CORRELATION BETWEEN FIELD AND LABORATORY MEASUREMENTS ON EARTH DAMS

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by

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ABSTRACT

Three topics will be discussed in this thesis. The first is a description of instruments currently used in Great Britain, including designs with which the Author was involved viz. an inverted pendulum, hydraulic settlement gauge and a wire and plate lateral deformation gauge.

In the second part, the intake factors for cylindrical piezometer tips were evaluated by solving Laplace's equation numerically on a digital computer. These results are then compared to data obtained elsewhere using an electrolytic water tank analogue.

For the unsteady-state case analytical and approximate numerical solutions were obtained for one, two and three-dimensional consolidation, for the constant-head and falling-head tests. In addition the effect of using non-linear theories in these solutions instead of the standard Terzaghi theory was investigated.

In the third part of this thesis a correlation is made between negative pore pressures, coefficients of swelling, consolidation and permeability as obtained from field measurements and laboratory tests.

CONTENTS

PART III

CORRELATION BETWEEN FIELD AND LABORATORY MEASUREMENTS

CHAPTER 1

INTRODUCTION

(1.1) Historical background to the construction of dams and embankments in the world

The construction of dams and embankments goes back a long way in history as man realised the need for water storage and flood protection as basic requisites for the maintenance and development of his civilisation. One of the earliest known dams to be built in the world is the Sadd Al-Kafara* constructed in Egypt around 3000 B.C. The remains are clearly visible to-day (Hellstrom, 1952; Murray, 1955). It had an earth-fill core supported by rubble shoulders with masonry facing, and seems to have failed by overtopping during the first filling season probably because there was no provision for a spillway.

Another famous old dam which was built around 1000 B.C. and whose failure was mentioned in the Koran is the Marib dam in the Yemen. This again probably suffered from inadequate spillway design. It seems that although ancient engineers produced reasonably good designs for the earth structures, their knowledge of the hydrology of the sites was inadequate to prevent catastrophes

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* In Arabic it means "Dam of the Pagans".

to their dams.

Hathaway (1958) in a very interesting paper on the building of dams in the world throughout the ages mentions the Sadd Al-Kafara and Marib dams and then goes on to describe the more successful and intricate systems of irrigation and water storage in ancient Babylonia, India and Ceylon. He then describes the very advanced techniques of the Moorish engineers in Spain during the occupation period by the Arabs and their later influence on Spanish engineers who practised their well acquired skills in earth dam building in South America in the 16th century (see also Rudolph, 1936).

More recently a World Register of Dams (1964) was published which lists the dams built in the world. Fig. 1.1 shows an interesting survey made by Guthrie Brown, the editor, on the number of dams built to different designs and of different construction materials for the period 1964-1965. The survey clearly shows the big increase in demand for dams in the world as industrial, agricultural and individuals' demands for more water and power continue to rise following the general improvement in the standard of living.

Another interesting feature of this survey is the high proportion of earth and rock fill dams of the total number. It clearly indicates an increasing preference for earth and rock-fill dams to concrete dams. This may be due to several factors one of which is probably the diminishing number of geologically suitable sites for concrete dams as the better sites have already been utilised. Another factor is the rising cost of labour which especially affected the cost of shuttering for concrete dams but which in the case of earth dams was partly compensated for by the introduction of cheap and efficient diesel earth-moving plant. A third possible factor may be the military advantage of an earth or rockfill dam over a concrete dam especially in the case of a high dam upstream of a densely populated area.

(1.2) The advantages and limitations of the application of the principles of Soil Mechanics to earth dam design

The usual questions to be asked in the design of earth dams are (assuming adequate spillway provisions):

(i) whether the earth structure will suffer excessive deformation or failure during construction, impounding, possible drawdowns and earthquakes.

(ii) whether the structure and also the impounded valley will be reasonably water-tight.

(iii) whether the change in the flow-net in the sides of the valley due to impounding would precipitate failures in the natural slopes of the valley sides especially in the downstream end of the dam near the outlet works thus endangering the safety of the dam.

The application of the principles of Soil Mechanics to these problems has gone a long way towards providing safe and economical solutions. A detailed discussion of these considerations is beyond the scope of this thesis but reference can be made to a few important publications. Regarding the first question of stability, Bishop (1952), (1957), Bishop and Bjerrum (1960) have dealt with the cases of construction, impounding and drawdown; and Ambraseys (1960) and Newmark (1965) dealt with the danger of earthquakes to earth dams.

Regarding the second question of water-tightness, Casagrande (1961) made an interesting contribution which dealt with the effectiveness of grouting in the foundation on seepage losses.

Bishop and Vaughan (1962) in a paper on Selset Dam gave a very good example of the beneficial application of the modern principles of Soil Mechanics to earth dam design, where numerous complicated problems of geology and adverse weather were treated successfully and efficiently. They also refer to the third question of the development of high pore pressures in the impounded valley and the remedial measures taken.

Thus with the present knowledge of the mechanics of soil, earth and rockfill dams are currently built to such heights and with such speed and confidence as was hardly the case even a few decades ago.

But although this advance in our understanding of the mechanics of soils is remarkable since the material dealt with is quite difficult to sample, test and correlate its behaviour with that in the prototype structure, it still does not provide answers to some rather important questions. For example, the simple problem of predicting the stress-strain relationship of soils has yet to be solved adequately although recent research work at Cambridge (see Roscoe, and Poorooshasb, 1963 and other publications) has gone some way towards the solution. This is why in the case of earth dams stability calculations have to be based on the limiting strength of the soil, where the shear stresses along an assumed critical surface is compared to the shear strength of a laboratory specimen tested under similar conditions to those obtaining in the field. But even our present knowledge of the shear strength of soils is not completely adequate; for apart from the usual difficulties in obtaining representative undisturbed samples, duplicating the stress conditions in the field, etc., it seems that the simple question of sample size has, for fissured clays in any case, an appreciable effect on the measured strength. Bishop and Little (1967) suggest that an insitu shear test on a 2' x 2' sample of Blue London Clay may yield an undrained shear strength value a third less than that obtained from the commonly used 3" x 1½" sample tested in the laboratory.

It was mentioned earlier that due to the lack of a general stress-strain theory for soils earth dams are designed on a semi-empirical basis. A factor of safety which is the ratio of the measured shear strength in the laboratory to the calculated maximum shear stress in the field is used to limit the deformation of the dam to an acceptable value.

There exists at present two approaches to the problem, one utilising total stresses and the second effective stresses. If the soil in the field undergoes negligible drainage such as during the construction stage with a fairly impervious fill then either method can be used although the effective stress approach might still be preferred. If drainage does take place then the effective stress method only is used.

The disadvantage of the total stress approach is the strong dependence of the laboratory measured shear strength on the stress path and also the impossibility of conducting any field check on the value of the shear strength assumed in the design calculations. Using effective stresses, on the other hand, the shear parameter c' and ϕ' are reasonably free from the stress-path and can be estimated with reasonable accuracy for the practical range of soils encountered in the field. The chief uncertainty in this approach is the pore pressure ratio r_u, defined as the ratio of the piezometric water level at a point to the vertical overburden pressure at that point which plays an important part in the stability calculations as shown in Fig. 1.2 for a typical design problem. It is seen that an increase in the value of r_u from 0.5 to 0.7 causes an alarming decrease in the factor of safety from an acceptable value of 1.5 to a near failure state. The value of r_u can be estimated from laboratory measurements but placement conditions in the field can be so variable as to render these forecasts quite unreliable.

Another uncertainty in the design of earth dams is the speed with which the compacted fill can dissipate its excess pore pressure due to construction or drawdown. Here again laboratory results may be somewhat misleading. A third factor concerns the deformation of the dam, for since a low factor of safety (usually 1.5) is accepted in earth dam design, it is important to know how much deformation the dam will suffer, so that, for example, due allowance can be made to the free-board.

Partly to resolve these uncertainties and partly to acquire a better knowledge of the actual behaviour of the soil in the field resort had to be made to instrumentation of the prototype structures.

(1.3) Contents of this thesis

The author has been very fortunate to work with Professor A. W. Bishop, Dr. P. R. Vaughan and Dr. N. Morgenstern in performing numerous field seepage tests, the design of one or two field instruments, installation of some hydraulic piezometers and other instrumentation, etc., in the following earth dams and embankments:

(i) Balderhead Dam, Co. Durham

(ii) Diddington Dam, Huntingdonshire

(iii) Peterborough Ash Disposal Scheme, West Perry

(iv) Fiddlersferry Ash Disposal Scheme, near Warrington

(v) M6 Trial Embankment, near Kendal

(vi) Muirhead Dam, near Glasgow

Field records of pore water pressure, deformation, etc. were kindly made available to the Author by the various consulting firms to which the proper acknowledgements are made at the end of the thesis.

The research work reported in this thesis is concerned with the behaviour of currently used field instrumentation in earth dams and the correlation between some measurements in the field with those in the laboratory.

Vaughan (1965) with whom the Author had the pleasure of working with for two years, covered similar grounds to those discussed here and in addition reported some very interesting field case-records. This important work will often be referred to in this thesis.

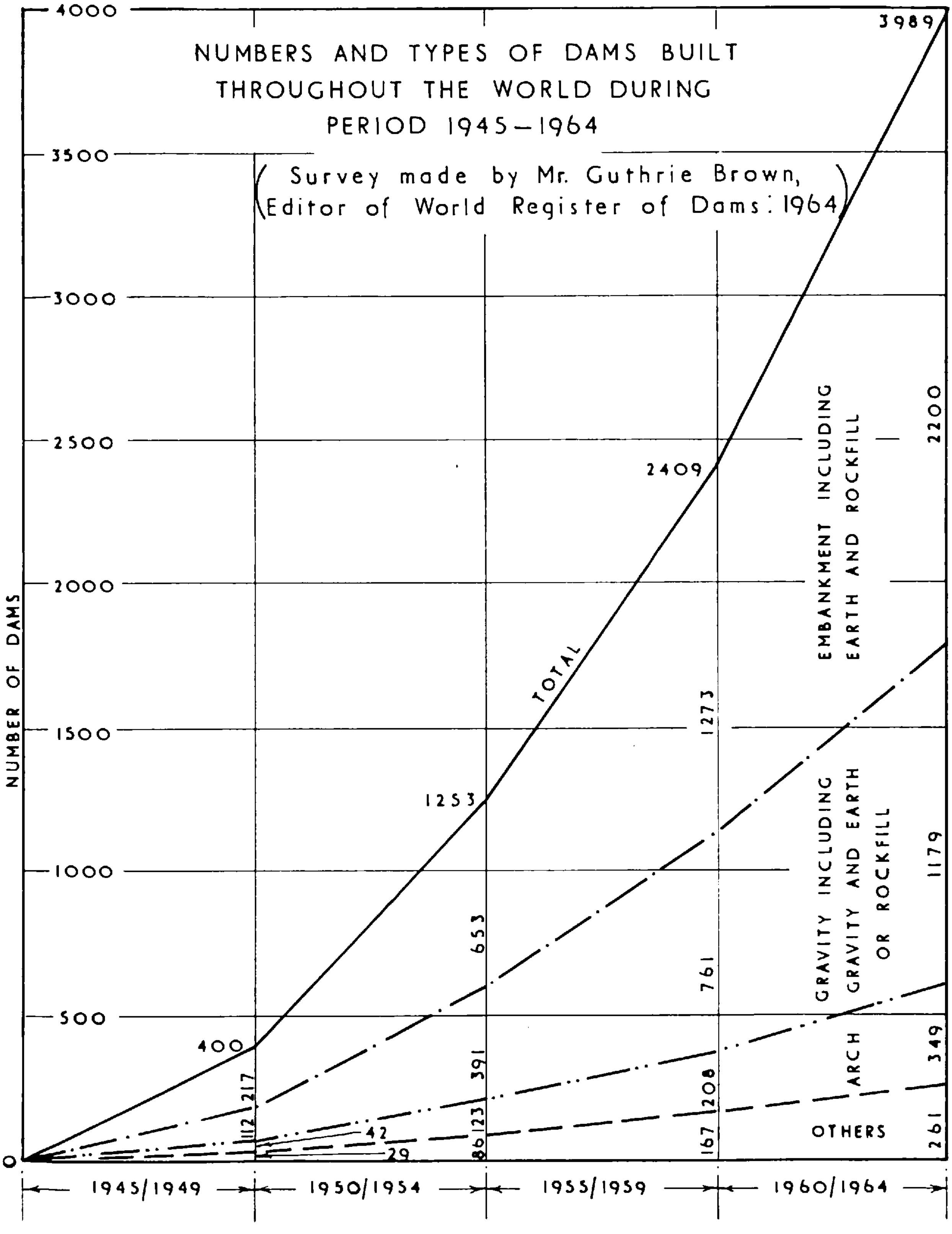
No attempt has been made in this thesis to correlate all the properties of the soil in the field to those in the laboratory; instead, attention was fixed on a few interesting properties, e.g. coefficients of permeability and consolidation, initial pore water pressures in freshly compacted soils, and $\bar{B}$ values for construction and drawdown cases, etc. Other properties like density, total stresses, etc. were not treated in any detail; not because they are unimportant but simply because to have included them would have been quite outside the scope of this thesis. One subject which was included in this thesis was an extensive approximate mathematical treatment on a digital computer of the problem of flow water to piezometers in compressible soils to clarify some uncertainties regarding the correlation between the values of the coefficients of permeability and consolidation obtained in the field with those from the laboratory.

This thesis is divided into four parts. The first part deals with instrumentation currently used in earth dams, and mainly with British practice. It includes one or two instruments which the author helped to design and use. A more thorough treatment of most of the instruments is given by Vaughan (1965) but for the sake of completeness and to record personal experiences with these instruments they were also included in this thesis.

The second part of this thesis deals with the specific problem of the flow of water from (or to) piezometers in saturated soils. This work was inspired by the excellent paper dealing with spherical piezometers by Gibson (1963). The remarkably simple form of the equation for the constant-head test enabled the Author to conduct numerous field tests on widely different types of soils with the twin aim of checking the validity of the equation and measuring the two very important soil parameters: the coefficients of permeability "K" and consolidation "C_v" (or swelling "Cs").

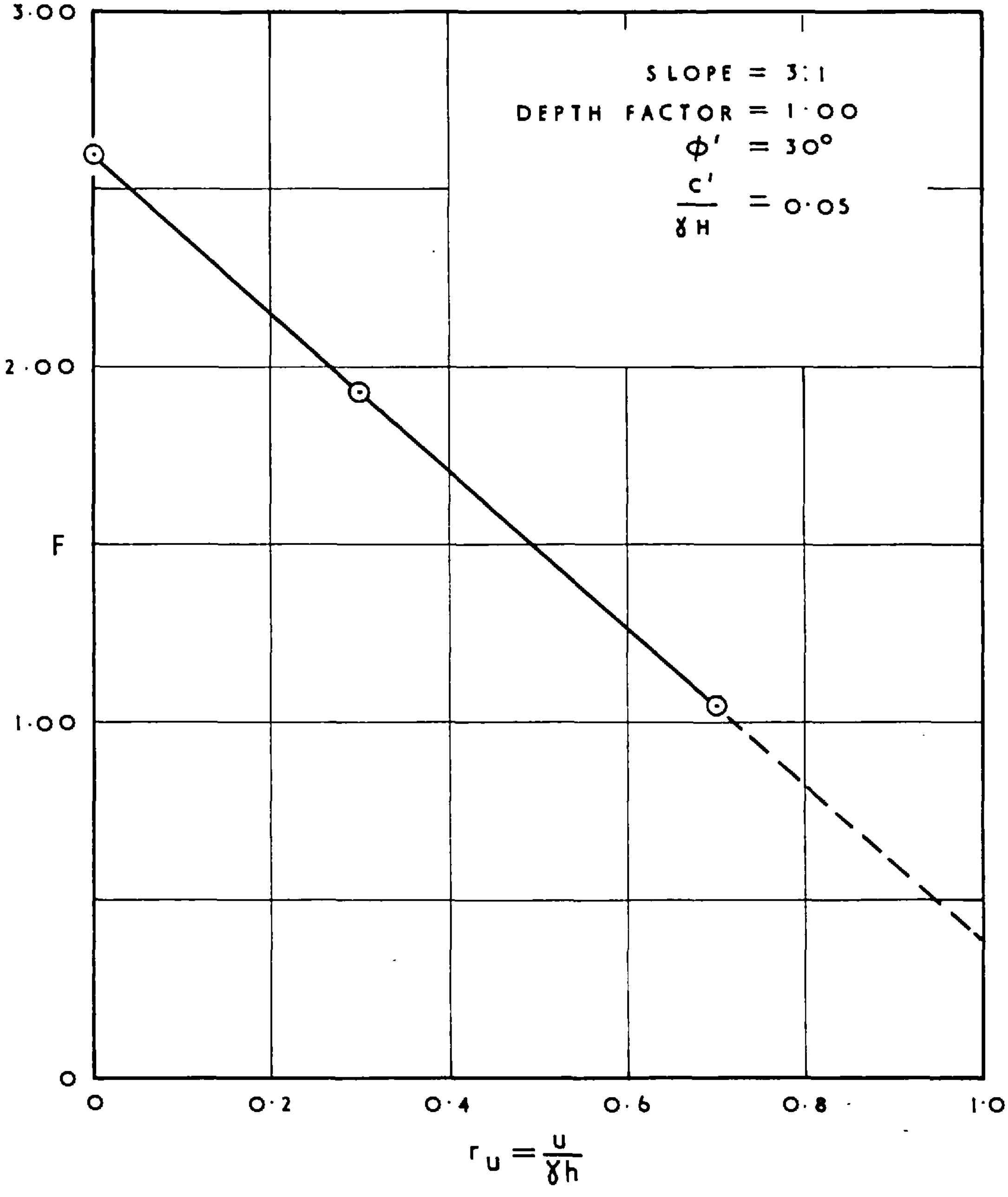
But, as the piezometers actually used in the field are usually either cylindrical or conical in shape rather than spherical, and as it is very difficult to treat the cylindrical and conical shapes theoretically, it was decided to solve the problem by numerical analysis on a digital computer. Only the cylindrical shape was treated for which the steady state intake factors were first evaluated and then the unsteady state was solved for both constant and falling-head tests. The results are given in Part II of this thesis and the numerical analysis and computer programme are discussed in Part IV.

Part III of this thesis contains the bulk of the field and laboratory results. In the first Chapter of this section the problem of suction in soils is treated with special reference to initial pore water pressures in freshly compacted relatively dry soils. In the next Chapter field records of construction and in one case drawdown pore pressures are given. A correlation is made between values of $\bar{B}$, Cv and K from the field and the laboratory where available data exist. In addition this part contains the results from field seepage tests.



NUMBERS AND TYPES OF DAMS BUILT THROUGHOUT THE WORLD DURING PERIOD 1945—1964
(Survey made by Mr. Guthrie Brown, Editor of World Register of Dams: 1964)
NUMBER OF DAMS
0
500
1000
1500
2000
2500
3000
3500
4000
1945/1949
1950/1954
1955/1959
1960/1964
TOTAL
400
1253
2409
3989
EMBANKMENT INCLUDING EARTH AND ROCKFILL
GRAVITY INCLUDING GRAVITY AND EARTH OR ROCKFILL
ARCH
OTHERS
217
112
42
29
653
391
123
86
1273
761
208
167
2200
1179
349
261

FIG. 1:1



The linear relationship between factor of safety and pore pressure ratio for a slope or cut in cohesive soil
(After Bishop: 1955)

FIG.1:2

PART I

INSTRUMENTATION IN EARTH DAMS

CHAPTER 2

PORE PRESSURE PIEZOMETERS AND FIELD IN-SITU SEEPAGE-TEST APPARATUS

(2.1) Introduction

The first part of this thesis deals with currently used instrumentation in earth dams mostly in Britain. The relevance of instrumentation to design calculations for earth dams was briefly mentioned in the introduction; a fuller treatment is given by Vaughan (1965).

Penman (1960a) gives an interesting historical background to the development of pore-water pressure meas ring apparatus for field use. Walker and Daehn (1948) reported the first measurements in the U.S.A. In 1949 the Building Research Station introduced this technique to this country. Since then a remarkable advance has occurred in this field and nowadays hardly any major earth dam is built in the U.K., at any rate, without some pore pressure measurement.

Fig. 2.1 shows schematically the development of piezometers in earth dams since open wells were first used mainly to measure seepage pressures after impounding (see Gould, 1959). As high pore pressure resulting from rapid construction started to be appreciated a responsive type of piezometer was adopted. This is the Casagrande type of piezometer which consisted of a porous hollow cylindrical tip sealed in the bottom of a borehole and connected to a vertical stand-pipe in which the level of the water can be observed. This had the advantage over the open-well piezometer in that measurement of the pore water pressure at a particular point can be made. This type of piezometer is still in use but extra methods of locating the water level in the stand-pipe have been adopted, e.g. electrical probe and air bubbling technique using small-bore plastic tubing.

To obtain an even more responsive system and to be able to record pore pressures higher than ground level a closed system was adopted where the stand-pipe was filled with water and connected to a pressure measuring device. Later when gas was found to accumulate at the top of the stand-pipe, and also to prevent frost damage, two plastic leads were connected to the buried top of the stand-pipe and measurement of the water pressure was made at the free ends of the plastic tubes outside the dam. Water under pressure can then be circulated through the system and any gas bubbles flushed out. A further recent development is the three-lead system (Vaughan, 1965) shown in Fig. 2.1. In deep boreholes where the water table is more than 28 ft. below the ground level hydraulic continuity in the closed system cannot be maintained. All three leads are then cleared of water and air pressure is applied to the middle-lead and collected through either side-leads through a bubble chamber. The air pressure at which the returning air ceases to flow is then equivalent to the water head "h". This method works quite well in practice as was found by Vaughan and by the Author. A small correction might be made for the amount of water flushed out of the middle tube during the measurement but this is usually not very significant.

The other advantage of the third lead is that when the water table is high the whole system can readily be flushed with de-aired water through the middle-lead and out of the other two side-leads.

The development of piezometer tips for use in compacted fill is also shown in Fig. 2.1 where the Building Research Station disc and cylindrical tips were later replaced by the Imperial College conical high air-entry value piezometer tip. Also shown is an electrical piezometer where a vibrating-wire pressure transducer situated at the tip measures the pore pressure which is then relayed to the outside through electric cables. The advantages and disadvantages of the different types of piezometers will be discussed in sections (2.4) and (2.5).

(2.2) Errors in the calculation of effective stresses in partly saturated soils when only one fluid pore pressure is used

In compacted fill in earth dams the soil tends to be partly saturated except in very wet regions. Due to capillary action and other factors (see for example Bishop, Alpan, Blight and Donald, 1960; Lambe, 1961) the air pressure is always higher than the water pressure; the difference in pressure $(u_a - u_w)$ depending on the size of voids, moisture content, etc*.

Up to a few years ago no distinction was made between the measurement of pore air and water pressures in compacted fill in earth dams or in the laboratory (Bishop, Kennard and Vaughan, 1965). Low air-entry value filters were used on pore water piezometer tips in partly saturated soils which tended to measure pore air pressure rather than pore water pressure when the term $(u_a - u_w)$ in the soil exceeded the air-entry value of the filter. A good example of this is shown in Fig. 7.1 where measurement from Building Research Station disc type low-air entry tip is compared to that by a high air-entry value Imperial College tip, although as will be explained in Section (7.3.1) the Imperial College tip was probably not recording the true suction either.

Bishop et al. (1965) discuss this problem and analyse the

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* This will be discussed in more detail in Chapter 7.

error in the calculation of effective stresses in partly saturated fills when air pressure or water pressure alone is used.

Terzaghi's effective stress equation for fully saturated soils is:

$$\sigma' = \sigma - u \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.1)$$

where σ' is the effective stress

u the pore water pressure

and σ the total stress.

For partly saturated soils Bishop (1959) suggested the following expression:

$$\sigma' = \sigma - u_a + x\,(u_a - u_w) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.2)$$

where u_w is the pore water pressure

u_a is the pore air pressure

x is a parameter closely related to the degree of saturation "S" and depending on other factors such as the soil type and the cycles of wetting and drying leading to a particular value of S. For more discussion of this equation see Bishop, Alpan, Blight and Donald (1960), Matyas (1963) and Jennings and Burland (1962).

Eq. (2.2) can be rewritten as:

$$\sigma - u_w = \sigma' + (1 - x)(u_a - u_w) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.3)$$

or

$$\sigma - u_a = \sigma' - x(u_a - u_w) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.4)$$

Thus, effective stress calculated on the basis of the measurement of pore water pressure alone will exceed the correct value by $(1 - x)(u_a - u_w)$. On the basis of air pressure alone the effective stress will be underestimated by $x(u_a - u_w)$. The magnitude of these errors depends on the value of x and $(u_a - u_w)$ and hence on the clay fraction of the soil, its degree of saturation, etc.

Records of specific measurements of air-pressure in compacted fill are very scarce. Vaughan (1965) designed an air piezometer which he installed in Balderhead, Peterborough and Diddington dams, but the instrument was found to be unsatisfactory due to its poor response and mechanical weakness. But from measurements in soils with a typical degree of saturation of 85 - 90% using low air-entry tips which presumably recorded air pressures and high-air entry Imperial College tips presumably recording water pressures the errors introduced by using u_w alone is, although on the unsafe side, much smaller than that when u_a alone is used which might lead to a fictitiously low value for the effective stress, see Bishop et al. (1965).

(2.3) Basic requirements for pore-water piezometers

The three basic requirements for any instrument used in earth dams are that it should (apart from robustness and long-term stability):

(i) unambiguously record the property of the soil it is designed for.

(ii) cause the minimum of interference with the conditions in the soil around it during installation and operation.

(iii) have as short a response time as possible.

The relative importance of these three requirements will vary with the soil property measured. For a pore-water piezometer the first requirement is the most important as was discussed in the last section. The second requirement does not affect pore-water pressure measurement but it does have a great influence on the results of seepage tests on the piezometer (see Chapter 6). The third requirement is only significant in the case of open borehole or large diameter stand-pipe piezometers in fairly impervious soils. For fill-type piezometers the time-lag of typical piezometer systems is given in Chapter 8 where it is seen to be quite acceptable (see also Penman, 1960).

(2.4) A comparison between the electrical and high air-entry piezometer tips

Partly to overcome the difficulty in the measurement of negative pore-water pressure in partly saturated fill and partly to reduce the amount of tedious de-airing necessary with the low-air-entry tip a special high-air-entry piezometer tip was designed at Imperial College (see Bishop, Kennard and Penman, 1960 and Little and Vail, 1960). The tip is shown schematically in Fig. 2.1 and also on plate 2.1 which also shows two other low-air-entry cylindrical tips.

The filter used in the Imperial College tip has an air-entry value of approximately 30 p.s.i. and permeability of 3×10^{-6} cm./sec. as checked by the Author. Its tapered shape improves the initial contact with the soil during installation.

Electrical piezometer tips have also been widely used in earthdams. Their chief advantages over hydraulic piezometers are that they may be used independently of the position of the piezometer house so that for example they would be ideal in very deep boreholes. Secondly, they have a very short response time but this, as was mentioned earlier, has little significance in field measurements. Thirdly, they require a much more compact piezometer house than that necessary with hydraulic tips.

However, they have three important disadvantages. Firstly, once they are installed and buried there are no means of checking the instrument for zero drift or mal-functioning as can be done with a twin-lead hydraulic piezometer. Secondly, it is impossible to conduct in-situ seepage tests on them. Thirdly, and perhaps most important, if used in partly saturated fill it is impossible to circulate water through the tip to flush out gas bubbles which would have no difficulty in entering it especially since most electrical tips are fitted with a low-air-entry filter of 1 or 2 p.s.i. A discussion of the behaviour of an electrical piezometer fitted with a thick high-air-entry filter will be given in the next section.

The most popular electrical piezometer tip in this country is the vibrating-wire type described by Muhs and Campbell-Allen (1955), and Penman (1960) and supplied by Maihak, A-G. These tips are supplied with low-air-entry filters. The Author had some experience with them in Diddington dam, details of which will be given in Chapter 7. Some of these instruments together with Imperial College tips were installed in the road embankment constructed prior to the dam as a road diversion and also as a trial embankment.

Fig. 2.2 shows the pore pressure readings obtained from the neighbouring Imperial College and Maihak gauges in the road embankment. It is quite obvious that the Maihak gauges cavitated shortly after installation and ceased to record pore water pressures, and were most probably recording air pressures. However, other Maihak instruments installed in the fully saturated foundations of the road embankment and the dam seem to have been functioning quite satisfactorily (see Chapter 7). Hydraulic

Imperial College piezometers installed near them ceased to function shortly after installation but one tip did give readings which were quite in accord with those from the neighbouring Maihak tip.

Vaughan (1965) gives records of discrepancies between measurements using Maihak and Imperial College tips in Walton dam which agrees with those given for the trial embankment at Diddington.

Matyas (1963) tested a Maihak gauge in the laboratory by installing it in a 4" x 8" partly saturated sample in a large triaxial cell and made measurements of the pore water and air pressure in the sample together with those recorded by the Maihak tip. He concluded that the tip was most probably measuring air-pressure, and in fact at the end of the test when he dismantled the tip air bubbles were seen to come out of the tip cavity.

Evidence by Hosking and Hilton (1963) and Pinkerton and McConnell (1964) point to a different source of trouble with Maihak gauges, viz. the interference with the measurements of stray electrical charges in the earth.

A recent interesting paper on the use of electrical gauges is by Silva (1966) who reports the use of both strain-gauge and vibrating-wire tips in four earth dams in Brazil. It seems from the data given that negative pore pressures exceeding one atmosphere (and in one or two cases around -20 p.s.i.) were read by these instruments for periods up to six months. In addition some of these negative pore pressures tended to become even more negative with an increase in the height of the fill. These measurements seem to be quite unreliable to the Author since it is normally very difficult to sustain such large suctions for such lengths of time. It is interesting to note that when Matyas subjected a Maihak gauge to a vacuum pressure and then to a positive pressure, a slight change in calibration resulted, indicating that Maihak gauges should not be subjected to negative pore pressures.

The Author had similar experiences with Maihak gauges in Diddington. These were installed in a piezometer man-hole and relayed pore pressures recorded by hydraulic piezometers to a more comfortable house some distance away. The Maihak tips were pressurised and calibrated against a mercury manometer. Errors of -0.5 to -1.5 p.s.i. were recorded at a pressure of 15 p.s.i. This could be due to zero shift, or possibly to change in atmospheric pressure since they were calibrated first. When subjected to a negative pressure of -7.5 p.s.i. the error was from -0.5 to +0.5 p.s.i.; but it must be mentioned here that the accuracy with which these instruments are read is within $\pm$ 0.5 p.s.i. Nevertheless a hydraulic piezometer situated near a Maihak tip in the foundation of the dam gave higher readings by about 1.5 p.s.i. which may be explained by the errors quoted above.

To sum up it seems that electrical gauges should not be used in compacted fills and should be restricted to use in fully saturated foundations where especially in deep boreholes they may be more suitable than hydraulic gauges.

(2.5) Rate of diffusion of air into initially water-filled piezometer tips

The Norwegian Geotechnical Institute (1964) have designed an electrical vibrating-wire piezometer with a 10 cms. long solid filter having an air-entry value of approximately 15 p.s.i. It was intended to be used in the partly saturated fill of earth dams constructed in regions where the temperature was too low for using the hydraulic Imperial College tips.

They have assumed that when the tip is recording a suction less than its air-entry value the water in the tip cavity will not cavitate, but it would slowly be replaced by air diffusing through the thick filter from the soil outside. They then argue that while there is still some water in the tip cavity the pressure recorded will be that of water rather than air since the permeability of the saturated filter to water is much greater that that to the diffusion of air through it.

They then conduct an interesting approximate calculation of the time required to empty the 1 $cm.^3$ cavity next to the pressure diaphragm in the tip, thus:

Using the diffusion Law for the flow of air through the saturated filter:

$$\frac{dQ}{dt} = K \frac{dc}{dx} \times A$$

where

$\frac{dQ}{dt}$ is the rate of mass diffusion of air

$\frac{dc}{dx}$ is the concentration gradient across the 10 cm. filter

K is the diffusion coefficient for air in the water contained in the pores of the saturated ceramic

and A is the cross-sectional area of the filter, in this case = 3.14 $cm.^2$

The value of the constant "K" is not known but was assumed by them to be very roughly 0.02 $cm.^2$/day on the basis of observed rates of diffusion of air in water inside the pores of clay.

The concentration gradient (of air in solution) will depend on the pressure of air at both ends of the filter. On the soil side this will be the pore-air pressure which can be, for example, taken as 3Kg./$cm.^2$ On the inside it can be taken equal to the pore water pressure, for example 2Kg./cm.^{2} The solubility of air in water at +5°C, 3Kg./cm.^{2} air pressure is approx. 76 cm.^{3}/litre and at 2Kg./cm.^{2} approx 50 cm.^{3}/litre, so

$$\frac{dc}{dx} = \left(\frac{76}{1000} - \frac{50}{1000}\right) \frac{1}{10}$$

$$\therefore \quad \frac{dQ}{dt} = 1.64 \times 10^{-4} \text{ cm.}^3/\text{day}$$

If the volume of the initially water filled cavity is taken as 1 cm.^{3} then to emp y it will require 17 years! A similar calculation for an ordinary Imperial College tip gave 13 years. Although these calculated times seem to be rather on the high side especially to those who had the misfortune of being in charge of de-airing tips, field evidence by Vaughan (1960) and by the Author indicates that when an Imperial College tip is situated in a soil exhibiting positive pore water pressure the amount of air collected in the tip is very small indeed. Vaughan mentions two tips in Selset where he collected only 0.5 cc. in nearly 9 months.

When, however, a tip is situated in a soil exhibiting large suctions then the time for complete cavitation would be a matter of a few weeks only. Fig. 7.4 shows a good example of this in Peterborough Embankment. Here there are other mechanisms for cavitation besides the diffusion of air through the filter. The first is the cavitation of water contained in the pores of the filter when its air-entry value is exceeded. Matyas (1963) concluded from a series of tests on typical fills at typical moisture contents that appreciable initial suctions existed in the laboratory compacted samples (see also Chapter 7), which would easily be greater than the 30 p.s.i. air-entry value of the Imperial College tip.

The other mechanism is the cavitation of the water inside the tip. This depends on how well the water was de-aired prior to filling the tip but in any case cavitation will occur at suctions exceeding -12 p.s.i., where dissolved air and water vapour will come out of solution and displace the water in the tip and out to the soil. This process is self-deccelerating since the flow of water out of the tip causes a drop in the suction in the soil which with its typically low permeability and the high compressibility causes an appreciable time-lag until the whole tip is cavitated.

Some measurements of the amounts of air flushed out of Imperial College tips in the shoulders of Peterborough Embankment were made by the Author. These piezometers were reading appreciably high negative pore pressures (see Fig. 7.4), and they tended to cavitate roughly 2 - 4 months after each de-airing operation.

Nearly all the piezometers in the shoulders were de-aired at the end of Feb. 1965. The last previous date on which they were de-aired was about a year earlier. Just prior to de-airing on Feb. 1965 these piezometers were recording pore pressures ranging from -4 p.s.i. to +2 p.s.i.

The volume of gas collected from each tip, corrected for the pressure difference between the tip and piezometer house were in the range 45-60 cc. for 8 piezometers out of 16, the volume of the cavity inside an Imperial College tip being 50 cc. For the rest of the tips the amount of air collected was between 0 to 30 cc. but most of these tips did not record large suctions, either because they were in a wetter fill, or were near the up-stream face where they were affected by the reservoir level.

Here it is very difficult to give a quantitative analysis of the rate of cavitation of the tip, except where extreme simplifications are made. For example it may be assumed that an Imperial College tip installed in a soil having a true suction of, say, -2 atmospheres, is capable of withstanding -1 atmosphere without appreciable loss of water to the soil. Suddenly cavitation of the water in tip occurs, and the tip starts losing water to the soil but keeping its pressure at -1 atmosphere until it is completely cavitated. Using Gibson's constant-head equation (5.13) the time taken for 50 cc. of water to leave the tip can be calculated.

The values of "K" and "C_s" to be used are from constant-head tests in Peterborough reported in Chapter 7, viz. $K = 10^{-9}$ cm./sec., and $C_s = 5 \times 10^{-6}$ cm.^{2}/sec. The time takes is seen to be 4 days only. This is much shorter than the 3 to 4 month periods reported earlier, the obvious reason being the very crude assumption that the suction in the soil just before cavitation was uniform and 1 atmosphere below that inside the tip. The second factor may well be the time lag necessary for the diffusion of air into the tip and its coming out of solution inside it.

(2.6) Hydraulic piezometers - layout in earth dams

Generally there are two classes of problems of pore water pressure measurement (see for example Bishop and Bjerrum, 1960).

(a) problems where the pore pressure is an independent variable and is controlled either by ground water level or by the flow pattern of impounded or underground water, for example, and (b) problems in which the magnitude of the pore pressure depends on the magnitude of the stress tending to lead to instability, as in rapid construction or drawdown in soils of low permeability.

In dam construction both problems occur. Piezometers are installed in the rock and soil foundation of the dam, in the valley sides especially in the downstream end, and in other appropriate locations in order to establish the water table in the different strata and also to measure their permeabilities. During construction they would provide data on construction pore pressures in the foundation. After impounding, these piezometers together with others in the fill would indicate the efficiency of the impermeable barrier in the dam and the tightness of the foundation.

In the compacted fill the number and location of piezometers depend on the problems likely to be encountered. For an analysis of construction failure and rapid drawdown, special attention must be paid to the weakest zones in the fill. Examples of the lay-out of piezometers in recently built earth dams in the U.K. are given in Chapters 7 and 8.

(2.6.1) Installation of hydraulic piezometers

A main cause of difficulty in obtaining a satisfactory performance from a hydraulic piezometer (where the cause of damage is obviously not large shear deformation of the soil) is simply due to unsatisfactory installation of the tip in the first place, and/or not having the piezometer-house ready before installation of the tips, the significance of which will be seen later.

In Diddington dam, for example, several piezometers were tested with the constant-head test and found to have an incomplete soil seal around them. Where the expected permeabilities of the soil were of the order of 10^{-8} cm./sec. at most, permeabilities much higher than these were measured which were very nearly that of the Imperial College tip itself, 3×10^{-6} cm./sec. The pore pressures recorded by these piezometers were naturally erratic (see also Section 7.5.2). In Balderhead dam half a dozen tips were installed with the wrong type of screw-connections and again gave unsatisfactory performance.

A procedure recommended to the Author by Dr. P.R. Vaughan for installing both fill and borehole piezometers was found to be very satisfactory, and will be outlined in the next two sections. It was applied by the Author and the engineers in Peterborough embankment and gave a 100% success for all 29 fill piezometers and 6 borehole piezometers.

(2.6.2) Installation of borehole piezometers

For foundation piezometers it is recommended that the diameter of the borehole should be not less than 6" to enable satisfactory grouting of the hole.

When the borehole had reached its required depth and the stratum in question had been located, samples of the soil or rock were taken for later identification. An accurate measurement of the depth of the borehole was made. Pea gravel or similar material was then poured in to a depth of about 6". The tip securely attached to a $\frac{3}{4}$" heavy duty p.v.c. stand-pipe (or 3/16" nylon tubing in the case of shallow holes) was then lowered, and some more gravel was poured to about 6" above the tip. Next. sand was poured in to form a 6" layer and was allowed to settle if the borehole was wet. An accurate measurement of the top of the sand layer is made and hence the length of pervious plug is known for future use in seepage tests. Generally, the length of the pervious plug depends on the strata involved. If, for example. interest is centred on a thin layer of clay sandwiched between two sandy layers, then the length of the plug should be as short as possible. On the other hand in deep rock formation the length of the plug will be determined by the distance between the fissures and may be 10' or more.

To seal the plug, first dry bentonite in nylon mesh bags was lowered down the hole to a depth of 6" above the sand, and then grout slurry was pumped down with the hose outlet near the dry bentonite to start with and gradually being withdrawn, but always remaining below the rising grout surface to avoid trapping air bubbles. The casing, if any, was withdrawn gradually as the grout came up the hole. When dry bentonite is wetted by the slurry or the natural water in the ground it swells and forms the first line of defence against leakage. The grout mixture used in Peterborough was roughly 1:1:5 cement: bentonite:water.

Extra care was taken when making the top connection between the stand-pipe and the plastic tubing, if any, running to the piezometer house. A small amount of concrete around and on top of the junction served to protect it when there was an inadequate height of fill above it. When the three-lead system shown in Fig. 1.1 was used the middle lead was marked clearly for easy identification in the piezometer house.

Vaughan (1965) gives an approximate analysis of the error in the measurement of the water pressure using a borehole piezometer when the permeability of the grout is higher than that of the surrounding soil. This may be the case for soils of extremely low permeability or when an imperfect grout seal is obtained. Reference to this point will again be made in Section 7.6.

(2.6.3) Installation of fill piezometers

The most common type of tip currently used in earth fills is the Imperial College conical high air-entry value tip (Fig. 2.1). When installing it the aim was to have the whole piezometer system completely free from air and leaks before insertion in the soil.

The tip was dismantled and reassembled under water to avoid trapping air between the central spindle and the bottom brass-end. It was then soaked in water for a couple of days and flushed with de-aired water for at least 15 minutes. It was then tested for leaks by applying an air pressure to the tip while submerged in water. The air pressure used must naturally be less than 30 p.s.i., the air-entry value of the filter, and in fact 20 p.s.i. is quite adequate. If there were any leaks, they were put right and the tip retested. Quite often the leaks occurred between the ceramic and the rubber seal. A simple wet grinding and polishing of the ceramic overcame this difficulty.

The plastic tubing normally used was the polythene coated 3/16" nylon tubing which will be discussed in the next section. It, too, was soaked in water for a few days, as nylon tubing imbibes water and may affect the readings later on (see Vaughan, 1965 for a discussion on the water absorption of nylon tubing, also Penman, 1960).

The type of trenches required to carry the leads back to the piezometer house varied with the soil. If the trenches were in a rock fill they were dug at least 3' deep and 3' wide. 6 to 9 inches of sand below and on top of the leads was used to protect them against large boulders. In more clayey fills a wide shallow trench dug with a scraper with a V-groove in one side was found to be satisfactory. When leads were laid across a clay core soft clay lumps were kneaded below and on top of the leads except for the last 4' near the tip where natural material was used to avoid disturbing the local conditions there.

A useful procedure for de-airing the piezometer system before installation will be given now. The two leads belonging to one tip were laid alongside the trench and bound together with identifying sticking tape of different colours or combination of colours every 50' or so. This served to identify the leads if by accident a machine ripped the leads before an adequate cover of fill was obtained. This in fact did happen in Peterborough and Balderhead.

The leads were then connected to their appropriate manometers in the piezometer house and de-aired water was flushed through them. One lead was then connected to the tip and any air inside flushed out. While the pressure was still on, the second lead was connected and the tip was allowed to 'sweat' for say 5 minutes to de-air the filter. The pressure was then shuttoff and with the tip immersed in a bucket of water measurements on the manometers were taken and if the difference in readings was more than 0.1 ft. the piezometer system was flushed again. A hole was then made in the fill with the shaped mandrel without unduly disturbing the soil. A sample of the soil was then taken for soil classification tests. The tip was then wiped dry, installed in the hole, and covered with well compacted soil. The leads from the other tips were then laid in the trench and covered.

(2.6.4) Piezometer house equipment

It was mentioned earlier that a disadvantage of using hydraulic piezometers is the need for large and suitably placed piezometer houses. The level of the house should be about the same level as the lowest set of tips to prevent cavitation of the water in the leads. One of the first deeply buried piezometer houses was built in Selset dam (see Kennard and Kennard, 1962). Since then the size and amenities, e.g. light, heating, etc., have gradually progressed. It is in the Author's view quite essential to have a warm well ventilated and lit instrument house to encourage the engineer in charge to obtain more frequent readings and to carry out maintenance work if necessary.

The de-airing equipment currently used is an improvement on the original Building Research Station design (see Penman, 1960a and Vaughan, 1965). Plate 2.2 shows the unit used in Balderhead dam.

The pressure measuring gauges are now mostly mercury manometers which replaced the originally used Bourdon gauges. The latter were found to deteriorate with time due to the chemical action between its different components.

There has also been some progress in the choice of valves. Needle valves were used first, and were later replaced by the more reliable Klinger valves. The latter, however, were found to be too bulky, and tended to 'stick' after a long period of not being used. They were replaced by grease-packed Simplifix cocks which are much more compact (see Vaughan, 1965). However, when used in Peterborough they were found very troublesome as the rubber seals kept coming off their spindles, leaving the valves permanently on the off-position. A third type was introduced which was a compact 3-way valve block on the Klinger valve principle but key-operated. This has been used in several installations and was found to be satisfactory.

(2.6.5) Some properties of the plastic tubing

Penman (1960) and Vaughan (1965) describe some of the properties of the plastic tubing. Penman found that polythene tubing is pervious to air. Nylon tubing is slightly permeable to water, largely when subject to differences in vapour or osmotic potential. This suggested the use of the polythene coated nylon tubing commonly used at present (Vaughan, 1965).

A calibration was made by the Author on the deformation of polythene coated 3/16"*[1] nylon 11 tubing under internal water pressure and the results were compared to Penman's test on the uncoated 3/16" nylon 66. This is shown in Fig. 2.3 which also shows the results from a similar test on 5/16" Saran tubing used in Diddington dam. It is seen that the two nylon tubings have practically the same deformation characteristics and have a volume factor*[2] $\frac{1}{3}$ of that of 5/16" Saran tubing.

Fig. 2.4 shows the creep deformation of the coated 3/16" nylon tubing under increasing pressure.

Fig. 2.5 shows the resistance to flow of water in 3/16" coated nylon 11 and 5/16" Saran tubing. In these two tests a constant pressure was applied to one end of the tubing and the other end was subjected to atmospheric pressure, where also the flow rate was measured. Correcting for the very small velocity

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*[1] All diameters are O.D.

*[2] Defined as the volume change per unit pressure change.

potential, the net pressure drop across the tubing was found. It is seen that the velocity of flow for a certain pressure differential across the 5/16" Saran tubing is 2.4 times faster than that in the 3/16" nylon. This point will be further discussed in the next Section dealing with de-airing of hydraulic piezometers. An advantage of Saran tubing is its very low permeability to the diffusion of air or water through its walls. Its disadvantage, apart from its high cost, is brittleness. The Author had witnessed a complete severance of Saran tubing when water inside it had frozen.

(2.6.6) De-airing of hydraulic piezometers

An essential advantage of the hydraulic piezometer is the ability to circulate de-aired water to flush gas bubbles out of the piezometer system. The rate of diffusion of air into the tips has been discussed in Section (2.5) together with some typical field records.

It was also stated that where the piezometers were recording positive pressures (and assuming that the piezometer tip is higher than the piezometer house) very small amounts of air diffused into the tip. Nevertheless, it was found a good practice to de-air the piezometers at least once every six months to ensure water continuity in the system. In the case of piezometers recording suctions more frequent de-airing was necessary. The most obvious sign of the presence of air in the piezometer system was a difference between the two mercury manometers greater than, say, 0.5 ft. of water pressure. In addition piezometers were de-aired when they gave unexpected readings.

The pressures chosen for de-airing were those recommended by Little and Vail (1960) where the net pressure at the tip during de-airing was made to be approximately the same as that in the soil outside, so as to cause as little disturbance as possible. In fact, the pressure at the tip during de-airing was made about 5 ft. higher than the pore water pressure in the soil to prevent aerated water from entering the tip, and also to flush the ceramic filter.

Assuming that the undrained shear strength of the soil is small, the pressures used must not exceed the overburden pressure at the tip or else cracking of the soil will occur. This did occur accidentally in one or two cases during de-airing and seepage tests, but the cracks in the soil seemed to have 'healed' after a few weeks.

The most tedious tips to de-air were those in the upstream end of the dam under a shallow overburden pressure, where for a typical lay-out up to 30 minutes or more were required. Here and in other cases of long distances between the tip and the house $\frac{1}{4}$" or 5/16" tubing is recommended. The use of a large suction on the return lead was avoided unless it was applied in the beginning of the de-airing operation and released half-way.

The Author found it useful to measure and record the amounts of air collected during de-airing to isolate troublesome piezometers and to obtain a better interpretation of inconsistent pore pressure records.

(2.7) Constant-head permeability apparatus

A very useful advantage of hydraulic piezometers is the ability to conduct in-situ seepage tests on the compacted fill and foundation strata of earth dams. These tests serve two purposes, firstly they provide a check on the soil or grout seal around the tip, blockages and leaks in the piezometer system, and secondly they provide in-situ values for the coefficients of permeability and consolidation (or swelling).

There are two types of tests, the constant-head and falling-head tests. Hvorslev (1951) derived the theory for both tests for incompressible soils. Gibson (1963, 1966) treated the more general case of compressible soils using Terzaghi's theory of consolidation. But of the two tests Gibson's solution for the constant-head test proved to be much easier to use for the interpretation of field data than that for the falling-head test. The latter test has other disadvantages which limit its use to a qualitative test only and where much less reliable values of K and C_v can be obtained than from the constant-head test. A further discussion of this point will be given in Section (8.2.2). The theory for the constant-head and falling-head tests will be discussed in Chapter 5 and 6.

In order to conduct a constant-head test on a piezometer it is necessary to have an apparatus which will maintain a constant pressure on the piezometer tip and will have an accurate volume gauge. There are several methods for supplying a constant pressure, e.g. the conventional two mercury pots and spring, a syphon from a large area water tank, a pressure regulated air-water cylinder, etc. The flow can be measured by the conventional water-paraffin gauge, or by direct measurement on the water gained or lost from the pressure tank, or by introducing an air-bubble into the system and timing its rate of travel, etc. The choice of a constant pressure source and volume gauge depends on the soil to be tested. The difference in the flow rates of two soils of permeabilities of 10^{-9} and 10^{-6} cm./sec. is 1000 times! For tests on soils with a high permeability the hydraulic losses in the apparatus and the connecting tubing must be kept to a minimum; for as the flow changes with time (see Chapter 4) so will the hydraulic losses and this in turn will affect the 'constant-head' applied to the tip. For tests on soils of low permeability an accurate measurement of the flow is required and also no large fluctuations in the pressures can be tolerated. Also the connecting tubes must be small in diameter to avoid errors introduced by temperature changes.

Two units were used by the Author, the first for use in soils having a permeability range of 10^{-8} to 10^{-6} cm./sec., and the second for the range 5×10^{-7} to 5×10^{-5} cm./sec.

(2.7.1) Constant-head low-permeability unit

This unit is shown on plate (2.3) and schematically in Fig. 2.6. The constant-head was supplied by the conventional two mercury pots and spring (see Bishop and Henkel, 1962), and the flow was measured on the paraffin-water interface in three 50 cc. burettes so that simultaneous tests can be conducted on three piezometers through valves 1, 2, 3. The connections between the glass burettes and the $\frac{1}{4}$" nylon tubing were made with compression fittings with rubber 'olives'.

The maximum pressure obtainable from this unit is approximately 30 p.s.i. with the lower pot resting on floor level. This is probably near the limiting strength of the glass burettes. But this range of pressure was found adequate for most of the tests conducted. Jacketed burettes with interchange valves were not used because of their expense and extra weight and space required.

The unit shown on plate (2.3) was, in fact, manufactured by Soil Instruments Ltd., and is very similar to an earlier unit built by the Author which had 10 cc. burettes and the conventional Klinger valves.

Vavle 4 connects the unit to a 2 gallon ordinary plastic container suitably placed to supply a pressure exactly that of the mercury pots. When the paraffin had run through the length of any of the three burettes it was pushed back by opening valve 4 and the appropriate valves in valve-block (B), and by slightly depressing (or lifting) the upper mercury pot. When no flow measurements were required or when the tests were left running over night the valves (A) were closed and the constant pressure was then supplied by the water container.

If it was found impossible to find a suitable place for the container, then the de-airing panel was used by setting it to the right pressure and connecting it to the unit through valve 4. This arrangement was found less satisfactory than the water container as it was difficult ot keep the pressure in the de-airing unit constant due to air leaks, temperature changes, etc.

The unit shown in Fig. 2.6 is mobile and can easily be carried by two men. It has been used in several of the dams mentioned in Chapter 1. Another unit was built by Soil Instruments Ltd. and used in the M6 trial embankment near Kendal. Both instruments gave satisfactory performance. Results obtained using this unit will be given in Chapter 8.

One source of temporary scatter of results especially in the low permeability range was when the paraffin had to be pushed back after it had reached its end of travel, and when the unit was subjected to a high flow-rate at the start of a test on another burette. The first difficulty could not be overcome completely but the rate at which the paraffin was pushed back was made as slow as possible. The other difficulty was overcome where possible by starting a new test on the container and leaving it on it for a few minutes before switching back to the mercury pots. When used outside its permeability range, a slight scatter of results was obtained when the permeability was less than 10^{-8} cm./sec., due to the difficulty in reading the meniscus and also to temperature changes. When used for soils of permeabilities higher than 10^{-6} cm./sec. the hydraulic losses in the unit became significant, but in any case the hydraulic losses in the tubes connecting the tip to the house would then be even more appreciable. The test pressure at the tip was measured on the other manometer and this was the actual pressure used in the calculation, but for the purpose of setting the mercury pots roughly at the required pressure the vertical graduated scale was used.

(2.7.2) Constant-head high-permeability unit

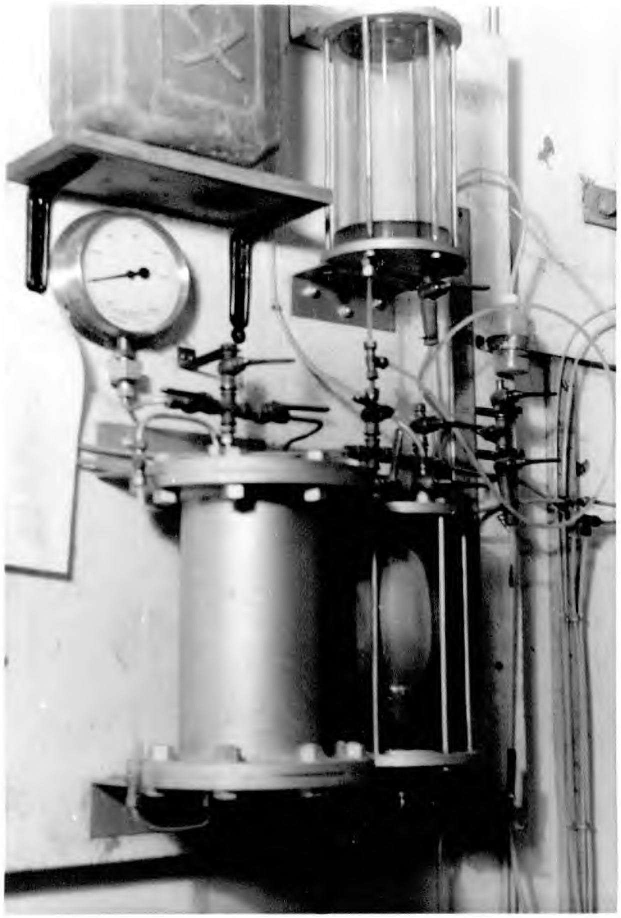
This unit was built at Imperial College and used in Muirhead dam for tests on ½" alkathene open stand-pipe piezometers where previous tests by Wimpeys indicated a permeability range 10^{-7} tp 10^{-5} cm./sec. The unit is shown in Fig. 2.7 where the pressure head was supplied by a 3' x 6' x 5' deep water tank on a scaffolding. The flow rate was measured on the paraffin-water interface in two cylindrical perspex containers where the flow can be reversed as shown.

To minimise hydraulic losses, similar tubing to that of the stand-pipe was used to supply the pressure from the tank to the stand-pipe. Where connections had to be made between the alkathene and 3/16" nylon tubing special tapered brass adapters 6" long were used. To obtain a good seal between the adapter and alkathene tubing hot water was first poured on the tubing and the adapter pushed in as far as possible. A jubilee-clip was then tightened around it. The square end of the adapter facilitated the use of spanners for twisting and extracting the adapter after the test.

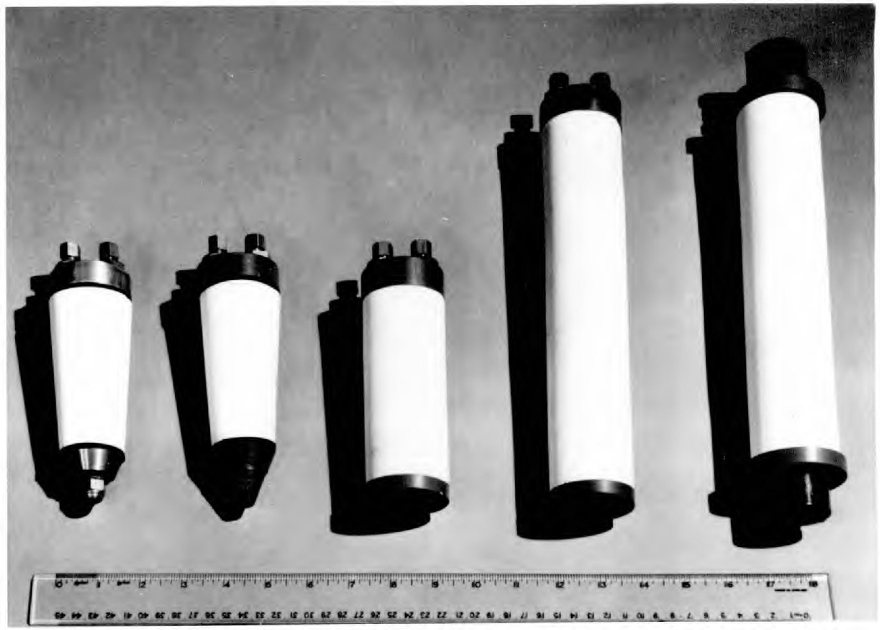
Three valve blocks "C" (Fig. 2.7) were made so that three simultaneous tests could be conducted, whereby the flow unit could be 'plugged-in' to valves 2 and 3. With valve 1 closed and the appropriate valves in Block (D) open, measurements of the flow rates were taken.

An air-bubble trap was later added to measure the amount of air (or gas) collected in consolidation tests where a water container situated below the water level in the stand-pipe, provided the necessary drop in pressure. This unit also behaved satisfactorily for the relevant range of permeability. Some of the results obtained from it will be given in Chapter 8.

PLATES. 2:1 & 2:2



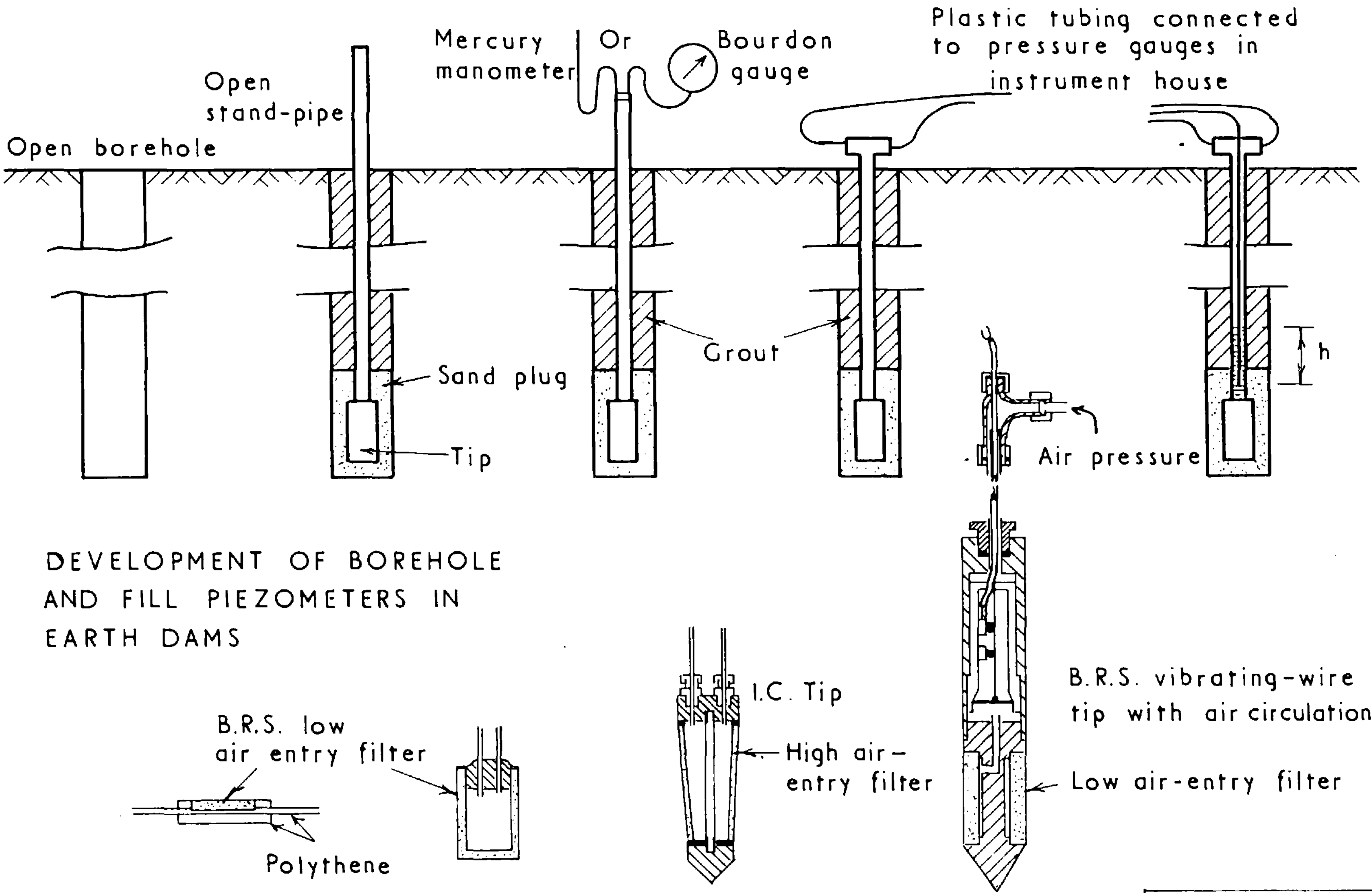
2:1 DE – AIRING SYSTEM



2:2 I.C. AND CASEGRANDE'S PIEZ. TIPS

2 : 3 LOW-PERMEABILITY C.H.T. APPARATUS

PLATE. 2 : 3



DEVELOPMENT OF BOREHOLE AND FILL PIEZOMETERS IN EARTH DAMS

FIG. 2:1

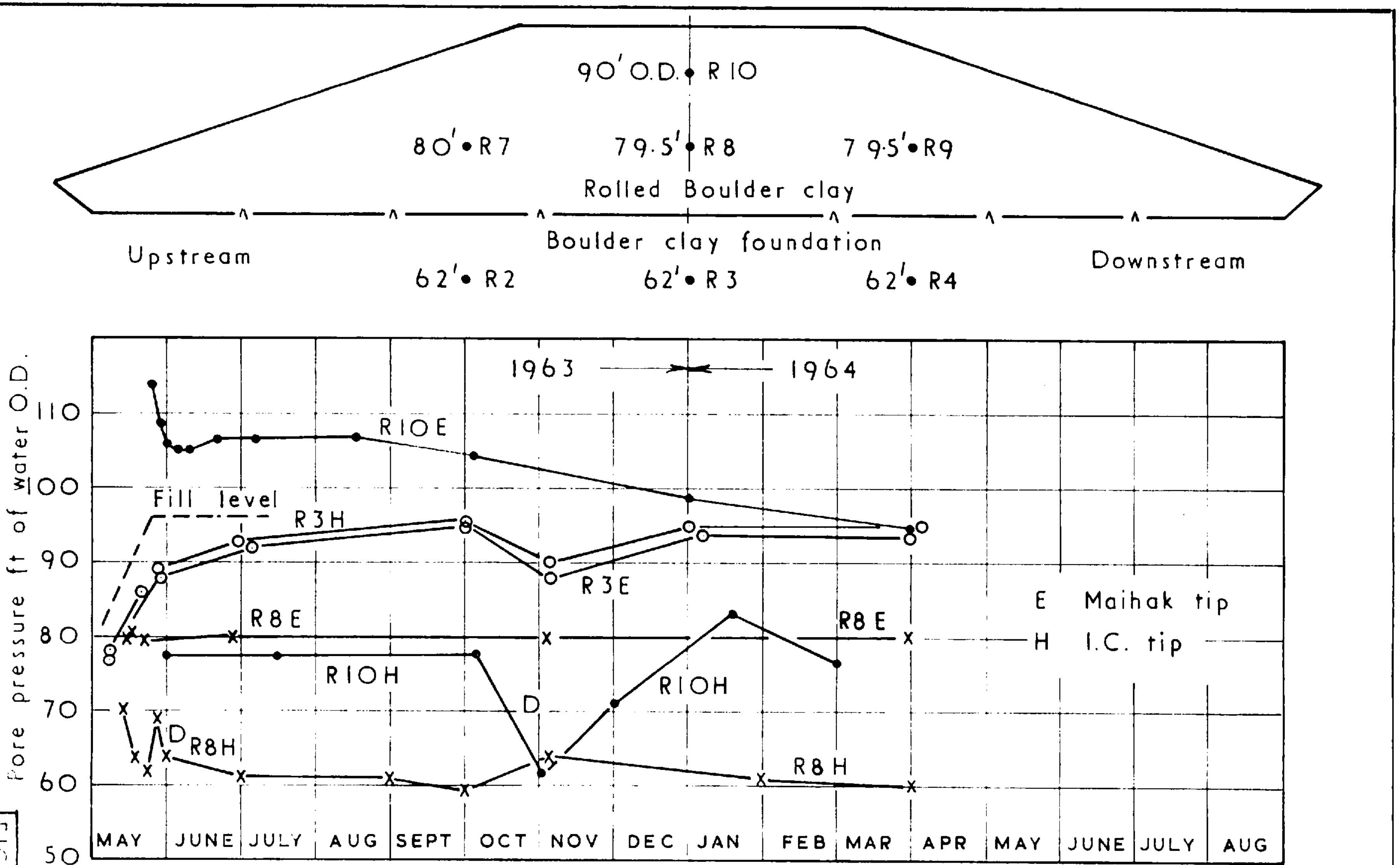
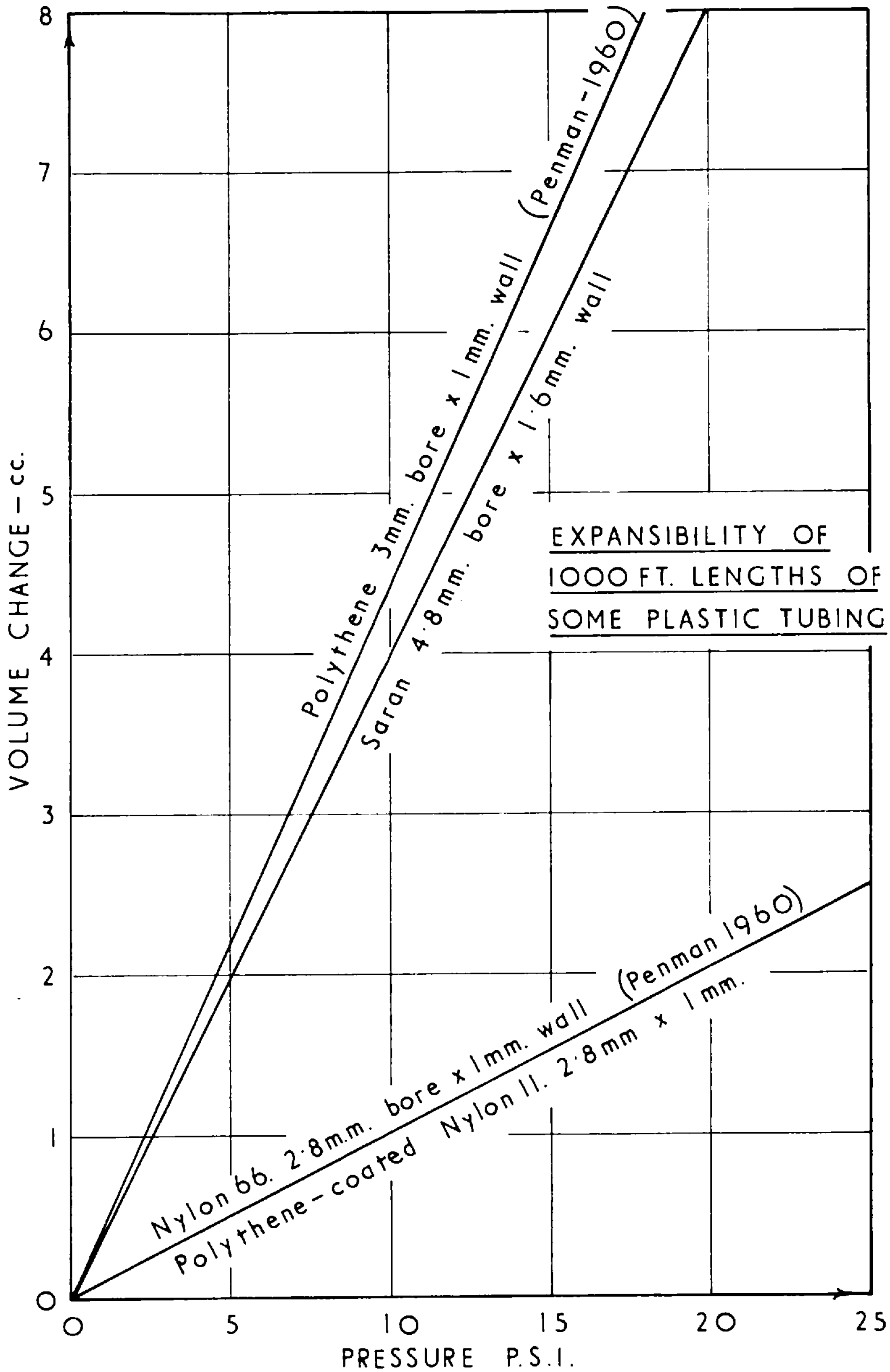


FIG 2·2

COMPARISON BETWEEN READINGS FROM MAIHAK & I.C. TIPS IN ROAD EMBANKMENT — DIDDINGTON



VOLUME CHANGE – cc.
PRESSURE P.S.I.
0
1
2
3
4
5
6
7
8
0
5
10
15
20
25
Polythene 3mm. bore x 1 mm. wall (Penman-1960)
Saran 4·8mm. bore x 1·6mm. wall
Nylon 66. 2·8m.m. bore x 1mm. wall (Penman 1960)
Polythene-coated Nylon 11. 2·8mm x 1mm.
EXPANSIBILITY OF
1000 FT. LENGTHS OF
SOME PLASTIC TUBING

FIG. 2.3

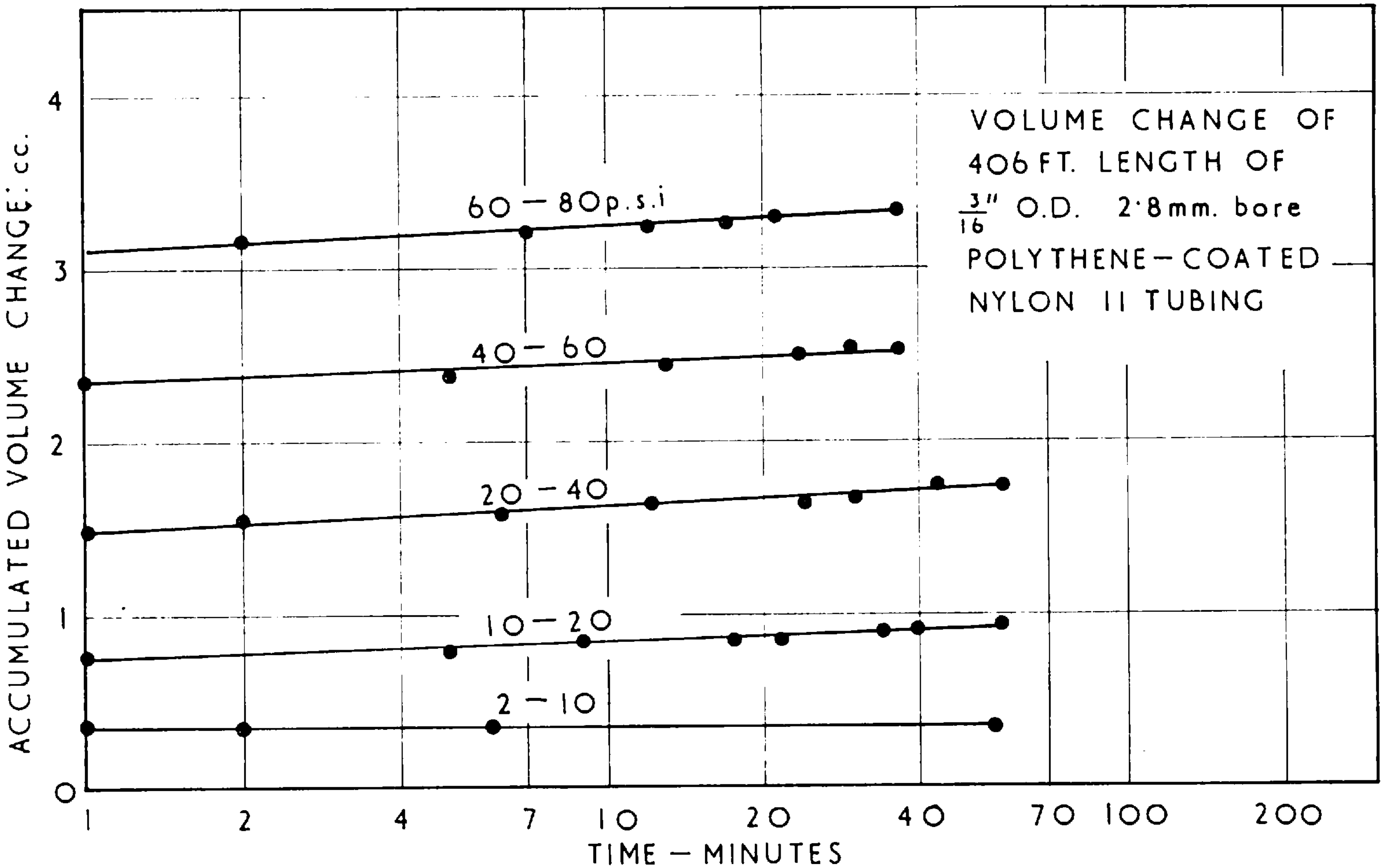
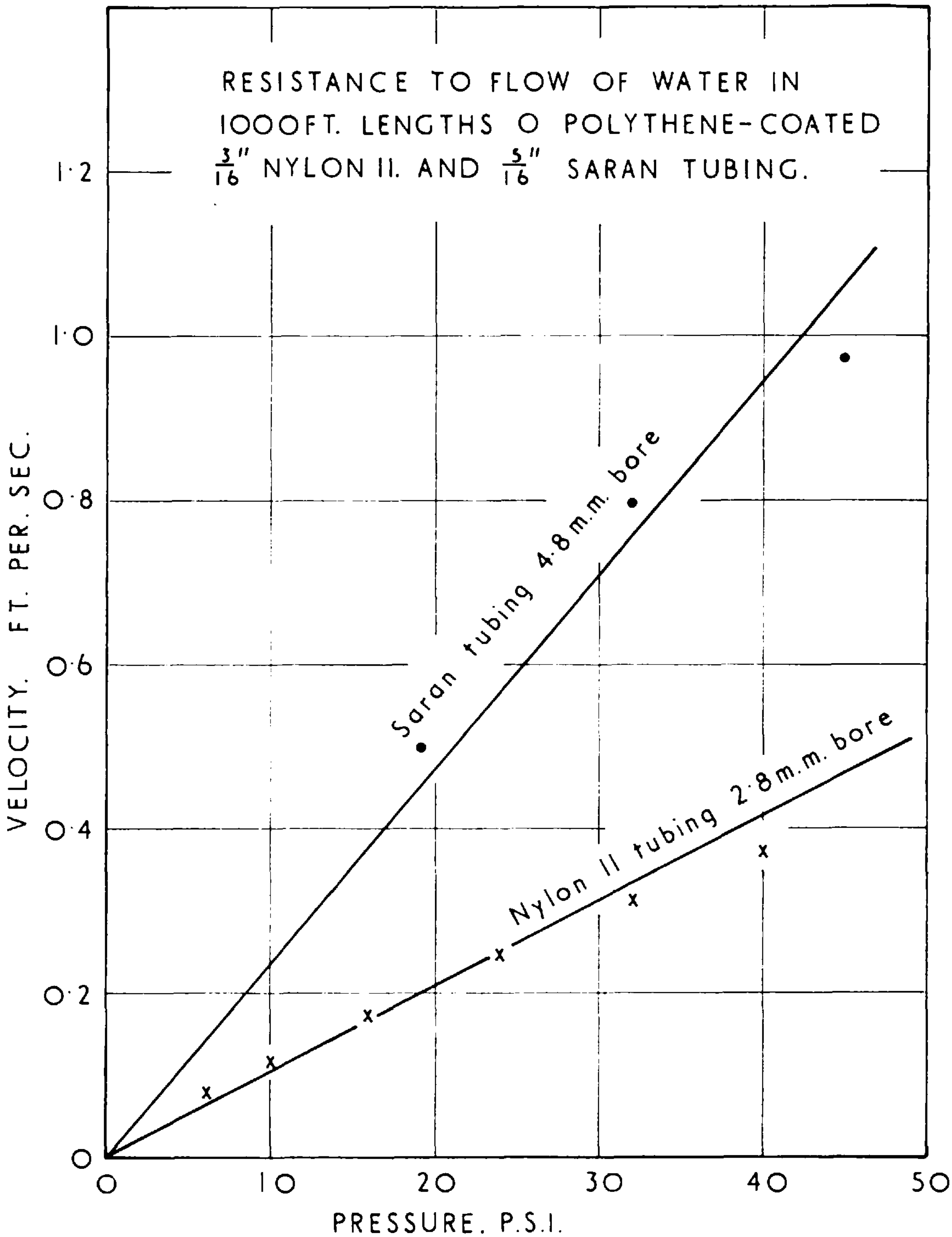
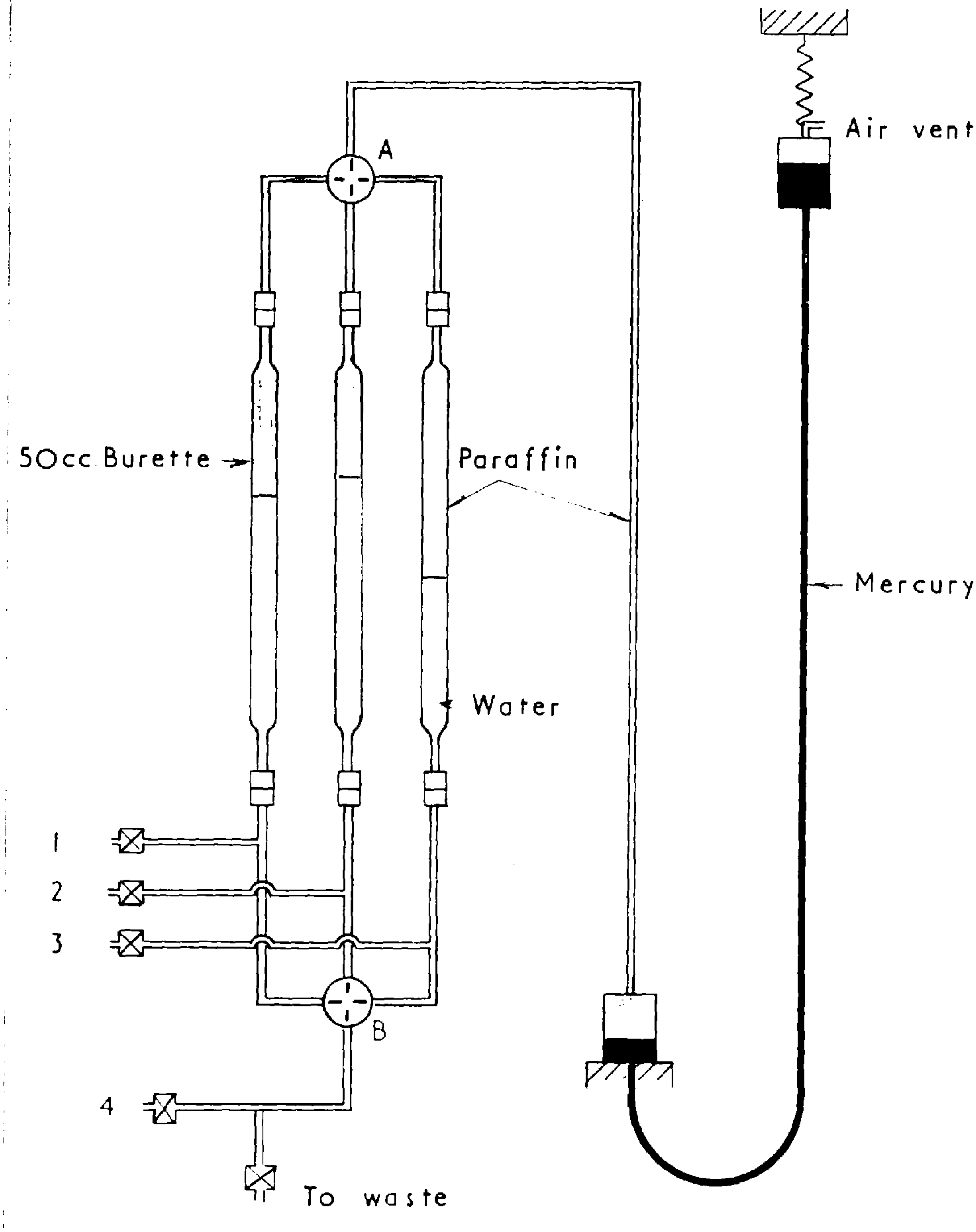

VOLUME CHANGE OF 406 FT. LENGTH OF $\frac{3}{16}$" O.D. 2·8 mm. bore POLYTHENE-COATED NYLON 11 TUBING
ACCUMULATED VOLUME CHANGE: cc.
TIME - MINUTES
60 - 80 p.s.i
40 - 60
20 - 40
10 - 20
2 - 10
0
1
2
3
4
1
2
4
7
10
20
40
70
100
200

FIG. 2.4



RESISTANCE TO FLOW OF WATER IN
1000FT. LENGTHS O POLYTHENE-COATED
3/16" NYLON 11. AND 5/16" SARAN TUBING.
VELOCITY. FT. PER. SEC.
PRESSURE. P.S.I.
Saran tubing 4·8 m.m. bore
Nylon 11 tubing 2·8 m.m. bore
0
0·2
0·4
0·6
0·8
1·0
1·2
0
10
20
30
40
50

FIG. 2.5



LOW PERMEABILITY CONSTANT-HEAD APPARATUS

FIG. 2 6

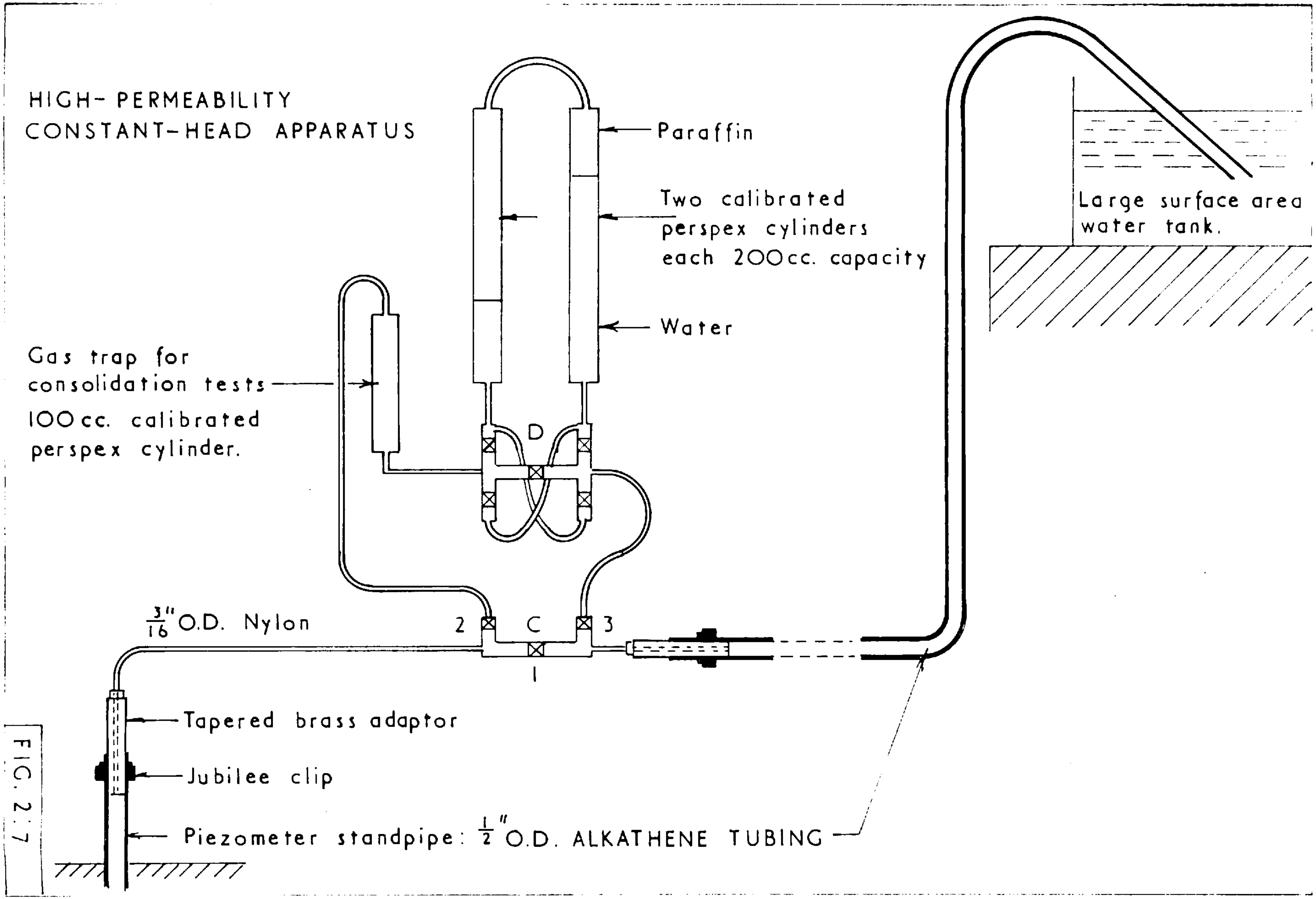

HIGH- PERMEABILITY
CONSTANT-HEAD APPARATUS
Paraffin
Two calibrated perspex cylinders each 200cc. capacity
Water
Large surface area water tank.
Gas trap for consolidation tests
100 cc. calibrated perspex cylinder.
D
C
1
2
3
$\frac{3}{16}$" O.D. Nylon
Tapered brass adaptor
Jubilee clip
Piezometer standpipe: $\frac{1}{2}$" O.D. ALKATHENE TUBING

FIG. 2.7

CHAPTER 3

DEFORMATION AND TOTAL STRESS GAUGES

(3.1) Deformation gauges - Introduction

The introduction of deformation gauges to earth dam instrumentation has been somewhat slower than that of pore water pressure gauges, and, whereas the latter gauges have been very nearly standardised (see Chapter 2), experiments with different designs of deformation gauges are still going on to find cheap, robust and reliable instruments for measuring vertical and horizontal deformation. The usefulness of deformation gauges in earth dam design has been perhaps less obvious than that of pore water pressure gauges, and the problems encountered in their design are rather more difficult to solve.

