# LIMIT DESIGN OF REINFORCED CONCRETE SKELETAL STRUCTURES

by

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A Thesis submitted for the degree of Doctor of Philosophy in the Faculty of Engineering of the University of London.

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London.

July 1966

## ABSTRACT

This thesis examines the general concepts of limit design of reinforced concrete structures, in relation to the European Concrete Committee (C.E.B) recommendations and current design practice. The investigations have been divided into four parts representing different aspects of limit design.

The basic properties of reinforced concrete members, particularly effective flexural stiffness, plastic rotation capacity and ultimate strength are discussed in Part (2). Simplified design calculations are suggested which have also been compared with experomental results to determine the relative error involved.

The limit design of continuous beams as a special category of structures is fully investigated in Part (3). It appears that the degree of redistribution could be used as a link between ultimate load and working load states. Simple methods of detailing to ensure minimum limit requirements have been developed.

The last part of the thesis deals with the application of limit methods to skeletal structures. A general method of superposition of load systems to obtain the adverse load combinations in skeletal structures have been developed.

The method of ultimate load design using plastic hinge systems by a trial and adjustment procedure has been investigated. The conditions under which an assumed hinge system would be considered satisfactory for inelastic compatibility analysis , have been discussed in relation to statically admissible release systems.

It is recognised that instability effects would play an important role in frame analysis. The limits within which an elasto-plastic design may have advantages over an elastic design, have been derived using an approximate method. This is illustrated by a worked example.

# ACKNOWLEDGEMENTS

This work was carried out under the supervision of Prof. A.L.L.Baker, to whom the author is indebted for his advice and encouragement.

The author also wishes to acknowledge his debt of gratitude to the following :-

Dr. C.W.Yu for his advice and suggestions throughout the laboratory tests and in the preparation of the thesis,

Dr. J.Munro and Dr. A.D.Edwards for their valuable comments on Chapters 14 and 15,

Messrs.M.T.Soliman, P.K.Neogi, D.O.Paranagama, and R.G.Matheson for their help with experimental work and contributions by way of discussions,

Messrs. H.Wilson, P.J.Jellis, J.Turner, R.Loveday and C. Mortlock for their technical assistance,

Miss. J. Gurr and Mrs. B. White for taking the photographs and typing the tesis.

Lastly the author wishes to express his appreciation of the financial assistance of the Department of Scientific and Industrial Research* who sponsored this project.

*** now Science Research Council**

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# NOTATION

The notation given below are used with a general set of suffixes ( one or more suffixes may be used at the same time) indicated by the following.

| Suffix | General Meaning | Example |
|---|---|---|
| a | tension reinforcement | $A_a$ |
| $'_a$ | compression reinforcement | $A'_a$ |
| b | concrete | $e_b$ |
| 0 | cracking limit - $L_o$ | $e_{b0}$ |
| 1 | idealised limit - $L_1$ | $e_{b1}$ |
| 2 | idealised limit - $L_2$ | $e_{b2}$ |
| y | yield conditions | $e_{ay}$ |
| u | ultimate load conditions | $M_u$ |
| e | elastic stage | $(EI)_e$ |
| * | design value | $\sigma^*_b$ |
| g | permanent load | $\gamma_g$ |
| q | superimposed load (vertical) | $\gamma_q$ |
| v | superimposed load (lateral) | $\gamma_v$ |

## General Notation

| Symbol | General Meaning | Suffixes used |
|---|---|---|
| A | area | a, $'_a$, b |
| b | width of rectangular section | |
| h | effective depth of tension reinforcement | |
| h' | effective depth of compression reinforcement | |
| $h_t$ | total depth | |
| x | neutral axis depth parameter | 1, 2 |
| $\gamma$ | lever arm parameter | 1, 2 |
| $\alpha$ | mean compressive stress parameter | 1, 2 |
| p'' | $\frac{100.\text{Volume of binders per unit length}}{bh}$ | |

## General Notation ctd.

| Symbol | General Meaning | Suffixes used |
|---|---|---|
| $I$ | second moment of area of section | |
| $l$ | length of span | |
| $\sigma$ | stress | a, $'_a$, b, 0, 1, 2, y, u, * |
| $\sigma'_b$ | standard cylinder strength of concrete | |
| $\sigma''_b$ | maximum concrete stress in it's idealised stress strain relation | |
| e | strain (or eccentricity of axial load) | same as for |
| E | elastic modulus | a, b, e, *, 1 |
| $\gamma$ | coefficient of variation (partial safety factor) | a, b, g, q, v |
| $\delta$ | standard deviation | a, b, g, q, v |
| EI | flexural stiffness | e, 1, * |
| $\xi$ | $EI/\sigma'_b bh^3$ | e, 1, * |
| G | permanent load | |
| Q | superimposed load (vertical ) | |
| V | superimposed load ( lateral) | |
| $\gamma_{q1}$ | over load coefficient for Q under G+Q | |
| $\gamma_{q2}$ | " " " Q " G+Q+V | |
| $\gamma_{v1}$ | " " " V " G+V | |
| $\gamma_{v2}$ | " " " V " G+Q+V | |
| $\gamma_0$ | work load coefficient | |
| $\lambda_c$ | crack width parameter | |
| $\lambda_y$ | yield safety parameter | |
| $\lambda_s$ | serviceability parameter | |
| $\lambda$ | load factor | |
| M | moment | 1, 2, *, u, y |
| N | axial force | 1, 2, *, u, y |
| T | shear force | * |
| m | $M/\sigma'_b bh^2$ | 1, 2, *, u, y |
| n | $N/\sigma'_b bh^2$ | 1, 2, *, u, y |
| r | $\sqrt{m^2 + n^2}$ | 1, 2, *, u, y |

## General Notation ctd.

| Symbol | General Meaning | Suffixes used |
|---|---|---|
| $\overline{\omega}$ | $A_a / \sigma'_b bh$ | |
| $\overline{\omega}'$ | $A'_a / \sigma'_b bh$ | |
| $\phi$ | curvature ( or diameter of bars) | 1, 2, u |
| $\theta$ | rotation | 1, 2, u |
| $\theta_p$ | plastic rotation | |
| D | ductility ratio | |
| $\Delta$ | deflection | |
| R | degree of redistribution of moments | |
| X | release moment or force | |
| $P_i$ | applied load system | |
| $f_{ij}$ | influence coefficient | |
| $\delta_{ij}$ | " | |

Any other symbols used are defined on the first time they occur.

# PART I

## Chapter 1

## Introduction

The use of reinforced concrete as a structural material has grown enormously in the last two or three decades, in which period considerable improvement in the methods of construction, workmanship, and quality control has been affected. At present extensive informatio is available on both the properties of materials and the functional requirements of the structures. The main problems dealt with in current research concern ways and means of bringing in these two aspects together to arrive at a rationalised design p'ocedure encorporating safety, serviceability and economy expressed interms of the random variation in the properties of materials and design loads.

The concept of limit design first developed in in the U.S.S.R. and currently investigated in greater detail under the European Concrete Committee may be considered as a logical development in this direction. The preliminary recommendations on the principles of limit design have already been published under the European Concrete Committee[14]. The main object of this thesis is to consider the application of limit design concepts to skeletal structures.

A qualitative assesment of the basic criteria in the limit design is outlined in the first part of this thesis. In view of the radical change in the design concepts, it has been found necessary to consider the basic properties of materials and loads as limit conditions subject to individual variations. The structural properties of reinforced concrete members in relation to moment-rotation characteristics are discussed in Part 2. These are illustrated by the test results reported by the European Concrete Committee recently.[9]

The application of limit concepts to the design of statically indeterminate structures has been treated in two parts. The design of continuous beams to conform to specified limit requirements in terms of yield safety, crack width, deflection, ultimate shear and inelastic compatibility is outlined in Part 3. The results of eight three-span continuous beams carried out at the Imperial Coolege laboratory are discussed with particular reference to the above design methods.

Part 4 deals with the design of multistorey skeletal frames. The basic concept of elementary load systems is used to derive a general priciple of combined loading which is aimed at obtaining the ultimate load configuration for a given set of loads, thus reducing safety analysis to a minimum. This would have similar advantages to limit design as the principle of superposition in ordinary elastic design.

The suitability of release systems in an ultimate load design as suggested by Baker[5] is discussed as a special case of inelastic compatibility. A simple method of ultimate load design for multistorey structures which encorporates instability effects is put forward as a particular limit application of inelastic compatibility.

# Chapter 2

## Reveiw of Design Methods.

### 2.1 Early developments.

In the early stages of the development of reinforced concrete design, from about 1880's to 1920's, Koenen, Empreger, and many others have laid down the foundation of what may be termed as the first attempts at ultimate load design. The foundamental design concepts were mainly empirical, in which the safety of the structure was based on the ultimate strength as determined by tests on simple structures. The safety as such was similar in concept to the load factor of safety as it appears in current practice[25].

But as a method of analysis of complex structures no suitable ultimate load design procedure was available, hence the method was necessarily restricted to simple structures. A rigourous method of analysis of complex structures was to await the development of the elastic theory of bending in relation to statically indeterminate structures, which is also refered to as the permissible stress method of design.

### 2.2 Permissible stress method.

The application of the linear elastic theory to the design of statically indeterminate structures was to change the design concepts from ultimate strength to that of permissible stress derived from idealised elastic properties of reinforced concrete members. In the permissible stress method the safety of the structure was defined interms of the " stress factor of safety ".

As a method of design, the elastic theory offered great advantages in the superposition of stresses due to combined loading and simplified analytical means based on slope deflection (1918), moment distribution (Hardy Cross Method - 1930) and other methods of relaxation.

However, it was known that the elastic idealisation of the deformation characteristics of reinforced concrete members was very approximate, and that in reality reinforced concrete undergoes considerable inelastic deformation before failure. Experimental work by Glanville and Thomas[1], Whitney[2] and many others showed that continuous beams and simple portal frames could carry much higher loads than those estimated under the permissible stress method. The additional loads in these indeterminate structures were seem to be possible due to the plastic rotation at critical sections which helps to " redistribute " the moments to those sections which are still elastic, until mechanism conditions were attained or local failure takes place. This was to show the shortcomings of the permissible stress method to predict the actual safety inherent in elasto-plastic structures.

The permissible stress method of analysis was partly modified to take into account of the above observations by the introduction of the concept of " redistribution of moments " applied to the conventional elastic analysis. In its application to the design of indeterminate structures a maximum of 15% redistribution has been permitted by the British Code of Practice since 1939, where as the Russian, Danish and some other European Codes have permitted larger amounts of redistribution under particular circumstances. The main difficulty in obtaining a quantitative limit for the degree of redistribution was due to its dependence on the requirements of serviceability conditions under working load and the limited degree of plastic rotation observed in reinforced concrete members. In contrast, it must be noticed that these difficulties have not been encountered to the same extent in steel structures for which an idealised plastic analysis has been developed subsequently.

## 2.3 Rigid - plastic analysis.

The experimental investigations into the plastic behaviour of steel structures in the 1930's by Leibnitz and later by J.F.Baker and others[3] have laid the foundation for a rational basis of safety analysis based on the collapse state. This has been later developed into the Rigid -Plastic Theory, which is also refered to as a " Limit Design " due to the upper and lower bound limits of the collapse load factor.[3,50,52]

This theory is extermely simple in its application, and where the preliminary assumptions are satisfied, the accuracy of the analysis is quite reliable as in the case of continuous beams and portal frames etc[3]. However, subsequent research has shown that in multistorey structures and those that contain members that carry large axial loads, the instability effects may seriously affect the limits on the collapse load factor, as the collapse mode may be altered due to the deterioration of the structural stiffness with the formation of a few local hinges before complete mechanism conditions are reached [51]. The seriousness of the problem has been illustrated by Wood[51] in a typical example of a four storey single bay frame, in which the collapse load factor has been reduced from 2.21 ( as estimated by the rigid-plastic theory ) to about 1.70 due to instability effects ( 23 % reduction ). Some approximate methods of correcting for the instability effects have been later suggested by Heyman[45], Holmes and Ghandhi[53].

The main difficulty in extending this method to reinforced concrete design was the uncertainty of the degree of plasticity of reinforced concrete members.It was known already due to early tests, that when concrete commences to crush at critical sections due to excessive strain, the strength of the section tends to decrease, which was contrary to the basic assumptions in the plastic theory. A method of reconsiling the limited plasticity in reinforced concrete has been put forward by . . .

A.L.L.Baker in the Ultimate Load Theory[5].

## 2.4 Ultimate Load Theory.

In 1949 Baker [5] has suggested that the classical Muller Bresleau equations for hinge rotations in a statically determinate structure could be used as a basis for an elasto-plastic method of design for reinforced concrete structures. Statical determinancy in a hyperstatic structure was attained by the introduction of suitably placed " plastic hinges ". The method of design was formulated to an ultimate theory for reinforced concrete and prestressed concrete structures in 1956[5].

The ultimate load theory is based on an elasto-plastic idealisation of the moment rotation characteristics of reinforced concrete members and the maximum load at collapse which is referred to as the ultimate load. The design procedure involves trial and adjustment of the plastic hinge moments to obtain compatible hinge rotations, which must also lie within specified permissible limits.

Extensive research has been carried out on the properties of plastic hinges in reinforced doncrete members [8,28,34,23] and the flexural stiffness characteristics[27]. The basic problems involved in the determination of the ultimate load and hinge compatibility in general frame design are considered in greater detail in Part 4 of this thesis.

## 2.5 Limit Design Method.

It must be remembered that in an ultimate load method of design aimed at greater economy, whether a plastic hinge method or an idealised elastic method with subsequent redistribution of moments is used, the nett effect is to reduce the over-all safety.This may also result in higher stresses at critical sections ; larger deflections and crack widths at working load which would necessarily call for more stringent checks for unserviceability than hitherto recognised.

In Russian design practice[10,11] where ultimate load analysis with redistribution of moments upto 30 % has been allowed for a considerable time, the problem of unserviceability at working load has been investigated in greater detail. Since 1955, the method of design for reinforced concrete structures has been based on the concept of limit requirements. The safety and serviceability requirements which are defined interms of the different modes of failure and causes of unserviceability are considered as limit conditions. These conditions are related to the probability of overload, variation in the properties of materials, errors in the design assumptions and methods of construction in terms of individual design coefficients, which replace the concept of overall safety factors that are being used in the load factor method of design and the stress factor of safety .

The following limit conditions have been recommended in the Russian Specifications[10] as the most important factors to be considered in the limit design of general structures.

(1). Ultimate strength based on the probable load bearing capacity of the structure which must be sufficient to withstand the specified load.

(2). Excessive deflection at working load depending on the type of structure and its utility.

(3). Excessive crack width at working load subject to enviromental and aesthetic considerations.

The introduction of the limit concepts to both design and specifications helps to reduce the "universal factor of safety " as applied in the permissible stress method and the load factor method to its constituent basic causes of unserviceability and partial or overall failure. This would enable the individual limits to be investigated thoroughly so that they may be provided for with a reasonable degree of probability which would be compatible with the minimum requirements of the structure and the type of loading.

The application of limit concepts to the design of reinforced concrete structures has given rise to considerable interest in a statistical study of the interlinking parameters affecting the design methods. The investigations by Tichy[15] illustrate the advantages in the limit design methods in evluating the actual safety in structures. Cohn and Petcu [12,13] have recently suggested the application of the limit concepts to obtain an " optimum solution " for continuous beams.

The proposed European Concrete Committee recommendations [14] for an International Code of Practice are based on the limit design approach in its specifications of the material properties and the design principles. It further suggests that further investigations should be carried out so as to determine suitable design methods of ensuring the desired limit conditions.

# Chapter 3

## Limit Design of Reinforced Concrete Structures

### 3.1 Limit design criteria

The application of the limit concepts to the design of reinforced concrete structures is closely related to the deformation characteristics of reinforced concrete members and the minimum requirements of the structures. These may be broadly classified into two limit categories depending on the loads at which they are to be investigated.

#### (a) Failure Criteria or Collapse Limit.

The ultimate strength required of the structure could be defined by the probability of the over load and the possible modes of collapse such as,

(1) formation of partial or complete mechanisms of collapse in the structure,

(2) excessive shear resulting in local failure,

(3) failure due to insbility effects either in the elasti stages or under elasto-plastic conditions.

It must be noted that each of the above modes of failure must take into account the probable distributions and the adverse effects of the combination of the live loads and the dead load.

#### (b) Serviceability Criteria or Service Limits.

The minimum service requirements for different structures may be specified depending on the type of structure and its utility. Structurally it would be required to satisfy these minimum conditions for all the possible combinations of the superimposed loads and the permanent load which together comprises the working load. A structural design method that ensures both the limiting collapse and service requirements is termed a limit design method.

## 3.2 Basis of Collapse Limit Analysis

Recent investigations into the basic criteria of limit analysis[11,38] indicate that it could be separated into the following three stages,

(1) Load Analysis
(2) Material Analysis
(3) Structural Analysis

Each of the above investigations may be carried out independently and the minimum requirements could be easily specified individually. Thus the limit design procedure really consists of ensuring the limit requirements with a reasonable degree of probability based on the above analysis, which replaces the concept of universal safety completely.

### (1) Load Analysis.

The degree of accuracy with which any structure could be analysed for safety would not exceed the degree of accuracy with which the applied loads are known, however precise the method of analysis may be. Hence a thorough analysis of the applied load systems including the mean working loads and the degree of variation of each of the loads within a given period of time would be absolutely vital.

The over load coefficient ( or partial safety factor for load ) would depend on the probability of the specified load being exceeded within the lifetime of the structure. Even then, the most adverse effect of the loads may occur due to different combinations of the permanent load with the superimposed loads. Thus the over load coefficient when applied to groups of loads must also take into account the probability of their acting together.

The loads acting on structures in general may be devided into four categories based on their characteristics.
(a) Permanent load or dead load, (b) Superimposed load or live load

(c) Lateral load due to aerodynamic forces or earthquake movements, and (d) Transient loads.

(a) Permanent load or dead load consists of the weight of the structure and permanent fixures. These have very small coefficients of variation due to change in the moisture content, density and errors in the size of members; but in general they remain constant throughout the life of the structure. In most cases they could also be determined accurately. In U.S.S.R. it has been found(11) that the dispersion in the permanent loads varies in the range 0 - 0.15 . In taking these into account, the Russian specifications provide for a small over load coefficient of 1.1. Although the British Code does not recognize the distinction between the dead and the live loads in safety analysis, it provides an equivalent load factor of 1.8 (25). Similarly the ACI Code of practice provides for a factor of 1.5 under dead and live load and a factor of 1.25 under dead load, live load and wind load.

(b) Superimposed load or live load. The moveable loads that the structure is intended to carry during its lifetime could be termed as the superimposed load. Naturally it could be expected that the variation in the superimposed load to be greater than in the previous case. This may also depend on the type of structure and the load itself. For example it has been found that the coefficient of variation in the superimposed loads in private buildings is about 0.10 and in industrial buildings it was about 0.15. Hence the over load coefficient would be defined according to the type of structure and the loads anticipated. A list of comparative over load coefficients and specified loads is given in Table 3.1.

(c) Lateral loads. These are subject to large variations depending on the locality, nature of building and its environment. Hence in structures where the lateral loads are of primary concern, considerable precautions must be taken to safeguard against their unduly large variations. But in structures in which it is not of primary concern,

TABLE 3.1 Superimposed loads and over load coefficients *

| Type of structure | B.S.(17) | A.S.A.(58) | Russian(11) |
|---|---|---|---|
| 1. Apartments | 30 (1.8) | 40 (1.8) | 32 (1.4) |
| 2. Offices , dormitaries | 50 (1.8) | 80 (1.8) | 42 (1.4) |
| 3. Offices and dormitary halls | 70 (1.8) | 100(1.8) | 63 (1.3) |
| 4. Dinning halls, restaurants, auditoriums, stairways. | 80 (1.8) | 100(1.8) | 63 (1.3) |
| 5. Theatre halls, places of public gathering. | 100(1.8) | 100(1.8) | 85 (1.2) |
| 6. Light storage (minimum) | 150(1.8) | 125(1.8) | 85 (1.2) |
| 7. Minimum for book storage and warehouses in commercial and industrial buildings. | 200(1.8) | 250(1.8) | 105(1.2) |
| 8. Hydraustatic pressure of liquids.+ | - | - | - (1.1) |
| 9. Crane loads + | - | - | - (1.3) |
| 10. Pressure of granular materials.+ | - | - | - (1.2) |

* The figures represent the specified loads in lbs/sq.ft. The figures within the brackets refer to the over load coefficient or to its equivalent partial load factor implied in the safety analysis.

+ In these cases the actual loads must be considered.

it could be assumed that the probability of the simultaneous occurance of loads in categories (b) and (c) at their peak values are much less, so that the over load coefficients when these are considered together may be reduced. A similar consideration is given in the stress factor method of design, where the permissible stress may be exceeded by as much as 40 % due to the wind loads[17]. Table 3.2 shows a comparative study of the over load coefficients.

Table 3.2.

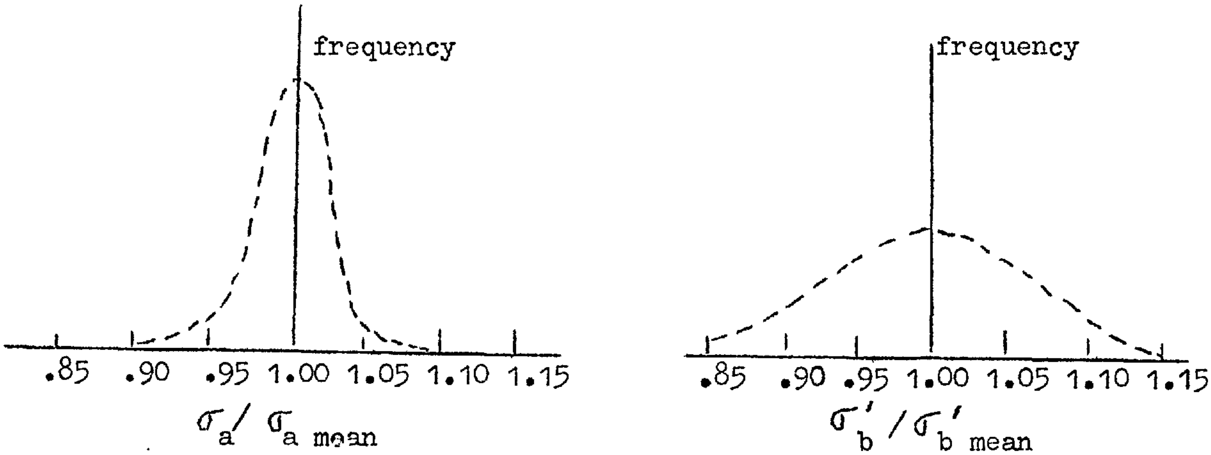
| Type of Load | Over load coefficients | | |
|---|---|---|---|
| | B.S. | A.C.I. | Russian |
| G + Q | 1.8(G+Q) | 1.5G + 1.8Q | 1.1G + $\gamma_q$Q |
| G + Q + V | 1.3(G+Q+V)* | 1.25(G+Q+V) | 1.1G + $\gamma_q$Q + $\gamma_v$V |
| G + V | - | 0.9G + 1.1V | - |

* This is based on an allowance of 40 % increase in the stress when wind load acts.

(d) Transient loads. Special loads that may act on the structure at different times although it has not been designed primarily for these loads may be considered as transient loads. Some examples are constructional loads and loads due to flooding in particular areas and due to variation in temperature and creep in ordinary structures. These loads cannot be assesed accurately but must be allowed for in the design so that no permanent damage may result due to them.

(2) Material Analysis

The basic properties of concrete and reinforcing steel vary considerably depending on the conditions under which they are being manufactured. Thus the actual properties could only be denoted by their statistical mean values and the respective coefficients of variation. Fig 3.1 shows the typical variations in the strength of specified samples of steel and concrete[10].




(a) Variation in the yield strength of reinforcing steel.

(b) Variation in the crushing strength of concrete.

Fig. 3.1

The frequency distribution of the ' strength ' of both reinforcing steel and concrete approximates to a normal curve and the respective coefficients of variations are about 0.05 and 0.10-0.20, the latter depending on the degree of quality control.

The differences in the probability of failure of steel and concrete are taken into account in the limit design of members by the introduction of coefficients of variation as in the Russian Specifications [11] or by the use of partial safety factors as in the C.E.B. recommendations[14]. Table 3.3 gives the comparative reduction factors for the mean strength (coefficients of variation or the inverse of the partial safety factor) to be used in limit design. The coefficients of variation are obtained by using the standard formula,

$$\gamma = 1 - k\delta$$

where $\delta$ is the standard deviation as obtained from distribution curves as above and k is a factor based on the desired risk of failure.

TABLE 3.3 Coefficients of variation

| Grade of concrete+ | Russian | | C.E.B. |
|---|---|---|---|
| | 35-200 | 300-600 | |
| Concrete in compression (A)* | 0.60 | 0.65 | 0.67 |
| " (B) | 0.55 | 0.60 | 0.67 |
| Concrete in tension (A) | 0.45 | 0.50 | 0.67 |
| " (B) | 0.40 | 0.45 | 0.67 |
| Mild steel | 0.90 | | 0.90 |
| Cold worked steel | 0.80 | | 0.90 |

+ The grade of concrete refers to the specified strength in $kg/cm^2$.

* (A) and (B) refer to the concrete obtained under factory and site conditions.

## (3) Structural Analysis

The methods of structural analysis available at present whether conventional elastic, elasto-plastic, or rigid plastic are based on idealised properties of members which may be considered necessary to obtain simple methods of analysis. However it could be seen that some of the idealisations are in greater error than others, leading to 'safe' and 'unsafe' results as the case may be. If these factors inherent in the methods of anlysis are taken into account, it would be possible to associate a coefficient with each of the methods of anlysis based on any particular idealisation representing the reliability of the method of anlysis. This would enable the design to be related to the actual structure on a similar basis of probability of collapse irrespective of the simplified idealisations. Such a coefficient could also take into account incidental errors due to variable phinomena like diffenential settlement of foundation, vibrations, partial fixity at footings etc.

The work load coefficient suggested by Goldenblatt[11] serves some of the above purposes. A similar suggestion has been made by Wood amounting to as increase of 25 % in the ultimate load factor in conjunction with the rigid-plastic method of analysis which ignores instability effects. In this respect the work load coefficient serves as a 'safety factor' on the method of analysis and must be determined for individual categories of structures in relation to the simplified methods of analysis used in the design.

## 3.3 Application of limit concepts to design

In limit design, the results of the material and load analysis are used individually to determine the respective coefficients of variation in the material properties and the over load coefficients[10,14]. Thus the following coefficients may be assumed to be known.

| | |
|---|---|
| Coefficient of variation of concrete | $\gamma_b = 1-k\,\delta_b$ |
| " " steel | $\gamma_a = 1-k\,\delta_a$ |
| Over load coefficient for permanent load | $\gamma_g = 1+k\,\delta_g$ |
| " " superimposed load (vertical) | $\gamma_q = 1+k\,\delta_q$ |
| " " superimposed load (lateral ) | $\gamma_v = 1+k\,\delta_v$ |
| Work load coefficient | $\gamma_o$ |

where k and $\delta$ etc. depend on the degree of control and acceptable risks. In the case of over load coefficients, the value of k also depends on the probability of combined load when different systems of loads are considered together.

The design loads are then given by, $G^* = \gamma_g G$, $Q^* = \gamma_q Q$ etc.

The forces in the members of the structure are obtained by using any particular method of analysis as a function of the design loads, which may be expressed in the form,

$$M = F(G^*, Q^*, \text{etc.}) \qquad \text{.... (A)}$$
$$N = F(G^*, Q^*, \text{etc.})$$

Similarly the design stresses of the materials are obtained as follows,

$$\sigma_b^* = \gamma_b \, \sigma_b$$
$$\sigma_a^* = \gamma_a \, \sigma_a$$

where $\sigma_a$ , $\sigma_b$ refer to the mean strengths.

The strength of members M*, N*, etc. may be calculated from the stress strain characteristics of the materials (idealised) , and the member properties. The work load coefficient is encorporated to allow for the deviation in the actual member properties from the idealised properties assumed in the method of analysis (A) above. Then the strength of the members are given by,

$$M^* = \gamma_0 \, F(\sigma_b^*, \sigma_a^*, b, h, \text{etc.}) \qquad \text{....(B)}$$
$$N^* = \gamma_0 \, F(\sigma_b^*, \sigma_a^*, b, h, \text{etc.})$$

The compatibility of (A) and (B) produces a limit design which when all conditions of loading are considered could be regarded as a sufficient safeguard against all modes of failure. Thus this method this method provides an ideal collapse limit design

The detail anlysis of service limits are discussed in Chapter 11 with particular reference to continuous beams.

# Chapter 4

## Stress strain characteristics of bound concrete

### 4.1. Basic properties of concrete

Extensive research has been carried out in the recent years to study the basic characteristics of structural concrete as it forms the primary material used in the construction industry. The stress strain characteristics of plain concrete under varying loading conditions show that it is a brittle material, but under carefully controlled test conditions, some plastic properties could be detected[21,29]. It has been found that the maximum stress in uniaxial tests on plain concrete varies from 0.8 times the standard cylinder strength to about the cylinder strength depending on the rate of loading[21]. The failure strain which is much more influenced by the nature of loading and the test machine may vary from about 0.0015 to 0.004 [19,21,22].

The effect of reinforcement on concrete is to change some of the above characteristics, so that reinforced concrete shows more marked plastic properties. Richart et al[19] and others[23,23] have shown that the effective strength of axially loaded columns could be increased by the use of binders as given in the following empirical equation,

$$N = A_b \sigma_b'' + 2.1 A_a'' \sigma_{ay} \qquad \ldots . (4.1)$$

where $A_a''$ is the area of binders per unit length of the column. They also found that the ultimate concrete strain i.e. strain in the extreme compression fibres in concrete just before the applied load starts decreasing, was raised from 0.0015 to about 0.015 due to the presence of closely spaced binders. Similar results have been observed in bound columns subjected to axial load and bending[8,9]. Lately it has been shown that binding is one of the many parameters influencing the restraining sffects of concrete, thus effectively increasing its ultimate stress and strain[8,28,31] The other parameters that influence the ultimate strain are bending moment gradient and curvature at the section. An approximate empirical relation to evaluate the effects of the above parameters on the ultimate strain is derived in the next section.

## 4.2 Ultimate strain in bound concrete.

Consider the forces acting on the concrete in the compression zone near the critical section of a beam which is subjected to bending as in Fig. 4.2. Let the area of binders per unit length of the beam at the section be $A''_a$, let $F_x$ be the axial force in the compression stress block and $F_y$ be the lateral restraining force per unit length as indicated. The lateral restraining force $F_y$ is a function of the axial force, the curvature at the section and the force exerted by the binders. The latter is mainly due to poissons ratio effect of the concrete under compression. Thus if $F_b$ be the force in the binders, it may be regarded as proportional to the ratio of the depth of binders embeded in the compression zone and the depth of the compression zone.

$$\text{i.e.} \quad F_b = B \left( \frac{xh - d'}{xh} \right) A''_a \sigma_{ay}$$

where B may be regarded as an empirical constant.

The curvature at the section is given by $e_b/xh$, and the force $F_x$ is equal to the force in the tension reinforcement. Then the lateral restraining force $F_y$ could be given by the semi-empirical relation,

$$\begin{aligned} F_y &= F_x e_b / xh + F_b \\ &= A_a \sigma_{ay} \frac{e_b}{xh} + B \left( \frac{xh - d'}{xh} \right) A''_a \sigma_{ay} \qquad \ldots\ldots \quad (4.2) \end{aligned}$$

where $A_a$ is the area of the tension reinforcement.

If $e_{bo}$ be the ultimate strain in the concrete due to axial load in the absence of any lateral restraint, the the increased ultimate strain value $e_{b2}$ when the concrete is under the action of a restraining force $F_y$ may be given in the form

$$e_{b2} = e_{bo} + f(F_y) \qquad ..... \quad (4.3)$$

Considering a simple function of the type 4.3, the following expression for the ultimate strain is obtained,

$$e_{b2} = e_{bo} + k F_y \qquad ..... \quad (4.4)$$

where k is a constant. Substituting for $F_y$ from 4.2, $e_{b2}$ is given by,

$$e_{b2} = e_{bo} + k\left( \frac{e_b A_a \sigma_{ay}}{xh} + B \frac{xh - d'}{xh} A''_a \sigma_{ay} \right) \quad ...$$

$$= e_{bo} + k \sigma_{ay} bh \left( \frac{e_{b2}\, p}{xh} + B p'' - \frac{B p'' d'}{xh} \right)$$

$$= e_{bo} \quad 1 + k_1 p'' + ( k_2 - k_3 p'' ) \frac{1}{x} \qquad ....( 4.4a)$$

where $e_{bo}$, $k_1$, $k_2$, $k_3$ could be considered as approximate constants which may be obtained from test results where each of the parameters are varied in tern.

The effect of variation of the neutral axis depth on the ultimate strain when the amount of binders is kept constant has been studied by the author in an earlier series of tests as shown in Fig. 4.1b.(8,34) The stress strain curves for four axially loaded short columns in which the amount of binders are varied from 0 - 3.5% are shown in Fig. 4.1a. From these results the following approximate values of the empirical constants in equation 4.4a are obtained

$$e_{bo} = 0.0015, \quad k_1 = 1.5, \quad k_2 = 0.7, \quad k_3 = 0.1.$$

Substituting for these values in the original equation, the ultimate strain in bound concrete subject to bending could be given by the simple semi-empirical formula,

$$e_{b2} = 0.0015 \left[ 1 + 1.5 p'' + ( 0.7 - 0.1 p'' ) \frac{1}{x_2} \right] \qquad ..... ( 4.5)$$

Equation 4.5 seems to explain the observations that have been reported by Chan[23], Richart et al[19]. It may be noted that when the axial load is large, the neutral axis depth is also large. Then the ultimate strain in concrete with little or no binding could be as low as 0.0015 as in the case of plain concrete specimens. In beams where $x_2$ is generally less than 0.5, the C.E.B. recommendation of $e_{b2}$ = 0.0035 seems to be a safe limit. But in very under-reinforced beams higher values of the ultimate strain as reported by Bremner[34] and the author may be obtained.

## 4.3. Maximum stress in concrete under flexure.

The concrete subjected to flexure as in Fig. 4.2 also shows an increase in the crushing stress due to the biaxial nature of the stresses around the concrete in the compression zone. Thus in beams the maximum stress before concrete starts spalling may be as high as the standard cylinder strength or even the cube strength, depending on the nature of the restraining force $F_y$ as explained earlier

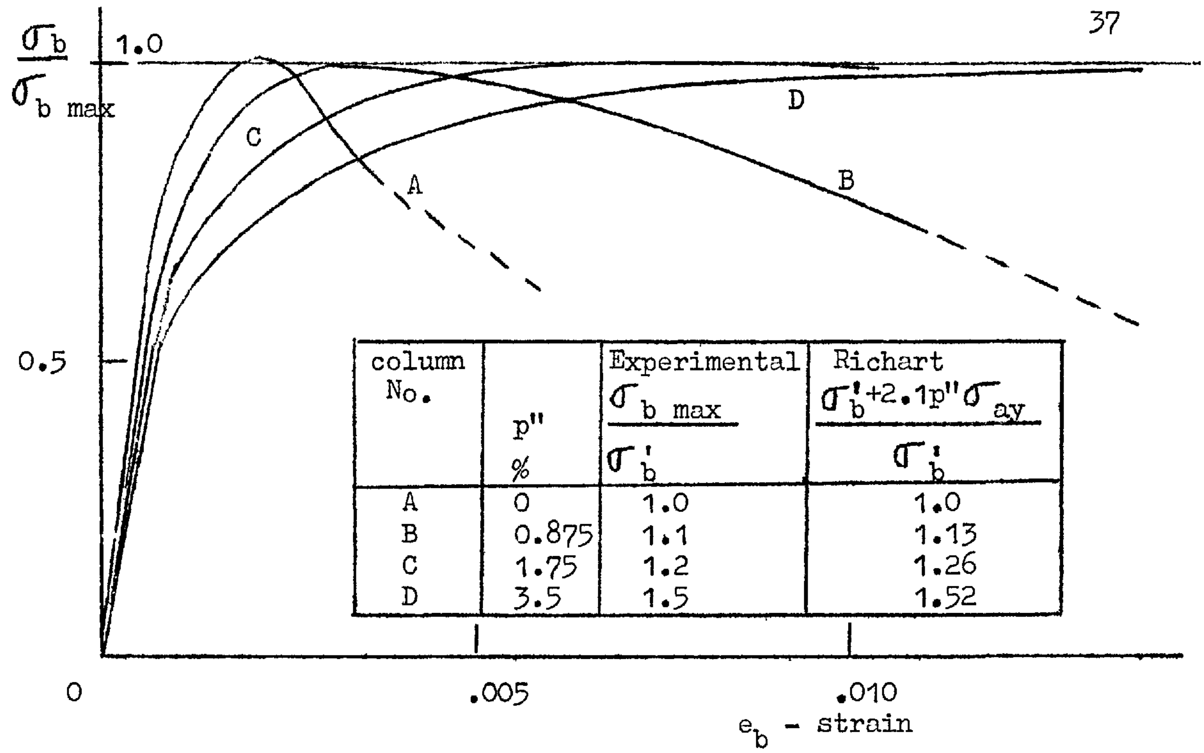
Considering the neutral axis depth as the most important parameter influencing the ultimate stress, the following approximate expression for the maximum stress in concrete has been suggested by the author, which is applicable for rectangular beams and columns as shown by test results.

$$\sigma''_b = \sigma'_b ( 0.8 + \frac{0.1}{x_2} ) \qquad ..... (4.6)$$

where $\sigma_b'$ is the cylinder stress. For $x_2 < 0.5$, the upper limit of $\sigma_b''$ is assumed to be equal to $\sigma_b'$. As a reasonable approximation the maximum stress in the concrete in axially loaded columns may be assumed to 0.8 of the cylinder stress and in under reinforced beams it may be assumed to be equal to the cylinder stress.

Considering the separate relations for the ultimate stress and strain, the idealised stress strain curve for concrete subjected to bending is given in Fig. 4.4. The variation of $e_{b2}$ with the neutral axis depth and the amount of binders is shown in a diagramatic form in Fig. 4.3. The properties of the stress block in a rectangular section are given in Fig. 4.6 where $\alpha$ and $\gamma$ are the usual stress block parameters. Since the ultimate strain and the neutral axis depth are now inter related, the actual properties could only be obtained by trial and error. But it could be seen from Fig. 4.6 that when $e_{b2}$ is greater than 0.004, the change in both $\alpha$ and $\gamma$ is very small.

It may be noticed that in beams the strength calculations are relatively unaffected even if an approximate value for the ultimate strain is assumed. However the neutral axis depth thus determined may be used to obtain a better approximation for the value of the ultimate strain to be used in the permissible rotation calculations.




| column No. | p" % | Experimental $\frac{\sigma_{b\ max}}{\sigma'_b}$ | Richart $\frac{\sigma'_b + 2.1p''\sigma_{ay}}{\sigma'_b}$ |
|---|---|---|---|
| A | 0 | 1.0 | 1.0 |
| B | 0.875 | 1.1 | 1.13 |
| C | 1.75 | 1.2 | 1.26 |
| D | 3.5 | 1.5 | 1.52 |

Fig. 4.1(a). Ef ective stress strain curves for four short reinforced concrete columns

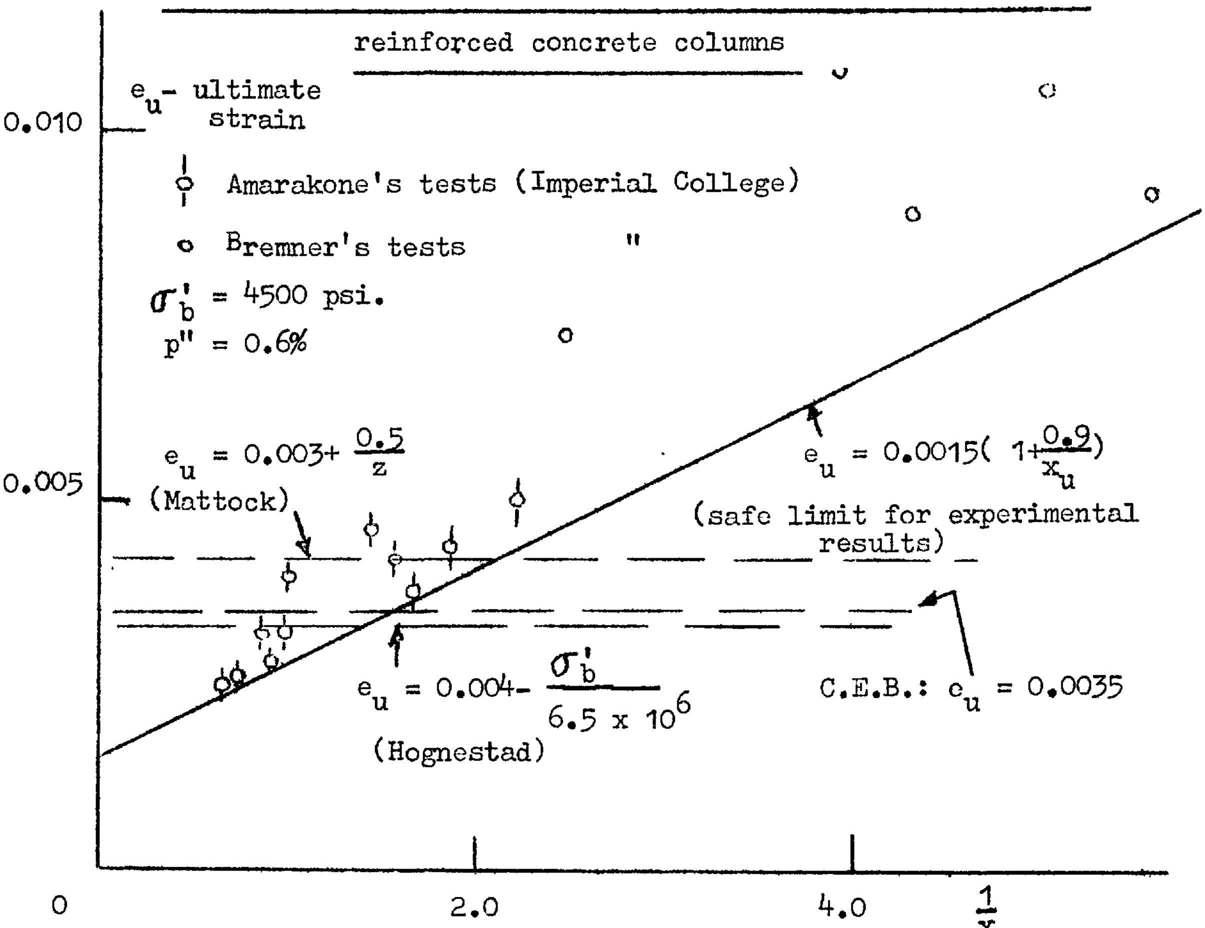



Fig. 4.1(b). Variation of ultimate strain with neutral axis depth

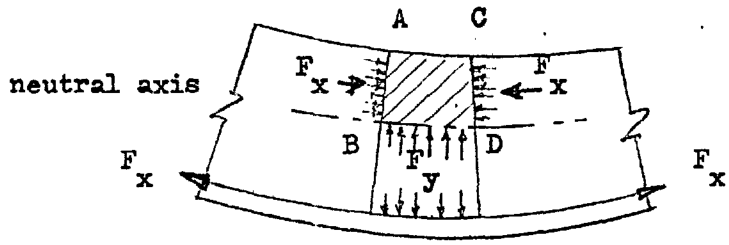



Fig. 4.2 State of Stress in Concrete Under Flexure

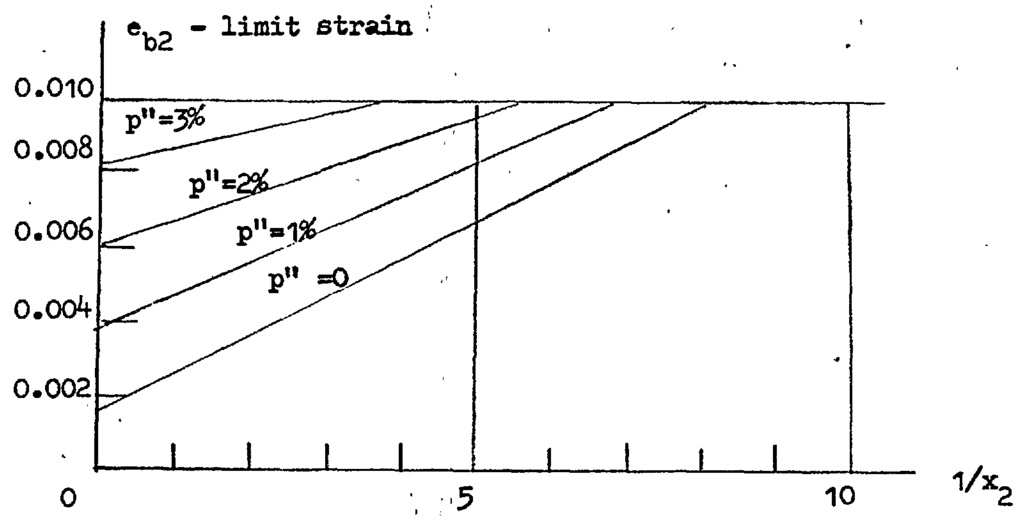



Fig. 4.3 Limit Strain in Bound Concrete

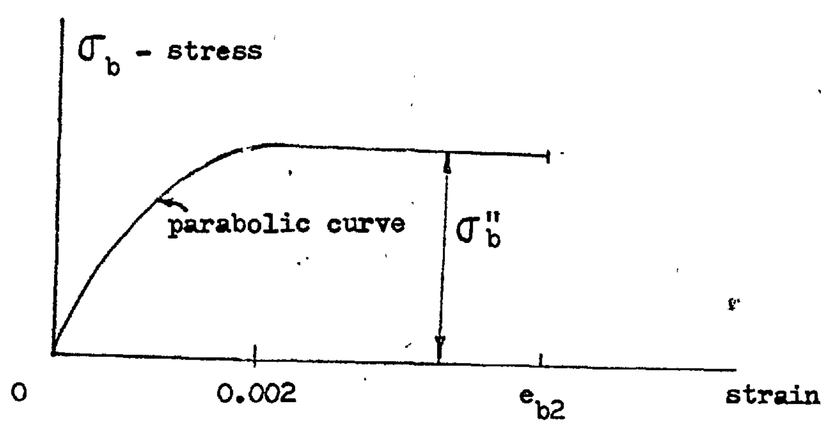



Fig. 4.4 Idealised Stress Strain Relation for Bound Concrete

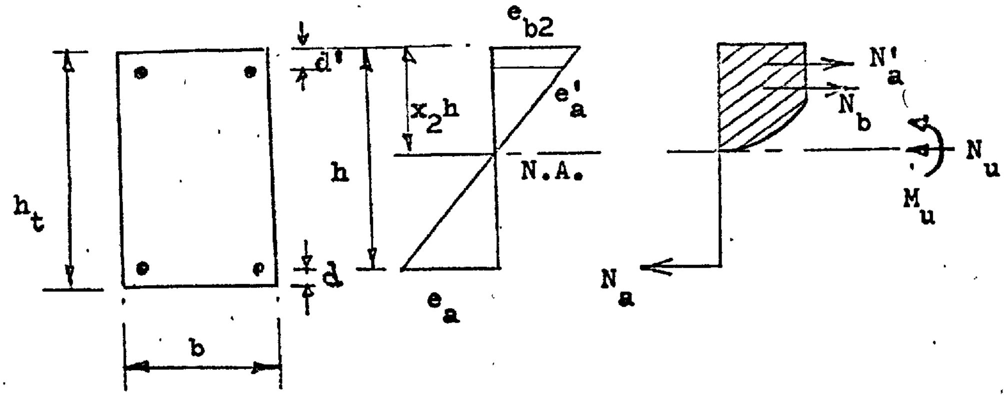



Fig. 4.5 Column Section

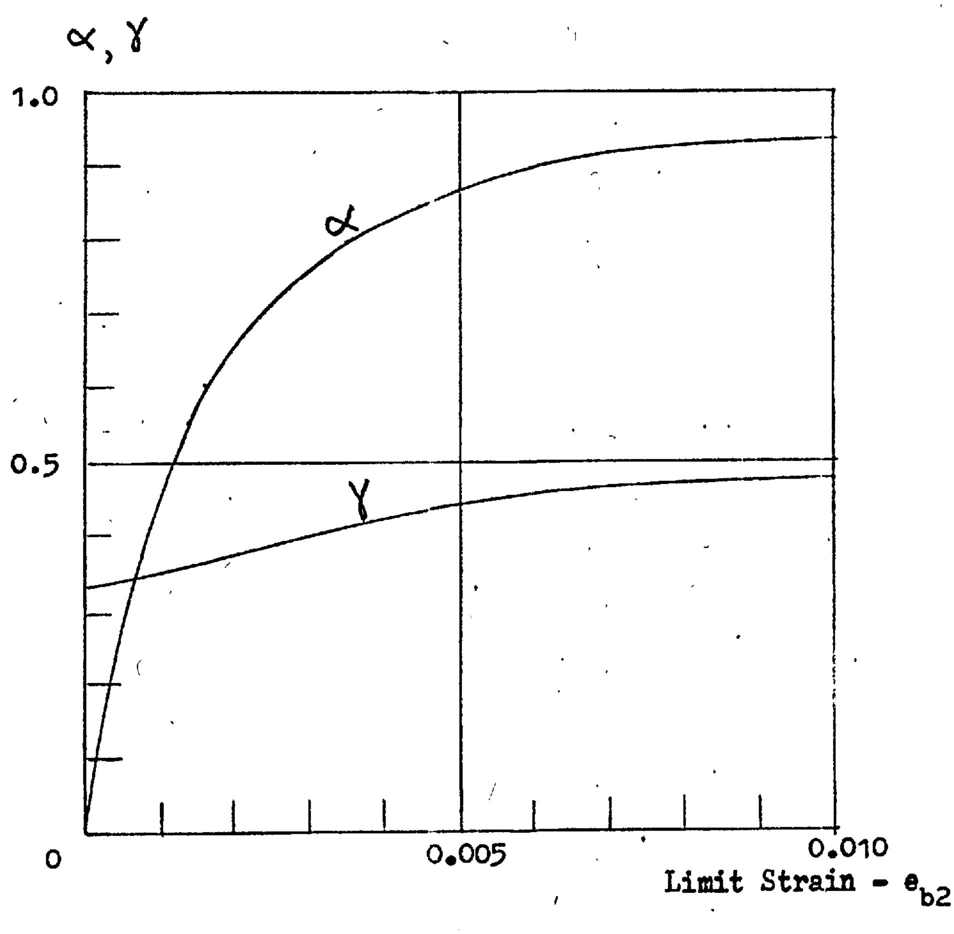



Fig. 4.6 Stress Block Parameters

# CHAPTER 5

# DESIGN OF SECTIONS

## 5.1. INTERACTION CURVES FOR COLUMNS.

Consider a rectangular section subjected to axial load and bending as shown in Fig. 4.5. Let $A_a$ and $A'_a$ be the area of tension and compression reinforcement and $x_2h$ be the depth of the neutral axis from the extreme compression fibres at limit $L_2$. Then for equilibrium of axial load and moment in the section,

$$N_2 = N_b + N'_a - N_a \qquad \ldots (5.1)$$

$$M_2 = -x_2 h\, N_b - N'_a d' + N_a h + \frac{N_2 h_t}{2} \qquad \ldots (5.2)$$

where the forces $N_a$, $N'_a$, $N_b$ are given by

$$N_a = A_a \sigma_a = \frac{\omega_a bh \sigma_b'}{\sigma_{ay}}$$

$$N'_a = A_a' \sigma_a' = \frac{\omega_a' \; bh \sigma_b'}{\sigma_{ay}'}$$

$$N_b = \alpha x_2 \, bh \, \sigma_b''.$$

$\sigma_a$, $\sigma_a'$ depend on the strain in the tension and compression reinforcement which could be easily obtained in terms of the ultimate strain in concrete and the depth of the neutral axis. The stress block parameters $\alpha$ and $\gamma$ could be read off from Fig. 4.6 corresponding to the ultimate strain. Hence the axial load $N_2$ and moment $M_2$ at limit $L_2$ could be expressed in terms of the single parameter $x_2$.

Equations 5.1 and 5.2 may be non-demensionalised by dividing them by $\sigma_b' \, bh$ and $\sigma_b' \, bh^2$ respectively substituting $n_2 = N_2 / \sigma_b' \, bh$, and $m_2 = M_2 / \sigma_b' bh^2$

$$n_2 = \frac{\alpha \sigma_b''}{\sigma_b'} + \frac{\bar{\omega}' \sigma_a'}{\sigma_{ay}'} - \frac{\bar{\omega} \sigma_a}{\sigma_{ay}} \quad \ldots (5.3)$$

$$m_2 = \frac{\bar{\omega} \sigma_a}{\sigma_{ay}} + \frac{n_2 h_t}{2h} - \frac{\alpha \sigma_b'' \gamma x_2}{\sigma_b'} - \frac{\bar{\omega}' \sigma_a' d'}{\sigma_{ay}' h} \quad \ldots (5.4)$$

The interaction curves in Fig. 5.1 are obtained by plotting $n_2$ against $m_2$ for varying percentages of reinforcement. These curves also show the values of $x_2$ which in tern would determine the value of the ultimate strain in concrete to be used in inelastic compatibility calculations.

## 5.2 Simplified calculations for beams

In practice reinforced concrete beams are under-reinforced and their design could be greatly simplified as the changes in stress-block parameters for variation in the amount of reinforcement has very little influence on the ultimate moment when the actual amount of reinforcement is small.

When $(\bar{\omega} - \bar{\omega}') < 0.3$, the average values of $\gamma = 0.41$ and $\alpha = 0.85$ have been used to obtain the following simple expressions for $x_2$ and $m_2$ at limit $L_2$,

$$x_2 = 1.18 \, (\bar{\omega} - \bar{\omega}') \quad \ldots (5.5)$$

$$m_2 = (\bar{\omega} - \bar{\omega}') \left[ 1 - 0.48 \, (\bar{\omega} - \bar{\omega}') \right] + \bar{\omega}' \left(1 - \frac{d'}{h}\right) \quad \ldots (5.6)$$

$x_2$ and $m_2$ are plotted against $\overline{\omega} - \overline{\omega}'$ in Fig 5.2 for a value of $\frac{d'}{h} = 0.10$. Similar design charts may be obtained for other values of $\frac{d'}{h}$. It may be noticed that the grade of steel is already taken into account in $\overline{\omega}$, hence the charts are applicable for all grades of steel.

5.3 Comparison of theoretical calculations with CEB test results.

The experimental results of an extensive series of tests to determine the moment - rotation characteristics of reinforced concrete beams and columns carried out under the European Concrete Committee have been reported recently (ref.9). The results of 80 beam tests and 32 column tests have been analysed using the stress-strain relation suggested in chapter (4) and the results are presented in Tables. 5.1 and 5.2, the details of the beams are given in table 2 in reference (9) (see Appendix I).

The beam calculations based on the proposed stress block were compared with the similar calculations based on other stress blocks suggested by the European Concrete Committee (ref 14) which allows for a rectangular - parabolic stress block (CEB. R.P.) or a parabolic stress block (CEB.P) and the stress block suggested by Hognestad (ref 22). The essential properties of the different stress blocks are expressed in terms of the $\alpha$, $\gamma$ parameters as in the table below.

TABLE 5.1

| Beam No. | Ultimate Strength | | Stiffness | | Ductility | |
|---|---|---|---|---|---|---|
| | $x_2$ | $\frac{M_{exp}}{M_{cal}}$ | $\xi_{cal}$ | $\frac{\xi_{exp}}{\xi_{cal}}$ | $D_{exp}$ | $D_{cal}$ |
| Imperial College 1 | .145 | 0.98 | 46.3 | 0.97 | - | - |
| " 2 | .196 | 1.06 | 53.8 | 1.00 | - | - |
| " 3 | .372 | 1.08 | 74.1 | 1.20 | 10 | 7 |
| " 4 | .431 | 1.11 | 84.2 | 1.14 | 7 | 5 |
| " 5 | .489 | 1.04 | 88.7 | 0.96 | 10 | 5 |
| " 6 | .435 | 1.03 | 86.5 | 1.00 | 10 | 6 |
| " 7 | .381 | 0.98 | 91.9 | 1.14 | 22 | 4 |
| " 8 | .582 | 0.97 | 107.0 | 0.97 | 11 | 4 |
| Madrid 6 -1 | .374 | 1.16 | 73.7 | 1.60 | - | - |
| " 6 -2 | .362 | 1.18 | 72.5 | 1.60 | - | - |
| " 6 -3 | .371 | 1.09 | 69.6 | 1.48 | 8 | 4 |
| Paris(IRABA) A2 | .050 | 1.18 | 27.3 | 1.17 | 55 | 29 |
| " A5 | .240 | 1.08 | 60.4 | 1.06 | 6 | 8 |
| " A8 | .074 | 1.21 | 34.2 | 1.29 | 49 | 25 |
| " A11 | .038 | 1.21 | 24.5 | 1.00 | 48 | 29 |
| " B2 | .048 | 1.01 | 26.8 | 1.09 | 30 | 29 |
| " B5 | .250 | 1.10 | 62.0 | 1.08 | 20 | 8 |
| " B8 | .083 | 1.07 | 35.8 | 1.26 | 33 | 27 |
| " B11 | .038 | 1.21 | 24.4 | 1.18 | 38 | 27 |
| Porto B4 | .394 | 0.95 | 76.5 | 1.30 | - | - |
| " B6 | .238 | 1.04 | 58.6 | 1.23 | 17 | 8 |
| " B7 | .147 | 1.11 | 46.1 | 1.21 | 8 | 16 |
| " B9 | .065 | 1.18 | 31.4 | 1.43 | 72 | 26 |
| " B10 | .078 | 1.06 | 33.6 | 1.01 | 42 | 25 |
| " B12 | .032 | 1.28 | 21.6 | 1.06 | 110 | 27 |
| Torino A6 | .197 | 1.18 | 54.5 | 1.23 | 11 | 10 |
| " A9 | .068 | 1.23 | 30.3 | 1.62 | - | - |
| " A12 | .037 | 1.17 | 21.9 | 1.46 | 28 | 26 |
| " D5 | .621 | 1.21 | 77.2 | 1.31 | - | - |
| " D11 | .119 | 1.41 | 30.4 | 1.15 | 27 | 12 |
| " D8 | .138 | 1.36 | 32.7 | 1.32 | 22 | 10 |

ctd.

TABLE 5.1 ctd.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| C&CA | A4 | .411 | 1.16 | 77.5 | 1.42 | - | - |
| " | A7 | .144 | 1.01 | 45.9 | 1.57 | - | - |
| " | A10 | .077 | 1.10 | 33.5 | 1.08 | - | - |
| Imperial College | 9 | .124 | 1.11 | 28.1 | 0.91 | 9 | 11 |
| " | 10 | .176 | 1.09 | 33.4 | 0.96 | 12 | 8 |
| " | 11 | .266 | 1.12 | 41.1 | 0.97 | 9 | 6 |
| " | 12 | .296 | 1.25 | 43.4 | 0.99 | 4 | 6 |
| " | 13 | .507 | 1.10 | 58.1 | 1.00 | 3 | 4 |
| " | 14 | .197 | 1.13 | 35.4 | 0.98 | 15 | 14 |
| " | 15 | .389 | 1.15 | 49.7 | 0.98 | 11 | 12 |
| Torino | F4 | .447 | 0.95 | 65.9 | 0.97 | - | - |
| " | L4 | .568 | 1.07 | 74.2 | 0.96 | 4 | 4 |
| Paris (IRABA) | E6 | .452 | 1.16 | 58.2 | 1.35 | 2 | 2 |
| " | E9 | .138 | 1.08 | 32.1 | 1.22 | 10 | 12 |
| " | F6 | .316 | 1.07 | 44.1 | 1.23 | 3 | 4 |
| " | F9 | .078 | 1.05 | 23.3 | 1.03 | 13 | 14 |
| " | H2 | .097 | 1.24 | 28.2 | 1.03 | 16 | 8 |
| " | H5 | .525 | 1.04 | 63.9 | 1.18 | 2 | 2 |
| " | H8 | .179 | 1.09 | 39.9 | 0.94 | 15 | 10 |
| " | H11 | .090 | 1.10 | 26.9 | 0.90 | 10 | 15 |
| " | R4 | .516 | 1.03 | 79.5 | 1.31 | - | - |
| " | R5 | .323 | 1.09 | 63.0 | 1.11 | 3 | 3 |
| " | R6 | .296 | 1.15 | 47.1 | 0.96 | 5 | 5 |
| " | N2 | .186 | 1.17 | 39.9 | 1.23 | 21 | 8 |
| " | N5 | .649 | 1.05 | 75.7 | 1.33 | 2 | 2 |
| " | N8 | .170 | 1.22 | 39.6 | 1.21 | 7 | 9 |
| " | N9 | .115 | 1.05 | 30.1 | 1.15 | 8 | 12 |
| Porto | C6 | .430 | 1.01 | 63.4 | 1.17 | 4 | 4 |
| " | C7 | .240 | 1.00 | 46.7 | 1.00 | 14 | 9 |
| " | C9 | .077 | 1.23 | 29.1 | 1.20 | 27 | 19 |
| " | C10 | .119 | 1.07 | 31.5 | 0.95 | 25 | 13 |
| " | C12 | .042 | 1.36 | 21.4 | 0.90 | 28 | 19 |

ctd.

TABLE 5.1 ctd.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Porto | M9 | .079 | 1.34 | 29.5 | 0.95 | 18 | 19 |
| " | M10 | .107 | 1.07 | 30.9 | 0.63 | 11 | 14 |
| " | M12 | .040 | 1.38 | 20.9 | 0.76 | 22 | 19 |
| Mexico | A464C | .165 | 1.36 | 38.2 | 0.86 | 14 | 11 |
| " | A413A | .132 | 1.34 | 33.7 | 0.62 | 5 | 13 |
| " | A4192D | .150 | 1.31 | 34.8 | 0.83 | 19 | 12 |
| " | A400B | .163 | 1.24 | 36.3 | 0.83 | 19 | 8 |
| " | A264G | .083 | 1.24 | 25.5 | 0.83 | 10 | 15 |
| " | A2127F | .087 | 1.27 | 25.7 | 0.90 | 15 | 14 |
| " | A2192H | .072 | 1.34 | 23.8 | 0.80 | 10 | 15 |
| C&CA | C4 | .557 | 0.96 | 77.8 | 1.24 | - | - |
| | C5 | .517 | 1.07 | 73.4 | 1.22 | - | - |
| | C8 | .131 | 0.94 | 36.4 | 0.89 | - | - |
| | C11 | .073 | 1.27 | 27.2 | 0.89 | - | - |
| | L5 | .485 | 1.07 | 71.2 | 1.20 | - | - |
| | M5 | .158 | 1.36 | 40.0 | - | - | - |
| | M11 | .078 | 1.28 | 28.2 | 1.35 | - | - |
| Mean | | | 1.14 | | 1.11 | | |
| Standard Deviation | | | 0.117 | | 0.21 | | |

TABLE 5.2. COLUMN TESTS

| Column No. | | $x_2$ cal | $m_2$ cal | $n_2$ cal | $r_2$ cal | $r_u$ exp | $\frac{r_u \text{ exp}}{r_2 \text{ cal}}$ |
|---|---|---|---|---|---|---|---|
| Imperial College | A1 | .50 | .28 | .40 | .488 | .456 | 0.935 |
| " | A2 | .65 | .28 | .53 | .599 | .596 | 0.995 |
| " | A3 | .71 | .26 | .59 | .645 | .644 | 1.000 |
| " | A4 | .92 | .31 | .84 | .904 | 1.033 | 1.140 |
| " | A5 | 1.10 | .11 | .98 | .987 | .996 | 1.010 |

ctd..

TABLE 5.2 ctd.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Imperial College | A6 | 1.09 | .09 | 1.03 | 1.003 | 1.071 | 1.070 |
| " | B1 | .55 | .31 | .46 | .555 | .555 | 1.000 |
| " | B2 | .60 | .27 | .50 | .570 | .546 | .958 |
| " | B3 | .75 | .25 | .64 | .688 | .658 | .957 |
| " | B4 | .97 | .18 | .86 | .880 | .875 | .995 |
| " | B5 | 1.00 | .08 | 1.04 | 1.042 | .990 | .951 |
| " | B6 | - | .08 | 1.07 | 1.072 | 1.204 | 1.118 |
| Torino | A2 | .81 | .17 | .57 | .594 | .660 | 1.111 |
| " | D2 | .90 | .21 | .55 | .589 | .760 | 1.290 |
| " | F1 | .80 | .18 | .62 | .645 | .657 | 1.020 |
| " | F2 | .82 | .16 | .67 | .640 | .664 | 1.040 |
| " | F3 | .60 | .20 | .48 | .522 | .536 | 1.028 |
| " | G1 | - | .12 | .83 | .839 | .991 | 1.182 |
| " | H2 | .78 | .21 | .54 | .580 | .670 | 1.155 |
| " | L1 | 1.05 | .13 | .80 | .811 | .902 | 1.110 |
| " | A3 | .65 | .21 | .51 | .551 | .571 | 1.040 |
| C&CA | A1 | .89 | .11 | .74 | .750 | .678 | .905 |
| " | A2 | .38 | .16 | .32 | .362 | .364 | 1.000 |
| " | C1 | .94 | .13 | .81 | .820 | .785 | .958 |
| " | C2 | .38 | .21 | .39 | .445 | .425 | .958 |
| " | C3 | .18 | .15 | .15 | .211 | .234 | 1.110 |
| " | E1 | .75 | .10 | .84 | .845 | .606 | .718 |
| " | E2 | .54 | .25 | .45 | .517 | .520 | 1.010 |
| " | E3 | .18 | .17 | .16 | .234 | .246 | 1.050 |
| " | G2 | .33 | .18 | .32 | .367 | .326 | .890 |
| " | L2 | .45 | .22 | .40 | .459 | .429 | .935 |
| " | M2 | .45 | .38 | .60 | .711 | .439 | - |
| Mean | | | | | | | 1.020 |
| Standard Deviation | | | | | | | 0.105 |

Table 5.3 stress-block Parameters.

| Parameter | CEB. R.P. | CEB.P. | Hognestad | Proposed |
|---|---|---|---|---|
| $\sigma''/\sigma_b'$ | 0.8 - 0.82 | 1.0 | 0.85 | 1.0 |
| $\alpha$ | 0.81 | 0.67 | 0.79 | 0.85 |
| $\gamma$ | 0.41 | 0.375 | 0.43 | 0.41 |

The standard deviation in the ratio of ultimate moment to calculated limit moment due to the four methods are 12.5% (CEB. R.P), 11.8% (CEB.P), 11.7% (Hognestad) and 11.7% (proposed method). The corresponding mean values were 1.16, 1.15, 1.17 and 1.14. These results show that the differences in the assumptions in the above methods have little influence on the ultimate strength calculation of beams. The error in the calculated moment was in all casss on the safe side, which may be due to the safe limit assumptions of the ultimate stress of reinforcing bars.

However the proposed method yields smaller values for the neutral axis depth than the other three methods. These results agree well with the experimental values as shown in Fig. 5.3.

