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External beam–column joints: design to Eurocode 2

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In practice external beam–column joints are seldom designed for monotonic loading. The current authors believe that this is an oversight which should be addressed. This paper presents a simple strut and tie model for the analysis and design of external reinforced concrete beam–column joints. The strut and tie model is developed from first principles using the concrete design strengths given in Eurocode 2. The main difficulty in developing strut and tie models for beam–column joints is in determining the node dimensions. The novel feature of the authors' analysis is that the joint strength is related to the flexural capacity of the beam at the face of the column which is defined in terms of the maximum moment which can be transferred through the joint into the upper and lower columns. The model is shown to give better predictions of joint shear strength than existing simple design models. A case study is presented which shows that it is often sufficient to provide only minimum shear reinforcement in beam column joints.

Notation

A_{sw}	area of links in one plane within the top 5/8ths of the beam depth below the tensile beam reinforcement
b_c	column width
b_e	effective joint width which is assumed to equal the average width of the beam and column as commonly assumed $\leqslant 0{\cdot}85/0{\cdot}6b_b$
d_b	beam effective depth
d_c	effective depth of the column
f_{ywd}	design strength of the links
h_c	column depth
h_b	beam depth
L_b	distance to the point of contraflexure in the beam from the column face
L_c	distance between the points of contraflexure in the upper and lower columns
M_b	moment in the beam at the face of the column
M_{col}	moment in the column resisted by concrete at the top and bottom of the beam
s	spacing of the links within the joint
$SI = A_s f_{yd}/(\nu f_{cd} b_e h_c)$	the stirrup index
STM	strut and tie model
T_b	design force in the beam tension reinforcement at the face of the column
$T_{syd} = A_{sw} f_{yd}$	design yield capacity of the joint shear reinforcement within the top 5/8^{ths} of the beam depth below the tensile beam reinforcement
V_b	shear force in the beam
V_{col}	shear force in the column
V_j	shear force in the joint

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Introduction

The joint of a beam–column assembly sometimes limits the strength of a structure, with failures in shear observed in tests and earthquakes even when the members are adequately designed and the member reinforcement is detailed to pass right through the joint.[1] Therefore, the current authors believe that all beam–

column joints should be checked for shear. Links should normally be provided within beam–column joints in reinforced concrete structures to restrain the column bars against buckling. Links may also be occasionally required to increase the joint shear strength. Even if the joint shear strength is not critical, nominal links are advisable to increase the ductility of the joint and to provide crack control.

In practice external beam–column joints are seldom designed unless subject to seismic loading. This is no doubt the result of the lack of design guidance in Codes of Practice such as British Standard (BS) 8110[2] and Eurocode (EC)2.[3] American Concrete Institute (ACI) standard ACI 318-05[4] specifies a minimum area of stirrups to be provided in external beam–column joints but gives no limits on the joint shear stress. More detailed design guidelines are given by ACI/American Society of Civil Engineers (ASCE) Committee 352.[5]

The current paper presents a rational strut and tie model for the design of external beam–column joints which is consistent with the recommendations of EC2.[3] The model is a considerable improvement on Vollum and Newman's[6] earlier stut and tie model (STM) for external beam column joints which dimensioned the struts using non-rational calibration factors derived from back analysis of test data. The method is validated with test data and is shown to give consistent and safe designs. It is shown that the proposed STM gives significantly better estimates of joint shear strength than the design methods given by ACI/ASCE Committee 352[5] and in EC2[3] for shear in beams.

General principles

The member forces are usually determined from an elastic analysis of the frame using factored loads. In a braced frame, moment redistribution may be used to reduce the design hogging bending moments in the beams providing the span bending moments are also appropriately adjusted. The area of reinforcement required in the top of the beam should be based on the bending moment at the face of the column. It is not good practice, or necessary, to design the beam tension steel for the moments at the centre of the column.

Design shear force within a joint

The shear force within a joint may be calculated from the resultant forces acting on it at the joint boundaries. For an edge column, this is the design force in the beam flexural reinforcement minus the column shear above the joint.

$$V_j = T_b - V_{col} \tag{1}$$

where

V_j is the shear force in the joint

T_b is the design force in the beam tension reinforcement at the face of the column

V_{col} is the design shear force in the column above the joint

Strut and tie model for external beam–column joints

EC2[3] includes general recommendations on concrete strength in strut and tie models which are applicable to the design of beam–column joints. Fig. 1 shows an idealised strut and tie model for a beam–column joint without stirrups in which the stresses in the concrete are assumed to be equal on all faces of the nodes (i.e. hydrostatic). The STM is applicable to beam–column joints with aspect ratios h_b/h_c between 1 and 2. This restriction is of no consequence in practice since joints with $h_b/h_c < 1$ are likely to fail in flexure and joints with $h_b/h_c > 2$ can be designed using the variable truss method given in EC2 since the contribution of the direct strut disappears as h_b/h_c is increased above 2 to 2·5. The tensile force in the beam reinforcement is assumed to be transferred into the back of the column through a rigid plate whereas, in reality, the beam reinforcement is usually anchored with an L or U bar. The consequences of this assumption, which simplifies the analysis of the top node, are examined later but are not believed to be significant providing the beam reinforcement is bent down into the column with an adequate radius to avoid bearing failure and it is fully anchored past the beginning of the bend. The joint shear strength can be expressed in terms of the node dimensions in Fig. 1 as follows

$$V_j = b_e k \nu' f_{cd}(x - y) \tag{2}$$

where

$k\nu' f_{cd}$ is the concrete design strength given in EC2[3] which is given by

$$k\nu' f_{cd} = k(1 - f_{ck}/250) f_{ck}/\gamma_c \tag{3}$$

γ_c is the material factor of safety for concrete which is taken as 1·5 in EC2[3]

EC2 gives $k = 0{\cdot}6$ for concrete struts in cracked compression zones, $k = 0{\cdot}85$ for compression–tension nodes with anchored ties in one direction and $k = 0{\cdot}75$ for compression-tension nodes with anchored ties in more than one direction. In this paper, k is taken as 0·6 throughout for reasons discussed below and $k\nu'$ is replaced by $\nu = 0{\cdot}6(1-f_{ck}/250)$.