The significance of the measurement of deformation in earth dams was briefly mentioned in Chapters 1 and 2.

In this chapter only internal measurements of deformation will be dealt with, as for external measurements the usual methods of surveying apply. An interesting and perhaps very useful possible addition to surveying instruments is a 'Mekometer' working on the principle of the interference of light waves which is presently being developed in the Building Research

Station as reported to the Author by Mr. A. D. M. Penman. It would have an accuracy of $\frac{1}{2}$ cm. over a distance of $\frac{1}{2}$ mile or more.

The criteria for the satisfactory performance of an earth dam installation were stated in Section (2.3). Here the most important requirements are (i) and (iii) regarding satisfactory performance under large deformation, and least interference with the local soil conditions. The latter is especially relevant in the case of the stand-pipe vertical deformation gauges as will be mentioned in the next section.

There are, at present, three approaches to the design of vertical and horizontal gauges, viz. mechanical, hydraulic and electrical, or combinations of them. Descriptions and sketches or drawings of instruments currently used in mostly British dams, with which the Author is familiar, will be given together with a discussion of their advantages and disadvantages. A useful discussion is also made by Binnie (1963).

(3.2) Vertical deformation gauges - mechanical type

One of the simplest designs for a vertical deformation gauge is shown in Fig. (3.1a) comprising a vertical solid rod passing freely through a tubular protective sheath with its bottom end attached to a buried concrete slab (see for example

U.S.B.R. Earth Manual, 1960; Vinzi and Nicolai, 1957; and Lewis, 1956). The main advantage of this system is its cheapness, and it would probably function satisfactorily when measuring deformation at shallow depths, for example that of a soft foundation under a shallow height of rigid pervious fill. Its disadvantages are that

(i) it measures deformation at only one point

(ii) it requires accurate surveying every time a measurement is made

(iii) it forms a drainage path in impervious soils and hence leads to errors in the measurements, as well as affecting readings from nearby piezometers, if any

(iv) it is quite likely to be accidentally damaged by construction plant.

This system was used at the M6 Trial Embankment* where it was installed after completion to replace the unsuccessful Road Research Laboratory mercury-settlement gauges, but the readings obtained were inconsistent with the known behaviour of the fill, as will be discussed in Chapter 8.

An alternative to this arrangement is the U.S.B.R. cross-arm method (U.S.B.R. Earth Manual, 1960) where measurement

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* For some details of this embankment and other embankments and dams mentioned in this Chapter, see Chapter 8.

of settlement at several points down a vertical tube can be made, Fig. 3.1. This system comprises a telescopic steel tubing to which steel cross-arms are attached at intervals to anchor the tubing to the soil. To take measurements a "torpedo" with spring-loaded pawls, attached to a calibrated tape is lowered down the tubing. Measurements on the tape are then taken when the pawls rest on the bottom end of the smaller tubing. When the torpedo reaches the bottom of the telescopic tubing it is dropped whereby the pawls close down to facilitate its withdrawal.

This system was used in Diddington dam where, to avoid interference with construction work, the tubing was capped and buried. When the fill was 5' above it a hole 5' by 5' was dug, readings were taken and the tube extended. The disadvantage with this routine was the difficulty, especially in adverse weather conditions, of stopping dirt falling into the tube before an extension was made. This, in fact, did happen in Diddington as well as the jamming and loss of one or two torpedos. Fortunately in this case the dirt and jammed torpedos were successfully rammed down to the bottom.

Here again accurate surveying is required to establish the level of the top of the tubing every time a measurement is taken, unless the lowest point in the tubing is taken as a fixed datum when it is, for example, resting on a rock bed whose deformation is negligible.

(3.2.1) Electrical type

To overcome the risk of jamming and loss of the torpedos in the U.S.B.R. cross-arm method a modification was made jointly by Imperial College and the Building Research Station whereby the location of steel plates buried around telescopic o-ring sealed plastic tubing was made by lowering an electrical sounding device whose inductance changed when it passed through a steel plate, Fig. 3.1 (see Vaughan, 1965). This system was used in the clay cores of Selset and Balderhead dams. Vaughan mentions the difficulties encountered in Selset dam in obtaining straight lengths of the initially coiled plasticised tubing used there. This was overcome in Balderhead by the use of un-plasticised tubing manufactured in short straight lengths.

In both dams the tubing was left standing above the fill and well protected from construction plant. Compaction of the fill immediately next to the tube was achieved by the use of a small motorised hammer. A special difficulty arose in the top narrow section of the clay core in Balderhead where construction plant could only pass on one side thus shifting the tubing some 2 ft. off line. Another difficulty in the case of Balderhead dam was the extension of the ~~cloth tape~~ electric cable used to lower the sensing device. The ~~tape~~ cable was later calibrated in the deep vertical shaft of the bell-mouth spillway and corrections applied to the previous readings.

Recently the surface equipment required for monitoring the change in inductance has been made very compact, and is now being commercially made by Soil Instruments Ltd. to a design by the Building Research Station.

(3.2.2) Hydraulic type

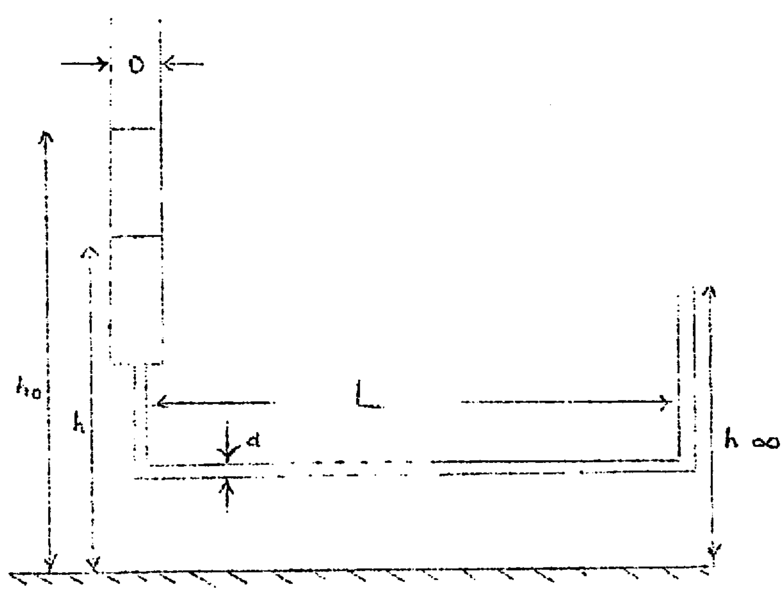
The hydraulic type settlement gauge obviates the need for a vertical pipe through the fill, and it is designed on the simple principle of a U-tube, Fig. 3.1. Skempton and Bishop (1955) report one of the earliest uses of this instrument in earth dams. Vaughan (1965) describes the units used in Selset and Balderhead dams.

Fig. 3.2 shows a detailed drawing of a cell designed by the Author working with Dr. P. R. Vaughan where an ordinary U-4 sample tube and standard fittings were used. The 3/16" nylon lead is the U-tube itself while the other two $\frac{3}{8}$" N.B. tubes are air and drain leads. The former ensures that the air pressure inside the cell is equal to the atmospheric pressure outside, and the latter is used when flushing the system with water or gas. This unit was used in Fiddlersferry Embankment as will be referred to later in this section.

The outside limb of the U-tube is simply a graduated glass tube open to the atmosphere and with connections to a water supply. This together with the air and drain leads are housed in a suitable man-hole on the slope of the dam at the same level of the cell.

Theoretically if the settlement point is settling faster than the gauge house then the reading on the glass tube is correct. However, because of possible meniscus error it is advisable to fill the graduated glass tube with water and allow it to dissipate to the cell. The speed with which equalisation occurs is an exponential function of time, the theory of which will be given now. It is very similar to that of the falling-head seepage test on a piezometer, given in Chapter 4 (see also Vaughan, 1965).

Schema (3.1)



Referring to Schema (3.1)

If the boundary conditions are

at $t = 0$, $h = ho$

at $t = \infty$, $h = h_\infty$

Then using Poiseuille's equation for the average velocity $\bar{u}$ in a small bore tubing of length L and diameter d:

$$\bar{u} = \frac{\rho . g . d^2 (h - h\ )}{32 . \mu . L} \quad (3.1)$$

and putting $\frac{dh}{dt} = \bar{u}\ \frac{d^2}{D^2}$

we have $$\frac{h - h}{ho - h} = e^{-Nt} \quad (3.2)$$

where $$N = \frac{\rho\ g\ d^4}{32 . \mu L . D^2} \quad (3.3)$$

As an example the times required for 95% and 99% equilisation when L = 100', d = 2.8 mm. (ordinary 3/16" nylon) and D = 0.25" are 37.8 and 58.2 minutes respectively at 15°C.

To obtain a rough but quick value of h without waiting for complete equilibrium, an alternative method can be used where three readings h_1, h_2 and h_3 are taken at two equal intervals of time and using eq. (3.2) the following relationship emerges:

$$h = \frac{h_1\ h_3 - h_2^2}{h_1 + h_3 - 2h_2} \quad (3.4)$$

Accurate measurements have to be taken in this case as this formula is very sensitive to errors. Fig. 3.3 shows two equilisation curves for a settlement point in the upstream shale shoulder in Balderhead dam on two different dates during which the height of fill above the settlement point rose by about 3.5 ft. The graduated observation glass in this case was made of two different diameters (see Vaughan, 1965) and hence the change of slope of the curves. The readings obtained clearly show the sensitivity of the system even to very small changes in vertical deformation.

An alternative and very useful method of obtaining a quick and accurate value of h is given by Vaughan where use is again made of eq. (3.2) thus:

$$\frac{dh}{dt} = N.h_{\infty} - Nh \qquad (3.5)$$

By plotting $\frac{dh}{dt}$ against h, the intercept on the h - axis when $\frac{dh}{dt} = 0$ gives h_{∞}.

Hydraulic settlement gauges were used in two dams with which the Author was familiar, Balderhead dam and Fiddlersferry embankment. Vaughan reported the behaviour of the gauges in Balderhead, where six units were installed in the shale shoulders. On the whole they behaved quite satisfactorily and required infrequent flushing to maintain water continuity in the U-tube. However, after impounding, a difficulty in obtaining readings was encountered with one upstream cell where it was obvious that, perhaps due to a leak, it was communicating with the reservoir level. This came to light when water started overflowing from the observation glass tube when the drain and air leads were deliberately blocked. One of the earlier difficulties with this instrument, for example in Selset dam, was the presence of air locks in the air lead which led to a back pressure on the cell resulting in erratic readings. In Balderhead dam this problem was avoided by laying the three leads with a sufficient fall to keep the air lead free of water.

The units used in Fiddlersferry were made to the design shown in Fig. (3.2) and were laid on two sections of the embankment at foundation level. In one section where the foundation was fairly rigid these units behaved satisfactorily and gave consistent readings. On the other section, however, where the foundation consisted of some 15' - 20' of soft silty clay and peat all six units ceased to function at varying times during the construction of the embankment. Water introduced into the observation glass tubes refused to dissipate to the cells. Nitrogen under pressures up to 80 p.s.i. was applied to the air and drain leads in an effort to clear any obstructions but without success. It soon became apparent that it was quite possible that the leads from the cell were severed completely at some point under the embankment due to the large lateral shear deformation of the soft foundation. In fact two small slips did occur during the construction which severed some pore water piezometer stand-pipes.

This difficulty was later encountered with the Road Research Laboratory mercury settlement gauges used in M6 Trial Embankment as will be discussed in the next section where suggestions for overcoming it will also be given.

(3.2.3) Road Research Laboratory mercury type

This cell was designed by the Road Research Laboratory (see Irwin, 1964) and used over the last few years in road embankment construction. Its introduction to earth.dam installation has been very recent probably starting with the Khaingi dam in Nigeria where Dr. P. R. Vaughan is the Resident Engineer. It has also been recently used in the M6 Trial Embankment near Kendal.

It is essentially another U-tube arrangement where gas pressure is applied to one limb to balance a mercury column in the other limb. The balancing operation is carried out by sending an electrical current through an insulated wire to a contract point in the settlement unit buried in the fill and using the mercury as the return conductor. From the gas pressure recorded the level of the settlement point is found (see Fig. 3.1 for a diagrammatic layout).

Accurate bleed values are used to control the gas pressure with a feed-back from the electrical circuit to shut-off when contact is established. The outside end of the mercury tube is coiled horizontally with its end open to the atmosphere. The gas pressure applied "h" is then equal to the difference in mercury head between the settlement point and the outside fixed horizontal datum. The settlement point is earthed to enable readings to be taken even when the wire is damaged. Several settlement points can be served from the same outside unit.

Irwin (1964) claims that the main advantage of this instrument over the hydraulic type described in Section (3.2.2) is the much increased reliability and accuracy in the measurements taken since it does not suffer as seriously from air locks. Even if the mercury column is broken then this can be detected more easily than an air lock in the hydraulic unit. Another advantage is that the gauge house for the mercury cell does not have to be exactly at the same level as the buried cell, and can be, in fact, a few feet higher or lower, allowing more freedom in its positioning.

Four lines each consisting of five cells were installed in the 30' high M6 Trial Embankment by the Road Research Laboratory. Two lines were installed on the rigid partly saturated boulder clay foundation and the other two in the middle of the very wet boulder clay fill. During the initial part of construction when deformations were very small all four lines gave consistent readings repeatable to within 0.1 cm. Later on when more fill was placed and, in fact, a small slip occurred, one middle line was completely out of action while the other gave erratic readings, probably due to the breaking up of the mercury column. The two foundation lines, however, continued to behave very satisfactorily measuring very small deformations. Vaughan in a private communication to the Author mentioned similar trouble with the breaking up of the mercury column in the cells he is using in the Khainji dam, Nigeria. He also mentioned the very slow response of the system and that readings were only repeatable to within $\pm \frac{1}{2}$".

These difficulties are similar to those encountered with the hydraulic cells mentioned in the last section. It is quite obvious to the Author that the root of the trouble is simply the inability of the tubing connecting the cells to the gauge house to withstand the large shear deformations encountered in both embankments. This difficulty is not encountered with hydraulic piezometers since the tubing used is most commonly the very robust and small bore 3/16" nylon which is capable of withstanding up to 100% extension without serious "necking". In the hydraulic and mercury gauges bigger and much less ductile tubing was used. The obvious solution seems to be either to use some tubing as rigid as say P.V.C. but also as ductile as nylon, or better to enclose all the leads in rigid O-ring sealed telescopic tubing similar to that used in the vertical induction settlement gauge. The edges of the tubing should be bevelled on the inside and outside to ease the slipping of O-rings during assembling and also to protect the inner tubes from damage when the telescopic tubing has been pulled apart. The lengths of the tubing must be of such dimensions as to resist fracture by tension failure when the soil is undergoing large shear deformation.

As an example, the case of the hydraulic gauges in Fiddlersferry will be analysed, assuming, for convenience, that the frictional stress on the surface of the tubing is equal to the undrained shear strength of the soil, and that the cell does not move.

If the undrained shear strength of the soil is C_u, the maximum tensile stress of the tubing t, the diameter of the tubing d and the thickness of the wall is h, then L, the maximum permissible length of the tubing is given by

$$Cu \times L \times \pi \times d = t \times \pi \times d \times h \qquad (3.6)$$

Putting Cu = 4 p.s.i., d = 0.68", t = 3000 p.s.i. and h = 0.16", L is found to be 10 ft.

In Fiddlersferry continuous lengths between 60 ft. and 180 ft. were used which, as was mentioned earlier, all failed at different times during construction.

To guard against small slips with maximum relative movements of say 1 or 2 ft. a similar calculation can be carried out. Taking one length of the telescopic tubing and assuming that it is cantilevered at one end by the stable mass of the soil and resisting shear fracture due to the vertical load imposed by the sliding mass, a suitable length 'L' can be found assuming that the vertical loading on the upper face of the tubing is say 6 times the shear strength of the soil. This calculation would yield much smaller acceptable values of 'L' than that from eq. (3.6). However, it is in the Author's opinion, quite essential to conduct such calculations for future installation of hydraulic and mercury settlement gauges especially in soft soils, even if it was found necessary to use very short lengths of rigid telescopic tubing; otherwise the chances of obtaining readings of large deformations, where interest lies most, are almost negligible.

(3.3) Lateral deformation gauges: wire and plate method

Here again there are at present three approaches viz. mechanical, hydraulic and electrical. The cheapest and simplest method is the wire and plate method (see Hoskin and Hilton, 1963). Fig. 3.4 shows such a unit designed by the Author working with Dr. Vaughan. The wire is attached to the plate by unscrewing the ordinary compression fittings against the chucks. The wire is greased and passed through O-ring sealed plastic telescopic tubing, and kept taut by tensioning it outside the dam using a spring or a pulley and weight system. Up to three wires can be accommodated in the same telescopic tubing as shown.

A similar arrangement was used at Selset dam but was not very successful, probably due to the "snaking" of the thinner wire and improper tensioning at the end. However, the arrangement shown in Fig. 3.4 is currently being used in the Khainji dam by Dr. Vaughan who privately reported to the Author that the instruments are at this early stage of construction recording zero lateral deformation to within $\pm \frac{1}{8}''$. These measurements are in accord with external surveys which also show negligible deformation.

(3.3.1) Vibrating wire strain gauge extensometer

This was developed by the Building Research Station and Imperial College and used in the clay core at Balderhead dam. Vaughan (1965) gives a sketch of the instrument and describes it thus:

"It measures (strain) between two steel joists 6 ft. apart. In principle it consists of a telescopic tube. The elongation of the tube extends a spring which in turn loads a proving ring. The small deflection of the proving ring is measured by a vibrating wire strain gauge".

He also mentions that all three instruments installed behaved satisfactorily. The maximum reading recorded by these instruments was one inch over the 6 ft. gauge length when the dam reached full height. Kennard, Penman and Vaughan (1967) give more details of this instrument and the total stress gauges used in Balderhead dam.

(3.3.2) Building Research Station proposed electrical-hydraulic type

An interesting development of a new instrument is now taking place in the Building Research Station where vertical and horizontal deformation can be measured in one horizontal tube buried in the dam, similar to the instrument used in the Gepatsch dam (see Lauffer and Schober, 1964).

In this arrangement a trolley is moved in the horizontal tube either by a wire and pulley system or pneumatically. Measurements of lateral deformation can be obtained by the inductance coil method by attaching the coil to the trolley to detect the positions of vertical steel plates buried around the tubing. To obtain measurements of vertical deformation the trolley is used as one end of a water filled U-tube, as in the ordinary hydraulic settlement gauge.

Mr. Penman has privately told the Author that work on this instrument is going on in the Building Research Station and he hoped that it will be soon used in the Scammonden dam near Huddersfield, the construction of which will start in the Spring of 1967.

Nevertheless, several difficulties have yet to be solved before complete success can be expected. The first relates to the already discussed problem of the fracture of the tubing by tension or shear when the soil surrounding the tubing undergoes large shear deformation or slipping. The second problem is making sure that the tubing does not form depressions where water might accumulate and affect the readings from the U-tube arrangement. This difficulty can easily be overcome by ensuring a sufficient fall on the tubing when installed. The third problem lies in the choice of the propelling method and ensuring free passage of the trolley even when the tubing is slightly out of shape under the load of the dam.

(3.3.3) The inverted pendulum

This is a recent useful addition to lateral strain gauges in earth dams, but earlier it has been frequently used to measure lateral deformations at rock bases of concrete dams. Roberts, Wilson and Wiltshire (1965) describe the unit used in the Monar dam, Scotland where one end of the wire was anchored at the bottom of a deep borehole and the other end attached to an annular float in an oil bath. Readings were taken on the top of the float with a travelling microscope.

An opportunity arose for the design and manufacture of such an instrument for use in the Muirhead dam for the measurement of lateral deformation of the upstream face of the dam during a rapid drawdown test. The instrument was to be installed in the 60 ft. deep Valve Tower where measurements of the relative lateral deformation between the crest of the dam and the Valve Tower could be made across the connecting foot-bridge. Normally an ordinary pendulum could have been used but to avoid the necessity of taking readings at the bottom of the Tower the inverted pendulum was used. Fig. 3.5 shows a drawing of the instrument designed by the Author under the supervision of Professor A. W. Bishop. In the design procedure the first thing was to decide on the force required to keep the wire straight and taut. A figure of 50 lbs. was chosen acting on a stainless steel wire 16 S.W.G. The next step was to decide on the clearance between the float and the tank, and this was made 3" each way. If larger deformations occurred then the whole tank could be re-centred again. The tank could be filled with oil or mercury, the latter reducing the size of the unit considerably. A calculation was then made of the amount of mercury required and the price was found to be too prohibitive (the price of mercury was then £3 a lb.). But even with $\frac{1}{2}$" travel the cost of mercury alone would be more than £100. It was then decided to use oil and accept the increase in the size of the unit.

The unit was manufactured in the laboratories of Imperial College by Messrs. D. Evans and L. Spall. The float and tank made of thin tinned iron sheets which were machine cut to size, rolled and spot-welded. Further soft soldering sealed the units.

An important point in the design was the necessity of attaching the wire to a point well below the centre of buoyancy of the float to stabilise it, but even this did not stop the float from tilting slightly to one side, probably due to an uneven distribution of weight. This was remedied quite simply by resting a 5 lb. balance load on the float.

The whole float was completely immersed in oil to protect it against corrosion and to avoid effects of surface tension.

The unit was mounted on a 30" high wooden pedestal in the

Valve Tower and the wire passed through the grills in the flooring, and anchored at the bottom. Measurements were taken on the wire itself by taping the distance from a fixed point on the crest of the dam to the wire. Ideally a calibrated scale travelling at right angles to another calibrated fixed scale can be used which would give a two dimensional picture of the lateral deformation.

The pendulum was assembled and calibrated first, in the staircase of the Civil Engineering Department using water instead of oil. A graduated marker was attached to the wire below the float and observed through a cathometer. When pulled to one side a distance of about one inch and let go it oscillated about the mean position five or six times in a few minutes, and then came back to its exact equilibrium position. It was calibrated again in the Valve Tower in Muirhead using oil. This time it did not oscillate but it still came back to its exact position in 3 or 4 minutes. It thus seems to be functioning satisfactorily although there has been no measurable movements yet.

At the moment Mr. Robertshaw of Soil Instruments Ltd. is making similar units to the same design for use in other dams. He told the Author that he may use a plastic float unit to avoid the risk of leakage of oil into the float which may have lately affected the unit at Muirhead where the float was tilting badly to one side but still not touching the sides of the tank.

(3.4) Measurement of total stresses

The measurement of total stresses in earth dams is again a very recent development and in fact the only case in the U.K. with which the Author is familiar is the Balderhead dam where five Building Research Station-Imperial College rigid diaphragm vibrating wire cells were installed in the clay core, to study the effect of arching and rotation of the principal stresses.

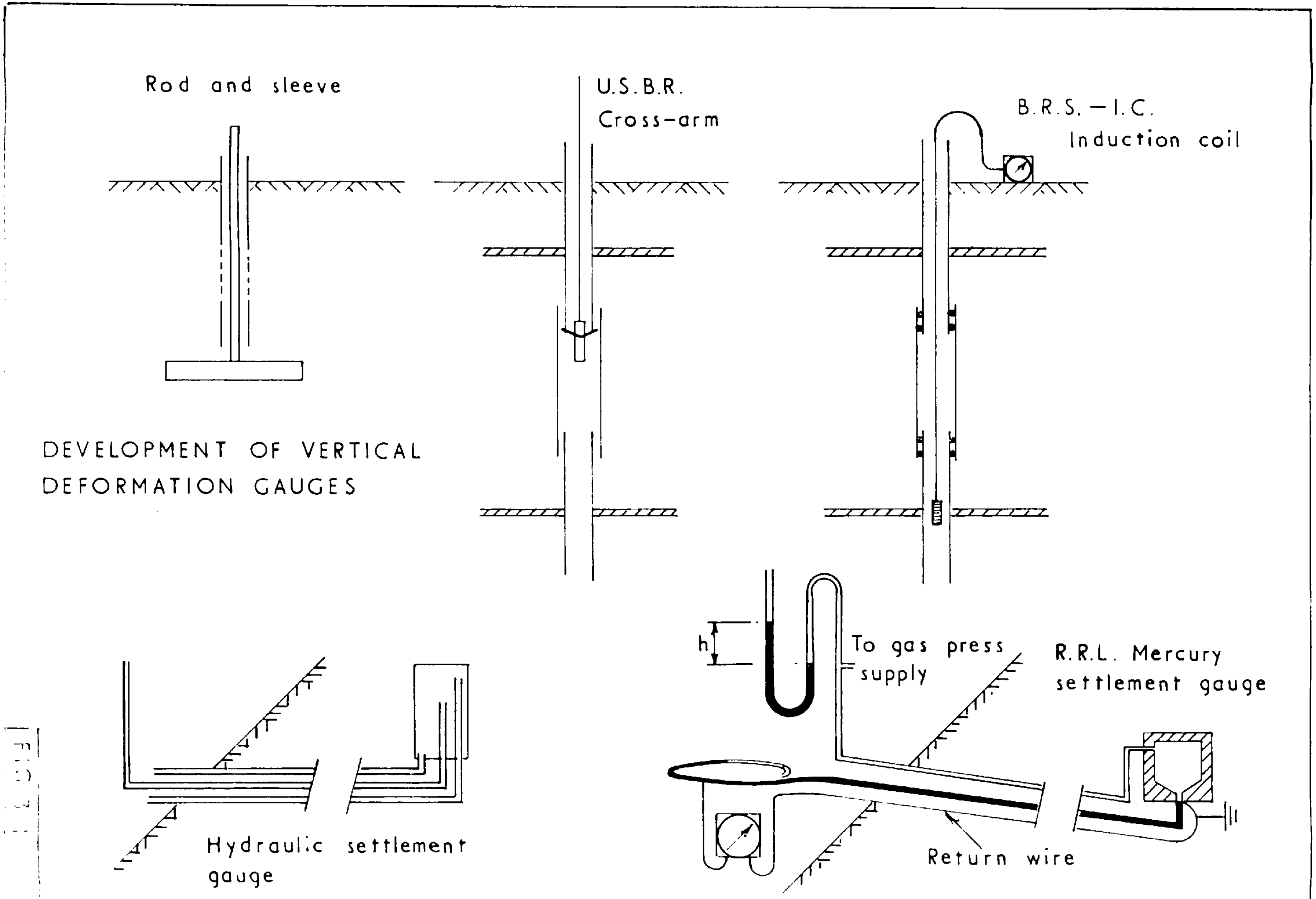
The design of most total stress gauges incorporates a rigid thin diaphragm, the deflection of which is measured using hydraulic-pneumatic, electrical devices, etc. A useful discussion of total stress cells is given by Arthur (1963). Vaughan (1965) describes the cells used in Balderhead and reports very satisfactory performance.

Rowe and Briggs (1961) describe a hydraulic system where the change of volume of water in a small cavity behind the diaphragm is measured in a small bore capillary tube. A dummy cavity and capillary tube is also incorporated for temperature corrections. This system would be difficult to use in an earth dam where the volume change in the connecting tubing would probably have a significant influence on the measurements.

Wikramaratna (1961) describes a pneumatic system where the pressure drop in a continuous air flow across an orifice whose dimension is controlled by the movement of the diaphragm is measured. This system is again not very suitable for earth dams because of the possible temperature effects on the large volume of air in the leads, and also possible long term instability of the system.

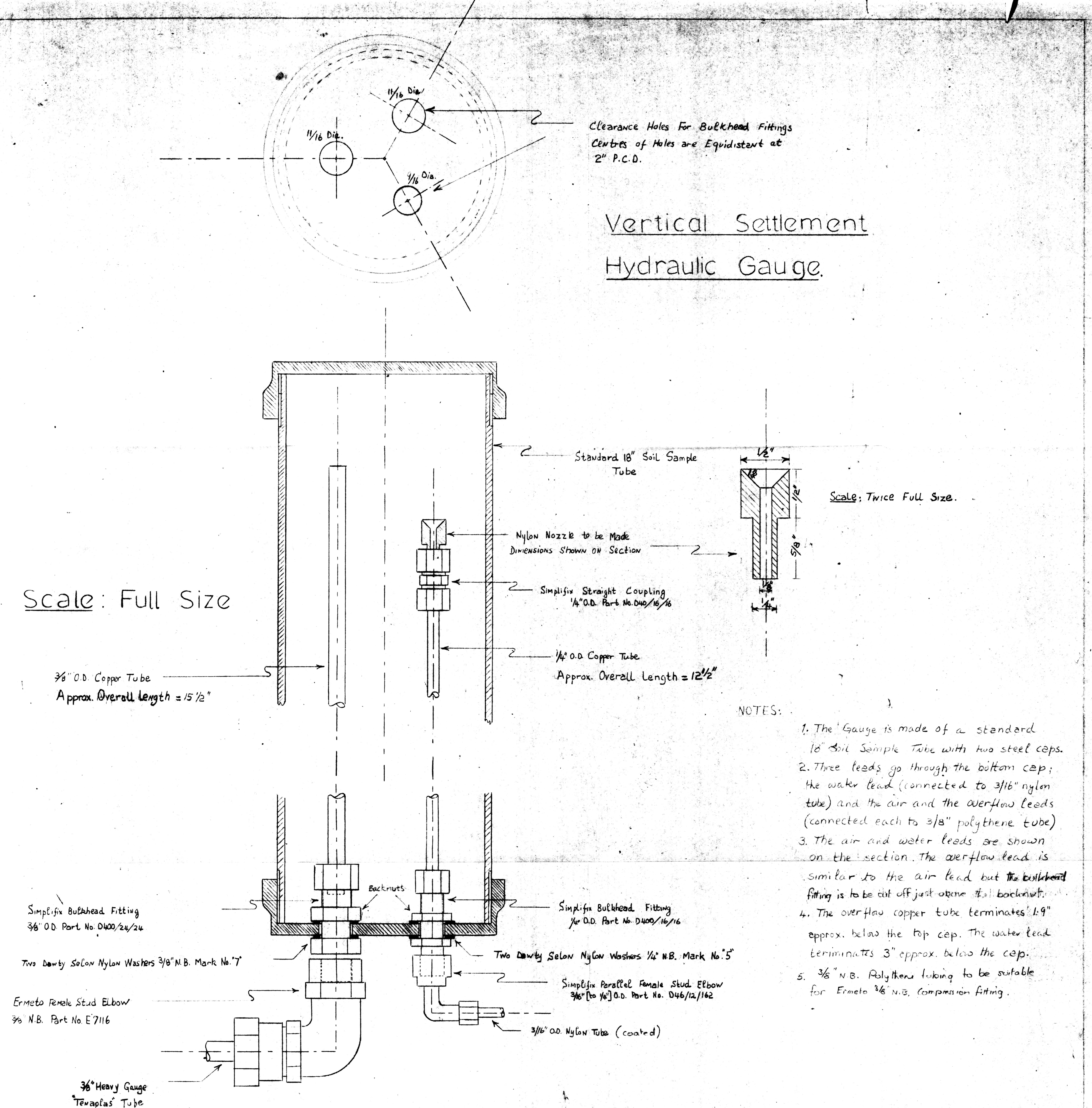
Direct measurement of the deflection of the diaphragm can be achieved by an electrical inductance method (Rowe, 1954) electrical resistance strain gauges (Plantema, 1953; Arthur and Roscoe, 1961) or by vibrating wire strain gauges (Ward, 1955).

In the Author's opinion a useful alternative to the rigid diaphragm method may be the use of a wide but thin flexible metal 'bag' filled with a mixture of oil and nitrogen such that the compressibility of the cell is comparable to that of the soil, to avoid errors in the measurements due to the actual presence of the gauge. Readings of the total stress can then be relayed to the outside of the dam through a rigid small bore tubing or by attaching a pressure transducer to the 'bag'.



DEVELOPMENT OF VERTICAL DEFORMATION GAUGES

Vertical Settlement Hydraulic Gauge.



NOTES:

1. The Gauge is made of a standard 18" Soil Sample Tube with two steel caps.
2. Three leads go through the bottom cap; the water lead (connected to 3/16" nylon tube) and the air and the overflow leads (connected each to 3/8" polythene tube)
3. The air and water leads are shown on the section. The overflow lead is similar to the air lead but the bulkhead fitting is to be cut off just above the backnut.
4. The overflow copper tube terminates 1.9" approx. below the top cap. The water lead terminates 3" approx. below the cap.
5. 3/8" N.B. Polythene tubing to be suitable for Ermeto 3/8" N.B. Compression fitting.

FIG. 3:2

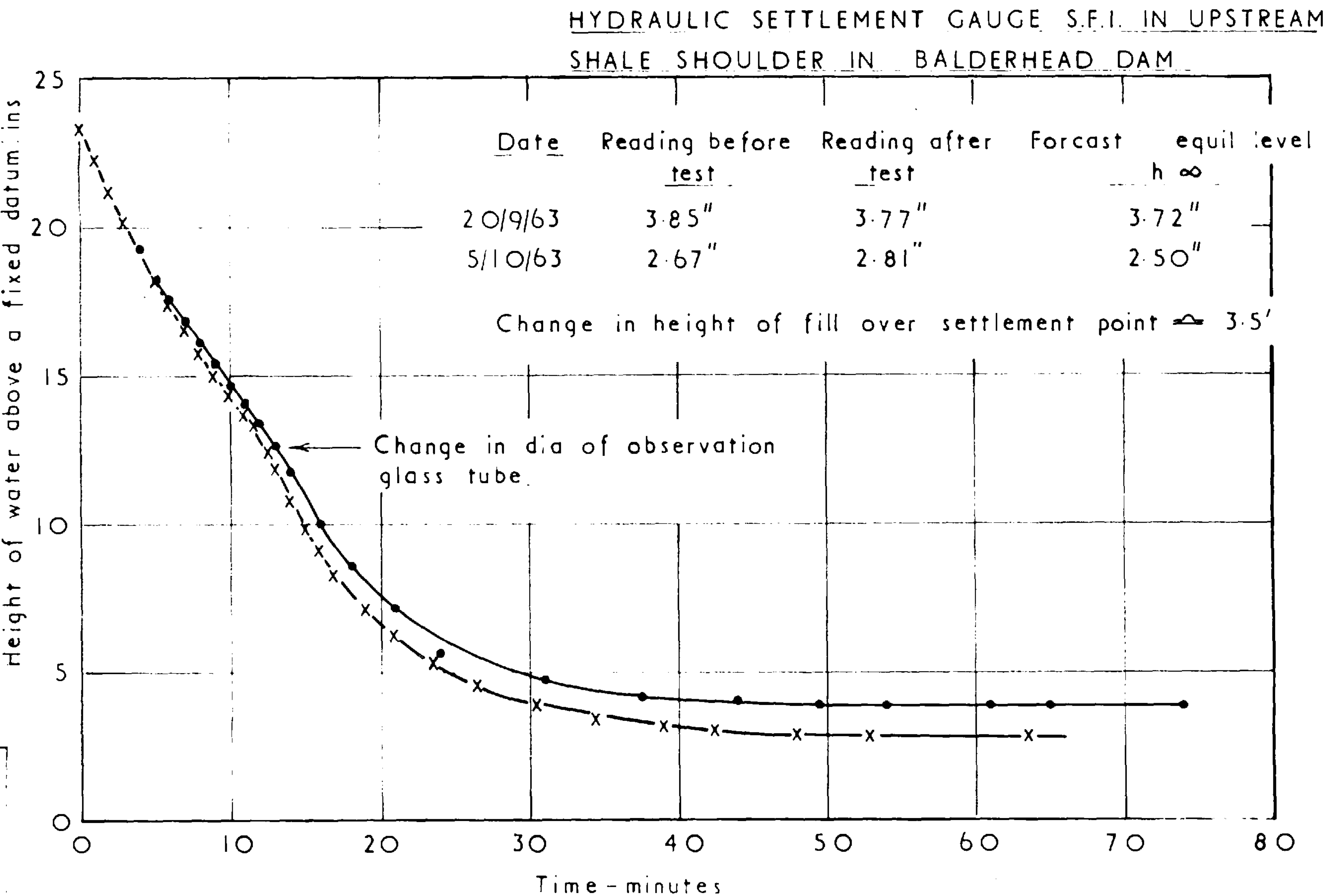
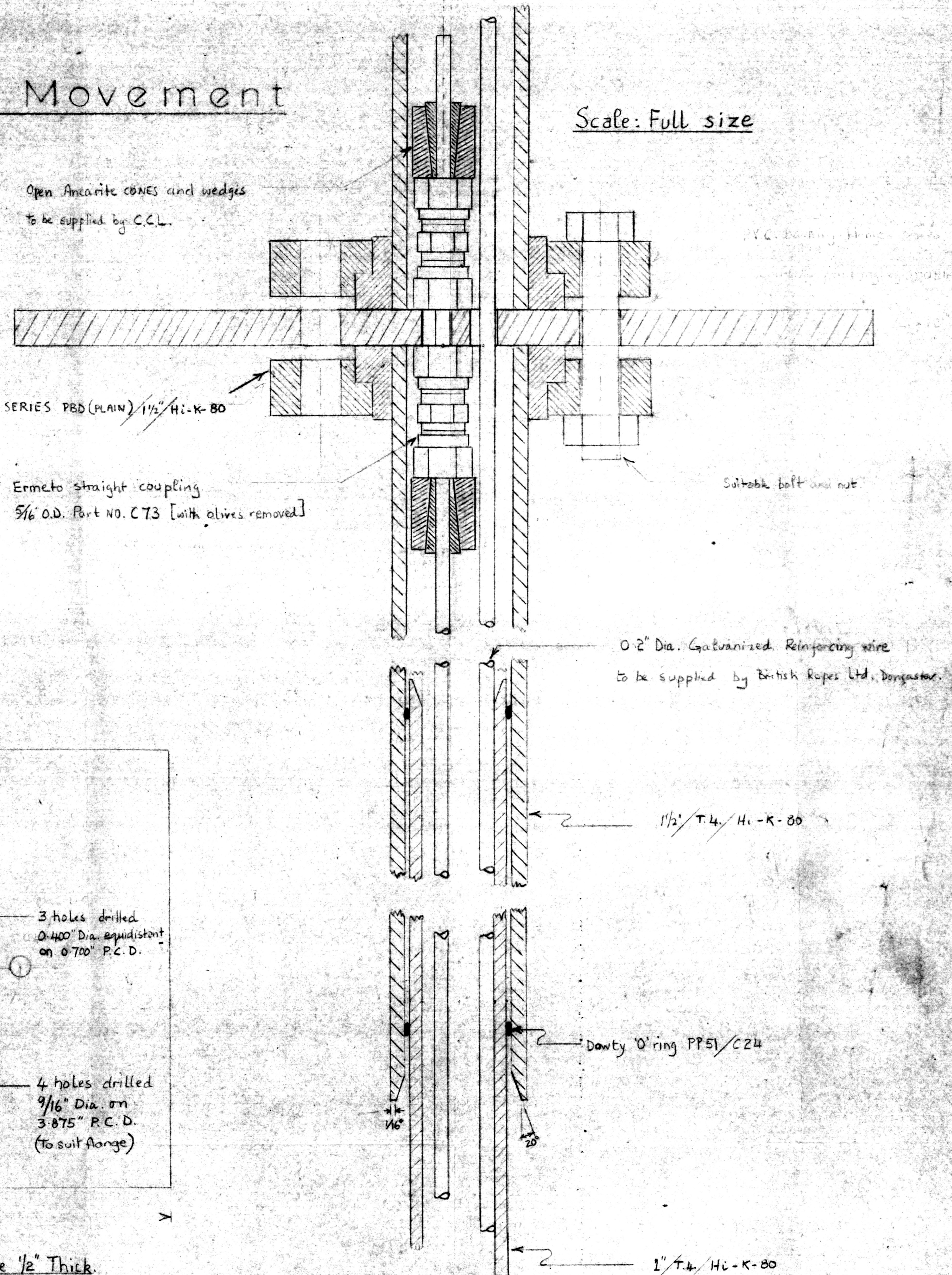
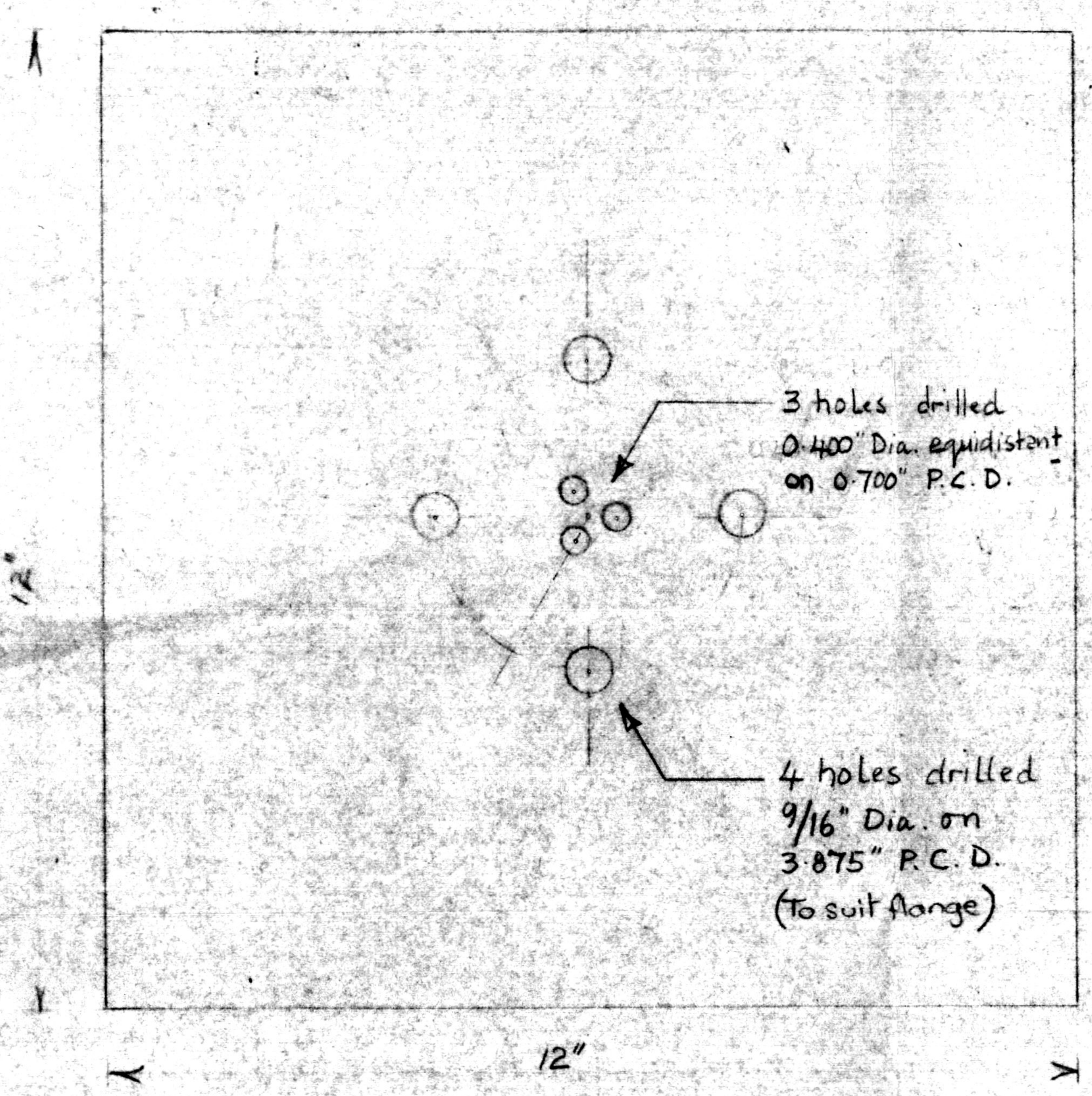

HYDRAULIC SETTLEMENT GAUGE S.F.I. IN UPSTREAM SHALE SHOULDER IN BALDERHEAD DAM
Date
Reading before test
Reading after test
Forcast equil level h ∞
20/9/63 3·85" 3·77" 3·72"
5/10/63 2·67" 2·81" 2·50"
Change in height of fill over settlement point ≏ 3·5'
Change in dia of observation glass tube
Height of water above a fixed datum ins
Time - minutes
0
5
10
15
20
25
0
10
20
30
40
50
60
70
80

FIG 3.3

Horizontal Movement Gauge





Mild Steel Plate 1/2" Thick.

Scale 1/2 Full size

Section.

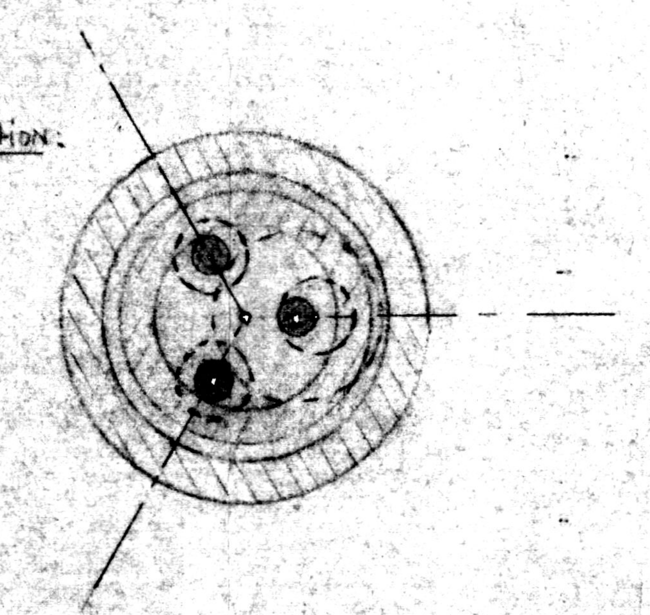
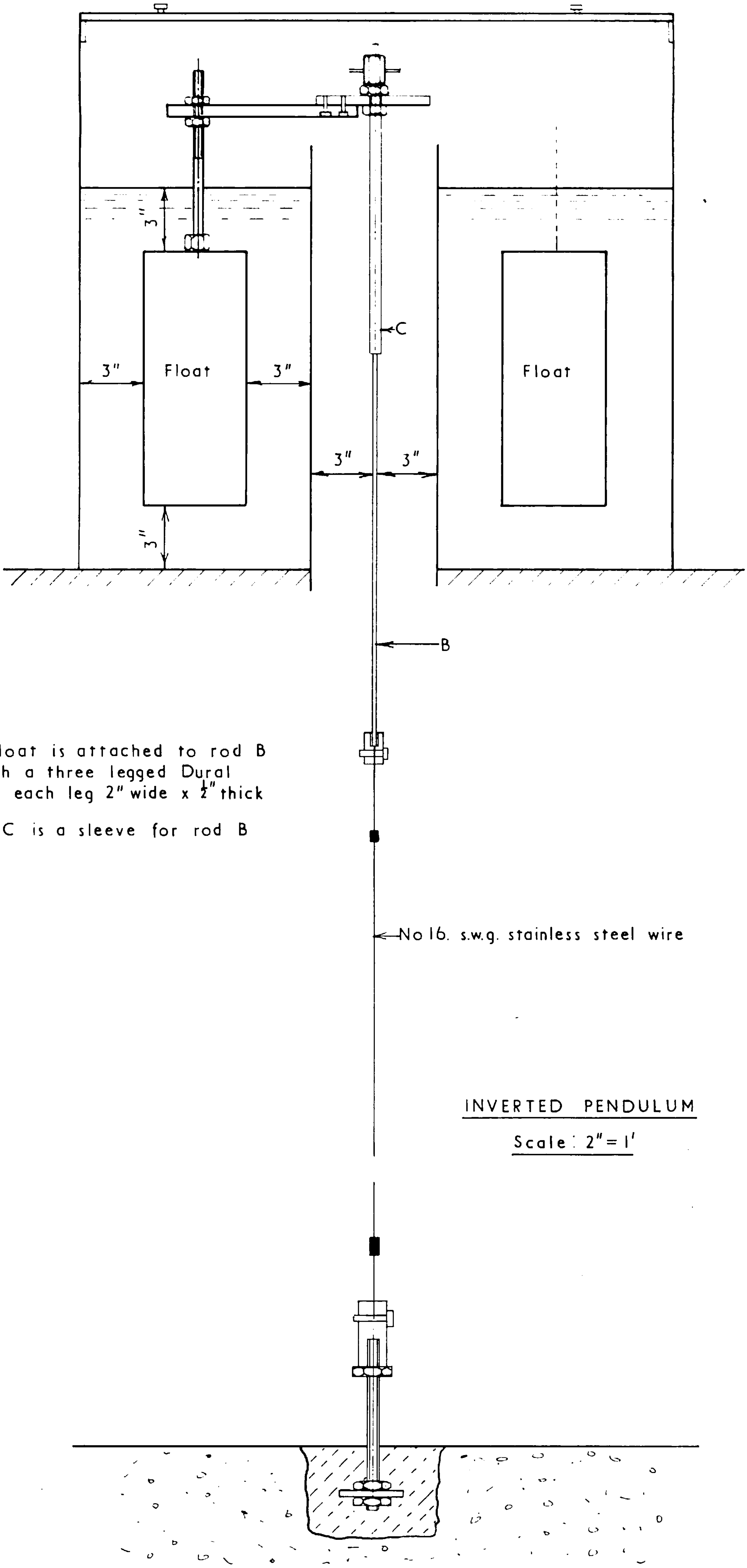


FIG. 3:4



Notes

1. The float is attached to rod B through a three legged Dural spider each leg 2" wide x ½" thick
2. Tube C is a sleeve for rod B

INVERTED PENDULUM

Scale: 2" = 1'

FIG 3.5

PART II

PROBLEM OF FLOW OF WATER TO PIEZOMETER TIPS IN FULLY-SATURATED SOILS

CHAPTER 4

ANALYTICAL AND APPROXIMATE SOLUTIONS FOR THE STEADY STATE PROBLEM

(4.1) Introduction

The usefulness of pore water piezometers in earth dams has been discussed in Chapters 1 and 2. It was also mentioned in Chapter 2 that an ideal piezometer should not interfere with the soil conditions around it, e.g. degree of compaction, stress distribution, moisture content, etc.; and furthermore it should operate on a null system so that no water need flow in or out of the piezometer tip when registering pore water pressure changes in the surrounding soil. In practice it is very difficult to satisfy both requirements completely and in fact some departures can be tolerated without much loss in the accuracy of the measurements.

The technique for installing piezometers so as to cause the minimum disturbance to the surrounding soil has been discussed in sections (2.6.2) and (2.6.3). The second requirement can be satisfied by having a null system or at least a very rigid transducer in the piezometer tip. However, to the Author's knowledge, no tip incorporating a null system has yet been successfully used, but tips with rigid transducers are in common use, e.g. the vibrating wire electrical tip. But the disadvantages of an electrical piezometer were outlined in section (2.4.), the most important being the inability to flush gas bubbles from the tip when installed in partly saturated soil. Thus in modern practice the most popular piezometer is the hydraulic type with an Imperial College tip for compacted fills and a Casagrande tip for borehole piezometers in fully saturated foundations.

The hydraulic piezometer operates strictly on a non-null basis since a definite flow of water from the soil to the tip is required to pressurise the connecting tubing and the pressure gauge; the latter at present being most commonly a mercury manometer. With such a system a time-lag exists between the actual pressure change in the soil and that recorded by the piezometer.

This second part of the thesis is concerned with studying the problem of flow of water to* a piezometer system from the surrounding saturated soil. Two practical aspects of the problem are considered in detail:

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* Subject to the assumptions made in this and the next chapter this problem is mathematically identical to that of the flow in the opposite direction, i.e. from the tip to the soil.

(i) the evaluation of the time-lag in the measurement of pore water pressure, knowing before hand the flexibility of the piezometer system and the permeability and compressibility of the soil,

(ii) the evaluation of the permeability and compressibility of the soil from time-lag measurements.

The two aspects are of course, mathematically identical. Hvorslev (1951) treated the problem for incompressible soils, while Josselin de Jong (1953) using Biot's theory and Gibson (1963, 1966) using Terzaghi's theory derived solutions for the more general case of compressible soils. The latter's solutions are mathematically identical to those of the analogous problem of the conduction of heat in solids (see for example Carlsaw and Jaeger, 1959).

Penman (1960) reported laboratory measurements of time-lag associated with different piezometer systems. The problem is also discussed by Whitman, Richardson and Healy (1961), and Perloff, Nair and Smith (1965).

It was already mentioned in Section (2.3.) that Penman's (1960) results together with those by Vaughan (1965) and by the Author (Chapter 8) indicate that from an engineering point of view the time-lag in the measurement of pore water pressure in the field is hardly significant. It is the evaluation of the coefficients of permeability, k, and consolidation, C_v, of the soil from time-lag measurements which is far more useful, as the results obtained are in-situ values and can be compared to those obtained in the laboratory or in the field.

These time-lag measurements can take two forms, falling-head and constant-head seepage tests. Gibson gave analytical solutions for these in the case of a spherical piezometer cavity. For convenience these two tests will now be referred to as F.H.T. and C.H.T.

During this research work the author conducted numerous in-situ seepage tests, especially of the C.H.T. type, on piezometers in several dams the results of which are reported in Chapters 7 and 8. However, there was an uncertainty regarding the application of Gibson's solutions to the more practical shapes of piezometer tips, viz. cylindrical and conical. Some results from C.H.T. on Imperial College and Casagrande tips deviated from the pattern predicted by Gibson's solution and it was thought that that may have been due to the difference in behaviour of conical and cylindrical tips from that of the idealised spherical tip treated by Gibson. Another uncertainty lay in the values of the intake factor to be used for cylindrical and conical tips where three sets of experimental results exist but which differed by as much as 10% from each other.

To resolve these uncertainties the Author embarked on a numerical solution for both the steady and unsteady state cases for cylindrical tips. The conical tip was not studied as will be noted in Section (10.).

The steady state case covers C.H.T. on incompressible soils, or compressible soils at very large values of time, and will be dealt with in this Chapter. The unsteady state case covers C.H.T. on compressible soils and F.H.T. on compressible and incompressible soils, as will be discussed in the next chapter. In Chapter 6 a study is made of the effects of some departures from Terzaghi's theory on the solutions for the C.H.T. on a spherical piezometer cavity.

(4.2) Steady state analytical solutions for a spherical tip and an infinite sand drain.

In this section standard solutions (see for example Muskat, 1937; Carslaw and Jaeger, 1959) for the steady state case for a spherical cavity and an infinite sand drain will be given as a useful introduction to the approximate solutions by the Author and other workers for the finite cylindrical tip.

Here Laplace's equation can be used if the following assumptions are made,

(i) soil is saturated and incompressible

(ii) water and soil grains are incompressible

(iii) Darcy's law is valid.

The equation can take three forms depending on the co-ordinate system chosen, viz. cartesian, cylindrical or spherical. If the soil is assumed to be isotropic then its permeability is independent of the co-ordinate system and the three forms of the equation are as shown in Fig. (4.1). In this chapter a further assumption is made. The permeability of the soil is assumed to be very much smaller than that of the tip filter and sand plug, if any.

(4.2.1) Spherical tip

With symmetry about Θ and $\emptyset$, Laplace's equation in spherical co-ordinates (Fig. 4.1) reduces to the following simple form:

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} = 0 \qquad (4.1)$$

where u is the excess pore water pressure outside the sphere. Let the boundary conditions be:

$$\left.\begin{array}{ll}\text{(i) at } r = r_1, \text{ (the radius of the cavity),} & u = u_1 \\ \text{(ii) at } r = r_2, \text{ (radius of the outside boundary)} & u = u_2\end{array}\right)(4.2)$$

i.e. a constant pressure u_1 is maintained at the cavity and u_2 at the outside boundary which is another sphere of radius r_2.

A useful substitution in the solution of eq. (4.1) is to put v = u.r. which reduces it to

$$\frac{d^2v}{dr^2} = 0 \tag{4.3}$$

The solution of equation (4.3) with the boundary conditions (4.2) gives

$$u = u_1 - \left(\frac{u_1 - u_2}{1 - r_1/r_2}\right) . \left(1 - r_1/r\right) \tag{4.4}$$

If $u_2 = 0$ this equation reduces to

$$u = \frac{u_1\ (r_1/r - r_1/r_2)}{1 - r_1/r_2} \tag{4.5}$$

and if $r_2 = \infty$ also,

$$u = \frac{u_1 . r_1}{r} \tag{4.6}$$

Eqs. (4.4), (4.5) and (4.6) give the familiar hyperbolic distribution of the pore pressure away from the spherical piezometer tip.

To find the constant flow rate of water q, eq. (4.4) is differentiated w.r.t. r:

$$q = \frac{4\pi\, r_1}{(1 - r_1/r_2)} \cdot \frac{k}{\delta_w} \cdot (u_1 - u_2) \tag{4.7}$$

where k is the isotropic permeability of the soil, (LT^{-1}).

If $r_2 = \infty$ then the more familiar equation emerges

$$q = 4\pi.\ r_1 \cdot \frac{k}{\gamma_w} (u_1 - u_2) \tag{4.8}$$

The value of q as given in equation (4.7) and (4.8) is seen to be a function of the pressure difference $(u_1 - u_2)$, the permeability of the soil, and a third parameter which in the case of eq. (4.7) is $\frac{4\pi r_1}{1 - r_1/r_2}$ and in the case of eq. (4.9) is $4\pi r_1$. These two terms have the dimension of length only and are functions of the size of the tip and the outside boundary alone. They are commonly called the intake factors, shape factors, A - functions, etc. (see Hvorslev, 1951; Kirkham, 1946; Penman, 1960). Their significance is obvious as once they are known for any shape of tip, then from a simple C.H.T. in which q is measured the permeability of the soil can easily be calculated.

(4.2.2) Infinite sand drain in a semi-infinite mass of incompressible soil

Writing Laplace's equation in cylindrical co-ordinates and noting the symmetry about Θ and Z (Fig. 4.1) we obtain:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) = 0 \tag{4.9}$$

Let the boundary conditions be

$$\left.\begin{aligned} \text{at} \quad r &= r_1 \text{ (radius of the drain)}, \quad u = u_1 \\ r &= r_2 \text{ (radius of outside boundary) } u = u_2 \end{aligned}\right) \tag{4.10}$$

The solution of eq. (4.9) with the boundary conditions (4.10) gives

$$u = u_1 - \frac{(u_1 - u_2)}{\ln(r_2/r_1)} \ln (r/r_1) \tag{4.11}$$

The constant flow rate of water q per unit length of the drain is then:

$$q = \frac{2\pi}{\ln(r_2/r_1)} \cdot (u_1 - u_2) \cdot \frac{k}{\gamma_w} \tag{4.12}$$

Here the quantity $\frac{2\pi}{\ln(r_2/r_1)}$ is the intake factor or shape factor per unit length of the infinite cylinder.

If $r_2 = \infty$, then the flow rate $q = 0$. This is an interesting point for whereas with the spherical cavity a definite flow occurs when $r_2 = \infty$ (eq. 4.9), this is not so with the infinite cylinder (see Muskat, 1937, p. 155).

(4.3) Hvorslev's summary of intake factors for various shapes of tips

Hvorslev (1951) made a very useful summary of the intake factors of different piezometer tips commonly used in practice, as shown in Fig. (4.3). He quotes analytical solutions (as in Section 4.2) for the spherical and semi-spherical tips by Dachler and for case (3) by Forchheimer (1930) and Dachler. For case (4) an approximate solution from experiments was obtained by Harza (1935), and by Taylor (1948) using flow-nets. The formulas for cases (5) and (6) are approximate and are derived by the addition of the losses in piezometric pressure head outside the casing to that in the soil column inside the casing.

The formulas for cases (7) and (8) are derived by Dachler who assumed a line source and semi-ellipsoidal surfaces, as will be referred to again in Section (4.4.1).

The formula for case (9) is exact and is the same as that given for the infinite sand drain in the last section.

(4.4) Past results for the intake factors of cylindrical tips

To the Author's knowledge there are at present no analytical solutions for this case, but there exist five sets of approximate solutions as will be discussed shortly.

The difficulty in obtaining analytical solutions can easily be seen by stating the boundary conditions. Fig. (4.2) shows this for a foundation piezometer where the tip is the cylindrical sand plug, its permeability being much higher than that of the surrounding soil.

The differential equation governing the pore water pressure 'u' distribution around this tip is Laplace's equation which in cylindrical co-ordinates and with symmetry about θ (Fig. 4.1) takes the form:

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial Z^2} = 0 \qquad (4.13)$$

Considering case (1b) Fig. (4.2) the boundary conditions are:

$$\begin{array}{llll}
\text{at} & r = r_1, & |Z| \leqslant Z_1 & u = u_1 \\
 & Z = -Z_1, & 0 \leqslant r \leqslant r_1 & u = u_1 \\
 & r = r_1, & Z_1 < Z < Z_2 & \frac{\partial u}{\partial r} = 0 \\
 & r = r_2, & & u = u_2 \\
 & |Z| = Z_2 & & u = u_2
\end{array} \qquad (4.14)$$

The general solution of eq. (4.13) is most commonly expressed in terms of Bessel functions, but to find a particular solution suitable for this problem and satisfying all the boundary conditions listed in eq. (4.14) is very difficult. Carslaw and Jaeger (1959) treat similar problems but not with exactly the same boundary conditions.

It was mentioned earlier that there are at present a few approximate solutions for evaluating the intake factor of cylindrical tips. These solutions were obtained for different boundary conditions as shown in Fig. (4.2) for the case where the bottom of the tip is pervious as well as the sides, case (1) and where the top and bottom are impervious, case (2).

For case (1) four approximate solutions exist, viz. by Dachler (1936), Frevert and Kirkham (1948), Smiles and Young (1965), and by the Author.

For case (2) there are two sets of results, by Kadir (1951) and by the Author, For the Imperial College tip there is at present no solution.

(4.4.1) Dachler's solution

Hvorslev (1951) quotes Dachler's (1936) approximate formula for the intake factor of a cylindrical tip where:

$$F = \frac{2\pi L}{\ln\left\{\frac{L}{D} + \sqrt{\left(\frac{L}{D}\right)^2 + 1}\right\}} \tag{4.15}$$

L and D being the length and diameter of the tip. This formula has been widely used by soil engineers (see, for example, Penman, 1960; Bishop, Kennard and Penman, 1960) to evaluate the intake factor without discrimination for cases (1), (2) (Fig. 4.2) and for the Imperial College tip.

Dachler published his book in German, and from a rough English translation of the proof of this formula the Author found that it was by no means rigerous. He represented the tip in case (7) (Fig. 4.3) by a line source and assumed semi-ellipsoidal equipotential surfaces. By approximating the shape of the tip to a semi-ellipsoid of diameter D and length L he obtained the formula quoted there. Next by assuming horizontal symmetry he obtained eq. (4.15) for case (8), Fig. (4.3).

Fig. (4.2) shows the value of the intake factor (for cases 1 and 2 Fig. 4.2) as calculated from Dachler's formula and as obtained by other workers. An arbitrary length of 4" is chosen for the tip and the value of F is plotted against the diameter of the tip. The intake factor calculated using Dachler's formula is surprisingly more accurate in the region L/D = 1, than at higher values of L/D in contrast to Hvorslev's remarks who thought it would be more accurate for high L/D ratios.

(4.4.2) Frevert and Kirkham's results

Frevert and Kirkham (1948) evaluated the intake factor for case (1a) (Fig. 4.2) using an electrolytic tank analogue where the flow of electrical current from a probe (representing the tip) to the tank is mathematically identical to the steady state flow of water from a piezometer cavity to the surrounding saturated incompressible soil. The boundary conditions they chose were from a soil science point of view where a water table exists above the piezometer and an impermeable boundary below it.

The tank used was a wooden tank 68" in diameter and 20" high. A copper plate was attached to the bottom of the tank representing the water table, while a cylindrical brass rod coated along part of its length with non-conductive plastic coating was fed through the bottom of the tank to represent the piezometer tip. The surface of the water in the tank represented the impermeable boundary.

Ordinary tap water was used as the electrolyte through which an alternating current (to prevent polarisation) was passed between the probe and the bottom copper plate. The resistance measured together with the specific conductivity of the water gave the intake factor of the probe.

The values of F given by Kirkham and Frevert are plotted in Fig. (4.2) for tips whose dimensions are much smaller than those of the outside boundaries. In addition to evaluating the intake factor they plotted the equipotential surfaces around the tips by probing the electrolyte with a thin conductive rod set at a given potential and noting the position when no current flowed between the rod and the water.

(4.4.3) Smiles and Young's results

Smiles and Young (1965) repeated Frevert and Kirkham's experiment and obtained, as they suspected, consistently higher values of the intake factor by about 11% as shown in

Fig. (4.2). They suggest that a very probable cause of this consistent error may have been an inaccurate determination of the specific conductivity of the water in Frevert and Kirkham's tests.

(4.4.4) Kadir's results

Kadir (1951) also repeated Frevert and Kirkham's tests to find the intake factor for case (2A) (Fig. 4.2) where the top and bottom of the cylindrical tip are impervious. Kadir mentioned that he checked his experimental procedure by finding the intake factor for case (1A) and his results were some 4% lower than those by Frevert and Kirkham. Kadir's results for case (2A) are plotted in Fig. (4.2) and are very probably at least 15% on the low side.

(4.5) Numerical solution for the intake factor of cylindrical tips

Fig. (4.2) shows some of the combinations of the boundary conditions associated with cylindrical tips where the bottom and/or top may be impervious. However, the numerical solution for all these cases is essentially the same and involves the approximate solution of Laplace's equation (4.13) to satisfy the relevant oundary conditions. Once the pore pressure distribution outside the tip has been established then it is an easy matter to compute the constant flow rate which in turn gives the intake factor of the tip. The results for cases (1C) and (2B) obtained by the Author are shown in Fig. (4.2).

In this chapter only certain general aspects affecting the accuracy of these results will be discussed; details of the numerical analysis and the computer programming will be given in Part IV of this thesis. These aspects involve the effects on the computed values of the intake factor of:

(i) different boundary conditions

(ii) the proximity of the outside boundaries

(iii) mesh size

(iv) singularity point S, Fig. (4.4)

(4.5.1) The effect of boundary conditions

Two types of tips were analysed the borehole, and fill piezometer tips (cases 1 and 2, Fig. 4.2). Frevert and Kirkham (1948) and Smiles and Young (1965) considered case (1A) while Kadir (1951) obtained results for case (2A). All three solutions were obtained from a soil science point of view where a water table exists above the piezometer and an impervious layer below it. From a Soil Mechanics point of view cases (1B) (1C) and (2B) are more interesting as they represent piezometer tips in, to all practical purposes, an infinite mass of soil where at some distance away from the piezometer tip the pressure in the soil is hardly affected by a seepage test on the tip. To check the accuracy of his solution the Author made his computer programme flexible so as to solve any set of the boundary conditions given in Fig. (4.2).

As expected the effect on the value of the intake factor of replacing $du/dr = 0$ by $u = 0$ at the outside boundaries was negligible provided that r_2/r_1 and Z_2/Z_1 were say $\geq$ 50. In Fig. (4.2) a corresponding theoretical comparison is shown for a spherical tip where the upper curve was computed from eq. (4.7) while the lower one was obtained from Kirkham's (1946) solution who used the method of images to solve case (1A) for a spherical tip. The difference in the intake factor at $r_2/r_1 = Z_2/Z_1 = 50$ is only 1%. This is about the same difference obtained by the Author for the cylindrical tip. However, changing the outside boundary conditions from $u = 0$ to $du/dr = 0$ manifestly slowed down the rate of convergence of the solutions as will be discussed in section (10.33).

Next the effect of the presence or absence of the impermeable borehole was studied and again the effects on the intake factor were negligible. For an L/D ratio of 2 the intake factor for case (1B) is only 0.1% less than that of case (1C) while for a ratio of 1 the corresponding difference is 0.4%. This is hardly surprising since the presumably more serious effect of sealing the bottom of the cavity only produced a few percent drop in the intake factor as mentioned by Kadir (1951) and as found by the Author (see Fig. 4.2).

(4.5.2) The effect of the proximity of the outside boundaries

In any numerical analysis of the type discussed here the position of the outside boundary has to be fixed, and if it is desired to find the intake factor corresponding to an infinite mass of soil then this position has to be varied and the 'infinity' value of the intake factor extrapolated. This was done for case (1C) for an L/D ratio of 2. Fig. (4.2) shows the results together with the corresponding theoretical values for a spherical tip as governed by eq. (4.7).

The values of the intake factor for cases 1C and 2B given in Fig. (4.2) were all corrected to correspond to $r_2 = Z_2 = \infty$, using equation (4.7) for the spherical tip. However, the values of r_2/r_1 and Z_2/Z_1 actually used were such that these corrections amounted to 1 or 2% only. A certain ambiguity arose as to whether to use r_2 , Z_2 or their average in eq. (4.7) but a few runs on the computer showed that provided Z_2 was not less than r_2, the latter was far more predominant for all the L/D ratios considered. This is again hardly surprising since most of the flow is in the radial direction anyway.

(4.5.3) The effect of mesh size

The first programme written to solve the steady state problem utilised a linear mesh size in the r and Z directions. However, it soon became apparent that the storage capacity of the computer would not be adequate if a reasonably small mesh size was to be used together with large r_2/r_1 and Z_2/Z_1 ratios. In addition it was storage and time wasting to have the same mesh size everywhere especially far away from the tip where the pressure gradients are small. Thus the programme was changed to accommodate the new axes $R = \ln(1 + r)$ and $Z = \ln(1 + Z)$. For roughly the same mesh size near the tip this resulted in a saving of 75% of computer storage and time.

The affect of the mesh size on the computed values of the intake factor is very difficult to assess in this case, for when the mesh size is changed the effects of the boundary conditions at the singularity point will change as will be discussed in the next section.

(4.5.4) The effect of the singularity point

This proved to have the most critical influence on the computed values of the intake factor. At the singularity point S, Fig. (4.4), two ~~three~~ boundary conditions have to be satisfied simultaneously, viz. [illegible], $\partial u/\partial Z = 0$ and $u = u_1$! In a numerical analysis this is clearly impossible and a compromise has to be sought. If the singularity point is set at $u = u_1$ then the length of the tip is effectively extended by $\frac{1}{2}$ the mesh size above S. On the other hand if $\partial u/\partial Z$ was specified there the length of the tip is then reduced by $\frac{1}{2}$ Z (Fig. 4.4), together with extending the impermeable top, half the radial mesh size outside the tip.

Several runs were made with different mesh sizes for case 1C and 2B, for different L/D ratios where the length of the tip was set at 6" and $r_2 = Z_2 = 23"$ approximately. Since the mesh size was not linear it was difficult to maintain the same exact value of r_2 and Z_2, but care was taken to ensure that they were nearly the same for all the runs, and corrections of the order of only $\frac{1}{2}\%$ were made using eq. (4.8).

In the first series of the runs for L/D = 2, S was set at $u = u_1$. The intake factor decreased with increasing mesh size as shown in Fig. (10.4). In the next series S was set at $\partial u/\partial Z = 0$. Here the intake factor increased with decreasing mesh size in very nearly a linear fashion. For this geometry the increase in intake factor with decreasing mesh size can be quantitatively explained by the effective increase in the length of the tip when the mesh size was decreased. For other goemetries, however, and for case (2A) this simple explanation does not quite suffice. For the latter case the actual increase in the intake factor with decreasing mesh size was more than twice that for case (1C) as shown in Fig. (4.2). Obviously other factors were involved apart from the obvious one of the increased accuracy due to the decreasing mesh size, especially near the piezometer where the pressure gradients are highest.

The final computer values of the intake factors are given in Fig. (4.2) for cases (1C) and (2B) were calculated thus. For case (1C) runs were made for each geometry at r_2/r_1 and Z_2/Z_1 values of 50 or more with the singularity point set at $\partial u/\partial Z = 0$. Correction factors were then obtained from Fig. (4.2) by extrapolating the very nearly linear plots of F v . δZ to $\delta Z = 0$, and applying these corrections to the first set of results where $r_2/r_1 > 50$.
Next 'infinity' corrections were made as described in section (4.5.2). For case (2B) no runs were made for $r_2/r_1 > 50$, and instead the ratios of the extrapolated values of F for cases (1C) and (2B) at $r_2/r_1 > 8$ were used to obtain directly

the 'infinity' values for case (2B). However, one run for case (2B) at $r_2/r_1 = 50$ for $L/D = 2$ confirmed that this procedure was accurate to within $\frac{1}{2}$%.

(4.6) Discussion of results

Fig. (4.2) shows the six different sets of approximate results for the intake factor of cylindrical tips. In spite of the somewhat serious effect of the singularity point on the Author's solution, his results agree remarkably well with those by Smiles and Young (1965). This agreement perhaps confirms the suggestion that Frevert and Kirkham's (1948) results are at least 10% too low, and those by Kadir (1951) probably more than 15% on the low side. From a practical point of view these errors will not produce any tangible effects on the values of the permeability calculated from seepage tests. However, in assessing the numerical results for the unsteady state discussed in the next Chapter such errors would have a significant effect on the solutions obtained.

CARTESIAN CO-ORDINATES

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

SPHERICAL CO-ORDINATES

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\,\frac{\partial u}{\gamma r} + \frac{1}{\gamma^2}\,\frac{\partial}{\gamma \mu}\left\{(1-u^2)\frac{\partial u}{\partial \mu}\right\} + \frac{1}{\gamma^2(1-\mu^2)}\,\frac{\partial^2 u}{\partial \phi^2} = 0$$

Where $\mu = \cos\theta$

$x = r.\sin\theta \cos\phi$

$y = r.\sin\theta \sin\phi$

$z = r.\cos\theta$

CYLINDRICAL CO-ORDINATES

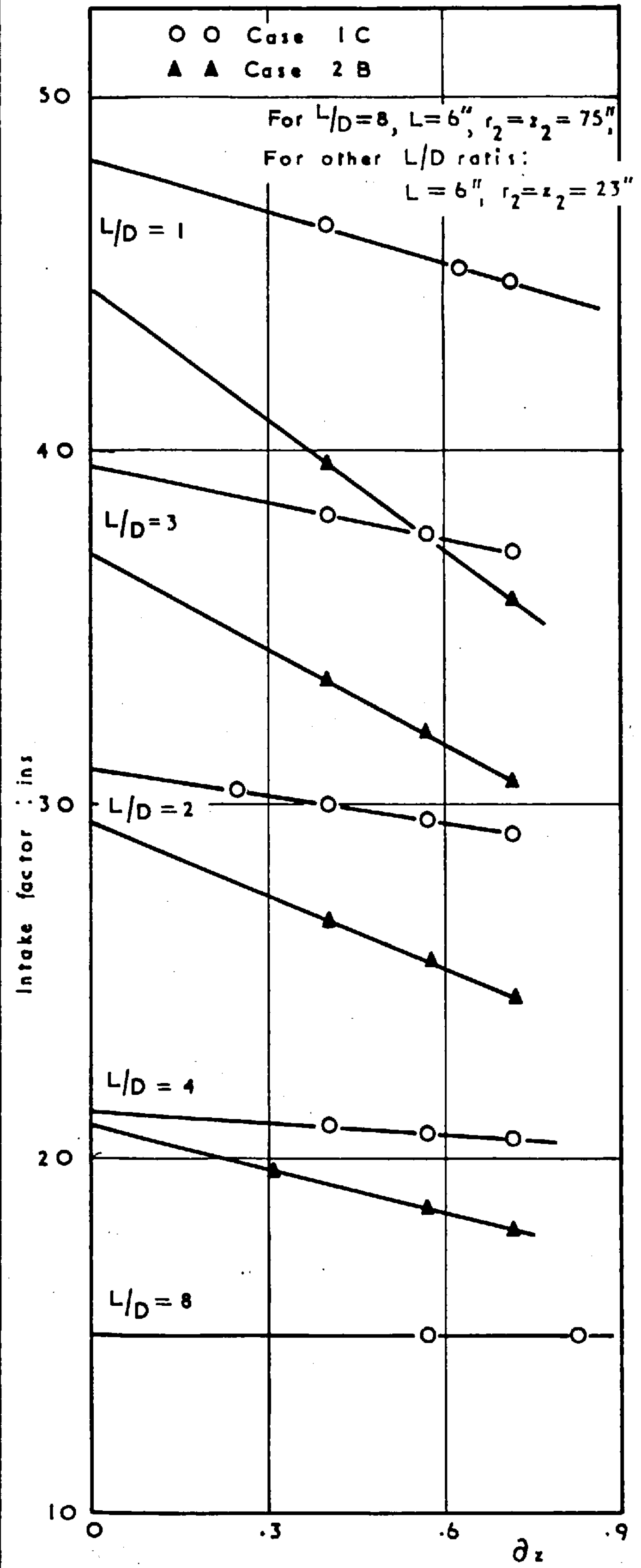
$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\,\frac{\partial u}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\frac{1}{r}\,\frac{\partial u}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(r\,\frac{\partial u}{\partial z}\right)\right] = 0$$

Where $x = r.\cos\theta$

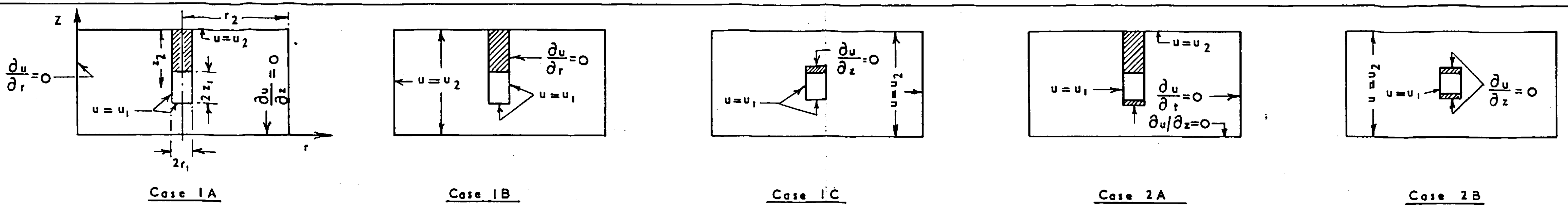
$y = r.\sin\theta$

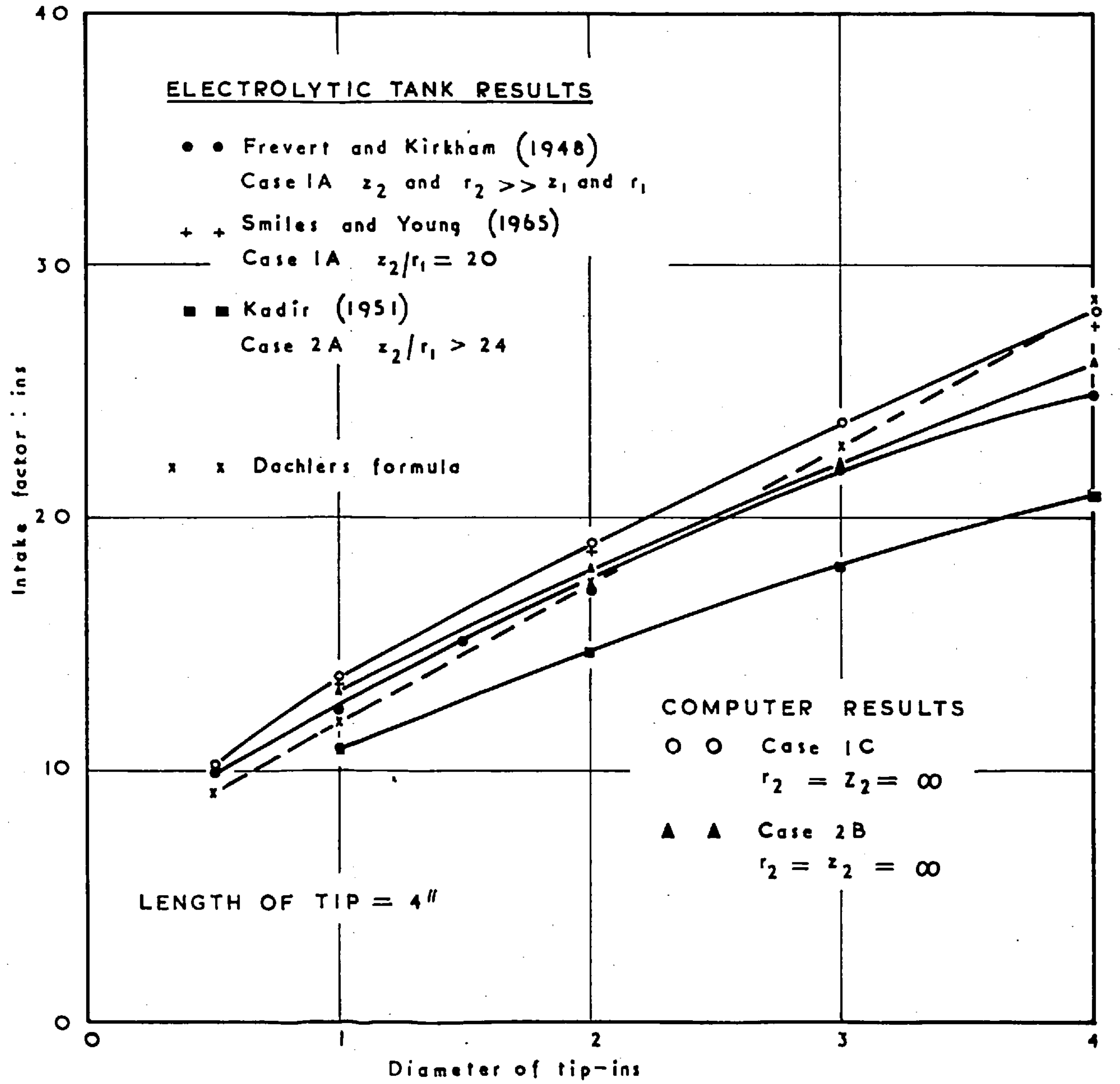
THE THREE FORMS OF LAPLACE'S EQUATION

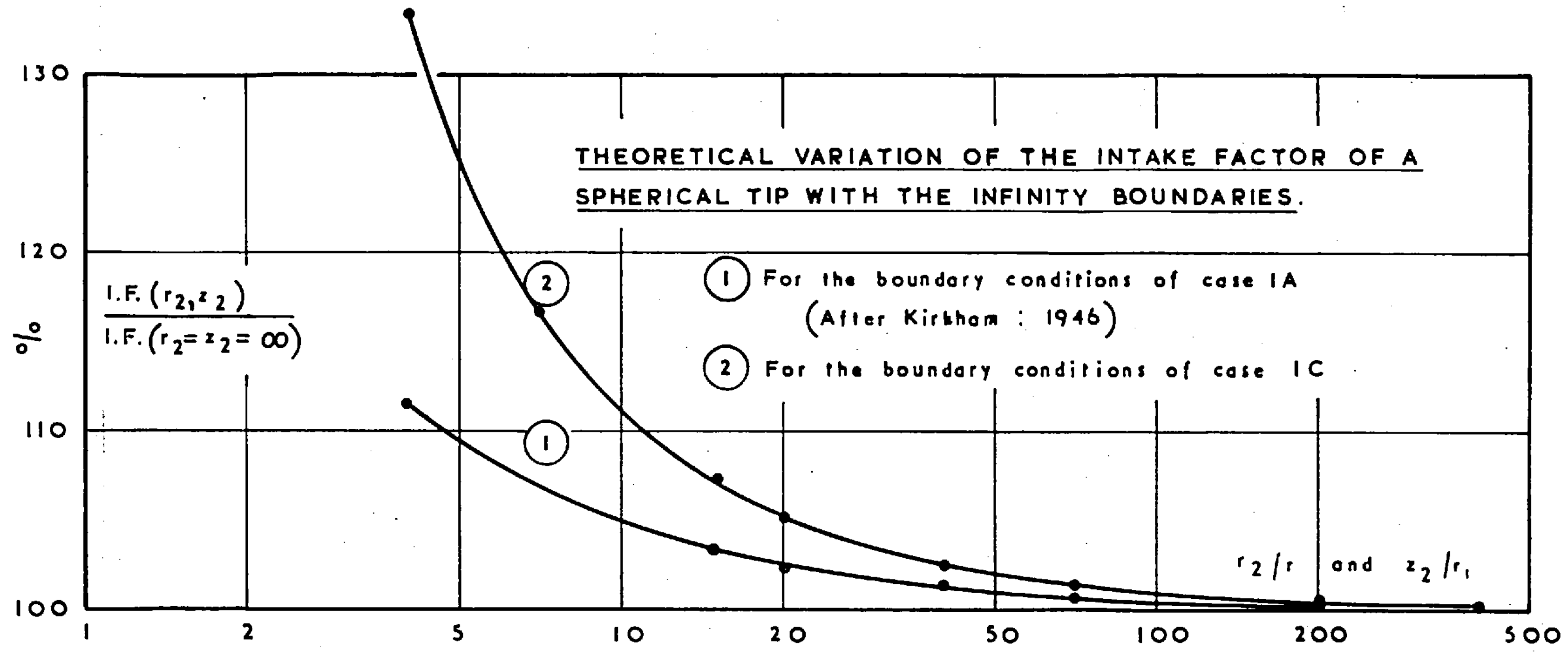
FIG 4:1

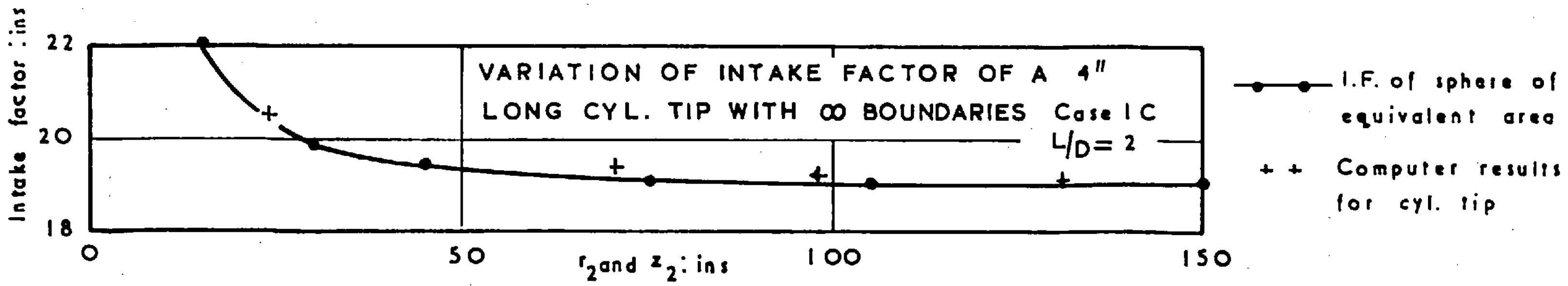


THE EFFECT OF MESH SIZE AND SINGULARITY POINT ON INTAKE FACTOR









COMPUTER RESULTS $L = 4''$ $r_2 = z_2 = \infty$		Length / Diameter					
		1/2	1	4/3	2	4	8
Intake factor inches	Case 1C	43·8	28·2	23·7	19·0	13·5	10·0
	Case 2B		26·2	22·1	18·0	13·2	

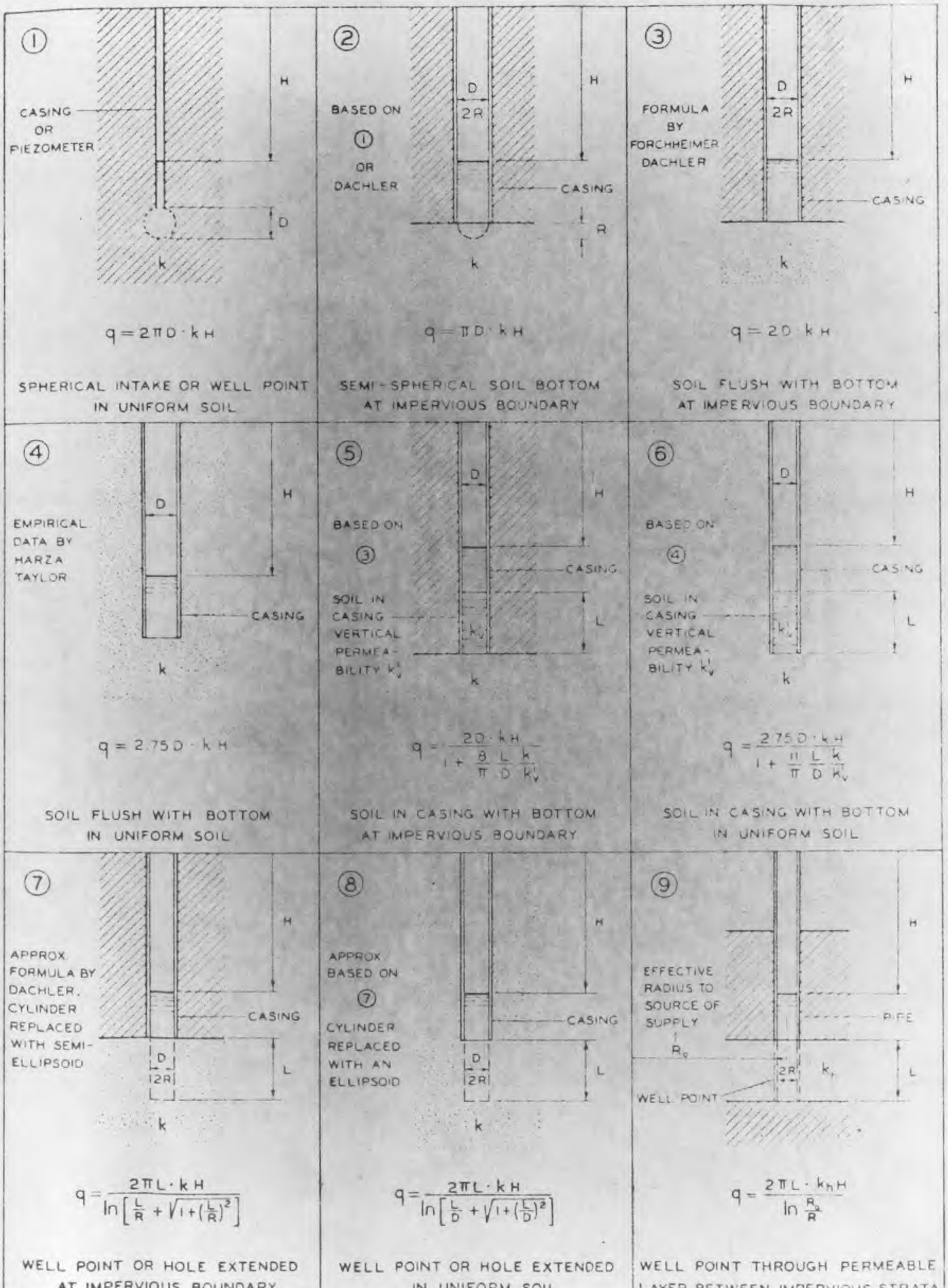
NOTE

From computer results for case 2B an approx. intake factor for an I.C. tip = 41·4 cms

INTAKE FACTORS OF CYLINDRICAL TIPS

FIG. 4.2

After Hvorslev (1951)

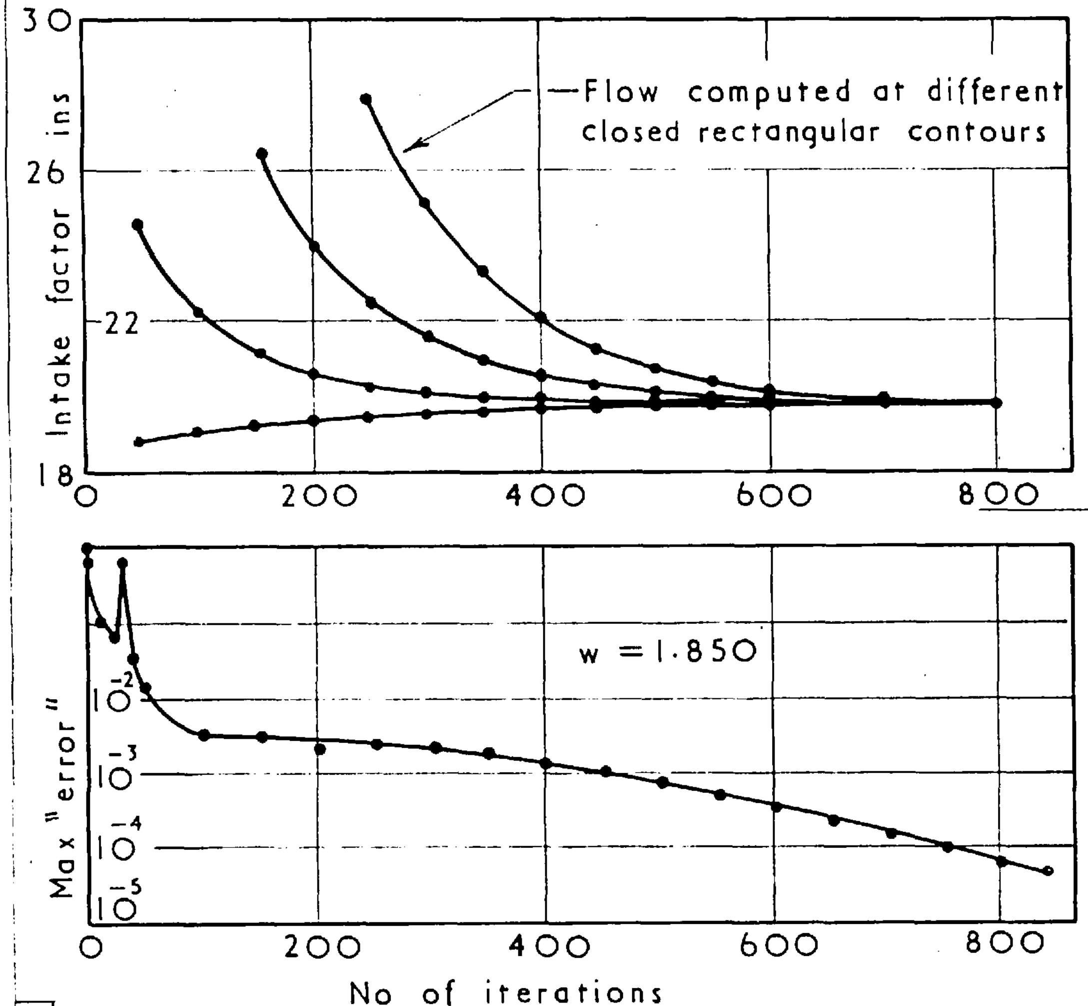


q = RATE OF FLOW IN cm^3/SEC, H = HEAD IN cm, k = COEF. OF PERMEABILITY IN cm/SEC, $\ln = \log_e$, DIMENSIONS IN cm

CASES 1 TO 8: UNIFORM PERMEABILITY AND INFINITE DEPTH OF PERVIOUS STRATUM ASSUMED

FORMULAS FOR ANISOTROPIC PERMEABILITY GIVEN IN TEXT

Fig. 4:3



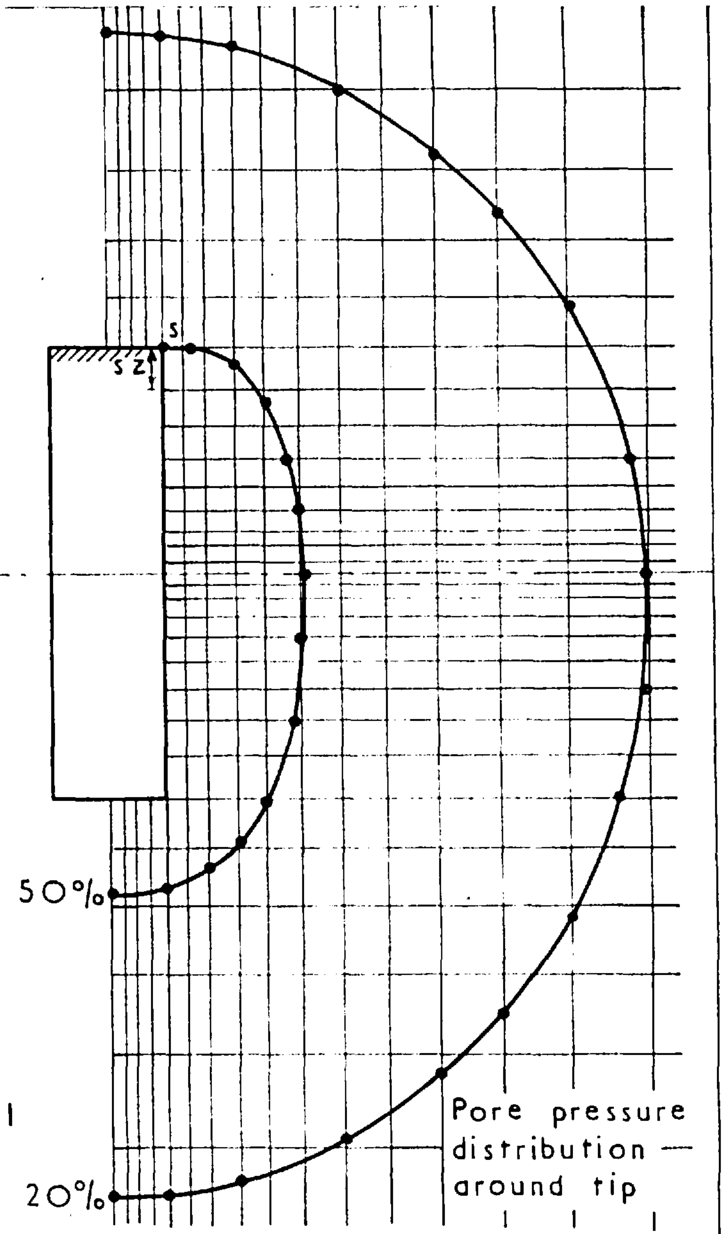


FIG. 4.4 Numerical solution for the intake factor of a cylindrical tip using the Gauss – Seidel overrelaxation method
$r_1 = 0.75''$, $z_1 = 3.0''$, $r_2 = z_2 = 100''$, Case I.C.

