ASPECTS OF SCATTERING THEORY

by

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PREFACE

The work described in this thesis was carried out under the supervision of Professor P.T.Matthews in the Department of Physics, Imperial College, London between October 1967 and June 1970.

Except where stated in the text the work described is original and has not been submitted in this or any other University for any other degree.

The author wishes to thahk Professor P.T.Matthews, Professor Abdus Salam and Dr.Delbourgo for their advice and encouragement. He also expresses his gratitude to many members of the Theoretical Physics Department, in particular to Q.Shafi and D.G.Fincham with whom he collaborated for parts of this and similar work.

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CONTENTS

ABSTRACT

The work presented in this thesis is divided into two distinct parts.

The first part deals with the theory of non-linear Lagrangians. Techniques are presented for computing S-matrix elements for non-polynomial scalar field Lagrangians with derivative interactions. To second order in the interaction Lagrangian it is shown that all the dependence arising from the derivative part is completely separated out as operators acting on integrals identical to those obtained in a non-derivative theory. The Fourier transforms of self-energy graphs for a class of non-local interaction Lagrangians are taken in the massless case. The on-mass-shell contributions are determined by the analytic continuation of the coefficients appearing in the series expansion of the Lagrangian. As special examples two Lagrangians which are iso-scalar analogues of chiral SU(2) x SU(2) Lagrangians are treated. The possible equivalence of on-mass-shell matrix elements for Lagrangians related by non-linear field transformations is discussed. The extension of these techniques for Lagrangians with iso-spin and hence for the chiral SU(2) x SU(2) Lagrangians is also presented.

The second part is phenomenological. The absorption model is applied to photoproduction processes at intermediate energies assuming vector dominance. U(6,6) symmetry is used to uniquely determine the couplings of the exchanged pseudo-scalar and vector mesons.

An approximate extension of this model using phenomenological form factors to describe three particle final state reactions is given in the three papers included. The reactions considered are $K^- p \rightarrow \bar{K}^{*0} \pi^- p$, $K^- p \rightarrow K^{*-} \pi^+ n$ and $K^- p \rightarrow K^- \pi^- \Delta^{++}$. The first two of these papers were essentially presented by Dr.J.L.Schonfelder in his thesis.

PART I

NON-LINEAR FIELD THEORY

"It often happens that objectively the masses need a certain change."

Chairman Mao Tse-Tung
"The United Front in Cultural Work"
October 30th, 1944.

TO CAROL

CHAPTER 1

In this chapter a general review of non-linear field theory and of the formulation of chiral Lagrangians is given

1) Introduction

Lagrangians of interacting quantised fields were originally classified [1] into those of the first and those of the second kind. The first kind are renormalisable and the second kind unrenormalisable. A non-linear Lagrangian expressible as an infinite power series in the field variable

$$\mathcal{L}(\varphi) = g\, v(\varphi)$$

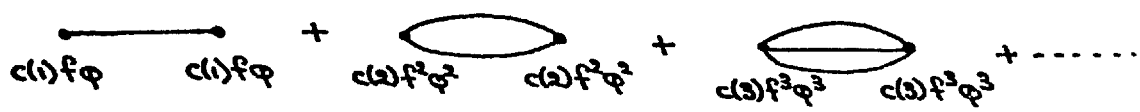
$$= g \sum_{r=0}^{\infty} c(r)\, f^{r}\, \varphi^{r} \tag{1.1}$$

was therefore considered to be a mixture of the two kinds. The terms φ^{r} with $r \leq 4$ are renormalisable whereas the terms with $r > 4$ are not. As early as 1954 in a much neglected paper Okubo [2] showed, for the example of $v(\varphi)$ being an exponential of φ, that a non-linear Lagrangian could be renormalisable. The major step forward was not to treat the infinite series by normal perturbation methods. In 1962 the problem of dealing with non-linear Lagrangians was revived by Efimov [3] and by Fradkin [4]. Although it was stressed that finite results could be obtained the work was considered of academic rather than of direct physical interest until only a few years ago.

One of the recent advances of particle physics has been the formulation of chirally invariant Lagrangians [5] using non-linear realisations of the chiral group. These non-linear Lagrangians can be given a direct physical meaning. There has already been reasonable success in using the chiral Lagrangians by expanding them as a power series in φ and then evaluating the lowest order contributions (tree diagrams) [6] to the appropriate amplitudes. Clearly this can only be considered as a first step and one is next interested in evaluating the closed loop contributions and higher.

In the following section a review is given of the formulation of some chiral SU(2) x SU(2) Lagrangians. These Lagrangians possess the algebraic complexities of containing derivatives of the field and, naturally, of involving iso-spin. The above-mentioned authors have restricted themselves to Lagrangians of the form given in (1.1). One of the main objectives of this work is to develop methods for dealing with the physical non-linear chiral Lagrangians. These techniques can then be used to evaluate the contributions to self-energies and scattering amplitudes. This will be dealt with in the thesis of Q.Shafi with whom I have collaborated for much of the work. Self-consistency relations will be given there which give rise to a <u>finite</u> prediction for the pion mass.

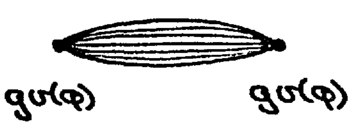
For simplicity we first look at the situation for a non-polynomial Lagrangian given by equation (1.1) i.e. we neglect the derivative part and the iso-spin. This is sufficient to see the general features. We see that two coupling constants appear in equation (1.1). The coupling g is termed the major coupling constant and determines the order of any contribution. To each order in g there are an infinite number of terms to all orders in f. Thus the second order vacuum contributions can be represented diagramatically as the infinite sum.



i.e. the r'th diagram corresponds to there being a Lagrangian $c(r)\, f^r \varphi^r$ at each vertex. The number of r'th order diagrams is r! and hence we must expect the expansion to be a divergent series. Also from conventional held theory we know that each diagram for $r > 4$ is unrenormalisable in the usual sense of the word. The ultraviolet divergences therefore get progressively worse in the perturbation expansion.

The crucial point is that although each term in the series is divergent a summation can be performed which either eliminates or at least greatly reduces the number of ultraviolet infinities. Summation methods for divergent series always give rise to the problem of uniqueness, however it has been shown that the self-energy diagrams obtained this way do satisfy the requirements of analyticity and Landau-Cutkosky unitarity (3).

Once the summation has been performed we represent the diagramatic divergent series above by just one superpropagator, i.e. by



The obvious next step would be to perform the summation in the major coupling constant g. So far as the author is aware this is still a very open problem.

The techniques for dealing with non-polynomial Lagrangians yield matrix elements in x-space. A review of the methods introduced by Efimov and Fradkin is given in the following chapter. These techniques are then extended for the case of Lagrangians containing derivatives of the scalar field. Although p-space is of more physical interest, x-space is useful for studying the overall ultra-violet divergences which may occur. This is also looked at in chapter 2.

The Fourier transforms of the second order self-energy diagrams are taken in chapter 3 assuming zero mass fields. A comparison of the on-mass-shell self-energy contributions is then made for two particular Lagrangians.

The final chapter deals with extending the techniques to include iso-spin and thus allowing the chiral Lagrangians to be treated.

A need for extending the techniques for dealing with non-linear Lagrangians also arises from weak and gravitational interaction theories. Einstein's gravitational Lagrangian is

$$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-g}\cdot g^{\mu\nu}\left(\Gamma^{\lambda}_{\mu\rho}\Gamma^{\rho}_{\nu\lambda} - \Gamma^{\lambda}_{\mu\nu}\Gamma^{\rho}_{\lambda\rho}\right) \tag{1.2}$$

with the Christoffel symbol

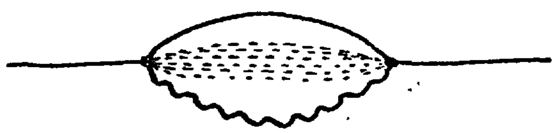
$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho}\left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu}\right) , \tag{1.3}$$

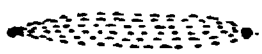
κ is the gravitational constant and $g = \det g_{\alpha\beta}$. The covariant components $g_{\mu\nu}$ can be given as a ratio of two polynomials in the contravariant components $g^{\mu\nu}$ thus giving rise to non-linearity in the field variable. Of more interest is the coupling of the gravitational field to other physical fields where it seems likely that the gravitational constant κ has the effect of being a cut-off parameter and thus suppresses infinities quite naturally [7]. For example, the electromagnetic interaction

Lagrangian for the electron could be taken as

$$\mathcal{L} = e\bar{\psi}\not{A}\psi\sqrt{-g} \tag{1.4}$$

The second order self-energy for the electron is then diagramatically given by



where  is the graviton superpropagator. The author is at present working on the evaluation of diagrams including gravitons. The electron's self-mass arising from the above diagram is

$$\frac{\delta m}{m} = \frac{3\alpha}{2\pi}\left[\log\left(\frac{1}{\kappa m}\right) + O(1)\right] \tag{1.5}$$

where $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ and $\kappa m \sim 10^{-22}$. The terms of O(1) are indeed negligible and hence

$$\frac{\delta m}{m} \simeq \frac{1}{6} \tag{1.6}$$

<u>Thus the inclusion of the gravitons as above yields a finite self-mass for the electron without the necessity of putting in a cut-off parameter.</u>

An example of non-linearity arising in a weak Lagrangian is to consider an intermediate neutral vector meson U_μ interacting with quarks Q. In Stückelberg's representation [3] $\left(U_\mu = A_\mu + \frac{1}{\kappa}\partial_\mu B\right)$ the interaction Lagrangian can be written in the form.

$$\mathcal{L} = f\,\bar{Q}\,\gamma_\mu(1+\gamma_5)\,Q\,A_\mu + m\,\bar{Q}\left(e^{i\gamma_5 f B/\kappa} - 1\right)Q \tag{1.7}$$

which is clearly non-linear in B.

2) Chiral Lagrangians

We shall review here the simplest case of non-linear chiral SU(2) x SU(2) Lagrangians with derivative couplings (5). Mesons of the $(\frac{1}{2},\frac{1}{2})$ representation can be described by the field matrix

$$\Delta = \sigma + i\,\underset{\sim}{\tau}\cdot\underset{\sim}{\varphi}\,\Lambda(\underset{\sim}{\varphi}\cdot\underset{\sim}{\varphi}) \tag{1.8}$$

where $\Lambda(\underset{\sim}{\varphi}\cdot\underset{\sim}{\varphi})$ is, at least for the time being, to be regarded as an arbitrary isoscalar function of the $\underset{\sim}{\varphi}$ fields. Imposing the unitary constraint

$$\Delta\Delta^{+} = 1 \tag{1.9}$$

we obtain the non-linear relation

$$\sigma^2 + \underset{\sim}{\varphi}\cdot\underset{\sim}{\varphi}\,\Lambda^2(\underset{\sim}{\varphi}\cdot\underset{\sim}{\varphi}) = 1 \tag{1.10}$$

Thus the σ field may be eliminated and the field matrix becomes

$$\Delta = \left[1 - \underset{\sim}{\varphi}\cdot\underset{\sim}{\varphi}\,\Lambda^2(\underset{\sim}{\varphi}\cdot\underset{\sim}{\varphi})\right]^{1/2} + i\,\underset{\sim}{\tau}\cdot\underset{\sim}{\varphi}\,\Lambda(\underset{\sim}{\varphi}\cdot\underset{\sim}{\varphi}) \tag{1.11}$$

The total Lagrangian for the $\underline{\varphi}$ fields is now taken as

$$\mathcal{L}_{\text{Total}} = \frac{1}{4}\Lambda^{-2}(0)\,\mathrm{Tr}\left[\partial_\mu \Delta\, \partial_\mu \Delta^+\right] \tag{1.12}$$

It has in fact been shown [9] that this prescription for a chiral SU(2) x SU(2) invariant Lagrangian containing only two derivatives is unique. Using equation (1.11) we explicitly obtain

$$\mathcal{L}_{\text{Total}} = \frac{1}{2}\partial_\mu\underline{\varphi}.\partial_\mu\underline{\varphi}\,\frac{\Lambda^2}{\Lambda^2(0)}$$

$$+\frac{(\underline{\varphi}.\partial_\mu\underline{\varphi})(\underline{\varphi}.\partial_\mu\underline{\varphi})}{2\Lambda^2(0)(1-\Lambda^2\,\underline{\varphi}.\underline{\varphi})}\left[\Lambda^4+4\Lambda\Lambda'+4\Lambda'^2\underline{\varphi}.\underline{\varphi}\right] \tag{1.13}$$

The interaction Lagrangian may now be obtained by subtracting off the free part, i.e.

$$\mathcal{L}_{\text{int}} = \mathcal{L}_{\text{Total}} - \frac{1}{2}\partial_\mu\underline{\varphi}.\partial_\mu\underline{\varphi} \tag{1.14}$$

Before considering various choices for the function $\Lambda(\underline{\varphi}.\underline{\varphi})$ we shall look at the ultra-violet behaviour of the Lagrangian. Assuming that large $\varphi \sim M$, $\partial_\mu\underline{\varphi} \sim M^2$ and $\Lambda(\underline{\varphi}.\underline{\varphi}) \to \varphi^k \sim M^k$ then

$$\mathcal{L}_{\text{Total}} \sim M^{2k+4} + \frac{M^6}{1-M^{2+2k}}\left[M^{4k}+M^{2k-2}+M^{2k-2}\right] \tag{1.15}$$

We note that both parts $\sim M^{2k+4}$. Since $\partial_\mu\underline{\varphi}$ is coupled isotopically differently in both parts this leading order could not be cancelled out. Thus we must have

$$\mathcal{L}_{Total} \sim M^{2k+4} \tag{1.16}$$

Since $\frac{1}{2}\partial_\mu \underline{\phi}.\partial_\mu \underline{\phi} \sim M^4$ we see that the ultra-violet behaviour of the interaction Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{int} &\sim M^{2k+4} \qquad & k>0 \\ &\sim M^4 \qquad & k\leq 0 \end{aligned} \tag{1.17}$$

Assuming the usual Dyson power counting method for estimating divergences (10) holds for non-linear as well as for the conventional polynomial type Lagrangians we can divide the chiral Lagrangians into three classes. A polynomial Lagrangian with the behaviour

$$\mathcal{L} \sim M^n$$

can be renormalised for $n \leq 4$ with a finite number of renormalisation constants. For non-polynomial Lagrangians such theories are called normal since it is hoped they can be renormalised in a similar way. A renormalisation procedure has been given (11) for model non-linear Lagrangians with n = 2 and 3 however it is now questionable as to the renormalisability of a non-linear M^4 theory. For $n<2$ there should be no overall infinities arising at all and these theories are classified as supernormal. The term abnormal is given to theories with $n>4$ since here difficulties arise in polynomial Lagrangians and one therefore expects similar, if not worse, difficulties to arise in non-linear theories. In fact the Dyson power counting method has to be slightly amended as we shall see in chapter 2 but the above classification is still useful. Consequently, using this classification we see that the non-linear realisations of chiral groups for the meson field $\underline{\varphi}$ yield only normal and abnormal interaction Lagrangians. If we restrict ourselves to normal theories some divergences still arise. In certain cases these can be avoided by using, in a self-consistent manner, the total Lagrangian (12) which is supernormal for $k<-1$ and normal for $k = 0$ or -1.

If we assume the basic equivalence theorem to be correct then the question of the significance of the abnormal parametrisations arises. The equivalence theorem states that if a local point transformation of fields is made such that the physical spectrum associated with these fields is unaltered and consequently the Hilbert spaces of in- and out- states also remain the same then the on-mass-shell S-matrix elements are identical for the original and the transformed Lagrangians. This theorem[13] was first stated by Chisholm, Kamefuchi, O'Raifeartaigh and Salam who, together with Borchers, have all proved it to varying degrees of restrictiveness on the field transformations and rigour. It has also been extended by Coleman, Wess and Zumino [9] who claim that the result applies to diagrams with equal numbers of closed loops. The abnormal parametrisations of the chiral Lagrangians can be obtained from normal ones by a (non-linear) point transformation from the one set of co-ordinates $\underline{\varphi}$ to the other $\underline{\varphi}'$. The difficulty in applying the theorem lies in a lack of criteria for determining what transformations leave unchanged the in- and out- states of the interpolating fields.

Three important parametrisations are now given. In each case λ is to be taken as a constant but different in each case.

a) Gasiorowicz - Geffen Co-ordinates

$$\Lambda(\underline{\varphi}.\underline{\varphi}) = \lambda$$

$$\mathcal{L}_{Total} = \frac{1}{2}\partial_\mu\underline{\varphi}.\partial_\mu\underline{\varphi} + \frac{1}{2}\frac{\lambda^2(\underline{\varphi}.\partial_\mu\underline{\varphi})(\underline{\varphi}.\partial_\mu\underline{\varphi})}{(1-\lambda^2\underline{\varphi}.\underline{\varphi})}$$

$$\sim M^4 \tag{1.18}$$

b) Schwinger Co-ordinates

$$\Lambda(\underline{\varphi}.\underline{\varphi}) = \frac{\lambda}{(1+\lambda^2\underline{\varphi}.\underline{\varphi})^{1/2}}$$

$$\mathcal{L}_{Total} = \frac{1}{2}\frac{\partial_\mu\underline{\varphi}.\partial_\mu\underline{\varphi}}{(1+\lambda^2\underline{\varphi}.\underline{\varphi})} - \frac{1}{2}\frac{\lambda^2(\underline{\varphi}.\partial_\mu\underline{\varphi})(\underline{\varphi}.\partial_\mu\underline{\varphi})}{(1+\lambda^2\underline{\varphi}.\underline{\varphi})^2}$$

$$\sim M^2 \tag{1.19}$$

c) Stereographic Co-ordinates

$$\Lambda(\underline{\varphi}.\underline{\varphi}) = \frac{2\lambda}{(1+\lambda^2\underline{\varphi}.\underline{\varphi})}$$

$$\mathcal{L}_{Total} = \frac{1}{2}\frac{\partial_\mu\underline{\varphi}.\partial_\mu\underline{\varphi}}{(1+\lambda^2\underline{\varphi}.\underline{\varphi})^2} \tag{1.20}$$

$$\sim M^0$$

CHAPTER 2

Techniques for calculating the x-space contributions for general second order graphs are given. The discussion includes Lagrangians with derivative couplings but iso-spin is not included. Overall ultra-violet behaviour is discussed.

1) Non-Derivative Lagrangians.

In this section we briefly review the techniques for dealing with non-derivative Lagrangians without iso-spin as were first given by Efimov (3) and Fradkin (4). The Lagrangian is assumed to have an infinite series expansion in the field variable i.e. we consider the Lagrangian

$$\sigma(\varphi) = g \sum_{r=0}^{\infty} c(r)\, f^r \varphi^r \tag{2.1}$$

The normal ordering of $\sigma(\varphi)$ can now be defined by normally ordering each term in the expansion (2.1). The second order term in the S-matrix expansion

$$S = \sum_{n=0}^{\infty} \frac{i^n}{n!} S^{(n)} \tag{2.2}$$

is given by

$$S^{(2)} = g^2 \int d^4 z_1\, d^4 z_2\; T\left\{ \sigma(\varphi(z_1))\, \sigma(\varphi(z_2)) \right\} \tag{2.3}$$

The Feynman propagator Δ is defined in the usual way,

$$\Delta(x_1 - x_2) = \langle T\{\varphi(x_1)\,\varphi(x_2)\}\rangle \tag{2.4}$$

then expanding $S^{(2)}$ into normal ordered products by Hori's lemma (14) one obtains

$$S^{(2)} = g^2 \int d^4z_1 d^4z_2 \sum_{m,n} I_{m,n}(\Delta) : \frac{\varphi(z_1)}{m!}\ \frac{\varphi(z_2)}{n!} : \tag{2.5}$$

where

$$\begin{aligned} I_{m,n}(\Delta) &= \exp\left(\frac{\partial}{\partial\varphi_1}\Delta\frac{\partial}{\partial\varphi_2}\right)\left(\frac{\partial}{\partial\varphi_1}\right)^m\left(\frac{\partial}{\partial\varphi_2}\right)^n \sigma(\varphi_1)\,\sigma(\varphi_2)\Big|_{\varphi_1=0=\varphi_2} \\ &= \exp\left(\frac{\partial}{\partial\varphi_1}\Delta\frac{\partial}{\partial\varphi_2}\right) F(\varphi_1,\varphi_2)\Big|_{\varphi_1=0=\varphi_2} \end{aligned} \tag{2.6}$$

with

$$\varphi_1 = \varphi(z_1) \qquad \varphi_2 = \varphi(z_2)$$

Using the Efimov-Fradkin lemma (3)(4) we now give an integral representation of Hori's exponential operator:

$$\begin{aligned} &\exp\left(\frac{\partial}{\partial\varphi_1}\Delta\frac{\partial}{\partial\varphi_2}\right) F(\varphi_1,\varphi_2) \\ &\quad = \frac{1}{\pi}\int du\,du^*\ \exp\left(-uu^* + u\Delta\frac{\partial}{\partial\varphi_1} + u^*\frac{\partial}{\partial\varphi_2}\right) F(\varphi_1,\varphi_2) \\ &\quad = \frac{1}{\pi}\int du\,du^*\ \exp(-uu^*)\ F(\varphi_1+\Delta u,\ \varphi_2+u^*) \end{aligned} \tag{2.7}$$

The integrations are taken over the whole of the complex u-plane. Thus on substituting (2.7) into equation (2.6) and using equation (2.1),

$$I_{m,n}(\Delta) = \frac{1}{\pi}\int du\,du^* \; e^{-uu^*}\left[\frac{\partial}{\partial(\Delta u)}\right]^m\left[\frac{\partial}{\partial u^*}\right]^n \sigma(\Delta u)\,\sigma(u^*)$$

$$= \frac{1}{\pi}\int du\,du^* \; e^{-uu^*}\sum_{r,s=0}^{\infty} c(r+m)\frac{(r+m)!}{r!}\; c(s+n)\frac{(s+n)!}{s!}\,(f\Delta u)^r\,(fu^*)^s\, f^{n+m}$$

$$= \sum_{r=0}^{\infty} c(r+m)\,c(r+n)\;\frac{(r+m)!\,(r+n)!}{r!}\;(\Delta f^2)^r\; f^{n+m} \tag{2.8}$$

The last step requires the assumption that integration and summation may be interchanged and then use is made of the orthogonality identity

$$\frac{1}{\pi}\int du\,du^*\; e^{-uu^*}\; u^m\, u^{*n} = \delta_{mn}\, n! \tag{2.9}$$

An alternative approach to derive a form for $I_{m,n}(\Delta)$ is to use the Laplace transform $\tilde{\sigma}(\zeta)$ of $\sigma(\varphi)$

$$\sigma(\varphi) = \int_0^{\infty} d\zeta\; e^{-\varphi\zeta}\;\tilde{\sigma}(\zeta) \tag{2.10}$$

defined by analytic continuation where necessary. Noting that

$$\frac{\partial}{\partial\varphi}\,\sigma(\varphi) = \int_0^{\infty} d\zeta\; e^{-\varphi\zeta}\,(-\zeta)\;\tilde{\sigma}(\zeta) \tag{2.11}$$

equation (2.6) may be immediately written as

$$I_{m,n}(\Delta) = \int_0^\infty\int_0^\infty d\zeta_1 d\zeta_2\, e^{\zeta_1\Delta\zeta_2}(-\zeta_1)^m(-\zeta_2)^n\,\tilde{\sigma}(\zeta_1)\,\tilde{\sigma}(\zeta_2) \tag{2.12}$$

Once the Laplace transform of $\sigma(\varphi)$ is known this method will yield, on performing the ζ_1 and ζ_2 integrations, a closed form for $I_{m,n}(\Delta)$ equivalent to having performed the summation, by Borel technique, in the last line of equation (2.8). The series form can be obtained from equation (2.12) by using

$$\sum_{r=0}^{\infty} c(r)\,\varphi^r = \sum_{r=0}^{\infty} c(r)\int_0^\infty d\zeta\; e^{-\varphi\zeta}\left(\frac{\partial}{\partial\zeta}\right)^r \delta(\zeta) \tag{2.13}$$

which implies

$$\tilde{\sigma}(\zeta) = g\sum_{r=0}^{\infty} c(r)\, f^r\left(\frac{\partial}{\partial\zeta}\right)^r \delta(\zeta) \tag{2.14}$$

This equation may be substituted into equation (2.12), the integrations can then be trivially performed by partial integration and the last line of equation (2.8) is obtained.

This method of using Laplace transforms is more suitable for extending to Lagrangians with derivative couplings whether iso-spin is included or not. It is also more suitable for extending to higher orders. Reference 15 gives the rules for a general n'th order (in the major coupling constant) graph. It can be seen that the n'th order extension of equation (2.8) contains n(n-1) integration variables (two for each superpropogator) whereas the extension of equation (2.12) contains only n integration variables (one for each vertex).

2) Derivative Lagrangians

Delbourgo, Salam and Strathdee [15] have outlined a method for treating non-polynomial Lagrangians containing derivative interactions by extending the techniques of Efimov [3] and Fradkin [4] given in the previous section. In this section the techniques required for calculating the S-matrix elements for these Lagrangians are given.

We consider a one component scalar field Lagrangian given by

$$\mathcal{L}(\varphi,\partial_\mu\varphi) = \tfrac{1}{2}:\partial_\mu\varphi\,\partial_\mu\varphi: - \tfrac{1}{2}m^2:\varphi^2: + \mathcal{L}_{int}(\varphi,\partial_\mu\varphi) \qquad (2.15)$$

where the interaction Lagrangian is of the form

$$\mathcal{L}_{int}(\varphi,\partial_\mu\varphi) = h:u(\varphi): + g:\partial_\mu\varphi\,\partial_\mu\varphi\,\sigma(\varphi): \qquad (2.16)$$

For the time being $u(\varphi)$ and $\sigma(\varphi)$ are taken to be arbitrary functions of the field φ which have a Taylor series expansion about $\varphi = 0$, g and h are the major coupling constants. The normal ordering in equation (2.16) is again defined by expanding $u(\varphi)$ and $\sigma(\varphi)$ and then normally ordering each term.

We first derive the general second order matrix elements for the derivative part of (2.16)

$$g\mathcal{L}_I(\varphi,\partial_\mu\varphi) = g:\partial_\mu\varphi\,\partial_\mu\varphi\,v(\varphi): \tag{2.17}$$

The contributions from the product of $\mathcal{L}_I(\varphi,\partial_\mu\varphi)$ with the non-derivative part

$$h\mathcal{L}_i(\varphi) = h:u(\varphi): \tag{2.18}$$

can then be simply deduced by the same methods, and are given towards the end of this section.

For notational convenience the concept of a " 5-vector" is introduced defined as

$$\begin{aligned}\varphi_N(z) &\equiv \left(\varphi(z), \frac{\partial}{\partial z_\mu}\varphi(z)\right)\\ &\equiv (\varphi(z), \varphi_\mu(z))\end{aligned} \tag{2.19}$$

The S-matrix expansion is still given by equation (2.2) but the second order term must be amended to

$$S^{(2)} = g^2\int d^4z_1\,d^4z_2\,T^*\left\{\mathcal{L}_I(\varphi_N(z_1))\,\mathcal{L}_I(\varphi_N(z_2))\right\} \tag{2.20}$$

where the modified time ordering operator T^* is introduced which is defined such that the order of time ordering and differentiation is inverted in taking vacuum expectation values of the following kind (16)

$$\langle T^*\{\varphi_\mu(x_1)\,\varphi(x_2)\}\rangle = \Delta_{\mu_1}(x_1-x_2) \equiv \frac{\partial}{\partial x_{1\mu}}\Delta(x_1-x_2)$$

$$\langle T^*\{\varphi_\mu(x_1)\,\varphi_\nu(x_2)\}\rangle = \Delta_{\mu_1\nu_2}(x_1-x_2) \equiv \frac{\partial}{\partial x_{1\mu}}\frac{\partial}{\partial x_{2\nu}}\Delta(x_1-x_2) \qquad (2.21)$$

where $\Delta(x_1-x_2)$ is the Feynman propagator defined in equation (2.4).

Expanding $S^{(2)}$ into normal ordered products equation (2.5) is extended to

$$\begin{aligned} S^{(2)} = g^2\int d^4z_1\,d^4z_2 \sum_{m,n=0}^{\infty} \Big\{ & S_{m;n}(\Delta(z_1-z_2)) : \frac{\varphi^m(z_1)}{m!}\,\frac{\varphi^n(z_2)}{n!} : \\ & + 2\,S_{m+1,\mu;n}(\Delta(z_1-z_2)) : \varphi_\mu(z_1)\,\frac{\varphi^m(z_1)}{m!}\,\frac{\varphi^n(z_2)}{n!} : \\ & + S_{m+1,\mu;n+1,\nu}(\Delta(z_1-z_2)) : \varphi_\mu(z_1)\,\frac{\varphi^m(z_1)}{m!}\,\varphi_\nu(z_2)\,\frac{\varphi^n(z_2)}{n!} : \\ & + 2\,S_{m+2,\mu\rho;n}(\Delta(z_1-z_2)) : \varphi_\mu(z_1)\varphi_\rho(z_1)\,\frac{\varphi^m(z_1)}{m!}\,\frac{\varphi^n(z_2)}{n!} : \\ & + 2\,S_{m+2,\mu\rho;n+1,\nu}(\Delta(z_1-z_2)) : \varphi_\mu(z_1)\varphi_\rho(z_1)\,\frac{\varphi^m(z_1)}{m!}\,\varphi_\nu(z_2)\,\frac{\varphi^n(z_2)}{n!} : \\ & + S_{m+2,\mu\rho;n+2,\nu\sigma}(\Delta(z_1-z_2)) : \varphi_\mu(z_1)\varphi_\rho(z_1)\,\frac{\varphi^m(z_1)}{m!}\,\varphi_\nu(z_2)\varphi_\sigma(z_2)\,\frac{\varphi^n(z_2)}{n!} : \end{aligned}$$

(2.22)

The coefficient functions in this normal product expansion can be written, as for equation (2.5), in the form

$$S_{mK;nL}(\Delta) = \exp\left(\frac{\partial}{\partial\varphi_{1,M}}\Delta_{MN}\frac{\partial}{\partial\varphi_{2,N}}\right)\left(\frac{\partial}{\partial\varphi_{1,K}}\right)^m\left(\frac{\partial}{\partial\varphi_{2,L}}\right)^n \mathcal{L}_I(\varphi_{1,M})\mathcal{L}_I(\varphi_{2,N})\Bigg|_{\substack{\varphi_{1,M}=0\\ \varphi_{2,N}=0}} \tag{2.23}$$

where Δ_{MN} is the 5 x 5 matrix

$$\Delta_{MN} = \begin{pmatrix} \Delta & \Delta_{\nu_2} \\ \Delta_{\mu_1} & \Delta_{\mu_1\nu_2} \end{pmatrix} \tag{2.24}$$

The indices m, n refer to the number of external lines (including derived scalar lines) at the vertices "1" and "2" and K, L are the "5-vector" labels.

We may now proceed by either extending the Efimov-Fradkin lemma or by using Laplace transforms. The generalisation for derivative interactions of the second method is the more straightforward but we shall give both methods here.

Introducing the "5-vector" integration variables u_N and its complex conjugate u^*_N the exponential operator in equation (2.23) can be represented by, in analogy to equation (2.7),

$$\exp\left(\frac{\partial}{\partial\varphi_{1,M}}\Delta_{MN}\frac{\partial}{\partial\varphi_{2,N}}\right)$$

$$= \frac{1}{\pi^5}\int d^5u_{(K)}\, d^5u^*_{(K)}\, \exp\left(-u_N u^*_N + \frac{\partial}{\partial\varphi_{1,M}}\Delta_{MN}u_N + u^*_N\frac{\partial}{\partial\varphi_{2,N}}\right) \tag{2.25}$$

Making the usual argument shifts in the Lagrangians all second order contributions to the S-matrix are given by

$$S_{m,K;n,L}(\Delta)$$

$$= \int d^{10}\Lambda \left(\frac{\partial}{\partial \varphi_{1,K}}\right)^m \left(\frac{\partial}{\partial \varphi_{2,L}}\right)^n \mathcal{L}_I(\varphi_1 + \Delta u + \Delta_{\nu_2} u_\nu ; \varphi_{1,\mu} + \Delta_{\mu_1} u + \Delta_{\mu_1 \nu_2} u_\nu) \cdot$$

$$\cdot \mathcal{L}_I(\varphi_2 + u^* ; \varphi_{2,\nu} + u^*_\nu) \Big|_{\substack{\varphi_{1,M}=0 \\ \varphi_{2,N}=0}} \tag{2.26}$$

where

$$d^{10}\Lambda = \frac{1}{\pi^5} d^5 u_{(N)} d^5 u^*_{(N)} \, e^{-u_N u^*_N}$$

$$= \frac{1}{\pi^5} du du^* \, d^4 u_{(\mu)} d^4 u^*_{(\mu)} \, e^{-uu^* - u_\mu u^*_\mu} \tag{2.27}$$

Taking $\mathcal{L}_I$ to be as given in equation (2.17) we have, for the case of no external derived scalar lines,

$$S_{m;n}(\Delta) = \int d^{10}\Lambda \left[\frac{\partial}{\partial(\Delta u)}\right]^m \left[\frac{\partial}{\partial u^*}\right]^n (\Delta_{\mu_1} u + \Delta_{\mu_1 \nu_2} u_\nu)^2 \cdot$$

$$\cdot \sigma(\Delta u + \Delta_{\nu_2} u_\nu) \, u^*_\nu u^*_\nu \, \sigma(u^*) \tag{2.28}$$

In order to perform the eight vector intergrations we expand the integrand in a Taylor series about u_μ and retain only the second order term. The remaining terms in the series vanish with the use of identities like

$$\frac{1}{\pi^4}\int d^4u_{(\mu)}\, d^4u^*_{(\mu)}\, e^{-u_\mu u^*_\mu}\; u_\alpha u_\beta\, u^*_\nu u^*_\nu = 2g_{\alpha\beta}$$

$$\frac{1}{\pi^4}\int d^4u_{(\mu)}\, d^4u^*_{(\mu)}\, e^{-u_\mu u^*_\mu}\; u_\alpha u_\beta\, f(u_\nu u^*_\nu) = 0 \qquad (2.29)$$

The integral (2.28) then reduces to

$$S_{m;n}(\Delta) = \frac{2}{\pi}\int du\, du^*\; e^{-uu^*}\, X(u)\cdot \sigma(u^*) \qquad (2.30)$$

where $X(u)$ is the contribution from the second order term of the Taylor series and is given by

$$X(u) = \frac{1}{2}\frac{\partial}{\partial u_\nu}\frac{\partial}{\partial u_\nu}\left\{(\Delta_{\mu_1}u + \Delta_{\mu_1\nu_2}u_{\nu_2})^2\left[\frac{\partial}{\partial(\Delta u)}\right]^m \sigma(\Delta u + \Delta_{\nu_2}u_\nu)\right\}_{u_j=0}$$

$$= \frac{1}{2}\left\{2\Delta_{\mu_1\nu_2}\Delta_{\mu_1\nu_2} + 4\Delta_{\mu_1}\Delta_{\mu_1\nu_2}u\frac{\partial}{\partial u_\nu} + \Delta_{\mu_1}\Delta_{\mu_1}u^2\frac{\partial}{\partial u_\nu}\frac{\partial}{\partial u_\nu}\right\}\cdot$$

$$\cdot\left[\frac{\partial}{\partial(\Delta u)}\right]^m \sigma(\Delta u + \Delta_{\nu_2}u_\nu)\bigg|_{u_j=0}$$

$$= \frac{1}{2}\,(n)\left[\frac{\partial}{\partial(\Delta u)}\right]^m \sigma(\Delta u) \qquad (2.31)$$

where the operator Θ is defined by

$$\Theta = 2\,\Delta_{\mu_1\nu_2}\Delta_{\mu_1\nu_2} + 4\,\Delta_{\mu_1}\Delta_{\mu_1\nu_2}\Delta_{\nu_2}\frac{\partial}{\partial\Delta} + \Delta_{\mu_1}\Delta_{\mu_1}\Delta_{\nu_2}\Delta_{\nu_2}\left(\frac{\partial}{\partial\Delta}\right)^2 \tag{2.32}$$

Substituting back into equation (2.30) we have

$$\begin{aligned} S_{m;n}(\Delta) &= \Theta\,\frac{1}{\pi}\int du\,du^*\,e^{-uu^*}\left[\frac{\partial}{\partial(\Delta u)}\right]^m\left[\frac{\partial}{\partial u^*}\right]^n \sigma(\Delta u)\,\sigma(u^*) \\ &= \Theta\, I_{m;n}(\Delta) \end{aligned} \tag{2.33}$$

The second order contribution has now been expressed as an operator acting on the integral $I_{m;n}(\Delta)$ already obtained in equation (2.8) as the second order contribution for a non-derivative Lagrangian.

This same result can also be obtained by using the Laplace transform method. Substituting the 5-dimensional Laplace transform $\tilde{\mathcal{L}}_I(\zeta_M)$ of $\mathcal{L}_I(\varphi_M)$, where

$$\mathcal{L}_I(\varphi_M) = \int_0^\infty d^5\zeta_{(M)}\, e^{-\varphi_M\zeta_M}\,\tilde{\mathcal{L}}_I(\zeta_M)\,, \tag{2.34}$$

into equation (2.23) and noting that

$$\frac{\partial}{\partial\varphi_M}\mathcal{L}_I(\varphi_M) = \int_0^\infty d^5\zeta_{(M)}\,(-\zeta_M)\,\tilde{\mathcal{L}}_I(\zeta_M)\,e^{-\varphi_M\zeta_M} \tag{2.35}$$

we obtain

$$S_{m,K;n,L}(\Delta) = \int_0^\infty \int_0^\infty d^5 \zeta_{1,(M)}\, d^5 \zeta_{2(N)} \left(-\zeta_{1,K}\right)^m \left(-\zeta_{2,L}\right)^n \cdot$$
$$\cdot \tilde{R}_I(\zeta_{1,M})\, \tilde{R}_I(\zeta_{2,N}) \exp(\zeta_{1,M} \Delta_{MN} \zeta_{2,N}) \tag{2.36}$$

which is the obvious extension of equation (2.12).
Again taking R_I to be given by equation (2.17) gives

$$\tilde{R}_I(\zeta_M) = \tilde{\sigma}(\zeta)\left[\frac{\partial^2}{\partial \zeta_\mu\, \partial \zeta_\mu}\ \delta^4(\zeta_\mu)\right] \tag{2.37}$$

where $\tilde{\sigma}(\zeta)$ is the Laplace transform of $\sigma(\varphi)$.
Substituting into equation (2.36) we then obtain for the case of no external derived scalar lines

$$S_{m;n}(\Delta) = \int_0^\infty \int_0^\infty d\zeta_1 d\zeta_2\, d^4 \zeta_{1(\mu)}\, d^4 \zeta_{2(\nu)} \left(-\zeta_1\right)^m \left(-\zeta_2\right)^n \cdot$$
$$\cdot\ \tilde{\sigma}(\zeta_1)\, \tilde{\sigma}(\zeta_2)\left[\frac{\partial^2}{\partial \zeta_{1,\mu} \partial \zeta_{1,\mu}}\ \delta^4(\zeta_{1,\mu})\right]\left[\frac{\partial^2}{\partial \zeta_{2,\nu} \partial \zeta_{2,\nu}}\ \delta^4(\zeta_{2,\nu})\right] \cdot$$
$$\cdot \exp\left(\zeta_1 \Delta \zeta_2 + \zeta_{1\mu} \Delta_{\mu_1} \zeta_2 + \zeta_2 \Delta_{\nu_2} \zeta_{2\nu} + \zeta_{1\mu} \Delta_{\mu_1 \nu_2} \zeta_{2\nu}\right) \tag{2.38}$$

The vector integrations are performed by partial integration to give

$$S_{m;n}(\Delta) = \int_0^\infty \int_0^\infty d\zeta_1 d\zeta_2 \, (-\zeta_1)^m (-\zeta_2)^n \, \tilde{\sigma}(\zeta_1) \, \tilde{\sigma}(\zeta_2) \, e^{\zeta_1 \Delta \zeta_2} \, .$$

$$\cdot \left[2\Delta_{\mu_1\nu_2}\Delta_{\mu_1\nu_2} + 4\Delta_{\mu_1}\Delta_{\mu_1\nu_2}\Delta_{\nu_2}\zeta_1\zeta_2 + \Delta_{\mu_1}\Delta_{\mu_1}\Delta_{\nu_2}\Delta_{\nu_2}(\zeta_1\zeta_2)^2 \right]$$

$$= \Theta \int_0^\infty \int_0^\infty d\zeta_1 d\zeta_2 \, (-\zeta_1)^m (-\zeta_2)^n \, \tilde{\sigma}(\zeta_1) \, \tilde{\sigma}(\zeta_2) \, e^{\zeta_1 \Delta \zeta_2}$$

$$= \Theta \, I_{m;n}(\Delta) \qquad (2.39)$$

i.e. the same result as obtained by the previous method and given by equation (2.33).

