perpendicular to the line OP joining the centre of the sphere O to the observation point P. Any point within the disc is defined by its polar co-ordinates (ℓ, λ) relative to the centre, the reference line $\lambda = 0$ lying in the plane OPF.

The sphere travels in the positive 1 direction and θ and α are the spherical polar co-ordinates of the observation point P. If the distance OQ is s, then the range of integration is

$$\left.\begin{aligned} &0 \le \lambda \le 2\pi \\ &0 \le \ell \le (a^2 - s^2)^{1/2} \\ &-a \le s \le a \end{aligned}\right\} \qquad (2.6)$$

The advantage of performing the integral in this form is that since the observer is many sphere diameters from the surface, we can regard the retarded time as constant throughout the disc and in fact to be taken at

$$t - \frac{|\underline{x}| - s}{c}.$$

The direction cosines are

$$\left.\begin{aligned} y_1 &= s\cos\theta - \ell\cos\lambda\sin\theta \\ y_2 &= \sin\alpha\,(s\sin\theta + \ell\cos\lambda\cos\theta) - \ell\sin\lambda\cos\alpha \\ y_3 &= \cos\alpha\,(s\sin\theta + \ell\cos\lambda\cos\theta) + \ell\sin\lambda\sin\alpha \end{aligned}\right\} \qquad (2.7)$$

The integrals are then straightforward and yield the following approximate forms for the sound field

$$p_{\text{sound}} = \frac{\rho U\cos\theta\, c}{2|\underline{x}|}\left(ct - |\underline{x}|\right)\left(\frac{ct - |\underline{x}|}{a} - 1\right) H\left(a - |ct - |\underline{x}||\right). \qquad (2.8)$$

This corresponds to a total energy radiation of

$$E = \frac{\rho\pi a^3 U^2}{3} \cdot \frac{16}{15}. \qquad (2.9)$$

Thus the estimate for the total power is too high by a factor of only 16/15. Comparison of equations (2.8) and (1.3) shows that our approximation predicts the correct 'switch-on' value of the pressure field. However the model predicts that the sound pressure is non zero, only for a finite time, in fact the time taken for a sound wave to cross the sphere. The exact field decays exponentially. This is only to be expected. Ray theory is correct only instantaneously following the initial impulse. It is probably the effects of source spreading which account for the discrepancies.

In the second example where the sphere is unconstrained we require knowledge of the time-history of the sphere's motion. The only self-consistent approach is to use the ray theory estimate of pressure to calculate the drag on the sphere. The ray theory pressure at a point defined by the polar angle Ψ is

$$p = \rho c v_n = \rho c \, U(t) \cos\Psi . \tag{2.10}$$

The equation of motion of the sphere is thus

$$-\rho c \, 2\pi a^2 U(t) \int_{\Psi=0}^{\pi} \cos^2\Psi \sin\Psi \, d\Psi = \frac{4\pi}{3} \rho_0 a^3 \frac{\partial U}{\partial t} .$$

with the solution

$$U(t) = U_0 \exp\left\{ \frac{-2c\beta t}{a} \right\} H(t). \tag{2.11}$$

One obvious draw-back of this simple scheme is that it implies that the sphere eventually comes to rest. This is overcome, as we shall see later, by including the effects of ray front curvature in the model.

The integration necessary to obtain the pressure field is performed as before, and after some labourious algebra we obtain

$$p(\underline{x},t) = \frac{\rho \cos\theta \, U_0 c a}{4|\underline{x}|\beta^2} \left\{ (4\beta^2 + 3\beta + 1) \exp\left(-\frac{2c\beta}{a}\left(t - \frac{|\underline{x}| - a}{c}\right)\right) - \left(1 + \beta\left(1 - 2\frac{ct - |\underline{x}|}{a}\right)\right) \right\} H(a - |ct - |\underline{x}||)$$

$$+ \frac{\rho \cos\theta \, U_0 c a}{4|\underline{x}|\beta^2} \left\{ (4\beta^2 + 3\beta + 1) \exp\left(-\frac{2c\beta}{a}\left(t - \frac{|\underline{x}| - a}{c}\right)\right) - (\beta - 1) \exp\left(-\frac{2c\beta}{a}\left(t - \frac{|\underline{x}| + a}{c}\right)\right) \right\} H\left(t - \frac{|\underline{x}| + a}{c}\right)$$

