The Development of Improved Timing Systems for Rapid and Precise Astronomical Fixation in Mining Exploratory Surveying

by

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Abstract

The absolute position and orientation of survey schemes in exploration for mineral resources and other purposes can be established by astronomical observation.

The precise timing of optical observations constitutes a major problem in field astronomy.

The employment of a special form of portable crystal chronometer for the measurement of time in the field for the purpose of achieving a precision unattainable with devices governed by mechanical clocks, is investigated. Results show that the error in comparing the local chronometer with a radiated time signal is reduced to a negligible amount. The calibration and performance of the crystal chronometer is dealt with, also its reliability during survey operations. Various methods of time measurement are investigated.

The specifications of the design of the frequency standard and the investigations conducted by the author and described here indicate that the performance, ranging from long term to instantaneous stability, has proved to be of the order of $\pm$ 1 millisecond. A practical and rapid field test for ascertaining the proper functioning of the crystal chronometer is given.

One of the outputs of the crystal chronometer provides a marker signal, operated by a microswitch, which enables the timings to be measured of consecutive transits of a star across a special five line graticule. Simultaneous measurements of vertical and horizontal angles are made possible with the graticule, and the accuracy achieved is dealt with.

The recording and storing on magnetic tape of all field data including frequency pulses from U. T. transmissions, pulses from the crystal chronometer, and pulses from optical observations released manually, are described. The pulses form characteristic patterns on the tape which are made visible with a special fluid and metallic powder. Investigations into suitable chemicals and microsized metal particles are reported.

The methods of calibrating the crystal chronometer against U. T. time signals are found sufficiently accurate to detect variations in emission times of frequency pulses published by the Royal Greenwich Observatory and the Bureau International de l'Heure.

Reference is made to the instruments employed; tests of their performance are described. The personal error is dealt with and the design of an arrangement for its detection is given.

The result of theoretical investigations and practical field work show that for obtaining an astronomical fix, the chronometer error will be only a small portion of the total error, including those due to theodolite operation, personal error, refraction etc.

Computation methods for position fix are analysed; investigations into semi-graphical solution of field observations are described.

A special design of illuminated telescope sight of transparent synthetic resin and its use in the field for lining up the stars is reported.

The author believes that the method of timing, recording and evaluation of optical observations together with the instrumental modifications described in this thesis offer a considerable saving in time and a higher accuracy for the determination of absolute position and of azimuth, and are the essential supplement to recent developments of other survey methods and instruments.

The investigations and the field work have been carried out at the Royal School of Mines, Imperial College, London, and at the Imperial College Field Station, Tywarnhale Mine, Cornwall.

1. Introduction

To keep pace with the increasing demand for raw materials extensive search for minerals in unexplored territory is in progress. Geologically unknown areas are mostly in remote parts of the world of which few or no maps exist. It is a prerequisite in development projects involving the discovery of mineral deposits to survey the countryside for producing the necessary maps and plans.

The development of electronic distance measuring instruments and the ever growing use of aerial photography have, in their application to surveying, revolutionized this science, particularly as regards to preliminary surveys, which are of great importance in the field of mining exploration.

Preliminary survey schemes and also local survey systems for a mine or for major engineering projects can nowadays be carried out in a fraction of the time which was required for the same work only a few decades ago.

Basically a survey system consists of monuments or marks, which are placed on ground, related to each other in terms of angular and linear measurements, expressed in some form of co-ordinates and these constitute references, or define legal rights. A survey system with no positional relationship to others or without information of absolute position and orientation should be regarded as being incomplete.

The absolute location of a point on the earth's surface is effected independently from any other points on it.

A satisfactory absolute position and the necessary control for azimuth can be provided by astronomical observations. The alternative, the establishment of control points obtained from continuous ground survey connected to an existing geodetic frame which can be a considerable distance away, is often prevented by time and cost.

Once astronomical control has been established, any surveyed

1, cont.

points expressed in plane co-ordinates, as is invariably the case with local systems, can be converted into geographical co-ordinates.

The latitude - longitude system has the great advantage of being unrestricted in area, permitting indefinite extension, assisting the re-establishment of lost monuments, retracement of boundary lines etc., and contains a ready available azimuth direction, a useful item which should not be overlooked at the increasing employment of electronic instruments for distance measurement, and at the present day trend of replacing triangulation schemes by traverses. Another independent method of obtaining azimuth is by the use of gyroscopic instruments which are still rather expensive.

The order of accuracy normally achieved with positional astronomy providing latitude and longitude, and the amount of calculation it involves discourages its application in all but the necessary circumstances.

The practice of solving the problem of absolute location of a local survey scheme has not changed a great deal, considering the great development of other ground survey methods. Time and effort would be well spent by improving conventional procedures necessary to achieve higher accuracy and greater speed.

The various astronomical methods for position fixing - location of a field survey station by astronomical latitude, longitude and azimuth - are restricted in solving the astronomical triangle. The parts which can be measured from the earth's surface are:

(1) the co-altitude or zenith distance of the heavenly body (90° - h), i. e. the arc distance from the astronomical zenith of the observer to the apparent position of the heavenly body, and

(2) the hour angle (t), which is the angle subtended at the celestial pole included between the celestial meridian of the observer and the declination circle of the heavenly body.

The former can be measured with various instruments, the most

1. cont.

common of which is the surveyor's theodolite. The latter is the link between the celestial sphere and the earth, which is time, and is therefore measured with the aid of a clock. Time cannot be represented yet as an arbitrarily selected and permanently preserved piece of unit as a "yard-stick" of a standard second. The ultimate practical reference standard of time is the period of axial rotation of the earth. The determination of time is an astronomical process and is not discussed here.

The results of the measurements of angles and of time taken at survey stations are dependent, to a large extent, apart from general field and weather conditions, on the accuracy which can be achieved with present day field instruments. Modern glass circle theodolites, which are easily transportable, allow a direct reading up to one second of arc of both the horizontal and vertical circle. Advances in the accuracy of subdividing the circle will enable the maker to incorporate into the engineer's theodolite a direct reading system for an even smaller part of the arc, down to one or two tenths of a second. The measurement of time, in the field, would require a clock indicating intervals of about one hundredth of a second of time, or less, to meet the continuous improvements of precision of optical instruments. The maximum star movement in one second of time as seen with a telescope of 30 magnification corresponds to 7.5 minutes of arc.

At the survey station the observation consists of timing and recording the instant an event (star's crossing of the wire) takes place. This instant of observation can be timed for instance with a stop watch in conjunction with a box chronometer, or can be registered on chronographs driven by a mechanical clock etc. Mechanical clocks, other than pendulum clocks, which are not considered to be used in the field, have been developed to a remarkable degree of precision, but have not attained the small subdivision of time which can be attained with crystal chronometers. A most convenient supply of time for the surveyor in

1. cont.

the field could obviously be the radio time signals, and also the high precision signals associated with standard frequency transmissions. The term signal denotes the emitted pulse from the transmitting station. This time is based directly on astronomical observation at observatories and is subdivided either by crystal frequency or by an atomic clock, and is available through the national time service.

In practice, however, the direct application of time signals, received with a portable radio set for timing astronomical observations in the field, is inadequate.

The various forms of standard time signals are radiated at selected times only. Most of the continuous frequency transmissions are interrupted at periodical intervals. Although some are partly overlapping in time, a poor reception might prevent their use. Experience has shown that in most cases, when good visibility existed for a sight to be taken to a star, there was only a faint reception of time signals, and sometimes none at all.

For rapid and precise timing of optical observations of a repetitive nature, the employment of an instrument, which for shorter periods maintains an extremely steady rate, for the measurement of time intervals in terms of an accepted unit to a high order of precision, is indispensable. Furthermore, the locally measured time intervals have to be referred to an epoch, before they can be used for the absolute location of the observer's station, as intended.

It is essential that instruments intended for field use should be portable and light in weight. The procedure and methods to be adopted for field observations and for the evaluation of data have to be rapid yet reliable.

It is considered advisable to treat the investigations of interconnected items, contributing to an astronomical fix, instruments, timing, optical observations, recording of data, computations etc.,

1. cont.

under separate headings. It involves an occasional restatement of some facts, but it enables one to present the investigations and their phases more fully and without too many cross references through the whole text. It necessitates also presenting the research undertaken partly in form of a report and submitting important conclusions relevant to the advancement in field astronomy amongst the individual subjects investigated.

Interpretation of the various investigations are enhanced by diagrams and tables.

Existing methods which have direct bearing on the subject-matter are outlined with a critical examination. New techniques and methods seldom replace the conventional methods completely. The investigated field techniques of timing, recording, and evaluation of optical observations, and the precise time link established by comparing the crystal chronometer with radiated time signals, together with the instrumental modifications described in this thesis will certainly supplement existing methods.

Incorporated in this thesis is the work described in the article "Accurate Time for Field Astronomy" published by the author in Nature, Vol. 199, No. 4889, July 13, 1963., and also material which will be presented in a paper to be read at the Royal Institution of Chartered Surveyors in 1966.

The responsibility for any statements of fact or opinion expressed, rests solely with the author.

2. Scope of Research

The scope of the research is to investigate to what extent and to what order of accuracy a subdivision of time interval from an independent source could be achieved and made available in the field.

Further, the research is concerned with the correlation of time from an independent source with standard frequency transmissions or with time signal transmissions, and the precision at which this can be effected.

It is the purpose of this research to develop methods of measuring time intervals, which are of short duration, in terms of an accepted unit, at a very high order of precision.

This includes a detailed investigation into the factors governing the performance and calibration constants of time measuring equipment, as well as an analysation of errors contributing to fluctuations of the observed calibration values. Since the calibration values are not instrumental constants, their determination once and for all is not sufficient, and readily applicable methods of standardization of calibration values have to be established.

The research is directed to reveal possibilities of utilising a high precision time source for timing optical observations which are used for providing rapid and precise location of survey stations in terms of latitude, longitude and azimuth.

The research embraces the examination of existing methods of timing and of recording optical observations and is aimed at instrumental improvements. This includes the investigation into the possibility of simultaneous measurements of parts of the astronomical triangle which are accessible to measurement from the surface of the earth.

The research also necessitates the analysation of computation methods and the investigation into semi-graphical solutions for a position fix.

2. cont.

In his research, the author is restricted by the necessity of working with the representative field survey instruments and equipment that are available, their conventional uses and purposes, and their fundamental features of lightness, compactness, portability and low costs. Hence the research is directed at supplementing conventional methods and instruments and introducing justified replacements having equal basic features.

3. Crystal Chronometer

3.1. Reason for Employment of a Crystal Chronometer.

The succession of complete revolutions of the earth about its axis represents a periodic occurrence, the duration of a single cycle of which is the practical standard for the measurement of time, or of its reciprocal, the frequency, which is expressed by the number of cycles occurring within a given length of time.

As the accurate interval of time of a single cycle and its subdivision to 24 x 60 x 60 parts is available through the national time service within the accuracy of 1 part in 10^9, it is only necessary to have a chronometer or an auxiliary frequency standard which maintains its rate unchanged throughout a time interval during which the national time service is unobtainable.

3.2. General Description and Uses of a Crystal Chronometer.

Portable crystal chronometers are now available with an accuracy approaching that of large stationary frequency or time standards installed in laboratories. The crystal chronometer indicates the time by output pulses from an oscillating circuit controlled by the piezoelectric property of a quartz crystal. The natural frequency of vibration of the crystal depends on its size and shape, and can be maintained nearly constant if appropriate precautions are taken against temperature and pressure changes.

Crystal controlled time standards are designed and manufactured by various research institutes and laboratories, broadly to be used as

(a) a pulse source or frequency control for the automatic programming and operation of instruments and machines,

(b) for visual time indication and control of time recorders.

From a theoretical aspect, specific features of the crystal chronometer were considered, which should be of great advantage for its employment in field astronomy. These specifications for a special crystal chronometer to be used in field astronomy are given in Section 3.3. Analysis of the results of tests and field experiments indicates success in achieving higher accuracies in time comparisons, in the extraction of time differences, elimination of ambiguities between pulses, and a greater ease in operation, etc. This is in consequence of the specifications, which proved to be in conformity with the theory.

3.3. Specifications of the Crystal Chronometer.

A portable frequency standard, built by Messrs. Communication Systems Ltd., according to specification, is used as a pulse source, and kept in operation by a 12 volt battery. An arrangement for interchanging batteries during charging provides for uninterrupted running. Basically the frequency standard comprises a crystal oscillator which includes a flexural-mode bar crystal element, mounted in vacuo, and which can be thermally compensated. The working frequency of 16 657.066 c/s - adjustable to compensate for long term drift - is converted:

(1) to give 61 pulses per minute, of which 60 are audible as a tone output, of about 1041 impulses per second of 1/16 second duration.

(2) by means of a pulse feedback arrangement to produce one pulse per minute audible as a tone output of about 260 i.p.s., of $\frac{1}{4}$ second duration, which is also used to operate the minute clock,

(3) to provide a marker signal also as a tone output of 520 i.p.s., operated manually by a key via a relay.

The current at 12 volt, supplied by a battery, is considered adequate for a mobile standard. When carried in a small van the chronometer can be kept running on the car battery which will also power the circuit of ancillary equipment, and even of the theodolite illumination.

3.3. cont.

The temperature compensation of the crystal is essential when operating the equipment outdoors. The oven heating circuit can be switched off when not required.

The subdivision of the minute into 61 parts constitutes a time vernier, placed at the receiving end of the time signals, and is used as an approximate coincidence meter when comparing the crystal chronometer against standard frequency transmissions, in the absence of rhythmic time signal reception. The advantage of the time vernier in general, and over an electromechanical phase shifter is outlined under evaluation of data.

Table 3.3.-1 has been prepared to convert the numbers of crystal chronometer beats into values of U. T. seconds, thus showing nominal time of the crystal chronometer. (1st and 2nd column). The 3rd column in both groups gives times of vernier coincidences.

The specified 1041c/s tone to which the other output tones are arranged in intervals of octaves for distinction, has been chosen to be almost 4 times the standard pitch of $2^4 \times 16$ c/s. The output pulses can be put directly on tape or recorded as 'sound waves' via loudspeaker and microphone. Recorded on magnetic tape and played back at two or four times lower speed, all output tones are therefore adequately within the audible range of the human ear. The frequency and, mainly depending on it, the pitch of the output tones is in that part of the spectrum, about 1 000 c/s, where the intensity range of perceptible sound is practically at its maximum for the average person. This is also an advantage when recording both chronometer pulses and pulses on standard frequency transmissions simultaneously. The latter, when recorded at different volume, for reasons of reception conditions, which requires the setting of a corresponding recording volume, could otherwise render the chronometer pulses inaudible during play-back at lower level, or could cause difficulties in erasing the tape. This is of course bearing in mind when the distances from the loudspeakers to

(I) VALUES OF CRYSTAL CHRONOMETER BEATS IN U.T. sec.'
= NOMINAL TIME OF CRYSTAL CHRONOMETER IN U.T. sec.' = $\frac{60}{61} \times n$

(II) VALUES OF CRYSTAL CHRONOMETER BEAT-COINCIDENCES WITH U.T. sec.'
= TIME VERNIER COINCIDENCE IN U.T. sec.' = $\frac{n}{61}$

n	$\frac{60}{61} \times n$ $0·983\,607 \times n$	$\frac{n}{61}$ $0·016\,393 \times n$	n	$\frac{60}{61} \times n$ $0·983\,607 \times n$	$\frac{n}{61}$ $0·016\,393 \times n$
0	0·0000	·0000			
1	0·9836	·0164	31	30·4918	·5082
2	1·9672	·0328	32	31·4754	·5246
3	2·9508	·0492	33	32·4590	·5410
4	3·9344	·0656	34	33·4426	·5574
5	4·9180	·0820	35	34·4262	·5738
6	5·9016	·0984	36	35·4098	·5902
7	6·8852	·1148	37	36·3934	·6066
8	7·8689	·1311	38	37·3770	·6230
9	8·8525	·1475	39	38·3607	·6393
10	9·8361	·1639	40	39·3443	·6557
11	10·8197	·1803	41	40·3279	·6721
12	11·8033	·1967	42	41·3115	·6885
13	12·7869	·2131	43	42·2951	·7049
14	13·7705	·2295	44	43·2787	·7213
15	14·7541	·2459	45	44·2623	·7377
16	15·7377	·2623	46	45·2459	·7541
17	16·7213	·2787	47	46·2295	·7705
18	17·7049	·2951	48	47·2131	·7869
19	18·6885	·3115	49	48·1967	·8033
20	19·6721	·3279	50	49·1803	·8197
21	20·6557	·3443	51	50·1639	·8361
22	21·6393	·3607	52	51·1475	·8525
23	22·6230	·3770	53	52·1311	·8689
24	23·6066	·3934	54	53·1148	·8852
25	24·5902	·4098	55	54·0984	·9016
26	25·5738	·4262	56	55·0820	·9180
27	26·5574	·4426	57	56·0656	·9344
28	27·5410	·4590	58	57·0492	·9508
29	28·5246	·4754	59	58·0328	·9672
30	29·5082	·4918	60	59·0164	·9836
			61 = 0	60·0000	1·0000

n = Nº of Crystal Chronometer Beat.
U.T. sec.' = Seconds of Mean Solar Time.

TABLE 3.3.-1.

3.3. cont.

the microphone are maintained equal. The audible pulses can be recorded and reproduced by any commercially available recording equipment at their speeds provided. A coincidence of marker and chronometer pulse will produce a distinct frequency on the overlapping part. The 61 audible output pulses make a second hand somewhat redundant and simplify the clock construction. The vernier, as well as the duration and frequency of the output tones, has also been specified to assist the extraction of data after tape development.

The crystal chronometer registers not Sidereal Time but Universal Time (U. T.), which is displayed by hands on a clock face in hours and minutes for convenience in comparing it with observatory radio time signals, and for time indication in the field. The minute clock can be retarded or advanced by stopping the minute pulse output or raising it to one pulse per second. Ultra-fine adjustments of time rate are regulated by a vernier control dial which can be locked.

Delays in the electric circuits of the crystal chronometer are usually small enough to be neglected. The chronometer is unaffected by vibrations and can operate in practically any position. An overturning of the chronometer will only affect the run of the clock hands. A chronometer registering sidereal time would save the calculation of conversion of U. T. to G. S. T., but would be at a disadvantage by having only one approximate coincidence with time signals of which the broadcasts last for five minutes. (e. g. H. B. N. transmissions). Also, the small amount mean solar seconds differ from sidereal seconds (0.00274 sec.) and would cause a considerable number of signals to overlap in the vicinity of approximate coincidence, reducing the precision of scaling their recorded distances after tape development. Likewise a crystal chronometer indicating units of arc instead of time units would have no further advantage, but would eliminate only the conversion of time to arc.

SCHEMATIC DIAGRAM OF CRYSTAL CHRONOMETER AND RECORDING UNIT

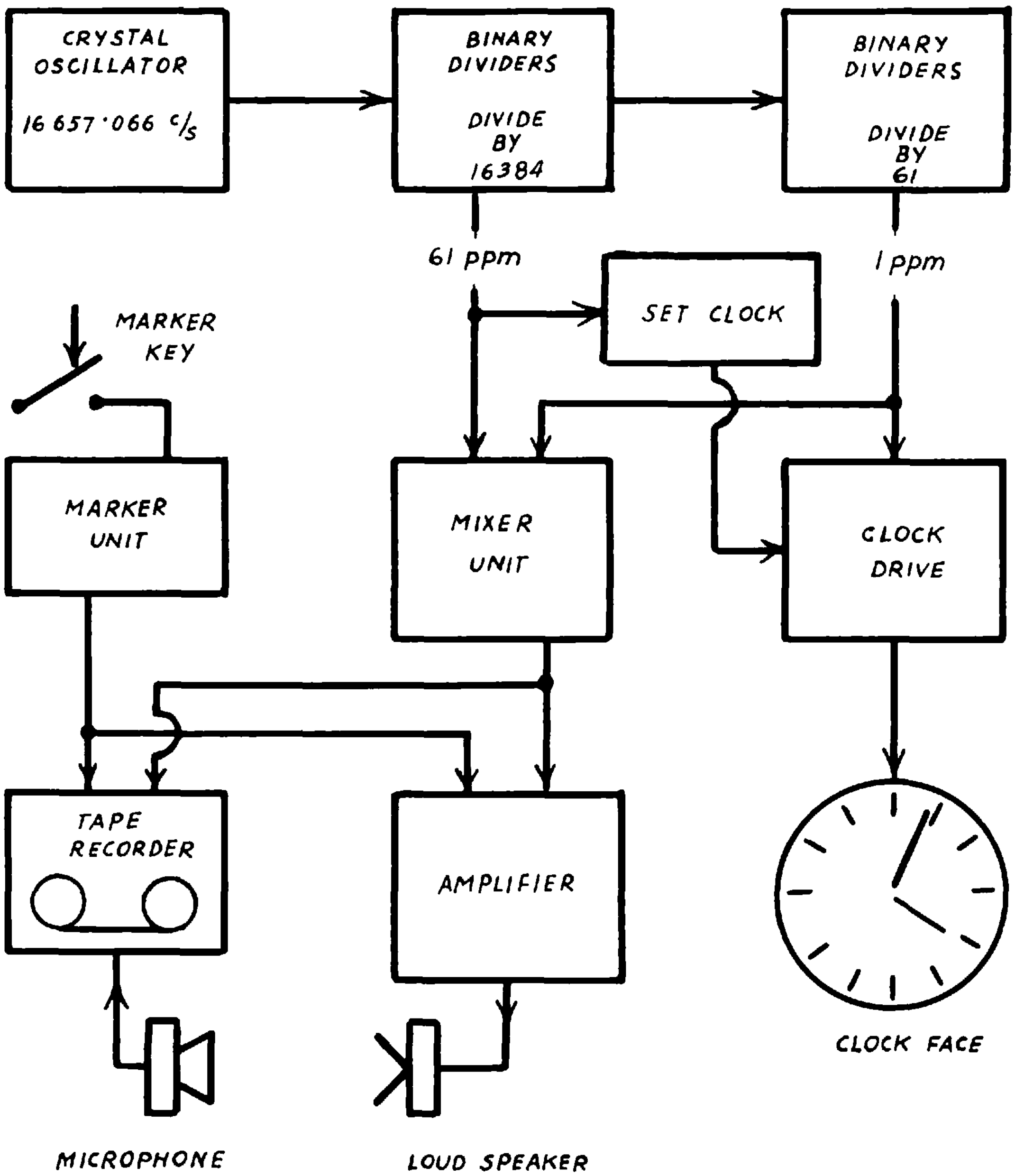


Fig. 3.3-1

3.3. cont.

The employment of a marker signal and its recording on magnetic tape, instead of using a stopwatch in conjunction with a chronometer to registe r the instant the star crosses the wire, still requires the personal error to be taken into consideration, which is the reaction of the surveyor when observing the star and pressing the key; but there is the advantage of eliminating any further personal error, of the observer or booker, in abstracting the time from the stopwatch. The marker signal can be operated by a microswitch situated near the clock face, or with a remote key. The output pulse of the marker can also be used for automatic recording of star crossings.

The essential sections of the crystal chronometer are shown in Fig. 3.3.-1.

3.4. Requirements of the Crystal Chronometer

The field surveyor's frequency standard is in operation during the time of astronomical observations, which normally may take two to four hours, but in some cases may take up to two or three days. This is contrary to the requirements of the astronomer, who expects a continuous run of several years from his clocks.

Comparison with standard frequency transmissions may not be possible during observations ; the crystal chronometer may have to be compared either before or after the observations.

Consequently, whatever the fundamental frequency of the secondary standard. used in the field, it must be capable of:

3.4. cont.

(1) a daily frequency stability closely predictable, i.e. a definite number of oscillation cycles occurring, or expected to occur, in a time interval,

(2) an hour-to-hour stability, or short period stability,

(3) a minute-to-minute stability, or very short term stability, and

(4) an instantaneous stability over intervals of seconds of time.

The accuracy of short interval stability of portable crystal chronometers is nowadays approaching 2 parts in 10^9. Further:

(5) a short "running-in" time to allow settling down. This is of great importance because of the frequent switching on and off which is the ordinary practice with battery-operated field equipment. This last requirement, namely "stability in interrupted service", is practically non-existent for observatory clocks.

Portability and robustness were the essential technical factors which controlled the design of the chronometer used. Physical and chemical stability of the material is expected as an important item to prevent frequency losses and to reduce the frequency-ageing rate.

3.5. Performance of the Crystal Chronometer

3.5.1. General, Rate, Errors.

The rate of a chronometer is the amount expressed in hours, minutes and seconds by which the time indicated by the chronometer, after a specified interval, differs from the true elapsed time, after the same specified interval. It is therefore the amount gained, rate negative, or lost, rate positive, over a specified length of time.

The isochronism error of a chronometer is the amount the rate differs from uniformity within a specified interval over which the rate is

3.5.1 Cont.

determined.

In general chronometers are tested to determine their precision for isochronism; this is the deviation and recovery of the rate due to temperature and position, hereby distinguishing between the travelling rate and standing rate. For chronometers mounted on gimbals, the tests are normally restricted to either the horizontal or vertical position. The magnitude of the rate is of no consequence for the purposes of calibration and time keeping.

For the assessment of its performance, the crystal chronometer under discussion was subjected to the above tests with additional testing to determine the required running-in time. The tests were carried out over a period of three years to investigate whether the chronometer satisfied the requirements listed in section 3.4. or not.

3.5.2. Rate Formulae.

It is known that mechanical chronometers follow a parabolic law, the law of linear motion. This is best shown graphically by plotting the amount of time gained or lost against the time indicated on the clock face.

In ordinary practice, because the time is referred to the rotation of the earth, the approximate performance of any clock is expressed in the form:

$$T_1 = t_o + A\, t_1 + B\, t_1^2$$

or:

$$T_1 = t_o + A\, t_1 + B\, t_1^2 + C\, t_1^3$$

Where T_1 is the future predictable time the clock will indicate after the elapsed time t_1, which has elapsed since the epoch t_o.

A, B, C, are clock parameters, depending on mechanical coefficients, temperature etc., and have to be determined from individual clock readings.

With reference to linear motion, the expression $A\ t_1$ can be considered as the velocity or as the rate of the clock, $B\ t_1^2$ the acceleration during time t_1, and $C\ t_1^3$ the drift of the acceleration. The solution of this equation of motion is possible by e. g. successive approximations.

The application of either of the above equations, for an approximation to a smooth curve joining the plotted observational results, is immediately apparent from an examination of the crystal clock performance diagrams. (See Section 3.5.4. and 3.5.5.)

A logarithmic or an exponential function of the form

$$T = t_o + A\ t_1 + B \ln\ t_1$$

$$T = t_o + A\ t_1 + B\ e^t$$

respectively, would obviously achieve no fundamentally different fit, because these functions can be represented as a power series of t. In the case of a nearly linear clock rate, only small errors would be left in its interpolation when using such an elementary mathematical expression.

The operational performance of the crystal clock - although not pertaining to the subject of mechanics - can be considered as an oscillation because there are periodic forces which effect its running. These oscillations occur in a certain interval of time, hence the formulae of the various types of oscillations as derived for the dynamics of points and rigid bodies could be applied, provided its constants and variables are appropriately interpreted. The question of which type of oscillation is possible in a dynamic system is closely connected with the question of the stability of its equilibrium, or the stability of

3.5.2 cont.

motion of the system. The oscillations of the quartz crystal, its proper oscillations, produce the forced oscillations of the electric magnetic circuit. Whether the crystal clock can be regarded as a special case of the free damped oscillation or damped forced oscillation depends on the magnitudes taken into consideration. In a resonant circuit the magnitudes to consider are: the capacity of the condensor inductance, resistance (it can hardly be a frictionless case), voltage, temperature of the oven, etc. There are the following relations between voltage U, resistance R, induction L during the interval of time dt, for the discharge of a condensor which is not a one time performance, but a sequence of oscillations:

$$U - L\frac{dI}{dt} - RI = O$$

Differentiating the above equation with respect to time:

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I = O$$

gives an expression between the associated quantities which has the same form as the equation for harmonic oscillation:

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega^2x = O$$

In this equation of motion the acceleration term $\frac{d^2x}{dt^2}$ of the oscillating point in the direction of the x-axis takes the place of the force in the energy equation.

$2k\frac{dx}{dt}$ is the frictional term, 2k its constant

ω = circular frequency, i.e. the number of vibrations in 2π units of time, of the restoring force.

3.5.2. cont.

The "path" is obtained by integrating the above equation of motion. For $k > \omega$ i.e. if the damping factor is greater than the constant of the restoring force, the path will approach the x-axis asymptotically without oscillation. Such a system with kinetic and potential energies moving in accordance with the above equation ultimately tends to rest. In addition to the damping force represented by the frictional term $-2k \frac{dx}{dt}$ and in addition to the restoring force, there is the force which acts on to the oscillating circuit, periodically with respect to time, originating from the oscillation of the quartz crystal. The properties of the quartz crystal, its temperature, and the length of time the crystal chronometer is operating, together with external parameters, temperature and voltage, will shape the curve obtained from the plotted values:- clock errors against time. The equation of motion will then be in the form of the differential equation of the damped forced oscillation:

$$\frac{d^2x}{dt^2} + 2k \frac{dx}{dt} + \omega^2 x = c \sin \nu t$$

The right hand term is the acceleration, resulting from the oscillation of the quartz crystal which is assumed to change with time following a sin law, ν is its frequency, c a constant representing the amplitude of vibrations. In conformity with mechanics, x, the path, as the result of the motion produced by the vibratory force, will also change periodically with the time t. When ν, the frequency of the quartz crystal, approaches the number of vibrations of the undamped harmonic oscillation ω, the condition of resonance, will be obtained. Any change in temperature of the oven will change the resonance condition and consequently there will be a drift in the rate of the crystal chronometer. The graphical representation of the rate in this case will show a typical curve of a function in

3.5.2 cont.

dependence of temperature. Generally any oscillating motion is a combination of the forced oscillations and of the natural frequency oscillations of the system which may gradually die out, being absorbed by the damping system.

The determination of the crystal clock parameters, which are in practice functions of time and material, on the basis of the mentioned formulae:

(1) linear motion

(2) logarithmic function

(3) exponential function

(4) free and forced damped and undamped oscillations

would leave only small residuals, in view of the uniform behaviour of the rate of the crystal chronometer tested. Therefore the theoretical investigation as to which formula would represent the true law of performance would be a lengthy effort, and would contribute little in improving the forecast of the chronometer's behaviour.

Fairly accurate frequency prediction and interpolation is possible from graphical representation of the clock performance, subject to the possibility of adequate comparison with available standard frequencies.

3.5.3. Calibration of the Crystal Chronometer

(A) Possibilities of Calibrating Crystal Chronometers.

No portable crystal chronometer can have, or can maintain, a completely constant frequency. Therefore testing the chronometer for its rate and change of rate is a normal procedure which cannot be dispensed with.

The rate of a crystal chronometer can be determined by comparing it in terms of a measurable interval between natural or artificial events. The selection of the type of interval and type of events is, for practical purposes, restricted to the following:

(1) The natural resonance of a molecule = nuclear decay process;

(2) The period of the earth's rotation, which contains secular, periodic, and irregular fluctuations (The irregular fluctuations are the most difficult to ascertain as their vector has to be determined afresh from current observations.).

(3) The periodic movements of stars, having long term changes,

(4) The time signal transmissions.

At the present time, the only practical solution to the problem of calibrating a portable secondary standard of frequency is to choose the interval marked by the time signal transmissions. (No. 4 above).

This can be accomplished in various ways, which are described below.

(B) - Methods of Calibrating Crystal Chronometers.

It is not intended to mention all feasible methods of calibrating a crystal chronometer, or to deal with techniques in detail. Only those methods are listed which are relevant to the field engineer. In particular, the methods described are those which were investigated and are new as regards their application to field astronomy. New methods and techniques seldom completely replace the well-established conventional methods, but they undoubtedly supplement them and contribute to the

3.5.3. cont.

perfection in the achievement of practical results.

The process of calibration can be performed by the following methods:

(1) Pulse Counter

The cycles of oscillation of the crystal chronometer can be counted over an interval determined by standard time signals, e.g. with a pulse counter, etc.; obviously, the longer the interval, the higher the accuracy of determination of the rate. The relative error of the rate derived, or of the numbers of cycles counted, is preserved by the registering apparatus, since the multiplication factor, governed by the technique of measurement, has absolutely no influence on the error. This method requires a special instrument which may not be easily available to the average field engineer; it is mentioned here, because in the near future portable pulse counters will be obtainable which will have the required frequency range.

(2) Frequency Comparison.

The time interval between two pulses of nearly equal frequencies (i.e. crystal chronometer and time signal) can be measured to a high order of accuracy in a short interval of time. Mixed in a non-linear circuit, the system produces a frequency which is equal to the difference between the two frequencies.

This method requires additional equipment and is therefore only briefly mentioned.

(3) Electro-mechanical Phaseshifter.

The employment of an electro-mechanical phase shifter allows the crystal chronometer to be set in synchronism with time signals received by radio. However, this can be used effectively only with crystal chronometers registering U.T. U.T. second contacts of the crystal chronometer are arranged so that they short circuit the radio time signals. No audio signal can be heard if the crystal chronometer and radio time pulses are exactly synchronized. An actual measurement of frequency difference cannot be executed.+) It follows that the rate and the change of rate of the crystal chronometer cannot be ascertained. It is therefore necessary to set the crystal chronometer against time signals immediately before observations are registered, and to rely on the precision quoted by the manufacturer. In this way the chronometer is much too dependent on reception conditions. Further, the electro-mechanical phase shifter is not suited for timing of optical observations and of no assistance in the investigation of the performance of the crystal chronometer.

(4) Stopwatch

The time interval between crystal chronometer pulses, or between chronometer pulse and radio time pulse, can be measured in the easiest way with a stopwatch. This method is applicable for the calibration of any chronometer registering any types and any subdivisions of time. The registering instrument, the stopwatch, is presumed to maintain a known rate. The accuracy depends mainly on the type of stopwatch employed. The length of time interval is not restricted, but no repeated measurements of any one time interval are possible. Information on the behaviour of the chronometer in intervals of minutes are readily obtained, but are not of primary importance. The performance during intervals of seconds can be measured only to the order of accuracy of the stopwatch.

This method does not achieve the accuracy required for the investigations into timing methods and measurements of short intervals of time, as treated in this thesis. The method is listed here in order to satisfy the needs of an average field engineer.

+ The frequency difference can be measured with additional equipment or with e.g. digital read-out of the e.m.-phase shifter.

3.5.3. cont.

(5) Paper Strip Chronograph

Both signals, crystal chronometer and time transmission, are recorded on a paper band chronograph (or on any mechanical or electrical chronograph) with a pen recorder. The time difference is abstracted by scaling or by the use of various forms of graphs and the chronometer rate is then deduced therefrom.

This conventional method is described in most text books and is still much in use. The time interval of one second is usually represented as a distance of 10 mm.

Generally, with this method, the accuracy in time comparison is accepted to be in the neighbourhood of 0.02 seconds, presuming a scaling accuracy of 0.1 mm. The time of optical observations recorded on a paper band chronograph and determined from measurements taken on it, is also affected by a possible drag error.

There are high speed pen recorders operating from mains supply or battery, with paper speeds of ½, 1, 2, 4, 8 up to 16 cms/sec. They weigh 35 to 40 lbs (without battery) and cost over £ 350, which imposes limitations on field use.

Methods of Calibration adopted for the Employment of the Crystal Chronometer in Field Astronomy.

No measuring method can be regarded as being free of systematical errors; such errors can be detected by comparing results obtained from different methods employed.

As a rule, relative measurements of physical quantities can be executed at a higher degree of precision than their absolute measurements.

The methods of calibration which were investigated consisted of accurate measurements of time intervals.

3.5 3. cont.

(6) Time Vernier

The 61 subdivisions of one minute which are audible as output pulses of the crystal chronometer, together with the seconds pulses superposed on standard frequency transmissions, constitute a time vernier. This time vernier was used as an approximate coincidence meter for the calibration of the crystal chronometer.

The aural determination of a vernier coincidence, which consists of two signals of different duration and frequency, is essentially the same as the aural test of detecting the vanishing signal coincidence. In practice, no signal ever synchronizes. The task, therefore, is to estimate the smallest time gap between chronometer and radio pulses. This can be done directly during reception of time transmissions. The accuracy which can be achieved depends on the observer's ear; it is therefore affected by the personal error, by the quality of the reception and by the type of time pulses received. The approximate coincidence, occuring once a minute, can be observed five or ten times during any one period of reception of co-ordinated time signals, since most of them are transmitted over a period of five or ten minutes. Tests have shown that the uncertainty in judgement of a near coincidence can be limited to three or to two seconds pulses. Thus the accuracy achieved is 3 to 4 x $\frac{1}{61}$ seconds = 0.05 to 0.07 seconds, for a single coincidence.

Although the time vernier has found some application in certain types of transmitted time signals, its employment at the receiving end is believed to be new and has not been found in any reference published hitherto.

(7) Slow Replay

The crystal chronometer pulses were recorded at high speed ($7\frac{1}{2}$ in./sec.) on magnetic tape recorder, via loud speaker, as audio pulses, or by direct feeding-in, and were played back at half speed ($3\frac{3}{4}$ in./sec)

3.5.3. cont.

and/or quarter speed (1^{7}/8 in./sec.) The interval between pulses was timed with a stopwatch at slow replay. Information about the rate and its variations in terms of U. T. can be calculated if the stopwatch is in synchronism with U. T., or if its rate is known. Differences of the running speed of the tape recorder have to be taken into consideration.

Generally speaking, the crystal chronometer can be considered as a time keeper much superior in quality to a stopwatch. The rate and the indication of U. T. of a stopwatch and also the constancy of the tape speed of a recorder are of limited reliability. Therefore standard frequency pulses w were used as a reference standard and were recorded together with the crystal chronometer pulses on the same tape track. The interval between crystal chronometer pulses and pulses of standard frequency transmissions was measured with a stopwatch at replay of the tape at half and at quarter speed. Thus the stopwatch was required to "carry" time intervals of four seconds duration only. Hence its synchronization with U. T. is not required and its rate becomes negligible.

This method allows for repeated measurements of the same time interval and produces a permanent record. Furthermore, this method can be used for comparing any type of chronometer with standard frequency transmissions, for testing the chronometer rate and change of rate, for the timing of optical observations and for tests of the stability of the running speed of the tape recorder.

The recordings were made on a Fi-Cord tape recorder and simultaneously on a Vortexion. This was intended as an independent check. The running speed of the Fi-Cord was tested against MSF standard frequency pulses and found to be sufficiently constant for the purposes required (Section 6.3.)

Time differences, during slow replay, could be measured at an accuracy of 0.03 seconds. This value was established from repeated observations of the same time interval and from the calculation of the r. m. s. error of the unit distance of time concerned.

3.5.3. cont.

This method is restricted by the quality and type of the stopwatch and by the capacity of the human ear to distinguish time intervals which are in the neighbourhood of 0.02 seconds.

The slow replay method used in conjunction with a time vernier which can be placed at the receiving end (crystal chronometer registering 61 subdivisions of a minute) or at the transmitter (System of Rhythmic Time Signals) permits a more accurate determination of a near coincidence.

It has been found that a near coincidence to a single beat could be determined during slow replay ($1\ ^7/8$ in./sec.) of the pulses previously recorded on high speed. ($7\frac{1}{2}$ in/sec.) Thus the above accuracy of 0.02 seconds could be achieved in comparing the crystal chronometer with standard frequency transmissions. The time interval between successive near coincidences is 1/61 seconds, which is almost beyond the average human ear capacity.

(8) Visible Time-Pulses

The author endeavoured to obtain more precise time comparison of the crystal chronometer with standard frequency transmissions and higher accuracy in measurements of time intervals, surpassing the accuracies achieved with methods (6) and (7). The most effective way of obtaining higher accuracy was thought to be possible by the elimination of the human ear in the process of time measurement. Therefore, the audio signals from the crystal chronometer and radio time transmissions, recorded on magnetic tape, were made temporarily visible by using special fluids and metallic powders. The time differences between crystal chronometer pulses, pulses superposed on standard frequency transmissions and observation signals were thus converted to a measure of length. The distances between the pulses could be scaled with an ordinary scale.

This method can be used for calibration of the chronometer for timing of optical observations and renders the recorded time intervals accessible

3.5.3. cont.

for repeated measurements. The time vernier is still very desirable for the successful application of this method. It helps to reduce the personal error in scaling and avoids the possibility of having permanently overlapping signals.

The procedure of making the pulses visible (i. e. "Tape Development") and the manner of scaling their distances is described in Sections 7.2 and 7.3

The accuracy of extracting time differences with this method is far superior to the accuracies achieved with any one method outlined before. A scaling accuracy of 0.00005 seconds was achieved with a glass scale graduated to 0.1 mm and a 5 x or 10 x magnifying glass. The smallest scale division was estimated to one tenth.

The above scaling accuracy could be achieved because every single cycle of the crystal chronometer or of any standard frequency pulse was visible after tape development.

This method has been employed for the performance tests executed as it has, besides higher accuracy, accumulated advantages: no personal error, except where estimation is involved, also no drag error. Further, the method gives great assistance in the identification of the type of signal recorded, by visual inspection, and provides, if wanted, a permanent record.

Methods (7) and (8) are not described in any textbook. Both methods are also suited for timing of optical observations, but are not mentioned for this purpose in any literature. Their application in field astronomy is believed to be new.

(C) Procedure of Performance Tests and of Calibration of the Crystal Chronometer for Astronomic Observations.

Information on the behaviour of the crystal chronometer during the length of time it is required for astronomic observations is obtained with sufficient accuracy by comparing the extracted U. T. of its

3.5.3. cont.

individual beats (Table 3.3.-1) with the U.T. of emission of radio time signals over the period of time during which both are received at the field station.

The average daily stability of emission times of radio time signals is within one millisecond. It follows that a correction for change in delay time of signal emission is unnecessary over the intervals of performance tests. The total delay time of emission, also, need not be taken into consideration. Further, no correction need be applied for the travel times of signals to the receiver, as these can be regarded as constant when neglecting changes in atmospheric conditions. Obviously the receiver is presumed to be stationary.

The radio receiver delay can also be disregarded and presumed to remain constant. (Section 4.4.) The tape recorder delay plays no part during performance tests.

The above procedure is justified, because only constancy of the time interval and its stability is the subject of calibration and of the performance tests. Over longer periods of time, or wherever it was considered advisable for unbiased assessment of the performance, appropriate corrections, e.g. for emission delays, were applied and duly noted.

It was found convenient to keep the crystal chronometer an adequate amount either slow or fast (six to seven seconds). This was also helpful in detecting an accidental application of a wrong sign.

The number of pulse intervals that it is advisable to measure depends on the quality of the reception. In the average case five to eight are enough.

The procedure adopted for testing the performance of the crystal chronometer is shown on the form sheet "Calibration of Crystal Chronometer", and a specimen calculation is included. (Tables 3.5.3.-1 and 3.5.3.-2).

Calibration of Crystal Chronometer

Time Difference between Crystal Chronometer Beat and

____________ Pulse, f _______ kc/s   ______ Pulse, f ______ kc/s

at Observation Station: _______ Date: ___________ approx. Time: __________

C.C. operating since: __________ Oven: on, off   Battery No.: __________

Field Temp.: ___ °F., Pressure: ___ "Hg., Voltage of C.C. Battery: ____ v.

Recording Instrument: _________ Tape No.: ______ Track: _____ Speed: ______

developed with: ___________________________________ Date: _________

$$\frac{\textit{Distance betw. std. f. pulse \& C.C. beat}}{\textit{" " two relevant std. f. pulses}} =$$

= Ratio to be added to U.T. of std. f. pulse.

− nominal time of C.C. beat (Table: 3.3.-1)
+ U.T. of C.C. beat, scaled from std. f. pulses

∓ Crystal Chronometer { fast / slow

Description	Measurements, Scale:		
C.C.			
C.C.			
C.C.			
C.C.			
C.C.			
C.C.			
C.C.			
C.C.			
C.C.			
C.C.			

TABLE: 3.5.3-1

Calibration of Crystal Chronometer

Time Difference between Crystal Chronometer Beat and

M.S.F. Pulse, f 5000 kc/s ______ Pulse, f ______ kc/s

at Observation Station: LONDON Date: FRI. 19.1.62 approx. Time: 20^{06}-07

C.C operating since: 19.1.62 13^{30} Oven: on, ~~off~~ Battery No.: A.

Field Temp.: 70 °F, Pressure: 30.5 "Hg., Voltage of C.C Battery: 11.75 v.

Recording Instrument: Vortexion Tape No.: L01 Track: A Speed: 7.5"/s

developed with: iron powder & $C_6H_4(CH_3)_2$ Date: 26.1.1962

$\frac{\text{Distance betw. std. f. pulse \& C.C. beat}}{\text{" " two relevant std. f. pulses}}$ = Ratio to be added to U.T. of std. f. pulse.

− nominal time of C.C beat (Table: 3.3.-1)
+ U.T. of C.C beat, scaled from std. f. pulses
∓ Crystal Chronometer { fast / slow

Description	Measurements, Scale: 1 mm/...		Length of Pulse		
U.T. = $20^h\ 06^m\ 20^s$	+0020.0			$\frac{1012.0}{1921.0}$	22.6230
C.C. 23	0992.0	1112.0	0120.0		20.5268
	1901.0				−2.0962
21^s	+0022.0	0013.2	0009.0	$\frac{0978.8}{1919.2}$	23.6066
C.C. 24	0956.6				21.5100
	1897.0				−2.0966
22^s	+0021.0	0012.3	0008.7	$\frac{0948.0}{1919.9}$	24.5902
C.C. 25	0927.0	1045.5	0118.5		22.4938
	1898.9				−2.0964
23^s	+0019.5	0010.4	0009.1	$\frac{0916.2}{1919.6}$	25.5738
C.C. 26	0896.7	1015.2	0118.5		23.4773
	1900.1				−2.0965
24^s	+0027.8			$\frac{0885.1}{1920.0}$	26.5574
C.C. 27	0857.3	0976.3	0119.0		24.4610
	1892.2				−2.0964
25^s	0018.7			$\frac{0853.7}{1919.8}$	27.5410
C.C. 28	0835.0				25.4447
	1901.1				−2.0963
26^s	0018.7	0005.2	(re meas.)	$\frac{0822.6}{1920.3}$	28.5246
C.C. 29	0803.9	0817.1		$\frac{0822.3}{1920.1}$	26.4284 (3)
	1901.6	1914.9			−2.0962 (3)
27^s	0012.1			$\frac{0791.1}{1920.3}$	29.5082
C.C. 30	0779.0				27.4120
	1908.2				−2.0962
C.C.					
C.C.					

TABLE 3.5.3.-2

3.5.3. cont.

(D) Errors of Calibration.

The results of calibration, calculated as presented in the specimen can be plotted, so that errors related to measurements and their connection to it can be shown by a graph.

The calibration error incorporates errors from various sources. These can be sought in the following items which are listed below. A theoretical estimate of their magnitudes is included.

(1) Scaling pulse distances on the developed magnetic tape

Scale subdivision: 0.1 mm.
Estimated interval: 0.05 mm.
Recording speed: $7\frac{1}{2}$ inch/sec.

(Section 7.2 and 7.3) . . . . $\pm$ 0.3 milliseconds.

(2) Quality of magnetic tape, uneven residual elongation . . . . $\pm$ 0.1 "

(2b) Variations of running speed of the recording instrument (Section 6.3) . . $\pm$ 0.1 "

(3) Radio receiver delays . . . . $\pm$ 0.4 "

(4) Crystal chronometer contacts . . . . . . $\pm$ 0.1

The total error is about $\pm$ 0.5 milliseconds and may approach $\pm$ 1 millisecond. It is derived with the use of the law of propagation of errors. Here, the presumption is made that the individual errors listed follow a Gaussian distribution about their true values. Whether this conception is correct is difficult to say, because the result, the frequency difference, is obtained from measurements taken with the aid of various instruments having different classes of working tolerance.

It would require a detailed analysis of the errors involved to improve the calibration accuracy. (Electronics problem)

The reliability of calibration can be considerably reduced when transmissions traverse the ionosphere. The variations in ionization of the layers of the ionosphere alter the velocity of propagation and distort

3.5.3. cont.

the path of the transmissions. This can cause the received frequency to differ from that transmitted by several milliseconds.

3.5.4. Performance of the Crystal Chronometer operating from the mains supply, with the heating circuit switched off.

(a) Long term performance and daily stability

Fig. 3.5.4.-1 shows the performance of the crystal chronometer kept in a sufficiently constant temperature of about 72°F, and operated with the driving batteries connected via the battery charger to the mains supply.

The plotted points are average values of time differences in U. T., between chronometer signals and two seconds pulses superposed on standard frequency transmissions. The total number of crystal chronometer signals used to obtain one average value is shown in brackets near the point plotted. The number is, of course, dependent on reception conditions. The standard deviation of each average value in milliseconds is also shown. The measurements of time differences are about 24 hours apart, and their plotted averages are connected by a smooth curve. Corrections from the Time Service Circular of the Royal Greenwich Observatory are applied. The measured deviations represented by the graphs are referred to whichever standard frequency reception was available:
M. S. F., 2.5 and 5 Mc/s., H. B. N. 2.5 and 5 M/cs, W. W. V. 2.5 and 5 Mc/s, O. M. A. 2.5. Mc/s. The corrected and uncorrected time differences are shown in Fig. 3.5.4.-2, where the uncorrected mean values of frequency deviation are joined by a dashed curve. The distances from the corrected to the uncorrected mean values are of different magnitudes, since various frequency receptions are used.

The standard frequency transmissions are in terms of U. T. 2, which takes account of polar and seasonal variations. The summary (Table 3.5.4.-1) contains details of recording and calculation of data, which are used to compile the graph for the period 17.1.1961 to 25.1.1961. The resulting time-error curve (Fig. 3.5.4-1) consisting of four separate

3.5.4 cont.

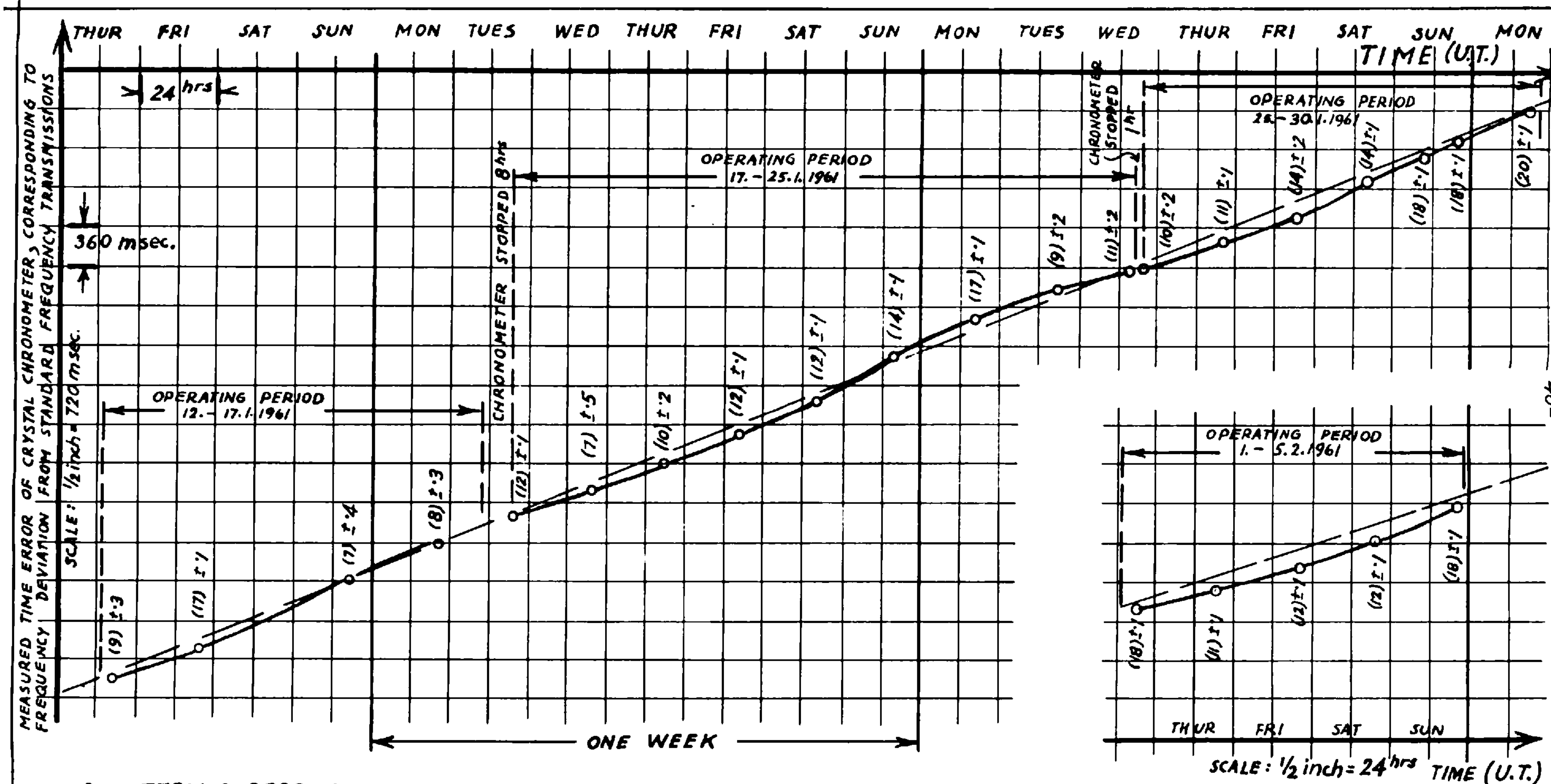
periods of crystal chronometer running, covers the time interval of about three weeks. The curve indicates a rate of the chronometer, which can be adjusted if required, but for comparing subsequent tests, the frequency drift has not been altered.

When a crystal chronometer is switched on, its rate is actually unknown and has to be presumed to be variable, although from past performance a very fair estimate is possible. After a temporary suspension of the running of the crystal chronometer, even for a short period of time, e.g. one hour, the rate, may or may not, come back. The small difference between the average rates of the chronometer during two consecutive periods of operation can be seen on the diagram (Fig. 3.5.4.-2), which shows the chronometer performance at a larger scale.

The small differences between the average rates of the three separate periods would not be noticable at the plotting scale in Fig. 3.5.4.-1. Further, these first three periods are not symmetrical to each other and are of unequal length, and graphs representing their average rates would be unsuited for comparison. The dashed line, therefore, is plotted to indicate the average rate over the three periods. For clarity of representation the initial chronometer settings are plotted, in each case, with reference to the extrapolated chronometer rate of the previous period.

The ordinates between the dashed and full line are a measure of the isochronism error in each case.

There is, as can be seen from the diagrams, a periodic change of frequency deviation. This is mainly due to voltage variations in the mains, depending on the period of varying power consumption in the locality where the crystal chronometer is operating. The duration of this cycle, of the particular time-error curve shown, is seven days,



LONG TERM PERFORMANCE
OF THE CRYSTAL CHRONOMETER
OPERATING ON MAINSPOWER SUPPLY
OVEN HEATING CIRCUIT SWITCHED OFF

DATE: JAN.-FEB. 1961
LOCATION: LONDON R.S.M. INST. ROOM.

PLOTTED POINTS = ARITHMETIC MEANS OF MEASUREMENTS OF TIME DIFFERENCES BETWEEN CRYSTAL CHRONOMETER AND SECONDS PULSES SUPERPOSED ON STANDARD FREQUENCY TRANSMISSIONS.

FULL LINE = ISOCHRONISM CURVE OF CRYSTAL CHRONOMETER

DASHED LINE = AVERAGE CRYSTAL CHRONOMETER PERFORMANCE

THE THREE SEPARATE PERIODS OF OPERATION ARE PLOTTED WITH REFERENCE TO THE AVERAGE RATE.

Fig. 3.5.4.-1

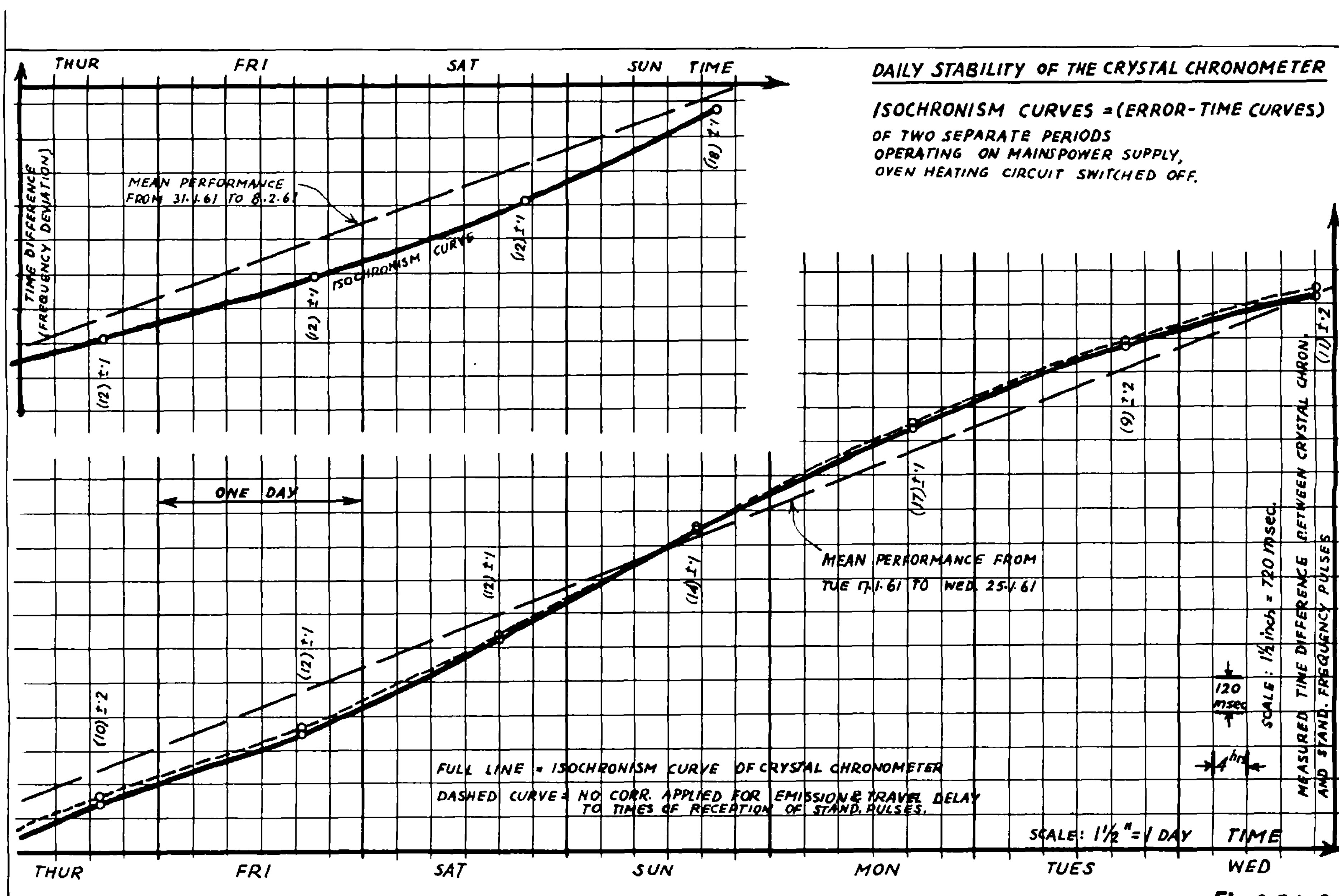

DAILY STABILITY OF THE CRYSTAL CHRONOMETER
ISOCHRONISM CURVES =(ERROR-TIME CURVES)
OF TWO SEPARATE PERIODS
OPERATING ON MAINSPOWER SUPPLY,
OVEN HEATING CIRCUIT SWITCHED OFF.
THUR
FRI
SAT
SUN
TIME
TIME DIFFERENCE (FREQUENCY DEVIATION)
MEAN PERFORMANCE FROM 31.1.61 TO 8.2.61
ISOCHRONISM CURVE
ONE DAY
MEAN PERFORMANCE FROM TUE 17.1.61 TO WED. 25.1.61
FULL LINE = ISOCHRONISM CURVE OF CRYSTAL CHRONOMETER
DASHED CURVE = NO CORR. APPLIED FOR EMISSION & TRAVEL DELAY TO TIMES OF RECEPTION OF STAND. PULSES.
SCALE: 1½" = 1 DAY
MEASURED TIME DIFFERENCE BETWEEN CRYSTAL CHRON. AND STAND. FREQUENCY PULSES
SCALE: 1½ inch = 720 msec.
120 msec
4 hrs
MON
TUES
WED

Fig.3.5.4.-2

CALIBRATION OF THE CRYSTAL CHRONOMETER

POWER SUPPLY: MAINS, OVEN: OFF, AT: R.S.M., LONDON.

CRYSTAL CHRONOMETER OPERATING FROM 17.1.1961 $18^h 26^m$.

DATE	PULSES RECORDED FROM / TO	PULSES AT ARITHM. MEAN	№ OF PULSES	CRYSTAL CHRONOM.' + SLOW − FAST CORR.	STANDARD DEVIATION SINGLE OBSERV.' ±	STANDARD DEVIATION ARITH.' MEAN ±	TIME INTERVAL BETWEEN TRANSM.' PULSES	RATE PER HOURS	PULSE SOURCE
	h m s	h m s		sec.	sec.	sec.	h m s	sec.	
TUE. 17.1.1961	18 47 52 48 07	18 47 59·5	12	$-2{\cdot}068_6$ $+{\cdot}046_6$	${\cdot}000_4$	${\cdot}000_1$			H.B.N. 5 Mc/s
WED. 18.1.1961	18 38 55 39 05	18 39 00	7	$-2{\cdot}280_7$ $+{\cdot}046_4$	${\cdot}001_4$	${\cdot}000_5$	23 51 00_5	${\cdot}008_9$	H.B.N 2.5 Mc/s
THU. 19.1.1961	17 01 56 02 06	17 02 01	10	$-2{\cdot}506_7$ $+{\cdot}012_8$	${\cdot}000_7$	${\cdot}000_2$	22 23 01	${\cdot}010_1$	M.S.F. 5 Mc/s
FRI. 20.1.1961	16 52 50 53 02	16 52 56	12	$-2{\cdot}755_9$ $+{\cdot}013_1$	${\cdot}000_4$	${\cdot}000_1$	23 50 55	${\cdot}010_4$	"
SAT. 21.1.1961	15 45 51 46 04	15 45 57_5	12	$-3{\cdot}082_0$ $+{\cdot}013_7$	${\cdot}000_4$	${\cdot}000_1$	22 53 01	${\cdot}014_3$	"
SUN. 22.1.1961	15 34 53 35 08	15 35 00_5	14	$-3{\cdot}472_2$ $+{\cdot}013_6$	${\cdot}000_3$	${\cdot}000_{08}$	23 49 03	${\cdot}016_4$	"
MON. 23.1.1961	16 48 52 49 13	16 49 02_5	17	$-3{\cdot}823_8$ $+{\cdot}013_8$	${\cdot}000_3$	${\cdot}000_{08}$	25 14 02	${\cdot}013_9$	"
TUE. 24.1.1961	17 50 58 51 08	17 51 03	9	$-4{\cdot}105_1$ $+{\cdot}014_0$	${\cdot}000_5$	${\cdot}000_2$	25 02 01	${\cdot}011_2$	"
WED. 25.1.1961	16 04 52 05 02	16 04 57	11	$-4{\cdot}290_6$ $+{\cdot}014_4$	${\cdot}000_7$	${\cdot}000_2$	22 13 54	${\cdot}008_3$	"

CHANGE OF DAILY RATE/WEEK = $\pm 0{\cdot}024$ sec.

CHANGE OF HOURLY RATE/DAY = $\pm 0{\cdot}002_9$ sec.

CHANGE OF HOURLY RATE/WEEK = $\pm 0{\cdot}001$ sec.

TABLE 3.5.4.-1

3.5.4 cont.

with the highest negative or highest positive rate on Saturday and Sunday, the lowest negative or positive rate on Wednesday; consequently the minimum or maximum of the weekly period is on Friday night for negative or positive rate respectively.

The average daily stability of the chronometer, working at an input voltage of 12.5 v, is 3 parts in 10^7, as the average amplitudes of the cycles scaled from diagrams is about 150 msecs. With reference to the measured voltage variation during the week, the frequency stability amounts to $\pm$ 0.03 msec/day/$\frac{1}{2}$ volt.

There is no reason to believe that these sinusoidal undulations can be attributed to the recording equipment, because, for that very reason, as mentioned in section 3.5.3 (6), two tape recorders were used simultaneously, one battery operated and one on the mains. To eliminate any possible effect of irregularities of standard frequency receptions, every effort was made to obtain time signals from various transmitting stations.

The length of time during which the crystal chronometer is out of operation, i.e. the quartz crystal not oscillating, is of no consequence to its general performance and no apparent running-in time can be noticed.

There is no necessity to have the crystal chronometer, which is specified for field astronomy, operating on mains supply during the period of a whole week. The reason for testing its long term performance is to ascertain if it could be used also as stationary time standard for small observatories.

As the results show, its general behaviour and accuracy during uninterrupted running on the main with the heating circuit switched off, is little inferior to large observatory standards.

3.5.4. cont.

(b) Short Term Performance = Hourly Stability of the Crystal Chronometer

operating from the mains supply with the oven heating circuit switched off.

Fig. 3.5.4.-3 shows the performance of the crystal chronometer over a period of several hours. The chronometer is kept at a constant temperature of about 72°F; operating from the mains; the oven is switched off.

The values plotted are the means of time differences of several crystal chronometer signals, against signals of standard frequency receptions. The number of signals used is shown near each point plotted. Full data are given in table 3.5.4.-2. The diagram reveals that the performance of the crystal chronometer approximates closely to a linear form. The two parallel lines drawn at a distance of 10 milliseconds about the mean rate emphasize the near linear behaviour of the chronometer over the specified time interval. No corrections for travel delay, emission time etc., have been applied.

As can be seen, simultaneous receptions of two standard frequency transmissions were possible and these confirm the chronometer performance.

The time-error curve is approximately of parabolic shape. The isochronism error at any instant for the time interval shown, could be scaled off as ordinate from the mean rate. The mean rate would be a straight line almost coinciding with the time-error curve, and is not shown in order not to confuse the diagram. The two parallel lines can be used to obtain the isochronism error accordingly.

As expected, no running-in time can be noticed, which would show up in the curve as a distinct interval of characteristic shape. The reason for its absence is that the chronometer was kept sufficiently long under the same conditions for the quartz crystal to acquire

3.5.4. cont.

the same temperature as its environment. Therefore, no changes in temperature would be likely to arise which would upset the constancy of oscillation.

The insert in the upper left corner of Fig. 3.5.4.-3 shows the same isochronism curve of the chronometer, plotted at a reduced scale for general presentation.

The hour to hour stability determined from tests over a period of more than three years has proved to be consistent. Fig. 3.5.4.-4 shows observations for the hourly performance taken about one year later than those displayed in Fig. 3.5.4.-3. In the meantime, the rate of the crystal chronometer has been neither changed nor adjusted. Between the three periods of operation presented, the chronometer has been switched off as indicated. Throughout the time of its running the chronometer was maintained at room temperature of about 72°F, with the oven switched off, etc., purposely to establish the same set of conditions that prevailed during the previous tests. It can be seen from the graph and from the table 3.5.4.-3 containing the relevant data, that the hour-to-hour behaviour is very nearly the same, and that the rate is within the range given further on.

Evidently, there is no effect which could be attributed to the frequency aging rate of the quartz crystal. It has been the purpose to discover this; and therefore the last mentioned tests have been executed after the chronometer has been used extensively in the field under most adverse conditions as regards to temperature, transport etc.

On account of the uniform chronometer performance, time prediction and interpolation over five to six hours can be made to an accuracy of a few milliseconds. The accuracy of the hourly stability and of interpolation within an interval of about one hour, marked by two comparisons of the chronometer with standard frequency transmissions, can be accepted to be about $\pm$ 0.5 milliseconds.

The quoted accuracy of chronometer calibration could only be

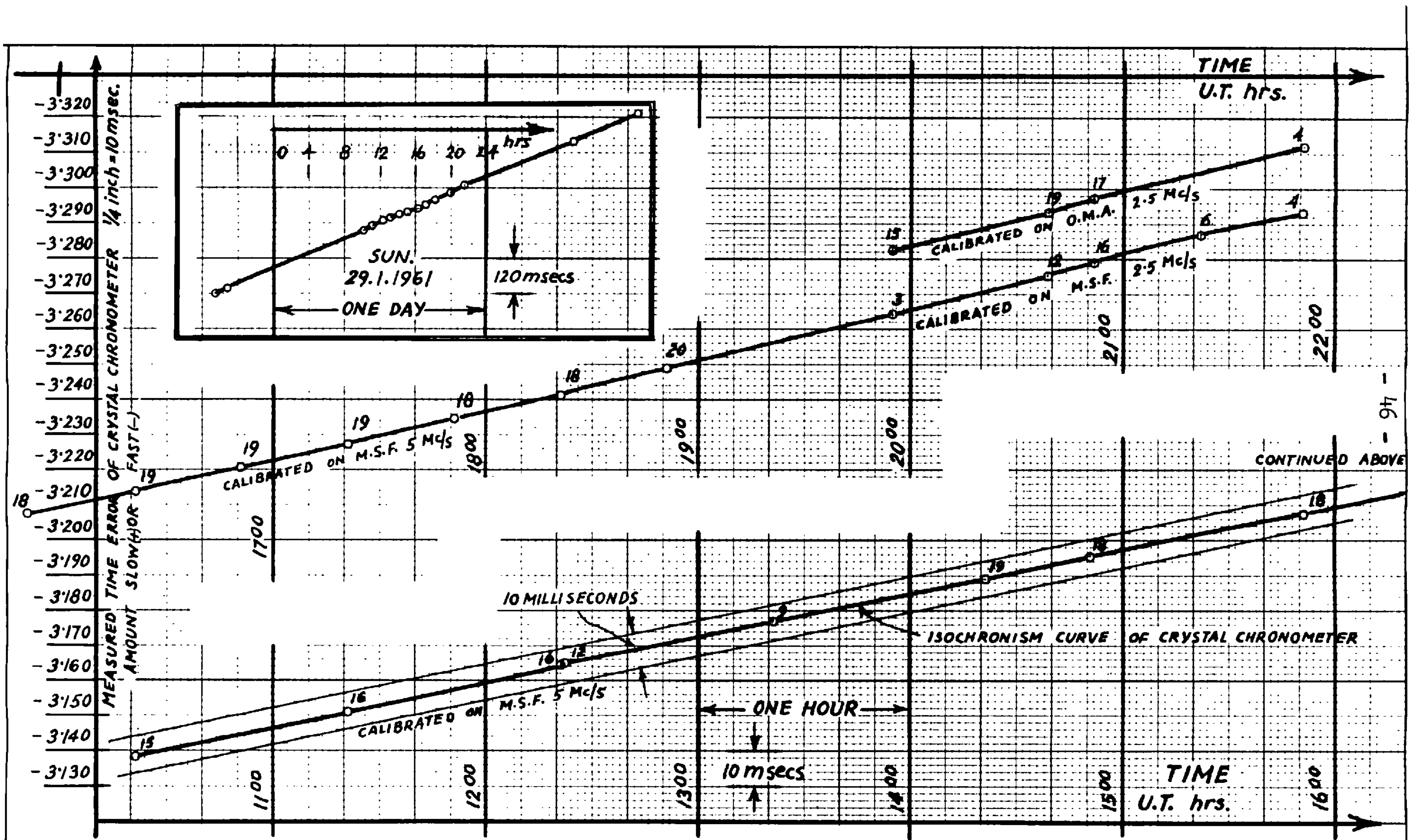
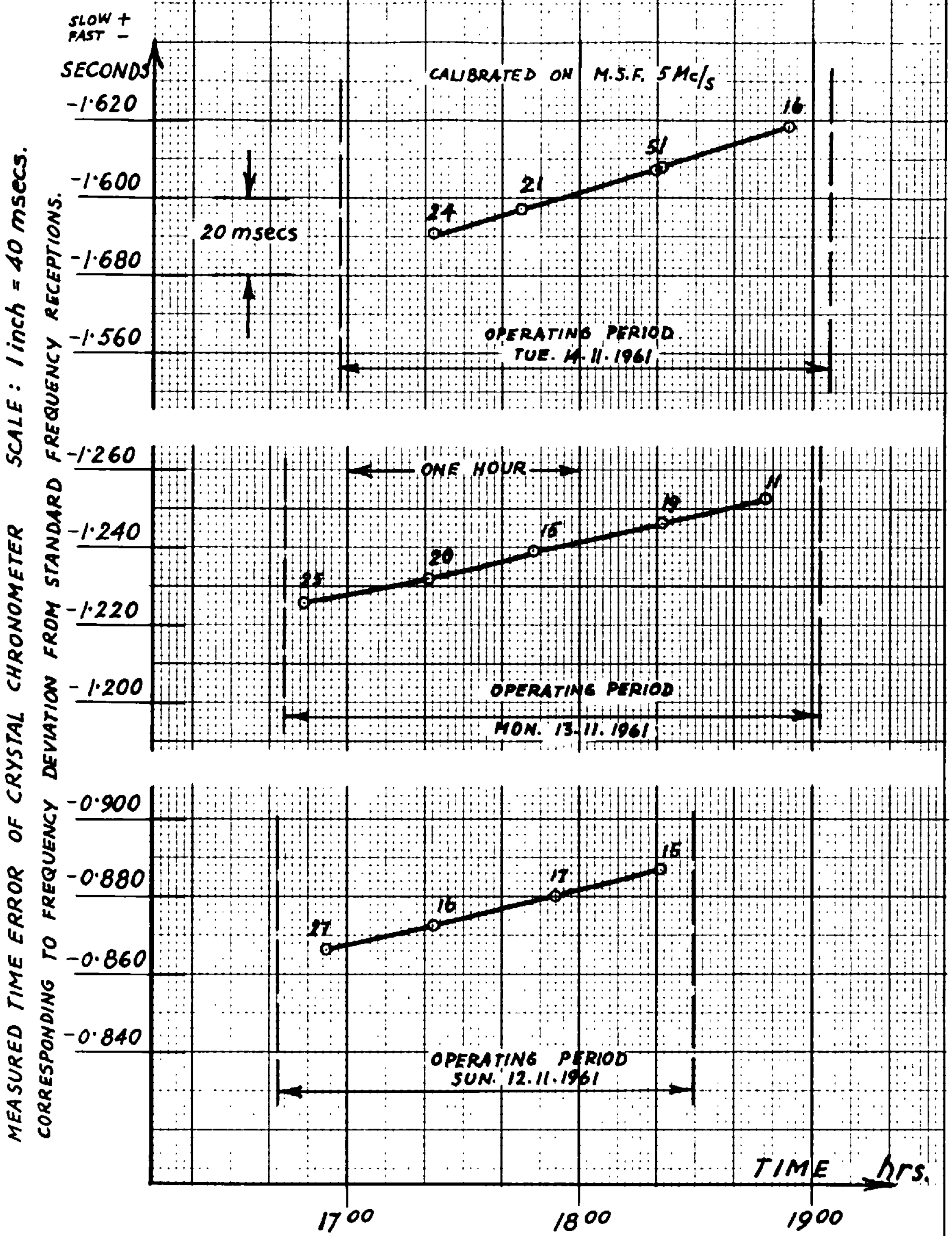


Fig. 3.5.4.–3

DATE: SUN. 29.1.1961
LONDON
R.S.M. INST. ROOM

SHORT TERM PERFORMANCE OF CRYSTAL CHRONOMETER

OPERATING ON MAINSPOWER SUPPLY
OVEN HEATING CIRCUIT SWITCHED OFF



SHORT TERM PERFORMANCE OF CRYSTAL CHRONOMETER

HOURLY STABILITY DURING THREE SEPARATE PERIODS.

DATE: NOV. 1961
LONDON.
R.S.M. INST. ROOM.

OVEN: OFF.
POWER: MAINS

Fig. 3.5.4.-4

CALIBRATION OF THE CRYSTAL CHRONOMETER

POWER SUPPLY : MAINS, OVEN: OFF, AT: R.S.M., LONDON.

DATE	PULSES RECORDED FROM / TO h m s	No. OF PULSES	CRYSTAL CHRONOMETER + SLOW − FAST sec.	STANDARD DEVIATION SINGLE OBSERV? ± sec.	STANDARD DEVIATION ARITH? MEAN ± sec.	TIME INTERVAL BETWEEN PULSES h m s	RATE PER 30 MINUTES sec.	PULSE SOURCE	REM?
SUN. 29.1.1961	15 50 50 / 51 09	18	$-3{\cdot}207_6$	$\cdot000_3$	$\cdot000_{08}$			M.S.F. 5 Mc/s	E.M.I. TAPE
	16 20 52 / 21 12	19	$-3{\cdot}214_0$	$\cdot000_8$	$\cdot000_2$	00 30 02	$\cdot006_4$	"	"
	50 49 / 51 08	19	$-3{\cdot}220_6$	$\cdot000_3$	$\cdot000_{08}$	29 57	$\cdot006_6$	"	"
	17 20 48 / 21 09	19	$-3{\cdot}227_6$	$\cdot000_5$	$\cdot000_1$	30 00	$\cdot007_0$	"	"
	50 54 / 51 12	18	$-3{\cdot}234_3$	$\cdot000_4$	$\cdot000_{09}$	30 04	$\cdot006_7$	"	"
	18 20 51 / 21 10	18	$-3{\cdot}241_6$	$\cdot000_7$	$\cdot000_{11}$	29 58	$\cdot007_3$	"	"
	50 46 / 51 06	20	$-3{\cdot}248_6$	$\cdot000_4$	$\cdot000_{09}$	29 55	$\cdot007_0$	"	"
	19 54 49 / 54 57	3	$-3{\cdot}264_2$	$\cdot001_3$	$\cdot000_{08}$	01 03 57	$\cdot007_3$	M.S.F. 2·5 Mc/s	"
	54 48 / 55 11	15	$-3{\cdot}282_6$	$\cdot001_0$	$\cdot000_3$			O.M.A. 2·5 Mc/s	"
	20 38 47 / 39 08	12	$-3{\cdot}275_4$	$\cdot000_7$	$\cdot000_2$	44 04·5	$\cdot007_6$	M.S.F. 2·5 Mc/s	"
	38 47 / 39 08	19	$-3{\cdot}293_8$	$\cdot001_1$	$\cdot000_3$	43 58	$\cdot007_6$	O.M.A. 2·5 Mc/s	"
	51 45 / 52 07	16	$-3{\cdot}278_9$	$\cdot000_9$	$\cdot000_2$	12 58·5	$\cdot007_2$	M.S.F. 2·5 Mc/s	"
	51 45 / 52 07	17	$-3{\cdot}297_0$	$\cdot000_8$	$\cdot000_2$	12 58·5	·0074	O.M.A. 2·5 Mc/s	"
	21 21 46 / 22 00	6	$-3{\cdot}286_8$	$\cdot001_6$	$\cdot000_7$	29 57	$\cdot007_9$	M.S.F. 2·5 Mc/s	"
	50 51 / 51 05	4	$-3{\cdot}293_2$	$\cdot000_7$	·0004	29 05	$\cdot006_6$	"	"
	50 51 / 51 04	4	$-3{\cdot}311_9$	$\cdot000_6$	·0003	59 01·5	$\cdot007_6$	O.M.A. 2·5 Mc/s	

TABLE 3.5.4.−2

CALIBRATION OF THE CRYSTAL CHRONOMETER

POWER SUPPLY : MAINS, OVEN : OFF, AT: R.S.M. LONDON.

DATE	PULSES RECORDED FROM / TO h m s	No OF PULSES	CRYSTAL CHRONOMETER + SLOW − FAST sec.	STAND DEVIATION ± sec.	TIME INTERVAL BETW. PULSES h m s	RATE PER 30 MINUTES sec.	PULSE SOURCE	REM.S
SUN. 12.11.1961	16 54 24 / 55 00	27	− ·866$_2$	·000$_1$		−(·007$_3$)	M.S.F. 5 Mc/s	Scotch TAPE Vortexion
	17 21 43 / 22 00	16	·872$_6$	·000$_2$	00 27 09·5	·007$_1$	″	″
	53 40 / 53 58	17	·879$_6$	·000$_2$	31 57·5	·006$_6$	″	″
	18 20 47 / 21 04	15	·886$_9$	·000$_3$	27 06·5	·008$_1$	″	″
MON. 13.11.1961	16 48 28 / 49 02	25	−1·225$_2$	·000$_1$		−(·007$_5$)	″	E.M.I.
	17 21 19 / 21 39	20	1·231$_6$	·000$_2$	00 32 44	·006$_0$	″	″
	47 49 / 48 05	15	1·239$_2$	·0004	26 28	·008$_6$	″	″
	18 20 23 / 21 01	19	1·246$_2$	·000$_1$	32 45	·006$_4$	″	″
	47 51 / 48 03	11	1·252$_8$	·000$_2$	27 13	·007$_3$	″	″
TUE. 14.11.1961	17 22 06 / 22 30	24	−1·591$_2$	·000$_0$		−(·007$_5$)	″	″
	45 24 / 45 47	21	1·597$_2$	·0002	23 17·5	·007$_7$	″	″
	18 20 16 / 21 47	51	1·608$_1$	·000$_1$	35 26·0	·009$_2$	″	″
	53 58 / 54 14	16	1·618$_1$	·000$_2$	33 04·5	·009$_1$	″	″

TABLE 3.5.4.−3

3.5.4. cont.

achieved by using the developed method No. 8 (Section 3.5.3.) The rate can be ascertained within 30 minutes to an accuracy of about one millisecond; and the time required to determine the change of rate is about one and a half to two hours.

The human ear would require recorded vernier measurements between the interval of at least two to three hours to detect the amount of hourly rate to the nearest hundredth of a second at slow re-play. The determination of the change of rate with the last method takes about six hours. It is doubtful whether adequate time signal reception over this total length of time can be expected or not.

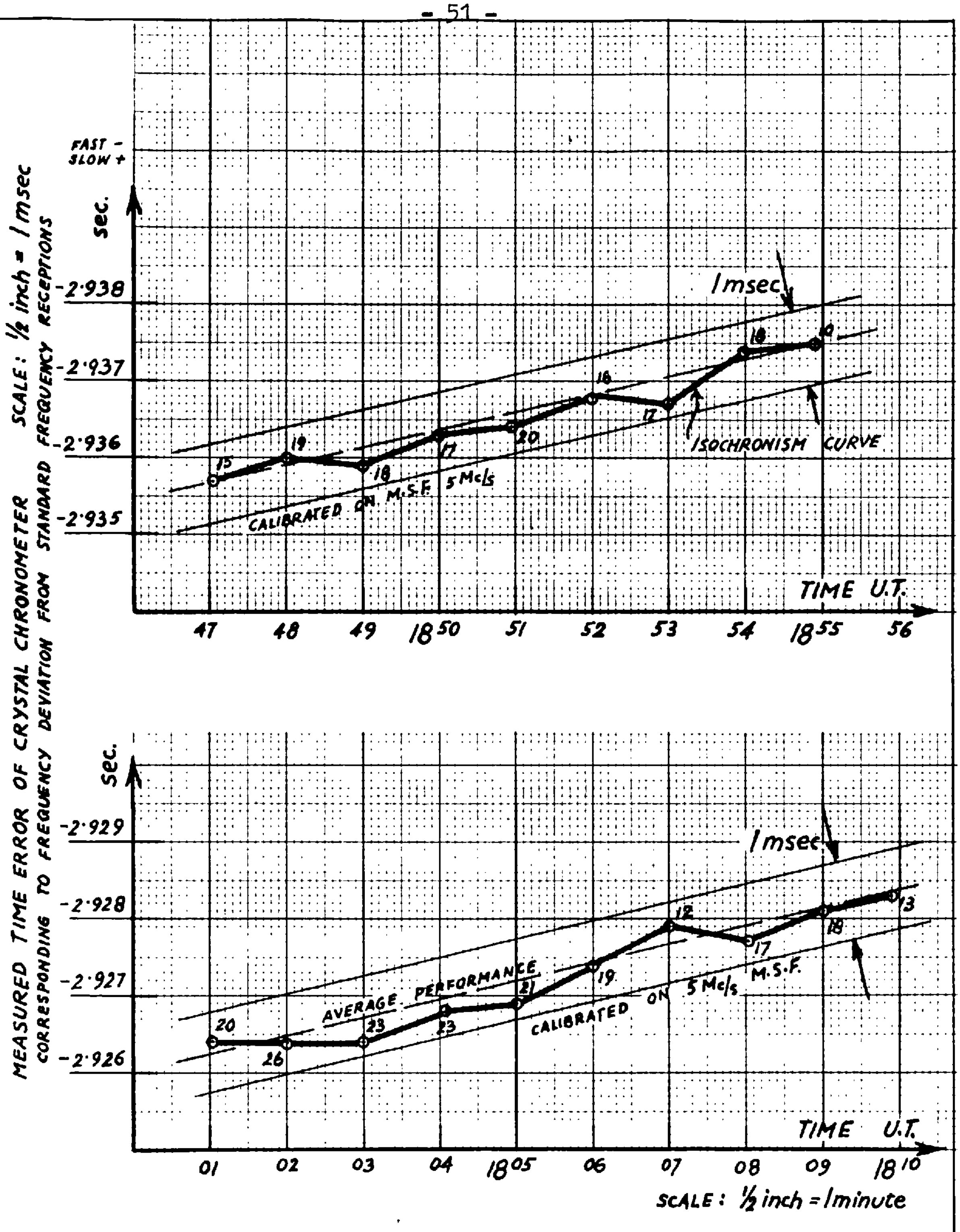
(c) Very Short Term Performance = Minute-to-Minute Stability of the Crystal Chronometer

operating on mains power supply, with the oven heating circuit switched off.

The chronometer stability from minute to minute is shown in Fig. 3.5.4.-5. As in previous diagrams, the points plotted are groups of chronometer comparisons with standard frequency transmissions. The number of signals forming the arithmetic mean is shown near the points.

The two parallel lines subtending a distance of one millisecond are drawn symmetrically about the mean rate, relevant to the space of time plotted.

The isochronism "curve" is represented by the zigzag line joining the plotted average measurements grouped near consecutive minutes. The average rate is shown by a dashed line; and the ordinate distance from it to the individual points is their isochronism error, which is, as can be clearly seen, in the order of 0.5 milliseconds. The minute stability of the crystal chronometer is approaching the accuracy



VERY SHORT TERM PERFORMANCE OF CRYSTAL CHRONOMETER

MINUTE-TO-MINUTE STABILITY

DATE: JAN. 1961

LONDON, R.S.M. INST. ROOM

OVEN: OFF

POWER: MAINS

Fig. 3.5.4.-5

3.5.4. cont.

which can be achieved with the calibration method No. 8 (Section 3.5.3) or vice versa. The error of claibration (section 3.5.3(D), is therefore greatly affecting the appearance of the isochronism curve. (The scaling of the magnetic tape was done with a box wood -rule graduated to 0.5 mm. Section 7.3)

The minute-to-minute stability can be regarded to be well within ± 1 millisecond. There is no reason to expect inconsistency in the minute-to-minute performance during the running-in time, which needs not to be considered when dealing with minute intervals. Its magnitude will no doubt embrace the whole time space presented in the diagram and will consequently not show up. Instruments employing crystal oscillators have normally a waiting period or running-in time approaching 20 to 30 minutes.

3.5.4. cont.

(d) Performance of the crystal chronometer during intervals of seconds = instantaneous stability of the crystal chronometer operating from the mains supply, with the oven heating circuit switched off.

The performance of the crystal chronometer during the interval of 60/61 solar seconds, marked by two successive pulses, is of utmost importance for its intended use.

Fig. 3.5.4.-6 shows plotted results of typical chronometer behaviour determined from standard frequency comparisons under average reception conditions as prevail in the instrument room of the R.S.M. building. Each plotted point represents one chronometer signal. Its time in terms of U.T. can be read off the abscissa. The ordinate of each point is the actual amount of time (U.T.) the chronometer signal was slow or fast with respect to its nominal time. This value is obtained from measured time differences of the chronometer signal to

3.5.4. cont.

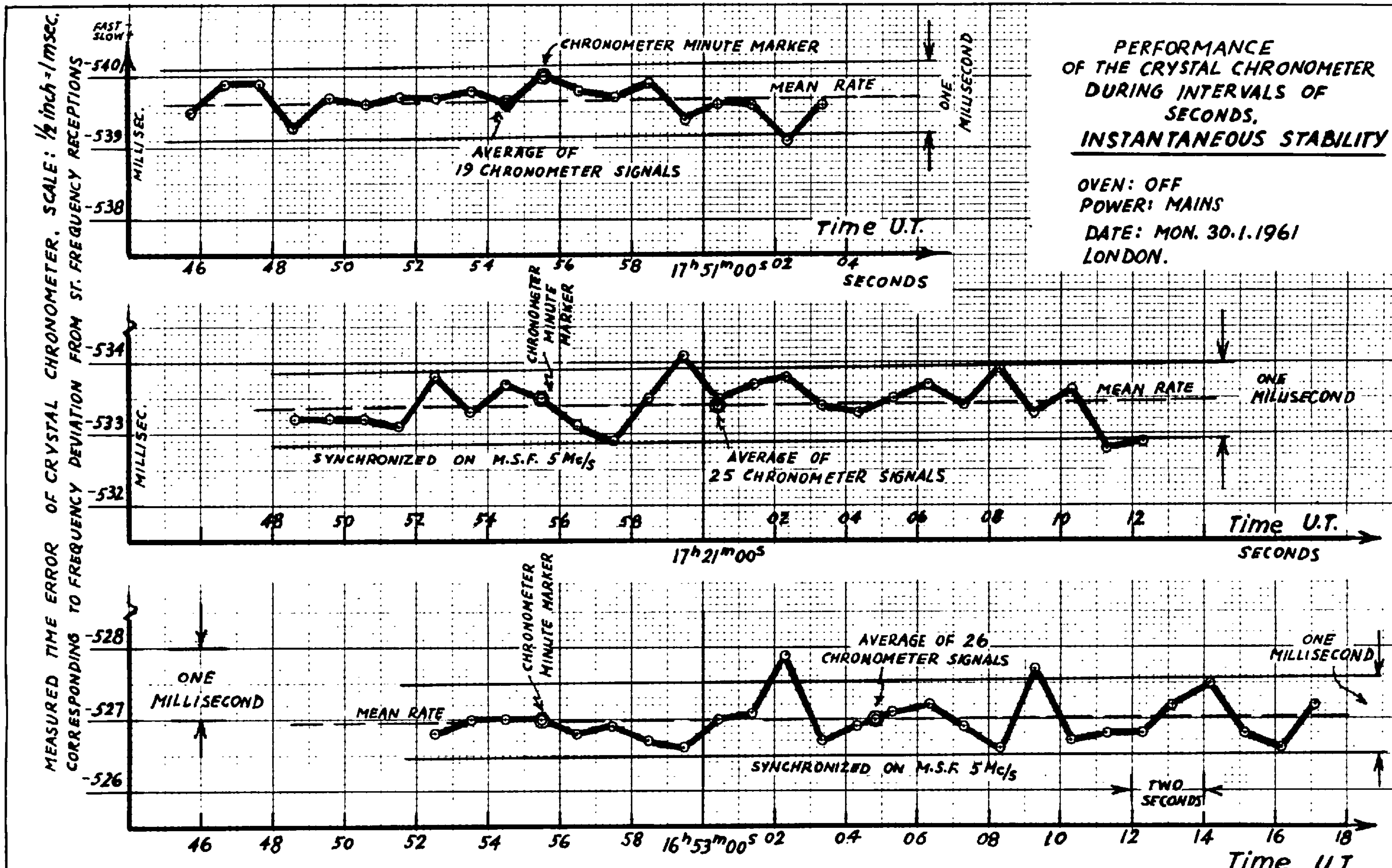
two second signals of standard frequency transmission received. The three consecutive sets of time comparison made on 5 Mc/s (M.S.F.), in the late afternoon, are grouped about the lengthened 60th second pulse of the transmitted frequency. Amongst the crystal chronometer pulses the chronometer minute marker is also included. The recording of the minute markers of both frequencies to be compared is advisable for accurate indication of the time. Alternatively, time indication can be obtained by counting the seconds pulses from the last minute marker.

The dashed line is the mean rate during the time interval concerned, and is obtained by joining the points representing group averages.

It can be seen that each individual chronometer pulse is less than one millisecond off the mean. The accuracy in calibration is emphasized by the two parallel lines, subtending one millisecond, drawn parallel to the mean rate. The column four of table 3.5.4.-4 contains the standard deviation of one single chronometer pulse and column five the standard deviation (or mean square error) of the arithmetic mean of the total group of signals.

Extensive tests have shown that the average mean square error of one single pulse is in the order of about $\pm$ 0.5 milliseconds and of the arithmetic mean about $\pm$ 0.1 millisecond, and is mainly due to calibration errors.

For timing optical observations the stability of the individual chronometer pulse is of significance. For time indication or for calibration, the standard deviation of the arithmetic mean will reveal the chronometer stability. Obviously only the developed calibration method No. 8 can be employed to deal with time intervals and accuracies of the quoted order.



PERFORMANCE
OF THE CRYSTAL CHRONOMETER
DURING INTERVALS OF
SECONDS.
INSTANTANEOUS STABILITY
OVEN: OFF
POWER: MAINS
DATE: MON. 30.1.1961
LONDON.
MEASURED TIME ERROR OF CRYSTAL CHRONOMETER. SCALE: 1/2 inch = 1 msec.
CORRESPONDING TO FREQUENCY DEVIATION FROM ST. FREQUENCY RECEPTIONS
FAST +
SLOW −
MILLISEC.
-540
-539
-538
CHRONOMETER MINUTE MARKER
MEAN RATE
ONE MILLISECOND
AVERAGE OF
19 CHRONOMETER SIGNALS
Time U.T.
46
48
50
52
54
56
58
17h51m00s 02
04
SECONDS
-534
-533
-532
CHRONOMETER MINUTE MARKER
MEAN RATE
ONE MILLISECOND
SYNCHRONIZED ON M.S.F. 5 Mc/s
AVERAGE OF
25 CHRONOMETER SIGNALS
48
50
52
54
56
58
02
04
06
08
10
12
Time U.T.
17h21m00s
SECONDS
-528
-527
-526
ONE
MILLISECOND
CHRONOMETER MINUTE MARKER
MEAN RATE
AVERAGE OF 26
CHRONOMETER SIGNALS
ONE
MILLISECOND
SYNCHRONIZED ON M.S.F. 5 Mc/s
TWO
SECONDS
46
48
50
52
54
56
58
16h53m00s 02
04
06
08
10
12
14
16
18
Time U.T.

Fig. 3.5.4.-6

Calibration of the Crystal Chronometer.

Power supply: Mains, Pulse source: MSF 5 Mc/s.,
Oven: off,
At: London, R.S.M.,
Date: Mon., Jan. 30, 1961

Pulses recorded from - to (h m s)	No. of Pulses	+slow -fast (sec.)	Standard Deviation of single Pulse ± sec.	Standard Deviation of ar. mean ± sec.	Time Interval (h m s)	Rate per 30 min.
09 50 47 51 06	18	-3.442_7	$.000_6$	$.000_1$	11 59 58	$.006_2$
16 52 52 53 18	26	-3.527_0	$.000_6$	$.000_1$	07 02 08	$.006_0$
17 20 48 21 13	25	-3.533_4	$.000_6$	$.000_6$	27 56	$.006_7$
50 45 51 04	19	-3.5397	.000[illegible]	$.000_{09}$	29 54	$.006_3$
18 20 47 21 07	20	-3.546_2	$.000_4$	$.000_{08}$	30 02	$.006_5$
19 20 53 21 07	14	-3.560_0	$.000_5$	$.000_1$	01 00 03	$.006_9$

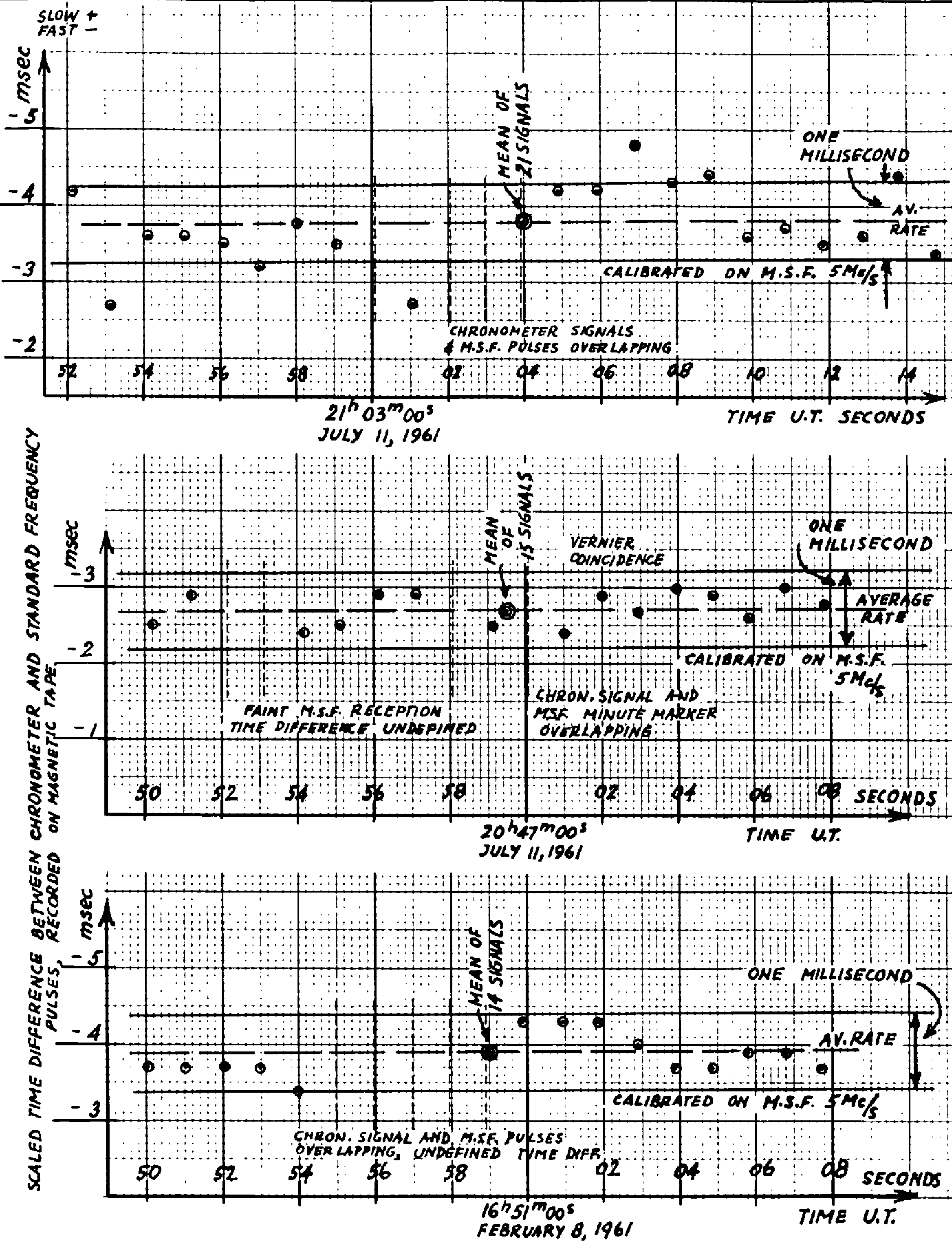
Table 3 .5.4. - 4

The three observation sets (Fig. 3.5.4.-6) contain no overlapping signals of chronometer and frequency reception. Apart from reception conditions, the ever present instrumental errors, tape distortion and scaling, there is no further source of errors to be considered.

In Fig. 3.5.4.-7 three separate calibration sets are given. The reception of chronometer signals and standard frequency pulses is in the vicinity of the time vernier coincidence; some of the signals are partly and some completely overlapping. In the presence of radio noise the overlapping signal may be rather undefined. Hence it depends on reception conditions whether this part, which is limited to about four or five pulses, can be utilized or not. Coincidence of pulses near the minute pulse of time signal transmissions can be easily avoided, by retarding or advancing the chronometer at its initial setting.

The plotted calibration results for the period July 11, $21^h\ 03^m 00^s$, Fig. 3.5.4.-7, reveal overlapping chronometer pulses and transmitted minute pulse of time signals, during poor reception conditions. The time difference at the 60^{th} second is undefined, since the chronometer pulse is on the minute whistle, and lacking a distinct shape. The following three time differences are undetermined since the overlapping M.S.F. pulses are too faint.

The group recorded at $20^h\ 47^m\ 00^s$ on July 11, (center part of the graph) shows adequate time extraction at and near vernier coincidences, obviously due to absence of radio noise. The time difference is undefined for the 60^{th} second pulse, because the chronometer signal is covered by the minute whistle of the time transmission. Faint reception of frequency pulses renders three time differences unobtainable. The graph at the bottom, period of 8.2., $16^h\ 51^m 00^s$, shows vernier coincidence and overlap of chronometer signals and seconds pulses superposed on standard frequency transmissions



INSTANTANEOUS STABILITY OF CRYSTAL CHRONOMETER.

CRYSTAL CHRONOMETER AND STANDARD FREQUENCY PULSES IN THE VICINITY OF VERNIER COINCIDENCE.

OVEN : OFF
POWER: MAINS

Fig. 3.5.4.-7

3.5.4. cont.

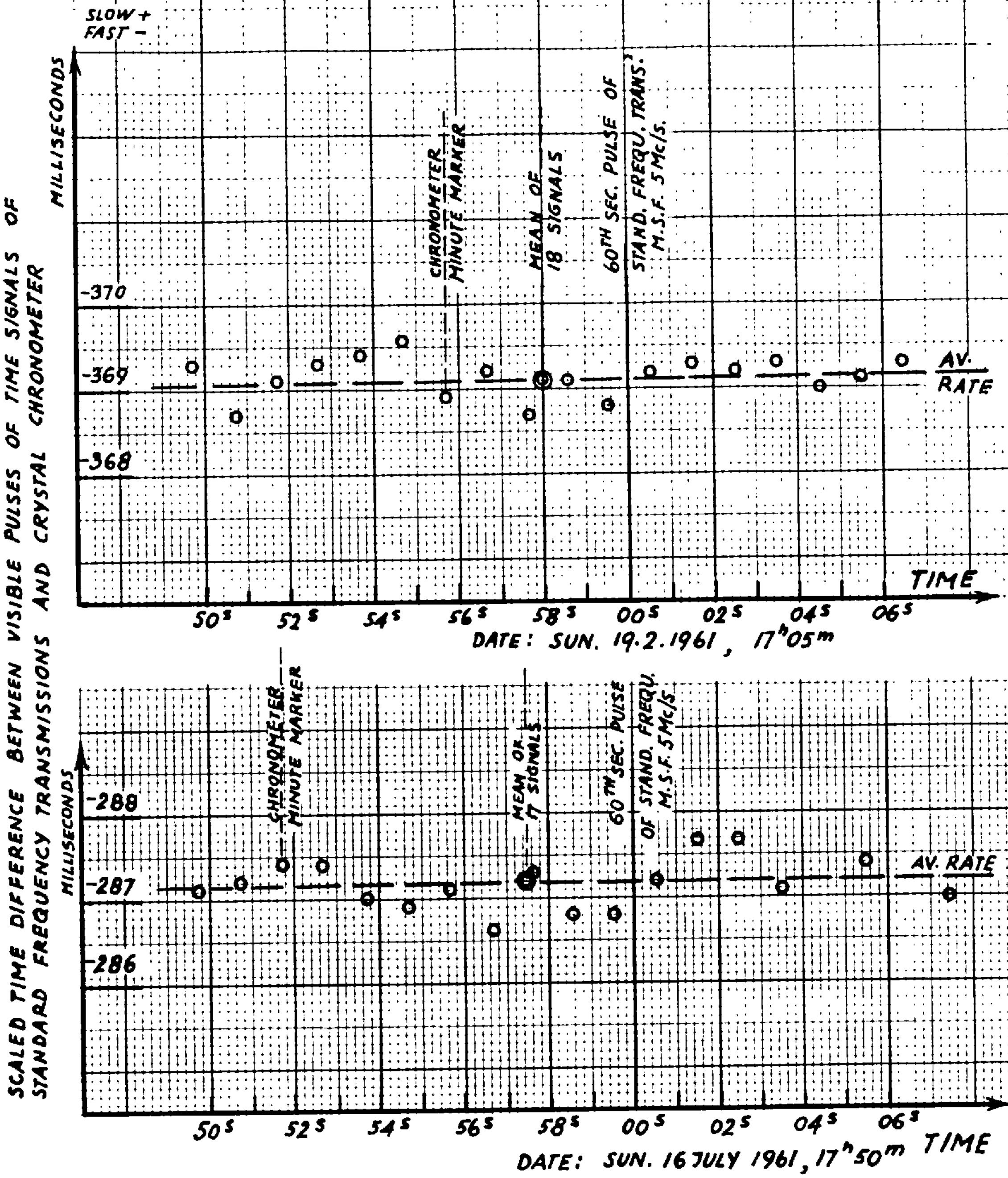
already gone by, when the minute marker comes in.

The calibration accuracy is unaffected by the choice of any particular group in the $61 \times \frac{60}{61}$ interval of solar seconds; i.e. the presence of the chronometer minute marker of of the lengthened second pulse at the 60^{th} second time signal, is meaningless, and the use of both for calibration purposes show results of equal accuracy as the other frequency pulses. (Fig. 3.5.4.-8)

The inadequate reception of one standard frequency pulse, as a rule, affects the extraction of time difference of two consecutive chronometer pulses. The diagram in Fig. 3.5.4.-9 shows the chronometer pulses No. 59 and 60; No. 11 and 12; No. 20 and 21; No. 30 and 31; which are clearly in error due to reception conditions. The error of the pulse No. 61 cannot be attributed to poor reception of standard frequencies from the diagram alone. Its timed position, half way between frequency pulses may obscure scaling errors. In fact, the 61^{st} chronometer pulse is affected by the faint reception of the 29^{th} second time pulse, resulting in an interpretation error.

The amount, to which scaled time differences between chronometer and time pulses can be in error, is illustrated by the three sets of observations plotted in Fig. 3.5.4.-10. Atmospheric noise is largely responsible for the poor reception during the period of recording. Nevertheless, all individual chronometer signals are fluctuating within less than one millisecond from the average rate. The determination of the rate was little affected by the reception conditions, as can be seen from the dotted line which represents the average rate.

Fig. 3.5.4.-11 contains two isochronism curves resulting from plotted arithmetic means of three and of five consecutive signals respectively. The arithmetic means of time errors are plotted at signal distance. The top part of the diagram shows the individual chronometer signals synchronized on standard frequency transmissions.

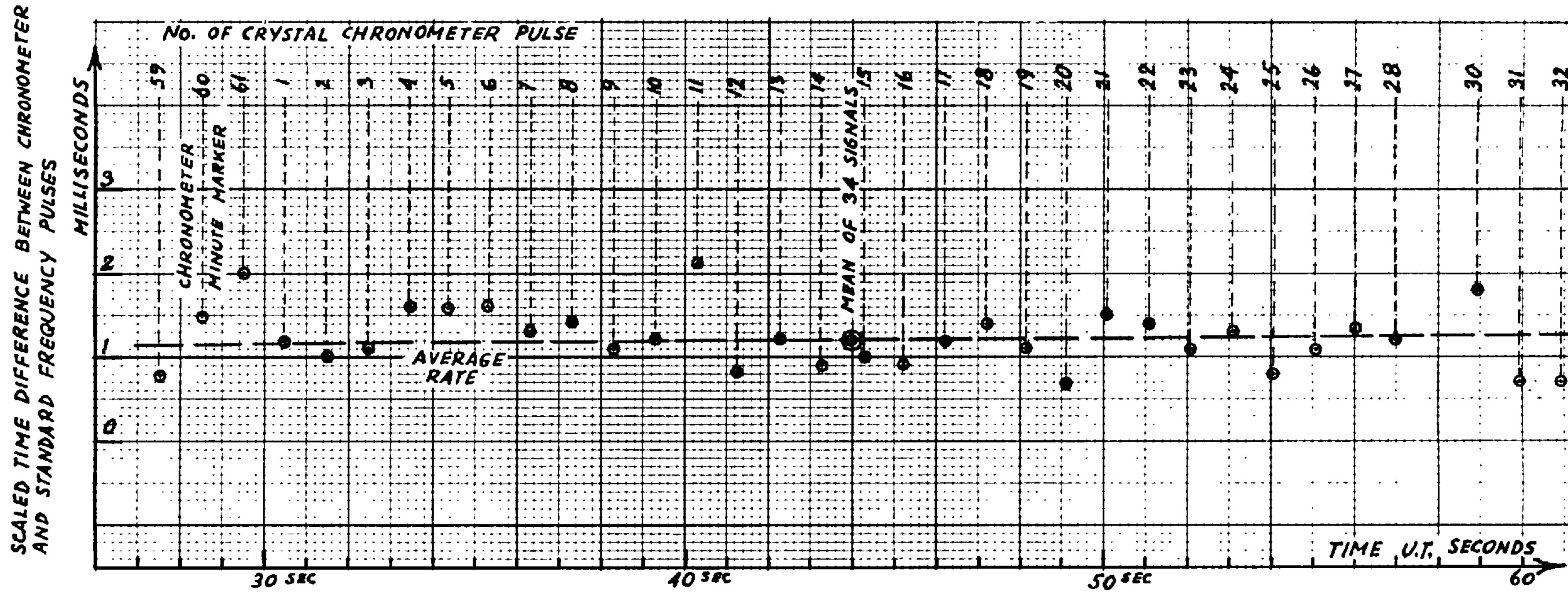


INSTANTANEOUS STABILITY OF CRYSTAL CHRONOMETER

MAGNETIC TAPE EVALUATION IN THE VICINITY OF CHRONOMETER MINUTE MARKER AND LENGTHENED MINUTE PULSE OF STANDARD FREQUENCY TRANSMISSIONS.

POWER : MAINS
OVEN : OFF

Fig. 3.5.4.-8



SYNCHRONIZED ON M.S.F. 5 Mc/s

INSTANTANEOUS STABILITY OF CRYSTAL CHRONOMETER.

SCALING OF CHRONOMETER PULSES AFFECTED BY INADEQUATE RECEPTION OF STANDARD FREQUENCY PULSES.

POWER: MAINS
OVEN : OFF
LONDON, NOV. 11, 1961

Fig. 3.5.4.-9

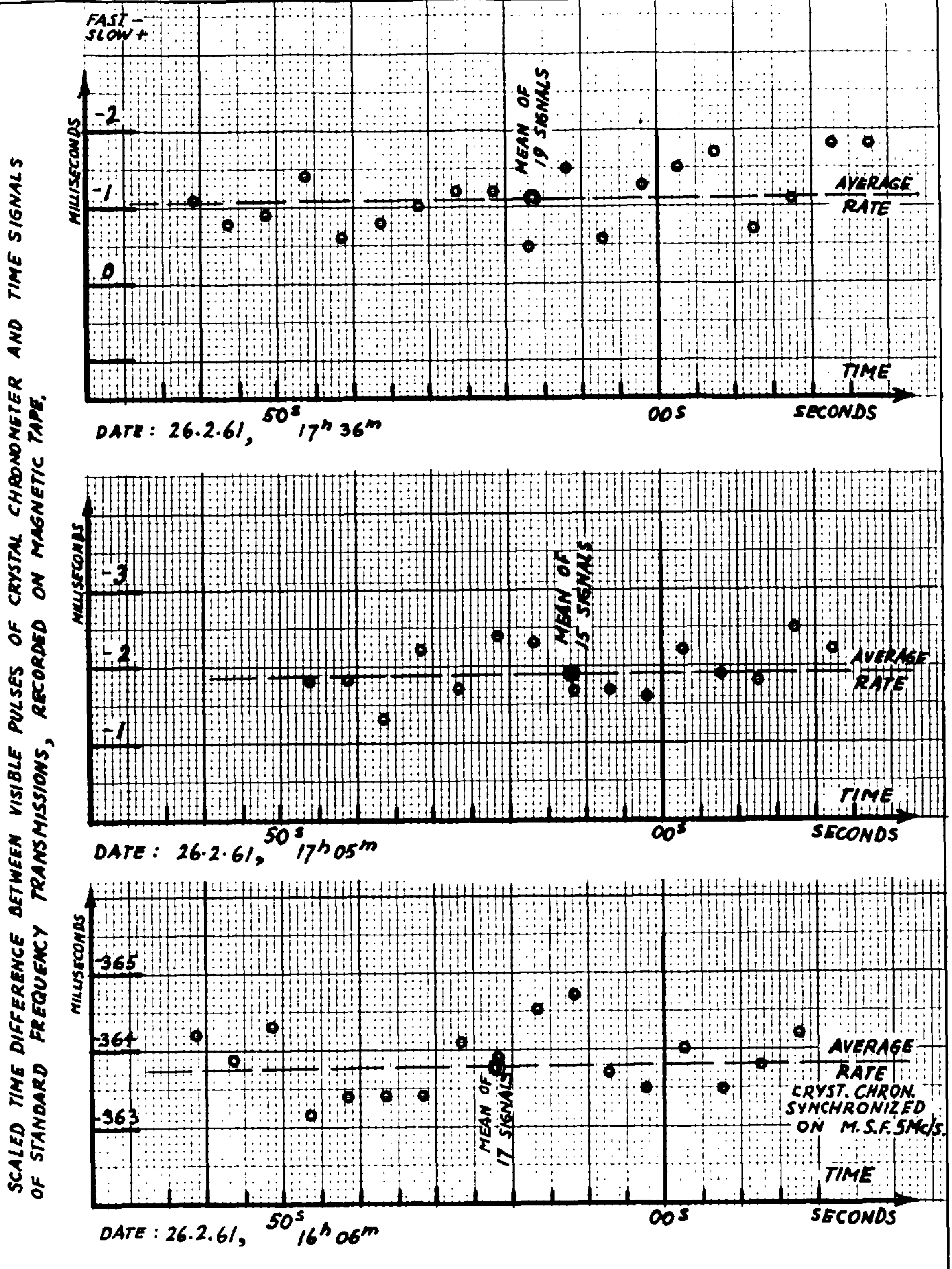
3.5.4. cont.

The time error of the chronometer signals with respect to the average rate can be obtained by scaling the ordinate. The isochronism curve joining averages of five signals (bottom part) is undulating about the average rate with an amplitude of less than 0.4 milliseconds. From this value the number of - crystal chronometer pulses required to be compared with standard frequency pulses for calibration purposes, can be deduced. This also shows the accuracy which can be achieved with the calibration method developed. An isochronism curve from arithmetic means of seven signals plotted at signal distance would almost coincide with the average rate and is therefore not shown. The two parallel lines subtending a distance of one millisecond help to visualize the rapid approach of the isochronism curves towards the average rate, with increasing number forming the arithmetic mean. The average rate is shown by a dashed line.

Position tests and the investigation into the chronometer perform ance during transport to determine its travelling rate etc. were not considered when the chronometer was running on the mains supply.

The indoor location of the crystal chronometer, where sudden changes of temperature are not likely to occur, showed that tests of its performance when thermostatically controlled and running on the mains, are rather pointless.

It is believed that it is not possible to simulate field conditions in the laboratory for performing tests implicating the practical application of specially designed equipment. Many items, wind, humidity, solar influence etc., are interrelated, not successive, and cause considerable trouble to instrument makers. The practical tests of chronometer performance outdoor were executed at the Field Station, Tywarnhale Mine, Cornwall.



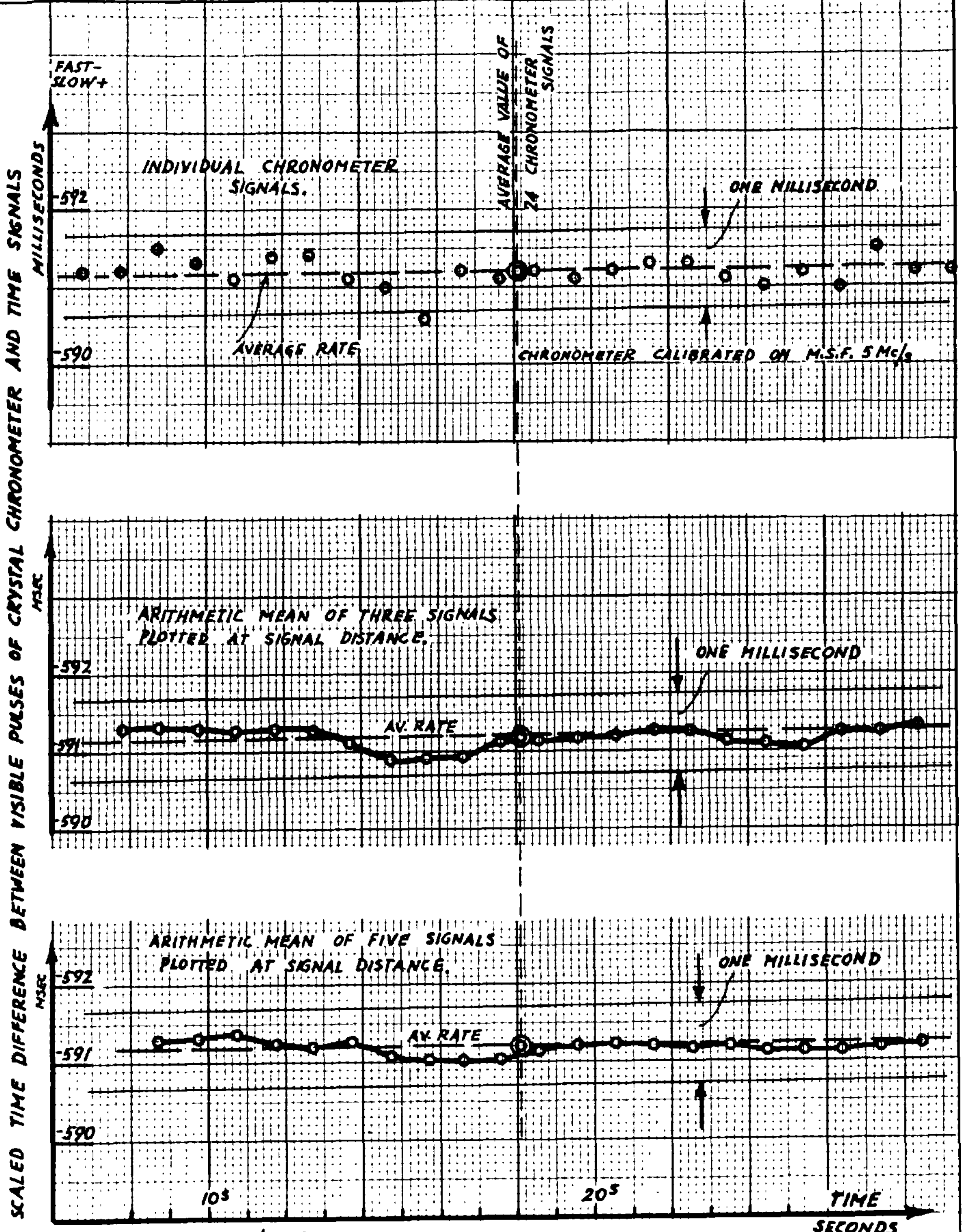
INSTANTANEOUS STABILITY OF CRYSTAL CHRONOMETER

SCALING OF DISTANCES ON TAPE AFFECTED BY VERY POOR RECEPTION CONDITIONS OF STANDARD FREQUENCY TRANSMISSIONS.

POWER: MAINS

OVEN : OFF

Fig. 3.5.4.-10



DATE: 14.11.1961, $17^h 22^m$

INSTANTANEOUS STABILITY OF CRYSTAL CHRONOMETER

ISOCHRONISM CURVES FORMED BY ARITHMETIC MEANS OF GROUPS OF SIGNALS.

POWER: MAINS
OVEN: OFF

Fig. 3.5.4.-11

3.5.5. Performance of the Crystal Chronometer, thermostatically controlled, battery driven.

(a) Hourly stability and running-in time.

The graph presented in Fig. 3.5.5.-1 shows the performance of the crystal chronometer with the heating circuit in operation, driven on two sets of batteries. The oven is temperature controlled to $\pm$ 0.5° C at 40° C over the range 0° C to + 30°C. The time required to reach working termperature is about 30 minutes. Generally, after this time, the chronometer is ready for use. During the following 30 minutes the change of rate follows a parabolic law, which imposes a distinct shape on the isochronism curve. These two periods totalling about one hour are designated as the running-in time. Thereafter, the crystal chronometer displays a nearly constant rate. The running-in time depends on the aging of the quartz crystal, the quality of the electronic components, the electrically heated oven, field temperature, warming-up ability of the quartz, etc.

The performance of the chronometer during the running-in time has proved to consist of a characteristic rate and change of rate. It follows that the chronometer can be used during the running-in period, and rate prediction and interpolation are possible.

The behaviour of the crystal chronometer when acquiring working temperature during the running-in time, as the graphical representation reveals, is in perfect agreement with the known fact that the frequency of quartz crystals increases or decreases in a parabolic curve with temperature.

No cyclic variation of the frequency was encountered and was not expected, since the crystal is mounted in vacuo, which, it is believed, eliminates the effect of vibrations of air waves. The vibrations are generated by a change in the velocity of the air waves with temperature.

After the running-in time the rate depends on the stability of the input voltage. The chronometer can function properly between 11 and 14v.

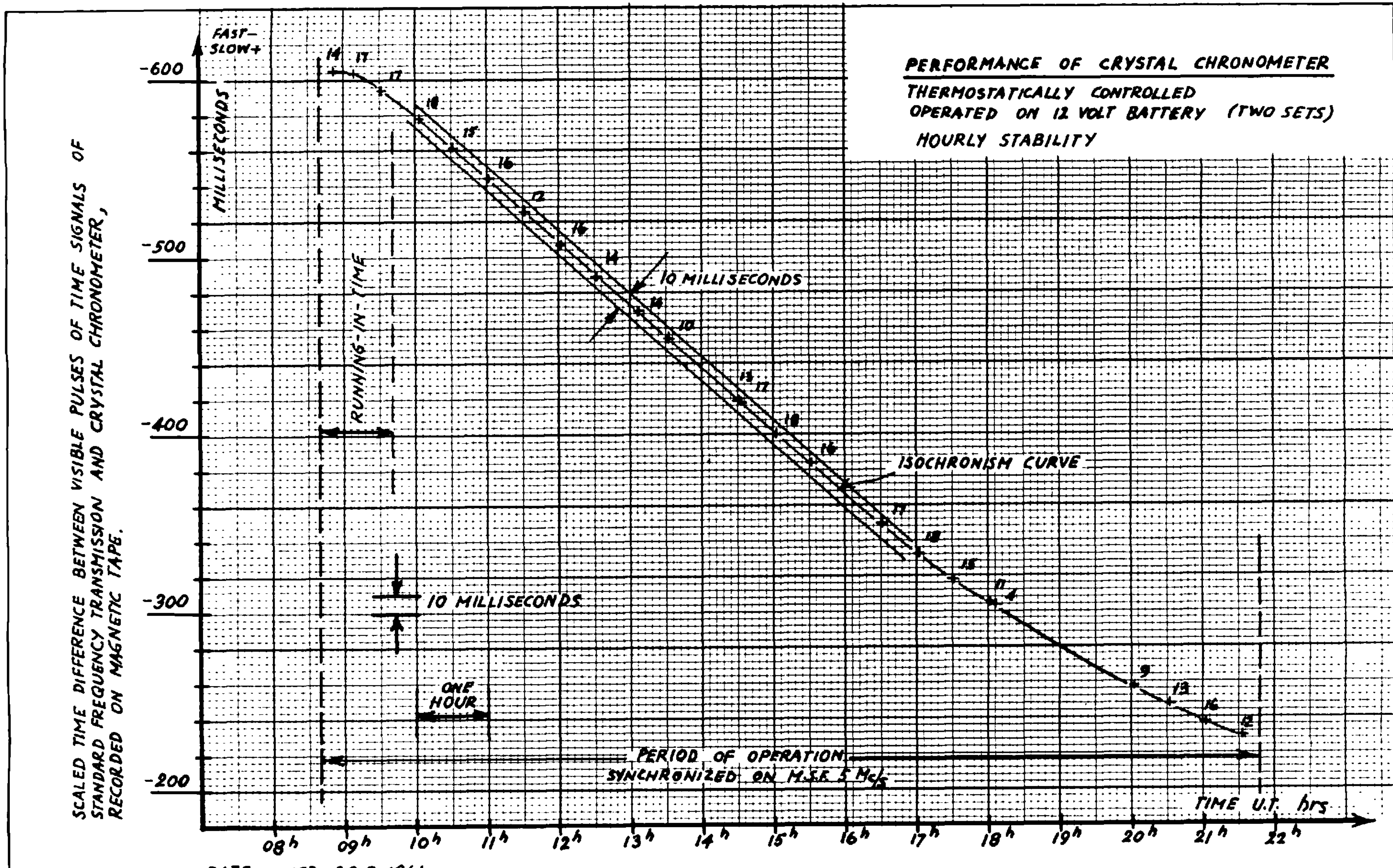

PERFORMANCE OF CRYSTAL CHRONOMETER
THERMOSTATICALLY CONTROLLED
OPERATED ON 12 VOLT BATTERY (TWO SETS)
HOURLY STABILITY
FAST–
SLOW+
MILLISECONDS
-600
-500
-400
-300
-200
SCALED TIME DIFFERENCE BETWEEN VISIBLE PULSES OF TIME SIGNALS OF
STANDARD FREQUENCY TRANSMISSION AND CRYSTAL CHRONOMETER,
RECORDED ON MAGNETIC TAPE.
RUNNING-IN-TIME
10 MILLISECONDS
ISOCHRONISM CURVE
10 MILLISECONDS
ONE HOUR
PERIOD OF OPERATION
SYNCHRONIZED ON M.S.F. 5 Mc/s
TIME U.T. hrs
08h
09h
10h
11h
12h
13h
14h
15h
16h
17h
18h
19h
20h
21h
22h
DATE: WED. 29.3.1961
TYWARNHALE, CORNWALL

Fig. 3.5.5.–1

Performance of the Crystal Chronometer

thermostatically controlled,
operated on two 12 volt batteries.

At: Tywarnhale, Cornwall
Date: Wed., 29.3.1961
Chronometer synchronized on: M.S.F. 5,0 Mc/s.

	U.T. h m s	No. of Pulses	Rate per 30 min. sec.	
	10 03 57	18		
	31 58	15	$+.017_6$	
	11 01 56	16	$+.018_5$	
	31 54	12	$+.018_5$	
	12 01 58	16	$+.018_0$	
	31 55	14	$+.018_0$	
	13 06 56	14	$+.017_5$	
	31 56	10	$+.017_5$	
	14 31 56	13	$+.017_5$	
	34 01	17		
	15 02 58	18	$+.017_5$	
	31 57	16	$+.017_0$	
	16 31 59	17	$+.017_0$	
	17 01 57	18	$+.016_7$	

Table 3.5.5. -1

In the diagram, the 10 millisecond time space is drawn symmetrically about the isochronism curve along the portion of near linear performance. The table No. 3.5.5.-1 contains the evaluated data of time comparison. In column 3 is listed the rate per 30 minutes, determined from time comparisons spaced about 30 minutes apart; two intervals amount to one hour approximately. The rate is quoted to the nearest half millisecond. The data show clearly that a precision of $\pm$ 1 millisecond can be accepted for linear interpolation of time over an interval up to six hours after the running-in period. Obviously, time comparisons before and after this interval are required. The quoted accuracy can be achieved, provided the voltage is maintained within the given limits.