It is easy to see how equation (2.38) may be generalised for higher order S-matrix elements. Performing the required partial integrations, however, becomes far more complex algebraically and hence the generalisation of the operator Θ to n'th order is by no means trivial.

All other graphs may be derived in a similar manner but they are more immediately obtained by partial differentiation of $S_{m;n}(\Delta)$ with respect to the Δ_{μ_1}, Δ_{ν_2} and $\Delta_{\mu_1\nu_2}$ propagators. Note here that it becomes important that we have distinguished between Δ_{μ_1} and Δ_{ν_2}. For the partial differentiation they must be regarded as independent variables and only after this has been performed may one use

$$\Delta_{\mu_1} = - \Delta_{\mu_2} \qquad (2.40)$$

From the general formula (2.23) it follows that

$$S_{m+1,\mu;n+1}(\Delta) = \frac{\partial}{\partial \Delta_{\mu_1}} S_{m;n}(\Delta)$$

$$S_{m+1,\mu;n+1,\nu}(\Delta) = \frac{\partial}{\partial \Delta_{\mu_1\nu_2}} S_{m;n}(\Delta)$$

$$S_{m+2,\mu\rho;n+2}(\Delta) = \frac{\partial}{\partial \Delta_{\mu_1}} \frac{\partial}{\partial \Delta_{\rho_1}} S_{m;n}(\Delta)$$

$$S_{m+2,\mu\rho;n+2,\nu}(\Delta) = \frac{\partial}{\partial \Delta_{\mu_1\nu_2}} \frac{\partial}{\partial \Delta_{\rho_1}} S_{m;n}(\Delta)$$

$$S_{m+2,\mu\rho;n+2,\nu\sigma}(\Delta) = \frac{\partial}{\partial \Delta_{\mu_1\nu_2}} \frac{\partial}{\partial \Delta_{\rho_1\sigma_2}} S_{m;n}(\Delta) \qquad (2.41)$$

In addition there is the symmetry relation

$$S_{m,k;n,l}(\Delta) = S_{n,l;m,k}(\Delta) \qquad (2.42)$$

These propagator differentiations act only on the $\textcircled{$\sim$}$ operator and so may be easily performed to give the following formulae for all second order contributions.

$$S_{m,\mu;n}(\Delta) = \left(4\Delta_{\mu_1\nu_2}\Delta_{\nu_2} + 2\Delta_{\mu_1}\Delta_{\nu_2}\Delta_{\nu_2}\frac{\partial}{\partial\Delta}\right) I_{m;n}(\Delta)$$

$$S_{m,\mu\rho;n}(\Delta) = 2 g_{\mu\rho}\, \Delta_{\nu_2}\Delta_{\nu_2}\, I_{m;n}(\Delta)$$

$$S_{m+1,\mu;n+1,\nu}(\Delta) = 4\left(\Delta_{\mu_1\nu_2} + \Delta_{\mu_1}\Delta_{\nu_2}\frac{\partial}{\partial\Delta}\right) I_{m;n}(\Delta)$$

$$S_{m+1,\mu\rho;n+1,\nu}(\Delta) = 4\, g_{\mu\rho}\, \Delta_{\nu_2}\, I_{m;n}(\Delta)$$

$$S_{m+2,\mu\rho;n+2,\nu\sigma}(\Delta) = 4\, g_{\mu\rho}\, g_{\nu\sigma}\, I_{m;n}(\Delta) \qquad (2.43)$$

In practice it is also very useful to note the identity

$$S_{m+\ell;\ell} = \left(\frac{\partial}{\partial\Delta}\right)^{\ell} S_{m;0} \qquad (2.44)$$

which again follows from the general formula (2.23).

We now make the observation that all the second order graphs are written in the form of an operator acting on $I_{m;n}(\Delta)$ integrals. These integrals are identical to those which one obtains for second order graphs with m external lines at one vertex and n at the other using a non-derivative Lagrangian $\upsilon(\varphi)$. Thus all the dependence coming from the derivative part of the Lagrangian has been completely separated out. Use of the identity (2.44) means in practice that we need only evaluate $I_{m;0}(\Delta)$ and from this we may calculate all possible second order graphs corresponding to a Lagrangian given by equation (2.17).

To take into account the contributions from the product of the two Lagrangians $\mathcal{L}_i(\varphi_1)$ and $\mathcal{L}_I(\varphi_2, \varphi_{2,\mu})$ defined in equations (2.18) and (2.17) respectively, we again expand the modified time ordering operator into a series of normal ordered products. The corresponding coefficient functions in the expansion are $\hat{S}_{m;n}(\Delta)$, $2\hat{S}_{m+1;n+1,\nu}(\Delta)$ and $\hat{S}_{m+2;n+2,\nu\rho}(\Delta)$ where

$$\hat{S}_{m;n}(\Delta) = \exp\left\{\frac{\partial}{\partial\varphi_1}\left(\Delta\frac{\partial}{\partial\varphi_2}+\Delta_{\nu_2}\frac{\partial}{\partial\varphi_{2,\nu}}\right)\right\}\left(\frac{\partial}{\partial\varphi_1}\right)^m\left(\frac{\partial}{\partial\varphi_2}\right)^n \cdot$$

$$\cdot\, \mathcal{L}_i(\varphi_1)\, \mathcal{L}_I(\varphi_{2,\mu})\Big|_{\substack{\varphi_1=0\\ \varphi_{2,\mu}=0}}$$

$$= \Delta_{\nu_2}\Delta_{\nu_2}\int_0^\infty\int_0^\infty d\zeta_1 d\zeta_2 (-\zeta_1)^{m+2}(-\zeta_2)^n\, \tilde{u}(\zeta_1)\, \tilde{v}(\zeta_2)\, e^{\zeta_1\Delta\zeta_2}$$

$$= \Delta_{\nu_2}\Delta_{\nu_2}\hat{I}_{m;n}(\Delta)\ , \tag{2.45}$$

$\tilde{u}(\zeta_1)$ being the Laplace transform of $u(\varphi)$,

$$\hat{S}_{m+1;n+1,\nu}(\Delta) = \frac{\partial}{\partial\Delta_{\nu_2}}\hat{S}_{m;n}(\Delta)$$

$$= 2\Delta_{\nu_2}\hat{I}_{m;n}(\Delta) \tag{2.46}$$

and

$$\hat{S}_{m+2;n+2,\nu\rho}(\Delta) = 2g_{\nu\rho}\,\hat{I}_{m;n}(\Delta) \tag{2.47}$$

There are also similar contributions with $\mathcal{L}_i(\varphi_1)$ and $\mathcal{L}_I(\varphi_2,\varphi_{2,\mu})$ interchanged. The $\hat{I}_{m;n}$ integrals are again identical to those which occur in second order for a non-derivative Lagrangian but correspond to a Lagrangian $u(\varphi)$ at one vertex and $v(\varphi)$ at the other. For the remainder of this work we only consider the contributions from the derivative interaction Lagrangian.

3) Ultra-violet Behaviour

So far no restrictions have been placed on the form of $v(\varphi)$, the non-derivative part of the Lagrangian, other than the condition that it can be expanded as an infinite power series in φ . To be able to make definite statements concerning ultra-violet behaviour and about the Fourier transforms we must impose certain restrictions on the coefficients appearing in the series expansion of $v(\varphi)$. We shall restrict the discussion to the class of Lagrangians $\mathcal{L}_I(\varphi,\partial_\mu\varphi)$ defined in equation (2.17) where $v(\varphi)$ is defined as a linear combination of expressions of the form

$$w(\varphi^2) = \frac{(f^2\varphi^2)^\alpha}{(1-f^2\varphi^2)^\beta} \tag{2.48}$$

with α and β being integers. From chapter 1 we see that the iso-scalar analogues of the chiral Lagrangians fall into this class. The restriction $\beta>\alpha\geq 0$ is also made since with this condition we shall see that we meet no difficulties with overall ultra-violet divergences. Any expression of the form (2.48) but with $\beta\leq\alpha$ can always be written as a sum of terms satisfying the condition $\beta>\alpha\geq 0$ together with a polynomial in φ^2. This polynomial in φ^2 can then be treated separately.

Expanding $w(\varphi^2)$ binomially we have

$$w(\varphi^2) = \sum_{r=0}^{\infty} \frac{(\beta-\alpha+r-1)\ldots\ldots(r-\alpha+1)}{(\beta-1)!}\,(f^2\varphi^2)^r \tag{2.49}$$

Since $\upsilon(\varphi)$ is some linear combination of the $\omega(\varphi^2)$'s we deduce that

$$\upsilon(\varphi) = \sum_{r=0}^{\infty} c(r) \; (f^2 \varphi^2)^r \tag{2.50}$$

where $c(r)$ is a polynomial in r.

We shall now look in detail at the second order self-energy contributions $S_{1;1}$, $S_{1,\mu;1}$, $S_{1,\mu;1,\nu}$, $S_{2;0}$, $S_{2,\mu;0}$ and $S_{2,\mu\nu;0}$ which are given by equations (2.39) and (2.43). Explicitly we have

$$S_{1;1}(\Delta) = \Theta \frac{\partial}{\partial \Delta} I_{0;0}(\Delta)$$

$$S_{1,\mu;1}(\Delta) = \left(4\Delta_{\mu\nu_2}\Delta_{\nu_2} + 2\Delta_{\mu_1}\Delta_{\nu_2}\Delta_{\nu_2}\frac{\partial}{\partial \Delta}\right)\frac{\partial}{\partial \Delta} I_{0;0}(\Delta)$$

$$S_{1,\mu;1,\nu}(\Delta) = 4\left(\Delta_{\mu\nu_2} + \Delta_{\mu_1}\Delta_{\nu_2}\frac{\partial}{\partial \Delta}\right) I_{0;0}(\Delta)$$

$$S_{2;0}(\Delta) = \Theta \, I_{2;0}(\Delta)$$

$$S_{2;\mu;0}(\Delta) = \left(4\Delta_{\mu\nu_2}\Delta_{\nu_2} + 2\Delta_{\mu_1}\Delta_{\nu_2}\Delta_{\nu_2}\frac{\partial}{\partial \Delta}\right) I_{2;0}(\Delta)$$

$$S_{2,\mu\nu;0}(\Delta) = 2 g_{\mu\nu}\Delta_{g_2}\Delta_{g_2} \, I_{2;0}(\Delta) \tag{2.51}$$

where, from equation (2.8),

$$I_{2n;0}(\Delta) = f_1^{2n} \sum_{r=0}^{\infty} c(r+n)\, c(r)\, (2r+2n)! \; f^{4r} \Delta^{2r} \tag{2.52}$$

with $f^4 = f_1^2 f_2^2$ and for later convenience we have distinguished between the couplings at each vertex by the subindices "1" and "2".

To give meaning to the divergent series (2.52) and to study their overall ultra-violet behaviours, Borel's method of summation is applied. Since the coefficients, $c(r)$, are polynomials in r the summation (2.52) can be written as a linear combination of series of the form

$$\mathcal{J}_{2n+k}(\Delta) = \sum_{r=0}^{\infty} (2r+2n+k)!\, f^{4r} \Delta^{2r} \tag{2.53}$$

Borel's method of summation is now to write the factorial coefficients in (2.53) as integrals,

$$\mathcal{J}_{2n+k}(\Delta) = \sum_{r=0}^{\infty} \int_0^{\infty} d\varsigma\; e^{-\varsigma}\; \varsigma^{2r+2n+k}\, f^{4r} \Delta^{2r}, \tag{2.54}$$

and invert the order of integration and summation to give, on performing the summation,

$$\mathcal{J}_{2n+k}(\Delta) = \int_0^{\infty} d\varsigma\, \frac{e^{-\varsigma}\, \varsigma^{2n+k}}{1 - f^4 \Delta^2 \varsigma^2} \tag{2.55}$$

The asymptotic behaviour $(\Delta \to \infty)$ of this integral is

$$\mathcal{J}_{2n+k}(\Delta) = (2n+k-2)!\left(\frac{1}{f^4\Delta^2}\right) + O\left(\left(\frac{1}{f^4\Delta^2}\right)^2\right) \tag{2.56}$$

Thus, a priori, we may expect $I_{2n;0}(\Delta)$ to also have this asymptotic behaviour. However, it can be shown that the linear combination of $\mathcal{J}_{2n+k}(\Delta)$ integrals is such that the behaviour $\left(\frac{1}{f^4\Delta^2}\right)^{\rho}$ vanishes if $c(-\rho)$ is zero. From equation (2.49) we see that $c(-\rho)$ vanishes for

$$\rho = 1, 2, \ldots\ldots, \beta - d - 1 \tag{2.57}$$

and thus the asymptotic behaviour $(\Delta \to \infty)$ of (2.52) is given by $r = -(\beta - d)$ and hence

$$\begin{aligned} I_{2n;0}(\Delta) &\sim (f^4\Delta^2)^{d-\beta} \\ &\sim (M^4)^{d-\beta} \end{aligned} \tag{2.58}$$

An easier way to derive this result is to write the summation (2.52) as a Sommerfeld-Watson contour integral

$$I_{2n;0}(\Delta) = \frac{1}{2i}\int_C f_1^{2n}\,\frac{c(z+n)\,c(z)}{\sin \pi z}\,\Gamma(2z+2n+1)\,(-f^4)^z\,\Delta^{2z} \tag{2.59}$$

where the contour C is about the positive half of the real axis in the complex z-plane. In the following chapter this Sommerfeld-Watson method will again be employed for taking the Fourier transform and there the uniqueness of the analytic continuation of the coefficients $c(z)$ is important. Here it is only necessary that their continuation agrees at the integer points. It can be shown (17) that the contour can be opened up and then collapsed about the negative real axis. The poles on the negative real axis can then be picked up to yield an infinite series in inverse powers of Δ and hence the overall ultra-violet behaviour is given by the leading pole. Due to the vanishing of $c(-\rho)$ for $\rho < \beta - d$ we see that the first possible pole is at $z = -(\beta - d)$. Although, for n = 1 say, the coefficient $c(z+n)$ vanishes at this point the gamma function in (2.59) may give rise to a simple pole. Hence we again have the asymptotic behaviour given in equation (2.58).

We note that applying the Dyson power counting method to the Lagrangian (2.48) we have

$$\omega(\varphi^2) \sim (M^2)^{d-\beta} \tag{2.60}$$

The integral $I_{2n;0}(\Delta)$ represents a graph with 2n external lines at one vertex and none at the other and hence a naive Dyson power count would indicate a behaviour $(M^2)^{d-\beta-n}$ at one vertex and $(M^2)^{d-\beta}$ at the other and hence an overall behaviour of $(M^2)^{2d-2\beta-n}$ which we see is not correct. The Dyson rule for non-linear Lagrangians is therefore to take the worst behaviour occurring at either vertex and square the result to give a behaviour in agreement with equation (2.58).

This method naturally only yields the worst behaviour but cancellations of the leading divergences may take place. We see now from equation (2.51) that we expect the following overall behaviours for the second order self-energy contributions

$$S_{1;1}(\Delta) \sim M^{6+4\alpha-4\beta}$$

$$S_{1,\mu;1}(\Delta) \sim M^{5+4\alpha-4\beta}$$

$$S_{1,\mu;1,\nu}(\Delta) \sim M^{4+4\alpha-4\beta}$$

$$S_{2;0}(\Delta) \sim M^{8+4\alpha-4\beta}$$

$$S_{2,\mu;0}(\Delta) \sim M^{7+4\alpha-4\beta}$$

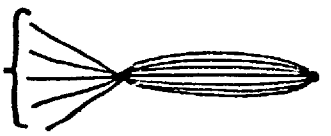
$$S_{2,\mu\nu;0}(\Delta) \sim M^{6+4\alpha-4\beta} \qquad (2.61)$$

Thus $S_{2;0}(\Delta)$ yields the worst behaviour and with the restriction $\beta > \alpha$ is, at worst, M^4. Taking the Fourier transform this would give rise to an overall ultra-violet log divergence. However, we shall see that this leading order is in fact cancelled out and consequently with Lagrangians whose non-derivative parts are given by equation (2.49) (with $\beta > \alpha \geqslant 0$) we shall meet no difficulties with overall ultra-violet divergences for the second order self-energies.

4) Conclusions

The techniques required to calculate second order diagrams for non-linear scalar Lagrangians with derivative couplings have been explicitly given. It has been shown how the dependence arising from the derivative part can be factored out as operators acting on integrals identical to those arising in a non-derivative theory. The worst ultra-violet behaviour expected can be estimated by modifying Dyson's power counting method. It is noted that if the vacuum diagram $S_{0;0}$ gives rise to an overall ultra-violet divergence then, in general, the tadpole-like diagrams

$$S_{n;0}(\Delta) = n$$



will have the same behaviour.

CHAPTER 3

Fourier transforms of the self-energy contributions derived in the previous chapter are taken for the case of zero mass fields. Iso-scalar analogues of the chiral Lagrangians, discussed in chapter 1, are treated as special examples.

1) Fourier Transforms

The method of taking Fourier transforms for theories with massive fields will be discussed in the thesis of Q.Shafi. Here we shall consider the zero mass case for which an elegant technique for taking Fourier transforms exists. The case of non-derivative Lagrangians has been discussed in great detail by Efimov (3), by Volkov (17), and by Salam and Strathdee (18)*. The Fourier transform is first taken in the Symanzik region in p-space ($p^2<0$) and the results obtained are then analytically continued to time-like values of p^2. For $p^2<0$ one continues the propagators $\Delta(x)$ into Euclidean x-space. Hence in the zero mass case one obtains

$$\Delta(x) = \frac{-1}{4\pi^2 x^2} \tag{3.1}$$

where $x^2=-x_0^2-\underline{x}^2$. From this equation it follows that

$$\begin{aligned} \Delta_{\mu_1}(x) &= \tfrac{1}{2}(4\pi)^2 \Delta^2 x_\mu \\ \Delta_{\mu_1\nu_2}(x) &= -\tfrac{1}{2}(4\pi)^2 \Delta^2\left[g_{\mu\nu} - \frac{4x_\mu x_\nu}{x^2}\right] \end{aligned} \tag{3.2}$$

* This reference shows that the method is equivalent to a regularisation of the massless propagator.

Hence the Θ operator defined in equation (2.32) takes on the following form

$$\Theta = (4\pi)^4 \Delta^4 \left[6 + 6\Delta \frac{\partial}{\partial \Delta} + \Delta^2 \left(\frac{\partial}{\partial \Delta} \right)^2 \right]$$

$$= (4\pi)^4 \Delta^4 \left(3 + \Delta \frac{\partial}{\partial \Delta} \right) \left(2 + \Delta \frac{\partial}{\partial \Delta} \right) \qquad (3.3)$$

Thus

$$S_{1;1}(\Delta) \equiv S_{1;1}(\Delta, f^4)$$

$$= \Theta \sum_{r=0}^{\infty} c(r)^2 \, 2r \, (2r)! \, f^{4r} \Delta^{2r-1}$$

$$= (4\pi)^4 \sum_{r=0}^{\infty} c(r)^2 \, 2r \, (2r+2)! \, f^{4r} \Delta^{2r+3}$$

$$= \frac{(4\pi)^4}{2i} \int_C \frac{dz}{\sin \pi z} \, c(z)^2 \, 2z \, \Gamma(2z+3) \left(e^{\pm i\pi} f^4 \right)^z \Delta^{2z+3} \qquad (3.4)$$

where we have written the sum as a Sommerfeld-Watson integral with the contour C taken counter clockwise around the poles on the positive z-axis including the point z = 0. Details of such a procedure may be found in references 17 and 18. Volkov has discussed the restrictions imposed on the coefficients $c(z)^2$.

The invalidity of Carlson's Theorem for the formal power series (3.4) implies that the analytic continuation of the coefficients $c(r)^2$ from the positive integers to complex values z is not, in general, unique. For example additional terms of the form

$$d(r) \sin \pi r \, \Delta^r$$

with undetermined coefficients $d(r)$ may be added. Following Volkov it is, however, possible in certain cases to obtain a unique continuation of $c(r)^2$. For instance the requirement

$$\overline{\lim_{r\to\infty}} \; (2r)^{-a} \left| (2r)! \, c(r)^2 \right|^{\frac{1}{2r}} = A \qquad (3.5)$$

with $0 \leq a < 2$ and $A > 0$ determines $c(z)^2$ uniquely and sets $d(z) \equiv 0$. The condition (3.5) determines a class of non-local interactions [17] and with our restriction that $c(r)$ is a polynomial in r the Lagrangians, and in particular the chiral Lagrangians, that we are considering fall into this class. In the case of more general coefficients the presence of the terms $d(r) \sin \pi r \, \Delta^r$ would lead, after taking the Fourier transforms, to an undetermined entire function in the energy for the self-energy graphs. These coefficients $d(r)$ probably play the role of an infinite set of renormalisation constants. It is of extreme importance that these are identically zero for non-local interactions satisfying condition (3.5), in particular for chiral Lagrangians not written in exponential co-ordinates.

The integral (3.4) has a cut in g^4 from 0 to $+\infty$. The Fourier transform will first be taken for negative values of g^4 and the result then analytically continued to positive physical values of g^4 with an averaging procedure determined by unitarity. For the massless case the Fourier transform of $\Delta^z(x)$ is given by the Gel'fand-Shilov formula [18]

$$D(p^2,z) = i\int d^4x\, e^{ipx}\, \Delta^z(x)$$

$$= \frac{-(16\pi)^{-1}}{\sin\pi z\,\Gamma(z)\,\Gamma(z-1)} \left(\frac{-p^2}{16\pi^2}\right)^{z-2} \tag{3.6}$$

which is valid initially in the strip $0 < \mathrm{Re}\, z < 2$ and outside it by analytic continuation. In order to take the Fourier transform of $S_{1;1}(\Delta,-g^4)$ the Sommerfeld-Watson contour is deformed to lie in the strip $1 < \mathrm{Re}(2z+3) < 2$ along the imaginary axis. This can be done without picking up additional pole contributions since no overall ultra-violet divergences are present. The validity of this deformation also depends on being able to write the gamma function coefficients as integrals as was outlined in the previous chapter. This is discussed in references 17 and 18. One obtains for $s = p^2 < 0$ and in the $\mathrm{Re}(g^4) > 0$ half of the $(-g^4)$-plane

$$F(s,-g^4) = i\int d^4x\, e^{ipx}\, S_{1;1}(\Delta(x),-g^4)$$

$$= -8i\pi^3 \int_{d+i\infty}^{d-i\infty} \frac{dz}{\sin\pi z}\; \frac{c(z)^2\, 2z}{\sin 2\pi z\,\Gamma(2z+3)}\; f^{4z} \left(\frac{-s}{16\pi^2}\right)^{2z+1} \tag{3.7}$$

where $-1 < \mathrm{Re}\, d < -\frac{1}{2}$. We next collapse the contour back around the positive real axis and pick up the residues of the first and second order poles to obtain

$$F(s,-f^4) = -c(0)^2 s + 8\pi^3 \sum_{r=0}^{\infty} (-)^r \frac{(2r-1)\, c(r-1/2)^2 f^{4r-2}}{(2r)!} \left(\frac{s}{16\pi^2}\right)^{2r}$$

$$-s \sum_{r=1}^{\infty} \frac{(-)^r (2r)\, c(r)^2}{(2r+1)!} \left(\frac{f^2 s}{16\pi^2}\right)^{2r} \left\{ \frac{1}{2}\log f^4 + \log\left(\frac{-s}{16\pi^2}\right) + \frac{1}{2}\frac{d}{dz}\log c(z)^2\Big|_{z=r} - \psi(2r+2) + \frac{1}{2r} \right\} \tag{3.8}$$

Here we point out that the original x-space sum (3.4) contains only odd powers of Δ while the evaluation of the Sommerfeld-Watson contour integral, after taking the Fourier transform, also yields terms arising from initially non-existent even powers of Δ. The mathematical reason for this is that the Fourier transform (3.7) of $\Delta^2(x)$ has itself simple poles at the integers $z = 2,3,4,\ldots$ and therefore changes - as a renormalisation - the original simple poles under the Sommerfeld-Watson integral (3.4) into a series of double poles while introducing simple poles for the even powers of Δ.

Analytic continuation of $F(s,-f^4)$ to positive values of the coupling constant f^4 from below and above the cut in the f^4-plane determines the physical amplitude to be

$$F(s,f^4,b) = \alpha F(s,-f^4e^{i\pi}) + \beta F(s,-f^4e^{-i\pi}) \tag{3.9}$$

where

$$\alpha+\beta = 1 \quad \text{and} \quad \mathrm{Re}(\alpha-\beta) = 0 \tag{3.10}$$

The second equation in (3.10) follows from unitarity. Thus

$$\alpha = \tfrac{1}{2}(1-ib) \quad \text{and} \quad \beta = \tfrac{1}{2}(1+ib) \tag{3.11}$$

where b is an arbitrary real constant. Therefore the Fourier transform of the self-energy diagram $S_{1;1}(\Delta)$ is given by

$$\begin{aligned} &F(s,f^4,b) \\ &= -c(0)^2 s + b\,\frac{8\pi^3}{f^2}\,c(-\tfrac{1}{2})^2 - b.8\pi^3 \sum_{r=0}^{\infty} \frac{(r+1)\,c(\tfrac{r}{2}+\tfrac{1}{2})^2}{(r+2)!\,f^2}\left(\frac{f^2 s}{16\pi^2}\right)^{r+2} \\ &\quad - s\sum_{r=1}^{\infty} \frac{(2r)\,c(r)^2}{(2r+1)!}\left(\frac{f^2 s}{16\pi^2}\right)^{2r}\left\{\log\left(\frac{-sf^2}{16\pi^2}\right) + \frac{1}{2}\frac{d}{dz}\log c(z)^2\Big|_{z=r} \right. \\ &\qquad\qquad \left. - \psi(2r+2) + \frac{1}{2r}\right\} \end{aligned} \tag{3.12}$$

where $f^2 = +(f_1^2 f_2^2)^{1/2}$.

The amplitude $F(s,\rho^4,b)$ may be written in the form

$$F(s,\rho^4,b) = F_1(s,\rho^4) + b\, F_2(s,\rho^4) \tag{3.13}$$

where $F_2(s,\rho^4)$ is an entire function of s. In the limit $s \to 0$, i.e. on mass-shell, one obtains

$$bF_2(s,\rho^4) = b \cdot \frac{8\pi^3}{\rho^2} \cdot c(-1/2)^2 \qquad \text{at} \quad s=0 \tag{3.14}$$

while

$$F_1(s,\rho^4) = 0 \qquad \text{at} \quad s=0 \tag{3.15}$$

The p-space contributions from $2\,S_{1,\mu;1}(\Delta)$ and $S_{1,\mu;1,\nu}(\Delta)$ may be evaluated in a similar manner. We note that the p-space contribution from $2\,S_{1,\mu;1}(\Delta)$ is

$$\begin{aligned} F'(s,\rho^4) &= i p_\mu \cdot i \int d^4x\, e^{ipx}\, 2\, S_{1,\mu;1}(\Delta(x),\rho^4) \\ &= -i \int d^4x\, e^{ipx}\, 2 \frac{\partial}{\partial x_\mu} S_{1,\mu;1}(\Delta(x),\rho^4) \\ &= -i \int d^4x\, e^{ipx}\, \partial_\mu \left(8\Delta_{\mu_1\nu_2}\Delta_{\nu_2} + 4\Delta_{\mu_1}\Delta_{\nu_2}\Delta_{\nu_2} \frac{\partial}{\partial\Delta} \right) \frac{\partial}{\partial\Delta} I_{0;0}(\Delta) \end{aligned} \tag{3.16}$$

For the massless propagator

$$\partial^2 \Delta(x) = \delta^4(x) \tag{3.17}$$

and hence on performing the ∂_μ differentiation

$$F'(s,\rho^4) = -i\int d^4x\ e^{ipx}\left[4S_{1;1}(\Delta(x)) - 8ip_\nu\delta(x)\partial_\nu I_{0;0}(\Delta(x)) - 4\delta(x)\,\partial_\nu\partial_\nu I_{0;0}(\Delta(x))\right]$$

(3.18)

From equation (2.58)

$$I_{0;0}(\Delta) \sim (\rho^4\Delta^2)^{d-\beta}$$

$$\sim x^{4\beta-4d} \quad \text{for small } x \qquad (3.19)$$

Hence

$$\partial_\nu I_{0;0}(\Delta) \sim x^{4\beta-4d-1}$$

$$\partial_\nu\partial_\nu I_{0;0}(\Delta) \sim x^{4\beta-4d-2} \qquad (3.20)$$

and since $\beta \geqslant d+1$ both must vanish at x = 0.
Consequently equation (3.18) reduces to

$$F'(s,\rho^4) = -4i\int d^4x\ e^{ipx}\ S_{1;1}(\Delta,\rho^4)$$

$$= -4F(s,\rho^4,b) \qquad (3.21)$$

The p-space contribution from $S_{1,\mu;1,\nu}(\Delta)$ is

$$F''(s,\rho^4) = ip_\mu . -ip_\nu . i\int d^4x\ e^{ipx}\,S_{1,\mu;1,\nu}(\Delta(x),\rho^4)$$

(3.22)

which, in an identical manner, reduces to

$$F''(s,\rho^4) = 4F(s,\rho^4,b) \tag{3.23}$$

Thus the contributions from $2S_{1,\mu;1}(\Delta)$ and $S_{1,\mu;1,\nu}(\Delta)$ cancel. We stress that without our restrictions on the coefficients $c(r)$ the relations (3.21) and (3.23) are not necessarily valid since the $\delta-$ function integrations will not in general vanish.

The self-energy contribution $S_{2;0}(\Delta)$ is a p^2 independent constant. From equation (2.51)

$$S_{2;0}(\Delta) = \rho_1^2 \Theta \sum_{r=0}^{\infty} c(r+1)\, c(r)\, (2r+2)!\; \rho^{4r} \Delta^{2r}$$

$$= \rho_1^2 (4\pi)^4 \sum_{r=0}^{\infty} c(r+1)\, c(r)\, (2r+2)(2r+3)!\; \rho^{4r} \Delta^{2r+4} \tag{3.24}$$

This is the contribution that in the previous chapter appeared to have the worst overall ultra-violet behaviour, i.e. log divergent if $\beta = d+1$. However, the asymptotic behaviour for large Δ is given by the coefficient at r = -1 which in fact vanishes due to the factor (2r + 2) which has arisen by application of the Θ operator. Hence the asymptotic behaviour is lower than Δ^2 and so no overall divergences occur. Performing as before the Fourier transform by Sommerfeld-Watson technique, only the energy independent term has to be taken into account. We immediately obtain the expression

$$G(-\rho^4) = 8i\pi^3 \rho_1^2 \int_{d+i\infty}^{d-i\infty} \frac{dz}{\sin\pi z} \frac{c(z+1)\,c(z)\,(2z+2)}{\sin 2\pi z\;\Gamma(2z+4)}\; \rho^{4z}\left(\frac{-s}{16\pi^2}\right)^{2z+2}\Bigg|_{s=0}$$

$$= -16\pi^2 \rho_1^2\, c(0)\, c(-1)\, \rho^4 \qquad (3.25)$$

Hence

$$G(\rho^4) = \frac{16\pi^2}{\rho_2^2}\, c(0)\, c(-1) \qquad (3.26)$$

is the Fourier transform of $S_{2;0}(\Delta)$. We note that here no ambiguity parameter arises.

The contribution from $S_{2,\mu;0}(\Delta)$ always vanishes trivially in x-space since it is of the form

$$\delta^4(x)\int S_{2,\mu;0}(\Delta(z))\, d^4z \qquad (3.27)$$

where

$$S_{2,\mu;0}(\Delta(z)) = z_\mu\, f(z^2) \qquad (3.28)$$

with $f(z^2)$ some function of z^2 and the integral (3.27) then vanishes identically by symmetry.

Finally we have from equation (2.51)

$$S_{2,\mu\nu;0}(\Delta) = 2 g_{\mu\nu}\, \Delta_{\rho_2}\Delta_{\rho_2}\, \rho_1^2 \sum_{r=0}^{\infty} c(r+1)\, c(r)\, (2r+2)!\; \rho^{4r}\, \Delta^{2r}$$

$$= 2g_{\mu\nu}\, 16\pi^2 f_1^2 \sum_{r=0}^{\infty} c(r+1)\,c(r)\,(2r+2)!\; f^{4r}\,\Delta^{2r+3} \tag{3.29}$$

which as shown in the previous chapter exhibits no overall ultra-violet divergences. For the p-space contribution we have the expression

$$H(-p^4) = -p^2 . i\pi f_1^2 \int_{\alpha+i\infty}^{\alpha-i\infty} \frac{dz}{\sin\pi z}\, \frac{c(z+1)\,c(z)}{\sin 2\pi z\, \Gamma(2z+3)}\, f^{4z} \left(\frac{-s}{16\pi^2}\right)^{2z+1} \Bigg|_{s=0}$$

$$= -p^2 \pi\, c(1/2)\,c(-1/2)\, \frac{f_1^2}{f^2} \tag{3.30}$$

and analytically continuing in p^4 using equation (3.9)

$$H(p^4) = s.b\pi\, c(1/2)\,c(-1/2)\, \frac{f_1^2}{f^2} \tag{3.31}$$

which clearly vanishes on the mass-shell.

All final results are to be evaluated with $f_1^2 = f_2^2$. Then with the definition $f^2 = +(f_1^2 f_2^2)^{1/2}$ it follows that

$$\frac{f_i^2}{f^2} = \pm 1 \qquad \text{for } f_i^2 \gtrless 0 \qquad i = 1 \text{ or } 2 \tag{3.32}$$

and our results hold for positive and negative values of the coupling f^2.

Thus the self-energies from all second order diagrams for Lagrangians falling into the class that we have considered may be simply determined by substituting into equations (3.12), (3.26) and (3.31) the coefficients $c(r)$ appearing in the expansion of $v(\varphi)$, the non-derivative part of the Lagrangian. We remark, especially with reference to equations (3.26) and (3.31), that the use of the Sommerfeld-Watson method in taking Fourier transforms of divergent series has a formal character. However, the results given here can also be obtained using other methods [19].

The Lagrangian we are considering can be shown to be equivalent to a free field Lagrangian. If we therefore wish to require that the sum of all second order self-energy graphs vanishes on mass-shell for zero mass particles then this implies

$$b\,F_2(f^4) + G(f^4) = 0 \qquad \text{at} \quad s = 0 \tag{3.33}$$

and allows the ambiguity parameter b to be uniquely determined as

$$b = -\frac{2}{\pi}\,\frac{c(0)\,c(-1)}{c(-1/2)^2}\,\frac{f^2}{f_2^2} \tag{3.34}$$

It is seen that zero self-energy (to second order) on mass-shell implies, in general, $b \neq 0$. An extremely interesting point to note is the coefficient $c(-1)$ appears in the numerator of the expression for b. From equation (2.49) we know that this coefficient vanishes if $\beta - 2 \geqslant \alpha \geqslant 0$ i.e. for a theory where the scalar part of the Lagrangian $\sim M^{-4}$ (or better) applying the usual power counting method of Dyson.

When the self-energy graphs contain additional ultra-violet divergences then it is not possible to deduce from equation (3.33) a unique value for b. This will be the case for the chiral Lagrangians.

2) Special Examples

We now apply the results to the scalar (no iso-spin) analogues of the chiral Lagrangians described in chapter 1. These are of the form

$$\mathcal{L}(\varphi,\partial_\mu\varphi) = \tfrac{1}{2} : g(\varphi)\,\partial_\mu\varphi\,\partial_\mu\varphi : \tag{3.35}$$

where $g(\varphi)$ is a metric on the circle S^1. We consider two different co-ordinate systems of S^1. Co-ordinate system I is obtained by restricting the co-ordinates of the plane $\mathbb{R}^2$ to a circle of radius $1/\lambda$ giving

$$g^{I}(\varphi) = \frac{1}{1-\lambda^2\varphi^2} \tag{3.36}$$

which is the scalar analogue of Gasiorowicz co-ordinates. For co-ordinate system II we take the stereographic co-ordinate system on S^1 to obtain

$$g^{II}(\varphi) = \frac{1}{(1+\frac{\lambda^2}{4}\varphi^2)^2} \tag{3.37}$$

which is the scalar analogue of both Weinberg and Schwinger co-ordinates.

In the massless case these scalar Lagrangians can be reduced to the usual free massless scalar field Lagrangian

$$\mathcal{L}(\psi) = \frac{1}{2} : \partial_\mu \psi \, \partial_\mu \psi : \qquad (3.38)$$

by the transformation

$$\psi = G(\varphi) \qquad (3.39)$$

where

$$G(\varphi) = \int [g(\varphi)]^{1/2} d\varphi \qquad (3.40)$$

The Lagrangians $\mathcal{L}^{I}$ and $\mathcal{L}^{II}$ may be generated from the free Lagrangian $\mathcal{L}(\psi)$ of equation (3.38) by the respective transformations

$$\psi = -\frac{1}{\lambda} \sin^{-1}(\lambda\varphi) \qquad (3.41)$$

and

$$\psi = \frac{2}{\lambda} \tan^{-1}\left(\frac{\lambda\varphi}{2}\right) \qquad (3.42)$$

These transformations from a free field theory are, of course, not possible for the chiral SU(2) x SU(2) theory. The two Lagrangians $\mathcal{L}^{I}, \mathcal{L}^{II}$ are also related by a co-ordinate transformation of the field since Lagrangian $\mathcal{L}^{II}(\varphi)$ can be obtained from the Lagrangian $\mathcal{L}^{I}(\varphi)$ by the transformation

$$\varphi \longrightarrow \frac{\varphi}{1+\frac{\lambda^2}{4}\varphi^2} \tag{3.43}$$

Subtracting from the total Lagrangian the free part one obtains the following two interaction Lagrangians

$$\mathcal{L}^{\mathrm{I}}_{int}(\lambda^2,\varphi) = \frac{1}{2} : \partial_\mu\varphi\,\partial_\mu\varphi\left\{\frac{1}{1-\lambda^2\varphi^2} - 1\right\}: \tag{3.44}$$

and

$$\mathcal{L}^{\mathrm{II}}_{int}(\kappa^2,\varphi) = \frac{1}{2} : \partial_\mu\varphi\,\partial_\mu\varphi\left\{\frac{1}{(1+\kappa^2\varphi^2)^2} - 1\right\}: \tag{3.45}$$

where

$$\kappa^2 = \frac{1}{4}\lambda^2 \tag{3.46}$$

Note that the two interaction Lagrangians are also related by differentiation with respect to the coupling constant. We have

$$\mathcal{L}^{\mathrm{II}}_{int}(\kappa^2) = \frac{\partial}{\partial\kappa^2}\,\kappa^2\,\mathcal{L}^{\mathrm{I}}_{int}(-\kappa^2) \tag{3.47}$$

This relation would enable one to deduce, in perturbation theory, all the Green's functions of $\mathcal{L}^{\mathrm{II}}_{int}$ to any order in the major coupling constant from the corresponding Green's functions of $\mathcal{L}^{\mathrm{I}}_{int}$ and is hence another reason for having distinguished between the couplings arising at each vertex.

We see that in both co-ordinate systems the total Lagrangians can be dealt with as special cases of the previous chapter. Simply by noting that $g^{I}(\varphi) \sim M^{-2}$ and $g^{II}(\varphi) \sim M^{-4}$ we can immediately tell that the b independent contribution to the on-mass-shell self-energy, i.e. that of $S_{2;0}$, will be zero for co-ordinate system II but non zero for co-ordinate system I. Explicitly the respective coefficients and couplings are

$$
\begin{aligned}
c^{I}(r) &= \tfrac{1}{2} \qquad & f_1^2 = f_2^2 = \lambda^2 \\
c^{II}(r) &= \tfrac{1}{2}(r+1) \qquad & f_1^2 = f_2^2 = -k^2
\end{aligned}
\tag{3.48}
$$

and from equations (3.14) and (3.26) the second order contributions for the total Lagrangians are

$$
\begin{aligned}
\tilde{S}^{I}(s=0,\lambda^2) &= \frac{2\pi^3 b}{\lambda^2} + \frac{4\pi^2}{\lambda^2} \\
\tilde{S}^{II}(s=0,k^2) &= \frac{\pi^3 b}{2k^2} \left(= \frac{2\pi^3 b}{\lambda^2}\right)
\end{aligned}
\tag{3.49}
$$

Note that, since $\lambda^2 = 4k^2$ the $S_{1;1}$ on-mass-shell contributions are equal in both co-ordinate systems.

