Influence of beam cross-section, loading arrangement and aggregate type on shear strength

J. Sagaseta
École Polytechnique Fédérale de Lausanne (EPFL), Lausanne, Switzerland

R. L. Vollum
Department of Civil and Environmental Engineering, Imperial College London, London, UK

This paper describes 14 tests on simply supported and continuous reinforced concrete beams which were designed to investigate the influences of aggregate type, loading arrangement and shear reinforcement ratio on shear strength. The beams were rectangular in cross-section with $a/d \sim 3.5$ (where a is the shear span and d the effective depth). Marine dredged gravel was used in ten beams (two without shear reinforcement) and limestone in the remaining four (two without shear reinforcement). The cracks typically passed around the gravel aggregate but through the limestone aggregate. Aggregate fracture was found only to reduce the shear strength of the beams without shear reinforcement. The factors of safety for shear failure implicit in Eurocode 2, BS 8110 and CSA A.23.3-04 are examined with data from this project and elsewhere. Eurocode 2 is shown to provide the lowest factor of safety for most beams. This is particularly significant for rectangular sections where Eurocode 2 can give lower factor of safety for shear than flexural failure which is undesirable. An amendment is proposed to address this.

Notation

a	maximum aggregate size
b	beam width
b_f	width of compression flange
C	cohesion
d	effective depth
EC2	Eurocode 2
FOS	factor of safety (FOS for shear = V_{design} divided by overall load factor)
f'_c	concrete cylinder strength
f_{cu}	concrete cube strength
f_y	yield strength of reinforcement (subscript k = characteristic value)
h_f	depth of compression flange
P_{cr}	load at which first cracks originated
P_{ult}	ultimate failure load
SI	stirrup index SI = $\rho_w f_y / (v f'_c)$
s	stirrup spacing
V	shear force
V_c	concrete component of shear resistance
V_{cz}	shear carried by the compression zone
V_{design}	design ultimate shear resistance divided by overall load factor
V_{Rdc}	EC2 design shear resistance for beams without shear reinforcement
V_s	stirrup contribution to shear resistance
VSI	EC2 variable strut inclination design method
z	lever arm for shear ($0.9d$ unless noted otherwise)
γ_c, γ_s	partial factors for concrete and steel respectively
θ	inclination of compressive stress field to the longitudinal axis of the beam
μ	coefficent of friction along crack plane
ν	strength reduction factor for concrete cracked in shear $v = 0.6(1 - f'_c/250)$
ρ_l	longitudinal reinforcement ratio $\rho_l = A_{sl}/(bd)$
ρ_w	shear reinforcement ratio $\rho_w = A_{sw}/(bs)$
σ_n	normal stress to crack plane
τ	shear stress along crack plane

Introduction

Shear is resisted in beams through the combined contributions of the compression zone at the head of the shear crack, aggregate interlock, dowel action and stirrups if present. The contribution of each mechanism has been keenly debated since pioneering work by Taylor (1970) and others showed that aggregate interlock contributes up to 50% of the shear strength of beams without stirrups. This paper considers the influences of aggregate fracture and beam cross-section geometry on the factors of safety (FOS) for shear implicit in EC2 (Eurocode 2) (BSI, 2004), BS 8110-1: 1997 (BSI, 2007) and CSA A.23.3-04 (CSA, 2004). These design methods are refinements of the classical 45° truss analogy in which the shear strength is taken as $V = V_c + V_s$ where V_c and V_s represent the contributions of the concrete and shear reinforcement respectively. BS 8110-1:1997 (BSI, 2007) takes V_c as the

Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

shear strength of beams without stirrups and calculates V_s with a 45º truss. EC2 (BSI, 2004) uses a variable strut inclination method in which the shear force is assumed to be entirely resisted by the shear reinforcement (i.e. $V_c = 0$). The Canadian code CSA A.23.3-04 (CSA, 2004) is derived from the modified compression field theory (MCFT) of Vecchio and Collins (1986). The latter two methods relate V_c to the strain in the flexural reinforcement as described by Collins et al. (2008).

Influence of aggregate fracture on shear strength

In high-strength (HSC) and lightweight aggregate concrete (LWAC), the bond between the aggregate particles and the cement paste can be strong enough for the aggregate to fracture at cracks, as shown in Figure 1. Aggregate fracture results in smoother crack surfaces and can reduce shear transfer through aggregate interlock. A similar situation arises in self-compacting concretes (SCC) where cracks are relatively smooth owing to the reduced content of coarse aggregates (Lachemi et al., 2005).

The literature provides conflicting views on the effect of aggregate fracture on shear strength. For example, Walraven and Stroband (1994) found that aggregate fracture reduced the shear strength of their HSC push-off specimens. Conversely, Walraven and Al-Zubi (1995) and Walraven and Stroband (1999) tested LWAC and HSC beams with stirrups and found no reduction in shear strength owing to aggregate fracture. They explained this behaviour in terms of irregularities in the crack surface at a macro-level. On the other hand, Hamadi and Regan (1980) found that the shear strength of their beams with stirrups made from expanded clay LWAC was reduced by aggregate fracture. More recently, Regan et al. (2005) tested a series of beams without stirrups and found that the shear strength was reduced if the

coarse aggregate fractured at cracks. They speculated that the shear strength of beams with stirrups would be reduced by aggregate fracture but were unable to quantify the reduction owing to insufficient experimental data.

Code provisions for influence of crack roughness on shear strength

Regan's work (Regan et al., 2005) led to the UK national annex to EC2 (BSI, 2005) limiting the maximum allowable concrete cylinder strength to be used in the shear provisions of the code to 50 MPa unless justified otherwise. More rational alternatives to limiting the concrete strength are the MCFT of Vecchio and Collins (1986) and the critical shear crack theory of Muttoni and Fernández Ruiz (2008) which are incorporated into the Canadian CSA A.23.3-04 (CSA, 2004)) and Swiss SIA262 (SIA, 2003) codes respectively. Both these methods express the crack roughness in terms of the maximum aggregate size (a) and account for aggregate fracture by reducing the aggregate size linearly to zero for concrete cylinder strengths between 60 MPa and 70 MPa (Angelakos et al., 2001). Even so, the MCFT neglects the influence of previous cracks since it is a fully rotational crack approach in which shear stresses are checked along fictitious cracks which are assumed to be oriented parallel to the principal compressive stress field.

Contribution of compression flange to shear strength

EC2, BS 8110 and CSA A.23.3-04 assume that the shear force is entirely carried by the web in 'I' and 'T' sections and attribute the contribution of the compression flange to the web. Therefore, it is striking that the variable strut inclination (VSI) method in EC2 was validated with data from tests on highly stressed simply supported beams with 'I' and 'T' sections (Walraven, 2004) in which the compression flange contributed significantly, but
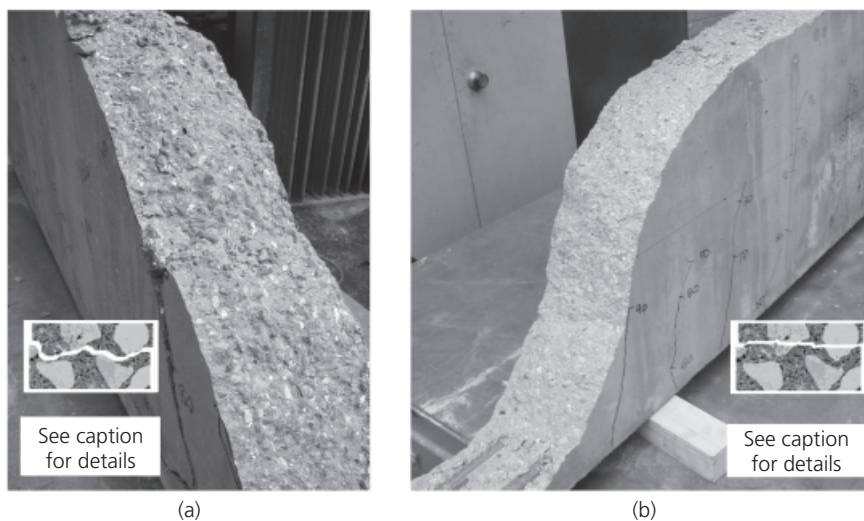

(a)

(b)

Figure 1. Failure surface at critical shear crack in beams without stirrups: (a) rough crack surface in beam BG0 ($f'_c = 80.2$ MPa) with gravel aggregate; and (b) smooth crack surface due to aggregate fracture in beam BL0 ($f'_c = 68.4$ MPa) with limestone aggregate

Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

indeterminately, to shear strength. There is a scarcity of tests on continuous beams which is unfortunate since continuous beam tests have several advantages over simply supported beam tests as noted by Leonhardt and Walther (1961). First, the compression zone contributes less to shear strength in continuous than simply supported beams. It follows that tests on simply supported 'T' and 'I' sections are not directly applicable to continuous beams owing to the reversal of bending moment within the critical section for shear. Second, the stress state in continuous beams is closer to pure shear than in simply supported beams. Third, shear failure is obtainable at much greater shear reinforcement ratios in continuous than simply supported beams with rectangular sections. The greater M/V ratio in simply supported beams necessitates the use of 'T' or 'I' cross-sections with relatively thin webs at high shear reinforcement ratios. The present paper examines the consequences of not explicitly considering the contribution of the compression flange to shear strength by comparing measured and predicted shear strengths for rectangular, 'T'- and 'I'-shaped cross-sections with similar shear reinforcement ratios.

Experimental investigation

The authors tested 14 beams (four without shear reinforcement) to investigate the influences of aggregate fracture, beam cross-section and loading arrangement on shear strength. Marine dredged gravel was used in ten beams and limestone in the remaining four. The maximum size of the aggregate was 10 mm in all the beams. The cracks typically passed around the gravel aggregate but through the limestone aggregate. The primary aim of the tests was to determine whether (a) the shear strength of beams with shear reinforcement is significantly reduced by aggre-

gate fracture and (b) to assess the safety of code guidelines for shear in highly stressed beams without compression flanges.

Beams without shear reinforcement

Four beams were tested without stirrups to determine whether the reduction in shear strength owing to aggregate fracture was comparable to that found by Regan *et al.* (2005). Figure 2 shows the geometry of the tested beams and the loading configuration. Two beams were cast from concrete with marine (siliceous) gravel aggregate and two with limestone (calcareous) crushed aggregate. The main characteristics of the beams and their failure loads are summarised in Table 1 in which 'G' denotes gravel and 'L' denotes limestone aggregate. The concrete cylinder strengths were 80·2 MPa and 68·4 MPa for the BG0 and BL0 beams respectively. The ratio of the shear span to the effective depth (a/d) was 3·44. The flexural reinforcement was the same in all the beams and consisted of 2T20 bars ($\rho_l = 1\%$) with a yield strength of 580 MPa.

