The Use of Radiosonde Data to Examine Lee Waves and Other Small-Scale Motions in the Atmosphere

by

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Abstract

Radiosonde data have been examined for evidence of small-scale motions. The rate of switching produced by the windmill has been used to separate variations in the rate of decrease of pressure due to air motions from those due to changes of the rate of ascent of the radiosonde balloon through the air. Variations of this latter type do occur mainly because of changes of the drag coefficient of the radiosonde balloon during a sounding but are usually small. Rates of ascent as large as 15 m sec^{-1} in the upper troposphere which have occurred may be due to small-scale turbulence in the atmosphere. Other disturbances appear to be mainly caused by the passage of the radiosonde through deep systems of lee waves, present not only in the troposphere but also occasionally in the lower stratosphere. The records from Shanwell (on the east coast of Scotland) for 1964 show that the streamline displacements in the waves there exceeded 100 m about 10% of the time at 2 km and about 2% of the time at 16 km. The frequency of lee waves at 2 km appears to be smaller than theories indicate and the inferred wavelengths in the stratosphere are shorter than expected. Regions of the atmosphere where the parameter (gBU^{-2}) is constant with height appear to favour the development of the waves. This feature and the correspondence between the wave period and the Brunt-Vaisala period suggest that pronounced lee waves occur in systems with vertical phase-lines. In several respects pronounced lee waves conflict with ideas about the 'trapping' of wave energy. Departures of the air flow in lee waves from the conditions normally assumed to be present may be important.

List of Contents

List of Contents (continued)

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Chapter 1 - Introduction

1.1 Introduction

Corby (1957) showed that radiosonde data could be used to detect departures of the vertical velocity of the instrument, of a few m sec^{-1}, from a mean value. Occasionally marked periodic variations of the vertical velocity along the path of the balloon suggested the presence of atmospheric gravity waves, almost certainly due to orography, extending through a considerable depth of the atmosphere. Corby limited his study to levels below 40,000 ft; however, on some occasions the waves which he inferred maintained considerable vertical motions up to this level and may have extended considerably higher. In the present investigation improved radiosonde data have been examined for indications of small-scale air motions at all levels. Some emphasis has been placed on motions in the stratosphere, which have considerable practical interest in view of the imminent operation of supersonic aircraft there.

The routine soundings have been used to provide an estimate of the frequency of small-scale air motions. An equally interesting application of radiosonde data has been found in the examination of the association of lee waves and the conditions of wind and temperature in the atmosphere. It appears that the value of these parameters measured by the radiosonde is influenced by the vertical air motions; however, Corby has shown that the correction to the temperature is usually small. When the stability and the wind are measured as averages in layers a few kilometers deep the corrections are further diminished. The vertical distribution of the parameter l^2 which is important in the theory of lee waves may be accurately inferred in such layers and used to discuss factors which appear to favour the lee waves. Soundings launched from Shanwell, on the east coast of Scotland, during 1964, have provided the bulk of the data, but other records showing special features have also been used. For comparison with the Shanwell soundings some records from Weather Ships I and J, far from any orographic feature, have been inspected. Six months data from Muharraq in the Persian Gulf have provided comparison with a site at which there are no marked orographic features but the winds are strong at high levels due to the presence of the subtropical jet stream near this latitude in some months of the year.

1.2 The radiosonde and ancillary equipment

The Meteorological Office radiosonde (Mark 2B) is contained in a cylindrical case from which protrude the three shielded meteorological sensors and the windmill. The pressure measuring instrument is a steel aneroid capsule clamped on one side and attached to a small armature on the other. The temperature measuring element is a bimetallic strip rolled into a cylinder and attached to a similar armature. The humidity measuring instrument, a small piece of gold beater's skin, is likewise attached to an armature. The movement of each armature varies the air gap with the pole pieces of an inductor core and each inductor core is switched successively into an audio-frequency circuit. The rate of switching is determined by the rotation of the windmill which will be regarded as a simple anemometer mounted in a vertical plane on the outside of the radiosonde.

The movement of each armature varies the inductance in the audio-frequency circuit. For example, a pressure change of 1000mb produces a movement of about 2 mm and produces a frequency change of about 200 c sec^{-1} in the audio-frequency. This signal modulates a high radio-frequency carrier wave and is transmitted to the ground where each successive audio-frequency is measured. In the data examined by Corby the frequency was measured using an oscilliscope and recorded manually; however, in present radiosounding practice the measurement is recorded automatically as a function of time on a paper chart.

In flight the radiosonde is suspended about 30m below a hydrogen-filled balloon about 2m in diameter. A small parachute is usually connected immediately below the balloon and below it the radar reflector, a large open-mesh structure which has very little drag. The balloons used in most of the soundings had a mass of 1 kgm but in some more recent soundings high-altitude balloons of mass 2 kgm were used.

The radar which is used to track the reflector and determine the winds measures the elevation angle, the azimuthal angle, and the slant range of the radiosonde at the end of each minute interval. In normal practice the horizontal range and the azimuth are plotted on a chart and the speed and direction of the wind are then determined as an average over two or three minutes. The measurements may, however, be readily used to calculate the wind and vertical velocity at one minute intervals providing average values over layers about 400 m deep.

The radar in use at Shanwell during 1964 was of the

G.L.3 type which was adapted from an anti-aircraft set by extending the range and increasing the sensitivity. The instrument itself is capable of measuring the range to $\pm$ 10 m and the angles to the nearest 0.1°; however, the accuracy is dependent on a clear signal and with normal reception the range is accurate to about $\pm$ 30 m and the angles to $\pm$ 0.1°. In more recent soundings launched from Shanwell a Cossor radar was used; the instrument tracks the balloon and is set to read out the azimuth and elevation angles and the range automatically at one minute intervals. The range is accurate to about $\pm$ 15 m and the angles are accurate to the nearest 0.07°.

1.3 The radiosonde as a detector of small-scale air motions

The radiosonde ascends through the air at about 6 m sec^{-1} and the rotation of the windmill switches between the pressure, temperature, and relative humidity sensors at such a rate that each successive measurement is transmitted for about 6 sec. The individual pressure measurements which normally lie on a smooth curve on the paper chart are therefore about 20 sec apart, corresponding to a vertical ascent of the radiosonde about 100 m. Evidence of small-scale air motions can be more readily and accurately inferred from changes in the slope of this curve than by converting the individual pressure measurements into mb. This is because the pressure-measuring instrument is chiefly subject to systematic errors and evaluation of each pressure measurement introduces an error of a few mb whereas direct measurement of the rate of decrease of pressure largely avoids this error. It appears that changes of the vertical velocity of the radiosonde by 1 m sec^{-1} or more produce detectable changes of slope on the pressure record.

The radar measurements of height may be used to evaluate the vertical velocity of the radiosonde independently during 1 minute intervals, or in layers about 400 m deep. Unlike the pressure measurements the radar measurements are subject mainly to random errors. These appear in the height chiefly because the elevation angle is rounded off to the nearest 0.1^{o} and in many cases is only accurate to 0.2^{o} or less. The error in the height is about $\pm$ 100 m and at long ranges the error in the vertical velocity which is measured as a difference between successive heights is greater than the magnitude of the vertical velocity. Nevertheless, the radar measurements can often be used to confirm changes of vertical velocity indicated by the pressure trace, and by measuring between radar heights a few km apart a useful and accurate mean value of the vertical velocity can be obtained.

The pressure and radar measurements are capable of indicating small-scale air motions with vertical velocities of a few m sec^{-1}. In an isotropic eddy, for example, horizontal perturbation velocities of a similar magnitude to the vertical velocities would be expected and it is possible that the radar data might show these features also. The error in the measurements of horizontal wind depends on the error in the measured azimuth angle and the error in the range. These two errors have different magnitudes and it has been found useful to express the horizontal wind in two components: a component radially away from the station which has a constant error of about 1 m sec^{-1} during the sounding, and an azimuthal component in which the error increases with range and is usually considerably more. It appears that small-scale motions might be detected in the radial component of the wind if they have horizontal velocities of a few m sec^{-1} present in layers not less than 400 m deep, although even then it may be difficult to separate them from changes of the wind with height.

Chapter 2 - The use of radiosonde data to detect small-scale air motions in the atmosphere

2.1 Introduction

The vertical velocity of a radiosonde may be measured either as the rate of change of radar height or from the rate of decrease of pressure. The paper chart on which the pressure values are displayed may be conveniently used to detect small departures of the vertical velocity from a mean value. The range of scales detectable is such that vertical air motions of 1 m sec^{-1} or more, present in phenomena such as cumulus convection or lee waves, should be readily visible on the pressure record. It is possible that some incorrect behaviour of the aneroid instrument might form similar disturbances on the pressure record but the radar measurements can often be used to detect these. A more important source of uncertainty in the interpretation of disturbances on the pressure record is likely to be variations of the rate of ascent of the radiosonde through the air. These variations which might conceivably produce disturbances on the pressure record similar to those produced by small-scale air motions are the principal subject of this chapter.

2.2 Factors affecting the rate of ascent of a radiosonde through the air

In radiosounding practice it is desirable to achieve an almost constant rate of ascent of the radiosonde through the air in order to maintain the ventilation of the meteorological units and to simplify the lag correction applied to the temperature measurements. This rate of ascent is normally about 6 m sec^{-1} and is determined principally by the free lift, L, of the balloon (the difference between the total buoyancy, B, and the total weight, M, of the balloon, radiosonde, and accessories). A free lift of 2.5 kgm is normally used because a greater value would lower the height at which the balloon burst, while the rate of ascent produced is large enough for the complete sounding to be made within an hour.

A constant rate of ascent is also desirable for the recognition of small-scale motions as departures from a mean vertical velocity. The free lift of a balloon ascending steadily is balanced by the drag, D, so that L is a function of

$$L = \tfrac{1}{2} \rho_a A W_a^2 C_d \qquad (1)$$

the rate of ascent through the air, W_a, the air density, ρ_a, the horizontal cross-section of the balloon, A, and the drag coefficient, C_d. The rate of ascent is therefore a function with several variables and also depends on departures from the steady state L = D. These latter do not, however, affect W_a much: for example, an increment to W_a of 1 m sec^{-1} above its equilibrium value of 6 m sec^{-1} increases the drag force by about 500 gm. This reduces the increment of W_a to $1/e$ of its initial value in about 1 sec so that for practical purposes W_a is determined by equation 1.

Variations in L during a sounding may be produced only by changes of the buoyancy. This is determined by the mass of air displaced by the balloon:

$$B = (\rho_a - \rho_h)\ V \qquad (2)$$

where ρh is the density of hydrogen and V is the volume. Because the pressure inside the balloon is barely more than outside and provided the temperature is the same inside and out the densities remain in constant proportion during a sounding. The buoyancy therefore also remains constant because the mass of hydrogen inside the balloon remains constant.

In practice, however, as the balloon expands with height the hydrogen inside tends to cool and solar radiation warms it. Any consequent difference between the temperature inside and outside the balloon produces changes in the free lift, for example a temperature difference of 20°C would change the free lift by about 10% and the rate of ascent by about ½ m sec^{-1}. Solar radiation can typically heat the balloon at 10^5 cal min^{-1}; however, it can be shown that conduction through the thin skin of the balloon on the shaded side could keep the temperature difference down to 1°C or less. The situation will be more complex than this with the gas filling of the balloon warmer on the sunlit side of the balloon; nevertheless, the rate of ascent is not likely to change suddenly and be confused with small-scale air motions. The effect of expansion of the balloon on the temperature of the gas filling can be compensated for by a flow of heat less than 10^4 cal min^{-1} through the skin of the balloon; this is possible with a very small temperature difference and the effect is therefore negligible.

The density of the air affects the rate of ascent through two factors: the density of air obstructing the passage of the balloon decreases with height but the volume and the cross-section increase. These variables are related in equation 1 to W_0, C_d and the constant L. The cross-section of the balloon is related to the volume through the radius of the balloon, R, and from equation 2 it appears that R is proportional to $\rho_a^{-\frac{1}{3}}$. Substituting for A in equation 1 shows that the rate of ascent is proportional to $\rho_a^{-\frac{1}{6}}$. The rate of ascent of radiosonde balloons should therefore increase with height.

Although the dependence on density is small most radiosonde ascents pass through a considerable depth of the atmosphere and the rate of ascent could be expected to increase by a factor of about 2. A comparison of the expected rate of ascent and observed vertical velocity in two soundings is made in figures 1 and 2. In both cases the vertical velocity increases steadily up till twenty minutes after launch in fair agreement with the expected behaviour. After twenty minutes the vertical velocity first becomes larger than expected then rapidly becomes smaller and reasonably steady and other factors must be important.

The shape of a balloon influences the rate of ascent through the cross-sectional area A and the drag coefficient C_d. A reduction of both these factors may be produced by an elongation of the balloon and it is possible that a large rate of ascent may be caused in this manner. At launch, radiosonde balloons are visibly pear-shaped, particularly the 2 kgm balloons intended for high ascents, but as they rise in the atmosphere and expand the shape should become more spherical. In figures 1 and 2 the behaviour of the rate of ascent in the earlier part of the flight is consistent with the density effect alone and there does not appear to be any need to consider a change of shape.

The flow around a sphere is dependent on the Reynold's number R_e

$$R_e = 2 \rho_a R W_a \mu^{-1} \qquad (3)$$

where μ is the coefficient of viscosity of air. The Reynold's number of a radiosonde balloon is typically in the range 10^6 to 10^5 and decreases during a sounding. In this range of Reynold's number the flow around a smooth sphere is observed to change from a supercritical condition when the boundary layer is fully turbulent to a subcritical condition when it is not. The drag coefficient is greater in the latter condition because the sphere has a broad wake behind it. It appears that radiosonde balloons do experience such a critical change and that the corresponding increase of drag reduces the rate of ascent in the stratosphere below what might be expected on the basis of the density effect.

The critical change for a smooth solid sphere in a wind tunnel is quite abrupt but the Reynold's number at which the change takes place varies in the range $1X10^5$ to $6X10^5$ (Hoerner 1958), and for a smooth sphere depends mainly on the amount of turbulence in the airstream (Goldstein, 1938). Values of the drag coefficient of a smooth sphere are typically 0.45 on the subcritical side of the change and 0.08 on the supercritical side increasing to over 0.1 at slightly greater Reynold's numbers. Changes of the drag coefficient of this magnitude in radiosonde balloons could evidently produce changes of the rate of ascent by a factor of 2.4.

2.3 Two case-studies involving large changes in the rate of ascent

The case represented in figure 1 was the sounding launched from Shanwell at midnight preceding the 20 January 1964. It was exceptional amongst the year's records in having a very marked decrease in the slope of the pressure trace immediately above the tropopause. However, plotting the vertical velocity measured over one-minute intervals showed that there was a similar increase in the vertical velocity in the upper troposphere; although, it was not so abrupt. The radar measurements of vertical velocity also showed this but had a much larger scatter of values. The rate of switching between the pressure, temperature, and humidity elements, produced by the windmill, indicated that the rate of passage of air past the radiosonde behaved in a similar manner.

The potential temperature lapse rate and two components of the wind are also shown in figure 1. The stability evidently decreases with height and becomes quite small in the upper troposphere where the vertical velocity reaches its maximum value of about 9 m sec^{-1}. The tropopause is reached 29 minutes after launch and the stability increased much more than usual. As was frequently found among the Shanwell records, a strong wind shear, confirmed by the two independently measured components, was present near the tropopause. Other quite pronounced wind changes were encountered during the sounding and using the indicated values places where the Richardson number was less than 1 are marked by a single line. Places where the Richardson number was less than $\frac{1}{4}$ are shown by a double line. The wind was from a SW'ly direction below the tropopause at about 12 km and above it veered.

The second case, in figure 2, has been treated in a similar manner. A discontinuity in the slope of the pressure trace near 38 minutes indicated that the vertical velocity decreased suddenly near this level. The radar measurements also showed this and have been plotted in this figure because they are relatively free of random errors and may probably provide a better estimate of the true vertical velocity than the pressure measurements. The vertical velocity evidently increased in accord with the density effect alone up to nearly 30 minutes at 184 mb. Subsequently, it rises to the exceptionally large value of 15 m sec^{-1} near the tropopause. The rate of switching behaves similarly except for an anomalously high value at 33 minutes.

The potential temperature lapse rate shows that the troposphere was rather unstable except near the tropopause where a number of stable layers occurred. The wind was from a generally SE'ly direction in the troposphere, but again changed at the tropopause and backed strongly in the stratosphere. A number of layers of low values of the Richardson number were present in the lower troposphere but there were also some in the upper troposphere.

In the six months of data from Muharraq in the Persian Gulf covering the period October 1967 to March 1968 inclusive, 6 cases were found where there was a decrease of the vertical velocity greater than that present in the Shanwell case. Although the balloon used in the second case study was of the 2 kgm high-altitude type this does not appear to be an important factor because 4 of the 6 Muharraq cases used 1 kgm balloons similar to those normally used at Shanwell. The decrease of vertical velocity in all cases occurred near the tropopause and it appears that the change of stability there must be important. The greater frequency at

Muharraq suggests that some climatic feature may be important.

The two case-studies appear to be examples of particularly pronounced changes of the rate of ascent of radiosonde balloons. It seems likely that these changes were produced by variations of the drag coefficient and assuming that the balloons were spherical some values have been calculated in figure 3. The Reynold's number and the drag coefficient have been calculated after 10 minute intervals and also before and after the abrupt decreases of rate of ascent near the tropopause. For comparison, some mean values are shown for ascents with 1.25 kgm balloons, with free lifts of 2.1 kgm, taken from the Handbook of Meteorological Instruments. The curves representing the change of drag coefficient for a solid sphere in a wind tunnel, which are also shown, correspond to critical changes at the greatest and least values of the Reynold's number given by Hoerner (1958).

The critical change for radiosonde balloons is evidently less abrupt than for solid spheres and usually involves a much smaller change of drag coefficient. The drag coefficient is usually much greater than for solid spheres but, exceptionally, for the Muharraq case it drops to near the value for solid spheres before the critical change, and rises almost the full amount expected for solid spheres at the critical change. In both case-studies the critical change involves a considerable decrease of Reynold's number because the rate of ascent decreases. The Reynold's number for rising balloons is not the independent variable of the experiment but depends on the rate of ascent and ultimately on the drag coefficient.

The critical change for radiosonde balloons occurs in the same range of Reynold's number as it does for solid spheres. Departures of the rate of ascent from the steady increase which might be expected from the decrease of density, may reasonably be attributed to changes of the drag coefficient. The critical change is usually not very pronounced for radiosonde balloons, possibly because the Reynold's number is not an independent variable for the flow but depends on the rate of ascent and finally Cd. The remarkably large changes of drag coefficient which occur occasionally appear to occur near the tropopause. The influence of turbulence in an airstream on the position of the critical change for a solid sphere, has already been noted and it is possible that a smaller intensity of small-scale turbulence above the tropopause is important. It is also possible that a large intensity of small-scale turbulence may triyger the onset of the large rate of ascent occasionally observed in the upper troposphere.

Some low values of the Richardson number are evident in figure 1 near the level where the rate of ascent takes up its highest value, although these are by no means the lowest values encountered. There are, however, difficulties in measuring the Richardson number accurately because of the inaccuracies of the wind data. Moreover, turbulence of a scale which is likely to affect flow over the surface of a radiosonde balloon probably results from a cascade from larger scales and need not necessarily be accompanied by a value of the Richardson number below $\frac{1}{4}$. Some values below 1 are also present in figure 2 near where the rate of ascent increases markedly. The lowest values in this case occur in the lower troposphere where the rate of ascent is evidently unaffected by any perturbation.

There does not appear to be any conclusive evidence that turbulent layers did exist or even that they could have affected the radiosonde in the way observed. However, the high tropopause at Muharraq (near 17 km in the cases noted) suggests that a low air density which brings the balloon towards a large rate of ascent may be significant as an initial factor. The mean wind shear in the upper troposphere in winter, which is larger at Muharraq than at Shanwell, is possibly the climatic feature which favours the observed balloon behaviour and indicates that turbulence is probably the direct cause.

2.4 Self induced balloon motions

Recent work using precision radars has made it clear that rising balloons undergo short-period motions not caused by any atmospheric factor. These motions are not caused by changes of balloon shape (Murrow and Henry, 1965) but appear to be related to changes of the aerodynamic lift force, which acts primarily in a horizontal direction (Scoggins, 1965). For balloons in the subcritical regime the motion appears to be fairly regular but near the critical change large erratic lateral motions appear which obscure much of the small-scale detail of the wind. For radiosonde balloons the hanging load will modify the lateral motion in a variety of ways, involving especially the coupling between the balloon lateral motions and the swing of the radiosonde. Scoggins shows that the American rawindsonde system is a poorer wind detector than free balloons and fails completely to detect wind changes in a layer less than 500 m deep.

Many of the wind changes in figures 1 and 2 do take place over height intervals of about 500 m. Often they are part of a wind change, probably real, over a deeper layer, such as near the tropopause in the two cases, but sometimes, as in the stratosphere in figure 1, they may be spurious. The rate of switching which could conceivably be affected by lateral motion of air past the radiosonde, also shows some variability of a similar period in the stratosphere in figure 1. The slope of the pressure trace does not usually show any variability over intervals longer than about 20 sec. The rate of switching does appear to vary on smaller time scales and is possibly related to motions induced by the swinging of the radiosonde below the balloon, which has a period about 10 sec. The wind measurements must evidently be used cautiously when the wind shear is measured over shallow layers.

2.5 The calculation of the rate of ascent of radiosondes

The correspondence between the rate of switching of the radiosonde and the vertical velocity in figure 1 and 2 suggests that the windmill switch could be calibrated to provide a measure of the rate of ascent of the radiosonde relative to the air. This value could provide a very valuable means of calculating the vertical velocity present in small-scale air motions, by subtraction of the rate of ascent from the vertical velocity measured from the pressure or the radar readings. The simple construction of the windmill switch may lead to difficulties because a calibration between the rate of switching and the air speed past one radiosonde may not be applicable to all radiosondes. Moreover, the friction in the gears may change during the ascent because of wearing of the metal. On the other hand, because changes of the vertical velocity over quite long time intervals are fairly well reflected in the rate of switching, it is reasonable to suppose that changes of the rate of ascent through the air over shorter time intervals should be as well or better represented by the rate of switching.

A wind tunnel was used to calibrate two radiosondes: one was an old model with 3 cups on the windmill, and the second was a more recent model which had the 6 cups normally used at present in radiosounding practice. The results are shown in figure 4 where they are compared with a line representing a constant anemometer factor of 2.5. This factor is the ratio of the speed of the air past the windmill to the linear speed of the cups and is nearly constant for anemometers used in practice. Calibration at air speeds above 9 m sec^{-1} showed that the rate of switching did not increase. This was attributed to a sideways deflection from their plane of rotation of the cups of the windmill, by air forced outwards by the measuring elements. Removal of the measuring elements produced switching rates which increased above 9 m sec^{-1} but it appeared that at high air speeds the radiosonde did restrict the airflow through the windtunnel. Moreover, from soundings it appeared that there was a smaller rate of switching produced by a given rate of ascent high in the atmosphere than at low levels. For these reasons a less fundamental approach suggested by Hawson (1967) was adopted.

If the vertical velocity is measured using the radar heights at 10 minute intervals, not only are random errors to which the radar measurements are subject insignificant, but also, the effect of any small-scale air motions present will probably be removed. These values may be considered to be estimates of the rate of ascent and may be compared with the rate of switching averaged over the same 10 minute intervals. Because it is normally found that the rate of ascent reaches a maximum near the middle of each sounding and subsequently decreases, positions on each side of the maximum may be located where the rate of ascent is the same and the ratio of the rates of switching may be calculated. The decrease may be considered to depend on the air density which also decreases, and the ratio may be compared with the ratio of the densities at the two positions, on logarithmic graph paper. It is normally found then that points for several comparisons normally lie near a straight line and the rate of switching depends on a small positive power of the density. Correction of the rate of switching to an equivalent rate of switching at ground level then allows calibration points to be plotted, some examples of which are shown in figure 4. The rate of switching measured over 1 minute intervals may be similarly adjusted for the effect of decreasing density with height. Reference to a curve drawn through the appropriate calibration points then yields values which may be regarded as reasonable estimates of the rate of ascent of the radiosonde through the air.

2.6 Some examples of non-periodic air motions

On radiosonde records inspected it is usual for all the pressure values to define a curve (called the 'pressure trace') whose smoothness frequently makes a wave-like form obvious. With the aid of a straight-edge the wave-like form is especially easily detected and disturbances with vertical velocities substantially less than 1 m sec^{-1} may be inferred; they are probably caused by lee waves. Other disturbances are rare and it is remarkable that the vertical air motions in cumulus clouds and clear air turbulence which are frequently present in the atmosphere are not represented on the pressure trace.

It appears that these must normally involve air motions which are too small in extent or amplitude to be detected. Occasionally, however, radiosondes do seem to abruptly encounter substantial vertical air motions; two examples are subsequently discussed.