To consider the interaction Lagrangians (3.44) and (3.45) we need only evaluate the additional contributions resulting from the subtraction of the free part from the total Lagrangian. Explicitly the integrals for the non-derivative part of the interaction Lagrangian reduce to

$$I_{0;0}(\Delta) = \sum_{r=0}^{\infty} c(r)^2 (2r)!\, f^{4r} \Delta^{2r} - 1$$

$$I_{2;0}(\Delta) = f_1^2 \sum_{r=0}^{\infty} c(r+1)\, c(r)\, (2r+2)!\, f^{4r} \Delta^{2r} - f_1^2 c(1) \tag{3.50}$$

where the additional contributions from the free part are just the single terms subtracted off from the infinite sums. These sums simply yield the results already given in equation (3.49). Applying the appropriate operators as defined in equation (2.51) to obtain expressions for the x-space second order contributions we see that the additional terms only contribute to $S_{1,\mu;1,\nu}(\Delta)$, $S_{2;0}(\Delta)$ and $S_{2,\mu\nu;0}(\Delta)$. The respective expressions for these additional terms are

$$S'_{1,\mu;1,\nu}(\Delta) = -\Delta_{\mu,\nu_2} \tag{3.51}$$

$$\begin{aligned} S'_{2;0}(\Delta) &= -2f_1^2 c(1)\, \Delta_{\mu,\nu_2} \Delta_{\mu,\nu_2} \\ &= -6 f_1^2 (4\pi)^4 c(1)\, \Delta^4 \end{aligned} \tag{3.52}$$

and

$$\begin{aligned} S'_{2,\mu\nu;0}(\Delta) &= -2 g_{\mu\nu} f_1^2 c(1)\, \Delta_{\rho_2} \Delta_{\rho_2} \\ &= 2 g_{\mu\nu} f_1^2 (4\pi)^2 c(1)\, \Delta^3 \end{aligned} \tag{3.53}$$

It is here that we notice the appearance of overall ultra-violet divergences. In equation (3.52) there is a quartic divergence and in equation (3.53) a quadratic divergence. A renormalisation procedure for certain non-polynomial scalar Lagrangians without derivative couplings yielding overall log and quadratic divergences has been discussed in reference 11. A renormalisation will not be attempted here but in removing the divergences in equations (3.52) and (3.53), finite parts will remain which we denote by C_4 and C_2 respectively. Thus the Fourier transforms of the expressions (3.51), (3.52) and (3.53) together yield the following additional contribution to the self-energy

$$\tilde{S}'(s) = s + C_4 + C_2 s \tag{3.54}$$

Thus on mass-shell we are left with the undetermined constant C_4 arising from the removal of the quartic divergence. Clearly this constant is not necessarily the same in both co-ordinate systems. Hence equations (3.49) must be amended to

$$\begin{aligned}\tilde{S}^{\mathrm{I}}(s=0,\lambda^2) &= \frac{2\pi^3 b}{\lambda^2} + \frac{4\pi^2}{\lambda^2} + C_4\\ \tilde{S}^{\mathrm{II}}(s=0,\kappa^2=\tfrac{1}{4}\lambda^2) &= \frac{2\pi^3 b}{\lambda^2} + C_4'\end{aligned}\tag{3.55}$$

and are the final second order on-mass-shell self-energy contributions for the two special interaction Lagrangians considered.

3) Conclusions

The Fourier transforms of the second order self-energy graphs, evaluated in chapter 2, have been taken assuming massless fields. The on-mass-shell contributions to the self-energy are then determined by the analytically continued expansion coefficients $c(z)$ to the critical points $z=-1,-1/2,0$ and yield, for a theory with no overall ultra-violet divergences, the general self-mass contribution

$$\delta\mu^2 = -g^2\left\{ b\frac{8\pi^3}{(\rho_1^2\rho_2^2)^{1/2}}c(-1/2)^2 + \frac{16\pi^2}{\rho_2^2}c(0)\,c(-1)\right\} \tag{3.56}$$

where b is a real parameter. Restricting ourselves to the class of non-local interactions defined by equation (2.49) it is clear that the second order self-mass will be an invariant for those field transformations

$$\varphi \longrightarrow \varphi' = G(\varphi) \tag{3.57}$$

which leave unchanged

$$\frac{c(-1/2)^2}{(\rho_1^2\rho_2^2)^{1/2}} \quad \text{and} \quad \frac{c(0)\,c(-1)}{\rho_2^2}$$

In general, however, there are additional ultra-violet divergences which introduce extra renormalisation parameters and then this statement becomes more complicated.

In the case of the scalar analogues of the chiral Lagrangians considered the finite self-energy graph $S_{1,1}(\Delta)$ gives the same result on mass-shell for both co-ordinate systems; the invariance of $c(-1/2)^2/(\rho_1^2\rho_2^2)^{1/2}$ is quite remarkable. However the $S_{2;0}(\Delta)$ graphs are infinite. Hence in this model, using the Efimov-Fradkin method of partial summation of perturbation theories, the theorem of Coleman, Wess and Zumino [(9)] that co-ordinate transformations leave invariant the on-mass-shell results of S-matrix elements with a fixed number of loops, cannot be checked directly because the S-matrix elements are infinite to each order in $\mathcal{L}_{int}(\varphi, \partial_\mu\varphi)$. This theorem must be implemented by the requirement of a co-ordinate independent choice of the parameter b and the renormalisation parameters. Co-ordinate independence to second order can be guaranteed by a suitable choice of the renormalisation parameters C_4 and C_4'.

CHAPTER 4

Techniques for calculating contributions from chiral SU(2) x SU(2) Lagrangians are presented.

1) x-space Methods for the Chiral Lagrangians

We shall now look at extending the techniques already presented so as to take into account iso-spin and thus treat correctly the chiral Lagrangians described in chapter 1. The chiral SU(2) x SU(2) meson Lagrangians may always be written in the form

$$\mathcal{L}(\underline{\varphi}, \partial_\mu \underline{\varphi}) = g : \partial_\mu \varphi_k \partial_\mu \varphi_\ell \; \sigma_{k\ell}(\underline{\varphi}) \tag{4.1}$$

where

$$\sigma_{k\ell}(\underline{\varphi}) = \delta_{k\ell}\, \sigma^{(A)}(\underline{\varphi}.\underline{\varphi}) + f^2 \varphi_k \varphi_\ell\, \sigma^{(B)}(\underline{\varphi}.\underline{\varphi}) \tag{4.2}$$

Latin letters are used for the iso-spin labels and Greek letters for the Lorentz labels. Defining the Feynman propagator

$$\langle T\{\varphi_i(x_1)\;\varphi_j(x_2)\}\rangle = \Delta_{ij}(x_1 - x_2) \tag{4.3}$$

and derivatives thereof by the modified time ordering operator T*

$$\langle T^*\{\varphi_{i,\mu}(x_1)\;\varphi_j(x_2)\} = \Delta_{i,\mu;j}(x_1 - x_2)$$

$$\equiv \frac{\partial}{\partial x_{1,\mu}} \Delta_{i;j}(x_1 - x_2)$$

$$\langle T^{*}\{\varphi_{i,\mu}(x_1)\ \varphi_{j,\nu}(x_2)\}\rangle = \Delta_{i,\mu;j,\nu}(x_1-x_2)$$

$$\equiv \frac{\partial}{\partial x_{1,\mu}}\frac{\partial}{\partial x_{2,\nu}}\Delta_{i;j}(x_1-x_2) \qquad (4.4)$$

either technique described in chapter 2 can be extended to obtain, for the case of m external non-derived scalar lines with iso-spin labels $i_1,\ldots\ldots,i_m$ at one vertex and n lines with iso-spin labels $j_1,\ldots\ldots,j_n$ at the other,

$$S_{i_1,\ldots,i_m;j_1,\ldots,j_n}(\Delta) = \tilde{\Theta}_{k_1l_1;k_2l_2}\, I_{i_1,\ldots,i_m;j_1,\ldots,j_n:k_1l_1;k_2l_2}(\Delta) \qquad (4.5)$$

where

$$\tilde{\Theta}_{k_1l_1;k_2l_2} = 2\Delta_{k_1,\mu;k_2,\nu}\,\Delta_{l_1,\mu;l_2,\nu}$$

$$+4\Delta_{k_1,\mu;p}\,\Delta_{l_1,\mu;l_2,\nu}\,\Delta_{q;k_2,\nu}\frac{\partial}{\partial\Delta_{q;p}}$$

$$+\Delta_{k_1,\mu;p}\,\Delta_{l_1,\mu;p'}\,\Delta_{q;k_2,\nu}\,\Delta_{q';l_2,\nu}\frac{\partial}{\partial\Delta_{q;p}}\frac{\partial}{\partial\Delta_{q';p'}} \qquad (4.6)$$

and

$$I_{i_1,\dots,i_m;j_1,\dots,j_n:k_1l_1;k_2l_2}(\Delta)$$

$$= \int_0^\infty d^3\underline{\zeta}^1\, d^3\underline{\zeta}^2\, (-)^{m+n}\, \zeta^1_{i_1} \cdots\cdots \zeta^1_{i_m}\, \zeta^2_{j_1} \cdots\cdots \zeta^2_{j_n}\, \tilde{\sigma}_{k_1l_1}(\underline{\zeta}^1)\, \tilde{\sigma}_{k_2l_2}(\underline{\zeta}^2) \cdot$$

$$\cdot \exp\left[\zeta^1_k \Delta_{kt} \zeta^2_t\right]$$

$$= \frac{1}{\pi^3} \int d^3\underline{u}\, d^3\underline{u}^*\, e^{-\underline{u}\cdot\underline{u}^*} \frac{\partial}{\partial(\Delta_{i_1a_1}u_{a_1})} \cdots\cdots \frac{\partial}{\partial(\Delta_{i_ma_m}u_{a_m})} \frac{\partial}{\partial u^*_{j_1}} \cdots\cdots \frac{\partial}{\partial u^*_{j_n}} \cdot$$

$$\cdot\, \sigma_{k_1l_1}(\Delta_{ab}u_b)\, \sigma_{k_2l_2}(\underline{u}^*) \qquad (4.7)$$

As before all other second order graphs can be simply obtained by the appropriate propagator differentiations. Thus in analogy to equation (2.43) we obtain

$$S_{i_1,\dots,i_{m-1},i_m(\mu);j_1,\dots,j_n}(\Delta)$$

$$= \left[4\Delta_{k_1\mu;l_2\nu}\Delta_{q;k_2\nu} + 2\Delta_{k_1\mu;p'}\Delta_{q;k_2\nu}\Delta_{q';l_2\nu}\frac{\partial}{\partial\Delta_{q';p'}}\right] \cdot$$

$$\cdot\, I_{i_1,\dots,i_{m-1},q;j_1,\dots,j_n:k_1i_m;k_2l_2}(\Delta)$$

$$S_{i_1,\dots,i_{m-1},i_m(\mu);j_1,\dots,j_{n-1},j_n(\nu)}(\Delta)$$

$$= 4\left[\Delta_{k_1\mu;k_2\nu} + \Delta_{k_1\mu;p}\Delta_{q;k_2\nu}\frac{\partial}{\partial\Delta_{q;p}}\right] I_{i_1,\dots,i_{m-1};j_1,\dots,j_{n-1}:k_1i_m;k_2j_n}(\Delta)$$

$$S_{i_1,\dots,i_{m-2},i_{m-1}(\rho),i_m(\mu);\,j_1,\dots,j_n}(\Delta)$$

$$= 2\, g_{\mu\rho}\, \Delta_{q;k_2,\nu}\, \Delta_{q';l_2,\nu}\, I_{i_1,\dots,i_{m-2},q',q;j_1,\dots,j_n:i_{m-1},i_m;k_2 l_2}(\Delta)$$

$$S_{i_1,\dots,i_{m-2},i_{m-1}(\rho),i_m(\mu);\,j_1,\dots,j_{n-1},j_n(\nu)}(\Delta)$$

$$= 4\, g_{\mu\rho}\, \Delta_{q;k_2,\nu}\, I_{i_1,\dots,i_{m-2}q;j_1,\dots,j_{n-1}:i_{m-1}i_m;k_2 j_n}(\Delta)$$

$$S_{i_1,\dots,i_{m-1}(\rho),i_m(\mu);\,j_1,\dots,j_{n-1}(\sigma),j_n(\nu)}(\Delta)$$

$$= 4\, g_{\mu\rho}\, g_{\nu\sigma}\, I_{i_1,\dots,i_{m-2};j_1,\dots,j_{n-2}:i_{m-1}i_m;j_{n-1}j_n}(\Delta) \tag{4.8}$$

where the suffix $i_m(\mu)$ denotes an external derived scalar line with iso-spin i_m . In analogy to equation (2.44) we also have

$$S_{i_1,\dots,i_{2n+m};\,j_1,\dots,j_m}(\Delta) = \prod_{\alpha=1}^{m} \frac{\partial}{\partial \Delta_{i_{2n+\alpha};j_\alpha}} \cdot S_{i_1,\dots,i_{2n};0}(\Delta) \tag{4.9}$$

Thus we again conclude that all second order graphs are written in the form of an operator acting on integrals which are identical to those arising for second order graphs using a non-derivative "Lagrangian" $U_{kl}(\underline{\varphi})$. We are now left with the difficulty of solving the integral

$I_{i_1,\ldots,i_{2n};0:k_1\ell_1;k_2\ell_2}(\Delta)$ which must, in general, be operated on by an arbitrary number of $\Delta_{i;j}$ differentiations. Once these differentiations have been performed we can use the fact that $\Delta_{i;j}$ is diagonal in its iso-spin labels, i.e.

$$\Delta_{i;j}(x_1-x_2) = \delta_{ij}\,\Delta(x_1-x_2) \tag{4.10}$$

and similarly for the derived propagators.

Taking the non-derivative part of the Lagrangian to be given by equation (4.2) and assuming the expansions

$$\delta_{k\ell}\, v^{(0)}(\underline{\varphi}.\underline{\varphi}) = \sum_{r=0}^{\infty} a(r)\, f^{2r}\, (\underline{\varphi}.\underline{\varphi})^r\, \delta_{k\ell}$$

$$f^2 \varphi_k \varphi_\ell\, v^{(0)}(\underline{\varphi}.\underline{\varphi}) = \sum_{r=0}^{\infty} b(r+1)\, f^{2r+2}\, \varphi_k\varphi_\ell\, (\underline{\varphi}.\underline{\varphi})^r \tag{4.11}$$

then the integral (4.7) splits up into 4 parts:

$$\begin{aligned} I_{i_1,\ldots,i_{2n};0:k_1\ell_1;k_2\ell_2}(\Delta) &= I^{AA}_{i_1,\ldots,i_{2n};0:k_1\ell_1;k_2\ell_2}(\Delta) + I^{BB}_{i_1,\ldots,i_{2n};0:k_1\ell_1;k_2\ell_2}(\Delta) \\ &\quad + I^{AB}_{i_1,\ldots,i_{2n};0:k_1\ell_1;k_2\ell_2}(\Delta) + I^{BA}_{i_1,\ldots,i_{2n};0:k_1\ell_1;k_2\ell_2}(\Delta) \end{aligned} \tag{4.12}$$

The result of operating on these with

$$\prod_{d=1}^{\ell} \frac{\partial}{\partial \Delta_{i_{2n+d}};j_{2n+d}} \tag{4.13}$$

is as follows

$$\left[\prod_{\alpha=1}^{\ell} \frac{\partial}{\partial \Delta_{i_{2n+\alpha};j_{2n+\alpha}}}\right] I^{AA}_{i_1,\ldots,i_{2n};0:k_1\ell_1;k_2\ell_2}(\Delta)$$

$$= \delta_{k_1\ell_1}\delta_{k_2\ell_2}\int \frac{d^3\underline{u}\,d^3\underline{u}^*}{\pi^3}\, e^{-\underline{u}\cdot\underline{u}^*}\, u_{j_{2n+1}}\cdots u_{j_{2n+\ell}}\left[\prod_{\alpha=1}^{2n+\ell}\frac{\partial}{\partial \Delta_{i_\alpha a}u_a}\right]\cdot$$

$$\cdot\, \sigma^{(A)}(\Delta_{ab}u_b\Delta_{ac}u_c)\;\sigma^{(A)}(\underline{u}^*\cdot\underline{u}^*)$$

$$= \delta_{k_1\ell_1}\delta_{k_2\ell_2}\sum_{s=0}^{[n+\ell/2]}\;\sum_{d_1<d_2\cdots d_{2s}\le 2n+\ell}\delta_{i_{d_1}\cdots i_{d_{2s}}}\int\frac{d^3\underline{u}\,d^3\underline{u}^*}{\pi^3}\, e^{-\underline{u}\cdot\underline{u}^*}\cdot$$

$$\cdot\, u_{i_{d_{2s+1}}}\cdots u_{i_{d_{2n+\ell}}}\, u_{j_{2n+1}}\cdots u_{j_{2n+\ell}}\,\Delta^{2n-2s+\ell}\left[2\frac{\partial}{\partial(\Delta^2\underline{u}\cdot\underline{u})}\right]^{2n+\ell-s}\cdot$$

$$\cdot\,\sigma^{(A)}(\Delta^2\underline{u}\cdot\underline{u})\;\;\sigma^{(A)}(\underline{u}^*\cdot\underline{u}^*)$$

$$= \delta_{k_1\ell_1}\delta_{k_2\ell_2}\sum_{s=0}^{[n+\ell/2]}\;\sum_{d_1<d_2\cdots<d_{2s}\le 2n+\ell}\frac{\delta_{i_{d_1}\cdots i_{d_{2s}}}\,\delta_{i_{d_{2s+1}}\cdots i_{d_{2n+\ell}}j_{2n+1}\cdots j_{2n+\ell}}}{3.5.\cdots\cdots(2n+2\ell-2s+1)}\,.$$

$$\cdot\int\frac{d^3\underline{u}\,d^3\underline{u}^*}{\pi^3}\, e^{-\underline{u}\cdot\underline{u}^*}\,(\Delta^2\underline{u}\cdot\underline{u})^{n+\ell-s}\,\Delta^{-\ell}\left[2\frac{\partial}{\partial(\Delta^2\underline{u}\cdot\underline{u})}\right]^{2n+\ell-s}\sigma^{(A)}(\Delta^2\underline{u}\cdot\underline{u})\;\sigma^{(A)}(\underline{u}^*\cdot\underline{u}^*)$$

$$= \delta_{k_1 \ell_1} \delta_{k_2 \ell_2} \sum_{s=0}^{[n+\ell/2]} \sum_{d_1 < d_2 \cdots < d_{2s} \leq 2n+\ell} \frac{\delta_{i_{d_1} \cdots i_{d_{2s}}} \delta_{i_{d_{2s+1}} \cdots i_{d_{2n+\ell}} j_{2n+1} \cdots j_{2n+\ell}}}{3.5. \cdots \cdots .(2n+2\ell-2s+1)} \,.$$

$$. f_1^{2n} \sum_{r=0}^{\infty} a(r+n)\, a(r)\, (2r+2n)(2r+2n-2) \cdots\cdots (2r-2n-2\ell+2s+2)\, (2r+1)! \,.$$

$$. f^{4r} \Delta^{2r-\ell} \qquad (4.14)$$

The last two lines are shown in detail in the appendix. The notation used is $[n+\ell/2]$ for the integral part of $n+\ell/2$, $d_1, \ldots\ldots, d_{2n+\ell}$ are some permutation of the numbers $1, \ldots\ldots, 2n+\ell$ and $\delta_{i_1 \ldots\ldots i_{2r}}$ is the symmetric combination of all kronecker δ's e.g.

$$\delta_{i_1 i_2 i_3 i_4} = \delta_{i_1 i_2} \delta_{i_3 i_4} + \delta_{i_1 i_3} \delta_{i_2 i_4} + \delta_{i_1 i_4} \delta_{i_2 i_3} \qquad (4.15)$$

$$\left[\prod_{d=1}^{\ell} \frac{\partial}{\partial(\Delta_{i_{2n+d}\, j_{2n+d}})} \right] I^{BB}_{i_1, \ldots i_{2n}; 0 : i_{2n+2\ell+1}, i_{2n+2\ell+2}; k_2 \ell_2}(\Delta)$$

$$= f^4 \int \frac{d^3\underline{u}\, d^3\underline{u}^*}{\pi^3}\, e^{-\underline{u}.\underline{u}^*}\, u_{j_{2n+1}} \cdots u_{j_{2n+\ell}} \left[\prod_{d=1}^{2n+\ell} \frac{\partial}{\partial(\Delta_{i_d a} u_a)} \right] .$$

$$. \Delta_{i_{2n+\ell+1} a} u_a \Delta_{i_{2n+\ell+2}; b} u_b\, \sigma^{(B)}(\Delta_{ab} u_b \Delta_{ac} u_c)\, u^*_{k_2} u^*_{\ell_2}\, \sigma^{(B)}(\underline{u}^*.\underline{u}^*)$$

$$= \sum_{s=0}^{[n+\ell/2+1]} \sum_{d_1<d_2\cdots<d_{2s}\le 2n+\ell+2} \tilde{\delta}_{i_{d_1}\cdots i_{d_{2s}}} f^4 \int \frac{d^3\underline{u}\, d^3\underline{u}^*}{\pi^3} e^{-\underline{u}\cdot\underline{u}^*} .$$

$$\cdot u_{i_{d_{2s+1}}} \cdots u_{i_{d_{2n+\ell+2}}} u_{j_{2n+1}} \cdots u_{j_{2n+\ell}} \Delta^{2n-2s+\ell+2} \left[2 \frac{\partial}{\partial(\Delta^2 \underline{u}\cdot\underline{u})} \right]^{2n+\ell-s} .$$

$$\cdot \sigma^{(B)}(\Delta^2 \underline{u}\cdot\underline{u})\, u^*_{k_2} u^*_{\ell_2}\, \sigma^{(B)}(\underline{u}^*\cdot\underline{u}^*)$$

$$= \sum_{s=0}^{[n+\ell/2+1]} \sum_{d_1<d_2\cdots d_{2s}\le 2n+\ell+2} \tilde{\delta}_{i_{d_1}\cdots i_{d_{2s}}} \int \frac{d^3\underline{u}\, d^3\underline{u}^*}{\pi^3} e^{-\underline{u}\cdot\underline{u}^*} .$$

$$\cdot u_{i_{d_{2s+1}}} \cdots u_{i_{d_{2n+\ell+2}}} u_{j_{2n+1}} \cdots u_{j_{2n+\ell}} u^*_{k_2} u^*_{\ell_2} \sum_{r=0}^{\infty} f_1^{2n} f^{4r+4} b(r+n+1)\, b(r+1) \cdot$$

$$\cdot \Delta^{2n-2s+\ell+2} (2r+2n)(2r+2n-2)\cdots(2r-2n-2\ell+2s+2)\, (\Delta^2 \underline{u}\cdot\underline{u})^{r-n-\ell+s} (\underline{u}^*\cdot\underline{u}^*)^r$$

$$= \sum_{s=0}^{[n+\ell/2+1]} \sum_{d_1<d_2\cdots d_{2s}\le 2n+\ell+2} \tilde{\delta}_{i_{d_1}\cdots i_{d_{2s}}} f_1^{2n} \sum_{r=0}^{\infty} b(r+n+1)\, b(r+1)\, f^{4r+4} \Delta^{2r+2-\ell} .$$

$$\cdot \frac{(2r+2n)(2r+2n-2)\cdots(2r-2n-2\ell+2s+2)}{3.5.\cdots(2n+2\ell-2s+5)} \cdot (2r+1)!\, (2r+3) \cdot$$

$$\cdot \Big\{ (2r+5)\, \delta_{k_2\ell_2 i_{d_{2s+1}}\cdots i_{d_{2n+\ell+2}} j_{2n+1}\cdots j_{2n+\ell}}$$

$$- (2n+2\ell-2s+5)\, \delta_{k_2\ell_2}\, \delta_{i_{d_{2s+1}}\cdots i_{d_{2n+\ell+2}} j_{2n+1}\cdots j_{2n+\ell}} \Big\} \qquad (4.16)$$

where $\tilde{\delta}_{i_{\alpha_1}\cdots i_{\alpha_{2s}}}$ is as defined before except that the term $\delta_{i_{2n+1}i_{2n+2}}$ must not occur e.g.

$$\tilde{\delta}_{i_1 i_2 i_{2n+1} i_{2n+2}} = \delta_{i_1 i_{2n+1}}\delta_{i_2 i_{2n+2}} + \delta_{i_1 i_{2n+2}}\delta_{i_2 i_{2n+1}}$$

$$\tilde{\delta}_{i_1 i_2 i_3 i_4} = \delta_{i_1 i_2 i_3 i_4} \qquad \text{for} \quad 2n+\ell \geq 4 \tag{4.17}$$

$$\left[\prod_{\alpha=1}^{\ell}\frac{\partial}{\partial(\Delta_{i_{2n+\alpha}j_{2n+\alpha}})}\right] I^{AB}_{i_1,\dots,i_{2n};0:k_1\ell_1;k_2\ell_2}(\Delta)$$

$$= f_2^2\,\delta_{k_1\ell_1}\int\frac{d^3\underline{u}\,d^3\underline{u}^*}{\pi^3}\, e^{-\underline{u}\cdot\underline{u}^*}\, u_{j_{2n+1}}\cdots u_{j_{2n+\ell}}\left[\prod_{\alpha=1}^{2n+\ell}\frac{\partial}{\partial(\Delta_{i_\alpha a}u_a)}\right]\cdot$$

$$\cdot\,\sigma^{(A)}(\Delta_{ab}u_b\Delta_{ac}u_c)\, u^*_{k_2}u^*_{\ell_2}\,\sigma^{(B)}(\underline{u}^*\cdot\underline{u}^*)$$

$$= \delta_{k_1\ell_1}\sum_{s=0}^{[n+\ell/2]}\;\sum_{\alpha_1<\alpha_2<\cdots<\alpha_{2s}\leq 2n+\ell}\tilde{\delta}_{i_{\alpha_1}\cdots i_{\alpha_{2s}}}\, f_2^2\int\frac{d^3\underline{u}\,d^3\underline{u}^*}{\pi^3}\, e^{-\underline{u}\cdot\underline{u}^*}\cdot$$

$$\cdot\, u_{i_{\alpha_{2s+1}}}\cdots u_{i_{\alpha_{2n+\ell}}}\, u_{j_{2n+1}}\cdots u_{j_{2n+\ell}}\;\Delta^{2n-2s+\ell}\left[2\frac{\partial}{\partial(\Delta^2\underline{u}\cdot\underline{u})}\right]^{2n+\ell-s}\cdot$$

$$\cdot\,\sigma^{(A)}(\Delta^2\underline{u}\cdot\underline{u})\, u^*_{k_2}u^*_{\ell_2}\,\sigma^{(B)}(\underline{u}^*\cdot\underline{u}^*)$$

$$= \delta_{k_1 l_1} \sum_{s=0}^{[n+l/2]} \sum_{d_1<d_2\cdots d_{2s}\le 2n+l} \delta_{i_{d_1}\cdots i_{d_{2s}}} \int \frac{d^3\underline{u}\, d^3\underline{u}^*}{\pi^3}\, e^{-\underline{u}\cdot\underline{u}^*}\, u^*_{k_2} u^*_{l_2} \cdot$$

$$\cdot u_{i_{d_{2s+1}}} \cdots u_{i_{d_{2n+l}}} u_{j_{2n+1}} \cdots u_{j_{2n+l}} \sum_{r=0}^{\infty} f_1^{2n} f^{4r+4} a(r+n+1)\, b(r+1) \cdot$$

$$\cdot \Delta^{2n-2s+l} (2r+2n+2)(2r+2n)\cdots(2r-2n-2l+2s+4)\,(\Delta^2\underline{u}\cdot\underline{u})^{r-n-l+s+1} (\underline{u}^*\cdot\underline{u}^*)^r$$

$$= \delta_{k_1 l_1} \sum_{s=0}^{[n+l/2]} \sum_{d_1<d_2\cdots d_{2s}\le 2n+l} \delta_{i_{d_1}\cdots i_{d_{2s}}} f_1^{2n} \sum_{r=0}^{\infty} a(r+n+1)\, b(r+1)\, f^{4r+4} \Delta^{2r+2-l} \cdot$$

$$\cdot \frac{(2r+2n+2)(2r+2n)\cdots(2r-2n-2l+2s+4)}{3\cdot 5\cdot \cdots\cdots (2n+2l-2s+3)} (2r+1)!\,(2r+3)\,\cdot$$

$$\cdot \Big\{ (2r+5)\, \delta_{k_2 l_2 i_{d_{2s+1}}\cdots i_{d_{2n+l}} j_{d_{2n+1}}\cdots j_{d_{2n+l}}}$$

$$- (2n+2l-2s+3)\, \delta_{k_2 l_2}\, \delta_{i_{d_{2s+1}}\cdots i_{d_{2n+l}} j_{d_{2n+1}}\cdots j_{d_{2n+l}}} \Big\} \tag{4.18}$$

and finally

$$\left[\prod_{d=1}^{l} \frac{\partial}{\partial(\Delta_{i_{2n+d}\, j_{2n+d}})} \right] I^{BA}_{i_1,\cdots i_{2n};0\,:\,i_{2n+l+1}\, i_{2n+l+2}\,;\,k_2 l_2}(\Delta)$$

$$= f_1^2 \delta_{k_2 l_2} \int \frac{d^3\underline{u}\, d^3\underline{u}^*}{\pi^3}\, e^{-\underline{u}\cdot\underline{u}^*}\, u_{j_{2n+1}} \cdots u_{j_{2n+l}} \left[\prod_{d=1}^{2n+l} \frac{\partial}{\partial(\Delta_{i_d a} u_a)} \right] \cdot$$

$$\cdot \Delta_{i_{2n+l+1};a}\, \Delta_{i_{2n+l+2};b}\, u_a u_b\, \upsilon^{(B)}(\Delta_{ab} u_a \Delta_{ac} u_c)\, \upsilon^{(A)}(\underline{u}^*\cdot\underline{u}^*)$$

$$= \delta_{k_2 l_2} \sum_{s=0}^{[n+l/2+1]} \sum_{d_1<d_2\cdots d_{2s}\le 2n+l+2} \tilde{\delta}_{i_{d_1}\cdots i_{d_{2s}}}\, f_1^2 \int \frac{d^3\underline{u}\, d^3\underline{u}^*}{\pi^3}\, e^{-\underline{u}\cdot\underline{u}^*} \cdot$$

$$\cdot u_{i_{d_{2s+1}}} \cdots u_{i_{d_{2n+l+2}}}\, u_{j_{2n+1}} \cdots u_{j_{2n+l}}\, \Delta^{2n-2s+l+2} \left[2 \frac{\partial}{\partial(\Delta^2 \underline{u}\cdot\underline{u})} \right]^{2n+l-s} \cdot$$

$$\cdot\, \upsilon^{(B)}(\Delta^2 \underline{u}\cdot\underline{u})\, \upsilon^{(A)}(\underline{u}^*\cdot\underline{u}^*)$$

$$= \delta_{k_2 l_2} \sum_{s=0}^{[n+l/2+1]} \sum_{d_1<d_2\cdots d_{2s}\le 2n+l+2} \tilde{\delta}_{i_{d_1}\cdots i_{d_{2s}}} \int \frac{d^3\underline{u}\, d^3\underline{u}^*}{\pi^3}\, e^{-\underline{u}\cdot\underline{u}^*} \cdot$$

$$\cdot\, u_{i_{d_{2s+1}}} \cdots u_{i_{d_{2n+l+2}}}\, u_{j_{2n+1}} \cdots u_{j_{2n+l}} \sum_{r=0}^{\infty} f_1^{4n} f^{4r+4}\, b(r+n+1) \cdot$$

$$\cdot\, a(r+1)\, \Delta^{2n-2s+l+2}\, (2r+2n)(2r+2n-2) \cdots\cdots (2r-2n-2l+2s+2) \cdot$$

$$\cdot\, (\Delta^2 \underline{u}\cdot\underline{u})^{r-n-l+3}\, (\underline{u}^*\cdot\underline{u}^*)^{r+1}$$

$$= \delta_{k_2 l_2} \sum_{s=0}^{[n+l/2+1]} \sum_{d_1<d_2\cdots d_{2s}\le 2n+l+2} \tilde{\delta}_{i_{d_1}\cdots i_{d_{2s}}}\, f_1^{2n} \sum_{r=0}^{\infty} b(r+n+1)\, a(r+1)\, f^{4r+4} \cdot$$

$$\cdot\, \Delta^{2r+2-l}\, (2r+3)!\, (2r+2n)(2r+2n-2) \cdots\cdots (2r-2n-2l+2s+2) \cdot$$

$$\cdot\, \frac{\delta_{i_{d_{2s+1}}\cdots i_{d_{2n+l+2}} j_{2n+1}\cdots j_{2n+l}}}{3.5.\cdots\cdots(2n+2l-2s+3)} \tag{4.19}$$

2) Conclusions

It has been shown that, although the principles for dealing with Lagrangians containing iso-spin are the same as for the iso-scalar Lagrangians, the notation and algebra is somewhat more complicated. Use of the formulae given does save considerable time and energy in, for example, calculating second order corrections to $\pi\pi$ scattering amplitudes using Weinberg's co-ordinates.

The general form for the on-mass-shell self-energies in the massless case could be found in an identical manner to that used in chapter 3. However, since the SU(2) x SU(2) chiral Lagrangians are not equivalent to a free field theory such a calculation is only of academic interest so we shall not go into the details. Assuming no overall ultra-violet divergences to be present, the self-energy contributions for an interaction Lagrangian given by equation (4.1) can be shown to be of the same general form as given in equation (3.56) for the case of no iso-spin.

Similar techniques to those presented may also be used for gravitational and weak interaction theories. A full mastery of non-linear methods could well lead to the disappearance of the infinities which are normally so prevalent in Field theories.

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APPENDIX

This appendix contains the details for computing integrals arising in chapter 4. They are computed by termwise integration as in the case of no iso-spin. We shall make use of the orthogonality condition

$$\frac{1}{\pi}\int du\, du^* \, e^{-uu^*} u^m u^{*n} = \delta_{mn} \cdot n! \tag{A.1}$$

and the double sum identities

$$\sum_{m=0}^{s} \sum_{n=0}^{m} \frac{s!(2r-2\ell+s-m)!(2\ell-2k+s-n)!(2k+n)!}{(s-m)!(m-n)!n!}$$

$$= \sum_{m=0}^{s} \frac{s!(2r-2\ell+s-m)!}{(s-m)!} \cdot \frac{(2\ell+m+1)!(2k)!(2\ell-2k)!}{m!(2\ell+1)!}$$

$$= \frac{(2r+s+2)!}{(2r+2)!} (2r-2\ell)!(2\ell-2k)!(2k)! \tag{A.2}$$

and

$$\sum_{\ell=0}^{r} \sum_{k=0}^{\ell} \binom{r}{\ell}^2 \binom{\ell}{k}^2 (2r-2\ell)!(2\ell-2k)!(2k)!$$

$$= \sum_{\ell=0}^{r} \binom{r}{\ell}^2 (2r-2\ell)! \, 2^{2\ell} \ell! \ell!$$

$$= (2r+1)! \tag{A.3}$$

These identities are derived with the use of the addition theorem for binomial coefficients (see Edmonds[20], appendix 1).

Consider the integration

$$H=\int\frac{d^3\underline{u}\,d^3\underline{u}^*}{\pi^3}\,e^{-\underline{u}.\underline{u}^*}\sum_{r=0}^{\infty}\sum_{r'=0}^{\infty}A(r)\,(\underline{u}.\underline{u})^r\;B(r')\,(\underline{u}^*.\underline{u}^*)^{r'}\,(\underline{u}.\underline{u}^*)^s \tag{A.4}$$

Expanding binomially we have

$$H=\sum_{r=0}^{\infty}\sum_{l=0}^{r}\sum_{k=0}^{l}\sum_{r'=0}^{\infty}\sum_{l'=0}^{r'}\sum_{k'=0}^{l'}\sum_{m=0}^{s}\sum_{n=0}^{m}\int\frac{d^3\underline{u}\,d^3\underline{u}^*}{\pi^3}\,e^{-u_1u_1^*-u_2u_2^*-u_3u_3^*}\,.$$

$$\cdot A(r)\,B(r')\binom{r}{l}\binom{l}{k}\binom{r'}{l'}\binom{l'}{k'}\binom{s}{m}\binom{m}{n}\,u_1^{2r-2l+s-m}\,u_2^{2l-2k+m-n}\,.$$

$$\cdot u_3^{2k+n}\,u_1^{*\,2r'-2l'+s-m}\,u_2^{*\,2l'-2k'+m-n}\,u_3^{*\,2k'+n} \tag{A.5}$$

Using the orthogonality condition (A.1) gives

$$H=\sum_{r=0}^{\infty}\sum_{l=0}^{r}\sum_{k=0}^{l}\sum_{m=0}^{s}\sum_{n=0}^{m}A(r)\,B(r)\binom{r}{l}^2\binom{l}{k}^2\binom{s}{m}\binom{m}{n}\,\cdot$$

$$\cdot(2r-2l+s-m)!\,(2l-2k+m-n)!\,(2k+n)! \tag{A.6}$$

Summing over m and n using the identity (A.2) gives

$$H=\sum_{r=0}^{\infty}\sum_{l=0}^{r}\sum_{k=0}^{l}A(r)\,B(r)\binom{r}{l}^2\binom{l}{k}^2\,\frac{(2r+s+2)!}{(2r+2)!}\,(2r-2l)!\,(2l-2k)!\,(2k)! \tag{A.7}$$

and finally summing over l and k using the identity (A.3) results in

$$H = \sum_{r=0}^{\infty} A(r)\,B(r)\;\frac{(2r+2s+2)!}{(2r+2)!}\;(2r+1)! \tag{A.8}$$

In equations (4.14) and (4.19) we are interested in an integral of the form

$$J_{i_1,\ldots,i_{2n}} = \int \frac{d^3\underline{u}\,d^3\underline{u}^*}{\pi^3} \sum_{r=0}^{\infty}\sum_{r'=0}^{\infty} A(r)\,B(r')\;(\underline{u}.\underline{u})^{r}\,(\underline{u}^*.\underline{u}^*)^{r'}\,(\underline{u}.\underline{u}^*)^{s}.$$

$$\cdot u_{i_1}\cdots u_{i_{2n}} \tag{A.9}$$

Clearly $J_{i_1,\ldots,i_{2n}}$ must be symmetric in the iso-spin labels and hence

$$J_{i_1,\ldots,i_{2n}} = C\;\delta_{i_1\ldots i_{2n}} \tag{A.10}$$

Multiplying equation (A.10) by $\delta_{i_1i_2}\,\delta_{i_3i_4}\cdots\cdots\delta_{i_{2n-1}i_{2n}}$ and using equation (A.8) one obtains

$$\begin{aligned} C\,.\,(2n+1)(2n-1).\cdots\,.5.3 &= \int \frac{d^3\underline{u}\,d^3\underline{u}^*}{\pi^3} \sum_{r=0}^{\infty}\sum_{r'=0}^{\infty} A(r)\,B(r')\;(\underline{u}.\underline{u})^{r+n}\,(\underline{u}^*.\underline{u}^*)^{r'}\,(\underline{u}.\underline{u}^*)^{s} \\ &= \sum_{r=0}^{\infty} A(r)\,B(r+n)\;\frac{(2r+2n+s+2)!}{(2r+2n+2)!}\;(2r+2n+1)! \end{aligned} \tag{A.11}$$

and consequently

$$J_{i_1 \ldots i_{2n}} = \frac{\delta_{i_1 \ldots i_{2n}}}{3.5.\ldots.(2n+1)} \sum_{r=0}^{\infty} A(r)\, B(r+n)\, \frac{(2r+2n+s+2)!}{(2r+2n+2)!}\, (2r+2n+1)! \tag{A.12}$$

Equations (4.16) and (4.18) involve an integral of the form

$$K_{i_1 \ldots i_{2n+2}; j_1 j_2} = \int \frac{d^3\underline{u}\, d^3\underline{u}^*}{\pi^3} \sum_{r=0}^{\infty} \sum_{r'=0}^{\infty} A(r)\, B(r')\, (\underline{u}.\underline{u})^r (\underline{u}^*.\underline{u}^*)^{r'} u^*_{j_1} u^*_{j_2} \cdot u_{i_1} \ldots u_{i_{2n+2}} \tag{A.13}$$

Again looking at the iso-spin symmetry of the integral we see that $K_{i_1 \ldots i_{2n+2}; j_1 j_2}$ is of the form

$$K_{i_1 \ldots i_{2n+2}; j_1 j_2} = D\, \delta_{j_1 j_2 i_1 \ldots i_{2n+2}} + E\, \delta_{j_1 j_2} \delta_{i_1 \ldots i_{2n+2}} \tag{A.14}$$

Multiplying equation (A.14) by $\delta_{j_1 j_2}$ and using equation (A.12) gives

$$\left[(2n+5)D + 3E\right] \delta_{i_1 \ldots i_{2n+2}}$$

$$= \int \frac{d^3\underline{u}\, d^3\underline{u}^*}{\pi^3} \sum_{r=0}^{\infty} \sum_{r'=0}^{\infty} A(r)\, B(r')\, (\underline{u}.\underline{u})^r (\underline{u}^*.\underline{u}^*)^{r'+1} u_{i_1} \ldots u_{i_{2n+2}}$$

$$= \frac{\delta_{i_1 \ldots i_{2n+2}}}{3.5.\ldots.(2n+3)} \sum_{r=0}^{\infty} A(r)\, B(r+n)\, (2r+2n+3)! \tag{A.15}$$

Multiplying equation (A.14) instead by $\delta_{j_1 i_{2n+1}} \delta_{j_2 i_{2n+2}}$ and using equation (A.12) gives

$$\left[(2n+5)D+E\right](2n+3)\,\delta_{i_1\ldots i_{2n}}$$

$$=\int\frac{d^3\underline{u}\,d^3\underline{u}^*}{\pi^3}\sum_{r=0}^{\infty}\sum_{r'=0}^{\infty}A(r)\,B(r')\,(\underline{u}.\underline{u})^r(\underline{u}^*.\underline{u}^*)^{r'}(\underline{u}.\underline{u}^*)^2\,u_{i_1}\ldots u_{i_{2n}}$$

$$=\frac{\delta_{i_1\ldots i_{2n}}}{3.5.\ldots.(2n+1)}\sum_{r=0}^{\infty}A(r)B(r+n)(2r+2n+4)(2r+2n+3)(2r+2n+1)! \qquad (A.16)$$

Equations (A.15) and (A.16) can now be used to find D and E, hence we have

$$K_{i_1\ldots i_{2n+2};j_1j_2}=\sum_{r=0}^{\infty}A(r)\,B(r+n)\;\frac{(2r+2n+3)(2r+2n+1)!}{3.5.\ldots.(2n+5)}\;.$$

$$.\left\{(2r+2n+5)\,\delta_{i_1\ldots i_{2n+2}j_1j_2}-(2r+5)\,\delta_{j_1j_2}\delta_{i_1\ldots i_{2n+2}}\right\} \qquad (A.17)$$

PART II

U(6,6) AND THE ABSORPTION MODEL FOR PHOTOPRODUCTION PROCESSES

"Let there be light: and there was light."