All the beams failed in shear as shown in Figure 3. The critical shear crack initiated at a previous flexural-shear crack and propagated suddenly to the load and support plates. Figure 3(c) shows that the angle between the longitudinal reinforcement and the critical diagonal crack was steeper in the BL than the BG specimens. The top halves of the beams were removed after testing to inspect the crack surfaces. Figure 1 shows that the aggregate fractured completely in the BL beams ($f'_c = 68\cdot4$ MPa) but not in the gravel beams ($f'_c = 80\cdot2$ MPa) where the crack went through around 30% of the aggregate. Figure 1 shows that the crack
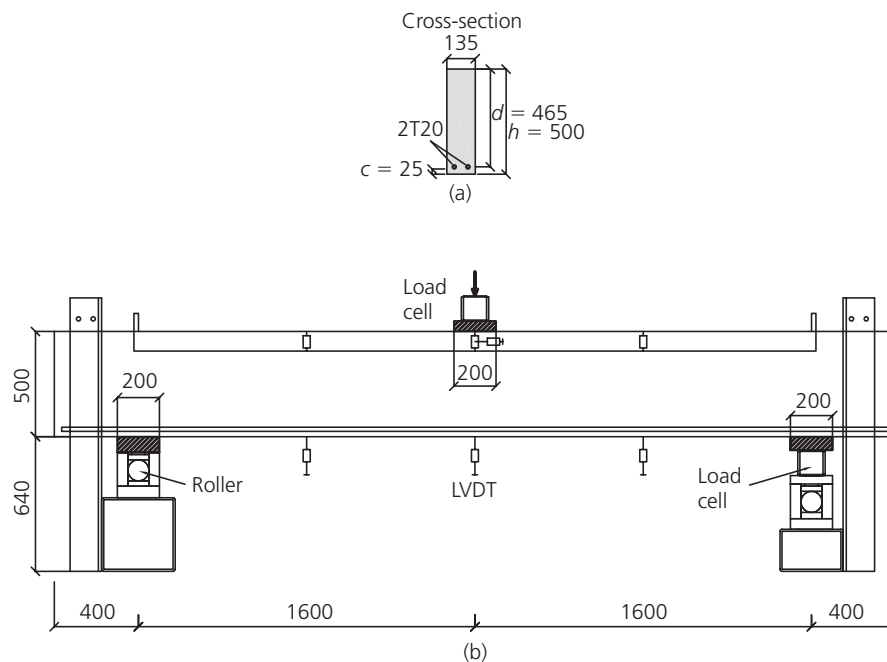

Figure 2. Test arrangement for beams B0 (dimensions in mm):
(a) cross-section; and (b) geometry and instrumentation

Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

Beam	f'_c: MPa	f_y (long.): MPa	ρ_1: %	a/d	δ_{centre}: mm	P_{cr}: kN	P_{ult}: kN
BG01	80·20	580	1	3·46	4·14	56·2	122·63
BG02	80·20	580	1	3·46	4·70	50·0	126·22
BL01	68·44	580	1	3·46	3·58	50·0	93·72
BL02	68·44	580	1	3·46	4·27	50·0	108·14

Notes: δ^*_{centre} central deflection at failure load; P_{cr} load at which cracks were first observed; all beams failed in diagonal tension

Table 1. Experimental data for beams without shear
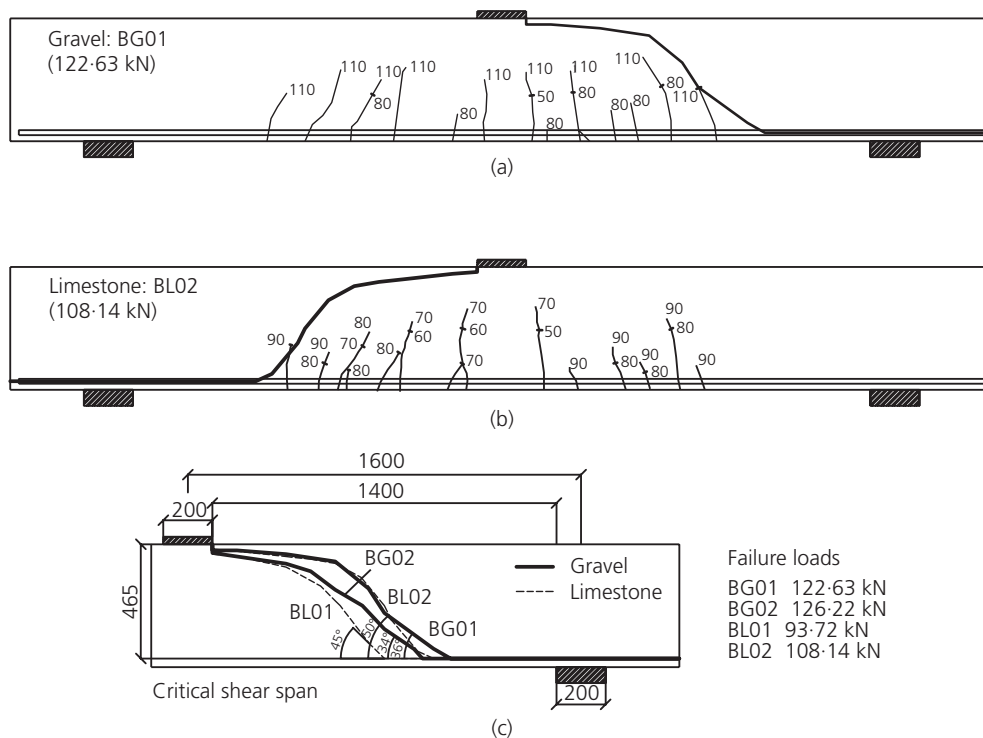reinforcement

Figure 3. Crack patterns in beams B0 (the numbers along the
cracks denote the load at which the cracks were first observed in
kN; critical cracks are highlighted in bold): (a) BG01; (b) BL02;
and (c) relative position of main diagonal shear cracks

roughness depends on the aggregate type and not just the concrete
strength as commonly assumed in codes of practice.

Comparison between measured and predicted shear strengths of beams BG0 and BL0

The shear strengths of the BG0 and BL0 beams are not directly
comparable due to unintended differences in concrete strength.
Therefore, the effect of aggregate fracture was determined
indirectly by comparing the strengths of the beams with the
predictions of various design methods including Eurocode 2
(BSI, 2004) which takes the shear strength as

$$1. \quad V_{Rd,c}(EC2) = \frac{0·18}{\gamma_c}(100\rho_1 f'_c)^{1/3}\left(1 + \sqrt{200/d}\right)bd$$

where ρ_1 is the longitudinal reinforcement ratio, f'_c is the cylinder
compressive strength, d is the effective depth, γ_c is the partial

Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

factor for concrete which is taken as 1·5 for design, and b is the beam width.

The measured and calculated shear strengths are compared in Table 2 which gives strengths calculated with the partial factor γ_c equal to both 1·0 and 1·5 with the latter values shown in parentheses. V_{Rdc} was calculated with both the actual concrete strength and the maximum strength of $f_{ck} = 50$ MPa allowed by the UK national annex to EC2. Table 2 shows that V_{test}/V_{Rdc} (with $\gamma_c = 1·0$) was less than 1 for all the beams and furthermore, V_{test}/V_{Rdc} was 16% lower for the limestone than for the gravel beams. Limiting the concrete cylinder strength to 50 MPa noticeably improved the predictions for the gravel beams, which had a concrete strength of 80·2 MPa. This was less so for the limestone beams where the concrete strength of 68·4 MPa is closer to the cut-off of 50 MPa in the UK national annex to EC2. Table 2 also shows that limiting the concrete cylinder strength to 50 MPa gives safe results when $\gamma_c = 1·5$. The main drawback with limiting the concrete strength is that the influence of aggregate type is neglected.

Regan *et al.* (2005) used the interpolated surfaces ($V_{test}/V_{calc} - f_c - d$) shown in Figure 4 to illustrate the worsening predictions of Equation 1 for their beams, with limestone and gravel aggregates, with increasing member depth (d) and concrete strength (f'_c). Figure 4 shows V_{test}/V_{calc} is lower for the authors' beams with limestone than gravel aggregate and the authors' data lie close to the interpolated surfaces of Regan *et al.* (2005).

The shear strengths of the B0 beams were also calculated with the MCFT using the programme Response 2000 (Bentz, 2000). Shear strengths were calculated with maximum aggregate sizes of $a = 10$ mm and $a = 0$. Table 2 shows that the MCFT predicted the shear strengths of the tested beams more accu-

rately than Equation 1 from EC2 but V_{test}/V_{calc} was less than 1 for all the beams tested by the authors. The percentage improvement in V_{test}/V_{calc} owing to taking $a = 0$ was similar for all the beams which was not the case when the concrete cylinder strength was limited to 50 MPa in Equation 1 from EC2. The strengths calculated with $a = 10$ mm are theoretically appropriate for beams BG0 since the aggregate did not fracture. It is interesting to note that the V_{test}/V_{calc} ratios obtained with the MCFT are very similar for beams BG0 and BL0 if a is taken as 10 mm for the gravel and zero for the limestone beams, as implied by the observed behaviour.

Beams with shear reinforcement
Experimental investigation
Ten beams were tested to investigate the effect of varying the aggregate type, loading arrangement and shear reinforcement ratio on the shear strength and cracking pattern. Six continuous and four simply supported beams were tested. Figure 5 gives details of the loading arrangement for the continuous beams which were statically determinate with a point of contraflexure at the centre of the critical shear span. Table 3 and Figure 6 give details of the beam dimensions, material properties and reinforcement. The width of beams C and D was increased to 160 mm to accommodate the 10 mm diameter stirrups used in these beams. The flexural reinforcement in the continuous beams consisted of two layers of 2T25 bars placed at the top and bottom of the section to resist the anti-symmetric bending moment distribution shown in Figure 5(d). The corresponding flexural reinforcement ratios ($100A_{sl}/bs$) were 3·32% in beams B and 2·80% in beams C. A third layer of flexural reinforcement was provided in the simply supported D series to prevent flexural failure which reduced the effective depth d from 437 mm to 412 mm. The stirrup spacing in the critical shear spans varied between 90 mm and 300 mm as shown in Figure 6 which corresponds to stirrup ratios ($100A_{sw}/$

Beam	V_{test}: kN	V_{flex}: kN	EC2 (Equation 1)		MCFT: Response 2000	
			V_{test}/V_{calc}	V_{test}/V_{calc} ($f'_c \leqslant 50$ MPa)	V_{test}/V_{calc} ($a = 10$ mm)	V_{test}/V_{calc} ($a = 0$)
BG0-1	61·31	100	0·76 (1·14)	0·89 (1·33)	0·88 (1·34)	0·94 (1·44)
BG0-2	63·11	100	0·78 (1·17)	0·92 (1·37)	0·91 (1·38)	0·97 (1·48)
BL0-1	46·86	98·5	0·61 (0·91)	0·68 (1·02)	0·71 (1·08)	0·76 (1·16)
BL0-2	54·07	98·5	0·71 (1·06)	0·78 (1·18)	0·82 (1·25)	0·88 (1·34)
	Avg.	BG0	0·77 (1·15)	0·90 (1·35)	0·89 (1·36)	0·95 (1·46)
		BL0	0·66 (0·98)	0·73 (1·09)	0·76 (1·16)	0·82 (1·25)