The effective joint width, b_e, is assumed to equal the average width of the beam and column as commonly assumed[5,7] but $\leqslant 0{\cdot}85/0{\cdot}6 b_b$ to limit compressive stresses in the beam to $0{\cdot}85(1-f_{ck}/250)f_{ck}/\gamma_c$ as required in EC2.[3]

The main difficulty in determining the joint shear strength is in determining the node dimensions and appropriate design concrete strengths at the node boundaries. The coefficient k in equation (3) is taken as 0·6 in the analysis of the STM to limit the stress in the direct strut to $0{\cdot}6(1-f_{ck}/250)f_{cd}$ in accordance with EC2.[3] The

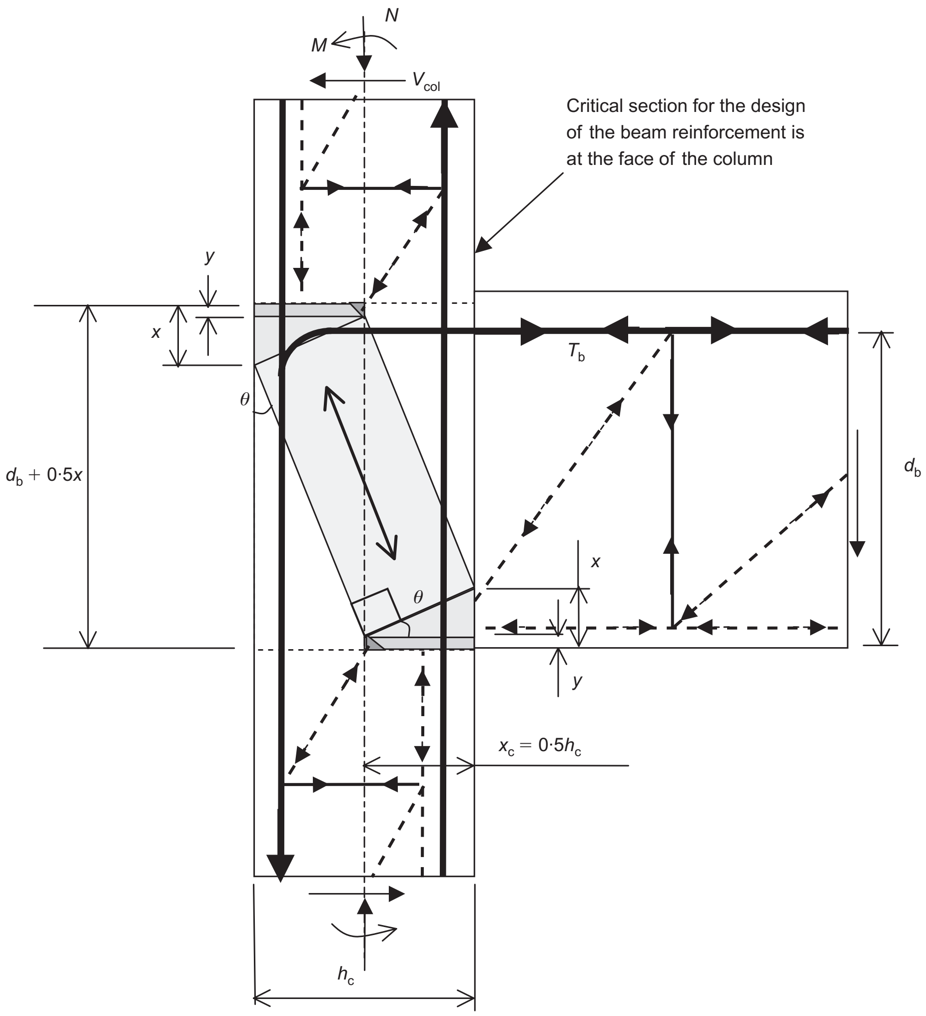

Fig. 1. Strut and tie model for external beam–column joint without joint stirrups

novel feature of the present authors' analysis is that the joint shear strength is related to the maximum moment that can develop in the beam at the face of the column, which is defined in terms of the maximum moment which can be transferred through the joint into the upper and lower columns. In the model presented, the width of the node (x_c in Fig. 1) is taken as half the column width to maximise the moment transferred into the columns through the concrete at the joint boundaries. It is also assumed that any transfer of vertical force between the column bars and the direct strut occurs behind the nodes within the column. This assumption is shown to be broadly in line with the available test data in the discussion of column bar forces later in this paper. It is assumed for simplicity in the development of the model below that the moments in the upper and lower columns are equal at the joint boundaries. It follows that in the absence of joint shear reinforcement, the maximum moment that can be transferred through the joint into the columns above and below the beam is given by

$$M_{col} = 0{\cdot}125 b_e h_c^2 \nu f_{cd} \tag{4}$$

The shear force in the beam V_b is assumed to be transferred into the lower node at the face of the column as shown in Fig. 2. The eccentricity of V_b with respect to the column centreline gives rise to an out-of-balance moment which is equilibrated by equal and opposite shear forces in the upper and lower columns equal to $0{\cdot}5V_b h_c/L_c$ (where L_c is the column length between the points of contra-flexure). The column shear forces of magnitude $0{\cdot}5V_b h_c/L_c$ are balanced by horizontal forces, resulting from flexure in the beam, at the top and bottom of the joint. It follows that the out-of-balance moment $0{\cdot}5V_b h_c/L_c$ only introduces vertical forces within the joint, which are assumed to act at the centroid of the column bars as shown in Fig. 2. The geometry of the strut and tie model is independent of the axial load in the column since axial equilibrium is maintained by adjusting the forces in the column bars. This assumption is consistent with the experimental data, which show no consistent relationship between joint shear strength and column axial load.[7] The depth of the node, which determines the joint shear strength, is determined by considering the geometry and equilibrium of the joint.

