PARTICLE PRODUCTION IN STRONG INTERACTIONS

by

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ABSTRACT

The work contained in this thesis is clearly divided into two distinct parts.

In Part I the production and decay of arbitrary spin and parity resonant states are discussed within the context of a peripheral or one-meson exchange model. General results are obtained for the production angular distribution and the decay angular correlations of the resonant states assuming that the overall process is a quasi two body inelastic scattering process, mediated by the exchange of either a spin zero or a spin one meson, followed by the free decay of the resonant state or states. A field theoretic formalism is used to determine the propagators involving arbitrary spin particles and the results are expressed in terms of the most general possible three-particle couplings involving arbitrary coupling constants and their associated form factors. The usefulness of the model as a means of calculating coupling constants and form factors and of carrying out spin determinations is also discussed.

In Part II a model to describe the production of Ξ particles in $\bar{p}p$ collisions is set up in which the pole contributions of the Λ and Σ particles and the contributions of two-particle intermediate states are considered. These latter states are approximated by Y^* resonances in the s and u channels and by a D-particle resonance, a boson of strangeness 2, in the t channel. The two alternatives of spin 1/2 and spin 3/2 Ξ are considered. A comparison with experimental data indicates that the spin

of the Ξ is not 3/2 and that the production process is mediated by fermion exchange. The data can best be fitted by the parity combinations P($\Lambda\Sigma$) even, P(KΛN) odd and P(K$\Lambda\Xi$) odd.

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PREFACE

The work described in this thesis was developed in the Department of Physics, Imperial College of Science and Technology, University of London between October 1961 and September 1964 under the supervision of Professor P.T. Matthews.

Except where otherwise stated the material contained in this thesis is original and has not been previously presented for a degree in this or any other university.

The author wishes to thank Professor Matthews for his guidance, encouragement and advice during the pursuit of this work. The author would also like to express his gratitude to many other members of the department who have contributed so much to this work through helpful discussions.

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PART I

The peripheral model of the production and subsequent decay of resonances having arbitrary spin.

1. INTRODUCTION

Experiments in which beams of high energy mesons collide with nucleons have resulted in the discovery of a large number of resonant states. These resonances are often produced together with other particles and their subsequent decays lead to multiparticle final states. However in many cases it is possible to describe the overall process as a two body inelastic collision followed by the free decay of the unstable states. It has been found that the production process favours events with small momentum transfer. This has led to the consideration of a peripheral or one-meson exchange model involving either pseudoscalar[1-5] or vector meson[6-10] exchanges. In general such a model does not account for the extreme peripheralism observed in experiment without the introduction of very severe form factors to allow for the off-mass-shell nature of the exchanged particle.[11,12] These form factors are empirical functions but a prediction of the model is that they are functions only of the square of the 4-momentum transfer. This must be verified experimentally but until any energy dependence is exhibited the model provides a useful phenomenological basis for the description of the production process.

If the resonant states decay as free particles of well defined spin the decay angular distributions are determined by the spin alignment of the resonance and this alignment is itself determined by the production mechanism. In general for a resonance of any particular spin the peripheral model provides some restriction in the decay distribution and in

certain cases the distribution may be uniquely defined. Examination of the decay distributions may therefore be used both to test the validity of the peripheral model[13-15] and to determine the spin of the resonance. It should be emphasised that such a spin determination is model dependent but it may nevertheless be a useful guide in cases where more rigorous tests[16-27] are rendered inconclusive by the insufficiency of experimental data.

The purpose of this paper is to present in a systematic manner the predictions of the peripheral model for the production and subsequent decay of resonances of arbitrary spin produced in quasi-two body collisions of pseudoscalar particles with nucleons. To this end it is assumed that the resonances each have a unique spin and a narrow width. Cross sections and decay distributions will be presented in terms of arbitrary coupling constants and all possible couplings will be included.

Throughout this paper the resonances are assumed to decay through strong interactions and a state is said to be stable if it does not decay strongly.

If resonances are classified by their decay products five distinct types have been observed to date. These are denoted by F, B_1, B_2, B_3 and B_4. Their characteristic decay modes are as follows:

$$F \rightarrow N + P \tag{1.1}$$

$$B_1 \rightarrow P + P \tag{1.2}$$

$$B_2 \rightarrow P + P + P \tag{1.3}$$

$$B_3 \rightarrow P + V_1 \tag{1.4a}$$

$$V_1 \rightarrow P + P \tag{1.4b}$$

$$B_4 \rightarrow P + V_2 \tag{1.5a}$$

$$V_2 \rightarrow P + P + P \tag{1.5b}$$

where N denotes a stable spin-half particle, P denotes a stable pseudoscalar particle, V_1 and V_2 denote unstable vector particles which decay into two and three stable pseudoscalar particles respectively. In this paper those resonances are discussed whose decay mechanisms involve only three point functions and which thus have decay widths completely determined by coupling constants. Thus only the fermion resonance F, and the boson resonances B_1 and B_3 are discussed.

The following are the general quasi two body production processes considered:

$$P + N \rightarrow N + B_1 \tag{1.6}$$

$$P + N \rightarrow N + B_3 \tag{1.7}$$

$$P + N \rightarrow F + P \tag{1.8}$$

$$P + N \rightarrow F + B_1 \tag{1.9}$$

The one-meson exchange diagrams used as the basis of the calculation of the production cross-sections and decay distributions for the above processes are shown in Fig. 1. The exchange meson E may be any one of four spin-parity types: scalar (0+), pseudoscalar$^{(0-)}$, vector (1-), or pseudovector (1+).

The resonances of arbitrary spin are described in terms of a field theoretic formalism involving the usual tensor representation of a field with integral spin j, i.e. the (j/2, j/2) representation, and the Rarita-Schwinger$^{(28,29)}$ spinor-tensor representation of a field with half integral spin J, i.e. the $[(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})] \otimes \left(\frac{2J-1}{4}, \frac{2J-1}{4}\right)$ representation. As the resonances are assumed to have unique spin certain subsidiary conditions are needed to reduce the number of independent field components to (2j + 1) and (2J + 1) for integral and half-integral spin respectively. It should be noted that when these subsidiary conditions are included the general results are independent of the particular representation used even though the form of the interactions depends on the choice of representation. Choosing different representations merely leads to parametrisation in terms of different couplings each with its associated form factors.

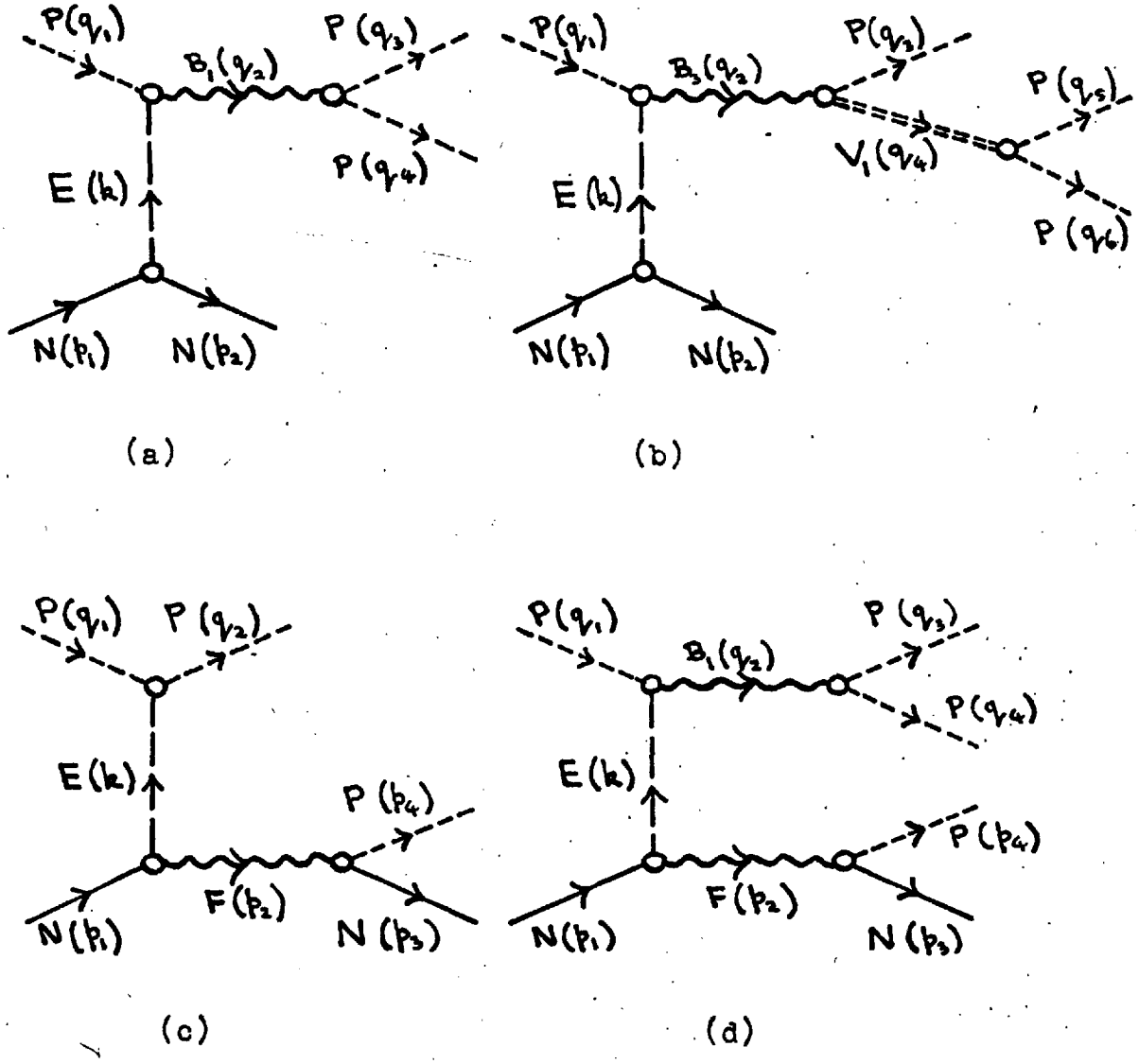
In section 2 the notation for the kenematics is established. The phase space factors involved in the calculation of the production cross sections and decay distributions for the processes shown in

Fig. 1 are evaluated in section 3. In section 4 a summary is given of the field theoretic formalism used for dealing with arbitrary spin resonances and the propagators for such states are discussed. The vertex functions are determined in section 5 for the most general possible couplings of arbitrary spin and parity resonances with the appropriate incoming and exchange particles and with the appropriate decay products. In section 6 the partial widths corresponding to the decay modes (1.1), (1.2) and (1.4) for the arbitrary spin resonances are calculated in terms of the coupling constants defined in section 5. The results for the production cross sections and decay correlations of the reactions (1.6), (1.7), (1.8) and (1.9) are given in sections 7, 8, 9 and 10 respectively and each of these sections has subdivisions in which a particular spin and parity combination for the exchange meson is considered. In section 11 the analysis of all the previous sections is used to tabulate results for the production cross sections and decay correlations of resonances having specific spins and specific parities. In section 12 the results are discussed in the context of testing the peripheral model and of carrying out spin, parity and coupling constant determinations.

The Appendix A, contains a summary of the relationships involving the arbitrary spin projection operators which are used to perform the calculations of sections 6 - 10.

FIG.1

Feynman diagrams for the peripheral production of boson and fermion resonances and their subsequent decay.



2. KINEMATICS

For each of the two body reactions (1.6) - (1.9) the 4-momenta of the incident pseudoscalar particle and the target nucleon are denoted by q_1 and p_1 and the 4-momenta of the outgoing boson and fermion are denoted by q_2 and p_2 respectively. For the subsequent decay processes the notation is established by the equations:

$$B_1\ (q_2) \rightarrow P\ (q_3) + P\ (q_4) \tag{2.1}$$

$$B_3\ (q_2) \rightarrow P\ (q_3) + V_1\ (q_4) \tag{2.2}$$

$$V_1\ (q_4) \rightarrow P\ (q_5) + P\ (q_6) \tag{2.3}$$

$$F\ (p_2) \rightarrow N\ (p_3) + P\ (q_4) \tag{2.4}$$

where the 4-momentum of each particle has been inserted in brackets immediately following the symbol for that particle. This notation is exhibited in the four diagrams of Fig. 1.

It is necessary to consider several coordinate systems and the following notation[30] is used: $p_i = (\epsilon_i, \underline{p}_i)$ and $q_i = (\omega_i, \underline{q}_i)$ stand for the 4-momenta of the particles indicated by the subscripts i. When all these particles are on the mass shell the 4-momenta satisfy the invariant relations

$$p_i^2 = \epsilon_i^2 - \underline{p}_i^2 = m_i^2 \qquad (2.5a)$$

$$q_i^2 = \omega_i^2 - \underline{q}_i^2 = \mu_i^2 \qquad (2.5b)$$

where m_i and μ_i are the masses of the appropriate fermion and boson respectively.

The components of any 4-momentum vector in a particular coordinate system are designated with upper-case subscripts. The subscripts L , B, V, W and X refer to the laboratory system ($\underline{p}_{1\,L} = 0$), the overall barycentric system ($\underline{p}_{1B} + \underline{q}_{1B} = 0$), the centre of mass system for the outgoing boson ($\underline{q}_{2V} = 0$), the centre of mass system for the outgoing fermion ($\underline{p}_{2W} = 0$) and the centre of mass system for the decay product vector meson V_1 ($\underline{q}_{4X} = 0$). For any frame of reference K we use the notation $p = (\epsilon_K, \underline{p}_K)$ and $q = (\omega_K, \underline{q}_K)$ and the magnitude of the 3-momenta in the frame K are given by $p_K = |\underline{p}_K|$ and $q_K = |\underline{q}_K|$.

For the production process the usual Mandelstam variables are given by:

$$s = (p_1 + q_1)^2 = (p_2 + q_2)^2 \qquad (2.6a)$$

$$t = (p_1 - p_2)^2 = (q_1 - q_2)^2 \qquad (2.6b)$$

$$u = (p_1 - q_2)^2 = (q_1 - p_2)^2 \qquad (2.6c)$$

In addition the following invariants are defined:

$$v = q_2^2 = (q_3 + q_4)^2 \qquad (2.6d)$$

$$w = p_2^2 = (p_3 + p_4)^2 \qquad (2.6e)$$

$$x = q_4^2 = (q_5 + q_6)^2 \qquad (2.6f)$$

The total energy of the complete system in the overall barycentric system is E_B where $s = E_B^2$. Defining the 4-momentum $k = (p_1 - p_2) = (q_2 - q_1)$, the square of the 4-momentum transfer for the production process is given by $t = k^2$. The energies of the boson and fermion resonances in their own centre of mass systems are V and W respectively where $v = V^2$ and $w = W^2$.

If the resonances are on the mass shell, that is $v = \mu_2^2$ and $w = m_2^2$, it is possible to define a number of useful quantities as follows. In the two body processes (1.6) - (1.9) the magnitudes of the 3-momenta of the initial and final state particles in the frame of reference B are given by:

$$p_B^2 = p_{1B}^2 = q_{1B}^2 = [\, s - (m_1 + \mu_1)^2]\, [\, s - (m_1 - \mu_1)^2 \,] \,/\, 4s \qquad (2.7a)$$

$$q_B^2 = p_{2B}^2 = q_{2B}^2 = [\, s - (m_2 + \mu_2)^2 \,]\, [\, s - (m_2 - \mu_2)^2] \,/\, 4s \qquad (2.7b)$$

the corresponding energies are given by:

$$\epsilon_{1B} = (E_B^2 + m_1^2 - \mu_1^2) / 2E_B \qquad \epsilon_{2B} = (E_B^2 + m_2^2 - \mu_2^2) / 2E_B \qquad (2.7c)$$

$$\omega_{1B} = (E_B^2 + \mu_1^2 - m_1^2) / 2E_B \qquad \omega_{2B} = (E_B^2 + \mu_2^2 - m_2^2) / 2E_B \qquad (2.7d)$$

In the frames V and W the relevant 3-momenta are given by:

$$k_V^2 = q_{1V}^2 = [-t + (\mu_1 + \mu_2)^2] [-t + (\mu_1 - \mu_2)^2] / 4\mu_2^2 \qquad (2.8a)$$

$$q_V^2 = q_{3V}^2 = q_{4V}^2 = [\mu_2^2 - (\mu_3 + \mu_4)^2][\mu_2^2 - (\mu_3 - \mu_4)^2] / 4\mu_2^2 \qquad (2.8b)$$

$$k_W^2 = p_{1W}^2 = [-t + (m_1 + m_2)^2] [-t + (m_1 - m_2)^2] / 4m_2^2 \qquad (2.9a)$$

$$p_W^2 = p_{3W}^2 = p_{4W}^2 = [m_2^2 - (m_3 + m_4)^2] [m_2^2 - (m_3 - m_4)^2] / 4m_2^2 \qquad (2.9b)$$

In the frame B the scattering angle θ_B is defined to be the angle between the incoming and outgoing bosons which is of course the same as the angle between the incoming and outgoing fermions. An azimuthal angle ϕ_B may be formally defined but it is a feature of the quasi two body reactions that ϕ_B is indeterminate since the reaction is confined to a plane, the production plane. It is to be noted that the direction of the normal to the production plane is an invariant under all Lorentz transformations between the frames of reference L, B, V and W.

In terms of θ_B

$$t = m_1^2 + m_2^2 - 2\epsilon_{1B}\,\epsilon_{2B} + 2p_{1B}\,p_{2B}\cos\theta_B \qquad (2.10a)$$

$$= \mu_1^2 + \mu_2^2 - 2\omega_{1B}\,\omega_{2B} + 2q_{1B}\,q_{2B}\cos\theta_B \qquad (2.10b)$$

The above definitions may be generalised to cover the situation in which the resonances are off the mass shell by replacing μ_2 by V and m_2 by W in all the formulae of this section. The only off mass shell quantities which arise in this paper are those involved in the calculation of phase space factors. These quantities are designated in the same way as the corresponding on-mass shell quantities but it is understood that the above replacements are made in their definitions.

3. PHASE SPACE

Quite apart from the particular mechanism which leads to the production of resonances in collisions of pseudoscalar mesons with nucleons it is necessary to discuss the variables which are appropriate to the complete description of the production process and the subsequent decays. The choice of variables is not unique but it will be shown that the variables used in the following to describe phase space are particularly suited to the peripheral model.

The differential cross-sections for the processes described by the diagrams (a), (b), (c) and (d) of Fig. 1 are given by $d\sigma$ (a), $d\sigma$ (b), $d\sigma$ (c) and $d\sigma$ (d) as follows:

$$d\sigma(a) = \frac{1}{4E_B p_B} (2\pi)^4 \delta^4(p_1 + q_1 - p_2 - q_3 - q_4) \frac{1}{[2(2\pi)^3]^3}$$

$$\times \frac{d^3p_2}{\epsilon_2} \frac{d^3q_3}{\omega_3} \frac{d^3q_4}{\omega_4} |\langle p_2 q_3 q_4 | T | p_2 q_1 \rangle|^2_{AV} \qquad (3.1)$$

$$d\sigma(b) = \frac{1}{4E_B p_B} (2\pi)^4 \delta^4(p_1 + q_1 - p_2 - q_3 - q_5 - q_6) \frac{1}{[2(2\pi)^3]^4}$$

$$\times \frac{d^3p_2}{\epsilon_2} \frac{d^3q_3}{\omega_3} \frac{d^3q_5}{\omega_5} \frac{d^3q_6}{\omega_6} |\langle p_2 q_3 q_5 q_6 | T | p_1 q_1 \rangle|^2_{AV} \qquad (3.2)$$

$$d\sigma(c) = \frac{1}{4E_B p_B} (2\pi)^4 \delta^4(p_1 + q_1 - p_3 - p_4 - q_2) \frac{1}{[2(2\pi)^3]^3}$$

$$\times \frac{d^3p_3}{\epsilon_3} \frac{d^3p_4}{\epsilon_4} \frac{d^3q_2}{\omega_2} |\langle p_3 p_4 q_2 | T | p_1 q_1 \rangle|^2_{AV} \qquad (3.3)$$

$$d\sigma(d) = \frac{1}{4E_B p_B} (2\pi)^4 \delta^4(p_1 + q_1 - p_3 - p_4 - q_3 - q_4) \frac{1}{[2(2\pi)^3]^4}$$

$$\times \frac{d^3p_3}{\epsilon_3} \frac{d^3p_4}{\epsilon_4} \frac{d^3q_3}{\omega_3} \frac{d^3q_4}{\omega_4} |\langle p_3 p_4 q_3 q_4 | T | p_1 q_1 \rangle|^2_{AV} \qquad (3.4)$$

where $|\langle \cdots | T | p_1 q_1 \rangle|^2_{AV}$ represents the average over initial spin states and the sum over final spin states of the product of the appropriate scattering amplitude with its hermition conjugate. The factor $4E_B p_B$ is the invariant flux of the incoming particles evaluated in the frame B. This quantity is equal to $4m_1 q_{1L}$ when evaluated in the laboratory frame . The remaining factor in each expression is called the phase space factor.

In case (a) the phase space factor reduces, apart from numerical factors, to the product of two Lorentz invariant factors as follows:

$$\left(\frac{d^3p_2}{\epsilon_2}\right)\left[\,\delta^0(\epsilon_1+\omega_1-\epsilon_2-\omega_3-\omega_4)\,\frac{d^3q_3}{\omega_3\omega_4}\,\right] \tag{3.5}$$

The first factor evaluated in the frame B gives:

$$\frac{q_B}{2E_B}\quad dv\quad d\cos\theta_B\quad d\phi_B \tag{3.6}$$

where θ_B is the scattering angle between $\underline{p}_1$ and $\underline{p}_2$ in the frame B and ϕ_B is an azimuthal angle which merely serves to define the production plane. The second invariant factor is evaluated in the frame V and after integration over dq_{3V} to remove the delta function yields:

$$\frac{q_V}{V}\quad d\cos\theta_{3V}\quad d\phi_{3V} \tag{3.7}$$

where θ_{3V} is the angle between $\underline{q}_3$ and some polar axis in the frame V and ϕ_{3V} is an azimuthal angle. It is convenient to choose the direction of the incident meson as the polar axis since this is the natural axis of quantisation to use when describing the collision of this incident meson and the virtual meson exchanged in the peripheral model. This follows from the fact that the component of angular momentum of the resonant state along this axis must be the same as the spin component of the exchanged particle in the same direction. In particular for spin zero exchange the component of

the spin of the resonant state along this polar axis is zero and in the distribution of the decay products there is rotational symmetry about this axis. This is in accordance with the Trieman-Yang[13] test. The quasi two body inelastic collision process defines a production plane and the azimuthal angle ϕ_{3V} may be defined in terms of the direction of the normal to this production plane. The angle ϕ_{3V} is then the complement of the usual Trieman-Yang angle. The notation used is shown in Fig. 2a where $\underline{I}$, the polar axis, and $\underline{N}$, the normal to the production plane, are given by:

$$\underline{I} = \underline{q}_{1V} / k_V \tag{3.8}$$

$$\underline{N} = (\underline{p}_{1V} \wedge \underline{p}_{2V}) / |\underline{p}_{1V} \wedge \underline{p}_{2V}| \tag{3.9}$$

The angular variables in (3.7) are then uniquely defined as follows:

$$\underline{I} \cdot \underline{q}_{3V} = q_V \cos\theta_{3V} \tag{3.10}$$

$$\underline{N} \cdot \underline{q}_{3V} = q_V \sin\theta_{3V} \cos\phi_{3V} \tag{3.11}$$

With these definitions substitution of (3.6) and (3.7) in (3.1) gives:

$$d\sigma(a) = \frac{1}{(4\pi)^4} \frac{q_B}{4E^2_B p_B} \frac{q_V}{2\pi V} |\langle p_2 q_3 q_4 |T| p_1 q_1 \rangle|^2_{AV}$$

$$\times \; dv \; d\cos\theta_B \; d\phi_B \; d\cos\theta_{3V} \; d\phi_{3V} \tag{3.12}$$

In case (b) the phase space factor reduces as in case (a) to the product of two Lorentz invariant factors as follows:

$$\left(\frac{d^3p_2}{\epsilon 2}\right) \left[\delta^o(\epsilon_1 + \omega_1 - \epsilon_2 - \omega_3 - \omega_5 - \omega_6) \frac{d^3q_3\, d^3q_5}{\omega\omega_3\,\omega_5\,\omega_6}\right] \qquad (3.13)$$

The first factor evaluated in the frame $\mathbb{P}$, gives as before (3.6). The second invariant factor may be evaluated in the frame V and gives:

$$\delta^o\,(V - \omega_{3V} - \omega_{5V} - \omega_{6V}) \frac{q^2_{3V}\, dq_{3V}\, d\Omega_{3V}\, q^2_{5V}\, dq_{5V}\, d\Omega_{5V}}{\omega_{3V}\,\omega_{5V}\omega_{6V}} \qquad (3.14)$$

where $d\Omega_{3V} = d\cos\theta_{3V}\, d\phi_{3V}$ and $d\Omega_{5V} = d\cos\theta_{5V}\, d\phi_{5V}$ and θ_{3V} and θ_{5V} are the angles $\underline{q}_{3V}$ and $\underline{q}_{5V}$ make with some polar axis in the frame V and ϕ_{3V} and ϕ_{5V} are the corresponding azimuthal angles.

All the final state particles are on the mass shell and thus $\omega_{3V}\, d\,\omega_{3V} = q_{3V}\, dq_{3V}$ etc. By the conservation of momentum $\underline{q}_{3V} + \underline{q}_{5V} + \underline{q}_{6V} = \underline{q}_{2V} = 0$ and hence:

$$\omega^2_{6V} - \mu^2_6 = q^2_{3V} + q^2_{5V} + 2q_{3V}q_{5V}\cos\theta_{35V} \qquad (3.15)$$

where θ_{35V} is the angle between $\underline{q}_{3V}$ and $\underline{q}_{5V}$. This angle is completely determined by Ω_{3V} and Ω_{5V} and for fixed ω_{3V} and ω_{5V}

it follows that:

$$\omega_{6V}\, d\omega_{6V} = q_{3V}\, q_{5V}\, d\cos\theta_{35V} \tag{3.16}$$

thus the invariant (3.14) can be rewritten as:

$$d\omega_{3V}\, d\omega_{5V}\, d\omega_{6V}\ \delta^{o}(V - \omega_{3V} - \omega_{5V} - \omega_{6V})\left[d\Omega_{3V} d\Omega_{5V} / d\cos\theta_{35V}\right]$$

Integration over ω_{6V} then removes the delta function and yields:

$$d\omega_{3V}\, d\omega_{5V}\, [\, d\cos\theta_{3V}\, d\phi_{3V}\, d\cos\theta_{5V}\, d\phi_{5V} / d\cos\theta_{35V} \tag{3.17}$$

It is advantageous to exhibit directly the two stage nature of the decay process by using the variables x, $\cos\theta_{3V}$ and ϕ_{3V} where as before the angular variables are defined with respect to the axes $\underline{I}$ and $\underline{N}$ by equations (3.8) - (3.11). The variable $x = q_4^2$ is related to ω_{3V} by the expression

$$x = v + \mu_3^2 - 2V\,\omega_{3V} \tag{3.18}$$

and therefore:

$$d\omega_{3V} = -dx / (2V) \tag{3.19}$$

To remove the denominator from (3.17) it is necessary to transform the differential variable $d\Omega_{5V}$ into the product of $d\cos\theta_{35V}$ and some other differential variable dependent on the direction of $\underline{q}_{5V}$ in the frame V. The choice of this phase space

variable must be such that the Jacobian of the transformation is not a complicated function of the variables x, $\cos\theta_{3V}$ and ϕ_{3V}. This ensures that the behaviour of the scattering amplitude as a function of these variables may be readily determined from experimental data. It should be noted that, in the same way, although the variable ω_{5V} is related to the angle θ_{35V} the transformation from one to the other introduces unnecessary complications. Moreover it is shown in section 8 that it is the variable ω_{5V} which appears in the scattering amplitude so that it is convenient to treat this as an independent phase space variable.

In the frame V the resonant state B_3 decays into three particles all moving in the same plane. The normal to this decay plane can be defined by:

$$\underline{M} = (\underline{q}_{3V} \wedge \underline{q}_{5V}) \; / \; | \underline{q}_{3V} \wedge \underline{q}_{5V} | \qquad (3.20)$$

and the angle between $\underline{I}$ and $\underline{M}$ is denoted by β_V and the corresponding azimuthal angle measures with respect to $\underline{N}$ by $\phi_{\beta V}$. Thus:

$$\underline{I} \cdot \underline{M} = \cos\beta_V \qquad (3.21)$$

$$\underline{N} \cdot \underline{M} = \sin\beta_V \cos\phi_{\beta V} \qquad (3.22)$$

With this notation it is easily shown that:

$$d\Omega_{5V} = d\cos\theta_{5V} \; d\cos\theta_{35V} \; / \; \sin\theta_{35V} \cos\beta_V \qquad (3.23)$$

where

$$(\sin\Theta_{35V}\cos\beta_V)^2 = 1 - \cos^2\Theta_{3V} - \cos^2\Theta_{5V} - \cos^2\Theta_{35V} +$$

$$2\cos\Theta_{3V}\cos\Theta_{5V}\cos\Theta_{35V} \qquad (3.24)$$

the complicated dependence of the denominator on $\cos\Theta_{3V}$ and $\cos\Theta_{5V}$ clearly precludes the choice of $d\cos\Theta_{5V}$ as a differential variable. Calculation of the transformation from the variable $\cos\Theta_{5V}$ to the variable $\cos\beta_V$ then yields the result:

$$d\Omega_{5V} = d\cos\beta_V \, d\cos\Theta_{35V} / (\sin^2\Theta_{3V} - \cos^2\beta_V)^{\frac{1}{2}} \qquad (3.25)$$

Once again there is a complicated dependence of the denominator on $\cos\Theta_{3V}$. However it is clear from (3.25) that

$$d\Omega_{5V} = d\psi_{3V} \, d\cos\Theta_{35V} \qquad (3.26)$$

where the angle ψ_{3V} is defined by:

$$\cos\beta_V = -\sin\Theta_{3V}\cos\psi_{3V} \qquad (3.27a)$$

The equation (3.27a) does not furnish a unique definition of ψ_{3V} but it may be defined by noting that successive rotations through the Euler angles[31] $+\phi_{3V}$, $-\Theta_{3V}$ and $+\psi_{3V}$ align two of the initial set of axes $\underline{I}$ and $\underline{N}$ with the directions of $\underline{q}_{3V}$ and $\underline{M}$ respectively as shown in Fig. 2b. It can then be shown using (3.21) and (3.22) that:

$$\sin\beta_V \cos\phi_{\beta V} = \cos\psi_{3V} \cos\theta_{3V} \cos\phi_{3V} - \sin\psi_{3V} \sin\phi_{3V} \quad (3.27b)$$

$$\sin\beta_V \sin\phi_{\beta V} = \cos\psi_{3V} \cos\theta_{3V} \sin\phi_{3V} + \sin\psi_{3V} \cos\phi_{3V} \quad (3.27c)$$

It may also be shown from the definitions of this section that:

$$\cos\theta_{5V} = \cos\theta_{35V} \cos\theta_{3V} + \sin\theta_{35V} \sin\theta_{3V} \sin\psi_{3V} \quad (3.28a)$$

$$\sin\theta_{5V} \cos\phi_{5V} = \cos\theta_{35V} \sin\theta_{3V} \cos\phi_{3V} + \sin\theta_{35V} \times$$

$$\times (\cos\psi_{3V} \sin\phi_{3V} - \sin\psi_{3V} \cos\theta_{3V} \cos\phi_{3V}) \quad (3.28b)$$

$$\sin\theta_{5V} \sin\phi_{5V} = \cos\theta_{35V} \sin\theta_{3V} \sin\phi_{3V} + \sin\theta_{35V} \times$$

$$\times (\cos\psi_{3V} \cos\phi_{3V} - \sin\psi_{3V} \cos\theta_{3V} \sin\phi_{3V}) \quad (3.28c)$$

Thus using (3.6), (3.17) and (3.26) it follows that (3.2) gives the result[32]:

$$d\sigma(b) = \frac{1}{(4\pi)^6} \frac{q_B}{4E^2_B p_B} \frac{1}{4\pi^2 V} |\langle p_2 q_3 q_5 q_6 | T | p_1 q_1 \rangle|^2_{AV}$$

$$\times \; dv\, dx\, d\omega_{5V}\, d\cos\theta_B\, d\phi_B\, d\cos\theta_{3V}\, d\phi_{3V}\, d\psi_{3V} \quad (3.29)$$

Case (c) is exactly analogous to case (d). In the frame W the natural axis of quantisation to use when describing the

collision of the target nucleon and the virtual meson exchanged in the peripheral model is the direction of the target nucleon. Once again the normal to the production plane is used to define the azimuthal angle. The notation used is shown in Fig. 2c. Where $\underline{I}$, the polar axis, and $\underline{N}$, the normal to the production plane, are given by:

$$\underline{I} = \underline{p}_{1W} / k_W \tag{3.30}$$

$$\underline{N} = (\underline{q}_{1W} \wedge \underline{q}_{2W}) / |\underline{q}_{1W} \wedge \underline{q}_{2W}| \tag{3.31}$$

The angular variables used are then defined by:

$$\underline{I} \cdot \underline{p}_{4W} = p_W \cos\Theta_{4W} \tag{3.32}$$

$$\underline{N} \cdot \underline{p}_{4W} = p_W \sin\Theta_{4W} \cos\phi_{4W} \tag{3.33}$$

With the above definitions it follows as before that (3.3) gives:

$$d\sigma(c) = \frac{1}{(4\pi)^4} \frac{q_B}{4E^2_B p_B} \frac{p_W}{2\pi W} |\langle p_3 p_4 q_2 | T | p_1 q_1 \rangle|^2_{AV}$$

$$dw\, d\cos\Theta_B\, d\phi_B\, d\cos\Theta_{4W}\, d\phi_{4W} \tag{3.34}$$

In case (d) the phase space factor can once again be written as the product of two Lorentz invariant factors:

$$\left[\frac{d^3p_3}{\epsilon_3} \quad \frac{d^3p_4}{\epsilon_4}\right]\left[\delta^0(\epsilon_1+\omega_1-\epsilon_3-\epsilon_4-\omega_3-\omega_4)\,\frac{d^3q_3}{\omega_3\omega_4}\right] \tag{3.35}$$

The first factor may be transformed into

$$\frac{d^3p_2}{\epsilon_2} \quad \frac{d^3p_4}{\epsilon_4} \quad \frac{\epsilon_2}{\epsilon_3} \tag{3.36}$$

and evaluating in the frame B the invariant d^3p_2/ϵ_2 gives the factor (3.6) whilst it may be shown that the remaining terms in (3.36), when transformed to the frame W give[4]:

$$\frac{p_4}{2W}\; dw \;\; d\cos\Theta_4 \;\; d\phi_{4W} \tag{3.37}$$

where Θ_{4W} and ϕ_{4W} are defined by equations (3.30) - (3.33). The second factor in (3.35) is evaluated in the frame V and after integration over dq_{3V} to remove the delta function yields the expression (3.7).

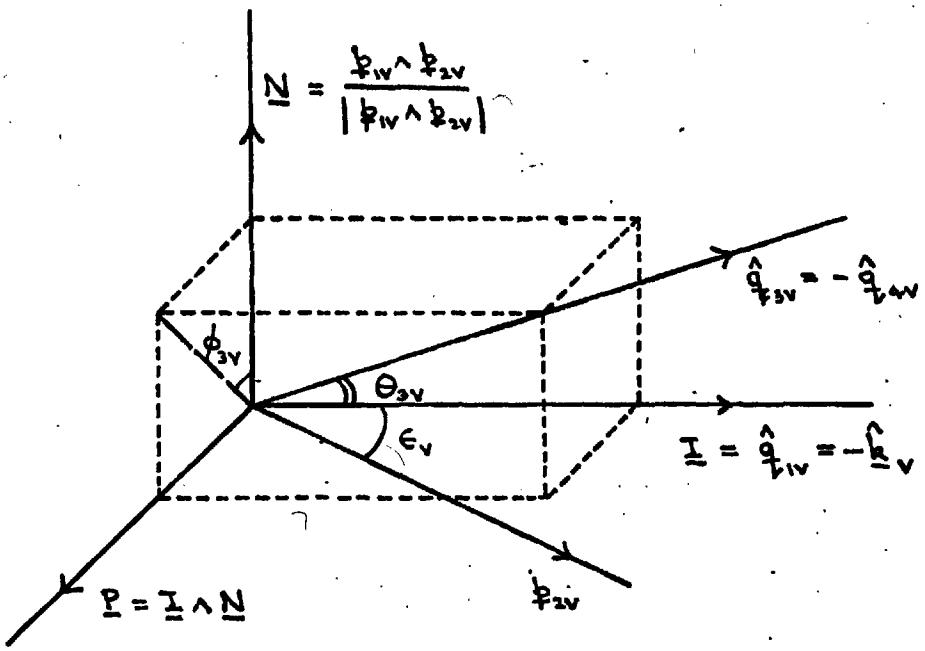
Thus substituting (3.6), (3.7) and (3.37) into (3.4) gives[33]:

$$d\sigma(d) = \frac{1}{(4\pi)^6}\;\frac{q_B}{4E_B^2p_B}\;\frac{q_V}{2\pi V}\;\frac{p_W}{2\pi W}\;|\langle p_3p_4q_3q_4 \mid T \mid p_1q_1\rangle|^2_{AV}$$

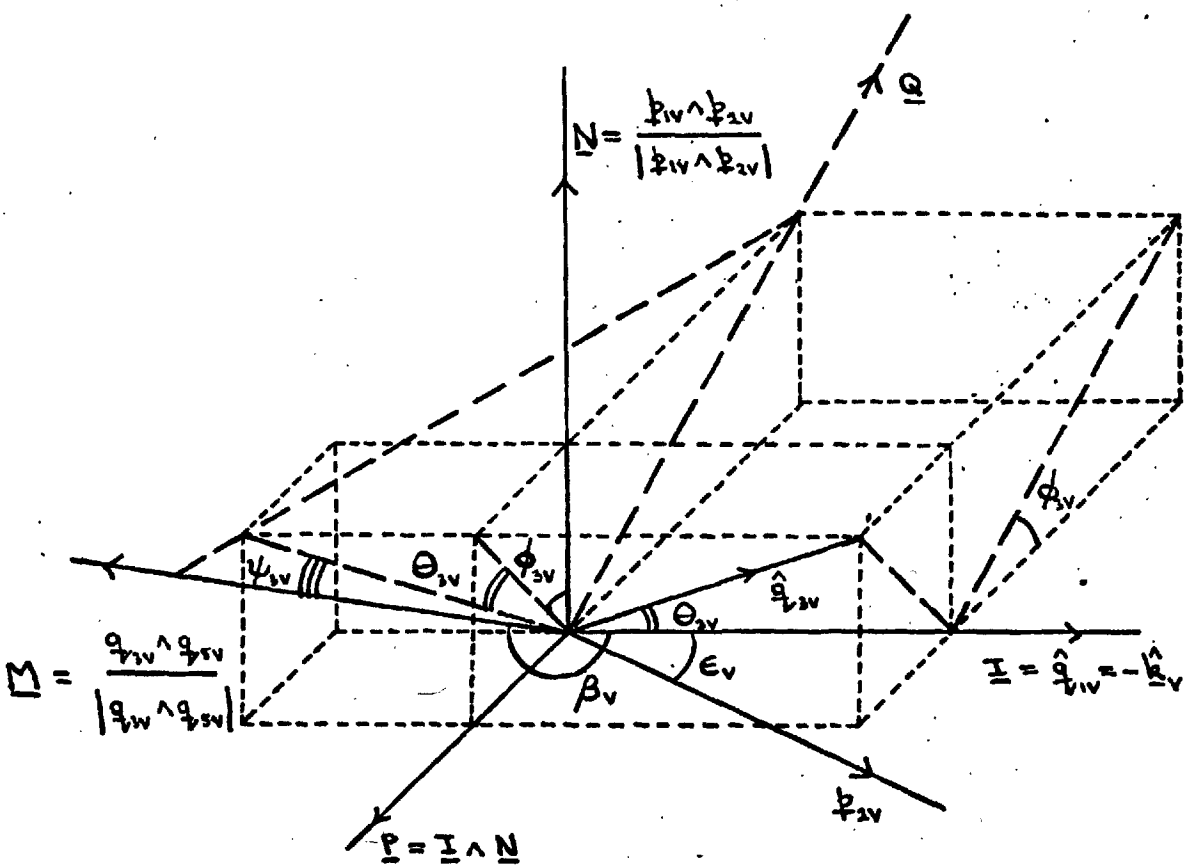
$$dv\; dw\; d\cos\Theta_B\; d\phi_B\; d\cos\Theta_{3V}\; d\phi_{3V}\; d\cos\Theta_{4W}\; d\phi_{4W} \tag{3.38}$$

FIG.2

Decay angular distribution variables.

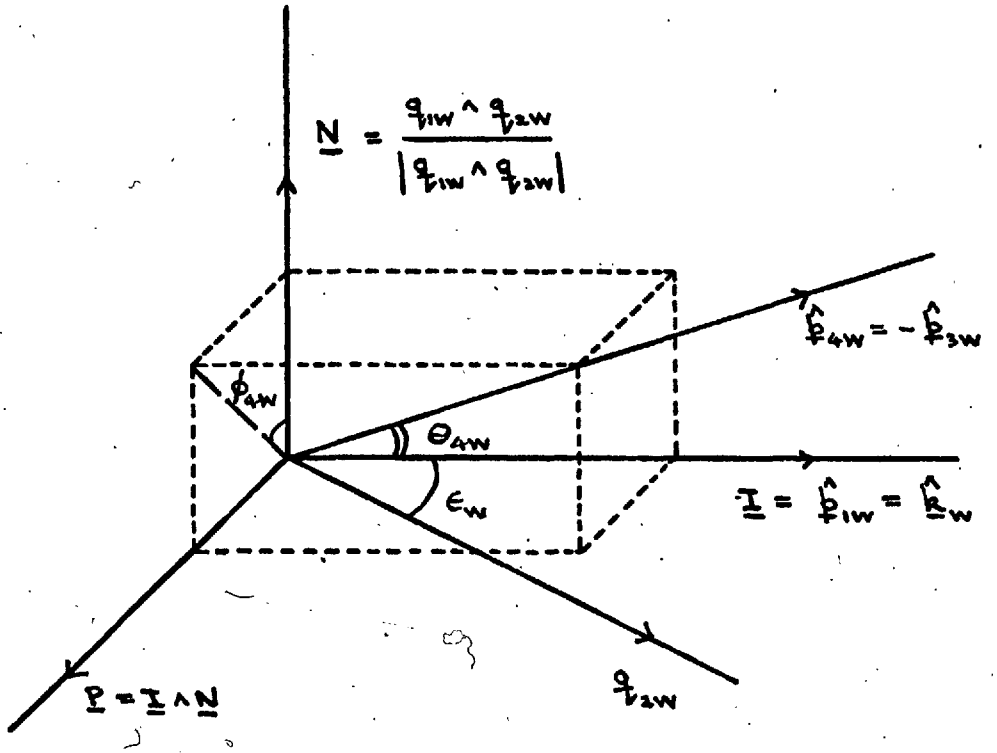


(a)



(b)

The axes $\underline{I}$ and $\underline{N}$ are aligned with the directions of $\underline{q}_{b3V}$ and $\underline{M}$ by successive rotations through the angles ϕ_{3V}, $-\theta_{3V}$ and ψ_{3V} about the axes $\underline{I}$, $\underline{Q}$ and $\hat{\underline{q}}_{b3V}$ respectively.


$\underline{N} = \frac{\underline{q}_{1w} \wedge \underline{q}_{2w}}{|\underline{q}_{1w} \wedge \underline{q}_{2w}|}$
$\hat{\underline{p}}_{4w} = -\hat{\underline{p}}_{3w}$
ϕ_{4w}
θ_{4w}
ϵ_w
$\underline{I} = \hat{\underline{p}}_{1w} = \hat{\underline{k}}_w$
$\underline{P} = \underline{I} \wedge \underline{N}$
$\underline{q}_{2w}$

(c)

4. FIELD THEORETIC FORMALISM AND THE PROPAGATORS FOR ARBITRARY SPIN PARTICLES

Using the usual tensor representation[34,35] a free boson field of spin j, mass μ and 4-momentum q may be described in momentum space by means of a wave function $\chi^{(q)}_{\lambda_1\lambda_2\ldots\lambda_n}$ where j = n. This wave function satisfies the wave equation:

$$(q^2 - \mu^2)\, \chi^{(q)}_{\lambda_1\lambda_2\ldots\lambda_n} = 0 \tag{4.1}$$

and the subsidiary conditions:

$$\chi^{(q)}_{\lambda_1\ldots\lambda_i\ldots\lambda_j\ldots\lambda_n} = \chi^{(q)}_{\lambda_1\ldots\lambda_j\ldots\lambda_i\ldots\lambda_n} \tag{4.2}$$

for i, j = 1, 2,... n and

$$g_{\lambda_1\lambda_2}\, \chi^{(q)}_{\lambda_1\lambda_2\ldots\lambda_n} = 0 \tag{4.3}$$

$$q_{\lambda_1}\, \chi^{(q)}_{\lambda_1\lambda_2\ldots\lambda_n} = 0 \tag{4.4}$$

Consider the operator $\phi^{(q,n)}_{\lambda_1\lambda_2\ldots\lambda_n\rho_1\rho_2\ldots\rho_n}$ which is symmetric and traceless with respect to any pair of the indices $\lambda_1, \lambda_2, \ldots \lambda_n$ so that:

$$\phi_{\lambda_1 \ldots \lambda_i \ldots \lambda_j \ldots \lambda_n \rho_1 \rho_2 \ldots \rho_n}^{(q,n)} = \phi_{\lambda_1 \ldots \lambda_j \ldots \lambda_i \ldots \lambda_n \rho_1 \rho_2 \ldots \rho_n}^{(q,n)} \tag{4.5}$$

for $i,j = 1,2,\ldots n$ and

$$\sigma_{\lambda_1 \lambda_2} \phi_{\lambda_1 \lambda_2 \ldots \lambda_n \rho_1 \rho_2 \ldots \rho_n}^{(q,n)} = 0 \tag{4.6}$$

and which satisfies

$$q_{\lambda_1} \phi_{\lambda_1 \lambda_2 \ldots \lambda_n \rho_1 \rho_2 \ldots \rho_n}^{(q,n)} = 0 \tag{4.7}$$

and

$$\phi_{\lambda_1 \lambda_2 \ldots \lambda_n \rho_1 \rho_2 \ldots \rho_n}^{(q,n)} \phi_{\rho_1 \rho_2 \ldots \rho_n \sigma_1 \sigma_2 \ldots \sigma_n}^{(q,n)} = \phi_{\lambda_1 \lambda_2 \ldots \lambda_n \sigma_1 \sigma_2 \ldots \sigma_n}^{(q,n)} \tag{4.8}$$

Such an operator is clearly a projection operator for the spin n component of any field with n tensor indices. For a pure spin state wave function it can be shown that:

$$\sum_{\text{Pol.}} \chi_{\lambda_1 \lambda_2 \ldots \lambda_n}^{(q)} \chi^{+(q)}_{\rho_1 \rho_2 \ldots \rho_n} = (-1)^n \phi_{\lambda_1 \lambda_2 \ldots \lambda_n \rho_1 \rho_2 \ldots \rho_n}^{(q,n)} \tag{4.9}$$

where $^{+}$ denotes a hermitian conjugate and the summation has been carried out over all polarisation states. Furthermore $\phi(q,n)$ is uniquely defined by the conditions (4.5) - (4.8). In particular for n = 1 it is easy to show that the spin one projection operator is given by:

$$\phi_{\lambda_1 \rho_1}(q,n) = \phi_{\lambda_1 \rho_1}(q) = g_{\lambda_1 \rho} - \frac{q_{\lambda_1} q_{\rho}}{q^2} \tag{4.10}$$

In terms of this operator it can be shown that:

$$\phi_{\lambda_1 \lambda_2 \cdots \lambda_n \rho_1 \rho_2 \cdots \rho_n}(q,n) = \frac{1}{(n!)^2} \sum_{\substack{P(\lambda) \\ P(\rho)}} \left[\sum_{r=0}^{n'} \left\{ a_r^{\,n} \prod_{k=1}^{r=1} \phi_{\lambda_{2k} \lambda_{2k+1}}(q) \prod_{l=1}^{r-1} \phi_{\rho_{2l} \rho_{2l+1}}(q) \prod_{m=2r}^{n} \phi_{\lambda_m \rho_m}(q) \right\} \right] \tag{4.11}$$

where

$$a_r^{\,n} = (-1)^r \frac{n!}{r!(n-r)!} \quad \frac{(2n-2r)!}{(n-2r)!\, n!} \quad \frac{n!\, n!}{(2n)!} \tag{4.12}$$

and $n' = n/2$ for n even and $n' = (n-1)/2$ for n odd and where the first summation is carried out over all permutations of both sets of indices $\lambda_1, \lambda_2, \ldots \lambda_n$ and $\rho_1 \rho_2 \cdots \rho_n$

Similarly using the Rarita-Schwinger formalism a free fermion field of spin J, mass m and 4-momentum p may be described in momentum space by means of a wave function $u^{(p)}_{\lambda_1\lambda_2\ldots\lambda_n}$ where $J = n + \frac{1}{2}$ and to each tensor index λ_i corresponds a Dirac 4-component spinor. The wave function satisfies the generalised Dirac wave equation:

$$(\not{p} - m)\ u^{(p)}_{\lambda_1\lambda_2\ldots\lambda_n} = 0 \tag{4.14}$$

and the subsidiary conditions:

$$u^{(p)}_{\lambda_1\ldots\lambda_i\ldots\lambda_j\ldots\lambda_n} = u^{(p)}_{\lambda_1\ldots\lambda_j\ldots\lambda_i\ldots\lambda_n} \tag{4.15}$$

for $i,j = 1,2\ldots n$ and

$$g_{\lambda_1\lambda_2}\ u^{(p)}_{\lambda_1\lambda_2\ldots\lambda_n} = 0 \tag{4.16}$$

$$p_{\lambda_1}\ u^{(p)}_{\lambda_1\lambda_2\ldots\lambda_n} = 0 \tag{4.17}$$

$$\gamma_{\lambda_1}\ u^{(p)}_{\lambda_1\lambda_2\ldots\lambda_n} = 0 \tag{4.18}$$

Consider the operator $\theta^{(p,n)}_{\lambda_1\lambda_2\ldots\lambda_n\rho_1\rho_2\ldots\rho_n}$ which is symmetric and traceless with respect to any pair of the indices

$\lambda\lambda\ \ \lambda$ so that:
$1\ 2\ldots\ n$

$$\theta^{(p,n)}_{\lambda_1\ldots\lambda_i\ldots\lambda_j\ldots\lambda_n\rho_1\rho_2\ldots\rho_n} = \theta^{(p,n)}_{\lambda_1\ldots\lambda_j\ldots\lambda_i\ldots\lambda_n\rho_1\rho_2\ldots\rho_j} \tag{4.19}$$

for $i,j = 1,2\ldots n$ and

$$\varepsilon_{\lambda_1\lambda_2\ldots\lambda_n\rho_1\rho_2\ldots\rho_n} = 0 \tag{4.20}$$

and which satisfies

$$p_{\lambda_1}\theta^{(p,n)}_{\lambda_1\lambda_2\ldots\lambda_n\rho_1\rho_2\ldots\rho_n} = 0 \tag{4.21}$$

$$\gamma_{\lambda_1}\theta^{(p,n)}_{\lambda_1\lambda_2\ldots\lambda_n\rho_1\rho_2\ldots\rho_n} = 0 \tag{4.22}$$

and

$$\theta^{(p,n)}_{\lambda_1\lambda_2\ldots\lambda_n\rho_1\rho_2\ldots\rho_n}\ \theta^{(p,n)}_{\rho_1\rho_2\ldots\rho_n\sigma_1\sigma_2\ldots\sigma_n} = \theta^{(p,n)}_{\lambda_1\lambda_2\ldots\lambda_n\sigma_1\sigma_2\ldots\sigma_n} \tag{4.23}$$

Such an operator is clearly a projection operator for the spin J ($J = n + \frac{1}{2}$) component of any spinor-tensor field with n tensor indices to each of which corresponds a 4-spinor. For a pure spin state wave function it can be shown that:

$$\sum u^{(p)}_{\lambda_1\lambda_2\ldots\lambda_n}\ \bar{u}^{(p)}_{\rho_1\rho_2\ldots\rho_n} = (-1)^n\ \theta^{(p,n)}_{\lambda_1\lambda_2\ldots\lambda_n\rho_1\rho_2\ldots\rho_n}(\not{p}' + m)$$

$$= (-1)^n\ (\not{p} + m)\ \theta^{(p,n)}_{\lambda_1\lambda_2\ldots\lambda_n\rho_1\rho_2\ldots\rho_n} \qquad (4.24)$$

where $\bar{u}(p) = u^{+}(p)\gamma_0$ and the summation is carried out over all polarisation states.

Once again $\theta(p,n)$ is uniquely defined by the conditions (4.19) - (4.23) and this operator can be expressed in terms of the integral spin projection operator $\phi(p,n+1)$ as follows:

$$\theta^{(p,n)}_{\lambda_1\lambda_2\ldots\lambda_n\rho_1\rho_2\ldots\rho_n} = \frac{(2J+1)}{4(J+1)}\gamma_\lambda\ \gamma_\rho\ \phi^{(p,n+1)}_{\lambda\lambda_1\ldots\lambda_n\rho\rho_1\ldots\rho_n} \qquad (4.25)$$

where $J = n + \frac{1}{2}$.

Strictly speaking all the above considerations apply only to free, stable particles. However with the assumption that the resonances produced peripherally decay freely it is appropriate in the description of these resonant states to use the above projection operators $\phi(q,n)$ and $\theta(p,n)$ in conjunction with the usual Breit-Wigner modification to the stable particle propagators. Thus for the spin j boson resonances B_1 and B_3 of mass μ_2, width Γ_V and 4-momentum q_2 the corresponding propagators are given by:

$$(-1)^n \phi^{(q_2,n)}_{\lambda_1\lambda_2\ldots\lambda_n\rho_1\rho_2\ldots\rho_n} / [(v-\mu_2^2) + i\mu_2\Gamma_V] \qquad (4.26)$$

where $j = n$.