2.6.1 The sounding launched from Crawley at 1130 GMT on the 23rd August 1964

On this sounding the slope of the pressure trace indicated that the vertical velocity of the radiosonde abruptly decreased for 12 seconds when the radiosonde was nearly 11 km above the ground. The slope of the temperature trace was similarly affected, as might be anticipated. The humidity trace was unaffected consistently with a high lag. The readings were regularly spaced showing that malfunctioning of the chart recorder was not responsible for the change of slope of the traces. The rate of switching was not measurably diminished during the 12 seconds but was clearly increased over a longer period. It appears that the radiosonde encountered a downward air current of about $4\frac{1}{2}$ m sec^{-1}, during which time the radiosonde rose about 36 m and drifted horizontally about 600 m.

At the bottom right of figure 5 the vertical velocity determined from each reading of pressure is plotted. The detail on the temperature trace has also been used to measure the average value of $\partial\theta/\partial z$ during 12 second intervals. Because the radiosonde rises through the air at W_a m sec^{-1} the observed value of the parameter, $(\partial\theta/\partial z)'$ must be corrected for the effect of the vertical air speed (W m sec^{-1}) by the equation:

$$\frac{\partial\theta}{\partial z} = \frac{W_a + W}{W_a}\left(\frac{\partial\theta}{\partial z}\right)' \qquad (4)$$

The uncorrected value of the parameter $(\partial\theta/\partial z)'$ is shown by the circle in figure 5. The wind between 27 and 31 minutes is represented by the radial and tangential components, measured as averages over 1 minute intervals.

The disturbance was at about the level of a NW'ly jet-stream and above a layer in which the wind shear was quite large. The levels marked (H) and (M) are the heights which Ludlam (1967) inferred for high-level and medium-level billow clouds observed during the late morning on the 23rd August, 1964, from Ascot. The radiosonde was launched from Crawley about 50 km downwind of Ascot, and it is possible that it intersected a billow-wave. From the conventional smoothed wind data the disturbance appears to be above the layer in which the wind shear is largest. However, using the components of the wind averaged over 1 minute intervals the Richardson number is less than 0.25 and a point of inflexion occurs in the wind profile between 27 and 29 minutes, fulfilling the known criteria for the occurrence of Kelvin-Helmholtz instability. Thus it is plausible that an air motion caused by Kelvin-Helmholtz instability should be detected in this layer. No disturbance is visible on the radiosonde record near the height of the medium-level billows nor is the Richardson number near this level small.

The rate of ascent of the radiosonde through the air, estimated from the rate of switching, is compared with the rate of change of radar height at the top of figure 5. The random scatter of the radar measurements is remarkably small and slight changes of the vertical velocity correspond on the whole to similar changes of the rate of ascent. A small increase of the vertical velocity between

28 and 29 minutes is also present in the rate of ascent and in the vertical velocity measured from the pressure trace (not shown) but in this latter case over a longer time interval. This increment probably represents an increase of the rate of ascent through the air rather than an air motion and its position in the upper troposphere suggests that it may be related to a temporary reduction of the drag coefficient. It is possible that Kelvin-Helmholtz instability led to the presence of turbulent motion on very small scales which reduced the drag coefficient of the surface of the balloon.

2.6.2 The sounding launched from Muharraq at 2330 GMT on the 7th November, 1967

Among 350 Muharraq soundings inspected there were 3 cases with abrupt changes of slope of the pressure trace probably caused by air motions. One of these occurred in the upper troposphere and two in the stratosphere. An example which occurred about 45 minutes after launch, when the radiosonde was about 17 km above the ground, is shown in figure 6. The vertical velocity measured from the pressure trace decreased and then increased before returning to the former value. The rate of switching changed slightly but it does appear that the greater part of the disturbance on the pressure trace was caused by an air motion.

The vertical velocity measured from the slope of the pressure trace is plotted against time after launch at the top of figure 6 and on an expanded scale at the right of the diagram. The rate of ascent of the radiosonde through the air is also shown at the top of the diagram. Values of the wind and temperature are plotted at the standard levels and it is evident that the vertical air motion lies in a layer in which the wind decreases markedly with height. The parameter $\partial\theta/\partial z$ is plotted in detail at the right of the diagram and is corrected for the effect of the air motion. The wind is plotted in the form of radial and tangential components and the Richardson number is measured as an average over 1 minute intervals.

The form of the disturbance, implying a change in the vertical speed of the air from 3 m sec^{-1} downwards to 3.5 m sec^{-1} upwards, suggests that the radiosonde passed through a wave-like motion present in only a shallow layer. The stability between 44 and 48 minutes is smaller than is usual in the stratosphere except in a shallow layer just before 46 minutes. The Richardson number using the indicated values is minimum near 47 min, however, the wind shear is probably modified by the disturbance and could have been much larger when the disturbance was initiated. It is difficult to imagine what other than Kelvin-Helmholtz instability could be responsible, and the observations suggest that the criteria for this instability were probably fulfilled.

Chapter 3 - The analysis of periodic disturbances in the pressure record

3.1 Introduction

The frequent periodic disturbances on the pressure trace appear to be the same as those detected by Corby (1957) and are probably lee waves. Occasionally these disturbances occur in the stratosphere and are so well marked and their character so similar to those occurring in the troposphere that they are strongly suggestive of lee waves in the stratosphere.

Of other possible causes, small-scale convection, intermittent precipitation and intermittent icing of the balloon are evidently not relevant in the stratosphere. A plausible cause of wave-like disturbances might be small departures from the normally smooth functioning of the pressure aneroid capsule. Perhaps a more important source might be periodic fluctuations in the rate of ascent of the balloon through the air related to changes of the drag coefficient caused by a succession of turbulent layers. Another important consideration in the stratosphere is the temperature effect on the pressure sensor. This might also produce wave-like disturbances on the pressure trace if the balloon rises through layers of varying stability.

3.2 <u>The calculation of the vertical air speed</u>

For the above reasons a procedure to calculate the vertical air speed encountered by the radiosonde as a function of time has been developed.

3.2.1 The radiosonde record

Figure 7 is a photograph of a section of a record for the sounding launched from Shanwell at 1130 GMT on the 15th June, 1964. The sounding begins at the left of the figure and successive minutes are marked by heavy lines. The humidity marks are the most variable, except beyond 15 minutes where the lag becomes important. The temperature marks are less variable and have a distinctly wavelike form. The pressure marks begin near the middle of the chart and change regularly. The individual marks are connected by pencil lines on the record and significant points are marked according to conventional criteria. This is a particularly pronounced example in which the form of the disturbance on the pressure trace strongly suggests the passage of the balloon through lee waves. It is evident that the time taken between successive crests on the pressure trace is related to the wavelength of the lee waves and that the departure of the pressure trace from a mean undisturbed path is related to the displacements of the streamlines in the waves. The gradient of the pressure trace may be measured and used to calculate the vertical velocity of the balloon.

3.2.2 The 'Q' correction

The changing temperature during a sounding produces a variation of the permeability of the mumetal cores of the inductances which control the transmitted frequency from the three meteorological sensors of the radiosonde. There is a smaller effect of temperature on the resistance of the coils in the inductance, and a compensating factor for both these effects, called the 'Q' correction, has to be applied to the pressure measurements. A similar correction for the temperature unit is unnecessary since it is included in the calibration and none is applied to the humidity measurements because they are not considered sufficiently accurate.

The 'Q' correction varies from one instrument to another and is largest at small pressures, sometimes producing very large corrections to the height of a radiosonde in the stratosphere. In radiosonde practice the 'Q' correction is combined with the sensitivity of the pressure unit and the so-called modified 'Q' factor is formed. Tables then give the correction in millibars to be added to or subtracted from each pressure reading as a function of the factor and the temperature.

Changes of the slope of the temperature trace act in such a way as to produce apparent changes of slope of the pressure trace. In some cases when the atmosphere is stratified in such a way that the rising radiosonde encounters an approximately periodic variation of lapse rate, wave-like disturbances might be inferred from associated periodic variations of the slope of the pressure trace. Fortunately, the sign of the effect for most radiosondes is such that wave-like disturbances on the pressure trace are produced by disturbances with opposite phase on the temperature trace. These may be distinguished from lee waves which produce disturbances with the same phase on the temperature and pressure traces. No cases where pronounced wave-like disturbances were caused by the temperature dependence of the pressure unit were found using this criterion in the 1964 Shanwell records. The 'Q' correction of many radiosondes was small and large temperature oscillations infrequent, so that it does not appear that this was a significant source of wave-like disturbances.

Nevertheless, the vertical velocity inferred from the slope of the pressure trace must be corrected for the effect of temperature. The procedure which has been adopted is to plot all the significant points defining the temperature as a function of time. The temperature change per minute may then be determined and each value multiplied by the change of 'Q' correction (mb) per ^{o}C at the appropriate temperature. These values in mb min^{-1} may be converted by means of the hydrostatic equation into a correction C_p to the vertical velocity W_p' ($m\,min^{-1}$) inferred from the recorded rate of change of pressure.

Figure 8 illustrates a case in which the correction was important and had a wavelike variation; it amounted to as much as 100 m min^{-1} in the stratosphere. The modified 'Q' factor of the pressure instrument decreased from 1.80 to 1.40 during the sounding. The uncorrected vertical velocity from the pressure measurements, the correction C_p in mb min^{-1}, the temperature, and the corrected vertical velocity W_p are shown in figure 8. The correction in m min^{-1} is not important until after 20 min and introduces new wave-like features after 44 min. Other features are altered in form by the correction.

3.2.3 Vertical air motions in the atmosphere

Two methods of measuring the vertical velocity of a radiosonde relative to the ground have been described in Chapter 2. An important correction for that derived from the pressure measurements has been described above and this corrected vertical velocity may usefully be compared with the rate of change of radar height. Such a comparison is made in figure 9 for the sounding at 2330 GMT on the 4th September, 1966, when the improved (Cossor) radar was in use at Shanwell.

The entirely independent measures of vertical velocity agree satisfactorily in the period up to about 20 minutes after the launch, and indicate that the pressure element was operating correctly. Subsequently a small peak of ascent speed between 20 and 21 minutes appears on both records and is probably real; the next peak at 24 minutes is well defined. The minima at about 28, 34 and 42 minutes, and peaks at about 31, 37 and 44 minutes are probably real.

The gradual deterioration after 20 minutes in the agreement between the two measures is accompanied by an increase in the scatter of the vertical velocities indicated by the radar. This scatter may be attributed to errors in the indicated elevation angle E, which is read only to the nearest 0.1^{o}. The magnitude of the error in the vertical velocity which may thus be incurred is plotted as a function of time in figure 9 and is compared with the difference between the two measures. This difference can evidently be explained simply as the error incurred by rounding off the elevation angle indicated by the radar.

The error to which the vertical velocity inferred from the pressure trace is liable is also shown (by the pecked lines) in figure 9 and is $\pm$ 40 m min^{-1} throughout the ascent. It has been assessed as a 10% random error in measuring the slope of the pressure trace on the paper record. Up to 20 minutes after launch this error may evidently be the more important; subsequently the vertical velocity inferred from the pressure trace is the better estimate of the true vertical velocity of the radiosonde.

It appears that the vertical motion of the radiosonde does undergo real wave-like variations throughout almost the whole ascent. It does not appear that the rate of ascent of the radiosonde through the air, indicated by the rate of switching, was similarly affected and the observed disturbances must therefore have been caused by real air motions. Values of the rate of ascent are included in figure 9 and the result of subtracting these values from the vertical velocities inferred from the pressure trace, which can be regarded as a representation of the actual vertical velocities of the air relative to the ground, is also shown. The form of the variation and the nature of the meteorological situation and of the airstream strongly suggest that the encountered disturbances were due to the passage of the balloon through lee waves present to high levels.

3.3 The analysis of wave-like disturbances on the pressure trace

The many more numerous cases of smaller wave-like disturbances are not so readily analysed in the above manner because their vertical air speeds are about the same as the errors in measuring them. It is advantageous to treat them in a simpler manner to derive their amplitudes.

Much of the subsequent discussion will refer to 'wave-like disturbances'. Some precise meaning must be given to the term 'wave-like' to ensure, as far as possible, that the disturbances are due to real waves in the atmosphere rather than any other air motion or irregular behaviour of the radiosonde or the balloon. It has been found convenient to define a wave-like disturbance as a disturbance of the pressure trace of smooth character which shows two or more wave troughs. Where more than two troughs are visible a regular periodicity is also required.

Wave-like disturbances have two well-defined properties: a period, the time between two consecutive troughs or crests, and an amplitude, which is the maximum deviation in Cintel-paper divisions of the departure of the pressure trace from a mean undisturbed path estimated with a straight edge. Radiosonde data from Shanwell have been systematically examined for disturbances on the pressure trace and the properties of the wave-like features found is the basis of some following sections.

There is some indirect evidence that the majority of the wave-like disturbances are due to lee waves. Thus, no cases of wave-like disturbances have been found on the records when the wind approached Shanwell from the North Sea. Further evidence has been obtained from an investigation of radiosonde records from Weather Ships. It was found that there were no wave-like disturbances at all on records from Weather Ship J, west of Ireland, during a period when disturbances occurred very frequently on the Shanwell records. Some disturbances were present on other records from Weather Ship I, but they were much less frequent than at Shanwell and usually lacked the distinct periodicity. These may have been caused by travelling gravity waves associated with weather systems.

3.4 The amplitude of lee waves

The amplitude of the wave-like disturbances is measured in divisions of the vertical coordinate of the Cintel recording paper. This coordinate for the pressure trace corresponds to pressure and it is evident that deviations of the trace from a mean can be expressed directly as a pressure deviation or height interval. A radiosonde rising through a system of lee waves in which the phase-lines are vertical, or nearly so, will rise and fall about the path it would have taken had the waves been absent, and it can reasonably be assumed that the maximum deviation will correspond to an average vertical displacement of the lee-wave streamlines over a layer a few kilometres deep.

Two difficulties arise in measuring the amplitude: first, a curvature of the pressure trace, which is greatest near and above the tropopause, hinders the identification of wave-like disturbances in this region and makes reliable measurement of amplitude impossible. Secondly, the sensitivity of the pressure aneroid to changes of height decreases upwards and lee wave streamline displacements of 100 m, which can produce readily visible disturbances in the troposphere, can have no detectable influence on the pressure trace if they occur above about the 50 mb level.

In the troposphere, although the sensitivity of the radiosonde aneroid to changes of height decreases with the pressure, an increase in the rate of change of frequency occurs and fully compensates for the former effect. Accordingly, up to about 300 mb a change of height of 25 m typically produces a change of about $\frac{1}{2}$ Cintel paper division in the pressure trace. Wave-like disturbances of this amplitude are usually readily visible on the pressure trace and so a fairly complete climatology of lee waves of amplitude greater than 25 m can be constructed.

The frequency of occurrence of wave-like disturbances on 500 Shanwell records for the year 1964 has been assessed in two parts of the radiosonde record, corresponding to heights of about 2 km and 16 km (figure 10). As the radiosonde may rise a few kilometers while passing through one complete wavelength the inferred amplitude must necessarily be an average over such a distance, and lee waves confined to shallow layers are likely to escape detection. The two heights chosen appear to be representative of the levels at which lee waves most frequently have their maximum amplitude in the troposphere and lower stratosphere respectively. At 16 km the sensitivity of the radiosonde is still adequate to detect waves of amplitude 50 m or greater.

Owing to the difficulty in estimating the amplitude of a lee wave in the stratosphere directly from the pressure trace, it has been assumed that all 44 cases (out of 500) where a distinct periodicity was evident in the pressure trace at about 16 km, represent cases of lee waves of amplitude greater than 50 m. Three of these contained especially marked disturbances and from the period and the vertical velocity, the amplitude of the waves was estimated as about 150 m. In the same way it appeared that there were 21 records indicating lee waves with amplitudes exceeding 75 m. This evidence is the basis for the line (b) drawn in the frequency diagram figure 10.

If it is assumed that the frequency of amplitudes greater than A is represented by similar lines in figure 10, at other heights and in other locations, further statistics can be prepared. Thus for an intermediate height, say 10 km, Corby's (1957) data on the heights of the maximum amplitudes may be used to draw another line; this would be below both (a) and (b) because the frequency is about 1/7 that at 2 km. At Muharraq, the frequency of the waves in the six months of data analysed may be used to construct similar lines. No waves were detected in the stratosphere but in the lower troposphere it may be inferred that the streamline displacement exceeds 100 m about 0.3% of the time. At Weather Ship I inspection of the data for the months December and January 1964 showed that wave-like features, possibly gravity waves, have streamline displacements exceeding 100 m about 0.8% of the time at 2 km and 0.5% of the time at 16 km.

The frequency of pronounced wave-like disturbances has been estimated over periods of 3 months at Shanwell using data between 1964 and 1967. The data for 1965 was prepared by Stewart (1966) and no frequencies for waves in the troposphere are available. Rougher statistics were prepared for 1966 and 1967 and the proportion of soundings with pronounced wave-like disturbances are shown in Table 1. For this table a pronounced wave is understood to be one which has either a large amplitude, or extends through a large depth of the atmosphere.

Pronounced waves occur most frequently in the winter months, both in the troposphere and in the stratosphere. During the summer the frequency in the stratosphere decreases more markedly than in the troposphere; this indicates that the strong W'ly winds in the stratosphere during the winter favour lee waves there. The seasonal frequency from year to year varies markedly.

3.5 The period of lee waves

A most distinctive feature of wave-like disturbances is the regularity of the time interval between successive crests or troughs on the pressure trace. This corresponds to the time taken for a given air parcel to traverse one wave-length - the wave period - if the wave crests and troughs lie in vertical planes and the period remains constant in the layer under consideration. In a real case, the period of a wave-like disturbance will be approximately the period of the wave if the two conditions are nearly met.

The periods of the wave-like disturbances represented in figure 10 have been noted, and the frequency of occurrence of individual periods (to the nearest whole minute) has been expressed as the histogram of figure 11. The data at 2 km contain a considerable proportion of waves of amplitude between 25 and 50 m, not represented in the data at 16 km.

The most frequent periods at 2 km lie between 6 and 10 minutes and correspond to typical values of the Brunt-Vaisala period ($2\pi(g\beta)^{-\frac{1}{2}}$). The high correlation between the wavelength and the mean wind speed observed by Corby (1957) also shows that the periods in his data cluster about 8 minutes with a maximum value of 10 minutes. The most frequent period at 16 km is 5 minutes which corresponds to the Brunt-Vaisala frequency when the temperature increases with height at about $2^{\circ}C\ km^{-1}$.

3.6 Some case-studies of pronounced lee waves in the atmosphere

3.6.1 The sounding launched from Shanwell on the 5th September, 1966

This radiosonde was launched from Shanwell at 2330 GMT on the 4th September, 1966. Wave-like disturbances were present on the pressure trace throughout almost the whole ascent. The variation of the vertical air speed encountered during the ascent has been calculated in section 3.2.

At the time of launch scattered small cumulus and some altocumulus were observed. The wind was westerly, about 10 knots at the surface but increasing to nearly 50 kts below the 900 mb level. A NW'ly jet stream of about 150 kts was present at 250 mb, and remained near Shanwell for a few days following this ascent. Corby (1966) has described wave-like features on the ascent from Shanwell 36 hours later and has associated them with lee wave clouds visible in a satellite picture. Numerous reports of pronounced lee waves were made during this period by aircraft pilots.

Figure 12 shows the distribution of the vertical air motions with time and height, and the stratification of the atmosphere and the winds indicated by the sounding. The correction to the temperature for the effect of the vertical air motions is not included because it does not significantly alter the form of the temperature profile. Moreover, in the study of lee waves, shallow layers in which the correction to the lapse rate may be important seem less significant than deeper layers in which the effect of the upward and downward air motions tend to cancel out. The correction to the stability in layers 2 km deep may be conveniently estimated using equation 4 and in typical pronounced lee waves amounts to about $1^{o}C\ km^{-1}$.

The strong jet stream with a maximum speed of over 70 $m\ sec^{-1}$ at about 10 km is a striking feature. The most intense waves appear to occur a little below 3 km, not in a region of strong wind shear as would commonly be anticipated, but in a region where the wind speed is almost constant with height. The waves become weaker at higher levels but at 7 km quite pronounced waves appear and are present throughout much of the remainder of the ascent.

The fairly regular periodicity characteristic of many wave-like disturbances is well illustrated in this case. The time taken for the radiosonde to pass from one region of upward air motion to the next has a tendency to decrease with height: thus, the interval between the first two updraught maxima is 9 minutes, and that between the first two downdraught maxima is 8 minutes while at high levels it is about 7 minutes.

An important function in the theory of lee waves is l^2 which has dimensions of $[\text{length}^{-2}]$:

$$l^2 = g\beta U^{-2} - U''U^{-1} \qquad (5)$$

where g is the acceleration due to gravity, B is the stability $\frac{1}{\theta}\frac{\partial\theta}{\partial z}$ and U'' is $\partial^2 U/\partial z^2$. The function l^2 varies rapidly with height but horizontal variations are usually only important on scales greater than those affecting lee waves. Of the two principal terms of l^2, ($g\beta U^{-2}$) varies greatly when measured in shallow layers but varies less and more regularly, when measured over layers 1 km or more deep. The term ($U'' U^{-1}$) also varies rapidly in shallow layers, and cannot be measured in layers less than about ½ km deep using conventional wind observations. If estimated over greater depths its magnitude varies more regularly but tends to zero as the smoothing reduces the curvature of the wind profile.

The parameter which has been plotted in figure 12 is ($2\pi U (g\beta)^{-\frac{1}{2}}$) which will here be referred to as m. It is related to l^2 thus: $l^2 = 4\pi^2 m^{-2} - U''U^{-1}$

If the period of the lee waves is the Brunt-Vaisala period this parameter is simply the wavelength. It is estimated over depths of 2 km, thereby removing the influence of irregularities of wind and temperature over smaller depths, which probably are not significant for lee waves. This parameter is nearly constant in a deep layer in the lower troposphere on this occasion.

3.6.2 The sounding from Shanwell on 31st January 1964

The radiosonde was launched from Shanwell at 2330 GMT on 30th January, 1964. At the time of launch small amounts of low cloud and some dense cirrus, but no medium-level clouds, were reported.

An intense NW'ly jet stream was present over Scotland with its axis near Shanwell; the maximum wind speed encountered over Shanwell was 70 m sec^{-1}. Wave-like disturbances were visible over much of the pressure trace and those occurring in the stratosphere were as marked as any found in the 1964 records. In common with all wave-like disturbances occurring on the 1964 records their amplitude appeared to have a minimum near the level of the jet stream.

During 1964 a G.L.3 radar was in use at Shanwell and the measurements of height made with it were too inaccurate to be useful. The temperature correction for the pressure unit was negligibly small in this case, and the very distinct periodicity of the wave-like disturbances suggests that malfunctioning of the pressure aneroid was unlikely. The rate of ascent of the radiosonde through the air can again be estimated from the rate of switching. Figure 13 shows the derived vertical air velocities, which are larger in the stratosphere than in the troposphere.

The stratification of the atmosphere, the wind distribution and values of m $(= 2\pi U (g\beta)^{-\frac{1}{2}})$ are also shown in figure 13. A major feature is again a strong jet stream in the upper troposphere. On this occasion there is a shallower layer in the lower troposphere in which the wind is approximately constant with height; there are two values of about 30 m sec^{-1} obtained well above the jet stream before the radiosonde passed out of radar range. Evidence from other radiosonde stations suggests that the wind speed did not change much at greater heights, up to 20 km. It is probable therefore that m was nearly constant in the stratosphere up to 20 km and this may have been important for the pronounced lee waves present there.

It is remarkable that the interval between the waves in the stratosphere is clearly about 6 minutes, whereas in the troposphere it appears to be about 10 minutes, values which are close to the Brunt-Vaisala frequencies. The equation

$$\frac{1}{W}\cdot\frac{\partial^2 W}{\partial z^2} + (\ell^2 - k^2) = 0 \qquad (6)$$

($k = 2\pi/L$, L is the horizontal wavelength) used in the theory of lee waves implies that the crests and troughs are aligned vertically if $l = k$. In regions where the wind shear is small this implies that the wave period is given by the Brunt-Vaisala frequency which is the frequency of a simple inertial oscillation of a parcel given a small vertical displacement. It seems reasonable that lee waves should have a frequency about this value; moreover, this is consistent through equation 6 with the frequency detected by the radiosonde.

It might alternatively be suggested that the horizontal wavelength is much longer and the wavefronts tilt upstream so that the rising radiosonde intersects successive crests about 6 minutes apart in the stratosphere. Consideration of similar triangles shows that the difference between the true horizontal wavelength and that inferred from radiosonde data cannot exceed 20% of the wavelength unless $l^2 - k^2$ exceeds 0.40 km^{-2}. This is impossible with typical cases of pronounced lee waves because l^2 itself is usually less than .40. In fact, much ambiguity only arises when m is less than about 5 km. In figure 13 the wavelength in the stratosphere is about 11.5 km.

3.6.3 The sounding at Shanwell on the 4th December, 1964

This radiosonde was launched from Shanwell at about midnight before the morning of the 4th December, 1964. Wave-like disturbances were exceptionally frequent during this month, mainly on the night soundings. At the time of launch no clouds were reported. A strong jet stream was present in the upper troposphere but in this case also, there was a layer in the lower troposphere where the wind was less strong and approximately constant with height.

Figure 14 shows the vertical air velocity as a function of time and height. The balloon was out of radar range below 14 km and no confirmation could be obtained of the disturbances inferred from the pressure trace above this height. The 'Q' factor was again small and the changes of slope of the pressure trace probably accurately represented the air motions present.