leading to a total radiation

$$E = \frac{\rho a^3 \pi U^2}{24\beta^5} \left\{ 8\beta^4 - \frac{8\beta^3}{3} - \beta^2 + 1 - (4\beta^3 + 7\beta^2 + 4\beta + 1) e^{-4\beta} \right\}. \tag{2.13}$$

as $\beta \to 0$ the total radiation can be written

$$E = \frac{\rho a^3 \pi U^2}{3} \left\{ \frac{16}{15} - \frac{4\beta}{9} + O(\beta^2) \right\}. \tag{2.14}$$

Once again we have recaptured the result corresponding to steady motion in the limit $\beta = 0$.

Taylors problem is given by $\beta = \frac{1}{2}$, and our model gives the approximate result as

$$E = \frac{2}{9}\rho\pi a^3 U^2\left\{\frac{1}{2}\left(11 - 63\exp(-2)\right)\right\} \qquad (2.15)$$

a result which is too high by some 24%. Once again this failure is due to effects at large time where ray theory is inappropriate and it is for this reason that we suggest our second approximation scheme.

In this scheme we take account of the effects of local curvature, and hence some account of the spreading of the pressure field away from the sphere.

The pressure is approximated by the pressure in the neighbourhood of a radially growing sphere. The potential in the fluid surrounding a radially growing sphere is

$$\phi = -\frac{Va^2}{r}\left\{1 - \exp\left(-\frac{c}{a}\left(t - \frac{r-a}{c}\right)\right)\right\} H\left(t - \frac{r-a}{c}\right). \qquad (2.16)$$

where V is the (constant) radial velocity of the surface. We have applied the boundary condition $\frac{\partial\phi}{\partial r} = V$ at the initial position of the surface $r = a$, rather than the true position, $r = a + Vt$. Provided we are restricting our analysis to cases of small subsonic speeds, the difference will be negligible since once again we expect that all the characteristics of the sound field are determined on a time scale (a/c).

Locally, the radial velocity at a point on the surface defined by the polar angle Ψ is $U\cos\Psi$. Thus the pressure here is, from (2.16)

$$p(a,\theta,t) = \rho U c \cos\theta \exp\left\{-\frac{ct}{a}\right\} H(t). \qquad (2.17)$$

Green's theorem is again employed to convert the surface integrals required in expression (2.1) into integrals throughout the volume of the sphere. We obtain

$$p_{sound} = \frac{\rho a c\cos\theta U}{2|\underline{x}|}\Bigg[\left\{5\exp\left\{-\frac{c}{a}\left(t - \frac{|\underline{x}|-a}{c}\right)\right\} + \left(\frac{ct-|\underline{x}|}{a} - 2\right)\right\} H\left(a - |ct - |\underline{x}||\right) +$$
$$+\left\{5\exp\left\{-\frac{c}{a}\left(t - \frac{|\underline{x}|-a}{c}\right)\right\} - \exp\left\{-\frac{c}{a}\left(t - \frac{|\underline{x}|+a}{c}\right)\right\}\right\} H\left(t - \frac{|\underline{x}|+a}{c}\right)\Bigg]. \qquad (2.18)$$

with the total radiated energy

$$E = \frac{\rho U^2 \pi a^3}{3}\left\{\frac{5 - 15\exp(-2)}{3}\right\} \tag{2.19}$$

The result (2.19) now provides an estimate of the total radiation which is accurate to within 1%. The pressure profile (2.18) agrees very closely with the exact result since not only are the 'switch-on' values identical but so also are the instantaneous pressure decay rates. The estimate also features an exponentially decaying tail.