Notes:
Design values in parentheses calculated with (EC2: $\gamma_c = 1·5$; CSA: $\gamma_c = 1·53$).
Response 2000: base curve (Popovic–Thorenfeld–Collins), comp. softening (Vecchio and Collins, 1986), tension stiffening (Bentz, 2000), crack spacing (auto), f_{ct} and ε'_c (auto)

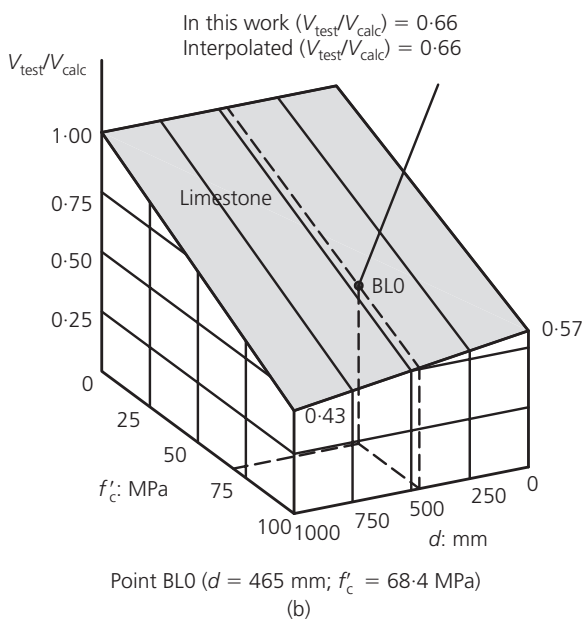**Table 2.** Measured and predicted shear strengths for beams without shear reinforcement

Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

In this work (V_{test}/V_{calc}) = 0·77
Interpolated (V_{test}/V_{calc}) = 0·73

Point BG0 (d = 465 mm; f_c' = 80·2 MPa)
(a)

In this work (V_{test}/V_{calc}) = 0·66
Interpolated (V_{test}/V_{calc}) = 0·66
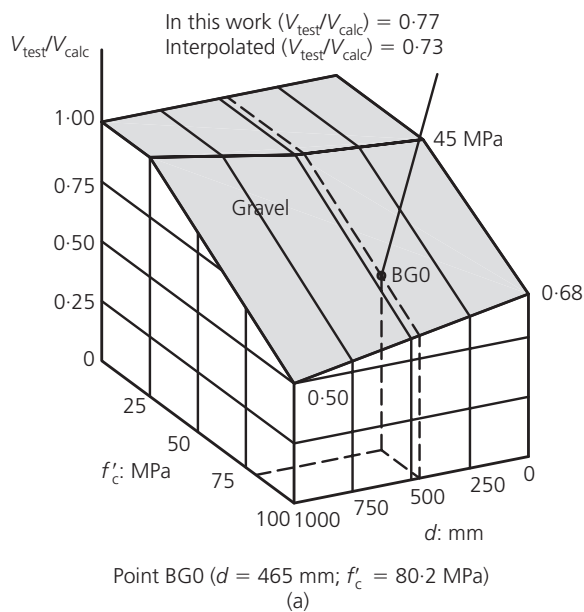
Point BL0 (d = 465 mm; f_c' = 68·4 MPa)
(b)

Figure 4. Comparison of V_{test}/V_{calc} for B0 beams with predictions
of Regan *et al.* (2005) (note: V_{calc} is calculated with EC2 with no
restriction on f_c'): (a) gravel aggregate specimens; and (b)
limestone aggregate specimens

bs) between 0·33% and 0·83%. Sufficient shear reinforcement
was provided in the shorter shear spans of the continuous beams
to prevent shear failure. The shear reinforcement ratio and stirrup
spacing were the same in the critical shear spans of the beams in
the C1 and D1 series. Gravel aggregate was used in all the beams
except the BL series in which limestone aggregate was used. The

CA, DA, CB and DB beams were cast together and can be
regarded as duplicate sets A and B. The BL and BG beams were
intended to have the same concrete strength to assess the
influence of aggregate type on shear strength. In reality, the
concrete cylinder strength was significantly less in the BG
(31·7 MPa) than the BL beams (53·1 MPa) which are comparable
in strength to the CA and CB beams (49·35 MPa).

Instrumentation
Deflections were measured with linear variable differential
transformers (LVDTs) and digital photogrammetry. Crack open-
ing and sliding displacements were also derived from displace-
ments measured between crosses of Demec points and LVDTs
positioned as shown in Figure 5(a). The beams were pre-cracked
before the instrumentation was positioned along the critical shear
cracks. Transverse and longitudinal strains were also measured in
the surface of the concrete between Demec points. Strains were
measured in the flexural reinforcement in tests BG2 and BL2 at
the position of maximum span moment.

Test results
Table 3 summarises the loads at failure and when shear cracks
first formed. The maximum tensile strains in the flexural rein-
forcement were 1·79‰ and 2·5‰ respectively in beams BG2 and
BL2, which were the most heavily loaded. Full details of the
strain and displacement measurements are given elsewhere
(Sagaseta, 2008). The shear cracks developed differently in the
continuous and simply supported beams as shown in Figure 7
which highlights the critical shear cracks at failure. Figure 7(a)
shows that shear cracks initially formed near mid-height at 45° in
the simply supported beams and that, subsequently, additional
shear cracks formed at flatter angles mobilising more stirrups.
Strain measurements showed that the flatter cracks formed after
the stirrups yielded. The 'crack rotation' evident in Figure 7(a) is
well documented and is implicit in the plasticity truss used in
EC2. The crack development was similar in all the continuous
beams up to failure with little if any evidence of crack rotation.
The shear cracks initially formed in the continuous beams around
an effective depth from the loading points. Subsequently, addi-
tional cracks formed near the loading points in a fan shape as
shown in Figures 7(b) and (c). Cracks only formed in the centre
of the shear span in beams BG2, BL2, CA2 and CB2 towards the
end of the test. The lack of crack rotation in the continuous
beams would seem to suggest that aggregate interlock plays a
greater part in transferring shear stresses through pre-existing
cracks in simply supported than continuous beams.

The failure mode of the continuous beams was clearly influenced
by the stirrup index SI = $\rho_w f_y/(v f_c')$ where $\rho_w = A_{sw}/(bs)$, s is
the stirrup spacing and $v = 0 \cdot 6(1 - f_c'/250)$. The failure of the
beams with the lowest SI (CA1, CB1 and BL1) appears to have
been triggered by loss of aggregate interlock resulting from the
development of excessive strain in the stirrups crossing the
critical diagonal crack. The critical shear crack in these beams
(see Figure 7(b)) extended to the loading point, at the peak load,
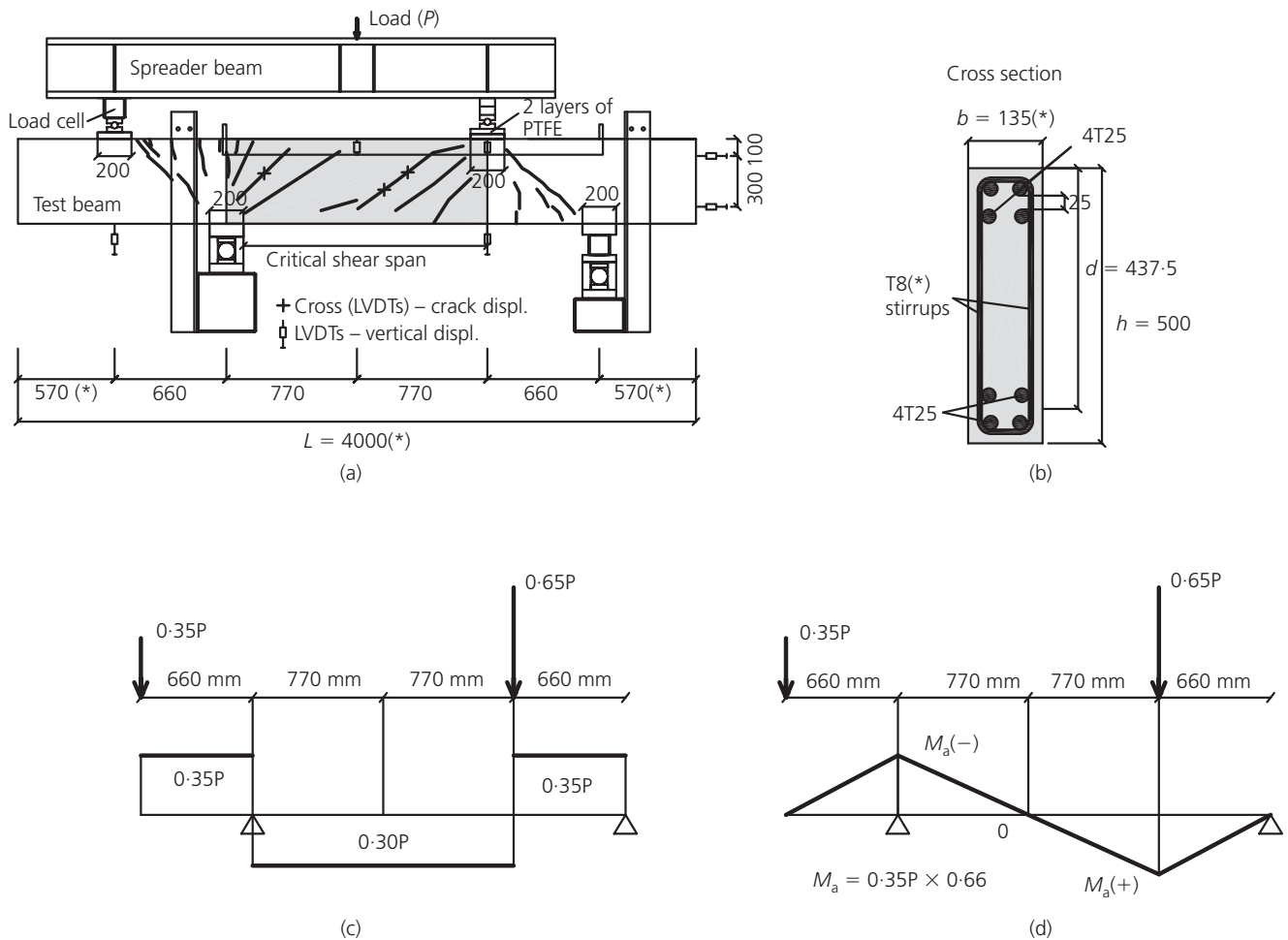
Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

Figure 5. Loading arrangement for continuous beams: (a) testing
rig; (b) cross-section; (c) shear force diagram; and (d) bending
moment diagram. Notes: (*) values for beams B; in beamc C:
$L = 4500$ mm, $b = 160$ mm and 10 mm stirrups

and widened considerably under almost constant load. Beams
CA2 and CB2 behaved similarly with several shear cracks
widening simultaneously at failure (see Figure 7(c)). Reducing
the stirrup spacing from 300 mm in beam CB1 to 200 mm in
beam CB2 led to a significant reduction in the spacing of the
inclined cracks as shown in Figure 7(c). The maximum width of
the critical shear crack in beam CB1 was 1·2 mm, which was
approximately twice that in CB2. Beams BG2 and BL2 had the
highest SI and failed in shear-compression. The diagonal strut
split in the out-of-plane direction in beam BL2 leading to the
concrete spalling at the top and bottom of the beam. The main
crack extended along the flexural reinforcement towards the ends
of the beam immediately after failure in beams BG1 and BG2.
Analysis of the beams in the next section provides no evidence
that the longitudinal cracking, which is indicative of bond failure,
precipitated failure. This conclusion is consistent with Hamadi

and Regan's (1980) observation that bond failure frequently
occurs in combination with shear failure.