CHAPTER 5

ANALYTICAL AND APPROXIMATE SOLUTIONS FOR THE UNSTEADY STATE PROBLEM

(5.1) Introduction

In Chapter 4 analytical and approximate solutions were given for the steady flow of water from various shapes of piezometer tips to the surrounding soil. In this Chapter the unsteady state problem will be discussed for a spherical tip, infinite sand drain and finite cylindrical tip in incompressible and compressible soils.

For the spherical tip and infinite sand drain in incompressible soils the analytical solutions given can be found in standard text books on the subject, for example Muskat (1937) and Carslaw and Jaeger (1959). For the infinite sand drain in an infinite or semi-infinite mass of compressible soil the solutiohs given using Terzaghi's theory of consolidation, are taken from Carslaw and Jaeger (1948, 1959). The solution for a spherical cavity in an infinite mass of compressible soil is given by Gibson (1963, 1966), again based on Terzaghi's theory*

* In Chapter 6 the Author gives other analytical solutions for the constant-head test on a spherical tip where some departures from Terzaghi's theory are considered.

As for the steady state there are at present no analytical solutions for the finite cylindrical piezometer cavity. However, the Author obtained approximate solutions by solving Terzaghi's equation numerically on a digital computer. In addition a similar analysis was carried out for a spherical tip where Gibson's exact solutions exist as a check on the numerical procedure adopted for the cylindrical tip.

(5.2) Analytical solutions for the unsteady state flow of water from a spherical tip and an infinite sand drain to incompressible soils

This problem is the falling-head test on piezometers in incompressible soils as treated by Hvorslev (1950) who derived the following relationship for the equalisation ratio 'E':

$$E = \frac{H - H_2}{Ho - H_2} = e^{-\frac{Fk}{A}t} \qquad (5.1)$$

where $(Ho - H_2)$ is the out of balance pressure between the piezometer and the surrounding soil at time t = 0, and

$H - H_2$ is the out of balance pressure at time t,

A is the cross-sectional area of the piezometer stand-pipe ($\equiv \lambda/\gamma_w$ where λ is the flexibility of a closed piezometer system).

F is the intake factor of the tip

and k is the isotropic permeability of the soil.

This formula will now be derived for the two simple cases of a spherical tip and an infinite sand drain.

(5.2.1) Spherical tip

The equation governing the pore pressure distribution in the soil is Laplace's equation (4.1) but the boundary conditions are now:

$$\begin{aligned}
&\text{at } r = r_1, \quad t > 0, \quad \left(\frac{\partial u}{\partial r}\right)_{r=r_1} \cdot 4\pi . r_1^2 \cdot \frac{k}{\gamma_w} = A\,\frac{dH}{dt} \\
&\text{at } r_1 < r \leqslant r_2, \quad t = 0, \quad u = u_2 = \gamma_w H_2 \\
&\text{at } r = r_1, \quad t = 0, \quad u = \gamma_w Ho \\
&\text{at } r = r_2, \quad t \geqslant 0, \quad u = \gamma_w H_2
\end{aligned} \tag{5.2}$$

The solution of eq. (4.1) with the boundary conditions (5.2) gives:

$$E = \frac{H - H_2}{Ho - H_2} = e^{-4\pi \frac{r_1}{1 - r_1/r_2} \cdot \frac{k}{A} \cdot t} \tag{5.3}$$

If $r_2 = \infty$

$$E = e^{-4\pi r_1 \frac{k}{A} t} \tag{5.4}$$

The quantities $\dfrac{4\pi r_1}{1 - r_1/r_2}$ and $4\pi r_1$ appear again and are of of course the intake factors of the spherical tip as in Section (4.2.1) for the steady state case.

(5.2.2) Infinite sand drain

Let the boundary conditions in this case be:

$$\left.\begin{aligned}
&\text{at}\quad r = r_1,\quad t>0,\quad \left(\frac{\partial u}{\partial r}\right)_{r = r_1} \cdot \frac{k}{\gamma_w} \cdot 2\pi \cdot r_1 = \frac{dH}{dt} \cdot A\\
&\text{at}\quad r = r_2,\quad t\geqslant 0,\quad u = u_2 = \gamma_w H_2 \,..\, \text{a constant}\\
&\text{at}\quad r = r_1,\quad t = 0,\quad H = Ho\\
&\quad r_1 < r \leqslant r_2,\quad t = 0,\quad u = u_2 = \gamma_w H_2
\end{aligned}\right\} \tag{5.5}$$

The solution of Laplace's euqation (4.9) together with boundary conditions eq. (5.5) give

$$E = e^{-\frac{2\pi}{\ln r_2/r_1} \cdot \frac{k}{A} t} \tag{5.6}$$

Here again the quantity $\frac{2\pi}{\ln r_2/r_1}$ appears and is the intake factor per unit length of the infinite sand drain.

(5.3) Analytical solutions for C.H.T. and F.H.T. tests on a spherical piezometer tip in a compressible soil

Josselin de Jong (1953) using Biot's theory of consolidation obtained a solution for the falling head test for a spherical cavity in a compressible soil, for the two cases where the cavity is unsupported, and when supported by a rigid pervious sphere. For the second case Gibson (1963) using Terzaghi's theory of consolidation derived a solution exactly the same as that by Josselin de Jong. This rather surprising result is discussed for a similar problem by Leeuw (1965) who outlines the conditions where the two formulations give the same result.

Gibson (1963) and later (1966) gave solutions for the more useful case of a C.H.T. on a spherical piezometer cavity. In his first paper he assumed that the permeability of the tip filter and surrounding sand plug (if any) were much greater than those of the soil, but in his later solution he derived the more general solution taking into account the permeability of the filter and the sand plug. This latter solution will be given in the next section.

(5.3.1) Gibson's solution for the C.H.T. on a spherical piezometer cavity

Using Terzaghi's theory the differential equation governing the pore pressure distribution in the soil is the well known heat-conduction equation

$$c\left(\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r}\right) = \frac{\partial u}{\partial t} \qquad (5.7)$$

For the pore pressure distribution in the ceramic of the tip and in the sand plug (assuming the sand to be incompressible) Laplace's equation operates where

$$\left.\begin{aligned} \frac{\partial^2 u_1}{\partial r^2} + \frac{2}{r}\frac{\partial u_1}{\partial r} = 0 \\ \frac{\partial^2 u_2}{\partial r^2} + \frac{2}{r}\frac{\partial u_2}{\partial r} = 0 \end{aligned}\right) \qquad (5.8)$$

The boundary conditions are

$$\left.\begin{array}{ll} \text{as} & r \to \infty, \quad u_1 \to 0 \\ \text{at} & t = 0, \quad u_1 = 0 \quad \text{for } r > r_1 \\ \text{at} & r = r_1, \; u_2 = u_1 \text{ and } k_2 \frac{\partial u_2}{\partial r} = k_1 \frac{\partial u_1}{\partial r} \\ \text{at} & r = r_2, \; u_3 = u_2 \text{ and } k_3 \frac{\partial u_3}{\partial r} = k_2 \frac{\partial u_2}{\partial r} \\ \text{at} & r = r_3, \; u = \Delta u, \quad t > 0 \\ & \text{and} \quad q(r_3,t) = -4\pi a_3^2 \, k_3 \left(\frac{\partial u_3}{\partial r}\right)_{r = r_3} \end{array}\right) \qquad (5.9)$$

where r_3, r_2 and r_1 are the radii of the tip cavity, the ceramic-sand plug interface and the sand plug-soil interface; k_3, k_2, k_1 being the permeabilities of the ceramic, the sand plug and the soil. The solution of eq. (5.7) combined with (5.8) and the boundary conditions (5.9) give for the rate of flow out of the tip

$$q(t) = \frac{4\pi r_1 k_1 u_w}{(1 + \lambda)} \left\{1 + \frac{1}{\lambda} \exp\left[T(1 + \lambda^{-1})^2\right] \operatorname{erfc}\left[(1 + \lambda^{-1})T^{\frac{1}{2}}\right]\right\} \qquad (5.10)$$

where $T = ct/r_1^2$

$$\text{and} \qquad \lambda = \frac{k_1}{k_2}\left(\frac{r_1}{r_2} - 1\right) + \frac{k_1}{k_3}\left(\frac{r_1}{r_3} - \frac{r_1}{r_2}\right) \qquad (5.11)$$

when $k_2 \gg k_1$ and $k_3 \gg k_1$ i.e. when $\lambda \to 0$, Gibson's (1963) earlier solution can be retrieved.

Putting, $Z = T^{\frac{1}{2}} (1 + \frac{1}{\lambda})$ eq. (5.10) can be rewritten as:

$$q = \frac{4\pi r_1 k_1 \Delta u}{(1 + \lambda)} \left[1 + \frac{1}{T^{\frac{1}{2}}} Z \exp(Z^2) \operatorname{erfc}(Z) - \exp(Z^2) \operatorname{erfc}(Z)\right] \quad (5.10a)$$

As $\lambda \to 0$, $Z \to \infty$, exp. (Z^2) erfc $(Z) \to 0$ $(T > 0)$

and Z exp. (Z^2)erfc (Z) $\frac{1}{\sqrt{\pi}}$

(see Abramowitz and Stegun, 1965).

Eq. (5.10a) thus reduces to Gibson's earlier solution:

$$q = 4\pi r_1 \frac{k}{\gamma_w} \Delta u \left(1 + \frac{1}{\sqrt{\pi T}}\right) \quad (5.12)$$

where k now is the permeability of the soil. Furthermore, if the soil is incompressible or compressible but at very large values of time, $T \to \infty$, and eq. (5.13) reduces to the steady state solution, eq. (4.8)

$$q = 4\pi r_1 \frac{k}{\gamma_w} u \quad (5.13)$$

Gibson (1966) gave plots of the flow rate q against $1/\sqrt{T}$ for different values of λ, where at $\lambda = 0$ a straight line is obtained. Otherwise the plots are curved. As an important example Gibson takes the Imperial College tip with values of λ = .05, 0.5 and 5 corresponding to ratios of the permeability of the soil to that of the filter of 0.1, 1, 10. A reasonably straight plot is obtained between q and $1/\sqrt{T}$ at λ = .05 but for higher values the plots are clearly curved. Some field examples illustrating the latter case are given in Chapter 8.

(5.3.2) Gibson's analytical solution for the F.H.T. on a spherical piezometer tip in an infinite mass of compressible soil

Gibson (1963) used Terzaghi's theory and assumed that the permeability of the tip is much higher than that of the surrounding soil. The boundary conditions are:

$$\left.\begin{aligned} &\text{at} \quad t = 0\,, \quad u = h(\infty)\,\gamma w \\ &\qquad u \to h(\infty)\,\gamma w \text{ as } r \to \infty \\ &\text{at} \quad r = r_1, \quad \left(\frac{\partial u}{\partial r}\right)_{r = r_1} \cdot \frac{k}{\gamma w}\, 4\pi r_1^2 = \frac{dh}{dt} \cdot (t)\ .\ A \\ &\qquad u = ho\ w\,\gamma \text{ at } r = r_1, \quad t = 0 \end{aligned}\right\} \quad (5.14)$$

where r_1 is the radius of the spherical tip and $h(t)$ is the height of the water in the piezometer stand-pipe at time t. The solution of eq. (5.7) with the boundary conditions (5.14) gives:

$$E = \frac{h(\infty) - h(t)}{h(\infty) - h(o)} = \frac{1}{\eta_1 - \eta_2}\Big[\eta_1 \exp\,(\eta_1^2 T)\ \mathrm{erfc}\,(\eta_1 T^{\frac{1}{2}}) - \eta_2 \exp\,(\eta_2^2 T)\ \mathrm{erfc}\,(\eta_2 T^{\frac{1}{2}})\Big] \quad (5.15)$$

where

$$\left.\begin{aligned}\eta_1 \\ \eta_2\end{aligned}\right\} = \tfrac{1}{2}\left[\mu \pm (\mu^2 - 4\mu)^{\frac{1}{2}}\right] \quad (5.16)$$

and

$$\mu = 4\pi r_1^2\, m\, \frac{\gamma_w}{A} \quad (5.17)$$

m being the coefficient of compressibility (or swelling).

Gibson plotted E against μT and $\mu^2 T$ for various values of μ including the limiting case when $\mu = 0$ corresponding to Hvorslev's solution (eq. 5.4) for incompressible soils.

This solution, although valuable for estimating the time-lag in pore pressure measurements, is not very useful for the more important and reverse process of estimating k and Cv(or Cs) from falling-head seepage tests, see section (8.2.2).

(5.4) Analytical solutions for the C.H.T. on an infinite sand drain in a compressible soil

Solutions for this problem are given by Carslaw and Jaeger (1948, 1959) for the two cases when the soil is infinite and semi-infinite in the radial direction. These solutions are especially interesting as they represent the limiting case for the finite cylindrical tip with high L/D ratios as discussed in Section (5.5.3).

For the F.H.T. analytical solutions are given by Carslaw and Jaeger (1959) and Richart (1957) ~~for infinite system flexibility~~, but these will not be pursued here. Instead the more interesting case of the C.H.T. will be dealt with.

)5.4.1) Infinite sand drain in a semi-infinite mass of soil*

Terzaghi's equation in a cylindrical co-ord. is

$$c\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) = \frac{\partial u}{\partial t} \tag{5.18}$$

Let the boundary conditions be

$$\left.\begin{array}{lll} \text{at} \quad r = r_2, & u = 0, & t \geqslant 0 \\ t = 0, & u = 0 & \text{at } r_1 < r \leqslant r_2 \\ r = r_1, & u = u_1 & t > 0 \end{array}\right) \tag{5.19}$$

The solution of eq. (5.17) together with the boundary conditions (5.19) is given by Carlsaw and Jaeger (1959) who treat the analogous problem of heat-conduction in solids:

$$\frac{u}{u_1} = \pi \sum_{n=1}^{\infty} \left[\frac{Jo(r_1\alpha n)\,.\,Jo(r_2\alpha n)}{Jo^2(r_1\alpha n) - Jo^2(r_2\alpha n)}\,.\,Uo\,(r\alpha n)\,.\,e^{-c\alpha^2 nt}\right] + \frac{\ln(r_2/r)}{\ln(r_2/r_1)} \tag{5.20}$$

where

$$o(\alpha r) - Jo(\alpha r)\,.\,Yo(\alpha r_2) - Jo(\alpha r_2)\,.\,Yo(\alpha r) \tag{5.21}$$

and Jo is Bessel's function of zero order, first kind,

Yo is Bessel's function of zero order, second kind,

and α_n are the positive roots of the equation

$$Jo(\alpha r_1)\,.\,Yo(\alpha r_2) - Jo(\alpha r_2)\,.\,Yo(\alpha r_1) = 0 \tag{5.22}$$

The flow rate q at $r = r_1$ per unit length of the cylinder is

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* Infinite in the z direction, but not in the r-direction

$$q = -2\pi r_1 \left(\frac{\partial u}{\partial r}\right)_{r = r_1} \cdot \frac{k}{\gamma_w} \qquad (5.23)$$

where k is the permeability of the soil. Differentiating eq. (5.20) w.r.t. r, writing $\beta_n = \alpha_n r_1$ and using the relationship

$$\left(r\frac{\partial U_0}{\partial r}\right)_{r = r_1} = -\frac{2}{\pi}\,\frac{J_0(\alpha r_2)}{J_0(\alpha r_1)} = -\frac{2}{\pi\rho}$$ say, we find

$$q(r_1,t) = \frac{2\pi k u_1}{\gamma_w \ln(r_2/r_1)}\left[1 + 2\ln(r_2/r_1)\sum_{n=1}^{\infty}\frac{e^{-\beta^2_n T}}{\rho^2 - 1}\right] \qquad (5.24)$$

and for the flow rate per unit length at $r = r_2$

$$q(r_2,t) = \frac{2\pi k u_1}{\gamma_w \ln(r_2/r_1)}\left[1 + 2\ln(r_2/r_1)\sum_{n=1}^{\infty}\frac{\rho}{\rho^2 - 1}\,e^{-\beta^2_n T}\right]$$

where $T = ct/r_1^2$.

Fig. (5.1) shows plots q v. $1/\sqrt{T}$ for r_2/r_1 ratios of 2, 5, 10 and 50 as well as that for the limiting case of $r_2/r_1 = \infty$ given in the next section.

(5.4.2) Infinite sand drain in an infinite mass of soil

Carslaw and Jaeger (1948) give this solution where the boundary conditions are now:

$$\left.\begin{array}{llll} \text{at} & r = r_1, \; u = u_1 & t > 0 & \text{(a)} \\ & r \to \infty, \; u \to 0 & & \text{(b)} \\ & t = 0, \; u = 0 & r > r_1 & \text{(c)} \end{array}\right) \qquad (5.25)$$

The solution is obtained using a Laplace transform of eq. (5.18) and the boundary condition eq. (5.25c):

$$\bar{u} = \frac{Ko(qr)}{pKo(qa)} \tag{5.26}$$

where $\bar{u}$ = Laplace transform of $u = \int_0^\infty u \cdot e^{-pt}\, dt, \quad q^2 = p/c$

Ko is the modified Bessel function of zero order of the second kind. Using the inversion theorem the solution of eq. (5.26) for u is:

$$\frac{u}{u_1} = 1 + \frac{2}{\pi}\int_0^\infty e^{-cv^2t}\,\frac{Jo(ur)\,Yo\,(ua) - Yo\,(ur)\,Jo\,(ua)}{Jo^2\,(ua) + Yo^2\,(ua)}\,\frac{dv}{v} \tag{5.27}$$

The flow rate q at $r = r_1$ per unit length of the cylinder is:

$$q(r_1,t) = -\left(\frac{\partial u}{\partial r}\right)_{r\,=\,r_1} \cdot 2\pi r_1 \cdot \frac{k}{\gamma_w} \tag{5.28}$$

Differentiating equation (5.27) w.r.t. and substituting in eq. (5.28):

$$q(r_1,t) = 4 \cdot u_1 \cdot \frac{k}{\gamma_w}\int_0^\infty e^{-Z^2T}\,\frac{J_1(Z).Yo(Z) - Y_1(Z)\,Jo(Z)}{Jo^2(Z) + Yo^2(Z)}\,dZ \tag{5.29}$$

where $Z = v.a$

But $J_1(Z).\ Yo(Z) - Y_1(Z)\ Jo(Z)$ is the Wronskian

$$W\left[J\ (Z)\ ,\ Y\ (Z)\right] = \frac{2}{\pi z} \tag{5.30}$$

$$\therefore\ q(r_1,t) = \frac{8}{\pi}\,u_1\frac{k}{\gamma_w}\int_{-\infty}^{\infty} e^{-Z^2T} \cdot \frac{1}{Jo^2(Z) + Yo^2(Z)}\ d\left[\ln(Z)\right] \tag{5.31}$$

A simple computer programme was written to evaluate eq. (5.31) using the library subroutines for Bessel functions, and the results are plotted as $\frac{q}{u_1\ k/\gamma_w}$ per unit length v . $1/\sqrt{T}$, Fig. (5.1).

The function inside the integral was found to converge very quickly for positive values of ln(Z), but was very slow to converge for the negative values. Two mesh sizes for ln(Z) were used, 0.05 and 0.1 but there was practically no difference in values of q computed, using Simpson's rule. The limits of integration were chosen as the maximum $|\ln(Z)|$ at which the value of the function inside the integral was less than 10^{-6}.

Carslaw and Jaeger (1947) also give an approximate solution for equation (5.26) suitable for small times. Expanding and dividing Ko(qr) and Ko(qa) and inverting term by term they obtain:

$$u = \frac{r_1^{\frac{1}{2}}}{r^{\frac{1}{2}}} \text{ erfc } \left(\frac{r - r_1}{2\sqrt{ct}}\right) + \frac{(r - r_1)\sqrt{ct}}{4\, r_1^{\frac{1}{2}} r^{3/2}} \; i^1 \text{ erfc } \frac{(r - r_1)}{2\sqrt{ct}} -$$

$$\frac{9 r_1^2 - 2r_1 r - 7r^2}{32 r_1^{3/2} r^{5/2}} \cdot ct \cdot i^2 \text{ erfc } \frac{r - r_1}{2\sqrt{ct}} + \quad (5.32)$$

Differentiating eq. (5.32) w.r.t. r to obtain q at $r = r_1$ (per unit length):

$$\frac{q(r_1,t)}{u_1 \frac{k}{\gamma_w}} = \pi \left[1 + \frac{2}{\sqrt{\pi T}} - \frac{\sqrt{T}}{4\Gamma(1.5)} + \frac{\sqrt{T}}{4\Gamma(2)} + \ . \ . \ . \right] \quad (5.33)$$

where again $T = ct/r_1^2$, Γ is the gamma function and i^n erfc is the repeated integral of the complementary error function, and using the relationship,(see Abramowitz and Stegun, 1965):

$$i^n\mathrm{erfc}(o) = \frac{1}{2^n \Gamma(n/2 + 1)}$$

Values of q calculated from equation (5.33) are plotted in Fig. (5.1) and are seen to agree remarkably well with the exact solution of eq. (5.31).

The plots in Fig. (5.1) show that the q v . $1/\sqrt{T}$ plots are very nearly linear as for the case of the spherical piezometer treated by Gibson (1963). For $r_2/r_1 = \infty$ the plot changes slopes downwards at large values of T since it must head towards the origin as $T \to \infty$, according to eq. (4.12). For other values of r_2/r_1 the plots join the $r_2/r_1 = \infty$ plot for part of the way and then branch off towards their steady state flows.

It is interesting to compare Gibson's solutions for a spherical tip with that of the infinite cylinder. The slope of the straight line portion in Fig. (5.1) is very nearly $2\sqrt{\pi}$, in fact it is $2\sqrt{\pi}$ + 1%. Neglecting this 1% difference the flow rate q per $2r_1$ length of the cylinder for all r_2/r_1 ratios and for relatively small values of time, may be very nearly represented by the following empirical equation:

$$\frac{q(r_1,t)}{k/\gamma_w \cdot u_1} = A + 4\pi r_1 \frac{1}{\sqrt{\pi T}} \tag{5.34}$$

The value of A as extrapolated on the q - axis Fig. (5.4) is 5.336 r_1 per $2r_1$ length. Hence eq. (5.34) may be rewritten as:

$$\frac{q}{4\pi r_1 \cdot k/\gamma_w \cdot u_1} = 0.424 + \frac{1}{\sqrt{\pi T}} \qquad (5.35)$$

This equation has a very similar structure to eq. (5.12) derived by Gibson (1963) for the spherical tip. It implies that at small values of time the flow rate out of an infinite cylinder per $2r_1$ length is very nearly the same as that from a sphere with the same radius and hence the same surface area.

One useful aspect of eq. (5.35) is in analysing C.H.T. results on a clay sample with a drilled central hole and confined in an apparatus similar to the standard oedometer. A constant pressure may be applied to the central hole while the outside radius of the sample is maintained at the same uniform pressure before the test. The value of k can then be calculated from the steady state flow rate, and that for Cv (or Cs) from eq. (5.35).

In Section (5.6.3) further use is made of this equation when discussing numerical results from C.H.T. tests on finite cylindrical tips with high length/diameter ratios.

(5.5) Analytical solution for one dimensional C.H.T. on an oedometer sample

During his research work on the C.H.T. on field piezometers the Author noticed the absence of a solution for one dimensional C.H.T. on oedometer or triaxial samples. The solution for the infinite sand drain given in the last section showed similar behaviour to that of Gibson's spherical tip, and it was thought that the same might apply to the one dimensional case.

Terzaghi's equation for one dimensional consolidation is:

$$c \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \tag{5.36}$$

Let the boundary conditions be (see Fig. 5.2)

$$\left.\begin{array}{lll} 0<x\leq L & u = 0 & t = 0 \\ x = 0 & u = \Delta u & t>0 \\ x = L & u = 0 & t\geqslant 0 \end{array}\right) \tag{5.37}$$

Eq. (5.36) can be solved using the Laplace transformation thus:

$$\frac{d^2\bar{u}}{dx^2} - q^2\bar{u} = 0 \tag{5.38}$$

where q^2 is again $= p/c$.

The solution of eq. (5.38) together with the boundary conditions of eq. (5.37) give

$$\bar{u} = \frac{\Delta u}{p} \cdot \sum_{n=o}^{\infty} \left\{ e^{-q(x + 2nL)} - e^{-q(2L(1 + n) - x)} \right\} \tag{5.39}$$

$$\therefore \frac{u}{\Delta u} = \sum_{n=o}^{\infty} \left\{ \text{erfc}\left(\frac{x + 2nl}{2\sqrt{ct}}\right) - \text{erfc}\left(\frac{2L(1 + n) - x}{2\sqrt{ct}}\right) \right\} \tag{5.40}$$

The flow rate per unit area of the sample

$$q(o,t) = \frac{\Delta u}{L}\frac{k}{\gamma_w}\frac{1}{\sqrt{\pi T}} \sum_{n=o}^{\infty} \left\{ e^{-(n+1)^2/T} + e^{-n^2/T} \right\} \tag{5.41}$$

and $$q(O,L) = \frac{\Delta u}{L}\frac{k}{\gamma_w}\frac{2}{\sqrt{\pi T}} \sum_{n=o}^{\infty} e^{-\frac{(n+0.5)^2}{T}} \tag{5.42}$$

where $T = ct/L^2$

For the case when the initial pore pressure distribution in the sample is triangular the boundary conditions are

$$\left.\begin{array}{lll} t = 0, & u = \Delta u\left(\frac{L - x}{L}\right) & 0 < x \leq L \\ t > 0 & u = 0 \ \text{at}\ x = 0 & \\ u = 0 & x = L \quad t \geq 0 & \end{array}\right) \tag{5.43}$$

The flow rate is given then by

$$q(o,t) = \frac{k}{\gamma_w}\frac{\Delta u}{L}\left\{\frac{1}{\sqrt{\pi T}} \sum_{n=0}^{\infty} \left[e^{-(n+1)^2/T} + e^{-n^2/T}\right] - 1\right\} \tag{5.44}$$

Fig. (5.2) shows a plot of $q(o,t)/q(o,\infty)$ and $q(L,t)/q(L,\infty)$ against $1/\sqrt{T}$, where q^{∞} is the steady state flow rate.

These solutions* are identical to those of the analogous problems of heat conduction in solids given by Carlsaw and Jaeger (1959) who also give the solution for $L = \infty$:

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* These solutions were derived by a colleague, Mr. Tan Swan-Beng.

$$q(o,t) = \Delta u \frac{k}{\gamma_w} \frac{1}{\sqrt{\pi c t}} \tag{5.45}$$

This plot is also shown in Fig. (5.2) and as for the case of the sand drain. The plots for L finite and infinite are almost the same down to $1/\sqrt{T} \doteq \sqrt{\pi}$ after which the flow rate for L finite curves sharply towards the steady state value.

These solutions have the same significance as those by Gibson (1963) for the spherical piezometer cavity, as they provide a value for Cv or Cs as well as k from a C.H.T. Perhaps more important is the ability to locate the steady state more accurately.

A C.H.T. test has been performed on remoulded London Clay at an initial m/c of 53%. An oedometer ring was filled with the remoulded clay taking care to exclude air bubbles. The top and bottom of the sample were covered with filter paper and coarse porous stones. The ring, sealed at the bottom, was then placed in a large ordinary triaxial cell. The sides of the ring were lightly smeared with silicone grease to minimise friction and possible leakage. The top of the sample was in free contact with water inside the triaxial cell, while the bottom was sealed and connected by a $\frac{1}{8}$" saran tubing to an ordinary reversible 5 c.c. paraffin-water volume gauge, and a null-indicator for pore pressure measurement.

At zero cell pressure the pore pressure at the bottom of the sample was also zero and when the cell pressure was raised to 15 p.s.i. the pore pressure responded immediately and recorded the same amount.

The test was then started by maintaining the cell pressure at 15 p.s.i. and the bottom of the sample at 5 p.s.i. The flow rate measured is plotted against $1/\sqrt{t}$ in Fig. (5.3), and is seen to be in remarkable agreement with the theoretical prediction of eq. (5.41).

The values of the coefficients of permeability and consolidation (or swelling) are given by:

$$k = \frac{q_{\infty} \cdot \gamma_w}{A \Delta u / L} \qquad (5.46)$$

and

$$Cv = \frac{q^2_{\infty} L^2}{n^2 \pi} \qquad (5.47)$$

where q_{∞} is the measured steady-state flow rate, and A is the cross-sectional area of the sample. Eq. (5.45) assumes that the theoretical slope of the q v . $1/\sqrt{T}$ plot for L finite is equal to $1/\sqrt{\pi}$ which is true to better than 99%.

For the test conducted k was found to be 3.4×10^{-8} cm./sec. and Cv = 3.7 ft.^{2}/year, which are in agreement with other data on London Clay. In these calculations the original value of L was used, but in a more sophisticated apparatus the deformation can be measured and the average value of L might provide more accurate results.

An alternative but probably less accurate method for calculating Cv would be to note the time at which the steady state is obtained, which from Fig. (5.2) should be at $1/\sqrt{T} \doteqdot \sqrt{\pi}$. For the test conducted the value of Cv calculated using this approach gave 4.6 ft^2/year, which is not in bad accord with that deduced from the slope of the q v . $1/\sqrt{t}$ plot.

When enough readings were obtained at the steady state, the test was terminated by closing the bottom drainage valve. The sample then swelled back to the original 15 p.s.i. pressure in a few hours.

It was then decided to repeat the test and study the effect of void ratio-effective stress path on the values of k and Cv measured in subsequent tests. This was inspired by the field results obtained by the Author from repeated C.H.T. on the same piezometers.

The results for the repeat tests are also shown in Fig. (5.3) where they clearly show the remarkable increase in Cv after the first test, while the value of k remained nearly unchanged. This will be discussed more fully in Chapter 8.

Although it is difficult to judge on the basis of results of one test only, it seems that the technique is encouraging in that a more accurate value of k can be obtained as well as a value for Cv.

In order to compare the values obtained from such tests with those from normal oedometer tests the following procedure might be useful. The sample should be subjected to a high load increment, say 50 p.s.i. The bottom is then opened to 40 p.s.i. and the values of k and Cv calculated. Next the top should be opened to 40 p.s.i. and the flow rate measured at the top, from which only Cv can be calculated. The cycle can then be repeated by setting the bottom to 30 p.s.i. and then the top, etc. This procedure will ensure a gradual decrease of the void ratio as in the normal oedometer test.

Finally although these tests strictly apply to oedometer samples, since the solutions obtained are for one dimensional consolidation only, they may be applied to the more convenient tests on triaxial samples where the errors introduced by the slight lateral consolidation might probably be acceptable.

(5.6) Numerical solutions to the unsteady flow of water from spherical and cylindrical tips to compressible soils

Using Terzaghi's equation in cylindrical co-ordinates we have for the cylindrical tip

$$c\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial Z^2}\right) = \frac{\partial u}{\partial t} \qquad (5.48)$$

For the C.H.T. the boundary conditions chosen corresponded to case (1C) for the steady state Fig. (4.2)

$$\begin{aligned}
&\text{at } r > r_1 \text{ and } |Z| > Z_1 \; u = 0 \text{ at } t = 0 \quad \text{(a)} \\
&\left.\begin{aligned}&\text{at } r = r_2 \\ &\text{and } |Z| = Z_2\end{aligned}\right\} u = 0 \quad \text{for } t > 0 \quad \text{(b)} \\
&\left.\begin{aligned}&\text{at } r = r_1, \; 0 \leqslant |Z| < Z_1 \\ &0 \leqslant r \leqslant r_1, \; Z = -Z_1\end{aligned}\right\} u = u_1 \text{ at } t > 0 \quad \text{(c)} \\
&\left(\frac{\partial u}{\partial Z}\right)_{Z = Z_1} = 0 \text{ for } 0 < r \leqslant r_1, \; t \geqslant 0 \quad \text{(d)}
\end{aligned} \qquad (5.49)$$

For the F.H.T. boundary conditions (5.49c) is replaced by

$\frac{dh}{dt} A$ = rate of flow of water from the tip to the soil, h and A as defined in Section (5.2).

Equation (5.48) was put into a finite difference form and solved numerically using an explicit method, on a digital computer. As for the steady state case, details of the numerical analysis and computer programming will be given in Part IV of this thesis, but some general aspects of the problem will be discussed here.

For the C.H.T. the procedure was to evaluate the pore pressure distribution outside the tip at different values of time and from that calculate the flow rate of water out of the tip. For the latter, three methods were considered:

(i) by integrating the flow rate at the piezometer tip itself,

(ii) by integrating the flow rate of water across successive closed rectangular contours in the soil and adding to that the swelling rate of the soil enclosed by the relevant contour,

(iii) This is the limiting case of (ii) where a large enough rectangular contour was chosen far away from the tip where no flow occurred across it, and the swelling rate of all the soil affected by the relaxation process was evaluated.

For the F.H.T. the pressure at the tip h was kept constant through the time interval δt and then adjusted at the end of the interval as follows:

$$h(t) = h(t - \delta t) - q_{(t - \frac{\delta t}{2})} \cdot \frac{\delta t}{A} \qquad (5.50)$$

where $q_{(t - \frac{\delta t}{2})}$ is the average flow rate during the interval $t = t - \delta t$ to $t = t$.

The first computer runs for the C.H.T. showed that these three methods did not agree very well especially at small values of time. Other irregularities arose when the mesh size was changed, partly due to the singularity point (see section 4.5.4) and partly because of the difficulty in evaluating accurately the pore pressures and swelling rate near the piezometer tip where the gradients were very high.

It was then decided to write programmes for the spherical tip using both explicit and implicit methods and comparing the results to Gibson's (1963) analytical solutions.

(5.6.1) Numerical solution for the C.H.T. - spherical tip

As was anticipated the first few runs for the spherical cavity showed for small values of time, similar irregularities to those encountered for the cylindrical tip in that the mesh size had an exaggerated effect on the solutions for both the C.H.T. and F.H.T. This could not be explained in terms of increased accuracy alone. Of the three methods of integration discussed in the last section the first was found completely unsatisfactory as it depended greatly on the pore pressure distribution near the tip, while the second was again unsuitable where the contour was near the tip, for the same reason. The third method was found to give the most accurate answers

. But even with this last method the flow rate calculated for the C.H.T. fell appreciably short of the theoretical value at small values of time.

Two possible sources of errors are involved here; the evaluation of the pore pressures and the integration of the flow rate. The computed pore pressures were not in very good agreement up to the first or second interval of time, but from then on they agreed quite well with the theoretical values, see Fig. (5.4). It was in fact the evaluation of the flow rate which gave rise to most of these discrepancies. This was confirmed by running programmes using an implicit method and very small mesh sizes. It was realised that the mesh point at the surface of the sphere was giving most of the trouble, similar to that encountered with the singularity point S for the steady state solutions (Fig. 4.4).

At small values of time most of the swelling occurs near the tip, at the first few mesh points. Initially the mesh point, at the surface of the tip was set at the boundary condition $u = u_1$ for all t. This meant that the soil occupying half a mesh size next to the tip would not contribute anything to the total swell rate. Yet it is this particular part of the soil which is swelling most, at small values of time. A remedy was sought by setting this mesh point at $u = u_1$, and its swell rate at half of that of the first mesh point away from the tip. This compromise greatly improved the solutions although up to the first or second time interval the computed flow rates still fell slightly short of the theoretical values.

Fig. (5.4) shows the numerical results for the flow rate q plotted against $1/\sqrt{T}$, T being the time factor. The letter k refers to the number of mesh intervals into which the radius of the sphere was divided. As was mentioned earlier the method of computing the flow rate at the surface of the sphere proved unsatisfactory. However, calculating the total swell rate with the correction for the $\frac{1}{2}$ mesh size gave satisfactory values.

(5.6.2) Numerical solution for the F.H.T. spherical tip

For the F.H.T. the flow rate at small values of time must be calculated accurately so that the head 'h' can be adjusted at the end of the time interval according to eq. (5.50). For a small value of μ, say 0.04 E changes very little with time at the outset and hence the error due to the $\frac{1}{2}$ mesh size near the tip hardly affects the solution. However, for large values of μ, say 10, this error becomes appreciable and if no correction was made for the $\frac{1}{2}$ mesh size the computed E was found to be as much as 10% higher than the theoretical value.

The procedure adopted here was to allow the $\frac{1}{2}$ mesh size to swell to the full u_1 pressure after the first interval of time. For the subsequent intervals of time the total swell rate was calculated by first assuming that the $\frac{1}{2}$ mesh size suffers zero consolidation. Next the head was adjusted according to equation (5.50). Finally a correction was made to the head by allowing the $\frac{1}{2}$ mesh size to consolidate by the average change in pressure at the tip and the first mesh point outside it. Fig. (5.5) shows the computed results together with Gibson's theoretical values, and the agreement is seen to be quite good.

(5.6.3) Numerical solution for the C.H.T. and F.H.T. cylindrical tip

The same procedure was adopted here as for the spherical tip. Programmes were run for different L/D ratios for the C.H.T. and F.H.T. as shown in Fig. (5.6 - 5.10), where k refers to the number of mesh intervals into which the radius of the tip was divided and L to that corresponding to half the height of tip.

For the F.H.T. it is seen that the numerical results for all the L/D ratios considered show that the behaviour of a cylindrical tip is very similar to that of the equivalent sphere; unlike the C.H.T. results where for high L/D ratios some departures are observed as will be discussed now.

For the C.H.T. the results are plotted as q/q_{∞} (the current rate of flow divided by the steady state flow rate q_{∞}) against $1/\sqrt{T}$. The value of q_{∞} was taken from the Author's computed values of the intake factor, Fig. (4.2). For the time factor $T(= ct/r^2)$ the equivalent radius 'r' may be estimated by equating the intake factors of the cylindrical tip and the equivalent sphere:

$$r_f = \frac{\text{Intake factor of cylindrical tip}}{4\pi} \tag{5.51}$$

or, by equating the surface areas of the two:

$$r_s = \left(\frac{\text{Surface area of the cylindrical tip}}{4\pi}\right)^{\frac{1}{2}} \tag{5.52}$$

The values of 'r' calculated using both equations are given below:

Table 5.1

L = 6"	L/D ratio				
	$\frac{1}{2}$	1	2	4	8
r_f"	5.23	3.37	2.26	1.61	1.20
r_s"	5.19	3.36	2.25	1.55	1.08
%Diff.	1%	$\frac{1}{2}$%	$\frac{1}{2}$%	4.0%	10.0%

For L/D ratios less than 4 there is little to chose between the two values. Using 'r_f' Fig. (5.6, 5.7 and 5.8) were plotted for L/D = $\frac{1}{2}$, 1 and 2. For these geometries the cylindrical tip behaves in a very similar fashion to Gibson's spherical tip for both the C.H.T. and F.H.T.

For the C.H.T. the computed results for L/D = 1 and 2 are seen to cross Gibson's line very slightly and then fall back on it. This is similar to the numerical results obtained for the spherical tip (Fig. 5.4), and is very probably due to the $\frac{1}{2}$ mesh size correction discussed in Section (5.6.1).

For the L/D ratios of 4 and 8 the computed values of q/q_∞ for the C.H.T. (Figs. 5.9 and 5.10) are compared to those of the equivalent Gibson's sphere, eq. (5.12), and to those of the infinite sand drain, eq. (5.35). For an L/D ratio of 4 and using 'r_s' close agreement is obtained between the computed values and those of Gibson's sphere. However, for L/D = 8 a better agreement is obtained by assuming the cylindrical tip to behave as an infinite sand drain. This is hardly surprising since at such a high L/D ratio most of the flow is in the radial direction.

Another interesting point emerging from the numerical solution for the L/D ratio of 8 is that when the straight portion of the $\frac{q}{q_\infty}$ v. $1/\sqrt{T}$ plot is extrapolated towards the $\frac{q}{q_\infty}$ axis it intersects it at a value of around 1.3 instead of 1.0. This is similar to the behaviour of the infinite sand drain, Fig. (5.1). For example, for r_2/r_1 ratio of 50 the plot is very nearly linear down to a value of $1/\sqrt{T}$ of approximately one after which it curves downwards towards the steady state flow rate. For an L/D ratio of 4 this is less pronounced but even then the q/q_∞ v. $1/\sqrt{T}$ plot intersects the q/q_∞ axis at a value of 1.1.

The errors introduced into the calculation of k and c from field tests on a cylindrical tip when its behaviour is approximately that of a sphere or an infinite sand cylinder, will now be discussed.

For the spherical case (eq. 5.12) it is convenient to calculate k and c from field tests using the relationships:

$$k = \frac{q_\infty \cdot \gamma_w}{F \times \Delta u} \qquad (5.53)$$

and

$$c = \frac{q_\infty^2 \cdot r^2}{\pi n^2} \qquad (5.54)$$

where q is the extraploated steady state flow rate, and n is the slope of the q v. $1/\sqrt{t}$ plot. For the L/D = ½, 1 and 2 these two relationships can be used with good accuracy.

For the higher L/D ratios if the tests are not carried near enough to the q axis, say to $1/\sqrt{T} = 1$ the value of k calculated from the apparent q_∞ will be overestimated. For the L/D ratio of 4 the error is about 10% while for L/D = 8 it is 30% as shown by the computer results.

In the calculation for C if the apparent q_∞ is used in eq. (5.54), the true value of c would not only be overestimated by the difference of slopes of the q/q_∞ v. $1/\sqrt{T}$ plots (Fig. 5.9 and 5.10). For L/D = 4 the true c would be overestimated by 6½% and 11½% when r_s or r_f is used, while for L/D = 8 the corresponding figures are 19% and 45.5%.

To calculate k and c comparing the cylindrical tip to an infinite sand drain, use can be made of the approximate equation (5.35)

$$k = \frac{q_\infty \cdot \gamma_w}{5.336 r_1 \cdot L/D \cdot \Delta u} \qquad (5.55)$$

and

$$c = \frac{1}{(0.424)^2 \cdot \pi} \quad \frac{q_\infty^2}{n^2} \quad r_1^2 \qquad (5.56)$$

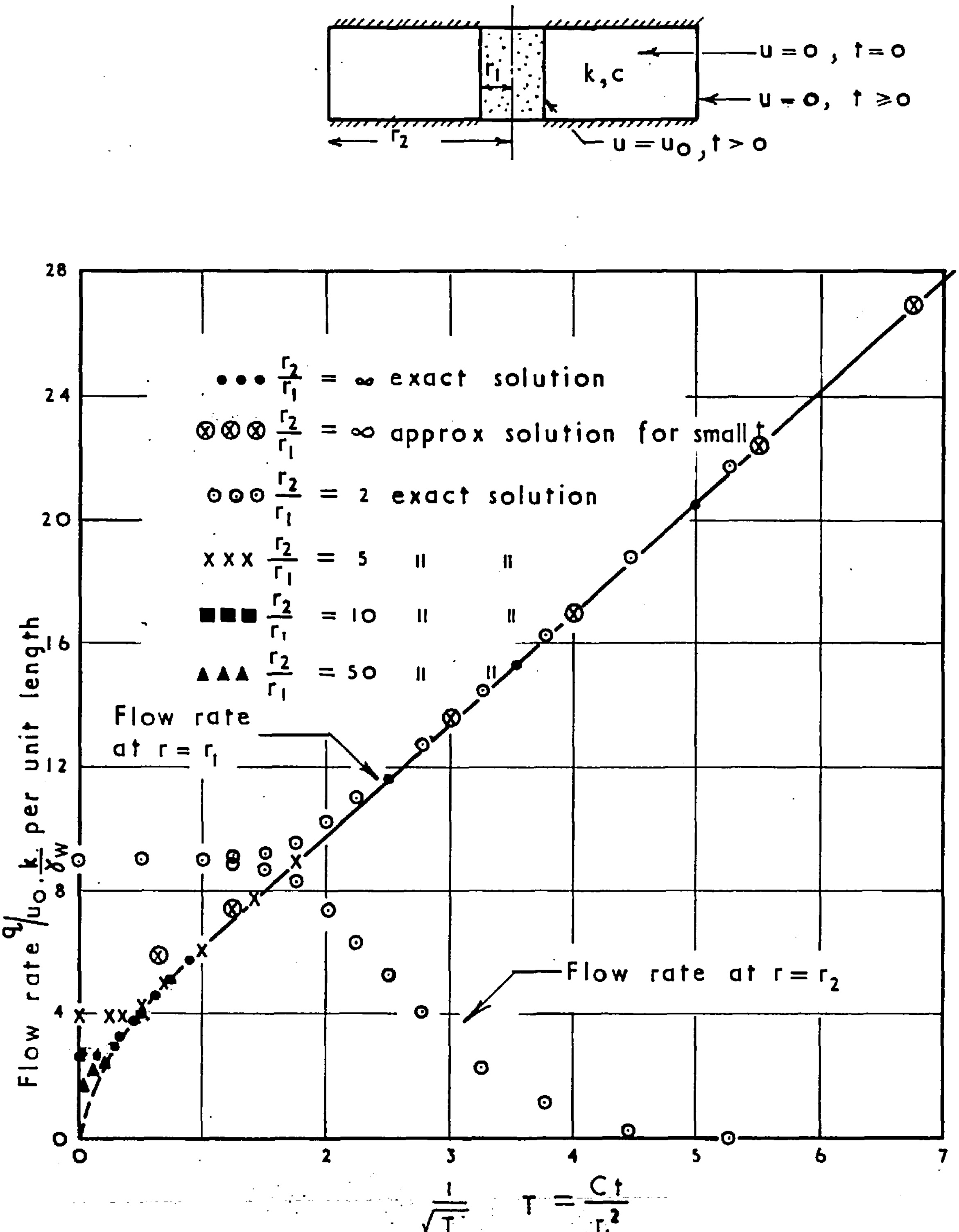
where n is again the slope of the q v. $1/\sqrt{t}$ plot obtained in a field test. In equation (5.55) the quantity 5.336 r_1.L/D replaces the true intake factor, and for an L/D ratio of 8 it exceeds it by 6½%. It was mentioned earlier that if the q v .$1/\sqrt{t}$ plot is not carried near enough to the q axis, the apparent q would be some 30% higher than the true q_∞. Thus the true value of k would be again overestimated by 22% as compared to 30% when the tip was equated to a sphere. In the calculation for c, using eq. (5.56) where q_∞ is the apparent value, this will lead to an overestimation of the true value of c by 6½%.

(5.6) Summary and discussion

In this Chapter the constant-head and falling-head seepage tests (C.H.T. and F.H.T.) are discussed from a theoretical point of view. Gibson's analytical solutions for a spherical tip are mentioned, together with similar solutions for an infinite sand drain in an infinite or semi-infinite mass of soil and for one-dimensional constant-head test on an oedometer sample. For finite cylindrical tips numerical solutions are given for different L/D ratios.

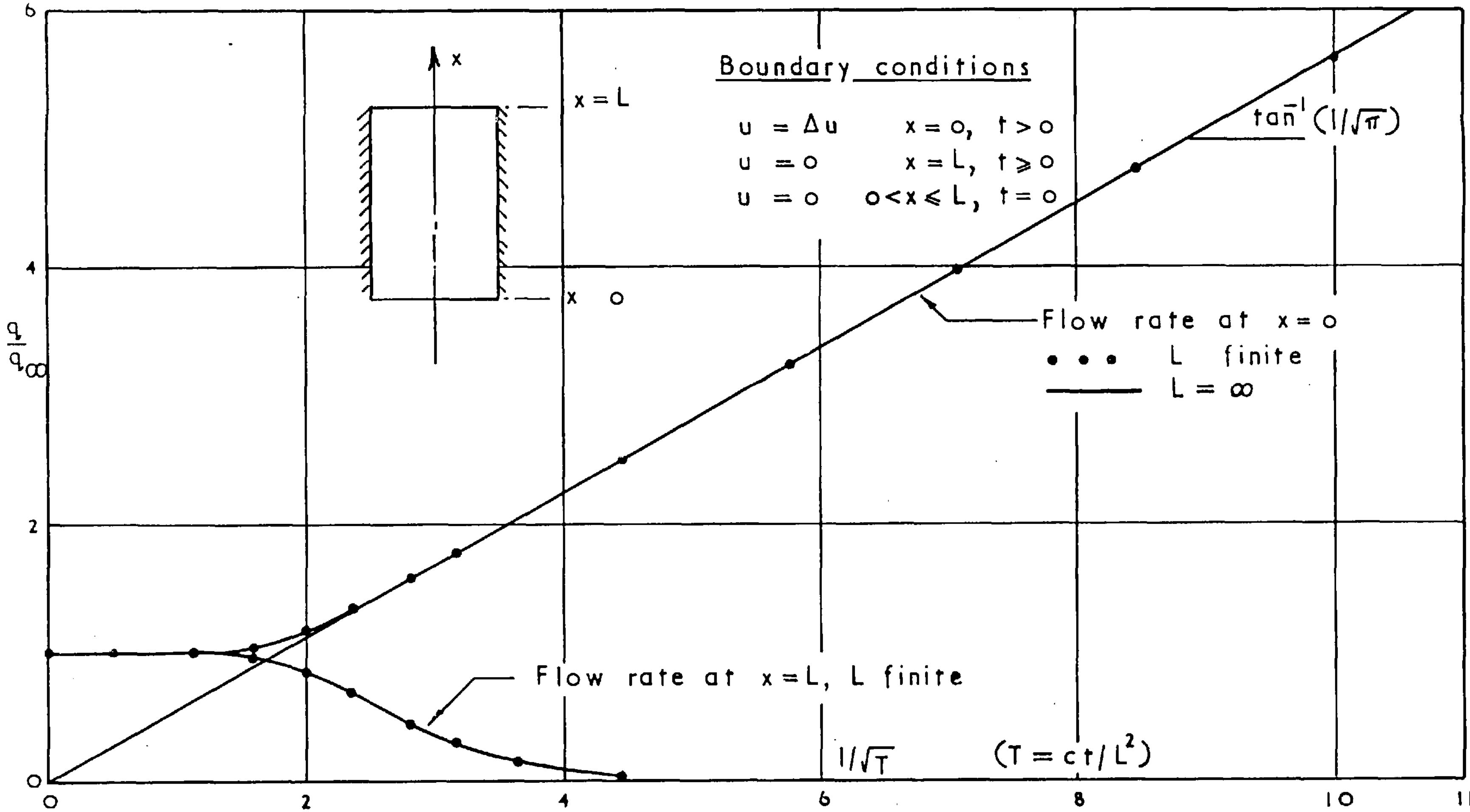
For the C.H.T. and for L/D ratios $\geqslant 4$ Gibson's solutions for the spherical tip may be used with adequate accuracy especially if the equivalent radius "a" of the cylindrical tip is calculated by equating the surface areas of the cylindrical tip and sphere. For an L/D ratio of 8 and higher the computed behaviour of the cylindrical tip is in better agreement with that of an infinite sand drain.

For the F.H.T. Gibson's analytical solutions are not very sensitive to the value μ and hence to 'a' and thus close agreement is obtained between the behaviour of the cylindrical tip and equivalent sphere for all L/D ratios considered.



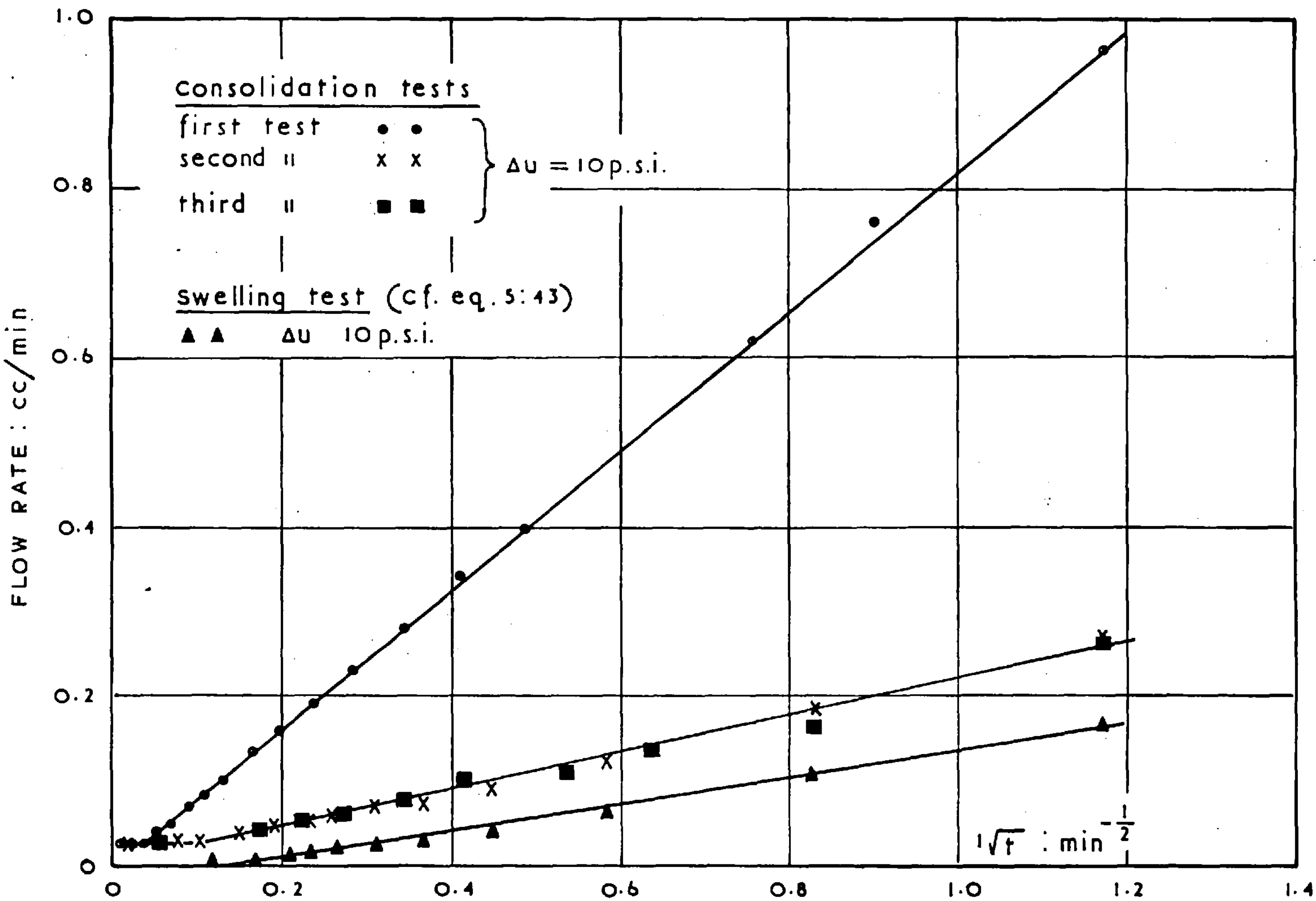
CONSTANT HEAD FLOW RATE OF WATER FROM AN INFINITE SAND DRAIN IN A COMPRESSIBLE SOIL : ANALYTICAL SOLUTION

FIG. 5.1



ANALYTICAL SOLUTIONS FOR ONE DIMENSIONAL C.H.T.

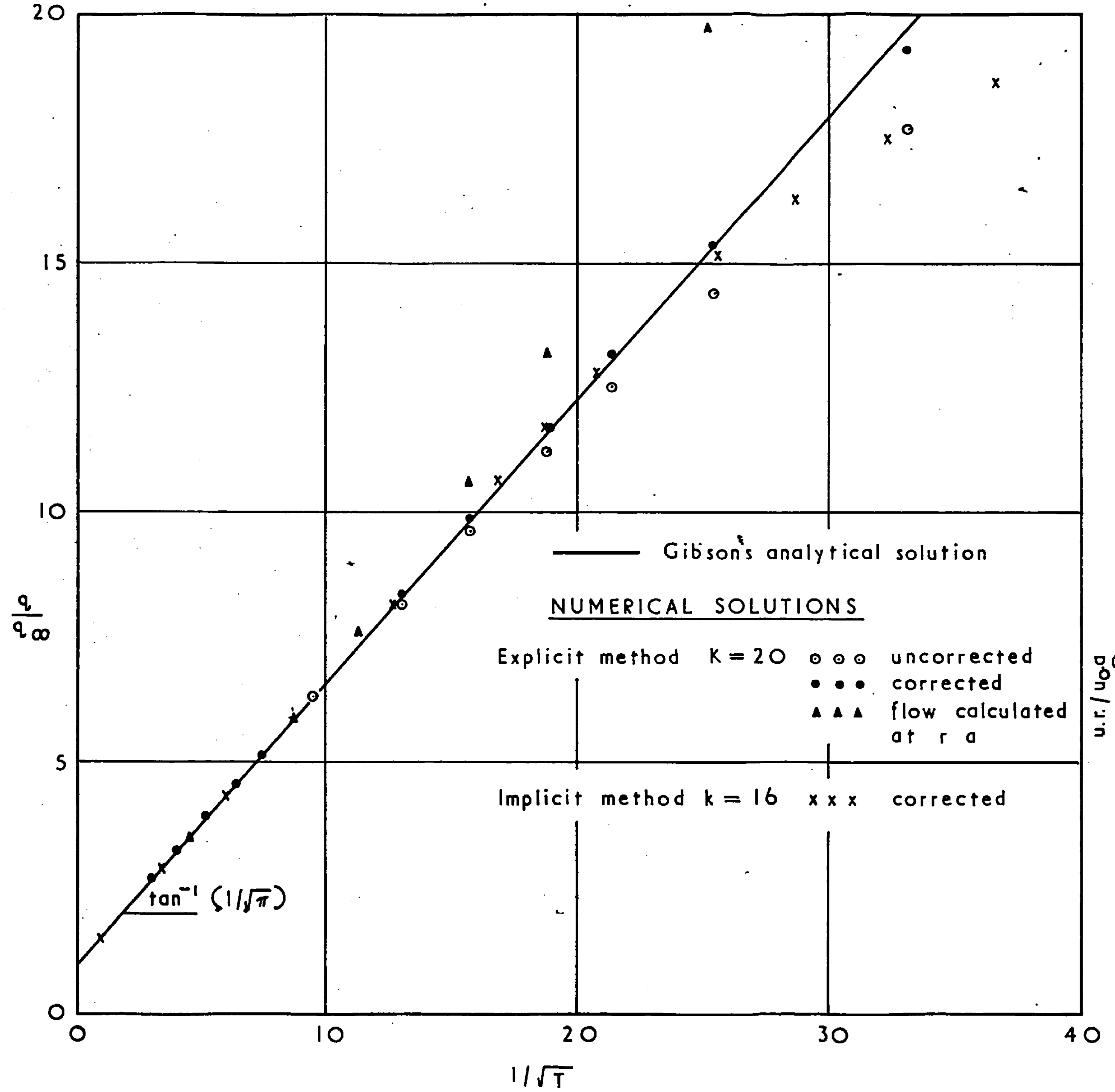
FIG. 5.2



I-D C.H.T. ON A SAMPLE OF REMOULDED LONDON CLAY

FIG. 5:3

NUMERICAL SOLUTION FOR C.H.T. FOR A SPHERICAL PIEZ. CAVITY



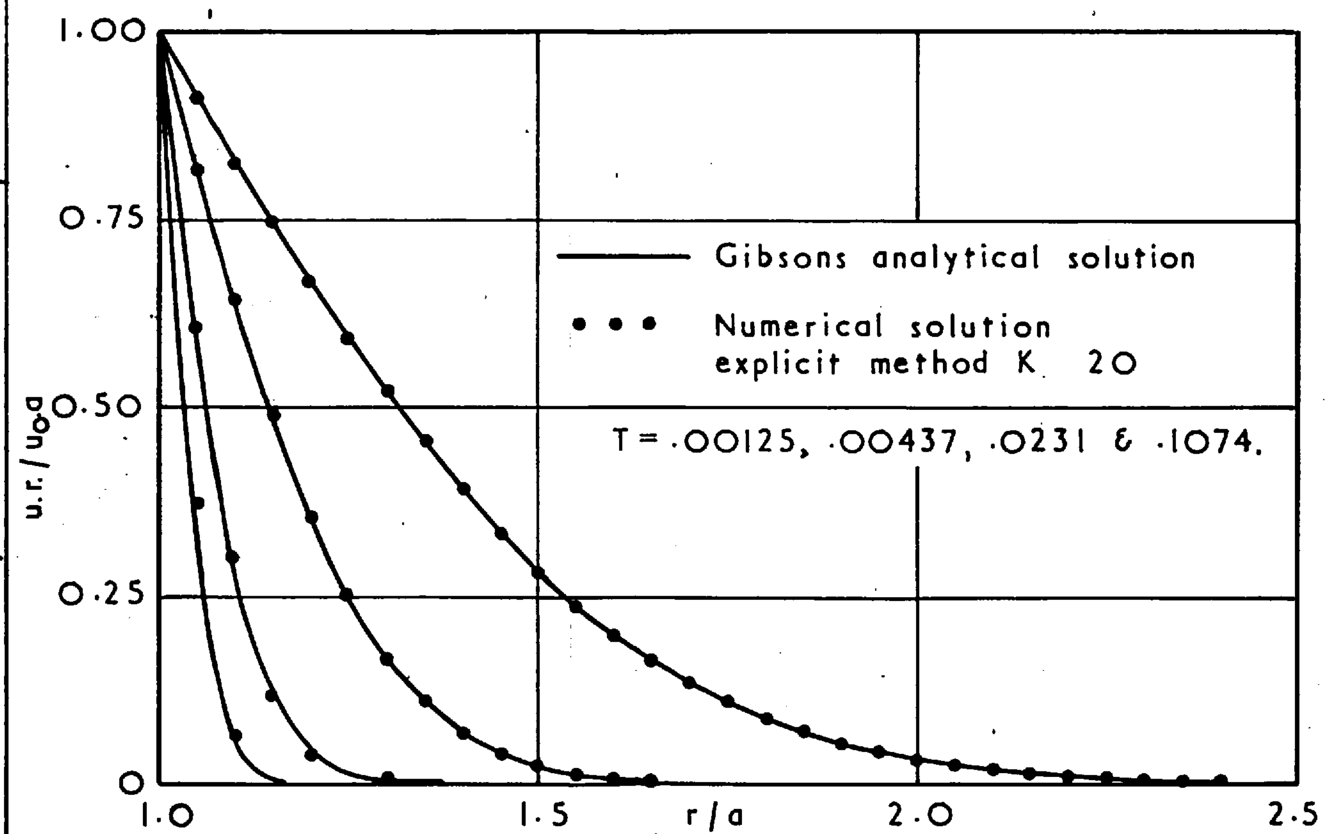
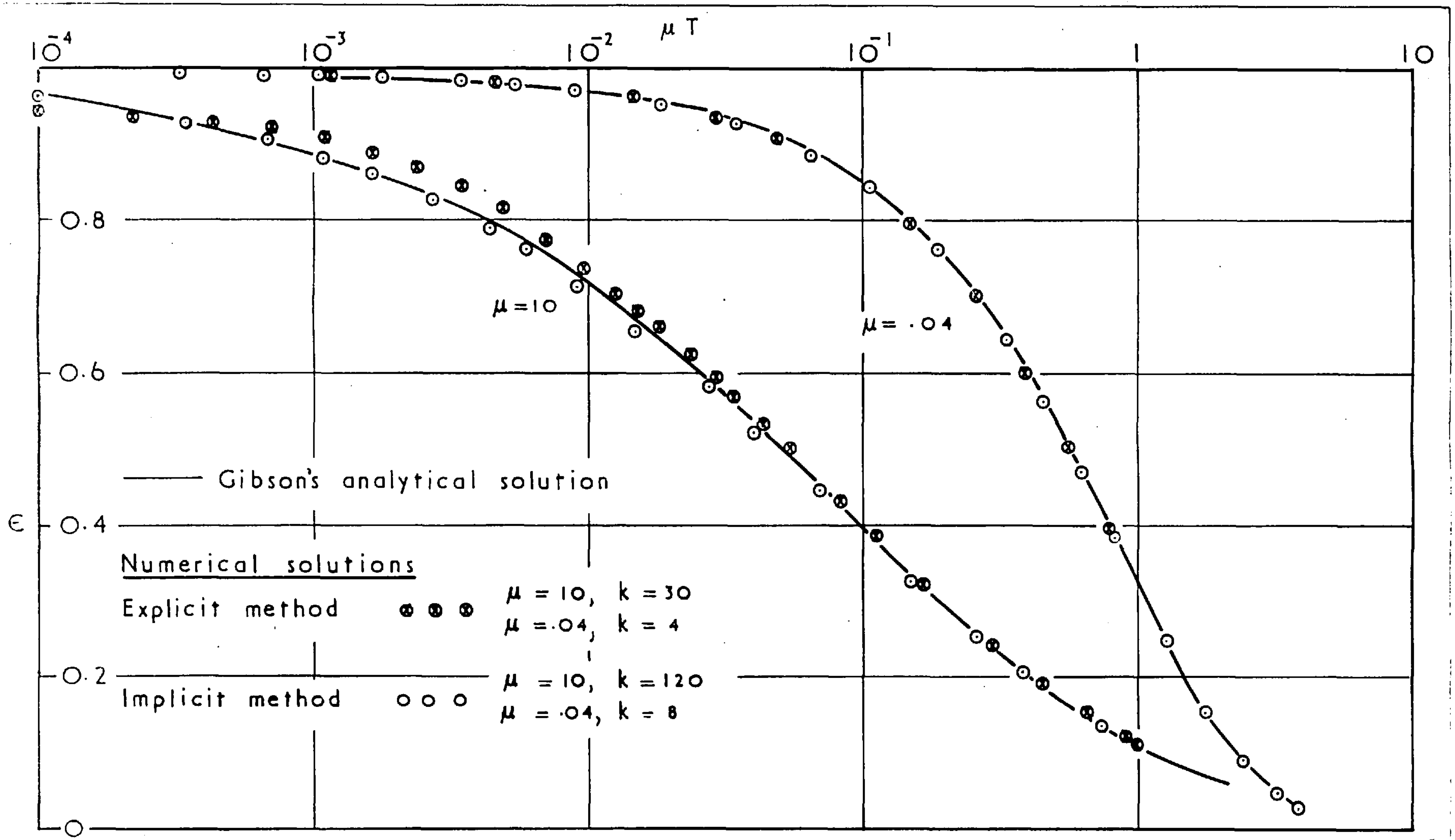


FIG. 5.4



NUMERICAL SOLUTION FOR F.H.T. ON SPHERICAL PIEZ CAVITY

FIG 5·5

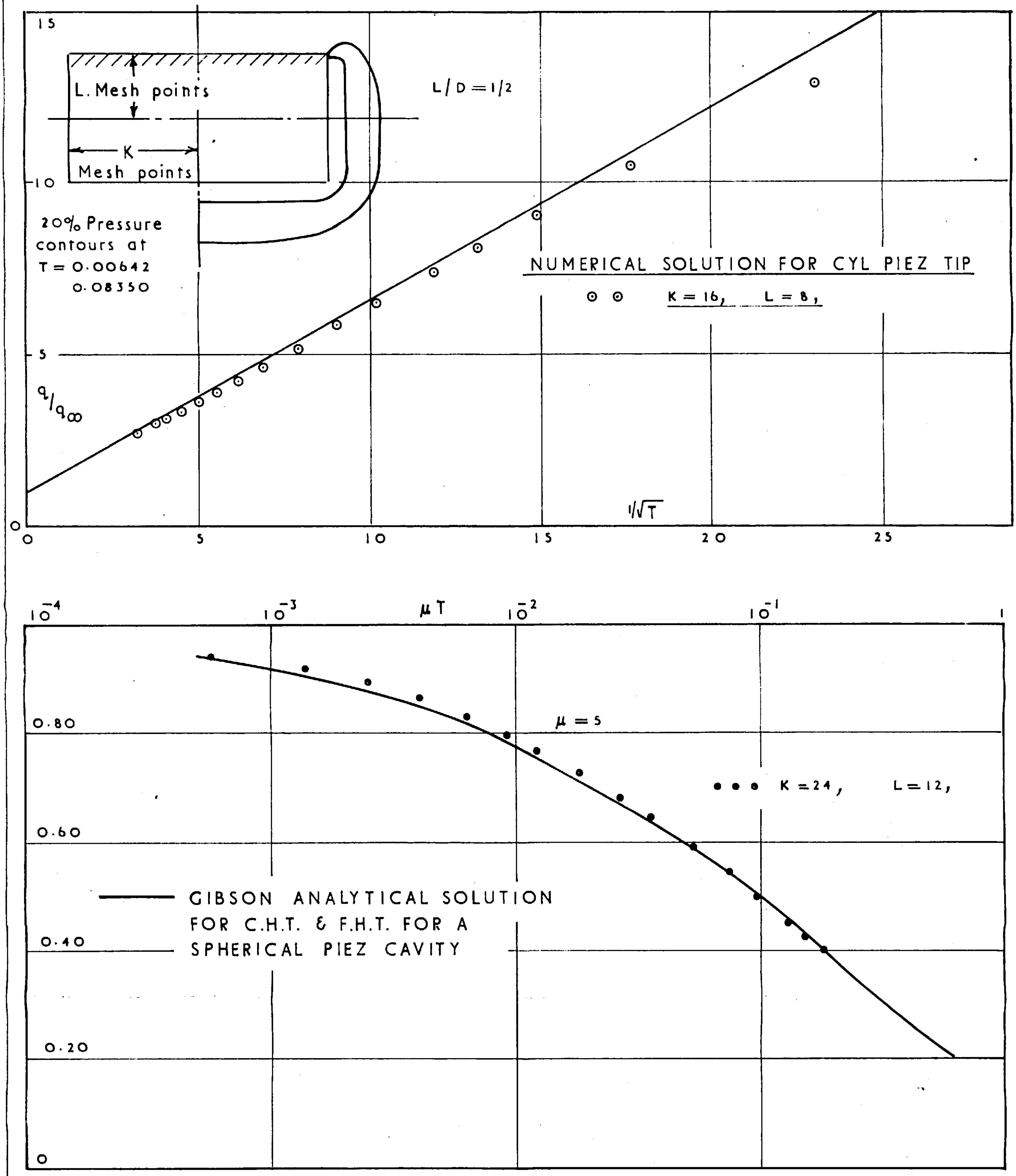

L. Mesh points
K
Mesh points
L/D = 1/2
20% Pressure
contours at
T = 0·00642
0·08350
NUMERICAL SOLUTION FOR CYL PIEZ TIP
K = 16, L = 8,
q/q∞
1/√T
μT
μ = 5
K = 24, L = 12,
GIBSON ANALYTICAL SOLUTION
FOR C.H.T. & F.H.T. FOR A
SPHERICAL PIEZ CAVITY

FIG. 5.6

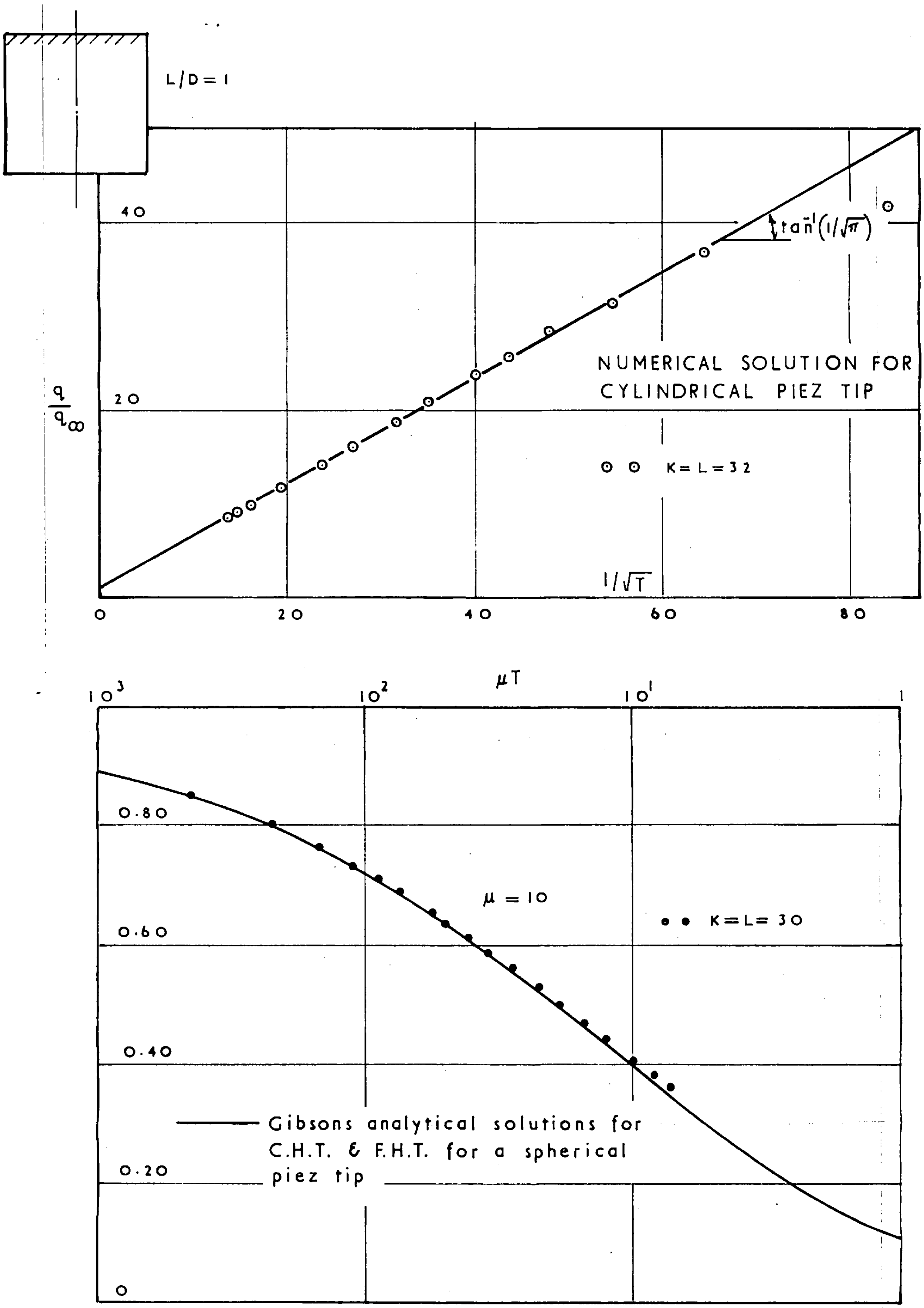
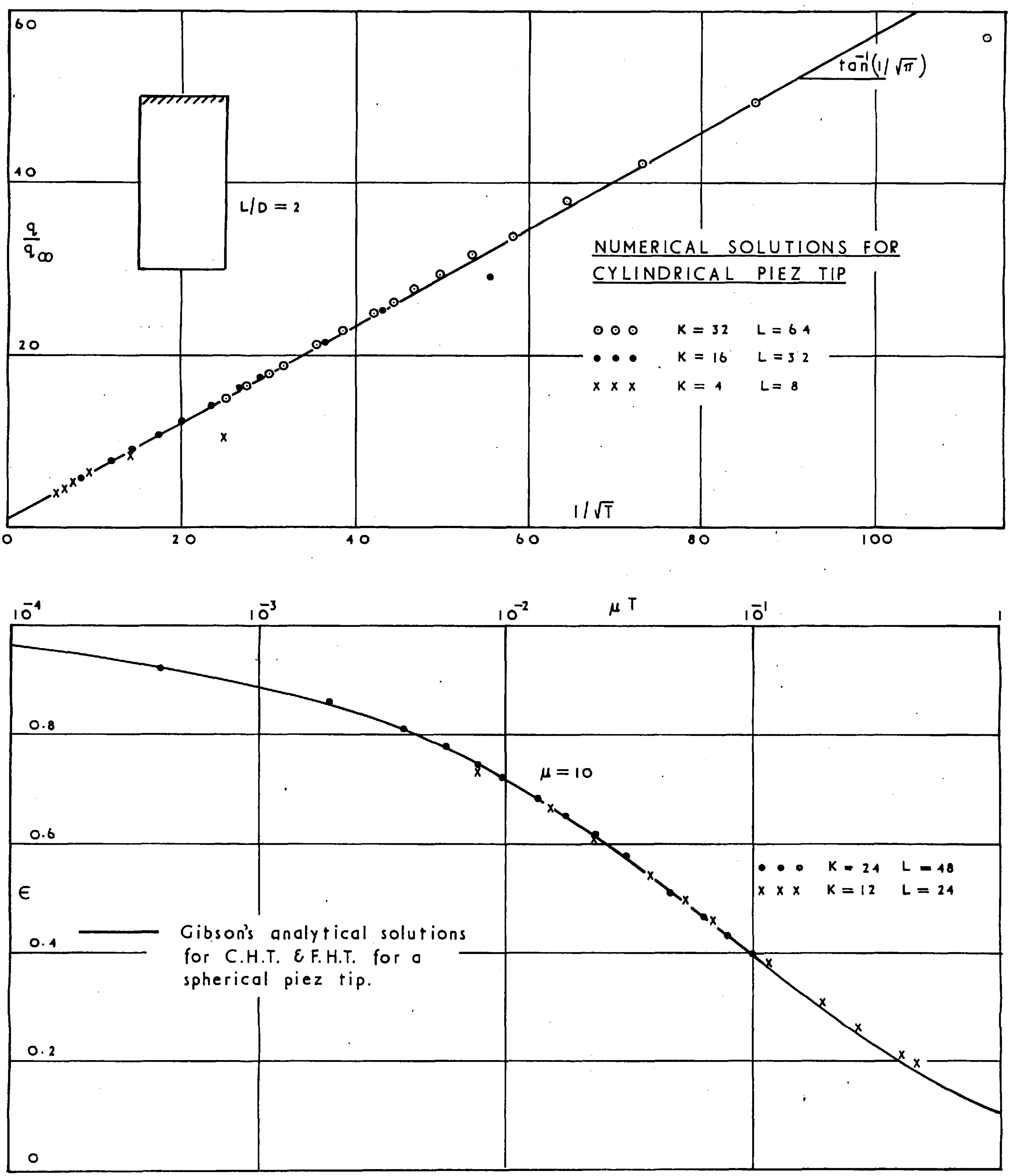


FIG. 5.7

NUMERICAL SOLUTIONS FOR C.H.T. & F.H.T. FOR A CYLINDRICAL PIEZ TIP.