The stratification of the atmosphere, the wind speed and the direction expressed to the nearest 10°, and the height are also shown in figure 14. The parameter m is plotted for layers 2 km deep except between 4 and 5 km in an inversion layer in which the term $(U''U^{-1})$ is large. The disturbances are more regular in the stratosphere than in the troposphere and have a period about 5 minutes.

3.6.4 The sounding at Shanwell on 31st December, 1964

This radiosonde was launched from Shanwell at 2330 GMT on the 30th December, 1964. At the time of launch only some scattered stratocumulus and cumulus were reported. There was no jet stream present in the upper troposphere and the maximum wind speed measured was near the highest point reached during the flight.

The rate of change of radar height was used to confirm the vertical velocity measured from the pressure trace. The temperature correction for the pressure unit was again small and no 'Q' correction was applied to the vertical velocities. The vertical air velocities, inferred from the pressure measurements and the rate of switching, are shown in figure 15. The disturbances are more regular in the stratosphere than in the troposphere and have a period about 6 minutes.

The stratification, wind and values of m are included in figure 15. The tropopause is relatively low and the maximum value of m occurs below this level. In the stratosphere m increases a little with height.

Chapter 4 - The relation of the observations to theories of lee waves

4.1 Introduction

It has been shown in Chapter 3 that lee waves are only rarely very pronounced and it is the aim of this chapter to isolate the factors which appear to favour their development. Simple interpretations of theories of lee waves which are used as a basis for forecasting lee waves also indicate conditions which favour lee waves. Comparison between observation and theory in this and other respects shows that some discrepancies are present.

4.2 The frequency of occurrence of lee waves

Pronounced lee waves are thought to occur in conditions under which the wave energy is 'trapped' in the lower troposphere, an idea developed by Scorer (1949). Such lee waves have been termed 'resonant' and their existence depends on a sufficiently large decrease with height of the parameter $l^2 (=gBU^{-2} - U''U^{-1})$. This decrease is most readily produced by an increase of wind speed with height. Two other conditions of a reasonably self-evident nature have been recognised: that the wind direction should not change greatly with height, and that the wind velocity should have a component perpendicular to the ridge(s) producing the waves.

Inspection of mean monthly data shows that these several conditions must be fulfilled a substantial proportion of the time over Scotland. The wind speed often increases up to 10 km, usually with only a small change of direction. The prevalent westerly airstreams pass over high ground before reaching Shanwell in its lee. We might reasonably expect to observe lee waves over Shanwell on over half the radiosonde records, with amplitudes comparable with the magnitude of the height of the high ground (several hundred m). However, lee waves of amplitude 100 m or more appear to occur on only about 10% of the radiosonde records.

The numerical studies of Sawyer (1960) show that in an airstream with typical variations of wind speed and potential temperature with height lee waves exist with maximum streamline displacements comparable with the height of the ridge producing the waves. Insofar as the assumptions of the model allow a comparison with the atmosphere it does appear that quite pronounced waves should be very frequent, occurring not only when the wave energy is 'trapped' but also in conditions which cannot be so simply interpreted.

It might be argued that resonances with the prominent details of the orography are important for the formation of pronounced waves in the lee of mountains, but the wide variety of wind directions on the occasions of strong lee waves make this seem unlikely.

Moreover, the regularly spaced bands of cloud sometimes visible on satellite photographs which may be ascribed to lee waves (e.g. Corby (1966)) extend laterally for hundreds of km and for these, details of the orography can have only a minor influence. The very complex structure of the topography may be viewed as having only a statistical effect (Bretherton (1969)) - the orography generates a wide range of frequencies in the airstream but only discrete values attain a significant amplitude in the atmosphere.

The small frequency of occurrence of pronounced lee waves may indicate a real defect in the assumptions of the theories. This is given some support by the comparisons of Davis (1969) between predicted system of lee waves and those observed in a tank containing a stably stratified liquid. These experiments were similar to those of Long (1954) but contained the improvements that a continuous stratification was used and the theory incorporated the exact shape of the ridge producing the disturbance. The theory used was non-linear and the usual restriction to small amplitude waves was not present; other assumptions were similar to those in the linear theory of Sawyer. Typically in all the experiments performed, the wave amplitudes downstream were smaller than predicted. When the theory predicted complex lee wave systems there was not even qualitative agreement with observation and Davis attributed much of the discrepancy to turbulence in the liquid. It is not certain how far the fluid flow in the tank does represent flow in the atmosphere; however, it is clear that a deficiency of this non-linear theory which produces more pronounced lee waves than are observed could equally well be present in the similar and less complete theories applied to the atmosphere.

4.3 Conditions which favour the development of lee waves

In order to make some comparisons of the conditions in which pronounced lee waves occur with simple interpretations of theory some observed values of $(g\beta U^{-2})$ are shown in figure 16. This first term of l^2 makes the more important contribution to the variation through the depth of the troposphere, and between 0 and 1 km where the term $(U''U^{-1})$ is almost certainly dominant the parameter is not plotted. Otherwise it is plotted for layers 2 km deep, with a logarithmic horizontal scale to show the variation of the parameter more clearly.

The values of $(g\beta U^{-2})$ shown in figure 16 correspond to three soundings made during 1964 at Shanwell. On the one launched at 1130 GMT on the 15th June very pronounced waves were present in the lower troposphere but not at higher levels up to 20 km. The second set of values are for the sounding launched at 2330 GMT on the 30th January on which occasion lee waves were especially pronounced in the lower stratosphere and less so in the troposphere. The third case provides a comparison when lee waves were pronounced neither in the troposphere nor in the stratosphere, and corresponds to the sounding launched at 2330 GMT on the 16th November.

The wind was strong in each case and a jet stream was present in the upper troposphere; the wind direction did not vary much with height and, in fact, varied less in the case (c) when no waves were present than in the case (b). The wind direction in each case was reported as 290° at about the 900 mb level near the tops of the hills of Scotland and the airstreams probably traversed similar topography. The three cases have very different systems of lee waves and serve to emphasise the importance of the frequencies to which the airstream is responsive as opposed to the frequencies generated by the ranges. The vertical distribution of (gBU^{-2}) in cases (b) and (c) is similar and very small values are present in the upper troposphere. This condition is favourable for 'trapping' the wave energy in the troposphere yet one case had no pronounced lee waves there and the other had pronounced lee waves above the level at which the energy should have been 'trapped'. Moreover, the case (a) in which the lee waves were particularly pronounced in the lower troposphere had a comparatively small decrease of the parameter with height and should not have been a particularly favourable case.

The mean monthly data shows that it is unusual for l^2 not to decrease upwards in the troposphere, but they also show that it is unusual for l^2 not to increase upwards in the stratosphere, and so the observed existence of lee waves in this latter region of the atmosphere seems to conflict with the idea of 'resonant' waves. It appears that lee waves occur most frequently in the stratosphere during December and January at Shanwell and the presence there of the strong winds associated with lower part of the polar night jet stream is almost certainly important. In the regions of the stratosphere where radiosondes detect lee waves the mean December and January data show that the wind is nearly constant with height. It is possible that the associated small variation of l^2 with height may be a factor favouring the occurrence of lee waves in the stratosphere and perhaps also in the troposphere.

Some regularly spaced routine soundings made at Shanwell in December 1964 also suggest this. This month was chosen for study because although the day-time soundings showed no unusual incidence of lee waves the night-time soundings showed exceptionally frequent lee waves in the stratosphere. Their presence, often in very deep layers, can be used to show clearly an associated relative constancy of m with height.

The variation of m ($=2\pi U(gB)^{-\frac{1}{2}}$) with height is shown in figure 17 for twelve occasions during December, 1964. The first four day-time soundings were on occasions when a pronounced jet stream was present in the upper trosphere. The wind direction changes little with height and indicates that the airstreams passed over hills before reaching Shanwell. The increase of m with height corresponds to a decrease of l^2 with height and on the basis of the presently accepted criteria for the occurrence of lee waves all four should be favourable situations. Yet lee waves with amplitudes greater than 50 m were not present. The last two day-time ascents show small values of m corresponding to the low wind speeds present. The airstreams for very substantial depths were approaching Shanwell from the North Sea and the absence of lee waves was not remarkable.

The midnight soundings in figure 17 show more nearly uniform profiles of m. The occurrence of waves of amplitude greater than 50 m in the stratosphere in four out of six cases was remarkable; they seem to appear on occasions when the variation of m with height is least. Waves were present throughout the ascent on the 25th December, in the exceptional circumstance that this parameter was nearly constant.

It is interesting to discuss the four cases of strong stratosphere lee waves described in Chapter 3 in the light of the above evidence. In three of the four cases there were large changes of m; however, in each of the three cases there was a region of small wind shear and constant m in the lower tropsophere, and because of the larger density the:: conditions in this layer may be important. The larger vertical air velocities of Case 1 were associated with a greater vertical extent of this region in Case 1 than in the other cases. In Case 4 no jet stream was present and m is fairly constant in the stratosphere and in a shallower layer in the troposphère. The vertical velocities in Case 2 and Case 4, which were larger in the stratosphere than in the troposphere, may have been associated with the extension of the region of constant wind and m through a greater depth in the stratosphere. The interesting correspondence of m in the lower troposphere and in the stratosphere in Case 2 may also be significant. The discussion of the lee waves extending through the jet stream in Case 1 is postponed till section 4.4.

It has been suggested above that conditions in which m does not vary much with height are the most favourable for the development of lee waves. This condition appears to be normally only present when the wind speed does not change much with height, and is in conflict with the accepted view that considerable wind shear is a necessary condition for pronounced lee waves in the lower troposphere. However, in such cases among the Shanwell records the shear had often been confined to a small depth including an inversion, and the regions above and below have contained relatively small shears.

An example in which lee waves occurred in similar atmospheric conditions is shown in figure 18. The vertical air motions were inferred by Corby (1957) and the maximum velocities are shown in the figure.

Above and below the inversion the wind speed is nearly constant with height, but in the inversion layer between $1\frac{1}{2}$ and 2 km m (curve a) varies considerably becoming quite small and has not been calculated in this interval, nor close to the ground where the lapse rate was superadiabatic. Elsewhere it was calculated as an average over a depth of 2 km.

It is evident that m increases by very little across the inversion layer producing a layer in which m is nearly constant. This case contained the largest vertical air speeds found by Corby in 1440 radiosonde records, and is without parallel in the 1964 Shanwell records. This exceptional nature is accompanied by quite a small increase of m throughout the tropsophere; indeed, the increase is proportionately less than that occurring in the mean April conditions (b in figure 18).

Another case, which occurred in similar atmospheric conditions during April 1967, in a period of strong winds and frequent lee waves, is shown in figure 19. The amplitude (about 300 m) was comparable with the largest values which occur at Shanwell more than once a year (figure 10). Here, rather more stable conditions and a higher inversion allowed a reliable estimate of m to be made in the lowest 2 km of the troposphere. Remarkably, this is almost equal to the value immediately above the inversion and again suggests that the deep layer with nearly constant m is perhaps the most important feature favouring the most pronounced lee waves. This feature, dependent upon the presence of a stronger wind and a more stable stratification above an inversion, is remarkably similar to that which appears to be associated with the pronounced waves in the stratosphere in Cases 2 and 4 in Chapter 3.

It can be further shown that in general the wind shear in the troposphere is not large on occasions of pronounced lee waves. The 19 soundings from Leuchars (near Shanwell) from which Corby inferred pronounced lee waves (with vertical velocities exceeding $1\frac{1}{2}$ m sec^{-1}) with distinct horizontal wavelengths are represented in Table 2. The wind speeds and directions at four levels in the troposphere are compared with six-monthly mean values of the wind speed, based on data for the months October to April for the years 1951 to 1955. The mean wind speeds for the 19 cases are shown at the bottom of the table.

At all levels the mean wind speed for the occasions of waves is greater than the six-month mean, as might be anticipated. However, this difference is greater at low levels than at 300 mb and, in fact, the increase of wind from 850 mb to 300 mb is only about half of that which occurs in the mean. Moreover, four of the cases had a substantial increase of wind at a stable layer between 850 and 700 mb and this appears to have been partly responsible for the quite large increase of wind speed in the lower troposphere, not present in the mean. Above 700 mb the increase of wind speed in the cases of waves is in absolute terms about half of that occurring in the mean and proportionately is much less. It is in this region of the atmosphere that we might expect the major increases of m which should 'trap' the lee waves. Because of the rather small increase of wind speed a constant value of m therefore again appears as a factor which may favour pronounced lee waves.

Another well known occasion of very pronounced lee waves was that of the Sheffield Gale on the 16th February, 1962. Winds near the ground in the Sheffield area at about 0600 GMT averaged 40 kts and gusts to over 80 kts were recorded. Many buildings were severely damaged by the high winds, which have been ascribed to local concentrations of wind speed near the ground under the troughs of lee waves, pronounced at higher levels (Aanensen, 1965). An aircraft report of lee waves with vertical velocities of about 10 m sec^{-1} at 8000 ft. in the vicinity of Sheffield was made during the gale. It appears that the particular arrangement of the ridges immediately upwind of Sheffield may have contributed to the amplitude of the waves.

Analyses of radiosonde records for 0000, 1200 and 2400 GMT on the 16th February from Aughton (near Liverpool) and Hemsby (near the Norfolk coast and about 130 miles downwind of Sheffield) do not show pronounced lee waves. Lee waves in the lower troposphere with a maximum streamline displacement of about 100 m were present downwind of Aughton at 1200 GMT on the 16th February and rather more pronounced lee waves were present on the same record just above the tropo pause when the radiosonde was about 60 miles downwind of Aughton and nearly above Sheffield. The temperature, wind, and the parameter m are shown in figure 20 for 0000 GMT and 1200 GMT at Aughton on the 16th February and the wind at 0600 GMT is also shown. During this period there was a general cooling of the atmosphere, most pronounced at about 2 km, a strengthening of the wind at low levels until 0600, followed by a slight weakening, and a veering of the wind.

The increase of wind above the boundary layer was at no time large and at 0600 GMT when the gale was at its height the wind increased by about 30% between 1 and 5 km. The small stability beneath a strong inversion and the more stable conditions above are features which were also present in the cases of very pronounced lee waves in figure 18 and figure 19. Here the value of m in the layer above the inversion did not increase much during the 12 hours covering the storm. However, due largely to the decreasing stability of the layer near the ground m increased greatly (from 13 to 28 km) during the period of the storm and at 0600 when the lee waves were probably most pronounced the stability and the wind may have produced a layer in which m was nearly constant.

4.4 The variation of amplitude with height

The basic hydrodynamic equations may be used to derive the equation for flow in the x-z plane,

$$\frac{1}{W}\frac{\partial^2 W}{\partial z^2} + (\ell^2 - k^2) = 0 \qquad (6)$$

which is valid, for airmotions in the range of scales typically found in lee wave systems, provided that the flow is laminar, steady, frictionless and isentropic. The equation relates the vertical variation of the amplitude to the function l^2 and the horizontal wavelength ($2\pi/k$). In layers of the atmosphere in which l^2 and k^2 can be supposed constant with height the amplitude varies exponentially ($W \propto \exp(k^2-\ell^2)^{\frac{1}{2}}z$) or harmonically ($W \propto \exp i(\ell^2-k^2)^{\frac{1}{2}}z$) with height according to whether l^2 is less or greater than k^2.

Queney (1947) has considered lee waves in a model which has a constant value of l^2 throughout the depth of the atmosphere. The problem was made 2-dimensional by considering flow over an infinitely long ridge and was linearised by keeping the ridge low. Of the infinite number of wave solutions which had zero amplitude at the ground only one had waves confined to the lee of the ridge. The amplitude of these lee waves decreased downstream but increased upwards due to the decrease of density with height; in fact, the wave energy $\frac{1}{2}\rho_3 W^2$ decreased in both directions as r^{-1}, corresponding to cylindrical radiation behind the mountain. The amplitude of the lee waves in this model varies harmonically with height at all levels and the lines of crests (and troughs) slope upstream.

Pronounced lee waves in the lower troposphere have some properties inconsistent with waves in Queney's model: they do sometimes extend for many wavelengths downwind of mountain ranges without diminishing much in amplitude, they do not increase in amplitude up to stratospheric levels in the atmosphere, and they often appear to have vertical phases. Scorer (1949) has shown that in a model in which a layer of constant l^2 overlies a layer with a sufficiently larger value of l^2 lee waves with amplitudes which increase upwards to a maximum in the lower troposphere then decrease above could exist. The waves have vertical phases and do not decrease in amplitude downstream; in these respects they are similar to pronounced lee waves in the lower troposphere. The extension of the theory, however, to more frequent situations where such a simple two-layer representation of the atmosphere is not possible would suggest that all the frequent occasions when the wind speed increases substantially with height should be favourable for pronounced lee waves; in fact, this does not appear to be true.

Model atmospheres with three or more discrete layers may also contain lee waves but interpretation is not as easy as for the 'trapped' waves in Scorer's two-layer model. In general, there are a number of solutions with different horizontal wavelengths which satisfy equation 6 and have zero amplitude at the ground. All are valid solutions and the degree to which each is excited in a given air-stream depends on the width of the ridge. Corby and Sawyer (1958) have shown that some solutions reach their maximum amplitude in the upper troposphere or stratosphere, and that these solutions have relatively long wavelengths. The strong influence of l^2 at high levels on these solutions is shown in figure 21 for a system of lee waves computed by Sawyer (1960) in a model atmosphere in which l^2 varies continuously with height. This case illustrates the sort of complication encountered in the real atmosphere and has a continuous variation of apparent horizontal wavelength with height, corresponding to a continuous change of the dominant solution with height.

In section 4.3 observations have been interpreted as implying that a constant value of m, and hence of $g\beta\bar{U}^{-2}$, in a deep layer is favourable for the development of lee waves. This would suggest that a single-layer model might be appropriate. However, a maximum amplitude which typically occurs in the lower troposphere on occasions of pronounced lee waves is not a feature of the one-layer model but is a feature of the two-layer model which in other respects appears inconsistent with the observations. For these reasons the variation of amplitude with height in some real cases has been considered in terms of the equation 6.

4.4.1 The balance of terms for lee waves in the stratosphere

The sounding referred to as Case 2 has been used to calculate the values shown in Table 3. The values of m in figure 13 have been used to calculate $(g\beta U^{-2})$ in layers 2 km deep and values of the wind speed close to those measured have been used to estimate the term $(U'' U^{-1})$. The maximum vertical air velocities (W_{max}) in the waves have been used to calculate $(W'' W^{-1})$ in the lower troposphere and in the lower stratosphere assuming vertical trough lines. At intermediate heights the function has not been calculated because radiosonde evidence suggests that the wavelength varied with height.

The value of k^2 used corresponds to a wavelength of 11.5 km which is seen to be present below 5 km and above 11 km. This is based at low levels on the time between successive wave crests which is about 10 min and the wind speed which is about 19 m sec^{-1}; at high levels the radar measurements of the horizontal range of successive wave crests (or troughs) show that the wavelength is near 11.5 km. The ambiguity in the wavelength which occurs in radiosonde data because the instrument rises about 2 km while passing between successive crests has been shown in section 3.6.2 to be unimportant because of the small values of l^2 which occur. Between 5 and 11 km the horizontal range between successive wave features increases up to 40 km; it is not possible that the horizontal wavelength was 11.5 km because a very large downstream tilt would be needed for the radiosonde to intersect the crests at the observed intervals; this is impossible because of the negative values of l^2-k^2 in the upper troposphere. It appears that the wavelength in the troposphere may have varied in a manner similar to that present in figure 21.

The equation 6 requires that the terms in the last two columns of Table 3 should be equal and opposite. In this and subsequent tables this is only approximately true as might be anticipated with values obtained on real occasions. The term $(W'' W^{-1})$ is probably the most inaccurate because of uncertainties about the curvature of Wmax with height and about the vertical disposition of the waves.

The basic wind and temperature measurements from which (gBU^{-2}) and $(U''U^{-1})$ are inferred contain evidence of stable layers and shear zones several hundred meters deep. These produce considerable irregularities in the l^2 profile; however, these irregularities are not directly reflected in the change of amplitude or wavelength with height in computed systems of lee waves (Sawyer, 1960). In Table 3, it seems reasonable that the 2 km depth of the layers which is a small proportion of the wavelength (11.5 km), should include many of the important atmospheric features affecting the change of amplitude with height. The discrepancies in the last two columns of Table 3 perhaps reflect the extent to which this is not true.

The largest value of $(W''W^{-1})$ occurs at 2 km where there is a large convex curvature of the amplitude profile. It is balanced in equation 6 partly because of the convex shape of the wind profile in the boundary layer, and partly by the large value of (gBU^{-2}) (although m only increases from 9 to 11 km between 2 and 4 km, (gBU^{-2}) diminishes by a much larger proportion). At 4 km $(W''W^{-1})$ is balanced mainly by $(U''U^{-1})$ and the increase of amplitude near 12 km also seems to occur because of the concave wind profile.

The values in Table 4 are based on the sounding referred to as Case 1. The unique occurrence of quite pronounced lee waves through the jet stream on this occasion suggests that the situation was unusually complex. In fact, the break in the periodicity of the vertical air speed (figure 12) near 20 min at a height of about 6 km suggests that above this height another system of lee waves becomes dominant. The radar measurements of range also indicate this: up to 20 min the distance between successive crests is constant and about

10.5 km but between 20 and 30 min it increases to over 20 km. Between 30 and 45 minutes the distance between successive crests is remarkably constant and near 10.5 km. The small values of l^2 up to 16 km show that the wavelength was approximately equal to the distance between crests encountered by the radiosonde, and it seems likely that the upper system of the waves above 12 km is developed because the wavelength coincides with the value in the lower troposphere. This also appears to be true of Cases 2 and 4.

The values of k^2 correspond to wavelengths of 10.5 km and 25 km which give values of l^2-k^2 consistent up to 16 km with the term $(W'' W^{-1})$ in equation 6 estimated for two systems of lee waves each with vertical crests. The existence of the lee waves within the jet stream can be related to the large positive value of l^2-k^2 at 10 km which makes a steady system of lee waves confined to the jet stream possible. At 16 km l^2 is positive and large enough so that the lines of crests and troughs could slope considerably; the evidence of waves is less definite at this height but those indicated in figure 12 after 45 min are nearly consistent with a system with a horizontal wavelength about 10 km and which slope upstream with a gradient about 0.7.

4.4.2 The variation of amplitude in the troposphere

Table 5 contains values for the sounding launched from Shanwell at 1130 GMT on the 19th April 1967. The waves were unusually pronounced in the lower troposphere but none were detected in the stratosphere and the conditions which appeared to favour the waves have already been noted. The values of m in figure 19 in layers 2 km deep have been used to infer the values of (gBU^{-2}) except for the value at 2 km which is measured to include the stable layer. The terms $(U'' U^{-1})$

and $(W'' W^{-1})$ have been calculated from the values of U and Wmax shown and k^2 corresponds to a wavelength of 16 km estimated from the radar data.

Remarkably, at 2 km the wind profile is concave and even with the relatively large value of $(gB U^{-2})$ the curvature of the amplitude is small. The maximum amplitude occurs rather higher than usual, near 4 km; a similar situation occurs in figure 18 and it seems that the convex portion of the wind profile above the inversion, which produces a positive value in l^2-k^2, is a very important factor. The term $(W'' W^{-1})$ is of a large magnitude at 12 km because of the small value of W max.

Some conjectured values of the amplitude for the Sheffield Gale are shown in Table 6. The aircraft report of maximum vertical velocities of about 10 m sec^{-1} in the lee waves at 8000 ft. has been used to fix the amplitude at 2 km and the amplitude has been assumed to be zero at the ground. The other values of amplitude have been obtained by integrating equation 6 upwards. The winds measured at Aughton at 0600 GMT and the stability at 0600 GMT estimated from the 0000 GMT and the 1200 GMT Aughton soundings have been used to calculate $(U'' U^{-1})$ and $(gB U^{-2})$. The wavelength of 22 km computed by Aanensen (1965) has been used to determine k^2. The values of l^2-k^2 determine the amplitude up to 14 km.

The values of l^2-k^2 are small in the troposphere due to the generally small stability and the high winds. The relatively large value at 2 km is due partly to the inversion and partly to the curvature of the wind profile in the boundary layer but the value is small when compared with those typical of the lower troposphere. As a result the convex curvature of the amplitude profile is not large and the amplitude increases a little above 2 km. At other heights up to 12 km l^2-k^2 and hence the curvature of the amplitude profile are small so that the amplitude remains large. The observation of mother-of-pearl clouds from Manchester indicates that the amplitude remained large up to about 30 km. The sounding from Aughton at 1200 GMT shows evidence of lee waves with vertical velocities of about 6 m sec^{-1} at 16 km and nearly over Sheffield; even larger amplitude waves may have been present earlier in the day. Above 14 km the amplitude may have varied periodically with height because of the larger values of l^2 present.

The amplitudes in Table 6 suggest that very pronounced lee waves were present throughout the troposphere and other evidence shows that they extended well into the stratosphere. The conditions which favoured them included strong winds and small stability and have been observed before to be associated with very pronounced lee waves. This case, however, represents a more extreme example of these conditions so that the values of l^2-k^2 are very small and the amplitude remains large through the troposphere.