The following statements can be derived from the results of the investigations; (specimen data are given in the above table).

The performance of the thermostatically controlled crystal chronometer can be adequately ascertained by time comparisons spaced about 30 minutes apart. The developed method of calibration (No. 8) can meet the accuracy of the chronometer. The hourly stability of the chronometer in the field for the interval of time concerned is far higher than the stability of any mechanical chronometer.

These, and the following experiments carried out in the field since 1961, employing the equipment described, are original, and no published or unpublished references are known to exist elsewhere.

If necessary, the chronometer can run on a gradually decreasing voltage, to as low as 9 v., should uninterrupted field operations require its extended service. Adequate time comparison is then essential to obtain sufficiently accurate results. The length of time interval, during which linear interpolation is possible, depends therefore on battery conditions, and on the initial voltage when the chronometer was switched on. Linear interpolation can be extended over longer periods if the rate of the chronometer is kept fairly constant, which requires the voltage of 12v to be maintained within the specified limits. This can be achieved by a fresh supply of batteries, should they be available in the

3.5.5. cont.

field, or by voltage control. Provision is made to insure continuity of operation when replacing a set of batteries with fully charged units.

Fig. 3.5.5.-1 shows the chronometer operating over a period of 13 hours. This length of running time may be necessary to satisfy the requirements of an extensive observation programme, or of time comparison during unfavourable frequency receptions.

The part of the isochronism curve displaying near linear behaviour and also the portion containing the running-in time are drawn on larger scale in Fig. 3.5.5.-2. The selected plotting scale is quite sufficient for graphical linear interpolation to $\pm$ one millisecond.

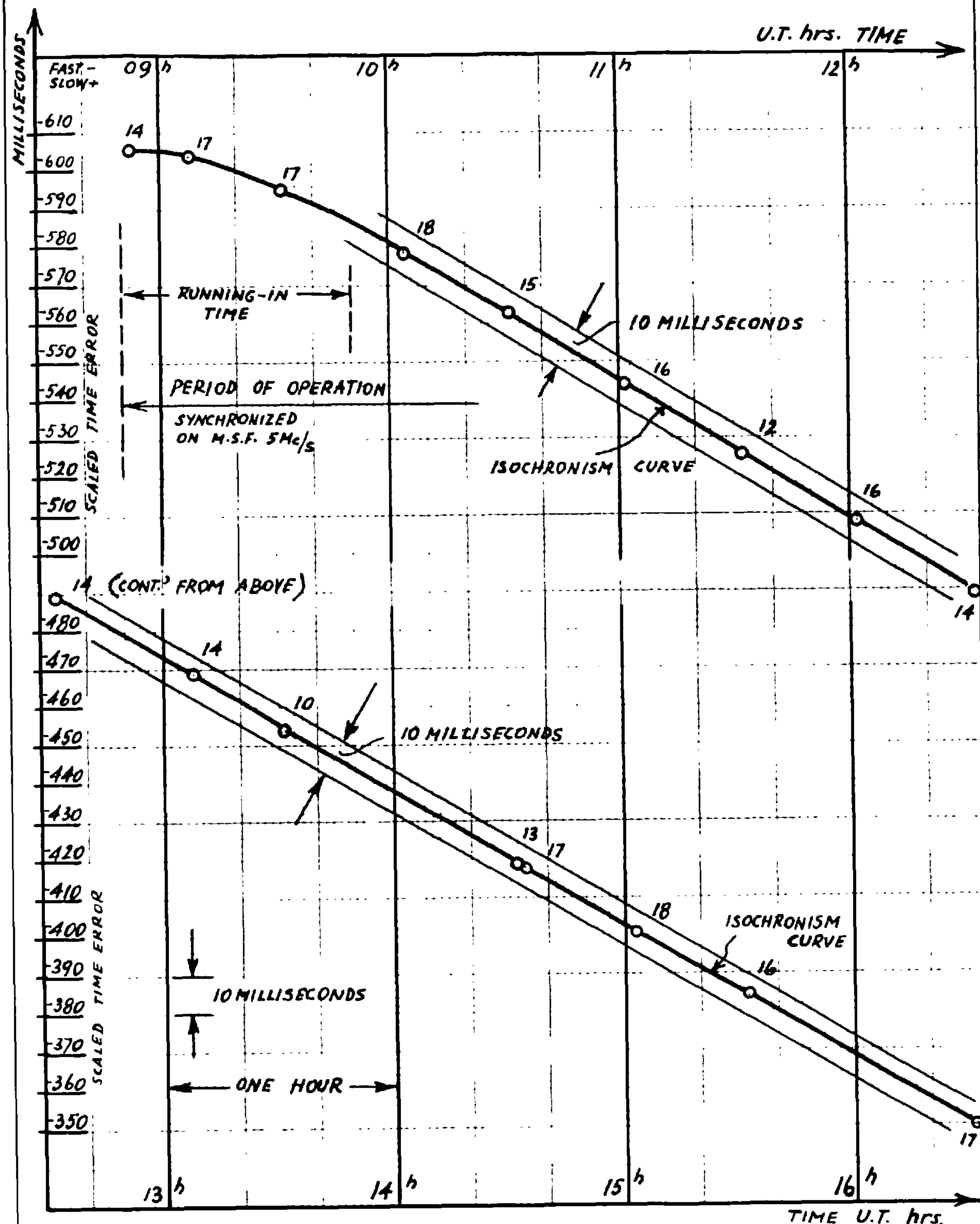
The same procedure, as outlined in previous paragraphs, has been applied for comparing the crystal chronometer with standard frequency transmissions, as well as for the evaluation and plotting of data.

To reduce the weight, the standard procedure adopted for future tests was to run the chronometer on one battery alone. This proved to be perfectly compatible with field requirements.

Fig. 3.5.5.-3 illustrates the performance of the thermostatically controlled chronometer driven with one battery. It can be seen from the diagram and from the data, Table 3.5.5.-2, that the quoted stability exists for the first six hours after running-in, presuming average field conditions and adequate input voltage. This is the maximum time interval between comparisons with seconds pulses on standard frequency transmissions to achieve an accuracy of $\pm$ 1 millisecond by interpolation.

The values of rate and change of rate, after switching the chronometer on and off, may or may not come back; differences in performance depend on power supply; and are very small and predictable after return to operating temparature under average battery conditions. Herein lies the advantage of the electrically heated oven.

A standstill of the crystal chronometer from two hours up to several days does not appreciably alter its rate over the one hour period of running-in; but, as stated, battery conditions and,or, initial voltage will influence its rate after the running-in time, which can in adverse

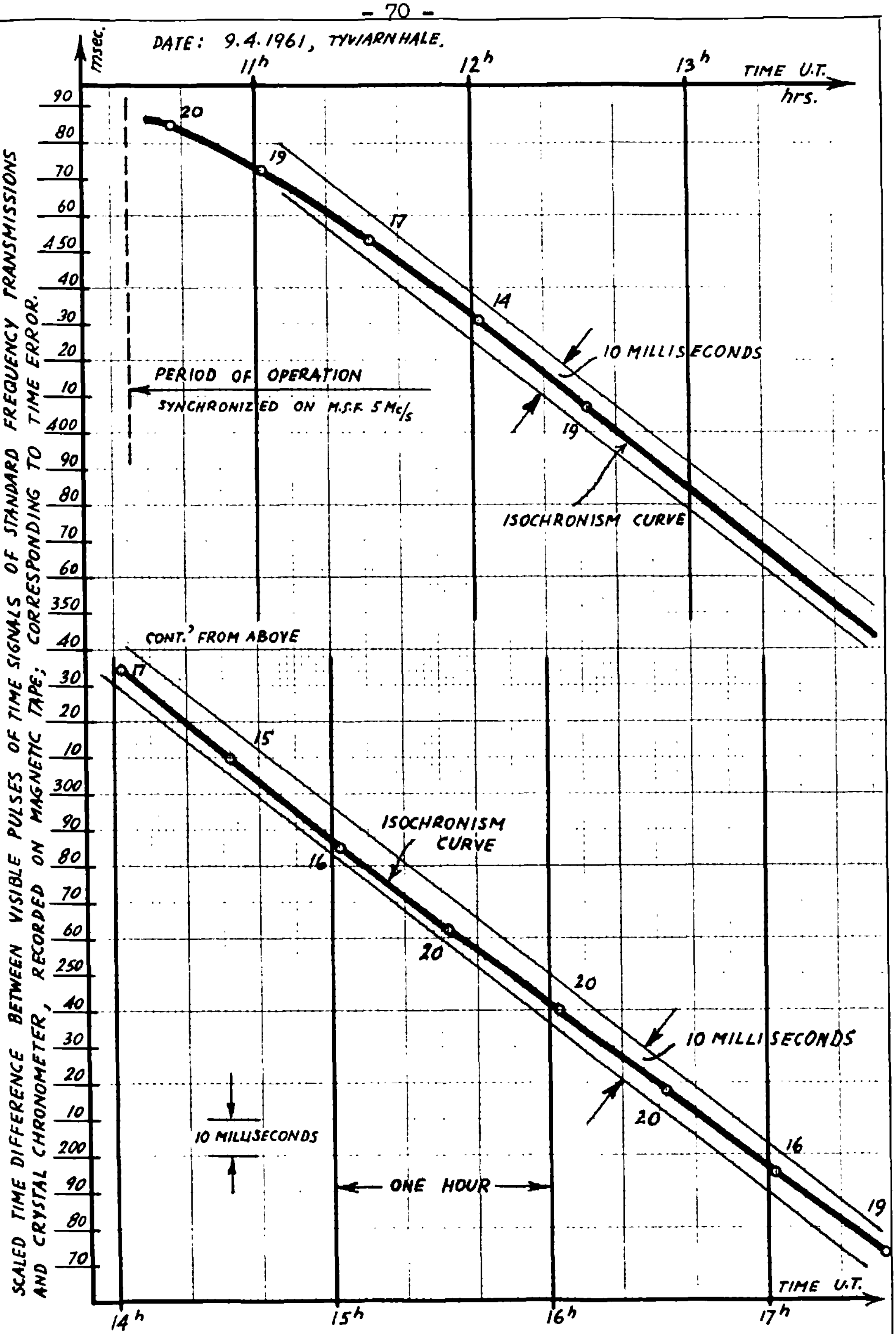


PERFORMANCE OF CRYSTAL CHRONOMETER

THERMOSTATICALLY CONTROLLED, SYNCHRONIZED ON M.S.F. 5Mc/s.,
OPERATED ON 12 VOLT BATTERY (TWO SETS)
HOURLY STABILITY

WED. 29.3.1961
TYWARNHALE, CORNWALL

Fig. 3.5.5.-2



PERFORMANCE OF CRYSTAL CHRONOMETER

THERMOSTATICALLY CONTROLLED, SYNCHRONIZED ON M.S.F. 5 Mc/s.
OPERATED ON 12 VOLT BATTERY (ONE SET)
HOURLY STABILITY.

Fig. 3.5.5.-3

Performance of the Crystal Chronometer
thermostatically controlled,
operated on 12 volt battery (one set)

At: Tywarnhale Mine, Cornwall
Date: April 9, 1961
Crystal chronometer working from $10^h\ 25^m$,

U.T. h m s	No. of Pulses	Rate per 30 min ± sec.	Standard Deviation ± sec.	Pulse Source
10 36 57	20		.000 13	M.S.F. 5Mc/s
11 01 59	19	$.015_5$	.000 06	
11 31 58	17	$.019_0$	.000 07	
12 01 58	14	$.022_5$	.000 12	
12 31 57	19	$.023_5$	.000 16	
14 01 58	17	$.024_5$	.000 07	
14 31 58	15	$.024_5$	.000 11	
15 01 56	16	$.024_0$	.000 13	
15 31 57	20	$.023_0$	.000 19	
16 01 59	20	$.023_0$	.000 25	
16 31 59	20	$.022_0$	.000 23	
17 01 59	16	$.022_0$	.000 26	
17 32 02	19	$.022_0$	.000 16	

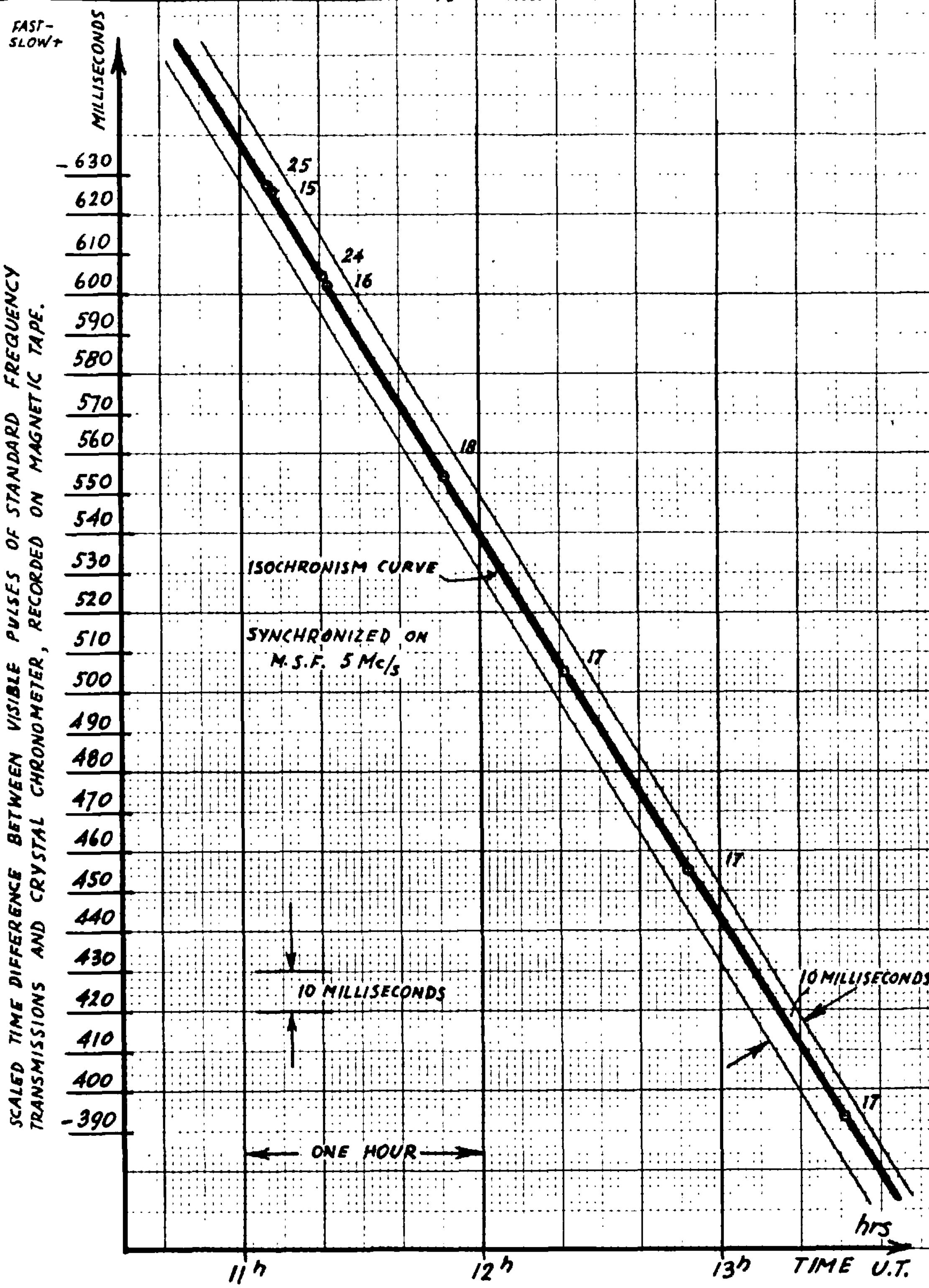
Table 3.5.5. - 2

conditions differ from one period of operation to another by as much as $\pm$ 15 msec/h.

The performance of the chronometer is not affected by advancing or stopping of the minute clock, nor is its linear performance over the six consecutive hours during individual operation periods affected by interruption of its service over longer time intervals.

The records of the chronometer performance after about one year of service, shown in Figs. 3.5.5.-4 and 3.5.5.-5, reveal its unchanged time keeping property. In the mean time, the rate of the chronometer has been changed deliberately with no adverse effect on its performance. The total operating period was five days. Care was taken to maintain constant input voltage. The thermostatically controlled chronometer was rated regularly; the two graphs indicate at once the nearly identical rate at different times of the operating period. The two parallel lines accompanying the isochronism curves at ten milliseconds distance show the constancy of the hourly rate in each case. There is only a small change of rate amounting to $\pm 5 \times 10^{-5}$ sec/min/h, which will not affect linear interpolation. The results of this test, plotted in Fig. 3.5.5.-6, show that it is impossible to eliminate completely variations in the rate over an extended period. These variations can be attributed to small changes in operating conditions. Hence, frequency comparisons are necessary at the beginning and end of the required operating period.

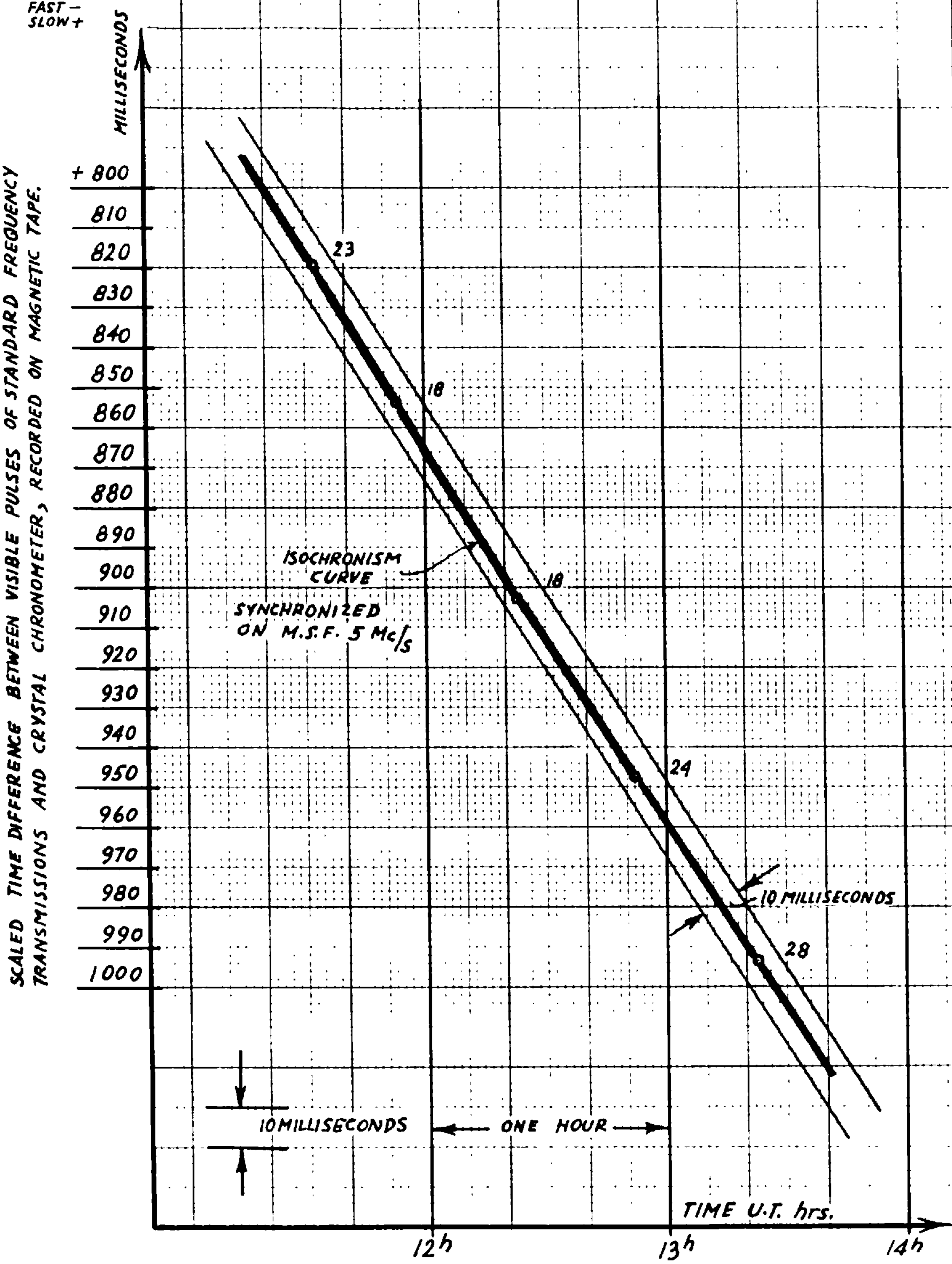
Synchronizing the thermostatically controlled chronometer during the first ten minutes of operation is insignificant, since its rate is not sufficiently stable yet, and the quartz crystal has not acquired adequate working temperature. It can be done, should a low order of accuracy of $\pm$ 0.01 second be acceptable for time extrapolation during the running-in time, and an accuracy up to $\pm$ 0.015 sec/h thereafter.



PERFORMANCE OF CRYSTAL CHRONOMETER

THERMOSTATICALLY CONTROLLED
FIVE DAY PERIOD OF OPERATION
DATE: JAN. 20, 1962
HOURLY STABILITY.

Fig. 3.5.5.-4



PERFORMANCE OF CRYSTAL CHRONOMETER

THERMOSTATICALLY CONTROLLED
FIVE DAY PERIOD OF OPERATION
DATE: JAN. 21, 1962
HOURLY STABILITY.

Fig. 3.5.5-5

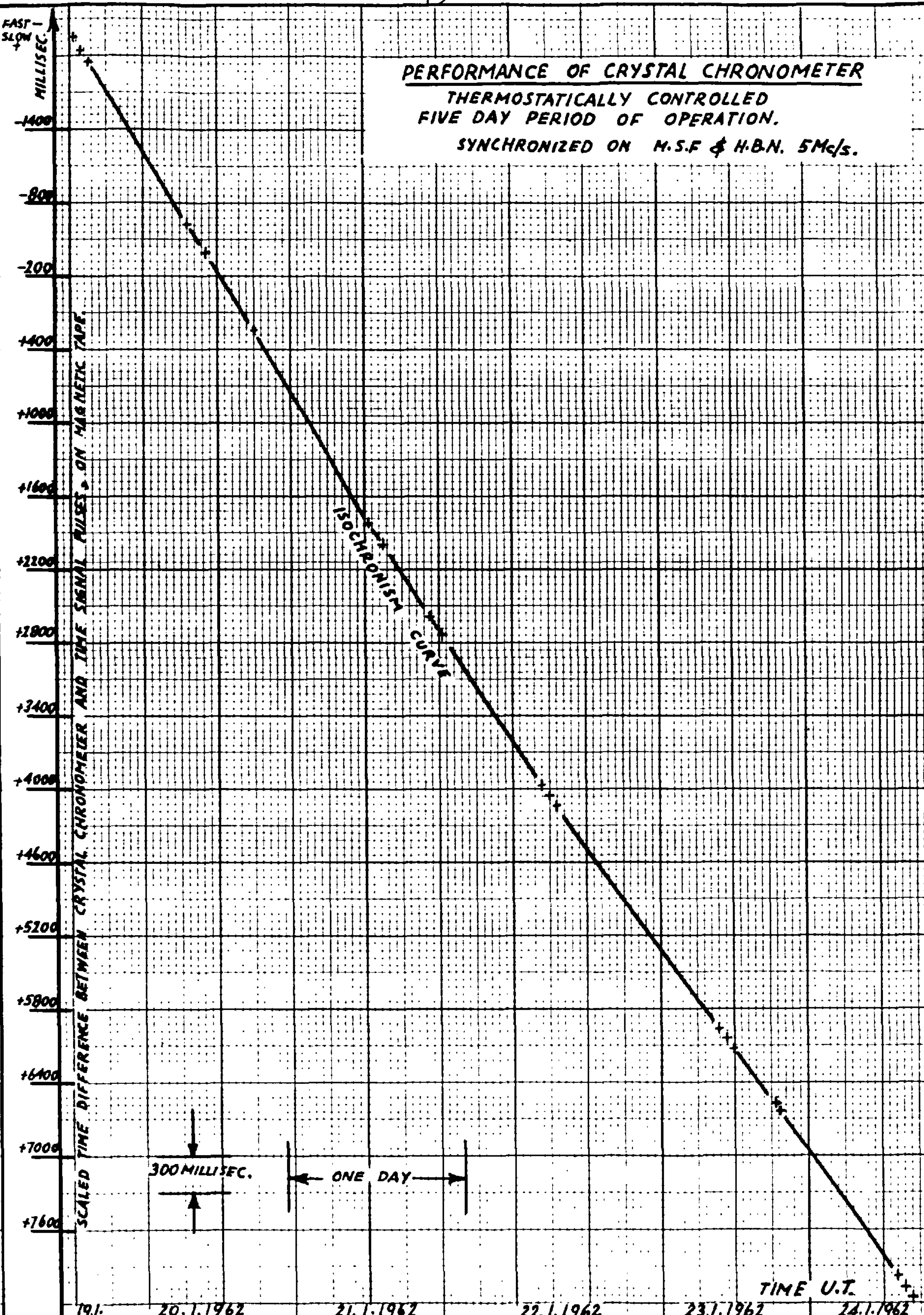

PERFORMANCE OF CRYSTAL CHRONOMETER
THERMOSTATICALLY CONTROLLED
FIVE DAY PERIOD OF OPERATION.
SYNCHRONIZED ON M.S.F & H.B.N. 5Mc/s.
FAST −
SLOW +
MILLISEC.
SCALED TIME DIFFERENCE BETWEEN CRYSTAL CHRONOMETER AND TIME SIGNAL PULSES ON MAGNETIC TAPE.
-1400
-800
-200
+400
+1000
+1600
+2200
+2800
+3400
+4000
+4600
+5200
+5800
+6400
+7000
+7600
ISOCHRONISM CURVE
300 MILLISEC.
ONE DAY
TIME U.T.
19.1.1962
20.1.1962
21.1.1962
22.1.1962
23.1.1962
24.1.1962
18
0
6
12
18
0
6
12
18
0
6
12
18
0
6
12
18
0
6
hrs

Fig. 3.5.5.-6

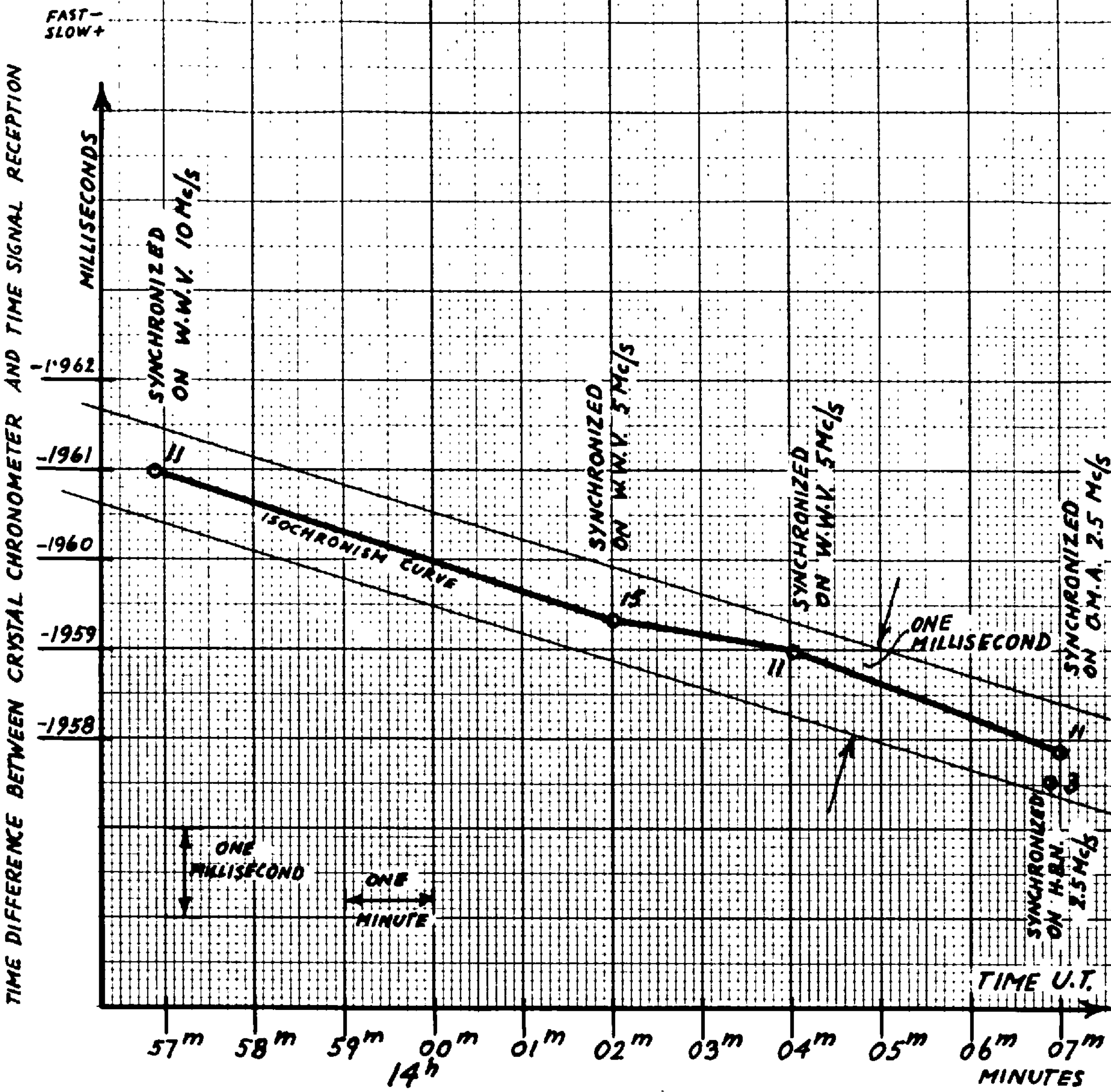
3.5.5. cont.

(b) Very short term performance = minute-to-minute stability of the thermostatically controlled crystal chronometer, battery driven.

The stability from minute to minute is within $\pm$ 1 millisecond during all operating periods of the chronometer. No erratic behaviour could be detected during the interval of minutes.

The very short term performance during the running-in time is shown in Fig. 3.5.5.-7. The chronometer was rated 12 minutes after it was switched on. The isochronism curve resulting from time comparisons with various transmitted frequencies shows reliable chronometer performance over the interval of several minutes, namely in the order of $\pm$ 1 millisecond. It is therefore justified, if necessary, to make use of the running-in time of the chronometer. If urgently required, the chronometer can be rated forthwith (after 10 minutes, which is the minimum waiting time as explained above) and subsequently it can be employed for timing observations, which, provided time comparisons covering the running-in period are possible, are then controlled by chronometer time intervals of the quoted accuracy. Since the plotted values of time differences are derived from synchronizations with various frequency transmissions, corrections for emission and travel delays are applied.

Fig. 3.5.5.-8 contains the isochronism curve plotted from arithmetic means of groups of time comparisons spaced about 15 minutes apart. The time interval presented in the graph is about 3 hours after the running-in period. The two lines at 5 millisecond distance run parallel to the mean rate of the operating period. It is quite obvious, when looking at the graph, that linear interpolation can be executed to an accuracy of $\pm$ 1 millisecond, when using two time comparisons spaced as close as 15 minutes. Further, the rate of the chronometer can be determined from arithmetic means of groups of time comparisons at intervals of about 15 minutes, should frequency receptions be feasible.

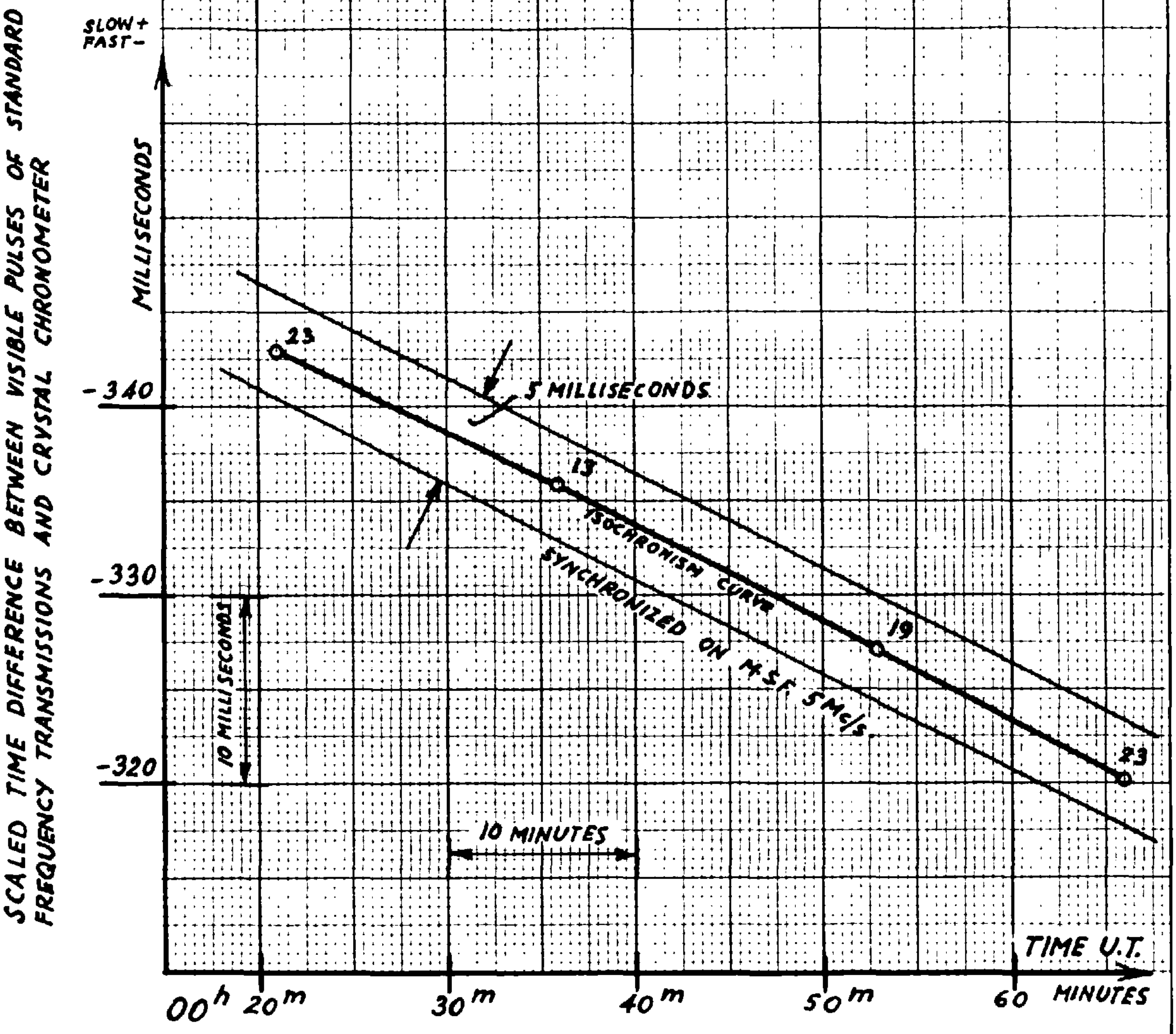


PERFORMANCE OF CRYSTAL CHRONOMETER

THERMOSTATICALLY CONTROLLED
OPERATING ON 12 VOLT BATTERY (ONE SET)
VERY SHORT TERM STABILITY
DATE: MARCH 26, 1961
TYWARNHALE.

PERIOD OF OPERATION: $13^h 45^m$ to $14^h 10^m$.
CORRECTIONS FOR TRAVEL AND EMISSION DELAYS ARE APPLIED.

Fig. 3.5.5.-7



PERFORMANCE OF CRYSTAL CHRONOMETER

THERMOSTATICALLY CONTROLLED
OPERATED ON 12 VOLT BATTERY (ONE SET)
VERY SHORT TERM STABILITY
DATE : TUE. 18TH APRIL 1961,
TYWARNHALE, CORNWALL.

Fig. 3.5.5-8

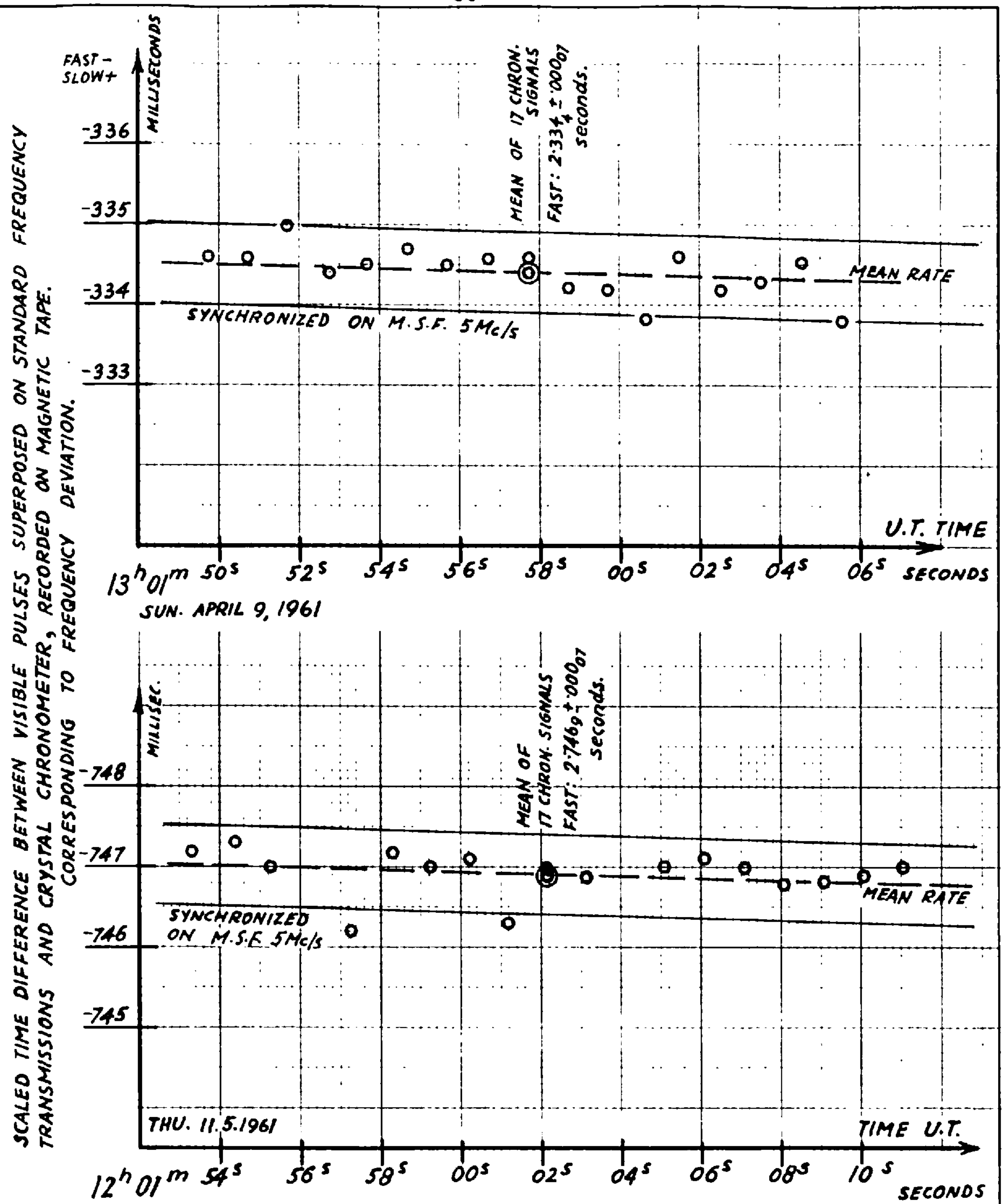
3.5.5. cont.

(c) Performance of the thermostatically controlled crystal chronometer during intervals of seconds = instantaneous stability.

The lengths of the time intervals indicated by consecutive chronometer pulses and evaluated by the calibration method described in Section 3.5.3.(8) normally differ only fractions of milliseconds. Causes which contribute to this instantaneous frequency fluctuation are to be sought in easing of stresses in the quartz crystal, small changes in supply voltage which disturb the thermal equilibrium, and erratic behaviour of the material of the electronic components. In addition, the quality of reception of standard frequency transmissions, the recording and the evaluation method may complicate and obscure the assess ment of the chronometer performance.

The graph in Fig. 3.5.5.-9 shows that the variations of the isochronism error are well within the limits of $\pm$ one millisecond. The plotted time differences during the period April 9, 1961 (top part of the diagram), were extracted by scaling the tape with a box wood rule subdivided to 0.5 mm., the time differences during the period May 11, 1961, with a glass scale subdivided to 0.1 mm. respectively. The scaling of distances is described in section 7.3. The Table 3.5.5.-3 contains the evaluated data. In column 4 are noted the deviations of the lengths of the time intervals, marked by two consecutive chronometer pulses, from the theoretical value. The diagrams and tables are specimens of numerous measurements from which it follows that all 60/61 seconds intervals of mean solar time marked by two consecutive chronometer signals are accurate to $\pm$ one millisecond. Therefore it is necessary to scale the magnetic tape and to calculate the time differences to the fourth decimal of a second, to avoid rounding off errors.

The broken lines in Figs. 3.5.5.-10 and 3.5.5.-11, representing the isochronism polygons, are obtained from time comparisons with M.S.F. and H.B.N. transmissions respectively. They display the instantaneous stability of the chronometer after one year of operation,



U.T. INDICATED BY SECONDS PULSES SUPERPOSED ON STANDARD FREQUENCY TRANSMISSIONS: M.S.F. 5 Mc/s, AT THE INSTANT OF RECORDING ON MAGNETIC TAPE.

CRYSTAL CHRONOMETER PULSES ARE PLOTTED AT U.T., SCALED FROM STANDARD FREQUENCY PULSES.

PERFORMANCE OF CRYSTAL CHRONOMETER

THERMOSTATICALLY CONTROLLED
OPERATING ON 12 VOLT BATTERY

INSTANTANEOUS STABILITY

DURING 60/61 SECOND INTERVAL OF MEAN SOLAR TIME.

1961

Fig. 3.5.5.-9

Performance of the crystal chronometer

thermostatically controlled,
operated on 12 volt battery (one set)

No of Chronometer Signal	Scaled U T. of Chronometer Signal, with reference to MSF reception	Length of Time Interval between two Chron. Signals from scaled UT seconds	Difference from theoretical time interval (=.9836 seconds) ± milliseconds
	$13^h\ 01^m$		
53	49.7965^s		
54	50.7802	.9837	- .1
55	51.7634	.9832	+ .4
56	52.7476	.9842	- .6
57	53.7311	.9835	+ .1
58	54.7145	.9834	+ .2
59	55.6983	.9838	- .2
60	56.6818	.9835	+ .1
61	57.6654	.9836	0.0
1	58.6494	.9840	- .4
2	59.6330	.9836	0.0
	$13^h\ 02^m$		
3	00.6170^s		
4	01.5998	.9828	+ .8
5	02.5838	.9840	- .4
6	03.5673	.9835	+ .1
7	04.5507	.9834	+ .2
8	05.5351	.9844	- .8

Date: April 9, 1961 Table 3 .5.5. - 3

Performance of the crystal chronometer,
thermostatically controlled,
operated on 12 volt battery (one set)

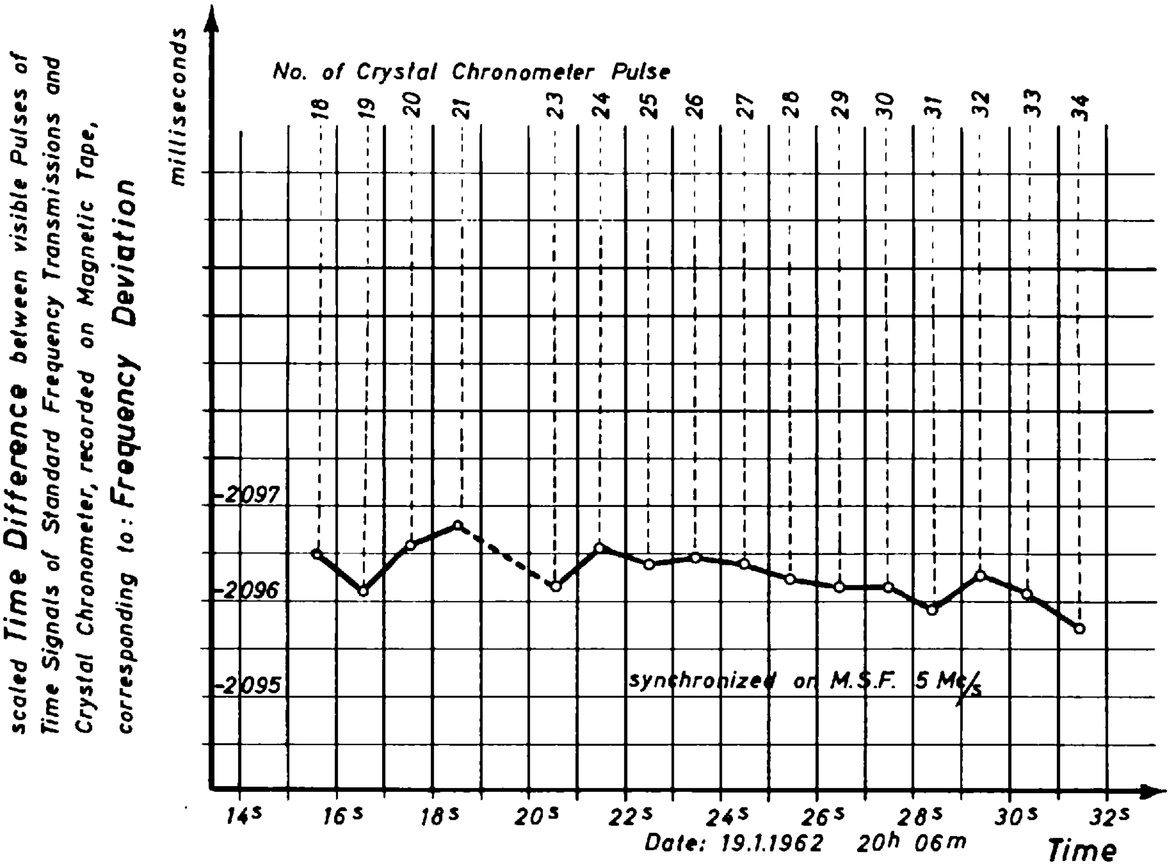
No. of Chronometer Signal	Scaled U.T. of Chronometer Signal, with reference to MSF reception	Length of Time Interval between two Chron. Signals from scaled UT seconds	Difference from Theoreticall Time Interval (=.9836 sec.) ± millisec.
	$12^h\ 01^m$		
57	53.3184^s		
58	54.30 19	.9835	+ .1
59	55.2858	.9839	- .3
60			- .4 } average
61	57.2538		- .4 } average
1	58.2364	.9826	+1.0
2	59.2202	.9838	- .2
3	$12^h\ 02^m$ 00.2037^s	.9835	+ .1
4	1.1881	.9844	- .8
5	2.1710	.9829	+ .7
6	3.1547	.9837	- .1
7			0.0 } average
8	5.1219		0.0 } average
9	6.1054	.9835	+ .1
10	7.[illegible]	.9837	- .1
11	8.[illegible]	.9838	- .2
12	9.0565	.9836	0.0
13	10.0400	.9835	+ .1
14	11.0235	.9835	+ .1

Date: May 11, 1961

Table 3.5.5.-3 cont.

Performance of Crystal Chronometer

Power Supply : 12 volt Battery
thermostatically controlled

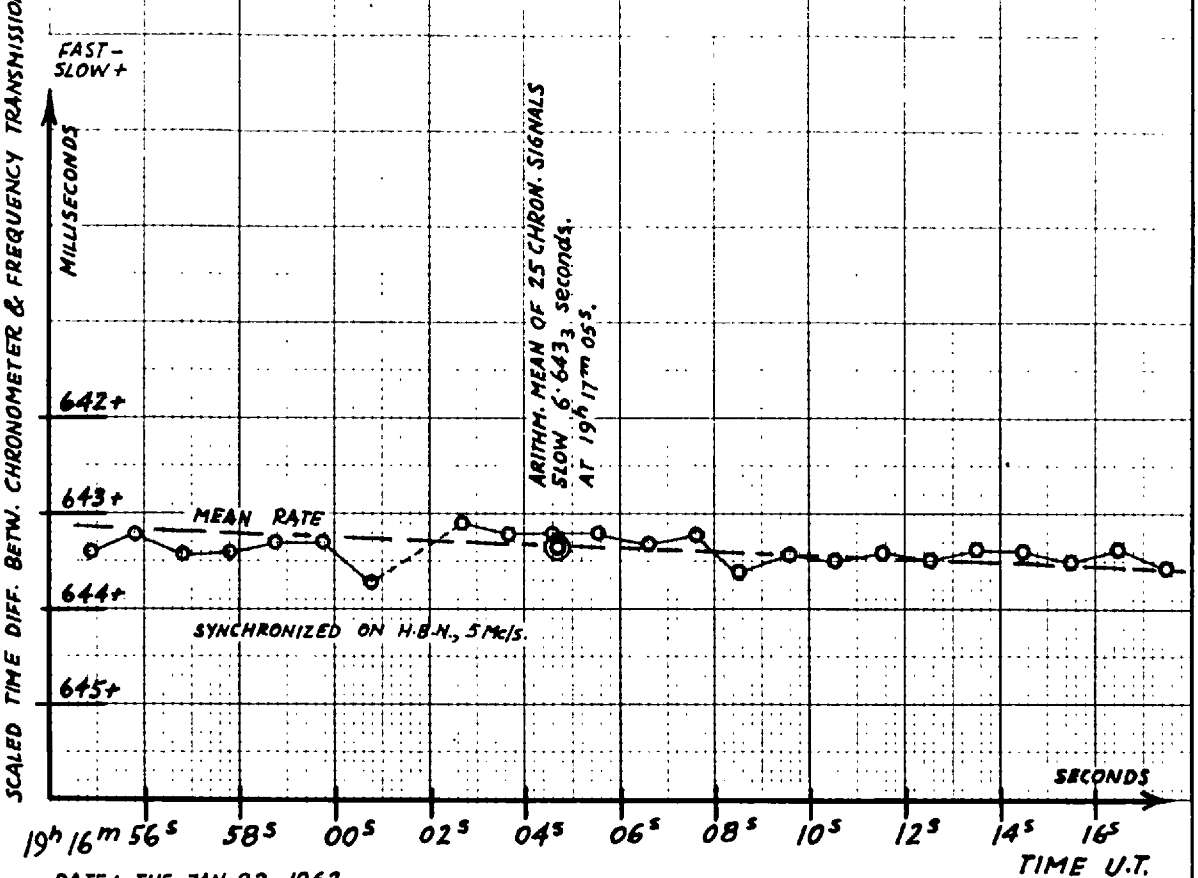


U.T. indicated by Seconds Pulses superposed on Standard Frequency Transmission : M.S.F, at the instant of recording on magnetic tape.

Instantaneous Stability of Crystal Chronometer during the $60/61$ second interval of mean solar time.

Crystal Chronometer pulses are plotted at U.T., scaled from standard frequency pulses.

Fig. 3.5.5.-10



DATE: TUE. JAN. 23, 1962

U.T. INDICATED BY SECONDS PULSES SUPERPOSED ON STANDARD FREQUENCY TRANSMISSION: H.B.N., 5 Mc/s, AT THE INSTANT OF RECORDING ON MAGNETIC TAPE.

CRYSTAL CHRONOMETER PULSES ARE PLOTTED AT U.T., SCALED FROM STANDARD FREQUENCY PULSES.

PERFORMANCE OF CRYSTAL CHRONOMETER.

THERMOSTATICALLY CONTROLLED
OPERATING ON 12 VOLT BATTERY

INSTANTANEOUS STABILITY

DURING THE 60/61 SECOND INTERVAL OF MEAN SOLAR TIME.

DATE: JAN. 23, 1962

Fig. 3.5.5.–1)

3.5.5. cont.

and in Fig. 3.5.5.-12 after two years. All the time-error polygons exhibit the same quality of performance, and no signs of aging of the quartz crystal or material can be noticed. It will be found that when the time differences between pulses of the crystal chronometer and pulses superposed on standard frequency transmissions are plotted as arithmetic means of five or six in the time error diagram, they will lie approximately on a straight line, representing the mean rate of the chronometer. From the slope of this line the drift parameter may be deduced with sufficient accuracy.

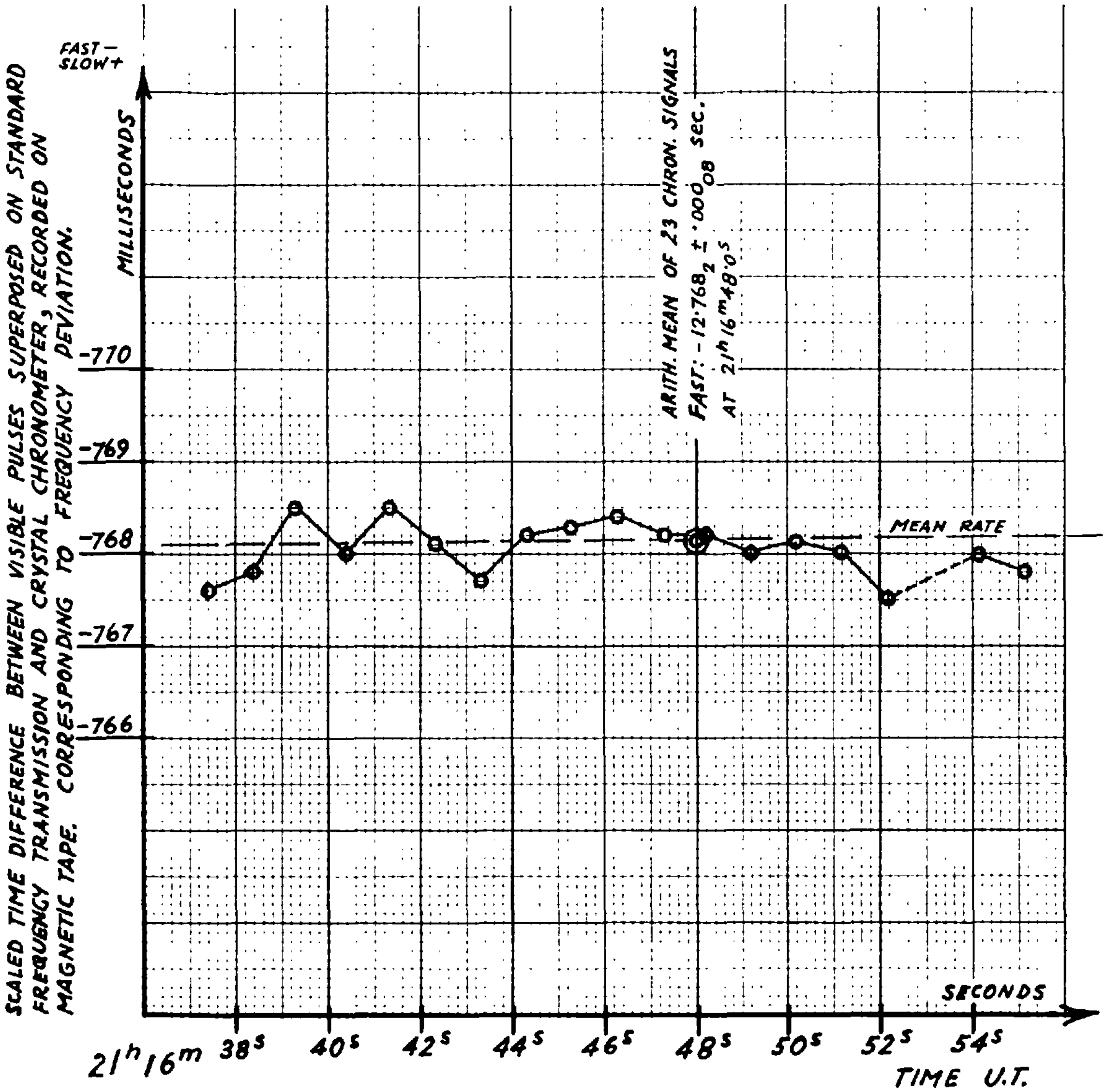
Table 3.5.5.-4 contains the relevant data used in Fig. 3.5.5.-11. In column 4 are listed again the deviations of the time space between individual chronometer beats from the 60/61 solar second time interval; these incidentally could be read off the graphs: a downward slope of the isochronism "curve" indicates a pulse interval which is less than its theoretical value, an upward slope a pulse time interval of longer duration.

The results obtained from scaling the tape are listed in the tables, and shown in Fig. 3.5.5.-13. The relative variations of the lengths of the time intervals clearly indicate their high degree of constancy. It will be noticed that there is no periodicity in the variation of the lengths of the 60/61 seconds intervals; this may be accounted for in terms of random errors, which are due mainly to calibration, recording and scaling.

It must be stressed that the accuracy quoted is obtained from calibration against various standard frequency transmissions: M.S.F., H.B.N., O.M.A., W.W.V., and therefore gives a reliabee precision of the instantaneous stability of the chronometer.

During field trials the chronometer was exposed to the sun's rays, to wind, humid atmosphere, and to temperatures from freezing to about 80°F. No departure from its mean performance after reaching operating temperature was met.

The chronometer in general behaves in a very uniform manner, and



U.T. INDICATED BY SECONDS PULSES SUPERPOSED ON STANDARD FREQUENCY TRANSMISSIONS : H.B.N., 5 Mc/s, AT THE INSTANT OF RECORDING ON MAGNETIC TAPE.

CRYSTAL CHRONOMETER PULSES ARE PLOTTED AT U.T., SCALED FROM STANDARD FREQUENCY PULSES.

PERFORMANCE OF CRYSTAL CHRONOMETER

THERMOSTATICALLY CONTROLLED
OPERATING ON 12 VOLT BATTERY

INSTANTANEOUS STABILITY

DURING 60/61 SECOND INTERVAL OF MEAN SOLAR TIME.

DATE : MARCH 31, 1963.

Fig. 3.5.5.-12

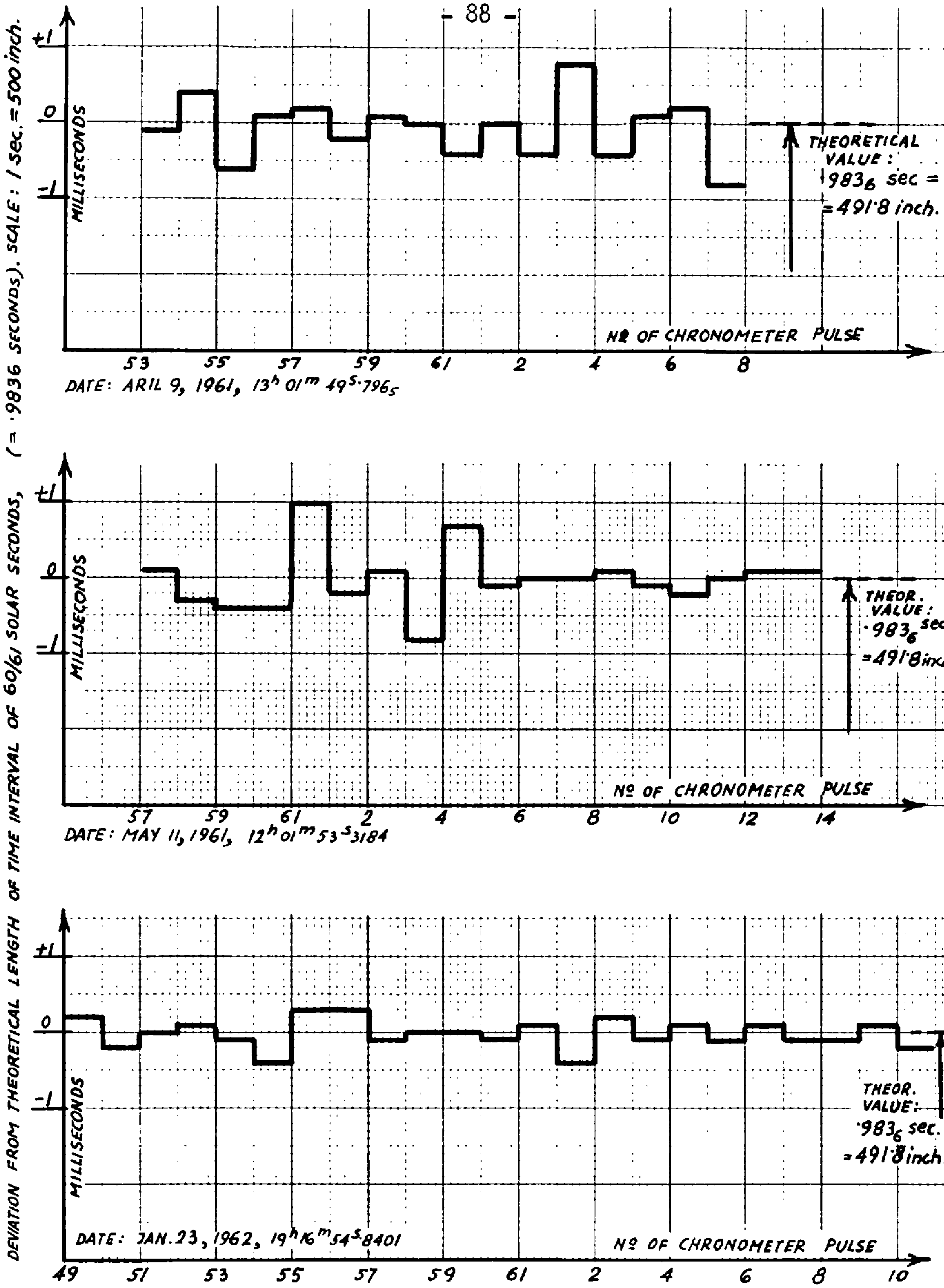
Performance of the Crystal Chronometer.

Thermostatically controlled,
operated on 12 volt battery,

No. of Chronometer Signal	Scaled U.T. of Chronometer Signal with reference to HBN reception	Length of Time Interval between two Chron. signals from scaled U.T.	Difference from theoretical Time interval (=.9836 sec.) ±milliseconds
	$19^h\ 16^m$	sec.	
49	54.8401^s		
50	55.8235	.9834	+ .2
51	56.8[illegible]73	.9836	- .2
52	57.7909	.9836	0.0
53	58.7744	.9835	+ .1
54	59.7581	.9837	- .1
55	00.7421	.9840	- .4
56			+ .3 average
57	02.7087		+ .3
58	03.6924	.9837	- .1
59	04.6760	.9836	0.0
60	05.6596	.9836	0.0
61	06.6433	.9837	- .1
1	07.6268	.9835	+ .1
2	08.6108	.9840	- .4
3	09.5942	.9834	+ .2
4	10.5779	.9837	- .1
5	11.5614	.9835	+ .1
6	12.5451	.9837	- .1
7	13.5286	.9835	+ .1
8	14.5123	.9837	- .1
9	15.4960	.9837	- .1
10	16.4795	.9835	+ .1
11	17.4633	.9838	- .2

Date: Jan., 23, 1962

[illegible] 3 .5.5. - 4.



PERFORMANCE OF CRYSTAL CHRONOMETER

THERMOSTATICALLY CONTROLLED
OPERATING ON 12 VOLT BATTERY.

LENGTH OF TIME INTERVALS BETWEEN TWO
CHRONOMETER PULSES.

Fig. 3.5.5.-13

since its performance over specified lengths of time has been determined, time interpolation and extrapolation can be carried out with high precision. The chronometer complies with the requirements of timing of optical observations in field astronomy.

The timing of a chronometer with reference to the instantaneous astronomical time, i.e. rotation of the earth, cannot be effected to a higher degree than $\pm$ .008 seconds. This is the mean correction of the fluctuation of the astronomical unit of time. It follows that the mean square error of the daily rate of a chronometer is approximately 0.012 sec/day, as referred to one comparison with the astronomical length of the day, which is determined astronomically from the knowledge of stellar positions and motions.

3.5.6. Calculation of Curve Fitting to Observational Results of Crystal Chronometer Performance.

Information on time may be required from the crystal chronometer during running-in and thereafter, between intervals of, or subsequent to its comparison with standard frequency transmission.

From the investigations described in the foregoing sections, linear interpolation within intervals of seconds and minutes can be executed whenever required, should either nominal chronometer time be sufficient or time comparisons available, and over a period of six to eight hours following the running-in time, when time comparisons at the beginning and at the end are possible.

Linear interpolation for time, shortly after setting the chronometer in operation, and for longer time intervals, would mean a failure to make full use of its characteristic performance. For this and for extrapolation, curve fitting has been investigated as follows.

The smooth curves obtained when joining the plotted average results of the measured time errors, represented in the performance diagram, suggest an approximation of:

$$Y = \text{function } X$$

of the second or higher order, where:

Y = amount the crystal chronometer is fast or slow on Universal Time

and

X = the time the crystal chronometer is operating.

In other words this means, making a general assumption, that Y is a power series in X.

The results from the observations X_i, Y_i, should satisfy the second order equation:

$$Y = A + BX + CX^2$$

A, B, C are the chronometer parameters to be determined.

The observations are conveniently referred to an auxiliary co-ordinate system, with the co-ordinate axes x, y, parallel to X, Y, their origin being the centre of gravity of the observation set.

Thus, the co-ordinates of the origin of the auxiliary system are:

3.5.6. cont.

$$X_m = \frac{[X]}{n} \qquad\qquad Y_m = \frac{[Y]}{n}$$

$[\ ]$ denotes the Gauss's symbol for addition.

and: $x_i = X_i - X_m \qquad y_i = Y_i - Y_m$

it follows that:

$$[x] = 0 \qquad \text{and} \qquad [y] = 0$$

the observations $x_i\ y_i$ should satisfy the equations:

$$y = a + bx + cx^2$$

There are the following observation equations, the number of which is greater than the number of the unknowns:

$$v_i = a + b\,x_1 + c\,x_1^2 - y_1$$

$$\vdots$$

The v's are residuals, expressed as functions of the unknown constants, a, b, c.

The observations could have been weighted according to the number of crystal chronometer signals recorded during the individual observations of comparison with standard frequency transmission; this has been omitted because the average number of signals making up each mean result varies very little.

The introduction of a third order term x^3 for the approximation would show little improvement. The observations are taken at equal time intervals i.e. the x's are distributed equally and, as they are chosen to be of an odd number, the first and third constant (a, and c) of both, the second and third order approximations are identical.

The graphs show clearly the shape of an even order function, and unless a fourth term is introduced the addition of a third order term x^3 will produce only a small alteration of the fitted curve.

3.5.6. cont.

The Gauss's condition for the best approximation is:

$$[vv] = \text{minimum}$$

The minimum is obtained by partial differentiation, with respect to the constants, of the sum of the squares of the observation equations using partial coordinates, from the "centre of gravity", of the plotted observations.

The partial derivatives set equal to zero, as required for the condition of minimum, constitute the normal equations.

The most probable values of the constants a, b, c, which are sought, are then obtained from the solution of the normal equations. Thus:

$$a = -c \cdot \frac{[x^2]}{n} \qquad b = \frac{[xy]}{[x^2]}$$

$$c = \frac{[x^2 y]}{[x^4] - \frac{[x^2]^2}{n}}$$

n = number of observation equations.

The introduction of a third order term would produce:

$$a = -c. \frac{[x^2]}{n} \qquad b = \frac{[xy] - d. [x^4]}{[x^2]}$$

$$c = \frac{[x^2 y]}{[x^4] - \frac{[x^2]^2}{n}} \qquad d = \frac{[x^3 y]\,[x^2] - [x^4]\,[xy]}{[x^2][x^6] - [x^4]^2}$$

It will be noticed that the terms $[x^3]$ and $[x^5]$ do not appear, because they are = 0, as the result of the symmetrical spacing of the X 's.

3.5.6. cont.

After the constants a, b, c, have been determined, the residuals v_i are calculated from n individual observations.

The degree of accuracy of a single observation is expressed by the size of its mean square error, which gives, what is essential, a critical estimate of the reliability of the constants a, b, c.

Mean square error: $$m = \pm\sqrt{\frac{[vv]}{n-3}}$$

The denominator is made up of number of observations equations, less number of unknowns.

Relation of time error and operating time of the chronometer during the running-in period.

The chronometer constants a, b, c, have been calculated, using the above equations, from means of separate periods of operation, compared with the following standard time transmissions:

MSF 2, MSF 3, HBN 2, 5 Mc/s, and 5 Mc/s, OMA 2, 5 Mc/s.
Full calculations are shown further on.

The following equation has been deduced, for interpolating and extrapolating the amount the crystal chronometer was slow, or fast, during the running-in time, when operating on one set of batteries with the heating circuit switched on:

$$Y_1 = Y_o - .076\,(t_1 - t_o) + .0056\,(t_1^2 - t_o^2) \quad \text{. . . . .} \quad (1)$$

Y_1 is the unknown amount the crystal chronometer is slow or fast in milliseconds, at the time t_1, in minutes from the instant the chronometer is started.

Y_o is the known amount the chronometer is slow or fast, obtained from comparison with standard time signals, at time t_o.

t_o = elapsed time in minutes from the instant the chronometer is started.

3.5.6. cont.

This equation is intended for use in circumstances when time comparison is effected after the chronometer has run about a quarter of an hour, i.e. $t_o > 15$ minutes, so that the warming-up period is under way.

In the event , where only one comparison could be executed, approximately ten minutes after switching on the crystal chronometer, the following equation was used for the determination of Y_1 during the running-in time:

$$Y_1 = Y_{10} + .613 - .076\, t_1 + .0056\, t_1^2 \quad \ldots\ldots\ldots\ldots \quad (1A)$$

Y_1 is the unknown amount the chronometer is slow or fast in milliseconds, at the time t_1, in minutes, from start of operation.

Y_{10} is the known amount in milliseconds the chronometer is slow or fast, obtained from comparison with standard time signals, ten minutes after starting the crystal chronometer.

Approximation equations which fit observational data, obtained with instruments permitting individual operating periods, contain additive constants which take care of the initial setting. The above equations (1) and (1A) contain Y_0 and Y_{10} respectively, for due allowance of the time discrepancy of the chronometer when it starts operating.

<u>Relation of time error and operating time of the chronometer after the running-in period</u>:

The extrapolation of the amount the chronometer was slow or fast, after the running-in time, was carried out with the following equation, which was deduced from the chronometer performance with one set of batteries, with temperature control and, as in the previous case, without disturbing the adjustment for calibration.

$$Y_1 = Y_o + 49.22\,(T_1 - T_o) + .1629\,(T_1^2 - T_o^2) \quad \ldots\ldots\ldots \quad (2)$$

3.5.6. cont.

Y_1 is the unknown amount in msec, the chronometer is slow or fast, with respect to standard time transmission , at some future time T_1, from start of operation, in hours.

Y_o is the known amount in msec the chronometer is slow or fast, at time T_o, from start of operation and after the running-in time has elapsed, in hours.

In the event of one time comparison being executed one hour after the chronometer has started to operate, the equation below can be used:

$$Y_1 = Y_o - 52.947 + 49.22\, T_1 + .1629\, T_1^2 \ldots\ldots\ldots\ldots (2A)$$

Y_1 is the unknown amount the chronometer is slow or fast in msec, after T_1 hours of operation, in hours.

Y_o is the known amount in msec the chronometer was slow or fast one hour after it has started to operate.

The equations (2) and (2A) are intended to be used for time extrapolation, when no further receptions of time signals are possible.

No approximation equations have been calculated for the performance of the crystal chronometer when operating on main power supply, with or without temperature control, and over a long term period. These approximations are trigonometrical functions and are obviously of no practical use for a chronometer designed primarily for field use.

Equation (2) represents a mean expression of a series of chronometer runnings, referred to the same initial setting Y_o. Although it could be used for interpolation, should further comparisons of time be available, graphical interpolation for the period after the running-in time over an area of four to five hours, not covered by time comparisons, is convenient to use and the individual signals of the chronometer can be extracted with a precision of $\pm$ 1 msec., as mentioned in previous sections.

The same result can be obtained by calculating the chronometer parameters a, b, c, afresh for every individual period the chronometer is operating.

3.5.6. cont.

It follows that the crystal chronometer has to be compared with standard time signals at least once (half to one hour) after start of operation, and at least once after about seven hours, to achieve the quoted accuracy in time interpolation.

The following calculations illustrate the procedure for obtaining chronometer constants from various sets of observations which, for this purpose, are reduced to mean values. These mean values form the mean isochronism curve. The mean isochronism curve during the running-in time is given in Fig. 3.5.6.-1.

It will be noticed that observed mean values of time differences of various periods of chronometer running are plotted in relation to the mean isochronism curve.

In column two of Table 3.5.6.-1 are listed the times to which are referred the time comparisons between chronometer pulses and pulses of standard frequency transmissions; the intervals are equally spaced, deliberately, to reduce the amount of calculation. Column three contains the scaled time differences from the graph in Fig. 3.5.6.-1.

These observations marked with X and Y represent the Cartesian co-ordinates of a set of points, which indicates a relationship of the form:

$$Y = A + B \,.\, X + C \,.\, X^2$$

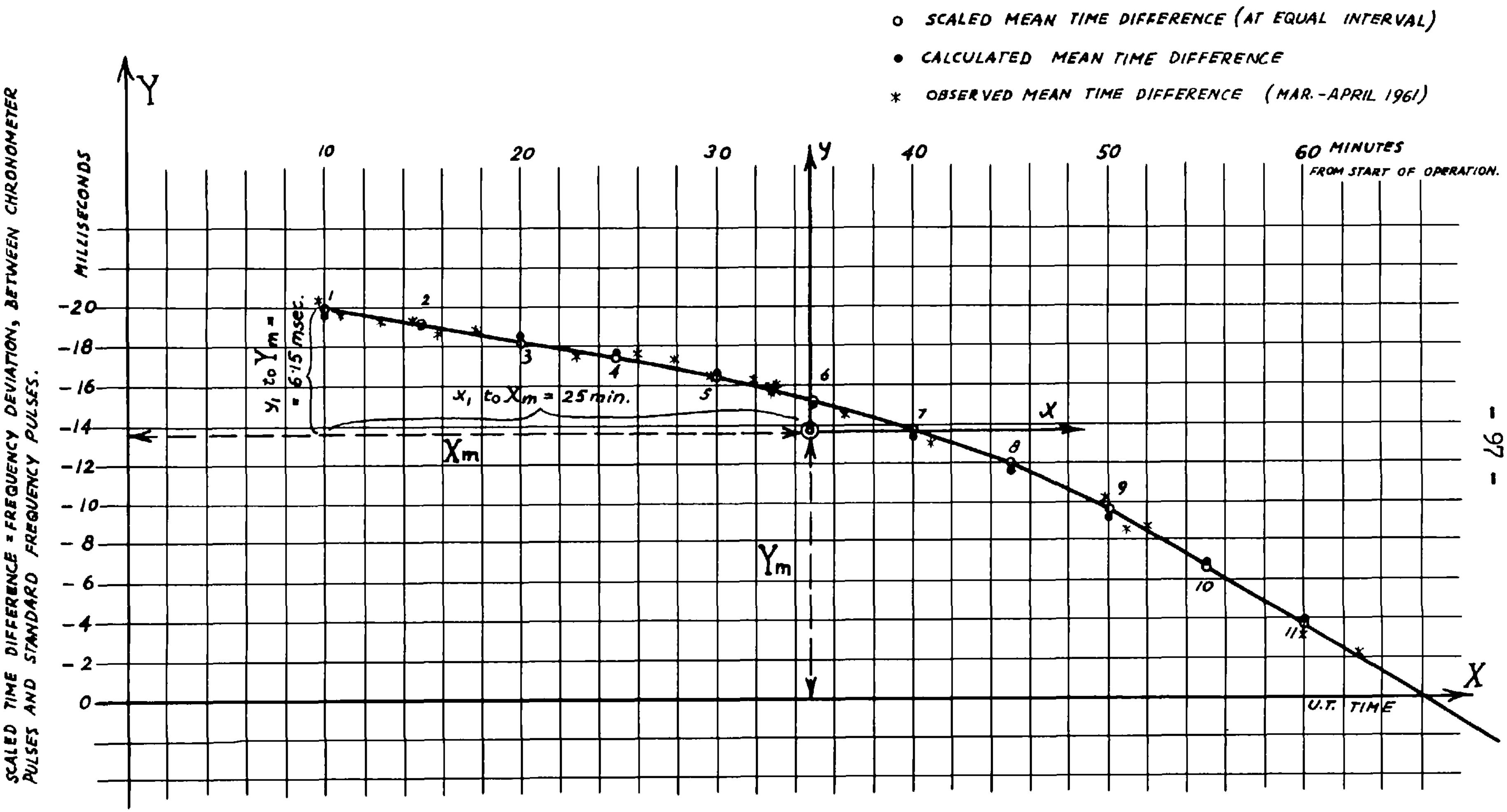
The subsequent columns show the values of the observations, referred to an auxiliary co-ordinate system, and the evaluation of the constants.

From the formulae are obtained:

$$c = \frac{+\,2\,985.00}{+\,1\,223\,750 - \dfrac{2\,750^2}{11}} = +.005\ 56$$

$$b = \frac{+\,861.50}{+\,2\,750} = +.\ 3132$$

$$a = -\,c,\ \frac{+\,2\,750}{11} = -\,1,3916$$



MEAN ISOCHRONISM CURVE DURING RUNNING-IN TIME

Fig. 3.5.6.-1

Curve Fitting to Observational Results
during Running-in Time.

Observations No.	Mean values X	Y	$X-X_m$ = x	$Y-Y_m$ = y	x^2	x^4
	minutes	msec.	minutes	millisec		
1	10	-20.0	- 25	- 6.15	625	390 625
2	15	-19.3	- 20	- 5.45	400	160 000
3	20	-18.2	- 15	- 4.35	225	50 625
4	25	-17.6	- 10	- 3.75	100	10 000
5	30	-16.4	- 5	- 2.55	25	625
6	35	-15.3	0	- 1.45	0	0
7	40	-13.8	+ 5	+ .05	25	
8	45	-12.0	+ 10	+ 1.85	100	
9	50	- 9.6	+ 15	+ 4.25	225	
10	55	- 6.5	+ 20	+ 7.35	400	
11	60	- 3.7	+ 25	+10.15	625	
[ ]=11			0	+ .05	2750	1223 750
$\frac{[\ ]}{11}$ =	X_m = 35	Y_m = -13.85				

Table 3.5.6. - 1

Column No. 1	8	9	10	11	12
	x^6	xy	x^2y	x^3y	x^3
1		+ 153.75	- 3 843.75	+ 96 093.75	-15 625
2		+ 109.00	- 2 180.00	+ 43 600.00	- 8 000
3		+ 65.25	- 978.75	+ 14 681.25	- 3 375
4		+ 37.50	- 375.00	+ 3 750.00	- 1 000
5		+ 12.75	- 63.75	+ 318.75	- 125
6		0.00	0.00	0.00	0
7		+ 0.25	+ 1.25	+ 6.25	+ 125
8		+ 18.50	+ 185.00	+ 1 850.00	+ 1 000
9		+ 63.75	+ 956.25	+ 14 343.75	+ 3 375
10		+ 147.00	+ 2 940.00	+ 58 800.00	+ 8 000
11		+ 253.75	+ 6 343.75	+158 593.75	+15 625
[ ]	+641 093 750	+ 861.50	+ 2 985.00	+392 037.50	0

Table 3.5.6. - 1 cont.

3.5.6. cont.

It follows the equation of the approximation curve in the auxiliary system x, y:

$$y = -1.392 + .313\,x + .0056\,x^2$$

Hence, the equation referred to the time when the chronometer was set in operation, provided one observation was taken ten minutes after starting:

In the X Y system:

$$Y = Y_{10} + .613 - .076\,X + .0056\,X^2$$

Y = unknown amount the chronometer is slow (+) or fast (-) in milliseconds at time X .

Y_{10} = observed amount the chronometer is slow (+) or fast (-) in milliseconds, ten minutes after it has started to operate.

X = time in minutes, after it has started to operate.

From the above, the standard equation for the chronometer during the running-in time is obtained:

e.g. for time t_1:

$$Y_{(at\ t_1)} = Y_{10} + .613 - .076\,X_{(at\ t_1)} + .0056\,X^2_{(at\ t_1)}$$

and analogous for t_o:

therefore:

$$Y_1 = Y_o - .076\,(t_1 - t_o) + .0056\,(t_1^2 - t_o^2) \quad \ldots\ldots\ldots \quad (1)$$

as stated:

Y_1 is the unknown amount the crystal chronometer is slow or fast in milliseconds, at time t_1, in minutes after the chronometer has started to operate.

Y_o is the known amount the chronometer is slow or fast, in milliseconds, obtained from comparison with standard frequency pulses, at time t_o.

t_o = elapsed time in minutes after the chronometer has started to operate.

3.5.6. cont.

The remaining residuals v_i (i = 1, 2, 3, 4 . . .) and $[vv]$ are calculated by making use of the numerical values of the constants, given in column 13 to 17.

The calculated time differences in the auxiliary system and in the system XY, namely (y_i) and (Y_i) respectively, are obtained at the same time: column 15 and 18.

The mean square error of one scaled time difference is then:

$$m = \pm \sqrt{\frac{[vv]}{11 - 3}} = \pm .35 \text{ milliseconds.}$$

The denominator is made up from eleven observations less three unknowns which have to be determined from the observations. The mean square error shows the precision of curve fitting to the mean performance of the crystal chronometer during the interval of time concerned. Throughout the calculation more decimal places were used than the precision warranted. This involved no extra work, because a desk calculator was employed, having transfer from the product register to the setting register.

The calculated time differences (Y_i) are plotted in Fig. 3,5.6.-1, to illustrate the result obtained analytically.

The agreement between the calculated and the scaled time differences is just about what might be expected from a second order approximation to a smooth parabolic curve.

It would not be justified to take more than two terms in the approximation formula, not only on account of the labour involved in determining the constants, but also in view of the second differences of the observed values Y, (column 3) which are nearly of the same order, about $\pm \frac{1}{2}$ msec. Since the X's are taken at equal intervals, the values of the second differences indicate that the required polynomial is most likely of the second order.

Column No. 1	13	14	15	16	17	18
	bx	cx^2	$a+bx+cx^2$ (y)	v (y)-y	vv	$(y)+Y_m$ (Y)
1	- 7.832	+ 3.479	- 5.745	- .41	.1681	- 19.60
2	- 6.265	+ 2.226	- 5.430	- .02	.0004	- 19.28
3	- 4.699	+ 1.252	- 4.839	+ .49	.2391	- 18.69
4	- 3.133	+ .557	- 3.968	+ .22	.0475	- 17.82
5	- 1.566	+ .139	- 2.819	+ .27	.0729	- 16.67
6	0	0	- 1.392	- .06	.0036	- 15.24
7	+ 1.566	+ .139	+ .313	- .26	.0676	- 13.54
8	+ 3.133	+ .557	+ 2.298	- .45	.2025	- 11.55
9	+ 4.699	+ 1.252	+ 4.559	- .31	.0961	- 9.29
10	+ 6.265	+ 2.226	+ 7.099	+ .25	.0625	- 6.75
11	+ 7.832	+ 3.479	+ 9.919	+ .23	.0529	- 3.93
[ ]	0			+ .05	1.0132	

Table 3.5.6. - 1 cont.

3.5.6. cont.

The observational results for the determination of the chronometer constants of the extrapolation equations (2) and (2A) to be used after the running-in time, and the residuals, are shown in Table 3.5.6.-2.

Extrapolation is rather hazardous, and the accuracy of the result obtained cannot be ascertained. Therefore, a long distance in the X direction was preferred to the accuracy in the Y direction, thus sacrificing the closest fit. The second order equation was calculated to cover eight hours' running of the chronometer.

The mean isochronism curve has been obtained by passing a smooth curve through mean observation groups of various operating periods. The scaled time differences are again spaced at equal time intervals. The observations are referred to an auxiliary co-ordinate system, as before.

Scaled results are given in Fig. 3.5.6.-2. The calculated results are not plotted because their difference from the scaled values will not show up at the adopted plotting scale.

The constants are then obtained:

$$b = \frac{[xy]}{[x^2]} = \frac{+3\,548.00}{+70.00} = +50.686$$

$$c = \frac{[x^2 y]}{[x^4] - \frac{[x^2]^2}{n}} = \frac{+42.00}{+584.50 - \frac{4900.00}{15}} = +0.1629$$

$$a = -c\,\frac{[x^2]}{n} = -0.1629\;\frac{70.00}{15} = -0.760$$

and the approximation equation in the auxiliary system referred to its origin is:

$$y = -0.760 + 50.686\,x + 0.1629\,x^2$$

substituting for x and y the respective values in the X Y system:

Curve Fitting to Mean Performance of the Crystal Chronometer

after the Running-in Time.

Chronometer thermostatically controlled,
operated on one set of batteries.

Column No. 1	2	3	4	5	6	7
No. of Obs.	Mean Values of Observations X hours	Y msec.	$X - X_m$ = x hrs.	$Y - Y_m$ = y msec.	x^2	x^4
1	1.0	- 360	- 3.5	- 172.6	12.25	150.0625
2	1.5	- 339	- 3.0	- 151.6	9.00	81.0000
3	2.0	- 317	- 2.5	- 129.6	6.25	39.0625
4	2.5	- 290	- 2.0	- 102.6	4.00	16.0000
5	3.0	- 266	- 1.5	- 78.6	2.25	5.0625
6	3.5	- 240	- 1.0	- 52.6	1.00	1.0000
7	4.0	- 212	- 0.5	- 24.6	0.25	0.0625
8	4.5	- 187	0	+ 0.4	0	0
9	5.0	- 162	+ 0.5	+ 25.4	0.25	
10	5.5	- 136	+ 1.0	+ 51.4	1.00	
11	6.0	- 110	+ 1.5	+ 77.4	2.25	
12	6.5	- 86	+ 2.0	+ 101.4	4.00	
13	7.0	- 61	+ 2.5	+ 126.4	6.25	
14	7.5	- 35	+ 3.0	+ 152.4	9.00	
15	8.0	- 10	+ 3.5	+ 177.4	12.25	
[ ]	X_m=	Y_m=	0	0	+70.00	+ 584.50
[ ] / 15	+4.5	- 187.4				

Table 3.5.6. - 2

Curve Fitting to Mean Performance of the Crystal Chronometer

after the Running-in Time.

Chronometer: Thermostatically controlled,
operated on one set of batteries.

Column No.					
1	8	9	10	11	12
	x^3	x^6	xy	x^2y	x^3y
1	-42.875	1 838.265 625	+ 604.10		
2	-27.000	729.000 000	+ 454.80		
3	-15.625	244.140 625			
4	- 8.000	64.000 000			
5	- 3.375	11.390 625			
6	- 1.000	1.000 000			
7	- 0.125	.015 625		-4 934.75	
8	0			+4 976.75	
9	+ 0.125				
10	+ 1.000				
11	+ 3.375				
12	+ 8.000				
13	+15.625				
14	+27.000				
15	+42.875	2 887.812 5000			
[ ]	0	+5 775.625 000	+3 548.00	+ 42.00	+29 495.640

Table 3.5.6. - 2 cont.

Curve Fitting to Mean Performance of the Crystal Chronometer after the Running-in Time.

Chronometer: Thermostatically controlled,
operated on one set of batteries.

Column No. 1	13	14	15	16	17	18
	bx	cx^2	$a+bx+cx^2$ = (y)	(y)-y v	vv	$(y)+Y_m$ (Y)
1	-177.4	+ 2.0	-176.2	-3.56	12.67	363.6
2	-152.1	+ 1.5	-151.4	+0.24	0.06	338.8
3	-126.1	+ 1.0	-125.9	+3.74	13.99	313.3
4	-101.4	+ 0.6	-101.6	+1.04	1.08	289.0
5	- 76.0	+ 0.4	- 76.4	+2.24	5.02	263.8
6	- 50.7	+ 0.2	- 51.3	+1.34	1.80	238.7
7	- 25.3	0	- 26.1	-1.46	2.13	213.5
8	0	0	- 0.8	-1.16	1.35	188.2
9	+ 25.3	0	+ 24.5	-0.86	0.74	162.9
10	+ 50.7	+ 0.2	+ 50.1	-1.26	1.59	137.3
11	+ 76.0	+ 0.4	+ 75.6	-1.76	3.10	111.8
12	+101.4	+ 0.6	+101.2	-0.16	0.03	86.2
13	+126.1	+ 1.0	+126.3	-0.06	0.01	61.1
14	+152.1	+ 1.5	+152.8	+0.44	0.19	34.6
15	+177.4	+ 2.0	+178.6	+1.24	1.54	8.8
[ ]	0	+11.4	- 0.6	0	45.28	

Table 3.5.6. - 2 cont.

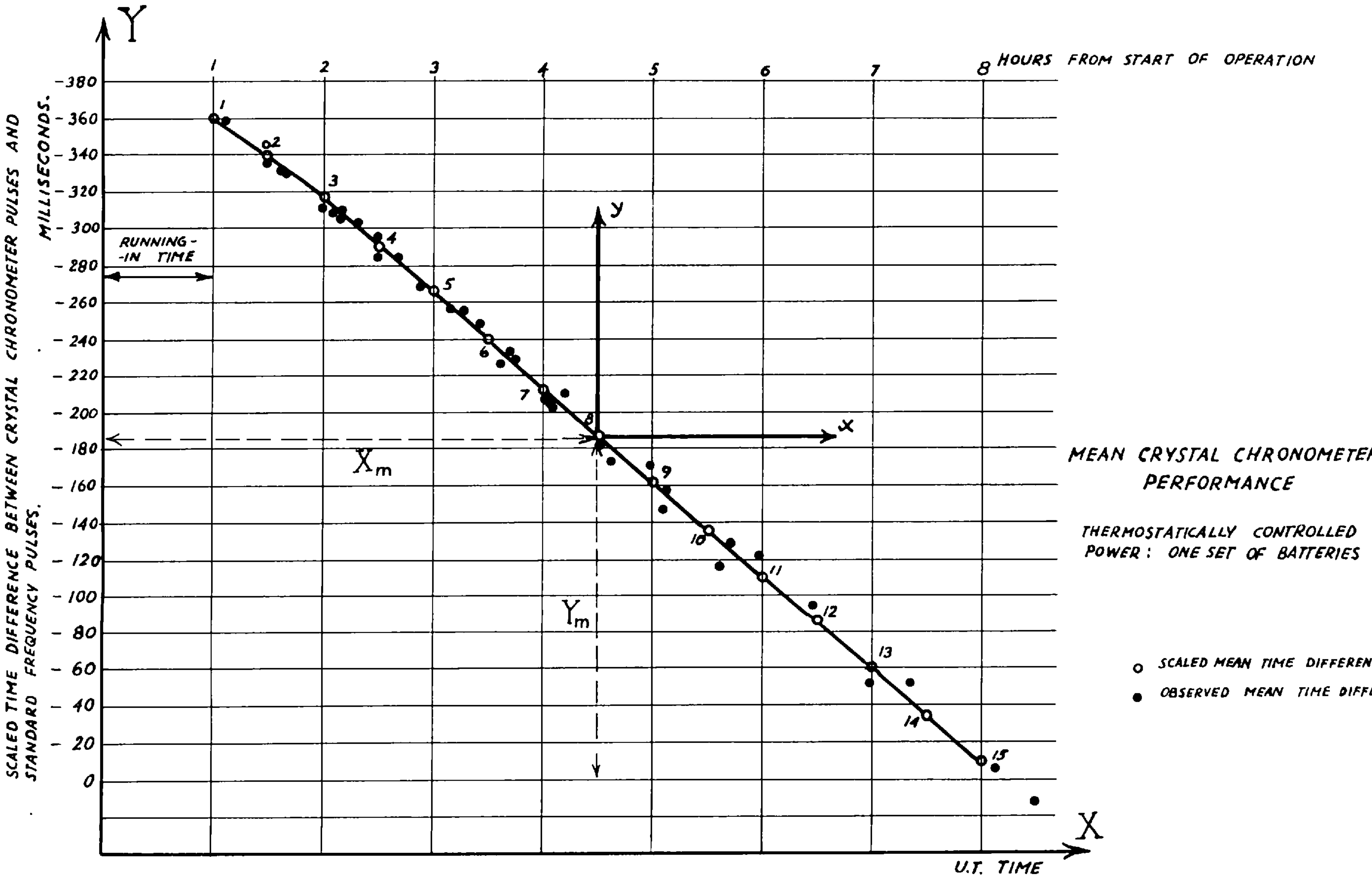

Y
HOURS FROM START OF OPERATION
1
2
3
4
5
6
7
8
-380
-360
-340
-320
-300
-280
-260
-240
-220
-200
-180
-160
-140
-120
-100
-80
-60
-40
-20
0
SCALED TIME DIFFERENCE BETWEEN CRYSTAL CHRONOMETER PULSES AND MILLISECONDS.
STANDARD FREQUENCY PULSES.
RUNNING-
-IN TIME
y
x
X_m
Y_m
X
U.T. TIME
MEAN CRYSTAL CHRONOMETER
PERFORMANCE
THERMOSTATICALLY CONTROLLED
POWER : ONE SET OF BATTERIES
SCALED MEAN TIME DIFFERENCE
OBSERVED MEAN TIME DIFFERENCE

Fig. 3.5.6.-2

3.5.6. cont.

$$Y + 187.4 = -.760 + 50.686 (X + 4.5) + .1629 (X - 4.5)^2$$

Y in msec. = amount the chronometer is slow or fast at time X, X in hours.

Substituting in the above equation successively:

Y_1, T_1, and Y_o, T_o and subtracting:

$$Y_1 = Y_o + 49.22(T_1 - T_o) + .1629 (T_1^2 - T_o^2) \ldots\ldots \quad (2)$$

Y_1 = unknown amount the chronometer is slow or fast in msec, at time T_1 in hours.

Y_o = known amount in msec. the chronometer is slow or fast at time T_o in hours.

The r.m.s. error of one scaled time difference is derived from values obtained in columns 13 to 17:

$$m = \pm\sqrt{\frac{45.28}{12}} = \pm 1.9 \text{ msec.}$$

The r.m.s. error indicates that scaling the time deviations from the mean isochronism curve to the nearest milli second should be aimed at. The same result for the r.m.s. error is obtained from the calculated time differences in the X Y system; since the residuals v_i (1, 2, 3, 4 . . .) are identical in both systems:

$$v = (y) - y \ldots\ldots\ldots\ldots \text{ xy system}$$

$$v = (Y) - Y \ldots\ldots\ldots\ldots \text{ XY system.}$$

It is: $(y) + Y_m = (Y) \ldots\ldots\ldots$ column 18

and:

$$Y - Y_m = y \ldots\ldots\ldots \text{ column 5}$$

substituting:

$$v = (Y) - Y_m - Y + Y_m$$

hence:

$$v = (Y) - Y.$$

3.5.6. cont.

The necessary checks of the calculations are the following:

$$[v] = n \cdot a + b[x] + c[x^2] - [y]$$

It is:

$$[x] = 0 \quad \text{and} \quad [-y] = 0$$

because: x_i and y_i are referred to the "centre of gravity".

Therefore:

$$[v] = n \cdot a + c[x^2]$$

$$= -15 \cdot 0.760 + 0.163 \cdot 70 = 0.$$

further:

$$[v] = [(y)] - [y]$$

$$[v] = [(y)]$$

the difference - 0.6 (column 15) resulting from rounding-off errors, can be disregarded.

From the observation equations:

$$v_i = a + b\, x_i + c\, x_i^2 - y_i$$

It follows:

$$[vv] = [yy] - \frac{[xy]^2}{[x^2]} - \frac{[x^2y]^2}{[x^4] - \frac{[x^2]^2}{n}}$$

or:

$$[vv] = [yy] - b[xy] - c[x^2y]$$

$$= 179\ 879.60 - 179\ 839.74 \cong 40.$$

The precision of the constants a, b, c, is expressed by their r.m.s. errors, which are obtained from their weights.

3.5.6. cont.

The application of the equations (1) and (2) is shown in Table 3.5.6.-3, where eight time comparisons with standard frequency transmissions, taken during a running period of about four hours, form an isochronism curve. Subsequently, the amount the chronometer was slow or fast, during the running-in period and for the time thereafter, was also extrapolated with equation (1) and (2) respectively.

The graphical representation of the results is given in Fig. 3.5.6.-3, which shows the performance of the crystal chronometer determined by:

(a) observations for comparison with time signals superposed on standard frequency transmissions, and

(b) the use of the mathematical model (equation (1) and (2).

Any lack of fit results from systematic and accidental errors.

Systematic errors can be detected and removed; the remaining errors are unavoidable or accidental.

Systematic errors, which influence the observed and plotted chronometer performance, i.e. the frequency stability, are due to variations of voltage and temperature, and due to aging.

Systematic errors of the model, equations (1) to (2A), are caused by the degree of imperfection of its assumption.

Fig. 3.5.6.-4 illustrates the extraction of the amount of time the chronometer was slow or fast, with respect to pulses superposed on standard frequency transmissions, for the purpose of correcting the timing of the field observations taken at Carn Marth on May 9, 1961.

Linear graphical interpolation, and results of analytical extrapolation are presented, to show their relative values.

For field work, graphical interpolation will produce a most acceptable answer. The corrections for time difference for most of the field observations were obtained graphically, (linear or parabolic).

No. of Observat!	U.T. of Observations from / to h m s	Mean h m s	No. of Pulses	+ slow - fast observed values millisec.	Chronometer operating time minutes	+ slow - fast calculat. values millisec.	Diff. + - milli-sec.
1	14 32 48 33 04	14 32 56	13	-2 934.2	10.9		
2	14 47 45 48 03	14 47 54	16	-2 933.9	25.9	-2 932.5	+1.4
3	15 02 47 03 03	15 02 55	15	-2 928.5	40.9	-2 927.8	+0.7
4	15 32 49	15 32 57	13	-2 908.2	71	-2 (10.9	-2.7

Egcuation (1):

$$y_{25.9} = -2\,934.2 - 0.076(25.9 - 10.9) + 0.0056(25.9^2 - 10.9^2) = \\ = -\,2\,932.5$$

4				-2 908.2	72		
5	16 01 48 02 04	16 01 56	15	-2 885.4	102	-2 883.5	+1.9
6	16 32 50 33 14	16 33 02	24	-2 858.6	132	-2 858.4	+0.2

Table 3.5.6. - 3

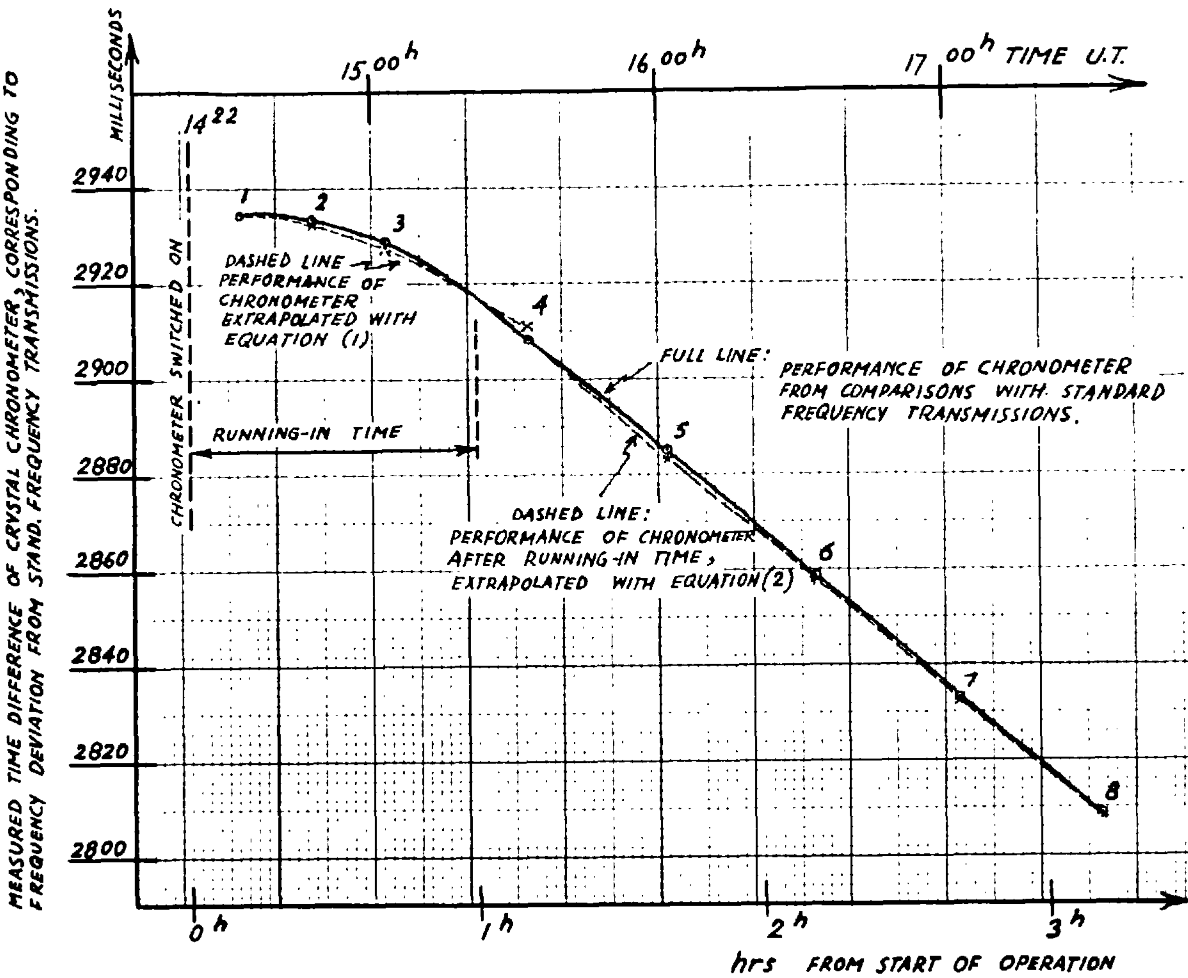
No. of Observat.	U.T. of Observations from / to h m s	Mean h m s	No. of Pulses	+ slow - fast observed values millisec.	chronometer operating time hrs.	+ slow - fast calculat. values millisec.	Diff. + - milli-sec.
7	17 02 44 03 04	17 02 54	20	-2 833.7	2.7	-2 833.3	+0.4
8	17 32 48 33 05	17 32 57	17	-2 808.4	3.2	-2 808.3	+0.1

Equation (2):

$$y_{1.7} = -2\,908.2+49.22(1.7-1.2)+0.1629(1.7^2-1.2^2) = -2\,883.5$$

$$y_{3.2} = -2\,908.2+49.22(3.2-1.2)+0.1629(3.2^2-1.2^2) = -2\,808.3$$

3.5.6. - 3 cont.



PERFORMANCE OF THE CRYSTAL CHRONOMETER

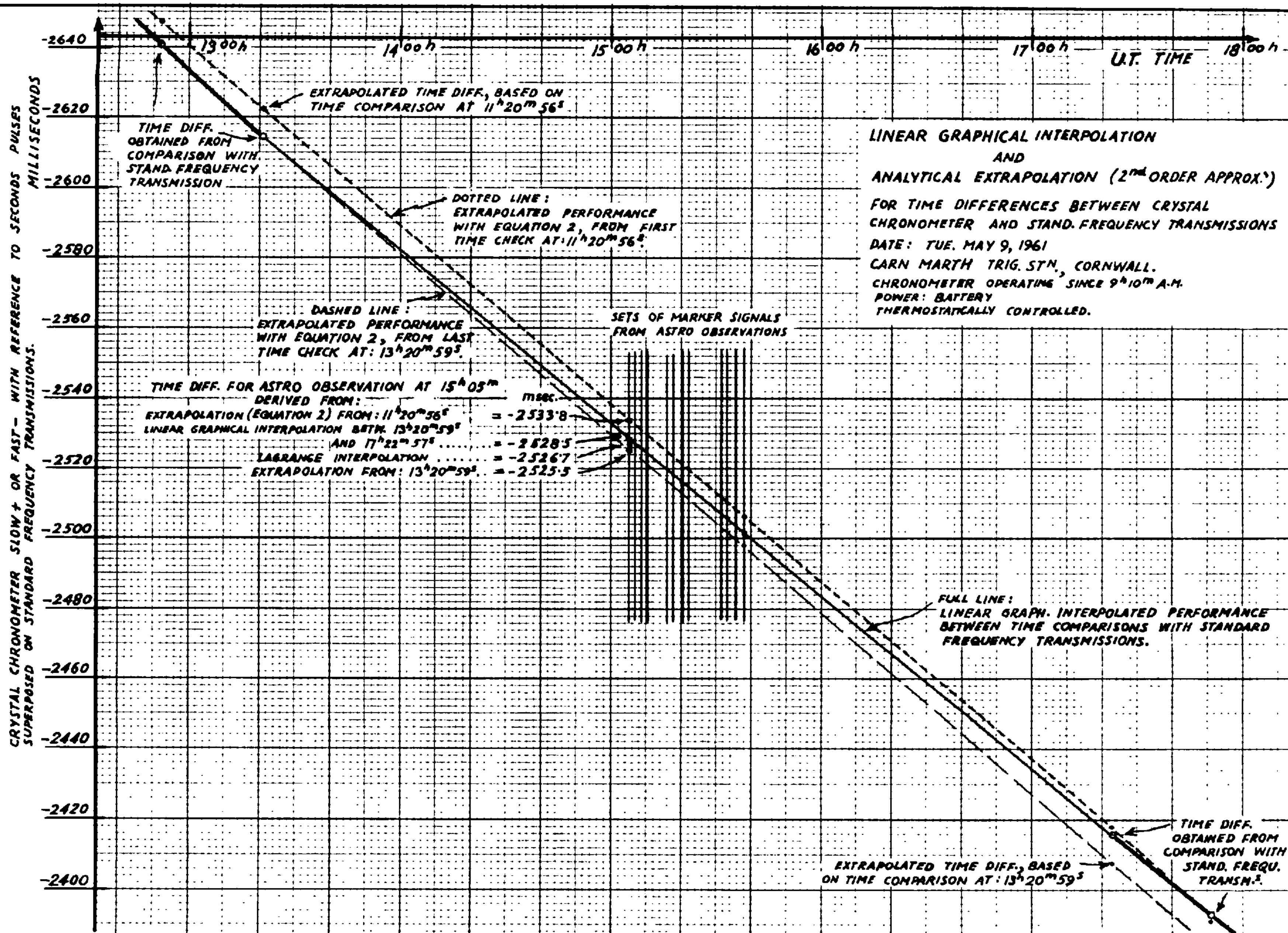
DETERMINED FROM :

COMPARISONS WITH STAND. FREQUENCY TRANSMISSIONS ∘
EXTRAPOLATION EQUATION (1) ×
EXTRAPOLATION EQUATION (2) ✳

POWER: ONE SET OF BATTERIES
THERMOSTATICALLY CONTROLLED

DATE: SAT. APRIL 22, 1961
TYWARNHALE
M.S.F. 5 Mc/s
Fi-cord, Scotch Tape.

Fig. 3.5.6.-3



LINEAR GRAPHICAL INTERPOLATION
AND
ANALYTICAL EXTRAPOLATION (2nd ORDER APPROX.^{n})
FOR TIME DIFFERENCES BETWEEN CRYSTAL CHRONOMETER AND STAND. FREQUENCY TRANSMISSIONS
DATE: TUE. MAY 9, 1961
CARN MARTH TRIG. ST^N., CORNWALL.
CHRONOMETER OPERATING SINCE 9^{h}10^m A.M.
POWER: BATTERY
THERMOSTATICALLY CONTROLLED.
U.T. TIME
13^{00h}
14^{00h}
15^{00h}
16^{00h}
17^{00h}
18^{00h}
CRYSTAL CHRONOMETER SLOW + OR FAST − WITH REFERENCE TO SECONDS PULSES SUPERPOSED ON STANDARD FREQUENCY TRANSMISSIONS.
MILLISECONDS
-2640
-2620
-2600
-2580
-2560
-2540
-2520
-2500
-2480
-2460
-2440
-2420
-2400
EXTRAPOLATED TIME DIFF., BASED ON TIME COMPARISON AT 11^{h}20^{m}56^s
TIME DIFF. OBTAINED FROM COMPARISON WITH STAND. FREQUENCY TRANSMISSION
DOTTED LINE: EXTRAPOLATED PERFORMANCE WITH EQUATION 2, FROM FIRST TIME CHECK AT: 11^{h}20^{m}56^s.
DASHED LINE: EXTRAPOLATED PERFORMANCE WITH EQUATION 2, FROM LAST TIME CHECK AT: 13^{h}20^{m}59^s.
SETS OF MARKER SIGNALS FROM ASTRO OBSERVATIONS
TIME DIFF. FOR ASTRO OBSERVATION AT 15^{h}05^m DERIVED FROM: msec.
EXTRAPOLATION (EQUATION 2) FROM: 11^{h}20^{m}56^s = -2533·8
LINEAR GRAPHICAL INTERPOLATION BETW. 13^{h}20^{m}59^s AND 17^{h}22^{m}57^s = -2528·5
LAGRANGE INTERPOLATION = -2526·7
EXTRAPOLATION FROM: 13^{h}20^{m}59^s. = -2525·5
FULL LINE: LINEAR GRAPH. INTERPOLATED PERFORMANCE BETWEEN TIME COMPARISONS WITH STANDARD FREQUENCY TRANSMISSIONS.
TIME DIFF. OBTAINED FROM COMPARISON WITH STAND. FREQU. TRANSM.^{s}.
EXTRAPOLATED TIME DIFF., BASED ON TIME COMPARISON AT: 13^{h}20^{m}59^s

Fig. 3.5.6-4

3.5.6. cont.

In the diagram, analytical extrapolation is based on available chronometer intercomparisons with time transmissions which are effected

(a) two hours

(b) four hours

previous to the timing of optical field observations. The results differ from the graphical interpolation only by -3 and +5 milliseconds respectively.

Lagrange's formula of interpolation.

The time error at $15^h\ 05^m$ obtained with Lagrange's formula of interpolation, based on the four observed time comparisons shown, is also plotted on the diagram.

The reliability of a value obtained with the use of Lagrange's formula connot be estimated and the amount of work involved in the computation is too large to justify its application for calculating time errors. The calculation is shown in table 3.5.6.-4.

The Binomial Theorem cannot be applied for interpolation, because the observed quantities, i.e. the time errors resulting from comparisons of the chronometer with time transmissions, as a rule are not spaced at equal intervals.

The precision of intermediate values obtained by graphical or analytical interpolation depends on the accuracy of the initial data entering into the plot or calculation respectively.

Interpolation of Time Error

with

Lagrange's Formula.

At. Carn Marth, Cornwall
Date. May, 9, 1961

Required: Time error of Crystal Chronometer
at: $15^h\ 05^m\ 00^s$

Time (U.T.) of Comparison of Crystal Chronometer with Time Transmissions			Observed Quantities Chronometer slow + fast -	
	h m s	reduced -12^h minutes		milliseconds
x_o	12 51 57	51.95	y_o	- 2 640.2
x_1	13 20 59	80.98	y_1	- 2 614.1
x_2	17 [illegible] 56.5	322.94		- 2 415.9
x_3	17 50 56.5	350.94	y_3	- 2 392.8
At:				
x	15 05 00	185.00	y	required.

Table 3.5.6. - 4

Interpolation of Time Error

with

Lagrange's Formula

$$y = \frac{(185.00-80.98)(185.00-322.94)(185.00-350.94)}{(51.95-80.98)(51.95-322.94)(51.95-350.94)} \cdot -2\,640.2$$

$$+ \frac{(185.00-51.95)(185.00-322.94)(185.00-350.94)}{(80.98-51.95)(80.98-322.94)(80.98-350.94)} \cdot -2\,614.1$$

$$+ \frac{(185.00-51.95)(185.00-80.98)(185.00-350.94)}{(322.94-51.95)(322.94-80.98)(322.94-350.94)} \cdot -2\,415.9$$

$$+ \frac{(185.00-51.95)(185.00-80.98)(185.00-322.94)}{(350.94-51.95)(350.94-80.98)(350.94-322.94)} \cdot -2\,392.8$$

$= -2\,526.7$ milliseconds.

Table 3.5.6. - 4 cont.

3.5.7. Crystal Chronometer Performance affected by Decrease in Voltage of the Driving Battery.

The observed time differences between crystal chronometer and transmitted frequency pulses do not only depend on time, as the approximation equations 1 to 2A, derived in the previous section. show, but are subjected to the influence of a variety of factors, some of which are unknown, and, what is even worse, cannot be eliminated.

The calculated relationship (equation 1 to 2A) can therefore at most represent a probable interdependence.

The time difference or time error between chronometer pulse and transmitted frequency pulse, as well as the decrease of the voltage of the driving battery, vary with the elapsed time from the start of operation. Hence, both magnitudes, time error and voltage decrease, can be represented as a function of time. The representation of the former as a function of time is already given in Section 3.5.6., equation 1 to 2A.

In addition to the above, it can be presumed that there is also a relationship between both the time error and the voltage, so that the time error of the chronometer could be expressed as a function of the voltage. From the knowledge of this relationship, it will be possible to predict, to a certain extent, the frequency deviation with voltage drop.

It is important to distinguish between:

(a) Immediate changes in supply voltage which will produce instantaneous frequency fluctuations. These are caused mainly by unpredictable sources, such as imperfect connections, faulty components etc., and are not suited for mathematical treatment; they are therefore not considered further.

(b) Progressive decrease of voltage during discharge of the battery, with possible after effect on the thermal equilibrium, exhibiting a distinct performance of gradually changing frequency.

The performance of the battery-driven chronometer during the respective intervals of hours, minutes and seconds investigated, and the equations for extrapolation are based on an operating voltage of 12v.

In Fig. 3.5.7.-1 is shown the isochronism curve of the chronometer and the voltage-time curve. The graph is drawn at the same scale as the performance shown in previous diagrams. The rate of the chronometer has been altered, so that the resulting performance curve runs nearly parallel to the x-axis.

The changed rate permits convenient graphical interpolation and is better suited for plotting several performance curves in the same space.

From the graph it can be seen that the chronometer displays the usual near-linear performance when driven at the specified voltage.

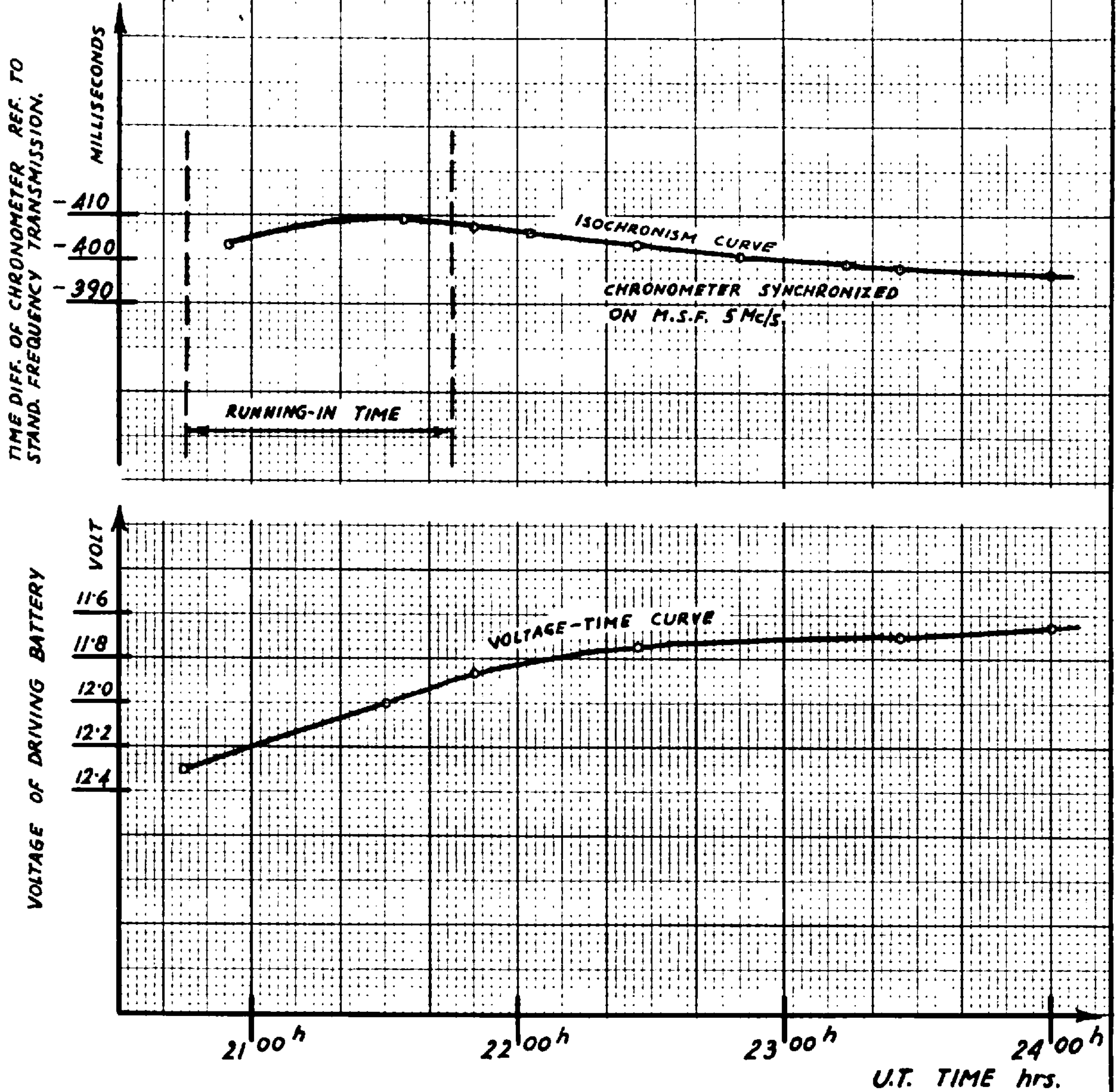
Linearity of the time-error/voltage variation within the limits of adequate voltage can be treated as time-error/ running-time variation, or also as the variation of voltage drop/running-time. In which case the relation of time-error/running-time may have greater ease of application. (Formulae 1 to 2A, Section 3.5.6.)

In the field it is rather cumbersome to take frequent hydrometer readings from a set of six cells; therefore, voltage readings are taken to obtain information of the condition of the battery.

The chronometer could function at an input voltage as low as 9 v., and it could be kept running in the absence of stand-by batteries, should its employment be absolutely necessary. Difficulty may arise in charging the battery again.

There will be a parabolic change of the chronometer's time error: the frequency increases due to decrease in voltage, which must also theoretically cause a decrease in temperature. The temperature of the oven of the chronometer under discussion is controlled by means of a mercury column contact thermometer and an associated reed relay.

Throughout the tests and during its employment in the field, when the terminal voltage was approaching 10.7, the chronometer was



TIME-ERROR CURVE OF CRYSTAL CHRONOMETER
THERMOSTATICALLY CONTROLLED
BATTERY DRIVEN

AND

TIME-VOLTAGE CURVE OF DRIVING BATTERY

DATE: APRIL 1963

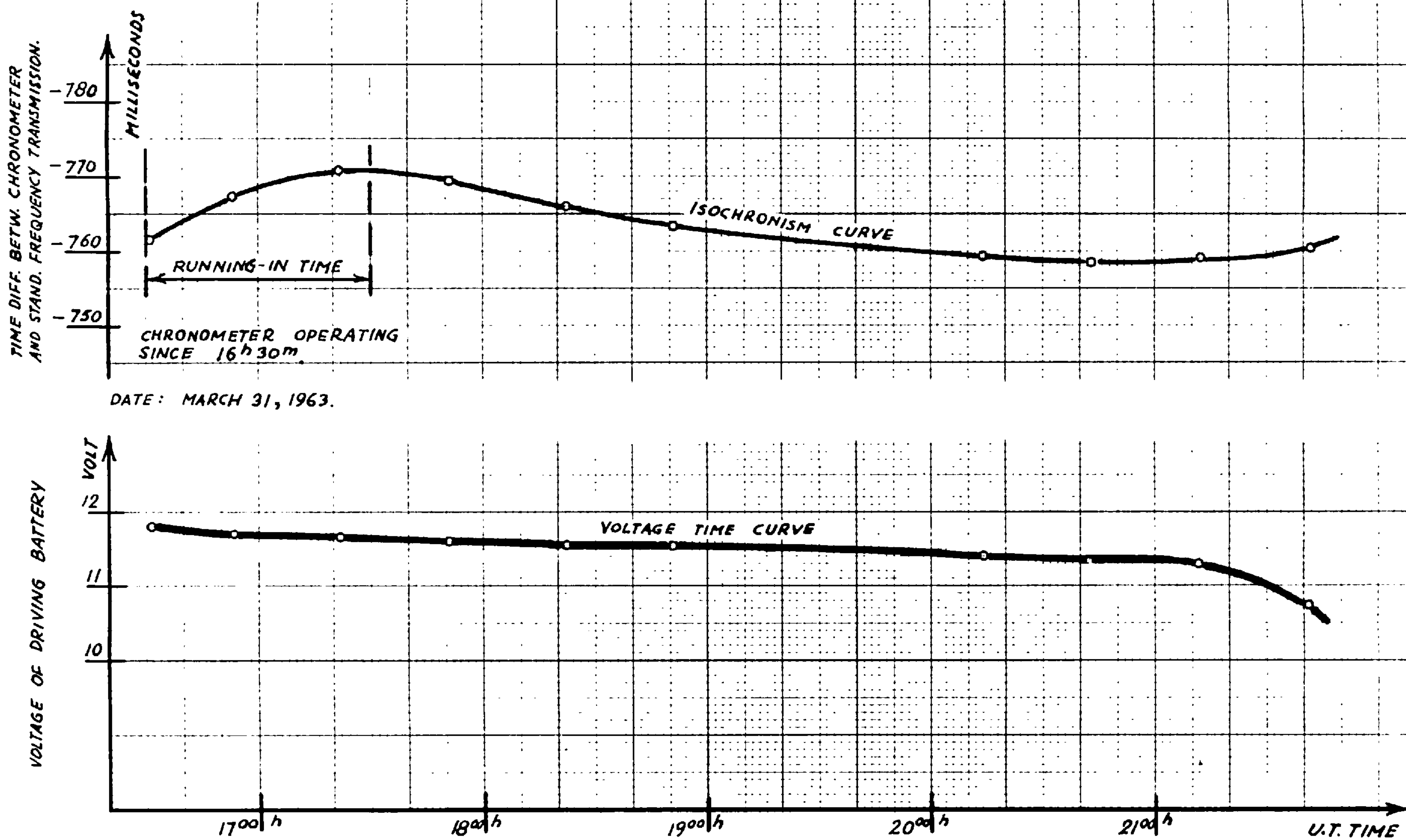
Fig. 3.5.7.-1

3.5.7. cont.

stopped, or the battery replaced. This was done to avoid an effect on the chronometer's behaviour caused by a drop in voltage, and also to achieve long service life of the cells without injury. It is standard practice to avoid reaching the limit of about $10\frac{1}{2}$ v. Research into the performance of the chronometer operating on less than $10\frac{1}{2}$ v. is therefore redundant.

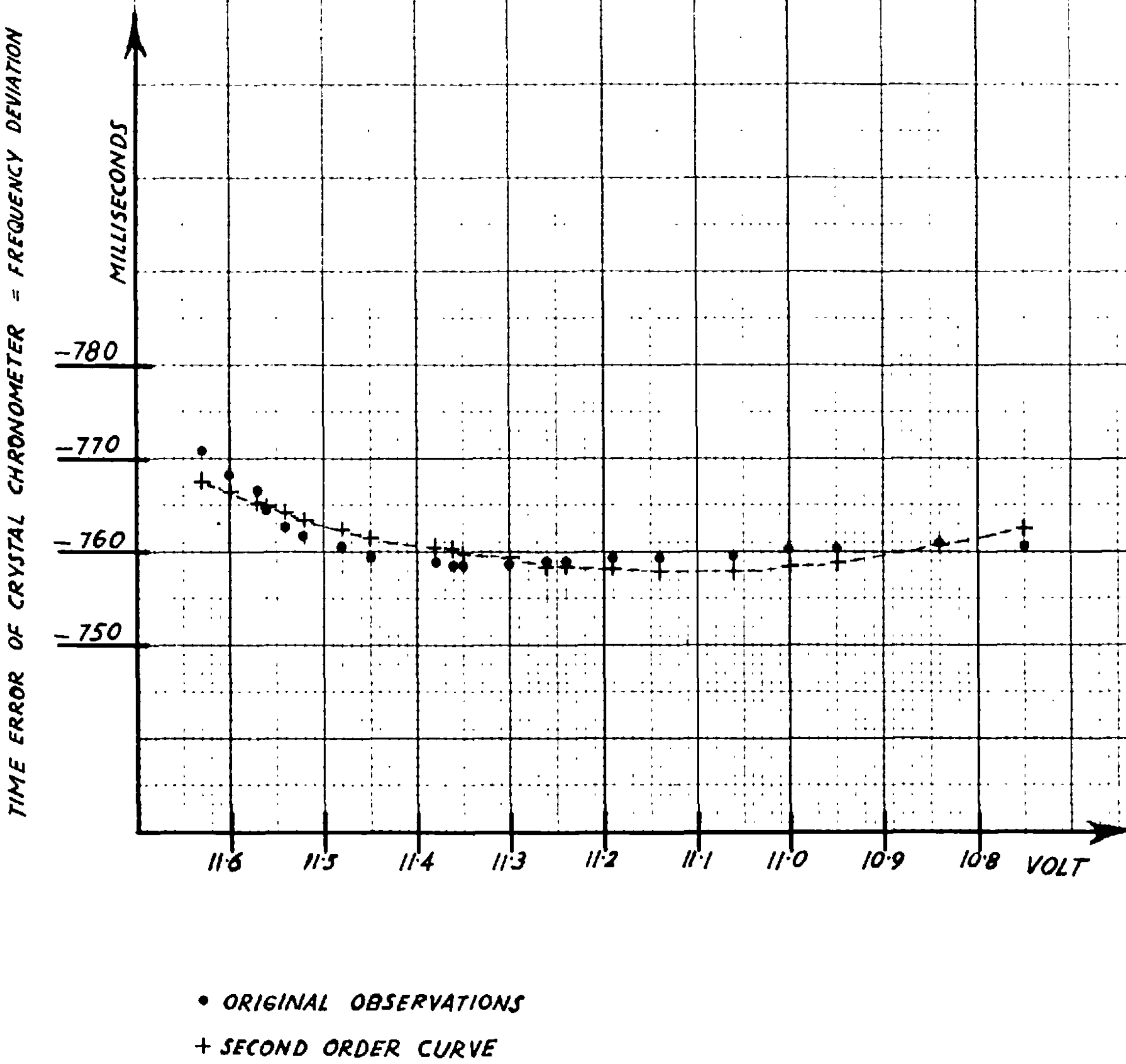
The following is the investigation into the chronometer performance when driven on marginal voltage, lower than 11.7 and obviously only to $10\frac{1}{2}$ v. Driven at the mentioned voltage, an interdependence of voltage decrease and time error can be anticipated. In Fig. 3.5.7.-2 is given a specimen of the chronometer performance and a record of voltage readings during operation. It can be seen that about 11.8 v. is reached soon after setting the chronometer in operation. This depends on battery conditions. After the running-in time, the isochronism curve shows, as expected, a parabolic shape. For better illustration of the resulting curve, the ordinate scale has been multiplied by two. It is quite obvious that there is a definite relationship between decrease of voltage and time-error.