The column tests were analysed using the proposed method and interaction curves of the type described earlier. The methods of comparison of the strength of columns subjected to axial load and bending that are often used are found to be misleading due to the

need to compare the axial load and moment results at the same time as they are inter-related. (ref.59)
This was over come by comparing the column strength in terms of the radius sector of the interaction curves given by

$$r_2 = \sqrt{m_2^2 + n_2^2}$$

to the corresponding experimental results. The above values of $m_2$ and $n_2$ were determined for the same values of eccentricity as in the test results.

The calculations are presented in table 5.2 and Fig. 5.4. The mean value of $r_{exp}/r_{cal}$ for 31 tests was 1.02 and its standard deviation was about 10%. The calculated neutral axis depths of the columns are compared with the observed values in Fig.5.3. These results indicate that the proposed method forms a satisfactory basis of calculation of the short term strength and ultimate load charactistries of beams and columns.

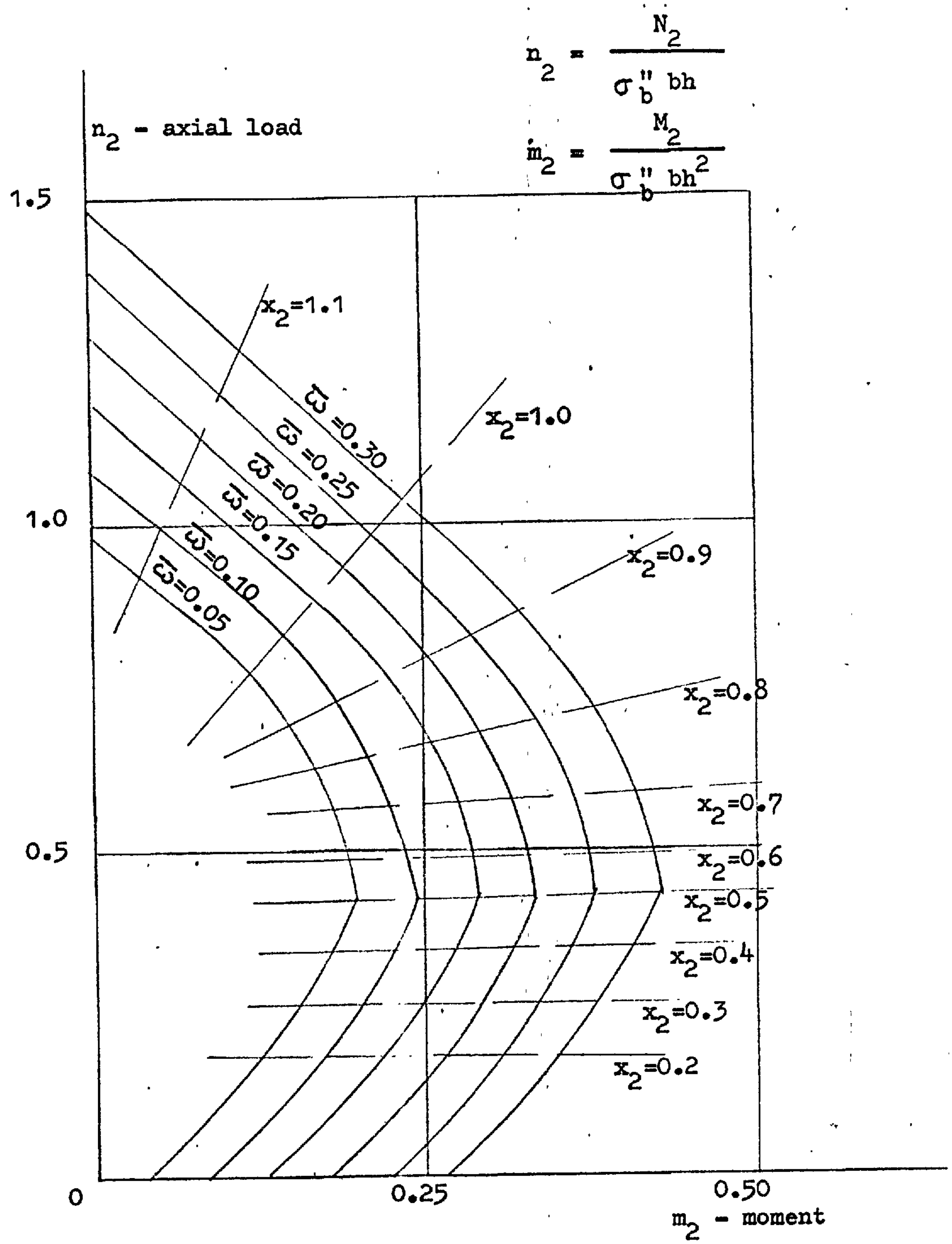



Fig. 5.1 Interaction Curves for Columns

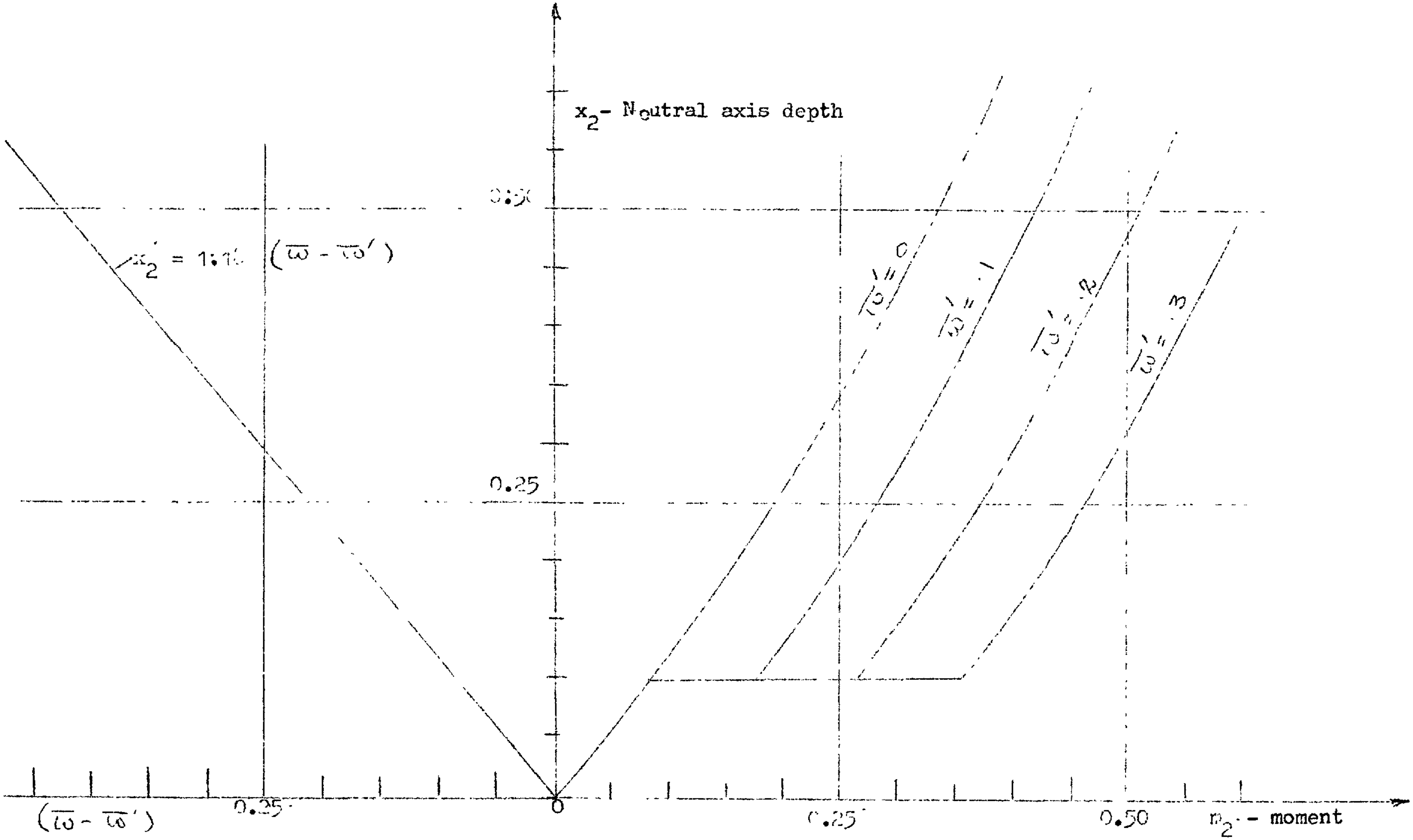



Fig. 5.2. Computation Curves for Beams at Limit $L_2$

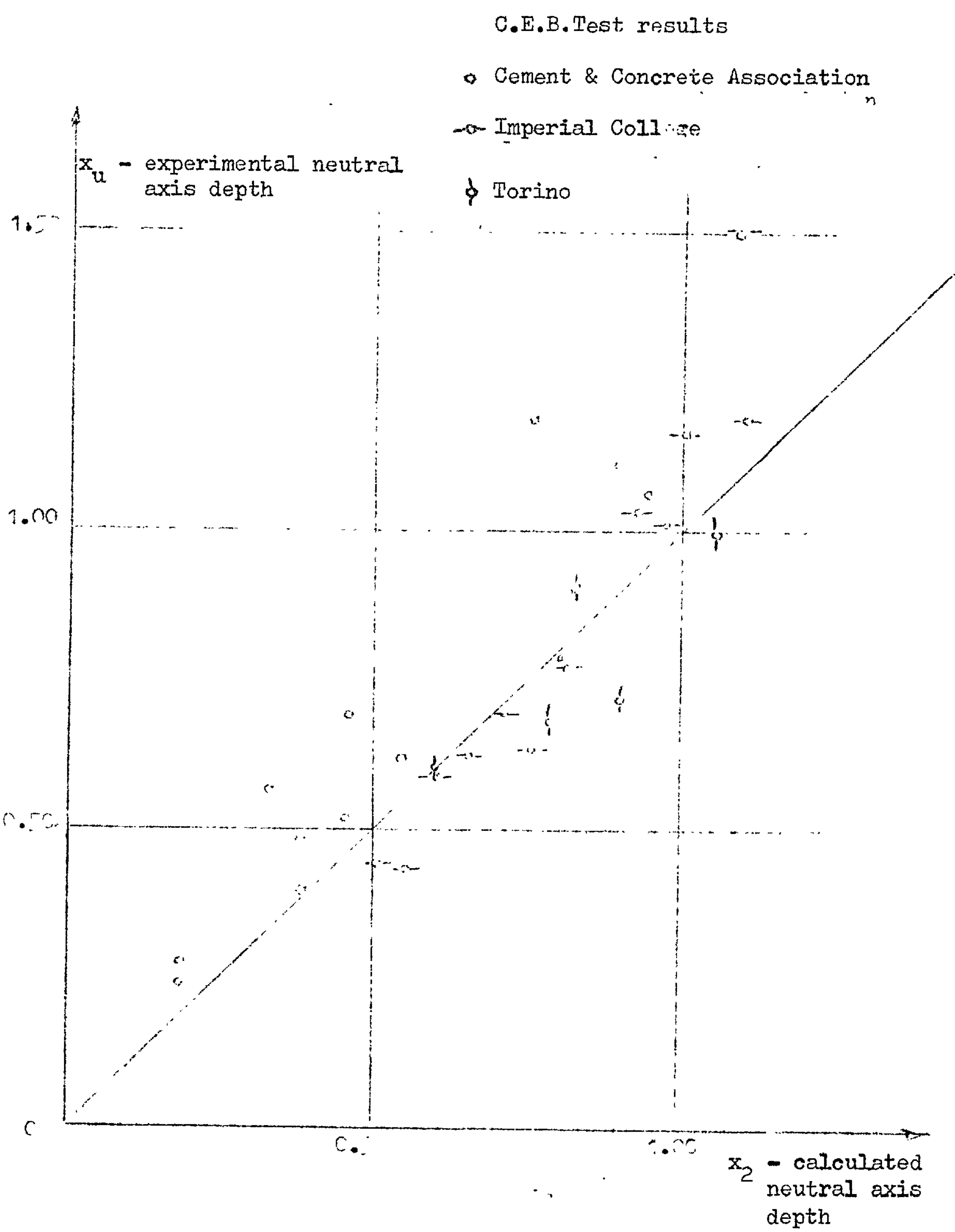



Fig. 5.3 Neutral Axis Depth Results.

C.E.B.Test Results

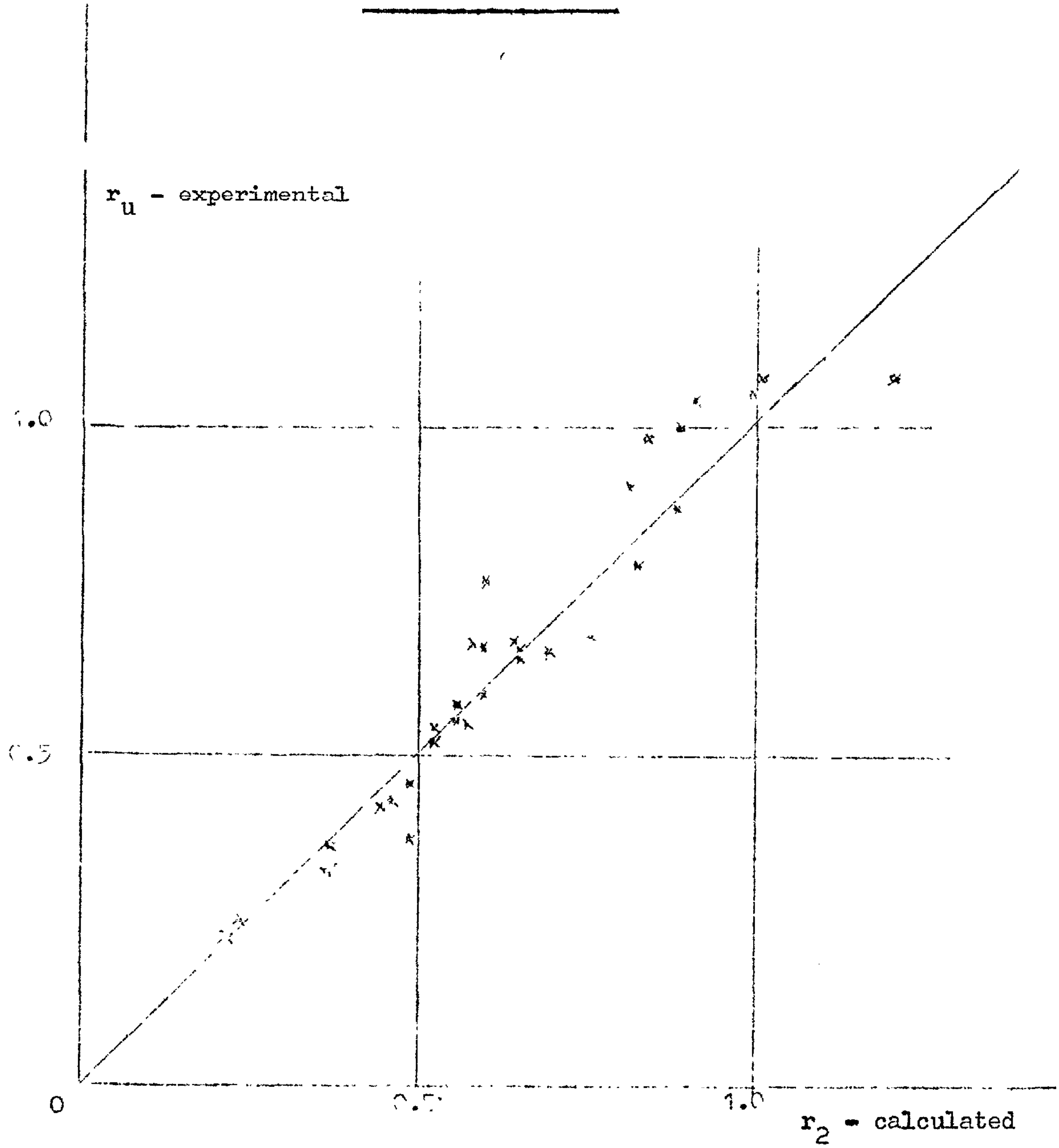



Fig. 5.4. Column Results (Nos. 31 Tests)

# CHAPTER 6

## Flexural Stiffness of Reinforced Concrete Members in Limit Design

### 6.1 Idealised flexural stiffness

In the stress and deformation analysis of reinforced concrete structures, the flexural stiffness of members forms an extremely important factor influencing the calculated stress state. But it is not unusual to formulate and attempt a rigorous analysis based on very approximate stiffness values which in turn could create large errors in the final results. In classical elastic analysis, reinforced concrete members are assumed to be homogeneous and elastic. They are then treated in the same way as any other elastic material. Thus most designers may use the formula for EI involving the second moment of the area of entire section and the elastic modulus of concrete (a reduced value of $E_b$ is normally employed to allow for creep and cracking, etc.) where the area of reinforcement is completely ignored. Other provisions included in the British Code[25] allow the reinforcement to be included in the above calculations on the basis of the modular ratio.

The differences in the EI values calculated by the different assumptions are quite large. In the analysis of multistorey structures where the relative error in the EI values between the beams and columns is more important than their absolute values, the basic assumptions for the determination of EI may give rise to large differences in the design values. It has been shown that under extreme conditions, the error in the stress analysis, due to different assumptions for the EI values, could be as much as 40%[24]. In limit design methods, where the degree of safety is more critical, it is considered essential to base the stiffness calculations on safe limit assumptions.

The results of extensive experimental investigations into the study of moment-curvature and moment-rotation characteristics[8,9,23,34] show that essentially reinforced concrete is inelastic but the moment-rotation characteristics could be closely approximated to a bilinear[5] or a trilinear curve[26].

Baker in introducing the ultimate load method of analysis,[5] has suggested a bilinear relation for the moment-rotation characteristics of reinforced concrete members. A lower limit EI value is based on the idealised limit $L_1$ as given by equation

$$EI = \frac{M_1 x_1 h}{e_{b1}} \quad \ldots \quad (6.1)$$

where the suffix 1 denotes the stress state at the critical section (referred to as limit $L_1$), when either reinforcing steel reaches the elastic limit (or 0.1% off set strain in the case of cold worked steel) or the concrete reaches a strain of 0.002 . EI may then be calculated based on the stress-strain relations for concrete and reinforcement.

For an under-reinforced rectangular section, the following limit $L_1$ values for $e_{b1}$, $M_1$ may be derived in terms of the neutral axis depth.

$$e_{b_1} = \frac{x_1}{1-x_1} e_{a_1} \qquad \text{.... (6.2 )}$$

$$\frac{A_a \sigma_{ay}}{b'} = \overline{\omega} = 1.13\, x_1 \left[ \frac{x_1}{1-x_1} \left( \frac{e_{a_1}}{0.002} \right) - \frac{1}{2} \left( \frac{x_1}{1-x_1} \right)^2 \left( \frac{e_{a_1}}{0.002} \right)^2 \right] \quad \text{.(6.3)}$$

$$m_1 = \frac{M_1}{b'bh^2} = \overline{\omega} \left[ 1-\gamma_1 x_1 \right] \qquad \text{... (6.4)}$$

$\gamma_1$ has a value ranging between 0.33 and 0.375

From Fig.6.1 $m_1$ and $x_1$ may be obtained for a known value of $\overline{\omega}$, hence the EI value could be evaluated.

The limit calculations for EI values of under-reinforced and over-reinforced beams have been considered in greater detail by the author in a recent publication[27].

The resultant bilinear representation of the moment-curvature and moment-rotation curves for typical members are shown in Fig. 6.2 and 6.3 An idealised bilinear representation as shown in Fig.6.2 as suggested by Chan[23] and Macchi[26] seem to be a closer approximation to experimental results than the bilinear assumptions. However, the advantages in the bilinear idealisation may have to be weighed against the fact that a departure from a bilinear representation of the moment-rotation relation makes stress analysis extremely complicated even in very simple structures. The relative error involved in the two methods may be compared easily for simple cases as in the next section.

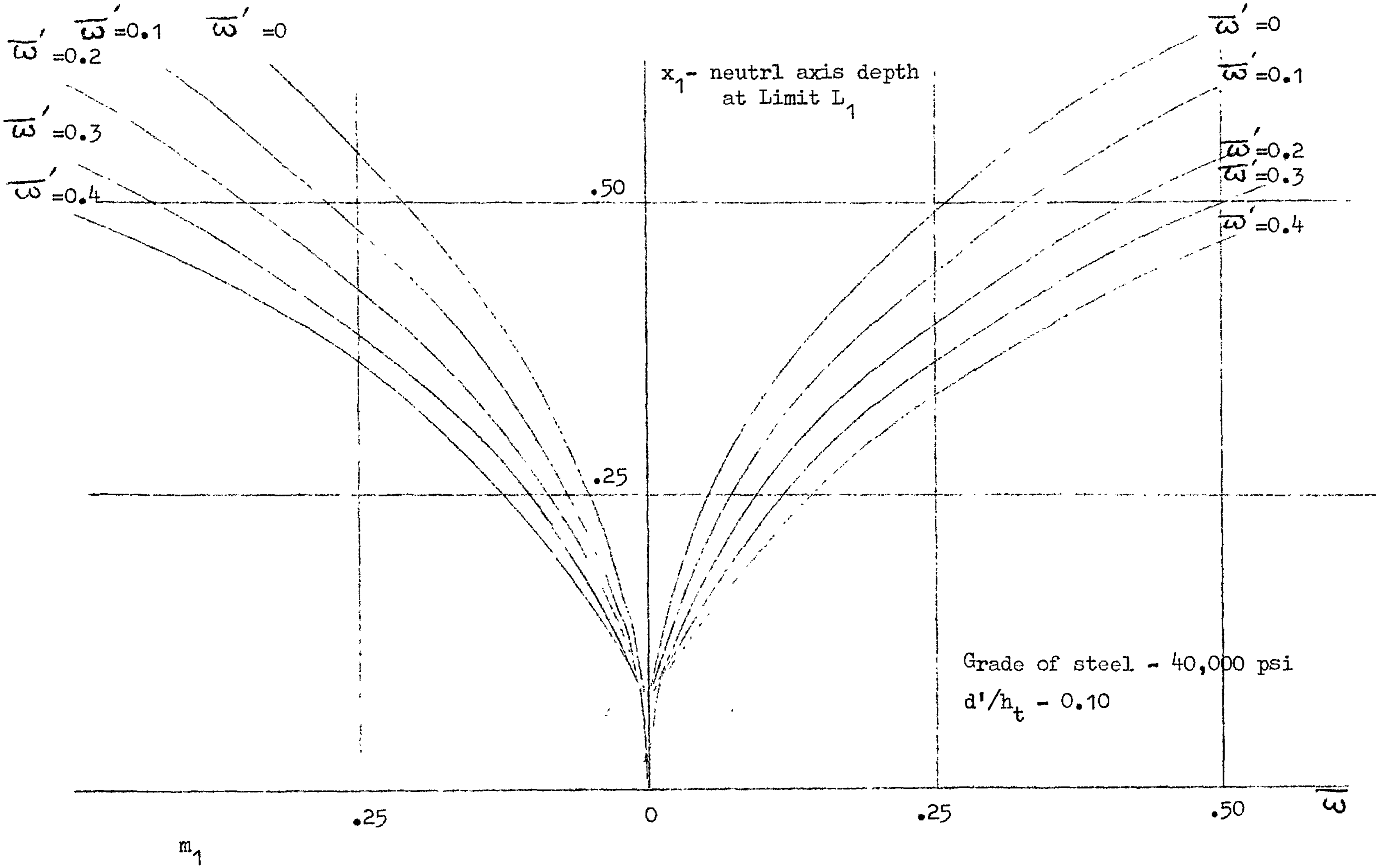



Fig. 6.1. Limit $L_1$ Properties for Beams

## 6.2 Relative error in total rotation due to bilinear and trilinear assumptions.

Consider a simply supported uniform beam subjected to a central point load as in Fig.6.4. The curvature distribution at limit $L_1$ corresponding to the bilinear and the trilinear assumptions are shown in (c) and (d). The total rotations could be easily obtained by integrating the area of the curvature distribution diagrams. Let $(EI)_e$ and $(EI)_1$ be the values of flexural stiffness in the uncracked elastic stages and cracked stages respectively. These are given by the idealised assumptions,

$$(EI)_e = \frac{1}{12} \quad E_b.bh^3$$

$$(EI)_1 = \frac{M_1 \, x_1 \, h}{e_{b_1}}$$

The total rotation $\theta_1^B$ due to bilinear assumptions is given by

$$\theta_1^B = \int_0^{M_1} \frac{M}{EI} \, ds = \frac{M_1 l}{2(EI)_1} \qquad \ldots\ldots \quad (6.5)$$

The corresponding total rotation $\theta_1^T$ due to the trilinear assumptions, is

$$\theta_1^T = \int_0^{M_1} \frac{M}{EI} \, ds = \frac{M_1 l}{2} \left[ \frac{c^2}{(EI)_e} + \frac{(1-c^2)}{(EI)_1} \right] \ldots \quad (6.6)$$

where $c = \frac{Mo}{M_1}$

and Mo is the cracking moment.

From equations 6.5 and 6.6,

$$\frac{\theta_1^B}{\theta_1^T} = \frac{1}{c^2 \frac{(EI)_1}{(EI)_e} + 1 - c^2} \qquad \ldots\ldots \quad (6.7)$$

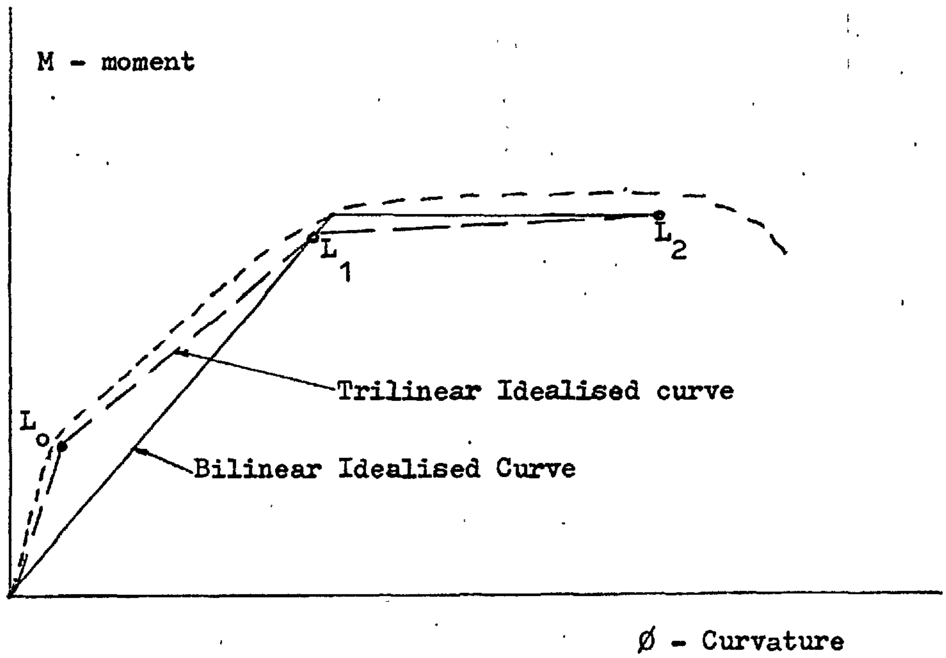



Fig. 6.2 Typical Moment Curvature Diagram for Reinforced concrete section

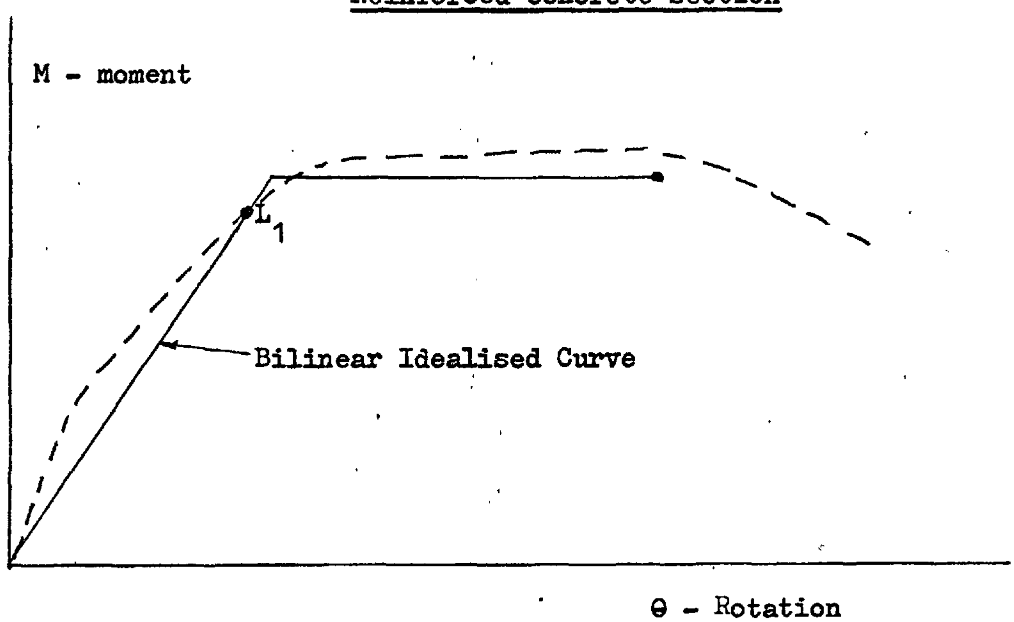
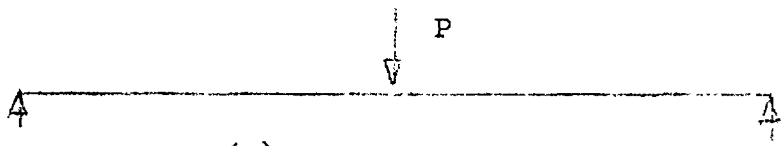
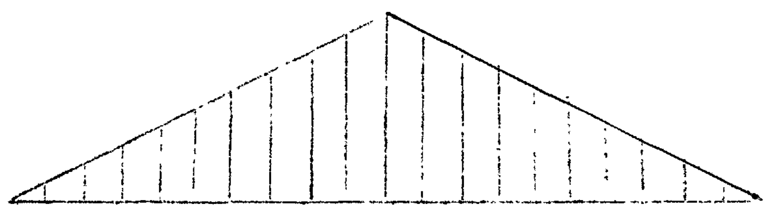



Fig. 6.3 Typical Moment Rotation Diagram for Reinforced concrete section

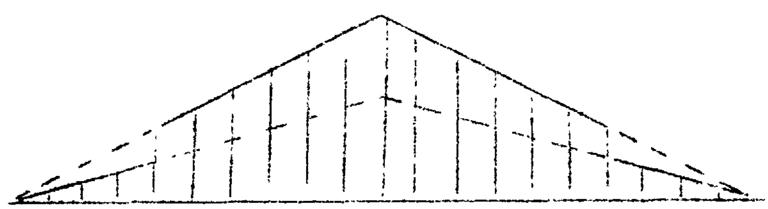



(a) Loading

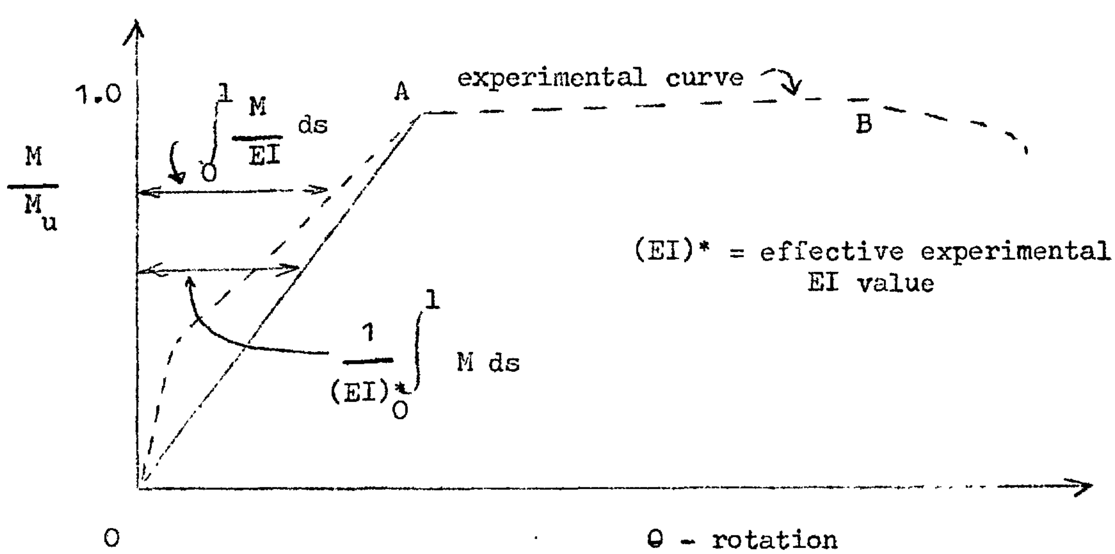

(b) Moment



(c) Idealised curvature distribution (bilinear assumption)



(d) Idealised curvature distribution ( trilenear assumption)




(e) Typical experimental moment - rotation curve

Fig. 6.4. Simply supported Beam Tests

$(EI)_1$ increases with the increase in the tension reinforcement in beams[27]. Thus equation 6.7 shows that the error in the bilinear assumption compared to the trilinear assumption varies with the ratio of the cracking moment to the ultimate moment and the degree of reinforcement. Table 6.1 gives the calculations for typical beams where $(EI)_1/(EI)_e$ may vary from 0.10 to 1.0.

Table 6.1 Comparison between bilinear and trilinear assumptions.

| $\frac{(EI)_1}{(EI)_e}$ | $\theta_1^B / \theta_1^T$ | | | |
|---|---|---|---|---|
| | R.C. | Pre-stressed concrete | | |
| | C = 0.3 | C = .6 | C = 0.7 | C = 0.8 |
| .10 | 1.09 | 1.47 | 1.79 | 2.36 |
| .25 | 1.07 | 1.37 | 1.59 | 1.92 |
| .50 | 1.05 | 1.22 | 1.33 | 1.47 |
| .75 | 1.02 | 1.11 | 1.14 | 1.20 |
| 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

For reinforced concrete beams where the average value of C is about 0.3 the maximum difference in the EI values due to the two assumptions is about 9%. However in the prestressed concrete where c depends on initial prestress the differences vary widely with a maximum of 136% for c = 0.8 and $(EI)_1/(EI)_e$ = 0.1.

Thus it is clear that the bilinear assumptions are perfectly adequate for reinforced concrete design while in prestressed concrete design, the trilinear assumptions or an equivalent must be used.

6.3 Semi-empirical relation for the EI value of beams

The determination of the limit EI value from equation 6.1, even when assisted by diagrams of the form Fig.6.1, remains difficult and subject to large error due to small inaccuracies in all secondary terms like $M_1$, $x_1$ or $e_{b_1}$, which must be first calculated from the section properties.

However, if the sections are under-reinforced (or has compression reinforcement to enable tension steel to yield before concrete), the calculation of limit EI may be simplified.

Let $\xi$ represent the fluxural stiffness reduced to non-dimentional terms given by

$$\xi = \frac{EI}{\sigma_h' \cdot bh^3} \qquad \ldots\ldots (6.8)$$

Then from equations 6.2, 6.3 and 6.4, $\xi$ may be expressed in terms of the single parameter, $X_1$ representing the neutral axis depth at limit $L_1$ given by,

$$\xi = \left(\frac{565 - 12500\, e_{ay}}{1 - x_1}\right)(1 - \gamma_1 x_1) x_1^2 \qquad \ldots\ldots (6.9)$$

The neutral axis depth in under-reinforced beams could be related to the degree of reinforcement as in equation 6.3, but it is not possible to express $\xi$ directly in terms of the degree of reinforcement. Hence $\xi$ may be obtained graphically as in Figs. 6.5, 6.6 and 6.7 for different grades of steel.

However, in considering the idealised calculations and actual test results it was found that the fluxural stiffness factor $\xi$ may be given in the approximate form

$$\xi = (\alpha - \beta\, e_{ay})\sqrt{\omega} \qquad \ldots\ldots (6.10)$$

where $\alpha \doteq 175$, $\beta \doteq 31200$. This expression has the advantage that the flexural stiffness is expressed directly in terms of the section properties and the grade of steel. This is discussed with reference to 80 beam tests carried out under the European Concrete in the next section.

## 6.4 Experimental results

The mean experimental fluxural stiffness in reinforced concrete beams could be obtained from the moment rotation diagrams as in Fig.6.4. Table 5.2[1] gives the stiffness results determined for 80 beam tests[9] the properties of which are given in Table 2 in reference 9. $\xi_{exp}$ indicates the mean experimental fluxural stiffness value as described above. $\xi_{cal}$ represents the calculated value of $\xi$ using equation 6.10. $\xi_{exp}$ have been plotted against $\xi_{cal}$ in Fig.6.8 and the frequency distribution of $\xi_{exp}/\xi_{cal}$ is given in Fig.6.9.

The mean value of $\xi_{exp}/\xi_{cal}$ for 80 test results has been found to be 1.11 and the standard deviation was 0.21. It is known that the actual stiffness of beams is dependent on the duration and nature of loading and the creep characteristics, hence the values predicted by the approximate equation 6.10 represents as an accurate measure of the mean fluxural stiffness in short term test results as may be expected.

The above tests also cover a wide range of steels, varying from mild steel of yield strength 40,000 psi to cold worked steel of 0.1% proof strength of about 85,000 psi. The expression 6.10 for $\xi$ seem to adequately take into account the effect of different grades of steel. This was considered in greater detail by selecting the test results for beams with the same grade of steel. Figs. 6.5, 6.6, 6.7 show the variation of $\xi_{exp}$ with the degree of reinforcement for three grades of steel.

These results may be compared with the bilinear idealisation as predicted by Baker's equation 6.1 and the simple empirical equation 6.10. In mild steel beams the values predicted by both these methods aggree very closely and forms a lower limit of the experimental results. In both grades of cold worked steel, the results predicted by equation 6.10 is slightly larger than those given by 6.1 but agrees well with the experimental results.

The value of EI obtained by the present code[25] method assuming a modular ratio of 15 is also shown in the above diagrams. The actual variation of EI is not reflected at all by the provisions in the Code rules, but in the range of $\bar{\omega}$ ranging from 0.1 to 0.2 which is most common in design practice, the Code provisions may be considered reasonable.

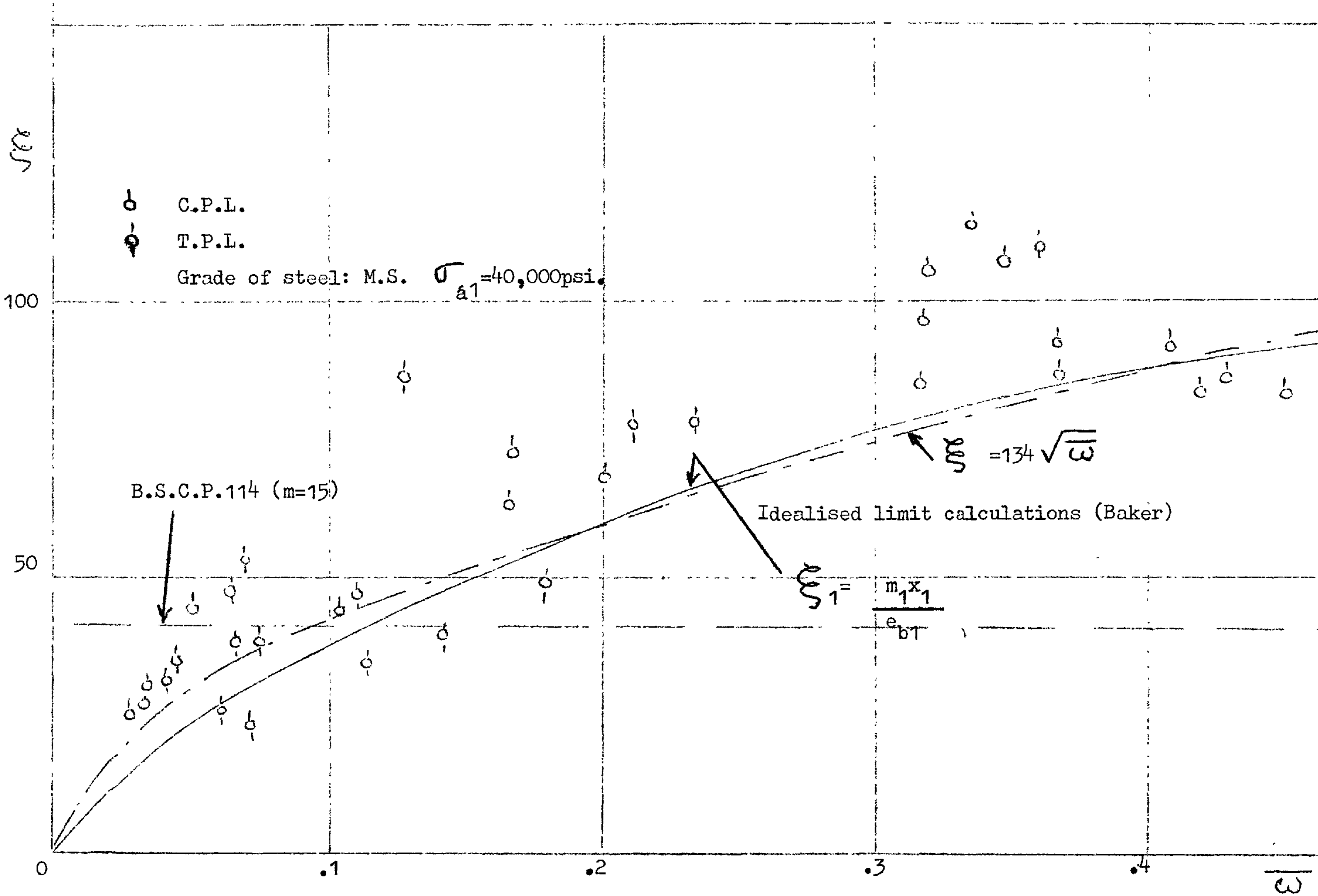



Fig. 6.5. Effective flexural stiffness of reinforced concrete beams

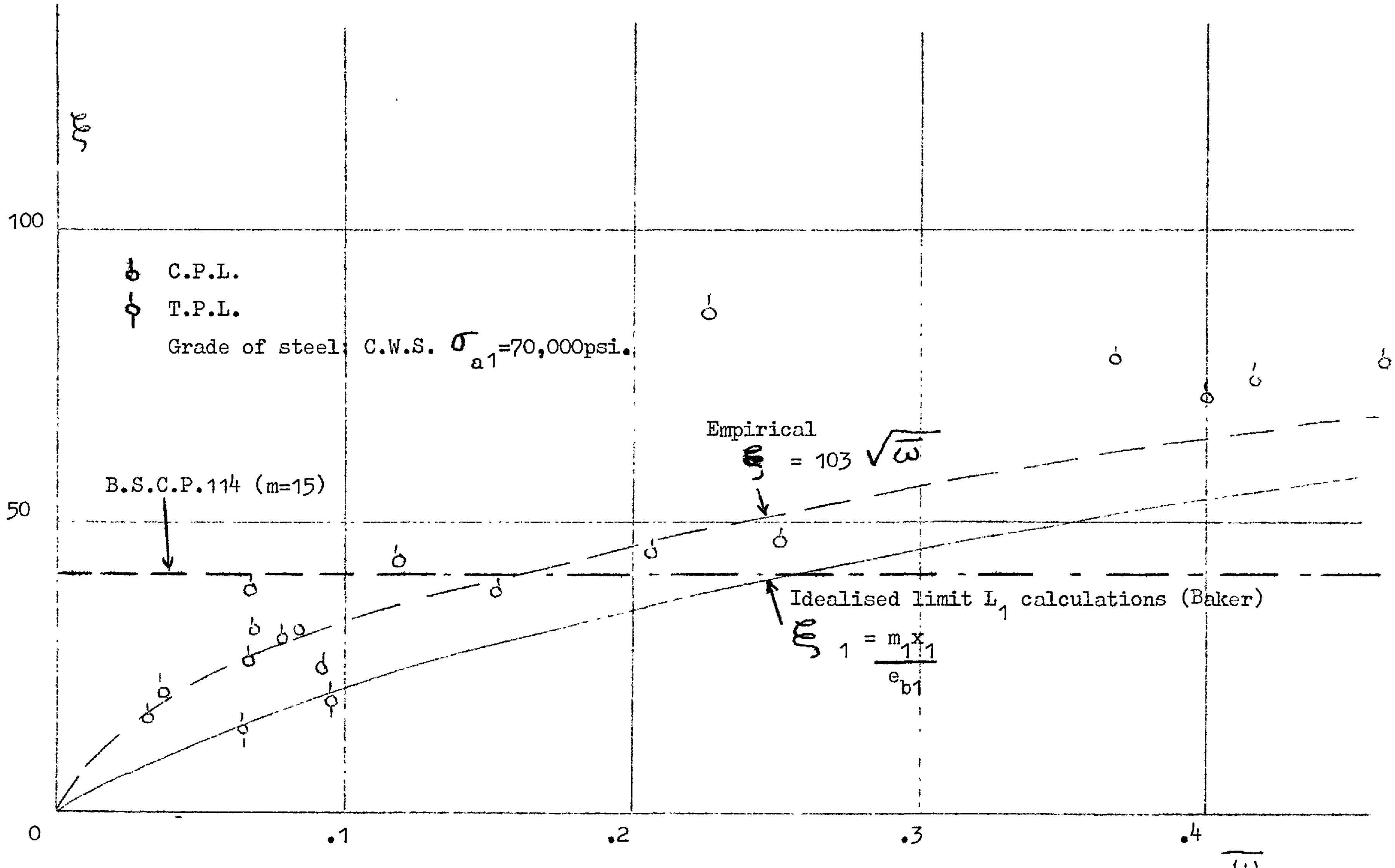



Fig. 6.6. Effective stiffness of reinforced concrete beams

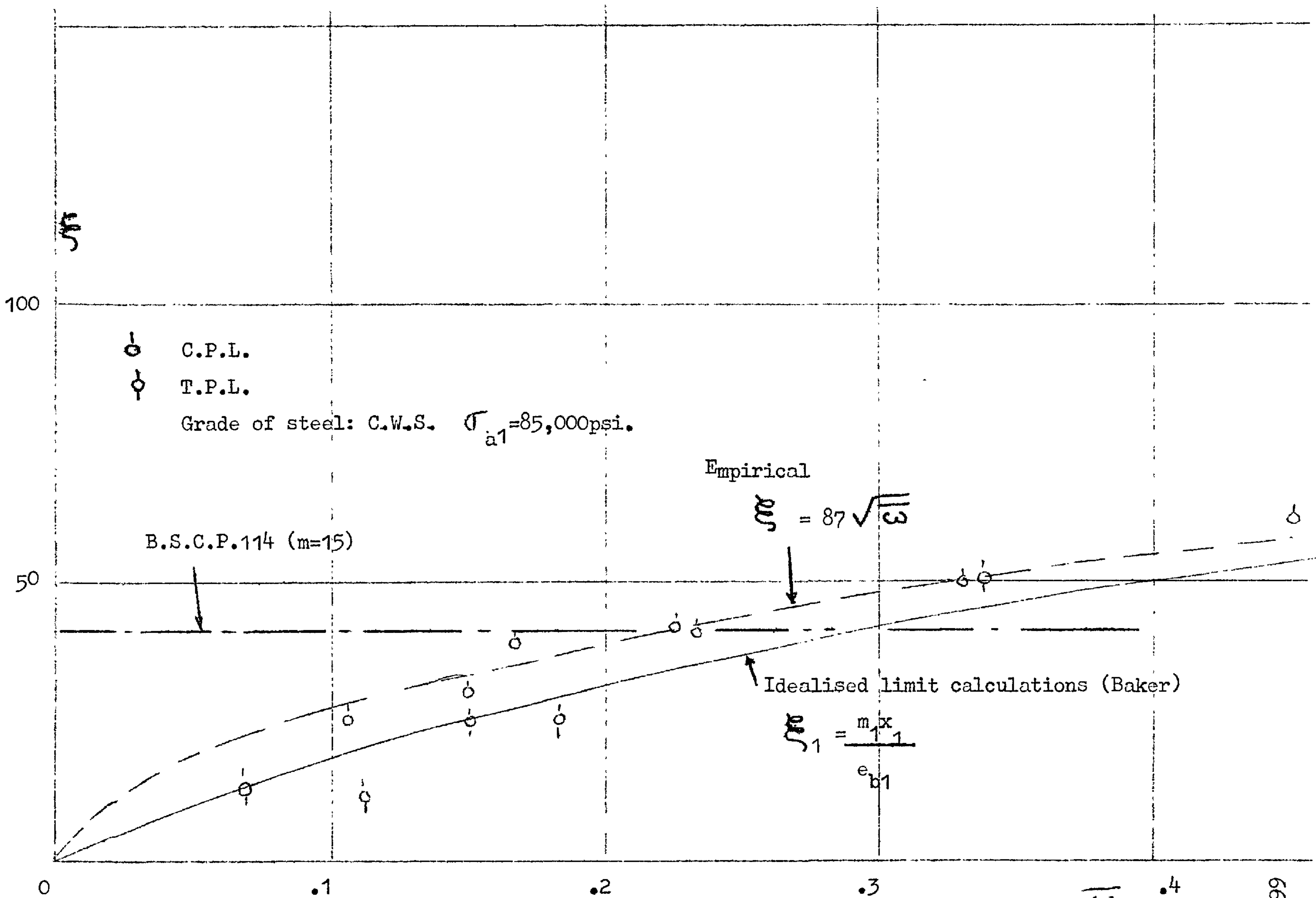





Fig. 6.7. Effective flexural stiffness of reinforced concrete beams

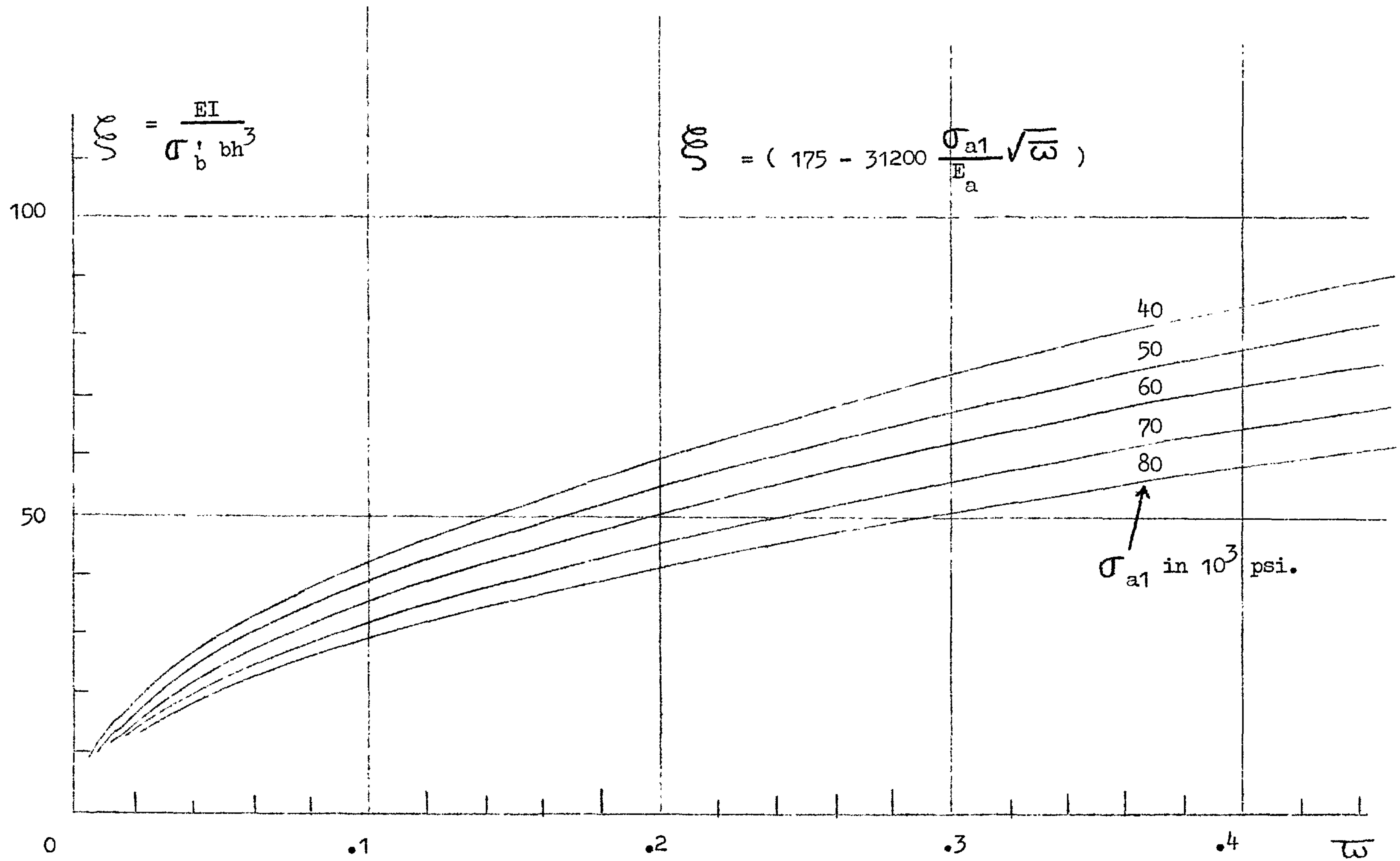



Fig. 6.8(a) Empirical relationship between flexural stiffness of reinforced concrete beams and the tension reinforcement

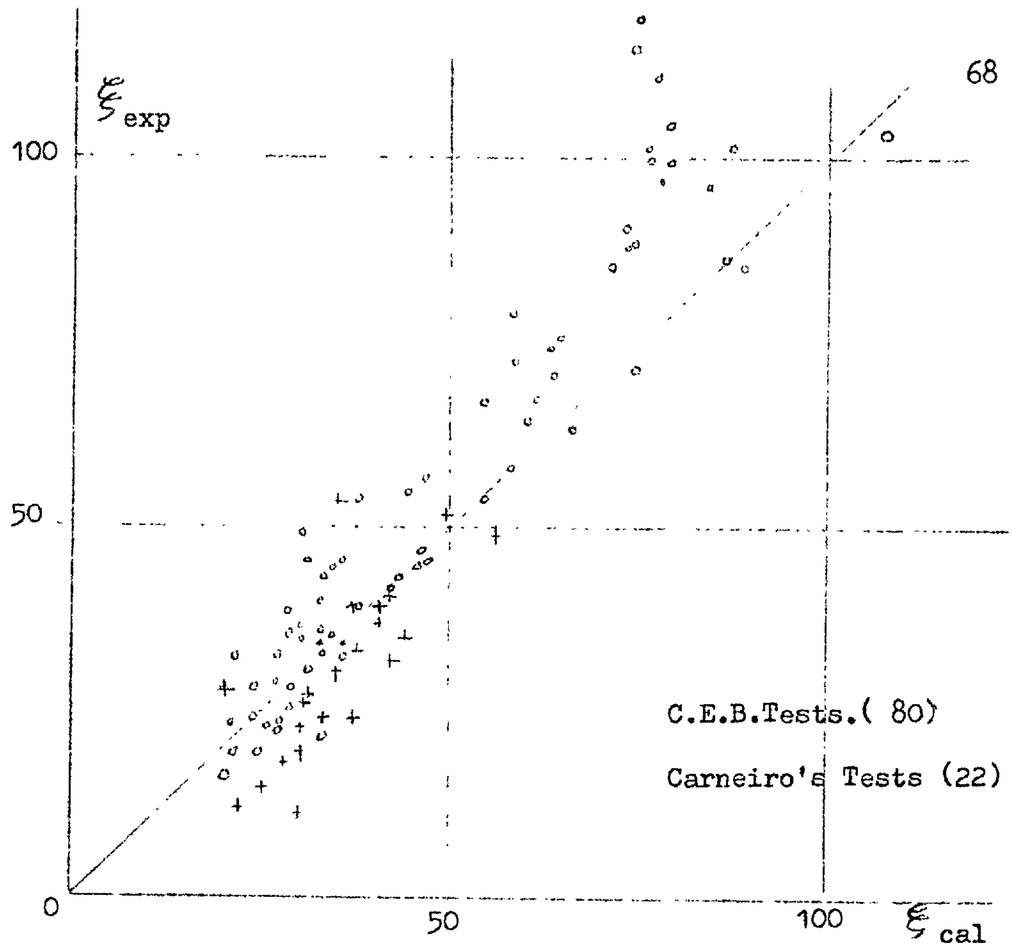



Fig. 6.8. Flexural Stiffness of Reinforced Concrete Beams

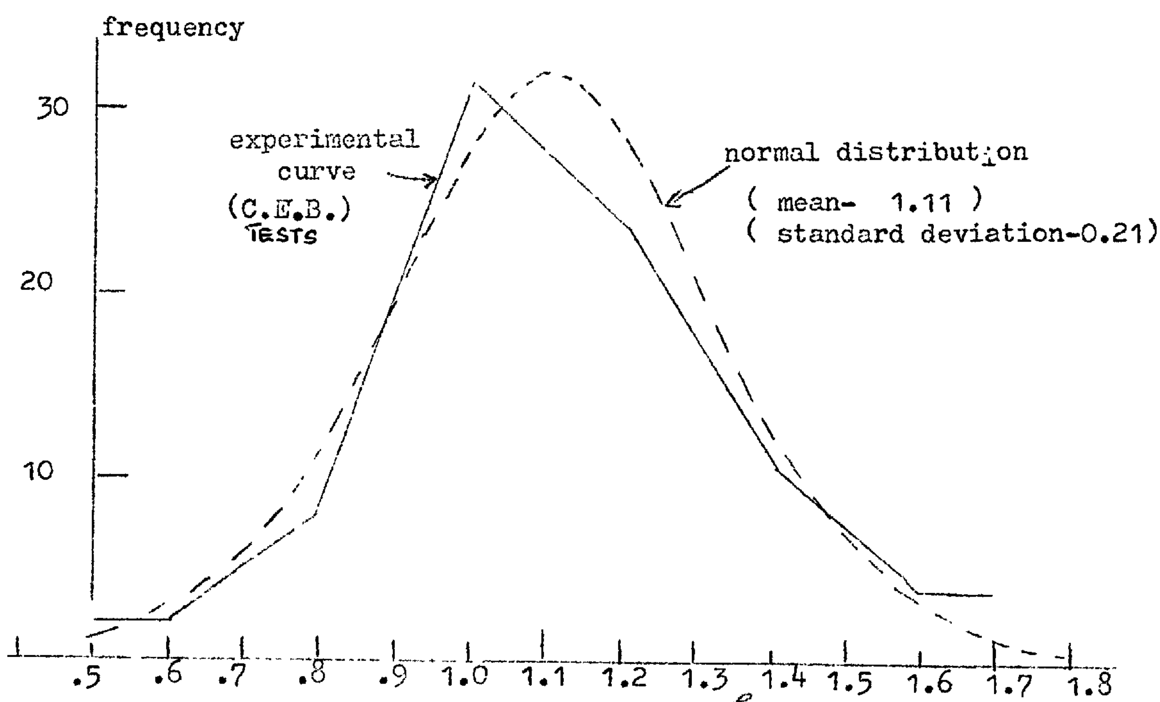



Fig. 6.9. Distribution of $\frac{\xi_{exp}}{\xi_{cal}}$.

## 6.5 Flexural stiffness of Columns

In considering the relative EI values between beams and columns, Baker has pointed out that the EI Values of columns must also be determined from the limit properties as in equation 6.1.

The values of $M_1$, $e_{b_1}$, for columns at limit $L_1$ could easily be determined in terms of the neutral axis depth $x_1$ as for the strength calculations. Fig. 6.10 shows a plot of EI for columns against e/h where e is the effective eccentricity at the critical section.

If $\xi_{col}$ represents the flexural stiffness factor for a column under axial load and bending and if $\xi_b$ refers to the extreme case of $\xi_{col}$ when $e/h \rightarrow \infty$, i.e. the column with no axial load and failing in fluxure, the ratio $\xi_{col}/\xi_b$ gives a measure of the increase in the fluxural stiffness due to the axial load. Using the previous calculations based on limit $L_1$, $\xi_{col}/\xi_b$ is plotted against $\overline{\omega}$ and e/h in Figs. 6.11 and 6.12. These show that $\xi_{col}/\xi_b$ as calculated do not change appreciably over the whole range of $\overline{\omega}$ and e/h considered, the mean value being 1.51. Thus based on Baker's limit $L_1$ assumptions for columns, the EI for columns with equal tension and compression reinforcement may be regarded as 1.51 times that for beams with the same amount of tension reinforcement. However it may be expected that with larger axial load, the degree of cracking in columns would be reduced and the effective stiffness would be appreciably increased. This aspect cannot be taken into account in the idealised limit calculations as the cracked length could not be well defined. An empirical approach to a closer approximate value of the column stiffness may be preferred under these conditions.

The mean effective EI for 26 short column tests reported in the CEB tests[9] and four tests by Soliman[28], which were obtained from moment-rotation results in the same way as described for beams are plotted against the axial load shown in Fig.6.13. The results clearly indicate that $\xi_{col}$ increases with the axial load and the test results could be expressed by the empirical equation 6.11.

$$\xi_{col} = (1 + 1.8\, n_u)\, \xi_b \qquad \ldots (6.11)$$

The axial load $n$ varies from zero to about 1.2 depending on the degree of reinforcement and the ratio of the axial load to moment. Hence $\xi_{col}$ may vary from $\xi_b$ to about $3\xi_b$. The value predicted by the idealised limit $L_1$ calculations is a mean of these variations.

Substituting for $\xi_b$, from equation 6.10 the mean effective flexural stiffness of columns may then be written as

$$\xi_{col} = (1 + 1.8\, n_u)(175 + 31200\, e_{ay})\sqrt{\omega} \qquad \ldots 6.12$$

The expression 6.12 for the stiffness of columns has the same advantage as the corresponding expression of beams as it depends only on the primary variables assumed in the design. The charts given in Fig.6.8(a) could be used to determine $\xi_{col}$ as $n_u$ in the column is known separately.

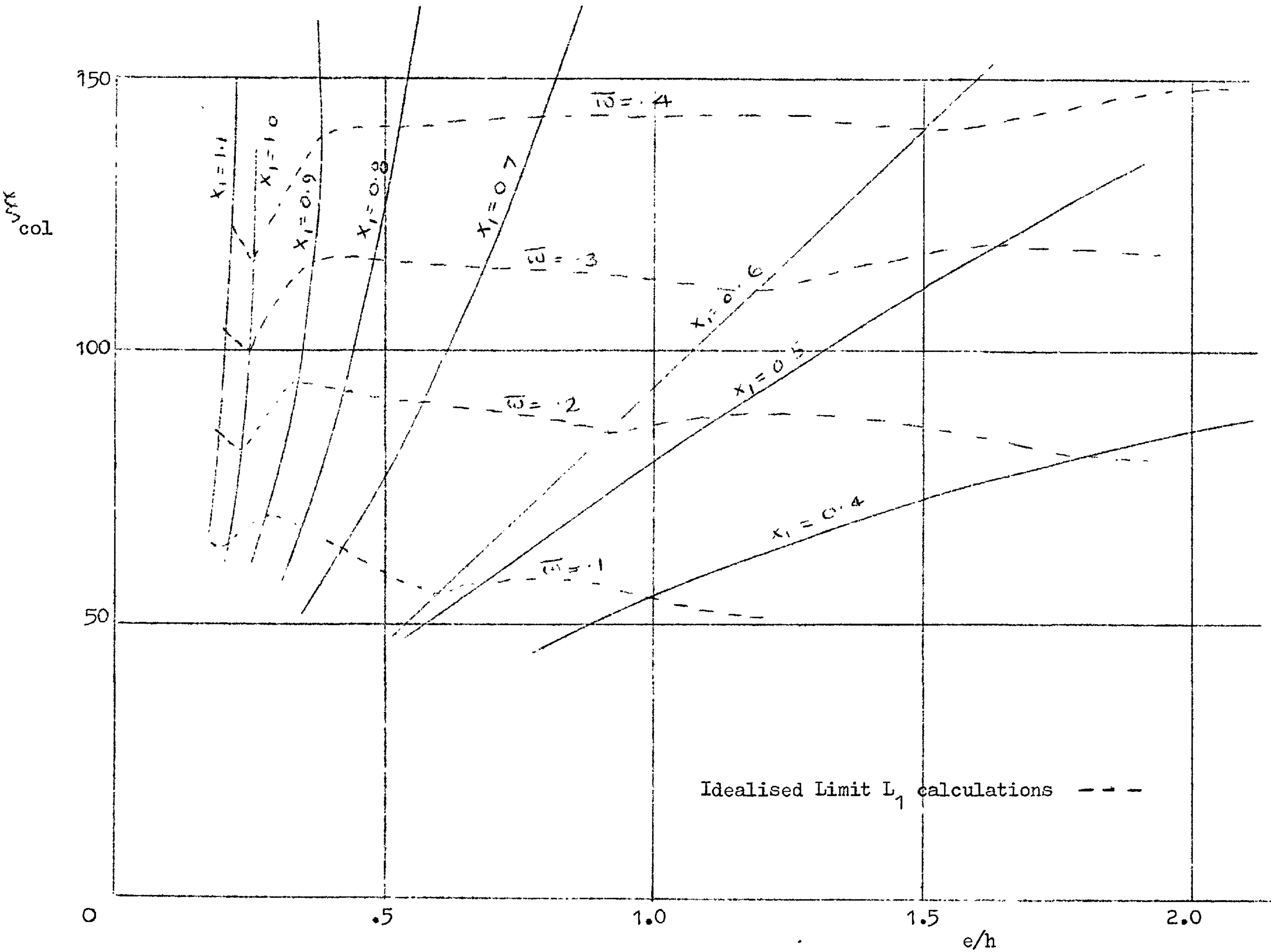



Fig. 6.10. Limit Calculations for the Flexural Stiffness of Reinforced concrete columns

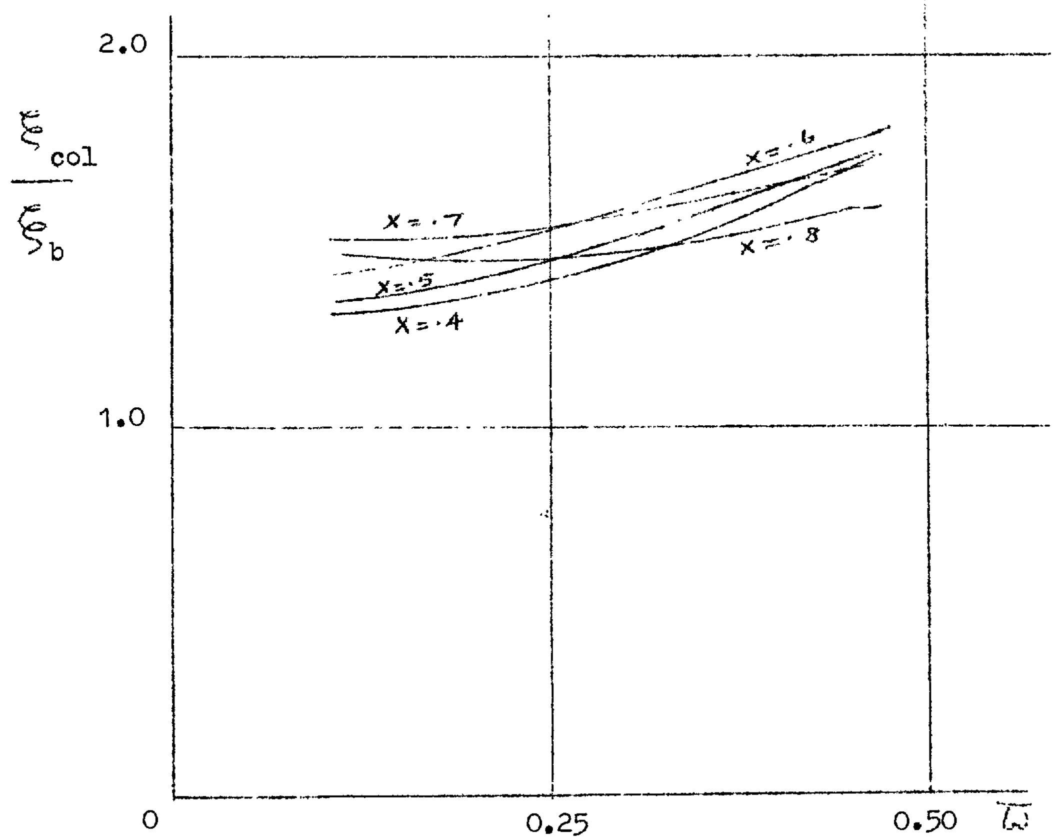



Fig. 6.11

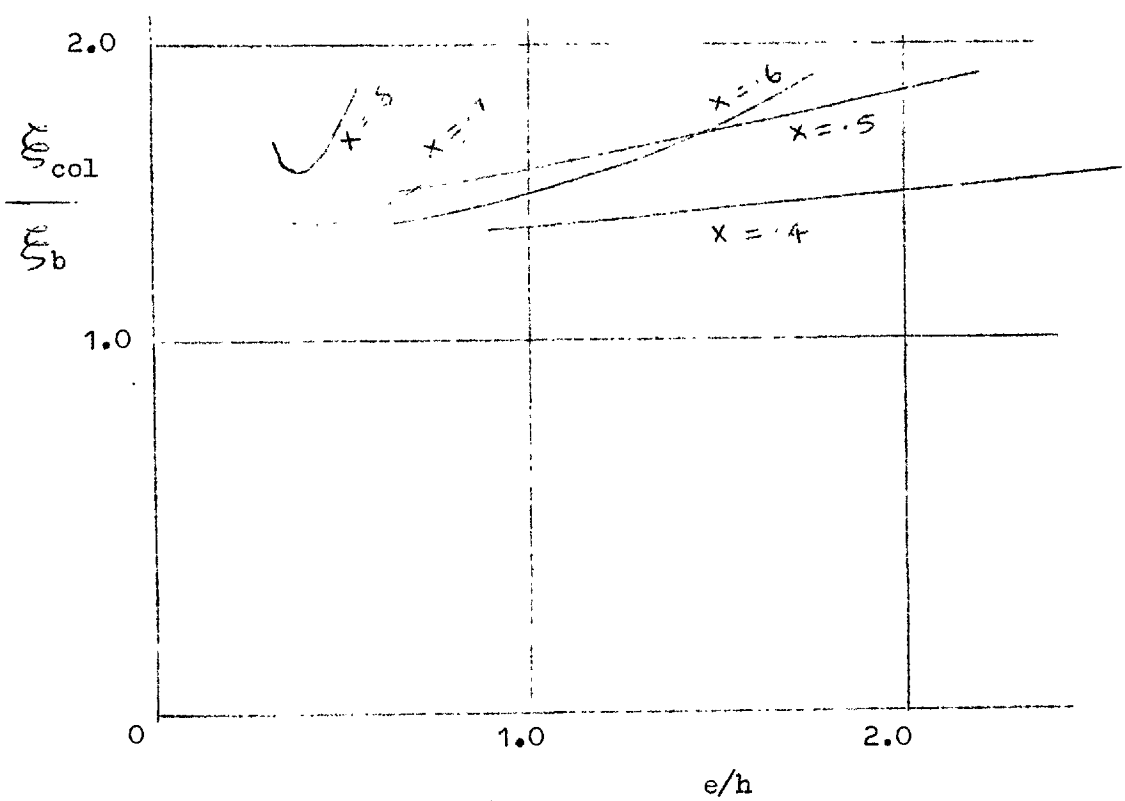



Fig. 6.12.