The special case of $j = 1$ then yields for the spin one resonance V_1 of mass μ_4, width Γ_x and 4-momentum q_4 a propagator given by:

$$(-1)\ [g_{\sigma\tau} - q_{4\sigma}q_{4\tau}/q_4^2] \ /\ [(x-\mu_4^2) + i\mu_4\Gamma_x] \qquad (4.27)$$

For the spin J fermion resonance F of mass m_2, width Γ_W and 4-momentum p_2 the propagator is given by:

$$(-1)^n \theta^{(p,n)}_{\lambda_1\lambda_2\ldots\lambda_n\rho_1\rho_2\ldots\rho_n} (\not{p}+m_2) / [(w-m_2^2) + im_2\Gamma_W] \qquad (4.28)$$

where $J = n + \frac{1}{2}$.

In contrast to this the exchanged particles E(k) in the diagrams of Fig. 1 are not on the mass shell and even if these particles are unstable it is a good approximation to use the unmodified propagators appropriate to the description of stable particles. The propagators for a spin zero particle and a spin one particle both of mass μ and with 4-momentum k are given by:

$$1/(k^2 - \mu^2) \qquad (4.29)$$

and

$$(-1)\left[g_{\alpha\beta} - k_\alpha k_\beta /\mu^2 \right] / \left(k^2 - \mu^2\right) \qquad (4.30)$$

respectively. By rewriting the latter propagator as:

$$(-1)\left[g_{\alpha\beta} - k_\alpha k_\beta /k^2 \right] / \left(k^2 - \mu^2\right) + \left[k_\alpha k_\beta /k^2 \right] /\mu^2 \qquad (4.31)$$

it can be seen that the off-mass shell spin one particle corresponds to a mixed state of spin zero and spin one since the first term contains the spin one projection operator and the second term the spin zero projection operator appropriate to a tensor field with just one index. It should be noted however that only the spin one term has a pole at $k^2 = \mu^2$. It is this fact, when generalised to higher spin, which permits the above description of the resonant states to be used.

It is convenient at this stage to simplify the notation by writing

$$\phi^{(q,n)}_{\lambda_1\lambda_2\ldots\lambda_n\rho_1\rho_2\ldots\rho_n} = \phi^{(q,n)}_{\lambda\rho} \qquad (4.32)$$

and

$$\theta^{(p,n)}_{\lambda_1\lambda_2\ldots\lambda_n\rho_1\rho_2\ldots\rho_n} = \theta^{(p,n)}_{\lambda\rho} \qquad (4.33)$$

where λ and ρ stand for the two sets of indices $\lambda_1\lambda_2\ldots\lambda_n$ and $\rho_1\rho_2\ldots\rho_n$ respectively.

5. VERTEX FUNCTIONS INVOLVING PARTICLES OF ARBITRARY SPIN AND PARITY.

To evaluate the peripheral diagrams of Fig. 1 it is necessary to consider the possible couplings of various combinations of three particles any one of which may have arbitrary spin and parity. As baryon number is conserved there are only two types of three particle coupling namely the coupling of three bosons (boson coupling) and the coupling of two fermions with one boson (fermion coupling). Since only strong interactions are considered, the vertex functions and the associated particle wave functions must together form a scalar which is invariant under Lorentz transformations and the parity transformation.

In the coupling of three bosons, as shown in Fig. 3, with 4-momenta p, q and r such that $p^2 = x$, $q^2 = y$ and $r^2 = z$ the most general Lorentz invariant is an arbitrary function of x, y and z only since energy and momentum are conserved at the vertex. If any one of the particles is on the mass shell say p^2 = constant, then the vertex function is a function of two invariants y and z. If any two of the particles are on the mass shell say p^2 = constant and q^2 = constant then the vertex function is a function of a single invariant z.

Similar results hold in the case of fermion coupling except that there may be an additional dependence of the vertex functions on any two of the three linearly dependent Lorentz invariant $\not{p}$, $\not{q}$ and $\not{r}$. It is convenient to choose $\not{p}$ and $\not{q}$ if p and q are the 4-momenta of the two fermions. Since $\not{p}\,\not{p} = x$ and $\not{q}\,\not{q} = y$ the vertex function is at most linear in each of the matrix quantities $\not{p}$ and $\not{q}$. If any one of the fermions is on the mass shell say p^2 = constant then by virtue of equation (4.4) $\not{p}$ = constant and the vertex function is a function only of the three invariants y, z and $\not{q}$. If both fermions are on the mass shell then the vertex function is a function of a single invariant z.

The tensor quantities with which the vertex functions are constructed consist of the metric tensor $g_{\mu\nu}$, the completely antisymmetric pseudotensor $\epsilon_{\mu\nu\lambda\rho}$ and any two independent 4-momentum tensors which are linear combinations of p_μ, q_μ and r_μ. In the case of fermion couplings the matrices γ_μ may also be used.

In general a tensor with n indices has parity $(-1)^n$. Thus for a particle whose wave function has n tensor indices it is convenient to say that the intrinsic parity of the particle is normal if it has parity $(-1)^n$ and abnormal if it has parity $(-1)^{n+1}$. If all three particles coupled together have normal parity or if one

has normal parity and the other two have abnormal parity then it can be said that the coupling has normal parity since no pseudo-tensors need to be included in the construction of a scalar vertex function. On the other hand if all three particles have abnormal parity or if one has abnormal parity and the other two have normal parity then the coupling is said to have abnormal parity since a pseudotensor must in this case be included if the interaction is to be invariant under parity transformations. It should be noted that the most general pseudotensor is linear in $\epsilon_{\mu\nu\lambda\rho}$ since the product of two antisymmetric pseudotensors can be written as a sum of products of the metric tensor $g_{\mu\nu}$ by means of the identity

$$\epsilon_{\mu\nu\lambda\rho}\epsilon_{\mu'\nu'\lambda'\rho'} = - \begin{vmatrix} g_{\mu\mu'} & g_{\mu\nu'} & g_{\mu\lambda'} & g_{\mu\rho'} \\ g_{\nu\mu'} & g_{\nu\nu'} & g_{\nu\lambda'} & g_{\nu\rho'} \\ g_{\lambda\mu'} & g_{\lambda\nu'} & g_{\lambda\lambda'} & g_{\lambda\rho'} \\ g_{\rho\mu'} & g_{\rho\nu'} & g_{\rho\lambda'} & g_{\rho\rho'} \end{vmatrix} \qquad (5.1)$$

In the case of abnormal parity boson coupling no spinors are involved in the interaction and the indices of the pseudotensor must be contracted with either 4-momentum indices or wave function

indices. However in the case of abnormal parity fermion coupling spinors are involved and the indices of the pseudotensor may be contracted with γ - matrix, 4-momentum or wave function indices. A particularly useful pseudotensor quantity is the invariant matrix γ_5 which may be written as:

$$\gamma_5 = \frac{1}{4!} \epsilon_{\mu\nu\lambda\rho} \gamma_\mu \gamma_\nu \gamma_\lambda \gamma_\rho \qquad (5.2)$$

All abnormal fermion couplings may be obtained from the most general normal fermion couplings by the inclusion of a factor γ_5. This factor is, for convenience, always inserted next to the spinor wave function of the spin half particle involved in the coupling which is on the mass shell.

To evaluate the diagrams of Fig. 1 for spin zero and spin one exchange it is necessary to consider the vertices shown in the diagrams (a), (b), (c) and (d) of Fig. 4. The subsidiary conditions (4.2) - (4.4) and (4.15) - (4.18) greatly restrict the allowed couplings involving arbitrary but pure spin states even if these states are not on the mass shell. The most general possible couplings are given in Table 1 for both normal and abnormal parity couplings. The notation used is such that the quantities g, g_1, g_2, g_3, g_0, f, f_1, f_2, f_3, f_4 and f_5 are dimensionless coupling constants and the quantities G, G_1, G_2,

G_3, G_0, F, F_1, F_2, F_3, F_4 and F_5 are the corresponding form factors which all reduce to unity when all three coupled states are on the shell, that is when $v = \mu_2^2$, $w = m_2^2$, $\not{p}_2 = m_2$ and $t = \mu^2$. where μ is the mass of the exchanged particle. A mass characteristic of the three interacting particles should be chosen for the arbitrary quantities m_x where x is the appropriate coupling constant.

By making full use of the relationships (5.1) and (5.2) and of the useful identity

$$\epsilon_{\mu\nu\lambda\rho}\gamma_\rho\gamma_5 = -\left[\gamma_\mu\gamma_\nu\gamma_\lambda - g_{\mu\nu}\gamma_\lambda + g_{\mu\lambda}\gamma_\nu - g_{\nu\lambda}\gamma_\mu\right] \qquad (5.3)$$

it is easy to show that the most general normal fermion coupling can be expressed in a form which contans no pseudotensors. Thus the terms of the fermion couplings involving f_5 are really superfluous since they can be written as a sum of terms involving only f_1, f_2, f_3 and f_4. However the particular form of the couplings involving f_5 is of particular physical significance, as will be shown in later sections, so it is included here in Table 1.

It should be noted that the couplings involving g_3 and f_4 contain a factor k_μ and this factor removes the pole term in the exchange diagrams of Fig. 1 because of the form of the propagator for a spin one particle as given in equation (4.31).

Using the couplings defined in Table 1 it is possible to write down all the vertex functions appearing in the diagrams of Fig. 1. In general all particles in these diagrams may be different and in this paper the notation used is such that the coupling constants e, f, g h and i, with subscripts where appropriate, are associated with the vertices at which the three coupled particles have 4-momenta given by: (p_2, p_3, p_4), (p_1, p_2, k), (q_1, q_2, k), (q_2, q_3, q_4) and (q_4, q_5, q_6) respectively.

At the two vertices involving the fermion resonance F the alternative parity cases may be considered simultaneously by inserting the factors γ_e and γ_f at the vertices with associated coupling constants e and f respectively. These quantities are both defined to be the unit matrix I in the case of normal parity coupling and to be the matrix γ_5 in the case of abnormal parity coupling.

With the assumption that the resonances produced peripherally have propagators containing a Breit-Wigner resonance term it is shown in the sections which follow that the only arbitrary functional dependence of the form factors appearing in the final expression for the various differential cross sections is a dependence upon t, the square of the 4-momentum transfer. It is thus convenient to introduce the notation:

$$\bar{x} = \frac{x\, X(t)}{(m_x)^{n_x}} \tag{5.4}$$

where n_x = (j-1), (j-2), j, j, j, (J-½), (J-3/2), (J-½), (J+½), (J+½) and (J+½) for x = g, g_1, g_2, g_3, g_0, f, f_1, f_2, f_3, f_4 and f_5 respectively and where j is the spin of the boson resonance and J is the spin of the fermion resonance.

It is also convenient to introduce a great simplification in the notation for the tensor factors constructed from the 4-momentum k_λ in the vertex functions of Table 1 by writing:

$$k_{\lambda_1} k_{\lambda_2} \cdots k_{\lambda_n} = (k_\lambda)^n \tag{5.5a}$$

$$k_{\lambda_2} k_{\lambda_3} \cdots k_{\lambda_n} = (k_\lambda)^{n-1} \tag{5.5b}$$

. . . . . .

. . . . . .

$$k_{\lambda_{m+1}} k_{\lambda_{m+2}} \cdots k_{\lambda_n} = (k_\lambda)^{n-m} \tag{5.5c}$$

This notation is readily generalised so that any product of identical 4-momentum tensors with consecutive indices may be written in a shortened manner.

FIG.3

Vertex Diagram

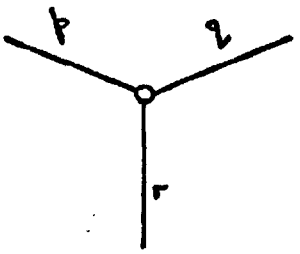
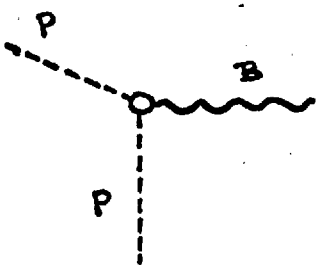
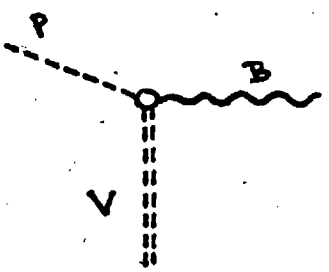


FIG.4

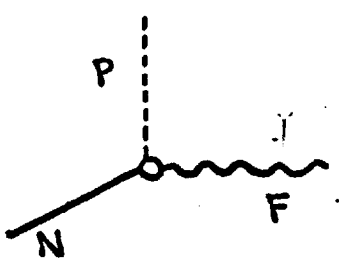
Vertex diagrams involving the coupling of one arbitrary spin particle.



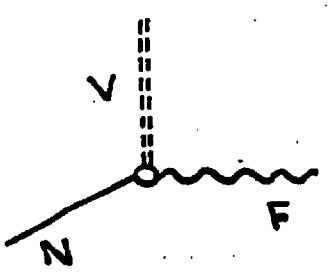
(a)



(b)



(c)



(d)

TABLE 1

The general form of the vertex functions for the diagrams (a),(b),(c) and (d) of Fig 4.

(4a) Coupling of particles of spin (0,0,j) where j = n

Normal parity couplings.:

$$g \frac{G(t,v)}{(m_g)^{n-1}} k_{\lambda_1} k_{\lambda_2} \cdots k_{\lambda_n}$$

Abnormal parity coupling:

COUPLING FORBIDDEN

(4b) Coupling of particles of spin (0,1,j) where j = n

Normal parity coupling:

$$\left[\frac{g_1 G_1(t,v)}{(m_{g_1})^{n-2}} g_{\mu\lambda_1} + \frac{g_2 G_2(t,v)}{(m_{g_2})^{n}} q_{2\mu} k_{\lambda_1} + \frac{g_3 G_3(t,v)}{(m_{g_3})^{n}} k_\mu k_{\lambda_1} \right] *$$

$$* k_{\lambda_2} k_{\lambda_3} \cdots k_{\lambda_n}$$

Abnormal parity coupling:

$$\frac{g_0 G_0(t,v)}{(m_{g_0})^{n}} \epsilon_{\mu\alpha\beta\lambda_1} k_\alpha q_{2\beta} k_{\lambda_2} k_{\lambda_3} \cdots k_{\lambda_n}$$

(4c) Coupling of particles of spin $(0,\frac{1}{2},J)$ where $J = n+\frac{1}{2}$

$$\frac{f\ F(t,w,\vec{p}_2)}{(m_f)^n}\ \gamma_f\ k_{\lambda_1} k_{\lambda_2} \cdots k_{\lambda_n}$$

Normal parity coupling: $\gamma_f = I$

Abnormal parity coupling: $\gamma_f = \gamma_5$

(4d) Coupling of particles of spin $(1,\frac{1}{2},J)$ where $J = n+\frac{1}{2}$

$$\left[\frac{f_1 F_1(t,w,\vec{p}_2)}{(m_{f_1})^{n-1}}\ g_{\mu\lambda_1} + \frac{f_2 F_2(t,w,\vec{p}_2)}{(m_{f_2})^n}\ \gamma_\mu k_{\lambda_1} + \frac{f_3 F_3(t,w,\vec{p}_2)}{(m_{f_3})^{n+1}}\ p_{2\mu} k_{\lambda_1} + \right.$$

$$\left. \frac{f_4 F_4(t,w,\vec{p}_2)}{(m_{f_4})^{n+1}}\ k_\mu k_{\lambda_1} + \frac{f_5 F_5(t,w,\vec{p}_2)}{(m_{f_5})^{n+1}}\ \epsilon_{\mu\alpha\rho\lambda_1} k_\alpha p_{2\rho} \gamma_5 \right] \times$$

$$\gamma_f\ k_{\lambda_2} k_{\lambda_3} \cdots k_{\lambda_n}$$

Normal parity coupling: $\gamma_f = I$

Abnormal parity coupling: $\gamma_f = \gamma_5$

6. THE DECAY PARTIAL WIDTHS OF ARBITRARY SPIN AND PARITY RESONANCES.

The differential cross sections for the processes described by the diagrams of Fig. 1 are all proportional to the decay probability of the arbitrary spin and parity resonances. This is explicitly shown in the sections which follow and it is useful to determine these decay probalities as functions of the coupling constants defined in section 5. The decay processes (2.1) - (2.4) are each considered in turn.

(i) From the discussion of section 5 and the results expressed in Table I it is clear that a boson resonance which decays into two pseudoscalar particles must necessarily have normal parity. The partial width corresponding to the decay of such a boson resonance $B_1(q_2)$ into two pseudoscalar particles $P(q_3)$ and $P(q_4)$ when all these particles are on the mass shell is denoted by $\Gamma_V(2 \to 3,4)$ and is given by:

$$d\Gamma_V(2 \to 3, 4) = \frac{1}{2\mu_2} (2\pi)^4 \delta^4(q_2 - q_3 - q_4) \frac{1}{[2(2\pi)^3]^2} \times \frac{d^3q_3}{\omega_3} \frac{d^3q_4}{\omega_4} \left| \langle q_3 q_4 | T | q_2 \rangle \right|^2_{AV} \qquad (6.1)$$

The invariant phase space factor reduces, after

integration over d^3q_4 and dq_{3V} to remove the delta function, to:

$$\frac{q_V}{\mu^2} \; d\cos\theta_{3V} \; d\phi_{3V} \tag{6.2}$$

and the decay amplitude for the spin j resonance in the notation of sections 4 and 5 is given by:

$$\langle q_3 \; q_4 \mid T \mid q_2 \rangle = \frac{h}{(m_h)^{n-1}} \; (q_{4\lambda})^n \; \times \; (q_2)_{\lambda_1\lambda_2\cdots\lambda_n} \tag{6.3}$$

where j = n. Thus from equation (4.9) it follows that:

$$\left| \langle q_3 q_4 \mid T \mid q_2 \rangle \right|^2_{AV} = \frac{1}{(2j+1)} \; \frac{h^2}{(m_h)^{2n-2}} \; (-1)^n \; (q_{4\lambda})^n$$

$$\phi_{\lambda\rho}(q_2, n) \; (q_{4\rho})^n \tag{6.4}$$

Using the result (A.14) of the appendix and integrating over the redundant variables $\cos\theta_{3V}$ and ϕ_{3V} the partial width can be written as:

$$\Gamma_V(2 \to 3,4) = \frac{h^2}{4\pi} \; \frac{q_V^{2j+1}}{\mu_2^2 (m_h)^{2j-2}} \left[\frac{2^j \, j! \, j!}{(2j)!} \right] \frac{1}{(2j+1)} \tag{6.5}$$

(ii) A boson resonance which decays into a vector particle and a pseudoscalar particle may have either normal or abnormal parity. In both cases the partial width corresponding

to the decay of the boson resonance B_3 (q_2) into a pseudoscalar particle P (q_3) and a vector particle V_1 (q_4) when all these particles are on the mass shell is again denoted by Γ_V $(2 \to 3,4)$ and is given by (6.1). The phase space factor is again given by (6.2) but the decay amplitude for the spin j resonance now depends upon the parity of the resonance. The alternatives (a) normal parity and (b) abnormal parity are considered as follows:

(a) The decay amplitude for the spin j, parity $(-1)^j$ resonance is given by:

$$\langle q_3\, q_4 \,|\, T \,|\, q_2 \rangle = \frac{h_o}{(m_{h_o})^n} \; (q_{4\lambda})^{n-1} \epsilon_{\lambda_1 \mu \nu \sigma} \, q_{4\mu} \, q_{3\nu}$$

$$\chi^{+}_{\sigma}(q_4)\, \chi_{\lambda_1 \lambda_2 \ldots \lambda_n}(q_2) \qquad (6.6)$$

where j = n. Thus from equations (4.9) and (4.10) it follows that:

$$\left| \langle q_3 q_4 \,|\, T \,|\, q_2 \rangle \right|^2_{AV} = \frac{1}{(2j+1)} \; \frac{h^2_o}{(m_{h_o})^{2n}}$$

$$\epsilon_{\lambda_1 \mu \nu \sigma} \, q_{4\mu} \, q_{3\nu} \, \epsilon_{\rho_1 \alpha \beta \tau} \, q_{4\alpha} \, q_{3\beta} \quad \times$$

$$\left[g_{\sigma\tau} - \frac{q_{4\sigma} \, q_{4\tau}}{\mu^2_4} \right] (-1)^{n+1} \; (q_{4\lambda})^{n-1} \; \phi_{\lambda\rho}(q_2, n) \; (q_{4\rho})^{n-1} \qquad (6.7)$$

Using the result (A.15) and integrating over the redundant variables $\cos\theta_{3V}$, the partial width can be written as:

$$\Gamma_V\,(2\to 3,4) = \frac{h_o^2}{4\pi}\;\frac{q_V^{2j+1}}{(m_{h_o})^{2j}}\;\frac{2^j\,j!\,j!}{(2j)!}\;\frac{(j+1)}{2j(2j+1)} \qquad (6.8)$$

(b) The decay amplitude for the spin j, parity $(-1)^{j+1}$ resonance is given by:

$$\langle q_3 q_4 \mid T \mid q_2\rangle = \left[\frac{h_1}{(m_{h_1})^{n-2}}\;g_{\sigma\lambda_1} + \frac{h_2}{(m_{h_2})^{n}}\;q\;{}_{\sigma}q_{4\lambda_1} + \frac{h_3}{(m_{h_3})}\;q_{4\sigma}\,q_{4\lambda_1}\right]$$

$$(q_{4\rho})^{n-1}\;\chi^{+}_{\sigma}(q_4)\;\chi_{\lambda_1\lambda_2\ldots\lambda_n}(q_2) \qquad (6.9)$$

where j = n. Thus from equations (4.9) and (4.10) it follows that:

$$\left|\langle q_3 q_4 \mid T \mid q_2\rangle\right|^2_{AV} = \frac{1}{(2j+1)}\left[\frac{h_1}{(m_{h_1})^{n-2}}\;g_{\sigma\lambda_1} + \right.$$

$$\left.\frac{h_2}{(m_{h_1})^{n}}\;q_{2\sigma}\,q_{4\lambda_1}\right]\left[g_{\sigma\tau} - \frac{q_{4\sigma}q_{4\tau}}{\mu^2_4}\right]\left[\frac{h_1}{(m_{h_1})^{n-2}}\;g_{\tau\rho_1} + \right.$$

$$\left.\frac{h_2}{(m_{h_2})^{n}}\;q_{2\tau}\,q_{4\rho_1}\right](-1)^{n+1}\,(q_{4\lambda})^{n-1}\,\phi_{\lambda\rho}(q_2,n)\,(q_{4\rho})^{n-1} \qquad (6.10)$$

Using the result (A.15) and integrating over the redundant variables $\cos\Theta_{3V}$ and ϕ_{3V} the partial width can be written as:

$$\Gamma_V(2 \to 3,4) = \frac{1}{4\pi} \frac{q_V^{2j-1}}{2\mu^2} \frac{2^j\, j!\, j!}{(2j)!} \frac{1}{j(2j+1)} \times$$

$$\left[\frac{h_1^2}{(m_{h_1})^{2j-4}} (j+1) + \left\{ \frac{h_1}{(m_{h_1})^{j-2}} \frac{\omega_{4V}}{\mu_4} + \frac{h_2}{(m_{h_2})^{j}} \frac{\mu_2 q_V^2}{\mu_4} \right\}^2 j \right] \qquad (6.11)$$

(iii) The partial width corresponding to the decay of the vector meson $V_1(q_4)$ into two pseudoscalar particles $P(q_5)$ and $P(q_6)$ when all these particles are on the mass shell is denoted by $\Gamma_X(4 \to 56)$ and is given, by analogy with the special case of $j = 1$ in equation (6.5), by

$$\Gamma_X(4 \to 5,6) = \frac{i^2}{4\pi} \frac{q_5^3}{2\mu_4^2} \frac{1}{3} \qquad (6.12)$$

(iv) A fermion resonance which decays into a spin half particle and a pseudoscalar particle may have either normal or

abnormal parity. These two cases may be dealt with simultaneously by making use of the factor γ_e defined in section 5. In both cases the partial width corresponding to the decay of the fermion resonance $F(p_2)$ into a spin half particle $N(p_3)$ and a pseudo-scalar particle $P(p_4)$ when all these particles are on the mass shell is denoted by $\Gamma_W\ (2 \to 3,4)$ and is given by:

$$d\Gamma_W\ (2 \to 3,4) = \frac{1}{2\,m_2}\ (2\pi)^4\ \delta^4(p_2 - p_3 - p_4)\ \frac{1}{\left[2(2\pi)^3\right]^2}$$

$$\times\ \frac{d^3p_3}{\epsilon_3}\quad \frac{d^3p_4}{\epsilon_4}\quad \left|< p_3\ p_4\ |\ T\ |\ p_2 >\right|^2_{AV} \qquad (6.13)$$

The invariant phase space factor reduces to:

$$\frac{p_W}{m_2}\quad d\cos\theta_{4W}\quad d\phi_{4W} \qquad (6.14)$$

and the decay amplitude for the spin J resonance in the notation of sections 4 and 5 is given by:

$$< p_3\ p_4\ |\ T\ |\ p_2 > = \bar{u}\ (p_3)\ \gamma_e\ \frac{e}{(m_e)^{n-1}}\ (p_4\lambda)^{n-1}\ \underset{\lambda_2\lambda_3\ldots\lambda_n}{u\ (p_2)} \qquad (6.15)$$

where $J = (n - \frac{1}{2})$. In order to deal with both parity cases simultaneously it is convenient to define m'_3 such that:

$$m'_3 = + m_3 \quad \text{if } \gamma_e = I \tag{6.16a}$$

$$m'_3 = - m_3 \quad \text{if } \gamma_e = \gamma_5 \tag{6.16b}$$

Thus from equations (4.24) and (4.25) it follows that:

$$\left| < p_3\ p_4 \mid T \mid p_2 > \right|^2_{AV} = \frac{1}{(2J+1)} \quad \frac{e^2}{(m_e)^{2n-2}} \quad x$$

$$2(p_2\ p_3 + m_2\ m'_3) \quad \frac{2n}{(2n+1)} \quad g_{\rho_1 \lambda_1} \quad x$$

$$(-1)^{n+1} \quad (p_{4\lambda})^{n-1} \quad \phi_{\lambda\rho}(p_2, n) \quad (p_{4\rho})^{n-1} \tag{6.17}$$

Using the result (A.15) of the appendix and integrating over the redundant variables $\cos \theta_{4W}$ and ϕ_{4W} the partial width may be written as:

$$\Gamma_W (2 \rightarrow 3,4) = \frac{e^2}{4\pi} \quad \frac{p_W^{2n-1}}{2m^2_2 (m_e)^{2n-2}} \quad 2\ (p_2\ p_3 + m_2\ m'_3) \quad x$$

$$\frac{2^n\ n!\ n!}{(2n)!} \quad \frac{1}{n} \tag{6.18}$$

where $J = (n - \frac{1}{2})$. It should be noted that as the particle P (p_4) is a pseudoscalar and as the only stable spin half particles, N(p_3), observed to date appear to have positive parity the decay width of a spin J resonance into such particles is given by (6.18) with $m'_3 = - m_3$ for a normal parity resonance and $m'_3 = + m_3$ for an abnormal parity resonance.

7. BOSON RESONANCES DECAYING INTO TWO PSEUDOSCALAR PARTICLES

As pointed out in section 6 a boson resonance which decays into two pseudoscalar particles must necessarily have normal parity. In the peripheral production of such a resonance the exchanged particle can be a pseudoscalar, a vector or a pseudovector particle but not a scalar particle. For each of these three allowed possibilities the differential cross section is calculated for the process described by Fig. 1 (a) in which the resonance B_1 has spin j and normal parity $(-1)^j$. This is done by constructing the appropriate invariant matrix element from the vertex functions of Table 1.and the propagators of section 4. In particular the propagator for the spin j resonance is given by (4.26) where $j = n$. The resulting expression for the matrix element is then substituted in the formula (3.12) for $d\sigma$ (a).

(i) Pseudoscalar particle exchange.

The invariant matrix element is given by:

$$\langle p_2 q_3 q_4 \mid T \mid p_1 q_1 \rangle = \bar{u}(p_2)\, f\, F(t)\, \gamma_f\, u(p_1)\, \frac{1}{(t-\mu^2)} \times$$

$$\times (-1)^n \frac{g\, G(t,v)}{(m_g)^{n-1}} (k_\lambda)^n \phi_{\lambda\rho}(q_2,n) \frac{1}{(v-\mu_2^2) + i\mu_2 \Gamma_V} \times$$

$$x \quad \frac{h\,H(v)}{(m_h)^{n-1}} \qquad (q_4\rho)^n \tag{7.1}$$

Averaging over the initial spin states and summing over final spin states:

$$\left|\langle p_2 q_3 q_4 \mid T \mid p_1 q_1 \rangle\right|^2_{AV} = \left|\frac{f\,F(t)\;g\,G(t,v)\;h\,H(v)}{(m_g)^{n-1}\;(m_h)^n}\right|^2 \quad x$$

$$\frac{\left[-t + (m'_1+m_2)^2\right]}{(t-\mu^2)^2} \qquad \frac{1}{(v-\mu^2_2)^2 + \mu^2_2 \Gamma_V^2} \qquad x$$

$$\left[(k_\lambda)^n \phi_{\lambda\rho}(q_2,n) \quad (q_4\rho)^n\right]^2 \tag{7.2}$$

where

$$m'_1 = +m_1 \quad \text{if} \quad \gamma_f = I \tag{7.3a}$$

$$m'_1 = -m_1 \quad \text{if} \quad \gamma_f = \gamma_5 \tag{7.3b}$$

Assuming that the resonance has a narrow width i.e. $\Gamma_V << \mu_2$ then:

$$\frac{1}{(v-\mu^2_2)^2 + \mu^2_2 \Gamma_V^2} \approx \frac{\pi}{\mu_2 \Gamma_V}\,\delta(v-\mu^2_2) \tag{7.4}$$

With this approximation and the result (A.8) given in the appendix, substitution of (7.2) in the expression (3.12) for $d\sigma$ (a) and integration over dv gives the result:

$$d\sigma(a) = \frac{f^2}{4\pi} \frac{g^2}{4\pi} \left| \frac{F(t)\, G(t)}{(m_g)^{j-1}} \right|^2 \frac{q_B}{4E^2_B\, p_B} \quad x$$

$$x \frac{\left[-t + (m'_1 + m_2)^2\right]}{(t - \mu^2)^2} \frac{2^j\, j!\, j!}{(2j)!} k_V^{2j} \; d\cos\theta_B \, d\phi_B$$

$$x \frac{\Gamma_V (2 - 3,4)}{\Gamma_V} \; x \frac{(2j + 1)}{4\pi} \left[P_j (\cos\theta_{3V}) \right]^2 d\cos\theta_{3V} \, d\phi_{3V} \qquad (7.5)$$

where (6.5) has been used to separate out the factor $\Gamma_V(2 - 3,4)/\Gamma_V$ where Γ_V (2 - 3,4) is the partial width and Γ_V is the total width of the resonance. This factor is thus the branching ratio of the decay process (1.2) relative to all other possible decay modes of B_1.

The first factor in the expression (7.5), made up of all terms preceding the branching ratio factor, is in fact the spin averaged differential cross section for the production of a spin j, parity $(-1)^j$, resonance in the quasi two body process (1.6) either assuming that the resonance does not decay or equivalently summing over all possible decay modes.

The final factor in (7.5) gives the distribution of the decay products and is normalised so that integration over $\cos\Theta_{3V}$ and ϕ_{3V} gives unity. It should be noted that the distribution is independent of ϕ_{3V} and there is thus rotational symmetry of the decay products about the polar axis $\underline{I}$ in the frame V.[13] Furthermore the decay distribution as a function of $\cos\Theta_{3V}$ is independent of the production process variables. In particular this distribution is independent of the arbitrary form factors F(t) and G(t).

The spin averaged differential cross section for the production of a spin j, parity $(-1)^j$ resonance in a process mediated by pseudoscalar particle exchange is obtained from (7.5) by integrating over $\cos\Theta_{3V}$ and ϕ_{3V} and putting $\Gamma_V(2-3,4) = \Gamma_V$. The result obtained, which confirms the statement made above concerning the first factor of (7.5) is:

$$d\sigma(a) = \frac{f^2}{4\pi}\,\frac{g^2}{4\pi}\left|\frac{F(t)\,G(t)}{(m_g)^{j-1}}\right|^2 \frac{q_B}{4E^2_B\,p_B} \times \frac{\left[-t+(m'_1+m_2)^2\right]}{(t-\mu^2)^2}\,\frac{2^j\,j!\,j!}{(2j)!}\,k_V^{2j}\,d\cos\Theta_B\,d\phi_B \qquad (7.6)$$

It should be noted that this result is completely independent of the decay mode of the boson resonance which might

for instance decay into three pseudoscalar mesons or into a pseudoscalar meson and a vector meson rather than into two pseudoscalar mesons. Thus (7.6) applies to the general process:

$$P + N \rightarrow N + B \tag{7.7}$$

where B is any boson having spin j and normal parity provided that this production process (7.7) is mediated by the exchange of a pseudoscalar meson.

(ii) Vector particle exchange

The invariant matrix element is given by:

$$\begin{aligned} \langle p_2\, q_3\, q_4 \mid T \mid p_1 q_1 \rangle = \bar{u}(p_2) \Big[& f_2 F_2(t)\, \gamma_\mu + \\ & \frac{f_3 F_3(t)}{(m f_3)}\, p_{2\mu} + \frac{f_4 F_4(t)}{(m f_4)}\, k_\mu \Big] \gamma_f\, u(p_1) \quad x \\ x\, (-1) \Big[& g_{\mu\nu} - \frac{k_\mu k_\nu}{\mu^2} \Big] \frac{1}{(t-\mu^2)} \frac{g_o\, G_o(t,v)}{(m_{g_o})^n} \quad x \\ x\, \epsilon_{\nu\sigma\tau\lambda_1}\, & k_\sigma\, q_{2\tau}\, (k_\lambda)^{n-1} \quad (-1)^n \phi_{\lambda\rho}(q_2, n) \quad x \\ x\, & \frac{1}{(v-\mu_2^2) + i\mu_2 \Gamma_V} \quad (q_{4\rho})^n \quad \frac{h\, H(v)}{(m_h)^{n-1}} \end{aligned} \tag{7.8}$$

Making full use of the antisymmetric nature of the pseudotensor in this expression, substituting (7.8) in (3.12), averaging over initial spin states, summing over final spin states and integrating over dv gives, with the notation of (7.3) and the approximation (7.4), the result:

$$d\sigma(a) = \frac{1}{(4\pi)^3} \frac{q_B}{4E^2_B\, p_B} \frac{1}{(t-\mu^2)^2} \frac{h^2}{4\pi} \frac{q_V}{2\mu^2_2 (m_h)^{2j-2}} \frac{1}{\Gamma_V} \times$$

$$\times \left[M_{p_2 p_2}(t)\ (p_2 r)\ (p_2 r^+) - M(t)\ (r \cdot r^+) \right] d\cos\Theta_B\, d\phi_B$$

$$d\cos\Theta_{3V}\, d\phi_{3V} \qquad (7.9)$$

where

$$r_\mu = \bar{g}_o \epsilon_{\mu\sigma\tau\lambda 1}\ k_\sigma\, q_{2\tau}\ (-1)^{j+1}\ (k_\lambda)^{j-1} \phi_{\lambda\rho}(q_2, j)\ (q_{4\rho})^j \qquad (7.10)$$

and

$$M_{p_2 p_2}(t) = \left\{ |\bar{f}_2|^2\ 4 + 2\,\mathrm{Re}\,(\bar{f}_2 \bar{f}_3^+)\ 2(m'_1 + m_2) + \right.$$

$$\left. + |\bar{f}_3|^2 \left[-t + (m'_1 + m_2)^2 \right] \right\} \qquad (7.11a)$$

$$M(t) = \left\{ |\bar{f}_2|^2 \left[-t + (m'_1 - m_2)^2 \right] \right\} \qquad (7.11b)$$

It follows from (A.9) that:

$$p_2 r = \bar{g}_o \left[\frac{2^j\, j!\, j!}{(2j)!} \quad \frac{1}{j} \quad (E_B\, p_B\, q_B \sin\theta_B) \qquad x \right.$$

$$\left. k_V^{j-1}\, q_V^j \quad \cos\phi_{3V} \quad P_j^1 \,(\cos\theta_{3V}) \right] \tag{7.12}$$

where use has been made of the fact that the angle between p_{2V} and q_{1V}, denoted by ϵ_V as shown in Fig. 2(a), satisfies the relationship:

$$\mu_2\, p_{2V}\, k_V \sin\epsilon_V \;=\; E_B\, p_B\, q_B \sin\theta_B \tag{7.13}$$

Similarly it may be shown that:

$$r.r^+ = - \left|\bar{g}_o\right|^2 \quad \frac{2^j\, j!\, j!}{(2j)!} \quad \frac{1}{j} \quad (\mu_2\, k_V)\, k_V^{j-1}\, q_V^j\, P_j^1\, (\cos\theta_{3V})^2 \tag{7.14}$$

Substituting (7.12) and (7.14) into the expression (7.9) for $d\sigma(a)$ gives:

$$d\sigma(a) = \frac{1}{(4\pi)^2} \quad \left|\bar{g}_o\right|^2 \quad \frac{q_B}{4E_B^2\, p_B} \quad \frac{1}{(t-\mu^2)^2} \quad \frac{2^j\, j!\, j!}{(2j)!} \qquad x$$

$$\frac{(j+1)}{j} \quad k_V^{2j-2} \left[M_{p_2 p_2}(t) \quad \left(E_B p_B q_B \sin\theta_B \right)^2 \quad \cos^2\phi_{3V} \;+\right.$$

$$\left. M(t) \quad (\mu_2\, k_V)^2 \right] d\cos\theta_B\, d\phi_B \qquad \frac{\Gamma_V(2-3,4)}{\Gamma_V} \qquad x$$

$$\frac{(2j+1)}{2j(j+1)} \left[P_j^1 (\cos\theta_{3V}) \right]^2 d\cos\theta_{3V} \frac{1}{2\pi} d\phi_{3V} \qquad (7.15)$$

where (6.5) has again been used to separate out the factor $\Gamma_V(2-3,4) / \Gamma_V$ which, as explained above, is the branching ratio for the decay process (1.2) relative to all other possible decay modes of the resonance B_1.

The distribution of the decay products as a function of $\cos\theta_{3V}$ is clearly independent of the production process variables and is given by:

$$\frac{(2j+1)}{2j(j+1)} \left[P_j^1 (\cos\theta_{3V}) \right]^2 d\cos\theta_{3V} \qquad (7.16)$$

On the other hand the decay distribution as a function of ϕ_{3V} depends not only on the production process variables but also on the ratio of the coupling constants f_2 and f_3 and their associated form factors. The distribution is of the general form:

$$\frac{1}{2\pi} \left[A(s,t) + B(s,t) \cos^2\phi_{3V} \right] d\phi_{3V} \qquad (7.17)$$

where A(s,t) and B(s,t) are specific functions of s but arbitrary functions of t. For the situation in which $\sin\theta_B = 0$,

that is the case of forward and backward production of the resonance in the process (1.6), the decay distribution is of course independent of ϕ_{3V} since this angle is then indeterminate. It should be noted that the number of events in the forward direction gives a measure of the coupling constant f_2 independent of f_3.

The spin averaged differential cross section for the two body production process (1.6), independent of the decay of the resonance, is found by putting $\Gamma_V\,(2-3,4) = \Gamma_V$ in (7.15) and integrating over $\cos\Theta_{3V}$ and ϕ_{3V}. The result is:

$$d\sigma(a) = \frac{g_o^2}{4\pi}\left|\frac{G_o(t)}{(m_{g_o})^j}\right|^2 \frac{q_B}{4E_B^2\, p_B}\,\frac{1}{(t-\mu^2)^2}\;\times$$

$$\times\;\frac{2^j\, j!\, j!}{(2j)!}\;\frac{(j+1)}{j}\;k_V^{2j-2}\Bigg[\Bigg\{\frac{f_2^2}{4\pi}\left|F_2(t)\right|^2 4 +$$

$$\frac{f_2 f_3}{4\pi}\;\frac{F_2(t)\,F_3^{+}(t) + F_3(t)\,F_2^{+}(t)}{(m_{f_3})}\;2(m_1' + m_2) +$$

$$\frac{f_3^2}{4\pi}\;\frac{\left|F_3(t)\right|^2}{(m_{f_3})^2}\left[-t + (m_1' + m_2)^2\right]\Bigg\}\;\left(E_B\, p_B\, q_B \sin\Theta_B\right)^2\,\frac{1}{2}\cdot$$

$$+\;\Bigg\{\frac{f_2^2}{4\pi}\left|F_2(t)\right|^2\left[-t + (m_1' - m_2)^2\right]\Bigg\}\left(\mu_2\, k_V\right)\Bigg]\, d\cos\Theta_B\, d\phi_B$$

(7.18)

Once again this result is completely independent of the decay mode of the boson resonance. Thus (7.18) applies to the general process (7.7) provided that this process is mediated by the exchange of a vector meson.

(iii) Pseudovector particle exchange.

The invariant matrix element is given by:

$$\langle p_2\, q_3\, q_4 \,|\, T \,|\, p_1\, q_1 \rangle = \bar{u}(p_2) \left[f_2 F_2(t)\, \gamma_\mu + \frac{f_3 F_3(t)}{(m f_3)}\, p_{2\mu} + \frac{f_4 F_4(t)}{(m f_4)}\, k_\mu \right] \gamma_f \; u(p_1) \; x$$

$$x\,(-1)\left[g_{\mu\nu} - \frac{k_\mu k_\nu}{\mu^2} \right] \frac{1}{(t-\mu^2)} \left[\frac{g_1 G_1(t,v)}{(m_{g_1})^{n-2}}\, g_{\nu\lambda_1} + \frac{g_2 G_2(t,v)}{(m_{g_2})^{n}}\, q_{2\nu}\, k_{\lambda_1} + \frac{g_3 G_3(t,v)}{(m_{g_3})^{n}}\, k_\nu\, k_{\lambda_1} \right]$$

$$x\,(k_\lambda)^{n-1}\,(-1)^n\, \phi_{\lambda\rho}(q_2,n)\, (q_4)_\rho^{\,n} \frac{1}{\left[(v-\mu_2^2) + i\mu_2 \Gamma_V \right]} \frac{h\,H(v)}{(m_h)^{n-1}} \qquad (7.19)$$

Substituting (7.19) in (3.12), averaging over initial spin states, summing over final spin states and integrating over dv gives with the notation of (7.3) and the approximation (7.4) the result:

$$d\sigma(a) = \frac{1}{(4\pi)^3} \quad \frac{q_B}{4E^2_B\, p_B} \quad \frac{1}{(t-\mu^2)^2} \quad \frac{h^2}{4\pi} \quad x$$

$$\frac{q_V}{2\mu^2_2\,(m_h)^{2j-2}\Gamma_V} \quad \Big[M_{p_2p_2}(t)\,(p_2\cdot r)\,(p_2\cdot r^+) + M_{p_2k}(t)\,(p_2 r)(k\cdot r^+) +$$

$$M_{kp_2}(t)\,(k\cdot r)\,(p_2\cdot r^+) + M_{kk}(t)\,(k\cdot r)\,(k\cdot r^+) - M(t)\,(r\cdot r^+)\Big] \quad x$$

$$x \quad d\cos\Theta_B \; d\phi_B \; d\cos\Theta_{3V} \; d\phi_{3V} \qquad (7.20)$$

where

$$r_\mu = \Big[g_{\mu\nu} - \frac{k_\mu k_\nu}{\mu^2} \Big] \Big[\bar{g}_1 \quad g_{\nu\lambda_1} + \bar{g}_2\, q_{2\nu}\, k_{\lambda_1} + \bar{g}_3 \quad k_\nu k_{\lambda_1} \Big] \; x$$

$$x\,(-1)^{j+1} \quad (k_\lambda)^{j-1}\, \phi_{\lambda\rho}(q_2, j) \quad (q_{4\rho})^j \qquad (7.21)$$

and

$$M_{p_2p_2}(t) = \quad |\bar{f}_2|^2\, 4 + 2\,\mathrm{Re}\,(\bar{f}_2\,\bar{f}_3^+) \quad 2\,(m_1' + m_2) + |\bar{f}_3|^2 \quad x$$

$$x \quad \big[-t + (m_1' + m_2)^2 \big] \qquad (7.22a)$$

$$M_{p_2k}(t) = \quad |\bar{f}_2|^2 \quad 2 + (\bar{f}_2\,\bar{f}_3^+)\, 2m_2 + (\bar{f}_2\,\bar{f}_4^+)\, 2\,(m_1' + m_2) +$$

$$(\bar{f}_3\,\bar{f}_4^+) \quad \big[-t + (m_1' + m_2)^2 \big] \qquad (7.22b)$$

$$M_{kp_2}(t) = M^{+}_{p_2 k}(t) \tag{7.22c}$$

$$M_{kk}(t) = 2\,\mathrm{Re}\,(\bar{f}_2 \bar{f}_4^{+})\, 2m_2 + |f_4|^2 \left[-t + (m_1' + m_2)^2 \right] \tag{7.22d}$$

$$M(t) = |\bar{f}_2|^2 \left[-t + (m_1' + m_2)^2 \right] \tag{7.22e}$$

Denoting the variable $\cos\theta_{3V}$ by C_V it follows from the definitions (7.21) and the result (A.8) that:

$$k.r = -\left[\frac{2^j j! j!}{(2j)!} \frac{1}{j} k_V^{j-2} q_V^j \right] \bar{g}_k k_V^2 j P_j^o (C_V) \tag{7.23}$$

and from (A.8) and (A.9)

$$p_2 r = -\left[\frac{2^j j! j!}{(2j)!} \frac{1}{j} k_V^{j-2} q_V^j \right] \left[\bar{g}_{p_2} k_V^2 j P_j^o (C_V) \right.$$

$$\left. - \bar{g}_1 p_{2V} k_V \left\{ \cos\epsilon_V j P_j^o (C_V) + \sin\epsilon_V \sin\phi_{3V} P_j^1 (C_V) \right\} \right] \tag{7.24}$$

and

$$r.r^{+} = \left[\frac{2^j j! j!}{(2j)!} \frac{1}{j} k_V^{j-2} q_V^j \right]^2 \left[|\bar{g}_r|^2 k_V^4 \left\{ j P_j^o (C_V) \right\}^2 \right.$$

$$\left. - |\bar{g}_1|^2 k_V^2 \left\{ \left[j P_j^o (C_V) \right]^2 + \left[P_j^1 (C_V) \right]^2 \right\} \right] \tag{7.25}$$

where

$$\bar{g}_k = \left(1 - \frac{k^2}{\mu^2}\right) \left[\bar{g}_1 + \bar{g}_2 k q_2 + \bar{g}_3 k^2 \right] \tag{7.26a}$$

$$\bar{g}_{p_2} = \left[-\bar{g}_1 \frac{p_2 k}{2} + \bar{g}_2 \left(p_2 \cdot q_2 - \frac{p_2 \cdot k \; k \cdot q_2}{\mu^2}\right) + \bar{g}_3 \; p_2 \cdot k \left(1 - \frac{k^2}{\mu^2}\right) \right] \qquad (7.26b)$$

$$|\bar{g}_r|^2 = |\bar{g}_2|^2 \mu_2^2 + 2 \, \mathrm{Re} \left\{ \left(\bar{g}_1 + \bar{g}_2 \; k \cdot q_2\right) \quad x \right.$$

$$\left. \left[-\bar{g}_1^{+} \frac{1}{\mu^2} - \bar{g}_2^{+} \frac{k \cdot q_2}{\mu^2} + \bar{g}_3^{+} \left(1 - \frac{k^2}{\mu^2}\right) \right] \right\}$$

$$+ \left| \left[-\bar{g}_1 \frac{1}{\mu^2} - \bar{g}_2 \frac{k \cdot q_2}{\mu^2} + \bar{g}_3 \left(1 - \frac{k^2}{\mu^2}\right) \right] \right|^2 k^2 \qquad (7.26c)$$

and where the angle ϵ_V is given by (7.13). In fact

$$\mu_2 \, p_{2V} \, k_V \sin\epsilon_V = E_B \, p_B \, q_B \sin\theta_B \qquad (7.27a)$$

$$p_{2V} \, k_V \cos\epsilon_V = -\left[\left(s - m_2^2 - \mu_2^2\right) \left(-t + \mu_1^2 - \mu_2^2\right) \right.$$

$$\left. + 2\mu_2^2 \left(-t + m_1^2 - m_2^2\right) \right] / 4\mu_2^2 \qquad (7.27b)$$

Substitution of (7.23) - (7.25) in (7.20) gives:

$$d\sigma\,(a) = \frac{1}{(4\pi)^2} \frac{q_B}{4E_B^2 \, p_B} \frac{1}{(t-\mu^2)^2} \frac{2^j j! j!}{(2j)!} k_V^{2j-4}$$

$$x \left[\bar{M}_{p_2 p_2} \, p_{2V}^2 \, k_V^2 \left\{ \cos\epsilon_V \; j \, P_j^0 (C_V) + \sin\epsilon_V \sin\theta_{3V} \, P_j^1 (C_V) \right\} \right]^2$$

$$+ 2\,\mathrm{Re}\,(\Gamma_{p_2 k})\; p_{2V}\, k_V^3 \left\{ \cos\epsilon_V \; j\, P_j^0 (C_V) + \sin\epsilon_V \sin\phi_{3V}\, P_j^1 (C_V) \right\}$$

$$\times \; j\, P_j^0 (C_V) \quad + \bar{M}_{kk}\, k_V^4 \left\{ j\, P_j^0 (C_V) \right\}^2 + \bar{M} \left\{ \left[j\, P_j^0 (C_V) \right] \frac{2}{+} \left[P_j^1 (C_V) \right]^2 \right\} \Big]$$

$$\times \; d\cos\theta_B \, d\phi_B \; \frac{\Gamma_V (2-3,4)}{\Gamma_V} \times \frac{(2j+1)}{2j^2} \; d C_V \, \frac{1}{2\pi} \, d\phi_{3V} \qquad (7.28)$$

where the various $\bar{M}_{..}$ depend in a complicated way on the coupling constants and their associated form factors. Because of this complexity it is unlikely that measurements of the decay distributions will furnish any information on the couplings involved in the production process.

Integration over ϕ_{3V} gives the decay distribution as a function of $\cos\theta_{3V}$:

$$\frac{(2j+1)}{2j^2} \left[A(s,t) \left\{ j\, P_j^0 (\cos\theta_{3V}) \right\}^2 + B(s,t) \left\{ P_j^1 (\cos\theta_{3V}) \right\}^2 \right] d\cos\theta_{3V} \qquad (7.29)$$

and integration over $\cos\theta_{3V}$ gives the decay distribution as a function of ϕ_{3V}:

$$\frac{1}{2} \left[C(s,t) + D(s,t) \sin^2\phi_{3V} \right] d\phi_{3V} \qquad (7.30)$$

where A, B, C and D depend on the coupling constants and their associated form factors which are in turn arbitrary functions of t.

These distributions are the most general distributions for the decay of a resonance of spin j produced by a peripheral mechanism involving the exchange of a spin one particle.

The spin averaged differential cross section for the quasi two body production process (1.6) independent of the decay mode of the resonance is found by putting $\Gamma_V(2-3,4) = \Gamma_V$ in (7.28) and integrating over the physical region of C_V and ϕ_{3V}. The result obtained is:

$$d\sigma(b) = \frac{1}{(4\pi)^2} \frac{q_B}{4E_B^2 p_B} \frac{1}{(t-\mu^2)^2} \frac{2^j j!j!}{(2j)!} k_V^{2j-4} \times$$

$$\times \left[\bar{M}_{p_2p_2} p_{2V}^2 k_V^2 \left\{ \cos^2\epsilon_V + \sin^2\epsilon_V \frac{(j+1)}{2j} \right\} + \right.$$

$$\left. 2\,\mathrm{Re}\,(\bar{M}_{p_2k})\, p_{2V} k_V^3 \cos\epsilon_V + \bar{M}_{kk} k_V^4 + \bar{M} \frac{(2j+1)}{j} \right] d\cos\theta_B\, d\phi_B \qquad (7.31)$$

This result is again completely independent of the decay mode of the boson resonance and thus applies to the general process (7.7) provided that this process is mediated by the exchange of a pseudovector meson.

8. BOSON RESONANCES DECAYING INTO A VECTOR PARTICLE AND A PSEUDOSCALAR PARTICLE.

A boson resonance of spin j which decays into a vector particle and a pseudoscalar particle may have either normal or abnormal parity, that is parity $(-1)^j$ or $(-1)^{j+1}$ respectively. These cases are considered in turn:

a) Normal parity

In the peripheral production of such a resonance the exchanged particle may be a pseudoscalar, a vector or a pseudovector particle. The differential cross section for the process described by Fig. 1(b) is calculated for each of these three possibilities as follows:

(i) Pseudoscalar particle exchange.

The invariant matrix element is given by:

$$\langle p_2 q_3 q_5 q_6 | T | p_1 q_1 \rangle = \bar{u}(p_2)\, f\, F(t) \gamma_5\, u(p_1)\, \frac{1}{(t-\mu^2)} \quad x$$

$$x\ (-1)^n\ g\ \frac{G(t,v)}{(m_g)^{n-1}}\ (k_\lambda)^n \phi_\rho(q_2,n)\ \frac{1}{(v-\mu_2^2)+i\mu_2\Gamma_V} \quad x$$

$$x\ \frac{h_o H_o(vx)}{(m_{h_o})^n}\ (q_{4\rho})^{n-1} \epsilon_{\rho_1 \mu \nu \sigma}\ q_{4\mu}\, q_{3\nu} \left[g_{\sigma\tau} - \frac{q_{4\sigma} q_{4\tau}}{\mu_4^2} \right] x$$

$$\frac{1}{(x-\mu_4^2)+i\mu_4\Gamma_x}\ i\, I(x)\ q_{5\tau} \qquad (8.1)$$

Averaging over the initial spin states and summing over final spin states gives:

$$\left| \langle p_2 q_3 q_5 q_6 \mid T \mid p_1 q_1 \rangle \right|^2_{AV} = \left| \frac{fF(t)\; g\; G(t,v)\; h_o H_o(v,x)\; i\; I(x)}{(m_g)^{n-1} \qquad (m_{h_o})^n} \right|^2 \quad x$$

$$x \frac{\left[-t + (m_1' + m_2)^2\right]}{(t-\mu^2)^2} \quad \frac{1}{(v-\mu_2^2)^2 + \mu_2^2 \Gamma_V^2} \quad \frac{1}{(x-\mu_4^2)^2 + \mu_4^2 \Gamma_X^2} \quad x$$

$$x \left[(k_\lambda)^n \phi_{\lambda\rho}(q_2, n)\; (q_4)^{n-1}_\rho \epsilon_{\rho_1 \mu\nu\sigma} q_{4\mu}\, q_{3\nu}\, q_{5\sigma} \right]^2 \qquad (8.2)$$

where the notation of (7.3) has been used.