Analysis of test results

Influence of aggregate fracture

Table 3 shows beams CA2 and CB2, with gravel aggregate, had
similar concrete strengths and shear reinforcement ratios to beam
BL1 with limestone aggregate. Aggregate fracture does not
appear to have reduced the shear strength of beam BL1 since its
shear strength ($V/bd = 5·94$ MPa) is very similar to that of beams
CA2 (6·12 MPa) and CB2 (5·98 MPa). Interestingly, push-off
tests (Sagaseta and Vollum, 2011) showed that significant shear
stresses could be transferred through cracks in the limestone
concrete even though the aggregate fractured. Sagaseta and
Vollum (2011) has shown that significant shear stresses were

145

Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

Loading	Beam	f'_c: MPa	b: mm	ρ_l: %	$\rho_w f_y$: MPa	δ^*_{centre}: mm	P_{cr}: kN	P_{ult}: kN	Failure mode
Continuous beam	BG1	31·70	135	3·32	2·73	5·29	300	950·6	SC(*)
($V = 0.3P$)	BG2	31·70	135	3·32	4·55	5·43	300	1074·1	CW(*)
	BL1	53·11	135	3·32	2·73	4·90	400	1169·1	CW
	BL2	53·11	135	3·32	4·55	6·02	300	1593·9	SC
	CB1	49·35	160	2·80	1·96	3·14	400	1029·3	CW
	CB2	49·35	160	2·80	2·94	5·99	450	1429·0	CW
	CA1	49·35	160	2·80	1·96	4·23	400	979·8	CW
	CA2	49·35	160	2·80	2·94	4·76	450	1395·5	CW
Simply	DA1	49·35	160	4·46	1·96	14·87	200	622·7	CW
supported	DB1	49·35	160	4·46	1·96	13·84	200	598·4	CW

Notes:
f_y (stirrups) = 550 MPa; f_y (longitudinal) = 580 MPa
Failure modes: shear compression (SC); crack widening (CW)
(*) shear failure accompanied by bond failure
δ^*_{centre} vertical deflection under the central load at failure load
P_{cr} load at which shear cracks were first observed

Table 3. Experimental data for beams with shear reinforcement

transferred through the cracks in beam BL2 despite the aggregate
fracturing.

Contribution of compression flange

Figure 8 compares the shear strengths of the tested beams with
the predictions of EC2 and the shear strengths of beams with
rectangular, 'T'- and 'I'- shaped cross-sections tested by others.
Figure 8 is similar to the figure used to demonstrate the validity
of the EC2 design method in the background document for shear
in EC2 (Walraven, 2004). The main difference between the
figures is that Walraven (2004) only included data from 'I' and
'T' sections which fail at significantly higher shear stresses than
the rectangular sections shown in Figure 8. The beams from
Watanabe and Lee's (1998) database in Figure 8 were tested in
Japan in the late 1980s and failed in shear. The beams were
continuous and rectangular in cross-section with similar dimen-
sions and flexural reinforcement ratios to the B and C series of
beams tested in this programme. The concrete cylinder strengths
varied between 20·4 MPa and 139·5 MPa. The yield strengths of
the flexural ($870 < f_y < 1070$) and shear reinforcement
($225 < f_y < 1478$) were unusually high in these tests.

Figure 8 shows that the authors' results are broadly consistent
with those of Watanabe and Lee (1998) despite the unusually
high reinforcement strengths in the latter. The increased shear
strength of the 'T' and 'I' beams, relative to the rectangular
beams, is attributed to the contribution of the compression flange
which can be estimated as follows (Placas, 1969)

$$2. \qquad V_{cz} = 0 \cdot 1 f_{cu}^{2/3} (b_w + 1 \cdot 5 \lambda h_f) h_f$$

where

$$\lambda = 1 \cdot 0 \text{ if } [(b_f - b_w) > 3 h_f]$$

and

$$\lambda = (b_f - b_w)/3 h_f \text{ if } [(b_f - b_w) < 3 h_f]$$

Equation 2 implies that the compression flange increased the
shear strength of the 'I' and 'T' beams in Figure 8 by between
10% and 20%. This estimate is consistent with the findings of
Leonhardt and Walther (1961), Regan (1971) and Pansuk and
Sato (2007) and is of the same order as the difference between
the shear strengths of comparable flanged and rectangular
sections in Figure 8. It is also striking that the shear strengths of
the authors' simply supported beams DA1 and DB1 were very
similar to those of the equivalent continuous beams CA1 and
CB1. This suggests that the difference between the strengths of
the rectangular and flanged sections in Figure 8 is principally
attributable to the reduced contribution of the compression zone
in the rectangular beams.

Evaluation of shear strength with EC2

The variable strut inclination (VSI) method in EC2 was used to
assess the shear strength of all the rectangular beams in Figure 8.
The VSI method is based on the work of Nielsen and Baestrup
(1976) and Muttoni *et al.* (1989) among others. The method
assumes that the shear force is resisted by a truss consisting of
concrete struts equilibrated by shear reinforcement with $V_c = 0$.
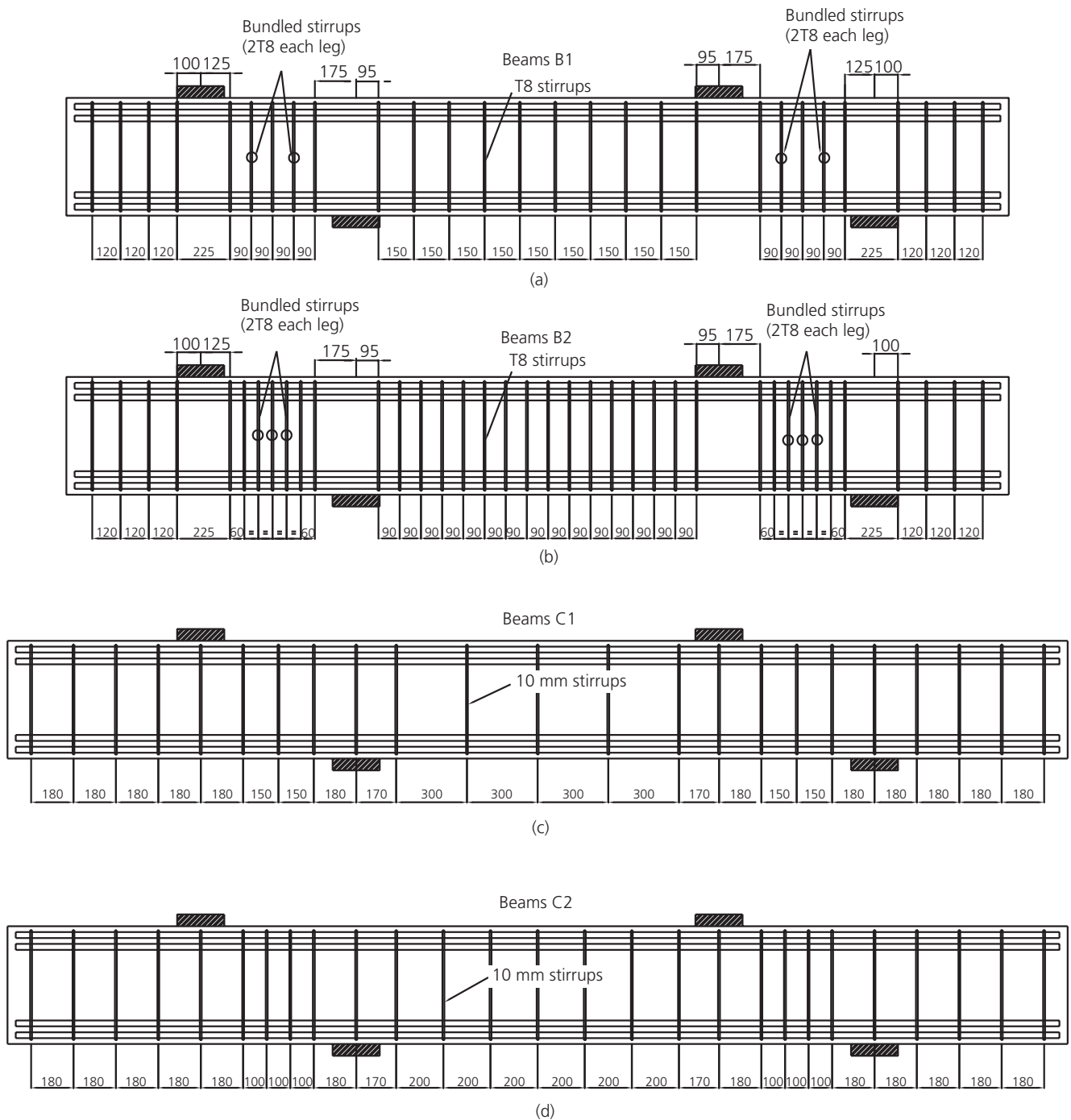The angle between the concrete struts and the longitudinal axis of

Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

Figure 6. Reinforcement arrangement in continuous beams:
(a) beams BG1/BL1; (b) beams BG2/BL2; (c) beams CA1/CB1; and
(d) beams CA2/CB2

the beam is allowed to vary between 21·8° to 45° depending upon
the applied shear force. In reality, the angle of the compression
field in the truss is steeper than assumed in EC2 as part of the
shear force is resisted by V_c. For members with vertical shear
reinforcement, the design value of the shear strength is given by

3. $\quad V_{Rd,s} = A_{sw} z f_{ywd} \cot \theta / s$

where A_{sw} is the area of shear reinforcement, $f_{ywd} = f_{yk}/\gamma_s$ is the
design yield strength of the shear reinforcement, s is the stirrup

Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
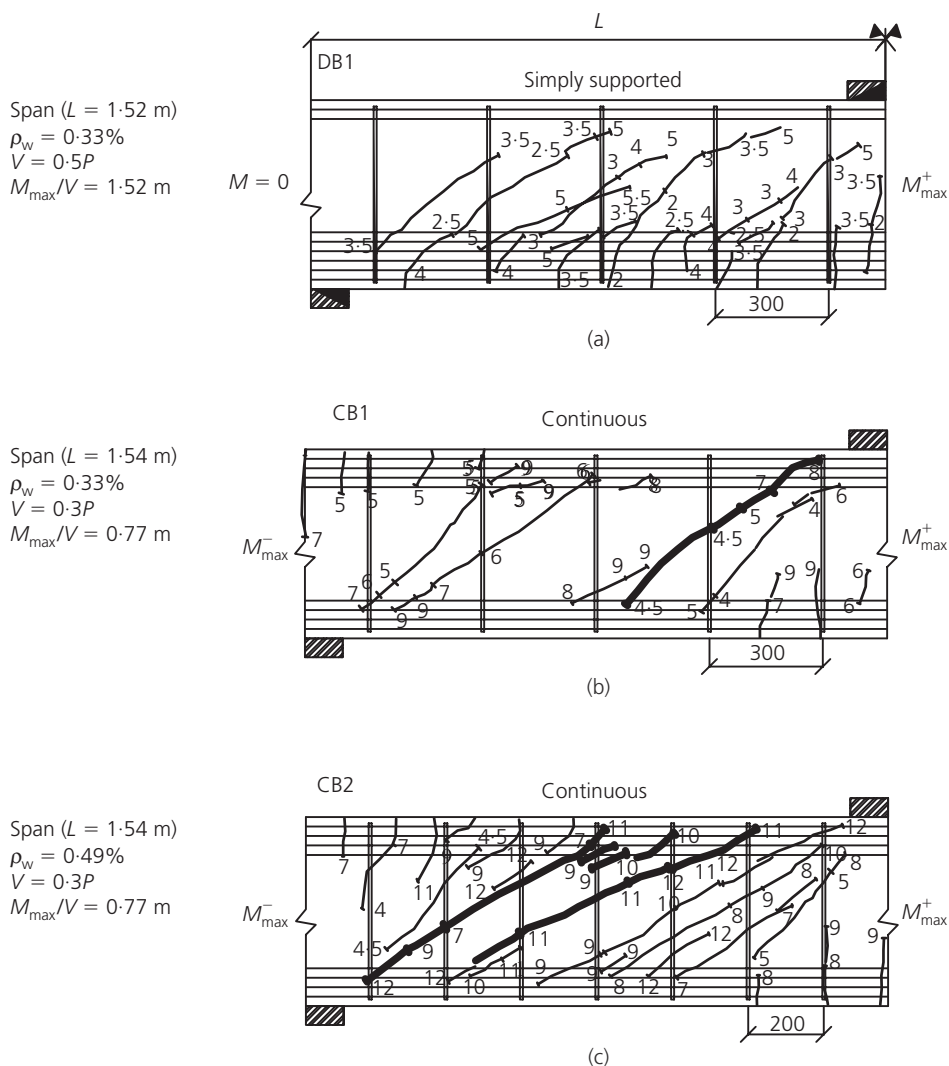strength
Sagaseta and Vollum

Figure 7. Typical crack pattern in beams: (a) DB1 (simply
supported); (b) CB1 (continuous); and (c) CB2 (continuous).
Note: numbers refer to load ($\times$ 100 kN) at which cracks were
first observed

spacing, θ is the angle between the compressive stress field and
the longitudinal axis of the beam, $1 \leqslant \cot\theta \leqslant 2 \cdot 5$ and z is the
lever arm for shear for which EC2 recommends a value of $0 \cdot 9d$.
EC2 defines the maximum shear capacity as follows for beams
with vertical stirrups

4. $\qquad V_{\mathrm{Rd,max}} = b_{\mathrm{w}} z \nu f_{\mathrm{cd}} / (\cot\theta + \tan\theta)$

where $\nu = 0 \cdot 6(1 - f_{\mathrm{ck}}/250)$ is a strength reduction factor for
concrete with skew cracks and $f_{\mathrm{cd}} = f_{\mathrm{ck}}/\gamma_{\mathrm{c}}$ is the design concrete
strength.

Table 4 shows that the shear strengths of the tested beams were
typically less than predicted by EC2 with $z = 0 \cdot 9d$ and partial

material factors γ_{c} and $\gamma_{\mathrm{s}} = 1 \cdot 0$. For example, the shear
strengths of beams BG1 and BG2, for which $\cot\theta$ is less than
$2 \cdot 5$, are overestimated by 15% and 22% respectively. Figure 8
shows that EC2 systematically underestimates the shear strength
of rectangular beams with SI $= \rho_{\mathrm{w}} f_{\mathrm{y}} / \nu f_{\mathrm{c}}' > \sim 0 \cdot 05$ when the
material partial factors equal 1. However, EC2 gives a reason-
able lower bound to the test data when shear strengths are
calculated with partial factors of $\gamma_{\mathrm{c}} = 1 \cdot 5$ for concrete and
$\gamma_{\mathrm{s}} = 1 \cdot 15$ for steel.

Comparison of shear strength predictions of EC2, BS 8110, MCFT and CSA A.23.3-04

The relative performance of the design methods in EC2, BS 8110
and CSA A.23.3-04 (CSA, 2004) was evaluated, with and without
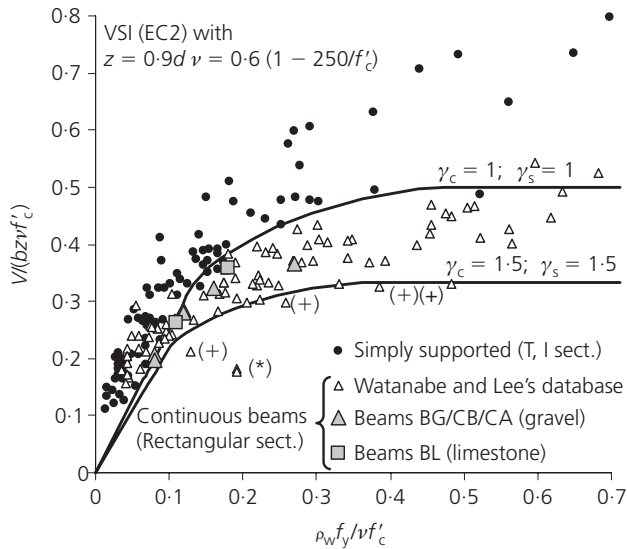material partial factors, for the beams tested in this programme

Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

Figure 8. Comparison of test data with shear strengths calculated
with EC2 for simply supported beams with 'T' and 'I' sections and
continuous beams with rectangular sections. Tests marked with
(*) failed prematurely due to bonding. Tests marked with (+) had
a very high concrete strength $f'_c = 140$ MPa. Simply supported
beams: Sörensen (1974), Regan and Reza-Jorabi (1987), Placas
(1969), Leonhardt and Walther (1961), Moayer and Regan (1974),
Hamadi and Regan (1980), Muhidin and Regan (1977), Levi and
Marro (1993), Walraven and Stroband (1999)

and 62 continuous beam tests from Figure 8 where sufficient data
were available to apply the CSA method. The shear strength of
the authors' beams was also calculated with the MCFT using
Response 2000 (Bentz, 2000). The 62 beams consisted of 54 from
Watanabe and Lee's database (Watanabe and Lee, 1998), seven
from the normal-weight concrete tests by Ramirez *et al.* (2004)
and one from Regan (1971). The concrete cylinder strengths in
the 62 beam tests considered varied from 21·2 MPa to 75·2 MPa.
The maximum possible design shear strength was taken as $0\cdot8\sqrt{f_{cu}}$
in BS 8110 but not more than 8 MPa as recommended in Concrete
Society technical report 49 (Concrete Society, 1998).

Table 4 presents results for the beams tested in this programme.
It shows that the MCFT predicted the strengths most accurately
and BS 8110 most conservatively. As expected, the predictions of
CSA A.23.3-04 were more conservative than the rigorous im-
plementation of the MCFT in Response 2000 (Bentz, 2000).
Reducing the aggregate size to zero in Response 2000 only
reduced the predicted strength of the BL beams with limestone
aggregate by around 2% since the concrete component of shear
resistance (V_c) was relatively small compared with the stirrup
component (V_s). These results are consistent with the authors' test
data which, as previously discussed, show no noticeable influence
of aggregate fracture on the shear strengths of beams with shear
reinforcement.

Figures 9(a) and 9(b) show results for all the beams with material
partial factors equal to 1. The figures show that BS 8110 and
CSA A.23.3-04 performed reasonably well independently of the
stirrup index but EC2 overestimated the strength of a significant
number of beams. Figure 9(c) suggests that the accuracy of the
shear strength predictions is relatively independent of the stirrup
yield strength for strengths as high as 1478 MPa. Table 5(a)
shows that EC2 (with $\gamma_c = \gamma_s = 1\cdot0$) underestimates the strength
of 67% of the beams with SI < 0·5 which is the maximum SI at
which stirrups yield in EC2. Furthermore, BS 8110 and CSA
A.23.3-04 performed significantly better giving similar values for
V_{test}/V_{calc}. Table 5(a) and Figure 9 also show that the EC2
predictions improve significantly for SI > 0·05 if the lever arm z
is taken as 0·8d in Equations 3 and 4. The corresponding flexural
lever arms were calculated for comparison using the rectangular
parabolic stress block in EC2 and were found to be 0·86d for the
BL and BG beams. The mean flexural lever arm was 0·84d for
the beams of Watanabe and Lee (1998) with a coefficient of
variation of 2·4%.

Comparison of design shear strengths

A direct comparison of the shear capacities without partial factors
is somewhat misleading since their effect varies between the
codes and depends on the proportion of shear stress resisted by
the shear reinforcement. For example, the shear strength is
independent of the concrete strength in EC2 if $\cot\theta = 2\cdot5$, and is
only reduced by the multiple $1/\gamma_s = 0\cdot87$ (where γ_s is the partial
factor for steel) when partial material factors are introduced. The
reduction in strength due to partial factors can be significantly
greater in CSA A.23.3-04 and BS 8110 since V_c reduces by
multiples of 0·65 and 0·8 respectively. Therefore, notional overall
FOS equal to V_{test}/V_{design} (where V_{design} is the design ultimate
shear resistance divided by the overall load factor) were calcu-
lated for all the continuous beams in Figure 9 using EC2, BS
8110 and CSA A.23.3-04 assuming equal dead and imposed
loads. The results are plotted against the stirrup index in Figure
10 which lists the partial load and material factors used for each
code. Table 5(b) shows that EC2 gives significantly lower FOS
than BS 8110 or CSA A.23.3-04 for 0·05 < SI < 0·25 if z is
taken as 0·9d as recommended in the code.