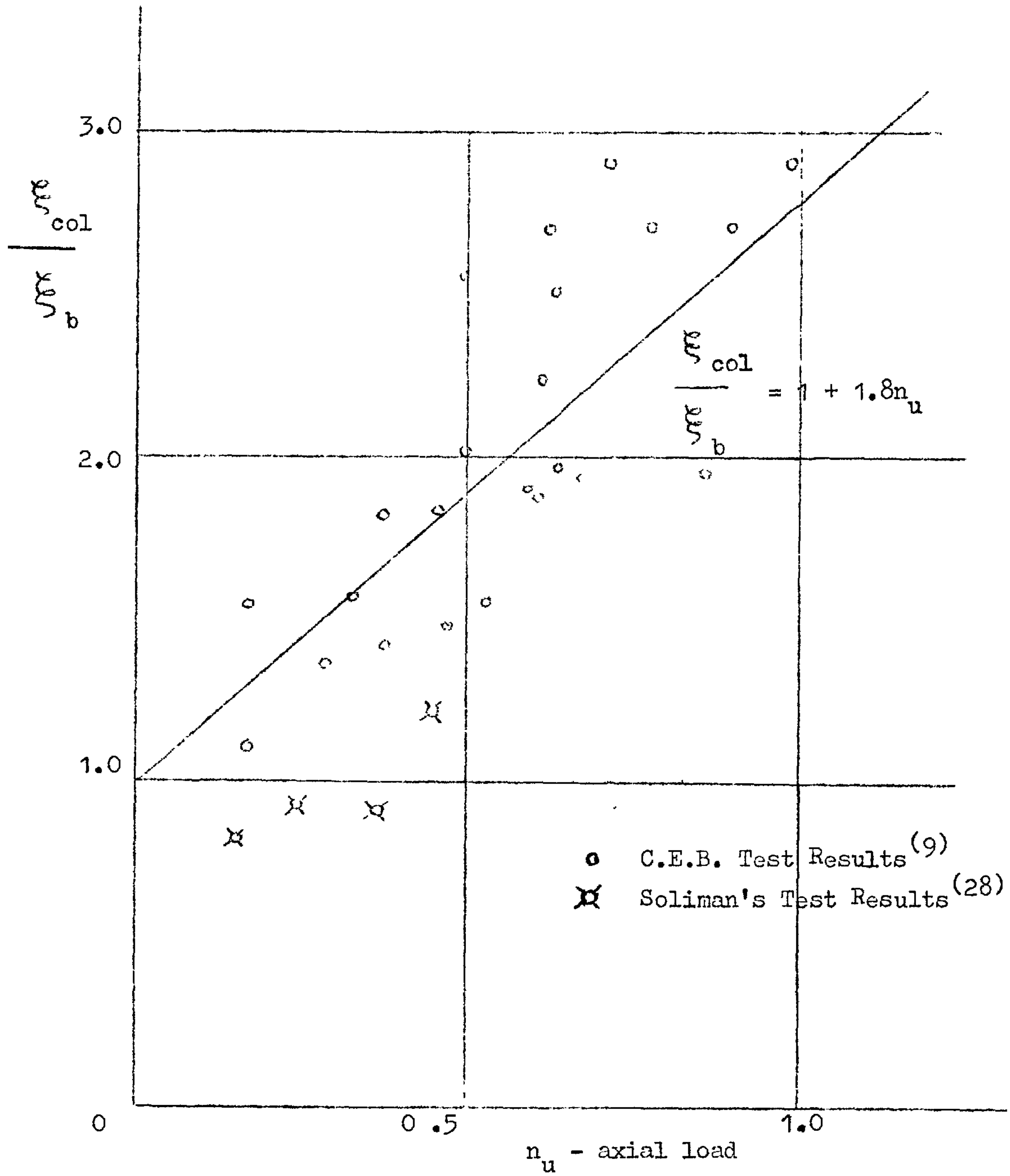



Fig. 6.13. Variation of Flexural Stiffness of Reinforce concrete columns with axial load

# CHAPTER 7.

## Inelastic rotation capacity of reinforced concrete members.

### 7.1. Plastic rotation.

Experimentally it has been observed that the plastic rotation, that a reinforced concrete member may undergo without effectively reducing the carrying capacity, is of the same order as the elastic rotation[9]. The limit of the plastic rotation is primarily controlled by the ultimate strain in the concrete as discussed in Chapter 4, or in extremely under-reinforced members where the reinforcement may have brittle characteristics, the strain capacity of reinforcement itself may determine the maximum plastic rotation. These limits are referred to as Limit $L_2$[5].

Extensive experimental observations[5, 8, 23, 31, 34] on simple reinforced concrete members, show that the plastic rotation capacity ($\theta_p$) is subject to large fluctuations, even when the members are tested under similar conditions. Hence, in calculations involving plastic rotations, it would only be possible to use approximate values of $\theta_p$, which may be considered to be safe values as compared to experimental results.

In 1956, Baker[5] has suggested the following empirical formulae for $\theta_p$,

Tension hinges i.e. under-reinforced beam hinges

$$\theta_p = \frac{0.01}{X_2}$$

<u>Compression hinges</u> i.e. column hinges or over-reinforced beam hinges

$\theta_p$ = 0.01 for well bound sections.

$\theta_p$ = 0.001 for unbound sections.

A more comprehensive formula for $\theta_p$, which incorporates the influence of section parameters is given by[20]

$$\theta_p = k_1 k_2 k_3 \,(0.0035 - e_{b1}) \left(\frac{z}{h}\right)^{\frac{1}{4}} \qquad \ldots (7.1)$$

where $k_1$, $k_2$, $k_3$ are factors which take into account the influence of grade of steel, axial load and grade of concrete respectively. z refers to the length of the member between critical section and point of contra-flexure, h is the effective depth of section, and $e_{b1}$ is the concrete strain at limit $L_1$.

The values of $\theta_p$ predicted by equation (7.1) has been previously compared by the author with moment-rotation characteristics for beam and column tests carried out under the European Concrete Committee[9]. The large scatter in the test results and the variation in the ultimate strain in concrete as shown in Chapter 4, show that the main parameters affecting $\theta_p$ are the degree of binding and the depth of concrete in the compression zone at the critical section as expressed by the neutral axis depth. The latter also takes into account the axial load if the hinge section happens to be a column hinge. Thus the author has suggested the simple empirical formula

$$\theta_p = 2.4\,(e_{b2} - e_{b1}) \qquad \ldots (7.2)$$

where $e_{b2}$ is the ultimate strain in concrete as given by equation (4.5), and $e_{b1}$ corresponds to the concrete strain at the limit $L_1$, for which an upper limit of 0.002 may be assumed as a further simplification. Fig.7.1 shows a plot of $\theta_p$ as given by equation (7.2) against $x_2$ for varying amounts of binding.

Recent experimental research by Soliman[28] shows, that the spacing of binders must not be greater than about 12 times the diameter of binders, if they are to be effective in restraining concrete. It has also been shown by Base and Read[61] that helical binders in the compression zone may be more effective, so that even larger rotations than indicated by the above formula may be obtained. Thus it appears that by suitable detailing, particularly in beams, the ductility in reinforced concrete members may be increased as required for design purposes. But in most cases the use of large amounts of binders to increase the ductility must be compared with the actual advantages gained by the extra redistribution, particularly from the point of view of economy.

7.2. Experimental results.

In the application of the plastic rotation capacity of reinforced concrete members as a limit criterion in the design of indeterminate structures, the hinge rotations are obtained as a function of the idealised elastic properties of the members.

Thus the ratio of the plastic rotation to the elastic rotation of the member may be used to enable a comparative estimation of the ductility of the members.

In a simply supported beam, as in Fig.6.4, the elastic rotation $\theta_e$ is given by,

$$\theta_e = \int \frac{Ml}{EI} \, ds$$

substituting for $EI = M_1 x_1 h/e_{b1}$ and assuming that $M_1 \doteqdot M_2$,

$$\theta_e = A \frac{e_{b1}}{x_1} \frac{l}{h} \qquad \ldots (7.3)$$

where A is a dimensionless factor representing the shape of the bending moment diagram, having the value $\frac{1}{2}$ for a single point load, $\frac{2}{3}$ for a third point load or a uniformly distributed load.

Using equation (7.2) for $\theta_p$, the ratio of $\theta_p/\theta_e$ is given by

$$\frac{\theta_p}{\theta_e} = \frac{2.4 x_1 (e_{b2} - e_{b1}) h}{A \, e_{b1} l}$$

$$= D \frac{h}{Al} \qquad \ldots (7.4)$$

$$\text{where } D = 2.4 \, x_1 \left( \frac{e_{b2}}{e_{b1}} - 1 \right) \qquad \ldots (7.5)$$

The factor D gives the ratio of the plastic rotation at the critical section to the elastic rotation of the member in terms of the section parameters. It may be noted that D is independent of the length, effective depth and the shape of the bending moment diagram. Thus this value may be used to compare the test results obtained for beams with varying span to depth ratios and different types of loading. For convenience D is referred to as the Ductility or the Ductility Ratio of the member.

Table 5.2 gives the calculated and experimental values of the ductility ratio for the C.L.B. test results quoted earlier[9]. The results for 58 beam tests are plotted in Fig.7.2. The scatter in the results are as expected, but it may be noted that the calculated values of the ductility ratio are a reasonable safe limit as compared to the experimental results.

The direct application of the ductility ratio in the determination of suitable detailing in continuous beams are discussed in Chapter 7.

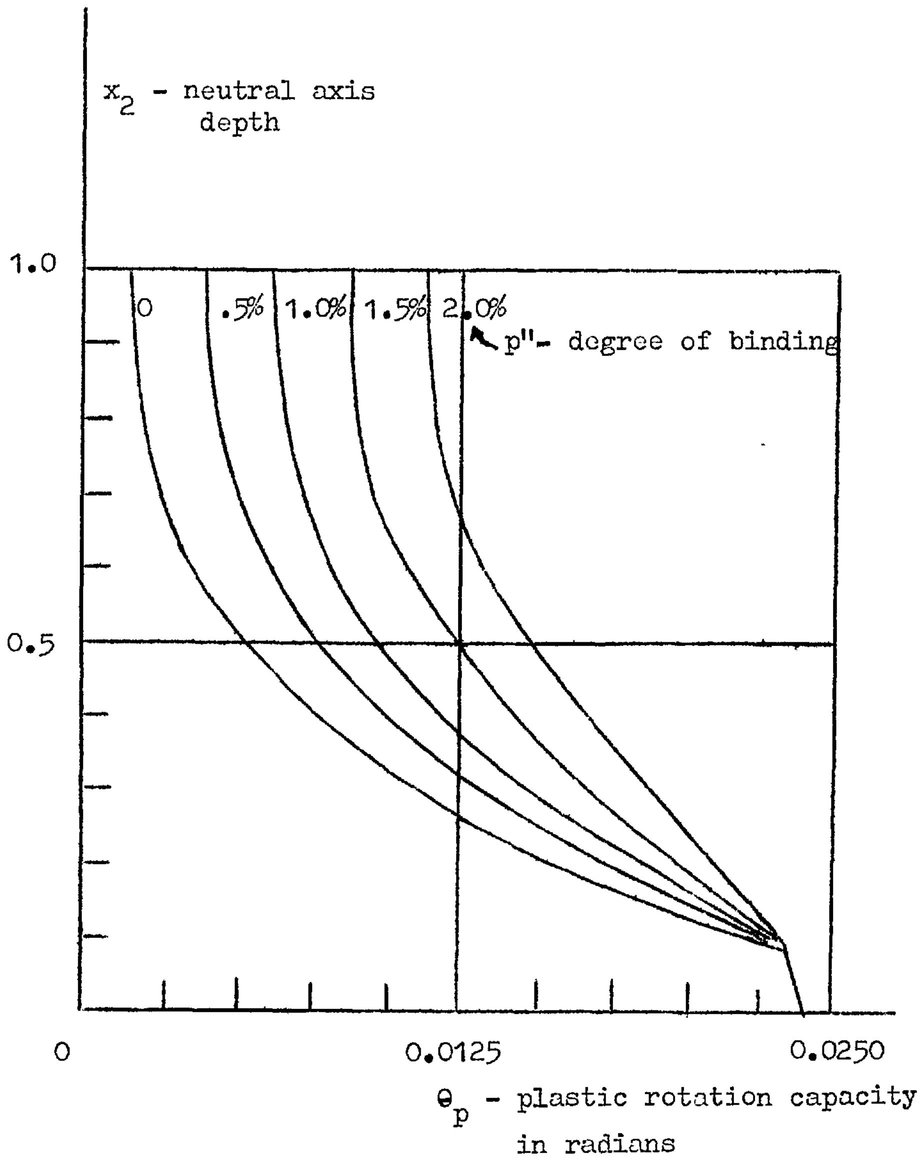



Fig. 7.1. Idealised plastic rotation capacity of reinforced concrete members

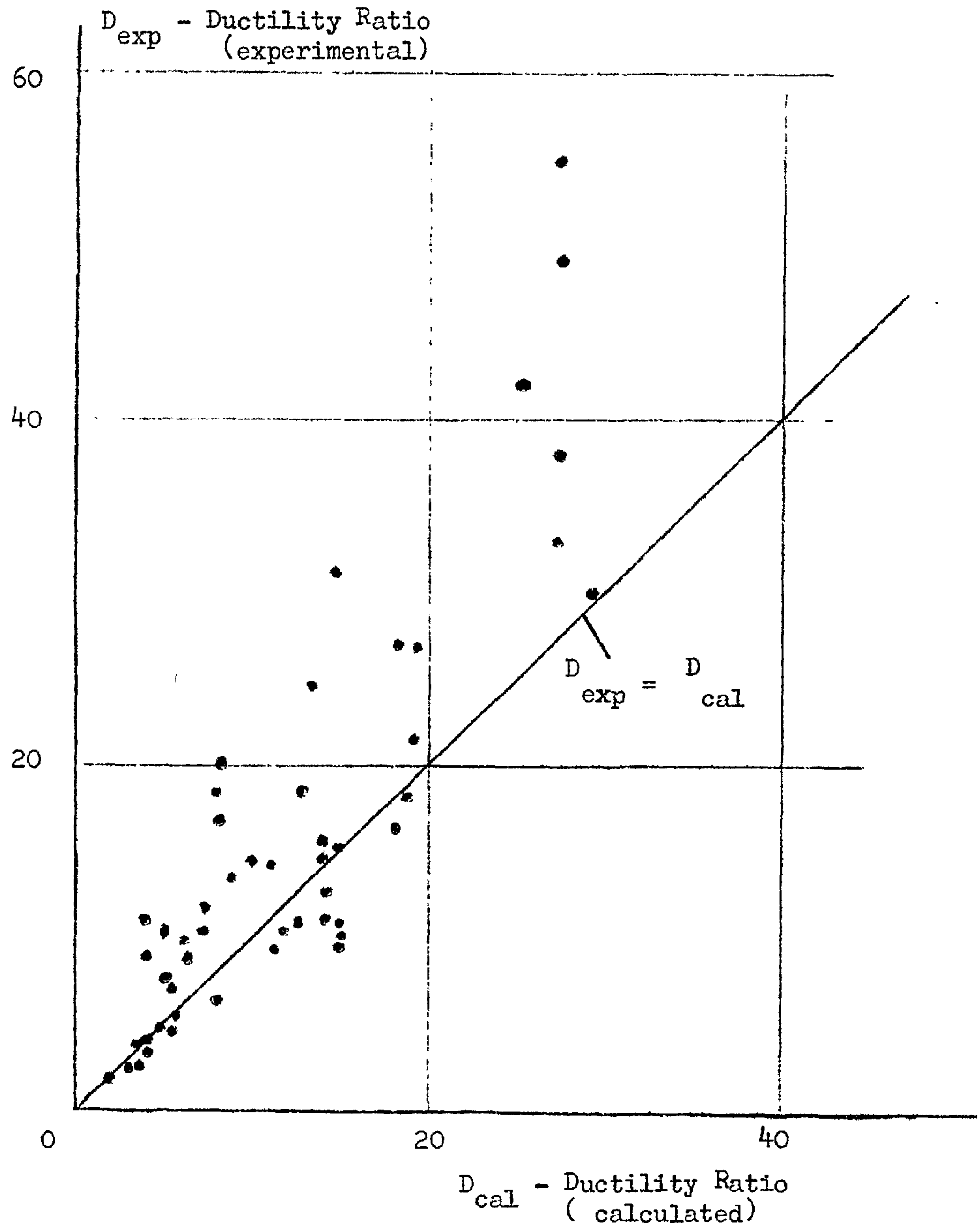



Fig. 7.2. Ductility Ratio for Beams

# PART III

## CHAPTER 8.

## LIMIT DESIGN OF REINFORCED CONCRETE CONTINUOUS BEAMS.

### 8.1. Ultimate load on continuous beams

In the design of continuous beams, the primary mode of failure could be assumed to be due to beam mechanism. Fig 8.1 shows the collapse mechanism in a typical span. Let G and Q be the permanent and super-imposed loads on any span and $\gamma_g$ and $\gamma_q$ be the corresponding over-load coefficients. Then the equilibrium condition in the beam mechanism is given by

$$\frac{(l-x)}{l} M'_n + \frac{x}{l} M'_{n+1} + M_n = F_g \gamma_g Gl + F_q \gamma_q Ql \quad \ldots \qquad (8.1)$$

where $M'_n$ $M'_{n+1}$ are moments at supports n, n+1.

$M_n$ is the moment at span hinge, x is the distance of span hinge from support n, $F_g$, $F_q$ are free bending moment coefficients which depend on the distribution of the loads and the distance x.

Since the beam mechanism in each span is independent of the loads in other spans, the collapse load factor for the whole structure is determined entirely by the weakest span. Given that the super-imposed loads in the separate spans are equally probable either individually or in combination, the configuration of the load corresponding to all spans been loaded with the maximum super-load would incorporate the condition of loading corresponding to the collapse load factor. This may be termed the ultimate load configuration for continuous beams as it forms a special arrangement of a given set of loads having the largest probability of causing collapse of the whole structure or part of it.

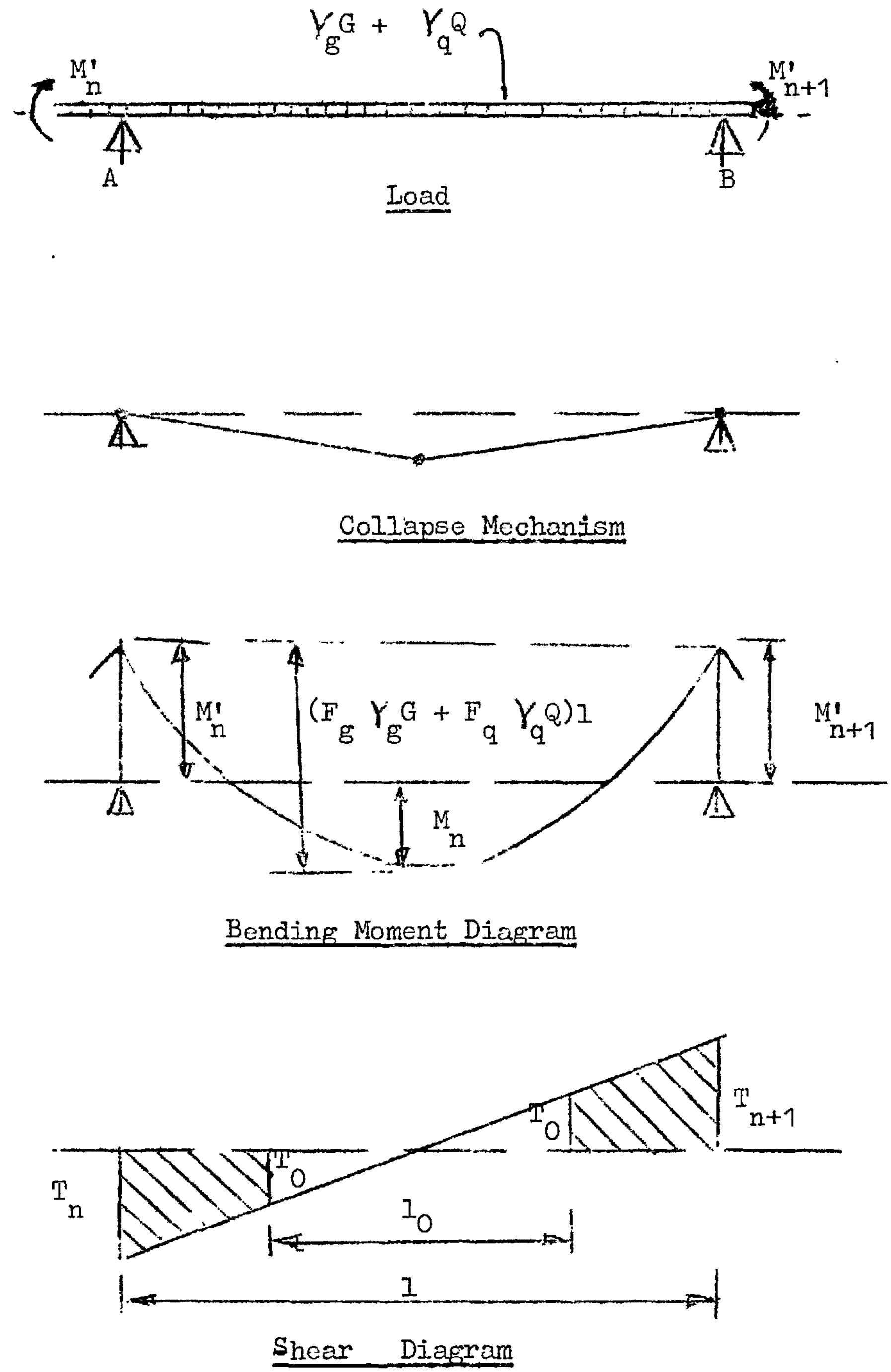



Fig. 8.1 Typical Span in Continuous Beam

( see also chapter 14 on the principle of combined loading). Limit design of continuous beams, and redistribution of moments could now be introduced with respects to the ultimate load as defined above.

## 8.2 Redistribution of moments.

Consider a continuous beam subjected to ultimate load as defined in section 8.1 ( all spans loaded). For equilibrium in each span, any arbitrary distribution of moments that satisfies equation (8.1) could be assumed with the single provision that the moments at hinges at sections n, n', n'+1 are compatible with respect to their rotation i.e. the rotations $\theta_n$, $\theta'_{n+1}$ must be of the same sign in regard to the moments $M_n$, $M'_n$, $M'_{n+1}$. Suppose the special solution that reduces the hinge rotations $\theta_n$, $\theta'_n$, $\theta'_{n+1}$ to zero be represented by the additional suffix e; then the degree of redistribution R at support n' may be defined by (8.2)

$$R = \frac{M_{ne'} - M_{n'}}{M'_{ne}} \quad . \; . \; . \qquad (8.2)$$

Now $M'_{ne}$ could be given in the form

$$M'_{ne} = \beta g \, \gamma g \, Gl + \beta q \, \gamma q \, Ql \quad . \; . \; . \qquad (8.3)$$

where $\beta g$, $\beta q$ are elastic coefficients corresponding to the distribution of the load, which could be easily obtained from any standard hand book. Then the least value of $M'_n$ is determined by the maximum value of R.

The above definition of the degree of redistribution of moments although similar in concept to the general ideas of redistribution as presently used in design practice, differs fundamentally from them in that it is applied to a special loading configuration as defined by the ultimate load. Thus R depends only on the elastic properties of the beam and the plastic moment capacity. This enables the degree of redistribution to be used as a general parameter which could however be related to other loading configurations such as those that cause unserviceability at critical sections during working load.

## 8.3. Secondary modes of failure

There are other modes of failure in continuous beams not incorporated in the definition of collapse discussed in the earlier sections. They may be due to (4) shear failure (2) bond failure in the reinforcement (3) excessive rotation at plastic hinges (4) buckling of compression reinforcement particularly during plastic stages as the concrete may start crushing (5) lateral buckling of beam.

In beams most of the above requirements could be catered for by suitable detailing and choice of size of members. Safeguards against shear failure and excessive plastic rotation have been discussed in greater detail in subsequent sections. The other modes of failure may be prevented by empirical rules for detailing as in the present codes of practice.

## 8.4 Limit design of continuous beams.

The general requirements in the limit design of reinforced concrete structures have been discussed in chapter 3. As a particular group of structures, the design of continuous beams would be based on the following:-

(1) Safety requirements expressed in terms of the load carrying capacity with special consideration for inelastic

compatibility and limiting shear at ultimate load.

(2) Serviceability at working load expressed in terms of (a) deflections, (b) crack width and (c) yield safety.

Each of the above limits are related to the material properties and the load configuration that produces the critical limit conditions, which in general have to be evaluated separately. Since these conditions do not give rise to a unique solution, an optimising criteria based on the total cost of the structure may be used to obtain an ideal limit design.

In the next three chapters an attempt is made to evaluate the relationship between the degree of redistribution R and the following criteria in limit design.

1. Economic criterian based on an idealised concept of total cost.

2. Inelastic compatibility at ultimate load based on the bilinear moment rotation characteristics discussed in Part II.

3. Serviceability of structure under working load based on yield safety and permissible crack width.

The application of limiting deflection and permissible shear stress to continuous beams have been discussed in chapter 11 and 12.

# CHAPTER 9

## Criteria of Economic Design of Reinforced Concrete continuous beams

### 9.1 Economic design.

The economic aspect of reinforced concrete design has been given very little consideration so far, although it cannot be denied that it is one of the most important factors to be considered in a satisfactory design. Under the assumption of brittle failure in the classical elastic theory, the actual design procedure consisted of determining the worst stress condition at critical sections, and then designing each section as economically as possible. The criterion for economic design of reinforced concrete sections is generally interpreted as balanced design with respect to steel and concrete stresses.

The discussion of the elasto-plastic properties of reinforced concrete members in the preceding sections show that moments in the structure oould be more equitably distributed provided that limits on the hinge rotations are not exceeded. This was used as a basis of economic design by Baker (ref 20) in suggesting, that the support moments may be made equal to the span moments under oonditions of ultimate load; thus producing an "eoonomic distribution of moment".

However in practical terms the economy of the structure must be related to the total cost of design and construction.

This would depend on Volume of material, cost of fabrication, formwork, labour rates and many other factors which may even vary for individual cases depending on the circumstances. The volume of concrete in itself is dependent on the sizes of members which are subject to architectual and other requirements. However in most cases the nett economy is not appreciably affected by the size of members, as the reduction in size is always accompanied by increase in the total quantity of reinforcement, while the cost of shutters and labour involved in placing the concrete remains almost unaffected. On the other hand, the cost of fabrication of reinforcement is quite large. An increase in the volume of reinforcement is also accompanied by an increase in the labour required in placing and compaction of concrete due to larger conjestion of reinforcement. A process of ultimate load design by minimising the amount of shear and tension steel at critical sections has been outlined by Peredy and Vizy (ref 40), Kalinsky (ref 41) has extended the above method to obtain a theoretical solution for beams and slabs based on the total volume of steel, which has been expressed by an approximate quadratic function. The results show that the economic solution for a continuous beam of given external dimensions and requiring the minimum quantity of reinforcement is the same as a "special elastic solution" in which the shear and flexural rigidities are expressed by special terms.

The following investigation into the economic design of continuous beams as a particular aspect of limit design is based on the following assumption:

(1) Ultimate load as the unique conditon of load at collapse as shown in chapter 8

(2) Uniform size of members.

(3) Limit properties of reinforced concrete as discussed in Part II.

(4) Least total volume of reinforcement as the criterion of minimum cost.

## 9.2 Volume of shear reinforcement.

Let T be the shear force at any section in a continuous beam. Then the shear stress s at the section, may be given in the form (ref 42)

$$s = \frac{T}{(1 - \gamma_x)bh} \qquad \ldots \quad (9.1)$$

where $1 - \gamma_x$ is the lever arm factor at the section.

If $\sigma^*_{bt}$ be the maximum shear stress permissible in plain concrete, then no shear reinforcement is required if $s < \sigma^*_{bt}$. When $s \geq \sigma^*_{bt}$ we may assume that the totäl shear force is taken by the shear reinforcement. Then any span in a continuous beam may be divided into distinct zones where shear reinforcement is required and where it is not required as shown in Fig. 8.1.

In general, shear is catered for either by stirrups or by bent up bars. However expressed in terms of the equivalent area of vertical stirrups the area of shear reinforcement per unit length is given by,

$$a_t = \frac{T}{\sigma_{ay}} \quad \ldots \quad (9.2)$$

where $\sigma_{ay}$ is the yield stress of shear reinforcement. The volume of shear reinforcement over a length dl assuming the stirrups are vertical bars of length h is given by $dV_t$ where,

$$dV_t = \frac{Th}{\sigma_{ay}} dl \quad \ldots \quad (9.3)$$

Then the total volume of shear reinforcement in the span is given by

$$V_t = \int_{s \geq \sigma^*_{bt}} \frac{Th}{\sigma_{ay}} dl \quad \ldots \quad (9.4)$$

9.1 <u>Typical Values of stress block parameters</u>

| $\bar{\omega}$ | $x_1$ | $\gamma$ | $1-\gamma x_2$ | $1-\gamma x_2$ |
|---|---|---|---|---|
| 0.05 | .25 | .327 | .918 | 1.22 |
| 0.10 | .33 | .330 | .891 | 1.34 |
| 0.15 | .40 | .335 | .866 | 1.44 |
| 0.20 | .45 | .350 | .845 | 1.54 |
| 0.25 | .49 | .355 | .826 | 1.62 |
| 0.30 | .53 | .360 | .809 | 1.68 |

Table 9.1 gives the variation in $1-\gamma_x$ with the amount of reinforcement. Over the practical range of $\bar{\omega}$ and allowing for cut off in tension reinforcement upto 2/3 of maximum value the variation in the lever arm is less than 10%. Thus assuming all stirrups are at its yield limit, $V_t$ is given by the simple function,

$$V_t = \frac{h}{\sigma_{ay}} \int_{s \not> \sigma^*_{bt}} T \; dl$$

$$= \frac{h}{\sigma_{ay}} \left[ \; ( W - W_o ) \; \right] \qquad \ldots 9.5$$

where W is total load on span , and

$W_o$ is the load on length $l_o$ over which no shear reinforcement is required.

For uniformly distributed permanent and super imposed load W and $W_o$ may be easily worked out.

$$W = (\gamma g \; G + \gamma q \; Q)$$

$$l_o = \frac{2 \; (1-\gamma x) \sigma^*_{bt} . bh \; l}{\gamma g \; G + \gamma q \; Q}$$

$$W_o = (\gamma g \; G + \gamma q \; Q) \; \frac{l_o}{l}$$

$$= 2 \; (1-\gamma x) \sigma^*_{bt} . \; bh$$

Hence, $$V_t = \frac{(\gamma g \; G + \gamma q \; Q) \; h}{\sigma_{ay}} - \frac{2 \sigma^*_{bt} (1-\gamma_x) bh^2}{\sigma_{ay}} \qquad \ldots (9.6)$$

Equation (9.6) shows that the total volume of shear reinforcement is independent of the bending moment at either end of the span as lo is independent of the end shears. $V_t$ depends only on b and h which in most beams could only be varied with in narrow limits. Thus for design purposes the volume of shear reinforcement remains an invariant for all arbitrary distributions of the support moment.

## 9.3 Volume of tension reinforcement.

Consider an intermediate span AB as in Fig. 8.1. If $M'_n$, $M'_{n+1}$ and $M_n$ are the limit moments at ultimate load, then equilibrium condition is given by equation (8.1). Ideally if the beam is reinforced so as to fully utilise the tension reinforcement the stress in reinforcement must equal yield value at all sections. In practice a close approximation may be made by cutting off the tension reinforcement at as many sections as possible. Let $A_a$ be the area of tension reinforcement at any section. Then the volume of tension reinforcement over a length dl is given by,

$$dv = A_a \, dl,$$

$$= \bar{\omega} . \frac{\sigma_b}{\sigma_{ay}} . \, bh \, dl \qquad \ldots \quad (9.7)$$

However in terms of the limit properties of beams as discussed in part II the energy due to bending in the length dl could be expressed as,

$$dU = \frac{M^2}{EI} \, dl \qquad \ldots (9.8)$$

where EI and M are given by $EI = \frac{M\,(1-x_1)\,h}{e_{ay}}$.

$$M = \bar{\omega}\,(1 - \gamma x_2)\,\sigma_b^* \, bh^2$$

Substituting for M, and EI in (9.8), and replacing $\bar{\omega}$ in terms of dv from 9.7,

$$dU = \frac{1 - \gamma x_1}{1 - x_2} \, \frac{e_{ay}}{\sigma_{ay}} \cdot \frac{\sigma_b^*}{E_a} \, bh . \, dV$$

$$= \frac{(1 - \gamma x_1)}{1 - x_2} \, \frac{\sigma_b^*}{Ea} \, bh \, dV \qquad \ldots (9.9)$$

Table (9.1) shows that for a variation of $\bar{\omega}$ by a factor of 2,(a reasonable amount for cut off) the stress block parameter $\frac{1 - \gamma x_1}{1 - x_2}$ varies only by about 10%. Hence it appears that this factor may be regarded as a constant for a particular span. Then the total energy in the beam due to bending for any arbitrary distribution of bending moment could be expressed as the energy in each span (say i) obtained by integrating equation (9.9) and summing it up over the total number of spans (say N) i.e. $U = \sum_N \frac{(1 - x_1)}{(1 - x_2)} \frac{\sigma_b^*}{E_a} bh \int_i dv$

$$= \sum_N \frac{(1 - x_1)}{(1 - x_2)} \frac{\sigma_b^*}{E_a} bh \; V_i$$

$$= \sum_N K_i V_i \qquad \ldots (9.10)$$

where $K_i = \frac{1 - \gamma x_1}{1 - x_2} \frac{\sigma_b^*}{E_a} bh$

which is a constant for each span, and

$V_i$ is the total volume of tension reinforcement in each span.

From the theory of elasticity it is well known that the state of minimum bending energy in a structure corresponds to the unique distribution of moments given by the elastic equilibrium state. In terms of an elasto-plastic analysis this corresponds to the case when the plastic discontinuities at the releases are zero. The correspondence between the volume of tension reinforcement and the total energy due to bending in equation (9.10) implies that the limiting elastic distribution of moments also corresponds to the least volume of tension reinforcement.

Thus under ultimate load conditions, the least total volume of reinforcement (shear and tension reinforcement) is given under the equilibrium state where the plastic hinge rotations are zero i.e. the spans are elastically continuous. The degree of redistribution (R) as defined in section 8.2.

under these conditions is zero. In practice it may be necessary to redistribute moments particularly for convenience in detailing over supports, etc. But such a procedure seems to involve in an increase in the total volume of reinforcement in contrast to currently held assumptions.

# CHAPTER 10

# INELASTIC COMPATIBILITY

## 10.1 Introduction

In the earlier chapter, the optimum conditions that must be satisfied to obtain an economic solution has been discussed. However, this ideal economic solution has been entirely based on the safety requirements at ultimate load. In practice, the difficulties arising out of detailing at support sections, and minimum serviceability requirements may lead to modification of this solution. Under these conditions, moments at supports may be redistributed to mid-span ssctions making use of the yield characteristics of the "plastic hinges" thus formed.

Unlike in steel structures, it has been emphasised (ref: 20) that the actual rotation in the hinges due to the redistribution of moments must be compared with the inelastic rotation capacity so as to prevent a possible reduction in the moment capacity of the hinges. If $\theta$ be the actual rotation in a plastic hinge due to a set of equilibrium forces, and $\theta_p$ be the permissible inelastic rotation for the hinge as defined in chapter 7, then the inelastic compatibility requirement may be expressed by $\theta \leqslant \theta_p$. This inequality in general implies two conditions. (1) $\theta$ must be compatible with the yield moment at the hinge i.e. the hinge opens in the tension side (2) $\theta$ does not exceed the limit inelastic rotation capacity. The term "inelastic compatibility" is used by

Baker to mean an investigation of both of the above properties (ref 18).

Connected with the problem of limit design it may be assumed that $\theta$ and $\theta_p$ are both variable based on the initial assumptions for the plastic moments and section properties. Hence a considerable amount of discretion could be applied by the engineer as in the trial and error method put forward by Baker. (ref.4)
However in continous beams, the problem of inelastic compatibility is enormously reduced, due to the fact that the condition (1) stated above is always satisfied, when the degree of redistribution as defined in chapter 8 is possitive i.e. support moments are less than the elastic moments (see chapter 15).

The condition (2) could be easily ensured by suitably designing the hinge sections so to provide minimum ductility as discussed in chapter 7.

Some attempts have already been made in this direction to determine suitable limits for detailing of sections in specific cases where the maximum degree of redistribution is restricted. The Institution of Civil Engineers Research Committee (ref.20) has suggested that in continuous beams consisting of four or more spans, each span length differing by not more than 15%, and where the live load does not exceed the dead load by more than 50%, the degree of redistribution upto 25% may be permitted, provided that the tension reinforcement satisfies the condition

$$\overline{\omega} \not> \sqrt{\frac{h}{10\,l}} \qquad \ldots\ldots(10.1)$$

In practice this may restrict the maximum value of $\overline{\omega}$ to about 10%. However under the above conditions the Russian code (ref.10) requires that the neutral axis depth at hinge sections be less than 0.55h which allows $\overline{\omega}$ to be as much as about 30 %. In "optimum design of reinforced concrete beams, Cohn[43] has suggested an upper limit of reinforcement corresponding to 2% of the effective section area. Then he derived that the degree of redistribution for moments due to super-load be limited to 20%, 25%, 30% for support sections of beams with 2,3,4 or more spans and with free ends, and 15%, 20% and 25% for support sections of beams with 2,3 and 4 or more spans with fixed ends.

In general the rotation of plastic hinges in continuous beams could be related directly to the degree of redistribution enabling inelastic compatibility to be expressed in terms of a single parameter. The limit requirements under uniformly distributed loads are discussed in detail below. Other types of loads could be treated similarly or could be represented by their equivalent distributed loads.

## 10.2. Particular cases of inelastic compatibility.

### Intermediate span under uniformly distributed load

Consider an intermediate span AB of a continous beam as in Fig 8.1. Assuming that the end conditions at A and B are similar, ~~then~~ the hinge moments $M'_n$, $M'_{n+1}$ at ultimate load could be given by

$$M'_n = M'_{n+1} = (1 - R)\, M'_{ne} \quad \ldots \qquad (10.2)$$

where $M'_{ne}$ corresponds to the fixed end moment given by

$$M'_{ne} = \frac{1}{12} \left( \gamma_g G + \gamma_q Q \right) l \quad \ldots \qquad (10.3)$$

G and Q are the total permanent and super-imposed loads on the span and R is the degree of redistribution.

The inelastic rotation $\theta$ at the hinges is given by

$$\theta = \frac{R\, M'_{ne} \quad l}{2\, EI} \quad \ldots \qquad (10.4)$$

where EI is based on the idealised cracked stiffness of beams given in equation (6.10)

As limit $L_1$ moments in beams are approximately equal to the limit $L_2$ moment, EI could be expressed as

$$EI = (1-R)\, M'_{ne} \cdot \frac{x_1 h}{e_{bl}} \quad \ldots \qquad (10.5)$$

From 10.4, and 10.5, $\theta$ is given by

$$\theta = \frac{R}{1 - R} \; \frac{e_{bl}\; l}{2\, x_1 h} \quad \ldots \qquad (10.6)$$

If $\theta_p$ be the permissible rotation for the hinge the minimum requirement for inelastic compatibility would be satisfied if,

$$\theta_p \geqslant \frac{R}{1 - R} \quad \frac{e_{bl}\; l}{2 x_1 h} \quad \ldots \qquad (10.8)$$

Substituting for $\theta_p$ from equation 7.4 and expressing the inelastic properties in terms of the ductility ratio as defined in chapter 7, equation 10.8 can be reduced to

$$D \geqslant \frac{R}{1-R} \frac{1}{2h} \quad \ldots \quad 10.9$$

Equation 10.9 could be used to determine the maximum permissible redistribution for a limiting value of the ductility ratio or it may be used to determine the minimum value of ductility ratio so as to obtain a required degree of distribution. As the ductility ratio depends only on the section parameters such as percentage of reinforcement, binding ratio, and the grade of steel, the permissible redistribution corresponding to the detailing conditions could be easily determined. Figs 10.1, 10.2, and 10.3 gives the relationship between ductility ratio and percentage of reinforcement for three typical grades of steel.

End spans in a continous beam under uniformly distributed load

The degree of redistribution in the end span of a continuous beam depends on the pen-ultimate support moment which is affected by the adjoining spans. A two-span beam with freely supported ends forms the limiting case of this category. Hence in limit design the inelastic compatibility derived for this case would be considered satisfactory for other end span conditions as well.

As before let $M_n'$ be the support moment, R and $\theta$ be the degree of redistribution and the inelastic rotation at hinge, then the elastic

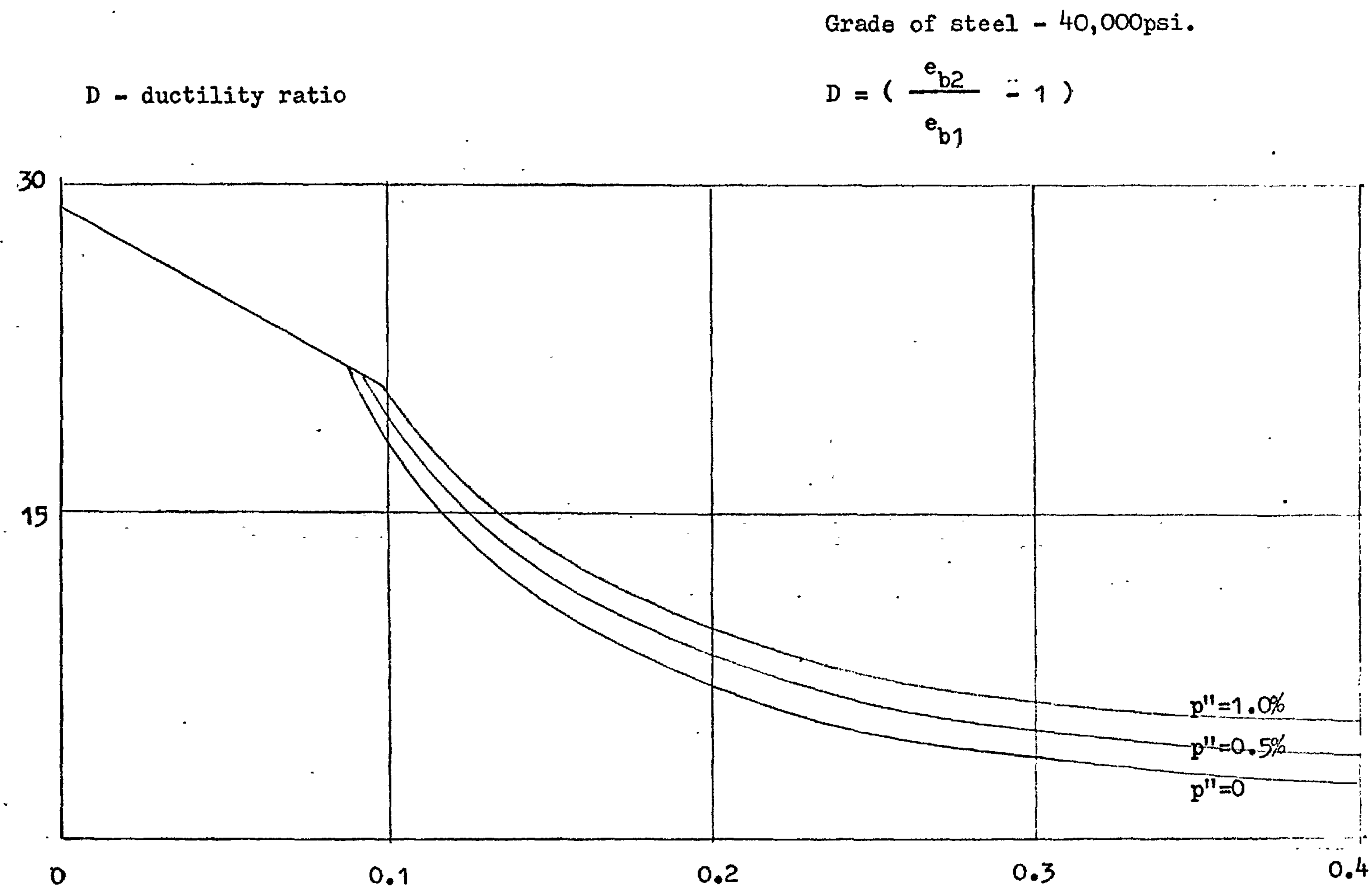



Fig. 10.1 Variation of Ductility Ratio with Reinforcement

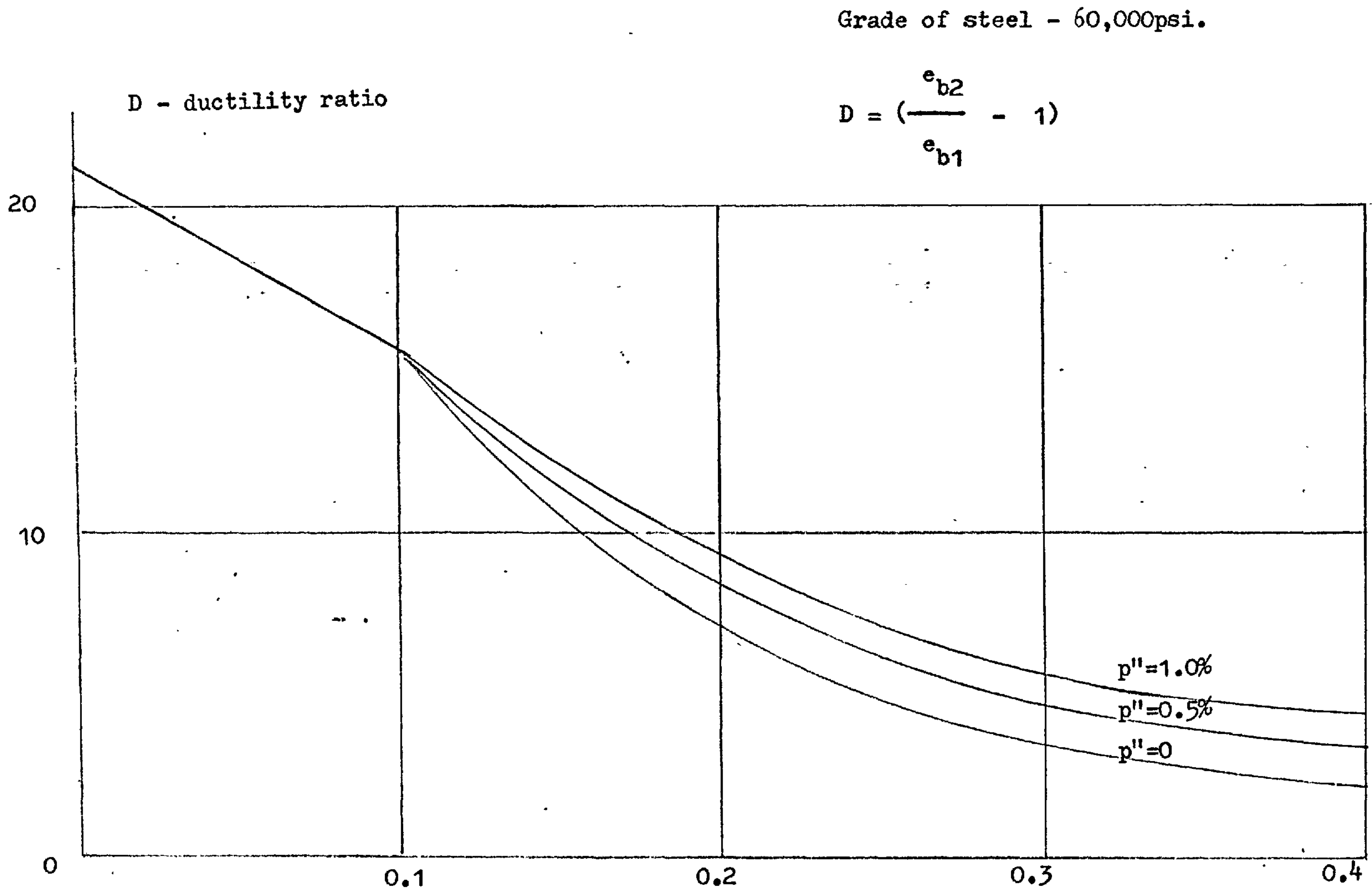



Fig. 10.2 Variation of Ductility Ratio with Reinforcement

$$D = \left( \frac{e_{b2}}{e_{b1}} - 1 \right)$$

Grade of steel - 80,000psi.

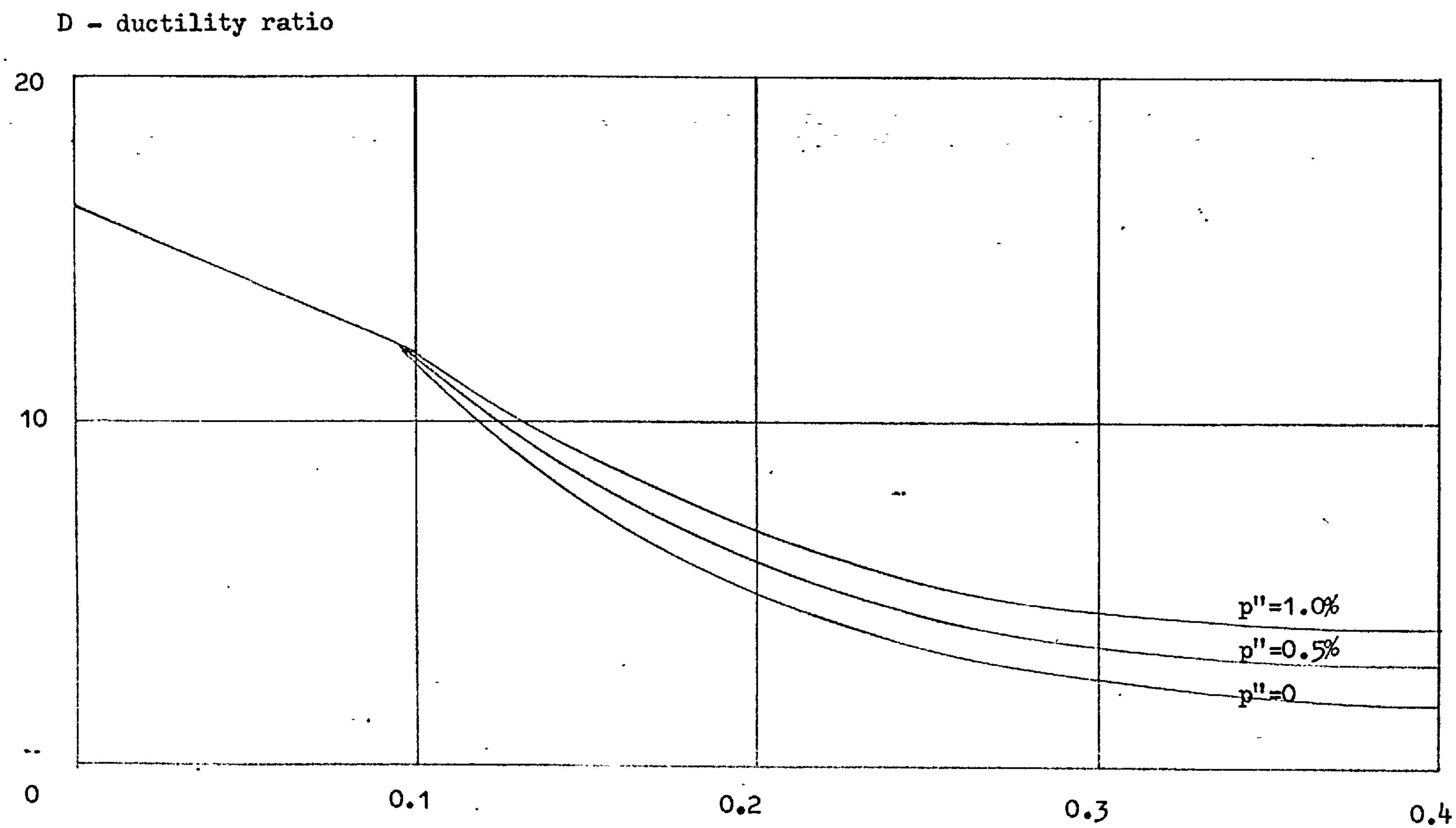



Fig. 10.3 Variation of Ductility Ratio with Reinforcement

moment $M'_{ne}$ is given by,

$$M'_{ne} = \frac{1}{8} \left( \gamma_g G + \gamma_q Q \right) l \qquad \ldots (10.10)$$

The inelastic rotation is given by

$$\theta = \frac{2 R M'_{ne} l}{3 EI} \qquad \ldots (10.11)$$

Substituting for EI, as in 10.5, the minimum requirement for inelastic compatibility is given by

$$\theta_p \geqslant \frac{R}{1 - R} \cdot \frac{2}{3} \cdot \frac{e_{bl}\, l}{x_1 h} \qquad \ldots (10.12)$$

When $\theta_p$ is expressed in terms of ductility ratio, the inequality 10.12 reduces to,

$$D \geqslant \frac{R}{1-R} \cdot \frac{2\, l}{3\, h} \qquad \ldots (10.13)$$

Thus 10.13 defines the limiting case of inelastic Compatibility for end spans and could be used in the same way as 10.9.

The inelastic compatibility conditions given in equations 10.9 and 10.13 are presented in graphical form in Figs 10.4 and 10.5. The minimum ductility ratio corresponding to particular values of span, effective depth and degree of redistribution could be read out as in the illustration. Then the reinforcement detailing may be obtained from the relation between ductility ratio and the percentage reinforcement as given in Figs. 10.1, 10.2, and 10.3.

Limiting cases. - (a) Internal spans

For a maximum redistribution of 25% as suggested by the Institution of Civil Engineers research committee (ref.20) the limiting ductility ratio in intermediate spans is given by

$$D \geqslant \frac{l}{6h}$$

Assuming an extreme value of $^{l}/_{h}$ = 30, the maximum ductility required under such circumstances is found to be 5. Figs. 3.2, 3.3 and 3.4 show that in all cases this may be satisfied with no extra binders if $(\bar{\omega} - \bar{\omega}') \leqslant 0.20$. However in most beams $^{l}/_{h}$ would be much smaller. Thus in practice $\bar{\omega} - \bar{\omega}' = 0.20$ may be regarded as an upper limit for the percentage of reinforcement below which no checks for inelastic compatibility would be required in the intermediate spans, of continuous beams. For larger percentages of reinforcement, the limit redistribution could be determined from equation 10.9.

(b) End Span

The degree of redistribution necessary to equalise the span and support moments in an end span under uniformly distributed load as discussed earlier is about 33%. Hence for the extreme case of $^{l}/_{h}$ = 30, the value of ductility ratio from equation 10.13 is given by $D \geqslant 10$. From Figs 10-1, 10-2 and 10-3, this condition could be satisfied by $(\bar{\omega} - \bar{\omega}') \leqslant 0.15$ without additional binders. This value of reinforcement may then be regarded as a limit below which no inelastic compatibility checks are required in end spans.

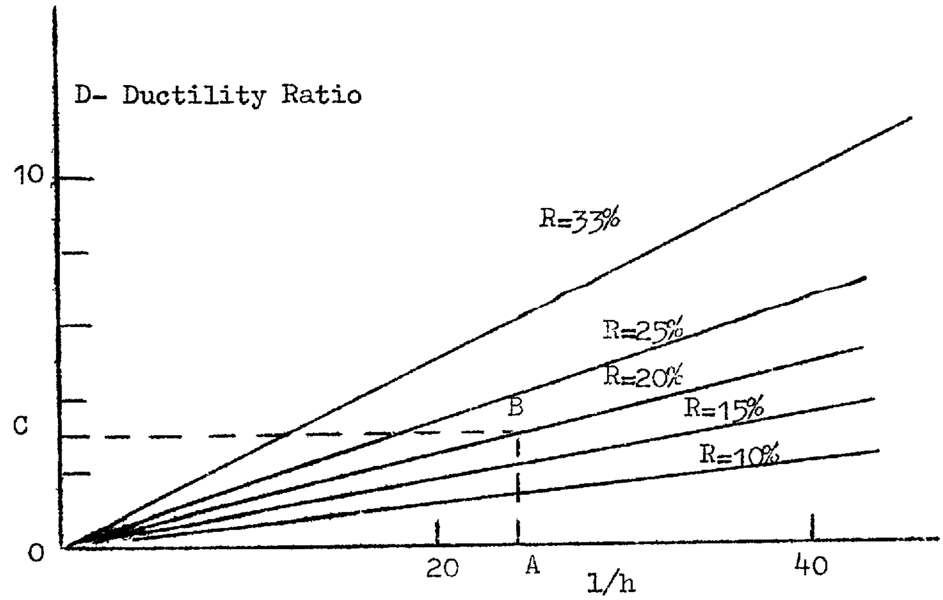



Fig 10.4 Inelastic Compatibility for Intermediate Beams

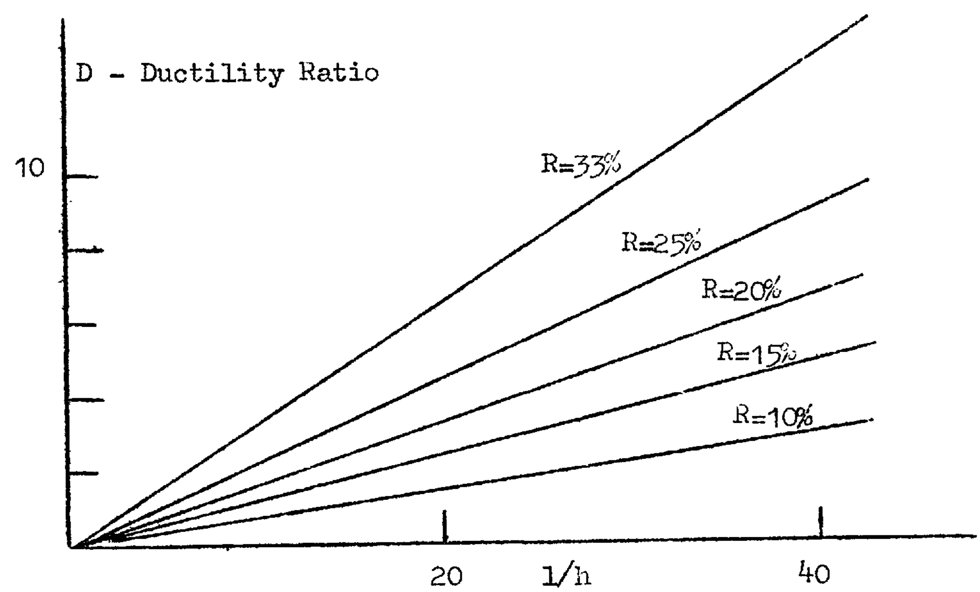



Fig. 10.5 Inelastic Compatibility for End Spans

# CHAPTER 11

## SERVICEABILITY LIMITS

### 11.1 Serviceability criteria.

The most important factors that determine unserviceability in reinforced concrete structures have been discussed in Chapter 3. In the limit design of continuous beams the following conditions have been discussed.

1. Excessive stress at critical sections under working load, which gives rise to long term creep and deflection. The type of structure and the nature of the super-imposed load would determine the maximum stress that could be allowed under working load conditions.
2. Excessive crack width
3. Large deflections, due to super-imposed load which may render the structure unserviceable or physically unsound.

### 11.2. Yield Safety.

The maximum permissible stress under working load is generally specified in relation to the yield stress of either the tension reinforcement or concrete (14) and it is generally assumed that there must be no plastic hinges in any part of the structure under working load conditions.

In practice, almost all the beams are under-reinforced, i.e. tension reinforcement yields before concrete reaches maximum stress, hence the degree of safety against yield could be represented by the ratio of yield stress of the reinforcement to the maximum permissible stress at working load.

i.e. $$\lambda_y = \frac{\sigma_a^*}{\sigma_{a\omega}} \qquad \text{... (11.1)}$$

where $\sigma_a^*$ is the specified yield stress

$\sigma_{a\omega}$ is the permissible working load stress.

Then $\lambda_y$ is referred to as the yield safety parameter

The C.E.B. recommendations suggest values varying between 1.0 and 1.3 for the yield safety parameter[14].

However since very little experimental information is available on the requirement of structures from the point of view of yield safety, this parameter may be assumed as a variable greater than unity whose value must be specified depending on the circumstances.

11.3. Limit crack width.

The maximum crack width in structures, though significant as an important serviceability requirement can only be described in relatively broad limits. However it is known that the type of structure and the environment in which it is situated may influence the permissible limit for the crack widths. For example, close to the sea or in an industrial area with corrosive waste gases, the atmospheric conditions may have adverse effects on the reinforcement if the crack widths are large enough to expose the reinforcement to weather. In the interior of structures, the maximum crack width may be limited by aesthetic considerations.

The values given in table (11.1) have been recommended by the European Concrete Committee as suitable limits for design purposes.

Table 11.1. C.E.B. Recommendations on permissible crack width.

| Type | Description. | Permissible crack width. |
|---|---|---|
| 1. | Structures in aggressive atmosphere. | 0.1 mm. |
| 2. | Unprotected exterior of structure. | 0.2 mm. |
| 3. | Protected structure or interior. | 0.3 mm. |
| 4. | Limit of harmful cracks. | 0.4 mm. |

The relation between the crack width ($\omega$) and the stress in the tension reinforcement ($\sigma_a$) could be given by the following approximate empirical formula due to Brice[36].

$$\omega = A \cdot \frac{\phi}{\mu'} \sigma_a \qquad \ldots (11.2)$$

where $\phi$ = diameter of bar

$\mu'$ = percentage of reinforcement based on the equivalent tie bar

$$= \frac{100 A_a}{2b(h_t - h)}$$

A = an empirical constant which has the following values.

Smooth bars : $A = 2.25 \times 10^{-3}$ mm$^2$/kg.

Deformed bars: $A = 1.40 \times 10^{-3}$ mm$^2$/kg.

The crack width parameter ($\lambda_\omega$) is defined as the ratio of the yield stress in tension reinforcement to the limit stress as given by (11.2), then $\lambda_\omega$ is given by

$$\lambda_\omega = \frac{\sigma_a^*}{\sigma_a}$$

$$= \frac{A\,\phi}{\omega\mu'}\quad \sigma_a^* \qquad \ldots (11.3)$$

where $\sigma_a^*$ is the specified yield stress in tension reinforcement.

11.4. Serviceability parameter.

The minimum service requirements of limiting stress and crack width are expressed in terms of the yield stress in the tension reinforcement. But to simplify the design procedure, let $\lambda_s$ be defined as the serviceability parameter given by

$$\lambda_s = \frac{\text{Yield moment at critical section}}{\text{Maximum permissible moment under work-load.}}$$

$$= \frac{M_y}{M_\omega} \qquad \ldots (11.4)$$

Considering under-reinforced sections, the yield moment ($M_y$) and the permissible work-load moment ($M_\omega$) could be expressed in terms of the yield stress ($\sigma_a^*$) and permissible limit stress ($\sigma_{a\omega}$) as follows.

$$M_y = A_a \sigma_a^* (1 - \gamma_1 x_1) d$$

$$M_\omega = A_a \sigma_{a\omega} (1 - \gamma x)\, d$$

Substituting in (11.4),

$$\lambda_s = \frac{\sigma_a^*(1 - \gamma_1 x_1)}{\sigma_{a\omega}(1 - \gamma x)} \qquad \ldots (11.5)$$

In the limit conditions the stress at working load is close to that at yield, hence the lever arm factor is nearly equal to that at yield (c.f. Fig.4.6) . Then equation (11.5) may be simplified to the form (11.6) which is similar to the definitions of yield safety parameter and crack width parameter.

$$\lambda_s \doteqdot \frac{\sigma_a^*}{\sigma_{a\omega}} \qquad \ldots (11.6)$$

From (11.6) it could be seen that the minimum value of the serviceability parameter is equal to either the yield safety parameter or crack width parameter whichever is greater.