Assuming that the resonances B_3 and V_1 both have narrow widths i.e. $\Gamma_V \ll \mu_2$ and $\Gamma_X \ll \mu_4$ then:

$$\frac{1}{(v-\mu_2^2)^2 + \mu_2^2 \Gamma_V^2} \approx \frac{\pi}{\mu_2 \Gamma_V}\, \delta(v - \mu_2^2) \qquad (8.3a)$$

$$\frac{1}{(x-\mu_4^2)^2 + \mu_4^2 \Gamma_X^2} \approx \frac{\pi}{\mu_4 \Gamma_X}\, \delta(x - \mu_4^2) \qquad (8.3b)$$

With these approximations and the result (A.10) substitution of (8.2) in the expression (3.29) for $d\sigma$ (b) and integration over d v and d x yields the result:

$$d\sigma(b) = \frac{f^2}{4\pi}\,\frac{g^2}{4\pi}\,\left|\frac{F(t)\,G(t)}{(m_g)^{j-1}}\right|^2\,\frac{q_B}{4E_B^2\,p_B}\quad x$$

$$x\,\frac{\left[-t+(m_1'+m_2)^2\right]}{(t-\mu^2)^2}\quad\frac{2^j j!\,j!}{(2j)!}\quad k_V^{2j}\quad d\cos\Theta_B\,d\phi_B$$

$$x\,\frac{\Gamma_V(2-3,4)}{\Gamma_V}\;x\,\frac{(2j+1)}{2j(j+1)}\left[P_j^1(\cos\Theta_{3V})\right]^2 d\cos\Theta_{3V}\;x\,\frac{1}{2\pi}\,d\phi_{3V}$$

$$x\,\frac{\Gamma_X(4-5,6)}{\Gamma_X}\;x\,\frac{1}{\pi}\cos^2\psi_{3V}\,d\psi_{3V}\;x\,\frac{3}{4}\,\frac{\mu_4}{q_{5X}^3\,q_{3V}^3}\quad x$$

$$x\;q_{3V}^2\,q_{5V}^2\;\sin^2\Theta_{35V}\;d\omega_{5V} \qquad (8.4)$$

where (6.8) and (6.12) have been used to separate out the factors $\Gamma_V(2-3,4)/\Gamma_V$ and $\Gamma_X(4-5,6)/\Gamma_X$. These factors are the branching ratios of the decay process (1.4a) relative to all possible decay modes of B_3 and of the decay process (1.4b) relative to all possible decay modes of V_1 respectively.

The first factor in the expression (8.4) is identical with that appearing in (7.5) and is in fact the spin averaged differencial cross section for the production of a spin j,

parity $(-1)^j$ resonance in the process (7.7) mediated by pseudo-scalar particle exchange.

The remaining factors in (8.4) give the distributions of the decay products as functions of the variables $\cos\Theta_{3V}$, ϕ_{3V}, ψ_{3V} and ω_{5V}. Each of these distribution terms is normalised so that integration over the complete physical region appropriate to each variable gives unity. The distribution of the decay products as functions of these variables are all independent of each other and of the production process variables. In particular the distributions are independent of the arbitrary form factors F(t) and G(t). This makes comparison of the model with experimental data relatively easy.

Despite the fact that the factor $\sin^2\Theta_{35V}$ arises in thesquare of the matrix element it is not convenient to measure the distribution with respect to the variable Θ_{35V} because of the complicated nature of the relationship between Θ_{35V} and the variable ω_{5V} which appeared naturally in the phase space factor evaluated in section 3. The relationship is such that:

$$q_{3V}^2\, q_{5V}^2 \sin^2\Theta_{35V} = \left[\mu_4^2\mu_5^2 - \mu_5^2\,(\mu_2^2+\mu_4^2-\mu_3^2)^2 / 4\mu_2^2 - (\mu_4^2+\mu_5^2-\mu_6^2)^2/4\right.$$

$$\left. + (\mu_2^2+\mu_4^2-\mu_3^2)\,(\mu_4^2+\mu_5^2-\mu_6^2)\,\omega_{5V} / 2\mu_2 - \mu_4^2\omega_{5V}^2\right. \qquad (8.5)$$

where of course q^2_{3V} is given by (2.8b) and $q^2_{5V} = \omega^2_{5V} - \mu^2_5$. The distribution of the decay products as a function of the usual Dalitz plot variable ω_{5V} is thus of the above form. By carrying out the integration over ω_{5V} and making use of the fact that the limits of the physical region are defined by $\sin\Theta_{35V} = o$ it is possible to show that the final term in (8.4) reduces to unity as required.

The spin-averaged differential cross section for the production of a spin j, parity $(-1)^j$, resonance and the decay of this state into a pseudoscalar meson and a vector meson is obtained from (8.4) by integrating over $d\psi_{3V}$ and $d\omega_{5V}$ and assuming $\Gamma_X\ (4-5,6) = \Gamma_X$. The result obtained is then:

$$d\sigma(b) = \frac{f^2}{4\pi}\ \frac{g^2}{4\pi}\ \left|\frac{F(t)\ G(t)}{(m_g)^{j-1}}\right|^2\ \frac{q_B}{4E_B^2\ p_B}$$

$$x\ \frac{\left[-t + (m_1' + m_2)^2\right]}{(t-\mu^2)^2}\qquad \frac{2^j j! j!}{(2j)!}\qquad k_V^{2j}\ d\cos\Theta_B\ d\phi_B$$

$$x\ \frac{\Gamma_V(2-3,4)}{\Gamma_V}\ x\ \frac{(2j+1)}{2j(j+1)}\ \left[P_j^1\ (\cos\Theta_{3V})\right]^2\ d\cos\Theta_{3V}\ \frac{1}{2\pi}\ d\phi_{3V} \qquad (8.6)$$

It should be noted that this result is completely independent of the decay mode of the vector particle which might for instance decay into three pseudoscalar particles rather than into two such particles. Thus (8.6) applies to both of the following two stage processes:

$$P + N \rightarrow N + B_3 \tag{8.7a}$$

$$B_3 \rightarrow P + V_1 \tag{8.7b}$$

and

$$P + N \rightarrow N + B_4 \tag{8.8a}$$

$$B_4 \rightarrow P + V_2 \tag{8.8b}$$

provided the boson resonances have normal parity and the production processes are mediated by pseudoscalar particle exchange.

(ii) Vector particle exchange.

The invariant matrix element is given by:

$$\langle p_2 q_3 q_5 q_6 \mid T \mid p_1 q_1 \rangle = \bar{u}(p_2) \left[f_2 F_2(t) \gamma_\mu + \frac{f_3 F_3(t)}{(m_{f_3})} p_{2\mu} + \frac{f_4 F_4(t)}{(m_{f_4})} k_\mu \right] \gamma_f u(p_1) \times (-1) \left[g_{\mu\nu} - \frac{k_\mu k_\nu}{\mu^2} \right]$$

$$\frac{1}{(t-\mu)^2} \quad \frac{g_o G_o(t,v)}{(m_{g_o})^n} \epsilon_{\nu\alpha\beta\lambda_1} \; k_\alpha q_{2\beta} \; (k_\lambda)^{n-1} \; x \; (-1)^n \phi_{\lambda\rho} \; (q_2,n)$$

$$\frac{1}{(v-\mu_2^2) + i\mu_2\Gamma_V} \quad (q_4)^{n-1}_\rho \; \frac{h_o H_o(v,x)}{(m_{h_o})^n} \epsilon_{\rho 1\gamma\delta\sigma} \; q_{4\gamma} \; q_{3\delta} \; x$$

$$x \left[g_{\sigma\tau} - \frac{q_{4\sigma} q_{4\tau}}{\mu_4^2} \right] \quad \frac{1}{(x-\mu_4^2) + i\mu_4\Gamma_X} \quad i \; I(x) \; q_{5\tau} \qquad (8.9)$$

Substituting (8.9) in (3.29), making use of the anti-symmetric nature of the pseudotensors, averaging over initial spin states, summing over final spin states and integrating over d v and d x gives, with the notation of (7.3) and the approximations (8.3), the result:

$$d\sigma(b) = \frac{1}{(4\pi)^4} \quad \frac{q_B}{4E_B^2 \; p_B} \quad \frac{1}{(t-\mu^2)^2} \quad \frac{h_o^2}{4\pi} \quad \frac{1}{2\mu_2^2 \; (m_{h_o})^{2j} \Gamma_V} \quad x$$

$$x \quad \frac{i^2}{4\pi} \quad \frac{1}{2\mu_4\Gamma_X} \quad x \left[M_{p_2 p_2}(t) \; (p_2 r) \; (p_2 r^+) - M(t) \; (r.r^+) \right] \quad x$$

$$x \quad d\cos\theta_B \; d\phi_B \; d\cos\theta_{3V} \; d\phi_{3V} \; d\psi_{3V} \; d\omega_{5V} \qquad (8.10)$$

where

$$r_\mu = \bar{g}_o \epsilon_{\mu\alpha\beta\lambda_1} k_\alpha q_{2\beta} \epsilon_{\rho_1\gamma\delta\tau} q_{4\gamma} q_{3\delta} q_{5\tau} (-1)^{j+1} \times$$

$$\times (k_\lambda)^{j-1} \phi_{\lambda\rho}(q_2, j) (q_{4\rho})^{j-1} \tag{8.11}$$

and $M_{p_2 p_2}(t)$ and $M(t)$ are given by (7.9).

Denoting $\cos\Theta_{3V}$ by C_V and $\sin\Theta_{3V}$ by S_V it follows from (A.11) and the definitions (3.27) that:

$$p_2 \cdot r = -\bar{g}_o \left[\frac{2^j j! j!}{(2j)!} \frac{1}{j^2} (E_B p_B q_B \sin\Theta_B) \mu_2 k_V^{j-1} q_V^{j-1} \times \right.$$

$$\left. q_{3V} q_{5V} \sin\Theta_{35V} \right] \times \left[\cos\psi_{3V} \cos\phi_{3V} \left\{ j(j+1) P_j^o(C_V) - \right. \right.$$

$$\left. \left. \frac{C_V}{S_V} P_j^1(C_V) \right\} - \sin\psi_{3V} \sin\phi_{3V} \left\{ \frac{1}{S_V} P_j^1(C_V) \right\} \right] \tag{8.12}$$

where use has been made of the fact that the angle between p_{2V} and q_{1V}, denoted by ϵ_V as shown in Fig. 2(b), satisfies equation (7.11).

Similarly it may be shown that:

$$r \cdot r^+ = -|\bar{g}_o|^2 \left[\frac{2^j j! j!}{(2j)!} \frac{1}{j^2} (\mu_2 k_V) \mu_2 k_V^{j-1} q_V^{j-1} q_{3V} q_{5V} \sin\Theta_{35V} \right]^2 \times$$

$$\times \left[\cos^2\psi_{3V} \left\{ j(j+1) P_j^o(C_V) - \frac{C_V}{S_V} P_j^1(C_V) \right\}^2 + \right.$$

$$+ \sin^2\psi_{3V} \left\{ \frac{1}{S_V} P_j^1 (C_V) \right\}^2 \Bigg] \qquad (8.13)$$

Substituting (8.12) and (8.13) into the expression (8.10) for $d\sigma(b)$ gives:

$$d\sigma(b) = \frac{1}{(4\pi)^2} |\bar{g}_o|^2 \frac{q_B}{4E_B^2 p_B} \frac{1}{(t-\mu^2)^2} \frac{2^j j! j!}{(2j)!} \times$$

$$\frac{(j+1)}{j} k_V^{2j-2} \Bigg[M_{p_2 p_2}(t) \left(E_B p_B q_B \sin\theta_B \right)^2 \Bigg\{ \cos\psi_{3V} \cos\phi_{3V} \times$$

$$\times \left[j(j+1) P_j^o (C_V) - \frac{C_V}{S_V} P_j^1 (C_V) \right] - \sin\psi_{3V} \sin\phi_{3V} \left[\frac{1}{S_V} P_j^1 (C_V) \right] \Bigg\}^2$$

$$+ M(t) \left(\mu_2 k_V \right)^2 \Bigg\{ \cos^2\psi_{3V} \left[j(j+1) P_j^o (C_V) - \frac{C_V}{S_V} P_j^1 (C_V) \right]^2$$

$$+ \sin^2\psi_{3V} \left[\frac{1}{S_V} P_j^1 (C_V) \right]^2 \Bigg\} \Bigg] d\cos\theta_B \, d\phi_B \times \frac{\Gamma_V (2-3,4)}{\Gamma_V} \times$$

$$\times \frac{(2j+1)}{2j^2 (j+1)^2} d C_V \frac{1}{2\pi} d\phi_{3V} \times \frac{\Gamma_X (4-5,6)}{\Gamma_X} \times \frac{1}{\pi} d\psi_{3V} \times$$

$$\frac{3}{4} \frac{\mu_4}{q_{5X}^3 q_{3V}^3} q_{3V}^2 q_{5V}^2 \sin^2\theta_{35V} \, d\omega_{5V} \qquad (8.14)$$

where use has been made of (6.8) and (6.12) to separate out the branching ratio factors.

To determine the decay distribution as a function of any of the variables $\cos\Theta_{3V}$, ϕ_{3V} or Ψ_{3V} it is necessary to integrate over the other variables. These decay distributions are of the form:

$$\frac{(2j+1)}{2j^2(j+1)^2}\left[\left\{ j(j+1)\, P_j^0(\cos\Theta_{3V}) - \frac{\cos\Theta_{3V}}{\sin\Theta_{3V}} P_j^1(\cos\Theta_{3V})\right\}^2 + \right.$$

$$\left. + \left\{\frac{1}{\sin\Theta_{3V}}\, P_j^1(\cos\Theta_{3V})\right\}^2 \right] d\cos\Theta_{3V} \qquad (8.15)$$

$$\frac{1}{2\pi}\left[A(s,t) + B(s,t)\,\frac{1}{2j(j+1)}\left\{(2j+1) + (j^2-j-1)\,2\cos^2\phi_{3V}\right\}\right] d\phi_{3V} \qquad (8.16)$$

$$\frac{1}{2\pi}\left[\frac{1}{j(j+1)}\left\{(2j+1) + (j^2-j-1)\,2\cos^2\Psi_{3V}\right\}\right] d\Psi_{3V} \qquad (8.17)$$

where A(s,t) and B(s,t) are specific functions of s but arbitrary functions of t. The distribution of the decay products as a function of $\cos\Theta_{3V}$ and as a function of Ψ_{3V} are both independent of the production process variables whereas the distribution as a

function of ϕ_{3V} depends not only on the production process variables but also on the ratio of the coupling constants f_2 and f_3 and their associated form factors.

The distribution of the decay products as a function of ω_{5V} is independent of all the other variables and is given by (8.5) as in the case of pseudoscalar particle exchange.

Integrating over $d\psi_{3V}$ and $d\omega_{5V}$ and assuming $\Gamma_X(4-5,6) = \Gamma_X$ gives the spin averaged differential cross section for the production of a spin j, normal parity resonance and the decay of this state into a pseudoscalar meson and a vector meson. The result obtained is then:

$$d\sigma(b) = \frac{g_o^2}{4\pi} \left| \frac{G_o(t)}{(m_{g_o})^j} \right|^2 \frac{q_B}{4E_B^2 p_B} \frac{1}{(t-\mu^2)^2} \frac{2^j j! j!}{(2j)!} \times$$

$$\times \frac{(j+1)}{j} k_V^{2j-2} \times \left[\left\{ \frac{f_2^2}{4\pi} |F_2(t)|^2 4 + \frac{f_2 f_3}{4\pi} \times \right.\right.$$

$$\times \frac{2\mathrm{Re}\left[F_2(t)\, F_3^+(t)\right]}{(m_{f_3})} 2(m_1' + m_2) + \frac{f_3^2}{4\pi} \left.\left. \frac{|F_3(t)|^2}{(m_{f_3})^2} \left[-t+(m_1'+m_2)^2\right] \right\} \times \right.$$

$$\times \left(E_B p_B q_B \sin\theta_B\right)^2 \left\{ \cos^2\phi_{3V} \left[j(j+1) P_j^o(C_V) - \frac{C_V}{S_V} P_j^1(C_V) \right]^2 + \right.$$

$$+ \sin^2\phi_{3V}\left[\frac{1}{S_V} P_j^1(C_V)\right]^2\Big\} + \Big\{\frac{f^2}{4\pi}\,|F_2(t)|^2\left[-t+(m_1'-m_2)^2\right]x$$

$$(p_2 k_V)^2 \; x\Big\{\Big[j(j+1)\,P_j^0(C_V) - \frac{C_V}{S_V}\,P_j^1(C_V)\Big]^2 +$$

$$+\Big[\frac{1}{S_V}\,P_j^1(C_V)\Big]^2\Big\}\Big]\; d\cos\theta_B\, d\phi_B \quad x$$

$$x\;\frac{\Gamma_V(2-3,4)}{\Gamma_V}\;\frac{(2j+1)}{2j^2(j+1)}\;d\,C_V\;\frac{1}{2\pi}\;d\phi_{3V} \qquad (8.18)$$

As for (8.6) this result is completely independent of the decay mode of the vector meson. Thus (8.18) applies to both (8.7) and (8.8) provided the boson resonance of spin j has normal parity and the production processes are mediated by vector particle exchange.

The spin averaged differential cross section for the general two body production process (7.7) mediated by vector meson exchange is found by putting $\Gamma_V(2-3,4) = \Gamma_V$ in (8.18) and integrating over C_V and ϕ_{3V}. The result is of course given by (7.18)

(iii) Pseudovector particle exchange.

The invariant matrix element is given by:

$$< p_2 q_3 q_5 q_6 \mid T \mid p_1 q_1 > = \bar{u}(p_2) \Big[f_2 F_2(t) \gamma_\mu + \frac{f_3 F_3(t)}{(m_{f_3})} p_{2\mu} +$$

$$\frac{f_4 F_4(t)}{(m_{f_4})} k_\mu \Big] \gamma_f \; u(p_1) \times (-1) \Big[g_{\mu\nu} - \frac{k_\mu k_\nu}{\mu^2} \Big] \frac{1}{(t-\mu^2)} \quad \times$$

$$\Big[\frac{g_1 G_1(t,v)}{(m_{g_1})^{n-2}} g_{\nu\lambda_1} + \frac{g_2 G_2(t,v)}{(m_{g_2})^n} q_{2\nu} k_{\lambda 1} + \frac{g_3 G_3(t,v)}{(m_{g_3})^n} k_\nu k_{\lambda 1} \Big]$$

$$\times (k_\lambda)^{n-1} (-1)^n \phi_{\lambda\rho}(q_2, n) \frac{1}{(v-\mu_2^2) + i\mu_2 \Gamma_V} \frac{h_o H_o(v,x)}{(m_{h_o})^n} (q_4{}_\rho)^{n-1} \quad \times$$

$$\times \epsilon_{\rho\gamma\delta\sigma} q_{4\gamma} q_{3\delta} \Big[g_{\sigma\tau} - \frac{q_{4\sigma} q_{4\tau}}{\mu_4^2} \Big] \frac{1}{(x-\mu_4^2) + i\mu_4 \Gamma_X} \; i\, I(x)\, q_{5\tau}$$

(8.19)

Substituting (8.19) in (3.29), averaging over initial spin states, summing over final spin states and integrating over dv and dx gives, with the notation of (7.3) and the approximations (8.3), the result:

$$d\sigma(b) = \frac{1}{(4\pi)^4} \frac{q_B}{4E_B^2 p_B} \frac{1}{(t-\mu^2)^2} \frac{h_o^2}{4\pi} \frac{1}{2\mu_2^2 (m_{h_o})^{2j} \Gamma_V} \quad \times$$

$$\frac{i^2}{4\pi} \quad \frac{1}{2^4 \Gamma_X} \quad x \left[M_{p_2 p_2}(t)\ (p_2.r)\ (p_2.r^+) + M_{p_2 k}(t)\ (p_2.r)(kr^+) + \right.$$

$$\left. M_{k p_2}(t)\ (k.r)\ (p_2.r^+) + M_{kk}(t)\ (k.r)\ (k.r^+) - M(t)(r.r^+) \right] \quad x$$

$$d\cos\theta_B\ d\phi_B\ d\cos\theta_{3V}\ d\phi_{3V}\ d\psi_{3V}\ d\omega_{5V} \tag{8.20}$$

where

$$r_\mu = \left[g_{\mu\nu} - \frac{k_\mu k_\nu}{\mu^2} \right] \left[\bar{g}_1\ g_{\nu\lambda_1} + \bar{g}_2\ q_{2\nu}\ k_{\lambda_1} + \bar{g}_3\ k_\nu k_{\lambda_1} \right] \quad x$$

$$\epsilon_{\rho_1 \alpha\beta\gamma}\ q_{2\alpha}\ q_{3\beta}\ q_{5\gamma}\ x\ (-1)^{j+1}\ (k_\lambda)^{j-1}\ \phi_{\lambda\rho}(q_2, j)\ (q_4)^{j-1}_\rho \tag{8.21}$$

and the various $M_{..}(t)$ are given by (7.22).

Denoting the variable $\cos\theta_{3V}$ by C_V it follows from (A.10) and (3.27) that, with the notation of (7.26)

$$k.r = -\left[\frac{2^j j! j!}{(2j)!}\ \frac{1}{j^2}\ k_V^{j-2}\ q_V^{j-1}\ \rho_2\ q_{3V}\ q_{5V}\ \sin\theta_{35V} \right] \quad x$$

$$\left[\bar{g}_k\ k_V^2\ \cos\psi_{3V}\ j\ P_j^1(C_V) \right] \tag{8.22}$$

and from (A.10) and (A.11) it follows that:

$$p_2 \cdot r = -\left[\frac{2^j j! j!}{(2j)!} \frac{1}{j^2} k_V^{j-2} q_V^{j-1} \mu_2 q_{3V} q_{5V} \sin\Theta_{35V} \right] \times$$

$$\times \left[\bar{g}_{p_2} k_V^2 \cos\psi_{3V} \, j \, P_j^1(C_V) - \bar{g}_1 p_{2V} k_V \left\{ \cos\epsilon_V \cos\psi_{3V} \, j \, P_j^1(C_V) - \right.\right.$$

$$- \sin\epsilon_V \left[\cos\psi_{3V} \sin\phi_{3V} \left\{ j(j+1) P_j^0(C_V) - \frac{C_V}{S_V} P_j^1(C_V) \right\} + \right.$$

$$\left.\left.\left. + \sin\psi_{3V} \cos\phi_{3V} \left\{ \frac{1}{S_V} P_j^1(C_V) \right\} \right] \right\} \right] \qquad (8.23)$$

and

$$r \cdot r^+ = \left[\frac{2^j j! j!}{(2j)!} \frac{1}{j^2} k_V^{j-2} q_V^{j-1} \mu_2 q_{3V} q_{5V} \sin\Theta_{35V} \right]^2$$

$$\left[|\bar{g}_r|^2 k_V^4 \cos^2\psi_{3V} \left\{ j \, P_j^1(C_V) \right\}^2 - |\bar{g}_1|^2 k_V^2 \left\{ \cos^2\psi_{3V} \times \right.\right.$$

$$\times \left[j \, P_j^1(C_V) \right]^2 + \cos^2\psi_{3V} \left[j(j+1) P_j^0(C_V) - \frac{C_V}{V} P_j^1(C_V) \right]^2$$

$$\left.\left. + \sin^2\psi_{3V} \left[\frac{1}{S_V} P_j^1(C_V) \right]^2 \right\} \right] \qquad (8.24)$$

where the angle ϵ_V satisfies the relationships (7.27).

Substitution of (8.22) - (8.25) in (8.20) gives

$$d\sigma(b) = \frac{1}{(4\pi)^2} \frac{q_B}{4E_B^2 p_B} \frac{1}{(t-\mu^2)^2} \frac{2^j j! j!}{(2j)!} k_V^{2j-4} \quad x$$

$$x \Big[\bar{M}_{p_2 p_2} p_{2V}^2 k_V^2 \Big\{ \cos\epsilon_V \cos\psi_{3V} j P_j^1(C_V) - \sin\epsilon_V \quad x$$

$$\Big[\cos\psi_{3V} \sin\phi_{3V} \Big\{ j(j+1) P_j^0(C_V) - \frac{C_V}{S_V} P_j^1(C_V) \Big\} + \sin\psi_{3V} \cos\phi_{3V}$$

$$\Big\{ \frac{1}{S_V} P_j^1(C_V) \Big\} \Big] \Big\}^2 + 2\mathrm{Re}(\bar{M}_{p_2 k}) p_{2V} k_V^3 \Big\{ \cos\psi_{3V} j P_j^1(C_V) \Big\} x$$

$$\Big\{ \cos\epsilon_V \cos\psi_{3V} j P_j^1(C_V) - \sin\epsilon_V \Big[\cos\psi_{3V} \sin\phi_{3V} \quad x$$

$$\Big\{ j(j+1) P_j^0(C_V) - \frac{C_V}{S_V} P_j^1(C_V) \Big\} + \sin\psi_{3V} \cos\phi_{3V} \Big\{ \frac{1}{S_V} P_j^1(C_V) \Big\} \Big] \Big\}$$

$$+ \bar{M}_{kk} k_V^4 \Big\{ \cos\psi_{3V} j P_j^1(C_V) \Big\}^2 + \bar{M} \Big\{ \cos^2\psi_{3V} \Big[j P_j^1(C_V) \Big]^2$$

$$+ \cos^2\psi_{3V}\left[j(j+1)\, P_j^o(C_V) - \frac{C_V}{S_V}\, P_j^1(C_V)\right]^2 + \sin^2\psi_{3V} \quad x$$

$$\left[\frac{1}{S_V}\, P_j^1(C_V)\right]^2\Bigg\}\Bigg] \, d\cos\theta_B \, d\phi_B \; x \; \frac{\Gamma_V(2-3,4)}{\Gamma_V} \; \frac{(2j+1)}{2j^3(j+1)} \quad x$$

$$d C_V \; \frac{1}{2\pi} \, d\phi_{3V} \; \frac{\Gamma_X(4-5,6)}{\Gamma_X} \; x \frac{1}{\pi} \, d\psi_{3V} \; x \frac{3}{4} \; \frac{\mu_4}{q_5^3 \, X \, q_{3V}^3} \quad x$$

$$q_{3V}^2 \, q_{5V}^2 \sin^2\theta_{35V} \; d\omega_{5V} \tag{8.25}$$

where the various $\bar{M}_{..}$ are the functions appearing in (7.28) which depend in a complicated way on the coupling constants and their associated form factors.

The decay distribution as a function of any one of the variables $\cos\theta_{3V}$, ϕ_{3V} or ψ_{3V} is obtained by integration over the other two variables. The results are as follows:

$$\frac{(2j+1)}{2j^3(j+1)}\Bigg[A(s,t)\left\{ j\, P_j^1(\cos\theta_{3V})\right\}^2 + B(s,t)\Bigg\{\Big[j(j+1)\, P_j^o(\cos\theta_{3V}) -$$

$$- \frac{\cos\theta_{3V}}{\sin\theta_{3V}} P_j^1(\cos\theta_{3V})\Big]^2 + \left[\frac{1}{\sin\theta_{3V}} P_j^1(\cos\theta_{3V})\right]^2\Bigg\}\Bigg] d\cos\theta_{3V} \tag{8.26}$$

$$\frac{1}{2\pi}\left[C(s,t) + D(s,t)\sin^2\phi_{3V}\right] d\phi_{3V} \qquad (8.27)$$

$$\frac{1}{\pi}\left[E(s,t) + F(s,t)\sin^2\psi_{3V}\right] d\psi_{3V} \qquad (8.28)$$

where A, B, C, D, E and F depend on the coupling constants and their associated form factors which are of course arbitrary functions of t.

The distribution of the decay products as a function of ω_{5V} is independent of all other variables and is once again given by (8.5).

The spin averaged differential cross section for the production of a spin j, normal parity resonance and the decay of this state into a pseudoscalar meson and a vector meson is obtained from (8.25) by integrating over $d\psi_{3V}$ and $d\omega_{5V}$ and putting $\Gamma_X(4-5,6) = \Gamma_X$. The result obtained is then:

$$d\sigma(b) = \frac{1}{(4\pi)^2}\,\frac{q_B}{4E_B^2\,p_B}\,\frac{1}{(t-\mu^2)^2}\,\frac{2^j j!\,j!}{(2j)!}\,k_V^{2j-4} \qquad x$$

$$x\Big[\,\bar{M}_{p_2p_2}\,p_{2V}^2 k_V^2\Big\{\Big[\cos\epsilon_V\; j\; P_j^1(C_V) - \sin\epsilon_V\,\sin\phi_{3V} \qquad x$$

$$\Big\{j(j+1)\,P_j^o(C_V) - \frac{C_V}{S_V}P_j^1(C_V)\Big\}\Big]^2 + \sin^2\epsilon_V\cos^2\phi_{3V} \qquad x$$

$$\left[\frac{1}{S_V} P_j^1 (C_V)\right]^2 \Big\} + 2 \text{ Re } (\bar{M}_{p_2 k}) \, p_{2V} k_V^3 \Big\{ j \, P_j^1 (C_V) \Big\} \quad x$$

$$\Big\{ \cos\epsilon_V \, j \, P_j^1 (C_V) - \sin\epsilon_V \, \sin\phi_{3V} \left[j(j+1) \, P_j^o (C_V) - \frac{C_V}{S_V} P_j^1 (C_V) \right] \Big\}$$

$$+ \bar{M}_{kk} \, k_V^4 \Big\{ j \, P_j^1 (C_V) \Big\}^2 + \bar{M} \Big\{ \Big[j \, P_j^1 (C_V) \Big]^2$$

$$+ \left[j(j+1) \, P_j^o (C_V) - \frac{C_V}{S_V} P_j^1 (C_V) \right]^2 + \left[\frac{1}{S_V} P_j^1 (C_V) \right]^2 \Big\} \Big] d \cos\Theta_B \, d\phi_B \, x$$

$$x \frac{\Gamma_V(2- 3,4)}{\Gamma_V} \quad x \quad \frac{(2j+1)}{2j^3(j+1)} \quad d C_V \quad x \quad \frac{1}{2\pi} \, d\phi_{3V} \qquad (8.29)$$

As for (8.6) and (8.18) this result is once again independent of the decay mode of the vector meson and thus applies to both (8.7) and (8.8) provided the boson resonance of spin j has normal parity and the production processes are mediated by pseudovector particle exchange.

The spin averaged differential cross section for the general two body production process (7.7) mediated by pseudovector meson exchange is found by putting $\Gamma_V(2- 3,4) = \Gamma_V$ in (8.29) and integrating over C_V and ϕ_{3V}. The result is of course given by (7.31).

b) Abnormal parity

In the peripheral production of such a resonance the exchanged particle may be a scalar, a vector or a pseudovector particle. The differential cross section for the process described by Fig. 1(b) is calculated for each of these three possibilities as follows:

(i) Scalar particle exchange.

The invariant matrix element is given by:

$$\langle p_2 q_3 q_5 q_6 \mid T \mid p_1 q_1 \rangle = \bar{u}(p_2)\, f\, F(t) \gamma_{\delta f}\, u(p_1)\, \frac{1}{(t-\mu^2)} \quad x$$

$$x\,(-1)\,\frac{g\,G(t,v)}{(m_g)^{n-1}}\,(k_\lambda)^n\,\phi_{\lambda\rho}(q_2,n)\,(q_{4\rho})^{n-1}\,\frac{1}{\left[(v-\mu_2^2)+i\mu_2\Gamma_v\right]}$$

$$x\left[\frac{h_1 H_1(v,x)}{(m_{h_1})^{n-2}}\,g_{\sigma\rho 1}+\frac{h_2 H_2(v,x)}{(m_{h_2})^{n}}\,q_{2\sigma}q_{4\rho_1}+\frac{h_3 H_3(v,x)}{(m_{h3})^{n}}\,q_{4\sigma}q_{4\rho 1}\right.$$

$$x\,(-1)\left[g_{\sigma\tau}-\frac{q_{4\sigma}q_{4\tau}}{\mu_4^2}\right]\frac{1}{\left[(x-\mu_4^2)+i\mu_4\Gamma_x\right]}\,i\,I(x)\,q_{5\tau} \qquad (8.30)$$

Substituting (8.30) in the expression (3.29) for $d\sigma(b)$, averaging over initial spin states and summing over final spin states and integrating over dv and dx gives, with the notation of

(7.3) and the approximations (8.3), the result:

$$d\sigma(b) = \frac{1}{(4\pi)^6} \frac{q_B}{4E_B^2 p_B} \left| \frac{f\, F(t)\, g\, G(t)\, i}{(m_g)^{j-1}} \right|^2 \frac{\left[-t + (m_1' + m_2)^2\right]}{(t-\mu^2)^2}$$

$$x \frac{1}{2\gamma_2^2 \Gamma_V} \frac{1}{2\gamma_4 \Gamma_X} \left[(-1)^{j+1} (k_\lambda)^j \phi_{\lambda\rho}(q_2, j) (q_{4\rho})^{j-1} \right.$$

$$x \left\{ \frac{h_1}{(m_{h_1})^{j-2}} \left(q_5 p_1 - \frac{q_4 \cdot q_5}{\mu_4^2} q_4 p_1 \right) + \frac{h_2}{(m_{h_2})^j} \right. \quad x$$

$$\left. \left. x \left(q_2 q_5 - \frac{q_2 \cdot q_4 \; q_4 \cdot q_5}{\mu_4^2} \right) q_4 p_1 \right\} \right]^2 \quad d\cos\theta_B \, d\phi_B \, d\cos\theta_{3V} \quad x$$

$$x \, d\phi_{3V} \, d\psi_{3V} \, d\omega_{5V} \qquad (8.31)$$

Using the relationships (3.28), (A.8) and (A.10) the above result may be written as:

$$d\sigma(b) = \frac{f^2}{4\pi} \frac{g^2}{4\pi} \left| \frac{F(t)\, G(t)}{(m_g)^{j-1}} \right|^2 \frac{q_B}{4E_B^2 p_B}$$

$$x \frac{\left[-t + (m_1' + m_2)^2\right]}{(t-\mu^2)^2} \frac{2^j\, j!\, j!}{(2j)!} \qquad k_V^{2j} \; d\cos\Theta_B \; d\phi_B$$

$$x \frac{\Gamma_V(2-3,4)}{\Gamma_V} \left[(\bar{h}_1)^2 (j+1) + (\bar{h}_2)\, j \right]^{-1} x \frac{\Gamma_X(4-5,6)}{\Gamma_X} \qquad x$$

$$x \left[(\bar{h}_1) \; Q_1 (\omega_{5V}) \sin\psi_{3V} P_j^1 (\cos\Theta_{3V}) + (\bar{h}_2) \, Q_2 (\omega_{5V}) \, j \, P_j^o(\cos\Theta_{3V}) \right]^2$$

$$x \frac{(2j+1)}{4\pi^2 j} \; d\cos\Theta_{3V} \; d\phi_{3V} \; d\psi_{3V} \; \frac{3}{4} \; \frac{\mu_4}{q_{3V}^3 q_{5X}^3} \; d\omega_{5V} \qquad (8.32)$$

where

$$\bar{h}_1 = \frac{h_1}{(m_{h_1})^{j-2}} \qquad (8.33a)$$

$$\bar{h}_2 = \frac{h_1}{(m_{h_1})^{j-2}} \frac{\omega_{4V}}{\mu_4} + \frac{h_2}{(m_{h_2})^j} \frac{\mu_2 q_V^2}{\mu_4} \qquad (8.33b)$$

and

$$Q_1(\omega_{5V}) = q_{3V} \, q_{5V} \sin\Theta_{35V} \qquad (8.34a)$$

$$Q_2(\omega_{5V}) = (\omega_{5X}\omega_{4V} - \mu_4 \omega_{5V}) \qquad (8.34b)$$

and where (6.11) and (6.12) have been used to separate out the branching ratio factors.

The first factor in the expression (8.32) is identical with that appearing in (7.5) and is in fact the spin averaged differential cross section for the production of a spin j, abnormal parity resonance in the process (7.7) mediated by scalar meson exchange.

The remaining factors in (8.32) give the distributions of the decay products as functions of the variables $\cos\theta_{3V}$, ϕ_{3V}, ψ_{3V}, and ω_{5V}, normalised so that integration over all these variables gives unity. The decay distributions as a functions of a single variable are given by:

$$\left[(\bar{h}_1)^2 (j+1) + (\bar{h}_2)^2 j \right]^{-1} \frac{(2j+1)}{2j} \left[(\bar{h}_1)^2 \left\{ P_j^1 (\cos\theta_{3V}) \right\}^2 + \right.$$

$$\left. + (\bar{h}_2)^2 \left\{ j P^o (\cos\theta_{3V}) \right\}^2 \right] d\cos\theta_{3V} \qquad (8.35)$$

$$\frac{1}{2\pi} d\phi_{3V} \qquad (8.36)$$

$$\left[(h_1)^2 (j+1) + (h_2)^2 j \right]^{-1} \frac{1}{\pi} \left[(\bar{h}_1)^2 (j+1) \sin^2\psi_{3V} + \right.$$

$$\left. + (\bar{h}_2)^2 \frac{1}{2} j \right] d\psi_{3V} \qquad (8.37)$$

$$\left[(h_1)^2\,(j+1) + (h_2)^2\,j\right]^{-1} \left[(\tilde{h}_1)^2\,(j+1)\,(q_{3V}\,q_{5V}\sin\theta_{35V})^2 + \right.$$

$$\left. + \;(\tilde{h}_2)^2\,2j\,(\omega_{5X}\omega_{4V} - \nu_4\omega_{5V})^2\right] \frac{3}{4}\,\frac{\nu_4}{q_{3V}^3\,q_{5X}^3}\,d\omega_{5V} \qquad (8.38)$$

Apart from the distribution as a function of ϕ_{3V}, which is isotropic in accordance with the Trieman-Yang test, these distributions depend upon the ratio of h_1 and h_2. This limits to a certain extent the use of these distributions as a test of the spin and parity of the resonance. However the distributions of the decay products as functions of the above variables are all independent of the production process variables and of the arbitrary form factors. This makes a test of the model comparatively easy.

Integrating over $d\psi_{3V}$ and $d\omega_{5V}$ and putting $\Gamma_X(4-5,6) = \Gamma_X$ gives the spin averaged differential cross section for the production of a spin j, abnormal parity resonance and the decay of this state into a pseudoscalar particle and a vector particle. The result is:

$$d\sigma(b) = \frac{f^2}{4\pi}\,\frac{g^2}{4\pi}\,\left|\frac{F(t)\,G(t)}{(m_g)^{j-1}}\right|^2\,\frac{q_B}{4E_B^2\,p_B}$$

$$\times \frac{\left[-t + (m_1' + m_2)^2\right]}{(t-\mu^2)^2} \quad \frac{2^j\, j!\, j!}{(2j)!} \quad k_V^{2j} \quad d\cos\theta_B \quad d\phi_B$$

$$\times \frac{\Gamma_V(2-3,4)}{\Gamma_V} \quad \frac{1}{\left[(\bar{h}_1)^2 (j+1) + (\bar{h}_2)^2 j\right]}$$

$$\times \left[(\bar{h}_1)^2 \left\{ P_j^1(\cos\theta_{3V}) \right\}^2 + (\bar{h}_2)^2 \left\{ j\, P_j^0(\cos\theta_{3V}) \right\}^2 \right]$$

$$\times \frac{(2j+1)}{2j} \quad d\cos\theta_{3V} \quad \frac{1}{2\pi} \quad d\phi_{3V} \qquad (8.39)$$

Once again this result is independent of the decay mode of the vector meson and thus applies to both (8.7) and (8.8) provided that the boson resonance of spin j has abnormal parity and the production processes are mediated by scalar meson exchange.

The spin averaged differential cross section for the general two body production process (7.7) mediated by scalar meson exchange is found by putting $\Gamma_V(2-3,4) = \Gamma_V$ in (8.39) and integrating over $\cos\theta_{3V}$ and ϕ_{3V}. The result is of course given by (7.6).

(ii) Vector particle exchange.

The invariant matrix element is given by:

$$\langle p_2 q_3 q_5 q_6 \mid T \mid p_1 q_1 \rangle = \bar{u}(p_2)\left[f_2 F_2(t)\gamma_\rho + \frac{f_3 F_3(t)}{(m_{f_3})} p_{2\rho} + \right.$$

$$\left. \frac{f_4 F_4(t)}{(m_{f_4})} k_\mu \right] \gamma_f \; u(p_1) \times (-1)\left[g_{\mu\nu} - \frac{k_\mu k_\nu}{\mu^2} \right] \frac{1}{(t-\mu^2)}$$

$$\times \left[\frac{g_1 G_1(t,v)}{(m_{g_1})^{n-2}} g_{\nu\lambda_1} + \frac{g_2 G_2(t,v)}{(m_{g_2})^{n}} q_{2\nu} k_{\lambda_1} + \frac{g_3 G_3(t,v)}{(m_{g_3})^{n}} k_\nu k_{\lambda_1} \right.$$

$$\times (k_\lambda)^{n-1} (-1)^n \phi_{\lambda\rho}(q_2,n) \frac{1}{\left[(v-\mu_2^2) + i\mu_2\Gamma_v\right]} (q_4)_\rho^{n-1}$$

$$\times \left[\frac{h_1 H_1(v,x)}{(m_{h_1})^{n-2}} g_{\sigma\rho_1} + \frac{h_2 H_2(v,x)}{(m_{h_2})^{n}} q_{2\sigma} q_{4\rho_1} + \right.$$

$$\left. \frac{h_3 H_3(v,x)}{(m_{h_3})^{n}} q_{4\sigma} q_{4\rho_1} \right] \times (-1)\left[g_{\sigma\tau} - \frac{q_{4\sigma} q_{4\tau}}{\mu_4^2} \right] \times$$

$$\times \frac{1}{(x-\mu_4^2) + i\mu_4\Gamma_x} \; i\, I(x)\, q_{5\tau} \qquad (8.40)$$

Substituting (8.40) in the expression (3.29) for $d\sigma(b)$, averaging over initial spin states, summing over final spin states

and integrating over dv and dx gives, with the notation of (7.3) and the approximations (8.3) the result:

$$d\sigma(b) = \frac{1}{(4\pi)^5} \frac{q_B}{4E_B^2 p_B} \frac{1}{(t-\mu^2)^2} \frac{1}{2\mu_2^2 \Gamma_V} \frac{i^2}{4\pi} \frac{1}{2\mu_4 \Gamma_X} \quad x$$

$$x \left[M_{p_2p_2}(t)\ (p_2\cdot r)\ (p_2\cdot r^+) + M_{p_2k_2}(t)\ (p_2\cdot r)\ (k\cdot r^+) + M_{kp_2}(t)(k\cdot r)(p_2 r^+) \right.$$

$$\left. + M_{kk}(t)\ (k\cdot r)\ (k\cdot r^+) - M\ (t)\ (r\cdot r^+) \right] d\cos\theta_B\ d\phi_B\ d\cos\theta_{3V} \quad x$$

$$d\phi_{3V}\ d\psi_{3V}\ d\omega_{5V} \tag{8.41}$$

where

$$r_\mu = \left[g_{\mu\nu} - \frac{k_\mu k_\nu}{\mu^2} \right] \left[\bar{g}_1\ g_{\nu\lambda_1} + \bar{g}_2\ q_{2\nu} k_{\lambda_1} + \bar{g}_3\ k_\nu k_{\lambda_1} \right]$$

$$x\ (-1)^j\ (k_\lambda)^{j-1} \phi_{\lambda\rho}\ (q_2\ j\ )\ (q_4\rho)^{j-1} \left[\frac{h_1}{(m_{h_1})^{j-2}} \left(q_{5\rho 1} - \frac{q_4\cdot q_5}{\mu_4^2} q_{4\rho 1} \right) \right.$$

$$\left. + \frac{h_2}{(m_{h_2})^j} \left(q_2\cdot q_5 - q_2\cdot \frac{q_4\ q_4\cdot q_5}{\mu_4^2} \right) q_{4\rho 1} \right] \tag{8.42}$$

and the various M(t) are given by (7.22).

Denoting $\cos\Theta_{3V}$ by C_V and $\sin\Theta_{3V}$ by S_V and with the notation of (7.26), (8.33) and (8.34) it follows from (A.8) and (A.10) that:

$$k.r = -\left[\frac{2^j j!j!}{(2j)!} \quad \frac{1}{j} \; k_V^{j-2} \; q_V^{j-2}\right] \bar{g}_k \; k_V^2 \qquad x$$

$$x \left[(\bar{h}_1) \; Q_1(\omega_{5V}) \sin\psi_{3V} \; P_j^1 \; (C_V) + (\bar{h}_2) \; Q_2 \; (\omega_{5V}) \; j \; P_j^o \; (C_V)\right] \quad (8.43)$$

and from (A.8) - (A.11) it may be shown that:

$$p_2.r = -\left[\frac{2^j j!j!}{(2j)!} \; \frac{1}{j} \; k_V^{j-2} \; q_V^{j-2}\right]$$

$$\left\{\bar{g}_{p_2} \; k_V^2 \left[\bar{h}_1 \; Q_1(\omega_{5V}) \sin\psi_{3V} \; P_j^1 \; (C_V) + \bar{h}_2 \; Q_2(\omega_{5V}) \; j \; P_j^o \; (C_V)\right]\right.$$

$$- \bar{g}_1 \; p_{2V} \; k_V \left[h_1 Q_1(\omega_{5V}) \left\{ \cos\epsilon_V \sin\psi_{3V} \; P_j^1 \; (C_V) + \sin\epsilon_V \; \frac{1}{j}\right.\right.$$

$$\left[\cos\psi_{3V} \cos\phi_{3V} \frac{1}{S_V} \; P_j^1(C_V) - \sin\psi_{3V} \sin\phi_{3V} \left\{ j(j+1) \; P_j^o \; (C_V) - \right.\right.$$

$$\frac{C_V}{S_V} P_j^1 (C_V)\Big\}\Big]\Big\} + \bar{h}_2 Q_2(\omega_{5V}) \Big\{ \cos\epsilon_V \; j \; P_j^o (C_V) + \sin\epsilon_V \sin\phi_{3V} P_j^1 (C_V)\Big\}\Big]\Big\} \tag{8.44}$$

and

$$r.r^+ = -\left[\frac{2^j j! j!}{(2j)!} \frac{1}{j} k_V^{j-2} q_V^{j-2}\right]^2 \Big\{ |\bar{g}_r|^2 k_V^4 \times \Big[\bar{h}_1 Q_1(\omega_{5V}) \sin\psi_{3V} P_j^1(C_V) + \bar{h}_2 Q_2(\omega_{5V}) \; j \; P_j^o (C_V)\Big]^2$$

$$- |\bar{g}_1|^2 k_V^2 \Big[\Big\{\bar{h}_1 Q_1(\omega_{5V})\Big\}^2 \Big\{ \frac{1}{j^2}\Big[\frac{1}{S_V} P_j^1 (C_V)\Big]^2 + \sin^2\psi_{3V} \frac{(j^2-1)}{j^2}\Big[\Big\{ P_j^1 (C_V)\Big\}^2 - \frac{j}{(j-1)} P_j^o (C_V) P_j^2 (C_V)\Big]\Big\}$$

$$- 2\Big\{ \bar{h}_1 Q_1(\omega_{5V})\Big\}\Big\{ \bar{h}_2 Q_2(\omega_{5V})\Big\} \sin\psi_{3V} \frac{1}{j} P_j^1(C_V) \Big\{ j \; P_j^o(C_V) - \frac{C_V}{S_V} P_j^1(C_V)\Big\} + \Big\{\bar{h}_2 Q_2(\omega_{5V})\Big\}^2 \Big\{ \Big[j \; P_j^o(C_V)\Big]^2 + \Big[P_j^1 (C_V)\Big]^2\Big\}\Big]\Big\} \tag{8.45}$$

where the angle ϵ_V satisfies the relationships (7.27).

Substitution of (8.43) - (8.45) into (8.41) gives:

$$d\sigma(b) = \frac{1}{(4\pi)^2} \frac{q_B}{4E_B^2 \beta_B} \frac{1}{(t-\mu^2)^2} \frac{2^j j! j!}{(2j)!} k_V^{2j-4} \quad x$$

$$x \Big[\bar{M}_{p_2 p_2} p_{2V}^2 k_V^2 \Big\{ \bar{h}_1 Q_1(\omega_{5V}) \Big[\cos\epsilon_V \sin\psi_{3V} P_j^1(C_V) +$$

$$\sin\epsilon_V \frac{1}{j} \Big\{ \cos\psi_{3V} \cos\phi_{3V} \frac{1}{S_V} P_j^1(C_V) - \sin\psi_{3V} \sin\phi_{3V}$$

$$x \Big[j(j+1) P_j^0(C_V) - \frac{C_V}{S_V} P_j^1(C_V) \Big] \Big\} \Big] + \bar{h}_2 Q_2(\omega_{5V}) \Big[\cos\epsilon_V j P_j^0(C_V) +$$

$$\sin\epsilon_V \sin\phi_{3V} P_j^1(C_V) \Big] \Big\}^2 + 2 \mathrm{Re}(\bar{M}_{p_2 k}) p_{2V} k_V^3 \quad x$$

$$x \Big\{ \bar{h}_1 Q_1(\omega_{5V}) \sin\psi_{3V} P_j^1(C_V) + \bar{h}_2 Q_2(\omega_{5V}) j P_j^0(C_V) \Big\} \quad x$$

$$x \Big\{ \bar{h}_1 Q_1(\omega_{5V}) \Big[\cos\epsilon_V \sin\psi_{3V} P_j^1(C_V) + \sin\epsilon_V \frac{1}{j} \quad x$$

$$\left\{\cos\psi_{3V}\ \cos\phi_{3V}\ \frac{1}{S_V}\ P_j^1(C_V) - \sin\psi_{3V}\sin\phi_{3V}\left[\ j(j+1)\ P_j^0(C_V) - \frac{C_V}{S_V}\ P_j^1(C_V)\right]\right\}\Bigg] + \bar{h}_2\ Q_2(\omega_{5V})\left[\ \cos\epsilon_V\ j\ P_j^0(C_V) + \sin\epsilon_V\sin\phi_{3V}\ x\right.$$

$$\left.x\ P_j^1(C_V)\right]\Bigg\} \quad + \bar{M}_{kk}\ k_V^4\left\{\ \bar{h}_1\ Q_1(\omega_{5V})\sin\psi_{3V}\ P_j^1(C_V)\ +\right.$$

$$\left.\bar{h}_2\ Q_2(\omega_{5V})\ j\ P_j^0(C_V)\right\}^2 \quad + \bar{M}\left\{\left[\bar{h}_1\ Q_1(\omega_{5V})\right]^2\left[\sin^2\psi_{3V}\ x\right.\right.$$

$$\left.x\left\{P_j^1(C_V)\right\}^2 + \frac{1}{j^2}\left\{\cos^2\psi_{3V}\left[\frac{1}{S_V}\ P_j^1(C_V)\right]^2 + \sin^2\psi_{3V}\ x\right.\right.$$

$$\left.\left.\left[j(j+1)\ P_j^0(C_V) - \frac{C_V}{S_V}\ P_j^1(C_V)\right]^2\right\}\right] - 2\left[\bar{h}_1 Q_1(\omega_{5V})\cdot\bar{h}_2 Q_2(\omega_{5V})\right]\ x$$

$$\sin\psi_{3V}\ \frac{1}{j}\ P_j^1(C_V)\left[\ j\ P_j^0(C_V) - \frac{C_V}{S_V}\ P_j^1(C_V)\right] + \left[\bar{h}_2\ Q_2(\omega_{5V})\right]^2$$

$$\left.x\left[\left\{j\ P_j^0(C_V)\right\}^2 + \left\{P^1{}_j(C_V)\right\}^2\right]\right\}\Bigg]\ d\cos\Theta_B\ d\phi_B$$

$$\frac{\Gamma_V\,(2-3,4)}{\Gamma_V}\quad \frac{1}{\left[(\bar{h}_1)^2\,(j+1)+(\bar{h}_2)^2\,j\right]}\quad \frac{(2j+1)}{2j}\quad d\,C_V\quad \frac{1}{2\pi}\quad d\phi_{3V}$$

$$\frac{\Gamma_X\,(4-5,6)}{\Gamma_X}\quad \frac{1}{\pi}\quad d\psi_{3V}\;x\;\frac{3}{4}\quad \frac{\mu_4^3}{q_{3V}^3\,q_{5X}^3}\quad d\omega_{5V} \qquad (8.46)$$

where the various $\bar{M}_{..}$ are the functions appearing in (7.28) which depend in a complicated way on the coupling constants and their associated form factors.

The decay distribution as a function of any one of the variables $\cos\theta_{3V}$, ϕ_{3V}, ψ_{3V} or ω_{5V} is obtained by integration over the other three variables. The results are as follows:

$$\left[(\bar{h}_1)^2\,(j+1)+(\bar{h}_2)^2\,j\right]^{-1}\quad \frac{(2j+1)}{2j}\left[A(s,t)\left\{(\bar{h}_1)^2\quad x\right.\right.$$

$$\left[P_j^1\,(\cos\theta_{3V})\right]^2+(\bar{h}_2)^2\left[j\,P_j^0\,(\cos\theta_{3V})\right]^2\Big\}+B(s,t)\quad x$$

$$\left\{(\bar{h}_1)^2\;\frac{1}{j^2}\left[\left\{\frac{1}{S_V}\;P_j^1\,(C_V)\right\}^2+\left\{j(j+1)\,P_j^0\,(C_V)-\frac{C_V}{S_V}\;P_j^1\,(C_V)\right\}^2\right]\right.$$

$$\left.\left.+(\bar{h}_2)^2\left[P_j^1\,(\cos\theta_{3V})\right]^2\right\}\right]\,d\cos\theta_{3V} \qquad (8.47)$$

$$\frac{1}{2\pi}\left[C(s,t) + D(s,t)\sin^2\phi_{3V}\right] d\phi_{3V} \qquad (8.48)$$

$$\frac{1}{\pi}\left[E(s,t) + F(s,t)\sin^2\psi_{3V}\right] d\psi_{3V} \qquad (8.49)$$

$$\left[(\bar{h}_1)^2(j+1) + (\bar{h}_2)^2 j\right]^{-1}\left[(\bar{h}1)^2(j+1)(q_{3} \; q_{5V}\sin\theta_{35V})^2 + \right.$$

$$\left. + (\bar{h}_2)^2 2j(\omega_{5X}\omega_{4V} - \mu_4\omega_{5V})^2 \; \frac{3}{4} q \frac{4}{3 \; q_3} d\omega_{5V} \right. \qquad (8.50)$$

Where A, B, C, D, E, and F depend in a complicated way upon the coupling constants and their associated form factors which are themselves arbitrary functions of t.