Genesis chapter I verse 3.

TO MY PARENTS

CHAPTER 1

1) Introduction

Calculations are presented for two-body and quasi-two-body photoproduction processes using the absorption model[1] and incorporating the U(6,6) symmetry scheme [2] [3] to determine the coupling strengths.

For any two-body production process one can start by writing down all possible one particle exchange t-channel graphs assuming only 0^- and 1^- meson exchanges. These Born graphs are hopefully a good approximation at the higher energies where s-channel resonances can be neglected and in the near forward direction where the u-channel exchanges, dominant in the backward direction, may also be neglected. In fact the Born amplitudes alone do not yield good results and Sopkovich [1] proposed that scattering of the incoming particles could take place before the meson is exchanged and similarly afterwards for the outgoing particles. These absorption corrections are approximated by assuming elastic scattering to be the dominant mechanism. They have the effect of greatly reducing the large contribution arising from the low partial waves of the Born terms. Both 0^- and 1^- exchange amplitudes are therefore improved however the bad energy dependence of the vector exchanges still remains. Consequently the model is useful for reactions dominated by pseudoscalar exchange but since most such reactions do permit the exchange of one or more vector particles, we should only expect the model to work in the intermediate energy region, ideally 4-8 GeV incident momentum, owing to the poor asymptotic behaviour of the vector exchange graphs.

The couplings at the vertices of the Born diagrams can sometimes be obtained by experimentally measuring appropriate decay widths. However they are frequently inaccessible so we calculate them assuming U(6,6) symmetry. U(6,6) allows all the meson-baryon-baryon (MBB) and meson-meson-meson (MMM) couplings for 0^- and 1^- mesons and for $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons to be related to just two coupling constants. Thus once these couplings are fixed we are left with a parameter free theory for all the production processes involving $0^-, 1^-, \frac{1}{2}^+$ and $\frac{3}{2}^+$ particles. In particular the couplings for strange particles will be determined from non-strange effects. This model has been previously applied with considerable success to strong interaction processes [4].

The photoproduction reactions fall into four categories, namely

a) $\gamma N \rightarrow 0^- \frac{1}{2}^+$

b) $\gamma N \rightarrow 0^- \frac{3}{2}^+$

c) $\gamma N \rightarrow 1^- \frac{1}{2}^+$

d) $\gamma N \rightarrow 1^- \frac{3}{2}^+$

where N denotes the target proton or neutron. Much of the photoproduction experimental data is for processes a) and this category will be treated in full in the thesis of D.G.Fincham. There has recently been an increase of data for processes b) and c) and no doubt data on processes d) will be forthcoming soon. Here we shall be dealing with these last three categories.

CHAPTER 2

A general description of the U(6,6) absorption model applied to photoproduction processes is presented. The unmodified helicity amplitudes are given in section 2.

1) U(6,6) and Vector Dominance

We assume the vector dominance model [5]; the u-spin scalar transformation property of the photon gives, for the amplitude for any meson-baryon final state,

$$T(\gamma N \to MB) = X_\rho \left[T(\rho^0_{Tr} N \to MB) + \frac{1}{\sqrt{3}} T(\omega_{8_{Tr}} N \to MB) \right] \tag{2.1}$$

thus relating the photoproduction amplitude to purely strong amplitudes for the scattering of transversely polarised vector mesons off a nucleon target. Taking the SU(6) prediction for $\omega - \varphi$ mixing, i.e.

$$|\omega_8\rangle = \frac{1}{\sqrt{3}} |\omega_{physical}\rangle + \sqrt{\frac{2}{3}} |\varphi_{physical}\rangle \tag{2.2}$$

we obtain

$$T(\gamma N \to MB) = X_\rho T(\rho^0_{Tr} N \to MB) + X_\omega T(\omega_{Tr} N \to MB) + X_\varphi T(\varphi_{Tr} N \to MB) \tag{2.3}$$

where

$$X_\rho^2 : X_\omega^2 : X_\varphi^2 = 9 : 1 : 2 \tag{2.4}$$

The ρ -photon coupling X_ρ is given by

$$e = X_\rho \, g_{\rho\pi\pi} \tag{2.5}$$

where

$$\frac{g^2_{\rho\pi\pi}}{4\pi} = 1.8 \tag{2.6}$$

and

$$\frac{e^2}{4\pi} = \frac{1}{137} \tag{2.7}$$

Alternatively X_ρ may be evaluated using the decay $\omega \rightarrow \gamma\pi$. In either case we obtain

$$X_\rho^2 \simeq .004 \tag{2.8}$$

We need now only consider the Born graphs for the purely strong reactions $V^0_\pi N \rightarrow MB$. These graphs involve the exchange of $\underline{35}$ mesons which through U(6,6) couple uniquely to $\overline{\underline{56}} \times \underline{56}$ or $\overline{\underline{35}} \times \underline{35}$. Thus the three particle vertices are written as

$$L_{\bar{B}BM} = g \, \bar{B}^{ABC} B_{ABD} M_C{}^D$$

$$L_{MMM} = h \, M_A{}^B M_B{}^C M_C{}^A \tag{2.9}$$

where A, B, C and D are the U(6,6) labels. Just retaining those parts of the currents in which we shall be interested we have, for the pseudoscalar current,

$$j_5 = j_5(O) + j_5(D) + j_5(P) + j_5(V) \tag{2.10}$$

where

$$j_5(O) = g\left(1+\frac{2m}{\mu}\right)\frac{P^2}{4m^2}\left(\bar{N}\gamma_5 N\right)_{D+\frac{2}{3}F-S}$$

$$j_5(D) = g\left(1+\frac{2m}{\mu}\right)\frac{q_\lambda}{m}\left(\bar{D}_\lambda N\right)_G$$

$$j_5(P) = 3h\, q_\lambda \left(\bar{\varphi}_5 \varphi_\lambda\right)_F$$

$$j_5(V) = -\frac{3h}{2\mu}\,\varepsilon_{\kappa\lambda\mu\nu}\, q_\kappa P_\lambda \left(\bar{\varphi}_\mu \varphi_\nu\right)_D \tag{2.11}$$

and, for the vector current,

$$j_\mu = j_\mu(O) + j_\mu(D) + j_\mu(P) + j_\mu(V) \tag{2.12}$$

where

$$j_\mu(O) = g\frac{P_\mu}{2m}\left(1+\frac{q^2}{2m\mu}\right)\left(\bar{N}N\right)_{F+3S} + g\left(1+\frac{2m}{\mu}\right)\left(\bar{N}\frac{\Gamma_\mu}{4m^2}N\right)_{D+\frac{2}{3}F-S}$$

$$j_\mu(D) = -g\frac{1}{2m^2}\left(1+\frac{2m}{\mu}\right)\varepsilon_{\mu\nu\kappa\lambda} P_\nu q_\kappa \left(\bar{D}_\lambda N\right)_G$$

$$j_\mu(P) = -\frac{3h}{2\mu}\,\varepsilon_{\mu\kappa\lambda\nu} P_\lambda q_\nu \left(\bar{\varphi}_5 \varphi_\kappa\right)_D$$

$$j_\mu(V) = h\Big[\frac{1}{\mu^2} P_\mu q_\nu q_\lambda (\overline{\varphi}_\nu \varphi_\lambda)_F - \Big(1+\frac{q^2}{2\mu^2}\Big) P_\mu (\overline{\varphi}_\lambda \varphi_\lambda)_F$$

$$+ 3 q_\nu (\overline{\varphi}_\nu \varphi_\mu - \varphi_\nu \overline{\varphi}_\mu)_F\Big] \qquad (2.13)$$

The following notation has been used

p = momentum of incoming particle

p' = momentum of outgoing particle

$q = p - p'$

$P = p + p'$

$\Gamma_\mu = \varepsilon_{\mu\nu\kappa\lambda} P_\nu q_\kappa \gamma_\lambda \gamma_5$

μ = meson mass

m = baryon mass

F, D and S are the familiar anti-symmetric, symmetric and singlet SU(3) couplings respectively. We have also introduced explicitly a G coupling for the meson-octet-decuplet vertex defined as

$$(\overline{D}NM)_G = \sqrt{2}\ \overline{D}^{rst} M_t^{\ u} \varepsilon_{usv} N_r^{\ v} \qquad (2.14)$$

where M, N and D are the usual meson and baryon fields [3]. The $\sqrt{2}$ arises in equation (2.14) as we have normalised to $\overline{p} p \pi^0$.

The two coupling constants which appear in the currents given in equations (2.11) and (2.13) are calculated from Chew-Low πN scattering, giving

$$\frac{g^2_{NN\pi}}{4\pi} = 14.9 = \frac{g^2}{4\pi}\left(1+\frac{2m}{\mu}\right)^2\left(\frac{5}{3}\right)^2 \tag{2.15}$$

and from the Novisibirsk (6) experiment for the $\rho \rightarrow 2\pi$ decay width, giving

$$\frac{g^2_{\rho\pi\pi}}{4\pi} = 1.8 = \frac{(3h)^2}{4\pi} \tag{2.16}$$

Thus by using U(6,6) the coupling of the decuplet is completely specified. For the mesons the D:F ratios and the ratio of the charge to moment couplings are also completely determined. The only arbitrariness in the model now lies in the choice of the U(6,6) masses μ and m. We adopt the following prescription: m = mean mass of $\frac{1}{2}^+$ octet in $j_S(O)$ and $j_\mu(O)$; m = mean mass of $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet in $j_S(D)$ and $j_\mu(D)$; μ = mean mass of 0^- octet in $j_S(O)$ and $j_S(D)$; μ = mean mass of 1^- nonet in $j_\mu(O), j_\mu(D), j_S(V), j_\mu(P)$ and $j_\mu(V)$. We denote these masses by m, m', μ and μ' respectively. In the propagators the experimental masses of the particles are taken.

2) The Unmodified Helicity Amplitudes

It is now a simple matter to write down the Born amplitudes, in terms of these currents, for the processes in which we are interested. Denoting the pseudoscalar and vector exchange amplitudes by T_p and T_v respectively and the exchanged pseudoscalar and vector masses by μ_P and μ_V respectively, we have, for $0^- \frac{1}{2}^+$ production,

$$T_P^{(A)} = j_5(O)\frac{1}{t-\mu_P^2}\,j_5(P) \tag{2.17}$$

$$T_V^{(A)} = j_\mu(O)\,\frac{[-g_{\mu\nu}+q_\mu q_\nu/\mu_V^2]}{t-\mu_V^2}\,j_\nu(P)\ , \tag{2.18}$$

for $0^-\frac{3}{2}^+$ production,

$$T_P^{(B)} = j_5(D)\frac{1}{t-\mu_P^2}\,j_5(P) \tag{2.19}$$

$$T_V^{(B)} = j_\mu(D)\,\frac{[-g_{\mu\nu}+q_\mu q_\nu/\mu_V^2]}{t-\mu_V^2}\,j_\nu(P)\ , \tag{2.20}$$

for $1^-\frac{1}{2}^+$ production,

$$T_P^{(C)} = j_5(O)\frac{1}{t-\mu_P^2}\,j_5(V) \tag{2.21}$$

$$T_V^{(C)} = j_\mu(O)\,\frac{[-g_{\mu\nu}+q_\mu q_\nu/\mu_V^2]}{t-\mu_V^2}\,j_\nu(V) \tag{2.22}$$

and, for $1^-\frac{3}{2}^+$ production,

$$T_P^{(D)} = j_5(D)\frac{1}{t-\mu_P^2}\,j_5(V) \tag{2.23}$$

$$T_V^{(D)} = j_\mu(D)\,\frac{[-g_{\mu\nu}+q_\mu q_\nu/\mu_V^2]}{t-\mu_V^2}\,j_\nu(V) \tag{2.24}$$

The metric is taken as $g_{\mu\nu} = (+1,-1,-1,-1)$.

The $0^-\frac{1}{2}^+$ production processes will be dealt with by D.G.Fincham in his thesis so we shall restrict ourselves here to the remaining reactions. Using equations (2.11) and (2.13) we obtain the following expressions for the amplitudes

$$T_P^{(B)} = B_1\, p_{1\lambda} \bar{u}_\lambda(p_3)\, u(p_1)\, p_{4\mu} \varphi_\mu(p_2) \tag{2.25}$$

where

$$B_1 = -\frac{3gh\chi_\rho}{m'}\left(1+\frac{2m'}{\mu}\right) F' G \frac{1}{(t-\mu_\rho^2)} , \tag{2.26}$$

$$T_V^{(B)} = B_2\, \varepsilon_{\mu\nu\kappa\lambda}\, p_{3\nu} p_{1\kappa} \bar{u}_\lambda(p_3)\, u(p_1)\, \varepsilon_{\mu\rho\sigma\tau}\, p_{4\sigma} p_{2\tau} \varphi_\rho(p_2) \tag{2.27}$$

where

$$B_2 = -\frac{3gh\chi_\rho}{\mu' m'^2}\left(1+\frac{2m'}{\mu'}\right) D' G \frac{1}{t-\mu_V^2} , \tag{2.28}$$

$$T_P^{(C)} = C_1 \bar{u}(p_3)\, \gamma_5\, u(p_1)\, \varepsilon_{\kappa\lambda\mu\nu}\, p_{2\kappa} p_{4\lambda} \bar{\varphi}_\mu(p_4) \varphi_\nu(p_2) \tag{2.29}$$

where

$$C_1 = -\frac{3gh\chi_\rho}{\mu'}\left(1+\frac{2m}{\mu}\right)\left(1-\frac{\mu^2}{4m^2}\right) D'\left(D+\frac{2}{3}F-S\right)\frac{1}{t-\mu_P^2} , \tag{2.30}$$

$$T_V^{(c)} = \bar{u}(p_3)\Big\{C_2(p_1+p_3)_\mu - C_2\frac{(m_1^2-m_3^2)}{\mu_V^2}(p_{1\mu}-p_{3\mu})$$

$$+C_3\gamma_\mu - C_3\frac{(m_1-m_3)}{\mu_V^2}(p_{1\mu}-p_{3\mu})\Big\}u(p_1)\cdot$$

$$\cdot\Big\{(p_2+p_4)_\mu\Big[\frac{1}{\mu'^2}p_{2\lambda}\bar{\varphi}_\lambda(p_4)\,p_{4\kappa}\varphi_\kappa(p_2) + \frac{3}{2}\bar{\varphi}_\lambda(p_4)\varphi_\lambda(p_2)\Big]$$

$$-3\varphi_\mu(p_2)\,p_{2\lambda}\bar{\varphi}_\lambda(p_4) - 3\bar{\varphi}_\mu(p_4)\,p_{4\lambda}\varphi_\lambda(p_2)\Big\} \tag{2.31}$$

where

$$C_2 = \frac{g h X_\rho}{2m}\Big\{\Big(1+\frac{\mu'}{2m}\Big)(F+3S) - \Big(1+\frac{2m}{\mu'}\Big)\Big(D+\frac{2}{3}F-S\Big)\Big\}F'\,\frac{1}{t-\mu_V^2}$$

$$C_3 = g h X_\rho\Big(1+\frac{2m}{\mu'}\Big)\Big(1-\frac{t}{4m^2}\Big)\Big(D+\frac{2}{3}F-S\Big)F'\,\frac{1}{t-\mu_V^2} \tag{2.32}$$

and

$$T_P^{(D)} = D_1\cdot p_{1\alpha}\bar{u}_\alpha(p_3)\,u(p_1)\,\varepsilon_{\kappa\lambda\mu\nu}\,p_{2\kappa}\,p_{4\lambda}\,\bar{\varphi}_\mu(p_4)\,\varphi_\nu(p_2) \tag{2.33}$$

where

$$D_1 = -\frac{3 g h X_\rho}{\mu' m'}\Big(1+\frac{2m'}{\mu'}\Big)G D'\,\frac{1}{t-\mu_\rho^2} \tag{2.34}$$

$$T_\gamma^{(D)} = D_2 \varepsilon_{\mu\nu\kappa\lambda} p_{3\nu} p_{1\kappa} \bar{u}_\lambda(p_3) u(p_1) \Big\{ 3\varphi_\mu(p_2) p_{2\alpha} \bar{\varphi}_\alpha(p_4)$$

$$+ 3\bar{\varphi}_\mu(p_4) p_{4\alpha} \varphi_\alpha(p_2) - (p_2+p_4)_\mu \Big[\frac{1}{\mu'^2} p_{2\alpha} \bar{\varphi}_\alpha(p_4) p_{4\beta} \varphi_\beta(p_2)$$

$$+ \frac{3}{2} \bar{\varphi}_\alpha(p_4) \varphi_\alpha(p_2) \Big] \Big\} \quad (2.35)$$

where

$$D_2 = -\frac{gh\chi_\rho}{2m'^2}\left(1 + \frac{2m'}{\mu'}\right) G F' \frac{1}{t - \mu_v^2} \quad (2.36)$$

We have distinguished between the incoming nucleon and photon and the outgoing baryon and meson by the labels "1","2","3" and "4" respectively. m_i and p_i then denote the mass and 4-momentum of particle "i". F' and D' are the SU(3) couplings at the MMM vertex; F,D,S and G are the couplings at the $M\bar{B}B$ vertex.

To include absorption corrections the matrix elements T are diagonalised in the helicity representation of Jacob and Wick (7). We define, for the general two body reaction 1 + 2 → 3 + 4,

$$\varphi_i = \langle \lambda_3 \lambda_4 | T | \lambda_1 \lambda_2 \rangle \quad (2.37)$$

where $T = T_p + T_v$ and the index i specifies the helicity dependence. The restriction imposed by parity reduces the number of independent helicity amplitudes to half which are then evaluated in the centre of mass frame. Explicit expressions for these amplitudes are long so only those corresponding to present available data are given viz. $0^- \frac{3}{2}^+$ production and $1^- \frac{1}{2}^+$ (pseudoscalar exchange only) production.

The helicity dependence of the amplitudes for $0^- \frac{3}{2}^+$ production is shown in Table 1. We obtain the following expressions for the helicity amplitudes

$$\varphi_1^B = -\frac{1}{2\sqrt{c}} B_1 \eta_- QK \sin^2\theta \cos\theta/2 \quad + B_2 B_3 \eta_- \cos\theta/2$$

$$\varphi_2^B = -\frac{1}{2\sqrt{3c}} B_1 Q\left[\frac{2}{m_3}\eta_-(E_1 Q - E_3 K\cos\theta)\cos\theta/2 + \eta_+ K \sin\theta \sin\theta/2\right]\sin\theta$$

$$+ B_2\left[\frac{1}{\sqrt{3}} B_3 \eta_+ \sin\theta/2 \quad + B_5 \eta_- \cos\theta/2\right]$$

$$\varphi_3^B = \frac{-1}{2\sqrt{3c}} B_1 Q\left[\frac{2}{m_3}\eta_+(E_1 Q - E_3 K\cos\theta)\sin\theta/2 - \eta_- K\sin\theta\cos\theta/2\right]\sin\theta$$

$$+ B_2\left[\frac{1}{\sqrt{3}} B_4 \eta_- \cos\theta/2 \quad + B_5 \eta_+ \sin\theta/2\right]$$

$$\varphi_4^B = \frac{1}{2\sqrt{c}} B_1 \eta_+ QK \sin^2\theta \sin\theta/2 \quad + B_2 B_4 \eta_+ \sin\theta/2$$

$$\varphi_5^B = \frac{1}{2\sqrt{c}} B_1 \eta_+ QK \sin^2\theta \sin\theta/2 \quad - B_2 B_3 \eta_+ \sin\theta/2$$

$$\varphi_6^B = \frac{1}{2\sqrt{3C}} B_1 Q\Big[\frac{2}{m_3}\eta_+ (E_1 Q - E_3 K\cos\theta)\sin\theta/2$$

$$- \eta_- K\sin\theta\cos\theta/2\Big]\sin\theta$$

$$+ B_2\Big[\frac{1}{\sqrt{3}} B_3 \eta_- \cos\theta/2 - B_5 \eta_+ \sin\theta/2\Big]$$

$$\varphi_7^B = \frac{-1}{2\sqrt{3C}} B_1 Q\Big[\frac{2}{m_3}\eta_- (E_1 Q - E_3 K\cos\theta)\cos\theta/2$$

$$+ \eta_+ K\sin\theta\sin\theta/2\Big]\sin\theta$$

$$- B_2\Big[\frac{1}{\sqrt{3}} B_4 \eta_+ \sin\theta/2 - B_5 \eta_- \cos\theta/2\Big]$$

$$\varphi_8^B = \frac{1}{2\sqrt{C}} B_1 \eta_- QK\sin^2\theta\cos\theta/2 \quad + B_2 B_4 \eta_- \cos\theta/2 \tag{2.38}$$

where E_i is the energy of particle "i", K and Q are the magnitudes of the incoming and outgoing 3-momentum in the centre of mass frame and θ is the centre of mass scattering angle. Also

$$C = (E_1 + m_1)(E_3 + m_3)$$

$$\eta_\pm = C \pm QK$$

$$B_3 = \frac{K}{2\sqrt{C}}\Big\{\big[QE_1(Q+E_4) + KE_3(E_4 - Q\cos\theta)\big](1-\cos\theta)$$

$$+ Q^2 K\sin^2\theta\Big\}$$

$$B_4 = \frac{K}{2\sqrt{C}}\Big\{\big[QE_1(Q-E_4) + KE_3(E_4 - Q\cos\theta)\big](1+\cos\theta)$$

$$+ Q^2 K\sin^2\theta\Big\}$$

$$B_s = \frac{m_3}{\sqrt{3C}} \left(Q\cos\theta - E_4 \right) K^2 \sin\theta \tag{2.39}$$

Table 2 shows the helicity dependence for $\Gamma\,\frac{1}{2}^+$ production and we obtain the following expressions for the pseudoscalar exchange part of the helicity amplitudes

$$\varphi_1^c = -\varphi_8^c = -C_1 H_-(E_2Q - E_4K)\cos^3\theta/2$$

$$\varphi_2^c = \varphi_7^c = -C_1 H_+(E_2Q - E_4K)\sin\theta/2\cos^2\theta/2$$

$$\varphi_3^c = -\varphi_{10}^c = -\sqrt{2}\,C_1 H_- m_4 K\sin\theta/2\cos^2\theta/2$$

$$\varphi_4^c = \varphi_9^c = -\sqrt{2}\,C_1 H_+ m_4 K\sin^2\theta/2\cos\theta/2$$

$$\varphi_5^c = -\varphi_{12}^c = C_1 H_-(E_2Q + E_4K)\sin^2\theta/2\cos\theta/2$$

$$\varphi_6^c = \varphi_{11}^c = C_1 H_+(E_2Q + E_4K)\sin^3\theta/2 \tag{2.40}$$

where

$$H_\pm = \frac{1}{\sqrt{C}}\left[K(E_3+m_3) \pm Q(E_1+m_1)\right] \tag{2.41}$$

The helicity amplitudes for $T_v^{(C)}$, $T_p^{(D)}$ and $T_v^{(D)}$ are embarassingly long and so are not given here. They are evaluated using standard techniques and patience.

3) Absorption Effects

As vector dominance relates electromagnetic interactions to strong interactions, the two incoming particles can be considered to be a nucleon and a zero charged vector meson. This model allows strong elastic scattering to take place in the initial as well as in the final state as for the usual strong interaction absorption model originally proposed by Sopkovich [1]. This model has been widely discussed [1] so we restrict ourselves here to just a brief summary.

The production partial wave amplitude from channel α to channel β is related to the Born amplitude $T^{j}_{\beta\alpha}$ by

$$\langle\beta_1\beta_2|T'^{j}_{\beta\alpha}|\alpha_1\alpha_2\rangle$$

$$= \frac{1}{2}\Big\{\sum_{\beta_1'\beta_2'}\langle\beta_1\beta_2|S^{j}_{\beta\beta}|\beta_1'\beta_2'\rangle\langle\beta_1'\beta_2'|T^{j}_{\beta\alpha}|\alpha_1\alpha_2\rangle$$

$$+\sum_{\alpha_1'\alpha_2'}\langle\beta_1\beta_2|T^{j}_{\beta\alpha}|\alpha_1'\alpha_2'\rangle\langle\alpha_1'\alpha_2'|S^{j}_{\alpha\alpha}|\alpha_1\alpha_2\rangle \qquad (2.42)$$

where $\alpha_1, \alpha_2, \beta_1$ and β_2 are the helicities of the particles. $S^{j}_{\alpha\alpha}$ and $S^{j}_{\beta\beta}$ are the S-matrix elements for elastic scattering in the initial and final states respectively. This elastic scattering is assumed to be pure non-spin flip. Its form is parametrised in terms of a real Gaussian model of radius R(s) and opacity C(s), thus giving

$$\langle \lambda_1 \lambda_2 | S^j | \lambda_1 \lambda_2 \rangle = 1 - C_\lambda(s) \exp\left[-\ell(\ell+1)/R_\lambda^2(s) K^2\right] \qquad (2.43)$$

where $\ell = j - \frac{1}{2}$ and K is the magnitude of the 3-momentum in the centre of mass frame. The parameters R(s) and C(s) describing the elastic scattering are, in general, not known. In the initial state we take R(s) from πp elastic scattering and C(s) = 1. If the final state contains no strange particles we use the same values for R(s) and C(s). For strange particle production we take R(s) from Kp elastic scattering and again C(s) = 1. Elastic scattering data gives $R(s)^{-1} = .26\ \mathrm{GeV}^{-1}$ for $\pi^\pm p$ and $K^- p$, and $R(s)^{-1} = .32\ \mathrm{GeV}^{-1}$ for $K^+ p$ at the intermediate energies that we consider

The partial wave projections of the Born helicity amplitudes were obtained by numerical integration

$$T_i^j(s) = \frac{1}{2}\int_{-1}^{+1} \varphi_i\, d^j_{\lambda\mu}(\theta)\, d(\cos\theta) \;, \qquad (2.44)$$

modified according to equation (2.42), and the new partial wave series re-summed

$$\varphi_i' = \sum_j (2j+1)\, T_i'^{\,j}(s)\, d^j_{\lambda\mu}(\theta) \qquad (2.45)$$

to give the modified amplitudes. The differential cross-section according to the absorption model was then obtained by

$$\frac{d\sigma}{dt} = \frac{1}{64\pi K^2 s}\ \frac{1}{2} \cdot \sum_i |\varphi_i'|^2 \qquad (2.46)$$

where the summation is over the independent helicity amplitudes. K is the magnitude of the centre of mass 3-momentum of the particles in the initial state.

For completeness the usual method of obtaining the spin density matrix elements for the decay of outgoing resonances is outlined [8]. In the centre of mass system they are given by

$$\rho_{\lambda_3'\lambda_3} = \frac{1}{N}\sum_{\lambda_1\lambda_2\lambda_4} \langle \lambda_3'\lambda_4 | T' | \lambda_1\lambda_2 \rangle \langle \lambda_3\lambda_4 | T' | \lambda_1\lambda_2 \rangle^* \tag{2.47}$$

and similarly for $\rho_{\lambda_4'\lambda_4}$ where

$$N = 2\sum_i |\varphi_i'|^2 \tag{2.48}$$

These distributions are then transformed to the rest frame of the decaying resonance. In this frame the z-axis is taken parallel to the incident momentum and the y-axis perpendicular to the production plane. With respect to these new axes the density matrix elements are given by

$$\bar{\rho}_{\mu_3'\mu_3} = \sum_{\text{all } \lambda} d^{S_3}_{\mu_3'\lambda_3'}(-\psi_3)\, \rho_{\lambda_3'\lambda_3}\, d^{S_3}_{\mu_3\lambda_3}(-\psi_3) \tag{2.49}$$

where ψ_3 is the angle between the directions of the incident particle and particle "4" as seen in the rest frame of particle "3". The expression for $\bar{\rho}_{\mu_4'\mu_4}$ is obtained in an analogous manner.

Thus given the helicity amplitudes evaluated in the previous section we are now able to compute the relevant differential cross-sections and spin density matrix elements. However, we shall see in the following chapter that these amplitudes may need to be amended due to the requirements of gauge invariance.

CHAPTER 3

A dicussion of the theoretical predictions of the model is given.

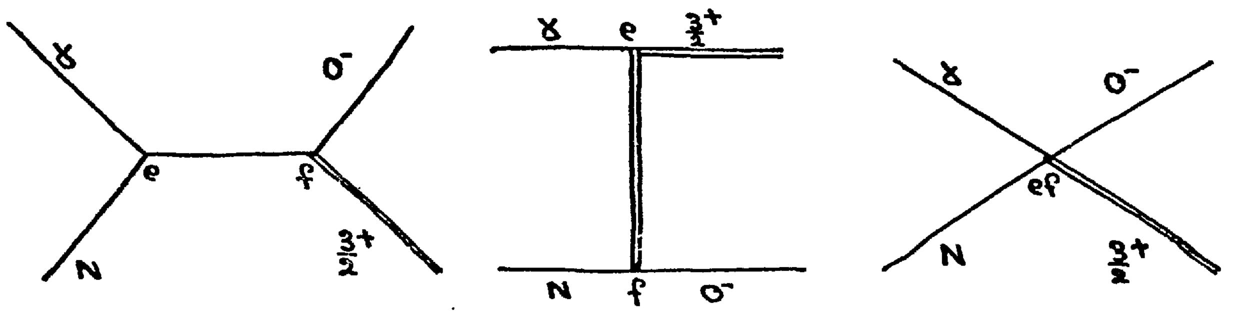
1) Gauge Invariance and $\gamma N \rightarrow 0^- \frac{3}{2}^+$

Continuity of the electric current implies that the matrix element for a photoproduction process must obey the condition

$$K_\mu j_\mu = 0 \tag{3.1}$$

where K_μ is the 4-momentum of the photon. Each photoproduction amplitude is written in the form $\varphi_\mu j_\mu$ where φ_μ is the polarisation vector of the photon. Thus replacing φ_μ by K_μ one must obtain zero for a gauge invariant theory. We immediately see that for the processes in which we are interested $T_v^{(B)}$, $T_p^{(C)}$ and $T_p^{(D)}$ are gauge invariant due to the presence of the term $\varepsilon_{\mu\kappa\lambda\nu} P_\lambda q_\nu \varphi_\kappa$ in $j_5(V)$ and $j_\mu(P)$ defined in equations (2.11) and (2.13).

A gauge invariant extension to $T_p^{(B)}$ can be obtained by including parts of three other graphs (9)(10) viz.



The gauge invariant amplitude then contains contributions from the t-channel, s-channel, u-channel and contact graphs. The relative contributions from these three additional graphs depend on what basic assumptions are taken.

Our t-channel matrix element is such that the photon interacts with the "orbital current" of a moving charged particle, the total iso-spin is equal to 1 and only the vector part of the photon couples. In both reference 9 and 10 a gauge invariant extension is defined such that the first property is maintained. Thus only the "orbital parts" of the s-channel and u-channel graphs are retained giving, for $\gamma N \rightarrow \pi\Delta$,

$$T^{I} = ef \; \frac{\varphi_\lambda(p_2)\, p_{4\lambda}}{t - m_\pi^2} \; \bar{u}_\mu(p_3)\, p_{1\mu}\, u(p_1)$$

$$T^{II} = ef \; \frac{\varphi_\lambda(p_2)\, p_{1\lambda}}{s - m_N^2} \; \bar{u}_\mu(p_3)\, p_{4\mu}\, u(p_1)$$

$$T^{III} = ef \; \frac{\varphi_\lambda(p_2)\, p_{3\lambda}}{u - m_\Delta^2} \; \bar{u}_\mu(p_3)\, p_{4\mu}\, u(p_1)$$

$$T^{IV} = -\tfrac{1}{2} ef \; \bar{u}_\mu(p_3)\, \varphi_\mu(p_2)\, u(p_1) \tag{3.2}$$

where T^{I}, T^{II}, T^{III} and T^{IV} are the t-channel, s-channel, u-channel and contact graph contributions respectively. Comparing T^{I} with equation (2.24) and remembering to introduce the ρ-photon coupling X_ρ we see that

$$ef = -\frac{3gh\chi_\rho}{m'}\left(1+\frac{2m'}{\mu}\right)F'G$$

In reference 9 only those parts of T^{II} and T^{III} are retained that correspond to the iso-spin I = 1 amplitude in the t-channel and to the iso-vector part of the photon. It is pointed out in reference 10 that, for example, in the case of $\gamma p \rightarrow \pi^+\Delta^0$ the u-channel graph does not vanish as it should corresponding to the electromagnetic orbital current of the neutrally charged Δ. We therefore follow reference 10 and consider the full iso-scalar and iso-vector contributions. Consequently the gauge invariant extensions for the four $0^- \frac{3}{2}^+$ production processes in which we shall be mainly interested are given by

$$T^G = T^I + T^{IV} + aT^{II} + bT^{III}$$

where $(a,b) = (-1,-2), (1,0), (0,-1)$ and $(0,-1)$ for $\gamma p \rightarrow \pi^-\Delta^{++}, \gamma p \rightarrow \pi^+\Delta^0, \gamma n \rightarrow \pi^-\Delta^+$ and $\gamma n \rightarrow \pi^+\Delta^-$ respectively.

In the centre of mass frame $T^{II} = 0$. The helicity amplitudes for T^{III} and T^{IV} are easily evaluated so that the φ_i^B given in equation (2.37) are amended to

$$\Phi_1^B = \varphi_1^B - B_6\eta_- \cos^3\theta/2$$

$$\Phi_2^B = \varphi_2^B - \frac{1}{\sqrt{3}}B_6\left(\eta_+ + 2\eta_- \frac{E_3}{m_3}\right)\sin\theta/2\cos^2\theta/2$$
$$+ b\, B_7\eta_- \sin\theta\cos\theta/2$$

$$\Phi_3^B = \varphi_3^B - \frac{1}{\sqrt{3}} B_6 \left(\eta_- + 2\eta_+ \frac{E_3}{m_3}\right) \sin^2\theta/2 \cos\theta/2$$

$$+ b B_7 \eta_+ \sin\theta \sin\theta/2$$

$$\Phi_4^B = \varphi_4^B - B_6 \eta_+ \sin^3\theta/2$$

$$\Phi_5^B = \varphi_5^B + B_6 \eta_+ \sin\theta/2 \cos^2\theta/2$$

$$\Phi_6^B = \varphi_6^B - \frac{1}{\sqrt{3}} B_6 \left(\eta_- \cos^2\theta/2 - 2\eta_+ \frac{E_3}{m_3} \sin^2\theta/2\right) \cos\theta/2$$

$$- b B_7 \eta_+ \sin\theta \sin\theta/2$$

$$\Phi_7^B = \varphi_7^B + \frac{1}{\sqrt{3}} B_6 \left(\eta_+ \sin^2\theta/2 - 2\eta_- \frac{E_3}{m_3} \cos^2\theta/2\right) \sin\theta/2$$

$$+ b B_7 \eta_- \sin\theta \cos\theta/2$$

$$\Phi_8^B = \varphi_8^B - B_6 \eta_- \sin^2\theta/2 \cos\theta/2 \tag{3.5}$$

where

$$B_6 = -\frac{3ghX_\rho}{m'}\left(1 + \frac{2m'}{\mu}\right) F'G \frac{1}{\sqrt{C}}$$

$$B_7 = -\frac{3ghX_\rho}{m'}\left(1 + \frac{2m'}{\mu}\right) F'G \frac{1}{\sqrt{C}} \frac{1}{(u - m_\Delta)} \cdot \frac{2}{\sqrt{3}} \frac{Q^2}{m_3} (E_3 + E_\Delta) \tag{3.6}$$

where m_Δ is the mass of the exchanged decuplet in the u-channel and all other notation is as in chapter 2.

Gauge invariant extensions to the helicity amplitudes for $\gamma p \rightarrow K^+ Y_1^{*0}$ and $\gamma n \rightarrow K^+ Y_1^{*-}$ are identical to those for $\gamma p \rightarrow \pi^+ \Delta^0$ and $\gamma n \rightarrow \pi^+ \Delta^-$ with K and Y_1^* replacing π and Δ respectively.

The six channels already mentioned, $\gamma p \rightarrow \pi^\mp \Delta^{++}_{0}$, $\gamma n \rightarrow \pi^\mp \Delta^\pm$, $\gamma p \rightarrow K^+ Y_1^{*0}$ and $\gamma n \rightarrow K^+ Y_1^{*-}$ are the only $0^- \frac{3}{2}^+$ photoproduction processes which allow pseudoscalar exchange. The other charge states of $\gamma N \rightarrow \pi \Delta$ do not allow pion exchange by charge conjugation and the F coupling for K exchange at the MMM vertex is zero for the other charge states of $\gamma N \rightarrow K Y_1^*$. The couplings of the pseudoscalar and vector exchanges for the six channels are shown in table 3.

The terms added to the helicity amplitudes to ensure guage invariance are seen to be approximately S-wave. Since we are applying absorption to the initial and final states the contributions from these additional amplitudes are very small particularly near the forward direction.

The theoretical results for the differential cross-section at energies from 3 to 16 GeV are shown in figures 1, 2, 3(a) and 4(a). The data shows the very interesting feature, pointed out by Richter [11], that in the near forward direction the differential cross-sections appear to fall as e^{12t}. This slope rapidly decreases at around t = -0.2 and then becomes similar to that for $\gamma p \to \pi^+ n$. The contribution from π exchange alone is also shown and we see that the steep slope in the forward direction is not accounted for. However, the inclusion of ρ exchange improves this particularly at the higher energies. The larger $|t|$ behaviour is quite pleasing although the change in the slope is not sharp enough at t = - 0.2 and, in fact, becomes almost negligible at 16 GeV, the slope in the larger $|t|$ region then being too steep. The energy dependence is good with the normalisation consistently a little too low. The correct turn-over in the forward direction is obtained.

At present there is only preliminary data available at 16 GeV for the remaining three charge states of $\gamma N \to \pi \Delta$. The theoretical predictions at 11 GeV are shown in figures 3(b), (c) and (d) and comparison is made with experiment at 16 GeV in figures 4 (b), (c) and (d). From table 3 for the couplings we see that the π exchange contributions to $\gamma p \to \pi^- \Delta^{++}$ and $\gamma n \to \pi^+ \Delta^-$ are equal and three times those for $\gamma p \to \pi^+ \Delta^0$ and $\gamma n \to \pi^- \Delta^+$. However, the equality (up to this factor of 3) of all four reactions is slightly broken by the gauge invariant extensions being different in each case, and also broken by a difference in the relative sign between the π exchange and ρ exchange amplitudes. The constructive interference in the wider angle region for $\gamma p \to \pi^+ \Delta^0$ and $\gamma n \to \pi^+ \Delta^-$ gives too large a differential cross-section whereas the destructive interference in this region for $\gamma p \to \pi^- \Delta^{++}$ and $\gamma n \to \pi^- \Delta^+$ gives too small a prediction. This is not surprising since the predicted ρ exchange contribution is probably too large at 16 GeV. We have stressed that the energy dependence for vector exchange amplitudes is known to be wrong and is why we should only expect the model to work at intermediate energies. It can, however, be seen that the relative sign between the ρ and π exchange amplitudes does appear to be correct. Perhaps more may be said when data at lower energies becomes available.

The spin density matrix elements for the decay of the Δ resonance for $\gamma p \to \pi^- \Delta^{++}$ at 3 and 4.65 GeV, are shown in figure 5. Being sensitive to the ρ exchange contribution more experimental data here would prove extremely useful.