## 11.5. Correlation between ultimate load analysis and serviceability parameter.

Strict serviceability analysis could generally be more detailed than collapse analysis due to the fact that in the former case various combinations of the super-imposed load producing the most critical condition at each section of the elastic structure must be investigated, whereas in the latter only the ultimate load at the collapse state of the structure need be **considered.** However, in practice, these calculations need be only approximate and may be simplified to obtain suitable limits for the degree of redistribution.

Consider the continuous beam at ultimate load as defined in Chapter 8. The elastic moments in a similar beam could be given in the form,

$$M_e' = (\beta_g \, \gamma_g G + \beta_q \, \gamma_q \, Q)l \qquad \ldots (11.7)$$

where $\beta_g$, $\beta_q$ are elastic coefficients which depend only on the distribution of load and the flexural stiffness properties of the beam.

If R is the degree of redistribution, and the yield moment at support is $M'_y$, then

$$M'_y = (1 - R)\ (\beta_g\ \gamma_g G + \beta_q \gamma_q\ Q) l \qquad \dots (11.8)$$

Similarly the moment at the same section under working load could be given in the form

$$M' = (\beta_g G + \bar{\beta}_g\ Q)\ l \qquad \dots (11.9)$$

where $\bar{\beta}_g$ is an elastic coefficient depending on the distribution of super-imposed load corresponding to the most critical configuration of loading.

Substituting for $M'_y$ and $M'_\omega$ in (11.4) in terms of (11.8) and (11.9), $\lambda_s$ may be given in the form

$$\lambda_s = (1 - R)\ \lambda_o \qquad \dots (11.10)$$

where $$\lambda_o = \frac{\beta_g\ \gamma_g\ G + \beta_q\ \gamma_q\ Q}{\beta_g\ G + \bar{\beta}_q\ Q}$$

$\lambda_o$ could be easily determined for the specified over-load coefficients and the type of load. Experience in elastic design show that in practice $\bar{\beta}$ may be determined only for the adjacent spans loaded, as this gives rise to near critical conditions at the support(25).

The limit on the degree of redistribution corresponding to the serviceability parameter $\lambda_s$ may be given by

$$R \leqslant \frac{\lambda_o - \lambda_s}{\lambda_o} \qquad \dots (11.12)$$

This gives the maximum degree of redistribution of moment at ultimate load without causing unserviceability in terms of excessive stress and crack width.

Particular case - Uniformly distributed load.

The two special cases considered in Chapter 10 for inelastic compatability also provide the extreme examples that would be encountered in serviceability calculations. The following approximate values for $\lambda_o$ is easily obtained by substituting for $\beta_g$, $\beta_g$ and $\bar{\beta}_q$.

Intermediate spans: $\lambda_o = \dfrac{\gamma_g G + \gamma_q Q}{G + 1.25 Q}$

End span $\lambda_o = \dfrac{\gamma_g G + \gamma_q Q}{G + 1.05Q}$

As $\lambda_s$ is defined by either the crack width limit or permissible stress, the maximum permissible redistribution corresponding to the given over-load coefficients could now be calculated from (11.12).

Example I. If $\gamma_g = \gamma_q = 1.75$, $G = Q$ and $\lambda_s = 1.1$,

Then, $\lambda_o = 1.56$ and $R \leqslant 26\%$ for intermediate spans and

$\lambda_o = 1.71$ and $R \leqslant 35\%$ for end spans.

Example II. If $\gamma_g = \gamma_q = 1.5$, $G = Q$, $\lambda_s = 1.1$

Then, $\lambda_o = 1.32$ and $R \leqslant 15\%$ for intermediate spans and

$\lambda_o = 1.46$ and $R \leqslant 25\%$ for end spans.

11.6. Midspan sections.

The minimum requirements at midspan (or the critical section close to midspan) under working load is given by

$$M = \lambda_s \, ( \alpha_g \, G + \alpha_q \, Q) \, l \qquad \ldots (11.13)$$

where M is the midspan design moment

$\alpha_g$, $\alpha_q$ are elastic coefficients corresponding to the permanent load G and the critical distribution of the super-imposed load Q, which may be obtained from design tables. In general the critical distribution of the super-load Q corresponds to the case where alternate spans are loaded.

The design moment at midspan could now be based on the service limit in (11.13) above or the minimum equilibrium condition given by equation (8.1).

Particular case - Uniformly distributed load.

The minimum service conditions at midspan under U.D.L. for typical spans are given by the approximate values,

$M = \lambda_s(0.046G + 0.086Q)\, l$ for intermediate spans

$M = \lambda_s(0.078G + 0.10Q)\, l$ for end spans.

11.7. Limit Deflection.

The deflection of beams under working load must be taken into consideration in view of the safety requirements of partition walls, peeling of plaster, and other conditions depending on the general utility of the structure (e.g. beams carrying crane loads, etc.). Under these conditions, only the relative deflection due to the super-imposed load (without the use of over-load coefficients) need be considered[14],[10]. The initial deflection due to the permanent load takes place during construction and may be ignored as it generally occurs before the structure is put into use.

Specific limits for deflection are not specifically provided in the British Code, but the values given in Table 11.2 have been recommended in the Russian structural standards and Regulations[10].

Table 11.2 Limit deformations of slabs and beams.

| Designation of element. | Limit deformation. |
|---|---|
| 1. Beams supporting cranes, hand-operated cranes. | $\frac{1}{500}$ |
| 2. Same for electric cranes. | $\frac{1}{500}$ |
| 3. Deck elements and stairs with ribbed slabs : (a) for $l \leqslant 5$m. | $\frac{1}{200}$ |
| (b) 5m $l < 7$m. | $\frac{1}{300}$ |
| (c) $l > 7$ m. | $\frac{1}{400}$ |
| 4. Flat slabs and roofs (a) $l < 7$ m. | $\frac{1}{200}$ |
| (b) $l \geqslant 7$ m. | $\frac{1}{300}$ |

where l = length of span.

The limit requirements that are necessary to ensure that the deflections are not exceeded could be derived as follows.

Consider the central deflection $\Delta$ in a span under the action of the critical live load (the condition of alternate spans loaded is generally the most critical) given in the form (11.14)

$$\Delta = \beta \frac{Q\, l^3}{EI} \qquad \ldots (11.14)$$

where $\beta$ depends on the nature and position of the live load, which could be obtained from design tables, and Q is the total live load. Substituting for the lower limit value of EI as in (6.13) where the yield moment of support section is used, is given by

$$\Delta = \frac{\beta \quad Q\, e_{ay}\, l^2}{(1-x_1)(1-R)(\gamma_g \beta_g G + \gamma_q \beta_q Q)h}$$

$$\text{i.e.} \quad \frac{\Delta}{l} = \beta \, \frac{e_{ay}}{(1-x_1)} \, \frac{Q}{(1-R)(\gamma_g \beta_g G + \gamma_q \beta_q Q)} \quad \frac{l}{h} \qquad \ldots (11.15)$$

If the permissible limit of deflection is represented by $\left(\frac{\Delta}{l}\right)^*$, the deflection limit may be given by

$$\frac{l}{h} \ngtr \frac{1}{\beta} \, \frac{(1-x_1)}{e_{ay}} \, \frac{(1-R)(\gamma_g \beta_g G + \gamma_q \beta_q Q)}{Q} \left(\frac{\Delta}{l}\right)^* \qquad \ldots (11.16)$$

Equation (11.16) could be used to obtain the minimum depth of beam that is necessary to satisfy the deflection requirement, depending on the reinforcement and degree of redistribution.

Particular case - Uniformly distributed load.

In the case of uniformly distributed super-load on an intermediate span,

$$\beta_g = \beta_q = \frac{1}{12}, \quad \beta = \frac{5}{768}$$

The lower limit of $\frac{1 - x_1}{e_{ay}}$ is given by

$$\frac{1 - x_1}{e_{ay}} = \frac{x_1}{e_{bl}} = \frac{1}{e_{ay} + e_{bl}}$$

where $e_{bl}$ is taken as 0.002.

Substituting in equation (11.6) the limit of $\frac{1}{h}$ is given by

$$\frac{1}{h} \ngtr \Delta_i (1-R) \frac{(\gamma_g G + \gamma_q Q)}{Q} \cdot \left(\frac{\Delta}{l}\right)^{*} \qquad \ldots (11.17)$$

where $\Delta_i = \frac{64}{5} \frac{1}{(e_{ay} + 0.002)}$

$\Delta_i$ depends only on the grade of reinforcement.

Similarly for an end span $\beta_g = \beta_q = \frac{1}{8}$

$\beta = \frac{1}{96}$, which gives the following limit

$$\frac{1}{h} \ngtr \Delta_1 \frac{(1-R)(\gamma_g G + \gamma_q Q)}{Q} \left(\frac{\Delta}{l}\right)^{*} \qquad \ldots (11.18)$$

where $\Delta_1 = \frac{12}{e_{ay} + 0.002}$

Other types of loads could either be treated similarly or represented by their equivalent uniformly distributed load.

# CHAPTER 12.

## Criteria of limiting shear

Although the exact nature or the mechanism of shear failure in reinforced concrete beams is still under intensive investigation, the importance of this type of failure is well established particularly as it is preceeded by very little or no warning of failure. The present methods of design for shear is based on empirical results most of which have been evaluated in terms of the well known formula attributed to Mörsch mentioned in chapter 9, where the nominal shear stress in a rectangular section is defined in terms of the shear force, the lever arm and width of section.

$$\text{i.e.} \quad s = \frac{T}{(1-\gamma x)bh} \qquad 12.1$$

In the latest A.C.I. recommendations however, the lever arm factor $(1-\gamma x)$ in the above equation has been omitted in the determination of the nominal shear stress.

In general, if s is less than the permissible shear stress in plain concrete ($\sigma^{*}_{bt}$), no shear reinforcement is required, while if it lies between this value and an upper limit which may be defined as permissible shear stress in reinforced concrete ($\sigma^{**}_{bt}$), the shear may be resisted by additional reinforcement. If the shear stress is above $\sigma^{**}_{bt}$, then it is considered excessive and the beam section must be redesigned.

The actual values of $\sigma_{bt}^{*}$ and $\sigma_{bt}^{**}$ and the method of reinforcing for shear are subject to variations. The current B.S. code (ref. 25) suggests a value of approximately $\frac{1}{10}$ of the permissible compression stress in bending for $\sigma_{bt}^{*}$ while $\sigma_{bt}^{**}$ is given by $4\sigma_{bt}^{*}$. In recent research, Leonhardt and Walther (ref 42) has recommended, $\sigma_{bt}^{*} = \sigma_{b}^{*} / 9$ , $\sigma_{t}^{**} = 5\sigma_{bt}^{*}$. They have also suggested a different basis for reinforcing when the shear stress lies in the intermediate range. The CEB recommendations (ref.14) differ from both of the above methods in its approach to the design for shear but on the same basis of comparrison $\sigma_{bt}^{**}$ has been increased to $5\sigma_{bt}^{*}$ for rectangular beams and $6\sigma_{bt}^{**}$ for beams with a compression flange. ACI standard (ref. 57) recommends a conservative value of $2.7\sqrt{\sigma_{b}^{*}}$ for $\sigma_{bt}^{**}$. Thus it appears that the permissible shear stress in reinforced concrete may lie in the range $4\sigma_{bt}^{*}$ to $6\sigma_{bt}^{*}$ where $\sigma_{bt}^{*}$ varies from $\sigma_{b}^{*}/9$ to $\sigma^{*}/10$ and in no case must $\sigma_{bt}^{**}$ exceed the above limits. The limits of the section parameters that is necessary to ensure the shear requirements at ultimate load could be easily determined as an integral part of a limit design procedure.

Consider a beam loaded with the permanent and super-imposed loads G and Q associated with load factors $\gamma_g$ and $\gamma_q$. The shear force T could be given in the form

$$T = X_g \gamma_g G + X_q \gamma_q Q \qquad 12.2$$

where $\chi_g$, $\chi_q$ are constants depending on the loads.

Using equation 12.1, shear stress s is given by

$$s = \frac{\chi_g \gamma_q G + \chi_q \gamma_q Q}{(1-\gamma_x)\ b h} \qquad \text{———} \qquad 12.3$$

But the support section parameters could be expressed in terms of the plastic moment as in the earlier chapters which may be given in the form,

$$\bar{\omega} \quad (1-\gamma_x) \quad \sigma_b^* \ bh^2 = M' = (1-R)\ (\beta_g \gamma_g \ G + \beta_q \gamma_q \ Q)\ l \qquad 12.4$$

where R is the degree of redistribution, $\beta_g$, $\gamma_q$ are elastic constants as defined in chapter 8.

From 12.3 and 12.4, the following relation for l/h could be obtained.

$$\frac{l}{h} = \frac{(\chi_g \ \gamma_q \ G + \chi_q \ \gamma_q \ Q)\ \bar{\omega}}{(\beta_g \ \gamma_g \ G + \beta_q \ \gamma_q \ Q)\ (1-R)} \qquad \frac{(\sigma_b^*)}{s} \qquad \text{———} \qquad 12.5$$

Since the maximum limit of s is given by $\sigma_{bt}^{**}$, the limit based on shear can be given by

$$\frac{l}{h} \not< \frac{(\chi_g \gamma_q \ G + \chi_q \gamma_q \ Q)\ \bar{\omega}}{(\beta_g \gamma_g \ G + \beta_q \gamma_q \ Q)\ (1-R)} \qquad \frac{(\sigma_b^*)}{\sigma_{bt}^{**}} \qquad \text{———} \qquad 12.6$$

Equation 12.6 may be used in the limit design to obtain the minimum value of l/h compatible with the shear requirements

Particular case - Uniformly distributed load

In a uniformly distributed beam, the following values can be easily derived

Intermediate span. $\chi_g = \chi_q = 0.5$,

$$\beta_g = \beta_q = \frac{1}{12}$$

$$\therefore \frac{1}{h} \nless \frac{6\,\overline{\omega}}{1 - R} \left(\frac{\sigma_b^*}{\sigma_{bt}^{**}}\right) \qquad \text{———} \qquad 12.7$$

<u>End span</u> $\quad \chi_g = \chi_q = 0.6$

$\beta_g = \beta_q = \frac{1}{8}$

$$\therefore \frac{1}{h} \nless \frac{4.8\,\overline{\omega}}{1 - R} \left(\frac{\sigma_b^*}{\sigma_{bt}^{**}}\right) \qquad \text{———} \qquad 12.8$$

Assuming $\sigma_{bt}^{**} = \dfrac{4\,\sigma_b^*}{9}$, equation 12.6 and 12.7 reduce to

$$1/h \nless 13.5 \;.\; \frac{\overline{\omega}}{1-R} \quad \text{and} \quad \frac{1}{h} \nless 10.8 \;.\; \frac{\overline{\omega}}{1 - R}$$

Considering a maximum value of $\overline{\omega} = 0.3$ and $R = 0.25$, for internal spans and $R = 0.33$ for end spans, the shear limit reduces to $1/h \nless 5.4$ for internal spans and, $1/h \nless 4.9$ for end spans.

These two values may be considered to be extreme cases which may not be exceeded under practical design conditions.

# CHAPTER 13

## Experimental Investigations on Reinforced Concrete Continuous Beams

### 13.1 Introduction

The following test programme was set out in order to study the serviceability and ultimate load characteristics in continuous beams, as discussed in the preceding chapters, in relation to the idealised member properties as derived from simple beam tests.

A three span continuous beam was chosen as a suitable test specimen as it encorporates the extreme types of span conditions that may be encountered in a wide range of structures. The size of beam was mainly determined by the available test facilities in the laboratory but it was considered large enough to prevent any scale effects. The test programme covered eight beams, six of which were reinforced with mild steel bars and the others reinforced with cold worked steel bars.

### 13.2 Materials and Fabrication

#### (a) Aggregates

Ordinary Portland cement was used to obtain a mix of approximate strength 5000 psi on 28 days. The coarse aggregates consisted of $\frac{3}{8}$" maximum size crushed Thames Valley river gravel. The fine aggregates were from the same source.

#### (b) Steel

Mild steel reinforcement used in the beams varied in diameter from $\frac{1}{4}$" to 5/8". In the case of cold worked steel only $\frac{1}{4}$" diameter bars were used. The details of the reinforcement are given in Fig.13.1 and the section areas are given in Table 13.1.

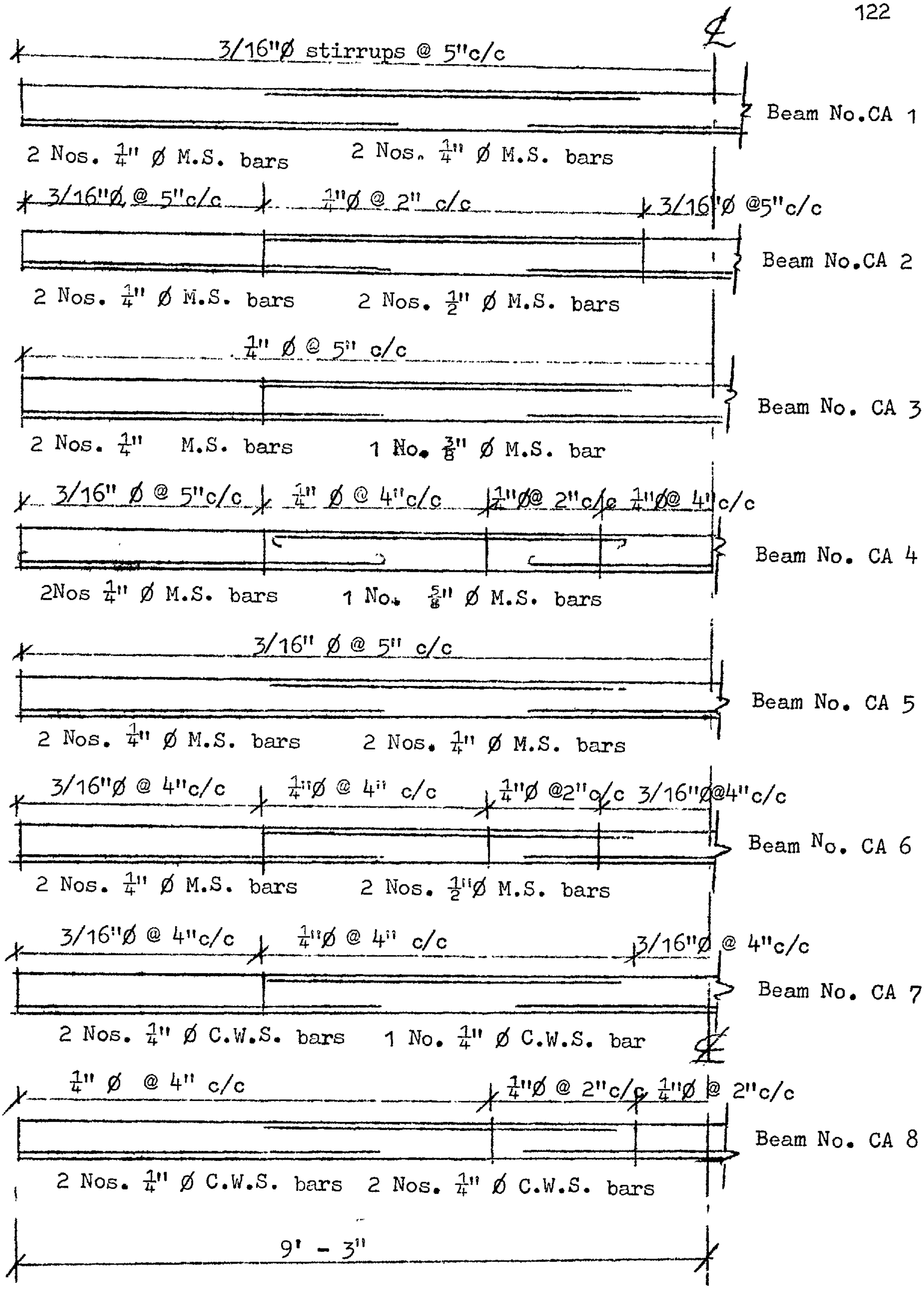



Fig. 13.1 (a). Main Reinforcement Details of Continuous Beams

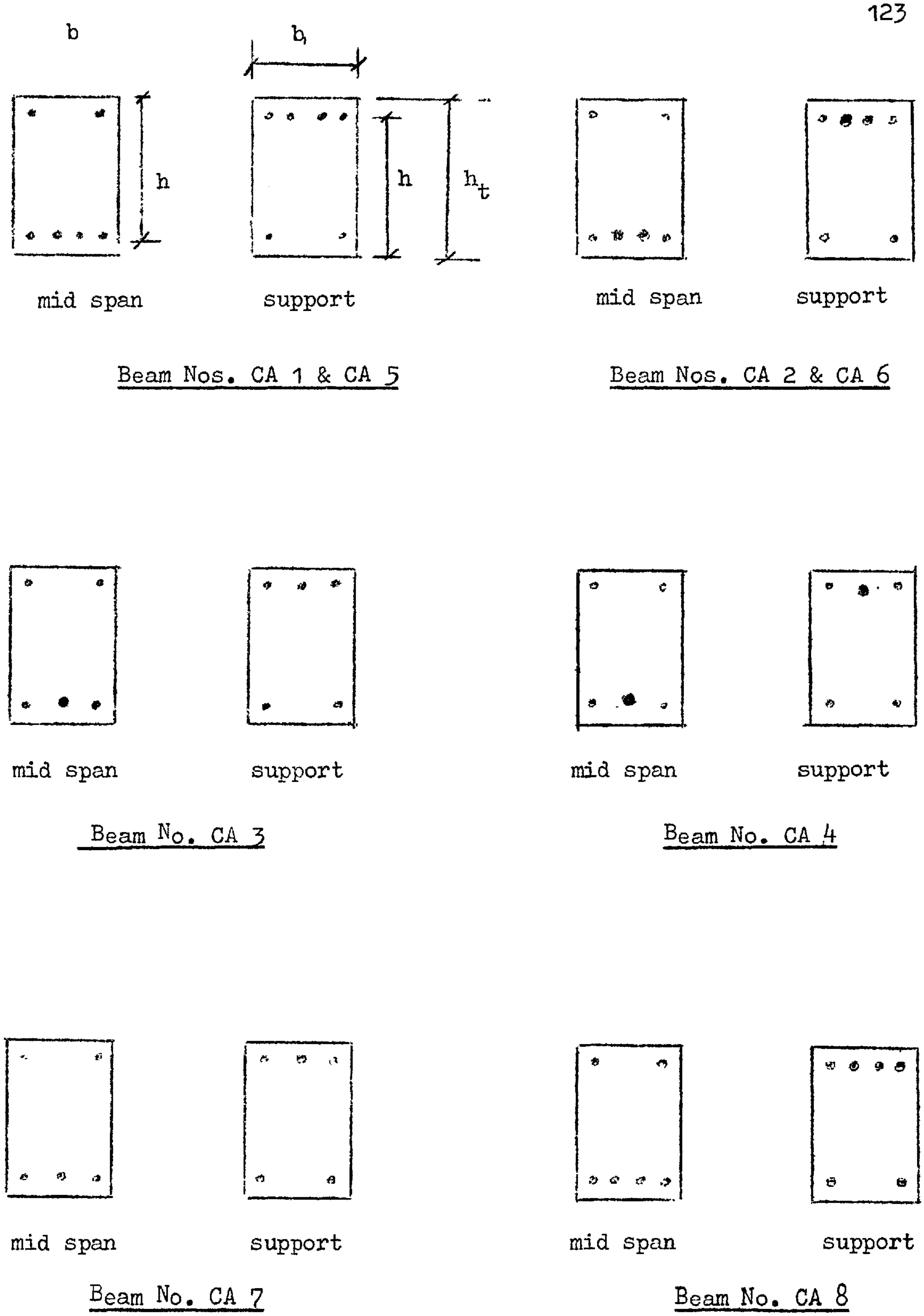



Fig. 13.1 (b). Sectional Detaiis of Beams

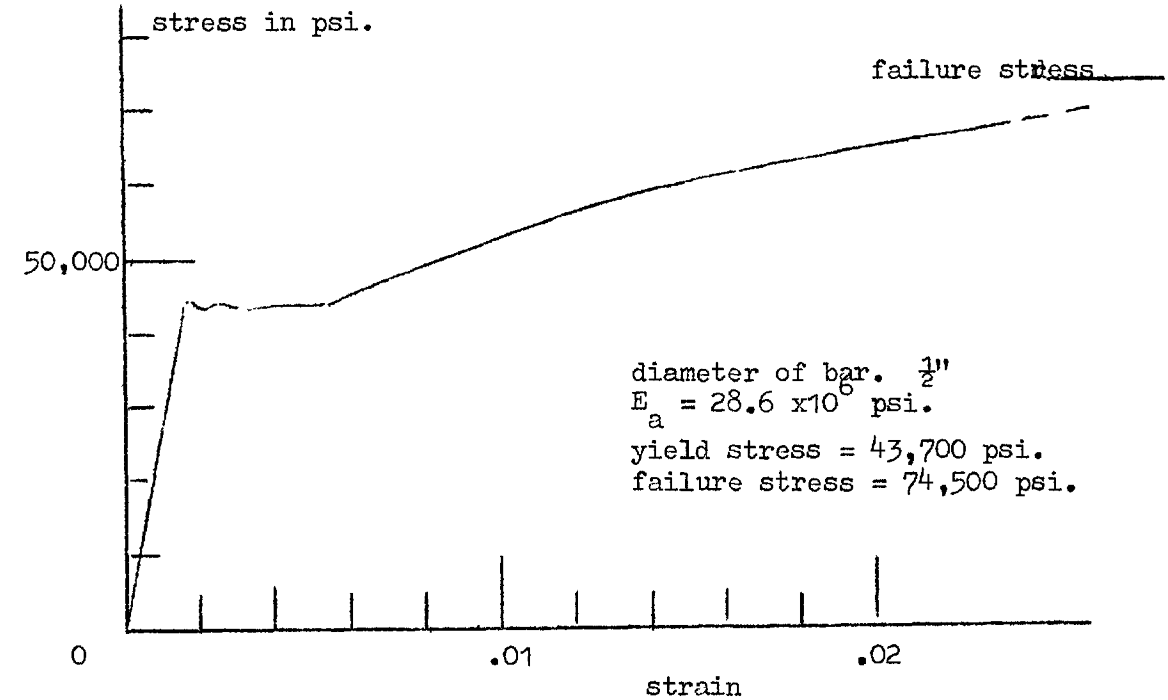



Fig. 13.2 (b). Typical stress strain curve for mild steel reinforcement bars

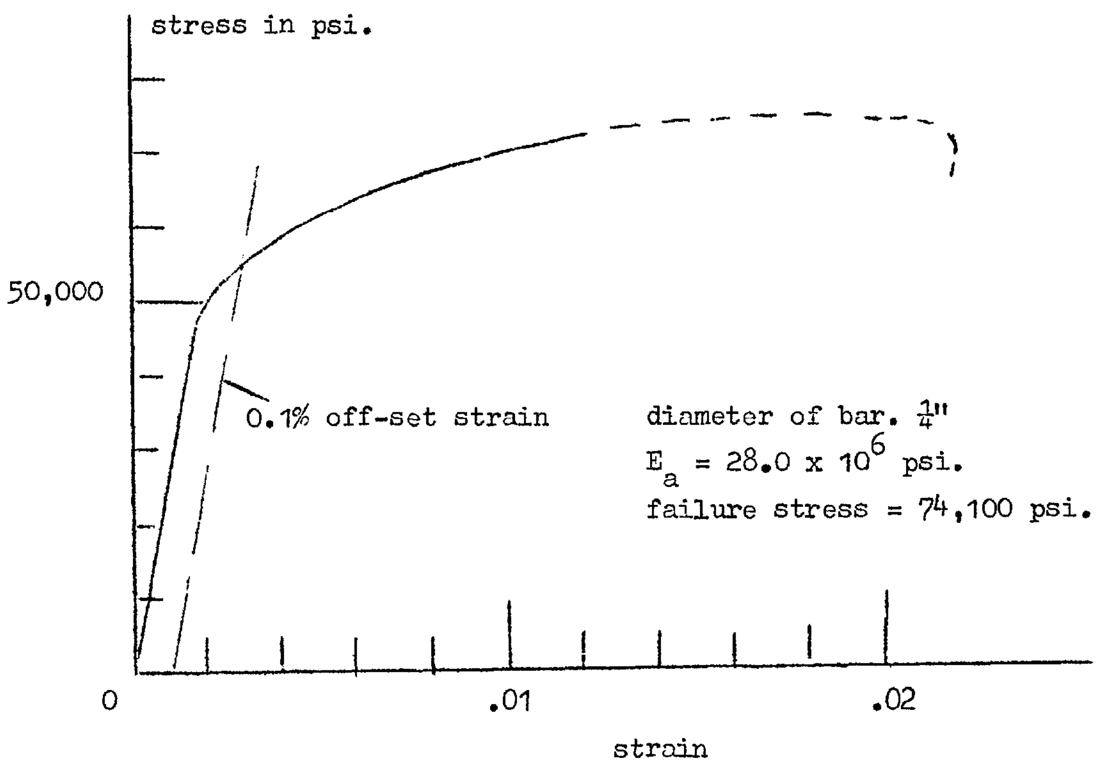



Fig. 13.2 (a). Typical stress strain curve for cold worked steel reinforcement bars

The yield characteristics of the reinforcement bars were determined from the stress-strain curves of at least three random samples of each grade of steel; typical examples of which are given in Fig.13.2. The yield stress in cold worked steel was defined by 0.1% off set strain. Strain hardening effect of tension reinforcement were not taken into account in the calculation of the moments.

In all the beams mild steel stirrups were used, which were lap welded and tied to the main reinforcement using steel wire.

(c) Casting and Curing

The beams were cast in steel form work which consisted of three sections, each of 6' 6" long, bolted together with the joints sealed with plasticene. Two batches of the same mix were used in each beam and the concrete was placed in four layers, each layer being well compacted using a shutter Vibrator mounted on the top of the formwork.

Three standard 6" cylinders and three 6" cubes were cast as control test specimen for each beam. The moulds were stripped after one day, and the specimens were cured under wet hessian for 7 days before allowing to dry out under standard laboratory conditions for a further three weeks. The beams were tested at an approximate age of four weeks.

(d) Test rig

The details of the test rig are shown in Figs.13.3 and 13.4 The 19' 0" long beam was mounted on three roller bearings and one rocker bearing the details of which may be seen from Fig.13.5, each of which was supported on similar electrical resistance gauge type load

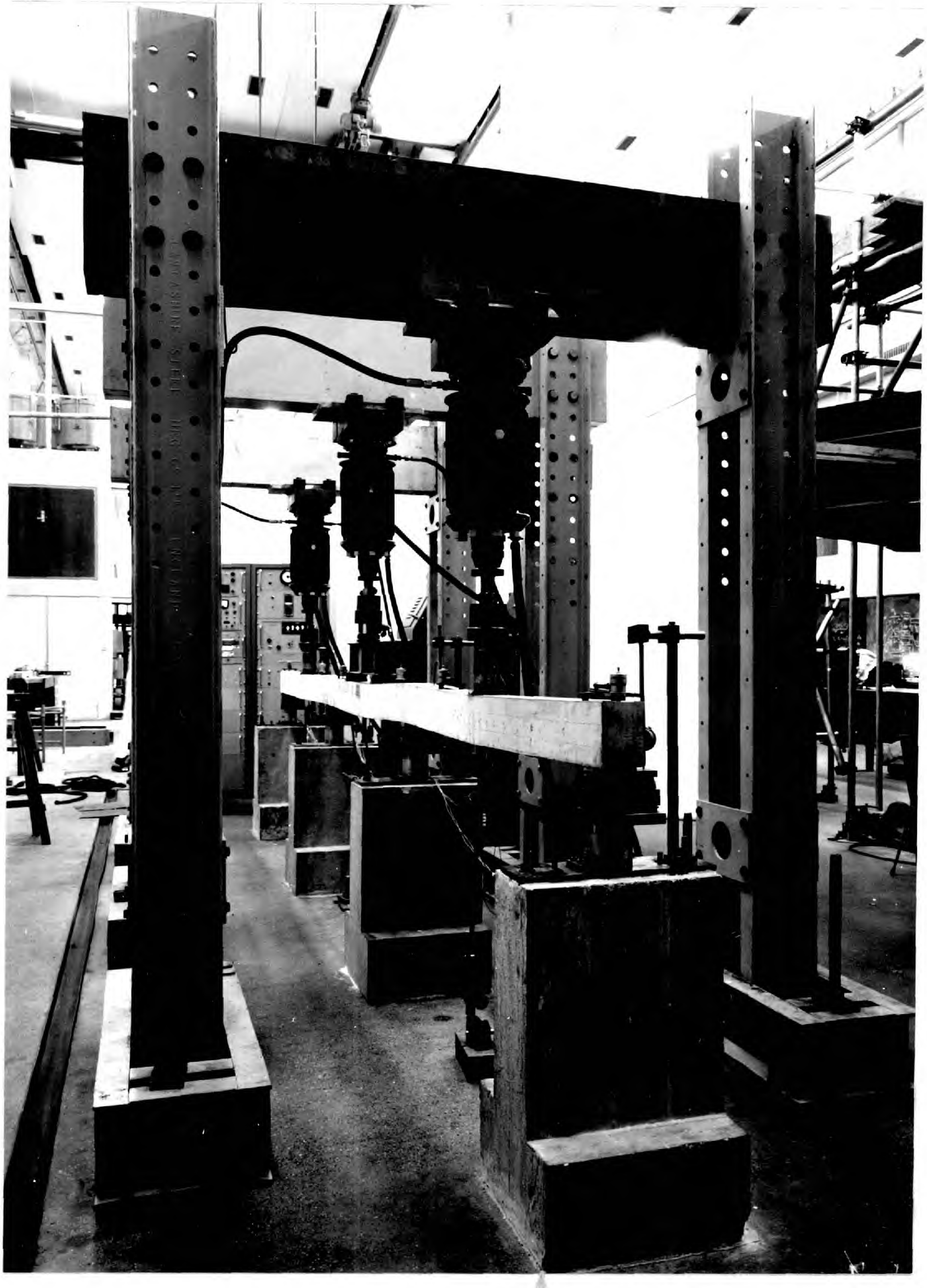
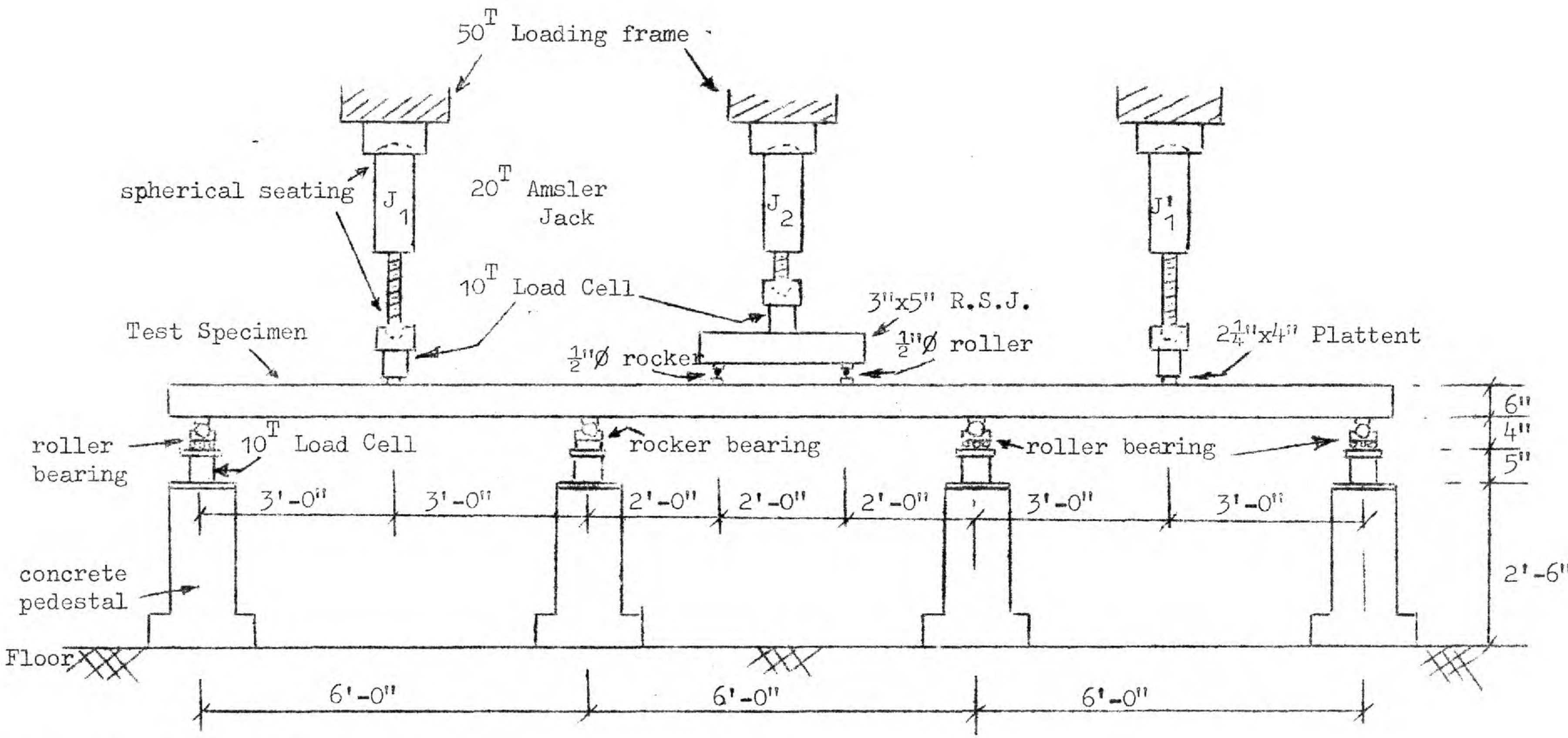
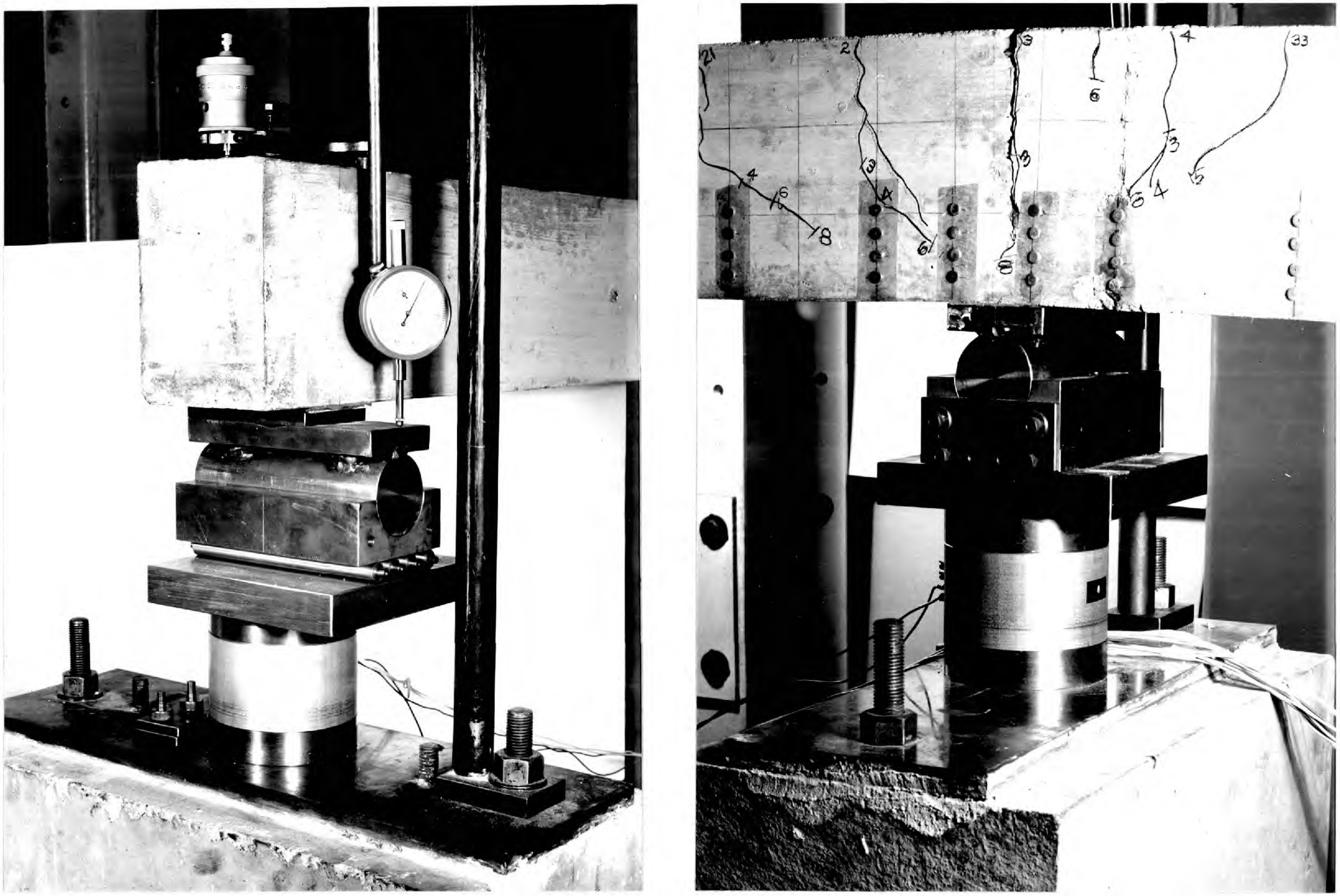


Fig. 13.3 Test Rig for Three Span Continuous Beams.




Note: The Amsler Jacks $J_1$ and $J'_1$ were connected in series to one loading cabinet. $J_2$ was connected to a separate loading cabinet.

Fig. 13.3 (a) General Layout for Three Span Continuous Beam Tests.



(a) Roller Bearing

(b) Rocker Bearing

Fig 13.4 Support Details

cells which were used to measure the support reactions.

The loads on the beam were applied by three 20T Amsler jacks mounted on 50T load frames reacting against the laboratory floor. The outer jacks were coupled in series to a loading cabinet. The centre load was applied independently using a separate loading cabinet. The loading platens as well as the support platens consisted of steel plates $2\frac{1}{4}" \times 4" \times \frac{1}{2}"$, Similar to those that have been used in the simple beam tests quoted earlier[8, 34]. The applied loads were measured by the loading cabinets as well as by electrical resistance gauge type load cells.

(e) Instrumentation

The strain in the concrete compression zone was measured along the length of the beam using 4" Demec Strain gauges and 30mm electrical resistance strain gauges as shown in the layout diagram Fig.13.5(a). The readings close to the critical sections were taken on Demec points spaced at 2" apart, so that the local variations could be better extrapolated.

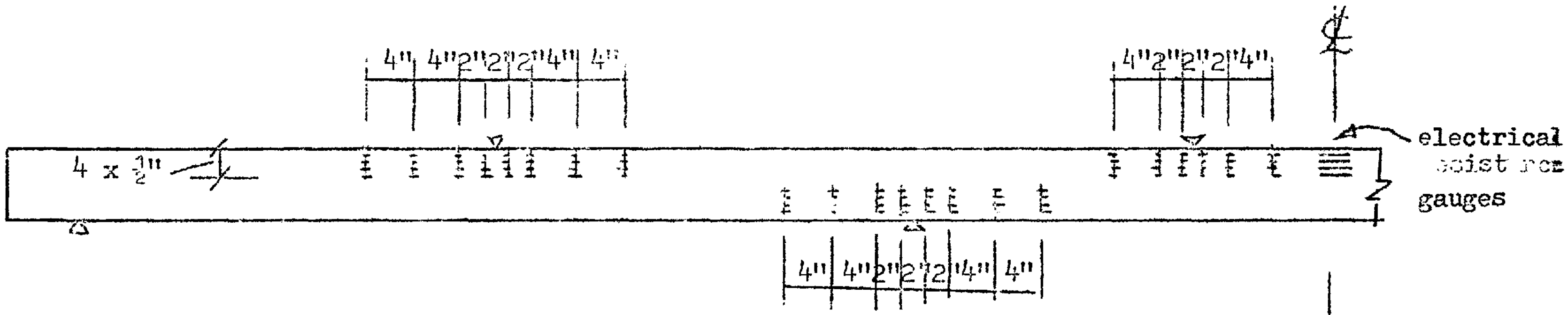
The strain gauge layout was designed to obtain :

(a) the strain at the extreme compression fibres at sections close to the critical points.

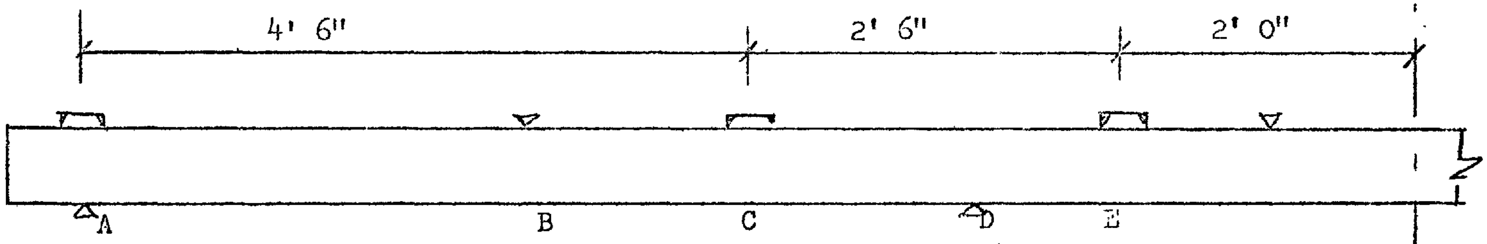
(b) neutral axis depth at each of the above positions.

(c) the curvature profile along the beam.

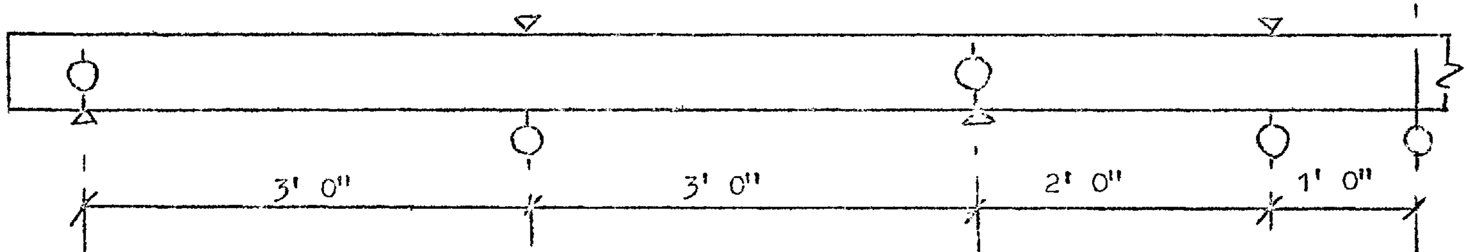
The reliability of the electrical gauge system was ± 1 micro strain and that of Demec gauges was ± 10 micro strains. But at each section four sets of gauges were used, so that the effects




(a). Demec and Electrical resistance Strain Gauge Layout




(b) Clinometer Layout ( 6 Nos. )




(c) Deflection Gauge Layout ( 9 Nos. )

Fig. 13.5 INSTRUMENTATION.

of random errors in individual readings could be reduced to a minimum.

The rotation in the beam was measured at six positions using clinometers as shown in Fig.13.5(b). The clinometers[34] had an accuracy of $\pm$ 25 x $10^{-6}$ radians. The total rotation of any part of the beam enclosing a critical section could be determined using the clinometer readings.

The deflections at the critical sections, as in Fig.13.5(c), were measured using mechanical deflection gauges reading to 0.001". The settlement of the supports were measured using similar gauges, the details of which could be seen from Fig.13.3(c).

The maximum crack width corresponding to each significant crack were measured using a gauge reading to the nearest 0.001".

(f) Setting up of beam

The strain gauges were mounted on the test beam after the beam had dried out for about two weeks, but at least two days in advance of testing. In the setting up of the beam it was found that the base of the beam was not straight, in some cases the out of alignment being as much as $\frac{1}{8}$". This was corrected in the initial alignment using additional packing until approximately equal reactions were registered at the outer and the inner supports.

(g) Test procedure

Six of the eight beams were tested under incremental loading, where the loads were increased in 12-15 stages. The first four load increments were approximately 15% of the calculated ultimate load, and each of the subsequent increments was about 5% of the ultimate load. On the average each load stage required about 15 minutes before all the readings could be taken. This was similar to the test procedure in the simple beam tests conducted under the European Concrete Committee[9].

The beams CA5 and CA6 which were similar to CA1 and CA2 were subjected to repeated loading corresponding to yield safety factors, 1.6, 1.2 and 1.1, each load being repeated four times from zero to maximum.

In each of the tests the applied loads were kept approximately constant while the instrument readings were taken.

## 13.3 Theoretical calculations

The loading on the three span continuous beams consisted of a single point load on the end span and third point load in centre span as shown in Fig.13.6. Assuming constant EI value for all the spans, the moments at the critical sections during the elastic stage of the beam are given by

$$\left.\begin{aligned} M_1 &= \frac{17}{80} P_1 l - \frac{P_2 l}{30} \\ M_2 &= \frac{3}{40} P_1 l + \frac{P_2 l}{15} \\ M_3 &= \frac{1}{10} P_2 l - \frac{3}{40} P_1 l \end{aligned}\right\} \qquad 13.1$$

Where $P_1$ is the load on the end span and $P_2$ is the total load on the centre span. Substituting for the experimental values $P_2/P_1 = 2.0$ and $l = 72''$, the above moments reduce to

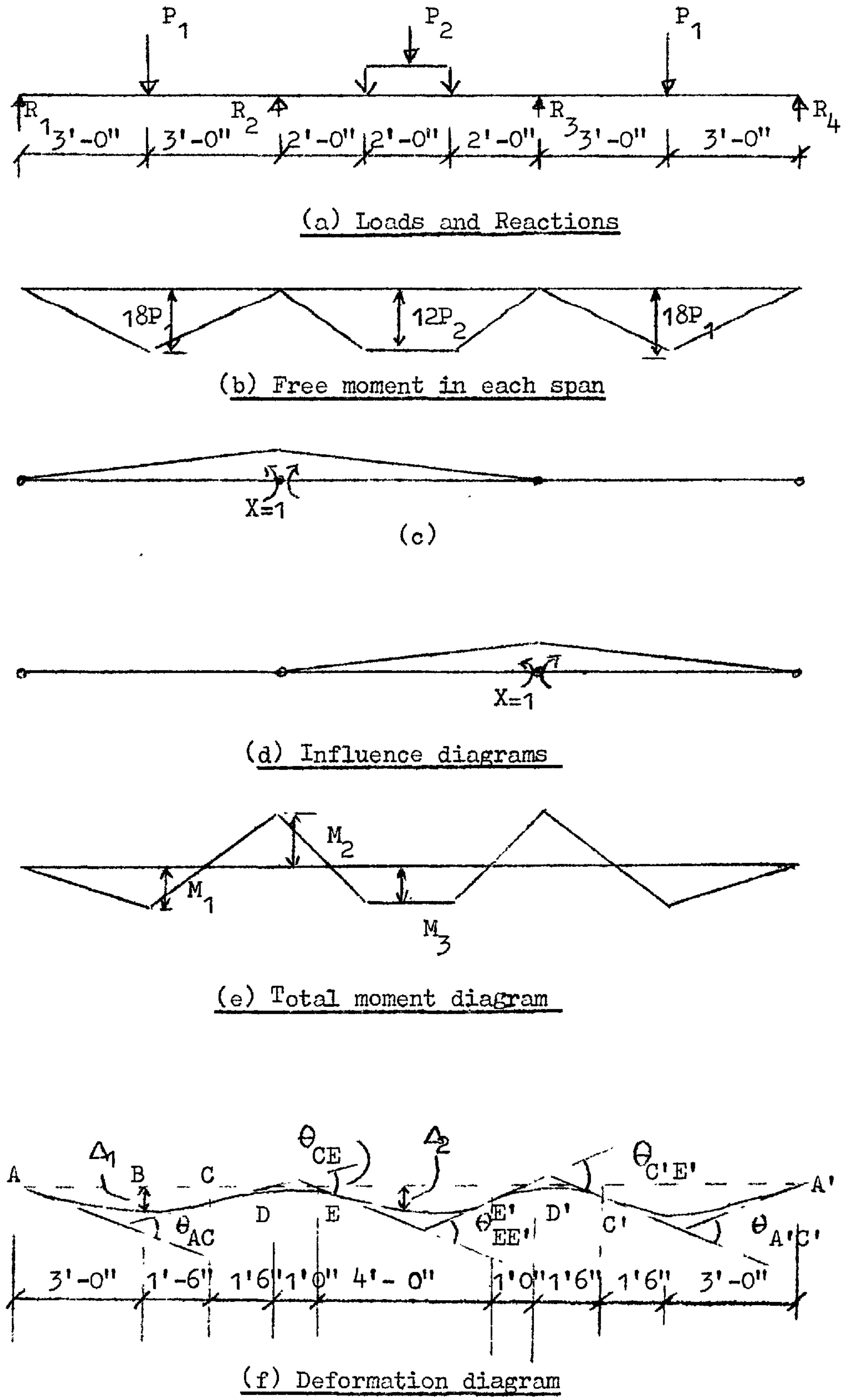

P1
P2
P1
R1
R2
R3
R4
3'-0"
3'-0"
2'-0"
2'-0"
2'-0"
3'-0"
3'-0"
(a) Loads and Reactions
18P
12P2
18P1
(b) Free moment in each span
X=1
(c)
X=1
(d) Influence diagrams
M2
M1
M3
(e) Total moment diagram
A
B
C
D
E
E'
D'
C'
A'
Δ1
Δ2
θCE
θC'E'
θAC
θEE'
θA'C'
3'-0"
1'-6"
1'6"
1'0"
4'- 0"
1'0"
1'6"
1'6"
3'-0"
(f) Deformation diagram

Fig. 13.6

$$M_1 = 10.5\,P_1$$
$$M_2 = 15\,P_1 = 7.5\,P_2 \qquad 13.2$$
$$M_3 = 4.5\,P_2$$

The elastic rotation in the lengths AC, CE, EE' (Fig.13.6) could now be obtained by integrating the area of the bending moment diagram. If $O_{AC}$, $O_{CE}$, $O_{EE'}$ represent the elastic rotation in the above sections of the beam corresponding to the critical moments $M_1$, $M_2$, and $M_3$,

$$O_{AC} = \frac{1}{16EI}\,(7M_1 - M_2)$$
$$O_{CE} = \frac{1}{48EI}\,(15M_2 - 3M_1 - 2M_3) \qquad 13.3$$
$$O_{EE'} = \frac{1}{24EI}\,(14M_3 - 2M_2)$$

Similarly the central deflection in the end span ($\Delta_1$) and centre span ($\Delta_2$) before the supports yielded are given by

$$\Delta_1 = (6M_1 - 1.5M_2)\,\frac{l^2}{72EI}$$
$$\Delta_2 = (23M_3 - 4M_2)\frac{l^2}{216EI} \qquad 13.4$$

After the supports yield,the increase in deflection in each span is given by the same expressions when $M_1$ and $M_3$ are substituted by the increase in the free span moment and $M_2$ is equated to zero.

<u>Correction for settlement of support</u>

If $\Delta_D$ and $\Delta_{D'}$ be the settlement of the supports D and D', relative to A and A', then the virtual reactions at the supports D and D' are given by

$$R_D = \frac{6}{5}\frac{EI}{l^3}[8\Delta_D - 7\Delta_{D'}] \quad \Bigg) \quad 13.5$$
$$R_D = \frac{6}{5}\frac{EI}{l^3}[-7\Delta_D + 8\Delta_{D'}] \quad \Bigg)$$

Hence the moments at the critical sections due to the settlement of supports could be calculated.

## 13.4 Discussion of experimental results

### (a) Load-moment curves

The applied loads, reactions, critical moments and the moments due to settlement of supports were evaluated from the observed results using a computor programme. The settlement of supports in general were less than 0.020" and the correction required was less than 1% of the actual moments, which was smaller than the accuracy of the load measuring devices. Hence if sufficiently rigid supports are used in continuous beam tests it may be concluded that the support settlements could be ignored.

Table 13.2 gives the ultimate moments of critical sections obtained experimentally compared with the calculated values. It was found that at the ultimate load, the moments at the support sections were consistantly larger than those at the mid-span sections even though they were designed to have similar moments. This was similar to the observations made by Macchi[49], and could be attributed to strain hardening of the tension steel at the support hinges, due to the large rotation taken place before the ultimate load was reached. In some beams, the mid-span

TABLE 13-1

| Beam No | Support Section | | | | | | Midspan Section | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | b ins | h ins | $\overline{\omega}$ | $\overline{\omega}'$ | $\sigma_b$ psi | $\sigma_{bu}$ psi | b ins | h ins | $\overline{\omega}$ | $\overline{\omega}'$ |
| CA1 | 4 | 5.37 | .1012 | .0506 | 4680 | - | 4 | 5.25 | .1040 | .0520 |
| CA2 | 4 | 5.25 | .2155 | .0505 | 4800 | 5907 | 4 | 5.12 | .2210 | .0520 |
| CA3 | 4 | 5.32 | .1188 | .0583 | 4100 | 5750 | 4 | 5.20 | .1220 | .0600 |
| CA4 | 4 | 5.22 | .2005 | .0553 | 4410 | 5950 | 4 | 5.10 | .2035 | .0565 |
| CA5 | 4 | 5.37 | .0950 | .0475 | 5000 | 6300 | 4 | 5.25 | .0965 | .0464 |
| CA6 | 4 | 5.25 | .2050 | .0480 | 5030 | 7030 | 4 | 5.12 | .2012 | .0402 |
| CA7 | 4 | 5.37 | .0745 | .0495 | 5070 | 6450 | 4 | 5.25 | .0759 | .0506 |
| CA8 | 4 | 5.37 | .1030 | .0515 | 4650 | 6650 | 4 | 5.25 | .1102 | .0551 |

TABLE 13-2

| Beam | Support Section | | | | | Midspan Section | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| No. | $M_2$ in-lbs | $M_u$ in-lbs | $x_2$ | $x_u$ | $M_u/M_2$ | $M_2$ in-lbs | $M_u$ in-lbs | $x_2$ | $x_u$ | $M_u/M_2$ |
| CA1 | 54600 | 61000 | .120 | .12 | 1.12 | 53500 | 61000 | .120 | .190 | 1.14 |
| CA2 | 103000 | 112000 | .255 | .19 | 1.09 | 101000 | 100000 | .260 | .23 | 0.99 |
| CA3 | 54700 | 57000 | .140 | .23 | 1.04 | 53200 | 50000 | .144 | .22 | 0.94 |
| CA4 | 89000 | 99000 | .236 | .25 | 1.11 | 85000 | 92000 | .240 | .30 | 1.08 |
| CA5 | 51800 | 62000 | .112 | .18 | 1.07 | 50700 | 60000 | .114 | .28 | 1.20 |
| CA6 | 103000 | 125000 | .242 | .29 | 1.21 | 97800 | 110000 | .238 | .27 | 1.12 |
| CA7 | 42000 | 60000 | .088 | .13 | 1.43 | 42000 | 54000 | .089 | .15 | 1.29 |
| CA8 | 53000 | 70000 | .122 | .12 | 1.32 | 53000 | 58000 | .130 | .134 | 1.09 |

TABLE 13-3

| Beam No. | $P_{1cal}$ lbs | $P_{2cal}$ lbs | $P_{1exp}$ lbs | $P_{2exp}$ lbs | $\frac{P_{1exp}}{P_{1cal}}$ | $\frac{P_{2exp}}{P_{2cal}}$ | $(EI)_1$ $10^6$ psi | $(EI)_e$ $10^6$ psi |
|---|---|---|---|---|---|---|---|---|
| CA1 | 4500 | 9010 | 4800 | 9900 | 1.07 | 1.10 | 113 | 250 |
| CA2 | 8470 | 17000 | 8800 | 17000 | 1.04 | 1.00 | 166 | 251 |
| CA3 | 4470 | 9000 | 4600 | 8500 | 1.03 | - | 106 | 240 |
| CA4 | 7190 | 14500 | 7600 | 15100 | 1.06 | 1.04 | 145 | 246 |
| CA5 | | 8550 | 4800 | 9800 | 1.12 | 1.15 | 105 | 254 |
| CA6 | 8300 | 16700 | 9500 | 19500 | 1.14 | 1.17 | 169 | 254 |
| CA7 | 3580 | 7200 | 4700 | 9600 | 1.31 | 1.33 | 100 | 254 |
| CA | 4620 | 9200 | 5200 | 10100 | 1.13 | 1.10 | 107 | 250 |

moment had not increased very much beyond the limit $L_1$, but larger ultimate loads than calculated were observed as a result of the increase in the support moments.

The moment at critical sections were plotted against the span load as in Figs.13.7(a) - 13.14(a). These may be compared with the theoretical curves which are also shown. In beams CA1, CA7, and CA8 which were reinforced with about 1% tension reinforcement, there was some transfer of moment from support to mid-span in the 'elastic' stages due to cracking and reduction of the flexural stiffness over supports. In the other beams the theoretical curves correspond very closely to the experimental results.

Post yield redistribution of moments could be clearly seen from the load-moment curves for beams with mild steel reinforcement. These are similar to the predicted behaviour based on idealised yield properties. However after the mid-span sections reached yield limit, the support moments had increased due to strain hardening as explained earlier, which is clearly indicated in Figs.13.7(a), 13.11(a) and 13.12(a). In beams CA7 and CA8, which were reinforced with cold worked steel, a definite yield stage could not be detected from the load-moment curves. The support moments continued to increase after the idealised yield limit as defined by the 0.1% off set strain. The final collapse load was 10-30% higher than predicted. However, the deflections in the post yield stage in these beams were quite large.

Marked diagonal cracking due to shear could be seen in beams CA2, CA4 and CA6 (Figs. 13.27, 13.29, 13.31). These were accompanied by noticeable increase in deflections and creep at higher loads.

Moment-curvature results

The curvatures at similar sections were plotted against the moment as in Figs. 13.15 - 13.22. These clearly show the scatter in the test results both in the 'elastic' and in the 'inelastic' stages which is similar to observations in simple beam tests[8, 34]. In general, the sections remain uncracked upto about 20-30% of the ultimate moment, the stiffness is then reduced until the section yields at limit $L_1$.

The 'elastic' stages in the moment-curvature curves could be compared with the calculated bilinear curves based on the effective flexural stiffness of the beams. The curve marked $(EI)_1$ is based on the semi-empirical formula 6.12 discussed in Section 6. The conventional elastic calculations are indicated by the curve marked $(EI)_e$ where the Young's modulus was assumed as suggested by Hognestad et al[22] i.e.

$$E_b = \frac{30 \times 10^6}{6 + \frac{10,000}{\sigma_b'}}$$

and I was the second moment of the entire concrete area.
The curve $(EI)_1$ agree closely with the test results, where as the conventional elastic EI under estimate the actual curvature in all tests. The behaviour is very similar to that observed in

simple beam tests.

The curvatures corresponding to a particular load stage are plotted along the length of the beam in Figs.13.23(a) - 13.23(h). These indicate the spread of plastic hinges at different load stages at mid-span and support sections. At ultimate load the 'length of the plastic hinge' does not seem to have any relation to the point of contraflexure. Thus the inelastic rotation at each hinge may be best indicated by a single expression as in equation 7.4.

Load-deflection curves

The observed central deflection in the end and centre spans are plotted against the load in Figs. 13.7(b) - 13.14(b). These indicate that with the gradual increase in load, the stiffness of beam decreases, which in turn increases the rate of deflection. Beyond the load corresponding to the yield of support, the deflection tends to increase faster until the mid-spans yield, when the deflection increases rapidly. At the latter stages, considerable creep deflection takes place.

The experimental curves may be compared with the calculated curves based on the effective flexural stiffness of the beams as predicted by equation 6.12. As may be expected the calculations over-estimate the deflections in the 'elastic' stages, but close to first yield (at supports), the predicted values agree closely with the experimental results. The behaviour of beams after yield is closely parallled by the calculated curves except for the increase in the ultimate load due to strain hardening and other effects not accounted for in the calculations.

In beams CA5 and CA6, the repeated loads at $\lambda_s$ equal to 1.6, 1.2, 1.1 had very little influence on the deflection. However when $\lambda_s$ was less than 1.0, considerable permanent deflection has taken place. Thus, first yield may be considered as a minimum serviceability requirement from the point of view of deflection.

Observations on total rotation

An experimental verification of the total rotation in indeterminate structures offers considerable difficulty in comparing the measured results with the idealised calculations. Upto first yield the calculated total rotation in segments AC, CE, EE' (Fig.13.6) given by equations 13.3 are compared with the corresponding experimental values in Figs.13.25(a) - 13.25(h). These are indicated by the full lines. Beyond the first yield, the amount of inelastic rotation at a hinge at any load cannot be calculated using the bilinear idealisation. Thus the dotted line indicates the measured total rotation after first yield plotted against the calculated elastic rotation in between the plastic hinges. The relative deviation of the curves from the lines $\theta_{exp} = \theta_{cal}$ could be regarded as a measure of the inelastic rotation at the hinge. In none of the beams, has the inelastic rotation exceeded the permissible limit.

These diagrams show that in general, all the beams are stiffer than estimated at early stages of loading, but close to first yield, the calculated total rotations are very close to those observed. This behaviour of continuous beams is similar to that of simple beams[8, 34] and demonstrates the validity of the idealised assumptions in the theoretical calculations presented in Chapter 6.