NUMERICAL SOLUTIONS FOR C.H.T. & F.H.T. FOR A CYLINDRICAL PIEZ TIP.

FIG. 5.8

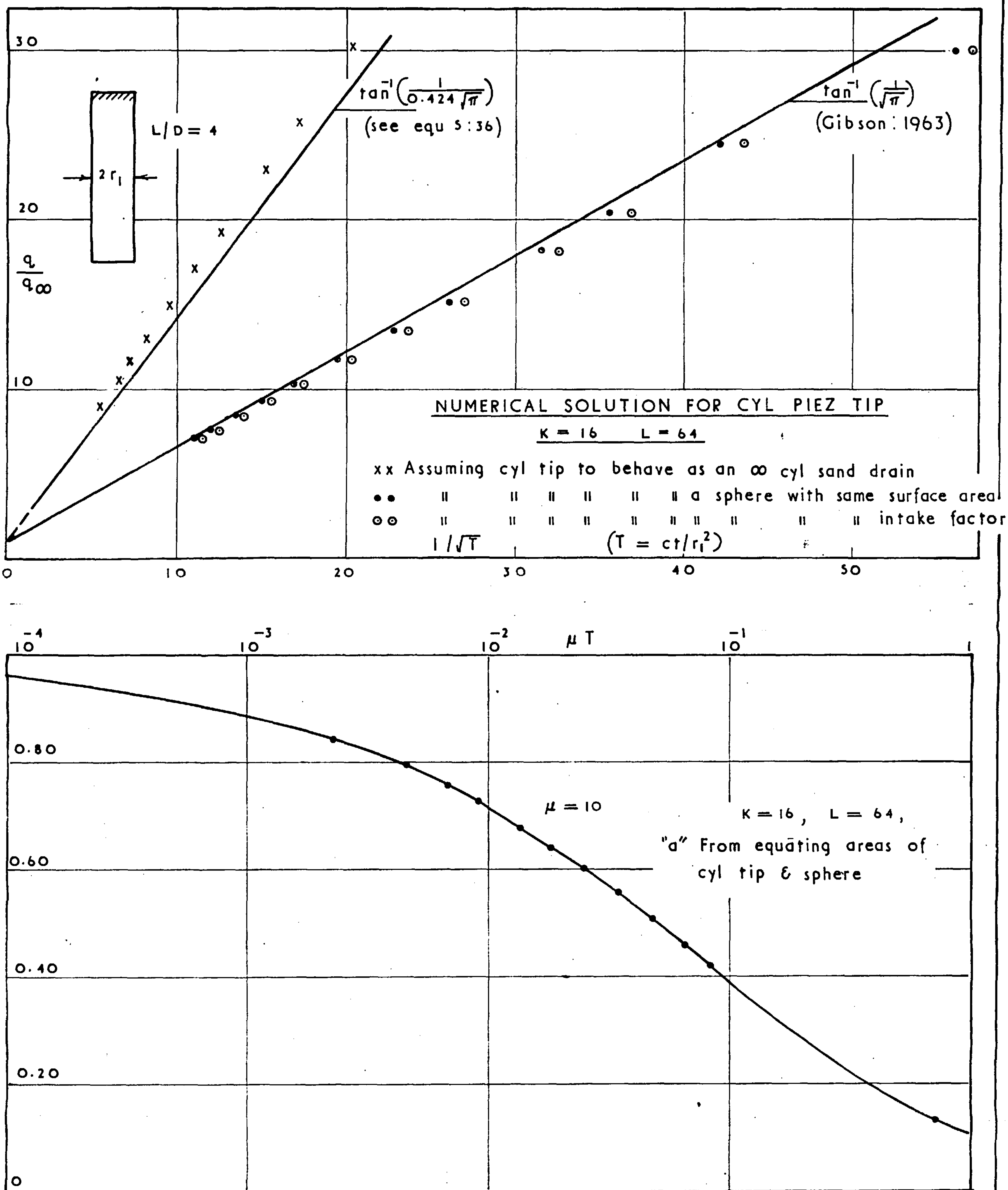
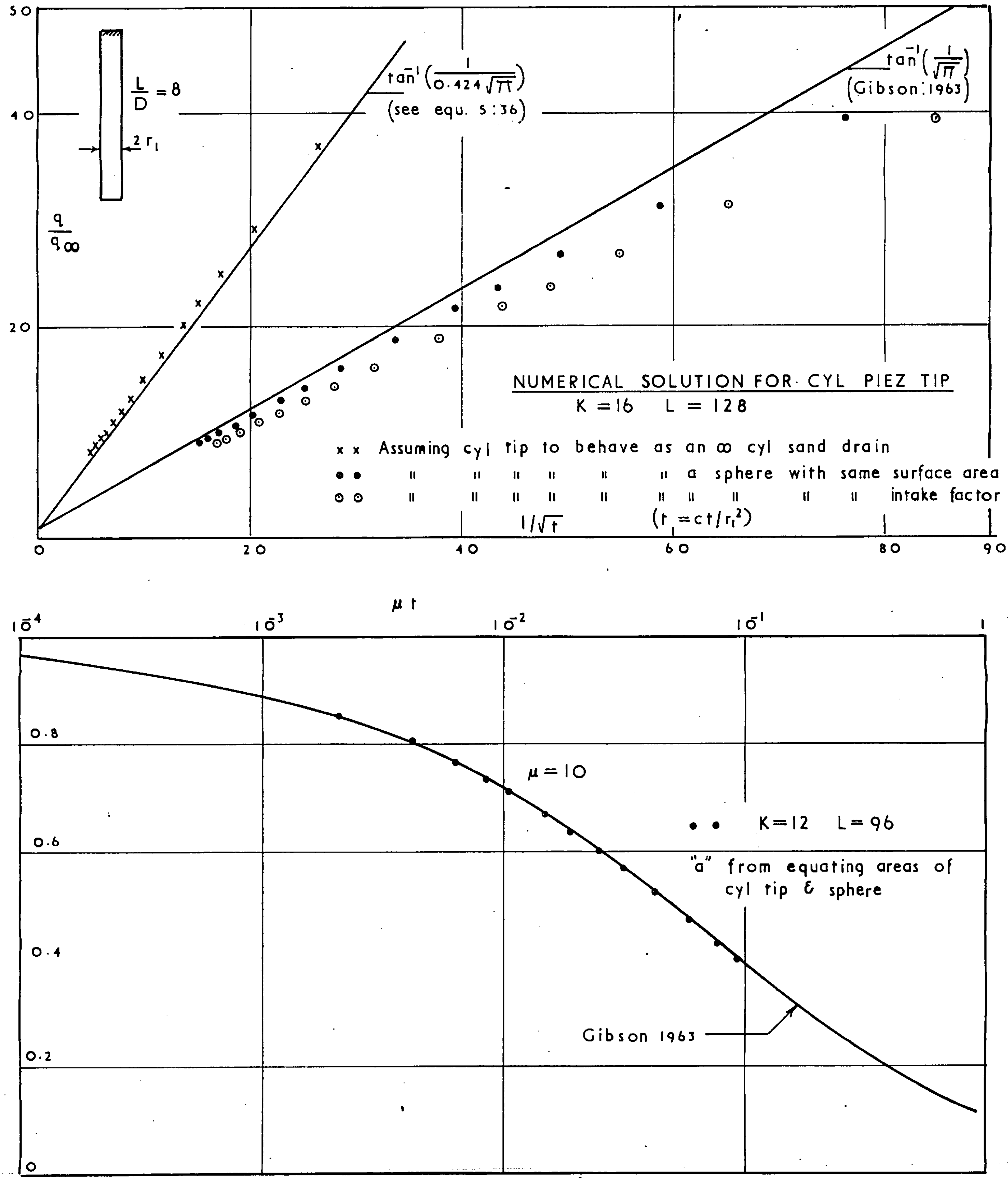


FIG: 5:9

NUMERICAL SOLUTIONS FOR C.H.T. & F.H.T.
FOR A CYLINDRICAL PIEZ TIP



NUMERICAL SOLUTIONS FOR O.H.T. & F.H.I.
FOR A CYL PIEZ TIP

FIG. 5.10

CHAPTER 6

SOLUTIONS FOR THE CONSTANT-HEAD TEST ON A SPHERICAL PIEZOMETER CAVITY IN COMPRESSIBLE SOILS

(6.1) Introduction

In Chapter 5 Gibson's analytical solutions based on Terzaghi's theory of consolidation were given for the constant-head test (C.H.T.) and falling-head test (F.H.T.) on a spherical piezometer cavity together with the Author's approximate numerical solutions for a cylindrical tip. For the latter it was shown that for a length/diameter ratio of 4 or less the tip behaved like a spherical tip, whereas for higher ratios its behaviour approximated to that of an infinite sand drain. Nevertheless, for all the length/diameter ratios considered the q v. $1/\sqrt{t}$ plots were found to be linear.

Since Gibson published his first solution in 1963 there are very few published records of C.H.T. and F.H.T. conducted with Gibson's theory in mind. One such record is by Bishop, Kennard and Vaughan (1964). The Author in the last three years has conducted many C.H.T. and a few F.H.T. on piezometers in five earth dams and embankments. The Author's results (reported in Chapters 7 and 8), those by Bishop et al. (1964), and other unpublished data collected by the Author from other workers suggest clearly that the form of Gibson's equation for the C.H.T. (eq. 5.12) is closely followed in practice, i.e. when the flow rate q is plotted against $1/\sqrt{t}$ a reasonably straight line plot is obtained for at least part of the plot.

For F.H.T. the usefulness of Gibson's solution for evaluating k and c is severely limited by the need for a trial and error solution by fitting the experimental results to Gibson's curves. A further complication is the initial drop in head due to the expansion of the plastic tubing which is difficult to estimate, as was also found by Vaughan (1965). Nevertheless someF.H.T. results will be given in Chapter 9 as well as a correlation of the value of k and c obtained from these tests with those from corresponding C.H.T.

It is the intention in this Chapter to concentrate on the more useful C.H.T. and study the effects of some departures from Gibson's solution on the interpretation of test results.

This study was prompted by two questions; the first being the reasons for the upward or downward curvature of some of the q v.$1/\sqrt{t}$ plots at small values of time, and the second and perhaps most important being the sometimes much larger values of C_v obtained from C.H.T. as compared to laboratory and field dissipation values. Naturally these two questions may not be independent of each other.

However, some of the answers to the first problem are quite obvious. For example plastic creep of the connecting tubing will produce an initial upward curvature of the q v.$1/\sqrt{t}$ plot especially when the permeability of the soil is relatively low (see Fig. 8.2). Another factor which produces a downward curvature is the head-loss in the connecting tubing, the tip ceramic and sand plug when the permeability of the soil is relatively high, (see Gibson, 1966, and Fig. 8.15 as an example). When the permeability of the soil is very low, temperature changes of the water and plastic tubing, together with the difficulty in obtaining accurate flow readings will produce erratic plots (see Fig. 7.6). Other possible factors are presence of gas bubbles in the piezometer system, disturbance to the pore pressure distribution around tip by previous de-airing, leaks, C.H.T. testing, etc.

Possible answers for the second question, and perhaps for the first also, are the lack of validity of some of the assumptions in Terzaghi's theory and the effect of disturbance to the soil immediately next to the piezometer tip on the values of k and C_v obtained from the tests.

These latter possibilities will now be investigated in detail in the following sections.

(6.2) Field records of C.H.T.; a better method for plotting q v. $1/\sqrt{t}$

Before discussing departures from Gibson's solution, a more correct method of plotting q v. $1/\sqrt{t}$ will be mentioned. Previously (Vaughan, 1965), the flow rate q was plotted against $1/\sqrt{t_1 + \Delta t/2}$, whereas a more correct plot would be against $2/(\sqrt{t_1} + \sqrt{t_2})$, where t_1 and t_2 are the beginning and end of the time interval Δt over which the flow rate q was calculated. This can be proved mathematically:

Gibson's equation for the C.H.T. (eq. 5.12) can be re-written as:

$$q = A + n\frac{1}{\sqrt{t}} \tag{6.1}$$

where A and n are constants. Integrating w.r.t. t between t_1 and t_2, the total flow Q is

$$Q_{t_1 \to t_2} = A(t_2 - t_1) + 2n(t_2^{\frac{1}{2}} - t_1^{\frac{1}{2}}) \tag{6.2}$$

Thus the average flow rate over the interval $(t_2 - t_1)$ is then:

$$q_{AV} = A + n\frac{2}{t_1^{\frac{1}{2}} + t_2^{\frac{1}{2}}} \tag{6.3}$$

Thus the average flow rate corresponds exactly to $2/(t_1^{\frac{1}{2}} + t_2^{\frac{1}{2}})$ and not to $1/\sqrt{t_1 + \frac{\Delta t}{2}}$. Of course one drawback in the above argument is the assumption that the field results will fall on a straight line plot of q v. $1/\sqrt{t}$ but as was mentioned earlier this is reasonable.

The two methods of plotting are nearly identical at large values of t, but for small t and large Δt the more correct method would even out some of the upward curvature of the flow rate plot often encountered in field tests.

(6.3) The effects of the assumption in Terzaghi's theory on solutions for the C.H.T.

These assumptions are listed for example by Taylor (1948) and can be written thus:

- (i) homogeneous soil
- (ii) saturation by incompressible fluid
- (iii) incompressible soil grains
- (iv) validity of Darcy's Law at small gradients
- (v) one dimensional drainage and consolidation
- (vi) small strains
- (vii) constant k and c.
- (viii) the void ratio 'e' a function of the effective stress σ' only.

Recently Barden (1966) made a useful summary of the recent developments in consolidation theory and testing technique. He quoted references to published work dealing with the effect on Terzaghi's theory of the invalidity of one or more of these assumptions. In his own view assumption (iv) might be the most controversial but in this discussion on C.H.T. on field piezometers assumption (iv) will be upheld, since evidence to the contrary is not conclusive (see Hansbo, 1960; Matyas, 1963; Normand, 1964).

Gibson (see Normand, 1964) derived a general differential equation for one dimensional consolidation where assumptions (vi) and (vii) were not required. He also upheld assumption (iv) regarding Darcy's Law but he clarified it by stating that it is the <u>relative</u> velocity of the water to the soil grains and not the absolute velocity of the water which is proportional to the hydraulic gradient.

In this discussion,however, the strains suffered by the soil next to a piezometer tip undergoing a seepage test will be assumed to be small, and furthermore the absolute velocity of the water will be assumed proportional to the hydraulic gradient in the soil. In addition Terzaghi's theory will be assumed valid for three-dimensional consolidation and no distinction will be made between the three-dimensional compressibility measured in the field and the usual one-dimensional value measured on laboratory samples.

(6.4) Effect of non-homogeniety of the soil

Although compacted fills in earth dams and embankments are usually fairly homogeneous, some variation in their properties between the horizontal and vertical directions must exist due to the rolling and compacting action. There are to the Author's knowledge few data on this subject although Bishop and Vaughan (1962) suggest that for the rolled fill in Selset dam the horizontal permeability was probably not very mcuh greater than that in the vertical direction. But this effect will perhaps be more pronounced for foundation strata.

The effect of unisotropy on the values of k and c obtained from C.H.T. field results was not studied in his research work, although the Author's computer programme for cylindrical tips (Section 5.6.3) may be modified to deal with the problem. Naturally the steady state programme would also have to be modified to give the new intake factors of the tip. However, it was felt that the effects of remoulding and previous testing were more important as will be shown later on in this chapter.

(6.5) Effect of non-saturation of the soil

Although the majority of C.H.T. conducted by the Author were on tips in fully saturated or nearly fully saturated foundations and compacted fills, a few swelling tests were performed on piezometers in partly saturated soils some of which recorded negative pore water pressures of -5 to -10 p.s.i., see Figs. (7.5) (7.6). These plots suggest that a straight line plot of q v.$1/\sqrt{t}$ might also be obtained for partly saturated soils, at least for swelling tests.

Wilson and Luthin (1963) give an interesting possible explanation by suggesting that since the soil enclosed by the water front advancing away from the tip is nearly fully saturated and since all the flow occurs within this front then the results obtained would be close to those on an initially fully saturated soil.

Few consolidation tests were attempted on tips registering negative pore water pressures. The release of gas bubbles from the aerated water drawn from the soil invalidated the readings.

To obtain a more quantitative picture of this problem a differential equation for volume change of partly saturated soils has to be used, such as that suggested by Barden (1965a) or Gibson (see Matyas, 1963). Such a solution might also be useful in analysing the effect of the release of gas bubbles from the pore water in consolidation C.H.T. However, if the pore pressure before the test is relatively high and Δu is small then there is perhaps little likelihood of this occurring. This problem was not pursued any further in this thesis.

(6.6) Effect of secondary consolidation

Assumption (viii) (section 6.3) states that the void ratio is a function of the effective stress only. This is probably a fair assumption for the majority of the soils tested by the Author except a few in a peaty layer in the foundation of Fiddlersferry embankment (section 8.3) where a definite upward curvature of the q v.$1/\sqrt{t}$ plots was obtained on all three tips tested. It will be shown in sections (6.7.1) and (6.7.2) that this effect is not due to variation of k and c_v with effective stress and is quite likely due to the secondary consolidation behaviour of the peaty clay. To analyse this case some model has to be assumed, for example that by Gibson and Lo (1961) or Barden (1965b), but this is outside the scope of this thesis.

(6.7) Effect of 'k' and 'Cv' varying with effective stress

In this section the soil surrounding a piezometer tip will be assumed homogeneous and isotropic before the test, but during the test k and Cv are allowed to vary with the effective stress.

The first approach to this problem will be made using Davies and Raymond (1965) modified Terzaghi's equation, while in the record Barden and Berry's (1965) more general treatment will be adopted.

(6.7.1) Derivation of C.H.T. equation based on Davies and Raymond's modified Terzaghi's theory of consolidation

Davies and Raymond utilised the well known experimental finding that results from oedometer tests on normally consolidated soils have shown that the following empirical equation is approximately valid.

$$e = e_o - I_c \log_{10}(\sigma'/\sigma_o') \qquad (6.4)$$

where e is the void ratio at an effective stress σ' and e_o is the void ratio at σ_o', the latter being the point 'O' on $e - \log_{10}\sigma'$ plot and I_c is the compression index.

For our case we may perhaps assume that eq.(6.4) is also valid for compacted saturated fills and normally consolidated or slightly over-consolidated foundations.

The coefficient of compressibility m_v is defined as:

$$m_v = -\frac{1}{1+e}\frac{de}{d\sigma'} \qquad (6.5)$$

assuming the void ratio e to be a function of the effective stress σ' only.

Hence

$$m_v = \frac{0.434\, I_c}{(1+e)}, \qquad (6.6)$$

If small strains are assumed then (1 + e) can be assumed constant, i.e.

$$m_v = A/\sigma' \qquad (6.7)$$

where A is a constant. If C_v is assumed to remain constant then the coefficient of permeability k is given by:

$$k = C_v \frac{A}{\sigma'} \tag{6.8}$$

Although C_v was assumed to remain constant here, this is quite an advance on Terzaghi's theory since we are allowing k and m_v to vary with the effective stress, albeit in a prescribed manner.

Considering an element of soil δr in thickness at a distance r ($r > r_1$) from the centre of the spherical cavity of radius r_1, the equation of continuity of the pore water is:

$$4\pi r \, \delta r(2v + r \frac{\partial v}{\partial r}) = 4\pi r^2 \, \delta r \frac{1}{1 + e} \frac{\partial e}{\partial t} \tag{6.9}$$

where v is the velocity of the pore water entering the element.

Now
$$\frac{\partial e}{\partial t} = \frac{de}{d\sigma'} \cdot \frac{\partial \sigma'}{\partial t} \tag{6.10}$$

Further, using the concept of effective stress and assuming that the total stress does not vary with the radius r nor with time t:

$$\frac{\partial \sigma'}{\partial t} = - \frac{\partial u}{\partial t} \tag{6.11}$$

where u is an excess pore pressure (as defined in section 8.4.4)

Substituting eqs. (6.7), (6.8) and (6.11) into eq. (6.9) gives:

$$-C_v \left\{ \frac{2}{\sigma'} \frac{\partial u}{\partial r} + r \left[(\frac{1}{\sigma'})^2 (\frac{\partial u}{\partial r})^2 + \frac{1}{\sigma'} \right] \frac{\partial^2 u}{\partial r^2} \right\} = \frac{r}{\sigma'} \frac{\partial \sigma'}{\partial t} \tag{6.12}$$

Using the very useful substitution

$$w = \ln \sigma' \tag{6.13}$$

eq. (6.12) is simplified to

$$C_v\left\{\frac{\partial^2 w}{\partial r^2}+\frac{2}{r}\ \frac{\partial w}{\partial t}\right\}=\frac{\partial w}{\partial t} \tag{6.14}$$

which has exactly the same structure as that derived by using Terzaghi's equation (eq. 5.12).

Choosing the same boundary conditions as Gibson (1963):

$$\begin{cases} u = u_o & \text{at } t = o \quad r > r_1 \\ w = w_o & \text{at } t = o \quad r > r_1 \end{cases} \qquad (a)$$

$$\begin{cases} u \rightarrow u_o & \text{as } r \longrightarrow \infty \\ w \rightarrow w_o & \text{as } r \longrightarrow \infty \end{cases} \qquad (b) \tag{6.15}$$

$$\begin{cases} u\ (r_1,t) = u_o + \Delta u & t > o \\ w\ (r_1,t) = \ln(\sigma_i{}' - \Delta u) & t > o \end{cases} \qquad (c)$$

where $\sigma_i{}'$ is the effective stress in the soil prior to the test and Δu is the pressure decrement applied in the test (-ve for consolidation tests), and r_1 is the radius of the tip cavity.

We now proceed to solve eq. (6.13)

$$\text{let } v = w.r \tag{6.16}$$

Eq. (6.14) then simplifies further to

$$C_v\,\frac{\partial^2 v}{\partial r^2}=\frac{\partial v}{\partial t} \tag{6.17}$$

The Laplace transformation of eq. (6.17) is

$$\frac{d^2\bar{v}}{dr^2}-q^2\bar{v}+\frac{w_o r}{c}=0 \tag{6.18}$$

where

$$\bar{v}\ (r,p) = \int_0^\infty v(r,t)\ e^{-pt}\ dt, \text{ and } q^2 = p/c \qquad (6.19)$$

The solution of eq. (6.18) is

$$\bar{w} = \frac{1}{r}\ A(p)\ e^{-qr} + \frac{1}{r}\ B(p)\ e^{qv} + \frac{w_o}{p} \qquad (6.20)$$

where A(p) and B(p) are constants to be determined from the boundary conditions. From (6.15b) B(p) must be zero and using (6.15.c) eq. (6.20) becomes:

$$\bar{w} = \frac{r_1}{r}\ \ln(1 - \frac{\Delta u}{\sigma_i'})\ \frac{1}{p}\ e^{q(r_1 - r)} + \frac{w_o}{p} \qquad (6.21)$$

Hence

$$w = \ln \sigma' = \frac{r_1}{r} \ln(1 - \frac{\Delta u}{\sigma_i'})\ \mathrm{erfc}(\frac{r - r_1}{2\sqrt{ct}}) + \ln \sigma_i' \quad (6.22)$$

The flow rate 'q' at the piezometer tip is given by

$$q(r_1,t) = -4\pi r_1^2\ (\frac{\partial u}{\partial r})_{r = r_1} \cdot k_{r = r_1}\ \frac{1}{\gamma_w} \qquad (6.23)$$

Writing $\sigma' = (\sigma - u)$, diff. eq.(6.22) w.r.t. γ, and putting $\gamma = r_1$ we have finally:

$$q(r_1,t) = 4\pi r_1 k_{(r = r_1)}\ \frac{\Delta u}{\gamma_w}\ (\ 1 - \ \frac{\sigma_i'}{\Delta u}\ )\ \ln(\ 1 - \frac{\Delta u}{\sigma_i'})(1 + \frac{1}{\sqrt{\pi T}}) \quad (6.24)$$

where again $T = C_v t/r_1^2$. Equation (6.24) is seen to have a very similar structure to Gibson's equation (5.12) in that again predicts a linear relationship between q and $1/\sqrt{t}$.

To analyse field results using equation (6.24) the permeability k and coefficient of consolidation C_v are given by:

$$k_{(r = r_1)} = \frac{q\,(r_1,\infty)}{4\pi\frac{\Delta u}{\gamma_w}\,(1 - \frac{\sigma_i'}{\Delta u})\,\ln(1 - \frac{\Delta u}{\sigma_i'})} \tag{6.25}$$

and
$$C_v = \frac{q^2(r_1,\infty).r_1^2}{\pi\, n^2} \tag{6.26}$$

where $q(r_1\infty)$ is the extrapolated steady state flow rate, n is the slope of the q v.$1/\sqrt{t}$ plot obtained from a field test and r_1 is the equivalent radius of the tip (see section 5.6.3). In analysing field results using Gibson's equation (5.12) the value of C_v is often calculated using the relationship eq. (6.26), where the results obtained would be identical to those based on eq. (6.24). However, the value of k calculated using eq. (6.25) will be different from that using Gibson's solution, by the term $(1 - \frac{\sigma_i'}{\Delta u})\ln(1 - \frac{\Delta u}{\sigma_i'})$. The magnitude of this difference depends on σ_i' and Δu. An example of this difference is shown in Fig. (8.17) for C.H.T. on piezometers in Fiddlersferry.

One experimental method of comparing Gibson's solution (eq. 5.12) to eq. (6.24) would be to conduct a few C.H.T. on preferably the same piezometer to eliminate other effects, e.g. non-homogeneity, etc. Also it would be desirable to have the same initial effective stress σ_i' before each test. This means that the tests should be conducted on a piezometer where the total stress and pore pressure are not changing with time.

If the flow rate for the first test is q_o at $\Delta u = \Delta u_o$, and q_n for subsequent tests at $\Delta u = \Delta u_n$ then according to Gibson's equation (5.12)

$$\frac{q_n(r_1,t)}{q_o(r_1,t)} = \frac{\Delta u_n}{\Delta u_o} \tag{6.27}$$

Eq. (6.24) however gives

$$\frac{q_n(r_1,t)}{q_o(r_1,t)} = \frac{k_{n(\ r\ =\ r_1)}[\Delta u_n - \sigma_i'] \ [\ln(1 - \Delta u_n/\sigma_i')]}{k_{o(r\ =\ r_1)}[\Delta u_o - \sigma_i'] \ [\ln(1 - \Delta u_o/\sigma_i')]} \tag{6.28}$$

But since it was assumed (eq. 6.8) that

$$k \cdot \sigma' = \text{a constant,}$$

eq.(6.28) reduces to

$$\frac{q_n(r_1,t)}{q_o(r_1,t)} = \frac{\ln(1 - \Delta u_n/\sigma_i')}{\ln(1 - \Delta u_o/\sigma_i')} \tag{6.29}$$

To compare eqs. (6.27) and (6.29) an initial value for $\Delta u_o/\sigma_i'$ must be assumed and the ratio $q_n(r_1,t)/q_o(r_1,t)$ is plotted against $\Delta u_n/\Delta u_o$ remembering that σ_i' was assumed to remain constant. Fig. (6.1) shows this plot for values of $\Delta u_o/\sigma_i'$ of 0.1 and 0.2

The plots show that for consolidation tests, as Δ_u increases eq. (6.24) predicts smaller flow rates than those predicted by Gibson's equation (5.12).

So far, the derivation of eq.(6.24) and the comparison with Gibson's equation have dealt with consolidation tests only. But if eq.(6.4) is assumed to apply for the rebound part of the e -log p plot, eq.(6.24) would also apply for swelling type C.H.T. in which Δu has a positive value. Fig. (6.1) shows plots of $q_n(r_1,t)/q_o(r_1,t)$ v. $\Delta u_n/\Delta u_o$ for swelling tests where again σ_i' is assumed to be constant. Here eq.(6.24) predicts that as Δ_u increases, higher flow rates will be obtained than those given by Gibson's equation (5.12). At $\Delta u_n = \sigma_i'$ eq.(6.24) predicts an infinite value of q (cf. eqs. 6.7 and 6.8).

As will be discussed in Section (6.8) a difficulty arises when attempting to verify the above arguements experimentally. This is because of the disturbance to the soil caused by repeated testing which is especially significant for consolidation tests.

Eq. (6.24) was derived in the latter part of the Author's research programme and hence there was not sufficient time to verify it in any detail. However, three sets of repeated swelling C.H.T. tests were conducted on piezometers in Peterborough embankment where σ_i' was in fact constant. The results for piezometer 14 which showed the least scatter are shown in Fig. (6.2) where it is seen that they do not seem to follow either Gibson's solution (eq. 5.12) or the Author's modified solution, eq. (6.24). This will be discussed in more detail in section (6. 8).

Fig. (6.2) also shows a set of C.H.T. repeated swelling tests on a piezometer in the Guma dam, Sierra Leone, kindly made available to the Author by Messrs. Howard Humphreys and Sons (Consulting Engineers). Here the results are more consistent but may have been affected by the rather small time interval allowed for equalisation after each test. Here again the results do not show any conclusive evidence for preference of eq.(6.24) over eq.(5.12).

(6.7.2) Solution for C.H.T. based on Barden and Berry's theory of consolidation

Barden and Berry (1965) formulated a more general non-linear theory for one-dimensional consolidation of saturated normally consolidated clays, in which k, m_v and C_v were allowed to vary with the effective stress in some prescribed manner. They assumed initial homogeneity of the saturated soil, validity of Darcy's law at small gradients, small strains, and that e is a function of σ' only. This approach will now be utilised to derive a differential equation suitable for the C.H.T. on a spherical piezometer cavity.

First, the e -log σ' will again be assumed linear and hence eq. (6.7) follows.

For the permeability k it is assumed that the ln k - e plot is linear (see Fig. 6.3) and hence

$$\ln (k) = \ln (kf) + m(e - ef) \tag{6.30}$$

where ln is the logarithm to the natural base, kf is the permeability at the minimum void ratio and hence at the maximum effective stress encountered in the consolidation process, and m is the slope of the ln(k) - e plot defined as $d[\ln(k)]/de$. But from Fig. 6.3

$$e_f = e + C_c(\ln \sigma_f' - \ln \sigma') \tag{6.31}$$

where C_c is another definition for Ic (eq. 6.7), define here as $de/d(\ln \sigma')$. Combining eqs. (6.30) and (6.31) we have finally:

$$k = k_f \left(\frac{\sigma_f'}{\sigma'}\right)^{-m \cdot C_c} \tag{6.32}$$

For the compressibility m_v, eq.(6.7) gives

$$m_v = mv_f\left(\frac{\sigma_f'}{\sigma'}\right) \tag{6.33}$$

where mv_f corresponds to e_f and σ_f'. Hence

$$C_v = Cv_f \left(\frac{\sigma_f'}{\sigma'}\right)^{-m \cdot C_c - 1} \tag{6.34}$$

Here $k_o m_v$ and C_v are all functions of the effective stress σ'. If $- m. C_c - 1$ is put equal to zero, we recover Davies and Raymond's assumption that C_v remains constant throughout the consolidation process. However, by setting $(- m C_c - 1)$ not equal to zero, we may vary C_v with the effective stress.

Raymond (1966) derives equations similar to eqs. (6.32) and (6.33) but he extrapolates the plots in Fig. (6.3) to the point e = o, where his k_f and mv_f correspond to e = o.

It must also be emphasised here that eqs. (6.32, 6.33 and 6.34) may only be assumed valid over the range of lnk - e and e - ln σ' plots where they can be approximated by straight lines.

Barden and Berry (1965) write eq. (6.32) in the form

$$k = k_f \left(\frac{\sigma'_f}{\sigma'}\right)^a \qquad (6.35)$$

where a corresponds to $- m.C_c$. They then approximate this equation by the following relationship:

$$k = k_f(1 + b.u^n) \qquad (6.36)$$

where b and n are constants, and u is the excess pore pressure.

Proceeding with the derivation of the differential equation governing the consolidation of the soil surrounding a piezometer tip we again have equation (6.9)

$$\frac{\partial v}{\partial r} + \frac{2}{r} v = \frac{\partial}{\partial t} \left(\frac{e}{1 + e}\right) \qquad (6.37)$$

Rewriting eq. (6.36) in a form suitable for our problem we have

$$k(r,t) = k(r_1,t) \left[1 + b(u')^n\right] \qquad (6.38)$$

where $\quad u' = u - (u_o - \Delta u) \qquad (6.39)$

and u is an excess pore pressure (the gravity potential being neglected), u_o the initial uniform pore pressure before the test and Δu is the pressure decrement applied in the test. The term u' varies from zero at $r = r_1$ to Δu as $r \rightarrow \infty$. $k(r_1,t)$ is the permeability of the soil immediately next to the tip, where it remains constant for $t > o$.

In eq. (6.39) Δu must always be positive to avoid obtaining complex numbers. If swelling tests are to be considered, Δu is kept positive, but the sign before the term b is made negative. However, in the present discussion only consolidation C.H.T. will be analysed.

Assuming small strains, (1 + e) may be assumed to remain constant throughout the test, and hence the R.H.S. of eq. (6.37) becomes:

$$\frac{\partial}{\partial t}\left(\frac{e}{1+e}\right) = \frac{1}{1+e}\frac{\partial e}{\partial t} = \frac{1}{1+e}\frac{de}{d\sigma'}\frac{\partial \sigma'}{\partial t},$$

since e is assumed to be a function of the effective stress only.

Using eq. (6.31) and the assumption that the total stress remains constant, we may write the L.H.S. of eq. (6.37) as:

$$\frac{\partial}{\partial t}\left(\frac{e}{1+e}\right) = \frac{C_c}{1+e_o}\frac{1}{\sigma'}\frac{\partial u}{\partial t}$$

For the L.H.S. of eq. (6.37) we use Darcy's Law:

$$v = \frac{k}{\gamma_w}\frac{\partial u}{\partial r} \qquad (6.40)$$

Hence eq. (6.37) can be rewritten:

$$\frac{\partial}{\partial r}\left\{\frac{k_f}{\gamma_w}(1+bu'^n)\frac{\partial u}{\partial r}\right\} + \frac{2}{r}\left\{\frac{k_f}{\gamma_w}(1+bu'^n)\frac{\partial u}{\partial r}\right\} = \frac{C_c}{1+e_o}\frac{1}{\sigma'}\frac{\partial u}{\partial t} \qquad (6.41)$$

Let $v = u'/\Delta u$, $x = \gamma/\gamma_1$ and $\beta = b\Delta u^n$. (6.42) where v now varies between zero at $r = r_1$ and one as $r \to \infty$, and β is a measure of the change in permeability during the test. Eq. (6.41) can then be rewritten as:

$$(1 + \beta v^n) \frac{\partial^2 v}{\partial x^2} + \left(\frac{\partial v}{\partial x}\right)^2 (\beta . n . v^{n-1}) + \frac{2}{x} (1 + \beta v^n) \frac{\partial v}{\partial x} =$$
$$\frac{\gamma_w}{k_f} r_1^2 \frac{C_c}{1 + e_o} \frac{1}{\sigma'(r_1, t)} \frac{\partial v}{\partial t} \qquad (6.43)$$

Now $\sigma'(r_1, t) = \sigma'(\infty, t) + \Delta u$ (6.44)

where $\sigma'(r_1, t)$ is constant for $t > o$, and $\sigma'(\infty, t)$ is again not affected by the test.

Let $\theta = \sigma'(r_1, t)/\Delta u$ (6.45)

Hence $$\frac{1}{\sigma'(r, t)} = \frac{1}{\Delta u(\theta - v)} \qquad (6.46)$$

∴ we obtain finally

$$(1 + \beta v^n) \frac{\partial^2 v}{\partial x^2} + n\beta v^{n-1} \left(\frac{\partial v}{\partial x}\right)^2 + \frac{2}{x} (1 + \beta v^n) \frac{\partial v}{\partial x} = \frac{1}{\theta - v} \frac{\partial v}{\partial T} \qquad (6.47)$$

where $$T = \frac{k_f}{\gamma_w} \frac{1 + e_o}{C_c} \Delta u \frac{t}{r_1^2} \qquad (6.48)$$

T here is non-dimensional and is analogous to the usual term used in Terzaghi's theory.

Eq. (6.47) is non-linear and cannot be solved at present analytically. However, Barden and Berry have solved this equation numerically using an explicit method and small values of the stability $\delta T/\delta x^2$ (see section 10.4.1).

Using an explicit method eq. (6.47) was programmed for the computer in the same manner adopted for Terzaghi's equation. From the distribution of v outside the cavity the flow rate out of the tip was calculated using the method of 'total swelling rate', see sections (5.6).

Two low values of θ were chosen, viz 2.5 and 1.5 as these represented the more interesting cases where the effective stresses changed most during the tests (see eq. 6.45). For β the values chosen were 0.25, 1, 3, 7, and 15. For n the value of 0.5 recommended by Barden and Berry was used, but in one case n was put equal to 1.0. The computed results are shown in Figs. 6.4 and 6.5.

The initial downward curvature of the plots is similar to that obtained when solving Terzaghi's equation, and is again improved by decreasing the mesh size (see section 5.6.1 for more details). In these calculations no correction was made for the $\frac{1}{2}$ mesh size next to the tip.

The results show that for the ranges of θ, β and n considered that a linear relationship between q and $1/\sqrt{t}$ obtains. This is not altogether surprising since eq. (6.24) based on Davies and Raymond non-linear consolidation theory also predicts this linear relationship. Moreover it is a well known experimental fact that for a good many clays tested in the oedometer a plot of the settlement against $\sqrt{t}$ produces a straight line for the initial part of the plot in spite of clear evidence of variability of k and C_v with σ'. Remembering that the flow rate q is proportional to the differential w.r.t. t of the settlement, it is to be expected that a plot of q v. $1/\sqrt{t}$ should be also linear. In fact it was already shown that for one-dimensional C.H.T. (see section 5.5), which is very similar at small t to the standard Terzaghi one-dimensional consolidation problem, a linear plot between q and $1/\sqrt{T}$ is obtained down to the steady state flow rate.

The numerical results obtained using Barden and Berry's consolidation equation are not very suitable for analysing field seepage tests. This is because the values of n, β and k_f have to be known before hand.

(6.8) Numerical solution for the effect of disturbance of the soil next to a piezometer tip on C.H.T. results.

In a C.H.T. the values of 'k' and 'C' obtained are, as will be shown, very much influenced by the soil enclosed within a very short distance from the piezometer tip, and hence the results obtained may very well be representative of the soil mass as a whole.

The soil next to the piezometer could be disturbed in several ways. The first and most obvious is during installation of the tip, but perhaps more important is the effect of de-airing and C.H.T. and F.H.T. testing.

Fig. (6.6) shows diagramatically the effective stress paths taken by c an element of soil immediately next to the tip, and that of D far away from the tip where it is hardly affected by seepage tests on the tip. Here it is assumed that the soil tested is normally consolidated or lightly overconsolidated.

Now even before the tip is ever de-aired or is subjected to a C.H.T. some difference in the void ratio may exist between the points c and D, for two reasons; firstly, because of the process of remoulding when the piezometer tip is installed in the soil, and secondly, because of the volume change of the soil necessary to pressurise the piezometer system. But to simplify the argument we shall assume here that the elements c and D lie on the same positions on the $e - \sigma'$ plot, before the tip is even de-aired or subjected to a C.H.T., (c_1 and D_1, Fig. 6.6). We shall then consider the effects of repeated consolidation C.H.T. on the values of k and C_v obtained from the tests.

During the first test at Δu_1 c moves from c_1 to c_2 while D of course stays at D_1. Points between c and D move part way down $c_1 \rightarrow c_2$ depending on their positions relative to the tip.

After the test c swells to c_3 as shown.

If a repeat test is conducted with $\Delta u_2 < \Delta u_1$, the compressibility 'm_v' of the soil next to the tip may be much less than that of the soil far away from it. The permeability k may be lower too but perhaps by a much smaller factor, leading to a measured value of C_v much bigger than that measured in the first test.

If $\Delta u_2 > \Delta u_1$ the point c will travel from c_3 to c_2 and move down the virgin consolidation curve, with the result that the measured value of C_v will be a function of the two compressibilities. This may also perhaps give rise to non-linearity of the q v.$q/\sqrt{t}$ plots.

To avoid obtaining non-representative values of C_v two things must be done. Firstly, during the first and any other subsequent seepage tests, Δu must be as small as practically possible. Secondly, the tip must not be tested again until the soil en mass has consolidated from a point D_1 to D_2 so that the disturbed soil next to the piezometer would have rejoined the virgin consolidation curve.

Unfortunately the significance of these two rules was not appreciated by the Author until recently, with the consequence that some of the values of C_v measured from repeated tests on the same piezometers were clearly unrepresentative of soil tested as a mass. This will be discussed further in Chapter 8.

An interesting experimental evidence in support of this argument was given in Section (5.5) for one dimensional constant-head test.

To show the great influence of the properties of the soil immediately next to the tip on the values of C_v and k measured in field tests, the model shown in Fig. (6.6 B) was studied using, for convenience, Terzaghi's equation for consolidation.

Let the boundary conditions be:

$$C_{v1}\left(\frac{\partial^2 u_1}{\partial r^2} + \frac{2}{r}\frac{\partial u_1}{\partial r}\right) = \frac{\partial u_1}{\partial t}\ , \quad r_1 < r < r_2 \tag{6.49}$$

$$C_{v2}\left(\frac{\partial^2 u_2}{\partial r^2} + \frac{2}{r}\frac{\partial u_2}{\partial r}\right) = \frac{\partial u_2}{\partial t}\ , \quad r > r_2 \tag{6.50}$$

$$\text{at } r = r_1\ ,\ q(r_1,t) = -4\pi r_1^{\,2}\left(\frac{\partial u_1}{\partial r}\right)_{r = r_1}\frac{k_1}{\gamma_w} \tag{6.51}$$

$$\text{at } r = r_2\ ,\ u_1 = u_2 \quad \text{and } k_1\frac{\partial u_1}{\partial r} = k_2\frac{\partial u_2}{\partial r} \tag{6.52}$$

The above problem can perhaps be solved analytically using for example Laplace transformation, but such a solution would be tedious to obtain algebraically. Instead a numerical solution using an explicit method was used.

At the point $r = r_2$ the boundary conditions (6.52) have to be satisfied. The method described by Abbott (1960) was used. Figs. 6.7 to 6.11 show the computed results for some combinations of r_2/r_1, C_{v_2}/C_{v_1} and k_2/k_1. The results are plotted in dimensionless form. The terms 'intact' and 'disturbed' refer to whether the values of k and C_v for the disturbed or intact soil have been used to compute the time factor T and the dimensionless $q(r_1,t)/4\, r_1 \frac{k}{\gamma_w} \Delta u$. The intercept on the flow rate flow rate/axes corresponding to $1/\sqrt{T} = o$ were calculated using eq. 5.10. The results show quite remarkably the great dependence of the flow rate from the piezometer tip on the properties of the soil immediately next to the tip. Marked upward or downward initial curvatures of the plots are caused by differences in k and C_v in the disturbed and intact soil outside the piezometers when the thickness of the 'disturbed' soil is only 1/10 or 1/5 of the radius of the tip.

Finally it must be mentioned that the analysis given above is only meant to serve as an indication of the effect of soil disturbance on the results of C.H.T. Owing to the limitations of the explicit method (see section 10.4.1) the plots could only be computed for a maximum value of T of about $\frac{1}{4}$. For an Imperial College tip in a soil having a typical value of C_v of 50 ft.^{2}/yr. the actual time t corresponding to $T = \frac{1}{4}$ is 30 mins. only. Also, it would be more representative to assume a gradual change in k and C_v away from the tip instead of the step-function assumed in the present analysis. However, this would have made the numerical procedure much more tedious.

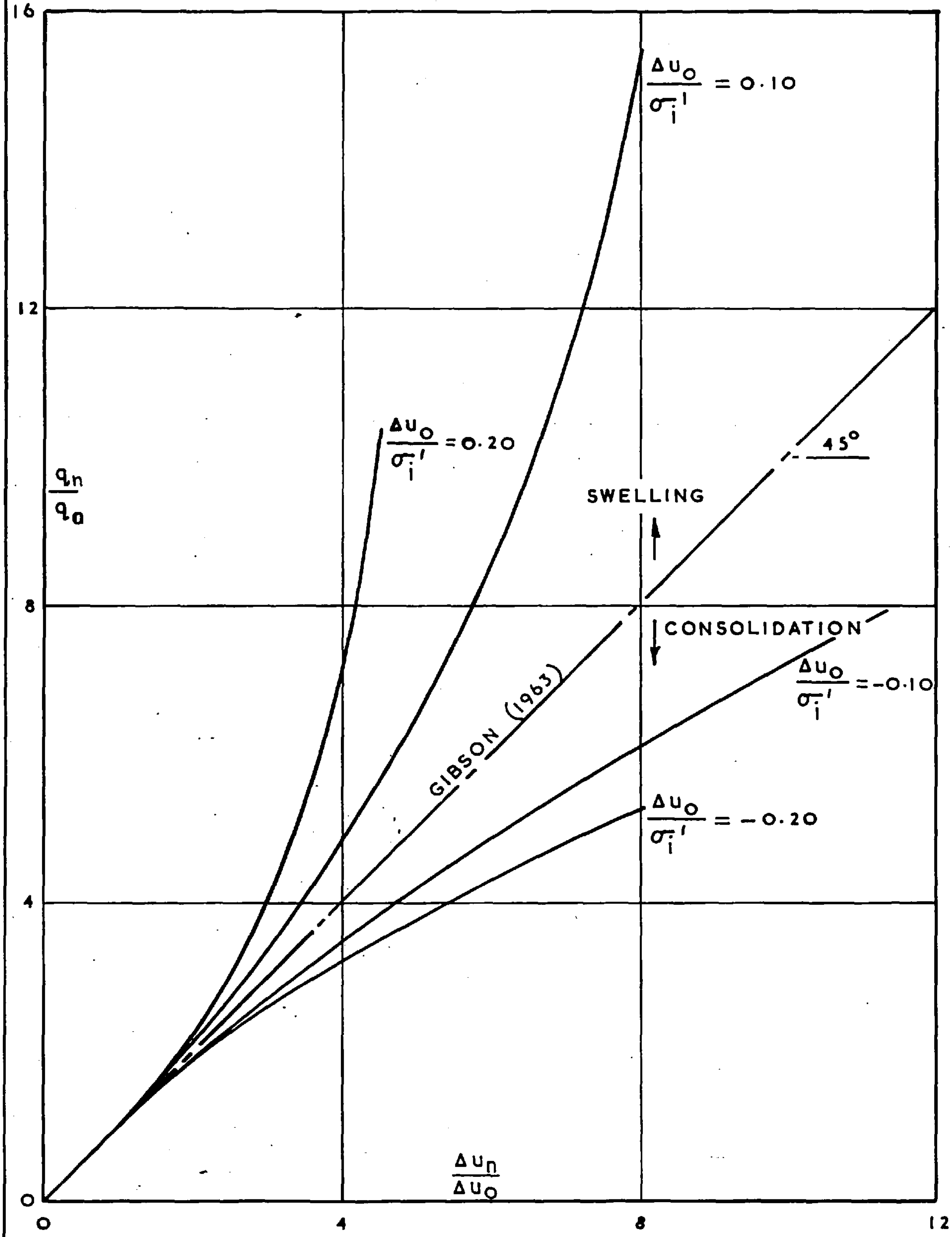
(6.9) Discussion and summary

In this chapter a discussion was given of some of the factors affecting results from constant-head seepage tests on field piezometers. Here two questions had to be answered. The first is why do some field plots of q v. $1/\sqrt{t}$ tend to curve either upwards or downwards, especially at small values of time, inspite of the fact that it was shown in Chapter 5 that field piezometer tips should behave like Gibson's (1963) spherical tip. The second and perhaps more important question is why do the calculated values of C_v sometimes tend to be too high as compared to, for example, values deduced from field pore pressure dissipation. The two questions may obviously be interrelated.

Several possible factors were discussed. Plastic creep and head-loss in the connecting tubing, head-loss in the tip ceramic and sand plug, temperature changes, etc. may cause initial curvatures of the q v. $1/\sqrt{t}$ plots.

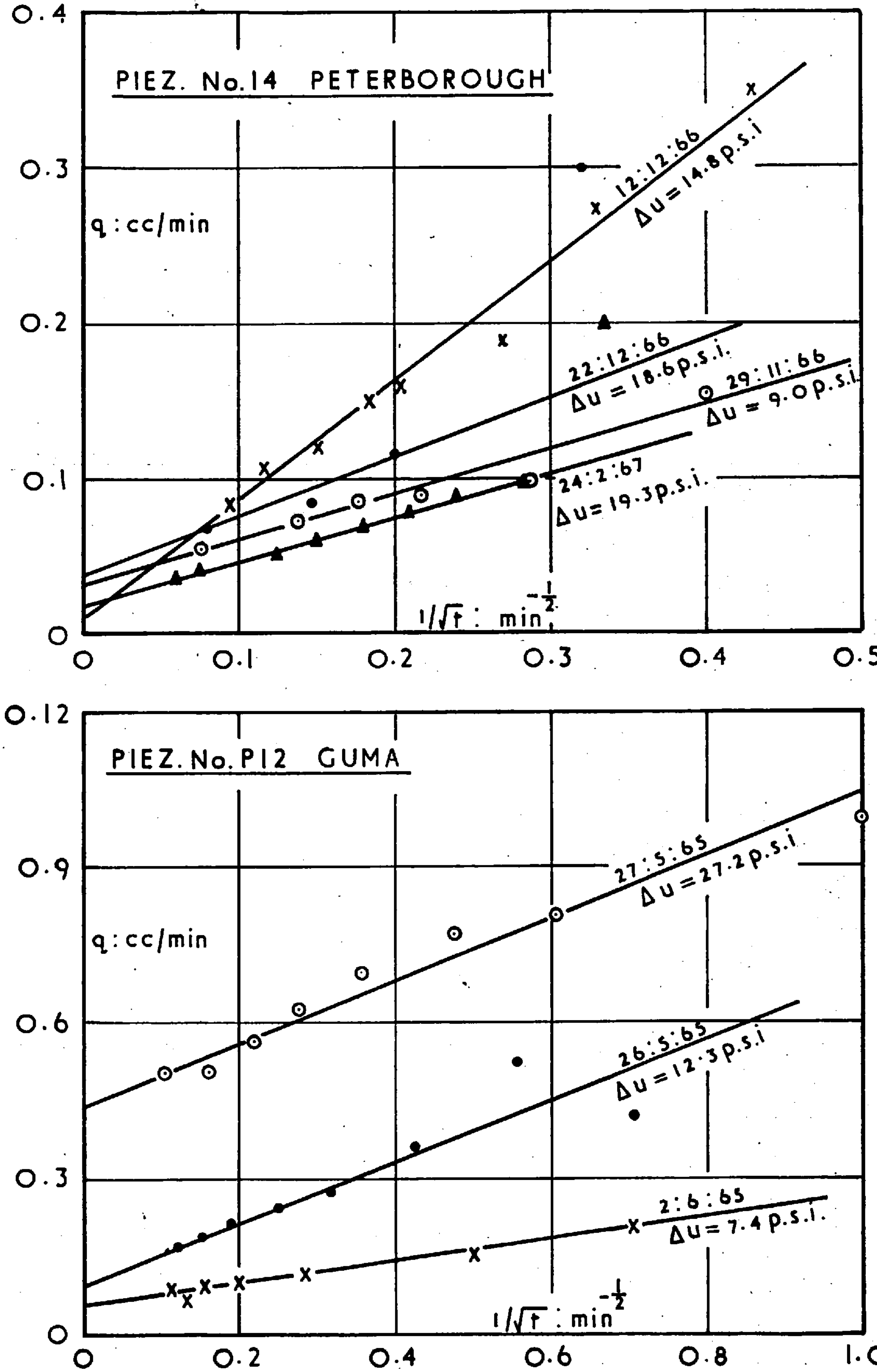
Next Davies and Raymond's (1965) and Barden and Berry's (1965) non-linear consolidation approaches were adopted for the C.H.T., but the solutions obtained still predict a linear relationship between q and $1/\sqrt{t}$. However, the values of k and C_v calculated from field results are obviously affected by the approach adopted, but this still did not explain the very high values of C_v sometimes obtained from field seepage tests using Gibson's solution.

Finally a study was made of the effect of disturbance to the soil immediately next to the piezometer tip and its effects on the calculated k and C_v from C.H.T. tests. This disturbance may be caused during installation of the tip, de-airing, seepage testing, etc. This was found to provide likely answers for some non-linearities in the q v . $1/\sqrt{t}$.



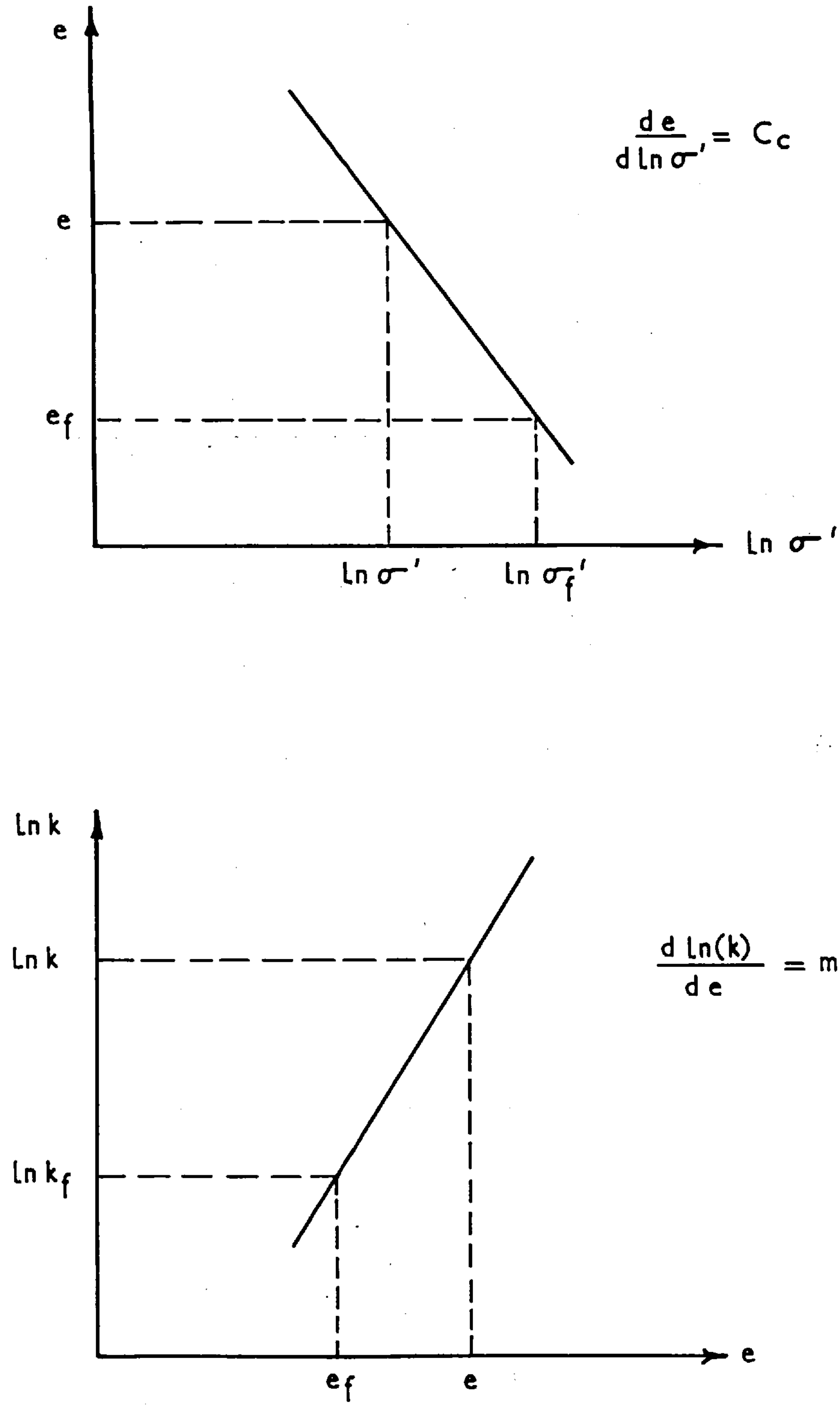
COMPARISON BETWEEN GIBSON'S (1963) SOLUTION FOR C.H.T. SPHERICAL PIEZ. CAVITY AND EQ (6:24) BASED ON DAVIES & RAYMOND (1965) THEORY

FIG. 6:1



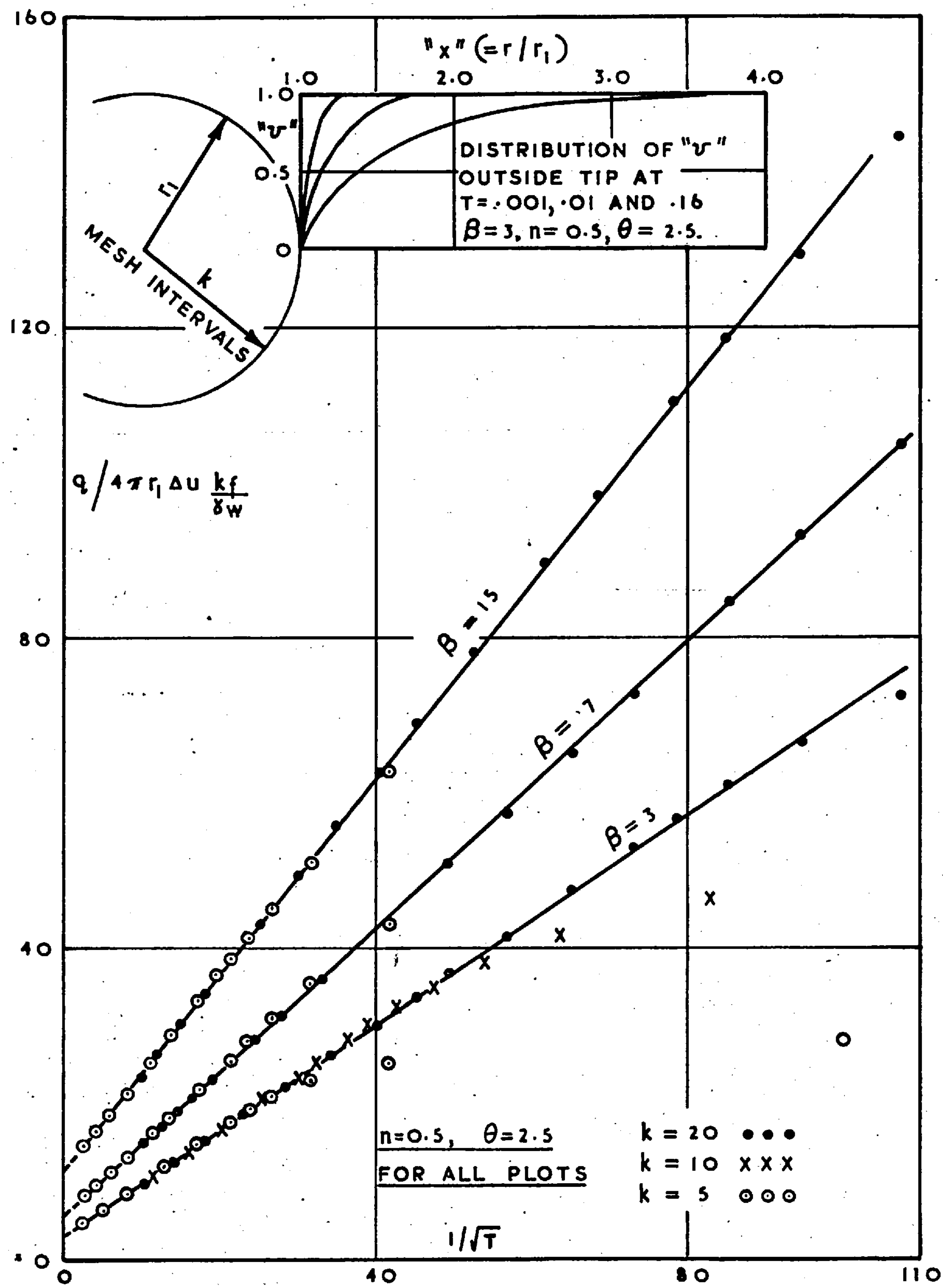
REPEATED SWELLING C.H.T.

FIG. 6:2



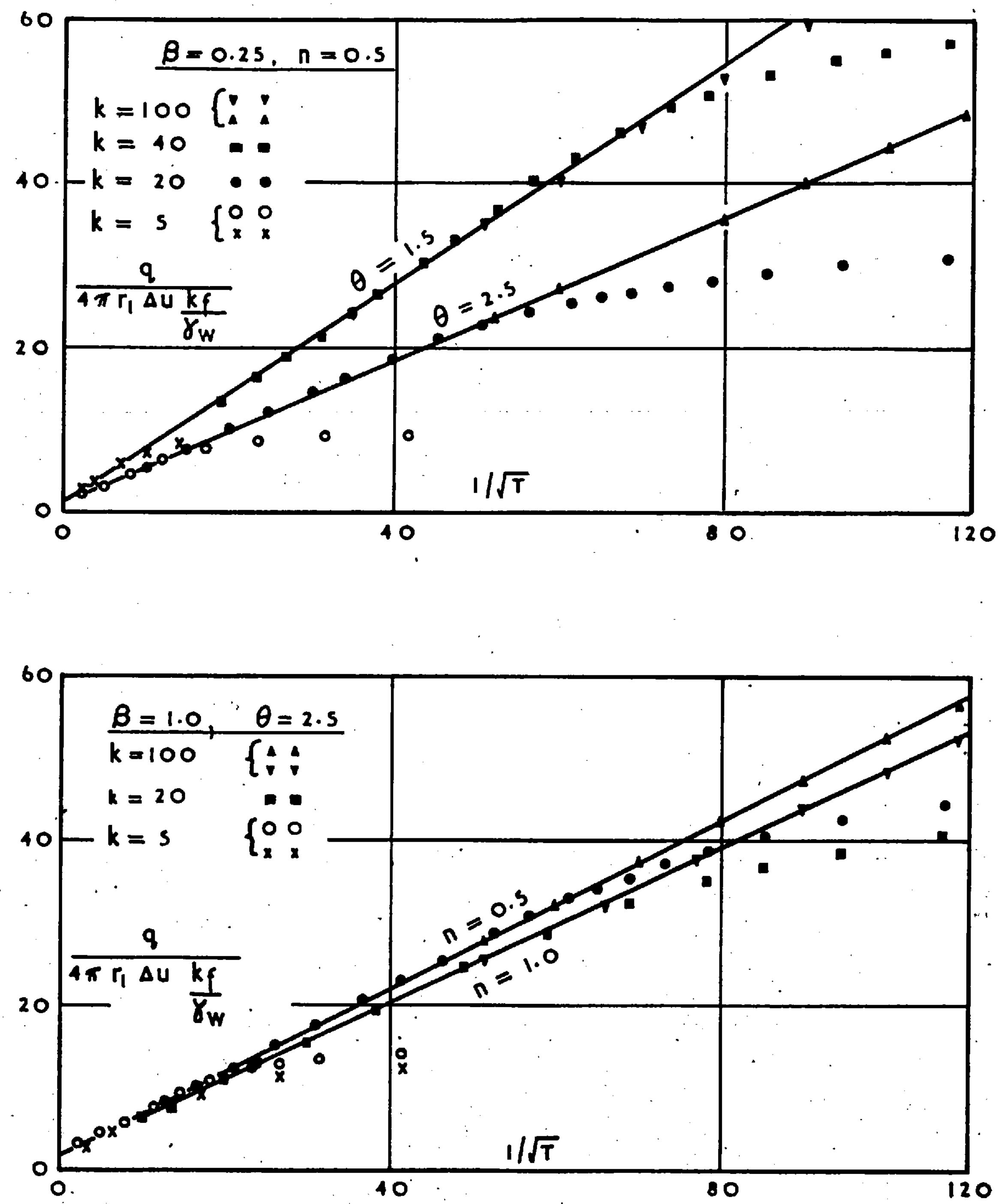
SCHEMATIC REPRESENTATION OF THE IDEALIZED RELATIONSHIPS BETWEEN e σ' and k.

FIG. 6:3



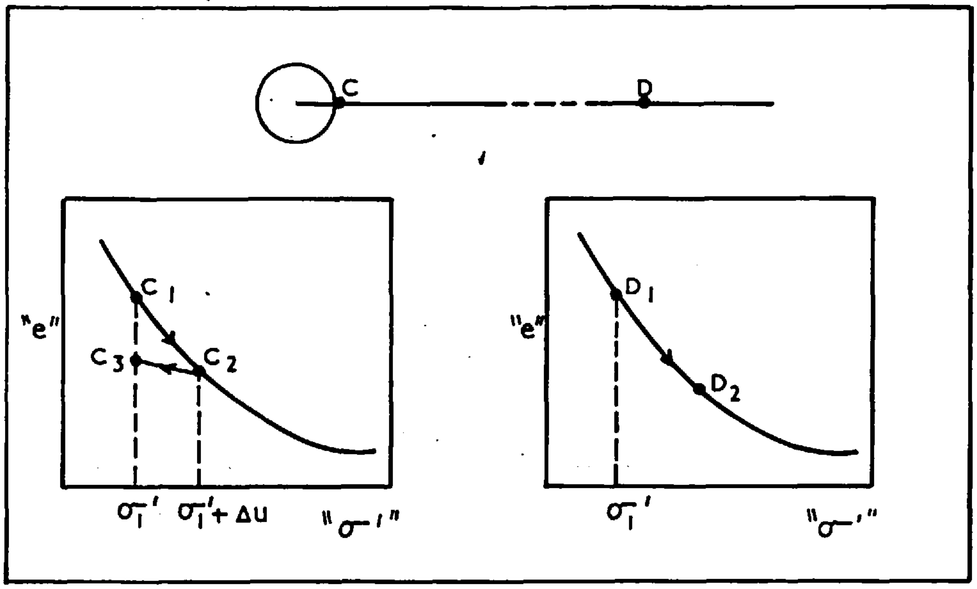
NUMERICAL SOLUTION FOR C.H.T. CONSOLIDATION ON SPHERICAL PIEZ. TIP USING BARDEN AND BERRY'S (1965) NON-LINEAR DIFFERENTIAL EQUATION.

FIG. 6:4



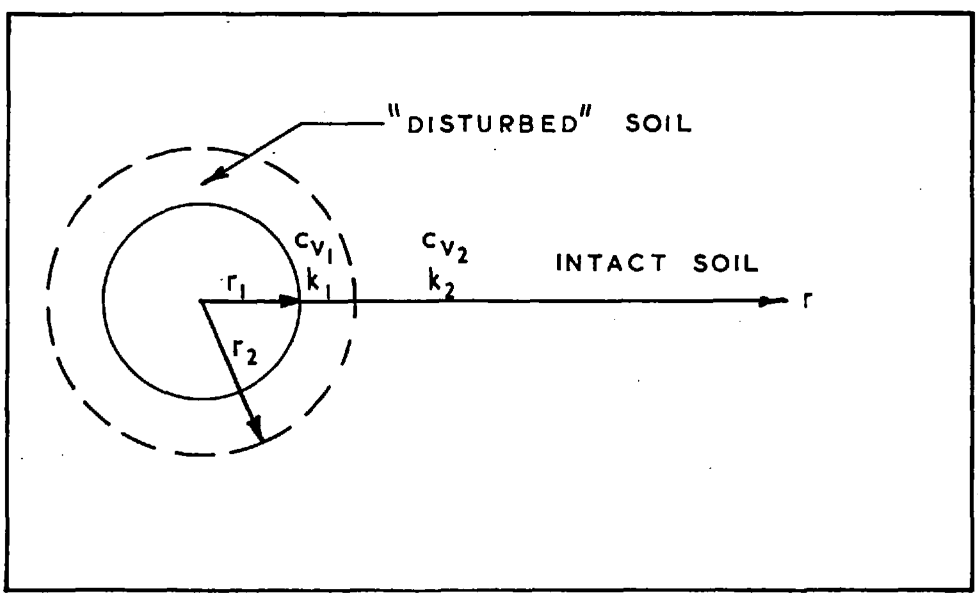
NUMERICAL SOLUTION FOR C.H.T. CONSOLIDATION ON SPHERICAL PIEZ. TIP USING BARDEN & BERRY'S 1965 NON-LINEAR DIFFERENTIAL EQUATION.

FIG. 6:5



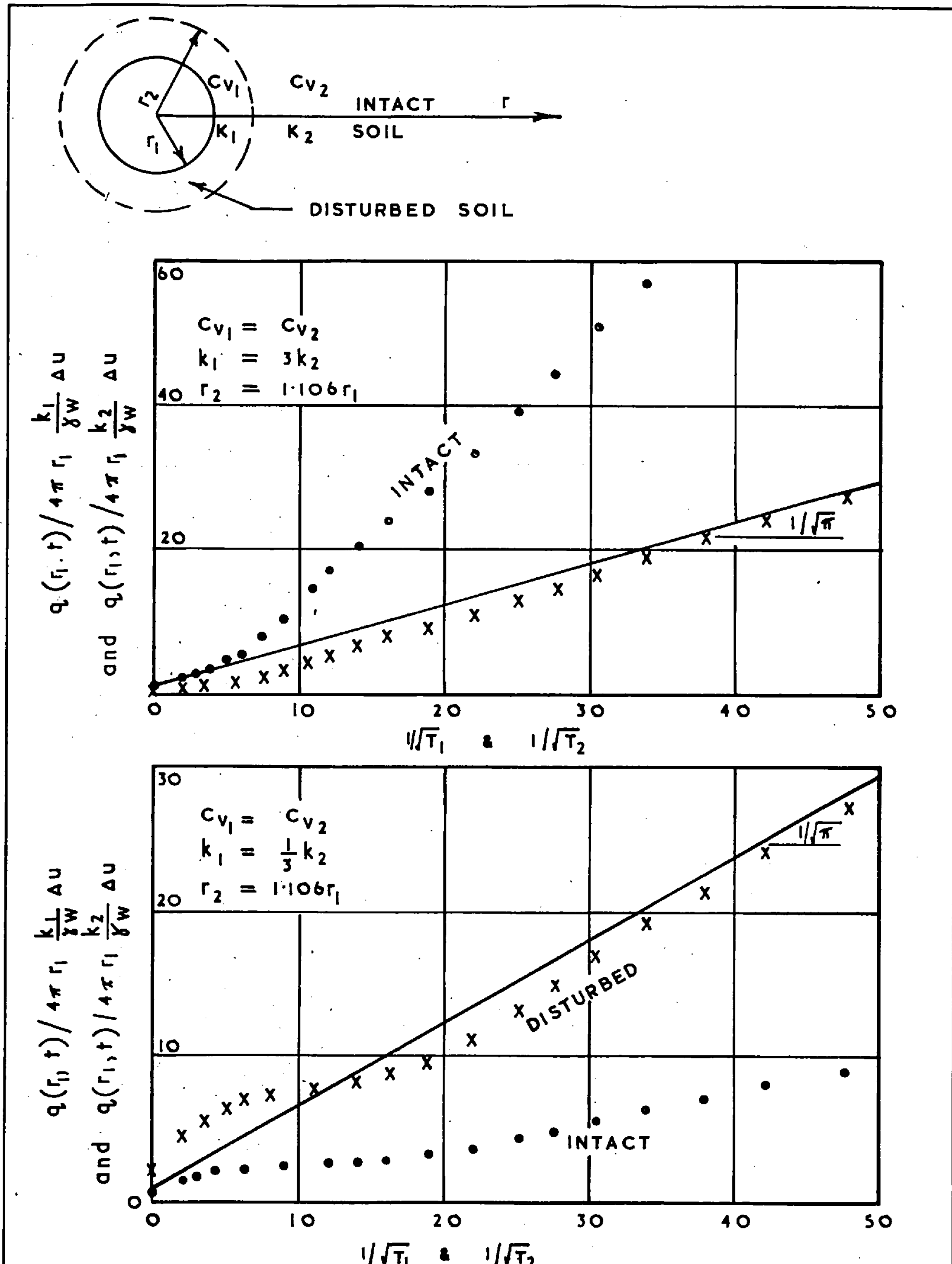
(A)

SCHEMATIC REPRESENTATION OF EFFECT OF EFFECTIVE STRESS PATH ON SOIL SURROUNDING A PIEZ. TIP.



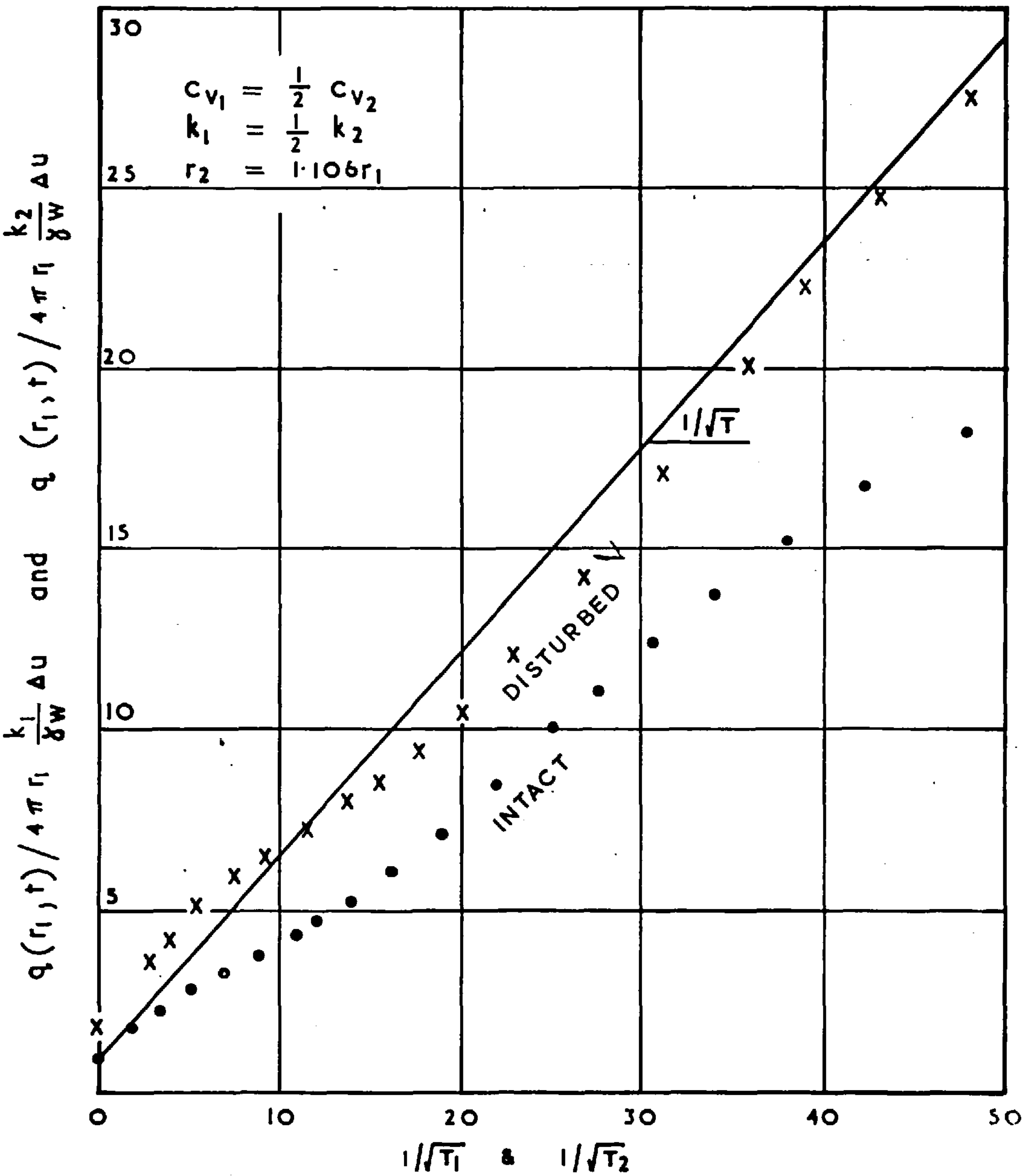
(B)

FIG. 6 : 6



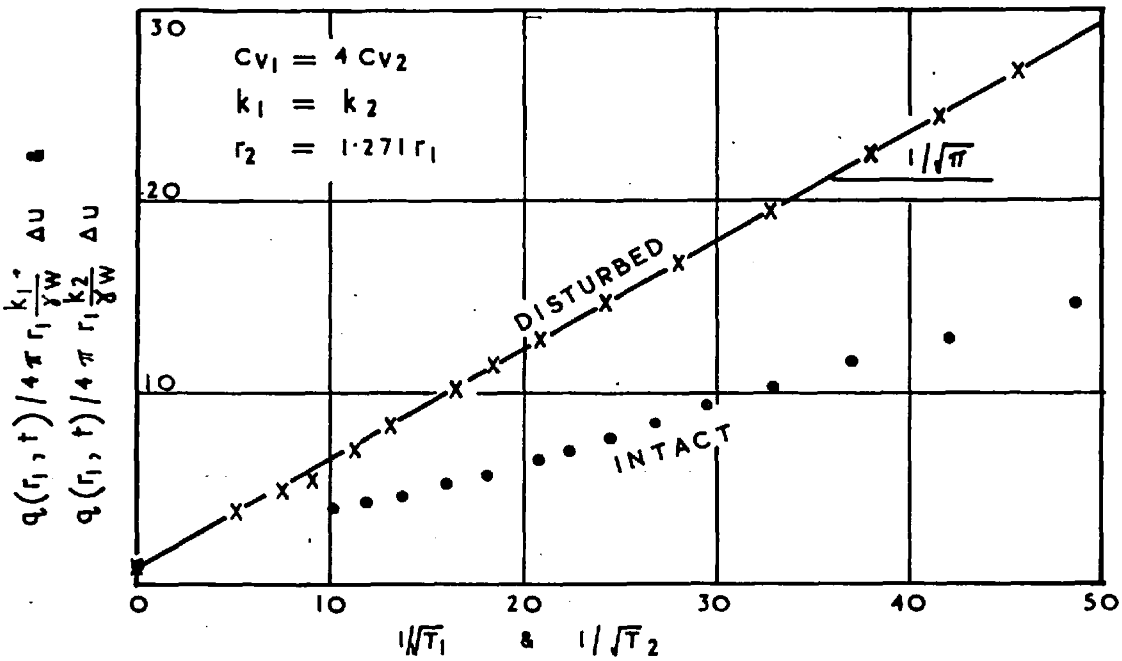
NUMERICAL SOLUTION FOR EFFECT OF DISTURBANCE OF SOIL NEAR SPHERICAL PIEZ. TIP ON C.H.T. RESULTS

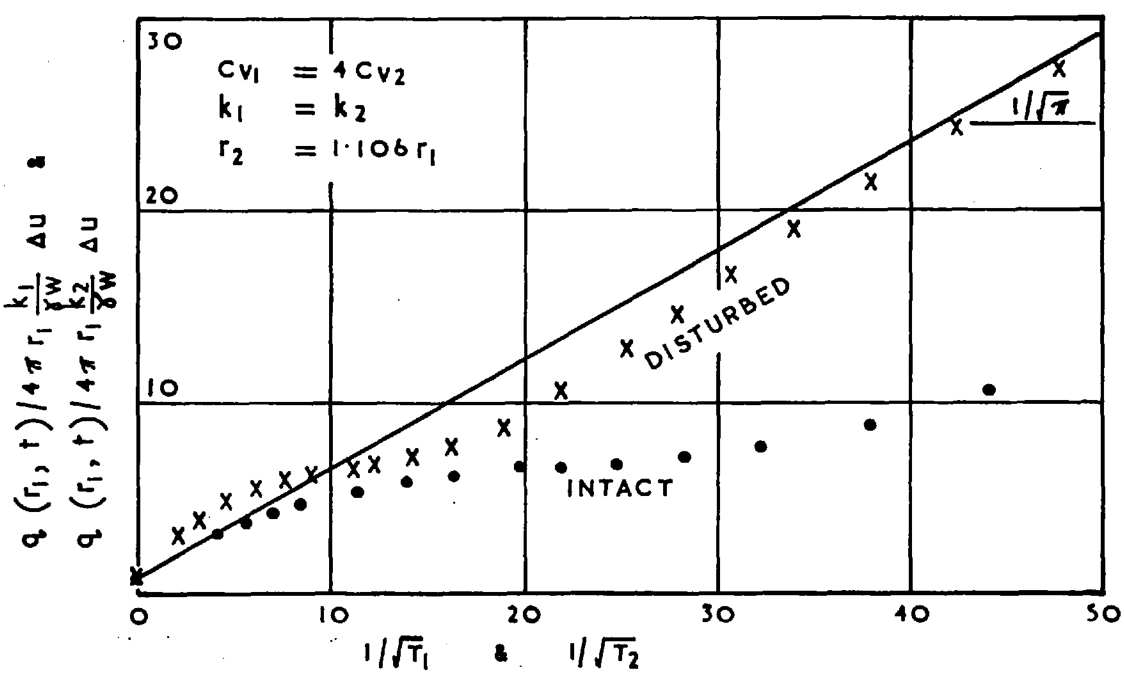
FIG: 6 : 7



NUMERICAL SOLUTION FOR EFFECT OF DISTURBANCE
OF SOIL NEAR SPHERICAL PIEZ. TIP ON C.H.T. RESULTS

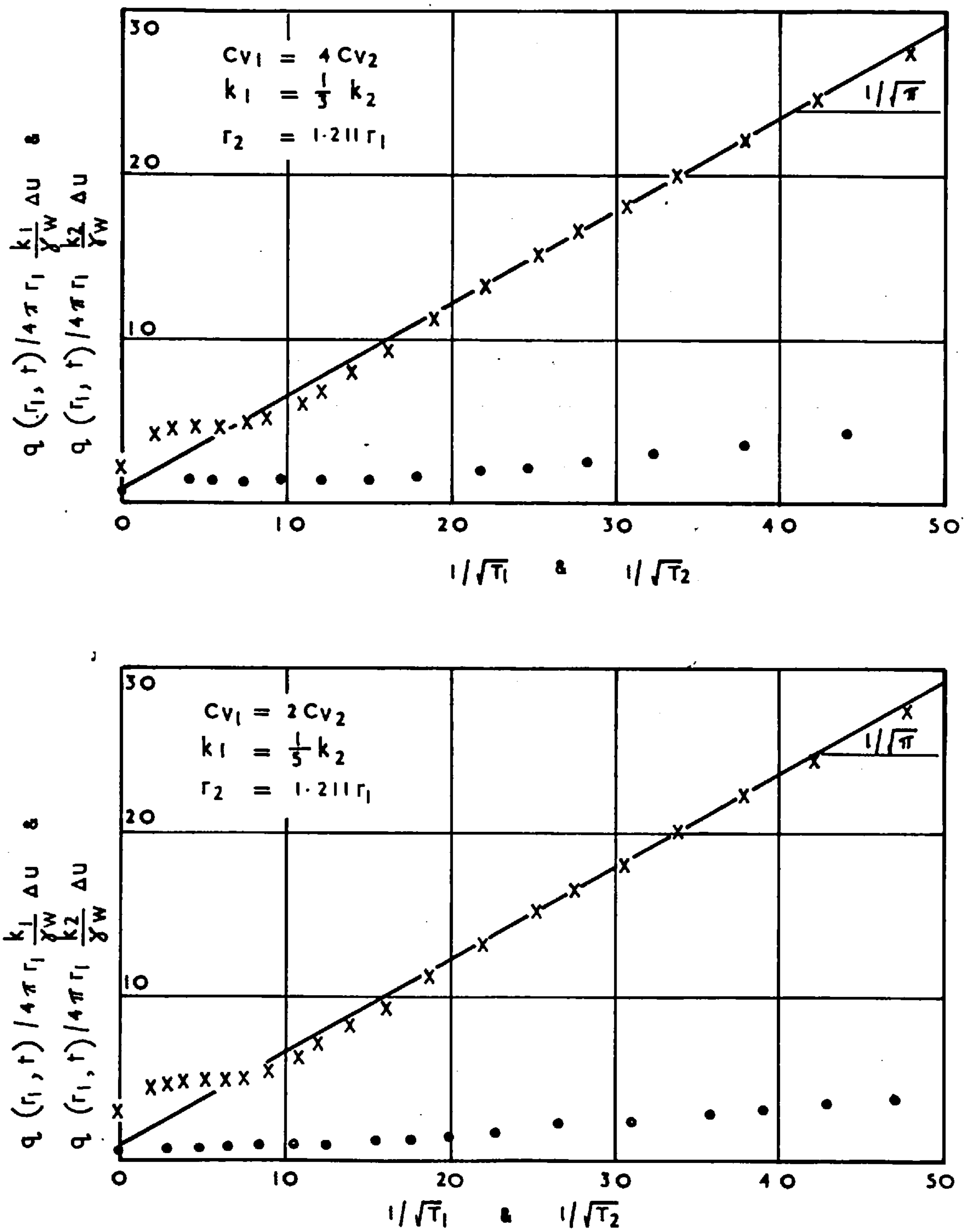
FIG. 6 : 8





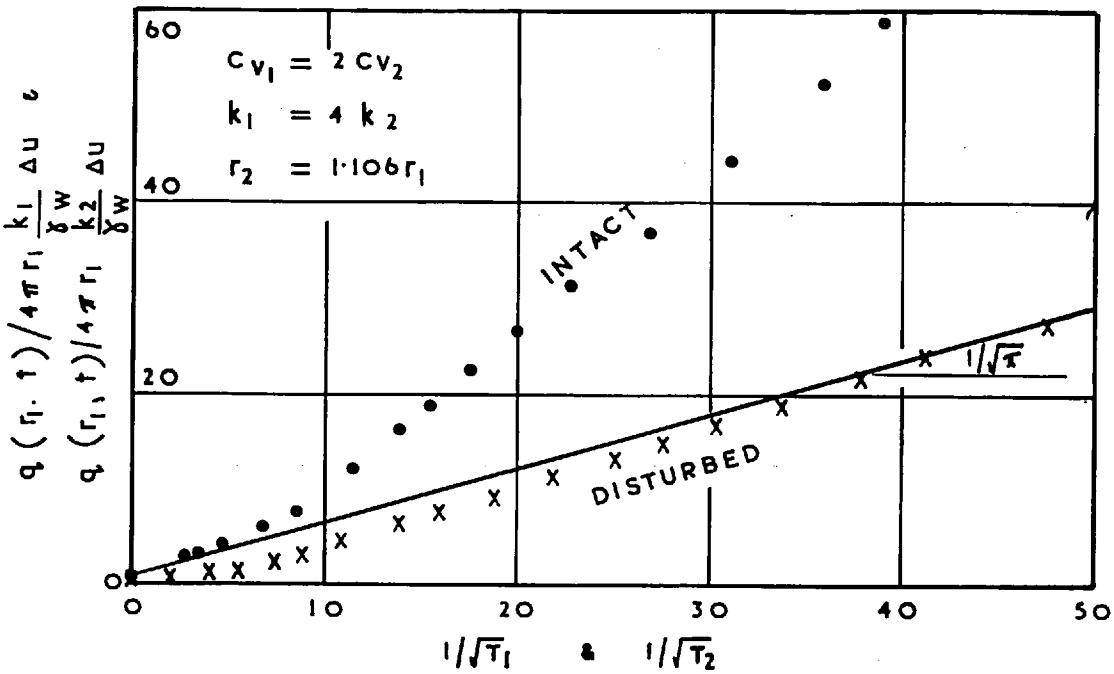
NUMERICAL SOLUTION FOR EFFECT OF DISTURBANCE OF SOIL NEAR SPHERICAL PIEZ. TIP ON C.H.T. RESULTS

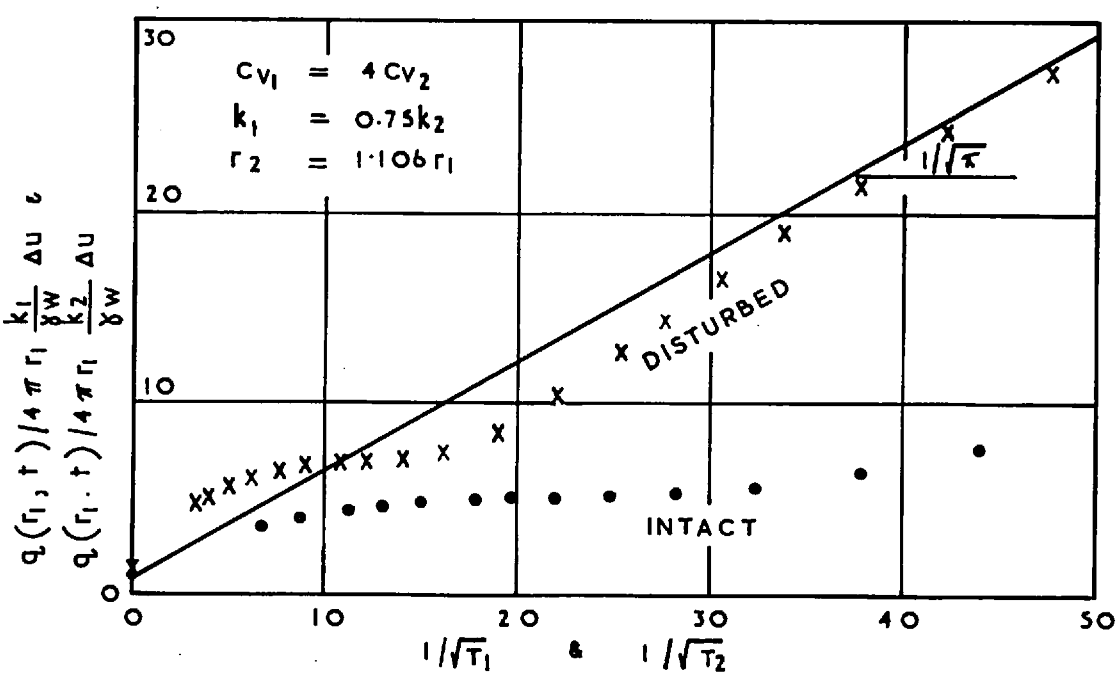
FIG. 6:9



NUMERICAL SOLUTION FOR EFFECT OF DISTURBANCE OF SOIL NEAR SPHERICAL PIEZ. TIP ON C.H.T. RESULTS

FIG. 6:10





NUMERICAL SOLUTION FOR EFFECT OF DISTURBANCE OF SOIL NEAR SPHERICAL PIEZ. TIP ON C.H.T. RESULTS.

