APPLICATIONS OF GROUP THEORY TO FUNDAMENTAL PARTICLES AND SCATTERING

by

Toshar Kanti GUJADHUR

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Department of Mathematical Physics,
Imperial College,
University of London.
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ABSTRACT

We study the classification and the production of 2^+ and 1^+ Mesons and $5/2^+$ Baryons within U (6,6) and U $(6,6) \otimes O(3)$ groups, using the Born term Reggeised, Absorbed and K Matrix Model.

TO MY MOTHER

PREFACE

The work presented in this thesis was carried out in the Department of Mathematical Physics, Imperial College, of the University of London between December 1966 and July 1969, under the supervision of Dr. S. Hochberg. The author is indebted to him and to Professor P.T. Matthews for their help and encouragement during the course of this work. I would also like to take this opportunity for expressing my gratitude for the numerous helpful discussions I have had, to my colleagues and members of the staff, especially Dr. R. Delbourgo, of the Theoretical Physics Group of Imperial College.

Except where stated in the text, the work described is original and has not been submitted in this or any other University for any other degree. The thesis is based partly upon the following papers which are in preparation:

1 The $U(6,6) \otimes O(3)$, Peripheral Absorption Model and Production of 2^+ and 1^+ Mesons, by T.K.Gujadhur, J.H.R. Migneron and K.J.M. Moriarty.

2 An $U(6,6) \otimes O(3)$ Absorbed K - Matrix Model for Spin 2 Exchange in the Process $\pi^- p \longrightarrow \eta n$, by T.K. Gujadhur, J.H.R. Migneron and K.J.M. Moriarty

3 The $U(6,6) \otimes O(3)$ Peripheral Absorption Model and Production of the $5/2^+$ Nucleon Resonance, by T.K.Gujadhur, J.H.R. Migneron and K.J.M. Moriarty.

4 Production of 2^+ and 1^+ Mesons in a Reggeised $U(6,6) \otimes O(3)$ Model, by T.K.Gujadhur, J.H.R.Migneron and O.Shafi.

I would like to thank the co-authors, especially Dr.R.Migneron and Dr. K.Moriarty for permission to include the joint work in this thesis. The latter is thanked also for a critical reading of the manuscript.

Applied Mathematics is a compromise. The questions we deal with are usually too hard to be treated by rigorous argument but we have the added interest that the theorems are observable.

Professor STEWARTSON.[1]

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CHAPTER 1

Quantum Electrodynamics is not a consistent theory - in fact not a theory in the proper sense because its equations are in contradiction to each other.

E. WIGNER[2]

1.1 High Energy Phenomenology

The situation in strong interaction physics is worse than in Quantum Electrodynamics. Whereas Quantum Electrodynamics characterised by a coupling constant $e^2/4\pi = 1/137$ makes certain low order term predictions which are truly remarkable[3], quantised strong interaction field theory has hardly any redeeming feature. For instance there exist now more than 100 elementary particles[4] and a massive amount of data, supplied in ever copius amounts from high-energy collisions, to be explained and correlated. As a consequence one has to rely on a large number of models, to extrapolate techniques and results of non-relativistic quantum mechanics and to obtain as much as we can from the theory of quantized fields, in spite of its weak foundation, in order to explain this great diversity of empirical facts[5].

As in this thesis we are concerned mainly with the classification and production of resonances within the Global and Supermultiplet Symmetry Schemes we shall confine our discussion only to those theories of scattering in which two particles or quasi-particles are produced in the final state. See fig. 1.1.