The distribution of the decay products as a function of ω_{5V} depends only on the ratio of $\bar{h}_1$ and $\bar{h}_2$ and is once again given by (8.38).

The spin averaged differential cross section for the production of a spin j, abnormal parity resonance and the decay of this state into a pseudoscalar meson and a vector meson is obtained from (8.46) by integrating over $d\psi_{3V}$ and $d\omega_{5V}$ and putting $\Gamma_X(4\to5,6) = \Gamma_X$. The result obtained is

$$d\sigma(b) = \frac{1}{(4\pi)^2} \; \frac{q_B}{4E_B^2 p_B} \; \frac{1}{(t-\mu^2)^2}\left[\frac{2^j j!\, j!}{(2j)!}\right] k_V^{2j} \; -4 \; x$$

$$\left[M_{p_2 p_2} p_{2V}^2 k_V^2 \left\{(\bar{h}_1)^2\left[\left\{\cos\epsilon_v \, P^1{}_j(C_V) - \sin\epsilon_v \sin\phi_{3V} \right. \right. \right. \right. \; x$$

$$\times \frac{1}{j}\left[j(j+1)\, P_j^0(C_V) - \frac{C_V}{S_V}\, P_j^1(C_V)\right]\Big\}^2$$

$$+\Big\{ \sin\epsilon_V \cos\phi_{3V}\, \frac{1}{j}\, \frac{1}{S_V}\, P_j^1(C_V)\Big\}^2\Big] + (\bar{h}_2)^2\Big[\cos\epsilon_V jP_j^0(C_V)$$

$$+ \sin\epsilon_V \sin\phi_{3V}\, P_j^1(C_V)\Big]^2\Big\} + 2\,\mathrm{Re}\,(\bar{M}_{p_2k})p_{2V}k_V^3 \quad \times$$

$$\Big\{(\bar{h}_1)^2\, P_j^1(C_V)\Big[\cos\epsilon_V\, P_j^1(C_V) - \sin\epsilon_V \sin\phi_{3V} \quad \times$$

$$\frac{1}{j}\Big\{ j(j+1)\, P_j^0(C_V) - \frac{C_V}{S_V}\, P_j^1(C_V)\Big\}\Big] + (\bar{h}_2)^2 jP_j^0(C_V) \quad \times$$

$$\times\Big[\cos\epsilon_V\, jP_j^0(C_V) + \sin\epsilon_V \sin\phi_{3V}\, P_j^1(C_V)\Big]\Big\} +$$

$$+ \bar{M}_{kk}\, k^4{}_V \Big\{ (\bar{h}_1)^2 \left[P_j^1(C_V)\right]^2 + (\bar{h}_2)^2 \left[jP_j^0(C_V)\right]^2\Big\} +$$

$$+ \bar{M}\Big\{ (\bar{h}_1)^2\Big[\Big\{P_j^1(C_V)\Big\}^2 + \frac{1}{j^2}\Big\{\Big[j(j+1)\, P_j^0(C_V) -$$

$$- \frac{C_V}{S_V}\, P_j^1(C_V)\Big]^2 + \Big[\frac{1}{S_V}\, P_j^1(C_V)\Big]^2\Big\} + (\bar{h}_2)^2\Big[\Big\{jP_j^0(C_V)\Big\}^2 +$$

$$+ \Big\{P_j^1(C_V)\Big\}^2\Big]\Big\}\Big]\, d\cos\Theta_B\, d\phi_B \times \frac{\Gamma_V(2\text{-}3,4)}{\Gamma_V} \quad \times$$

$$\times\Big[(\bar{h}_1)^2(j+1) + (\bar{h}_2)^2 j\Big]^{-1} \frac{(2j+1)}{2j}\, d\, C_V\, \frac{1}{2\pi}\, d\phi_{3V} \qquad (8.51)$$

As for (8.39) this result is independent of the decay mode of the vector meson and thus applies to both (8.7) and (8.8) provided the boson resonance of spin j has abnormal parity and the production processes are mediated by vector particle exchange.

The spin averaged differential cross section for the general two body production process (7.7) mediated by vector particle exchange is found by putting $\Gamma_V(2 \text{ --}3,4.) = \Gamma_V$ in (8.51) and integrating over C_V and ϕ_{3V}. The result is of course given by (7.31)

(iii) Pseudovector particle exchange.

The invariant matrix element is given by:

$$< p_2 q_3 q_5 q_6 \mid T \mid p_1 q_1 > = \bar{u}(p_2) \Big[f_2 F_2(t) \gamma_\mu +$$

$$+ \frac{f_3 F_3(t)}{(m_{f_3})} p_{2\mu} + \frac{f_4 F_4(t)}{(m_{f_4})} k_\mu \Big] \gamma_f \, u(p_1) \, (-1) \Big[g_{\mu\nu} - \frac{k_\mu k_\nu}{\mu^2} \Big]$$

$$\frac{1}{(t-\mu^2)} \frac{g_o G_o(t,v) \epsilon_{\nu\alpha\beta\lambda 1} \, k_\alpha q_{2\beta} \, (k_\lambda)^{n-1} \, (-1)^n}{(m_{g_o})^n} x$$

$$\phi_{\lambda\rho}(q_2,n) \frac{1}{\left[(v-\mu_2^2) + i\mu_2 \Gamma_V\right]} (q_{4\rho})^{n-1} \Big[\frac{h_1 H_1(v,x)}{(m_{h_1})^{n-2}} g_{\sigma\rho 1} +$$

$$\frac{h_2 H_2(v,x)}{(m_{h_2})^n} q_{2\sigma} q_{4\rho 1} + \frac{h_3 H_3(v,x)}{(m_{h_3})^n} q_{4\sigma} q_{4\rho 1} \Big] x$$

$$x\ (-1)\left[g_{\sigma\tau} - \frac{q_{4\sigma}\, q_{4\tau}}{\mu_4^2} \right] \frac{1}{\left[(x-\mu_4^2) + i\mu_4\Gamma_X\right]}\ i\ I(x)\ q_{5\tau} \qquad (8.52)$$

Substituting (8.52) in the expression (3.29) for $d\sigma(b)$, averaging over initial spin states, summing over final spin states and integrating over dv and dx gives, with the notation of (7.3) and the approximations (8.3) the result:

$$d\sigma(b) = \frac{1}{(4\pi)^5}\ \frac{q_B^2}{4E_B p_B}\ \frac{1}{(t-\mu^2)^2}\ \frac{1}{2\mu_2^2\Gamma_V}\ \frac{i^2}{4\pi}\ \frac{1}{2\mu_4\Gamma_X}$$

$$x\left[M_{p_2p_2}(t)\ (p_2.r)(p_2.r^+) - M(t)\ (r.r^+) \right]$$

$$x\ d\cos\theta_B\ d\phi_B\ d\cos\theta_{3V}\ d\phi_{3V}\ d\psi_{3V}\ d\omega_{5V} \qquad (8.53)$$

where

$$r_\mu = \bar{g}_o\left[\epsilon_{\mu\alpha\beta\lambda_1} k_\alpha q_{2\beta}\ (-1)^j (k_\lambda)^{j-1} \phi_{\lambda\rho}(q_2, j)(q_{4\rho})^{j-1} \right]$$

$$x\left[\frac{h_1}{(m_{h_1})^{j-2}} \left(q_{5\rho_1} - \frac{q_4 \cdot q_5}{\mu_4^2}\ q_{4\rho_1} \right) \right.$$

$$\left. + \frac{h_2}{(m_{h_2})^j} \left(q_2 \cdot q_5 - \frac{q_2 . q_4\ q_4 \cdot q_5}{\mu_4^2} \right) q_{4\rho_1} \right] \qquad (8.54)$$

and $M_{p_2p_2}(t)$ and $M(t)$ are given by (7.9)

Denoting $\cos\Theta_{3V}$ by C_V and $\sin\Theta_{3V}$ by S_V it follows from (A.9) and (A 11) that, with the notation of (8.33) and (8.34),

$$p_2.r = \bar{g}_o\left[\frac{2^j j!\,j!}{(2j)!}\ \frac{1}{j^2}\ (E_B p_B q_B \sin\Theta_B)\ k_V^{j-1}\ q_V^{j-2}\right] x$$

$$x\Big[\bar{h}_1\ Q_1(\omega_{5V})\Big\{\cos\psi_{3V}\ \sin\phi_{3V}\ \frac{1}{S_V}\ P_j^1(C_V) -$$

$$- \sin\psi_{3V}\ \cos\phi_{3V}\left[j(j+1)P_j^0\ (C_V) - \frac{C_V}{S_V}\ P_j^1\ (C_V)\right]\Big\} +$$

$$+ \bar{h}_2 Q_2(\omega_{5V})\ \cos\phi_{3V}\ jP_j^1(C_V)\Big] \qquad (8.55)$$

and

$$r.r^+ = -|\bar{g}_o|^2\left[\frac{2^j j!\,j!}{(2j)!}\ \frac{1}{j^2}\ (\nu\, 2^{k_V})^2 k_V^{j-1}\ q_V^{j-2}\right]^2\ x$$

$$x\Big[\Big\{\bar{h}_1 Q_1(\omega_{5V})\Big\}^2\Big\{\cos^2\psi_{3V}\left[\frac{1}{S_V}\ P_j^1\ (C_V)\right]^2 +$$

$$+ \sin^2\psi_{3V}\left[j(j+1)P_j^o(C_V) - \frac{C_V}{S_V}\ P_j^1\ (C_V)\right]^2 -$$

$$- 4\ \sin\psi_{3V}\ \cos\psi_{3V}\ \sin\phi_{3V}\ \cos\phi_{3V}\ \frac{1}{S_V}\ P_j^1\ (C_V)\ x$$

$$x\left[j(j+1)P_j^o\ (C_V) - \frac{C_V}{S_V}\ P_j^1\ (C_V)\right]\Big\} + 2\Big\{\bar{h}_1 Q_1(\omega_{5V})\ \bar{h}_2 Q_2(\omega_{5V})\Big\} x$$

$$x\ jP_j^1(C_V)\Big\{ \sin\psi_{3V}\left[j(j+1)P_j^o(C_V) - \frac{C_V}{S_V} P_j^1(C_V)\right] +$$

$$+ \cos\psi_{3V} \sin\phi_{3V} \cos\phi_{3V} \frac{1}{S_V} P_j^1(C_V)\Big\} + \left\{\bar{h}_2 Q_2(\omega_{5V})\right\}^2 x$$

$$x\left\{jP_j^1(C_V)\right\}^2 \Big] \qquad (8.56)$$

Substituting (8.55) and (8.56) into the expression (8.53) gives:

$$d\sigma\ (b) = \frac{1}{(4\pi)^2} |\bar{g}_o|^2 \frac{q_B}{(4E_B^2 p_B)} \frac{1}{(t-\mu^2)^2} \frac{2^j j!\, j!}{(2j)!} \frac{(j+1)}{j}$$

$$x\ k_V^{2j-2}\Big[M_{p_2 p_2}(t)\left\{E_B p_B q_B \sin\Theta_B\right\}^2 \left\{\bar{h}_1 Q_1(\omega_{5V})\right.\ x$$

$$x\Big[\cos\psi_{3V} \sin\phi_{3V} \frac{1}{S_V} P_j^1(C_V) - \sin\psi_{3V} \cos\phi_{3V}\ x$$

$$x\left\{j(j+1)P_j^o(C_V) - \frac{C_V}{S_V} P_j^1(C_V)\right\}\Big] + \bar{h}_2 Q_2(\omega_{5V}) \cos\phi_{3V}\ jP_j^1(C_V)\Big\}$$

$$+\ M(t)\left\{\mu_2 k_V\right\}^2 \left\{\left[\bar{h}_1 Q_1(\omega_{5V})\right]^2 \Big[\cos^2\psi_{3V}\left\{\frac{1}{S_V} P_j^1(C_V)\right\}^2\right.$$

$$- 4 \sin\psi_{3V} \cos\psi_{3V} \sin\phi_{3V} \cos\phi_{3V} \frac{1}{S_V} P_j^1(C_V)\ x$$

$$x\left\{j(j+1)\ P_j^o(C_V) - \frac{C_V}{S_V} P_j^1(C_V)\right\} +$$

$$+ \sin^2\psi_{3V}\left\{j(j+1)\,P_j^o(C_V) - \frac{C_V}{S_V}\,P_j^1(C_V)\right\}^2 +$$

$$+ 2\left[\bar{h}_1 Q_1(\omega_{5V})\,\bar{h}_2 Q_2(\omega_{5V})\right] jP_j^1(C_V)\left[\sin\psi_{3V} \quad x\right.$$

$$x\left\{j(j+1)P_j^o(C_V) - \frac{C_V}{S_V}\,P_j^1(C_V)\right\} + \cos\psi_{3V}\sin\phi_{3V}\cos\phi_{3V}$$

$$\left. x\,\frac{1}{S_V}\,P_j^1(C_V)\right] + \left[\bar{h}_2 Q_2(\omega_{5V})\right]^2\left[jP_j^1(C_V)\right]^2\Big\}\Big] \quad x$$

$$x\; d\cos\Theta_B\, d\phi_B\, \frac{\Gamma_V(2\text{-}3,4)}{\Gamma_V}\left[(\bar{h}_1)^2(j+1) + (\bar{h}_2)^2 j\right]^{-1}$$

$$x\; \frac{(2j+1)}{2j^2(j+1)}\; d\,C_V\,\frac{1}{2\pi}\,d\phi_{3V}\,\frac{\Gamma_X(4\text{-}5,6)}{\Gamma_X}\,\frac{1}{\pi}\,d\psi_{3V} \quad x$$

$$x\,\frac{3}{4}\,\frac{\rho_4}{q_{3V}^3\,q_{5X}^3}\,d\omega_{5V} \qquad (8.57)$$

where use has been made of (6.11) and (6.12) to separate out the branching ratio factors.

To determine the decay distribution as a function of any one of the variables $\cos\Theta_{3V}$, ϕ_{3V}, ψ_{3V}, or ω_{5V} it is necessary to integrate over the other three variables. The results are as follows:

$$\left[(\bar{h}_1)^2(j+1) + (\bar{h}_2)^2 j\right]^{-1}\left[(\bar{h}_1)^2\left\{\left[\frac{1}{\sin\Theta_{3V}} P_j^1(\cos\Theta_{3V})\right]^2 + \right.\right.$$

$$+\left[j(j+1)P_j^0(\cos\Theta_{3V}) - \frac{\cos\Theta_{3V}}{\sin\Theta_{3V}} P_j^1(\cos\Theta_{3V})\right]^2\Big\}+$$

$$\left.\left. + (\bar{h}_2)^2\left\{\left[jP_j^1(\cos\Theta_{3V})\right]^2\right\}\right] \frac{(2j+1)}{2j^2(j+1)} \, d\cos\Theta_{3V} \qquad (8.58)$$

$$\frac{1}{2\pi}\left[A(s,t) + B(s,t)\left\{(\bar{h}_1)^2(j+1) + (\bar{h}_2)^2 j\right\}^{-1} \; x\right.$$

$$x\left\{(\bar{h}_1)^2 \frac{1}{2j}\left[(2j+1) + (j^2-j-1)2\cos^2\phi_{3V}\right] \; + \right.$$

$$\left.\left. + (\bar{h}_2)^2\left[j\cos^2\phi_{3V}\right]\right\}\right] d\phi_{3V} \qquad (8.59)$$

$$\frac{1}{2\pi}\left[(\bar{h}_1)^2(j+1) + (\bar{h}_2)^2 j\right]^{-1}\left[(\bar{h}_1)^2 \frac{1}{j}\left\{(2j+1) + \right.\right.$$

$$\left.\left. + (j^2-j-1)2\sin^2\psi_{3V}\right\} + (\bar{h}_2)^2 j\right] d\psi_{3V} \qquad (8.60)$$

$$\left[(\bar{h}_1)^2(j+1) + (\bar{h}_2)^2 j\right]^{-1}\left[(\bar{h}_1)^2 (j+1)(q_{3V}q_{5V}\sin\Theta_{35V})^2 + \right.$$

$$\left. + (\bar{h}_2)^2 2j(\omega_{5X}\omega_{4V} - \mu_4\omega_{5V})^2\right]^{\frac{3}{4}} \frac{\mu_4}{q_{3V}^3 q_{5X}^3} d\omega_{5V} \qquad (8.61)$$

where A(s,t) and B(s,t) are the functions appearing in (7.17) and (8.16). It is to be noted that these distributions all depend upon the ratio of $\bar{h}_1$ and $\bar{h}_2$.

The spin averaged differential cross section for the production of a spin j, abnormal parity resonance and the decay of this state into a pseudoscalar meson and a vector meson is obtained from (8.57) by integrating over $d\psi_{3V}$ and $d\omega_{5V}$ and putting $\Gamma_X(4\text{-}5,6) = \Gamma_X$. The result obtained is:

$$d\sigma(b) = \frac{g_o^2}{4\pi}\left|\frac{G_o(t)}{(m_{g_o})^j}\right|^2 \frac{q_B}{4E_B^2 p_B}\frac{1}{(t-\mu^2)^2}\frac{2^j j!\,j!}{(2j)!}\ x$$

$$x\ \frac{(j+1)}{j}\,k_V^{j-2}\left[\left\{\frac{f_2^2}{4\pi}|F_2(t)|^2\,4 + \frac{f_2 f_3}{4\pi}\frac{2\,\mathrm{Re}\left[F_2(t)F_3^+(t)\right]}{(m_{f_3})}\right.\right. x$$

$$\left.\left. x\ 2(m_1' + m_2) + \frac{f_3^2}{4\pi}\frac{|F_3(t)|^2}{(m_{f_3})^2}\left[-t + (m_1'+m_2)^2\right]\right\}\right]^2 x$$

$$x\left\{E_B p_B q_B \sin\theta_B\right\}^2\left\{(\bar{h}_1)^2\left[\cos^2\phi_{3V}\left\{j(j+1)P_j^o(C_V) - \right.\right.\right.$$

$$\left.\left.\left. - \frac{C_V}{S_V}P_j^1(C_V)\right\}^2 + \sin^2\phi_{3V}\left\{\frac{1}{S_V}P_j^1(C_V)\right\}^2\right] +\right.$$

$$\left. + (\bar{h}_2)^2\left[\cos^2\phi_{3V}\left\{jP_j^1(C_V)\right\}^2\right]\right\} + \left\{\frac{f_2^2}{4\pi}|F_2(t)|^2\ x\right.$$

$$x\left[-t+(m_1'+m_2)^2\right]\left\{\mu_{2k_V}\right\}^2\left\{(\bar{h}_1)^2\left[\left\{j(j+1)P_j^o(C_V)-\right.\right.\right.$$

$$\left.\left.\left.-\frac{C_V}{S_V}P_j^1(C_V)\right\}^2+\left\{\frac{1}{S_V}P_j^1(C_V)\right\}^2\right]+(\bar{h}_2)^2\left[jP_j^1(C_V)\right]^2\right\}\Bigg]x$$

$$x\,d\cos\Theta_B\,d\phi_B\,\frac{\Gamma_V(2\text{-}3,4)}{\Gamma_V}\left[(\bar{h}_1)^2(j+1)+(\bar{h}_2)^2 j\right]^{-1}\quad x$$

$$x\,\frac{(2j+1)}{2j^2(j+1)}\quad d\,C_V\,\frac{1}{2\pi}\,d\phi_{3V} \tag{8.62}$$

As for (8.39) and (8.51) this result is independent of the decay mode of the vector meson and thus applies to both (8.7) and (8.8) provided the boson resonance of spin j has abnormal parity and the production processes are mediated by pseudovector particle exchange.

The spin averaged differential cross section for the general process (7.7) mediated by pseudovector particle exchange is found by putting $\Gamma_V(2\text{-}3,4) = \Gamma_V$ in (8.62) and intergrating over C_V and ϕ_{3V}. The result is of course given by (7.18).

9. FERMION RESONANCES DECAYING INTO A SPIN-HALF PARTICLE AND A PSEUDOSCALAR PARTICLE.

In the peripheral production of a fermion resonance and a pseudoscalar particle the exchanged particle must have normal parity as can be seen from the discussion of section 5. For each of the two allowed possibilities, that is scalar and vector particle exchange the differential cross section is calculated for the process described by Fig. 1(c) in which the resonance has spin J. The various parity cases for the fermion fields are treated simultaneously by making use of the factors γ_e and γ_f defined in section 5. The invariant matrix element is in each case constructed from the vertex functions of Table 1, and the propagators of section 4. In particular for the spin J resonance the propagator used is given by (4.28). The resulting expression for the matrix element is then substituted in the formula (3.34) for d (c)

(i) Scalar particle exchange.

The invariant matrix element is given by:

$$\langle p_3 p_4 q_2 \mid T \mid p_1 q_1 \rangle = \bar{u}(p_3)\,\gamma_e\,\frac{eE(w,p_2)(p_4\rho)^n(-1)^n\Theta_{\rho\lambda}(p_2,n)}{(m_e)^n}\times$$

$$\times(\not p_2+m_2)\,\frac{1}{\left[(w-m_2^2)+im_2\Gamma_w\right]}(k_\lambda)^n\,\frac{fF(t,w,\not p_2)}{(m_f)^n}\,\gamma_f u(p_1)\times\frac{1}{(t-\mu^2)}\frac{gG(t)}{(m_g)^{-1}}$$

where $J=(n+\frac{1}{2})$ (9.1)

In order to deal with all parity cases simultaneously it is convenient to define m_1' and m_3' such that

$$m_1' = + m_1 \text{ if } \gamma_f = I \tag{9.2a}$$

$$m_1' = - m_1 \text{ if } \gamma_f = \gamma_5 \tag{9.2b}$$

$$m_3' = + m_3 \text{ if } \gamma_e = I \tag{9.2c}$$

$$m_3' = - m_3 \text{ if } \gamma_e = \gamma_5 \tag{9.2d}$$

Assuming that the resonance has a narrow width i.e. $\Gamma_W \ll m_2$ then

$$\left[\frac{1}{(w-m_2^2)^2 + (m_2^2\,\Gamma_W^2)}\right] \approx \frac{\pi}{m_2\,\Gamma_W}\,\delta(w-m_2^2) \tag{9.3}$$

Substituting (9.1) in (3.34), averaging over initial spin states, summing over final spin states and integrating over dw gives, with the notation of (9.2) and the approximation of (9.3) the result:

$$d\sigma(c) = \frac{\bar{f}^2}{4\pi}\,\frac{\bar{g}^2}{4\pi}\,\frac{q_B}{4E_B^2\,p_B}\,\frac{\left[-t + (m_1' + m_2)^2\right]}{(t-\mu^2)^2} \times$$

$$\times\ \frac{1}{4}\ \mathrm{Tr}\left\{\gamma_{\beta_1}\gamma\,\gamma_1\gamma_{\rho_1}\gamma_{\lambda_1}\right\} \times$$

$$\times \frac{\Gamma_W(2-3,4)}{\Gamma_W}\left[\frac{2^n\, n!\, n!}{(2n)!}\,\frac{(2n+1)^2}{n^3}\,p_W^{2n-2}\right]^{-1}$$

$$\times\left[(-1)^{n+1}(p_4\rho)^{n-1}\phi_{\rho\lambda}(p_2,n)\,(k_\lambda)^{n-1}\right]\left[(-1)^{n+1}(k_\beta)^{n-1}\phi_{\beta\gamma}(p_2,n)\times(p_4\gamma)^{n-1}\right]$$

$$\times\frac{1}{4\pi}\,d\cos\Theta_B d\phi_B\, d\cos\Theta_{4W} d\phi_{4W} \qquad (9.4)$$

where now $J = (n-\frac{1}{2})$. Use has been made of the fact that $\not{p}_2$ commutes with $\Theta(p_2,n)$ and (4.25) has been used in order to facilitate the evaluation of the trace. In addition (6.18) has been used to separate out the factor $\Gamma_W(2-3,4)/\Gamma_W$ where $\Gamma_W(2-3,4)$ is the partial width and Γ_W is the total width of the resonance. This factor is thus the branding ratio of the decay process (1.1) relative to all other possible decay modes of F.

Using (A.21), (A.24) and (A.26) this expression may be rewritten as:

$$d\sigma(c) = \frac{f^2}{4\pi}\,\frac{g^2}{4\pi}\left|\frac{F(t)\,G(t)}{(m_f)^{n-1}(m_g)^{n-1}}\right|^2 \frac{q_B}{4E_B^2 p_B}$$

$$\times\left[\frac{-t+(m_1+m_2)^2}{(t-\mu^2)^2}\right]\frac{2^n\, n!\, n!}{(2n)!}\,k_W^{2n-2}\, d\cos\Theta_B\, d\phi_B$$

$$\frac{\Gamma_W(2-3,4)}{\Gamma_W}\,\frac{1}{4\pi n}\left[\left\{nP_n^0(\cos\Theta_{4W})\right\}^2+\left\{P_n^1(\cos\Theta_{4W})\right\}^2\right]d\cos\Theta_{4W}\,d\phi_{4W} \qquad (9.5)$$

The three factors in the above expression are completely analogous to those appearing in (7.5). The first factor is independent of ϕ_B of course and is in fact the spin averaged differential cross section for the production of a spin J ($J=n-\frac{1}{2}$) resonance in the quasi two body process (1.8) assuming that the resonance does not decay or equivalently summing over all possible decay modes.

The final factor in (9.5) gives the distribution of the decay products and is normalised so that integration over $\cos\Theta_{4W}$ and ϕ_{4W} gives unity. The distribution is independent of ϕ_{4W} in agreement with the Trieman-Yang test[13] and the distribution of the decay products as a function of $\cos\Theta_{4W}$ is independent of the production process variables and of the arbitrary form factors F(t) and G(t). This distribution is of course the Adair distribution[36] for the decay of a spin J resonance produced in the forward direction of a quasi two body production process. The Adair distribution is independent of the production mechanism and the distributions obtained in this section all reduce to the Adair distribution when events in the forward or backward directions are considered.

(ii) Vector particle exchange

The invariant matrix element is given by:

$$\langle p_3 p_4 q_2 | T | p_1 q_1 \rangle = \bar{u}(p_3) \gamma_e \frac{e E(w, \not{p}_2)}{(m_e)^n} (p_4 \rho)^n (-1)^n \Theta_{\rho\lambda}(p_2, n) (\not{p}_2 + m_2)$$

$$\times \frac{1}{\left[(w - m_2^2) + i m_2 \Gamma_W \right]} (k_\lambda)^{n-1} \left[\frac{f_1 F_1(t, w, \not{p}_2) g_{\mu\lambda_1} + f_2 F_2(t, w, \not{p}_2) \gamma_\mu k \lambda_1}{(m_{f_1})^{n-1}} \right.$$

$$+ \frac{f_3 F_3(t, w, \not{p}_2)}{(m_{f_3})^{n+1}} p_{2\mu} k\lambda_1 + \frac{f_4 F_4(t, w, \not{p}_2)}{(m_{f_4})^{n+1}} k_\mu k \lambda_1 + \frac{f_5 F_5(t, w, \not{p}_2)}{(m_{f_5})^{n+1}} \times$$

$$\left. \epsilon_{\mu\lambda_1\sigma\tau} k_\sigma p_{2\tau} \gamma_5 \right] \not{k} u(p_1) (-1) \left[g_{\mu\nu} - \frac{k_\mu k_\nu}{\mu^2} \right] \frac{1}{(t - \mu^2)}$$

$$\times \left[g_2 G_2(t) q_{2\nu} + g_3 G_3(t) k_\nu \right] \qquad (9.6)$$

where $J = (n + 1/2)$

Substituting (9.6) in (3.34), averaging over initial spin states, summing over final spin states and integrating over dw gives, with the notation of (9.2) and the approximation (9.3), the result:

$$d\sigma(c) = \frac{1}{(4\pi)^3} \frac{q_B}{4 E_B^2 p_B} \frac{1}{(t - \mu^2)^2} \frac{\Gamma_W(2 - 3,4)}{\Gamma_W} \left[\frac{2^n n! n!}{(2n)!} \frac{(2n+1)^2 p_W^{2n-2}}{n^3} \right]^{-1} \times$$

$$\times T(r) d\cos\Theta_B d\phi_B d\cos\Theta_{4W} d\phi_{4W} \qquad (9.7)$$

where now $J = (n - 1/2)$ and

$$T(r) = \Big[\Big\{ M_{kk} {}^{k}\beta_2 {}^{k}\lambda_2 + M_{kr} {}^{k}\beta_2 {}^{r}\lambda_2 + M_{rk} r^{+}_{\beta_2} {}^{k}\lambda_2 + M_{rr} r^{+}_{\beta_2} {}^{r}\lambda_2 +$$

$$+ M_{ss} s^{+}_{\beta_2} {}^{s}\lambda_2 \Big\} \frac{1}{4} \mathrm{Tr} \Big\{ \gamma_{\beta_1} \gamma_{\gamma_1} \gamma_{\rho_1} \gamma_{\lambda_1} \Big\} +$$

$$+ \Big\{ N_{kk} {}^{k}\beta_2 {}^{k}\lambda_2 + N_{kr} {}^{k}\beta_2 {}^{r}\lambda_2 \Big\} \frac{1}{4} \mathrm{Tr} \Big\{ (\not{p}_1 \not{r}^{+} - \not{r}^{+} \not{p}_1) \gamma_{\beta_1} \gamma_{\gamma_1} \gamma_{\rho_1} \gamma_{\lambda}$$

$$+ \Big\{ N^{+}_{kk} {}^{k}\beta_2 {}^{k}\lambda_2 + N^{+}_{kr} r^{+}_{\beta_2} {}^{k}\lambda_2 \Big\} \frac{1}{4} \mathrm{Tr} \Big\{ (\not{r}\not{p}_1 - \not{p}_1 \not{r}) \gamma_{\beta_1} \gamma_{\gamma_1} \gamma_{\rho_1} \gamma_{\lambda_1} \Big\}$$

$$+ \Big\{ O_{kk} {}^{k}\beta_2 {}^{k}\lambda_2 \Big\} \frac{1}{4} \mathrm{Tr} \Big\{ (\not{r}\not{r}^{+} - \not{r}^{+}\not{r}) \gamma_{\beta_1} \gamma_{\gamma_1} \gamma_{\rho_1} \gamma_{\lambda_1} \Big\}$$

$$+ \Big\{ P_{ks} {}^{k}\beta_2 {}^{s}\lambda_2 + P_{sk} s^{+}_{\beta_2} {}^{k}\lambda_2 + P_{rs} r^{+}_{\beta_2} {}^{s}\lambda_2 + P_{sr} s^{+}_{\beta_2} {}^{r}\lambda_2 \Big\} \quad x$$

$$x \quad \frac{1}{4} \mathrm{Tr} \Big\{ \gamma_5 \gamma_{\beta_1} \gamma_{\gamma_1} \gamma_{\rho_1} \gamma_{\lambda_1} \Big\}$$

$$+ \Big\{ Q_{ks} {}^{k}\beta_2 {}^{s}\lambda_2 + Q_{sk} s^{+}_{\beta_2} {}^{k}\lambda_2 + Q_{rs} r^{+}_{\beta_2} {}^{s}\lambda_2 + Q_{sr} s^{+}_{\beta_2} {}^{r}\lambda_2 \Big\} \quad x$$

$$x \quad \frac{1}{4} \mathrm{Tr} \Big\{ \gamma_5 (\not{p}_1 \not{p}_2 - \not{p}_2 \not{p}_1) \gamma_{\beta_1} \gamma_{\gamma_1} \gamma_{\rho_1} \gamma_{\lambda_1} \Big\}$$

$$+ \Big\{ R_{ks} {}^{k}\beta_2 {}^{s}\lambda_2 \Big\} \frac{1}{4} \mathrm{Tr} \Big\{ \gamma_5 (\not{p}_1 \not{r}^{+} - \not{r}^{+} \not{p}_1) \gamma_{\beta_1} \gamma_{\gamma_1} \gamma_{\rho_1} \gamma_{\lambda_1} \Big\}$$

$$+ \Big\{ R^{+}_{ks} s^{+}_{\beta_2} {}^{k}\lambda_2 \Big\} \frac{1}{4} \mathrm{Tr} \Big\{ \gamma_5 (\not{r}\not{p}_1 - \not{p}_1 \not{r}) \gamma_{\beta_1} \gamma_{\gamma_1} \gamma_{\rho_1} \gamma_{\lambda_1} \Big\}$$

$$+\left\{S_{ks}{}^{k}{}_{\beta_2}{}^{s}{}_{\lambda_2}\right\}\frac{1}{4}\,\mathrm{Tr}\left\{\gamma_5(\not r^{+}\not p_2-\not p_2\not r^{+})\gamma_{\beta_1}\gamma_{\gamma_1}\gamma_{\rho_1}\gamma_{\lambda_1}\right\}$$

$$+\left\{S^{+}_{ks}{}^{s}{}_{\beta_2}{}^{+}{}^{k}{}_{\lambda_2}\right\}\frac{1}{4}\,\mathrm{Tr}\left\{\gamma_5(\not p_2\not r-\not r\not p_2)\gamma_{\beta_1}\gamma_{\gamma_1}\gamma_{\rho_1}\gamma_{\lambda_1}\right\}\Bigg]\ x$$

$$x\left[(-1)^{n+1}\,(k_{\beta})^{n-2}\,\phi_{\beta\gamma}\,(p_2,n)\,(p_{4\gamma})^{n-1}\right]\quad x$$

$$x\left[(-1)^{n+1}\,(p_{4\rho})^{n-1}\phi_{\rho\lambda}\,(p_2,n)\,(k_{\lambda})^{n-2}\right]\qquad(9.8)$$

in which

$$r_{\mu}=(-1)\left[g_{\mu\nu}-\frac{k_{\mu}k_{\nu}}{\mu^{2}}\right]\left[\bar{g}_2\,q_{2\nu}+\bar{g}_3\,k_{\nu}\right]\qquad(9.9)$$

and

$$s_{\mu}=\epsilon_{\mu\nu\sigma\tau}p_{2\nu}k_{\sigma}r_{\tau}\qquad(9.10)$$

The notation used is such that:

$$M_{kk}=\left\{|\bar{f}_2|^2\,4+2\mathrm{Re}\,(\bar{f}_2\bar{f}^{+}_3)2(m_1'+m_2)\right.$$

$$\left.+\ |\bar{f}_3|^2\left[-t+(m_1'+m_2)^2\right]\right\}(p_2.r)\,(p_2.r^{+})$$

$$+\left\{|\bar{f}_2|^2\,2+(\bar{f}_2\bar{f}^{+}_3)2m_2+(\bar{f}_2\bar{f}^{+}_4)2(m_1'+m_2)\right.$$

$$\left.+\ (\bar{f}_3\bar{f}^{+}_4)\left[-t+(m_1'+m_2)^2\right]\right\}(p_2.r)\,(k.r^{+})$$

$$+\left\{|\bar{f}_2|^2\,2+(\bar{f}_3\bar{f}^{+}_2)2m_2+(\bar{f}_4\bar{f}^{+}_2)2(m_1'+m_2)\right.$$

$$\left.+\ (\bar{f}_4\bar{f}^{+}_3)\left[-t+(m_1'+m_2)^2\right]\right\}(k.r)\,(p_2.r^{+})$$

$$+ \left\{ 2\,\mathrm{Re}\,(\bar{f}_2\bar{f}_4^{+})2m_2 + |\bar{f}_4|^2\left[-t + (m_1' + m_2)^2\right]\right\}(k.r)\,(k.r^{+})$$

$$- \left\{ |\bar{f}_2|^2\left[-t + (m_1' + m_2)^2\right]\right\}(r.r^{+}) \qquad (9.11a)$$

$$M_{kr} = \left\{(\bar{f}_1\bar{f}_2^{+})\,2(m_1' + m_2) + (\bar{f}_1\bar{f}_3^{+})\left[-t + (m_1' + m_2)^2\right]\right\}(p_2.r^{+})$$

$$+ \left\{(\bar{f}_1\bar{f}_2^{+})\,2\,m_2 + (\bar{f}_1\bar{f}_4^{+})\left[-t + (m_1' + m_2)^2\right]\right\}(k.r^{+}) \qquad (9.11b)$$

$$M_{rk} = M_{kr}^{+} \qquad (9.11c)$$

$$M_{rr} = \left\{|\bar{f}_1|^2\left[-t + (m_1' + m_2)^2\right]\right\} \qquad (9.11d)$$

$$M_{ss} = \left\{|\bar{f}_5|^2\left[-t + (m_1' + m_2)^2\right]\right\} \qquad (9.11e)$$

$$N_{kk} = \left\{|\bar{f}_2|^2 + (\bar{f}_3\bar{f}^{+}{}_2)m_2\right\}(p_2.r) + \left\{(\bar{f}_4\bar{f}^{+}{}_2)m_2\right\}(k.r) \qquad (9.11f)$$

$$N_{kr} = \left\{(\bar{f}_1\bar{f}^{+}{}_2)\,m_2\right\} \qquad (9.11g)$$

$$O_{kk} = \left\{|\bar{f}_2|^2\left[-t + (m_1' + m_2)^2\right]\right\} \qquad (9.11h)$$

$$P_{ks} = \left\{(\bar{f}_5\bar{f}_2^{+})2(m_1' + m_2) + (\bar{f}_5\bar{f}_3^{+})\left[-t + (m_1' + m_2)^2\right]\right\}(p_2.r^{+})$$

$$+ \left\{(\bar{f}_5\bar{f}_2^{+})2m_2 + (\bar{f}_5\bar{f}_4^{+})\left[-t + (m_1' + m_2)^2\right]\right\}(k.r^{+}) \qquad (9.11i)$$

$$P_{sk} = \left\{ (\bar{f}_2 \bar{f}_5^+) 2(-m_1' + m_2) + (\bar{f}_3 \bar{f}_5^+) \left[t - (m_1' - m_2)^2 \right] \right\} (p_2.r)$$

$$+ \left\{ (\bar{f}_2 \bar{f}_5^+) \, 2m_2 + (\bar{f}_4 \bar{f}_5^+) \left[t - (m_1' - m_2)^2 \right] \right\} (k.r) \qquad (9.11j)$$

$$P_{rs} = \left\{ (\bar{f}_5 \bar{f}_1^+) \left[-t + (m_1' + m_2)^2 \right] \right\} \qquad (9.11k)$$

$$P_{sr} = \left\{ (\bar{f}_1 \bar{f}_5^+) \left[t - (m_1' + m_2)^2 \right] \right\} \qquad (9.11l)$$

$$Q_{ks} = \left\{ (\bar{f}_5 \bar{f}_3^+) \, (p_2.r^+) + (\bar{f}_5 \bar{f}_4^+) \, (k.r^+) \right\} \qquad (9.11m)$$

$$Q_{sk} = - Q_{ks}^+ \qquad (9.11n)$$

$$Q_{rs} = \left\{ (\bar{f}_5 \bar{f}_1^+) \right\} \qquad (9.11o)$$

$$Q_{sr} = - Q_{rs}^+ \qquad (9.11p)$$

$$R_{ks} = \left\{ (\bar{f}_5 \bar{f}_2^+) \, m_2 \right\} \qquad (9.11q)$$

$$S_{ks} = \left\{ (\bar{f}_5 \bar{f}_2^+) \, m_1' \right\} \qquad (9.11r)$$

Using the definition (9.9) the following functions of the 4-momentum r_μ which appear in the expression T(r) may be readily evaluated:

$$k.r = - \left[\bar{g}_2 \, (k.q_2) + \bar{g}_3 k^2 \right] \left(1 - \frac{k^2}{\mu^2} \right) \qquad (9.12)$$

$$p_2.r = - \left[\bar{g}_2 \left(p_2.q_2 - \frac{p_2.k \; k.q_2}{\mu^2} \right) + \bar{g}_3 p_2.k \left(1 - \frac{k^2}{\mu^2} \right) \right] \qquad (9.13)$$

$$r.r^{+} = |\bar{g}_2|^2\left[q_2^2 - \frac{(k.q_2)^2}{\mu^2} - \frac{(k.q_2)^2}{\mu^2}\left(1 - \frac{k^2}{\mu^2}\right)\right]$$

$$+ \left[2\,\mathrm{Re}\,(\bar{g}_2\bar{g}_3^{+})\,(k.q_2) + |\bar{g}_3|^2\,k^2\right]\left(1 - \frac{k^2}{\mu^2}\right)^2 \qquad (9.14)$$

It is convenient to denote the angle in the frame W between $\underline{q}_{2W}$ and $\underline{k}_W$ by ϵ_W as shown in Fig 2c. This angle satisfies the relationships:

$$m_2 q_{2W} k_W \sin\epsilon_W = E_B p_B q_B \sin\Theta_B \qquad (9.15)$$

and

$$q_{2W} k_W \cos\epsilon_W = \frac{\left[(s-m_2^2-\mu_2^2)\,(-t+m_1^2-m_2^2)+2m_2^2(-t+\mu_1^2-\mu_2^2)\right]}{4m_2^2} \qquad (9.16)$$

Using this notation the following relationships may be derived:

$$\underline{r}_W.\underline{k}_W = -\bar{g}_2\, q_{2W} k_W \cos\epsilon_W + \left[\bar{g}_2\frac{(k.q_2)}{\mu^2} - \bar{g}_3\left(1-\frac{k^2}{\mu^2}\right)\right] k^2_W \qquad (9.17)$$

$$\frac{(\underline{r}_W \wedge \underline{k}_W).(\underline{p}_{4W} \wedge \underline{k}_W)}{k_W p_W} = -\bar{g}_2 (q_{2W} k_W \sin\epsilon_W)\,\sin\Theta_{4W}\sin\phi_{4W} \qquad (9.18)$$

$$(\underline{r}_W \wedge k_W).(\underline{r}_W^{+} \wedge \underline{k}_W) = |\bar{g}_2|^2\,(q_{2W} k_W \sin\epsilon_W)^2 \qquad (9.19)$$

$$\frac{p_{4W}\cdot(\underline{r}_W \wedge \underline{k}_W)}{p_W} = -\bar{g}_2(q_{2W}k_W\sin\epsilon_W)\sin\Theta_{4W}\cos\phi_{4W} \qquad (9.20)$$

$$\frac{1}{p_W}(\underline{r}_W \wedge \underline{k}_W)\cdot(\underline{r}_W^+ \wedge \underline{p}_{4W})\, k_W = |\bar{g}_2|^2 (q_{2W}k_W\sin\epsilon_W)^2\cos\Theta_{4W} +$$

$$+ \bar{g}_2(q_{2W}k_W\sin\epsilon_W)\left\{ -\bar{g}_2^+ q_{2W}k_W\cos\epsilon_W + \right.$$

$$\left. + \left[\bar{g}_2^+\frac{(k\cdot q_2)}{\mu^2} - \bar{g}_2^{\ddagger}\left(1-\frac{k^2}{\mu^2}\right)\right] k_W^2 \right\} \sin\Theta_{4W}\sin\phi_{4W} \qquad (9.21)$$

and

$$\frac{(\underline{r}_W \wedge \underline{k}_W)\cdot(\underline{p}_{4W} \wedge k_W)}{p_W}\, k_W(p_2\cdot r^+) + \frac{(\underline{r}_W \wedge \underline{k}_W)\cdot(\underline{r}_W^+ \wedge \underline{p}_{4W})}{p_W} \times$$

$$\times\ k_W\,(p_2\cdot k) = |\bar{g}_2|^2 \left[(q_{2W}k_W\sin\epsilon_W)^2(p_2\cdot k)\cos\Theta_{4W} \right.$$

$$\left. - (q_{2W}k_W\sin\epsilon_W)(p_2\cdot k\ k\cdot q_2 - k^2 p_2\cdot q_2)\sin\Theta_{4W}\sin\phi_{4W} \right] \qquad (9.22)$$

Using the results (A.33)-(A.42) which give the various factors appearing in T(r) it follows from (9.7) that:

$$d\sigma(c) = \frac{1}{(4\pi)^2}\frac{q_B}{4E_B^2 p_B}\frac{1}{(t-\mu^2)^2}\frac{2^n n!n!}{(2n)!}\, k_W^{2n-6} \times$$

$$\times \left[A \left\{ \left[nP_n^o(c_W)\right]^2 + \left[P_n^1(c_W)\right]^2 \right\} \right.$$

$$\left. + B\sin\phi_{4W}\frac{(-1)}{(n-1)} P_n^1(c_W)\left\{ nP_n^o(c_W) - \frac{c_W}{s_W} P_n^1(c_W)\right\}\right.$$

$$+ C \cos\phi_{4W} \frac{(-1)}{(n-1)} P_n^1(c_W) \left\{ nP_n^o(c_W) - \frac{c_W}{s_W} P_n^1(c_W) \right\}$$

$$+ D \left\{ \frac{1}{(n-1)^2} \left[nP_n^o(c_W) - \frac{c_W}{s_W} P_n^1(c_W) \right]^2 \right.$$

$$\left. + \sin^2\phi_{4W} \left[\left\{ P_n^1(c_W) \right\}^2 - \frac{nP_n^o(c_W)P_n^2(c_W)}{(n-1)} \right] \right\}$$

$$+ E \left\{ \frac{1}{(n-1)^2} \left[nP_n^o(c_W) - \frac{c_W}{s_W} P_n^1(c_W) \right]^2 \right.$$

$$\left. + \cos^2\phi_{4W} \left[\left\{ P_n^1(c_W) \right\}^2 - \frac{nP_n^o(c_W)P_n^2(c_W)}{(n-1)} \right] \right\}$$

$$+ F \left\{ \frac{-1}{(n-1)} nP_n^o \left[nP_n^o(c_W) - \frac{c_W}{s_W} P_n^1(c_W) \right] \right.$$

$$\left. + \sin^2\phi_{4W} \left[\left\{ P_n^1(c_W) \right\}^2 - \frac{nP_n^o(c_W)P_n^2(c_W)}{(n-1)} \right] \right\}$$

$$+ G \left\{ \frac{-1}{(n-1)} nP_n^o \left[nP_n^o(c_W) - \frac{c_W}{s_W} P_n^1(c_W) \right] \right.$$

$$\left. \left. + \cos^2\phi_{4W} \left[\left\{ P_n^1(c_W) \right\}^2 - \frac{nP_n^o(c_W)P_n^2(c_W)}{(n-1)} \right] \right\} \right] \quad x$$

$$\times \; d\cos\theta_B d\phi_B \times \frac{\Gamma_W(2-z,4)}{\Gamma_W} \times \frac{1}{4\pi n} \; d c_W d\phi_{4W} \qquad (9.23)$$

where

$$A = M_{kk}k_W^4 + 2Re\left\{ N_{kr}\left(\frac{r.p_2 \; k.p_2}{p_2^2} - r.k\right)\right\} k_W^2 +$$

$$+ M_{rr} \left|\left(\frac{r.p_2 \; k.p_2}{p_2^2} - r.k\right)\right|^2 \qquad (9.24)$$

$$B = -2Re\left\{ \bar{g}_2(q_{2W}k_W\sin\epsilon_W)\left[M_{kr}k_W^2 + M_{rr}\left(\frac{r^+.p_2 \; k.p_2}{p_2^2} - r^+.k\right)\right.\right.$$

$$+ Q_{rs} \; 2m_2^2k_W^2 \left(\frac{r^+.p_2 \; k.p_2}{p_2^2} - r^+.k\right) + S_{ks}2m_2^2\left(\frac{r^+.p_2 k.p_2}{p_2^2} - r^+.k\right)$$

$$\left.\left. + R_{ks} \; 2(p_2.k \; k.q_2 - k^2p_2q_2) \; \bar{g}_2^+ \; (q_{2W}k_W\sin\epsilon_W)\right]\right\} \qquad (9.25)$$

$$C = -2Re\left\{ \bar{g}_2(q_{2W}k_W\sin\epsilon_W) \; Q_{ks} \; 2m_2^2k_W^2 \right\} \qquad (9.26)$$

$$D = |\bar{g}_2|^2 \; (q_{2W}k_W\sin\epsilon_W)^2 \left\{ M_{rr} + Q_{rs} \; 2m_2^2k_W^2 \right\} \qquad (9.27)$$

$$E = |\bar{g}_2|^2 \; (q_{2W}k_W\sin\epsilon_W)^2 \left\{ M_{ss} \; m_2^2k_W^2 \right\} \qquad (9.28)$$

$$F = -|\bar{g}_2|^2 \; (q_{2W}k_W\sin\epsilon_W)^2 \; 2Re\left\{ N_{kr}2k_W^2 + Q_{rs}2m_2^2k_W^2\right\} \qquad (9.29)$$

$$G = -|\bar{g}_2|^2 \; (q_{2W}k_W\sin\epsilon_W)^2 \; 2Re\left\{ R_{ks}(-2p_2.k) + S_{ks}2m_2^2\right\} \qquad (9.30)$$

The factor $\Gamma_W(2-3,4)/\Gamma_W$ is the branching ratio of the decay process (1.1) relative to all other possible decay modes of F.

The decay distribution is clearly a very complicated function of the coupling constants and their associated form factors. Integration over ϕ_{4W} gives the decay distribution as a function of $c_W = \cos\Theta_{4W}$.

$$\frac{1}{2n}\bar{A}\left\{\left[nP_n^0(\cos\Theta_{4W})\right]^2 + \left[P_n^1(\cos\Theta_{4W})\right]^2\right\}$$

$$+\bar{B}\left\{\frac{1}{(n-1)^2}\left[nP_n^0(\cos\Theta_{4W}) - \frac{\cos\Theta_{4W}}{\sin\Theta_{4W}}P_n^1(\cos\Theta_{4W})\right]^2\right.$$

$$\left.+\frac{1}{2}\left[\left\{P_n^1(\cos\Theta_{4W})\right\}^2 - \frac{nP_n^0(\cos\Theta_{4W})P_n^2(\cos\Theta_{4W})}{(n-1)}\right]\right\}\Bigg]d\cos\Theta_{4W} \qquad (9.31)$$

and integration over c_W gives the decay distribution as a function of ϕ_{4W}:

$$\frac{1}{2\pi}\left[\bar{C} + \bar{D}\sin^2\phi_{4W}\right]d\phi_{4W} \qquad (9.32)$$

where $\bar{A}, \bar{B}, \bar{C}$ and $\bar{D}$ depend on the production process. variables s and t in a very complicated way.

It is to be noted that for the production of the resonance F in the forward or backward direction for which $\sin\Theta_B = 0$, the decay distribution is uniquely defined as:

$$\frac{1}{4\pi n} \; A\left\{\left[nP_n^0(\cos\Theta_{4W})\right]^2 + \left[P_n^1(\cos\Theta_{4W})\right]^2\right\} d\cos\Theta_{4W} d\phi_{4W} \qquad (9.33)$$

This distribution is of course the Adair distribution for the decay of a spin $(n-\frac{1}{2})$ resonance produced in the forward direction.

The spin averaged differential cross section for the production of a spin J ($J = n - \frac{1}{2}$) resonance in the quasi two body process (1.8) assuming that the resonance does not decay or equivalently, summing over all possible decay modes is obtained from (9.23) by putting $\Gamma_W(2 - 3,4) = \Gamma_W$ and integrating over c_W and ϕ_{4W}.

The result is:

$$d\sigma(c) = \frac{1}{(4\pi)^2} \; \frac{q_B}{4E_B^2 p_B} \; \frac{1}{(t-\mu^2)^2} \; \frac{2^n n!\, n!\, k_W^{2n-6}}{(2n)!}$$

$$\left[\left\{A + \frac{1}{2}(F + G)\right\} + \frac{n}{(n-1)} \; \frac{1}{2}(D + E)\right] d\cos\Theta_B d\phi_B \qquad (9.34)$$

where A, D, E, F and G are given above.

10. SIMULTANEOUS PRODUCTION OF A BOSON RESONANCE AND A FERMION RESONANCE.

In this section a boson resonance decaying into two pseudoscalar particles and a fermion resonance decaying into a pseudoscalar particle and a spin half particle are considered. The boson resonance must necessarily have normal parity and in the peripheral production process the exchanged particle may be a pseudoscalar, a vector or a pseudovector particle but not a scalar particle. For each of the three allowed possibilities the differential cross section is calculated for the process described by Fig. 1 (d) in which the boson resonance has spin j, and the fermion resonance has spin J. As in section 9 the various parity cases for the fermion fields are treated by making use of the factors γ_e and γ_f defined in section 5. The invariant matrix element in each case is constructed from the vertex functions of Table 1. and the propagators of section 4. The resulting expression is then substituted in the formula (3.38) for $d\sigma(d)$.

(i) Pseudoscalar particle exchange.

The invariant matrix element is given by:

$$\langle p_3 p_4 q_3 q_4 \mid T \mid p_1 q_1 \rangle = \bar{u}(p_3) \frac{\gamma_e e E(w, p_2)}{(m_e)^n} (p_4 \rho)^n (-1)^n \quad x$$

$$\Theta_{\rho\lambda}(p_2,n)\ (\not{p}_2 + m_2)\ x\ \frac{1}{(w-m_2^2) + i\, m_2 \Gamma_W}\ (k_\lambda)^n \qquad x$$

$$\frac{f\ F(t,w,\not{p}_2)}{(m_f)^n}\ \gamma_f\ u(p_1)\ \frac{1}{(t-\mu^2)}\ \frac{g\ G(t,v)}{(m_g)^{j-1}}\ (-1)^j\ (k_\mu)^j\ x$$

$$\phi_{\mu\nu}(q_2,j)\ \frac{1}{(v-\mu_2^2) + i\mu_2 \Gamma_V}\ (q_{4\nu})^j\ \frac{h\ H(v)}{(m_h)^{j-1}} \qquad (10.1)$$

where $J = (n + \frac{1}{2})$.