So far there is no experimental data available on the differential cross-sections for $\gamma p \to K^+ Y_1^{*0}$ and $\gamma n \to K^+ Y_1^{*-}$. In figure 6 we show the predicted angular distribution at 4.65 GeV. The cross-section for $\gamma p \to K^+ Y_1^{*0}$ out to $|t| = 0.6$ is calculated as $0.16\ \mu b$. This value is compatible with the experimental value of $0.22 \pm .10\ \mu b$ (12) for the total cross-section for this reaction.

2) Elastic Scattering and $\gamma p \to V^0 p$ $(V^0 = \rho^0, \omega, \varphi)$

C parity excludes vector exchanges in the reactions $\gamma N \to V^0 N$ thus leaving only π^0 and η exchanges.

These pseudo-scalar exchange amplitudes are already gauge invariant. There is only experimental data for the reaction when the nucleon is a proton and hence we shall restrict ourselves to this case although all the discussion will also go through for the case of a target neutron.

From equation (2.3) vector dominance allows us to write

$$T(\gamma p \to V^0 p)$$
$$= X_\rho T(\rho^0_{Tr} p \to V^0 p) + X_\omega T(\omega_{Tr} p \to V^0 p) + X_\phi T(\phi_{Tr} p \to V^0 p) \tag{3.7}$$

Consequently for each production process two parts of the amplitude allow, in general, π^0 and η exchanges and one part is elastic scattering of $V^0_{Tr} p \to V^0 p$ multiplied by the appropriate factor. To include this part of the total amplitude we take values of the total cross-sections for ρp, ωp and ϕp from the Quark Model predictions since these are so far in agreement with experiment. These predictions are [13]

$$\sigma_T(\rho p) = \sigma_T(\omega p) = \tfrac{1}{2}\left[\sigma_T(\pi^+ p) + \sigma_T(\pi^- p)\right]$$

$$\sigma_T(\phi p) = 2\,\sigma_T(K^+ p) + \sigma_T(\pi^- p) - \sigma_T(\pi^+ p) \tag{3.8}$$

giving

$$\sigma_T(\rho p) = \sigma_T(\omega p) = 28 \text{ mb}$$

$$\sigma_T(\varphi p) = 11.5 \text{ mb} \tag{3.9}$$

From the Optical Theorem we obtain the differential cross-sections in the forward direction to be

$$X_\rho^2 \frac{d\sigma}{dt}(\rho_0 p \to \rho_0 p)\Big|_{t=0} = 158 \ \mu b \ (GeV/c)^{-2}$$

$$X_\omega^2 \frac{d\sigma}{dt}(\omega p \to \omega p)\Big|_{t=0} = 17.5 \ \mu b \ (GeV/c)^{-2}$$

$$X_\varphi^2 \frac{d\sigma}{dt}(\varphi p \to \varphi p)\Big|_{t=0} = 5.9 \ \mu b \ (GeV/c)^{-2} \tag{3.10}$$

For this diffractive part of the differential cross-sections we use the form

$$\frac{d\sigma}{dt} = A \exp(Bt + Ct^2) \tag{3.11}$$

In all three processes we take B = 9 and C = 2.5 which are obtained from πp elastic scattering, A is, of course, determined from equation (3.10). So our diffractive differential cross-sections are

$$\frac{d\sigma}{dt}(\gamma p \to \rho_o p) = 158 \exp(9t + 2.5t^2) \ \mu b\ (GeV)^{-2}$$

$$\frac{d\sigma}{dt}(\gamma p \to \omega p) = 17.5 \exp(9t + 2.5t^2) \ \mu b\ (GeV)^{-2}$$

$$\frac{d\sigma}{dt}(\gamma p \to \phi p) = 5.9 \exp(9t + 2.5t^2) \ \mu b\ (GeV)^{-2}$$

(3.12)

Since the diffractive amplitude is purely imaginary and the exchange amplitude is purely real there is no interference between the two mechanisms. We thus evaluate the differential cross-sections arising from the pseudoscalar exchange amplitudes given in equation (2.39) using the absorption corrections and then add the diffractive contribution.

The SU(3) couplings for π^0 and η exchanges are shown in table 4. The final results for the differential cross-sections for $\gamma p \to \rho^0 p$ at 4 GeV, $\gamma p \to \omega p$ at 2.15 GeV and 4.15 GeV and $\gamma p \to \phi p$ at 4.15 GeV are shown as continuous curves in figures 7, 8(a), 8(b) and 9 respectively. Without the inclusion of the diffractive part the results are given as dotted lines in these figures.

The good fit for $\gamma p \to \rho^0 p$ at 4 GeV tell us nothing more than that we have chosen the correct parameters for the diffractive part since in this process the meson exchange contribution is negligible in comparison. We could have started from here and then obtained the diffractive differential cross-sections for ωp and ϕp production.

Both diffractive and exchange contributions are seen to be important for $\gamma p \rightarrow \omega p$ at 2.15 and 4.15 GeV. These combine to give extremely good results at both energies.

The reaction $\gamma p \rightarrow \phi p$ at 4.15 GeV does not allow π^0 exchange and the η exchange contribution is small. The evaluated differential cross-section appears to be slightly too large in the forward direction, and then falls off too rapidly with increasing $|t|$. It is hardly surprising that we do not obtain good agreement here since our assumption that the ϕp elastic scattering differential cross-section is similar to the πp is probably unreasonable.

In figure 10 we show the spin density matrix elements for the decay of the ρ meson in $\gamma p \rightarrow \rho^0 p$ at 3 and 4.65 GeV. The inclusion of an exchange contribution for this reaction has more effect here than in the differential cross-section. However, the difference is not large enough to ascertain whether an improvement has been obtained.

Figure 11 shows the decay matrix elements for $\gamma p \rightarrow \omega p$ at 1.95 and 4.15 GeV. The results are an improvement on those for diffraction only and are in good agreement with experiment.

No data appears to be available for the decay matrix elements for $\gamma p \rightarrow \phi p$.

3) <u>Remaining Processes</u>

Restricting ourselves again to the target baryon being a proton, the photoproduction of $\rho^+ n$, $K^{*+}\Lambda$, $K^{*+}\Sigma^0$, $K^{*0}\Sigma^+$, $\rho^-\Delta^{++}$, $\rho^+\Delta^0$, $\rho^0\Delta^+$, $\omega\Delta^+$, $\phi\Delta^+$, $K^{*0}Y_1^{*+}$ and $K^{*+}Y_1^{*0}$ all allow 0^- exchanges. Except for the reactions $\gamma p \rightarrow V^0\Delta^+$, $\gamma p \rightarrow K^{*0}\Sigma^+$ and $\gamma p \rightarrow K^{*0}Y_1^{*+}$, 1^- exchanges are also allowed. The vector amplitudes are not gauge invariant but since no experimental data is available for the differential cross-sections and since we expect the pseudoscalar exchanges to dominate we shall not consider this problem here. The couplings for $1^- \frac{1}{2}^+$ production are given in table 5 and for $1^- \frac{3}{2}^+$ production in tables 6 and 7.

Figure 12 shows the predictions for the angular distributions of $\gamma p \rightarrow \rho^-\Delta^{++}$ and $\gamma p \rightarrow \rho^+\Delta^0$ at 4.65 GeV. The cross-section for $\gamma p \rightarrow \rho^-\Delta^{++}$ in the region $\cdot 0 < |t| \leq \cdot 3$ is calculated to be $\cdot 63\mu b$. This is in good agreement with the experimental value of $\cdot 6 \pm \cdot 3\ \mu b$ (14) for this part of the cross-section.

At 5 GeV the predictions for $\rho^+ n$ production are shown in figure 13, for $K^{*+}\Lambda$, $K^{*+}\Sigma^0$ and $K^{*0}\Sigma^+$ in figure 14, for $\rho^0\Delta^+$, $\omega\Delta^+$ and $\phi\Delta^+$ in figure 15 and for $K^{*+}Y_1^{*0}$ and $K^{*0}Y_1^{*+}$ in figure 16. No data is so far available on the cross-sections for these processes.

4) Conclusions

For processes dominated by pion exchange the model reproduces well the overall characteristics of the data, though detailed agreement is not always good. However, it must be remembered that the model is parameter free. It is an interesting point to investigate whether the general success of the absorption model for pion exchange can be extended to reactions proceeding by kaon exchange. Calculations for the reactions $\gamma p \rightarrow K^+\Lambda, K^+\Sigma^0$ (15) have been found to be not very encouraging in contrast to the excellent fits obtained for $p\bar{p} \rightarrow Y\bar{Y}$ (4). Unfortunately there is no experimental data yet available for the strangeness exchange processes we have considered here to allow investigation of this point. It would also be very useful to have better data on the decay density matrix elements in such reactions as $\gamma N \rightarrow \pi\Delta$ allowing a more accurate test of the U(6,6) predictions for the vector exchange contributions.

An extension of the model is to apply the absorption corrections to Reggeised amplitudes. Some extremely good fits have been obtained (16) using this model for strong interactions but for photoproduction difficulties present with our model are not always eliminated. The answer to this may lie in a wrong assumption about vector dominance.

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$\lambda_3\ \lambda_4 \backslash \lambda_1\ \lambda_2$	$1\ \ \frac{1}{2}$	$1\ \ -\frac{1}{2}$	$-1\ \ \frac{1}{2}$	$-1\ \ -\frac{1}{2}$
$0\ \ \frac{3}{2}$	φ_1	φ_5	φ_8	$-\varphi_4$
$0\ \ \frac{1}{2}$	φ_2	φ_6	$-\varphi_7$	φ_3
$0\ \ -\frac{1}{2}$	φ_3	φ_7	φ_6	$-\varphi_2$
$0\ \ -\frac{3}{2}$	φ_4	φ_8	$-\varphi_5$	φ_1

TABLE 1.

HELICITY DEPENDENCE OF AMPLITUDES FOR $\gamma N \rightarrow 0^- \frac{3}{2}^+$

$\lambda_3\ \lambda_4$ \ $\lambda_1\ \lambda_2$	$1\ \ \frac{1}{2}$	$1\ \ -\frac{1}{2}$	$-1\ \ \frac{1}{2}$	$-1\ \ -\frac{1}{2}$
$1\ \ \frac{1}{2}$	φ_1	φ_7	φ_{12}	$-\varphi_6$
$1\ \ -\frac{1}{2}$	φ_2	φ_8	$-\varphi_{11}$	φ_5
$0\ \ \frac{1}{2}$	φ_3	φ_9	$-\varphi_{10}$	φ_4
$0\ \ -\frac{1}{2}$	φ_4	φ_{10}	φ_9	$-\varphi_3$
$-1\ \ \frac{1}{2}$	φ_5	φ_{11}	φ_8	$-\varphi_2$
$-1\ \ -\frac{1}{2}$	φ_6	φ_{12}	$-\varphi_7$	φ_1

TABLE 2.

HELICITY DEPENDENCE OF AMPLITUDES FOR $\gamma N \to 1^- \frac{1}{2}^+$

REACTION	F'	D'	G
$\gamma p \rightarrow \pi^- \Delta^{++}$	2	$\frac{2}{3}$	$-\sqrt{2}$
$\gamma p \rightarrow \pi^+ \Delta^0$	-2	$\frac{2}{3}$	$\sqrt{\frac{2}{3}}$
$\gamma n \rightarrow \pi^- \Delta^+$	2	$\frac{2}{3}$	$-\sqrt{\frac{2}{3}}$
$\gamma n \rightarrow \pi^+ \Delta^-$	-2	$\frac{2}{3}$	$\sqrt{2}$
$\gamma p \rightarrow K^+ Y_1^{*0}$	-2	$\frac{2}{3}$	$\frac{1}{\sqrt{3}}$
$\gamma n \rightarrow K^+ Y_1^{*-}$	-2	$\frac{2}{3}$	$\sqrt{\frac{2}{3}}$

TABLE 3.

COUPLINGS FOR $\gamma N \rightarrow 0^- \frac{3}{2}^+$

REACTION	π^0 EXCHANGE		η EXCHANGE	
	D'	$D+\frac{2}{3}F-S$	D'	$D+\frac{2}{3}F-S$
$\gamma p \to \rho^0 p$	$\frac{2}{3}$	$\frac{5}{3}$	$\frac{2}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$
$\gamma p \to \omega p$	2	$\frac{5}{3}$	$\frac{2\sqrt{3}}{9}$	$\frac{1}{\sqrt{3}}$
$\gamma p \to \phi p$	0	$\frac{5}{3}$	$\frac{-4\sqrt{6}}{9}$	$\frac{1}{\sqrt{3}}$

TABLE 4.

COUPLINGS FOR $\gamma p \to V^0 p$

REACTION	D'	F'	$D+\frac{3}{8}F-S$	$F+3S$
$\gamma p \to \rho^+ n$	$\frac{2}{3}$	-2	$\frac{5}{3}\sqrt{2}$	$\sqrt{2}$
$\gamma p \to K^{*+}\Lambda$	$\frac{2}{3}$	-2	$-\sqrt{3}$	$-\sqrt{3}$
$\gamma p \to K^{*+}\Sigma^0$	$\frac{2}{3}$	-2	$\frac{1}{3}$	-1
$\gamma p \to K^{*0}\Sigma^+$	$-\frac{4}{3}$	0	$\frac{\sqrt{2}}{3}$	$-\sqrt{2}$

TABLE 5.

COUPLINGS FOR $\gamma p \to 1^- \frac{1}{2}^+$

REACTION	π^0 EXCHANGE		η EXCHANGE	
	D'	G	D'	G
$\gamma p \to \rho^0 \Delta^+$	$\frac{2}{3}$	$\frac{1}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$	$-\frac{1}{3}$
$\gamma p \to \omega \Delta^+$	2	$\frac{1}{\sqrt{3}}$	$\frac{2\sqrt{3}}{9}$	$-\frac{1}{3}$
$\gamma p \to \varphi \Delta^+$	0	$\frac{1}{\sqrt{3}}$	$-\frac{4\sqrt{6}}{9}$	$-\frac{1}{3}$

TABLE 6.

COUPLINGS FOR $\gamma p \to V^0 \Delta^+$

REACTION	D'	F'	G
$\gamma p \to \rho^- \Delta^{++}$	$\frac{2}{3}$	2	$-\sqrt{2}$
$\gamma p \to \rho^+ \Delta^0$	$\frac{2}{3}$	-2	$\sqrt{\frac{2}{3}}$
$\gamma p \to K^{*+} Y_1^{*0}$	$\frac{2}{3}$	-2	$\frac{1}{\sqrt{3}}$
$\gamma p \to K^{*0} Y_1^{*+}$	$-\frac{4}{3}$	0	$-\sqrt{\frac{2}{3}}$

TABLE 7.

COUPLINGS FOR $\gamma p \to 1^- \frac{3}{2}^+$

FIGURE CAPTIONS

Figure 1. Differential cross-section for $\gamma p \to \pi^- \Delta^{++}$ at 3 and 4.65 GeV. The data is from DESY (17).

Figure 2. Differential cross-section for $\gamma p \to \pi^- \Delta^{++}$ at 5 and 8 GeV. The data is from SLAC (18).

Figure 3. Differential cross-section for $\gamma p \to \pi^- \Delta^{++}$ $\gamma p \to \pi^+ \Delta^0$, $\gamma n \to \pi^+ \Delta^-$ and $\gamma n \to \pi^- \Delta^+$ at 11 GeV. The data is from SLAC (18).

Figure 4. Differential cross-section for $\gamma p \to \pi^- \Delta^{++}$ $\gamma p \to \pi^+ \Delta^0$, $\gamma n \to \pi^+ \Delta^-$ and $\gamma n \to \pi^- \Delta^+$ at 16 GeV. The data is from SLAC (18) (19).

Figure 5. Spin density matrix elements for the decay of the Δ in $\gamma p \to \pi^- \Delta^{++}$ at 3 and 4.65 GeV. The data is from DESY (17).

Figure 6. Differential cross-section for $\gamma p \to K^+ Y_1^{*0}$ and $\gamma n \to K^+ Y_1^{*-}$ at 4.65 GeV.

Figure 7. Differential cross-section for $\gamma p \to \rho^0 p$ at 4 GeV. The data is from DESY (17).

Figure 8. Differential cross-section for $\gamma p \to \omega p$

at a) 2.15 GeV
b) 4.15 GeV

The data is from DESY (17).

Figure 9. Differential cross-section for $\gamma p \to \phi p$ at 4.15 GeV. The data is from DESY [17].

Figure 10. Spin density matrix elements for the decay of the ρ in $\gamma p \to \rho^0 p$ at 3 and 4.65 GeV. The data is from DESY [17].

Figure 11. Spin density matrix elements for the decay of the ω in $\gamma p \to \omega p$ at 1.95 and 4.15 GeV. The data is from DESY [17].

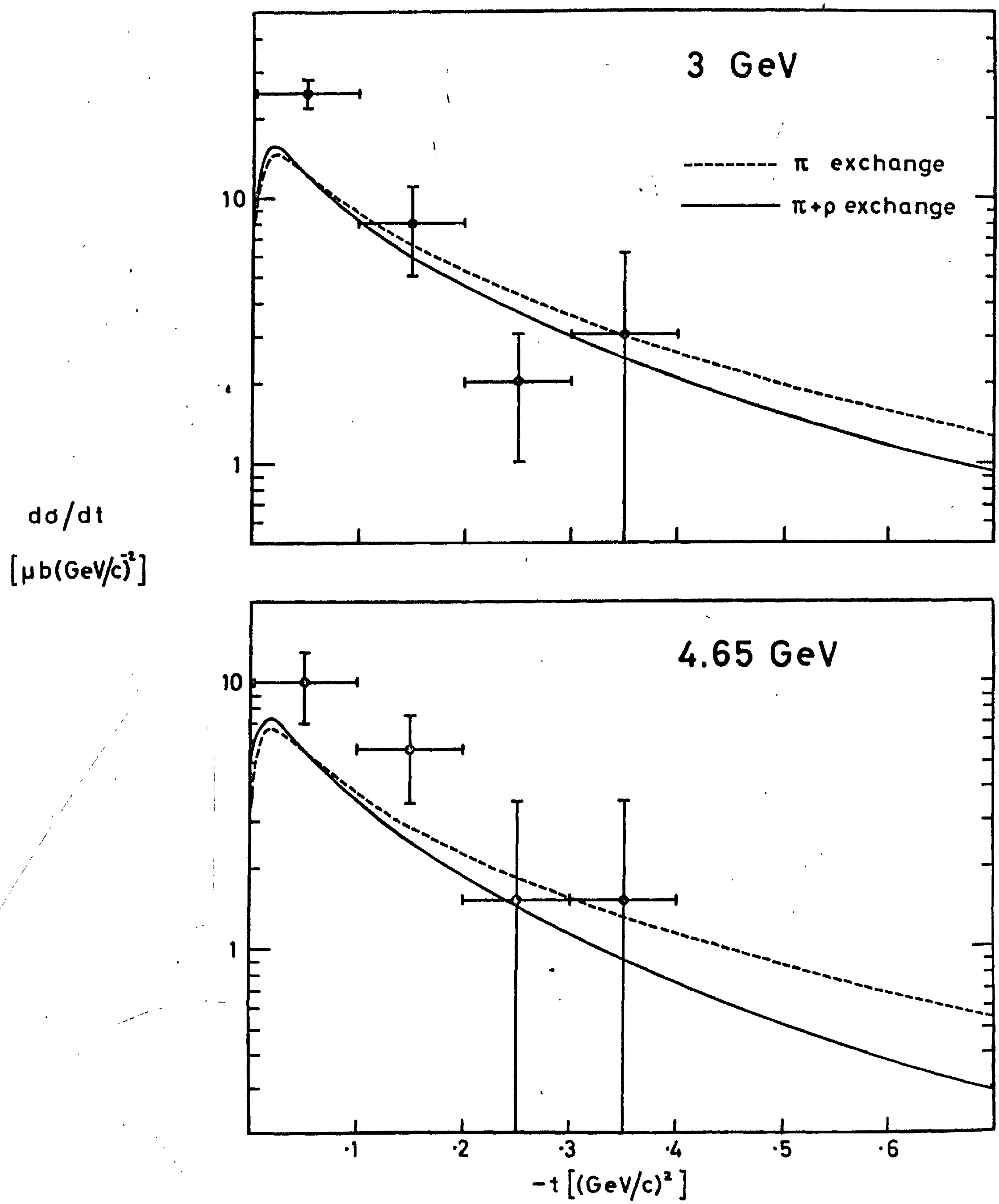
Figure 12. Differential cross-section for $\gamma p \to \rho^- \Delta^{++}$ and $\gamma p \to \rho^+ \Delta^0$ at 4.65 GeV.

Figure 13. Differential cross-section for $\gamma p \to \rho^+ n$ at 5GeV.

Figure 14. Differential cross-section for $\gamma p \to K^{*+} \Lambda$, $\gamma p \to K^{*+} \Sigma^0$ and $\gamma p \to K^{*0} \Sigma^+$ at 5 GeV.

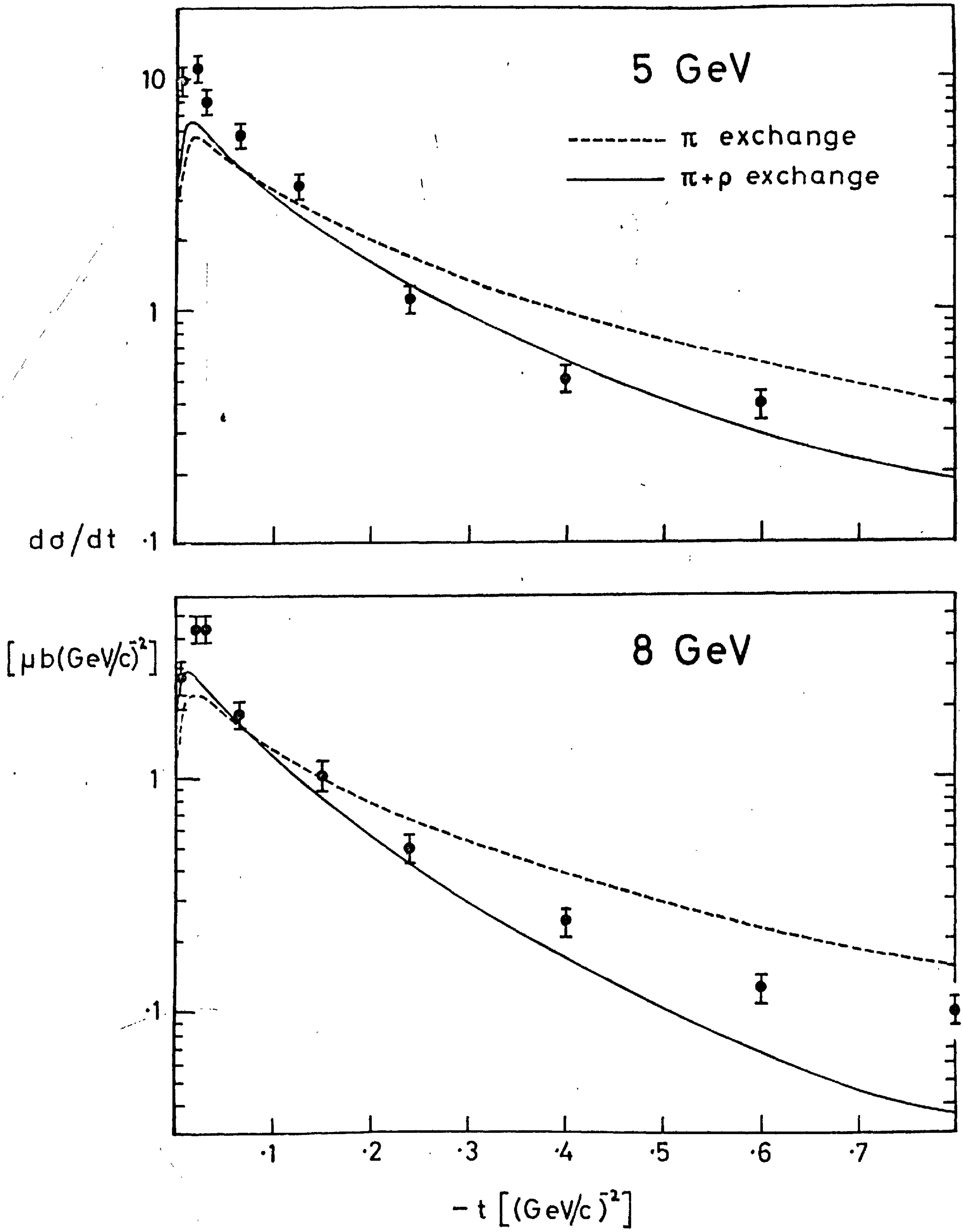
Figure 15. Differential cross-section for $\gamma p \to \rho^0 \Delta^+$ $\gamma p \to \omega \Delta^+$ and $\gamma p \to \phi \Delta^+$ at 5 GeV.

Figure 16. Differential cross-section for $\gamma p \to K^{*0} Y_1^{*+}$ and $\gamma p \to K^{*+} Y_1^{*0}$ at 5 GeV.



3 GeV
π exchange
π+ρ exchange
10
1
dσ/dt
[μb(GeV/c)⁻²]
4.65 GeV
·1
·2
·3
·4
·5
·6
−t [(GeV/c)²]

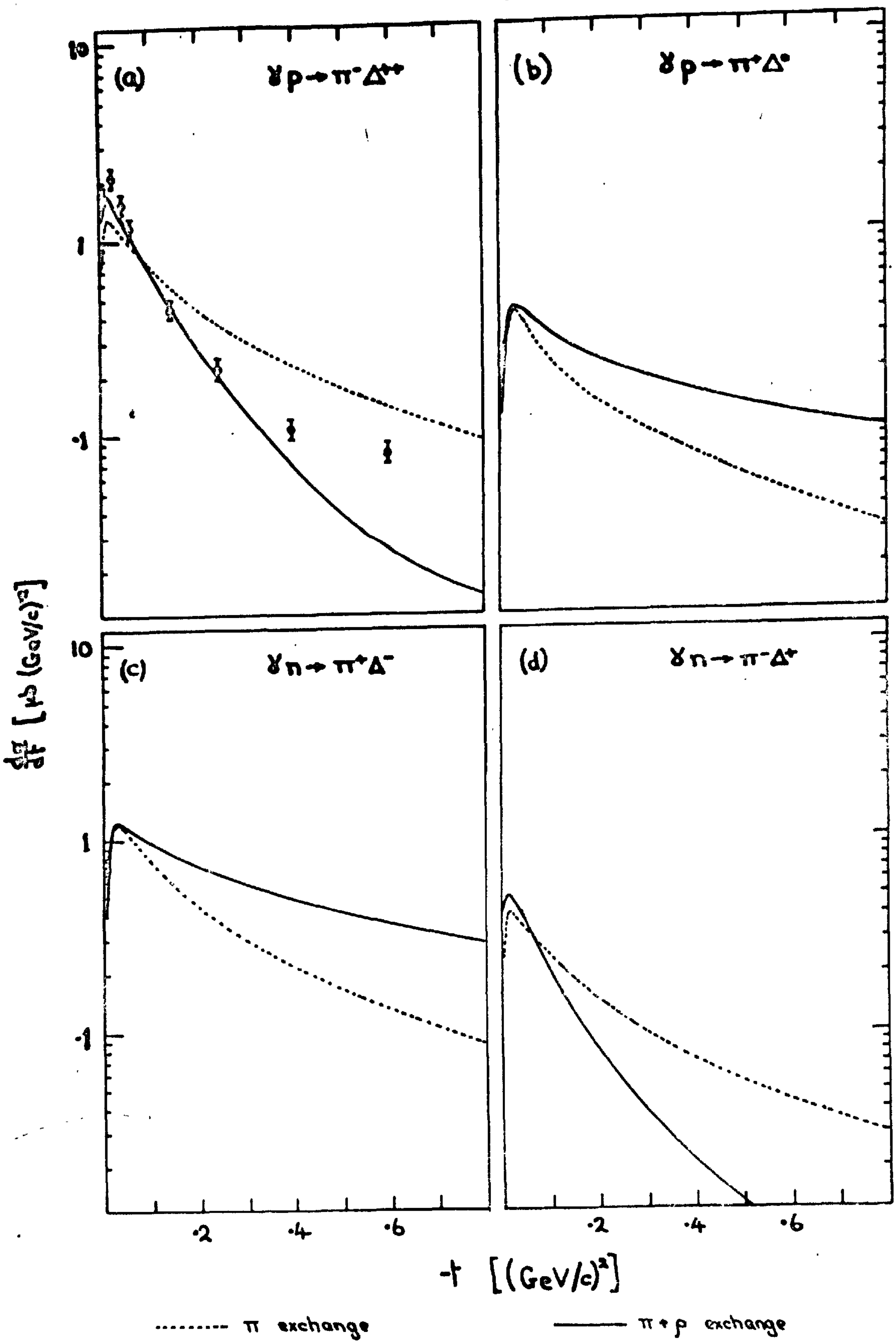
Fig.1.



5 GeV
π exchange
π+ρ exchange
10
1
·1
dσ/dt
[μb(GeV/c)$^{-2}$]
8 GeV
1
·1
·1
·2
·3
·4
·5
·6
·7
− t [(GeV/c)$^{-2}$]

Fig 2.

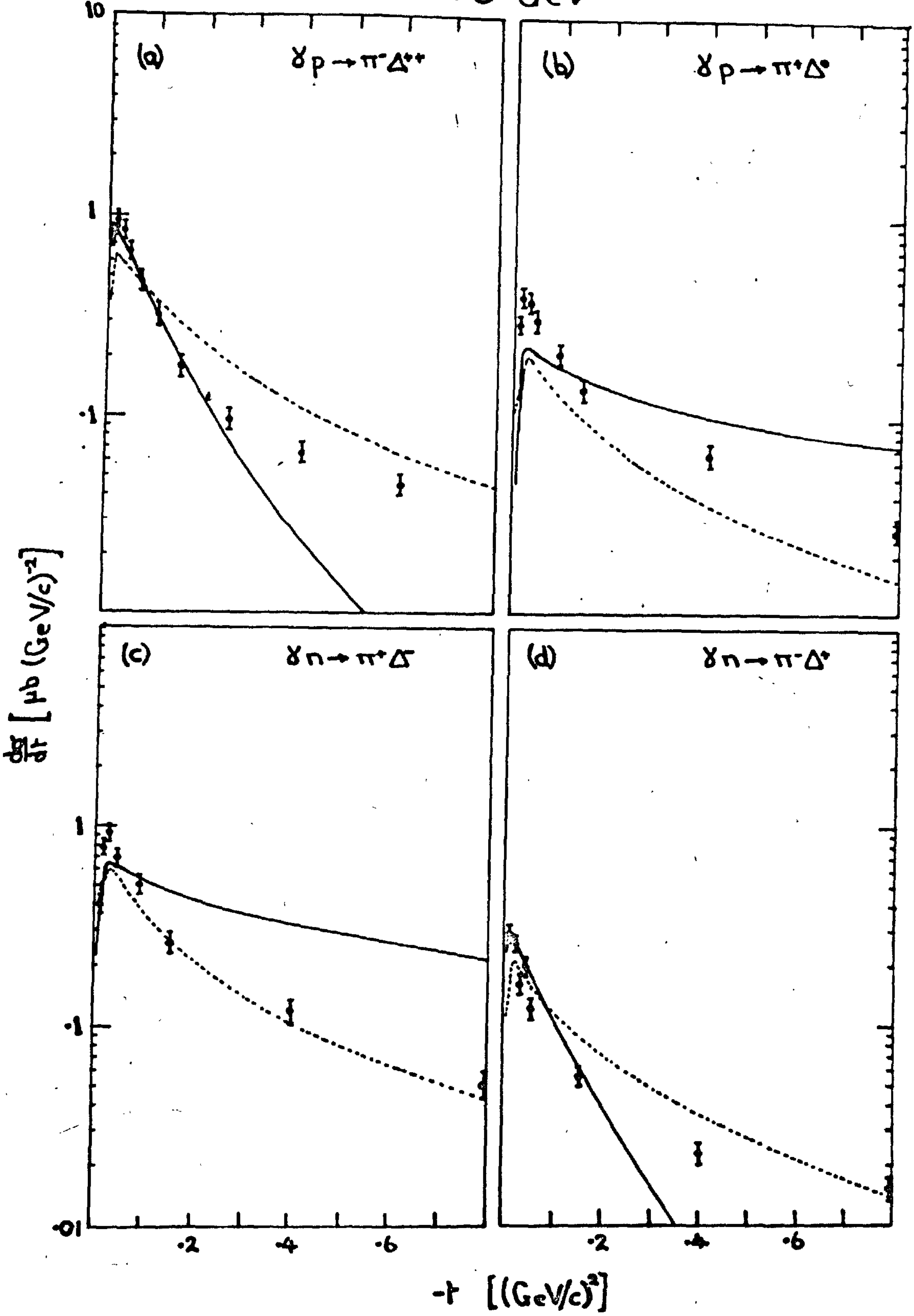
11 GeV



(a) $\gamma p \to \pi^- \Delta^{++}$
(b) $\gamma p \to \pi^+ \Delta^0$
(c) $\gamma n \to \pi^+ \Delta^-$
(d) $\gamma n \to \pi^- \Delta^+$
10
1
·1
$\frac{d\sigma}{dt}$ [μb (GeV/c)$^{-2}$]
·2
·4
·6
$-t$ [(GeV/c)^2]
.... π exchange
——— π + ρ exchange

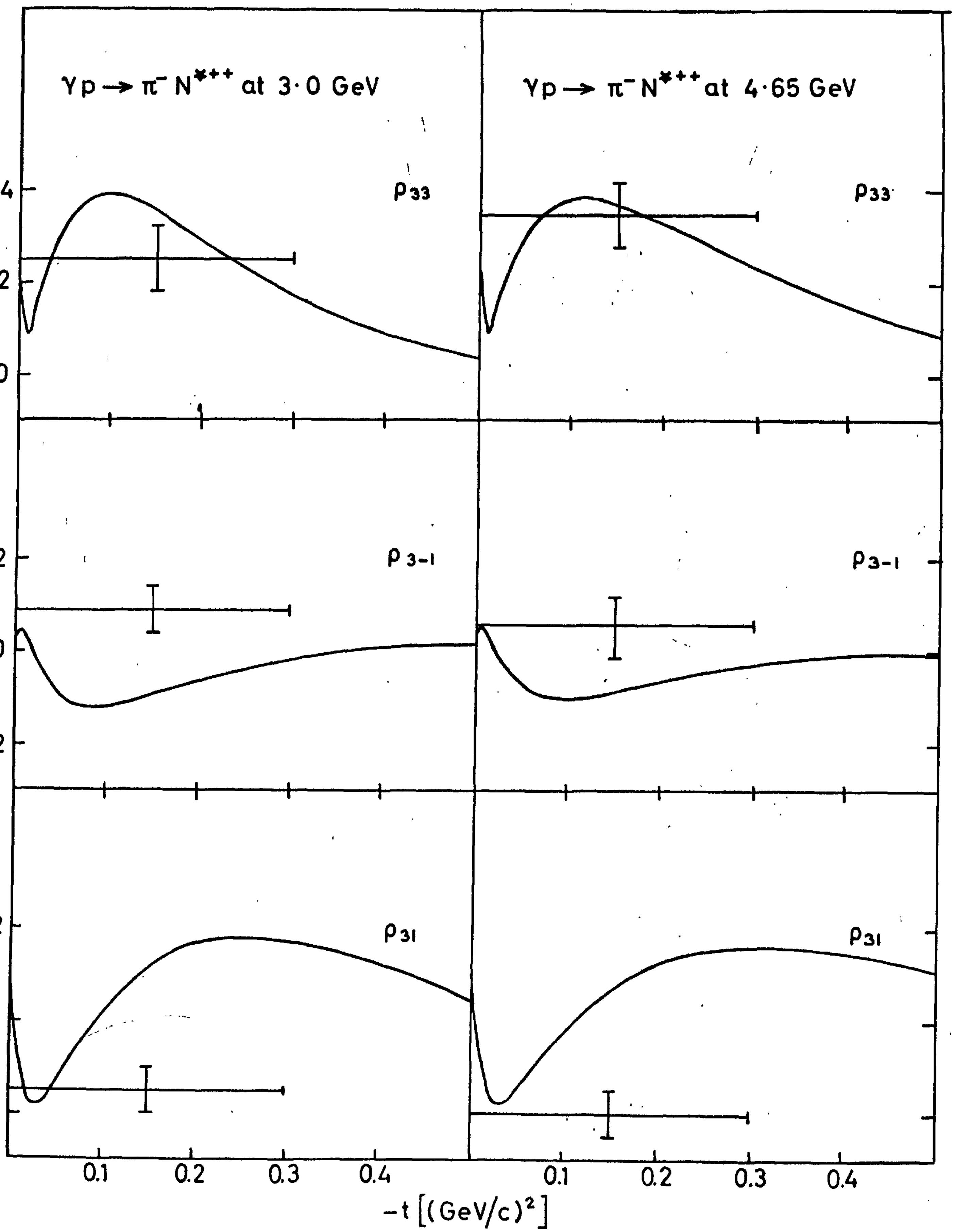
Fig. 3.

16 GeV



·········· π exchange ———— π + ρ exchange

Fig. 4.



γp → π⁻ N*++ at 3·0 GeV
γp → π⁻ N*++ at 4·65 GeV
ρ33
ρ33
ρ3-1
ρ3-1
ρ31
ρ31
0.4
0.2
0
0.2
0
-0.2
0.2
0
-0.2
0.1
0.2
0.3
0.4
0.1
0.2
0.3
0.4
−t [(GeV/c)²]

Fig. 5.