The STM is modified by the presence of joint shear

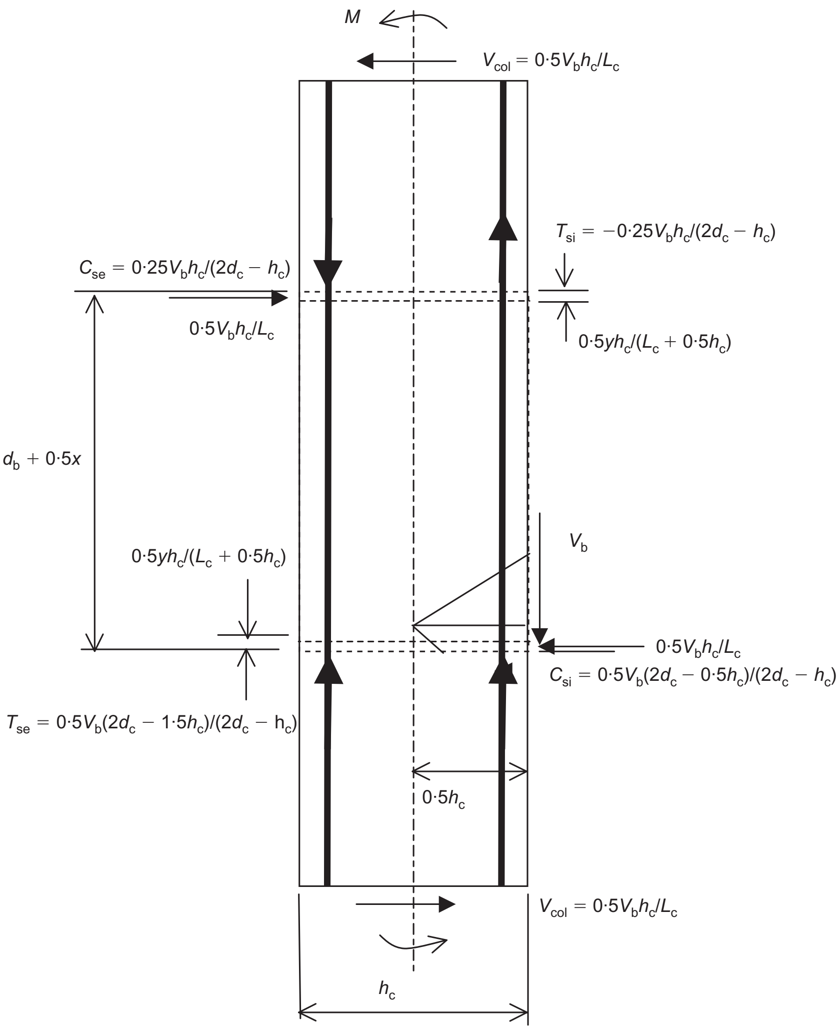

Fig. 2. Transfer of beam shear force into the column

reinforcement as shown in Fig. 3. Theoretically, stirrups increase joint shear strength in the STM if positioned within the central zone shown in Fig. 3 between the flexural compressive stress blocks in the beam. The experimental work of Hamil[8] and Reys de Ortiz[9] suggests this is too onerous a restriction and that that stirrups are effective in increasing joint shear strength if positioned between the tensile reinforcement and the top of the flexural compressive zone in the beam. Therefore, joint stirrups are considered to be effective in increasing joint shear strength if placed within the top 5/8$^{\text{ths}}$ of the beam depth below the tensile beam reinforcement as previously recommended by Vollum and Newman.[6,7] This assumption is justified in the context of the STM as

(*a*) the stirrup force is transferred into the joint through the column bars which allow stirrups to be mobilised even if not within the central region shown in Fig. 3

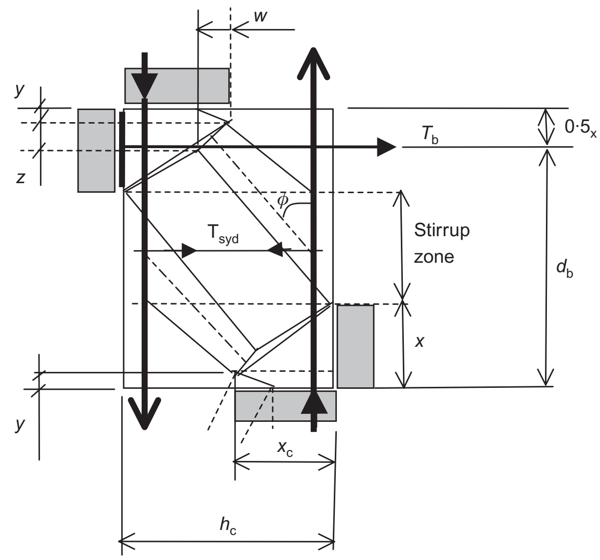

Fig. 3. Strut and tie model of idealised external beam–column joint with stirrups

(*b*) the efficiency of the direct strut is likely to be enhanced by the provision of stirrups within the depth of the radius of the bend in the beam reinforcement.

Joint stirrups increase joint shear capacity in the STM by increasing the maximum moment that can be transferred into the columns through the mobilisation of the column bars. The depth of the stress block in the beam is assumed to increase proportionately to maintain the hydrostatic state of stress in the nodes. Analysis shows that the predicted joint shear strength is almost insensitive to the position of the centroid of the stirrup force. Therefore, it is assumed for simplicity, in equation (5), that the centroid of the joint stirrups is at the mid-height of the beam. Joint stirrups increase the moment capacity of the column at the top and bottom of the beam by

$$\Delta M = (2d_c - h_c)\Delta T \tag{5}$$

where ΔT (see Fig. 3) is half the vertical force transferred from the inclined strut into the internal and external column bars. This assumes a symmetrical joint in which the column geometry does not change through the joint. The force ΔT is given by

$$\Delta T = 0{\cdot}5T_{syd}\cot\phi \tag{6}$$

where

$$T_{syd} = A_{sw}f_{yd}$$

where A_{sw} is the area of joint shear reinforcement within the top 5/8^{ths} of the beam depth below the tensile beam reinforcement and

$$\cot\phi = (d_b + 0{\cdot}5x - 2y - z)/(2d_c - h_c + w) \tag{7}$$

where

$$x = d_b(1 - \sqrt{(1 - 2M_b/(b_e d_b^2 \nu f_{cd}))}) \leqslant 0{\cdot}5h_b \tag{8}$$

$$y = M_b(1 + 0{\cdot}5h_c/L_b)/(L_c b_e \nu f_{cd}) \tag{9}$$

$$w = 2\Delta T/(b_e \nu f_{cd}) \leqslant 0{\cdot}5h_{col} \tag{10}$$

$$z = T_{syd}/(b_e \nu f_{cd}) \tag{11}$$

where L_c is the distance between the points of contraflexure in the upper and lower columns, L_b is the distance to the point of contraflexure in the beam from the column face and M_b is the moment in the beam at the column face. In accordance with clause 6·2·3 (8) of EC2[3] (which is concerned with the shear strength of beams with concentrated loads close to supports), the maximum permissible joint shear force is limited to

$$V_{jdmax} \leqslant 0{\cdot}45\nu f_{cd} b_c d_c/\gamma_m \tag{12}$$

where b_c is the full width of the column through the joint and d_c is the effective depth of the column within the joint.