FIG. 6 : 11

PART 3

CORRELATION BETWEEN FIELD AND LABORATORY MEASUREMENTS

CHAPTER 7

INITIAL SUCTIONS AND NEGATIVE PORE WATER PRESSURES*[1] IN RELATIVELY DRY COMPACTED FILLS

(7.1) Introduction

It was mentioned in Chapter 2 that the interest taken by Soil physicists and road construction engineers in the measurement of negative pore water pressures was much earlier than that of soil mechanics engineers, where the somewhat recent replacement of low air entry B.R.S. tips by the Imperial College (I.C.) high air entry tips made such measurements possible.

In this Chapter field and laboratory measurements of negative pore water pressures will be given for a rolled boulder clay used as the clay core in Balderhead dam*,[2] and rolled Oxford clay and boulder clay used as fill for the Peterborough

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*[1]Here the nomenclature suggested by Croney and Coleman (1960) is used where suction is defined as 'the pressure deficiency (below atmospheric pressure) measured in a small sample of the soil free from external stress. The term negative pore water pressure is reserved for any pressure deficiency (below atmospheric pressure) measured in-situ or in the laboratory with the soil subject to the stress regime associated with the particular loading conditions under consideration'.

*[2]For some details of Balderhead dam see Vaughan (1965). For details of Peterborough embankment and Diddington dam see Sections (7.4) and (7.5).

embankment and Diddington dam.

These records together with other published field and laboratory data (for field records see, for example, Hilf, 1956; Bishop, Kennard and Penman, 1960; Vaughan, 1965, and for laboratory data, Matyas, 1963; Lamb, 1961; Olson and Longfelder, 1965) indicate beyond doubt that most engineering soils compacted within, say, $\pm$ 2% of their optimum moisture content would exhibit suction, the magnitude of which as well as the subsequent value of $\bar{\bar{B}}$ depends on many factors as will be discussed in Section (7.2).

The significance of the correct measurement of negative pore water pressures has been dealt with in Section (2.2) regarding the error in the calculation of the effective stress if the pore air pressure (which was very probably the pressure recorded by the low air entry tips) alone is used in Terzaghi's effective stress equation. This error can cause a considerable drop in the calculated effective stress, a field example of which will be given in Section (7.5.1).. It was also mentioned in Section (2.2) that in most practical cases the use of the pore water pressure alone, although on the unsafe side, gives a much better estimate of the true effective stress (see Bishop, Kennard and Vaughan, 1964.).

This recent awareness of the presence of negative pore water pressures in relatively dry compacted fills has had an interesting and beneficial effect on earth dam design where at least on a short time basis use is made of the much increased effective stresses whereby steeper slopes and faster rates of construction can be accepted (see Little, 1965 and Blight, 1963).

(7.2) Factors controlling initial suctions in compacted fills

Lamb (1961) gave a very useful discussion of the factors affecting the magnitude of the initial suctions in compacted soils in terms of particle orientation and electrical forces between the particles and the pore water. A more dispersed structure (i.e. better parallel arrangement of the particles) prior to compaction would give a lower suction* after compaction, since the rebound of the soil is less than that when it has a more random orientation.

He gives the factors leading to a more dispersed structure as increase in moisture content, increase in compactive effort, kneading instead of static compaction, and pre-wetting the sample and then drying it before compaction. He also discusses the effect of temperature on suction and concludes that (i) a colder sample before compaction would have a more dispersed

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* 'Lower suction' means less negative, e.g. - 5 p.s.i. is lower than - 10 p.s.i.

structure, and (ii) cooling a compacted free standing sample would increase the suction as a result of altering the electrical attraction between the soil particles and the pore water.

Olson and Langfelder (1965) in an important contribution to this subject give laboratory records of the effects of moisture content, and kneading or static compaction on suctions measured on five clays. They conclude that the moisture content was by far the most important parameter. But a more important finding was the magnitude of the suctions measured, as low as - 250 p.s.i. for a clay compacted at only 5% dry of optimum. They also give a very interesting discussion on the relationship of the water pressure measured with a probe to the actual water pressure in the pores of the soil, using the concept of total head which includes in addition to the capillary pressure, the 'osmotic' and 'absorptive' heads generated by the electrical forces between the soil particles and the pore water.

(7.3) A comparison between field and laboratory measurements of suctions

Croney and Coleman (1960) gave an excellent review of the different methods of the measurement of suction in the laboratory and the field, where in the laboratory suctions as high as -10^6 p.s.i. can be measured, but in-situ measurement in the field is limited to about - 1 atmosphere. The maximum in-situ suction recorded by the Author was - 29 ft. of water (= -12.5 p.s.i.) in Peterborough and Diddington (see Sections 7.4 and 7.5).

Vaughan (1965) referred to some anomalies in the measurement of pore water pressures between field and laboratory compacted boulder clays from Selset and Balderhead. For the same m/c the pore pressure measurements obtained using I.C. tips were consistently higher than those predicted by Matyas (1963) on laboratory compacted samples. In Selset where the average m/c was opt. + 1% (Kennard and Kennard, 1962) Vaughan (1960) reported initial suctions of - 8 p.s.i. recorded by two I.C. tips. Matyas' corresponding tests for laboratory compacted samples gave - 10 to - 20 p.s.i. However, a bigger discrepancy occurs in the values of $\bar{B}$ for whereas the actual fill exhibited positive pore water pressures at an overburden pressure of only 20 p.s.i., the corresponding laboratory all round pressure was 40 to 50 p.s.i.

Vaughan (1965) mentions some of the factors which may be responsible for these discrepancies, the obvious one being errors in the determination of the moisture content where an error of only 2% would completely account for these differences. The Selset boulder clay has a high stone content where 30% is retained on a No. 18 sieve (Matyas, 1963). Vaughan (1965) made a correction for the m/c of the clay by assuming that the m/c of the stones retained on an arbitrarily chosen sieve size (usually No. 18 or 36) is 3% and hence by finding the dry weight of the stones a correction made to the m/c of the clay matrix. Such a correction would typically be of the order of 5% for an uncorrected m/c of 15%. In addition, the optimum m/c was found to vary by up to 4% from one batch of clay to another, and thus it seems quite likely that errors in the m/c determination may have played an important role in the discrepancies mentioned above.

To resolve the first uncertainty regarding the magnitude of initial suctions in freshly compacted fills the Author conducted two in-situ suction tests in Balderhead and Diddington dams and tested some undisturbed samples in the laboratory. The results are given in the next two sections (7.3.1 and 7.3.2), whilst a discussion of the results is given in Section (7.3.3). In all these tests the m/c was corrected for the stone content retained on No. 18 sieve assuming a 3% m/c for the stones.

(7.3.1) Suction measurements on boulder clay from Balderhead dam

Three tips were installed simultaneously and within a few feet of each other in the freshly compacted boulder clay forming the clay core of Balderhead dam. The first two were an Imperial College tip and a Building Research Station low air entry disc tip connected to two 2.8 mm. bore mercury manometers by about 200 ft. of polythene coated 3/16" nylon tubing to a clear polythene mercury manometer, where the limbs of the manometer could be adjusted to act as a null device. The other connection to the tip was blocked off. A calibration of the volume change in the tubing for the null tip was made before the test. The tip was disconnected and the end of the nylon tubing blocked. Under a suction of - 22 ft. of water a volume change of nearly 1 c.c. took place mainly in the soft polythene manometer tubing. During the in-situ test the position of the null point was changed to compensate for this volume change.

Fig. (7.1) shows the suctions recorded by all three tips, together with that recorded by C24 a normal piezometer installed 500 ft. from C13 at the same level but a day earlier. This had 400 ft. of polythene coated 3/16" nylon tubing.

As expected, the Building Research Station disc tip cavitated almost immediately giving a constant reading of - $1\frac{1}{2}$ ft. of water which is probably its air entry value.

U4 samples were obtained within a few feet from the test area and sealed. Within a few days measurement of the suction in the free standing 'undisturbed' sample was made in the standard large triaxial cell fitted with a 70 p.s.i. air entry ceramic at the base. Fibre glass strands were laid on top and down the sides of the sample which was then sealed in a rubber membrane. Air pressure was applied to the cell and the sample. Some water was introduced into the base of the cell as an extra precaution against drying of the sample (see Bishop, 1960). The validity of the elevated air pressure technique has been proved by many writers (see, for example. Olson and Longfelder, 1965) as was also verified by the Author in this test.

The air pressure in the cell was maintained by a 'Sunvic' valve but this unit used in this and subsequent tests was not very accurate and in fact the air pressure varied by up to ± 2 p.s.i. at a pressure of only 20 p.s.i. However, this was not critical in this test.

After measuring the initial suction, water was added to the sample through the base by applying a back pressure slightly in excess of the air pressure. The sample was then allowed to come to equilibrium taking typically 4 to 5 days. It was then taken out and quickly recompacted in the standard proctor machine, reinstalled in the triaxial cell and the same procedure repeated.

Fig. 7.2 shows the results obtained. The suction measured on the undisturbed sample was - 44.5 ft. of water compared to the maximum in-situ value of - 18.5 ft. and the subsequent value of - 12 ft. (Fig. 7.1). Recompaction of the sample at constant m/c caused a drop in suction from - 12 ft. to - 39 ft. The sensitivity of suction changes in m/c is also shown where an increase of m/c of $1\frac{1}{2}$% in the undisturbed sample caused the suction to drop to $\frac{1}{4}$ of its initial value.

The other half of the U4 sample was tested a year later. Its m/c was found to be very similar to the first sample. The initial suction measured was - 44 ft. of water confirming the first measurement.

(7.3.2) In-situ suction test in Diddington dam

This test was conducted on the rolled boulder clay used as the clay core in Diddington dam.

Here a 25 ft. continuous length of $\frac{1}{8}$" saran tubing was used, one end of which was connected to the tip while the other end served as a mercury manometer. The tip was installed in a freshly dug 3 ft. deep trench and two m/c samples were taken, before and after the test, of shavings of the clay from the sides of the hole made to accommodate the tip. These measurements revealed an increase of 2.8 % in m/c after the test. Fig. 7.3 shows the suction recorded which reached a maximum of - 29 ft. of water (= -12.5 p.s.i.) in 24 hrs. The system was then very 'spongy' and an air bubble 2 cms. long was observed at the top of the manometer where the absolute pressure was only one or two feet of water.

A 10" block sample and some U4 samples were obtained of the same rolled clay, but a few weeks later and at a different elevation. Unfortunately these were not tested until some 18 months later. However, m/c samples taken from different corners of the block and the middle showed a uniform distribution at about 1% dry of the average placement m/c (see Section 7.5). The U4 samples, however, appeared to have lost 2 - 3%, the sealing wax having lost contact with the U4 tubes.

Two simple tests were then conducted to study the effect of sampling on suction, and also to compare the measurements taken using an Imperial College tip to that using the base of a triaxial cell.

A $1\frac{1}{2}$" sample was obtained from the clay block by hammering a sample tube into it. The sample was extruded and installed in a small triaxial cell fitted with a 70 p.s.i. air entry ceramic at the base. The sample was not jacketed, but a small amount of water was introduced at the base of the cell to prevent drying of the sample*. The 24 hr. equilibrium suction measured using the

* It could be argued that this procedure would in fact increase the m/c of the sample as the air around it gets saturated with water vapour, but in this particular experiment the duration of the test was relatively short and hence this point was probably irrelevant.

elevated air pressure technique was - 34.2 p.s.i.

The block sample was trimmed and an Imperial College tip installed in the hole left by the sample tube. The trimmed sample was then placed in the large triaxial cell. The tip was connected to the same continuous length of saran tubing used for the in-situ test. The same procedure was adopted as for the 1½" sample. The air pressure applied was 25 p.s.i., less than the 30 p.s.i. air entry value of the Imperial College tip. The 24 hour equilibrium suction measured at the base of the block sample was - 35 p.s.i. Moisture content checks before and after the tests on the base of the block and the 1½" samples showed that they were very nearly the same at about 19% and were not affected by the tests. Due to an unfortunate leak no measurement on the Imperial College tip could be obtained.

A further test was conducted on a U4 sample which had a m/c of 17%. Again a 1½" sample was obtained by hammering a sample tube into the 4" sample while it was still inside the U4 tube. Suction measurements on the extruded 1½" and 4" samples gave - 63 and - 64 p.s.i. respectively. But these measurements may well have been lower, as after the tests the ceramics were found to have cavitated in both cells. In addition, moisture content checks on the base of the samples before and after the test showed an increase of 1%.

The Imperial College null tip was then installed in the U4 sample inside the large triaxial cell but with no air pressure. The maximum suction recorded with this tip was - 10.5 p.s.i. after 25 hours. Two small air bubbles were seen in the manometer tubing each 0.25 cms. long. When the tip was de-aired no air bubbles were seen in the tip. The moisture content of the clay around the tip after the test was found to be 1% higher than that before the test.

(7.3.3) Discussion and conclusions from the suction measurements on the boulder clay from Balderhead and Diddington dams

Vaughan (1965) who also reported the in-situ test in Balderhead concluded that since the readings of the null tip and the ordinary piezometer C13 converged after 3 days the measured suction of about - 12 ft. of water (Fig. 7.1) was probably the correct value, although the suction measured in the laboratory on the undisturbed U4 samples gave a much lower value of - 44 ft. of water.

This difference could easily be explained by the drying out of the U4 sample, for as was mentioned earlier a m/c increase of only 1½% decreased the suction from - 44 ft. to - 12 ft. However, the U4 samples were very carefully sealed using grease instead of wax, and tested within a few days of the in-situ test.

The next factor to consider is the effect of sampling and extension of the sample from the U4 tube, but the simple tests on the Diddington boulder clay showed that even when the $1\frac{1}{2}$" was obtained by hammering in the sample tube, this made little difference to the suction measured.

The third possibility is that the suction measured in the laboratory is the correct value implying that the null tip must have lost some water to the soil. In fact two small air bubbles were observed at the top of the manometer in the field test, but when the tip was de-aired after the test no air bubbles were seen to come from the tip. However, since de-airing was conducted at a pressure a few feet above atmospheric any bubbles which may have existed at an absolute pressure of, say, + 5 ft. of water would have decreased to $\frac{1}{4}$ of their size at + 20 ft. absolute and would have been difficult to see.

Another factor to be considered here is the flexibility of the manometer system, although, as mentioned previously, a correction to the null position was made during the in-situ test this probably did not stop some water escaping to the soil. In addition it is quite likely that the water in the pores of the ceramic cavitated which would not be detected by de-airing.

The manometer unit used in the Diddington dam test was very much more rigid than that used in Balderhead dam. The maximum suction recorded was - 12.5 p.s.i. (Fig. 7.3) but the absolute pressure at the top of the manometer was only + 1 ft. of water. The presence of air bubbles in the manometer and the 2.8% increase in m/c of the soil after the test indicate quite unmistakingly that the - 12.5 p.s.i. measured was not the true suction. The test on the block sample which was 2% wetter gave a suction of -35 p.s.i. The test on the U4 sample showed that whereas the true suction of the soil was -63 p.s.i. or more, the Imperial College tip managed to record -10.5 p.s.i. only.

It can thus be concluded that the two in-situ tests in Balderhead and Diddington damd did not record the true suctions in the field, and, further that the laboratory measurements on the undisturbed samples probably gave the correct values.

Matyas (1963) reported some measurements of initial suctions on undisturbed U4 samples of boulder clay obtained at a depth of 10 ft. below the foundation of Balderhead dam. The suctions measured were -20 p.s.i. at a m/c of 14.7% and -30 p.s.i. at 13.8% which confirm the Author's measurements in the laboratory.

(7.4) Negative pore pressures in Peterborough embankment

This embankment was constructed in the summer of 1962 as a part of the first stage of the Peterborough Ash Disposal Scheme which involves building water reservoirs and divider embankments in pits of varying sizes, dug in the Oxford clay by brick manufacturers. Flyash brought by trains from a Midlands power station would then be mixed with water and pumped to the various pits thus achieving the dual purpose of disposing of the flyash, and the reclamation of valuable land.

The main reservoir embankment, oval in plan, was constructed with a 'callow' clay core and a mixture of Oxford clay and Kellaway ~~sands~~ sands for the shoulders. The callow clay is the local name for the weathered top of the Oxford clay dumped back by the brick manufacturers, its m/c being a few percent higher than that of the intact Oxford clay.

One section of the embankment was instrumented with Imperial College hydraulic piezometers, mainly to detect high pore pressures during steady seepage and rapid drawdown. Fig. 7.4 shows the instrumented section where 23 Imperial College tips were installed in the fill and 6 borehole piezometers in the foundation strata, all connected to mercury manometers.

Tests on samples taken from the holes made to accommodate the Imperial College tips gave for the Oxford clay a L.L. of 55 to 65%, and a P.L. of 20 to 25%. These figures are very similar to those measured for the Oxford Clay at Diddington, where the clay fraction ($< 2\mu$) is 40% or more.

Since the fill was a mixture of Oxford clay and Kellaway sands it is difficult to compare the average m/c of the fill to its optimum m/c, but it is quite certain that the clay was placed and rolled at a m/c at least 1 or 2% dry of optimum.

Fig. 7.4 also shows some typical pore water pressure records together with plots of the pore pressure against vertical overburden pressure, conveniently grouped according to the plastic index of the soil. An interesting feature here is the almost unanimous behaviour of the piezometers in recording initial suctions to start with, and the subsequent increase in pore pressure with the increase in the overburden pressure as forecast by laboratory tests (see for example Matyas, 1963). The dependence of the value of $\bar{B}$ on the m/c is also clearly shown (see Vaughan, 1965 who also reports these records).

(7.4.1) Cavitation of the Imperial College tips

Tip 14 in the callow clay core and 5 in the foundation gave consistent positive readings and did not require frequent de-airing. Most of the other tips in the shoulders, however, for example 11, 15 and 21, recorded large suctions of -20 to

-25 ft. of water for a few weeks and then cavitated in a similar period of time. As was mentioned in section (2.5) when the tips were de-aired, the amount of air collected indicated that air had completely displaced the water to the soil. A week or two after de-airing the tips reverted to measuring very nearly the previous suctions, and the cycle repeated.

An interesting point here is whether the ceramic filter of an Imperial College tip cavitates throughout these cycles or not. Simple laboratory tests show that once the filter is cavitated, by blowing air through it at a pressure in excess of its air entry value, it is difficult to resaturate it under 2 or 3 p.s.i. water pressure typically used in de-airing operations (Section 2.6.6). However, in a simple test where a saturated tip was subjected to a suction of -12.5 p.s.i. for 12 hours, the tip cavity lost most of its water by cavitation, but the ceramic was found to be still saturated. This was again confirmed by the laboratory test on the U4 sample from Diddington (Section 7.3.2) where the Imperial College ceramic remained saturated even after the water inside the tip had obviously cavitated.

This might explain the fact that in the field when piezometers were de-aired, they were again capable of recording large suctions for a relatively long period of time. The calculation in Section (2.5) showed that if the ceramic was cavitated then in conditions similar to those in Peterborough the tip should lose all its water to the soil in only 4 days assuming that the true suction in the soil is -2 atmospheres.

Regarding the entry of air into the tip it seems that apart from the diffusion of air through the ceramic and its release from solution inside the tip, the compression fittings ordinarily used in these tips might be unsuitable when they are recording negative water pressures. If this is so then it confirms the idea that the ceramic remains saturated even after the tip has lost its water to the soil.

(7.4.2) Constant-head and falling-head tests on piezometers recording negative pore water pressures

Although Gibson's solutions for the constant-head (C.H.T.) and falling-head tests (F.H.T.) were derived for fully saturated soils (see Chapter 5) it was interesting to see whether they may be applied to partly saturated soils.

Wilson and Luthin (1963) discuss this point regarding the flow of water in partly saturated sands. They argue that the permeability measured is essentially that of the fully saturated soil (at the same void ratio) since most of the resistance to the flow is offered in the zone already saturated by the advancing water front.

Using the apparatus described in Section (2.7.1) constant-head tests were performed on seven piezometers reading negative pore pressures ranging from -10 to -24 ft. of water, and the results are given in Table 7.1. The measured permeability 'k' varied between 1 to 7 x 10^{-9} cm./sec. for the clayey fill, and 2 to 5 x 10^{-8} cm./sec. for the sandy fill. The corresponding values of the coefficient of swelling 'c_s' were 5 to 7 x 10^{-6}, and 2 to 20 x 10^{-3} cm.^{2}/sec.

The flow rate q v. $1/\sqrt{t}$ plots for some of the tests are given in Figs. 7.5 and 7.6 where it is seen that apart from some scatter due to the very small flow rates measured and other errors possibly due to temperature changes, the results obtained are similar to those from fully saturated soils (Chapter 8).

The very low coefficient of swelling 6 x 10^{-6} cm.^{2}/sec. (0.2 ft.^{2}/year) for the clayey soil is in accord with laboratory tests on other partly saturated soils (see for example Matyas, 1963) where the compressibility is high under large suctions. A comparison of these values will be made to those obtained at Diddington dam, Section (7.6).

In addition to the constant-head tests, falling-head tests were conducted on two piezometers recording suctions. The results are given in Fig. 7.7. In both tests the excess pressure was held for a period of 5 minutes to pressurise the tubing, as will be referred to again in Chapter 8. It is seen that the 95% equilisation times were one week for tip 19 installed in a sandy soil and nearly three weeks for tip 24 installed in a more clayey soil.

Gibson's (1963) curves were used to obtain rough values of c_s and k which confirmed the low values obtained from the constant-head tests given in Table 7.1

(7.5) Negative pore water pressures in Diddington dam

This 75 ft. dam was constructed in 1963-1964 as part of the Great Ouse Supply Scheme which involves water storage, purification and pumping, serving the South Midlands and North London areas. Hammond and Winder (1967) describe the Scheme and give details of the construction of the dam and the road diversion embankment which was constructed first, downstream of the dam and served as a trial embankment.

Fig. 7.8 shows the instrumented section of the dam. The soil strata in the valley consisted of about 120 ft. of boulder clay overlying about 100 ft. of Oxford clay, below which lay the Kellaway beds. Economically therefore there was no choice but to use the boulder clay as fill material. The site investigation tests on the boulder clay gave L.L. = 60%, P.L. = 20%, op m/c = 19%, natural m/c around optimum, k = 10^{-9} cm./sec., and

C_v = 2 ft.^{2}/year. The very plastic nature of the fill and its very low coefficients of permeability and consolidation indicated that difficulties might be encountered regarding construction pore pressures, and long term softening of the clay which would especially effect the stability of the dam under rapid drawdown conditions. A further factor was that the construction programme stipulated placement of the fill in twelve months including the winter season. As a safeguard against these uncertainties drainage blankets at 10 ft. centres were incorporated in the design. Professor A. W. Bishop who was then called to advise on this project suggested that for the blankets to have any effect at all on dissipating possible high pore pressures they would have to be placed at 5 ft. centres because of the very low values of k and C_v. The design was then altered to confirm to this new spacing, as shown in Fig. 7.8.

(7.5.1) Records from piezometers in the road embankment

Another uncertainty lay in the choice of the hydraulic piezometers. Binnie & Partners who were the consultants on this scheme had then had some difficulties with the hydraulic piezometers in Treweryn dam, North Wales. There nylon 6 was used and this proved to be unsatisfactory due to the blocking of the tubing by crystals of the plasticiser material. They thus decided to use the vibrating wire Maihak piezometers.

A comparison was then made between the behaviour of the Maihak and Imperial College tips by installing a few pairs of them in the road embankment on two sections. Fig. 2.2 shows the instruments in the main section together with some typical readings.

The Maihak and Imperial College tips in the foundation gave very nearly the same readings although those from the Maihak tips were 0.5 to 1.5 p.s.i. lower than those recorded by the Imperial College tips. As was mentioned in Section (2.2) the reason for this was very probably 'zero shift' in the Maihak tips.

In the fill, however, Maihak gauges 7 and 8 read consistently zero pressure while no. 9 read a positive pressure of $7\frac{1}{2}$ ft. of water. Only one Maihak gauge, in the other instrumented section, recorded a negative pore water pressure. Initially it read -13 ft. of water, but during the subsequent six weeks the reading gradually moved to zero pressure. The corresponding Imperial College piezometers were nearly all recording negative pressures of -10 to -20 ft. of water.

The record from R10 is interesting. The two tips were installed in a wet patch of fill with a m/c of 26% but even then the hydraulic piezomete recorded a negative water pressure of

-12 ft. of water (Fig. 2.2). The Maihak tip recorded an initial pressure of +24 ft. which then dissipated very slowly. Assuming that the Maihak tips were recording air pressure rather than water pressure, this confirms laboratory findings (see for example Matyas, 1963) that air permeability in partly saturated clays is very small at high degrees of saturation.

Three air piezometers designed by Vaughan (1965) were installed in the road embankment near R8, R9 and R10. All three recorded zero pressures, although Maihak gauges R9 and R10 which presumably were recording air pressures gave positive readings of $7\frac{1}{2}$ and 24 ft. of water. This discrepancy may have been due to the very poor response of the air piezometers, and perhaps also to cracking of the soil around them when large air pressures were used to 'de-water' the air tips.

The hydraulic piezometers have unfortunately not been de-aired lately and it is interesting to note that their readings after a period of some months tended to coincide with those from the Maihak gauges recording zero pressures. It may be that the negative pore pressures in the embankment have been equalised but experience at Peterborough suggests that this is hardly the case, and in fact the hydraulic piezometers may well have cavitated.

(7.5.2) Records from piezometers in the compacted fill of the dam

From the experience gained in the trial embankment it was decided to use Imperial College hydraulic piezometers in the compacted fill of the dam. Saran tubing was used instead of the normal 3/16" nylon 11 or 66 because of the previous difficulties with nylon 6 as mentioned already. The leads were brought first to a manhole upstream of the road embankment and later extended to a manhole on the downstream side where they were joined to the redundant Maihak gauges, the latter serving to relay the readings to a more comfortable instrument house.

The instrumented section is shown in Fig. 7.8. The clay core and shoulders were both of rolled boulder clay. Hammond and Winder (1967) give the average m/c of the core as 1% higher than that of the shoulders which were at optimum m/c. Thus, as in Peterborough negative pore water pressures were expected as, in fact, recorded by many of the piezometers. Examples are given in Fig. 7.8 which also shows plots of the initial negative pore pressures recorded against m/c of the samples taken near the tips, as well as the subsequent values of $\bar{B}$. The plot of the initial pore pressures shows a marked density of readings around -20 to -25 ft. of water. This confirms the pattern in

Peterborough strongly suggesting that at a m/c of opt. and below the readings obtained with these piezometers may very well be not the true suctions in the soil, as confirmed by laboratory measurements (Section 7.3.3).

A plot of the rise of pore pressure with overburden pressure is given in the bottom right hand corner of Fig. 7.8 Results from the only one test reported in the site investigation agrees well with those recorded by piezometers C4 and C9.

During construction some of the piezometers gave readings at zero pressure $\pm$ 1 or 2 p.s.i. which suggested that they may not be recording the true pore water pressure. The Author conducted constant-head tests on the suspected piezometers and compared the values of k and c_s obtained with those from other more consistent piezometers. The results are given in Table 7.2 and are shown for some of the tests on Figs. 7.9 and 7.10.

The measured values of k for piezometers U8, C1, C8, D1, D6 were some 1000 times higher than those measured for the other piezometers, and from tests in Peterborough. When first calculating these high permeabilities the value of the intake factor used was 40 cms. This gave values of k in the range of 10^{-5} cm./sec. which assuming that the piezometer systems were free of leaks, is theoretically impossible since it exceeded the permeability of the filter, 3 x 10^{-6} cm./sec. Assuming that the soil did not offer any back pressure a different value of the intake factor was used, viz. the surface area of the filter divided by its thickness giving about 200 cms. This reduced the calculated permeabilities to the more reasonable figure of 2×10^{-6} cm./sec. This showed that the source of trouble was very probably improper compaction of the soil around the tips during installation.

The value of k calculated for D6 using 200 cms. for the intake factor was still too high at 10^{-5} cm./sec. This suggested a leak in the piezometer system, and in fact upon inspection a leak was found in a coupling in the manhole upstream of the road embankment. When this was mended and the tip retested the value of k obtained was 5×10^{-9} cm./sec.

(7.6) Comparison of 'k' and 'c' from field and laboratory measurements for Peterborough and Diddington

At Diddington the site investigation provided useful laboratory data especially of k and c_v which will now be compared to field values measured in Diddington and Peterborough from constant-head tests and from records of pore pressure dissipation in the foundation of Diddington dam.

Fig. 7.11 shows the results plotted against an approximate average of the effective stress.

For the permeability of the Oxford clay the laboratory results on the undisturbed samples from Diddington confirm the low values of around 2×10^{-9} cm./sec. measured on the compacted fill in Peterborough. The permeability of the boulder clay used as fill in Diddington varies between 1×10^{-9} to 30×10^{-9} cm./sec. probably depending on the proportion of the Kellaway sands mixed with the Oxford clay during the process of remoulding under the advancing ice sheet.

There are no available field data for the coefficient of consolidation 'C_v' of the Oxford clay as all the constant-head seepage tests were of the swelling type since most piezometers were reading negative pore water pressures. However, the field values of the coefficient of swelling 'c_s' measured on the compacted fill at Peterborough are slightly lower than those of C_v measured on undisturbed saturated samples from Diddington. These very low values of c_s, around 1 ft.^{2}/year were also measured from seepage tests on the compacted boulder clay fill in Diddington. A test on a laboratory compacted boulder clay sample gave C_v = 1.8 ft.^{2}/year. These results reflect the decrease in k and probably greater increase in m_v or m_s due to the presence of air in partly saturated soil, as observed by other workers (Matyas, 1963; Blight, 1961).

A calculation of the value of C_v was made from pore pressure dissipation in the foundation of Diddington dam as recorded by Maihak piezometers situated in the boulder clay just above the 100 ft. thick Oxford clay bed. Here Taylor's (1948) curves were not convenient to use and instead use was made of Bishop's (1948) curves based on Terzaghi theory but for an infinite clay layer under a uniform stress increment. The value of C_v obtained, around 10 ft.^{2}/year correlates well with other laboratory data for boulder clay as shown in Fig. 7.11. The value of C_v derived from Bishop's curve was then checked by solving Terzaghi's equation numerically for a 100' thick bed with double drainage (for more details of the method see Section 8.4.5). This gave 10 to 12 ft.^{2}/year.

(7.7) Settlement records from Diddington dam

Three U.S.B.R. cross-arm settlement gauges were installed in Diddington dam, two in the clay core and one in the upstream shoulder. As mentioned in Section (3.2) apart from some difficulties with jamming and loss of the torpedo unit these behaved quite satisfactorily. Fig. 7.8 shows the readings from the unit in the clay core on ch. 2010 together with direct measurements of the vertical deformation of a 23 ft. service conduit buried under the dam at foundation level. The record from the second U.S.B.R. unit in the clay core is very similar to the one shown in Fig. 7.8 while the set in the upstream shoulder has thw lowest cross-arm 30 ft. higher than foundation level and that was recording on Feb. 1966 a settlement of 1.45 ft. compared to the 0.9 ft. recorded by the lowest cross-arms at foundation level of the other two sets. This ties in with other observations which suggest that the lower part of the core was probably on the dry side and not very well compacted.

A rough calculation can now be made of the theoretical settlement of the dam using the measured values of c_v, m_v, etc.

For the stress distribution in the foundation, Bishop's (1952) solution for the elastic distribution of stresses in a foundation under a triangular load suggests that the vertical stress in the 100 thick Oxford clay bed would be nearly uniform at 0.9 x γH, γ being the density of the fill and H the height of the dam; while the horizontal stresses will vary with depth. That some shear stresses developed under the dam is confirmed by the $\bar{B}$ values obtained from foundation piezometers as shown in Fig. 7.8 where the average value of $\bar{B}$ is about 0.7. This of course may have been due to other factors, e.g. consolidation, or leakage through the sealing grout in the boreholes, (see Vaughan, 1965).

The settlement record of the lowest cross-arms (Fig. 7.8) appears to indicate an 'elastic' deformation of about 0.4 ft.

This is not confirmed by direct measurements on the buried conduit, but the latter measurements were atarted a few months later and were probably upset by the disturbance to the stress distribution in the underlying soil caused by the actual presence of the tunnel.

For a theoretical calculation of the elastic settlement, values of Young's modulus 'E' and Poisson's ratio 'ν' are required as well as the ratio of the horizontal stress to the vertical stress, = 'a' say. For a rough calculation ν can be put equal to 0.5 while 'a' may be assumed to be uniform at 0.7. The value of E required to give 0.4 ft. settlement is then found to be 370 ton/sq.ft. For London clay (Ward, Marsland and Samuels, 1965) the value of E is around 600 ton/sq.ft. Unfortunately there are no records of undrained triaxial tests on undisturbed samples of Oxford clay from which E can be calculated.

For the consolidation settlement of the foundation of the dam the value of m_v as quoted in the site investigation report can be used. For the relevant pressure increment of 4 tons/sq. ft. the value of m_v is 0.006 ft.^{2}/ton. This would correspond to $k = 1 \times 10^{-9}$ cm./sec. and $c_v = 6$ ft.^{2}/year which is in agreement with results from triaxial dissipation tests (Fig. 7.11).

If the actual consolidation settlement between Feb. 1964 and Feb. 1966 (Fig. 7.8) is taken as 0.5 ft., then using Taylor's curves for the average degree of consolidation in a clay layer with double drainage would give a value for c_v of 35 ft.^{2}/year. This is higher than the measured values in the laboratory and if the analysis is correct it may point to the presence of more permeable strata in the Oxford clay layer, as was in fact observed in Peterborough 20 miles away.

The settlement records for the clay core indicate that apart from an initial settlement depending on the local m/c there has been little deformation. However, the negative pore pressures measured on C2 and C6 indicate that the fill should swell with time as in fact can be seen from the settlement records. Theoretically, if the value of c_s (Fig. 7.11) is taken as 2 ft.^{2}/year and $k = 2 \times 10^{-9}$ cm./sec., then for a 10 ft. layer an increase in pore pressure from -15 p.s.i. to 0 should give a heave of 0.4 ft. The maximum recorded heave so far is 0.15' for the 70 to 80' O.D. layer. Piezometer C2 situated at the top of this layer has recorded an increase in pressure of 5 p.s.i. giving a theoretical heave of 0.13 ft.

(7.8) Summary

In this Chapter a comparison was made between field and laboratory measurements of negative pore water pressures in relatively dry compacted fills. Field measurements are limited to -1 atmosphere whereas in the laboratory the elevated air pressure and other techniques enable measurement of higher suctions.

Field records of negative pore pressures in Diddington and Peterborough were given. Theae dams are only 70 and 50ft. high respectively and were constructed some 20 miles apart in a relatively dry area 70 miles north of London.

Many of the high initial suctions recorded by the Imperial College piezometers in Diddington, Peterborough and Balderhead dams, where the fill was not more than 2% dry of optimum, were in the region of -10 to -12 p.s.i. compared to laboratory measurements on undisturbed samples of -20 to -30 p.s.i. This suggests that at least some of these initial records are not giving the true values of the negative pore water pressures. If it is desired to measure the correct values other apparatus such as the electrical resistance gauge described by Croney and Coleman (1960) has to be developed and used.

Finally a comparison was made between the observed and calculated settlement in Diddington dam and a field record of 'heave' in the clay core was given where the negative pore pressures have partly equalised.

PIEZ NO	Nature of fill around tip	P.I.	HEIGHT OF FILL ABOVE TIP (ft.)	Pore pressure before test (ft. of water)	Δh = Excess press applied (ft. of water)	k cm/sec	c_s cm^2/sec
12	Mixture of Oxford clay and Kellaway sands	16	25	-16.6	6.3	2×10^{-9}	-
13	Clayey	20	40	- 3.0	7.5	7×10^{-9}	$=7 \times 10^{-6}$ (2.4)
15	Clayey	28.8	40	-18.0	8.9	0.9×10^{-9}	5.4×10^{-6} (1.8)
16	Mixture	21.4	25	-19.7	40.9	-	-
18	Sandy	19.1	7	-10.3	37.3	-	-
19	Very sandy	1.1	17	- 7.6	33.6	2×10^{-8}	2.1×10^{-2}
27	Sandy	18.7	13	-24.5	6.2	5×10^{-8}	1.8×10^{-3}

TABLE (7.1)

Constant-head test results on piezometers reading suctions: Peterborough

PIEZ NO	TIP LEVEL O.D.	HEIGHT OF FILL ABOVE TIP O.D.	PORE PRESS. O.D.	TEST PRESS O.D.	k cm/sec	c_s cm^2/sec
R2	62.0	93.5	98.4	118.7	3.3×10^{-9}	2.2×10^{-4}
U2	75.0	98.8	75.0	81.9	1.2×10^{-8}	1.1×10^{-5}
U7	85.0	99.0	80.0	95.4	5.4×10^{-9}	7.1×10^{-5}
U8	86.0	102.0	86.5	95.4	$(\quad 10^{-7})$	-
C1	68.0	118	58.0	84.2	(1.8×10^{-6})	-
C2	76.0	118	57.5	84.2	$3 \quad \times 10^{-8}$	2.6×10^{-5}
C3	76.0	118	81.7	100.1	1.7×10^{-9}	2.1×10^{-5}
C4	91.0	118	117.5	100.1	1.5×10^{-9}	$c_v = 4 \times 10^{-5}$
C8	73.0	11.8	73.0	102.9	(1.4×10^{-6})	-
C9	88.0	97	80.5	95.4	2.2×10^{-9}	1.8×10^{-5}
D1	77.5	111.7	72.0	100.1	$(4 \quad \times 10^{-7})$	-
D6	88.0	120.0	82.7	102.9	(10^{-5})	-

(TABLE 7.2)

Constant-head tests on piezometers in Diddington dam

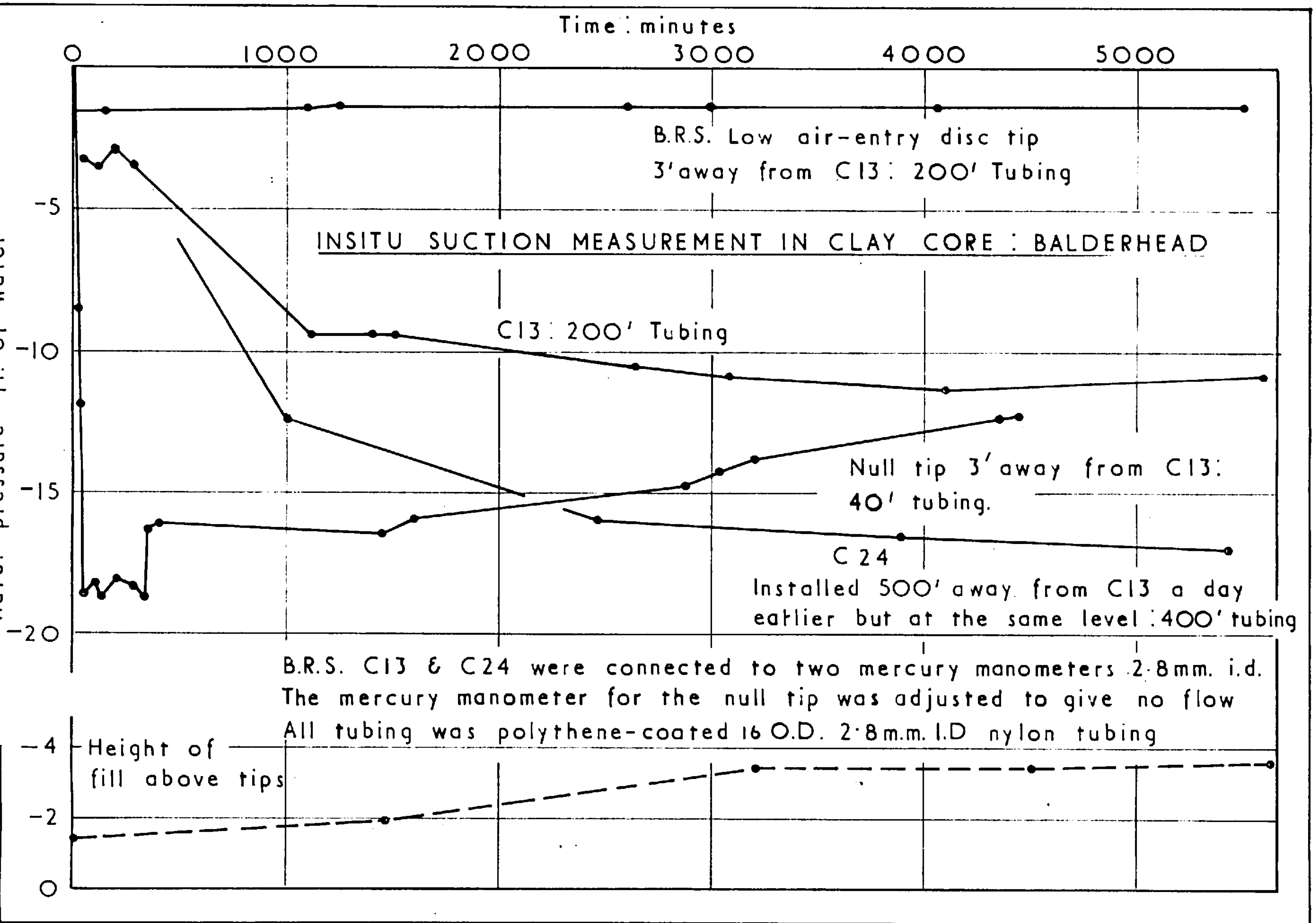
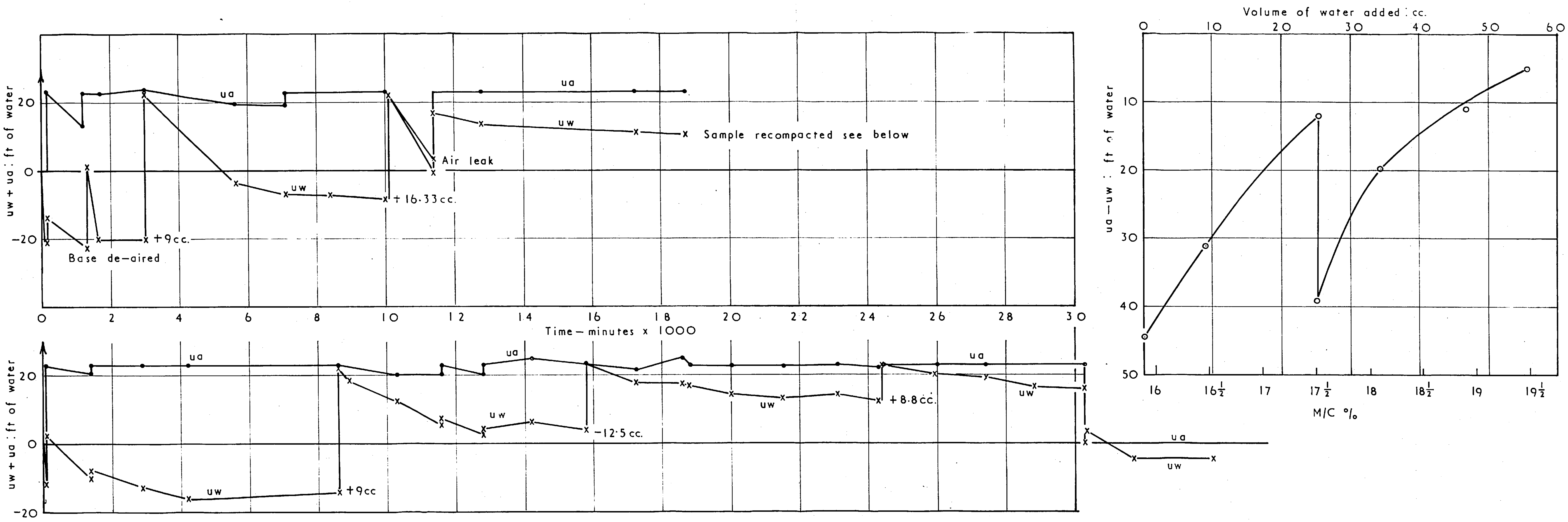
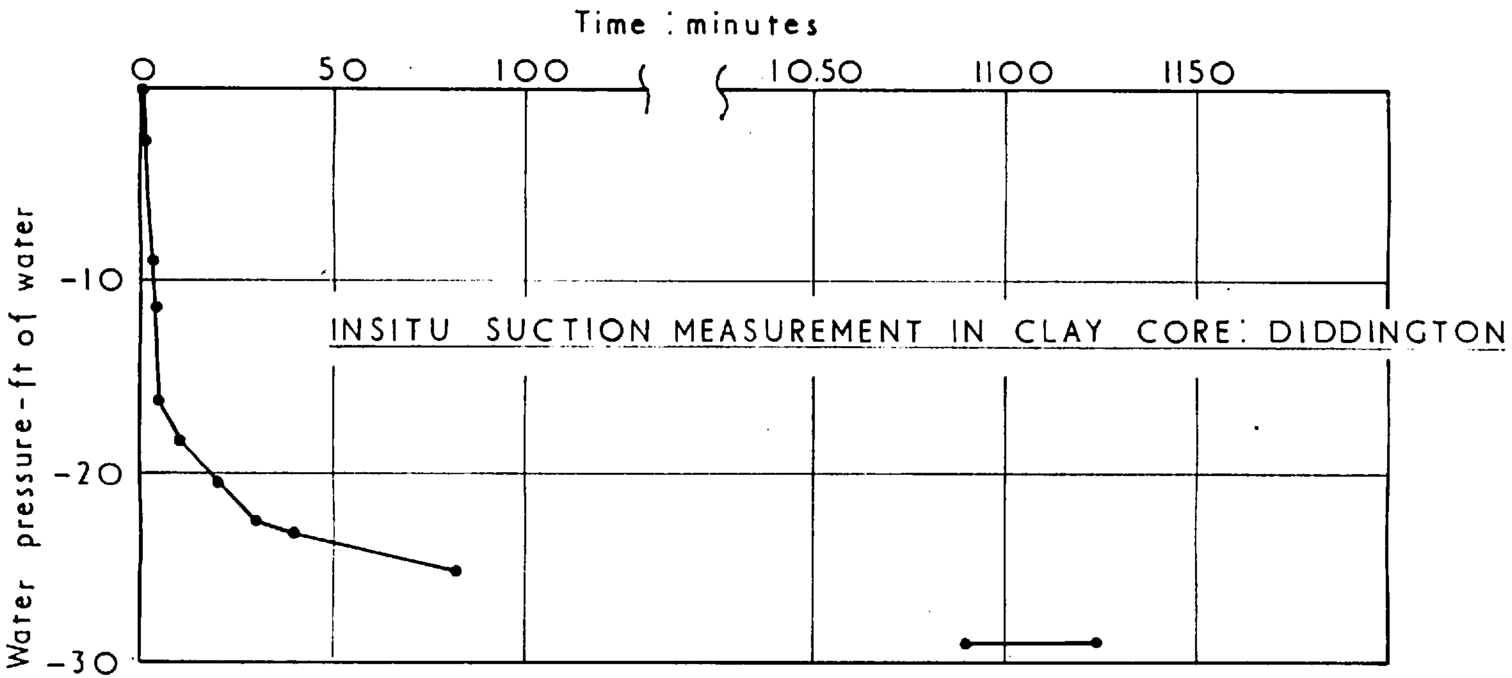

Time : minutes
0
1000
2000
3000
4000
5000
Water pressure – ft. of water
-5
-10
-15
-20
B.R.S. Low air-entry disc tip
3' away from C13 : 200' Tubing
INSITU SUCTION MEASUREMENT IN CLAY CORE : BALDERHEAD
C13 : 200' Tubing
Null tip 3' away from C13 :
40' tubing.
C 24
Installed 500' away from C13 a day
earlier but at the same level : 400' tubing
B.R.S. C13 & C24 were connected to two mercury manometers 2·8mm. i.d.
The mercury manometer for the null tip was adjusted to give no flow
All tubing was polythene-coated 16 O.D. 2·8m.m. I.D nylon tubing
Height of
fill above tips
-4
-2
0

FIG. 7.1



SUCTION MEASUREMENTS ON A U.4. BOULDER CLAY SAMPLE FROM BALDERHEAD DAM

FIG. 7 : 2



INSITU SUCTION MEASUREMENT IN CLAY CORE: DIDDINGTON.

Null tip was connected to 20′ of $\frac{1}{8}''$ Saran tubing

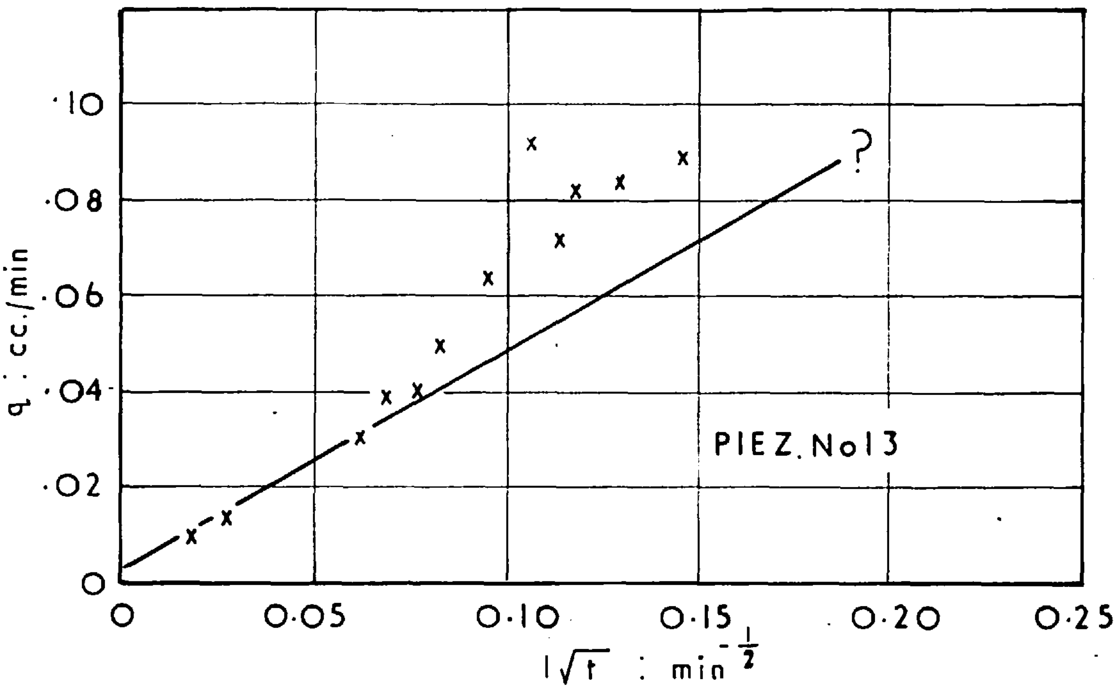
M/C of clay shavings around tip before test = 17·1%

After test = 19·9%

FIG. 7.3


PORE WATER PRESSURE RECORDS
PETERBOROUGH
SCALE:- 1 INCH 50FT.
OXFORD CLAY
KELLAWAY SAND
KELLAWAY CLAY
CORNBRASH LIMESTONE
OXFORD CLAY & KELLAWAY SAND
OXFORD CLAY & KELLAWAY SANDS
REDEPOSITED CLAY
PIEZ. HOUSE
DRAIN
TOE DRAIN
FILL LEVEL Nos. 5.14.15.25.
TIP LEVEL
PORE WATER PRESSURE FT OF WATER ABOVE O.D.
40'O.D.
20'O.D.
MAY 63
JAN 64
JUNE
JAN 65
JAN 66
JAN 67
LEAK
FILL LEVEL 21
FILL LEVEL 11
RESERVOIR LEVEL
PIEZ. No M.C. P.I.
22 28·2 38·7
26 27·7 30·3
20 26·1 30·5
21 21·3 38·5
No M.C. P.I.
14 31·1 29·1
17 30·4 29·2
15 20·5 28·8
VERTICAL OVERBURDEN PRESSURE σ1 FT. OF WATER.
PORE WATER PRESSURE: u. FT. OF WATER.
P.I. 30 → 40
u: FT. OF WATER
ARCHING?
CONSOLIDATION?
PROBABLY MUCH LOWER
P.I. 25 → 30
σ1 : FT. OF WATER
No M.C. P.I.
5 18·1 15·8
23 13·0 7·7
19 12·5 1·1
No M/C P.I.
10 18·7 21·4
24 18·2 23·0
25 17·6 20·3
16 16·5 21·4
8 13·5 ≃ 22
DE-AIRED
P.I. 20 TO 25
No M/C P.I.
13 18·1 20·0
12 17·1 ≃16
11 15·2 16·3
18 14·1 19·1
27 13·2 18·7
P.I. 15 TO 20
SHEAR STRAIN?
P.I. 0 TO 15
45°

FIG. 7.4



CONSTANT HEAD TESTS PIEZOMETERS
RECORDING SUCTIONS — PETERBOROUGH.

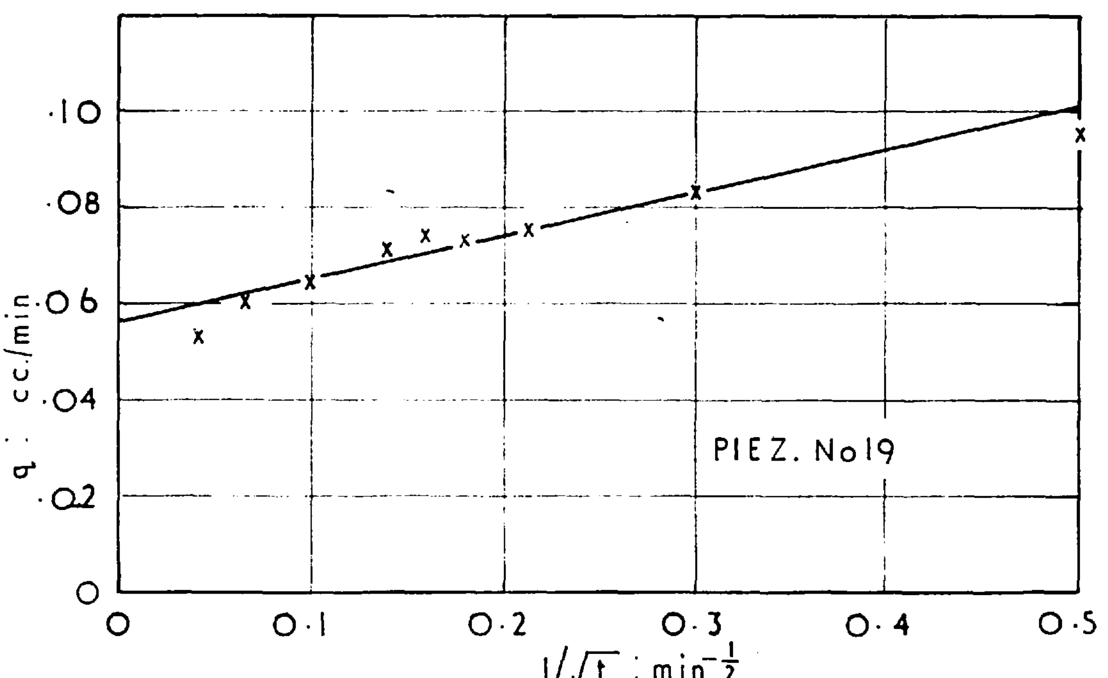
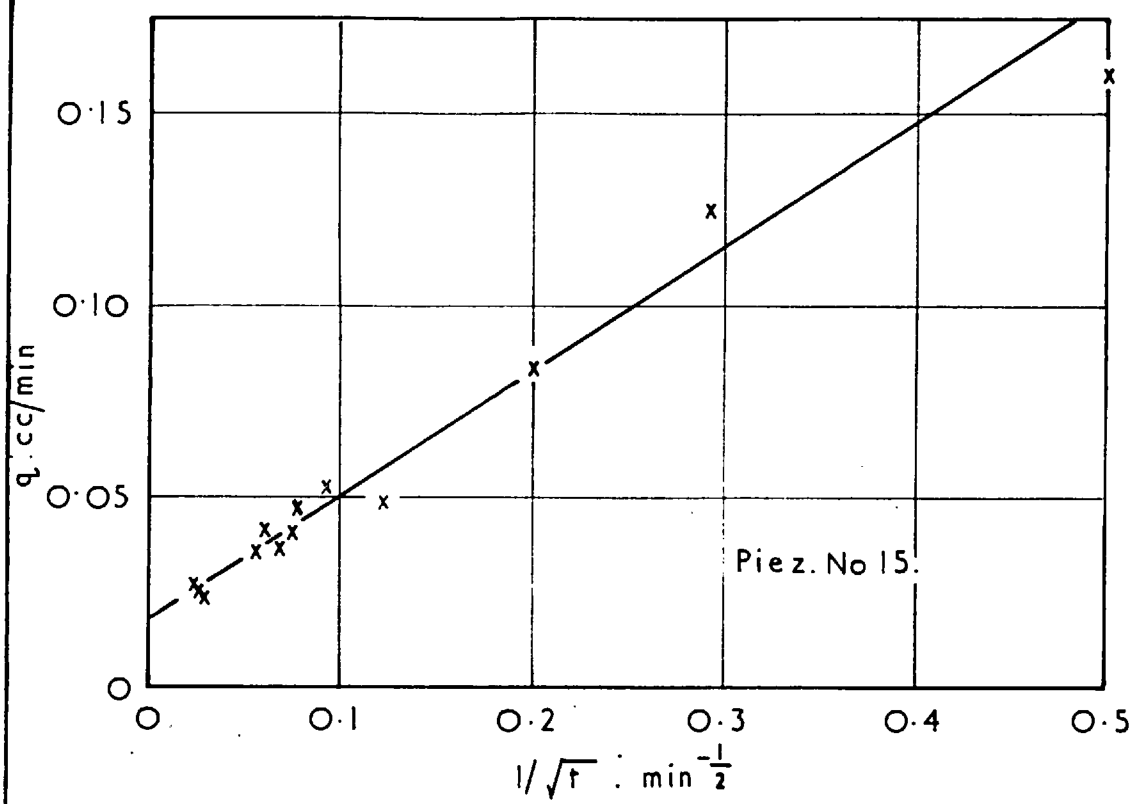


FIG. 7.5



CONSTANT HEAD TESTS ON PIEZOMETERS
RECORDING SUCTIONS — PETERBOROUGH

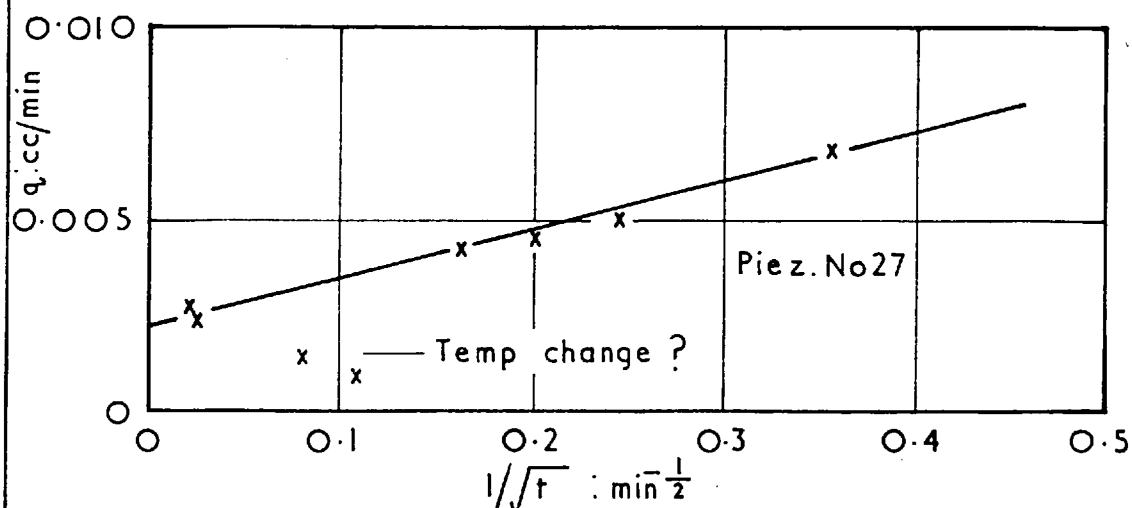


FIG. 7:6

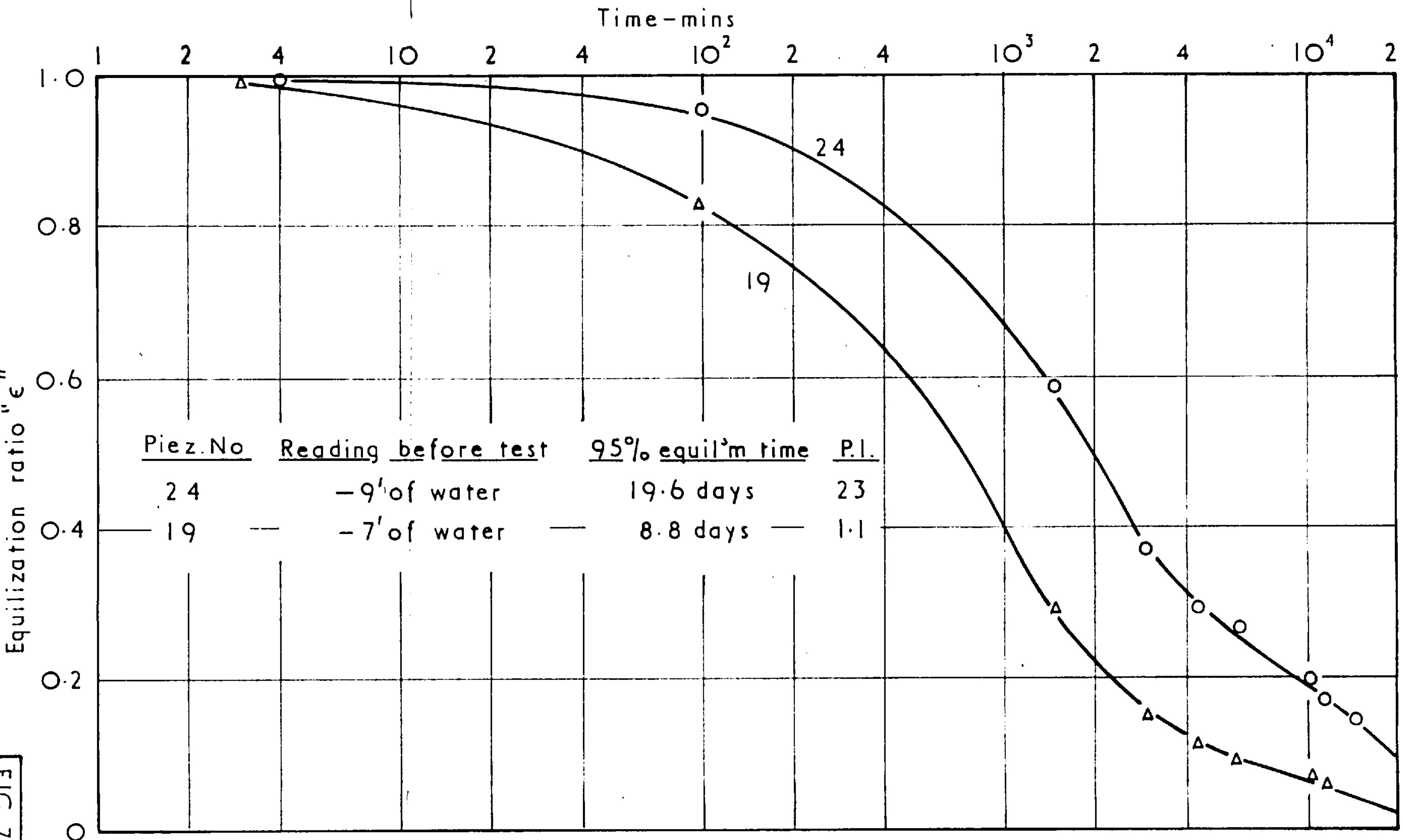
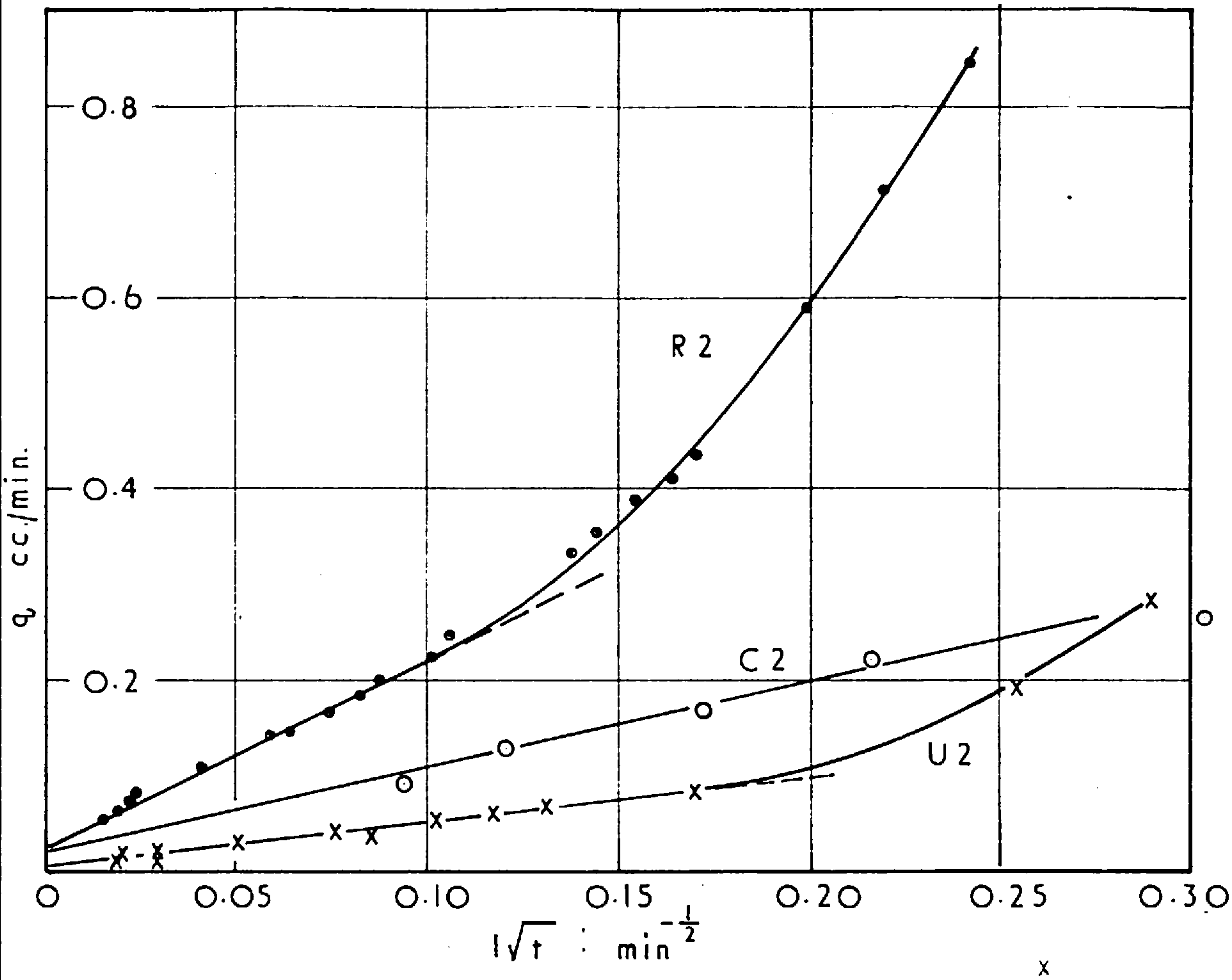


FIG. 7.7

FALLING-HEAD TESTS ON PIEZOMETERS RECORDING NEGATIVE PORE WATER PRESSURE – PETERBOROUGH

PORE PRESSURE AND SETTLEMENT RECORDS: DIDDINGTON DAM.

FIG. 7.8



CONSTANT HEAD TESTS—DIDDINGTON

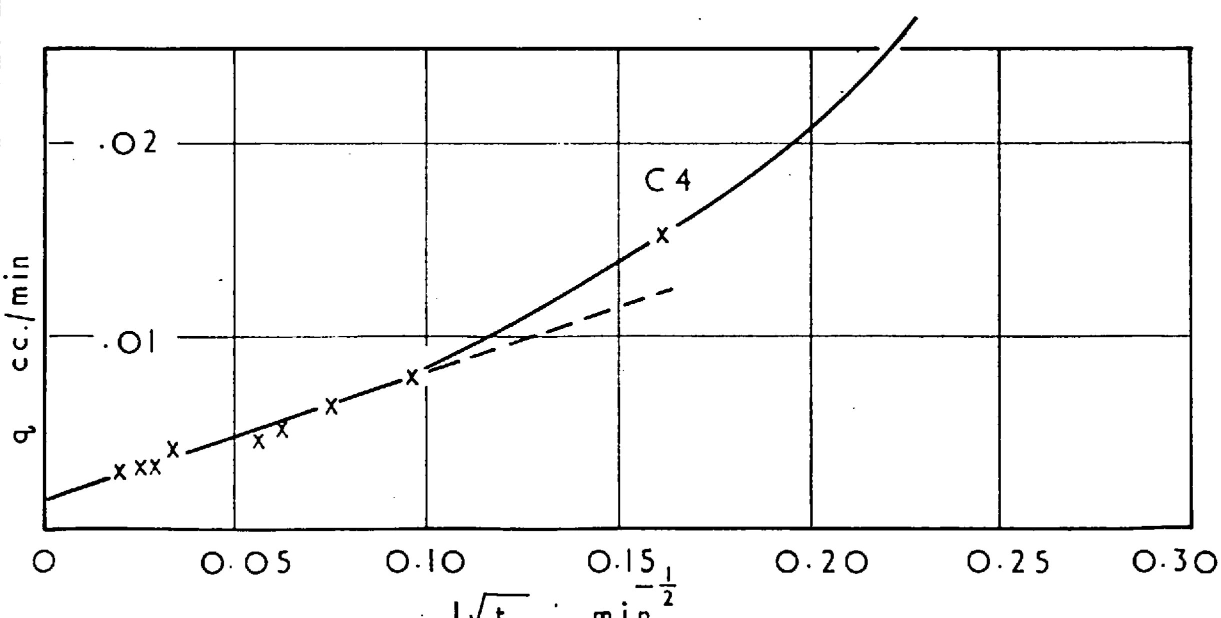
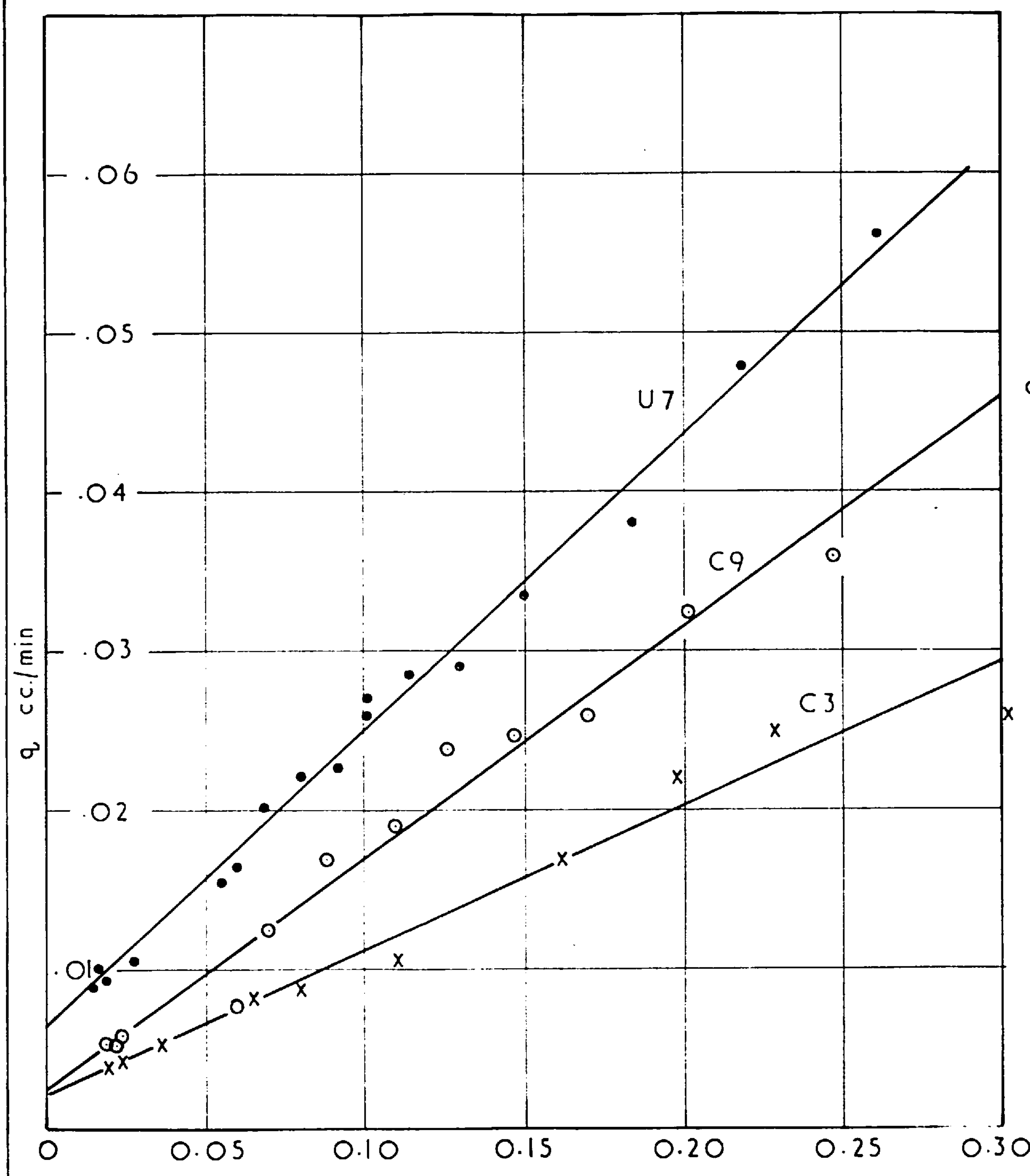
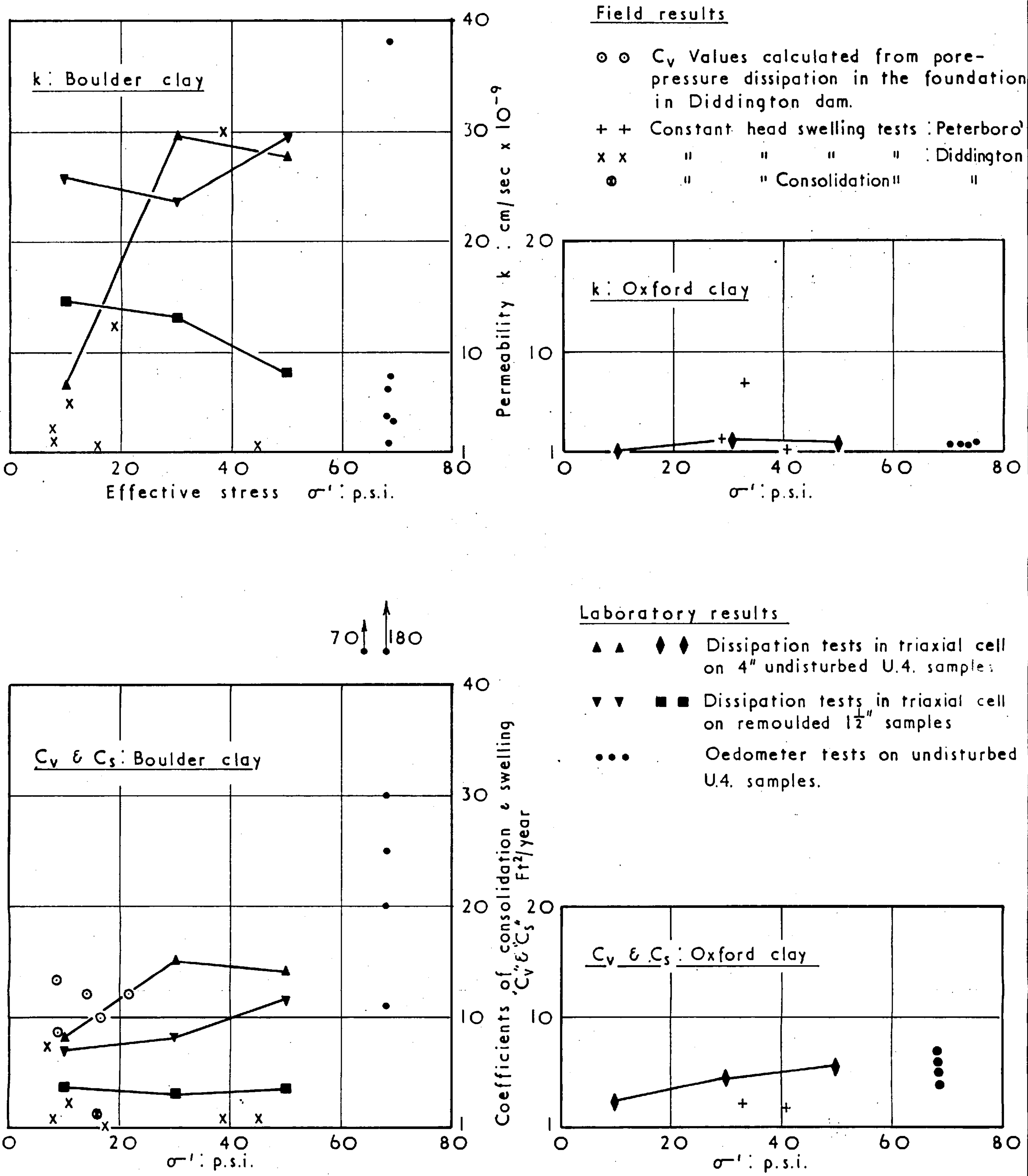


FIG. 7:9



CONSTANT HEAD TESTS—DIDDINGTON

FIG. 7.10



COMPARISON BETWEEN FIELD AND LABORATORY VALUES

OF K, C_v & C_s : DIDDINGTON AND PETERBOROUGH.

FIG. 7 : 11

CHAPTER 8

CORRELATION BETWEEN THE VALUES OF Cv AND k OBTAINED FROM FIELD AND LABORATORY MEASUREMENTS IN FOUR EARTH DAMS AND EMBANKMENTS

(8.1) Introduction

In this chapter values are given for Cv and k from field seepage tests on piezometers in Balderhead dam, Fiddlersferry embankment, M.6 Trial Embankment and Muirhead dam. These results are compared to available laboratory data and to other values deduced from field pore pressure dissipation and settlement records.

The Author did not perform any laboratory tests; the data given are mostly from commercial laboratories. Unfortunately in the case of the M.6 Trial Embankment hardly any laboratory information exists, restricting the correlation between results obtained from field observations only.

Although the primary object of this study is to find how well the field and laboratory values of k and Cv agree, another aim is to establish from field results, the extent of the variation of k and Cv with the effective stress.

(8.2) Balderhead dam

This 175 ft. high dam was constructed in 1962-1964 in Teesdale, 10 miles from Barnard Castle, Co. Durham. It has a narrow rolled boulder clay core supported by shale shoulders, and rests on a thin blanket of boulder clay overlying the rock foundation. The boulder clay is very similar in nature to that in the nearby Selset dam, some properties of which are given by Matyas (1963).

Balderhead is one of the best instrumented dams the Author is familiar with. Apart from the usual I.C. and Casagrande piezometer tips and hydraulic settlement gauges, it had the then newly designed vertical and lateral deformation gauges described in Chapter 3. Vaughan (1965) and Kennard, Penman and Vaughan (1967) give cross-sections of the instrumented sections together with pore pressure, deformation and total stress records, to which the reader may refer.

(8.2.1) In-situ C.H.T. tests

The Author conducted C.H.T. on most of the piezometers in the clay core. In particular three sets of C.H.T. were performed on C4 and C5 during the period 1963-1966. The results are shown in Figs. 8.1 and 8.2. These piezometers are on the centre line of the dam, about 50 ft. above the foundation level, near the cluster of total stress gauges. Fig. 8.3 shows similar plots for piezometers C21 while Fig. 8.4 shows the rest of the results for the other piezometers.

The values of k and Cv deduced from these tests are given in Table 8.1 together with other relevant data, where $p\omega$ is the piezometeric pressure before the test, δu is the applied increment of decrement of pressure and σ_v' is effective overburden pressure. For σ_v' the density of the soil was conveniently assumed to be twice that of water. Eq. (5.12) was used in the calculation for k and Cv. Using the readings from the total stress gauges additional values of k were calculated using eq. 6.24, where σ_i' was assumed to be the average of the effective vertical and two horizontal stresses and labelled σ_{Av}'. Here σ_{Av}' was found to be about one half of σ_v' for the three sets of tests.

Vaughan (1963) and Kennard, Penman and Vaughan (1967) give a discussion of the records obtained from these total stress gauges. Apart from a slight difficulty in the choice of the correction factor (this is a calibration factor by which the readings have to be adjusted to correct for the stress disturbance caused by the actual presence of the gauge in the soil) the readings were satisfactory, and indicated that there was no appreciable rotation of the major principal stresses, nor any evidence of extensive arching of the clay core against the shale shoulders. The vertical total stress measured was about 80% of the over-burden pressure. The two horizontal effective stresses were greater than the vertical effective stress to start with, but then decreased, with increasing load, to less than half of the vertical effective stress.

Fig. 8.9a and b show the C.H.T. results for k and Cv plotted against σ_v'. In plots c and d of the same figure a comparison is shown of the values of k as deduced from Eq. (5.12) and (6.24) from the test results on piezometers C4 and C5. The use of the latter equation produces a slightly better plot, but still does not even out the results for C5. But the consistency of the values obtained for k from nearly all the piezometers is quite remarkable, with a definite trend for decreasing k with increasing effective stress.

