The role of spanwise forcing on vortex shedding suppression in a flow past a cylinder

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Abstract Controlling the wake vortex dynamics of bluff bodies efficiently is a fundamental problem in many applications. Earlier direct numerical simulations [3] of three-dimensional bluff bodies demonstrated that the introduction of a spanwise waviness at both the leading and trailing surfaces suppresses the vortex shedding and reduces the amplitude of the fluctuating aerodynamic forces. Under this motivation, starting from a fully developed shedding, a sufficiently high spanwise forcing is introduced on the surface of the cylinder, in the regions where separation effects occur, resulting in the stabilisation of the near wake in a time-independent state, similar to the effect of a sinusoidal stagnation surface. Stability analysis of the linearised Navier-Stokes equations was then performed on the three-dimensional flows to investigate the role of the spanwise modulation on the absolute instability associated with the von Karman street.

1 Introduction

Controlling vortex shedding is an important problem in many engineering applications due to the relevant amount of drag, vibrations and noise that it generates. Several techniques aimed at suppressing the von Kárman street in two-dimensional wakes, have been studied; these methods can be classified in two categories respectively: two-dimensional and three-dimensional controls. Two-dimensional controls are characterised by a constant control input in the spanwise direction; this type of control has been extensively studied and some examples are the end plate

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(Nishiko & Sato 1974; Stansby 1974), base bleed (Bearman, 1967; Wood, 1967), splitter plates (Roshko 1955; Bearman 1965; Kwon & Choi, 1996), secondary small cylinders (Strykowski & Sreenivasan 1990). Three-dimensional control techniques present instead a variation of the actuation property in the spanwise direction; some examples are the segmented trailing edge (Tanner, 1972; Rodriguez, 1991), the wavy trailing edge (Tombazis & Bearman, 1997), the wavy stagnation face (Bearman & Owen, 1998; Szewczyk & Bearman, 2000; Darekar & Sherwin, 2001),the spanwise-periodic blowing/suction (Kim & Choi, 2005) and a small vertical tab (Park et al, 2006). Despite 3D control techniques are found to be more efficient than two-dimensional ones, the physical mechanisms underlying the suppression of the vortex shedding remains poorly understood. Hwang et al (2013) presented the first explanation of the most relevant phenomena by means of the linear stability analysis. It was found that the application of a spanwise waviness to a flow past a bluff body is able to stabilise the near-wake absolute instabilities; the flow was assumed to be parallel and the Monkewitz model was used to describe the profile of the wake. However, real configurations may be strongly non-parallel and this might significantly affect the dynamics of the absolute instabilities, as the authors point out. Therefore, the complex structure of the wake associated with a bluff body subject to three-dimensional control methods leads us to a question: how do the three-dimensional modifications of the base flow affect the stabilisation of the global instabilities? In the present study the attention is focused on the suppression of the vortex shedding in a flow past a cylinder at the supercritical regime ($Re = 60$) using two different types of spanwise forcing. The effect of the the role of spanwise modulation on the absolute instability was then investigated by computing the fully three-dimensional global mode.

2 Three-dimensional base flow

The spatial discretisation of the three-dimensional Navier-Stokes equations was performed using a quasi-3D method; in the spanwise direction a Fourier spectral method was used, while a spectral/*hp* element method was adopted in the streamwise and transverse directions. The time-integration of the equations was performed using a stiffly stable splitting scheme (Karniadakis, Israeli & Orszag, 1991).

The suppression of vortex-shedding was achieved by 3D distributed forcing applied to two slots, respectively at the top and bottom surface of the cylinder. The forcing is the product of two components: $F_{st}(x,y)$ and $F_{sp}(z)$, where F_{st} is a gaussian function, mapped on to the surface of the cylinder and centred at about 20° from the transverse direction just before the separation points of the boundary layer. Concerning the spanwise forcing F_{sp}, two different functions were applied: a sinusoidal $F_{sp}^{(1)}$ and a gaussian one $F_{sp}^{(2)}$:

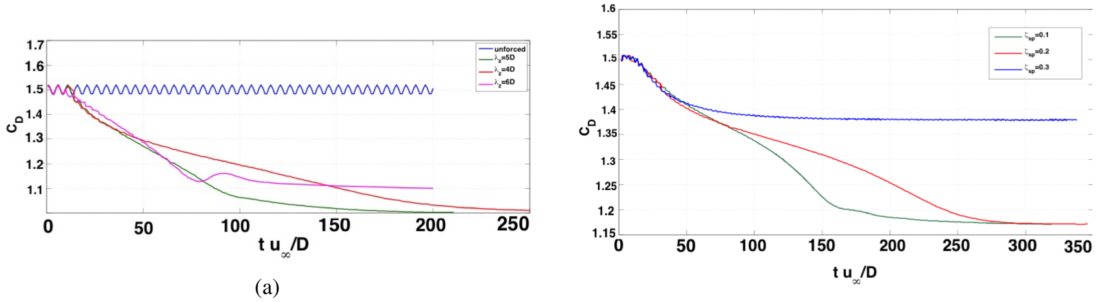
$$F_{st} = \exp\left[-\left(\frac{\arctan^2\left(\frac{x}{|y|}\right)+\frac{\pi}{9}}{2\zeta_{st}^2}\right)\right] \quad \text{where} \quad \zeta_{st} = 0.1. \tag{1}$$

$$F_{sp}^{(1)} = \sin\left(\frac{2\pi}{\lambda_z}z\right) \tag{2}$$

$$F_{sp}^{(2)} = \exp\left[-\left(\frac{z-\frac{L_z}{2}}{2\zeta_{sp}}\right)^2\right] \tag{3}$$