Analysis to determine joint moment capacity

If the stirrup force T_{sy} is known, equation (6) can be expressed in the form

$$\Delta T = 0{\cdot}5(-\mathrm{b} + \sqrt{(\mathrm{b}^2 - 4\mathrm{c})}) \tag{13}$$

where

$$b = 0{\cdot}5(2d_c - h_c)\nu f_{cd} b_e \tag{14}$$

$$c = -0{\cdot}25T_{syd}(h^* - z)\nu f_{cd} b_e \tag{15}$$

where

$$h^* = d_b + 0{\cdot}5x - 2y \tag{16}$$

The maximum moment which can be transferred into the joint at the beam face M_b is given by

$$M_b = 2(M_{col} + \Delta M)/ (1 - (1 + 0{\cdot}5h_c/L_b)(d_b + 0{\cdot}5x - y)/L_c) \tag{17a}$$

$$\sim 2(M_{col} + \Delta M)/(1 - (1 + 0{\cdot}5h_c/L_b)d_b/L_c) \tag{17b}$$

where M_{col} is given by equation (4), ΔM by equation (5) and L_c is the distance between the points of contraflexure in the upper and lower columns. M_b can be calculated iteratively as follows.

Step 1: calculate ΔT with equation (13) assuming $h^* - z = d_b$ in equation (15).
Step 2: calculate $M_{col} + \Delta M$ with equations (4) and (5) respectively.
Step 3: calculate M_b with equation (17b) in the first iteration and with equation (17a) subsequently.
Step 4: calculate the node dimensions x and y with equations (8) and (9) respectively in terms of M_b from step 3.
Step 5: recalculate ΔT with equation (13) and the current value of h^*.
Step 6: repeat steps 2–5 until values of M_b from successive iterations converge.

Step 1 gives a reasonably accurate estimate of the joint moment capacity unless the joint is heavily reinforced in shear.

Design procedure for beam–column joints

Design joint shear reinforcement is required if

$$\Delta M_{design} = 0{\cdot}5M_{b\,design}(1 - (1 + 0{\cdot}5h_c/L_b)d_b/L_c) - 0{\cdot}125 b_e h_c^2 \nu f_{cd} b_e \geqslant 0 \tag{18}$$

where ΔM_{design} is the required increment in column moment capacity and $M_{bdesign}$ is the design moment in the beam at the column face. The required increment in column bar force ΔT is found by equating ΔM_{design} to ΔM which is defined in terms of ΔT in equation (5).

The design joint stirrup force can be calculated with equation (6), which relates the stirrup force to ΔT and $\cot\phi$. Substituting into equation (6) for $\cot\phi$ from equation (7) and rearranging gives

$$T_{\mathrm{syd}} = 0{\cdot}5(-b - \sqrt{(b^2 - 4c)}) \tag{19}$$

where

$$b = -h^* \nu f_{\mathrm{cd}} b_{\mathrm{e}} \tag{20}$$

$$c = 2\Delta M_{\mathrm{design}}(2d_{\mathrm{c}} - h_{\mathrm{c}} + w)\nu f_{\mathrm{cd}} b_{\mathrm{e}}/(2d_{\mathrm{c}} - h_{\mathrm{c}}) \tag{21}$$

where w and h^* can be calculated directly with equations (10) and (16) respectively.

It is assumed in equation (18) that the distances to the points of contra-flexure in the upper and lower columns from the centreline of the beam are equal and that the axial force in the beam is zero. If this is not the case, the maximum column moment at the top or bottom of the beam should not exceed $M_{\mathrm{col}} + \Delta M$ where M_{col} and ΔM are given by equations (4) and (5) respectively. This can be achieved by replacing L_{c} by $2L_{\mathrm{c}}^*$ where L_{c}^* is the minimum distance to the point of contraflexure in upper or lower column from the column centreline.

If the design joint shear force exceeds V_{jdmax} (see equation (12)), it can be reduced by moment redistribution. Alternatively, the column size or the concrete strength can be increased.

Vertical equilibrium

The design tensile force in the internal column bars immediately above the joint (see Fig. 2) is given by

$$\begin{aligned} T_{\mathrm{si}} = {} & 0{\cdot}5(N - 0{\cdot}5 b_{\mathrm{e}} h_{\mathrm{c}} \nu f_{\mathrm{cd}}) \\ & - 0{\cdot}25 V_{\mathrm{b}} h_{\mathrm{c}}/(2d_{\mathrm{c}} - h_{\mathrm{c}}) - \Delta T \end{aligned} \tag{22}$$

where ΔT is found by rearranging equation (5), N is the compressive force (positive) in the upper column and V_{b} is the shear force in the beam.

Comparison with test data and other design methods

The current authors' STM was validated with a data base of 38 beam–column joint specimens,[3] that are believed to have failed in joint shear, tested by Ortiz,[9] Taylor,[10] Scott,[11] Hamil,[8] Parker and Bullman[12] and Kordina[13] in which the beam reinforcement was anchored with L bars. All the specimens were similar in geometry to Fig. 1. Specimens with U bars were omitted from the data base since previous research[7] indicated that their joint shear strength was around 20% less than that of similar specimens with L bars. Twenty six of the specimens were reinforced with joint stirrups. Details of all the specimens except five tested by Hamil[8] are given in Table 1 of Vollum *et al.*[7] Details of the additional five specimens (C4PLN0, C7LN0, C7LN1, C7LN3 and C7LN5) are given by Hamil.[8] The geometry of Hamil's[8] C4 and C7 series of specimens, which was identical with that of Scott's[11] C4 and C7 series, is also given in Table 1 of Vollum and Newman.[7] The notation N0, N1 and so on defines the number of joint stirrups provided over the full depth of the beam. The joint aspect ratio $h_{\mathrm{b}}/h_{\mathrm{c}}$ was 1·4 in the C4 series and 2 in the C7 series. The partial material factors of safety for steel and concrete were taken as 1 throughout. Experimental joint shear strengths were calculated for the test specimens from reinforcement bar forces derived with a parabolic stress block.

Theoretical failure loads were calculated for all the specimens in the data base using the authors STM, the recommendations of ACI/ASCE Committee 352,[5] the empirical design method of Vollum and Newman,[7] the minimum energy model of Parker and Bullman[12] and the design methods for shear given in EC2[3] for beams with and without stirrups. Theoretical joint shear strengths were calculated for specimens without joint shear reinforcement using equation 6·2(a) in EC2[3] with a material factor of safety of 1 for concrete. The joint shear strength was increased by a factor $2d/a_{\mathrm{v}}$ (where $d = d_{\mathrm{c}}$ and $a_{\mathrm{v}} = 0{\cdot}8d_{\mathrm{b}}$) in accordance with clause 6·2·2 (6) in EC2. Joint shear strengths were calculated for specimens with joint shear reinforcement using the variable strut inclination method (VSI) in EC2 (equations (6·8) and (6·9)) with the largest permissible value of $\cot\theta$. The stirrup spacing was defined as $s = 0{\cdot}9d_{\mathrm{b}}/n$ where n is the number of stirrups in the column within the depth of the beam. The effective width of the joint was taken as the column width in all the analyses with EC2.