4.5 The length of lee waves

Making the further assumptions, besides those made in section 3.5 to infer the wave period, that the waves are stationary with respect to the ground and lie across the wind, multiplication of the wave period by the wind speed in the layer in which the period is measured yields the wavelength. Typical values shown in figure 21, are of the magnitude observed in lee wave clouds but correspond in detail to the wavelengths inferred from routine midnight soundings made during January 1964 at Shanwell. The wavelength is shown as a function of period and lines of constant proportion, the wind speeds through the waves are marked in m sec^{-1}.

The plotted points have a much greater spread than the values of the period in figure 11 and it appears that in general as well as in particular cases the wavelength inferred from radiosonde records varies with height. The range of values lies chiefly within the wind speeds 10-40 m sec^{-1} and evidently implies that the occurrence of lee waves is favoured by winds stronger than the mean January wind speed averaged over the troposphere (line a). On the other hand, in the stratosphere the points are more evenly distributed about the mean January wind speed between 12 and 20 km (line b). The wavelengths are shorter than tropospheric wavelengths for this reason as well as because of their shorter periods.

The assumptions which lead to these values of the wavelength are not unreasonable. Waves which are orographic in origin might be expected to be stationary. Lee wave clouds in satellite photographs do usually lie across the wind direction. The assumption of little change of period within each layer is also reasonable in so far as the periods of the wave-like disturbances observed vary only slowly with height. The assumption that the crests and troughs in the lee waves are aligned approximately vertically has been shown to be true if the inferred wave period is near the Brunt-Vaisala period and l^2 is sufficiently small; these conditions are often fulfilled in pronounced lee waves.

4.5.1 The length of lee waves in the stratosphere

The periodicity of the wave-like disturbances on the pressure trace even when the wind changes markedly with height suggests that it is usual for the dominant lee wavelength to change with height. The system of lee waves in figure 21 shows that computations that represent the usually complex structure of the atmosphere also predict that the horizontal wavelength changes with height. The dominant wavelength, however, tends to increase with height at all levels; whereas wavelengths inferred from radiosonde records, although often long in the upper troposphere, tend to be shorter in the stratosphere.

From considerations of the propagation of gravity waves in the atmosphere the presence of shorter wavelengths in the stratosphere is surprising. Eliassen and Palm (1960) have used a three-layer model of the atmosphere to show that low values of l^2 in the upper troposphere make it act as a reflecting barrier to upward propagating gravity waves. The degree of reflection depends on the depth of the reflecting layer and the difference between l^2 in the two lowest layers. In the cases of stratospheric waves discussed in Chapter 3 the lower troposphere, the upper troposphere and the lower stratosphere do roughly correspond to layers of constant l^2. The reflection coefficient of the upper troposphere for the wavelengths found in the lower troposphere (which are evidently close to those in the stratosphere) is about .98 in Cases 1, 2 and 3 and about .70 in Case 4 so the observed amplitudes are on this basis impossible. Moreover, only wavelengths longer than the maximum value of m in the upper troposphere can propagate freely into the stratosphere.

Nevertheless, further evidence indicates that short waves often occur in the lower stratosphere. In months when the wind speeds are considerably less than those typical during January over Scotland lee waves with lengths of only a few km might be expected. Such waves might often be present but not detectable in radiosonde data.

Occasionally paroxysmal volcanic eruptions inject into the lower stratosphere dust clouds which persist for some months and spread over most of a hemisphere, producing abnormal twilight phenomena. Recent examples of such eruptions were those of Mt. Spurr, in Alaska (9th July, 1953); Bezymianny, in Kamchatka (30th March, 1956), and Mt. Agung in Bali (17th March, 1963).

The dust itself may be visible as a haze layer with some streaky detail, although not usually in the day-time except for a while after extreme eruptions such as that of Krakatau (1883). However, even the more tenuous layers are often visible about sunrise and sunset, when the illumination is favourable, especially if the haze layer has a wavy form. Then at low elevations about the sun's azimuth the haze usually appears in faint waves approximately parallel to the horizon; two examples are shown in figures 23 and 24.

Near the end of July 1953 and during the following August such haze was frequently observed over England from the ground and from aircraft, which on a special flight found it in a thin layer at a height of 15 km; it was subsequently reported by pilots that the layers sometimes were slightly 'waved' or 'rolled' (Jacobs, 1954). The undulations were occasionally clearly visible from the ground (Ludlam, unpublished), and were observed to change in form and to move only very slowly, if at all. It therefore seems likely that the undulations were produced by lee waves. The wave-length varied somewhat with position in the sky, and from occasion to occasion; assuming a height of 15 km it varied between 1.4 and 4.2 km in 17 measurements on 5 separate occasions, with an average value of $2\frac{1}{2}$ km. Wave lengths of about or somewhat less than 4 km were typical also of similar haze clouds observed in southern England in May, 1956 (following the eruption in Kamchatka), and December, 1963 (following the eruption in Bali).

The undulations were ordinarily orientated approximately across the line of sight towards the setting sun, consistent with the W'ly direction of the winds at about 15 km. However, on 10th May and 27th May 1956 the undulations were across lines of sight towards NNW and N respectively, while the winds in the lower stratosphere were from the NNW and S respectively. Because of the uncertainty of the actual heights of the undulations it is hardly possible to compare the estimated wave-lengths and those implied by the Brunt-Vaisala period, but in the summers of 1953 and 1956 the wind speeds were about 10 m sec^{-1}, implying a wave-length of about 3 km, close to that usually observed. On 29th December, 1963 a rather stronger wind of about 20 m sec^{-1} implies a wave-length of $6\frac{1}{2}$ km, when the observed value was about 5 km.

These observations evidently indicate that waves which are virtually stationary, and therefore are probably lee waves, may rather frequently occur over large areas of southern England during both summer and winter. Their wave-length is a few km, and the observation that they are most clearly defined at an elevation of between about 7 and 10° ($\approx \tan^{-1} 0.15$) probably indicates that their usual amplitude is about $(0.15/2\pi)$ of the wave-length, or about 100 m.

4.5.2 The estimation of the wavelength

The magnitude of the horizontal wavelength is clearly related to the wind speed: the longer stratospheric wavelengths occur when the wind is strong and Corby (1957) has shown that there is a high correlation between the wavelength and the tropospheric mean wind speed. The reason appears to be that the wave periods tend to cluster round mean values which correspond to typical values of the Brunt-Vaisala frequency. In particular cases also the wave period is related to the stability of the atmosphere, so that in Tables 3 - 6 k^2 has similar values to (gBU^{-2}). In Tables 3 and 4 k^2 in the lower troposphere and lower stratosphere is approximately equal to (gBU^{-2}) in those regions of the atmosphere. In Tables 5 and 6 k^2 is intermediate between the greatest and least values of (gBU^{-2}) in the whole troposphere. In fact, for the case in Table 5, taking U as the mean wind speed between 0 and 10 km and B measured with the temperatures at these two heights, (gBU^{-2}) equals 0.16, just the value of k^2.

Some values of m (= $2\pi U (gB)^{-\frac{1}{2}}$) evaluated with U and B measured in the same way are shown in Table 7. They are compared, with values of the preferred horizontal wavelength computed by Wallington and Portnall (1958) using a multi-layer representation of the atmosphere, and wavelengths inferred from radiosonde data by Corby (1957). The preferred wavelength corresponded to one of the several solutions which had zero amplitude at the ground; an additional condition, that the solution should be relatively independent of conditions in the stratosphere, usually favoured one wavelength above others. In some cases, however, no such solution existed or the solution had a negligibly small amplitude; no wavelength is shown for these cases. The agreement between the observed wavelength and the two sets of estimated wavelengths is tested quantitatively and the results are shown in Table 7.

Wallington and Portnall calculated a preferred horizontal wavelength in 13 of the 19 cases giving good agreement with Corby's measured values. There is a similar though slightly less good agreement between Corby's values and m in the 13 cases; however, m gives an unambiguous value in the other 6 as well. The averaging of m over 10 km is fairly arbitrary and it is possible that in the cases when m is not a good estimate of the wavelength a smaller depth is more appropriate. For example, in the sounding on the 21st December, 1953 which accounts for over half of the value of χ_2^2 the lee waves were pronounced only below about 5 km and averaging U and B between 0 and 5 km gives a value of 10.0 km agreeing exactly with Corby's observed value.

It can be inferred from these results that the horizontal wavelength can be estimated simply from the Brunt-Vaisala period and the average wind speed in the troposphere. Cohen and Doron (1967) have compared similar estimates with wavelengths measured on satellite photographs. They found that the method using simply m was superior to theoretical values. To the extent that m provides a single estimate for the wavelength in every case in Table 7, it also appears to be superior than Wallington and Portnall's method. Moreover, the approximate correspondence between m and the observed wavelength in the stratosphere, where present theories indicate that the dominant wavelength should be much longer, suggests that this relation is more than an empirical coincidence. It appears, in fact, that only frequencies near the Brunt-Vaisala frequency attain a significant amplitude in the atmosphere.

4.6 Lee wave systems in the atmosphere

Regular radiosonde soundings indicate the conditions favourable for pronounced lee waves. The wind and temperature are disturbed by the air motions but are not much affected by the waves normally encountered. The waves found on the soundings from Shanwell are usually over the North Sea, many miles from the highest mountains in Scotland and are therefore true lee waves. The amplitude of the waves may be measured as a streamline displacement, accurate to about 25 m while the balloon is in the troposphere, or as a vertical velocity, accurate to about 1 m sec^{-1} in most soundings. Since the most pronounced lee waves have amplitudes about an order of magnitude greater than these values, nearly all waves will be detected. The wavelength may also be measured when several assumptions are made; these do not appear to be unreasonable.

The discussion of these aspects of the results in the preceding sections suggests an important general feature about lee wave systems. Thus, the horizontal wavelength is approximately equal to m averaged over a suitable depth. Moreover, a small variation of this parameter with height is desirable for large amplitude waves and it follows that lee waves are pronounced if the wavelength tends to be constant with height. It has also appeared that the crests align vertically in most pronounced lee waves and the oscillations in the waves therefore involve deep columns of the atmosphere. This is consistent with the observation of Larsson (1954) that deep piles of wave clouds are usually vertical.

In real cases detected by radiosondes the situation is often complex. In Case 2 the wavelength in the upper troposphere varies with height. However, the first pronounced crest after 30 min is exactly 6 wavelengths downwind of the first crest in the troposphere and the lee waves involve vertical columns except that the oscillations near the tropopause are small. A similar situation appears in Case 1 but with an additional system of pronounced lee waves present in the upper troposphere. In the other two cases considered in Section 4.4 m varies substantially with height in that layer in which the waves are pronounced. However, it has appeared in Section 4.3 that with the exclusion of the shallow inversion layer m is nearly constant with height in the lower troposphere. It is possible that this is a necessary initial condition for the development of the lee waves but because the amplitude of the oscillations is large the columns are affected by a greater depth of atmosphere. The wavelength in Table 5 corresponds to the value of m in columns 10 km deep whereas the larger amplitude oscillations in Table 6 have a wavelength depending on a layer about 15 km deep.

Airstreams normally approach Shanwell having passed over high hills causing displacements of a few hundred m, but the observed displacements near Shanwell are much smaller. This is supported by the observation by Meteorological Office personnel at Shanwell that wave clouds are frequently visible over the hills but are not visible overhead nor are lee waves pronounced on the radiosonde record. This situation is unfavourable for lee waves perhaps because the horizontal wavelength tends to change with height. The oscillations of columns in the waves cannot then be regular and mechanisms which dissipate the waves may operate in such systems.

Figure 25 shows isopleths of potential temperature deduced from aircraft flights in the Rockies W of Denver during the afternoon of the 15th February 1968 (about 0000 GMT 16th February). The usual assumptions of lee wave theory that the flow is steady, isentropic, laminar and 2 dimensional imply that the flow is along the isentropes. In fact, on the side of the ridge where the isentropes intersect the ground this is impossible and in the stratosphere where static instability occurs this is unlikely. In the lee waves the displacements are smaller and the assumptions are probably more nearly fulfilled. The wavelength in the low troposphere is about 16.5 km; in the upper troposphere it appears to be near 30 km. In the stratosphere the displacements are irregular and may be unsteady but the scale of the disturbances is small (about 3 km), and is typical of lee wavelengths inferred in this thesis.

The disturbance over the mountain is more pronounced than the lee waves. A radiosonde launched from a position comparable with the location of Shanwell in relation to the Scottish Highlands (several wavelengths downstream of the ridge) might intersect a few lee waves at low levels. These would not be classed as pronounced when their amplitude was compared with height of the mountain range. At higher levels it is doubtful whether any waves would be detected. In these respects this system of lee waves appears similar to those typically detected on soundings from Shanwell and can usefully be compared with Figure 21 which shows a typical system of lee waves predicted by a linear theory.

An important difference between the two systems of lee waves is the large irregular disturbance in the stratosphere over the range in the real case. A linear theory in which the amplitudes are small would not, however, be expected to predict this feature. Perhaps a more significant difference is the absence of lee waves with strongly sloping crests and troughs in the observed system, apparently because the amplitude in layers where the wavelength changes with height is small. A similar feature is suggested by radiosonde data and it seems that mechanisms dissipate waves with sloping phases more rapidly than any present in Figure 21. These must operate through small departures of the real flow from assumed conditions, possibly involving non-laminar flow. The experiments of Davis (1969) also suggest that mechanisms outside the scope of present theories dissipate lee waves and allow only simple systems to exist.

The short wavelengths in the stratosphere in Figure 25 are accompanied by clear air turbulence and both may be associated with a dissipation of the kinetic energy of the mean flow. This seems likely on thermodynamic grounds because departures of the air trajectories from the isentropes in the stratosphere would almost certainly lead to an irreversible transfer of kinetic energy to heat; the waves and turbulence possibly represent intermediate states of the energy. The energy for the most pronounced lee waves in the stratosphere detected by soundings from Shanwell may have originated in a similar manner. It seems unlikely that it can have propagated up from the lower troposphere partly because of the high reflection coefficient of the upper troposphere, and partly, because the strong westerly winds up to very high levels which favour lee waves in the stratosphere are the conditions least likely to lead to an accumulation of wave energy at a 'critical level'.

Chapter 5 - Conclusions

The Meteorological Office radiosonde (Mark 2 B), although principally an instrument to measure atmospheric parameters for synoptic-scale analysis, makes measurements of pressure, temperature, and relative humidity frequently enough to enable small-scale details in the sounding to be studied. The pressure measurements which are displayed as a function of time on a paper chart, decrease regularly as the balloon rises and it might be anticipated that rapid changes of the vertical velocity by 1 m sec^{-1} or more are readily detectable if they are present in a layer 100 m or more deep. A radar is used to track the balloon and determine the winds, and the measurements obtained may also be used to determine the height of the radiosonde after each minute interval. The rate of change of radar height, may be compared with the vertical velocity calculated from the rate of decrease of pressure, and frequently provides independent confirmation of departures of the vertical velocity of the radiosonde from a mean value. Such departures may be due to either a variation of the rate of ascent of the radiosonde through the surrounding atmosphere or to a change of the vertical speed of the air surrounding the radiosonde balloon.

It appears that the rate of switching between signals corresponding to measurements of pressure, temperature and relative humidity, governed by a windmill whose cups revolve in a vertical plane on the outside of the radiosonde, is chiefly determined by the vertical passage through the air and may be used to estimate the rate of ascent of the radiosonde. The rate of ascent does change in nearly all soundings due partly to variations of the drag coefficient of the balloon and partly to the decreasing density of the air. The resulting variations in the rate of ascent are small except in rare cases when large decreases are present near the tropopause from exceptionally large rates of ascent in the upper troposphere. The decrease may be ascribed to the critical change in the flow conditions around the surface of the radiosonde balloon which is expected at Reynold's numbers between 10^5 and 10^6. In these cases the increase of drag coefficient at the critical change approaches the magnitude of that experienced by solid spheres in a windtunnel and the close correspondence with the tropopause suggests that the flow over the surface of the balloon is influenced by the stability or possibly small-scale turbulence in the atmosphere. The behaviour of the drag coefficient of solid spheres near the critical change is known to depend mainly on the turbulence in the airstream and it seems likely that small-scale turbulence also produced the unusual behaviour of the balloons.

Other changes of the vertical velocity of the radiosonde appear to be mainly caused by vertical air motions; most take the form of smooth periodic disturbances on the pressure trace and are probably due to lee waves. Occasional irregular disturbances occur which affect the rising radiosonde only for short times and contain vertical air speeds of a few m sec^{-1}. It is not possible from the radiosonde data to infer the form of the disturbances; however, in one case independent evidence suggests that the radiosonde intersected a billow wave present quite extensively within a shallow layer. Similar disturbances are infrequent, occurring much more rarely than turbulent layers detected by gust-sondes, but may be more important for aircraft.

Lee waves inferred from the records from Shanwell (E. Scotland) for 1964 occupy layers some kilometers deep and on occasions extend into the lower stratosphere. It is then usual for the wave amplitude to become small in the upper troposphere and to increase again in the lower stratosphere. The streamline displacements in the waves exceed 100 m on about 10% of the records at 2 km and on about 2% of the records at 16 km. At 10 km only about 1% of the records have streamline displacements exceeding 100 m. The frequency of lee waves is greatest during the winter months; during the summer the frequency in the lower stratosphere decreases more markedly than in the troposphere. Records from Weather Ships I and J and Muharraq show that these locations have a much smaller frequency of waves (perhaps gravity waves) than Shanwell does.

The time taken for the radiosonde to pass from one region of upward (or downward) air motion to the next, which is the wave period if the crests and troughs lie in vertical lines, is found to correspond to typical values of the Brunt-Vaisala period. Observations of dust layers in the stratosphere suggest that lee waves with lengths of only a few kilometers do occur, consistently with wave periods about the Brunt-Vaisala period. This was also true for the most pronounced lee waves detected in the stratosphere; however, because of the strong winds which favour the waves, the wavelength was about 10 km. The vertical air speed experienced by a supersonic aircraft travelling in the stratosphere directly across these waves would change at a rate of about

$\frac{1}{2}$ m sec^{-2} (or g/20), with a period about 20 sec. In waves with lengths of 3 km and vertical velocities of $\frac{1}{2}$ m sec^{-1} similar accelerations could be produced but with a period of 5 sec. Such conditions could be frequently present because small amplitude waves are not detected by radiosondes in the stratosphere.

Pronounced lee waves in the lower troposphere are thought to occur in conditions under which the wave energy is 'trapped', an idea developed by Scorer (1949) from a two-layer model. The extension of the model to more typical atmosphere conditions indicates that a sufficiently large decrease of l^2 with height in the troposphere should be favourable for pronounced lee waves. Sawyer's (1960) computations which illustrate the sort of complication of l^2 encountered in the atmosphere, confirm that pronounced lee waves might be expected in conditions of decreasing l^2 in the troposphere. These conditions are, however, often present in the lower troposphere and are more frequent than pronounced lee waves.

The conditions which do, in fact, favour lee waves in the lower troposphere have a relatively small decrease of l^2 with height; this is usually associated with a strong wind which does not change much with height. The most pronounced lee waves tend to have long wavelengths and this factor together with the small magnitudes of l^2 which occur implies that the values of $l^2 - k^2$ are small. This also conflicts with the idea of 'trapped' waves for which a large negative value of $l^2 - k^2$ in the upper troposphere would more effectively confine the disturbance to lower levels. In one case of exceptionally pronounced lee waves the dominant wavelength appears hardly to be 'trapped' in the troposphere at all.

The existence of lee waves in the stratosphere is also inconsistent with notions of 'trapping' because it is unusual for l^2 to decrease with height in this region of the atmosphere. Moreover, the most pronounced stratospheric lee waves encountered in the Shanwell data have wavelengths for which the energy should have been 'trapped' in the lower troposphere. The waves are unlike systems of lee waves predicted by present theories in the respects that they often have shorter wavelengths than are typical in the troposphere, and have phases which are vertical.

The variations of amplitude with height which occur in lee waves can in practical cases be described by the equation:

$$\frac{1}{w}\frac{\partial^2 w}{\partial z^2} + (l^2 - k^2) = 0$$

The small values of $l^2 - k^2$ which occur in pronounced waves only suffice for the change of amplitude and show that systems of lee waves which have sloping phases are not favoured. The curvature of the wind profile is an important term in l^2 for the variation of amplitude with height, and the maximum amplitude which is most frequently present near 2 km may be associated with the change of wind in the boundary layer.

The length of lee waves inferred from radiosonde data corresponds to typical values of the Brunt-Vaisala period multiplied by the average wind speed in a layer. The depth of the layer which seems most significant for pronounced lee waves is typically about 10 km. The conditions in which pronounced lee waves occur favour a wavelength which tends to be constant with height and suggest that a regular disposition of crests in the atmosphere is necessary for the development of lee waves. Pronounced waves appear to normally have vertical crests and involve the oscillation of deep columns of the atmosphere. The dissipation of less regular systems of lee waves probably explains the small frequency of pronounced lee waves and may involve departures from the conditions normally assumed to be present in lee wave systems.

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Tables

TABLE 1 : The percentage of Shanwell records with pronounced waves. The winter frequency is based on the records for December of previous year, January, and February. The subsequent seasons are taken in periods of 3 months. Where less than 3 months records were available the percentage refers to the records used.

		Winter	Spring	Summer	Autumn
1964	Trop	14%	4%	11%	11%
	Strat	9%	3%	3%	6%
1965	Trop	17%	-	-	-
	Strat	16%	9%	2%	2%
1966	Trop	4%	7%	9%	10%
	Strat	4%	4%	2%	7%
1967	Trop	13%	16%	-	-
	Strat	9%	3%	-	-

TABLE 2 : The wind speed at 4 levels in the troposphere on occasions of lee waves. The time and date of 19 soundings at Leuchars from which Corby (1957) inferred pronounced lee waves and the wind speed and direction at 850,700,500 and 300 mb are shown.

Date	Time GMT	U(850)kts	U(700)kts	U(500)kts	U(300)kts
4-11-53	1400	281/35	285/45	273/50	272/56
18-11-53	1400	262/39	262/43	268/55	265/62
20-11-53	0400	268/34	251/15	270/31	280/52
30-11-53	0200	264/60	247/88	244/105	236/90
21-12-53	1400	305/38	306/54	309/49	304/94
3-1-54	0300	345/43	353/62	354/77	007/77
3-1-54	1400	014/47	019/63	007/77	358/76
4-1-54	0200	048/27	027/51	019/39	017/44
11-1-54	0200	336/34	321/28	319/37	337/23
11-1-54	1400	314/24	309/25	314/37	226/17
12-1-54	1600	240/38	227/42	232/38	249/36
14-1-54	0200	297/37	297/32	315/30	312/80
23-1-54	1400	201/21	198/24	218/20	210/24
24-1-54	1400	204/37	202/40	214/40	248/29
21-3-54	0200	262/41	265/44	261/57	272/58
11-4-54	1400	256/27	276/31	284/28	272/29
14-4-54	1500	299/54	308/54	314/60	312/84
15-4-54	0200	322/62	324/63	322/61	317/72
15-4-54	1400	349/25	358/56	350/62	344/76
Mean wind speed		39	45	50	57
Six month mean wind speeds		26	27	36	51

TABLE 3 : The terms of the equation $w''w^{-1}+(l^2-k^2) = 0$ in layers 2 km deep are shown for the sounding launched from Shanwell at 2330 GMT on the 30th January, 1964.

H	U	$U''U^{-1}$	gBU^{-2}	k^2	W max	$W''W^{-1}$	$l^2 - k^2$
0	5				0		
2	18	−.14	.49	.30	1.5	−.37	.33
4	22	.16	.34	.30	0.8	.12	−.12
6	40	.08	.08	−	0.6		
8	70	−.11	.02	−	0.6		
10	70	−.12	.04	−	0.7		
12	36	.20	.34	.30	1.0	.18	−.16
14	30	.02	.38	.30	2.0	−.12	.06
16	26				2.0		

TABLE 4 : As Table 3 but for the sounding at 2330 GMT on the 4th September, 1966

H	U	$U''U^{-1}$	gBU^{-2}	k^2		W max		$W''W^{-1}$		$l^2 - k^2$	
0	5					0					
2	23	−.22	.42	.36		2.5		−.38		.28	
4	20	.15	.42	.36	.06	1.2	0.0	.10		−.09	
6	29	.03	.15	.36	.06	0.4	0.5	.25	−.05	−.24	.06
8	41	.11	.06	.36	.06	0.0	0.9		.06		−.11
10	71	−.26	.04	.36	.06	0.0	1.5		−.27		.24
12	30	.30	.15	.36	.06	0.4	0.5	−.45	.25	−.51	−.21
14	24	−.04	.47	.36	.06	1.5	0.0	−.18		.15	
16	15	.07	1.2	.36		1.5				.27	
18	9										

TABLE 5 : As Table 3 but for the sounding at 1130 GMT on the 19th April, 1967.

H	U	$U''U^{-1}$	gBU^{-2}	k^2	W max	$W''W^{-1}$	1^2-k^2
0	16				0		
2	20	0.11	0.34	0.16	2	-.06	.07
4	33	-0.11	0.20	0.16	3.5	-.19	.15
6	31	0.00	0.10	0.16	2.4	0.02	-.06
8	30	0.02	0.06	0.16	1.5	0.07	-.12
10	32	0.02	0.11	0.16	1.0	0.07	-.07
12	36	-0.12	0.24	0.16	0.75	-0.16	+.20
14	24				0.0		

TABLE 6 : The terms of the equation estimated for about 0600 GMT on the 16th February, 1962 at Aughton, W is calculated by integrating $1^2 - k^2$ upwards.