Equation (2) represents the sinusoidal forcing, while equation (3) the gaussian one; λ_z represents the forcing wave-length, while ζ_{sp} the width of the Gaussian function. These two functions have been normalised such that the integral of the forcing over the domain is constant, so the same amount of energy is introduced into the system. The role of these two parameters has been been investigated assuming an amplitude of the forcing $A = 0.1u_\infty$, which was verified to lead to a complete stabilisation of the vortex shedding, in accordance with Kim and Choi (2005). The optimal wavelength for the sinusoidal forcing is around $\lambda_D \simeq 5$, with a drag reduction of about 20%, resembling the studies of Darekar & Sherwin (2001) and Kim & Choi (2005) (figure 1.(a)) . The effect of ζ_{sp} is reported instead in figure 1.(b); a decrease of the standard deviation results in a more efficient drag reduction. This behaviour is related to the increasing steepness of the gaussian function and a consequent larger amount of spanwise/transverse vorticity is introduced in the system. These additional components of the vorticity are responsible for preventing the rolling-up of the three-dimensional shear layers into the vortex street, as described by Darekar & Sherwin (2001). Figure 2 shows the temporal change of ω_x/ω_z and ω_y/ω_z in the point $(x,y,z) = (0,0.7,1.8)$, where z corresponds to the inflection point and y is within the shear layer. It can be seen that both these components grow in time and the gaussian forcing is generally more efficient. The sinusoidal forcing introduces in fact an amount of streamwise and transverse vorticity that is about 20% less than the gaussian forcing. This issue is reflected in the critical value of the amplitude able to suppress the vortex shedding, which is almost double for the sinusoidal forcing ($A_{cr} = 0.23$) with respect to the gaussian forcing ($A_{cr} = 0.12$). For this reason, the following sections will only discuss the gaussian forcing.

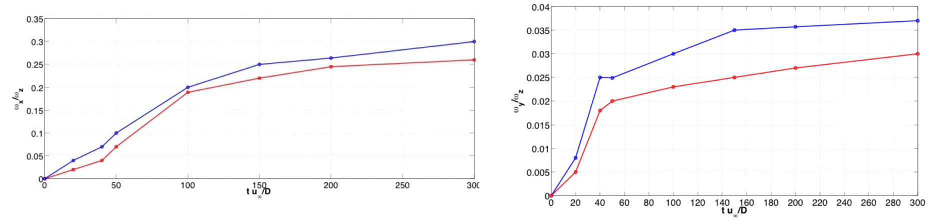
In figure 3.(a) the profile of the vortical structures is shown using the Q-criterion. The vortex shedding has been completely suppressed using a gaussian spanwise forcing at the critical amplitude $A = 0.12$. It can be seen that the iso-surface is symmetrical with respect to the centreline with both vertical and horizontal connection. The drag coefficient is subject to a reduction of 22%, while the Strouhal frequency and the lift coefficients are zero, pointing out the steadiness of the near-wake (figure 3.(b)).



(a)

(b)

Fig. 1: variation of the drag coefficient with the wave-length λ_D (sinusoidal forcing) and the standard deviation ζ_{sp} (gaussian forcing)



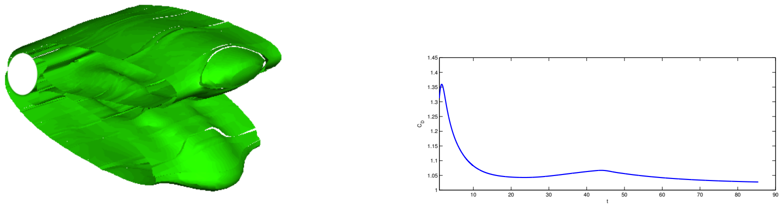
(a)

(b)

Fig. 2: Time evolution of: (a) the streamwise and (b) transverse vorticity. Blue lines represent the gaussian forcing, red lines the sinusoidal one.