Fig. 8.9b shows the results for Cv, where except for the repeat tests on C4 and C5 they seem to be reasonably consistent, remembering that the clay was variable in nature. The possible reasons for the high values of Cv obtained for C4 and C5 from the second and third sets of tests have already been discussed in Section (6.8).

Before the first C.H.T. on C4 and C5 σ_{Av}' as deduced from the total stress gauges was about 30 p.s.i. During the first set of tests Δu was -28 p.s.i. for C4 and -25 p.s.i. for C5. Referring to Fig. 6.6, point C_2 for C4 corresponded to $\sigma' = 58$ p.s.i. and C_3 to $\sigma' = 30$ p.s.i. The corresponding figures for C5 are 55 and 30 p.s.i. During the interval Sept. 1963 and March, 1965 the change in σ'_{Av} was 14 p.s.i. for both C4 and C5. This change was not sufficient to bring the point C back on the 'virgin' consolidation curve, which would explain the remarkable change in the value of Cv from 61 to 681 ft.^{2}/yr. for C5 for the test on March, 1965. However, the corresponding test for C4 gave a completely different picture where in fact Cv decreased from 95 to 42 ft.^{2}/yr. The reason for this discrepancy is not clear. The simplified picture presented using Fig. 6.6 may well be affected by other factors. One such factor is the duration and pressures used during de-airing operations in the intermediate period. Other tests, leaks etc. may have taken place of which unfortunately no complete records are available.

In addition genuine differences in the stress path and magnitude of effective stress may exist between the tips C4 and C5 although they are at the same level and only 20 ft. apart. A difference in average effective stress of only 10 p.s.i. would completely explain this discrepancy.

For the third set of tests on Sept. 1966, both piezometers and especially C4 gave high values of Cv. Here the change in σ'_{Av} between March, 1965 and Sept. 1966 was only 3 p.s.i., and clearly was not sufficient to offset the large Δu used in the 1965 tests, viz. 20 and 23 p.s.i.

A part of this increase in Cv with the effective stress may well be genuine. This is reflected in laboratory results discussed in Section (8.2.4), where probably with increasing effective stress the compressibility decreases at a faster rate than the permeability, leading to an overall increase in Cv.

The effect of the creep of the connecting plastic tubing on results from C.H.T. will now be discussed. Using the calibration plot, Fig. 2.4 for the volume change of the 3/16" ploythene coated nylon tubing used in Balderhead dam, an approximate quantitative analysis was made of the proportion of the measured flow rates for C4 and C5, which was due to the volume change of the tubing.

The length of the tubing used for the calibration test was 400 ft., which is approximately the distance from the piezometer house to tips C4 and C5. Thus the total length of tubing for each piezometer is around 800 ft.

For the tests in Sept. 1966 the tubing suffered a decrease in pressure from 40 p.s.i. down to 10 p.s.i. In the C.H.T. the elastic deformation of the tubing is irrelevant.

The plastic deformation may be very approximately calculated from Fig. 2.4. The results are shown in Fig. 8.1. It is seen that at $1/\sqrt{t} = 0.5$ the flow rate of water due to the contraction of the nylon tubing may have been comparable to that flowing from the soil, but at smaller values of $1/\sqrt{t}$, this effect diminishes. However, as it is obvious from Fig. 8.1 the actual volume change of the tubing during the tests must have been less than that shown. The tubing used for the calibration test was soaked for only a few days prior to the test, and may have imbibed more water during the test (see Vaughan, 1965). Penman (1960) showed that when the tubing is confined it suffers less deformation. The effect of repeated pressurizing and de-pressurizing during de-airing, C.H.T., etc. may diminish the plastic expansion of the tubing.

Nevertheless the plot given in Fig. 8.1 gives the order of the magnitude of the error in the measurement of the flow rate. This cóuld cause an appreciable error in the calculation for Cv, but will obviously hardly effect the value of k.

(8.2.2) In-situ F.H.T.

The theory for this test was discussed in Section 5.2 where both Hvorslev's (1950) solution for incompressible soils and Gibson's (1963) solution for compressible soils were given.

These two theories can be used to estimate the time-lag necessary for piezometers to record pore pressure changes in the soil. However, with the current types of piezometers used in earth dams this time-lag is seldom significant. More important is the use of this test as a check on the functioning of the piezometer, and in addition to obtain rough values for k and Cv.

This type of test is very simple to perform. For the case where a mercury manometer is the pressure measuring device, the valves to the tip are first closed, the mercury manometer 'disturbed' by the required amount, and the valves opened again. By plotting the equilization ratio E against log t a value for k may be obtained using Hvorslev's eq. (5.1); or better the experimental results can be fitted to Gibson's (1963) curves from which values of k and Cv (or Cs) may be obtained.

Unfortunately, there are several difficulties in interpreting the results when the tests are performed on closed hydraulic systems. The most serious being the loss in the applied pressure excess (or deficiency) due to the elastic and plastic expansion of the connecting tubing, where typically it may amount to as much as 30%. Another difficulty arises, when using Gibson's curves, in the necessity for a trial and error method. But even then these curves are not sensitive to the parameter μ and hence only give approximate values for Mv and Cv.

Vaughan (1965) discusses these difficulties and gives the following relationship for the loss in applied head due to the elastic deformation of the tubing:

$$p' = p\left(\frac{v_1}{v_1 + v_2}\right) \tag{8.1}$$

where p is the out of balance pressure applied to the manometer before opening the valve to the tip, p' is the net out of balance pressure after the tubing suffered elastic deformation, and v_1 and v_2 are the volume factors for the manometer and the connecting tubing respectively.

For a 3/16" nylon mercury manometer typically used in earth dams, $v_1 = 25 \times 10^{-4}$ cm^5/gm. For a typical tip 340 ft. away from the piezometer house connected by 3/16" nylon tubing, $v_2 = 10 \times 10^{-4}$ $cm^5/gm.$, (Penman, 1960). Thus from Eq. (8.1) $p'/p = 70\%$.

F.H.T. were performed on most of the tips in the clay core in the period Aug. 1963. For some tips two tests were run, a swelling test followed by a consolidation test. The results are shown in Figs. 8.5, 8.6, 8.7 and 8.8. In all the tests except those shown in Fig. 8.8 only one manometer was used.

When fitting Gibson's curves to the experimental results k was assured to be 6×10^{-9} cm./sec. and a value for Cv was chosen which gave the best fit. The value for h(o) (see Section 5.3.2) was taken as the reading at t = 15 secs. To estimate the error involved in making this assumption recourse can be made to eq. (5.15). However, it is more convenient to use eq. (5.12) and to assume that the pressure head remains constant from the time t = 0 to t = 15 secs. The calculated flow divided by the volume factor of the system would then give the approximate pressure fall during this period. For the cases considered here this amounted to 0.5 to 1.0 ft. of water which is less than 3% of the applied Δp.

The results given in Figs. 8.5, and 8.6 show that the experimental results correspond much closer to Gibson's (1963) curves than to Hvorslev's (1950) theory. Another interesting feature here is the difference in the E - log t plots for swelling and consolidation tests. Initially E drops more quickly in the swelling test, as it is governed by Cs, than for the consolidation test where it is governed by Cv. For the latter half of the plots both are governed by a value approximately equal to Cs.

Fig. 8.8 shows results for F.H.T. after a 5 minute and 24 hour C.H.T. where both manometers were used. These results confirm the high values of Cv obtained from the C.H.T.

(8.2.3) Calculation of Cv from field dissipation records

The field records of the pore pressure dissipation in the clay core during the winter shut-down seasons were analysed. The clay was assumed to drain in the horizontal direction only, to the much more permeable shale shoulder on the upstream side and the filter on the downstream side.

Ignoring the gravity potential Taylor's (1948) curves were used and the results are shown in Fig. 8.9b. An alternative to using Taylor's curves would be to solve Terzaghi's equation numerically (see Vaughan, 1965), but this is hardly justified here since there are at any level, only one or two piezometers across the whole width of the core.

(8.2.4) Laboratory values for Cv and k

There are a few laboratory results obtained from triaxial consolidation and direct permeability tests on undisturbed U4 samples from the rolled clay core. Two types of clay were identified, the more clayey soil with about 20% clay fraction and the more sandy boulder clay as from the foundation of the dam. The results are given in Fig. 8.9b.

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* These tests were performed by Mr. J. Smith, a site engineer.

(8.2.5) Comparison of results from the field and laboratory

Owing to the rather variable nature of the fill and placement water content direct correlation of the vlaues of k and Cv is not easy to make. However, some general trends can be seen where k decreases and Cv increases with increasing effective stress.

Another difficulty arises when comparing results. Ideally k and Cv should perhaps be plotted against the void ratio instead of the effective stress. But even then a variation in k and Cv may exist between say the horizontal and vertical directions, at the same void ratio. The plot shown in Fig. 8.9b is clearly then only a convenient method of plotting the results. It also perhaps serves to show the order of the change in k and Cv with increases in the effective stress.

The C.H.T. results for Cv and k are seen to agree reasonably well with other data except where the values of Cv are derived from repeated tests on the same piezometers, as was mentioned earlier in Section (8.2.1).

Vaughan (1965) makes a comparison between the value of mv obtained from averaging the C.H.T. results for C4 and C5 for the tests in Sept. 1964 with the value of mv deduced from the settlement gauge readings at roughly the same effective overburden pressure. The former gives $3.1 \times 10^{-6} cm^2/gm$ and the latter 4.8 x $10^{-6} cm^2/gm$ which show reasonable agreement. However, it must be mentioned here that the mv value from the settlement records is only approximate since no correction for the lateral plastic deformation of the core was made.

(8.3) Fiddlersferry Embankment

The 36 ft. high embankment was built in 1964-1965 as part of the ash disposal scheme next to a thermal power station which is to be completed in 1968. The scheme involved building lagoons for the disposal of the ash, as for Peterborough embankment. (Section 7.4). The construction of the power station and embankment is described by Smith and Rennie (1967).

The foundation of the embankment consisted of recently deposited layers of sands, silts, clays and peat. The embankment itself was constructed of silty clay won from the foundation of the power station and elsewhere.

There are two instrumented sections, one where the foundation strata contained clay in only 5'-10' thickness; and the other where the embankment was probably situated on the weakest part. Fig. 8.10 shows the latter section, and the location of the one I.C. tip in the fill and 16 Casagrande tips in the foundation. In addition to the piezometers, hydraulic settlement gauges of the type shown in Fig. 3.2 were installed at the same locations as the piezometer tips.

(8.3.1) Pore pressure records

Fig. 8.10 shows some of the high pore pressure which developed in the foundation due to construction load. The only drainage facilities were two narrow toe drains. This meant that the foundation clay and peat strata had to dissipate mainly downwards into the coarse sand layer where the piezometric pressure was nearly zero. Some dissipation must have occurred upwards into the fill, as will be discussed in Section (8.3.3).

(8.3.2) C.H.T. results

The author performed three sets of C.H.T. on the piezometers at approximately one year intervals. The results are shown in Figs. 8.11 to 8.15 and tabulated in Table 8.2. Values of k were calculated using both equations 5.12 and 6.24.

For the effective stress before the test, σ_i', the average of the effective vertical and two horizontal stresses was taken. Using elastic theory plane strain condition was adsumed, and the value of Poisson's ratio for the horizontal direction across the width of the dam was put equal to 0.5. Jurgensson's (1934) tables were used to evaluate the vertical and horizontal stresses due to the weight of the fill. These are only approximate since the horizontal shear stresses between the fill and the foundation are ignored. For the contribution to the effective stress due to self weight of the foundation strata above the piezometer tips, Ko conditions were assumed. The value of Ko was deduced from Jaky's empirical relationship (1948):

$$Ko = 1 - \sin \phi' \qquad (8.2)$$

Fig. 8.16 shows the C.H.T. results together with other field and laboratory data. In Fig. 8.17 the results for the tests on piezometers 19 and 22 are plotted again where it is seen that the agreement for k is remarkably good, whether calculated according to eq. (5.12) or (6.24). However, here again the values of Cv obtained from the repeat tests are too high as was the case for the tests in Balderhead, Section (8.2.1)

Referring to Fig. (6.6) point C for piezometer 22 moved down to an average effective stress, σ'_{Av} of 8.4 p.s.i. during the first C.H.T. in Nov. 1964. After the test it swelled back to σ'_{Av} = 3.0 p.s.i. (see Table 8.2). On Nov. 1965 the change in σ'_{Av} was only 1.7 p.s.i. which was not enough to bring back point C to the 'virgin' consolidation curve. This perhaps explains the high value of Cv obtained for the Nov. 1965 test. However, by Nov. 1966 σ'_{Av} increased to 10.7 p.s.i., which seems to have brought the point C back on the virgin consolidation curve and hence the low value of Cv for that test.

A similar analysis for piezometer 19 showed that the Δu's used were smaller and the increases in σ'_{Av} larger leading to more consistent values for Cv.

(8.3.3) Calculation of Cv from field dissipation records

Assuming that the clay and peat strata can be treated as one layer, Taylor's (1940) curves were used to deduce a value for Cv from the pore pressure dissipation records during the shut-down seasons. The C.H.T. on the tips near the peat layer gave values for k similar to those for the overlying clay layer, as shown in Table 8.2. However, the plots, Fig. 8.15, showed consistent upward curvature which may reflect the secondary consolidation of the peat layer. Nevertheless it is probably a fair assumption to treat the two layers as one stratum.

A more serious difficulty arose as whether to assume single or double drainage; and whereas the drainage condition at the lower boundary, the sand layer, was well defined, the drainage conditions at the interface with the fill is not specified. The C.H.T. on piezometer 36 gave a value for k at least 4 times that of the foundation strata. Also this piezometer showed (for clarity the records is not given in Fig. 8.10) an increase in pore pressure during the shut-down season, which would perhaps indicate that a back pressure existed in the fill which would have slowed down the upward dissipation of the pore pressure from the foundation. Two sets of values of Cv were calculated assuming single and double drainage. The results are tabulated in Table 8.3, and will be discussed further in Section (8.3.5).

(8.3.4) Laboratory values for k and Cv

Due to the nature of the foundation strata the initial site investigation was quite comprehensive. In addition to the tests to determine the strength of the soil, oedometer and triaxial dissipation tests were performed to evaluate Cv and k. The values of Cv obtained ranged from 5 to over 50 $ft.^2/yr.$ However, oedometer tests on samples from near the instrumented section shown in Fig. 8.10 gave consistent values for Cv of around 7 $ft.^2/year.$ The value for k deduced from oedometer tests ranged from 25 down to 2×10^{-8} cm./sec.

(8.3.5) Comparison between field and laboratory values of k and Cv

The values for k obtained from field tests show a consistent behaviour, Fig. 8.16, and agree well with the only set of laboratory data.

The average value for Cv deduced from C.H.T. is around 50 to 100 $ft.^2/yr.$, allowing for the effect of repeated testing on the piezometers. The laboratory values indicate a much lower value of Cv. Most of the triaxial dissipation tests gave 2 to 10 $ft.^2/yr.$ Oedometer tests on samples from near the instrumented section gave consistent values for Cv in the horizontal and vertical directions of 5 to 10 $ft.^2/yr.$ However, a set of oedometer tests on samples taken further away gave 30 to 60 $ft.^2/yr.$

The values deduced from the field dissipation of pore pressure show that if double drainage is assumed, i.e. the clay and peat layers drained to the lower sand layer and to the fill, then the Cv values obtained are in agreement with the low laboratory results. However, if single drainage is assumed then the values agree with those of the C.H.T. It is quite probable that the actual drainage conditions in the field are in between these two limits. That some drainage occurred upwards is beyond doubt, since the permeability of the fill is 4 times higher than that of the foundations. However, piezometer 36 showed an increase in pore pressure during the shut-down season. Due to the width of the embankment and the added side stabilizing berms this build-up of the pore pressure in the fill was probably inhibited from dissipating quickly to the outside. In fact just after completion of the embankment the piezometric pore pressure recorded by 36 was 20 ft. higher than foundation level. Allowing for the fact that this tip is 5 ft. higher than the foundation level, the actual pore pressure there could have been higher. From this it can perhaps be concluded that most of the drainage from the foundation strata must have occurred in the downward direction only.

To sum up, it seems that the field and laboratory values of k are in reasonable agreement. However, laboratory values for Cv seem to be on the low side compared to the field values. A very possible factor here is the question of sample size. The soil is a recent estuarine deposit. It is laminated and is a mixture of clay, silt and sands. It might be that the laboratory samples taken were more often from the intact clay, as they are usually easier to handle. The field C.H.T. results, on the other hand, represent bigger samples, and although they suffer from errors introduced by repeated testing, probably furnish more accurate values for Cv.

(8.4) M.6 Trial Embankment

This 30 ft. high embankment was constructed 6 miles from Kendal as a trial section for the proposed extension of the M.6 Motorway to the North. The proposed route passes through a hilly area covered with a blanket of boulder clay. For filling in depressions the possibility of utilizing all the clay including the wetter top layer was investigated.

A section 600 ft. long by 200 ft. wide was built of boulder clay and resting on a boulder clay foundation. For the study of worst conditions, the clay was compacted at the highest moisture content practically possible. Along half of the embankment a drainage blanket was placed at mid-height and a third of the height as shown in Fig. 8.18. 60 Imperial College tips were installed in close spacing to study the pore pressure distribution in the embankment. These were connected to mercury manometers in a piezometer house. To measure vertical deformation twenty cells, in four groups, of the Road Research Laboratory mercury settlement gauge type (see Section 3.2.3) were used. Two groups were at foundation level, one in section CC and the other below BB. These units behaved satisfactorily. The other two units in the fill ceased to work when the embankment experienced some lateral deformation, as already referred to in Section 3.2.3.

(8.4.1) Pore pressure records

Nearly all the I.C. tips behaved satisfactorily giving a very useful picture of the build-up and dissipation of the pore pressure. An interesting feature here was that more than half of the piezometers had recorded maximum pore pressures in excess of the overburden pressure. In some cases the value of $\bar{B}$ was 1.4, but it must be remembered that the pore pressure never exceeded the overburden pressure by more than 2 or 3 p.s.i. It was thought first that this may have been due to the presence of air bubbles in the tips, but careful de-airing revelaed no air bubbles. It was then thought that this effect might be due to the compaction action of the plant, where horizontal stresses are locked in the soil when the plant had moved away. This in fact was investigated by running a light plant on an area where a piezometer tip was situated at a depth of about 3 ft. When the plant was vertically above the tip the pore pressure increased by some 2 p.s.i., of which about 1 p.s.i. remained when the plant moved away.

This phenomenon is not new. Penman (1956) reported a similar observation on piezometers in Usk dam and came to the same conclusion regarding the effect of the compacting plant.

Some of the pore pressure records are shown in Figs. 8.24 to 8.31, from which values of Cv were calculated as will be discussed in Section 8.4.3.

(8.4.2) C.H.T. results

An apparatus of the type described in Section (2.7.1) was installed in the piezometer house to find the in-situ value for Cv and k. The test results as shown in Fig.s 8.19, 8.20 and 8.21 are tabulated in Table 8.4. A marked feature of these results is the downward curvature of the q v . $1/\sqrt{t}$ plots for small values of t. This cannot wholly be explained by head losses in the tubing and may in fact be due to soil disturbance caused by installation of the tips, de-airing, previous seepage testing etc., as discussed in Section (6.8).

(8.4.3) Calculation of Cv from pore pressure dissipation

The pore pressure records obtained from this embankment are perhaps some of the best the Author has seen. The fill was homogeneous and saturated for most of the embankment, and there were a large number of tips to monitor the development and dissipation of the pore pressure.

The dissipation records were utilized to obtain values for Cv. One dimensional drainage was assumed throughout. This is certainly a fair assumption for section BB for example, but perhaps less so for section CC.

Two methods were used. The first was Terzaghi's theory while the second was an adaptation of Barden and Berry's (1965) non-linear approach already discussed in Section (6.7.2).

The boundary conditions will now be considered. The embankment was constructed in the period June 5th to July 15th, 1966 during which time some dissipation of pore pressure must have taken place. However, it may be safely assumed that there were no change in the total stress after July 19th, 1966.

During the initial process of consolidation the compacted fill would drain both upwards and downwards, it already has been assumed that consolidation took place in the vertical direction only. If the top of the embankment and the drainage blankets are prevented from drying, then the local piezometric pressure there would be zero, relative to atmospheric pressure. Later on, and due to the gravity potential, the fill could cease to drain upwards and is restricted to the downwards direction only. To maintain the piezometric pressure at zero at the top of the clay bed a constant supply of water must be provided; otherwise capillary pressures could be set up. To avoid unnecessary complications in analyzing the data, the surface of the embankment was in fact covered with a plastic sheet shortly after completion. The balnkets themselves seem to have maintained their zero piezometric pressure to within 1/2 p.s.i. However, the top of the embankment must have suffered a slight drying out as indicated by piezometers near the surface. Nevertheless in this discussion it will be assumed that the piezometric pressure at all the drainage boundaries is zero, relative to atmospheric pressure.

(8.4.4) Use of Taylor's curves

Referring to Fig. 8.2.3 Terzaghi's equations for one dimensional consolidation is:

$$Cv \frac{\partial^2 u}{\partial Z^2} = \frac{\partial u}{\partial t} \qquad (8.3)$$

where u is an excess pore pressure related to the piezometric pressure ρ_w by the following relationships:

$$u = \rho_w + \gamma_w Z \qquad (8.4)$$

Here the datum is conveniently taken as the bottom drainage blanket. If ρ_w is used in Eq. (8.3) instead of u then the gravity potential is neglected. This would lead to an error whose magnitude depends in this case on the relative height of the bed under consideration to the overall height of the embankment.

As a first approximation use was made of Taylor's (1948) curves. It was assumed that the embankment was constructed instantaneously at an appropriate date, say end of June 1966. The final piezometric pressure will have to be assumed equal to zero. This as was already mentioned will only be true if the drainage boundaries are maintained throughout the consolidation process at zero piezometric pressure. Rather than use the pore pressures given by particular piezometers, an average degree of consolidation was used to derive values for Cv. These are shown in Fig. 8.22.

(8.4.5) Numerical solution for Terzaghi's equation

An alternative method is by solving Eq. 8.3 numerically. This method has two advantages over using Taylor's curves. Firstly, it does not require the assumption regarding the instantaneous construction of the dam. Secondly, actual pore pressures may be used as the initial values.

Here Eq. (8.3) is solved numerically using the marching forward method described in Section (10.4.1), with an appropriate value for β. From the number of steps taken and the chosen value of β a value for Cv may be calculated. For more details of this method the reader is referred to Vaughan (1965).

A simple computer programme was written to perform these calculations using small values for β. The sections analysed were WB, XB, YC and YB (Fig. 8.18). These sections gave consistent records which seem to indicate reasonable homogeneity of the fill.

The computer results for the pore pressure distribution are shown in Figs. 8.24, 8.26, 8.28 and 8.30. By comparing the average computed pore pressure distribution to the actual average distribution, values for Cv were calculated and plotted in Fig. 8.22.

(8.4.6) Use of Barden and Berry's (1965) non-linear approach

The values for Cv derived from the numerical analysis presented in the previous section showed quite clearly that Cv increased with increasing effective stress. Since an implicit assumption in Terzaghi's theory is that Cv should be constant, the results were naturally in some doubt.

To overcome this difficulty the numerical analysis of the last section can be modified to deal with a variable Cv. This may be done, for example, by dividing the clay layer into finite thicknesses each with a different assumed Cv. During a small change in the effective stress the value for Cv for a particular layer may be assumed to remain constant. By a trial and error method the initial guessed values for Cv may be modified until agreement is obtained between the calculated and observed pore pressures. Here it must be emphasised that the boundary condition (see Abbott, 1960),

$$k_I \left(\frac{\partial u}{\partial z}\right)_I = k_{I+1} \left(\frac{\partial u}{\partial z}\right)_{I+1}^{*} \qquad (8.5)$$

must be incorporated in the analysis between every mesh point and its neighbours, unless the permeability of the clay is

* I refers to the layer number

assumed to be a constant.

The above analysis is basically a linear approach to the problem where small thicknesses of the soil are assumed to obey Terzaghi's theory. A more direct method is the non-linear theory proposed by Barden and Berry (1965), as already described in Section (6.7.2).

This method was used to derive a non-linear differential equation to analyse the field results.

It will be remembered that the e - ln σ' plot and lnk - e plot are assumed to be linear, i.e. (c.f. Eqs. 6.4 and 6.30):

$$m_v = m_{vo} \left(\frac{\sigma'}{\sigma_o}\right) \qquad (8.6)$$

$$k = k_o \left(\frac{\sigma'}{\sigma_o}\right)^a \qquad (8.7)$$

where the subscript 'o' refers to the property of the soil at z = 0, Fig. 8.23.

For continuity of the pore fluid we have

$$\frac{\partial}{\partial z}\left(\frac{k}{\gamma_w}\,\frac{\partial u}{\partial z}\right) = \frac{\partial}{\partial t}\left(\frac{e}{1+e}\right) \qquad (8.8)$$

where u is the excess pore pressure, defined in Eq. (8.4). If we assume small strains we have.

$$\frac{\partial}{\partial z}\left(\frac{k}{\gamma_w}\frac{\partial u}{\partial z}\right) = \frac{e}{1 + e}\frac{\partial e}{\partial t} \tag{8.9}$$

Now,

$$\sigma' = \sigma - (u - z\gamma_w) \tag{8.10}$$

and if the total stress remains constant with respect to time, we have

$$\frac{\partial \sigma'}{\partial t} = -\frac{\partial u}{\partial t} \tag{8.11}$$

Moreover, assuming the void ratio to be a function of the effective stress only, and using Eq. (8.11), we may rewrite Eq. (8.9)

$$\frac{1}{\gamma_w}\left(\frac{\partial k}{\partial z}\frac{\partial u}{\partial z} + k\frac{\partial^2 u}{\partial z^2}\right) = -\frac{1}{1 + e}\frac{de}{d\sigma'}\frac{\partial u}{\partial t} \tag{8.12}$$

Now

$$\frac{\partial k}{\partial z} = \frac{dk}{d\sigma'}\frac{\partial \sigma'}{\partial z} \tag{8.13}$$

From Eq. (8.7)

$$\frac{dk}{d\sigma'} = -a\left(\frac{\sigma'_o}{\sigma'}\right)^a \frac{1}{\sigma'} \cdot k_o \tag{8.14}$$

Also

$$\sigma' = \gamma_s(T - H) - (u - \gamma_w z) \tag{8.15}$$

where T is the overburden pressure above the datum plane and H is the thickness of the layer under consideration. Here γ_s is the bulk density of the soil, and γ_w the density of water. It follows that

$$\frac{\partial \sigma'}{\partial z} = -\gamma_s + \gamma_w - \frac{\partial u}{\partial z} \tag{8.16}$$

substituting Eqs. (8.14) and (8.16) into Eq. (8.12) we have:

$$\frac{k_o}{\gamma_w}\left\{\frac{a}{\sigma'}\left(\frac{\sigma'_o}{\sigma'}\right)^a\left[(\gamma_s-\gamma_w)+\frac{\partial u}{\partial z}\right]\frac{\partial u}{\partial z}+\left(\frac{\sigma'_o}{\sigma'}\right)^a\frac{\partial^2 u}{\partial z^2}\right\}=$$

$$mv_o\,\frac{\sigma'_o}{\sigma'}\,\frac{\partial u}{\partial t} \qquad (8.17)$$

Using Cv_o which is the coefficient of consolidation at the datum plane, we arrive at the final form of the differential equation:

$$Cv_o\left\{\frac{a}{\sigma'}\left(\frac{\sigma'_o}{\sigma'}\right)^{a-1}\left[\gamma_s-\gamma_w+\frac{\partial u}{\partial z}\right]\frac{\partial u}{\partial z}+\left(\frac{\sigma'_o}{\sigma'}\right)^{a-1}\frac{\partial^2 u}{\partial z^2}\right\}=\frac{\partial u}{\partial t} \qquad (8.18)$$

Here Cv_o remains constant throughout the process of consolidation.

If we put $a = 1$ and $\gamma_s = \gamma_w$ in Eq. (8.18) the resulting expression is identical to that given by Davies and Raymond (1965) who treated laboratory consolidation where the gravity potential was neglected.

Eq. (8.18) was solved numerically on the computer, using values for β of up to 0.5, and with the clay bed divided into 10 or 12 mesh intervals. No oscillation was encountered although these figures are considerably higher than those recommended by Barden and Berry (1965). A possible explanation for this stability is that the initial pore pressure contours fed into the computer were smooth and continuous, unlike the step-function distribution obtaining just after the application of the total stress.

The pore pressure readings for the 19 July 1966 were fed into the computer. Other input data were the effective stress σ'_o, Cv_o and 'a'. The value for σ'_o was increased by about 2 to 3 p.s.i. to avoid obtaining negative effective stresses at the middle of the layer during the initial consolidation process. This problem arose because initially $\bar{B}$ was greater than one as already mentioned.

A more serious difficulty arose in the choice of values for Cv_o and 'a'. These were varied until the computed pore pressures approximated to the recorded pore pressures at some specified times. As a first approximation the relationship between Cv and σ' obtained using Terzaghi's equation and shown in Fig. (8.22) was used by putting Cv_o = 60 ft.^{2}/yr. and 'a' = 0.5. These values were then fed into the computer to analyse the results for section YB. The calculated results are not shown but they fell appreciably short of the recorded values. Several other runs were made with higher values of Cv_o and/or 'a'. Some of these results are shown in Figs. 8.25, 8.27, 8.29 8.30 and 8.31, and are discussed in the next section.

The factor 'a' will now be examined in more detail. It was shown in Section (6.7.2) that 'a' is given by:

$$a = -Cc.m$$

where Cc is the slope of the e - ln σ' linear plot, and m that of the lnk - e linear plot. Unfortunately in this case no laboratory data exist from which values of Cc and m may be obtained. However, using the results form C.H.T., approximate values for Cc may be calculated.

By definition:

$$m_v = -\frac{de}{d\sigma'} \cdot \frac{1}{1+e} = - Cc.\frac{1}{1+e} \cdot \frac{1}{\sigma'} \qquad (8.19)$$

A value for the void ratio e can be estimated using the relationship

$$w = Se/G \qquad (8.20)$$

where w in the water content, S the degree of saturation and G the specific gravity of the soil grains. In this case S is very nearly equal to one. The value of Cc thus deduced is of the of -0.02. For m there exists no laboratory or acceptable field record. However, for 'a' to be equal to one say m must be equal to 50. This means that if the void ratio decreases by 0.01 k is halved, which is somewhat unrealistic.

Raymond (1966) who adopted a very similar approach gives more realistic values for Cc, m and 'a', for four clays and silts. His values for Cc ranged from -0.12 to -0.28, and for m from 3.2 to 7.9 . These gave for 'a' three values at about 0.6 and one at 2.2. However, Raymond mentions that he obtained his values for k from F.H.T. results using Eq. 5.1.for incompressible soils, which must have influenced the accuracy of his results.

(8.4.7) Comparison of results

The comparison given here unfortunately lacks the laboratory data and field settlement records. Nevertheless it serves the important purpose of outlining the differences between Terzaghi's classical approach and the more recent theories of Barden and Berry (1965) and Davis and Raymond (1965).

Using Terzaghi's theory the value of Cv calculated increased from about 15 to 50 ft.^{2}/yr. as shown in Fig. (8.22). Using the non-linear Eq. (8.18) indicated higher values of Cv of around80 ft.^{2}/yr. with a value for 'a' of 0.70 to 1.0. However, neither approach was consistent when considering different sections of the embankment. For example, Terzaghi's theory gave good results for sections WB and YB, but less so for sections XB and YC. On the other hand using Cv_o = 70 to 90 ft.^{2}/yr. and 'a' = 0.70 to 1.0 which fitted sections YB and WB did not give a good fit for sections XB. A possible explanation for these inconsistencies is the inaccuracy of the pore pressure distribution fed into the computer to 'start' the numerical analysis; for although the piezometers were spaced as close as 1' - 6" for some of the sections, the pore pressures near the blankets were not clearly defined. In the numerical analysis small mesh sizes were used, some 6" and some 12", but the initial pore pressures had to be interpolated for the points between the piezometers. It is suggested here that for future field tests of this type the positioning of the tips should not be linear, i.e. not at the $\frac{1}{4}$ and $\frac{1}{2}$ points as in this case. Instead the outer tips should be moved nearer to the blankets to achieve a better definition of the pore pressure where the gradients are highest. In addition, the more compact Building Research Station disc type may be used fitted with a high air-entry ceramic which would narrow the range over which the pore pressure is measured.

The C.H.T. results for Cv, Fig. 8.22, give consistent results, in agreement with the values derived from the dissipation records, apart from a few high values. The increase in Cv with σ' here is not obvious as was the case for results calculated from the pore pressure dissipation using Terzaghi's equations.

(8.5) Draw down test on Muirhead dam: Introduction

A draw down test was performed on this dam in Sept. 1966, by Professor A. W. Bishop, Mr. A.D.M. Penman, the Author and Mr. G. Tombs helped by engineers of the Consultants on the project Messrs. Babtie, Shaw and Morton, Glasgow. Mr. A.D.M.Penman in a short private report to the Consultants described the test. In this discussion extracts from this report will be given. The Author helped in performing the draw down test, conducting field seepage tests and in analysing the draw down test results.

Muirhead dam was constructed during the period 1938 to 1943 across the Rye Water, about 5 miles east of the town of Largs in Renfrew to provide water for Paisley. The dam was constructed from boulder clay with a central puddled clay core and was to be 90' high. By the end of 1940, the bank was only about $\frac{1}{3}$rd. of its full height and because the need for water had become urgent due to wartime conditions, another contractor with more modern earth moving equipment was engaged. In September 1944 when the bank had reached a height of 70 ft. a shear failure occurred, as described by Banks (1948).

Analysis of this failure by the Building Research Station showed that the shear strength of the fill placed by the first contractor was not sufficient to allow the bank to be built to its full height without considerable widening which would not be possible because part of the spill-way and tunnel works, etc. had already been completed. As a remedial measure, rockfill was placed at the upstream toe of the clay and the crest moved upstream so as to reduce the weight of the downstream part; the final height being 73 ft. instead of the originally intended 90 ft. The dam in this form was completed in 1943 and the reservoir has since held water satisfactorily.

After 20 years the Corporation of Paisley asked if it was possible to raise the dam to its original height and as a result of this enquiry the advice of the Building Research Station was sought. In September 1963 boreholes were put down in the dam and samples obtained ot measure the shear parameters of the soil. At the same time 24 Casagrande type piezometers were placed at the various depths in the dam along its highest cross-section some of which are shown in Fig. 8.35.

This investigation revealed two very interesting features,
(1) the strength of the lower fill, which had been placed by the first contractor was found to be almost the same as at the time of the failure in 1941, and the strength of the upper fill placed by the second contractor was less than at the time of the 1941 failure.
(2) the piezometric level measured was above reservoir level; in fact, it was above the surface of the dam at its crest. However, it was later discovered that some of the piezometers recording these high pore pressures were blocked and may in fact have been acting as 'rain-gauges'.
Nevertheless these two factors indicated that not only could raising the dam be difficult but its stability at present might

be in question especially under a rapid draw down. The stability of the existing dam was analysed with the aid of a DEUCE Computer using the method of Bishop (1955), and Little and Price (1958). Shear strength parameters measured in the laboratory from samples taken from the dam together with the measured pore pressures were used in the analysis. The factor of safety when the reservoir is full was found to be 1.25, which apparently fell to 1.08 if the reservoir was suddenly drawn down 10 ft. However, subsequent calculations on the computer gave higher values.

(8.5.1) Permeability tests

The investigation carried out in 1963 was performed by Messrs. G. Wimpeys who issued a report on their investigations. They gave results from laboratory triaxial dissipation tests on undisturbed samples taken from the fill. The values of k obtained were of the order of 10^{-9} cm./sec.; and for Cv about 50 ft.^{2}/yr. However, a test on a sample from a depth of 45 ft. in the borehole for piezometer 25 gave values for k and Cv of about 10^{-6} cm./sec. and 1000-2000 ft.^{2}/yr. The report also gives results from field falling and constant head tests on the piezometers. The measured permeabilities were of the order of 10^{-6} cm./sec.

The Author repeated the constant head tests on most of the piezometers in 1965, using the unit described in Section (2.7.2). The tests performed were of the swelling type, care being taken not to exceed the overburden pressure above the tips. The results for k vary between 10^{-6} down to 10^{-7} cm./sec., while Cs varied between 200 and 1000 ft.^{2}/yr. For some of the tests the flow rate measured was in fact the maximum possible through the apparatus, and hence the permeability of the soil may have been even higher.

If the permeability of the fill is in fact of the order of 10^{-9} cm./sec. then some of the high pore pressure measured in the dam, if they were genuine, could conceivably be residual pore pressures left from the time of construction. On the other hand, if the permeability is of the order of 10^{-6} cm./sec. then these high pore pressures may be caused by artesian pressures developed by water running from the valley sides under the dam. Another possibility which is less probable is that these high water pore pressures may be caused by the continuous generation of organic gas within the fill.

It was felt that the only way to assess the overall permeability and to check on the existing stability of the dam was to carry out a drawdown test.

In order to measure the settlement and lateral movement of the crest of the dam, a number of monuments were established along the crest and on two cross-sections going part way down the downstream slope of the dam. The inverted pendulum described in Section (3.3.3) was installed in the valve tower with the bottom end of the wire connected to the rock foundation at the bottom of the tower, and used as a reference to show up any movements of the monuments.

(8.5.2) The draw down test

The water level in the reservoir before the test was sufficiently long at overflow level for a steady state to exist. The test started on September 5th, 1966, and on 9th September, the water level was lowered by about $9\frac{1}{2}$ ft. Piezometer readings were taken 3 times a day. The positions of the monuments were obtained by surveying methods about once in a day, although the weather conditions made some of the readings very difficult. Five lines of pins were also established on the upstream face of the dam by placing screws into the rocks forming the pitching as the level of the water went down. The distances between the pins were measured frequently to detect any movement of the pitching.

The pore pressures measured are shown in Fig. 8.32. Prior to the test stability analysis were carried out for a

10 ft. draw down. In this analysis the distribution of the pore pressure at the end of the draw down was assumed to vary linearly from that of the overflow level at the bottom of the foundation to the draw down level at the surface of the dam. This assumption was based on observations from similar tests on the Glen-Shira dam, described by Paton and Simple (1960), and Vaughan (1965). The assumed pore pressures were found to be in very good agreement with the measured values. The stability analysis gave a factor of safety of 1.44 for deep slips, but progressively less against shallower slip surfaces.

When the reservoir level dropped by about $9\frac{1}{2}$ ft. it was attempted to hold it constant to study the effects of dissipation. This was found difficult to do especially since it coincided with a heavy rainstorm as shown in Fig. 8.32. However, during the four days which it was held 'constant' the pore pressures in the upstream side of the dam continued to dissipate. Deformation measurements on the monuments and pins revealed no detectable movements.

(8.5.3) Theoretical analysis of results - Bishop's method

There are at present two theoretical methods for predicting pore pressures during a draw down. The first is Bishop's (1952) method where for a reasonably rapid draw down the fill is assumed to be impervious, and any changes in the pore pressures would be due to changes in the total stresses only. Use can then be made of Skempton's (1954) pore pressure coefficients A and B:

$$\frac{\Delta u}{\Delta \sigma_1} = \bar{B}\left[1 - (1 - A)\left(1 - \frac{\Delta \sigma_3}{\Delta \sigma_1}\right)\right] \tag{8.20}$$

$\Delta \sigma_1$ being the change in the major principal stress

$\Delta \sigma_3$ being the change in the minor principal stress

Δu being the change in the pore pressure.

Bishop then makes two further assumptions. The first is that the change in the vertical stress, due to the removal of the water load, is approximately equal to the change in the major principal stress. In the second assumption $\bar{B}$ is put equal to 1.0. Laboratory tests by Bishop (1954) and Bishop and Henkel (1962) suggest that the value for fully saturated soils is $\simeq$ 1.0 and for partly saturated soils is $<$ 1. Bishop (1952) analysed field results for the Alcova dam and found that the value of $\bar{B}$ is nearly equal to unity.

Fig. (8.33) shows the pore pressures, for Muirhead dam, in the upstream side plotted against the reservoir level. It is seen that the $\bar{B}$ values obtained in the field are consistently less than 1.0, as also shown in Table 8.5.

(8.5.4) Reinius' method

An alternative to Bishop's approach is the Reinius' (1948) method. Here it is assumed that the soil skeleton is incompressible and hence not affected by changes in the total stress. It is also assumed that the water level is drawn down instantaneously whereby a steady-state flow of water occurs from the upstream body of the dam to the upstream face and bottom drainage blanket, if any. It is also assumed that the impermeable barrier has a permeability infinitely lower than that of the upstream fill, and hence the water table in the fill before the draw down corresponds to the water level in the reservoir. After draw down this water table is assumed to remain at its place with a fictitious supply of water to replace that flowing into the upstream face and bottom drainage blanket. By draning a flow-net such as shown in Figs. 8.34 and 8.35, the piezometric pressures in the upstream face of the dam can readily be found. Reinius then discusses various factors affecting his method, and concludes that by far the most significant is the presence of gas in the pores of the soil which would considerably slow down the rate of dissipation.

In Muirhead no bottom drainage blanket was incorporated in the construction. However, the readings during the test from piezometer 25 seemed to indicate that at least in this locality the fill was almost in direct communication with the reservoir level, possibly through the rock bed.

To study the possibilities tow flow-nets were drawn as shown in Figs. 8.34 and 8.35. In case 1 the rock bed was assumed to act as a drainage blanket in direct communication with the reservoir. In case 2, the rock bed was assumed to have the same permeability as the fill. The third possibility of assuming the rock bed to be impervious was discarded, on the basis of the readings from piezometer 25.

The computed drops in pore pressure from the two flow-nets are given in Table 8.5.

(8.5.5) Discussion of results

From Table 8.5 it is seen that the measured drops in pore pressure are less than those predicted by either Bishop's (1954) method using a value for $\bar{B}$ of unity or Reinius' (1948) method; although the latter, and especially case 2, is in better agreement with the field results. These two conclusions were also reached by Widdis (1964) who analysed the results from the two draw down tests on Glen Shira dam.

It thus seems from the experience gained from Glen Shira and Muirhead dams that these two methods produce unacceptably optimistic forecasts for the drop in pore pressure after rapid draw down. Perhaps the most significant factor here is the effect of the presence of gas in the pores of the soil on the rate of dissipation of pore pressure, a conclusion reached earlier by Reinius, as already mentioned. This gas may have been in the pores of the soil before the test, or released during the test due to changes in the total stress and pore pressure.

Both methods suffer from oversimplifying assumptions. An alternative and more sophisticated method is called for; such a method was outlined by Vaughan (1965).

Using elastic theory the changes in the principal stress due to the removal of the water load may be calculated, in the same manner as adopted by Bishop (1952). From laboratory samples the values of A and B may be found following the same stress path in the field and for a range of the degree of saturation. Using Eq. 8.20 the drop in pore pressure due to the stress release can then be evaluated.

The second component of the drop in pore pressure is that due to consolidation of the fill during draw down, Vaughan (1965) derived a semi-rigorous differential equation for this. Earlier Gibson (see Matyas, 1963) derived a rigorous differential equation for the consolidation of partly-saturated soils. Here again solutions must be obtained for a range of the degree of saturation, stress level, etc. The two solutions may easily be obtained, by numerical analysis on a digital computer. A drawback in this approach is the uncertainty regarding the unknown degree of saturation before the draw down. This may perhaps be overcome by careful sampling and testing. Nevertheless and at worst this approach will outline the relative importance of the two components of the drop in pore pressure for typical degrees of saturation. Finally, field records suchas those from Muirhead and Glen Shira may be analysed as a check on the whole procedure.

Piez. No.	Date Tested	P_w p.s.i.	σ_v' p.s.i.	Δu p.s.i.	K cm/sec. x 10^{-9}	Cv or Cs $ft^2/yr.$
C4	27.9.63	47.2	27.4	-27.4	6.4	95
	30.3.65	42.3	64.0	-23.8	5.8	42
	16.9.66	41.9	64.4	-30.1	5.0	1078
C5	27.9.63	43.7	31.0	-24.6	6.9	61
	30.3.63	39.7	67.0	-21.2	6.9	680
	16.9.66	40.5	66.0	-29.0	5.0	334
C7	30.3.65	8.5	68.7	- 4.3	4.0	220
C8	18.9.66	11.0	62.3	-14.0	3.0	13
C9	17.9.66	12.9	46.5	-14.6	6.4	20
C10	20.9.66	5.6	36.9	+11.5	5.9	26
C11	19.9.66	1.8	40.7	+15.7	13.0	15
C12	20.9.66	6.0	21.0	+ 3.9	8.4	49
C21	3.7.66	1.7	41.3	+ 8.6	8.4	29
	22.9.66	13.1	44.7	- 6.5	5.0	113
C23	22.9.66	5.4	32.0	- 9.0	20	3

Table 8.1

C.H.T. Results: Balderhead

Piez. No.	Date test started	Final Δu p.s.i.	σ_v p.s.i.	Av. σ p.s.i.	Av. σ' p.s.i.	K (eq.5:12) cm/sec x 10^{-8}	K (eq.6:24) cm/sec x 10^{-8}	Cv ft^2/year
16	15.11.64 30.11.65 2.11.66	-2.5 +4.8 -1.1	6.6 12.5 12.5		3.0 8.1 8.5	2.4 to 4.0 3 7	1.8 to 3.0 5 6	10 to 64 Cs = 36 550
19	13.11.64 29.11.65 31.10.66	-5.8 -3.7 -5.8	19.9 19.0 21.9	18.6 19.1 22.6	3.9 6.4 7.9	9.3 4.8 4.2	6.1 3,8 3.3	62 85 77
20	3.11.66	-7.0	26.7	25.0	10.7	11.4		
22	13.11.64 29.11.65 1.11.66	-5.4 -10.2 -8.1	19.7 26.5 34.8	17.9 23.9 35.9	3.0 4.7 10.7	11.7 8.0 4.2	7.3 4.7 3.2	117 250 39
23	1.11.66	-7.6	40.0	35.4	8.1	6.0		
25	13.11.64 1.12.65 31.10.66	-4.1 -6.5 -5.3	18.2 19.7 21.5	18.6 19.1 22.6	5.2 8.1 8.0	67 61 26	51 46 21	45 to 63 430 102
26	1.12.65	-7.0	24.4	23.8	3.0	5.8		
34	2.12.65	+3.0	13.5	13.5	9.3	3.0	2.7	
36	30.11.65 3.11.66	+1.6 -3.1	16.8 25.2		14.8 19.1	217 30	234 28	Cs = 2180 51

Table 8.2

C.H.T. results: Fiddlersferry

Piez. No.	t_1	t_2	% consolidation	'Cv' if H = 8 ' ft^2/year	'Cv' if H = 16' ft^2/year	σ'_{AV} p.s.i.
16	7.4.65	1.6.66	87	28	186	7.0
19	19.8.64	9.3.65	32	14	100	3.9
	1.10.65	1.5.66	27	13	97	6.7
22	19.8.64	9.3.65	22	9	76	3.3
	1.11.65	1.5.66	7	4	31	4.1
25	7.10.64	15.6.65	33	12	78	7.0
	7.11.65	15.5.66	18	9	72	8.7
32	1.5.65	1.1.66	45	26	150	6.0
33	1.5.65	1.1.66	45	45	102	8.5
34	1.5.65	1.1.66	95	105	400	6.9
35	1.5.65	1.1.66	95	100	400	9.4

Table 8.3

Calculation of Cv from field

pore pressure dissipation records: Fiddlersferry

Piez. No.	Date tested	P_w p.s.i.	σ_v' p.s.i.	Δu p.s.i.	K cm/sec x 10^{-8}	Cv $ft^2/yr.$
11	2.8.66	10	5.0	- 6.4	3.4	254
13	19.8.66	11.9	2.0	- 8.7	5.4	264
14	19.7.66	14.9	-0.8	- 9.4	4.9	88
16	20.7.66	12.4	1.4	- 6.5	4.7	112
19	10.8.66	12.6	-0.4	-12.2	4.2	41
26	9.8.66	7.4	0.3	- 8.2	5.1	29
42	29.7.66	13.3	2.6	- 8.8	3.5	29
44	6.8.66	12.8	0.3	-10.3	5.0	69
47	7.8.66	10.2	1.6	- 8.1	6.9	30
50	20.8.66	7.5	1.4	- 7.9	6.3	48
54	1.8.66	8.9	-0.5	-8.7	2.9	53

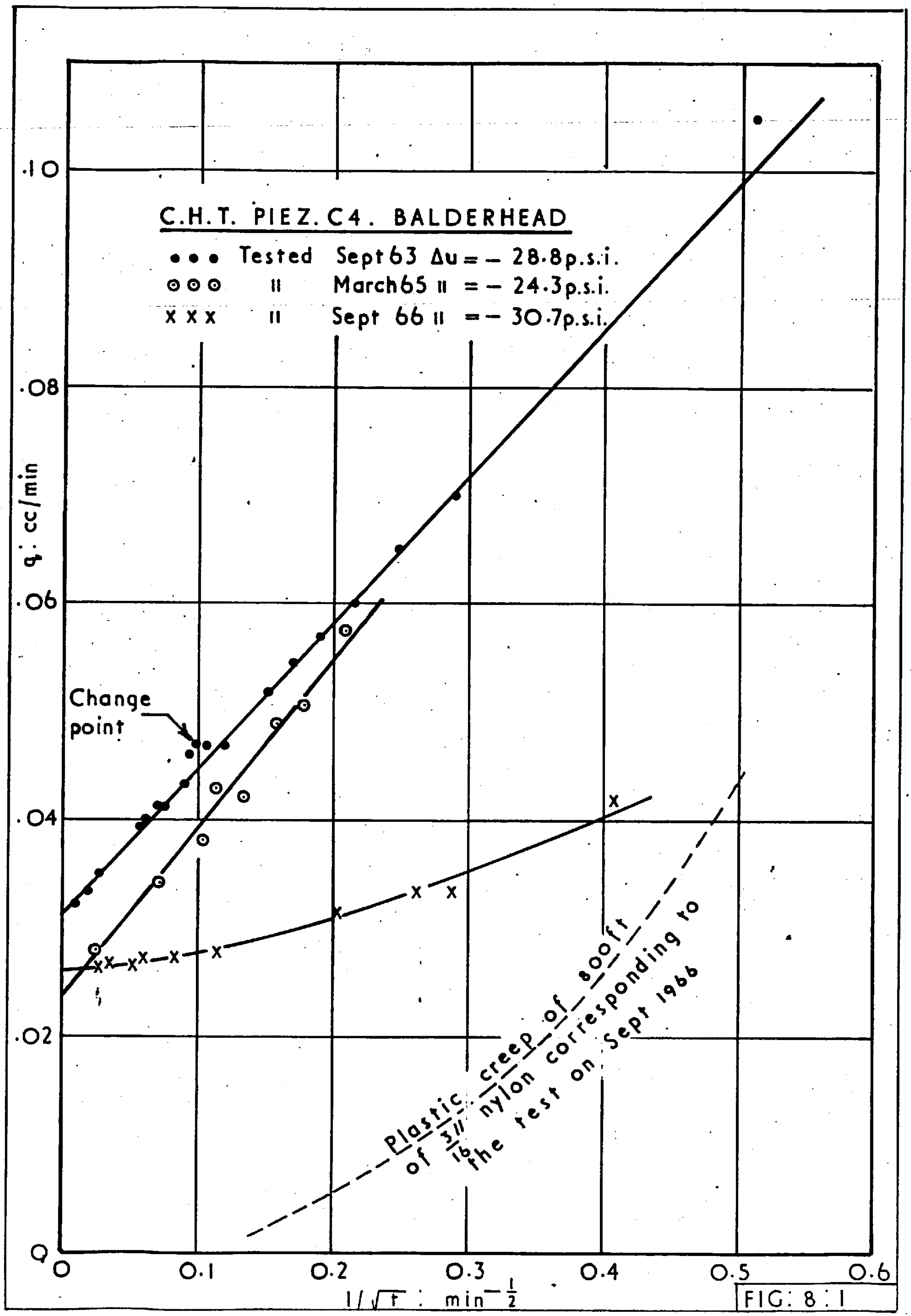
Table 8.4

C.H.T. results: M6 Trial Embankment

Piez. No.	measured drop in pore pressure Ft. of water	$\bar{B}$	Predicted drop in pore pressure using Reinius' method Ft. of water	Ft. of water
			Case 1	Case 2
22	1.94	0.20	9.50	9.20
21	5.73	0.61	9.50	8.50
7	6.78	0.72	9.00	7.00
20	5.23	0.55	8.50	6.90
19	0.71	0.09	8.70	6.20
18	0.35	0.05	9.20	6.00
6	1.87	3.70	6.20	4.10
15	0.08		6.50	3.00
16	0.48		5.0	2.20
17	0.65		2.0	1.10
25	9.08		8.50	3.00
5	0.02		5.0	1.30
4	0.39		0	0

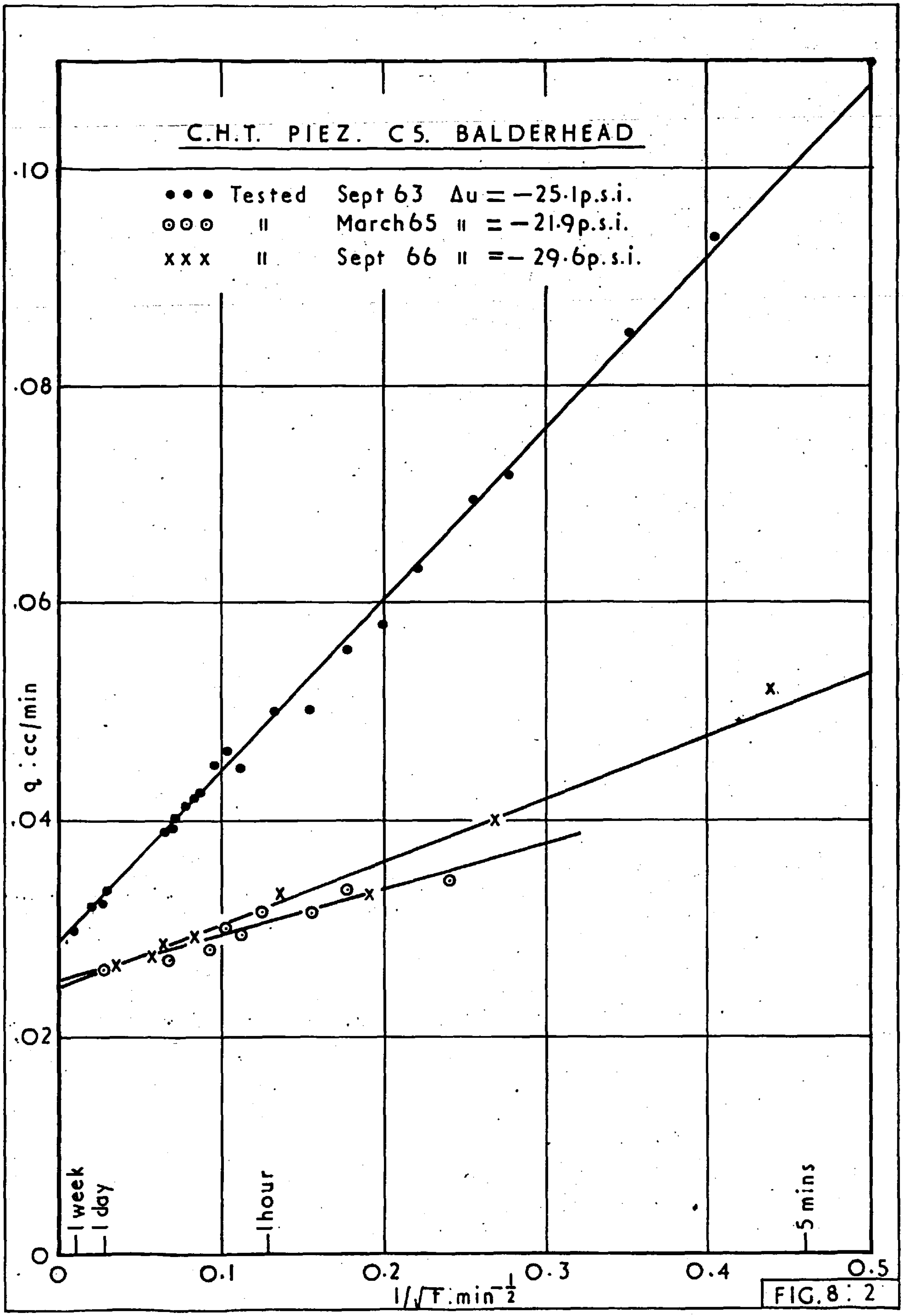
Table 8.5

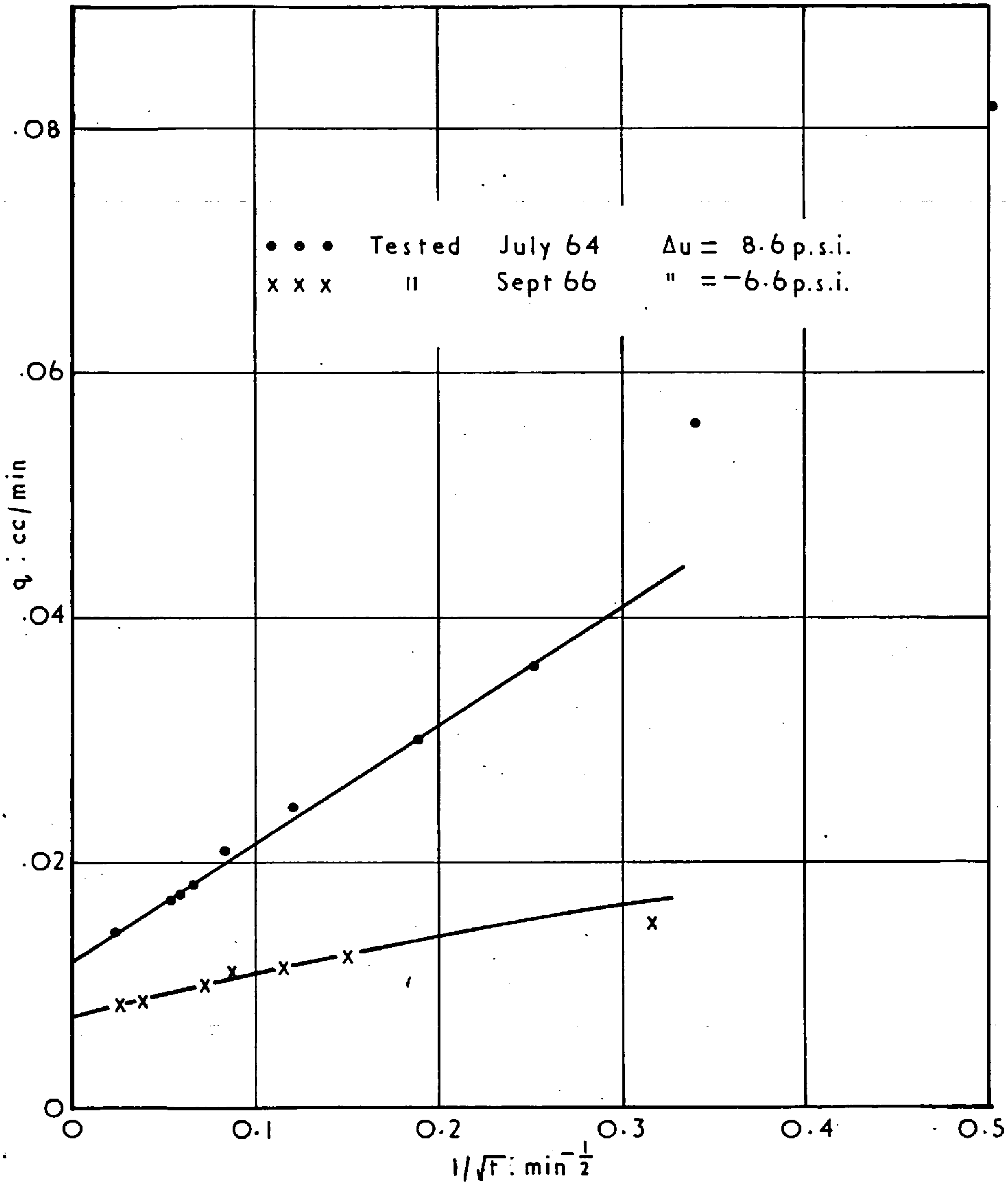
Measured and predicted pore pressure drop during draw down test: Muirhead dam



C.H.T. PIEZ. C4. BALDERHEAD
● ● ● Tested Sept 63 Δu = – 28.8 p.s.i.
⊙ ⊙ ⊙ " March 65 " = – 24.3 p.s.i.
x x x " Sept 66 " = – 30.7 p.s.i.
Change point
Plastic creep of 800 ft of 5/16" nylon corresponding to the test on Sept 1966
q : cc/min
·10
·08
·06
·04
·02
0
0
0.1
0.2
0.3
0.4
0.5
0.6
1/√t : min^-1/2

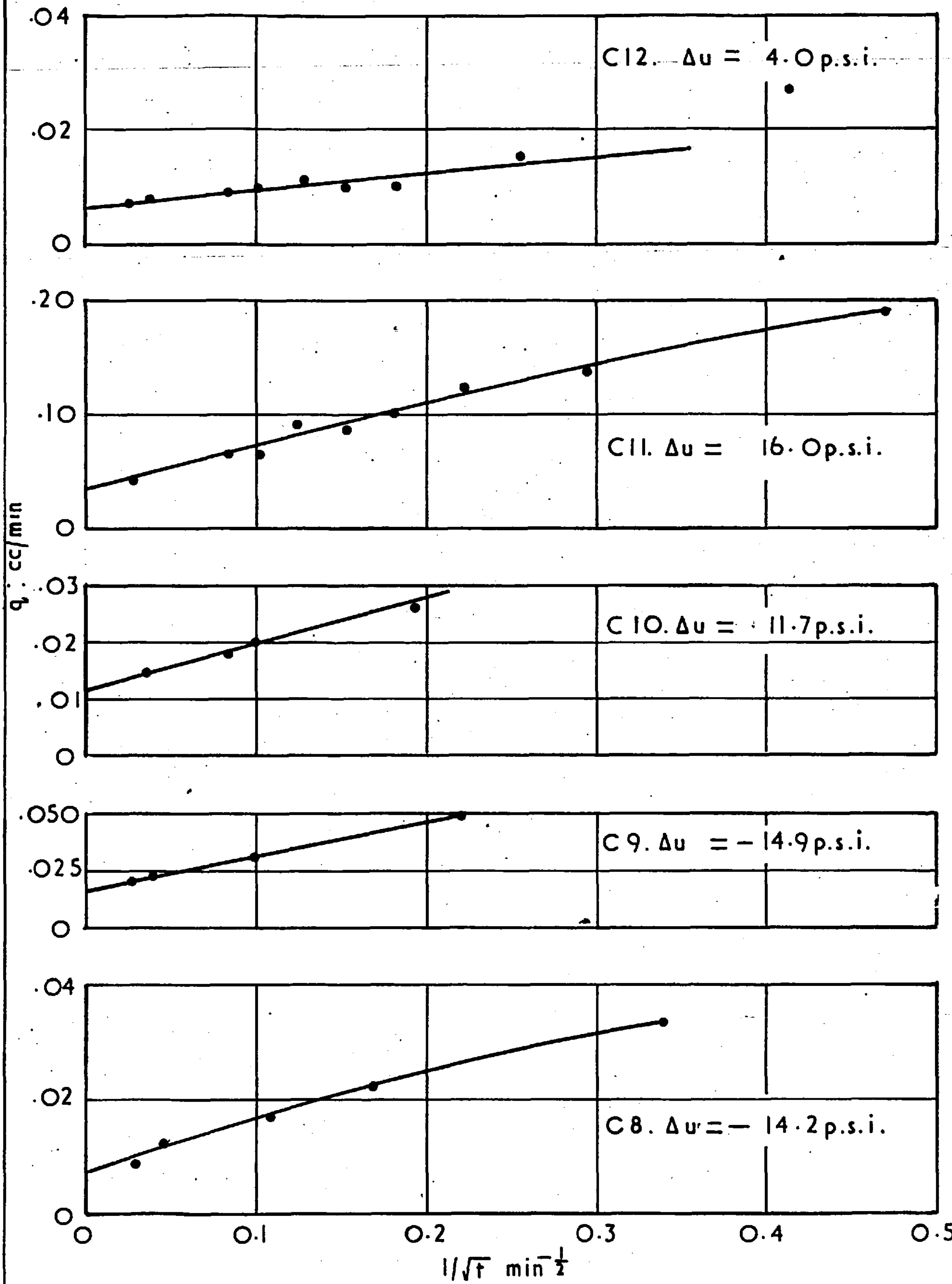
FIG: 8 : 1


C.H.T. PIEZ. C.S. BALDERHEAD
● ● ● Tested Sept 63 Δu = −25·1 p.s.i.
⊙ ⊙ ⊙ " March 65 " = −21·9 p.s.i.
x x x " Sept 66 " = −29·6 p.s.i.
q : cc/min
.10
.08
.06
.04
.02
0
1 week
1 day
1 hour
5 mins
0
0.1
0.2
0.3
0.4
0.5
1/√T . min^−½
FIG. 8 : 2



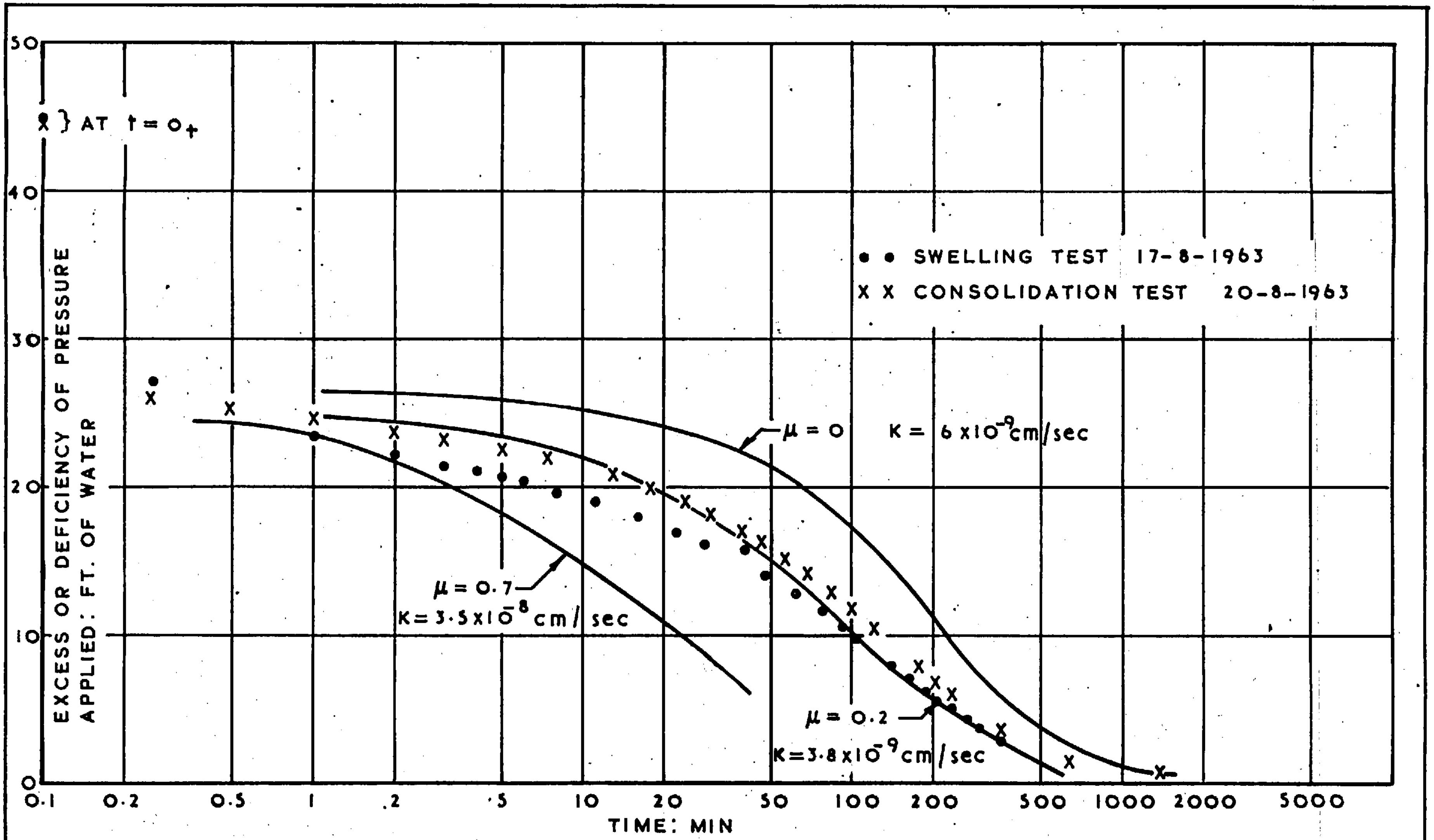
C.H.T. PIEZ 21 BALDERHEAD

FIG. 8 : 3



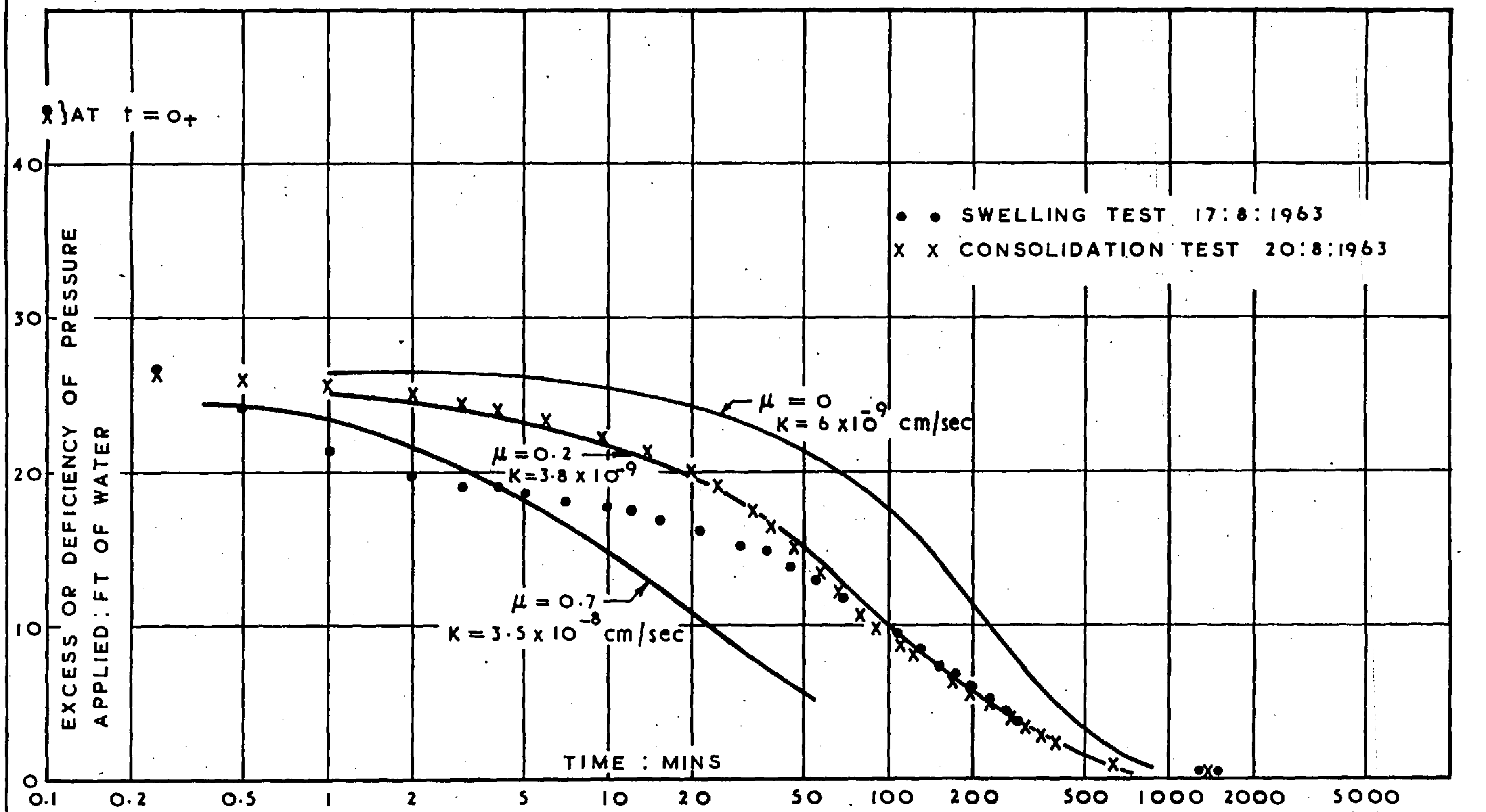
C.H.T. BALDERHEAD (SEPT. 1966)

FIG. 8.4



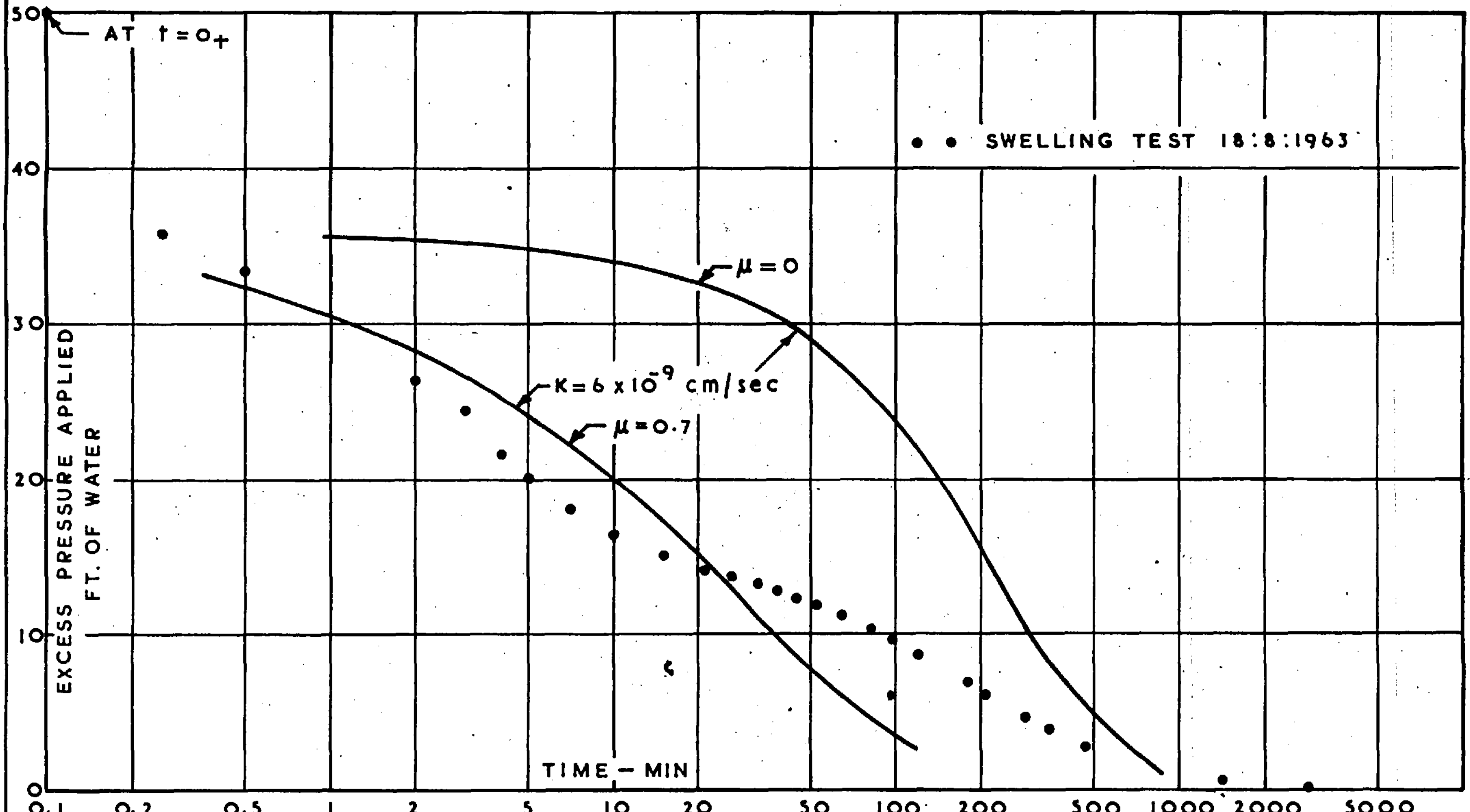
F.H.T. PIEZ C3 BALDERHEAD

FIG. 8:5



F.H.T. PIEZ. C4. BALDERHEAD

FIG. 8:6



F.H.T. PIEZ. C5 BALDERHEAD

FIG. 8 : 7

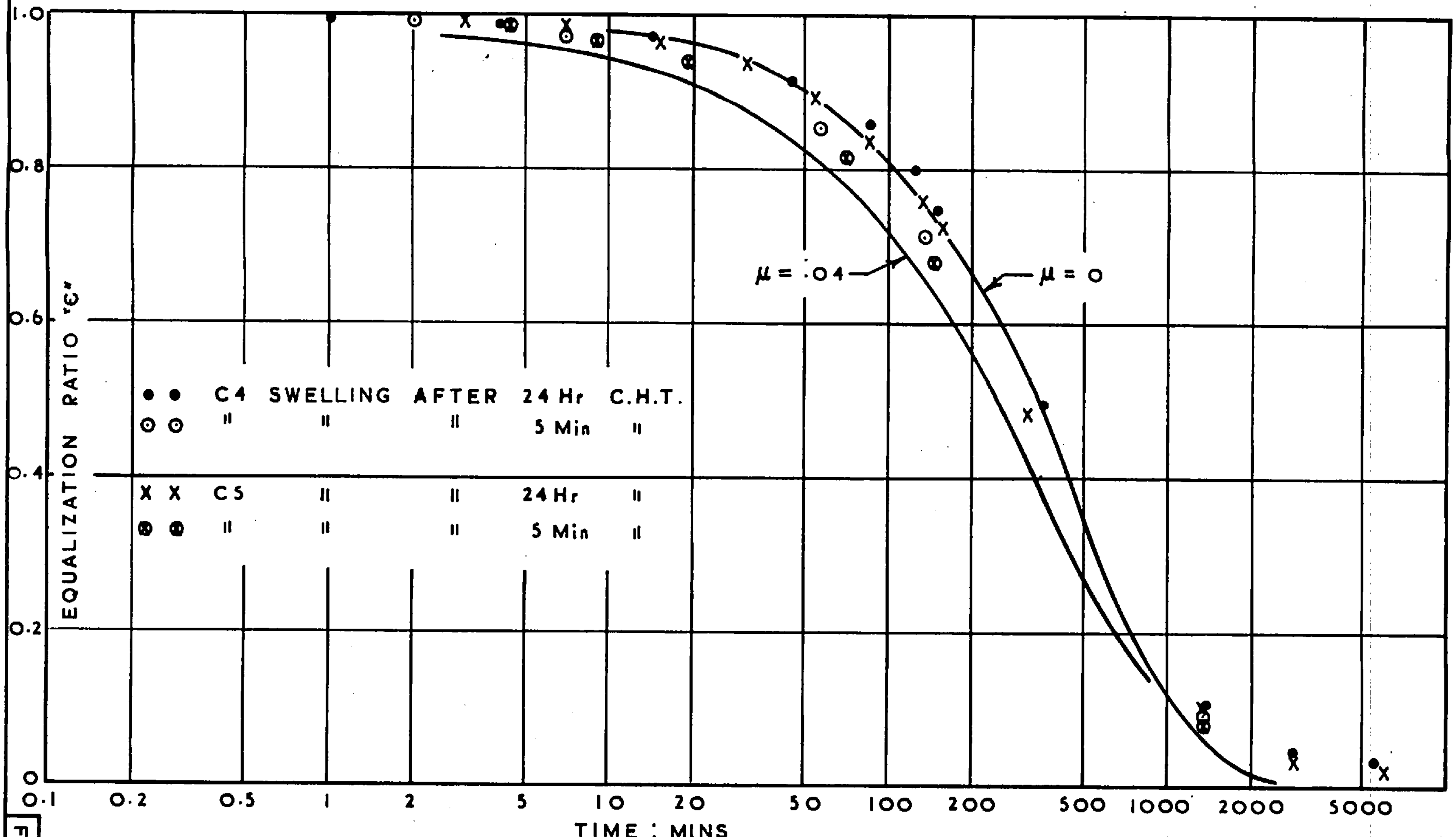

EQUALIZATION RATIO "ε"
1.0
0.8
0.6
0.4
0.2
0
0.1
0.2
0.5
1
2
5
10
20
50
100
200
500
1000
2000
5000
TIME : MINS
μ = ·04
μ = 0
C4 SWELLING AFTER 24 Hr C.H.T.
" " " 5 Min "
C5 " " 24 Hr "
" " " 5 Min "

FIG. 8:8

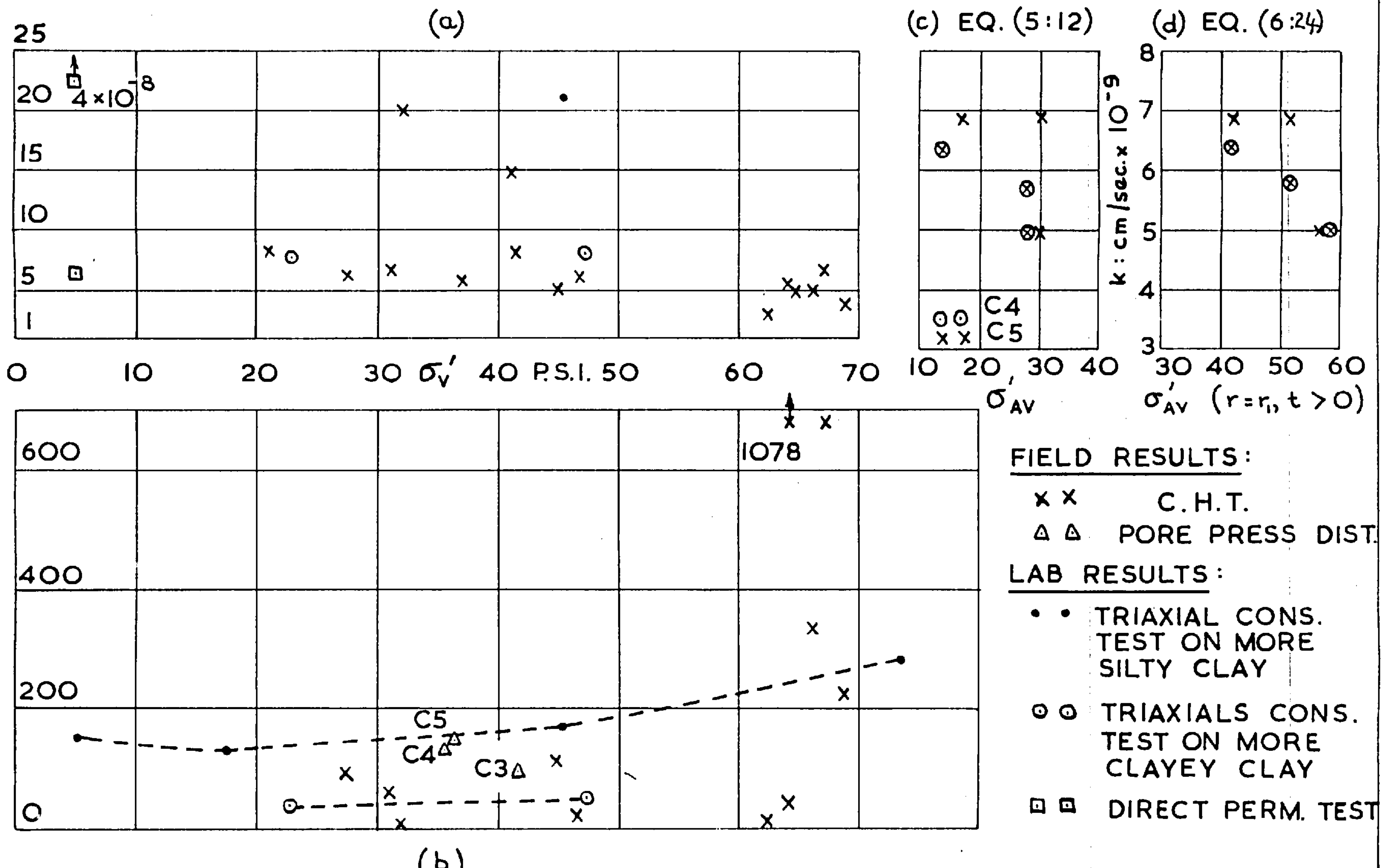
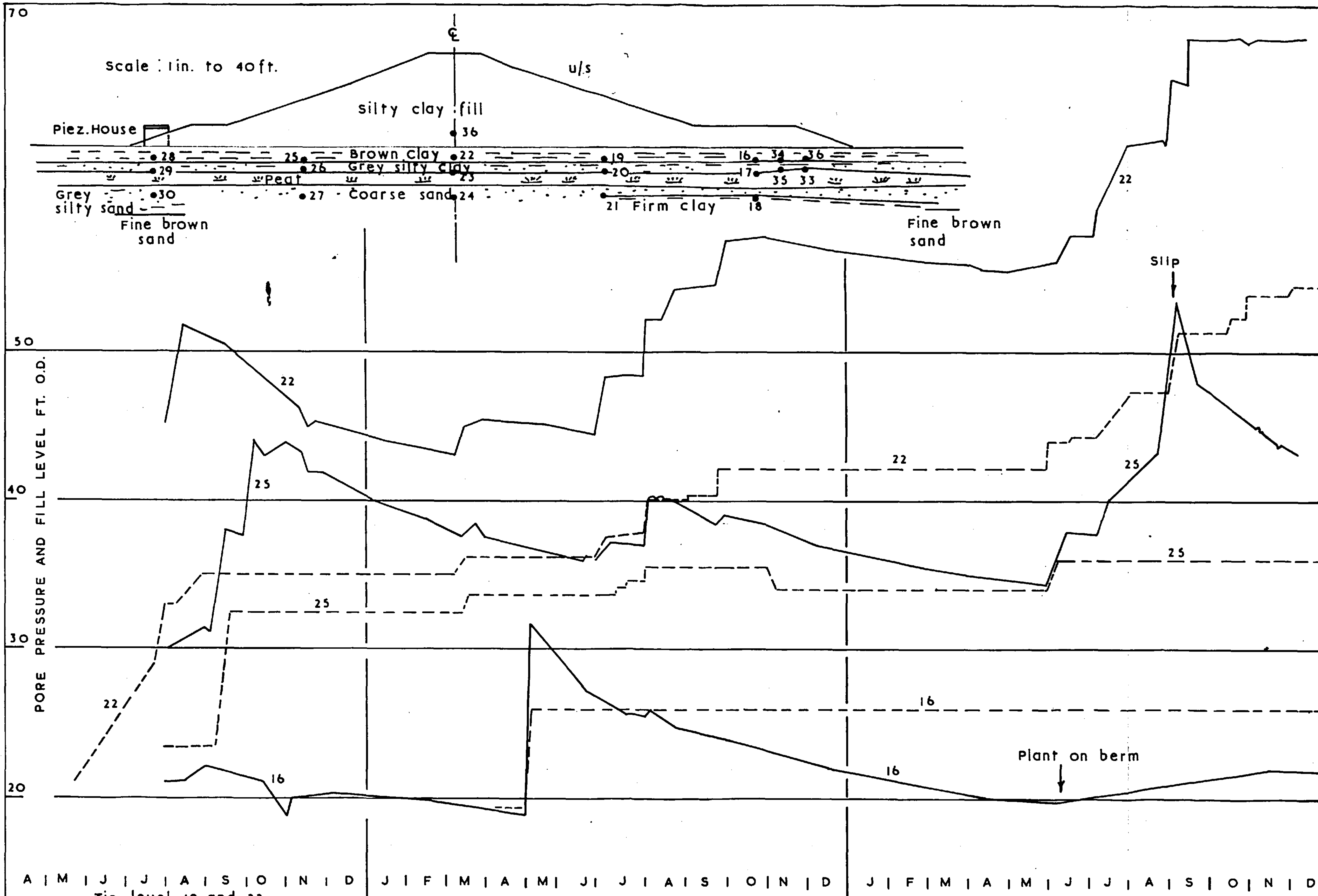