A statistical analysis of all the results is presented in Table 1, which compares the predictions of the current authors' STM with the predictions of ACI/ASCE Committee 352,[5] EC2[3] and the earlier models of Parker and Bullman[12] and Vollum and Newman.[7] All the models, tend on average to overestimate the strength of Parker and Bullman's[12] specimens 4b to 4f, which failed at comparatively low joint shear forces,[6,7,14] probably influenced by the high bearing stresses inside the bends. Separate statistical analyses are given in Table 1 for joints with joint stirrups and in Table 1 for joints without shear reinforcement excluding the specimens of Parker and Bullman,[12] which failed at comparatively low joint shear strengths. Table 1 shows that the accuracy of the STM proposed in the present paper is better than the authors' previous simple models[7,10] and considerably superior to the recommendations of EC2[3] for shear in beams and the recommendations of ACI/ASCE Committee 352.[5] The most significant feature of the STM proposed in this paper is that it was derived from first principals using the recommendations for strut and tie models given in EC2.[3]

Table 1. Statistical analysis

Analysis of all results

V_{jpred}/V_{jtest}	STM	Vollum and Newman[7]	Parker and Bullman[12]	ACI 352[5]	EC2[3]
Mean	0·95	0·98	0·83	1·17	0·52
SD*	0·13	0·26	0·16	0·41	0·21
COV†	0·14	0·26	0·20	0·36	0·40

Analysis of joints with stirrups

V_{jpred}/V_{jtest}	STM	Vollum[7]	Parker and Bullman[12]	ACI 352[5]	EC2[3]
Mean	0·94	0·90	0·80	0·99	0·53
SD	0·08	0·17	0·12	0·19	0·24
COV	0·08	0·19	0·15	0·20	0·46

Joints without stirrups (excluding Parker and Bullman[12])

V_{jpred}/V_{jtest}	STM	Vollum and Newman[7]	Parker and Bullman[11]	ACI 352[5]	EC2[3]
Mean	0·83	0·89	0·75	1·13	0·45
SD	0·09	0·07	0·11	0·12	0·12
COV	0·11	0·08	0·14	0·10	0·27

*Standard deviation
†Coefficient of variation

Discussion

Treatment of nodes

The most questionable assumption in the STM is the treatment of the top node where the tensile force in the beam reinforcement is treated as if it were transferred into the rear face of the column through a rigid plate. Despite this Table 1 shows that the STM gives reasonable predictions of joint shear strength. Hamil's[8] specimens C4PLN0 and C4ALN0 are particularly interesting in this regard. Both specimens were notionally identical except the beam reinforcement was anchored with a plate bearing onto the back face of the column in specimen C4PLN0 as assumed in the STM. Specimen C4PLN0 in which the reinforcement was anchored with a plate failed at a load 20% greater than specimen C4ALN0. The ratio of the measured and predicted failure loads was 0·75 for C4PLN0 and 0·89 for C4ALN0. It appears that adopting a relatively low concrete design strength of $0{\cdot}6(1-f_{ck}/250)f_{cd}$ compensates for the poor anchorage of the beam reinforcement at the top node. The influence of the radius of the bend on the joint shear strength was assessed by plotting the ratio of the straight length of reinforcement between the column face and the start of the bend to the column depth. The results are shown in Fig. 4, which does not show any evidence that the joint shear is influenced by the ratio of the radius of the bend to the column depth provided bearing failure does not occur as may have been the case in the specimens of Parker and Bullman.[12]

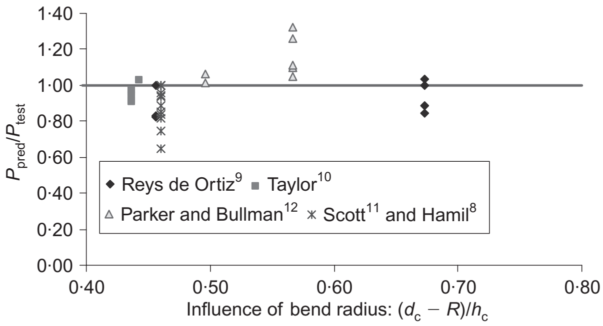

Fig. 4. Influence of radius of bend on accuracy of predicted failure loads given by STM

Analysis of the test data showed that the width of the indirect strut (i.e. for joints with stirrups) can be underestimated at the column reinforcement if the limiting concrete strength is assumed to be $0{\cdot}6(1-f_{ck}/250)f_{cd}$ on the vertical node boundaries. In these cases, the geometry of the STM is improved within the joint by increasing the limiting stress on the vertical node boundaries to $0{\cdot}85(1-f_{ck}/250)f_{cd}$ in which case the stress distribution within the nodes is no longer hydrostatic. In practice, this refinement is unnecessary if the joint shear strength is limited by equation (12) since increasing the permissible stress on the vertical node boundaries from $0{\cdot}6(1-f_{ck}/250)f_{cd}$ to $0{\cdot}85(1-f_{ck}/250)f_{cd}$ has almost no effect on the predicted failure load. Therefore, it is proposed that k is taken as 0·6 throughout in equation (3).

Vertical equilibrium

A comparison was made between the measured and predicted tensile forces in the internal column bars immediately above the joint for the test data of Ortiz,[9] Scott[11] and Hamil.[8] The measured forces were derived from strain measurements at or near joint failure. Analysis showed that the measured tensile strains in the internal column bars above the joint were significantly greater than predicted assuming plane sections remain plane. Figs 5 and 6 show that the proposed STM gives more realistic predictions of the tensile forces in the column bars above the joint, which are critical for design, than flexural analysis assuming plane sections remain plane. The STM does not provide any information on the compressive forces in the column bars at the nodes since the distribution of compressive force between the concrete and the reinforcement is indeterminate. However, the good correspondence between the measured and predicted tensile forces supports the assumption that no vertical force is transferred into the direct strut from the column bars within the joint depth.

Consistency of predictions of STM

The STM has been examined for consistency by plotting the ratio of the measured and predicted failure loads against the key parameters believed to influence joint shear strength which are the stirrup index $SI = A_{sw}f_y/(b_e h_c \nu f_{ck})$, the concrete strength and the joint aspect ratio h_b/h_c. The STM is shown to give realistic and consistent predictions of joint strength in Figs 7–9 in which the ratio P_{pred}/P_{test} is plotted against

(a) the stirrup index SI,
(b) the concrete strength and
(c) the joint aspect ratio.