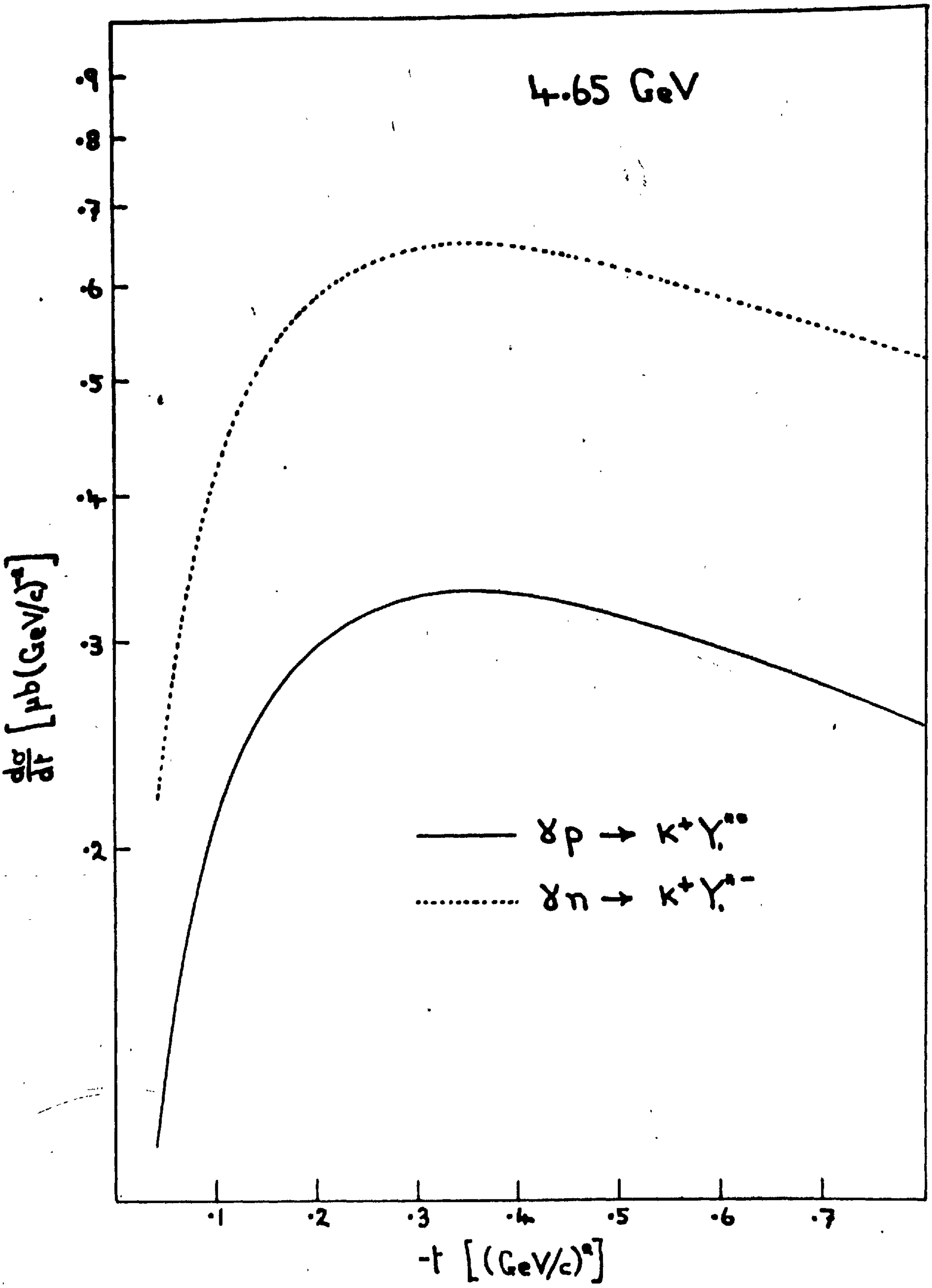
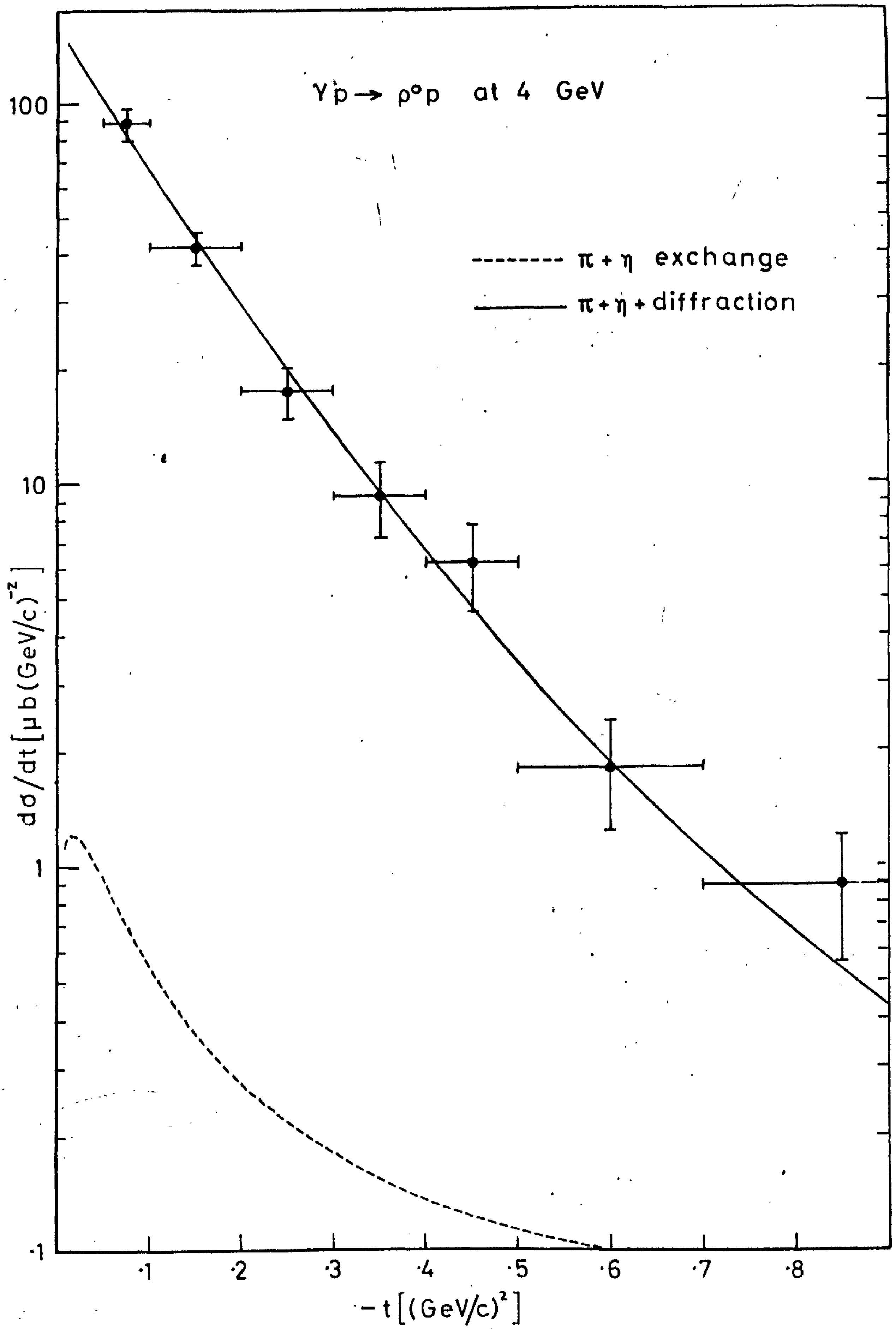

4.65 GeV
$\frac{d\sigma}{dt}$ [μb(GeV/c)$^{-2}$]
.9
.8
.7
.6
.5
.4
.3
.2
$\gamma p \to K^+ Y_1^{*0}$
$\gamma n \to K^+ Y_1^{*-}$
.1
.2
.3
.4
.5
.6
.7
$-t$ [(GeV/c)^2]

Fig. 6.



γp → ρ°p at 4 GeV
π + η exchange
π + η + diffraction
100
10
1
·1
·1
·2
·3
·4
·5
·6
·7
·8
dσ/dt [μb (GeV/c)⁻²]
−t [(GeV/c)²]

Fig. 7.

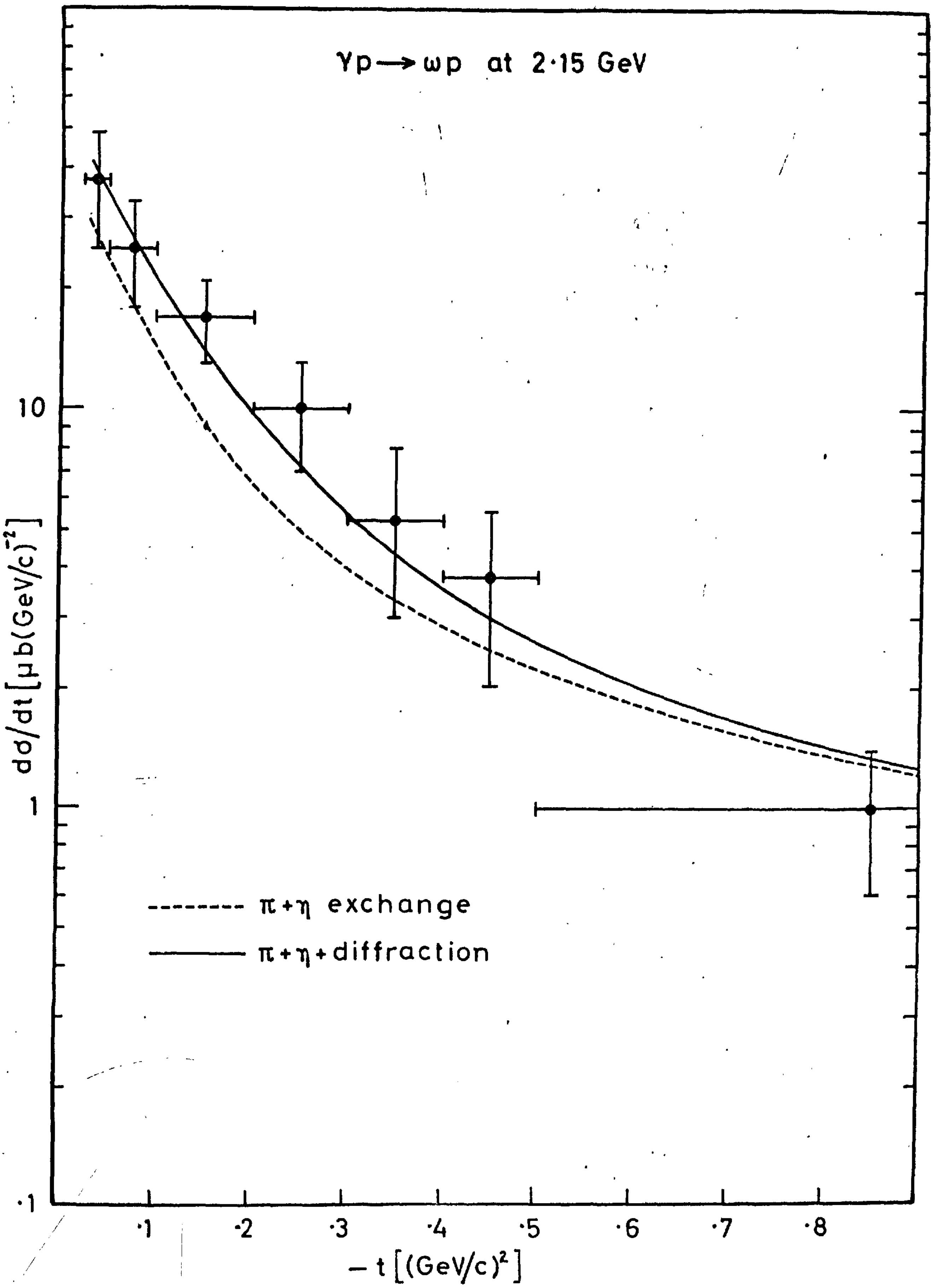

γp ⟶ ωp at 2·15 GeV
dσ/dt [μb(GeV/c)$^{-2}$]
10
1
·1
·1
·2
·3
·4
·5
·6
·7
·8
− t [(GeV/c)^2]
π + η exchange
π + η + diffraction

Fig. 8a.

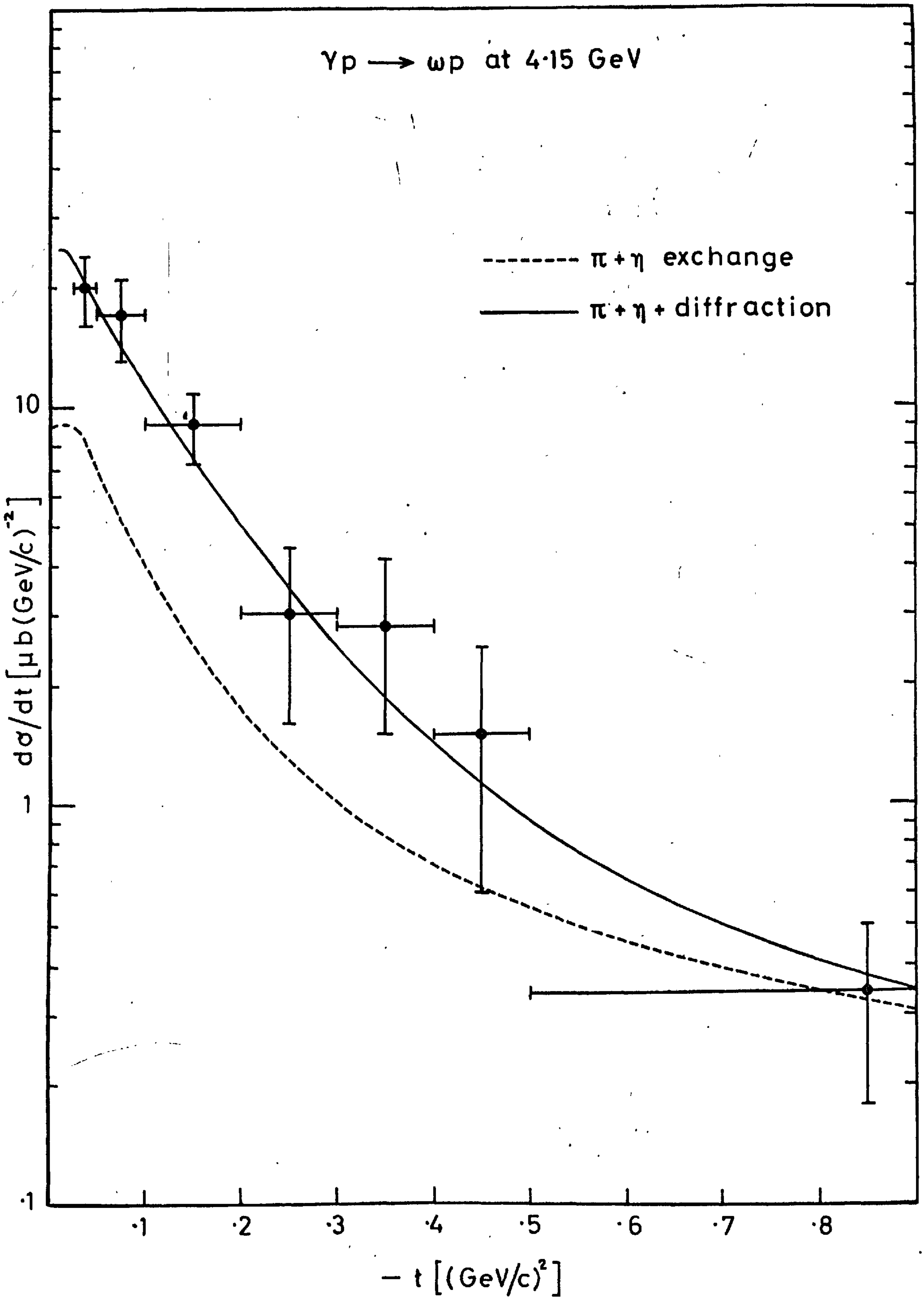

γp ⟶ ωp at 4·15 GeV
π + η exchange
π + η + diffraction
dσ/dt [μb (GeV/c)^-2]
10
1
·1
·1
·2
·3
·4
·5
·6
·7
·8
− t [(GeV/c)²]

Fig. 8b.

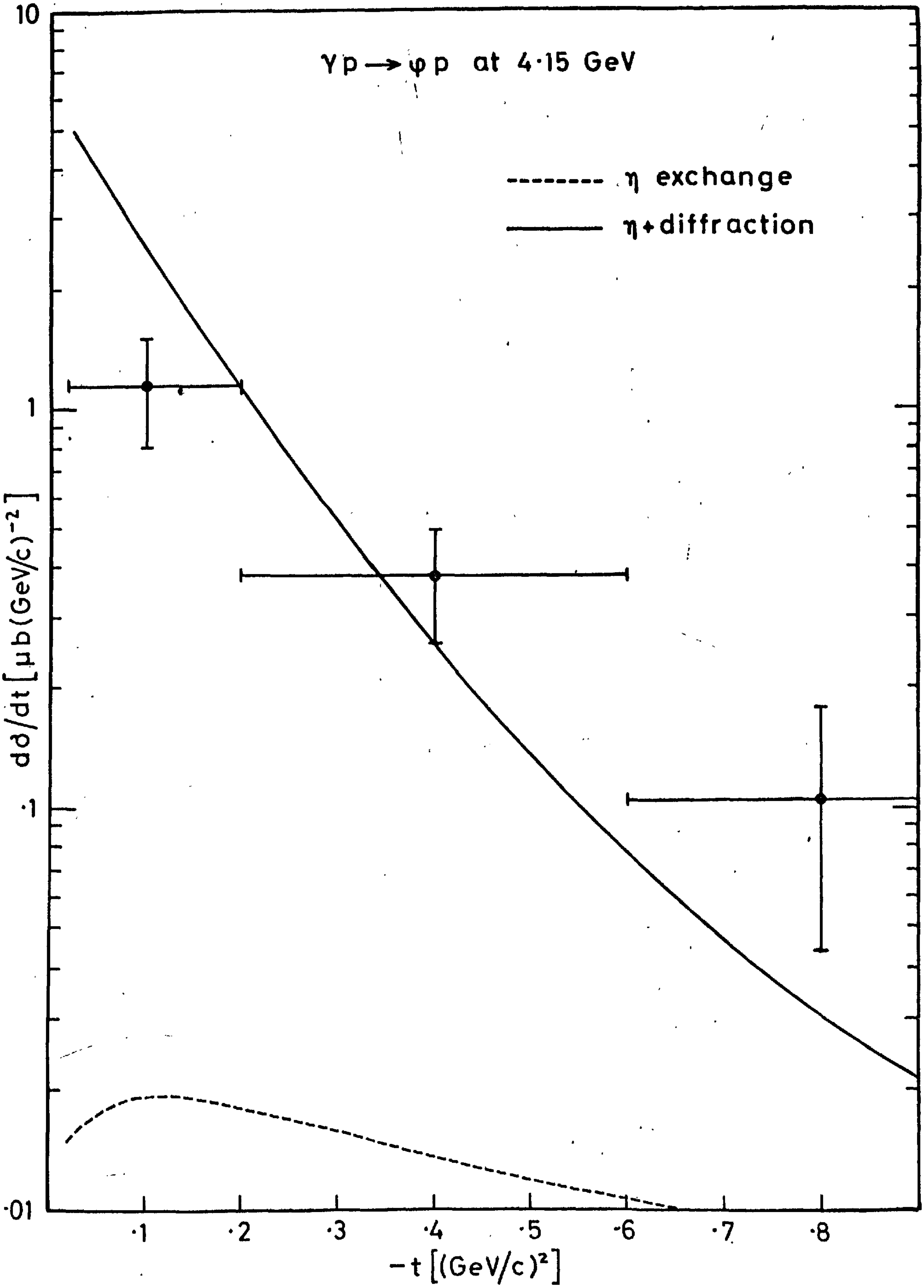

γp ⟶ φp at 4·15 GeV
η exchange
η+diffraction
dσ/dt [μb(GeV/c)⁻²]
−t [(GeV/c)²]
10
1
·1
·01
·1
·2
·3
·4
·5
·6
·7
·8

Fig. 9.

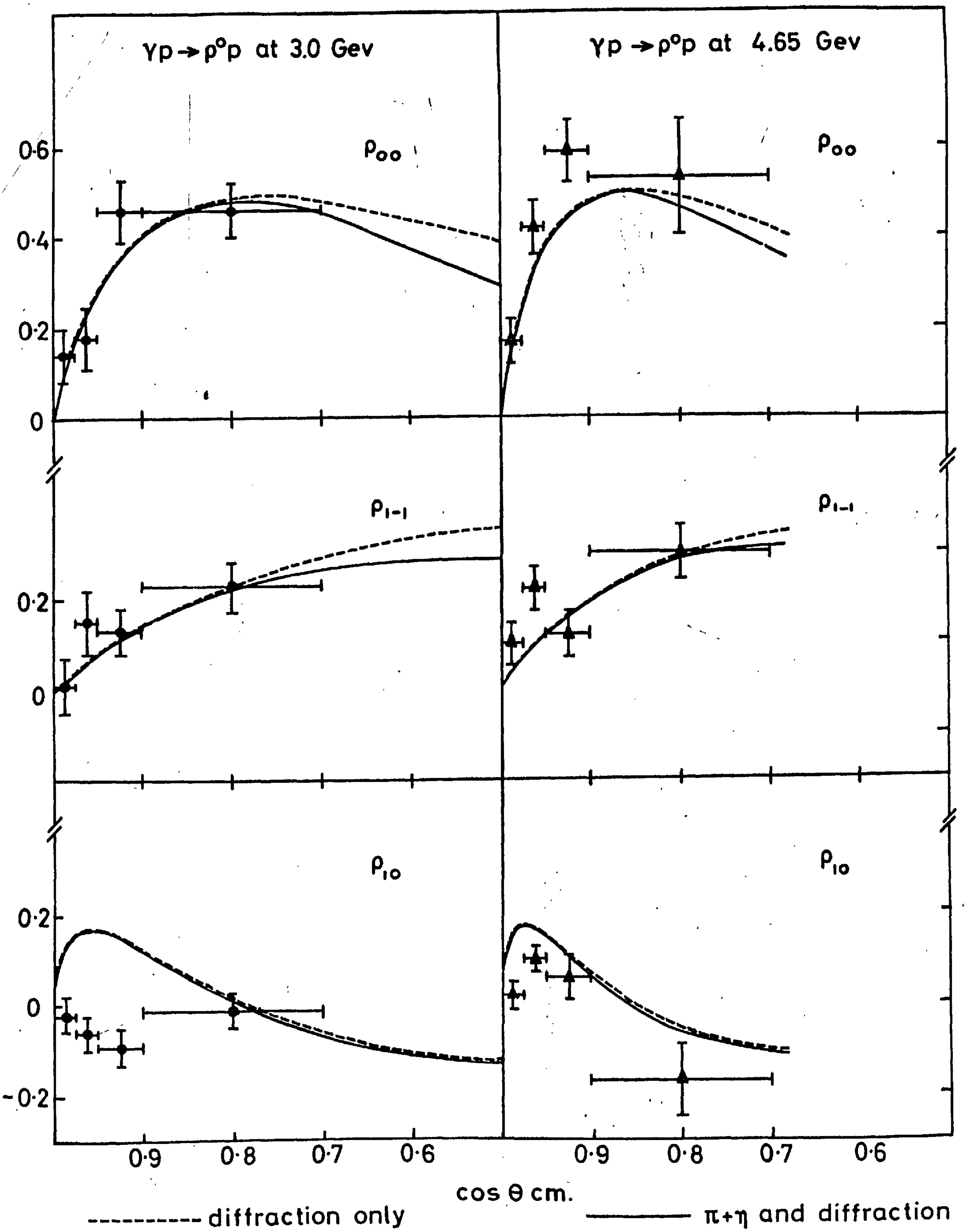

γp → ρ°p at 3.0 Gev
γp → ρ°p at 4.65 Gev
ρ00
ρ1-1
ρ10
0·6
0·4
0·2
0
-0·2
0·9
0·8
0·7
0·6
cos θ cm.
---------- diffraction only
——— π+η and diffraction

Fig. 10.

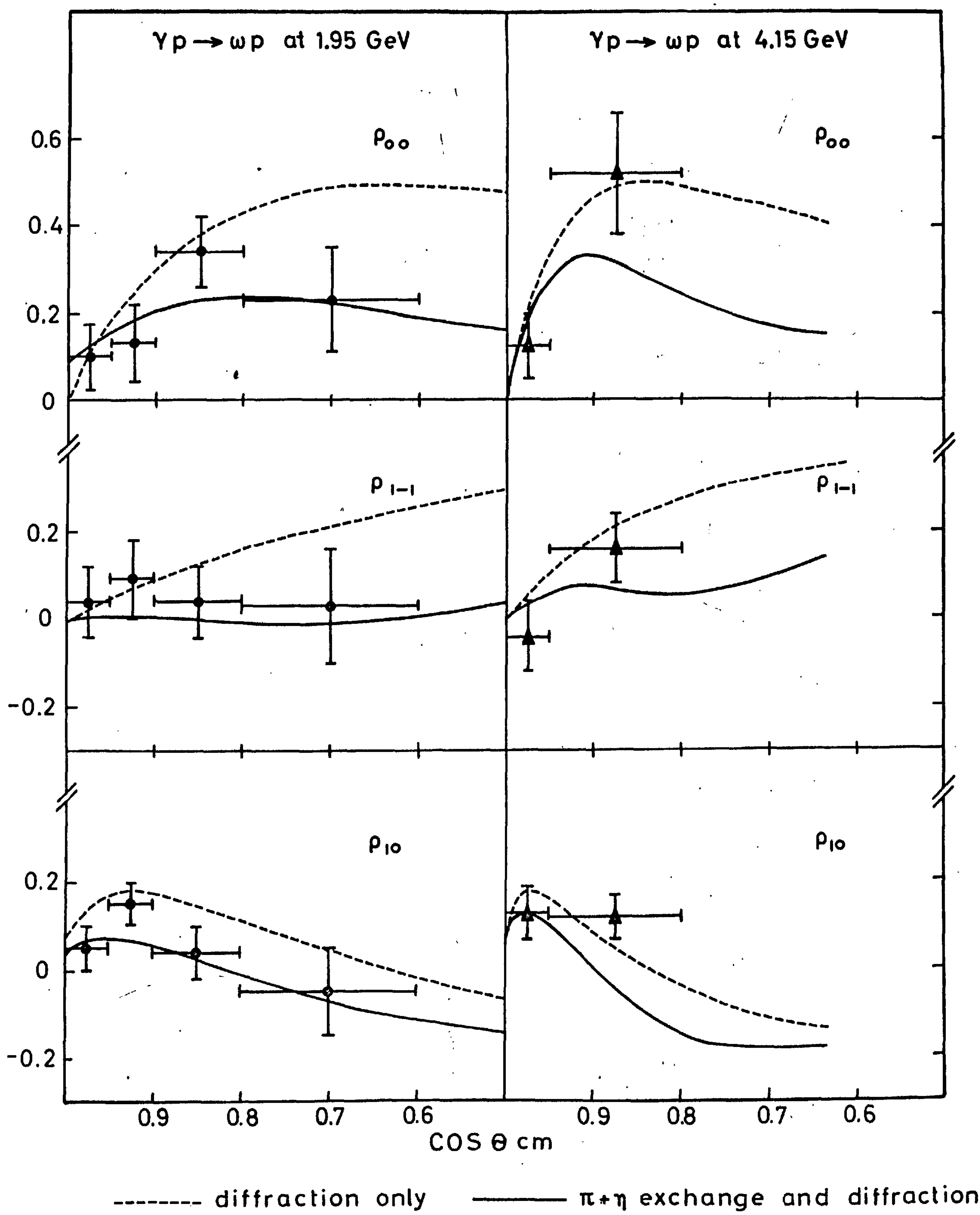

γp → ωp at 1.95 GeV
γp → ωp at 4.15 GeV
ρ00
ρ1-1
ρ10
0.6
0.4
0.2
0
-0.2
0.9
0.8
0.7
0.6
COS θ cm
diffraction only
π+η exchange and diffraction

Fig. 11.

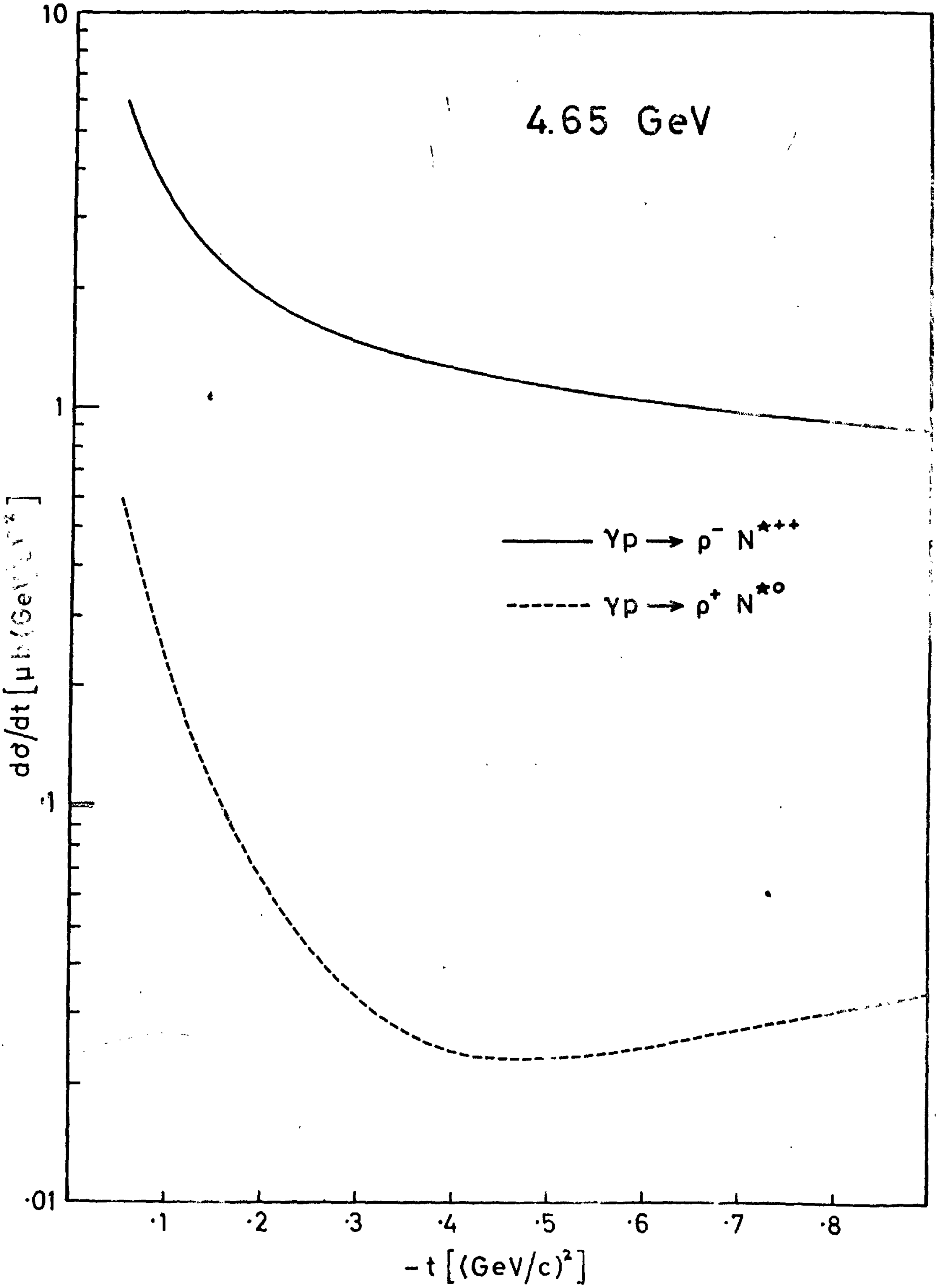


Fig. 12.

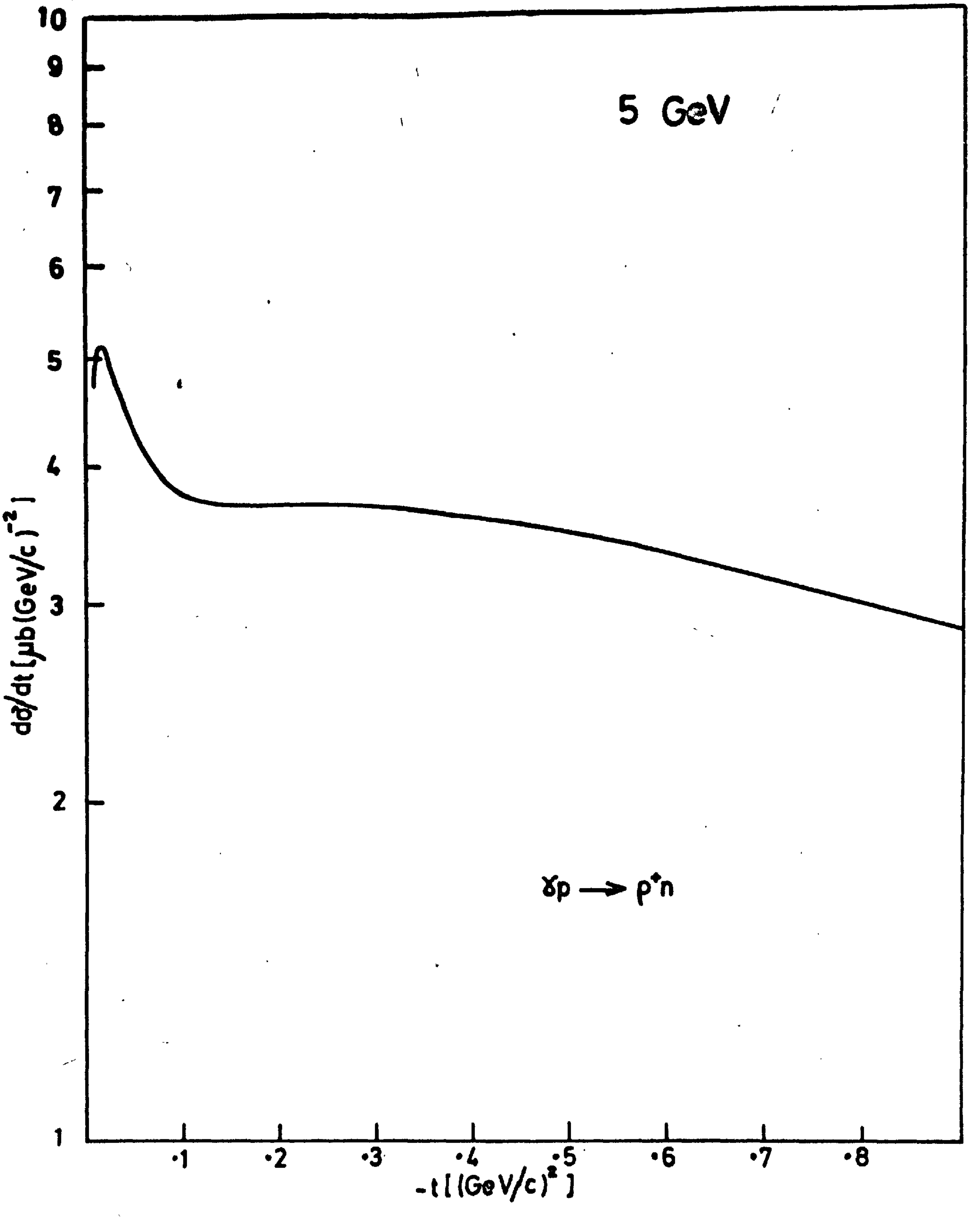

5 GeV
$\gamma p \longrightarrow \rho^+ n$
$d\sigma/dt\ [\mu b\,(GeV/c)^{-2}]$
$-t\ [(GeV/c)^2]$
10
9
8
7
6
5
4
3
2
1
·1
·2
·3
·4
·5
·6
·7
·8

Fig. 13.

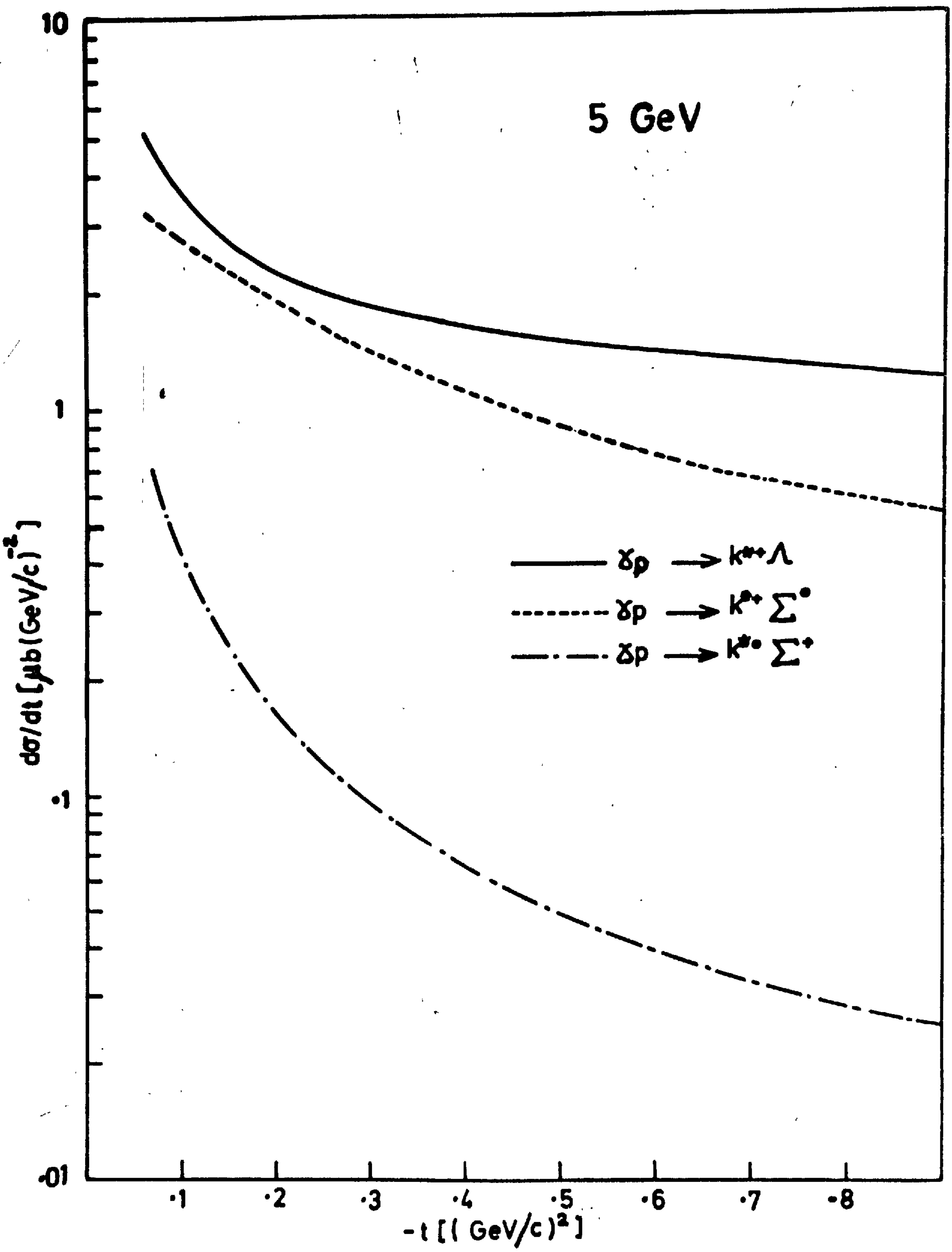
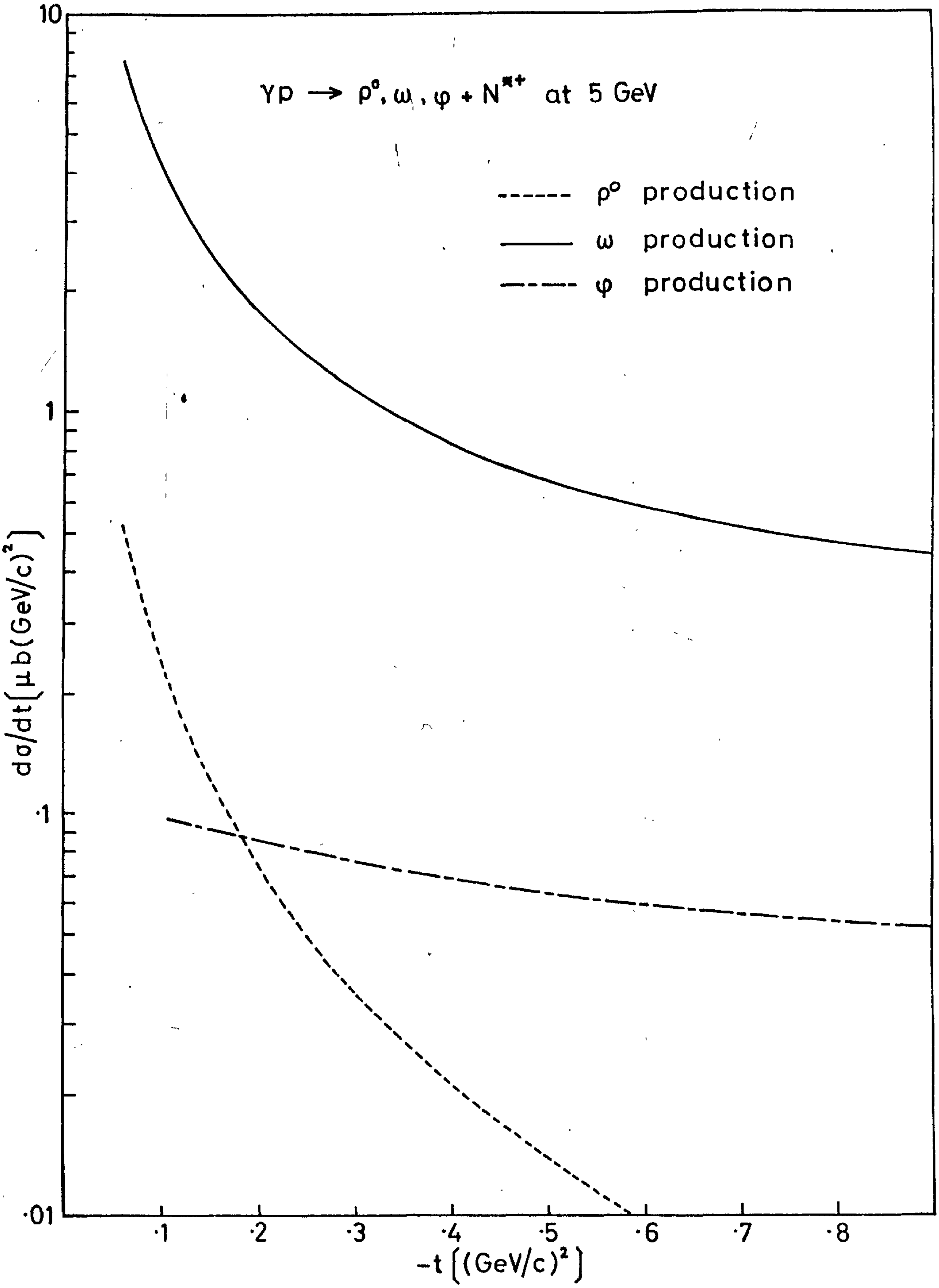
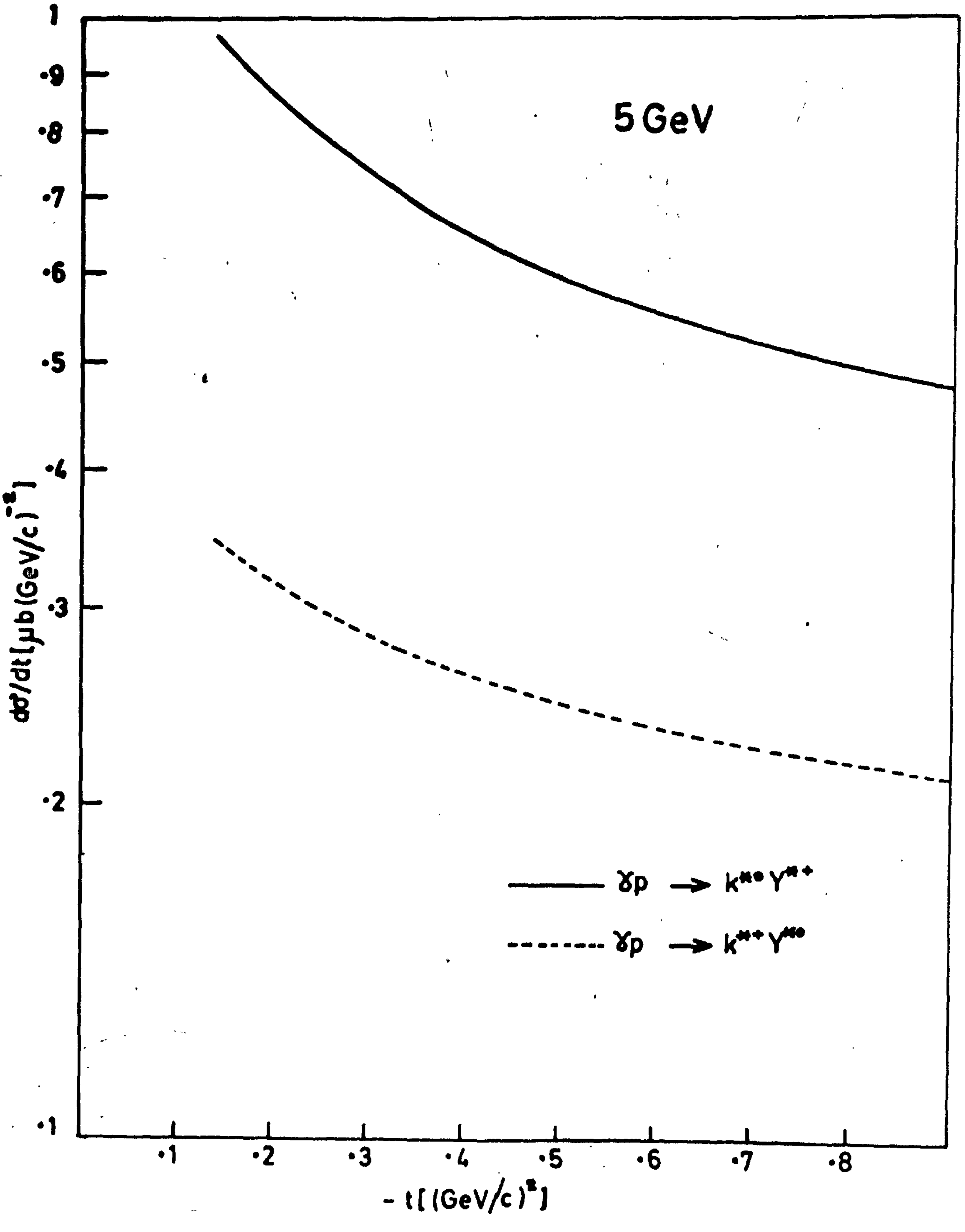


Fig. 14.