The matrix element clearly consists of two separate parts: one involving the boson resonance and one involving the fermion resonance. Thus using the results of sections 7 and 9 substituting (10.1) in (3.38), averaging over initial spin states, summing over final spin states and integrating over dv and dw gives, with the notation of (9.2) and the approximations (7.4) and (9.3), the result:

$$d\sigma(d) = \frac{f^2}{4\pi}\ \frac{g^2}{4\pi}\ \left| \frac{F(t)\ G(t)}{(m_f)^{n-1}\ (m_g)^{j-1}} \right|^2\ \frac{q_B}{4E_B^2\ p_B}\ \frac{\left[-t+(m_1' + m_2)^2\right]}{(t-\mu^2)^2}$$

$$x\ \frac{2^n\ n!\ n!}{(2n)!}\ k_W^{2n-2}\ \frac{2^j\ j!\ j!}{(2j)!}\ k_V^{2j}\ d\cos\theta_B\ d\phi_B$$

$$x \frac{\Gamma_W(2-3,4)}{\Gamma_W} \frac{1}{4\pi n}\left[\left\{n P_n^0(\cos\theta_{4W})\right\}^2 + \left\{P_n^1(\cos\theta_{4W})\right\}^2\right] x$$

$$x \; d\cos\theta_{4W} \; d\phi_{4W} \frac{\Gamma_V(2-3,4)}{\Gamma_V} \frac{(2j+1)}{4\pi} \left\{P_j^0(\cos\theta_{3V})\right\}^2 x$$

$$d\cos\theta_{3V} \; d\phi_{3V} \qquad (10.2)$$

where now $J = (n-\frac{1}{2})$ and where (6.5) and (6.18) have been used to separate out the two branching ratio factors $\Gamma_V(2-3,4)/\Gamma_V$ and $\Gamma_W(2-3,4)/\Gamma_W$ respectively. The first factor gives the spin averaged differential cross section for the two body production process (1.9) since the decay distribution factors are normalised so that integration over the distribution variables gives unity. The distribution of the decay products of the boson and fermion resonances are independent of each other and of the production process variables and the form factors F(t) and G(t). As expected there is no dependence on either ϕ_{3V} or ϕ_{4W} in accordance with the Trieman-Yang test.

(ii) Vector particle exchange.

The invariant matrix element is given by:

$$\langle p_3 p_4 q_3 q_4 \mid T \mid p_1 q_1 \rangle = \bar{u}(p_2)\,\gamma_e\,\frac{e\,E(w,\not{p}_2)}{(m_e)^n}\,(p_{4\rho})^n\,(-1)^n \quad x$$

$$\Theta_{\lambda\rho}(p_2,n)\,(\not{p}_2 + m_2)\,\frac{1}{(w-m_2^2) + i\,m_2\Gamma_W}\,(k_\lambda)^{n-1} \quad x$$

$$\left[\frac{f_1\,F_1(t,w,\not{p}_2)}{(m_{f_1})^{n-1}}\,g_{\alpha\lambda_1} + \frac{f_2\,F_2(t,w,\not{p}_2)}{(m_{f_2})^n}\,\gamma_\alpha\,k_{\lambda_1} +\right.$$

$$+ \frac{f_3\,F_3(t,w,\not{p}_2)}{(m_{f_3})^{n+1}}\,p_{2\alpha}\,k_{\lambda_1} + \frac{f_4\,F_4(t,w,\not{p}_2)}{(m_{f_4})^{n+1}}\,k_\alpha\,k_{\lambda_1} +$$

$$\left. + \frac{f_5\,F_5(t,w,\not{p}_2)}{(m_{f_5})^{n+1}}\,\epsilon_{\alpha\lambda_1\sigma\tau}\,k_\sigma\,p_{2\tau}\,\gamma_5\right]\,\gamma_f\,u(p_1)\,\frac{1}{(t-\mu^2)}\,(-1) \quad x$$

$$\left[g_{\alpha\beta} - \frac{k_\alpha k_\beta}{\mu^2}\right]\,\frac{g_o\,G_o(t,v)}{(m_{g_o})^j}\,\epsilon_{\beta\gamma\delta\mu}\,k_\gamma\,q_{2\delta}\,(k_\mu)^{j-1} \quad x$$

$$x\,(-1)^j\,\Phi_{\mu\nu}(q_2,j)\,\frac{1}{(v-\mu_2^2) + i\mu_2\Gamma_V}\,(q_{4\nu})^j\,\frac{h\,H(v)}{(m_h)^{j-1}} \qquad (10.3)$$

where $J = (n + \frac{1}{2})$.

Substituting (10.3) in (3.38), averaging over initial spin states, summing over final spin states and integrating over dv and dw gives, with the notation of (9.2) and the approximations (7.4) and (9.3), the result:

$$d\sigma(d) = \frac{1}{(4\pi)^4} \quad \frac{q_B}{4E_B^2\, p_B} \quad \frac{1}{(t-\mu^2)^2} \quad \frac{\Gamma_V(2-3,4)}{\Gamma_V} \quad \frac{\Gamma_W(2-3,4)}{\Gamma_W}$$

$$x \left[\frac{2^j\, j!\, j!}{(2j)!} \quad \frac{1}{(2j+1)} \quad q_V^{2j} \right]^{-1} \left[\frac{2^n\, n!\, n!}{(2n)!} \quad \frac{(2n+1)^2}{n^3} \quad p_W^{2n-2} \right]^{-1}$$

$$x \quad T(r)\, d\cos\theta_B\, d\phi_B\, d\cos\theta_{3V}\, d\phi_{3V}\, d\cos\theta_{4W}\, d\phi_{4W} \qquad (10.4)$$

where now $J = (n-\frac{1}{2})$ and $T(r)$ is given by (9.8) with r_μ given by (7.10) and s_μ by (9.10). The various coefficients M_{kk}, M_{kr}.... etc. appearing in (9.8) are given by (9.11) and the factors multiplying each of these coefficients are given by (A.33) - (A.34).

Using the definition (7.10) it follows that $k.r$ is zero and $p_2.r$ and $r.r^+$ are given by (7.12) and (7.14) respectively. Using the notation $C_V = \cos\theta_{3V}$, $S_V = \sin\theta_{3V}$, $C_W = \cos\theta_{4W}$ and $S_W = \sin\theta_{4W}$ the following relationships may be derived:

$$\underline{r}_W \cdot \underline{k}_W = \bar{g}_o \left[\frac{2^j\, j!\, j!}{(2j)!} \frac{1}{j} k_V^{j-1} q_V^j \right] P_j^1 (C_V) \frac{k \cdot p_2}{p_2^2} \quad x$$

$$x \left(\mu_2 p_{2V} k_V \sin\theta_V\right) \cos\phi_{3V} \qquad (10.5)$$

$$\frac{(\underline{r}_W \wedge \underline{k}_W)\cdot(\underline{p}_{4W} \wedge \underline{k}_W)}{p_W k_W} = \bar{g}_o \left[\frac{2^j\, j!\, j!}{(2j)!} \frac{1}{j} k_V^{j-1} q_V^j \right] P_j^1 (C_V) \quad x$$

$$x \left\{ \left(\mu_2 k_V k_W\right) \sin\phi_{3V} S_W \cos\phi_{4W} + \frac{(k \cdot p_2\, k \cdot q_2 - k^2\, q_2 \cdot p_2)}{m_2} \right. \quad x$$

$$\left. \cos\phi_{3V} \; S_W \; \sin\phi_{4W} \right\} \qquad (10.6)$$

$$(\underline{r}_W \wedge \underline{k}_W)\cdot(\underline{r}_W^+ \wedge \underline{k}_W) = \left|\bar{g}_o\right|^2 \left[\frac{2^j\, j!\, j!}{(2j)!} \frac{1}{j} k_V^{j-1} q_V^j \right]^2 \quad x$$

$$\left[P_j^1(C_V) \right]^2 x \left\{ \left(\mu_2 k_V k_W\right)^2 \sin^2\phi_{3V} + \frac{(k \cdot p_2\, k \cdot q_2 - k^2\, p_2 \cdot q_2)^2}{m_2^2} \right. \quad x$$

$$\left. x \; \cos^2\phi_{3V} \right\} \qquad (10.7)$$

$$\frac{p_{4W}\cdot(\underline{r}_W \wedge \underline{k}_W)}{p_W} = -\bar{g}_o \left[\frac{2^j\, j!\, j!}{(2j)!} \frac{1}{j} k_V^{j-1} q_V^j \right] P_j^1 (C_V) \quad x$$

$$x \left\{ (\mu_2 k_V k_W) \sin\phi_{3V} S_W \sin\phi_{4W} - \frac{(k\cdot p_2\, k\cdot q_2 - k^2\, q_2\cdot p_2)}{m_2} x \right.$$

$$\left. \cos\phi_{3V} S_W \cos\phi_{4W} \right\} \qquad (10.8)$$

$$\frac{(\underline{r}_W \wedge \underline{k}_W)\cdot(\underline{r}_W^+ \wedge \underline{p}_{4W})\, k_W}{p_W} = |\bar{g}_o|^2 \left[\frac{2^j\, j!\, j!}{(2j)!} \frac{1}{j} k_V^{j-1} q_V^j \right]^2 x$$

$$x \left[P_j^1 (C_V) \right] \left\{ \left[(\mu_2 k_V k_W)^2 \sin^2\phi_{3V} + \frac{(k\cdot p_2\, k\cdot q_2 - k^2\, p_2 q_2)^2}{m_2^2} x \right.\right.$$

$$\left.\left. x \cos^2\phi_{3V} \right] C_W - \frac{k\cdot p_2}{p_2^2} (\mu_2 p_{2V} k_V \sin\epsilon_V) \quad x \right.$$

$$\left[(\mu_2 k_V k_W) \cos\phi_{3V} \sin\phi_{3V} S_W \cos\phi_{4W} + \frac{(k\cdot p_2\, k\cdot q_2 - k^2\, p_2\cdot q_2)}{m_2} x \right.$$

$$\left. \left. x \cos^2\phi_{3V} S_W \sin\phi_{4W} \right] \right\} \qquad (10.9)$$

and

$$\frac{(\underline{r}_W \wedge \underline{k}_V)\cdot(\underline{p}_{4W} \wedge \underline{k}_W)}{p_W} \quad k_W \quad (p_2 \cdot r_W^+) + \frac{(\underline{r}_W \wedge \underline{k}_V)\cdot(\underline{r}_W^+ \wedge \underline{p}_{4W}}{p_W} \; k_W (p_2 \cdot k)$$

$$= |\bar{g}_o|^2 \left[\frac{2^j \, j! \, j!}{(2j)!} \; \frac{1}{j} \quad k_V^{j-1} \quad q_V^j \right]^2 \left[P_j^1 \, (C_V) \right]^2 \qquad x$$

$$x \left\{ (p_2 \cdot k) \quad \left[(\mu_2 \, k_V \, k_W)^2 \sin^2\phi_{3V} + \frac{(k \cdot p_2 \; k \cdot q_2 - k^2 \; q_2 \cdot p_2)^2}{m_2^2} \right. \right. \qquad x$$

$$x \quad \left. \left. \cos^2\phi_{3V} \right] \quad C_W \; - \; k^2 \, (\mu_2 \, p_{2V} \, k_V \sin\epsilon_V) \left[(\mu_2 \, k_V \, k_W) \right. \right. \qquad x$$

$$x \cos\phi_{3V} \sin\phi_{3V} \, S_W \quad \cos\phi_{4W} + \frac{(k \cdot p_2 \; k \cdot q_2 - k^2 \; q_2 \cdot p_2)}{m_2} \qquad x$$

$$\left. \left. x \quad \cos^2\phi_{3V} \quad S_W \quad \sin\phi_{4W} \right] \right\} \qquad (10.10)$$

Using these results it may be shown that:

$$d\sigma(d) \; = \left(\frac{1}{4\pi}\right)^2 \; |\bar{g}_o|^2 \; \frac{q_B}{4E_B^2 \, p_B} \; \frac{1}{(t-\mu^2)^2} \qquad x$$

$$x \quad \frac{2^j \, j! \, j!}{(2j)!} \quad \frac{(j+1)}{j} \quad k_V^{2j-2} \quad \frac{2^n \, n! \, n!}{(2n)!} \quad k_W^{2n-6} \quad x$$

$$x \left[\quad A \quad \left\{ \left[nP_n^o \, (C_W) \right]^2 + \left[P_n^1 \, (C_W) \right]^2 \right\} \right.$$

$$+ \quad B \quad (E_B \, p_B \, q_B \, \sin\Theta_B)^2 \quad \cos^2\phi_{3V} \left\{ \left[nP_n^o \, (C_W) \right]^2 + \left[P_n^1 \, (C_W) \right]^2 \right\}$$

$$+ \quad C \quad (E_B \, p_B \, q_B \, \sin\Theta_B) \quad \cos\phi_{3V} \left\{ a \quad \sin\phi_{3V} \cos\phi_{4W} + \right.$$

$$+ \quad b \quad \cos\phi_{3V} \quad \sin\phi_{4W} \quad x \quad \frac{(-1)}{(n-1)} \quad P_n^1 \, (C_W) \left\{ nP_n^o \, (C_W) - \frac{C_W}{S_W} P_n^1 \, (C_W) \right\}$$

$$+ \quad D \left\{ \left[a^2 \sin^2\phi_{3V} + b^2 \cos^2\phi_{3V} \right] \quad \frac{1}{(n-1)^2} \left[nP_n^o \, (C_W) - \frac{C_W}{S_W} P_n^1 \, (C_W) \right]^2 \right.$$

$$+ \left[a \sin\phi_{3V} \cos\phi_{4W} + b \cos\phi_{3V} \sin\phi_{4W} \right]^2 \left[\left\{ P_n^1 \, (C_W) \right\}^2 - \right.$$

$$\left. \left. - \quad \frac{nP_n^o \, (C_W) \, P_n^2 (C_W)}{(n-1)} \right] \right\} \quad +$$

$$+ \quad E \left\{ \left[a^2 \sin^2 \phi_{3V} + b^2 \cos^2 \phi_{3V} \right] \frac{1}{(n-1)^2} \left[nP_n^o (C_W) - \frac{C_W}{S_W} P_n^1 (C_W) \right]^2 \right.$$

$$+ \left[a \sin \phi_{3V} \sin \phi_{4W} - b \cos \phi_{3V} \cos \phi_{4W} \right]^2 \left[\left\{ P_n^1 (C_W) \right\}^2 - \right.$$

$$\left. \left. - \frac{nP_n^o (C_W) P_n^2 (C_W)}{(n-1)} \right] \right\} +$$

$$+ \quad F \left\{ \left[a^2 \sin^2 \phi_{3V} + b^2 \cos^2 \phi_{3V} \right] \frac{nP_n^o (C_W)}{(n-1)} \right. \qquad x$$

$$\left[nP_n^o (C_W) - \frac{C_W}{S_W} P_n^1 (C_W) \right] \quad (-1) \quad + \left[a \sin \phi_{3V} \cos \phi_{4W} + \right.$$

$$\left. + b \cos \phi_{3V} \sin \phi_{4W} \right]^2 \left[\left\{ P_n^1 (C_W) \right\}^2 - \frac{nP_n^o (C_W) P_n^2 (C_W)}{(n-1)} \right] \right\} +$$

$$+ G \left\{ \left[a^2 \sin^2 \phi_{3V} + b^2 \cos^2 \phi_{3V} \right] \frac{nP_n^o (C_W)}{(n-1)} \left[nP_n^o (C_W) - \right. \right.$$

$$\left. \left. - \frac{C_W}{S_W} P_n^1 (C_W) \right] \quad (-1) \quad + \left[a \sin \phi_{3V} \sin \phi_{4W} - b \cos \phi_{3V} \cos \phi_{4W} \right]^2 \right.$$

$$\times \left[\left\{ P_n^1 (C_W) \right\}^2 - \frac{n P_n^o (C_W)\, P_n^2 (C_W)}{(n-1)} \right] \right\} \times d\cos\Theta_B \; d\Phi_B \quad \times$$

$$\frac{\Gamma_V (2-3,4)}{\Gamma_V} \quad \frac{(2j+1)}{4\pi j(j+1)} \left[P_j^1 (C_V) \right]^2 \quad d C_V \; d\Phi_{3V} \quad \times$$

$$\times \frac{\Gamma_W (2-3,4)}{\Gamma_W} \quad \frac{1}{4\pi n} \quad d C_W \; d\Phi_{4W} \tag{10.11}$$

where $$a = (p_2 k_V k_W) \tag{10.12}$$

$$b = \frac{(k \cdot p_2 \; k \cdot q_2 - k^2 \, p_2 \cdot q_2)}{m_2} \tag{10.13}$$

and

$$A = |\bar{f}_2|^2 \left[-t + (m_1' - m_2)^2 \right] (p_2 k_V)^2 \tag{10.14}$$

$$B = \left\{ |\bar{f}_2|^2 \, 4 + 2 \, \mathrm{Re} \, (\bar{f}_2 \bar{f}_3^+) \; 2 (m_1' + m_2) + \bar{f}_3^{\,2} \right. \quad \times$$

$$\left. \times \left[-t + (m_1' + m_2)^2 \right] \right\} k_W^4$$

$$+ \left\{ 2 \, \mathrm{Re} \, (\bar{f}_1 \bar{f}_2^+) \; 2(m_1' + m_2) + 2 \, \mathrm{Re} \, (\bar{f}_1 \bar{f}_3^+) \right. \quad \times$$

$$\left. \times \left[-t + (m_1' + m_2)^2 \right] \right\} k_W^2 \quad \frac{k \cdot p_2}{p_2^2}$$

$$+ \left\{ |\bar{f}_1|^2 \left[-t + (m_1' + m_2)^2 \right] \right\} \left(\frac{k \cdot p_2}{p_2^2} \right)^2 \qquad (10.15)$$

$$C = \left\{ 2\,\mathrm{Re}\,(\bar{f}_1 \bar{f}_2^+)\, 2\,(m_1' + m_2) + 2\,\mathrm{Re}\,(\bar{f}_1 \bar{f}_3^+) \left[-t + (m_1' + m_2)^2 \right] \right\} k_W^2$$

$$+ \left\{ |\bar{f}_1|^2 \left[-t + (m_1' + m_2)^2 \right] \right\} 2 \frac{k \cdot p_2}{p_2^2}$$

$$+ \left\{ 2\,\mathrm{Re}\,(\bar{f}_5 \bar{f}_3^+) \right\} 2m_2^2 k_W^2 + \left\{ 2\,\mathrm{Re}\,(\bar{f}_5 \bar{f}_1^+) \right\} 2m_2^2 k_W^2 \frac{k \cdot p_2}{p_2^2}$$

$$- \left\{ 2\,\mathrm{Re}\,(\bar{f}_5 \bar{f}_2^+) \right\} 2 k_m^2{}_2 + \left\{ 2\,\mathrm{Re}\,(\bar{f}_5 \bar{f}_2^+) \right\} 2 m_2^2 m_1' \frac{k \cdot p_2}{p_2^2}$$

(10.16)

$$D = \left\{ |\bar{f}_1|^2 \left[-t + (m_1' + m_2)^2 \right] \right\} - \left\{ 2\,\mathrm{Re}\,(\bar{f}_5 \bar{f}_1^+)\; 2 m_2^2 k_W^2 \right\} \qquad (10.17)$$

$$E = \left\{ |f_5|^2 \left[-t + (m_1' - m_2)^2 \right] \right\} m_2^2 k_W^2 \qquad (10.18)$$

$$F = - \left\{ 2\,\mathrm{Re}\,(\bar{f}_1 \bar{f}_2^+) \right\} 2 m_2 k_W^2 + \left\{ 2\,\mathrm{Re}\,(\bar{f}_5 \bar{f}_1^+) \right\} 2 m_2^2 k_W^2 \qquad (10.19)$$

$$G = +\left\{2\ \mathrm{Re}\ (\bar{f}_5 \bar{f}^{+}_2)\right\} 2\ m_2\ (p_2.k) - \left\{2\ \mathrm{Re}\ (\bar{f}_5 \bar{f}^{+}_2)\right\} 2\ m_1'\ m_2^2 \qquad (10.20)$$

The factors $\Gamma_V(2-3,4)\ /\Gamma_V$ and $\Gamma_W(2-3,4)\ /\Gamma_W$ are the branching ratios of the decay processes (1.2) and (1.1) relative to all other possible decay modes of B_1 and F respectively.

The decay distribution of the boson resonance as a function of $\cos\Theta_{3V}$ is independent of all the other variables including the production process variables. This distribution is given by:

$$\frac{(2j+1)}{2j\ (j+1)}\ \left[\ P^1_j\ (\cos\Theta_{3V})\ \right]^2\ \ d\cos\Theta_{3V} \qquad (10.21)$$

As is to be expected the decay distributions as a function of Φ_{3V} and Φ_{4W} are both of the same form

$$\frac{1}{2\pi}\ \left[\ \bar{A}\ +\ \bar{B}\ \sin^2\Phi_{3V}\ \right]\ \ d\Phi_{3V} \qquad (10.22)$$

and

$$\frac{1}{2\pi}\ \left[\ \bar{C}\ +\ \bar{D}\ \sin^2\Phi_{4W}\ \right]\ d\Phi_{4W} \qquad (10.23)$$

The decay distribution of the fermion resonance as a function of $\cos\Theta_{4W}$ is obtained from (10.11) by integrating over C_V, Φ_{3V} and Φ_{4W}. The result is:

$$\frac{1}{2n}\left[\ \bar{E}\left\{\left[nP_n^o\ (\cos\Theta_{4W})\right]^2 + \left[P_n^1\ (\cos\Theta_{4W})\right]^2\right\}\right.$$

$$+\ \bar{F}\left\{\frac{1}{(n-1)^2}\left[\ nP_n^o\ (\cos\Theta_{4W}) - \frac{\cos\Theta_{4W}}{\sin\Theta_{4W}}\ P_n^1\ (\cos\Theta_{4W})\right]^2\right.$$

$$\left.\left.+\ \frac{1}{2}\left[\left\{P_n^1\ (\cos\Theta_{4W})\right\}^2 - \frac{nP_n^o\ (\cos\Theta_{4W})\ P_n^2\ (\cos\Theta_{4W})}{(n-1)}\right]\right\}\right]\quad x$$

$$d\cos\Theta_{4W} \qquad (10.24)$$

where $\bar{A}$, $\bar{B}$, ... $\bar{F}$ depend in a complicated way on the various coupling constants and their associated form factors. They thus depend on the production process variables and are, in particular, arbitrary functions of t.

It should be noted that the distributions (10.21) and (10.22) are independent of the decay mode of the fermion resonance and the distributions (10.23) and (10.24) are independent of the decay mode of the boson resonance. Thus the first two results apply to the general two stage process:

$$P + N \rightarrow F + B_1 \qquad (10.25a)$$

$$B_1 \rightarrow P + P \qquad (10.25b)$$

where F is an arbitrary spin fermion state, B_1 is a spin j boson resonance of normal parity and the production process is mediated by the exchange of a vector meson.

Similarly the last two results apply to the general two stage process:

$$P + N \rightarrow F + B \qquad (10.26a)$$

$$F \rightarrow N + P \qquad (10.26b)$$

where B is an arbitrary spin boson state of normal parity, F is a spin J, $(J = n - \frac{1}{2})$, fermion state of arbitrary parity and the production process is mediated by the exchange of a vector meson. The state B could be a stable particle or a resonance which might for instance decay into two or three pseudoscalar particles or into a vector particle and one pseudoscalar particle.

(iii) Pseudovector particle exchange.

The invariant matrix element is given by:

$$\langle p_3 p_4\, q_3 q_4 \mid T \mid p_1 q_1 \rangle = \bar{u}(p_3)\gamma_e \frac{eE(w,p_2)}{(m_e)^n} (p_4 \rho)^n (-1)^n \quad x$$

$$\theta_{\lambda\rho}(p_2,n)(\not{p}_2 + m_2) \; x \; \frac{1}{(w-m_2^2) + im_2\Gamma_W} (k_\lambda)^{n-1} \quad x$$

$$\left[\frac{f_1 F_1(t,w,\not{p}_2)}{(m_{f_1})^{n-1}} g_{\alpha\lambda_1} + \frac{f_2 F_2(t,w,\not{p}_2)}{(m_{f_2})^{n}} \gamma_\alpha k_{\lambda_1} + \frac{f_3 F_3(t,w,\not{p}_2)}{(m_{f_3})^{n+1}} \right. \quad x$$

$$x \quad p_{2\alpha} k_{\lambda_1} + \frac{f_4 F_4(t,w,\not{p}_2)}{(m_{f_4})^{n+1}} k_\alpha k_{\lambda_1} + \frac{f_5 F_5(t,w,\not{p}_2)}{(m_{f_5})^{n+1}} \quad x$$

$$\left. \epsilon_{\alpha\lambda_1\sigma\tau} k_\sigma p_{2\tau} \gamma_5 \right] \gamma_f u(p_1) \; x \; \frac{1}{(t-\mu^2)} \quad (-1)\left[g_{\alpha\beta} - \frac{k_\alpha k_\beta}{\mu^2} \right]$$

$$x\left[\frac{g_1 G_1(t,v)}{(m_{g_1})^{j-2}} g_{\beta\mu_1} + \frac{g_2 G_2(t,v)}{(m_{g_2})^{j}} q_{2\beta} k_{\mu_1} + \frac{g_3 G_3(t,v)}{(m_{g_3})^{j}} k_\beta k_{\mu_1} \right] \quad x$$

$$x \quad (k_\mu)^{j-1} (-1)^j \oint_{\mu\nu} (q_2, j) \frac{1}{(v-\mu_2^2) + i\,\mu_2 \Gamma_V} (q_4)^j \frac{hH(v)}{(m_h)^{j-1}} \qquad (10.27)$$

where $J = (n + \frac{1}{2})$.

Substituting (10.27) in (3.38), averaging over initial spin states, summing over final spin states and integrating over dv and dw gives, with the notation of (9.2) and the approximations (7.4) and (9.3) the result:

$$d\sigma(d) = \frac{1}{(4\pi)^4} \frac{q_B}{4E_B^2 p_B} \frac{1}{(t-\mu^2)^2} \frac{\Gamma_V(2-3,4)}{\Gamma_V} \frac{\Gamma_W(2-3,4)}{\Gamma_W} \quad x$$

$$x \left[\frac{2^j\, j!\, j!}{(2j)!} \; \frac{1}{(2j+1)} \; q_V^{2j} \right]^{-1} \left[\frac{2^n\, n!\, n!}{(2n)!} \; \frac{(2n+1)^2}{n^3} \; p_W^{2n-2} \right]^{-1} x$$

$$x \; T(r)\; d\cos\Theta_B \; d\phi_B \; d\cos\Theta_{3V} \; d\phi_{3V} \; d\cos\Theta_{4W} \; d\phi_{4W} \qquad (10.28)$$

where now $J = (n-\frac{1}{2})$ and $T(r)$ is given by (9.8) with r_μ given by (7.21) and s_μ by (9.10). The various coefficients M_{kk}, K_{kr}.... etc. appearing in (9.8) are given by (9.11) and the factors multiplying each of these coefficients are given by (A.33) - (A.34).

Using the definition (7.21) it follows that $k.r$, $p_2 r$ and $r.r^+$ are given by (7.23), (7.24) and (7.25) respectively. Using the notation $C_V = \cos\Theta_{3V}$, $S_V = \sin\Theta_{3V}$, $C_W = \cos\Theta_{4W}$ and $S_W = \sin\Theta_{4W}$ the following relationships may be derived:

$$\underline{r}_W \cdot \underline{k}_W = r_I \, k_W \qquad (10.29)$$

$$\frac{(\underline{r}_W \wedge \underline{k}_W).(\underline{p}_{4W} \wedge \underline{k}_W)}{p_W k_W} = \left[r_I \cos\phi_{4W} + r_D \sin\phi_{4W} \right] k_W \, S_W \qquad (10.30)$$

$$(\underline{r}_W \wedge \underline{k}_W).(\underline{r}_W^+ \wedge \underline{k}_W) = \left[|r_N|^2 + |r_P|^2 \right] k_W^2 \qquad (10.31)$$

$$\frac{\underline{p}_{4W} \cdot (\underline{r}_W \wedge \underline{k}_W)}{p_W} = -\left[r_N \sin\phi_{4W} - r_P \cos\phi_{4W} \right] k_W \, S_W \qquad (10.32)$$

$$\frac{(\underline{r}_W \wedge \underline{k}_W)\cdot(\underline{r}_W^+ \wedge \underline{p}_{4W})\ k_W}{p_W} = \Big\{ (|r_I|^2 + |r_N|^2)\ k_W^2\ C_W$$

$$- (r_N \cos\phi_{4W} + r_P \sin\phi_{4W})\ r_I^+\ k_W^2\ S_W \qquad (10.33)$$

and

$$(\underline{r}_W \wedge \underline{k}_W)\cdot(p_{4W} \wedge \underline{k}_W)\ \frac{k_W}{p_W}\ (p_2\cdot r^+) + (\underline{r}_W \wedge \underline{k}_W)(\underline{r}_W^+ \wedge \underline{p}_{4W})\ \frac{k_W}{p_W}\ (p_2\cdot k)$$

$$= \Big[(p_2\cdot k)\ (|r_N|^2 + |r_P|^2)\ k_W^2\ C_W$$

$$+ (r_N \cos\phi_{4W} + r_P \sin\phi_{4W})\ (p_2\cdot k\ r^+\cdot k - k^2\ p_2\cdot r^+)\ k_W S_W \Big] \qquad (10.34)$$

where

$$r_I = \left[\frac{2^j\ j!\ j!}{(2j)!}\ \frac{1}{j}\ k_V^{j-2}\ q_V^j \right] \times k_W$$

$$\times \Big[\Big\{ \bar{g}_k\ k_V^2 - \bar{g}_{p_2}\ \frac{k\cdot p_2}{p_2^2}\ k_V^2 + \bar{g}_1\ \frac{k\cdot p_2}{p_2^2}\ p_{2V}\ k_V \cos\epsilon_V \Big\}\ j\ P_j^o\ (C_V)$$

$$+ \Big\{ \bar{g}_1\ \frac{k\cdot p_2}{p_2^2}\ p_{2V}\ k_V \sin\epsilon_V \Big\}\ \sin\phi_{3V}\ P_j^1\ (C_V) \Big] \qquad (10.35)$$

$$r_N = -\left[\frac{2^j\ j!\ j!}{(2j)!}\ \frac{1}{j}\ k_V^{j-2}\ q_V^j \right] \left[\bar{g}_1\ k_V \cos\phi_{3V}\ P_j^1\ (C_V) \right] \qquad (10.36)$$

$$r_{\mathrm{P}} = -\left[\frac{2^j\, j!\, j!}{(2j)!}\ \frac{1}{j}\ k_V^{j-2}\quad q_V^j\right]\ \frac{1}{k_W}$$

$$\times\left[\left\{\bar{g}_1\ \frac{k\cdot q_2}{\mu_2^2} + \bar{g}_2\, k_V^2\right\}\ q_W k_W \sin\epsilon_W\quad j\, P_j^o\,(C_V)\right.$$

$$\left. -\ \left\{\bar{g}_1\ \frac{(k\cdot p_2\ k\cdot q_2 - k^2\ p_2\cdot q_2)}{m_2}\ \frac{1}{\mu_2}\right\}\ \sin\phi_{3V}\ P_j^1\,(C_V)\right] \qquad (10.37)$$

Using these results it may be shown that:

$$d\sigma(d) = \frac{1}{(4\pi)^2}\ \frac{q_B}{4E_B\ p_B^2}\ \frac{1}{(t-\mu^2)^2}\quad \frac{2^j\, j!\, j!}{(2j)!}\ k_V^{2j-4}\ \frac{2^n\, n!\, n!}{(2n)!}\ k_W^{2n-6}\ \times$$

$$\times\left[A\left\{\left[n\,P_n^o\,(C_W)\right]^2 + \left[P_n^1\,(C_W)\right]^2\right\}\right.$$

$$+\quad B\cos\phi_{4W}\ \frac{(-1)}{(n-1)}\ P_n^1\,(C_W)\left\{n\,P_n^o\,(C_W) - \frac{C_W}{S_W}\ P_n^1\,(C_W)\right\}$$

$$+\quad C\sin\phi_{4W}\ \frac{(-1)}{(n-1)}\ P_n^1\,(C_W)\left\{n\,P_n^o\,(C_W) - \frac{C_W}{S_W}\ P_n^1\,(C_W)\right\}$$

$$+\quad D\left\{\left[|r_N|^2 + |r_P|^2\right]\ \frac{1}{(n-1)^2}\left[n\,P_n^o\,(C_W) - \frac{C_W}{S_W}\,P_n^1\,(C_W)\right]^2\right.$$

$$\left.+\ |r_N\cos\phi_{4W} + r_P\sin\phi_{4W}|^2\left[\left\{P_n^1\,(C_W)\right\}^2 - \frac{n\,P_n^o\,(C_W)\,P_n^2\,(C_W)}{(n-1)}\right]\right\}$$

$$+ \quad E\left\{\left[|r_N|^2 + |r_I|^2\right] \frac{1}{(n-1)^2}\left[n P_n^o(C_W) - \frac{C_W}{S_W} P_n^1(C_W)\right]^2\right.$$

$$\left. + |r_N \sin\phi_{4W} - r_P \cos\phi_{4W}|^2 \left[\left\{P_n^1(C_W)\right\}^2 - \frac{n P_n^o(C_W) P_n^2(C_W)}{(n-1)}\right]\right\}$$

$$+ \quad F\left\{\left[|r_N|^2 + |r_P|^2\right] \frac{n P_n^o(C_W)}{(n-1)}\left[n P_n^o(C_W) - \frac{C_W}{S_W} P_n^1(C_W)\right](-1)\right.$$

$$\left. + |r_N \cos\phi_{4W} + r_P \sin\phi_{4W}|^2 \left[\left\{P_n^1(C_W)\right\}^2 - \frac{n P_n^o(C_W) P_n^2(C_W)}{(n-1)}\right]\right\}$$

$$+ \quad G\left\{\left[|r_N|^2 + |r_P|^2\right] \frac{n P_n^o(C_W)}{(n-1)}\left[n P_n^o(C_W) - \frac{C_W}{S_W} P_n^1(C_W)\right](-1)\right.$$

$$\left.\left. + |r_N \sin\phi_{4W} - r_P \cos\phi_{4W}|^2 \left[\left\{P_n^1(C_W)\right\}^2 - \frac{n P_n^o(C_W) P_n^2(C_W)}{(n-1)}\right]\right\}\right] x$$

$$x \left[\frac{2^j j! j!}{(2j)!} \frac{1}{j} k_V^{j-2} q_V^j\right]^{-2} \quad d\cos\theta_B \, d\phi_B$$

$$x \frac{\Gamma_V(2-3,4)}{\Gamma_V} \frac{(2j+1)}{4\pi j^2} \, d C_V \, d\phi_{3V} \; x \frac{\Gamma_W(2-3,4)}{\Gamma_W} \frac{1}{4\pi n} \, d C_W \, d\phi_{4W} \qquad (10.38)$$

$$A = M_{kk} k_W^4 + 2 \, \mathrm{Re}\,(M_{kr} r_I) k_W^3 + M_{rr} |r_I|^2 k_W^2 \qquad (10.39)$$

$$B = + 2\,\mathrm{Re}\,\Big\{ M_{kr}\, r_N\, k_W^3 + M_{rr}\, r_N\, r_I^+\, k_W^2 + Q_{ks}\, r_N\, 2m_2^2\, k_W^4$$

$$- Q_{rs}\, r_N\, r_I^+\, 2m_2^2\, k_W^4 + R_{ks}\, r_N\, k_W\, (p_2.k)\, (p_2.k\, r^+.k - k^2\, r^+.p_2)$$

$$+ S_{ks}\, r_N\, r_I^+\, 2m_2^2\, k_W^2 \Big\} \qquad (10.40)$$

$$C = + 2\,\mathrm{Re}\,\Big\{ M_{kr}\, r_P\, k_W^3 + M_{rr}\, r_P\, r_I^+\, k_W^2 + Q_{ks}\, r_P\, 2m_2^2\, k_W^4$$

$$- Q_{rs}\, r_P\, r_I^+\, 2m_2^2\, k_W^4 + R_{ks}\, r_P\, k_W\, (p_2.k)\, (p_2.k\, r^+.k - k^2\, r^+.p_2)$$

$$+ S_{ks}\, r_P\, r_I^+\, 2m_2^2\, k_W^2 \Big\} \qquad (10.41)$$

$$D = M_{rr}\, k_W^2 + Q_{rs}\, 2m_2^2\, k_W^4 \qquad (10.42)$$

$$E = M_{ss}\, m_2^2\, k_W^4 \qquad (10.43)$$

$$F = - 2\,\mathrm{Re}\Big\{ N_{kr}\, 2k_W^4 + Q_{rs}\, 2m_2^2\, k_W^4 \Big\} \qquad (10.44)$$

$$G = - 2\,\mathrm{Re}\,\Big\{ R_{ks}\, (-2)\, (p_2.k)\, k_W^2 + S_{ks}\, 2m_2^2\, k_W^2 \Big\} \qquad (10.45)$$

The factors $\Gamma_V(2-3,4)/\Gamma_V$ and $\Gamma_W(2-3,4)/\Gamma_W$ are the branching ratios of the decay processes (1.2) and (1.1) relative to all other possible decay modes of B_1 and F respectively.

The decay distribution as a function of any one of the variables C_V, ϕ_{3V}, C_W, and ϕ_{4W} is obtained from (10.38) by integrating over the other three variables. These distributions are of the form:

$$\frac{(2j+1)}{2j^2}\left[\bar{A}\left\{jP_j^0(\cos\Theta_{3V})\right\}^2 + \bar{B}\left\{P_j^1(\cos\Theta_{3V})\right\}^2\right] d\cos\Theta_{3V} \tag{10.46}$$

$$\frac{1}{2\pi}\left[\bar{C} + \bar{D}\sin^2\phi_{3V}\right] d\phi_{3V} \tag{10.47}$$

$$\frac{1}{2n}\left[\bar{E}\left\{\left[nP_n^0(\cos\Theta_{4W})\right]^2 + \left[P_n^1(\cos\Theta_{4W})\right]^2\right\} +\right.$$

$$+ \bar{F}\left\{\frac{1}{(n-1)^2}\left[nP_n^0(\cos\Theta_{4W}) - \frac{\cos\Theta_{4W}}{\sin\Theta_{4W}}P_n^1(\cos\Theta_{4W})\right]^2 +\right.$$

$$\left.\left.\left. + \frac{1}{2}\left[\left\{P_n^1(\cos\Theta_{4W})\right\}^2 - \frac{nP_n^0(\cos\Theta_{4W})\,P_n^2(\cos\Theta_{4W})}{(n-1)}\right]\right\}\right] x$$

$$x\, d\cos\Theta_{4W} \tag{10.48}$$

$$\frac{1}{2\pi}\left[\bar{G} + \bar{H}\sin^2\phi_{4W}\right] d\phi_{4W} \tag{10.49}$$

where $\bar{A},\bar{B},....\bar{H}$ depend in a very complicated way on the various coupling constants and their associated form factors. They thus depend on the production process variables and are, in particular, arbitrary functions of t.

As before the distributions (10.46) and (10.47) apply to the general process (10.25) provided that the production process is mediated by the exchange of a pseudovector meson. Similarly (10.48) and (10.49) apply to the general processes (10.26) provided once again that the production process is mediated by the exchange of a pseudovector meson.

11. RESULTS FOR SPECIFIC SPIN AND PARITY RESONANCES.

In this section the general results of the previous sections are applied to processes in which resonances of specific spin and parity are produced by a peripheral mechanism and subsequently decay freely. The results are tabulated as follows.

TABLE 2

The decay angular distribution of a normal parity boson resonance B_1 produced by pseudoscalar particle exchange in the process (1.6) or (1.9).

SPIN	PARITY	DECAY DISTRIBUTION $B_1 \rightarrow P + P$
0	+	$\frac{1}{2\pi}\,\frac{1}{2}$
1	−	$\frac{1}{2\pi}\,\frac{3}{2}\,c_V^2$
2	+	$\frac{1}{2\pi}\,\frac{5}{8}\,(3c_V^2-1)^2$
3	−	$\frac{1}{2\pi}\,\frac{7}{8}\,c_V^2\,(5c_V^2-3)^2$
j	$(-1)^j$	$\frac{1}{2\pi}\,\frac{(2j+1)}{2}\,\left[P_j^{\bullet}(c_V)\right]^2$

TABLE 3

The decay angular distribution of a normal parity boson resonance B_1 produced by vector particle exchange in the process (1.6) or (1.9).

SPIN	PARITY	DECAY DISTRIBUTION $B_1 \to P + P$
0	+	PRODUCTION FORBIDDEN
1	−	$\frac{1}{2\pi}(A\cos^2\phi_{3V} + B)\,\frac{3}{4}\,S_V^2$
2	+	$\frac{1}{2\pi}(A\cos^2\phi_{3V} + B)\,\frac{15}{4}\,S_V^2 C_V^2$
3	−	$\frac{1}{2\pi}(A\cos^2\phi_{3V} + B)\,\frac{21}{32}\,S_V^2(5C_V^2 - 1)^2$
j	$(-1)^j$	$\frac{1}{2\pi}(A\cos^2\phi_{3V} + B)\,\frac{(2j+1)}{2j(j+1)}\left[P_j'(C_V)\right]^2$

TABLE 4

The decay angular distribution of a normal parity boson resonance B_1 produced by pseudovector particle exchange in the process (1.6) or (1.9).

SPIN PARITY DECAY DISTRIBUTION $B_1 \to \rho + \rho$

$$0 \quad + \quad \frac{1}{2\pi}\,\frac{1}{2}$$

$$1 \quad - \quad \frac{1}{2\pi}\,\frac{3}{2}\left\{ A c_V^2 + B \sin\phi_{3V}\, s_V c_V + (C \sin^2\phi_{3V} + D)\, s_V^2 \right\}$$

$$2 \quad + \quad \frac{1}{2\pi}\,\frac{5}{8}\left\{ A (3c_V^2-1)^2 + B \sin\phi_{3V}\, s_V 3c_V (3c_V^2-1) + (C \sin^2\phi_{3V} + D)\, s_V^2\, 9 c_V^2 \right\}$$

$$3 \quad - \quad \frac{1}{2\pi}\,\frac{7}{8}\left\{ A c_V^2 (5c_V^2-3)^2 + B \sin\phi_{3V}\, s_V c_V (5c_V^2-3)(5c_V^2-1) + (C \sin^2\phi_{3V} + D)\, s_V^2 (5c_V^2-1)^2 \right\}$$

TABLE 4 (Contd.)

$$j \quad (-1)^j \quad \frac{1}{2\pi} \frac{(2j+1)}{2j^2} \left\{ A \left[j P_j^0(c_v) \right]^2 \right.$$

$$+ B \sin\phi_{sv} \left[j P_j^0(c_v) \right] \left[P_j^1(c_v) \right]$$

$$\left. + (C \sin^2\phi_{sv} + D) \left[P_j^1(c_v) \right]^2 \right\}$$

TABLE 5(a)

The decay angular distribution of a normal parity boson resonance B_3 produced by pseudoscalar particle exchange in the process (1.7).

SPIN	PARITY	DECAY DISTRIBUTION $B_3 \rightarrow V_1 + P$
0	+	DECAY PROCESS FORBIDDEN
1	−	$\frac{1}{2\pi}\frac{3}{4}s_V^2$
2	+	$\frac{1}{2\pi}\frac{15}{4}s_V^2 c_V^2$
3	−	$\frac{1}{2\pi}\frac{21}{32}s_V^2(5c_V^2-1)^2$
j	$(-1)^j$	$\frac{1}{2\pi}\frac{(2j+1)}{2j(j+1)}\left[P_j'(c_V)\right]^2$

TABLE 5(b)

The decay distribution with respect to the variables ψ_{3V} and ω_{5V} of a normal parity boson resonance B_3 produced by pseudoscalar particle exchange in the process (1.7).

SPIN	PARITY	DECAY DISTRIBUTION $B_3 \to P+P+P$
0	+	DECAY PROCESS FORBIDDEN
J	$(-1)^J$	$\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3V}^3} q_{3V}^2 q_{5V}^2 \sin^2\theta_{35V}\cos^2\psi_{3V}$

TABLE 6(a)

The decay angular distribution of a normal parity boson resonance B_3 produced by vector particle exchange in the process (1.7).

SPIN	PARITY	DECAY DISTRIBUTION $B_3 \rightarrow V_1 + P$
0	+	PRODUCTION FORBIDDEN
1	−	$\frac{1}{2\pi}\frac{3}{8}\left\{A\left[\cos^2\phi_{3V}\,c_V^2+\sin^2\phi_{3V}\right]+B\left[c_V^2+1\right]\right\}$
2	+	$\frac{1}{2\pi}\frac{5}{8}\left\{A\left[\cos^2\phi_{3V}\,(2c_V^2-1)^2+\sin^2\phi_{3V}\,c_V^2\right]+B\left[4c_V^4-3c_V^2+1\right]\right\}$
3	−	$\frac{1}{2\pi}\frac{7}{128}\left\{A\left[\cos^2\phi_{3V}\,c_V^2\,(15c_V^2-11)^2+\sin^2\phi_{3V}\,(5c_V^2-1)^2\right]+B\left[225c_V^6-305c_V^4+111c_V^2+1\right]\right\}$

TABLE 6(a) (Contd.)

$$j \quad (-1)^j \quad \frac{1}{2\pi} \frac{(2j+1)}{2j^2(j+1)^2} \left\{ \left[A \sin^2 \phi_{2v} + B \right] \left[\frac{1}{s_v} P_j^1(c_v) \right]^2 \right.$$

$$\left. + \left[A \cos^2 \phi_{2v} + B \right] \left[j(j+1) P_j^0(c_v) - \frac{c_v}{s_v} P_j^1(c_v) \right]^2 \right\}$$

TABLE 6(b)

The decay distribution with respect to the variables ψ_{3v} and ω_{5v} of a normal parity boson resonance B_3 produced by vector particle exchange in the process (1.7).

SPIN PARITY DECAY DISTRIBUTION $B_3 \longrightarrow P + P + P$

0	+	PRODUCTION FORBIDDEN
1	−	$\frac{1}{\pi} \frac{3\mu_4}{4 q_{5x}^3 q_{3v}^3} \left[Q_1(\omega_{5v})\right]^2 \frac{1}{4}(3 - 2\cos^2\psi_{3v})$
2	+	$\frac{1}{\pi} \frac{3\mu_4}{4 q_{5x}^3 q_{3v}^3} \left[Q_1(\omega_{5v})\right]^2 \frac{1}{12}(5 + 2\cos^2\psi_{3v})$
3	−	$\frac{1}{\pi} \frac{3\mu_4}{4 q_{5x}^3 q_{3v}^3} \left[Q_1(\omega_{5v})\right]^2 \frac{1}{24}(7 + 10\cos^2\psi_{3v})$
j	$(-1)^j$	$\frac{1}{\pi} \frac{3\mu_4}{4 q_{5x}^3 q_{3v}^3} \left[Q_1(\omega_{5v})\right]^2 \times \frac{1}{2j(j+1)} \left[(2j+1) + (j^2 - j - 1) 2\cos^2\psi_{3v}\right]$

$$Q_1(\omega_{5v}) = q_{3v} q_{5v} \sin\theta_{25v}$$

TABLE 7(a)

The decay angular distribution of a normal parity boson resonance B_3 produced by pseudovector particle exchange in the process (1.7).

SPIN	PARITY	DECAY DISTRIBUTION $B_3 \rightarrow V_1 + P$
0	+	DECAY PROCESS FORBIDDEN
1	−	$\frac{1}{2\pi}\frac{3}{4}\left\{A s_v^2 + B \sin\phi_{3v}\, s_v c_v + C\left[\sin^2\phi_{3v}\, c_v^2 + \cos^2\phi_{3v}\right] + D\left[c_v^2 + 1\right]\right\}$
2	+	$\frac{1}{2\pi}\frac{15}{16}\left\{A s_v^2 4c_v^2 + B \sin\phi_{3v}\, s_v 2c_v (2c_v^2 - 1) + C\left[\sin^2\phi_{3v}(2c_v^2-1)^2 + \cos^2\phi_{3v}\, c_v^2\right] + D\left[4c_v^4 - 3c_v^2 + 1\right]\right\}$

TABLE 7(a) (Contd.)

$$3 \quad - \quad \frac{1}{2\pi}\frac{7}{96}\Big\{ A s_v^2 9 (5c_v^2-1)^2$$

$$+B \sin\phi_{3v}\, s_v 3 c_v (5c_v^2-1)(15c_v^2-11)$$

$$+C\left[\sin^2\phi_{3v}\, c_v^2 (15c_v^2-11)^2 + \cos^2\phi_{3v}(5c_v^2-1)^2\right]$$

$$+D\left[225 c_v^6 - 305 c_v^4 + 111 c_v^2 + 1\right]\Big\}$$

$$j \quad (-1)^j \quad \frac{1}{2\pi}\frac{(2j+1)}{2j^3(j+1)}\Big\{ A\left[j P_j^1(c_v)\right]^2$$

$$+B \sin\phi_{3v}\left[j P_j^1(c_v)\right]\left[j(j+1)P_j^0(c_v) - \frac{c_v}{s_v} P_j^1(c_v)\right]$$

$$+\left[C\sin^2\phi_{3v}+D\right]\left[j(j+1)P_j^0(c_v) - \frac{c_v}{s_v} P_j^1(c_v)\right]^2$$

$$+\left[C\cos^2\phi_{3v}+D\right]\left[\frac{1}{s_v} P_j^1(c_v)\right]^2\Big\}$$

TABLE 7(b)

The decay distribution with respect to the variables ψ_{3v} and ω_{5v} of a normal parity boson resonance B_3 produced by pseudovector particle exchange in the process (1.7).

SPIN PARITY DECAY DISTRIBUTION $B_3 \longrightarrow \rho + \rho + \rho$

0 + Decay process forbidden

1 − $$\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3v}^3}\left[Q_1(\omega_{5v})\right]^2\left\{A\cos^2\psi_{3v}+B\frac{1}{2}(3-2\cos^2\psi_{3v})\right\}$$

2 + $$\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3v}^3}\left[Q_1(\omega_{5v})\right]^2\left\{A\cos^2\psi_{3v} + B\frac{1}{8}(5+2\cos^2\psi_{3v})\right\}$$

3 − $$\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3v}^3}\left[Q_1(\omega_{5v})\right]^2\left\{A\cos^2\psi_{3v} + B\frac{1}{18}(7+10\cos^2\psi_{3v})\right\}$$

j $(-1)^j$ $$\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3v}^3}\left[Q_1(\omega_{5v})\right]^2\left\{A\cos^2\psi_{3v} + B\frac{1}{2j^2}\left[(2j+1)+(j^2-j-1)2\cos^2\psi_{3v}\right]\right\}$$

$$Q_1(\omega_{5v}) = q_{3v}\,q_{5v}\sin\theta_{35v}$$

TABLE 8(a)

The decay angular distribution of an abnormal parity boson resonance B_3 produced by scalar particle exchange in the process (1.7).

SPIN	PARITY	DECAY DISTRIBUTION $B_3 \rightarrow V_1 + P$
0	$-$	$\frac{1}{2\pi}\,\frac{1}{2}$
1	$+$	$\frac{1}{2\pi}\,\frac{3}{2}\,\frac{\{(\bar{h}_1)^2 s_V^2 + (\bar{h}_2)^2 c_V^2\}}{\{(\bar{h}_1)^2 2 + (\bar{h}_2)^2\}}$
2	$-$	$\frac{1}{2\pi}\,\frac{5}{4}\,\frac{\{(\bar{h}_1)^2 s_V^2 9 c_V^2 + (\bar{h}_2)^2 (3c_V^2-1)^2\}}{\{(\bar{h}_1)^2 3 + (\bar{h}_2)^2 2\}}$
3	$+$	$\frac{1}{2\pi}\,\frac{21}{8}\,\frac{\{(\bar{h}_1)^2 s_V^2 (5c_V^2-1)^2 + (\bar{h}_2)^2 c_V^2 (5c_V^2-3)^2\}}{\{(\bar{h}_1)^2 4 + (\bar{h}_2)^2 3\}}$
j	$(-1)^{j+1}$	$\frac{1}{2\pi}\,\frac{(2j+1)}{2j}\,\frac{\{(\bar{h}_1)^2 [P_j'(c_V)]^2 + (\bar{h}_2)^2 [j P_j^*(c_V)]^2\}}{\{(\bar{h}_1)^2 (j+1) + (\bar{h}_2)^2 j\}}$

TABLE 8(b)

The decay distribution with respect to the variables ψ_{3V} and ω_{5V} of an abnormal parity boson resonance B_3 produced by scalar particle exchange in the process (1.7).

SPIN PARITY DECAY DISTRIBUTION $B_3 \longrightarrow p+p+p$.

0	−	$\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3V}^3} Q_2^2(\omega_{5V})$
1	+	$\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3V}^3}\frac{\{(\bar{h}_1)^2 2\, Q_1^2(\omega_{5V})\sin^2\psi_{3V}+(\bar{h}_2)^2 Q_2^2(\omega_{5V})\}}{\{(\bar{h}_1)^2 2+(\bar{h}_2)^2\}}$
2	−	$\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3V}^3}\frac{\{(\bar{h}_1)^2 3\, Q_1^2(\omega_{5V})\sin^2\psi_{3V}+(\bar{h}_2)^2 2\, Q_2^2(\omega_{5V})\}}{\{(\bar{h}_1)^2 3+(\bar{h}_2)^2 2\}}$
3	+	$\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3V}^3}\frac{\{(\bar{h}_1)^2 4\, Q_1^2(\omega_{5V})\sin^2\psi_{3V}+(\bar{h}_2)^2 3\, Q_2^2(\omega_{5V})\}}{\{(\bar{h}_1)^2 4+(\bar{h}_2)^2 3\}}$
j	$(-1)^{j+1}$	$\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3V}^3}\frac{\{(\bar{h}_1)^2(j+1)\, Q_1^2(\omega_{5V})\sin^2\psi_{3V}+(\bar{h}_2)^2 j\, Q_2^2(\omega_{5V})\}}{\{(\bar{h}_1)^2(j+1)+(\bar{h}_2)^2 j\}}$

$$Q_1(\omega_{5V}) = q_{3V} q_{5V} \sin\theta_{35V}$$

$$Q_2(\omega_{5V}) = (\omega_{5x}\omega_{4V} - \mu_4\omega_{5V})$$

TABLE 9(a)

The decay angular distribution of an abnormal parity boson resonance B_3 produced by vector particle exchange in the process (1.7).