3 Fully three-dimensional direct stability analysis

An insight into the mechanisms that lead to the suppression of the vortex shedding can be obtained by means of the hydrodynamic stability analysis. The distributed forcing on the surface of the cylinder has a prominent effect on the near-wake region,



(a) Wake topology at $tU_\infty/D = 100$

(b) Time evolution of the drag coefficient

Fig. 3

which was seen to be particularly sensitive to a force-velocity coupling [4]. Due to the variation of the flow in in all the spatial directions, a fully three-dimensional global stability analysis, often referred to as TriGlobal [7], is the appropriate approach to studying the dynamics of the instabilities. The stability of the problem was investigated by computing the eigenmodes of the Navier-Stokes equations linearised around the suppressed regime. An Arnoldi algorithm [1] was applied to calculate the eigenvalue and eigenvector of the system. The temporal growth rate is given by the imaginary part of the eigenvalue $\Im(\lambda_i)$, the frequency by the real part $\Re(\lambda_i)$. When the critical forcing amplitude was applied in order to stabilise the wake, the flow was found to be absolutely stable and the decay rate of order -3×10^{-4}, comparable with the value of a subcritical flow at $Re = 30$. An additional increase of the amplitude showed a further decrease of the decay rate despite the topology of the eigenmode did not change significantly. This decrease was observed to saturate when the amplitude of the forcing was about 30% above the critical value. Figure 4.(d) shows the profile of the eigenmode when $A = 1.3A_{cr}$ and the decay rate was found to be 3×10^{-5}. The structure of the mode is located close to the cylinder ($0.5D < x < 5D$) highlighting the crucial role of the perturbations in this region. Let us consider a cross-streamwise section at $x = 1$, where the perturbations were found to be most intense. Figure 4.(c) shows the contours of the magnitude of the perturbation $\|\mathbf{u}\| / \|\mathbf{u}_{max}\|$, overlapping with the iso-lines of the base flow magnitude at the same location. The mode is located in a relatively narrow spanwise range ($1 < z < 4$) in the region just above and below the cylinder. These regions are located at the same transverse location where the forcing was applied, where the perturbations are able to disrupt the vortex shedding. The normalised magnitude of the transverse and spanwise components of the mode have an important role in the dynamics of the transport of vorticity perturbations and their structures are reported in figure 4.(a) and (b). It can be seen that the transverse perturbation velocity is more intense at $y = 0$ and its intensity is more pronounced in the region where the base flow velocity is slower, in accordance with Hwang et. al (2013). The spanwise perturbation velocity is instead more intense at the extremities of the domain ($z = L/4$ and $z = 3/4L$) and in the region just above and below the cylinder, similar to the transverse component.

4 Conclusion

The present work investigated the effect of two different types of spanwise forcing to study the effect on the suppression of the vortex shedding in a flow past a cylinder at $Re = 60$. The gaussian forcing was found to be more efficient than the sinusoidal, in accordance with previous studies. This results in a noteworthy reduction of the drag, about 20%. Direct stability analysis allowed us to study the mechanism behind the suppression and the flow was confirmed to be absolutely stable with a decay rate similar to a flow at $Re = 30$. The eigenmode is located downstream from the cylinder, relatively close to the bluff body, up to about $5D$. The transverse component is more intense close to the axis $y = 0$ where the base flow is slower. The spanwise component is more intense at $z = L_z/4$ and $z = 3/4L_z$, following the profile of the base flow. No variation in the shape of the mode was detected using amplitudes over the critical value at this Reynolds number, although this is not necessarily expected to be valid at higher values.

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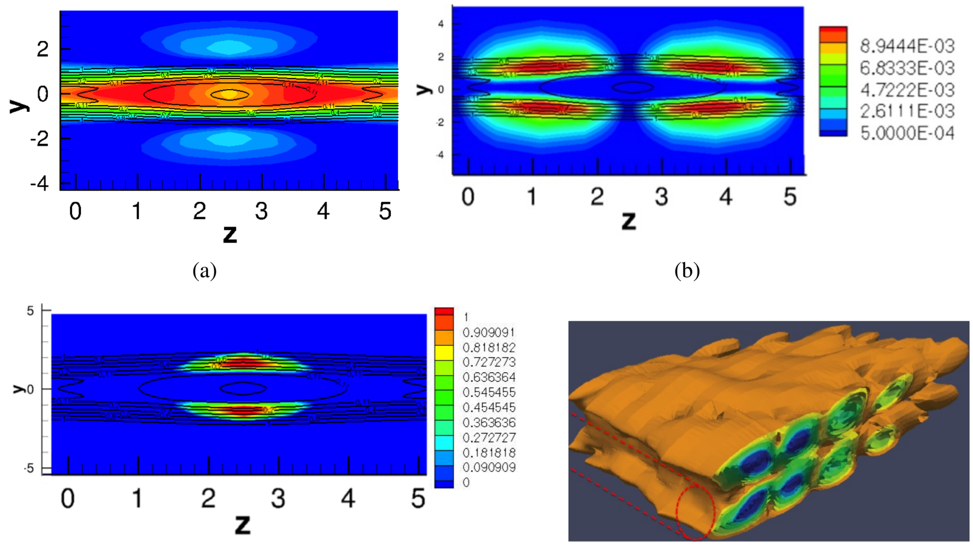

(a) (b)

(c) (d)

Fig. 4: (a) Transverse perturbation $|v|/|v_{max}|$, (b) Spanwise perturbation $|w|/|w_{max}|$, (c) Magnitude $\|\mathbf{u}\| \,/\, \|\mathbf{u}_{max}\|$, (d) Three-dimensional structure of the eigenmode.