In order to analyse this relation the observational results are arranged according to the increasing or decreasing values of one of the variables. The arrangement shown in Fig. 3.5.7.-3 thus represents the interdependence of time error or frequency deviation of the chronometer and voltage drop of the battery. The plotted values X and Y are derived from the original observations, which were also used to construct the diagram 3.5.7.-2. The problem now is to find an analytic function which will be a suitable expression of the casual relationship of the observed quantities. Since the diagram, Fig. 3.5.7.-3, indicates a non-linear relation, it is presumed that the interdependence could be adequately expressed as a 2nd order function. Only in exceptional cases will the calculated approximation equation give a perfect fit. This is because the observed values Y do not depend only on the observed values X, but are influenced by a variety



TIME-ERROR CURVE OF CRYSTAL CHRONOMETER
AND
TIME-VOLTAGE CURVE OF DRIVING BATTERY.

Fig. 3.5.7.-2



SECOND ORDER CURVE FITTED TO CORRELATED OBSERVATIONS
PERFORMANCE OF CRYSTAL CHRONOMETER
VOLTAGE AND TIME-ERROR

DATE: MARCH 1963
CHRONOMETER THERMOSTATICALLY CONTROLLED, BATTERY DRIVEN

Fig. 3.5.7.-3

3.5.7. cont.

of factors.

A measure of the quality of the fit is indicated by the correlation coefficient, which depends on the standard deviation of a single calculated observation and the standard deviation of a single observation from the arithmetic mean. The observations referred to are the quantities plotted in the ordinate-direction. The values observed are given in Table 3.5.7.-1, which contains also the evaluation of the constants by the method of Least Squares. It can be seen that there are systematic "errors" and not "accidental" errors, because $[xy]$ is not ≈ 0. (The sum of the products of the deviations ≈ 0.) This means that it is quite legitimate to assume a functional relationship between time error and voltage drop.

The calculations were carried out with an electric desk calculator. From the evaluated data (Table 3.5.7.-1) the constants of the quadratic approximation equation are obtained:

$$y = a + b.x + c.x^2$$

$$c = \frac{1,67 \; . \; 1,34 \; - \; 9,92 \; . \; -,18}{,19 \; . \; 1,34 \; - \; \frac{1,34^3}{21} \; - \; ,-18^2} = +35,7$$

$$a = -35,7 \; . \; \frac{1,34}{21} = -2,3$$

$$b = \frac{9,92 \; - \; 35,7 \; . \; -,18}{1,34} = +12,2$$

The equation which presumably fits the observations is therefore:

$$Y = +5\;176,9 - 794,8\,X + 35,7\,X^2$$

The mean square error of a single observation is:

$$\pm \sqrt{\frac{52,84}{21-3}} = \pm \; 1,7 \text{ msec.}$$

Curve Fitting to Results of Correlated Observations.

Column No. 1	2	3	4	5	6
No. of Observat.	Volt X	Time error Y msec.	$X-X_m=$ x	$Y-Y_m=$ y	xy
1	11.63	771.0	+ .34	+ 9.7	+ 3.30
2	11.60	768.3	+ .31	+ 7.0	+ 2.17
3	11.57	766.3	+ .28	+ 5.0	+ 1.40
4	11.56	764.5	+ .27	+ 3.2	+ .86
5	11.54	762.9	+ .25	+ 1.6	+ .40
6	11.52	761.9	+ .23	+ 0.6	+ .14
7	11.48	760.9	+ .19	- 0.4	- .08
8	11.45	759.8	+ .16	- 1.5	- .24
9	11.38	759.0	+ .09	- 2.3	- .21
10	11.36	758.5	+ .07	- 2.8	- .20
11	11.35	758.6	+ .06	- 2.7	- .16
12	11.30	758.8	+ .01	- 2.5	- .03
13	11.26	759.0	- .03	- 2.3	- .07
14	11.24	759.1	- .05	- 2.2	+ .11
15	11.19	759.2	- .10	- 2.1	+ .21
16	11.14	759.4	- .15	- 1.9	+ .29
17	11.06	759.9	- .23	- 1.4	+ .32
18	11.00	760.1	- .29	- 1.2	+ .35
19	10.95	760.2	- .34	- 1.1	+ .37
20	10.84	760.4	- .45	- 0.9	+ .41
21	10.75	760.5	- .54	- 0.8	+ .43
	X_m = 11.29	Y_m = 761.3	+ .08	+ 1.0	+ 9.92

Table 3.5.7. - 1

Curve Fitting to Results of Correlated Observations.

Column No. 1	7	8	9	10	11
No.	x^2y	x^2	x^3	x^4	y^2
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					
21					
[ ]	+1.67	+1.34	- .18	+ .19	+ 234.54

Table 3.5.7. - 1 cont.

Curve Fitting to Results of Correlated Observations.

Column No. 1		13	14	15	16	17
No.	- 794.78 X	+ 35.7 X^2	X^2	(Y)	(Y)-Y	vv
1				767.4	-3.6	
2				766.2	-2.1	
3				765.1	-1.2	
4				764.8	+ .3	
5				764.3	+1.4	
6				763.7	+1.8	
7				762.6	+1.7	
8				761.8	+2.0	
9				760.3	+1.3	
10				760.1	+1.6	
11				759.8	+1.2	
12				759.7	+ .4	
13				758.8	- .2	
14				758.6	- .5	
15				758.3	- .9	
16				758.0	-1.4	
17				758.0	-1.9	
18				758.5	-1.6	
19				759.0	-1.2	
20				760.6	+ .2	
21				762.8	+2.3	
[ ]						52,84

Table 3.5.7. - 1 cont.

3.5.7. cont.

and with respect to the arithmetic mean Y_m:

$$\pm \sqrt{\frac{234,5}{21 - 1}} = \pm\ 3,4 \text{ msec.}$$

and the correlation coefficient:

$$r^2 = 1 - \frac{1,7^2}{3,4^2} = ,75$$

and

$$r = ,87 \approx ,9$$

The correlation coefficient indicates that a relation between voltage drop and time error can be accepted with certainty.

Should there be a complete absence of any relationship between the two variables then the coefficient would be = 0. The calculated results are plotted in Fig. 3.5.7.-3.

But the best fit need not be produced by the customary empirical formula:

$$Y = A + B.X + C.X^2$$

Probably, in the present case, the quadratic approximation applied does not represent the actual relationship after all. The calculated curve has its minimum at 11.06 volt., which does not correspond to the actual occurrence, but the residuals listed in column 16 are small enough to accept the calculated approximation.

For general application of the derived formula, Y and X are replaced by Y_1, Y_o, and X_1, X_o respectively.

Hence the empirical formula according to which the time error varies with voltage drop (in the region of 11.6 to 10.8 volt) is:

$$Y_1 = Y_o - 794.8\,(X_1 - X_o) + 35.7\,(X_1^2 - X_0^2)$$

where:

Y_1 = unknown amount the chronometer is slow or fast in msec, corresponding to voltage reading X_1.

Y_o = known (previously determined or measured) amount in msec, the chronometer is slow fast, corresponding to voltage reading X_o.

To apply the formula effectively, the voltage readings X_1, X_o should be kept within limits of 1/3 v.

It is presumed that the chronometer, thermostatically controlled at 104°F, operates under average field conditions at 50°F, on one battery (six 2 v. cells). The average voltage drop during operation will then amount to about 0.2 v. over two to three hours. (Given average battery conditions).

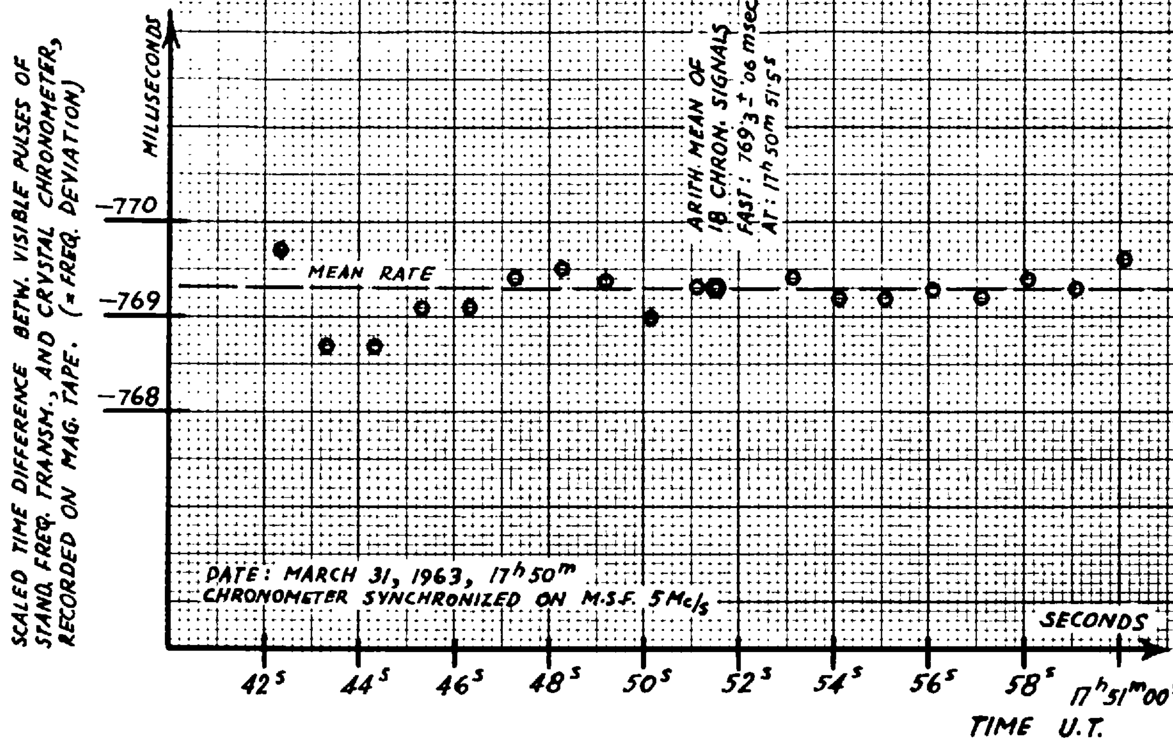
After reaching the limit of adequate voltage, a comparison for time difference with standard frequency transmission should be endeavoured, which enters in the formula as Y_o.

When plotting the pairs of observed quantities on logarithmic or semi-logarithmic paper, it was found that the points did not lie on a straight line. Hence it is unlikely that the observed data could be fitted by an exponential or logarithmic approximation. This is also apparent from inspection of columns 3 and 5, Table 3.5.7.-1.

Over a very short time interval the voltage drop will have a negligible effect on the chronometer performance.

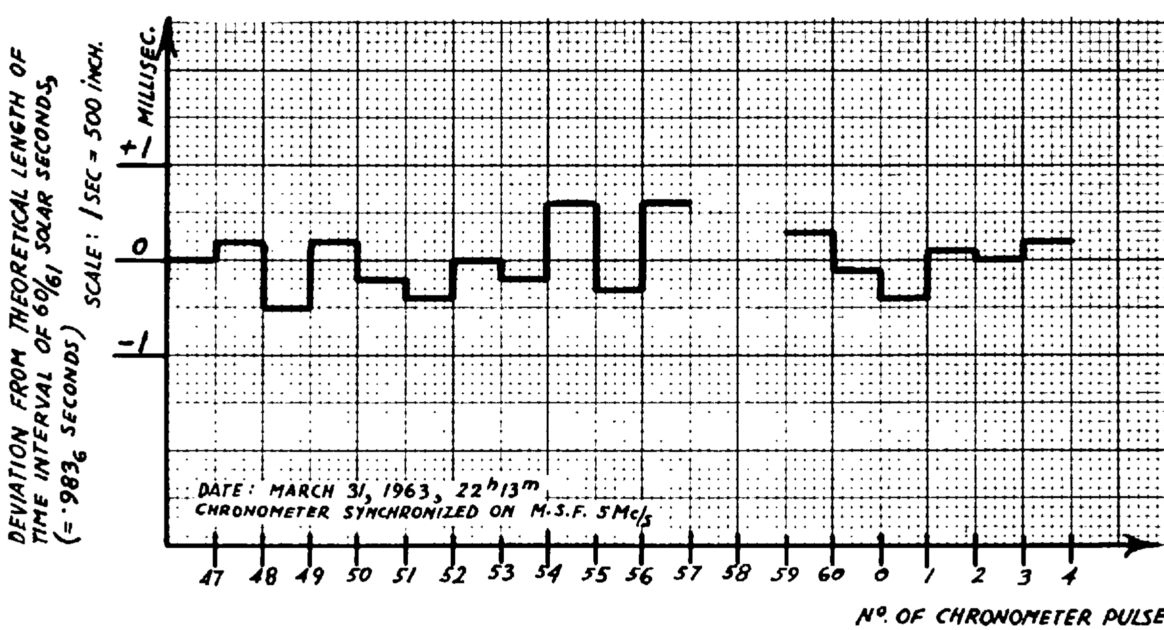
In Fig. 3.5.7.-4, it can be seen that the instantaneous stability (second-to-second) is within $\pm$ 1 msec.

The diagram in Fig. 3.5.7.-4 shows the length of time interval between individual chronometer pulses, namely: 60/61 solar seconds. The deviation from the standard length of .9386 sec is within $\pm$ 1 msec; the same deviation is encountered when the crystal chronometer is



INSTANTANEOUS STABILITY DURING 60/61 INTERVAL OF MEAN SOLAR TIME

CRYSTAL CHRONOMETER PULSES ARE PLOTTED AT U.T., SCALED FROM STAND. FREQ. PULSES.



LENGTH OF TIME INTERVAL BETWEEN TWO CHRONOMETER PULSES.

PERFORMANCE OF CRYSTAL CHRONOMETER

THERMOSTATICALLY CONTROLLED

OPERATING ON BATTERY: 10·8 TO 11·7 VOLT.

Fig. 3.5.7.-4

3.5.7. cont.

driven at specified voltage.

The power consumption of the chronometer when operated with the heating circuit switched off is so small that the voltage drop amounts to about 0.2 v. during a period of approximately 10 hours. In such circumstances, it is doubtful whether a practical case will arise where an empirical equation linking the relation between chronometer performance and voltage drop will be required. Therefore performance tests to collect observational data concerning the above specifications (heating circuit switched off) were not considered.

3.5.8. Performance of the Voltage Stabilized Crystal Chronometer

In June 1963 a voltage stabilizer was fitted to the oscillator unit of the chronometer. The purpose of this modification was to reduce the frequency/voltage variation to approximately $\pm$ 0.04 ppm/volt.

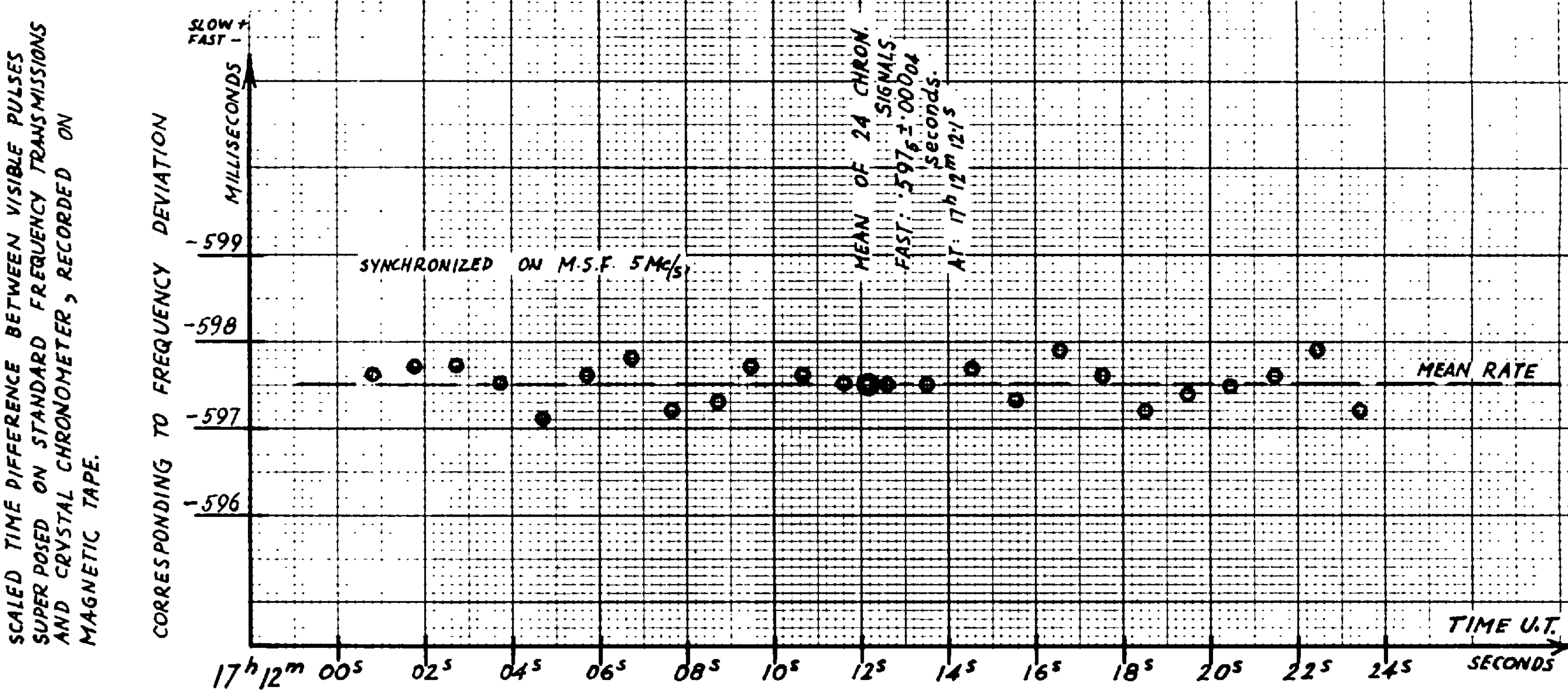
The performance of the chronometer with voltage stabilization, during the 60/61 second interval of mean solar time, is shown in Fig. 3.5.8.-1. Obviously no difference in the chronometer's behaviour can be noticed during the short interval of time shown (about twenty seconds) The isochronism curve (in the diagram not drawn up) is shaped mainly by calibration errors. The lengths of the time intervals marked by two consecutive chronometer pulses are affected by the isochronism error. The high degree of precision can be gathered from the fluctuations of the time lengths within $\pm$ 1 msec from their theoretical value of .9836 sec. The variations about the mean are random. This proves that it is unlikely that there are systematic errors present.

The hourly stability is given in Fig. 3.5.8.-2. The graph reveals that there is still a "running-in" time of a definite length, which indicates a negligible small change of rate.

It follows that the chronometer can be rated after 30 minutes. The hourly stability has been tested over periods of 12 hrs. running, and has proved to be within $\pm$ 1 msec, provided adequate voltage of 11 to 14 v. is maintained.

The replacement of a fully charged battery causes an abrupt change in the voltage, between 1 to 3 v. During normal field operation the voltage change is in the neighbourhood of 2 v. This produces a time error of less than $\pm$ 1 msec.

The recording and developing methods can detect time errors of this order, but the average reception of the standard frequency transmissions is not of the required precision, and therefore cannot be effectively employed when dealing with time errors of less than 1 msec. The



DATE: SAT. FEB. 8TH, 1964.

U.T. INDICATED BY SECONDS PULSES SUPERPOSED ON STANDARD FREQUENCY TRANSMISSION : M.S.F., 5 Mc/s, AT THE INSTANT OF RECORDING ON MAGNETIC TAPE.
CRYSTAL CHRONOMETER PULSES ARE PLOTTED AT U.T., SCALED FROM STANDARD FREQUENCY PULSES.

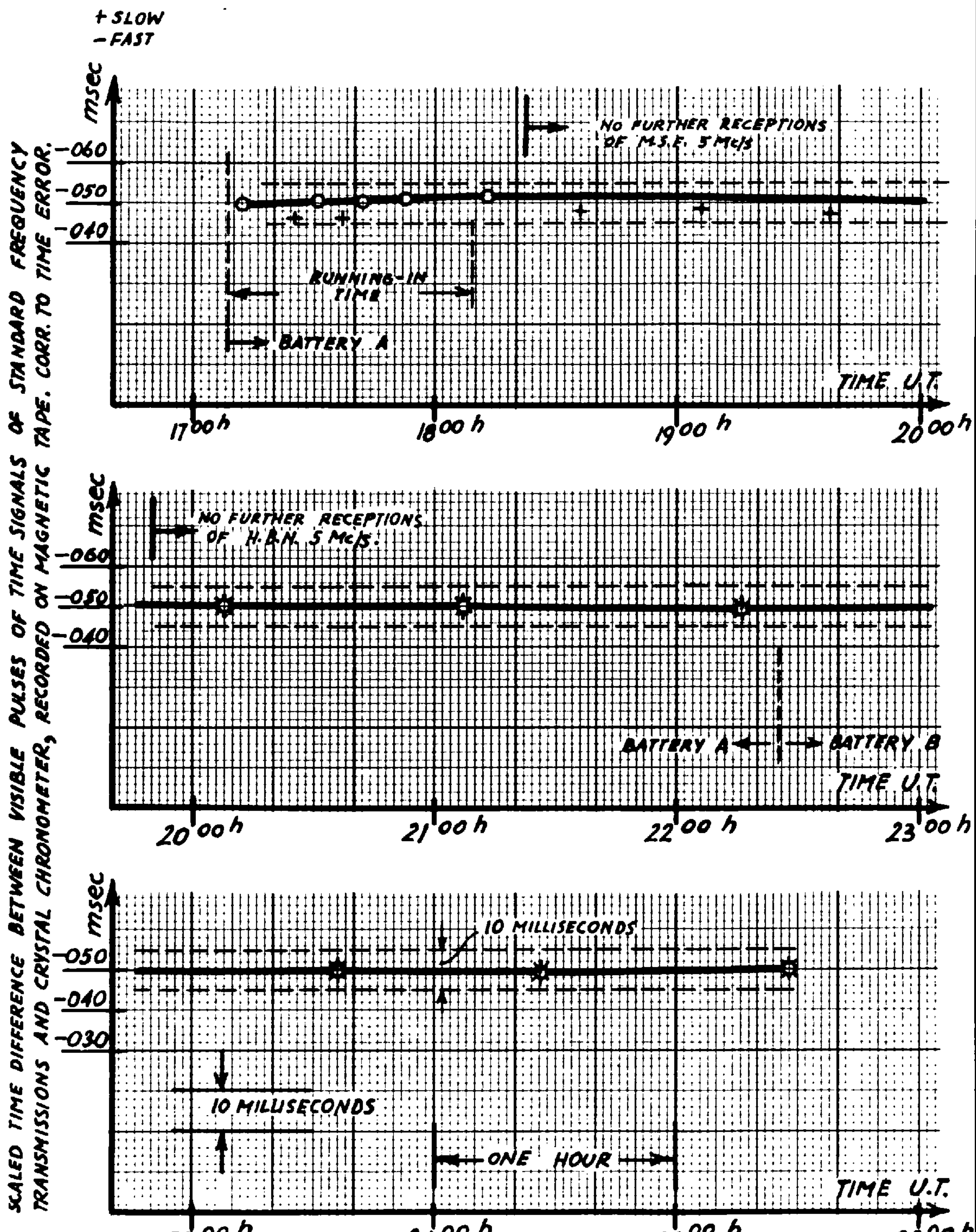
PERFORMANCE OF CRYSTAL CHRONOMETER

THERMOSTATICALLY CONTROLLED, VOLTAGE STABILIZED, OPERATING ON 12 VOLT BATTERY

INSTANTANEOUS STABILITY

DURING THE 60/61 SECOND INTERVAL OF MEAN SOLAR TIME.

Fig. 3.5.8.-1



FEB. 8, 1964.

PERFORMANCE OF CRYSTAL CHRONOMETER

THERMOSTATICALLY CONTROLLED

VOLTAGE STABILIZED

OPERATED ON 12 VOLT BATTERY, (ONE SET)

HOURLY STABILITY

CHRONOMETER SYNCHRONIZED ON:

○ M.S.F. 5 Mc/s, ☼ M.S.F. 2.5 Mc/s, + H.B.N. 5 Mc/s.

Fig. 3.5.8.-2

3.5.8. cont.

measurement of time intervals between any number of chronometer pulses is nothing more than an ordinary operation of accurate measurement of a physical quantity. To achieve this, a precisely defined standard which represents the unit must be available. In the present case, the practical unit is the time distance between superposed seconds pulses of frequency transmissions. The precision of the determination of the chronometer performance is therefore limited by the degree of precision of the reception of frequency transmissions.

The difficulties in synchronizing the chronometer and in ascertaining the time error could be overcome by employing a primary frequency standard, available in easy reach, for direct comparison; this is outside the scope of the research.

The specimen given in Fig. 3.5.8.-2 contains the performance of the crystal chronometer when synchronized on standard frequency transmissions which were available at that time. Corrections for differences in travel time are applied. As can be seen, there is undoubtedly a difference in emission time, in spite of the co-ordination of the time signal transmissions.

From results of performance tests, it can be deduced that one time comparison of the voltage stabilized chronometer with standard frequency transmissions is sufficient during any one period of operation. Great care must be taken to identify the transmitting station correctly, if the high precision of the crystal chronometer is to be effectively utilized. The small discrepancies in emission times of co-ordinated time signals can be detected with the equipment and methods employed. The stability of the voltage stabilized chronometer is emphasized in the diagram by the two parallel horizontal lines which accompany the isochronism curve, and subtend a distance of 10 msec, thus embracing a total time error from -45 to -55 msec from the initial setting of the chronometer. It can be seen that the absolute time error remains

3.5.8. cont.

unchanged over the total period of running, and fluctuations are within ± 1 msec.

The calculation of curve fitting to observational results is superfluous, because the isochronism curve is practically a straight line. The chronometer is adjusted to have zero rate. The labour involved for interpolation and extrapolation for time is therefore reduced to the addition or subtraction of the time difference, measured by comparing the chronometer with frequency receptions.

So far, no permanent change of rate has been found which might be expected after a period of suspended operation.

The quoted accuracy of time indication of the crystal chronometer conforms with the requirements for 1st order astronomical work.

Further improvements of the chronometer are not considered.

3.5.9. Effect of Voltage Fluctuation on the Performance of the Crystal Chronometer.

The frequency stability, before modification, was approximately ± 0.7 ppm/volt, within the limits of 11 to 14 v. and approximately ± 0.5 pmm/$^{\circ}$C.

The voltage fluctuation caused by the functioning of the thermostat produces a fluctuation of the frequency or time-error of about ± 10 microseconds; this is assuming a linear frequency/voltage variation over the range of one volt, and applying an average voltage drop of 0.25 v., measured, when the temperature is boosted under average field conditions at 60°F.

Furthermore, the temperature variations within $\pm \frac{1}{2}\,^{\circ}C$, which is the operating differential of the thermostat, will influence the frequency stability; assuming a linear frequency/temperature variation, this will cause a time error of about $\pm$ 5 to 10 microseconds.

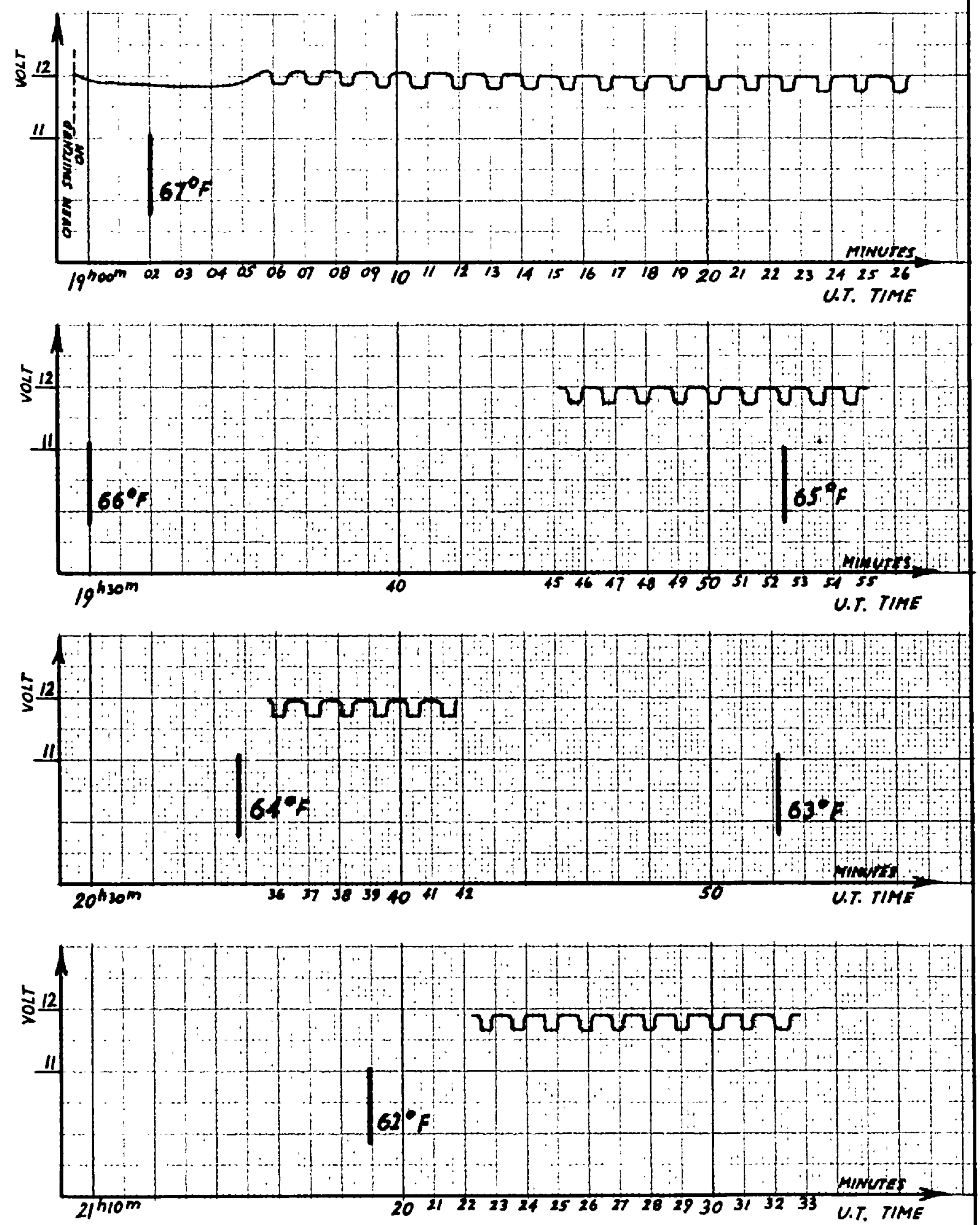
Since the voltage stabilizer reduces the frequency/voltage variations to approximately $\pm$ 0.04 ppm/volt, the time-error will only be about $\pm \frac{1}{2}$ microsecond.

The voltage variations with respect to time and out-door temperature resulting from the functioning of the thermostat are given diagrammatically in Fig. 3.5.9.-1. The length of time the oven can be off depends on the temperature of the environment.

The fluctuations indicated on the voltmeter in response to the operation of the thermostat are nearly instantaneous. The operating delay can be considered negligible compared with the period. The time required by the oven to reach its control temperature, at a given field temperature, can be read off the diagram. This does not mean that the crystal has acquired working temperature over the same length of time. The ratio of "seconds off"/"seconds on" increases according to a parabolic law from the start of the fluctuations over a certain length of time, which depends on the temperature of the environment. This parabolic increase is present whether the environmental temperature is rising or falling, and it contributes to the systematic parabolic chronometer performance during the running-in time. After the running-in time the ratio "heat off"/"heat on" is related to field temperature.

The current drawn when driving the minute clock also produces an instantaneous voltage drop, which is approximately 0.1 volt; its effect on chronometer performance is undetectable.

The quoted time errors, which do not exist when the heating circuit is switched off, are too small to be detected with the recording and evaluating equipment employed. Further, the precision of the standard



FLUCTUATIONS OF SUPPLY VOLTAGE CAUSED BY FUNCTIONING OF THE THERMOSTAT WITH DECREASING FIELD TEMPERATURE.

Fig. 3.5.9-1

3.5.9. cont.

frequency reception is also inadequate for making use of the international time service for this purpose.

The instantaneous stability as well as the long and short term performance of the chronometer are not affected by these small frequency fluctuations, which are within the accuracy required for field astronomy.

It is important to record the field temperature and the small cyclic changes of the voltage, because they can be used to check the proper working of the oven and the correct functioning of the thermostat, which are essential for the stability of the crystal chronometer.

In the absence of pilot lamps or alarm circuit, oven failure may pass unnoticed, until suspected from erratic survey data. Faulty conditions of the thermostat may cause the heat supply to remain on.

If the heater is not functioning properly, frequency deviation will depart greatly from the average. A breakdown of the thermostatically controlled oven will result in uncontrollable chronometer performance. Theoretically the thermostat will be permanently off when the field temperature reaches the peak for which the oven can be controlled; when the field temperature coincides with the lowest working temperature, the thermostat will be permanently on. When the controlled heat is permanently off, the temperature of the oven will increase at the same rate as the field temperature, and alternatively will decrease with the field temperature when the controlled heat is permanently on.

The working range of the thermostat limits the employment of the crystal chronometer.

In all field temperatures within the heating range there will be a cyclic temperature variation as a result of the operation of the thermostat. Due to the operating delay of the controlling device, the temperature when the oven heat is reduced is slightly higher,

3.5.9. cont.

and lower when the oven heat in increased.

It has been found that there is a near linear behaviour of the heating cycle: seconds "on" and seconds "off" with gradually decreasing or increasing field temperature, during any one period of chronometer running. (Fig. 3.5.9.-2(a).) The corresponding values of "on" and "off" time will produce a straight line when plotted on co-ordinate paper. The distance of this line from the origin will be influenced by battery conditions. The "off" time depends on type and material of the oven, and on the field temperature; since the former is a constant, the "off" time is therefore directly related to the field temperature. The distances between the plotted points are a measure of temperature differences; in other words represent a temperature scale.

Scaled off in both directions, the 104° F-point will lie at the intersection of the line with the "off"-axis and the 32° F-point on the intersection with the "on"-axis.

In Fig. 3.5.9.-2(a) the line is shown approaching assymptotically the "on" and "off" axes.

This method provides:

(a) a continuous check on oven conditions.

(b) a continuous check on whether the thermostat is functioning properly or not,

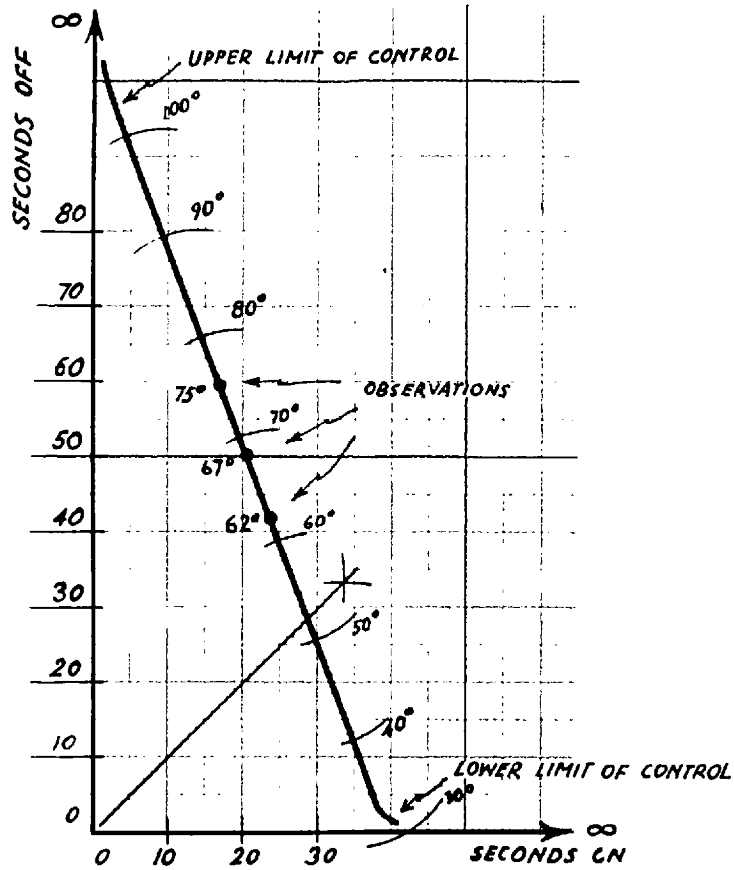
(c) a check on the oven information provided by the manufacturers, i.e. to ascertain the regulation of the thermostat or oven constants,

(d) a quick check if it has been omitted to switch the oven on.

The method can be used for any type of oven and thermostat.

From the diagram it can be seen that equal periods of "heat on" and "heat off" are at approximately 52°F field temperature, which is in agreement with test results. At this field temperature the temperature of the heater will vary according to a symmetrical triangular wave form, provided the temperature variations in the oven are linear, and the reaction of the thermostat instantaneous.

Fig. 3.5.9-2 (a)



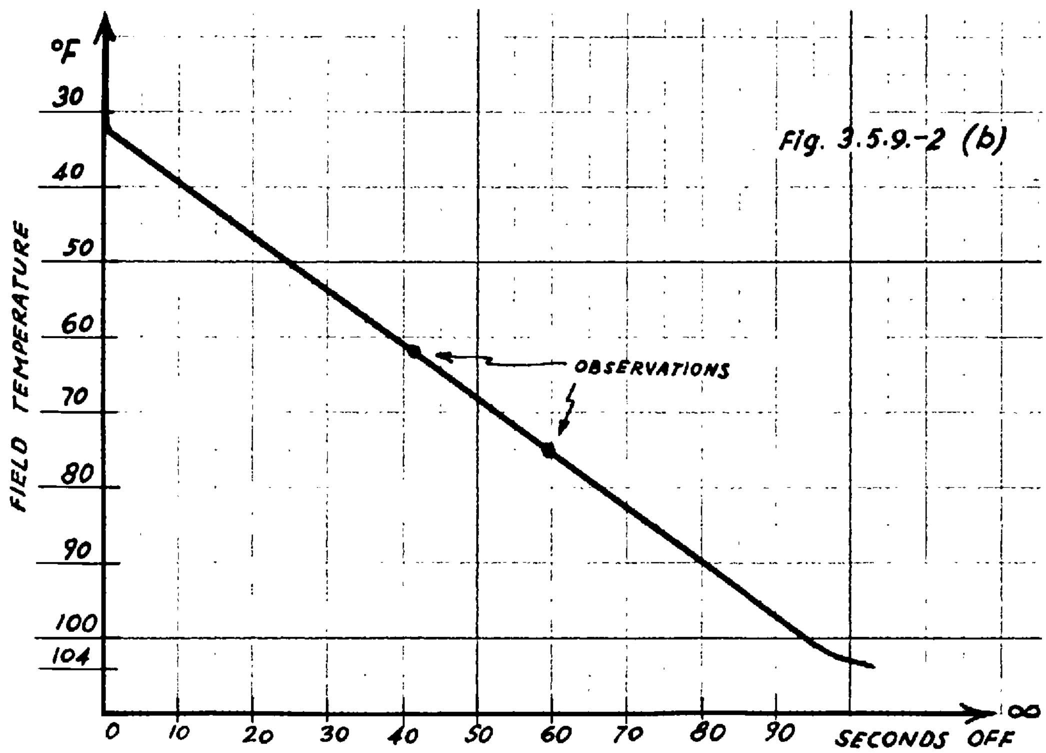


Fig. 3.5.9-2 (b)

CYCLIC OPERATION OF THE THERMOSTAT AFFECTED BY FIELD TEMPERATURE

3.5.9. cont.

The thermostat alone can be checked by the following method illustrated in Fig. 3.5.9.-2(b):

The pairs of observed values: length of "off"-time and field temperature are plotted on co-ordinate paper. The length of time "on" is not used. The resulting straight line (time - temperature line) will intercept on the temperature axis a length corresponding to the total control range of the oven. In the diagram the approach to the value of "time off" at control temperature 104° F is drawn assymptotically. Thus the graphical representation of the time - temperature line takes into consideration the time scale which is shown on the time axis.

Both diagrams could be used as "standard charts", or tables could be prepared from them for use in the field; but in the first case, as stated above, battery conditions will affect the distance of the time - temperature line from the origin, and in both cases adverse field conditions also will influence the slope.

It was observed that the presence of wind caused a differential thermal effect on the isothermal surface of the oven. The length of the "off" time is then changed, and the line, Fig. 3.5.9.-2(b), is rotated about its pivot, which remains fixed approximately at the point: 32° F field temperature, 0 = "off" time.

For practical application of both methods, a change in field temperature of a few degrees, as is mostly the case during astronomical observations at night, is quite adequate to obtain reliable information from voltmeter and thermometer readings for plotting the line. Sufficient readings of the voltmeter can be taken in four to five minutes.

The method, and its graphical evaluation, described above, of ascertaining the correct functioning of the thermostatically controlled crystal oven is believed to be new.

3.6. Employment of the Crystal Chronometer in the Field.

When the performance of the crystal chronometer is known, it provides, in a convenient form, an absolute standard of time.

The link to the practical standard of time, which is the rotation of the earth, can be established by comparing the time indicated by the crystal chronometer with the instants at which stars of known right ascension trans it the meridian, to which the time of the crystal chronometer is referred.

This operation, the determination of time for its use in field astronomy, can be dispensed with, since the crystal chronometer can be set on mean solar time, which should be available through the various standard time transmissions.

In some remote areas the reception is rather problematic, even over longer periods, amounting to several weeks. In such cases it is still advisable to have recourse to astronomical observations for the determination of time, to deduce the necessary link, instead of using any type of chronometer. A short discussion on mechanical chronometers follows in Section 3.8.

In the field the chronometer was carried up and down hill and subjected to a considerable amount of vibration and movement. No case of abrupt change of rate or of non-uniform behaviour has been encountered during transport, within the accurancy which can be determined by the methods employed, and which has been quoted in previous sections.

Tests have confirmed that even severe mechanical shocks produce no effect on the performance of the chronometer.

There is no difference between the chronometer's standing and travelling rate. The records of the performance show that the chronometer can operate in any position:

3.6. cont.

pendant up
pendant right
pendant left
dial up
dial down.

A rotation about any axis, clockwise or anticlockwise, does not alter the rate. The pulse clock mechanism is of the ratchet type. One advance movement of the minute hand drops out at a complete overturn of 360° about the x, or y-axis; but this is unlikely to happen in the field, since reasonable care has to be taken because of the battery. The drop-out of the hand movement does not affect the rate or the output of the chronometer.

The length the crystal chronometer can operate in the field depends mainly on the field temperature and on battery conditions. Given average battery conditions, the crystal chronometer can operate for 12 hrs. on one set of cells (= 12 volt battery) at field temperature of 50°, with the thermostat on.

3.7. Design of Chronometer case.

Figs. 3.7.-1 and 3.7.-2.

The crystal chronometer (1), as supplied by the manufacturers, is attached in supports to the frame (2) which forms part of the carrying case (3) of the batteries (4). The carrying case can hold two sets of six 2 v. cells, arranged in line, and can be transported by the handles (5). The two battery sets are wired in parallel (6) and are connected via a cable (7) to the charger (8) which is attached to the battery container. The charging equipment can be connected (9) to a permanent supply. The crystal chronometer can run whilst the batteries are being charged. In front of the battery case are the connections to power supply and tone outputs (10). The input cable (11) leads to the crystal chronometer. The control panel (12) carries the advance-retard key (13) and the switch (14) for the oven heating circuit. The marker button (15) is on the front of the chronometer cabinet and can be used instead of the remote marker key (16) which is wired (17) to the output points. The tone outputs can be heard from the loud speaker (18) which is plugged in a socket (19) on the front of the carrying case and can be switched off (20), if not wanted. The mercury thermometer (21) is shown near the aneroid barometer (22) which is removed from its case (23).

The overall dimensions comprising chronometer, charger and battery case are approximately: 1.9 feet high x 1.7 feet wide x 1.3 feet deep. The total weight of the carrying case with one battery charger and chronometer is about 62 lbs. In service it is necessary to use a rope (24) (shown unfastened) for transport, because the weight is not evenly distributed and the dimensions are rather awkward for the quipment to be handled by one person only. The chronometer has to be taken off the frame when replacing a battery. An occasional source of trouble is the somewhat loose fit of the

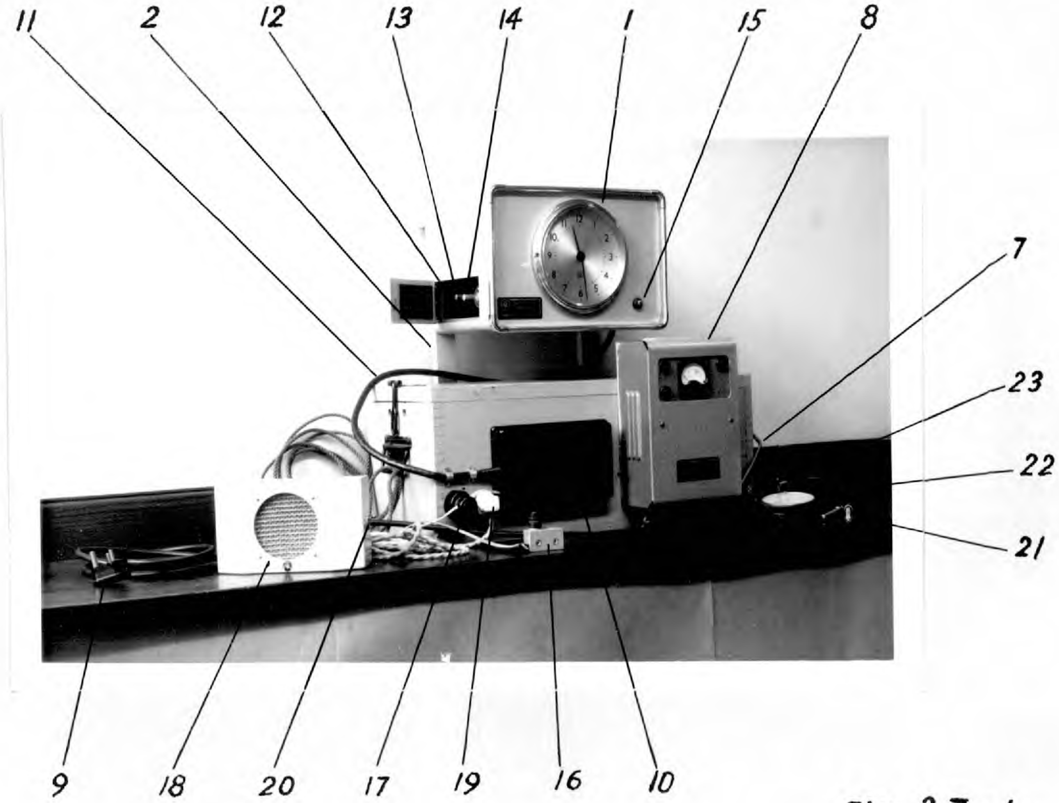


Fig. 3.7.-1.

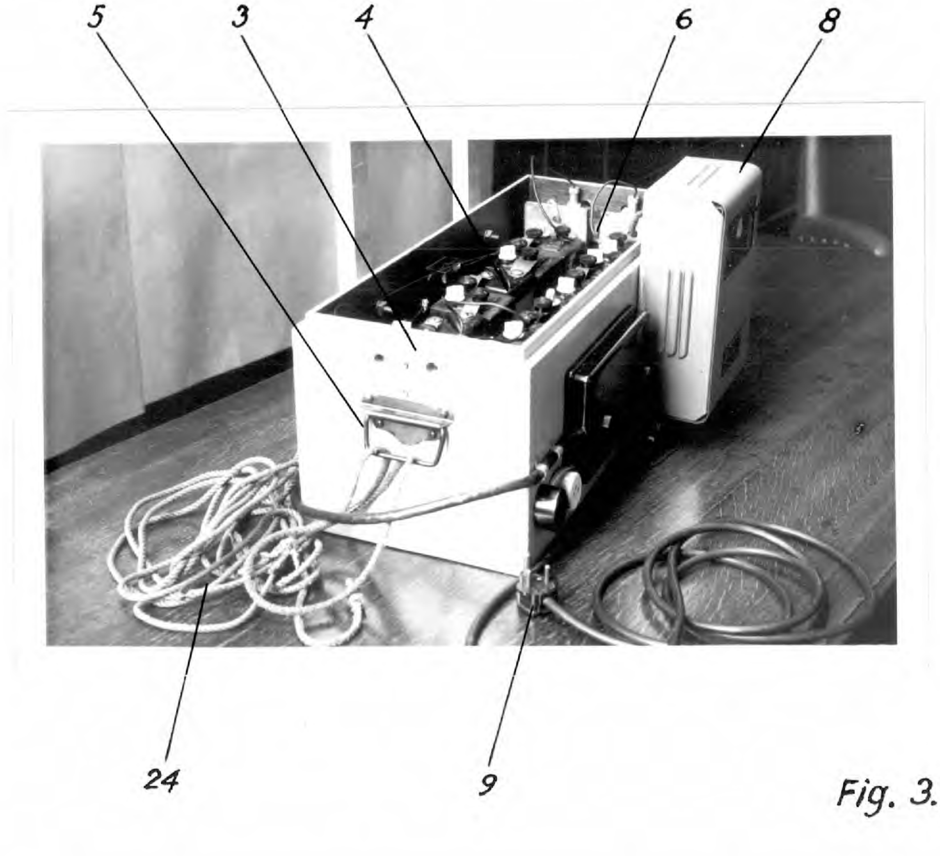


Fig. 3.7.-2.

3.7. cont.

batteries in the case; and the clips lacking a rigid connection with the terminals.

<u>The new design of the carrying case</u> is shown in Figs. 3.7.-3, 3.7.-4 and 3.7.-5.

Six 2v. cells are arranged to form a block (1). With the holder (2) the battery fits into the bottom part of the acid proof container (3) and is connected to the terminals (4). The terminals have marked plugs to insure the right-way-round connection of the battery. The front door (5) and the rear door (6) are shown open; the second battery with its holder (7) is standing in place, ready to be interchanged with battery 1. Arrangement is made for connecting battery (8) without interrupting the service with the aid of terminals (9) on the rear side of the container. Battery 1 is pulled out by the grips (10), and disconnected; battery (8), being already connected, can be moved in, and both doors when closed hold with stop bar (11), the battery fixed in position. The stop bars have rubber cushions for springy closure. This arrangement can easily be modified into a device to prevent relative movements of the cells, should the container be overturned completely in transport. The voltmeter (12) with a 4-way switch (13) indicates the voltage in the operating circuit, and the voltage of any battery before its connection (switch (14) and (15)) with the chronometer circuit. This is important because faulty connections amongst the six 2 v. cells can be detected before the supply circuit is disturbed.

The space on either side of the voltmeter is allocated to:

(a) the loud speaker (16) which can be taken out and connected to plug (17), or be kept running during transport,

(b) the remote key (18) to be connected to plug (19),

(c) barometer (20)

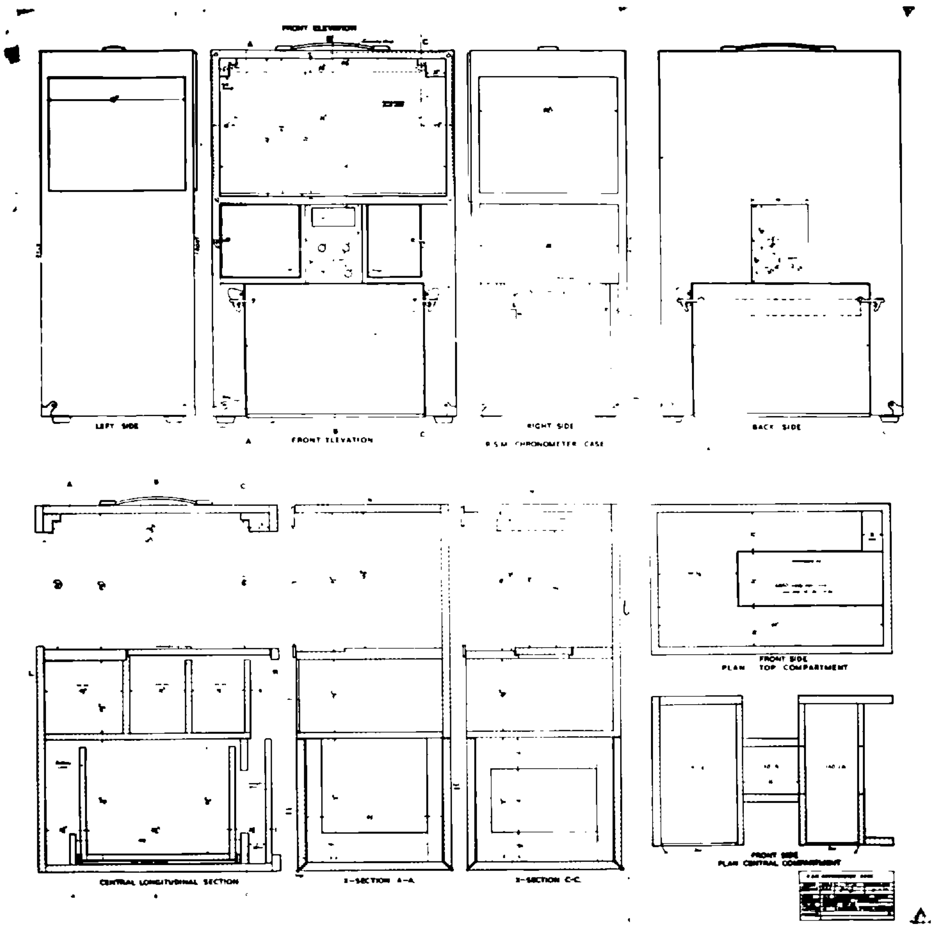

FRONT ELEVATION
LEFT SIDE
FRONT ELEVATION
RIGHT SIDE
BACK SIDE
CENTRAL LONGITUDINAL SECTION
X-SECTION A-A
X-SECTION C-C
FRONT SIDE
PLAN TOP COMPARTMENT
PLAN CENTRAL COMPARTMENT

Fig. 3.7.-3.

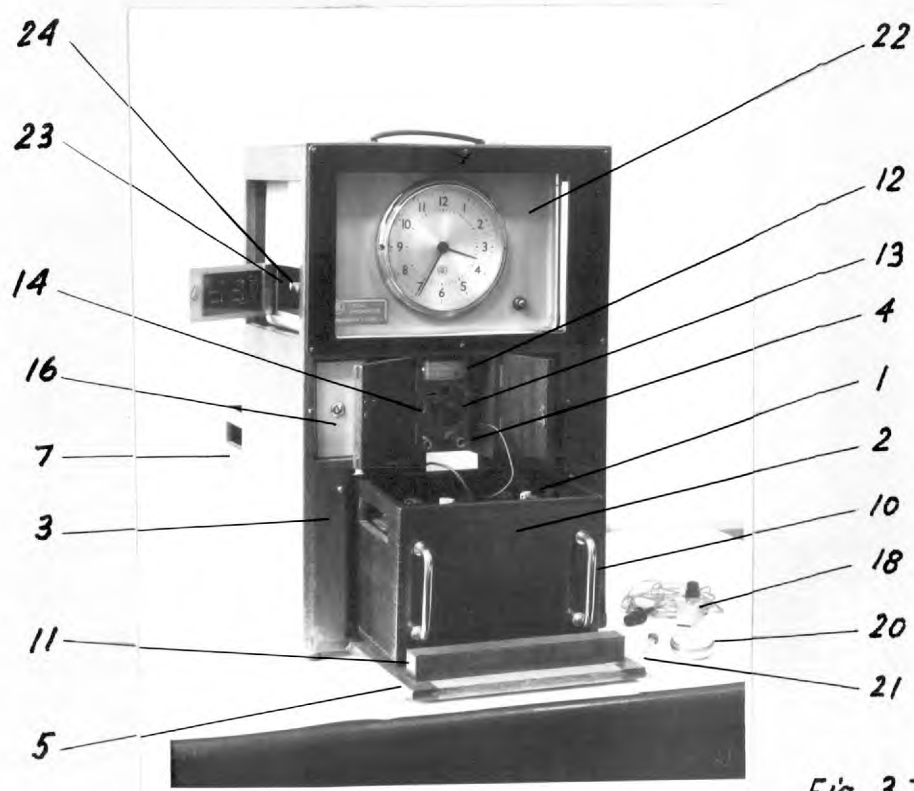


Fig. 3.7.-4

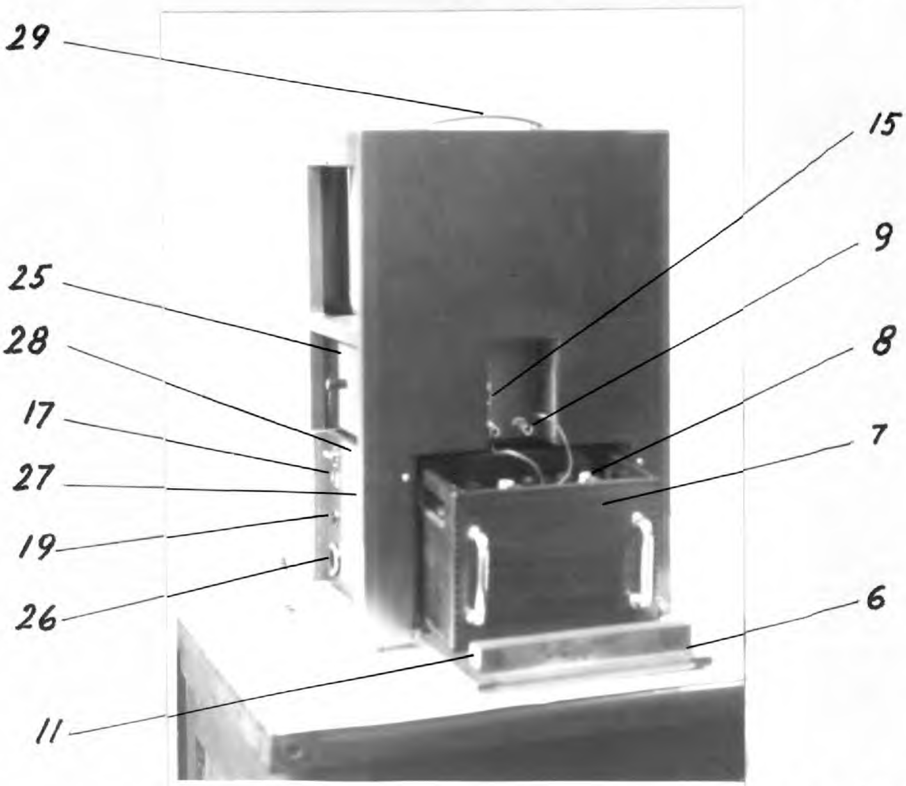


Fig. 3.7.-5

3.7. cont.

(d) thermometer (21) ;

(e) additional ancillary gadgets, as required.

The chronometer (22) is placed in the upper part of the case and is well protected against accidental damage.

Access to the oven heater switch (23) and to the advance-retard key (24) is provided on the left side of the carrying case. The output panel (25) is on the right side. The batteries can be charged whilst the chronometer is running, when connected, plug (26), via a charger to a permanent supply. The charger, which obviously has no use in the field, is not incorporated in this assembly. Provision is made for a third power input (27) with switch (28), should requirements call for it. Its voltage can be checked on the same voltmeter via the 4-way switch. Heat holes for the battery are substituted by the openings from the recess mounting of the connecting terminals, which at the same time gives them adequate protection.

The weight (including loudspeaker, barometer etc.) is approximately 54 lbs., and is evenly distributed in width and depth, with its greater portion in the lower part of the container to prevent overturning.

The new carrying case measures approximately 1.9 feet high x 1.2 feet wide x 0.8 feet deep. The measurements are suited for one-man transport. The leather grip (29) can be replaced by a carrying handle, or straps can be fitted for carrying the container over the shoulder.

3.8. Mechanical Chronometers.

In dealing with the following tests of mechanical chronometers, only the relevant items of chronometry are given which apply to accuracies which are nowadays aimed at in field astronomy.

Crystal chronometers have the advantage of being ready for use after the short time required to reach working temparature, and may be rated with high precision forthwith. Mechanical chronometers are, as a rule, not rated until 24 hours after winding.

In various textbooks, the timing of optical observations in field astronomy with mechanical chronometers is often quoted to two decimals of a second and sometimes to milliseconds.

The Department's mechanical chronometers were tested, to determine whether the quoted accuracy could be achieved with them or not.

The rate of a mechanical chronometer is supposed to be set to meet various conditions. Every effort is made by the maker to keep errors within limits of tolerance.

Winding at regular time intervals is essential to make full use of the adjustment or isochronism. This is equivalent to maintaining a constant operating voltage, when employing a crystal chronometer.

Normally, a mechanical chronometer is not thermostatically controlled and is adjusted for temperature changes so as to display the same performance at the maximum and minimum temperatures liable to be encountered.

The performance at intermediate temperatures can vary a considerable amount.

The surveying chronometers tested were not removed from their cases, and were left undisturbed as far as this was practical.

3.8. cont.

Dwerrihouse Chronometer No. $\frac{13097}{98}$

The Dwerrihouse chronometer is an eight-day chronometer, supported in gimbals to minimize the difference between the travelling and the standing rate; the former is determined in exceptional cases only; for example when the chronometer is required to run during transport and is liable to suffer shocks.

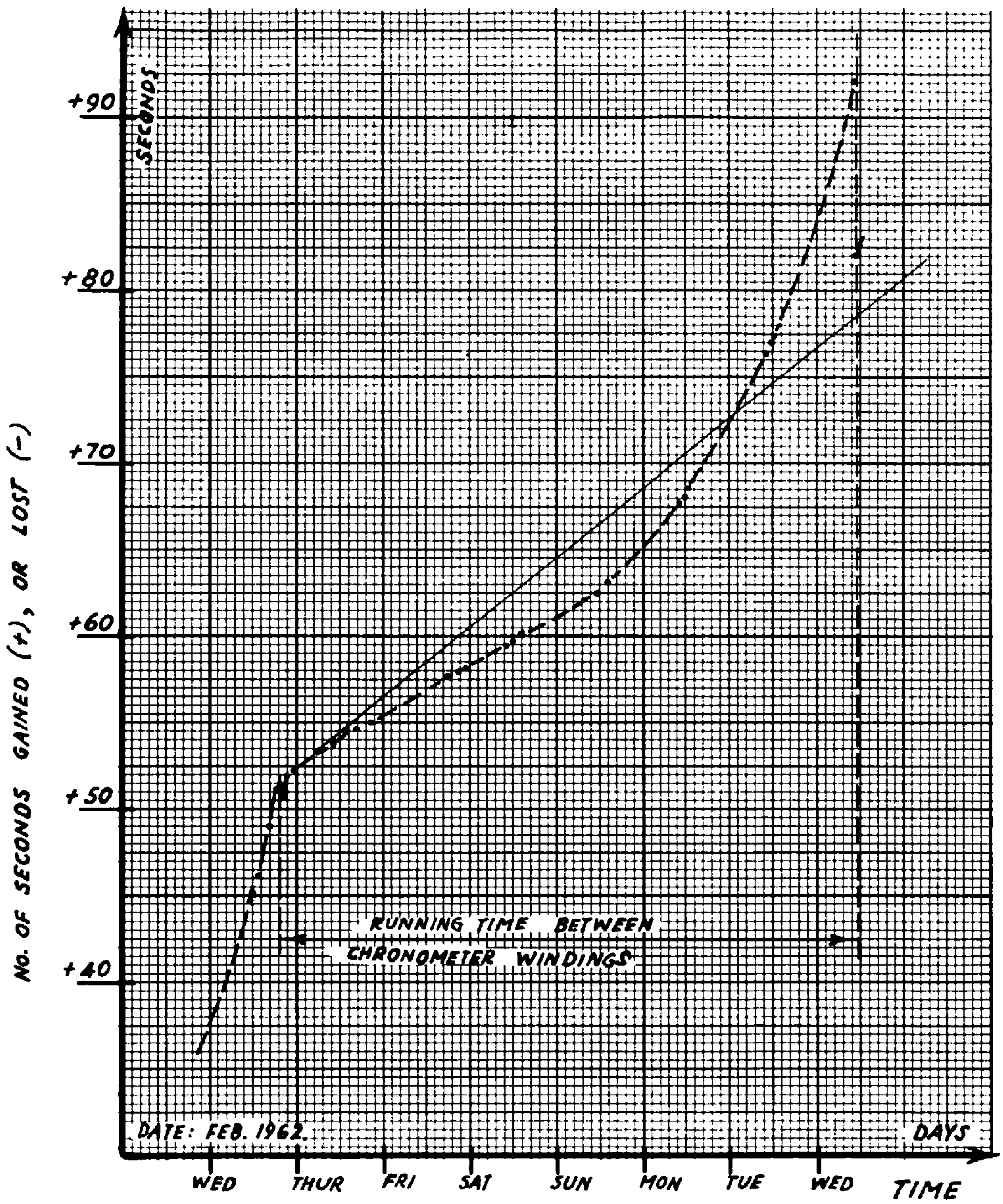
To conform with standard practice, the standing rate of the Dwerrihouse chronometer was tested in the horizontal position, i.e. "dial up" only.

The tests for isochronism are regarded as of primary importance; these include the determination of the mean rate, the consistency of the rate from day to day, and the overall drift of the rate. If the chronometer performance complies with the tolerance and if the time keeping property permits time extraction to the accuracy mentioned in textbooks, further tests have to be considered. These should be the determination of temperature compensation and tests to ascertain the influence of the state of winding on the performance.

The results of isochronism tests are shown in Fig. 3.8.-1, which is a specimen diagram of the observations taken during one period. Careful winding of the chronometer has to be attempted, otherwise the time indication by the minute-hand might be disturbed.

The various eight-day periods between windings produced consistent results; the difference between the mean rate in each period did not exceed $1\frac{1}{2}$ sec/day. Theoretically this would correspond to an hourly stability of about 0.06 sec.

During the first half of the running period, the mean daily rate, as can be seen, did not exceed the tolerance of 4.0 sec/day. The straight line drawn in Fig. 3.8.-1 shows an ideal rate of the prescribed limit of 4.0 sec/day, with no differences of rate between consecutive intervals; this would be the rate of a chronometer in perfect adjustment for isochronism. The sections on the ordinates



ISOCHRONISM CURVE OF DWERRIHOUSE CHRONOMETER $\frac{13097}{98}$

SYNCHRONIZED ON M.S.F. AND H.B.N. 5 Mc/s.

Fig. 3.8.-1

3.8. cont.

contained between curve and straight line represent the difference from the permissible isochronism error. It can be seen that the isochronism error of the chronometer is small for the first five days, after which time the error becomes larger. Hence, for the chronometer under test, a small adjustment, necessary for compliance with eight-day requirements, should be effected, to bring its isochronism error within the prescribed limits.

When the chronometer is used for field astronomy, its time keeping property over the time space required for optical observations is all that matters.

The average performance of the Dwerrihouse chronometer during the interval of four days, if maintained over the total eight-day period, would satisfy the appropriate requirements for a certificate to be issued by a recognized testing laboratory or competent governmental institution.

Four days represent an adequate time interval for astronomical observations to be taken.

Generally, the performance specifications of an eight-day chronometer are made up of requirements of a daily rate within certain limits, and of the stability of the rate determined from observations taken at regular intervals.

In each period the mean rate must not exceed 4.0 sec/day; in each period the difference between any two rates must not exceed 1.5 sec/day. There are further specifications about differences of rates in various periods limiting the variations of the rate to 2.0 sec/day.

A chronometer of assured quality for which a certificate has been issued may exhibit a steady daily rate over some longer intervals but at times within the 24 hours interval it may be in error by an amount proportionally in excess of the difference between two consecutive rates.

A satisfactory performance during intervals of hours and also a minute-to-minute stability of high quality are of great importance, in

3.8. cont.

the first case when time extraction is required to an accuracy of two decimals of a second, and secondly when facilities for comparison with some reliable standard of time are restricted to radio reception of transmitted time signals, known to be unobtainable at times.

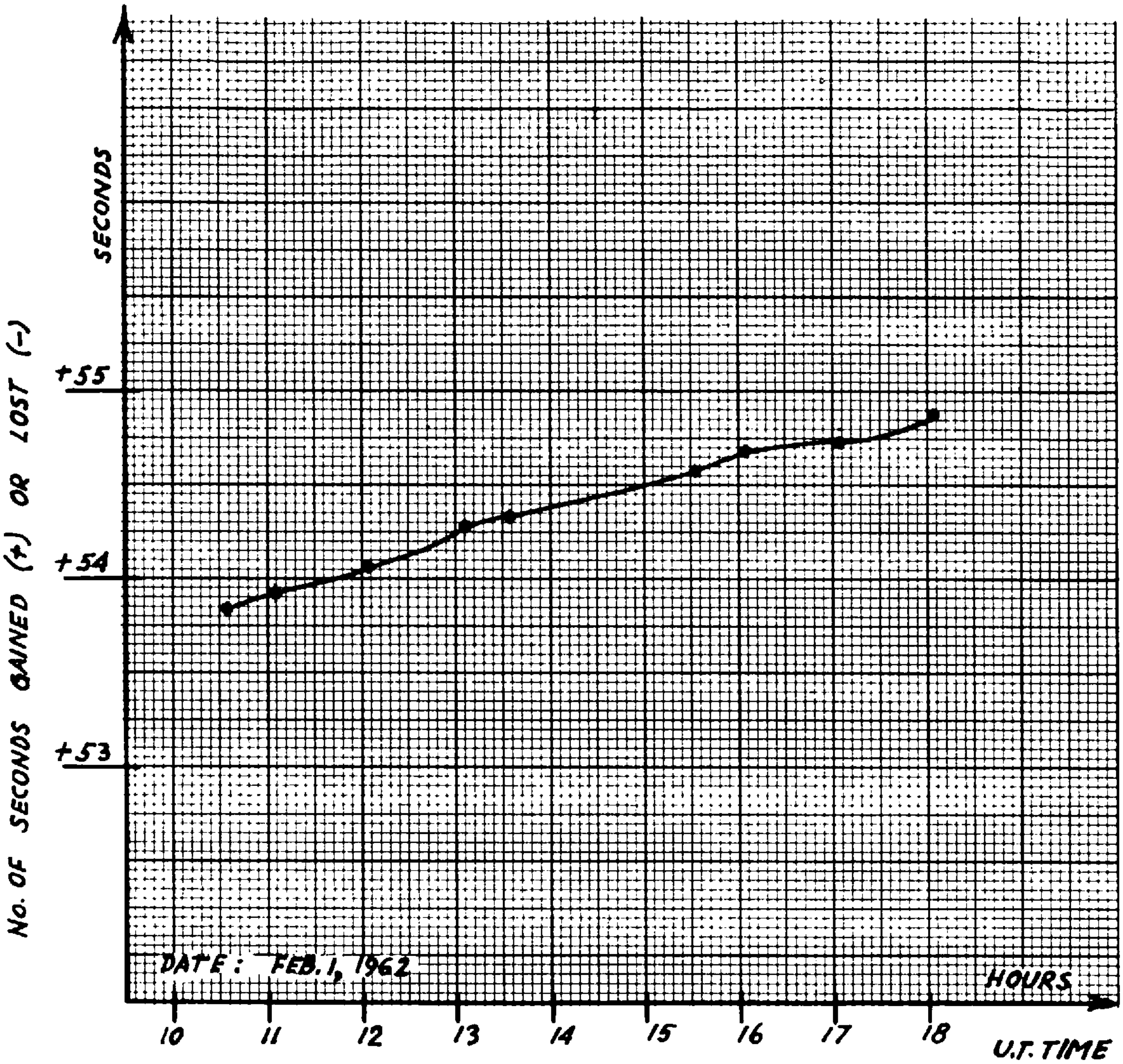
In Fig. 3.8.-2 is given the hourly stability of the Dwerrihouse chronometer determined from time comparisons with M. S. F. and H. B. N. transmissions, by the eye and ear method. The plotted points are means of about ten time comparisons at minute intervals, conveniently spaced to emphasize the hourly stability. Consecutive values are joined by a smooth curve with the assistance of intermediate observations which are not marked conspicuously. Increased accuracy in time comparison was achieved by the use of stopwatches. Two stopwatches were available: one Zenith stopwatch with five, and one split-seconds stopwatch with ten escapement beats per second. With them the time intervals between minutes, indicated by the seconds hand - half second beat - of the Dwerrihouse chronometer and signal reception, were measured by the eye and ear method.

First of all, both stopwatches were tested at normal room temperature in dial up position with the spring fully wound, to ascertain their general functioning, the action of the starting, stopping and recording mechanism.

These tests were regarded as essential rather than calibration tests, since only differences of measured time fractions were concerned. Furthermore, the stopwatches were not required to carry more than one minute which excluded testing the recorder dial. Care was taken in reading the watches to avoid a parallax which could have produced an error up to 0.3 and 0.4 seconds.

The isochronism curve in Fig. 3.8.-2 demonstrates clearly that the hourly rate varies a large amount, ranging from 0.05 to 0.20 seconds. The tendency to increase or decrease cannot be predicted; the chronometer's behaviour is seen to consist of erratic fluctuations.

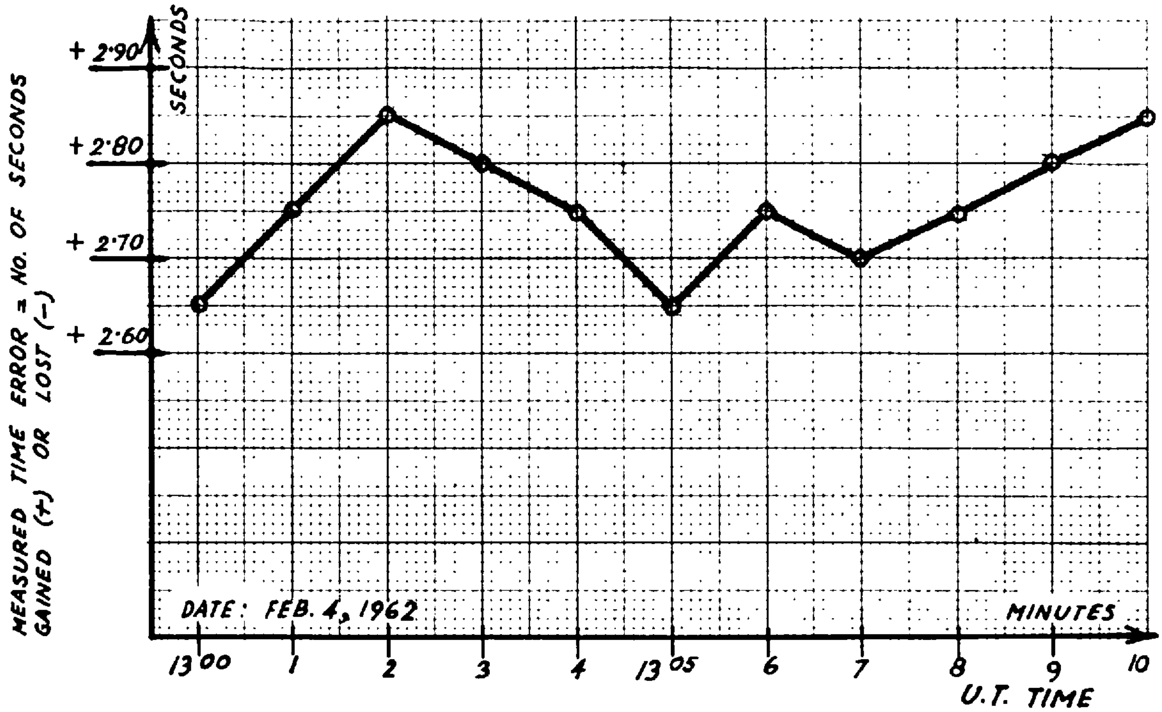
The minute-to-minute stability is shown in Fig. 3.8.-3. As in previous diagrams the isochronism error can be read off the ordinate



HOURLY STABILITY
OF
DWERRIHOUSE CHRONOMETER $^{13097}_{98}$

SYNCHRONIZED ON M.S.F. AND H.B.N. 5 Mc/s.

Fig. 3.8.-2.



MINUTE-TO-MINUTE STABILITY
OF
DWERRIHOUSE CHRONOMETER $\frac{13097}{98}$

SYNCHRONIZED ON M.S.F. AND H.B.N. 5 Mc/s.

Fig. 3.8.-3.

3.8. cont.

axis. The two illustrations taken from separate operating periods reveal a low order of stability of the chronometer during successive intervals of minutes.

Omega Chronometer No. 10 294 249 - 574

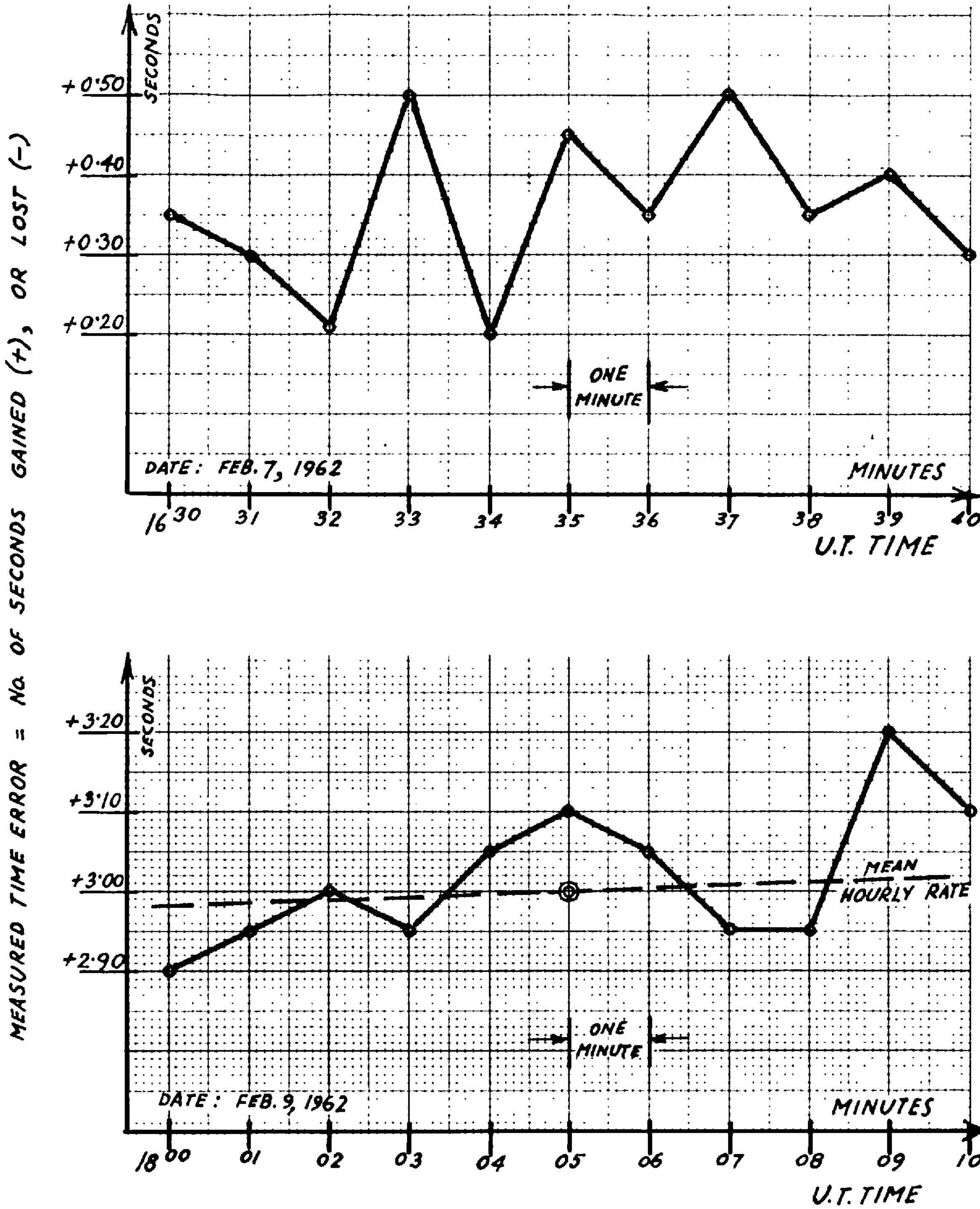
The Omega chronometer is a one-day chronometer, provided with a seconds hand which moves in five jumps per second. Before testing, the chronometer was kept running over a period of two weeks. It was wound daily, at regular intervals. After winding the chronometer can run over 36 hours which conforms with the requirements for a one-day chronometer.

The rate was determined over an interval of 24 hours; the difference of its mean value in any two consecutive periods was in the range of 0.5 sec; the difference of the time errors developed in two consecutive 12 hours intervals did not exceed 1.0 second.

The minute-to-minute stability shown in Fig. 3.8.-4 is of much the same quality as that displayed by the Dwerrihouse chronometer.

No effort was made to reduce the mean hourly rate, which is shown by a dotted line in Fig. 3.8.-4 (drawn through the mean value of the observations taken) to make it agree with the specification for a certificate because the consistency and the overall drift of the rate is well within the prescribed tolerance, for the chronometer to be accepted by a testing authority.

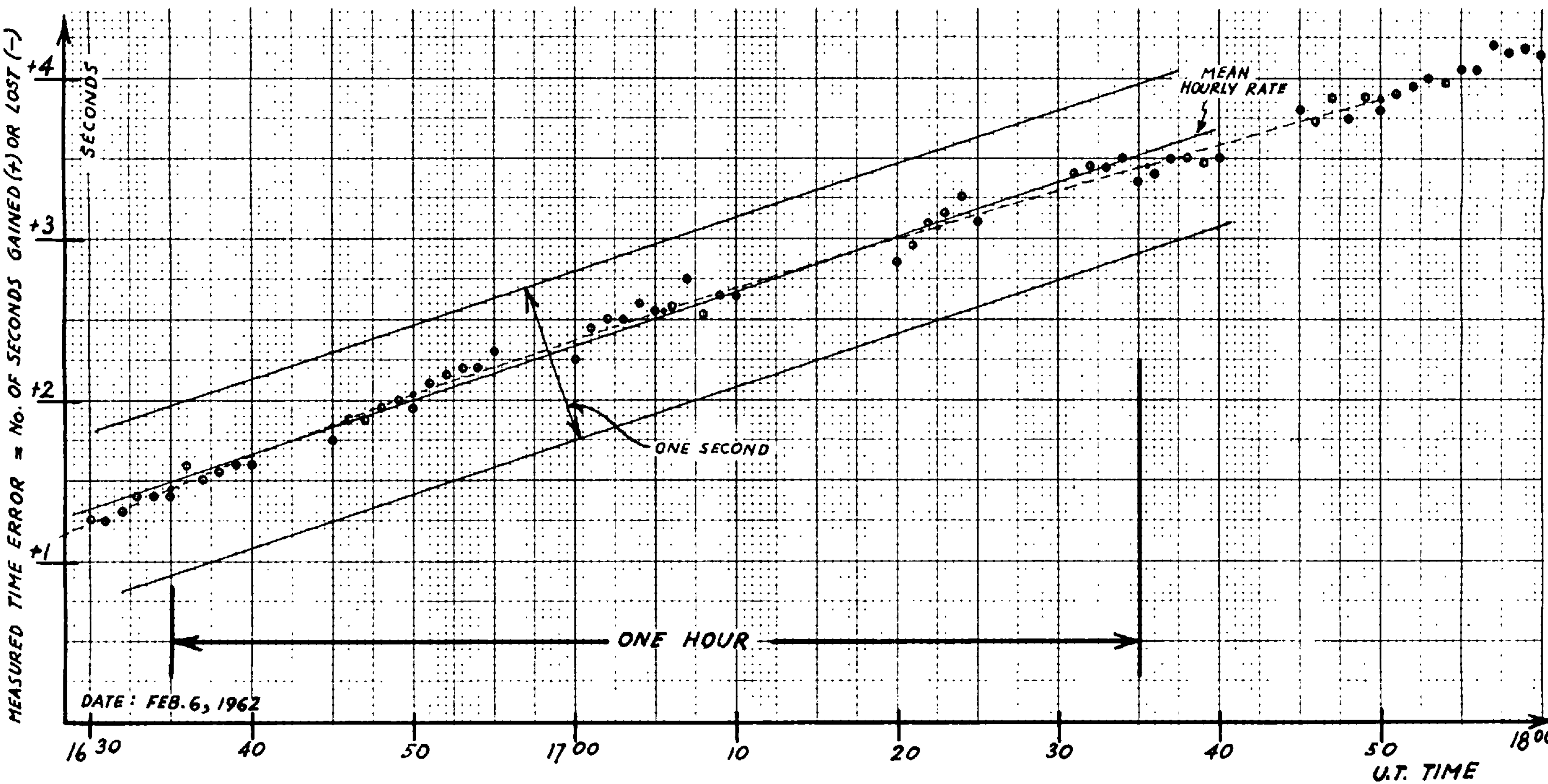
The chronometer's behaviour in the time space of one hour is presented in Fig. 3.8.-5. The points plotted are observations taken at minute intervals, depending on reception conditions and times of transmissions, the periods of which are evident from the diagram. The mean hourly rate shown by a straight line is derived from the average values of five groups of chronometer comparisons with standard frequency transmissions. Four of the five groups are made up of eleven observations. The straight line passing through the centre, defined by



MINUTE-TO-MINUTE STABILITY
OF
OMEGA CHRONOMETER 574

SYNCHRONIZED ON M.S.F., AND H.B.N. 5 Mc/s.

Fig. 3.8.-4.



PERFORMANCE OF OMEGA CHRONOMETER 574

Fig. 3.8.-5.

3.8. cont.

the arithmetic mean of the observations taken within one hour, slopes by an amount = $\frac{[xy]}{[yy]}$, where: x = time error of the group at mean time y, reduced to the average for the hour.

The graph shows that within one hour individual means of eleven consecutive observations at minute intervals can vary by 0.15 sec., or more. The group means are joined by a dotted line.

Although the stability of the rate over 24 hours conforms with the requirements of a certified chronometer, the uncertainty in the value of the hour and minute limits the accuracy of the measurement of a time interval of a length, as it is normally required in field astronomy, to at least $\pm$ 0.2 sec.

The Omega chronometer has an eccentricity of the seconds hand amounting to 0.1 second which was eliminated by making comparisons with the stopwatches at intervals of 30 seconds; this eliminated also a possible eccentricity of the seconds hands of the stopwatches. Since the ten seconds intervals are marked by heavy lines, comparison was made in pairs at the 5th and 35th, 15th and 45th, 25th and 55th seconds, for groups of 5 or 10 minutes. The observations were carried out with a magnifying glass.

The accuracy of subdividing time intervals with mechanical chronometers cannot be increased by the accumulation of time comparisons with standard frequency transmissions, because the determined time error of a specific minute instant, indicated by the mechanical chronometer, cannot be carried over to the next minute indication, due to the erratic behaviour of the chronometer.

The high precision measurement of a chronometer's time error is only effective if it can be used for bridging periods when comparisons are not available.

The low order of stability in intervals of hours and minutes is insufficient to effect a time measurement with mechanical chronometers

3.8. cont.

to two decimals of a second.

As proved with the foregoing tests, the subdivision of time intervals with the aid of mechanical chronometers in conjunction with a stopwatch may be obtained to an accuracy of at most $\pm$ 0.2 seconds. Hence the timing of observations in the field can be accurate only to this order, should time comparisons be available as near as possible to the instant of observation, i.e. one hour or less.

It follows that the necessary accuracy demanded from timing equipment to cope with the precision of modern lightweight theodolites cannot be expected from mechanical chronometers.

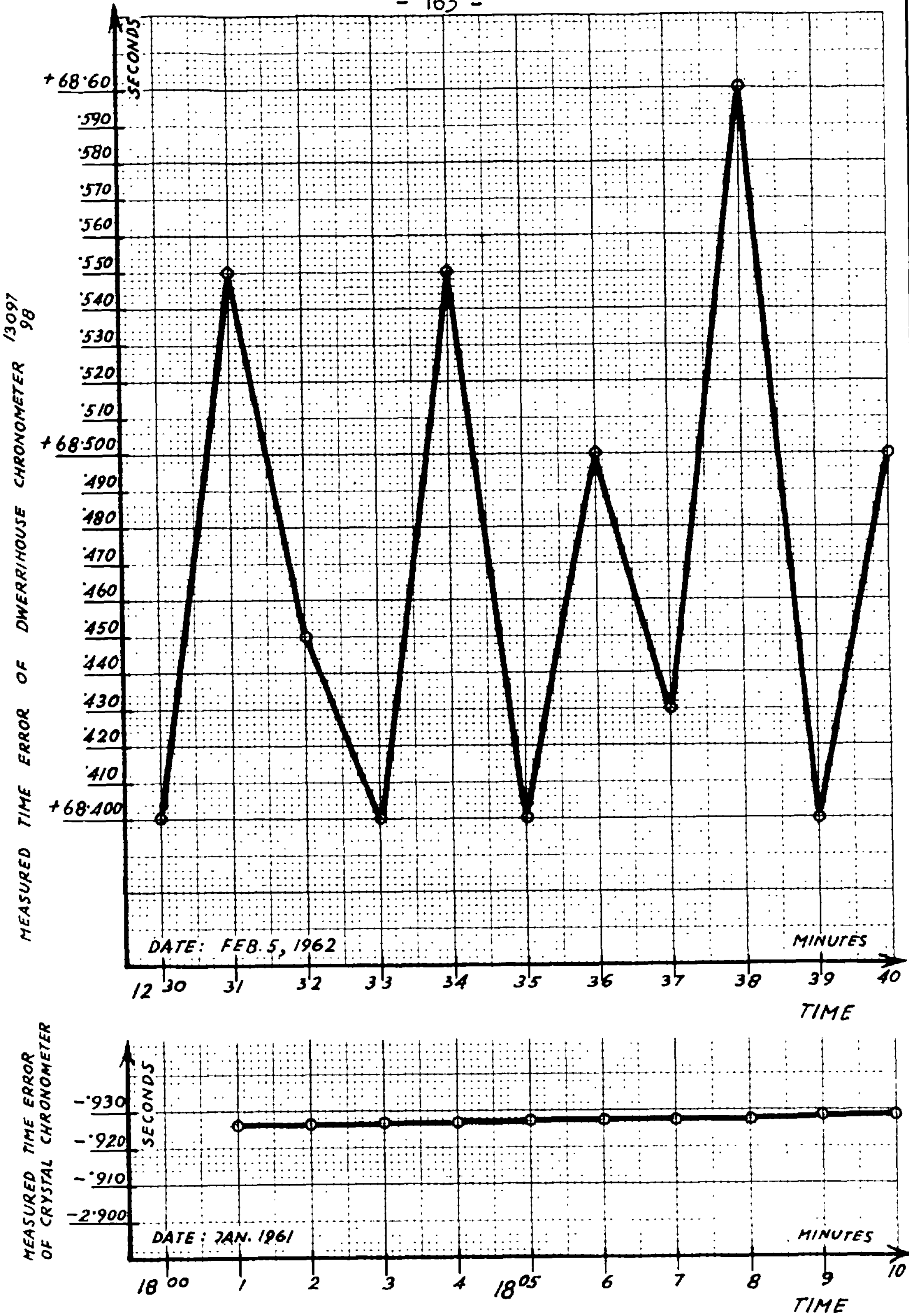
The tests also show how deceiving a result can be concerning chronometer performance over longer periods.

The behaviour of mechanical chronometers during intervals of single seconds cannot be tested by the eye and ear method in conjunction with a stopwatch, or time signals.

The chronometer clicks can be recorded on tape and the distances between developed magnetic impulses can be scaled; the information about the second-to-second behaviour so gained would produce a uselesss accuracy since the minute and hourly stability is of too low an order already.

The quality of the performance of a mechanical chronometer, the Dwerrihouse chronometer, and of the crystal chronometer can be compared from Fig. 3.8.-6. The diagram in the upper part shows the minute-to-minute stability of the Dwerrihouse chronometer while in the lower part is produced the minute-to-minute stability of the crystal chronometer, previously given in Fig. 3.5.4.-5. For plotting purposes the ordinate scale had to be divided by 20. The adopted small scale permits the representation of both isochronism polygons in the space provided.

The graphs enable recognition of the superior quality of the time keeping property of the crystal chronometer which approaches the



PERFORMANCE OF MECHANICAL CHRONOMETER
AND CRYSTAL CHRONOMETER
MINUTE-TO-MINUTE STABILITY

Fig. 3.8.-6.

3.8. cont.

hundred fold accuracy of a mechanical chronometer.

The survey returns dealing with timing of field observations with mechanical chronometers, frequently published in current literature, have to be regarded as being of limited reliability, should they contain figures quoted to an accuracy which is unlikely to be achieved with the instruments employed.

4. Time.

4.1 General.

The attempt to formulate a definition for time evokes the same embarrassment which inevitably arises when other fundamental physical concepts have to be explained, not in terms of experience or to justify them, but as a convenience and as a matter of tradition.

Every concept is supposed to meet particular requirements, namely to be valid, suitable and correct.

The common way out is to describe some properties of the fundamental physical concepts in terms of their mathematical relationships.

The prevailing idea during the past centuries was based on Galilei-Newton's concept of geometrical time. The greatest contribution in this line of thought comes from general relativity which criticizes the classical theory, and by introducing a frame of reference, presents different time perspectives for observers with different velocities, where the observations are irreversible processes.

Consequent on this interrelation of space and time, the "atomic time" has now replaced the geometrical time. Atomic is derived from Greek ατομος, meaning not divisible. The concept of atomic time requires the possibility of its resolving into smallest particles. The existence of the lowest limit of time, comparable with the absolute minimal parts of matter, i. e. the atomic unit of time, the chronon, is now a universally accepted idea. On this basis, time is accepted as being finitely divisible.

This does not prevent the free application of mathematical treatment, e. g. the common law of dividing magnitudes, of time differences as long as these are in a tolerable relation to the chronon, which means that the range of the applicability of numerical calculation is clearly defined and limited. In the opposite scale, this is comparable to the

4.1. cont.

velocity, namely the velocity of points and rigid bodies in relation to the velocity of light.

The origin of time - analogous with the origin of any co-ordinate system in geometry - is of no interest when the measurement of time intervals is concerned, as only time differences matter. The measurement of time differences between instants corresponds to the measurement of distances between points for which, in either case, a standard measure of the unit is required. For convenience and for scientific purposes, various standard measures of the unit of time intervals are in common use, to which measurements of physical magnitudes can be referred.

4.2. Time Systems

Any occurrence which repeats itself, for known or unknown reasons, can be used for the measurement of time intervals, provided the repeating performance possesses adequate exactitude and remains undisturbed by outside influence.

From the continuous successions of day and night originates the day time, which is the measurement of the position of a point on the surface of the earth in relation to the sun. From this relation in space, the day time reveals itself as solar time, and is therefore based on the rotation of the earth on its axis.

The mean solar time is derived directly from astronomical observations to stellar bodies, which give sidereal time via the true or apparent local sidereal time.

The apparent local sidereal time is referred to the instantaneous local meridian and is obtained from the immediately observed positions of stars in their diurnal circuit.

4.2. cont.

Apparent sidereal time = hour angle of the true equinox.

True equinox = intersection of true equator and ecliptic.

The mean sidereal time is derived from the apparent local sidereal time by applying a correction for nutation in right ascension.

It is:

Apparent sidereal time - mean sidereal time = equation of the equinoxes, due to the nutation.

Mean sidereal time = diurnal motion of mean equinox.

The mean solar time is then derived from the mean sidereal time from the relation:

Mean solar time = mean sidereal time - right ascension of (fictitious) mean sun - 12 hours.

The universal time (U. T.) is mean solar time for the meridian of Greenwich and is an empirical measure, internationally adopted for practical purposes. At present there are three kinds of U. T. in use: U. T. O., U. T. 1, and U. T. 2.

U. T. is defined as the Greenwich hour angle of the mean sun + 12 hours.

The unit of mean solar time or universal time (U. T.) is the mean solar day, or the second of mean solar time.

One mean solar second = 15''.041067 (sec. arc).

The limiting factor of the constancy of the length of the mean solar day is the precision of the rotation of the earth on its axis.

The U. T. is affected over long and short term periods by:

annual fluctuations

polar variations

progressive retardation

irregular changes

of the rotation of the earth on its axis.

The U. T. is thus a non-uniform mean solar time.

A measurement of time intervals can be based on the physical laws

4.2. cont.

of motions of planetary bodies. The laws of motion, expressed as differential equations, contain the time as independent argument. This time system is an inertial time system; and the time is uniform. No connection with it can be established from the empirically derived mean solar time. Thus the inertial time which takes account of the laws of celestial mechanics is theoretically obtained, and is denoted as ephemeris time (E. T.) when it is listed as argument in the various national ephemerides published.

The irregular changes in rotational speed of the earth are unpredictable and must be observed currently, since the factors causing them are outside our control as yet. - They are also somewhat obscured by annual fluctuations.

A uniform time system can be derived empirically from mean solar time if the latter is provided with corrections for all listed variations in the rotational speed of the earth.

This uniform time system is called ephemeris time (E. T.); it is based on a fictitious uniform orbital motion of the earth round the sun. The difficulty in obtaining the corrections is the reason that ephemeris time is not available at a high accuracy at the time of performance of observations, but after some months or even years, and is therefore of no practical use for precision work in field astronomy.

Ephemeris time is derived from the irregular rotation of the earth on its axis, which has to be deduced from current astronomical observations to the sun, appropriate planets and mainly to the moon; the corrections to the observations have to be applied in arrears. Methods other than astronomical observations are still not available which could bridge the long time intervals required with equal accuracy.

This time system is an approximation to a time system based on the rigid Newtonian laws of motion, and E. T. is used as a substitute for inertial time. Further, E. T. can be brought into numerical ...

4.2. cont.

relation to the non-uniform mean solar time (U. T.). Although the ephemeris time, an empirical measure of time, is used for the theories of planets, for field astronomy all observations are referred to mean solar time. It follows that very precise predictions must be referred to ephemeris time and very precise observations to universal time.

Ephemeris time is conventionally expressed in hrs. min. sec, but should be counted in years and decimals of a year.

The primary unit of ephemeris time is the tropical year and is defined by the duration of the tropical year 1900 aD, beginning at the fundamental epoch January 0, 12h E. T., and is equal to:

315 56925.9747 SECONDS.

This value is obtained from the number of mean solar seconds of one sidereal day multiplied by the number of sidereal days in the tropical year 1900.

The tropical year is the interval during which the sun's mean longitude increases 360° referred to the mean equinox.

Considering the derivation of the primary unit of ephemeris time it can be concluded that it is constituted of the total number of mean solar seconds during the year 1900;

Since by definition the tropical year 1900 is the primary unit of E. T., and also by definition E. T. is uniform, the seconds of which the primary unit is made up can be termed ephemeris seconds.

For astronomical purposes the length of the ephemeris second, the "Newtonian second" has been fixed by definition as equal to - strictly one mean solar second at 1900 - the fraction:

1/315 56925.9747 of the length of the tropical year 1900, and not 1/86 400 of a day.

Consequently one ephemeris day consisting of 86 400 ephemeris seconds is not equal to one actual day and not equal to one mean solar day.

One second of ephemeris time is now the internationally adopted unit of the theoretically uniform time and can be referred to in arrears.

The time correction Δ T which provides for the change over from mean solar time to ephemeris time is the difference between observed and corrected positions of the moon, sun and planets, caused by the variable period of rotation of the earth on its axis, and is determined for each elapsed time interval.

$$\Delta T = \text{ephemeris time} - \text{universal time.}$$

The following expression is used for the calculation of Δ T, which is here given to show that the empirically derived E. T. is practically inertial time.

$$\Delta T = + 24.349 \text{ sec} + 72.318 \text{ sec } T + 29.950 \text{ sec } T^2 + 1.2821 B$$

where: T = Julian centuries from 1900 Jan., 0.5, E. T., Julian dates are normally in U. T. counted from noon instead of midnight.

B = a constant, determined from moon observations in arrears.

The absolute term takes care that the commencement of the ephemeris day coincides with the start of the mean solar day; the length of the epehemeris day is given by the T-term. The last two terms give the approximation to the inertial time.

Definite values of Δ T are known about a year in arrears, after the calculation of the constant B.

For 1964.5 Δ T is about + 83.45 sec + 1.82144 B; this will amount to about + 35 sec., which figure shows the accumulated changes in the rotation of the earth.

The secular variations of the tropical year in solar days is equal to - 0.000 00614 T and analogous with the sidereal year + 0.000 000 11 T, where T is the number of Julian centuries equal to 36525 mean solar days elapsed since the Julian year 1900.

This variation is small enough to be ignored; there is no proper evidence yet of any variation in the amount of the secular variation. Therefore the primary unit of E. T. and the number of rotations of the earth on its axis during one complete revolution round the sun can be regarded as being a constant, or at least to remain constant for a considerable length of time. For the purpose of choosing a unit measure of time interval, the above constant can be presumed to possess the desired property of being invariable.

From the foregoing can be concluded that the number of complete rotations of the earth on its axis, or the number of sidereal days, or the number of mean solar days during one revolution of the earth round the sun, when used as a multiple of the period of the earth's rotation on its axis, constitutes no initial condition of a time system. By initial condition of a system is meant the physical statement as regards to its position and velocity in space. Here the space is not rigid but a physical concept, and with time nowhere more interrelated than in astronomy.

U. T. is solely based on the rotation of the earth on its axis; a multiple number of rotation cannot be identified as a dependency. E. T. is based on the revolution of the earth round the sun, derived from planetary motions, and cannot be related via a constant to another space-time system.

The statement by D. H. Sadler that "U. T. depends both upon the revolution of the earth round the sun and on the rotation of the earth on its axis" is not correct when relativistic effects are considered.*

It is advantageous to have a readily accessible unit of measure. Therefore, the atomic standard of time has been chosen, which is based on the natural resonant frequency of the caesium atom. As yet there is no evidence or reason to believe that the atomic frequency is constant in terms of atomic time.

The value of the caesium frequency in terms of the second E. T. is: 9 192 631 770 cycles of the caesium resonance per seconds E. T., at 1957.0, so chosen, to bring the atomic time scale into agreement with the U.T.2 time system in 1955 June.

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* D. H. Sadler, Superintendent H. M. Nautical Almanac Office, "Ephemeris Time", E. S. R., Vol. 13, No. 102, p. 367.

This numerical value can not remain constant, because it constitutes a ratio between two different systems.

The chosen frequency is referred to as the "nominal frequency of caesium".

The national standard of frequency in U.K. is the caesium atomic beam resonator at the N.P.L.

The time systems in use, ephemeris time and universal time, are specified on the assumption that the solar system is not expanding and not contracting, which results from the simplified theory that the gravitational solar field is static and symmetric.

In the age of radar distance measurement to members of the solar system, statistical evidence of any variation will undoubtedly effect the present practical measures of time, without necessarily curtailing their useful application.

It may be that a radar-time or space-time system, depending on observed planetary motion and distance, will become necessary, the unit of which will have to be referred to the invariable ratio of a recurrence interval to a specific length. No claim for general correctness of the above statement is made, which is based on kinematic relativity..

4.2.1. Universal Time

U.T.O. is universal time or Greenwich mean solar time, calculated from immediately observed sidereal times of transit of a number of stars across the instantaneous meridian of the observer. Before U.T.0, of various observers can be compared, it is reduced to the mean pole.

4.2.1. cont.

U. T. 1 is U. T. O. corrected for variations of the observers' meridian arising from polar motion. U. T. 1 is therefore U. T. O reduced to an invariable Greenwich meridian.

U. T. 2 is obtained from U. T. 1 by subtracting the correction for extrapolated seasonal variations (S. V.) in the rate of rotation of the earth on its axis. The correction is derived from a formula supplied by the Bureau International de l'Heure.

The time system U. T. 2 is the adopted time system obtained by smoothing an extended series of U. T. 2 observations and is practically free of periodic variations.

Individual values of observed U. T. systems are denoted by U. T. O. (o), U. T. 1(o), and U. T. 2(o). The U. T., as every standard of reference, is a statistical product and ultimately physically unattainable.

4.3. Time Signals

The seconds pulses superposed on standard frequency transmissions, used as time signals, are invariably based on extrapolated values of U. T. 2 and are radiated from various stations on the following frequencies and frequency limits of the H. F. band allocated by the Administrative Radio Conference at Atlantic City, 1947:

2.5 Mc/s $\pm$ 5 kc/s, 2.5 Mc/s $\pm$ 2 kc/s, 5 Mc/s $\pm$ 5 kc/s, 10 Mc/s $\pm$ 5 kc/s, 15 Mc/s $\pm$ 10 kc/s, 20 Mc/s $\pm$ 10 kc/s, and 25 Mc/s $\pm$ 10 kc/s.

Recently the frequency of 20 kc/s, 50 c/s wide on either side in the V. L. F. band has been added.

Several transmitting stations give, after the call sign in morse, further information, voice announcement, time etc.

The frequencies are kept within specified limits with reference to an atomic or molecular standard. The caesium standards are stable to about 1 part in 10^{10}.

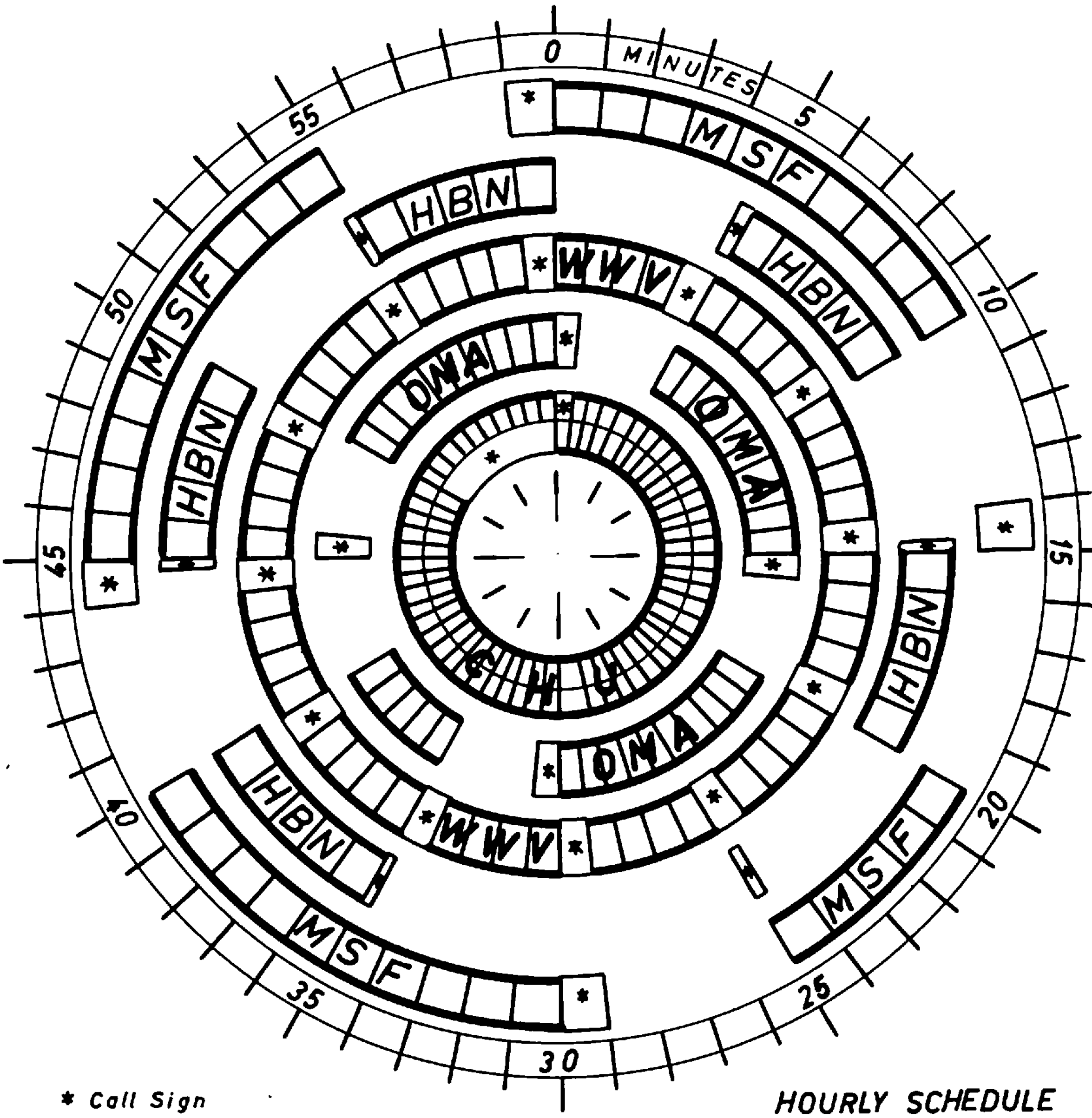
Step adjustments, when necessary, keep the radiated pulses in the immediate vicinity of U. T. 2.

The M. S. F. high frequency transmission was maintained at $\pm$ 2 parts in 10^{10} of their nominal values since May 1961.

Under normal reception conditions the accuracy of the time interval marked by two consecutive seconds pulses of standard frequency transmissions is known to be about $\pm$ 0.1 msec. When the ground wave is received, the time signals can be used to mark the epoch to $\pm$ one microsecond; the reception of the sky wave, under average ionospheric conditions, reduces the accuracy of defining the epoch to about 2 milliseconds. When frequency pulses are received at great distances, the transmission medium effects the accuracy of the time interval marked by consecutive signals, and ionospheric disturbances can effect distortions amounting to several milliseconds.

The diagrams Figs. 4.3.-1, 4.3.-2, and 4.3.-3 show in clock-face

Time Table
of
Seconds Pulses superposed on Standard Frequency Transmissions and Radio Time Signals, Time System: U.T. 2

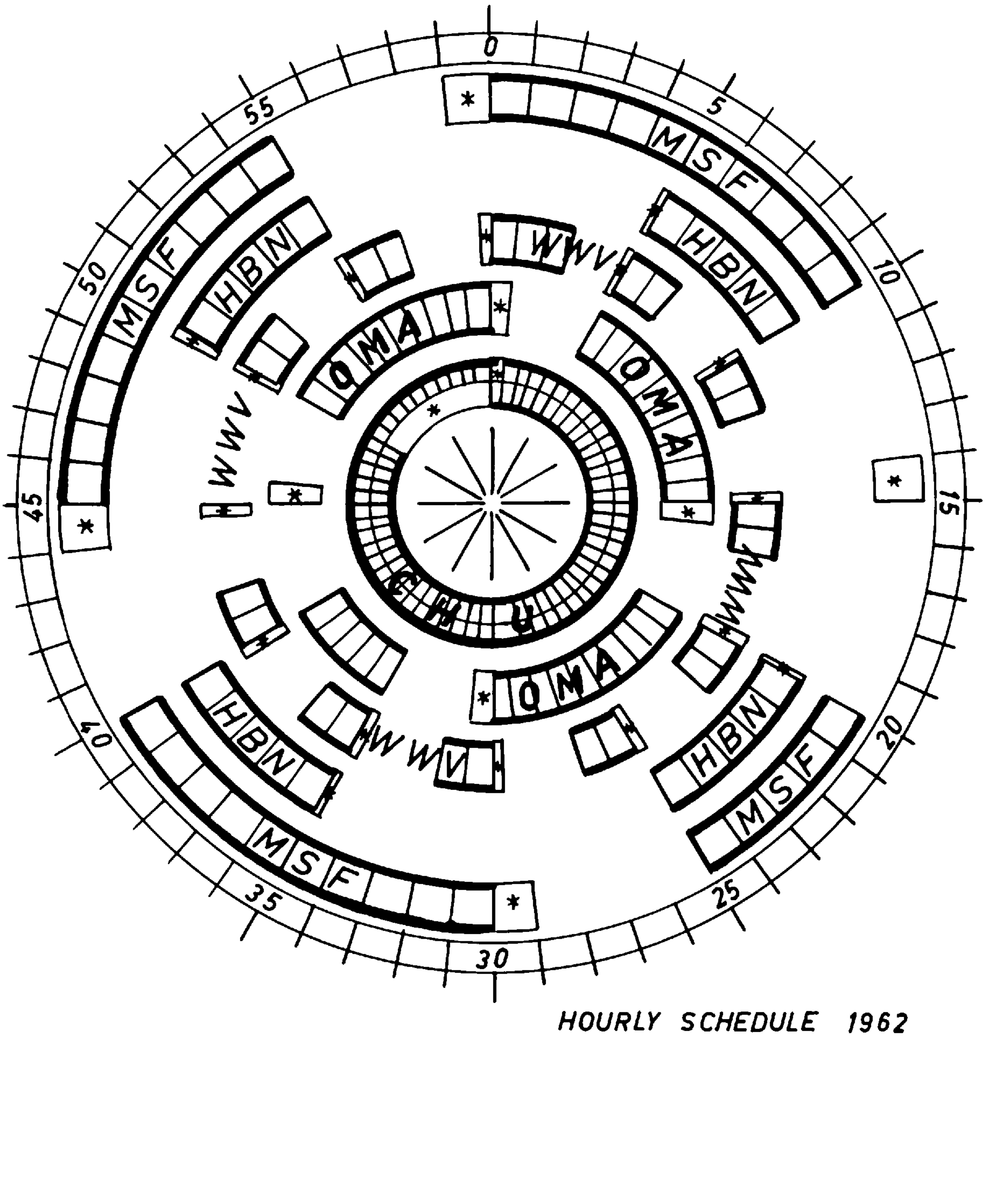


* Call Sign

HOURLY SCHEDULE
1960, 1961

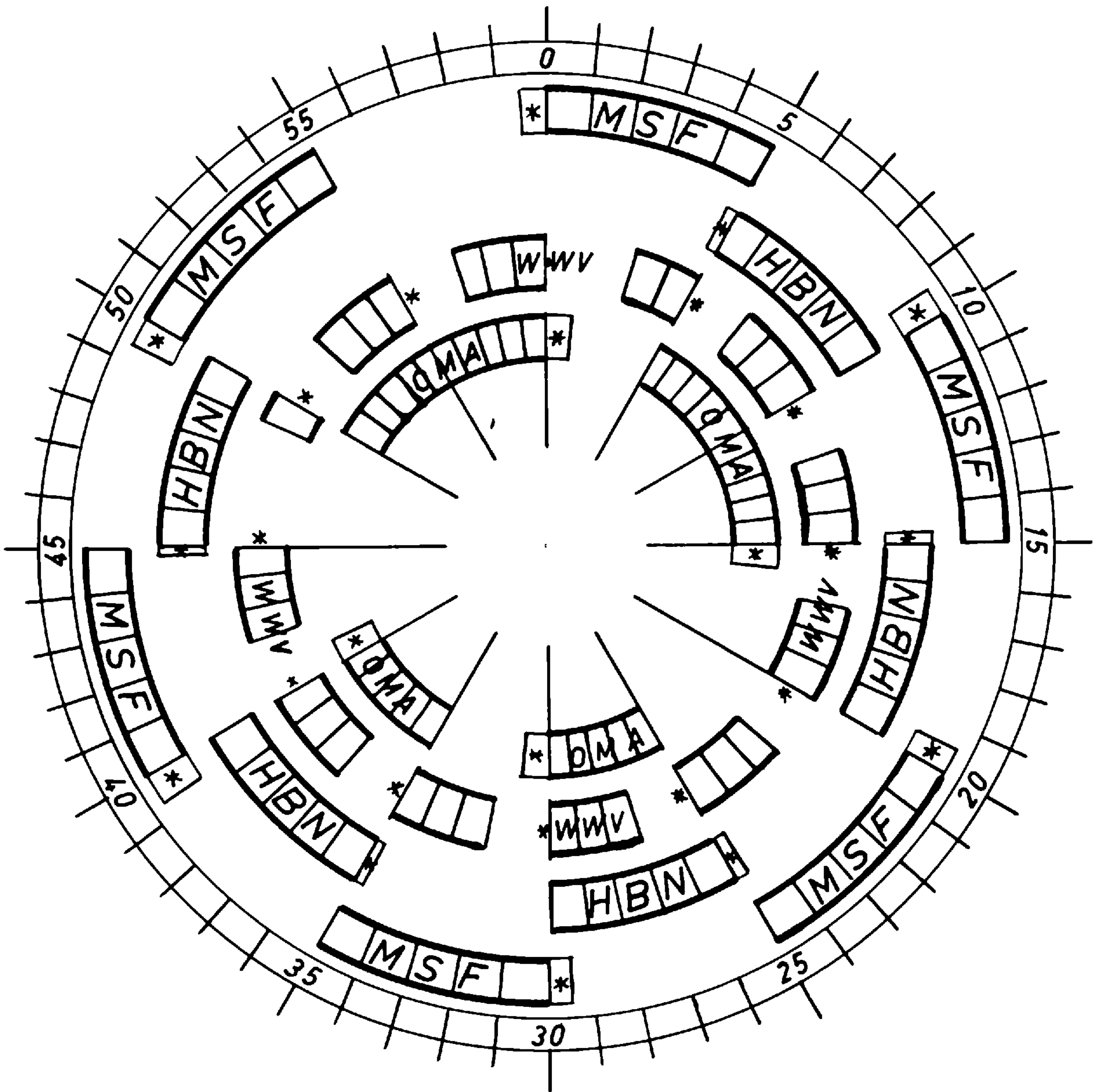
Station	Call Sign	Latitude	Longitude	Frequency
Rugby	MSF	52° 22' 10" N	01° 11' 15" W	2.5, 5, 10, Mc/s
Neuchatel	HBN	46° 58' N	06° 57' E	5 Mc/s Tues., Wed., Fri., Sat., 2.5 Mc/s Mon., Thur., Sun.,
Beltsville	WWV	39° 00' N	76° 51' W	2.5, 5, 10, 15, 20, 25 Mc/s
Prague	OMA	50° 07' N	14° 35' E	2.5 Mc/s
Ottawa	CHU	45° 17' 42" N	75° 45' 22" W	3.33, 7.35, 14.67 Mc/s

Fig. 4.3.-1



HOURLY SCHEDULE 1962

Fig. 4.3.-2



HOURLY SCHEDULE 1963

Fig. 4.3.-3

4.3. cont.

form the hourly schedules of various transmitting stations for 1960, 1961, 1962 and 1963 respectively. The period of one hour transmission is subdivided into minutes; in addition, for C. H. U. the period of one minute is subdivided into seconds. Its transmission pattern during the minute interval is incorporated in the diagram in the subdivision of the space allocated to it.

The diagrams contain, as intended, information relevant only for field use.

The manner of presenting the times of transmission, as can be seen in the diagrams, has been chosen on purpose and has proved to be of great assistance in selecting and identifying the transmitting station. In addition, times at which possible interference may be expected between one or several transmissions can easily be derived, and these unfavourable periods can be avoided. Alterations in the diagram to conform with changes in the time service constitute no difficulty.

Included also are details of call-sign, location and frequency of the transmitting stations.

Particulars about signals, voice announcements, modulation etc., not necessarily required in the field, can be found in the Admiralty List of Radio Signals, Vol. V, published by the Hydrographic Department, Admiralty, London.

Preliminary emission times of signals from various stations and provisional co-ordinates of the pole are given in the "Time Service Circular" of the Royal Greenwich Observatory.

Mean values of measured times of reception at the R. G. O., and times of emission of various radio time signals and seconds pulses superposed on standard frequency transmissions in terms of U. T. 2, are listed in the Royal Observatory Bulletins, London, in which are also included corrections for polar and seasonal variations. Time signal adjustments are announced in "Time Service Notice".

Times of emissions of signals are tabulated in the Bulletin

4.3. cont.

Horaire Ser., G, of the Bureau International de l'Heure, Paris.

The U.S. Naval Observatory Time Signal Bulletins contain final times of emissions. Changes in the time service, adjustments in time signal pulses etc., are published in "Time Service Notices" and in "Time Service Announcements".

4.4. Corrections to Time Signal Reception.

The local observations are referred to the instantaneous meridian of the survey station and to the instantaneous pole, if U. T. O. is used. Since the time signals and the seconds pulses superposed on standard frequency transmissions are radiated in terms of U. T. 2 at preliminary values of emission times, the reception times of signals have to be corrected for:

Seasonal variation of the rotational speed of the earth.
Polar variation
Travel time of transmitted pulses
Emission delay " "
Receiver delay at field station.

Correction for Seasonal Variation.

The definite time-signal correction for seasonal variation (S. V.) in the rate of rotation of the earth are based on interpolated values and are published in R. G. O. Bulletins at ten-day intervals, and daily values for Herstmonceux are also given. These corrections are available about one year in arrear.

Correction for Polar variation.

As stated, the uncorrected astronomical time U. T. O. refers to the observer's position, and is obtained from U. T. 1 by subtracting the

4.4. cont.

effects of polar motion.

In the R. G. O. Bulletin the correction for polar variation (P. V.) in seconds is given by the formula:

$$+\frac{1}{15} \cdot (x. \sin\lambda - y. \cos\lambda). \tan\phi$$

where: x and y are the co-ordinates in seconds of arc of the instantaneous pole referred to the mean pole,

ϕ the observer's latitude (+ N) and λ the observer's longitude, measured in the direction of star movement, i. e. positive west, from the meridian of Greenwich = 0° (clockwise) to 360°. This conforms with the Bureau International de l'Heure; alternatively the formula can be used for + E longitude from the meridian of Greenwich by substituting in the formula λ + west, with $(360^\circ - \lambda)$, thus:

$$+\frac{1}{15} \cdot (-x. \sin\lambda - y. \cos\lambda) . \tan\phi$$

Approximate corrections for current field observations can be calculated from extrapolated values of x and y published in advance in the Time Service Circular a. o., and preliminary values of λ and ϕ of the "Trial Point".

It is: U. T. O. + P. V. = U. T. 1.

and: U. T. O. + P. V. + S. V. = U. T. 2.

<u>Correction for Travel Time.</u>

The shortest distance from transmitter to receiver is approximately along a great circle and can be calculated from the cosine formula for the sides:

cos (arc distance) = $\sin\phi_1 . . \sin\phi_2 + \cos\phi_1 . \cos\phi_2 \cos\Delta\lambda = \cos d$

where: ϕ_1, ϕ_2 are the latitudes of the transmitting and receiving

4.4. cont.

stations respectively, $\Delta\lambda$ the difference in longitude between them. The travel time t is then obtained, using the mean velocities for H.F. transmissions:

$$t = \frac{\text{earth's semi axis (km)} \cdot \pi \cdot d^{\circ}}{\text{velocity of H.F. (km/sec)} \cdot 180^{\circ}}$$

In Table 4.4.-1 are shown the travel times for receptions at London and at the Field Station Tywarnhale Mine, using Clarke's 1858 values and velocities for H.F. transmission of 298 000 km/sec, and 280 000 km/sec for short waves along the shortest route, and 286 000 km/sec for the longer path round the earth.

The average delay in time signal propagation is about 1 millisecond per 186 miles of transmission path length.

If the field station is situated close to the time signal transmitter, ca. 100 to 200 miles, a case unlikely to be encountered in exploration surveys, the travel time of the transmissions becomes very uncertain. This is due to propagation anomalies. The time error of individual pulses received over short distances can amount to ten milliseconds.

Correction for Emission Delay.

The preliminary corrections for emission delay can be calculated from preliminary emission times which are published in advance in Time Service Circular a.o., and can be applied in the field straight away.

Final corrections for emission delay can be deduced from published times of reception at R.G.O. and at Paris, reduced by the calculated travel times to Herstmonceux and Paris respectively, which for this purpose are also shown in Table 4.4.-1.

This is important when using stations which are not participating in the co-ordinated time service; and still necessary for co-ordinated transmitters, since a considerable fluctuation of their emission time

TRAVEL TIMES OF SIGNALS

SIGNAL IS THE EMITTED SECOND PULSE SUPERPOSED ON STANDARD FREQUENCY TRANSMISSIONS, OR RADIO TIME SIGNAL.

FORMULAE USED:

$$\cos d = \sin\phi_1 . \sin\phi_2 + \cos\phi_1 . \cos\phi_2 . \cos\Delta\lambda$$

$$t = \cdot 000372\, d \qquad t = \cdot 000396\, d$$

d = arc distance between Transmitting and Receiving Stations.
ϕ_1 = Latitude of Transmitting St^n., ϕ_2 = Latitude of Receiving St^n.
$\Delta\lambda$ = Difference of Longitudes between Transmitting & Receiving Stations.
t = Travel Time of Signal.

CONSTANTS USED:

Effective Surface Speed of 298, & 280 km/msec for HF Signals
Average (MEAN) Diameter of the Earth.

TRANSMITTING STATIONS		RECEIVING STATIONS							
		PARIS		HERSTMONCEUX Lat.: 50°52'18"N Long: 00°20'16·53"E		LONDON R.S.M. Lat.: 51°25'10"N Long.: 00°02'30"W		TYWARNHALE CORNWALL Lat.: 50°16'48" N Long.: 05°13'34" W	
CALL SIGN Frequency	latitude Longitude	d^{km}	t^{sec}	d°	t^{sec}	d°	t^{sec}	d°	t^{sec}
MSF RUGBY 5, 2·5	52°22'10"N 01°11'15"W	465	$\cdot 002_7$	1·772 Rec. Lag.	– $\cdot 000_7$ $\cdot 000_3$	1·184	$\cdot 000_4$ $\cdot 000_5$	3·275	$\cdot 001_2$ $\cdot 001_3$
HBN NEUCHÂTEL 5 Mc/s	46° 59' N 06° 57'4 E	401	$\cdot 002_6$	5·856 Rec. Lag.	– $\cdot 002_3$ $\cdot 000_3$	6·375	$\cdot 002_4$ $\cdot 002_5$	8·691	$\cdot 003_2$ $\cdot 003_4$
WWV BELTSVILLE 10, 5, 2·5	38° 59'6 N 76° 50'9 W	6162	$\cdot 022_4$	53·340 Rec. Lag.	– $\cdot 021_2$ $\cdot 000_3$	52·947	$\cdot 019_7$ $\cdot 021_0$	50·162	$\cdot 018_7$ $\cdot 019_9$
WWVH HAWAII 10, 5, 2·5	20° 46' N 156° 28' W	12015	$\cdot 042_8$					104·542	$\cdot 038_9$ $\cdot 041_4$ $\cdot 095_0$*
OMA PRAGUE 2·5	50° 07' N 14° 35' E	898	$\cdot 003_8$	9·071	– $\cdot 003_6$	9·325	$\cdot 003_5$ $\cdot 003_7$	12·644	$\cdot 004_7$ ·005
LOL BUENOS AIRES 10, 5, 1·5	34° 37' S 58° 21' W	11023	$\cdot 039_4$					95·971	$\cdot 035_7$ $\cdot 038_0$ $\cdot 098_2$*
CHU OTTAWA 3·33, 7·335	45°17'42"N 75°45'22"W							37·049	$\cdot 013_8$ $\cdot 014_7$

* round the earth, via long path.

TABLE 4.4.-1

4.4. cont.

has been found from field receptions, exceeding the intended limit of one millisecond.

During the last five months of 1962 the preliminary emission times for signals from co-ordinated stations given by R. G. O. varied from the final corrected times of emission between 1 and 16 milliseconds.

Correction for Receiver Delay

From 1962 onwards the reception times at R. G. O. of seconds pulses superposed on standard frequency transmissions published in the R. G. O. Bulletins are corrected for receiver delay.

At the field station a correction for receiver lag has to be applied according to the type of receiver used.

This correction is unnecessary if preliminary times of emissions are the only ones that are available, since their accuracy does not justify the additional amount of calculation involved.

4.5. Universal Time (U. T.) from Crystal Chronometer Time.

An instant of crystal chronometer time is expressed in U. T. by adding the items 1 to 8 listed below.