H	U	$U''U^{-1}$	gBU^{-2}	k^2	$1^2-k^2=W''W^{-1}$	W m sec^{-1}
0	17					0
2	37	-.12	.20	.08	.24	10
4	40	0.00	.13	.08	.05	10.4
6	44	.01	.06	.08	-.03	8.8
8	50	-.04	.03	.08	-.01	8.2
10	49	-.03	.01	.08	-.04	8.0
12	43	.03	.04	.08	-.07	9.0
14	42	-.05	.16	.08	.13	12.4
16	32	.05	.32	.08	.20	
18	29					

TABLE 7 : A comparison of preferred wavelengths calculated by Wallington and Portnall (1958), those observed, and values of $2\pi U (gB)^{-\frac{1}{2}}$ (where B and U are simple averages over the lowest 10 km of the atmosphere).

Time and Date		Observed (O) Wavelength km	Wallington & Portnall (E_1)	$\frac{(O-E_1)^2}{E_1}$	$2\pi U(gB)^{-\frac{1}{2}}$ (E_2)	$\frac{(O-E_2)^2}{E_2}$
1400	4-11-53	13.3	14.5	0.10	12.0	0.14
1400	18-11-53	13.3	15.0	0.19	14.5	0.10
0400	20-11-53	8.9	8.9	0.00	9.0	0.00
0200	30-11-53	26.7	-	-	23.0	-
1400	21-12-53	10.0	13.0	0.70	16.0	2.24
0300	3-1-54	18.5	17.0	0.13	17.0	0.13
1400	3-1-54	18.5	18.0	0.02	17.0	0.13
0200	4-1-54	10.0	10.0	0.00	10.0	0.00
0200	11-1-54	12.0	-	-	9.0	-
1400	11-1-54	5.9	7.6	0.38	7.0	0.17
1600	12-1-54	8.9	11.0	0.40	12.5	1.04
0200	14-1-54	10.4	-	-	11.5	-
1400	23-1-54	5.6	5.7	0.00	6.0	0.03
1400	24-1-54	4.8	-	-	8.5	-
0200	21-3-54	8.7	-	-	14.0	-
1400	11-4-54	8.7	7.8	0.11	8.0	0.06
1500	14-4-54	24.1	-	-	20.5	-
0200	15-4-54	18.7	23.5	0.98	17.0	0.17
1400	15-4-54	14.4	11.5	0.73	14.5	0.00
$\chi^2 = \sum \frac{(O-E)^2}{E}$				3.84		4.21

Both E_1 and E_2 are significant estimates of O at the 5% level ($\chi^2 = 5.99$).

Figure Legends

Figure 1 The rate of switching, Rs, as a function of time, t, is compared with the vertical velocity inferred from the pressure measurements, W_p. The increase of potential temperature per kilometer, $\partial\Theta/\partial z$, and the winds measured over intervals of one minute are shown, the latter in the form of two components: one radially away from the radiosonde station, V_R, and one azimuthal component V_ϕ. The intervals when the Richardson number R_I was less than 1 are shown by a single line and when $R_I < \frac{1}{4}$ a double line is marked. The sounding was launched from Shanwell at 2330 GMT, 19th January 1964.

Figure 2 This sounding was launched at Muharraq at 1130 GMT, 27th October, 1967. The rate of switching, R_s, and the rate of change of radar height, W_r, are compared as functions of time after launch, t. The rate of increase of potential temperature, $\partial\Theta/\partial z$, the one-minute winds as radial, V_R, and azimuthal, V_ϕ, components, and the regions where the Richardson number R_I was less than 1 are also shown.

Figure 3 Some values of the drag coefficient, C_d, as a function of the Reynolds number, Re. The two curves represent the change of with Re for solid spheres in a wind tunnel. These two were the extremes of several experimental curves given by Hoerner (1958). The points (a) represent mean values of C_d for a number of ascents using 1.25 kgm balloons with free lifts of 2.1 kgm. The data was taken from the Handbook of Meteorological Instruments. The points (b) represent values during the sounding launched at 2330 GMT, 19th January, 1964. The values of C_d and Re were calculated at 10 minute intervals and also before and after the critical change. The points (c) correspond to similar values for the sounding at 1130 GMT, 27th October, 1967 from Muharraq.

Figure 4 A comparison of values of the rate of switching, R_s, with the rate of ascent, W_a, in a wind-tunnel and in the atmosphere. The points (a) represent values obtained in a wind-tunnel with a radiosonde with all elements in position and six cups on the windmill. The points (b) correspond to a similar situation but with all the elements removed. The points (c) are values for the second radiosonde with three cups on the windmill and all elements in position. The points (d) were derived from the sounding launched at Shanwell at 2330 GMT, 19th January 1964, the points (e) are from the Muharraq sounding on the 27th October 1967, and the points (f) from the Shanwell sounding on the 5th September 1966.

Figure 5 An irregular air motion encountered by the sounding launched from Crawley at 1115 GMT on the 23rd August, 1964. The vertical velocity from the radar measurements W_r, the rate of ascent W_a, and the height H above the ground are plotted against the time after launch, t. The wind direction and speed U, are plotted at the standard levels, the heights (H) and (M) which Ludlam (1967) inferred for billow clouds and the Richardson number are also shown. At the bottom right of the diagram the pressure vertical velocity W_p, and the parameter $\partial\theta/\partial z$ are plotted in detail on an expanded scale; the components V_r, V_ϕ of the wind and the Richardson number are shown at 1 minute intervals.

Figure 6 An irregular air motion encountered by the sounding launched from Muharraq at 2330 GMT on the 7th November 1967. The vertical velocity from the pressure measurements W_p, the rate of ascent W_a, and the height H above the ground are plotted against the time after launch t. The wind direction and speed U and the temperature T are plotted at the standard levels. The vertical velocity W_p, the parameter $\partial\theta/\partial z$, and the wind components V_R, V_ϕ are plotted against t on an expanded scale.

Figure 7 A section of a radiosonde record received at Shanwell from the sounding launched at 1130 GMT, 15th June 1964. The individual markings by the pen on the chart represent successive values of: pressure, temperature, and relative humidity. The pressure records begin near the middle of the record and change scale after 7 minutes. The marks have been connected by pencil lines on the record in accordance with radiosounding practice.

Figure 8 The 'Q' correction for values of the vertical velocity inferred from the slope of the pressure trace, W_p'. The radiosonde was launched from Shanwell at 2330 GMT on the 4th September 1966. The correction, C_p, the temperature, T, and the corrected vertical velocity, W_p, are also shown. All quantities are averages over 1 minute intervals and are plotted against the time after launch, t.

Figure 9 Features of the vertical motion of the radiosonde launched from Shanwell at 2330 GMT 4th September, 1966, plotted against t, the time after launch.

Top: a comparison of the corrected vertical velocity W_p inferred from the pressure trace and that inferred from the radar, W_r.

Second from top: a comparison of the difference between W_p and W_r (circles), the expected maximum errors in the derivation of the vertical velocity from the pressure trace, e_p (pecked line), and from the radar observations, e_r (continuous line).

Third from top: the rate of ascent of the radiosonde through the air, W_a, derived from the rate of switching.

Bottom: the vertical air speed W encountered by the radiosonde, deduced from the difference between W_p and W_a.

Figure 10 The fractional frequency F of waves with maximum streamline displacements exceeding A, (a) at low levels (about 2 km) (b) in the lower stratosphere (at about 16 km); the points plotted are based on an analysis of 500 soundings at Shanwell in 1964.

Figure 11 The fractional frequency F of waves of a given period (expressed to the nearest whole minute), (a) at low levels (about 2 km), (b) in the lower stratosphere (at about 16 km), based on an analysis of 500 soundings at Shanwell in 1964.

Figure 12 Wave motions and the atmospheric structure inferred from The Shanwell sounding at 2330 GMT, 4th September, 1966. The vertical air speed, W, is plotted at the top of the diagram against t, the time after launch. The height of the radiosonde measured by the radar, H, is also shown against time after launch; the profiles of wind direction and speed U, and the temperature T, appear on the left of the diagram. The profile of the parameter $2\pi U(gB)^{-\frac{1}{2}}$ ($B = (\partial\theta/\partial z)/\theta$ where θ is the potential temperature), is shown on the right of the diagram.

Figure 13 Wave motions and the atmospheric structure inferred from the Shanwell sounding at 2330 GMT, 30th January, 1964. The vertical air speed, W, and the height H, are plotted against t, the time after launch. The other parameters: wind direction and speed U, temperature T, and $2\pi U(gB)^{-\frac{1}{2}}$ are plotted against height and a suitable horizontal scale.

Figure 14 Wave motions and the atmospheric structure inferred from the Shanwell sounding at 2330 GMT, 3rd December, 1964. The vertical air speed, W, and the height of the radiosonde H, are plotted against t, the time after launch. The vertical structure of the wind direction and speed U, the temperature T, and the parameter $2\pi U(gB)^{-\frac{1}{2}}$ are also shown.

Figure 15 Wave motions and the atmospheric structure inferred from the Shanwell sounding at 2330 GMT, 30th December, 1964. The vertical air speed, W, and the height of the radiosonde H, are plotted against t, the time after launch. The vertical structure of the wind direction and speed U, the temperature T, and the parameter $2\pi U(gB)^{-\frac{1}{2}}$ are also shown.

Figure 16 A comparison of the vertical distribution of (gBU^{-2}) on three occasions during 1964. The parameter is measured as an average over layers 2 km deep, and the wind directions in whole degrees are shown at regular intervals. The three sets of values (a), (b) and (c) correspond respectively to the soundings from Shanwell at 1130 GMT on the 15th June 1964, 2330 GMT on the 31st January 1964 and 2330 GMT on the 16th November 1964.

Figure 17 The vertical distribution of the parameter $2\pi U(gB)^{-\frac{1}{2}}$ at intervals during December 1964. The 1130 GMT soundings commencing on the 2nd December, are represented in the upper portion of the diagram and the 2330 GMT soundings, commencing on the 4th December, are represented in the lower portion. The parameter is measured as an average over layers 2 km deep, and the wind directions in whole degrees are shown approximately at the standard levels. Regions where the wave amplitude A exceeded 50 m are shown by a thicker line, drawn continuously where the amplitude exceeded 100 m, but otherwise pecked.

Figure 18 The maximum vertical air speed W_{max} encountered on the sounding from Lenchars at 1400 GMT, 15th April, 1954 (Corby, 1957). The profiles of wind speed U, temperature T and the parameter $2\pi U(gB)^{-\frac{1}{2}}$ (line a) are also shown. The profile of $2\pi U(gB)^{-\frac{1}{2}}$ corresponding to mean April data for the years 1951 - 1955 is shown by the line (b).

Figure 19 Wave motions and the atmospheric structure inferred from the Shanwell sounding at 1130 GMT, 19th April 1967. The vertical air speed W at the top of the diagram and the height H of the radiosonde are plotted as functions of the time after launch, t. The profiles of the wind speed, U, the temperature T, and the parameter $2\pi U(gB)^{-\frac{1}{2}}$ are also shown.

Figure 20 The atmospheric structure on the occasion of the Sheffield Gale. The wind direction and speed U, the temperature T, and the parameter $2\pi U(gB)^{-\frac{1}{2}}$ are shown for 2330 GMT on the 15th February 1962, and for 1130 GMT on the 16th February 1962. The wind at 0530 on the 16th February is also represented.

Figure 21 Displacement of the stream lines computed for a typical airstream (after Sawyer 1960). The left-hand section of the diagram shows the assumed potential temperature, θ, wind speed U, and the parameter l^2. The vertical displacement of the stream lines is plotted for each level on the same scale as the mountain profile at the bottom of the figure, this scale is 4 times greater than the scale for the vertical separation of the curves. Above 16 km constant values of l^2, $= 6\ km^{-2}$ (continuous lines), $= 0\ km^{-2}$ (pecked lines), are assumed.

Figure 22 The wavelength L of the waves inferred from all night soundings at Shanwell in January, 1964. The abscissa is the wave period and the lines of constant ratio are labelled with the mean wind speeds (m sec^{-1}) in the layers in which the wavelengths are inferred. The pecked lines represent the mean January wind speed based on data from the years 1951 - 1955 (a) in the troposphere (b) in the layer 12-20 km. Wavelengths in the troposphere are shown by crosses and wavelengths in the stratosphere are shown by circles.

Figure 23 and 24 Haze undulations photographed from Dunstable, looking west. Figure 23 was taken at 1920 GMT and Figure 24 at 1942 GMT on the 17th August 1953. A contrail which is white in Figure 23 is dark and further right in Figure 24 and is evidently below the undulations. Similar billows were observed intermittently until 28th September, 1953. On this occasion, assuming a height of 15 km, wavelengths were measured as 1.8, 1.8, 4.0, 4.2, 3.8, 3.4, 4.0 km.

Figure 25 Isopleths of potential temperature deduced from aircraft flights (dotted lines) W of Denver on the 15th February 1968 (after Kuettner & Lilly, 1968). Balloon flights are shown by dashed lines and turbulent flight paths by jagged lines.

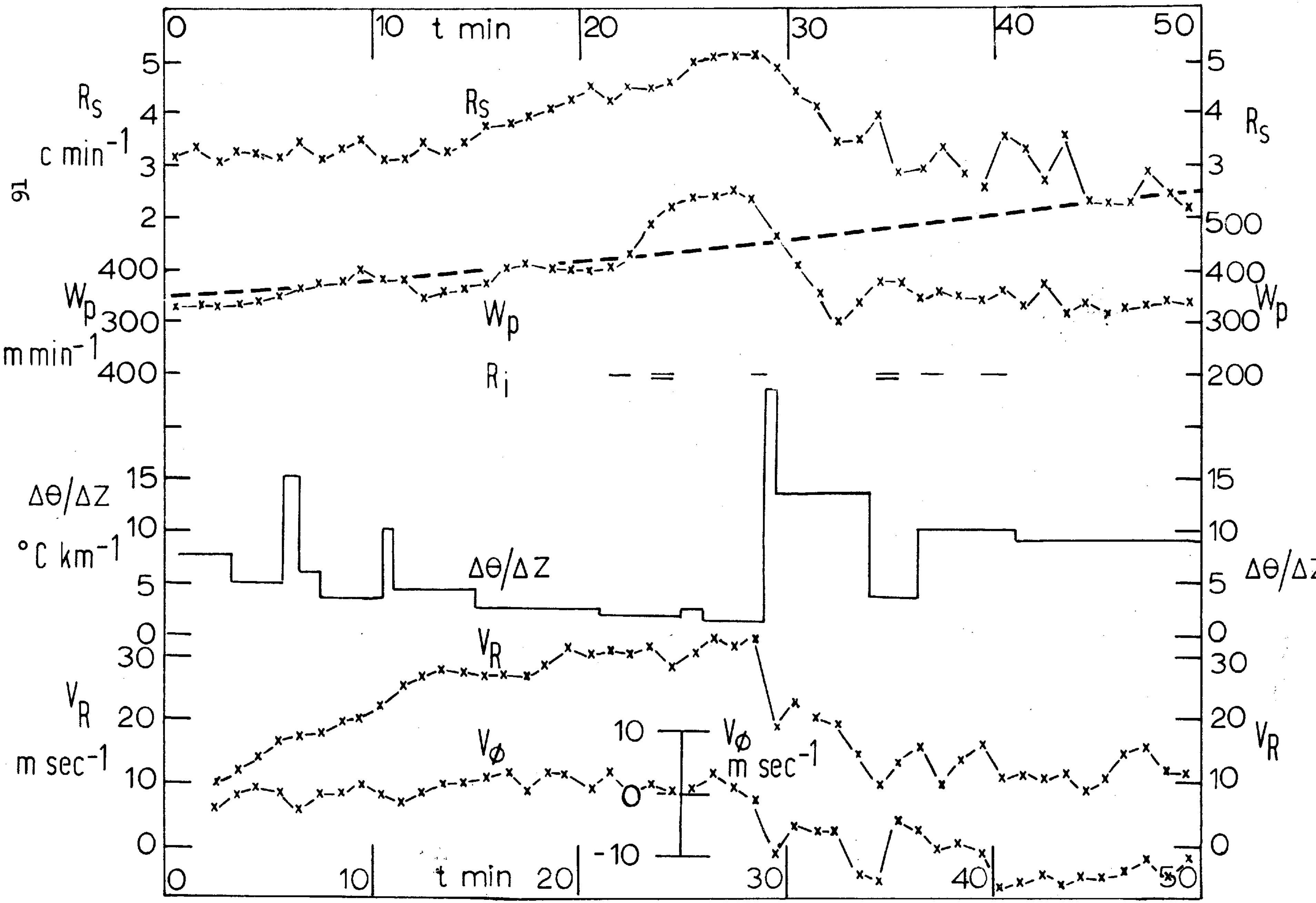

t min
R_S
c min^{-1}
W_p
m min^{-1}
R_i
$\Delta\theta/\Delta Z$
°C km^{-1}
V_R
m sec^{-1}
V_ϕ
m sec^{-1}
0
10
20
30
40
50
500
400
300
200
15
5
-10