The last stage of the approximation schemes is to apply the improved ray theory approximation to the unsteady problem. The potential due to a radially growing sphere of varying velocity, $V(t)$, is

$$\phi(r,t) = -\frac{ac}{r}\int_{\tau=0}^{t} V(t-\tau)\exp\left\{-\frac{c}{a}\left(\tau - \frac{r-a}{c}\right)\right\} H\left(\tau - \frac{r-a}{c}\right). \tag{2.20}$$

Once again we set up the equation of motion for the sphere and the solution for the time-history of the sphere is

$$U(t) = \frac{U_0}{1+2\beta}\left\{1 + 2\beta\exp\left(-\frac{ct}{a}(1+2\beta)\right)\right\} H(t). \tag{2.21}$$

The introduction of a term to include the effect of local curvature has produced a more realistic estimate of the time history of the sphere. The velocity decays to a non-zero value though unfortunately still not the correct value. In fact the predicted final velocity is too low by a factor

$$\frac{1+\beta}{1+2\beta} \sim (1-\beta) \quad \text{as } \beta \to 0$$

On performing the integrals over the surface of the sphere we find the radiated pressure

$$\begin{aligned} p_{sound} = {} & \frac{\rho c a \cos\theta\, U_0}{2|\underline{x}|(1+2\beta)^2}\left\{(5+12\beta+8\beta^2)\exp\left[-\frac{c}{a}(1+2\beta)\left(t - \frac{r-a}{c}\right)\right] + \right. \\ & \left. + \frac{c}{a}\left(t - \frac{r-a}{c}\right)(1+2\beta) - (3+4\beta)\right\} H\left(a - \left|t - \frac{r}{c}\right|\right) + \\ & + \frac{\rho c a \cos\theta\, U_0}{2|\underline{x}|(1+2\beta)^2}\left\{(5+12\beta+8\beta^2)\exp\left[-\frac{c}{a}(1+2\beta)\left(t - \frac{r-a}{c}\right)\right] - \right. \\ & \left. - (1+2\beta)\exp\left\{-\frac{c}{a}(1+2\beta)\left(t - \frac{r+a}{c}\right)\right\}\right\} H\left(t - \frac{r+a}{c}\right) \end{aligned} \tag{2.22}$$

corresponding to a total energy output

$$E = \frac{\rho a^3 \pi U_0^2}{3(1+2\beta)^5}\left[\frac{8(1+2\beta)^3}{3} + \frac{(5+12\beta+8\beta^2)^2}{2} - 4(5+12\beta+8\beta^2)(1+2\beta) + \right. \tag{2.23}$$
$$\left. + 2(1+2\beta)(3+4\beta) + \frac{(1+2\beta)^2}{3} - (5+12\beta+8\beta^2)(1+2\beta)\, e^{-2\beta(1+\beta)}\right]$$

which for small β can be expanded as

$$\frac{\rho a^3 \pi U^2}{3}\left\{\frac{5}{3} - 5\exp(-2) - \beta\left(\frac{20}{3} - 48\exp(-2)\right) + 0(\beta^2)\right\}$$

Once more we see that the limit $\beta \to 0$ recaptures the result for the steady problem. The estimate of the total energy for Taylors' problem is about 10% high.

The results of this section are summarised by the graph (Figure 2) which shows the true radiated pressure profile against the results of both of the approximation schemes in the steady case.