SPIN PARITY DECAY DISTRIBUTION $B_3 \longrightarrow V_1 + P$

0^- $\quad \frac{1}{2\pi}\,\frac{1}{2}$

1^+ $\quad \frac{1}{2\pi}\,\frac{1}{\{(\bar{h}_1)^2 2 + (\bar{h}_2)^2\}}\,\frac{3}{2}\Big\{ A\left[(\bar{h}_1)^2 s_V^2 + (\bar{h}_2)^2 c_V^2\right]$

$+ B\left[(\bar{h}_1)^2 \sin\phi_{3V}\, s_V c_V + (\bar{h}_2)^2 \sin\phi_{2V}\, s_V c_V\right]$

$+ C\left[(\bar{h}_1)^2\{\sin^2\phi_{3V}\, c_V^2 + \cos^2\phi_{3V}\} + (\bar{h}_2)^2 \sin^2\phi_{2V}\, s_V^2\right]$

$+ D\left[(\bar{h}_1)^2\{c_V^2 + 1\} + (\bar{h}_2)^2 s_V^2\right]\Big\}$

2^- $\quad \frac{1}{2\pi}\,\frac{1}{\{(\bar{h}_1)^2 3 + (\bar{h}_2)^2 2\}}\,\frac{5}{4}\times$

$\times\Big\{ A\left[(\bar{h}_1)^2 s_V^2\, 9 c_V^2 + (\bar{h}_2)^2 (3c_V^2 - 1)^2\right]$

$+ B \sin\phi_{3V}\, s_V 3 c_V \left[(\bar{h}_1)^2 \tfrac{3}{2}(2c_V^2 - 1) + (\bar{h}_2)^2 (3c_V^2 - 1)\right]$

$+ C\left[(\bar{h}_1)^2 \tfrac{9}{4}\{\sin^2\phi_{3V}\,(2c_V^2 - 1)^2 + \cos^2\phi_{2V}\, c_V^2\}\right.$

$\left. \quad + (\bar{h}_2)^2 \sin^2\phi_{3V}\, s_V^2\, 9 c_V^2\right]$

$+ D\left[(\bar{h}_1)^2 \tfrac{9}{4}\{4c_V^4 - 3c_V^2 + 1\} + (\bar{h}_2)^2 s_V^2\, 9 c_V^2\right]\Big\}$

TABLE 9(a) (Contd.)

$$3 \quad + \quad \frac{1}{2\pi}\left\{\frac{1}{(\bar{h}_1)^2 4 + (\bar{h}_2)^2 3}\right\}\frac{21}{8} \times$$

$$\times\Big\{ A\left[(\bar{h}_1)^2 s_V^2 (5c_V^2-1) + (\bar{h}_2)^2 c_V^2 (5c_V^2-3)\right]$$

$$+ B \sin\phi_{3V} s_V c_V (5c_V^2-1)\left[(\bar{h}_1)^2 \tfrac{1}{3}(15c_V^2-11) + (\bar{h}_2)^2 (5c_V^2-3)\right]$$

$$+ C\left[(\bar{h}_1)^2\left\{\sin^2\phi_{3V}\tfrac{1}{9}c_V^2(15c_V^2-11)^2 + \cos^2\phi_{3V}\tfrac{1}{9}(5c_V^2-1)^2\right\}\right.$$

$$\left. + (\bar{h}_2)^2 \sin^2\phi_{3V} s_V^2 (5c_V^2-1)^2\right]$$

$$+ D\left[(\bar{h}_1)^2\tfrac{1}{9}\left\{225c_V^6 - 305c_V^4 + 111c_V^2 + 1\right\} + (\bar{h}_2)^2 s_V^2 (5c_V^2-1)^2\right]\Big\}$$

$$j \quad (-1)^{j+1} \quad \frac{1}{2\pi}\left\{\frac{1}{(\bar{h}_1)^2(j+1) + (\bar{h}_2)^2 j}\right\}\frac{(2j+1)}{2j} \times$$

$$\times\Big\{ A\left[(\bar{h}_1)^2\left\{P_j^1(c_V)\right\}^2 + (\bar{h}_2)^2\left\{j P_j^0(c_V)\right\}^2\right]$$

$$+ B \sin\phi_{3V} P_j^1(c_V)\left[(\bar{h}_1)^2\tfrac{1}{j}\left\{j(j+1)P_j^0(c_V) - \frac{c_V}{s_V}P_j^1(c_V)\right\}\right.$$

$$\left. + (\bar{h}_2)^2 j P_j^0(c_V)\right]$$

$$+\left[C\sin^2\phi_{3V} + D\right]\left[(\bar{h}_1)^2\tfrac{1}{j^2}\left\{j(j+1)P_j^0(c_V) - \frac{c_V}{s_V}P_j^1(c_V)\right\}^2\right.$$

$$\left. + (\bar{h}_2)^2\left\{P_j^1(c_V)\right\}^2\right]$$

$$+\left[C\cos^2\phi_{3V} + D\right]\left[(\bar{h}_1)^2\tfrac{1}{j^2}\left\{\tfrac{1}{s_V}P_j^1(c_V)\right\}^2\right]\Big\}$$

TABLE 9 (b)

The decay distribution with respect to the variables ψ_{3V} and ω_{5V} of an abnormal parity boson B_3 produced by vector particle exchange in the process (1.7).

SPIN PARITY DECAY DISTRIBUTION $B_3 \longrightarrow P+P+P$

0 $-$ $\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3V}^3} Q_2^2(\omega_{5V})$

1 $+$ $\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3V}^3}\frac{1}{\{(\bar{h}_1)^2 2+(\bar{h}_1)^2\}}\times$

$\times\{A[(\bar{h}_1)^2 2\,Q_1^2(\omega_{5V})\sin^2\psi_{3V}+(\bar{h}_2)^2 Q_2^2(\omega_{5V})]$

$+B[(\bar{h}_1)^2 Q_1^2(\omega_{5V})(3-2\sin^2\psi_{3V})+(\bar{h}_2)^2 Q_2^2(\omega_{5V})2]\}$

2 $-$ $\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3V}^3}\frac{1}{\{(\bar{h}_1)^2 3+(\bar{h}_2)^2 2\}}\times$

$\times\{A[(\bar{h}_1)^2 3\,Q_1^2(\omega_{5V})\sin^2\psi_{3V}+(\bar{h}_2)^2 2\,Q_2^2(\omega_{5V})]$

$+B[(\bar{h}_1)^2 Q_1^2(\omega_{5V})\frac{3}{8}(5+2\sin^2\psi_{3V})+(\bar{h}_2)^2 Q_2^2(\omega_{5V})3]\}$

3 $+$ $\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3V}^3}\frac{1}{\{(\bar{h}_1)^2 3+(\bar{h}_2)^2 2\}}\times$

$\times\{A[(\bar{h}_1)^2 4\,Q_1^2(\omega_{5V})\sin^2\psi_{3V}+(\bar{h}_2)^2 3\,Q_2^2(\omega_{5V})]$

$+B[(\bar{h}_1)^2 Q_1^2(\omega_{5V})\frac{2}{9}(7+10\sin^2\psi_{3V})+(\bar{h}_2)^2 Q_2^2(\omega_{5V})4]\}$

TABLE 9(b) (Contd.)

$$j \qquad (-1)^{j+1} \frac{1}{\pi} \frac{3\mu_4}{4 q_{sx}^3 q_{sv}^3} \frac{1}{\{(\bar{n}_1)^2(j+1) + (\bar{n}_2)^2 j\}} \times$$

$$\times \Big\{ A\left[(\bar{n}_1)^2(j+1) Q_1^2(\omega_{sv}) \sin^2\psi_{sv} + (\bar{n}_2)^2 j\, Q_2^2(\omega_{sv})\right]$$

$$+ B\left[(\bar{n}_1)^2 Q_1^2(\omega_{sv}) \frac{(j+1)}{2j^2}\{(2j+1) + (j^2-j-1) 2\sin^2\psi_{sv}\}\right.$$

$$\left. + (\bar{n}_2)^2 Q_2^2(\omega_{sv})(j+1)\right] \Big\}$$

TABLE 10 (a)

The decay angular distribution of an abnormal parity boson resonance B_3 produced by pseudovector particle exchange in the process (1.7).

SPIN PARITY DECAY DISTRIBUTION $B_3 \rightarrow V_1 + P$

0 − PRODUCTION FORBIDDEN

1 +

$$\frac{1}{2\pi} \frac{1}{\{(\bar{h}_1)^2 2 + (\bar{h}_2)^2\}} \frac{3}{4} \times$$

$$\times \Big\{ A \big[(\bar{h}_1)^2 \{ \cos^2\phi_{3V}\, c_V^2 + \sin^2\phi_{3V} \} + (\bar{h}_2)^2 \cos^2\phi_{3V}\, s_V^2 \big] + B \big[(\bar{h}_1)^2 (c_V^2 + 1) + (\bar{h}_2)^2 s_V^2 \big] \Big\}$$

2 −

$$\frac{1}{2\pi} \frac{1}{\{(\bar{h}_1)^2 3 + (\bar{h}_2)^2 2\}} \frac{15}{8} \times$$

$$\times \Big\{ A \big[(\bar{h}_1)^2 \{ \cos^2\phi_{3V} (2c_V^2 - 1)^2 + \sin^2\phi_{3V} c_V^2 \} + (\bar{h}_2)^2 \cos^2\phi_{3V}\, s_V^2\, 4c_V^2 \big] + B \big[(\bar{h}_1)^2 (4c_V^4 - 3c_V^2 + 1) + (\bar{h}_2)^2 s_V^2\, 4c_V^2 \big] \Big\}$$

TABLE 10 (a) (Contd.)

$$3 \quad + \quad \frac{1}{2\pi} \frac{1}{\{(\bar{h}_1)^2 4 + (\bar{h}_2)^2 3\}} \frac{7}{32} \times$$

$$\times \Big\{ A \Big[(\bar{h}_1)^2 \{ \cos^2\phi_{3v} c_v^2 (15c_v^2-11)^2 + \sin^2\phi_{3v} (5c_v^2-1)^2 \}$$
$$+ (\bar{h}_2)^2 \cos^2\phi_{3v} s_v^2 \, 9 (5c_v^2-1)^2 \Big]$$
$$+ B \Big[(\bar{h}_1)^2 (225 c_v^6 - 305 c_v^4 + 111 c_v^2 + 1)$$
$$+ (\bar{h}_2)^2 s_v^2 \, 9 (5c_v^2-1)^2 \Big] \Big\}$$

$$j \quad (-1)^{j+1} \quad \frac{1}{2\pi} \frac{1}{\{(\bar{h}_1)^2 (j+1) + (\bar{h}_2)^2 j\}} \frac{(2j+1)}{2j^2(j+1)} \times$$

$$\times \Big\{ \Big[A\cos^2\phi_{3v} + B \Big] \Big[(\bar{h}_1)^2 \Big\{ j(j+1) P_j'(c_v) - \frac{c_v}{s_v} P_j'(c_v) \Big\}^2$$
$$+ (\bar{h}_2)^2 \Big\{ j P_j'(c_v) \Big\}^2 \Big]$$
$$+ \Big[A \sin^2\phi_{3v} + B \Big] \Big[(\bar{h}_1)^2 \Big\{ \frac{1}{s_v} P_j'(c_v) \Big\}^2 \Big] \Big\}$$

TABLE 10(b)

The decay distribution with respect to the variables ψ_{3V} and ω_{5V} of an abnormal parity boson resonance B_3 produced by pseudovector particle exchange in the process (1.7)

SPIN	PARITY	DECAY DISTRIBUTION $B_3 \to P+P+P$
0	−	PRODUCTION FORBIDDEN.
1	+	$\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3V}^3}\frac{1}{\{(\bar{n}_1)^2 2+(\bar{n}_2)^2\}}\times$ $\times\left[(\bar{n}_1)^2 Q_1^2(\omega_{5V})\frac{1}{2}(3-2\sin^2\psi_{3V})+(\bar{n}_2)^2 Q_2^2(\omega_{5V})\right]$
2	−	$\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3V}^3}\frac{1}{\{(\bar{n}_1)^2 3+(\bar{n}_2)^2 2\}}\times$ $\times\left[(\bar{n}_1)^2 Q_1^2(\omega_{5V})\frac{1}{4}(5+2\sin^2\psi_{3V})+(\bar{n}_2)^2 2 Q_2^2(\omega_{5V})\right]$
3	+	$\frac{1}{\pi}\frac{3\mu_4}{4q_{5x}^3 q_{3V}^3}\frac{1}{\{(\bar{n}_1)^2 4+(\bar{n}_2)^2 3\}}\times$ $\times\left[(\bar{n}_1)^2 Q_1^2(\omega_{5V})\frac{1}{6}(7+10\sin^2\psi_{3V})+(\bar{n}_2)^2 3 Q_2^2(\omega_{5V})\right]$

TABLE 10(b) (Contd.)

$$j \qquad (-1)^{j+1} \cdot \frac{1}{\pi} \frac{3\mu_4}{4 q_{5x}^3 q_{3v}^3} \frac{1}{\{(\hbar_1)^2 (j+1) + (\hbar_2)^2 j\}} \times$$

$$\times \left[(\hbar_1)^2 Q_1^2(\omega_{5v}) \frac{1}{2j} \{(2j+1) + (j^2 - j - 1) 2 \sin^2 \psi_{3v}\} \right.$$

$$\left. + (\hbar_2)^2 j \, Q_2^2(\omega_{5v}) \right]$$

TABLE 11

The decay angular distribution of a fermion resonance F produced by spin zero exchange in the process (1.8)or(1.9).

SPIN	DECAY DISTRIBUTION $F \rightarrow N + P$
$\frac{1}{2}$	$\frac{1}{2\pi}\ \frac{1}{2}$
$\frac{3}{2}$	$\frac{1}{2\pi}\ \frac{1}{4}\ (3c_w^2+1)$
$\frac{5}{2}$	$\frac{1}{2\pi}\ \frac{3}{8}\ (5c_w^4-2c_w^2+1)$
$\frac{7}{2}$	$\frac{1}{2\pi}\ \frac{1}{32}\ (175c_w^6-165c_w^4+45c_w^2+9)$
$J=(n-\frac{1}{2})$	$\frac{1}{2\pi}\ \frac{1}{2n}\left[\{nP_n^0(c_w)\}^2+\{P_n^1(c_w)\}^2\right]$

TABLE 12

The decay angular distribution of a fermion resonance F produced by spin one exchange in the process (1.8)or(1.9).

SPIN DECAY DISTRIBUTION $F \longrightarrow N + P$

$\frac{1}{2}$ $\quad \frac{1}{2\pi} \frac{1}{2}$

$\frac{3}{2}$ $\quad \frac{1}{2\pi} \frac{1}{4} \{ A(3c_w^2+1) + [B\sin\phi_{4w} s_w + C\cos\phi_{4w} s_w] 3c_w$
$+ D(3\sin^2\phi_{4w} s_w^2 + 1) + E(3\cos^2\phi_{4w} s_w^2 + 1)$
$+ F(3\sin^2\phi_{4w} s_w^2 + 3c_w^2 - 1)$
$+ G(3\cos^2\phi_{4w} s_w^2 + 3c_w^2 - 1)\}$

$\frac{5}{2}$ $\quad \frac{1}{2\pi} \frac{3}{8} \{ A(5c_w^4 - 2c_w^2 + 1)$
$+ [B\sin\phi_{4w} s_w + C\cos\phi_{4w} s_w] c_w (5c_w^2 - 1)$
$+ D[\sin^2\phi_{4w} s_w^2 (5c_w^2 + 1) + c_w^2]$
$+ E[\cos^2\phi_{4w} s_w^2 (5c_w^2 + 1) + c_w^2]$
$+ F[\sin^2\phi_{4w} s_w^2 (5c_w^2 + 1) + c_w^2 (5c_w^2 - 3)]$
$+ G[\cos^2\phi_{4w} s_w^2 (5c_w^2 + 1) + c_w^2 (5c_w^2 - 3)]\}$

$\frac{7}{2}$ $\quad \frac{1}{2\pi} \frac{1}{32} \{ A(175c_w^6 - 165c_w^4 + 45c_w^2 + 9)$
$+ [B\sin\phi_{4w} s_w + C\cos\phi_{4w} s_w] 5c_w (5c_w^2 - 1)(7c_w^2 - 3)$
$+ D[\sin^2\phi_{4w} s_w^2 \, 5(35c_w^4 - 6c_w^2 + 3) + (5c_w^2 - 1)^2]$
$+ E[\cos^2\phi_{4w} s_w^2 \, 5(35c_w^4 - 6c_w^2 + 3) + (5c_w^2 - 1)^2]$
$+ F[\sin^2\phi_{4w} s_w^2 \, 5(35c_w^4 - 6c_w^2 + 3) + (5c_w^2 - 1)(35c_w^4 - 30c_w^2 + 3)]$
$+ G[\cos^2\phi_{4w} s_w^2 \, 5(35c_w^4 - 6c_w^2 + 3) + (5c_w^2 - 1)(35c_w^4 - 30c_w^2 + 3)]\}$

TABLE 12 (Contd.)

$$J=(n-\tfrac{1}{2}) \quad \frac{1}{2\pi}\frac{1}{2n}\Big\{ A\Big[\{nP_n^0(c_w)\}^2+\{P_n^1(c_w)\}^2\Big]$$

$$+\Big[B\sin\phi_{4w}+C\cos\phi_{4w}\Big]P_n^1(c_w)\frac{(-1)}{(n-1)}\Big[nP_n^0(c_w)-\frac{c_w}{s_w}P_n^1(c_w)\Big]$$

$$+\Big[D+E\Big]\Big[\frac{1}{(n-1)}\Big\{nP_n^0(c_w)-\frac{c_w}{s_w}P_n^1(c_w)\Big\}\Big]^2$$

$$+\Big[F+G\Big]\Big[nP_n^0(c_w)\frac{(-1)}{(n-1)}\Big\{nP_n^0(c_w)-\frac{c_w}{s_w}P_n^1(c_w)\Big\}\Big]$$

$$+\Big[(D+F)\sin^2\phi_{4w}+(E+G)\cos^2\phi_{4w}\Big]\times$$

$$\times\Big[\{P_n^1(c_w)\}^2-\frac{nP_n^0(c_w)P_n^2(c_w)}{(n-1)}\Big]\Big\}$$

With regard to the above tables giving the distribution of the decay products of resonances produced peripherally it should be noted that the results in each table are obtained by integrating over all decay process variables except those explicitly given in that table.The quantities A,B,C....G depend only on the coupling constants involvedd in the production process,their associated form factors and the variables s and t.

The cross section for the production of resonances of specific spin and parity are easy to write down from the results of section 7-10.These cross sections are tabulated as follows in a way which stresses the spin dependent factors.

TABLE 13

The production cross-section for the processes (1.6)and(1.7) mediated by pseudoscalar or scalar particle exchange for the case of a normal or abnormal parity boson resonance respectively.

SPIN PRODUCTION CROSS SECTION

$$j \quad \left(\frac{f^2}{4\pi}\right)\left(\frac{g^2}{4\pi}\right)\left|\frac{F(t)\,G(t)}{(m_g)^{-1}}\right|^2 \frac{q_B}{4E_B^2 q_B}\,\frac{[-t+(m_1+m_2)^2]}{[-t+\mu^2]^2}\times$$

$$\times\left[\frac{2^j j!\,j!}{(2j)!}\right]\left\{\frac{[-t+(\mu_1+\mu_2)^2][-t+(\mu_1-\mu_2)^2]}{4\mu_2^2}\,\frac{1}{m_g^2}\right\}^j$$

$j = 0, 1, 2, 3 \ldots$

TABLE 14

The production cross-section for the processes (1.6) and (1.7) mediated by vector or pseudovector particle exchange for the case of a normal or abnormal parity boson resonance respectively.

SPIN	PRODUCTION CROSS SECTION
0	PRODUCTION FORBIDDEN
j	$\frac{1}{4\pi}\left(\frac{g_0^2}{4\pi}\right)\left\|\frac{G_0(t)}{m_{j_0}^2}\right\|^2 \frac{q_B}{4E_B^2 p_B} \times$ $\times\left\{M_{p_2 p_2}\frac{1}{2}(E_B p_B q_B \sin\theta_B)^2 + M(\mu_2 k_\nu)^2\right\} \times$ $\times\left[\frac{2^j j!\, j!}{(2j)!}\right]\frac{(j+1)}{j}\left\{\frac{[-t+(\mu_1+\mu_2)^2][-t+(\mu_1-\mu_2)^2]}{4\mu_2^2}\frac{1}{m_{j_0}^2}\right\}^{j-1}$

$j = 1, 2, 3, \ldots$

TABLE 15

The production cross section for the processes (1.6)and(1.7) mediated by pseudovector or vector particle exchange for the case of a normal or abnormal parity boson resonance respectively.

SPIN PRODUCTION CROSS SECTION

$$
\begin{aligned}
j \quad & \left(\frac{1}{4\pi}\right)^2 \frac{q_B}{4E_b^2 p_b} \frac{1}{[-t+\mu^2]^2} \times \\
& \times \left[\frac{2^j j! j!}{(2j)!}\right] \left\{\frac{[-t+(\mu_1+\mu_2)^2][-t+(\mu_1-\mu_2)^2]}{4\mu_2^2}\right\}^j \times \\
& \times \frac{1}{k_v^4} \Big[\left\{ M_{p_2 p_2} \left| -\bar{g}_{p_2} k_v^2 + \bar{g}_1 p_{2v} k_v \cos\epsilon_v \right|^2 \right. \\
& \quad + 2\,\mathrm{Re}\left[M_{p_2 k} (-\bar{g}_{p_2} k_v^2 + \bar{g}_1 p_{2v} k_v \cos\epsilon_v)(-\bar{g}_k^{\dagger} k_v^2) \right] \\
& \quad \left. + M_{kk} |\bar{g}_k|^2 k_v^4 - M |\bar{g}_r|^2 k_v^4 \right\} \\
& \quad + \left(\frac{j+1}{2j}\right) \left\{ M_{p_2 p_2} |\bar{g}_1|^2 (p_{2v} k_v \sin\epsilon_v)^2 \right\} \\
& \quad + \left(\frac{2j+1}{j}\right) \left\{ M |\bar{g}_1|^2 k_v^2 \right\} \Big]
\end{aligned}
$$

$j = 0, 1, 2, 3, \ldots$

TABLE 16

The production cross section for the process (1.8) mediated by the exchange of a scalar particle.

SPIN	PRODUCTION CROSS SECTION

$$J=(n-\tfrac{1}{2}) \quad \left(\frac{f^2}{4\pi}\right)\left(\frac{g^2}{4\pi}\right)\left|\frac{F(t)\,G(t)}{(m_g)^{-1}}\right|^2 \frac{q_{1B}}{4E_B^2 p_B}\,\frac{[-t+(m_1'+m_2)^2]}{[-t+\mu^2]^2}\times$$

$$\times\left[\frac{2^n n!\,n!}{(2n)!}\right]\left\{\frac{[-t+(m_1'+m_2)^2][-t+(m_1'-m_2)^2]}{4m_1^2}\,\frac{1}{m_g^2}\right\}^{n-1}$$

$n = 1, 2, 3, 4 \ldots$

$J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$

TABLE 17

The production cross section for the process (1.8) mediated by the exchange of a vector particle.

SPIN	PRODUCTION CROSS SECTION

$$J=(n-\tfrac{1}{2}) \quad \left(\frac{1}{4\pi}\right)^2 \frac{q_B}{4E_B^2 p_B} \frac{1}{[-t+\mu^2]^2} \times$$

$$\times \left[\frac{2^n n!\, n!}{(2n)!}\right] \left\{\frac{[-t+(m_1+m_2)^2][-t+(m_1-m_2)^2]}{4m_2^2}\right\}^n \times$$

$$\times \frac{1}{k_W^4}\left[A + \tfrac{1}{2}(F+G) + \frac{n}{2(n-1)}(D+E)\right]$$

$n = 1, 2, 3, 4 \ldots$

$J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2} \ldots$

12. DISCUSSION AND CONCLUSIONS

One of the most striking discoveries in the realm of strong interaction physics has been that of the existence of a large number of resonant states. Quite apart from the problem of determining the dynamical origin of these states it is of great interest to solve the semi-phenomenological problems of determining the precise quantum numbers of these states, the nature and strength of their interactions with other states and the dynamics of their production processes.

The facts that the quantum numbers of the resonant states most copiously produced in quasi two body processes are those which are consistent with the exchange of a single particle or a known resonance and that the production of these resonant states is considerably enhanced for events with small 4-momentum transfer both suggest that a peripheral model might be a good approximation to the real mechanism of the production process. In the previous sections of this paper the full consequences of adopting such a single particle exchange model have been determined for the exchange of a spin zero or a spin one particle. It is clear from the results obtained that it is possible both to test the validity of the model and to gain some information relevant to the problems mentioned above.

A test of the spin of the exchanged particles is provided by the observation of the distribution of the decay products of resonances with respect to the azimuthal angle associated with a polar axis in

the direction of the appropriate incoming particle as measured in the centre of mass system of the decaying resonance. For spin zero and spin one exchange these distributions are given by:

$$a_0 \qquad (12\cdot1)$$

and

$$a_0 + a_1 \cos^2\phi \qquad (12\cdot2)$$

respectively. The former test distribution is just the well known Trieman-Yang[13] isotropic distribution and the latter is a rather obvious generalisation of this result. In fact it is easy to see that the most general decay distribution with respect to the azimuthal angle ϕ corresponding to the exchange of a spin j particle is of the form

$$\sum_{n=0}^{j} a_n \cos^{2n}\phi \qquad (12\cdot3)$$

for arbitrary spin and parity boson and fermion resonances.

From the results of section 5 as given in Table 1 it follows that if any of the processes (1·6) - (1·9) are mediated by the exchange of a spin zero particle then the parity of the outgoing boson state is normal or abnormal according as the exchanged particle is pseudoscalar or scalar respectively. The corresponding decay correlations of the final state particles are then given by the results expressed in Tables 2,5,8 and 11. These decay distributions are all independent of the production process variables and of the mass and

coupling strengths of the exchanged particles. Thus the same decay distributions are obtained even if any number of such spin zero particle exchanges contribute to the scattering amplitude or if indeed there is an s-wave contribution to the exchange amplitude involving an integration over a cut in the variable t. Furthermore it is to be noted that there is no correlation between the boson resonance decay and the fermion resonance decay in the reaction (1·9) in the case of the exchange of a spin zero particle or any number of such exchanges.[15]

If any of the production processes (1·6) - (1·9) are mediated by spin zero exchange it is straightforward to determine the spin of the resonant states by a comparison of the experimental data with the results of Tables 2,5,8 and 11. In the case of the resonance B_3 any evidence in the decay distribution with respect to ω_{3V} of a term in $Q_2^2(\omega_{3V})$ indicates of course that the resonance has abnormal parity, but if the distribution is consistent with a pure $Q_1^2(\omega_{3V})$ distribution then the parity of the state may best be found by examining the distribution with respect to ψ_{3V} which is necessarily pure $\cos^2\psi_{3V}$ for a normal parity resonance.

It should be stressed that these spin and parity tests also form a series of necessary tests of the validity of the hypothesised spin zero exchange model.

The production process (1·6) may be mediated by both vector and pseudovector particle exchange. In the case of vector particle exchange the decay distribution with respect to the variable $\cos\theta_{3}$ is a unique function of the resonance spin as given in Table 3. Thus the vector particle model may be readily tested and a resonance spin determination carried out.

Unfortunately the corresponding distributions for a pseudovector particle exchange model as given in Table 4 depend upon the production process variables. Moreover they include terms of the form corresponding to vector particle exchange and there is not even any special value of θ_{B} which gives a distribution independent of the relative magnitude of the various production process coupling constants. This means that the model may not be tested as rigorously as the pseudoscalar and vector particle exchange models and it may prove difficult to distinguish between vector and pseudovector particle exchange. In addition a spin determination may not be straightforward in as much as it is possible for a decay distribution to be obtained which corresponds to the resonance having either spin j or spin $(j-1)$. This possibility may arise for an arbitrary spin resonance because of the existence of the identity:

$$(jP_j^0)^2 + (P_{j-1}^1)^2 = (jP_{j-1}^0)^2 + (P_{j-1}^1) \qquad (12\cdot4)$$

The decay distributions of Tables 3 and 4 are also obtained if the production mechanism is generalised to include the exchange of any number of spin one particles all of the same parity but of differing masses and coupling strengths.

The possibility of interference between spin zero and spin one exchanges giving contributions to the scattering amplitude of opposite parity has in a sense been included in the calculations of section 7 by the admissions of the couplings associated with g_3 and f_4 which only couple to the spin zero part of the propagator. From these results it follows that there can be no scalar particle exchange and that the interference between pseudoscalar and pseudovector particle exchanges gives a decay distribution of the general form:

$$\frac{1}{2\pi}\frac{(2j+1)}{2j^2}\left\{A\left[jP_j^0\left(c_V\right)\right]^2 + B\sin\phi_{3V}\left[jP_j^0\left(c_V\right)\right]\left[P_j^1\left(c_V\right)\right]\right\} \qquad (12\cdot5)$$

This form is already contained in Table 4.

It is easy to see that in the case of spin zero and spin one exchanges of the same parity contributing to the scattering amplitude no interference term arises[10] This is because the fermion couplings of the pseudoscalar and vector exchange particles are of opposite parity and evaluation of the trace resulting from summation over the spin states of the fermions gives zero for the interference term since this term contains an overall factor of γ_5 which forms a product with no more than three γ matrices. Thus the pseudoscalar and

vector exchange contributions to the process (1·6) add incoherently.

The remaining possibility of an interference term arising occurs when both vector and pseudovector particle exchanges contribute to the scattering amplitude. Such an interference term is only present if in each case the couplings associated with f_2 are present. A calculation then shows that the decay distribution associated with this term is of the form

$$\frac{1}{4\pi}\frac{(2j+1)}{2j^2}\left\{ B \sin\phi_{3V}\left[jP_j^0(c_V)\right]\left[P_j^1(c_V)\right] + D\left[P_j^1(c_V)\right]^2\right\} \qquad (12\cdot6)$$

This form is also contained in Table 4 so that it is difficult to distinguish between pure pseudovector particle exchange and a combination of pseudovector and pseudoscalar and/or vector particle exchanges.

The production process (1·7) may be mediated by both vector and pseudovector particle exchanges and the corresponding decay distributions are given for a normal parity resonance in Tables 6 and 7, and for an abnormal parity resonance in Tables 9 and 10. As for spin zero exchange any evidence in the decay distribution with respect to ω_{5V} of a term in $Q_2^2(\omega_{5V})$ indicates that the resonance has abnormal parity but if the distribution is pure $Q_1^2(\omega_{5V})$ then the parity of the resonance may only be found by examining the decay distribution with respect to ψ_{3V}.

The decay distributions with respect to $\cos\theta_{3V}$ and ψ_{3V} are only independent of the production process variables if the boson coupling in the production process has abnormal parity. In such cases the spin one exchange model is therefore relatively easy to test and spin determinations may be carried out with some confidence. However if the boson coupling has normal parity as in the case of vector particle exchange leading to the production of an abnormal parity boson resonance these distributions depend on the production process variables in a way which makes spin and parity determinations rather difficult. The decay distributions with respect to $\cos\theta_{3V}$ and ϕ_{3V} are of the same general form for vector and pseudovector particle exchange production of an abnormal and a normal resonance respectively. Moreover a decay distribution may be obtained which corresponds to the resonance having either spin 1 or 2 since:

$$(4c_V^4 - 3c_V^2 + 1) + 4s_V^2c_V^2 = c_V^2 + 1. \qquad (12\cdot7)$$

The remarks made concerning possible interference effect in the production process (1·6) apply equally well to the process(1.7). In particular there is no interference between pseudoscalar and vector particle exchanges or between scalar and pseudovector particle exchanges.

If the production process (1·8) is mediated by the exchange of a spin one particle such a particle must necessarily be a vector

particle. The corresponding decay correlations are given in Table 12. In the special case of events in the forward direction the decay distribution reduces to the Adair distribution[36]:

$$\frac{1}{2n}\left[\left\{nP_n^0\left(c_V\right)\right\}^2+\left\{P_n^1\left(c_V\right)\right\}^2\right] \qquad (12\cdot8)$$

In the past it has proved difficult to determine the spins of fermion resonances by means of the Adair test. This may be because the production mechanism is such that the cross section falls off quite rapidly in the immediate neighbourhood of the forward direction. However if the vector particle exchange model is valid the decay distribution as a function of $\cos\theta_{4W}$ is the sum of only two independent terms so that it should prove possible to test the model and carry out spin determinations.

Since the production process (1·8) can only be mediated by the exchange of normal parity particles no difficulties are introduced by including in the scattering amplitude the contributions of any number of scalar and vector particle exchanges. In fact by the admission of the couplings associated with g_3 and f_4 the scalar particle exchange and interference terms have effectively been included in the vector particle exchange calculations.

The production process (1·9) may be mediated by vector or pseudovector particle exchanges. The decay distributions are given for the boson resonance in Tables 3 and 4 and for the fermion resonance in Table 12. In the case of vector particle exchange the model may be relatively easily tested since the boson decay distribution with

respect to $\cos\theta_{3V}$ is independent of all the other variables in the problem.

It should be noted that for fermon resonances of spin greater than one half the interference between pseudoscalar and vector particle exchanges is non zero. The contribution of such a term to the decay distribution is of the form:

$$\left[A \sin\phi_{3V} \sin\phi_{4W} + B \cos\phi_{3V} \cos\phi_{4W}\right]\left[jP_J^0(c_V)P_J^1(c_V)\right]$$
$$\frac{(-1)}{(n-1)} P_h^1(c_W)\left[nP_h^0(c_W) - \frac{\sigma_T}{s_T} P_h^1(c_W)\right] \qquad (12\cdot9)$$

Clearly such a term makes no contribution to the decay distribution of Tables 3 and 12.

In order to gain the maximum confidence in any test of the peripheral model and in any spin determinations carried out on the basis of that model it is necessary to examine the decay distribution with respect to the variables $\cos\alpha$, $\cos\beta$, $\cos\gamma$ and the corresponding azimuthalangle variables ϕ_α, ϕ_β, ϕ_γ. These variables are, for boson and fermion resonances the direction cosines of $\underline{q}_{3V}$ and $\underline{p}_{4W}$ with respect to the systems of axes $\underline{I}$, $\underline{N}$ and $\underline{I} \wedge \underline{N}$ in the frames V and W respectively. Theazimuthal angles are defined in the usual way so that the relationships between these variables and those variables used in this paper are for boson resonances

$$\cos\alpha = \cos\theta_{3V} \qquad \phi_\alpha = \phi_{3V} \qquad (12\cdot10)$$

and for fermion resonances:

$$\cos\alpha = \cos\theta_{4W} \qquad \phi_\alpha = \phi_{4W} \tag{12·11}$$

There are only two independent variables and the relationships between the three pairs of variables are given by:

$$\cos\alpha = \sin\beta\,\sin\phi_\beta = \sin\gamma\,\cos\phi_\gamma \tag{12·12a}$$

$$\sin\alpha\,\cos\phi_\alpha = \cos\beta = \sin\gamma\,\sin\phi_\gamma \tag{12·12b}$$

$$\sin\alpha\,\sin\phi_\alpha = \sin\beta\,\cos\phi_\beta = \cos\gamma \tag{12·12c}$$

For any particular event all these variables are well defined but in measuring a decay distribution with respect to one variable in each pair an integration is effectively carried out over the other variable. In general this results in a loss of information which can only be made up by examining the decay distributions with respect to each of the variable defined above. If this is done it may be possible to distinguish in a quantitative manner between the contributions of individual couplings to a production cross section.[7,87]

To date the peripheral model has only been examined in detail for processes in which spin one and/or spin 3/2 resonances are produced. In the case of spin one production[7,10,87,88] it has been possible to fit the decay distributions by assuming some combination of pseudoscalar and vector particle exchanges. The model used with considerable success to describe the production of spin 3/2 resonances in a number of

processes[39-42] has been that of Stodolsky and Sakurai[6,8] which uses a ρ-photon analogy to predict a decay distribution of the form

$$(1 + 3 \sin^2\theta_{4W} \cos^2\phi_{4W}) \qquad (12\cdot13)$$

As can be seen from Table 12 this suggests that the dominant coupling of the exchanged vector particle with the isobar is that associated with the coupling constant f_5. Since such a coupling gives rise to a production cross section which goes to zero in the forward direction, this provides a natural explanation for the failure of the Adair test.

Processes in which simultaneous production of spin 1 and spin 3/2 resonances takes place have also been considered and the decay distributions have been found to be consistent with pseudoscalar particle exchange.[43]

If the spin of resonant state can be established from the decay distribution together with a knowledge of the dominant production mechanism then the model may be compared with experiment in a quantitative manner to obtain some information on the coupling strengths of the resonant state. It should be pointed out that before carrying out such an investigation the isotopic spin factors which have been

omitted from the results of this paper should be inserted. The total width of a resonance, as measured experimentally, will, on comparison with the results of section 6, yield information on the decay process coupling constants and the production process cross section will give information on the production process coupling constants and their associated form factors. The production cross sections corresponding to the single particle exchange model are given in Tables 13-17. The main feature of these results are the factors $(k_V^2)^j$ and $(k_\pi^2)^{n-1}$ which appear in the cross sections for the production of bosons of spin j and fermions of spin $(n - \frac{1}{2})$ respectively. These factors are the generalisations of the off-mass-shell correction of Selleri[11] which are necessary when determining the partial wave amplitudes of $\pi\pi$-scattering from a one pion exchange model. In fact his results correspond exactly to the results of section 7 for pseudoscalar particle exchange since $P_j^0(c_V)$ is the projection operator for the j-th partial wave.

These factors work in opposition to the pole term in the scattering amplitude in that, with constant form factors, the single particle exchange model results in production cross sections which are not peripheral in appearance. That is to say there is no pronounced forward peaking of the cross section. This becomes more apparent the higher the spin of the resonant state and the only means of compensating

for this effect is to assume that the form factors have a very strong dependence on the variable t. Various attempts have been made to fit the production cross section behaviour using one parameter form factors. At present this has reduced a dynamical analysis of such production processes to a purely phenomenological level since no really satisfactory explanation has as yet been given as to why these form factors are so strongly dependent on t. However the peripheral model may still prove useful as long as the form factors exhibit no energy dependence and there is no violation of the decay distribution predictions of the model. It would be particularly interesting to confront the experimental data on the production and decay properties of the B_3, (pseudoscalar-vector) resonances and the higher mass isobars with the predictions of the model.

Although as has been pointed out the decay distributions provide a test of the validity of the peripheral model some of these decay distributions can be associated with other models. That they are not all exclusive to the spin zero and spin one exchange models has been pointed out by Gottfried and Jackson[44] in an analysis of vector meson production. It can also be shown that the results for the boson resonance production processes (1·6) and (1·7) contain terms which correspond to a coherent, no-spin-flip model of the production process. Some of the results of Berman and Drell[45] for coherent production are generalised to the production of arbitrary spin resonances as follows.

A no-spin-flip amplitude for the production of boson resonances may be obtained by considering scalar particle exchange or the f_3 coupling in vector particle exchange for the situations in which the boson coupling allows scalar or vector particle exchanges to take place. Thus the decay angular distribution of a normal parity boson resonance, produced by coherent scattering of pseudoscalar mesons on nucleons, decaying into two spin zero particles of the same parity is obtained from Table 3 by putting B = 0. The distribution is of the form:

$$\frac{1}{2\pi}\ \frac{(2j+1)}{2j(j+1)}\cos^2\phi_{3V}\left[P_j^1\left(c_V\right)\right]^2 \qquad (12\cdot14)$$

The corresponding result for an abnormal parity resonance decaying into two spin zero particles of opposite parity is given in Table 2. The distribution is of the form:

$$\frac{1}{2\pi}\ \frac{(2j+1)}{2}\left[P_j^0\left(c_V\right)\right]^2 \qquad (12\cdot15)$$

Similarly the result for a normal parity resonance decaying into a spin zero and a spin one particle of the same parity is obtained from Table 6 by putting B = 0. The distribution is of the form:

$$\frac{1}{2\pi}\ \frac{(2j+1)}{2j^2(j+1)^2}\left\{\cos^2\phi_{3V}\left[j(j+1)P_j^0\left(c_V\right)-\frac{c_V}{s_V}P_j^1\left(c_V\right)\right]^2\right.$$

$$\left.+\ \sin^2\phi_{3V}\left[\frac{1}{s_V}P_i^1\left(c_V\right)\right]^2\right\} \qquad (12\cdot16)$$

The result for an abnormal parity resonance decaying into a spin zero and a spin one particle of the same parity is given in Table 8(a). The distribution is of the form:

$$\frac{1}{2\pi}\,\frac{1}{[(\bar{h}_1)^2(j+1)+(\bar{h}_2)^2 j]}\,\frac{(2j+1)}{2j}\left\{(\bar{h}_1)^2\left[P_j^1(c_V)\right]^2+(\bar{h}_2)^2\left[jP_j^0(c_V)\right]^2\right\} \qquad (12\cdot17)$$

Clearly none of the above distributions may be associated exclusively with the peripheral model.

Although there has been some success in comparing the angular correlations of the decay products of resonances with the predictions of the peripheral model it may be possible that a completely different production mechanism is responsible for these distributions. The peripheral model can only be an approximation to the production mechanism since it does not incorporate unitarity in any way and leads to production cross sections which increase with the incident particle energy unless once again very drastic form factors are incorporated in the model. One attempt to insert unitarity into the model has been to make use of the distorted-wave Born approximation[46-48]. This approach to the problem may be more realistic.

A. APPENDIX

To evaluate the differential cross sections for the production and subsequent decay of arbitrary spin resonances it is necessary to consider in some detail the quantities constructed by contraction of some of the indices of the projection operator (4·11) with various 4-momentum indices. One set of indices is contracted with the indices of the 4-momentum k and the other set with the indices of the 4-momentum q_4 or p_4 for boson or fermion resonances respectively.

For boson resonances the following expressions evaluated in the frame V are useful:

$$k_{\lambda_1} k_{\lambda_2} \phi_{\lambda_1 \lambda_2}(q_2) = -k_V^2 \qquad (A\cdot1)$$

$$\phi_{\rho_1 \rho_2}(q_2) q_{4\rho_1} q_{4\rho_2} = -q_V^2 \qquad (A\cdot2)$$

$$k_{\lambda_1} \phi_{\lambda_1 \rho_1}(q_2) q_{4\rho_1} = -k_V \, q_V \cos \theta_{3V} \qquad (A\cdot3)$$

and for fermion resonances the corresponding terms evaluated in the frame W are:

$$p_{4\rho_1} p_{4\rho_2} \phi_{\rho_1 \rho_2}(p_2) = - p_W^2 \qquad (A\cdot4)$$

$$\phi_{\lambda_1 \lambda_2}(p_2) k_{\lambda_1} k_{\lambda_2} = - k_W^2 \qquad (A\cdot5)$$

$$p_{4\rho_1} \phi_{\rho_1 \lambda_1}(p_2) k_{\lambda_1} = - p_W k_W \cos \theta_{4W} \qquad (A\cdot6)$$

It should be noted that the coefficients a_r^n given by (4·12) are such that the Legendre polynomial $P_n(\cos\theta)$ may be written as:-

$$P_n(\cos\theta) = \left[\frac{(2n)!}{2^n n!n!}\right] \sum_{r=0}^{n'} a_r^n (\cos\theta)^{n-2r} \qquad (A\cdot7)$$

where $n' = n/2$ for n even and $n' = (n-1)/2$ for n odd.

Making use of (A·1), (A·2) and (A·3) it follows from (4·11) and (A·7) that:

$$(k_\lambda)^n \phi_{\lambda\rho}(q_2, n)(q_{4\rho})^n = \left[\frac{2^n n!n!}{(2n)!}\right] k_V^n q_V^n P_n(\cos\theta_{3V}) \qquad (A\cdot8)$$

Summing over all terms in (4·11) and using both (A·7) and its first derivative which defines the associated Legendre polynomial $P_n^1(\cos\theta)$ it can be shown that:

$$(-1)^{n+1}(k_\lambda)^{n-1}\phi_{\lambda\rho}(q_2, n)(q_{4\rho})^n = \left[\frac{2^n n!n!}{(2n)!}\frac{1}{n} k_V^{n-1} q_V^n\right] x$$

$$x\left[\frac{\{\phi(q_2)q_4\}}{q_V}\lambda_1 \frac{1}{s_V} P_n^1(c_V) + \frac{\{k\phi(q_2)\}}{k_V}\lambda_1\left\{n P_n^0(c_V) - \frac{c_V}{s_V} P_n^1(c_V)\right\}\right] \qquad (A\cdot9)$$

and

$$(-1)^{n+1}(k_\lambda)^n \phi_{\lambda\rho}(q_2, n)(q_{4\rho})^{n-1} = \left[\frac{2^n n!n!}{(2n)!}\frac{1}{n} k_V^n q_V^{n-1}\right] x$$

$$x\left[\frac{\{k\phi(q_2)\}}{k_V}\rho_1 \frac{1}{s_V} P_n^1(c_V) + \frac{\{\phi(q_2)q_4\}}{q_V}\rho_1\left\{n P_n^0(c_V) - \frac{c_V}{s_V} P_n^1(c_V)\right\}\right] \qquad (A\cdot10)$$

where $c_V = \cos\theta_{3V}$ and $s_V = \sin\theta_{3V}$.

Similarly using (A·7) and its first and second derivatives it can be shown that:

$$(-1)^{n+1}(k_\lambda)^{n-1}\phi_{\lambda\rho}(q_2,n)(q_{4\rho})^{n-1} = \left[\frac{2^n n!n!}{(2n)!}\frac{1}{n^2}k_V^{n-1}q_V^{n-1}\right]x$$

$$x\left[\phi_{\lambda_1\rho_1}(q_2)\frac{1}{s_V}P_n^1(c_V) - \left\{\frac{[\phi(q_2)q_4]}{q_V}\lambda_1\frac{[k\phi(q_2)]}{k_V}\rho_1 + \frac{[k\phi(q_2)]}{k_V}\lambda_1\frac{[\phi(q_2)q_4]}{q_V}\rho_1\right\}\frac{P_n^2(c_V)}{s_V^2}\right.$$

$$-\left\{\frac{[k\phi(q_2)]}{k_V}\lambda_1\frac{[k\phi(q_2)]}{k_V}\rho_1 + \frac{[\phi(q_2)q_4]}{q_V}\lambda_1\frac{[\phi(q_2)q_4]}{q_V}\rho_1\right\}$$

$$\left\{(n-1)\frac{P_n^1(c_V)}{s_V} - \frac{c_V}{s_V^2}P_n^2(c_V)\right\}$$

$$\left.-\left\{\frac{[k\phi(q_2]}{k_V}\lambda_1\frac{[\phi(q_2)q_4]}{q_V}\rho_1\right\}\left\{n^2P_n^0(c_V) - (2n-1)\frac{c_V}{s_V}P_n^1(c_V) - P_n^2(c_V)\right\}\right] \qquad (A\cdot11)$$

The notation used is such that

$$[k\phi(q_2)]_\mu = k_\nu\phi_{\nu\mu}(q_2) \qquad (A\cdot12)$$

and

$$[\phi(q_2)q_4]_\nu = \phi_{\nu\mu}(q_2)q_{4\mu} \qquad (A\cdot13)$$

In section 6 it is necessary to consider the special cases of (A·8) and (A·11) in which a single 4-momentum is used to contract both sets of indices of the projection operator. It is easy to show that:

$$[P_n(\cos\theta)]_{\cos\theta = 1} = 1$$

$$\left[\frac{1}{\sin\theta} P_n^1(\cos\theta)\right]_{\cos\theta = 1} = \frac{n(n+1)}{2}$$

$$\left[\frac{1}{\sin^2\theta} P_n^2(\cos\theta)\right]_{\cos\theta = 1} = \frac{3(n+2)!}{4!(n-2)!}$$

Using these relationships it follows from (A·8) that:

$$(-1)^n (q_{4\lambda})^n \phi_{\lambda\rho}(q_2, n)(q_{4\rho})^n = \left[\frac{2^n n!n!}{(2n)!}\right] q_V^{2n} \qquad (A\cdot14)$$

and if follows from (A·9) that:

$$(-1)^n (q_{4\lambda})^{n-1} \phi_{\lambda\rho}(q_2, n)(q_{4\rho})^{n-1} = \left[\frac{2^n n!n!}{(2n)!} \frac{1}{n^2} q_V^{2n-2}\right] x$$

$$x \left[\phi_{\lambda_1\rho_1}(q_2) \frac{n(n+1)}{2} - \frac{\{q_4\phi(q_2)\}}{q_V} \lambda_1 \frac{\{q_4\phi(q_2)\}}{q_V} \rho_1 \frac{n(n-1)}{2}\right] \qquad (A\cdot15)$$

In sections 9 and 10 in which fermion resonances are discessed it is necessary to evaluate four traces. It is possible to write these as follows:

$$\frac{1}{4}\mathrm{Tr}\left\{\gamma_{\beta_1}\gamma_{\gamma_1}\gamma_{\rho_1}\gamma_{\lambda_1}\right\} = g_{\beta_1\gamma_1}g_{\rho_1\lambda_1} + \left[g_{\beta_1\lambda_1}g_{\gamma_1\rho_1} - g_{\beta_1\rho_1}g_{\gamma_1\lambda_1}\right] \quad (A\cdot16)$$

$$\frac{1}{4}\mathrm{Tr}\left\{(\not{a}\not{b} - \not{b}\not{a})\,\gamma_{\beta_1}\gamma_{\gamma_1}\gamma_{\rho_1}\gamma_{\lambda_1}\right\} = 2g_{\beta_1\gamma_1}\left(a_{\lambda_1}b_{\rho_1} - b_{\lambda_1}a_{\rho_1}\right)$$

$$+ 2g_{\rho_1\lambda_1}\left(a_{\gamma_1}b_{\beta_1} - b_{\gamma_1}a_{\beta_1}\right) + \frac{1}{16}\mathrm{Tr}\left\{(\not{a}\not{b} - \not{b}\not{a})\left(\gamma_{\beta_1}\gamma_{\gamma_1} - \gamma_{\gamma_1}\gamma_{\beta_1}\right)\left(\gamma_{\rho_1}\gamma_{\lambda_1} - \gamma_{\lambda_1}\gamma_{\rho_1}\right)\right\} \quad (A\cdot17)$$

$$\frac{1}{4}\mathrm{Tr}\left\{\gamma_5\gamma_{\beta_1}\gamma_{\tau_1}\gamma_{\rho_1}\gamma_{\lambda_1}\right\} = -\epsilon_{\beta_1\gamma_1\rho_1\lambda_1} \quad (A.18)$$

$$\frac{1}{4}\mathrm{Tr}\left\{\gamma_5(\not{a}\not{b} - \not{b}\not{a})\,\gamma_{\beta_1}\gamma_{\gamma_1}\gamma_{\rho_1}\gamma_{\lambda_1}\right\} = -2g_{\beta_1\gamma_1}\,\epsilon_{\mu\nu\rho_1\lambda_1}a_\mu b_\nu$$

$$- 2g_{\rho_1\lambda_1}\epsilon_{\mu\nu\beta_1\gamma_1}a_\mu b_\nu + \frac{1}{16}\mathrm{Tr}\left\{\gamma_5(\not{a}\not{b} - \not{b}\not{a})\left(\gamma_{\beta_1}\gamma_{\gamma_1} - \gamma_{\gamma_1}\gamma_{\beta_1}\right)\left(\gamma_{\rho_1}\gamma_{\lambda_1} - \gamma_{\lambda_1}\gamma_{\rho_1}\right)\right\} \quad (A\cdot19)$$

where each expression has been written as a sum of terms which are either proportional to $g_{\beta_1\gamma_1}$ or antisymmetric under the interchange of β_1 and γ_1 and are also either proportional to $g_{\rho_1\lambda_1}$ or antisymmetric under the interchange of ρ_1 and λ_1. These forms are particularly useful in the following calculations.

The expression analogous to (A·11) which is of use when dealing with fermion resonances may be written as:

$$(-1)^{n+1}(k_\beta)^{n-1}\,\phi_{\beta\gamma}(p_2,n)\,(p_1{}_\gamma)^{n-1} = \left[\frac{2^n n!n!}{(2n)!}\frac{(2n+1)}{n^2}\,k_W^{\,n-1}\,p_W^{\,n-1}\right]\left(A_{\beta_1\gamma_1} + B_{\beta_1\gamma_1}\right) \quad (A\cdot20)$$

where $A_{\beta_1\gamma_1}$ is symmetric under the interchange of β_1 and γ_1. It may easily be shown that, with the notation $\cos\theta_{4W} = c_W$ and $\sin\theta_{4W} = s_W$,

$$g_{\beta_1\gamma_1}(A_{\beta_1\gamma_1} + B_{\beta_1\gamma_1}) = [c_W\, n\, P_n^0(c_W) + s_W\, P_n^1(c_W)] \qquad (A\cdot21)$$

and

$$B_{\beta_1\gamma_1} = -\left\{\frac{[k\phi(p_2)]_{\beta_1}}{k_W}\,\frac{[\phi(p_2)p_4]_{\gamma_1}}{p_W}\right\}[n\,P_n^0(c_W) - \frac{c_W}{s_W}\,P_n^1(c_W)] \qquad (A\cdot22)$$

Similarly:

$$(-1)^{n+1})(p_{4\rho})^{n-1}\,\phi_{\rho\lambda}(p_2,n)(k_\lambda)^{n-1} = \left[\frac{2^n n!n!}{(2n)!}\,\frac{(2n+1)}{\pi^2}\,k_W^{n-1}p_W^{n-1}\right](A_{\rho_1\lambda_1} + C_{\rho_1\lambda_1}) \qquad (A\cdot23)$$

where as before $A_{\rho_1\lambda_1}$ is symmetric under the interchange of ρ_1 and λ_1. As before

$$g_{\rho_1\lambda_1}(A_{\rho_1\lambda_1} + C_{\rho_1\lambda_1}) = \left[c_W\, n\, P_n^0(c_W) + s_W\, P_n^1(c_W)\right] \qquad (A\cdot24)$$

and

$$C_{\rho_1\lambda_1} = -\left\{\frac{[p_4\phi(p_2)]_{\rho_1}}{p_W}\,\frac{[\phi(p_2)k]_{\lambda_1}}{k_W}\right\}\left[n\,P_n^0(c_W) - \frac{c_W}{s_W}\,P_n^1(c_W)\right] \qquad (A\cdot25)$$

The terms in $B_{\beta_1\gamma_1}$ and $C_{\rho_1\lambda_1}$ are the only parts of (A·20) and (A·23) which give any contribution when contracted with the antisymmetric parts of (A·16). It is clear that:

$$\left[g_{\beta_1\lambda_1}g_{\gamma_1\rho_1} - g_{\beta_1\rho_1}g_{\gamma_1\lambda_1}\right]B_{\beta_1\gamma_1}C_{\rho_1\lambda_1} = [s_W\, n\, P_n^0(c_W) - c_W\, P_n^1(c_W)]^2 \qquad (A\cdot26)$$

Summing over all the terms in (4·11) and using (A·7) and its derivatives it may be shown that:

$$(-1)^{n+1} e_{\beta_2}(k_\beta)^{n-2}\phi_{\beta\gamma}(p_2,n)(p_{4\gamma})^{n-1} = \left[\frac{2^n n!n!}{(2n)!}\ \frac{(2n+1)}{n^2}\ e_W k_W^{n-2} p_W^{n-1}\right] \left[D_{\beta_1\gamma_1}(e) + E_{\beta_1\gamma_1}(e)\right] \tag{A·27}$$

where $D_{\beta_1\gamma_1}(e)$ is symmetric under the interchange of β_1 and γ_1 and

$$g_{\beta_1\gamma_1}\left[D_{\beta_1\gamma_1}(e) + E_{\beta_1\gamma_1}(e)\right] = \left\{\cos\theta_{ekW}\left[nc_W P_n^0(c_W) + s_W P_n^1(c_W)\right] - \left[\cos\theta_{epW} - c_W\cos\theta_{ekW}\right]\ \frac{n}{(n-1)}\ \left[nP_n^0(c_W) - \frac{c_W}{s_W}P_n^1(c_W)\right]\right\} \tag{A·29}$$

and

$$E_{\beta_1\gamma_1}(e) = \left[-\frac{\{k\phi(\phi_2)\}}{k_W}\beta_1\left\{\cos\theta_{ekW}\frac{(n-2)}{(n-1)}\left[nP_n^0(c_W) - \frac{c_W}{s_W}P_n^1(c_W)\right] + \left[\cos\theta_{epW} - c_W\cos\theta_{ekW}\right]\ \frac{1}{(n-1)}\left[(n-1)\frac{P_n^1(c_W)}{s_W} - \frac{c_W}{s_W^2}P_n^2(s_W)\right]\right\} - \frac{\{e\phi(p_2)\}}{e_W}\beta_1\left\{\frac{1}{(n-1)}\left[nP_n^0(c_W) - \frac{c_W}{s_W}P_n^0(c_W)\right]\right\}\right]\frac{\{\phi(p_2)p_4\}}{p_W}\gamma_1 \tag{A·29}$$

where e_μ is an arbitrary 4-momentum such that θ_{ekW} and θ_{epW} are the angle in the frame W between $\underline{e}_W$ and $\underline{k}_W$ and between $\underline{e}_W$ and $\underline{P}_{4W}$ respectively.