Fig. 1

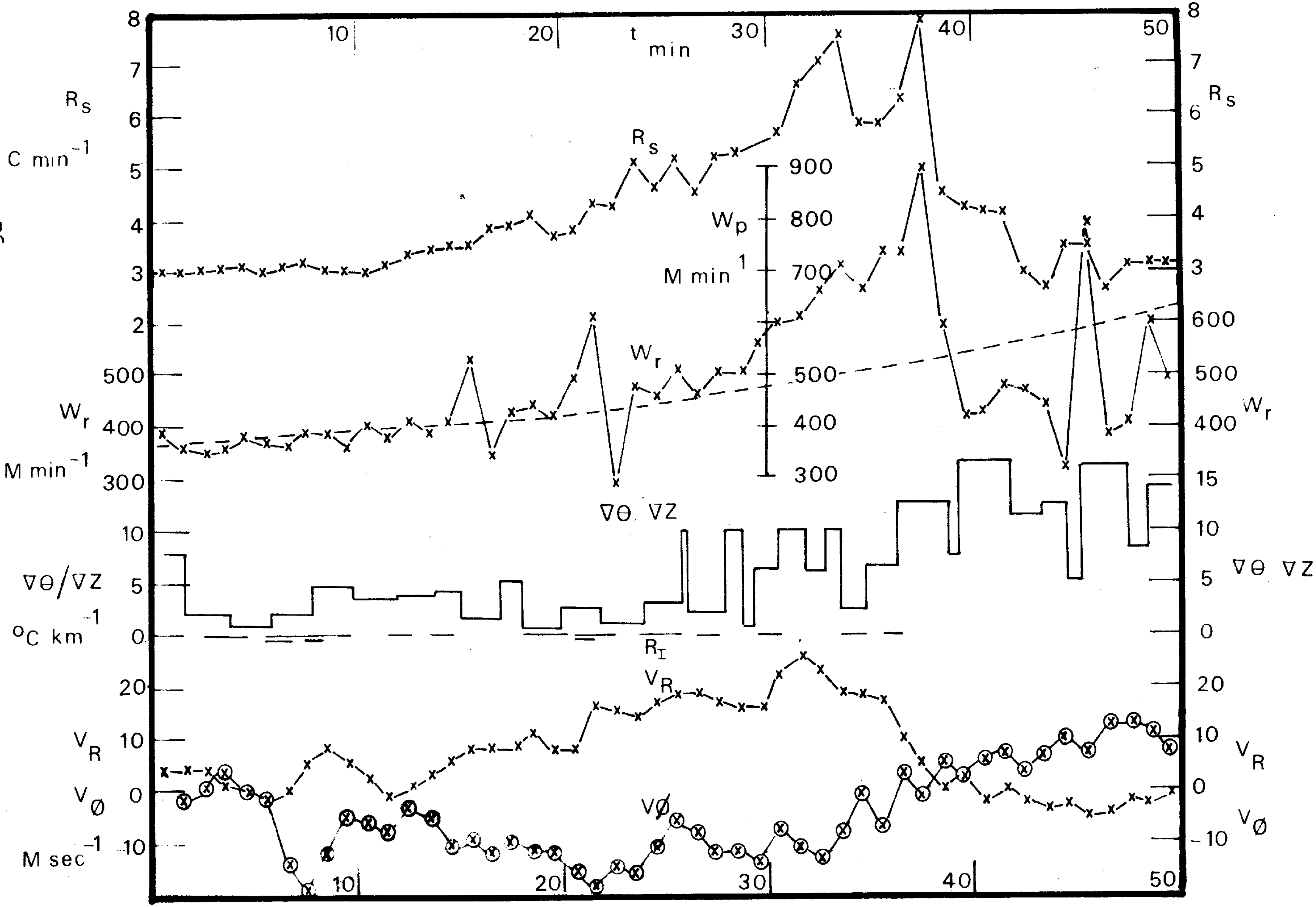


Fig. 2

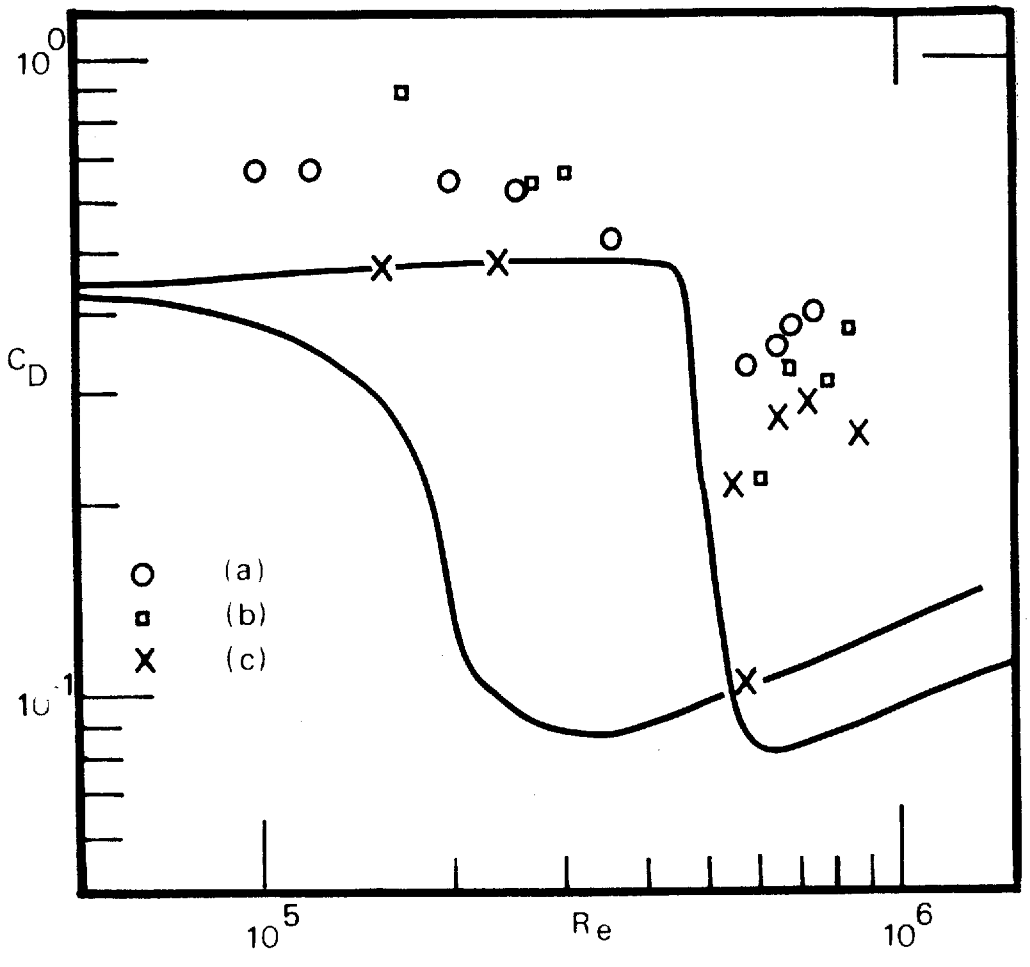


Fig. 3

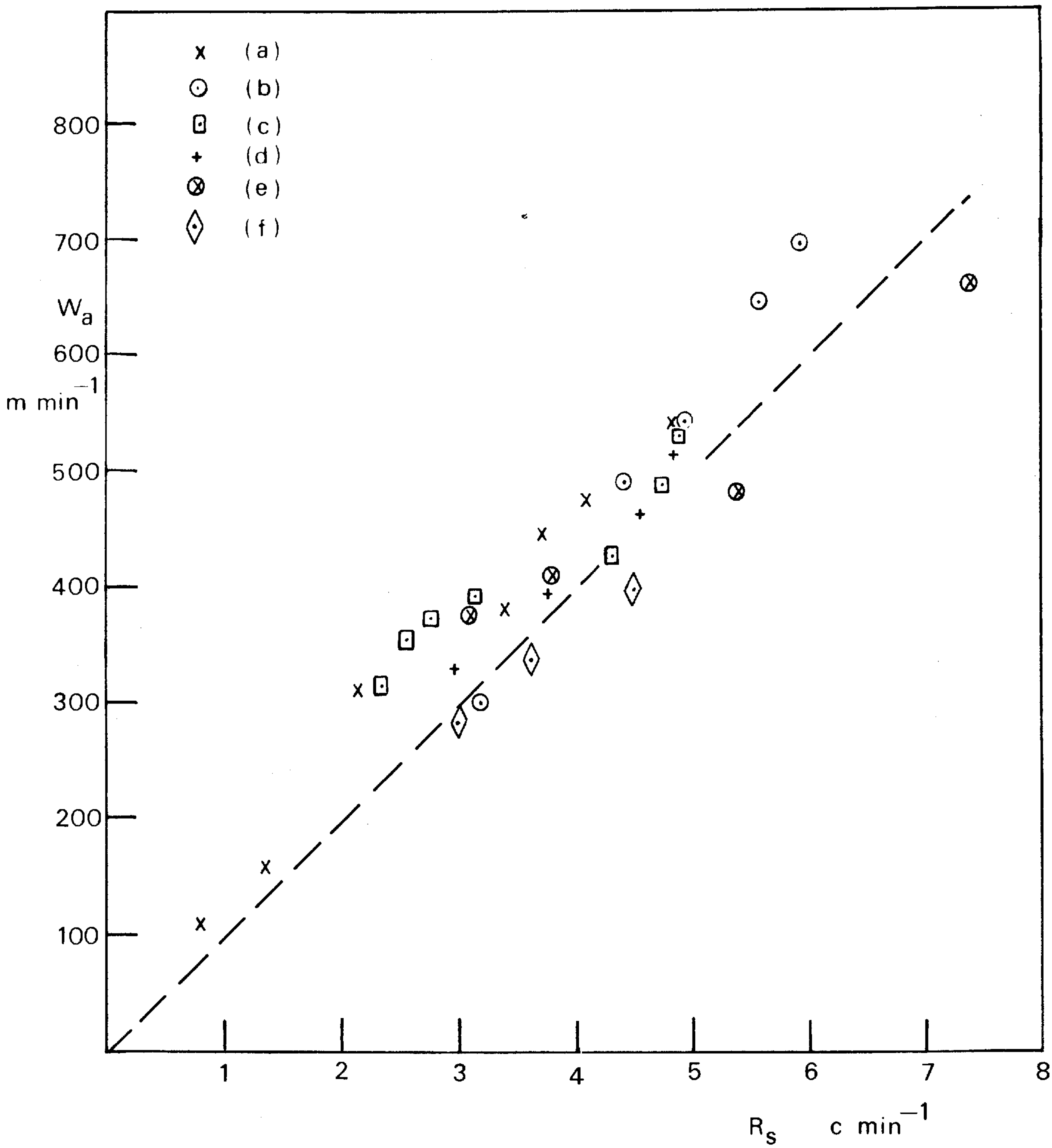
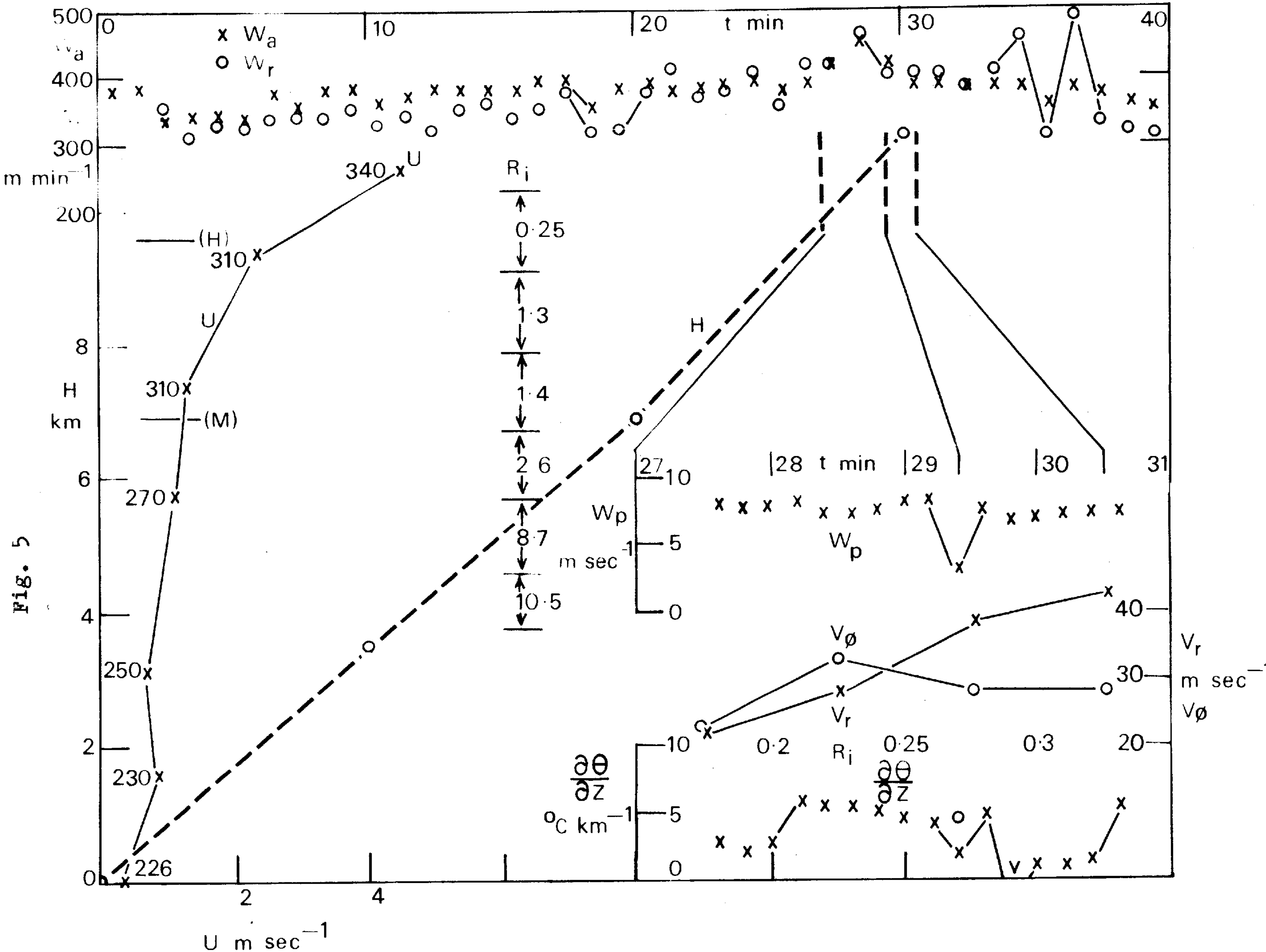

x (a)
⊙ (b)
⊡ (c)
+ (d)
⊗ (e)
◊ (f)
800
700
W_a
600
m min^{-1}
500
400
300
200
100
1
2
3
4
5
6
7
8
R_s c min^{-1}

Fig. 4



t min
x Wa
o Wr
m min⁻¹
Ri
0·25
1·3
1·4
2 6
8·7
10·5
(H)
(M)
U
H
H km
U m sec⁻¹
28 t min 29 30 31
Wp
m sec⁻¹
Vø
Vr
Vr m sec⁻¹ Vø
0·2 Ri 0·25 0·3
∂θ/∂z
°C km⁻¹
226
230
250
270
310
310
340

Fig. 5

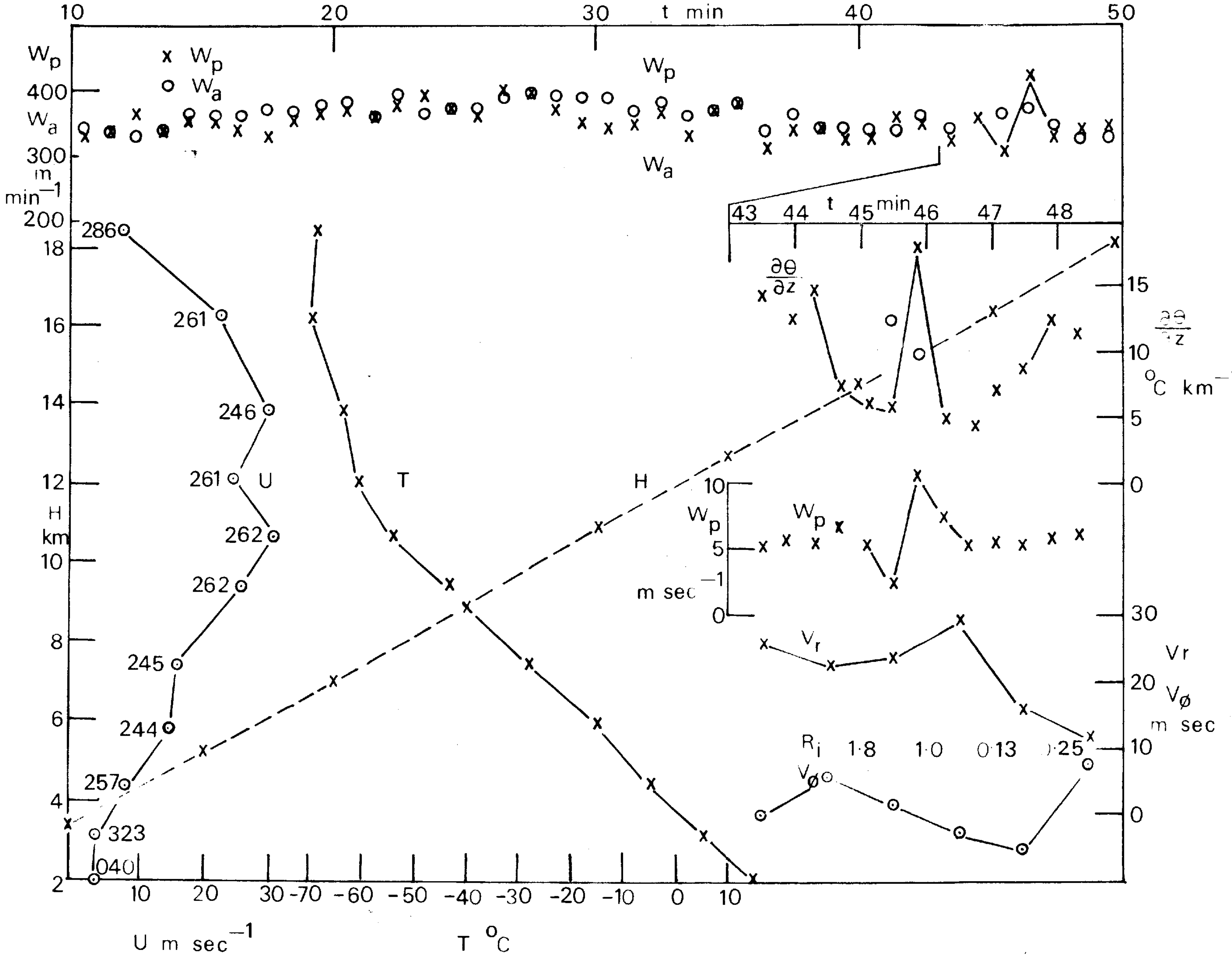

t min
W_p
W_a
m min^{-1}
H km
U
T
H
$\frac{\partial\theta}{\partial z}$
^oC km^{-1}
W_p
m sec^{-1}
V_r
Vr
$V_ø$
m sec^{-1}
R_i 1·8 1·0 0·13 0·25
U m sec^{-1}
T ^oC

Fig. 6

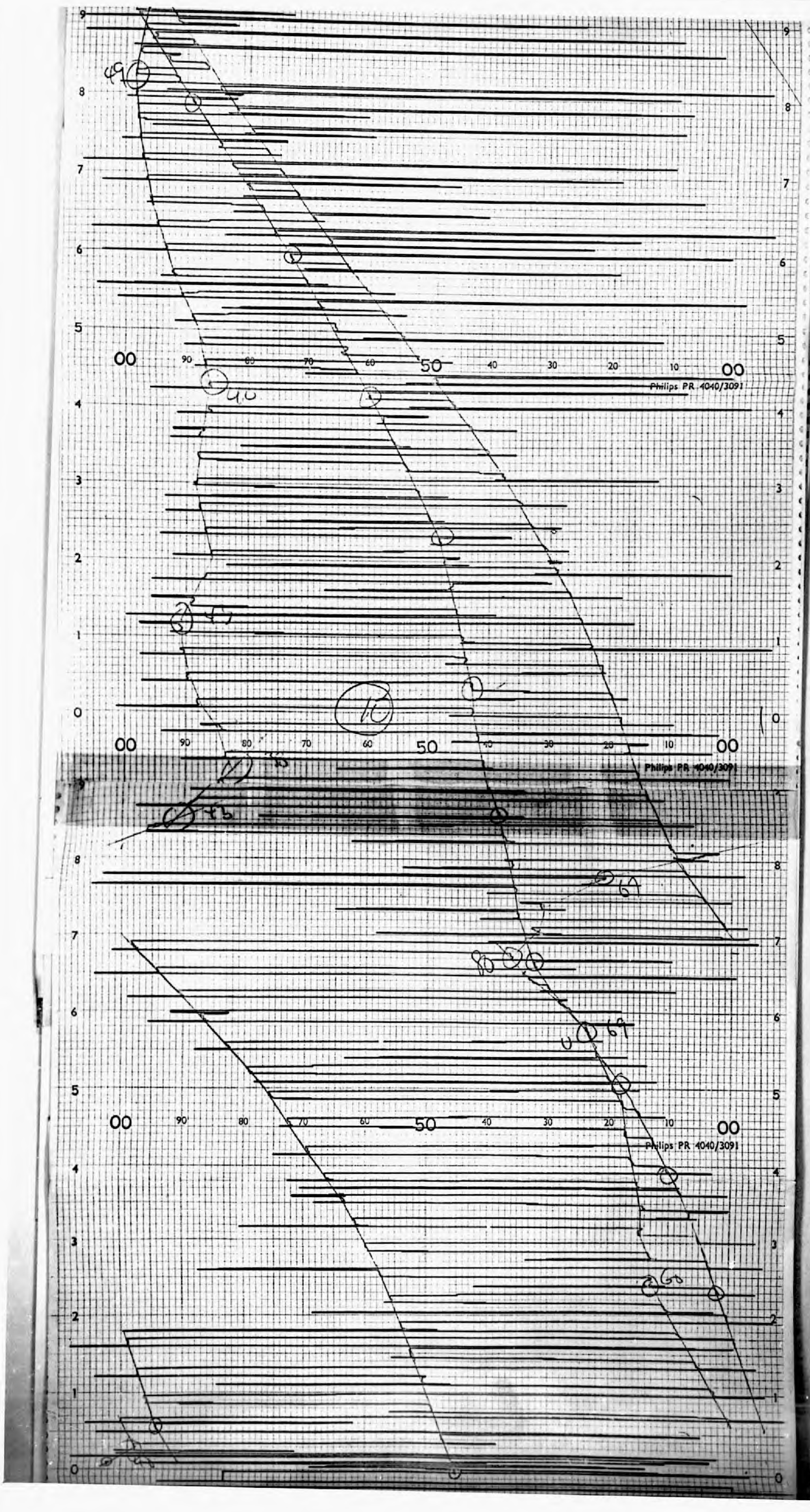

Philips PR 4040/3091
Philips PR 4040/3091
Philips PR 4040/3091

Fig. 7

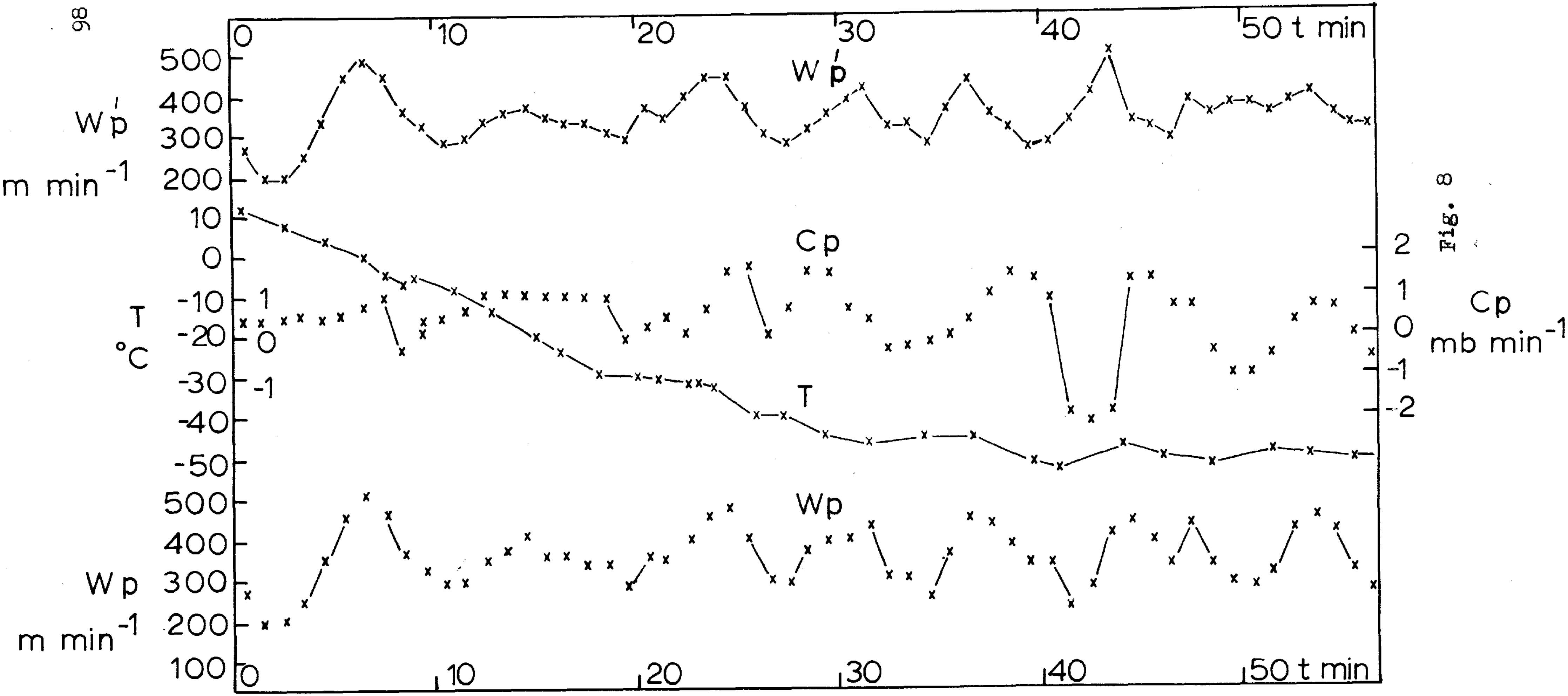

0
10
20
30
40
50 t min
W p'
500
400
300
200
m min^-1
10
0
-10
-20
-30
-40
-50
T
°C
1
0
-1
Cp
2
1
0
-1
-2
Cp
mb min^-1
T
Wp
500
400
300
200
100
W p
m min^-1
0
10
20
30
40
50 t min