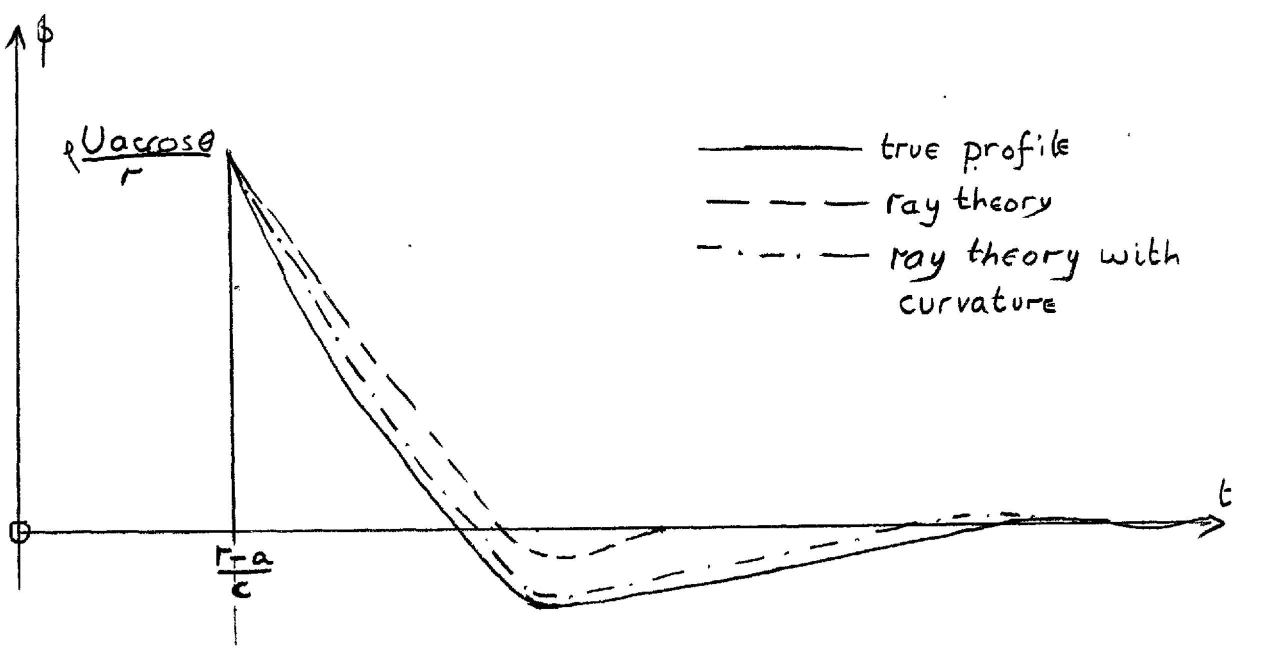

FIG 2.

APPROXIMATION SCHEME: APPLICATION TO THE MOTION OF A SEMI-INFINITE CYLINDER

If we are to make good use of the scheme we should now use the scheme in a situation where an exact solution is impossible to find. A geometry which is obviously of practical interest is that of a piston moving impulsively along its axis. Not only does this model describe an inevitable element of several physically important noise generating mechanisms, such as piledriver noise, but, armed with both the spherical and this plane problem we should have a good 'feel' for other impact problems by noting the types of changes induced by geometrical factors.

We expect that the duration of the pulse will be the time taken for sound to cross the face of the piston. Thus effects from either end of the piston will be independant provided its length is much greater than its diameter, and in that case we may as well study the impulsive motion of a semi-infinite cylinder. Contributions from the side walls is negligible, since there is no discontinuity in the motion here and in any case the first order ray theory term implies that the pressure is proportional to the (zero) normal velocity.

In the direction of travel, linear ray theory predicts a pressure pulse which is not decaying as $|\underline{x}|^{-1}$ but is of constant amplitude. This is an obvious failure of our model, since a beam of this type can only be supported by a high frequency oscillating piston. In this problem the beam must collapse. Off this central axis, the noise field can be estimated in the normal way.

The piston of radius a is given an impulsive velocity $U\,H(t)$ in the positive 1 direction. Using plane ray acoustics to calculate the pressure term in Curle's theory the radiation at an observation point $C(r_0,\theta,t)$ can be determined to be

$$p = \frac{\rho}{4\pi|\underline{x}|}\frac{\partial}{\partial t}\int l_i[v_i]\,dS(\underline{y}) - \frac{\rho c}{4\pi|\underline{x}|}\frac{\partial}{\partial x_i}\int l_i[l_j v_j]\,dS(\underline{y}) \qquad (3.1)$$