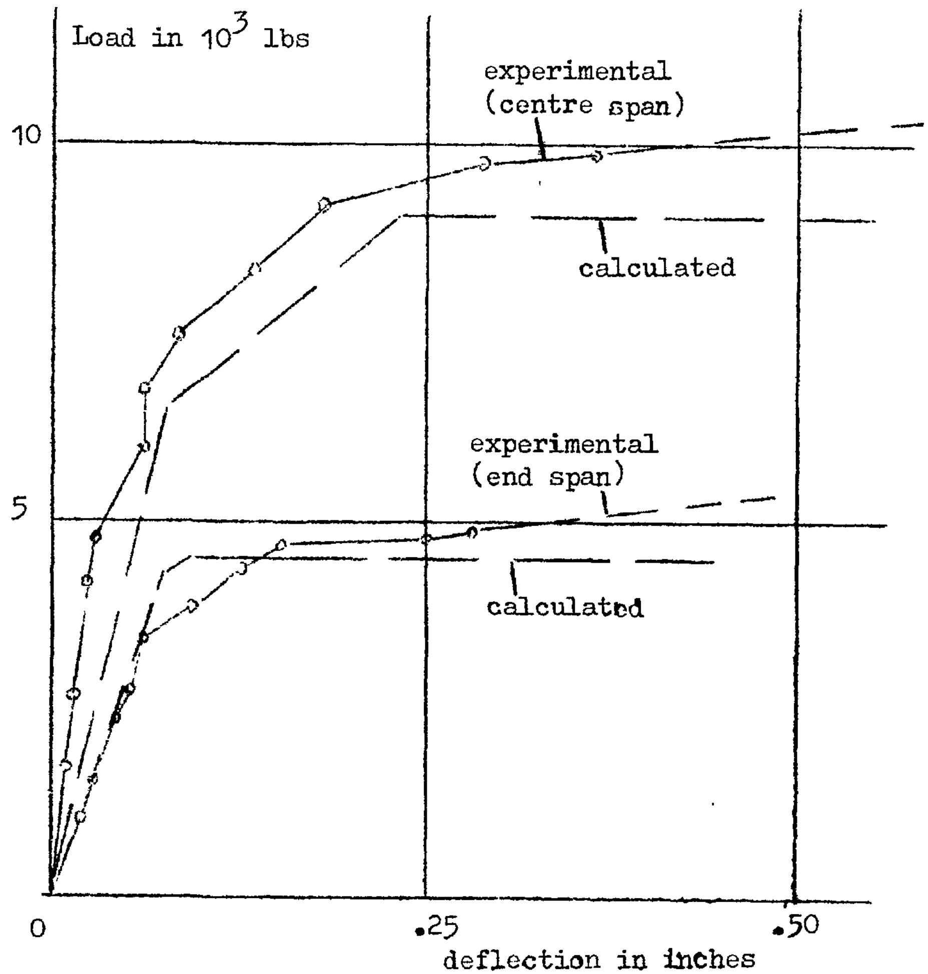



Fig. 13.7 (b). Beam No. CA 1

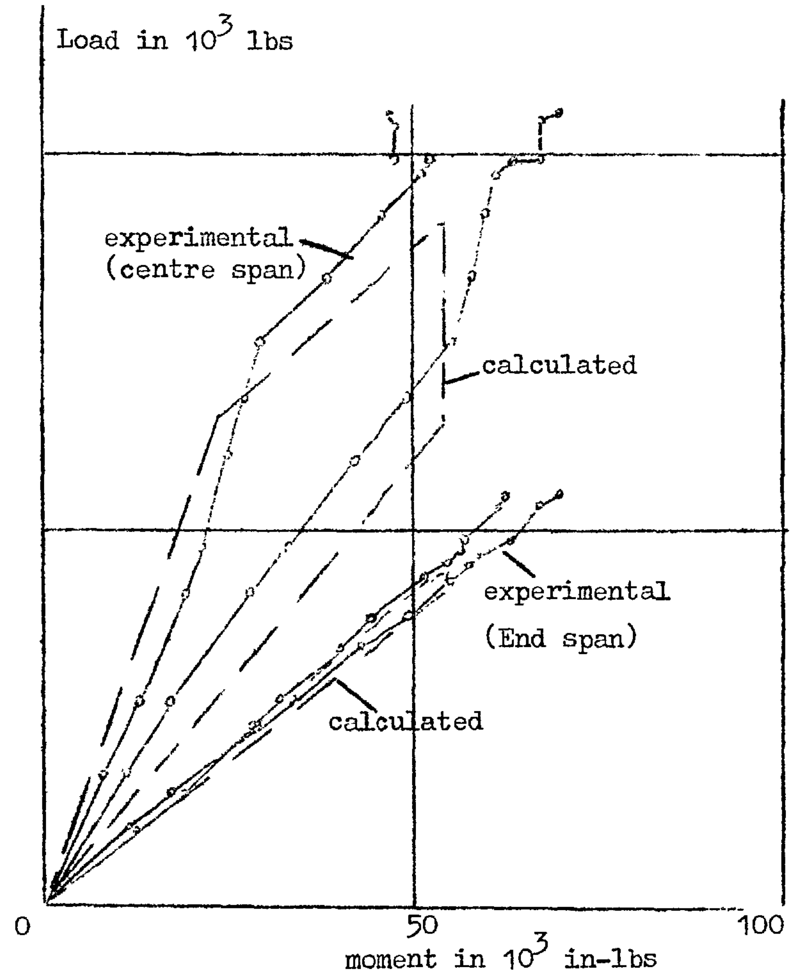



Fig. 13.7 (a). Beam No. CA 1

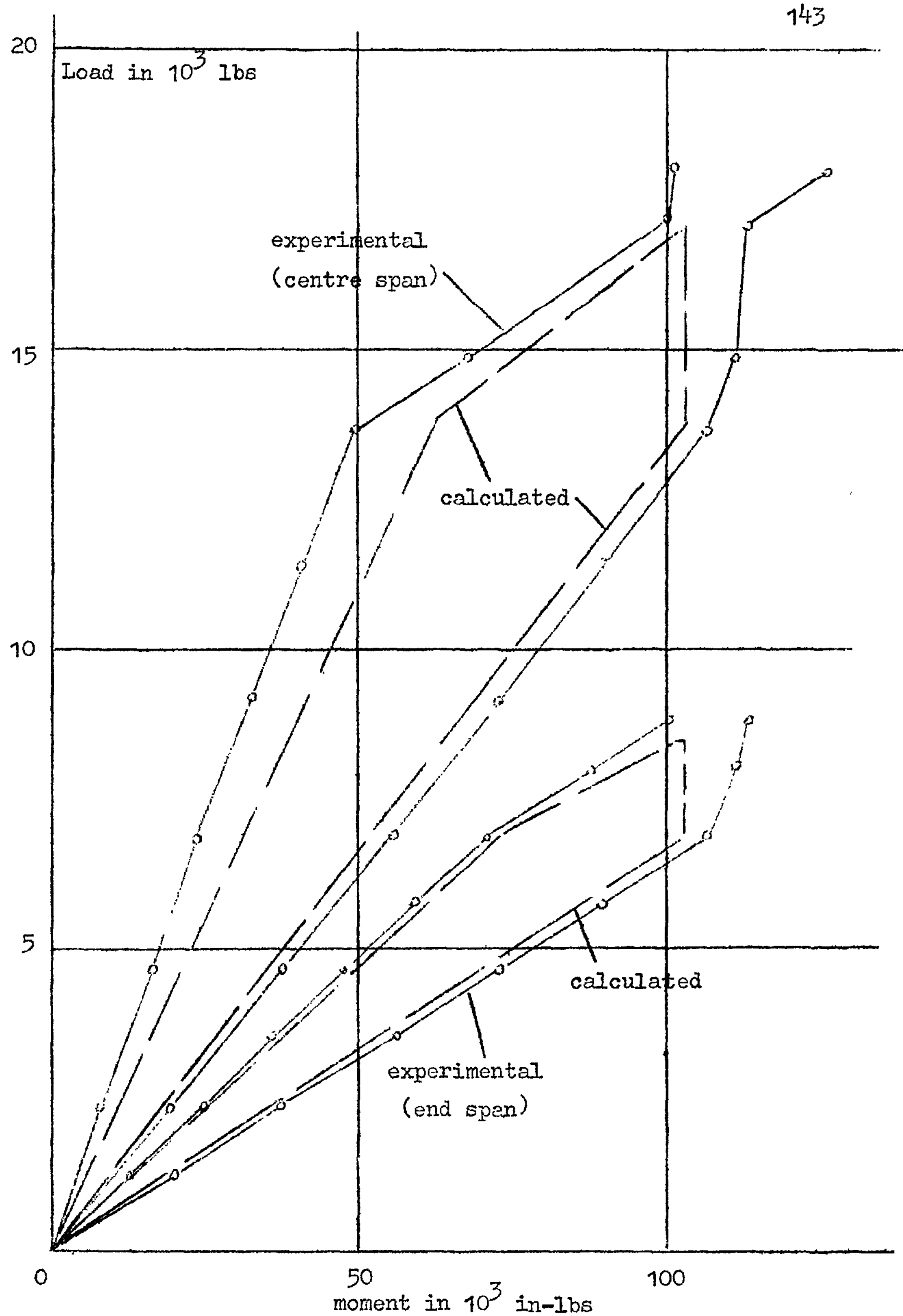



Fig. 13. 8 (a). Beam No. CA 2

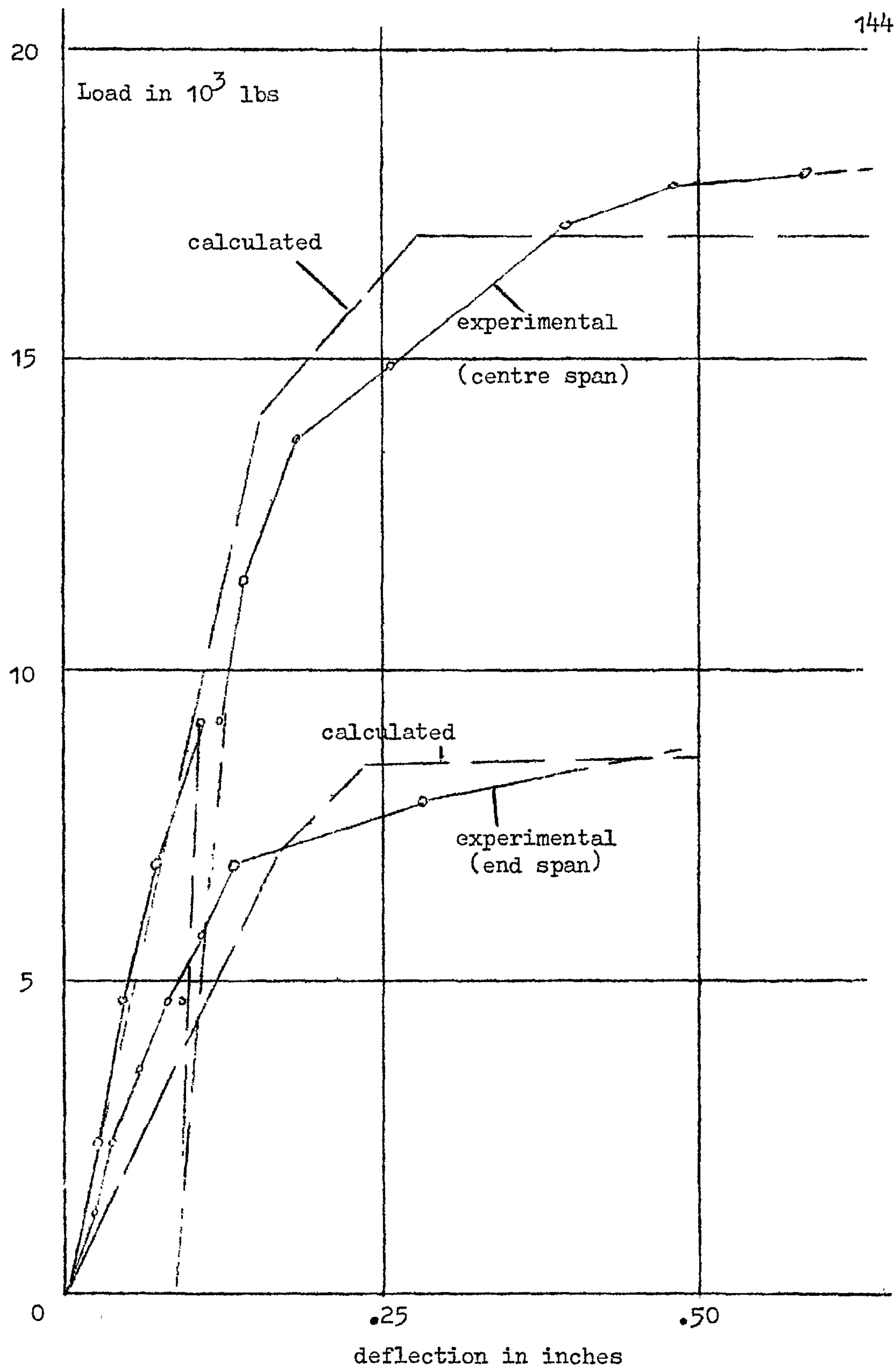



Fig. 13.8 (b) Beam No. CA 2

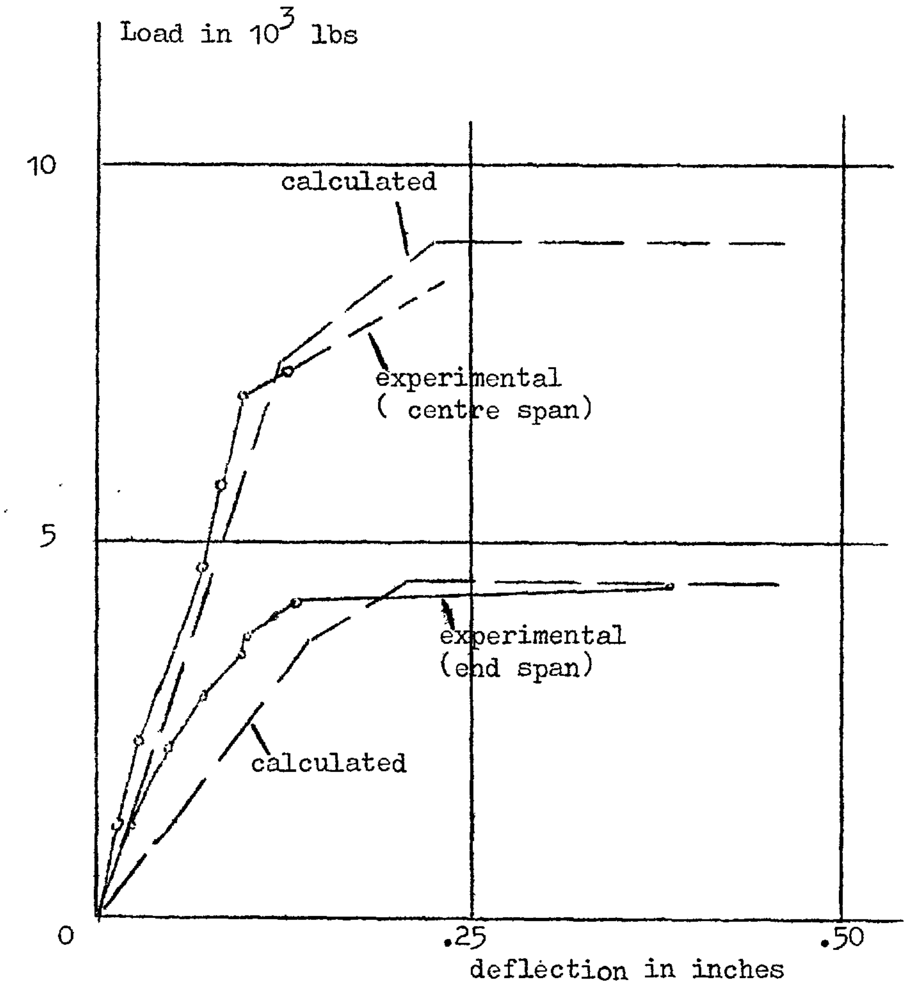



Fig. 13.9 (b). Beam No. CA 3

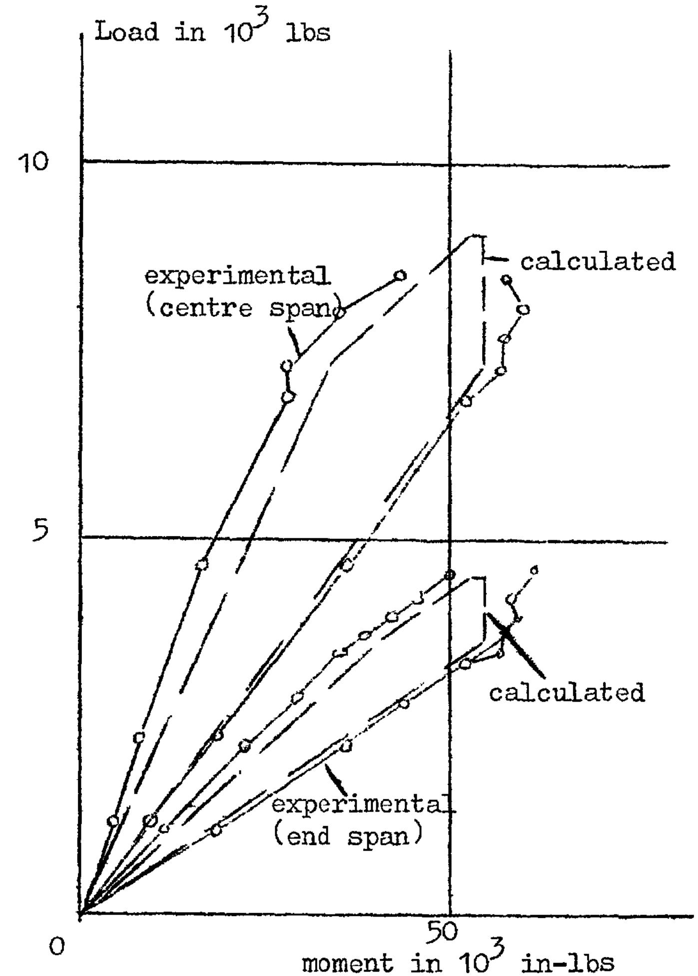



Fig. 13.9 (a). Beam No. CA 3

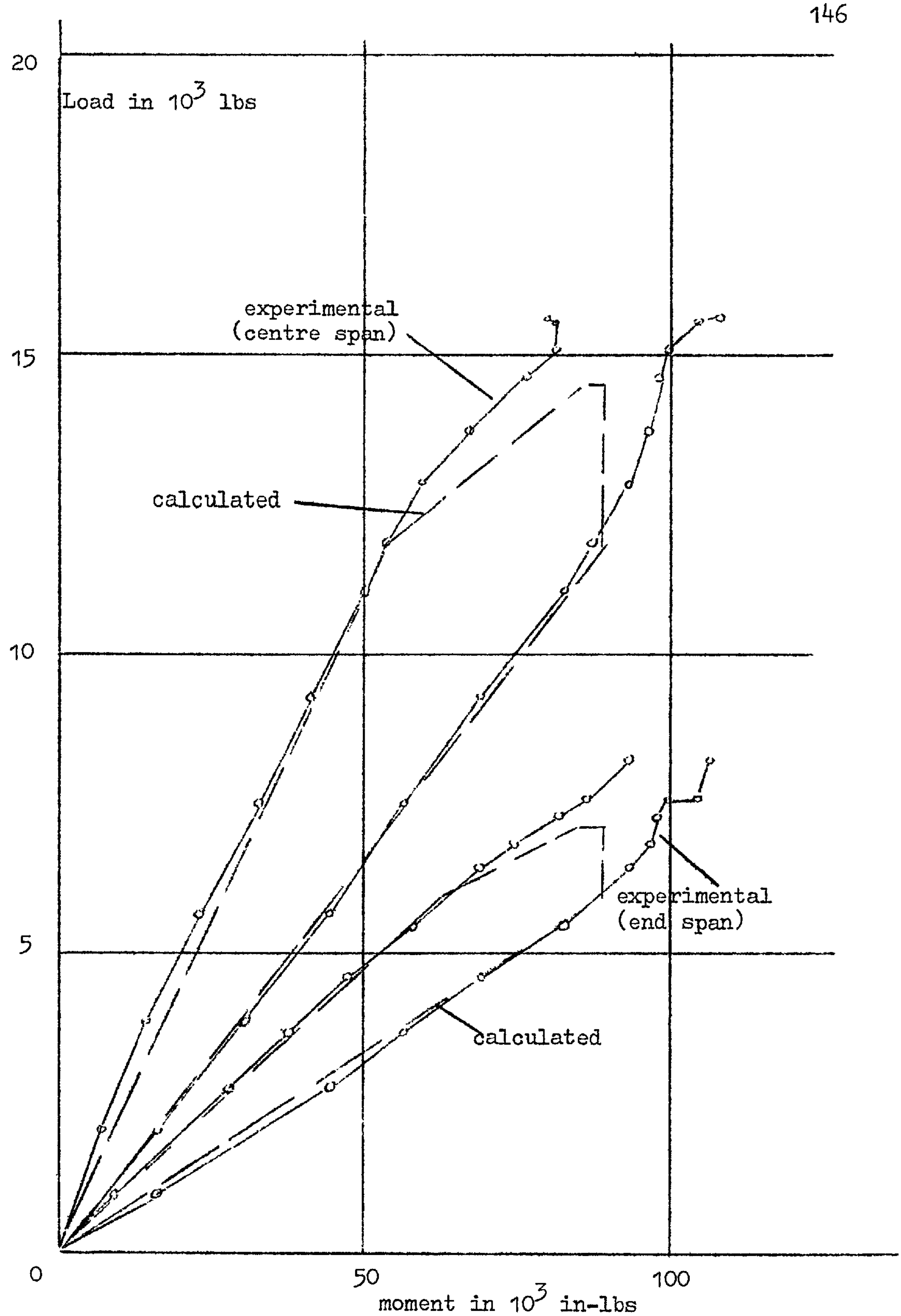



Fig. 13.10 (a). Beam No. CA 4.

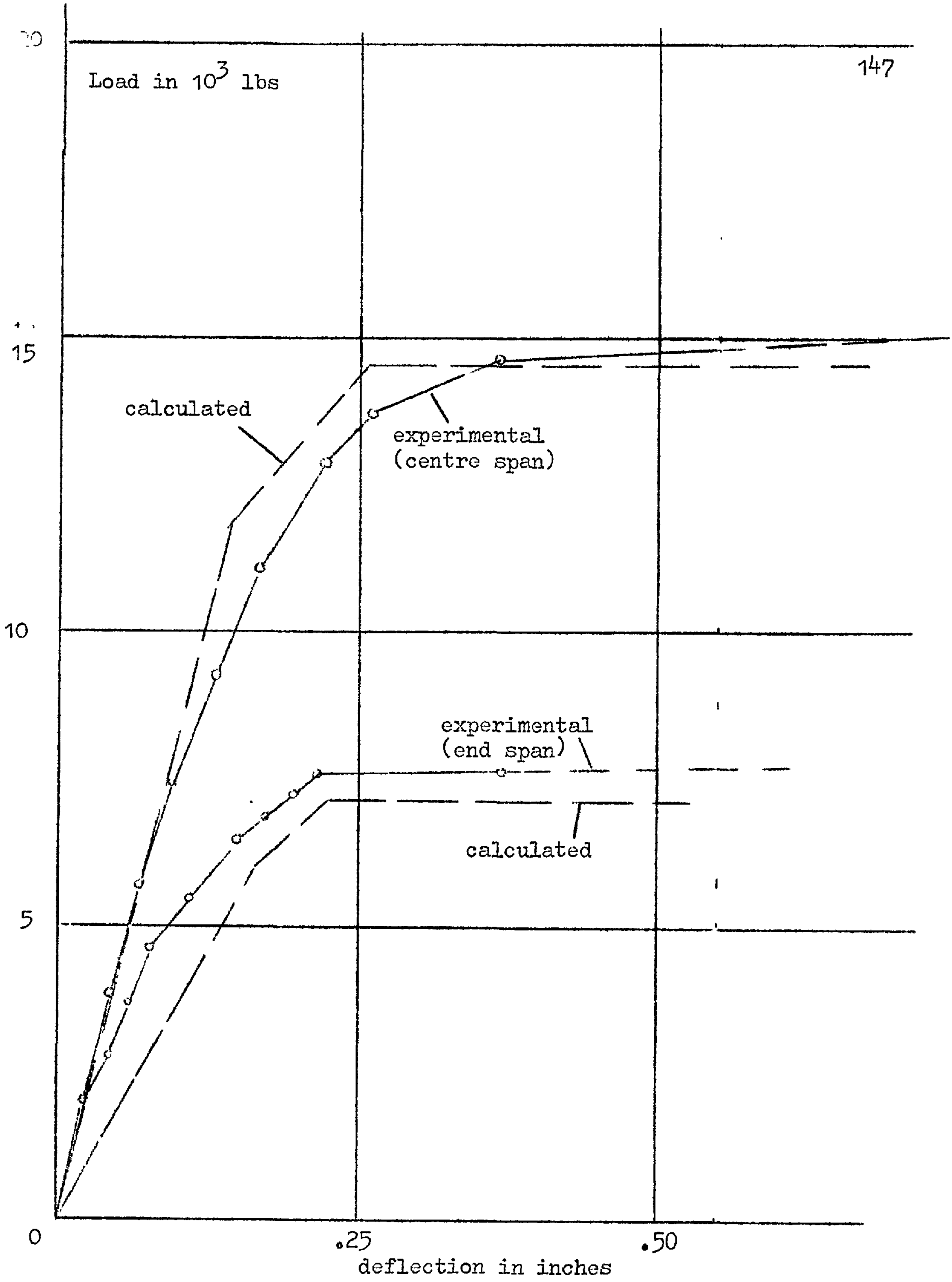



Fig. 13.10 (b). Beam No. CA 4.

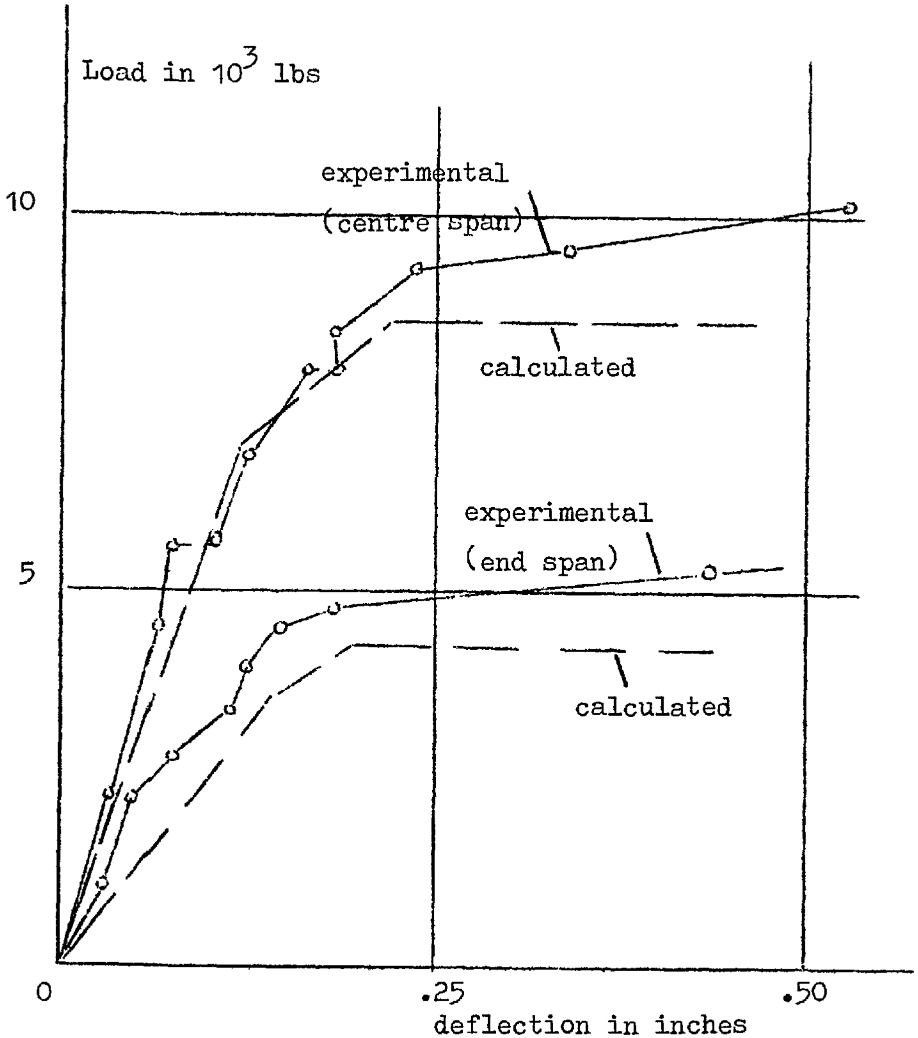



Fig. 13.11 (b). Beam No. CA 5.

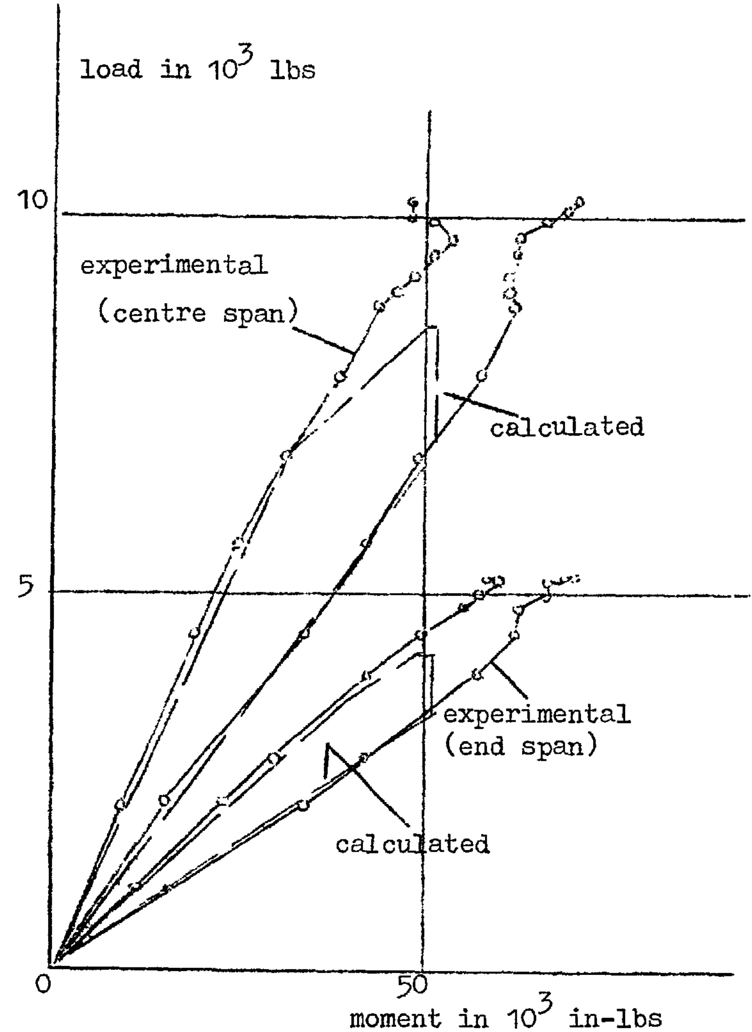



Fig. 13.11 (a). Beam No. CA 5.

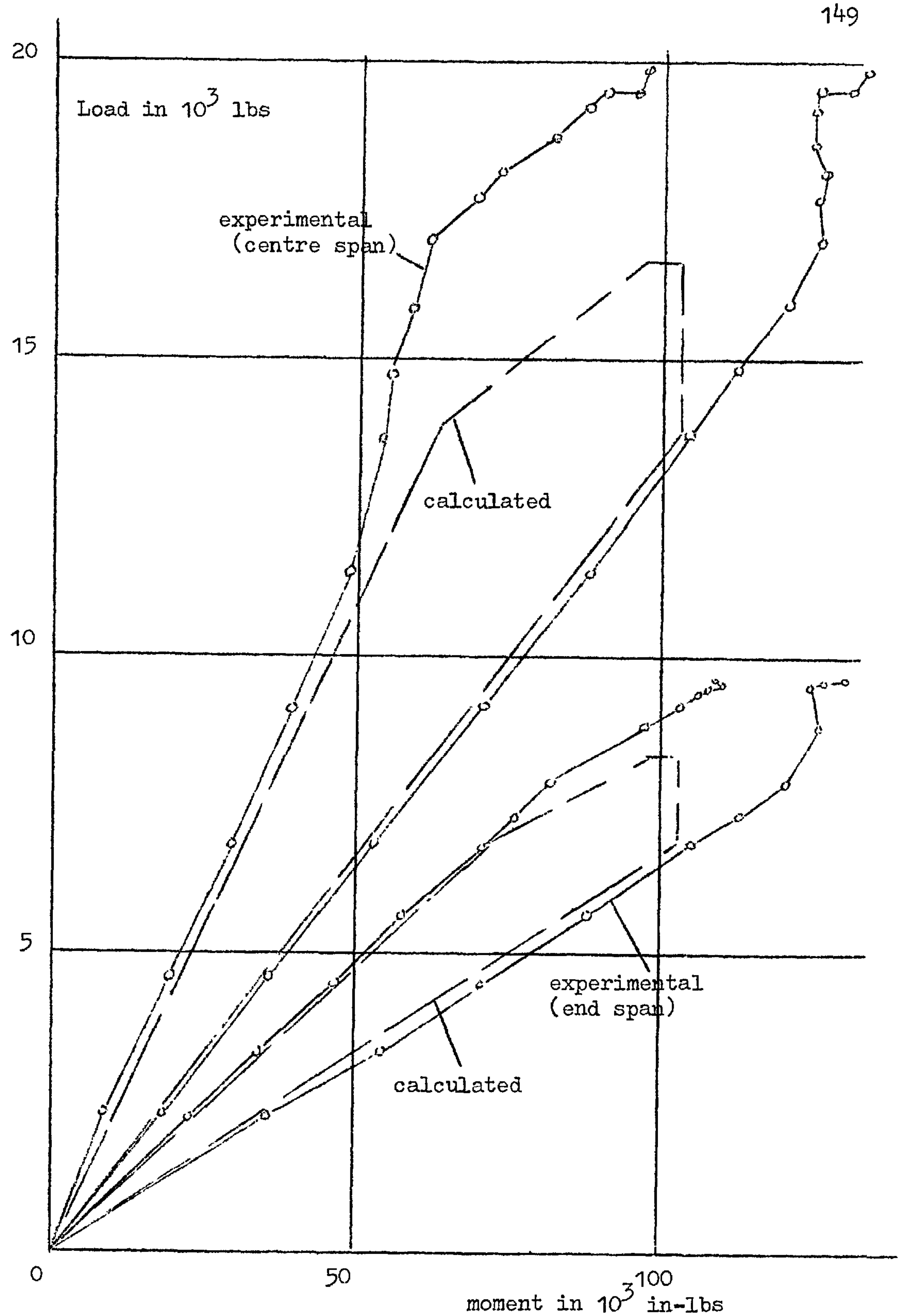



Fig. 13.12 (a). Beam No. CA 6.

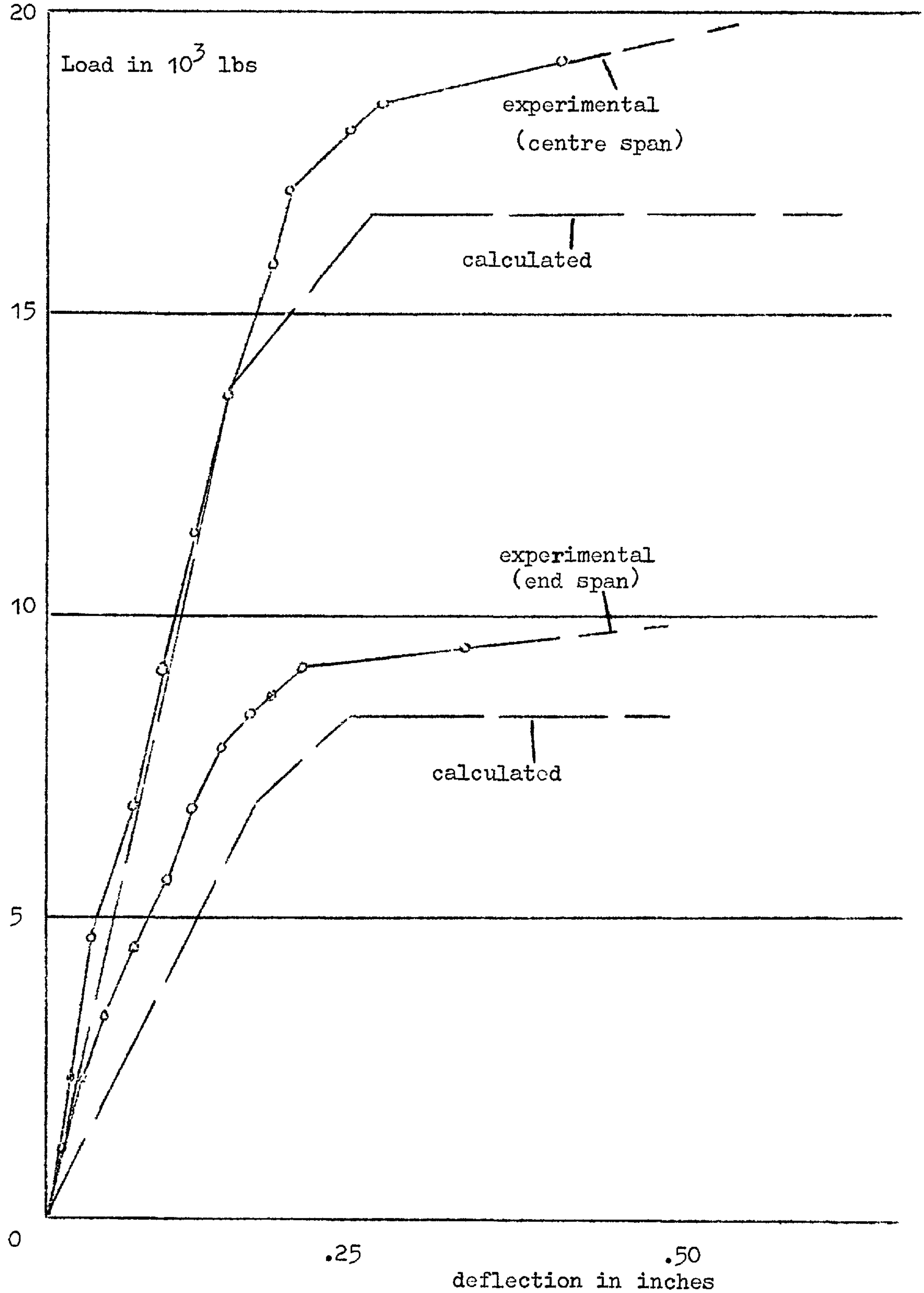



Fig. 13. 12 (b) Beam No. CA 6.

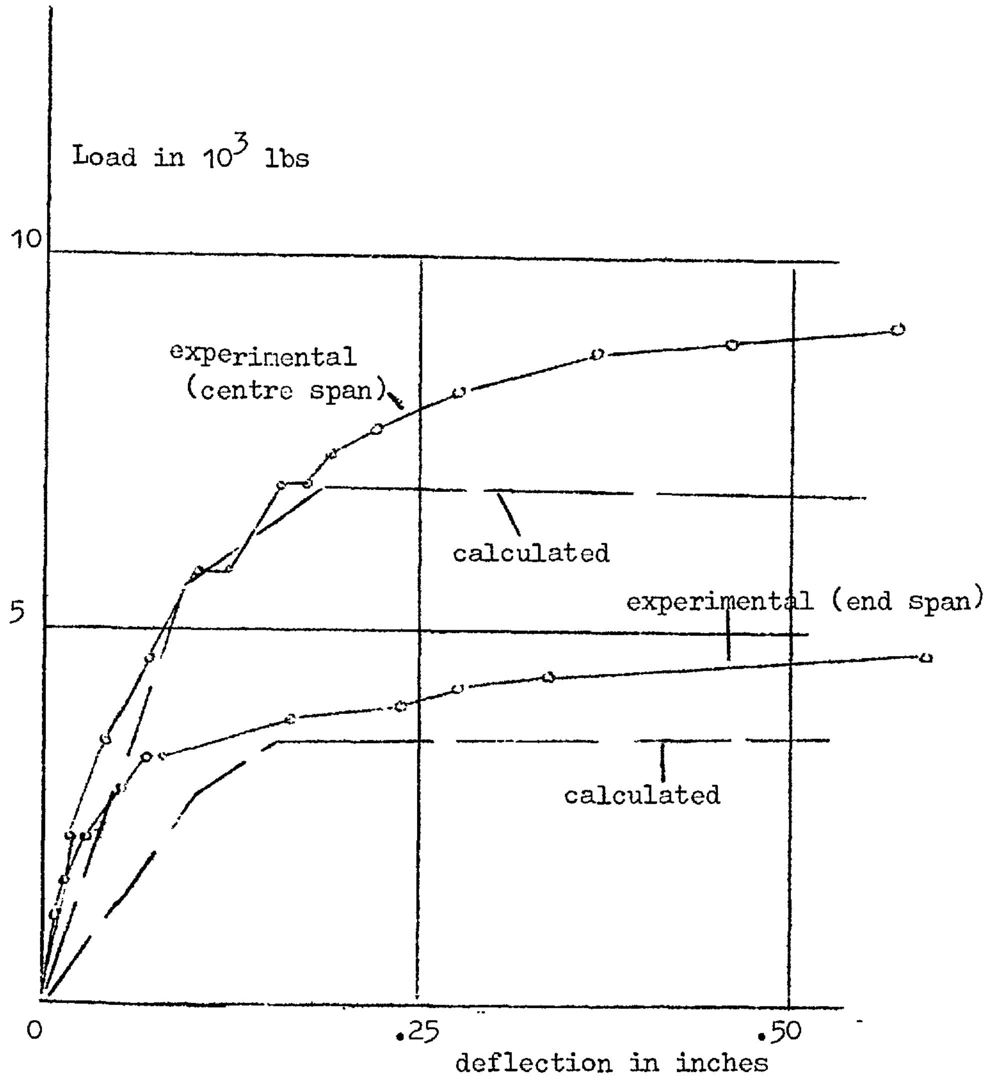



Fig. 13.13 (b). Beam No. CA 7.

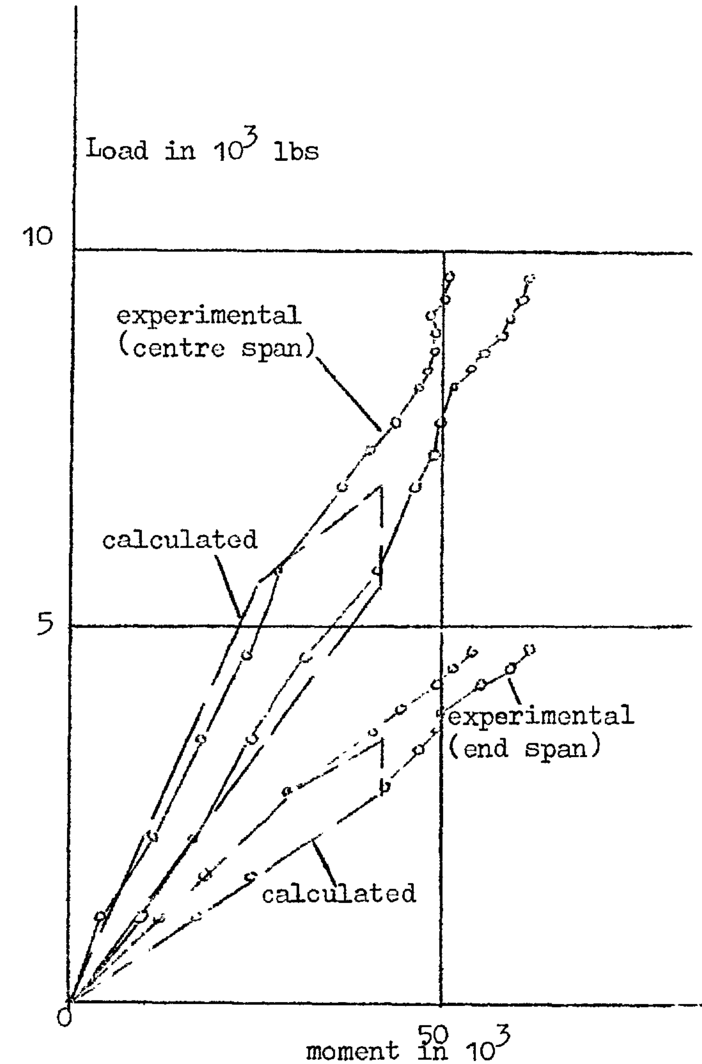



Fig. 13.13 (a). Beam No. CA 7.

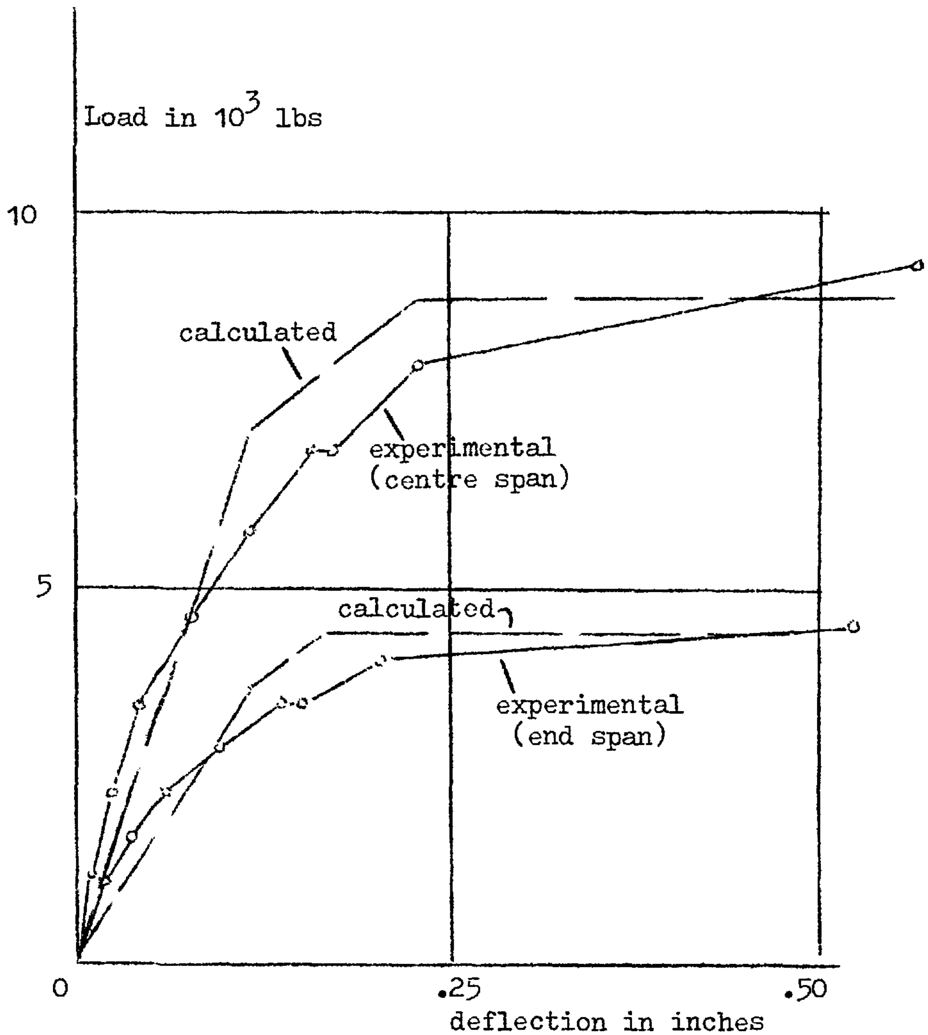



Fig. 13.14 (b). Beam No. CA 8.

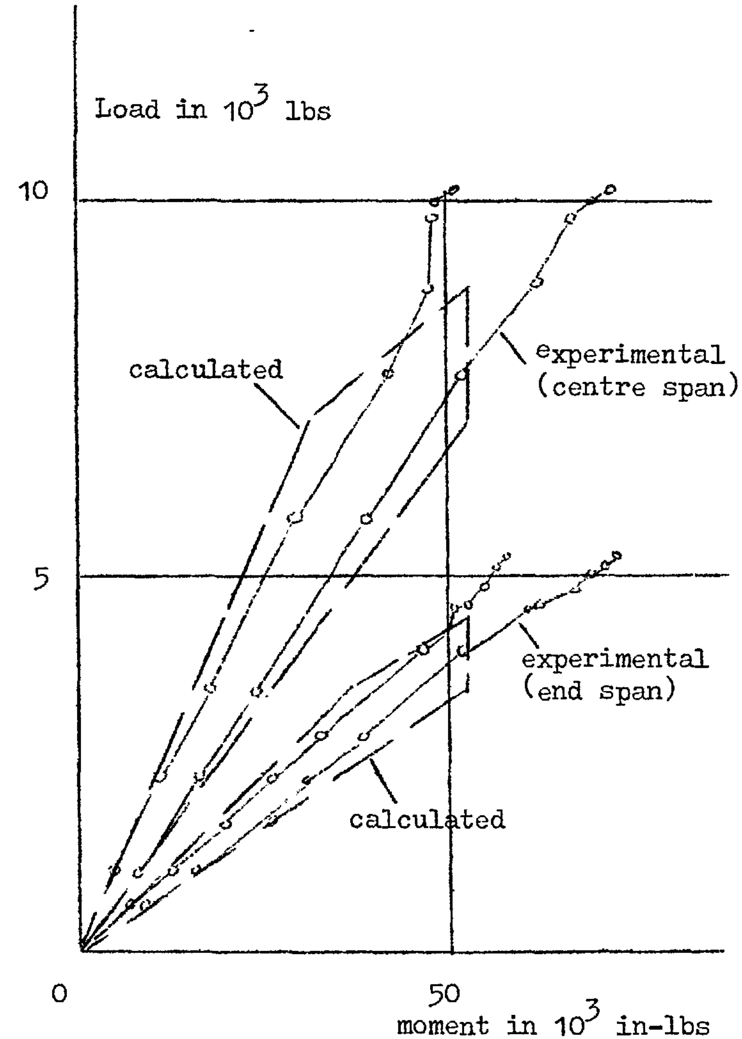



Fig. 13.14 (a). Beam No. CA 8.

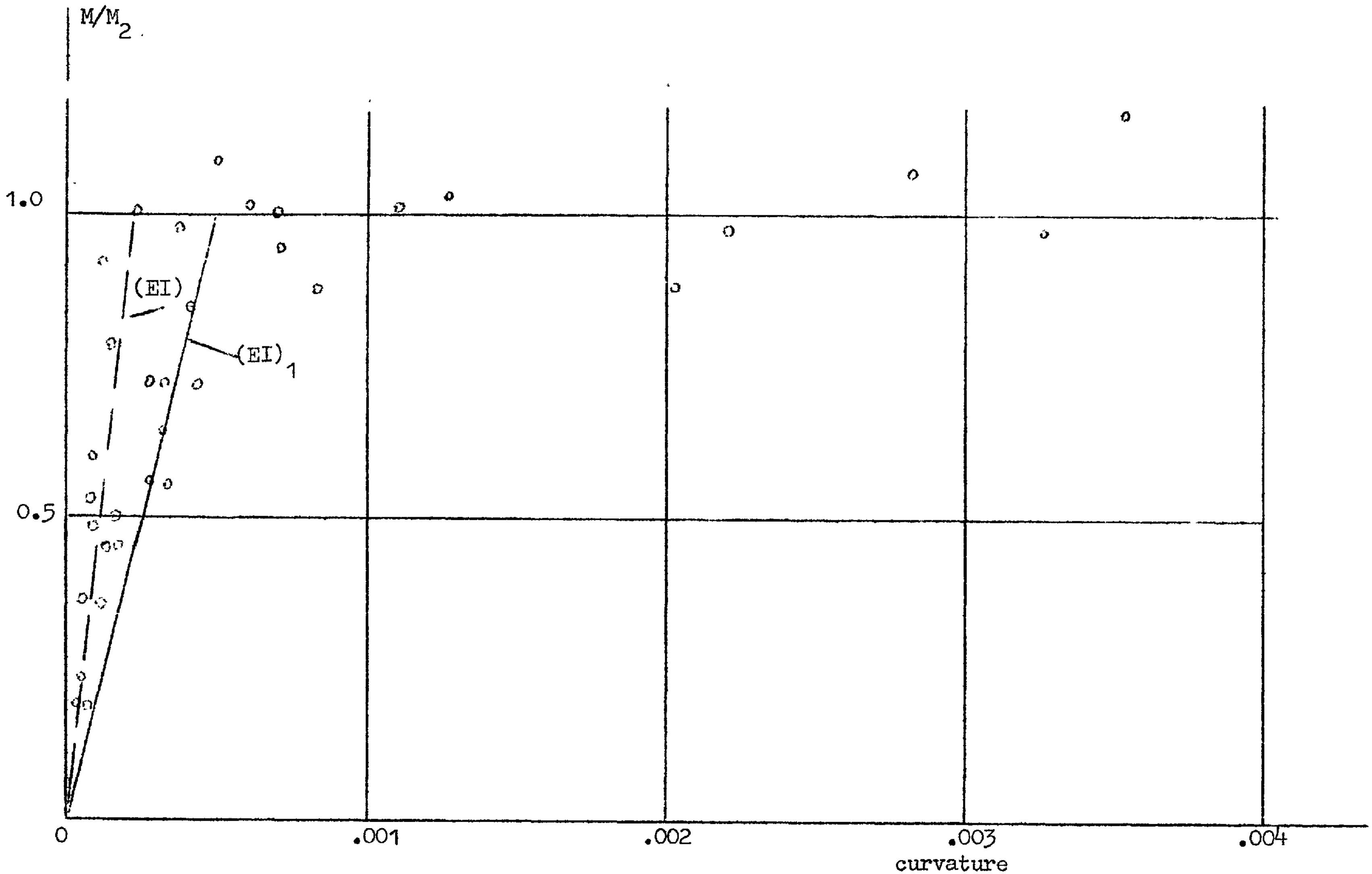



Fig. 13.15. Moment Curvature Diagram. Beam No. CA 1.

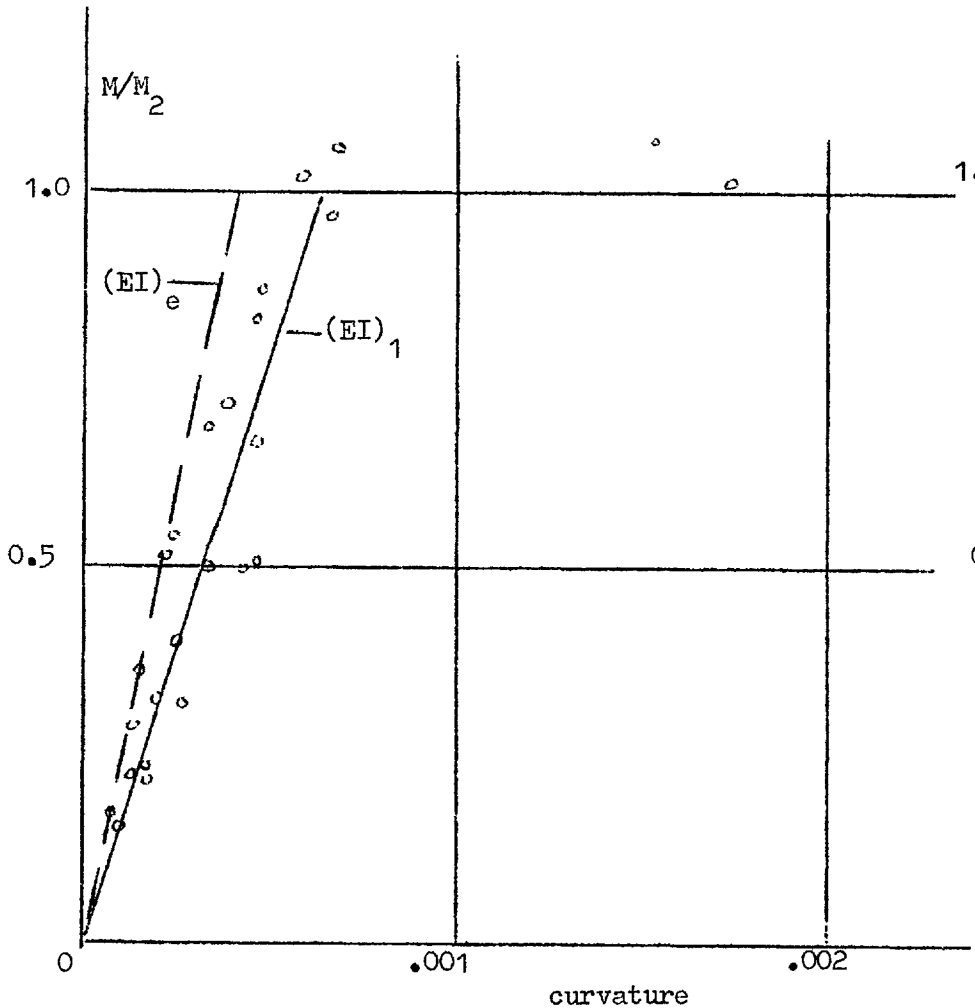



Fig. 13.16. Moment Curvature Diagram. Beam No. CA 2.

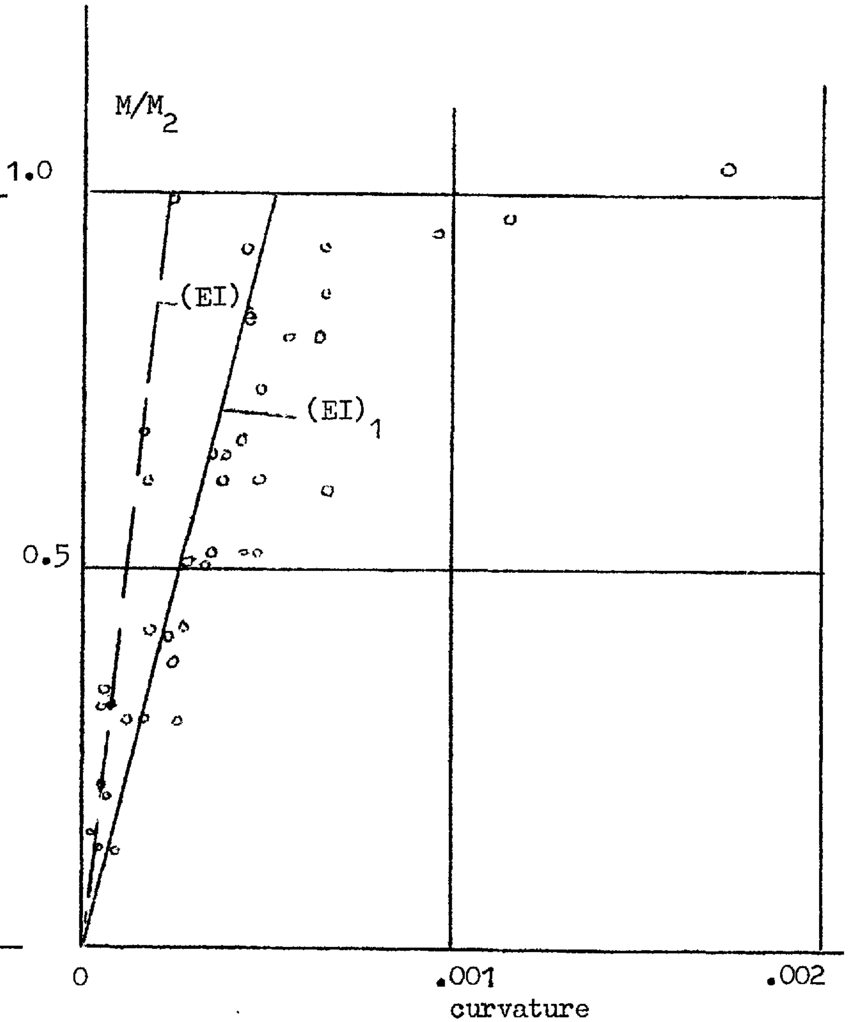



Fig. 13.17. Moment Curvature Diagram. Beam No. CA 3.

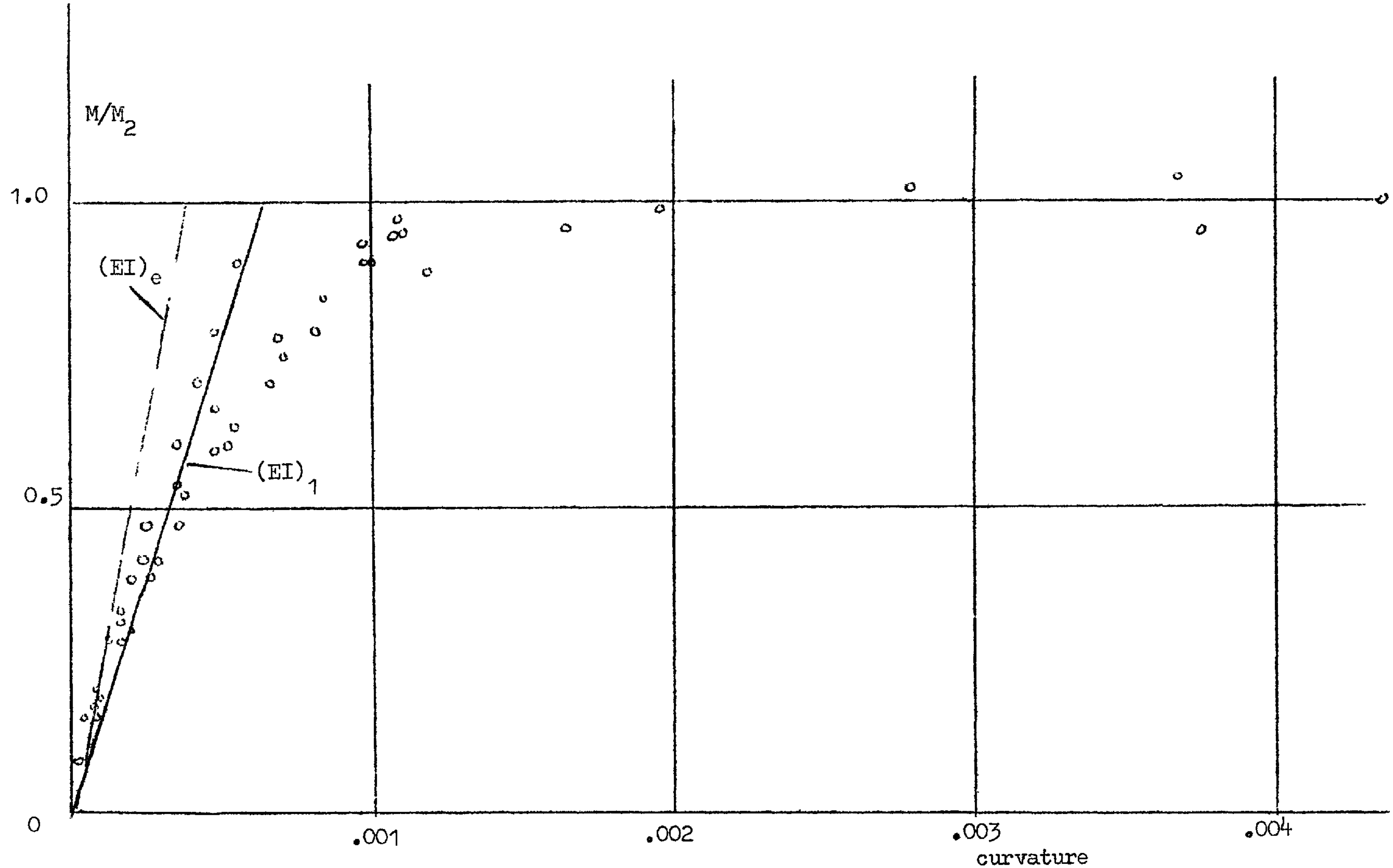



Fig. 13.18. Moment Curvature Diagram. Beam No. CA 4.

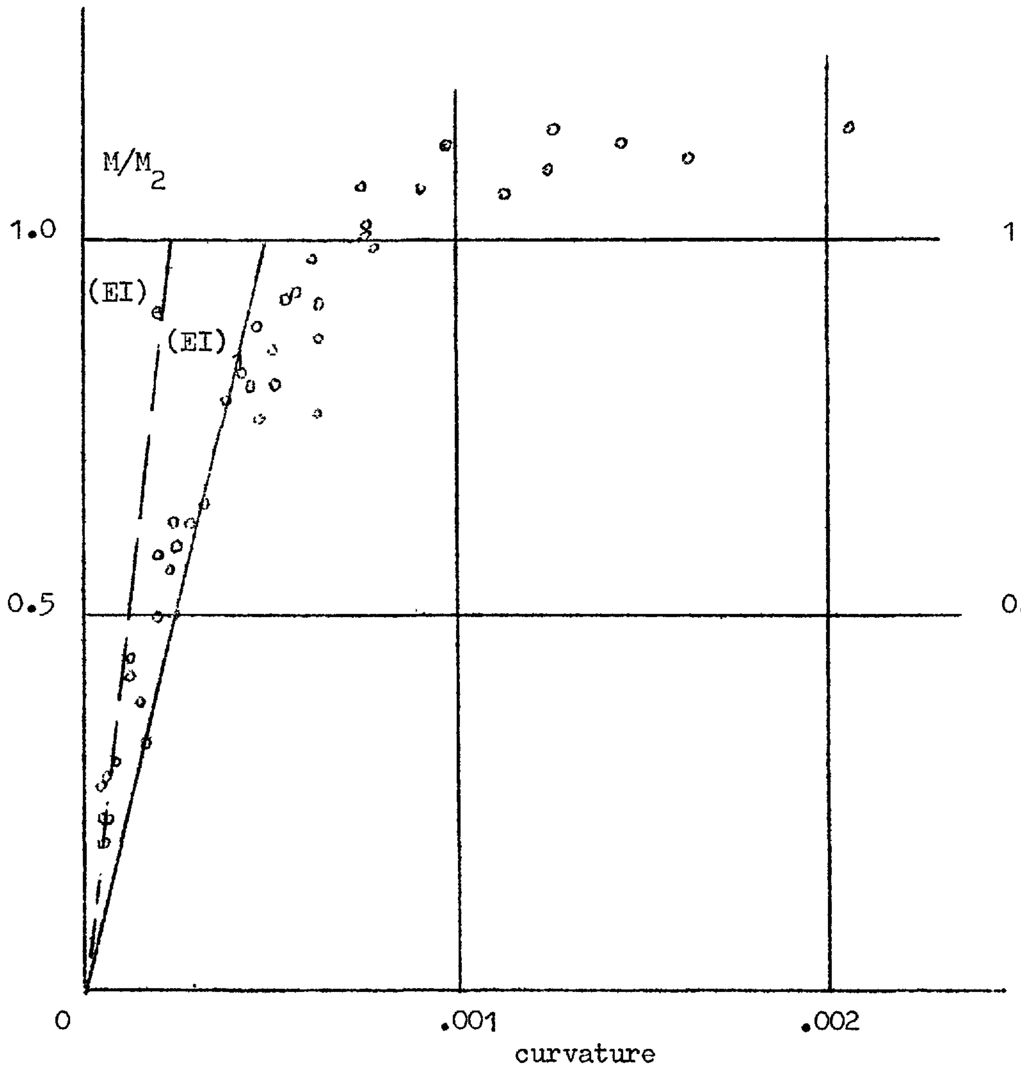



Fig. 13.19. Moment Curvature Diagram. Beam No. CA 5.