Fig. 8

99

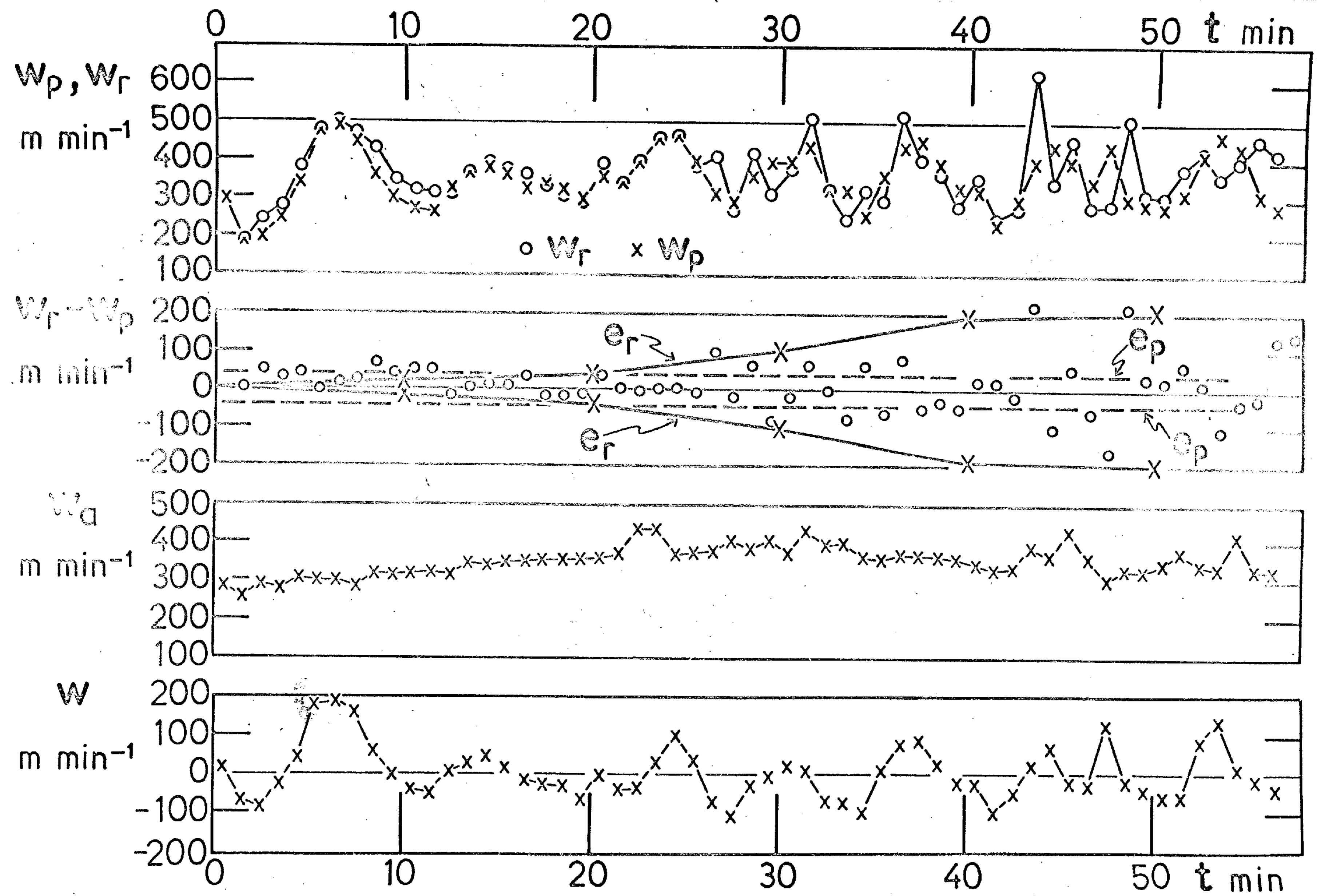


Fig. 9

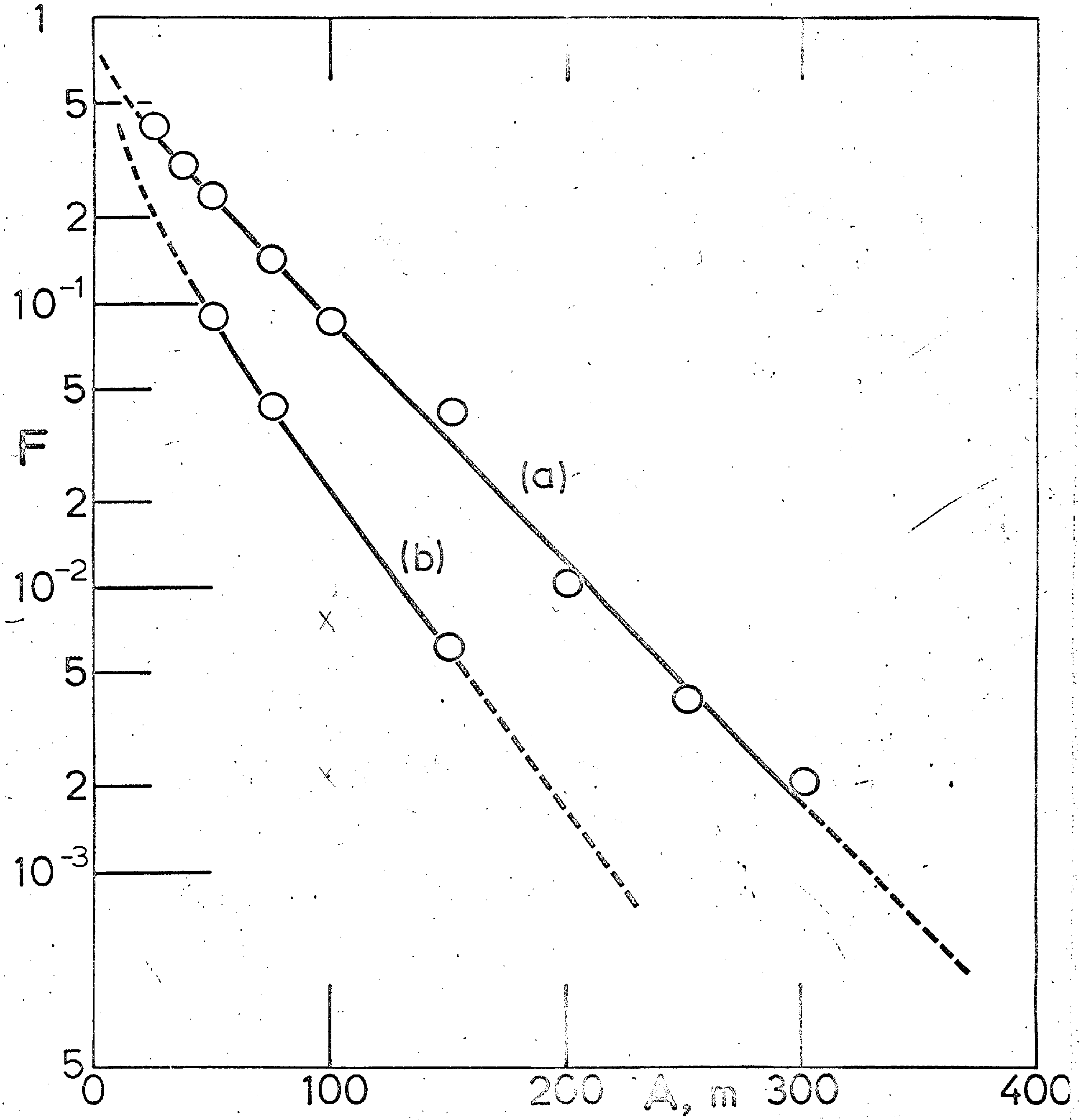


Fig 10

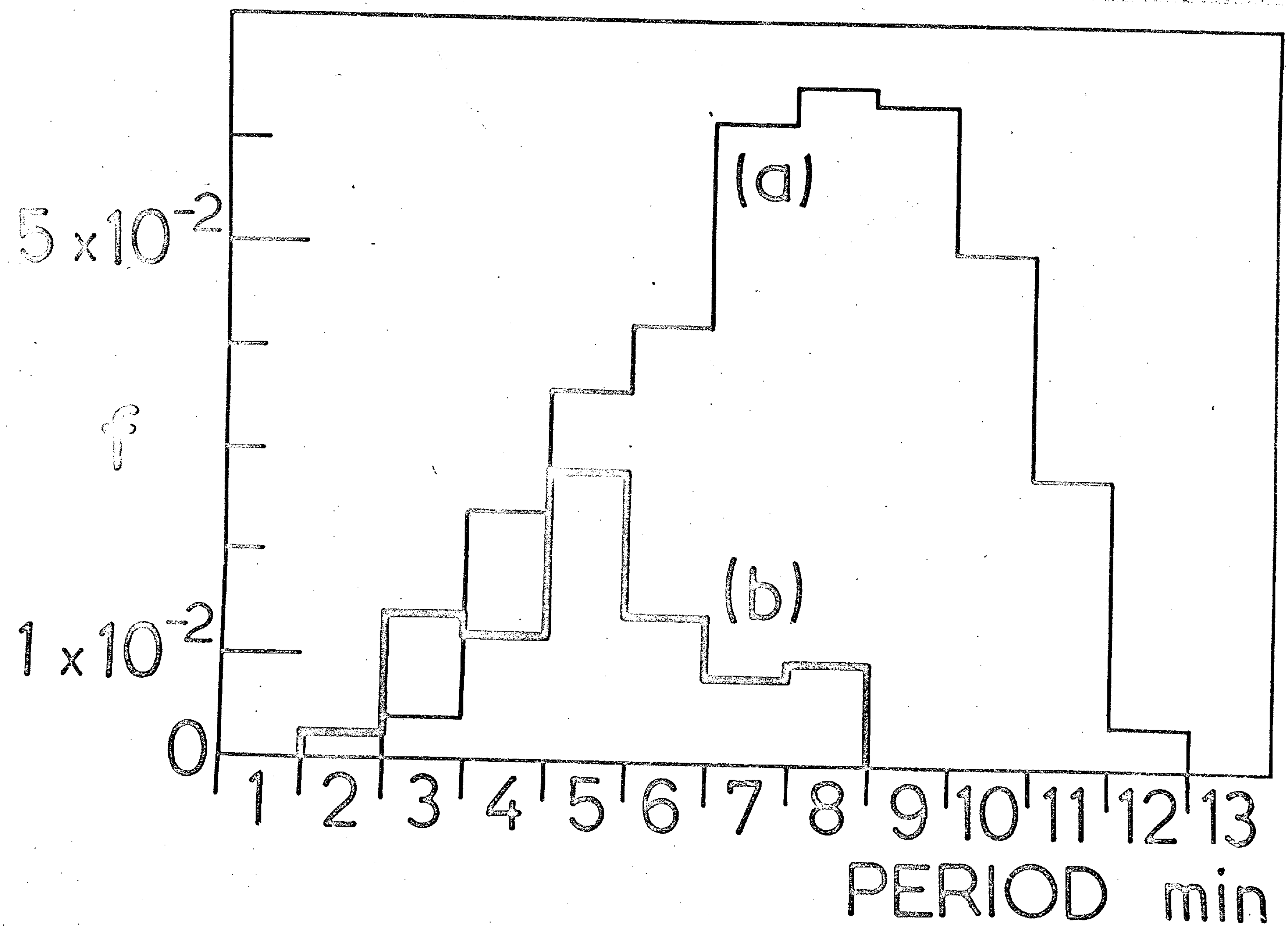
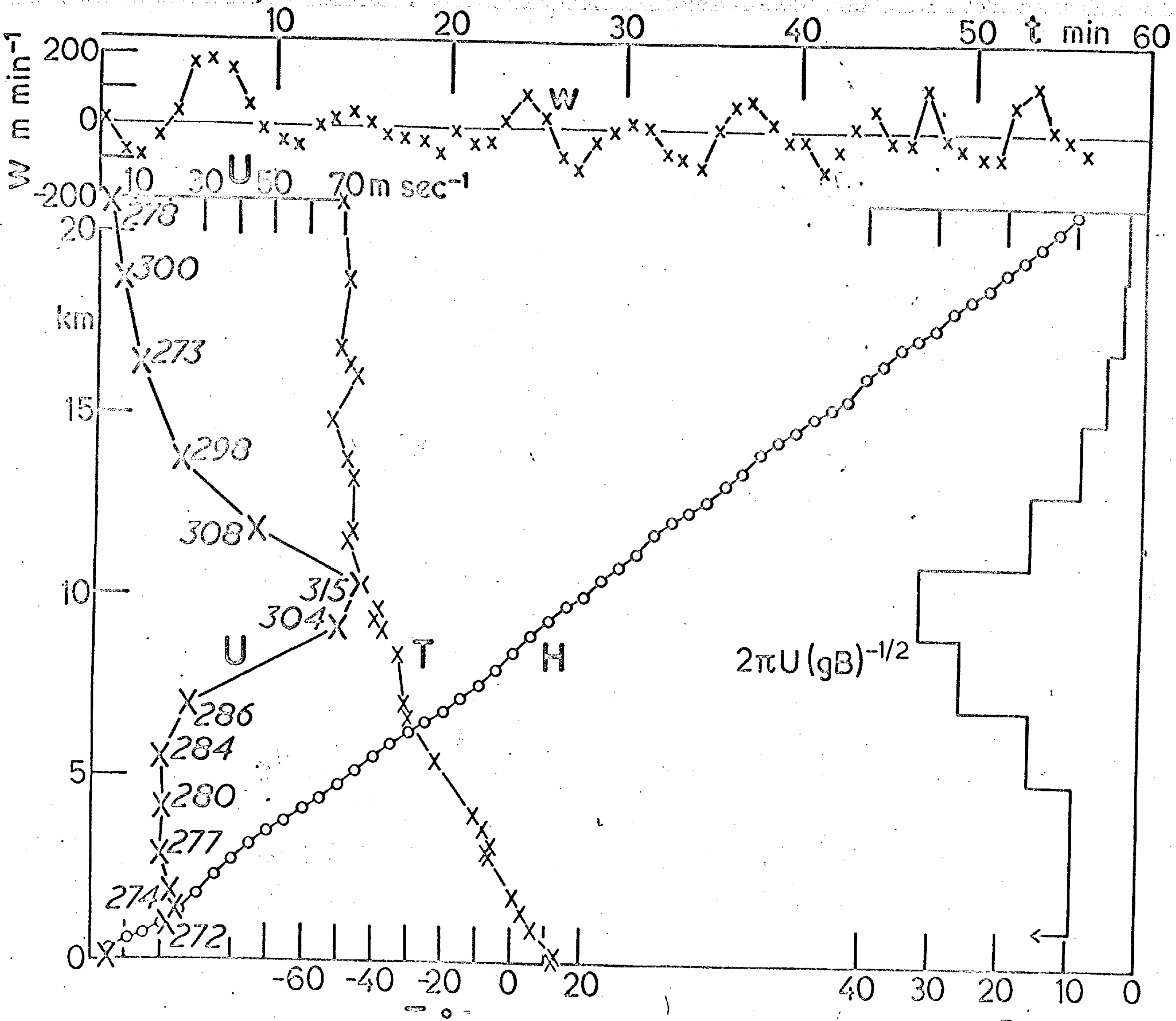
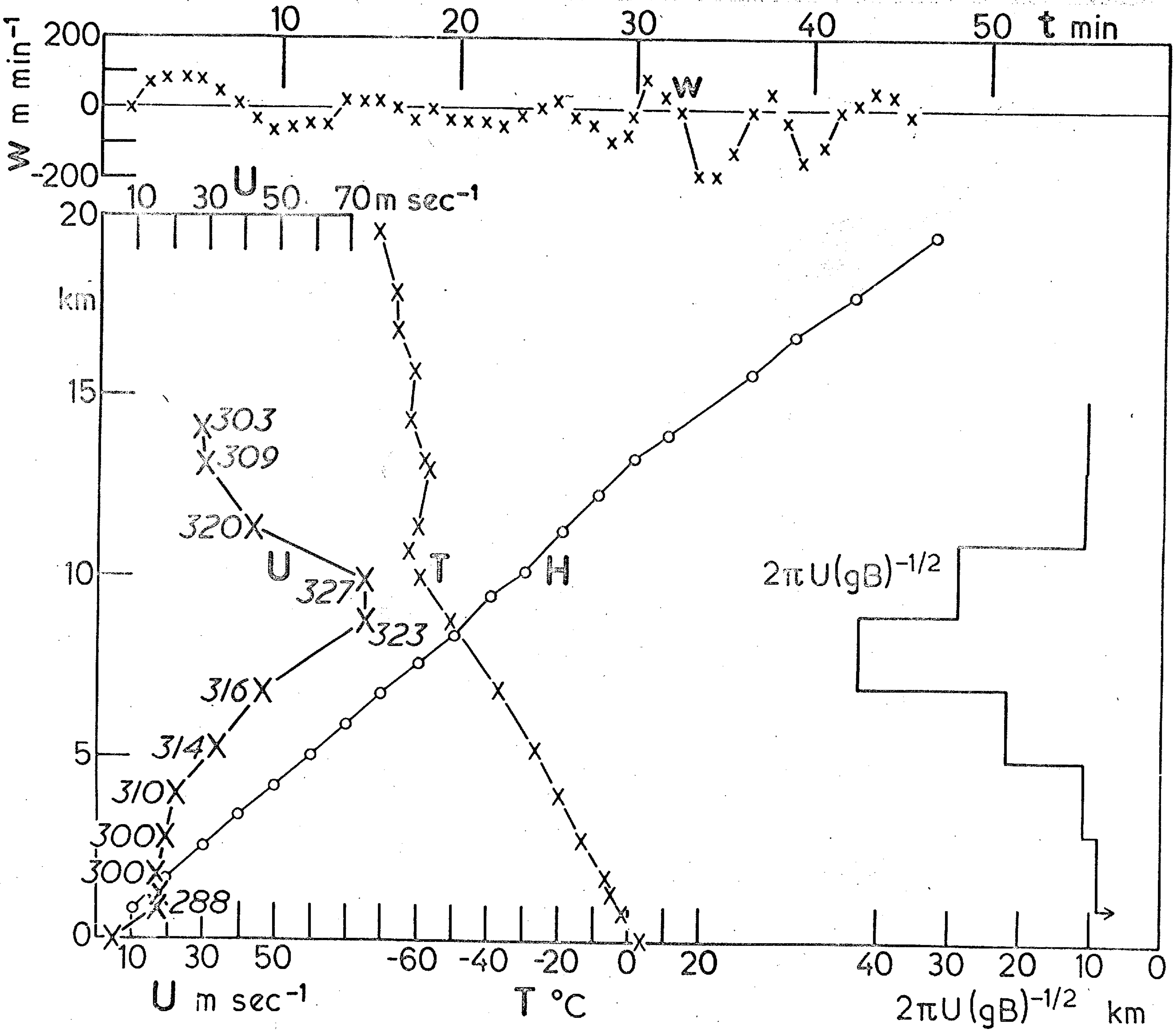


Fig. 11



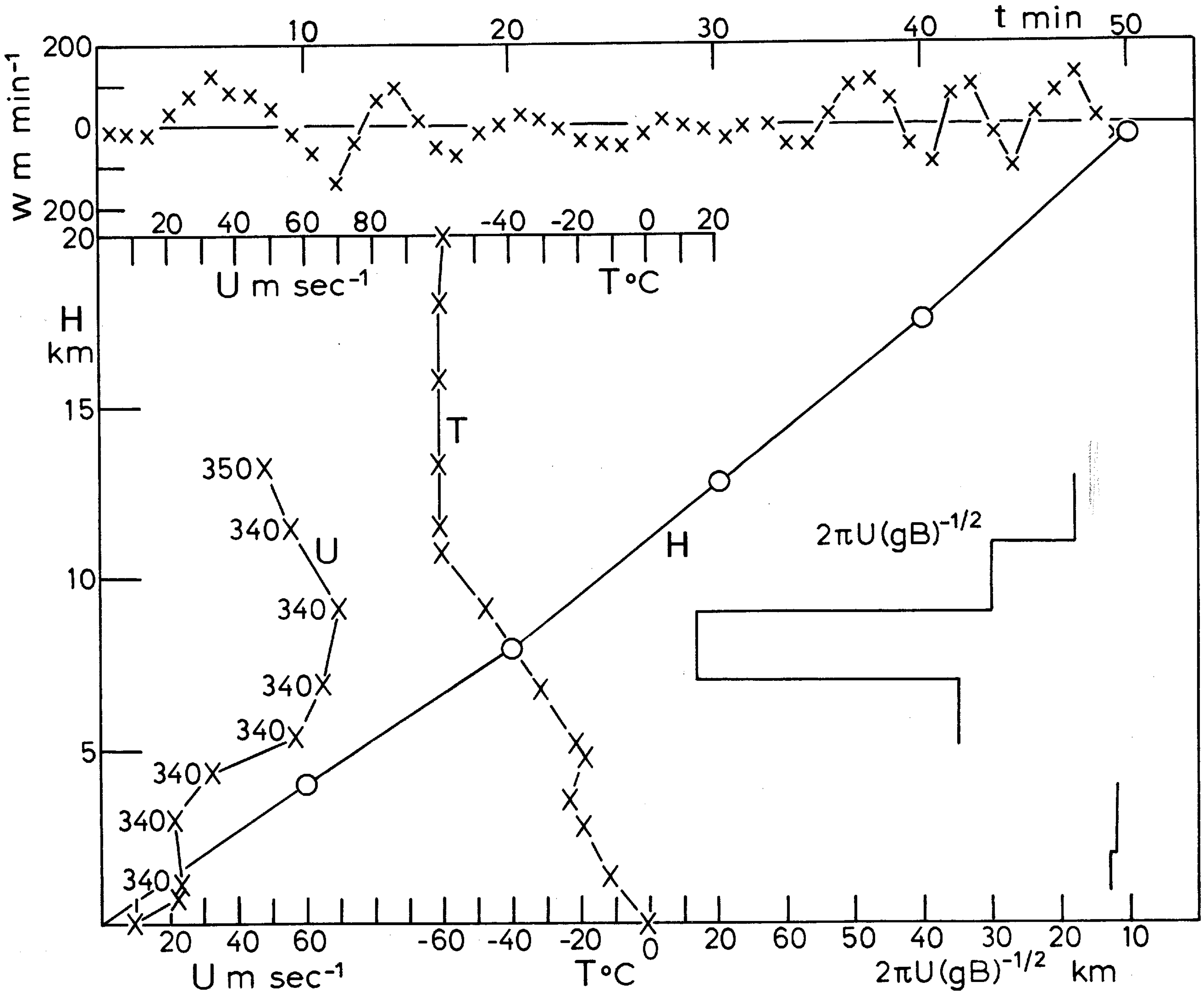
W m min^{-1}
200
0
-200
10
20
30
40
50
t min
60
W
10 30 U 50 70 m sec^{-1}
km
20
15
10
5
0
278
300
273
298
308
315
304
U
286
284
280
277
274
272
T
H
$2\pi U\,(gB)^{-1/2}$
-60 -40 -20 0 20
T °
40 30 20 10 0

Fig. 12



W m min⁻¹
t min
W
U m sec⁻¹
km
U
T
H
T °C
2πU(gB)^-1/2
2πU(gB)^-1/2 km
303
309
320
327
323
316
314
310
300
300
288

Fig. 13



t min
10
20
30
40
50
200
0
200
W m min⁻¹
20
40
60
80
-40
-20
0
20
U m sec⁻¹
T°C
H
km
15
10
5
T
U
H
350
340
340
340
340
340
340
340
$2\pi U(gB)^{-1/2}$
20
40
60
-60
-40
-20
0
20
60
50
40
30
20
10
U m sec⁻¹
T°C
$2\pi U(gB)^{-1/2}$ km

Fig. 14

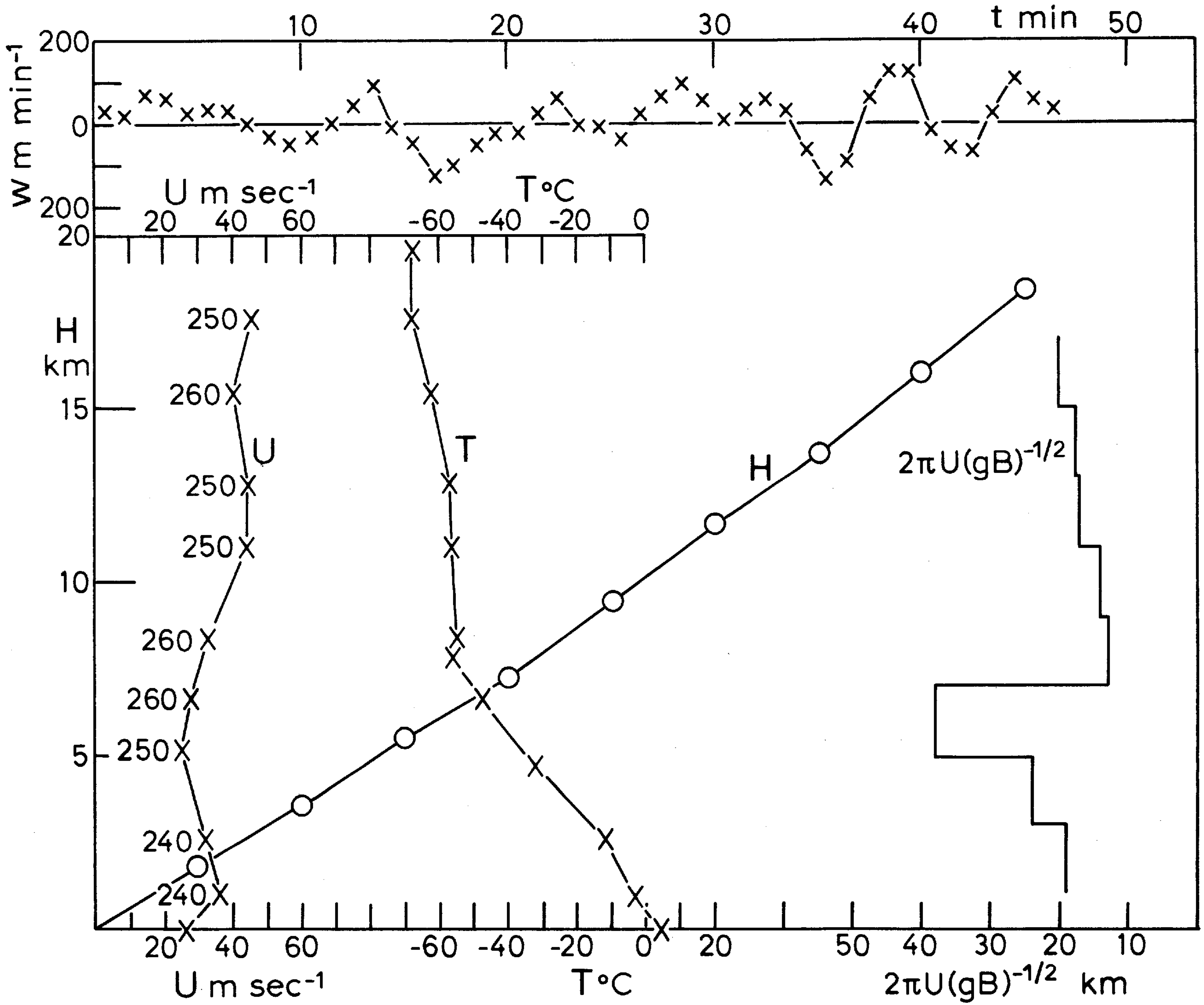

t min
W m min⁻¹
U m sec⁻¹
T°C
H km
U
T
H
2πU(gB)^-1/2
2πU(gB)^-1/2 km
250
260
250
250
260
260
250
240
240

Fig. 15

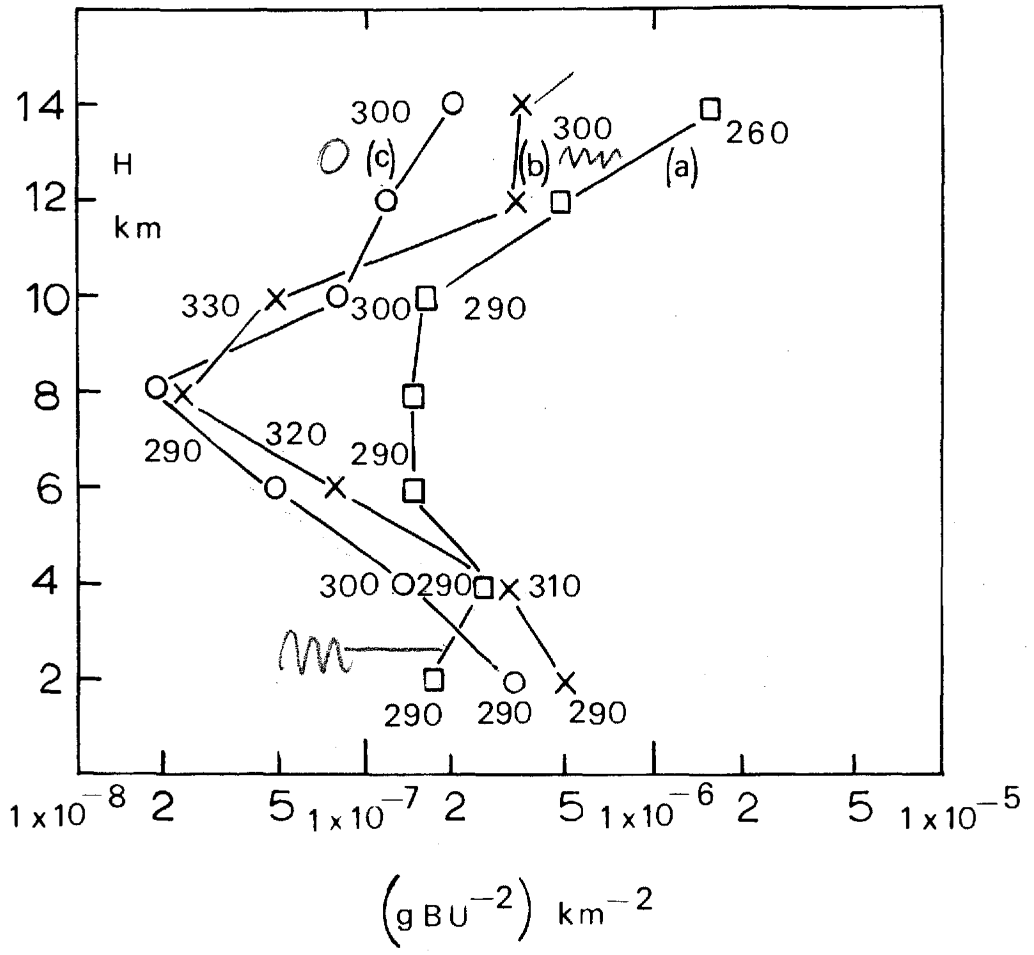

14
12
10
8
6
4
2
H
km
300
(c)
300
(b)
(a)
260
330
300
290
290
320
290
300
290
310
290
290
290
1 x 10^{-8} 2 5 1 x 10^{-7} 2 5 1 x 10^{-6} 2 5 1 x 10^{-5}
$(g\,BU^{-2})\ km^{-2}$