Similarly:

$$(-1)^{n+1}(p_{4\rho})^{n-1}\phi_{\rho\lambda}(p_2,n)(k_\lambda)^{n-2}f_{\lambda_2} = \left[\frac{2^n n!n!}{(2n)!}\frac{(2n+1)}{n^2} f_W k_W^{n-2} p_W^{n-1}\right] \left[D_{\rho_1\lambda_1}(f) + F_{\rho_1\lambda_1}(f)\right] \quad (A\cdot30)$$

where once again $D_{\rho_1\lambda_1}(f)$ is symmetric under the interchange of ρ_1 and λ_1 and

$$g_{\rho_1\lambda_1}\left[D_{\rho_1\lambda_1}(f) + F_{\rho_1\lambda_1}(f)\right] = \left\{\cos\theta_{fkW}\left[nc_W P_n^0(c_W) + s_W P_n^1(c_W)\right] - \left[\cos\theta_{fpW} - c_W\cos\theta_{fkW}\right]\frac{n}{(n-1)}\left[nP_n^0(c_W) - \frac{c_W}{s_W}P_n^1(c_W)\right]\right\} \quad (A\cdot31)$$

and

$$F_{\rho_1\lambda_1}(f) = \frac{\{p_1\phi(p_2)\}}{p_W}\rho_1\left[-\frac{\{\phi(p_2)k\}}{k_W}\lambda_1\left\{\cos\theta_{fkW}\frac{(n-2)}{(n-1)}\left[nP_n^0(c_W) - \frac{c_W}{s_W}P_n^1(c_W)\right] + \left[\cos\theta_{fpW} - c_W\cos\theta_{fkW}\right]\frac{1}{(n-1)}\left[\frac{1}{(n-1)}\frac{P_n^1(c_W)}{s_W} - \frac{c_W}{s_W^2}P_n^2(c_W)\right]\right\} - \frac{\{\phi(p_2)f\}}{f_W}\lambda_1\left\{\frac{1}{(n-1)}\left[nP_n^0(c_W) - \frac{c_W}{s_W}P_n^1(c_W)\right]\right\}\right] \quad (A\cdot32)$$

where f_μ is an arbitrary 4-momentum such that θ_{fkW} and θ_{fpW} are the angles in the frame W between $\underline{f}_W$ and $\underline{k}_W$ and between $\underline{f}_W$ and $\underline{p}_{4W}$ respectively.

The term in $E_{\beta_1\gamma_1}(e)$ is the only part of (A·27) which gives a non-zero contribution when contracted with the terms in (A·16) - (A·19) which are antisymmetric under the interchange of β_1 and γ_1. Similarly the term in $F_{\rho_1\lambda_1}(f)$ is the only part of (A·30) which gives a non-zero contribution when contracted with the terms in (A·16) - (A·19) which are antisymmetric under the interchange of ρ_1 and λ_1. Thus the expression T(r) given by (9·8) may be evaluated using the results (A·16) - (A·19), (A·28), (A·29), (A·31) and (A·32).

It is convenient to separate out from each term in the expression T(r) the factor:

$$\left[\frac{2^n n!n!}{(2n)!}\ \frac{(2n+1)}{n^2}\ k_W^{n-3}\ p_W^{n-1}\right]^2 \qquad (A\cdot33)$$

The remaining factors in each term are as follows:

$$M_{kk}\ k_W^4\left[\left\{n\ P_n^0(c_W)\right\}^2 + \left\{P_n^1(c_W)\right\}^2\right] \qquad (A\cdot34)$$

$$M_{kr}\ k_W^2\left[(\underline{r}_W\cdot\underline{k}_W)\left\{\left[n\ P_n^0(c_W)\right]^2 + \left[P_n^1(c_W)\right]^2\right\}\right.$$

$$\left. -\frac{1}{k_W p_W}(\underline{r}_W\wedge\underline{k}_W)\cdot(\underline{p}_{4W}\wedge\underline{k}_W)\ \frac{1}{(n-1)}\ \frac{1}{s_W}\ P_n^1(c_W)\left\{nP_n^0(c_W) - \frac{c_W}{s_W}\ P_n^1(c_W)\right\}\right] \qquad (A\cdot35)$$

The term in M_{rk} is the hermitian conjugate of that in M_{kr}

$$M_{rr}\Big[\,|(\underline{r}_W\cdot\underline{k}_W)|^2\left\{\left[n\,P_n^0(c_W)\right]^2+\left[P_n^1(c_W)\right]^2\right\}$$

$$-2\,\mathrm{Re}\left\{(\underline{r}_W\cdot\underline{k}_W)\,\frac{[(\underline{r}_W^*\wedge\underline{k})\cdot(\underline{p}_{l+W}\wedge\underline{k}_W)]}{k_W p_W}\right\}\frac{1}{(n-1)}\frac{1}{s_W}P_n^1(c_W)\left\{n\,P_n^0(c_W)-\frac{c_W}{s_W}P_n^1(c_W)\right\}$$

$$+\left|\frac{(\underline{r}_W\wedge\underline{k}_W)\cdot(\underline{p}_{l+W}\wedge\underline{k}_W)}{k_W p_W}\right|^2\left\{\left[\frac{P_n^1(c_W)}{s_W}\right]^2-\frac{n\,P_n^0(c_W)\,P_n^2(c_W)}{(n-1)\,s_W^2}\right\}$$

$$+(\underline{r}_W\wedge\underline{k}_W)\cdot(\underline{r}^*\wedge\underline{k}_W)\,\frac{1}{(n-1)^2}\left\{n\,P_n^0(c_W)-\frac{c_W}{s_W}P_n^1(c_W)\right\}^2\Big] \qquad (A\cdot36)$$

$$M_{ss}\,m_2^2k_W^2\Big[(\underline{r}_W\wedge\underline{k}_W)\cdot(\underline{r}_W^*\wedge\underline{k}_W)\,\frac{1}{(n-1)^2}\left\{n\,P_n^0(c_W)-\frac{c_W}{s_W}P_n^1(c_W)\right\}^2$$

$$+\left|\frac{\underline{p}_{l+W}\cdot(\underline{r}_W\wedge\underline{k}_W)}{p_W}\right|^2\left\{\left[\frac{P_n^1(c_W)}{s_W}\right]^2-\frac{n\,P_n^0(c_W)\,P_n^0(c_W)}{(n-1)\,s_W^2}\right\}\Big] \qquad (A\cdot37)$$

The terms in N_{kk} and N_{kk}^+ are both zero.

$$2\,N_{kr}k_W^2\Big[(\underline{r}_W\wedge\underline{k}_W)\cdot(\underline{r}_W^*\wedge\underline{k}_W)\,n\,P_n^0(c_W)\,\frac{1}{(n-1)}\left\{n\,P_n^0(c_W)-\frac{c_W}{s_W}P_n^1(c_W)\right\}$$

$$-\left|\frac{(\underline{r}_W\wedge\underline{k}_W)\cdot(\underline{p}_{l+W}\wedge\underline{k}_W)}{p_W k_W}\right|^2\left\{\left[\frac{P_n^1(c_W)}{s_W}\right]^2-\frac{n\,P_n^0(c_W)\,P_n^2(c_W)}{(n-1)\,s_W^2}\right\}\Big] \qquad (A\cdot38)$$

The term in N_{kr}^+ is the hermitian conjugate of that in N_{kr} .

The term in O_{kk} is zero.

The terms in P_{ks}, P_{sk}, P_{rs} and P_{sr} are all zero.

$$2\,Q_{ks}\,m_2^2\,k_W^2\left[-\frac{(\underline{r}_W\wedge\underline{k}_W).(\underline{p}_{4W}\wedge\underline{k}_W)}{p_W}\,\frac{1}{(n-1)}\,\frac{P_n^1(c_W)}{s_W}\left\{n\,P_n^0(c_W)-\frac{c_W}{s_W}\,P_n^1(c_W)\right\}\right] \qquad (A\cdot39)$$

The term in Q_{sk} is the hermitian conjugate of that in Q_{ks}.

$$-2\,Q_{rs}\,m_2^2\,k_W^2\left[(\underline{r}_W\wedge\underline{k}_W).(\underline{r}_W^+\wedge\underline{k}_W)\,\frac{1}{(n-1)^2}\left\{n\,P_n^0(c_W)-\frac{c_W}{s_W}\,P_n^1(c_W)\right\}^2\right.$$

$$+\frac{[(\underline{r}_W\wedge\underline{k}_W).(\underline{p}_{4W}\wedge\underline{k}_W)]}{p_W k_W}\,(\underline{r}_W^+.\underline{k}_W)\,\frac{1}{(n-1)}\,\frac{P_n^1(c_W)}{s_W}\left\{n\,P_n^0(c_W)-\frac{c_W}{s_W}\,P_n^1(c_W)\right\}$$

$$+\left|\frac{(\underline{r}_W\wedge\underline{k}_W).(\underline{p}_{4W}\wedge\underline{k}_W)}{p_W k_W}\right|^2\,\frac{1}{(n-1)^2}\,\frac{P_n^2(c_W)}{s_W}\left\{n\,P_n^0(c_W)-\frac{c_W}{s_W}\,P_n^1(c_W)\right\}$$

$$\left.+\left|\frac{\underline{p}_{4W}.(\underline{r}_W\wedge\underline{k}_W)}{p_W}\right|^2\,\frac{1}{(n-1)^2}\,\frac{P_n^2(c_W)}{s_W^2}\left\{n\,P_n^0(c_W)-\frac{c_W}{s_W}\,P_n^1(c_W)\right\}\right] \qquad (A\cdot40)$$

The term in Q_{sr} is the hermitian conjugate of that in Q_{rs}.

$$- 2\,R_{ks}\left[\,(p_2\cdot k)(\underline{r}_W\wedge\underline{k}_W)\cdot(\underline{r}^+_W\wedge\underline{k}_W)\,\frac{1}{(n-1)}\left\{n\,P_n^0(c_W)-\frac{c_W}{s_W}P_n^1(c_W)\right\}^2\right.$$

$$+\frac{(\underline{r}_W\wedge\underline{k}_W)\cdot(\underline{p}_{4W}\wedge\underline{k}_W)}{p_W}\,k_W(p_2\cdot r^+)\,\frac{1}{(n-1)}\left\{n\,P_n^0(c_W)-\frac{c_W}{s_W}P_n^1(c_W)\right\}\frac{P_n^1(c_W)}{s_W}$$

$$+\frac{(\underline{r}_W\wedge\underline{k}_W)\cdot(\underline{r}^+_W\wedge\underline{p}_W)}{p_W}\,k_W(p_2\cdot k)\,\frac{1}{(n-1)}\,\frac{P_n^1(c_W)}{s_W}\left\{n\,P_n^0(c_W)-\frac{c_W}{s_W}P_n^1(c_W)\right\}$$

$$\left.-(p_2\cdot k)\left|\frac{\underline{p}_{4W}\cdot(\underline{r}_W\wedge\underline{k}_W)}{p_W}\right|^2\left\{\left[\frac{P_n^1(c_W)}{s_W}\right]^2-\frac{n\,P_n^0(c_W)\,P_n^2(c_W)}{(n-1)\,s_W^2}\right\}\right] \qquad (A\cdot41)$$

The term in R^+_{ks} is the hermitian conjugate of that in R_{ks}.

$$+2\,S_{ks}\,m_2^2\left|(\underline{r}_W\wedge\underline{k}_W)\cdot(\underline{r}^+_W\wedge\underline{k}_W)\,\frac{1}{(n-1)}\left\{n\,P_n^0(c_W)-\frac{c_W}{s_W}P_n^1(c_W)\right\}^2\right.$$

$$+k_W\frac{(\underline{r}_W\wedge\underline{k}_W)\cdot(\underline{r}^+_W\wedge\underline{p}_{4W})}{p_W}\,\frac{1}{(n-1)}\,\frac{P_n^1(c_W)}{s_W}\left\{n\,P_n^0(c_W)-\frac{c_W}{s_W}P_n^1(c_W)\right\}$$

$$\left.-\left|\frac{\underline{p}_{4W}\cdot(\underline{r}_W\wedge\underline{k}_W)}{p_W}\right|^2\left\{\left[\frac{P_n^1(c_W)}{s_W}\right]^2-\frac{n\,P_n^0(c_W)\,P_n^2(c_W)}{(n-1)\,s_W^2}\right\}\right]$$

The term in S^+_{ks} is the hermitian conjugate of that in S_{ks}.

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PART II

Production of Cascade Particles.

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Production of Cascade Particles.

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Summary. — A model to describe the production of cascade particles in Kp collisions is set up in which the pole contribution of the Λ and Σ particles and the contributions of two-particle intermediate states are considered. These latter states are approximated by Y* resonances in the s and u channels and by a D-particle or resonance, a boson of strangeness 2, in the t channel. The two alternatives of spin $\frac{1}{2}$ and spin $\frac{3}{2}$ Ξ are considered. A field-theoretical technique is used in which all spin $\frac{3}{2}$ states are described by the Rarita-Schwinger formalism. The differential and total cross-sections for the production process are calculated at various energies for all possible parity combinations. A comparison with the experimental data indicates that the spin of the cascade particle is not $\frac{3}{2}$. The production process is anti-peripheral in the sense that the dominant mechanism is fermion exchange in the u channel. The data can best be fitted by the parity combinations $P(\Lambda\Sigma)$ even, $P(\mathrm{K}\Lambda N)$ odd and $P(\mathrm{K}\Lambda\Xi)$ odd. In this case it is also required that $g_{\mathrm{K}\Lambda N}g_{\mathrm{K}\Lambda\Xi} \approx g_{\mathrm{K}\Sigma N}g_{\mathrm{K}\Sigma\Xi}$. There is also some evidence for a peripheral process mediated by a D-particle or KK resonance. The analysis indicates that such a state, with isospin 1, should have spin 0 and even parity. The mass cannot be established but a mass of 1000 MeV and a width of 130 MeV is consistent with cascade production in both Kp and $\bar{\mathrm{p}}\mathrm{p}$ collisions.

1. – Introduction.

Recently a large number of cascade particles have been produced in Kp collisions and both the total cross-section and angular distribution for this

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production process have been measured at several energies ([1-5]). The threshold lies at $P_L = 1.06$ GeV/c (P_L is the laboratory momentum of the incident K^--meson). The total cross-section for the process

$$K^- + p \rightarrow \Xi^- + K^+ \tag{1}$$

rises from a value of 18 μb at $P_L = 1.17$ GeV/c to a value of 200 μb at $P_L = 1.6$ GeV/c. Above this energy the cross-section decreases to about 100 μb at $P_L = 2.2$ GeV/c. The main feature of the observed angular distribution is that the cascade particles are produced predominantly in the forward direction with respect to the incident K^--meson in the centre-of-mass system of the initial particles. However, production outside this forward peak is not negligible and indeed at $P_L = 1.81$ GeV/c there is evidence of an additional small backward peak.

We set up a model to describe the production process (1) in which we consider the pole contributions of the Λ and Σ particles in the direct and one of the crossed channels. In addition we calculate the contributions of two-particle intermediate states in the direct and both crossed channels. In order to do this we make the simplifying assumptions that the $\pi\Lambda$, $\pi\Sigma$ and $\bar{K}N$ intermediate states can be approximated by the Y* resonances and the KK intermediate states by a boson of strangeness 2 which we shall denote by D ([6]).

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([6]) The existence of such a particle has been proposed by a number of theorists and T. Yamanouchi (*Phys. Rev. Lett.*, **3**, 480 (1959)) presented experimental evidence for a D-particle of mass 720 MeV. Further search by Y. Eisenberg, M. Friedmann, G. Alexander and D. Kessler: *Phys. Rev.*, **120**, 1021 (1960); V. Cook, D. Keefe, L. T. Kerth, P. G. Murphy, W. A. Wenzel and T. F. Zipf: *Phys. Rev.*, **123**, 655 (1961) and B. A. Nikol'skii, V. L. Surkova, A. A. Varfolomeev and M. M. Sulkovskaya: *Soviet Physics JEPT*, **15**, 631 (1962) has produced no evidence in support of such a particle. However, these experiments could only detect a D-particle of mass less than 780 MeV and any KK resonance would have been undetected.

The necessity of introducing such a particle must stand or fall by the requirement that our model should fit the experimental data.

In Section **2** the notation for the kinematics is established.

Since the spin of the Ξ-particle is not well determined we consider the two alternatives of spin $\frac{1}{2}$ and spin $\frac{3}{2}$ in Sections **3** and **4**, respectively. We use a field-theoretical technique to describe the interactions of the particles and resonances with which we are concerned, making use of the Rarita-Schwinger formalism ([7,8]) to describe particles of spin $\frac{3}{2}$. We consider all possible parity combinations. We calculate both the differential and the total cross-sections for process (1) at various energies and make a comparison with experimental data in Section **6** after inserting isotopic spin factors which are discussed in Section **5**.

In Section **7** we discuss the consequence of our analysis as applied to the production of cascade particles in high-energy pion-proton and anti-proton-proton collisions.

In Section **8** a summary of our conclusions is given.

2. – Kinematics.

Consider the production process

$$\overline{\mathrm{K}} + \mathcal{N} \to \Xi + \mathrm{K} .$$

Let q_1 and p_1 denote the 4-momenta of the incident $\overline{\mathrm{K}}$ and $\mathcal{N}$ and q_2 and p_2 those of the outgoing K and Ξ, respectively. We define the invariants

$$s = (p_1 + q_1)^2 = (p_2 + q_2)^2 ,$$

$$t = (p_1 - p_2)^2 = (q_1 - q_2)^2 ,$$

$$u = (p_1 - q_2)^2 = (p_2 - q_1)^2 .$$

In the centre-of-mass system of the s-channel, that is the channel in which s represents the total energy squared, we shall define our 4-momenta as follows

$$p_1 = (\varepsilon_1, \boldsymbol{p}) , \qquad p_2 = (\varepsilon_2, \boldsymbol{q}) ,$$

$$q_1 = (\omega_1, -\boldsymbol{p}) , \qquad q_2 = (\omega_2, -\boldsymbol{q}) ,$$

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thus

$$s = (\varepsilon_1 + \omega_1)^2 = E^2 ,$$

$$t = m_{\mathcal{N}}^2 + m_{\Xi}^2 - 2\varepsilon_1\varepsilon_2 - 2pq\cos\vartheta ,$$

$$u = m_{\mathcal{N}} + m_{\mathrm{K}}^2 - 2\varepsilon_1\omega_2 + 2pq\cos\vartheta ,$$

where we have defined the scattering angle ϑ to be the angle between the incident $\overline{\mathrm{K}}$ and the outgoing Ξ in the centre-of-mass system.

The centre-of-mass momenta and energies are given in terms of the invariants by

$$p^2 = [(s - (m_{\mathcal{N}} + m_{\mathrm{K}})^2][s - (m_{\mathcal{N}} - m_{\mathrm{K}})^2]\frac{1}{4s} ,$$

$$q^2 = [s - (m_{\Xi} + m_{\mathrm{K}})^2][s - (m_{\Xi} - m_{\mathrm{K}})^2]\frac{1}{4s} ,$$

$$\varepsilon_1 = \frac{s + (m_{\mathcal{N}}^2 - m_{\mathrm{K}}^2)}{2\sqrt{s}} , \qquad \varepsilon_2 = \frac{s + (m_{\Xi}^2 - m_{\mathrm{K}}^2)}{2\sqrt{s}} ,$$

$$\omega_1 = \frac{s - (m_{\mathcal{N}}^2 - m_{\mathrm{K}}^2)}{2\sqrt{s}} , \qquad \omega_2 = \frac{s - (m_{\Xi}^2 - m_{\mathrm{K}}^2)}{2\sqrt{s}} .$$

The quantum numbers associated with single-particle exchange in each of the channels are for the s channel $B=1$, $S=-1$, for the u channel $B=1$, $S=-1$, and for the t channel $B=0$, $S=-2$, where B and S are baryon number and strangeness respectively.

3. – Formalism for Ξ spin $\frac{1}{2}$.

Assuming that the cascade-particle, like the nucleon, has spin $\frac{1}{2}$ and the K-meson has spin zero we can write an effective Lagrangian describing our system of particles as follows:

$$\mathscr{L} = \mathscr{L}_F + \mathscr{L}_I .$$

$\mathscr{L}_F$ is the free Lagrangian of each particle and if the form factors at all vertices are approximated by constants we have an effective-interaction Lagrangian (9),

(9) The form of the interaction Lagrangian we have adopted for coupling a spin $\frac{3}{2}$ field to a spin $\frac{1}{2}$ and a boson field is not the most general. This is because the subsidiary conditions on spin $\frac{3}{2}$ field *i.e.*, $\gamma_\mu\psi_\mu = 0$, $\partial_\mu\psi_\mu = 0$ do not apply to an intermediate state. For a discussion of this see Y. FUJII: *Prog. Theor. Phys.*, **24**, 1013 (1960). Our Lagrangian corresponds to the case when the spin $\frac{3}{2}$ state may be an external particle field.

$\mathscr{L}_I$, given by

$$\mathscr{L}_I=[g_{\mathcal{N}\mathrm{Y}\mathrm{K}}\bar{\psi}_{\mathcal{N}}\Gamma_1\psi_{\mathrm{Y}}\varphi_{\mathrm{K}}+g_{\Xi\mathrm{Y}\mathrm{K}}\bar{\psi}_{\Xi}\Gamma_2\psi_{\mathrm{Y}}\varphi_{\bar{\mathrm{K}}}+g_{\mathcal{N}\Xi\mathrm{D}}\bar{\psi}_{\mathcal{N}}\Gamma_3\psi_{\Xi}\varphi_{\mathrm{D}}+m_{\mathrm{K}}g_{\mathrm{DKK}}\varphi_{\mathrm{D}}\varphi_{\mathrm{K}}\varphi_{\mathrm{K}}+$$
$$+g_{\mathcal{N}\Xi\mathrm{D}}\bar{\psi}_{\mathcal{N}}\gamma_\mu\Gamma_4\psi_{\Xi}\varphi_{\mathrm{D}^*\mu}+g_{\mathrm{D}^*\mathrm{K}\mathrm{K}}\varphi_{\mathrm{D}^*\mu}(\varphi_{\mathrm{K}}\partial_\mu\varphi_{\mathrm{K}}-\partial_\mu\varphi_{\mathrm{K}}\varphi_{\mathrm{K}})+$$
$$+\frac{g_{\mathcal{N}\mathrm{Y}^*\mathrm{K}}}{m_{\mathcal{N}}}\bar{\psi}_{\mathrm{Y}^*\mu}\Gamma_5\psi_{\mathcal{N}}\partial_\mu\varphi_{\bar{\mathrm{K}}}+\frac{g_{\Xi\mathrm{Y}^*\mathrm{K}}}{m_{\Xi}}\bar{\psi}_{\mathrm{Y}^*\mu}\Gamma_6\psi_{\Xi}\partial_\mu\varphi_{\mathrm{K}}]+\text{h.c.}$$

The fields ψ_{Y}, $\psi_{\mathrm{Y}^*\mu}$, φ_{D} and $\varphi_{\mathrm{D}^*\mu}$ correspond to any particles having the following quantum numbers: Y: $B=1$, $S=1$, $J=\frac{1}{2}$; Y*: $B=1$, $S=-1$, $J=\frac{3}{2}$; D: $B=0$, $S=-2$, $J=0$; D*: $B=0$, $S=-2$, $J=1$, where B, S, J are baryon number, strangeness and spin, respectively.

The Γ_j ($j=1, 2, ..., 6$) are I or γ_5 according to the parity of the particles involved.

With this interaction Lagrangian we have six pole terms to evaluate corresponding to Y exchange in the s and u channels, D and D* exchange in the t channel and Y* exchange in the s and u channels.

In general we may write our S-matrix as

$$S=I-i(2\pi)^4\delta(p_1+q_1-p_2-q_2)T\,. \tag{3.1}$$

Using this definition of the T-matrix, the differential cross-section in the centre-of-mass system for the production of Ξ in $\bar{\mathrm{K}}\mathcal{N}$ collisions is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{1}{(4\pi)^2}\frac{q}{4E^2p}|T_{qp}|^2\,, \tag{3.2}$$

where the T-matrix element may quite generally be written in the form

$$T_{qp}=\bar{u}_{\Xi}(p_2)\,\Gamma^{\Xi}[-A+\tfrac{1}{2}(\gamma\cdot q_1+\gamma\cdot q_2)B]\Gamma^{\mathcal{N}}u_{\mathcal{N}}(p_1)\,. \tag{3.3}$$

Our normalization is such that

$$\sum_{\text{spin}}u_{\mathcal{N}}(p_1)\bar{u}_{\mathcal{N}}(p_1)=(\gamma\cdot p_1+m_{\mathcal{N}})\,, \tag{3.4a}$$

$$\sum_{\text{spin}}u_{\Xi}(p_2)\bar{u}_{\Xi}(p_2)=(\gamma\cdot p_2+m_{\Xi})\,. \tag{3.4b}$$

Averaging over initial spin states of nucleon and summing over the final spin states of the cascade we have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{1}{(4\pi)^2}\frac{q}{4E^2p}F\,, \tag{3.5}$$

where

$$F = \tfrac{1}{2} \sum_{\text{spin } \mathcal{N}} \sum_{\text{spin } \Xi} |T_{qp}|^2 .$$

Writing $T_{qp} = \bar{u}_{\Xi}(p_2)\, T \bar{u}_{\mathcal{N}}(p_1)$ we can split T up into contributions from each of the six pole terms we are considering $T = \sum_{i=1}^{6} T_i$. Now F is a sum of 6 terms corresponding to the pole terms taken separately and 15 terms corresponding to interference between different pole terms. In an obvious notation $F = \sum_{i,j} F_{ij}$, where

$$F_{ij} = \tfrac{1}{2} \sum \sum \bar{u}_{\Xi}(p_2) T_i u_{\mathcal{N}}(p_1)\, \bar{u}_{\mathcal{N}}(p_1) \overline{T}_j u_{\Xi}(p_2) . \tag{3.6}$$

Assuming $\Gamma^{\mathcal{N}}_{i,j} = I$ we can express F_{ij} in terms of our invariants s, t and u as follows:

$$\begin{aligned} F_{ij} = {} & A_i A_j^+ [-t + (m_{\mathcal{N}} + m_{\Xi})^2] + B_i B_j^+ \{\tfrac{1}{4}(s-u)^2 - \tfrac{1}{4}[-t + (m_{\mathcal{N}} - m_{\Xi})^2] \cdot \\ & \cdot [-t + 4m_{\mathrm{K}}^2]\} - (A_i B_j^+ + A_j^+ B_i)\, \tfrac{1}{2}(s-u)(m_{\mathcal{N}} + m_{\Xi}) . \end{aligned} \tag{3.7}$$

The six terms F_{ii} can be readily evaluated for all parity combinations by making use of the following observations [10]. The general form of T_{qp} as given by eq. (3.3) is arrived at by making use of the fact that the nucleon and cascade particle wave-functions satisfy the Dirac equations $(\gamma \cdot p_1 - m_{\mathcal{N}}) \cdot u_{\mathcal{N}}(p_1) = 0$, $(\gamma \cdot p_2 - m_{\Xi})\, u_{\Xi}(p_2) = 0$. In evaluating A_i and B_i between spinors we make the identifications $\gamma \cdot p_1 = \pm m_{\mathcal{N}}$, $\gamma \cdot p_2 = \pm m_{\Xi}$ according as $\Gamma^{\mathcal{N}} = I, \dfrac{I}{\gamma_5}$, $\Gamma^{\Xi} = \dfrac{I}{\gamma_5}$ respectively. Writing $P_i = -A_i + \tfrac{1}{2}(\gamma \cdot q_1 + \gamma \cdot q_2) B_i$ and $P_i' = -A_i^+ + \tfrac{1}{2}(\gamma \cdot q_1 + \gamma \cdot q_2) B_i^+$ we have

$$F_{ii} = \tfrac{1}{2} \operatorname{Tr} \{P_i \Gamma_i^{\mathcal{N}} (\gamma \cdot p_1 + m_{\mathcal{N}}) \Gamma_i^{\mathcal{N}} P_i' \Gamma_i^{\Xi} (\gamma \cdot p_2 + m_{\Xi}) \Gamma_i^{\Xi}\} . \tag{3.8}$$

Since $m_{\mathcal{N}}$ and m_{Ξ} only enter the expressions for A_i and B_i through the above identifications, it is clear from eq. (3.8) that changing $\Gamma_i^{\mathcal{N},\Xi}$ from I to γ_5 is equivalent to replacing $m_{\mathcal{N},\Xi}$ by $-m_{\mathcal{N},\Xi}$ throughout F_{ii}. We make use of this in evaluating the various contributions to the differential cross-section of each of the pole terms taken separately for all possible parity combinations of the particles involved. For the interference terms no such simple rule is valid.

For the pole terms 1, 2, ..., 6 the propagator functions $T_i(k)$, $i = 1, 2, \ldots, 6$

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[10] A. SALAM: *Nucl. Phys.*, **5**, 687 (1958).

apart from coupling constants are given by

$$(3.9a)\quad T_1(k) = \frac{\gamma\cdot k + m_Y}{k^2 - m_Y^2} \qquad \text{where } k = (p_1 + q_1),\ k^2 = s,$$

$$(3.9b)\quad T_2(k) = T_1(k) \qquad \text{where } k = (p_1 - q_2),\ k^2 = u,$$

$$(3.9c)\quad T_3(k) = \frac{m_K}{k^2 - m_D^2}, \qquad \text{where } k = (p_1 - p_2),\ k^2 = t,$$

$$(3.9d)\quad T_4(k) = \gamma_\mu\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m_{D^*}^2}\right)(q_{1\nu} + q_{2\nu})\frac{1}{k^2 - m_{D^*}^2}, \quad \text{where } k = (p_1 - p_2),\ k^2 = t,$$

$$(3.9e)\quad T_5(k) = \left\{\frac{\gamma\cdot k + m_{Y^*}}{k^2 - m_{Y^*}^2}\left[q_2\cdot q_1 - \tfrac{1}{3}\gamma\cdot q_2 k\cdot q_1 - \frac{1}{3m_{Y^*}}(\gamma\cdot q_2 k\cdot q_1 - k\cdot q_2\gamma\cdot q_1) - \right.\right.$$

$$\left.\left. - \frac{2}{3m_{Y^*}^2}k\cdot q_2 k\cdot q_1\right] - \frac{1}{3m_{Y^*}^2}[(\gamma\cdot q_2 k\cdot q_1 - k\cdot q_2\gamma\cdot q_1) + (\gamma\cdot k + m_{Y^*})\gamma\cdot q_2\gamma\cdot q_1]\right\}\frac{1}{(m_N m_\Xi)},$$

$$\text{where } k = (p_1 + q_1),\quad k^2 = s.$$

$$(3.9f)\quad T_6(k) = T_5(k) \qquad \text{where } k = (p_1 - p_2),\ k^2 = u.$$

It is interesting to note that we have a crossing symmetry [11] whereby

$$(3.10a)\qquad A_{1,5}(s, t, u) = \quad A_{2,6}(u, t, s),$$

$$(3.10b)\qquad B_{1,5}(s, t, u) = -B_{2,6}(u, t, s).$$

In terms of invariants

$$(3.11a)\quad A_1 = -g_{NYK}g_{\Xi YK}[\tfrac{1}{2}(m_N + m_\Xi) + m_Y]\frac{1}{s - m_Y^2}, \quad B_1 = \quad g_{NYK}g_{\Xi YK}\frac{1}{s - m_Y^2},$$

$$(3.11b)\quad A_3 = -g_{N\Xi D}g_{DKK}\frac{m_K}{t - m_D^2}, \qquad B_3 = 0,$$

$$(3.11c)\quad A_4 = 0, \qquad B_4 = -g_{N\Xi D}g_{DKK}\frac{2}{t - m_{D^*}^2},$$

$$(3.11d)\quad \begin{cases} A_5 = g_{NY^*K}g_{\Xi Y^*K}\dfrac{a_5(s, t, u)}{m_N m_\Xi}\dfrac{1}{(s - m_{Y^*}^2)}, \\ B_5 = g_{NY^*K}g_{\Xi Y^*K}\dfrac{b_5(s, t, u)}{m_N m_\Xi}\dfrac{1}{(u - m_{Y^*}^2)}, \end{cases}$$

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[11] A. Ramakrishnan, R. Thunga, T. V. Radha, G. Bhanathi and S. Indumathi: *Nucl. Phys.*, **37**, 585 (1962).

where

$$(3.11e)\qquad a_5(s,t,u) = -\tfrac{1}{2}(m_\Xi - m_\mathcal{N})\cdot$$
$$\cdot\left\{\frac{s+m_\Xi m_{Y^*}}{3m_{Y^*}^2}[s+m_K^2-\tfrac{1}{2}(m_\Xi^2+m_\mathcal{N}^2)]-\frac{sm_K^2}{3m_{Y^*}^2}\right\}-[\tfrac{1}{2}(m_\Xi+m_\mathcal{N})+m_{Y^*}]\cdot$$
$$\cdot\left\{\tfrac{1}{2}(-t+2m_K^2)-\frac{2}{3m_{Y^*}^2}\frac{1}{4}(s-m_\Xi^2+m_K^2)(s-m_\mathcal{N}^2+m_K^2)\right\}-[s-\tfrac{1}{2}(m_\mathcal{N}^2+m_\Xi^2)]\cdot$$
$$\cdot\left\{\frac{1}{3m_{Y^*}}\tfrac{1}{2}(s-m_\Xi^2+m_K^2)-\frac{s(m_\Xi+m_{Y^*})}{3m_{Y^*}^2}\right\}+\frac{m_\Xi m_K^2}{3m_{Y^*}^2}\frac{1}{2}(s-m_\mathcal{N}^2+m_K^2)\,,$$

and

$$(3.11f)\qquad b_5(s,t,u) = \tfrac{1}{2}(-t+2m_K^2)-\frac{2}{3m_{Y^*}^2}\frac{1}{4}(s-m_\Xi^2+m_K^2)(s-m_\mathcal{N}^2+m_K^2)-$$
$$-\tfrac{1}{2}(m_\Xi^2-m_\mathcal{N}^2)\frac{1}{3m_{Y^*}^2}(s+m_\Xi m_{Y^*})-\frac{sm_K^2}{3m_{Y^*}^2}-$$
$$-\tfrac{1}{2}(m_\Xi+m_\mathcal{N})\frac{1}{3m_{Y^*}}(s-m_\Xi^2+m_K^2)-(m_\Xi+m_\mathcal{N})(m_\Xi+m_{Y^*})\frac{s}{3m_{Y^*}^2}\,.$$

Making use of eqs. (3.5), (3.7) and the above expressions for A_i and B_i, we can calculate the differential cross-section for cascade particle production.

4. – Formalism for Ξ spin $\frac{3}{2}$.

We make use of the Rarita-Schwinger formalism for a particle of spin $\frac{3}{2}$ and represent such a particle by a wave function $\psi_\mu(x)$ which satisfies the wave equations

$$(-i\gamma\cdot\partial+m)\psi_\mu(x)=0 \qquad\qquad \gamma_\mu\psi_\mu(x)=0$$

and the subsidiary condition $\partial_\mu\psi_\mu(x)=0$.

The wave function $\psi_\mu(x)$ is a 4-vector each component of which is a spinor with 4 components.

The effective Lagrangian is given by

$$\mathscr{L}_I = \Big[g_{\mathcal{N}YK}\bar\psi_\mathcal{N}\Gamma_1\psi_Y\varphi_K+\frac{g_{\Xi YK}}{m_\Xi}\bar\psi_{\Xi\mu}\Gamma_2\psi_\mathcal{N}\partial_\mu\varphi_K+\frac{g_{\mathcal{N}\Xi D}}{m_\Xi}\bar\psi_{\Xi\mu}\Gamma_3\psi_\mathcal{N}\partial_\mu\varphi_D+$$
$$+m_K g_{DKK}\varphi_K\varphi_D\varphi_K+g_{\mathcal{N}\Xi D^*}\bar\psi_{\Xi\mu}\Gamma_4\psi_\mathcal{N}\varphi_{D^*\mu}+g_{D^*KK}\varphi_{D^*\mu}(\varphi_K\partial_\mu\varphi_K-\partial_\mu\varphi_K\varphi_K)+$$
$$+\frac{g_{\mathcal{N}Y^*K}}{m_\mathcal{N}}\bar\psi_{Y^*\mu}\Gamma_5\psi_\mathcal{N}\partial_\mu\varphi_K+g_{\Xi Y^*K}\bar\psi_{\Xi\mu}\Gamma_6\psi_{Y^*\mu}\varphi_K\Big]+\text{h. c.}$$

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As before Γ_j $(j=1, ..., 6)$ are I or γ_5 according to the parity of the particles involved. We use this Lagrangian to evaluate the six pole terms.

Once again we have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{(4\pi)^2}\frac{q}{4E^2p}|T_{qp}|^2, \tag{4.1}$$

where we can now write the T matrix element in the general form

$$T_{qp} = q_{1\mu}\overline{U}_{\Xi\mu}(p_2)\Gamma^{\Xi}[-A+\tfrac{1}{2}(\gamma\cdot q_1+\gamma\cdot q_2)B]\Gamma^{\mathcal{N}}u_{\mathcal{N}}(p_1) + \\ + q_{2\mu}\overline{U}_{\Xi\mu}(p_2)\Gamma^{\Xi}[-C+\tfrac{1}{2}(\gamma\cdot q_1+\gamma\cdot q_2)D]\Gamma^{\mathcal{N}}u_{\mathcal{N}}(p_1). \tag{4.2}$$

Our normalization is such that

$$\sum_{\text{spin}} u_{\mathcal{N}}(p_1)\overline{u}_{\mathcal{N}}(p_1) = (\gamma\cdot p_1 + m_{\mathcal{N}}), \tag{4.3a}$$

$$\sum_{\text{spin}} U_{\Xi\mu}(p_2)\overline{U}_{\Xi\nu}(p_2) = \theta_{\mu\nu}(\gamma\cdot p_2+m_{\Xi}) = (\gamma\cdot p_2+m_{\Xi})\theta_{\mu\nu}, \tag{4.3b}$$

where [12,13]

$$\theta_{\mu\nu} = g_{\mu\nu} - \tfrac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p_2^2}(\gamma\cdot p_2\gamma_\mu p_{2\nu} + p_{2\mu}\gamma_\nu\gamma\cdot p_2).$$

Writing $T_{qp} = \overline{U}_{\Xi\mu}(p_2)T_\mu u_{\mathcal{N}}(p_1)$ we can split T up into contributions from each of the six pole terms we are considering $T_\mu = \sum_{j=1}^{6} T_{\mu j}$.

As in Section **3** we have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{(4\pi)^2}\frac{q}{4E^2p}F, \tag{4.4}$$

where

$$F = \tfrac{1}{2}\sum_{\text{spin}\mathcal{N}}\sum_{\text{spin}\Xi}|T_{qp}|^2 = \sum_{i,j}F_{ij}$$

and

$$F_{ij} = \tfrac{1}{2}\sum\sum\{\overline{U}_{\Xi\mu}(p_2)T_{i\mu}u_{\mathcal{N}}(p_1)\,\overline{u}_{\mathcal{N}}(p_1)\overline{T}_{j\nu}U_{\Xi\nu}(p_2)\}. \tag{4.5}$$

Assuming $\Gamma_{i,j}^{\mathcal{N},\Xi} = I$ we can express F_{ij} as follows

$$F_{ij} = \tfrac{1}{2}\,\mathrm{Tr}\{(-a_{i\mu}+\gamma\cdot Q\,b_{i\mu})(\gamma\cdot p_1+m_{\mathcal{N}})(-a_{j\nu}^{+}+\gamma\cdot Qb_{j\nu}^{+})(\gamma\cdot p_2+m_{\Xi})\theta_{\nu\mu}\}, \tag{4.6}$$

where $Q=\frac{1}{2}(q_1+q_2)$, $a_i = q_1A_i+q_2C_i$, and $b_i = q_1B_i+q_2D_i$.

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(12) R. BEHRENDS and C. FRONSDAL: *Phys. Rev.*, **106**, 345 (1957).

(13) C. FRONSDAL: *Suppl. Nuovo Cimento*, **9**, 416 (1958).

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For the pole terms taken separately we have the same rule as in Section 3 by which we relate the sign of the mass of the external spinor particle in the expression for F_{ii} to the relative parity of the particles coupled at the vertices of the corresponding Feynman diagram.

If we now consider the case $i=j$, $I_i^{\mathcal{N},\Xi}=I$ it can be shown that

$$
\begin{aligned}
(4.7)\qquad F_{ii} &= (p_1\cdot p_2 + m_{\mathcal{N}} m_\Xi)\left[\frac{4}{3}\,a_i^2 - \frac{4}{3m_\Xi^2}\,(a_i\cdot p_2)^2\right] + \\
&+ [2p_1\cdot Q p_2\cdot Q - Q^2\,(p_1\cdot p_2 - m_{\mathcal{N}} m_\Xi)]\left[\frac{4}{3}\,b_i^2 - \frac{3}{4m_\Xi^2}\,(b_i\cdot p_2)^2\right] - \\
&- [m_\Xi p_1\cdot Q + m_{\mathcal{N}}\cdot p_2\cdot Q]\left[\frac{2}{3}\,a_i b_i - \frac{2}{3m_\Xi^2}\,a_i\cdot p_2 b_i\cdot p_2\right] + \\
&+ \frac{1}{3m_\Xi}\,p_2\cdot p_2(a_i\cdot p_1 b_i\cdot Q - a_i\cdot Q b_i\cdot p_1) + \frac{1}{3m_\Xi}\,p_2\cdot p_1(a_i\cdot Q b_i\cdot p_2 - a_i\cdot p_2 b_i\cdot Q) + \\
&+ \frac{1}{3m_\Xi}\,p_2\cdot Q(a_i\cdot p_2 b_i\cdot p_1 - a_i\cdot p_1 b_i\cdot p_2)\,.
\end{aligned}
$$

Apart from coupling constants the propagator functions $T_{i\mu}$ $(i=1, 2, \ldots, 6)$ for each of our pole terms are given by

$$
(4.8a)\qquad T_{1\mu} = \frac{q_{2\mu}}{m_\Xi}\,\frac{\gamma\cdot k + m_\Upsilon}{k^2 - m_\Upsilon^2}\,, \qquad \text{where } k=(p_1+q_1),\ k^2=s,
$$

$$
(4.8b)\qquad T_{2\mu} = \frac{q_{1\mu}}{m_\Xi}\,\frac{\gamma\cdot k + m_\Upsilon}{k^2 - m_\Upsilon^2}\,, \qquad \text{where } k=(p_1-q_2),\ k^2=u,
$$

$$
(4.8c)\qquad T_{3\mu} = \frac{m_K}{m_\Xi}\,\frac{k_\mu}{k^2 - m_D^2}\,, \qquad \text{where } k=(p_1-p_2),\ k^2=t,
$$

$$
(4.8d)\qquad T_{4\mu} = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m_{D^*}^2}\right)\frac{q_{1\nu}+q_{2\nu}}{(k^2 - m_{D^*}^2)}\,, \qquad \text{where } k=(p_1-p_2),\ k^2=t,
$$

$$
\begin{aligned}
(4.8e)\qquad T_{5\mu} = \Bigg\{\left(\frac{\gamma\cdot k + m_{\Upsilon^*}}{k^2 - m_{\Upsilon^*}}\right)\Bigg[q_{1\mu} - \frac{1}{3}\,\gamma_\mu\gamma\cdot q_1 - \frac{1}{3m_{\Upsilon^*}}\,(\gamma_\mu k\cdot q_1 - k_\mu\gamma\cdot q_1) - \frac{2}{3m_{\Upsilon^*}^2}\,k_\mu k\cdot q_1\Bigg] - \\
- \frac{1}{3m_{\Upsilon^*}^2}\Big[(\gamma_\mu k\cdot q_1 - k_\mu\gamma\cdot q_1) + (\gamma\cdot k + m_{\Upsilon^*})\gamma_\mu\gamma\cdot q_1\Big]\Bigg\}\frac{1}{m_{\mathcal{N}}}\,, \\
\text{where } k=(p_1+q_1),\ k^2=s,
\end{aligned}
$$

$$
(4.8f)\qquad T_{6\mu}(k, q_1, q_2) = T_{5\mu}(k, -q_2, -q_1)\,, \qquad \text{where } k=(p_1-q_2),\ k^2=u\,.
$$

We now have the following crossing symmetry relations

$$
(4.9a)\qquad A_{1,2,5,6}(s, t, u) = -\,C_{2,1,6,5}(u, t, s)\,,
$$

$$
(4.9b)\qquad B_{1,2,5,6}(s, t, u) = +\,D_{2,1,6,5}(u, t, s)\,.
$$

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In terms of invariants

$$(4.10a)\qquad A_1 = B_1 = 0\,, \qquad C_1 = -\,g_{\mathcal{N}\mathrm{Y}\mathrm{K}}g_{\Xi\mathrm{Y}\mathrm{K}}\frac{[\frac{1}{2}(m_{\mathcal{N}}+m_{\Xi})+m_{\mathrm{Y}}]}{m_{\Xi}}\frac{1}{(s-m_{\mathrm{Y}}^2)}$$

$$D_1 = g_{\mathcal{N}\mathrm{Y}\mathrm{K}}g_{\Xi\mathrm{Y}\mathrm{K}}\frac{1}{m_{\Xi}}\frac{1}{(s-m_{\mathrm{Y}}^2)}\,,$$

$$(4.10b)\qquad B_3 = D_3 = 0\,, \qquad A_3 = g_{\mathcal{N}\Xi\mathrm{D}}g_{\mathrm{D}\mathrm{K}\mathrm{K}}\frac{m_{\mathrm{K}}}{m_{\Xi}}\frac{1}{(t-m_{\mathrm{D}}^2)}\,, \qquad C_3 = -A_3\,,$$

$$(4.10c)\qquad B_4 = D_4 = 0\,, \qquad A_4 = -\,g_{\mathcal{N}\Xi\mathrm{D}}g_{\mathrm{D}\mathrm{K}\mathrm{K}}\frac{1}{(t-m_{\mathrm{D}^*}^2)}\,, \qquad C_4 = A_4\,,$$

$$(4.10d)\qquad \begin{cases} A_5 = -\,g_{\mathcal{N}\mathrm{Y}^*\mathrm{K}}g_{\Xi\mathrm{Y}^*\mathrm{K}}\dfrac{[\frac{1}{2}(m_{\mathcal{N}}+m_{\Xi})+m_{\mathrm{Y}^*}]}{m_{\mathcal{N}}}\dfrac{1}{(s-m_{\mathrm{Y}^*}^2)} \\ B_5 = g_{\mathcal{N}\mathrm{Y}^*\mathrm{K}}g_{\Xi\mathrm{Y}^*\mathrm{K}}\dfrac{1}{m_{\mathcal{N}}}\dfrac{1}{(s-m_{\mathrm{Y}^*}^2)}\,, \\ C_5 = g_{\mathcal{N}\mathrm{Y}^*\mathrm{K}}g_{\Xi\mathrm{Y}^*\mathrm{K}}\left\{-\dfrac{2}{3m_{\mathrm{Y}^*}^2}\dfrac{1}{2}(s-m_{\mathcal{N}}^2+m_{\mathrm{K}}^2)\left[\dfrac{1}{2}(m_{\Xi}+m_{\mathcal{N}})+2m_{\mathrm{Y}^*}\right]-\right. \\ \left.-\dfrac{1}{2}(m_{\Xi}-m_{\mathcal{N}})\dfrac{(s-m_{\mathrm{Y}^*}m_{\Xi})}{3m_{\mathrm{Y}^*}^2}+\dfrac{1}{3m_{\mathrm{Y}^*}}[s-\tfrac{1}{2}(m_{\mathcal{N}}^2+m_{\Xi}^2)]\right\}\dfrac{1}{m_{\mathcal{N}}}\dfrac{1}{(s-m_{\mathrm{Y}^*}^2)}, \\ D_5 = g_{\mathcal{N}\mathrm{Y}^*\mathrm{K}}g_{\Xi\mathrm{Y}^*\mathrm{K}}\left\{-\dfrac{(s+m_{\mathcal{N}}m_{\mathrm{Y}^*})}{3m_{\mathrm{Y}^*}^2}-\dfrac{2}{3m_{\mathrm{Y}^*}^2}\dfrac{1}{2}(s-m_{\mathcal{N}}^2+m_{\mathrm{K}}^2)\right\}\dfrac{1}{m_{\mathcal{N}}}\dfrac{1}{(s-m_{\mathrm{Y}^*}^2)}. \end{cases}$$

In terms of the above variables

$$(4.11)\qquad F_{ii} = [-t+(m_{\mathcal{N}}+m_{\Xi})^2]\cdot$$

$$\cdot\frac{2}{3}\left\{(A_i-C_i)^2m_{\mathrm{K}}^2+A_iC_i(-t+4m_{\mathrm{K}}^2)-\frac{1}{m_{\Xi}^2}\left[\frac{1}{2}(m_{\Xi}^2+m_{\mathrm{K}}^2)(A_i-C_i)-\tfrac{1}{2}(A_iu-C_is)\right]^2\right\}+$$

$$+\left\{\frac{1}{4}(s-u)^2-\frac{1}{4}(-t+4m_{\mathrm{K}}^2)[-t+(m_{\mathcal{N}}-m_{\Xi})^2]\right\}\frac{2}{3}\left\{(B_i-D_i)^2m_{\mathrm{K}}^2+B_iD_i(-t+4m_{\mathrm{K}}^2)-\right.$$

$$\left.-\frac{1}{m_{\Xi}^2}\left[\frac{1}{2}(m_{\Xi}^2+m_{\mathrm{K}}^2)(B_i-D_i)-\frac{1}{2}(B_iu-D_is)\right]^2\right\}-$$

$$-(m_{\Xi}+m_{\mathcal{N}})(s-u)\frac{2}{3}\left\{(A_i-C_i)(B_i-D_i)m_{\mathrm{K}}^2+(A_iD_i+B_iC_i)\frac{1}{2}(-t+4m_{\mathrm{K}}^2)-\right.$$

$$\left.-\frac{1}{m_{\Xi}^2}\left[\frac{1}{2}(m_{\Xi}^2+m_{\mathrm{K}}^2)(A_i-C_i)-\frac{1}{2}(A_iu-C_is)\right]\left[\frac{1}{2}(m_{\Xi}^2+m_{\mathrm{K}}^2)(B_i-D_i)-\frac{1}{2}(B_iu-D_is)\right]\right\}-$$

$$-\frac{4}{3m_{\Xi}}(B_iC_i-A_iD_i)\left\{\frac{1}{16}(s-u)^2t+\frac{1}{16}(-t+4m_{\mathrm{K}}^2)\cdot\right.$$

$$\left.\cdot[-t+(m_{\mathcal{N}}+m_{\Xi})^2][-t+(m_{\mathcal{N}}-m_{\Xi})^2]\right\}.$$

Using this expression and eq. (4.6) we can calculate the differential cross-section due to the pole terms taken separately.

5. – Isotopic spin analysis.

To complete our analysis of the single-particle-exchange terms that we have discussed in Sections **3** and **4** we must insert the appropriate isotopic-spin factors at each vertex.

For the process

$$K^- + p \to K^+ + \Xi^-$$

the minimum values of isospin for particles exchanged in the s, u and t channels are 0, 0 and 1, respectively. Hence our comparison with experimental data for the reaction (1) will only yield information about a boson of strangeness 2 and isospin 1. We can make no inferences about a possible D-boson with zero isospin until experimental data for the processes

$$K^- + p \to K^0 + \Xi^0,$$

$$K^- + n \to K^0 + \Xi^-,$$

become available.

The definition of the coupling constants is to a certain extent arbitrary but we have taken them so that in isospin space we can write our rotationally invariant Lagrangian in the following form

$$\begin{aligned}\mathscr{L} = \{ & g_{N\Xi D_0}(\bar{p}\Xi^0 D_0^+ + \bar{n}\Xi^- D_0^+) + \\ & + g_{N\Xi D_1}(\sqrt{2}\,\bar{p}\Xi^- D_1^{++} + \sqrt{2}\bar{n}\Xi^0 D_1^0 + \bar{p}\Xi^0 D_1^+ - \bar{n}\Xi^- D_1^+) + \\ & + g_{\bar{K}KD_0}(\bar{K}^0 K^+ D_0^- + \bar{K}^- K^0 D_0^-) + \\ & + g_{\bar{K}KD_1}(\sqrt{2}\,\bar{K}^0 K^0 D_1^0 + \sqrt{2}\,\bar{K}^- K^+ D_1^{--} + \bar{K}^- K^+ D^- - \bar{K}^- K^0 D^-) + \\ & + g_{NY_0K}(\bar{p}K^+ Y_0^0 + \bar{n}K^0 Y_0^0) + \\ & + g_{NY_1K}(\sqrt{2}\,\bar{p}K^0 Y_1^+ + \sqrt{2}\,\bar{n}K^+ Y_1^- + \bar{p}K^+ Y_1^0 - \bar{n}K^0 Y_1^0) + \\ & + g_{\Xi Y_0 K}(\bar{\Xi}^0 K^0 Y_0^0 + \bar{\Xi}^- K^- Y_0^0) + \\ & + g_{\Xi Y_1 K}(\sqrt{2}\,\bar{\Xi}^0 K^- Y_1^+ + \sqrt{2}\,\bar{\Xi}^- K^0 Y_1^- + \bar{\Xi}^0 K^0 Y_1^0 - \bar{\Xi}^- K^- Y_1^0)\} + \text{h. c.}\end{aligned}$$

where D_0, D_1 (Y_0, Y_1) denote bosons (fermions) of strangeness -2 (-1) and isotopic spin 0, 1 (0, 1), respectively.