γp → ρ°, ω, φ + N** at 5 GeV
ρ° production
ω production
φ production
dσ/dt [μb (GeV/c)²]
10
1
·1
·01
·1
·2
·3
·4
·5
·6
·7
·8
−t [(GeV/c)²]

Fig. 15.



5 GeV
$\gamma p \rightarrow K^{*0} Y^{*+}$
$\gamma p \rightarrow K^{*+} Y^{*0}$
$d\sigma/dt\,[\mu b\,(GeV/c)^{-2}]$
$-t\,[(GeV/c)^2]$
1
·9
·8
·7
·6
·5
·4
·3
·2
·1
·1
·2
·3
·4
·5
·6
·7
·8

Fig. 16.

A DOUBLE PERIPHERAL AND VIRTUAL DIFFRACTION ANALYSIS OF $K^- p \to K^- \pi^- \Delta^{++}$ AT 10 GeV/C

by

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Abstract

Detailed calculations for the reaction $K^- p \rightarrow K^- \pi^- \Delta^{++}$ at 10 GeV/C, using the double peripheral and virtual diffraction models, are presented. The couplings are determined by using U(6,6) symmetry. "Off shell" and "absorption" effects are included in an approximate way by the use of phenomenological form factors. Good agreement with experiment is found for all the relevant one dimensional distributions.

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Introduction

In previous papers we presented detailed calculations using the "Deck" virtual diffraction model for the reaction $K^-p \rightarrow K^{*0}\pi^-p$ (1) and the double peripheral model (DPM) for $K^-p \rightarrow K^{*-}\pi^+n$ (2). This paper reports on calculations for $K^-p \rightarrow K^-\pi^-\Delta^{++}$ at 10 GeV/C including contributions from both mechanisms. As in the previous work we have taken account of "off shell" and "absorption" effects in an approximate way by use of phenomenological form factors. The number of free parameters in the model is significantly reduced by imposing U(6,6), symmetry (3) to relate the couplings involved.

The data for this reaction is notable for its apparent lack of any easily resolved quasi-two particle contributions. This is not wholly surprising as any resonances in the $K^-\Delta^{++}$ and $\pi^-\Delta^{++}$ systems would be weakly coupled higher baryon resonances. It is also noted that almost all events have the Δ scattered backwards in the overall centre of mass frame and the events are fairly evenly divided between the K scattered forward (π isotropic) and the π scattered forward (K isotropic). These features are strongly suggestive of double peripheral mechanisms with strange and non-strange meson exchanges.

In section 2 we present the matrix elements used and our choice of parameters. Our results and conclusions are discussed in section 3.

2. Model:

The DPM processes are represented diagramatically in figures 1a and 1b, the exchanges (I,II) are $(\rho^0,\pi^-),(\omega\,\rho^-)$ in 1a and $(\bar{K}^{*0},\pi^-),(\bar{K}^{*0},\rho^-)$ in 1b. In addition there is the possibility of the virtual diffraction diagrams represented in figures 2a and 2b. Since the data shows few events with the Δ scattered away from the backward direction no attempt has been made to include contributions involving baryon exchange to either the DPM or virtual diffraction processes.

Using U(6,6) symmetry the M-functions are written down in an identical manner to that used in references 1 and 2. For the DPM we have, for $(\rho^0,\pi^-),(\omega,\rho^-)$ and $(\bar{K}^{*0},\pi^-),(\bar{K}^{*0},\rho^-)$ exchanges respectively.

$$M_{A_\mu} = 9\sqrt{2}\, h_\kappa h_\rho g \left(1+\frac{2m}{\mu_P}\right)\frac{1}{m}\frac{1}{(t_{1b}-m_\rho^2)(t_{3a}-m_\pi^2)}\left[(p_2p_b)+(p_1p_3)\right]p_{a\mu}$$

$$M_{B_\mu} = 18\sqrt{2}\, h_\kappa h_\rho g \left(1+\frac{2m}{\mu_V}\right)\frac{1}{m^2\mu_m}\frac{1}{(t_{1b}-m_\omega)^2(t_{3a}-m_\rho^2)}\,.$$

$$\cdot\Big\{\left[(p_2p_3)(p_ap_b)-(p_2p_a)(p_bp_3)\right]p_{1\mu}$$

$$+\left[(p_1p_a)(p_bp_3)-(p_1p_3)(p_ap_b)\right]p_{2\mu}$$

$$+\left[(p_1p_3)(p_2p_a)-(p_1p_a)(p_2p_3)\right]p_{b\mu}\Big\}$$

$$M_{C\mu} = -\frac{9\sqrt{2}}{2} h_K^2 g \left(1+\frac{2m}{\mu_P}\right)\frac{1}{m}\frac{p_{a\mu}}{(t_{2b}-m_{K^*}^2)(t_{3a}-m_\pi^2)}\cdot$$

$$\cdot\Big\{m_b^2 - m_2^2 - 2(p_1p_2) - 2(p_1p_b)$$

$$+\frac{1}{m_{K^*}^2}\left[2(p_1p_b)-2(p_1p_2)-m_2^2-m_b^2+2(p_2p_b)\right](m_b^2-m_2^2)\Big\}$$

$$M_{D\mu} = 9\sqrt{2}\, h_K^2 g\left(1+\frac{2m}{\mu_V}\right)\frac{1}{m^2\mu_m}\frac{1}{(t_{2b}-m_{K^*}^2)(t_{3a}-m_g^2)}\cdot$$

$$\cdot\Big\{\left[(p_2p_3)(p_ap_b)-(p_2p_a)(p_bp_3)\right]p_{1\mu}$$

$$+\left[(p_1p_a)(p_bp_3)-(p_1p_3)(p_ap_b)\right]p_{2\mu}$$

$$+\left[(p_1p_3)(p_2p_a)-(p_1p_a)(p_2p_3)\right]p_{b\mu}\Big\} \qquad (2.1)$$

The indices a, b, 1, 2 and 3 refer to the target proton, incoming kaon, outgoing kaon, outgoing pion and outgoing Δ respectively. The U(6,6) couplings g and h_p are as previously used [(2)] and we have also introduced the coupling h_k for an MMM vertex containing two strange mesons, its value is 1.17 times that of h_p being fixed from the K^*(890) decay. The U(6,6) masses which appear are, in GeV,

m = 1.18 (mean mass of $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons)

μ_P = .417 (mean mass of 0^- octet)

μ_m = .633 (mean mass of 0^- octet and 1^- nonet)

μ_V = .850 (mean mass of 1^- nonet)

The Deck effect M-functions are, for $K\pi$ virtual diffraction and Kp virtual diffraction respectively

$$M_{E\mu} = ig\left(1+\frac{2m}{\mu_P}\right)\frac{1}{m}\,\frac{p_{a\mu}}{(t_{3a}-m_2^2)}\,4\pi\,\Delta^{1/2}(s_{12},m_1^2,m_2^2)\,\cdot$$

$$\cdot\, 7{\cdot}1\,\exp(3{\cdot}75\,t_{1b})$$

$$M_{F\mu} = ig\left(1+\frac{2m}{\mu_P}\right)\frac{1}{m}\,\frac{p_{2\mu}(\not{p}_2+\not{p}_3+m_a)\,4\pi\,\Delta^{1/2}(S_{ab},m_a^2,m_b^2)}{(S_{23}-m_a^2)\left\{2\left[(p_ap_3)+(p_ap_2)+m_a^2\right]\right\}^{1/2}}\,\cdot$$

$$\cdot\, 7{\cdot}1\,\exp(3{\cdot}75\,t_{1b}) \qquad (2.2)$$

where

$$\Delta(x,y,z) = \frac{x^2+y^2+z^2-2xy-2yz-2zx}{4x} \qquad (2.3)$$

and we have taken the differential cross-section for $K\pi$ and Kp elastic scattering as 50 exp (7.5t) $(\mathrm{GeV}/C)^{-4}$.

Phenomenological form factors of the form

$$f(t) = f(0)\,\exp(\lambda t) \qquad (2.4)$$

are now included for each of the particle exchanges and the values taken for the parameters $f(0)$ and λ are as used in references 1 and 2. Introducing these form factors into equations (2.1) and (2.2) to modify the matrix elements, the T matrix for the reaction is now taken as

$$T_{\lambda_a}^{\lambda_3} = \bar{u}_\mu(p_3,\lambda_3)\left\{M_A+M_B+M_C+M_D+M_E+M_F\right\}_\mu u(p_a,\lambda_a) \qquad (2.5)$$

where $\bar{u}_\mu(p_3,\lambda_3)$ is the $\frac{3}{2}$ spin wave function and $u(p_a,\lambda_a)$ the spin $\frac{1}{2}$ wave function with helicities λ_3 and λ_a respectively.

The relevant distributions can now be calculated from

$$A = \overline{\sum_{\lambda_3 \lambda_a}} \left| T_{\lambda_a}^{\lambda_3} \right|^2 \qquad (2.6)$$

where A is evaluated in the usual way.

Results and Discussion

The Data was taken from the Aachen - Berlin - CERN - London (I.C.) - Vienna collaboration's K^-p exposure at 10 GeV/C. The restriction $S(K\pi) > 2$ GeV/c was applied to the data and to the theoretical calculations since we have assumed a virtual diffraction parametrisation in the $K\pi$ subsystem. This parametrisation can only be expected to be valid at high energies (c.f. reference 1). Our overall normalisation was fixed by the data, the relative normalisations of the various contributing processes having been fixed by the model.

Our results for the mass-squared distributions $S(K\pi)$, $S(\pi\Delta)$ and $S(K\Delta)$ are shown in figures 3(a), (b) and (c) respectively. In all these plots we find good agreement between theory and experiment. The peaks in these distributions at high and low masses are the kinematic reflections of the forward/backward peaked angular distributions which are a feature of multiperipheral models. In particular the sharp peaking at low mass in the $\pi\Delta$ subsystem which is given correctly by the model results mainly from the strongly peaked virtual diffraction processes.

The momentum transfer squared distributions $t(p\Delta)$, $t(KK)$ and $t(K\pi)$ are shown in figures 4(a), (b) and (c) respectively. Comparatively poor agreement is found for the $t(p\Delta)$ distribution; however this would undoubtedly be improved by allowing for the finite decay width of the Δ . The t(KK) distribution is in good agreement showing strong forward peaking typical of virtual diffraction. The third plot, for $t(K\pi)$ is interesting in that it shows quite strikingly the presence of both the strange and non-strange exchanges. In this distribution the forward peak is due entirely to strange meson exchange and the large momentum transfer scattering which peaks at about $2(GeV/C)^2$ is due to the non-strange exchange and virtual diffraction processes. The ability of our model to obtain agreement for this plot demonstrates that we are able to predict correctly the relative contributions from these types of processes.

ACKNOWLEDGEMENTS

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FIGURE CAPTIONS

Figure 1. Double peripheral diagrams for

(a) non-strange exchange only, and

(b) strange and non-strange exchange.

Figure 2. Virtual diffraction diagrams for

(a) $K\pi$ diffraction, and

(b) Kp diffraction.

Figure 3. Mass-squared distributions.

Figure 4. Momentum transfer-squared distributions.

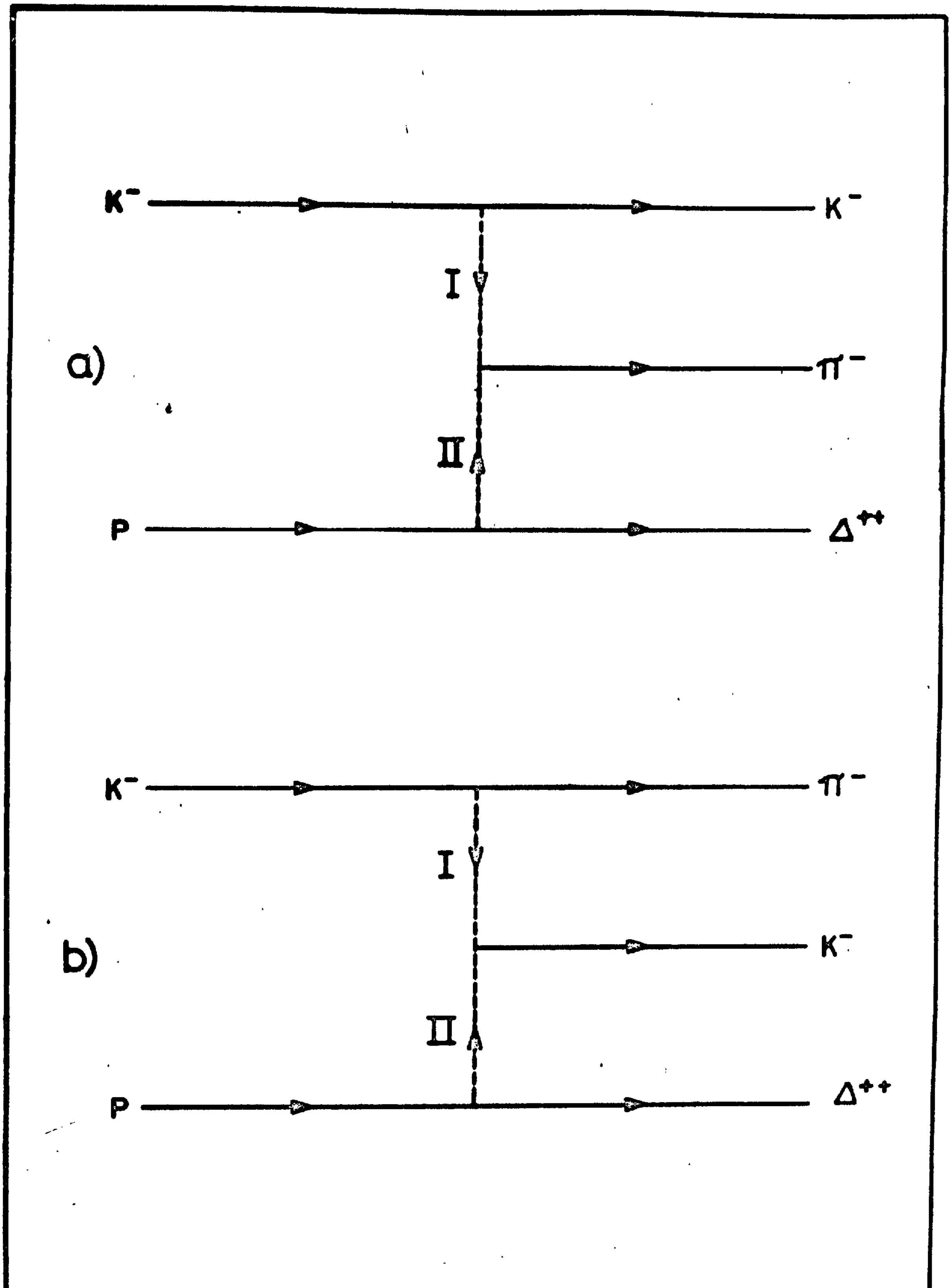


Fig. 1

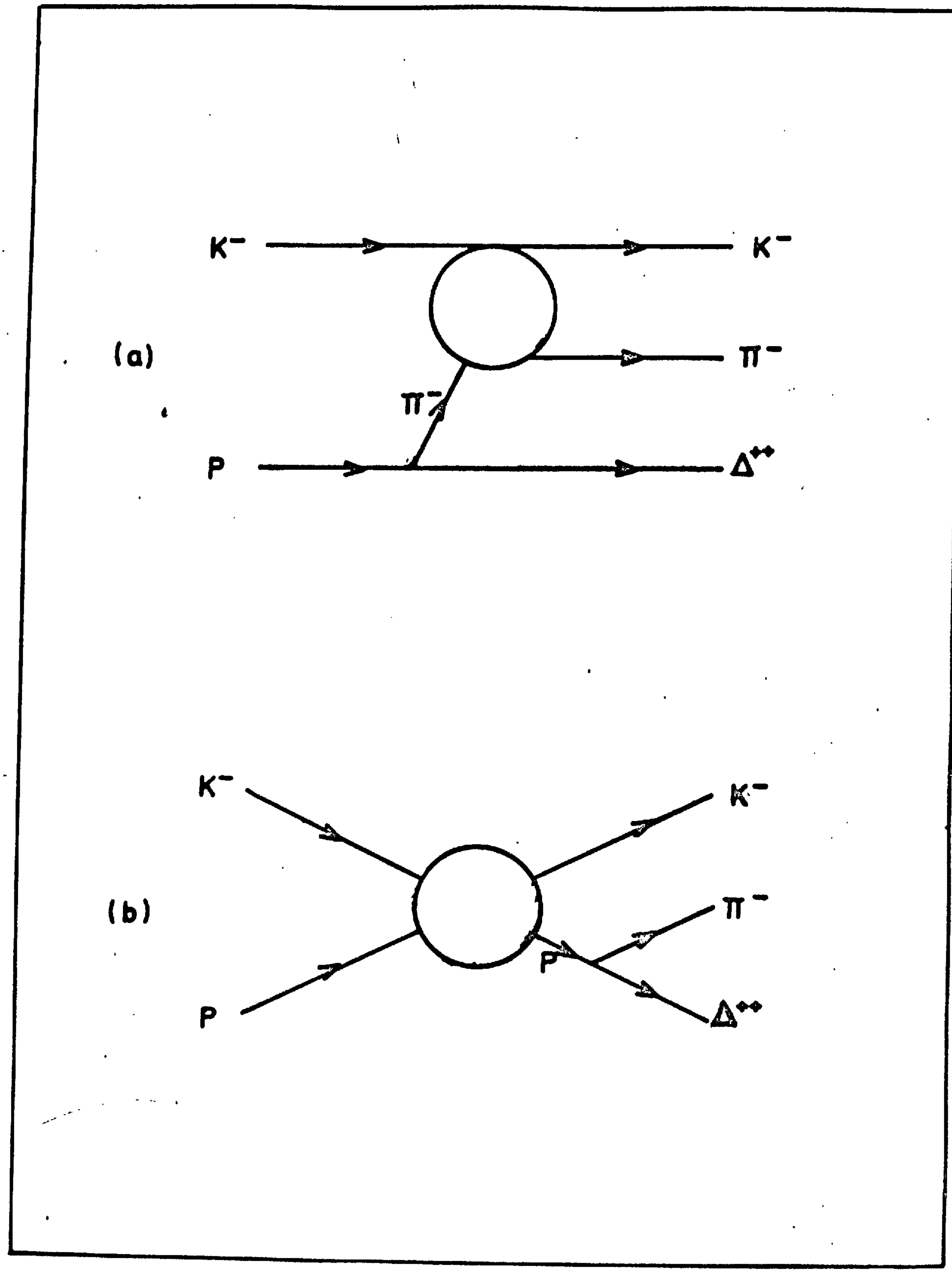
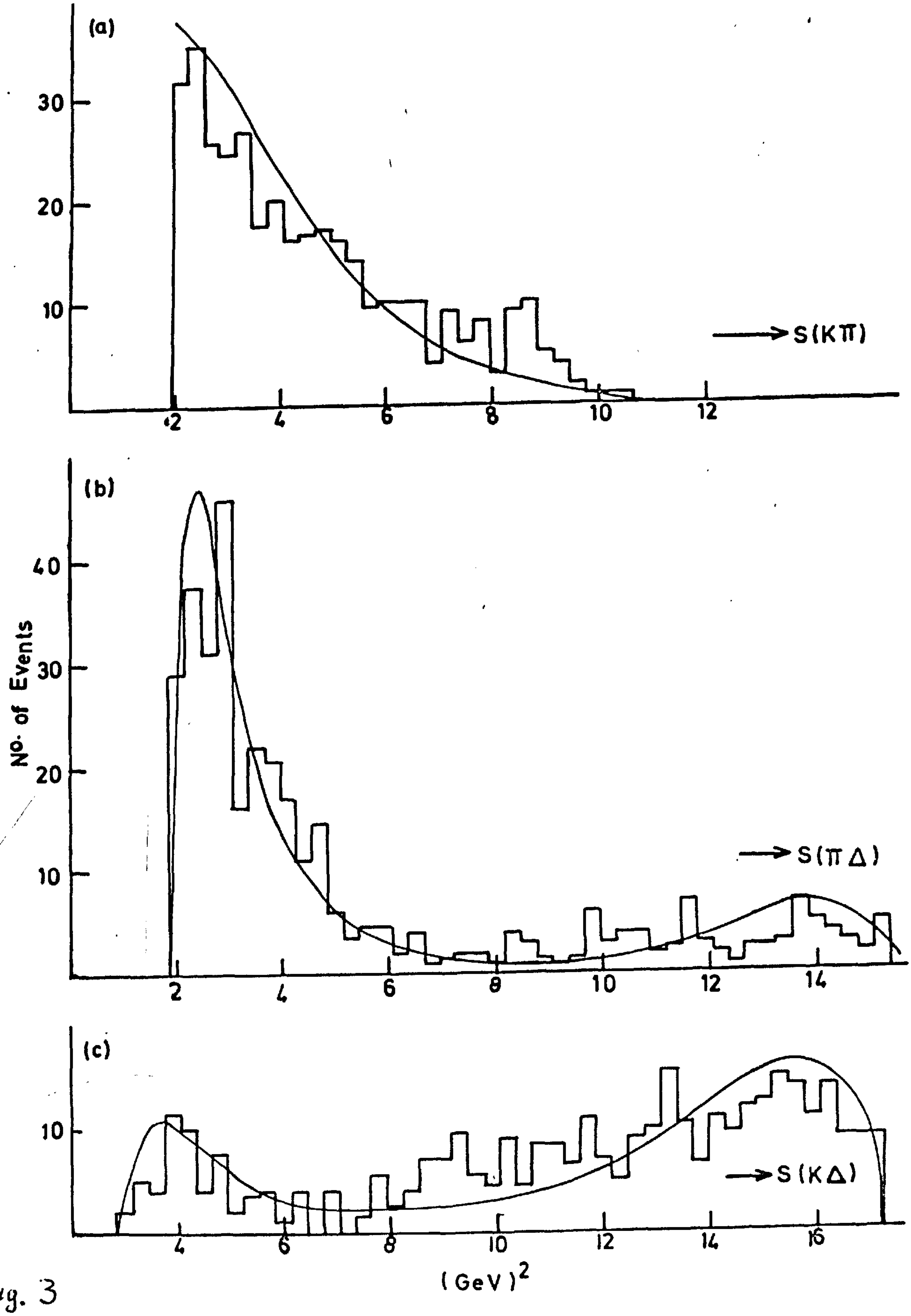
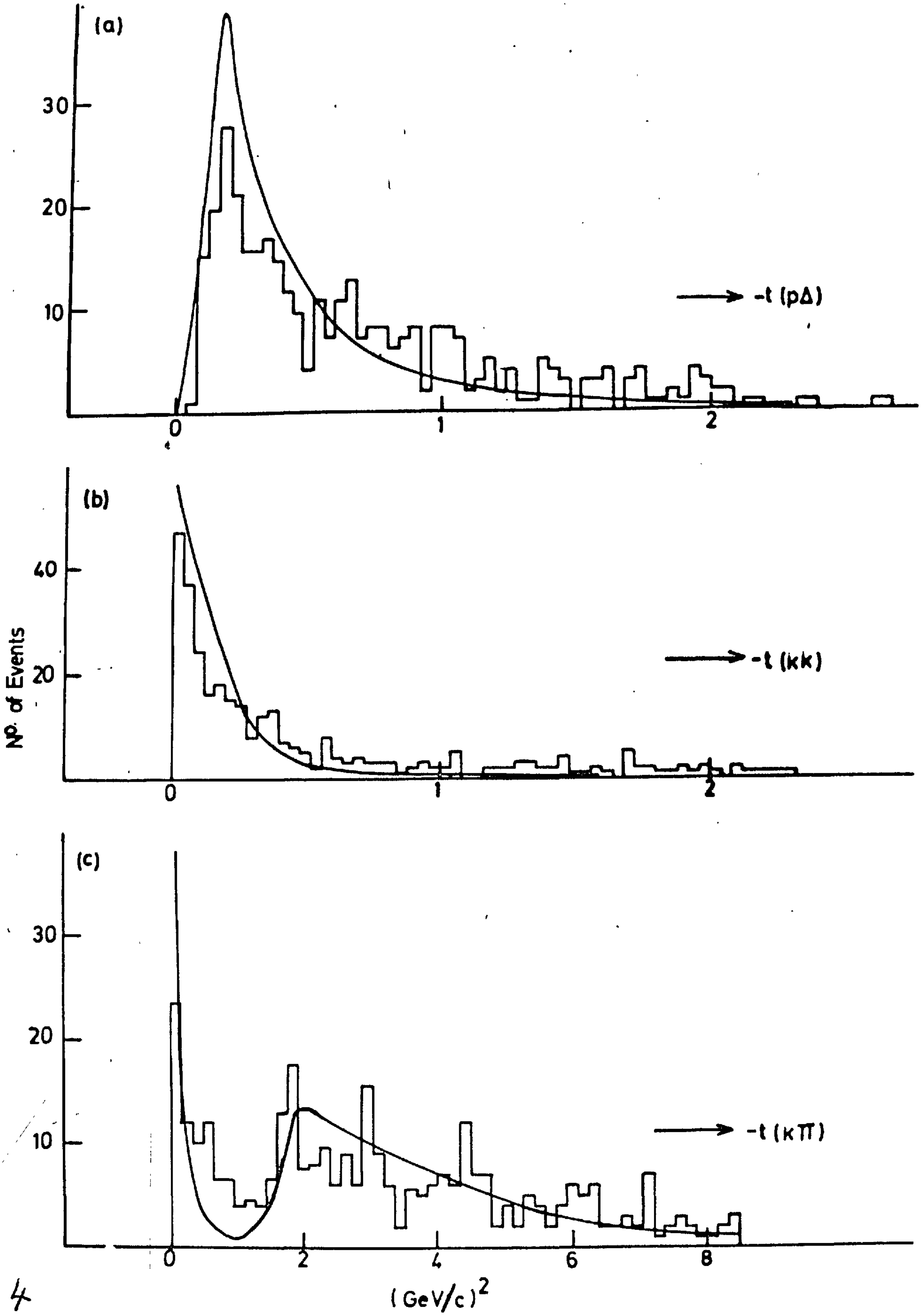


Fig. 2



(a)
30
20
10
→ S(Kπ)
2
4
6
8
10
12
(b)
No. of Events
40
30
20
10
→ S(π Δ)
2
4
6
8
10
12
14
(c)
10
→ S(K Δ)
4
6
8
10
12
14
16
(GeV)^2

Fig. 3



(a)
30
20
10
0
1
2
-t (pΔ)
(b)
40
20
No. of Events
0
1
2
-t (kk)
(c)
30
20
10
0
2
4
6
8
-t (κπ)
(GeV/c)^2

Fig. 4

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Virtual Diffraction and the Reaction $K^-p \rightarrow \bar{K}^{*0}\pi^-p$. - II (*)

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(ricevuto il 10 Marzo 1969)

Summary. — Detailed calculations of the « Deck » virtual diffraction background plus possible coherent diffractively produced resonances and the incoherent K^*_N resonance are presented at 6 and 10 GeV/c for the reaction $K^-p \rightarrow \bar{K}^{*0}\pi^-p$. « Off-shell » and « absorption » effects have been allowed for by the inclusion of phenomenological form factors. A simple parametrization for the diffractive production of the resonances in the $K^*\pi$ enhancement is employed and these amplitudes, which include the subsequent decay, interfere with the « Deck » background. A reasonable fit to experiment is obtained.

1. – Introduction.

In a previous paper under this title [1], hereafter referred to as I, we presented calculations with the simple « Deck » virtual diffraction model as suggested by FRASER and ROBERTS and also ROSS and YAM [2]. We demonstrated in I

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(1) J. L. SCHONFELDER: *Nuovo Cimento* **61** A, 114 (1969).

(2) G. FRASER and R. G. ROBERTS: *Nuovo Cimento*, **47**, 293 (1967); M. ROSS and Y. Y. YAM: *Phys. Rev. Lett.*, **19**, 546 (1967).

that this model was, at best, a model for background and we pointed out that, as the most likely resonances responsible for the $K^*\pi$ enhancement would interfere strongly with this background, it would be desirable to include such resonances explicitly in the calculation. We also concluded in I that neglecting « off-shell » and « absorption » effects was not strictly warranted and that some attempt to include these was also desirable.

Consideration of the relevant experimental data and the summarized particle properties (3) suggests that at least three resonances must be present in the $K^*\pi$ enhancement; the $K^*_A(1230)$, $K^*_A(1320)$ and $K^*_N(1420)$. All of these resonances have I-spin $\frac{1}{2}$, and $J^P = 1^+$, 1^+ and 2^+ are the most likely spin-parity assignments. The 1^+ mesons allow vacuum exchange in the t-channel and so are most likely to be produced by that process, *i.e.* diffractively. The 2^+ meson does not allow vacuum exchange but pion exchange is possible and we assume that this is the dominant production mechanism.

The inclusion of « off-shell » and « absorption » effects is by no means a trivial problem to do « correctly ». The inclusion of absorption effects by some technique like that suggested by SCHONFELDER (4) would involve considerable labour which the crudeness of the present model does not warrant. Consequently it was decided that, following the work of JOSEPH and PILKUHN and also GISLÉN (5), these effects could be taken account of in an approximate way by the use of phenomenological form factors. This is a far from satisfactory method of tackling the problem but as this is a somewhat exploratory calculation it was hoped that such a simple-minded approach would be sufficient.

In Sect. **2** of the paper we shall outline the way in which we have parametrized the backgound and resonance amplitudes. The results of our calculation are presented in Sect. **3**.

2. – The model.

For the background amplitude we use the same forms as in I except for the inclusion of a form factor for the two t-channel exchanges. The processes are represented diagrammatically in Fig. 1. The M-functions for these processes

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(3) A. H. ROSENFELD, N. BARASH-SCHMIDT, A. BARBARO-GALTIERI, L. R. PRICE, M. ROOS, P. SÖDING, W. J. WILLIS and C. G. WOHL: *Particle properties tables* (January, 1968), UCRL-8030.

(4) J. L. SCHONFELDER: *Nuovo Cimento*, **58** A, 221 (1968).

(5) J. JOSEPH and H. PILKUHN: *Nuovo Cimento*, **33**, 1407 (1964); L. GISLÉN: *Nuovo Cimento*, **54** A, 919 (1968).

are, from I,

$$(1)\quad \begin{cases} M_{A_\mu} = g_{K^*K\pi} p_{b_\mu} \cdot \dfrac{F_\pi(t_{1b})}{(t_{1b} - m_2^2)} \cdot A_A\,, \\[2ex] M_{B_\mu} = g_{K^*K\pi} p_{2_\mu} \cdot \dfrac{1}{(s_{12} - m_b^2)} \cdot A_B\,, \\[2ex] M_{C_\mu} = g_{K^*K\pi} p_{b\lambda} \cdot \dfrac{F_{K^*}(t_{2b})}{(t_{2b} - m_1^2)} \cdot \left\{ -g_{\lambda\nu} + \dfrac{\Delta_\lambda \Delta_\nu}{m_1^2} \right\} g_{\nu\mu} A_C\,, \end{cases}$$

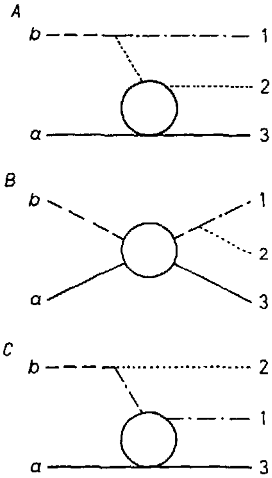

Fig. 1. – Deck effect for $K^-p \to \bar{K}^{*0}\pi^-p$. - - - π^-; — — — K^-; —·—·— $\bar{K}^{*0}(890)$; ——— p; ○ virtual diffraction.

where $\Delta = p_b - p_2$. As in ref. [5] we have assumed that the form factors factorize into the product of two functions, each dependent on one t-variable and we have taken the form as given in that reference, *viz.*

$$(2)\qquad F(t) = F(0)\, e^{\lambda t}\,.$$

No form factor is introduced explicitly for the vacuum exchanges since the vacuum exchange amplitudes « A » are calculated from experimental results and hence any form factors etc. are implicitly included. The treatment of process B we shall postpone until we have dealt with the resonance processes as we have treated all s-channel poles in a similar manner.

We now turn to the resonance amplitudes $D1$, $D2$ and E say, which are represented diagrammatically in Fig. 2. For the decay vertices we shall again use the covariants and couplings of SCADRON [6]. In the case of $D1$ and $D2$ where there are two possible couplings we shall assume lowest-order « angular-momentum barrier » coupling [7], *i.e.* we take only the coupling whose appropriate covariant contains the lowest number of momentum factors. The propagators for these unstable resonances we shall take as the numerators as given in ref. [6] and the denominators will be given a complex mass. The production process for E, taken as single-pion exchange, can be written down by quite standard techniques. The diffractive production amplitude is not so well defined. In order that we end up with an in-

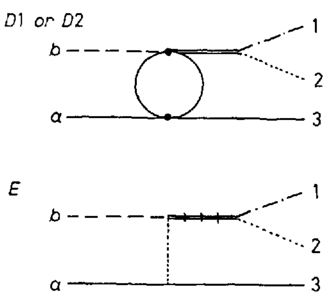

Fig. 2. – Resonance production for $K^-p \to \bar{K}^{*0}\pi^-p$. - - - π; — — — K^-; —·—·— $\bar{K}^{*0}(890)$; ——— p; ═══ K*(1230) or K*(1320); ═╪═ K*(1420); ○ inelastic virtual diffraction or vacuum exchange.

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[6] M. D. SCADRON: *Phys. Rev.*, **165**, 1640 (1968).

[7] T. J. WEARE: *Nuovo Cimento*, **56** A, 64 (1968).

variant T-matrix, we must have a vector index on the diffractive production amplitude which will contract out with the corresponding index on the spin-1 propagator for the resonance. The simplest assumption that one can make is that the K^- and the K^*_A are coupled to the vacuum exchange as if the vacuum exchange were a 0^+ object. The remaining scalar part of the amplitude can then be parametrized as for elastic scattering. Thus we obtain the following M-functions for resonance production and decay:

$$M_{D1\mu} = g_{K^*_{D1}K^*\pi} \cdot \frac{\mathscr{P}^1_{\mu,\lambda}(K)}{(s_{12} - m^2_{D1}) + i\Gamma_{D1} m_{D1}} \cdot p_{b_\lambda} A_{D1} ,$$

similarly for $D2$,

$$M_{E_\mu} = g_{K^*_E K^*\pi}\, \varepsilon_{\mu\beta\varrho\lambda}\, p_{1\varrho} K_\lambda p_{1\alpha} \cdot \frac{\mathscr{P}^2_{\alpha\beta,\gamma\delta}(K)}{(s_{12} - m^2_E) + i\Gamma_E m_E} \cdot g_{K^*_E K\pi} p_{b_\gamma} p_{b\delta} \cdot \frac{F_\pi(t_{3a})}{(t_{3a} - m^2_2)} \cdot \gamma_5 G_{\mathscr{N}\mathscr{N}\pi} , \tag{3}$$

where $\mathscr{P}^1(K)$ and $\mathscr{P}^2(K)$ are the spin-1 and spin-2 propagator numerators and $K = p_1 + p_2$. We have again introduced form factors for the t-channel exchanges, explicitly for the pion exchange process E and implicitly in the diffractive production amplitudes.

We now consider the energy dependence of the s-channel exchanges. The amplitude for two spinless particles scattering via a spin-J resonance in the direct channel can be written, in the centre-of-mass frame, as

$$M = \frac{g_A p^J_{\text{fi}} g_B p^J_{\text{in}} P_J(\cos\theta)}{s - m^2_R + i\Gamma_R m_R} . \tag{4}$$

It is well known from low-energy phenomenology that a reasonable energy dependence for such an amplitude can be obtained if the couplings and momentum factors in eq. (4) are replaced by the appropriate constant partial widths. This is essentially equivalent to evaluating these factors at the pole which is the method we have adopted.

The full T-matrix for the reaction is now taken as

$$T = \varepsilon_\mu(\lambda_1, p_1)\bar{u}(\lambda_3, p_3)[M_A + M_B + M_C + M_{D1} + M_{D2} + M_E]_\mu u(\lambda_a, p_a) \tag{5}$$

and the unpolarized cross-sections are calculated from

$$\overline{\sum_{\text{spins}}} |T|^2 = g_{K^*K\pi} \left\{ \frac{P_{AA} F^2_A}{(t_{1b} - m^2_2)^2} + \frac{P_{BB} F^2_B}{(s_{12} - m^2_b)^2} + \frac{P_{CC} F^2_C}{(t_{2b} - m^2_1)^2} + \right. \\ \left. + \frac{2P_{AB} F_A F_B}{(s_{12} - m^2_b)(t_{1b} - m^2_2)} + \frac{2P_{BC} F_B F_C}{(s_{12} - m^2_b)(t_{2b} - m^2_1)} + \frac{2P_{AC} F_A F_C}{(t_{1b} - m^2_2)(t_{2b} - m^1_1)} \right\} + \tag{6}$$

$$+\left\{\frac{P_{DD1}\Gamma_{D1}^2}{(s_{12}-m_{D1}^2)^2+\Gamma_{D1}^2 m_{D1}^2}+\frac{P_{DD2}F_{D2}^2}{(s_{12}-m_{D2}^2)^2+\Gamma_{D2}^2 m_{D2}^2}+\right.$$

$$\left.+\frac{2P_{DD12}F_{D1}F_{D2}[(s_{12}-m_{D1}^2)(s_{12}-m_{D2}^2)+\Gamma_{D1}m_{D1}\Gamma_{D2}m_{D2}]}{[(s_{12}-m_{D1}^2)^2+\Gamma_{D1}^2 m_{D1}^2][(s_{12}-m_{D2}^2)^2+\Gamma_{D2}^2 m_{D2}^2]}\right\}+$$

$$+\frac{2g_{K^*K\pi}F_{D1}(s_{12}-m_{D1}^2)}{(s_{12}-m_{D1}^2)^2+\Gamma_{D1}^2 m_{D1}^2}\left\{\frac{P_{AD1}F_A}{(t_{1b}-m_2^2)}+\frac{P_{BD1}F_B}{(s_{12}-m_b^2)}+\frac{P_{CD1}F_C}{(t_{2b}-m_1^2)}\right\}+$$

$$+\frac{2g_{K^*K\pi}F_{D2}(s_{12}-m_{D2}^2)}{(s_{12}-m_{D2}^2)^2+\Gamma_{D2}^2 m_{D2}^2}\left\{\frac{P_{AD2}F_A}{(t_{1b}-m_2^2)}+\frac{P_{BD2}F_B}{(s_{12}-m_b^2)}+\frac{P_{CD2}F_C}{(t_{2b}-m_1^2)}\right\}+\frac{P_{EE}F_E^2}{(s_{12}-m_E^2)^2+\Gamma_E^2 m_E^2},$$

where P_{AA} etc. are the factors resulting from the spin sums and F_A etc. are as in I except that they now include form factors where appropriate. (Detailed expressions for these quantities are recorded in the Appendix.)