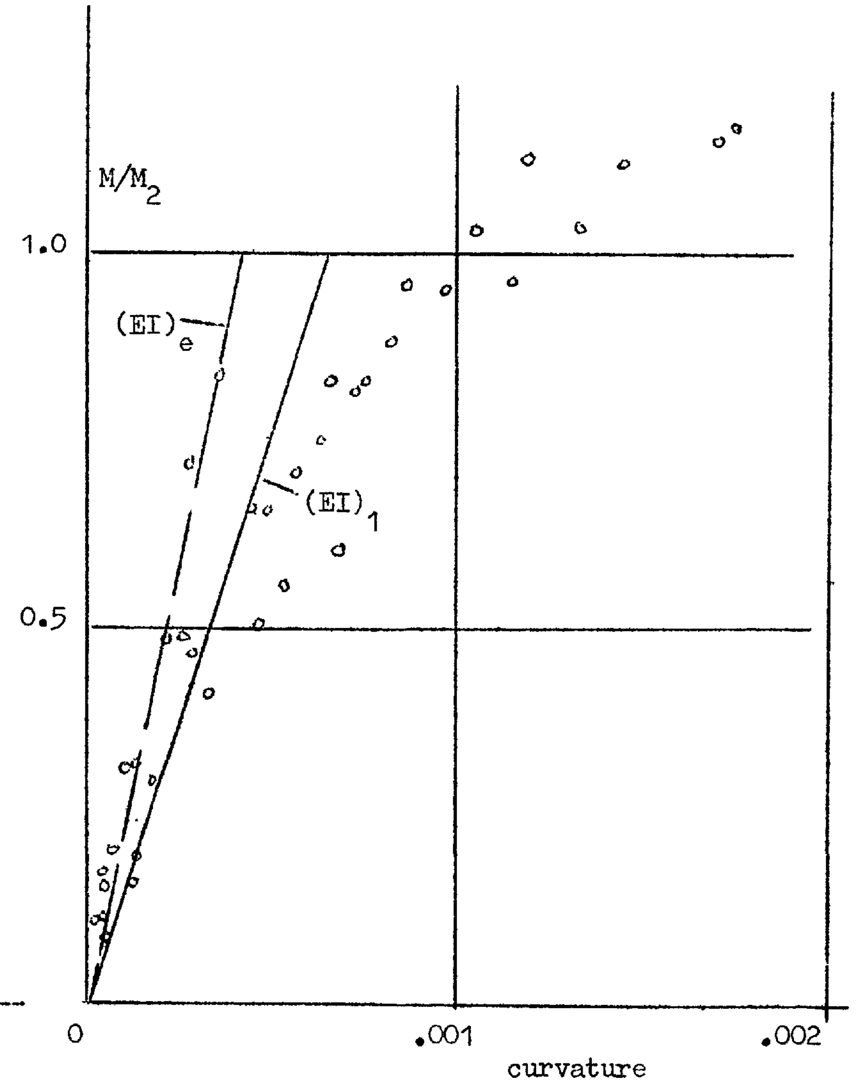



Fig. 13.20. Moment Curvature Diagram. Beam No. CA 6

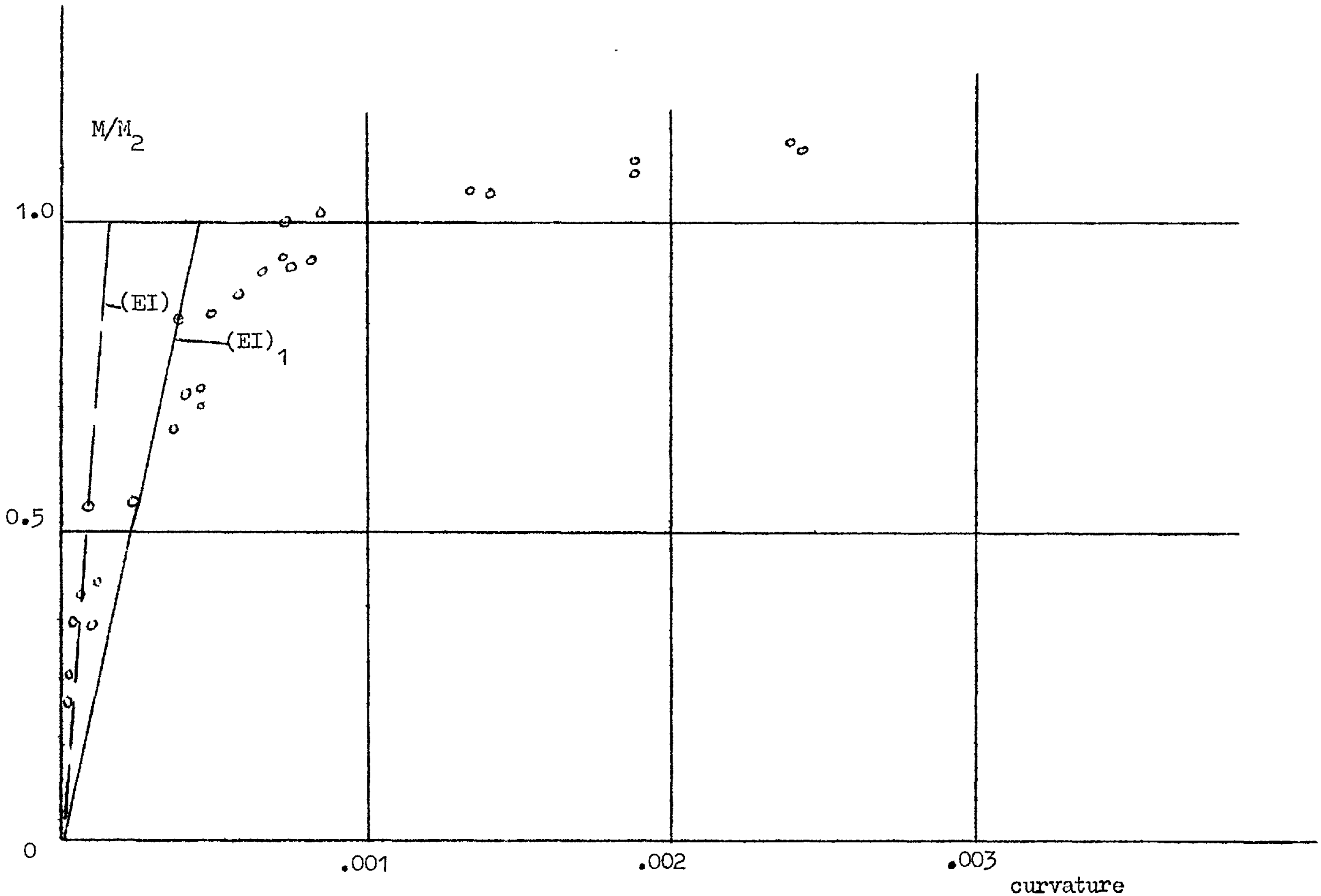



Fig. 13.21. Moment - Curvature Diagram. Beam No. CA 7.

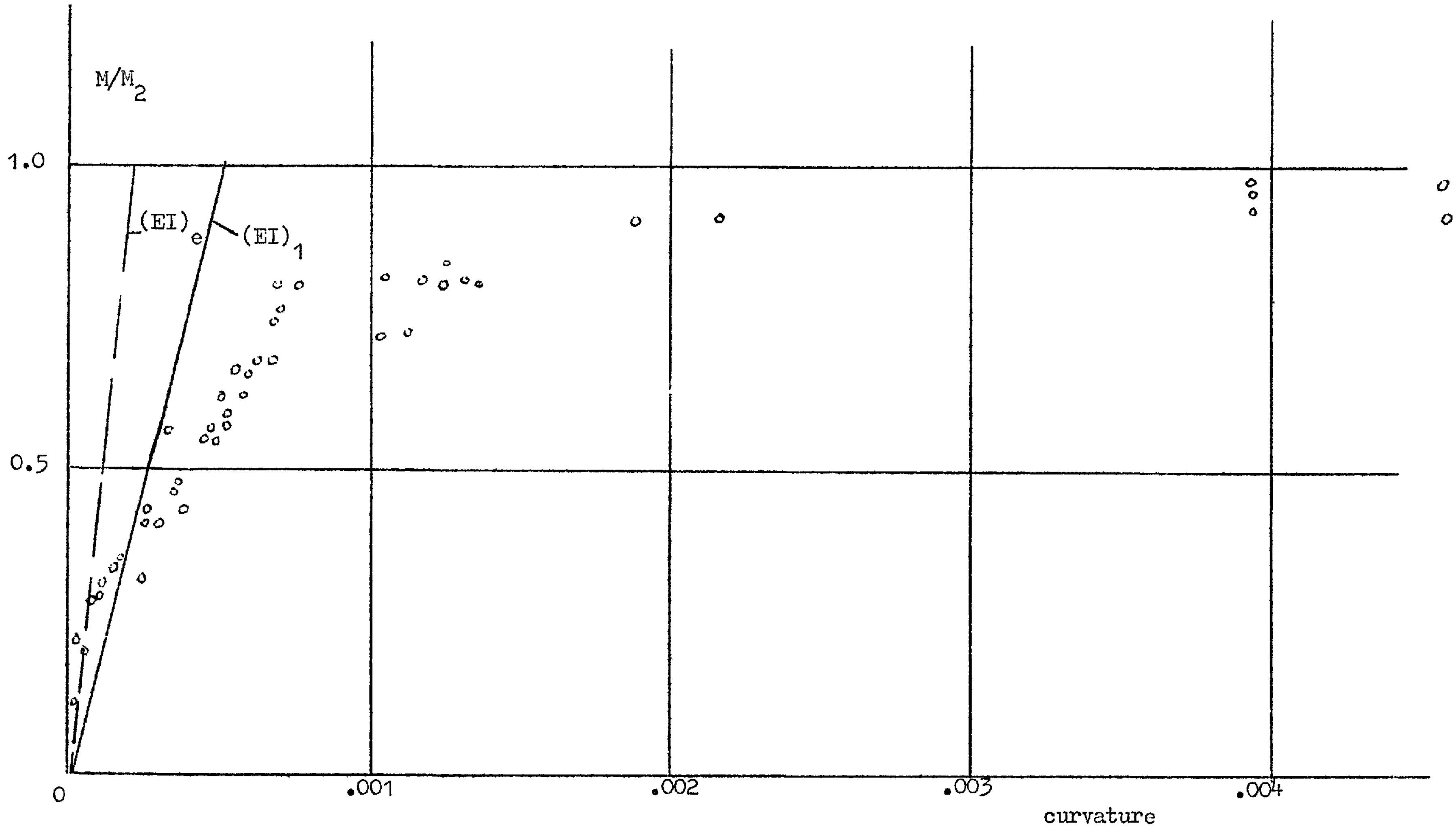



Fig. 13.22. Moment - Curvature Diagram Beam No. CA8.

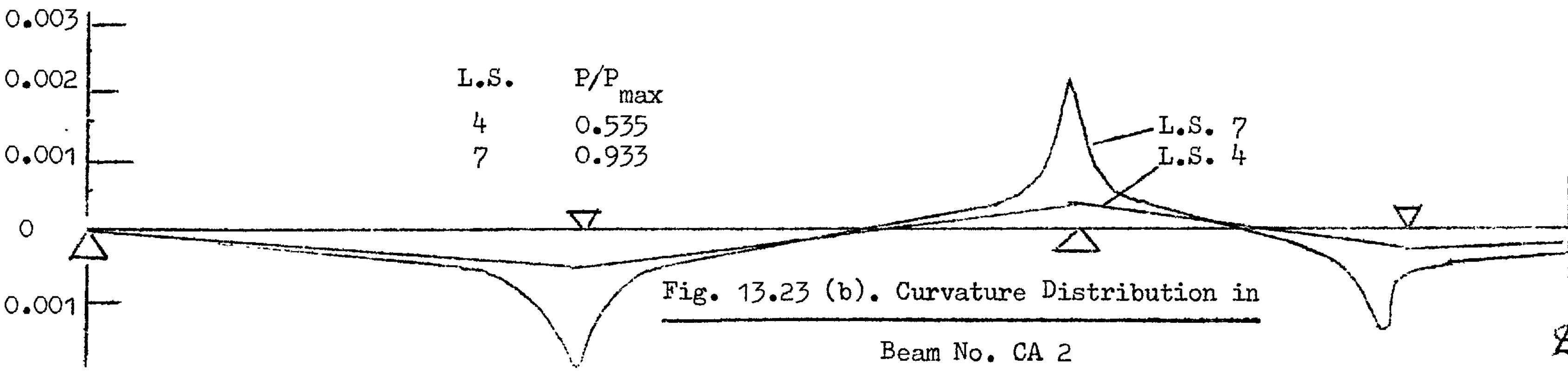



Fig. 13.23 (b). Curvature Distribution in Beam No. CA 2

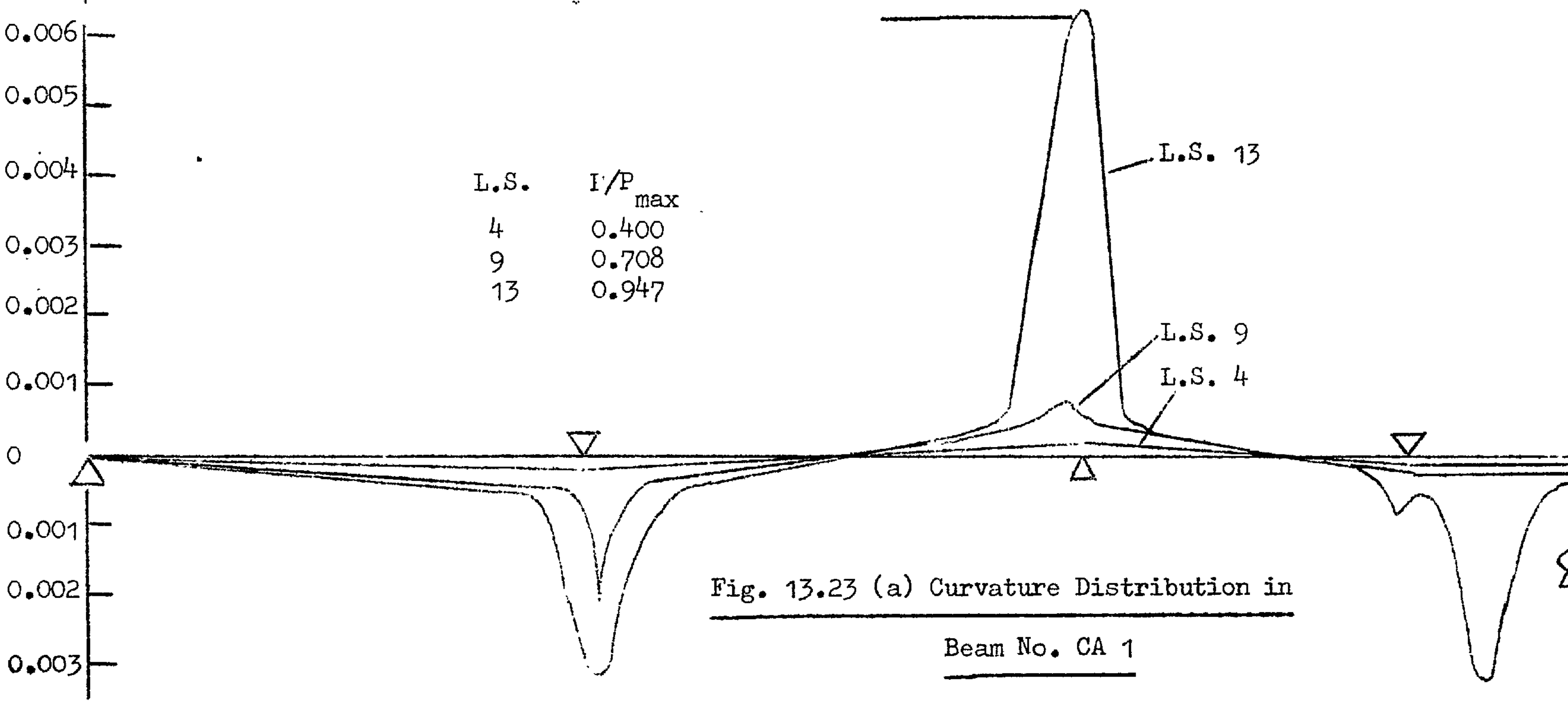



Fig. 13.23 (a) Curvature Distribution in Beam No. CA 1

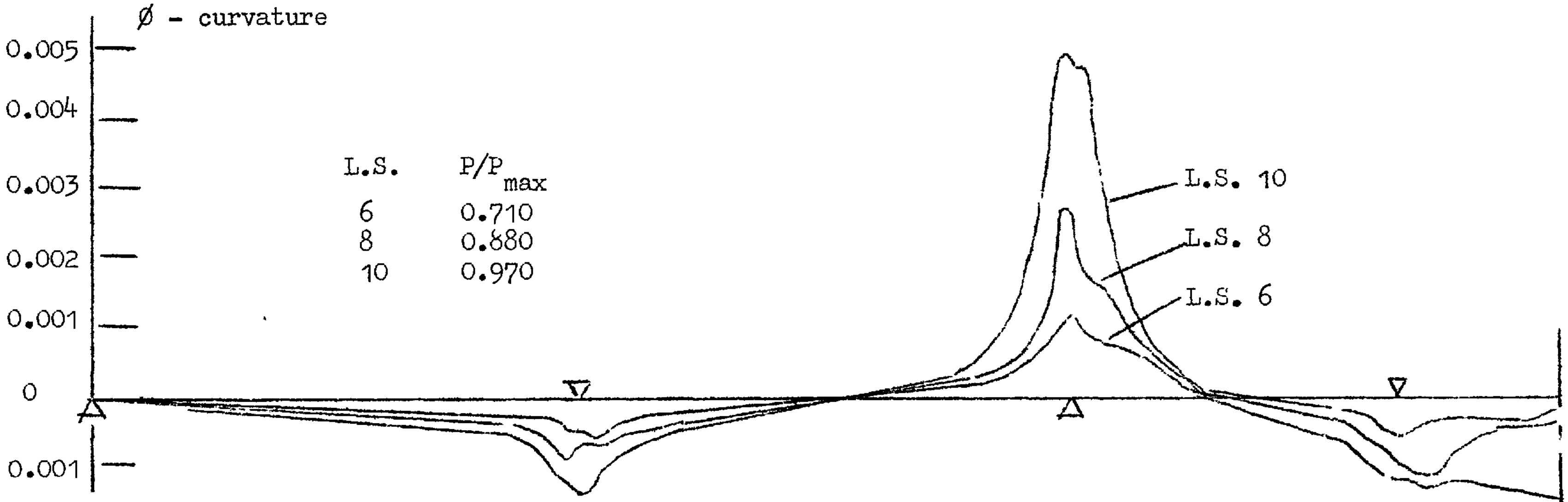



Fig. 13.23 (d) Curvature Distribution in Beam No. CA 4

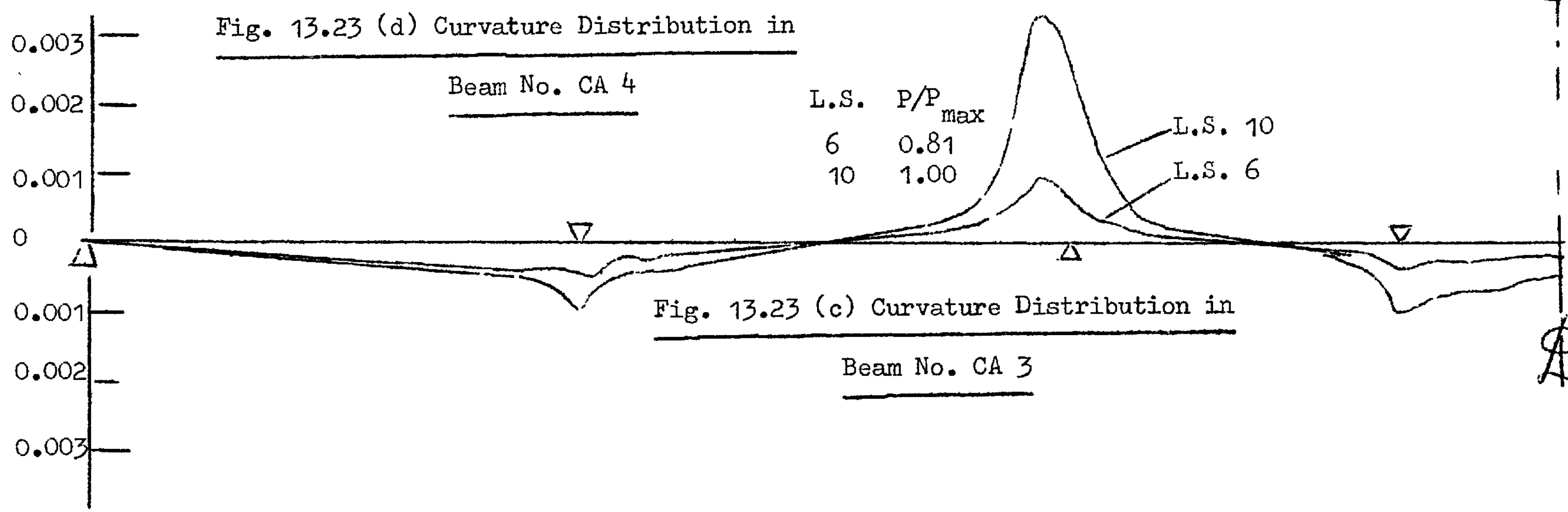



Fig. 13.23 (c) Curvature Distribution in Beam No. CA 3

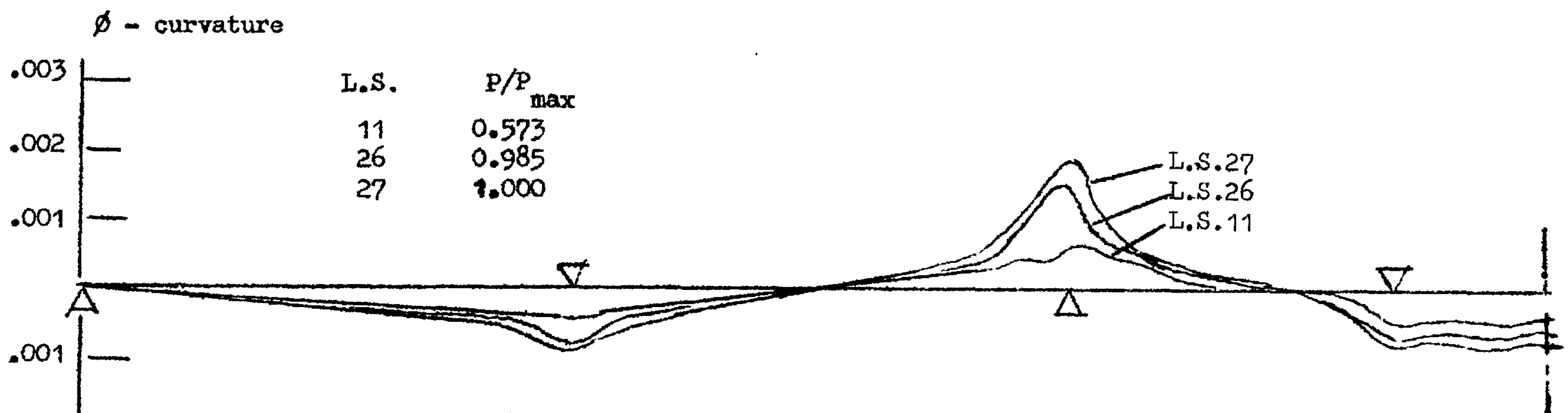



Fig. 13.23 (f). Curvature Distribution in Beam No. CA6.


Fig. 13.23 (e). Curvature Distribution in Beam No. CA5

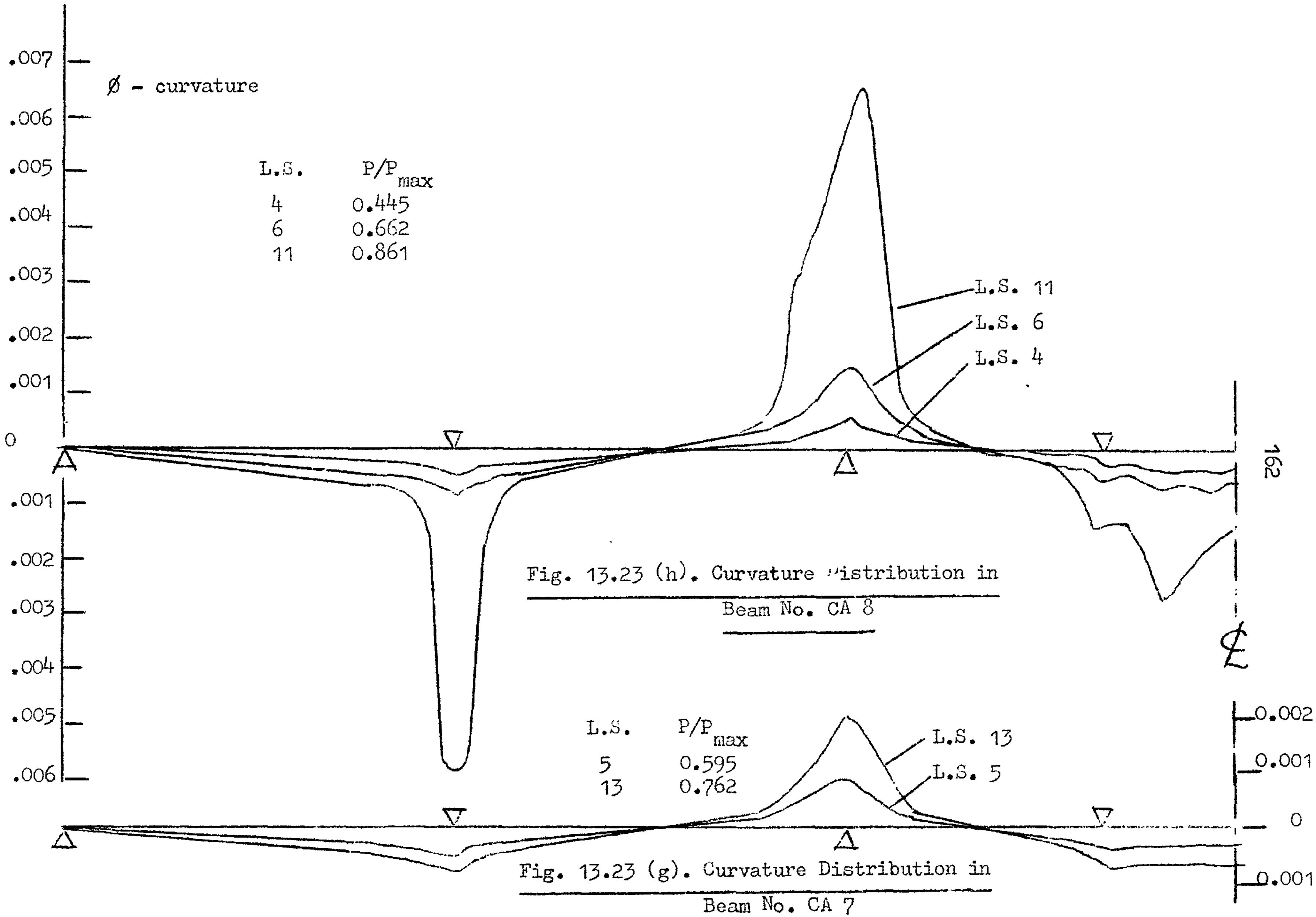



Fig. 13.23 (h). Curvature Distribution in Beam No. CA 8

Fig. 13.23 (g). Curvature Distribution in Beam No. CA 7

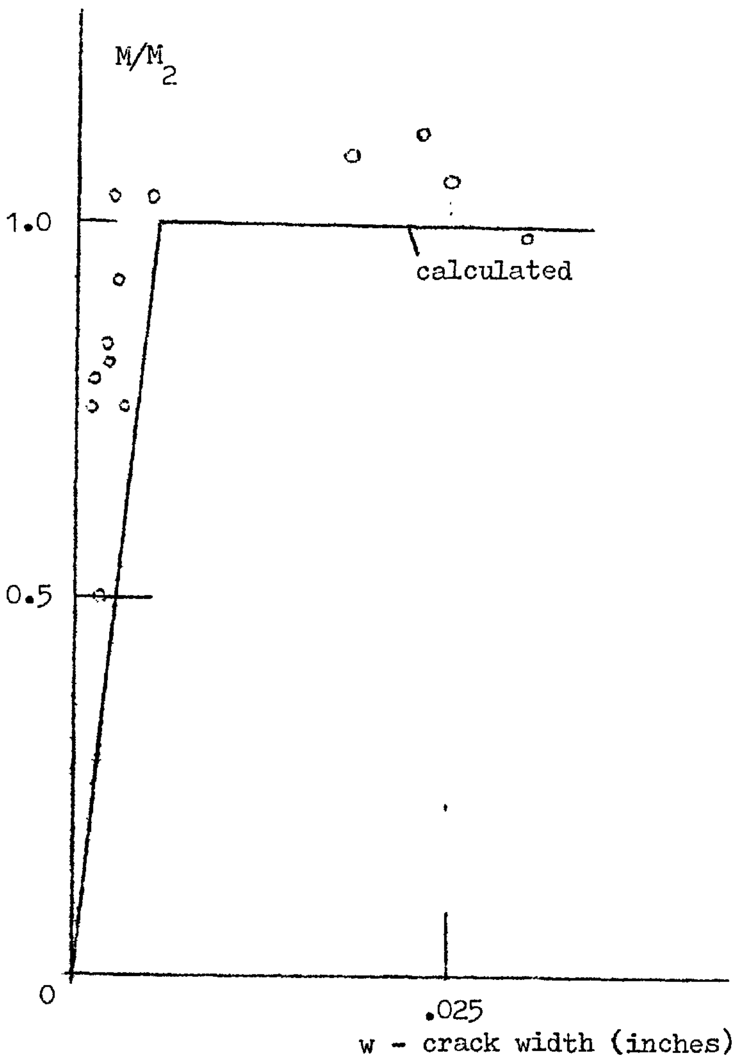



Fig. 13.24 (a). Beam No. CA 1.

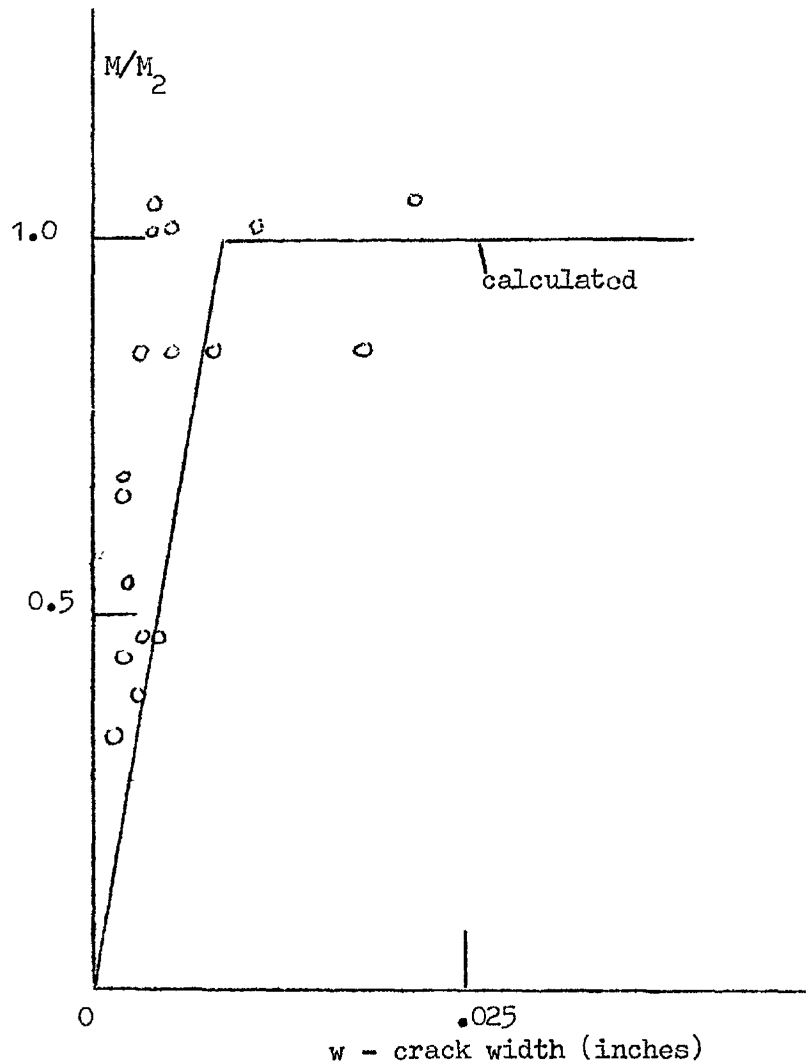



Fig. 13.24 (b). Beam No. CA 2.

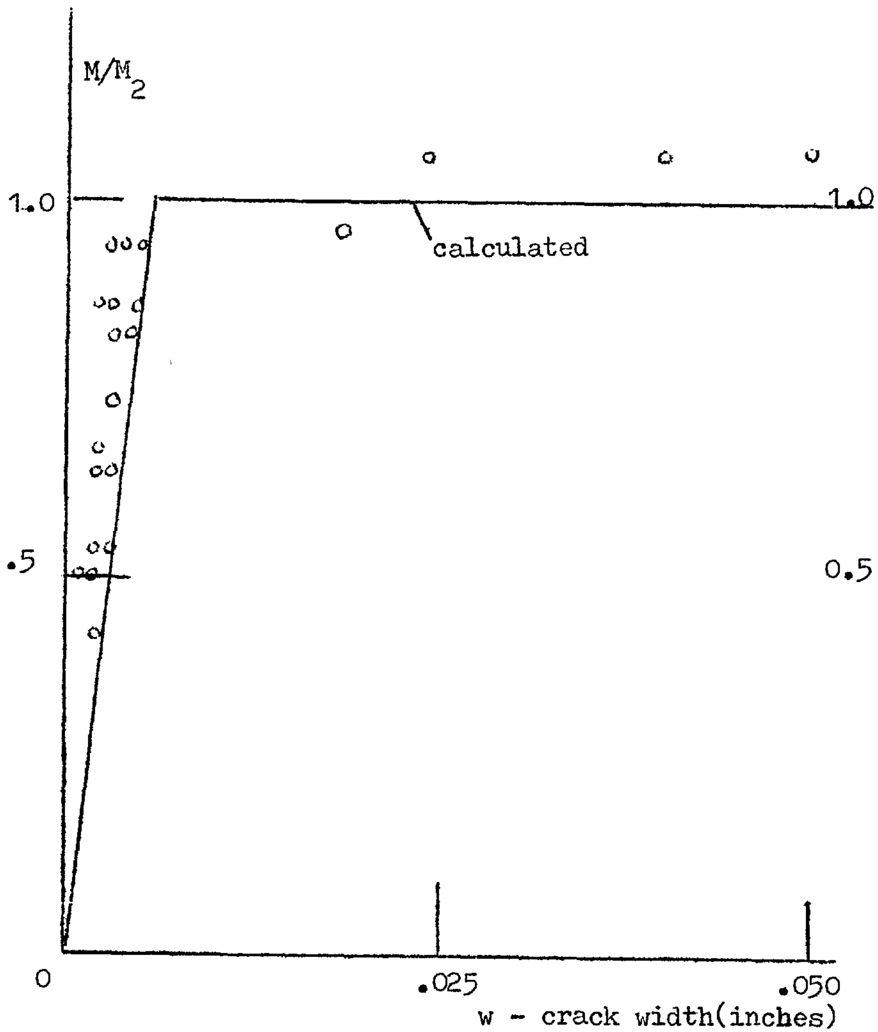



Fig. 13.24 (c). Beam No. CA 3.

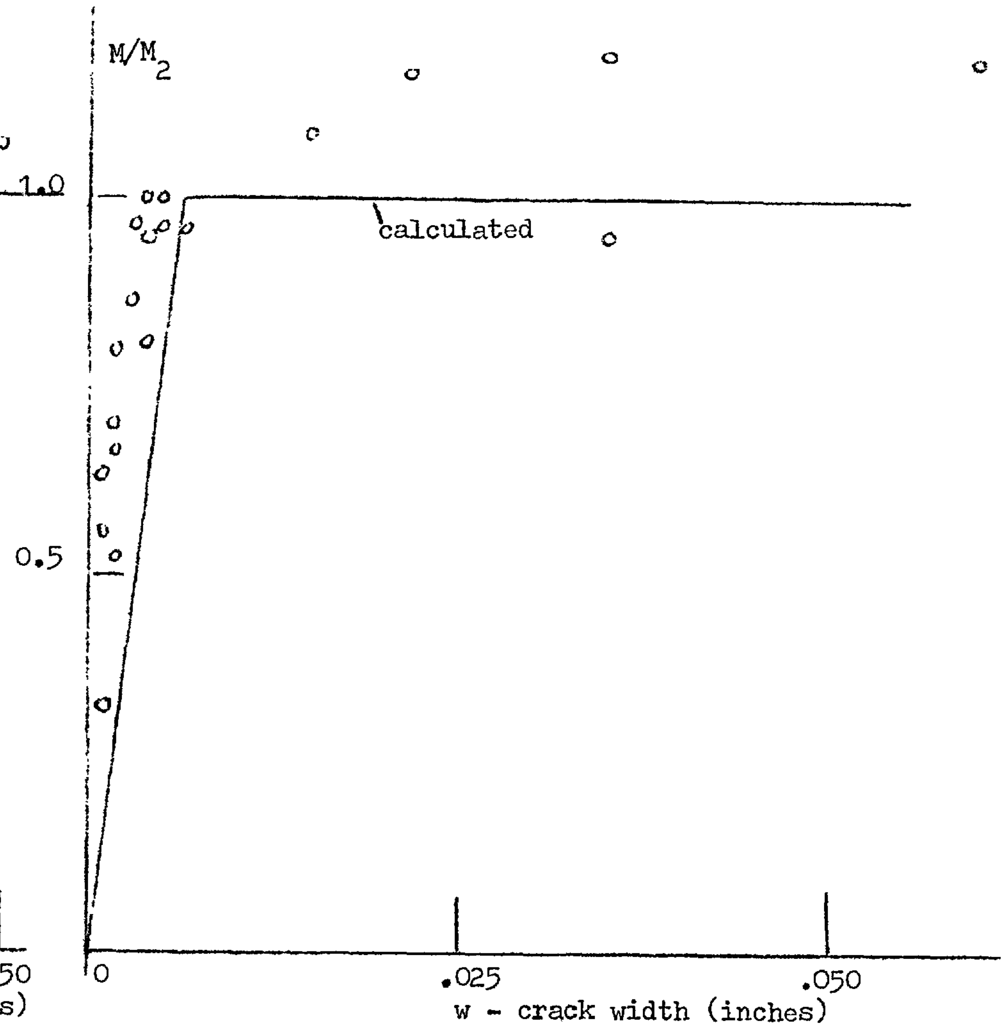



Fig. 13.24 (d). Beam No. CA 4.

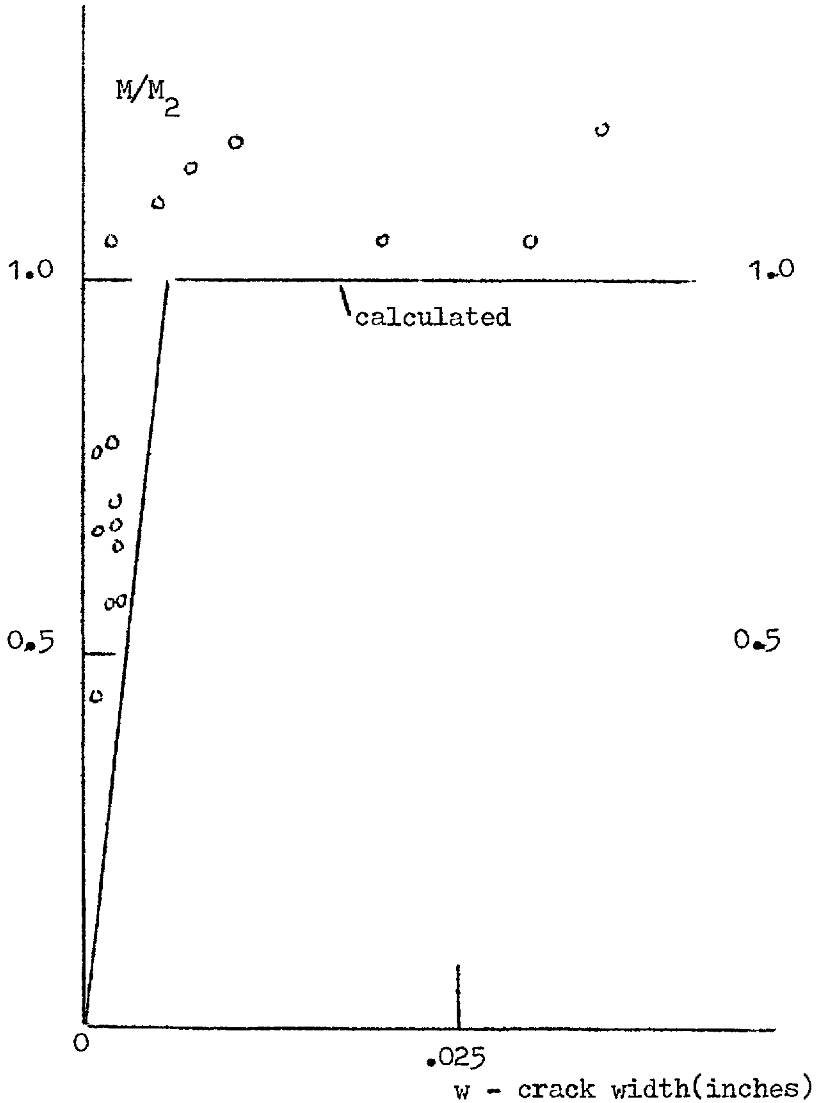



Fig. 13. 24 (e) Beam No CA 5.

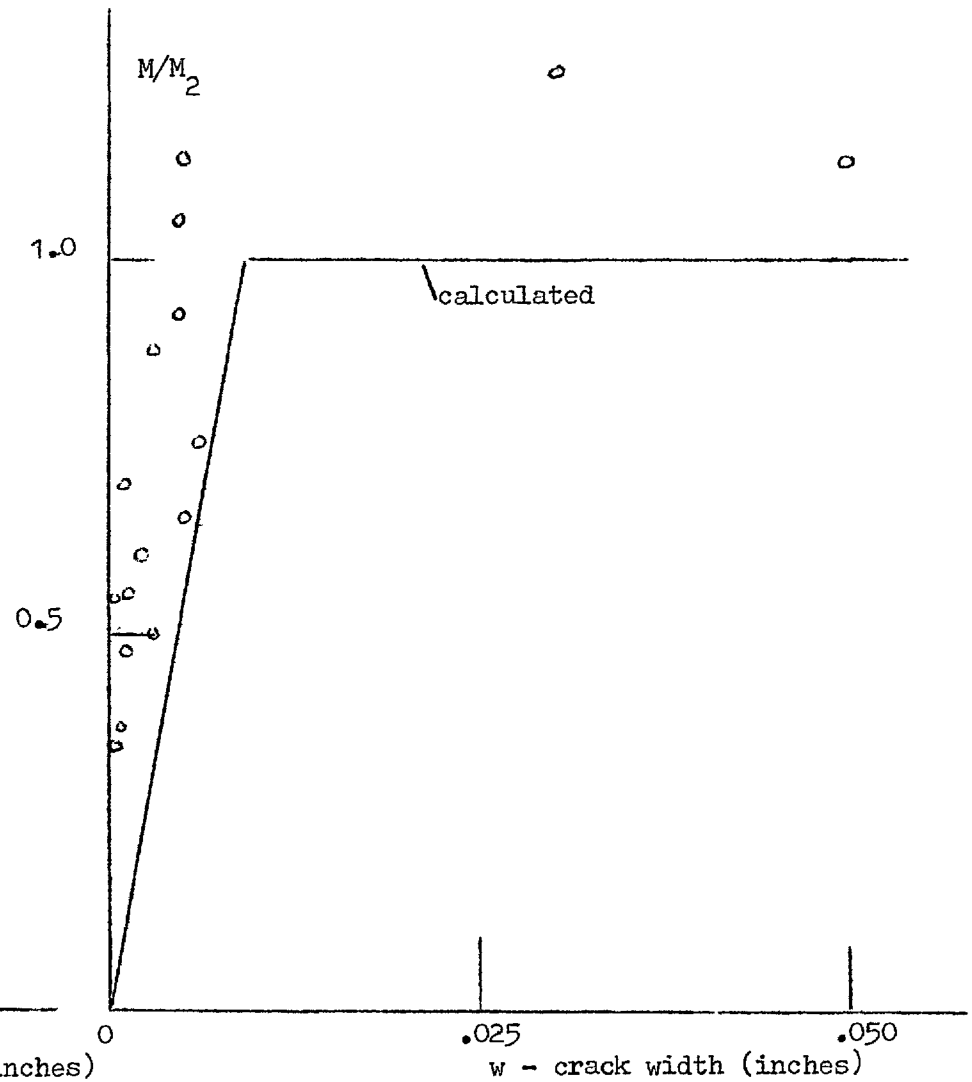



Fig. 13.24 (f). Beam No. CA 6.

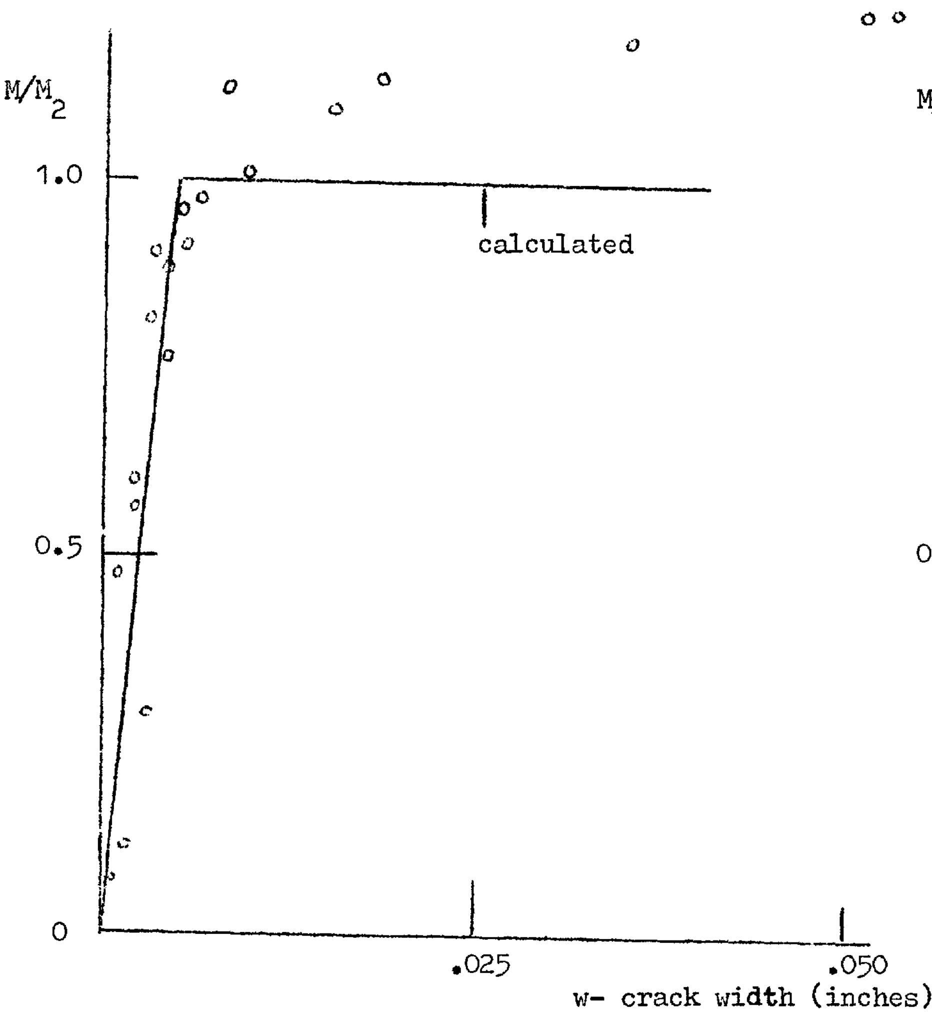



Fig. 13. 24 (g). Beam No. CA 7

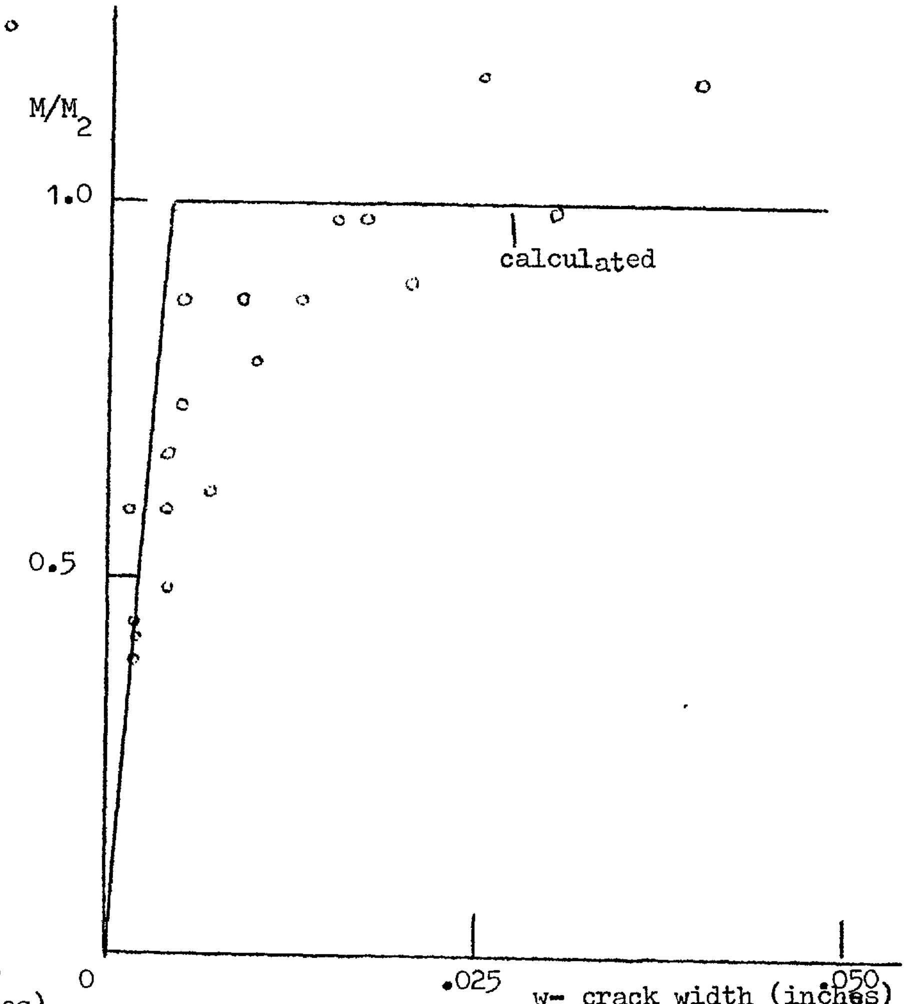



Fig. 13.24 (h). Beam No. CA 8

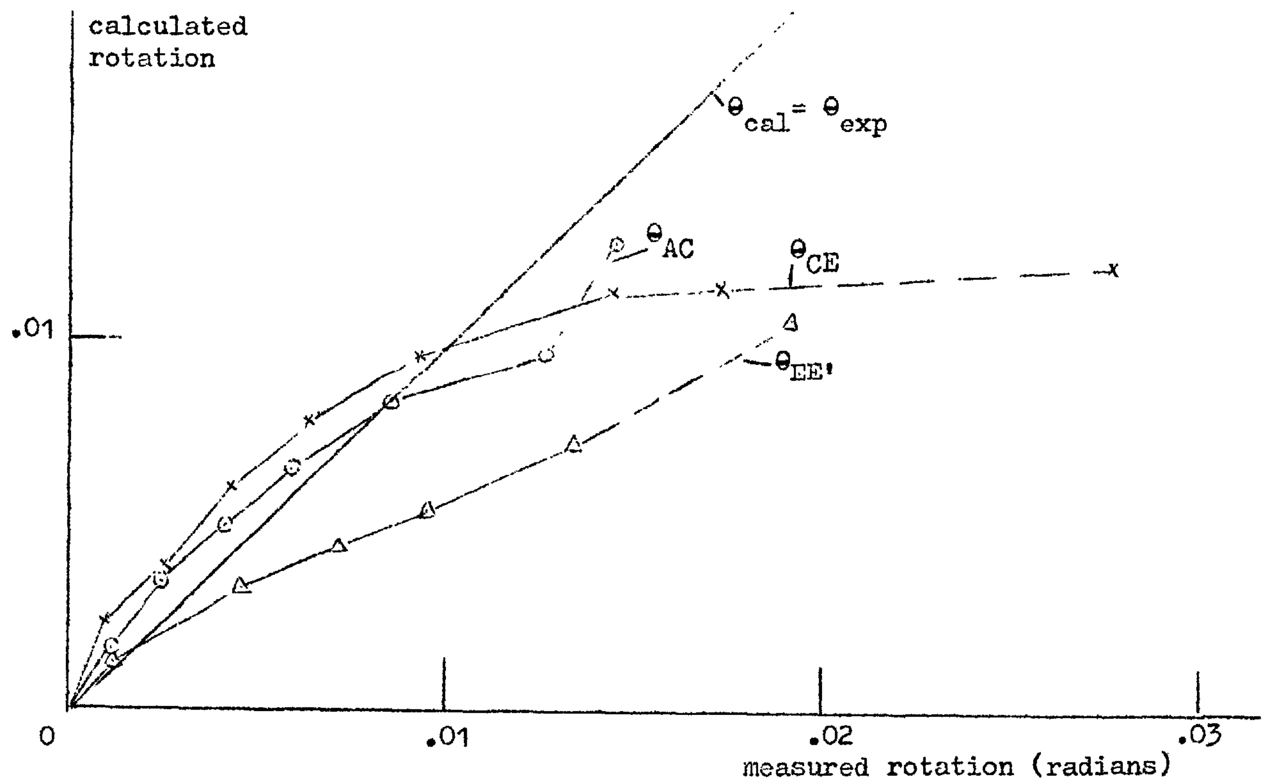



Fig. 13.25 (b). Beam No. CA 2.

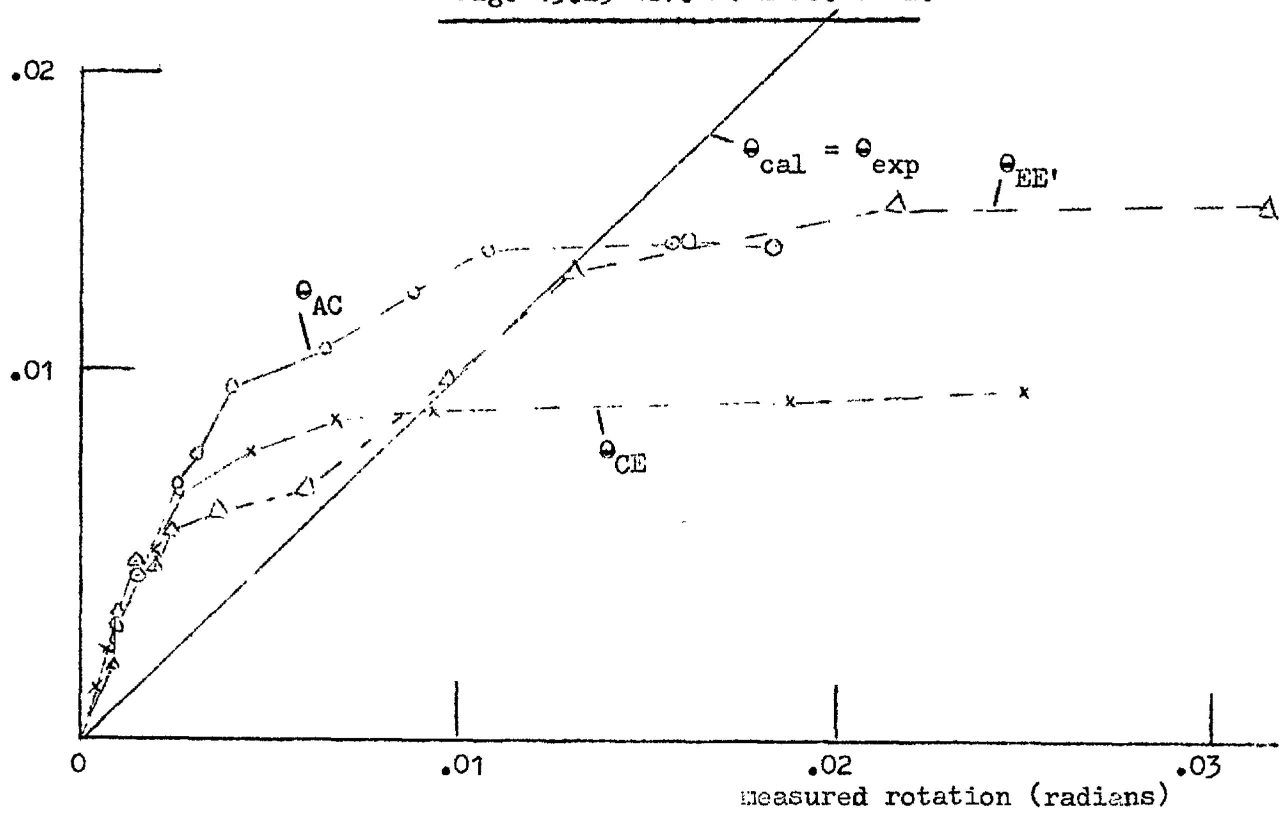



Fig. 13.25 (a). Beam No. CA 1.

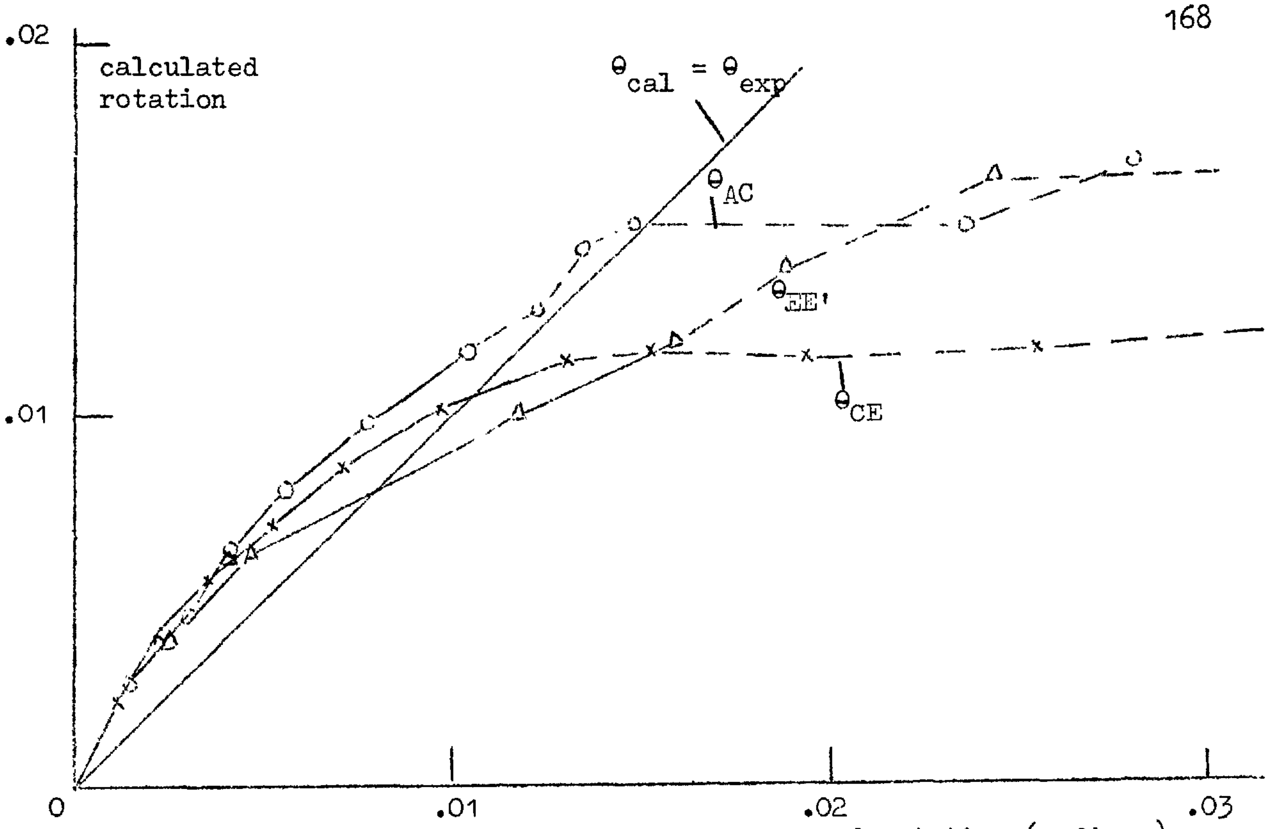



Fig. 13.25 (d). Beam No. CA 4.

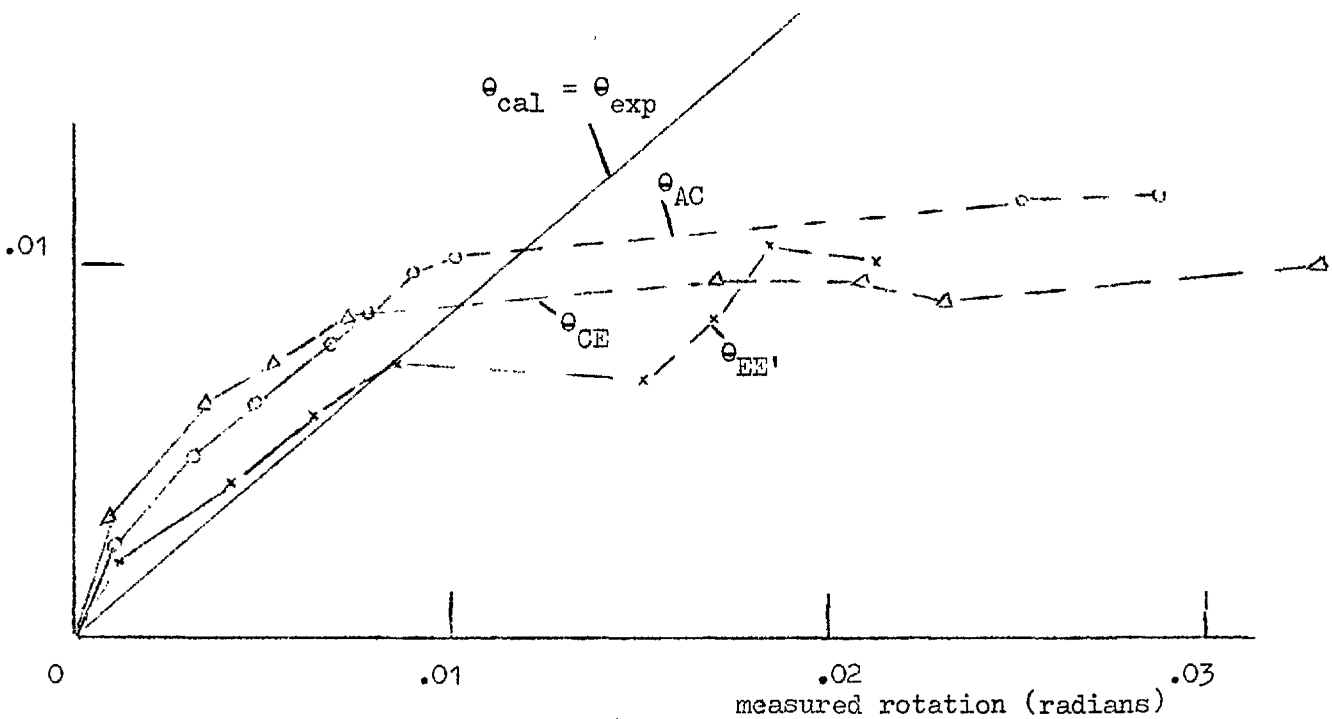



Fig. 13.25 (c). Beam No. CA 3.

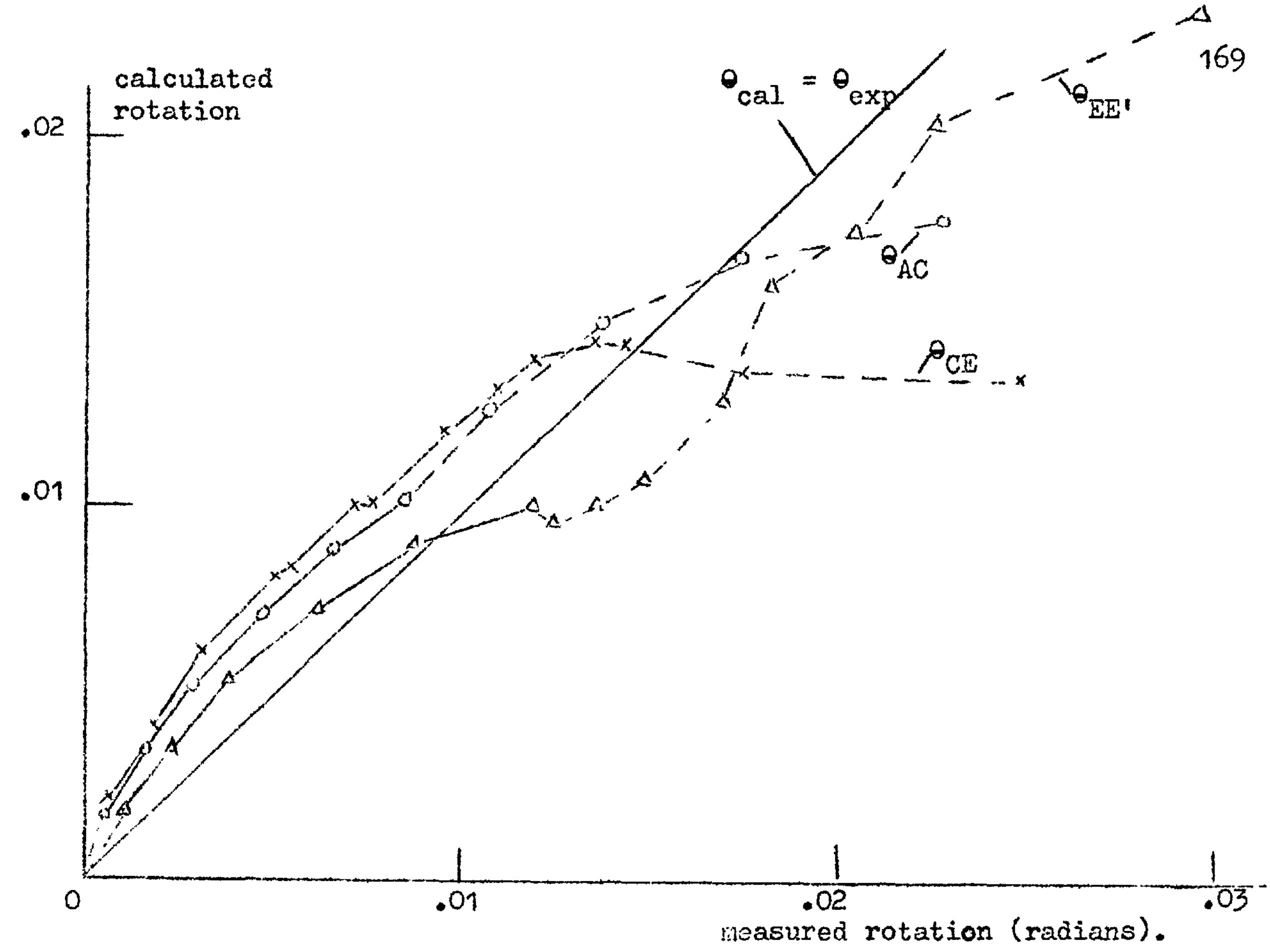



Fig. 13.25 (f). Beam No. CA 6.