A conversion vice versa from U. T. to crystal chronometer time is never required.

(1) The amount indicated on the chronometer clock face in hours and minutes, which is kept or set in synchronization with received standard frequency time signals, or via the telephone time service; (like conventional clocks the crystal chronometer is set to the nearest minute, slow or fast).

(2) The number of crystal chronometer signals multiplied by 60/61, in seconds and decimals of a second, since the last chronometer minute pulse, i. e. from pulse No. 0 = 61; Table 3.3.-1.

(3) The amount of time in decimals of a second elapsed since the last crystal chronometer pulse to the instant for which U. T. is

4.5. cont.

sought; obtained with methods (6) and (8) described in Section 3.5.3.

The time made up by (1), (2) and (3) is the nominal time of the crystal chronometer, and is expressed in mean solar seconds.

(4) The amount in seconds and decimals, the chronometer pulse, which immediately precedes the required instant, is slow or fast in relation to time signals of standard frequency transmissions (U.T.2). This can be obtained from methods described in Section 3.5.3. of which the most convenient are:
   (a) simultaneous recording of crystal chronometer and standard frequency pulses and subsequent scaling of their distances on magnetic tape;
   (b) time vernier (the reading of the time vernier is outlined below);
   (c) equations (1) to (2A), Section 3.5.2, combined with (a) or (b) above;
   (d) graphical interpolation, combined with (a) or (b).

(5) The delay time of signal emission; preliminary values are extracted from the Time Service Notice, a.o., definite values from the R.G.O. Bulletins, Bulletin Horaire, a.o. and Table 4.4.-1.

(6) The travel time of standard frequency transmissions obtained from the velocity of propagation of radio waves and distance travelled, Table 4.4.-1.

(7) The correction for receiver delay.

(8) The corrections for U.T.1 or U.T.O., as required.

Various possibilities for reading the time vernier are shown in Figs. 4.5.-1 and 4.5.-2. The reading is based either on chronometer pulses or on received time signals.

TIME VERNIER

CRYSTAL CHRONOMETER AT i^{th} PULSE FAST OR SLOW REF. TO U.T. :

(1) i^{th} CHRONOMETER PULSE RECORDED AFTER m U.T. sec $= m$ sec

VERNIER COINCIDENCE AT n^{th} U.T. sec
OR
VERNIER COINCIDENCE AT j^{th} CHRON. PULSE

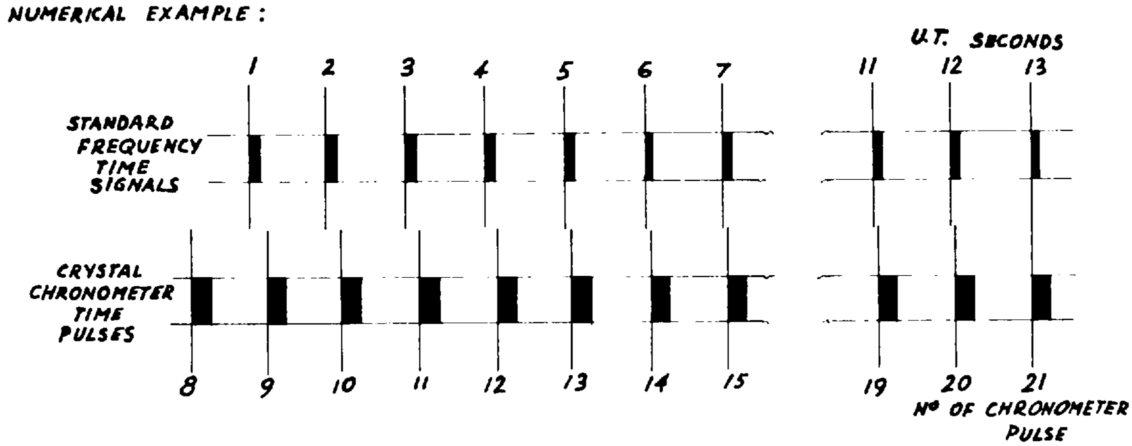
(2) $x = (n-m) \times \frac{1}{61} = (j-i) \times \frac{1}{61} =$ (TABLE 3.3.-1) $= x$ sec

(3) i^{th} CHRONOMETER PULSE : NOMINAL TIME $= i \times \frac{60}{61} \times$ (TABLE 3.3.-1) $= a$ sec

CRYSTAL CHRONOMETER AT i^{th} PULSE FAST (−) OR SLOW (+) : IN sec. U.T. :

$$= (m+x) - a$$
$$= m + (j-i)\frac{1}{61} - i\frac{60}{61} = m + j\frac{1}{61} - i$$
$$= m + (n-m)\frac{1}{61} - i\frac{60}{61} = m\frac{60}{61} + n\frac{1}{61} - i\frac{60}{61}$$

NUMERICAL EXAMPLE :



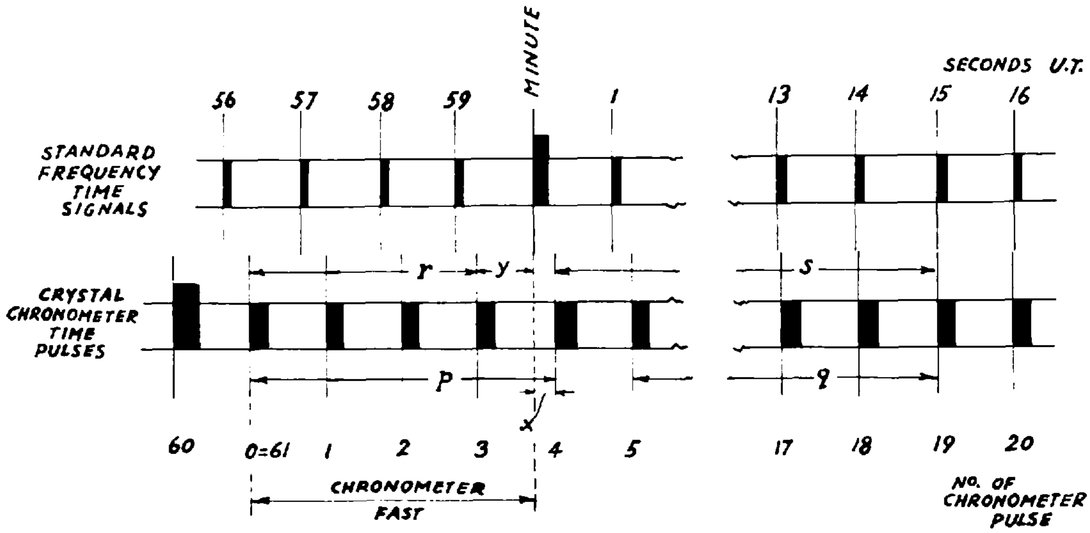
(1) 12^{th} (i^{th}) CHRONOMETER PULSE RECORDED AFTER 4 (m) U.T. sec. . . . . . . $4{\cdot}000_0$ sec
VERNIER COINCIDENCE AT 13^{th} U.T. sec ($= n$)
" " " 21^{st} CHRON. PULSE ($= j$)

(2) $x = (13-4) \times \frac{1}{61} = (21-12) \times \frac{1}{61}$ (TABLE 3.3.-1) . . . . $\cdot 147_5$ sec

(3) 12^{th} (i^{th}) CHRON. PULSE : NOMINAL TIME (TABLE 3.3.-1) . . . . $11{\cdot}803_3$ sec

CRYSTAL CHRON. AT 12^{th} PULSE FAST (REF. TO U.T.) . . . . $-7{\cdot}655_8$ sec.

Fig. 4.5.-1



r = 3rd CHRONOMETER PULSE = $2{\cdot}950_8$ (TABLE 3.3.-1)

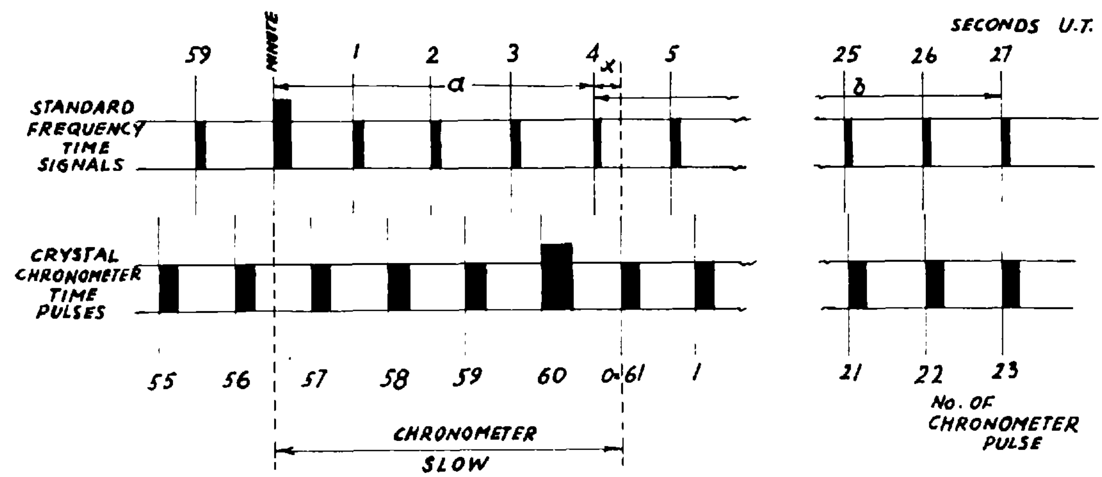
s : 16 CHRON. PULSES $+y$ = $\cdot737_7$ " = $\left(1 - \text{No. OF ELAPSED CHRON. PULSES AFTER MINUTE SIGNAL}\right) \times \frac{1}{61}$

CHRONOMETER FAST = $3{\cdot}688_5$ sec. U.T., AT EPOCH MARKED BY 0-CHRON. PULSE.

p = 4th CHRONOMETER PULSE = $3{\cdot}934_4$ (TABLE 3.3.-1)

q : 15 CHRON. PULSES $-x$ = $\cdot245_9$ = No. OF ELAPSED SECONDS AFTER MINUTE SIGNAL $\times \frac{1}{61}$

CHRONOMETER FAST = $3{\cdot}688_5$ sec. U.T.



a = $\left.\begin{array}{l}\text{No. OF SECONDS ELAPSED BETWEEN}\\ \text{MINUTE SIGNAL AND 0-CHRON. PULSE}\end{array}\right\}$ = $4{\cdot}000$ sec.

$x = b \times \frac{1}{61}$ = $\left.\begin{array}{l}\text{No. OF ELAPSED SECONDS FROM}\\ \text{0-CHRON. PULSE TO VERNIER}\\ \text{COINCIDENCE} \times \frac{1}{61}\end{array}\right\}$ = $0{\cdot}377_0$ " = $(27-4) \times \frac{1}{61}$ (TABLE 3.3.-1)

CHRONOMETER SLOW = $4{\cdot}377_0$ sec. U.T., AT EPOCH MARKED BY 0-CHRON. PULSE.

TIME VERNIER

THE LENGTH OF THE PULSES HAS BEEN EXAGGERATED FOR BETTER ILLUSTRATION.

Fig. 4.5.-2

4.5. cont.

According to the initial setting of the chronometer, slow or fast, and to the reception conditions of radio time signals; one method of time extraction from the vernier has some advantage over the other, and will be preferred by the field engineer. The diagrams are self-explanatory.

The precision with which time in terms of U.T.2 can be obtained from the time indicated by the crystal chronometer can be demonstrated by comparing the chronometer time with simultaneous receptions of two or more standard frequency transmissions.

Simultaneous receptions also reveal the accuracy of the recording and evaluation method and indicate the efficiency of the national time services as well as the precision of the time signal reception.

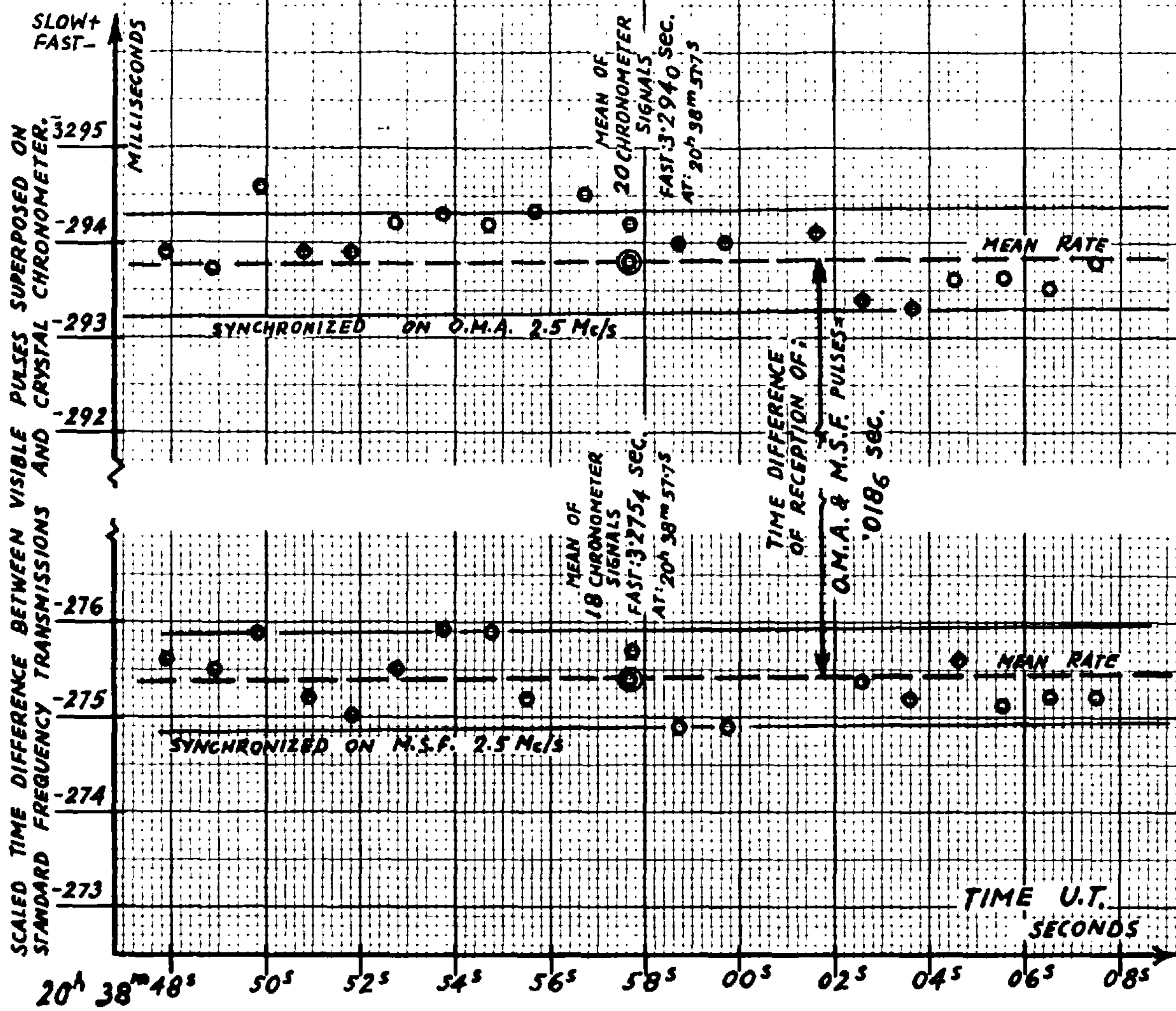
The time difference between pulses of two or more standard frequency transmissions may be used for the identification of the transmitting stations. Simultaneous receptions are therefore of great assistance in solving problems of time interpolation.

The synchronization of the crystal chronometer with various standard frequency time signals is given in Figs. 4.5.-3 to 4.5.-7 and in Tables 4.5.-1 and 4.5.-2.

The points plotted represent time errors or frequency deviations of the crystal chronometer with reference to frequency time signals received on 2.5 or 5 Mc/s governed by reception conditions.

Transmissions are reputed to be less affected by ionization anomalies during their daylight path than when travelling at night.

In the two diagrams (Fig. 4.5.-4 and Fig. 4.5.-6) chronometer comparisons are given with receptions in daylight on 2.5 and 5 Mc/s respectively. The scatter of the points about the mean depicting the effect of all errors is more or less of the same magnitude for all receptions and not greater for night travel. From neither of the various transmitting stations can a characteristic behaviour of their pulses be



U.T. INDICATED BY SECONDS PULSES SUPERPOSED ON STANDARD FREQUENCY TRANSMISSIONS : O.M.A. & M.S.F., 2.5 Mc/s, AT THE INSTANT OF RECORDING ON MAGNETIC TAPE.

CRYSTAL CHRONOMETER PULSES ARE PLOTTED AT U.T., SCALED FROM STANDARD FREQUENCY PULSES.

DATE : SUN. JAN. 29, 1961, $20^h 38^m$

FROM : ROYAL GREENWICH OBSERVATORY BULLETIN Nº 47

TIME OF RECEPTION ON U.T.2 OF O.M.A. 2·5 Mc/s, AT: $20{\cdot}5^h$	0345	
RECEIVER LAG AT R.G.O.	0003	
PULSE TRAVEL TIME TO R.G.O.	0036	
PULSE TRAV. TIME TO LONDON	0035	
TOTAL DELAY TIME		0341
TIME OF EMISSION ON U.T.2 OF M.S.F. 2·5 Mc/s	0149	
PULSE TRAVEL TIME TO LONDON	0004	
TOTAL DELAY TIME		0153
DIFFERENCE OF RECEPTION TIMES OF O.M.A. & M.S.F. AT LONDON		0188
DIFFERENCE OF RECEPTION TIMES SCALED FROM VISIBLE PULSES		0186
DISCREPANCY		·0002 sec

FROM : BULLETIN HORAIRE Nº 13 SER. G. BUREAU INTERNATIONAL DE L'HEURE

TIME OF EMISSION ON U.T.2 OF O.M.A. 2·5 Mc/s AT: 21^h	0321	
PULSE TRAV. TIME TO LONDON	·0035	
TOTAL DELAY TIME		0356
TIME OF EMISSION ON U.T.2 OF M.S.F. 2·5 Mc/s, AT: $20{\cdot}5^h$	0152	
PULSE TRAVEL TIME TO LONDON	·0004	
TOTAL DELAY TIME		0156
DIFFERENCE OF RECEPTION TIMES OF O.M.A. & M.S.F. AT LONDON		0200
DIFFERENCE OF RECEPTION TIMES SCALED FROM VISIBLE PULSES		0186
DISCREPANCY		·0014 sec.

U.T. FROM CRYSTAL CHRONOMETER TIME.

Fig. 4. 5.-3.

U.T. FROM CRYSTAL CHRONOMETER TIME.

26TH MAR. 1961, 13h 06m

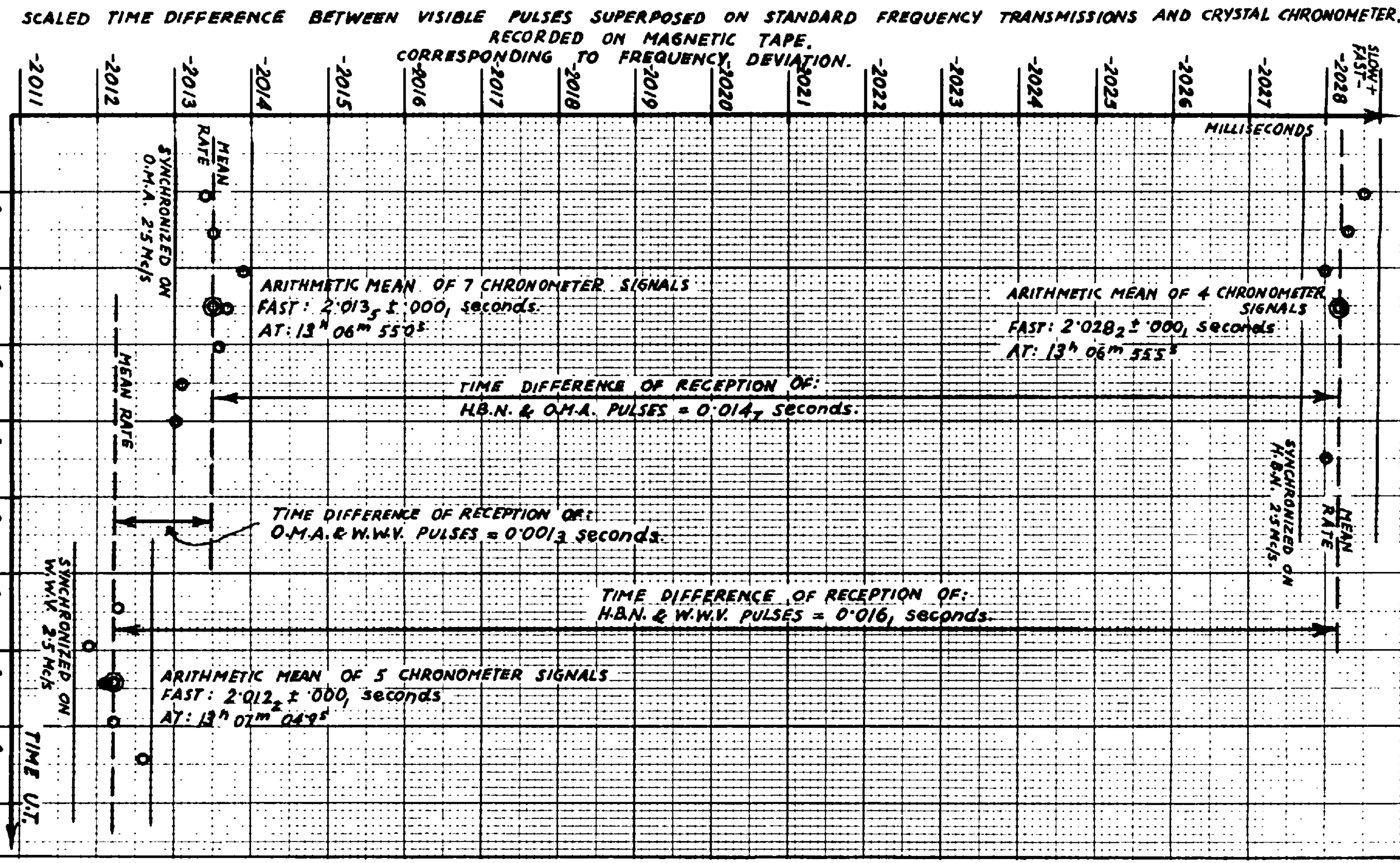


Fig. 4.5-4.

U.T. FROM CRYSTAL CHRONOMETER TIME.
(Fig. 4.5.-4.)

FROM: ROYAL GREENWICH OBSERVATORY BULLETIN Nº 41		H.B.N.	O.M.A.		W.W.V.	
TIME OF RECEPTION ON U.T.2	0690		0536		0562	
RECEIVER LAG AT R.G.O.	0003		0003		0003	
TRAVEL TIME TO R.G.O.	0023		0036		0212	
TIME OF EMISSION ON U.T.2		0664		0497		0347
TRAVEL TIME TO FIELD STN		0034		0050		0187
TOTAL DELAY TIME		0698		0547		0534
FROM: BULLETIN HORAIRE Nº 14 BUREAU INTERNATIONAL DE L'HEURE						
TIME OF EMISSION ON U.T. 2		0619		0456		0296
TRAVEL TIME TO FIELD STN		0034		0050		0187
TOTAL DELAY TIME		0653		0506		0483

DIFFERENCES BETWEEN RECEPTION TIMES OF TRANSMISSIONS AT THE FIELD STATION:

		O.M.A.	W.W.V.
H.B.N.	SCALED FROM VISIBLE PULSES	0147	0161
	R.G.O. BULLETIN	0151	0164
	BULLETIN HORAIRE	0147	0170
O.M.A.	SCALED FROM VISIBLE PULSES		0013
	R.G.O. BULLETIN		0013
	BULLETIN HORAIRE		0023

(TIMES ARE GIVEN IN DECIMALS OF A SECOND WITH THE DECIMAL POINT OMITTED)

CRYSTAL CHRONOMETER FAST ON :

$$\text{H.B.N.}: \; \begin{matrix} -2{\cdot}0282 \\ {\cdot}0698 \end{matrix} = -1{\cdot}958_4$$

$$\text{OMA}: \; \begin{matrix} -2{\cdot}0135 \\ {\cdot}0547 \end{matrix} = -1{\cdot}958_8$$

$$\text{W.W.V.}: \; \begin{matrix} -2{\cdot}0122 \\ {\cdot}0534 \end{matrix} = 1{\cdot}958_8 \text{ seconds.}$$

AT: $13^h 07^m$, MAR. 26, 1961, CRYSTAL CHRONOMETER FAST ON U.T.2 (REF. R.G.O.) :

$- 1{\cdot}958_6$ sec. (MEAN VALUE)

TABLE 4.5.-1

noticed which would show up in the scaled time difference from the crystal chronometer. This applies equally for transmissions during their night and daylight paths. Chronometer comparisons with standard frequency time signals on 2.5. and 5 Mc/s during their night path is given in Fig. 4.5.-3, 4.5.-5 and 4.5.-7.

Generally, the results of synchronization with the various combinations of time signal receptions are of nearly identical precision.

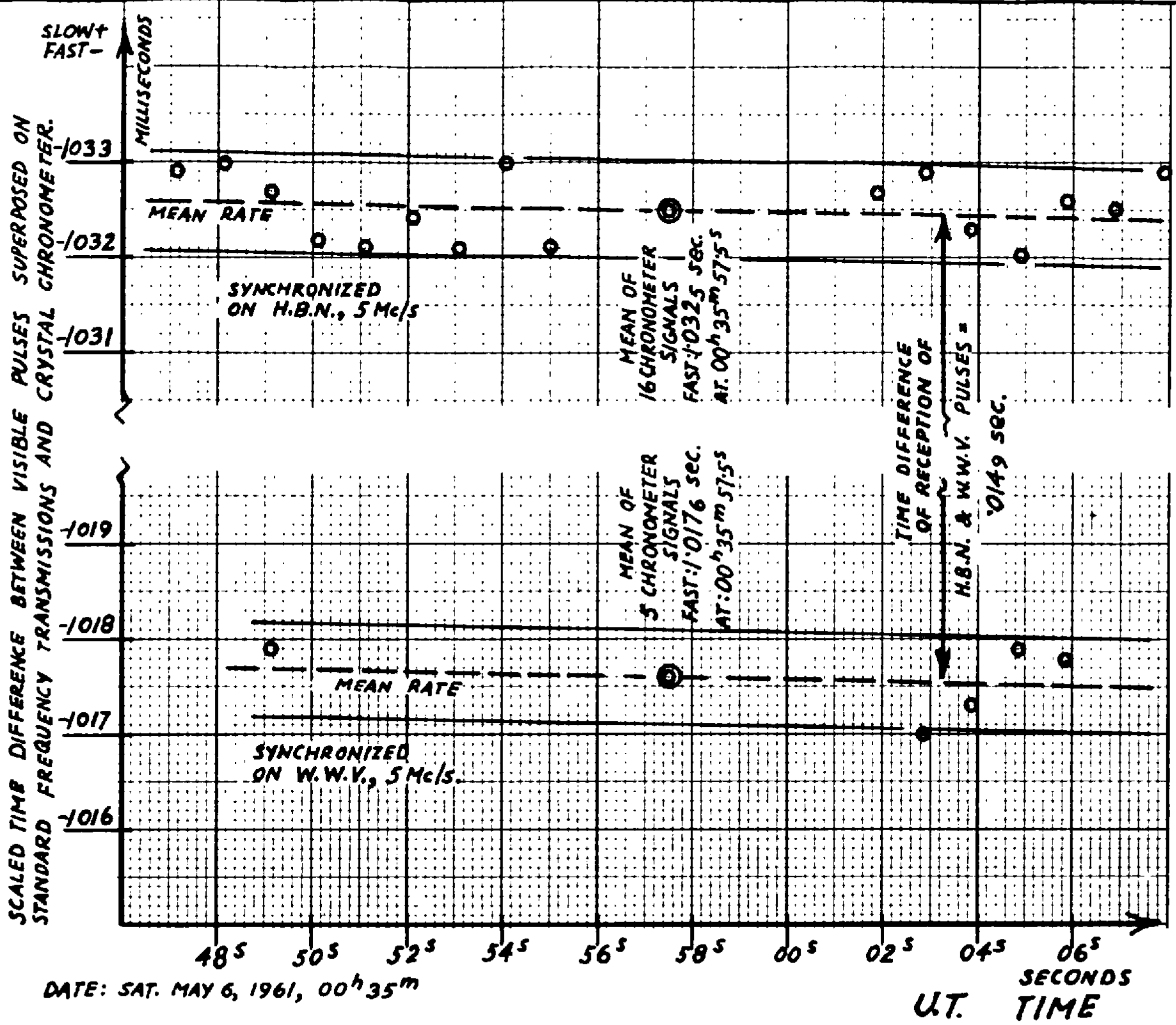
The differences of reception times of O.M.A. and M.S.F. signals, derived from R.G.O. Bulletin and Bulletin Horaire respectively and shown in Fig. 4.5.-3, vary by 1.2 msec.

Discrepancies of this order can arise because the various observatories refer the means of observed reception times to different day times; the receptions are also obtained at different times of the day, according to expediency.

The corrections applied to time measurements at observatories are certainly not free from the influence of personal judgement. Further, the reference point of time measurement can be the leading edge, the peak of the amplitude of the incoming pulse or any chosen cycle designated as commencement of the signal.

The diagrams clearly demonstrate that the time extraction in terms of U.T.2 from the crystal chronometer is obviously limited by reception conditions of time pulses.

It will be noticed also that discrepancies in differences of emission times deduced from published figures in R.G.O. Bulletin and Bulletin Horaire, and from scaling the magnetic tape, are mostly associated with O.M.A. pulses. The O.M.A. pulse made visible reveals an ill-defined starting edge; this lack of sharpness of the outlines of the pulse reduces the scaling accuracy. In general the reception of O.M.A. in the field during May 1961 was rather vague. In R.G.O. Bulletin No. 56 the daily reception times during May 1961 for O.M.A. are given only for nine days out of 31. In Fig. 4.5.-7 the field results and calculations deal with observations on May 12, 1961; for the



U.T. INDICATED BY SECONDS PULSES SUPERPOSED ON STANDARD FREQUENCY TRANSMISSIONS : H.B.N. & W.W.V., 5 Mc/s, AT THE INSTANT OF RECORDING ON MAGNETIC TAPE.

CRYSTAL CHRONOMETER PULSES ARE PLOTTED AT U.T., SCALED FROM STANDARD FREQUENCY PULSES.

FROM: BULLETIN Nº 56 ROYAL GREENWICH OBSERVATORY:

TIME OF RECEPTION ON U.T.2 OF H.B.N 5 Mc/s INTERPOLATED MAY 6, 00^h35^m	·0880	
RECEIVER LAG AT R.G.O.	·0003	
PULSE TRAVEL TIME TO R.G.O.	·0023	
PULSE TRAVEL TIME TO FIELD STATION	·0034	
TOTAL DELAY TIME		·0888
TIME OF RECEPTION ON U.T.2 OF W.W.V. 5 Mc/s INTERPOLATED MAY 6, 00^h35^m	·0752	
PULSE TRAVEL TIME TO R.G.O. & RECEIVER LAG	·0215	
PULSE TRAVEL TIME TO FIELD STATION	·0199	
TOTAL DELAY TIME		·0736
DIFF. OF RECEPTION TIMES OF HBN & WWV. AT FIELD STN		·0152
DIFF. OF RECEPTION TIMES OF HBN & WWV. SCALED FROM VIS. PULSES		·0149
DISCREPANCY		·0003 sec.

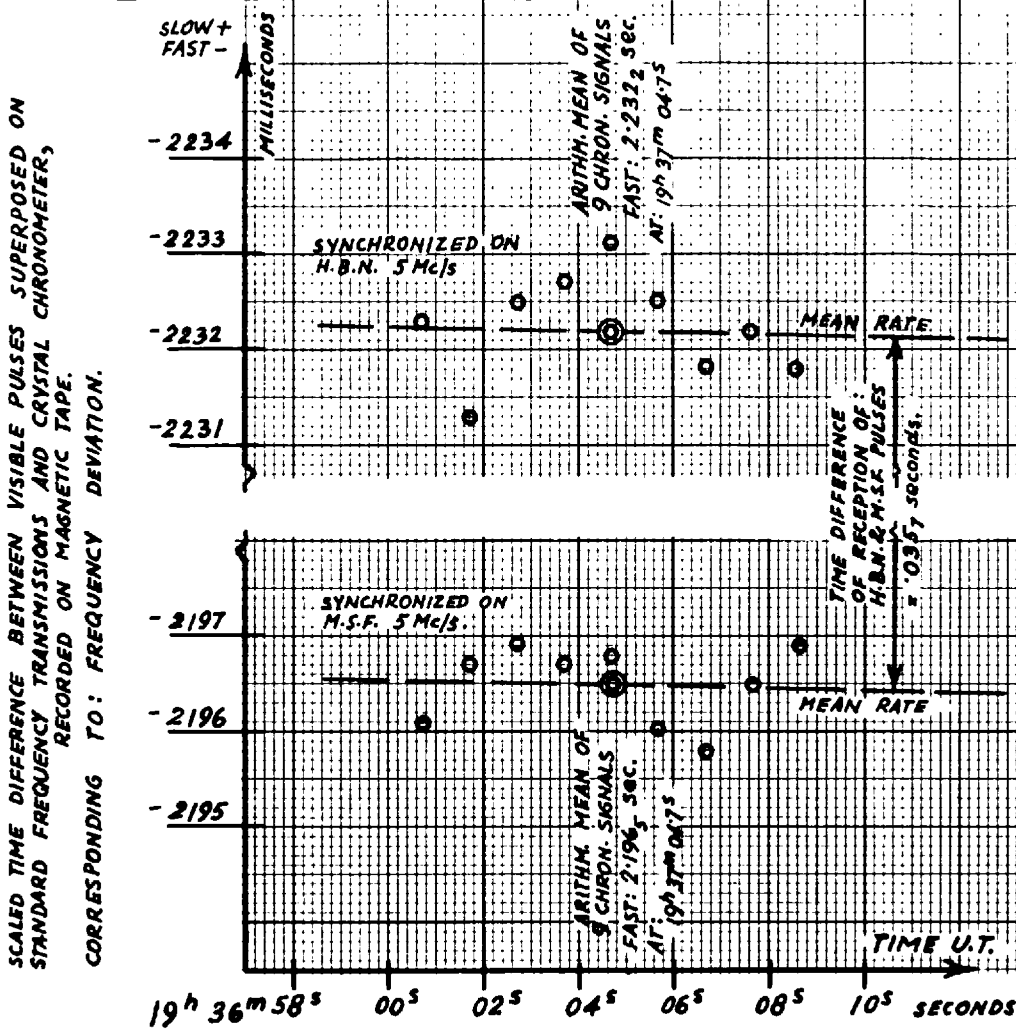
FROM: BULLETIN HORAIRE DU BUREAU INTERNATIONAL DE L'HEURE Nº 15 SERIE G:

TIME OF EMISSION ON U.T.2 OF H.B.N 5 Mc/s INT.d FOR MAY 6, 00^h35^m	·0806	
PULSE TRAVEL TIME TO FIELD STATION	·0034	
TOTAL DELAY TIME		·0840
TIME OF EMISSION ON U.T.2 OF W.W.V. 5 Mc/s INT.d FOR MAY 6, 00^h35^m	·0496	
PULSE TRAVEL TIME TO FIELD STATION	·0199	
TOTAL DELAY TIME		·0695
DIFFERENCE OF RECEPTION TIMES OF HBN & W.W.V. AT FIELD STATION		·0145
DIFFERENCE OF REC.N TIMES OF HBN & W.W.V. SCALED FROM VISIBLE PULSES		·0149
DISCREPANCY		·0004 sec.

CRYSTAL CHRONOMETER FAST ON U.T.2 (REF. R.G.O.) AT $00^h 35^m$, MAY 6, 1961 : $-0{\cdot}943_9$ sec.

U.T. FROM CRYSTAL CHRONOMETER TIME.

Fig. 4.5.- 5.



DATE: TUE. MAY 9, 1961

U.T. INDICATED BY SECONDS PULSES SUPERPOSED ON STANDARD FREQUENCY TRANSMISSIONS: H.B.N. & M.S.F. 5 Mc/s, AT THE INSTANT OF RECORDING ON MAGNETIC TAPE.

CRYSTAL CHRONOMETER PULSES ARE PLOTTED AT U.T., SCALED FROM STANDARD FREQUENCY PULSES.

FROM: ROYAL GREENWICH OBSERVATORY BULLETIN Nº 56

TIME OF RECEPTION ON U.T.2 OF H.B.N. 5 Mc/s	·0886
RECEIVER LAG AT R.G.O.	·0003
PULSE TRAVEL TIME TO R.G.O.	·0023
PULSE TRAV. TIME TO FIELD STN	·0034
TOTAL DELAY TIME	·0894
TIME OF RECEPTION ON U.T.2, M.S.F.	·0546*
TRAV. & EMISSION DELAY	·0010
PULSE TRAV. TIME TO FIELD STN	·0012
TOTAL DELAY TIME	·0548
DIFFERENCE OF RECEPTION TIMES OF H.B.N. & M.S.F. AT FIELD STN	·0346
DIFFERENCE OF RECEPTION TIMES SCALED FROM VISIBLE PULSES	·0357
DISCREPANCY	·0011 sec.

*PUBLISED RECEPTION TIMES SHOW A DIFF. OF ·0010 sec OVER 24h.

FROM: BULLETIN HORAIRE Nº 15 SER. G. BUREAU INTERNATIONAL DE L'HEURE

TIME OF EMISSION ON U.T.2 OF H.B.N. 5 Mc/s	·0819
PULSE TRAV. TIME TO FIELD STN	·0034
TOTAL DELAY TIME	·0853
TIME OF EMISSION ON U.T.2 OF M.S.F. 5 Mc/s	·0485
PULSE TRAV. TIME TO FIELD STN	·0012
TOTAL DELAY TIME	·0497
DIFFERENCE OF RECEPTION TIMES OF H.B.N. & M.S.F. AT FIELD STN	·0356
DIFFERENCE OF RECEPTION TIMES SCALED FROM VISIBLE PULSES	·0357
DISCREPANCY	·0001 sec.

U.T. FROM CRYSTAL CHRONOMETER TIME.

Fig. 4.5.–6.

4.5. cont.

following five days there are no O.M.A. reception times quoted in R.G.O. Bulletin.

When long distances are involved, the time taken by standard frequency time pulses to travel is longer at night, due to the changed height of the ionospheric layer. This has been taken into account in Table 4.5.-1 and in the calculation for the travel time of W.W.V. in day time and at night respectively. (Fig. 4.5.-5) There are no reception times quoted for M.S.F., H.B.N., O.M.A. and W.W.V. on May 6, 1961, in R.G.O. Bulletin. Consequently the values in Fig. 4.5.-5 are derived from linear interpolation.

The accuracy of the four decimals of the measured reception time of M.S.F. on 5 Mc/s quoted in R.G.O. Bulletin No. 56 (Fig. 4.5.-6) is somewhat doubtful because the reception times vary by 1 and 0 msec within the preceding and the following 24 hours.

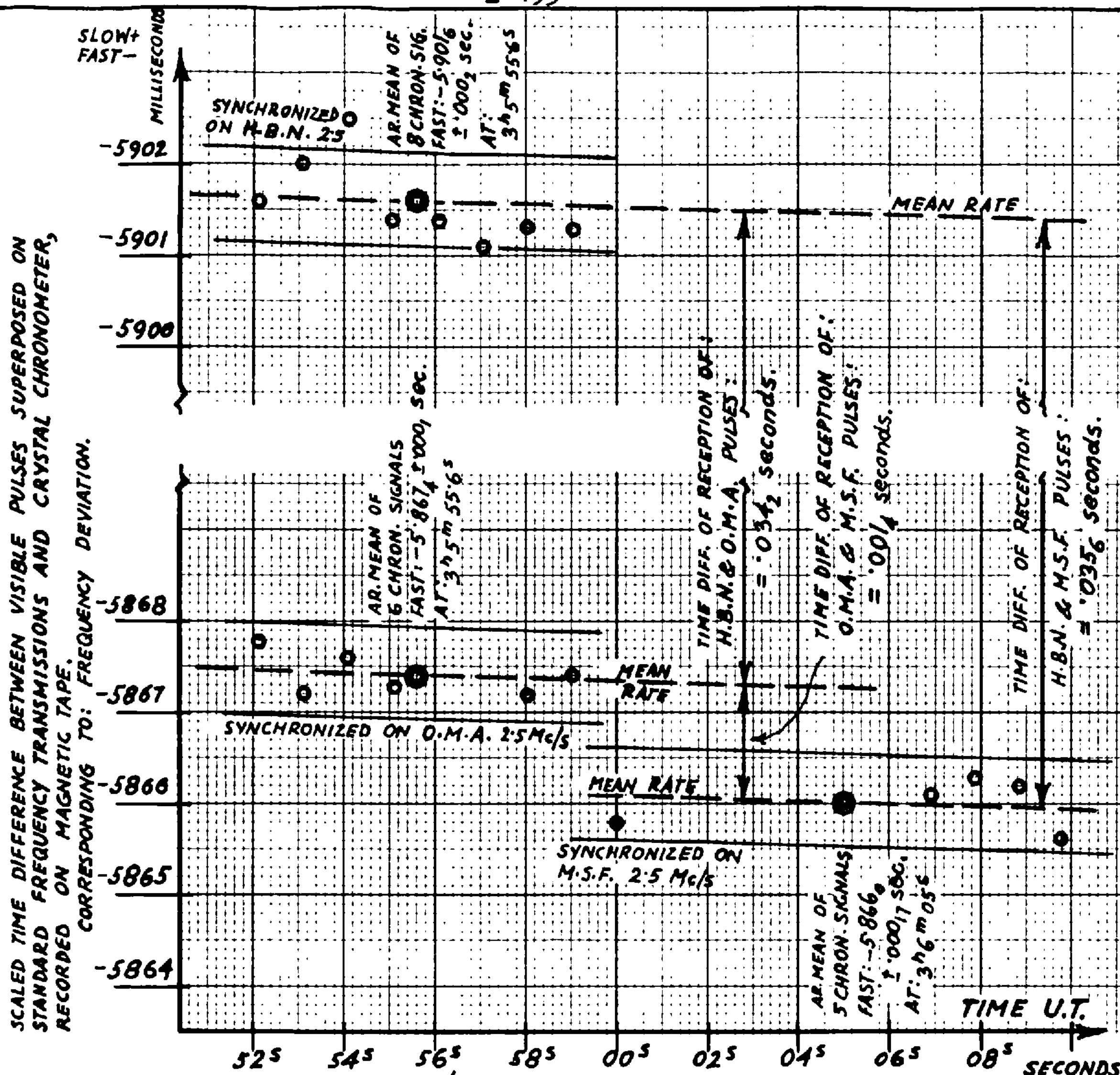
For the purpose of independent time comparisons with the crystal chronometer, transmissions which are separated by only a small time interval, a matter of minutes, can be used, instead of simultaneous receptions of standard frequency time signals, should the latter not be feasible.

The two stations M.S.F. and H.B.N. participating in the co-ordinated time signal service broadcast alternately every five minutes on the same wavelength.

In Fig. 4.5.-8 are given the synchronizations of the crystal chronometer with H.B.N. and M.S.F. receptions. These two consecutive receptions are about three minutes apart.

The reception of simultaneous transmissions and of transmissions which follow closely one another reveal also differences in emission times. One preliminary emission time common to all co-ordinated stations is published from extrapolated values in R.G.O. Time Service Circular.

Co-ordinated time signals are supposed to be synchronized within



DATE: FRI. MAY 12., 1961 $03^h\,05^m$

U.T. INDICATED BY SECONDS PULSES SUPERPOSED ON STANDARD FREQUENCY TRANSMISSIONS: H.B.N., O.M.A., & M.S.F., 2.5 Mc/s,
AT THE INSTANT OF RECORDING ON MAGNETIC TAPE.
CRYSTAL CHRONOMETER PULSES ARE PLOTTED AT U.T., SCALED FROM STANDARD FREQUENCY PULSES.

CRYSTAL CHRONOMETER FAST ON: U.T.2 (REF. R.G.O.),: ON MAY 12, 1961, AT: 3^h6^m

$$\text{H.B.N.}: \quad -5{\cdot}9016 \qquad 0900 \qquad = -5{\cdot}811_6$$

$$\text{O.M.A.}: \quad -5{\cdot}8674 \qquad 0579 \qquad = -5{\cdot}809_5$$

$$\text{M.S.F.}: \quad -5{\cdot}866_0 \qquad 0546 \qquad = -5{\cdot}811_4 \text{ seconds.}$$

U.T. FROM CRYSTAL CHRONOMETER TIME.

Fig. 4.5.-7.

U.T. FROM CRYSTAL CHRONOMETER TIME.
(Fig. 4.5.- 7.)

FROM: ROYAL GREENWICH OBSERVATORY BULLETIN Nº 56		H.B.N.	O.M.A.		M.S.F.	
TIME OF RECEPTION ON U.T.2	0892		0568		0544	
RECEIVER LAG AT R.G.O.	0003		0003		0003	
TRAVEL TIME TO R.G.O.	0023		0036		0007	
TIME OF EMISSION ON U.T.2		0866		0529		0534
TRAVEL TIME TO FIELD ST^{N}		0034		0050		0012
TOTAL DELAY TIME		0900		0579		0546

FROM: BULLETIN HORAIRE Nº15 BUREAU INTERNATIONAL DE L'HEURE	H.B.N.	O.M.A.	M.S.F.
TIME OF EMISSION ON U.T.2	0825	0492	0491
TRAVEL TIME TO FIELD ST^{N}	0034	0050	0012
TOTAL DELAY TIME	0859	0542	0503

DIFFERENCES BETWEEN RECEPTION TIMES OF TRANSMISSIONS AT THE FIELD STATION:

		O.M.A.	M.S.F.
H.B.N.	SCALED FROM VISIBLE PULSES	0342	0356
	R.G.O. BULLETIN	0321	0354
	BULLETIN HORAIRE	0317	0356
O.M.A.	SCALED FROM VISIBLE PULSES		0014
	R.G.O. BULLETIN		0033
	BULLETIN HORAIRE		0039

DATE: FRI. 12. MAY, 1961,

TIMES ARE GIVEN IN DECIMALS OF A SECOND WITH THE DECIMAL POINT OMITTED.

TABLE 4.5.-2

4.5. cont.

one msec; but from the difference in emission times given in the above diagram it is seen that the tolerance is exceeded by quite an amount.

If the reception times are far apart and the transmitting stations not properly identified, the small difference between the times of receptions might easily be overlooked and smoothed out in the time-error curve. The difference in reception times is not only due to unequal emission times but depends also on the location of the field station. Hence presumed interference between standard frequency time signals from various sources can occur at the receiving end regardless of whether the emission times are synchronized or not.

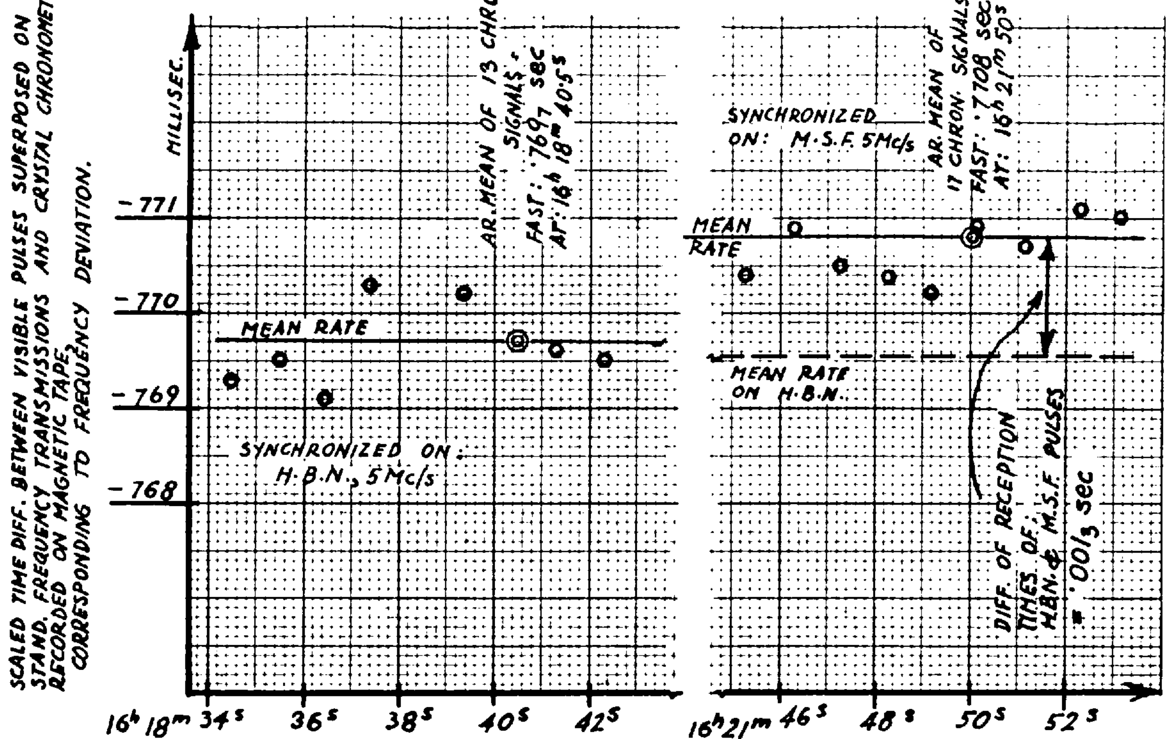
U. T. 2 extracted from crystal chronometer time with the aid of comparisons with standard frequency time signals from more than one source is obviously more reliable than a reference made to only one single transmitter. When time comparisons have to be based on, or are purposely referred to one transmitting station only, great care has to be exercised to identify the source with absolute certainty.

Results obtained from field experiments, specimens of which are given above, show that the scaled time differences and hence the calibration of the crystal chronometer can be obtained at an accuracy of $\pm$ one millisecond, or even better.

"For an auxiliary standard of frequency, a comparison accuracy of a few parts in 10^8, when great care is exercised", is quoted in H.M.S.O. "Quartz Vibrators", p. 176. Obviously by an "auxiliary standard of frequency" is meant a stationary standard of frequency.

To make use of the accuracy obtained in measuring time intervals, the unit of time, i.e. the length of one day, has to be measured to 0.001 second. When this precision is reached small variations which do exist in the unit of time have to be considered.

An accuracy of 1 part in 10^8 is at present the limit of the precision in obtaining the mean rotation of the earth on its axis over a -quarter year period. The accuracy of an astronomical time determination is limited



DATE: SUN. MARCH 31, 1963

U.T. INDICATED BY SECONDS PULSES SUPERPOSED ON STANDARD FREQUENCY TRANSMISSIONS: H.B.N., & M.S.F., 5 Mc/s, AT THE INSTANT OF RECORDING ON MAGNETIC TAPE. CRYSTAL CHRONOMETER PULSES ARE PLOTTED AT U.T., SCALED FROM STANDARD FREQUENCY PULSES.

AT FIELD ST^N TYWARNHALE:
CRYSTAL CHRONOMETER FAST ON:

AT:	H.B.N. 5 Mc/s	M.S.F. 5 Mc/s		
$16^h 18^m 40{\cdot}5^s$	7697 0001_6			
$16^h 21^m 50^s$	7695	7708		
PULSE TRAV. TIME TO FIELD ST^N	0034	0013		
TIMES OF EMISSION	7661	7695	DIFF.:	0034

R.G.O. BULLETINS Nº 79 AND 80:

TIMES OF RECEPTION (INTERPOLATED)	9789	9814		
PULSE TRAV. TIME	0023	0007		
TIMES OF EMISSION	9766	9807	DIFF.:	0041

BULLETIN HORAIRE Nº 2 SER. 6

	H.B.N. 15^h	18^h	20^h	M.S.F. 15^h	18^h	20^h		
PREL. RECEPTION TIMES (DEMI DEFINITIVE)	9756	9759	9756	9786	9787	9787		
PULSE TRAV. TIME	0026			0027				
TIMES OF EMISSION	9730	9733	9730	9759	9760	9760	DIFF.:	0029 0027 0030

TIMES ARE GIVEN IN FOUR DECIMALS OF A SECOND WITH THE DECIMAL POINT OMITTED.

Fig. 4.5.–8

by the influence of refraction. The astronomically determined time for one night may be in error by an estimated amount of $\pm$ 18 msec when the impersonal micrometer is employed, $\pm$ 12 msec with photo-electric registration, and $\pm$ 4 msec when obtained from the photographic zenith tube. At present higher accuracy in time determination cannot be obtained with astronomical methods.

It follows that in the field time intervals, in terms of the adopted U. T. (U. T. 2, U. T. 1, or U. T.0.) of an observatory, obtained from the crystal chronometer with the aid of standard frequency time signals, are of higher accuracy than U. T. arising from local astronomical observations. Astronomical time determinations at a field station obviously cannot compete with those from stationary observatories.

Time signals have been transmitted for four decades, and it is somewhat astonishing that there has not been a wider application of their use in field astronomy in conjunction with a portable crystal chronometer.

Users of the national time and frequency services engaged in physical research are more interested in accurate frequencies than U. T.; those concerned with surveying and astronomy require both accurate time intervals and precise U. T. First order longitude determinations can be effected if U. T. is available at an accuracy of a few milliseconds.

The crystal chronometer in its specified form and the methods of extracting U. T. from it provide a high precision tool for use in field astronomy.

Time intervals of higher accuracy obtainable from a portable frequency standard are not required for this purpose and not justified, for reasons set out below.

In the first place, the reception of time intervals marked by consecutive transmitted frequency time pulses is not more accurate than a few milliseconds. This is due to the recording of the reception and to the effects of the transmitting medium. In the second place the

4.5 cont.

published emission and the published reception times of standard frequency time pulses, which are the essential values for longitude determination, are average daily means. These smoothed-out values are obtained from various sets of receptions at observatories, and are referred to a standard of time. Related to means of stellar observations they are made available after a considerable length of time. Although quoted to four decimals of a second, their value can depart by several milliseconds at the instant astronomical observations are performed at a field station. At any time there may be a variation in the rotational speed of the earth of unpredictable and unknown duration. The values of transmitted time signals in terms of U. T. 2, which were available at the instant of field observations in 1962 and 1963, may have differed from the final corrected times by an amount of $\pm$ 17 milliseconds in 1962, $\pm$ 11 milliseconds in 1963, distributed over the year.

5. Observations for Astronomical Latitude, Longitude and Azimuth

5.1. General.

The absolute position of a point on the surface of the earth, defined by latitude and longitude, is accomplished by taking sights to stars which, for this purpose, are regarded as point targets. From directions so obtained the problem is solved in similar fashion to fundamental ground survey methods, namely resections and intersections, which are used to obtain relative position of points. Star targets and observation stations each belong to a system; and because of the movement of both systems with respect to each other, precise timing of observations, unfamiliar in terrestrial geodesy, becomes necessary.

The observations link together the direction of gravity, the co-ordinates of stars (right ascension and declination), the rotation of the earth (time), and produce the co-ordinates of the zenith of the observation station.

Latitude and azimuth can be obtained without a knowledge of time, but the determination of longitude is applied chronometry, the measurement of time which is basically a process of counting. (Described in previous sections).

5.2 Definitions.

Text and diagrams are kept to agree with the definitions of geoid, spheroid, geodetic and astronomical latitude, longitude and azimuth given in Bomford's Geodesy 3.02, 3.03, 3.04, 1962 edition.

5.3 Deviation of the Plumbline

Astronomical observations for latitude, longitude and azimuth are executed with reference to the direction of gravity at the point of observation, i.e. on the physical surface of the earth, and refer to the

5.3. cont.

instantaneous axis of rotation. For permanent records the observations are, as a rule, reduced to a mean pole.

The direction of gravity, which is the direction of the plumbline, is perpendicular to the horizontal tangent to the equipotential surface through the point of observation. It follows that the plumbline is defined physically from the potential theory (gravitational potential) and cannot be determined from the geometrical shape of the surface of the earth. It is the only perpendicular direction to the field of gravity which can be established and seen as a reality in nature. The direction of the plumbline is obtained with the aid of spirit bubbles or with other arrangements; thus, as an implication of a matter of fact in physics, the direction of gravity is introduced into geometrical methods of geodesy.

In geodesy, points of observation and hence survey systems are projected from the physical surface of the earth to a corresponding position on a reference surface - the spheroid of reference, by mathematical methods.

The normal to this reference surface, the spheroidal normal, in the projected points will generally deviate from the direction of the plumbline at the corresponding points on the physical surface. This difference in direction is called the relative deviation of the plumbline, and is defined by the difference between a geodetic and a corresponding astronomic set of angles. In other words, the deviation of the plumbline is the link between geodesy on the earth's surface and geodesy on the reference spheroid.

The absolute deviation of the plumbline is defined as the difference of the direction of the perpendicular at a point on the geoid, and the direction of the normal, in the corresponding point on the spheroid of reference.

The geoid as introduced by Stokes substitutes the figure of the earth by condensing the protruding topography vertically down until it coincides with sea-level.

5.3. cont.

It follows that, for geodetic and for astronomical observations, points on the physical surface of the earth are first projected with curved plumblines on to the geoid and from there with the spheroidal normal on to the spheroid of reference. The astronomically observed latitudes and longitudes require therefore a reduction to compensate for the influence of the curvature of the plumblines, which is the astronomic component. The amount of deviation of the plumbline depends on the selection of the type of reference surface, its form, orientation and position.

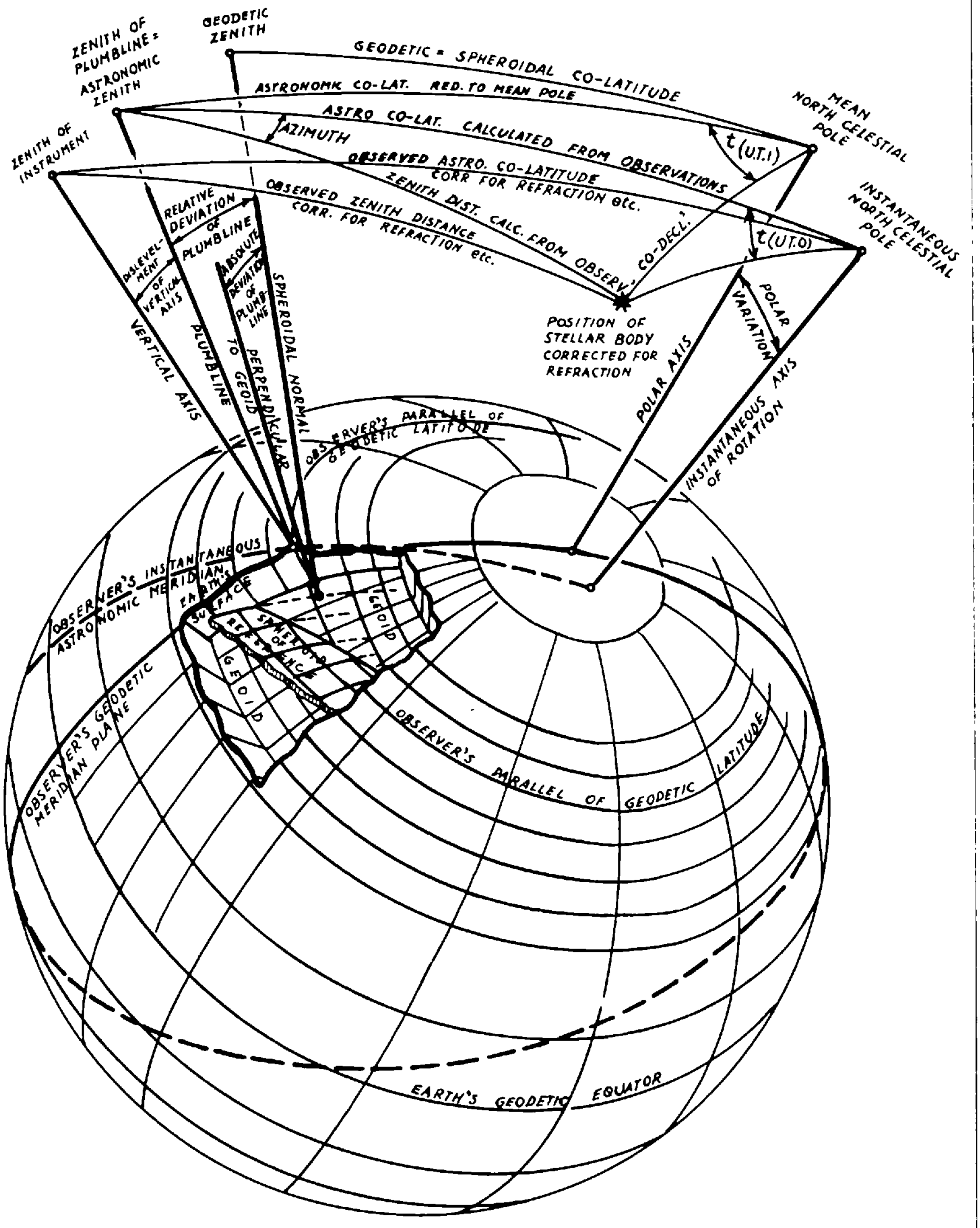
The choice of a particular spheroid of reference and its orientation can annul any deviation of the plumbline, even where irregularities of mass distribution are present; and vice versa, the reference surface can be chosen in such a way as to show plumbline deviations even in the absence of mass anomalies.

The answer to the question about the geometric possibility of the deviation of the plumbline at a particular station is obtained when applying the Laplace equation. Therein, in its application as condition equation or as a check at survey stations established by geodetic and astronomical observations, lies its importance.

As a rule, whenever a satisfactory result is obtained, the deviation is accepted as plausible. And, should the equation produce a value for the deviation which is not permissible and not appropriate to the area, then observational errors are suspected.

Various sources of errors, dislevelment of the vertical axis, irregularities of the pivots, of the graduation of the plate, micrometer or vernier, optical defects, temperature effects, etc., will cause a systematic distortion of observational results; other errors, e.g. collimation axis error, transit axis error, horizontality of the cross wires, will be largely compensated.

As shown in Fig. 5.3.-1, the astronomic meridian through the point of observation is referred to the astronomic zenith (i.e. zenith of the plumbline) and the instantaneous north celestial pole. The astronomic



GEODETIC ZENITH
ZENITH OF PLUMBLINE = ASTRONOMIC ZENITH
ZENITH OF INSTRUMENT
GEODETIC = SPHEROIDAL CO-LATITUDE
ASTRONOMIC CO-LAT. RED. TO MEAN POLE
ASTRO CO-LAT. CALCULATED FROM OBSERVATIONS
AZIMUTH
OBSERVED ASTRO. CO-LATITUDE CORR FOR REFRACTION etc.
ZENITH DIST. CALC. FROM OBSERV.
OBSERVED ZENITH DISTANCE CORR. FOR REFRACTION etc.
MEAN NORTH CELESTIAL POLE
t (U.T.1)
INSTANTANEOUS NORTH CELESTIAL POLE
t (U.T.0)
CO-DECL.
POSITION OF STELLAR BODY CORRECTED FOR REFRACTION
POLAR VARIATION
POLAR AXIS
INSTANTANEOUS AXIS OF ROTATION
RELATIVE DEVIATION OF PLUMBLINE
DISLEVEL-MENT OF VERTICAL AXIS
ABSOLUTE DEVIATION OF PLUMB-LINE
VERTICAL AXIS
PLUMBLINE
PERPENDICULAR TO GEOID
SPHEROIDAL NORMAL
OBSERVER'S PARALLEL OF GEODETIC LATITUDE
OBSERVER'S INSTANTANEOUS ASTRONOMIC MERIDIAN
EARTH'S SURFACE
SPHEROID OF REFERENCE
GEOID
OBSERVER'S GEODETIC MERIDIAN PLANE
OBSERVER'S PARALLEL OF GEODETIC LATITUDE
EARTH'S GEODETIC EQUATOR

Fig. 5.3.-1.

5.3. cont.

meridian may or may not go through the instantaneous north terrestrial pole.

In the diagram (Fig. 5.3.-1) it is presumed that the plumbline lies in the plane of the local meridian, and that the local meridian passes through the point of observation and the instantaneous terrestrial poles.

The hemisphere is represented in orthographic (horizontal) projection.

5.4. Motion of Stars in the Field of View of the Telescope.

Apparent motion of the stars on the celestial sphere and in the field of view of the telescope, as seen from the northern hemisphere.

The apparent motion of stars on the celestial sphere, nearer or at the meridian of the observer, south of the zenith, that is between zenith and celestial equator, and south of the celestial equator is from left to right, Fig. 5.4.-1 (a);

between zenith and pole, from right to left, Fig. 5.4.-1 (c), and north of the pole, also termed below the pole, from left to right, Fig. 5.4.-1 (e).

In the vicinity of, or at the prime vertical, east stars, i.e. stars to the east of the observer, and stars at east-elongation move upwards, Fig. 5.4.-2 (a); west stars near, or at the prime vertical, and stars at west elongation move downwards.

In the field of view of an inverting telescope the stars trace the same track as on the celestial sphere, but in the opposite direction. Therefore, the path of the star is seen inverted and left-right interchanged.

In the field of view of the diagonal eyepiece, which is invariably used in field astronomy, the path of the star is also seen inverted, but correct as to its left-right position.

Therefore stars nearer or at the meridian south of the zenith appear to move from left to right, Fig. 5.4.-1 (b), between zenith and pole appear to move from right to left, Fig. 5.4.-1 (d), and north of the pole appear to move from left to right, Fig. 5.4.-1 (f).

East stars at or near the prime vertical and stars at or near east elongation have an apparent movement downward, Fig. 5.4.-2 (b), west stars in the corresponding positions, upward.

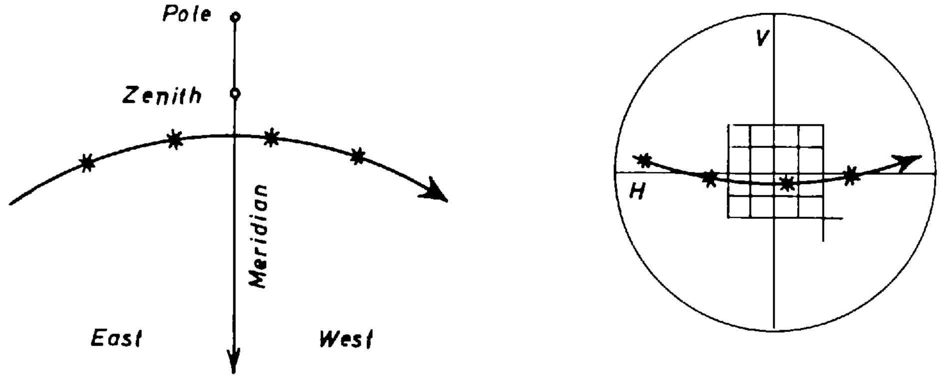
Briefly, the virtual image of the path of the star, seen with a diagonal eye peice, is rectified as to movement in azimuth and inverted as to

Apparent Motion of Stars nearer the Meridian of an Observer on the Northern Hemisphere

on the Celestial Sphere, as seen by the Observer.

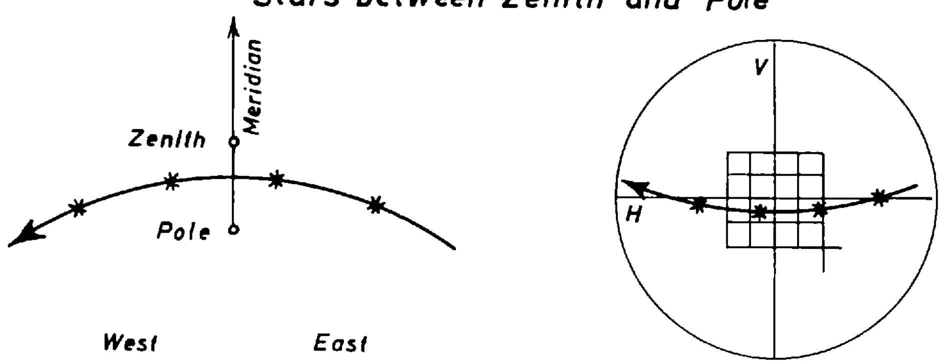
in the Field of View of the Diagonal Eye Piece. Circle left or Circle right

Stars South of the Zenith



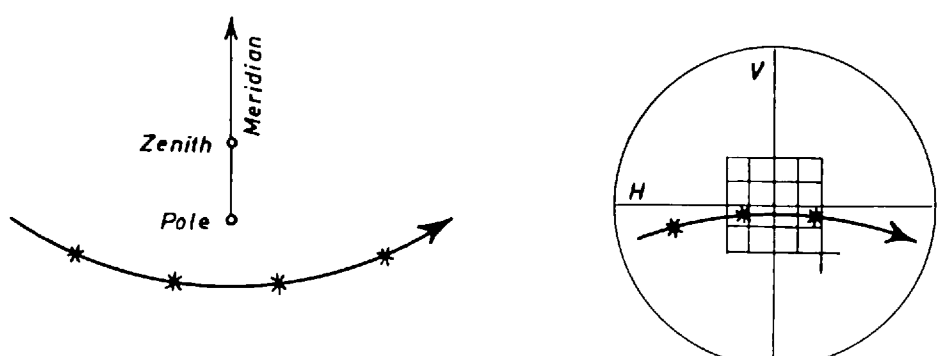
(a) (b)

Stars between Zenith and Pole



(c) (d)

Stars below the Pole



(e) (f)

V = central vertical wire
H = central horizontal wire

The curvature of the path is greatly exaggerated to illustrate the course of the star's motion. The telescope is fitted with a grid reticule.

Fig. 5.4.-1.

5.4. cont.

movement in altitude.

Sometimes, it is convenient to rotate the diagonal eye piece, in which case a rotation to either side up to about 90° impels the observer to face at right angles to the direction of the star sighted, and effects a rotation of the virtual image by about 180°. The apparent path of the star in the field of view is thereupon rectified as to movement in altitude and inverted as to movement in azimuth. Fig. 5.4.-2 (c).

Making use of the above changes of the direction of the star's movement in azimuth and altitude may be welcomed as a variety in routine observation, when taking several observations on to the same star, but does not eliminate the personal equation. Likewise it does not alter the choice of either altitude or azimuth bleeps.

Apparent paths of stars across the grid reticule in the field of view of the diagonal eye piece.

The star's path, when crossing the grid reticule (see Section 5.5.7) represents an arc of about two minutes, and is regarded as a straight line, since the small correction for curvature is negligible.

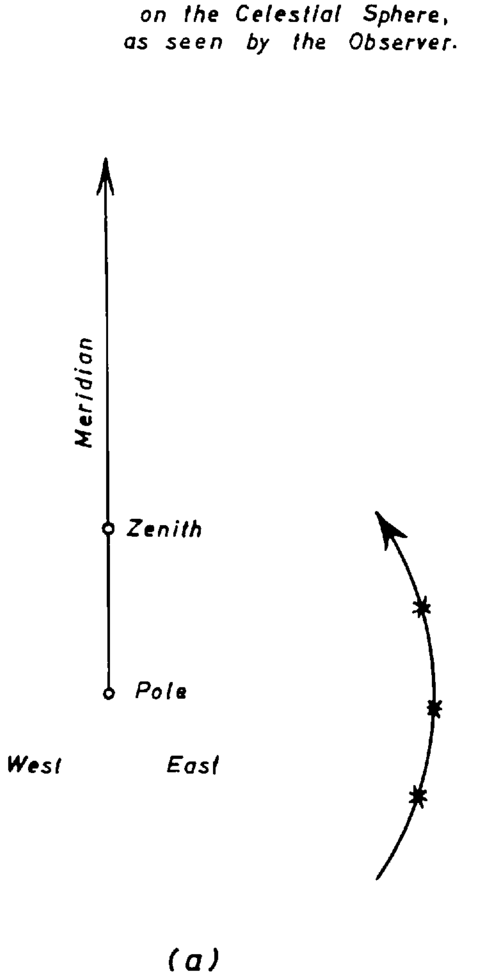
The slope angle formed by the track of the star and the horizontal wire, or wires, of the reticule is equal to the parallactic angle of the astronomical triangle, or to its supplement for anticlockwise angles.

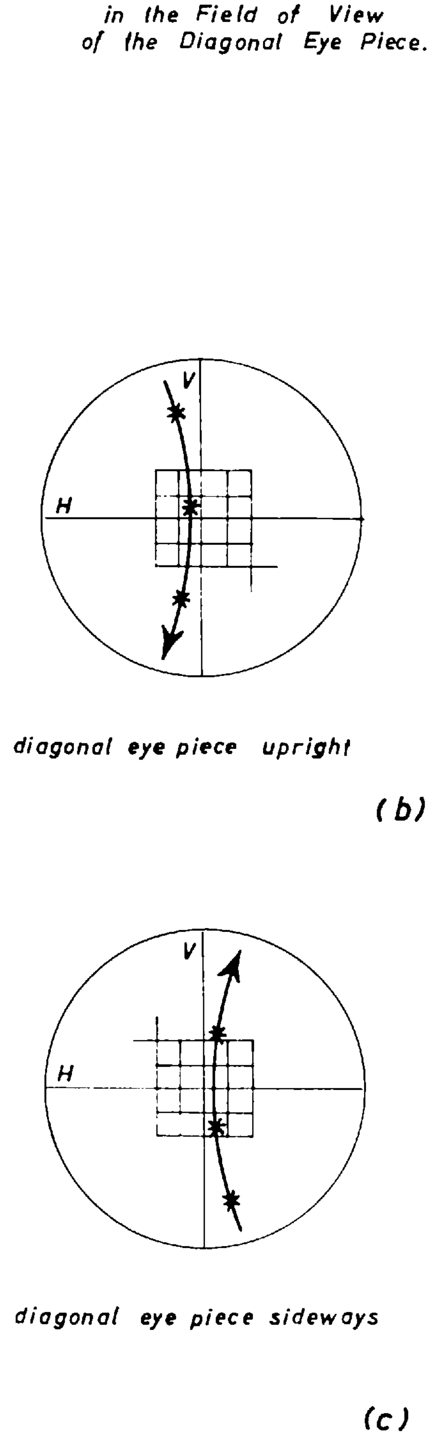
The amount of slope, or the size of the parallactic angle, will cause the star's path to intersect on the grid reticule: (Fig. 5.4.-3)

all parallel lines of one family	and	(a) all lines (b) some lines (c)(d) coinciding with one line (e)(f) none	of the other family

Case (a) will occur only when the parallactic angle is exactly 45°(135°) and one corner intersection of the grid lines is pointing towards the

Apparent Motion of Stars near East Elongation

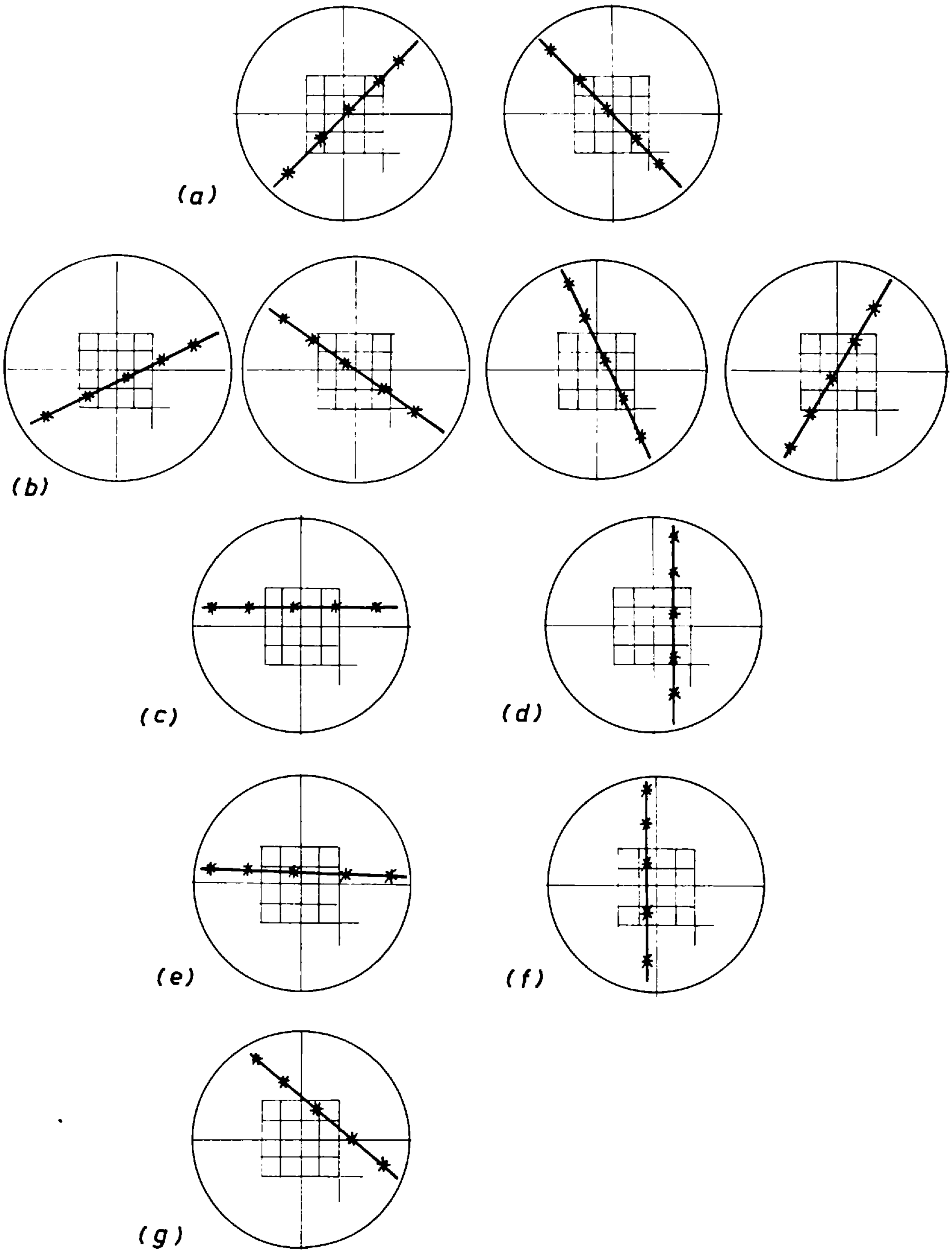




The curvature of the path is greatly exaggerated to illustrate the course of the star's motion. The telescope is fitted with a grid reticule.

Fig. 5.4.-2.

Apparent Paths of Stars in the Field of View of the Diagonal Eye Piece



The telescope is fitted with a grid reticule.

Fig. 5.4.-3.

5.4. cont.

path of the star.

Case (b) applies to stars which are neither at the meridian nor at elongation.

In case (c) the parallactic angle is 0° (180°); strictly, taking the curvature of the star's path into consideration, this case only applies when the telescope is pointing to the meridian and the star is on the celestial equator; and also for stars at the horizon when observed from the pole.

Case (d) happens when the parallactic angle is 90° (270°), and also when a star is at the celestial equator and the observer on the equator.

Case (e) is for stars very close to the meridian.

Case (f) for stars near elongation.

In case (g) the path of the star does not intersect all lines of any of the two families of parallel grid lines. This case can be avoided by lowering or raising the telescope in altitude, or rotating in azimuth.

Cases (a), (c) and (d) are rare, cases (b), (e) and (f) are most common, when selecting stars at random, without preparing a star programme.

5.5. Timing of Optical Observations

If the theodolite is in perfect adjustment the line of sight, (= line joining the eye of the observer and the intersection of the cross wires), the axis of collimation and the optical axis of the telescope are coinciding, and intersect the trunnion axis perpendicularly.

Further, the horizontal cross wire is perpendicular to the vertical cross wire and parallel to the trunnion axis.

All heavenly bodies or points on the celestial sphere which appear to lie on the great circle which is produced by the intersection of the celestial sphere with the plane containing the eye of the observer and the vertical cross wire, have equal azimuth. Heavenly bodies on the great circle produced by the intersection of the celestial sphere with the plane containing the eye of the observer and the horizontal cross wire have neither equal azimuth nor equal altitude. Exceptions to the above occur at particular pointings of the telescope, and/or at special observation places.

The transit of a star through the centre of the cross wires can be recorded on the horizontal and vertical circles which give the azimuth and altitude of the heavenly body, after the necessary corrections for the direction to the reference object, collimation error, index error, dislevelment, refraction, etc., have been applied.

Stellar crossings of the other parts of the vertical and/or horizontal cross wire require reduction to the centre, which can be expressed as a correction of the circle readings to represent azimuth and altitude values.

The azimuth and altitude readings can be related to time. The instant at which a heavenly body crosses the reticule lines can be recorded by various means: some of which can be used efficiently

5.5. cont.

by the field engineer and are therefore outlined in the following sections.

5.5.1. Timing of Optical Observations by the Eye and Ear Method.

Text books on field astronomy adequately describe the eye and ear method of timing stellar transits. It is also the most convenitonal method to use where the "true" time of the star's passage is estimated.

The employment of a chronometer with seconds beat and the estimation of the time interval to the nearest 0.1 second make timing errors in the order of 0.5 second common.

On the whole, the precision in timing which can be achieved with this method depends on the ability of the observer.

A similar method consists in "calling out" the star's transit, which requires an assistant.

The accuracy of the crystal chronometer can not be fully utilized by the eye and ear method and its variations, or by "calling out"; therefore, these methods were not applied in field experiments where the crystal chronometer was employed.

5.5.2. Timing of Optical Observations with Stopwatch and Chronometer

With this and the following methods the ear of the observer is redundant.

The process of estimating the time interval adopted in the foregoing method is substituted by the stopwatch, which is used for the measurement of time fractions marked by a mechanical, or crystal controlled, chronometer and the star's transit through a cross wire.

There are several means of extracting the transit time from the stopwatch and chronometer. The choice of one of them is a matter

5.5.2. cont.

of preference of the field engineer or of the booker, and has no significance as regards to accuracy. The precision of timing the stellar transit rests with the eye's share in optical perceptivity of the event, and with the reaction of the observer.

The quality of the stopwatch (Section 3.8.) and its number of escapement beats per second obviously are restrictions from the first. The employment of one single stopwatch limits the timing of stellar crossings to one at a time; measurements can be repeated only after intervals of several minutes.

It is rather doubtful whether an accuracy of about one tenth of a second, as it is generally referred to in the literature, can be relied upon or not. In field astronomy experiments can not be executed at a reduced scale of space and time in the laboratory, as can be the case in other branches of physics. Data provided from field work are in field astronomy the bulk on which to base a judgement of methods and of instruments employed. Analyses of field results can be influenced by opinion on what might appear or pretend to be an adequate method or procedure.

5.5.3. <u>Timing of Optical Observations with the Micrometer Screw.</u>

Various micrometers can be mounted in the focal plane of the objective and eye piece, or can be used in conjunction with the double image representation of the star object. These micrometers are mainly used to measure star co-ordinates, if need be from known or measured time differences, rather than for timing star transits.

A micrometer designed originally by Repsold and known as the impersonal micrometer or self recording micrometer is essentially used for timing optical observations. A hand operated micrometer screw enables the observer to place one single or a double thread on the star image, and to follow it during its path across the field of view. A steady coincidence of thread and star is maintained by moving smoothly the micrometer screw. The electric contacts on the micrometer drum mark the instants at which the star's passage attains definite positions in the field of -view. The contact times are registered on a chronograph. This arrangement eliminates the operating of a marker key by the observer, and avoids the personal timing error of stellar transits, but it does not completely remove the personal equation. The method can be regarded as a semi-automatic registration of optical observations.

The impersonal micrometer can be used perfectly well in conjunction with a crystal crronometer. The output pulses of the electric contacts of the impersonal micrometer can be recorded on a tape recorder simultaneously with the crystal chronometer beats.

The direction of the star's motion in the field of -view and the direction of the movement of the micrometer threads embracing the star, are equal only for meridian stars and for stars at elongation. For an observer on the equator or on the poles there are exceptions to this rule. The unequal direction of both movements when observing stars selected at random, causes some difficulty in manipulating the micrometer as regards to smoothness in the pursuit of a star. There is no possibility of eliminating the difference of the speeds of stars with the impersonal micrometer, and some observers do not find it easy to get readily acquainted with the manual operation of the micrometer knob when stars of different declination and right ascension are observed in quick succession.

An impersonal micrometer adapted for a telescope of 45 magnification and intended for highest precision, registers 120 contacts when the hair is carried across the field of view.

The accuracy in timing star transits with it, is accepted to be about 1/30 sec; but this figure refers to the timing of meridian stars

5.5.3 cont.

and of stars at elongation, which are easier to follow with the micrometer drive. Obviously this accuracy is higher than that obtained when timing stars selected at random in any quadrant.

The expression impersonal micrometer is not appropriate, because the eye of the observer has the basic task of estimating the star's velocity and the hand has to produce the wire movement; consequently sense impression and muscular reaction are causing a personal equation. (Section 5.7.) The amount of personal equation and of its error depends partly on the eye of the observer which is effectuating the alignment of the wires with the star; the alignment can not be of superior quality as the one deduced for the autocollimator in Section 5.8.

The employment of the impersonal micrometer requires permanent touching of the driving mechanism at the instant of observing; this is open to criticism with respect to the stability of the instrument.

5.5.4. Timing of Optical Observations with the Rapid Action Shutter.

Optical observations can be timed by interrupting the exposure, i.e. the visible target in the field of view, with a rapid action shutter.

The 'instant' the shutter is released, can be registered on a chronograph via a marker key, or recorded as an audio pulse on a tape recorder. The position of the star image with reference to the cross wires can be registered photographically (Section: 5.5.6), or visually against a scale placed in the field of view. The latter method is adopted by the Hunter shutter, which is released at regular intervals which can be recorded.

The cross wires can be illuminated from the direction of the objective, which renders them dark on a bright background, or the wires are illuminated sideways with reflected light, and appear as bright lines on a dark background. In the first case, the intensity of the background

5.5.4 cont.

illumination has to be such as to allow sufficient contrast for the star image; in the second case the brightness of the cross wires has to be properly balanced with the magnitude of the star observed.

The accuracy which can be achieved is about one or two hundredths of a second.

5.5.5. Timing of Optical Observations by Photo-electric Registration.

Essentially photo-electric registration consists of converting light into photo electrons.

The photo-electric cell on which the light from heavenly bodies or artificial satellites is directed can be placed in the optical axis of the telescope, constituting an attachment to the diagonal eye piece. (Fig. 5.5.5.-1) The response of the photo cell may be recorded as an audio signal on magnetic tape. If necessary amplifiers and filters could be used. The light from the heavenly body can be interrupted by a rapid action shutter, as outlined in the previous Section 5.5.4., or otherwise by the cross wires during the motion of the star. Thus, the stellar crossing of the reticule lines can be registered automatically at the highest or lowest output of the cell.

The width of the cross wires and the size of the open spaces formed, should the reticule contain a grid, (Section: 5.5.7.) can be such as to bear a definite relationship to the image of the star. This relationship, together with the quality and diameter of the star's image will have a modulating effect on the output of the photo cell. The quality and diameter of the star's image depend not only on the star's magnitude, but also on atmospheric conditions and on the properties and optics of the telescope. The telescope of a surveyor's transit of 20 to 30 magnification will produce from the transmitted light of the star, in perfect observing conditions, a clear and steady spot of

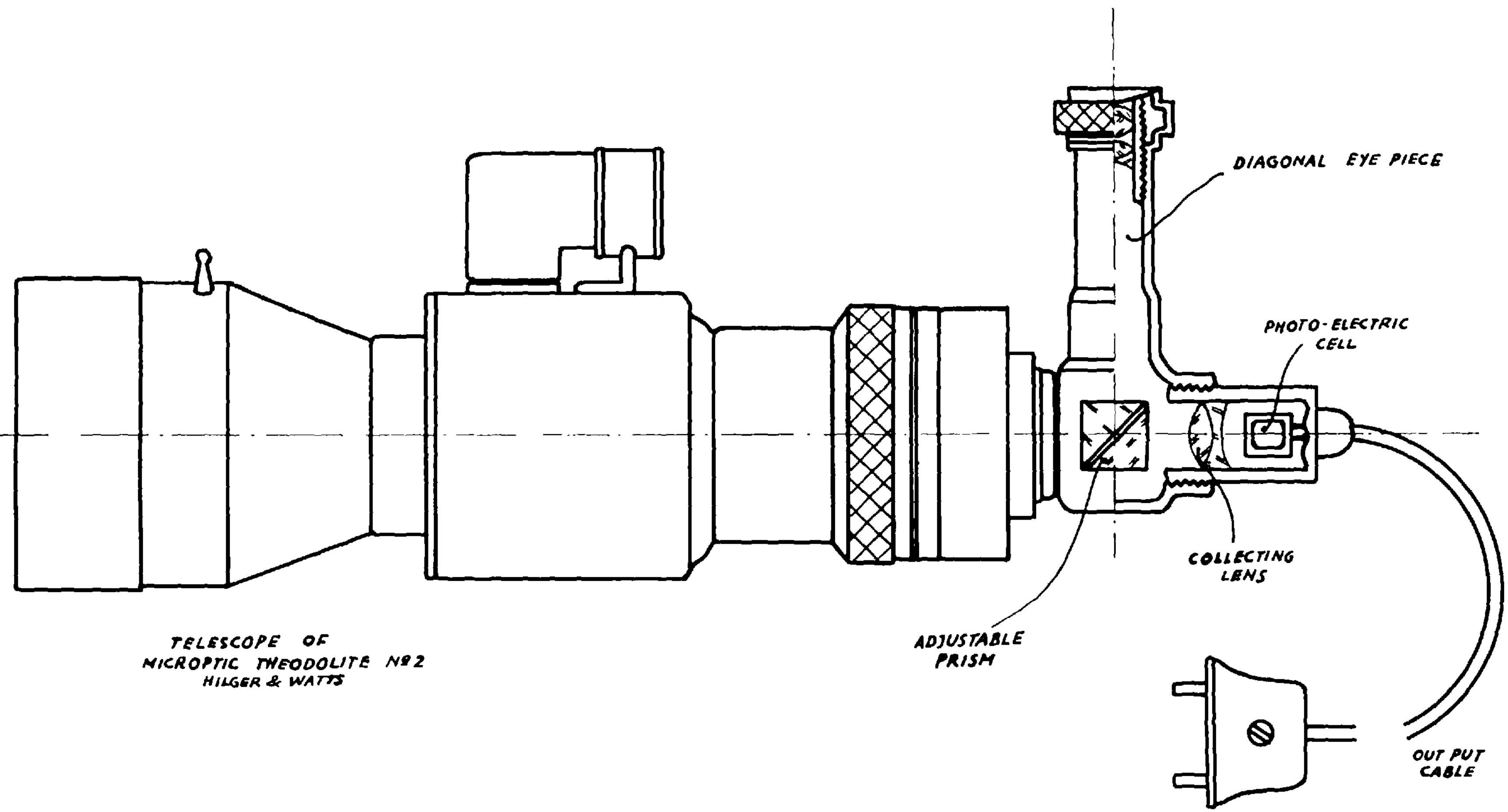

DIAGONAL EYE PIECE
PHOTO-ELECTRIC CELL
COLLECTING LENS
ADJUSTABLE PRISM
TELESCOPE OF MICROPTIC THEODOLITE Nº 2 HILGER & WATTS
OUT PUT CABLE

Fig. 5.5.5.-1

about 1" arc diameter or less. Hence, the width of the lines of an ordinary reticule will suffice to obscure the image completely.

The light from the heavenly body can also be split by a prism placed in the optical axis of the telescope, and reflected from it on to two cells. The difference of the output of the cells is a measure of the position of the stellar image with respect to the cross wire. This arrangement makes the cross wire redundant.

Turbulent layers of the upper atmosphere cause fluctuations of the intensity of the light from celestial bodies. This phenomenon, known as scintillation, and disturbances arising from vibrations of the telescope due to wind, will cause the star's image to appear brighter up to 2 or 3" arc and more. The light gathering power of the telescope, collecting stray light from the sky-background, is a further source of disturbance. It is obvious that the cell's output will be distorted. The definition of the output signal of the cell when using stars of low magnitude may not reach the requirements for field astronomy. Clear differences in output of the photocell are desired for accurate registration of the star's crossings. Therefore, a specific dimension is required for the width of the grid lines, if used, and for the size of the open spaces, if any, so that the star's image will be completely covered and uncovered respectively in all observing conditions for general astronomical field work.

A reticule consisting of one or more concentric rings of adequate width will cause, during the star's passage, characteristic time patterns of the variations in output of the photocell, which, with a knowledge of the star's parallactic angle, can be easily converted into altitude and azimuth corrections. Unless some simultaneous visual observation can be executed and recorded, ambiguities arise from symmetrical possibilities, except for stars on the meridian for azimuth correction, and for stars at elongation for altitude corrections. Here a grid pattern consisting of grid lines each having a different width can solve the problem, obviously after being calibrated.

If simultaneous observation during automatic registration should not be possible, a single thick cross wire subtending an angle of about 30" arc will give theoretically either two or three peaks of the response of the photocell. Altitude and azimuth corrections in this case can be obtained from an approximate value of the direction and velocity of the star's track in conjunction with the ratio of the time intervals of maximal output of the cell. Only stars on the meridian or at elongation, or in their immediate vicinity, would give no information for one of the corrections: altitude or azimuth respectively. (Fig. 5.5.5.-2).

Scintillation can disturb the output of the cell to such an extent that unreliable data could be obtained.

Defocussing the telescope will reduce the effect of scintillation, but the blurred image, which will also appear to be larger, will produce weak response of the photocell.

Should other information be desired, such as investigation into the amount of scintillation, rigidity of the observation station, site conditions for meteorological observations, refraction anamalies, etc., then it might be an advantage to have the star's image only partially covered, in which case the line width and open space distances should be kept less than the diameter of the star's image.

For analysing the recorded output of the cell and for separating the effects of scintillation from other image disturbances arising from field

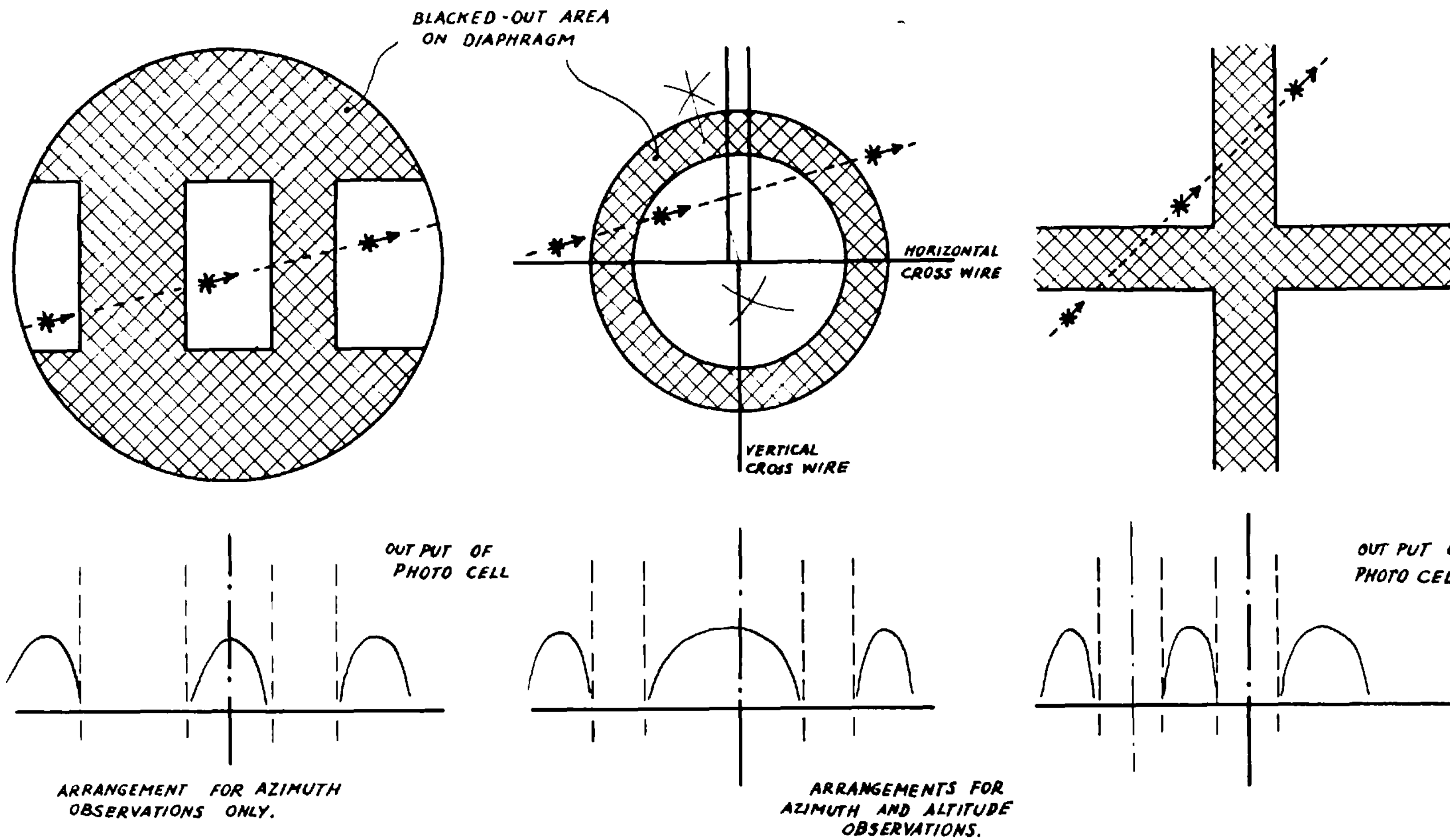


Fig. 5.5.5.-2 PHOTO-ELECTRIC REGISTRATION OF OPTICAL OBSERVATIONS.

5.5.5. cont.

conditions, the reticule could be put out of focus; -thus the photo cell will be affected mainly by scintillation alone; disturbances due to wind and vibrations of the set-up will have no influence. The response of the photo cell will also indicate the correct focussing position of the telescope and the quality of its optics.

This method of using the photo cell and the method described in the following Section are important, because they will find extensive application in the near future by utilizing flash lights released from geodetic satellites.

The flashes can be released from the satellite at predetermined times, and can be observed with a telescope in conjunction with a photo-electric ell or registered photographically. (Section: 5.5.6.)

This method is somewhat the inverse of the one using the rapid action shutter.

No higher accuracy in timing can be expected with flash lights from satellites, because the output and response of the photocell is the limiting factor. Other considerations, such as precision of the ephemeris of the satellite, atmospheric conditions, refraction anomalies, duration and intensity of the flash, indicate the sources of timing errors which limit the accuracy of the final result.

The arrangement shown in Fig. 5.5.5.-1 can be supplemented, if necessary, by a diaphragm, shutter, or a split-image prism.

5.5.6. Timing of Optical Observations by Photographic Registration.

Photographic registration consists of converting light into grains of silver in a photographic emulsion. This process is about one hundred times inferior to the process of converting light into electrons, outlined in the previous section.

The instant at which a photograph of a stellar image together with an illuminated pair of cross wires is taken can be recorded as a signal

5.5.6. cont.

on tape simultaneously with the output pulses of a crystal chronometer. When taking the photograph, the exact moment the star is transiting the cross wires may be missed, in spite of possible visual observation. The photograph will then show the position of the star in relation to the reticule lines. The necessary corrections for altitude and azimuth are obtained by measuring on the photograph the distances from the star image to both cross wires. Care has to be taken to determine the photo scale accurately.

The evaluation of the photograph is greatly facilitated by the use of a grid reticule, which is obviously calibrated. With it the photo scale can be kept at any ratio and there is no necessity of uniformity of the scales of subsequent enlargements when using a projector. The scale may vary for every picture taken. Film shrinkage is also eliminated by taking measurements from the star image to two parallel grid lines.

Special accessories for the theodolite for placing the photo sensitive material in the focal plane of the telescope are not essential. Of the wide range of cameras nowadays available, a single lens mirror reflex camera, permitting observation through the lens system, can be employed to advantage. The synchronized release operating the flash gun can be used to trigger off the marker pulse incorporated in the crystal chronometer, when taking the photograph.

The creation of a photographic image depends on the light intensity of the target, i.e. the properties of the star, the sensitivity of the film, the transmissivity of the atmosphere and of the lens system. Geometrical distortions arising from the optical trail, the mounting and construction of the camera, and film shrinkage are of minor consideration.

The accuracy of measuring the time of film exposure can reach a few 10^{-4} sec. The accuracy of timing star transits is limited by refraction, set-up and adjustment of the instrument, and the

5.5.6. cont.

measurement of -distances on the film or plate.

It can be expected that arithmetic means of groups of observations using the same star will fluctuate during one night and even more between successive nights.

The photographic method of timing star transits can be combined with photographic recording of the circle readings.

5.5.7. Timing of Optical Observations with the Grid Reticule and Marker Pulse

The methods of timing stellar transits by photo-electric and by photographic registration described in the foregoing sections require additional equipment; much of it will be unfamiliar to the average field engineer.

An efficient utilization of the precision of the crystal chronometer was thought to be possible by repeated timing of stellar transits within small time intervals, and their registration on a tape recorder. For this purpose a specially designed reticule was introduced into the telescope in place of the usual diaphragm. It consisted of a vertical and horizontal line centrally located with equally spaced parallel horizontal and vertical lines to form a grid. As the star moved across the grid the observer was able to time a series of transits across the vertical and horizontal grid lines with the aid of an external marker pulse, released manually.

The times of the grid line crossings can be referred to the central wires or to the centre of the cross wires; this requires a knowledge of the calibration values, i.e. the angular distances between the grid lines, and strictly also a correction for the change of refraction.

Theoretically a grid reticule with two families of five lines engraved on the telescope's diaphragm, each line subtending 30", should allow repeated measurements of both altitude and azimuth to be carried out, with adequate accuracy and efficiency. A series of five consecutive observations will reduce the uncertainty of the calculated mean square error of one single star transit to 35%. Twice the number of observations would give an additional reduction of only 10%. About 50 observations would be required to reduce the uncertainty of the mean square error of a single observation to 10%. The large area of the field of view taken up to accommodate 50 reticule lines will involve a great amount of risk of imposing on the observations errors due to optical defects.

5.5.7. cont.

The choice of a five line grid reticule makes the curved star path across the 2 minutes arc space replaceable by a straight line, and differences of refraction for the five consecutive altitude observations can be neglected.

The path of the star in the field of view as shown in Fig. 5.4.-3, Section 5.4., intersects, in the given cases (a) to (f), all five parallel lines of one family or the other. Hence, five marker bleeps can be triggered off by depressing a microswitch, at any one pointing to a star; these are either five azimuth or five altitude observations.

The observer can decide beforehand whether to release five azimuth or five altitude signals, because he should be able to estimate the approximate size of the parallactic angle in the sky, with the unaided eye; if not, he can obtain the necessary information about the parallactic angle from the inclination of the star's track toward the horizontal thread, roughly picked up, by taking a quick sight through the telescope. Thus, the observer gains an approximate knowledge of the orbit of the star in the field of view.

If the parallactic angle is less than 45° (case b), the star's path will intersect all five vertical threads and hence five "azimuth bleeps" can be released giving five azimuth observations; if the parallactic angle is greater than 45° all five horizontal threads are crossed by the star's track in which case the five "altitude bleeps" will mark five altitude observations.

In addition to the star's five line-crossings of one of the two families, the star, in its course over the grid reticule, will also intersect some threads of the other family, obviously at an acute angle. Marker bleeps could be triggered off giving azimuth observations, if the crossings of one complete set of five are altitude observations, and vice versa. This method could be used, except where these additional intersections of the star's path with the grid lines would be at such an acute angle, that the exact crossing point

5.5.7. cont.

would be too undefined. However, experience has shown that the pressing of the marker key should be limited when the star is crossing either the five azimuth lines or the five altitude lines. The precise reason is unknown but may be attributed to the psychology and physiology of vision.

Considering the intersection of the star's path with only one family of parallel lines of the grid reticule: The observer is watching the star's motion and has to devote his entire attention to a recurring event, consisting of five consecutive intersections of two lines at the same angle. The lines are in a definite position. One line is the thread of the grid reticule (of the same family), the other is formed by the movement of the star due to the persistence of vision. These consecutive crossings of symmetrically spaced grid lines and the almost constant velocity of the star produce the phenomenon of consciousness of a routine occurrence to which an observer becomes readily accustomed and from which he finds it not easy to release himself. This may explain why on several occasions the moment is missed when a star is crossing the threads of -alternative families of grid lines.

Viewed through an optical system, stars - being point objects - do not appear as point images, but as small bright discs, surrounded by diffraction rings. Due to the honeycomb-like structure of the retina, one single eye can perceive tiny disc-images commencing from a definite angular size. In the course of crossing the wire, the diffraction ring, which is of less brightness than the disc, is the first part of the star's image to appear on the other side of the wire, where it is subtending a tiny arc and might not be perceived by that particular part of the retina on which it is projected. During this small instant of time the partly split star image appears also of unchanged size. (Persistence of vision). Therefore, the star, when touching the wire, seems "to rest" or "its motion appears to be suspended". Not until the angular size of the advancing part is large and bright enough will it be seen, suddenly, and the remaining part of the split image which has not yet crossed the wire will simultaneously disappear. This together with refraction on small waterdrops on the lens system and scintillation (Section 5.5.5.) may explain the phenomenon of stars "jumping the wire".

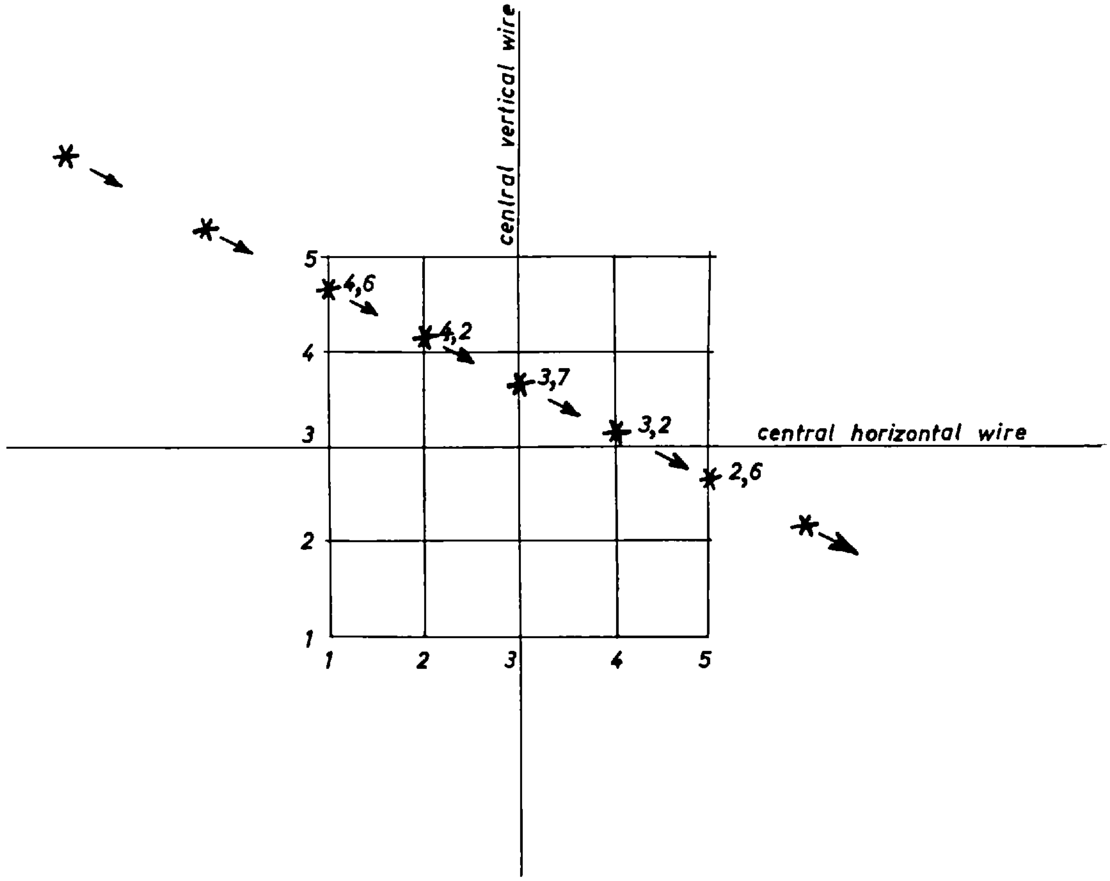
The illusion that the star rests on one side of the wire and an instant later has jumped to the other side causes a systematic error. The "jumping" is more pronounced when the star's crossings are observed on both families of grid lines, than on one family only, during any one observation.

Observations in field astronomy consist customarily of the measurement of one angle, either horizontal or vertical, with the recording of the time.

For the advantage of performing a combined measurement of both angles with the recording of the time, the following method was adopted in all field work.

At the instant when the apparent path of the star intersects each of the five parallel lines successively, the observer presses the marker key, thus recording the time of five observations. In addition the observer estimates the linear distance from the point of intersection to the line of the other family, thus obtaining a measurement for the other angle. Fig. 5.5.7.-1 shows azimuth bleeps and altitude readings. The distance between the grid lines is estimated to one tenth of its interval. The grid lines are presumed to be numbered from the bottom left corner upwards and to the right, regardless of the face position of the telescope. In Fig. 5.5.7.-1 the observer records e.g. at the third azimuth bleep 3.7, at the fourth 3.2, etc. The

Optical Observation with the Grid Reticule



Azimuth observations are released with a microswitch at the instant of the wire crossing

Altitude readings are estimated to one tenth of wire interval

Fig. 5.5.7.-1

5.5.7 cont.

measurements are spoken into the microphone after pressing the microswitch.

The accuracy of estimating a space interval depends, apart from the personal attitude of the observer, on the kind of picture presented by the original subdividing lines, the grid lines themselves, on their length and width in relation to their spacing.

An eventual effect of any remaining error in the verticality or horizontality of the wires is reduced by keeping the star's orbit symmetrical with respect to the centre of the field of view.

There are several possibilities of calibrating the grid reticule. The determination of the absolute value of angular subtense of the horizontal and vertical grid lines is not necessary if the path of the heavenly body is kept fairly symmetrical with respect to the centre of the cross wires. If so, relative measurements of the distances between parallel grid lines are adequate enough, because the corrections for altitude and azimuth are formed by arithmetic means of five observations.

Calibration values can be obtained from measured time intervals between successive grid line crossings, using the known speed of the observed star and its known path; a knowledge of the latitude of the place is necessary. - No additional apparatus is required; any method of timing star transits and its auxiliary equipment described above can be made use of.

The 30" grid reticule devised for the field experiments described here, was calibrated with the use of:

(a) Hilger and Watts 18"inch Autocollimator,

(b) micrometer head and target,

(c) theodolite circle readings and collimator.

The last method was adopted to measure also the width of the grid lines.

As expected the values obtained with the autocollimator - turned out

5.5.7. cont.

to be the most accurate ones. The r. m. s. error of one single measurement of the space interval

with the autocollimator is about $\pm$ .4"
with the micrometer head " $\pm$ 1.6"
with circle readings " $\pm$ 1.3"

In each case the error includes the error of sighting a target, setting and reading a micrometer.

The average angular subtense of the grid - lines is about 5.4". (i.e. "Width" of the grid lines.)

The timing error of stellar transits (dealt with later on), multiplied by $\sqrt{5}$ can be regarded as the r.m.s. error of a single measurement of time intervals between grid line crossings of a star, and therefore can be interpreted as the r.m.s. error of one single measurement of the angular subtense between grid lines for the purpose of calibration.

The methods of calibrating angular subtense of grid lines mentioned above employ stationary targets.

It is universally accepted that the accuracy of pointing with a telescope to stationary targets is three times superior to the accuracy of pointing to a moving target.

It is interesting to note that the calibration values obtained with the various methods are in accordance with judgement based on practical experience.

For the 1" Microptic theodolite it follows:

$$\sqrt{5} \cdot \begin{bmatrix} \text{timing error of one} \\ \text{pointing to star} \\ \text{(arc measure)} \end{bmatrix} \approx 3 \cdot \begin{bmatrix} \text{error in calibration} \\ \text{derived from circle} \\ \text{readings and stationary} \\ \text{target (arc measure)} \end{bmatrix}$$

or:

$$\sqrt{5} \cdot 1.3_5 \approx 3 \cdot 1.3'' \qquad \text{(Table 5.5.7.-6)}$$

5.5.7. cont.

and for the $^{1}/5''$ Microptic theodolite it follows:

$$\sqrt{5} \cdot 0.6_6'' \approx 3 \cdot 0.4''$$

The timing error of one pointing is about $0.6_6''$ (Table 5.5.7.-6) and the calibration error is about 0.4", derived from circle readings to a stationary collimator.

The conversion of the estimated fraction of the space between grid lines into angular subtense can be taken from Table 5.5.7.-1, which has been prepared for azimuth and altitude readings, for observations with the inverting telescope and with the diagonal eye-piece. To allow a wide range of its application, by taking into account individual practice of observers, the table gives conversions into angular measure of estimated intervals between grid lines and of intervals from centre to centre of grid lines. The maximum variation can amount to half the width of a grid line, about $2\frac{1}{2}''$.

The conversions are listed as altitude and azimuth corrections and are obtained from measurements with the 18 inch Hilger and Watts autocollimator, and with a stationary collimator. The angular distances between vertical wires were measured along the central horizontal wire, those between horizontal wires along the central vertical wire. The table is slightly extended to provide for observations taken just outside the area covered by the grid square.

The recorded altitude of a heavenly body observed outside the centre of the cross wires consists of the vertical circle reading, plus or minus the angular subtense derived from the estimated space interval from the central horizontal thread, or: plus or minus the angular subtense between horizontal grid lines should altitude marker bleeps be released.

If the line of sight is inclined, images along horizontal threads belong to objects having different altitudes. Therefore, the recorded altitude

CALIBRATION OF GRID RETICULE OF WATTS 1" MICROPTIC THEODOLITE.

Conversion of estimated Azimuth / Altitude interval (a) of space between grid lines (b) of space from ℄ of grid lines, into angular subtense, in sec. of arc, from central vertical / horizontal wire.

AZIMUTH								ALTITUDE							
EST. INT. invert. tele.' CR diag. eye p. CL −	CL CR +	corr.' a "	b "	EST. INT. invert. tele.' CR diag. eye p. CL +	CL CR −	corr.' a "	b "	EST. INT. invert. tele. or diag. eye p. CL +	CR −	corr.' a "	b "	EST. INT. invert. tele. or diag. eye p. CL −	CR +	corr.' a "	b "
·5	5·5	71	71					·5	5·5	73	73				
·6	5·4	68	68					·6	5·6	71	70				
·7	5·3	66	65					·7	5·7	69	68				
·8	5·2	64	62					·8	5·8	65	65				
·9	5·1	62	60					·9	5·9	63	62				
1·0	5·0	57·7		3·0	3·0	00		1·0	5·0	58·1		3·0	3·0	00	
1·1	4·9	53	55	3·1	2·9	05	03	1·1	4·9	53	55	3·1	2·9	06	03
1·2	4·8	51	52	3·2	2·8	07	06	1·2	4·8	51	52	3·2	2·8	08	06
1·3	4·7	48	49	3·3	2·7	10	08	1·3	4·7	48	49	3·3	2·7	11	09
1·4	4·6	46	46	3·4	2·6	12	11	1·4	4·6	46	46	3·4	2·6	13	12
1·5	4·5	44	44	3·5	2·5	14	14	1·5	4·5	43	43	3·5	2·5	15	15
1·6	4·4	41	41	3·6	2·4	16	17	1·6	4·4	41	40	3·6	2·4	18	18
1·7	4·3	39	38	3·7	2·3	18	20	1·7	4·3	38	38	3·7	2·3	20	21
1·8	4·2	37	35	3·8	2·2	21	22	1·8	4·2	34	35	3·8	2·2	22	23
1·9	4·1	35	32	3·9	2·1	23	25	1·9	4·1	33	32	3·9	2·1	24	26
2·0	4·0	29·6		4·0	2·0	28·1		2·0	4·0	28·7		4·0	2·0	29·3	
2·1	3·9	25	27	4·1	1·9	34	31	2·1	3·9	24	26	4·1	1·9	34	32
2·2	3·8	22	24	4·2	1·8	36	34	2·2	3·8	22	23	4·2	1·8	37	35
2·3	3·7	20	21	4·3	1·7	38	37	2·3	3·7	20	20	4·3	1·7	39	38
2·4	3·6	17	18	4·4	1·6	40	40	2·4	3·6	17	17	4·4	1·6	41	41
2·5	3·5	15	15	4·5	1·5	43	43	2·5	3·5	15	14	4·5	1·5	44	44
2·6	3·4	13	12	4·6	1·4	45	45	2·6	3·4	13	11	4·6	1·4	46	46
2·7	3·3	10	09	4·7	1·3	47	48	2·7	3·3	11	09	4·7	1·3	49	49
2·8	3·2	08	06	4·8	1·2	49	51	2·8	3·2	08	06	4·8	1·2	51	52
2·9	3·1	05	03	4·9	1·1	52	54	2·9	3·1	06	03	4·9	1·1	53	55
3·0	3·0	00		5·0	1·0	57·0		3·0	3·0	00		5·0	1·0	57·8	
				5·1	·9	62	59					5·1	·9	62	60
				5·2	·8	64	62					5·2	·8	64	62
				5·3	·7	66	65					5·3	·7	67	66
				5·4	·6	68	68					5·4	·6	69	69
				5·5	·5	71	71					5·5	·5	72	72

TABLE 5.5.7.-1

5.5.7. cont.

reqqiress correction for reduction to centre, to which the vertical circle readings are referred.

The approximate correction for reduction to centre or to the central vertical cross wire is:

$$\frac{-1}{2\rho''} \cdot (\text{angular subtense from vert. cent. wire})^2 \cdot \tan(\text{recorded altitude})$$

where: $\rho'' = 206\,265$.

For sights taken at an angular subtense of 1' from the central vertical thread (which is the maximal distance ruled on the grid reticule) to stars at an altitude of 70°, the correction is 0.024", and can therefore be neglected.

Further, the altitude has to be corrected for index error and refraction.

The recorded azimuth of a heavenly body observed outside the centre of the cross wires consists of the horizontal circle reading (oriented), plus or minus the angular subtense between vertical grid lines, divided by the cosine of the corrected altitude, when vertical wire crossings are timed, or: plus or minus the angular subtense derived from the estimated space interval from the central vertical thread, also divided by the cosine of the corrected altitude.

The second term of the recorded azimuth is identical with the correction for collimation axis error in pointing.

The angular subtense between parallel vertical or horizontal threads diminishes with the distance from the main horizontal or vertical thread respectively.

The resulting error for an altitude of 80°, for a star image on the corner of the grid square, is about 0.011" and may be disregarded.

Further, the azimuth has to be corrected for dislevelment of the horizontal plate, and for lateral refraction.

5.5.7 cont.

The arithmetic mean of the times of five line crossings does not correspond to the arithmetic mean of the five recorded altitudes, or five azimuth recordings, because a change in altitude or azimuth with time is not linear.

Azimuth or altitude recordings formed by arithmetic means of five observations are justified if the path of the star is a straight line. It depends on the time taken by the star to move across the grid square whether the curvature correction can be omitted or not. The amount of time required depends obviously on the star's declination.

The correction for curvature can be obtained by expressing the altitude and the azimuth as a function of time. The relation, [given in "Plane and Geodetic Surveying" by D. Clark, Vol. II, 1948, p.100:

$$\text{Diff. of meridian altitude} = \sin 2\delta \cdot \sin^2 \frac{t}{2} \rho'' \Big],$$

for the reduction to the meridian, can be used for computing the deviation of the star's path from a straight line.

Although it is not practical to take five grid line crossings of the pole star because of its slow motion, it is interesting to note that the deviation (of the pole star's path from a straight line) over the space occupied by the grid square would amount to approximately 0.3"; which is less than the r.m.s. error of one pointing of the telescope.

The curvature correction to the arithmetic mean height required for the height corresponding to the arithmetic mean of observed times can be calculated also from the relation:

$$\sin h = \sin \phi \,.\, \sin \delta + \cos \phi \,.\, \cos \delta \,.\, \cos t$$

and: $$\sin(h + \Delta h) = \sin\phi \,.\, \sin\delta + \cos \phi \,.\, \cos\delta \,.\, \cos (t + \Delta t)$$

by expanding: $(h + \Delta h)$ in a Taylor series.

For δ ursae minoris (declination $86\frac{1}{2}^{\circ}$) the correction Δh amounts to .003".

The total time taken by δ ursae minoris to travel across the grid square is about $\frac{1}{2}$ min.; it is therefore more convenient to make use

5.5.7. cont.

of only three grid line crossings with additional timing when the star is in the centre of individual grid squares, thus reducing the length of the track.

In practice, the paths of all stars accessible in field astronomy approximate closely to a straight line over the length contained in the square of the grid reticule; therefore the curvature correction can be disregarded.

The linearity of the path can be used to check the accuracy of estimating space intervals in altitude and azimuth, and to examine the attitude of personal judgement of different observers.

Since the path of the star across the grid reticule is accepted to be a straight line, the function which fits best the data of observation (estimated space intervals) is therefore linear. The equation of this "best fitting line" expressing the altitude or azimuth as a function of time has the form:

$$y = b.x + a$$

in a Cartesian co-ordinate system.

Every observation is represented by a point whose co-ordinates x_i and y_i are the time and altitude or azimuth. The constant b is the tangent of the angle with the axis of time, a is the intercept on the latitude or azimuth axis.

If the required function is linear, as in this case, the line has to pass through the point whose co-ordinates are the mean of all. (Centre of gravity).

The arithmetic mean of all five -line crossings therefore constitutes the data of one single observation.

For the determination of the accuracy of the observations, b and a have to be found. The requirement is that the sum of the squares of the distances of each observational result (altitude or azimuth) from the best fitting line, measured parallel to the y or x axis, should be a minimum.

5.5.7. cont.

Table 5.5.7.-2 shows calculations for deriving the accuracy of the estimation of space intervals. The observational values are taken from field work carried out in 1961, with a Watts Microptic theodolite No. 2, under very bad weather conditions. Thus, field experiments have provided data on which to base a judgement of the usefulness of the grid reticule.

Using the values from the table:

$b = 4.7$ and $a = 16.9$

The mean square error of one single observation (time assumed correct) is then:

$$[vv]_y = [yy] - \frac{[xy]^2}{[xx]}$$

$$m_{y\,Alt.} = \pm\sqrt{\frac{[vv]}{n-2}}$$ = r.m.s. error of one estimated fraction of grid - line interval

and:

$$m_{mean} = \pm\frac{m_y}{\sqrt{n}}$$ = r.m.s. error of the arithmetic mean of five estimated fractions of grid line interval.

Using the observational data given in the Table 5.5.7.-2:

$$m_{mean} \approx \pm 0.5''$$ (for this particular observation)

It is important to note that the above accuracy is achieved with one pointing of the telescope in one circle position, by assuming that the timing is correct, and only the estimated space intervals are in error.

The condition of linearity can be used also to derive the accuracy achieved in timing, by assuming that the estimated space intervals are correct. Although this assumption is exaggerated and the result not therefore of great practical value, nevertheless it is interesting to see that (using the figures from Table 5.5.7.-2) the r.m.s. error of the mean of five timings approximates to $\pm$ 0.1 sec.

The r.m.s. error of the time-observations has been worked out by

Table 5.5.7.-2

Accuracy of Estimating Space Fractions of the Grid Reticule.

Star: β Leonis, Date: May, 1961, At: Tywarhale,Cornwall, Azimuth Markers.

No.of Observation	Time seconds x $23^h\ 24^m\ 11^s$	$x_i - x_m$	Altitude: fraction	seconds	y_i $51^{\circ}\ 45'\ 50''$	$y_i - y_m$	xx	xy	yy
1	0.904	- 4.330	2.6	+ 13	63	+ 21			441
2	3.390	- 1.844	3.0	00	50	+ 08			64
3	5.137	- 0.097	3.1	- 06	44	+ 02			4
4	7.007	+ 1.773	3.6	- 18	32	- 10			100
5	9.730	+ 4.496	4.0	- 29	21	- 21			441
[ ]	26.168	0			210	0	45.516190	- 218.022	1050

$x_m = t_m = 5.234$ $\qquad Y_m = \text{Altitude}_m = 42$

If altitude only in error:

$$b = \tan \alpha = \frac{[xy]}{[xx]} = \text{coefficient of direction of star's path in the field of view.}$$

5.5.7. cont.

changing the co-ordinates:

$$[vv] = [xx] - \frac{[xy]^2}{[yy]}$$

$$m_{x}, \text{ time} = \pm\sqrt{\frac{vv}{n-2}}$$

$$m_{x}, \text{ mean} = \pm \frac{m_x}{\sqrt{n}}$$

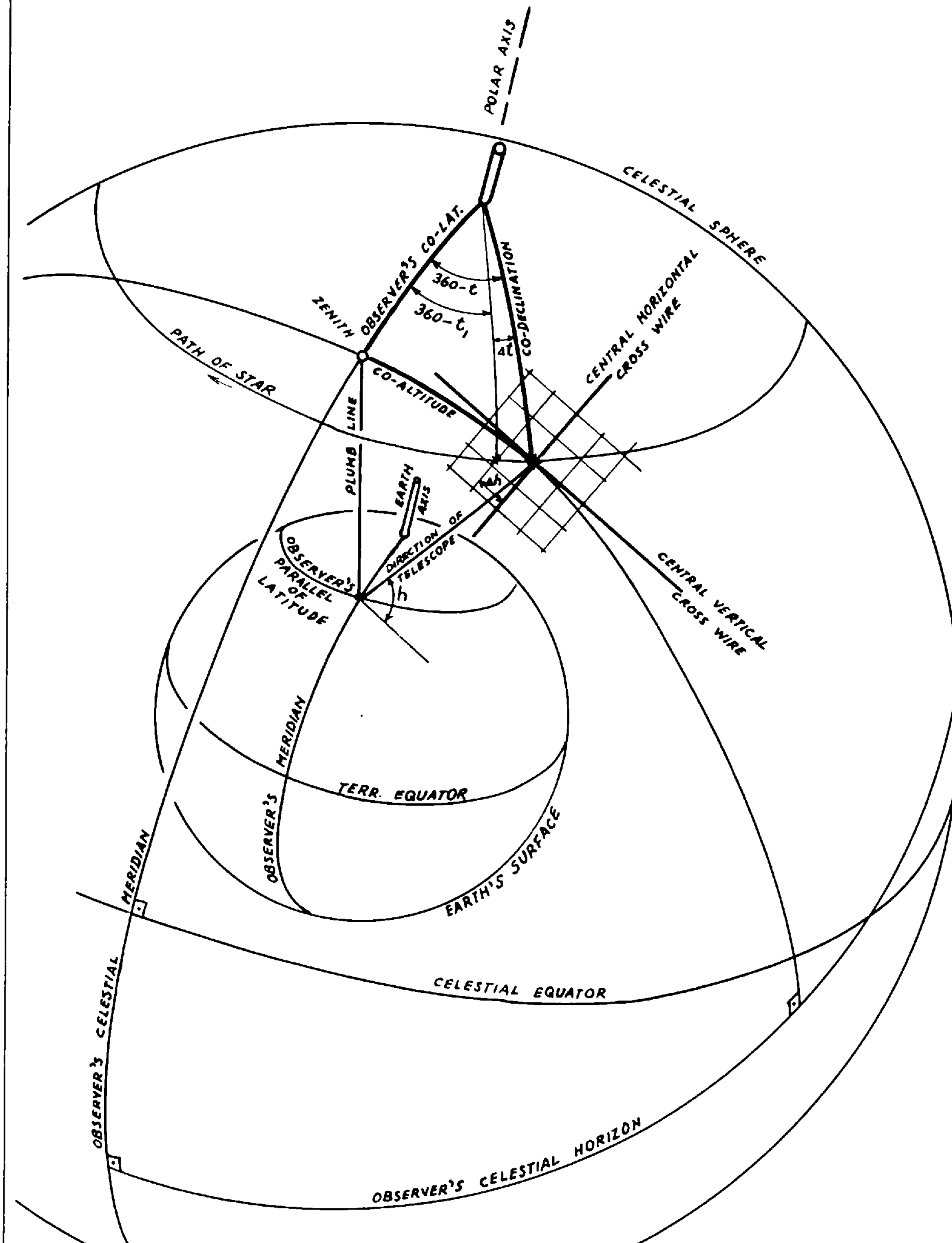
Obviously the r.m.s. error of the arithmetic mean of five grid line crossings is $\frac{1}{\sqrt{5}}$ times the r.m.s. error of one single wire crossing. This fact and the value obtained above can be used to judge theoretical considerations about the accuracy of timing stellar transits with other metho ds, especially with those methods restricted to one single wire crossing at a time. (e.g. method employing a stopwatch) The assessment of the accuracy of the methods adopting one wire crossing is rather cumbersome, obtainable as a rule via the final result of latitude and longitude, and so liable to be obscured by errors which have passed unnoticed.