The specimens of Parker and Bullman[12] which failed in flexure in the upper column are included in Fig. 7 to demonstrate the adequacy of equation (22) for calculating T_{si}. Fig. 10 shows that the STM gives good predictions of the influence of stirrups on the shear force carried by the direct strut in the tests of Scott[11] and

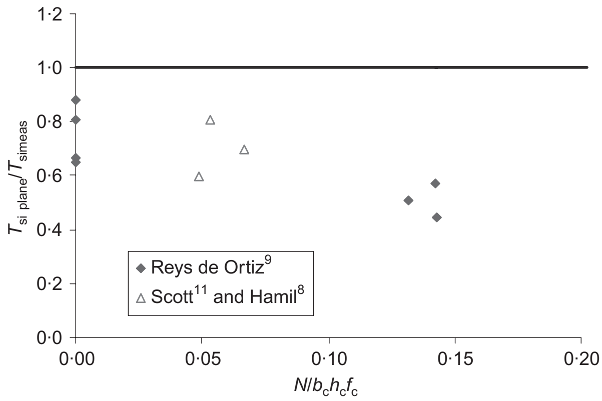

Fig 5. Comparison between measured column bar forces and forces calculated assuming plane sections remain plane

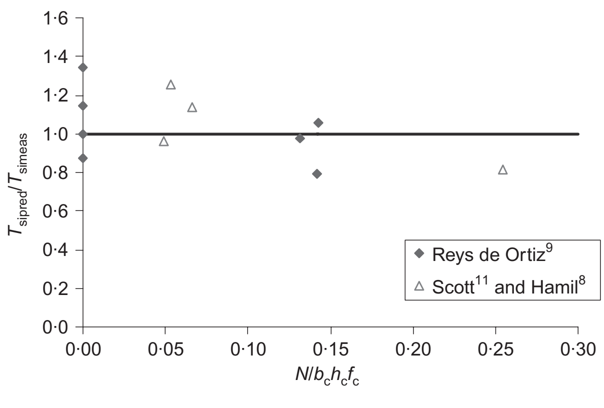

Fig 6. Comparison between measured column bar forces and forces calculated with STM

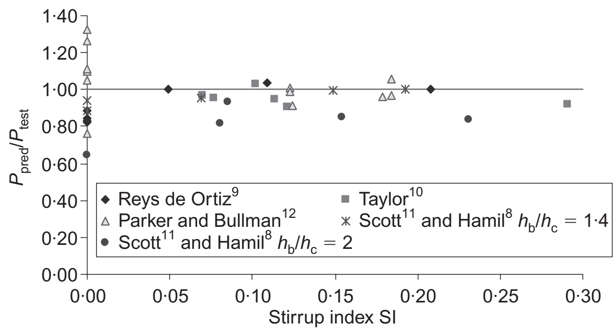

Fig. 7. Influence of stirrup index SI on accuracy of predicted failure loads given by STM

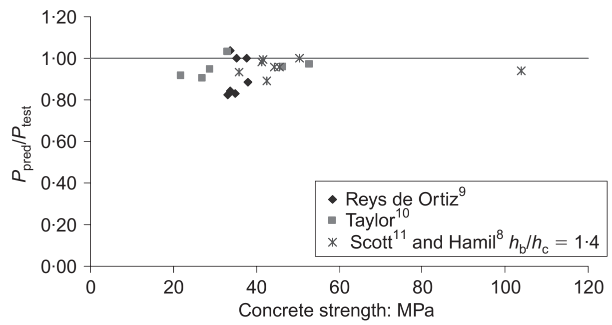

Fig. 8. Influence of concrete strength on accuracy of predicted failure loads given by STM

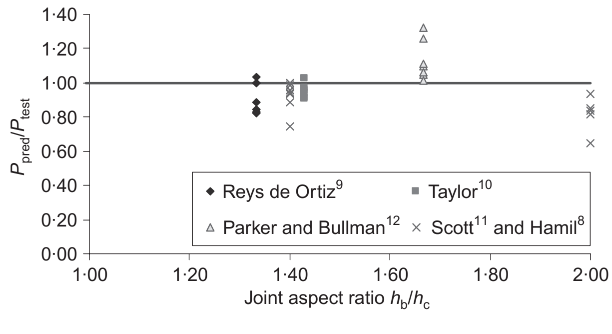

Fig. 9. Influence of joint aspect ratio on accuracy of predicted failure loads given by STM

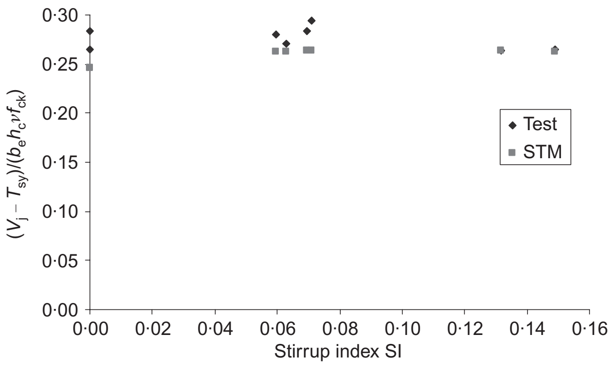

Fig. 10. Comparison of measured and predicted contributions of direct strut for STM for specimens tested by Scott[11] and Hamil[8]

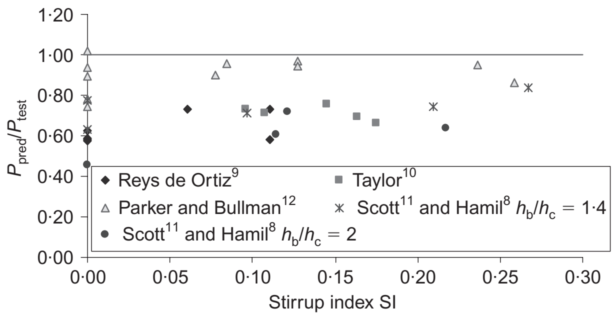

Fig. 11. Comparison of measured and design failure loads given by STM

Hamil,[8] which were typical. It is concluded that the STM provides a good description of the mechanics of the joint in addition to giving reasonable estimates of joint strength.