Influence of flexural reinforcement ratio on design shear strength

Shear strengths calculated with EC2 are independent of the
flexural reinforcement ratio ($\rho_l = A_s/bd$) for beams with shear
reinforcement unlike strengths calculated with BS 8110 or CSA
A.23.3-04. Shear strengths calculated with CSA A.23.3-04 reduce
with increasing strain in the flexural reinforcement at the critical
section, which is typically at a distance d from the loading point
for the beams considered in this paper. Figure 9(d) shows the
influence of the longitudinal strain at the mid-height of the
critical section (as defined in CSA A.23.3-04) on V_{test}/V_{calc} for
EC2 and CSA A.23.3-04. Figure 9(d) suggests that EC2 may
progressively overestimate shear strength as a result of neglecting
the influence of longitudinal strain but this was not the case for

Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

| Beam | $SI = \rho_w f_y / \nu f_c'$ | V_{test}: kN | VSI (EC2) | | BS 8110 | Resp. 2000 | MCFT (CSA) | |
			V_{test}/V_{calc}	$\cot\theta$	V_{test}/V_{calc}	V_{test}/V_{calc}	V_{test}/V_{calc}	$\cot\theta$
BG1	0·16	285·19	**0·87**	2·26	1·19	**0·98**	1·04	1·49
BG2	0·27	322·24	**0·82**	1·63	**0·93**	**0·82**	**0·84**	1·38
BL1	0·11	350·73	**0·97**	2·50	1·38	1·06	1·22	1·48
BL2	0·18	478·18	**0·93**	2·13	1·32	1·04	1·22	1·38
CA1	0·08	293·96	**0·95**	2·50	1·21	**0·97**	1·09	1·49
CB1	0·08	308·80	1·00	2·50	1·27	1·02	1·14	1·49
CA2	0·12	418·66	**0·90**	2·50	1·34	1·07	1·23	1·42
CB2	0·12	428·71	**0·93**	2·50	1·38	1·09	1·25	1·42
DA1*	0·08	311·37	1·07	2·50	1·35	1·07	1·29	1·43
DB1*	0·08	299·22	1·03	2·50	1·29	1·02	1·24	1·43
		Avg.	**0·95**		1·27	1·01	1·16	
		SD	0·07		0·13	0·08	0·14	
	COV: %		7·81		10·65	8·03	11·75	

Notes:
* All beams continuous except DA1 and DB1
No partial material factors were applied
EC2 (VSI), $z = 0.9d$
BS 8110, the cube strength was taken as $1.25 f_c'$ with no limitation on f_{cu}
MCFT: Response 2000, base curve (Popovics–Thorenfeldt–Collins); comp. softening (Vecchio and Collins, 1986); tens. stiffening (Bentz, 2000); aggregate size 10 mm; no strain hardening considered for steel
MCFT: CSA, crack spacing $s_{ze} = 300$ mm; $M_f \geq Vd_v$

Table 4. Measured and predicted shear strengths for beams with shear reinforcement (bold type signifies that V_{test}/V_{calc} is less than 1)

the authors' continuous beams, which are highlighted in Figure 9(d). It is, however, striking that the critical shear cracks in the authors' tests (see Figure 7) passed through the critical section assumed in CSA A.23.3-04 at d from the loading point. Further tests, in which just the area of flexural reinforcement is varied, are required to establish the influence of axial strain definitively.

The influence of the flexural reinforcement ratio on the code predictions was explored in a series of parametric studies on continuous beams with the dimensions used in the tests of (a) Ramirez et al. (2004) (with $f_{ck} = 46$ MPa) and (b) this programme (with $f_{ck} = 40$ MPa). The concrete strengths were chosen to be representative of those used in the corresponding test programmes. The following analyses were made for beams with each configuration. First, the flexural reinforcement was designed to yield at the design ultimate load which was assumed to be governed by the shear strength calculated with EC2. Second, the area of flexural reinforcement was increased by a multiple of 2 to replicate tests in which excess flexural reinforcement is provided. Third, shear strengths were calculated with the flexural reinforcement ratio of $100A_{sl}/bd = 3.3$ used in the BG and BL beams tested in this programme.

The results (for $\gamma_c = \gamma_s = 1.0$) are presented in Figures 11(a) and 11(b) along with test data from Ramirez et al. (2004) and this programme. The figures show that the difference between the EC2 and CSA A.23.3-04 predictions depends on the strain in the flexural reinforcement and is greatest when (a) the area of reinforcement provided for flexure equals that required for strength and (b) the calculated value of $\cot\theta$ equals 2·5 in EC2 (i.e. when $SI = \rho_w f_y / \nu f_c' \sim 0.14$). Figure 11(a) shows that EC2 predicts the shear strength of the Ramirez et al. (2004) beams, in which $SI = \rho_w f_y / \nu f_c'$ was less than 0·1, reasonably well. Figure 11(b) shows that EC2 tends to overestimate the shear strength of the authors' beams which were more heavily reinforced in shear and that furthermore the strength increased almost linearly in proportion with $SI = \rho_w f_y / \nu f_c'$ as predicted by CSA A.23.3-04. Figure 11(b) shows that the MCFT (Response 2000) accurately predicts the influence of SI on the shear strength of the tested beams with the exception of beam BG2 which had a surprisingly low strength. The inset to Figure 11(b) shows that the orientation of the inclined compressive stress field assumed in EC2 is typically skew to the cracks. It follows that the validity of the VSI model in EC2 is dependent on shear stresses being transferred through the crack. Sagaseta (2008) has shown that the trend of test data in Figure 11(b) is consistent with the predictions of the EC2 design model if the compressive strength of the concrete in the inclined stress field is limited by shear transfer through the cracks as illustrated diagrammatically in Figure 11(b).
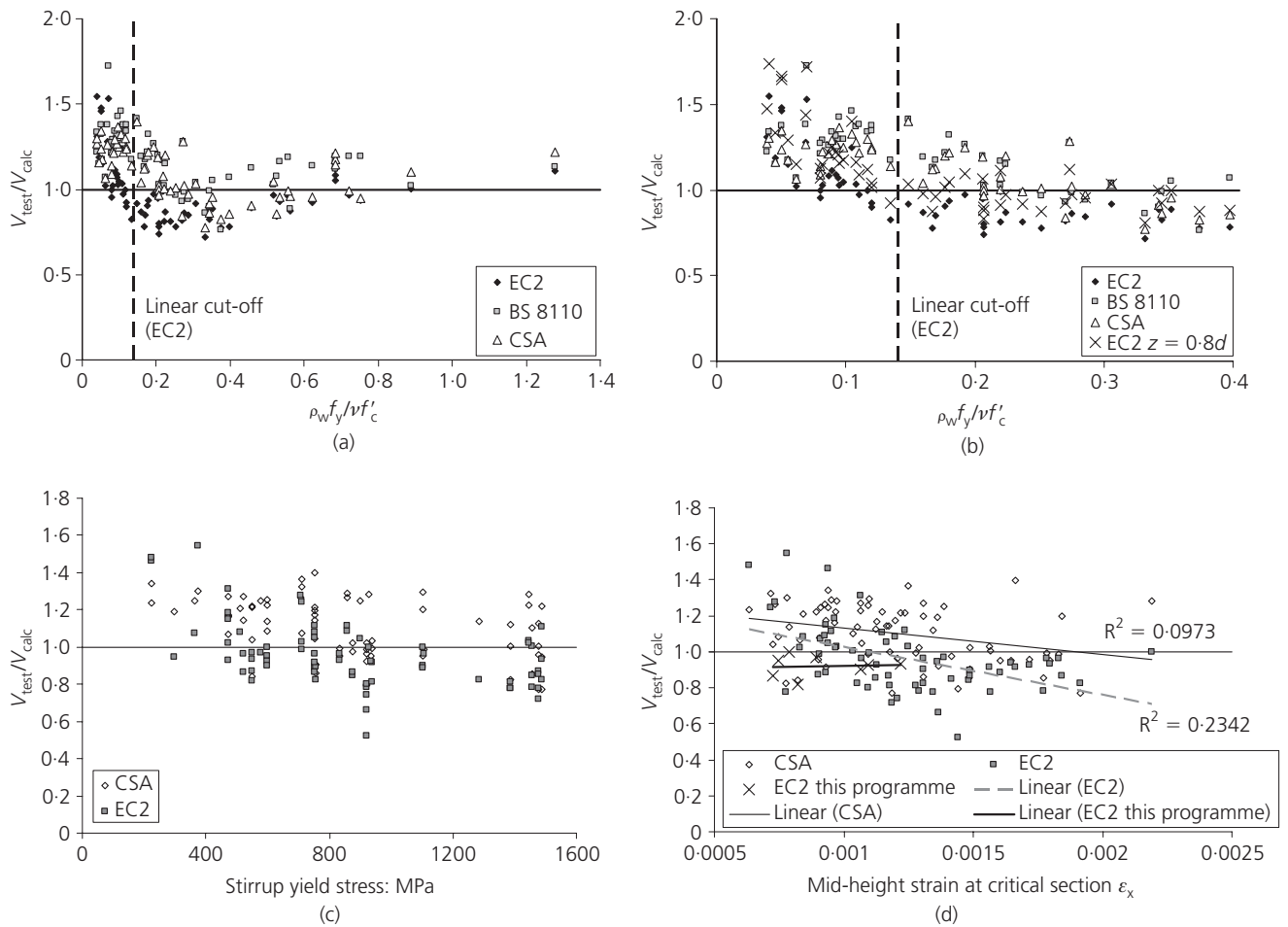
Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

Figure 9. Comparison of test data with shear strengths
calculated with EC2, BS 8110 and CSA for 62 continuous beams
($\gamma_c = \gamma_s = 1 \cdot 0$): (a) all beams (Watanabe and Lee (1998), Ramirez
et al. (2004), Regan (1971) and continuous beams tested in this
work); (b) practical subset of data from (a); (c) influence of stirrup
yield strength on V_{test}/V_{calc}; and (d) influence of mid-height axial
strain on V_{test}/V_{calc}

Figure 12 shows maximum permissible (i.e. unfactored) design
shear stresses calculated with each code for the beam considered
in Figure 11(b). The permissible shear stresses were calculated
for equal dead and imposed loads with the load factors appro-
priate to each code listed below Figure 10. The area of flexural
reinforcement was taken as (a) A_{sreq}, (b) $2A_{sreq}$ and (c)
$A_s = 0 \cdot 033bd$. Figure 12(a) shows that BS 8110 and CSA A.23.3-
04 tend to give significantly lower permissible shear stresses than
EC2 with $z = 0 \cdot 9d$ when $A_{sprov} = A_{sreq}$ (where A_{sprov} is the area of
flexural reinforcement provided and A_{sreq} is the area required for
strength) and $SI = \rho_w f_y / \upsilon f_c' > 0 \cdot 05$. Conversely, Figure 12(b)
(with $A_s/bd = 3 \cdot 3\%$) shows that EC2 (with $z = 0 \cdot 9d$) can give
lower design shear strengths than CSA A.23.3-04 or BS 8110
when $SI < 0 \cdot 05$.