The r.m.s. error of timing star transits through individual grid wires can be calculated also from residuals of the arithmetic mean obtained from all wire crossings, reduced to the central wire.

In Fig. 5.5.7.-2, t is the hour angle at the time of the star's transit through the central horizontal wire, corresponding to the observed altitude h.

t_1 is the hour angle at the time of the star's transit through the 1st parallel horizontal wire, corresponding to the observed altitude $h + \Delta h$.

Then: the difference of the hour angles $(t_1 - t) = \Delta t$ is equal to the recorded difference of the chronometer times (in sidereal time) at which the star was on the central horizontal wire and on the 1st



GRID RETICULE PROJECTED THROUGH THE TELESCOPE ONTO THE CELESTIAL SPHERE.
ORBIT OF STAR INTERSECTING HOR. & VERT. GRID LINES.

Fig. 5.5.7.-2

5.5.7. cont.

parallel horizontal wire.

The reduction from the parallel wire to the central wire is therefore equal to the difference of the hour angles = Δ t

The reduction can be computed by using the relation:

$$\sin h = \sin\phi \,.\, \sin\delta + \cos\phi \,.\, \cos\delta \,.\, \cos t$$

and $\sin (h + \Delta h) = \sin\phi \,.\, \sin\delta + \cos\phi \,.\, \cos\delta \,.\, \cos t_1$

subtracting, it follows:

$$2 \cos \left(h + \frac{\Delta h}{2}\right) . \sin \frac{\Delta h}{2} = 2 \sin \tfrac{1}{2}(t_1 - t) . \sin\tfrac{1}{2}(t_1 + t) . \cos\phi \,.\, \cos\delta$$

since: Δ h is 30" or 60" (angular subtense of grid lines)

and: $\Delta t = t_1 - t \geq$ 2 sec and $\geq$ 4 sec, in practice $<$ 30 sec;

the sine can be taken as being equal to the arc:

$$\Delta t = t_1 - t = \frac{\Delta h}{\cos\phi \,.\, \cos\delta} \cdot \frac{\cos\left(h + \frac{\Delta h}{2}\right)}{\sin \frac{t_1 + t}{2}}$$

for $\left(h + \frac{\Delta h}{2}\right)$ and for $\left(\frac{t_1 + t}{2}\right)$ the approximate values h and t respectively are quite sufficient; therefore:

$$\Delta t = \Delta h . \frac{\cos h}{\cos\phi \,.\, \cos\delta \,.\, \sin t}$$

since:

$$\sin A = \frac{\cos\delta \,.\, \sin t}{\cos h}, \text{ and: } \sin q = \sin A . \frac{\cos\phi}{\cos\delta}$$

the expression for time reduction becomes:

$$\Delta t = \frac{\Delta h}{\sin q \,.\, \cos\delta}$$

5.5.7 cont.

It can be seen that:

$$\frac{\Delta h}{\Delta t} = \sin q \cdot \cos \delta$$

is the rate of -change of altitude with respect to time, or the 1st derivative of the star's path in altitude, i.e. the velocity of a star in altitude; and in the same way the velocity of a star in azimuth is:

$$\frac{\Delta A}{\Delta t} = \cos q. \cos \delta$$

The formulae:

$$\Delta t\,(\text{sec}) = \frac{1''}{15.\ \sin q \cdot \cos \delta}.$$ (angular subtense of horizontal grid lines)

and

$$\Delta t\,(\text{sec}) = \frac{1''}{15.\cos q \cdot \cos \delta}.$$ (angular subtense of vertical grid lines)

were used to reduce the timing of grid line crossings to the central wires, for computing the r.m.s. error of timing stellar crossings of altitude and azimuth lines.

The formulae

$$t\,(\text{sec}) = \frac{1''}{15 \sin q \cdot \cos \delta}.$$ (estimated angular interval in altitude)

and

$$t\,(\text{sec}) = \frac{1''}{15 \cos q. \cos \delta},$$ (estimated angular interval in azimuth)

were used to compute the r.m.s. error of estimating space intervals, assuming correct timing.

The difference between sidereal time (hour angle) and U.T. (observed time of stellar crossing) has to be allowed for, when measured time

5.5.7. cont.

intervals between consecutive transits in terms of U. T. are compared with calculated time intervals.

The results, some of them being listed in Table 5.5.7.-3, show the quality of individual observations; accuracies of arithmetic means of five observations are given in Table 5.5.7.-4. Timing errors of stellar transits of vertical and horizontal cross wires are listed in Table 5.5.7.-4, column 4, and the corresponding error along horizontal and vertical grid lines in seconds of arc is listed as azimuth or altitude error in column 5. In column 7 are quoted the errors along horizontal or vertical grid lines as azimuth or altitude errors which result from estimated space fractions which are given in column 6.

The observations were executed with a Hilger and Watts Microptic theodolite No. 2, reading to 1". The telescope fitted with a long diagonal eye-piece has a magnification of about 23 x.

The values given in columns 4, 5, 6 and 7 are r.m.s. errors of the arithmetic means of five observations, i.e. of one pointing of the telescope; they are of course to a great extent shaped by the personal equation. More decimal places are carried in the calculation and quoted in the tables than the accuracy warrants, to avoid rounding-off errors.

From Table 5.5.7.-4 can be derived that a precision of almost the same order can be achieved in angular measurements for azimuth and altitude simultaneously, by timing star transits for one angle and estimating space fractions for the other, with the aid of the grid reticule and marker pulse.

Furthermore, timing of star crossings over five azimuth lines can be performed with equal accuracy as the timing of five altitude line crossings.

The accuracy of angular measurement obtained with the grid reticule and marker pulse matches the pointing and reading accuracy of the single second theodolite.

Timing of Stellar Transits
and
Estimation of Space Intervals

with Grid Reticule and Marker Pulse.

Date: May, 1961, At: "James" Trig, St^{n}., Tywarnhale, Cornwall

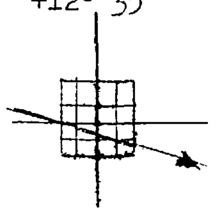
1	2	3	4	5	6	7
Star Magnitude Declination Movement in Field of View, Altitude	No. of Obs.	Angular Subtense to Central Wire seconds arc	Recorded Time seconds	Corr. to Central Wire seconds x 1.10708	Transit Time red. to Centre, seconds	Residual seconds
α Ophiuchi	1	-3.847 57.7	40.568	-4.259	44.827	+.206
2.14 +12° 35'	2	-1.973 29.6	43.271	-2.184	45.455	-.422
	3		44.715		44.715	+.318
	4	+1.873 28.1	47.188	+2.074	45.114	-.081
47° 42'	5	+3.800	49.265	+4.210	45.055	-.022
					45.033	$\pm.12_8$
		Est. Fraction "	Recorded Time	x .1803745 "		Residual of Fraction
	1	2.5= 15.0		2.706		$-.0_9$
	2	2.0 28.7		5.177		$-.1_3$
	3	1.4 46.0	a.a.	8.297		$+.1_5$
	4	1.0 58.1		10.480		$+.1_0$
	5	0.8 65.0		11.724		$-.0_4$
						$\pm.0_5$

Table 5.5.7. - 3

Timing Stellar Crossings of

Azimuth (Vertical) and Altitude (Horizontal) Grid Lines.

Date: May, 1961, At: Tywarnhale, Cornwall.

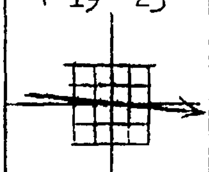
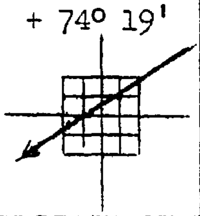
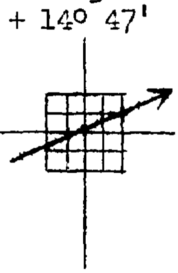
1	2	3	4	5	6	7	8
Star Magnitude Declination Movement in Field of View	Azimuth ° '	Altitude ° '	Timing Error Vertical Wire ± seconds	Azimuth Error ± " (arc)	Estim. Interval Altitude ± Fraction	Alt. Error ± " (arc)	appr. Total Time of Cross. sec.
α Bootis	156 06	57 19	0.09_5	1.3	0.0_4	1.4	8.4
0.24 + 19° 23'	156 51	25	$.10_8$	1.5	$.0_3$	0.9	
	157 51	34	$.10_0$	1.4	$.0_5$	1.8	
	158 33	40	$.08_9$	1.3	$.0_3$	1.0	
β Ursae Minoris	08 07	65 23	0.17_1	0.7	0.0_8	2.5	29.9
2 .24	07 39	27	$.13_2$	0.5	$.0_5$	1.7	
+ 74° 19'	06 52	33	$.12_4$	0.5	$.0_3$	1.0	
	06 23	36	$.17_8$	0.7	$.0_4$	1.2	
β Leonis	207 18	51 55	0.11_7	1.7	0.0_4	1.4	8.4
2.23 + 14° 47'	208 11	44	$.12_6$	1.8	$.0_2$	0.7	
	209 40	26	$.07_5$	1.1	$.0_5$	1.7	
	210 31	14	$.03_0$	0.4	$.0_4$	1.2	
	240 23	40 11	$.09_6$	1.2	$.0_5$	1.7	
	241 32	39 34	$.08_8$	1.1	$.0_5$	1.7	
	242 32	39 00	$.05_0$	0.6	$.0_3$	0.8	

Table 5.5.7. - 4

Timing Stellar Crossings
of
Azimuth(Vertical) and Altitude (Horizontal) Grid Lines.

Date: May, 1961 At: Tywarnhale,Cornwall.

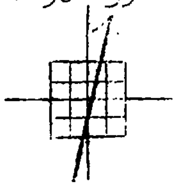
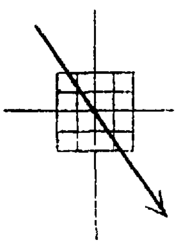
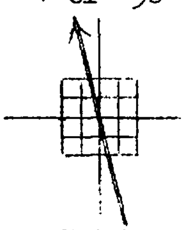
1	2	3	4	5	6	7	8
Star Magnitude Declination Movement in Field of View	Azimuth ° '	Altitude ° '	Timing Error Horizontal Wire ± seconds	Altitude Error ± " (arc)	Estim. Interval Azimuth ± Fraction	Az. Error ± " (arc)	appr. Total Time of Cross. sec.
β Ursae Majoris 2.44 + 56° 35'	304 41	53 34	0.11_3	0.9	0.0_4	1.3	15.5
	52	13	$.11_6$	0.9	$.0_4$	1.2	
	305 05	52 49	$.14_8$	1.2	$.0_3$	1.0	
	18	25	$.06_6$	0.5	$.0_3$	0.8	
α Lyrae 0.14 + 38° 44'	76 46	42 18	0.19_4	1.9	0.0_6	1.9	12.2
	77 04	42 36	$.04_6$	0.4	$.0_5$	1.8	
	77 53	43 24	$.25_5$	2.5	$.0_5$	1.6	
	79 22	44 50	$.08_6$	0.8	$.0_4$	1.3	
	90 03	54 34	$.26_8$	2.6	$.0_6$	2.1	
	91 50	55 58	$.06_9$	0.7	$.0_5$	1.5	
α Ursae Majoris 1.95 + 61° 58'	314 02	66 24	0.22_4	1.5	0.0_3	0.8	16.7
	313 41	65 42	$.07_4$	0.5	$.0_4$	1.4	
	316 01	65 22	$.15_5$	1.1	$.0_3$	0.9	
	331 26	65 06	$.11_4$	0.8	$.0_2$	0.7	

Table 5.5.7. - 4 cont.

5 . 5. 7. cont.

The small differences in the listed values suggest that timing of stellar transits appears to be more complicated for stars of higher magnitudes. Bright stars outshine the illuminated cross wires and the instant of crossing can be easily misjudged. The size of the star undoubtedly has also an influence on the estimation of space intervals.

Stars moving almost parallel to one family of grid lines cause some difficulty in judging fractions of space, because these differ only a little at consecutive timings, so that errors in estimation tend to be in one direction. It is therefore of advantage to have the star's path inclined to the grid line; this can be effected by employing quadrantal stars.

Table 5. 5. 7. -5 contains r. m. s. errors of azimuth and altitude measurements obtained with the grid reticule and marker pulse from some observations performed with a Hilger and Watts $^{1}/5''$ Microptic theodolite (Prototype). The telescope fitted with a long diagonal eye-piece has a magnification of about 30 x.

There are no theodolite readings involved in all the calculated results listed in Tables 5. 5. 7. -3, -4, and -5. Evidently the listed r. m. s. errors differ only small fractions from the r. m. s. errors of observations performed with the single second theodolite; but the differences are significant enough, and emphasize the higher accuracy achievable with the $^{1}/5''$ theodolite. The superior quality of the observations is due to the higher magnifying power of its optics.

No systematic corrections are applied to any observations and instrumental errors are irrelevant. The observations are not weighted; this could be done according to the magnitude of the star, its altitude and its direction of motion in the field of view. It is omitted, because the attribution of weight to an observation is rarely free of personal judgement; and detailed investigations into moderate variations of physical conditions, contributing small differences

Timing Stellar Crossings of Azimuth (Vertical) and Altitude (Horizontal) Grid Lines.

Date: May, 1963, At: Tywarnhale, Cornwall,
Instrument: Hilger and Watts, Microptic No.3 (Prototype)

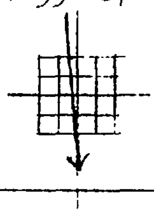
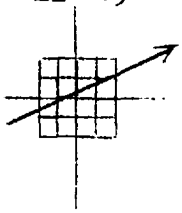
1	2	3	4	5	6	7	8
Star Magnitude Declination Movement in Field of View	Azimuth ° '	Altitude ° '	Timing Error Horizontal Wire ± seconds	Altitude Error ± " (arc)	Estim. Interval Azimuth ± Fraction	Az. Error ± " (arc)	appr. Total Time of Cross. sec.
ζ Ursae Majoris	60 59	59 38	0.09_{8}	0.8	0.0_{2}	0.6	14.2
2 .40 + 55° 07'	61 59	62 10	$.02_{6}$	0.2	$.0_{2}$	0.7	
	62 51	65 00	$.05_{2}$	0.4	$.0_{2}$	0.6	
	63 19	67 17	$.04_{9}$	0.4	$.0_{1}$	0.2	
			Timing Error Vertical Wire ± seconds	Azimuth Error ± " (arc)	Estim. Interval Altitude ± Fraction	Alt. Error ± " (arc)	
α Leonis 1.34 + 12° 09'	229 00	42 29	0.03_{6}	0.5	0.0_{2}	0.7	9.6
	230 51	41 42	$.04_{1}$	0.5	$.0_{2}$	0.5	
	231 53	41 14	$.05_{4}$	0.7	$.0_{3}$	0.8	
	233 28	40 30	$.04_{7}$	0.6	$.0_{2}$	0.5	

Table 5.5.7. - 5

5.5.7. cont.

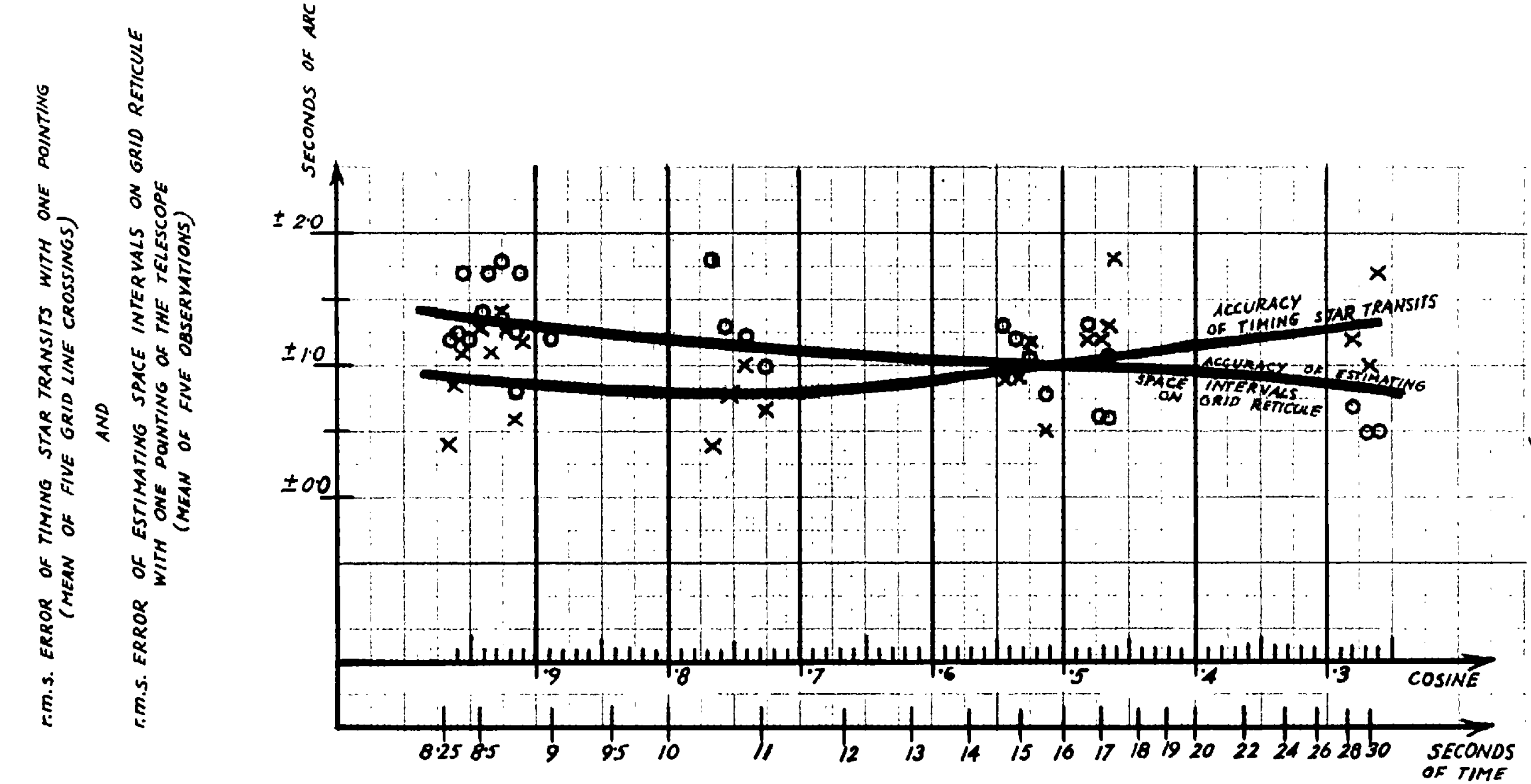
in observational precision, are outside the scope of this thesis.

In Fig. 5.5.7.-3 is shown the accuracy of angular subtense measurements (along horizontal and vertical grid lines), obtained from timing star transits and from estimating space intervals. The values plotted against stellar velocity or time are r.m.s. errors of observations to various stars, taken at about 50° latitude, with a Hilger and Watts 1" theodolite.

For better presentation the abscissa contains a cosine scale and the time a star requires to cross the grid square. The plotted values are summarized in form of accuracy curves, which are drawn by eye through the points to give the best fit to the various groups. This was possible because the values show a characteristic scatter. With the help of the diagram small differences in errors obtained from analysing field -results can be used for detecting relative merits and deriving advantage of timing transits of stars of various declinations, against estimation of space intervals on a grid. Obviously, results obtained from field experiments which were executed at totally different atmospheric conditions are not comparable. Therefore, the diagram is based on field returns from selected observations which were performed when similar weather conditions prevailed. A sufficiently large number of r.m.s. errors is used, for the purpose of eliminating the outcome of a distorted result due to observational selection or analytical treatment.

During all observations the field conditions were very unfavourable and therefore bigger variations in the r.m.s. errors of single pointings may not be expected than were deduced from the observational material obtained.

From the diagram can be derived that the accuracy in timing stellar transits is superior to the accuracy of estimating space



ACCURACY OF COMBINED OBSERVATIONS OF ALTITUDE AND AZIMUTH WITH GRID RETICULE & TIMING PULSE.

X ACCURACY OF TIMING STELLAR CROSSING WITH GRID RETICULE AND MARKER PULSE

O ACCURACY OF ESTIMATING SPACE INTERVALS WITH GRID RETICULE

Fig. 5.5.7.-3.

5.5.7. cont.

fractions when fast moving stars are concerned; obviously with slow moving stars the accuracy in estimating is increasing with the decrease in velocity; this would lead to the extreme case employing a stationary target; - the transit of which would be impossible to time, and its distance to the grid lines could be estimated with great care.

For slow moving stars it is therefore advisable to time the observations at the instant the star is midway between two parallel wires, and to estimate the distance to the wire of the other family. One more pointing, if necessary, can make up for the reduction from 5 to 4 individual observations.

Polaris and adjacent stars are conveniently sighted in the centre of the small squares formed by the grid lines.

It is interesting to note that the accuracy of timing transits is equivalent to the accuracy of estimating space fractions for stars having 50° to 60° declination.

The "timing-accuracy" curve has its minimum around $\cos\delta = 7\frac{1}{2}$, corresponding to a total time of about $11\frac{1}{2}$ seconds. Apparently this could mean that the highest accuracy in timing line crossings could be achieved with stars of 40° declination, and also with stars intersecting the grid lines at 45°.

In practice the slight differences between the two curves are of no importance in routine field work. Stars selected at random or adopted because of the presence of clouds will seldom belong only to the fast or only to the slow moving category. On the whole, practical field work adopting this method will produce angular measurements from timing star transits of equivalent accuracy to the angle obtained from estimating the grid distances; obviously disregarding atmospheric refraction dislevelment and errors of the instrument.

The tenth estimation of the angular subtense of grid lines is easily achieved, as the analyses indicate, and is therefore quite legitimate.

5.5.7. cont.

Unless stars are purposely selected in the immediate vicinity of the meridian, the altitudes of the five consecutive observations recorded with the aid of the grid reticule may require corrections for differences in atmospheric refraction.

Accepting a spherically stratified index of refraction, it follows that refraction will depend on elevation only and not on azimuth.

Usually, in field astronomy stars are observed at altitudes from 20° to 70°, in which range the accuracy of Bessel's formula is generally accepted to about 0.01 second of arc. The application of Bessel's formula requires a knowledge of the field temperature and air pressure. Humidity and wind direction which undoubtedly have an effect on refraction are not considered.

If:

h_o is the observed altitude when the heavenly body is on the middle horizontal wire, given by the reading of the vertical circle,

h_{o1}, h_{o2} the observed altitudes of the 1st and 2nd parallel horizontal wire respectively,

r_o the refraction corresponding to the altitude h_o

r_{o1}, r_{o2} " " " h_{o1}, h_{o2}

(r_o, r_{o1}, r_{o2}, etc. are the angular changes in the apparent positions of the star due to atmospheric refraction).

α_1 the angular distance in seconds of arc of the 1st horizontal wire from the middle wire,

α_2 the angular distance of the 2nd horizontal wire from the middle wire,

then the corrected altitude for the middle wire is:

h = corrected altitude $= h_o + r_o$ . . . . (1)

and the corrected altitude for the 1st horizontal wire:

$h_1 = h_{o1} + r_{o1}$ . . . . (2)

5.5.7. cont.

for the 2nd horizontal wire:

$$h_2 = h_{o2} + r_{o2}$$

i.e. the corrected altitude for the 1st horizontal wire is equal to the corrected altitude for the middle wire plus a corrective term, depending on the angular distance and refraction, thus:

$$h_1 = h + \Delta h_1$$

the corrective term Δh_1 is obtained by subtracting (1) from (2):

$$h_1 - h = \Delta h_1 = (h_{o1} - h_o) + (r_{o1} - r_o)$$

or:

$$\Delta h_1 = \frac{\alpha_1}{3600} + (r_{o1} - r_o)$$

and in the same way:

Δh_2 is the correction to be applied to the corrected altitude of the middle wire to obtain the corrected altitude of the 2nd parallel wire.

$$\Delta h_2 = \frac{\alpha_2}{3600} + (r_{o2} - r_o)$$

The term: $(r_{o1} - r_o)$ represents the difference of the refraction for different observed altitudes.

If Δr is the change of refraction in seconds of arc, per degree of altitude, obtainable from tables, then the difference in refraction is:

$$r_{o1} - r_o = \Delta r \cdot \alpha_1 \cdot \frac{1}{3600}$$

This term can be plus or minus as the altitude of the upper or lower parallel horizontal wire is used.

5.5.7. cont.

The corrective term Δh_1 in seconds of arc becomes:

$$\Delta h_1'' = \alpha''_1 \cdot \left(1 + \frac{\Delta r''}{3600}\right)$$

(Where: Δr is given in seconds of arc per degree.)

Δr varies with altitude.

at 20° altitude Δr is about 9" per 1°,
at 40° " " " $2\frac{1}{2}$" "
at 60° " " " 1.4" ".

The effect of different refractions for different horizontal wires is negligible as long as $\alpha'' \cdot \frac{\Delta r''}{3600}$ does not reach the size of observational errors. (i.e. unavoidable or accidental errors).

The smallest angular error in observing with the grid reticule is about 0.2". (tables 5.5.6-4 and -5, column 5.)

If then: $\alpha'' \cdot \frac{\Delta r''}{3600} \leqq$ minimum observational error (in seconds of arc)

the observed altitude 20°, and consequently

$\Delta r =$ 9" per 1°,

it follows that the condition of maximal angular subtense from the horizontal wire is:

$$\alpha'' = \frac{0.2'' \cdot 3600}{9''} \leqq 80''$$

The angular subtense from the middle wire was specified to be 30" and 60". This was done not only to render errors arising from off-centre observations consequent on using the grid reticule insignificant in relation to observational errors, but also to allow a wide range of applications of the grid reticule, should future higher magnification of the optics increase the accuracy in timing and estimating, and also for the possibility of using stars at very low altitudes.

5.5.7. cont.

The theoretical consideration, for deriving the maximum angular subtense of 60", aimed at keeping the observational errors in the order of $\frac{1}{\sqrt{5}}$ times the least reading of the most advanced engineer's theodolite. Obviously it is presumed that the pointing accuracy of the telescope matches the reading accuracy of the circle.

The data on which the specification for the grid reticule is based, are:

> The reading and pointing accuracy of Hilger and Watts Microptic theodolite No. 2, (Section 5.8.),
> the readings of the horizontal circle of the Geodetic Tavistock (Messrs. Cooke, Troughton and Simms),
> and the readings of both circles of the DKM 3 (Kern).

The following accuracy figures shown in Table 5.5.7.-6 are derived from observations executed in 1961 and 1963 at the Field Station, Tywarnhale Mine, Cornwall.

The results achieved are comparable in accuracy with those obtained from more elaborate methods and instruments, and approach the accuracy of timing star transits with the impersonal micrometer.

Timing with the grid reticule and marker pulse does not fully utilize the smallest subdivision of time intervals accessible via the crystal chronometer. Higher precision in timed simultaneous measurements of horizontal and vertical angles of star pointings can only be achieved with very different methods; with those impersonal methods employing electronic or photographic registration.

The basic idea of having a grid projected through the telescope on to the celestial sphere, the definite amount of angular subtense and the specific number of grid lines provide an easy way of measuring arc distances from a moving target to a reference line, which is comparable in efficiency with the timing arrangement.

Accuracy
of
Timing Star Transits and Estimating Space Intervals.

Instrument:	Hilger and Watts, Microptic Theodolite			
	No.2		No.3	
Telescope fitted with Diagonal eye-piece } Magnification:	23x		30x	
	seconds of time	arc distance "	seconds of time	arc distance "
Average of r.m.s. errors of Timing stellar crossings with one pointing of the telescope, (i.e. arithmetic mean of five observations) arc distance: along hor., or vert. grid lines.	$\pm 0.1_3$	$\pm 1.3_5$	$\pm 0.07_1$	$\pm 0.6_6$
	space fraction	arc distance "	space fraction	arc distance "
Average of r.m.s. errors of Estimating fractions of space between grid lines, with one pointing of the telescope, (i.e. arithmetic mean of five observations) arc distance: along hor., or vert. grid lines.	$\pm 0.04_2$	$\pm 1.3_5$	$\pm 0.02_6$	$\pm 0.9_5$

Table 5.5.7. - 6

5.5.7. cont.

Also the grid reticule and marker pulse provide adequate and uncomplicated means of testing the observer's ability to time line crossings and estimate fractions between grid lines.

The agreement between r.m.s. errors of angular measurements derived from timing star crossings and from estimating space intervals constitutes an excellent empirical check on the validity of the theoretical assumption.

<u>Summary of the advantages of the grid reticule in combination with the marker pulse, in field astronomy.</u>

(1) The stop-watch is made redundant.

(2) Grid reticule and marker pulse are cheaper than a stop-watch,

(3) Easier to handle than a stop-watch.

(4) Permits simultaneous measurements of both horizontal and vertical angles with an observation for time.

(5) Hence the possibility of star identification.

(6) Enables five combined observations with time to be performed within 8 to 20 seconds, instead of one observation only.

(7) The arithmetic mean of five observations gives one observation of both angles of improved accuracy, which is achieved with one pointing of the telescope.

(8) The observations for the measurement of an angle are about evenly balanced with respect to weight, i.e.

R.O. = stationary target;

star = moving target.

The sequence of observations in each circle position is:

R.O. - star - R.O., and the weight distribution is therefore = 6 : 5. (Stationary target: 3x the weight of moving target).

(9) The actual setting on the star and touching the slow motion screw when the time is noted is done away with. The star is only

5.5.7. cont.

observed in the field of view within the area occupied by the grid square and the transit times are marked without touching any parts of the instrument.

(10) The error of the personal equation of an observer, i.e. his ability to time transits and estimate space intervals can be determined.

(11) No corrections for errors due to observing off centre are required, (except corrections from Table 5.5.7.-1.).

(12) The booker is redundant, because timed crossings (= pulses), estimations of grid intervals and relevant data (= voice announcement) are recorded on tape.

(13) The grid reticule constitutes no special attachment to the telescope, and can be used instead of an ordinary cross wire, and

(14) It provides reference lines for the measurement of x and y distances when evaluating photographically registered star observations.

5.6. Eye's Share in Optical Perception

The observer's eye, unless replaced by an electric eye, forms an essential part of the optical instrument.

The visual acuity of the human eye sets a barrier to the perception of linear and angular separation, and governs the tolerances of any visual instrument.

Due to the wave nature of light, there is a limit to the forming of clear images of small detail with an optical system.

Perfect imagery is unachievable - the image of point objects is a diffraction pattern of measurable size. It depends a great deal on the quality of the eye of the observer to locate the centre point of the diffraction rings and to distinguish the proximity of the centre point to lines, as it is required for timing of transits. When observing, the eye has to look always straight and must be kept completely motionless. Slight movements of the observer's eye are unavoidable and are caused by muscular response to stimuli, produced by varying light conditions of the illuminated field of view, reflections on tiny water drops on the objective, stray light from the sky, etc. Movements of the head of the observer and the tendency of the eye to scan the field of view contribute to unsteadiness of the eye. A light ray which comes from the image point and passes through the centre of the pupil of the eye will pass, in the presence of ocular unrest, through outer zones of the pupil. On reaching the fovea centralis, different stimulation of the seeing sense is produced from rays passing through various zones of the pupil.

The visual acuity depends on the response of cones and rods in the retina, and the response depends to a major extent on the amount of light reaching the eye. Hence, the visual acuity is related to the brightness of the image. This relation is a constant over a wide range, because of the constuuction of the human eye, the adaptability of the eye-lens, chemical reactions in the retina, etc.

5.6. cont.

The image of a point-object (star) covers only a tiny part of the retina, the image of a line-object (e.g. R.O.) occupies more area. Hence the pointing accuracy of the telescope depends on the target, the quality of the target image, the eye of the observer and its ability to draw reasonable inferences from sense impressions.

5.7. Personal Equation and Error of the Personal Equation.

It is a biological fact that the responses to the impressions on the various senses are not equally fast. There are observers who proceed from hearing to seeing and others whose seeing perception comes before hearing. Further, there is also a time difference between sense impression and muscular reaction.

The impression of an event, on the seeing sense of an observer, produces a perception response which is either anticipated, simultaneous or delayed; the exact reason is as yet unknown.

The psychological phenomenon of placing an event mentally ahead, or too late, is the observer's peculiarity and can be measured. The reaction time-interval between event and response during which one or more senses, or one or more muscular actions are engaged, is commonly called the personal equation, and its deviations are termed the errors of the personal equation. The amount of the personal equation can be larger or smaller than the absolute value of its error.

The eye-ahd-ear method employs impressions on the hearing and on the seeing sense, and muscular reaction.

The timing of optical observations with the grid reticule and microswitch requires the concentration of the seeing sense only, and immediate muscular reaction.

Obviously, the personal equation peculiar to the latter timing method will not be identical to the personal equation of the eye-and-ear method.

Generally, the personal equation depends upon the observational methods; further, the personal equation is a function of properties of the observer, instrument and target, and of instantaneous conditions of the observer and of observing. Observing conditions pertain to the instrument and to the field. Habits are liable to produce a particular personal equation, e.g. a personal slowness, which may be considered as the property of the observer. The personal error of the observer is greatly affected by the head position and freedom from bodily strain;

5.7. cont.

tiredness will influence the sensual and muscular reactions. Mangification and optical quality are properties of the instrument and cannot be changed, but they can influence the personal equation; likewise the amount of illumination of the field of view and the proper focussing which can be altered. The type and size of the target, its brightness, the amount and direction of its movement, briefly its properties, and the instantaneous atmospheric conditions affecting the visibility will all contribute to a certain quality of the image. This is of significance to the impression produced on the retina and to the perception response.

Various forms of the personal equation result from the methods employed and conditions encountered.

The "polar equation" expressing the different reaction between sighting polar stars and equatorial stars has been studied since the end of the last century.

The "light equation" is based on the impressions that faint and bright stars create on the senses.

Unfamiliar direction of stellar motion and image distortions produce special personal equations.

The personal equation in timing stellar crossings of vertical grid lines can be considered to produce a collimation axis error, but of the same sign in both positions of the telescope, so that it is not eliminated with transiting.

The personal equation in timing stellar crossings of horizontal grid lines can be considered as an index error of the vertical circle, but again of the same sign in both positions of the telescope, so that it is not eliminated with transiting.

The anticipated or delayed sensation of an event influences the estimation of fractions of space between grid lines. This results in a personal equation which produces an equal collimation axis error or index error of the vertical circle, as mentioned above.

5.7. cont.

Another form of the personal equation is the peculiarity of some observers to wrongly estimate some particular fractions of space, This "scaling equation" can be detected from a sufficient number of observations, the analysis of which will show an unequal frequency of the decimal fractions. Assuming that all decimal fractions are equally likely to occur, the wrongly estimated fractions will appear at a higher or lower frequency than can be expected at equal distribution.

The personal equation is of no consequence when only time differences of stellar transits are required.

A knowledge of the personal equation and of its error is desirable for estimating its effect on the astronomical fixation of survey stations.

A practical way to determine the personal equation of an observer, in timing the instant of stellar crossings, which is mentioned in various publications, consists of making observations for longitude at a station, the position of which is well established, (e.g. primary beacon) and comparing the result.

Although undetected instrumental and observational errors may distort tbe result, the discrepancy obtained between observed and known longitudinal position, may be due to the personal equation alone; in which case the personal equation is deduced from observations which can be suspected to shape the personal equation according to the instrumental and field conditions encountered and stars sighted. One determination of longitude resulting from perhaps only one night's observations is not sufficient to derive a true picture of the personal equation. The result obtained for the longitude from one night's work fluctuates within groups of observations and results from successive nights fluctuate even more. The amount of fluctuation in one night of a longitudinal result obtained with a Watts 1" theodolite, grid reticule, marker pulse, crystal chronometer and tape recorder, is

5.7. cont.

not less than one to two seconds of arc, and values obtained on successive nights fluctuate double this amount.

Diverse opinions exist with respect to the size of the personal equation of the average type of observer, ranging from 0.1 to 0.3 seconds. There are observers with exceptional reaction times as high as $1\frac{1}{2}$ seconds either way; slow or fast.

Consequently, fluctuations of the results of longitude observations executed with the instruments mentioned above, cannot be attributed to the personal equation alone, and therefore the latter cannot be determined in this way with sufficient accuracy to justify its application.

For the measurement of reactions to a sense impression a personal equation machine can be used. This is an instrument which records both the observer's reaction to and event and the occurrence of the event to which the observer reacts.

The personal equation can be derived from measured time intervals between audible mechanical chronometer clicks and marker pulses released at definite time readings indicated by the second hand of the mechanical chronometer.

The chronometer clicks and marker pulses can be recorded on tape, and the time intervals can be measured with a stop-watch during slow replay. The tape can be developed and the time intervals measured with a scale. Any eccentricity of the second hand and parallax are eliminated by taking a series of readings at several intervals with a simple sighting device and magnifying glass. It is important that the clicks of the chronometer are in exact synchronization with the time indication of the second hand. This has to be arranged by the makers. If there is any consistent discrepancy between the audible clicks and the position of the second hand, then this method can be used only to determine the error of the personal equation.

In the course of testing mechanical chronometers, the clicks from

5.7. cont.

the second hand have been recorded on tape and made visible. This method is therefore feasible, but the number of escapement beats per second of the second hand governs the precision with which the personal equation can be determined. Further, since the subdivision of time obtained with a mechanical chronometer is of rather low accuracy (Section 3.8.), personal equations of observers ranging only from $\frac{1}{2}$ to 1 second or more can be ascertained with certainty, and obviously the error of the personal equation has to be at least comparable in size with the accuracy of the mechanical chronometer to be detectable.

The accuracy of timing stellar crossings with the grid reticule and marker pulse gives an indication of the error of the personal equation.

The error of the personal equation can be identified as the r.m.s. error of a single pointing of the telescope, which consists of five line crossings; its average size (for the observer concerned) is about $\pm 1.3_5$ sec. arc, or $\pm 0.1_3$ seconds of time, as already quoted. (Table 5.5.7.-6).

A device for measuring the personal equation and its error, suited to meet the requirements for accuracy combined with simplicity of manipulation, is shown in Fig. 5.7.-1.

Four equally spaced tiny holes (1) in a transparent, slowly rotating, dark disc (2) permit, when reaching a definite position in front of a slit (3) in a transparent, amber screen (4), the light of a lamp (5) to reach via a collecting lens (6) a photocell (7). The output of the photocell in the form of audio bleeps is recorded on magnetic tape. The exact spacing of the holes will be seen as exact spacing of the impulses after tape development. The disc, driven by a motor (12), can run at various speeds in both directions. The personal equation is tested by an observer, who is watching through the peep sight (8), and activates the lamp (5) with a microswitch (9) at the instant the first hole crosses the slit. After

DEVICE FOR TESTING
THE PERSONAL EQUATION
AND
THE ERROR OF THE PERSONAL EQUATION

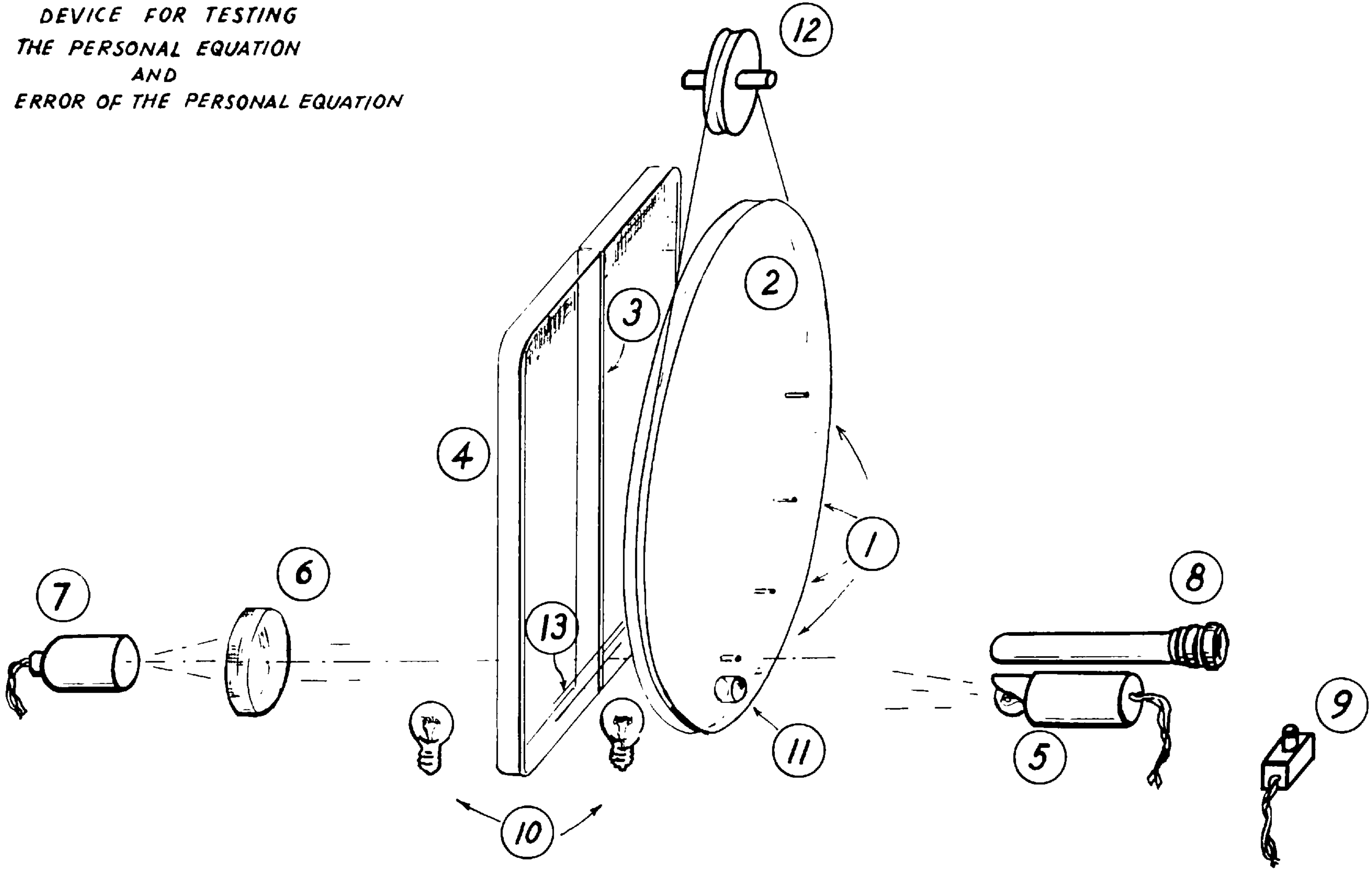


Fig.: 5.7.-1

5.7. cont.

the 4th hole has passed the slit, the lamp can be switched off, and the process can be repeated. The personal equation and its error are then determined from scaled distances on tape.

The equal spacing of the four holes, or the known distances between them, eliminates errors in the determination of the personal equation arising from changes in the rotational speed of the disc.

In the field of view of the peep sight the 1st hole (being observed) and the slit, both illuminated by dim-light lamps (10) create the impression of a star and illuminated cross wire respectively. The screen can be shifted and the light intensity of each lamp can be varied by a rheostat in the circuit, so that "stars" of different magnitude and brightness can be produced, and balanced against the illumination of the "cross wire". In this way "stars" can be kept brighter than the "cross wire", or vice versa.

Diametrically in line with the first hole, not visible through the peep sight, is the "timing hole" (11), a larger opening. The purpose of the timing hole is to eliminate distorted results. The timing hole is covered when the arrangement is being calibrated.

To obtain further detailed information about the personal equation, the peep sight, the screen with the slit, the collecting lens and the lamps can be shifted in their positions and lined up, so that it would be possible to produce in the field of view of the peep sight, the impression of any stellar movement likely to be encountered. The timing hole has to be replaced by a "timing bulb", situated between screen and collecting lens, for the purpose of giving a flash the instant the observer presses the key. After the 1st hole has passed by, the lamp (5) has to be illuminated to produce the three pulses to follow.

The scale error can be determined from estimation of space fractions. For this purpose two parallel lines (13) are drawn with a pencil on the screen, at right angles to the slit. These give the

5.7. cont.

impression of horizontal lines of the grid reticule. The correct space interval can be measured with an ordinary scale placed over the "star" against the two parallel lines when the disc is at rest.

For further investigation into the scale error, the width or position of the two parallel lines can easily be altered, or the disc raised or lowered. In this way the "star" is placed in all possible space fractions, that may occur during field observations.

The personal equation is a function of a variety of variables. For correcting an observation for the personal equation properly, it should be determined immediately before or after each pointing of the telescope, and it has to be assumed that the value obtained is a measure of the personal equation at the time of observation. This is of course impracticable because of the amount of time and work involved. Obviously the personal equation cannot be determined at actual field conditions. On the other hand, observing conditions in field astronomy cannot be imitated, and therefore every method of testing the personal equation and every personal equation machine will give inadequate information.

The value of the personal equation is rather uncertain. Consequently a figure based on practical experience which should be sufficiently accurate, is assumed.

As a rule, a skilled observer has some knowledge of his reaction time. Hence, the personal equation and its error, when recording the instant of an optical observation, can be estimated.

When the ear is not employed, stellar transits are timed with a manually operated key. A competent observer may be capable of estimating his personal equation by amounts accurate to the nearest tenth of a second, or perhaps even better.

The application of the impersonal micrometer does not completely eliminate, but reduces the size of the personal equation to a few hundredths of a second.

Timing stellar crossings with the grid reticule and manually operated key, or with the impersonal micrometer, does not fully utilize the precision with which U. T. is made accessible in the field with the crystal chronometer by the method described.

The development of improved timing methods from which the effect of the personal equation cannot be eliminated has to be considered to have reached a certain degree of perfection, when observational results obtained with these methods approach an accuracy which is comparable in size with the uncertainty of the results caused by the presence of errors which can neither be detected nor eliminated with adequate precision.

It becomes evident that further improvements on the instrument side, especially higher magnifying power of the telescope, will not improve the final result of the observations.

The advantage which can be derived by a replacement of the grid reticule and marker key with an impersonal micrometer consists mainly in reducing the personal equation. The higher reliability of the final result will be a minor point compared with the cost of a special piece of equipment.

For utilizing effectively the full precision of U. T., which is now available in the field, the method of timing stellar transits employing grid reticule and marker key can only be superceded by a method which replaces the eye of the observer with impersonal means of registration of stellar transits.

The device for testing the personal equation and its error, given in Fig. 5.7.-1, is believed to be original.

5.8. Theodolite Readings

If, in combined observations of altitude and azimuth, the crossing of successive grid lines by slow and fast moving stars can be timed to the high order of precision. quoted in Section 5.5.7., then any weakness in obtaining a position line must lie in the measurement of the other quantities; these consist of reading the vertical circle, where refraction is problematic, and of reading the horizontal circle which mostly requires a correction for its dislevelment.

Vertical and horizontal circle readings of micrometer theodolites contain errors of setting and reading the micrometers. Further, the accuracy of circle readings on which the precision of angular measurement depends, are affected by errors in the graduation of the circles, of the eccentricity of the circle, index errors, errors of collimation, transit axis error, errors due to dislevelment of the vertical axis, errors of the bubbles and the graduation errors of the bubbles, errors due to refraction, and errors caused by the personal equation and its error. (The last error is already dealt with in Section 5.7.)

A single measurement of an angle formed by a moving and a stationary target contains all the errors mentioned above, regardless of whether the single measurement is compensated or not, except in special cases.

A knowledge or elimination of these errors will contribute to the accuracy in angular measurements.

It is not possible to discover or to eliminate all the errors; certain errors can be determined and applied to angular measurements in form of corrections; these are the instrument constants, which, it is supposed, do not alter over a considerable length of time. Instrument constants are due to the fact that a theodolite can never be in perfect adjustment nor can the graduation of the circles or bubbles be perfect.

The instrument constants have been determined several days

5.8. cont.

before and also after the employment of the theodolites in the field. This was done because it was considered a necessity to collect accessible information on adjustments which can affect high precision observations.

A knowledge of the magnitude of these errors is essential for the application of a correction to individual circle readings, and also to make certain that their absolute amounts are so small that corrections can be calculated and errors eliminated separately.

The theodolites used were: Hilger and Watts No. 2, (single second), Hilger and Watts No. 3 (one fifth of a second), (Prototype), and Cooke, Troughton and Simms, "Tavistock" (single second).

The collimation axis error, transit axis error, index error of the vertical circle, and also the bubble error were found, as in most modern glass circle theodolites, to be very small, and constant over a long time.

The calibration values of the bubble graduations were obtained with the ingenious method invented by J. S. Sheppard, which is now widely adopted not only by field engineers, but also in optical workshops. Regrettably, J. S. S.'s method is not mentioned in the most recent editions of standard text books; the method is not contained in Bomford's Geodesy, although values of division of the bubble-scale - in seconds of arc enter into corrections given for local time computation on p. 291, and a general description of obtaining scale values and references are submitted on p. 20, 2nd edition, 1962; it is undoubtedly a serious omission that this method is also not described in D. Clark - J. Clendinning: "Plane and Geodetic Surveying".

The measuring accuracy achieved with the micrometers depends on their adjustment and on the setting and reading accuracy.

In the course of testing the micrometers for their measuring accuracies, information can be readily obtained of the quality of the

5.8. cont.

of the circle graduation.

The source of the errors of the circle graduation can often be traced to the subdividing process, for which the makers are responsible.

The total interval represented on the micrometer can be used to measure the angular distance subtended between circle divisions.

The procedure adopted to investigate the accuracy of the micrometers of the single second Hilger and Watts Microptic No. 2, is the following:

The relevant micrometer knob and slow motion screw are turned until the double lines straddle the single line, in the top aperture. This setting is repeated ten times in the immediate vicinity of 00' 00" and 10' 00" readings, i.e. the left-end of the micrometer and the right-end are set by operating the micrometer knob. The successive circle divisions brought forward with the slow - motion screw show up alternatively as one pair of double and single lines, and as two pairs of double and single lines, in the top window. The setting of both ends of the micrometer is performed on each circle division; if two pairs of double and single lines are visible, these settings can be performed on both pairs.

A refinement of setting the micrometer consists in bringing the single line to coincidence with each of the double lines by turning the micrometer screw.

A complete turning of the micrometer screw between each setting eliminates bias and possible minor accidental errors.

The readings are taken at the micrometer scale, with natural and artificial illumination.

In this way the space between circle divisions is measured with the interval contained by the micrometer; i.e. the intervals between circle divisions are expressed in terms of units of the micrometer, neglecting errors arising from mechanical defects of the rotating mechanism.

The interval between 00' 00" and 10' 00" subtended on the micrometer is a constant.

The difference of the readings taken at both ends of the micrometer, when set at consecutive circle divisions, will reveal the micrometer adjustment and the quality of the circle graduation.

The micrometer interval (= constant) can be larger, equal or smaller than the interval between the image of two circle divisions; this depends on the adjustment.

The variation of the difference between micrometer interval and interval of circle divisions depends on the precision of the circle graduation.

The magnitude of the r. m. s. error of setting the micrometer, and consequently of measuring the interval between circle divisions, and the amount of variation of the values obtained for the individual circle divisions in terms of micrometer units confirm the adjustment of the micrometer and the graduation accuracy of the circle, respectively.

Table 5. 8. -1 contains some of the measurements taken to determine the accuracy of the reading micrometer and its adjustment.

It is unnecessary to measure all 2160 circle divisions, unless as a matter of interest; the systematic errors of circle graduation are mostly periodic; their influence on the final result is reduced by a large number of field obseevations.

Readings of a few circle divisions distributed over 360° are quite adequate to expose the mean accidental uncertainty of the correct positioning of the micrometer and of its adjustment.

From Table 5. 8-1 it can be derived that the micrometer interval is larger than the circle divisions. The difference between micrometer interval and circle division is approximately 5 times the r. m. s. error

Measuring Accuracy and Adjustment of the Horizontal Circle Micrometer.

Instrument: Hilger and Watts Microptic Theodolite No.2

Date: 19.11.1961, Instrument Room, R.S.M., London.

Circle Readings:		00°00'	00°10'	00°10'	00°20'	10°00'	10°10'	20°00'	20°10'	30°00'	30°10'
Micrometer Readings: (Minutes of Micrometer Readings are omitted.)	1	00.0"	59.0"	+00.1"	58.1"	-02.0"	57.8"	-01.1"	57.8"	00.0"	59.2"
	2	00.0	59.6	00.0	58.0	-02.3	56.1	-01.0	57.8	-00.1	59.1
	3	-00.3	59.2	+00.8	59.7	-02.5	56.8	-01.1	57.3	-00.1	59.0
	4	00.0	58.2	-00.8	59.7	-01.9	57.1	-01.1	57.4	+00.2	59.2
	5	-00.8	59.1	+00.2	58.9	-02.1	56.9	-01.1	57.9	-00.3	59.0
	6	-00.1	60.1	-01.0	58.0	-01.9	56.4	-01.3	58.0	-00.1	59.0
	7	+00.1	59.2	-01.1	58.9	-02.0	57.1	-01.3	58.8	+00.8	59.6
	8	+00.9	59.2	-00.9	58.9	-02.6	56.5	-01.1	58.6	00.0	58.9
	9	00.0	58.3	-00.0	59.1	-0".0	56.8	-00.9	57.9	-00.2	58.8
	10	-00.4	58.8	-01.0	59.6	-02.0	56.8	-01.3	57.2	00.0	59.9
	$\frac{[\;]}{n}$	-00.0_6	59.0_7	-00.3_7	58.8_9	-02.1_3	56.8_3	-01.1_3	57.8_8	-00.1_4	59.1_7
r.m.s. error of single measurement (right - left)		$\pm 0.6_9$"		$\pm 0.9_3$"		$\pm 0.4_1$"		$\pm 0.5_6$"		$\pm 0.4_7$"	
Micrometer interv.		$60 - 59.1 \pm 0.2_2$		$60 - 59.3 \pm 0.2$[illegible]		$60 - 59.0 \pm 0.1_3$		$60 - 59.0 \pm 0.1_8$		$60 - 59.3 \pm 0.1_5$	
+ larger, - smaller than circle divisions		+0.87"		+0.74"		+1.04"		+0.99"		+0.69"	

Table 5.8. - 1

5.8. cont.

of the measurements. Consequently, it may be considered that a systematic error is not likely to be present. The adjustment of the micrometer is nearly as good as its measuring accuracy. The difference between micrometer interval and circle divisions is too large to be ignored; it may be identified as an "irregular error" of the circle graduations.

Because of the precision of the adjustment of the micrometer, no correction tables need to be prepared for reduction of micrometer readings, and the initial and final micrometer reading will not influence the size of the measured angle.

The information obtained about the micrometer accuracy is derived from tests under laboratory conditions; results of equivalent accuracy cannot be expected under adverse field conditions.

The graduations subdividing the total micrometer interval cannot be tested without additional equipment.

The graduation errors of the micrometer enter, of course, into the errors of angular measurements.

The pointing accuracy which reveals itself from analyses of angular measurements was determined under laboratory conditions, by measuring a 360° angle, clockwise and anticlockwise.

The target to define the 360° angle consisted of an electrically illuminated spider diaphragm of a collimator on tripod; the instrument, Hilger and Watts Microptic No. 2, was set up on an iron pillar.

The pointing tests for Hilger and Watts Microptic No. 3, prototype, were carried out as above and in addition to it the autocollimator arrangement, incorporated in the long diagonal eye-piece, was used to measure an additional angle of 360°; the target this time consisted of the mirror image of the theodolite's diaphragm. The mirror was set up on a tripod.

Both vertical and horizontal readings were recorded. Great care was taken in levelling the instrument; constant temperature was

5.8. cont.

maintained throughout the test; refocussing was not necessary; circle readings were discarded whenever a dislevelment of the vertical axis (indicated by the plate bubble) occurred between back-sight 0° and fore-sight 360°. When rotating the instrument care was taken to prevent unequal forces acting on the standards and to avoid torque affecting the instrument axis and its bearings. The slow-motion screw was turned in one direction only.

With these precautions, it is believed that the pointing accuracy can be determined from the amount the measurement falls short of 360° 00' 00"; further that the measurement of the 360° angle is exposed to errors of setting and reading the micrometer, to errors in sighting the target, errors due to manipulation of the instrument, together with errors caused by the clamping arrangements which are perhaps systematic, and to personal errors; to a minor extent (almost negligible) the 360° angle is also affected by errors of the adjustment of the micrometer, graduation of the plate and graduation of the micrometer. This is equivalent to saying that the pointing accuracy is affected by the above errors only, and that errors due to instrument constants, atmospheric conditions and dislevelment are prevented.

The r.m.s. errors of a single pointing, (i.e. resighting the target after a full rotation of the instrument) are given in Table 5.8.-2, as a summary of the tests. The accuracy of setting and reading the micrometers is obtained as a by-product of the tests this and the scatter of measurements about the mean are also given.

The results obtained for the uncertainty in pointing with the aid of the autocollimator are somewhat unexpected. The horizontal circle of the prototype No. 3 is about 100 mm in diameter, subdivided to 5', and the horizontal micrometer reads directly to $^{1}/5''$, whereas the vertical circle of about 75 mm diameter is subdivided to 10' and its micrometer reads to single seconds. From the above it could be presumed that higher accuracy in pointing could be achieved in azimuth

Pointing Accuracy of Theodolite,

Accuracy of Setting and Reading of

Horizontal and Vertical Micrometers,

Scatter of Measurements about the Mean.

Instrument: Hilger and Watts Microptic No.3
(Prototype)

Date: April 1963,Tywarhale,Cornwall.

	Target:	Vertical Circle	Horizontal Circle
m_p = r.m.s. error of single pointing	Spider Cross Hairs	± 2.8	± 2.4
Scatter of Measurements about the Mean	Spider Cross Hairs	3	2.5
m_p = r.m.s. error of single pointing	Auto-collimator	± 1.1	± 1.7
Scatter of Measurements about the Mean	Auto-collimator	1.5	2
m_s = r.m.s. error of single setting and reading of micrometer		± 0.4	± 0.3
Scatter of Measurements about the Mean		0.7	0.7

The figures are quoted in seconds of arc.

Table 5.8. - 2

5.8. cont.

than in altitude. The tests have shown that the direction of turning the instrument has no effect on the size of the 360° angle.

It has to be considered that the tests were carried out by one observer only; further, the prototype instrument is mounted on a three-tripod-base, which adds to height, and consequently to the effect of elasticity about the vertical axis. The spring of the slow motion screw was found to be rather weak, but no evidence could be traced that this contributed to detrimental effects.

The r.m.s. error of one single pointing with the autocollimator is deduced from ten measured angles; this is regarded as sufficient for obtaining data on which to base an estimate of the limitation of the instrument in conjunction with the human eye.

Laboratory tests of theodolites provide information which do not always conform with those gained from field experience.

On the whole, the pointing accuracy, in the neighbourhood of $1\frac{1}{2}$ second of arc (with the autocollimator) can be regarded as remarkably high, in view of the setting accuracy of the micrometer which is nearly $\frac{1}{2}$ second of arc.

The accuracy of alignment of the telescope can be deduced from the figures quoted in Table 5.8.-2.

Let:

m_p = r.m.s. error of pointing, deduced from angular measurement,

m_s = r.m.s. error of setting and reading the micrometer,

m_1 = r.m.s. error of alignment of the telescope (sighting the target),

m_m = r.m.s. error due to manipulating the instrument and due to mechanical defects,

m_e = r.m.s. error of the personal equation.

If: m_m and m_e are disregarded, then:

m_p depends only on m_s and m_1,

because m_p is obtained from angular measurement, therefore m_s and

5.8. cont.

m_1 enter twice into the relation:

$$m_p = \pm \sqrt{2\, m_s^2 + 2\, m_1^2}$$

with the relevant figures taken from Table 5.8.-2.
the average r.m.s. error of sighting, or the accuracy in alignment of the telescope in altitude and azimuth (with the autocollimator) is about $\pm$ 0.9".

The accuracy of the subdivisions of the micrometer can be examined by the following procedure: A standard angle of a size not less than four times the uncertainty of pointing with the theodolite, is measured over the total range of the micrometer.

This small angle is formed by two targets, rigidly connected to one another, mounted on a head which can be moved by a mechanical micrometer. The angle is measured alternatively with the theodolite and with the mechanical micrometer, by moving the twin-target forward on the micrometer head. The scatter of results of a series of repeated observations can be used to evaluate random errors of pointing with the theodolite, setting and reading the optical and mechanical micrometer and to study angular measurements performed on various parts of the optical micrometer, by comparing their variations with the values obtained from the mechanical micrometer.

The readings of the vertical circle contain the error of celestial refraction. The influence of refraction on the observations taken in the field was corrected with the use of Bessel's refraction tables, according to air pressure and temperature. The air pressure was measured with an aneroid barometer and with a barograph. The readings were verified by the R.A.F. Station St. Eval (Newquay), and reduced to the height above sea level of the observation station.

5.8. cont.

The temperatures were measured with two mercury thermometers graduated in degrees F.; the readings were estimated to a decimal fraction of the scale.

Air pressure and temperature were recorded in the immediate vicinity of the instrument. To conform with standard field practice, wind direction and wind velocity were not taken into account.

Obviously, the uncertainty of the value for refraction is caused by the unknown rate of change of the density of air with height.

Generally, refraction depends on the observer's position; presumably it does not affect the azimuth but reduces the zenith distance. Refraction causes a major problem, which will not be adequately solved until optical observations are possible from satellites outside the earth's atmosphere or from the moon.

6. Recording and Storing of Data.

6.1. General.

There are several possibilities of recording and storing of survey data. The most common way of keeping a record is by means of a field book with an up-to-date index, to facilitate access to the field returns.

The time of optical observation extracted from the stopwatch and chronometer is booked with all other data by the observer or his assistant.

The recording of chronometer and observation. signals can be performed by the employment of a chronograph as an attachment to the chronometer. Both types of signals can be recorded with the chronograph either on the same or on different paper strips.

The record can be inscribed with an ordinary pencil or fountain pen, with a sharp pin, or by electrical methods, e. g. high tension discharge that punch holes into the sheet, or by continuous circuits from the chronometer with breaks in it, or circuits to be established by the chronometer.

Ordinary paper will suffice for pen or pencil recordings, and will be quite adequate for storage over a certain length of time; wax coated paper, pressure sensitive material, abrasive coatings may be more satisfactory. The record can also be registered by photographic process.

6.2. Chronograph.

The chronograph has the possibility of converting the intervals between audible signals into visible distances.

The main advantage of a chronograph consists of replacing the stopwatch and the ear of the observer. Consequently, the personal equation resulting from the reaction to impressions on the senses, when comparing the stopwatch with the chronometer, is eliminated.

The measurements of time intervals with the stopwatch are replaced by the more accurate procedure of scaling distances on the paper strip. The timing accuracy achieved by depressing a marker key which effectuates the automatic inscription on the chronograph strip is superior to the accuracy obtainable with the eye-amd-ear method; but there is a scale difference between the time indicated by the chronometer and the instant the chronometer time is recorded on the chronograph strip. This results from delays in the chronometer mechanism and in the mechanism of the chronograph. The break-circuit wheel incorporated in the chronometer operates the release of a spring every even second which causes the closed circuit to be broken The break in the circuit suspends the action of the electromagnet of the chronograph whereupon the recording stylus, which is pressed against the rotating chronograph drum, produces the record of the circuit break. In some chronometers electric contacts are arranged so that the break circuit wheel breaks the closed circuit every second.

The running speed of most of the chronographs is 10 mm/sec, and can be altered to 20 mm/sec. A scaling accuracy of 0.1 mm would correspond to a measuring accuracy of 0.01 second. This scaling accuracy can hardly be achieved, because the inscriptions or the punched holes on the recording strip are not fine enough; consequently the precision of extracting the time of observation pulses will be about several hundredths of a second.

If the chronograph is driven by a falling weight, the speed between successive chronometer breaks can not be uniform, and the interval between breaks marked on the strip is not suited for linear subdivision. Further, the marks on the chronograph strip, if produced by mechanical means, approach a line width of $\frac{1}{2}$ mm, and therefore are inadequate to define a distance to the nearest $^{1}/10$ mm.

The mechanical device of inscribing the record on the paper strip causes a "drag error", which affects the observation breaks and also the time breaks; allowance for the drag error has to be made.

Interference may occur between observation breaks and chronometer breaks, in which case the instant of timing has to be interpolated over an interval of four seconds. Overlapping breaks produce a record from which no high scaling accuracy can be achieved nearer than one millimetre.

The chronometer breaks show up in a "line up", if the lengths of the time intervals are registered uniformly. This serves only for distinguishing chronometer breaks from observation breaks.

Headings and references have to be added to the chronograph record, if practicable, in the field; this constitutes additional work. All other data have to be entered into the field book in the ordinary way.

6.3. Tape recorder

Careful recording and handling of the amount of data associated with timing of optical observations, as in field astronomy, are effectively made by using a special or an ordinary tape recorder; thus, the tape recorder becomes a chronograph and field book simultaneously, replacing the conventional methods of booking by the observer or an assistant.

The crystal chronometer signals, the observation signals, and the radio time signals, as required, may be fed directly into the tape recorder, or alternatively, may be recorded over a loudspeaker via the microphone.

All relevant data, instrument readings etc., are spoken on to the tape; thereby considerably reducing the time required to put them down in writing. Both methods of recording can be used simultaneously on one tape track, i.e. direct feeding in of signals and recording the data through the microphone. Consequently, the check reading as well as the calling and repeating of figures, when another person does the booking, are eliminated. Special indicator words or description of the

6.3. cont.

data that follow, are not necessary.

Computations on field record sheets are seldom required; therefore, the magnetic tape, substituting the field book, is intended only for recording and for easy access to stored data.

The information contained on magnetic tape is approximately in the sequence in which they will be used during evaluation, as is the case with entries in the field book. Therefore, the magnetic tape is a satisfactory, but not a random storage medium, as the data have to be read in their consecutive order, regardless of whether all of them are required or not.

Access times, when using the tape as output device, are kept to a minimum by marking the tape at appropriate places with distinctive tags or paint. Chinese white diluted in water or in methylated spirit is conveniently applied with a brush and does not come off when running the tape through the instrument. The paint marks are easily removed with a dry cloth, if the tape is wanted for re-use. Indelible ink and grease pencil should be avoided, because they can contaminate the record head, or may cause a dropout on the tape.

Waste of time and tape is efficiently reduced if the recording instrument is capable of being stopped and started quickly, and regains operating speed readily, so that the maximum amount of data can be recorded on a given length of tape.

Magnetic tapes consist of a plastic base upon which is deposited a thin film of magnetic material, and are usually 0.25 to 1 inch wide, with normally 2 to 16 recording tracks which can be used separately or simultaneously. The recording of radio time pulses, chronometer pulses and observation pulses separately on different tracks has the disadvantage that it calls for a special tape recorder.

For the survey work described an ordinary tape recorder with a single record head is sufficient; there is no purpose in having more than one track engaged at one time; the necessity of inserting fresh

6.3. cont.

material in an existing recorded programme, composed of timed observations, is unlikely to be encountered.

The recording of all data on one single track simplifies also the procedure of calibrating the chronometer and the abstracting of time of optical observations. This also demands no extremely high accuracy in the performance of the recorder.

The standard running speeds of ordinary recording instruments are: $1\,^7/8$, $3\frac{3}{4}$, $7\frac{1}{2}$ and 15 in./sec.

When used as a chronograph the standard running speed of at least $3\frac{3}{4}$ in./sec. (or more) has to be used, to meet the quoted precision of time measurement, achieved with the crystal chronometer.

One millisecond at the running speed of $3\frac{3}{4}$ in./sec. corresponds to the linear distance of 0.1 mm, which can be scaled easily on the developed tape. (Secoion 7.3).

For synchronization of the crystal chronometer with standard frequency transmissions as well as for recording of optical observations in the field, it is more convenient to make use of a speed of $7\frac{1}{2}$ in./sec., for reasons to be outlined further on. Voice announcements can be recorded at any speed; for economy purposes the lowest speed provided on the recorder can be made use of.

The residual elongation of less than 1% of nearly all tapes when stretched, presumably uniformly distributed over a minimum length corresponding to one second of time, is of no consequence, as only ratios of measured or scaled tape distances are required; the same applies to variations in running speed, which are bound to occur with battery operated recorders out of doors.

A light-weight, portable battery operated tape recorder with speeds of $7\frac{1}{2}$ and $1\,^7/8$ in./sec. is most serviceable in the field, and can also serve to test the performance of the crystal chronometer. The lower speed is used for 4 times magnification of the time intervals at play back. The length of tape available to accommodate sufficient

6.3. cont.

observations on one track at high speed, is naturally restricted to the size of the recording instrument.

For the entire field research programme the Fi-Cord tape recorder weighing $4\frac{1}{2}$ lbs. was exclusively used and met well with the requirements. The recording time per track at $7\frac{1}{2}$ in./sec speed is about 9 minutes.

Up to 12 star observations could be recorded on one single track. The Fi-Cord motor was controlled by the stop-start switch situated at the microphone which was clipped to the surveyor's outfit, leaving the hands free to operate the theodolite. Most of the time the running speed remained within $\pm$ 0.1%. (Derived from evaluated data). In adverse conditions a steady drift in speed was encountered not exceeding the negligible amount of 0.65% during any one period of operation.

Irregularities of the running speed are easily detected from tape measurements; with the Fi-Cord tape recorder irregularities were seldom encountered, and, if present, hardly distorted the timing results or calibration values of the crystal chronometer.

The precision of the running speed of the tape recorder was determined under various sets of conditions likely to be encountered during astronomical observations.

Tests were made with magnetic tapes of various qualities:
Scotch Brand P.E. Base and Scotch Brand 111 Acetate,
Irish Tape Mylar (T), Irish Tape Mylar (A), Irish Tape
Cellulose Acetate, EMI Tape 77, 88 and 99.

The batteries were recharged at regular intervals and the recorder operated at different temperatures and in various positions.

Precise time intervals over the total length of one track are required for the tests. For this purpose the standard frequency transmissions available through the national broadcasting service are made use of. Because there is no adequate reception of continuous

6.3. cont.

transmissions over 36 minutes (to cover the total running time per track at 1 $^{7}/8$ in./sec.), the recordings of frequencies at high speed are used for marking time intervals at slow re-play. The relative accuracy of the measurement of time intervals with a stopwatch during slow re-play is thereby also increased.

Results of some of these tests are shown in Tables 6.3.-1, 6.3.-2, and 6.3.-3.

The figures quoted in the first column of each group are the times of M.S.F. pulses, (originally recorded at $7\frac{1}{2}$ in./sec.) measured during re-play at 1 $^{7}/8$ in./sec. The time interval, enlarged four times, was measured with a stopwatch; this was sufficiently accurate to determine any fluctuations of the running speed.

It is only necessary that the recorder maintains a uniform running speed during intervals of seconds; the absolute amount of the running speed may vary from one interval to the other by a few percent; this depends on the temperature of the surroundings and on battery conditions.

For each listed group the average stability of the running speed at 1 $^{7}/8$ in./sec. (coefficient of variation) during the interval of one minute is given in percentages.

A slow play-back of the same tape and record with various batteries reveals their influence on the running speed. It can be seen clearly that variations in the running speed depend mainly on battery conditions.

The running speed of the recorder is not affected by the quality of the tape used. The measurements expose the relation between the tape speed and the length of the tape, as well as between the tape speed and the decrease of the charge of the battery.

The recording instrument shows only few instances of erratic behaviour which can be attributed perhaps to the personal error of the observer in time measurement at slow replay; personal errors and

Running Stability of Recording Instrument.

Tape Recorder: Fi-Cord No.8321

Magnetic Tape: Scotch Brand P.E. base.

Stopwatch: Zenith 1/5 sec.

Minute interval of MSF pulses recorded at $7\frac{1}{2}$ in./sec.

Time measured during re-play at $1\frac{7}{8}$ in./sec.

Average Stability at $1\frac{7}{8}$ in./sec., during one minute.

Observer: I, BatteryNo.1, 50°F			Battery No.3, 50°F			Battery No.5, 50°F		
Time min sec	Interval min sec	v sec	Time min sec	Interval min sec	v sec	Time min sec	Interval min sec	v sec
5-24.8			2-23.0			5-19.9		
	4-13.2	+.7		4-13.9	+.7		4-14.3	+.1
9-38.0			6-36.9			9-34.2		
	13.8	+.1		14.3	+.3		15.1	-.7
13-15.8			10-51.2			13-49.3		
	14.2	-.3		14.8	-.2		15.1	-.7
18-0.60			15-06.0			18-04.4		
	14.0	-.1		14.9	-.3		14.1	+.3
22-20.0			19-20.9			22-18.5		
	14.0	-.1		14.6	.0		14.1	+.3
26-34.0			23-35.5			26-32.6		
	14.0	-.1		14.5	+.1		13.9	+.5
30-48.0			27-50.0			30-46.5		
	14.0	-.1		14.9	-.3		14.0	+.4
35-02.0			32-04.9			35-00.5		
	14.2	-.3		14.9	-.3		14.2	+.2
39-16.2			36-19.8			39-14.7		
av. 4x interv.: 4-13.9				4-14.6			4-14.4	
r.m.s.error=		±.32			±.34			±.48
av. stability during one minute = .13%					.14%			.20%

Table 6.3. - 1

Running Stability of Recording Instrument.

Tape Recorder: Fi-Cord No. 8321

Magnetic Tape: Scotch Brand P.E. base.

Stopwatch: Zenith 1/5 sec.,

Minute interval of MSF pulses recorded at $7\frac{1}{2}$ in./sec.,

Time measured during re-play at $1\frac{7}{8}$ in./sec.

Average Stability at $1\frac{7}{8}$ in./sec., during one minute.

Observer: II, Battery: No.4, 60°F, Time min sec	Interval min sec	v sec	Battery No.2, 60°F, Time min sec	Interval min sec	v sec	Battery No.5, 60°F, Time min sec	Interval min sec	v sec
2-02.4			4-12.1			5-21.0		
	4-14.0	+.2		4-12.8	+.5		4-13.5	-.5
6-16.4			8-24.9			9-34.5		
	13.9	+.3		13.0	+.3		13.3	-.3
10-30.3			12-37.9			13-47.8		
	14.3	-.1		13.1	+.2		12.7	+.3
14-44.6			16-51.0			18-00.5		
	14.2	.0		13.2	+.1		12.7	+.3
18-58.8			21-04.2			22-13.2		
	14.1	+.1		13.6	-.3		12.2	+.8
23-12.9			25-17.8			26-25.4		
	14.4	-.2		13.4	-.1		12.6	+.4
27-27.3			29-31.2			30-38.0		
	14.5	-.3		13.5	-.2		13.5	-.5
31-41.8			33-44.7			34-51.5		
	14.2	.0		13.6	-.3		13.5	-.5
35-56.0			37-58.3			39-05.0		
av. 4x interv.:	4-14.2			4-13.3			4-13.0	
r.m.s.error=		±.20			±.30			±.51
av. stability during one minute =		.08%			.13%			.23%

Table 6.3. - 1 cont.

Running Stability of Recording Instrument.

Tape Recorder : Fi-Cord No.8321

Magnetic Tape: Scotch Brand P.E. base

Stopwatch: Zenith 1/5 sec.,

Minute interval of MSF pulses recorded at $7\frac{1}{2}$ in./sec.,

Time measured during re-play at $1\frac{7}{8}$ in./sec.,

Average Stability at $1\frac{7}{8}$ in./sec., during one minute.

Observer I Battery: No. 5, 72°F,			Battery No.1, 70°F,			Battery No.4, 70°F,		
Time min sec	Interval min sec	v sec	Time min sec	Interval min sec	v sec	Time min sec	Interval min sec	v sec
0-16.0			0-38.8			0-39.8		
	4-10.5	-.3		4- 8.7	.0		4- 8.2	+.3
4-26.5			4-47.5			4-48.0		
	10.8	-.6		8.9	-.2		8.5	.0
8-37.3			8-56.4			8-56.5		
	11.0	-.8		8.9	-.2		8.5	.0
12-48.3			13-05.3			13-05.0		
	10.8	-.6		8.1	+.6		9.0	-.5
16-59.1			17-13.4			17-14.0		
	10.5	-.3		8.3	+.4		8.6	-.1
21-09.6			21-21.7			21-22.6		
	9.4	+.8		8.5	+.2		8.6	-.1
25-19.0			25-30.2			25-31.2		
	9.9	+.3		9.1	-.4		8.4	+.1
29-28.9			29-39.3			29-39.6		
	9.2	+1.0		8.7	.0		8.2	+.3
33-38.1			33-48.0			33-47.8		
	9.8	+.4		9.0	-.3		8.5	.0
37-47.9			37-57.0			37-56.3		
av. 4x interv.: 4 -10.2				4-8.7			4-8.5	
r.m.s.error =		±.65			±.33			±.24
av. stability during one minute =		.27%			.13%			.10%

Table 6.3.-1 cont.

Running Stability of Recording Instrument.

Tape Recorder: Fi-Cord No. 8321,

Magnetic Tape: Irish Tape Mylar,

Stopwatch: Zenith 1/5 sec.,

Minute Interval of MSF pulses recorded at $7\frac{1}{2}$ in./sec.,

Time measured during re-play at $1\frac{7}{8}$ in./sec.,

Average Stability at $1\frac{7}{8}$ in./sec., during one minute.

Observer: I
Battery: No.1 (charged), 70° F,

Time min sec	Interval min sec	v sec	Time min sec	Interval min sec	v sec	Time min sec	Interval min sec	v sec
0-14.4			0-31.8			0-21.9		
	4- 8.9	+.6		4- 9.4	+.7		4- 9.3	+.9
4-23.3			4-41.2			4-31.2		
	9.8	-.3		9.1	+1.0		9.8	+.4
8-33.1			8-50.3			8-41.0		
	9.9	-.4		10.1	.0		10.0	+.2
12-43.0			13-00.4			12-51.0		
	9.0	+.5		10.4	-.3		1[illegible].0	-.2
16-52.0			17-10.8			17-01.0		
	9.5	0		10.2	-.1		10.5	-.3
21-01.5			21-21.0			21-11.5		
	9.6	-.1		10.0	+.1		10.5	-.3
25-11.1			25-31.0			25-22.0		
	9.9	-.4		10.4	-.3		10.5	-.3
29-21.0			29-41.4			29-32.5		
	9.8	-.3		10.4	-.3		10.9	-.7
33-30.8			33-51.8			33-43.4		
	9.5	0		1[illegible].5	-.2		10.7	-.5
37-40.3			38-02.3			37-54.1		
av. 4x interv.:	4-9.5			4-10.1			4-10.2	
r.m.s.error =		$\pm.38$			$\pm.47$			$\pm.51$
av. stability during one minute =		.16%			.19%			.21%

Table 6.3. - 2

Running Stability of Recording Instrument.

Tape Recorder: Fi-Cord, No.8321

Magnetic Tape: Irish Tape Mylar,

Stopwatch: Zenith 1/5 sec.,

Minute Interval of MSF pulses recorded at $7\frac{1}{2}$ in./sec.,

Time measured during re-play at $1\frac{7}{8}$ in./sec.,

Average Stability during one minute at $1\frac{7}{8}$ in./sec.,

Observer: I,
Battery: No.5 (charged), 70°F,

Time min sec	Interval min sec	v sec	Time min sec	Interval min sec	v sec	Time min sec	Interval min sec	v sec
0-10.1			0-19.8			0-18.6		
	4- 8.9	+.6		4- 9.1	+1.4		4-10.7	+.5
4-19.0			4-28.9			4-29.3		
	8.3	+1.2		10.1	+.4		11.6	-.4
8-27.3			8-39.0			8-40.9		
	9.2	+.3		10.2	+.3		10.8	+.4
12-36.5			12-49.2			12-51.7		
	8.8	+.7		10.4	+.1		10.9	+.3
16-45.3			16-59.6			17-02.6		
	9.3	+.2		10.8	-.3		10.4	+.8
20-54.6			21-10.4			21-13.0		
	9.6	-.1		10.4	+.1		12.2	-1.0
25-04.2			25-20.8			25-25.2		
	10.7	-1.2		10.6	-.1		11.1	+.1
29-14.9			29-31.4			29-36.3		
	10.3	-.8		10.6	-.1		11.7	-.5
33-25.2			33-42.0			33-48.0		
	10.8	-1.3		12.0	-1.5		11.5	-.3
37-36.0			37-54.0			37-59.5		
av. 4x interv.:	4-9.5			4-10.5			4-11.2	
r.m.s.error =		±.88			±.77			±.57
av. stability during one minute =		.36%			.31%			.23%

Table 6.3. - 2 cont.

Running Stability of Recording Instrument.

Tape Recorder: Fi-Cord, No 3321,

Magnetic Tape: Irish Tape Mylar,

Stopwatch: Zenith 1/5 sec,

Minute Interval of MSF pulses recorded at $7\frac{1}{2}$ in./sec.,

Time measured during re-play at $1\frac{7}{8}$ in./sec.,

Average Stability during one minute at $1\frac{7}{8}$ in./sec.

Observer: I, Battery No. 3 (charged),								
Time min sec	Interval min sec	v sec	Time min sec	Interval min sec	v sec	Time min sec	Interval min sec	v sec
0-52.8			1-20.9			0-42.8		
	4-10.3	-.1		4- 9.8	+.6		4-10.5	+.2
5-03.1			5-30.7			4-53.3		
	10.2	0		10.1	+.3		10.8	-.1
9-13.3			9-40.8			9-04.1		
	10.5	-.3		10.3	+.1		10.9	-.2
13-23.8			13-51.1			13-15.0		
	9.6	+.6		10.6	-.2		10.1	+.6
17-33.4			18-01.7			17-25.1		
	9.6	+.6		10.3	+.1		10.2	+.5
21-43.0			22-12.0			21-35.3		
	10.2	0		11.0	-.6		10.7	0
25-53.2			26-23.0			25-46.0		
	10.6	-.4		10.5	-.1		10.8	-.1
30-03.8			30-33.5			29-56.8		
	10.3	-.1		10.6	-.2		11.4	-.7
34-14.1			34-44.1			34-08.2		
	10.4	-.2		10.5	-.1		11.2	-.5
38-24.5			38-54.6			38-20.0		
av. 4x interv.:	4-10.2			4-10.4			4-10.7	
r.m.s.error =		±.36			±.34			±.42
av. stability during one minute		.15%			.14%			.17%

Table 6.3. - 2 cont.

stopwatch errors have a minor effect on the measurement of time intervals of four minutes duration.

The running-in time of the recorder is about six to seven seconds. The first reading was taken after the tape had acquired average running speed. In some of the tests, the batteries were used up to 60% and more of their life time before re-charging. The running speed is still fairly consistent towards the end of the life time of the batteries. In the field changes in temperature and humidity will have some effect.

The r.m.s. errors quoted in the tables show the quality of the running stability at 1 $^7/8$ in./sec. over 4 times the enlarged interval of one minute recording. It has to be noted that the r.m.s. error is the error of one single measurement of one enlarged minute interval, containing errors due to variation in the running speed at the instant of recording and at play back, and errors from the measurement with the stopwatch.

The r.m.s. error of the arithmetic mean derived from 8 or 9 enlarged minute intervals is insignificant with respect to testing the running stability of the Fi-Cord for its intended employment.

The mean square error of one minute interval in seconds can be obtained approximately by taking one quarter of the mean square error of its four times enlarged value. This is a measure of the precision of the running speed during one minute and is quoted in percentages; its average can be taken to be between 0.1% and 0.3%. The percentage error indicates a "second interval stability" of the recorder of a few milliseconds.

Tables 6.3.-3 and 6.3.-3 cont. contain some of the tests to determine the change in tape velocity at $7\frac{1}{2}$ in./sec. running speed of the Fi-Cord tape recorder operated on batteries and of the Vortexion tape recorder operated on mains supply. The time intervals between M.S.F. pulses, recorded at 15 in./sec., measured with a stopwatch during re-play at $7\frac{1}{2}$ in./sec., are shown in the first column. For

Running Stability of Recording Instrument.

Tape recorder: Fi-Cord No.8321

Stopwatch: Zenith 1/5 sec.,

Minute interval of MSF pulses recorded at 15 in./sec.

Time measured during re-play at $7\frac{1}{2}$ in./sec.

Average stability at $7\frac{1}{2}$ in./sec.,during one minute in % .

Battery No.1, 70° F.,

Time min sec	min sec	v	min sec	min sec	v
00-32.4			00-22.5		
	02- 02.5	+.3		02-02.7	+.17
2-34.9			2-25.2		
	3.2	-.4		3.2	-.33
4-38.1			4-28.4		
	2.7	+.1		2.9	-.03
6-40.8			6-31.3		
	2.8	0		2.7	+.17
8-43.6			8-34.0		
av. 2x interval	02- 02.8			02-02.8	
r.m.s.error single interval		±.29			±.24
av. stability		.24%			.20%
00-07.6			01-19.3		
	02- 02.6	+.22		02-02.4	-.05
2-10.2			3-21.7		
	2.9	-.08		2.4	+.05
4-13.1			5-24.0		
	3.0	-.18		2.5	-.15
6-16.1			7-26.5		
	2.8	+.02		2.2	+.15
8-18.9			9-28.7		
av. 2x interval	02- 02.8			02-02.4	
r.m.s. error		±.17			±.22
av. stability		.14%			.18%

Table 6.3. - 3

Running Stability of Recording Instrument.

Tape recorder: Vortexion,

Stopwawtch: Zenith 1/5 sec.,

Minute interval of MSF pulses recorded at 15 in./sec.,

Time measured during re-play at 7½ in./sec.,

Average stability at 7½ in./sec., during one minute in % .

Power: Mains, 70° F.,

Time min sec	min sec	v	min sec	min sec	v
01-13.0			00-37.5		
	02-00.2	+.07		02-00.1	+.12
3-13.2			2-37.6		
	.3	-.03		.4	-.18
5-13.5			4-38.0		
	.3	-.03		.2	+.02
7-13.8			6-38.2		
	.3	+.03		.2	-.02
9-13.8			8-38.4		
av. 2x interval	02-00.27			02-00.22	
r.m.s. error single interval		±.14			±.13
av. stability		.12%			.11%
00-05.6			02-11.4		
	02-00.2	.0		02-00.1	+.15
2-05.8			4-11.5		
	.1	+.1		.2	+.05
4-05.9			6-11.7		
	.1	+.1		.4	-.15
6-06.0			8-12.1		
	.3	-.1		.2	+.05
8-06.3			10-12.3		
av. 2x interval	02-00.2			02-00.3	
r.m.s. error single interval		±.10			±.13
av. stability		.08%			.11%

Table 6.3. - 3 cont.

6.3. cont.

ease in comparing results of the two recorders, the re-play was measured over nine minutes, to conform with the total playing time per track when using the Fi-Cord at $7\frac{1}{2}$ in./sec., with a tape of standard thickness. The average stability of the running speed at $7\frac{1}{2}$ in./sec., during the interval of one minute, is given again in percentages; the r.m.s. error refers to the single measurement of one interval, i.e. the twice enlarged interval of one minute. Theoretically, the r.m.s. error of a single observation determined from a group of four observations only, does not give a reliable picture of the quality of the observation; but the number of observations which can be taken during one re-play is restricted by the capacity of the tape recorder.

The Fi-Cord recorder displays a coefficient of variation, change in tape velocity at $7\frac{1}{2}$ in./sec., which is very similar to the running stability at $1\ ^{7}/8$ in./sec. The decreasing charge of the batteries causes a progressive increase of the length of the enlarged minute intervals.

From data presented in Tables 6.3.-3 and 6.3.-3 cont., it can be seen that the stability of the running speed maintained by the Fi-Cord is not very inferior to the stability of the Vortexion. The percentage error clearly points out that the Fi-Cord is capable of a "second interval stability" of a few milliseconds at the running speed of $7\frac{1}{2}$ in./sec.

Higher precision of the running speed of the tape is not required. More detailed information about the performance of the tape recorder can be derived from scaled distances between pulses, after tape development (Section 7.3.); such information can be obtained as a by-product of the calibration of the crystal chronometer.

The graphs in Fig. 6.3.-1 show variations in the lengths of time intervals marked by consecutive M.S.F. seconds pulses recorded at $7\frac{1}{2}$ in./sec. on two tape recorders simultaneously.

The lengths of the time intervals, obtained from scaled distances after tape development, are represented in the form of a diagram, and reveal the precision of the stability of the running speed of both, the

6.3. cont.

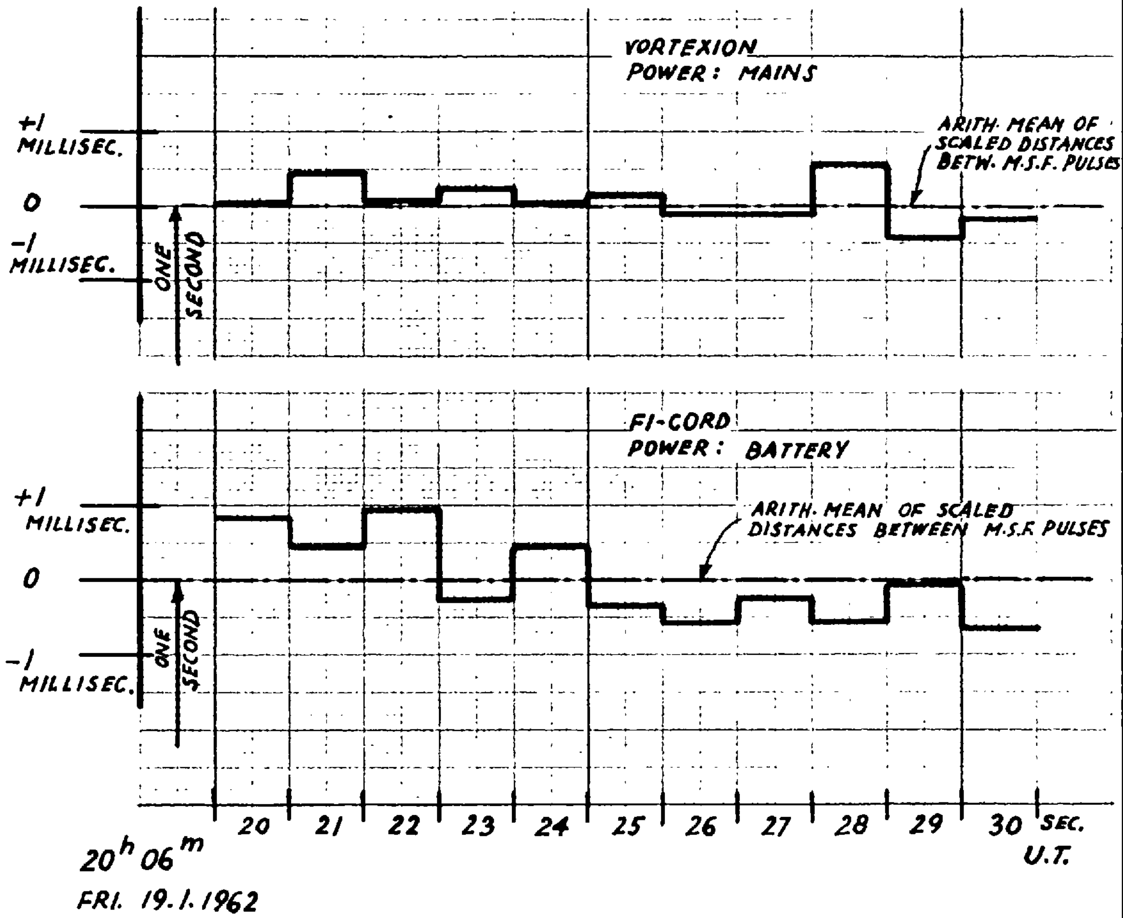
battery operated tape recorder and the tape recorder driven on mains power supply.

The variations of the time lengths about the arithmetic mean of the total time interval dealt with are plotted at a scale of 1 millisecond = 10 millimetres; for plotting purposes and for comparing the stability of the running speed of both recorders the arithmetic mean is set equal to one second U. T., in each case.

In Fig. 6.3.-1, the variations in the time lengths between successive pulses are all within $\pm$ one millisecond or better. Variations up to 5 milliseconds, which can be attributed to the running speed, have been encountered in adverse field conditions. The running stability is quite adequate for the employment of the tape recorder in field astronomy. Therefore, further investigations into the behaviour of the running stability during the intervals of one second of time are not considered.

Obviously, scaling errors and all other errors affecting reception, recording and evaluation all contribute to the variations of the time lengths shown in the diagrams, and are attributed to the running speed. Irregularities inflicted upon the transmission of standard frequencies during their path through the travel medium, and reception errors, would show up in similar shaped diagrams, if these errors were greater than the scaling errors and greater than the fluctuations of the running speed.

Scaling errors can be presumed to be random errors and of the same magnitude when scaling distances between pulses of identical receptions separately recorded on two tape recorders. Hence, from inspection of the diagrams, the variations in time intervals about the arithmetic mean can be attributed mainly to changes in tape speed. The order of accuracy of the stability of the running speed, which can be read off the diagrams, is in agreement with the accuracy derived from measured time intervals at re-play given in Tables 6.3.-1, -2, and -3.



STABILITY OF RUNNING SPEED OF TAPE RECORDERS

M.S.F. SECONDS PULSES, 5Mc/s, RECORDED AT 7½ in./sec ON FI-CORD AND VORTEXION TAPE RECORDERS SIMULTANEOUSLY.

TIME DISTANCE BETWEEN PULSES SCALED AFTER TAPE DEVELOPMENT.

Fig.: 6.3.-1.

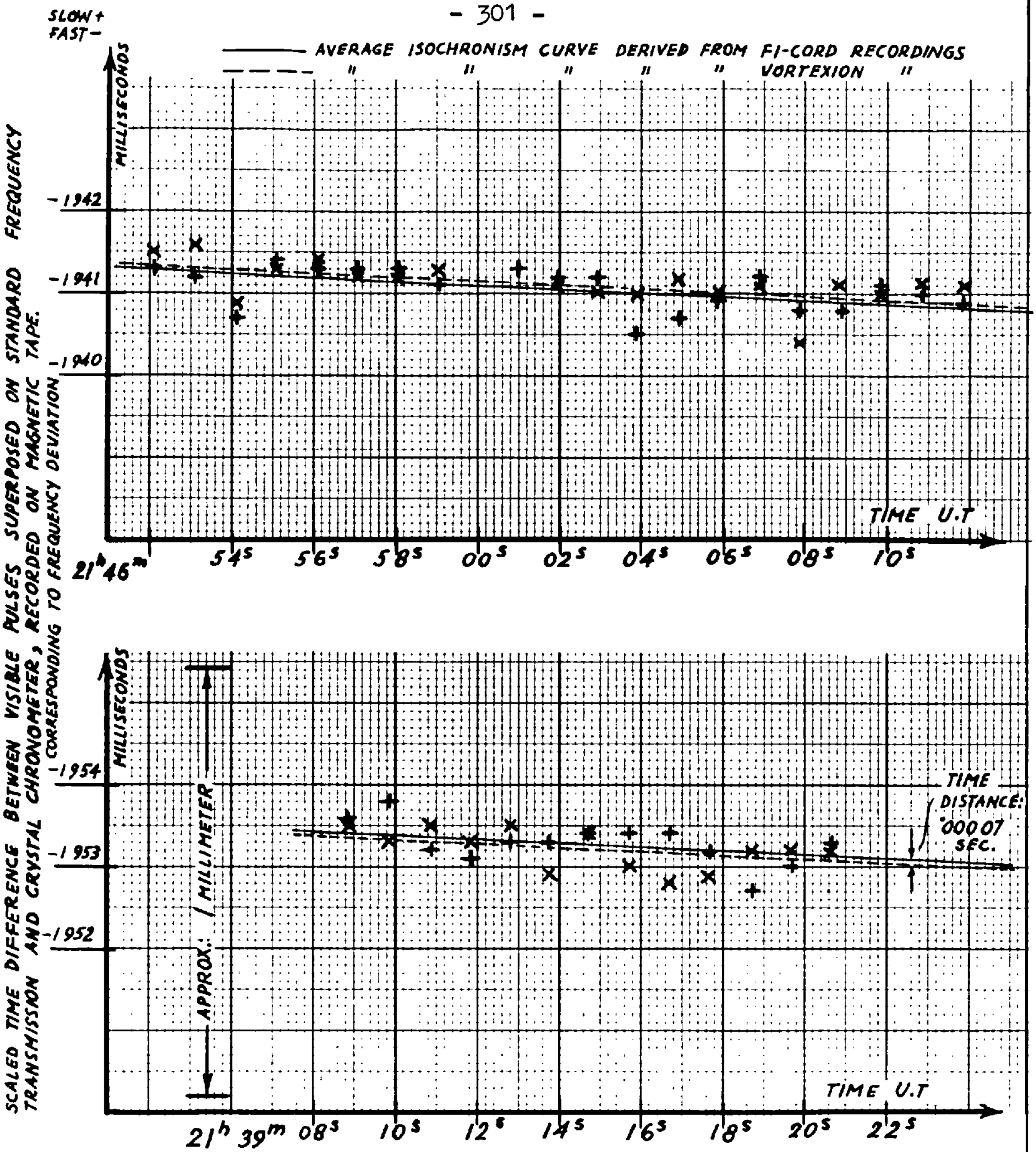
6.3. cont.

The isochronism curves of the crystal chronometer shown in Fig. 6.3.-2 are derived from independent tape development, and independent recording of the same sequence of frequency pulses, on two tape recorders. Submitted are two separate sets of observations. The recorders have been stopped and started between both series of frequency reception. Table 6.3.-4 contains the evaluation of the scaled distances for some of the plotted values. It will be noticed that the small fluctuations of the running speed (denominators in columns 3 and 4) have little or no effect on the calibration of the crystal chronometer (column 5 and 6).

The two average isochronism curves derived from recordings on Fi-Cord and on Vortexion differ a negligible amount, about 7×10^{-5} seconds of time, in each case. (Fig. 6.3.-2).

Information about the performance of the crystal chronometer, derived from independent recordings and tape development, reveal not only the precision with which the synchronization with frequency pulses can be executed, but also assist in recognizing the time keeping property of the crystal chronometer itself, and help to distinguish scaling errors from irregularities of received transmissions.

Reference to Fig. 6.3.-2 shows that the employment of a tape recorder as chronograph enables one to detect the crystal chronometer stability during seconds intervals in the order of fractions of milliseconds.



U.T. INDICATED BY SECONDS PULSES SUPERPOSED ON STANDARD FREQUENCY TRANSMISSION: H.B.N., 5 Mc/s, AT THE INSTANT OF RECORDING ON MAGNETIC TAPE.
CRYSTAL CHRONOMETER PULSES ARE PLOTTED AT U.T., SCALED FROM STANDARD FREQUENCY PULSES.
DATE: 19.1.1962

INDEPENDENT DETERMINATION OF

PERFORMANCE OF CRYSTAL CHRONOMETER WITH TWO TAPE RECORDERS

THERMOSTATICALLY CONTROLLED
OPERATING ON 12 V. BATTERY

INSTANTANEOUS STABILITY

DURING THE 60/61 SECOND INTERVAL OF MEAN SOLAR TIME.

AUDIBLE PULSES RECORDED ON VORTEXION . . . ×
" " " " FI-CORD. . . . . +

Fig.: 6.3.–2

INDEPENDENT SYNCHRONIZATION OF CRYSTAL CHRONOMETER WITH TWO TAPE RECORDERS

DATE: FRI. 19.1.1962
$21^h\ 46^m$ -

TABLE: 6.3.-4.

Nº OF CRYSTAL CHRONO-METER PULSE	U.T. sec. H.B.N. PULSE 5 Mc/s	SCALED DISTANCES BETWEEN VISIBLE PULSES OF CRYSTAL CHRONOMETER AND M.S.F. STAND. FREQUENCY PULSES $\frac{\text{DISTANCE : M.S.F. sec. to Chron.}}{\text{DISTANCE : M.S.F. sec. to M.S.F. sec.}}$		− NOMINAL TIME OF CRYSTAL CHRON. FROM TABLE 3.3.-1 + U.T. OF CRYSTAL CHRON. FROM RATIO OF SCALED DISTANCES ± CRYSTAL CHRON. {SLOW / FAST} ON H.B.N. RECEPTION	
		VORTEXION	FI-CORD	VORTEXION	FI-CORD
		MILLIMETER	MILLIMETER	SECONDS U.T.	SECONDS U.T.
55	52^s 53	$\frac{30.10}{191.90}$	$\frac{29.90}{190.34}$	-54.098_4 $+52.156_9$ -1.941_5	52.157_1 -1.941_3
56	53^s 54	$\frac{26.93}{191.84}$	$\frac{26.76}{190.06}$	55.082_0 53.140_4 -1.941_6	53.140_8 -1.941_2
57	54^s 55	$\frac{23.93}{191.83}$	$\frac{23.72}{189.86}$	56.065_6 54.124_7 -1.940_9	54.124_9 -1.940_7
58	55^s 56	$\frac{20.71}{191.90}$	$\frac{20.51}{190.26}$	57.049_2 55.107_9 -1.941_3	55.107_8 -1.941_4
59	56^s 57	$\frac{17.54}{191.86}$	$\frac{17.40}{190.19}$	58.032_8 56.091_4 -1.941_4	56.091_5 -1.941_3
60	57^s 58	$\frac{14.43}{191.84}$	$\frac{14.24}{189.65}$	59.016_4 57.075_2 -1.941_2	57.075_1 -1.941_3
61	58^s 59	$\frac{11.28}{191.84}$	$\frac{11.14}{189.74}$	60.000_0 58.058_8 -1.941_2	58.058_7 -1.941_3

7. Evaluation of Recorded Data.

7.1. General.

The data recorded by voice announcement are extracted by aural reception during re-play at recording speed, and are entered on calculation sheets. During the same re-play the start of a series of observation signals, or of a sequence of seconds pulses to be used for synchronization of the chronometer, can be marked with paint.

The information concerning time differences between recorded seconds pulses of standard frequency transmissions, radio time signals, chronometer seconds pulses and marker signals, or whichever it may be, are obtained with methods described in Section 3.5.3.; the most instructive of which is method No. 8, Section 3.5.3., enlarged upon in the following section.

Fig. 7.1.-1 shows the whole arrangement adopted for the extraction of the recorded data:

The section of tape between the winding reels has been just "developed" and the recorded pulses show up clearly. All the items used can be seen on the photograph: containers with various fluids and developing material, pestle and mortar, applicator next to the open flask (containing the fluid), magnifying glass, glass scale, a fine brush for marking the tape with white paint, and the tape recorder with spare batteries.

7.2. Tape Development

The purpose of "developing" the tape is to render the magnetic impulses recorded on the tape visible and thus to convert time intervals between audible pulses into linear distances between visible pulses.

The magnetized impulses recorded on the coated side of the tape are

Fig. 7.1.–1

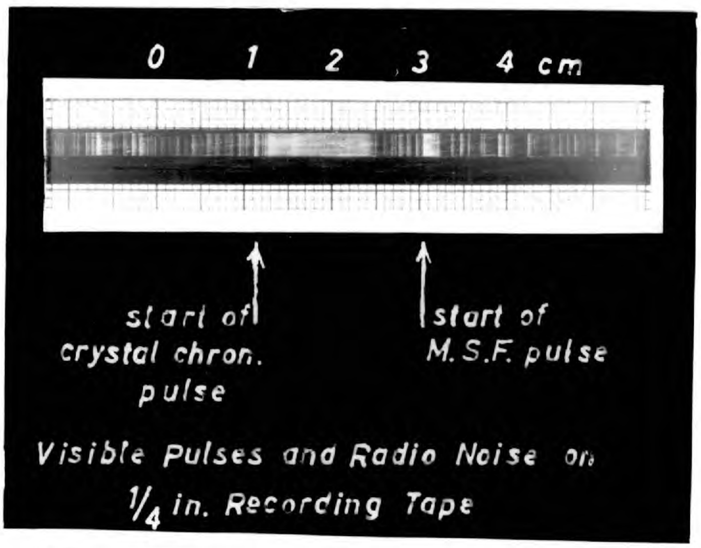


Fig. 7.2.–1

7.2. cont.

made temporarily visible with metallic powder and suitable liquid.

The tiny metal particles are kept in suspension in a volatile fluid and arrange themselves according to the pattern of the magnetized coating of the tape.

The shape of the pulses, defined by the metal particles which adhere to the tape, becomes visible to the unassisted eye, while the suspending medium evaporates.

The procedure of devel oping the tape consists of applying with a light touch the readily evaporable liquid saturated with powder, with a felt brush or cotton applicator to the track containing the recorded pulses, on the coated side of the tape.

Fig. 7.2.-1 shows a portion of developed $\frac{1}{4}$ inch recording tape, slightly enlarged (1: 1,1). The upper track contains a crystal chronometer pulse followed by a seconds pulse of M.S.F. standard frequency transmission, during atmospheric interference. (Radio noise). I ndividual cycles of the pulses can be recognized even on the photograph. (The tape reading is from left to right).

During development interference by magnetic objects in the vicinity should be prevented.

The tape is conveniently moved through two hand-type rewinds, spaced about four to five feet apart; the distance in between is subdivided into approximately $7\frac{1}{2}$ inches for locating the area of consecutive pulses.

The kind of metallic powder and its size to be used as developer varies with the type of suspending liquid. Both are chosen with the aim of achieving a specific result from recordings of definite frequencies and volumes, under a variety of conditions of reception, on tapes of different qualities.

The metallic material should not be strongly magnetized before coming into contact with the coating. Metallic powder with strong magnetic properties, or powder which has been strongly magnetized by contact with magnetic objects, can disturb the magnetic impulses on the

7.2. cont.

tape. The powder is required to possess only the properties of being readily attracted by the magnetic field of the tpae, and to adhere to its magnetized areas during the time the measurements between visible pulses are taken.

The suspending medium has to evaporate in a certain length of time during which the metal particles are attracted and set in position by the magnetic field. The fluid is supposed not to possess solvent properties, due to which the bonded iron oxide could be removed from the plastic base.

There are several metal powders which meet with the physical requirements; some are particularly suited to developing a tape containing only recorded crystal chronometer seconds pulses and observations signals in the absence of radio noise.

Carbonyl iron powder in a suspending medium has the advantage of its bright colour, which makes the magnetic pattern stand out clearly on the brown background of the coating, to which it sticks strongly. The powder, prepared from iron carbonyl, is commercially available. The size of the metal particles is sub-micron, but the effective size is about two to three microns, as the particles tend to agglomerate. The magnetic behaviour of iron carbonyl produces clear visible cycles, but renders a rather low contrast between individual pulses. It can be used to advantage for the development of recorded signals at medium amplitude with little atmospheric interference.

Fig. 7.2.-2 represents a crystal chronometer seconds pulse recorded at $7\frac{1}{2}$ in./sec., made visible with carbonyl iron powder. The 5x magnification enables one to distinguish every cycle. The photograph has been taken through a low power optical trail. A perfectly flat picture could not be achieved, and therefore some parts of the photograph are slightly out of focus. The fine metallic powder, applied thinly, adheres even to weak magnetic impulses.

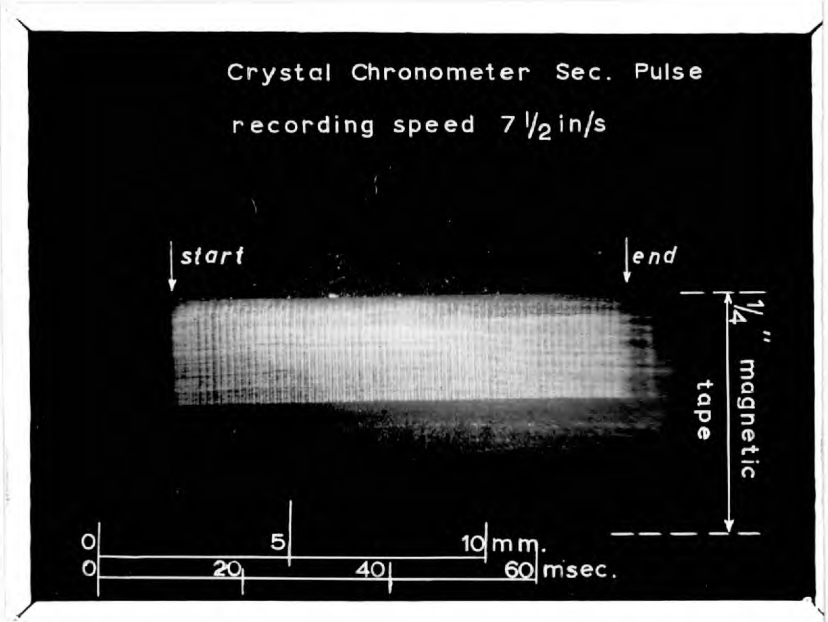


Fig. 7.2.-2

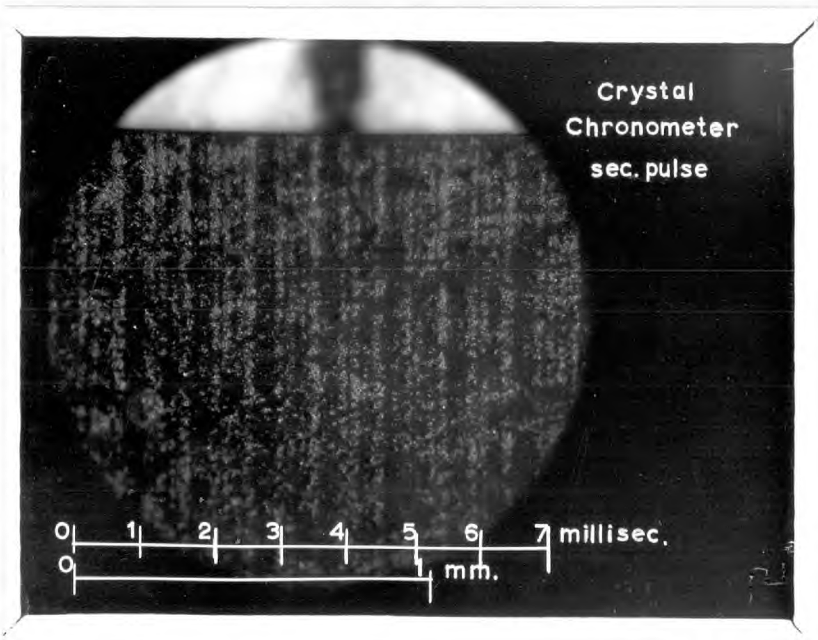


Fig. 7.2.-3

7.2. cont.

Fig. 7.2.-3 is a photograph taken through a microscope of 48x magnification, showing parts of the crystal chronometer seconds pulse, recorded at $7\frac{1}{2}$ in./sec. The metallic powder assembles itself in almost straight lines, following the cycle pattern, and even tiny irregularities of consecutive cycles, within fractions of a millisecond, can be noticed. Before being re-used, the portion of the tape treated with metallic powder has to be cleaned carefully, as its abrasive quality may cause considerable wear of the coating and can damage the record head.

Chromium flake in powder form has to be used with care; it is hard and brittle; therefore any pressure on the tape, during application of the powder, has to be avoided.

A suitable suspending medium for the metallic powders mentioned above is xylene $C_6H_4(CH_3)_2$, a colourless hydrocarbon, which is commercially available.

A coarse pattern of magnetized spots is preferably treated with unannealed electrolytic iron powder of 300 mesh. Temporary good results are achieved when it is brushed on to the tape with xylene. The powder can be administered to the tape equally well in a suspension of diluted trichloroethylene $C\,Cl_2 : CHCl$, which is also commercially available. The metallic powder can be left on the tape, so that the visible pulses may be used later, whenever required, and the tape may be wound on. Trichloroethylene applied in an overdose takes off the coated layer of the tape. After use, the metallic particles are removed easily with a brush or with cotton wool.

Almost any tape can be developed with iron filings, ground in a mortar to 300 mesh or finer. Iron powder is not very abrasive and is only slightly inferior to the micron-sized carbonyl iron powder as regards the response to the magnetic field of the tape.

Fig. 7.2.-4 shows parts of a crystal chronometer seconds pulse

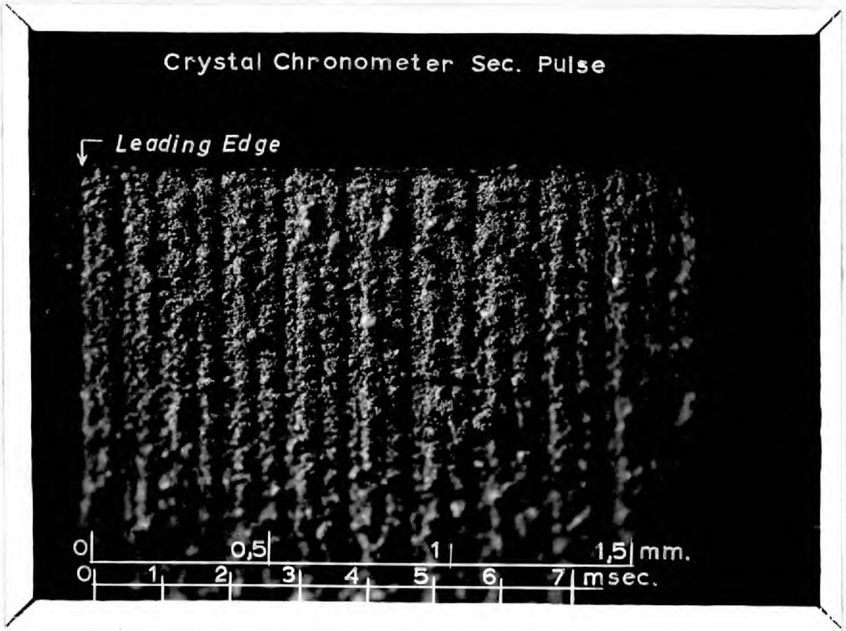


Fig. 7.2.-4

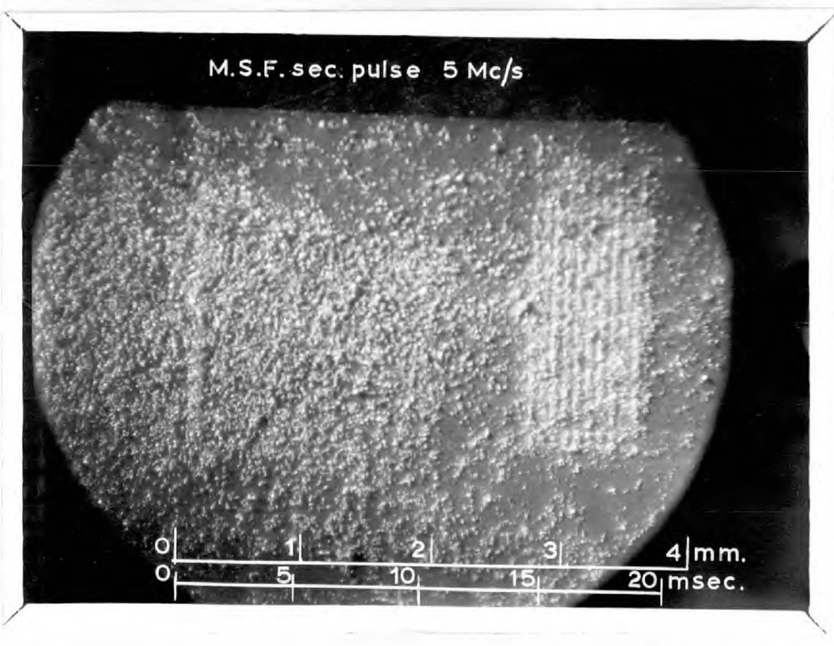


Fig. 7.2.-5

7.2. cont.

recorded at $7\frac{1}{2}$ in./sec, photographed through a microscope of 48x magnification. The developing material consists of iron powder (applied rather generously) suspended in toluene $C_6H_5CH_3$.

The cycles are visible and their shape stands out well. The leading edge is easily recognizable in spite of radio noise to which metal particles adhere (left edge of the photograph). The iron powder can be suspended equally well in xylene, or also in methylated spirit C_2H_5OH, which has to be diluted. Overlapping signals can be distinguished without effort when developed with an overdose of iron powder in the fluid.

The magnetic property of nickel makes nickel powder suitable for developing recordings in the presence of continuous atmospheric noises, as long as these are not of excessive volume.

In Fig. 7.2.-5 the portion of the tape shown contains atmospheric noise to which the nickel powder clings in a loose manner, and a well defined M.S.F. pulse on 5 Mc/s.

The weakly magnetized impulses produced by the radio noise failed to arrange the nickel particles. The M.S.F. pulse can be undoubtedly identified by the number of cycles and by its duration. Nickel powder is very brillliant and not too abrasive. Both qualities make it a very suitable developing agent. It requires properly magnetized areas of the tape to be attracted. The tape speed is again $7\frac{1}{2}$ in./sec. The photograph is taken at incident light through a microscope of about 17x magnification. Very fine nickel powder (micron-sized) produces equally good results as colloidal iron powder.

Undefined pattern of pulses associated with white noise can be treated also with cobalt powder suspended in toluene, which has to be mixed with alcohol and water to avoid the cellulose acetate base of the tape becoming too brittle.

Fig. 7.2.-6 shows a M.S.F. pulse developed with cobalt powder. The arrangement of the metal particles is quite distinct, and the cycles

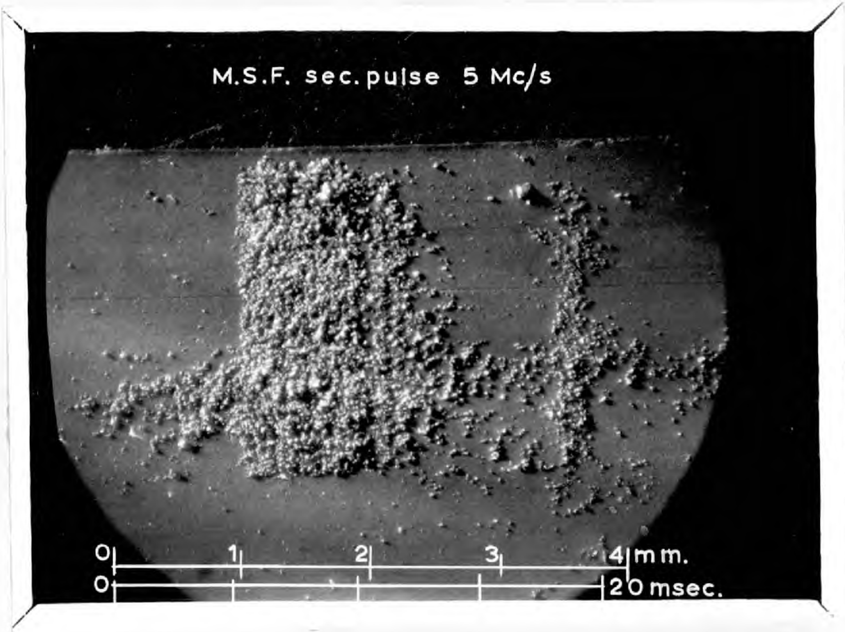


Fig. 7.2.-6

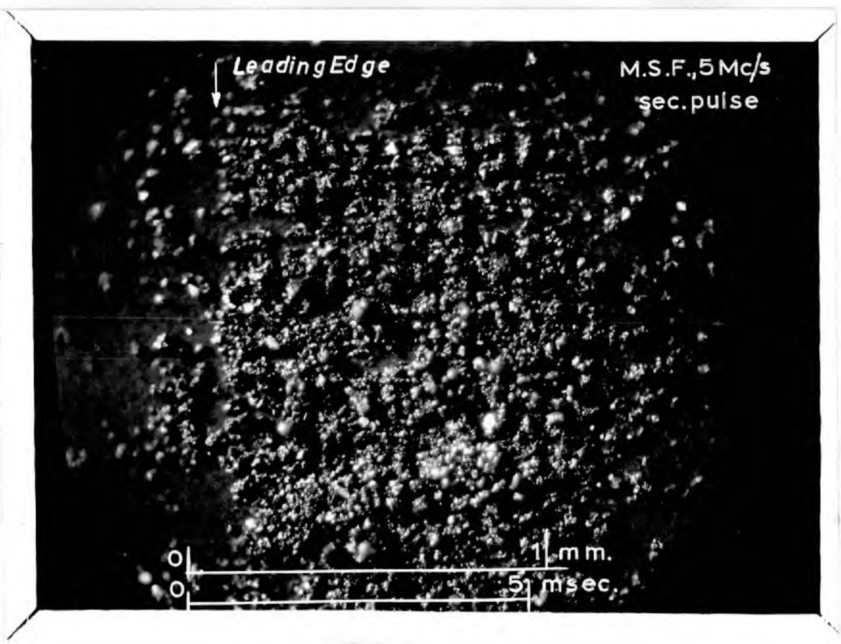


Fig. 7.2.-7

of the M.S.F. pulse can be recognized. Strong radio noise succeeded in attracting some metallic powder. The photograph is taken as in Fig. 7.2.-5. The cobalt powder used for the experiments is produced by Sheritt-Gordon Mines, Canada; it is of non-uniform size from ca. 100 to 300 mesh; pulverized finer, it would give similar results to nickel sponge.

Fig. 7.2.-7 represents the 49x magnification of a M.S.F. pulse made visible with cobalt powder; individual attraction of the metal particles according to their sizes can be noticed. The strong magnetic impulses effected a classification of the small particles in the drop of liquid before evaporation. Very fine particles cluster in globules and accumulate according to the strength of the magnetic field. It can be clearly seen that the maximum strength of the signal is not concentrated in its leading edge, and is gradually declining after the maximum peak is reached.

Records of faint receptions of signals fail to attract coarse metal powder. These pulses can be made visible with metallurgical fume in surplus of fluid. The fume is suited for tape development if it consists of very small solid particles which are as a rule a mixture of particles of elements and metallic compounds. In Fig. 7.2.-8, overlapping signals of standard frequency transmissions produced weak magnetic recordings. Parts of clear tape can be seen between the cycles, which are developed with iron oxide fume. The iron oxide fume is formed by condensation of vapour in the presence of carbon in the oxygen steel making process. The similar colours of the coating and developing material yield low contrast for the photograph, taken through a microscope of 17x magnification. When still wet, the iron fumes are almost black and produce distinct visible pulses. A suitable volatile fluid for the iron oxide particles is toluene, but alcohol CH_3CH_2OH can also be used. Saturated with trichloroethylene, the oxidized iron vapour can be glued to the tape; the fumes can be removed by washing the tape with trichloroethylene mixed with water and alcohol.

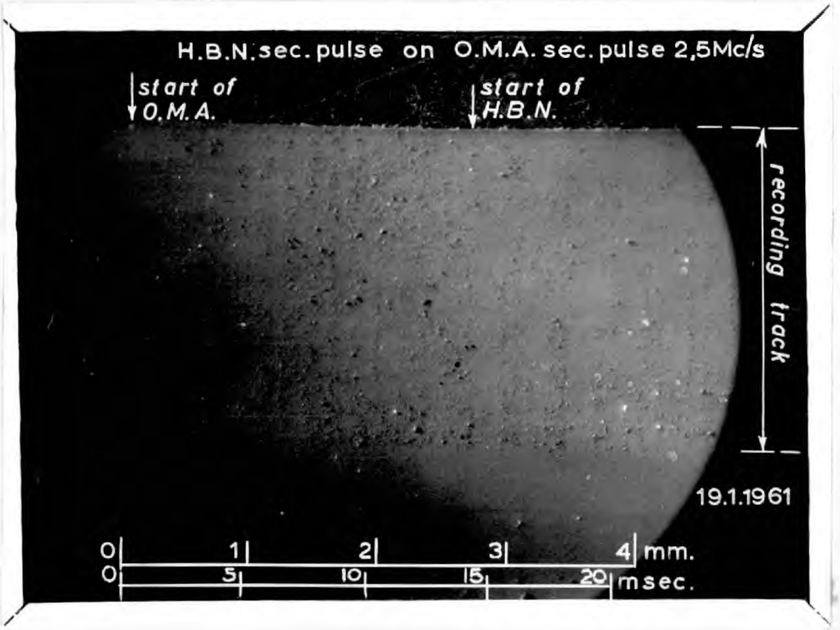


Fig. 7.2 -8

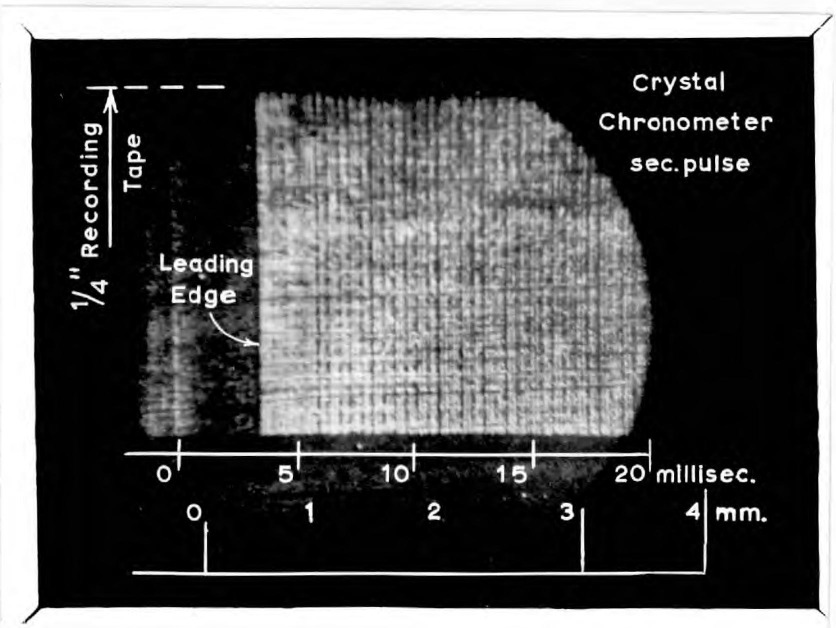


Fig. 7.3.-1

7.2. cont.

The plastic base of most of the tapes resists the treatment with hydrocarbons. Acetone affects the mylar base and has to be applied carefully. Methylated spirit, or any alcohol, causes wrinkling of some tape qualities; to prevent this, the fluid should be diluted with water or mixed with xylene, toluene, etc.

Ready made mixtures of metallic powders and fluids are commercially available and are used mainly for tape splicing, editing purposes, etc. A developing agent consisting of a mixture of metal powder and a lacquer diluent is produced by the Ampex Corporation, and is available under the trade name "Edivue". Edivue is best suited for the development of well defined magnetic impulses in the absence of strong radio noises. "Vulture", obtainable from Crow Co. Ltd., is well suited for making the magnetic impulses visible, and can be used on any tape. The fluid is non-inflammable and the metallic dust particles are of uniform fine grain.