Safety of STM for design of beam–column joints

The analysis of the test data was repeated, for the specimens that failed in joint shear, with the STM, the method of Parker and Bullman[12] and EC2[3] methods with material factors of safety of 1·5 for concrete and 1·15 for reinforcement. A statistical analysis of the results is given in Table 2 for all the specimens and for specimens with joint shear reinforcement. The ratio P_{design}/P_{test} is plotted against the stirrup index SI in Fig. 11 for all the specimens including those of Parker and Bullman,[12] which failed in flexure within the column or beam. It can be seen that the authors' STM safely predicts the failure load of all the specimens with an adequate factor of safety. The factor of safety is less for specimens that fail in flexure since their strength is less dependent on the concrete strength.

Table 2. Statistical analysis of design strengths

For all specimens			
$V_{j\text{pred}}/V_{j\text{test}}$	STM	Parker and Bullman[12]	EC2[3]
Mean	0·71	0·60	0·43
SD	0·12	0·12	0·16
COV	0·17	0·20	0·37
For all specimens with stirrups			
$V_{j\text{design}}/V_{j\text{test}}$	STM	Parker and Bullman[12]	EC2[3]
Mean	0·71	0·58	0·43
SD	0·09	0·09	0·17
COV	0·13	0·16	0·39

Influence of transverse beams

None of the specimens analysed in this paper had transverse beams. Many tests have been carried out on cyclically loaded joints which indicate that joint shear strength is increased by the presence of transverse edge beams framing into each side of the joint. This effect is included in the design recommendations of ACI/ASCE Committee 352[5] which increases the strength by 4/3 if transverse beams are present. Research[15] shows that the potential increase in joint strength owing to transverse beams depends on the beam cross-sectional area, area of longitudinal reinforcement and loading. The increase in joint shear strength arises through the combined effects of torsion and confinement of the concrete within the joint zone. The current authors believe that the effect of torsion is likely to be most significant in monotonically loaded joints where limited lateral expansion arises within the joint at failure. Vollum and Newman[16] carried out some tests on beam–column connections in which one of the beams was eccentric to the column. These tests showed that a lower bound to the maximum torque that can be transferred into the joint is given by

$$T_{\max} = \frac{2\nu f_{cd} A A_k}{U(\cot\theta + \tan\theta)} \tag{23}$$

where A is the cross-sectional area of the transverse beam, U is its perimeter and A_k is the area enclosed within the centreline of an equivalent thin walled tube with wall thickness $t = A/U$ and $\cot\theta = 1$. The reduction in torsional strength owing to shear can be estimated using the linear interaction equation in EC2. Torsion is transferred into the side face of the joint due to horizontal and vertical couples. It seems reasonable to assume that the maximum moment that can be transferred into the joint is increased by the couple corresponding to the vertical forces. It is follows that the maximum possible factor C by which the moment capacity of the joint can be increased by beams framing into each side of the joint is given by

$$C = 1 + (1 - V/V_{\max})T_{\max}/M_b \tag{24}$$

where M_b is given by equation (17), V is the shear force in the transverse beams and V_{max} is the maximum possible shear capacity given by EC2.[3] The same value of $\cot\theta$ is used to calculate V_{max} and T_{max}.

Equation (24) was evaluated for all the test specimens with L bars assuming $\cot\theta$ was the maximum permissible value of 2·5 and V/V_{max} was equal to 0·5. All the beams framing into the column were assumed to have the same depth and the width of the transverse beam was assumed to be the same as the column. The value of C varied between 1·32 and 1·49, which is of the same order as allowed by ACI/ASCE Committee 352.[5] The area of longitudinal reinforcement in the transverse beam should be increased for torsion as described in EC2.[3] Advantage can be taken of reinforcement already provided in the slab within the width of the effective flange defined in EC2.[3]

Detailing

In practice, the moment transferred into the joint at the column face is frequently less than that given by equation (17) with $\Delta M = 0$ and minimum joint stirrups are sufficient. Occasionally, it will be necessary to design shear reinforcement to increase the joint shear strength. The design stirrups required in the joint region should be provided between the beam tensile reinforcement and the top of the flexural compression zone in the beam which can be assumed to equal 3/8ths of the beam depth. It is suggested[12] that when the design joint shear force exceeds 2/3 of V_{jdmax} from equation (12), the link spacing should not exceed $0{\cdot}3d_c$. It is recommended that a minimum area of joint shear reinforcement should be provided in all external beam column joints as recommended in ACI 318-05.[4] It is suggested that the area of reinforcement should be taken as the minimum area of shear reinforcement required in beams in EC2,[3] which equals

$$A_{sw}/(b_c s) = 0{\cdot}08 f_{ck}^{0{\cdot}5}/f_{yk} \tag{25}$$

In practice, it can be physically difficult to position stirrups within the depth of the joint. A more practical alternative for monotonically loaded joints is to use horizontal U bars anchored in the beam instead of stirrups for joint shear reinforcement.

The beam reinforcement should be bent down into the column with an adequate radius to avoid bearing failure and should be fully anchored in the column past the beginning of the bend. In practice, it is often more convenient to anchor the beam reinforcement with U bars rather than L bars. Vollum and Newman[7] previously found that the joint shear strength of specimens with U bars was around 20% less than that of specimens with L bars probably owing to the U bars having an inadequate lap with the column bars in the tests considered.

Case study

A series of parametric studies were carried out to illustrate the impact of the proposed design recommendations on the design of the framed structure shown in Fig. 12, which is considered an onerous case. The structure consists of a one-way spanning slab supported on beams. The spans of the slab and beams were taken as 9 m and 8 m respectively. The design imposed load was taken as 4 kN/m^2. The slab thickness was taken as 275 mm, which is the minimum permissible thickness allowed by the EC2[3] span-to-depth rules with grade 30 concrete and 50% surplus flexural reinforcement in the span to control deflection. The beam was assumed to be 600 mm wide and its depth was chosen to be the minimum possible for a continuous beam over simple supports assuming either 0, 0·5% or 1·38% compression reinforcement at the first internal support. The resulting beam depths were 685, 582 and 484 mm respectively. The 484 mm deep beam just satisfies the span-to-depth rules in EC2 without the need for surplus flexural reinforcement to control deflections. The internal columns were assumed to be 600 mm square. The external columns were taken as 600 mm wide and their depth h_c was varied between 200 and 600 mm.