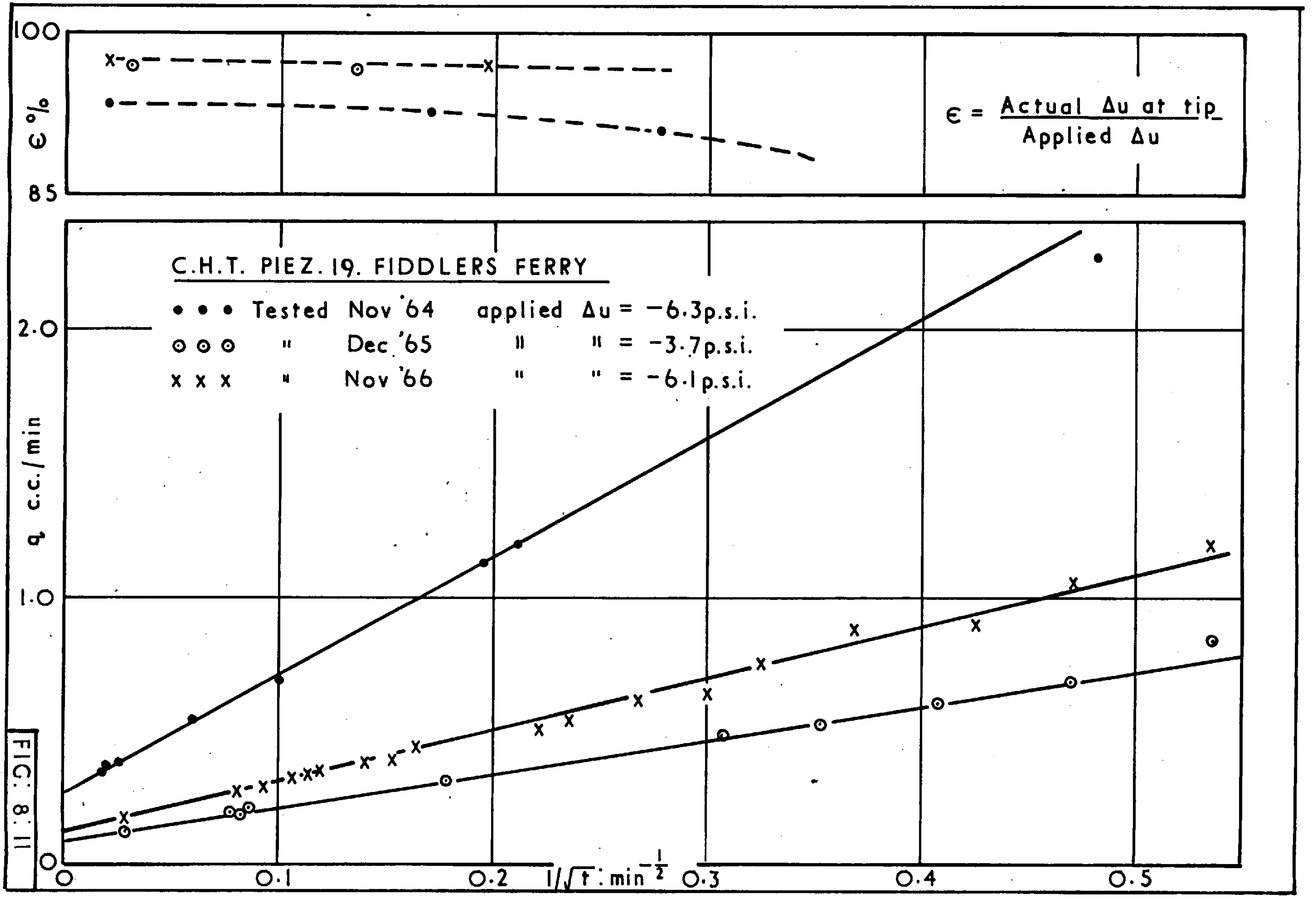
FIG. 8:9

LAB. AND FIELD VALUES K & C_v — BALDERHEAD DAM



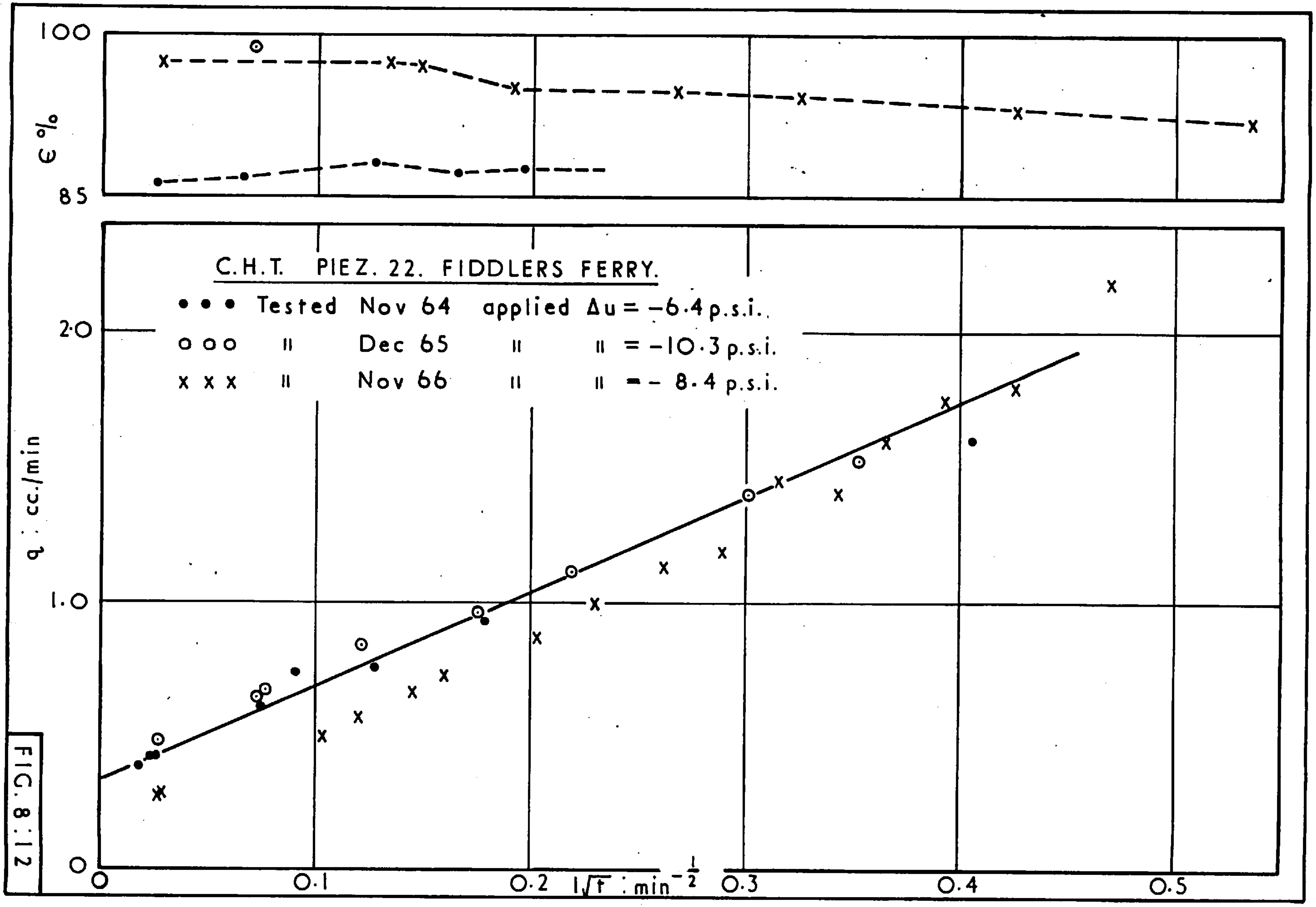
PORE PRESSURE RECORDS : FIDDLERS FERRY

FIG: 8:10



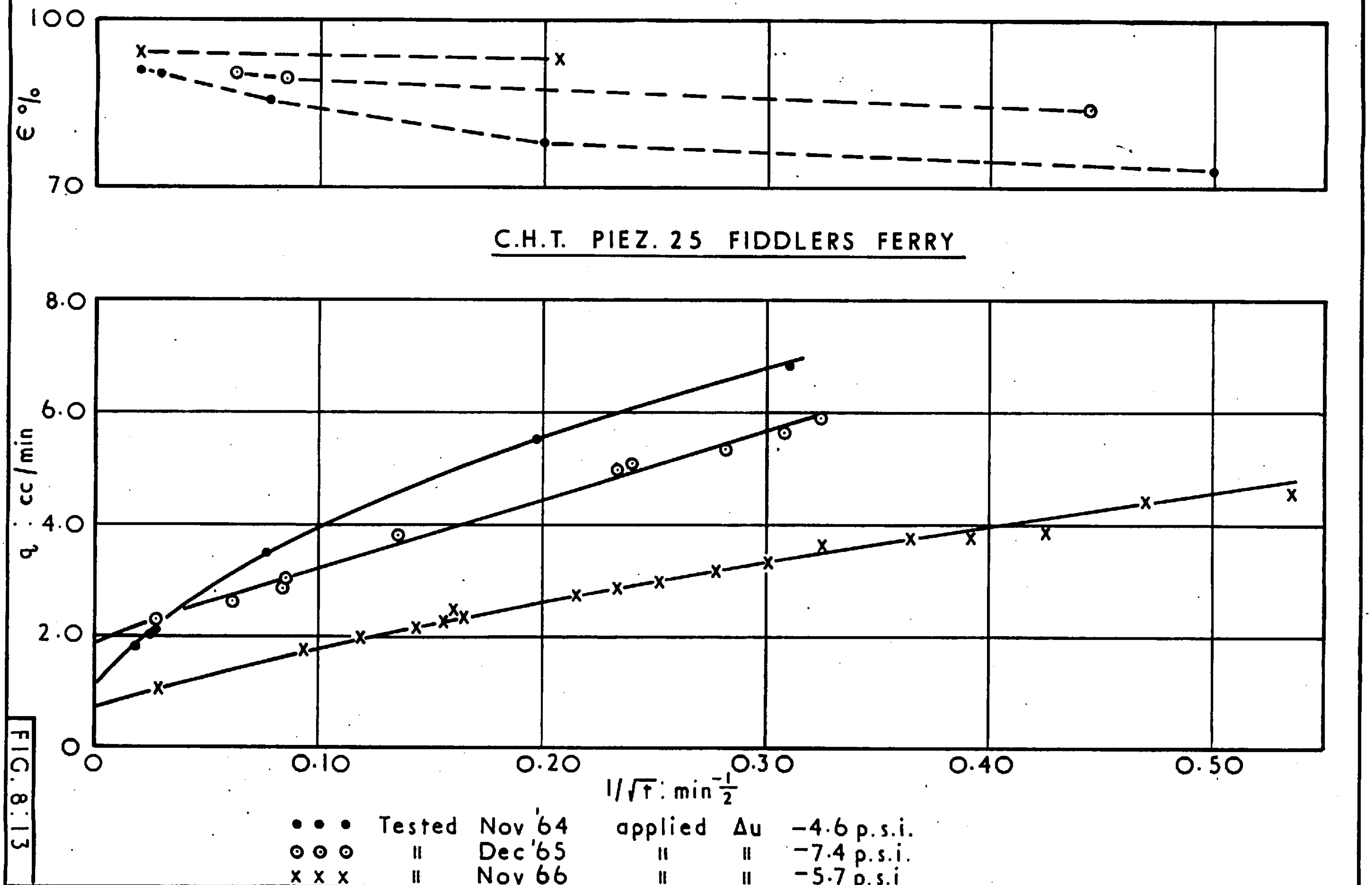
100
85
ε %
ε = Actual Δu at tip / Applied Δu
C.H.T. PIEZ. 19. FIDDLERS FERRY
• • • Tested Nov '64 applied Δu = −6.3 p.s.i.
⊙ ⊙ ⊙ " Dec '65 " " = −3.7 p.s.i.
x x x " Nov '66 " " = −6.1 p.s.i.
q c.c./min
2.0
1.0
0
0.1
0.2
0.3
0.4
0.5
1/√t : min^−½

FIG. 8.11



C.H.T. PIEZ. 22. FIDDLERS FERRY.
• • • Tested Nov 64 applied Δu = −6·4 p.s.i.
o o o " Dec 65 " " = −10·3 p.s.i.
x x x " Nov 66 " " = − 8·4 p.s.i.
ϵ %
100
85
q : cc./min
2·0
1.0
0
O
O·1
O·2
O·3
O·4
O·5
1/√t : min^-1/2

FIG. 8 : 12



100
70
∈ %
C.H.T. PIEZ. 25 FIDDLERS FERRY
8.0
6.0
4.0
2.0
0
q : cc/min
0
0.10
0.20
0.30
0.40
0.50
1/√t : min^-1/2
• • • Tested Nov '64 applied Δu −4.6 p.s.i.
⊙ ⊙ ⊙ " Dec '65 " " −7.4 p.s.i.
x x x " Nov 66 " " −5.7 p.s.i

FIG. 8.13

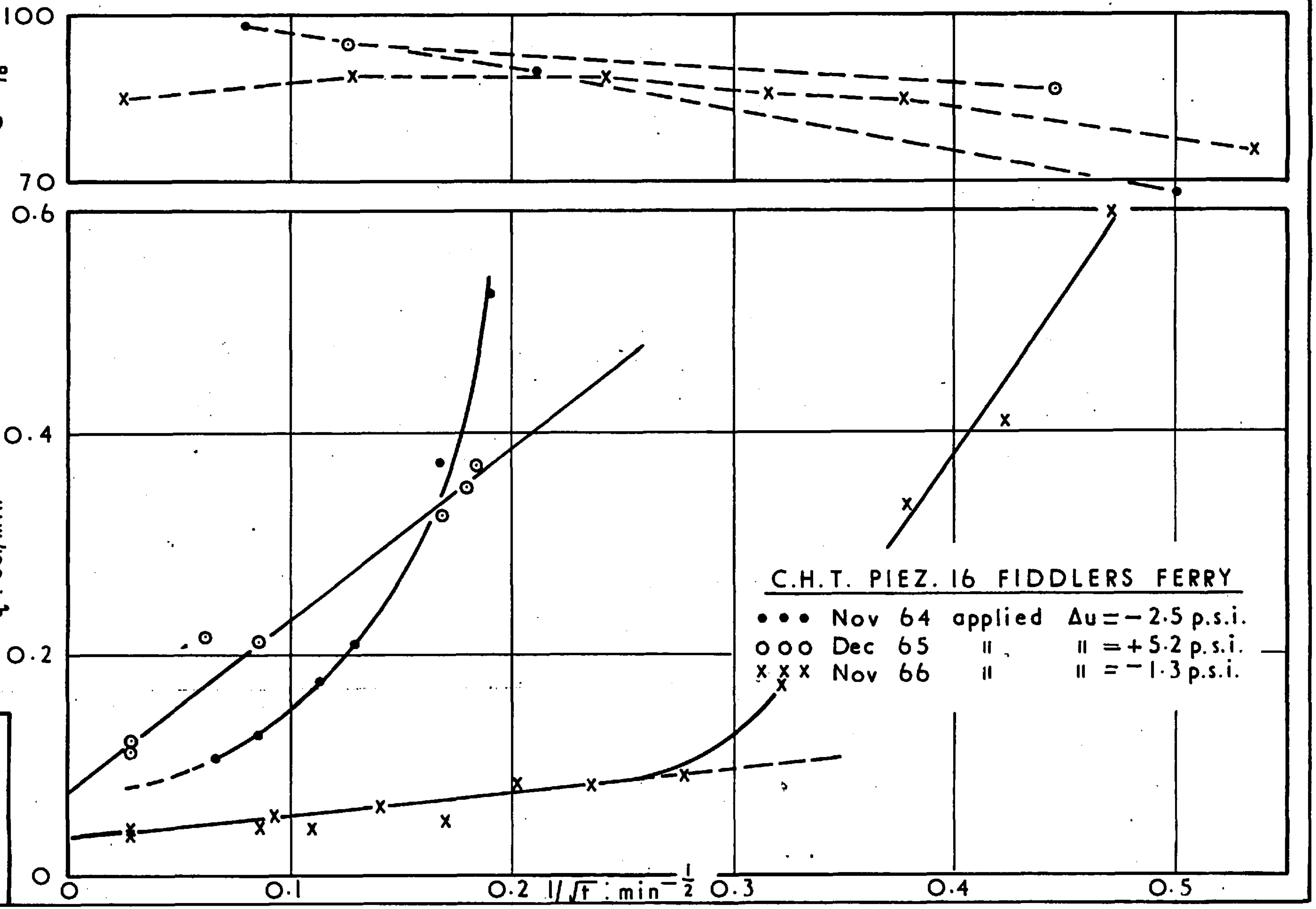

C.H.T. PIEZ. 16 FIDDLERS FERRY
• • • Nov 64 applied Δu = − 2.5 p.s.i.
o o o Dec 65 " " = + 5.2 p.s.i.
x x x Nov 66 " " = − 1.3 p.s.i.
ε %
100
70
q : cc/min
0.6
0.4
0.2
0
0
0.1
0.2
1/√t : min^−½
0.3
0.4
0.5

FIG. 8.14

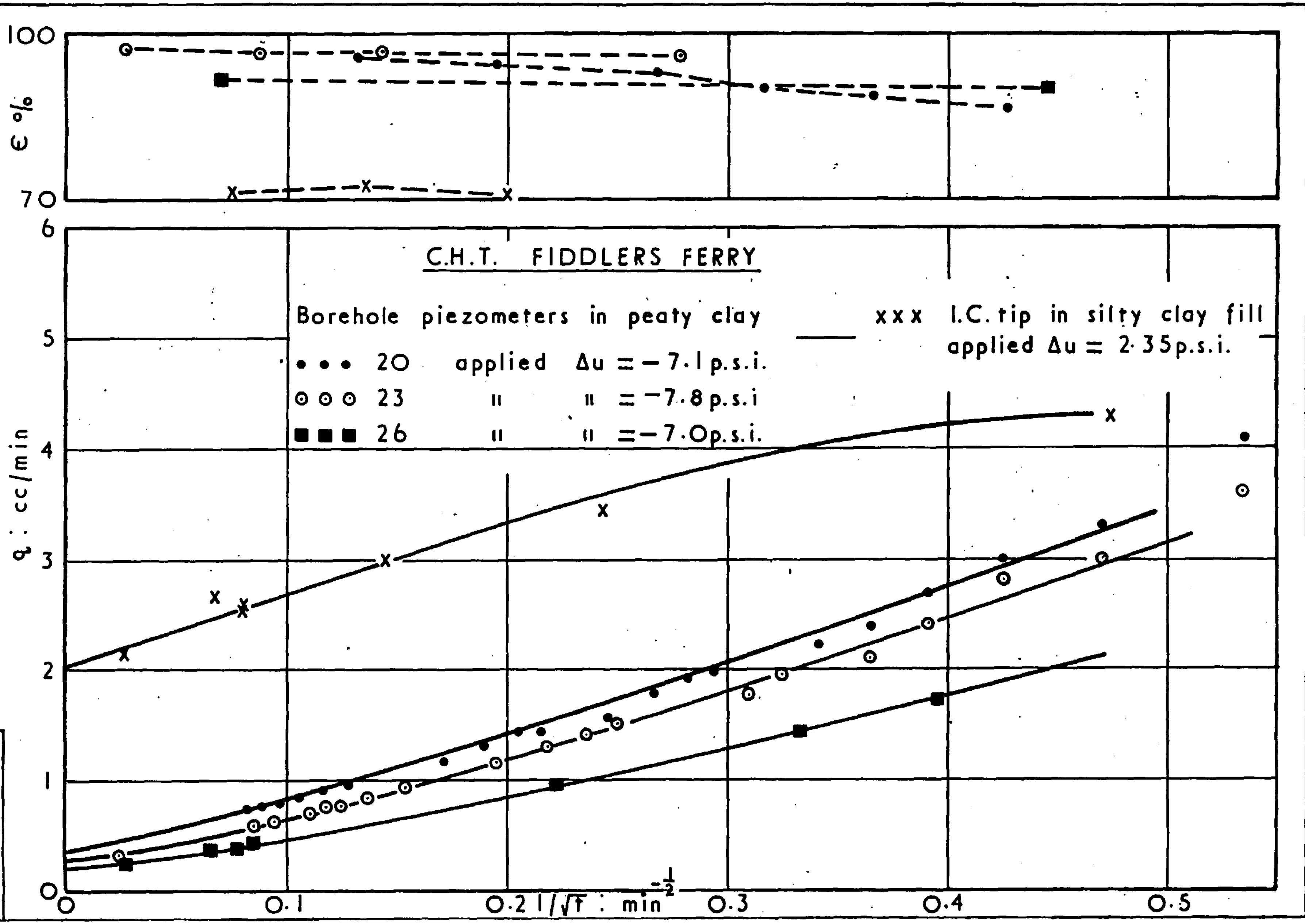
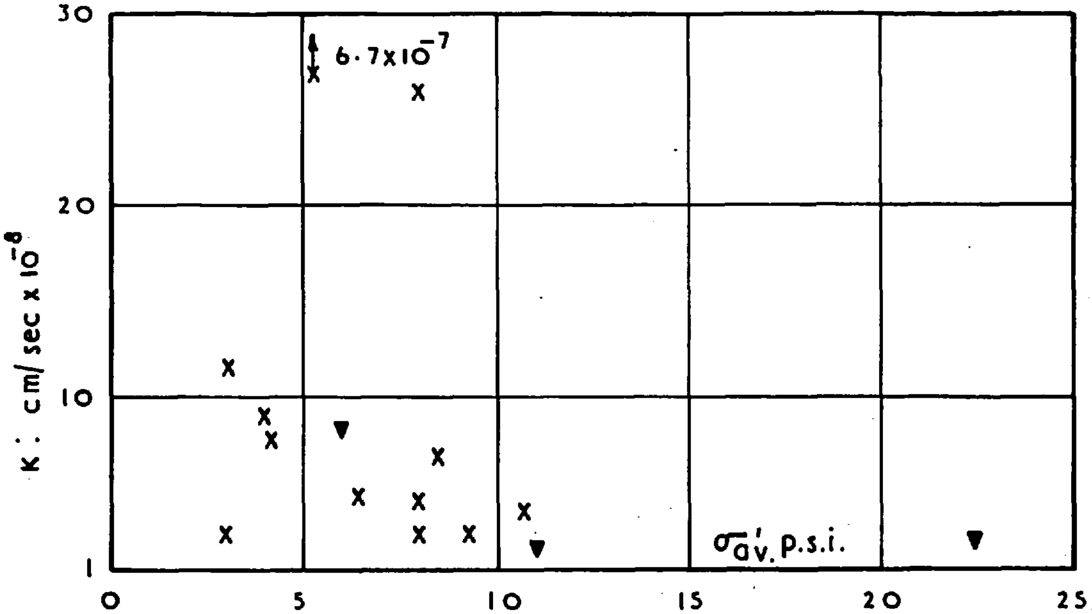

100
70
ε %
6
5
4
3
2
1
0
q : cc/min
C.H.T. FIDDLERS FERRY
Borehole piezometers in peaty clay
• • • 20 applied Δu = −7·1 p.s.i.
⊙ ⊙ ⊙ 23 " " = −7·8 p.s.i
■ ■ ■ 26 " " = −7·0 p.s.i.
x x x I.C. tip in silty clay fill
applied Δu = 2·35 p.s.i.
0
0·1
0·2
0·3
0·4
0·5
1/√t : min^-1/2

FIG. 8:15



COMPARISON BETWEEN FIELD & LAB. VALUES OF C_V & K : FIDDLERS FERRY.

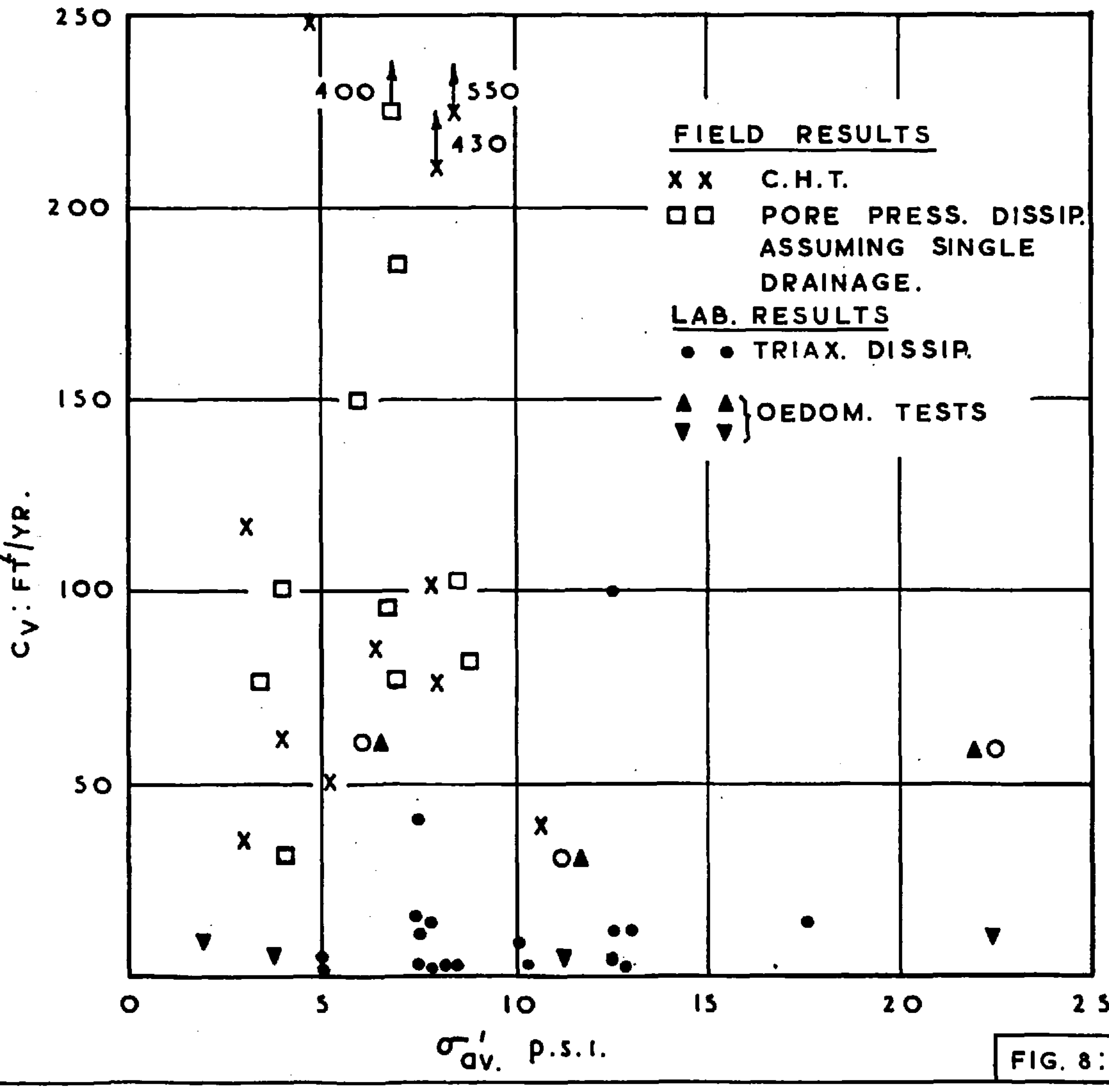
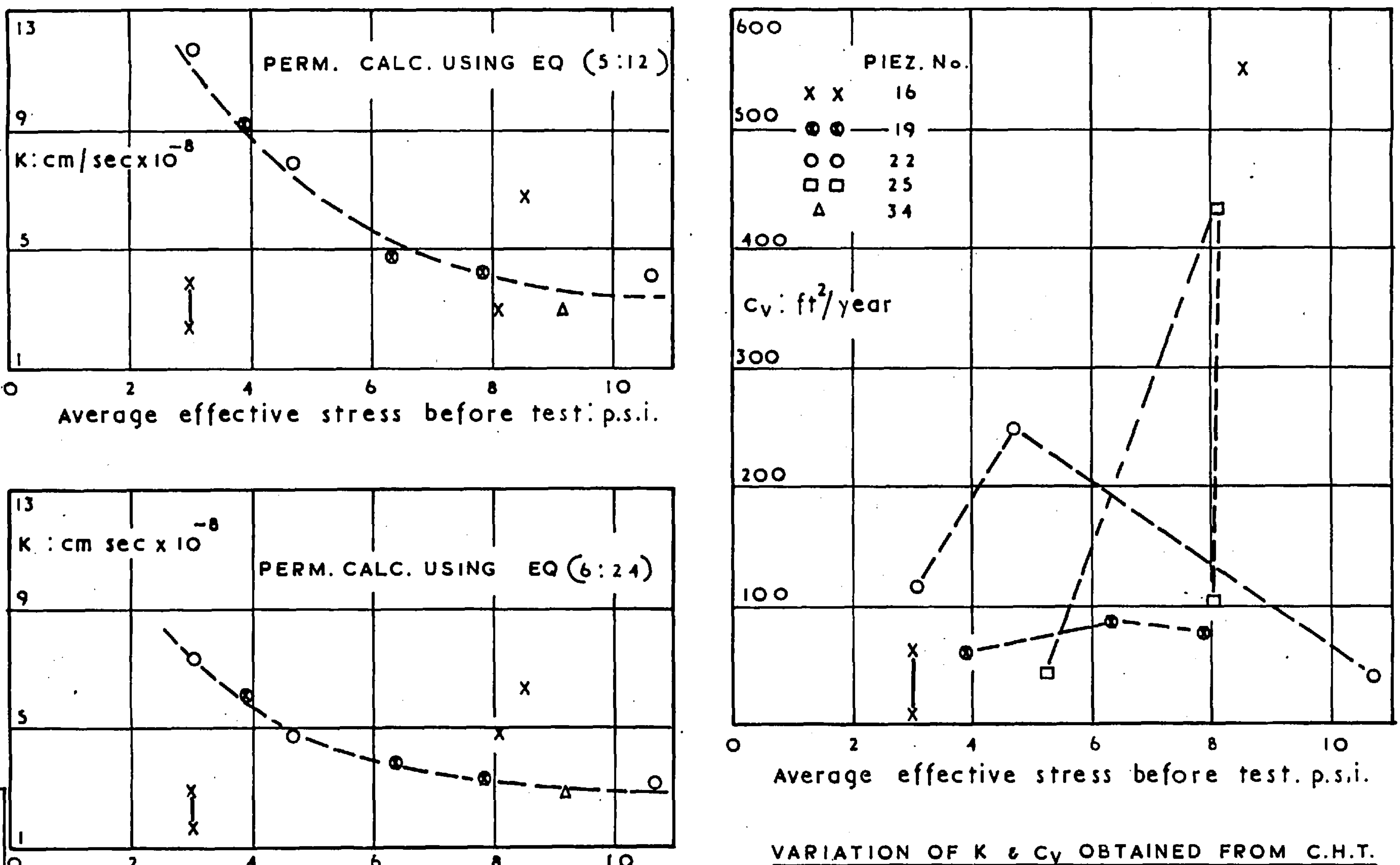
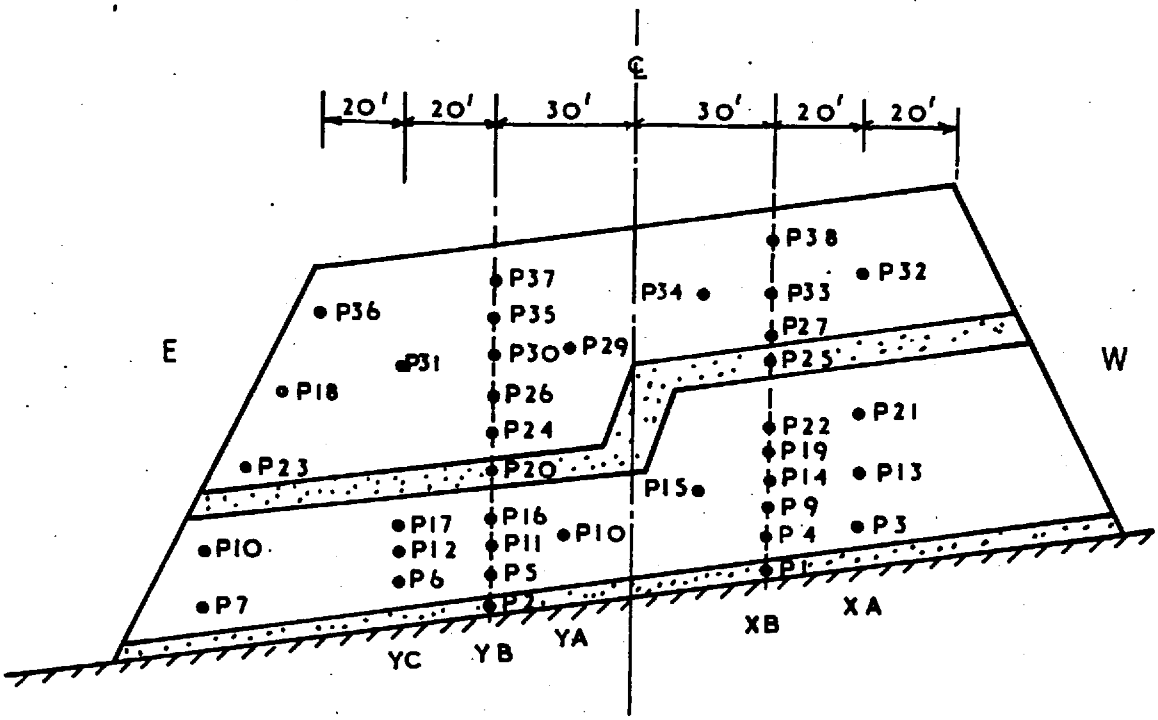


FIG. 8 : 16



VARIATION OF K & Cv OBTAINED FROM C.H.T.
WITH AVERAGE EFFECTIVE STRESS BEFORE TEST:
FIDDLERS FERRY

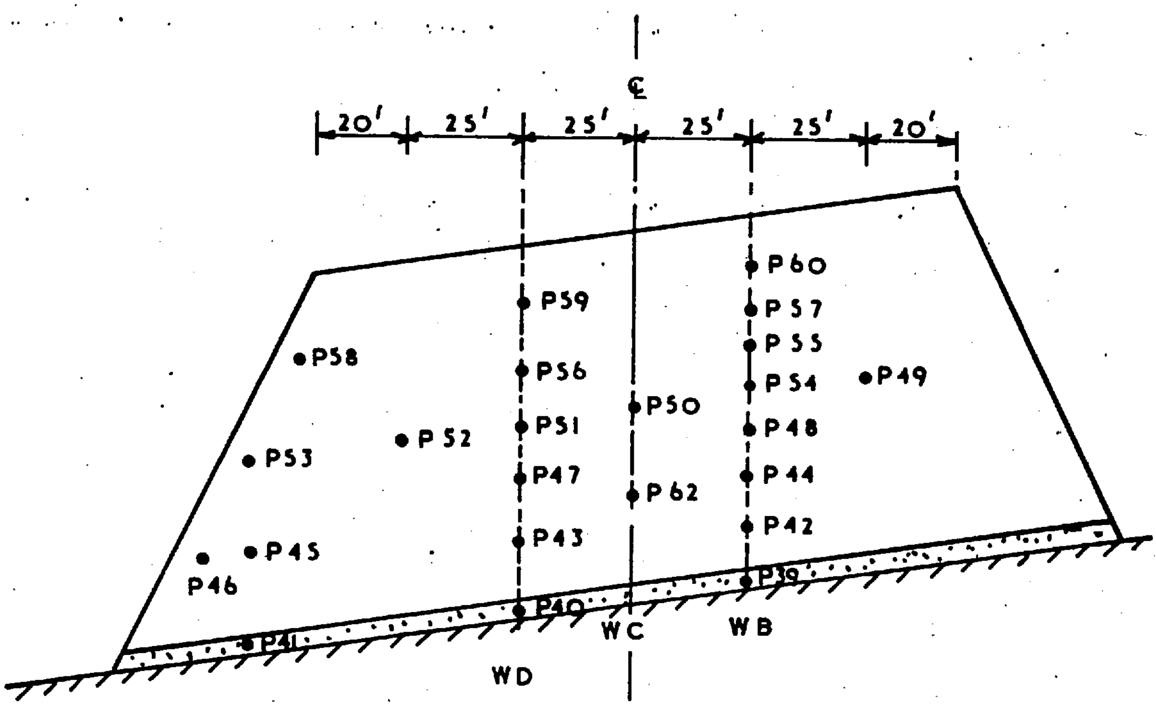
FIG. 8:17



SECTION BB

(LOOKING SOUTH)

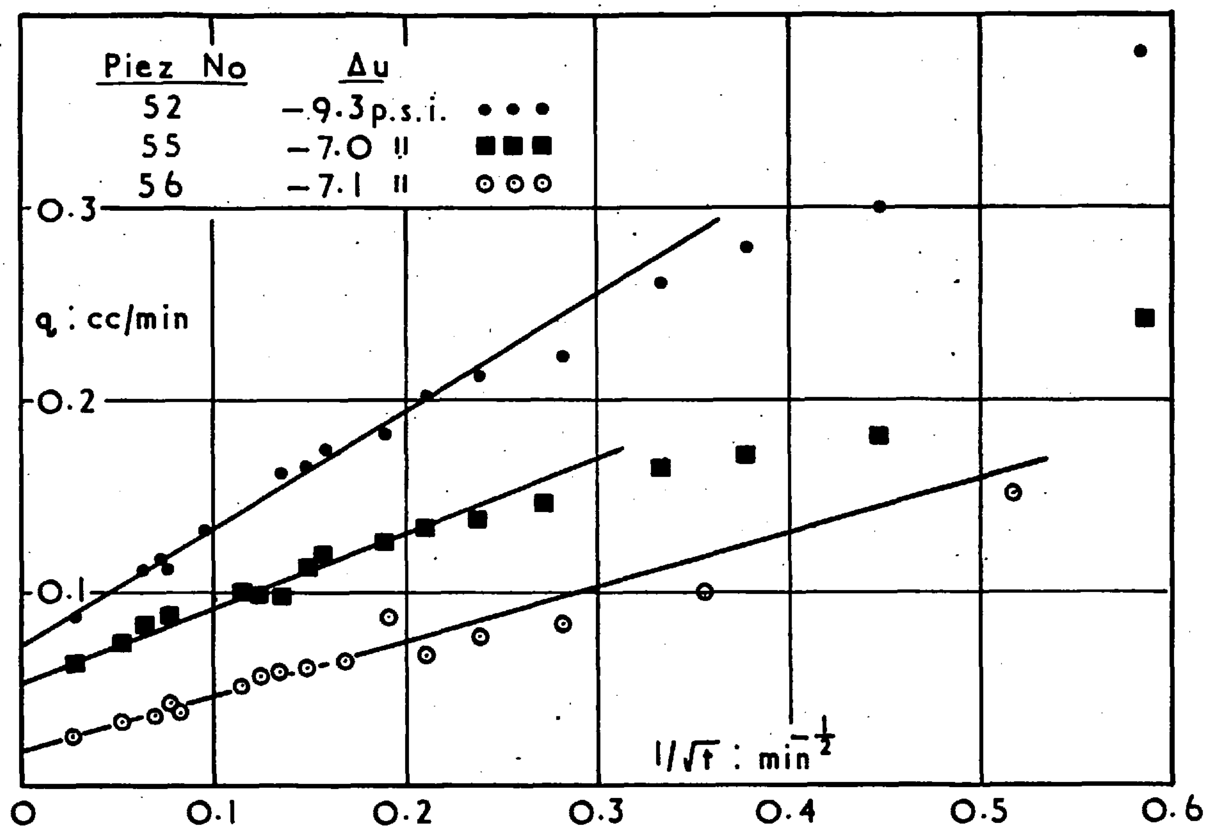
SCALES
1:500 HORIZ.
1:125 VERT.



SECTION CC

M 6 TRIAL EMBANKMENT

FIG. 8:18



C.H.T. M6. TRIAL EMBANKMENT

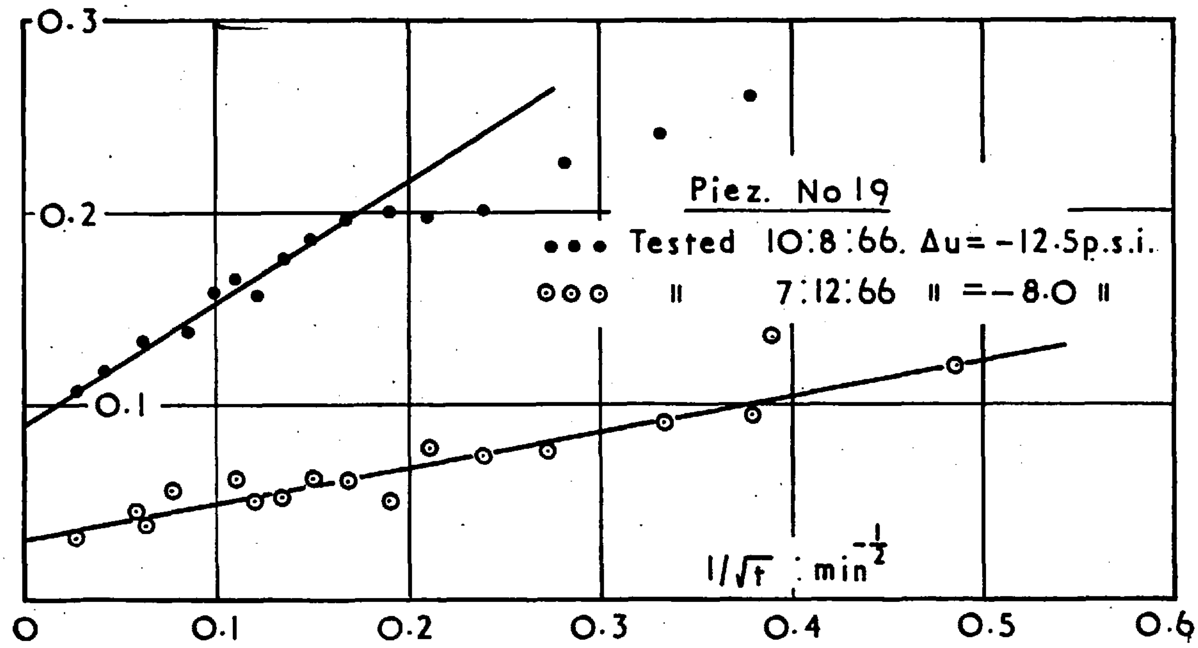
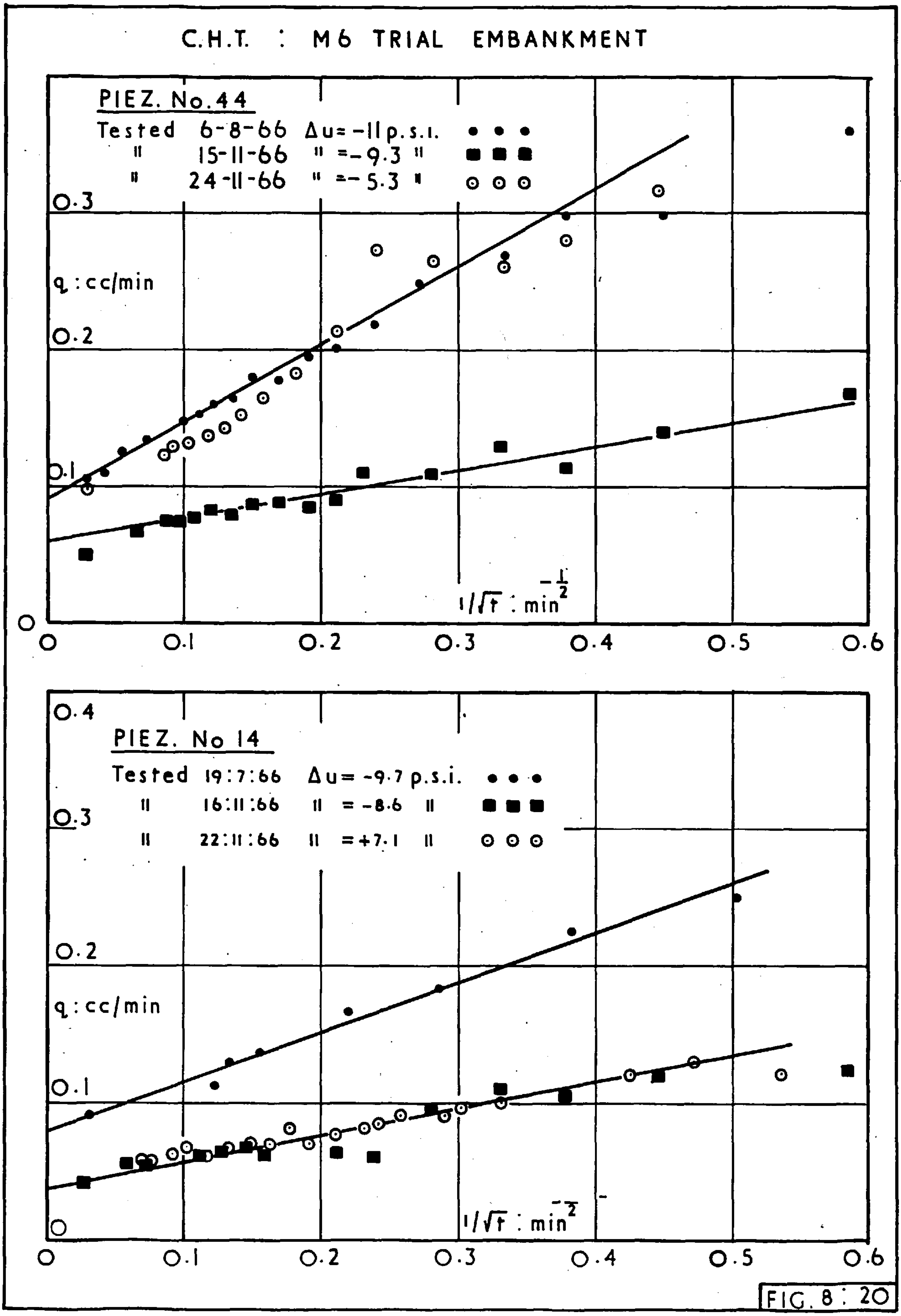
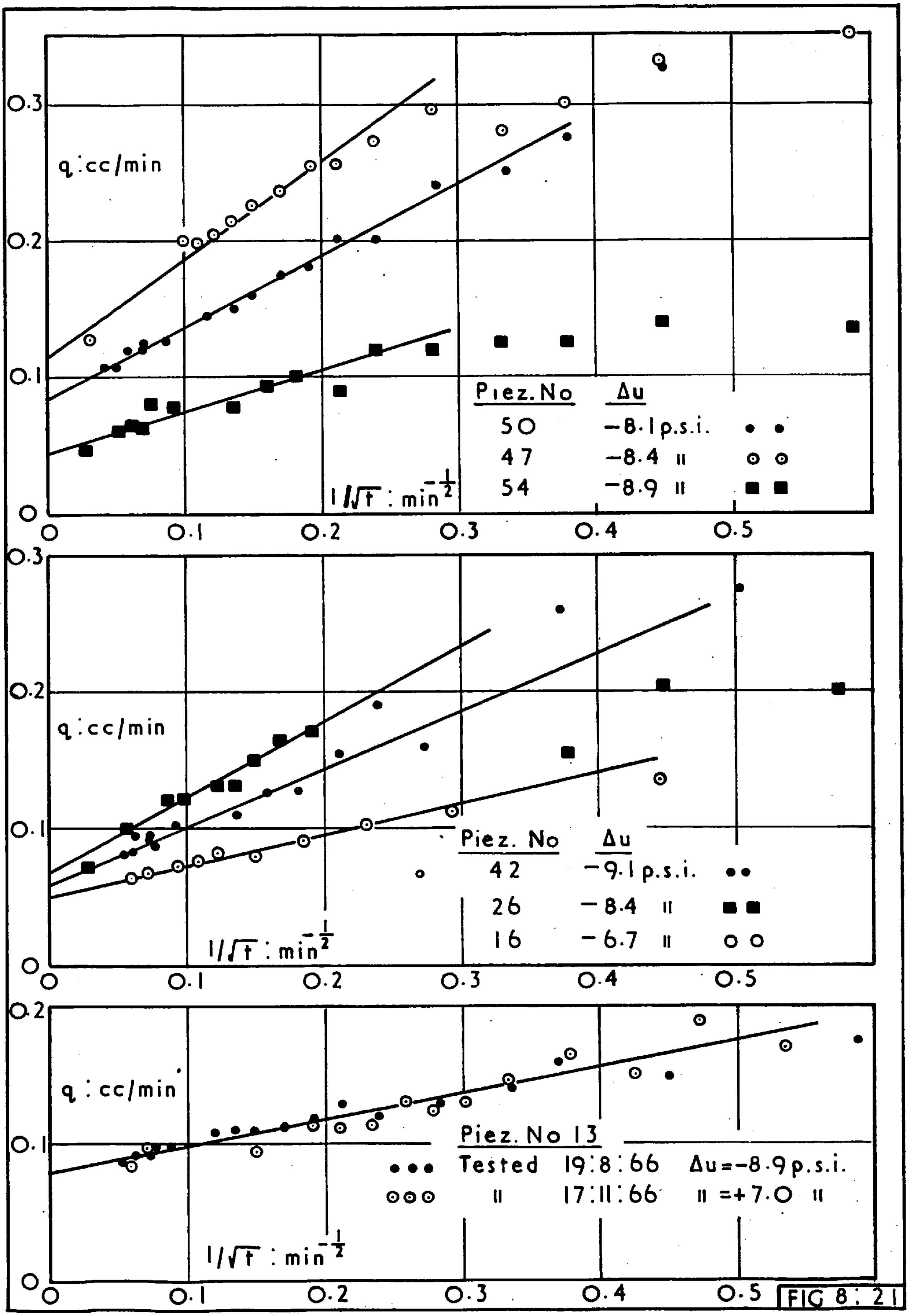


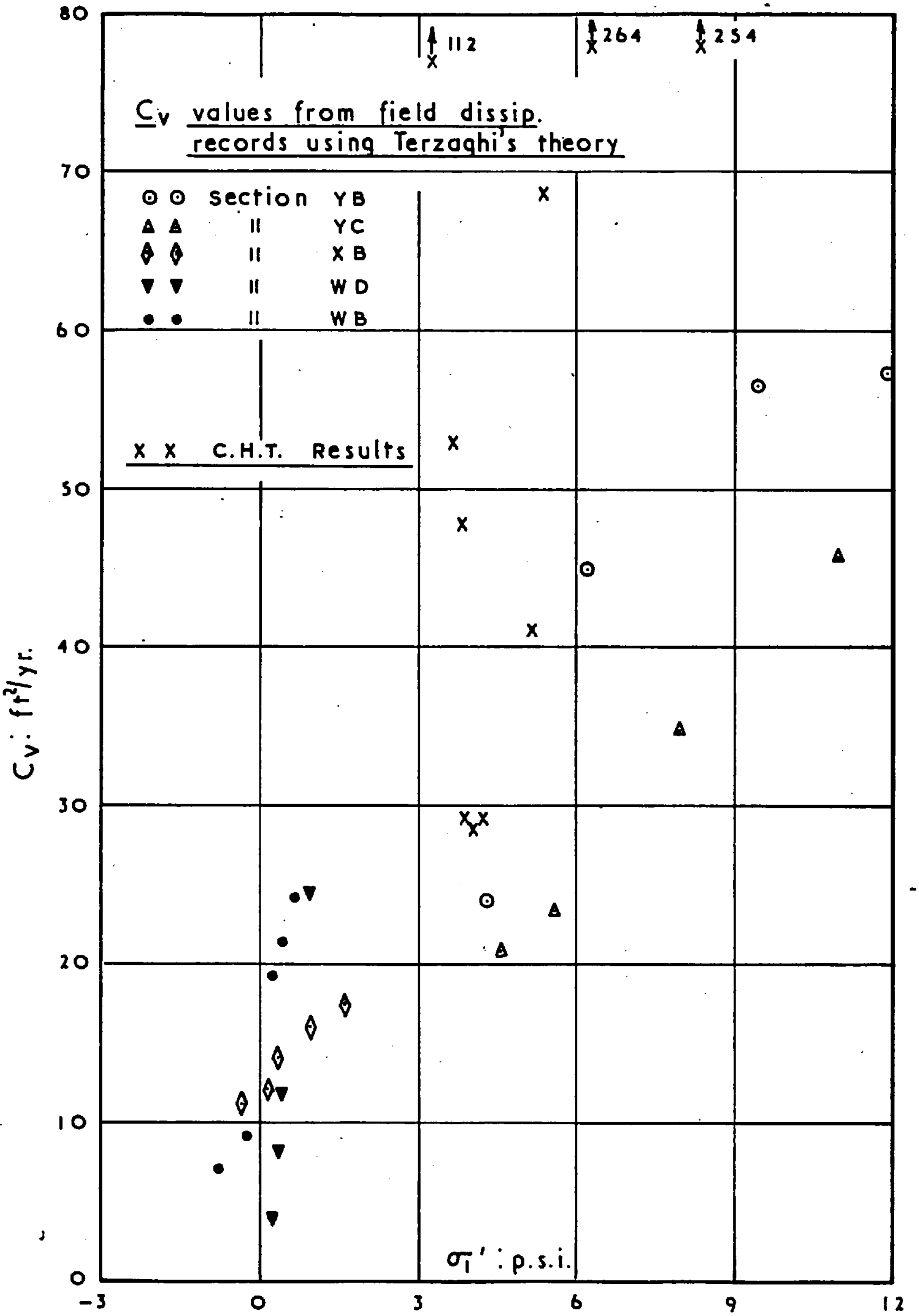
FIG. 8.19



C.H.T. : M6 TRIAL EMBANKMENT
PIEZ. No. 44
Tested 6-8-66 Δu= -11 p.s.i.
" 15-11-66 " = -9.3 "
" 24-11-66 " = -5.3 "
q : cc/min
1/√t : min^-1/2
PIEZ. No 14
Tested 19:7:66 Δu= -9.7 p.s.i.
" 16:11:66 " = -8.6 "
" 22:11:66 " = +7.1 "
q : cc/min
1/√t : min^-1/2

FIG. 8 : 20

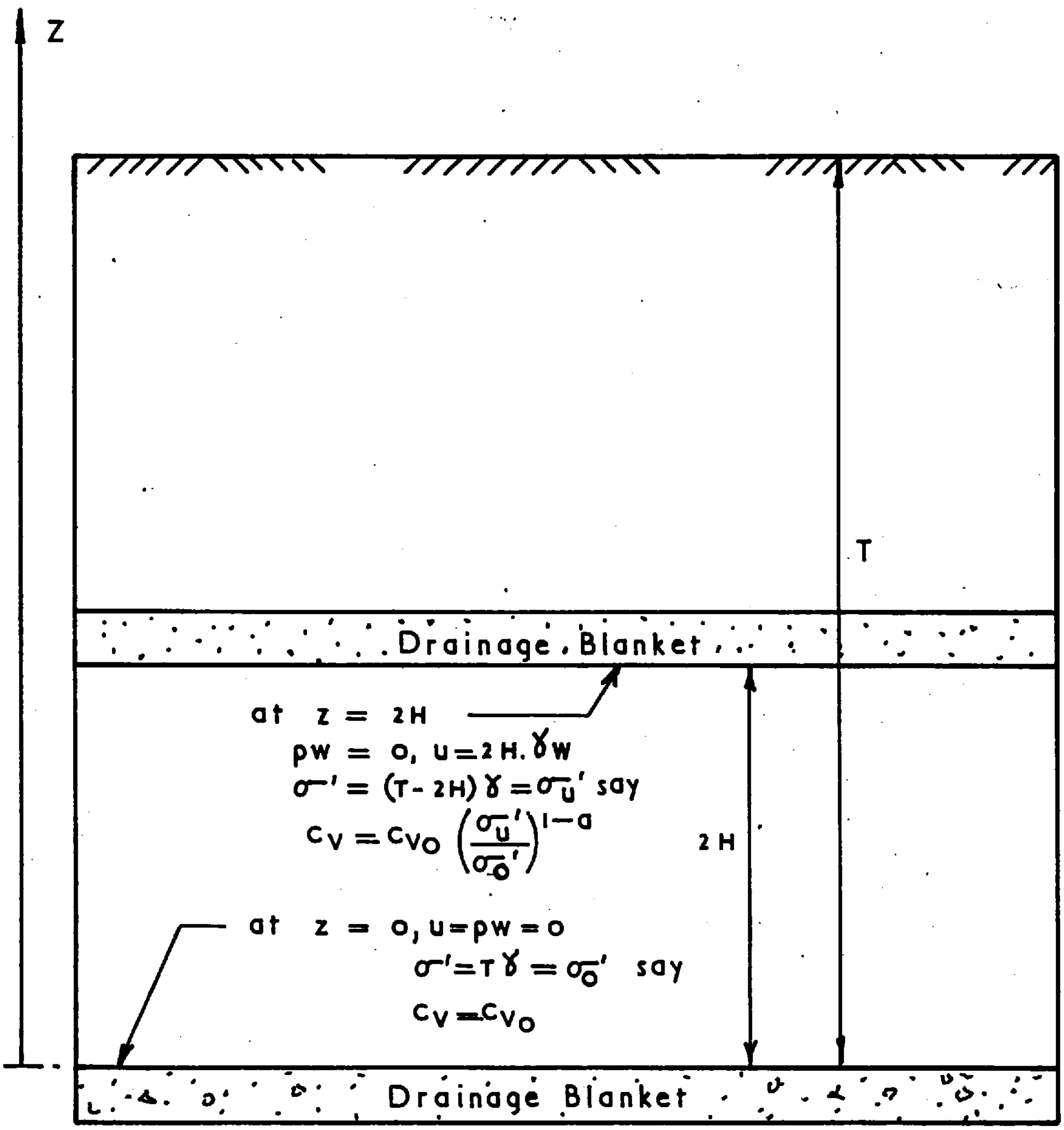

q : cc/min
1/√t : min^-1/2
Piez. No
Δu
50
−8·1 p.s.i.
47
−8·4 ıı
54
−8·9 ıı
Piez. No
Δu
42
−9·1 p.s.i.
26
−8·4 ıı
16
−6·7 ıı
Piez. No 13
Tested 19:8:66 Δu = −8·9 p.s.i.
ıı 17:11:66 ıı = +7·0 ıı
0
0·1
0·2
0·3
0·4
0·5
FIG 8: 21



FIELD VALUES OF C_v

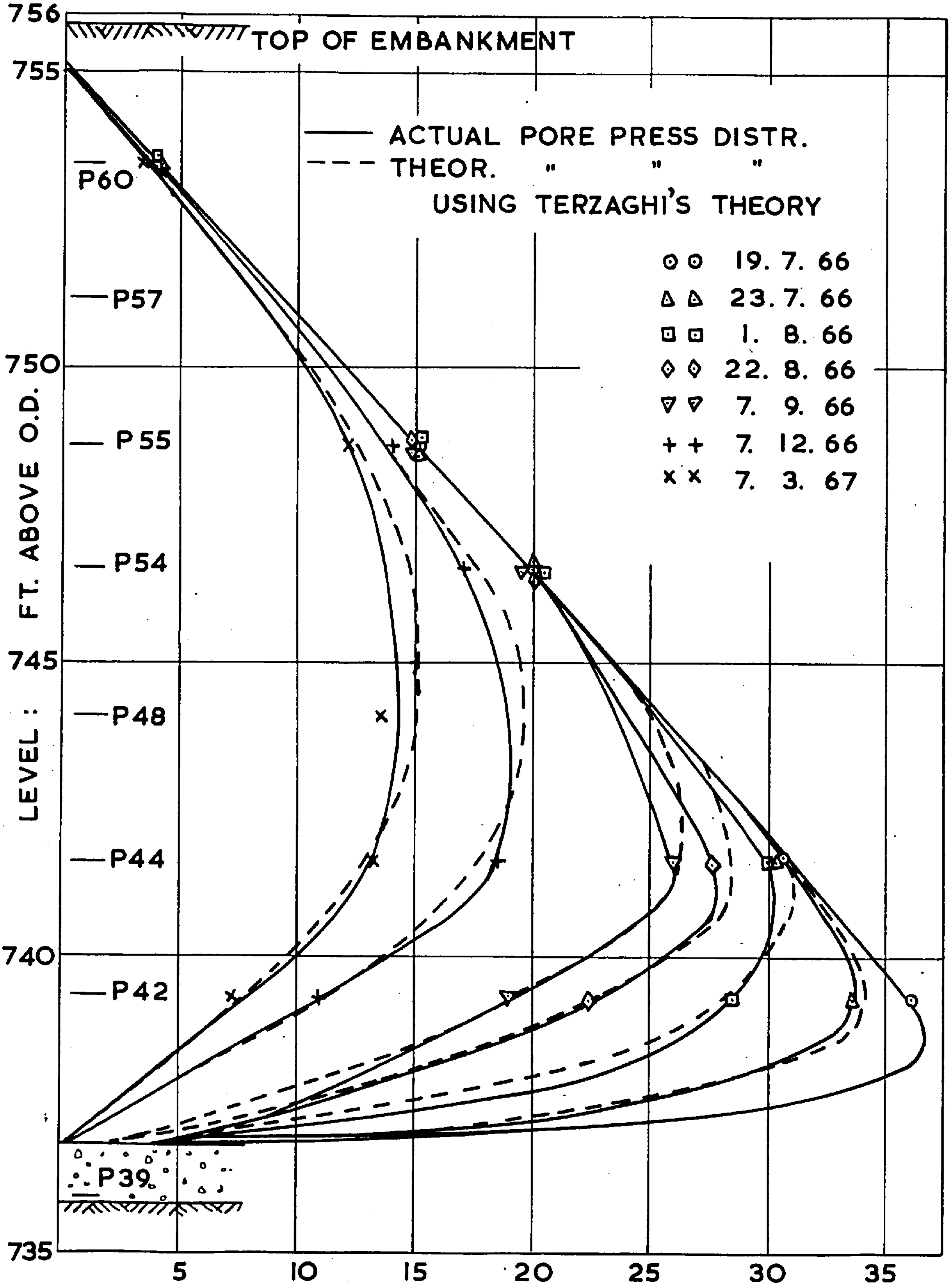
M6 TRIAL EMBANKMENT

FIG. 8 : 22



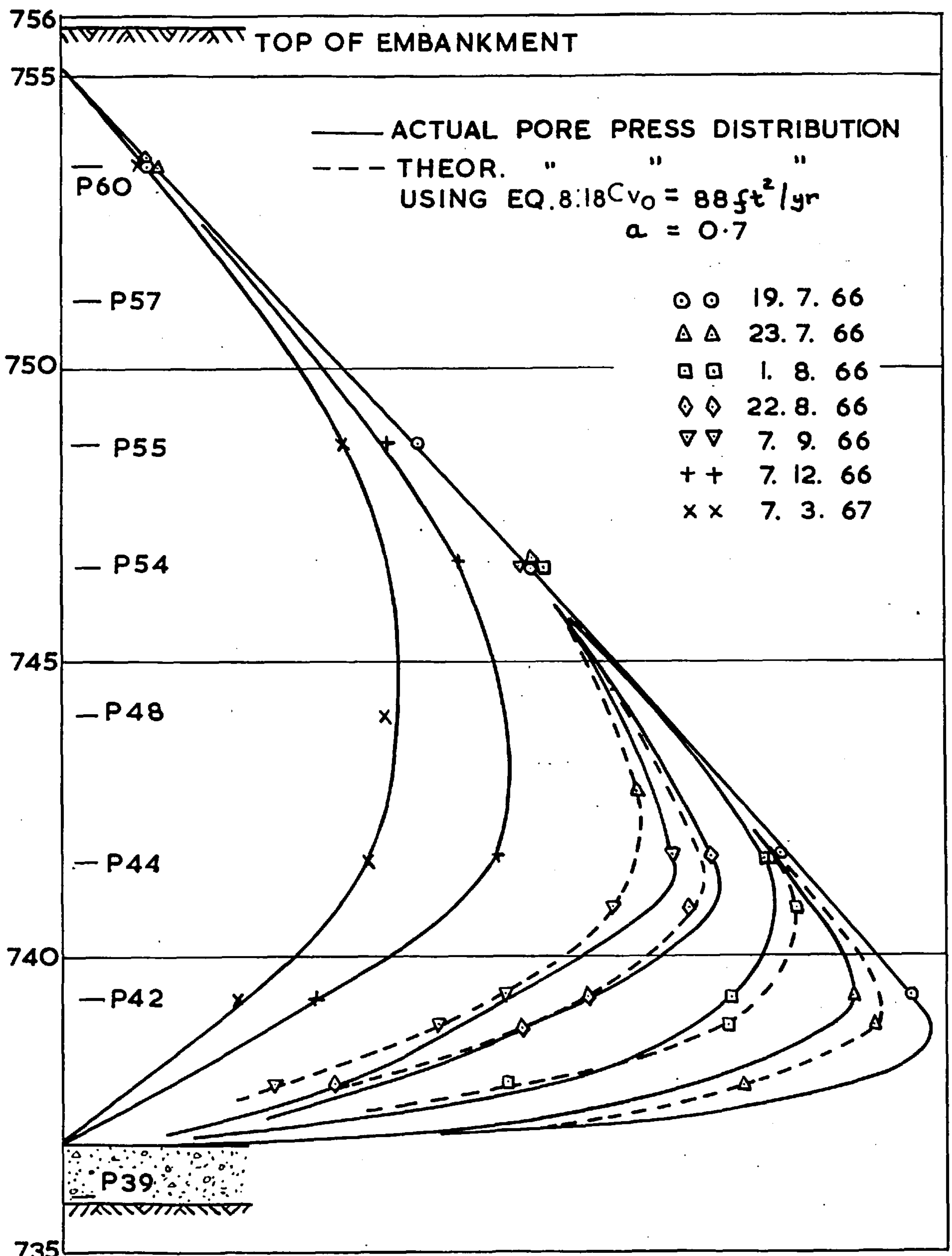
BOUNDARY CONDITIONS FOR THE CONSOLIDATION OF THE FILL IN THE M6 TRIAL EMBANKMENT

FIG. 8 : 23



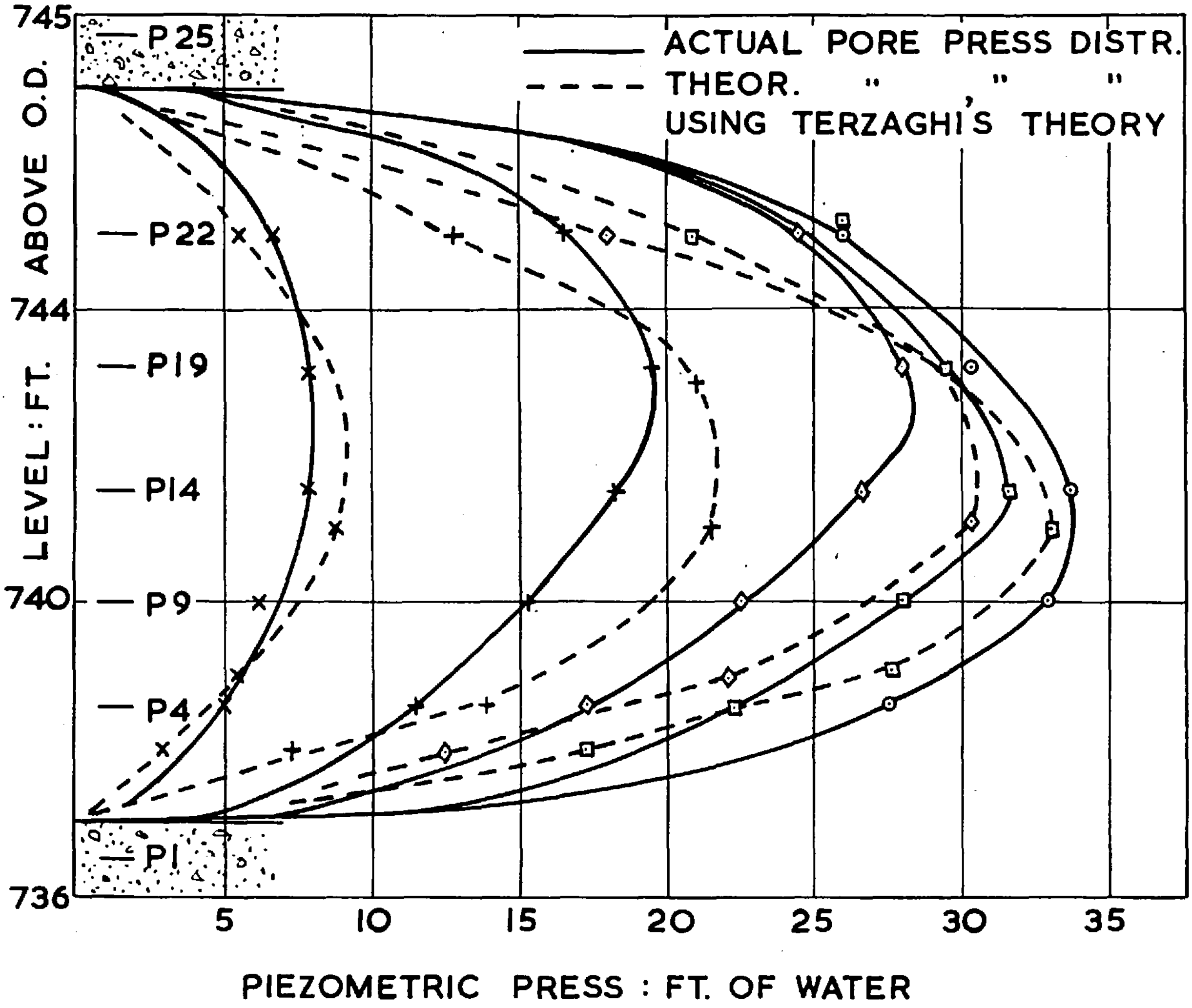
M6 TRIAL EMBANKMENT, SECTION WB

FIG. 8: 24



M6 TRIAL EMBANKMENT : SECTION WB

FIG.8: 25



○ ⊙	19. 7. 66
□ ⊡	1. 8. 66
◇ ◇	22. 8. 66
+ +	5. 10. 66
× ×	7. 12. 66

M6 TRIAL EMBANKMENT

SECTION XB

FIG. 8: 26

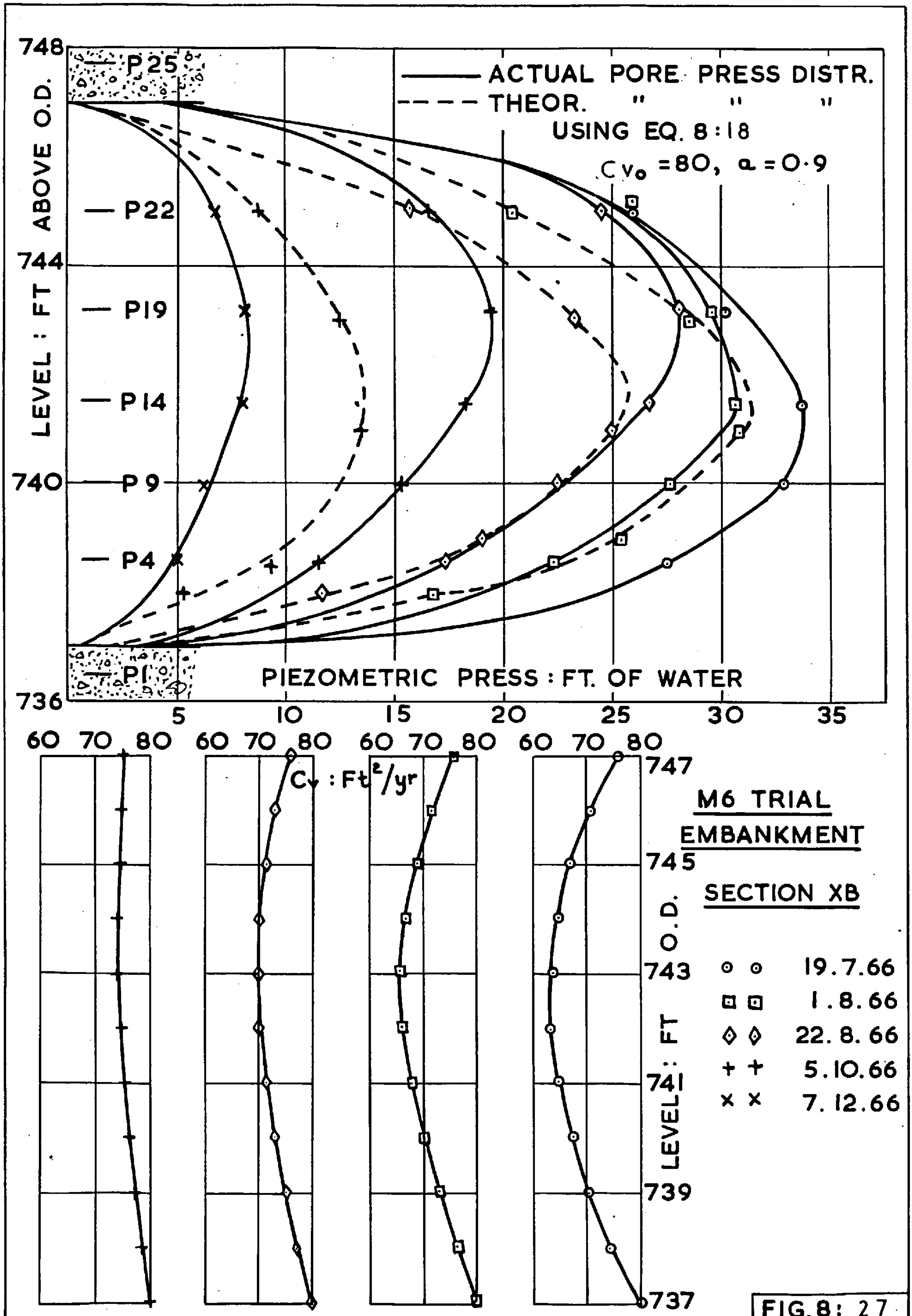
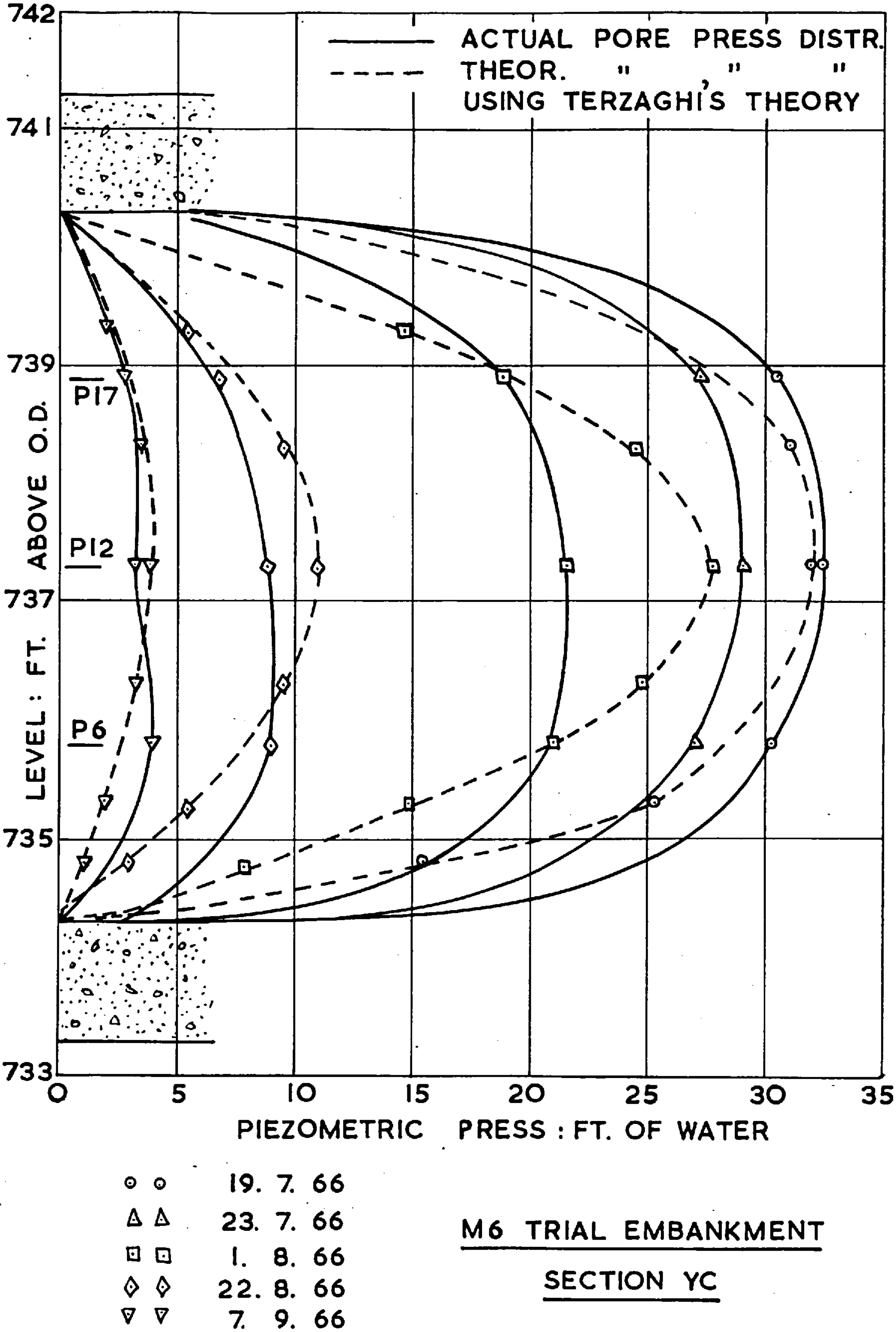

ACTUAL PORE PRESS DISTR.
THEOR. " " "
USING EQ. 8:18
Cvo =80, a=0·9
LEVEL : FT ABOVE O.D.
P25
P22
P19
P14
P9
P4
P1
PIEZOMETRIC PRESS : FT. OF WATER
Cv : Ft²/yr
LEVEL : FT O.D.
M6 TRIAL EMBANKMENT
SECTION XB
19.7.66
1.8.66
22.8.66
5.10.66
7.12.66

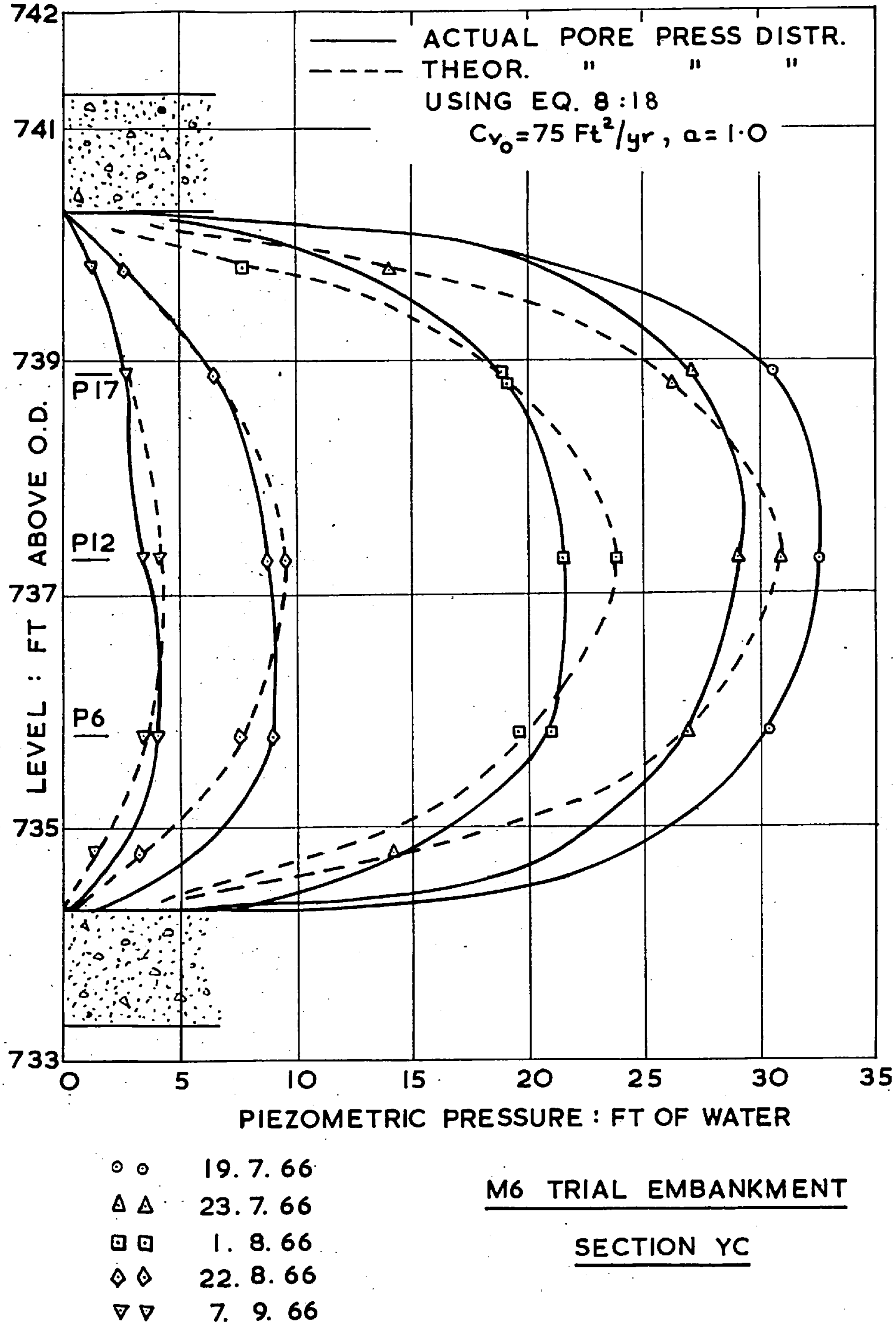
FIG. 8: 27



M6 TRIAL EMBANKMENT

SECTION YC

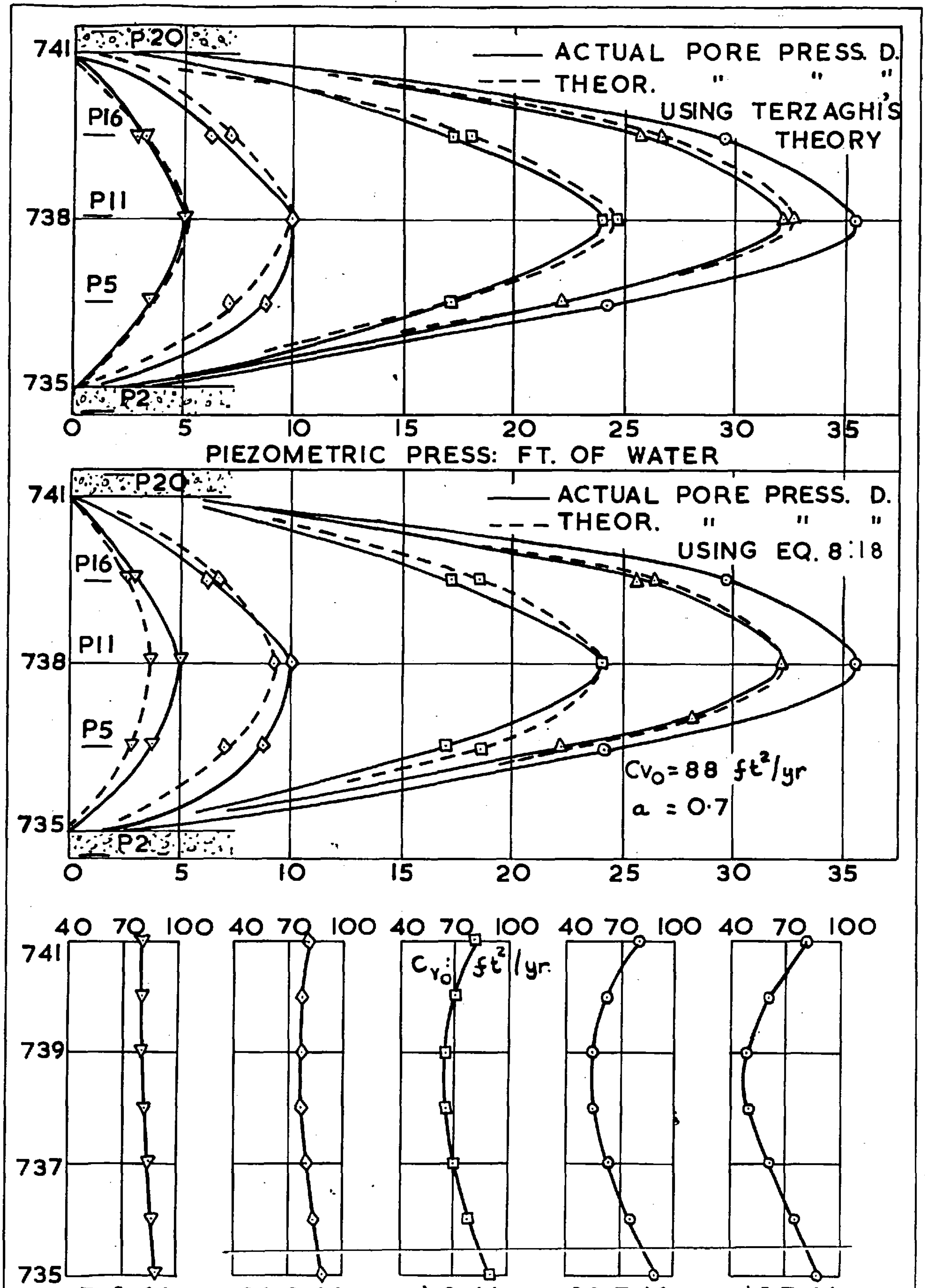
FIG. 8 : 28



M6 TRIAL EMBANKMENT

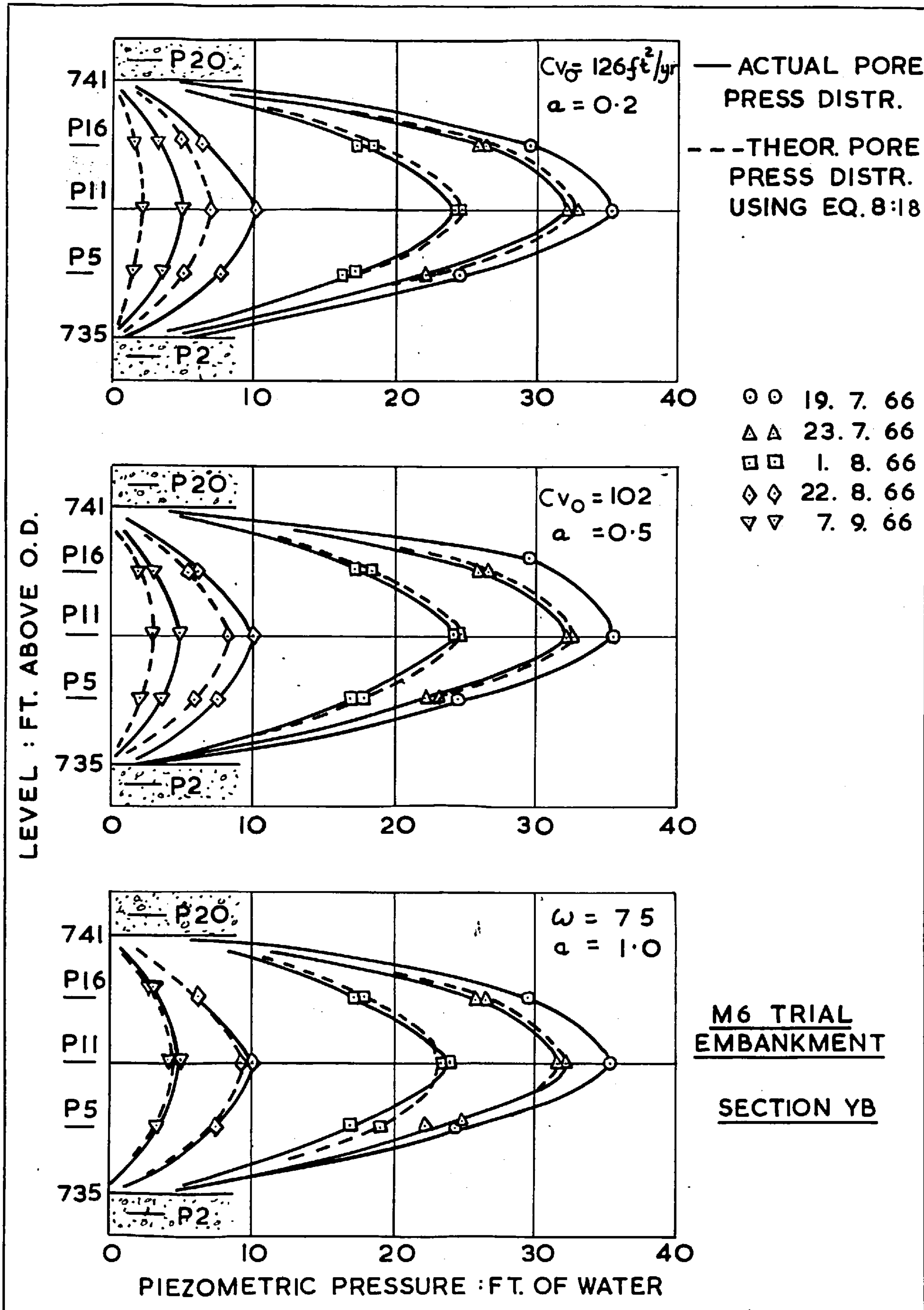
SECTION YC

FIG. 8:29



M6 TRIAL EMBANKMENT : SECTION YB

FIG. 8:30



$Cv_{\bar{\sigma}}$ = 126 ft²/yr
a = 0·2
— ACTUAL PORE PRESS DISTR.
- - - THEOR. PORE PRESS DISTR. USING EQ. 8:18
⊙ ⊙ 19. 7. 66
△ △ 23. 7. 66
⊡ ⊡ 1. 8. 66
◊ ◊ 22. 8. 66
▽ ▽ 7. 9. 66
Cv_0 = 102
a = 0·5
ω = 75
a = 1·0
P20
P16
P11
P5
P2
741
735
0
10
20
30
40
LEVEL : FT. ABOVE O.D.
PIEZOMETRIC PRESSURE : FT. OF WATER
M6 TRIAL EMBANKMENT
SECTION YB