The "Scotch Brand Magnetic Tape Viewer" contains colloidal iron powder enclosed in a transparent circular container of about one inch diameter. When placed on the magnetic tape the powder arranges itself according to the recorded magnetic impulses. The viewer can be used for studying the pulse pattern, but it is not suited for scaling distances longer than one inch.

The magnetic layer material, e.g. bonded gamma Fe_2O_3, is frequently removed from the plastic base by the diluent. This can be noticed during development by the decolouring effect, and damage to the tape can be prevented in time.

Re-development after wiping off the metal powder can be repeated any number of times, i.e. as long as the tape material resists the treatment, and obviously as long as the tape has not been cancelled.

Developing the tape does not affect the quality of the recording and neither deterioration nor difference can be noticed on back play. When

7.2. cont.

carefully cleaned, no stains remain on the tape except when permanent marks are produced.

The study of the pulse patterns confirmed the theoretical assumption that receiving and recording instruments do contribute to the final pattern of the magnetic impulses, visible after tape development, but the basic characteristics of the various pulses (frequency, amplitude, wave form etc.) are not affected.

7.3. Scaling Distances on Magnetic Tape.

The commencement of the crystal chronometer pulse is chosen as the reference point of time measurement. The tape travel of most of the tape recorders is from right to left, depending on the position of the recording head; therefore. the top track, when viewed from the coated side, is read from left to right and the left end of the pulse becomes its leading edge.

The Royal Greenwich Observatory defines the time at which the seconds dot of GBR 16 kc/s reaches about 40% of its peak amplitude as the reference point for measurement of the reception time. The average time interval from the start of the signal to this reference point of time measurement is about four milliseconds. The length of the time interval depends on the type of transmitter, and can amount to 12 or even 15 milliseconds. The reference point of the measured time of reception of the seconds pulse superposed on standard frequency transmissions is the commencement of the signal.

There is an advantage in having the leading edge as the reference point for time measurement: viz., there is a distinct line for scaling the distance between pulses and a general zero mark when dealing with a variety of signals.

The photograph (Fig. 7.3.-1) shows the crystal chronometer seconds pulse enlarged 16.5 x; the leading edge of the pulse is a clear fine line which represents a distinct index mark for scaling distances; the "strength" of the pulse is rapidly increasing and its regular structure can be seen to start after about $2\frac{1}{2}$ milliseconds.

The commencement of nearly all time pulses of standard frequency transmissions, when made visible, shows up in a sharp leading edge, which can be recognized also in the presence of strong radio noise.

From the photograph (Fig. 7.3.-2) of the HBN seconds pulse of 16.5 x magnification, it can be noticed that its leading edge is as well defined as its individual cycles. The transverse lines are only brush marks which remained after development.

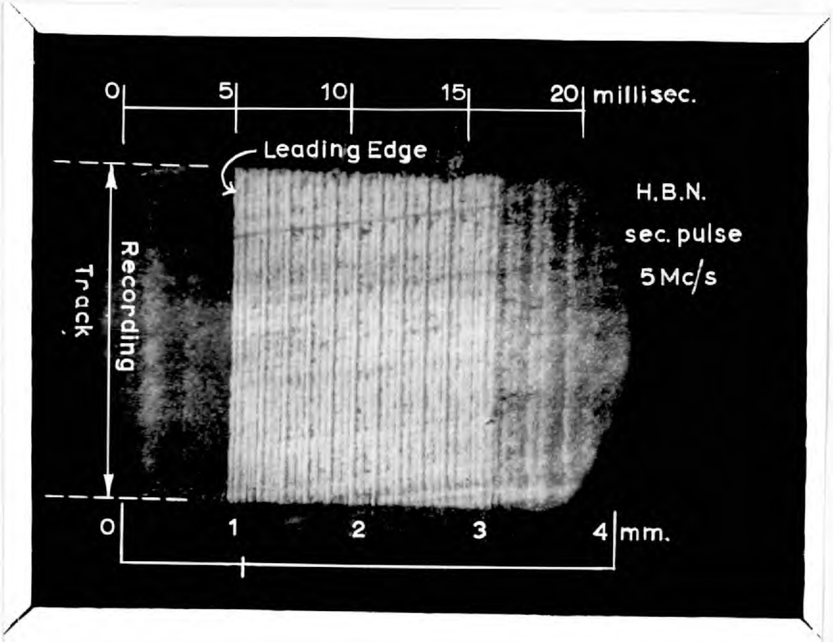


Fig. 7.3.-2

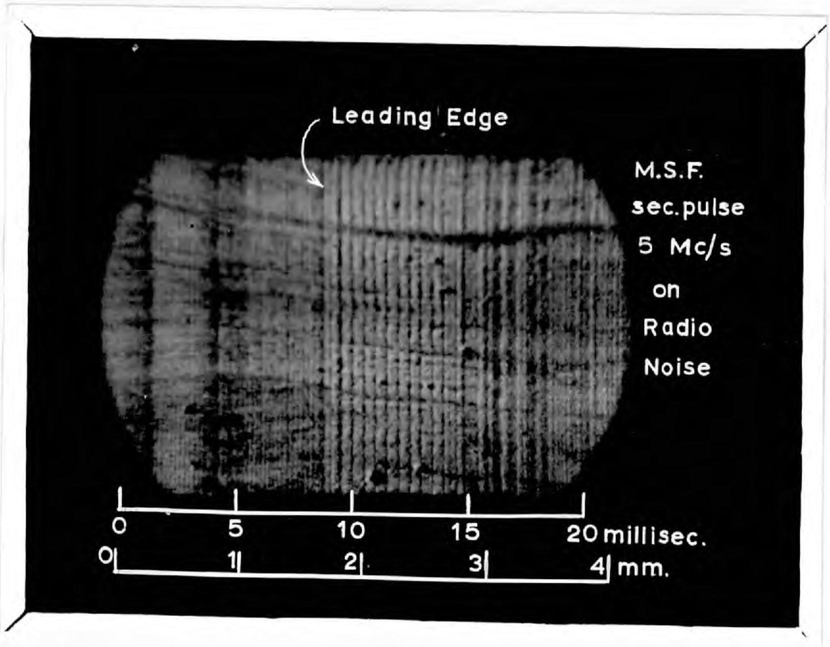


Fig. 7.3.-3

7.3. cont.

A clear distinction is possible between the leading edge of the M.S.F. second pulse (Fig. 7.3.-3) and the persistent radio noise throughout the reception.

Distances between visible pulses are measured with an ordinary box wood rule subdivided to $\frac{1}{2}$ mm, or to $^1/50$ inch. A scaling accuracy of one millisecond of time can easily be achieved at a running speed of $7\frac{1}{2}$ in./sec., with tenth estimation of the scale interval.

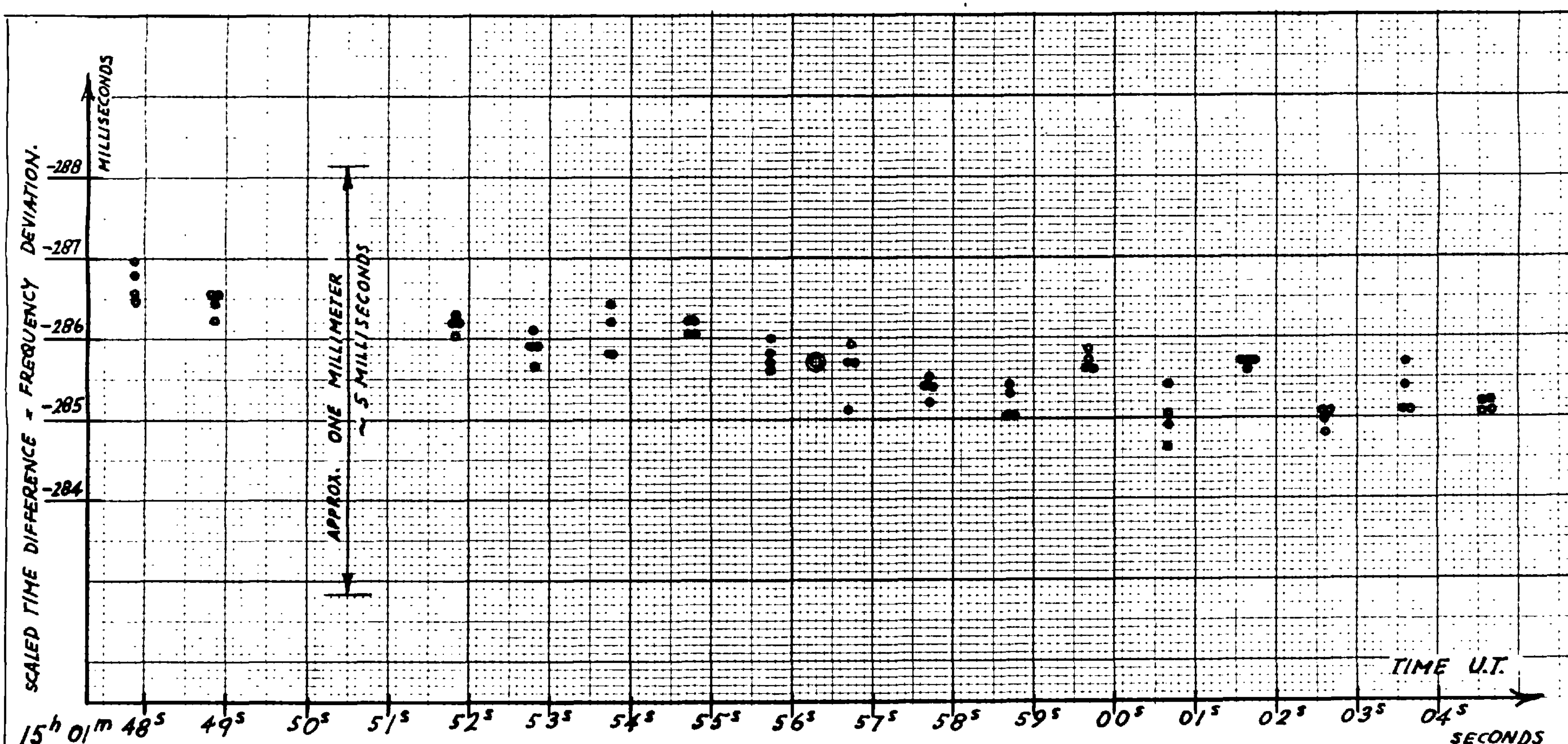
The calibration of the subdivisions of the scale is not necessary, since only ratios of distances are required.

The precision with which distances between visible pulses can be scaled is verified by repetition of the developing processs and by scaling the pulse distances again. Results of repeated measurements with a box wood rule subdivided into $\frac{1}{2}$ mm are submitted in Fig. 7.3.-4. Ratios of scaled distances between visible pulses corresponding to frequency deviations of the crystal chronometer are plotted against U.T. of frequency reception. The distances were scaled four times; each time the tape has been cleaned and re-developed. Thus, four independent scaling results are obtained for the time difference between the crystal chronometer pulses and standard frequency reception at consecutive crystal chronometer beats.

From the diagram, the precision of extracting data from tape development is quite conspicuous, and it will be noticed that the scatter of the results is well within one millisecond. Hence the scaling error alone will hardly shape the crystal chronometer isochronism curve on which is based the time interpolation for astronomical observations.

From the tests it can be concluded that scaling tape distances to one tenth of a millimetre is quite justified; to avoid rounding off errors it is customary to estimate the smallest scale interval to half or to quarter of its value.

The measurements required for the calibration of the crystal chronometer and for extraction of field data described, were taken with



DATE: APRIL 9, 1961

U.T. INDICATED BY SECONDS PULSES SUPERPOSED ON STANDARD FREQUENCY TRANSMISSION: M.S.F., 5 Mc/s, AT THE INSTANT OF RECORDING ON MAGNETIC TAPE, RECORDED ON FI-CORD.

TIME DIFFERENCE BETWEEN VISIBLE PULSES OF CRYSTAL CHRONOMETER AND OF STANDARD FREQUENCY TRANSMISSION OBTAINED FROM:

<u>SCALED DISTANCES ON MAGNETIC TAPE</u>

WITH BOXWOOD SCALE SUBDIVIDED TO 0·5 mm ≈ 2·6 MILLISEC.

DISTANCES SCALED INDEPENDENTLY FOUR TIMES.
FOUR RESULTS PLOTTED FOR EACH CHRONOMETER PULSE.

Fig. 7.3.–4

7.3. cont.

a glass scale made by Wild, Switzerland. The transparent scale of 220 mm length with deeply engraved lines, subdivided to 0.1 mm throughout, enabled readings to be taken which were almost free from parallax, since the side containing the divisions came into direct contact with the object.

Relative merits of the glass and box wood scales can be derived from Fig. 7.3.-5. Distances between developed pulses measured with the glass scale produced results which show slightly less deviation from the mean rate than those obtained with the box wood rule .

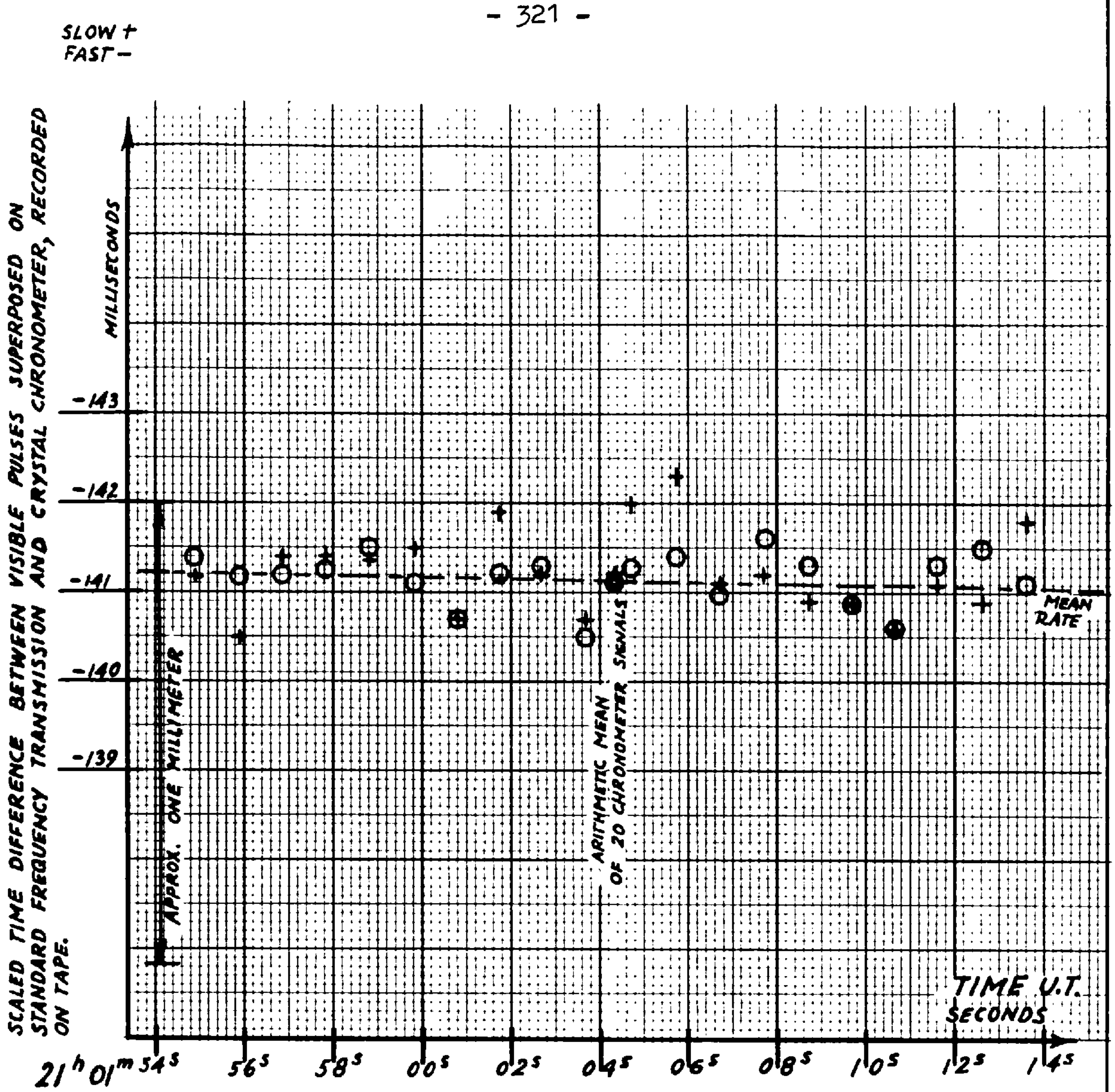
A lens of five to ten magnifications is quite adequate for reading the scale, since the unassisted eye can comfortably distinguish $\pm$ .15 mm at an observing distance of 250 mm. Scaled distances between pulses (normally taken from leading edge to leading edge) can be checked by measuring the distances between the ends of the pulses and adding their known lengths. The r.m.s. error of extracted time differences between chronometer and time reception is approximately 0.2 milliseconds , subject to a scaling and reading accuracy of 0.01 mm; this r.m.s. eeror is in agreement with the precision of the crystal chronometer.

The scaling accuracy surpasses the precision with which the time is represented by the received seconds pulses in the field.

Time measurements between individual pulses may be in error by more than 10 milliseconds, since the travel time of the H.F. transmission is very uncertain over longer distances, owing to back scatter. (Section 3.5.3.)

An effective velocity of 285 000 km/sec. can be used for the computation of the propagation time, for distances up to 5 000 miles, with an average uncertainty of $\pm$ 0.5 milliseconds.

However, receptions of seconds pulses of standard frequency transmissions which appeared to be in error of more than 2



DATE: TUE. MAY 9, 1961.

U.T. INDICATED BY SECONDS PULSES SUPERPOSED ON STANDARD FREQUENCY TRANSMISSION: M.S.F., 5 Mc/s, AT THE INSTANT OF RECORDING ON MAGNETIC TAPE.
CRYSTAL CHRONOMETER PULSES ARE PLOTTED AT U.T., SCALED FROM STANDARD FREQUENCY PULSES.

TIME DIFFERENCE BETWEEN VISIBLE PULSES OF CRYSTAL CHRONOMETER AND OF STANDARD FREQUENCY TRANSMISSION OBTAINED FROM:

SCALED DISTANCES ON MAGNETIC TAPE

○ WITH GLASS SCALE SUBDIVIDED TO 0·1 mm
\+ " BOXWOOD SCALE " " 0·5 mm

Fig. 7.3.-5

7.3. cont.

milliseconds, which happened rarely, were discarded when calibrating the crystal chronometer.

According to the statement of the Smithsonian Astrophysical Institution, time intervals in the order of one millisecond are sufficient for optical tracking of satellites. Likewise an overall accuracy of $\pm$ 2 milliseconds is required for the minitrack programme of the National Aeronautics and Space Administration.

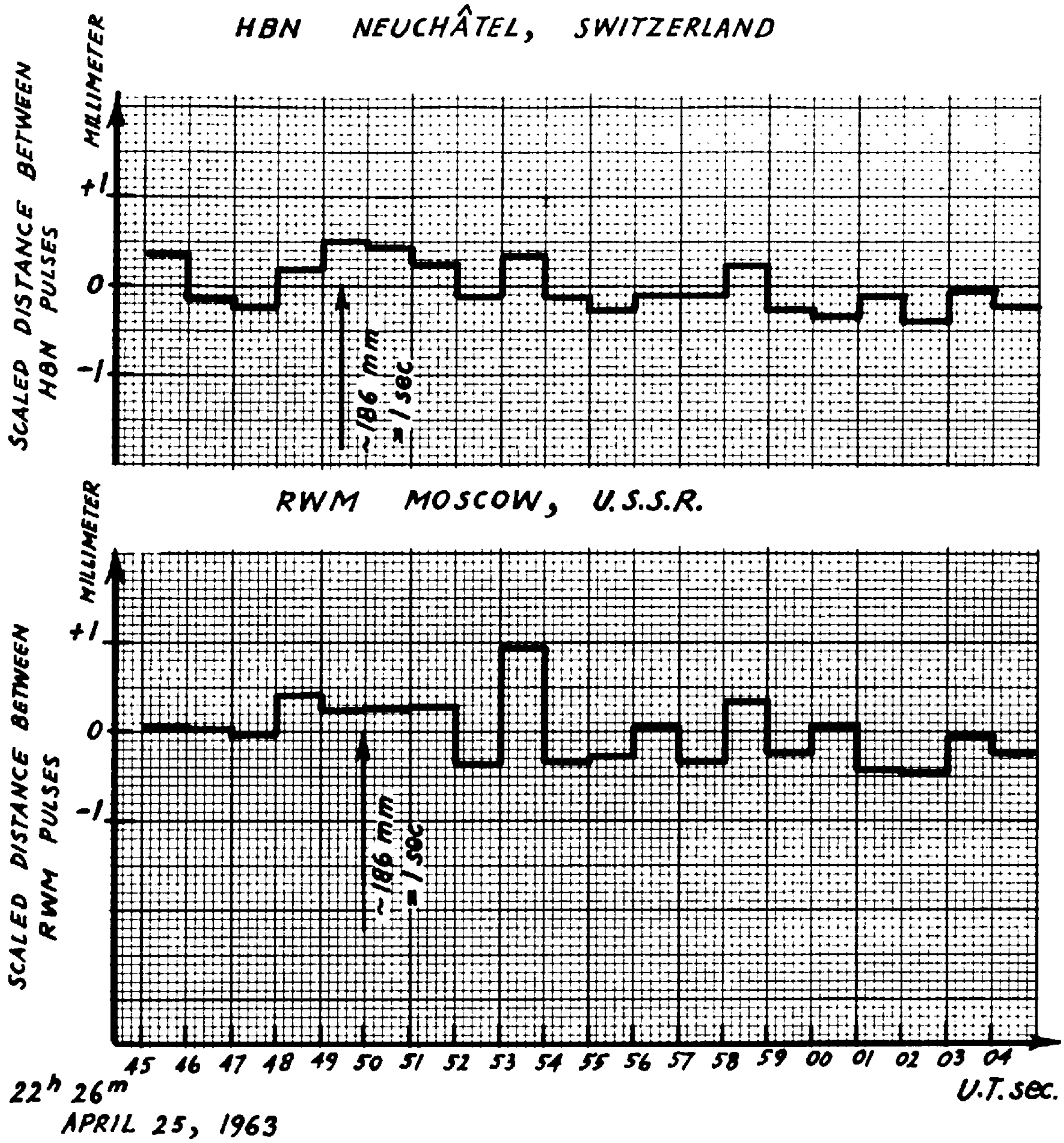
The great precision of extracting time intervals between pulses from magnetic tapes, after development, enables one to achieve high accuracy in timing optical observations, in testing and synchronizing the crystal chronometer and in the determination of the stability of the speed of tape recorders. These items are dealt with in previous sections.

Scaled pulse distances may be distorted by random errors, particularly by the personal error in scaling.

Isochronism curves of chronometers based on simultaneous receptions of several standard frequency transmissions should have similar shape, if there are adequate reception conditions and if the transmitted frequencies are of equal quality. The scaling errors which enter with the scaled distances into the time error contribute little to the general shape of the isochronism curve.

Irregularities of the tape velocity of recorders can be detected from scaled distances between visible pulses of standard frequency transmissions received simultaneously.

Fig. 7.3.-6 illustrates the fluctuations of the scaled lengths of the time intervals between consecutive pulses. The two standard frequency transmissions are H.B.N. and R.W.M. on 5 Mc/sec., received at the same time and scaled independently. Both diagrams indicate some similarity. Scaling errors may be responsible for the unequal appearance of some parts of the diagrams, but from their general trend the behaviour of the tape



SCALED TIME INTERVALS BETWEEN VISIBLE PULSES
OF STANDARD FREQUENCY TRANSMISSIONS
RECORDED ON MAGNETIC TAPE.

Fig. 7.3.-6

7.3. conti

recorder is clearly interpreted.

Errors caused by atmospheric interference are certainly present, but, because both diagrams are similarlyshaped, it is unlikely that the fluctuations are due solely to atmospheric influence; the directions and distances from transmitters to receiver are totally different.

7.4. Characteristics of Pulses and Advantages of Tape Development.

Significant properties of frequency pulses, their total length which depends on pulse duration, and the number of cycles per second, can be used for identifying the transmitting station.

The reception of a station's broadcast announcement or call sign preceding a standard frequency transmission is not a sufficient safeguard for accepting the audible superposed seconds pulses as originating from the same source.

Due to reception conditions, the seconds pulses may be completely covered by those of another station transmitting standard frequencies commencing without any announcement.

Definite frequencies are allocated to standard frequency transmissions which have usually the same type of time signal, constituted of 5 cycles of 1000c/s modulation. (Section 4.3).

In 1960 and 1961 there were two five minutes periods during each hour (Fig. 4.3.-1) when e.g. H.B.N. transmissions were not overlapping M.S.F. transmissions; and four periods of five minutes of M.S.F. transmissions not coinciding with H.B.N.

This schedule gave ample opportunity for selective reception, if wanted, of either of the two transmissions, and was therefore very convenient for aural and visual identification of pulses, especially at field stations where simultaneous receptions occurred. Due to changes in transmission pattern of H.B.N. (Fig. 4.3.-2), all H.B.N. seconds dots were radiated during M.S.F. transmissions, so that only three periods of five minutes of M.S.F. transmissions did not coincide with H.B.N. transmissions. Further changes of time service have established an alternative five minutes pattern between M.S.F. and H.B.N. (Fig. 4.3.-3) Both stations are now co-ordinated; their time pulses can be interchanged at the receiving station with proper allowance for travel delay.

The times of emission of radio time signals from stations which are now co-ordinated are kept at the same instant to within one millisecond

7.4. cont.

and with respect to U. T. 2 within limits of $\pm$ 50 milliseconds, for reasons of the interchangeability of time and for frequency reference.

The propagation time of the time signals makes it impossible to synchronize them at every point.

The proposed unification and co-ordination of transmission of conventional time signals and of the signals superposed on standard frequency transmission will no doubt establish a world coverage of signals of the kind appropriate for field requirements.

For full use of the high accuracy of the crystal chronometer and of the standard frequency transmissions the reliable identification of transmitted frequency pulses is of the utmost importance.

A transmission pattern of co-ordinated stations, which would provide for overlapping transmissions and also for periods of one transmission only, would enable identification of the various transmitting stations with the greatest of ease. Such a programme would also take care of reception conditions, and would be feasible since the number of participating authorities of the co-ordinated time and frequency transmission service is not excessive.

The time signal reception would be reasonably free of uncertainties if the times of emission could be kept constant and the pulses of the various transmitters would have distinct audible differences.

Particular characteristics of pulses are undetectable by the human ear, but are easily recognizable by visual inspection of the pattern of magnetized areas after tape development.

These characteristics consist, besides duration and frequency, of tiny irregularities of successive cycles, seen as unequal spacing and width; commencement and end of the pulse have also a peculiar shape.

The particular properties of the pulses depend on the transmission system and are therefore ever present, regardless of the developing material used and of the quantity with which the metal powder is administered.

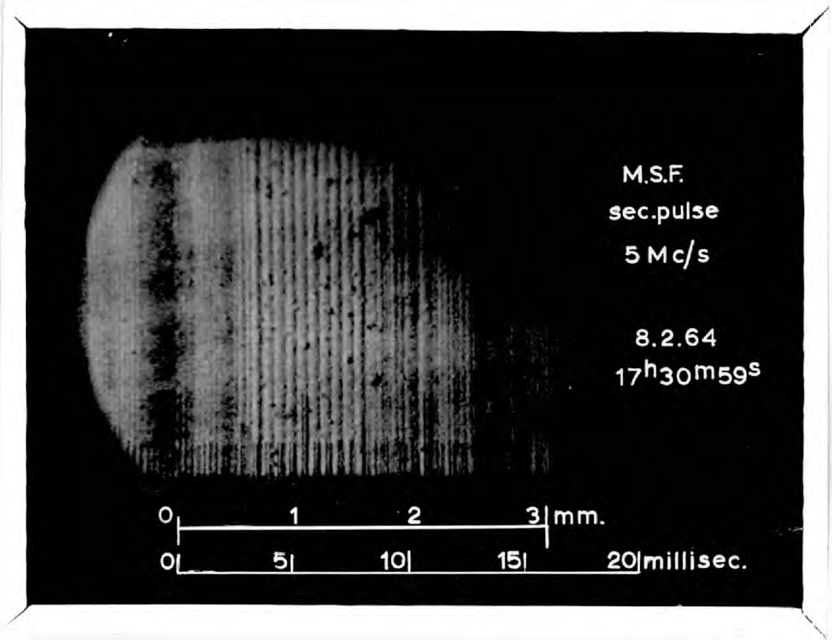


Fig. 7.4. -1

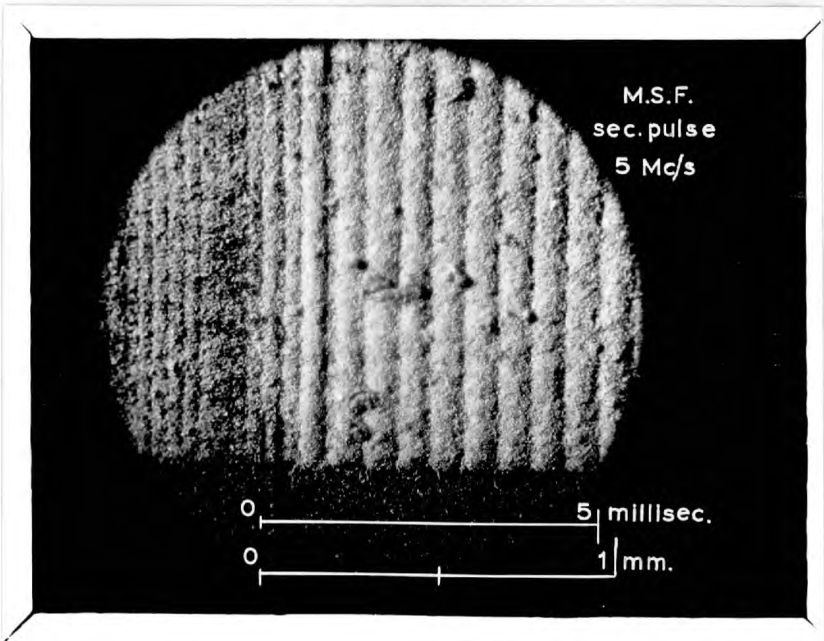


Fig. 7.4. -2

7.4. cont.

In Fig. 7.4.-1 the five cycles can be seen which constitute the MSF seconds pulse; very small differences between the cycles are revealed; these are quite distinct in spite of interference by radio noise.

The structure of the pulse and of the characteristic leading edge stand out clearly at 47 x magnification, shown in Fig. 7.4.-2. The pulse is audible as a tone of 1000 cycles per second. Owing to the short duration of the pulse the sound is rather hollow.

A recorded MSF seconds pulse made visible with metal powder applied sparingly can be seen in Fig. 7.4.-3. The identical form and the same peculiarities of the pulse, especially the unequal gaps between successive cycles, can be recognized without excessive magnification.

The elongated MSF minute pulse is audible as a whistle; it sounds entirely different from the MSF seconds pulse, although both pulses have an equal number of cycles per second. The developed minute pulse, illustrated in Fig. 7.4.-4 shows the same structure as the MSF seconds pulse and has also the identical amplitude. The reason for the apparent difference in sound is the resolving ability of the human ear. Owing to the short duration of the five cycles seconds pulse its components can not be recognized by the ear.

The HBN seconds pulse is audible as a tone of 1000 cycles per second and its sound can be distinguished from the MSF seconds pulse by the ear, subject to good reception of both transmissions, practical experience and training.

Fig. 7.4.-5 shows a seconds pulse of HBN transmission on the upper track of the tape, magnified 5 x. The properties stand out conspicuously, especially its leading edge followed by the individual cycles.

In Fig. 7.4.-6 the HBN pulse is presented at 17 x magnification and is somewhat deformed by radio noises. Nevertheless its structure can be seen to be entirely different from the preceding crystal chronometer seconds pulse, and from the MSF pulse when comparing Fig. 7.4.-6 with the MSF pulse in Fig. 7.4.-1.

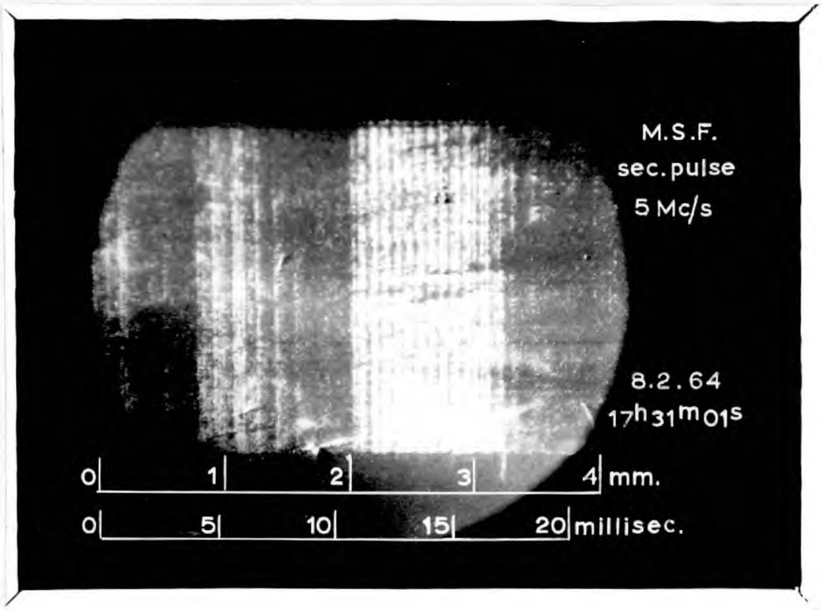


Fig. 7.4.-3

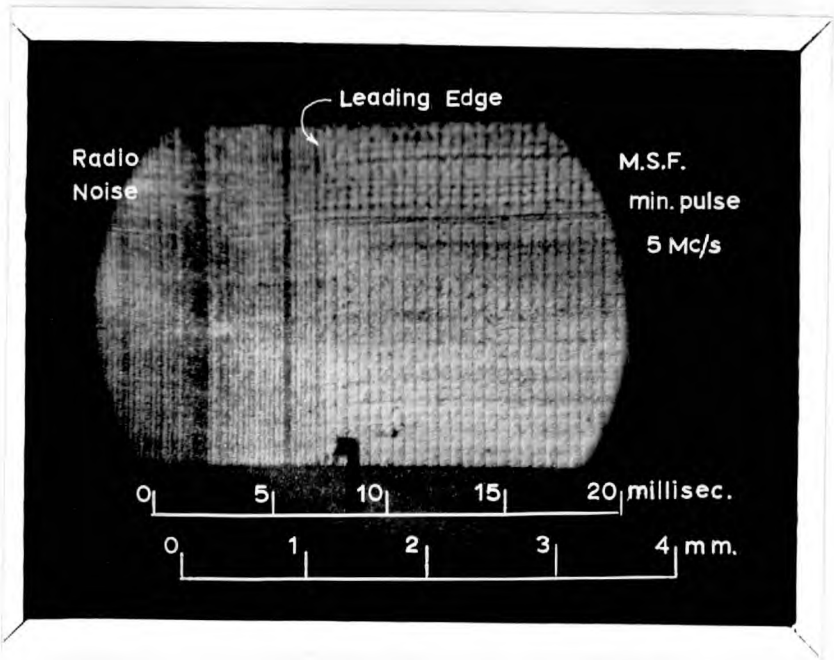


Fig. 7.4.-4

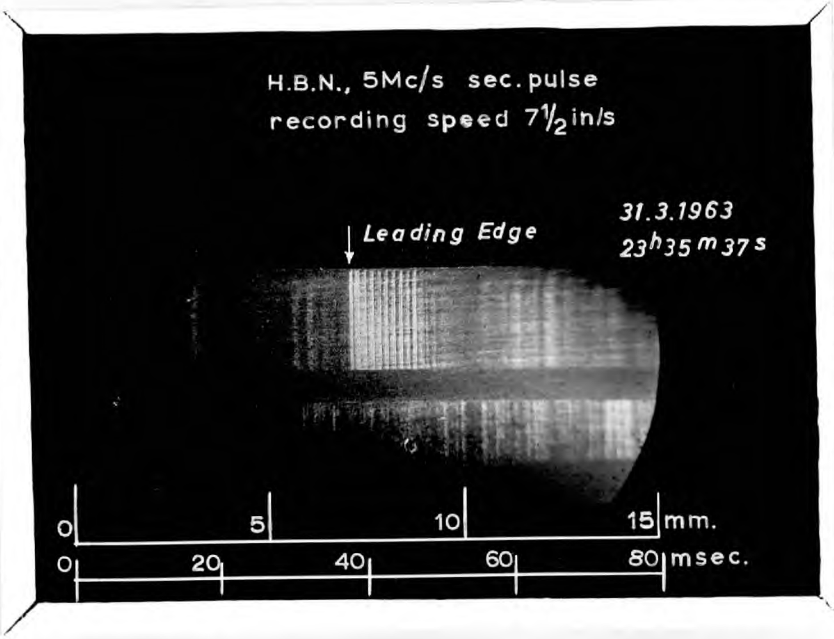


Fig. 7.4.-5

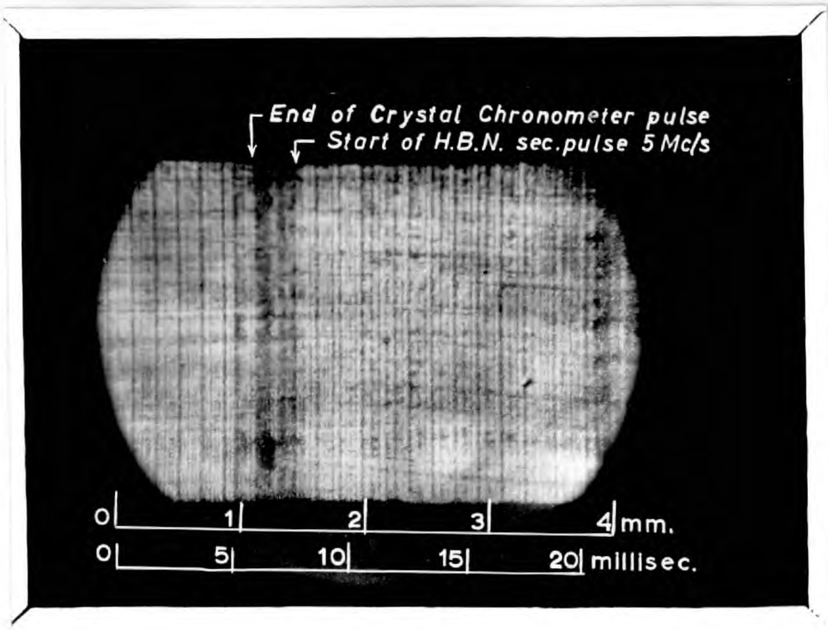


Fig. 7.4.-6

7.4. cont.

The HBN seconds pulse consists of five interruptions of one millisecond each transmitted at intervals of one millisecond. The audible pulse is therefore ten milliseconds long. On the photograph the five complete cycles are seen as clean cut lines, as is always the case when a train of waves is interrupted by the transmitter, or the frequency is altered.

OMA standard frequency transmissions are based on the English pattern, and their various pulses can hardly be distinguished aurally from other frequency pulses transmitted on the same system.

Fig. 7.4.-7 represents a part of - an OMA seconds pulse, as trans mitted during the last five minutes of each third year. The photograph, taken through a microscope of 17 x magnification, shows the characteristic pattern of the pulse. The imposed scale disarranged some of the metallic powder. The differences in the structure of the OMA and HBN pulses are quite obvious. The lengthened OMA minute pulse is shown in Fig. 7.4.-8. The underlying English pulse shape is unmistakable; slight irregularities of the width of the cycles nearer the leading edge constitute distinct marks. The OMA pulse can be easily distinguished from the typical English signal illustrated in Fig. 7.4.-9.

The uniform structure of the B.B. C. seconds dot and its clear commencement, which show up after tape development, prove the superior quality of the English time pulses

The familiar six B.B.C. pips are sent on continuous waves of which the successive oscillations are identical. The strokes of the telephone time service have the identical "structure".

Fig. 7.4.10 shows the pulse transmitted by WWV every second. The accurate five millisecond duration of the 1000c/s pulse and the "fading away" end are the permanent characteristics by which WWV can be recognized. The sharp cut lines indicate a pulse produced by interruption of a continuous wave. The drawback of a tick of short

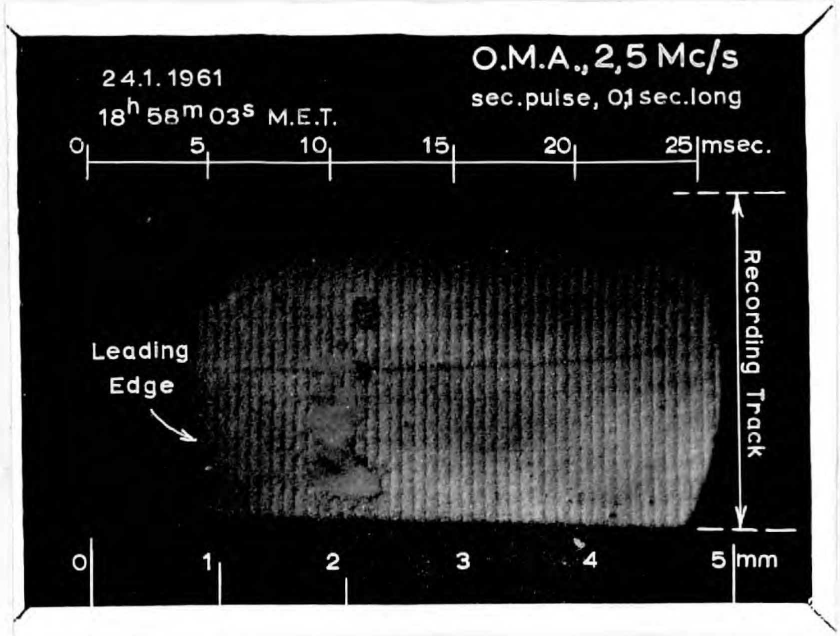


Fig. 7.4.-7

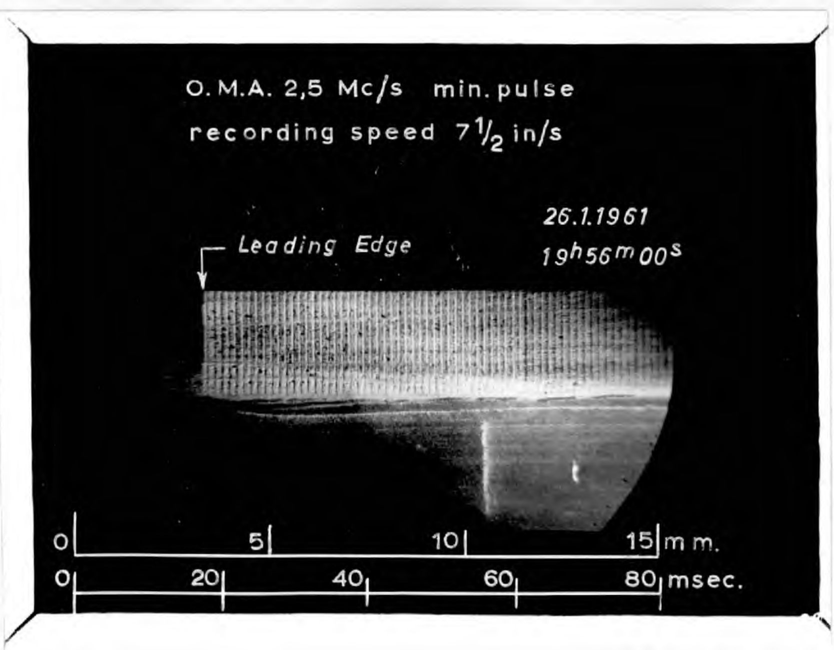


Fig. 7.4.-8

7.4. cont.

duration is that if the reception is very faint the pulse can be covered completely by heavy atmospheric interference, which makes the pulse inaudible and its cycles cannot be detected on tape.

The photograph of a RWM pulse (Fig. 7.4.-11) discloses the adopted English pulse type; the second pulses are 0.1 second long; at each minute the pulse is lengthened. The pulse (measured on the photograph) is transmitted at 1000 c/s. The leading edge consists of a fine line which is followed by a gap. Enlarged at slow re-play the leading edge lacks clearness for accurate ear comparison. Characteristic alternating pulse strengths will be noticed at intervals of five milliseconds; these produce fluctuating magnetic impulses on the tape.

Spark signals, now nearly obsolete for use as time signals or as pulses superposed on standard frequency transmissions are still locally employed by electrical time transmitters. Such time transmitters or master clocks operate seconds impulse dials by transmitting electrical impulses every second; the pulses can be used also for synchronization of chronographs etc. The spark signal consists of waves which are made up of successive trains.

The photograph of a developed spark signal released by the Imperial College of Science and Technology's master clock (Shortt) is shown in Fig. 7.4.-12. The amplitude of the oscillations reaches a maximum whereupon it decreases gradually. No regular pattern of cycles can be noticed as would be the case with continuous waves. The commencement has an undefined leading edge which in the presence of any atmospherics would be unrecognizable. The pulse is unsuited for time indication where higher accuracy is required. The duration of the College pulse is about 0.1 second. An open aerial near the College buildings or a wireless set operating on the College mains receive such signals. The developed I.C.S.T. pulse shown on the photograph is recorded via a tuned aerial on the roof of the R.S.M. building; the wireless set is operated on a 12 v car battery.

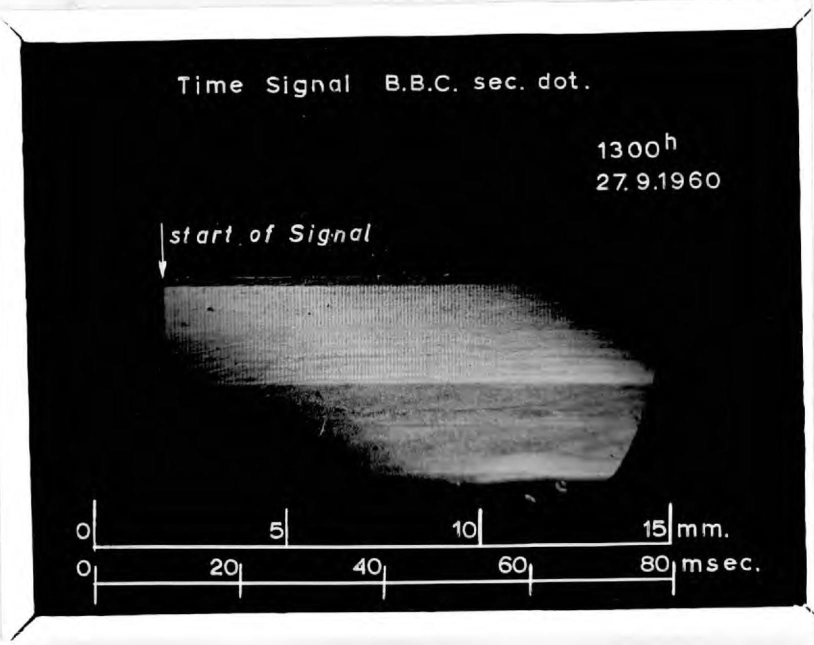


Fig. 7.4.-9

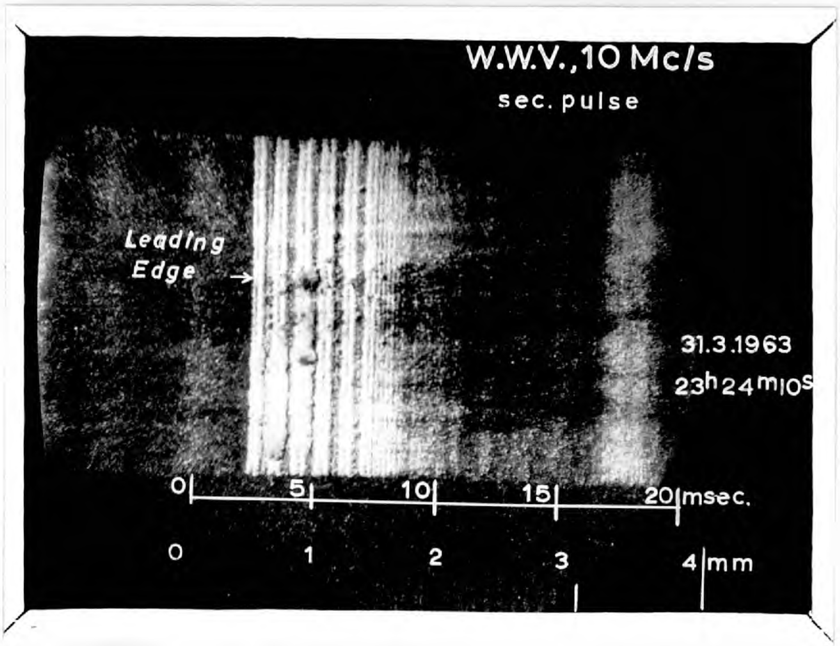


Fig. 7.4.-10

7.4. cont.

The intensity of reception of spark signals depends on the location of the receiver and its aerial. Selectivity in reception is required if spark signals are in exact synchronization with frequency time pulses.

The crystal chronometer second, minute and marker pulses have been specified (Section 3.3) to possess easily recognizable differences between them; further, to be distinguishable from all other transmitted time pulses. The duration of the crystal chronometer seconds pulse and the specified number of cycles per second constitute distinct properties which can be noticed with the unassisted eye. The crystal chronometer seconds pulse is shown in Fig. 7.4.-13, at small magnification, about 5 x; the pulse is contained in the lower track, which appears on the photograph inverted, and has to be read from right to left. The photograph of the developed seconds pulse, and also Fig. 7.2.-2, clearly demonstrate the characteristic structure by which the pulse can be distinguished from 1000 c/s pulses, (e.g. Fig. 7.4.-9, or Fig. 7.4.-11), and from radio noise on account of its uniform cycle pattern of considerable duration.

Fig. 7.4.-13 includes also a crystal chronometer marker pulse, of 1/25 second duration, which is visible on the upper track (track A). It has a similar characteristic leading edge to the seconds pulse, but its tone is an octave lower and consists of 520 c/s.

The crystal chronometer minute pulse (Fig. 7.4.-14) is photographed at the same magnification as the other crystal chronometer pulses. The sharp leading edge of the minute pulse permits accurate scaling. The arrangement in successive intervals of octaves of all crystal chronometer pulses, one above and one below the marker pulse, enables the overlapping part of pulses to unite. (Section 7.5.)

The tape velocity of $7\frac{1}{2}$ in./sec. has been adopted as the most convenient to achieve the scaling accuracy required and for the ease of visual identification of developed signals.

The space intervals between cycles of a 1000 c/s pulse can be

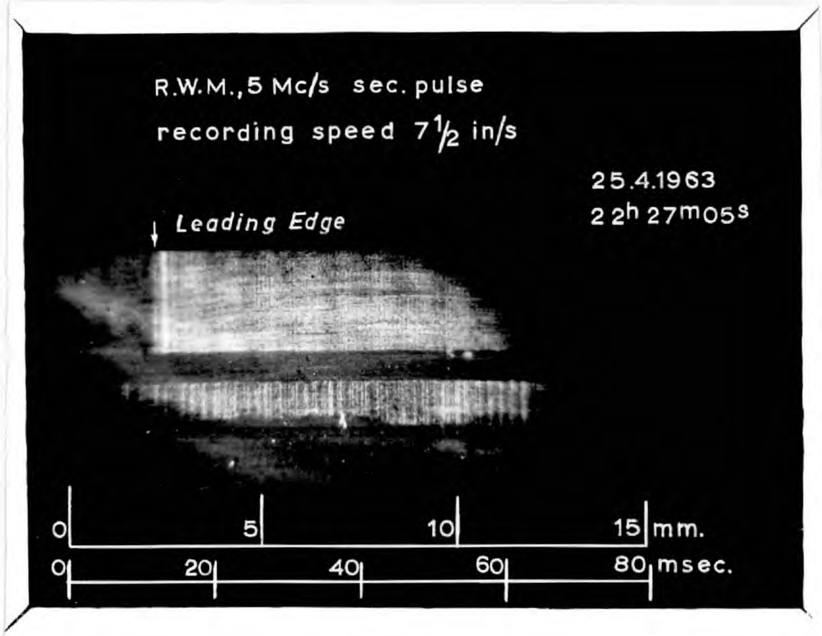


Fig. 7.4.–11

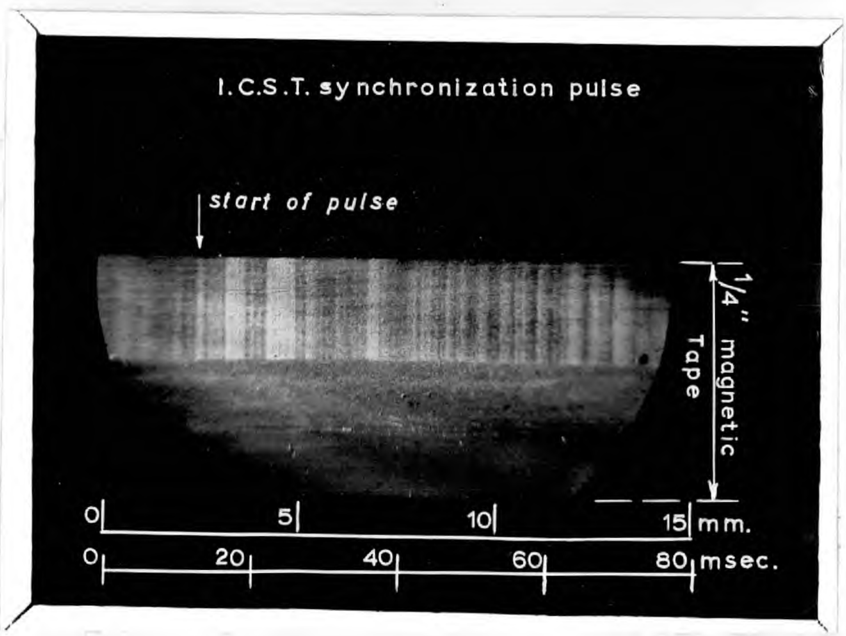


Fig. 7.4.–12

7.4. cont.

distinguished without a magnifying glass when recorded at the above tape speed. The next lower standard speed of $3\frac{3}{4}$ in./sec. would produce cycle intervals less than 1/10 mm wide; consequently individual cycles would be hardly recognizable and particular characteristics of the pulses would not show up, especially if coarse metal powder were used. Clear appearance of cycles is required particularly in the presence of radio noise. Atmospherics fail to produce regular or uniform magnetic pattern. (Fig. 7.3.-3, or also Fig. 7.4.-1). If any regularity of radio noise shows up after development, e.g. left end of photograph Fig. 7.4.-4, then such apparent regularity is of short duration, at the most a few milliseconds.

Machinery noise originating from vibration of solid bodies, e.g. ticks of mechanical chronometers, are recorded as "sound waves" on magnetic tape.

Fig. 7.4.-15 shows two consecutive beats of the half second beat Dwerrihouse chronometer. The irregular pattern of cycles is different for every tick. There is no well defined leading edge and the end of the beat disappears gradually in "silence", which is nearly as long as the beat itself. No radio noise is present; the paint marks on the tape are scattered metal particles.

Developed tape recordings of mechanical chronometer beats enable one to analyze chronometer behaviour within intervals of half seconds and seconds, independently from the human ear.

Changes of pulse characteristics are expected to occur, following alterations or improvements of the transmitting system; e.g., to overcome power limitation.

(Fig. 7.4.-16) The photograph taken through a microscope of 48 x magnification represents the MSF seconds pulse as transmitted in January 1961. The photograph may be compared with Fig. 7.4.-2, which shows the MSF seconds pulse transmitted in 1964.

It is quite obvious that modifications of the multivibrator

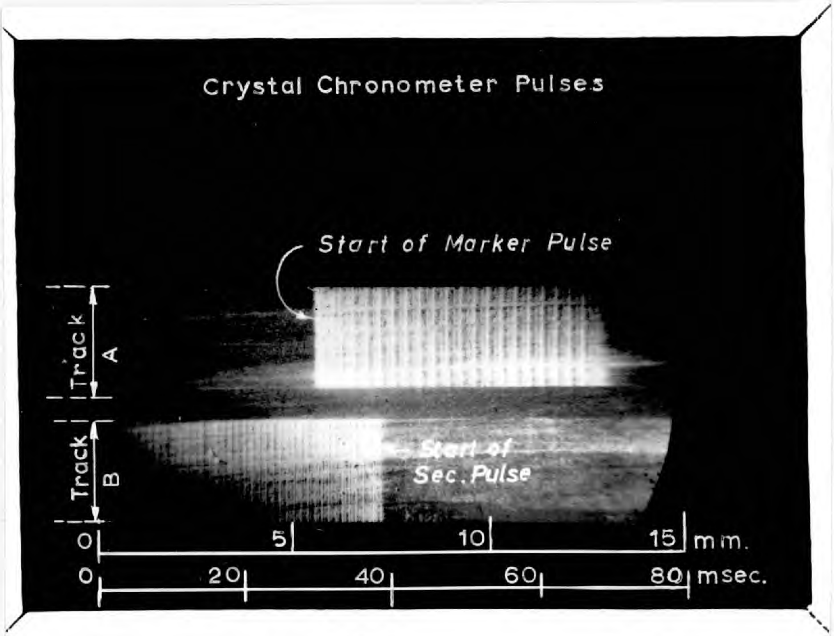


Fig. 7.4.-13

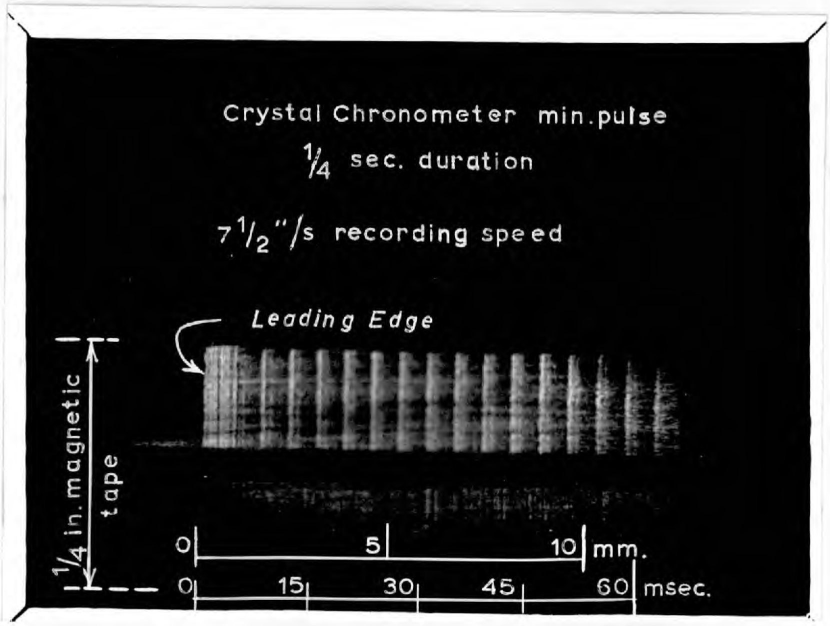


Fig. 7.4.-14

7.4. cont.

contributed to higher regularity of the cycles. The more uniform structure of the pulse can be noticed especially at the 2nd and 3rd cycles. The accuracy of the intervals marked by consecutive cycles is now approaching 1/10 microsecond. The powerful leading edge defining the epoch, the accuracy of which depends on the inertia of the transmitter, stands out more clearly now (Fig. 7.4.-2) than with the previous transmitting system.

Modifications of the transmitters are not likely to be the cause of confusion in visual interpretation of pulses. The various pulses retain their conspicuous structure, shaped by the prominent properties which are a function of the transmitters.

Fig. 7.4.-17 shows the HBN minute pulse transmitted in February 1964. The photograph is taken at the same magnification as the previous one. The gaps between the cycles have less magnetic attraction; the resulting structure of the lengthened minute pulse is identical to the HBN seconds pulse (Fig. 7.4.-6 or Fig. 7.3.-2), and supplies enough characteristic marks for the pulse to be distinguished from the MSF or other pulses.

Visual interoretation of developed signals is facilitated with artificial illumination at a low angle; if necessary the pulse relief produced by the metal powder is made more evident when viewed through a stereoscopic microscope.

A crystal chronometer can be synchronized with seconds pulses on standard frequency transmissions with an electromechanical phase shifter. In all practical circumstances, during such synchronization, the standard frequency pulses greatly overlap the crystal chronometer seconds pulses. The subsequent determination of the drift of the crystal chronometer will be somewhat problematic in view of the doubtful aural identification of the time signals and their transmitters.

The tests, as far as completed, indicate that tape development

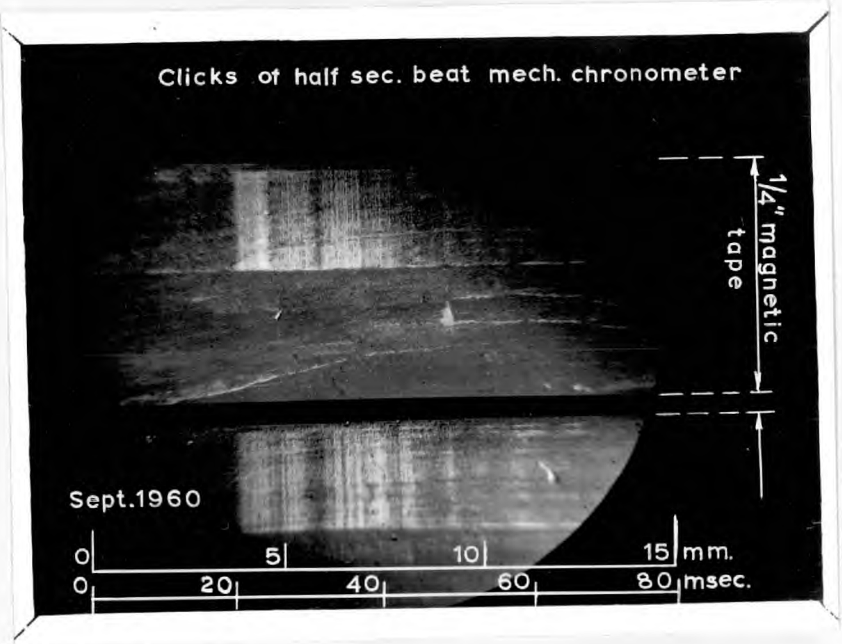


Fig. 7.4.–15

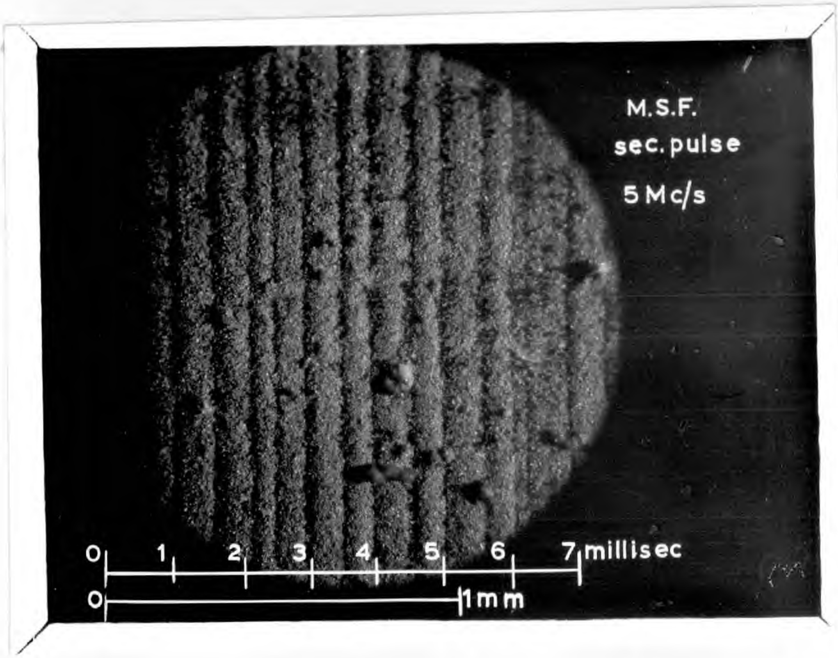


Fig. 7.4.–16

7.4. cont.

offers the advantage of reliable identification of the various time signals Recording via loudspeaker and microphone does not change the nature of the various types of signals.

In conjunction with tape development a time vernier consisting of 59 or 61 chronometer signals in the time interval of one minute attributes a definite distance between pulses, and therefore a number to each signal. These distances, arising from the time vernier, when scaled, are reasonably free of the personal error in scaling which error is analogous to the error of "reading" the clock. The time vernier also limits the number of overlapping pulses. This is convenient because a special tape recorder with more than one track, for recording chronometer and standard frequency signals separately, can be dispensed with.

Further, visibility of pulses has the great advantage over audibility that the reception of ill-defined or mutilated signals can still be used; their point of commencement can be established with great accuracy from the number of cycles and from their uniform and characteristic spacing.

The selection of particular pulses from a group of simultaneously received frequency transmissions is greatly facilitated when the cycles aof the pulses are made visible.

Confusion in aural interpretation of simultaneous receptions of time signals of various sources, caused by marginal differences in reception time, is eliminated by tape development. (Fig. 7.2.-8). A Small time gap between start of two consecutive signals as seen in Fig. 7.5.-5, is not perceptible by the human ear. As previously stated, the minimum time interval which is aurally detectable is about 0.02 second. Tape development enables over a hundred times smaller time intervals to be measured. Furthermore, interference between transmissions are rendered insignificant and ambiguities resulting from overlapping pulses can be eliminated.

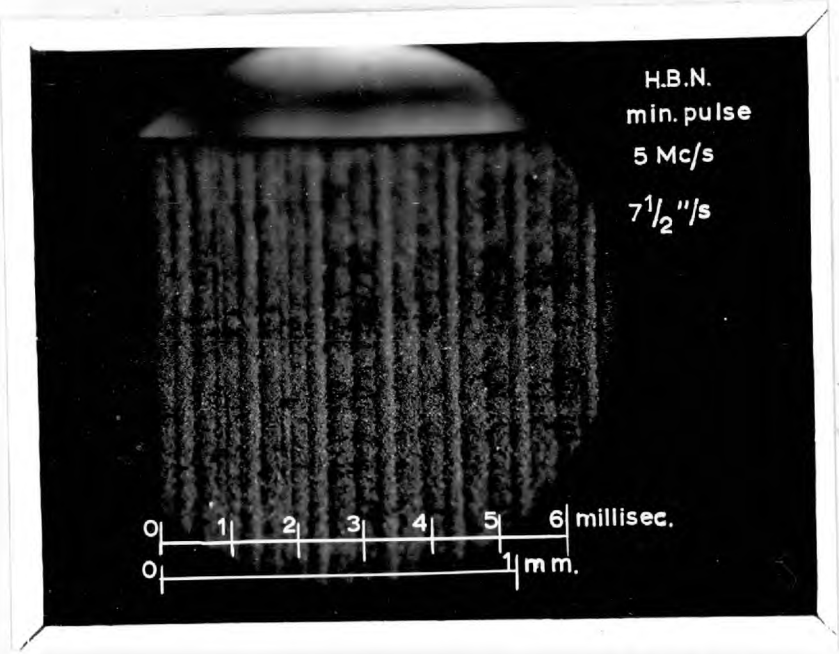


Fig. 7.4.–17

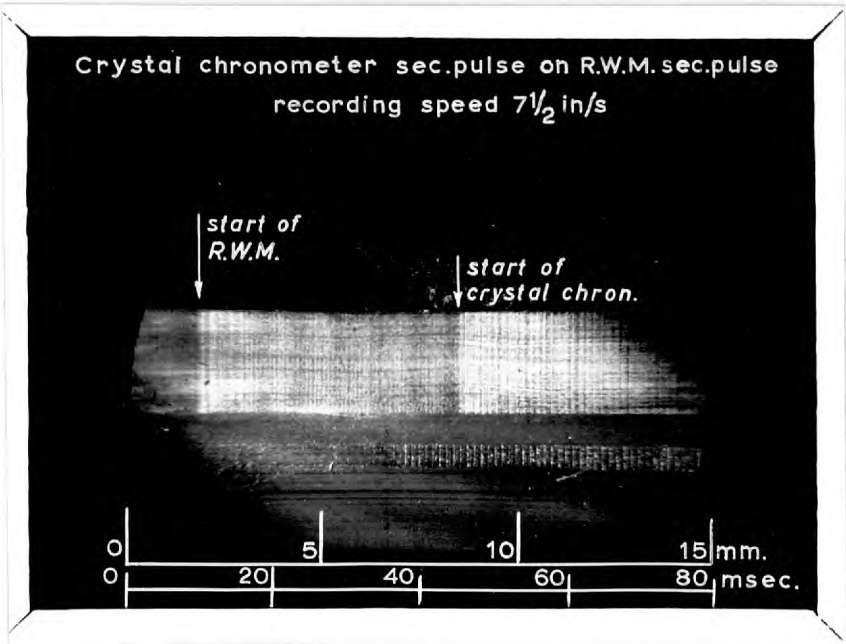


Fig. 7.5.–1

7.4. cont.

Tape development does not require great care, special skill, excessive length of time or expensive apparatus. Visibility of pulses is achieved rapidly and can be executed anywhere, at any time, and has no detrimental effect to the recordings.

No publications are known to exist on the application of tape development to field astronomy and of visual identification of time pulses. The subject treated is believed to be new.

7.5. Overlapping Pulses

Signals following each other in close proximity may have a time interval in common. The length of the time interval belonging equally to two pulses depends on the difference of the reception times and on the duration of the signals.

Fig. 7.5.-1 shows the crystal chronometer seconds pulse lying over a part of a RWM frequency pulse. The RWM pulse is recorded via the radio loudspeaker and the chronometer pulse via the clock loudspeaker. The pulses are received at different tone volume. This produces enough contrast on the tape for the overlapping part to be distinguished. The fluctuating volume of the RWM pulse, varying at regular intervals of about five milliseconds, can be traced even on the overlapping part where it causes some disturbance to the structure of the chronometer pulse. The definition of the leading edge of the crystal chronometer signal is by no means reduced, and the scaling accuracy can be accepted as unaltered.

Signals received at different volume and recorded on tape produce magnetic fields of different strength, which attract metal powder accordingly.

The visible record of a MSF pulse on top of a crystal chronometer seconds signal is illustrated in Fig. 7.5.-2. Both pulses, MSF and chronometer, are received at nearly equal volume. The MSF pulse succeeds in cutting out some of the chronometer cycles and produces well defined lines permitting the desired scaling accuracy. The fine film of metallic dust leaves blank tape areas between the cycles.

An overdose of metal powder produces a plastic appearance of the overlapping part, which can be made more prominent with oblique illumination. In Fig. 7.5.-3 are shown MSF pulses lying on crystal chronometer seconds signals. The thick layer of metal powder on the magnetized impulses results from the oversaturated liquid.

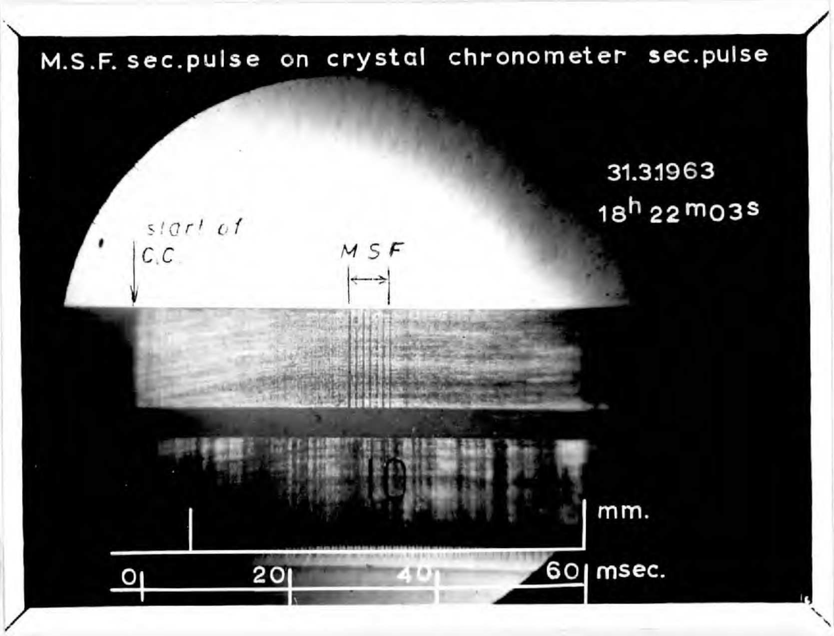


Fig. 7.5.-2

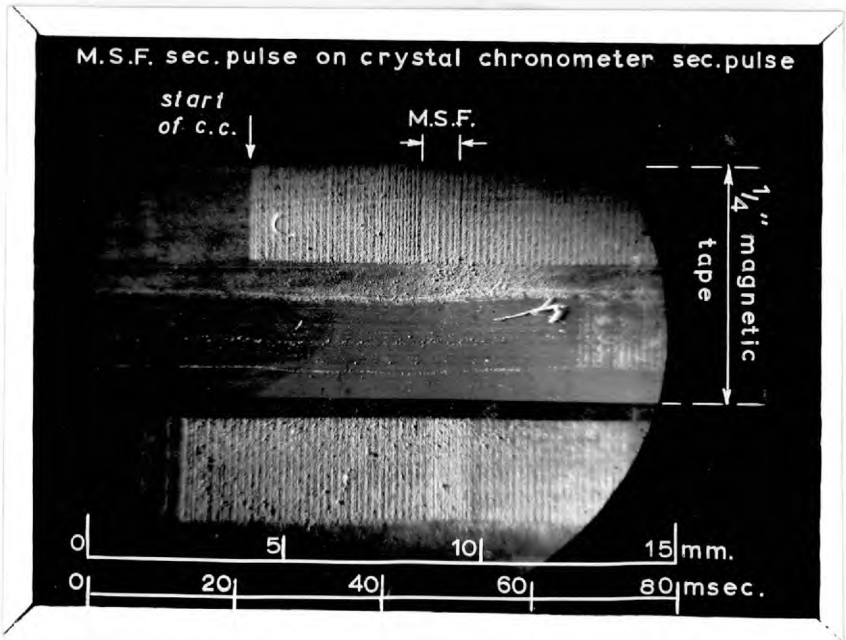


Fig. 7.5.-3

7.5. cont.

Every cycle and the leading edges stand out perfectly clear.

Fig. 7.5.-4 represents the crystal chronometer minute pulse overlapping two standard frequency pulses: MSF and HBN. The strong magnetized areas, produced by the pulses, and the wide spacing of the cycles of the chronometer minute pulse need only a small amount of metallic powder to become visible. The recording volume between 2 and 3 (on the Fi-Cord tape recorder) allows for easy cancellation of both the chronometer and the frequency pulses, of which the latter were received with higher volume. The leading edges of the two frequency pulses are very clear and the scaled time interval between them amounts to 0.034_7 seconds. The corresponding figures derived from the R. G. O. Bulletin and from the Bulletin Horaire are 0.032_0 and 0.035_6 seconds respectively. The characteristic structure of all the three signals can be recognized without a magnifying glass.

In Fig. 7.5.-5 the OMA seconds pulse is received during the crystal chronometer minute pulse. There is very little difference in the tone volume of the two signals. The structure of the chronometer signal can be noticed throughout the OMA seconds pulse. The same distinct appearance of the leading edge of the standard frequency pulse can again be noticed.

Fig. 7.5.-6 illustrates the visibility of the HBN seconds pulse when overlapping the crystal chronometer minute pulse in the presence of radio noise. The white noise has filled the wide spaces between the cycles of the minute marker. Nevertheless the cycles of both pulses are remarkably clear and neither the identification of the type of frequency pulse nor the definition of its start have suffered from the interference.

From the above it can be stated that in the absence of radio noise the exact start and end of the overlapping part can be seen and measurements can be taken from them with the same precision as from separate pulses. In the presence of radio noise or of other

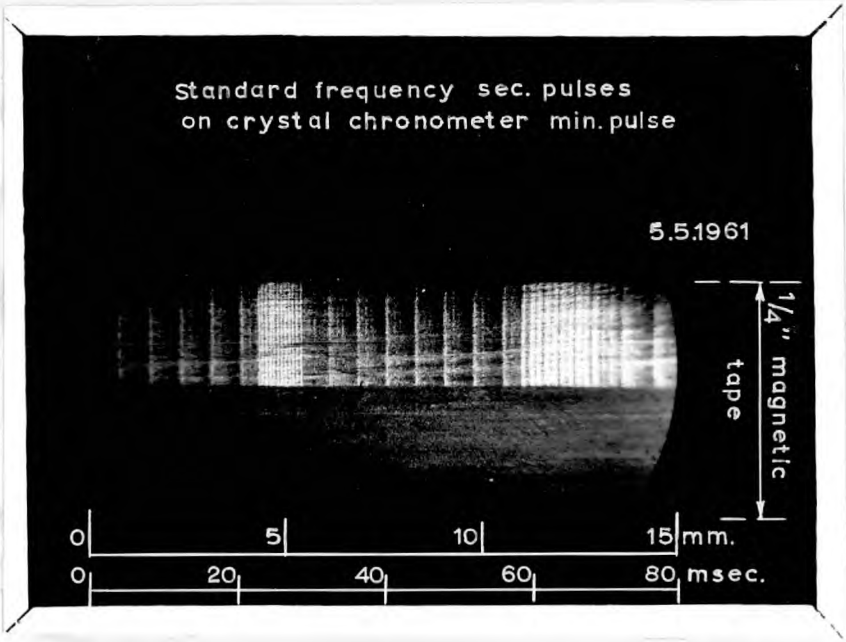


Fig. 7.5.-4

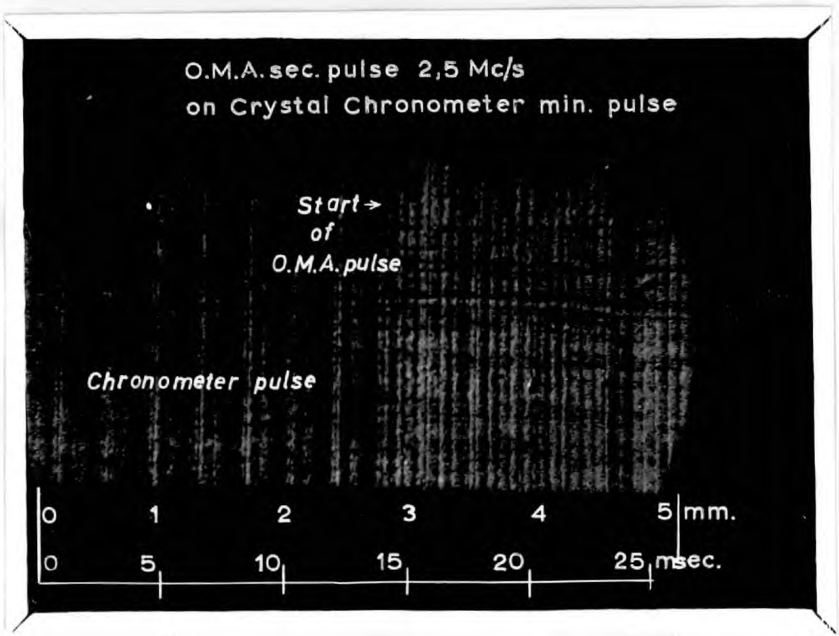


Fig. 7.5.-5

7.5. cont.

interferences, e.g. morse signals transmitted on the same frequency as the frequency pulses; noise from electrical machines connected to the same mains, etc.;, properly magnetized areas on tape can be obtained if the signals are received at higher volume than the background noise. Consequently the impulses on tape resulting from the time signals will attract a sufficient quantity of metal powder to stand out conspicuously.

The identification of overlapping pulses as shown in the above photographs is of great use when calibrating the crystal chronometer.; Furthermore it is essential to recognize overlapping pulses when the observation marker pulse is involved; otherwise the timing of that particular observation may be lost.

Electromagnetic waves of the same frequency may unite as one wave. The common time interval of overlapping signals, belonging to the same or to multiples of a basic frequency, has a definite number of cycles per second which corresponds to the frequencies of the signals. Consequently visible overlapping audio signals of the same basic frequency have a different pattern over the common time interval.

Fig. 7.5.-7 illustrates the overlap of the observation marker on a chronometer seconds pulse. Fig. 7.5.-8 shows the structure of both pulses and the common time interval. The three different structures stand out very clear. The start is well defined by the gap which the leading edge produces.

Fig. 7.5.-9 is the reverse case of Fig. 7.5.-8. This time the seconds pulse comes in later; its leading edge is quite clear, so is the common time interval and the end of the marker pulse.

The photographs demonstrate the validity of the theoretical assumption regarding the employment of chronometer signals chosen at a ratio of their basic frequency in order to be distinguishable over a possible common time interval. Different tone volumes for the

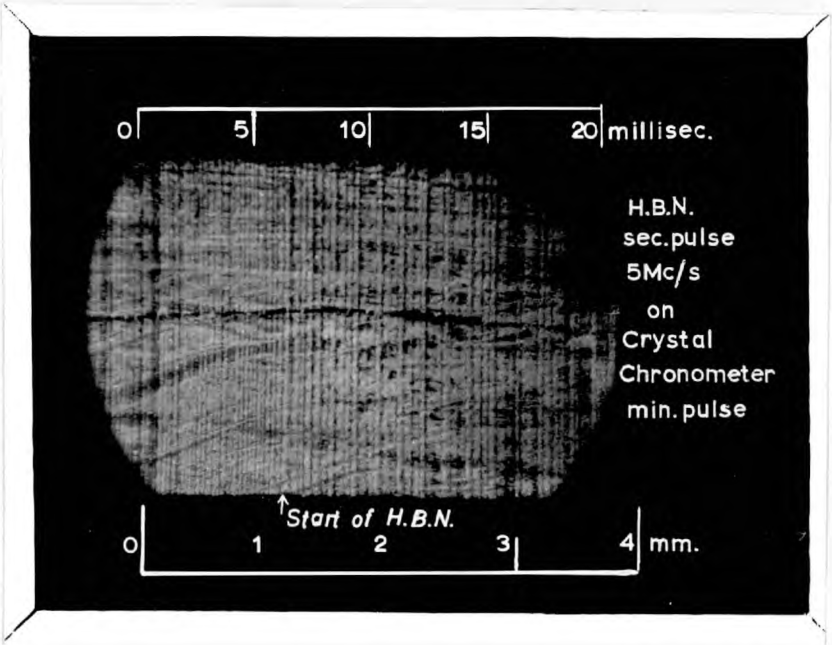


Fig. 7.5.-6

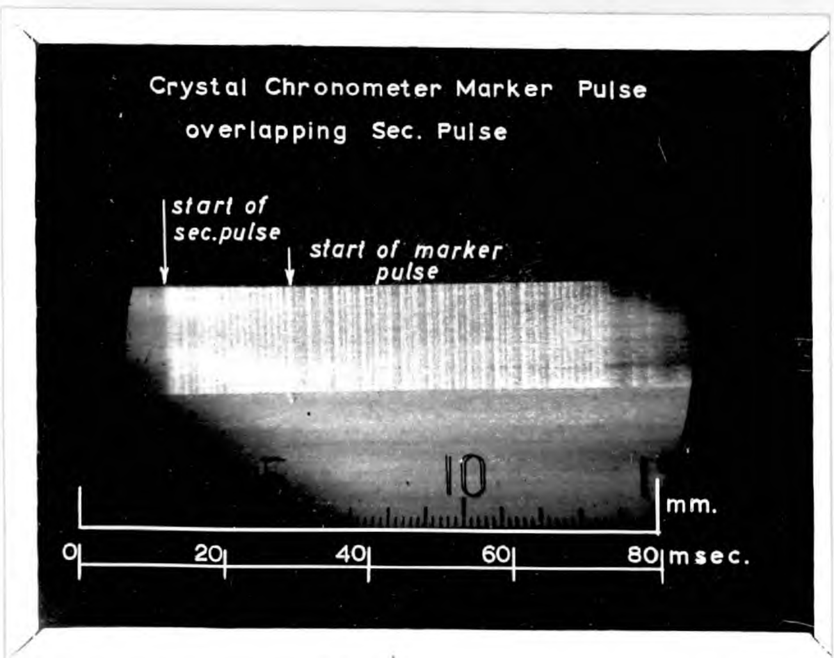


Fig. 7.5.-7

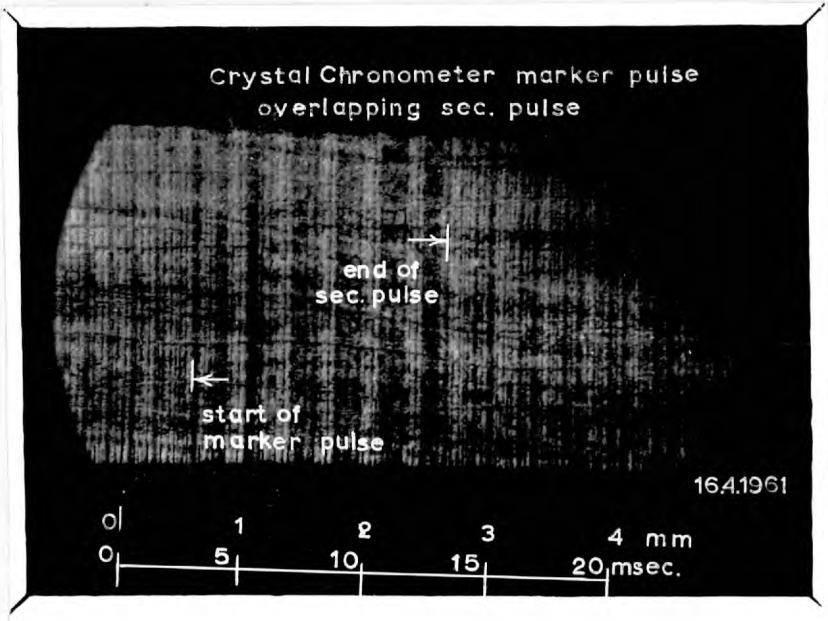


Fig. 7.5.-8

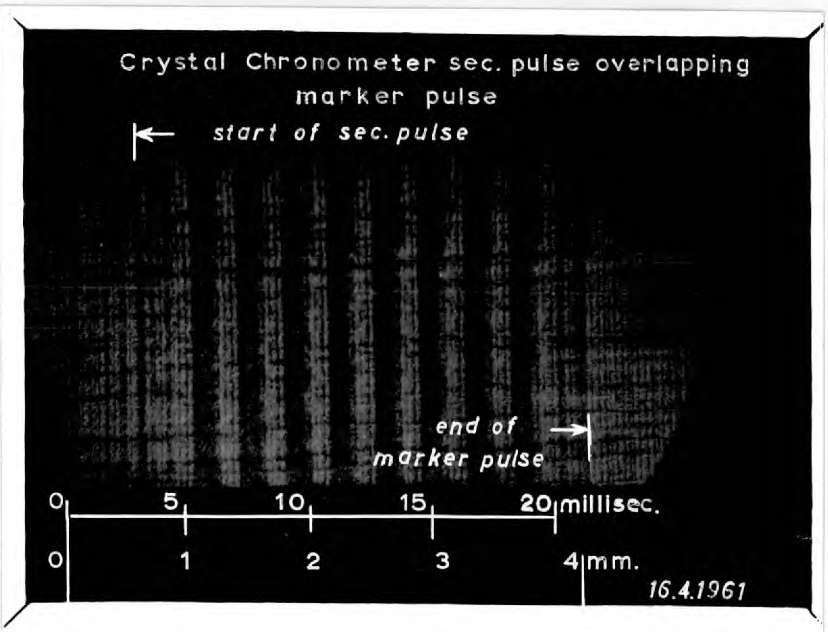


Fig. 7.5.-9

7.5. cont.

crystal chronometer signals are therefore not required.

Tape recorders offer a great advantage, namely that overlapping pulses can be distinguished after tape development, and thus definitely score over paper strip chronographs, unless the latter have more than one stylus, in which case the mechanical arrangement must be of the utmost precision.

8. Computation of Observations.

8.1. General.

Observations for astronomical latitude, longitude and azimuth were carried out at the Imperial College Field Station, Tywarnhale Mine, Porthtowan, Cornwall, to provide data on which to base a judgement of the practical application of the crystal chronometer, tape recorder, single second theodolite and timing outfit. Furthermore, the observations were intended to determine the accuracy of position fixing with field instruments when the subdivision of time intervals and U. T. were available to a high order of precision at the survey station. Position fixing is the determination of the co-ordinates of the observer's zenith; these are: sidereal time and zenith distance.

The area where the field tests took place is shown in Fig. 8.1.-1. Access to primary and secondary O.S. triangulation stations and a local mine triangulation frame tied to the National Grid were of great assistance.

The instruments used were:

(1) crystal chronometer and marker key,

(2) Fi-Cord tape recorder,

(3) short wave frequency receiver,

(4) Watts Microptic theodolite No. 2,
fitted with long diagonal eyepiece, lighting equipment, grid reticule, 25" plate bubble (one division = 2 mm),

(5) aneroid barometer,

(6) thermometer,

The observations consisted of timed combined measurements of both the horizontal and the vertical angles to heavenly bodies,

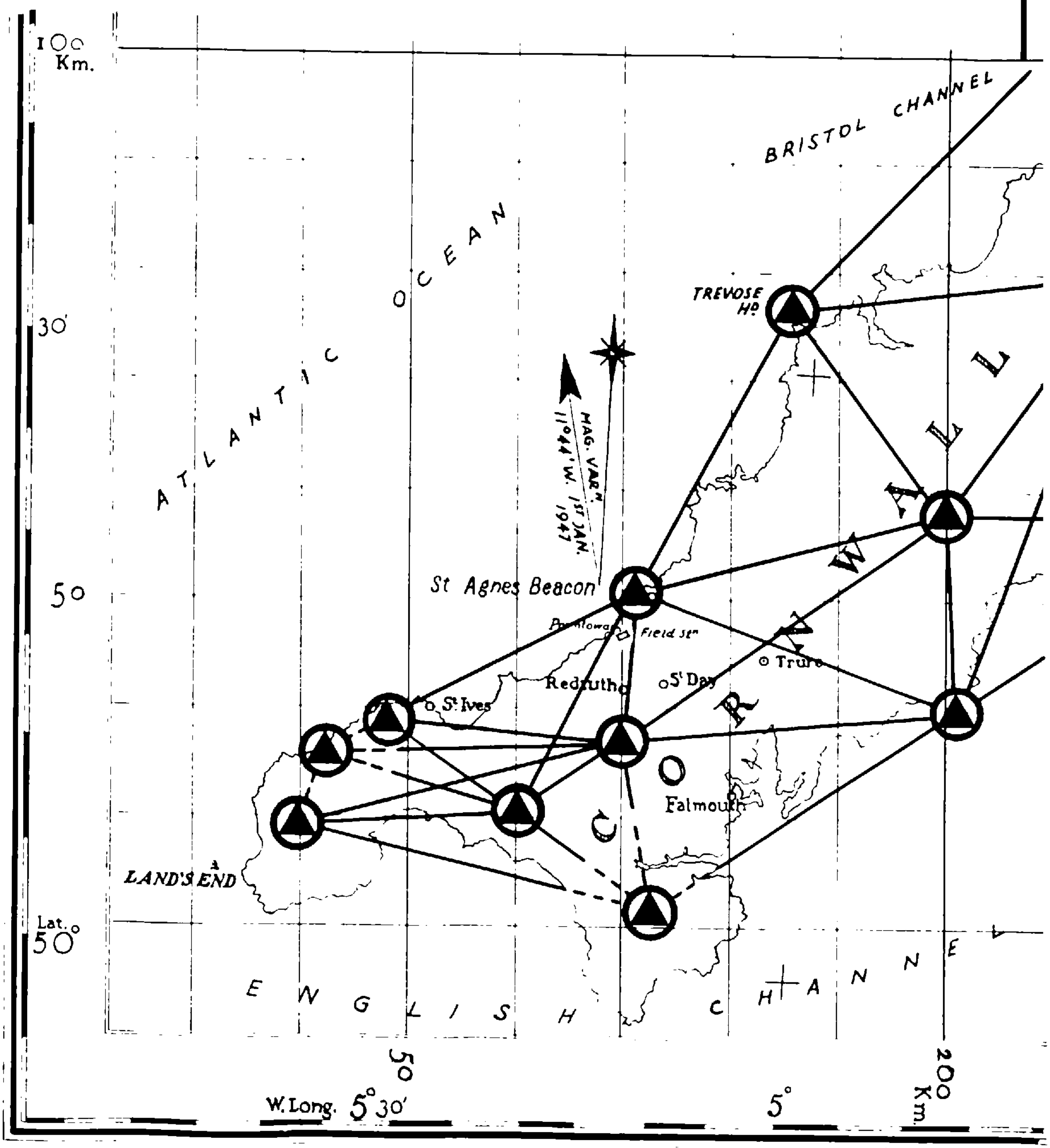
Fig.: 8.1.-1

ORDNANCE SURVEY TRIANGULATION

O.S. PRIMARY PILLAR STATION

IMPERIAL COLLEGE FIELD STATION

SCALE : 1/625 000 OR ABOUT TEN MILES TO ONE INCH.



8.1. cont.

and comparison of crystal chronometer time with U. T. at the observer's station.

The limiting factors of the precision of an astronomical fix are the personal equation, the rotation of the earth, the verticality of the vertical axis of the theodolite and the refraction. Therefore, differences in positional co-ordinates must be expected if these are determined from observations to stars employing accurate time based on standard frequency transmissions and stellar co-ordinates tabulated in A. P.; the differences can be attributed to variations in the length of the day, observational errors, instrumental imperfection, topography, and to the application of the incomplete theory of refraction.

The accuracy (r. m. s. error) of the determination of the astronomical azimuth at Laplace stations with 1st order instruments is accepted to be about $\pm$ 0.5", the longitude $\pm$ 0.03 sec = 0.45", and the latitude $\pm$ 0.3"; obviously disregarding the deviation of the plumb line.

The accuracies of the astronomical azimuths in the main net of Central Europe are about $\pm$ 0.35". Measurements at the turn of the century are quoted with the following r. m. s. errors: Bavaria: $\pm$ 0.51", Switzerland: $\pm$ 0.23", Austria: $\pm$ 0.47", recent measurements achieved the following accuracies: Czechoslovakia: $\pm$ 0.54", Bavaria: $\pm$ 0.44", Austria: $\pm$ 0.30".

Any star listed in A. P., or Ephemeris, at altitudes from 30° to 75° was regarded as suitable for observation.

Special methods of observing, e.g. prime vertical method, fixed altitude method etc., the selection of stars at special positions, e.g, at elongation, culmination etc., or methods of observing two stars simultaneously were not considered.

8.1. cont.

The selection of suitable pairs of stars or groups of stars is rather laborious and much time-consuming effort in preparing a star programme might be wasted by the sudden appearance of clouds.

The computations in astronomy consist in solving the astronomic triangle.

The selection of a method of computation depends on the known elements of the celestial triangle, and on the equipment, calculating machines, tables etc., available; furthermore, the number of stars observed influences the choice of the computation method for reasons of economy.

The declinations of celestial bodies are given in Apparent Places, Ephemeris, Nautical Almanac, etc.; the latitude and longitude of the survey stations are always sufficiently known; the altitude, azimuth, and the hour angle can be observed.

If there is an approximate knowledge of the final result (position of the observer's station) a method of successive approximations is conveniently used, instead of any of the various general methods. The approximate knowledge of the astronomic co-ordinates of the survey station is then expressed as "Trial Point". The final adjusted position is obtained by applying corrections to the co-ordinates of the trial point. The corrections can be evaluated analytically or semi-graphically.

Direct methods of computing the observer's position in exploratory surveys are considered to be outdated. Therefore, co-ordinates of the field stations in Cornwall are assumed and corrective terms are deduced from the field returns.

The final co-ordinates of the observation stations are obtained by the following methods:

8.1. cont.

(a) Method of Position Lines from Zenith Distances,

(b) Method of Position Lines from Horizontal Directions,

(c) Method of Least Squares, Astronomical Fixation from Horizontal Directions,

(d) Method of Position Planes from Horizontal Directions.

The method of position lines from zenith distances, with its semi-graphic solution, is most adequate; the computations involved are simple and rapid. Further, this method has the advantage of the possibility of utilizing any condition of observation. Identification of stars is possible, because with the grid reticule both circles are read. Normally, the precision of simultaneous sights (azimuth and altitude) of a moving target, i.e. sighting with vertical and horizontal wires simultaneously cannot be guaranteed; but with the use of the grid reticule both circles can be employed for accurate measurements at any one single pointing to a star.

The method of position lines from horizontal directions requires the identical and very little additional computations as are required by the position line method from zenith distances. Results of computation, deduced from observational data consisting of simultaneous recording of zenith distances and horizontal directions at noted instants of time, can be used in the graphical part of the solution of both types of position lines.

The superiority of semi-graphic methods over algebraic methods, or over conventional spherical trigonometrical methods, consists, on the analytical part, in the short cut of the calculation provided by uncomplicated formulae, in its checks on the graphical part and in the overall clearness of presentation of the whole solution.

In field astronomy, it is usually the case that more than two stars

8.1. cont.

are observed, especially where a position fix is sought. Hence, the final adjusted position can be determined by the rigorous least square method. This method is rather lengthy and causes some waste of effort, because the accuracies of the observed quantities are obviously of different quality, depending on atmospheric uncertainties, on the size of the star, its path and its velocity of movement in the field of view of the telescope, each of which is causing systematic errors.

A least square solution can be advocated when a large number of stars is observed. A position fix from timed horizontal directions is worked out by least squares mainly for the geometrical interpretation of the method. (Section 8.4.)

Astronomical fixation from horizontal directions can be regarded as a problem in three dimensional geometry; therefore its solution, the determination of the final position, can be found semi-graphically by the method of position planes.

So far as the writer is aware no worked out example of an astronomical fix using the submitted position line method from horizontal directions to stars at any altitude and azimuth, is to be found in any literature. Furthermore, no previous attempt has been made to work out a solution for an astronomical fix by the position plane method and with the use of the concept of duality in space. This method has been developed by the author mainly as an aid for illustrating the problem involved in position fix from horizontal directions. The application of the latter method entails a larger amount of analytical treatment than a solution by the other semi-graphical methods mentioned above would require.

8.2. Position Lines from Zenith Distances.

The position line method from measured zenith distances is described in various textbooks, and is therefore here omitted.

The vertical circle readings giving the altitude of the heavenly bodies are preferred quantities for observation, and easily accessible, because one direction, the plumb line, can be established without great effort.

The equations to be adopted for solving the celestial triangle link together the observed quantities and approximate values for the unknowns: latitude and longitude of the trial point.

Thus, the altitude h is expressed as a function of the latitude: $\phi_{TP} + d\phi$, and of the longitude: $\lambda_{TP} + d\lambda$;

$$h = h(\phi_{TP} + d\phi, \lambda_{TP} + d\lambda) = h_c + \left(\frac{\partial h}{\partial \phi}\right) d\phi + \left(\frac{\partial h}{\partial \lambda}\right) d\lambda \quad \ldots (1)$$

ϕ_{TP}, λ_{TP} are the latitude and longitude of the trial point, so chosen that second order terms can be neglected, $d\phi$ and $d\lambda$ are the corrections.

h_c is the calculated altitude for the trial point:

$$\sin h_c = \sin \phi_{TP} . \sin \delta + \cos \phi_{TP} . \cos \delta . \cos t \quad \ldots\ldots\ldots (2)$$

t = the hour angle, is made up of the observed time (U. T., or G.S.T.), a tabulated term (right ascension), and the longitude of the trial point.

The declination δ and the right ascension are tabulated, and accepted as being free of errors.

The aximuth, A_c, is calculated from the relation:

$$\cot A_c = \frac{\sin \phi_{TP} . \cos t - \cos \phi_{TP} . \tan \delta}{\sin t} \quad ; \ldots\ldots\ldots (3)$$

8.2 cont.

and a check is obtained from: $\sin A_c = \dfrac{-\cos\delta \cdot \sin t}{\cos h_{comp}}$; . . . . (3a)

Apparently, the azimuth is independent from the altitude of the star; but this is not so, because in the above relation the azimuth is again a function of (ϕ, λ), in the same sense as the altitude.

$$\frac{\partial h}{\partial \phi} = -\cos A_c \quad \text{and} \quad \frac{\partial h}{\partial t} = -\sin A_c \cdot \cos\phi_{TP}$$

Equations (2) and (3) are solved analytically, equation (1) is solved graphically.

Fig. 8.2.-1 shows the two unknowns $d\phi$ and $d\lambda$, the measured altitude h_o from the observer's station and the calculated altitude h_c for the trial point.

The difference between the observed and calculated altitude of the star is equal to the great circle distance between the two position circles through the trial point and through the observer's station respectively.

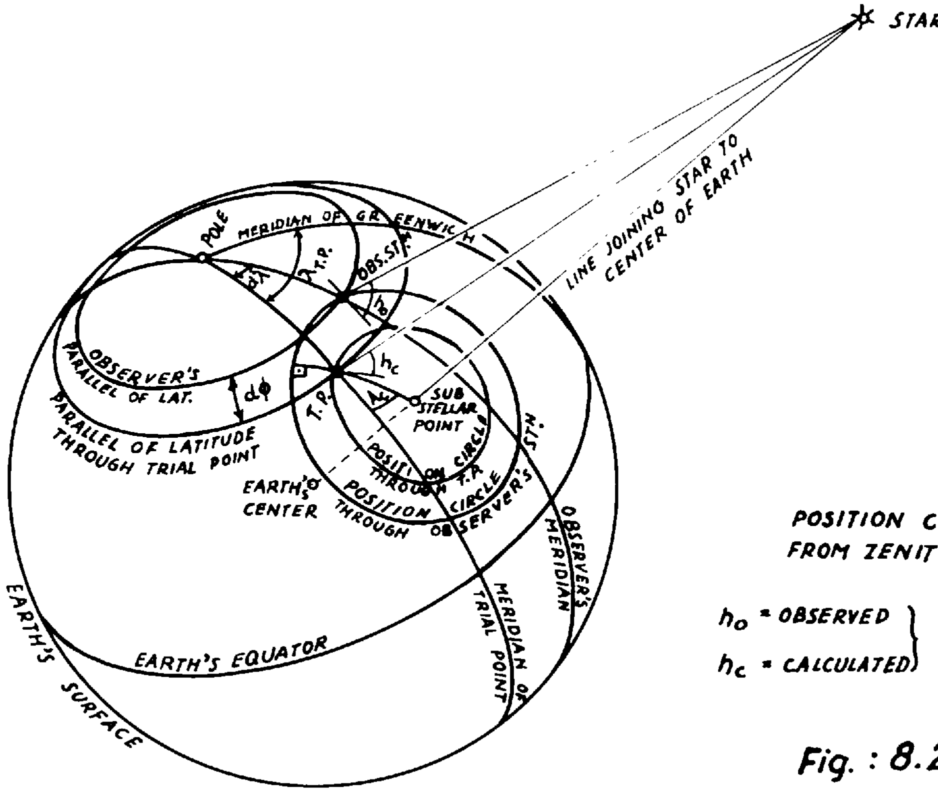
The position line substitutes the position circle by its tangent in the vicinity of the trial point and observer's station.

<u>Influence of errors of time and latitude measurement on positional co-ordinates.</u>

The time error Δt, which is the precision with which time is known, and the error of measuring the altitude, can be combined in one error and assigned to the altitude alone: Δh.

If then, the zenith distance $(90^\circ - h)$ contains the uncertainty Δh, the longitude, to be deduced, will be in error. (Fig. 8.2.-2.)

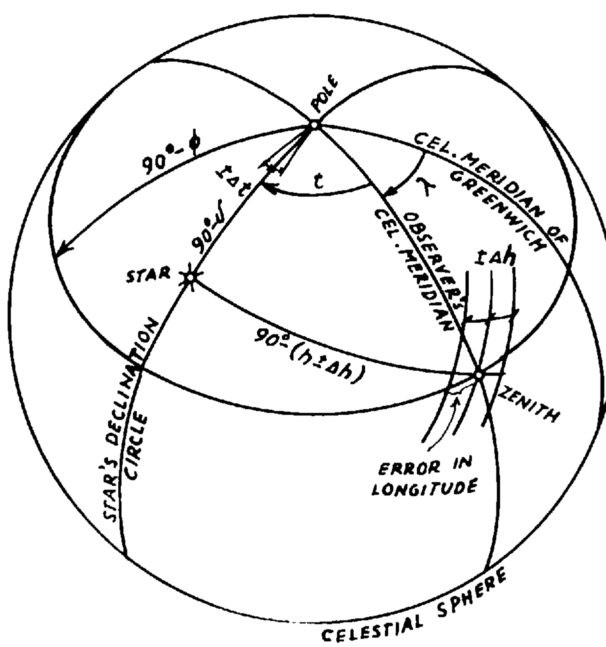
The error of the longitude caused by Δh is a minimum when the circle (radius = $90^\circ - h$, center = star) intersects the circle



POSITION CIRCLE
FROM ZENITH DISTANCE

h_o = OBSERVED } ALTITUDE
h_c = CALCULATED } OF STAR

Fig. : 8.2.-1



ACCURACY
OF
LONGITUDE DETERMINATION

GIVEN: δ, ϕ

MEASURED: h, t

Fig.: 8.2.-2

8.2 cont.

(radius = $90^\circ - \phi$, center = pole) at right angles.

The same answer is obtained by differentiating the altitude h with respect to time t (= longitude), in the equation:

$$\sin h = \sin\phi \cdot \sin\delta + \cos\phi \cdot \cos\delta \cdot \cos t$$

$$dh = \cos\phi \cdot \sin A \cdot dt = \sin q \cdot \cos\delta \cdot dt.$$

This means that the maximum influence the time error has on the altitude is when stars at or near the prime vertical are observed; the time error does not enter in full into the altitude error but is reduced by the factor $\cos\phi$. (Except on the equator). The time error has the minimum influence on the altitude for stars near the meridian of the observer, ($q = o^\circ$) and of course for the pole star. ($\delta = 89^\circ$).

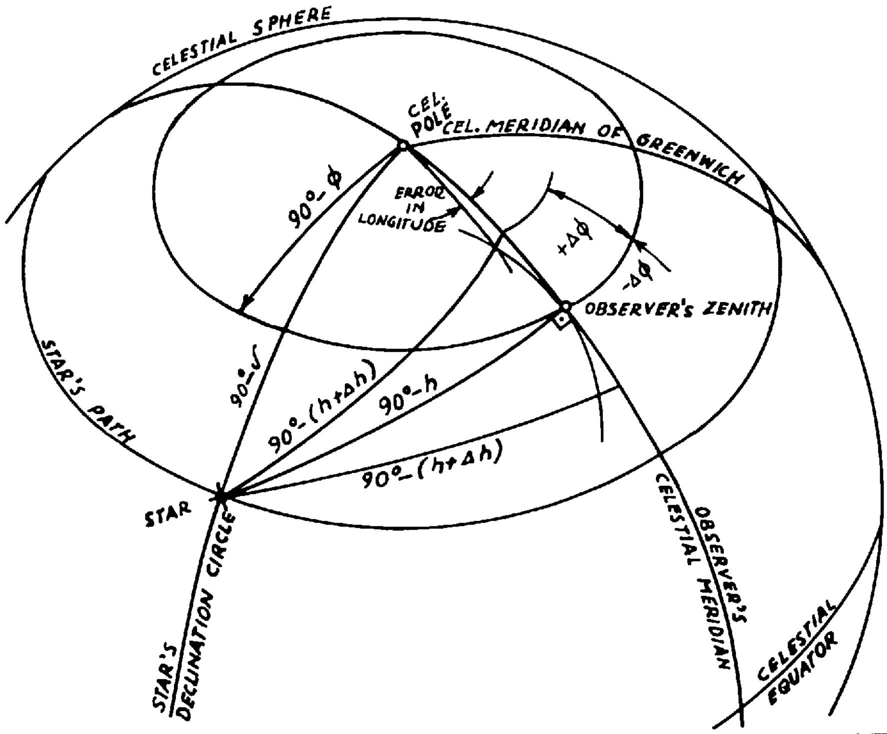
Further, the above differential relation:

$$dt = \frac{dh}{\cos\phi \cdot \sin A} = \frac{dh}{\sin q \cdot \cos\delta}$$

shows that the longitude obtained from altitude measurements is least affected by an altitude error when stars at or near the prime vertical are observed. $A = 90^\circ$, (270°); and longitude determination at the pole is impossible, since $\cos\phi$ is in the denominator; the denominator $\sin q \cdot \cos\delta$ means that the pole star and stars at or near the meridian of the observer are unsuited for longitude determination from measured altitudes.

An uncertainty of the latitude $\Delta\phi$, the latitude may have been determined previously, can also be assigned to the altitude error Δh. If so, the precision of longitude determination depends on the azimuth of the star observed. (Fig.: 8.2.-3)

For $A = 90^\circ$, (270°), the uncertainty of the latitude $\Delta\phi$ has the least influence on longitude determination. The same answer

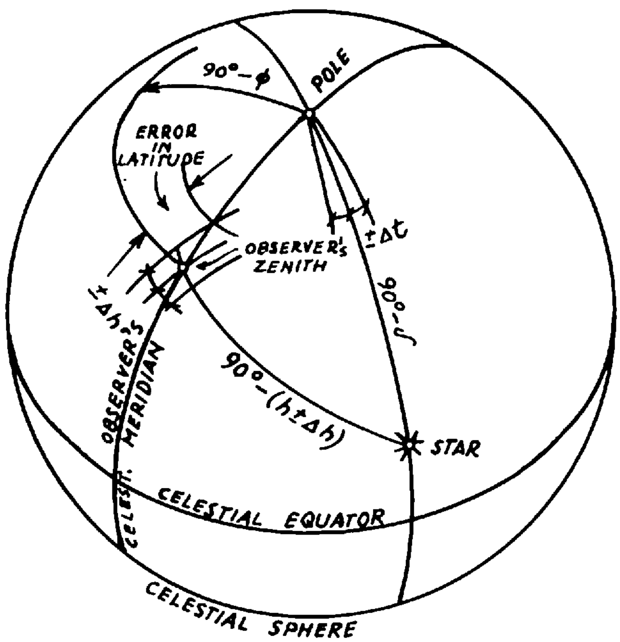


ACCURACY OF LONGITUDE DETERMINATION

GIVEN: δ, $\phi+\Delta\phi$

MEASURED: h, t

Fig.: 8.2.-3



ACCURACY OF LATITUDE DETERMINATION

MEASURED : h, t

Fig.: 8.2.-4

8.2 cont.

can be arrived at, when differentiating the altitude h with respect to ϕ :

$$dh = d\phi \, . \; \cos A$$

i.e. the azimuth for stars at or near the prime vertical is approaching 90° (270°) and will therefore reduce the influence of the latitude error on the altitude, presuming the time is correct; the resulting altitude error Δ h will therefore hardly influence the longitude determination.

From the above it follows that for accurate longitude determination from timed altitude measurements, stars at or near the prime vertical have to be selected; -in which case also an error of a previously determined latitude is negligible.

The time error Δ t which is contained in the altitude error Δ h causes also an uncertainty of the latitude determination. Fig.: 8.2.-4.

The influence of h on latitude determination is a minimum for $A = 180^\circ$ or 0°, or -for $q = 0^\circ$, i.e. when the circle (radius = 90° - h, centre = star) cuts the observer's meridian at right angles.

The differential relation of the altitude h with respect to the latitude ϕ can be written:

$$d\phi = \frac{dh}{\cos A}$$

which means that the altitude error dh, caused by the uncertainty of the known time, has least influence on the latitude determination when the azimuth is 0° or 180°; nevertheless, the altitude error is introduced at least in full into the latitude determination.

It follows, that for accurate latitude determination from observed altitudes, stars near the meridian of the observer have to be selected.

Any error in dislevelment of the theodolite distorts the result of position fixing. It is not possible to devise a method of eliminating dislevelment errors without the knowledge of the amount of dislevelment.

8.2. cont.

For altitude measurement the component of dislevelment of the vertical axis perpendicular to the transit axis is eliminated by levelling the alt-alidade bubble; it remains the other component in the direction of the transit axis, which is read on the plate bubble. (Section 5.8)

When observing movable targets, a knowledge of the central position of the plate bubble has some advantage for correcting the horizontal circle readings, for which the vertical circle readings are required.

The index error of the vertical circle can be assumed to be constant and of the same sign for observations taken during the same night.

8.2.1. Results obtained from Field Work

Astronomical Fixation from Vertical Angles, and Astronomical Azimuth of Terrestrial Line.

A specimen of the field record is shown in Table 8.2.1.-1, the data required in the heading can be entered before or after the field observations; the data concerned are: the instrument used; important instrument constants which can change; and the number of the recording tape. All other field data are filled in during re-play of the recording tape. (Lower part of Table 8.2.1.-1)

The majority of the star observations consisted of two pointings in each face position before re-sighitng the R.O. Two sights on each face give not only an additional position line but constitute also a check on the field data, viz.: estimations of grid reticule intervals, circle readings, and recorded times of transits have to be in relative agreement.

The times of observed stellar transits are obtained from tape development. (Table 8.3.1.-2.) The amount the crystal chronometer is slow or fast at the instant the marker signal is released, is determined graphically; the graphical interpolation is based on time differences between the crystal chronometer and time signal reception: Fig. 8.2.1.-1. U.T.O. of marker pulses are calculated using reception times at Herstmonceux published in Royal Observatory Bulletins.

The corrections applied to field measurements are shown in Table 8.2.1.-3. Conversions of grid reticule intervals into angular subtense are taken from Table 5.5.7.-1.

The horizontal circle reading is not corrected for the influence of the collimation axis error, because this error is considered to remain constant during one night and is obviously eliminated by C.L. and C.R. observations, even to movable targets, if sights are taken in quick succession.

FIELD RECORD

DATE	May, 11 & 12, 1961	INSTRUMENT	Watts No 2	PLATE BUBBLE	20·2"
AT	JAMES △ (Mine Trig. STN.)	EYE PIECE	long, diagonal	MID RUN	00"
R.O.	St. Agnes (O.S. Trig. STN.)	RETICULE	30" grid		
OBSERVER	W.A.S.	COLL. ERROR	see: May 10,	TAPE, TRACK	Scotch C09 Track A.
				REMARKS	B.S.T. = U.T. + 1^h

TRANSCRIPTION FROM RE-PLAY

SIGHT	C.L. C.R.	HORIZONTAL, VERTICAL CIRCLE		TEMP. °F.	PRESSURE " Hg.	PLATE BUBBLE C.L. L	C.L. R	C.R. L	C.R. R	AZIMUTH, ALTITUDE MARKER	SIGNAL	APPROX. TIME
			° ′ ″									
R.O.	C.R.	H	171 10 24									
"	C.L.	"	351 10 27									
α Bootis	C.L.	V	58 23 13	50	30·23							
		H	173 28 59			$0·9^{R}$	$1·6^{R}$	$1·0^{R}$	$1·4^{R}$	Az	↗ 3·0 3·1 3·2 3·3 3·4	00^{49} BST
"	C.R.	V	121 54 59	50	"							
		H	356 19 00			$0·7^{L}$	0·0	$0·7^{L}$	0·0	Az	↗ 3·4 3·9 4·0 4·2 4·4	00^{59} BST
R.O	CR											
"	C.L.											
1	2	3	4	5	6		7			8	9	10

TABLE: 8.2.1–1

Time Difference between Crystal Chronometer Pulse and
Marker Pulse observed: ~~Sun~~, Star Arcturus C.L., ~~C.R.~~
at Observation Station: James A Date: 11 & 12. 5. 1961 approx. Time: 0049 B.S.T.
C.C. operating since: 9³⁰ a.m. Oven: on, ~~off~~ Battery No.: 243

Recording Instrument: Fi-cord Tape No: Scotch C09 Track: A Speed: 7½ "/s
developed with: stainless steel flake & Xylene Date: 3.10.61

$$\frac{\text{Distance betw. Marker Pulse \& C.C. Pulse}}{\text{" " two relevant C.C. Pulses}} \times \frac{60}{61} =$$

= Amount to be added to nominal Time of preceding C.C. Pulse

+ nominal Time of C.C. Pulse (Table 3.3.-1)
+ Time Diff. betw. Marker Pulse & C.C. Pulse
∓ C.C. fast or ~~slow~~ on U.T.O − 6·0081
= U.T.O of Marker Pulse

Description	Measurements, Scale: 0·1 mm	(Check)	
U.T. = 23^h 50^m			
C.C. 50	0026·8	−0016·0	
1st marker	0208·9	0251·5	
C.C. 51	1853·9	1896·6	
C.C. 52	0020·2	0027·9	
2nd	0726·2	0719·0	
C.C. 53	1858·0	1850·3	
C.C. 54	0024·7	−0001·0	
3rd	1040·2	1065·9	
C.C. 55	1849·4	1874·9	
C.C. 56	0021·8	0027·3	
4th	1382·0	1376·3	
C.C. 57	1855·4	1849·7	
C.C. 58	0017·2	0028·1	
5th	1129·9	1119·1	
C.C. 59	1859·8	1849·0	
C.C.			
C.C.			
C.C.			
C.C.			
C.C.			
C.C.			

$\frac{0235·7}{1880·7} \times \frac{60}{61}$ +49·1803; + ·1232; −6·0081
$\frac{0235·5}{1880·6}$ (Check) $23^h - 50^m - 43·295^s$ ①

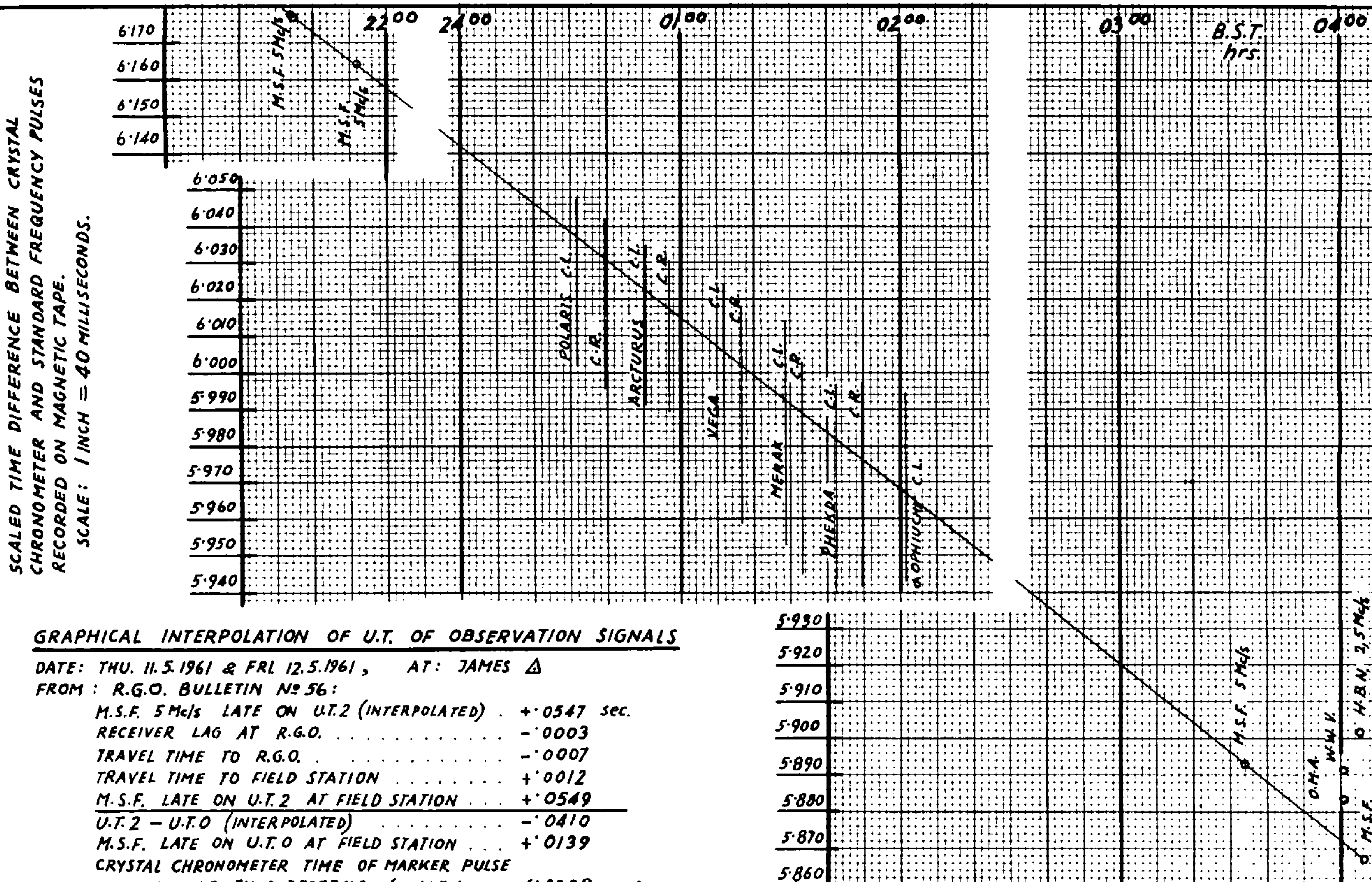
$\frac{0746·4}{1878·2}$ 51·1475; ·3910; 6·0081
$\frac{0746·9}{1878·2}$ 23 − 50 − 45·530 ②

$\frac{1064·9}{1874·1}$ 53·1148; ·5589; 6·0081
$\frac{1064·9}{1873·9}$ 23 − 50 − 47·666 ③

$\frac{1403·8}{1877·2}$ 55·0820; ·7355; 6·0081
$\frac{1403·6}{1877·0}$ 23 − 50 − 49·809 ④

$\frac{1147·1}{1877·0}$ 57·0492; ·6011; 6·0081
$\frac{1147·2}{1877·1}$ 23 − 50 − 51·642 ⑤

Table: 8 2.1.−2



GRAPHICAL INTERPOLATION OF U.T. OF OBSERVATION SIGNALS

DATE: THU. 11.5.1961 & FRI. 12.5.1961, AT: JAMES △

FROM: R.G.O. BULLETIN № 56:

M.S.F. 5 Mc/s LATE ON U.T.2 (INTERPOLATED) .	+·0547 sec.
RECEIVER LAG AT R.G.O.	−·0003
TRAVEL TIME TO R.G.O.	−·0007
TRAVEL TIME TO FIELD STATION	+·0012
M.S.F. LATE ON U.T.2 AT FIELD STATION . . .	+·0549
U.T.2 − U.T.0 (INTERPOLATED)	−·0410
M.S.F. LATE ON U.T.0 AT FIELD STATION . . .	+·0139
CRYSTAL CHRONOMETER TIME OF MARKER PULSE FAST ON M.S.F. FIELD RECEPTION (SCALED) . .	−6·0220 α BOOTIS
MARKER PULSE FAST ON U.T.0	−6·0081 sec. (α BOOTIS)

Fig.: 8.2.1.-1

8.2.1. cont.

Strictly, the altitude correction should not be obtained from the arithmetic mean of five estimations of reticule intervals, because the vertical reticule lines are not evenly spaced; the error introduced is negligible and beyond the accuracy of estimating gridline intervals. The altitude correction can be neglected when altitude marker pulses are released; this introduces an index error of the vertical circle which is eliminated when combining C. L. and C. R. observations.

The "Azimuth Correction x sec. Vertical Angle" is made up of the arithmetic mean of the angular subtense of the five reticule lines multiplied by the secant of the vertical angle. This correction can be neglected if azimuth marker pulses are released; in which case the azimuth correction contributes to the increase or decrease of the collimation axis error, and is eliminated by C. L. and C. R. observations.

If altitude marker pulses are released the slight unequal spacing of the horizontal reticule lines introduces an error to the azimuth correction; this error is so small that it is here neglected.

The corrected measurements enter into the calculation given in Table 8.2.1.-4. The layout of this table is arranged to be labour saving and is intended for the use of natural values of trigonometrical functions and a desk calculator. (Section 9.) In the heading are given the formulae adopted for computing the altitude and the azimuth which are required for plotting the position line.

Where stars are concerned, their declination and right ascension will not alter appreciably during the course of observation, because C. L. and C. R. pointings are only a few minutes of time apart. The numerical values of terms involving data which do not change during C. L. and C. R. observations are included in the heading, for easy access.

More decimal places are carried in the computation than are normally required, second order terms are included, and small adjustments are also applied, to avoid rounding off errors and inaccuracies from omitted corrections.

CORRECTIONS

AT *James Δ* DATE *May 11&12 1961*

STAR, ~~SUN~~ *Arcturus* No. *217 A.P.* MARKER PULSE *Azimuth →* TIME *00^{49} B.S.T.*

R.O. *St. Agnes C.L. - 351° 10′ 25″.5*

APPROX. AZIMUTH of R.O.	+ 13 09 00.0	VERT. CIRCLE (FIELD RECORD)	58 23 13
HOR. CIRCLE (FIELD RECORD)	173 28 59	ALTITUDE (uncorrected)	a.a.

AZIMUTH	1.0	−57.7″	ALTITUDE	3.0	00″	U.T. 0	23^h 50^m 43^s.295
estimated	2.0	−29.6	estimated	3.1	−06	of	45.530
grid interval and	3.0	00	grid interval and	3.2	−08	grid	47.666
correction to	4.0	+28.1	correction to	3.3	−11	reticule	49.809
℄ of reticule	5.0	+57.0	℄ of reticule	3.4	−13	crossings	51.642

i = Inclination of T.A. i × tan VERT. ANGLE	−40.0	REFRACTION $\frac{\cdot 991 \times \text{Pressure}}{460 + \text{Temp.}}$ × cot VERT. ANGLE	−36.2		
c = COLL. AXIS ERROR c × sec VERT. ANGLE	n.a. —				
−0.4 Azimuth Corr. × sec VERT. ANGLE	−00.7	ALTITUDE Correction	−07.6	mean U.T. 2	
observed AZIMUTH C.L. or C.R.	C.L. 195° 26′ 52″.8	corrected ALTITUDE (observed)	58° 22′ 29″.2	mean U.T. 0	23^h 50^m 47^s.588$_6$

TABLE 8.2.1.−3

ASTRONOMIC POSITION LINES

h = Altitude	A = Azimuth	Check:
$\sin h_c = \sin\phi.\sin\delta + \cos\phi.\cos\delta.\cos t$	$\cot A_c = \dfrac{\sin\phi.\cos t - \cos\phi.\tan\delta}{\sin t}$	$\sin A_c = \dfrac{-\cos\delta.\sin t}{\cos h_c}$
$\pm\sin\phi.\sin\delta =$ + 255 293 51		

$\pm\cos\phi.\cos\delta =$ + 602 804 75 $\qquad \pm\cos\phi.\tan\delta =$ + 224 840 51

T.P.	ϕ =	50°	16'	50"
	λ =	05°	13'	35" W
U.T. 0 of obs.		23h	50m	47·589s
app. G.S.T. at 0h U.T. 0		15	14	09·211
hrs., min.,			03	54·913
sec,				·130
equ.				− ·006
app. G.S.T. of obs.		15	08	51·837
app. Right ascension α		14	13	54·474
$n \times \Delta'_{\frac{1}{2}}$ = ·509$_4$ × +8				+ ·004
D = −59				+ ·004
$d\psi$ = +·056 $d\varepsilon$ = +·020				− ·001
Aberration				+ ·015
− α h, m, s		14	13	54·496
G.ST − α h,m,s		00	54	57·341
G.H.A. ° ' "		13	44	20·115
−W +E λ ° ' "		05	13	35
G.S.T. − α + λ t ° ' "		08	30	45·115
app. Declination δ ° ' "		+ 19	22	62·55
$n \times \Delta'_{\frac{1}{2}}$ = ·509$_4$ × 165				+ ·84
D = +04				—
$d\psi$ = −·33 $d\varepsilon$ = −·55				− ·00$_2$
Aberration				+ ·01
δ ° ' "		+ 19	23	03·40

$\sin\phi$	+ 769 182 73	
$\cos\phi$	+ 639 028 89	
h_o	58° 22' 29·2"	$h_o - h_c$
h_c	58 22 14·0	+ 15·2
$\sin h_c$	851 457 47	
$\cos h_c$	524 423 54	
A_c	195° 26' 31"	
Check $\sin A_c$	− 266 263 20	
$\sin A_c$	− 266 263 23	
$\cot A_c$	$\pm$ 3·620 104 41	
$\sin t \pm$	+ 148 025 71	
$\cos t \pm$	+ 988 983 51	
$\sin\delta \pm$	+ 331 902 29	
$\cos\delta \pm$	+ 943 313 77	
$\tan\delta \pm$	+ 351 847 18	

c	stands	for	calculated
o	"	"	observed
obs.	"	"	observation
app.	"	"	apparent

DATE: May 11&12, 1961 = 11·993606

STAR: α Bootis (Arcturus)

C.L.