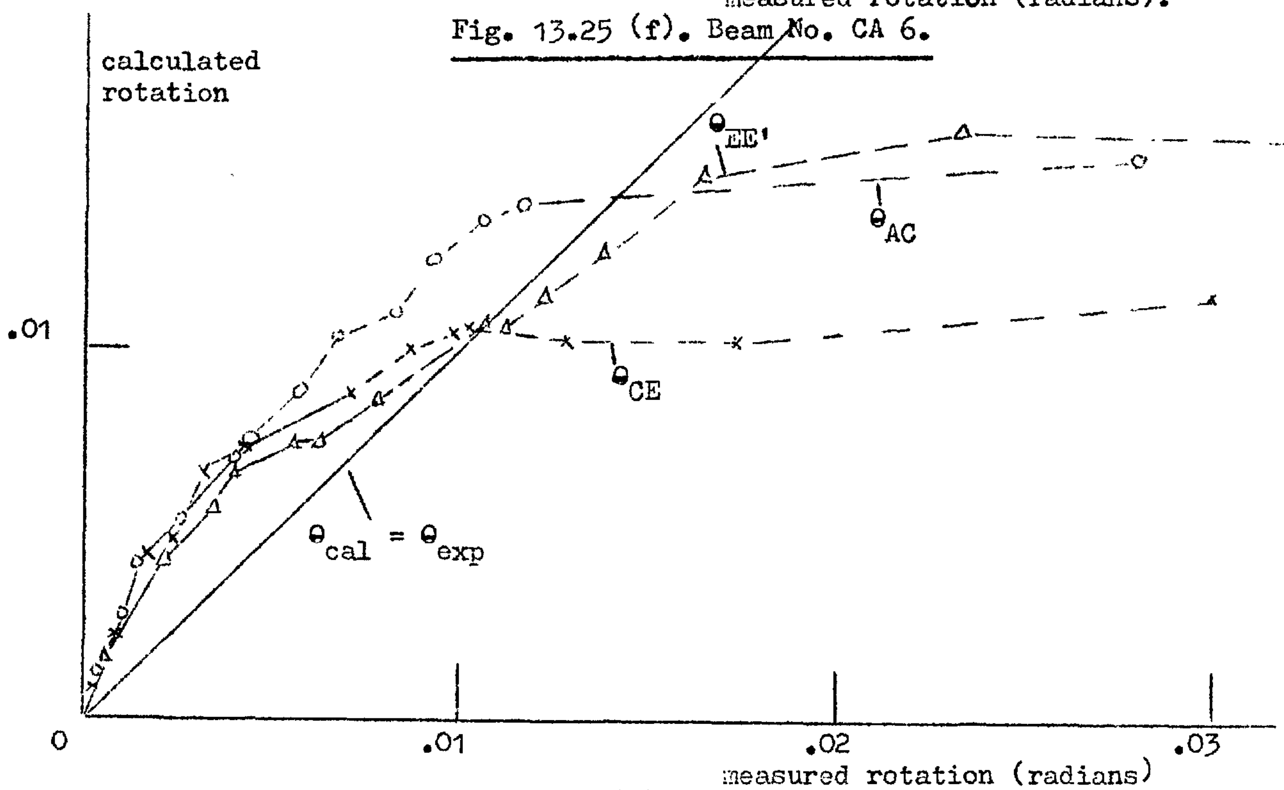



Fig. 13.25 (e). Beam No. CA 5.

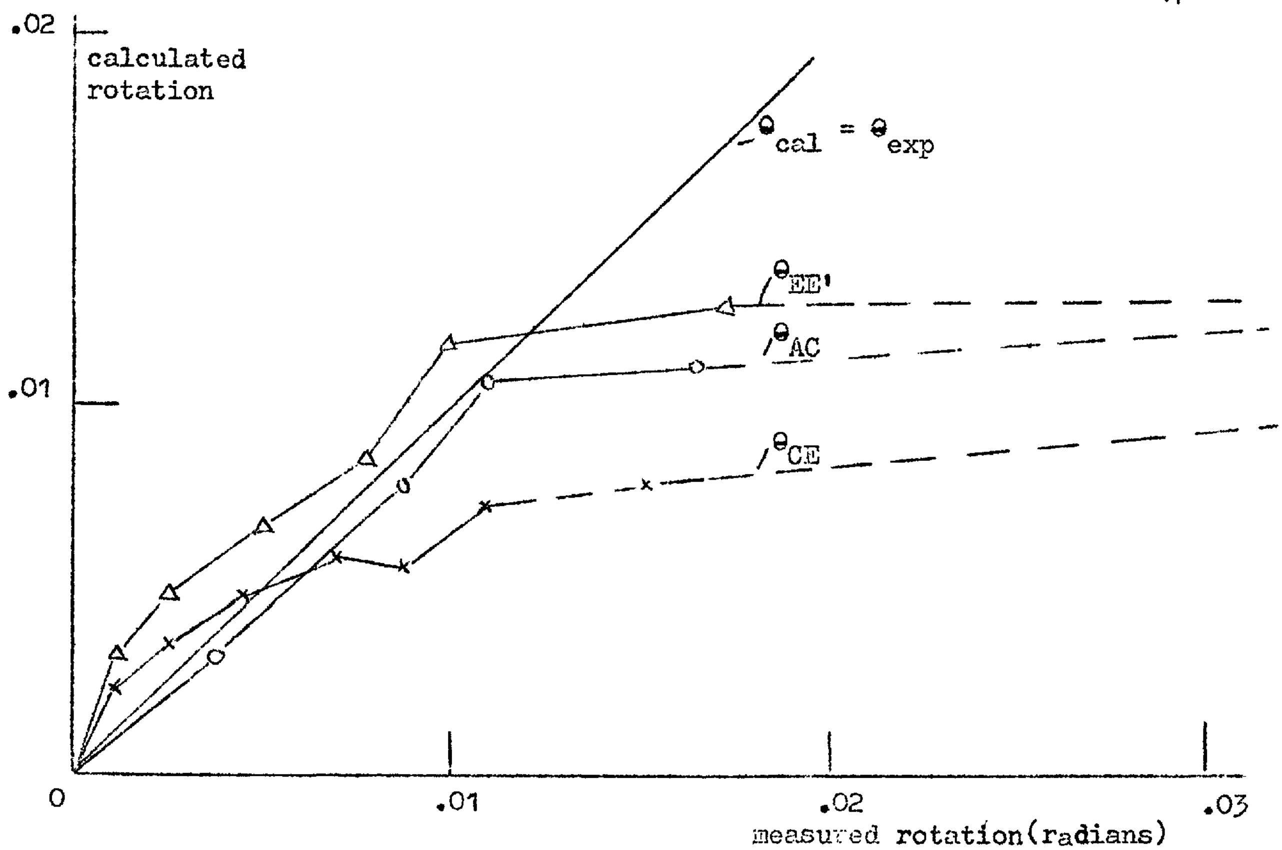



Fig. 13.25 (h). Beam No. CA 8.

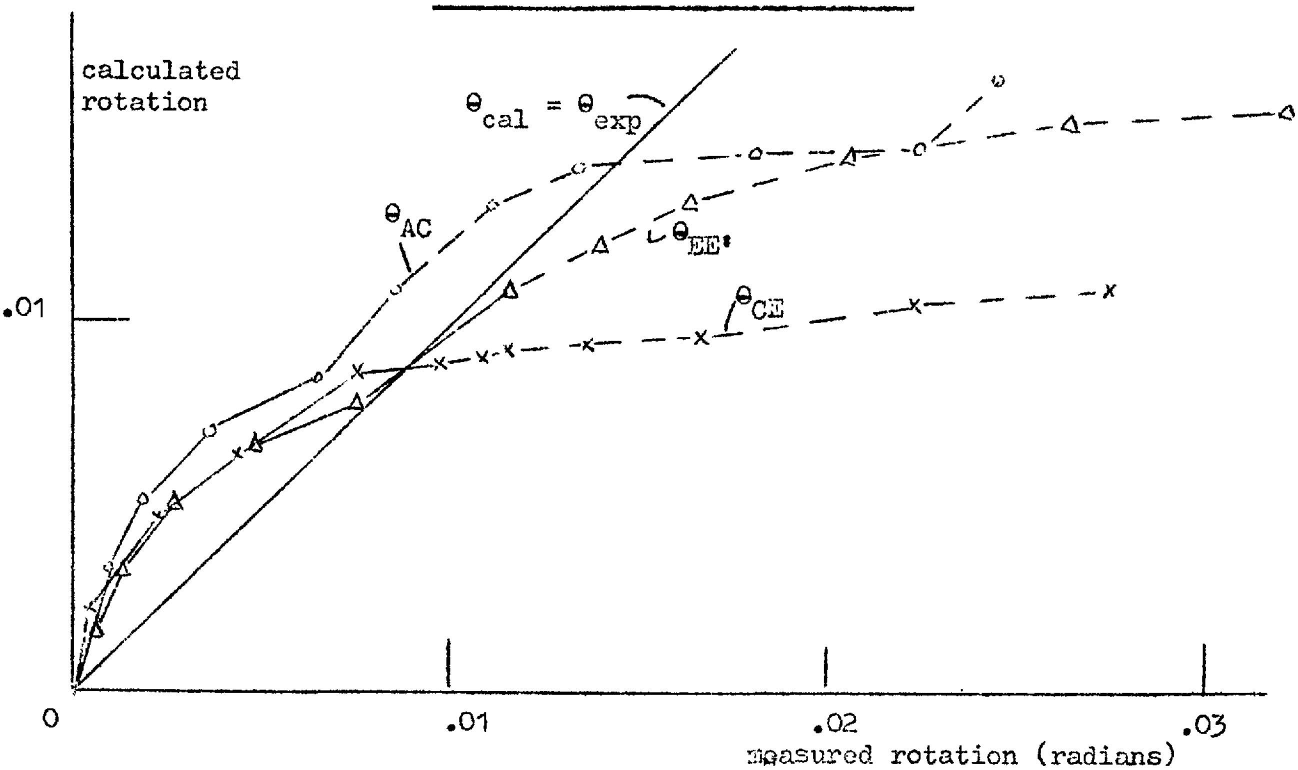
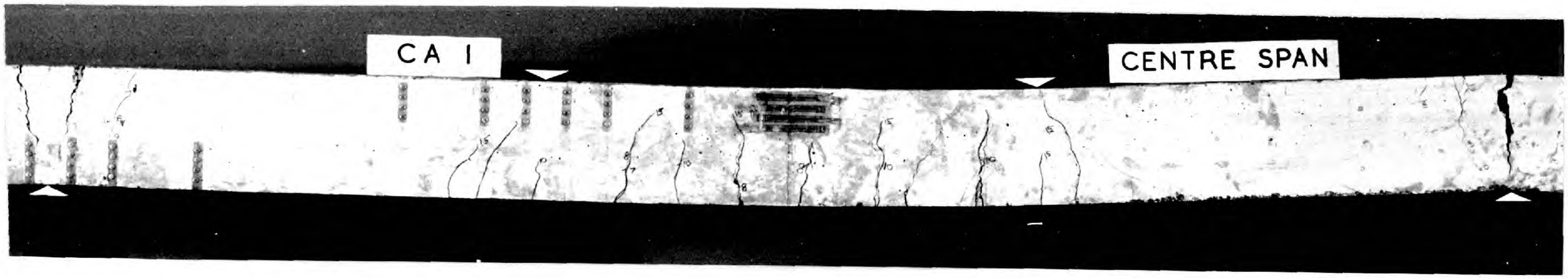



Fig. 13.25 (g). Beam No. CA 7.


CA I
END SPAN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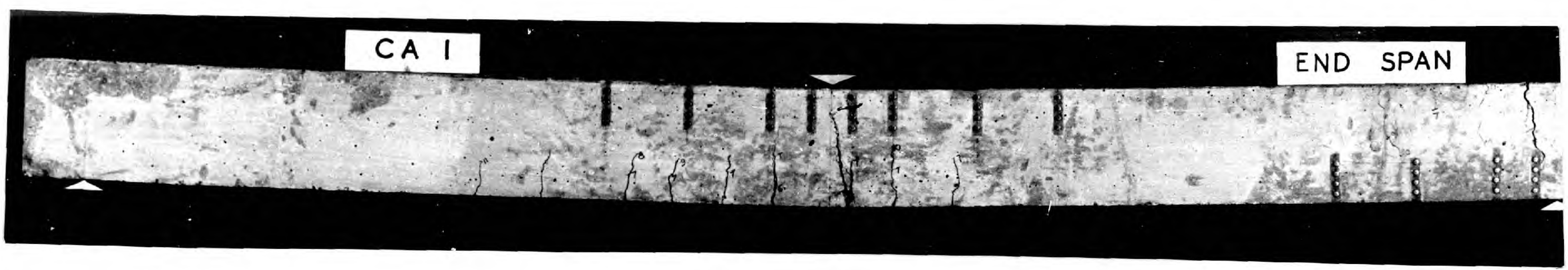
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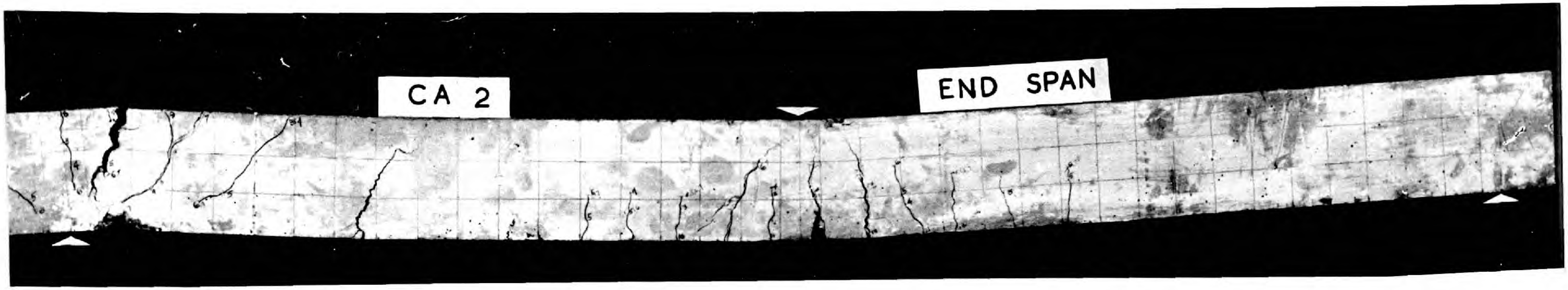
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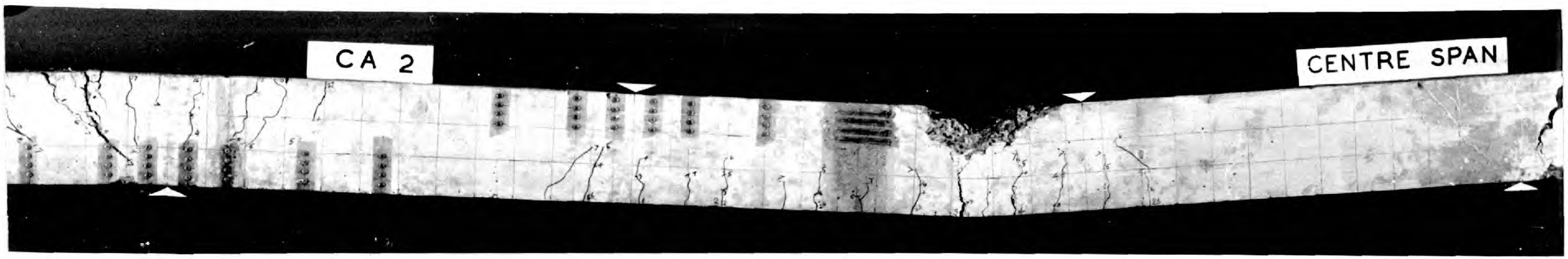
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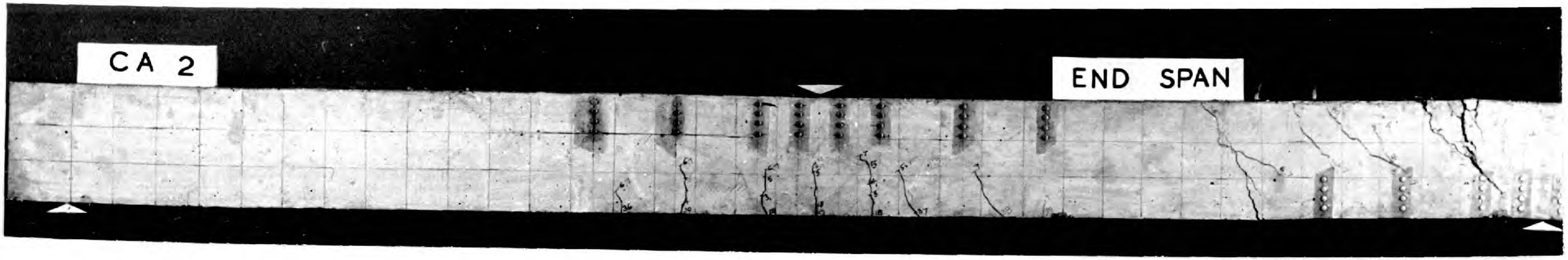
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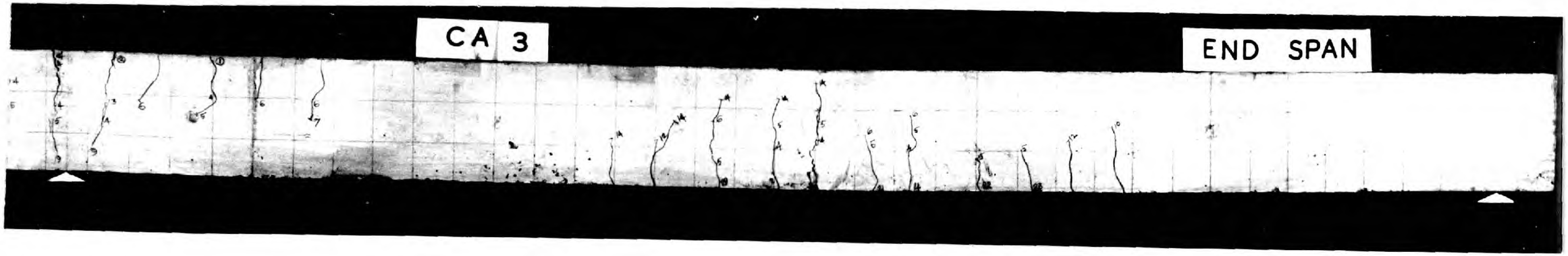
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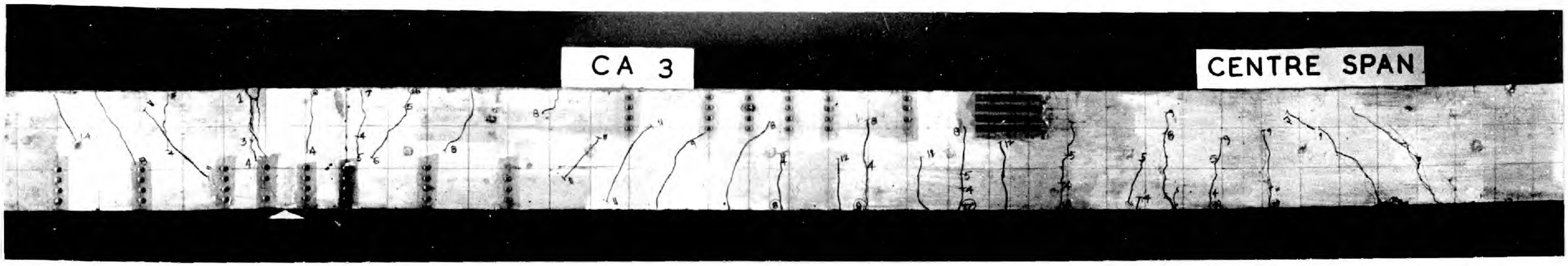
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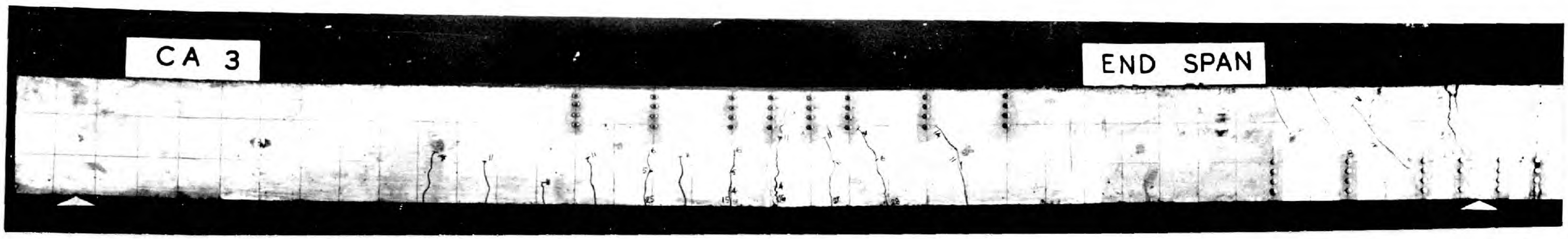
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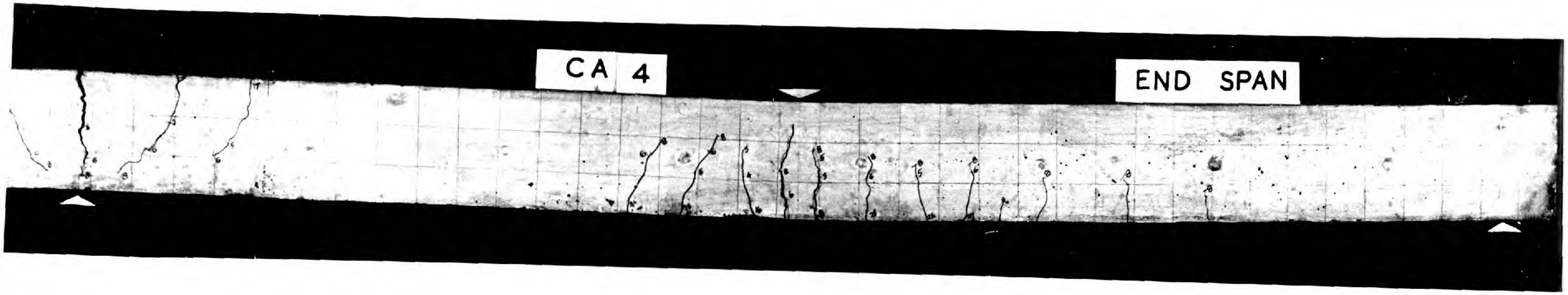
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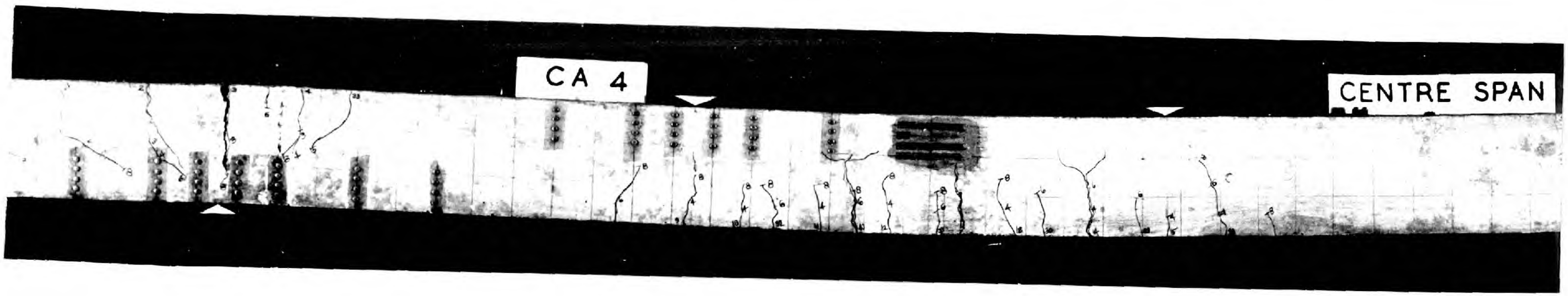
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CENTRE SPAN


CA 4
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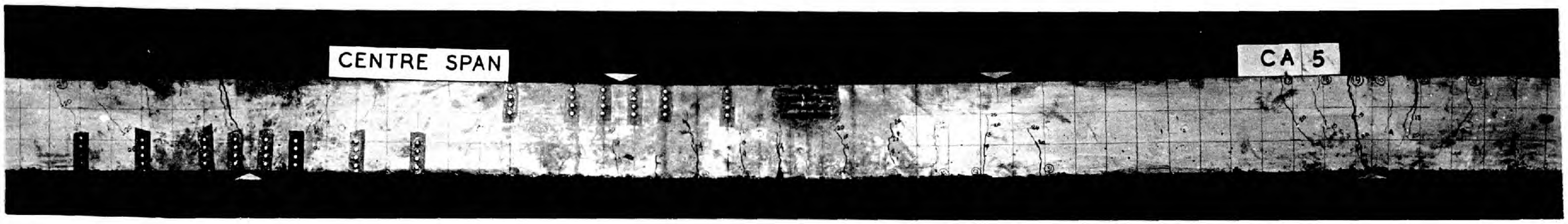
CA 5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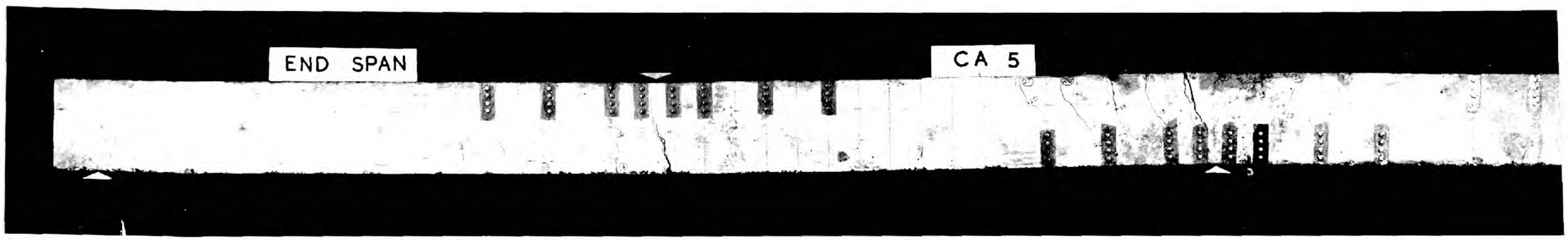
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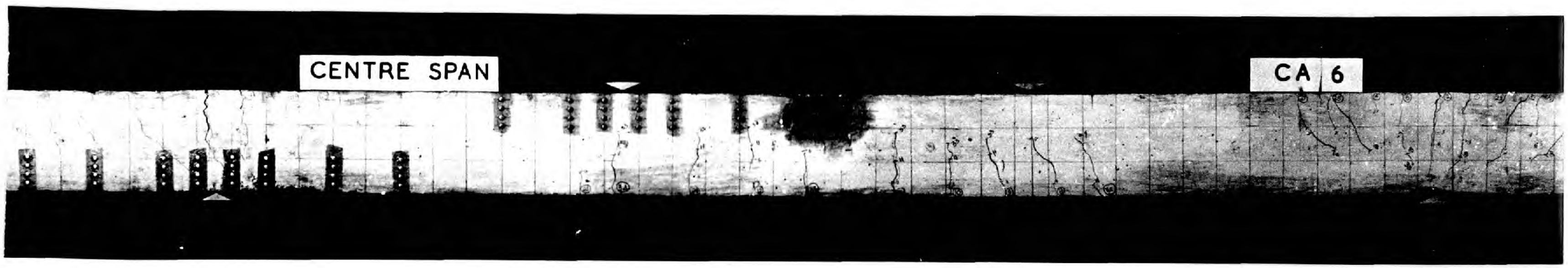
CA 6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CA 6


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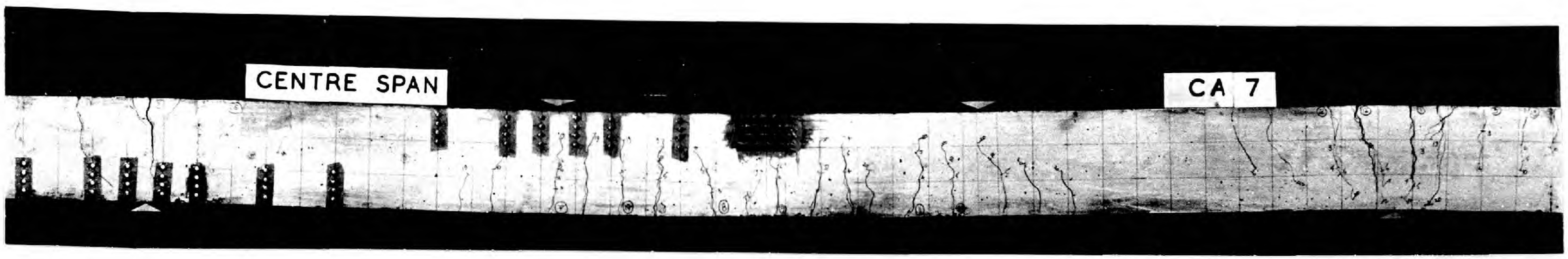
CA 7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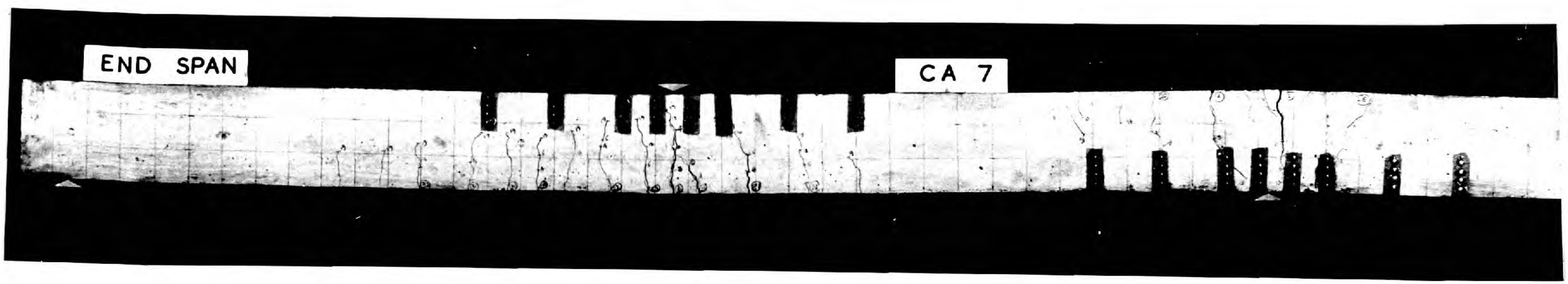
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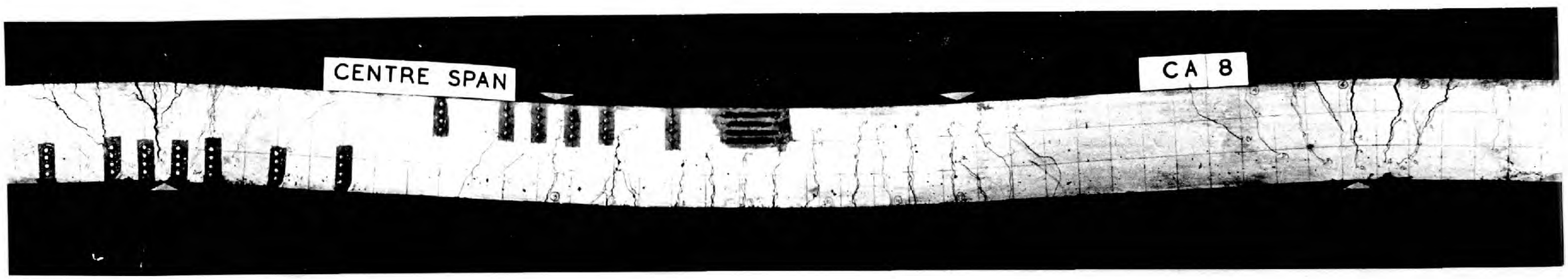
CA 8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CA 8


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CA 8

# PART IV

## Chapter 14

## Limit Design of Reinforced Concrete Skeletal Structures

### 14.1 Ultimate Load

The main loads acting on a structure as discussed in Chapter 3 belongs to two main categories (a) permanent loads consisting of the weight of the structure and permanent fixtures, (b) superimposed loads consisting of moveable loads, temporary fixtures, wind load and other transient loads. The magnitude of each of the loads in the limit design are defined in terms of the mean load and over load coefficients. In the case of the superimposed loads, the over load coefficients depend on the probability of each of the load acting alone or in combination with each other.

The following characteristics with respect to the loads may be assumed.

(1) The permanent load acting on the structure is defined by the mean load G and the over load coefficient $\gamma_g$ which is independent of the other loads.

(2) The superimposed loads are divided into two categories of independent loads.

(a) Vertical loads denoted by Q,

(b) Lateral loads denoted by V.

The over load coefficients for the vertical and lateral loads when acting individually are given by $\gamma_{q1}$ and

$\gamma_{v1}$. When they act together, they are given by $\gamma_{q2}$ and $\gamma_{v2}$. Each of the loads given above are considered as definable in terms of a single parameter ( incremental load parameter).

The ultimate load of a structure under a given set of specified loads is defined as the random combination of the superimposed loads with the permanent load that has the greatest probability of causing structural failure, the failure state being defined in terms of a collapse mechanism [3].

As an illustration consider the case of a continuous beam. In Chapter 8 it was shown that its collapse mechanism consists of partial collapse mechanisms for each of the spans, and that the equilibrium condition could be explicitly stated in terms of the moment at three critical sections in the span. Thus if the loads in the different spans could be stated independently, the ultimate load for the continuous beam corresponds to the state when all the spans are loaded with the maximum load for each span at the same time.

One of the first problems in the design of structures for safety is to determine the ultimate load as defined earlier for any structure and the specified loads. In elasto - plastic or rigid plastic design, this may be more important than in the elastic methods of design, as in the former, the combined effects of different loads cannot be determined by superposition of resultant stress due to each system of load. It must be remembered that, in elastic methods of design, the principle of super-position of stresses confines structural analysis to the investigation of individual cases of loading from which the combined effects could be easily derived. This process in general reduces the calculations enormously. So far a similar simplifying principle has not been available in limit design.

In plastic analysis Baker (3) and Prager (50)) have shown that for any structure subject to mechanism type of failure there is a unique collapse load factor associated with a particular mode of collapse. The probable collapse mechanism as derived in the above method of analysis is based on a quantitative investigation of combined elementary mechanisms, and the corresponding load systems. In design, a qualitative appreach on a similar basis may be used to obtain the properties of combined load systems which may help to determine the ultimate load configuration. The following principle of combined load is derived for a limited range of structures where the collapse modes corresponding to the elementary load systems could be specified and ensured by proper design.

## 14.2 Ultimate Load Theory

As in the plastic theory (3), the members at failure are assumed to have a constant moment at the "plastic hinges" which possess sufficient "plastic rotation capacity" so that a mechanism type of failure could be attained.

Definition (1) Let one or more independent load systems which could be represented in terms of a single load parameter be defined as an elementary load system. If $P_i$ denotes an elementary load system, and and $\lambda_i$ be the corresponding load parameter, then the load is defined by $\lambda_i P_i$.

Definition (2). The collapse mechanism corresponding to any elementary load system $\lambda_i P_i$ is defined as an elementary collapse mechanism, denoted by $S_i$.

Definition (3). Any two elementary collapse mechanisms are said to be independent if the mechanism displacement due to one system causes no displacement in the line of action of the load points in the other, while if there is any such displacement, they are said to be similar or dis-similar depending on whether this is similar or dis-similar to that caused by its own mechanism displacement. The corresponding elementary load systems are also referred to as similar or dis-similar accordingly.

The above definitions could be easily applied to any structure in which the modes of collapse are known or could be specified for purposes of design. For example, the individual span loads in a frame structure can be considered as elementary load systems. The resulting elementary beam mechanisms are independent with respect to each other as in continuous beams. Similarly the lateral wind load on an orthogonal structure can be considered as an elementary load system associated with a sway mechanism of collapse. In this case the sway mechanism is independent with respect to each of the beam mechanisms. However, if the structure is not orthogonal, the sway and beam mechanisms may be dependent. They could in this case be separated into similar or dis-similar categories depending on the direction of rotation of the common members.

In structures where the load systems could be classified under the above definitions, the following general principle of combined loading could be established.

"The collapse load factor of a structure is a minimum due to the combined action of all the independent and similar elementary load systems".

Proof.

Let $P_i$; $P_j$ be two elementary load systems and $S_i$, $S_j$ be the corresponding elementary collapse mechanisms. Let the equilibrium condition for the elementary collapse mechanisms be given by the equations,

$$\lambda_i^* \sum P_i \ \delta_i = \sum X_i \theta_i \quad \ldots \quad (14.1)$$

$$\lambda_j^* \sum P_j \delta_j = \sum X_j \theta_j \quad \ldots \quad (14.2)$$

where $\theta_i$, $\theta_j$ are the rotation of the hinges in the mechanisms due to arbitrary mechanism displacements, $\delta_i$, $\delta_j$ are the displacements at the points of application of the loads corresponding to mechanism displacement in the mechanisms.

$\lambda_i^*$, $\lambda_j^*$ are the collapse load factors corresponding to the elementary load systems $P_i$ and $P_j$ respectively. $X_i$, $X_j$ are the plastic hinge moments.

Let $\lambda_{i+j}^*$ be the collapse load factor corresponding to the loads $P_i$ and $P_j$ acting simultaneously. The failure mechanism under the combined load, denoted by $S_{i+j}$, may or may not be the same as ~~similar to~~ either of the elementary collapse mechanisms. But it could be shown that the combined load factor $\lambda_{i+j}^*$ is unique (3,50).

Consider the equilibrium of the mechanism $S_i$ under the action of the combined load $P_i + P_j$. Since the collapse load factor $\lambda_{i+j}^*$ is unique (corresponding to mechanism $S_{i+j}$), and as $S_i$ under these conditions is an arbitrary mechanism, the following inequality must be satisfied.

$$\lambda_{i+j}^* \sum P_i \ \delta_i + \lambda_{i+j}^* \sum P_i \ \delta_{ij} \leqslant \sum \theta_i X_i \quad \ldots (14.3)$$

where $\delta_{ij}$ is the corresponding displacement of the load system $P_j$ due to the arbitrary displacement of the mechanism $S_i$.

Eliminating $\sum X_i \theta_i$ from 14.1 and 14.3,

$$\lambda_{i+j}^* \sum P_i \delta_i + \lambda_{i+j}^* \sum P_j \ \delta_{ij} - \lambda_i^* \sum P_i \ \delta_i \leqslant 0$$

$$\text{i.e.,} \quad (\lambda_{i+j}^* - \lambda_i^*) \sum P_i \ \delta_i + \lambda_{i+j}^* \sum P_j \ \delta_{ij} \leqslant 0 \quad \ldots (14.4)$$

For mechanism $S_i$, $\sum P_i \delta_i > 0$ and from the definition of load systems, the displacement (if any) in the common members are similar.

$$\therefore \quad \sum P_j \, \delta_{ij} \geqslant 0.$$

Then equation 14.4 gives,

$$\lambda^*_{i+j} - \lambda^*_i \leqslant 0$$

i.e.,

$$\lambda^*_{i+j} \leqslant \lambda^*_i \qquad \ldots \qquad (14.5)$$

Similarly it could shown that

$$\lambda^*_{i+j} \leqslant \lambda^{**}_j \qquad \ldots \qquad (14.6)$$

The conditions given by 14.5 and 14.6 show that the collapse load factor due to the combined action of two independent or similar elementary load systems is less than either of the individual collapse load factors or in the limiting case it could equal the least of the collapse load factors.

Similarly, $P_{i+j}$ may now be treated as an elementary load system and the proof could be extended to cover all load systems which are mutually independent or similar.

The principle of combined loading may be applied generally to most common applied load systems and structural conditions. In all cases it is an advantage to treat smaller units of loads as elementary load systems, so that the dependence or the independence of the elementary collapse mechanisms may be checked by inspection. Then the ultimate load configuration under the combined loads could be easily established.

As an application of the principle of combined loading, the following useful corollaries may be easily derived.

<u>Corollary 1</u>. "The collapse load factor of a structure due to the combined action of dis-similar elementary load systems is greater than that due to each elementary load system applied separately."

The proof of this follows almost on the same lines as that of the principle of combined action of similar and independent load systems proved earlier, except that by definition the terms involving external work in the expression 14.4 are given by

$$\sum P_i \, \delta_{ij} < 0 \quad , \sum P_i \, \delta_i > 0$$

When substituted in the expression 14.4 as before the combined load factor is given by,

$$\lambda^*_{i+j} \geqslant \lambda^*_i \quad , \quad \lambda^*_j$$

<u>Corollary 2</u>. "The yield polygon due to the action of independent or similar elementary load systems lies within a surface bounded by planes normal to the axes and passing through the coordinates defined by the elementary collapse load factors".

This is illustrated in Fig. 14.1 and follows immediately from the principle of combined action of similar or independent load system as $\lambda^*_{i+j} \leqslant \lambda^*_i \; , \; \lambda^*_j$.

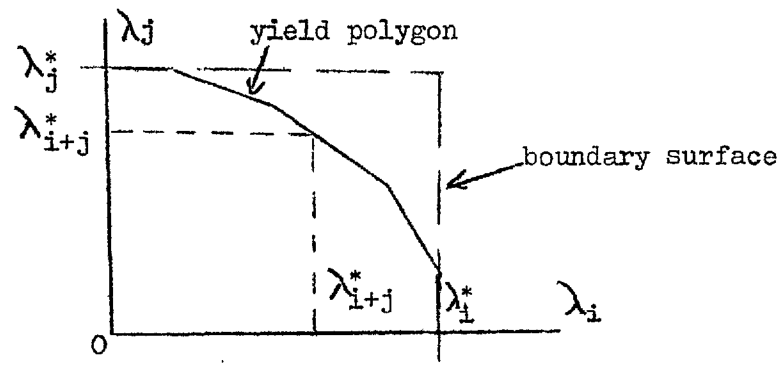



Fig. 14.1

Similarly the yield polygon due to the combined action of dis-similar elementary load systems may be shown to lie outside the boundary surface defined above. It may be noted that under both these conditions, the yield polygon remains convex (50).

Corollary 3. "In multistorey structures where the members are orthogonal, when only the permanent loads and the vertical super-loads are considered, the ultimate load configuration consists of all spans being loaded."

This condition follows from the principle of combined loading as the partial collapse mechanisms due to the vertical span loads consist of beam mechanisms which are mutually independent. The problem of continuous beams discussed in Part (3) of this thesis is a special case of this type. The ultimate load may then be defined by the load configuration where all the spans are loaded with $\gamma_g G + \gamma_{q1} Q$. Under these conditions, the limit design procedure for each storey beam is similar to that of continuous beams and the methods outlined earlier could be directly used in frames as well.

<u>Corollary 4</u>. "In orthogonal structures if the span loads and the lateral loads ( assumed to act at nodes ) are unidirectional, the ultimate load configuration consists of the maximum probable vertical and lateral load acting simultaneously."

The partial collapse mechanisms due to the span loads, where each of the span loads is considered as an elementary load system, consist of beam mechanisms. Similarly each of the lateral loads acting at the nodes may be considered as an elementary load system. If the lateral loads act in the same direction, as is usually the case in frame analysis where the wind pressure and earthquake forces form the lateral loads, the elementary collapse mechanisms consist of sway mechanisms which are independent or similar. In orthogonal structures the beam and sway mechanisms are mutually independent. Hence the direct application of the principle of combined loading yields the above result.

The ultimate load may then be represented by $\gamma_g G + \gamma_{q2} Q + \gamma_{v2} V$ and the load configuration consists of all the spans loaded and the lateral load acting at the same time. Baker (5) has used the above loading condition in the ultimate load design of reinforced concrete structures as the most critical case of loading to be considered. However in limit design, the safety analysis would be based on three cases of loading represented by,

(a) $\gamma_g G + \gamma_{q1} Q$

(b) $\gamma_g G + \gamma_{q2} Q + \gamma_{v2} V$

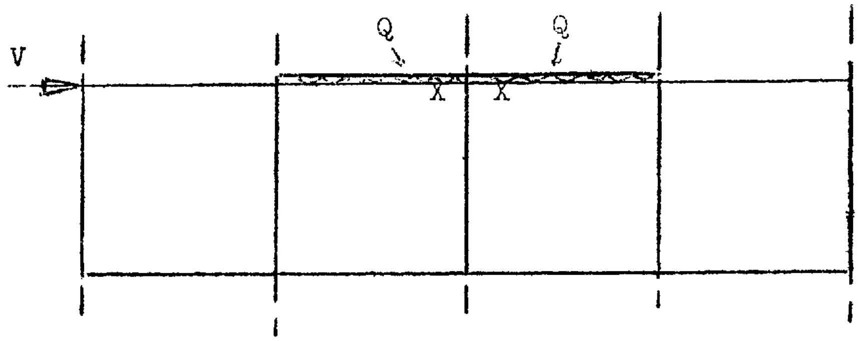
(c) $\gamma_g G + \gamma_{v1} V$

where the over load coefficients are as discussed in Chapter 3. Since the over load coefficients $\gamma_g$, $\gamma_{q1}$, $\gamma_{q2}$, $\gamma_{v1}$, $\gamma_{v2}$ are in general different from each other, each of the above cases of loading could give rise to critical safety conditions, and must be investigated separately.

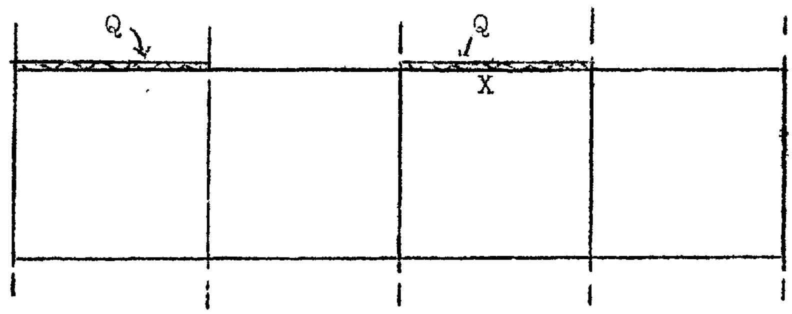
## 14.3. Serviceability requirements of framed structures

The minimum serviceability requirements discussed with respect to continuous beams in Chapter 11 are sufficiently general as to include framed structures. In the application of the serviceability requirements, they may be expressed in terms of the yield safety and crack width parameters as in Section 11.4. The minimum critical section moments required to satisfy serviceability may then be evaluated by an approximate elastic analysis. In this context the experience gained in the classical elastic design methods may prove quite useful. For example, the critical serviceability conditions in beam support sections occur when the adjacent spans are loaded, and that of midspan sections occur when alternate spans are loaded. In the case of columns the critical conditions occur when alternate bays are loaded as shown in Fig. 14.2.

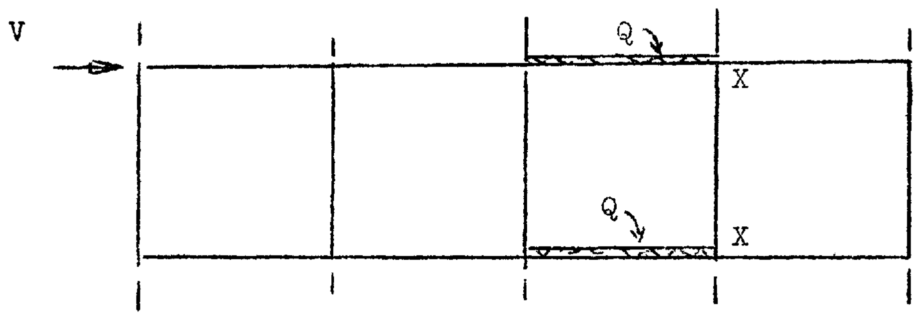
As in continuous beams, the minimum serviceability parameter in most cases may be close to unity and the limit EI values as discussed in Chapter 6 may be used in all limit calculations. The limits for the span to depth ratio derived for beams in Section 11.7 and in Chapter 12 with reference to limiting deflections and shear could also be applied to the beams in framed structures.




(a) Critical Load for Support Section X




(b) Critical Load for Midspan Section X




(c) Critical Load for Column Section X

Note. The permanent load (dead load) is assumed to act in addition to the above loading for all cases

Fig. 14.2 Critical Serviceability Conditions for a Typical Storey

# CHAPTER 15

## An Investigation into elasto - plastic design of skeletal Frames using plastic hinge systems

### 15.1 Statical Indeterminancy and elasto-plastic analysis.

In the conventional elastic design of skeletal structures, linear methods of structural analysis involving flexibility methods or stiffness methods could be used, (54, 55, 56, 60) both of which are easily adaptable for computor analysis. It could also be shown that the solution to the stress analysis problem is unique, hence any of the methods that is found most convenient for the problem may be used.

In the elasto-plastic analysis however, the flexibility method has a considerable advantage as the discontinuities at the "plastic hinges" could be taken into account in the analysis directly. By this method the post - yield stages of the structure under incremental load, may be analysed by treating it as if it was a reduced structure, where the yielded sections are replaced by actual hinge sections. It must however be noted that any of these sections that may undergo reversed rotations in the subsequent stages of loading in excess of the plastic rotation, may revert back to elastic conditions. The process may be repeated until an ideal mechanism condition of failure is reached (assuming no frame instability and other local modes of failure).

In this process of analysis the rotation of the plastic hinges at any stage of loading could be determined using the Muller Breslau virtual work equations or other means, as the compatible stress system at any stage of loading is known. The reduction in the strength of any of the sections that may undergo excessive plastic rotation as in reinforced concrete members could also be incorporated in the method, but in complicated structures a limit basis as suggested by Baker (5) may provide a reasonable safe limit for the collapse load.

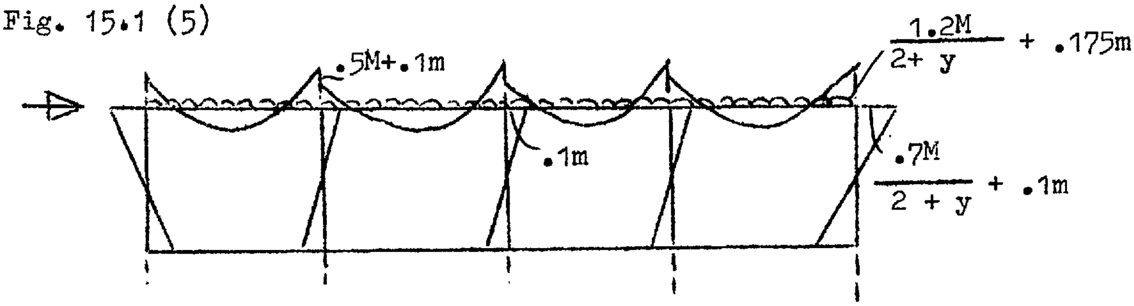
An elasto-plastic collapse analysis as described above could give rise to at most n plastic hinges in a structure that is n-times statically indeterminate, since the formation of one more hinge renders the structure unstable as a mechanism condition is reached. But in most cases, collapse may take place due to fewer plastic hinges, as partial mechanism conditions could take place . From the point of view of design of structures, the limiting case in which n plastic hinges could be envisaged corresponds to the maximum utilisation of the largest number of critical sections for the given loading system. But the existance of a solution of this nature, which is not unique if it exists, can be ascertained by compatibility and equilibrium. Under these conditions if a compatible stress system could be determined, the plastic rotation at the hinges is given by the virtual work equations as before.

A method of ultimate load design of multi-storey structures based on the above assumptions has been suggested by A.L.L. Baker (5). In this method of design a plastic hinge system is first assumed. The moments at the hinges are then checked for compatibility using the virtual equations as given by the set 15.1

$$\sum f_{ij} x_j \quad + \quad f_{oj} \quad = \quad -\theta_j \qquad \ldots \qquad (15.1)$$

where $i, j : 1 \rightarrow n$, and $f_{ij}$ and $f_{oj}$ are given by $\int \frac{M_i M_j \, ds}{EI}$ and $\int \frac{M_o M_j \, ds}{EI}$. respectively. For correspondence of stress and strain at the hinges $\theta_j$ and $X_j$ must be of the same sign. In reinforced concrete structures at which the theory is mainly aimed, it is also considered necessary to check that the plastic rotations are within "permissible limits". In the design procedure, the actual rotation at ultimate load may be taken into account in the detailing of the hinge sections or if they are too large, they must be reduced by selecting other compatible solutions.

However the main problem in this method of design is to arrive at a compatible solution for the position and direction of assumed hinges. The equation set 15.1 contains n arbitrary variables $X_j$ $(j: 1 \rightarrow n)$, which must satisfy the n compatibility conditions simultaneously. This condition that there are n plastic hinges with positive rotations so that the system remains in stable equilibrium at the ultimate load is generally referred to as inelastic compatibility (18). Baker has suggested a trial and adjustment method of obtaining a suitable moment set. In this method a preliminary guess of the moments at the hinges are made. These are then substituted in the compatibility equations and if any of the rotations are found to be incompatible with the assumed sense of the plastic moment, a fresh trial is made. A satisfactory moment set is obtained when all the hinge rotations are of the right sense. Some approximate values of the hinge moments for a typical multi-storey structure under a particular hinge system has been suggested by Baker as shown in Fig. 15.1 (5)




where m = storey sway moment

M = Free moment in span

y = Ratio of stiffness of beam to that of Column

Fig 15.1 Typical moment values suggested by Baker.

One of the implicit assumptions in this trial and adjustment method is the existence of an inelastic compatibility state with n plastic hinges at the positions as assumed.

For different load configurations and hinge position in certain categories of structures, these assumptions would be valid. For example continuous beams and portal frames, the existence of equilibrium conditions with sufficient number of plastic hinges to render the structure statically determinate could be verified as the failure mechanism conditions are comparatively simple. But in more complicated structures, particularly multistorey structures, it may not be easy to forsee that the n hinges at the sections as assumed could lead to a compatible stress state with the plastic rotations $\theta i$ $(i:1 \rightarrow n)$ being positive at each of the sections. Unlike in elastic analysis, a statically admissible hinge system may or may not be suitable as a basis of inelastic compatibility analysis depending on the possibility or the impossibility of achieving the final stress state as assumed in the trial and adjustment method. Hence in general, for a structure which is n times statically indeterminate, it would be necessary to establish the existence of a compatible state with n plastic hinges at the positions assumed as an "a priori" condition before any trial and adjustment method could be used to determine an actual compatible distribution of moments. A basis of investigating the suitability of an assumed hinge system for inelastic compatibility analysis is outlined in the next section. Under these conditions a hinge system for which a compatible solution may be shown to exist is referred to as a suitable hinge system otherwise it may be referred to as unsuitable.

## 15.2 Suitability of plastic hinge systems

Consider an n-times statically indeterminate structure. Suppose that it is possible to reduce the structure to a state of statical determinancy by the introduction of n plastic hinges, so that they sustain finite plastic rotations at ultimate load (considering the load factor and load configuration). The actual rotation could now be calculated using the virtual work equation as in 15.1.

In this state the structure must be in equilibrium if all the compatibility conditions are satisfied, provided that none of the critical sections other than those assumed have undergone yield. But in a limiting case, it would be possible to have one or more of the remaining critical sections to be at the yield limit. Thus consider any arbitrary critical section to be at its yield limit, which may be considered to be the last hinge to form before eventual collapse. Let this last section which reduces the structure to a mechanism with any further increase in the applied load be called the $(n+1)^{th}$ hinge. Thus the failure mechanism for the structure at ultimate load would consist of m+1 hinges ($m \leq n$) of which m hinges have undergone plastic rotation prior to ultimate load and the $(n+1)^{th}$ hinge is at its yield limit.

For convenience of nomenclature let the mechanism of collapse that may be initiated by the m+1 hinges as in the above limiting case be called a quasi-mechanism to distinguish it from an actual mechanism as the $(n+1)^{th}$ hinge included in the above quasi-mechanism has not undergone any plastic rotation. The m hinges that are common to the statical release system and have undergone plastic rotation at ultimate load could be referred to as the basic hinges of the quasi-mechanism.

Thus it could be stated that in the limiting equilibrium state, a compatible plastic hinge system must contain at least one quasi-mechanism. This may in practice be determined by inspection or by the properties of the influence coefficients characteristic to quasi-mechanisms. The latter may be investigated with respect to the basic properties of the quasi-mechanisms and statical release systems.

## 15.3 Properties of quasi-mechanisms

A quasi-mechanism as defined above represents the state of transformation of compatible elasto-plastic structure into a mechanism condition. Hence it may be expected to have properties similar to that of mechanisms as well as statical release systems. Thus a quasi-mechanism in its limiting state may be considered to have the following characteristics.

(1) An infinitesimal increase in the load factor causes large increase in the deformation (collapse)

(2) An infinitesimal reduction in the moment at any of the basic hinges given rise to large deformations (collapse).

In elasto-plastic analysis the collapse conditions may be expressed in terms of the hinge rotations. If $\theta_i$ be the plastic rotation at a hinge in the quasi-mechanism, $X_i$ be the correspon-ding moment, and $\lambda$ be the load factor, the collapse properties given in (1) and (2) above could be expressed as

$$(a)\frac{\partial \theta_i}{\partial \lambda} > 0 \qquad (b)\frac{\partial \theta_i}{\partial X_j} < 0 \qquad \ldots (15.2)$$

where i, j: $1 \longrightarrow m$.

The rotations at the hinges could be easily obtained as a function of the moments ($X_1 \ldots\ldots X_n$) and the load factor as in equation set (15.1). Suppose for simplicity the plastic rotation of the m basic hinges are given by the equations 1 to m. Then differentiating each of these m equations with respect to $\lambda$, $X_1$, $X_2$, ...... $X_m$

$$-\frac{\partial \theta_i}{\partial \lambda} = \frac{\partial}{\partial \lambda}\left[f_{i1}X_1 + \ldots + f_{ij}X_j \ldots\ldots f_{in}X_n + \lambda f_{io}\right]$$

$$= f_{io}$$

$$-\frac{\partial \theta_i}{\partial X_j} = \frac{\partial}{\partial X_j}\left[f_{i1}X_1 \ldots + f_{ij}X_j \ldots\ldots f_{in}X_n + \lambda f_{io}\right]$$

$$= f_{ij}$$

for i, j : $1 \longrightarrow m$.

As the m hinges are the basic hinges in a quasi-mechanism, the collapse properties given in (15.2) provide the following conditions.

$$f_{io} < 0 \qquad f_{ij} > 0 \qquad \ldots (15.3)$$

where i, j: $1 \longrightarrow m$.

These relations show that the influence coefficients due to the basic hinges must satisfy certain requirements which depend on the whole group of hinges. It may be remembered that in the plastic collapse theory, a similar assumption is made, as the moments and the rotations at any of the hinges must be such that the virtual work terms are positive. It may be possible to obtain the above conditions using the principles of energy as at the limiting case of a quasi-mechanism, compatibility conditions must still be satisfied and the complementary potential energy due to the total deformation must be a maximum (54). This would only be satisfied if none of the hinges close under the result of an increased deformation.

A necessary condition for a hinge system to be suitable for inelastic compatibility analysis assuming positive moments at all hinge sections is given by $f_{ij} > 0, f_{io} < 0$ for at least one group of hinges which forms the basic hinges in a quasi-mechanism.

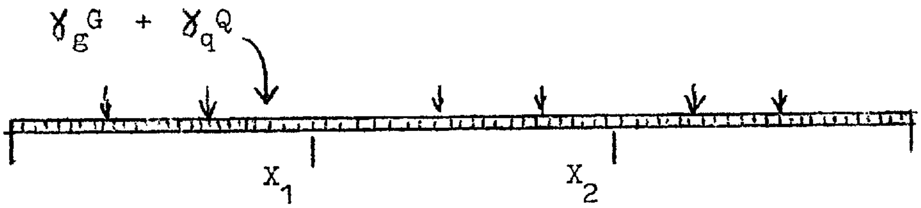
The case where $m = n$ may be referred to as an absolutely compatible hinge system, and if $m < n$, it may be said to conditionally compatible. In either case a compatible solution with n plastic hinges could be determined. But if the above necessary conditions cannot be satisfied for any group of hinges, then an equilibrium state as assumed does not exist and the corresponding hinge system is unsuitable for inelastic compatibility analysis.

A trial and adjustment method applied to such hinge systems would always provide a negative result.

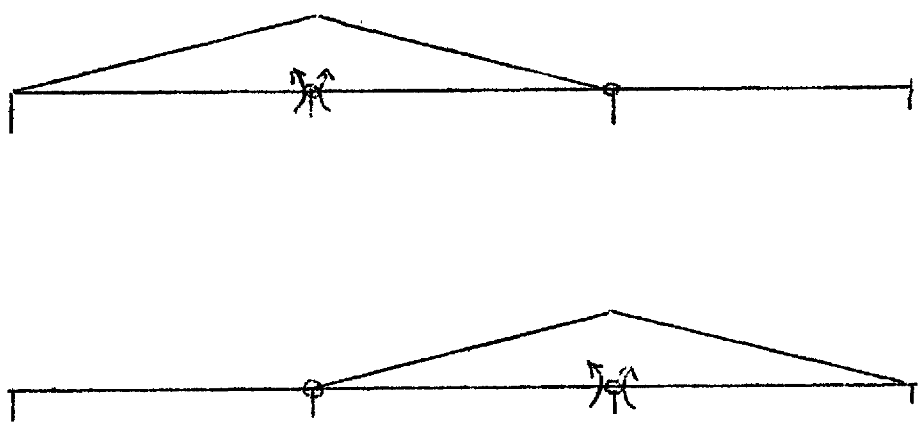
The reduction of the moments of any of the hinge sections in an absolutely compatible hinge system gives rise to a compatible set. This could be easily verified from the compatibility equations. In such cases the concept of moment redistribution as stated in Chapter 8 could be applied without a check for compatibility, provided that the permissible limits for the rotations of the hinges thus formed are not exceeded. However in conditionally compatible hinge systems any redistributions must be accompanied by a compatibility check. The significance of the latter may be easily seen from the subsequent examples, which are selected to primarily demonstrate the application of the suitability criteria to different hinge systems. The procedure may be extended to other structures so that it would be possible to obtain general hinge systems for different categories of structures that are suitable for inelastic compatibility analysis.

Example 1. Continuous Beams

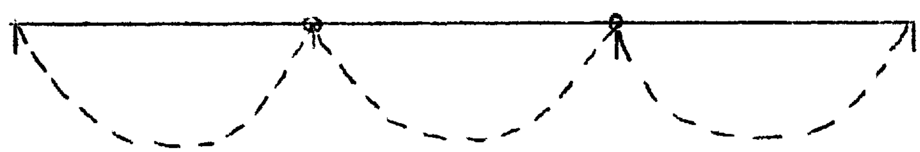
Consider a continuous beam and a typical load system as shown in Fig. 15.2. For a hinge system where the supports are chosen as the plastic hinges, the influence coefficients are of the type $f_{ij} > 0$, $f_{io} < 0$ for all the hinges. Thus the hinge system is absolutely compatible. Similarly in continuous beams other hinge systems involving midspans may be found which are also




(a) Ultimate Load



(b) Influence Diagrams



(c) Free Moment Diagrams

$f_{11}, f_{12}, f_{22} > 0 \qquad f_{10}, f_{20} < 0$

Fig. 15.2 A hinge System for a Typical Continuous Beam absolutely compatible. This would also confirm the validity of the concept of moment redistribution as suggested by Glanville and Thomas [1] in the design of continuous beams, as no check for compatibility is required except as a limit for the degree of redistribution to prevent unserviceability at working load or to prevent crushing of concrete.

Example 2. Portal Frames

A portal frame with typical vertical and lateral loads as shown in Fig. 15.3. Four possible hinge systems that are statically admissible are indicated in (b), (c), (d) and (e). The direction of rotation of the plastic hinges at each section is indicated in the diagram which must be assumed to start with as the same section may have different plastic rotation characteristics when hinging one way or the other, and the direction of rotation is also required before compatibility could be ascertained.

Considering the first three systems it could be seen that the influence coefficients are such that $f_{ij} > 0$, $f_{io} < 0$ where i, j: $1 \rightarrow 3$. Thus any of the hinge systems (b), (c) or (d) are suitable for compatibility analysis and infact they are absolutely compatible.

In the hinge system (e) which would be considered similar to (b) in elastic analysis, the influence coefficients are such that $f_{10} > 0$ while $f_{20}$, $f_{30} < 0$. $f_{12}$, $f_{13}$, $f_{21}$, $f_{31} < 0$ while the rest of the coefficients are positive. The group of hinges (2) and (3) which satisfy the condition $f_{10} < 0$, $f_{ij} > 0$ does not form the

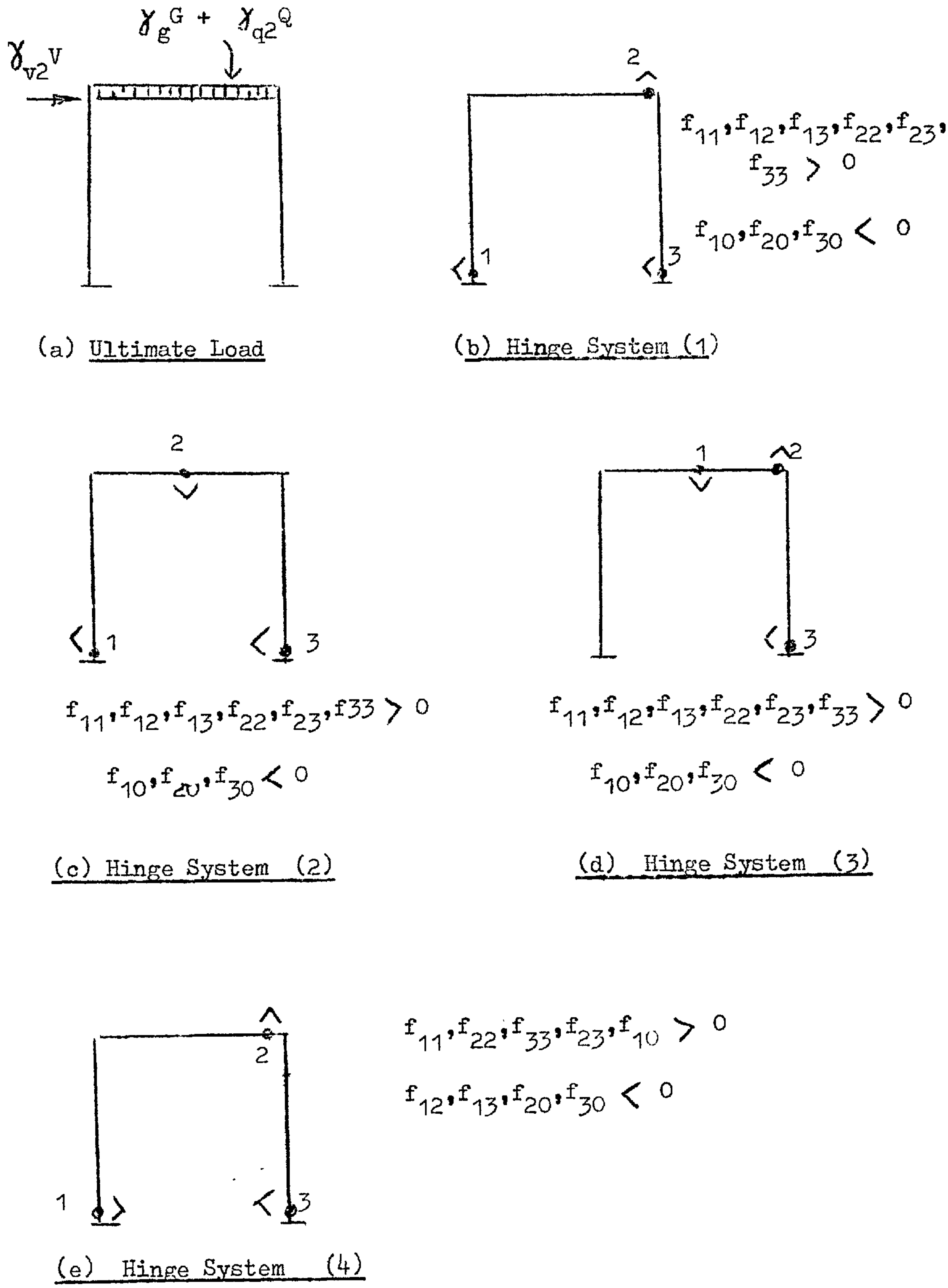



Fig. 15.3 Portal Frame

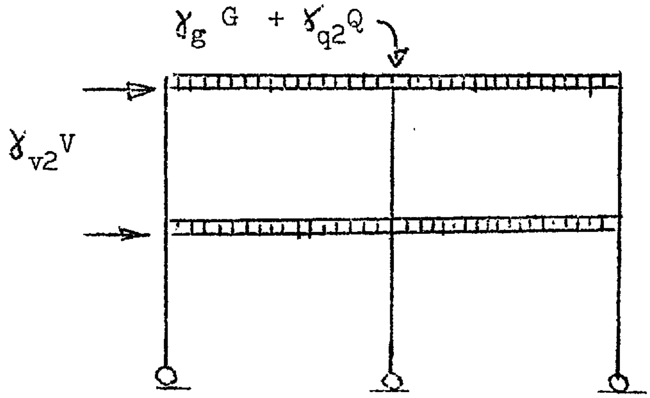
basic hinges of a quasi-mechanism. Hence this hinge system is unsuitable for an inelastic compatibility analysis.