These expressions were readily programmed and the required distributions generated using the programs for phase-space evaluation reported in I.

3. – Results and discussion.

Figures 3 and 4 show the results at 10 GeV/c, Fig. 5 and 6 the results at 6 GeV/c. The data are identical to those used in I. We have again restricted the kinematics and have only considered theoretical and experimental contributions for which $S_{23}>3$, $S_{13}>5$ and $|t_{3c}|<1$. In our choice of form factors we have followed GISLÉN (5) and have taken the parameters as

$$\left\{\begin{aligned} F_\pi(t) &= 0.95\exp[2\cdot 5t],\\ F_{K^*}(t) &= 0.8\ \ \exp[2t]. \end{aligned}\right. \tag{7}$$

With these factors and normalization taken from experimental elastic scattering we find that the absolute magnitude of the background as well as the shape is quite good. This can be seen in Fig. 3*a*) where in the region away from the resonances the cross-section is entirely due to background terms.

Initially we took the normalization for all resonance amplitudes from experiment. The contributions from the diffractively produced resonances were too large. This was to be expected since an inelastic diffraction is unlikely to be as strong as elastic diffraction. The K_N^*-resonance contribution came out to be too small. Again this was as expected since, for simplicity, we neglected η, ρ^0, ω and φ exchange production.

At 10 GeV/c we reduced both K_A^* amplitudes by a factor of 2 and increased the K_N^* amplitude by a factor of 3. At 6 GeV/c these factors were 2.8 and 1.5 respectively. We used no other parameters apart from the form factors given in eq. (7).

The masses and widths used to calculate the coupling strengths for these resonances were taken from ref. (3). These values are tentative especially for the K_A^*'s and consequently the effective coupling strengths used are within experimental-error bounds.

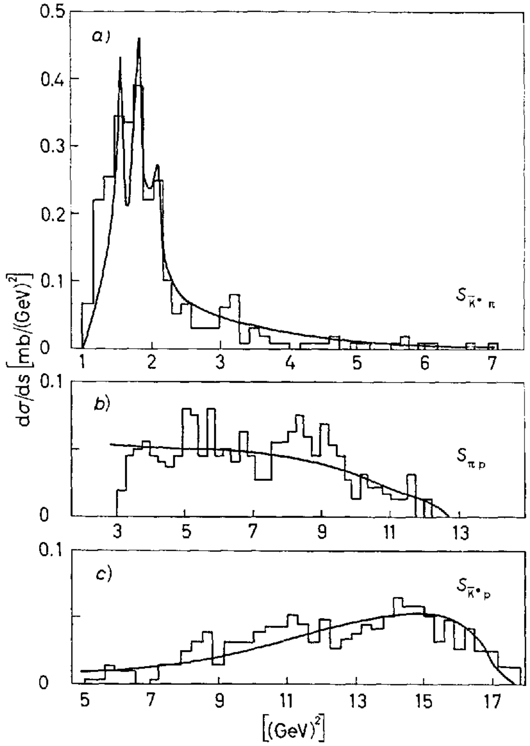

Fig. 3. – Mass-squared plots at 10 GeV/c.

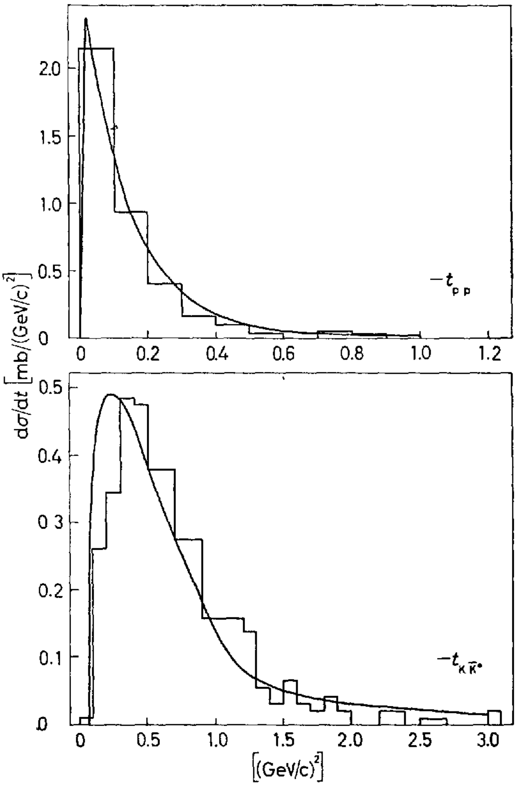

Fig. 4. – Momentum transfer plots at 10 GeV/c.

Unlike in I we now obtain fairly good agreement for all the mass-squared plots as can be seen in Fig. 3 and 5. The momentum-transfer distributions shown in Fig. 4 and 6 also show reasonable agreement although the t_{KK^*} distribution at 10 GeV/c, Fig. 4*b*), appears to peak at a slightly too small value of $|t|$. It is hardly surprising that some disagreement should occur in this distribution since it is the t_{lb} dependence of the matrix element that is most strongly related to the exact nature of the resonance production mechanism.

In conclusion then it would seem fair to say that these results demonstrate that the virtual diffraction model for background with coherent diffractively

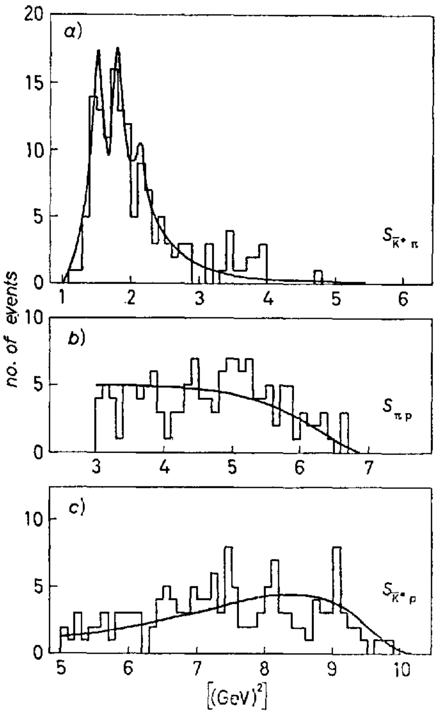

Fig. 5. – Mass-squared plots at 6 GeV/c. 144 events.

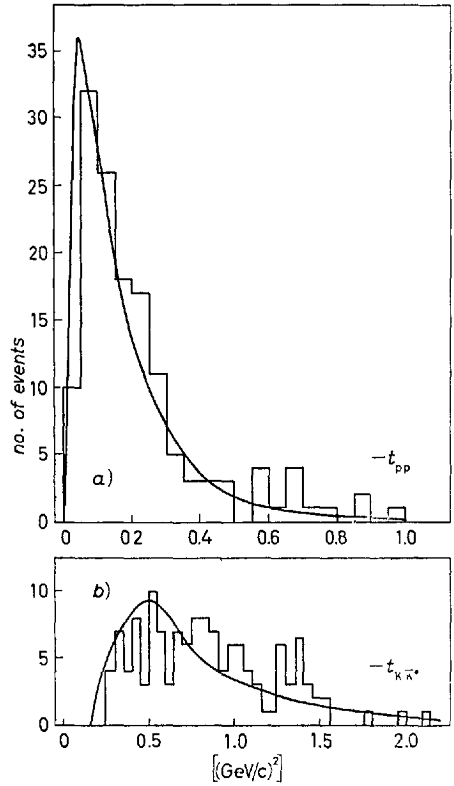

Fig. 6. – Momentum transfer plots at 6 GeV/c. 144 events.

produced resonances plus the incoherent $K_N^*(1420)$ can reasonably account for most of the cross-section in the reaction $K^-p \to \bar{K}^{*0}\pi^-p$ at high energies without drastic changes to calculated normalizations.

* * *

The authors would like to thank Prof. P. T. Matthews for his help and encouragement. Thanks are also due to Drs. M. Losty and M. E. Mermikides of the H.E.N.P. group at Imperial College who supplied the experimental data. One of us (J.L.S.) would like to acknowledge the financial assistance of a Scholarship from the Royal Commission for the Exhibition of 1851 which he held during the early part of doing this work.

APPENDIX

We present here the detailed expressions used in eq. (6) to calculate to unpolarized cross-sections. First we give the spin sum terms

$$
\begin{aligned}
P_{AA} &= (p_1 p_b)^2/m_1^2 - m_b^2\,,\\
P_{BB} &= (E_1^{(B)^2} - m_1^2)\,m_b^2/m_1^2\,,\\
P_{CC} &= (p_b\Delta)^2(p_1\Delta)^2/m_1^6 - [t_{2b}(p_b\Delta)^2 + 2\,(p_b\Delta)(p_1\Delta)(p_1p_b)]/m_1^4 +\\
&\qquad + [(p_1p_b)^2 + 2\,(p_b\Delta)^2]/m_1^2 - m_b^2\,,\\
P_{AB} &= -\,m_b E_b^{(3)} + (p_1p_b)\,m_b E_1^{(B)}/m_1^2\,,\\
P_{BC} &= m_b E_b^{(3)} - (p_1p_b)\,m_b E_1^{(B)}/m_1^2 - (p_b\Delta)(E_b^{(3)} - E_2^{(3)})\,m_b/m_1^2 + (p_b\Delta)(p_1p_b)\,m_b E_1^{(B)}/m_1^4\,,\\
P_{AC} &= (p_1p_b)(p_1\Delta)(p_b\Delta)/m_1^4 - [(p_1p_b)^2 + (p_b\Delta)^2]/m_1^2 + m_b^2\,,\\
P_{DD1} &= p_b^{(D1)^2} + p_1^{(D1)^2}p_b^{(D1)^2}\cos^2\theta/m_1^2\,,\\
P_{AD1} &= (p_1p_b)\,p_1^{(D1)}p_b^{(D1)}\cos\theta/m_1^2 - p_b^{(D1)}p_b^{(3)}\,,\\
P_{BD1} &= m_b E_1^{(B)}p_1^{(D1)}p_b^{(D1)}\cos\theta/m_1^2\,,\\
P_{CD1} &= p_b^{(D1)}p_b^{(3)} - (\Delta p_b)[p_b^{(D1)}p_b^{(3)} + p_b^{(D1)}p_1^{(3)}\cos\theta]/m_1^2 +\\
&\qquad + p_b^{(D1)}p_1^{(D1)}\cos\theta[(p_b\Delta)(p_1\Delta)/m_1^2 - (p_1p_b)]/m_1^2\,,
\end{aligned}
$$

similarly for P_{DD2}, P_{AD2}, P_{BD2}, P_{CD2},

$$
\begin{aligned}
P_{DD12} &= p_b^{(D1)}p_b^{(D2)} + p_b^{(D1)}p_b^{(D2)}p_1^{(D1)}p_1^{(D2)}\cos\theta\,,\\
P_{EE} &= m_E^2 p_1^{(E)^4}p_b^{(E)^4}\sin^2\theta\cos^2\theta\,.
\end{aligned}
$$

The notation we have used here is as in the Appendix of ref. [4] except that all the momentum and energy quantities which have process labels as superscripts are factors to be evaluated, in the (12) centre-of-mass frame, at the pole for the process designated by the label. θ is the angle between the three-vectors $\boldsymbol{p}_1^{(3)}$ and $\boldsymbol{p}_b^{(3)}$ in the (12) centre-of-mass frame. These quantities can be evaluated in terms of the invariants s and t

$$
\begin{aligned}
(p_1p_b) &= (m_1^2 + m_b^2 - t_{1b})/2\,,\\
(p_1\Delta) &= (m_1^2 + t_{1b} - t_{3a})/2\,,\\
(p_b\Delta) &= (m_b^2 + t_{1b} - m_2^2)/2\,,\\
E_1^{(3)} &= (s_{12} + m_1^2 - m_2^2)/2W_3\,,\\
p_1^{(3)} &= [(s_{12} + m_1^2 - m_2^2)^2 - 4m_1^2 s_{12}]^{\frac12}/2W_3\,,\\
E_b^{(3)} &= (s_{12} + m_b^2 - t_{3a})/2W_3\,,\\
p_b^{(3)} &= [(s_{12} + m_b^2 - t_{3a})^2 - 4m_b^2 s_{12}]^{\frac12}/2W_3\,,\\
\cos\theta &= [t_{1b} - m_1^2 - m_b^2 + 2E_1^{(3)}E_b^{(3)}]/2p_1^{(3)}p_b^{(3)}\,.
\end{aligned}
$$

The quantities $E_1^{(B)}$, $p_1^{(D1)}$, $p_b^{(D1)}$ etc., can be evaluated by taking $s_{12}=m_b^2$, m_{D1}^2, m_{D2}^2 or m_E^2 and $w_3=m_b$, m_{D1}, m_{D2} or m_E in the above expressions. The factors F_A, etc. are defined to include the common spin-$\frac{1}{2}$ trace factor

$$\begin{aligned}
F_A^2 &= 64\pi s_{23} p_3^{(1)^2} a_{\pi p} \exp[b_{\pi p} t_{3a}]\cdot[F_\pi(t_{1b})]^2 ,\\
F_B^2 &= 64\pi s_{ab} p_a^2 a_{Kp} \exp[b_{Kp} t_{3a}] ,\\
F_C^2 &= 64\pi s_{13} p_3^{(2)^2} a_{Kp} \exp[b_{Kp} t_{3a}]\cdot[F_{K^*}(t_{2b})]^2 ,\\
F_{D1}^2 &= 64\pi s_{ab} p_a^2 a_{D1} \exp[b_{D1} t_{3a}] ,
\end{aligned}$$

similarly for F_{D2}^2

$$F_E = a_E \cdot F_\pi(t_{3a}) ,$$

where the momentum factors occurring can be calculated by analogous expressions to the above and the parameters a_{D1}, a_{D2} and a_E are used to fix the normalizations of the resonances.

RIASSUNTO (*)

Si presentano calcoli dettagliati del fondo di diffrazione virtuale di « Deck » più risonanze prodotte diffrattivamente, coerenti, possibili e la risonanza K_N^* incoerente a 6 e 10 GeV/c per la reazione $K^-p \to \bar{K}^{*0}\pi^-p$. Si è tenuto conto degli effetti di « assorbimento » e « fuori strato » con l'inclusione di fattori di forma fenomenologici. Si usa una semplice parametrizzazione per la produzione diffrattiva delle risonanze nell'accrescimento $K^*\pi$ e queste ampiezze, che includono il decadimento susseguente, interferiscono col fondo di « Deck ». Si ottiene una ragionevole approssimazione agli esperimenti.

(*) *Traduzione a cura della Redazione.*

Виртуальная диффракция и реакция $K^-p \to \bar{K}^{*0}\pi^-p$. - II.

Резюме (*). — Для реакции $K^-p \to \bar{K}^{*0}\pi^-p$ при 6 и 10 ГэВ/с предлагаются подробные вычисления виртуального диффракционного фона « Дека » плюс возможных когерентных диффракционно рожденных резонансов и некогерентного резонанса K_N^*. Посредством включения феноменологических факторов были учтены эффекты « вне оболочки » и « поглощения ». Применяется простая параметризация для диффракционного рождения резонансов в $K^*\pi$ усилении, и эти амплитуды, которые включают последующий распад, интерферируют с фоном « Дека ». Получается разумное соответствие с экспериментом.

(*) *Переведено редакцией.*

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THE DOUBLE PERIPHERAL MODEL AND THE REACTION $K^-p \rightarrow K^{*-}\pi^+n$ AT 6 AND 10 GeV/c

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Abstract: Detailed calculations for the reaction $K^-p \rightarrow K^{*-}\pi^+n$ at 6 and 10 GeV/c are presented. The non-resonant background, which accounts for most of the cross section, is assumed to be due to double peripheral exchange mechanisms. The relative strengths of the couplings for the various exchanges involved are fixed by using U(6,6) symmetry. "Off-shell" and "absorption" effects are included in an approximate way by the use of phenomenological form factors. A contribution from the $K^*(1420)$ resonance is included explicitly and the production and decay amplitude for this process is allowed to interfere with the background. Encouraging agreement with experiment is found for all the relevant one dimensional distributions.

1. INTRODUCTION

In this paper we present the results of a double peripheral model [1] (DPM) analysis of the reaction $K^-p \rightarrow K^{*-}\pi^+n$ at 6 and 10 GeV/c. This reaction is notable for an apparent lack of strong quasi-two-particle intermediate state contributions. The only clearly resolved resonance is a $K^*_N(1420)$ in the $K^*\pi$ sub-system and this appears to be responsible for less than 20% of the total cross-section. Almost all the observed events have the baryon scattered backwards in the overall centre of mass frame and except for a few events, which mainly appear in the $K^*(1420)$ band, the $K^*(890)$ is scattered forward. The pion is mainly produced isotropically. These features are suggestive of a DPM with non-strange meson exchange.

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Our approach is similar to that of Gislén [2] differing in that we use the U(6, 6) symmetry scheme [3] to calculate the relative strengths of the couplings involved in the DPM processes. This method has had considerable success for the single particle exchange absorption model [4]. Also we include an amplitude for the peripheral production of the $K^*(1420)$ and its subsequent decay into a $K^*(890)$ and a π. This is done in the same way as in our previous work for the $\overline{K}^{*0}\pi^- p$ final state [5].

It is essential in such calculations to take account of "off-shell" and "absorption" effects. The most desirable approach would be to correct for absorption using a method like that suggested by Schonfelder [6]. However such a calculation is by no means a trivial one. As a first approximation we make use of the same phenomenological form factors as used in refs. [2, 5]. Even though this is a somewhat unsatisfactory procedure we do have significantly fewer parameters than would be present in a double-Regge model [7]. We show in the remainder of the paper that the model does account fairly well for the data and we therefore hope to begin a more thorough treatment using the suggestion of ref. [6] for the calculation of absorption effects.

In sect. 2 we outline the matrix elements used and give our choice of parameters. The results are presented and discussed in sect. 3.

2. MATRIX ELEMENTS

Strange meson exchanges would have to be doubly charged so we are restricted to considering DPM processes as represented in fig. 1a. The exchanged particles (I, II) are assumed to be members of the U(6, 6) supermultiplet $\underline{35}$ for mesons. Application of the various conservation laws at each vertex restricts the possible number of exchanges to four, viz. (ρ^0, π^+), (π^0, ρ^+), (ω, ρ^+) and (ϕ, ρ^+). SU(3) predicts zero for the ϕ-ρ-π coupling so the last of these is removed. We refer to the remaining three processes as A, B and C respectively.

We write down our matrix elements using U(6, 6) couplings [3] in the same manner as Migneron et al. [4]. The resulting M-functions for the three processes are

$$M_{A\mu} = A\,\epsilon_\mu(p_1, p_2, p_b)\gamma_5\,,$$

$$M_{B\mu} = B p_{b\mu} p_{2\nu}[G(p_a + p_3)_\nu + H\gamma_\nu]\,,$$

$$M_{C\mu} = C\,\epsilon_{\mu\nu}(p_1, p_b)\,\epsilon_{\nu\lambda}(p_3 - p_a, p_2)[G(p_a + p_3)_\lambda + H\gamma_\lambda]\,, \tag{1}$$

where

$$A = -30\sqrt{2}\,\frac{h^2 g}{\mu_V}\left(1 + \frac{2m_N}{\mu_P}\right)\left(1 - \frac{\mu_P^2}{4m_N^2}\right)\frac{1}{(t_{1b} - m_\rho^2)(t_{3a} - m_\pi^2)}\,,$$

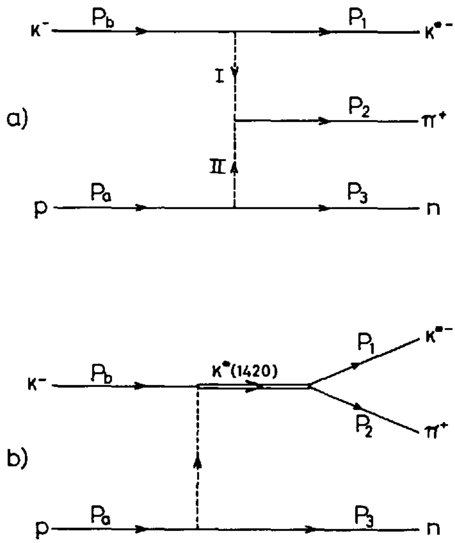

Fig. 1. (a) The double peripheral model and (b) the resonance production diagrams for $K^- p \to K^{*-} \pi^+ n$.

$$B = 18 h^2 g \frac{1}{(t_{1b} - m_\pi^2)(t_{3a} - m_\rho^2)},$$

$$C = \frac{-18 h^2 g}{\mu_V^2} \frac{1}{(t_{1b} - m_\omega^2)(t_{3a} - m_\rho^2)},$$

$$G = \frac{1}{\sqrt{2} m_N} \left[\left(1 + \frac{\mu_V}{2 m_N}\right) - \frac{5}{3}\left(1 + \frac{2 m_N}{\mu_V}\right) \right],$$

$$H = \tfrac{5}{3} \sqrt{2} \left(1 + \frac{2 m_N}{\mu_V}\right)\left(1 - \frac{\mu_V^2}{4 m_N^2}\right).$$

The masses which appear have the following values, measured in GeV,

$m_N = 0.938$,

$\mu_P = 0.417$ (mean mass of 0^- octet),

$\mu_V = 0.850$ (mean mass of 1^- nonet),

$m_\rho = 0.765$,

$$m_\pi = 0.138\,,$$

$$m_\omega = 0.783\,,$$

g and h are the U(6, 6) couplings for $\bar{\mathrm{B}}$BM and MMM vertices respectively and we use the values of ref. [4].

We now introduce the form factors. Following Joseph and Pilkuhn [1] we assume that the form factors factorise into two functions each depending on just one momentum transfer variable i.e. if $F(t_{1b}, t_{3a})$ is the required form factor then

$$F(t_{1b}, t_{3a}) = F(t_{1b})F(t_{3a})\,, \tag{2}$$

where we take

$$F(t) = F(0)\,\mathrm{e}^{\lambda t}\,. \tag{3}$$

These form factors are included in the matrix elements (1) by multiplying the quantities A, B and C by the factors appropriate to the exchanges involved. For π exchange we use $\lambda_\pi = 2.5\ (\mathrm{GeV}/c)^{-2}$ and $F_\pi(0)$ is evaluated by extrapolation to the pole giving a value of 0.95. For the vector exchanges we take $\lambda_\omega = \lambda_\rho = 2\ (\mathrm{GeV}/c)^{-2}$ and $F_\omega(0) = F_\rho(0) = 0.8$ since extrapolation to the pole is not likely to be valid over the larger distances now involved. This parameter is essentially free but the value of 0.8 is as we used for K^* exchange in ref. [5].

The matrix element for the production and decay of the $\mathrm{K}^*(1420)$ resonance, responsible for the observed peaking at 2 $(\mathrm{GeV})^2$ in the $s_{\mathrm{K}^*\pi}$ distribution, is assumed to be dominated by single-pion exchange as in ref. [5]. This is undoubtedly a more correct assumption as we are now dealing with charge exchange thus removing the possibility of η, ω and ϕ exchanges. The entire production and decay process is represented in fig. 1b. The normalization is fixed by calculating the couplings at the vertices A and B from the experimentally determined width and branching ratios [8] for the $\mathrm{K}^*(1420)$. The pion form factor is again included for the production exchange process and as in ref. [5], we evaluated the momentum factors for the resonant part of the amplitude "at the pole" (see appendix). The M-function for this process is now written as

$$M_{\mathrm{D}\mu} = g_{\mathrm{A}}\Lambda_{\mathrm{A}\alpha}\epsilon_{\beta\mu}(K\Lambda_{\mathrm{A}})\,\frac{\mathcal{P}^2_{\alpha\beta,\gamma\delta}(K)}{(s_{12} - m_{\mathrm{D}}^2) + i\Gamma_{\mathrm{D}}m_{\mathrm{D}}}\,g_{\mathrm{B}}\Lambda_{\mathrm{B}\gamma}\Lambda_{\mathrm{B}\delta}$$

$$\times \frac{1}{(t_{2a} - m_\pi^2)}\,g\left(1 + \frac{2m_{\mathrm{N}}}{\mu_{\mathrm{P}}}\right)\left(1 - \frac{\mu_{\mathrm{P}}^2}{4m_{\mathrm{N}}^2}\right)\tfrac{5}{3}\sqrt{2}\,\gamma_5\,, \tag{4}$$

where m_{D} and Γ_{D} are the mass and total width for the $\mathrm{K}^*(1420)$ and $\mathcal{P}^2(K)$ is the spin-2 propagator as defined by Scadron [9]. The various covariants used are defined as

$$\Lambda_{\mathrm{A}} = \tfrac{1}{2}(p_1 - p_2)\,, \qquad \Lambda_{\mathrm{B}} = \tfrac{1}{2}(p_{\mathrm{b}} - p_{\mathrm{a}} + p_3)\,, \qquad K = p_1 + p_2\,. \tag{5}$$

The full T-matrix for the reaction is now

$$T = \epsilon_\mu(\lambda_1, p_1)\bar{u}(\lambda_3, p_3)[M_A + M_B + M_C + M_D]_\mu u(\lambda_a, p_a). \tag{6}$$

To calculate the required mass and momentum transfer distributions we have to evaluate $\bar{\Sigma}_{\text{spins}} |T|^2$. Noticing that the only non-zero interference term is between A and D, we obtain

$$\bar{\sum_{\text{spins}}} |T|^2 = A^2 T_{AA} + \frac{2AD\,T_{AD}(s_{12} - m_D^2)}{(s_{12} - m_D^2)^2 + \Gamma_D^2 m_D^2} + \frac{D^2\,T_{DD}}{(s_{12} - m_D^2)^2 + \Gamma_D^2 m_D^2}$$

$$+ B^2[T_{BG}G^2 + T_{BGH}GH + T_{BH}H^2] + C^2[T_{CG}G^2 + T_{CGH}GH + T_{CH}H^2], \tag{7}$$

where the factors T are defined in the appendix. This expression is easily coded for use with the phase space programmes reported in ref. [3].

3. RESULTS AND DISCUSSION

The results for this calculation at 6 GeV/c are shown in figs. 2 and 3; at 10 GeV/c in figs. 4 and 5. The experimental data was supplied by the H.E.N.P. group at Imperial College and was taken from the Birmingham-Glasgow-London (I.C.)-Munich-Oxford collaborations K^-p exposure at 6 GeV/c and the Aachen-Berlin-CERN-London (I.C.)-Vienna collaborations K^-p exposure at 10 GeV/c.

Unfortunately we were unable to obtain an absolute normalisation for the experimental results and so we have normalised the theoretical curves to a cross section 90% of that for the data. This figure is chosen to allow approximately for processes not taken into account in the model. Looking at the t-distributions (figs. 3 and 5) we see that the results decrease too rapidly outside the forward peak. This is probably due to the presence of some non-peripheral processes and, possibly, an over drastic t behaviour of the form factors. The over rapid fall off is reflected kinematically into the mass-squared plots where it produces cross sections which are too small at large $s_{K^*\pi}$ and $s_{\pi n}$ and at small s_{K^*n}. This is a feature that may well be improved by the "correct" inclusion of absorption effects. Also, looking at the mass-squared plots for the πn sub-system, it appears that there is probably a contribution from the $\Delta(1236)$ resonance producing a peak at $s_{\pi n} = 1.5$ GeV^2 and possibly a small contribution from a higher mass resonance, perhaps the N'(1470). In the mass-squared plots for the $K^*\pi$ subsystem there is a suggestion of a contribution from the $K^*_A(1320)$, even though we are dealing with a charge exchange reaction so that diffractive production cannot occur. There is, however, no trace of the $K^*_A(1230)$, the other Q peak resonance.

One unsatisfactory feature of this model lies in the total energy dependence. At 10 GeV/c the relative magnitudes of the background and resonant contributions are given correctly by the model but at 6 GeV/c the background

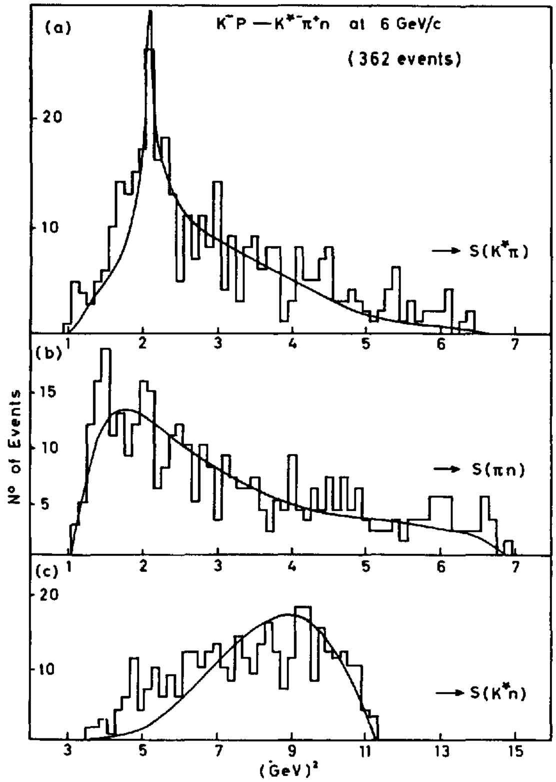

Fig. 2. Mass-squared distributions at 6 GeV/c.

is significantly too small relative to the resonance. To obtain the curves in figs. 2 and 3 the background amplitude was increased by a factor of 2 relative to the resonance. This is not wholly surprising since the resonance amplitude is assumed due to single pion exchange, which is known to give rise to a reasonable energy dependence, but the DPM terms all have vector exchanges which are known to predict an incorrect energy dependence. This poor energy dependence does not show up in the mass-squared distributions as these are dominated by the kinematic effect of the restricted momentum-transfer distributions, but causes the total energy dependence to be incorrect.

4. CONCLUSION

In conclusion we may say that these results suggest that the DPM gives a mechanism which can account well for most of the direct three particle cross section in high energy inelastic processes and that it would be desirable to perform a more thorough analysis taking account of "off-shell" and "absorption" effects in a more satisfactory manner. The major difficulty

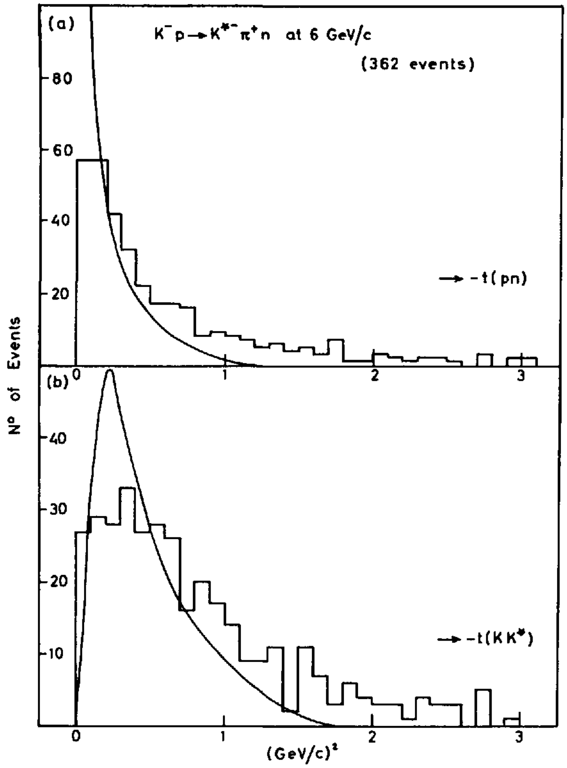

Fig. 3. Momentum transfer distributions at 6 GeV/c.

with this model, in common with the normal two particle peripheral model, is still the incorrect energy dependence of vector exchange amplitudes. Presumably this problem will find a similar solution in both the single and double peripheral models.

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APPENDIX

The explicit forms for the quantities not given fully in the text are as follows, where notation is as in refs. [5, 6],

$$D = \frac{10\sqrt{2}}{3} g_A g_B g \left(1 + \frac{2m_N}{\mu_P}\right)\left(1 - \frac{\mu_P^2}{4m_N}\right) \frac{F_\pi(0)\, e^{\lambda_\pi t_{3a}}}{(t_{3a} - m_\pi^2)} p_1^{(D)} p_b^{(D)} \cos\theta_{1b}^{(3)} ,$$

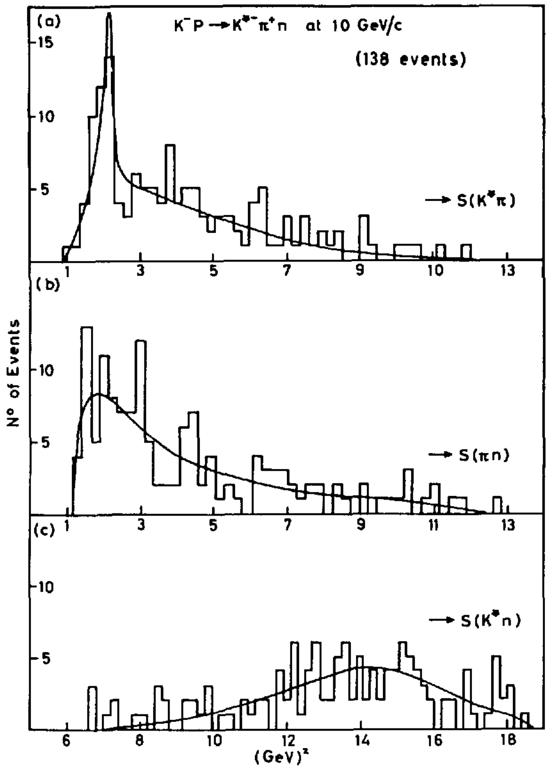

Fig. 4. Mass-squared distributions at 10 GeV/c.

$$T_{AA} = -t_{3a}[p_1^{(3)} p_b^{(3)} \sin\theta_{1b}^{(3)}]^2 s_{12},$$

$$T_{AD} = -t_{3a} p_1^{(3)} p_1^{(D)} p_b^{(3)} p_b^{(D)} \sin^2(\theta_{1b}^{(3)}) m_D W_3,$$

$$T_{DD} = -t_{3a}[p_1^{(D)} p_b^{(D)} \sin\theta_{1b}^{(3)}]^2 m_D^2,$$

$$T_{BG} = [\{p_1 p_b)^2/m_1^2\} - m_b^2][t_{3a} m_2^2 + 4(p_2 p_a)(p_2 p_3)],$$

$$T_{BGH} = 4m_a[\{p_1 p_b)^2/m_1^2\} - m_b^2][(p_2 p_a) + (p_2 p_3)]^2,$$

$$T_{BH} = [4m_a^2 - t_{3a}][\{(p_1 p_b)^2/m_1^2\} - m_b^2][(p_2 p_a) + (p_2 p_3)]^2,$$

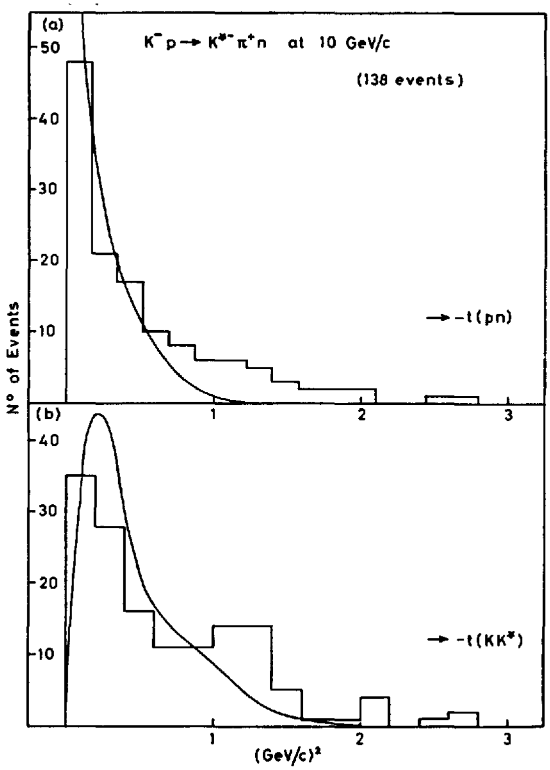

Fig. 5. Momentum transfer distributions at 10 GeV/c.

$$
\begin{aligned}
T_{\mathrm{CG}} = {} & [(p_1 p_b)\, t_{3a} + 2(p_1 p_a)(p_b p_3) + 2(p_a p_b)(p_1 p_3)] \\
& \times [D_1(p_1 p_2) + D_1(p_2 p_b) + D_2(p_1 \Delta) - D_3(p_b \Delta)] \\
& - [m_1^2 t_{3a} + 4(p_1 p_a)(p_1 p_3)][D_1(p_2 p_b) + D_2(p_b \Delta)] \\
& - 2D_1(p_b \Delta)[(p_1 p_2)\, t_{3a} + 2(p_1 p_a)(p_2 p_3) + 2(p_2 p_a)(p_1 p_3)] \\
& + 2D_1(p_1 \Delta)[(p_2 p_b)\, t_{3a} + 2(p_2 p_a)(p_3 p_b) + 2(p_a p_b)(p_2 p_3)] \\
& - [m_b^2 t_{3a} + 4(p_a p_b)(p_3 p_b)][D_1(p_1 p_2) - D_3(p_1 \Delta)] - 2D_1^2[t_{3a} + 2m_a^2],
\end{aligned}
$$

$$D_1 = (p_1 \Delta)(p_2 p_b) - (p_b \Delta)(p_1 p_2),$$

$$D_2 = m_2^2 (p_b \Delta) - (p_2 \Delta)(p_2 p_b),$$

$$D_3 = (p_1 p_2)(p_2 \Delta) - m_2^2 (p_1 \Delta),$$

$$\Delta = p_b - p_1;$$

$$T_{CGH} = 8m_a \{(p_1 \Delta)[(p_a p_b) + (p_3 p_b)] - (p_b \Delta)[(p_1 p_a) + (p_1 p_3)]\}$$

$$\times [-m_2^2 F_1 + (p_2 p_3) F_2 - (p_2 p_a) F_3]$$

$$+ 8m_a^2 D_1 \{F_1[(p_2 p_a) + (p_2 p_3)] - F_2[m_3^2 + (p_3 p_a)] + F_3[m_a^2 + (p_3 p_a)]\},$$

$$T_{CH} = -8\{F_1^2 m_2^2 + F_2^2 m_3^2 + F_3^2 m_a^2 - 2F_1 F_2 (p_2 p_3) + 2F_1 F_3 (p_2 p_a)$$

$$- 2F_2 F_3 (p_3 p_a)\} [m_a^2 + (p_3 p_a)],$$

where

$$F_1 = (p_1 p_3)(p_a p_b) - (p_1 p_a)(p_3 p_b),$$

$$F_2 = (p_1 p_2)(p_a p_b) - (p_2 p_a)(p_1 p_a),$$

$$F_3 = (p_1 p_2)(p_3 p_b) - (p_1 p_3)(p_2 p_b).$$

The momentum factors $p_1^{(3)}$ and $p_b^{(3)}$ are given by

$$p_1^{(3)} = \frac{[s_{12}^2 + m_1^4 + m_2^4 - 2s_{12} m_1^2 - 2s_{12} m_2^2 - 2m_1^2 m_2^2]^{\frac{1}{2}}}{2W_3},$$

$$p_b^{(3)} = \frac{[s_{12}^2 + m_b^4 + t_{3a}^2 - 2s_{12} m_b^2 - 2s_{12} t_{3a} - 2m_b^2 t_{3a}]^{\frac{1}{2}}}{2W_3},$$

and, $p_1^{(D)}$ and $p_b^{(D)}$ are the above momentum factors evaluated "at the pole", i.e. with $s_{12} = W_3^2 = m_D^2$

$$\cos\theta_{1b}^{(3)} = \frac{[t_{1b} - m_1 - m_b + 2E_1^{(3)} E_b^{(3)}]}{2p_1^{(3)} p_b^{(3)}},$$

$$(p_1 p_2) = \tfrac{1}{2}(s_{12} - m_1^2 - m_2^2) \text{ etc},$$

$$(p_3 p_a) = \tfrac{1}{2}(m_3^2 + m_a^2 - t_{3a}) \text{ etc}.$$

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