The design joint shear force was calculated at joint A (see Fig. 12) for external column depths between 200 mm and 600 mm. The required areas of joint shear reinforcement were found with equation (19) using material factors of safety of 1·5 for concrete and 1·15 for reinforcement as in EC2.[3] No increases were made to the joint shear strengths to take account of the presence of transverse beams. The resulting stirrup indices SI are plotted against the corresponding column depths in Fig. 13. Only minimum joint stirrups are required for the 685 mm deep beam which is the shallowest permissible beam without compression reinforcement at the first internal support. It can be seen that the required joint stirrup index SI increases significantly as the beam depth is reduced by the provision of compression reinforcement at the internal supports. Results are presented for the 484 mm deep beam even though the maximum joint shear strength given by equation (12) was generally exceeded. Fig. 14 shows that the demand for joint shear reinforcement is re-

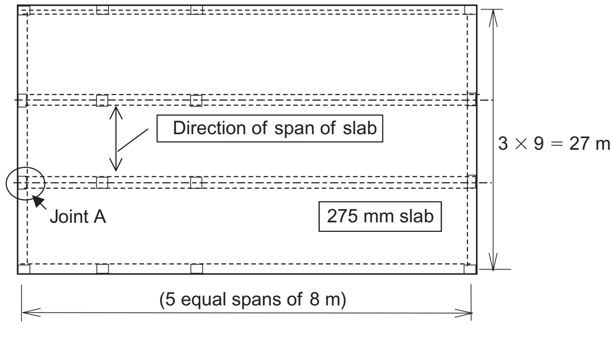

Fig. 12. Floor plate considered in case study

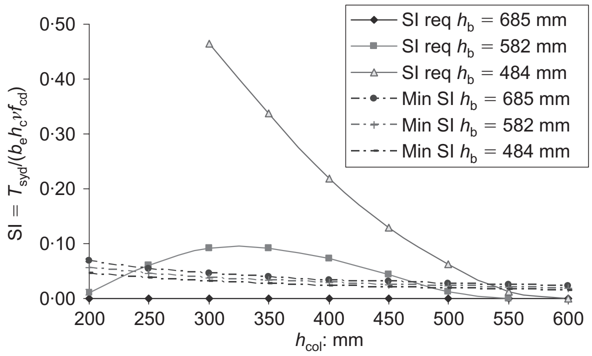

Fig. 13. Required joint shear stirrups with no moment redistribution

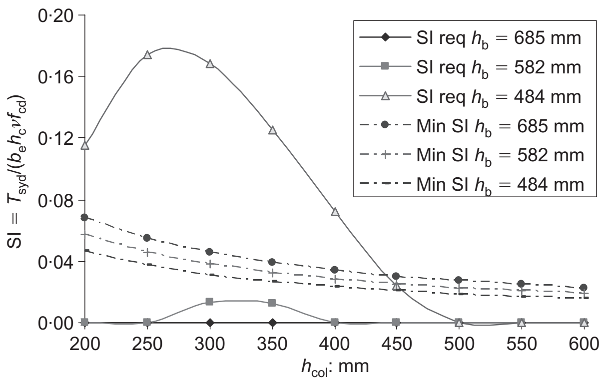

Fig. 14. Required joint shear stirrups with 20% moment redistribution at external column

duced significantly if the design bending moments are redistributed by 20% at the face of the external column.

Conclusions

An improved STM is presented for the design of external beam–column joints, which is shown to give better predictions of joint shear strength than existing simple design methods. The most significant aspect of the model is that it was developed from first principles using the design guidance given in EC2[3] for STM. The novel feature of the analysis presented is that the joint shear strength is limited by the maximum moment that can be transferred through the joint into the upper and lower columns. The STM is shown to predict many of the trends in behavior observed in laboratory tests. It is shown that minimum joint shear reinforcement will often be all that is required in framed structures, particularly if the design joint shear force is reduced by moment redistribution, unless beam depths are particularly shallow due to the provision of compression reinforcement. It is also shown that premature flexural failure can occur in the column above the joint in lightly loaded columns unless the flexural reinforcement is designed as described in the paper.

References

1. Park R. and Paulay T. *Reinforced Concrete Structures*. Wiley, New York, 1975.
2. British Standards Institution. *BS 8110. Part 1:1997: Structural use of concrete: Code of practice for design and construction.* BSI, London, 2005.
3. British Standards Institution. *BS EN 1992-1-1:2004. Eurocode 2: Design of concrete structures. Part 1-1: General rules and rules for buildings*. BSI, London, 2004.
4. American Concrete Institute. *ACI 318-5. Building Code Requirements for Structural Concrete and Commentary.* ACI, Farmington Hills Michigan, 2005.
5. ACI-ASCE Committee 352R-91. *Recommendations for the Design of Beam–column Joints in Monolithic Reinforced Concrete Structures*. American Concrete Institute, Detroit, 1998. ACI Manual of Concrete Practice, Part 3.
6. Vollum R. L. and Newman J. B. Strut and tie models for the analysis/design of external beam column joints. *Magazine of Concrete Research*, 1999, **51,** No. 6, 415–425.
7. Vollum R. L and Newman J. B. The design of external reinforced concrete beam–column joints. *The Structural Engineer,* 1999, **77**, Nos 23 and 24, 21–27.
8. Hamil S. J. *Reinforced Concrete Beam–column Connection Behaviour*. PhD Thesis, University of Durham, 2000.
9. Reys de Ortiz I. *Strut and Tie Modelling of Reinforced Concrete Short Beams and Beam–column Joints.* PhD Thesis, University of Westminster, 1993.
10. Taylor H. P. J. *The Behaviour of insitu Concrete Beam–column Joints*. Cement and Concrete Association, Wexham Springs, 1974, Technical Report 42·492.
11. Scott R. H. The effects of detailing on RC beam/column connection behaviour. *The Structural Engineer*, 1992, **70**, No. 18, 318–324.
12. Parker D. E. and Bullman P. J. M. Shear strength within reinforced concrete beam–column joints. *The Structural Engineer*, 1997, **75**, No. 4, 53–57.
13. Kordina K. *Bewehrungsfuhrung in Ecken und Rahmenendknoten*. Deutscher Ausschuss fur Stahlbeton, Berlin, Germany, 1984, Part 354, 93pp.
14. Bakir P. G. and Boduroglu H. M. Nonlinear analysis of beam–column joints using softened truss model. *Mechanics Research Communications,* 2006, **33**, No. 2, 134–147.
15. Eshani M. R. and Wight J. K. Effect of transverse beams and slab on behaviour of reinforced concrete beam-to-column connections. *ACI Structural Journal*, 1985, **82**, March–April, 188–195.
16. Vollum R. L. and Newman J. B. Towards the design of eccentric beam–column joints. *Magazine of Concrete Research*, 1999, **51**, No. 6, 397–407.

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