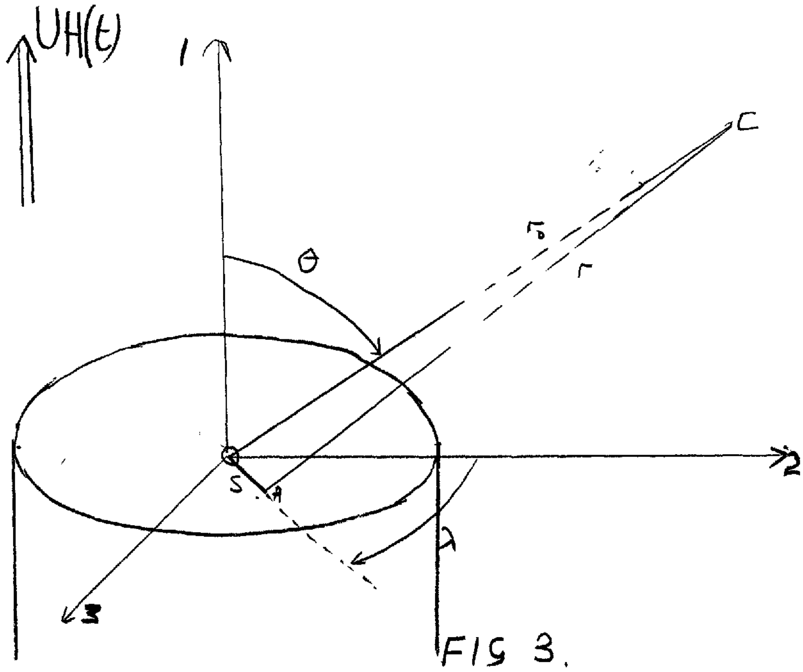

FIG 3.

THE AXIS SYSTEM

The reference axis $\lambda = 0$ is chosen arbitrarily to be in the direction of the observation point.

Over the face of the piston the direction cosines are simply

$$(l_1, l_2, l_3) = (1, 0, 0) \tag{3.2}$$

and the surface velocity

$$v_i = \delta_{1i} U H(t) \tag{3.3}$$

Finally, writing

$$\frac{\partial}{\partial x_1} = -\frac{\cos\theta}{c}\frac{\partial}{\partial t} \tag{3.4}$$

we obtain

$$p_{sound} = \frac{\rho U(1+\cos\theta)}{4\pi r_0}\frac{\partial}{\partial t}\int_S [H(t)]\, dS(y) \tag{3.5}$$

The integration over the face of the cylinder is carried out in terms of the polar variables (s, λ). It is easily seen that the retarded time is to be evaluated at

$$t - \frac{r_0 - s\sin\theta\cos\lambda}{c} \tag{3.6}$$

(except at $\theta = 0$)

and that

$$p_{sound} = \frac{\rho U(1+\cos\theta)}{4\pi r_0}\int_{s=0}^{a} s \int_{\lambda=0}^{2\pi} \delta\left\{t - \frac{r_0 - s\sin\theta\cos\lambda}{c}\right\} d\lambda$$

(3.7)

Concentrating on the λ integration and writing $T = t - \frac{r_0}{c}$, we have

$$\int_0^{2\pi} \delta\left\{T + \frac{s\sin\theta\cos\lambda}{c}\right\} d\lambda = 2\int_0^{\pi} \delta\left\{T + \frac{s\sin\theta\cos\lambda}{c}\right\} d\lambda$$

$$= 2\int_0^{\pi/2} \left(\delta\left\{T + \frac{s\sin\theta\cos\lambda}{c}\right\} + \delta\left\{T - \frac{s\sin\theta\cos\lambda}{c}\right\}\right) d\lambda$$

On making the transformation

$$\cos\lambda = \xi$$

we finally obtain

$$2\int_{-\infty}^{\infty} \left(\delta\left\{T + \frac{s\sin\theta\,\xi}{c}\right\} + \delta\left\{T - \frac{s\sin\theta\,\xi}{c}\right\}\right) \frac{H(\xi)H(1-\xi)}{(1-\xi^2)^{1/2}}\, d\xi$$

where H is the unit Heaviside function.

Now $\sin\theta \geq 0, \ \forall\theta$

so upon evaluating this integral we obtain

$$\frac{2c}{(s^2\sin^2\theta - c^2T^2)^{1/2}} H\left\{s^2\sin^2\theta - c^2T^2\right\}$$

The s integration may now be performed without difficulty giving

$$p_{sound} = \frac{\rho U(1-\cos\theta)^{-1}}{2\pi r_0} c \left\{a^2\sin^2\theta - c^2T^2\right\}^{1/2} H\left\{a^2\sin^2\theta - c^2T^2\right\}$$

(3.7)

The pressure profile is thus a pulse of finite duration. There is no associated shock since the pulse switches on and off with zero amplitude. The profile of the pulse is elliptical with a peak at

$$\frac{\rho U a c}{2\pi r_0} \quad \frac{\sin\theta}{1-\cos\theta}$$

(3.8)

The total radiated energy can be found simply and is in fact

$$E = \rho U^2 a^3$$

(3.9)