The fermion particles and resonances having the appropriate quantum numbers can be listed as

$I = 0$ Λ (mass 1.115 GeV) Y_0^* (1.405, 1.520, 1.815),

$I = 1$ Σ (mass 1.189 GeV) Y_1^* (1.385, 1.685).

If we invoke group theory and assume that the strong interactions are invariant under SU_3 then the D_0 meson belongs to the representation 10 and the D_1 meson to the representation 27. In the octet ([14-16]) model of SU_3 the pseudoscalar K-meson can be coupled to the baryons by D or F type couplings or an arbitrary mixing of the two.

For D type couplings

$$-\frac{1}{\sqrt{3}}\, g_{\mathcal{N}\Lambda\mathrm{K}} = g_{\mathcal{N}\Sigma\mathrm{K}} = -\frac{1}{\sqrt{3}}\, g_{\Xi\Lambda\mathrm{K}} = -\, g_{\Xi\Sigma\mathrm{K}} .$$

For F type couplings

$$-\sqrt{3}\, g_{\mathcal{N}\Lambda\mathrm{K}} = -\, g_{\mathcal{N}\Sigma\mathrm{K}} = \sqrt{3}\, g_{\Xi\Lambda\mathrm{K}} = -\, g_{\Xi\Sigma\mathrm{K}} .$$

The Sakata model gives no relations similar to the above since the Λ, Σ and Ξ particles belong to different representations of the unitary group.

Without any group-theoretic model all the coupling constants are independent of each other.

6. – Results and comparison with experiment.

Using the formalism of Sections 3 and 4 we have calculated the contribution to both the total and differential cross-sections of each of our 6 pole terms taken separately. We have used the mass values $m_{\mathcal{N}}=0.938$, $m_{\Xi}=1.320$, $m_{\mathrm{K}}=0.494$ GeV and considered the case $m_{\mathrm{D}}=m_{\mathrm{D}^*}=0.720$, 1.000, 1.440; $m_{\mathrm{Y}}=1.115$, 1.189; $m_{\mathrm{Y}^*}=1.385$ GeV. Calculations have been carried out for $P_{\mathrm{L}}=1.2$, 1.4, 1.6, ..., 3.0 GeV/c for all possible parity combinations.

We consider the two cases of Ξ having *a*) spin $\frac{3}{2}$, *b*) spin $\frac{1}{2}$ and in the latter case we discuss an interference effect between Λ and Σ pole terms.

a) Our results indicate that the data cannot be fitted in any way under the assumption of spin $\frac{3}{2}$ for the cascade particle since none of the terms we have calculated gives a large forward peak. Typical angular distributions are

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([14]) Y. NE'EMAN: *Nucl. Phys.*, **26**, 222 (1961).

([15]) M. GELL-MANN: *The eight-fold way: a theory of strong interaction symmetry*, California Institute of Technology Synchroton Laboratory Report CSTL-20 (1961) and *Phys. Rev.*, **125**, 1067 (1962).

([16]) J. J. SAKURAI: *New mesons and resonances in strong interaction physics-theoretical.* To be published in the *Proceedings of the International School of Physics «Enrico Fermi»* (Villa Monastero, Varenna, Como).

shown in Fig. 1 for exchange of a Λ particle in the u channel at $P_L = 1.8$ GeV/c. The rather obvious fact that kinematic factors in our spin $\frac{3}{2}$ projection operator give total cross-sections which increase rapidly and continuously with energy is additional evidence for the conclusion that the cascade particle is not a

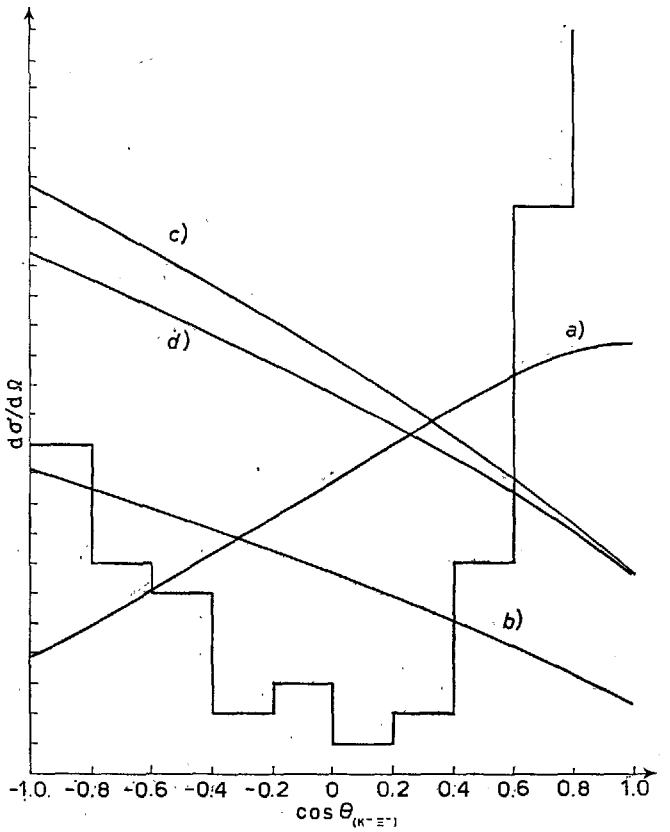

Fig. 1. – Contribution to the differential cross-section for process (1) due to exchange of a Y-particle (mass 1115 MeV, spin $\frac{1}{2}$) in the u channel at $P_L = 1.8$ GeV/c, assuming the Ξ-particle has spin $\frac{3}{2}$. Curves *a*), *b*), *c*) and *d*) correspond to the parity combinations even even, odd even, even odd and odd odd for $P(KY\mathcal{N})$ and $P(KY\Xi)$, respectively. The normalization is arbitrary. The histogram gives the experimental data of ref. [3].

spin $\frac{3}{2}$ particle satisfying the Rarita-Schwinger formalism. The total cross-section for cascade particle production is found experimentally to decrease with energy above $P_L = 1.6$ GeV/c as shown in Fig. 2.

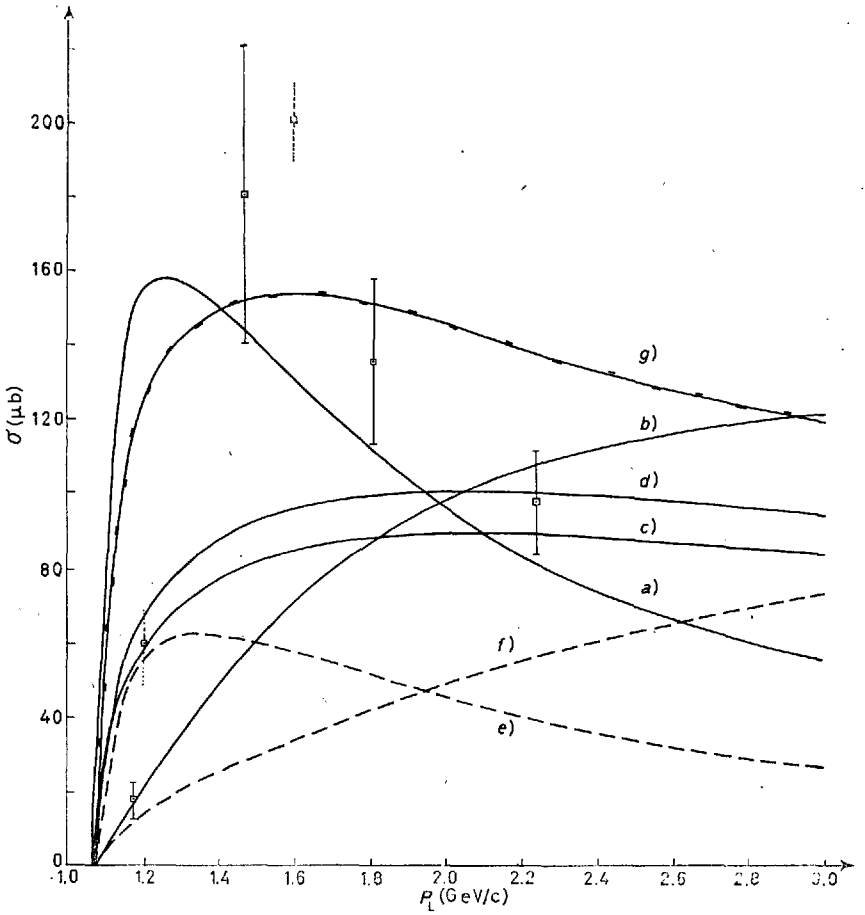

Fig. 2. – Variation of the total cross-section for process (1) with P_L due to exchange of a D-particle (mass 1000 MeV) in the t channel and of a Y-particle (mass 1115 MeV) in the u channel assuming the Ξ-particle has spin $\frac{1}{2}$. Curves *a*), *b*), *c*) and *d*) correspond to the similarly labelled curves of Fig. 5 and 6. Curves *e*) and *f*) correspond to the cases of D spin 0, $P(\mathcal{N}\Xi D)$ even and D spin 1, $P(\mathcal{N}\Xi D)$ odd, respectively. Curve *g*) gives the result of combining the curves *d*) and *e*). The experimental data are that given in ref. [1-5].

b) Since we define the scattering angle to be the angle between the incident K^--meson and the outgoing Ξ^--particle we can make the general comment that poles in the u and t channels lead to forward and backward peaking, respectively. We find that exchange of a spin 1 D-particle in the t channel gives a very sharp backward peak for both parity cases whilst exchange of a spin 0 D-particle gives a smaller backward peak for even $P(D\mathcal{N}\Xi)$ and a fairly uniform angular distribution for odd $P(D\mathcal{N}\Xi)$. Decreasing the mass of

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the exchanged particle gives sharper peaking in all cases. Typical angular distributions are shown in Figs. 3 and 4 for $m_D = 1.000$ at $P_L = 1.8$ GeV/c and 2.2 GeV/c, respectively.

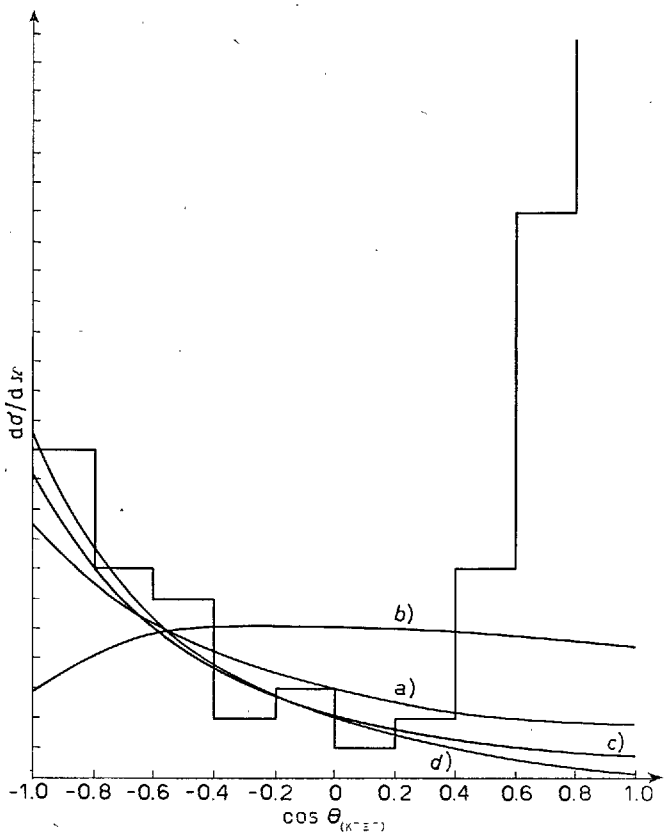

Fig. 3. – Contribution to the differential cross-section for process (1) due to exchange of a D-particle (mass 1000 MeV) in the t channel at $P_L = 1.8$ GeV/c assuming the Ξ-particle has spin $\frac{1}{2}$. Curves *a*) and *b*) correspond to D spin 0 with $P(\mathcal{N}\Xi D)$ even and odd, curves *c*) and *d*) correspond to D spin 1 with $P(\mathcal{N}\Xi D)$ even and odd. The normalization is such that $g_{\mathcal{N}\Xi D} g_{DKK}/4\pi = 0.37$, 0.94, 0.074 and 0.086 for the curves *a*), *b*), *c*) and *d*), respectively. The histogram gives the experimental data of ref. [3].

This mechanism might be responsible for the backward production of cascades observed at $P_L = 1.8$ GeV/c and at lower energies. The fact that this backward peak is not observed at $P_L = 2.2$ GeV/c suggests that the D-particle has spin zero for only in this case does the cross-section due to the exchange of such a particle decrease with energy. For a spin 1 D-particle the peak increases with energy. We have normalized the theoretical cross-section

to the data at $P_L = 1.8$ GeV/c assuming that this exchange process provides the mechanism for all the backward scattering. In this way we can set an upper limit to the coupling strength of the D-particle. For a D-particle of mass 1000 MeV, spin 0 and for $P(\mathrm{D}\mathcal{N}\Xi)$ even we find

$$\left(\frac{g_{\mathrm{KKD}}\, g_{\mathrm{D}\mathcal{N}\Xi}}{4\pi}\right) \leqslant 0.37 .$$

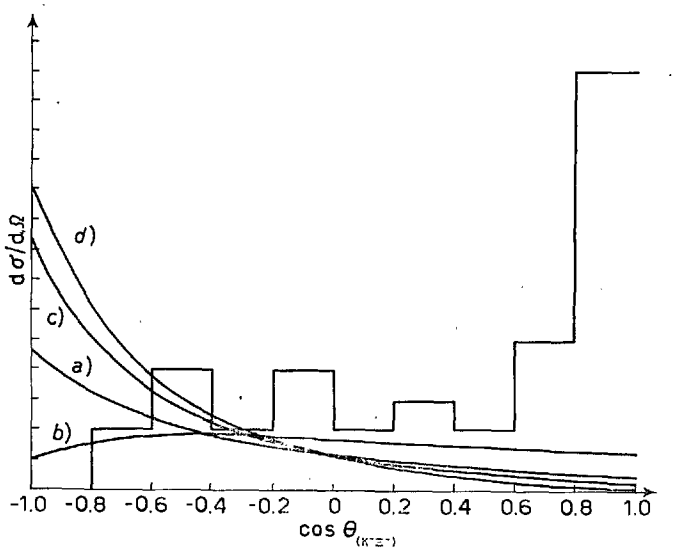

Fig. 4. – As in Fig. 3 except that $P_L = 2.2$ GeV/c. The histogram gives the experimental data of ref. ([5]).

Increasing the mass of the D-particle merely makes the backward peak somewhat broader and the decrease with energy of the cross-section becomes slower. For $m_D = 720$ MeV and 1440 MeV we find that the upper limits on the coupling constant are 0.25 and 0.62, respectively.

The angular distributions given by poles in the s channel do not exhibit any forward peaking for fermions of spin $\frac{1}{2}$ or spin $\frac{3}{2}$. The distribution is fairly uniform for most spin and parity cases although for a fermion of spin $\frac{3}{2}$ a small backward peak is given if $P(\mathrm{KY}^*\mathcal{N})$ and $P(\mathrm{KY}^*\Xi)$ are either even and odd respectively or both odd. Clearly therefore, the poles in the s channel do not provide the dominant mechanism for the production process although they may be responsible for a roughly uniform background term in the angular distribution upon which the peaks are super-posed. It is possible that the Y* pole term is responsible for the observed small backward peak but we find that kinematic factors in the propagator of such a pole term result in the backward peak increasing rapidly with energy in contrast to the experimental data.

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As is to be expected the exchange of a fermion in the u channel gives rise to a forward peak in the production of cascade particles. The size and shape of this peak depends on the spin of the exchanged particle and on the relative parity of the particles involved. The angular distribution for exchange of spin $\frac{1}{2}$ and spin $\frac{3}{2}$ particles are shown in Fig. 5 and 6 for $m_Y = 1.115$ and

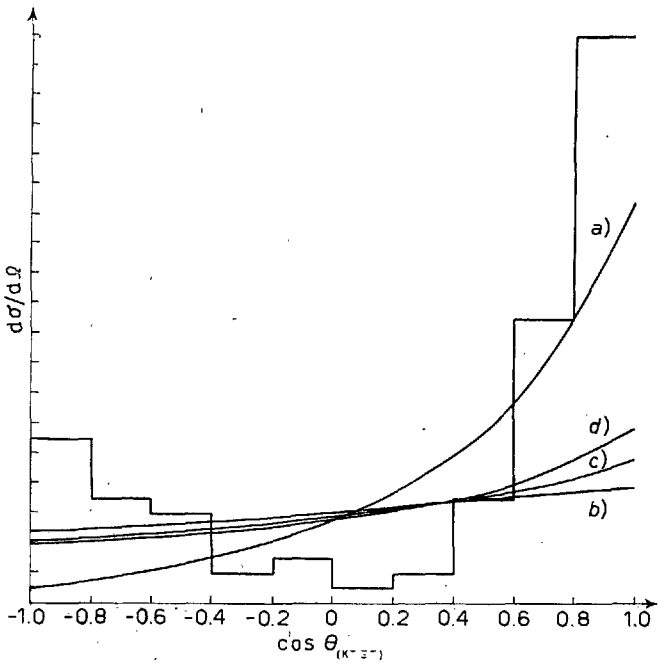

Fig. 5. – Contribution to the differential cross-section for process (1) due to exchange of a Y particle (mass 1115 MeV spin $\frac{1}{2}$) in the u channel at $P_L = 1.8$ GeV/c assuming the Ξ particle has spin $\frac{1}{2}$. Curves (*a*), (*b*), (*c*) and (*d*) correspond to the parity combinations even even, odd even, even odd, and odd odd for $P(\mathrm{KY}\mathcal{N})$ and $P(\mathrm{KY}\Xi)$, respectively. The normalization is such that $g_{\mathrm{KY}\mathcal{N}} g_{\mathrm{KY}\Xi}/4\pi = 0.40$, 0.72, 0.49 and 0.54 for the curves (*a*), (*b*), (*c*) and (*d*), respectively. The histogram gives the data of ref. [3].

$m_{Y^*} = 1.385$ GeV at $P_L = 1.8$ GeV/c. Once again kinematic factors in the propagator of a spin $\frac{3}{2}$ particle give rise to cross-sections which increase rapidly with energy, a result which is incompatible with the experimental data. For the exchange of spin $\frac{1}{2}$ particles there is no such effect and the total cross-section varies with energy as shown in Fig. 2. The angular distribution for exchange of spin $\frac{1}{2}$ particles at $P_L = 2.2$ GeV/c is shown in Fig. 7. Normalizing our results to the data we find $(g_{\mathrm{KY}\mathcal{N}} g_{\mathrm{K}\Xi\mathrm{Y}})/4\pi = 0.40$ and 0.54 for $P(\mathrm{KY}\mathcal{N})$ and $P(\mathrm{KY}\Xi)$ both even and both odd, respectively. Our results indicate that the best fit to the data is given by exchange of a Y particle such that $P(\mathrm{KY}\mathcal{N})$

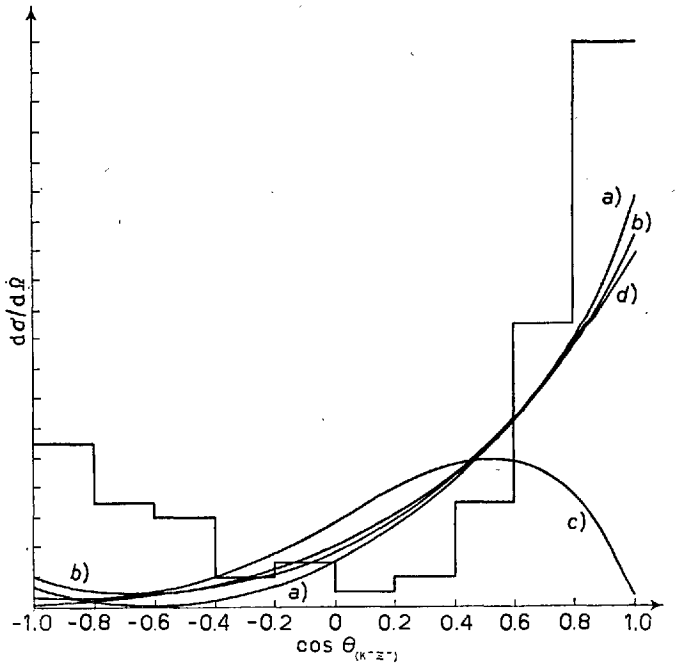

Fig. 6. – Contribution to the differential cross-section for process (1) due to exchange of a Y*-particle (mass 1385 MeV, spin $\frac{3}{2}$) in the u channel at $P_L = 1.8$ GeV/c assuming the Ξ-particle has spin $\frac{1}{2}$. Curves *a*), *b*), *c*) and *d*) correspond to the parity combinations even even, odd even, even odd and odd odd for $P(KY^*\mathcal{N})$ and $P(KY^*\Xi)$, respectively. The normalization is such that $g_{KY^*\mathcal{N}} g_{KY^*\Xi}/4\pi = 0.47$, 0.38, 2.16 and 1.49 for the curves *a*), *b*), *c*) and *d*), respectively. This histogram gives the experimental data of ref. (3).

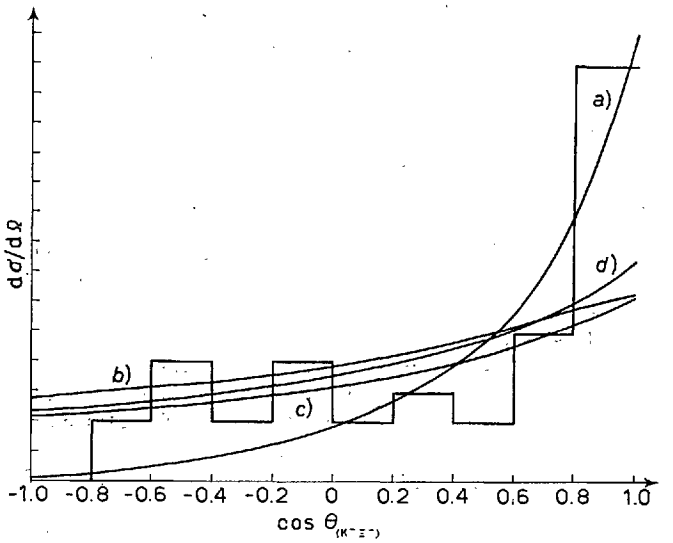

Fig. 7. – As in Fig. 5 except that $P_L = 2.2$ GeV/c. The histogram gives the experimental data of ref. (5).

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and $P(KY\Xi)$ are both even. For all other parity combinations the forward peak given by our theory is insufficiently large and narrow. This parity fit also gives the best fit to the energy-dependence of the production cross-section as shown in Fig. 2. However it is well established that $P(K\Lambda\mathcal{N})$ is odd ([17]) and if we just consider the Λ pole we clearly obtain a forward peak which is too broad to fit the data. If we also assume $P(K\Sigma\mathcal{N})$ is odd ([18,19]) and calculate the Σ-pole term we obtain a similar angular distribution but

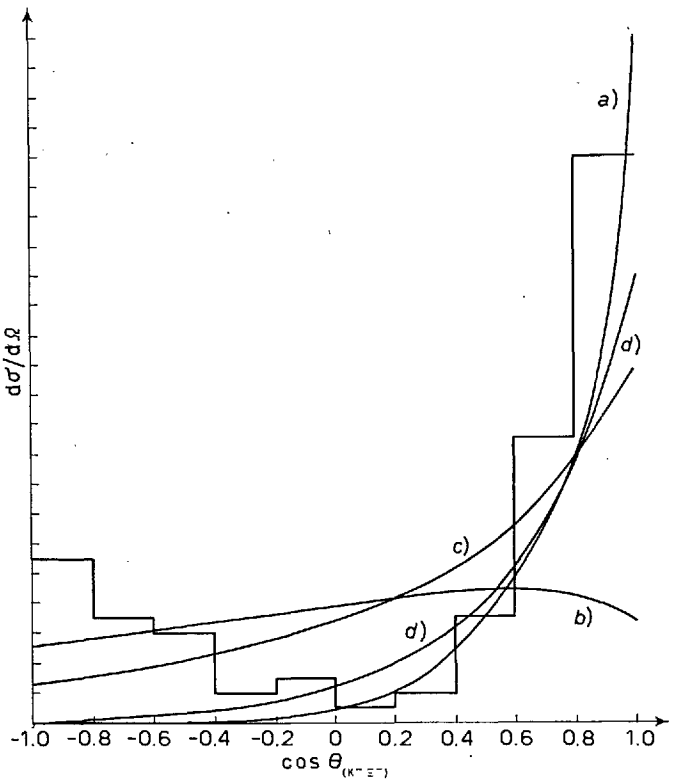

Fig. 8. – Contribution to the differential cross-section for process (1) due to exchange of Λ (mass 1115 MeV) and Σ (mass 1189 MeV) particles in the u channel at $P_L = 1.8$ GeV/c assuming the Ξ-particle has spin $\frac{1}{2}$. $P(\Lambda\Sigma)$ is even and curves *a*), *b*), *c*), *d*) correspond to the parity combinations even even, odd even, even odd, odd odd for $P(K\Lambda\mathcal{N})$ and $P(K\Lambda\Xi)$, respectively. The normalization is such that $g_{K\Lambda\mathcal{N}}g_{K\Lambda\Xi}/4\pi = g_{K\Sigma\mathcal{N}}g_{K\Sigma\Sigma}/4\pi = 5.3$, 10.6, 8.2 and 16.8 for the curves *a*), *b*), *c*), and *d*), respectively. The histogram gives the experimental data of ref. ([3]).

([17]) M. M. BLOCK, C. LENDINARA and L. MONARI: *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, p. 371.

([18]) R. D. TRIPP, M. B. WATSON and M. FERRO-LUZZI: *Phys. Rev. Lett.*, **8**, 175 (1962).

([19]) H. D. D. WATSON: *Hyperon Pair Production*. Preprint.

because of the slightly larger mass of the exchanged particle the peak is smaller and broader. From the isotopic spin analysis of Section 5 we see that the contribution to the scattering amplitudes of the Λ- and Σ-pole terms taken together depends upon the relative signs and magnitudes of the four coupling constants $g_{K\Lambda N}$, $g_{K\Sigma N}$, $g_{K\Sigma N}$, $g_{K\Sigma\Xi}$. In the octet model of SU_3 the Λ and Σ amplitudes have the same sign and their relative magnitude varies from 3:1 to 1:3 depending upon the mixing of F and D type couplings. Whatever this relative magnitude might be, the inclusion of both pole terms does not lead to any narrowing of the forward peak. However, provided $P(\Lambda\Sigma)$ is even, we can obtain a narrowing of the forward peak, for instance $(g_{K\Lambda N}g_{K\Lambda\Xi})=(g_{K\Sigma N}g_{K\Sigma\Xi})$. In this case the isotopic spin factors lead to a cancellation of the Λ and Σ contributions to the scattering amplitude which is most complete in the non-forward direction leaving a sharp forward peak. This interference effect is such that at higher energies the cancellation is even more complete and the theoretical cross-section decreases with increasing energy in agreement with the experimental data. We have calculated this effect for all the parity com-

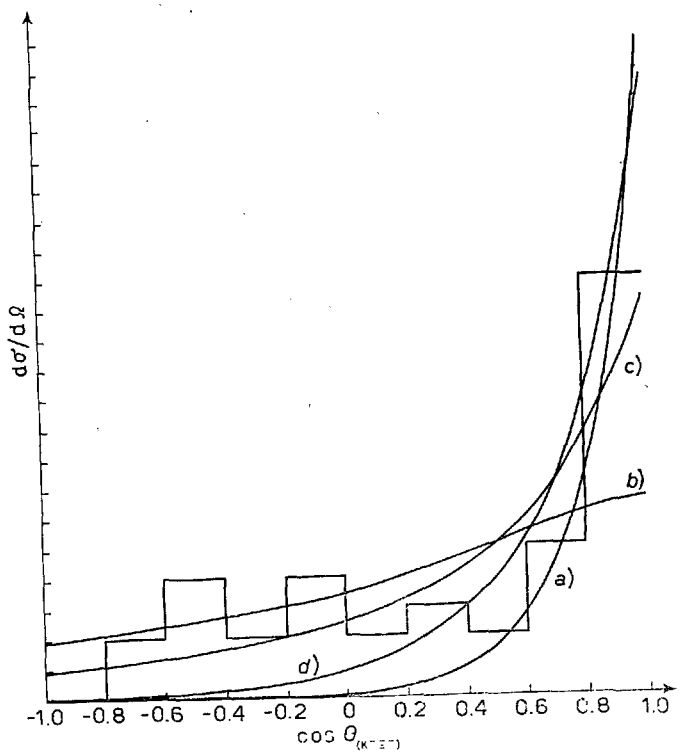

Fig. 9. – As in Fig. 8 except that $P_L=2.2$ GeV/c. The histogram gives the experimental data of ref. [5].

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binations at $P_L = 1.8$ and 2.2 GeV/c and the angular distributions we obtain are given in Fig. 8 and 9, respectively. Normalizing the results at $P_L = 1.8$ GeV/c in the case of $P(\Lambda\Sigma)$ even, $P(K\Lambda\mathcal{N})$ odd and $P(K\Lambda\Xi)$ odd we find $(g_{K\Lambda\mathcal{N}}g_{K\Lambda\Xi})/4\pi = (g_{K\Sigma\mathcal{N}}g_{K\Sigma\Xi})/4\pi = 17$. The only other parity combination which will fit the data is $P(\Lambda\Sigma)$, $P(K\Lambda\mathcal{N})$, $P(K\Lambda\Xi)$ all even.

If we now turn our attention to the s channel and examine the consequence of inserting both the Λ and Σ poles in the amplitude we again obtain some cancellation. For $P(K\Lambda\Xi)$ even the angular distribution is roughly uniform and the contribution to the cross-section is of the same magnitude as that due to poles in the u channel. However, for $P(K\Lambda\Xi)$ odd the s channel contribution is an order of magnitude smaller than that of the u channel and moreover in the case of $P(K\Lambda\mathcal{N})$ and $P(K\Lambda\Xi)$ both odd the s channel terms give rise to a large forward peak. This indicates that although interference between the s and u channels pole terms might give a significant contribution to the differential cross-section in most parity cases it is unlikely to alter the angular distribution given by the dominant u channel poles in the case of $P(\Lambda\Sigma)$ even and $P(K\Lambda\mathcal{N})$, $P(K\Lambda\Xi)$ both odd.

7. – Alternative production processes.

We now turn our attention to the production of cascade particles in πp and $\bar{p}p$ collisions and examine the consequences of our analysis as applied to the reactions

$$\bar{p} + p \to \bar{\Xi}^- + \Xi^- , \tag{2}$$

$$\pi^- + p \to \Xi^- + K^+ + K^0 . \tag{3}$$

The total cross-section ([20]) for process (2) is found to be 4 (± 2.5) μb at $P_L = 3.0$ GeV/c. This is an order of magnitude smaller than the cross-section for the production of baryons of strangeness one in $\bar{p}p$ collisions. No conclusions can be drawn about the angular distribution from the four events reported at $P_L = 3.0$ GeV/c but at higher energies there is some evidence that the process is peripheral ([21]). We set up a model to describe process (2) in which we consider only the peripheral diagram involving exchange of a D-

([20]) R. ARMENTEROS, E. FETT, B. FRENCH, C. MONTANET, V. NIKUTIN, M. SZEPTYCKA, CH. PEYROU, R. BOCK, A. SHIPIRA, J. BODIER, L. BLASKOVICZ, B. EEQUER, B. GREGORY, F. MULLER, S. J. GOLDSACK, D. H. MILLER, C. C. BUTLER, B. TALLINI, J. KINSON, L. RIDDIFORD, A. LEVÈQUE, J. MEYER, A. VERGLAS and S. ZYLBERACH: *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, p. 236.

([21]) H. GOLDBERG, S. NUSSINOV and G. YEKUTIELI: *Nuovo Cimento*, **28**, 446 (1963).

meson. From the value of the total cross-section we can determine $g_{\mathcal{N}\Xi D}$ for a given spin and parity case. For a D-particle of mass 1000 MeV, spin 0 with $P(\mathcal{N}\Xi D)$ even we find $(g^2_{\mathcal{N}\Xi D}/4\pi) = 0.04$. Using the results of Section 6 we then find $(g^2_{KKD}/4\pi) = 3.4$. This large value of the coupling of the D-meson with two K-mesons indicates that the D-meson has a large width. With our definition of the coupling constant g_{KKD} the width of the D resonance as observed in KK scattering is given by

$$\Gamma_D = \frac{g_{KKD}}{4\pi}\frac{m_K^2}{m_D^2}(m_D^2 - 4m_K^2)^{\frac{1}{2}}.$$

Using this relationship and the above value for g_{DKK} we find $\Gamma_D = 130$ MeV. Clearly more experimental data at high energies are needed to examine the validity of a peripheral model describing process (2).

The total cross-section [22-24] for process (3) rises with energy to a value of $10.4(^{+4.2}_{-3.2})$ μb at $P_L = 8.0$ GeV/c and at this energy 7 of the total of 8 events observed are such that $\cos\theta_{\pi\Xi} \geqslant 137°$ in the centre-of-mass system. This indicates that the process is peripheral and suggests that the mechanism for the reaction might involve the exchange of a D-meson. However, an alternative mechanism might be the exchange of a K*-meson.

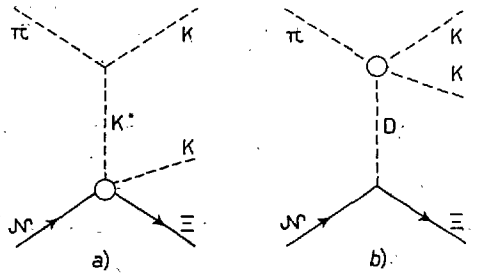

Fig. 10. – Feynman diagrams appropriate to the production process (3) corresponding to K* and D exchange.

The appropriate Feynman diagrams are shown in Fig. 10. To differentiate between these two models it will be necessary to observe the correlation of the associated K-mesons. It should also be stressed that the observation of these K-mesons offers the best possibility of examining the KK interaction for at sufficiently high energies the correlation in kinetic energies of the K-mesons would give direct evidence for or against the existence of a D « particle ».

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(22) W. B. Fowler, W. M. Powell and J. I. Shanle: *Nuovo Cimento*, **11**, 428 (1959).

(23) Wang Kang-ch'ang, Wang Tsu-tseng, N. M. Viryasov, Ting Ta-ts'ao Kin Hi Im, E. N. Kladnitskaya, A. A. Kuznetsov, A. Mikhul, Nguyen Dinh Tu, A. V. Nikitin and M. I. Solov'ev: *Žurn. Èksp. Teor. Fiz.*, **40**, 734 (1961); *Soviet Physics JEPT*, **13**, 512 (1961).

(24) A. Bigi, S. Brandt, R. Carrera, W. A. Cooper, A. de Marco, G. R. MacLeod, Ch. Peyrou, R. Sosnowski and A. Wroblewski: *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, p. 247.

8. - Conclusions.

Within the limitations of our model our calculations strongly suggest that the cascade particle is not a spin $\frac{3}{2}$ particle satisfying the Rarita-Schwinger formalism. This supports the tentative conclusion of ref. ([5]) in favour of spin $\frac{1}{2}$ for the cascade particle.

From our results it is clear that the mechanism dominating the production of cascade particles in Kp collisions is anti-peripheral in the sense that the exchange process involves fermions. The experimental fact that the production cross-section exhibits such a large forward peak can be accounted for by assuming some cancellation between the Λ- and Σ-pole contributions to the scattering amplitude. In the case of $P(\Lambda\Sigma)$ odd the interference effect is complicated and we do not have any obvious cancellation mechanism. For $P(\Lambda\Sigma)$ even and $P(\mathrm{K}\Lambda\mathcal{N})$ odd all the available data can be fitted provided $P(\mathrm{K}\Lambda\Xi)$ is also odd. It should be pointed out that the values of the coupling constants we obtain are very sensitive to the degree of cancellation we assume in order to fit the shape of the angular distribution. Thus the quantitative results are not to be taken too seriously.

The influence of the Y* resonances as approximations to two particle states appears to be masked by the Λ- and Σ-pole terms. However we have used an unrenormalizable field theory to describe the interactions of the spin $\frac{3}{2}$ particles and an alternative theory such as that of a Regge pole model would alleviate the difficulties our model meets at high energies.

In addition to the anti-peripheral process there is some evidence for a peripheral process mediated by a D-particle or KK resonance. Our analysis indicates that such a particle with isotopic spin 1 should be a spin-zero state of even parity. The mass cannot be established but a mass of 1 GeV with a width of 130 MeV is consistent with all the data for cascade particle production. Further evidence for this particle should be sought in high-energy production of cascade particles in πp collisions.

* * *

The author wishes to express his gratitude to Professor P. T. MATTHEWS for the suggestion of this work and for helpful discussions.

RIASSUNTO (*)

Per descrivere la produzione di particelle in cascata nelle collisioni Kp si costruisce un modello in cui si prendono in considerazione i contributi del polo delle particelle Λ e Σ ed i contributi degli stati intermedi di due particelle. Questi ultimi stati sono approssimati con risonanze Y* nei canali s ed u e con una particella o risonanza D, un bosone di stranezza 2, nel canale t. Si prendono in considerazione le due alternative di un Ξ di spin $\frac{1}{2}$ o di spin $\frac{3}{2}$. Si usa una tecnica di teoria dei campi in cui tutti gli stati di spin $\frac{3}{2}$ sono descritti dal formalismo di Rarita-Schwinger. Si calcolano le sezioni di urto totali e differenziali per il processo di produzione a varie energie per tutte le possibili combinazioni di parità. Un confronto con i dati sperimentali indica che lo spin della particella della cascata non è $\frac{3}{2}$. Il processo di produzione è anti-periferico nel senso che il meccanismo predominante è uno scambio di fermioni nel canale u. Si possono approssimare meglio i dati con le combinazioni di parità $P(\Lambda\Sigma)$ pari, $P(K\Lambda\mathcal{N})$ dispari e $P(K\Lambda\Xi)$ dispari. In questo caso si richiede anche che $g_{K\Lambda\mathcal{N}}g_{K\Lambda\Xi}\approx \approx g_{K\Sigma\mathcal{N}}g_{K\Sigma\Xi}$. Si ha anche qualche prova di un processo mediato da una particella D ossia risonanza KK. L'analisi indica che tale stato, di isospin 1, deve avere spin zero e parità pari. Non se ne può stabilire la massa ma una massa di 1000 MeV ed una ampiezza di 130 MeV concordano con la produzione in cascata nelle collisioni Kp e pp.

(*) *Traduzione a cura della Redazione.*

2802

DEPARTMENT OF MATHEMATICS

PROFESSORS: E. T. DAVIES
H. A. JAHN
B. THWAITES
H. B. GRIFFITHS

THE UNIVERSITY
SOUTHAMPTON
TEL. 56331

16th July 1965.

Professor P.T. Matthews,
Department of Physics,
Imperial College.
London S.W.7.

Dear Professor Matthews,

Thank you for your letter of the 13th July. The suggested date and time for my Ph.D. viva — Tues. 27th July 11 a.m — is quite convenient.

Yours sincerely,

R.C. King.

UNIVERSITY OF LONDON

SENATE HOUSE, WC1

TELEPHONE: MUSEUM 8000 TELEGRAMS: UNIVERSITY, LONDON

MAW

888 Physics 20 May 1965

Dear Sir,

I am directed to inform you that you have been appointed as Examiner in conjunction with J. C. Polkinghorne, Esq., M.A., Ph.D.,
Trinity College,
Cambridge.

for the thesis, a copy of which is enclosed, submitted by the following candidate for the Ph.D. Degree in the Faculty of Science as an Internal Student:

Name: R. C. King (I.C.)

Thesis: 'Particle production in strong interactions'.

I enclose the Instructions to Examiners for the Ph.D. Degree and would draw your attention to Sections 12 to 14, from which you will see that an oral examination should be held, unless the candidate is rejected without further test. I shall be glad if the Examiners will suggest a suitable time and place for the oral examination so that I may summon the candidate to attend.

The joint report of the Examiners should be written on the candidate's entry form which has been sent to your co-examiner.

Will you please return the thesis with the Examiners' report. I regret that I am no longer in a position to give permission to any Examiner to retain a copy of the thesis, in view of a scheme approved by the Senate with regard to the disposal of copies of theses after examination. This scheme makes no provision for the retention of theses by Examiners. Fewer copies are now submitted by candidates for the examination, and two of these copies must be bound, in accordance with the approved specification, before the conferment of the degree.

I enclose a copy of the Scheme of Payment to Examiners for Internal Students.

Yours faithfully,

James Henderson

Academic Registrar

Prof. P. T. Matthews, M.A., Ph.D.,
Imperial College,
Prince Consort Road,
South Kensington,
S.W.7.

UNIVERSITY OF LONDON

INSTRUCTIONS TO EXAMINERS FOR PH.D. DEGREE FOR INTERNAL STUDENTS IN THE FACULTIES OF THEOLOGY, ARTS, LAWS, MUSIC, MEDICINE, SCIENCE, ENGINEERING AND ECONOMICS

1. Boards of Studies shall recommend the names of two Examiners for each candidate but in the case of the Ph.D. Degree in the Faculties of Arts and Economics Boards of Studies are empowered whenever they consider it desirable to recommend the appointment of three Examiners, such Examiners to take equal parts in the examination.

2. At least one of the Examiners shall, whenever practicable, have had experience in examining for the Ph.D. Degree of the University, and one Examiner shall, if possible, be the Teacher, or one of the Teachers, under whose supervision the research of which the results are embodied in the thesis has been carried out.

3. Boards of Studies may, if they so desire, recommend the names of two of the candidate's Teachers instead of the one Teacher mentioned above.

4. In cases where only two Examiners are recommended to act in the first instance, Boards of Studies shall also recommend the name of an additional Examiner to act only if called upon by the Principal after the Examiners who have acted in the first instance shall have reported to him formally that they are unable to arrive at an agreement; and the additional Examiner shall act jointly with the other Examiners and not as a Referee. Whenever practicable this additional Examiner shall have had experience in examining for the Ph.D. of the University.

5. When theses are sent to the Examiners, the Academic Registrar shall request them to acquaint themselves with the standard of the University, and inform them that copies of the theses submitted by candidates who have presented themselves for the examination previously are available for this purpose.

6. The standard of the Ph.D. Degree is definitely higher than that of the M.A. and M.Sc. Degrees in the same subject.

7. The thesis must form a distinct contribution to the knowledge of the subject and afford evidence of originality, shown either by the discovery of new facts or by the exercise of independent critical power.

8. The thesis must be satisfactory as regards literary presentation, and if not already published in an approved form, must be suitable for publication, either as submitted or in an abridged form.

9. The candidate must indicate how far the thesis embodies the result of his own research or observation, and in what respects his investigations appear to him to advance the study of his subject.

10. The Degree of Ph.D. will not be conferred upon a candidate unless the Examiners certify that the thesis is worthy of publication as a 'Work approved for the Degree of Doctor of Philosophy in the University of London'.

11. A candidate will not be permitted to submit as his thesis a thesis for which a Degree has been conferred on him in this or in any other University, but a candidate shall not be precluded from incorporating work which he has already submitted for a Degree in this or in any other University in a thesis covering a wider field, provided that he shall indicate on his entry form and also on his thesis any work which has been so incorporated.

12. After the Examiners have read the thesis, they may, if they think fit, and without further test, recommend that the candidate be rejected.

13. Except as provided in paragraph 12 the Examiners after reading the thesis shall examine the candidate orally and at their discretion by printed papers or practical examinations or by both methods on the subject of the thesis, and, if they see fit, on subjects relevant thereto; provided that a candidate for the Ph.D. Degree in the Faculty of Arts who has obtained the Degree of M.A. in the same subject in this University shall in any case be exempted from a written examination.

14. If the thesis is adequate but the candidate fails to satisfy the Examiners at the oral, practical or written examination held in connection therewith, the Examiners may recommend the Senate to permit the candidate to re-present the same thesis and submit to a further oral, practical or written examination within a period not exceeding 18 months specified by them, and the fee on re-entry, if the Senate adopt such recommendation, shall be half the normal fee.

15. If the thesis though inadequate, shall seem of sufficient merit to justify such action the Examiners may recommend the Senate to permit the candidate to re-present his thesis in a revised form within 18 months from the decision of the Senate with regard thereto; and the fee on re-entry, if the Senate adopt such recommendation, shall be half the normal fee. Examiners shall not, however, refer any thesis without submitting the candidate to an oral examinaiton.

16. When a candidate for Ph.D. Degree in the field of Statistics has not a preliminary Degree in Statistics the Examiners are recommended to test the candidate's general knowledge of statistical method and theory outside the special field covered by his thesis.

17. Examiners are informed that it is not within their power to recommend the conferment of a Degree other than that for which the candidate has entered.

18. Copies of all successful theses, whether published or not, will be deposited for reference in the University Library.

UNIVERSITY OF LONDON

PAYMENTS TO EXAMINERS

8.—HIGHER DEGREE EXAMINATIONS

(1) Masters' Degrees (excluding M.S. and M.D.S.), Ph.D., D.Mus.

[*Note:*—Payments to Examiners for the 'one-year' M.Sc. (Econ.) Examination by written papers (under new Regulations) have not yet been prescribed, and the following fees are not at present applicable]

Setting Papers	£	s.	d.
For each paper set (fee divisible between the Examiners taking part)	10	0	0
[M.Sc. History and Philosophy of Science—Part I: A fixed fee of £45 is divisible between the External Examiners for their work in setting papers]			
Mathematics (including Mathematical Statistics) and Statistics papers, each ..	12	0	0
For transcribing the whole or the major part of a paper into an Oriental script for reproduction by photography	1	10	0
For modification of a paper for Overseas		15	0
Papers in connection with Practical Examinations: *see* Practical Examinations below.			
Marking Scripts			
For each of two markings or readings, for each script		7	6
At the D.Mus. Examination payment for three markings is allowed.			
Oral Examinations (and Practical Tests at M.A.)†			
Masters' Degree Examinations:			
Per candidate (fee payable for up to four orals on any one day) to each of two Examiners if required	3	10	0
If more than one Internal Examiner acts in addition to the External Examiner, £3 10s. 0d. per candidate is divisible among the Internal Examiners.			
The above fees are not payable in respect of orals given during practical examinations, if the Examiner(s) concerned receives fees for attendance at the practical examination.			
Ph.D. Degree Examinations:			
To each of two Examiners and to a third if called upon by the Principal, or to each of three Examiners appointed from more than one Faculty, per candidate (fee payable for up to four orals on any one day)	3	10	0
If three Examiners are appointed from the same Faculty to act in the first instance, the following fees will be payable:—			
Internal Examinations.—A fee of £7 per candidate shared equally between three teachers, or a fee of £3 10s. 0d. per candidate to the Examiner external to the University and a fee of £3 10s. 0d. per candidate shared equally between two teachers.			
External Examinations.—A fee of £3 10s. 0d. per candidate to each Examiner, provided that the appointment of the three Examiners has been specifically approved by the External Council after consideration of a special report of the appropriate Board of Studies.			
Practical Examinations (and Oral Examinations at D.Mus.)			
For each practical paper set (fee divisible between the Examiners taking part) ..	10	0	0
For a practical paper set to cover more than one day's practical at M.Sc. Examinations in Biochemistry (fee divisible between the Examiners taking part) ..	20	0	0
For attendance (including marking of candidates' work):—			
Whole day	7	0	0
Half-day	4	0	0
If oral examinations for the Intermediate Examination in Music, B.Mus. and D.Mus. Examinations are held consecutively on the same day, payment will be made for the whole time for which Examiners are present, and not for separate periods for each examination.			
If an External or Staff Examiner resides outside the University radius of 30 miles his attendance, if required for a single examination period only on any day, is to be reckoned as attendance for a day.			

† For conducting oral examinations at Part I of the M.Sc. Examination in Agriculture (selected subject Poultry Science) for candidates registering in and after October 1961, a fee of 10s. per candidate (minimum fee £4 per day) will be payable to each of two Examiners for each subject.

	£	s.	d.
Theses and Dissertations, etc.,			
Masters' Degree Examinations:			
For assessing material* submitted by a candidate, to each of two Examiners and to a third if called upon by the Principal	7	0	0
except in the following cases:—			
The fee for reading a thesis at the following examinations is £5 to each of two Examiners:—			
Internal M.A. Philosophy			
Internal M.A. and M.Sc. Psychology (if candidate is examined by thesis, oral, three papers and a practical)			
The fee for reading a dissertation at the following examinations is £5 to each of two Examiners:—			
M.A. Education examinations consisting of four papers, dissertation and oral			
M.Vet.Med. [the fee for a thesis is £7 to each of two Examiners]			
The fee for reading a Report at the Internal M.Sc. (Eng.) Examination, for candidates proceeding to the Degree by method of written papers and submission of Report, is £4 to each of two Examiners.			
At the M.Sc. Examination in Geophysics, without thesis, a fee of £5 per candidate is payable to each of two Examiners to cover reading the dissertation, examination of course-work and attendance at oral examination.			
If three Examiners read a thesis or dissertation, etc., in the first instance, the fee for two Examiners is divisible between them if they are all teachers of the University, or the full fee is payable to the Examiner external to the University and half the prescribed fee to each of the two teachers.			
Ph.D. Degree Examinations:			
For examination of a thesis, to each of two Examiners and to a third if called upon by the Principal, or to each of three Examiners appointed from more than one Faculty..	10	0	0
If Examiner is candidate's Supervisor	7	0	0
If three Examiners are appointed from the same Faculty to act in the first instance, the following fees will be payable:—			
Internal Examinations.—A fee of £6 13s. 4d. to each of three teachers, including the Supervisor, or a fee of £10 to the Examiner external to the University and half the prescribed fee to each of two teachers [£3 10s. 0d. to the Supervisor and £5 to the other teacher].			
External Examinations.—A fee of £10 to each Examiner, provided that the appointment of the three Examiners has been specifically approved by the External Council after consideration of a special report of the appropriate Board of Studies.			
No fee shall be payable to a Teacher of the University for reading and reporting on a Thesis or Dissertation submitted in the joint names of the candidate and himself.			
For re-reading a thesis or dissertation submitted for a Ph.D. Examination and re-submitted for a Master's Examination or for re-reading a thesis or dissertation re-presented in connection with further tests in oral, written or practical examinations, the fees payable are half the fees prescribed for the first reading.			
For re-reading a thesis or dissertation submitted in a revised form, the fees payable are as for the first reading.			
Design at M.A. (*Architecture*) under old Regulations—For assessing material submitted by a candidate, to each of two Examiners and to a third if called upon by the Principal	7	0	0
If three Examiners act in the first instance, the fee for two Examiners is divisible between them.			
Portfolio of Drawings at Internal M.A. or M.Sc. Architecture under new Regulations, per candidate, to each of two Examiners	5	0	0
Problem (*at Mathematics*) (fee divisible between the Examiners taking part)	12	0	0
Musical Exercise (*D.Mus.*) (fee divisible between the Examiners taking part)	21	0	0
Special ad hoc Qualifying Examination—fee per candidate (fee divisible between the Examiners taking part)	10	0	0
For a qualifying examination, for the M.Sc. Examination in Botany, consisting of an oral examination only or of an oral examination together with one or more papers of an existing examination, fee divisible among the Examiners taking part in the oral examination	5	0	0

*To include 'an approved piece of textual and editorial work' at M.A. Examination in English and Education.

Chairmen's Fees	£	s.	d.
M.Th. (old Regulations) (Common)	15	0	0
M.A.:			
Classics (Internal and External)	15	0	0
Classical Chinese (Internal and External)	12	0	0
English—December (Common)	9	0	0
May (Common)	15	0	0
French (Internal and External)	18	0	0
Indo-Aryan (External)	15	0	0
LL.M. (Common)	25	0	0
M.Sc.:			
Agriculture (Poultry Science) (Internal)*	9	0	0
Biochemistry, to the Chairman of a Board of more than two Examiners*	9	0	0
Crystallography (Internal)*	9	0	0
Geophysics (Internal)*	12	0	0
History and Philosophy of Science (Common)	15	0	0
Microbiology (Internal)*	9	0	0
Radiation Biology and Radiation Physics (Internal)	12	0	0
Meetings			
To each External or Staff Examiner, for attendance at each Meeting if summoned by the University (maximum fee £7 10s. 0d. per day)	2	10	0
No fee if held concurrently with a Practical Examination			
Minimum Fee to *ad hoc*, Staff or External Examiner	8	0	0

(2) D.D., D.Lit., LL.D., D.Sc., D.Sc.(Eng.), D.Sc.(Econ.)

To each Examiner who acts, inclusive fee, per candidate	15	15	0

Travelling Expenses

For each occasion on which an Examiner is required by the University to travel a distance of more than 30 miles from his usual residence he may claim in connection with attendance at Practical Examinations, Oral Examinations, or Examiners' Meetings, first-class return railway fare and the cost of travel by underground and/or public road transport (bus or coach) for all necessary journeys actually performed from his usual residence, and for any other necessary journeys performed while engaged on examination work, together with the following allowances:—

For necessary absence from home not involving a night:

For a period of 5–10 consecutive hours, 15s. 0d.

For a period of more than 10 consecutive hours, £1 10s. 0d.

For each necessary period of absence up to 24 hours involving a night away from home, £3.

For journeys to or from Berwick or Carlisle or stations in Scotland or Northern Ireland, an allowance of £1 per journey, in addition to the subsistence allowance of £3.

An Examiner external to the University may claim fares and allowances as set out above for all necessary journeys actually performed from his usual residence (irrespective of the 30-mile limit), and for any other necessary journeys performed while engaged on examination work.

In the event of travel from a vacation address, travelling expenses claimed may not exceed those from usual residence unless the University's request for attendance is made at short notice.

J. HOOD PHILLIPS
Secretary to the Senate

July 1964

* Fees cover any External candidates examined by the Board.