In the case of multiple portals under vertical and lateral loads, the pattern of hinges used in (d) for a single portal frame may be used in each bay to obtain a statically admissible system. In general such hinge systems may also be found to be absolutely compatible.

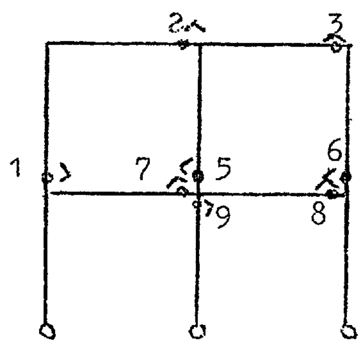
Example 2. Multi-Storey Frames

This example shows a more general structure of a two storey frame (Fig. 15.4). The loading diagram is shown in (a). Two of the possible hinge systems are shown in (b) and (c) which are used primarily to illustrate the application of the suitability criteria which may equally well be applied to other possible systems. The direction of the hinges are marked by the side of the hinges as in the previous case.

The hinge system (1) corresponds to the type of hinges suggested by Baker in order to separate storey sway. (5,9) The nature of the influence coefficients is shown in the accompaning diagram in which a question mark indicates that the influence coefficient may be affected by the magnitude of the stiffness values. In these cases the likely sign is given on the assumption that the structure is symmetrical, but in actual cases these values could be easily checked. From the influence coefficients it may be seen that this hinge system

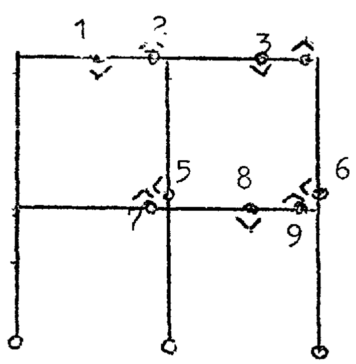



Ultimate Load




| | $f_{ij}$ 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | $f_{io}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | + | − | − | − | − | − | + | + | + | + |
| 2 | − | + | + | − | + | + | 0 | 0 | 0 | − |
| 3 | − | + | + | − | + | +? | 0 | 0 | 0 | − |
| 4 | − | − | − | + | + | + | 0 | 0 | 0 | −? |
| 5 | − | + | + | + | + | + | − | − | + | − |
| 6 | − | + | + | + | + | + | − | − | − | −? |
| 7 | + | 0 | 0 | 0 | − | − | + | + | − | − |
| 8 | + | 0 | 0 | 0 | − | − | + | + | + | − |
| 9 | + | 0 | 0 | 0 | + | − | − | + | + | − |

Hinge System (1)




| | $f_{ij}$ 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | $f_{io}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | + | + | + | + | + | + | + | + | + | − |
| 2 | + | + | + | + | +? | + | + | + | + | − |
| 3 | + | + | + | + | +? | + | + | + | + | − |
| 4 | + | + | + | + | + | + | + | + | + | − |
| 5 | + | +? | +? | +? | + | +? | − | − | − | −? |
| 6 | + | + | + | +? | + | + | − | − | − | −? |
| 7 | + | + | + | + | − | − | + | + | + | − |
| 8 | + | + | + | + | − | − | + | + | + | − |
| 9 | + | + | + | + | − | − | + | + | + | − |

Hinge System (2)

Fig. 15. 4 Two storey frame

is not absolutely compatible. Hence the separate groups of hinges for which $f_{ij} > 0$ , $f_{io} < 0$ may be checked for quasi-mechanism condition. In this case it may be found that not a single group of hinges satisfies quasi-mechanism condition, which shows that this hinge system could be considered as unsuitable for inelastic compatibility analysis as suggested. However if the hinge (1) is reversed in sign, the resulting hinge system may be found to be suitable as the group 1-4-3-5-6 under these conditions satisfies $f_{ij} > 0$, $f_{io} < 0$ and they also form the basis for a quasi-mechanism (leading to a sway type mechanism). But it must be noted that in multistorey structures this type of reversal may not be possible as it may give rise to cantilever effect on the first column with the resulting deterioration of its stiffness. In general it may be noted that the sway mechanism type of failure would also be the least likely for multistorey structures.

The hinge system (2) and the corresponding influence coefficients are shown in (b). As in the previous case these are not absolutely compatible. But out of these hinges, six groups of hinges may be selected for which $f_{ij} > 0$ , $f_{io} < 0$ and quasi-mechanism conditions are satisfied. These are given by 1-2, 2-3, 8-9, 7-8-9, 1-2-3-4-5-6 and 1-2-3-4-7-8-9. Of these the last quasi-mechanism leads to an over all collapse mechanism for the structure involving the largest number of hinges. Thus the hinge system (2) is suitable for inelastic compatibility analysis.

Any other statically admissible hinge systems for this structure may be analysed in the same way for its suitability. In multistorey structures the general type of hinges as in system (2) above may be suitable when the vertical and lateral loads are considered in the ultimate load. In these cases the possible quasi-mechanism would also include the over all type of collapse mechanism.

# CHAPTER 16.

## An approximate method of multistorey structures for ultimate load.

### 16.1. Elasto-plastic design for multistorey frames.

A method of combining elementary load systems derived in Chapter 14 shows that the different load configurations that must be considered in the design of multistorey structures consist of only two or three combinations of loads. Of these it has been shown that the design of the structure for the combined action of vertical loads is similar to that of continuous beams, for which the design methods have been completely discussed in Part 3. However, for the design of the structure for the combined action of the vertical and lateral loads, the whole structure must be considered, which may be based on an elastic or an elasto-plastic method. In the latter case, the reduction of the structural stiffness considerably increases the deflections prior to the ultimate load state, thus increasing the danger of premature failure due to instability[39]. This would make it inevitable that any elasto-plastic method must include provisions to safeguard against instability effects.

The following analysis shows that in general multistorey frames may be divided into two categories, depending on whether an elastic design or an elasto-plastic design would be more economical. In structures where the axial loads are large, the increased deflections that arise out of an elasto-plastic design, would outweigh the advantages derived from a reduction of the moments. Thus, in such cases, a deviation from the elastic design may lead to a more uneconomical design if instability effects are to be avoided.

16.2. <u>An approximate limit method</u>.

Consider the effects of the vertical and lateral loads acting on a multistorey frame. A typical storey is shown in Fig.16.1. The following simplifying assumptions would be made in order to derive general expressions for beam and column critical moments.

1. The columns are uniform in size and are equally reinforced on either face so that the positive and the negative moments are equal.

2. The lateral displacements of all the columns are equal and corresponds to the storey sway.

In general columns are designed so that the lateral forces could act from either side as in the case of wind forces, thus the assumption (1) could be considered as generally true in practical design. It may also be noted that in columns the reinforcements run through the length of each column. The assumption (2) could be considered as a first order approximation as long as no beam mechanisms are formed.

Now consider the release system for a typical storey shown in Fig.16.1. The compatibility requirements for each of the similar hinges may be derived in terms of the common sway angle for the storey. The limit moments for the beam and column sections for an intermediate and end panel are obtained in the following sections.

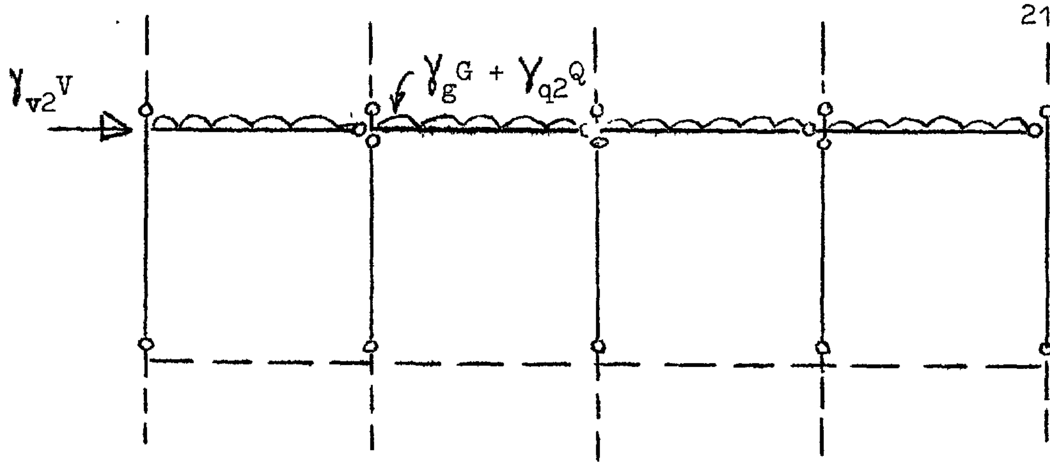



Fig. 16.1. Loads and Release System for Typical Storey

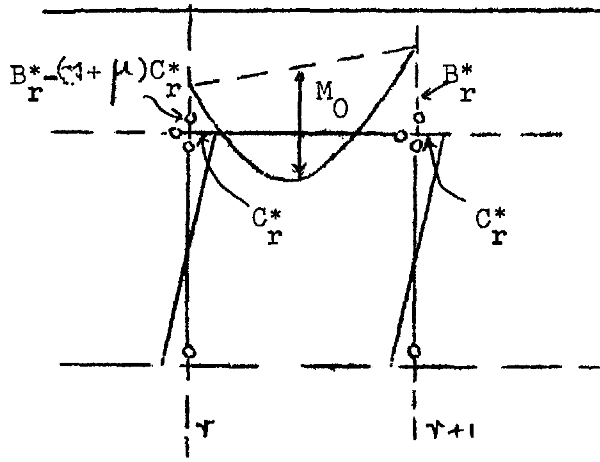



Fig. 16.2. Internal Panel

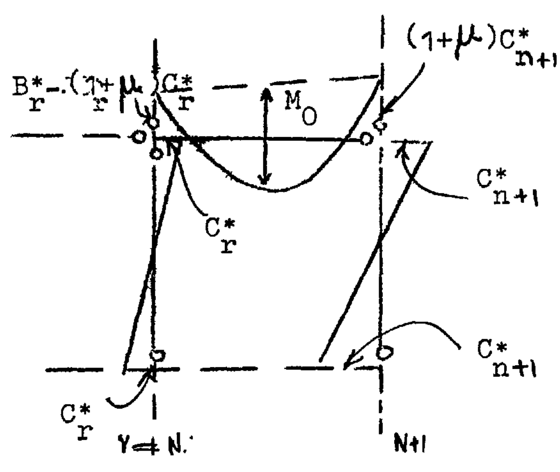



Fig. 16.3. End Panel

Internal panel in an intermediate storey.

The critical section moments for an internal panel are shown in Fig.16.2. Let $B_r^*$ and $C_r^*$ be the beam and column moments as shown. If $\psi$ denotes the storey sway, considering the end rotation of the members at the column hinge, the rotation of the column hinge $\theta_{rc}$ is given by,

$$-\left(\theta_{rc} - \psi\right) = \frac{\ell}{6EI}\left[2M_o - 2\left(B_r^* - (1+\mu)C_r^*\right) - B_r^*\right] + \frac{C_r^* h}{EI} \qquad \ldots (16.1)$$

where, $M_o$ = free span moment,

EI, EJ, = flexural stiffness of beam and column.

h = height of column.

$\mu$ = ratio of column moment in upper storey to the corresponding moment in this storey.

For compatibility, the rotation at the column hinge must satisfy the condition $\theta_{rc} \geq 0$. Thus denoting $\ell/6EI$ by k and $EIh/EJ\ell$ by y, the minimum beam moment to satisfy compatibility is given by,

$$B_r^* \geq \frac{2}{3} M_o + \frac{1}{3}(2 + 2\mu + y)\, C_r^* - \frac{\psi}{3k} \qquad \ldots (16.2)$$

An approximate value for the internal column moment $C_r^*$ could be obtained by distributing the storey moment in proportion to the column stiffness. The instability effect due to the axial load and sway deflection in the folumn is also taken into account in the column moment given by,

$$C_r^* = \frac{\gamma_{v2}\, Vh(EJ)_r}{2\sum_1^N (EJ)_i} + \frac{N_r h \psi}{2} \qquad \ldots (16.3)$$

where $N_r$ is the axial load in the $r^{th}$ column, N is the total number of columns, V is the storey sway force and $\gamma_{v2}$ is the overload coefficient for V.

It may be noted that in equation (16.3) the increase in the sway angle also increases $C^*_r$, which may not necessarily be uneconomical as the actual moment in the column may be larger for minimum service requirements. However an increase of $B^*_r$ due to larger sway would in all cases lead to uneconomic conditions.

Substituting for $C^*_r$ in equation (16.2),

$$B^*_r \geqslant \frac{2}{3} M_o + \frac{1}{3} (2 + 2\mu + y) \frac{\gamma_{v2} Vh(EJ)_r}{\sum_1^N (EJ)_i} + \psi' \left[\frac{1}{6} P_r h(2 + 2\mu + y) - \frac{1}{3k}\right] \quad \ldots (16.4)$$

$$\text{Let } P_{ro} = \frac{12 \text{ EI}}{\ell h(2 + 2\mu + y)} \quad \ldots (16.5)$$

From equation (16.4), it may be seen that if $P_r < P_{ro}$, then an increase in the sway angle decreases the beam moment, but if $P_r > P_{ro}$ an increase in sway also increases the beam moment. As the elastic conditions correspond to the least sway, the above limiting conditions represent the approximate limits for the axial load above which an elasto-plastic design would be uneconomical compared to an elastic design.

If the axial load in the column is less than the above limit, then a satisfactory value for the beam moment may be obtained by substituting a suitable value for the sway angle $\psi'$, which may enable the full column moments to be utilised while the effective beam moments are reduced. A limiting value for the sway angle may be obtained by considering the beams to be rigid, and the column to deflect elastically. Since all columns sway by the same amount, the corresponding $\psi_{min}$ is easily obtained as

$$\psi_{min} = \frac{\gamma_{v2}\ Vh^2}{12\sum_1^N (EJ)_i - h^2\sum_1^N P_r} \qquad \ldots (16.6)$$

In practice, the actual minimum value of the sway angle would be larger than the above as the beams are flexible. A value of $\psi$ greater than the $\psi_{min}$ may be substituted in equations (16.4) and (16.3) to obtain the corresponding values of the critical beam and column moments. If the minimum column moments required for serviceability are known beforehand, it would be possible to use as large a value of $\psi$ as would yield this value of the column moment. The corresponding beam moment may be found to be the most economical as any further increase in sway would increase the column moment, which may eventually prove to be uneconomical, as unlike in beams, the extra reinforcement that must be introduced in the columns are carried over the whole length of the column. Further it must be remembered, that even in the case of beams the minimum that the beam moments may be reduced with any effect in the actual design is given by the serviceability requirements.

End panel in an intermediate storey

As in the previous case the limit critical moment for the end column may be derived in terms of the compatibility conditions for the penultimate column hinge $\theta_{Nc}$. Consider the end panel of an intermediate storey as shown in Fig.16.3. Let $C^*_{N+1}$ be the end column moment and $(1+\mu)C^*_{N+1}$ be the moment at the last beam section as indicated in the diagram. The rotation $\theta_{Nc}$ is given by,

$$-(\theta_{Nc} - \psi) = \frac{1}{6EI}\left[2M_o - 2B^*_r - (1+\mu)C^*_r - (1+\mu)C_{N+1}\right] + \frac{C^*_r h}{6EJ} \quad \ldots (16.7)$$

For compatibility, $\theta_{Nc} \geqslant 0$. Then using the same notation for k and y as before the following condition for $C_{N+1}$ may be derived from 16.7.

$$C_{N+1} \geqslant \frac{2}{1+\mu}\left[M_o - B^*_r + (1+\mu+\frac{y}{2})\,C^*_r - \frac{\psi}{2k}\right] \quad \ldots (16.8)$$

The instability moment in the column must be added to $C_{N+1}$ to obtain a limit value of the column moment $C^*_{N+1}$ which is required to resist both the ultimate load and the instability effects due to sway. Thus if $P_{N+1}$ be the axial load in the end column, the minimum value of the column moment is given by,

$$C^*_{N+1} \geqslant \frac{2}{1+\mu}\left[M - B^*_r + (1+\mu+y)\,C^*_r + (P_{N+1}h - \frac{1}{k})\,\frac{\psi}{2}\right] \quad \ldots (16.9)$$

Substituting for $B_r^*$ from equation 16.2,

$$C_{N+1}^* \geq \frac{2}{1+\mu}\left[\frac{M}{3} - \frac{2-y}{6} \quad C_r^* + \left(P_{N+1}h - \frac{1}{3k}\right) \quad \frac{\psi}{2}\right] \qquad \ldots (16.10)$$

Since $C_r^*$ and $\psi$ are known from the previous calculations, $C_{N+1}^*$ may be determined. Then the minimum beam moment at the end span hinge section is given by $(1 + \mu)C_{N+1}^*$.

Top storey

Generally in the case of the uppermost storey, the instability effects would be small. Using a similar set of releases as in the case of the intermediate stories, the following limit conditions could be easily derived.

$$\psi_{min} = \frac{\gamma_{v2} \, V h^2}{12\sum_1^N (EJ)_i} \qquad \ldots (16.11)$$

$$C_r^* = \frac{\gamma_{v2} Vh(EJ)_r}{\sum_1^N (EJ)_c} \qquad \ldots (16.12)$$

$$B_r^* = \frac{2}{3} M_o + \frac{1}{6}(7 + 2y)C_r^* - \frac{\psi}{3k} \qquad \ldots (16.13)$$

$$C_{N+1}^* = \frac{2}{3} \; M_o + \frac{1}{6}(5 + 4y)C_r^* - \frac{2}{3}\frac{\psi}{k} \qquad \ldots (16.14)$$

It may be noted that in this case the beam moments are reduced by the increase of sway, but the serviceability requirements would invariably place an upper limit for the amount of restriction that may be allowed in each case.

A similar approach may be made in the case of the bottom storey depending on the nature of the foundation but, if the assumptions made in the intermediate storeys are applicable for the bottom storey, the same equations may be used.

Example

Consider a typical storey of a multistorey frame shown in Fig.16.4, for which the following design values are provided.

Axial loads in all columns = 400,000 lbs.

Free moment due to permanent load = 1,000,000 lbs.

" " " " super load = 1,000,000 lbs.

Storey sway moment Vh = 1,000,000 lbs.

Use overload coefficients $\gamma_g = \gamma_{q2} = \gamma_{v2} = 2.0$

Yield safety in beams $\lambda_s = 1.2$

" " " columns $\lambda_s = 1.0$

$\sigma_{ay} = 40,000$ p.s.i., $\sigma_b' = 4000$ p.s.i.

$\mu = 0.8$.

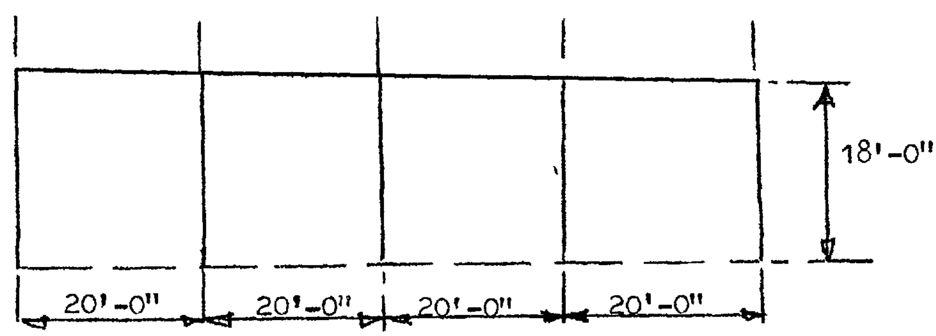



Fig.16.4. Typical Storey.

Select the following beam and columr sections.

| | | |
|---|---|---|
| Beams | b = 10", | h = 18". |
| Internal column | b = 12", | h = 15". |
| End column | b = 12", | h = 18". |

Assume EI for beams = $60 \sigma'_b bh^3$.

EI for int.col. = $150 \sigma'_b bh^3$,

EI for end col. = $100 \sigma'_b bh^3$.

(These values may be checked later).

$$\therefore EI = 60 \times 4000 \times 10 \times 18^3 = 1.39 \times 10^{10}$$

$$(EJ)_{INT} = 150 \times 4000 \times 12 \times 15^3 = 2.43 \times 10^{10}$$

$$(EJ)_{END} = 100 \times 4000 \times 12 \times 18^3 = 2.20 \times 10^{10}$$

$$y = \frac{EI}{1} \cdot \frac{h}{(EJ)_{INT}} = 0.38$$

From equation (16.5),

$$P_{ro} = \frac{12 \times 1.39 \times 10^{10}}{180 \times 240 \times 3.98} = 973{,}000 \text{ lbs.}$$

Thus the actual column loads are less than the above limiting case, so that an elasto-plastic solution may be adopted.

Consider the minimum sway angle in equation (16.6)

$$\psi_{min} = \frac{2 \times 10^6 \times 180}{12(3 \times 2.43 + 2 \times 2.20)10^{10} - 180^2 \times 5 \times 8 \times 10^5}$$

$$= 0.00028$$

<u>Case 1</u>. Consider, $\psi = 0.002$

Then from (16.3), $$C_T^* = \frac{2 \times 10^6 \times 2.43 \times 10^{10}}{2 \times 11.7 \times 10^7} + \frac{8\times10^5\times180\times0.002}{2}$$

$$= 208{,}000 + 144{,}000$$

$$= \underline{352{,}000 \text{ in lbs.}}$$

From equation (16.2),

$$B_r^* = \frac{2}{3} \times 4 \times 10^6 + \frac{1}{3} \times 3.98 \times 3.52 \times 10^5 - \frac{0.002 \times 6 \times 1.39 \times 10^{10}}{3 \times 240}$$

$$= \underline{2.90 \times 10^6}$$

The end column moment $C_{N+1}^*$ is given by equation (16.8)

$$C_{N+1}^* = 4 \times 10^6 - 2.90 \times 10^6 + 2.19 \times 3.52 \times 10^6 - \frac{0.002 \times 6 \times 1.39 \times 10^{10}}{2 \times 240}$$

$$= \underline{1,520,000 \text{ in lbs.}}$$

Case 2. Consider, $\psi = 0.004$

As in the previous case the following values for the limit moments could be obtained.

$\underline{C_r^* = 496,000 \text{ in lbs.}}$

$\underline{B_r^* = 2,867,000 \text{ in.lbs.}}$

$\underline{C_{N+1}^* = 1,490,000 \text{ in lbs.}}$

Thus the actual reduction of the beam moment in Case 2 is quite small as compared to Case 1, although the sway angle is doubled. Any further increase in the sway angle may increase the column moment beyond the minimum required for serviceability as shown below.

Approximate serviceability calculations.

The maximum moments in any internal column AX under working conditions are produced by unsymmetrical vertical load and lateral load as shown in Fig.16.5.

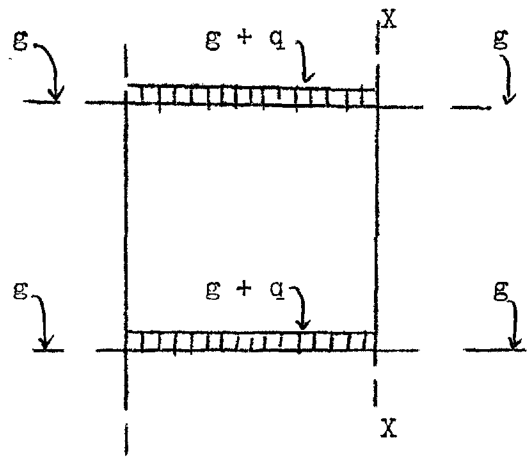



Fig.16.5. Critical loading for column XX.

The column moments may be determined by an approximate distribution of moment locally.

Thus, minimum column for serviceability = 250,000 + 100,000

= 350,000

As in continuous beams, the critical support moment in the frame are obtained when the alternate spans are loaded.

Thus, minimum moment at internal supports $\simeq \lambda_s\left(\frac{1}{12}\,gl^2 + \frac{1}{10}\,q.l^2 + \frac{Vh}{10}\right)$

$\simeq$ 1,880,000 in lbs.

Serviceability calculations for the above example show that in frames, the internal column moments may be governed by the service limits whereas the beam support and end column moments may be primarily governed by ultimate load limits. The service calculations also show that the Case 1, worked out earlier, yields the minimum column moments and any further decrease in beam moments increases the column moments which may eventually prove to be more uneconomical.

Check for the assumption for stiffness coefficients.

As in Case 1, if $B_r^* = 2.90 \times 10^6$,

$$n_u = \frac{B_r^*}{\sigma_b' bh^2} = \frac{2.90 \times 10^6}{4000 \times 10 \times 18^2} = .224$$

$\therefore$ From Fig. 5.2, $\bar{\omega} = 0.25$

From Fig. 6.8a $\xi = 65$

which is very close to the value assumed.

Internal columns: $n_u = \frac{800{,}000}{4000 \times 12 \times 18} = 0.93$

$$m_u = \frac{1{,}520{,}000}{4000 \times 12 \times 18^2} = 0.098$$

From Fig. 5.1, $\bar{\omega} = \bar{\omega}' = 0.14$

From Fig. 6.8a and equation 6.12 for column stiffness,

$$= (1 + 1.11 \quad 1.8)51$$

$$= 153$$

External columns ; $n_u = \frac{800000}{4000 \quad 12 \quad 18} = 0.93$

$$m_u = \frac{1520000}{4000 \quad 12 \quad 18^2} = 0.098$$

From Fig. 5.1, $= \quad = 0.11$

$$_{col} = (1 + 0.93 \quad 1.8)42$$

$$= 112$$

These values of effective flexural stiffness coefficients are very close to those already assumed and no correction need be made to the calculated moment values.

# CHAPTER 17

## Conclusions and suggestions for further research

### 17.1 . . Review of limit design criteria

The main conclusions that may be derived from the over all analysis of limit design methods in the foregoing chapters could be divided into two categories.

1. Limit design criteria.

2. Application of the limit design criteria to the design of skeletal structures.

The discussion in chapters 2 and 3 shows that the main conditions that must be considered in the limit design of reinforced concrete structures are based on the following:

1. Adequate safety against probable over load and variation in material properties.

2. Minimum safeguard against unserviceability of the structure for random combinations of the working load.

3. Highest economy in the over all design and construction of the structure within the scope of conditions 1 and 2 above.

Before a complete and satisfactory method of limit design could be adopted, each of the above conditions must be investigated in detail, to determine the minimum requirements for different types of structures, and to determine the means by which they may be ensured in the actual structure.

(1) Safety analysis

The investigation of safety in general, may be subdivided into three sections:

a. Load analysis

b. Material analysis

c. Structural analysis.

Of these, the first two sections are independent, while the third depends on the first two and the available methods and techniques of analysis.

a. Load analysis

The loads that are to be used in the limit design, may be specified by their characteristic or mean values, and respective over load coefficients or partial safety factors. These may take into account, the different probabilities of the loads being exceeded either individually or with possible combinations. The Russian and ACI codes of practice seem to have adopted a similar approach already.

b. Material analysis

The basic materials used in reinforced concrete structures possess widely different characteristics each subject to individual variations of different magnitude depending on methods of manufacture. These may be taken into account in the limit design by respective coefficients of variation or partial safety factors. These would also enable a rational approach to design which truly reflects the individual material characteristics. The CEB recommendations on the partial safety factors on concrete and reinforcement may be extended to take into account varying degrees of quality control attained in factory conditions as in precast members and in site conditions with varying degrees of quality control.

c. Structural analysis

The existing simple design methods for reinforced concrete structures incur different idealisations for the properties of materials, each of which is subject to varying error. The overall effect of these idealisations in relation to the actual structure may be evaluated to enable a study of the reliability of the different methods of analysis. A work load coefficient as in the Russian design practice may be attached to the method of analysis as a means of correlating the degree of safety or unsafety in the method of simple analysis to the actual structure. Thus similar structures that would be analysed by different methods would be expected to have similar over all safety when the respective work load coefficients are applied to them.

2. Serviceability Analysis

The functional and aesthetic consideration of structures under normal service loads could be expressed in terms of:

1. Yield safety which may ensure that the risk of repeated loading may not have any serious affect on the strength of

the structure.

2. Limit crack width which may not be exceeded without exposing the tension reinforcement to adverse weathering conditions or rendering the structure unsound.

3. Limit deflection which must not be exceeded for functional and other requirements.

The recent recommendations by the C.E.B. cover some of the above aspects, but detailed information for different types of structures must be available, as the serviceability limits are considerably important, in view of the fact that the over all safety factors in the limit design are less than those, that have been used in conventional design methods.

3. Economic design

While safety and serviceability requirements are minimum conditions that must be satisfied, it may be concluded that economic criteria based on the total cost or any other cost related function could be used as an optimising criteria to obtain the most economic design within the scope of the above limits. The total volume of reinforcement used in the structure could be used as a simple basis of evaluating the economic design conditions.

17.2 Application of limit design criteria

The conclusions derived from the experimental and theoretical investigations in Part II, III and IV may be divided into two sections:

1. Basic properties of reinforced concrete members

2. Design of indeterminate structures.

17.21 Basic properties of reinforced concrete members- Moment rotation characteristics

The moment-rotation characteristics of reinforced concrete members may be represented by a bilinear relation defined by the effective flexural stiffness, the ultimate moment and the plastic rotation capacity as discussed in Chapters 5,6 and 7. In reinforced concrete members, the error in the bilinear idealisation as compared to trilinear idealisation seems to be less than 10%, which is less than the variations in the experimental results. The following simple idealised bilinear characteristics may be used in limit design methods.

Effective Flexural stiffness of reinforced concrete members could be represented by Baker's idealised limit $L_1$ calculations or by the semi-empirical relation given below.

$$EI = \xi \sigma_b' b h^3$$

where, $$\xi = \left(175 + 31200 \frac{\sigma_{av}}{E_a}\right)(1 + 1.8 n_u)\sqrt{\omega}$$

which is applicable for both rectangular beams and columns. (see Fig.6.8a)

Plastic rotation capacity

This is subject to large variations, but a safe limit value is given by

$$\theta_p = 4.8( e_{c2} - e_{c1})$$

where $e_{c2} = 0.0015 \left[1 + 1.7p'' + (0.7 - 0.1p'') \frac{1}{x_2}\right]$

(see Fig. 7.1)

Ultimate strength of sections

The ultimate moment in beams depends largely on the properties of reinforcing steel, and in practice when the beams are under reinforced, the nature of the stress block has very little effect on the ultimate moment. Figs. 5.1 and 5.2 show graphical methods of obtaining the ultimate strength properties of beams and columns.

17.22 Design of indeterminate structures using elasto-plastic methods-ultimate load theory.

The ultimate load design of indeterminate structures would be greatly simplified by considering the particular combinations of elementary load systems which has the greatest probability of collapse, by using the principle of combined loading derived in Chapter 14.

2. Elasto-plastic design by trial and adjustment method

The design of indeterminate structures using inelastic compatibility conditions as a basis must be checked for suitability of the adopted hinge systems using the criteria derived in Chapter 15.

The concept of redistribution of moments without compatibility checks seems to be applicable for hinge systems that are absolutely compatible (Chapter 15) which is encountered only in continuous beams and portal frames (single or multiple portal frames). In multistorey skeletal structures, the redistribution of moments (i.e. elasto-plastic design) would require an eventual check for compatibility of the resultant moments.

3. Design of reinforced concrete continous beams

Continous beams with hinges at the support sections form an absolutely compatible hinge system. Thus the concept of redistribution could be used and no check for compatibility would be required except to determine if the hinge rotations are within permissible limits.

From the detailed discussion of elasto-plastic design of continous beams in Part II the following conclusions may be derived:

1. Under gravity loading, the ultimate load configuration consists of all the spans being loaded.

2. If $\lambda_s$ is the serviceability parameter defined either by yield safety or crack width, then the degree of redistribution is limited by R $\ngtr 1-\lambda_s/\lambda_0$ where $\lambda_0$ depends on the ratio of super-imposed load and elastic moment characteristics given in Chapter 11.

3. The permissible hinge rotation at any of the hinges under a given redistribution R may not be exceeded

provided the ductility of the hinge sections ( Chapter 7) exceeds the following limits:

$$D \leqslant \frac{R}{1-R}\,\frac{l}{2h} \text{ for intermediate spans}$$

$$D \leqslant \frac{R}{1-R}\,\frac{2l}{3h} \text{ for end spans.}$$

4. The maximum deflection and shear can be used as limit conditions to derive the upper and lower limits for $\frac{l}{h}$ of the beam as in Chapters 11 and 12.

5. Using the total volume of reinforcement as an optimising criteria, it has been found that the most economic design corresponds to the equilibrium state which involves the least redistribution of moments. (Chapter 9).

6. Experimental results for eight three-span continous beams are consistent with the predictions based on the idealised moment-rotation characteristics given in section 17.21. Thus the idealised bilinear relations form a reasonable basis for both safety and serviceability calculations. In continous beams, with under-reinforced sections, a yield safety factor of 1.0 did not have any adverse effect on the ultimate safety when the working load was repeated a few times.

4. Multistorey skeletal structures

When an elasto-plastic method of design is used in the design of multistorey skeletal structures, the instability effects due to axial loads must be taken into consideration. The approximate limit conditions within which an economic elasto-plastic design may be possible is given by the critical value of the axial load in columns as in equation 16.6. When the axial loads exceed this value, it was found that the most economic design corresponds with the elastic distribution of moments for ultimate load.

## 17.4 Suggestions for further research

As pointed out in the Part 1 of this thesis, the development of limit design methods would primarily depend on the reliability of the information available by which the specific limits could be defined. Particularly, the greatest advantage in the limit design methods is that, it lends itself easily to statistical investigations. Thus the nature of loads, inclusive of mean values and their variations for different types of structures, and the probability of different loads occuring separately and together must be available before the maximum advantage of the limit methods could be utilised.

The structural problems involved in the application of limit methods may be investigated in two steps:

1. Development of rigorous methods of analysis based on the non-linear characteristics of reinforced concrete members. These need not be practical methods.

2. Comparison of simple and practical methods of design based on idealised properties of members, with the rigorous methods as above. This would enable the degree

of error in the simple methods to be estimated, and the corresponding design method to be weighted accordingly using the work load coefficients as suggested in Chapter 3. This would also involve evaluation of safety factors based on statistical variations of the properties of materials and their relative influence in the over all design.

The use of elasto-plastic design methods, increases the risks of instability considerably, unless special precautions are adopted. The analysis in Chapter 16 shows that if the axial loads in the columns be greater than certain critical values, the use of elasto-plastic methods would always incur instability type of failure Further investigations on this aspect of design would be necessary to extend the above methods to different structural conditions.

# APPENDIX 1

## Details of C.E.B. Tests

Extract from: " Inelastic Hyperstatical Frames - Analysis and Application of International Correlated Tests"
By, Prof. A.L.L.Baker and A.M.N.Amarakone.
Proc. of the International Symposium on Flexural Mechanics of Reinforced Concrete. ACI-ASCE
Nov. 1964 Miami, Florida.

TABLE 1.— BEAM DETAILS

(a) Cord-worked steel reinforcement

| | BEAM NO. | | b | h | h′ | $h_t$ | $\sigma'_b$ | $\sigma_{ay}$ | $\bar{\omega}$ % |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Imperial College | 9 | 6 in. | 6.85 in. | - | 8 in. | 4910 psi | 85.0 ksi | 10.5 |
| 2 | Imperial College | 10 | 6 in. | 6.81 in. | - | 8 in. | 4915 psi | 85.0 ksi | 14.9 |
| 3 | Imperial College | 11 | 6 in. | 6.76 in. | 1.12 in. | 8 in. | 4895 psi | 85.0 ksi | 22.5 |
| 4 | Imperial College | 12 | 6 in. | 6.59 in. | 1.12 in. | 8 in. | 4890 psi | 85.0 ksi | 25.1 |
| 5 | Imperial College | 13 | 6 in. | 6.40 in. | 1.12 in. | 8 in. | 4390 psi | 85.0 ksi | 45.0 |
| 6 | Imperial College | 14 | 6 in. | 6.67 in. | 1.20 in. | 8 in. | 4470 psi | 85.0 ksi | 16.7 |
| 7 | Imperial College | 15 | 6 in. | 6.76 in. | 1.23 in. | 8 in. | 4450 psi | 85.0 ksi | 33.0 |
| 8 | Imperial College | 16 | 6 in. | 6.28 in. | 1.23 in. | 8 in. | 4550 psi | 85.0 ksi | 70.4 |
| 9 | Torino | F 4 | 15 cm | 24.5 cm | 2.5 cm | 28 cm | 390 kg/cm$^2$ | 47.5 kg/mm$^2$ | 39.8 |
| 10 | Torino | L 4 | 15 cm | 24.5 cm | 2.5 cm | 28 cm | 308 kg/cm$^2$ | 47.5 kg/mm$^2$ | 50.6 |
| 11 | Torino | D 8 | 15 cm | 25.5 cm | 2.5 cm | 28 cm | 374 kg/cm$^2$ | 50.4 kg/mm$^2$ | 11.0 |
| 12 | Torino | G 4 | 15 cm | 25.5 cm | 2.5 cm | 28 cm | 308 kg/cm$^2$ | 50.0 kg/mm$^2$ | 53.5 |
| 13 | Paris (IRABA) | E 6 | 15 cm | 24.5 cm | 2.5 cm | 28 cm | 258 kg/cm$^2$ | 55.8 kg/mm$^2$ | 22.5 |
| 14 | Paris (IRABA) | E 9 | 15 cm | 25.5 cm | 2.5 cm | 28 cm | 252 kg/cm$^2$ | 54.5 kg/mm$^2$ | 11.7 |
| 15 | Paris (IRABA) | F 6 | 15 cm | 24.5 cm | 2.5 cm | 28 cm | 415 kg/cm$^2$ | 60.5 kg/mm$^2$ | 23.3 |
| 16 | Paris (IRABA) | F 9 | 15 cm | 25.5 cm | 2.5 cm | 28 cm | 450 kg/cm$^2$ | 56.6 kg/mm$^2$ | 6.6 |
| 17 | Paris (IRABA) | H 2 | 25 cm | 25.5 cm | 2.5 cm | 28 cm | 308 kg/cm$^2$ | 51.6 kg/mm$^2$ | 8.2 |
| 18 | Paris (IRABA) | H 5 | 15 cm | 24.5 cm | 2.5 cm | 28 cm | 287 kg/cm$^2$ | 54.4 kg/mm$^2$ | 46.0 |
| 19 | Paris (IRABA) | H 8 | 15 cm | 25.5 cm | 2.5 cm | 28 cm | 295 kg/cm$^2$ | 48.9 kg/mm$^2$ | 15.2 |
| 20 | Paris (IRABA) | H 11 | 30 cm | 25.5 cm | 2.5 cm | 28 cm | 290 kg/cm$^2$ | 52.1 kg/mm$^2$ | 7.6 |
| 21 | Paris (IRABA) | R 4 | 15 cm | 24.6 cm | 2.5 cm | 28 cm | 292 kg/cm$^2$ | 48.1 kg/mm$^2$ | 59.0 |
| 22 | Paris (IRABA) | R 5 | 15 cm | 24.5 cm | 2.5 cm | 28 cm | 317 kg/cm$^2$ | 52.0 kg/mm$^2$ | 41.5 |
| 23 | Paris (IRABA) | R 6 | 15 cm | 24.5 cm | 2.5 cm | 28 cm | 387 kg/cm$^2$ | 54.5 kg/mm$^2$ | 25.1 |
| 24 | Porto | C 6 | 15.10 cm | 24.82 cm | 3.7 cm | 28.02 cm | 265 kg/cm$^2$ | 47.0 kg/mm$^2$ | 36.4 |
| 25 | Porto | C 7 | 15.04 cm | 25.60 cm | 3.15 cm | 28.07 cm | 303 kg/cm$^2$ | 48.0 kg/mm$^2$ | 20.3 |
| 26 | Porto | C 9 | 14.48 cm | 26.30 cm | 4.5 cm | 28.07 cm | 323 kg/cm$^2$ | 41.0 kg/mm$^2$ | 6.5 |
| 27 | Porto | C 10 | 30.31 cm | 25.57 cm | 3.58 cm | 28.37 cm | 307 kg/cm$^2$ | 51.0 kg/mm$^2$ | 10.1 |
| 28 | Porto | C 12 | 30.14 cm | 26.32 cm | 3.82 cm | 28.27 cm | 279 kg/cm$^2$ | 42.0 kg/mm$^2$ | 3.6 |
| 29 | Porto | M 7 | 15.20 cm | 25.48 cm | 3.60 cm | 27.96 cm | 341 kg/cm$^2$ | 49.0 kg/mm$^2$ | 18.0 |
| 30 | Porto | M 9 | 15.24 cm | 26.14 cm | 4.20 cm | 28.08 cm | 302 kg/cm$^2$ | 41.0 kg/mm$^2$ | 6.7 |
| 31 | Porto | M 10 | 30.14 cm | 25.52 cm | 3.35 cm | 28.14 cm | 341 kg/cm$^2$ | 48.9 kg/mm$^2$ | 9.1 |
| 32 | Porto | M 12 | 29.98 cm | 25.94 cm | 3.80 cm | 28.18 cm | 302 kg/cm$^2$ | 41.5 kg/mm$^2$ | 3.4 |

| | $\bar{\omega}'$ % | p″ % | $m_u$ | $\theta_1$ Cal. | $\theta_p$ Cal. | $\frac{m_1 \text{ Act.}}{m_1 \text{ Cal.}}$ | $\frac{m_2 \text{ Act.}}{m_2 \text{ Cal.}}$ | $\frac{\theta_p \text{ Act.}}{\theta_p \text{ Cal.}}$ | Remarks |
|---|---|---|---|---|---|---|---|---|---|
| 1 | - | - | .110 | .0270 | .035 | 1.09 | 1.11 | 0.8 | CPL L = 80 in. |
| 2 | - | - | .150 | .0324 | .027 | 1.04 | 1.09 | 1.8 | CPL L = 80 in. |
| 3 | 2.0 | .61 | .222 | .0330 | .024 | 1.08 | 1.14 | 1.5 | CPL L = 80 in. |
| 4 | 2.0 | .61 | .273 | .0311 | .019 | 1.19 | 1.14 | 1.7 | CPL L = 80 in. |
| 5 | 2.0 | .81 | .387 | .0260 | .017 | 1.18 | 1.34 | 1.2 | CPL L = 80 in. |
| 6 | 11.6 | .61 | .173 | .0320 | .032 | 1.06 | 1.13 | 0.9 | CPL L = 80 in. |
| 7 | 24.8 | 1.26 | .315 | .0331 | .029 | 1.09 | 1.12 | 0.8 | CPL L = 80 in. |
| 8 | 16.4 | 1.51 | .511 | .0259 | .024 | 1.26 | 1.27 | 2.5 | CPL L = 80 in. |
| 9 | 1.4 | .132 | .306 | .0228 | .011 | 1.17 | 1.00 | 1.0 | CPL L = 280 cm |
| 10 | 2.4 | .117 | .409 | .0210 | .010 | 1.12 | 1.12 | 2.8 | CPL L = 280 cm |
| 11 | 2.0 | .190 | .149 | .0191 | .036 | 1.20 | 1.32 | 2.0 | CPL L = 280 cm |
| 12 | 13.6 | .117 | .344 | .025 | .009 | - | 1.14 | - | CPL L = 280 cm |
| 13 | 1.67 | .176 | .373 | .0279 | .011 | 1.06 | 1.20 | 1.5 | CPL L = 280 cm |
| 14 | 1.67 | .176 | .119 | .0227 | .037 | 1.14 | 1.06 | 0.6 | CPL L = 280 cm |
| 15 | 1.04 | .176 | .246 | .0270 | .017 | 1.09 | 1.06 | 0.9 | CPL L = 280 cm |
| 16 | 0.96 | .176 | .067 | .0213 | .047 | 1.08 | 1.03 | 0.9 | CPL L = 280 cm |
| 17 | 3.17 | .051 | .097 | .0120 | .041 | 1.07 | 1.20 | 1.4 | CPL L = 280 cm |
| 18 | 1.50 | .062 | .370 | .0252 | .009 | 1.30 | 1.27 | 1.5 | CPL L = 280 cm |
| 19 | 1.47 | .062 | .153 | .0237 | .029 | 1.15 | 1.06 | 1.2 | CPL L = 280 cm |
| 20 | - | - | .080 | .0220 | .041 | 1.18 | 1.05 | 1.2 | (T) L = 280 cm |
| 21 | 15.3 | .176 | .490 | .0246 | .006 | 1.33 | 1.25 | 1.8 | (T) L = 280 cm |
| 22 | 14.1 | .176 | .394 | .0270 | .011 | 1.09 | 1.10 | 1.4 | (T) L = 280 cm |
| 23 | 6.6 | .176 | .262 | .0283 | .018 | 1.27 | 1.07 | 1.6 | (T) L = 280 cm |
| 24 | 3.0 | .25 | .298 | .025 | .013 | 1.26 | 1.09 | 1.8 | (T) L = 280 cm |
| 25 | 2.5 | .25 | .181 | .032 | .026 | 1.03 | 1.07 | 2.1 | (T) L = 280 cm |
| 26 | 2.5 | .25 | .077 | .0162 | .047 | 1.17 | 1.19 | 0.8 | (T) L = 280 cm |
| 27 | - | - | .102 | .025 | .039 | 1.04 | 1.05 | 4.7 | (T) L = 280 cm |
| 28 | - | - | .048 | .022 | .047 | 1.26 | 1.33 | 1.6 | (T) L = 280 cm |
| 29 | - | .25 | .124 | .0142 | .028 | - | - | - | CPL Shear Failure L = 140 |
| 30 | 2.5 | .25 | .087 | .0081 | .047 | 1.13 | 1.32 | 1.2 | CPL L = 140 cm |
| 31 | - | - | .093 | .0126 | .033 | .95 | 1.05 | 1.7 | (T) L = 140 cm |
| 32 | - | - | .046 | .0108 | .047 | 1.18 | 1.34 | 1.1 | (T) L = 140 cm |
| | | | | MEAN | | 1.14 | 1.15 | 1.53 | |

TABLE 1.—

(b) Mild steel

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Imperial College | 1 | 6 in. | 6.69 in. | - | 8 in. | 5130 psi | 41.3 ksi | 12.3 |
| 2 | Imperial College | 2 | 6 in. | 6.62 in. | - | 8 in. | 5760 psi | 41.3 ksi | 16.6 |
| 3 | Imperial College | 3 | 6 in. | 6.40 in. | 1.12 in. | 8 in. | 4180 psi | 41.3 ksi | 31.5 |
| 4 | Imperial College | 4 | 6 in. | 6.24 in. | 1.12 in. | 8 in. | 4150 psi | 41.3 ksi | 40.7 |
| 5 | Imperial College | 5 | 6 in. | 5.88 in. | 1.12 in. | 8 in. | 4790 psi | 45.1 ksi | 45.1 |
| 6 | Imperial College | 6 | 6 in. | 5.88 in. | 1.19 in. | 8 in. | 5030 psi | 41.3 ksi | 42.9 |
| 7 | Imperial College | 7 | 6 in. | 5.93 in. | 1.31 in. | 8 in. | 4420 psi | 41.3 ksi | 48.4 |
| 8 | Imperial College | 8 | 6 in. | 5.98 in. | 1.31 in. | 8 in. | 4300 psi | 41.3 ksi | 65.7 |
| 9 | Madrid | 6α- 1 | 15 cm | 24.5 cm | 2.5 cm | 28 cm | 179 $kg/cm^2$ | 33.5 $kg/mm^2$ | 34.6 |
| 10 | Madrid | 6α- 2 | 15 cm | 24.5 cm | 2.5 cm | 28 cm | 185 $kg/cm^2$ | 33.5 $kg/mm^2$ | 33.5 |
| 11 | Madrid | 6α- 3 | 15 cm | 24.5 cm | 2.5 cm | 28 cm | 202 $kg/cm^2$ | 34.2 $kg/mm^2$ | 31.4 |
| 12 | Paris (IRABA) | A 2 | 25 cm | 25.5 cm | 2.5 cm | 28 cm | 318 $kg/cm^2$ | 28.2 $kg/mm^2$ | 4.2 |
| 13 | Paris (IRABA) | A 5 | 15 cm | 24.5 cm | 2.5 cm | 28 cm | 338 $kg/cm^2$ | 27.6 $kg/mm^2$ | 20.3 |
| 14 | Paris (IRABA) | A 8 | 15 cm | 25.5 cm | 2.5 cm | 28 cm | 338 $kg/cm^2$ | 26.0 $kg/mm^2$ | 6.32 |
| 15 | Paris (IRABA) | A 11 | 30 cm | 25.5 cm | - | 28 cm | 327 $kg/cm^2$ | 25.7 $kg/mm^2$ | 3.27 |
| 16 | Paris (IRABA) | B 2 | 25 cm | 25.5 cm | 2.5 cm | 28 cm | 333 $kg/cm^2$ | 28.2 $kg/mm^2$ | 4.0 |
| 17 | Paris (IRABA) | B 5 | 15 cm | 24.5 cm | 2.5 cm | 28 cm | 310 $kg/cm^2$ | 27.2 $kg/mm^2$ | 21.2 |
| 18 | Paris (IRABA) | B 8 | 15 cm | 25.5 cm | 2.5 cm | 28 cm | 310 $kg/cm^2$ | 26.9 $kg/mm^2$ | 7.1 |
| 19 | Paris (IRABA) | B 11 | 30 cm | 25.5 cm | 2.5 cm | 28 cm | 310 $kg/cm^2$ | 26.5 $kg/mm^2$ | 3.23 |
| 20 | Porto | B 4 | 15 cm | 24.8 cm | 4.0 cm | 28 cm | 279 $kg/cm^2$ | 30.7 $kg/mm^2$ | 36.0 |
| 21 | Porto | B 6 | 15 cm | 25.1 cm | 4.0 cm | 28 cm | 306 $kg/cm^2$ | 34.4 $kg/mm^2$ | 23.4 |
| 22 | Porto | B 7 | 15 cm | 26.0 cm | 4.1 cm | 28 cm | 321 $kg/cm^2$ | 30.6 $kg/mm^2$ | 12.8 |
| 23 | Porto | B 9 | 15 cm | 26.0 cm | 4.1 cm | 28.3 cm | 287 $kg/cm^2$ | 27.6 $kg/mm^2$ | 5.0 |
| 24 | Porto | B 10 | 30 cm | 25.6 cm | - | 28.3 cm | 309 $kg/cm^2$ | 29.7 $kg/mm^2$ | 6.6 |
| 25 | Porto | B 12 | 30 cm | 26.0 cm | - | 28.1 cm | 284 $kg/cm^2$ | 29.2 $kg/mm^2$ | 2.7 |
| 26 | Torino | A 6 | 15 cm | 24.5 cm | 2.5 cm | 28 cm | 297 $kg/cm^2$ | 28 $kg/mm^2$ | 16.7 |
| 27 | Torino | A 9 | 15 cm | 25.5 cm | 2.5 cm | 28 cm | 297 $kg/cm^2$ | 28 $kg/mm^2$ | 4.9 |
| 28 | Torino | A 12 | 30 cm | 25.5 cm | 2.5 cm | 28 cm | 297 $kg/cm^2$ | 28 $kg/mm^2$ | 2.5 |
| 29 | Torino | D 5 | 15 cm | 24 cm | 2.5 cm | 28 cm | 224 $kg/cm^2$ | 28 $kg/mm^2$ | 32.2 |
| 30 | Torino | D 11 | 30 cm | 25.5 cm | 2.5 cm | 28 cm | 224 $kg/cm^2$ | 28 $kg/mm^2$ | 5.4 |

Note.—CPL = Central Point Load, TPL = Third Point Load, (T) = Tee Beam.



*CONTINUED*

reinforcements

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| - | - | .113 | .0129 | .037 | 1.00 | 1.07 | .- | CPL L = 80 in. |
| - | - | .161 | .0145 | .026 | .99 | 1.07 | - | CPL L = 80 in. |
| 4.0 | .61 | .284 | .0193 | .017 | 1.00 | 1.10 | 1.9 | CPL L = 80 in. |
| 4.2 | .81 | .367 | .0213 | .014 | 1.07 | 1.18 | 2.2 | CPL L = 80 in. |
| 3.7 | .97 | .371 | .0220 | .014 | 1.00 | 1.13 | 1.6 | CPL L = 80 in. |
| 6.0 | 1.22 | .357 | .0230 | .020 | .99 | 1.06 | 1.3 | CPL L = 80 in. |
| 16.1 | .46 | .385 | .0220 | .015 | .98 | 1.00 | 4.3 | CPL L = 80 in. |
| 16.4 | .55 | .480 | .0212 | .010 | .92 | 1.04 | 2.2 | CPL L = 80 in. |
| 2.9 | .176 | .337 | .0189 | .011 | 1.16 | 1.14 | - | CPL L = 280 cm |
| 2.8 | .176 | .335 | .0189 | .012 | 1.14 | 1.17 | 1.0 | CPL L = 280 cm |
| 2.6 | .176 | .288 | .0181 | .015 | 1.13 | 1.08 | 1.6 | CPL L = 280 cm |
| 4.2 | .132 | .049 | .0092 | .047 | 1.03 | 1.15 | 1.6 | CPL L = 280 cm |
| 1.2 | .172 | .197 | .0141 | .025 | .95 | 1.09 | 1.8 | CPL L = 280 cm |
| 1.2 | .165 | .074 | .0102 | .047 | 1.22 | 1.17 | 1.7 | CPL L = 280 cm |
| - | - | .038 | .0113 | .047 | 1.20 | 1.19 | 1.6 | (T) L = 280 cm |
| 4.0 | .132 | .040 | .0125 | .047 | .99 | 1.01 | - | TPL L = 280 cm |
| 1.2 | .165 | .208 | .0192 | .021 | 1.00 | 1.10 | 3.0 | TPL L = 280 cm |
| 1.2 | .165 | .073 | .0133 | .042 | 1.13 | 1.04 | 1.1 | TPL L = 280 cm |
| - | - | .038 | .0113 | .047 | 1.13 | 1.19 | 1.1 | (T) L = 280 cm |
| 1.6 | .26 | .274 | .0246 | .012 | .93 | 0.98 | - | (T) L = 280 cm |
| 1.6 | .26 | .188 | .0195 | .021 | .79 | 0.96 | 2.0 | (T) L = 280 cm |
| 1.6 | .26 | .130 | .0151 | .033 | .96 | 1.06 | 0.6 | (T) L = 280 cm |
| 1.6 | .26 | .063 | .0148 | .047 | 1.12 | 1.25 | 2.3 | (T) L = 280 cm |
| - | - | .067 | .0136 | .047 | .87 | 1.04 | 1.6 | (T) L = 280 cm |
| - | - | .034 | .0100 | .047 | .94 | 1.26 | 1.5 | (T) L = 280 cm |
| 1.5 | .132 | .180 | .0120 | .021 | 1.00 | 1.18 | 0.7 | CPL L = 280 cm |
| 1.4 | .132 | .069 | .0120 | .047 | - | 1.41 | - | CPL L = 280 cm |
| - | - | .036 | .0102 | .047 | 1.35 | 1.42 | - | CPL L = 280 cm |
| 2.0 | .132 | .486 | .023 | .012 | - | 1.10 | - | CPL L = 280 cm |
| 2.0 | - | .135 | .018 | .047 | - | 1.27 | 2.5 | (T) L = 280 cm |
| | | | MEAN | | 1.04 | 1.13 | 1.78 | |

## TABLE 2.—BEAM DETAILS

| | BEAM No. | b | $h$ | $h'$ | $h_t$ | $\sigma_b'$ | $\sigma_{ay}$ | $\varpi$% | $\varpi'$% | p'' |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Imperial College A1 | 6 in | 6.71 in | 1.29 in | 8 in | 4677 psi | 45.0 ksi | 14.6 | 14.6 | .61 |
| 2 | Imperial College A2 | 6 in | 6.71 in | 1.29 in | 8 in | 4457 psi | 45.0 ksi | 15.3 | 15.3 | .61 |
| 3 | Imperial College A3 | 6 in | 6.71 in | 1.29 in | 8 in | 4827 psi | 45.0 ksi | 14.2 | 14.2 | .61 |
| 4 | Imperial College A4 | 6 in | 6.71 in | 1.29 in | 8 in | 2600 psi | 45.0 ksi | 26.3 | 26.3 | .61 |
| 5 | Imperial College A5 | 6 in | 6.71 in | 1.29 in | 8 in | 4683 psi | 45.0 ksi | 14.6 | 14.6 | .61 |
| 6 | Imperial College A6 | 6 in | 6.71 in | 1.29 in | 8 in | 4256 psi | 45.0 ksi | 16.1 | 16.1 | .61 |
| 7 | Imperial College B1 | 6 in | 6.75 in | 1.25 in | 8 in | 4084 psi | 80.0 ksi | 17.0 | 17.0 | .60 |
| 8 | Imperial College B2 | 6 in | 6.75 in | 1.25 in | 8 in | 4758 psi | 80.0 ksi | 14.6 | 14.6 | .60 |
| 9 | Imperial College B3 | 6 in | 6.75 in | 1.25 in | 8 in | 4820 psi | 80.0 ksi | 14.4 | 14.4 | .60 |
| 10 | Imperial College B4 | 6 in | 6.75 in | 1.25 in | 8 in | 4414 psi | 80.0 ksi | 15.7 | 15.7 | .60 |
| 11 | Imperial College B5 | 6 in | 6.75 in | 1.25 in | 8 in | 4500 psi | 80.0 ksi | 15.4 | 15.4 | .60 |
| 12 | Imperial College B6 | 6 in | 6.75 in | 1.25 in | 8 in | 4100 psi | 80.0 ksi | 16.9 | 16.9 | .60 |
| 13 | Torino A2 | 25 cm | 25.5 cm | 2.5 cm | 28 cm | 297 kg/cm$^2$ | 28 kg/mm$^2$ | 4.1 | 4.1 | .14 |
| 14 | Torino D2 | 25 cm | 25.5 cm | 2.5 cm | 28 cm | 297 kg/cm$^2$ | 50.0 kg/mm$^2$ | 7.4 | 7.4 | .14 |
| 15 | Torino F1 | 25 cm | 25.5 cm | 2.5 cm | 28 cm | 398 kg/cm$^2$ | 53.0 kg/mm$^2$ | 5.9 | 5.9 | .14 |
| 16 | Torino F2 | 25 cm | 25.5 cm | 2.5 cm | 28 cm | 415 kg/cm$^2$ | 53.0 kg/mm$^2$ | 5.6 | 5.6 | .14 |
| 17 | Torino F3 | 25 cm | 25.5 cm | 2.5 cm | 28 cm | 398 kg/cm$^2$ | 53.0 kg/mm$^2$ | 5.9 | 5.9 | .14 |
| 18 | Torino G1 | 25 cm | 25.5 cm | 2.5 cm | 28 cm | 304 kg/cm$^2$ | 53.0 kg/mm$^2$ | 7.6 | 7.6 | — |
| 19 | Torino H2 | 25 cm | 25.5 cm | 2.5 cm | 28 cm | 304 kg/cm$^2$ | 53.0 kg/mm$^2$ | 7.6 | 7.6 | .05 |
| 20 | Torino L1 | 25 cm | 25.5 cm | 2.5 cm | 28 cm | 304 kg/cm$^2$ | 53.0 kg/mm$^2$ | 7.6 | 7.6 | .70 |
| 21 | Torino A3 | 25 cm | 25.5 cm | 2.5 cm | 28 cm | 296 kg/cm$^2$ | 28 kg/mm$^2$ | 4.1 | 4.1 | .14 |

| | BEAM NO. | b | $h$ | $h'$ | $h_t$ | $\sigma_b'$ | $\sigma_{ay}$ | $\varpi$% | $\varpi'$% | p''% |
|---|---|---|---|---|---|---|---|---|---|---|
| 22 | C & CA A1 | 10 ins | 10 ins | 1 ins | 11 ins | 3840 psi | 40.0 ksi | 4.08 | 4.08 | .21 |
| 23 | C & CA A2 | 10 ins | 10 ins | 1 ins | 11 ins | 4200 psi | 40.0 ksi | 3.74 | 3.74 | .21 |
| 24 | C & CA C1 | 10 ins | 10 ins | 1 ins | 11 ins | 3860 psi | 70.0 ksi | 8.70 | 8.70 | .21 |
| 25 | C & CA C2 | 10 ins | 10 ins | 1 ins | 11 ins | 4490 psi | 70.0 ksi | 7.45 | 7.45 | .21 |
| 26 | C & CA C3 | 10 ins | 10 ins | 1 ins | 11 ins | 3610 psi | 70.0 ksi | 9.25 | 9.25 | .21 |
| 27 | C & CA E1 | 10 ins | 10 ins | 1 ins | 11 ins | 4760 psi | 70.0 ksi | 7.05 | 7.05 | .21 |
| 28 | C & CA E2 | 10 ins | 10 ins | 1 ins | 11 ins | 2620 psi | 70.0 ksi | 12.42 | 12.42 | .21 |
| 29 | C & CA E3 | 10 ins | 10 ins | 1 ins | 11 ins | 3060 psi | 70.0 ksi | 11.00 | 11.00 | .21 |
| 30 | C & CA G2 | 10 ins | 10 ins | 1 ins | 11 ins | 5610 psi | 70.0 ksi | 5.96 | 5.96 | — |
| 31 | C & CA L2 | 10 ins | 10 ins | 1 ins | 11 ins | 4160 psi | 70.0 ksi | 8.05 | 8.05 | .94 |
| 32 | C & CA M2 | 6 ins | 10 ins | 1 ins | 11 ins | 4160 psi | 70.0 ksi | 22.9 | 22.9 | .26 |

$$^*P_u = \frac{P_u}{C_c\, bd}$$

FOR BIAXIAL LOADING

| $m_u$ | $n_u$ | $x_u$ | $\theta$ cal. 1 | $\theta$ cal. p | $\frac{m_1 \text{ act}}{m_1 \text{ cal}}$ | $\frac{m_2 \text{ act}}{m_2 \text{ cal}}$ | $\frac{p_2 \text{ act}}{p_2 \text{ cal}}$ | $\frac{\theta p \text{ act}}{\theta p \text{ cal}}$ | Grade of steel | REMARKS |
|---|---|---|---|---|---|---|---|---|---|---|
| .265 | .373 | .448 | .0266 | .019 | 1.2 | 0.90 | 1.28 | — | m.s | L=80 |
| .279 | .528 | .615 | .0193 | .016 | 1.00 | 1.02 | 1.29 | — | m.s | L=80 |
| .260 | .589 | .694 | .0171 | .013 | .96 | 0.95 | 1.27 | — | m.s | L=80 |
| .355 | .970 | 1.030 | .0106 | .009 | — | 1.65 | 1.03 | — | m.s | L=80 |
| .115 | .990 | 1.185 | .0100 | .006 | — | 0.93 | 1.00 | — | m.s | drift in n.w. position L=80 |
| .090 | 1.065 | 1.498 | .0080 | .006 | — | 0.93 | 0.96 | — | m.s | drift in n.w. position L=80 |
| .310 | .460 | .436 | .0272 | .019 | 1.26 | 1.29 | 1.88 | 0.8 | cws | L=80 |
| .260 | .480 | .593 | .0200 | .013 | 1.10 | 1.09 | 1.25 | 1.4 | cws | L=80 |
| .238 | .613 | .632 | .0130 | .012 | 1.12 | 1.11 | 1.45 | — | cws | L=80 |
| .184 | .854 | 1.010 | .0117 | .009 | .83 | 1.03 | 1.06 | — | cws | L=80 |
| .080 | .986 | 1.165 | .0092 | .008 | — | — | 1.06 | — | cws | drift in n.w. position L=80 |
| .095 | 1.20 | 1.553 | .0077 | .006 | — | — | 1.15 | — | cws | drift in n.w. position |
| .192 | .633 | .77 | .0143 | .006 | 1.21 | 1.16 | 1.22 | 1.5 | m.s | L=280 cm |
| .272 | .710 | .72 | .0153 | .006 | 1.33 | 1.35 | 1.40 | 2.5 | m.s | L=280 cm |
| .187 | .630 | .79 | .0139 | .006 | 1.04 | 1.17 | 1.08 | 1.5 | cws | L=280 cm |
| .157 | .645 | .90 | .0122 | .004 | 1.18 | 1.12 | 1.00 | — | cws | L=280 cm |
| .214 | .492 | .59 | .0127 | .009 | 1.07 | 1.21 | 1.22 | 1.4 | cws | L=280 cm |
| .145 | .980 | 1.02 | .0109 | .004 | 1.10 | 1.18 | 1.30 | 3.3 | cws | L=280 cm |
| .251 | .620 | .68 | .0162 | .007 | 1.48 | 1.40 | 1.29 | 2.4 | cws | L=280 cm |
| .151 | .890 | 1.00 | .0110 | .010 | 1.17 | 1.17 | 1.21 | 0.8 | cws | L=280 cm |
| .22 | .527 | .63 | .0143 | .006 | — | 1.01 | 1.02 | — | m.s | L=280 cm |
| | | mean | | | 1.14 | 1.14 | 1.21 | — | | |

| $m_u$ | $n_u$ | $x_u$ | $\theta$ cal. 1 | $2\theta$ cal. p | $\frac{m_1 \text{ act}}{m_1 \text{ cal}}$ | $\frac{m_2 \text{ act}}{m_2 \text{ cal}}$ | $\frac{p_2 \text{ act}}{p_2 \text{ cal}}$ | $\frac{\theta p \text{ act}}{\theta p \text{ cal}}$ | Grade of steel | REMARKS |
|---|---|---|---|---|---|---|---|---|---|---|
| .103 | .670 | 1.11 | .0108 | .0059 | .95 | .96 | 1.05 | — | m.s | L=117 ins |
| .167 | .324 | .396 | .0133 | .0175 | 1.05 | 1.20 | 1.18 | — | m.s | L=117 ins |
| .126 | .775 | 1.06 | .0113 | .0050 | .95 | .99 | 1.19 | — | cws | L=117 ins |
| .205 | .372 | .480 | .0250 | .0140 | 1.16 | 1.11 | 1.16 | — | cws | L=117 ins |
| .167 | .164 | .247 | .0200 | .028 | 1.33 | 1.08 | 1.05 | — | cws | L=117 ins |
| .085 | .600 | 1.180 | .0102 | .005 | .83 | .89 | 0.85 | — | cws | L=117 ins |
| .258 | .451 | .618 | .0194 | .0095 | 1.07 | 1.20 | 1.33 | — | cws | L=117 ins |
| .183 | .164 | .277 | .0193 | .026 | 1.28 | 1.02 | .91 | — | cws | L=117 ins |
| .158 | .285 | .559 | .0215 | .008 | .92 | 1.02 | .96 | — | cws | L=117 ins |
| .204 | .377 | .513 | .0234 | .0125 | .97 | 1.11 | 1.38 | — | cws | L=117 ins |
| .210 | .385 | .687 | .008 | .008 | — | 1.01 | .88 | — | cws | L=55 ins |

# APPENDIX 2

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