FIG. 8 : 31

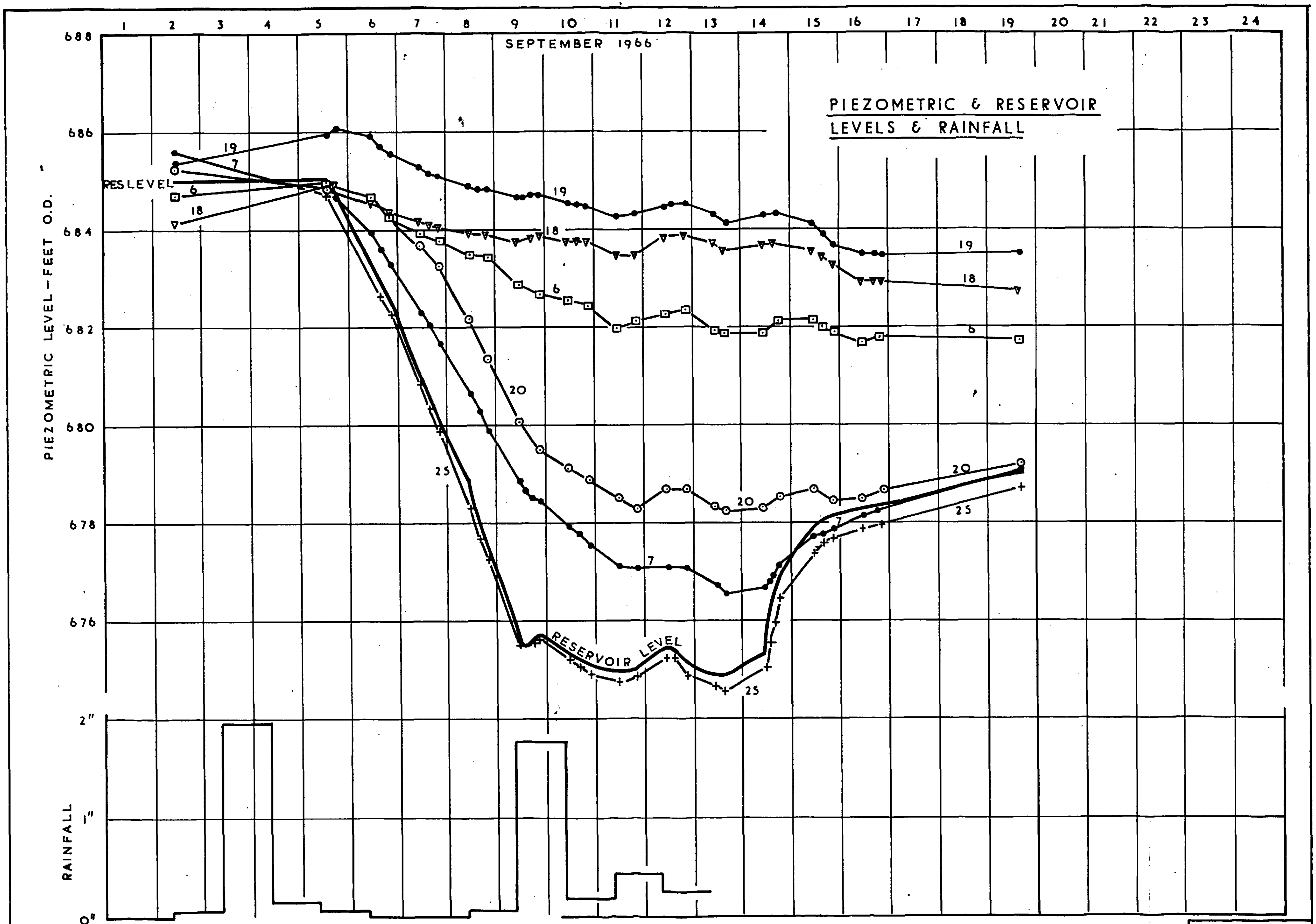
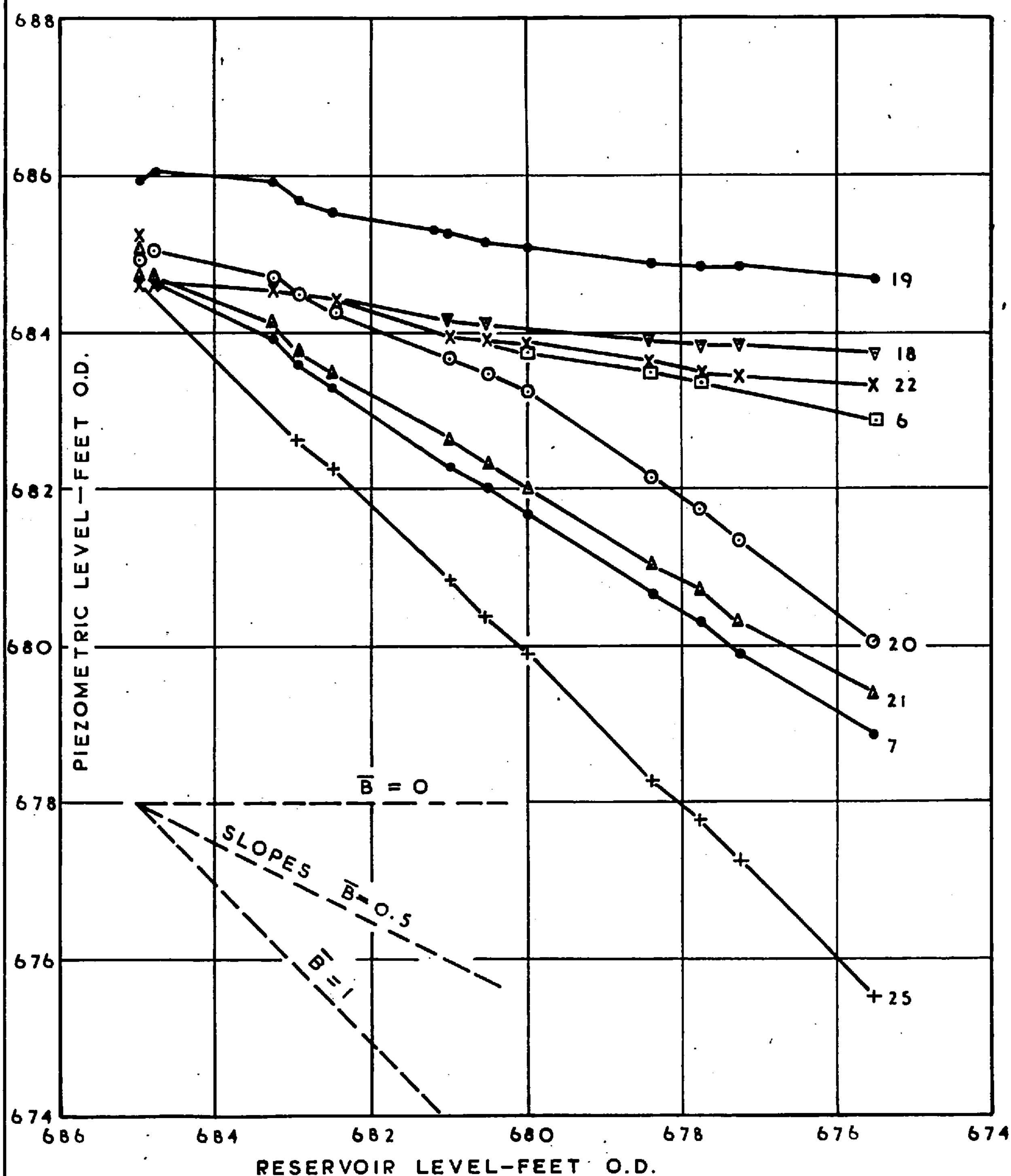
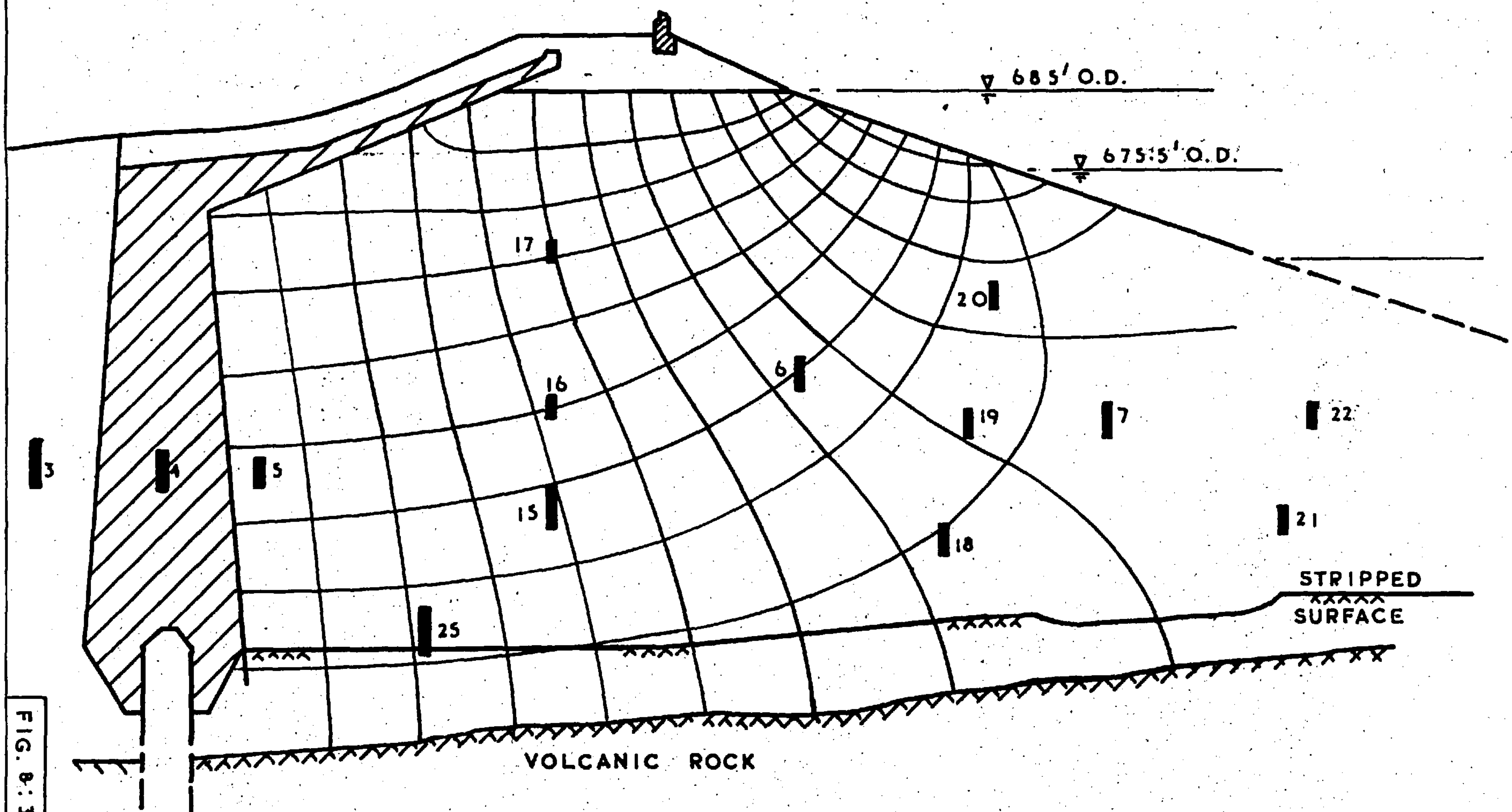

PIEZOMETRIC & RESERVOIR LEVELS & RAINFALL
SEPTEMBER 1966
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
688
686
684
682
680
678
676
PIEZOMETRIC LEVEL - FEET O.D.
RES LEVEL
RESERVOIR LEVEL
19
18
7
6
20
25
RAINFALL
2"
1"
0"

FIG 8:32



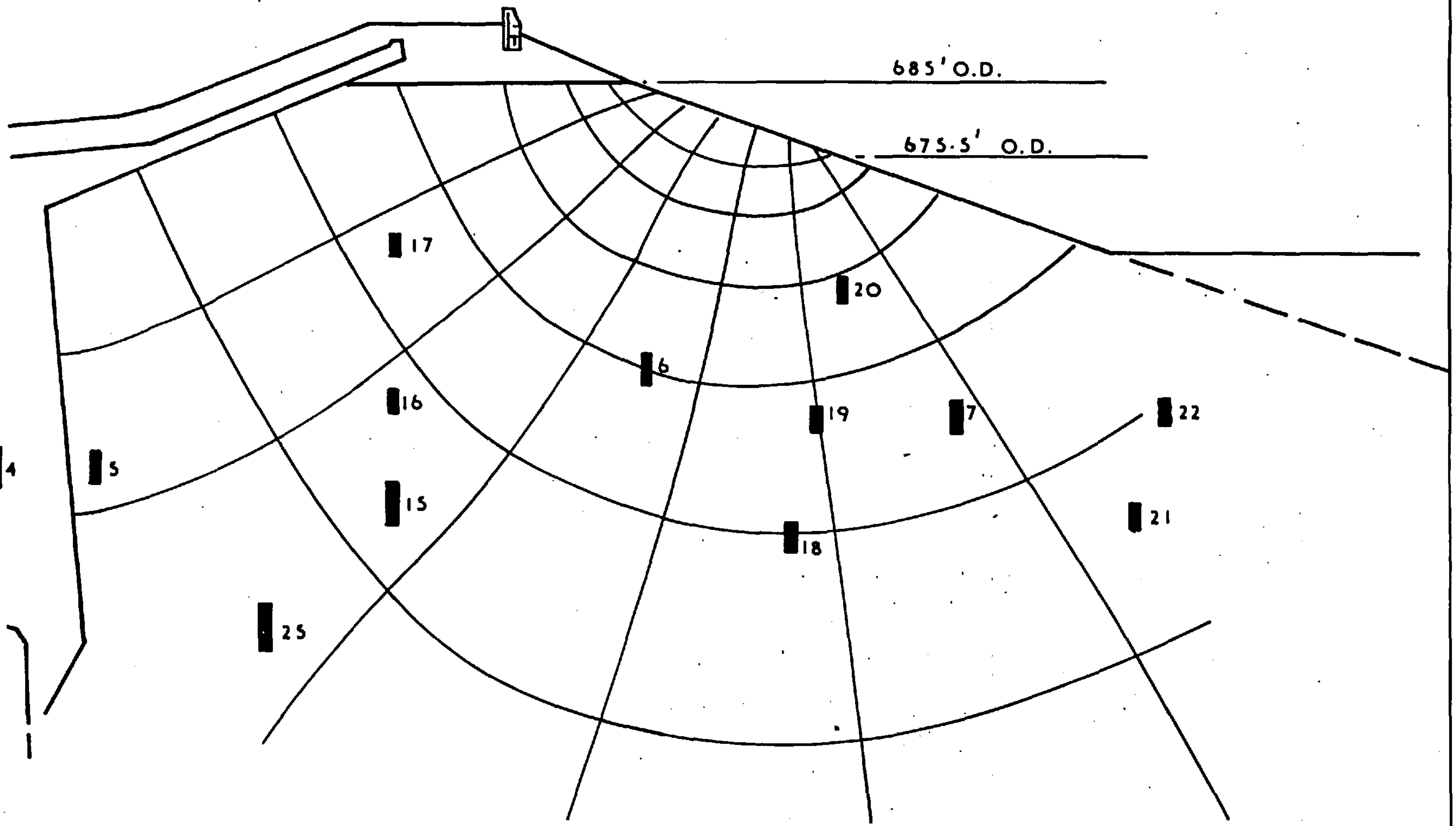
688
686
684
682
680
678
676
674
PIEZOMETRIC LEVEL — FEET O.D.
686
684
682
680
678
676
674
RESERVOIR LEVEL-FEET O.D.
19
18
22
6
20
21
7
25
$\bar{B}$ = 0
SLOPES
$\bar{B}$ = 0.5
$\bar{B}$ = 1

FIG. 8.33



FLOW-NET FOR DRAWDOWN TEST: MUIRHEAD DAM

FIG. 8: 34



FLOWNET FOR DRAWDOWN TEST : MUIRHEAD DAM

CASE 2

FIG: 8 : 35

CHAPTER 9

GENERAL SUMMARY AND DISCUSSION

(9.1) Instrumentation in earth dams

In the introduction to this thesis the vast increase in demand for more dams in the world was mentioned. In particular earth dams are currently favoured over other types, not only because of their economical viability but also because of the establishment of soil mechanics as a worthy quantitative science with which engineers can build dams with much more confidence than hitherto possible.

A major advance in this subject is the development of instruments to moniter the behaviour of the soil in the prototype structure, so that modifications to its design may be made if departures were noticed from the predicted behaviour.

In Chapters 2 and 3 the pore pressure and settlement monitoring equipment currently used in British earth dams were described. The pore water piezometers are now standardised, where I.C. tips are used in compacted fills and Casagrande tips in foundations. However, one problem remains unsolved, viz. the measurement of negative pore pressures exceeding say -8 p.s.i. Such measurements are not only important from an academic point of view, to enhance our knowledge of the behaviour of partly-saturated soils, but also are necessary if full advantage were to be taken of the much increased effective stresses in designing earth structures especially those for temporary use, (see Little, 1965; and Blight, 1963).

Regarding settlement gauges there is still some room for improvement. The U.S.B.R. and B.R.S.-I.C. telescopic vertical deformation gauges proved to be, in general, successful. However, the hydraulic and the Road Research Laboratory mercury vertical settlement gauges have been found to be less satisfactory. A common cause of the mal-functioning of the latter instruments is the failure of the tubes connecting the buried cells to the outside gauge houses. This trouble is avoided in the case of pore pressure piezometers since the plastic tubing used is much smaller and more ductile. It is suggested in Section 3.2.3 that this difficulty may be overcome by suing O-ring sealed telescopic tubing of suitable lengths.

For lateral deformation gauges two instruments which the Author helped to design were described in Chapter 3. These are a version of the wire and plate system and the inverted pendulum. Both instruments appear to be promising. The Building Research Station are currently manufacturing a dual system for measuring vertical deformation, by a hydraulic system, and horizontal movements by the inductance coil method.

The behaviour of the prototype total stress cells used at Balderhead dam and developed by the Building Research Station and Imperial College is very encouraging. However, more research is required to establish more accurately the error introduced into the readings due to the presence of the rigid cells in the soil. The Author in Section 3.4 suggests an alternative design, where the buried cell is a very flexible flat metal 'bag' filled with oil and nitrogen. The proportions of the two fluids may be so chosen as to make the compressibility of the cell comparable to that of the soil for the range of the total stress encountered in the dam.

Finally in this discussion concerning instrumentation the welcome acceptance of these instruments by most earth dam engineers is very encouraging. It is perhaps not an idle boast to say that these instruments together with recent advances in Soil Mechanics have played a vital role in preventing catastrophic failures of earth dams, of the type which befell coal tips, such as that tragically witnessed in the recent Aberfan disaster in South Wales.

(9.2) Intake factors for cylindrical tips

The intake factors obtained by solving Laplace's equation numerically on a digital computer, are in very close agreement with results given by Smiles and Youngs (1965) who used an electrolytic tank analogue. This close agreement confirms, perhaps, the error in Frevert and Kirkham's (1948) results of about 10%. While this error is not significant from an engineering point of view, accurate values were required when studying the unsteady state case.

The intake factor for the conical I.C. tip was not evaluated directly. Instead a probably accurate estimate was made by treating it as a cylindrical tip having its average diameter. The error introduced by this assumption is probably not much greater than that inherent in a direct solution, due to singularity point correction etc. (see Chapter 10).

(9.3) Analytical and approximate solutions for 1-D, 2-D and 3-D seepage tests in compressible soils

Considering first the more significant case of the constant-head test (C.H.T.) the analytical solutions using Terzaghi's theory, for a spherical cavity (by Gibson, 1963), an infinite cylinder and the one-dimensional case showed that a plot of the flow-rate at the pressure source is linear with $1/\sqrt{t}$, where t is the time elapsed since the start of the test.

Fig. 9.1 shows a comparison of the flow rates for the three cases, for the same surface area. The slope of the plot plots is $1/\sqrt{\pi}$ throughout, except for the 2-D case where at about $1/\sqrt{T} = 1$ the plot curves downwards towards the origin.

With this background it is not surprising that the Author's numerical solution for the C.H.T. for finite cylindrical tips gave again a linear relationship between the flow rate q and $1/\sqrt{t}$. Furthermore, this linearity would almost certainly apply for the I.C. conical tip.

For the cylindrical tip it was found that for length/diameter ratios of less than 4 say, the behaviour of the tip is almost the same as that of a spherical tip with same surface area or intake factor. However, for higher length/diameter ratios the behaviour of the cylindrical tip starts, quite understandably, to approximate to that of an infinite cylinder.

Solutions for the falling-head test (F.H.T.) for cylindrical tips showed again the close similarity to the behaviour of spherical tips. This similarity persisted over for high length/diameter ratios, probably because of the insensitivity of the solutions to the dimensionless parameter μ.

(9.4) Factors affecting field C.H.T. results

In Chapter 6 a discussion was given of the factors affecting the values of k and C_v from C.H.T. in the field. It was found that by far the most significant factor is the effect of previous testing on the value of C_v, which may cause as much as 5 or even 10 -fold increase in the true value. As a precaution against this, it was recommended that the test pressure used during the tests must be as small as practically possible; and furthermore the tips must not be tested again until the surrounding soil has undergone a change in the effective stress of at least that of the previous test pressure.

Another interesting point emerging from Chapter 6 is the fact that a C.H.T. on a spherical tip the linear relationship between q and $1/\sqrt{t}$ holds true for non-linear consolidation, such as that proposed by Davies and Raymond (1965) and Barden and Berry (1965). This would suggest that the fact that oedometer samples of a variety of clays giving a linear relationship between the settlement (which for small values of t is the integral w.r.t. of the flow-rate) and $\sqrt{t}$ does not mean that Terzaghi's theory is valid, a conclusion reached earlier by Normand (1964).

More research is clearly required in this field. It is felt that Barden and Berry's (1965) approach, although easily adaptable from a practical point of view, lays rigid conditions on the variation of k and C_v with the effective stress. It would perhaps be more helpful to use Gibson's more basic approach (see Normand, 1964), which allows for large deformations, and in addition places no restrictions on the variation of k and C_v with the effective stress. However, even the latter theory is not by any means perfect since it does not take account of non-homogeneity, nor secondary consolidation.

(9.5) Correlation between field and laboratory measurements

In Chapter 7 the discrepancy between the field and laboratory measurements of negative pore pressures was outlined. In addition, field examples were given of the very low permeability and high compressibility of partly-saturated fills.

In Chapter 8 a correlation was made between field and laboratory results for C_v and k. On the whole the values for k were quite consistent. However, for C_v and in very general terms, the laboratory results tended to be on the low side, and the field seepage results on the high side. Possible relevant factors here are:

(i) the inadequacy of the Terzaghi model to represent the soil. A field example in support of this argument was given from the M6 Trial embankment near Kendal,

(ii) errors in the C.H.T. due to previous testing, as discussed in the last section,

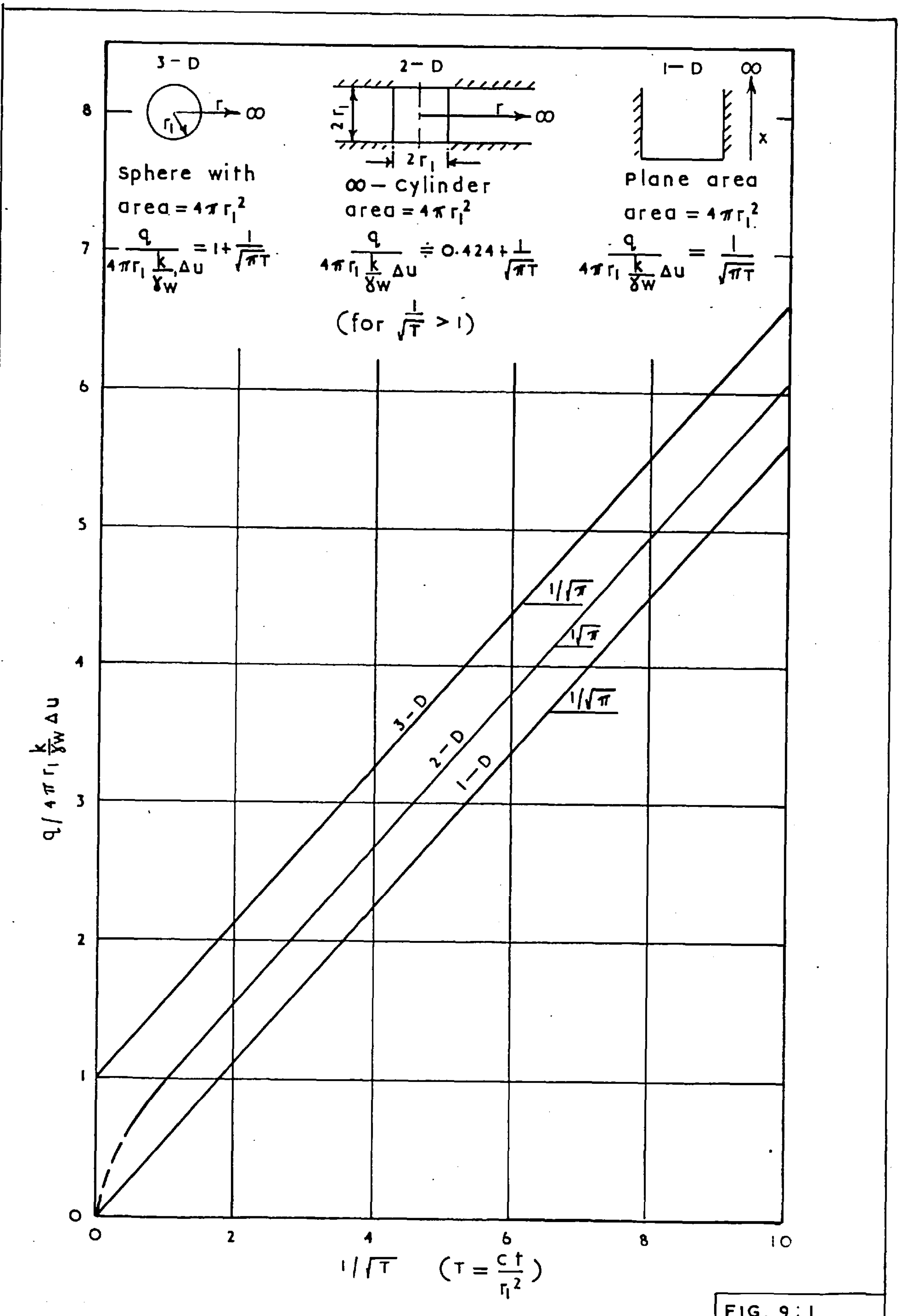
(iii) sampling and testing errors in the laboratory. Of primary significance here is sample size, especially for laminated or more generally non-homogeneous clays. With the current size of samples tested, viz. 4" for oedometer samples, and 1½" or 4" for triaxial samples the natural human tendency is to disregard the more silty and crumbly soil and to test the more clayey part which is easier to handle. The Author understands that recent research on laminated clays at Manchester University by Professor Rowe has shown that increasing the dimensions of the sample has caused an increase in C_v up to a certain limiting size, where the sample is probably then statistically representative of the original soil mass.

(9.6) Draw-down test on Muirhead dam

The results from this test confirmed Vaughan's (1965) conclusions about the tests on Glen Shira dam in that both Bishop's (1952) method, using $\bar{B} = 1$, and Reinius' (1948) approach grossly overestimated the drop in pore pressure with the decrease in reservoir level. This is in contrast to the earlier analysis by Bishop (1952) of the field results from the Alcova dam, where using his method and a value for $\bar{B}$ of unity gave satisfactory agreement between the actual and predicted values.

Nevertheless, it seems that there is urgent need for more basic research into this problem using, for example, the approach put forward by Vaughan (1965) and outlined in Section (8.5.5).

Here the two components in the drop in the pore pressure, viz. that due to the stress release and that due to consolidation during draw-down may be evaluated by a suitable numerical procedure on a digital computer.



3 - D
2 - D
1 - D
Sphere with
area = $4\pi r_1^2$
∞ - cylinder
area = $4\pi r_1^2$
Plane area
area = $4\pi r_1^2$
$\frac{q}{4\pi r_1 \frac{k}{\gamma_w} \Delta u} = 1 + \frac{1}{\sqrt{\pi T}}$
$\frac{q}{4\pi r_1 \frac{k}{\gamma_w} \Delta u} \doteq 0.424 + \frac{1}{\sqrt{\pi T}}$
$\frac{q}{4\pi r_1 \frac{k}{\gamma_w} \Delta u} = \frac{1}{\sqrt{\pi T}}$
(for $\frac{1}{\sqrt{T}} > 1$)
$1/\sqrt{\pi}$
$1/\sqrt{\pi}$
$1/\sqrt{\pi}$
$q / 4\pi r_1 \frac{k}{\gamma_w} \Delta u$
$1/\sqrt{T}$ $\left(T = \frac{ct}{r_1^2}\right)$

FIG. 9:1

CHAPTER 10

NUMERICAL ANALYSIS AND COMPUTER PROGRAMMING FOR THE STEADY AND UNSTEADY FLOW OF WATER INTO PIEZOMETER CAVITIES

(10.1) Introduction

In Chapters 4 and 5 results from numerical solutions for the steady and unsteady flow of water from cylindrical and spherical tips were given. In this Chapter the numerical analysis adopted, together with the computer programmes are presented.

In the following section the steady-state case will be treated. The numerical analysis is presented in an elementary way, without recourse to any matrix algebra. However, for a fuller understanding of the problem the reader should refer to standard works on this topic, e.g. Faddeeva (1959), Varga (1962).

(10.2) Steady state case, formulation of the problem

The object here was to evaluate the intake factor for cylindrical tips for a range of length/diameter ratios, and for different boundary conditions as shown in Fig. 4.2. To achieve this the pore pressure distribution outside the tip must be first found from eq. 4.13 together with the appropriate boundary conditions such as those given in eq. 4.14. From the pore pressure distribution the flow rate out of the tip may then be easily computed.

A closed-form analytical solution for this problem is probably not impossible to obtain, but is certainly very difficult. The Author's numerical solution using the finite difference method will now be outlined.

The first step was to represent the area or domain by a finite number of points so that at each mesh point represented a small region over which the pore pressure was sensibly constant. Naturally, the smaller the mesh size the more accurate would the answer be, but this is limited by obvious practical considerations.

The next step was to write eq. 4.13 in a finite difference form using, for convenience, Taylor's series:

$$u(y + \delta y) = u(y) + u'(y)\delta y + \frac{u''(y)\delta y^2}{2!} + \frac{u'''(y)\delta y^3}{3!} + \ldots \quad (10.1)$$

and
$$u(y - \delta y) = u(y) - u'(y)\delta y + \frac{u''(y)\delta y^2}{2!} - \frac{u'''(y)\delta y^3}{3!} + \ldots \quad (10.2)$$

Here u, which is some function of y, must be continuous and naturally, infinitely differentiable w.r.t. y.

The terms $\frac{\partial^2 u}{\partial r^2}$, $\frac{\partial u}{\partial r}$, and $\frac{\partial^2 u}{\partial z^2}$ in eq. 4.13 were then approximately represented by the first three terms in Taylor's series, thus

$$\frac{\partial u}{\partial r} \doteq \frac{u(r + \delta r) - u(r - \delta r)}{2\,r} \quad (10.3)$$

$$\frac{\partial^2 u}{\partial r^2} \doteq \frac{u(r+\delta r) + u(r-\delta r) - 2u(r)}{\delta r^2} \qquad (10.4)$$

$$\frac{\partial^2 u}{\partial Z^2} \doteq \frac{u(z+\delta Z) + u(Z-\delta Z) - 2u(Z)}{\delta Z^2} \qquad (10.5)$$

Had more terms in Taylor's series been used, the accuracy would have increased, but the simple formulae (10.3), (10.4) and (19.5) would have been much more complicated.

For the boundary conditions, case 1C Fig. 4.2 will be taken as an example. Due to the symmetry about the Z-axis only half the domain need be considered as shown in Fig. 10.1. Also, the impermeable top of the tip was assumed to have negligible thickness. In Section (4.5.1) this assumption was shown to be valid.

It is convenient at this stage to lable the mesh points with some numbering system as that shown in Fig. 10.1. From the boundary conditions, eq. (4.13) it follows

(i) at the points lying on the outside boundaries u is specified, and for convenience was put equal to zero.

(ii) along the centre-line at the mesh points 1 and 13: $\partial u/\partial r = 0$.

(iii) at 5: $\partial u/\partial r = \partial u/\partial Z = 0$.

(iv) at the boundaries of the tip u is specified and was put equal to 1000 units.

(v) at 6: $\partial u/\partial Z = 0$

(vi) at 7: $\partial u/\partial Z = 0$, and in addition u = 1000 units.

This is the singularity point S discussed in Section (4.5.4).

(vii) at all the remaining points there is flow in the r and z directions.

Taking 3 as an example of the latter case, eq. 4.13 may approximately be written (cf. eqs. 10.3, 10.4 and 10.5):

$$\frac{u(4) + u(2) - 2.u(3)}{\delta r^2} + \frac{1}{r}\,\frac{u(4) - u(2)}{2\delta r} + \frac{0 + u(7) - 2.u(3)}{\delta z^2} = 0 \qquad (10.6)$$

Where, of course, u(3), u(4) etc., refer to the pore pressure at these mesh points. Eq. (10.6) is the finite difference equation corresponding to eq. 4.13.

For the mesh points in category (ii) $\partial u/\partial r = 0$, and since $r = 0$ also, the limit of $1/r \; \partial u/\partial r$ as $r \to 0$ had to be found. Using L'houpital's rule:

$$\lim_{r \to 0} \frac{1}{r}\frac{\partial u}{\partial r} = \frac{\partial^2 u}{\partial r^2} \qquad (10.7)$$

For these mesh points, such as 1, the finite difference equation is:

$$4\,\frac{u(2) - u(1)}{\delta r^2} + \frac{0 + u(5) - 2.u(1)}{\delta z^2} = 0 \qquad (10.8)$$

For point 5 (category iii) we have:

$$2\,\frac{u(6) - u(5)}{\delta r^2} + \frac{u(1) - u(5)}{\delta z^2} = 0 \qquad (10.9)$$

For the mesh points such as 6 we have:

$$\frac{u(7) + u(5) - 2.u(6)}{\delta r^2} + \frac{1}{r}\,\frac{u(7) - u(5)}{2\delta r} + 2\,\frac{u(2) - u(6)}{\delta z^2} = 0 \quad (10.10)$$

For the singularity point 7, we may either assume $\partial u/\partial z = 0$ or u = 1000 units. If both conditions are specified the numerical analysis would break down as will be shown shortly.

Taking the boundary condition $\partial u/\partial z = 0$ only at 7 we may write the finite difference equations for all the 16 mesh points. This is shown in Fig. 10.1 where it is assumed for convenience that $\delta r = \delta z$. If we now, by definition, choose to write this set of equations in the manner shown in the upper right hand corner of Fig. 10.1, then the coefficients multiplying the unknowns are grouped together in the two dimensional array or matrix, A; the unknowns form the one dimensional matrix, or vector, u; and the known quantities then form the vector b. As a short-hand representation the 16 equations may thus conveniently be written as:

$$A.u = b \quad (10.11)$$

If we study the matrix A, we find that it is a square array of numbers, i.e. 16 x 16. This is because we had 16 equations for 16 unknowns, which is a necessary condition for obtaining a unique solution to the problem. Had we specified $\partial u/\partial Z = 0$ and u = 1000 units at the singularity point 5, we would then have 16 equations for 15 unknowns, from which we would not obtain one unique solution.

(10.2.1) Solution by exact method

To solve the equations (10.11) we may either employ an exact or an approximate method. In the former the well known method of elimination is used. For example, one unknown would be eliminated from the second equation,, two from the third, etc. This method will perhaps provide an answer provided that the determinant of the matrix A is not equal to zero (see for example Faddeeva, 1959). There are in fact in most of the larger digital computers built-in subroutines whereby the matrix A and the vector b may be fed in and the unknowns u evaluated .

However, this method (using the present standard subroutines) suffers from a severe disadvantage viz. the unacceptably large storage space required in the computer. In fact, to solve n simultaneous equations, $2n^2$ storage locations are required. Thus for 16 equations we require 512 locations; and if it is desired to solve the more realistic figure of 1000 equations we require 2×10^6 locations!

(10.2.2) Storage capacity of the 7090 I.B.M. computer

The storage capacity of the 7090 I.B.M computer used by the Author at Imperial College consists of two parts:

(i) the 'fast' storage which comprises about 22,000 locations.

(ii) the 'slow' storage which is practically unlimited.

The Author, in the early stages, made use of the magnetic tape facilities, but encountered several difficulties apart from the time wasted in writing and reading off the tape. Often due to human error the recorded results were obliterated following an incorrect mounting or dismounting of the tape on the computer. In addition if several arrays are required to be written or read on the tape, difficulties arose when it was decided to read, write or adjust one middle array without disturbing the rest. In fact, the Author recommends that unless it is absoluetly necessary to use the tape for extra storage, it is to be avoided and the programme should be limited to the fast storage facilities only.

Returning to the exact method of solving the set of simultaneous linear algebraic equations 10.11, we found that in order to solve 1000 equations we require 2 million storage spaces which is clearly well outside the limit of the fast storage. However, some saving in storage space may be possible by partitioning the matrix A (see Faddeeva, 1959) but this would still require large storage and is more difficult to programme.

An exception here is the case when the matrix A is symmetrical about the diagonal (see section 10.4.2) where a special direct elimination method may be used which requires very little storage.

(10.2.3) Solution by approximate methods, relaxation method

The relaxation technique for the solution of simultaneous linear algebraic equations was developed first by Southwell (1946) and was often used before the advent of efficient electronics computers. Basically, it entails guessing initial values of u in eq. 10.11, and then improving these values by relaxation.

Taking as an example mesh point 8, Fig. 10.1, the finite difference equation is:

$$u(8) = \left[u(7) \left\{ 1 - \delta r/2r \right\} + \delta r^2/\delta Z^2 \left\{ u(4) + u(9) \right\} \right] / \left[2 + 2\frac{\delta r^2}{\delta Z^2} \right] \quad (10.12)$$

Inserting the initially guessed values for the pore pressure at the points neighbouring mesh point 8, a new value will be obtained which is very likely to be different from the initially guessed value. This is repeated for all the mesh points and the magnitude of the differences between the old and new values are recorded. These are called the residuals. Next, the mesh points with the largest residuals, irrespective of the sign, are relaxed first, and the process continued until the maximum residual at any point is below a certain specified value.

The rate of convergence of this process may be accelerated by under or over-relaxation, which simply means replacing the old values by the new values $\pm$ a factor times the difference between the old and new values. For over-relaxation this factor is between 1 and 2, whereas for under relaxation it is between 1 and 0; but for more details about this and other techniques for efficient relaxation, see Allen (1954).

Although the relaxation process is perhaps the quickest approximate method, it is really meant for manual computation where human intelligence and experience play a vital role in deciding at a glance which mesh points should be relaxed first, by how much to over-relax or under-relax, etc. This is difficult to teach a computer to do, which prefers systematic computations with few choices, as is the case for the other approximate methods discussed in the next sections.

A very useful and obvious advantage of all these approximate methods is that over the direct elimination method any errors made during the computations, except when the final results are obtained, do not affect the final solution.

(10.2.4) Gauss' method

Here, as in the relaxation process, a start is made by guessing initial values, calculating new values using equations of the type of eq. 10.12 and recording the residuals.

In the next step or iteration, all the mesh points are systematically relaxed without regard to the largest residual. For example, in Fig. 10.1 the mesh points may conveniently be relaxed thus: 1, 2, 3, 4, - - - 13, 14, 15, 16. Also, none of the newly calculated values during the iteration are used for relaxing the neighbouring points. At the end of the iteration the largest residual is compared to some specified value. If it exceeds it, the old values are replaced by the new values and another iteration is made; if it does not, then the new values form the final solution.

(10.2.5) Gauss-Seidal method

This is similar to the Gauss method, but here use is made of the newly calculated values when relaxing their neighbours. Thus during an iteration immediately a mesh point is relaxed the residual is computed and compared to the maximum residual at the previously relaxed points. If it is greater it is stored and labelled the maximum, and if it is smaller or equal to it, it may be forgotten. At the end of the iteration, if the maximum residual is less than a specified value, the solution is taken as final; otherwise another iteration or more are required.

(10.2.6) Gauss-Seidel over-relaxation method (G.S.O.M.)

This is an improvement on the Gauss-Seidel method where over-relaxation is employed, as already discussed in 10.2.3. This is perhaps the most popular method and was used by the Author for the solution of the steady-state problem.

(10.2.7) Storage requirements and convergence rates for the three iterative methods.

For the solution of n simultaneous algebraic equations the three iterative methods require basically n spaces. In the case of the Gauss method an additional vector comprising n locations is also required for temporary storage of the newly calculated values along one row. Of the three methods the G.S.O.M. has been found generally to converge more quickly than the other two methods.

However, it has also been found that these iterative methods do not always yield solutions, i.e. after a certain number of iterations the residuals may start to oscillate and increase indefinitely. The convergence, or lack os it, depends on the matrix A.

Any detailed discussion of the convergence rates here is beyond the scope of this thesis, but, basically, is the matrix A has certain properties then it can be proved that a convergent solution may be obtained. Otherwise it may, or not converge. However, from the experience of a colleague, Mr. Phukan, it seems that the chances are that a matrix will converge, albeit slowly, even when obviously not satisfying the requirements. The Author's matrix for the steady-state case does satisfy the requirements, as will be mentioned in section 10.3.3. For more details about rate of convergence, etc. the reader is referred to Varga (1962) and Khabaza (1963).

(10.3) Computer programme for the steady-state case using the G.S.O.M.

As already mentioned in section 4.5.3 the Author's first programme utilised a linear mesh size for both the r and z co-ordinates, which proved to be wasteful of both time and storage. As a remedy the two co-ordinates were transformed thus:

$$x = \ln(1 + r) \tag{10.13}$$

and $$y = \ln(1 + z) \tag{10.14}$$

The quantities $\ln(1 + r)$ and $\ln(1 + z)$ were chosen instead of $\ln(r)$ and $\ln(z)$ to avoid the obvious difficulties at the mesh points lying on $r = 0$ and $z = 0$.

Using the transformation eq. (10.13), eq. (4.13) becomes:

$$\frac{\partial^2 u}{\partial x^2} + \frac{1}{r}\frac{\partial u}{\partial x} + (1 + r)^2 \frac{\partial^2 u}{\partial z^2} = 0 \tag{10.15}$$

It was found convenient to keep z in equation (10.15) to avoid obtaining the term $\partial u/\partial y$ if the y co-ordinate was used. However, the mesh sizes chosen were linear in x and y.

The finite difference relationships for $\partial^2 u/\partial x^2$ and $\partial u/\partial x$ are similar to those for $\partial^2 u/\partial r^2$ and $\partial u/\partial r$, but for $\partial^2 u/\partial z^2$ a new expression had to be derived using unequal mesh sizes along the z - axis.

Eq. (10.14) may be rewritten:

$$z = e^y - 1 \tag{10.16}$$

Consider three consecutive mesh points lying on the same value for r, at distances Z_1, Z_2, Z_3 above the Z = 0 plane. If y is constant, then:

$$\frac{z_3 - z_2}{z_2 - z_1} = e^{y} = \quad \text{say} \tag{10.17}$$

i δz increases in a geometric progression by the quantity ,
an example of which is shown in Fig. 4.4. The finite difference equation replacing the term $\partial^2 u/\partial z^2$ for mesh point 3 (Fig. 10.1) for example becomes:

$$\frac{\partial^2 u}{\partial z^2} \doteq 2 \left[\frac{0 + \ .u(7) - u(3)\ (1 + \alpha)}{\delta z_2(\delta z_2 + \delta z_1)} \right] \tag{10.18}$$

where δz_1 is the mesh size between 7 and 3, δz_2 is that between 3 and the top mesh point at the outside boundary, and $\alpha = \delta z_2/\delta z_1$
If $\alpha = 1$, eq. (10.18) degenerates into the form given in eq. (10.5)

Similarity for a mesh point below the z = 0 plane such as 12:

$$\frac{\partial^2 u}{\partial z^2} = 2\,\frac{u(16) + \alpha.u(11) - u(12).(1+\alpha)}{\delta z_2(\delta z_2 + \delta z_1)} \qquad (10.19)$$

where δz_1 is the mesh size between 11 and 12, δz_2 that between 12 and 16, and $\alpha = \delta z_2/\delta z_1$.

For points on the plane z = 0, such as 10:

$$\frac{\partial^2 u}{\partial z^2} = \frac{u(9) + u(11) - 2.u(10)}{\delta z0^2} \qquad (10.20)$$

where $\delta z0$ is the mesh size between 9 and 10 which is of course the same as that between 10 and 11, and is given by (see eq. 10.14):

$$\delta Zo = e^{y} - 1 \qquad (10.21)$$

We have now completed the derivation of the finite difference equations and may proceed to outline the computer programme utilising these logarithmic co-ordinates. A flow-chart is shown in Fig. 10.2.

(10.3.1) Data input

The data fed into the computer are as follows:

(i) a control number which determines whether the computer has to assume, initially. some values for u in eqs. (10.11), or read from a magnetic tape more accurate values previously calculated but which have not converged well enough, and where the computer was asked to stop after a specified time. However, as was mentioned in section (10.2.2) very little use was made of the magnetic tape facilities. With experience the number of iterations and hence time, required could be estimated quite accurately without having to use the magnetic tape for temporary storage.

(ii) the radius and half the height of the tip; for example 1.5 and 3.0 units, and the number of mesh intervals in each, say K and L respectively.

Knowing half the height of the tip and L the computer is asked to calculate δy from eq. 10.14, and from eq. 10.17 α may be computed. Eq. 10.21 then gives $\delta z0$.

(iii) the number of mesh intervals in r_2 and z_2, Fig. 4.2. From these r_2 and z_2 can be computed using eqs. 10.13 and 10.14.

(iv) the pore pressure at the surface of the tip, say 1000 units and at the outside boundaries, say zero.

(v) a control number to decide whether to relax the singularity point or set it and leave it at 1000 units of pressure.

(vi) the over-relaxation factor 'W', and the maximum number of iterations to be performed.

(vii) other control numbers to decide when and where to integrate to find the flow rate, when and how much data to be printed during the relaxation process and at the end, etc.

(10.3.2) Execution of the programme

The computer when performing the calculations, first computes and stores all the constants recurring repeatedly in the subsequent iterations. It then decides whether to assume some initial values or read off the magnetic tape. For guessing initial values the pore pressure was conveniently assumed to decrease linearly away from the tip along both the x and y co-ordinates.

The largest 'loop' is then entered by setting a symbol 'key' to zero. All the mesh points are then relaxed from left to right starting at the top. If the 'error', defined as the modulus of the difference between the new and old values divided by the new value, is more than say 10^{-5} for any mesh point, 'key' is set to one. At the end of the iteration if 'key' is zero, the final solution is obtained. If it is one, the computer is asked to write the number of the iteration and the pore pressure and location of the mesh point with the maximum error. After this another iteration is performed by setting 'key' to zero and proceeding as before. If after a certain maximum number of iterations the solution has still not converged, the computer is instructed either to write the latest results, forget them and stop; or to ask for a magnetic tape, record the results and stop. An additional feature of the programme is that every 50 or 100 iterations the rate of convergence is studied and if it is reasonable a reassuring message is pointed to the human operator. On the other hand if the solution is oscillating wildly the computer prints out the latest results and other relevant data and stops.

The flow rate, which is virtually the intake factor of the tip (see section 4.2.1) is computed after a specified number of iterations as governed by the control numbers. The pore pressure gradients are calculated across successive rectangular contours enclosing the tip but obviously not intersection each other. Laplace's equation 4.13 stipulates that the flow rate across such contours must be the same. Typical values of the computed flow rate are shown in Fig. 4.4 where they are seen to converge to better than 99.5% for a maximum 'error' of about 5×10^{-5}.

It was found convenient to use two methods for integrating the pore pressure gradients along the contours, viz. the trapezodial and Simpson's methods. Although the latter was more accurate by perhaps ½% it was likely to fail when, due to human error, the number of mesh points along the contour was set at an even and not an odd number.

(10.3.3) Rate of convergence of the solutions using the G.S.O.M.

It was stated in Section (10.2.7) that a detailed discussion of this topic is beyond the scope of this thesis. However, some general and practical aspects of the problem will now be noted.

Varga (1962) gives a relationship between a function of the rate of convergence and the over-relaxation factor w, the latter confined to the limits 1.0 to 2.0. Somewhere between these two limits an optimum value for w exists at which the fastest rate of convergence may be achieved. This optimum value can be assessed once the eigenvalues of the matrix A are computed. But the magnitude of computation required to evaluate the eigenvalues is comparable to that of obtaining the actual solution itself! This is why the optimum value of w is usually guessed, mainly from previous experience. From the relationship that Varga gives it seems that it is better to overestimate the optimum value than underestimate it (see Varga, 1962 p. 114).

In very general terms the Author found the following conditions to give a faster rate of convergence, i.e. fewer necessary iterations for a certain maximum 'error':

i) smaller domain and coarser mesh size

ii) extending z_2 and decreasing γ_2

iii) having the outside boundaries set at u = 0 instead of

$\partial u/\partial r = 0$.

iv) using a higher value for w for layer matrices than for smaller ones.

Perhaps the most significant item here is (ii) where the number of iterations required for a solution of the boundary conditions case 1A Fig. 4.2 was more than double that for case 1B.

To investigate the effect of w on the rate of convergence three runs were made with different values of w as shown in Fig. 10.3. For w = 1.9, which gave the fastest rate of convergence, the solution was oscillating throughout. The solution for w = 1.6 after a few initial oscillations settled down to a smooth linear relationship between the logarithm of the maximum error and the number of iterations. For w = 1.2 the solution is completely free of oscillations.

In the Author's solution the use of high values of w was avoided. This is because of the obvious difficulty in specifying the maximum allowable error. Thus to achieve a masimum error of 5×10^{-5} say, the solution must be carried down to 5×10^{-6} perhaps as shown in Fig. 10.3. Since it is difficult to estimate beforehand the magnitude of the oscillation, the lower limit of 5×10^{-6} would prove to be uneconomical if the solution turns out to be stable.

(10.3.4) Singularity point correction

The difficulties arising from the singularity point have been discussed in Section (4.5.4). To study the problem. several runs were made for case 1C and for a length/diameter ratio of 2. Four solutions with different mesh sizes were obtained, with the singularity point set at $\partial u/\partial z = 0$. Another four runs were made with the singularity point set at u = 1000 units. The computed intake factors are given in Fig. 10.4.

The choice of the particular value of δz (see Fig. 4.4) for plotting the computed intake factors was made for two reasons:

i) the intake factor values proved to be much more sensitive to δz than to δr even for a length/diameter ratio of $\frac{1}{2}$. This conclusion was arrived at from the results of several runs with various values for δz but with the same δr and vice versa.

ii) it conveniently gave a linear plot (see Fig. 4.2) for the case when the singularity point was set at $\partial u/\partial r = 0$.

The final results given in Fig. 4.2 were obtained by extrapolating the latter plots to the $\delta z = 0$ axis. This procedure is not entirely rigorous but perhaps not widely inaccurate as vindicated by the close agreement with the results obtained by Smiles and Young (1965).

(10.4) Unsteady state - C.H.T. and F.H.T. on a spherical tip in a compressible soil

In section (5.6) the difficulties initially encountered in the solutions for the cylindrical tip were outlined. This is why it was decided to solve the problem for a spherical tip and compare the numerical results to Gibson's (1963) analytical solutions to gain more in sight into the whole numerical procedure.

For any shape of tip the solution may be divided into two parts.

i) the evaluation of the pore pressure distribution at different values of time t.

and ii) the evaluation of the flow rate.

For the latter three different methods were considered as already mentioned in Section (5.6) of which the total swell method was found to be best.

Numerical solutions for the spherical tip were obtained using both explicit and implicit methods as discussed in the next two sections.

(10.4.1) Explicit method (or marching forward method)

Using Terzaghi's theory the differential equation in symmetrical spherical co-ordinates is:

$$c\frac{\partial^2 v}{\partial r^2} = \frac{\partial v}{\partial t} \qquad (10.22)$$

where v = u.r.

The C.H.T. case will be treated first.

Let the radius of the spherical tip be divided into 20 units say and let the boundary conditions be:

i) at the mesh point at the surface of the tip, 21, $u = 1000$ units at $t \geqslant 0_+$

ii) at a mesh point far away from the tip,

$$u = 0 \text{ for } t \geqslant 0$$

iii) at all other mesh points $u = 0$ at $t = 0$.

Thus at $t = 0_+$ the pore pressure is specified at all the mesh points. The object now is to determine the pore pressure at $t = 0 + \delta t$, $t = 0 + 2\delta t$, etc.

Taking mesh point 29, as an example, we may say that the time derivative $\partial v/\partial t$ at $t = \frac{1}{2}(0 + \delta t)$ is approximately equal to the space derivative $\partial^2 v/\partial r^2$ at $t = 0$. Using Taylor's series this may be written as

$$\frac{v_{29,t} - v_{29,t-\delta t}}{\delta t} \doteq \frac{c}{\delta r^2}\left\{v_{30,t-\delta t} + v_{28,t-\delta t} - 2v_{29,t-\delta t}\right\} \qquad (10.23)$$

Note the two approximations, the first when using Taylor's series to evaluate the derivations, and the second when assuming that the average time derivative during the interval t and $t + \delta t$ is equal to the space derivative at time t. Eq. (10.23) may be rewritten:

$$v_{29,t} = v_{29,t-\delta t} + \beta \left\{ v_{30,t-\delta t} + v_{28,t-\delta t} - 2v_{29,t-\delta t} \right\} \quad (10.24)$$

where all the quantities on the R.H.S. are known and may be used to evaluate the L.H.S. term. The dimensionless quantity β is given by

$$\beta = c\delta t/\delta r^2 \quad (10.25)$$

Using equations similar to 10.24 all the mesh points can be relaxed. The newly computed values are not used in the calculation of their neighbours. They are stored until all the points have been relaxed whereupon in one sweep the old values are replaced by the new values. The above procedure is then repeated for the desirable number of δt's.

This method is easy to programme, requires little storage space and was in fact found to be not much less accurate than the more sophisticated implicit method. However, it suffers from the disadvantage that a severe limit exists on the maximum value of to be used. If β is larger than 0.5, the solution will oscillate and diverge. While values of β of 0.5 and less are satisfactory at small values of time, they are not so later on when it is desired to march forward at a faster rate. Scott (1965) refers to this problem and also states that a value of β = 1/6 should give the most accurate answer.

(10.4.2) Implicit method

Here the average time derivative $\delta v/\delta r$ during the interval t and $t + \delta t$ is equated to the average of the two space derivations at time t and $t + \delta t$. Taking mesh point 29, as an example again we have:

$$\frac{v_{29,t} - v_{29,t-\delta t}}{\delta t} = \frac{1}{2}\frac{c}{\delta r^2}\Big\{ v_{30,t-\delta t} + v_{28,t-\delta t} - 2v_{29,t-\delta t} + v_{30,t} + v_{28,t} - 2v_{29,t} \Big\} \tag{10.26}$$

Grouping the known quantities on the R.H.S. and the unknown quantities on the L.H.S. we have

$$- v_{28,t} + v_{29,t}(2/\beta + 2) - v_{28,t} = v_{30,t-\delta t} + v_{28,t-\delta t} - 2v_{29,t-\delta t} + (2/\beta)v_{29,t-\delta t} \tag{10.27}$$

If we have n mesh points then we shall have n simultaneous linear algebraic equations for n unknowns. Writing all these equations in matrix form we have

$$A.v = b \tag{10.28}$$

where A is a square matrix which is tri-diagonal and symmetrical about the diagonal line. For this type of matrix a direct elimination method may be used without recourse to approximate iterative methods. This is the square-root method described for example by Faddeeva (1959). A flow chart for the programme used by the Author for the C.H.T. is given in Fig. 10.5, and the results are shown in Fig. 5.4

In this method there is no limit on the value of β to be used. However, at small values of t, when high pore pressure gradients exist near the tip, it was found that a sensibly small value had to be used to avoid inaccuracies due to the inadequacy of the first few terms in Taylor's series in representing the differential equation.

(10.4.3) Errors in the computation of the flow-rate

These errors have two sources, viz. the error in the determination of the pore pressure distribution and the error in the computation of the flow rate from the pore pressure distribution. For the former and at small values of t the error even when using an explicit method was only a few percent, as checked against Gibson's (1963) analytical solution. For larger values of time the accuracy improved as shown in Fig. 5.4

The errors in the calculation of the flow rate at small values of t proved to be much more significant even when using the total swell method. The major source of the error is the mesh interval immediately outside the tip. Even if Gibson's (1963) analytical solution for the pore pressure is used at the mesh points and Simpson's rule is employed to evaluate the total swell, the swelling of the soil enclosed within the half mesh near the tip would still be grossly underestimated. Yet it is this soil which is swelling most, at small values of time. Thus, however small the mesh size is made the initial flow rate would always be underestimated, see Fig. 5.4.

While this was not critical for the C.H.T. it was so for the F.H.T. where any initial errors in the flow rate are carried through the solution as already discussed in Section (5.6.2).

(10.5) Unsteady state case, cylindrical tip: C.H.T. and F.H.T.

From practical considerations the explicit method only could be used here. This is because the matrix A obtained for the implicit method is non-symmetrical and thus would have required either unacceptably large storage space if solved by elimination method, or unacceptably large time if solved for every δt using an iterative method. The flow-chart for the C.H.T. programme is shown in Fig. 10.6. The programme for the F.H.T. is not given but is basically the same (see Section 5.6).

The final programme used for obtaining the results shown in Fig. 5.6 to 5.10 was written so as to use the minimum space and time. At $t = 0 + \delta t$ only the mesh points next to the tip were relaxed. During the next interval these mesh points together with their immediate neighbours, outward from the tip, were relaxed, and so on.

To obtain adequate accuracy the mesh size were chosen to be very small which necessitated a very large number of mesh points. Nearly the whole of the fast storage, some 20,000 spaces were allocated for these mesh points. The relaxation process was carried as follows: the topmost horizontal layer was relaxed first and stored in an auxiliary one dimensional store. The swelling rates at every mesh point on this line were then calculated, integrated using the trapezodial and Simpson methods, and stored.

The next horizontal layer of mesh points was relaxed with the simultaneous process of transfering the contents of the auxiliary vector into the topmost layer, and replacing them by the newly calculated values for the second layer. When all the mesh points were relaxed, the stored flow rates for all the layers were then integrated using again the trapezodial and Simpson methods. The time taken for each computer solution was thus of the order of 5-8 minutes only.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

1 $-6.u(1) + 4.u(2) + u(5) = 0$

2 $0.5.u(1) - 4.u(2) + 1.5\,u(3) + u(6) = 0$

3 $0.75.u(2) - 4.u(3) + 1.25\,u(4) + u(7) = 0$

4 $\frac{5}{6}\cdot u(3) - 4.\,u(4) + u(8) = 0$

5 $u(1) - 3.u(5) + 2.u(6) = 0$

6 $2.u(2) + 0.5.u(5) - 4.u(6) + 1.5.u(7) = 0$

7 $2.u(3) + 0.75.u(6) - 4.u(7) + 1.25.u(8) = 0$

8 $u(4) + \frac{5}{6}u(7) - 4\,u(8) + u(9) = 0$

9 $u(8) - 4.u(9) + u(10) = -\frac{5000}{6}$

10 $u(9) - 4\,u(10) + u(11) = -\frac{5000}{6}$

11 $u(10) - 4.u(11) + u(12) = -\frac{5000}{6}$

12 $u(11) - 4.u(12) + u(16) = -\frac{5000}{6}$

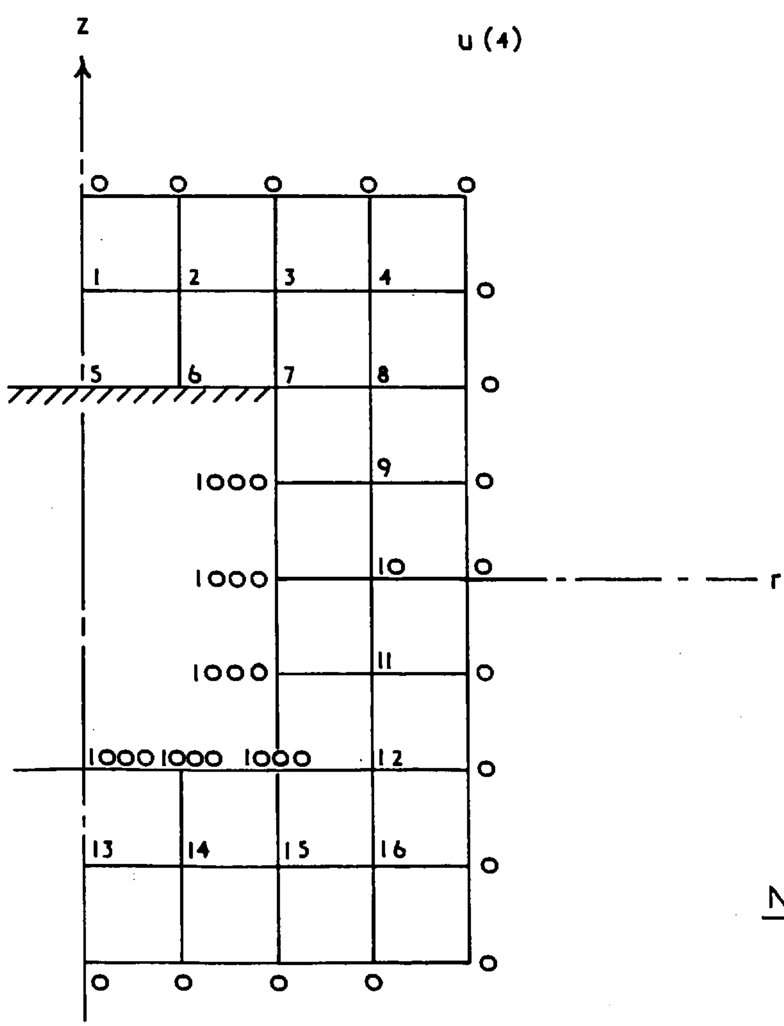
13 $-3.\,u(13) + 2.\,u(14) = -500$

14 $0.5.\,u(13) - 4.\,u(14) + 1.5.u(15) = -1000$

15 $0.75.\,u(14) - 4\,u(15) + 1.25\,u(16) = -1000$

16 $u(12) + \frac{5}{6}.u(15) - 4.\,u(16) = 0$

$$\begin{bmatrix} -6+4 & & +1 & & & & \\ 0.5-4+1.5 & & & +1 & & & \\ & \ddots & & & & & \\ & 1 & & +\frac{5}{6} & -4 & +1.0 & \\ & & & & & \ddots & \\ & & & & & & 0.75-4+1.25 \\ & & 1 & & & & +\frac{5}{6}\;-4 \end{bmatrix} \bullet \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_8 \\ \vdots \\ u_{15} \\ u_{16} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ -1000 \\ 0 \end{bmatrix}$$



NUMERICAL SOLUTION FOR THE STEADY-STATE FLOW INTO CYLINDRICAL TIP

FIG. 10:1

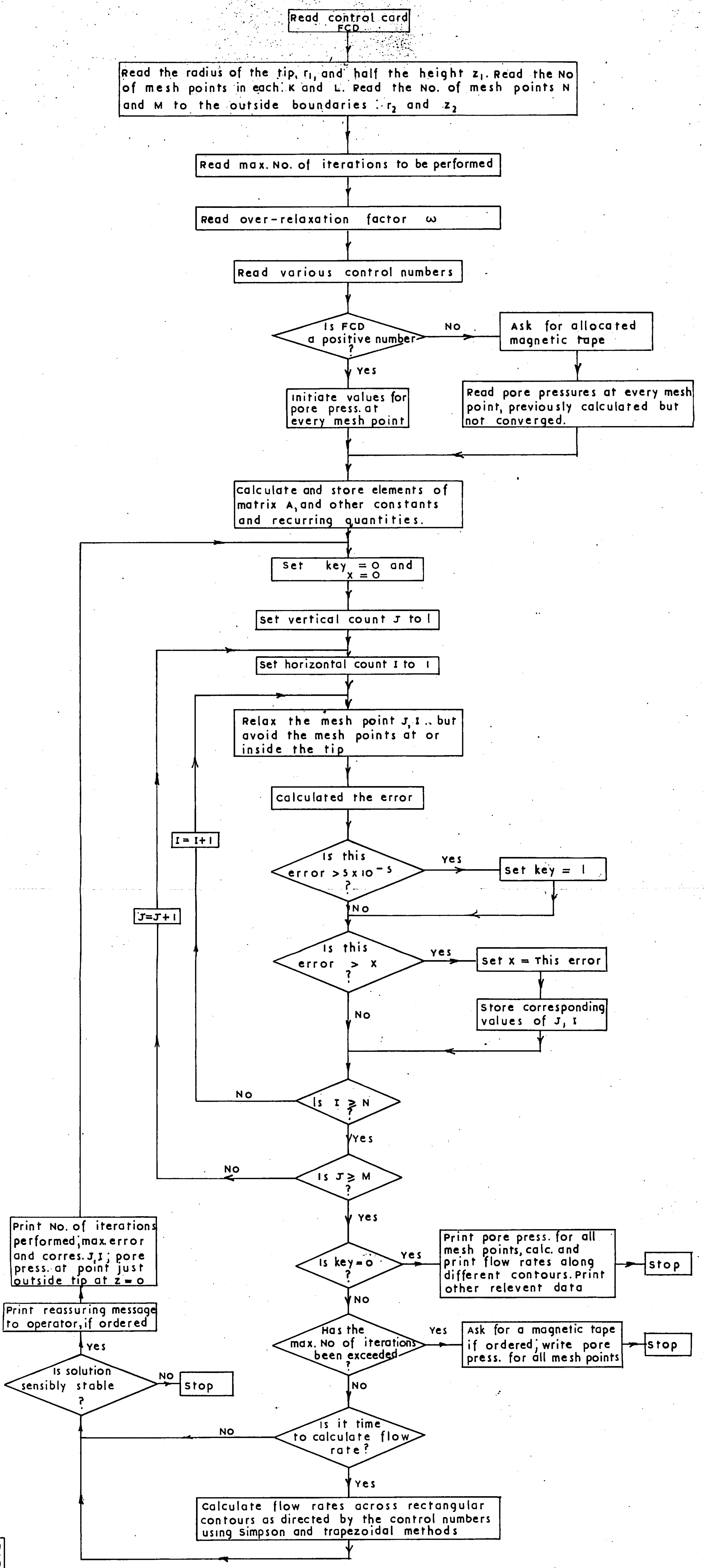
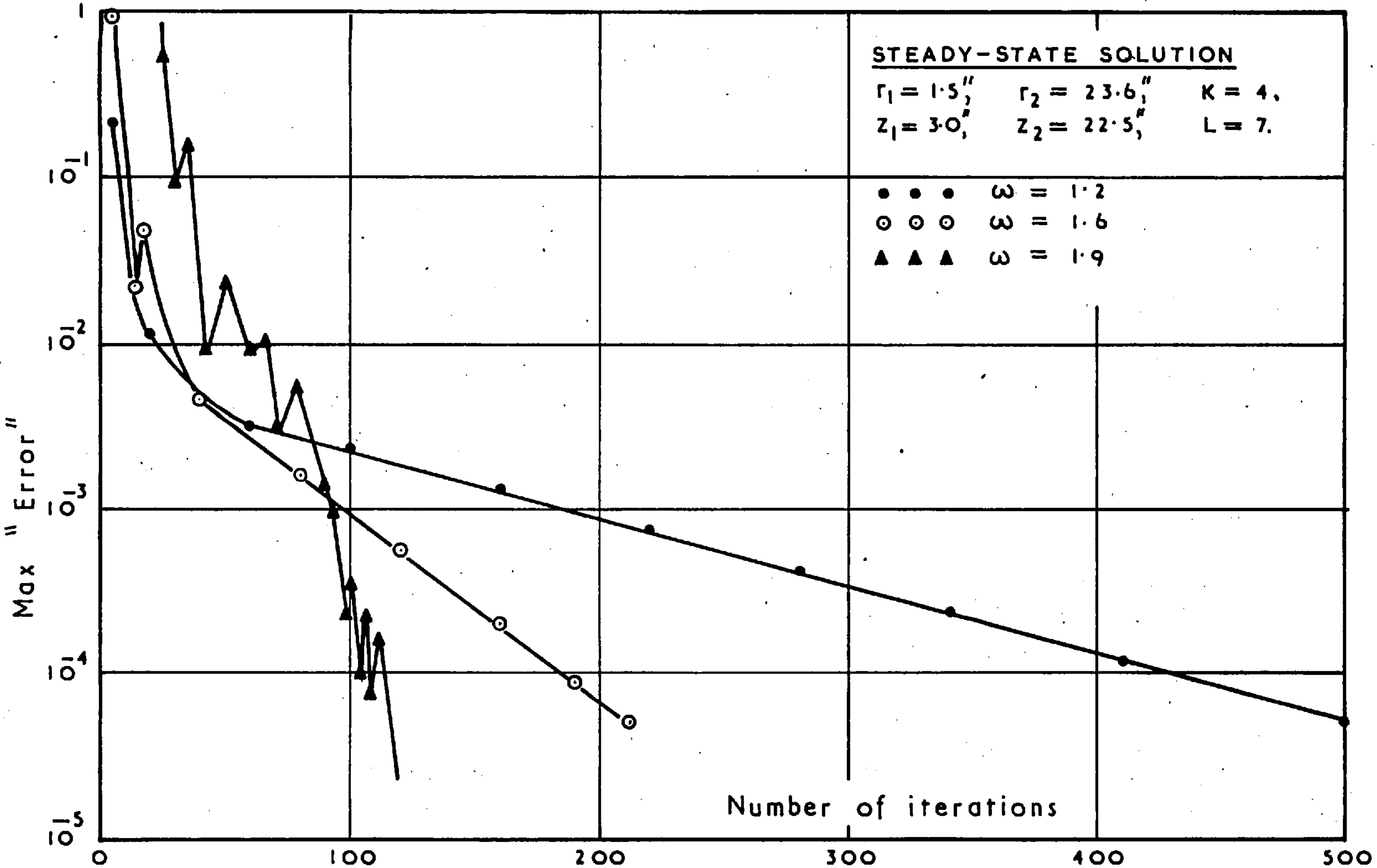


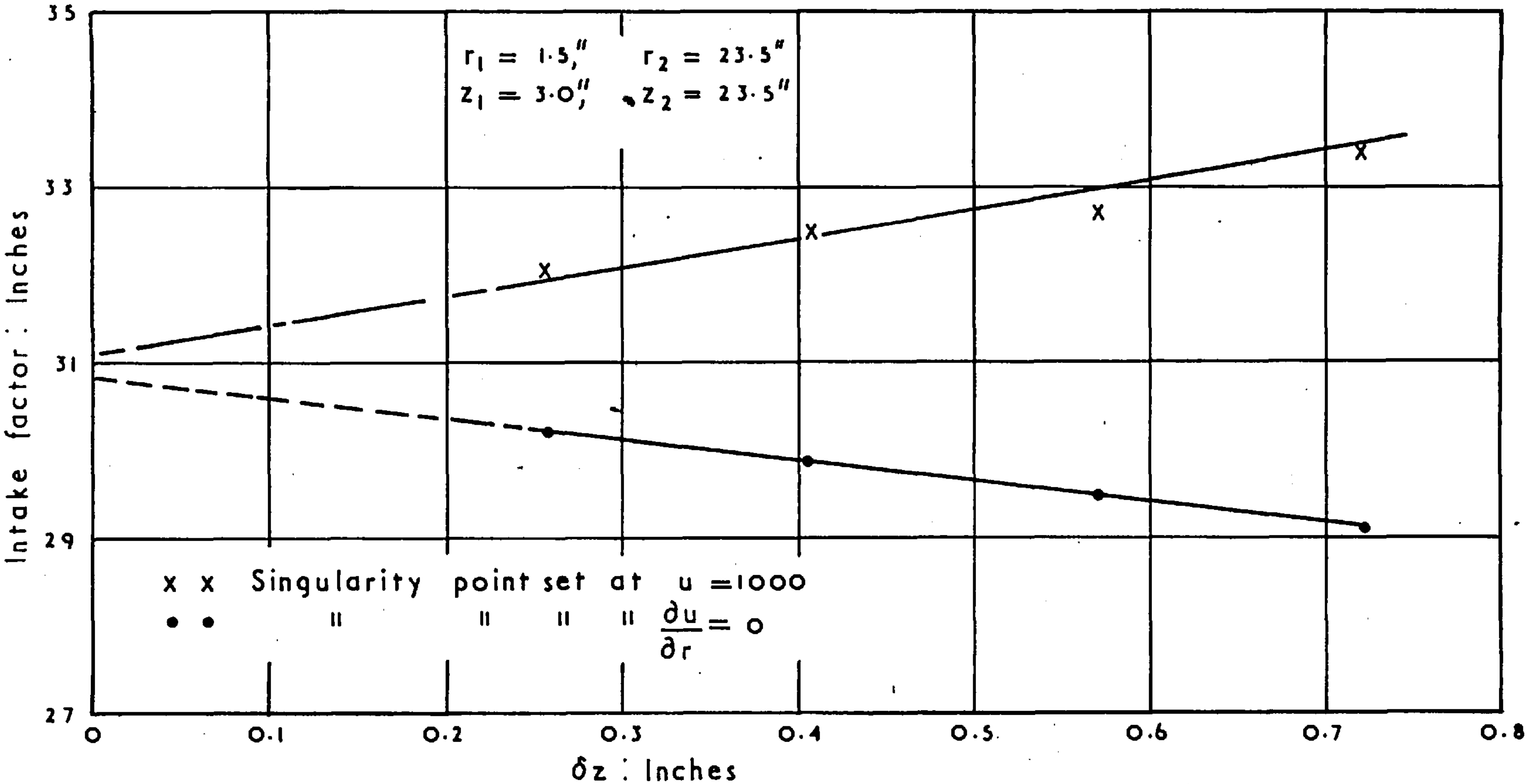
FIG. 10. 2

CONDENSED FLOW-CHART FOR COMPUTER PROGRAMME FOR STEADY STATE FLOW INTO CYLINDRICAL TIP



EFFECT OF THE OVERRELAXATION FACTOR "ω" ON THE RATE OF CONVERGENCE OF THE G.S.O.M.

FIG. 10: 3



STEADY – STATE NUMERICAL SOLUTION : EFFECT OF SINGULARITY POINT.

FIG: 10 : 4

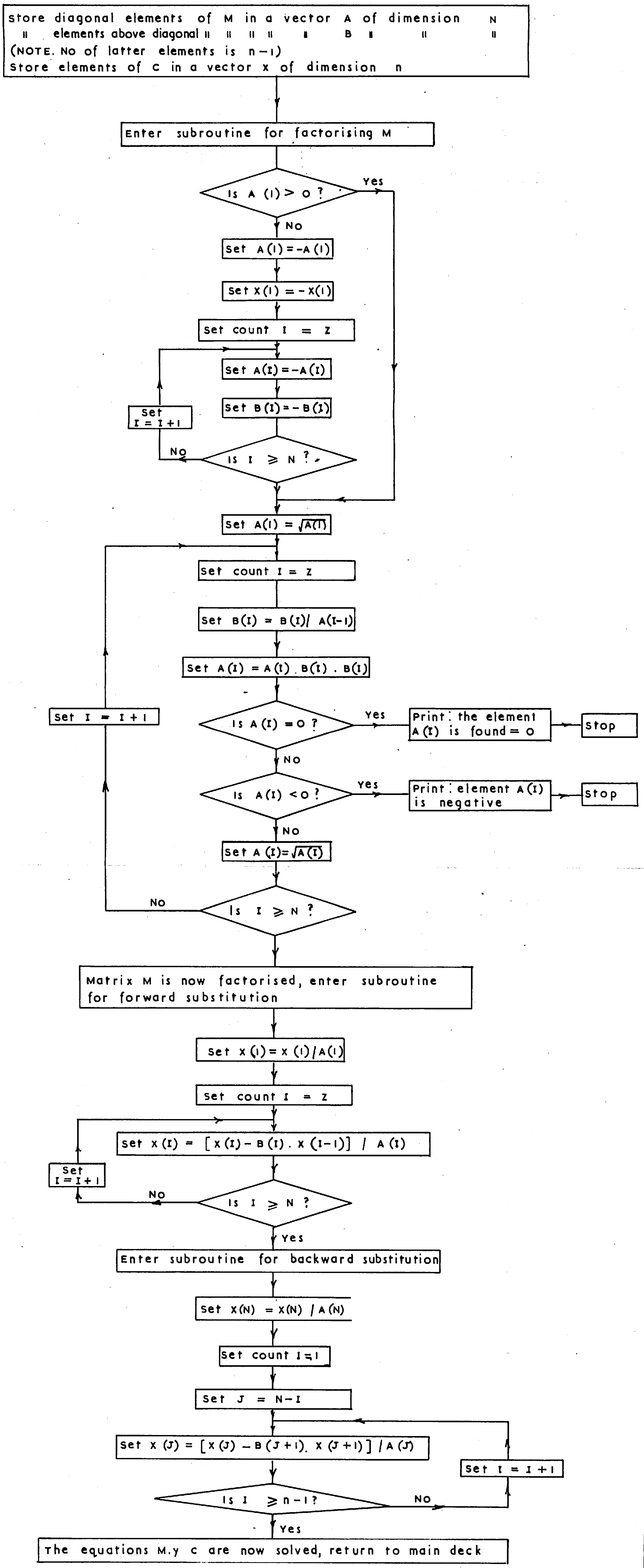
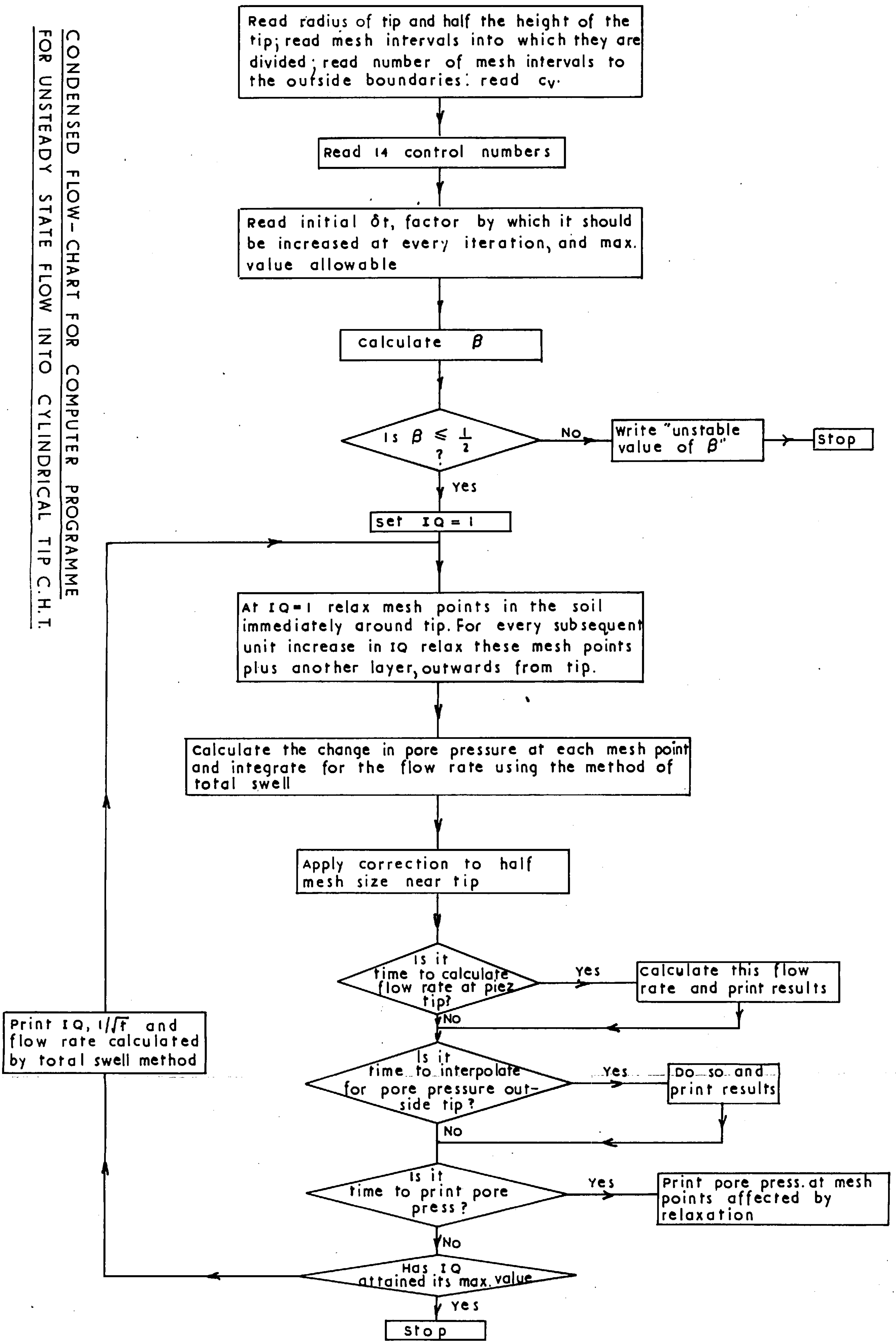


FIG: 10: 5

FLOW CHART FOR COMPUTER PROGRAMME FOR AN EXACT SOLUTION OF THE EQUATIONS M.y = C USING THE SQUARE-ROOT METHOD (SEE FADDIEV and FADDEEVA: 1963, PS. 144) WHERE M IS A TRIDIAGONAL SYMMETRIC MATRIX (n x n)



CONDENSED FLOW-CHART FOR COMPUTER PROGRAMME FOR UNSTEADY STATE FLOW INTO CYLINDRICAL TIP C.H.T.

FIG:10:6

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