Design recommendations for EC2

The EC2 design method for shear is safe for all the beams in
Figure 10 in the sense that the minimum FOS (calculated with
mean material strengths) is greater than the mean load factor of
$1 \cdot 43$. It is, however, striking that EC2 gives significantly lower
FOS for shear than BS 8110 or CSA A.23.3-04. The authors
believe that the FOS should be greater for shear than flexural
failure, when the areas of shear and flexural reinforcement equal
that required for strength, owing to the sudden and potentially
catastrophic nature of shear failure. CSA A.23.3-04 appears to
share this philosophy (Collins *et al.*, 2008).

The FOS for flexural failure in EC2 (FOS $= M_{test}/M_{design}$ where
M_{design} is the design ultimate moment of resistance divided by the

Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

V_{test}/V_{calc}	SI < 0.5	SI < 0.05		0.05 < SI < 0.14		0.14 < SI < 0.25		0.25 < SI < 0.5	
Code	% < 1.0[+]	Mean	Cov	Mean	Cov	Mean	Cov	Mean	Cov
EC2	67	1.35	0.14	1.08	0.17	0.87	0.09	0.85	0.09
EC2, $z = 0.8d$	38	1.52	0.14	1.21	0.17	0.97	0.09	0.95	0.09
CSA	22	1.24	0.06	1.23	0.06	1.12	0.11	0.94	0.14
BS 8110	18	1.24	0.08	1.31	0.10	1.15	0.11	0.99	0.13

Note: [+] % of specimens with $V_{test}/V_{calc} < 1.0$ for SI < 0.5

Table 5a. Statistical analysis of data in Figure 9 for beams with SI < 0.5 ($\gamma_c = \gamma_s = 1.0$)

FOS	SI < 0.5	SI < 0.5	SI < 0.05		0.05 < SI < 0.14		0.14 < SI < 0.25		0.25 < SI < 0.5	
Code	Min	% < 1.64*	Mean	Cov	Mean	Cov	Mean	Cov	Mean	Cov
EC2	1.42	35	2.21	0.14	1.79	0.16	1.67	0.09	1.72	0.09
EC2, $z = 0.8d$	1.60	5	2.48	0.14	2.02	0.16	1.89	0.09	1.93	0.09
CSA	1.45	16	2.11	0.07	2.04	0.07	1.80	0.11	1.73	0.10
BS 8110	1.44	5	2.21	0.05	2.34	0.10	2.03	0.11	1.85	0.13

Note: * percentage of specimens with FOS < 1.64

Table 5b. Statistical analysis of data in Figure 10 (with partial load and material factors) for beams with SI < 0.5
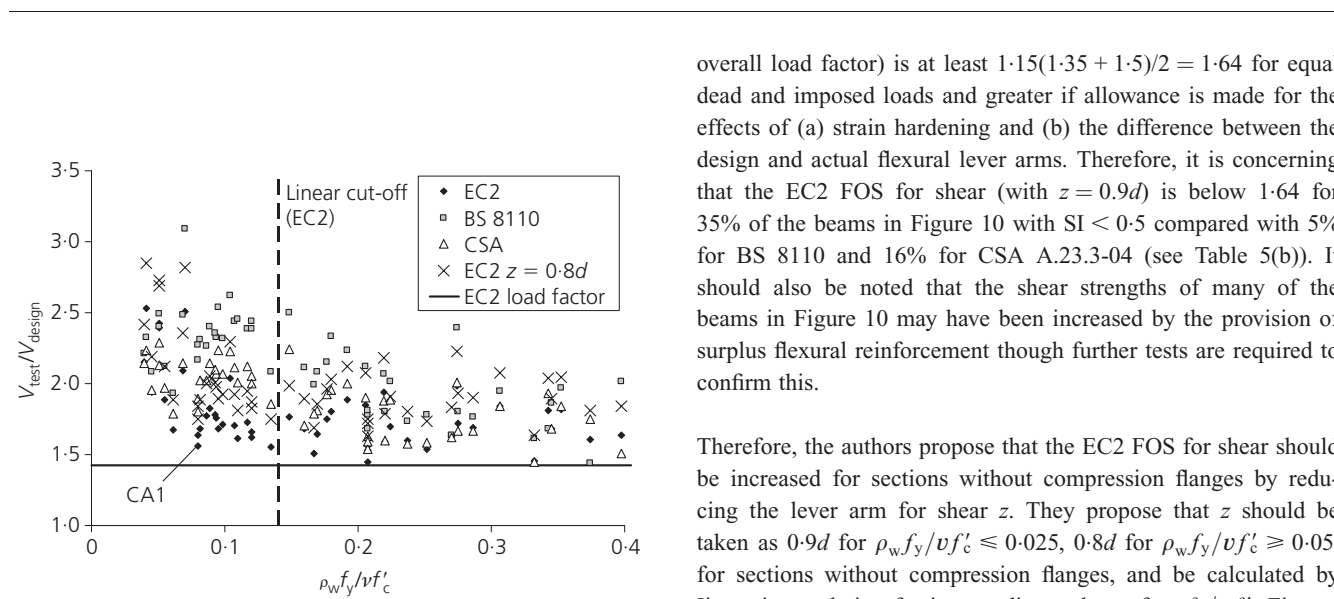

Figure 10. Overall factors of safety (V_{test}/V_{design}) for continuous beams from Figure 9 calculated with EC2, BS 8110 and CSA. Notes: Design values assumed. Materials factors of safety (γ_c/γ_s): EC2 (1.5/1.15); BS 8110 (1.25/1.05); CSA (1.54/1.18). Load factors of safety (dead load = live load): EC2 (1.35 + 1.5)/2; BS 8110 (1.4 + 1.6)/2; CSA (1.25 + 1.5)/2

overall load factor) is at least $1.15(1.35 + 1.5)/2 = 1.64$ for equal dead and imposed loads and greater if allowance is made for the effects of (a) strain hardening and (b) the difference between the design and actual flexural lever arms. Therefore, it is concerning that the EC2 FOS for shear (with $z = 0.9d$) is below 1.64 for 35% of the beams in Figure 10 with SI < 0.5 compared with 5% for BS 8110 and 16% for CSA A.23.3-04 (see Table 5(b)). It should also be noted that the shear strengths of many of the beams in Figure 10 may have been increased by the provision of surplus flexural reinforcement though further tests are required to confirm this.

Therefore, the authors propose that the EC2 FOS for shear should be increased for sections without compression flanges by reducing the lever arm for shear z. They propose that z should be taken as $0.9d$ for $\rho_w f_y/\upsilon f_c' \leqslant 0.025$, $0.8d$ for $\rho_w f_y/\upsilon f_c' \geqslant 0.05$, for sections without compression flanges, and be calculated by linear interpolation for intermediate values of $\rho_w f_y/\upsilon f_c'$. Figures 9 and 10 and Tables 5(a) and 5(b) show that this recommendation significantly improves the EC2 shear strength predictions for the rectangular beams studied in this paper. The effect of the proposed amendment is illustrated in Figures 11 and 12 which show that the proposed amendment to EC2 still allows greater shear strengths than CSA A.23.3-04 or BS 8110 when $A_{sprov} = A_{sreq}$ for flexure.
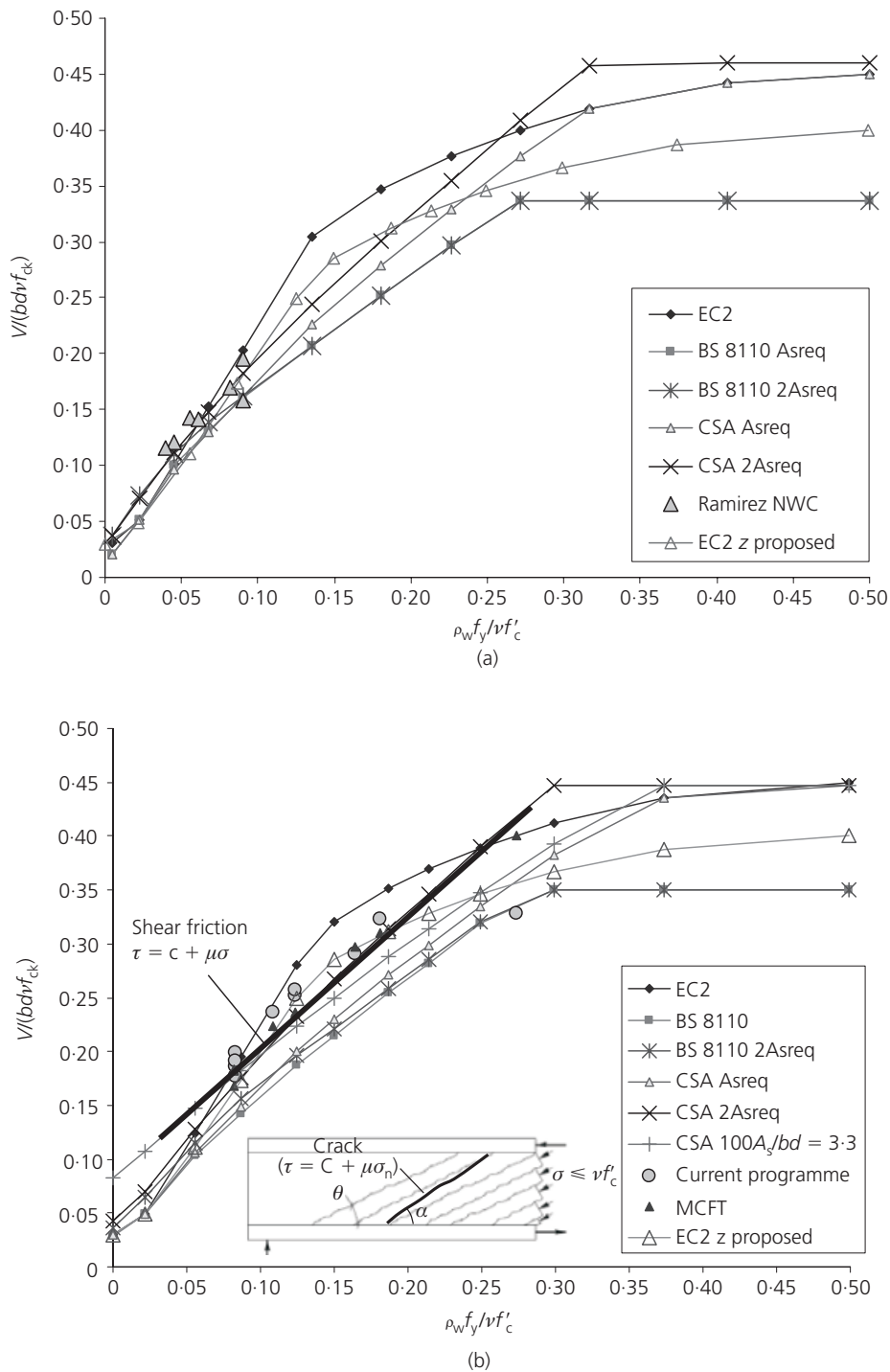
Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

Figure 11. Influence of stirrup index (SI $= \rho_w f_y / v f'_c$) on shear strength predicted by EC2, CSA and BS 8110 for beams with geometry used in tests of (a) Ramirez *et al.* (2004); and (b) this programme