In an attempt to verify the existance of a pressure pulse of the type indicated by expression (3.7) the following, crude, experiment was carried out by the author (with the assistance of Mr P Growcott of the Building Research Centre and Mr J Ludlaw of I.S.V.R). A steel cylinder, of diameter 20 cms and length 200 cms, was dropped on to a section of a pile embedded in sand. The resulting sound pressure profile was recorded on an oscilloscope. A similar experiment was carried out using a much smaller hammer, of approximately 2 cms diameter. The chief difficulty is the extremely small duration of the pulse, typically of the order of a millisecond for the larger hammer. In both cases the recorded pressure profile had the form indicated in Figure 4. The initial pulse showed the duration, peak pressure level and general profile appropriate to the dimensions of the hammer and in agreement with the expression (3.7). In both cases the pressure profile corresponding to ringing was also discernible, and always appeared following the completion of the impulse noise effect.

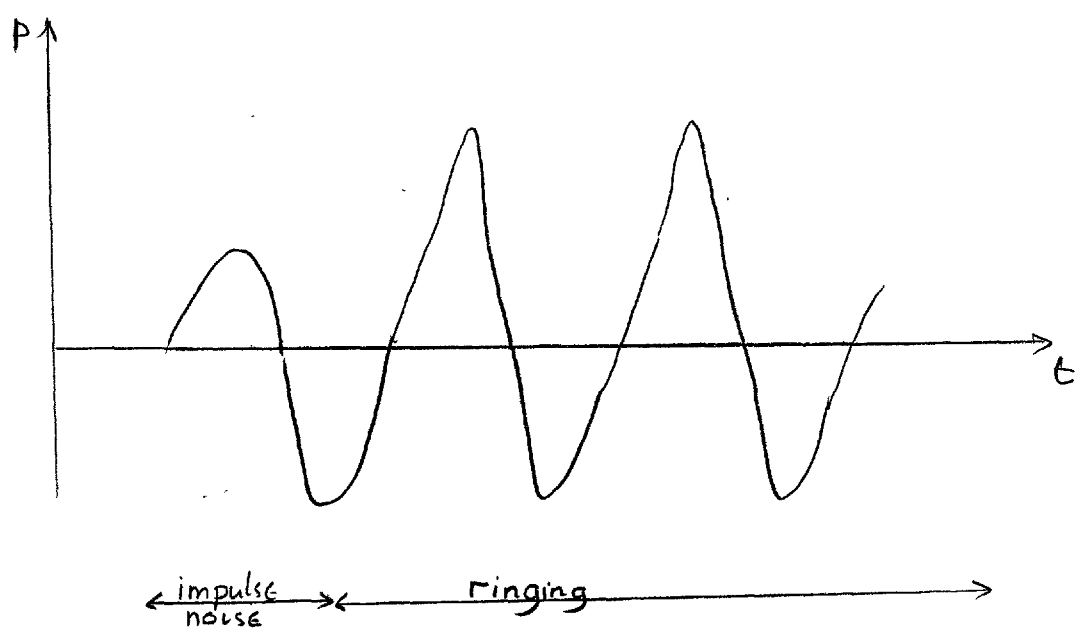

FIG 4. Profile obtained in pile-driving experiment.

CONCLUSIONS

The results of the second section of this work indicate that ray theory gives an accurate estimate of the sound field radiated by the impulsive motion of a single sphere. The application of this type of estimate to a general impact problem is not always straightforward. In the case of the piston and in fact for all plane surfaces impulsively moved, there is an inevitable difficulty in that an acoustic beam is formed. This is spurious and would not be found at large distances if the accelerations were finite. Results of our crude experiments appear to indicate that ray theory is giving a good estimate for the noise field at an observation point not on this axis. Also Sears, in a private communication with Professor JE Ffowcs Williams, has shown that the noise field from the impact of two spheres is given by a suitable combination of the fields from both of the impacting spheres taken independently. This indicates to us that the, at first sight complicated impulsive noise problems can be usefully approximated and we think that our method offers a powerful tool for the evaluation of noise fields in those situations where a high frequency limit is relevant. High frequency in this case means that events are changing instantaneously in comparison with the time taken for sound to travel a distance comparable to a typical length scale of the body. Noise generated by high speed rotating machinery, propeller noise, for example, could be estimated in this way.

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