Fig. 16

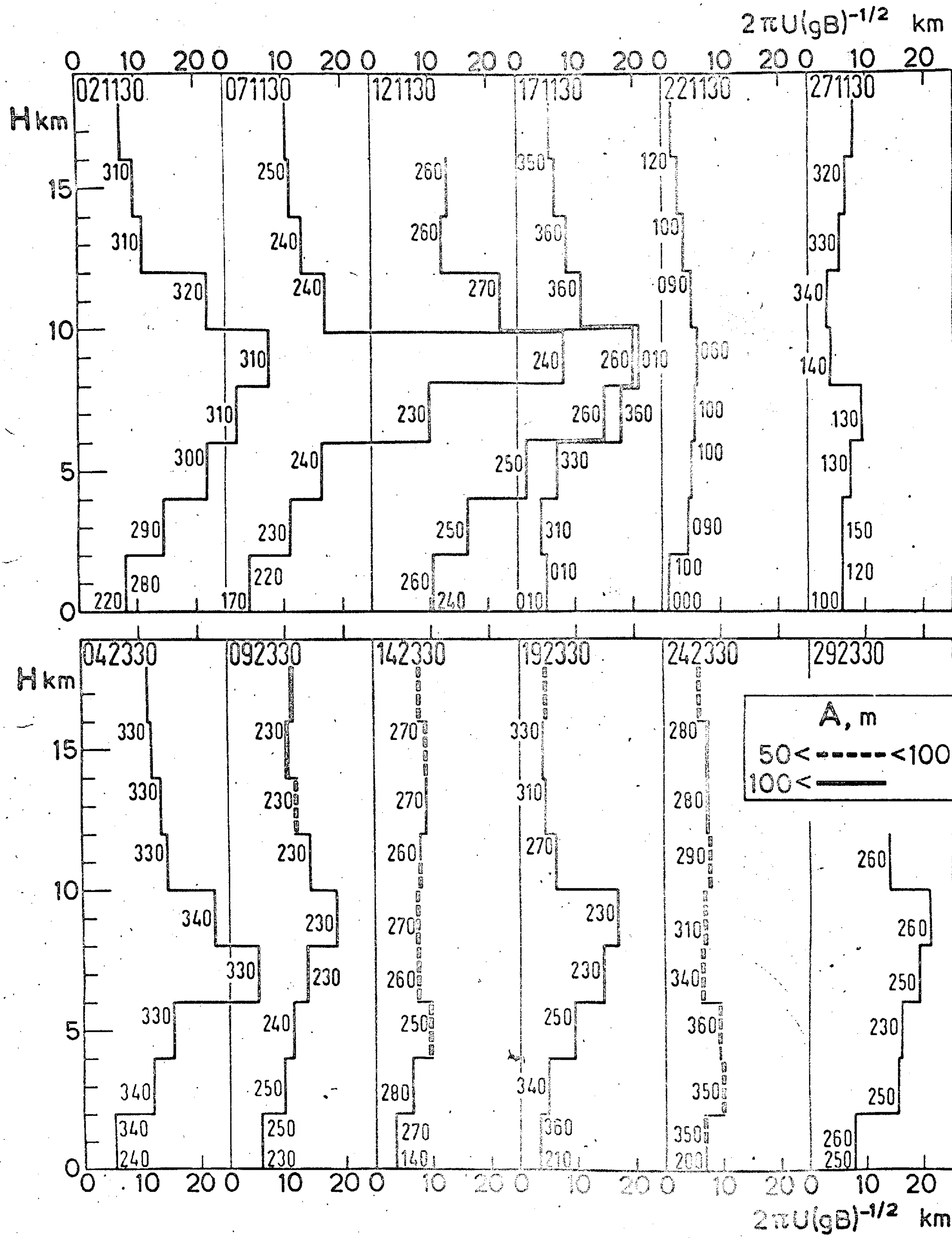
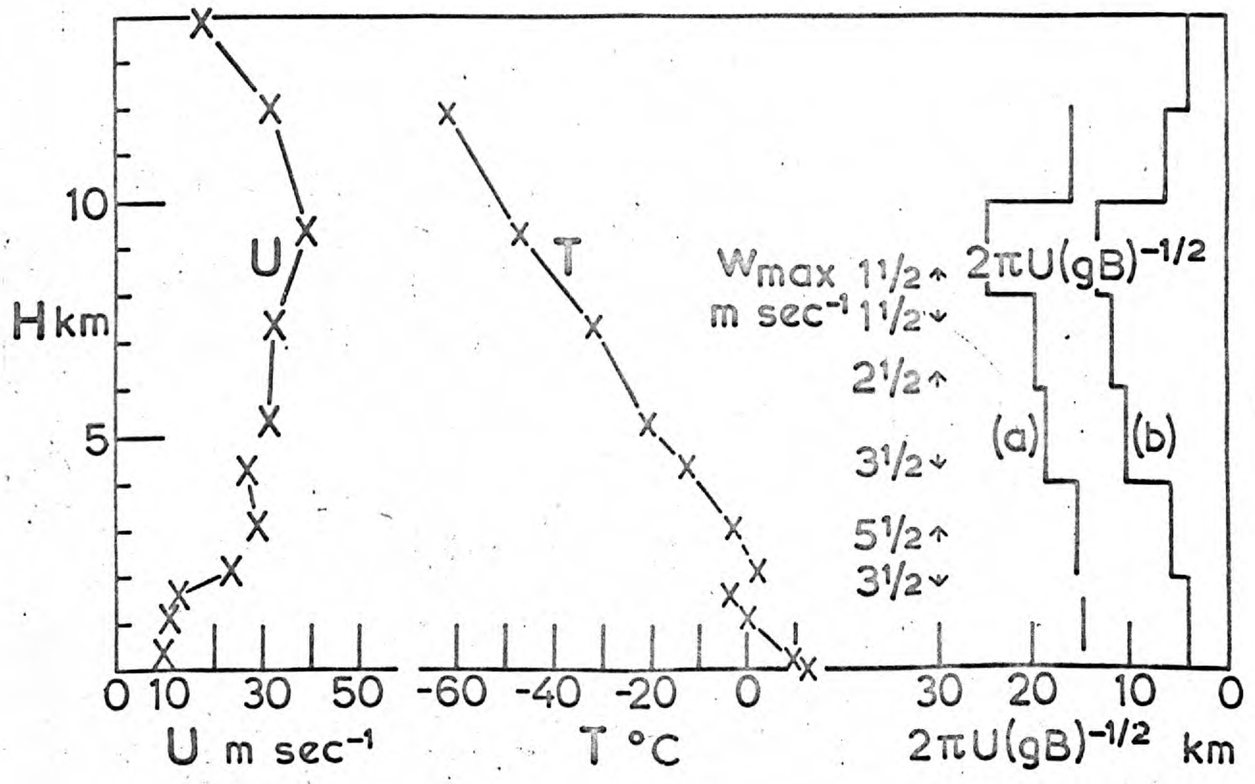
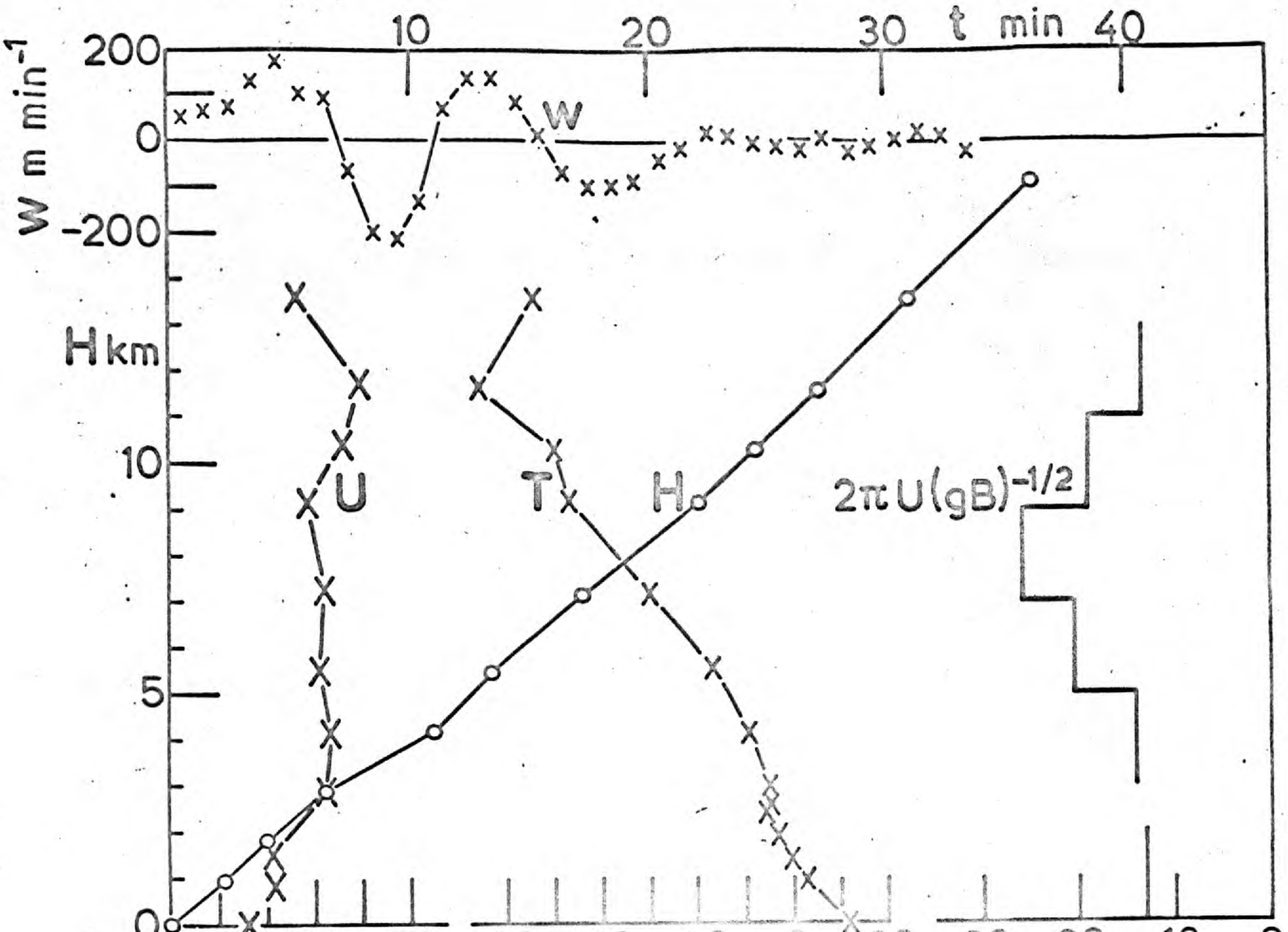

$2\pi U(gB)^{-1/2}$ km
H km
021130
071130
121130
171130
221130
271130
042330
092330
142330
192330
242330
292330
A, m
50< - - - - - <100
100<

Fig. 17



H km
10
5
0 10 30 50
U m sec⁻¹
U
-60 -40 -20 0
T °C
T
Wmax 1½↑
m sec⁻¹ 1½↓
2½↑
3½↓
5½↑
3½↓
2πU(gB)^-1/2
(a)
(b)
30 20 10 0
2πU(gB)^-1/2 km

Fig. 18



W m min⁻¹
200
0
-200
t min
10 20 30 40
W
H km
10
5
0
10 30 50
U m sec⁻¹
U
-60 -40 -20 0 20
T °C
T
H
2πU(gB)^-1/2
30 20 10 0
2πU(gB)^-1/2 km

Fig. 19

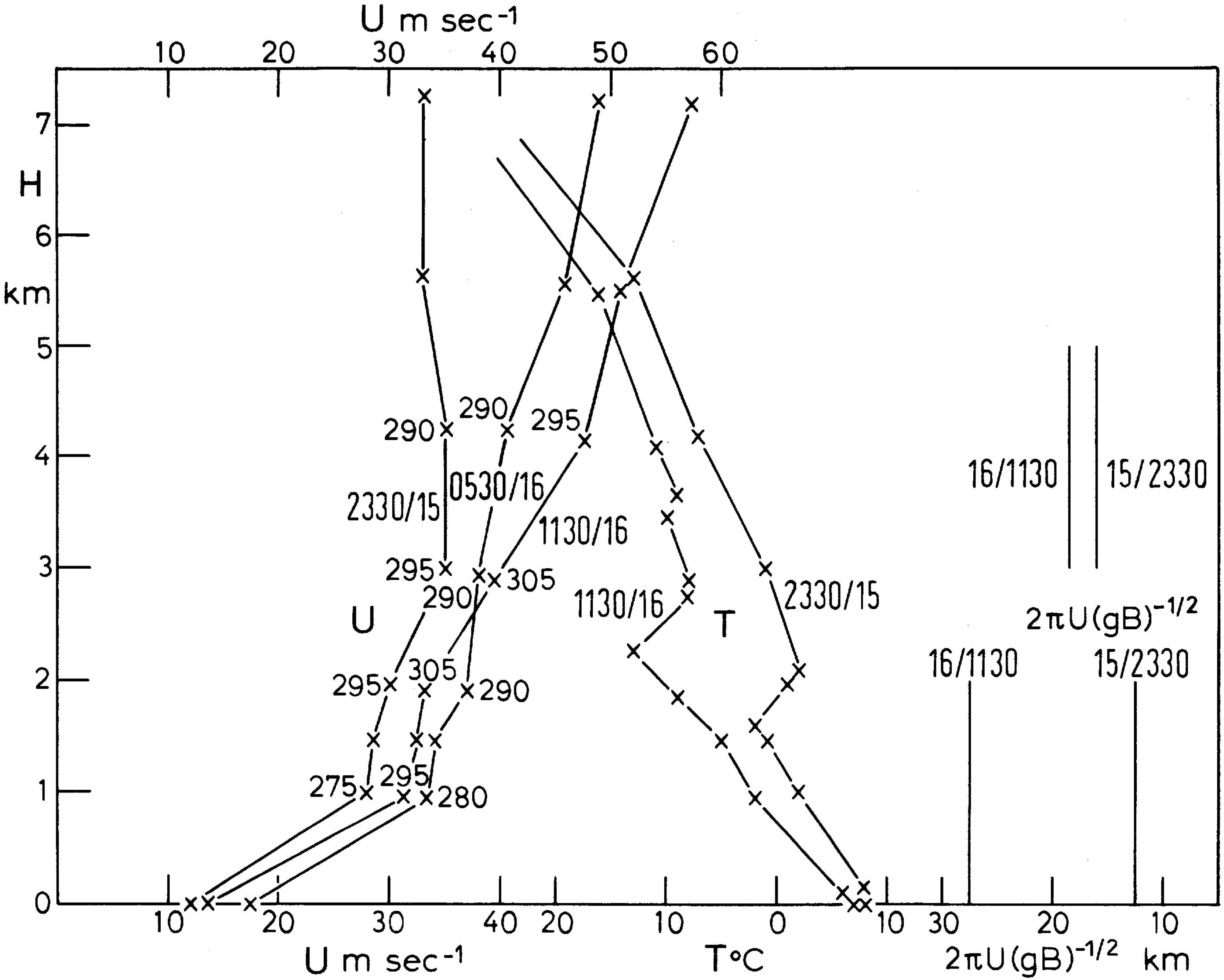

U m sec-1
10
20
30
40
50
60
H
km
0
1
2
3
4
5
6
7
290
290
295
2330/15
0530/16
1130/16
295
305
290
U
295
305
290
275
295
280
1130/16
T
2330/15
16/1130
15/2330
2πU(gB)-1/2
16/1130
15/2330
10
20
30
40
20
10
0
10
30
20
10
U m sec-1
T°C
2πU(gB)-1/2 km

Fig. 20

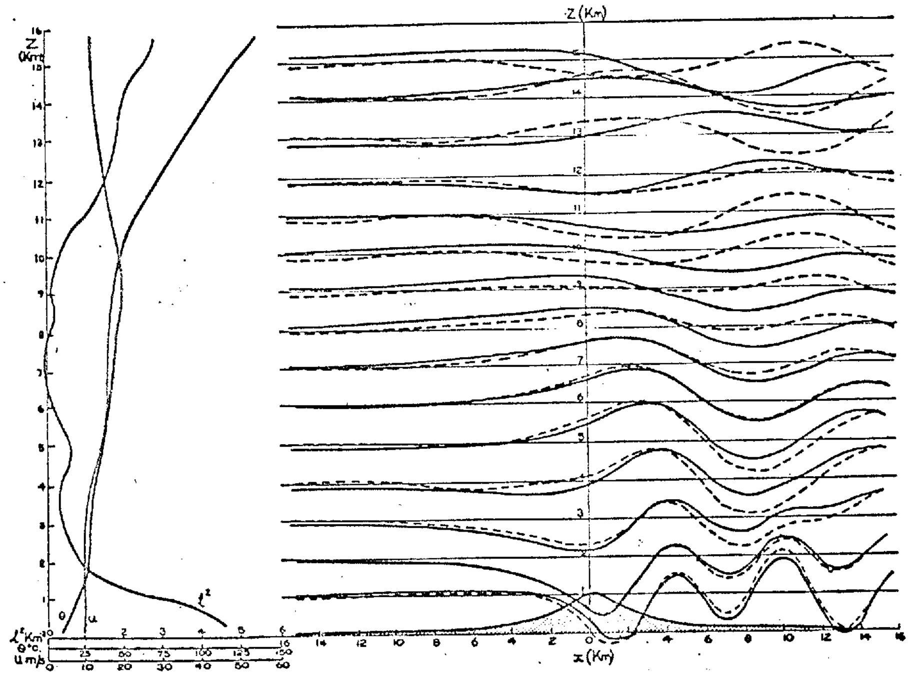


Fig. 21

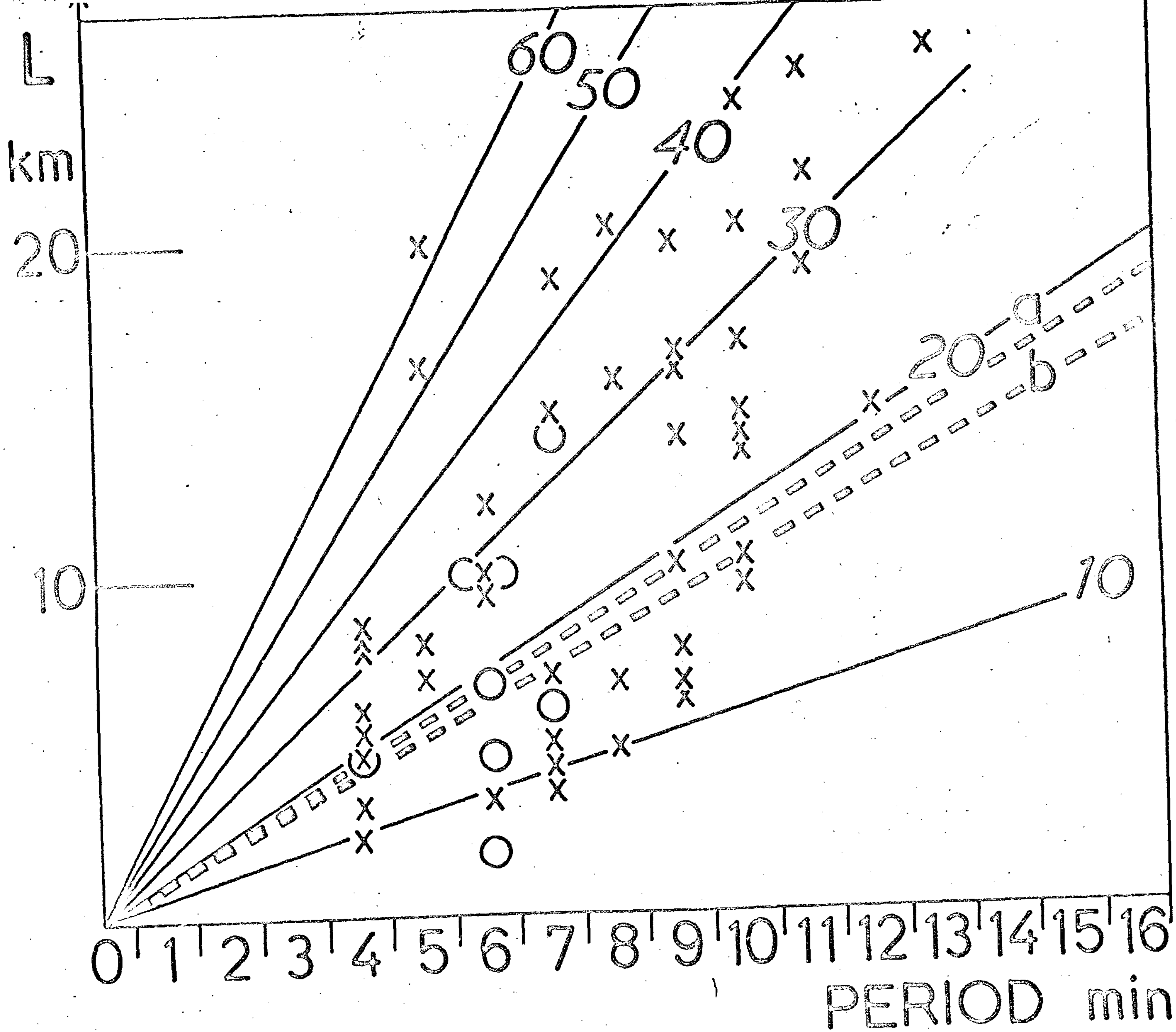

L
km
20
10
60
50
40
30
20
10
a
b
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
PERIOD min

Fig. 22

Fig. 23

Fig. 24

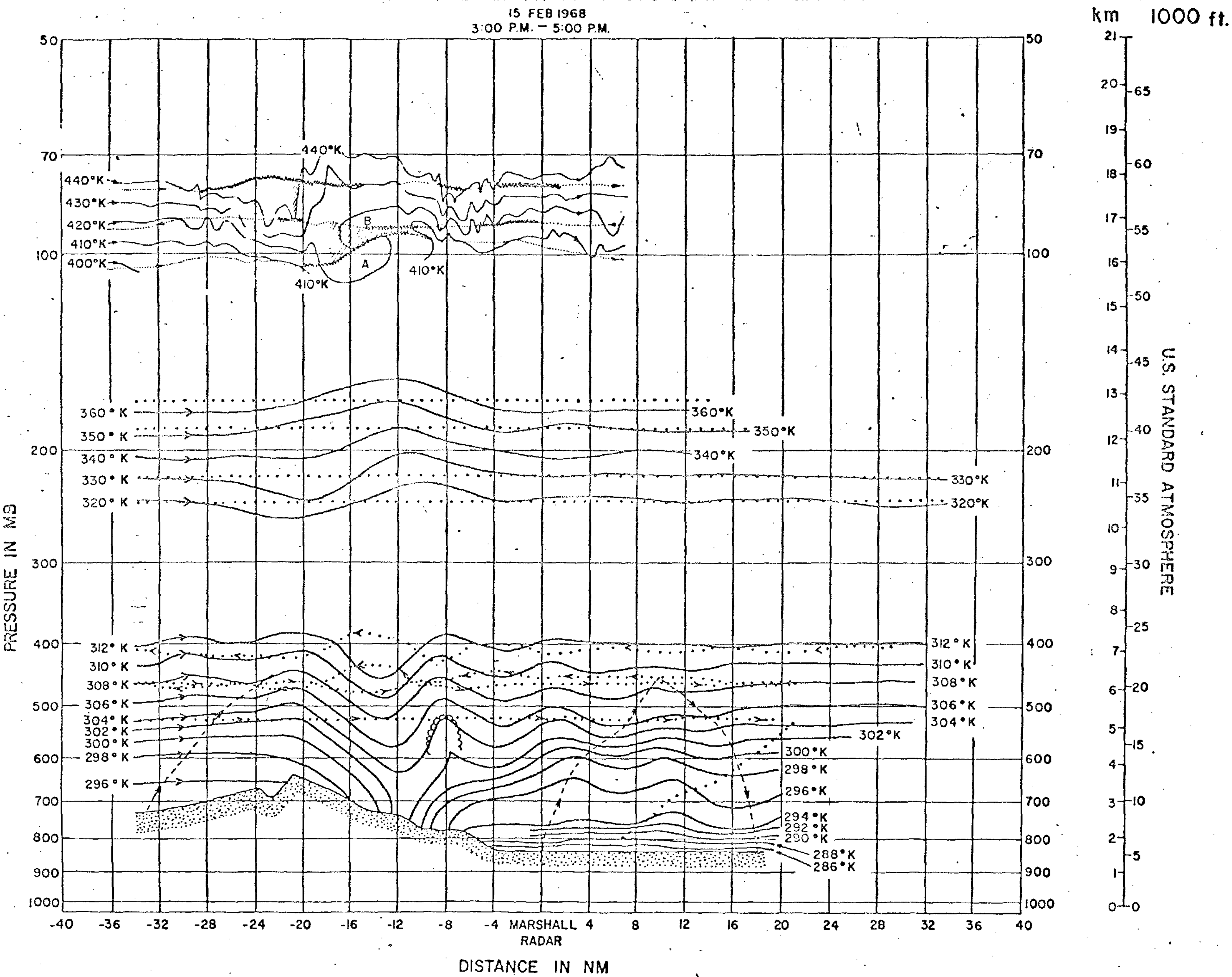

15 FEB 1968
3:00 P.M. — 5:00 P.M.
km
1000 ft.
PRESSURE IN MB
DISTANCE IN NM
MARSHALL RADAR
U.S. STANDARD ATMOSPHERE
440°K
430°K
420°K
410°K
400°K
A
B
360°K
350°K
340°K
330°K
320°K
312°K
310°K
308°K
306°K
304°K
302°K
300°K
298°K
296°K
294°K
292°K
290°K
288°K
286°K

Fig. 25