TABLE 8.2.1.−4

8.2.1. cont.

Star data are extracted from "Apparent Places of Fundamental Stars". (Section: 9.)

A numerical check of the calculated Azimuth (A_c) is included in the computations shown in Table 8.2.1.-4, viz.: the azimuth (A_c) from the trial point is obtained twice from relations involving identical quantities each time: ϕ_{TP}, λ_{TP}, declination δ; right ascension α, and observed time.

Less computational work is required to obtain altitude and azimuth from subsequent pointings to the same star in the same face position and from pointings after transiting because the co-ordinates of the trial point, the declination and right ascension remain unchanged. Altitude and azimuth calculations for the same star from C.R. observation are submitted in Table 8.2.1.-5. Computations to aid assumption of the co-ordinates of the trial point are not quoted because these are regarded irrevelant for the presentation of results from field experiments.

The precision of the observations governs the plotting scale. It has been found that results from observations taken with the Watts Microptic No. 2 (single second theodolite) are conveniently plotted at a scale - $1\frac{1}{4}$ inches = 10 seconds of arc, so that $\frac{1}{4}$ second of arc can be scaled from the graph. Results obtained with the Watts Microptic No. 3 (prototype, $^1/_5$ second theodolite) are plotted at a scale twice as large, viz.: $2\frac{1}{2}$ inches = 10 seconds of arc.

In Fig. 8.2.1.-2 are shown the position lines obtained from observations taken at "James" mine triangulation station in 1961. Table 8.2.1.-6 contains the relevant data from observations and computational results. From various possibilities of plotting position lines the plotting method using computed zenith distances and computed azimuths has been adopted for all observations.

It is evident from the plot and accompanying data that each position line is based on <u>one</u> pointing to the star, consisting of five

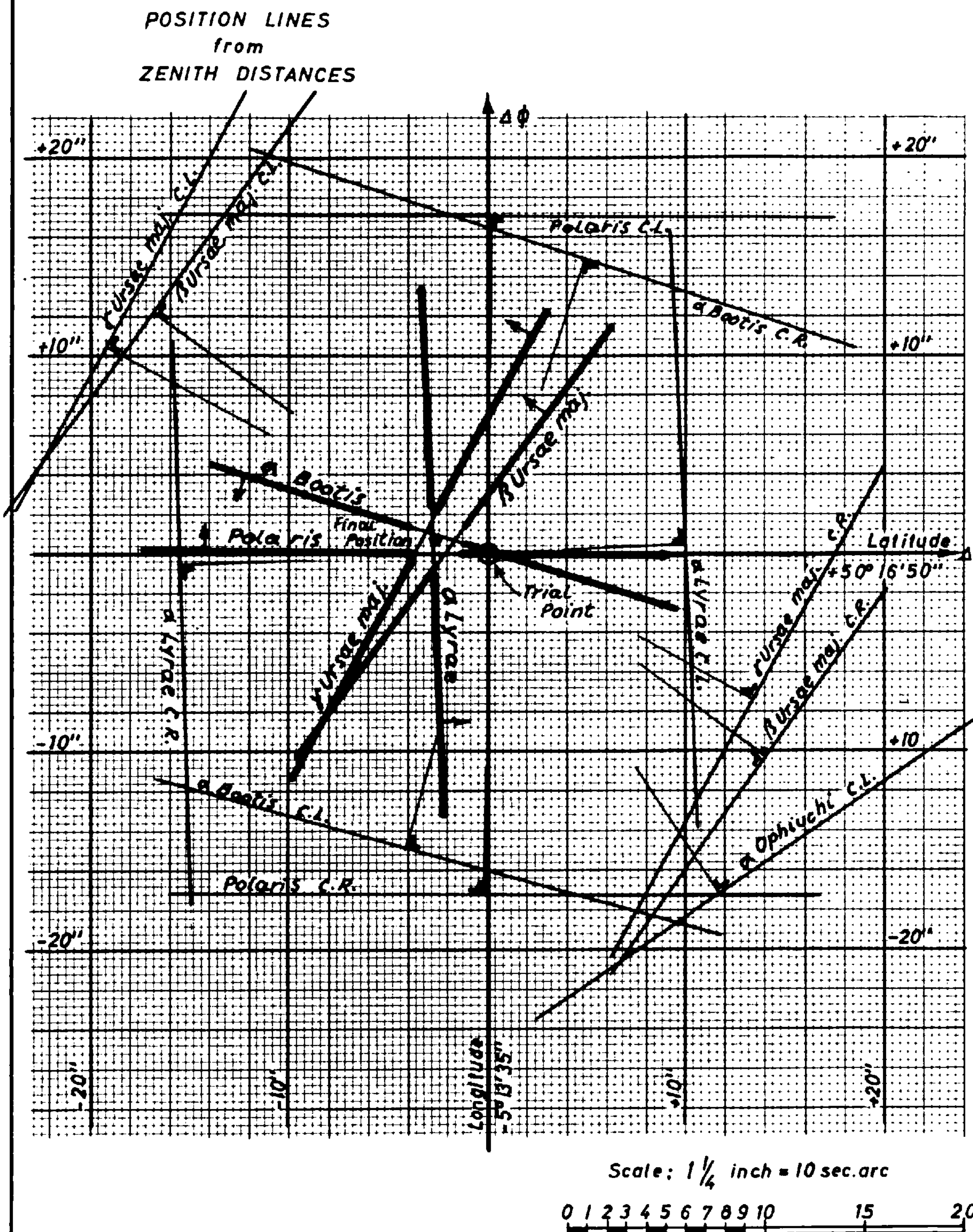
ASTRONOMIC POSITION LINES

h = Altitude	A = Azimuth	Check:
$\sin h_c = \sin\phi \cdot \sin\delta + \cos\phi \cdot \cos\delta \cdot \cos t$	$\cot A_c = \dfrac{\sin\phi \cdot \cos t - \cos\phi \cdot \tan\delta}{\sin t}$	$\sin A_c = \dfrac{-\cos\delta \cdot \sin t}{\cos h_c}$
$\pm \sin\phi \cdot \sin\delta =$		
$\pm \cos\phi \cdot \cos\delta =$	$\pm \cos\phi \cdot \tan\delta =$	

T.P. $\phi =$		$\sin\phi$			
$\lambda =$		$\cos\phi$			
U.T. 0 of obs.	23^h 57^m $15^s.573$	h_o	58° 03′ 56″.3	$h_o - h_c$	
app. G.S.T. at 0^h U.T.	0 15 14 09.211	h_c	58 04 11.9	−15.6	
hrs., min.,	03 56.063				
sec.,	.043	$\sin h_c$	848 694 68		
equ.	−.006				
app. G.S.T. of obs.	15 15 20.884	$\cos h_c$	528 883 11		
app. Right ascension α					
$n = \Delta'_{\frac{1}{2}} =$					
D =					
$d\psi =$ $d\varepsilon =$		A_c	198° 17′ 21″		
Aberration		Check $\sin A_c$	−313 811 88		
$-\alpha$ h, m, s		$\sin A_c$	−313 812 02		
G.S.T − α h,m,s	01 01 26.388				
G.H.A. ° ′ ″	15 21 35.82				
$-W$ $+E$ λ ° ′ ″		$\cot A_c$	± 3.025 650 77		
G.S.T. − α + λ t ° ′ ″	10 08 00.82	$\sin t$ ±	+ 175 943 37		
app. Declination δ ° ′ ″		$\cos t$ ±	+ 984 400 29		
$n = \Delta'_{\frac{1}{2}} =$		$\sin\delta$ ±			
D =					
$d\psi =$ $d\varepsilon =$		$\cos\delta$ ±			
Aberration					
δ ° ′ ″		$\tan\delta$ ±			

c stands for calculated	DATE May 11Ø12 1961 = 11.9980969
o " " observed	STAR α Bootis (Arcturus) Nº 217 A.P.
obs. " " observation	C.R.
app. " " apparent	

TABLE 8.2.1.–5



Trial Point:	scaled:	Final Position: "James" Δ
$\phi = +50°16'50''$	$\Delta\phi = +0{\cdot}7''$	$\phi = +50°16'50{\cdot}7''$
$\lambda = -05°13'35''$	$\Delta\psi = -2{\cdot}7''; \Delta\lambda = \dfrac{\Delta\psi}{\cos\phi} = -4{\cdot}2''$	$\lambda = -5°13'39{\cdot}2''$

Date: 11.5.1961

Fig: 8.2.1.– 2

AT "James" (Mine Trig. Stn.) DATE May 11, 12, 1961

TRIAL POINT ϕ_{TP} = +50° 16' 50" N λ_{TP} = -5° 13' 35" W

STAR	C L / C R	U.T. 0 Mean of five Marker Pulses h m s	h_o corrected observed Altitude ° ' "	$h_o - h_c$ obs. minus calc. Alt. "	calculated Azimuth from Trial Point ° '	Remarks
Polaris	C.L.	23 31 16·096	49 22 28·6	+17·0	00 12	
	C.R.	23 38 54·979	49 22 11·6	-17·3	00 15	
α Bootis	C.L.	23 50 47·589	58 22 29·2	+15·2	195 27	
	C.R.	23 57 15·573	58 03 56·3	-15·6	198 17	
α Lyrae	C.L.	00 11 43·349	52 13 56·4	+10·0	87 23	
	C.R.	00 16 35·426	53 00 16·0	-15·7	88 17	
β Ursae maj.	C.R.	00 29 01·837	51 22 25·3	-17·2	305 54	
	C.L.	00 33 27·066	50 48 42·4	+20·8	306 14	
γ Ursae maj.	C.L.	00 42 24·891	55 46 06·4	+21·9	298 37	
	C.R.	00 48 47·959	54 51 45·1	-15·4	299 08	
α Ophiuchi	C.L.	01 01 44·951	47 41 47·5	+20·8	144 40	
	C.R.					not recorded

TABLE 8.2.1 – 6

8.2.1. cont.

timed gridline crossings and of one vertical circle reading.

The mean position line for each star is obtained graphically. The final position is selected by inspection and "weighting up" the plotted position lines.

α Ophiuchi was observed through a small hole in the clouds and was not recognized. It had to be identified from measurements taken in C. L. position; C. R. observations were prevented by clouds and the field work had to be discontinued, because the sky was completely overcast.

The co-ordinates of the trial point, observed time and altitude, and approximate azimuth derived from horizontal circle readings and observations to recognised stars were used for the identification. The formulae linking the necessary quantities are:

$$\sin \delta = \sin \phi \,.\, \sin h + \cos \phi \,.\, \cos h \,.\, \cos A$$

$$\cot (GST - \alpha + \lambda) = \frac{\sin \phi \,.\, \cos A - \cos \phi \,.\, \tan h}{\sin A}$$

δ = star's declination

α = star's right ascension

ϕ, λ = latitude and longitude of the trial point

h = observed altitude

GST is obtained from observed U. T.

A = approximate azimuth.

The simultaneous measurements of stellar altitude and azimuth with the aid of the grid reticule made the identification easy. Furthermore the reading of both circles enables the determination of the astronomical azimuth of any terrestrial line of sight simultaneously with an astronomical fixation.

8.2.1i cont.

The astronomical azimuth (A_A) of the reference line

is derived from the horizontal circle readings to the R.O. and star, and from the relation:

$$\cot A_A = \frac{\sin\phi_A \cdot \cos(GST - \alpha + \lambda_A) - \cos\phi_A \cdot \tan\delta}{\sin(GST - \alpha + \lambda_A)}$$

where:

ϕ_A (astronomical latitude) and λ_A (astronomical longitude)

are the final co-ordinates of the station obtained from the graphical solution, GST . is already derived from the observed time, α and δ are the extracted tabulated values.

The resulting values of the astronomical azimuth from James mine triangulation station to St. Agnes O.S. pillar station are listed in Table 8.2.1.-7.

Whereas the azimuth (A_c) calculated from the trial point is checked numerically, with formulae containing basically the same quantities, the astronomical azimuth (A_A) is effectively checked with the aid of the observed altitude. The astronomical azimuth derived with the above cotangent formula is a function of :

$$A_A = f(\phi_A, \lambda_A, \alpha, \delta, \text{observed time})$$

for checking, the sine or cosine formula can be adopted:

$$\text{sine } A_A = -\frac{\cos\delta \;.\; \sin(GST - \alpha + \lambda_A)}{\cos h_o} \quad \ldots\ldots\ldots (1)$$

$$\cos A_A = \frac{\sin\delta - \sin\phi_A \;.\; \sin h_o}{\cos\phi_A \;.\; \cos h_o} \quad \ldots\ldots\ldots (2)$$

formula (1) is the quickest to use; any error of the observed height will enter into the azimuth multiplied by the product of tangent of the azimuth and tangent of the observed height. Formula (2) is employed whenever the azimuth is approaching 90° (270°). This check is not

AT "James" (Mine Trig. Stn.) DATE May 11, 12, 1961

FINAL POSITION $\phi_A = +50° 16' 50.7'' N$ $\lambda_A = -5° 13' 39.''2 W$

STAR	CL / CR	ASTRONOMICAL AZIMUTH FROM James Mine Trig. TO St. Agnes O.S. Beacon ° ′ ″	MEAN ° ′ ″	MEAN ° ′ ″	REMARKS
					check:
Polaris	C.L.	13 08 23.0	13 08 26.3		00.0″
	C.R.	29.6			
α Bootis	C.L.	13 08 30.8	13 08 28.6		− 00.7
	C.R.	26.5			
α Lyrae	C.L.	13 08 24.5	13 08 25.3		* − 00.2
		26.1			
β Ursae maj.	C.L.	13 08 17.3	13 08 18.0		+ 01.6
	C.R.	18.7			
γ Ursae maj.	C.L.	13 08 24.2	13 08 22.9		− 02.8
	C.R.	21.5			
α Ophiuchi	C.L.	13 08 24.5			
	C.R.				

Difference: A_A = Ast. Azimuth derived from:

$$\sin A_A = -\frac{\cos\delta \cdot \sin t}{\cos h_o}$$

or:

$$* \quad \cos A_A = \frac{\sin\delta - \sin\phi \cdot \sin h_o}{\cos\phi \cdot \cos h_o}$$

h_o = observed height

Mean : 13° 08′ 24.″2

TABLE 8.2.1 − 7

8.2.1. cont.

entirely independent because the final selected position (ϕ_A, λ_A) is a function of the observed zenith distances.

The differences between values of the azimuth derived from the cotangent equation and from the sine or cosine equation, as applicable, for each individual star are given in the remarks column, Table 8.2.1.-7

Atmospheric interference necessitated speed of observation, i.e. one single pointing in each face position, and stars could not be selected to suit optimal conditions for an astronomical fixation.

The accuracy of astronomical co-ordinates and of astronomical azimuths can not be judged from results of only one night's observations, regardless of the number of stars observed.

The personal equation and the constantly changing atmospherics, e.g. humidity, pressure, temperature, wind direction, etc., and also tide conditions influence the final result. Obviously, the method of position fixing discussed here is essentially based on the reception of electromagnetic waves; the reception is in the form of optical observation of light rays and in the form of the recording of transmitted time signals; light rays and time signal transmission are both equally affected by the properties and instantaneous conditions of their common travel medium.

Field data and results of observations for position fix and azimuth on three nights at the same survey station, viz.: "Cottage", mine triangulation station, in 1961, with the same instruments are given in Tables 8.2.1.-8 to -13. The graphical evaluations are shown in Figs. 8.2.1.-3, -4, and -5. It was endeavoured to observe the same stars each night.

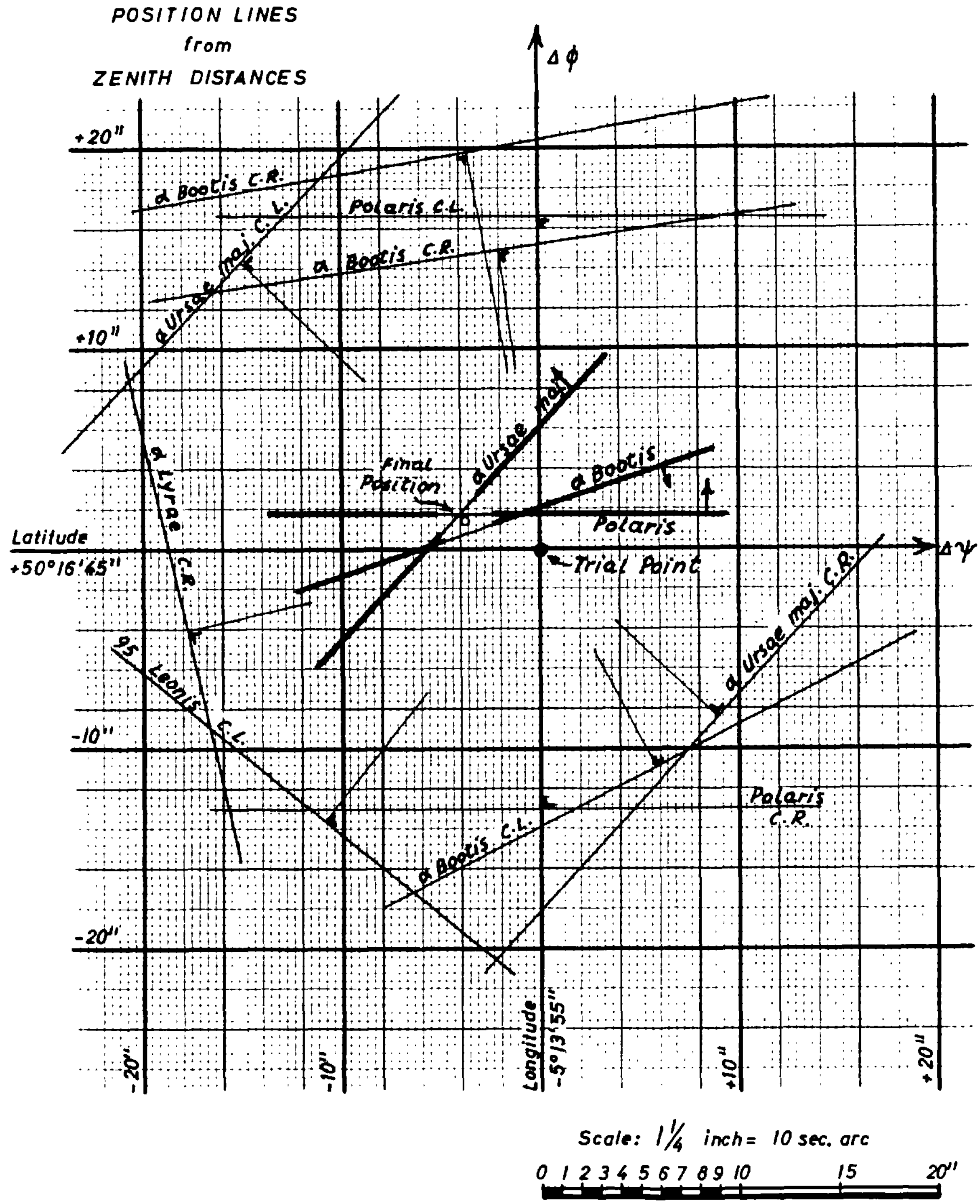
For better interpretation of observational results each position line has been calculated and plotted separately, and the mean position line resulting from C.L. and C.R. observations constructed graphically. The little arrow attached perpendicularly to each mean position line

AT " Cottage " DATE May 1st, 1961

TRIAL POINT ϕ_{TP} = +50° 16' 45" N λ_{TP} = -5° 13' 55" W

STAR	C L / C R	U.T. 0 Mean of five Marker Pulses h	m	s	h_o corrected observed Altitude °	'	"	$h_o - h_c$ obs. minus calc. Alt. "	calculated Azimuth from Trial Point °	'	Remarks
α Bootis	C.L.	22	55	15·870	56	48	35·6	+12·5	153	01	
Polaris	C.L.	23	04	33·737	49	22	27·7	+16·4	359	48	
95 Leonis	C.L.	23	15	09·617	50	12	49·0	+17·3	219	02	identified
α Ursae maj.	C.L.	23	23	47·762	65	21	49·6	+20·6	313	32	
Polaris	C.R.	23	29	13·291	49	21	26·1	-12·9	359	57	
α Bootis	C.R.	23	35	08·379	58	48	56·0	-20·0	170	22	
"	"	23	38	42·620	58	54	16·9	-15·2	171	59	
95 Leonis	C.R.										clouds
α Ursae maj.	C.R.	23	44	47·012	62	54	04·6	-12·1	312	50	
α Lyrae	C.R.	23	50	05·188	42	33	44·6	-18·1	77	02	identified
"	C.L.										clouds

TABLE 8.2.1 – 8



Trial Point:	scaled:	Final Position: "Cottage"
$\phi = +50°16'45''$	$\Delta\phi = +1{\cdot}5''$	$\phi = +50°16'46{\cdot}5''$
$\lambda = -5°13'55''$	$\Delta\psi = -3{\cdot}8''$, $\Delta\lambda = \dfrac{\Delta\psi}{\cos\phi} = -5{\cdot}9''$	$\lambda = -5°14'00{\cdot}9''$

Date: 1.5.1961

Fig.: 8.2.1. — 3

AT " Cottage" DATE May 1st, 1961

FINAL POSITION Φ_A = +50° 16' 46.5" N λ_A = -5° 14' 00.9" W

ASTRONOMICAL AZIMUTH

FROM " Cottage" TO Porthtowan R.O.

STAR	CL CR	°	'	"	MEAN °	'	"	MEAN °	'	"	REMARKS
α Bootis	C.L.	305	32	40.2	305	32	30.3				
	C.R.			19.7							
	"			21.3							
Polaris	C.L.	305	32	46.6	305	32	33.2				
	C.R.			19.7							
α Ursae maj.	C.L.	305	32	53.4	305	32	34.3				
	C.R.			15.2							
95 Leonis	CL	305	32	38.7							identified
	CR										clouds
α Lyrae	C.L.										clouds
	C.R.	305	32	22.5							identified
					Mean:			305°	32'	32.2"	

TABLE 8.2.1-9

8.2.1. cont.

indicates the direction to the star.

The majority of the mean position lines are within one second of arc from the final selected position, which is supposed to be the astronomical zenith. The distance of the position line from the final selected position is equal to the unknown errors affecting the observed quantities. If these errors are equal for all observations then all position lines are missing the final position at the same distance, either towards or away from the stars. Furthermore the position lines are then tangent to a circle with the astronomical zenith as centre and the errors as semidiameter.

In practice this is never the case; because as can be expected, observational errors are not equal for all stars, for reasons mentioned in Section 8.1. In addition, the causes originating the errors undoubtedly alter with time; different atmospheric displacement of a heavenly body, when re-observed after a time interval has elapsed, has been encountered in the presence of clouds or haze, during the field experiments.

Refraction does not only change with time, but it can hardly be assumed to remain at any time equal in all azimuths.

Examination of the graphical evaluations shown in Figs. 8.2.1. -3, -4, and -5 reveals regularity of the position lines pertaining to one star and also some regularity of the mean position lines in missing the final position, on any one night. The final position selected and scaled from the graphs varies slightly for each night.

Regularity of errors points towards the presence of systematic errors.

The first mentioned regularity which exhibits nearly equal distances between C.L. and C.R. position lines during one night,

8.2.1. cont.

is obviously due to an index error of the vertical circle and is accepted for convenience of plotting the individual lines.

The graphs also indicate that the index error fluctuates an amount of about 2 to 3 seconds of arc. Atmospherics could be the reason for these small fluctuations. Nevertheless, considering that the theodolite has been used by students and for sun observations in between its employment in field experiments, the results show good stability of the plumb line of the altalidade bubble.

Regularity of the mean position lines with respect to the final position is partly caused by its selection, but can be also attributed to instrumental error which is not eliminated by change of face or to errors due to physical conditions and anomalies of the environment, or also to the personal equation.

It is believed that the working out and plotting of each position line separately provides a criterion of the precision of the observation. The range of differences between single pointings and the agreement of the means provide a guide for acceptance of results or of rejection of doubtful observations and reasons for doing so. The adequate judgement of field results can not be clearly effected when taking the mean of observations of both circle positions and may be lost altogether if mean measurements are derived from pairs of stars.

Combining pairs of suitable stars, balanced in azimuth and altitude, for the purpose of producing mean position lines inevitably gives the illusion of "well established" results, which in fact may be distorted by one faulty observation.

In most textbooks it is recommended for latitude determination from altitudes of stars at any hour angle and specially at culmination "to pair north and south stars" and not to change face position. In this way the instrumental errors are eliminated or at least their effects minimized. This is equally true for position lines from

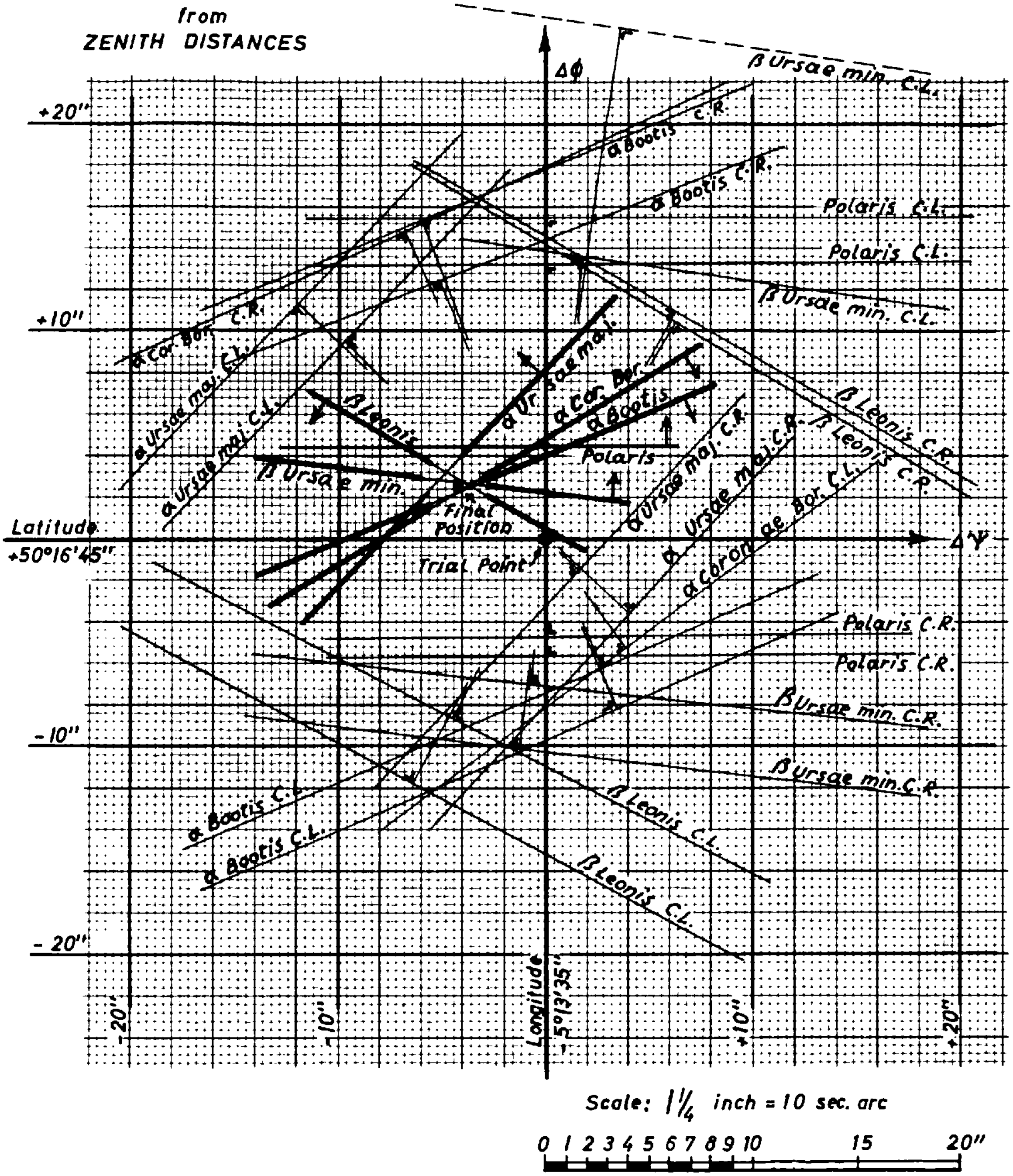
AT *"Cottage"* DATE *May 5th, 1961*

TRIAL POINT ϕ_{TP} = *+50° 16' 45" N* λ_{TP} = *-5° 13' 55" W*

STAR	CL CR	U.T. 0 Mean of five Marker Pulses h	m	s	h_o corrected observed Altitude °	'	"	$h_o - h_c$ obs. minus calc. Alt. "	calculated Azimuth from Trial Point °	'	Remarks
β Leonis	C.L.	22	21	53·241	51	55	31·7	+9·8	207	18	
	"		24	16·189		44	56	+13·4	208	11	
	C.R.		28	17·330		25	48·6	-12·7	209	40	
	"		30	35·425		14	43·6	-12·1	210	31	
Polaris	C.L.	22	35	25·433	49	22	57	+13·3	359	43	observed
	"		36	51·494			55	+15·4		43	through haze,
	C.R.		39	45·631			26	-05·7		44	discarded
	"		41	41·272			22	-04·8		45	
α Bootis	C.L.	22	46	58·303	57	19	14·5	+08·9	156	06	
	"		48	45·308		26	02·7	+06·7		52	
	C.R.		51	06·701		34	22·8	-16·4	157	52	
	"		52	43·688		40	11·8	-13·5	158	33	
α Ursae maj.	C.L.	22	58	58·669	66	24	47·3	+13·5	314	02	
	"	23	05	05·949	65	42	24·6	+16·5	313	41	
	C.R.		07	52·577		22	46·4	-02·1		33	
	"		10	13·233		06	22·1	-05·5		26	
β Ursae min.	C.L.	23	27	09·176	65	23	39·5	+24·9	08	07	haze, discarded
	"		30	12·341		27	30·0	+13·9	07	39	clouds
	C.R.		35	12·071		33	10·1	-10·2	06	52	"
	"		38	11·761		36	32·9	-07·0		24	"
α Coronae Borealis	C.L.	23	49	41·912	63	01	41·0	+06·5	143	48	inadequate theodolite
	C.R.	00	14	44·572	65	02	41·2	-16·4	155	46	illumination

TABLE 8.2.1 – 10

POSITION LINES
from
ZENITH DISTANCES



Trial Point:	scaled:	Final Position: "Cottage"
$\phi = +\ 50°16'\ 45''$	$\Delta\phi = +2{\cdot}4''$	$\phi = +\ 50°16'\ 47{\cdot}4''$
$\lambda = -\ 5°\ 13'\ 55''$	$\Delta\psi = -4{\cdot}2''; \Delta\lambda = \dfrac{\Delta\psi}{\cos\phi} = -6{\cdot}6''$	$\lambda = -\ 5°\ 14'\ 01{\cdot}6''$

Date: 5. 5. 1961

Fig.: 8.2.1. -4

AT " Cottage" DATE May 5th 1961

FINAL POSITION Φ_A = +50° 16' 47.4" N λ_A = -5° 14' 01.6" W

STAR	CL / CR	ASTRONOMICAL AZIMUTH FROM " Cottage" ° ' "	MEAN ° ' "	TO Porthtowan R.O. MEAN ° ' "	REMARKS
β Leonis	C.L.	305 32 52.1	305 32 32.5		
	"	47.5			
	C.R.	16.5			
	"	13.8			
Polaris	C.L.	305 32 52.6	305 32 35.2		
	"	49.5			
	C.R.	22.6			
	"	16.1			
α Bootis	C.L.	305 32 45.6	305 32 25.7		
	"	40.9			
	C.R.	12.1			
	"	04.0			
α Ursae maj	C.L.	305 32 56.4	305 32 35.6		
	"	53.4			
	C.R.	19.5			
	"	12.9			
β Ursae min.	C.L.	305 32 51.3	305 32 32.4	Azimuth referred to	
	"	52.4		initial R.O. reading	
	C.R.	11.7		poor visibility of	
	"	14.2		R.O. beacon Light.	
α Coronae Borealis	C.L.	305 32 54.5	305 32 35.2		
	C.R.	15.9			
			Mean of 4 :	305° 32' 32.3"	

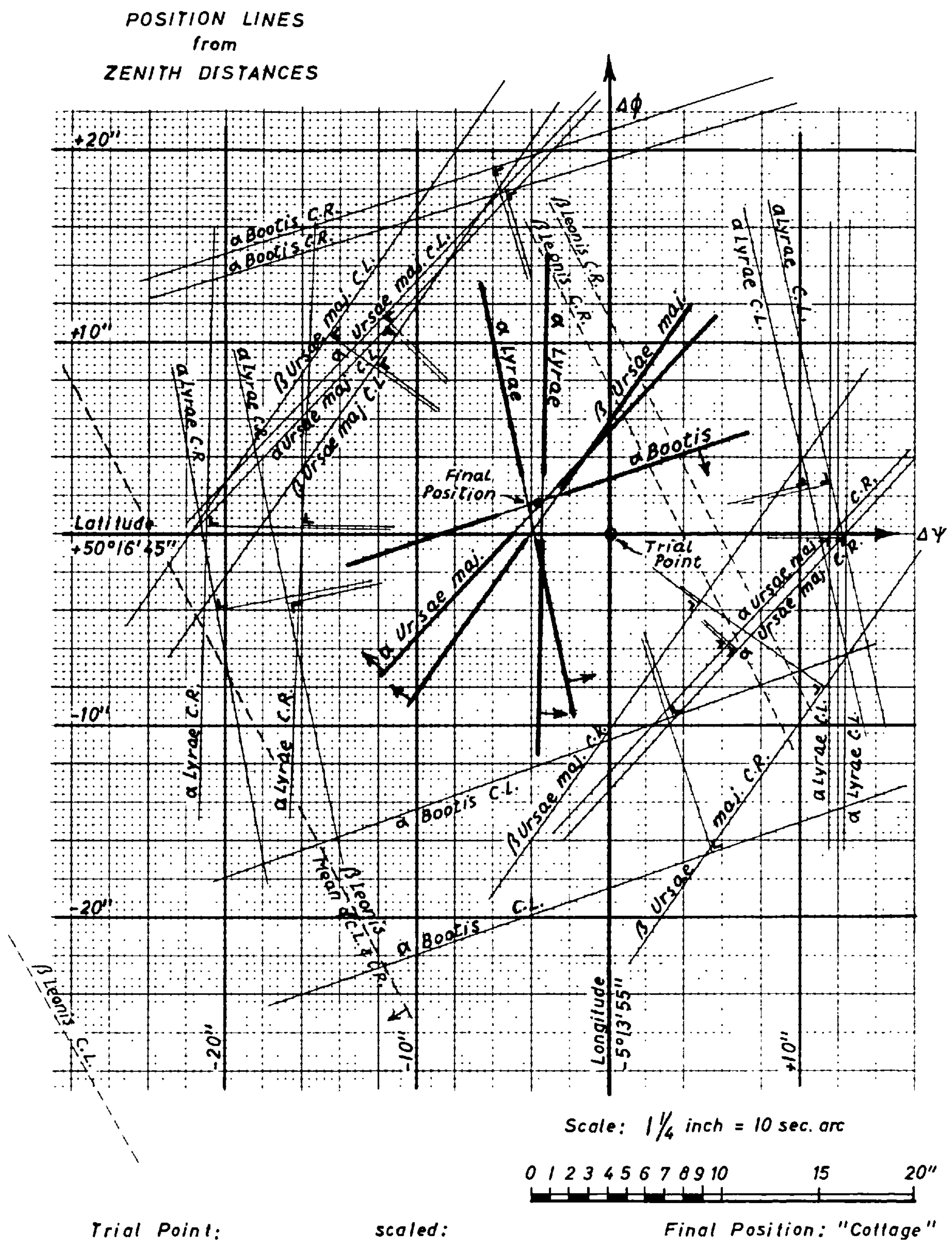
TABLE 8.2.1 — 11

AT "Cottage" DATE May 9th, 1961

TRIAL POINT ϕ_{TP} = +50° 16' 45" N λ_{TP} = −5° 13' 55" W

STAR	CL / CR	U.T. 0 Mean of five Marker Pulses h	m	s	h_o corrected observed Altitude °	'	"	h_o-h_c obs. minus calc. Alt. "	calculated Azimuth from Trial Point °	'	Remarks
α Bootis	C.L.	22	40	44·557	57	53	16·7	+10·1	160	10	
	"		42	48·810	58	00	00·6	+17·8	161	04	
	C.R.		45	39·908		08	00·0	−19·8	162	19	
	"		47	40·175		13	44·0	−18·6	163	11	
α Ursae maj	C.L.	22	53	23·674	65	14	23·0	+16·3	313	29	
	"		56	05·758	64	55	31·0	+15·8	313	22	
	C.R.	23	00	10·888		26	31·4	−8·6	313	12	
	"		02	58·783		06	54·0	−8·8	313	07	
α Lyrae	C.L.	23	16	53·744	42	17	57·3	+10·9	76	46	
	"		18	49·394		36	01·0	+11·9	77	04	
	C.R.		23	56·190	43	23	31·0	−16·9	77	53	
	"		33	04·827	44	49	37·1	−20·7	79	22	
β Leonis	C.L.	23	51	21·898	40	12	19·5	+37·5	240	24	identified
	"		55	49·053	39	35	08·9	+53·8	241	33	haze & clouds
	C.R.		59	44·275		00	54·7	−5·1	242	32	discarded
	"	00	01	16·341	38	47	51·0	−3·7	242	55	
β Ursae maj.	C.L.	00	20	09·571	53	34	26·8	+17·8	304	41	haze
	"		22	47·586		13	36·3	+14·9	304	52	& clouds
	C.R.		25	51·542	52	48	58·6	−13·9	305	05	
	"		28	52·229		25	27·0	−06·1	305	18	
α Lyrae	C.L.	00	34	11·467	54	34	05·5	+11·7	90	08	haze &
	"		36	00·567		51	35·0	+12·6	90	29	clouds
	C.R.		39	53·617	55	28	21·3	−20·9	91	15	
	"		42	53·616		57	17·5	−15·8	91	51	

TABLE 8.2.1 — 12



Trial Point:	scaled:	Final Position: "Cottage"
$\phi = +50°16'45''$	$\Delta\phi = +1{\cdot}6''$	$\phi = +50°16'46{\cdot}6''$
$\lambda = -5°13'55''$	$\Delta\Psi = -3{\cdot}8''$, $\Delta\lambda = \dfrac{\Delta\Psi}{\cos\phi} = -5{\cdot}9''$	$\lambda = -5°14'00{\cdot}9''$

Date: 9.5.1961

Fig.: 8.2.1.-5

AT "Cottage" DATE May 9th, 1961

FINAL POSITION ϕ_A = +50° 16' 46.6" N λ_A = −5° 14' 00.9" W

ASTRONOMICAL AZIMUTH FROM "Cottage" TO Porthtowan R.O.

STAR	CL / CR	° ' "	MEAN ° ' "	MEAN ° ' "	REMARKS
α Bootis	C.L.	305 32 43.1	305 32 29.2		
	"	46.1			
	C.R.	12.8			
	"	14.6			
α Ursae maj.	C.L.	305 32 54.7	305 32 36.4		
	"	50.5			
	C.R.	36.5			
	"	03.8			
α Lyrae	C.L.	305 32 44.2	305 32 35.7		
	"	49.6			
	C.R.	25.2			
	"	23.9			
β Leonis	C.L.	305 32 48.4	305 32 35.9		identified
	"	56.1			zenith Position Lines
	C.R.	20.3			discarded
	"	18.8			
β Ursae maj.	C.L.	305 32 47.5	305 32 29.6		haze & clouds
	"	47.1			
	C.R.	14.5			
	"	09.1			
α Lyrae	C.L.	305 32 49.3	305 32 32.9		haze & clouds
	"	44.1			
	CR	16.6			
	"	21.7			
			Mean :	305° 32' 33.3"	

TABLE 8.2.1—13

8.2.1. cont.

observed altitudes. Although this method is superior because it is quicker not to change face, nevertheless other errors, resulting from e.g. the difference in refraction due to direction and elapsed time, the different velocities and directions of the paths of stars, their magnitudes, etc., are not eliminated and hence obscure the final result. Instrumental errors of up-to-date theodolites are sufficiently constant during one period of operation; these errors are also easy to detect and can be corrected. .. .Pairing of stars of similar altitude in opposite azimuth can be advocated where instrumental errors are liable to change during the course of observation. The method of pairing is suitable for an astronomical fix when instrumental errors are expected to be larger than errors due to refraction, personal error etc.

The field observations on May 1st, 1961 were completed to all stars including to the R.O., first C.L., and thereafter C.R. Interference of clouds prevented stars being taken in the same sequence in both face positions, and obstructed the observations in changed face position to 95 Leonis and α Lyrae. The position lines obtained from these two stars (Fig. 8.2.1.-3) are not used for defining the final position, but their appearance on the plot contributes to the reliability of the final result.

The stability of instrumental constants mentioned above and the sudden interference in stellar seeing by clouds, justified the procedure of completing C.L. and C.R. observations to each star in succession.

The observations to Polaris and β Ursae minoris on May 5, 1961, were carried out through haze which covered fairly uniformly the area around the north celestial pole; the other parts of the sky were reasonably clear. The resulting position lines shown in Fig. 8.2.1.-4 are displaced; the reason for which is undoubtedly atmospheric distortion.

On May 9, 1961 slight mist obstructed the sight to β Leonis, which had to be identified because it was the only star visible in that area. The second observation on α Lyrae, during the same period of observation, was executed through a break in the clouds. The position lines from β Leonis, shown dotted in Fig. 8. 2. 1. -5, are displaced in the direction towards the star, which indicates that the observed altitude was too high.

The reason for the increment in altitude can be sought in the increased bending of the light ray consequent on the presence of water vapour in the air. During star observations, inevitably at night, there is no problematic heat source which could influence the refraction. Unlike the terrestrial refraction which causes the light ray to bend up or down according to the thermal conditions of the lower air layers in proximity to the earth's surface, the astronomical refractive index, as far as night observations are concerned, maintains the curvature of the ray in one direction i. e. the concave side of the light ray is directed towards the substellar point. No contrary evidence has been encountered; and can be hardly expected, unless the theodolite-set-up is so low that air layers immediately above the ground affect the observations.

It follows that alteration of the refractive index of the air in the presence of haze will shift the position line towards the star; intervening clouds can contribute to reflection of rays and hence to distortions in either direction, thus causing displacement of position lines towards or away from the star.

The difficult problem of applying an adequate correction for refraction when disturbances due to clouds, wind, haze, fine snow, are likely to occur, is solved by the rejection of observations which produce position lines that miss the final selected position.

If observations in C. L. and C. R. position follow in quick succession during which time diffused mist or haze does not drift away, the

8.2.1. cont.

position lines from all pointings in C.L. and C.R. to the star sighted, will be displaced uniformly; or alternatively a gradual displacement of position lines will result, where steady drifting water vapour partly affects the sights to stars.

Slight uniform displacement due to fine haze may pass unrecognised, in which case instrumental or personal errors are presumed. This is also true for distortions of sun altitudes during white out; these distortions are difficult to detect owing to the lack of balancing between opposite azimuths.

The small amount by which the mean position lines miss the final position, indicates that Bessel's formula gives adequate correction for refraction to observations in that particular part of the world.

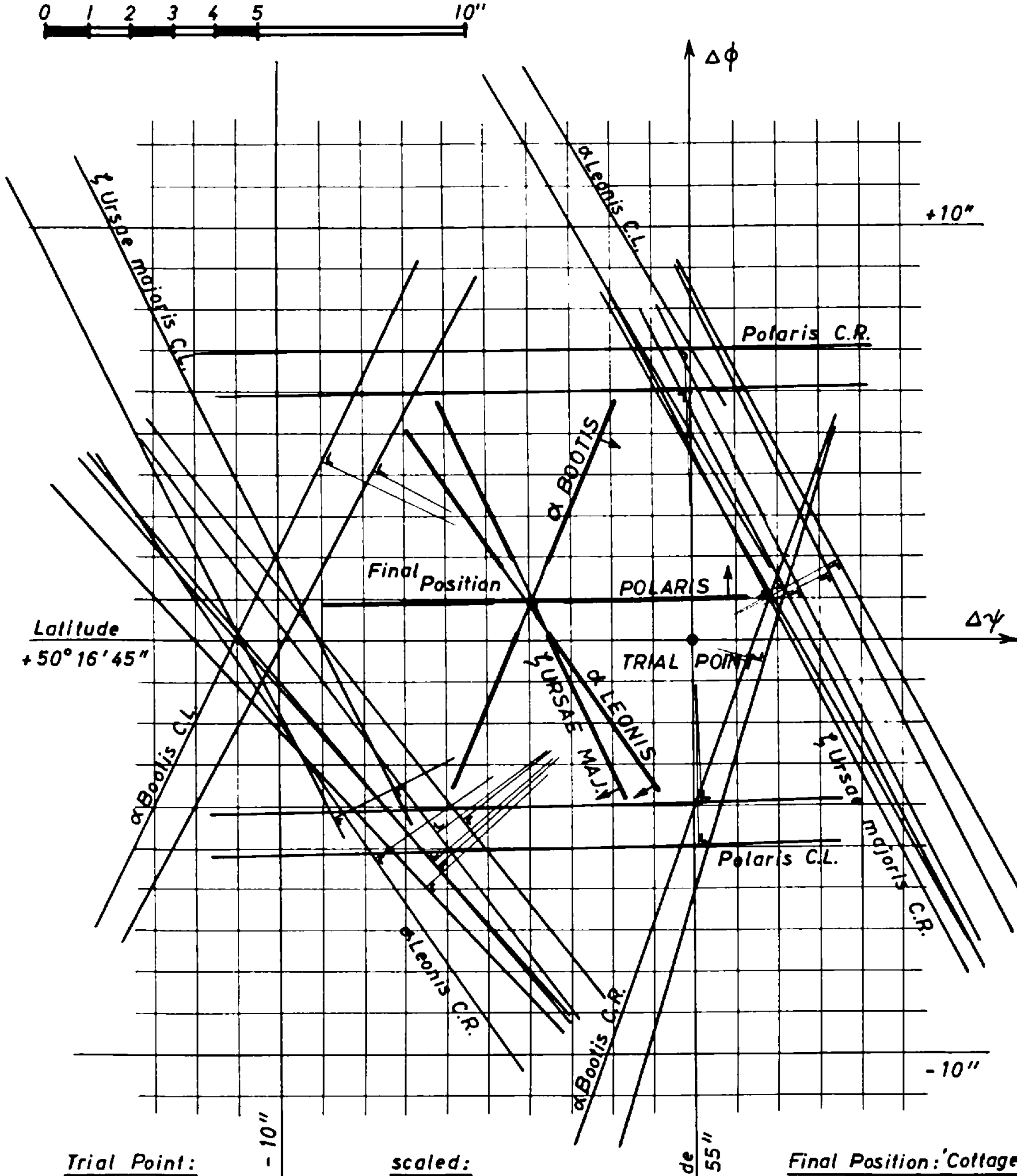
Tests for astronomical positions from zenith distances were continued in 1963 in Cornwall. Results obtained from field experiments with a Hilger and Watts geodetic theodolite (prototype) are plotted in Fig. 8.2.1.-6. Relevant data are given in Table 8.2.1.-14.

The spread of individual position lines from repeated pointings in the same face position to the same star is much smaller than in previous plots; in Fig. 8.2.1-7 are given, at a larger scale, the position lines from seven pointings to the same star in the same face position; the spread is about two seconds of arc. The position lines are numbered in the sequence in which the pointings to the star were taken. The higher precision achieved can be attributed mainly to the higher magnifying power of the telescope and hence to its increased accuracy in pointing. The tests in 1963 did not require additional equipment; as in previous field experiments only equipment to which an average field engineer has easy access was employed.

Table 8.2.1.-15 contains the astronomical azimuth from the final position of "Cottage" to "Porthtowan" R.O., derived from observations in 1963. Included in the table are the checks on the astronomical azimuths derived from the observed and corrected

POSITION LINES
from Zenith Distances

Scale: $2\frac{1}{2}$ inches = 10 sec. arc



Trial Point:	scaled:	Final Position: 'Cottage'
$\phi = +50°16'45''$	$\Delta\phi = +0{\cdot}9''$	$\phi = +50°16'45{\cdot}9''$
$\lambda = -5°13'55''$	$\Delta\psi = -3{\cdot}9'', \ \Delta\lambda = \dfrac{\Delta\psi}{\cos\phi} = -6{\cdot}1''$	$\lambda = -5°14'01{\cdot}1''$

Instrument: Watts $\frac{1}{5}''$ Optical Micrometer Theodolite. (Prototype)

Date: April 1963.

Fig.: 8.2.1.-6

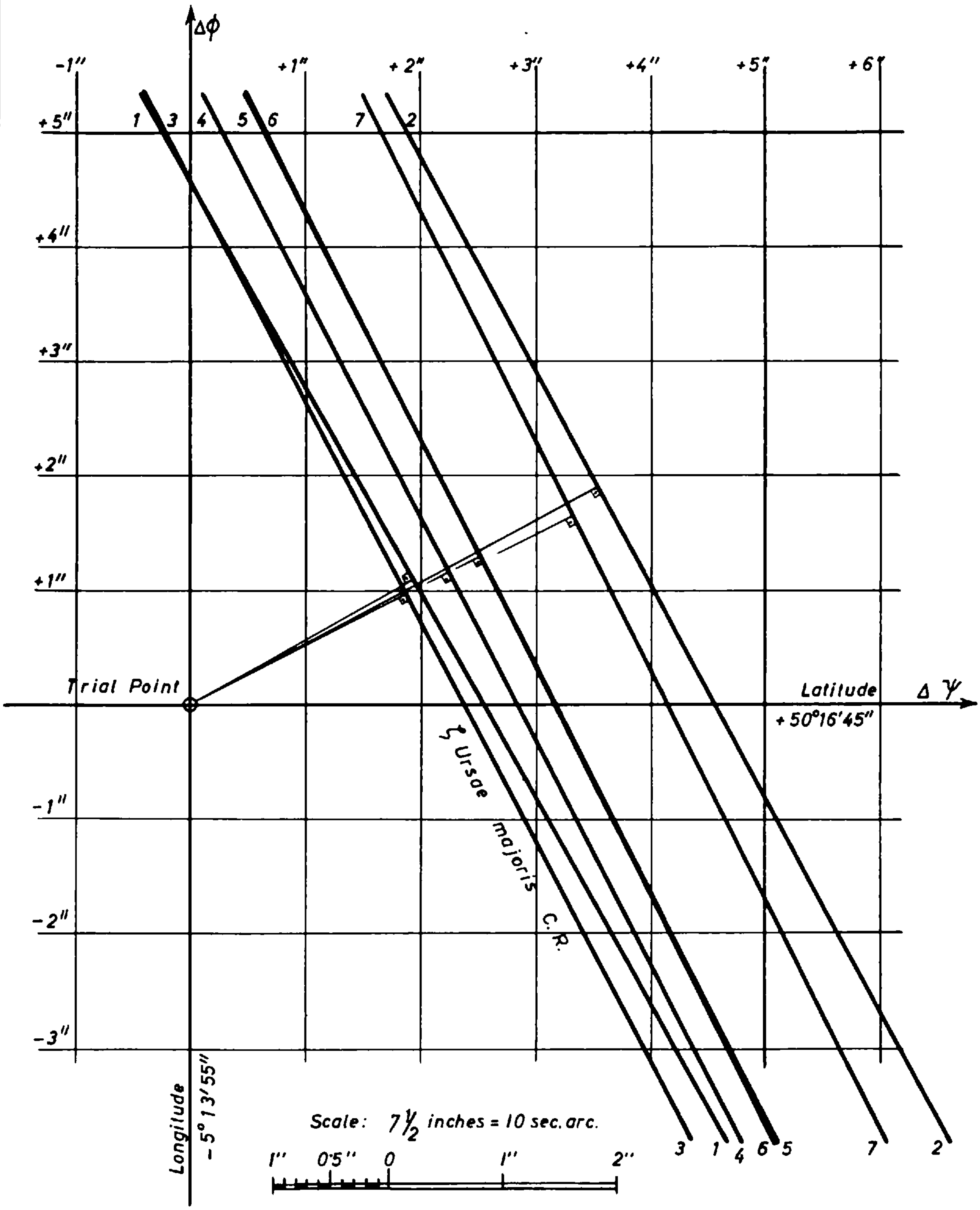
AT "Cottage" DATE April 1963

TRIAL POINT ϕ_{TP} = + 50° 16' 45" N λ_{TP} = − 5° 13' 55" W

STAR	CL CR	U.T. 0 Mean of five Marker Pulses h	m	s	h_o corrected observed Altitude °	'	"	$h_o - h_c$ obs. minus calc. Alt. "	calculated Azimuth from Trial Point °	'	Remarks
ζ Ursae maj.	C.R.	20	59	28·728	59	37	33·2	+02·2	60	59	
	"	21	17	31·069	62	09	57·5	+04·0	61	59	
	"		27	41·037	63	36	22·9	+02·1	62	28	
	"		37	26·853	64	59	45·5	+02·5	62	51	
	"		45	30·345	66	08	45·9	+02·8	63	07	
	"		53	25·040	67	16	39·3	+02·8	63	19	
	"	22	04	23·724	68	51	01·7	+03·7	63	28	
	C.L.		19	33·014	71	01	08·6	−09·6	63	26	
	"		27	07·932	72	06	18·2	−08·0	63	15	
α Bootis	C.R.	20	50	08·098	38	08	09·2	+01·7	106	34	
	"	21	03	17·946	40	08	25·0	+01·4	109	38	
	C.L.		27	12·559	43	40	22·7	−10·0	115	34	
	"		36	17·529	44	58	04·4	−08·6	117	57	
Polaris	C.L.	21	49	42·871	49	31	28·4	−04·0	359	13	
	"		55	18·839	49	30	43·4	−05·0	359	14	
	C.R.	22	15	01·736	49	28	34·2	+06·0	359	21	
	"		18	58·732	49	28	09·6	+07·0	359	22	
α Leonis	C.R.	22	26	07·320	43	13	41·2	+08·8	227	12	
	"		33	17·614	42	29	31·2	+08·2	229	00	
	"		39	44·165	41	42	07·9	+08·3	230	51	
	"		43	25·908	41	14	21·5	+07·0	231	53	
	"		49	09·254	40	30	37·7	+07·7	233	28	
	"		57	26·211	39	25	45·9	+09·2	235	43	
	CL	23	09	18·374	37	49	34·5	−03·7	238	50	
	"		15	00·447	37	02	21·2	−02·4	240	17	

TABLE 8.2.1.−14

POSITION LINES
from Zenith Distances



Instrument: Watts $\frac{1}{5}''$ Optical Micrometer Theodolite. (Prototype)

Date: April 1963.

Fig.: 8.2.1.-7

AT "Cottage" DATE April 1963

FINAL POSITION $\Phi_A = +50°16'45''.9$ N $\lambda_A = -5°14'01''.1$ W

STAR	CL / CR	ASTRONOMICAL AZIMUTH FROM "Cottage" TO Porthtowan R.O. ° ' "	MEAN ° ' "	MEAN ° ' " check:	REMARKS
ζ Ursae maj.	C.R.	305 32 28.7	305 32 29.8	305 32 25.3 *	
	"	29.1			
	"	27.9			* derived from
	"	29.6			observed altitude:
	"	30.7			$\sin A_A =$
	"	30.1			$= \frac{-\cos\delta . \sin t}{\cos h_o}$
	"	26.0			
	C.L.	30.5			h_o = corr.
	"	30.8			obs. altitude
α Bootis	C.R.	305 32 39.9	305 32 29.5	305 32 27.8 *	
	"	36.8			
	C.L.	23.0			
	"	18.2			
Polaris	C.L.	305 32 24.1	305 32 29.9	305 32 30.0 *	
	"	21.9			
	C.R.	37.5			
	"	36.1			
α Leonis	C.R.	305 32 40.2	305 32 30.3	305 32 29.5 *	
	"	39.3			
	"	41.2			
	"	40.4			
	"	41.1			
	"	39.7			
	C.L.	20.4			
	"	20.1			
		Mean:	305 32 29.9	305 32 28.2 *	

TABLE 8.2.1.–15

8.2.1. cont.

altitudes.

Equivalent field procedure was adopted for all observations in 1961 as well as in 1963. Altitude and azimuth observations were executed simultaneously; an error in timing, instrumental and personal errors would affect both measurements in a similar manner. In all the tables are also included the azimuths obtained from those observations which were rejected because the measured altitudes gave doubtful position lines. The azimuths from all observations show a remarkable constancy for each night. (Tables: 8.2.1.-9, -11, -13) Thus the agreement of results obtained in any one period of observation supports the presumption that the unpredictable refraction affects measurements mainly in the vertical plane. The differences between the observed azimuth derived from horizontal circle readings and the calculated azimuth are nearly equal for all stars, for each period of observation. The above indicates that the final position of the observer's station is selected and scaled from the graph with sufficient precision. Small differences between the calculated and observed azimuth can be attributed to quadrantal direction to stars and may be disregarded since they are not larger than allowable errors in pointing upon objects. The final scaled positions based on observations in 1961 and 1963 differ very little. The fluctuations in latitude and longitude are about one second of arc.

The discrepancy between the azimuth obtained in 1961 and in 1963 is attributed to the replacement of a wooden peg by a steel pipe, marking the observation station. The reconstruction of the survey station together with the rather short distance to the "Porthtowan" R.O., 810 feet, is detrimental to the comparison of the results.

The accuracy of the longitude determination hitherto depended on the problematic transfer of time. The employment of a crystal chronometer in conjunction with transmitted time signals and tape

8.2.1. cont.

recorder eleminated the time problem and reduced the longitude determination solely to an optical observation problem in the same sense as the latitude determination.

The employment of a striding level could not have achieved a result of higher accuracy. Speed in observation of moving objects is believed to be essential with field equipment. The time required for reversing the striding level and the time wasted in waiting until the bubble comes to rest has detrimental effects on observational results.; Furthermore a considerable part of the objective is covered by the supporting frame of the striding level vial when elevated sights are taken, and with some instruments sights over 60° are impossible with the striding level in position. The alternative, namely removing the striding level when steep sights are taken, thereafter lowering the telescope and replacing the stride, may introduce errors which would be too interwoven and complicated to be detected or compensated.

8.3. Position Lines from Horizontal Directions.

The co-ordinates of a survey station (ϕ , λ) can be obtained from readings of the horizontal circle pertaining to sights to heavenly bodies, taken at noted instants of time. In addition to sights to heavenly bodies, horizontal circle readings to a terrestrial target give the azimuth of the terrestrial line.

Horizontal circle readings used for the establishment of the meridian or azimuth have an awkward disadvantage as compared with readings of the vertical circle; the former cannot be oriented to a reference point or zero direction, which is readily available, or which can be established, physically, at any time.

The direction of the astronomic nadir and zenith to which zenith distances are referred, is governed by gravity.

The direction of the meridian, or of the azimuth is a function of the locality (ϕ , λ) and is obtained from observations to stars (α , δ).

$$A = f \; (\phi , \lambda , \alpha , \delta)$$

A = azimuth

ϕ , λ = latitude and longitude of the observer's station,

α , δ = right ascension and declination of the heavenly body.

Since the advent of radio direction finding, used mainly at sea, the position fix from measured azimuths has undergone a considerable amount of treatment.

The position fix from timed horizontal directions to stars in field astronomy is analogous to the geodetic problem of resection. Its solution with the aid of Collins' point is well known. Before Collins, Snellius and later Pothenot treated the same problem of resecting a survey station from angles measured to points of known position.

A description of radiogoniometry and azimuth position lines is given by M. F. da Costa. In his work: Radionavigation, Curves and Azimuth lines, published in 1927, he uses azimuth position lines in place of altitude position lines. His azimuth position lines are linear equations derived by differentiating the cotangent-equation of the azimuth, and contain horizontal circle readings to stars, R.O., and an approximate value for the azimuth. The unknowns in his equations are the corrections to the assumed latitude and longitude of the place, and a correction for the approximate azimuth of the R.O.

Later in 1935 F. Marguet describes the position fix by azimuth, mainly based on previous work done by M. Lecoq.

Basically, a position line is plotted at a distance from an assumed position, and perpendicularly to a straight line drawn through the assumed position. The straight line through the assumed position represents a great circle, and is drawn at its true azimuth. The distance along the straight line is plotted at its true scale.

The above constitutes the exact conditions for an azimuthal equidistant projection.

Any solution of the problem of locating a point on a sphere, where true distances and true azimuths from a given or chosen point (trial point) are used, leads to the azimuthal equidistant projection.

As far back as 1581, when Postel used this projection, to its discovery by Lambert in 1772, to its re-discovery by Cagnoli in 1799, the application of true azimuths and true distances from a point has been an attractive method for position fix, and has proved to be easily accessible for the reduction of field data.

The chief difficulty in a position fix from horizontal circle readings in field astronomy is to maintain the vertical axis of the theodolite consistently in the direction of the plumb line throughout the operation. A further disadvantage is the influence of the collimation axis error.

Automatic levelling of the instrument, i.e. the problem of keeping the vertical axis truly vertical during observations, is a present day field of instrumental research with various optical firms. V.E.B. Carl Zeiss, Jena, will produce a Geodetic-Astronomic Universal

8.3. cont.

Theodolite reading to 0.2". A compensator placed into the path of rays of its broken telescope will eliminate automatically the influence of dislevelment of the trunnion axis, regardless of whether this is causedby irregularities of the pivots or dislevelment of the vertical axis. With modern instrumental perfections, the position fix from horizontal directions will, no doubt, come into prominent use in the future.

For the field work under discussion, corrections for the dislevelment of the vertical axis were calculated from readings taken on the plate bubble. (See: References: "Precise Azimuths from Steep Sights", by J. S. Sheppard).

At the field station, horizontal circle readings are actually observed directions, rather than azimuths; because the zero orientation of the horizontal circle and the co-ordinates of the field station are at first unknown.

The equations to be adopted for solving the celestial triangle link together the approximate values for the unknowns: latitude and longitude of the trial point, and azimuth of the reference line; the observed quantities: horizontal circle reading and time of the optical observation; and tabulated quantities: the co-ordinates of the heavenly body.

Latitude and longitude of the trial point are so chosen that second order terms can be neglected. This is achieved by successive approximation. The tabulated quantities are accepted as being free of errors.

The unknown azimuth of the reference line ($A_{unknown}$) is equal to the sum of the assumed azimuth ($A_{assumed}$) plus a corrective term (dA) to make up for the deficiency of the assumption, for random errors, errors in sighting, and errors in reading the horizontal circle.

The assumed azimuth can be an approximation or can be obtained from observed and from calculated directions. Admittedly, there is a wide range of interpretation of the "assumed azimuth" and for its "corrective term". If necessary a suitable value for the assumed azimuth is derived by means of an adequate trial point, successively deduced by approximations.

<u>or</u>: <u>The unknown azimuth</u> of the reference line ($A_{unknown}$) is also equal to the sum of the azimuth calculated from the tri al point (A_{TP}), derived from measured time of astronomical observations, plus horizontal circle readings (c. r.), if necessary oorrected for curvature of the star's path, collimation and transit axis error etc., plus a corrective term to take up for the small difference of the co-ordinates between the trial point and the final position.

The corrective term is a differential of the computed azimuth with respect to the assumed latitude (ϕ_{TP}) and assumed longitude (λ_{TP}) of the trial point.

The above is rewritten in the following form:

(a) $A_{unknown} = A_{assumed} + dA_{corrective\ term}$

<u>or</u>:

(b) $A_{unknown} = A_{TP,\ computed,\ \&\ h.c.r.,\ etc.} +$ Change in computed A_{TP}

The left sides of (a) and (b) are equal, comparing (a) and (b),

$A_{assumed} + dA_{corr.} = A_{TP,\ comp.,\ \&\ h.c.r.,\ etc.} +$ Change in comp. A_{TP}

rearranged:

Change in comp. $A_{TP} - dA_{corr.} + (A_{TP,\ comp.\ \&\ h.c.r.,\ etc.} -$

$- A_{assumed}) = 0.$

or:

$$\left(\frac{\partial A}{\partial \lambda} d\lambda + \frac{\partial A}{\partial \phi} d\phi\right) - dA + (A_{TP,\ comp.,\ \&\ h.c.r.,\ etc.,} - A_{assumed})$$

$= 0.$ . . . . (1)

8.3. cont.

or:

Azimuth measured and computed + correction = Azimuth assumed.

The azimuth from the trial point is calculated from the relation:

$$\cot A_{TP} = \frac{\sin\phi_{TP} \cdot \cos(GST - \alpha + \lambda_{TP}) - \cos\phi_{TP} \cdot \tan\delta}{\sin(GST - \alpha + \lambda_{TP})} \quad \ldots (2)$$

$GST - \alpha + \lambda_{TP} = t$ = hour angle of the heavenly body with respect to the meridian of the trial point.

and: $d\lambda = dt$

The change of the azimuth with respect to λ is obtained by a differentiation of (2):

$$\frac{\partial A_{TP}}{\partial \lambda} = \sin\phi_{TP} - \cos\phi_{TP} \cdot \cos A_{TP} \cdot \tan h \quad \ldots\ldots\ldots (3)$$

The change of the azimuth with respect to ϕ :

$$\frac{\partial A_{TP}}{\partial \phi} = \sin A_{TP} \cdot \tan h \quad \ldots\ldots\ldots (4)$$

The altitude h can be observed, which is easily accomplished with the grid reticule, or can be calculated from the relation:

$$\sin h = \sin\phi_{TP} \cdot \sin\delta + \cos\phi_{TP} \cdot \cos\delta \cdot \cos(GST - \alpha + \lambda_{TP}) \quad \ldots\ldots\ldots (5)$$

and checked conveniently from the relation:

$$\cos h = \frac{-\cos\delta \cdot \sin(GST - \alpha + \lambda_{TP})}{\sin A_{TP}} \quad \ldots\ldots\ldots (5a)$$

The equations (2), (3), (4), (5) and (5a) can be found in slightly different notation and form in "Spherical and Practiaal Astronomy" by Chauvenet, and in various publications dealing with azimuth observations.

8.3. cont.

Substituting (3) and (4) in (1):

$$(\sin\phi_{TP} - \cos\phi_{TP} . \cos A_{TP} . \tan h) . d\lambda + \sin A_{TP} . \tan h . d\phi - dA + + (A_{TP}, \text{comp.} \& \text{ h. c. r. etc.}, -A_{assumed}) = 0 \ldots\ldots\ldots (6)$$

divided by tan h and rearranged:

$$+ d\phi . \sin A_{TP} + (-d\lambda . \cos\phi_{TP}) . \cos A_{TP} - \frac{-1}{\tan h}(A_{TP}, \text{comp.} \& \text{ h. c. r., etc.}, -A_{assumed}) = \frac{1}{\tan h}(dA - \sin\phi_{TP} . d\lambda) \ldots\ldots (7)$$

(6) and (7) are equations of the 1st degree in three variables, viz.:

$$\underline{d\phi, \ d\lambda, \ dA.}$$

with: $d\phi \equiv x$

$$-d\lambda . \cos\phi_{TP} \equiv y$$

$$\frac{-1}{\tan h}(A_{\text{comp.} \& \text{ h. c. r., etc.}}, -A_{assumed}) \equiv d$$

$$\frac{1}{\tan h}(d_A - \sin\phi_{TP} . d\lambda) \equiv n$$

$$A_{TP} \equiv \alpha$$

equation (7) corresponds to the form:

$$x . \sin\alpha + y . \cos\alpha - d = n \ldots\ldots\ldots\ldots (8)$$

Considering co-ordinate geometry in two dimensions: the left side of the equation (8) is the normal form of the equation of a straight line in a Cartesian system of co-ordinates, with ($x = 0$, $y = 0$) as origin. The perpendicular d, drawn from the origin to the straight line, is oriented in the direction $\alpha - 90^\circ$ clockwise from the x-axis; n is the distance to a point (x, y) from the line: $x . \sin\alpha + y . \cos\alpha - d = 0$.

8.3. cont.

It follows that the left side of the equation (7) can be represented as a straight line in a system of plane rectangular co-ordinates: $d\phi$ and $-d\lambda \cdot \cos\phi$.

Each such straight line represents a "position line" from timed horizontal direction. The term position line from horizontal direction is therefore preferred to the terms azimuth position line or horizontal angle position line.

The perpendicular distance from the origin to the position line =

$$\frac{-1}{\tan h} (A_{TP,\ comp.\ \&\ h.c.r.,\ etc.} - A_{assumed})$$

has to be drawn in the direction $A_{TP} - 90^\circ$ clockwise from the $d\phi$-axis.

A_{TP} is derived from equation (2).

The origin of the system has the co-ordinates of the trial point $(\phi_{TP},\ \lambda_{TP})$.

The right side of the equation (7) represents the distance to the observer's station from the position line.

Each observation, consisting of timed horizontal direction to a heavenly body and horizontal direction to a reference line, yields a relation of the form (1) and consequently one equation of the form (7).

From the above, it follows that the equation (7) defines the locus of an observer's position as a point at a distance from a straight line. (Obviously in a system of receangular co-ordinates.)

In the two-dimensional case, the locus so defined is a plane.

Unfortunately, in plane geometry a second term is missing to denote the basic element "plane".

It is evident that all points $(d\phi,\ d\lambda \cdot \cos\phi)$ of the plane satisfy the equation (7), as long as dA is unknown.

dA, caused by the unknown circle orientation, establishes a variable point $(d\phi,\ d\lambda \cdot \cos\phi)$, which occupies all points of the plane.

The left-hand members of two equations of the form (7), resulting

8.3. cont.

from timed horizontal directions to two heavenly bodies, establish two position lines; - in this case the locus of the observer's station consists of all those points to which the distances from the two position lines are proportional to the tangents of the zenith distances of the two observed heavenly bodies.

A locus of points, whose co-ordinates satisfy the condition of a fixed ratio of distances from two straight lines, is known to be an equation in two variables. In the above case, such an equation denotes a straight line passing through the intersection of the two position lines.

It follows that three position lines from timed horizontal directions to three heavenly bodies are required to obtain the final position of the observer's station, i.e. the point so chosen that its distances from the position lines correspond to the ratio of the tangents of the zenith distances of the observed stars.

The zero orientation of the horizontal circle is derived after the final position of the station is obtained.

Obviously for the method to be feasible three different heavenly bodies at different azimuths, or the same star at different azimuths, have to be observed. A solution becomes more complex when more than three stars are observed, because, strictly, each star observation produces a different set of corrective terms, $d\phi$, $d\lambda$, dA; hence, mean or weighted mean corrective terms have to be derived for the most probable answer.

Advantage can be gained from observations to stars, when in the same almucantar, and executed in one face position; but this may require a star programme.

The unknown distances to the final position from the position lines are the same for all stars in the same almucantar, and the influence of instrumental errors on the horizontal directions may be neglected.

The suggested procedure ofor treating the problem when timed

8.3. cont.

horizontal directions to at least three stars are available is as follows:

Equations (2), (5) and (5a) are solved analytically and are identical to the equations (2), (3) and (3a) Section 8.2., respectively. The latter are used in Section 8.2. for position lines from zenith distances.

Equation (1), in its re-arranged form (7), is solved semi-graphically.

Influence of errors of time and direction measurement on positional co-ordinates.

The accuracy of final position and azimuth depends on the precision of timing optical observations, and of measurement of horizontal directions.

Timing accuracies are dealt with in previous sections. Sources of errors in measurement of horizontal directions are mainly dislevelment of the vertical axis, instrumental errors, phase, and horizontal refraction.

The equation:

$$\cot A = \frac{\sin\phi \,.\, \cos t - \cos\phi \,.\, \tan\delta}{\sin t}$$

shows the relation between the measured quantities, the star's co-ordinates and the unknown observer's position.

A, ϕ, t are regarded as variable; the latitude of the star can be obtained from equation (5) or (5a).

Differentiating the above equation with respect to the measured quantities t, A, shows the effect of errors on the result ϕ :

$$d\phi = -\frac{\cot h}{\sin A}\, dA + \frac{\cos q.\ \cos\delta}{\sin h.\ \sin A}\, dt$$

The above differential relation shows that meridian stars will not give the latitude of the place by the horizontal direction method. The time error has little influence upon stars at or near elongation.

8.3. cont.

Observations to stars at zenith distances of less than 45° is advocated for latitude determination by the direction method.

The accuracy of the result λ, expressed as dt, depends on:

$$dt = \frac{\sin h \,.\, \sin A}{\cos q.\ \cos \delta} \, d\phi + \frac{\cos h}{\cos q.\ \cos \delta} \, dA$$

i.e. the precision of the previously determined latitude and measured azimuth.

The differential equation shows that the longitude of the station is preferably obtained from observations to stars at or near the meridian, where: $\cos q \cong 1$ and $\sin A \cong 0$; further, equatorial stars have the least influence on errors of previously obtained latitude and azimuth.

And finally, the differential relation for the azimuth:

$$dA = \frac{\cos q.\ \cos \delta}{\cos h} \,.\, dt - \sin A.\ \tan h.\ d\phi$$

shows that for the azimuth derived from stars nearer the meridian, an error of the previously determined latitude is negligible. Further, an error of the derived azimuth due to inaccurate time or timing is minimized for observations to stars at or near the prime vertical, or at or near elongation. These requirements are satisfied by stars close to the pole, i.e. north stars.

Stars at low altitude reduce the errors affecting the precision of the azimuth determination.

If latitude and longitude are known or determined from the horizontal direction method, higher precision of azimuth determination, if required, is achieved with additional observations to stars close to the horizon.

Considering the first term in the last differential equation, it would appear that a dependence of the azimuth dA, from the locality, ϕ, does not exist. But actually it does exist, because the altitude, h, and the parallactic angle, q., for a given star, declination δ, depend on the latitude.

Some textbooks advocate azimuth determination from one star only. Azimuth determination especially from Polaris is proposed, and examples are given where results are quoted to seconds of arc. Such results are doubtful because they are derived from measurements which are influenced by sighting conditions prevailing in one direction only.

Methods of obtaining astronomical positions (ϕ , λ) from measurements of horizontal directions should be superior to methods employing measurements of vertical angles, because of the uncertainty of astronomical refraction in the vertical plane.

Generally, the horizontal direction method derives latitude, longitude and azimuth as a function of the same timed horizontal circle readings; consequently the accuracy of the individual results, viz. latitude, longitude and azimuth, rests with the heavenly body selected for observation; i. e. on the position of the star and hence on its influence in shaping the celestial triangle.

8.3.1. Results obtained from Field Work.

Astronomical Fixation from Horizontal Directions, and Astronomical Azimuth of Terrestrial Line.

For greater simplicity the indirect method of observing horizontal directions has been applied throughout the field experiments. Thus, special equipment, e.g. micrometer eyepiece, additional terrestrial targets, and the working out of a star programme have been dispensed with.

The direct method of observing horizontal directions requires that both targets, the terrestrial R.O. and the heavenly body should be visible in the field of view of the telescope simultaneously; the arc distance between them is obtained with a special measuring device.

The indirect method as adopted here makes use of horizontal circle readings from each sight, taken separately.

The data of the same observations from which zenith position lines were deduced previously, are now evaluated by the method of position lines from horizontal directions.

Obviously this method requires observations linked to reliable R.O. readings; or an undisturbed zero of the horizontal circle.

Observations on May 5, 1961 to:

β Leonis,
Polaris,
α Bootis,
α Ursae majoris,

and on May 9, 1961 to:

α Bootis,
α Ursae majoris
α Lyrae,
β Leonis,
β Ursae majoris,
α Lyrae

8.3.1 cont.

which are referred to the same R.O., are suited for computation and plot by the method of position lines from horizontal directions.

Field record and reduction of observations as shown in Tables 8.2.1 -1 to 8.2.1 -5 and in Fig. 8.2.1. -1 are equally applicable for the method of position lines based on horizontal directions.

A fresh calculation of the hour angle, or of the azimuth from the trial point, is not necessary. Only the computation of the perpendicular distance "d" to the position line from the trial point is required in addition for plotting purposes.

The position line plot is used to derive the final corrections to the approximate co-ordinates chosen as trial point. The corrective term required for the astronomical azimuth of the reference line (observer's station to R.O.) is obtained from scaled distances to the position lines from the final position.

To simplify the plot by reducing the number of position lines, the arithmetic mean of the times of all observations in both face positions to each star is used. But the value so obtained does not correspond to the arithmetic mean of the horizontal circle readings. This would be correct only if the azimuth of a star were to change linearly with the transit time.

The rate of change of the azimuth with respect to time depends on the curvature of the path of the star. The effect of the curvature is compensated with a correction either to the mean of the observed transit times or to the mean of the readings of the horizontal circle.

The following formula for curvature correction given in Clark Vol. II, 1951, p. 108, has been used for correcting the mean horizontal circle reading:

$$\Delta A'' = - \frac{0.137}{1000} \cdot \frac{\sin A. \cos \phi}{\cos^2 h} (\cos h. \sin \delta - 2 \cos A. \cos \phi). [\Delta t^2]$$

8.3.1. cont.

The constant has been modified to take care of the average of four time measurements in solar seconds.

The azimuth and the altitude required in the above formula are already obtained from computation, and refer to the trial point whose latitude is also substituted for the latitude of the station.

The calculated azimuth from the trial point based on the mean U.T.O. of two observations in both circle positions, minus the observed azimuth of the star, based on approximate azimuth of reference line and horizontal circle readings, corrected for curvature, instrumental errors, etc., plus the approximate azimuth, give the "computed and observed" azimuth of the reference line. Its residual from the arithmetic mean, of from any assumed approximation, multiplied by the tangent of the zenith distance, is the directed distance from the trial point to the position line. A specimen of the above computation is given in Table 8.3.1.-1.

In routine work, the difference of computed and observed azimuth, for each sight, is obtained from the computations shown in Tables 8.2.-1 -3 and 8.2.1.-4 or -5.

Any value can be taken for the approximate azimuth of the reference line, because it cancels out in the computation.

The plot is made in a system of Cartesian co-ordinates, $d\phi$, and $d\lambda \cdot \cos\phi$ respectively, with the co-ordinates of the trial point (ϕ_{TP}, λ_{TP}) as origin.

The directed distance (+ or - d) is plotted from the trial point clockwise from the $d\phi$ - axis in the direction $(A_{TP} - 90^{\circ})$; the rule of sign, obviously, is the same as in analytic geometry.

Position lines from horizontal directions and plotting data are presented in Fig. 8.3.1.-1, and in Table 8.3.1.-2. The arrow of the position line indicates the direction to the star.

Equation (7), Section 8.3., clearly points out how to find the final position from the plot.

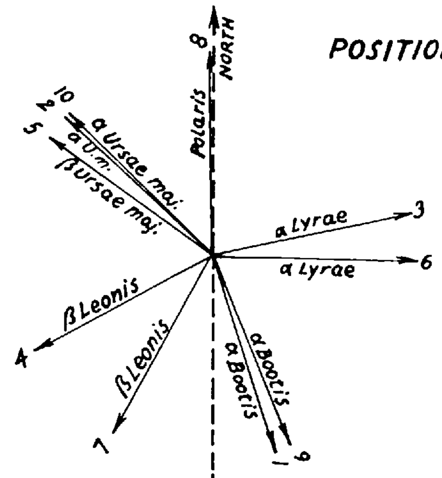
COMPUTATION
OF
DIRECTED DISTANCE
FROM
TRIAL POINT TO POSITION LINE

DATE: 9th May, 1961
AT: T.P. "Cottage"
STAR: α Lyrae

A_{TP} COMPUTED	90° 55' 14."0	based on: mean U.T.O., $\phi_{T.P.}$, $\lambda_{T.P.}$, α, δ,
OBSERVED AZIMUTH OF STAR (Mean of Four)	91° 22' 56."1	based on: approx. azimuth of Ref. Line, horizontal circle readings to Ref. Line and Star, corrections for instrumental errors,
CURVATURE CORRECTION (Time and Azimuth) ...	− 17."3	based on: $A_{T.P.}$, ϕ_{TP}, λ_{TP}, α, δ,
DIFFERENCE	− 27' 24."8	
APPROX. AZIMUTH of REF. LINE	306° 00' 00"	based on: e.g.: judgement, mag. orientation, etc.,
AZIMUTH OF REF. LINE (computed and observed)	305° 32' 35."2	derived from: observation to one Star, and computation from T.P.,
ARITHMETIC MEAN OF (COMPUTED AND OBSERVED) AZIMUTHS OF REFERENCE LINE = ASSUMED AZIMUTH	305° 32' 37."8$_5$	
DIFFERENCE	− 02."6$_5$	(RESIDUAL)
LENGTH OF PERPENDICULAR FROM TRIAL POINT TO POSITION LINE DIRECTED DISTANCE: d =	+ 01.84$_4$"	derived from: $-\tan(90° - h) \times -2.6_5$

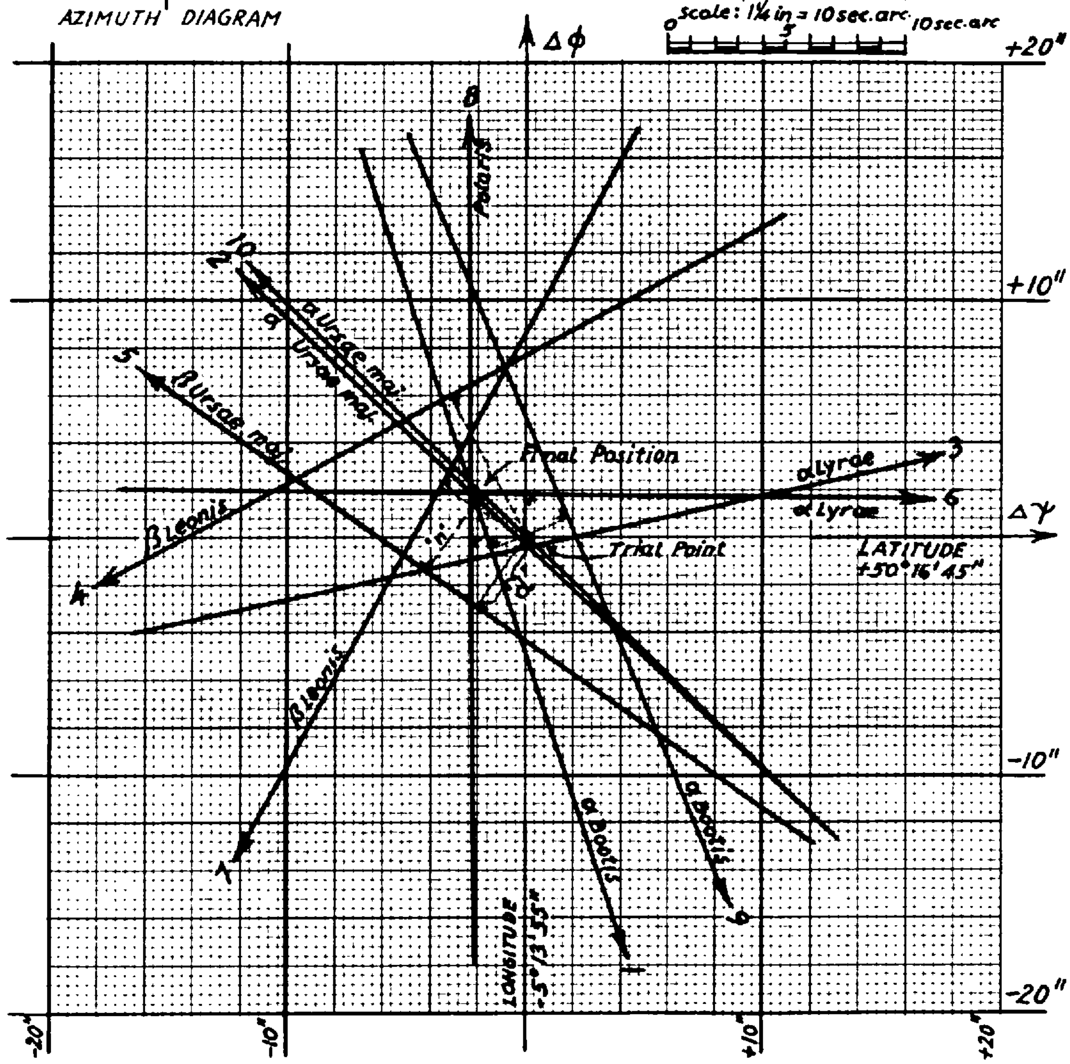
TABLE 8.3.1 – 1

POSITION LINES FROM HORIZONTAL DIRECTIONS



AZIMUTH DIAGRAM

STAR	A_{TP} CALCULATED AZIMUTH FROM TRIAL POINT ° '	CALCULATED DISTANCE "d" FROM TRIAL POINT TO POSITION LINE	SCALED DISTANCE "n" FROM FINAL POSITION TO POSITION LINE
1 α Bootis	161 41	$-1{\cdot}4_7''$	$+0{\cdot}1''$
2 α Ursae maj.	313 17	$+0{\cdot}1_7$	$0{\cdot}0$
3 α Lyrae	77 46	$-0{\cdot}4_8$	$+2{\cdot}5$
4 β Leonis	241 51	$-6{\cdot}7_5$	$+4{\cdot}3$
5 β Ursae maj.	304 59	$+3{\cdot}6_7$	$-3{\cdot}9$
6 α Lyrae	90 55	$+1{\cdot}8_4$	$-0{\cdot}2$
7 β Leonis	208 56	$-4{\cdot}1_6$	$+1{\cdot}6$
8 Polaris	359 44	$+2{\cdot}1_6$	$-0{\cdot}2$
9 α Bootis	157 21	$+1{\cdot}9_5$	$-3{\cdot}2$
10 α Ursae maj.	313 40	$+0{\cdot}0_9$	$+0{\cdot}1$



TRIAL POINT:

$\phi = +50°16'45''$

$\lambda = -5°13'55''$

$A = 305°\,32'\,37''.8_5$

Scaled:

$\Delta\phi = +1{\cdot}6''$

$\Delta\lambda = \frac{\Delta\psi}{\cos\phi} = -3{\cdot}1''$

$(\Delta\lambda\cdot\cos\phi = -2{\cdot}0'')$

Calculated:

$\Delta A = n\cdot\tanh + \sin\phi\cdot\Delta\lambda$

Mean:

$\Delta A = -2''.6_5$

FINAL POSITION: 'COTTAGE'

$\phi = +50°16'46{\cdot}6''$

$\lambda = -5°13'58{\cdot}1''$

$A = 305°32'35{\cdot}2''$

DATE: 5TH & 9TH MAY 1961

Fig.: 8.3.1—1

AT : "COTTAGE" DATE : MAY 1961

TRIAL POINT ϕ_{TP} +50° 16' 45" λ_{TP} – 5° 13' 55" AZIMUTH ASSUMED 305° 32' 37·"8$_5$

DATE	Nº	STAR	APPROXIMATE AZIMUTH OF REFERENCE LINE ° ' "	U.T. 0 MEAN OF C.L.&C.R. h m s	A_{TP} CALCULATED AZIMUTH FROM TRIAL POINT MEAN OF C.L.&C.R. ° ' "	OBSERVED AZIMUTH MEAN OF C.L.&C.R. ° ' "	CURVATURE CORRECTION "	DIFF. FROM ASSUMED AZIMUTH (RESIDUAL) "	CALCULATED OR OBSERVED ALTITUDE MEAN OF C.L.&C.R.
				SEE : TABLE 8.2.1–2 FIG. : 8.2.1.–1 TABLE 8.2.1–3 COL. 3	SEE : TABLE 8.2.1 – 4 OR " –5	SEE : TABLE 8.2.1 – 3 COL. 1		SEE : TABLE 8.3.1 – 1	SEE : TABLE 8.2.1 – 3 COL. 2
MAY 9 1961	1	α BOOTIS	306 00 00·0	22 44 13·362	161 40 38·3	162 08 13·3	– 15·2	$+2{\cdot}3_5$	FOR OBSERVATIONS IN 1961 SEE: TABLES 8.2.1–10 & –12 COL. 6
	2	α URSAE MAJ.		22 58 09·776	313 16 59·1	313 44 55·2	– 33·6	$-0{\cdot}3_5$	
	3	α LYRAE		23 23 11·039	77 46 12·7	78 13 52·0	– 17·6	$+0{\cdot}4_5$	
	4	β LEONIS		23 57 02·892	241 51 21·1	242 18 16·1	+ 21·6	$+5{\cdot}5_5$	
	5	β URSAE MAJ.		00 24 25·232	304 58 45·7	305 26 19·7	– 07·0	$-4{\cdot}8_5$	
	6	α LYRAE		00 38 14·817	90 55 14·0	91 22 56·1	– 17·3	$-2{\cdot}6_5$	
MAY 5 1961	7	β LEONIS	308 01 38·5	22 26 15·546	208 55 32·4	211 24 04·6	+ 23·2	$+5{\cdot}2_8$	
"	8	POLARIS		22 38 25·958	359 43 49·8	02 12 53·0	$-0{\cdot}0_5$	$-2{\cdot}5_2$	
"	9	α BOOTIS		22 49 53·500	157 20 31·3	159 49 46·1	– 11·1	$-3{\cdot}0_6$	
"	10	α URSAE MAJ.		23 05 32·607	313 39 40·3	316 09 30·3	– 49·1	$-0{\cdot}2_0$	

TABLE 8.3.1 – 2

8.3.1. cont.

The ratio of the distances to the final position from the position lines is equal to the ratio of the tangents of the zenith distances.

A rigid constructional procedure to obtain the final position is justified if the plot consists of three position lines only. Random errors may amount to about the same magnitude as the final corrections to the approximations, and may tend to prevent a clear issue by geometrical methods whenever more than three stars are observed.

From Fig. 8.3.1.-1 it will be noticed that the final position is obtained by geometrical construction and judgement.

The corrections to latitude and longitude are scaled from the plot. These corrections, obtained graphically, are mean values; because to each position line there belongs a definite set of corrections, $d\phi$, $d\lambda$, dA.

The distances "n" from the final position to the position lines are scaled on the graph; equal distances can be expected only for stars in the same almucantar.

The correction dA, for the azimuth, cannot be obtained from an average of the scaled distances "n" if stars are observed at different altitudes.

A simple calculation on the slide rule solves the right hand term of equation (7) Section 8.3., for dA:

$$dA = \pm n . \tan h + \sin \phi \; . \; d\lambda \; , \qquad \text{for each position line.}$$

The sign of n depends on the relative position of the final position and the position line.

n is $\pm$ if the final position is $\begin{matrix}\text{left}\\ \text{right}\end{matrix}$ from the position line, as seen in the direction to the star.

A check on the scaled distances n can be obtained by substituting the scaled values for $d\phi$ and $d\lambda$ in the left hand terms of equation (7) Section 8.3.

8.3.1. cont.

Fig. 8.3.1.-2 shows the plot from the identical original observations but with the position lines based on a different assumed azimuth. ($A_{assumed}$).

The assumed azimuth appearing in the last term of the left-hand side of equation (7) Section 8.3., is left to the discretion of the computer. A most convenient value is the arithmetic mean of all computed and observed azimuths of the reference line, derived from each star observed. But any other value of the assumed azimuth substituted for its arithmetic mean will yield a plot from which an equally reliable result for the final position and scaled values can be obtained without difficulties, as long as the selection of the stars is fairly well distributed in azimuth.

In Fig. 8.3.1.-2 are shown the construction lines for deriving the final position. The diagram in the top left hand corner of Fig. 8.3.1.-1 illustrates the distribution in azimuth of the stars observed.

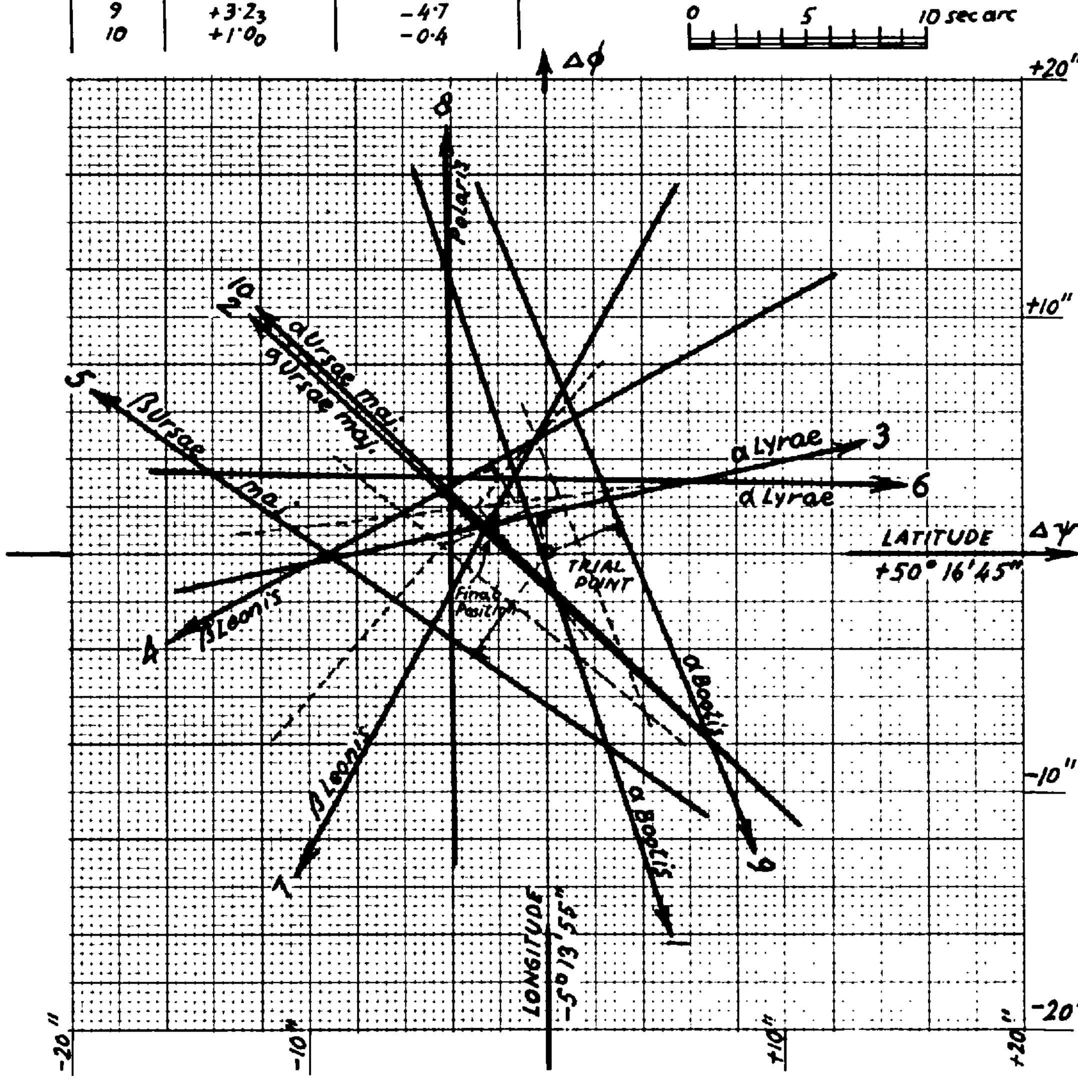
A different assumption for the azimuth will reduce or enlarge all directed distances from the trial point to the position lines, by an amount proportional to the tangents of the zenith distances. This means that each position line is shifted parallel and in the same direction with respect to the star.

The values of "n", scaled from the position lines to the final position are now different from those scaled on the previous plot. Obviously, because the corrective term dA, added to the new assumption, is expected to produce again the most probable answer. The negligible difference between the co-ordinates of the final position obtained from Fig. 8.3.1.-1 and Fig. 8.3.1.-2 respectively, is attributed to the limited accuracy of plotting and scaling, and to the judgement of the engineer. As already stated, the scale has been chosen to permit plotting of $\frac{1}{4}$ second of arc. Taking the unfavourable field conditions into account, the co-ordinates of the final position and the final azimuth of the reference line, as obtained by this method, are in satisfactory

POSITION LINES FROM HORIZONTAL DIRECTIONS

STAR №	CALCULATED DISTANCE "d" FROM TRIAL POINT TO POSITION LINE	SCALED DISTANCE "n" FROM FINAL POSITION TO POSITION LINE
1	$-0{\cdot}2_2$"	$-1{\cdot}4$"
2	$+1{\cdot}1_0$	$-0{\cdot}7$
3	$+1{\cdot}5_7$	$+0{\cdot}2$
4	$-4{\cdot}3_5$	$+2{\cdot}1$
5	$+5{\cdot}2_0$	$-5{\cdot}1$
6	$+3{\cdot}2_4$	$-1{\cdot}9$
7	$-2{\cdot}6_1$	$+0{\cdot}1$
8	$+3{\cdot}9_0$	$-1{\cdot}9$
9	$+3{\cdot}2_3$	$-4{\cdot}7$
10	$+1{\cdot}0_0$	$-0{\cdot}4$

Scale : 1¼ in. = 10 sec. arc.



TRIAL POINT:

$\phi = +50°16'45"$

$\lambda = -5°13'55"$

$A = 305°32'39\overset{"}{.}85$

Scaled:

$\Delta\phi = +1{\cdot}5"$

$\Delta\lambda.\cos\phi = -2{\cdot}2"$

$\Delta\lambda = -3{\cdot}4"$

calculated:

$\Delta A = n.\tan h + \sin\phi.\Delta\lambda$

Mean:

$\Delta A = -4\overset{"}{.}7_5$

FINAL POSITION: "COTTAGE"

$\phi = +50°16'46\overset{"}{.}5$

$\lambda = -5°13'58\overset{"}{.}4$

$A = 305°32'35\overset{"}{.}1$

DATE: 5TH & 9TH MAY, 1961

Fig.: 8.3.1 –2

8.3.1. cont.

agreement with results obtained from position lines based on zenith distances. The plot, presented in Figs. 8.3.1.-1 or -2, clearly indicates that none of the position lines are to be rejected. It follows that observations of timed horizontal directions give reliable results.

Observations taken in 1963 with the prototype geodetic theodolite (Watts $^1/5''$ microptic) were also reduced by the method of position lines from horizontal directions. The field work on April 25, 1963, consisted of observations to three stars. Results are given in Fig. 8.3.1.-3. Data for plotting the position lines and the azimuth correction obtained from scaled distances are submitted in Table 8.3.1.-3.

Differences between directed distances belonging to position lines from C. L. and C. R. observations respectively, are caused by collimation axis error, imperfect adjustment of the plate bubble, and random errors.

The mean position lines are again deduced by geometrical construction.

The evaluation of the final position is also obtained by construction which is shown on the diagram. The final position can be roughly estimated by observing the rules of graphical methods of resection, e.g. rules from plane tabling. When the three stars observed are evenly spaced over 360° in azimuth, the situation corresponds in plane tabling to: "the station inside the triangle formed by the points observed".

<u>The 1st Rule</u>: <u>In plane tabling</u>:

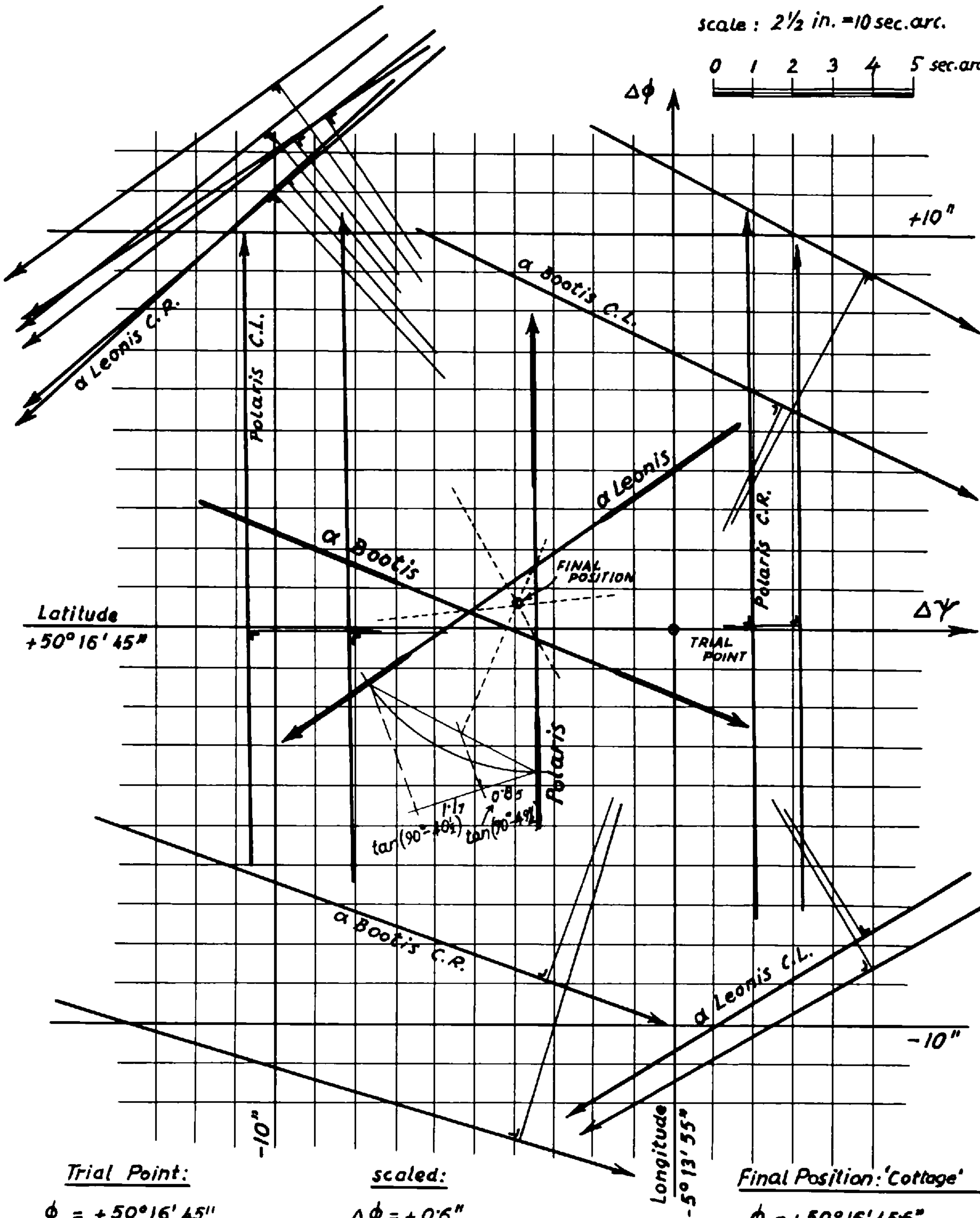
The final point is on the same side of all rays.

<u>In field astronomy</u>:

The final position is on the same side of all mean position lines.

The second rule is also identical, if stars are thought of as targets at finite distances, but it needs a modification to cope with infinite distances in astronomy:

POSITION LINES
from Horizontal Directions



Trial Point:

$\phi = +50°16'45''$

$\lambda = -5°13'55''$

scaled:

$\Delta\phi = +0{\cdot}6''$

$\Delta\psi = -3{\cdot}9''$, $\Delta\lambda = \dfrac{\Delta\psi}{\cos\phi} = -6{\cdot}1''$

Final Position: 'Cottage'

$\phi = +50°16'45{\cdot}6''$

$\lambda = -5°14'01{\cdot}1''$

Instrument: Watts 1/5" Optical Micrometer Theodolite. (Prototype)

Date: April 25, 1963.

Fig.: 8.3.1.–3

AT: "Cottage" DATE: April 25, 1963

TRIAL POINT ϕ_{TP} +50° 16' 45" λ_{TP} −5° 13' 55" AZIMUTH ASSUMED 305° 32' 34."8

STAR	C.L. OR C.R.	U.T.O MEAN OF FIVE MARKER PULSES	CALCULATED AZIMUTH FROM TRIAL POINT A_{TP}	OBSERVED AZIMUTH OF STAR	AZIMUTH OF REFERENCE LINE APPROX. AZIMUTH OF REF. LINE = 305° 32' 30."0	DIFFERENCE FROM ASSUMED AZIMUTH (RESIDUAL)	CALCULATED OR OBSERVED ALTITUDE h	CALCULATED DISTANCE d FROM TRIAL POINT TO POSITION LINE	SCALED DISTANCE n FROM FINAL POSITION TO MEAN POSITION LINE	dA $\pm n.\tan h + \sin\phi d\lambda$ $d\lambda$ scaled Fig.: 8.3.1−3 $\sin\phi.d\lambda = -4.7$
		h m s	° ' "	° ' "	° ' "	sec. arc.	° ' "	sec. arc.	sec. arc.	sec. arc.
α BOOTIS	C.R.	SEE: TABLE: 8.2.1−2 FOR OBSERVATIONS IN 1963: TABLE 8.2.1−14 COL. 3	SEE: TABLE: 8.2.1−4 OR −5 FOR OBSERVATIONS IN 1963: TABLE 8.2.1−14 COL. 6	SEE: TABLE: 8.2.1−3 COL. 1	305 32 44.8	+10.62	SEE: TABLE: 8.2.1−3 COL. 2 FOR OBSERVATIONS IN 1963: TABLE: 8.2.1−14 COL. 4	-13.5_4	+0.6	-4.1_5
	"				42.1	+ 7.92		-9.4_1		
	C.L.				28.2	− 5.98		+6.26		
	"				23.9	−10.28		$+10.2_7$		
POLARIS	C.L.				25.1	− 9.08		$+8.1_9$	+0.4	−4.22
	"				22.4	−11.78		$+10.6_2$		
	C.R.				37.9	+ 3.72		-3.1_8		
	"				36.4	+ 2.22		-2.0_0		
α LEONIS	C.R.				48.2	+14.02		-14.9_2	+0.6	−4.18
	"				46.8	+13.62		-14.8_6		
	"				48.6	+14.42		-16.2_0		
	"				47.9	+13.72		-15.6_5		
	"				48.7	+14.52		-17.0_1		
	"				47.0	+12.82		-15.6_0		
	C.L.				27.1	− 7.08		$+9.1_3$		
	"				26.7	− 7.48		$+9.9_1$		

[ ]/3 = −4.2

MEAN AZIMUTH OF REF. LINE: "COTTAGE" TO PORTHTOWAN R.O.: 305° 32' 30."0

TABLE: 8.3.1−3

8.3.1. cont.

2nd Rule: In plane tabling

The perpendicular distance of the final point from each ray is directly proportional to the distance: final point to point sighted.

In field astronomy:

The perpendicular distance of the final position from each mean position line is directly proportional to the tangent of the zenith distances.

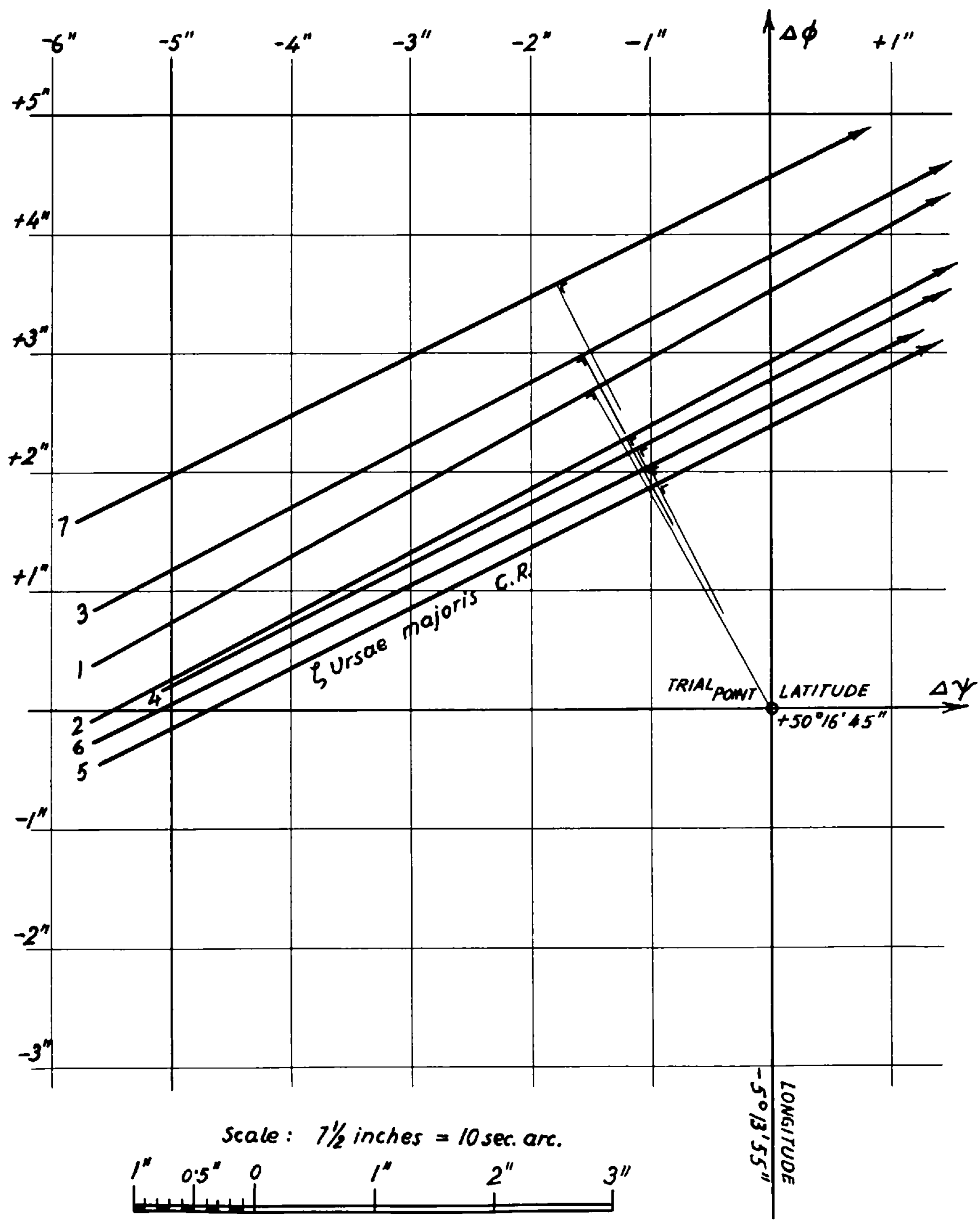
The spread of individual position lines obtained from repeated pointings in the same face position to the same star is illustrated in Fig. 8.3.1.-4, at a three times larger scale. Data are supplied in Table 8.3.1.-4. The observations were taken on April 12, 1963.

Figs. 8.3.1.-3 and -4 may be compared with Figs. 8.2.1.-6 and -7 respectively. From the diagrams it can be stated that practically equal pointing accuracy in altitude and azimuth is achieved with the equipment under discussion.

It is believed that it is perfectly admissible to use position lines, derived from observations at different days, on the same plot, for defining the final position. The observations may or may not have necessitated a new instrument set-up. Errors are introduced by alteration of the reference direction. The only direction which matters for position lines from zenith distances is the direction of the force of gravity; known to vary at regular and irregular intervals, its changes can be discovered from the result, but cannot be detected during observation or after a re-setup. Instrumental errors which can change from one setup to the next, are eliminated by change of face. This applies equally well for position lines from horizontal directions.

Timed horizontal directions are linked to a reference direction established by a terrestrial target. This terrestrial reference line creates a setup problem and a pointing problem; both are solved, to

POSITION LINES
from Horizontal Directions



Instrument: Watts 1/5" Optical Micrometer Theodolite (Prototype)

Date: April 12, 1963

Fig.: 8.3.1-4

AT: "Cottage" DATE: April 12, 1963

TRIAL POINT ϕ_{TP} $+50° 16' 45''$ λ_{TP} $-5° 13' 55''$ AZIMUTH ASSUMED $305° 32' 34.''1_8$

STAR	C.L. OR C.R	U.T.O MEAN OF FIVE MARKER PULSES	CALCULATED AZIMUTH FROM TRIAL POINT A_{TP}	OBSERVED AZIMUTH OF STAR	AZIMUTH OF REFERENCE LINE APPROX. AZIMUTH OF REF. LINE = $305° 32' 30.''0$	DIFFERENCE FROM ASSUMED AZIMUTH (RESIDUAL)	CALCULATED OR OBSERVED ALTITUDE h	CALCULATED DISTANCE d FROM TRIAL POINT TO POSITION LINE	REMARKS
		h m s	° ' "	° ' "	° ' "	"	° ' "	"	
ζ URSAE MAJ.	C.R.	SEE: TABLE: 8.2.1–2 FOR OBSERVATIONS IN 1963: TABLE 8.2.1–14 COL 3	SEE: TABLE: 8.2.1–4 OR –5 FOR OBSERVATIONS IN 1963: TABLE 8.2.1–14 COL. 6	SEE: TABLE: 8.2.1–3 COL. 1	305 32 28.9	-5.2_8	SEE: TABLE: 8.2.1–3 COL. 2 FOR OBSERVATIONS IN 1963: TABLE: 8.2.1–14 COL. 4	$+3.0_8$	
	"				29.3	-4.8_8		$+2.5_7$	
	"				27.4	-6.7_8		$+3.3_6$	
	"				28.9	-5.2_8		$+2.4_6$	
	"				29.4	-4.7_8		$+2.1_1$	
	"				28.7	-5.4_8		$+2.2_9$	
	"				24.0	-10.1_8		$+4.0_0$	
	C.L.				28.2	-5.9_8		$+2.0_5$	NOT SHOWN IN FIG. 8.3.1–4
	"				27.8	-6.3_8		$+2.0_6$	" "

TABLE: 8.3.1–4

a large extent, by the ability of the observer.

The method described in this section, consisting of computation and plot, is the easiest for making use of observations of timed horizontal directions for position fix and azimuth; and in conjunction with a position line plot from zenith distances it adds to the reliability of the result.

The simplicity of computation is based on the grid reticule, because it enables simultaneous measurements of vertical and horizontal angles of sights to moving targets.

The accurate subdivision of time and the possibility of referring optical observations to an adopted national time system is performed by the crystal chronometer; evidence of the efficiency of its performance is the consistency of results from field observations in 1961 and in 1963, exhibited in the graphs.

8.4. Least Squares Solution of Astronomical Fixation from Horizontal Directions.

The problem of position fix from timed horizontal directions to stars, as explained in Section 8.3. and presented in equation (6), contains three unknowns: $d\phi$, $d\lambda$, and dA.

A solution by algebraic methods is always possible if three stars are observed. If more than three stars are observed, as is usually endeavoured in field astronomy, a solution can be found by the method of least squares.

Each observation, or each mean of C. L. and C. R. observations, as may be, produces one equation of form (6), which reads:

$$(\sin\phi_{TP} - \cos\phi_{TP} \cdot \cos A_{TP} \cdot \tan h) \cdot d\lambda + \sin A_{TP} \cdot \tan h \cdot d\phi - dA +$$

$$+ (A_{TP}, \text{ comp. } \& \text{ h. c. r. etc.}, -A_{assumed}) = 0.$$

and is therefore <u>one observation equation</u>.

From all observation equations the normal equations can be formed and solved for the unknowns by any of the existing methods.

with:

$$(\sin\phi_{TP} - \cos\phi_{TP} \cdot \cos A_{TP} \cdot \tan h) \equiv a \quad \text{(coefficient of } d\lambda\text{)}$$

$$\sin A_{TP} \cdot \tan h \equiv b \quad \text{(coefficient of } d\phi\text{)}$$

$$-1 \equiv c \quad \text{(coefficient of dA)}$$

$$A_{TP}, \text{ comp. } \& \text{ h. c. r. etc. } -A_{assumed} \equiv l \quad \text{(absolute term) (residual)}$$

the observation equation of form (6) Section 8.3. becomes:

$$a \cdot d\lambda + b \cdot d\phi + c \cdot dA + l = 0 \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (1)$$

and the normal equations:

$$[aa] \; d\lambda + [ab] \; d\phi + [ac] \; dA + al = 0$$

$$\cdot$$

$$\cdot \qquad \ldots (2)$$

8.4. cont.

which are solved for the unknowns:

$$dA = +\frac{[cl.2]}{[cc.2]}$$

$$d\phi = +\frac{[bl.1]}{[bb.1]} - \frac{[bc.1]}{[bb.1]}\,dA$$

$$d\lambda = +\frac{[al]}{[aa]} - \frac{[ac]}{[aa]}\,dA - \frac{[ab]}{[aa]}\,d\phi$$

The r.m.s. errors are obtained from:

$$M = \pm\sqrt{\frac{[ll.3]}{m-n}}$$

and from the weights:

$$p_\lambda = [aa.2], \quad p_\phi = [bb.2], \quad p_A = [cc.2]$$

The precision of the values of the unknowns expressed as r.m.s. error will indicate whether a further approximation of the co-ordinates of the trial point is required or not.

A numerical answer obtained for the accuracy of a result constitutes an apparent advantage of least squares over semi-graphical methods. With some experience the precision of field results can be read from a plot just as well. Furthermore, the interpretation of errors and the detection of their origin is greatly facilitated by a solution obtained graphically. A diagram represents certain aspects of the work performed; in this sense it is an iconic model of the field work and permits analysation of individual observations.

8.4.1. Results obtained from Field Work.

Least Squares Solution of Astronomical Fixation from Horizontal Directions, and Astronomical Azimuth of Terrestrial Line.

The data of the same observations in 1961, shown in Table 8.3.1.-2, from which position lines from horizontal directions were deduced in Section 8.3.1, are, in this section, evaluated by the method of least squares.

The coefficients a, b, c and the absolute terms l of the observation equations have been computed as explained in Section 8.4., from values quoted in Table 8.3.1.-2.

Table 8.4.1.-1 contains the coefficients of the observation equations and of the normal equations. In Tables 8.4.1.-2 and -3 are given the solution of the normal equations, the precision of the unknowns, the co-ordinates of the final position and the final azimuth of the terrestrial line. Not all checks applied are shown in the tables.

To reduce calculation time, the arithmetic mean of ten azimuths listed was chosen as the assumed azimuth.

The r.m.s. errors of the unknowns are in about the same order of magnitude as the evaluated corrective terms for the co-ordinates and azimuth, which indicates that a further approximation to the co-ordinates of the trial point is not necessary.

The result obtained by least squares agrees well with results derived from position lines. It is interesting to note the little difference between the results obtained with each method, because the position line method is suspected by orthodox geodesists to be liable to create errors which are larger than observational errors.

The absolute term l (Table 8.4.1.-1, column 5) is based on the assumed azimuth of the reference line. It is evident from the nature of the least squares method that a different assumption of the azimuth

LEAST SQUARES SOLUTION OF ASTRONOMICAL FIXATION FROM TIMED HORIZONTAL DIRECTIONS.

AT: "COTTAGE" DATE: MAY 5TH & 9TH, 1961

Trial Point: $\phi = +50° 16' 45''$, $\lambda = -5° 13' 55''$, $A_{ASS.} = 305° 32' 37''.85$ (= ARITHMETIC MEAN)

No.	$d\lambda$ a	$d\phi$ b	dA c	l	s	aa	ab	ac	al	as	bb	bc	bl	bs	cc	cl	cs	ll	ls
1	+1·743	+0·504	−1	+2·35	+3·597														
2	−0·157	−1·539	−1	−0·35	−3·046														
3	+0·642	+0·920	−1	+0·45	+1·012														
4	+1·017	−0·724	−1	+5·55	+4·843														
5	+0·283	−1·088	−1	−4·85	−6·655														
6	+0·784	+1·439	−1	−2·65	−1·427														
7	+1·475	−0·610	−1	+5·28	+5·145														
8	+0·024	−0·005	−1	−2·52	−3·501														
9	+1·695	+0·605	−1	−3·06	−1·760														
10	−0·206	−1·598	−1	−0·20	−3·004														
[ ]	+7·300	−2·096	−10	±0·00	−4·796	+10·295	+2·249	−7·300	+9·216	+14·461	+10·539	+2·096	−5·158	+9·727	+10	±0·0	+4·796	+110·827	+114·886

COEFFICIENTS OF OBSERVATION EQUATIONS
AND
COEFFICIENTS OF NORMAL EQUATIONS

TABLE: 8.4.1−1

	$d\lambda$	$d\phi$	dA	l	s	CHECK	Red. F.	Q_1	Q_2	Q_3		Rem.?
A	+10·295	+2·249	−7·300	+9·216	+14·461	+14·461		−1			Q_1	
B		+10·539	+2·096	−5·158	+9·727	+9·727		0			Q_2	
		−0·491	+1·595	−2·013	−3·159		$-\frac{+2{\cdot}249}{+10{\cdot}295}\cdot A$	+0·218				
C			+10·000	±0·000	+4·796	+4·796		0			Q_3	
			−5·176	+6·535	+10·254		$-\frac{-7{\cdot}300}{+10{\cdot}295}\cdot A$	−0·709				
l				+110·827	+114·886	+114·885						
				−8·250	−12·945		$-\frac{+9{\cdot}216}{+10{\cdot}295}\cdot A$					
B.1		+10·048	+3·691	−7·171	+6·568	+6·568		+0·218	−1		Q_2	
C.1			+4·824	+6·535	+15·050	+15·050		−0·709	0		Q_3	
			−1·356	+2·634	−2·413		$-\frac{+3{\cdot}691}{+10{\cdot}048}\cdot B.1$	−0·080	+0·367			
L.1				+102·577	+101·941	+101·941						
				−5·118	+4·687		$-\frac{-7{\cdot}171}{+10{\cdot}048}\cdot B.1$					
C.2			+3·468	+9·169	+12·637	+12·637		−0·789	+0·367	−1	Q_3	
L.2				+97·459	+106·628	+106·628						
				−24·242	−33·411		$-\frac{+9{\cdot}169}{+3{\cdot}468}\cdot C.2 = dA$					
L.3				+73·217	+73·217							

$dA = -2{\cdot}64''$

$M = \pm\sqrt{\frac{73{\cdot}2}{10-3}} = \pm 3{\cdot}2_4''$

$d\phi = +0{\cdot}714 - 0{\cdot}367\, dA = +1{\cdot}68''$

$d\lambda = -0{\cdot}895 + 0{\cdot}709\, dA - 0{\cdot}218\, d\phi = -3{\cdot}13''$

SOLUTION OF NORMAL EQUATIONS

TABLE : 8.4.1 – 2

LEAST SQUARES SOLUTION CONT.'

$$Q_{33} = +\frac{1}{3{\cdot}46_8} = +0{\cdot}28_8$$

$$Q_{32} = \frac{-0{\cdot}36_7}{+3{\cdot}46_8} = -0{\cdot}10_6 = Q_{23}$$

$$Q_{31} = \frac{+0{\cdot}78_9}{+3{\cdot}46_8} = +0{\cdot}22_8 = Q_{13}$$

$$Q_{22} = \frac{+1}{+10{\cdot}04_8} - 0{\cdot}36_7 \times Q_{23} = +0{\cdot}13_8$$

$$Q_{21} = \frac{-0{\cdot}21_8}{+10{\cdot}04_8} - 0{\cdot}36_7 \times Q_{13} = -0{\cdot}10_5 = Q_{12}$$

$$Q_{11} = \frac{+1}{+10{\cdot}04_8} + 0{\cdot}70_9 \times Q_{13} - 0{\cdot}21_8 \times Q_{12} = +0{\cdot}28_1$$

$$m_A = \pm M.\sqrt{Q_{33}} = \pm 1{\cdot}7''$$

$$m_\phi = \pm M.\sqrt{Q_{22}} = \pm 1{\cdot}2''$$

$$m_\lambda = \pm M.\sqrt{Q_{33}} = \pm 1{\cdot}7''$$

FINAL POSITION "COTTAGE"

$\phi = +50^\circ\ 16'\ 46''{\cdot}7$

$\lambda = -5^\circ\ 13'\ 58''{\cdot}1$

ASTRONOMICAL AZIMUTH : COTTAGE TO PORTHTOWAN R.O

$305^\circ\ 32'\ 35''{\cdot}2$

DATE : 5TH & 9TH MAY, 1961

TABLE 8.4.1 – 3

8.4.1. cont.

will produce different numerical values of l, a different corrective term dA, an identical final result for the azimuth, and identical corrective terms for the co-ordinates of the trial point, and consequently an identical final position.

8.5. Position Planes from Horizontal Directions and Results obtained from Field Work

The problem of position fix from timed horizontal directions is accessible to semigraphical treatment in two- and in three-dimensional geometry.

The relation of the unknowns in form (7) Section 8.3. is interpreted as an equation of a variable point with reference to a straight line; this interpretation enables the problem to be treated in two dimensional geometry.

In three dimensional geometry an equation in one or more unknowns represents in general a surface or a system of surfaces. A linear equation in three unknowns is a plane or a point.

The relation of the three unknowns in form (6) Section 8.3., which reads:

$$(\sin\phi_{TP} - \cos\phi_{TP}.\cos A_{TP}.\tan h)d\lambda + \sin A_{TP}.\tan h.\, d\phi = dA +$$

$$+ (A_{TP}, \& \text{ h.c.r.} - A_{ass.}) = 0.$$

constitutes an equation of the 1st degree, involving three co-ordinates of a variable point.

In three dimensional geometry the above equation represents a plane or a system of planes which is the locus of the variable point.

With:

$$(\sin\phi_{TP} - \cos\phi_{TP}.\cos A_{TP}.\tan h) \equiv A$$

$$(\sin A_{TP}.\tan h) \equiv B$$

$$-1 \equiv C$$

$$(A_{TP}, \& \text{ h.c.r., etc.,} - A_{assumed}) \equiv D$$

equation (6) Section 8.3. can be written:

$$A\, d\lambda + B\, d\phi + C\, dA + D = 0 \quad . \; . \; . \; . \; . \; . \; . \quad (1)$$

8.5. cont.

substituting in the above equation:

$$d\lambda \doteqdot x$$
$$d\phi \doteqdot y$$
$$dA \doteqdot z$$

equation (6) Section 8.3. becomes:

$$A\,x + B\,y + C\,z + D = 0.$$

This is the general form of the equation of a plane in geometry of three dimensions.

The above equation (1) is the tangent plane on the surface $A(\lambda, \phi, \alpha, \delta) = 0$; the point of contact $(d\lambda, d\phi, dA)$ is the final position of the observer.

The surface is represented by the cylinder which is generated by the observation ray to the (given) star, as generatrix, and the line of equal azimuth on the earth's surface as directrix.

Each observation, consisting of timed horizontal direction to a heavenly body and horizontal direction to a reference line yields a relation of form (1) Section 8.3.. and consequently one equation of form (6).