Conclusions

The experimental results presented in this paper show that the shear strength of beams without shear reinforcement is reduced by aggregate fracture. Similar findings have been reported by Regan *et al.* (2005) among others. The UK national annex to EC2 (BSI, 2004) accounts for aggregate fracture by limiting the concrete cylinder strength to 50 MPa in the design equations for shear. This restriction was found to be necessary but only just
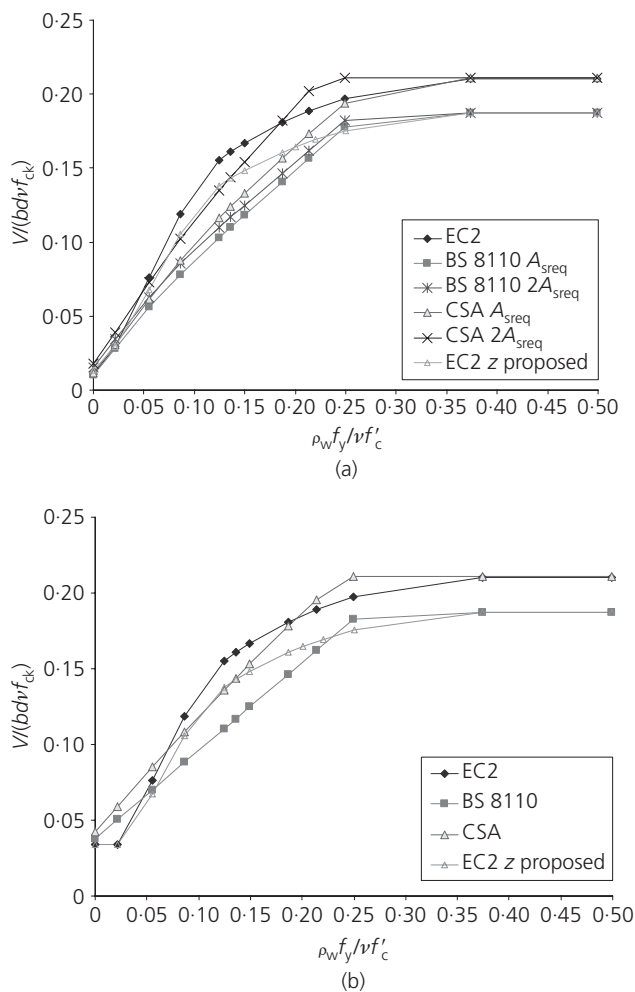
Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

Figure 12. Influence of stirrup index (SI $= \rho_w f_y / v f_c'$) on the
characteristic design shear strength for EC2, CSA and
BS 8110 with: (a) $A_{\mathrm{sprov}} = A_{\mathrm{sreq}}$ and $A_{\mathrm{sprov}} = 2A_{\mathrm{sreq}}$; and
(b) $100A_s/bd = 3.3\%$.

project if the lever arm for shear is taken as $0.9d$ as recom-
mended in the code.

Sixty-two continuous beams with rectangular cross-sections were
analysed to compare the factors of safety (FOS) for shear failure
implicit in EC2, BS 8110 and CSA A.23.3-04. The analysis
showed (see Table 5(b)) that EC2 can have a lower FOS for shear
in rectangular sections than flexure when SI $= \rho_w f_y / v f_c' > 0.05$
which seems undesirable. The authors believe that the FOS for
shear should be greater than for flexure (when the areas of
flexural and shear reinforcement equal that required for strength)
as shear failure occurs suddenly with little if any warning.
Therefore, the authors propose that the FOS for shear should be
increased in sections without compression flanges by reducing the
lever arm for shear from $z = 0.9d$ to $z = 0.8d$ for sections with
$\rho_w f_y / v f_c' \geq 0.05$. This amendment makes the FOS for shear in
EC2 more comparable to that in CSA A.23.3-04.

Acknowledgements

REFERENCES
Angelakos D, Bentz EC and Collins MP (2001) Effect on concrete
strength and minimum stirrups on shear strength of large
members. *ACI Structural Journal* **98(3)**: 290–300.

Bentz EC (2000) *Section Analysis of Reinforced Concrete
Members*. PhD thesis, University of Toronto.

BSI (British Standards Institution) (2004) European Standard
EN-1992-1-1:2004. Eurocode 2: Design of concrete
structures. Part 1: General rules and rules for buildings. BSI,
London.

BSI (2005) UK national annex to Eurocode 2: design of concrete
structures. Part 1: General rules and rules for buildings. BSI,
London.

BSI (2007) BS 8110, Part 1:1997: Structural use of concrete: code
of practice for design and construction. BSI, London.

Collins MP, Bentz EC, Sherwood EG and Xie L (2008) An
adequate theory for the shear strength of reinforced concrete
structures. *Magazine of Concrete Research* **60(9)**: 635–650.

Concrete Society (1998) *Design Guidance for High Strength
Concrete*. Concrete Society, Camberley, Technical report 49.

CSA (Canadian Standards Association) (2004) CSA A.23.3-04:
Design of concrete structures. CSA, Ontario.

Hamadi YD and Regan PE (1980) Behaviour of normal and
lightweight aggregate beams with shear cracks. *The
Structural Engineer* **58B(4)**: 71–79.

Lachemi M, Hossain KMA and Lambros V (2005) Shear resistance
of self-consolidating concrete beams – experimental
investigations. *Canadian Journal of Civil Engineering*
32(6):1103–1113.

Leonhardt F and Walther R (1961) *The Stuttgart Shear Tests
1961*. A translation of the articles that appeared in *Beton und
Stahlbetonbau*, 1961, 56, No. 12, and 1962, 57, No. 2, 3, 6, 7

sufficient for the safe prediction of the design shear strength of
the authors' beams without stirrups. The authors found no
evidence that the shear strength of their beams with stirrups was
reduced by aggregate fracture.

The authors tested ten rectangular beams with stirrups of which
eight were continuous. The shear strengths of the tested beams
are shown to be consistent with the strengths of similar beams
tested in Japan but significantly less than the strengths of the 'I'
and 'T' section beams used to validate the EC2 design method for
shear (Walraven, 2004). The increased strength of the 'I' and 'T'
sections (10–20%) results from the contribution of the compres-
sion flange which was not subtracted from the overall shear
capacity in the validation of EC2 (Walraven, 2004). This is
significant since EC2 assumes, like BS 8110 and CSA A.23.3-04,
that shear is entirely carried by the web. EC2 tends to over-
estimate the shear strength of the rectangular beams tested in this

Magazine of Concrete Research
Volume 63 Issue 2

Influence of beam cross-section, loading
arrangement and aggregate type on shear
strength
Sagaseta and Vollum

and 8. Cement and Concrete Association Library Translation
No. 111, Wexham Springs, UK, December 1964.

Levi F and Marro P (1993) Shear tests on HSC prestressed beams
– proposals of new interpretative models. *Proceedings of a
Conference on High Strength Concrete*, Lillehammer, pp.
293–305.

Moayer H and Regan PE (1974) Shear strength of prestressed and
reinforced concrete T-beams. *ACI Special Publication* **42(1)**:
183–221.

Muhidin NA and Regan PE (1977) Chopped steel fibres as shear
reinforcement in concrete beams. *Proceedings of Conference
on Fibre Reinforced Materials – Design and Engineering
Applications*, ICE, London, pp. 135–149.

Muttoni A and Fernández Ruiz M (2008) Shear strength of
members without transverse reinforcement as function of
critical shear crack width. *ACI Structural Journal* **105(2)**:
163–172.

Muttoni A, Schwartz J and Thürlimann B (1989) *Design and
Detailing of Reinforced Concrete Structures Using Stress
Fields*. Swiss Federal Institute of Technology, Zürich.

Nielsen MP and Baestrup MW (1976) *Plastic Shear Strength of
Reinforced Concrete Beams*. Technical University of
Denmark, Lyngby.

Pansuk W and Sato Y (2007) Shear mechanism of reinforced
concrete T-beams with stirrups. *Journal of Advanced
Concrete Technology* **5(3)**: 395–408.

Placas A (1969) *Shear Strength of Reinforced Concrete Beams*. PhD
thesis, Imperial College of Science and Technology. London.

Ramirez JA, Olek J and Malone BJ (2004) *Shear Strength of
Lightweight Reinforced Concrete Beams*. American Concrete
Institute, Farmington Hills, Michigan, ACI-Special
Publication SP218-5, 202 pp.

Regan PE (1971) *Shear in Reinforced Concrete – An
Experimental Study*. Construction Industry Research and
Information Association (CIRIA), London, report.

Regan PE and Reza-Jorabi H (1987) The shear resistance of
reinforced concrete I-beams. *Studi e Ricorche*, Politecnico de
Milano **19**: 305–321.

Regan PE, Kennedy-Reid IL, Pullen AD and Smith DA (2005)

The influence of aggregate type on the shear resistance of
reinforced concrete. *The Structural Engineer* **8(23/24)**: 27–32.

Sagaseta J (2008) *The Influence of Aggregate Fracture on the
Shear Strength of Reinforced Concrete Beams*. PhD thesis,
Imperial College London, London.

Sagaseta J and Vollum RL (2011) Influence of aggregate fracture
on shear transfer through cracks in reinforced concrete.
Magazine of Concrete Research, in press.

SIA (Swiss Society of Engineers and Architects) (2003) *SIA Code
262 for Concrete Structures*. SIA, Zurich.

Sörensen HC (1974) *Shear Tests on 12 Reinforced Concrete
T-beams*. Structural Research Laboratory, Technical
University of Denmark, Lyngby, Report R60.

Taylor HPJ (1970) *Investigation of the Forces Carried Across
Cracks in Reinforced Concrete Beams in Shear by Interlock
of Aggregate*. Cement and Concrete Association, London,
technical report 42.447.

Vecchio FJ and Collins MP (1986) The modified compression-field
theory for reinforced concrete elements subjected to shear.
ACI Journal **83(2)**: 219–231.

Walraven JC (2004) Shear, background document for prENV
1992-1-1:2001. In *Eurocode 2 Background Documents from
ENV to EN 1992-1-1* (Whittle R (ed.)). The Concrete Centre,
Camberley.

Walraven JC and Al-Zubi N (1995) Shear capacity of lightweight
concrete beams with shear reinforcement. *Proceedings of
Symposium on Lightweight Aggregate Concrete*, Sandefjord,
Norway, vol. 1, pp. 91–104.

Walraven JC and Stroband J (1994) Shear friction in high-
strength concrete. *Proceedings of the ACI International
Conference on High Performance Concrete*, Singapore, SP-
149, pp. 311–330.

Walraven JC and Stroband J (1999) Shear capacity of high
strength concrete beams with shear reinforcement.
Proceedings of a Symposium on High Strength Concrete,
Sandefjord, Norway, vol. 1, pp. 693–700.

Watanabe F and Lee JY (1998) Theoretical prediction of shear
strength and failure mode of reinforced concrete beams. *ACI
Structural Journal* **95(6)**: 749–757.