Geometrically, each such observation produces one "Position Plane":

$$A_i d\lambda + B_i d\phi + C_i dA + D_i = 0$$

in a system of three mutually perpendicular co-ordinate axes: $(d\lambda, d\phi, dA)$. The origin of the system has the co-ordinates of the trial point: $(\lambda_{TP}, \phi_{TP})$.

Two such equations, or position planes, together represent a line; namely a position line in space, or two position planes passing through a common line.

Observations to three stars produce three position planes. These will intersect in a point which is the final position of the observer's

8.5. cont.

station. The co-ordinates of the point of intersection are: $d\lambda$, $d\phi$, dA, which are the corrective terms required for the co-ordinates of the final position and for the final azimuth.

It would be erroneous to believe that the answer obtained from the intersection of the three planes is true, and gives the correct values for the unknowns.

The rigid solution from the trisection of three position planes represents at most an approximation to the corrective terms. The three stars concerned contribute each in a different way to the determination of the three unknowns; it could be said that each star has a preference for one particular corrective term and establishes its order of rank. In practice, whether position planes are seen to meet in a point or not will also depend on the size of the plotting scale in relation to the accuracy expected from observational results.

The nature of intersection of a set of position planes, if more than three stars are observed, depends on the quality of the observations and mainly on the position of the stars observed.

If all position planes intersect in one common point, then all sights taken are affected by errors of equal magnitude, and equal corrective terms are obtained from observations to all stars, regardless of their azimuth and altitude. In the discussion of errors, Section 8.3., it is shown that this is not possible.

Variation in observational results must be expected because, basically, the taking of a measurement is a statistical process associated with uncertainty, errors, etc.

Generally, n position planes will produce a polyhedron, consisting of the maximum of $\binom{n}{2}$ edges, and of the maximum of $\binom{n}{3}$ corners, of which four will not lie in one plane. A corner is formed by the intersection of three or more position planes. In practice, the body bounded by all position planes will be a rather intricate solid, or

8.5. cont.

"complex".

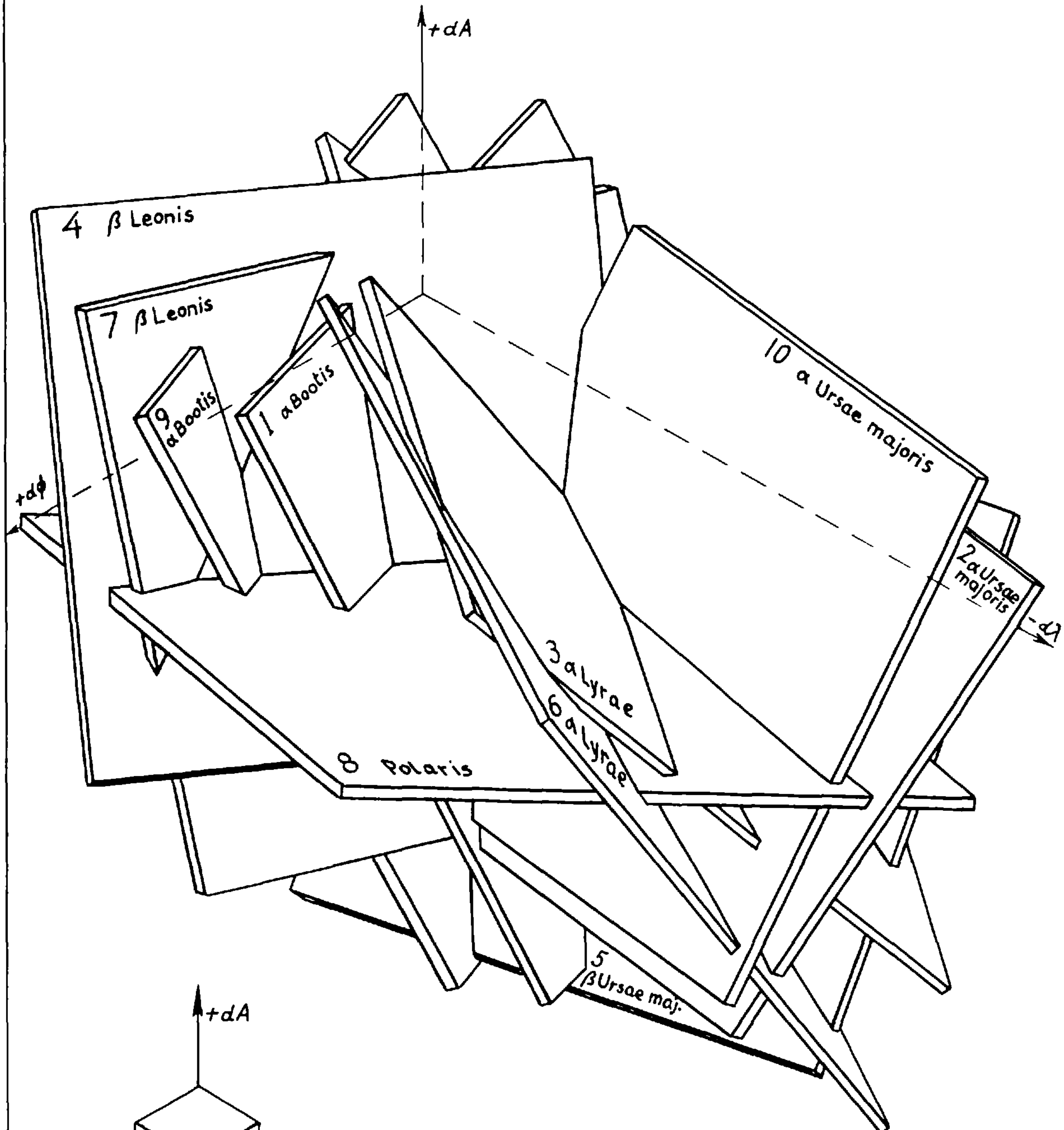
The spatial geometric relation of position planes obtained from observations on May 5th and 9th, 1961, is shown in Fig. 8.5.-1. Data for constructing the isometric view of the position planes are contained in Table 8.4.1.-1. The coefficients A, B, C, and the absolute term D of equation (1) above, are identical to the coefficients a, b, c, and the absolute term l of the observation equation, respectively. The intercepts of the position plane on the $d\lambda$ -, $d\phi$ -, and dA - axis are: $-\frac{D}{A}$, $-\frac{D}{B}$, and $-\frac{D}{C}$; these are used for constructing the diagram.

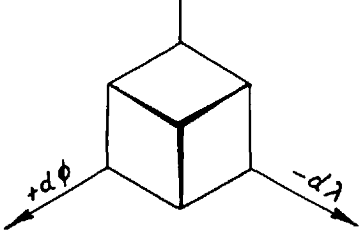
For the sake of clarity constructional lines are not included in the diagram. The co-ordinate axes are chosen as shown in the general arrangement. Consequently the trace of the position plane on the $(d\lambda, d\phi)$ - plane, which depends on the azimuth and zenith distance of the star, and on the latitude of the observer, is seen to be in conformity with the azimuth of the star as shown in the diagram inserted in Fig. 8.3.1.-1 in the top left corner. The intersection of the position planes with the $(d\lambda, d\phi)$ -plane, or "the line of strike" is positioned exactly in the star's azimuth for observations performed on the equator only.

The diagram emphasizes the properties of Polaris as a suitable azimuth star. Furthermore, the diagram clearly shows that the relative inclination of position planes corresponds to the zenith distances of the quadrantal stars observed.

Generally, the algebraic relations resulting from a geometric figure are solved with formulae linking measured with unknown quantities. The problem of position fix consists in solving a space geometrical figure. The problem can be treated analytically. The analytical treatment by the method of least squares and the result can be identified geometrically. A change over from analytical to geometrical treatment and a geometrical interpretation of an analytical method is effected with the aid of co-ordinates. The space problem of position fix, described

SPATIAL RELATION OF POSITION PLANES IN ISOMETRIC PROJECTION.





ARRANGEMENT OF CO-ORDINATE AXES

POSITION PLANES ARE DERIVED FROM OBSERVATIONS ON MAY 5TH & 9TH, 1961.

ISOMETRIC SCALE : 1 SEC. ARC = 0·215 INCHES

0 1 2 3 4 5 SEC. ARC

Fig.: 8.5.-1

8.5. cont .

above, deals with position planes:

$$A_i\, x + B_i\, y + C_i\, z + D_i = 0$$

Theoretically, the position planes have a common point (x_1, y_1) of intersection. Therefore its co-ordinates must satisfy the condition:

$$\frac{A_i x_1 + B_i y_1 + C_i z_1 + D_i}{\pm\sqrt{A_i^2 + B_i^2 + C_i^2}} = 0 \quad . \quad . \quad . \quad . \qquad (2)$$

If the point sought does not lie on the planes, as explained previously, the above condition is not zero, but represents the distance to the point (x_1, y_1) from the plane i.

The method of least squares requires that the position of the point satisfies the condition that the sum of the squares of the distances from the planes is a minimum.

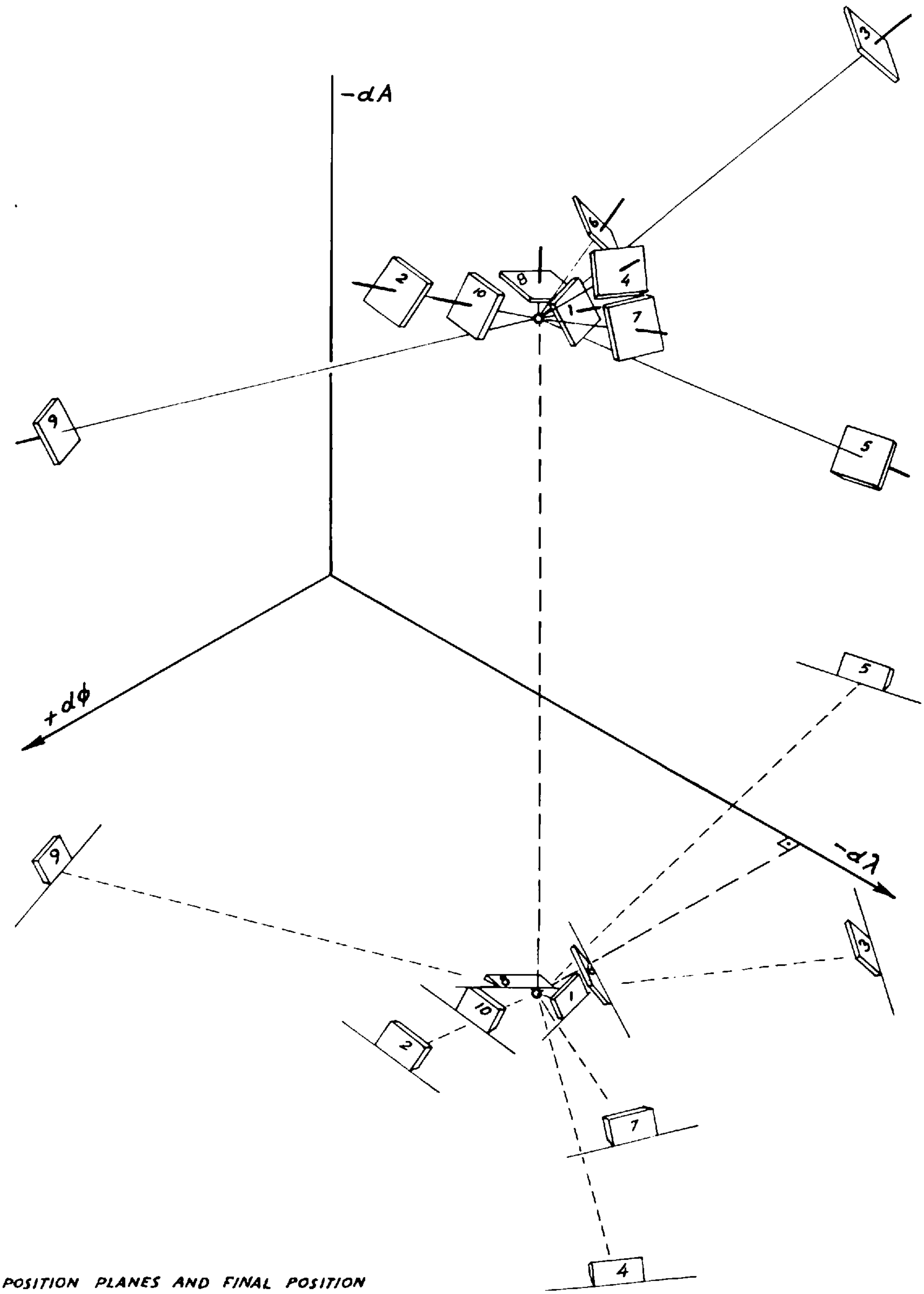
The observation equations are identical to the general equations of the position planes.

The normal equations deduced from the observation equations (= position planes) by squaring, adding and differentiation, fulfil the geometric condition of the sum of the squares of the distances, and the condition of minimum. The least squares method neglects the square root term in the above equation (2). This affects only the absolute term in the equation system of the least squares method. As already mentioned in a foregoing section, any alteration of the absolute term of the observation equations has no influence on the final result.

Fig. 8.5.-2 shows the geometric interpretation of the least squares result obtained from observations in May 1961. The scale is greatly exaggerated to enhance the illustration.

The upper part of the isometric diagram shows the final position of the observer's station and its distances to the position planes in space. The lower part is the projection of the space relation on to

GEOMETRIC INTERPRETATION OF LEAST SQUARES SOLUTION OF ASTRONOMICAL FIXATION FROM HORIZONTAL DIRECTIONS.



POSITION PLANES AND FINAL POSITION DERIVED FROM OBSERVATIONS ON MAY 5TH & 9TH, 1961, PRESENTED ISOMETRICALLY IN SPATIAL RELATION AND IN PROJECTION ON $(d\lambda, d\phi)$-PLANE.

ISOMETRIC SCALE : 1 SEC.ARC = 1·29 INCHES.

0 1 2 SEC.ARC

Fig.: 8.5.-2

8.5. cont.

the ($d\lambda$, $d\phi$) -plane. The variations of the corrective terms, $d\lambda$, $d\phi$, dA, as obtained from the individual position planes, can be scaled from the projections presented.

The minimum requirement as stipulated by the method of least squares means that the point sought is the centre of gravity of the body bounded by all position planes.

Therefore, the final position can be interpreted as the centre of a system of parallel forces which attract mass particles of equal weight.

The mass particles, forming a system, are the corners of the complex body and are obtained from all possible trisections of the position planes. The co-ordinates of the centroid of the system can be obtained by rules derived from the mechanics of points and rigid bodies.

The isometric diagram submitted in Fig. 8.5.-3 shows 18 out of $120 = \binom{n}{3}$, for: $n = 10$, trisections of position planes, from observations taken in May 1961. The diagram conforms with the representation of the position planes in Fig. 8.5.-1. For simplicity, only a few intersection lines of the position planes (dash-dotted) are included in the diagram, and the position planes are cut and shown in the vicinity of their trisections only.

The selection of position planes which are liable to give adequate trisections is governed by the skill of the surveyor, and is assisted by the azimuth diagram of the stars observed.

Not all position planes produce trisections which could lead to an acceptable solution.

The trisections of position planes from horizontal directions have to be treated in analogy to the intersections of position lines from zenith distances.

Fig. 8.5.-4 contains in isometric projection 34 trisections of position planes obtained from observations in May 1961. The numbers of the points of trisection refer to the numbers of the planes. Data are given in Table 8.4.1.-1. The position planes, expected to produce

TRISECTIONS OF POSITION PLANES
PRESENTED IN ISOMETRIC PROJECTION

POSITION PLANES ARE DERIVED
FROM
OBSERVATIONS ON MAY 5TH & 9TH 1961.

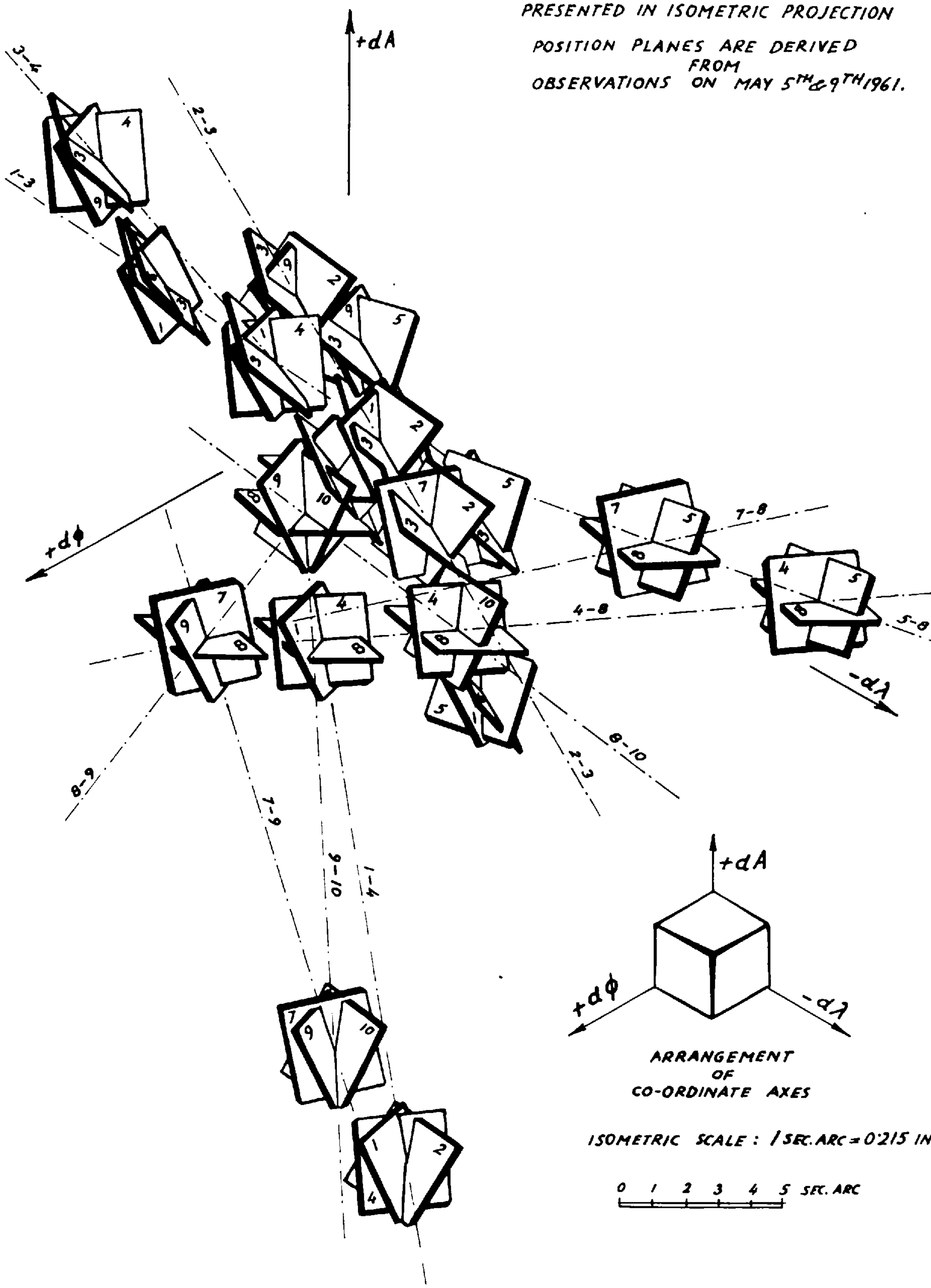


Fig.: 8.5.-3

8.5. cont.

feasible trisections, are selected with the aid of Fig. 8.5.-1 and of the diagram of the star azimuths in Fig. 8.3.1.-1.

34 trisections constitute about $\frac{1}{3}$ of the total possible trisections. Statistically, this amount is adequate to produce a reliable result.

The trisections are considered as fixed points on which parallel forces are acting. The resultant will act through the centre of the system. As known, the forces can be rotated about the points at which they are acting; keeping the forces parallel during rotation, their resultant also rotates about a point which is the centre of the parallel forces, or the centroid of the system of the elements of mass.

A graphical solution can be obtained by means of force and string polygons. For this purpose the points in space are projected on to the planes formed by the co-ordinate axes. For a complete solution any two of the three planes formed by the axes can be chosen.

Fig. 8.5.-5 shows the trisection-points of the previous diagram (Fig. 8.5.-4) in orthographic projection on the $(d\lambda, d\phi)$ -plane, and Fig. 8.5.-6 on the $(d\phi, dA)$ -plane.

The action lines are taken in two arbitrary directions; for convenience the directions are chosen at right angles to one another, as shown in the force diagram and indicated by the arrows attached to the trisection points.

In Fig. 8.5.-5 the distribution of the points causes a rather intricate string polygon belonging to the action lines drawn in horizontal direction. This can be avoided by alteration of the sequence of the forces in the force diagram, as shown in Fig. 8.5.-6.

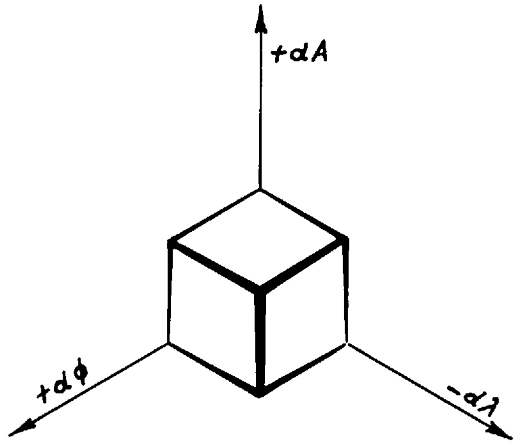
The corrective terms can be scaled from the intersection of the resultants to the co-ordinate axes.

In Fig. 8.5.-6 the resultant of the forces parallel to the dA-axis produces a check on the value for $d\phi$ obtained in Fig. 8.5.-5.

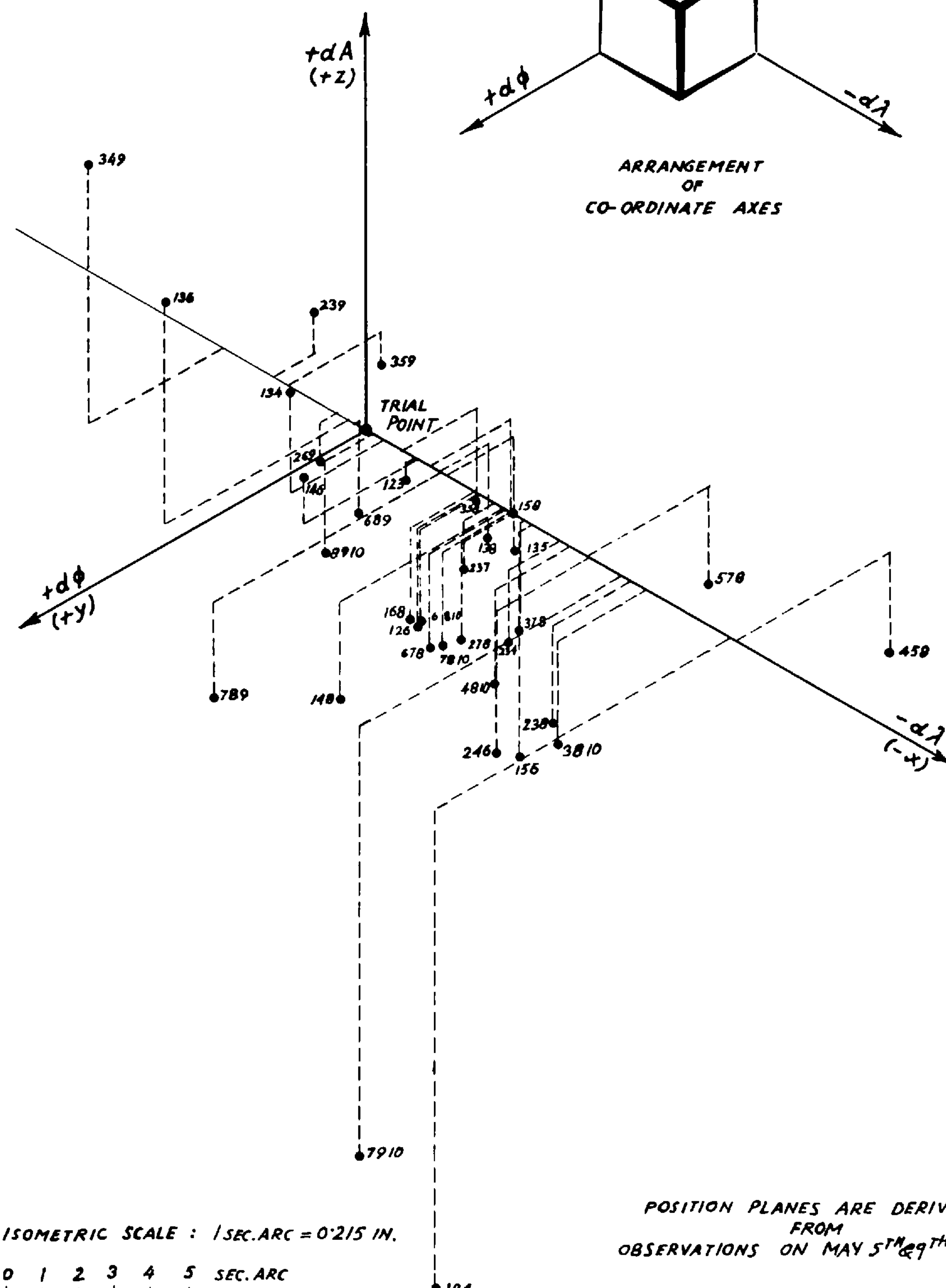
The final result is in agreement with the result obtained from least squares (Section 8.4.1) or from position lines (Sections 8.2.1 and 8.3.1)

ISOMETRIC DIAGRAM
OF
TRISECTIONS OF POSITION PLANES

TRISECTIONS ARE SHOWN AS FIXED POINTS.



ARRANGEMENT
OF
CO-ORDINATE AXES

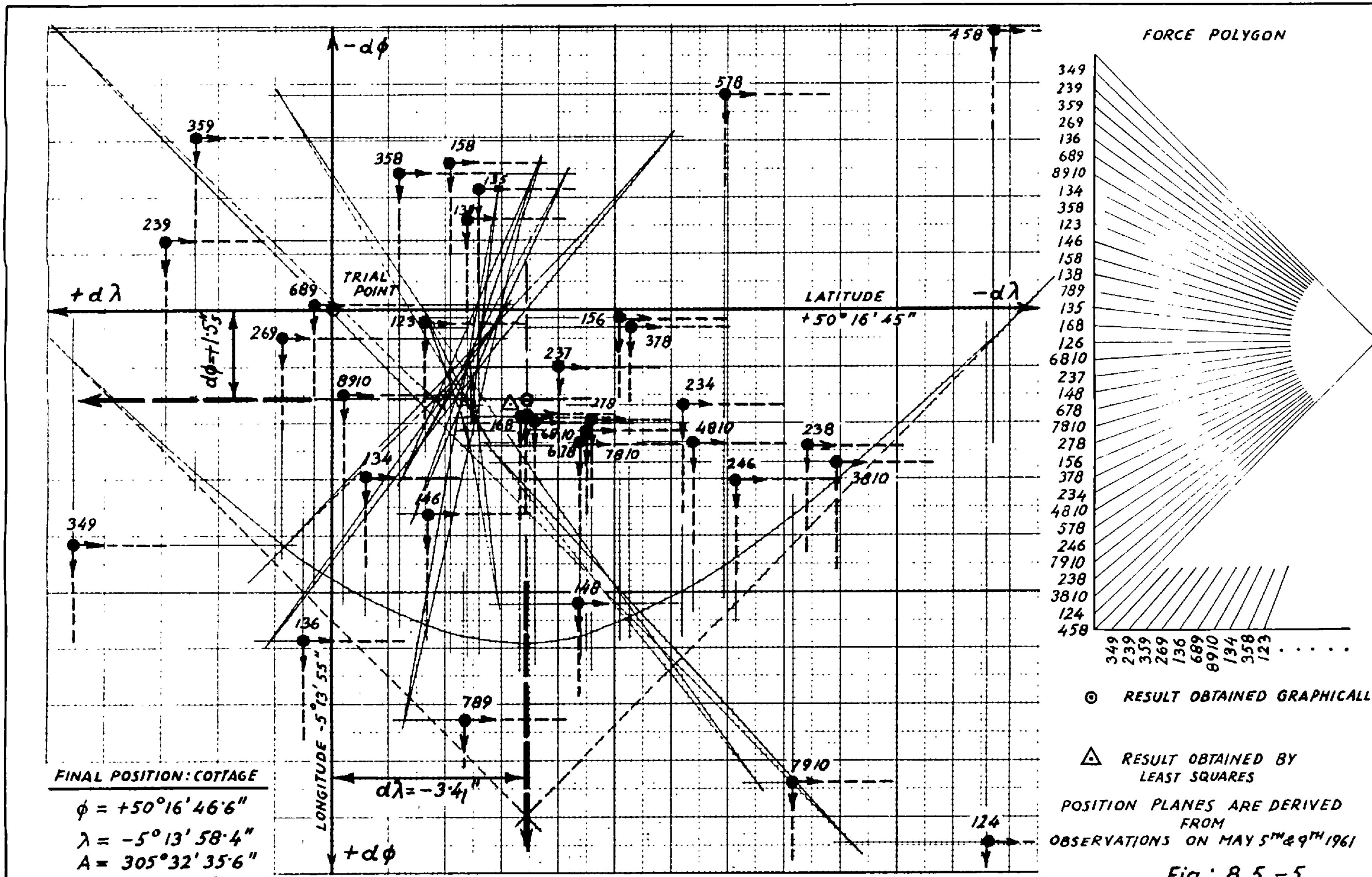


ISOMETRIC SCALE : 1 SEC. ARC = 0·215 IN.

0 1 2 3 4 5 SEC. ARC

POSITION PLANES ARE DERIVED
FROM
OBSERVATIONS ON MAY 5TH & 9TH 1961.

Fig.: 8.5.-4



FINAL POSITION DETERMINED GRAPHICALLY FROM TRISECTIONS OF POSITION PLANES

Fig.: 8.5.-5

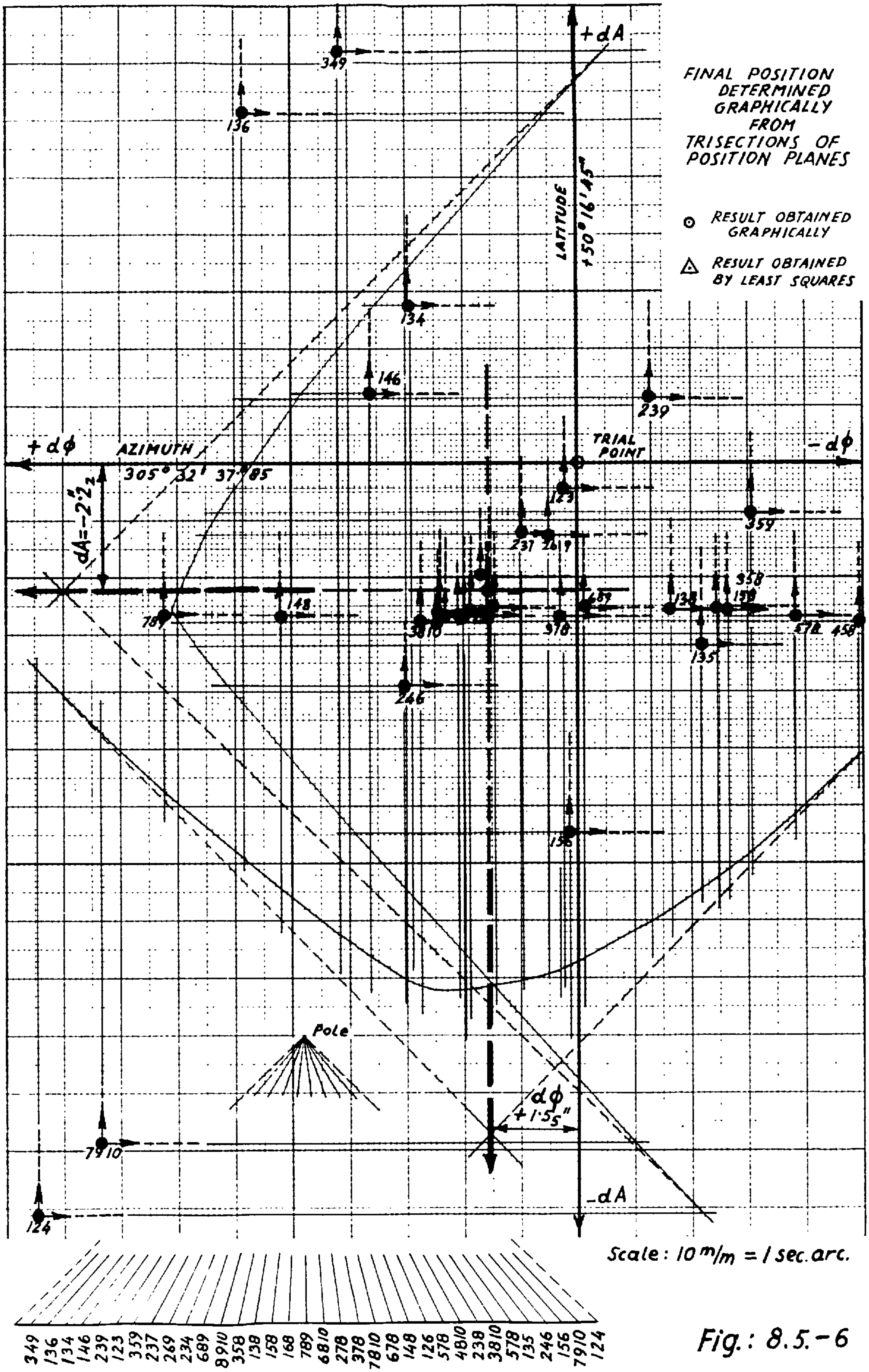

FINAL POSITION DETERMINED GRAPHICALLY FROM TRISECTIONS OF POSITION PLANES
RESULT OBTAINED GRAPHICALLY
RESULT OBTAINED BY LEAST SQUARES
+dA
-dA
+dφ
-dφ
AZIMUTH 305° 32' 37"85
dA = -2"2₂
LATITUDE +50° 16' 45"
TRIAL POINT
dφ +1.5₅"
Pole
349
136
134
146
239
123
359
237
269
789
148
3810
318
689
138
158
358
578
458
135
246
156
7910
124
349 136 134 146 239 123 359 237 269 234 689 8910 358 138 158 168 789 6810 278 378 7810 678 148 126 578 4810 238 3810 578 135 246 156 7910 124
Scale: 10 m/m = 1 sec. arc.

Fig.: 8.5.-6

8.5. cont.

In Section 8.3. and in the present Section, the problem of position fix is regarded as a problem in two- and in three-dimensional geometry respectively. Mean values for the three unknowns, i.e. corrections $d\lambda$, $d\phi$, and dA, are derived in each case simultaneously from graphs.

Each plotted position line, in plane geometry, e.g. Fig. 8.3.1.-1 and each plotted trisection in space geometry, e.g. Fig. 8.5.-5 and -6, reflect the interdependence of the three unknowns, as a set, which is obtained from each star observation.

The plot containing all position lines (Fig. 8.3.1.-1) shows their grouping around the final position, which is thereby indicated. The plot of trisections (Figs. 8.5.-5 and -6) reveals by the cluster of points where to select the most probable result.

An attempt could be made to solve the space problem semigraphically with the aid of only one plot in plane geometry. For this purpose linear equations in two unknowns are required, which represent the conditions for position fix.

One linear equation in two unknowns, derived by eliminating one of the three unknowns contained in a given pair of linear equations, represents in space geometry the projecting plane, passing through the intersection line of the given planes. Thus the projecting planes are found by eliminating, in turn, one of the three unknowns from the given pair of equations. The projecting planes are perpendicular to the co-ordinate planes; in particular, their equations in two unknowns each represent the trace of the projecting plane on the co-ordinate plane. Therefore, the trace is also the projection on the co-ordinate plane of the line of intersection of the given pair of planes.

The lines of intersection of position planes produce the edges of the complex. Their projections on to the co-ordinate planes can be used to derive the centroid of the complex, which is supposed to be the final position of the observer's station.

8.5. cont.

Fig. 8.5.-7 contains intersection lines projected on to the $(d\lambda, d\phi)$ co-ordinate plane. The intersection lines are derived from observations in May 1961, and are constructed graphically from data given in Table 8.4.1.-1. The numbers attached to the lines refer to the number of the position planes.

Briefly, the plot is a two-dimensional representation of the space problem.

The intersection lines shown belong to those position planes whose trisections are plotted in Figs. 8.5.-5 and -6. Incidentally, Fig. 8.5.-7 is the plot from which the above trisections are deduced.

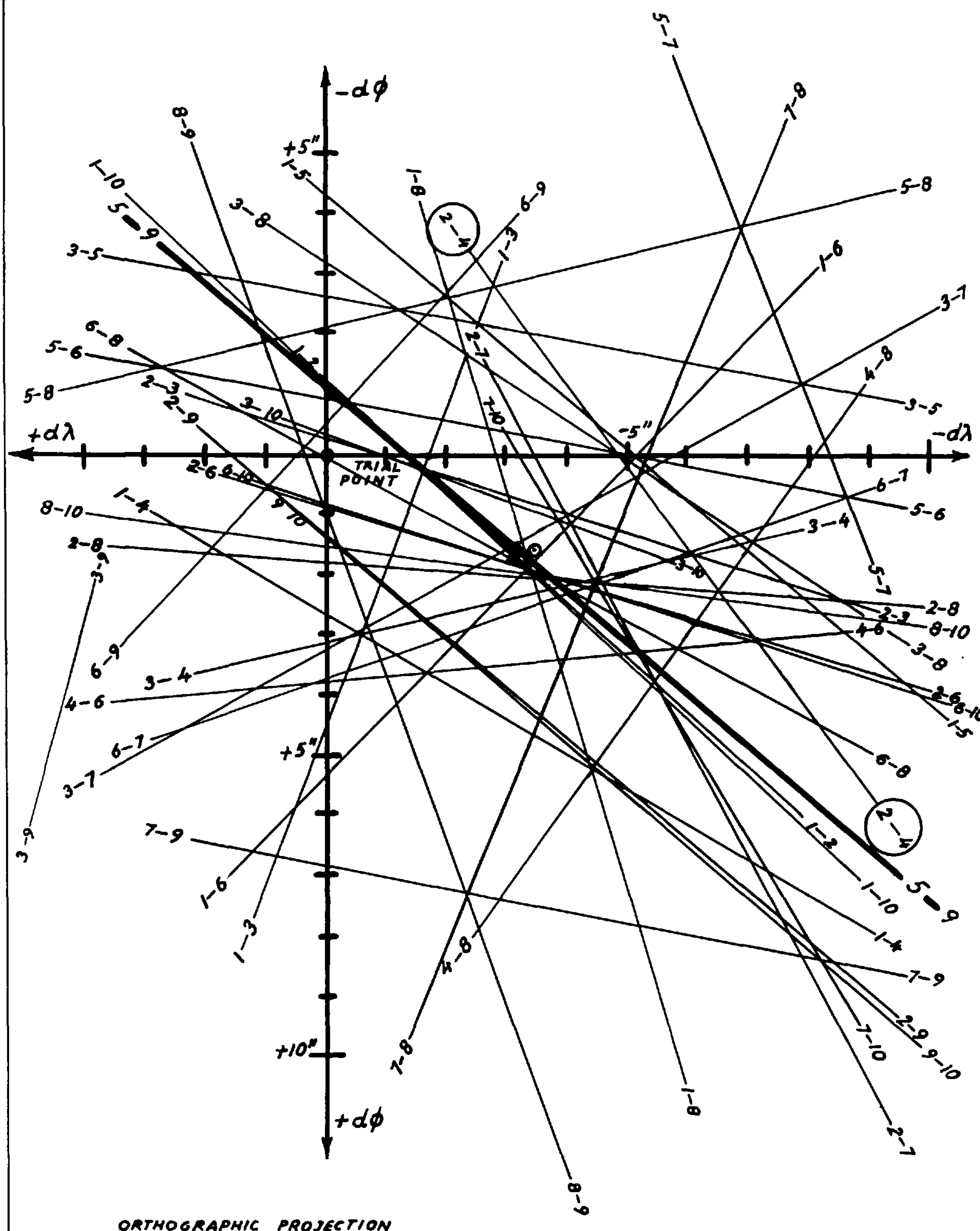
The intersections projected on to the $(d\lambda, d\phi)$ -plane do not give a clear indication where the final position could be expected, or how it could be derived, although pairs of planes are chosen with the aid of the azimuth diagram (Fig. 8.3.1.-1).

In the absence of the plotted results obtained by the least squares method and/or by the force and string polygons, it would hardly be possible to define the final position to more than $\pm$ 5 sec. arc, at the scale chosen.

The apparent difficulty of interpreting the line plot and of finding the final position is due to the confusion caused by the distribution of the projected intersection lines.

A sharp solution from a plot can be derived if projected intersection lines are deduced from stars which influence strongly the determination of either the longitude, latitude, or azimuth. The co-ordinate plane on which the intersection lines are to be projected has to be chosen accordingly.

Field data evaluated for Fig. 8.5.-7 contain also results from observations to Polaris, which influences the determination of azimuth. Intersections of the Polaris position plane, with position planes derived from south stars, will yield a clear picture as regards position of intersection lines projected on any of the two co-ordinate



ORTHOGRAPHIC PROJECTION
OF
INTERSECTIONS OF POSITION PLANES
ONTO : $(d\lambda, d\phi)$- PLANE

POSITION PLANES ARE DERIVED FROM
OBSERVATIONS ON MAY 5TH & 9TH, 1961
Scale: 10mm = 1 sec. arc.

Fig.: 8.5.-7.

8.5. cont.

planes containing the azimuth axis, i.e. on the ($d\lambda$, dA) or ($d\phi$, dA) -plane.

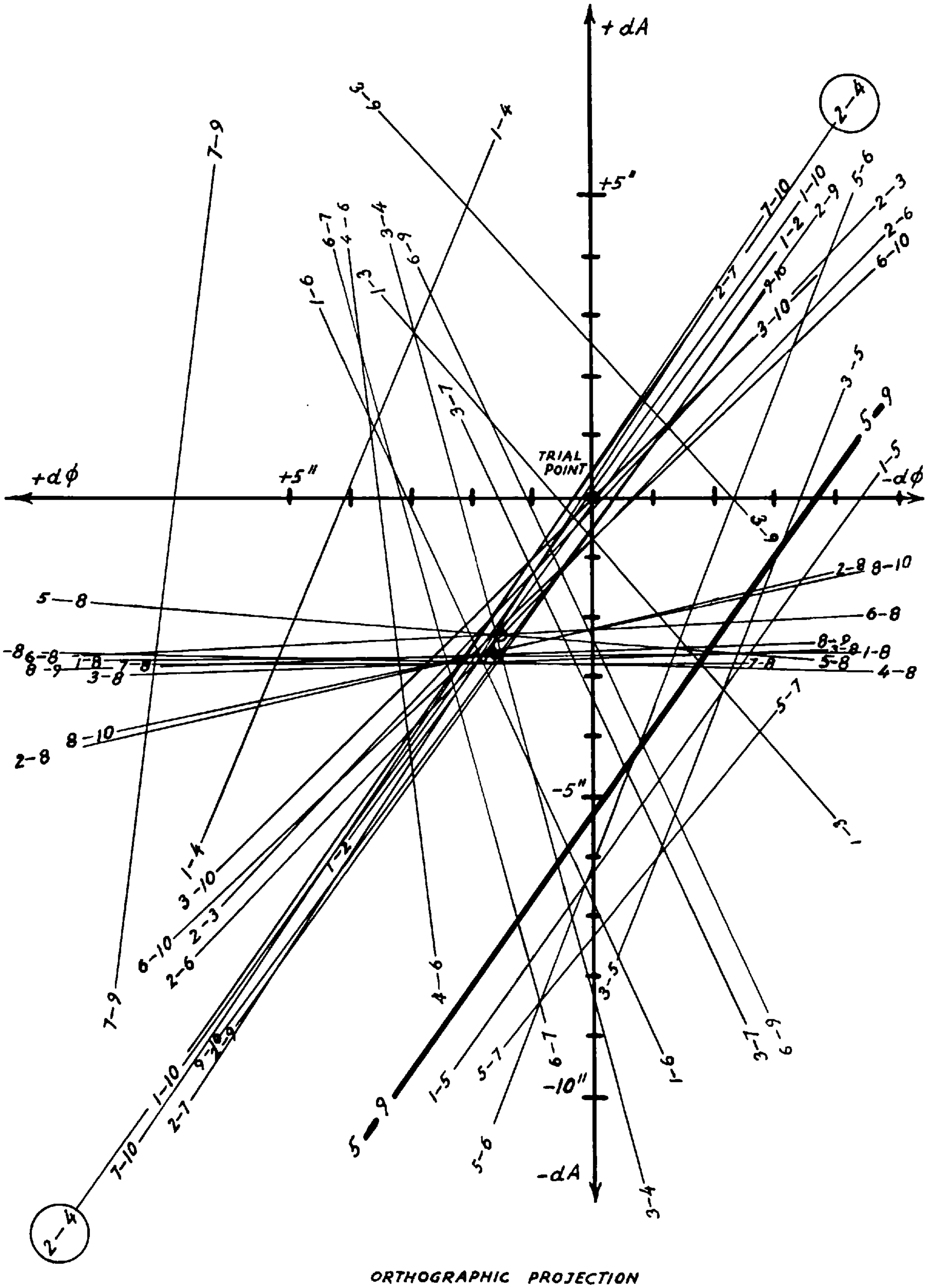
The same intersections which are shown in Fig. 8.5.-7 in orthographic projection on the ($d\lambda$, $d\phi$) -plane, are plotted in identical way in Figs. 8.5.-8 and 8.5.-9 on the ($d\phi$, dA) and on the ($d\lambda$, dA) -plane respectively. These last two diagrams reveal the influence of Polaris as an azimuth star; the azimuth correction dA can be scaled from both diagrams to an accuracy of $\frac{1}{4}$ second of arc. Further, it can be recognized that the stars observed produce a reliable answer for the latitude-, mainly stars in the N-E and N-W quadrants. The latitude correction, in combination with the azimuth correction (Fig. 8.5.-8) is defined by the intersection of the bundle of lines, and can be scaled to one second of arc, or even better. The longitude correction is the weakest result obtained, with a total spread of about 6 seconds of arc. (Fig. 8.5.-9).

The field work does not contain observations to stars balanced in 180° azimuth; nor any pair of stars which could be chosen for pronounced longitude or latitude determination. For these reasons the plot of intersection lines on the ($d\lambda$, $d\phi$) -plane fails to produce an answer.

These investigations into graphical treatment disclose that confinement to one plot in one specific co-ordinate plane should be avoided in the absence of a prominent pair of latitude, longitude-, or azimuth- stars. Furthermore, a better interpretation of the result is achieved from projections on to more than one co-ordinate plane.

Generally, position planes, considered as tangent planes, are presumed to miss the point of tangency (= final position) by a distance which is in proportion to permissible errors.

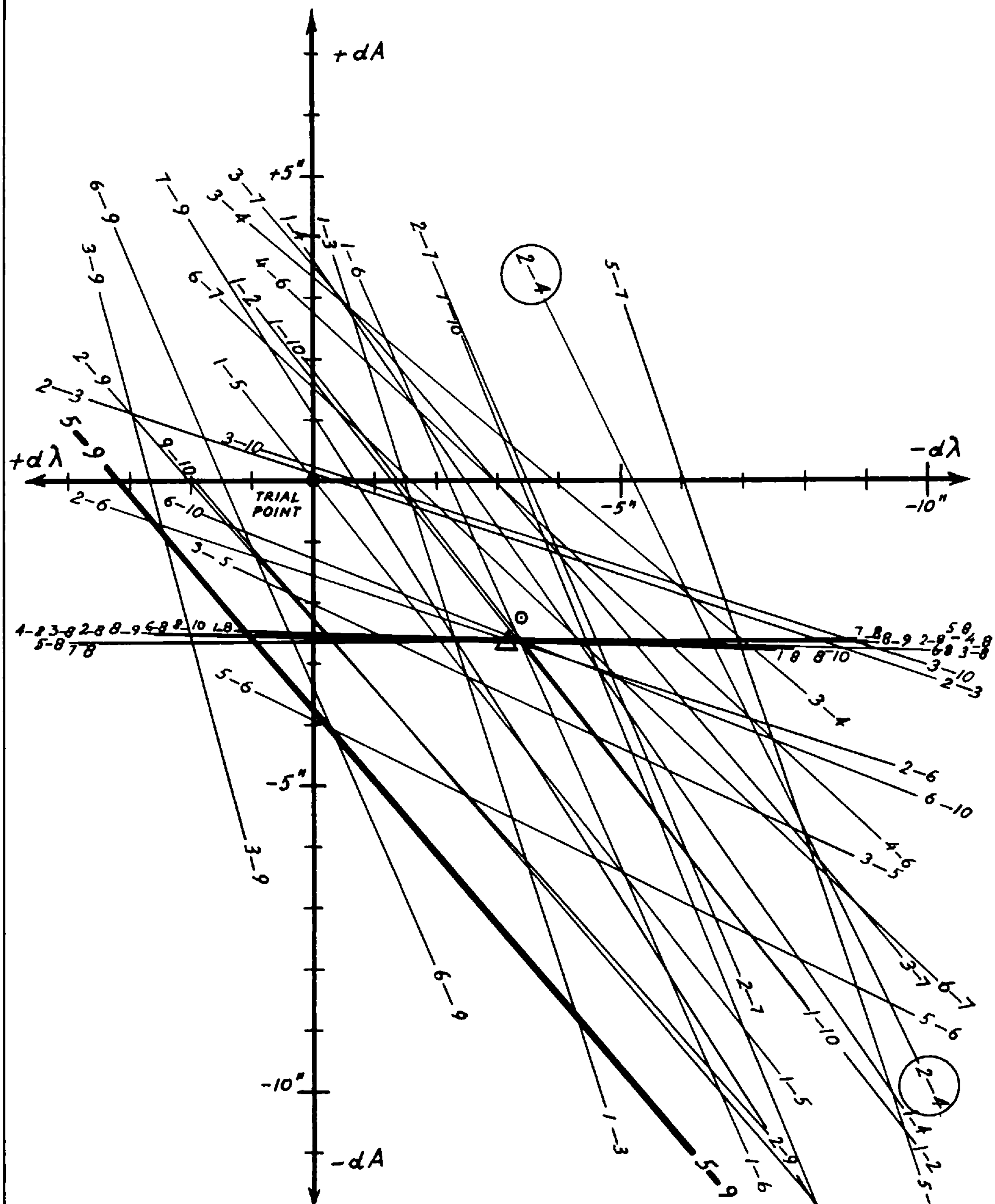
The intersection line of two position planes can miss the final position by the same or greater amount, as it is missed by the two planes. The projection of the intersection line on to any of the co-ordinate planes may give a distorted view of the relation between



ORTHOGRAPHIC PROJECTION
OF
INTERSECTIONS OF POSITION PLANES
ONTO : (dφ, dA)- PLANE

Scale : 10 mm = 1 sec.arc.

Fig.: 8.5.-8



ORTHOGRAPHIC PROJECTION
OF
INTERSECTIONS OF POSITION PLANES
ON TO : $(d\lambda, dA)$- PLANE

scale: 10mm = 1 sec. arc.

Fig.: 8.5.-9

8.5. cont.

final position and intersection line. Thus, the information derived from the graphical representation of the intersection lines on one particular co-ordinate plane only, is not sufficient to form an opinion about the quality of the observations.

e.g.: The projection of the intersection line of the planes 5 and 9 is seen to miss the final position in Figs. 8.5.-8 and 8.5.-9, and passes almost through it in Fig. 8.5.-7. For better interpretation the projections of the intersection line 5-9 are drawn three times as thick as the others. Similarly, the intersection line 2-4 (the number is encircled) passes through the final position in Fig. 8.5.-8 and not in the other two projections.

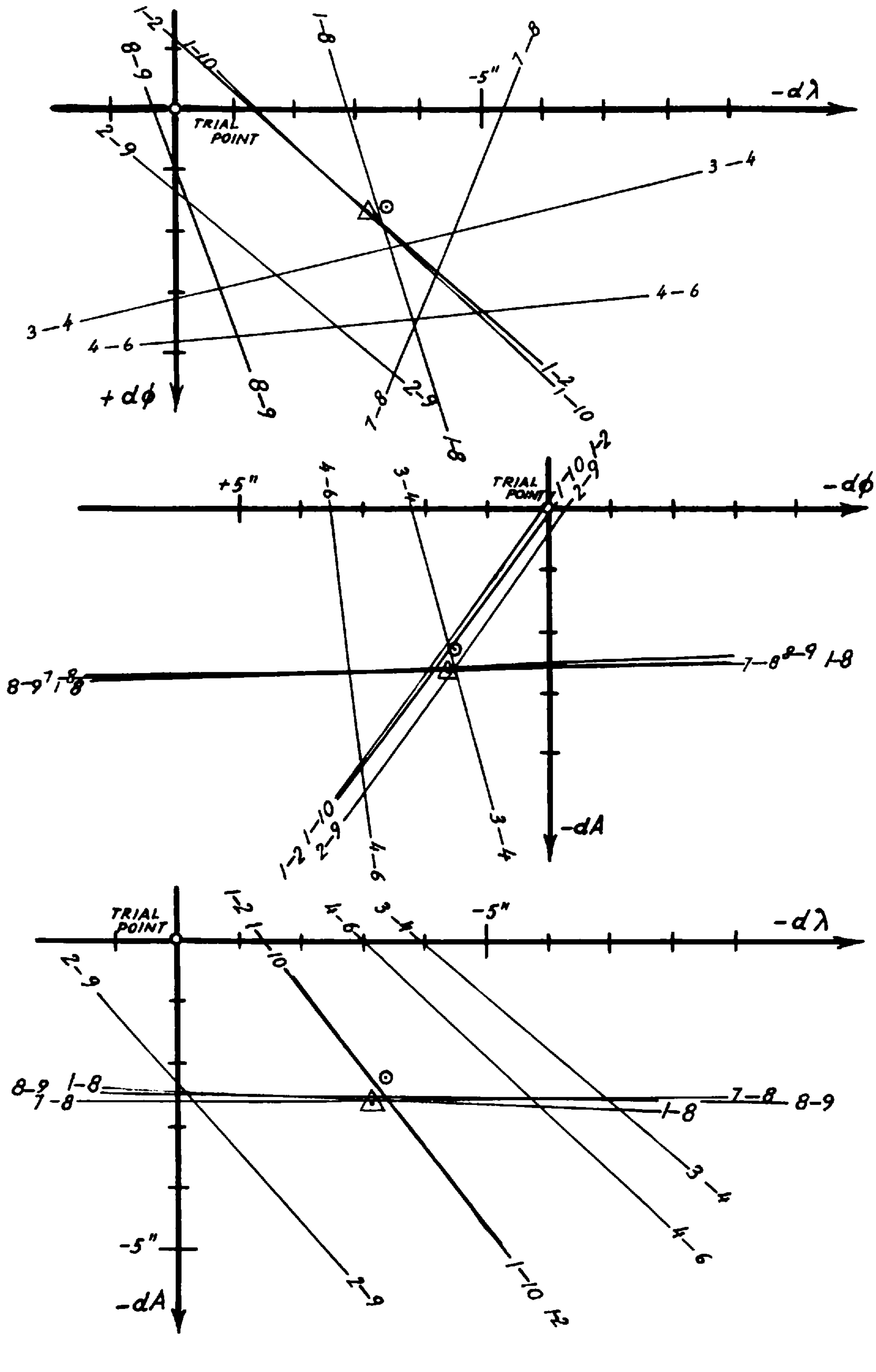
The projections of the intersection line 1-2, and of other planes, are seen to go through the final position in all three co-ordinate planes. This means that plane 1 and plane 2, and other planes, establish the final position with great accuracy, and equal values for the corrective terms are obtained from those planes.

The meeting point of several intersection lines appearing on all three co-ordinate planes may be adopted to advantage for defining the final position.

The interpretation of relative position of the intersection line and final result on all three diagrams simultaneously, and the comparison of it with Fig. 8.5.-1 show the conformity of the results obtained by the position plane-method and by the analytical method of least squares.

Projections of a few intersection lines are shown in Fig. 8.5.10. These are selected from Figs. 8.5.-7. -8. amd -9, in such a way as to belong to stars balanced as close as possible to 180° in azimuth.

Although the three diagrams do not give as clear an indication of the final result as Fig. 8.5.-5 or -6, nevertheless the final position can be derived from them and corrective terms can be scaled to the nearest second of arc.



INTERSECTIONS OF POSITION PLANES
PROJECTED ORTHOGRAPHICALLY
ONTO CO-ORDINATE PLANES.
SCALE : 10mm = 1 sec. arc.

Fig.: 8.5.-10.

8.5. cont.

In conclusion it can be stated that a plot of intersection lines projected on to one co-ordinate plane only (e.g. Fig. 8.5.-7) is almost meaningless. But even such a plot alone can be used for finding two of the corrective terms. The procedure to be adopted is to identify trisections of position planes; these are found from the intersections of intersection lines. Thereafter the corrective terms are derived with the aid of force and string polygons.

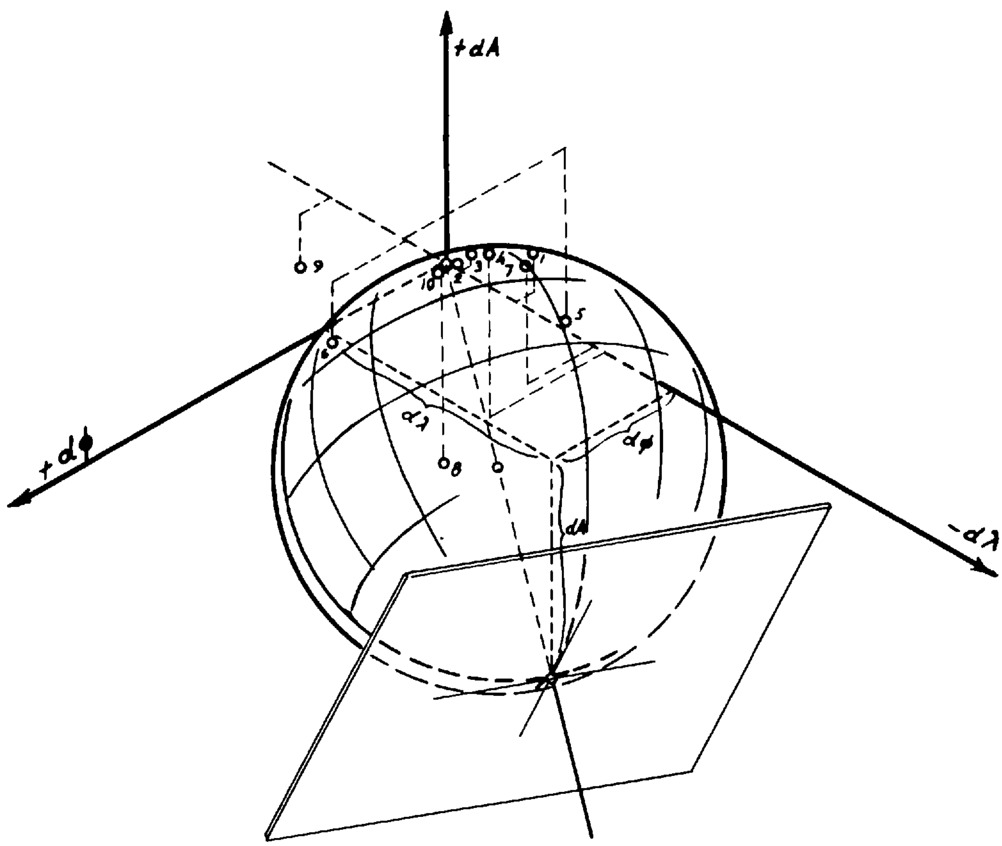
The solution of the problem of position fix from horizontal direction, if considered as a problem in space geometry, can be found by the application of the concept of duality .

Plane and point are reciprocal concepts in space. The equation of a position plane is, in its dual form, the equation of a point. The position of the point is given by its position vector; its distance and direction are equal to the distance and direction of the normal from the origin to the position plane.

The particular case in which the problem of the position fix is made up by only three position planes is presented in its dual form by three points, generating a directed trihedral. The plane tangent at the sphere circumscribed about the trihedral is the reciprocal to the point of trisection of the three position planes. The normal to the tangent plane is the diameter of the sphere drawn through the origin of the co-ordinate system. It is evident that the locus of the final position is identical to the apex of the supplementary trihedral.

The solution of the problem of position fix, presented by a set of n position planes can be obtained from the most probable length and direction of the diameter of the sphere which is circumscribed about all trihedrals, formed by the set of points which represent the dual to the position planes. Distance and direction of the diameter corresponds to distance and length of the centre line of gravity of the system of points.

Fig. 8.5.-11. shows the tangent plane, the circumscribed sphere and points numbered according to the position planes to which they



ISOMETRIC VIEW
OF
POINTS REPRESENTING POSITION PLANES
DERIVED FROM OBSERVATIONS IN MAY 1961.
CIRCUMSCRIBED SPHERE AND TANGENT-PLANE
ARE DEFINING THE FINAL POSITION.

ISOMETRIC SCALE : 1 SEC. ARC = 0·43 INCHES.

1 0 1 2 3 sec. arc.

Fig.: 8.5.-11.

8.5. cont.

constitute the reciprocal. The points are obtained from observations in May 1961. The data for construction are given in Table 8.4.1.-1. Obviously, since the circumscribed sphere is the best fitting approximation, it passes through some points and misses others. The centre of the sphere is derived from the approximate intersection of symmetry-lines which are drawn perpendicularly from the centres of circumscribed circles defined by three points.

An approximation for the centre and diameter of the sphere can be obtained by constructing the circumscribed circle about the direction-cosines of the points.

The final corrections can be scaled on the isometric diagram from the point of tangency to the co-ordinate axes.

8.6. Azimuth Position Lines

As already mentioned in Section 8.3., azimuth position lines are based upon timed horizontal circle readings. The linear equations, linking together horizontal circle readings, observed time, tabulated values of star co-ordinates, approximate values for the co-ordinates and azimuth of the station, and their unknown corrections, were originally called by M. F. da Costa (Section 8.3.): "rectas de azimute" = azimuth position lines. This name has been used ever since for linear equations defining the position of an observer from readings of the horizontal circle in combination with known and unknown parts of the celestial triangle.

The application of azimuth position lines for deriving astronomical latitude, longitude and azimuth in the field is also dealt with by Dr. T. L. Thomas, in his University of London Ph. D. thesis. Permission was obtained from the University of London Library to quote from his thesis, and Dr. Thomas graciously agreed.

The following is a brief account of his work, but it does not cover the whole subject of his thesis.

Essentially, the differential relation of the cotangent-equation of the azimuth, timed horizontal circle readings to a star and to a terrestrial R. O., approximate azimuth, and a correction for the unknown circle orientation, are the basic components of a linear equation, containing three unknowns: viz. $d\lambda$, $d\phi$, θ.

These unknowns are designated as the corrections for the longitude and latitude of the trial point, and for the true bearing of the terrestrial R. O., respectively. The unknown circle orientation θ, is eliminated by combining two linear equations. The resulting equation in two unknowns, termed Azimuth Position Line, is here given in Dr. Thomas's notation:

$$(\Delta B' - \Delta B) = (K_1' - K_1)\Delta\lambda + (K_2' - K_2)\Delta\phi$$

8.6. cont.

where:

ΔB = approximate observed minus approx. computed bearing for star No. 1.

$\Delta B'$ = " " " " " " star No. 2.

K_1 = $(\sin\phi - \cos\phi \cdot \cos A \cdot \tan H)$ . . . for star No. 1.

K_2' = $(\sin\phi - \cos\phi \cdot \cos A' \cdot \tan H')$ . . . for star No. 2.

K_2 = $\sin A \cdot \tan H$ . . . . for star No. 1.

K_2' = $\sin A' \cdot \tan H'$ . . . . for star No. 2.

ϕ = latitude of the trial point

λ = longitude of the trial point

H = altitude

A = computed azimuth from the trial point.

Each azimuth position line is obtained from two observations; the two stars chosen are at approximately equal altitude, and the azimuth difference is within the range $(180^\circ \pm 20^\circ)$.

The azimuth position line is identified as a straight line. Therefore the problem of finding the corrections $d\phi$ and $d\lambda$ is allocated to plane geometry.

The graphical solution is expected to be found in the intersection of the azimuth position lines, plotted in a Cartesian system of co-ordinates: $d\phi$, and $d\lambda$; the origin of the system has the co-ordinates of the trial point.

The plotted position lines should all meet in one point. The corrections $d\lambda$ and $d\phi$ have to be scaled from the point of intersection to the co-ordinate axes. Reductions of observational results are included, and the graphs show the agreement of the final position obtained from azimuth position lines as well as from zenith position lines.

Thereafter the correction θ for the circle orientation is evaluated from each pair of observations and mean values of all the θ s

8.6. cont.

give the correct mean result.

8.7. Astronomical Azimuth of a Terrestrial Line from timed Observations on the Sun at any Hour Angle.

The astronomical azimuth of a direction is obtained from the computation of:

(1) the azimuth of the sun at the instant of observation,

(2) the horizontal angle between the sun and the reference object, and from

(3) the combination of (1) and (2) by addition or subtraction.

Azimuth observations consisting of measuring the hour angle of the sun were carried out at two Ordnance Survey pillar stations and at two stations of the survey scheme of Tywarnhale Mine, on different days in 1961.

Stations of known geodetic positions were chosen for the purpose of comparing the observed astronomical azimuths with calculated geodetic azimuths. Furthermore, the fieldwork aimed at the determination of the accuracy of a sun azimuth, which could be obtained when time control with the crystal chronometer was available. The timing accuracy of the sun's transit across the grid reticule with the marker pulse incorporated in the crystal chronometer was determined at the same time.

The O.S. pillar stations and one station of the mine triangulation scheme enabled the observation of reciprocal sun azimuths.

It was expected that reciprocal azimuths would prove the reliability of azimuth determination from sun observations.

8.7. cont.

Furthermore, errors due to lateral refraction should also be reduced or detected from azimuth observations at both ends.

The national grid co-ordinates of the mine survey stations were obtained from the connection of the mine triangulation scheme to the Ordnance Survey system. The necessary observations had been carried out in previous years.

The geodetic latitudes and longitudes of the O.S. pillar stations and mine survey stations were calculated from the corresponding national grid co-ordinates and reconverted to national grid co-ordinates. For these and the following calculations, projection tables for the Transverse Mercator Projection, published by the Ordnance Survey, were used. At each station the convergence of the N-S line of the national grid with the geodetic meridian was calculated from both rectangular grid co-ordinates and from geodetic latitude and longitude.

A geodetic azimuth is a computed azimuth referred to the surface of reference and to the initial station of the survey system.

The geodetic azimuth and the reverse azimuth were obtained by combining the plane rectangular grid bearing, convergence and correction for the difference of geodetic and plane direction.

Results of the computations of geodetic azimuths are given in Table 8.7.-3, column 4. The difference of the reciprocal geodetic azimuth is listed in column 6.

Theoretically, in the computation of the observed sun azimuth, the astronomical latitude and the astronomical longitude should be used. At a station where the astronomical azimuth of a line is observed, simultatneous observations for astronomical longitude enable one to obtain the quantities required for the Laplace correction. Therefore, at stations selected for Laplace azimuth or when first order accuracy is required, the astronomical latitude and longitude are observed, but normally observations to stars and not to the sun will be taken.

Because the sun was observed, results of extremely high precision

8.7. cont.

were not expected, and therefore the astronomical longitude was not determined. Consequently, in the calculations for the sun azimuth, the astronomical longitude and latitude were replaced by the geodetic longitude and latitude respectively.

The differences of reciprocal astronomical azimuths between the stations concerned vary about three seconds of arc if geodetic latitude and longitude are used instead of astronomical latitude and longitude.

At each station the field work consisted of three sets of observations. One set was made up of two pointings on the sun in each face position and of sights being taken to the reference object before and after each pointing. The first pointing on the sun, in each face position, contained five timed grid line crossings of the leading edge; at the second pointing the crossing of the other edge was timed. The total time taken by one edge of the sun to cross the grid reticule is about nine seconds. Care was taken that the main horizontal wire bisected the sun at the instant its leading edge touched the main vertical wire. The vertical circle readings therefore referred to the main horizontal wire.

The observations followed in a quick succession, and it was not necessary to apply a correction for the collimation axis error.

The mean of the C. L. and C. R. pointings practically eliminates the influence of the collimation axis error in azimuth, neglecting small amounts (one or two tenths of a second of arc) resulting from the changed altitude of the sun.

Corrections to the sun observations, which can amount to one second of arc in azimuth, were not considered. The reduction to the mean pole would amount to about 0.2". The theodolite used was a Watts Microptic No. 2. No special targets were used; O. S. survey pillars were sighted on both sides.

A specimen of the field record is shown in Table 8.7.-1. Results of field work are quoted in Tables 8.7.-2 and 8.7.-3.

DATE May 9^{th}, 1961
AT Carn Marth O.S. Pillar
R.O. St. Agnes O.S. Pillar
OBSERVER W.A.S.

INSTRUMENT Watts Microptic 2
EYE PIECE Long, diagonal
RETICULE 30" grid
COLL. ERROR see: May 10,

PLATE BUBBLE 20·2"
MID RUN 00"
TAPE, TRACK
MARKER SIGNAL Azimuth

FIELD RECORD TRANSCRIPTION FROM RE-PLAY

SIGHT	C.L. / C.R.	BUBBLE L.	BUBBLE R.	HORIZONTAL CIRCLE °	'	"	BUBBLE L.	BUBBLE R.	VERTICAL CIRCLE °	'	"	TEMP. °F.	PRESS. " Hg.	APPROX. TIME h m	U.T. (mean of five) from Tape Development h	m	s	REMARKS
R.O.	C.R.			180	07	16 38												"ON"
"	C.L.			00	07	08 30												
Sun ON	C.L.	$1{\cdot}9^{R}$	$1{\cdot}9^{R}$	233	01	43	$2{\cdot}9^{R}$	$2{\cdot}9^{R}$	49	22	58	58	29·76	15^{16} BST	14	16	50·818	
OFF	"	$2{\cdot}0^{R}$	$2{\cdot}0^{R}$	232	48	32	$2{\cdot}0^{R}$	$2{\cdot}0^{R}$	49	11	39	"	"	15^{18} "		18	42·241	
ON	C.R.	$1{\cdot}0^{L}$	$1{\cdot}1^{L}$	54	20	26	$2{\cdot}0^{L}$	$2{\cdot}0^{L}$	131	06	58	58	29·76	15^{20} "		20	59·083	
OFF	"	$2{\cdot}0^{L}$	$2{\cdot}0^{L}$	54	08	58	$1{\cdot}4^{L}$	$1{\cdot}4^{L}$	131	19	46	"	"	15^{22} "		22	56·348	
R.O.	C.R.			180	07	16 36												"OFF"
"	C.L.			00	07	08 31												

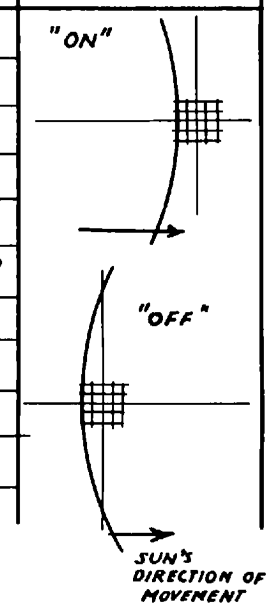


TABLE 8.7.–1

ASTRONOMICAL AZIMUTH OF TERRESTRIAL LINE

DERIVED FROM MEASURED HOUR ANGLE OF THE SUN

LINE	ST. AGNES – CARN MARTH			CARN MARTH – ST. AGNES			JAMES – ST. AGNES			'COTTAGE' – PORTHTOWAN R.O.		
DATE, TIME	MAY 8, 1961 13^h U.T.			MAY 9, 1961 14^h U.T.			MAY 11, 1961 14^h U.T.			MAY 10, 1961 13^h U.T.		
	°	′	″	°	′	″	°	′	″	°	′	″
C.L.	174	27	05·6	354	28	32·3	13	08	34·3	305	32	59·9
"		27	00·1		28	30·8		08	42·9		32	47·9
C.R.	174	26	32·3	354	26	01·1	13	08	18·1	305	32	44·1
"		26	49·4		26	05·4		08	08·7		32	44·8
C.L.	174	27	10·6	354	28	21·1	13	09	36·4	305	32	53·5
"		27	08·1		28	12·5		09	29·0		32	58·0
C.R.	174	26	40·6	354	26	30·3	13	07	42·9	305	32	23·8
"		26	36·8		26	29·0		07	08·5		32	24·9
C.L.	174	27	02·5	354	27	48·3	13	09	04·8	305	33	03·9
"		27	10·5		28	03·0		09	01·9		33	12·5
C.R.	174	26	22·1	354	26	30·9	13	07	56·9	305	32	23·0
"					26	45·4		07	58·8		32	20·7

OBSERVATIONAL RESULTS ARE QUOTED TO ONE DECIMAL OF A SECOND TO AVOID ROUNDING OFF ERRORS.

TABLE : 8.7. –2

	1 AT STATION – SIGHT TO	2 ASTRONOMICAL AZIMUTH FROM SUN OBSERVATIONS SET OF 2 C.L. & 2 C.R. OBSERVATIONS ° ′ ″	3 MEAN OF THREE SETS ° ′ ″	4 GEODETIC AZIMUTH CALCULATED FROM CO-ORDINATES ° ′ ″	5 RECIPROCAL AZIMUTH FROM SUN OBS. ″	6 RECIPROCAL AZIMUTH FROM CALC. GEOD. CO-ORDS ″	7 DIFFERENCE BETWEEN ASTRO & GEOD. AZ. ″	8 DIFFERENCE BETWEEN REC. ASTRO & REC. GEOD. AZ. ″
1	ST. AGNES – CARN MARTH	174 26 51·9 51·2 44·3	174 26 49·2	174 26 53·4	30·0	35·2	04·2	05·2
2	CARN MARTH – ST. AGNES	354 27 17·4 23·3 16·9	354 27 19·2	354 27 28·6			09·4	
3	JAMES – ST. AGNES	13 08 26·0 29·2 30·6	13 08 28·6	13 08 25·4	19·6	27·0	03·2	07·4
4	ST. AGNES – JAMES	OBSERVATIONS FROM 1	193 08 48·2	193 08 52·4			04·2	
5	CARN MARTH – JAMES	OBSERVATIONS FROM 2	345 51 32·2	345 51 46·3	49·6	62·2	14·1	12·6
6	JAMES – CARN MARTH	OBSERVATIONS FROM 3	165 50 42·6	165 50 44·1			01·5	

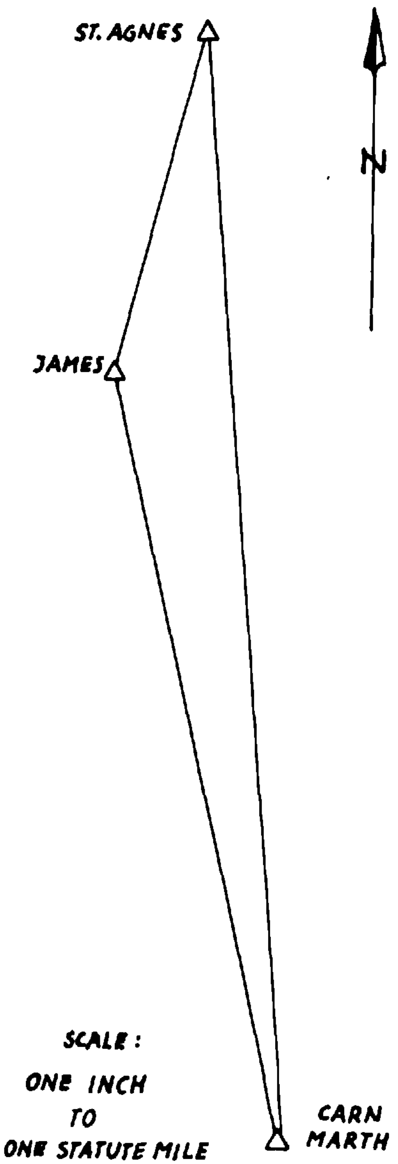


TABLE: 8.7 – 3

8.7. cont.

The azimuth from each pointing was worked out separately; corrections for dislevelment of the transit axis and reduction to the centre of the sun were applied. Sun data were extracted from the Astronomical Ephemeris. Results, given in the Tables, revealed that there was a collimation axis error and the plate bubble, which was adopted to determine the inclination of the transit axis, was undoubtedly affected by the sun's rays. The arithmetic mean of four pointings (2 C. L. and 2 C. R.) called one set, bracketed within two R. O. sightings, constitute one single azimuth observation. Results of individual sets and their means are given in Table 8.7.-3, column 2 and 3 respectively. The third set, observed at St. Agnes O. S. station, is made up of a total of only three pointings. The difference of reciprocal astronomical azimuths is listed in column 5. The difference between astronomical and geodetic azimuth (data in column 4 minus column 3) are shown in column 7, and reveal the accuracy which can be obtained from sun observations, and also the quality of angular measurement at the various stations.

It is quite obvious that an angular error of about 10" can be suspected at Carn Marth O. S. station. The source of the error may be sought either in the measurement of the angle or in its calculation from O. S. co-ordinates. The accuracy of the co-ordinates of Carn Marth, a third order pillar station, is about ± one metre.

Results quoted in column 6, minus those in column 5, are given in column 8; these are the differences between geodetic and astronomical azimuths. The average difference is slightly less than ten seconds of arc.

From the results it is obvious that the plumbline deviates to the West and North.

If the astronomical latitude and longitude had been used in the reduction of the sun observations, then the figures quoted in column 7 would have assumed larger values, and the differences shown in

8.7. cont.

column 8 would have been smaller.

The figures listed in column 7 and 8 are due not only to the different positions of the geodetic and astronomic zeniths, but also to the unequal elevation of the survey stations above sea level, to lateral refraction, motion of the pole, contraction of the horizontal semi-diameter of the sun, to the aberration of the sun, to instrumental and personal errors.

The evaluated field measurements indicate that a sun azimuth, accurate to 1/20 000 or better, can be obtained from the mean of three sets comprising a total of 12 pointings to the sun.

The twelve observations, each of five timed grid line crossings, can be taken in about 30 to 40 minutes, when it is possible to observe the sun. The observations are not restricted to specific times.

The value of this method is the observational speed and accuracy in obtaining a sun azimuth when observations to stars cannot be taken. The method can also be used where azimuth only is sought, as a check, and latitude and longitude are derived from ground survey work or scaled from a map, in which case only the geodetic position will be available.

Results of the azimuth observations (column 2) are not far off the specification of the U.S. Coast and Geodetic Survey for a Laplace azimuth, which demands that no observation should be accepted which gives a residual of 5" or greater from the mean.

The quality, or order of accuracy, of a sun azimuth obtained with ordinary engineering theodolites is generally believed to be in the neighbourhood of one minute of arc. None of the above field observations had to be rejected, and they are comparable in accuracy with results from observations made with larger instruments and even with those executed with stationary equipment installed in small observatories.

The accuracy in timing the sun's transit across the grid reticule was obtained in the same manner as for star transits (described in

8.7. cont.

Section 5.5.7). Sun observations at Carn Marth and at St. Agnes O.S. pillar stations proved that an average timing accuracy of $\pm$ 0.084 seconds of time corresponding to $\pm$ 1.25 seconds of arc along the main horizontal cross wire, was achieved at any one pointing on the sun. The above figure is the average r.m.s. error of five timed grid line crossings of the sun's left or right limb.

It would be misleading to endeavour to determine also the co-ordinates of the station simultaneously with the azimuth from the above observations, which were taken within a short period of time.

A position fix from timed horizontal directions to the sun requires the observations to be taken at least two hours apart. The possibility of the simultaneous measurement of vertical angles permits zenith position lines to be added on the plot. The procedure for a position fix from horizontal directions would be to use the superfluous observations and approximate values of the co-ordinates and azimuth. The three corrections required for the final value of latitude, longitude and azimuth expressed as differential changes of co-ordinates, and difference between observed and approximate azimuth, could be obtained graphically or analytically.

The above field observations for azimuth can be represented by a mathematical model made up by three normal equations, should a least-squares solution be adopted. The model would contain in each independent equation nearly identical ratios of the coefficients of the unknowns. In other words, there would be weak linear independence of the equations, and a possible solution would give an unreliable answer.

In a graphical representation of the solution the three unknowns constitute the axes of a three dimensional system. The three equations in the three unknowns, made linear, show up as three independent planes. Their intersection lines, projected on to the co-ordinate planes of the system, would have nearly identical sloping positions in the same direction. Consequently, the linearized equations in the

8.7. cont.

three unknowns would fail to intersect in a definite point.

8.8. Laplace Equation; Computation of the Deviation of the Plumb Line

Results of observations in field astronomy can be used for the assessment of geodetic measurements.

Geodetic measurements, linear and angular, aim at the relative fixation of points on the earth's surface.

Nowadays, distances up to 50 miles can be measured to an accuracy of $\pm$ 2 inches, with the application of electronic methods. An angular measurement to an accuracy of $\pm$ 0.1 second of arc over this distance would produce an uncertainty of $1\frac{1}{2}$ inches in position.

Field astronomy deals with angular measurements only; these are referred to the plumbline, and hence to the centre of the earth.

An astronomical position fix derived from angular measurements, executed to $\pm$ 0.1 second of arc, and presuming the earth's radius = 3 959 miles, contains an uncertainty of about 10 feet; this is approximately 80 times less accurate compared to a geodetic fix.

Consequently, results of astronomical observations cannot be used for comparing positions, but astronomical observations produce absolute orientations which cannot be obtained with geodetic methods.

The astronomically defined absolute orientation is used for comparing the propagated orientation in a geodetic system of triangulation or traverses. The element of comparison is the deviation of the plumbline, which is the angle formed by the direction of the plumbline at the observer's station, and the normal to the spheroid of reference in the corresponding point.

8.8. cont.

The correct term would be "deviation of the normal to the spheroid of reference".

The deviation of the plumb line is dealt with in Section 5.3. In practice, the relative deviation of the plumb line is expressed by its components; these are formed by the difference between the astronomical longitude, latitude and azimuth, reduced to the geoid, and the geodetic longitude, latitude and azimuth, reduced to the spheroid of reference, respectively.

The component of the relative deviation of the plumbline along the meridian is the difference between the latitudes, and the east-west component can be determined from the difference of the longitudes or azimuths. The possibility of a twofold determination of the east-west component establishes a restriction or condition, as far as observations are concerned.

The sine equation of the spherical triangle formed by the pole, the astronomical and geodetic zeniths, is known as the Laplace equation:

$$\Delta A - (\lambda_A - \lambda_G). \sin \phi = 0.$$

λ_A = astronomical longitude

λ_G = geodetic longitude

ΔA = angle between geodetic and astronomical meridian.

A_A = astronomical azimuth

A_G = geodetic azimuth.

ϕ = geodetic latitude.

ΔA can be substituted by the difference between astronomical and geodetic azimuth. Theoretically, this is permissible if both projections of the R.O. on to the geoid and on to the spheroid of reference are contained in the same vertical plane of the spheroid of reference. Furthermore, the above formula requires a 90° zenith

8.8. cont.

distance of the direction whose azimuth enters into the calculation.

The general Laplace equation:

$$(A_A - A_G) - (\lambda_A - \lambda_G) . \sin\phi \neq 0$$

expresses the discrepancy which is due to the azimuth-component of the deviation of the plumbline.

The theoretical expression for the azimuth-component requires astronomical and geodetic measurements to be free from errors and reduced to the geoid and spheroid of reference respectively. The evaluation of the answer obtained from the Laplace equation meets the task of distinguishing between errors of stellar observations and errors of angular measurements. The importance of the Laplace equation lies in its application as azimuth condition equation. Furthermore, all geometric solutions to the problem of defining the figure of the earth are effected be comparing astronomic and geodetic values, which are expressed as the deviation of the plumbline.

The Laplace azimuth $\equiv$ geodetic azimuth, derived from the observed astronomical azimuth:

$$A_G = A_A - (\lambda_A - \lambda_G) . \sin\phi$$

resulting from field observations in Cornwall, is given in Table 8.8.-1, column 9.

Discrepancies between Laplace azimuths and geodetic azimuths are listed in column 10. The consistency of the values in column 10 indicates that there is an error of about $1\frac{1}{2}$ seconds of arc; this error is most likely to be sought in the geodetic azimuth listed in column 8, which is obtained from mine triangulation observations and from O.S. co-ordinates. The discrepancy between the Laplace azimuth and the geodetic azimuth at St. Agnes pillar station is not excessive, considering that the astronomical azimuth is derived from sun observations. The deviation of the plumbline, or better, the deviation of the normal to the spheroid of reference, is generally accepted to be a fictitious value, because the spheroid of reference is an assumed

1	2	3	4	5	6	7	8	9	10	11
AT	DATE	OBSERVED STARS SUN	LATITUDE ϕ_A ASTRONOMICAL LATITUDE ϕ_G GEODETIC LATITUDE	LONGITUDE λ_A ASTRONOMICAL LONGITUDE λ_G GEODETIC LONGITUDE	$\Delta\lambda$ $\lambda_A - \lambda_G$	$\Delta\lambda . \sin\phi$	AZIMUTH A_A ASTRONOMICAL AZIMUTH A_G GEODETIC AZIMUTH	LAPLACE AZIMUTH $A_{G_L} = A_A - (\lambda_A - \lambda_G)\sin\phi$	A_G (col. 8) minus A_{G_L} (col. 9)	CALCULATION
JAMES			ϕ_G = +50°16′47″8439	λ_G = −5°13′33″793929			JAMES to S^t AGNES A_G = 13° 08′ 25″38			
	11.5.1961	STARS	ϕ_A = +50°16′50″7	λ_A = −5° 13′39″2	−5″4	−4″2	A_A = 13° 08′ 24″2	13° 08′ 28″4	−3″0	POSITION LINES FROM ZENITH DISTANCES
	11.5.1961	SUN					A_A = 13° 08′ 19″4	13° 08′ 23″6	+1″8	HOUR ANGLE
COTTAGE			ϕ_G = +50°16′46″611	λ_G = −5°13′56″19849			COTTAGE to PORTHT.^{n} RO. A_G = 305° 32′ 38″2			
	1.5.1961	STARS	ϕ_A = +50° 16′ 46″5	λ_A = −5° 14′ 00″9	−4″7	−3″6	A_A = 305° 32′ 32″2	305° 32′ 35″8	+2″4	POS. LINES FROM ZENITH DISTANCES
	5.5.1961	"	+50 16 47·4	−5 14 01·6	−5·4	−4·2	305 32 32·3	305 32 36·5	+1·7	"
	9.5.1961	"	+50 16 46·6	−5 14 00·9	−4·7	−3·6	305 32 33·3	305 32 36·9	+1·3	"
	5 & 9. MAY 1961	"	+50 16 46·6	−5 13 58·1	−2·0	−1·5	305 32 35·2	305 32 36·8	+1·4	POS. LIN. HOR. DIRECTION
	"	"	+50 16 46·7	−5 13 58·1			305 32 35·2			LEAST SQUARES HOR. DIRECTIONS
	"	"	+50 16 46·6	−5 13 58·4			305 32 35·6			POS. PLANES HOR. DIRECTION
	10·5·1961	SUN					A_A = 305 32 37·2	305 32 38·7	−0·5	HOUR ANGLE
S^t AGNES			ϕ_G = +50°18′24″241196	λ_G = −5°12′58″654687			S^t AGNES to JAMES A_G = 193° 08′ 52″41			
			ϕ_A = +50°18′25″3	λ_A = −5°13′02″2	−3″5	−2″7				
	8.5.1961	SUN					A_A = 193° 08′ 43″3	193° 08′ 46″0	+6·4	HOUR ANGLE

* DERIVED FROM MINE TRIANGULATION AND ORDNANCE SURVEY CO-ORDINATES

GEODETIC AZIMUTH, ASTRONOMICAL AZIMUTH, LAPLACE AZIMUTH.

TABLE 8.8.-1

8.8. cont.

surface.

The spheroid of reference may be considered as a surface of revolution which approximates the whole earth, with its main axes coinciding with the axes of the geoid. If so, the deviation of the plumbline represents the deviation of the geoid (its undulation) from the reference surface.

The new tendency aiming at the replacement of the geoid by the physical surface of the earth will in due course lead to the universal adoption of a world spheroid of reference, in consequence of which the deviation of the plumbline will have to be regarded as the undulation of the earth's surface, and will lose the odium of a geodetic fiction.

Fig. 8.8.-1 shows the direction of the astronomically determined deviation of the plumb line, not reduced to the spheroid of reference, obtained from field observations in Cornwall. Data are contained in Table 8.8.-1.

The component of the deviation along the meridian is:

$$\xi = \Delta\phi$$

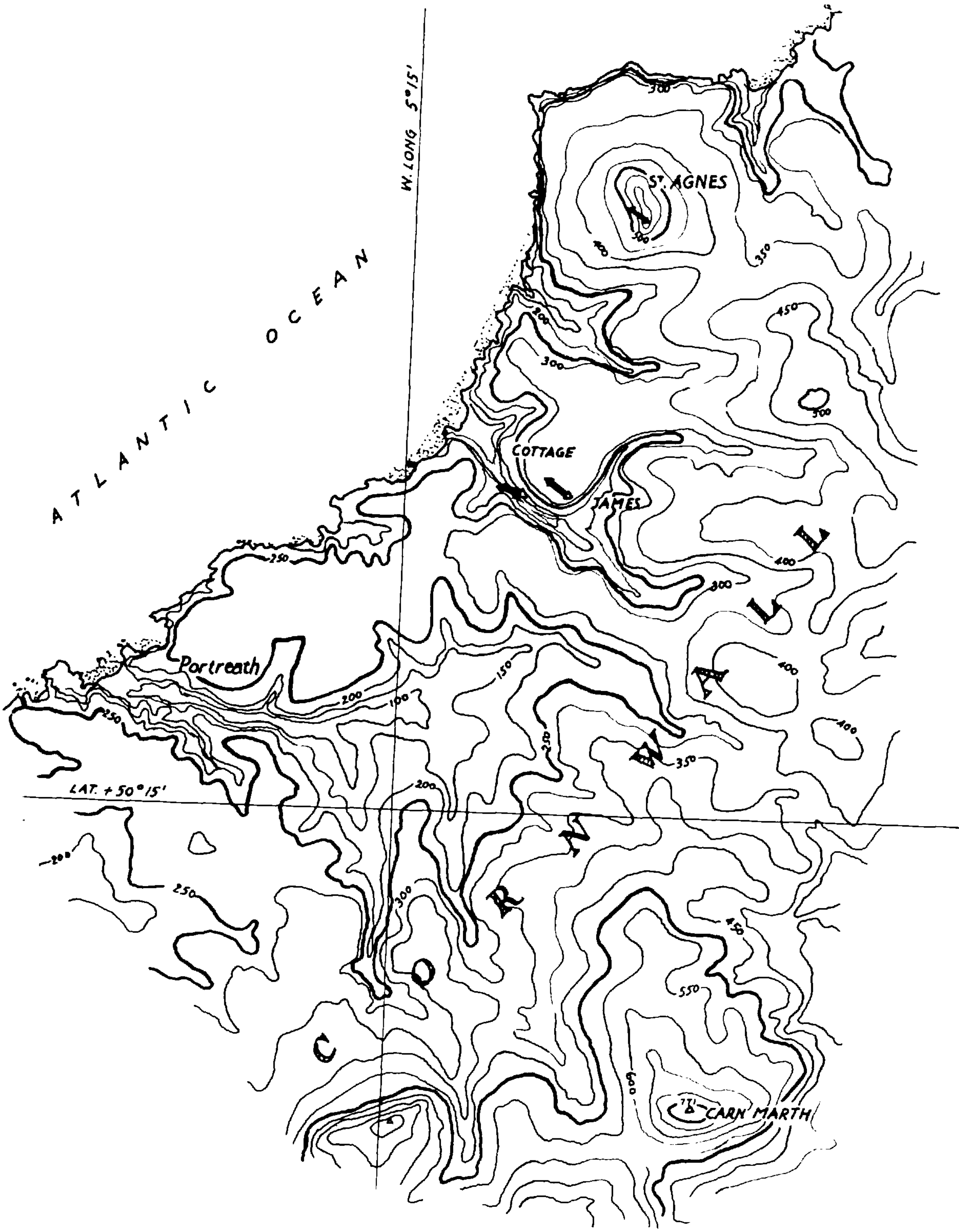
and the component in the prime vertical is:

$$\eta = \Delta\lambda . \cos\phi$$

In the diagram, the amount of the deviation is plotted twice as large, for better presentation.

The values quoted for the deviation of the plumb line can, to a certain extent, be verified by gravity measurements. It can be anticipated that gravity measurements will prove that the general direction of the deviation of the plumbline (shown in Fig. 8.8.-1) is correct. Gravity measurements were outside the scope of this thesis.

DEVIATION OF THE PLUMB LINE
DERIVED FROM ASTRONOMICAL OBSERVATIONS



SCALE: ONE INCH = ONE STATUTE MILE

Fig.: 8.8.-1

8.9. Corrections for Polar Motion.

Variations of the observer's meridian and parallel of latitude result from polar motion. Astronomical observations refer to the instantaneous axis of rotation and to the instantaneous equator.

Astronomical co-ordinates obtained from precise observations which are taken in time intervals of one or more years, can be reduced to a mean position. This is useful for permanent records and for better comparison of observational results spread over several years.

Corrections for variations in longitude and latitude of the place of observation are calculated from its co-ordinates and from the co-ordinates of the instantaneous pole.

Provisional values of the rectangular co-ordinates of the instantaneous pole are published in the Time Service Circulars of the Royal Greenwich Observatory, and final values are given in Royal Observatory Bulletins. The co-ordinates are referred to the mean pole, which is represented by zero co-ordinates in the Chart of the International Latitude Service. The positive direction of x is reckoned from the mean pole towards Greenwich and the positive direction of y is 90^0 west of Greenwich.

Fig. 8.9.-1 shows the position of the instantaneous pole in relation to the mean pole. The required correction for the latitude obtained from astronomical observations can be read off the graph:

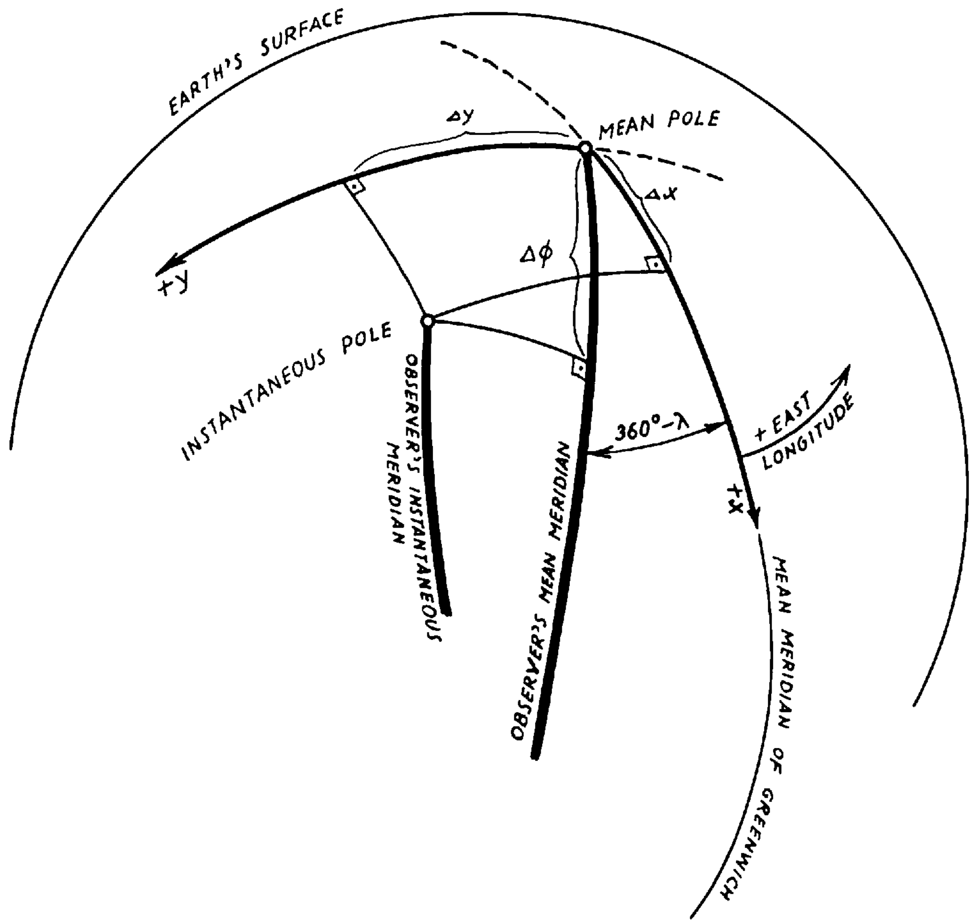
$$\Delta\phi = x.\ \cos\lambda\ -\ y.\ \sin\lambda$$

$$\phi_{mean} = \phi_{inst.} - \Delta\phi$$

where: x, y = co-ordinates of the instantaneous pole

λ = longitude of the observer's station measured positively eastward.

ϕ_{mean} = latitude of the observer's station referred to the mean pole



DISPLACEMENT OF THE POLE

Fig.: 8.9.-1

8.9. cont.

$\phi_{inst.}$ = latitude of the observer's station referred to the instantaneous pole.

$\Delta\phi$ = correction.

The correction for the longitude is:

$$\Delta\lambda = (x.\ \sin\lambda + \cdot y.\ \cos\lambda).\ \tan\phi \quad ;$$

the correction for the azimuth:

$$\Delta A = (x.\ \sin\lambda + y.\ \cos\lambda).\ \sec\phi$$

The correction to be applied to the co-ordinates and azimuths determined in 1961 and 1963, and their differences in seconds of arc, are given below:

Correction	1961	1963	Difference
Latitude	+ 0.03 "	- 0.01"	0.04"
Longitude	- 0.12 "	+ 0.16"	0.28"
Azimuth	- 0.15 "	+ 0.21"	0.36"

8.10. Corrections for Aberration.

The "Apparent Places of Fundamental Stars" contain the geocentric co-ordinates of the main stars, which are right ascension and declination expressed as functions of time. These co-ordinates are corrected for

(a) the precession of the pole of the ecliptic
(b) proper motion of stars.
(c) annual parallax, because the centre point of the star's sphere is shifted from the sun and placed in the centre point of the moving earth, and
(d) annual aberration, which is caused by the velocity of light.

Since the tabulated interval is about 10 days, the short period nutation cannot be taken care of and has to be worked out. As mentioned, the origin of the tabulated star co-ordinates is the centre of the earth; it follows that corrections have to be applied to co-ordinates which can be observed from, or deduced for, observation stations situated on the surface of the earth. These are:

(a) Diurnal parallax, = angle subtended at the heavenly body between observer and centre of the earth;
for stars: negligible;
for the sun and planets: the earth can be accepted as spherical.
for the moon: the earth's spheroidal form has to be taken care of.

(b) Diurnal aberration = the amount of stellar displacement at the instant of observation, which is caused by the relative movement of the observer's position with respect to the centre of the earth, and depends fundamentally on the finite velocity of light.

During the time the light takes to travel from the star to the observer, the system of reference - the observer's position - can be presumed to be in uniform rectilinear motion; consequently the Galilei - Newton

DIAGRAM SHOWING DIURNAL ABERRATION AS SEEN FROM OUT SIDE OF THE CELESTIAL SPHERE.

THE RATIO OF THE VELOCITY VECTORS IS NOT TO SCALE, AND DIMENSIONS ARE MISREPRESENTED FOR CLARITY.

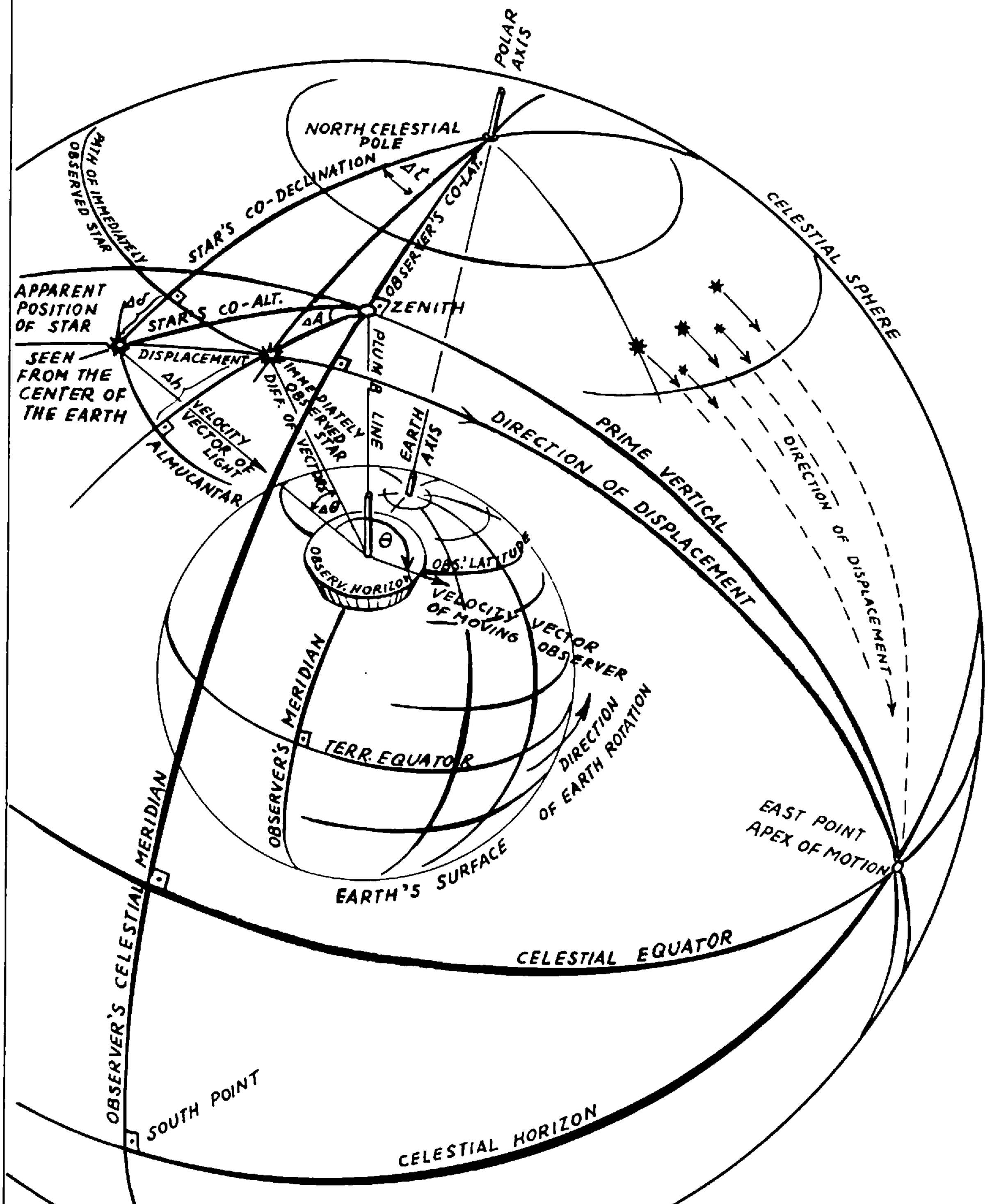


Fig. 8.10.-1

8.10 cont.

theory of mechanics can be applied.

As will be seen on the diagram (Fig. 8.10.-1), the displacement $\Delta\theta$ of the apparent position of a star can be calculated by solving the triangle which it forms with the velocity vectors. The ratio of the vectors is proportional to the sineof the angles:

$$\frac{v.t}{c.t.} = \frac{\sin \Delta\theta}{\sin(\theta - \Delta\theta)}$$

where:

v = velocity of the observer relative to the centre of the earth.

c = velocity of light.

t = light time (time required by the light to travel from the star to the observer.

θ = angle formed by the velocity vector of light and the velocity vector of the moving observer at the instant of observation.

$\Delta\theta$ = angular displacement during light time t.

The direction of movement of the observer is from west to east. The apex of motion is therefore the east point on the observer's horizon, where the direction of the displacement $\Delta\theta$ is pointing. The east point can thus be regarded as the vanishing point of the instantaneous displacement arcs. (= part of great circles passing through each star and east point at the instant of observation.) The right ascension of the apex of motion is equal to the instantaneous local sidereal time + 90°.

The effect of the displacement on azimuth and altitude, or on hour angle and declination, depends on the location of the observer and the position of the star. It can be calculated by solving the triangle formed by the displacement arc and zenith or pole respectively.. With approximation for small angles and replacing the geocentric latitude by the geodetic, the following formulae for corrections are obtained:

8.10 cont.

For topocentric co-ordinates:

Corrections to observations:

Displacement in azimuth: $\Delta A = 0.''32 \cos\phi \;.\; \cos A.\; \sec h$ . . (1)

" " altitude: $\Delta\, h = 0.''32 \cos\phi \;.\; \sin A.\; \sin h$ . . (2)

For equatorial co-ordinates:

Corrections to star places, or to observed time:

Correction to hour angle
or to observed time : $\Delta\, t = 0.''32 \cos\phi \;.\; \cos t.\; \sec\delta$ . . (3)

or to right ascension
(with opposite sign) : $\Delta\alpha = 0.^{s}0213 \cos\phi\,.\,.\cos t.\sec\delta$ . . (3)

Correction to declination: $\Delta\delta = -0.''32 \cos\phi \;.\; \sin t.\; \sin\delta$ . . (4)

These corrections are applicable where the azimuth is reckoned from N to E to S to W from 0° to 360°, and the hour angle from S to W to N to E also from 0° to 360°, or 0^{hrs} and $24^{hrs.}$

From the diagram, Fig. 8.10.-1, it can be seen that:
The correction in Azimuth for stars north of the zenith is negative.
" " " " " " south " " " " positive.
There is no correction in Azimuth for stars on the prime vertical.

The correction in altitude for stars east of the meridian is positive.
" " " " " " west " " " " negative.
There is no correction in altitude for stars on the meridian.

The correction for the hour angle between 18^h and 6^h is positive.
" " " " " " " 6^h " 18^h " negative.
" " " " right ascension has opposite sign.
There is no correction for the hour angle at $6^h0^m0^s$ and $18^h0^m0^s$
" " " " " right ascension " " " " " " "

8.10 cont.

The correction for the declination for east stars is positive.
" " " " " " west " " negative.
There is no correction for declination for stars on the meridian.

Although these are small corrections, it has been considered useful to include this somewhat detailed description, because the majority of text books on surveying and geodesy give little explanation of aberration. Most publications quote only formulae (1) and (3), the latter adapted for meridian transits, consequently omitting the correction for the declination. (See: footnote).

The brief treatment of the aberration in the literature, where mostly corrections for meridian transits are given, is due to the fact that the amount of correction required is always small. Furthermore, precise observations to heavenly bodies outisde the meridian, i.e. in any position, are as yet seldom adopted by field engineers.

Footnote:

Even advanced text books do not give a full account:

Clark, Vol. II, 3rd ed., p. 73,86 gives formulae (1) and (3) for meridian transits;
Clark, Vol. II, 4th ed., p. 101, 86, as above;
Ingram, Geodetic Surveying, quotes formula (3) only;
Chauvenet, Spherical and Practical Astronomy: formulae (3) and (4) for star places;
Bomford, Geodesy, quotes (3) and (4) for star places and (1) for azimuth;
Jordan, 8th ed., quotes (3) and (4) for star places, and (3) adapted for meridian transits;
In the Apparent Places of Fundamental Stars, table VII "Diurnal Aberration" gives the time correction (correction to right ascension) only for meridian transits, with arguments latitude and declination.

8.10 cont.

The procedure adopted to eliminate the influence of aberration to the observations taken, relevant to this thesis, is to correct:

(i) the hour angle (t), i.e. the time of observation. This is done by applying the correction from formula (3), with opposite sign, to the right ascension (α),

and

(ii) the declination (δ) with formula (4).

This is important for comparing each individual field measurement of altitude and azimuth with calculations. By reducing the star position, only two corrections have to be worked out; otherwise all measured quantities, time, altitude and azimuth would require a correction, which would necessitate additional work.

This procedure should also be adopted where the altitude has not been observed, or calculated. When lay-out work in the field is not immediately wanted, one final correction for the field station can be calculated for the mean result, if required.

Effect of aberration on the determination of azimuth and position of the observer.

The influence of aberration on the hour angle (Δ t) is equivalent to a timing error. For Polaris observations the timing error resulting from aberration can amount to 10" at latitudes 60° (depending on azimuth) increasing for lower latitudes, and approaching 20" nearer the equator.

This error for Polaris and for a dozen other stars listed in A. P. with $\delta > 86^{\circ}$ will be reduced by its sixtieth part (multiplied by the cosine of declination) when entering into the azimuth.

The azimuth error due to timing is obtained from differentiating the expression for azimuth as function of latitude, parallactic angle and declination, with respect to time.

$$dA = \frac{\cos \delta}{\cos h} \cdot \cos q.\ dt$$

It follows that the aberrational effect in azimuth (for all azimuths) for Polaris observations in all latitudes is about 0.32". (Except near the pole where the error is rather indeterminate.)

In practical field work the maximum aberrational azimuth error will approach 1" only nearer the equator (5° to 10° latitude) in the range of 60° altitude for stars nearer the meridian. Equatorial stars at meridian transits in low latitudes are normally avoided.

For all other latitudes the maximum azimuth error resulting from aberration, as can be seen when substituting Δt (timing error due to aberration) in the above formula, occurring at meridian transits, $q = 0^\circ$, is practically between the values 0.2" and 0.3".

Effect of Aberration on Latitude:

The aberrational timing error, as any timing error, will enter as a reduced amount into the error of the calculated altitude:

$$dh = \cos \phi \cdot \sin A.\ dt$$

and this in turn will enter the latitude error according to the star's position in azimuth:

$$d\phi = \cos \phi \cdot \tan A.\ dt,$$

which means that east and west stars are unsuited for latitude observations.

From the above differential relation is seen that for quadrantal stars at higher latitudes the resulting latitude error will not reach 0.3", and will be negligible for north and south stars.

8.10 cont.

Effect of aberration on Longitude:

Since longitude is time, the aberrational timing error will enter the longitude error with its full amount, which is for all practical observations 0.1" to 0.3".

Hence, for azimuth and position fixing aiming at an accuracy of one second of arc, the correction for aberration has to be applied.

9. Field Work, Instruments, Office Equipment.

The photograph (Fig. 9.-1) shows the arrangement in the early field experiments in Cornwall in 1961; the theodolite, Watts No. 2, is set up at "Cottage" mine survey station, for star observations. The crystal chronometer is mounted on the battery container to which the charging unit is attached. Audible chronometer and observation pulses are recorded via the loudspeaker (on the right) on to the Fi-Cord tape recorder. The microphone is placed near the loudspeaker. The voice can carry so far, for theodolite readings, announcements, etc., to be recorded at the same time. The rest of the field outfit, thermometer, barometer, are placed on the table, carrying cases stored nearby.

Apart from the usual care in setting up and levelling the instrument, starting the chronometer, recording transmitted time signals for the necessary time link, no preparations for the observations, such as star list or star programme, were necessary. No special arrangements were made in positioning or securing the telescopic tripod legs.

The observation stations, which belong to the mine triangulation scheme, are marked with iron rods concreted in, and are not likely to be destroyed or to move in the years to come.

The following sequence of recording or storing of field data on magnetic tape was adopted when observing for astronomical fixes:

1. Subject heading of observation.
2. Place of observation (triangulation beacon, survey station, etc.)
3. Instrument used, bubble value, etc.
4. Name of observer
5. Remarks on weather conditions.
6. Date of observation: day, month, year.
7. Name of reference object (R.O.)
8. Face position.
9. Horizontal circle readings to R.O.

9.(a) Vertical circle readings to R.O. and readings of the plate bubble,

9. cont.

in case of a steep sight to R. O.

Data for each star observed:

10. Name of star (if identified) and position east, west, etc.
11. Face position.
12. Type of marker signals, direction of star movement.
13. Readings of: thermometer
14. " " barometer
15. " " plate bubble
16. Date: hour and minutes, as indicated on the crystal clock face.
17. Signals from crystal chronometer (must contain the minute pulse and are recorded during 18).
18. Marker signals of star's crossings
 and:
19. Readings of wire crossings.
20. Readings of: plate bubble
21 " " vertical circle
22. " " alt. alidade bubble (if necessary)
23 " " horizontal circle
24. (15) to (23) are repeated in the above sequence.
25. Readings of horizontal circle to R.O.
 After transiting:
 (8) to (25) in the above order.

Star observations were performed with the Watts No. 2 Optical Micrometer theodolite, and Watts No. 3 Geodetic theodolite. The horizontal angles in the triangulation scheme of the mine were measured with a C. T. S. Tavistock theodolite reading to one second of arc, the directions of the O. S. triangulation net were observed with the Watts No. 2.

The instrument constants, where required, were determined with a micrometer head built by Hilger and Watts, with a C. T. S. collimator,

Fig.: 9.-1

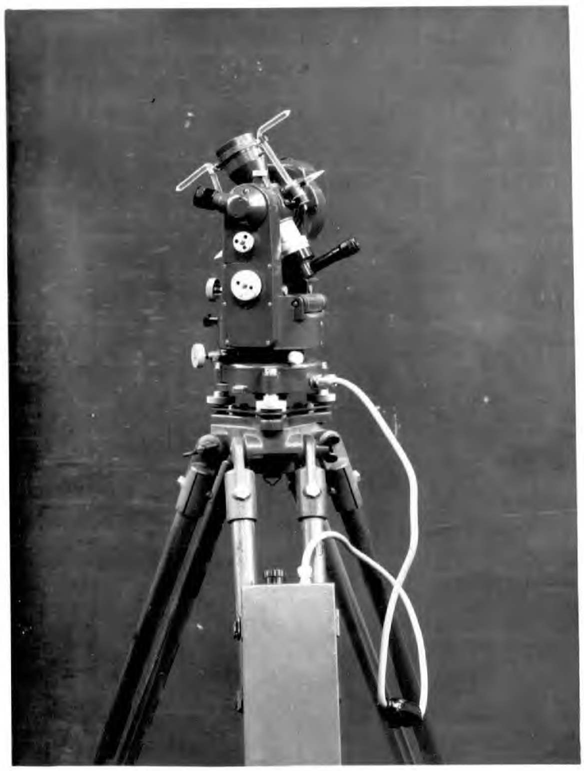


Fig.: 9.-2

9. cont.

and with an autocollimator at Messrs. Hilger and Watts' work shop.

Much time was wasted in early field experiments lining in the stars with the ordinary telescope sights provided by the makers. Great skill was required to point the telescope to stars, even at moderate angles of 30° to 45° because, invariably, the open telescope sights are too close to the telescope tube and too large in diameter, and no provision is made for their illumination.

The field procedure adopted for the experiments called for speed in observation; shortest time intervals between observations in changed face position are essential to the method of position fix, for reasons outlined in previous sections.

Efficiency in lining up the stars and improvement in operating speed was achieved with open sights of transparent synthetic resin, illuminated by the telescope lamp.

The idea of replacing the metallic telescope sight by light conducting rods, to cope with the requirement of illumination, originated from the well known application of translucent material in medicine for illuminating areas which are difficult of access. In solid rods, light - electromagnetic waves - is transmitted in the same way as electricity through cables.

Basically, the light from the telescope lamp, intended to illuminate mainly the cross wires, is also conducted by total reflection through flexible transparent media to four points of emergence. The rods are attached to the telescope from outside with steel clips. The material, ; synthetic resin, has a higher refractive index than the surrounding air; this makes "ideal total reflection" theoretically possible. The light conducting quality is therefore affected mainly by absorption by the material and by reflection losses on the resin-air surface.

Built-in light conductors which could illuminate the telescope sight, diaphragm, readers, bubbles etc., from a central light source, would have to be protected by a sheath of lower refractive index.

9. cont.

Fig. 9.-2 shows the illuminated star sight attached to the Watts Microptic No. 2 theodolite, as were used in the field. The plastic rods are positioned closely to the telescope lamp to provide for wide angular aperture on the entry end.

Field experiments decided adequate rod diameter, length of base subtended between fore and back sight; and height of the sights. A rod diameter of 0.2 inches is sufficiently wide to permit a clear sight of the night sky through the curved cylindrical surface; the rod forming a column of reflected dots of dim stray light. Very thin rods (diameter of few millimeters) are not robust enough, form an obstructing light-line against the sky, and should be avoided for safety reasons also.

Theoretically it should be possible to transmit light through a rod of any width, as long as its diameter is larger than the length of the light wave transmitted. Shiny and opaque rods and various shapes of the exit ends were tested. Polished rods with unpolished ends proved to be very adequate in the field.

The "exit cones" which can be recognised on the photograph are different for back and fore sight, for easier alignment, and form ideal pointers in the dark, when their unpolished surfaces are filled with "cold" light.

Provision for altering the light intensity to suit sighting conditions is usually provided by a rheostat built in the circuit. An advantage of the illuminated star sight is that at an elevated telescope position, enough light is thrown on to the plate bubble to enable the field engineer to read it. No publications are known to exist describing star sights of synthetic resins which can be illuminated by the telescope lamp. The application is believed to be new.

Temperatures were measured with two mercury thermometers.

At the mine office the air pressure was recorded on a barograph requiring interpolation between 5 millibars; at the field station the air pressure was registered on an aneroid barometer.

9 cont.

The reception of time signal transmission was obtained on a wireless receiver, U.S. Army Signal Corps Laboratories, Fort Monmouth, N.J., in the frequency range of 1500 to 18 000 kc/s, adapted to be operated from the mains, or from a 12 volt car battery.

The aerial for the wireless receiving set, 46 feet long, was stretched between mine buildings.

The tape recorders used are dealt with in previous sections.

The reference object, "Porthtowan" R.O., is a co-ordinated survey station of the mine survey scheme, and was illuminated by a kerosene beacon lamp

Field trials included also sights to a reference mirror, built by Hilger and Watts. The sights were taken with the aid of an autocollimator in the diagonal eye piece of the telescope.

In the absence of an illuminated reference object, or for replacing a reference object situated at a short distance, or requiring a steep sight, the mirror has, no doubt, some advantage, but it necessitates an additional angle to be measured. Further advantages of the mirror are that its height and position can be chosen to suit the theodolite set up and environmental conditions, e.g. wind direction. When sighting the mirror, the telescope can be kept focussed at infinity; no provision for the R.O. illumination is required. The mirror can be placed conveniently near the theodolite (maximum distance 50 feet) to eliminate the effect of ground haze at night.

If the mirror set up is disturbed during the night observations, the R.O. is lost, and the circle orientation has to be re-established.

The corrections considered in the computations were mostly calculated and eliminated separately, provided that each correction was so small that second order terms were negligible. Some of the corrections were calculated on a 12" slide rule.

In astronomical computations the number of digits involved is somewhat excessive. Slide rules and numerical tables are inadequate

9. cont.

to meet the required accuracy. Furthermore, calculating aids such as slide rules, logarithmic tables, numerical calculating tables etc., are far surpassed by calculating machines.

The mental work of addition, subtraction, multiplication and division, which produces fatigue in the long run, can be greatly assisted by the use of a desk calculator; a considerable amount of time is also saved. Division and multiplication with a modern calculating machine are automatically performed after the registers, the multiplier or the divisor, are set. The majority of the calculations were performed on a Monroe, Model 66 N, automatic desk calculator.

The formulae used throughout the computation are adapted for machine computing; natural values of trigonometrical functions were taken from Peter's eight figure tables. Interpolation, where necessary, was performed on the desk calculator.

Co-ordinates of star places and their corrections were extracted from Apparent Places of Fundamental Stars, sun data from Ephemeris.

Delays and corrections concerning transmitted time signals were obtained from Royal Greenwich Time Service Circulars, from the Royal Greenwich Bulletins, and from the Bulletin International de l'Heure.

Calculations, outlined in the various computation forms illustrated in this thesis, are easily set out on a programme for an electronic computer. The extraction of data from Apparent Places and of the time from tape development cannot yet be done by an electronic computer. The amount of time and work involved in the rest of the calculations, consisting of additions, subtractions, multiplications and divisions, does not necessarily require a computer programme.

10. Astronomical Fixations: Applications.

Astronomically determined latitudes and longitudes are extensively used in Geodesy for obtaining adequate Laplace control for a rigid survey frame which may or may not be connected to a national grid.

Electronic methods of distance measurement are replacing triangulation schemes and the work of angular measurement involved in it. Trilateration, where no angles are measured, requires information as regards initial position and initial direction, and control surveys for the deviation from the initial direction. Both can be provided adequately by the methods developed in this thesis.

Most tellurometer traverses, or other traverses adopting electronic survey equipment, still require the employment of a theodolite for the measurement of vertical angles. The astronomical orientation of such traverses and the astronomical co-ordinates of the initial station would require additional equipment at only a fraction of the cost of the initial expenditure.

Astronomical methods are also most economical for establishing the essential orientation in difficult terrain, where stations may not be intervisible, and their connection to an existing survey system may not be feasible with ground survey methods, for reasons of time and costs.

The task of staking out a parallel of latitude, for boundaries and other purposes, is essentially a problem of defining position and azimuth astronomically.

The determination of azimuth and of the absolute location is important in unexplored territory for protecting mining claims from encroachment when maps are not available. For reasons of competition, for adequate protection of future mining rights etc., long term ground survey programmes are in most cases not undertaken. In the absence of geodetic control and with lack of maps, air photographs become important and often constitute the only information available. Without control, the photo scale is rather unreliable, and the

10. cont.

orientation and the station error remain unknown.

It has become standard practice during the last decades to stake newly discovered mineral deposits by flying in surveying crews. Their task consists in defining the location and orientation of groups of claims. Accurate position fix has undoubtedly a great advantage to ensure the best location and maximum protection of future mining concessions with the minimum number of claims necessary to cover the prospective area.

In the dawn of the age of artificial members of the solar system with an expected permanent life time, precise and rapid astronomical methods for position fixing will be applied for the determination of the orbit of satellites from known positions on the earth's surface, should optical observation be feasible. And conversely, satellites, obviously outside the influence of the earth's atmosphere, can then be used, once their ephemerides are determined, for position fixing by astronomical methods.

The satisfactory results obtained in position fixing, with the application of the methods developed in this thesis, clearly indicate that the outcome of the research undertaken can also be usefully adopted wherever precise measurement of small time intervals, and the reference of an instant of time in terms of U. T., are required.

The application of the measurement of precise time intervals and precise timing is continuously increasing in the field of rock mechanics.

Geodetic astronomy is not only the science of surveying and portrayal of the earth's surface, as it used to be in the last century, but nowadays it deals also with the gravity potential of the earth's field for which accurate location of the observer's position is required.

11. Conclusions and Future Outlook.

Important results from theoretical investigations, laboratory experiments, field trials and observations are given in their relevant sections, with conclusions and new findings which appear to be original.

All observations, tests, calculations, drawings and diagrams carried out for this thesis were undertaken by the author unless otherwise stated.

The intention of this research, to approach the "lowest limit" of time subdivision and to make it available for precise work in the field, as well as its correlation to U. T. as determined by an observatory, has undoubtedly been achieved.

Results from field observations clearly indicate that the application of the method of timing, recording and evaluating, developed in this thesis, produced precise astronomical position and azimuth (within seconds of arc), in less time and with less effort than by conventional methods.

The crystal chronometer, built to specification, performs sufficiently well to be used as a secondary time standard in small observatories. The precision with which U. T. and the subdivision of time intervals are made available at the observer's station surpasses even the requirements of present day field astronomy, and can be utilized also for astronomical work of the first order, and for precision measurements of magnitudes related to time and epoch.

Standard methods of reduction of observations are analysed and the method of position planes, believed to be original, provides a satisfactory and reliable solution of position fixes; these contribute, due to their precision, to the knowledge of the geoid, as practical results indicate. The necessary corrections required in the reduction of observations are discussed, and a method of approach

11.cont.

to investigate the personal error is given. A more efficient use of accurate time in field astronomy will be possible when the star co-ordinates are fully corrected in the Apparent Places of Fundamental Stars, which can be anticipated by 1968.

The ultimate limitation of the method developed is governed by the personal equation. A link in the chain of instrumentation is the observer's eye, which cannot be dispensed with, if the high degree of precision in measurement is to be achieved. At present, results obtained by the use of the visual acuity of the eye, and the reaction to sense impressions, cannot be improved with impersonal methods suited to field astronomy.

Automatic recording of star transits (photomultiplier etc.) fail to produce a distinct and sharp record of the event, unless used with stationary equipment in an observatory.

The method of synchronizing the crystal chronometer, and of extracting the time of stellar transits by the process of tape development, is also ultimately restricted by the eye of the observer in identifying visible time pulses and in scaling their distances on tape, and cannot be replaced by automatic methods without curtailing the accuracy. Admittedly, automatic comparing of the time indicated by the portable crystal chronometer with time signal reception in the field can be done with electronic equipment, in various ways, some of which are outlined in Section 3. 5. 3. Electronic methods are most inadequate, mainly due to the interference of radio noise. Furthermore, overlapping pulses may be hard to identify or to distinguish by the electronic equipment and serious errors are liable to be introduced. If automatic time comparison in the field were feasible its adequate use could be made only with a portable chronometer indicating U. T., which lacks the advantage of the time vernier. The comparison of a U. T. chronometer by means of an

11 cont.

acoustical measurement with head phones can be effected only at a rather low order of accuracy; a maasurement of time phase difference, with digital read-out, produces very problematic results.

Specifications published by instrument makers claim accuracies of signal extinction and vernier coincidences of 0.008 seconds of time, using head phones. Such figures are unrealistic and their quotation reveals disregard to the subjective limitations of the human senses in observations and measurements.

The research work was intended as a service to the mining industry, to assist the pioneering spirit in exploring the earth's crust in uncultivated regions, until such time as the entire material supply is obtained out of the air or from the sea.

It is hoped that the advances achieved and presented in this thesis will commend themselves also to other workers in field astronomy.

The methods presented are a high precision tool also for carrying out detailod studies in astronomic refraction, and for research into frequency receptions, propagation anomalies, ionospheric disturbances, etc.

All measurements taken from the earth's surface are referred to and are valid in this particular inertial system. Units of time and length are not independent; nevertheless their dimensions are usually defined and applied separately.

It can be anticipated that in the near future it will be possible for astronomical observations to be taken from outside the effect of the earth's atmosphere or even from outside the earth's gravitational field. Already, time comparison of a portable crystal chronometer with time pulses from transmitters placed in satellites is feasible. Hence, it will become necessary to consider precise measurements of physical quantities as not being restricted to the environmental conditions of the earth's surface, which to date are regarded as

11 cont.

indispensable realities.

Basically, field astronomy is concerned with the reduction of relative positions of heavenly bodies into a pair of co-ordinates on the surface of the earth. The process by which this is done consists of timed observations of irrevocable events taken under definite conditions. Each astronomical observation and its reduction is therefore an irreplaceable unit with its predecessors and successors.

Consequently, any observational work carried out in field astronomy is unlikely to become obsolete, and, provided precise timed observations of star transits and frequency receptions from various sources were taken and recorded on magnetic tape, which makes them easily accessible at a later date, their re-evaluation under a different aspect and their relation to different physical concepts of time and space, might perhaps constitute a wealth of information which hitherto has passed unnoticed.

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