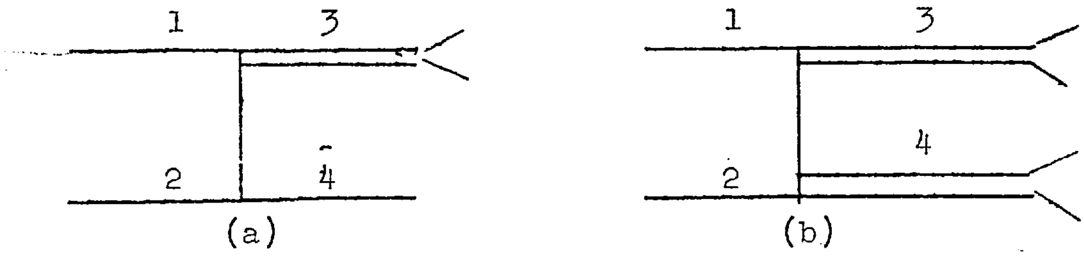


Fig.1.1 Production of (a) one and (b) two quasi-particles

We do this review of current techniques of high energy collision in order to show clearly the merits and drawbacks of the Absorption Model and Regge Pole theory which are the main tools we use in our dynamical consideration. At the same time we bring out the main experimental features of high energy scattering. The extent to which our two main group theoretical approaches explain these features will then be used as a criterion to decide which approach is better, if at all. The main facts that a model must attempt to explain are

(i) the magnitudes of the cross-sections,

(ii) the energy dependence of the cross-sections,

(iii) the momentum-transfer (t, or, u) dependence of the differential cross-section and any structure, such as 'dip' this dependence may exhibit. See fig1.2,

(iv) the polarizations of the particles or resonances,

(v) the ratio of real to the imaginary part of the amplitude in the forward direction.

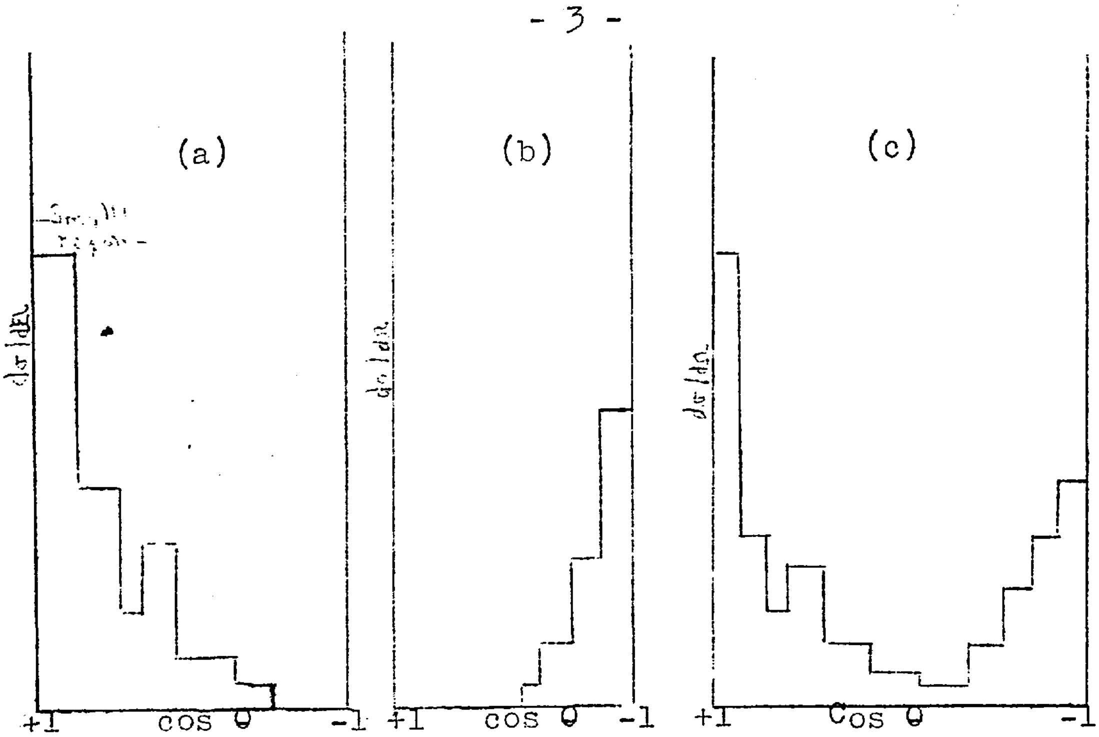


Fig.1.2 Representative production cross-sections

(a) A large forward peak is caused by meson exchange. Of the order of 100 - 500 μb/st

(b) Backward peak caused by baryone exchange. Of the order of 5 - 20 μb/st. They decrease rapidly with increasing incident momentum.

(c) The most frequent type in which both meson and baryon exchanges occur.

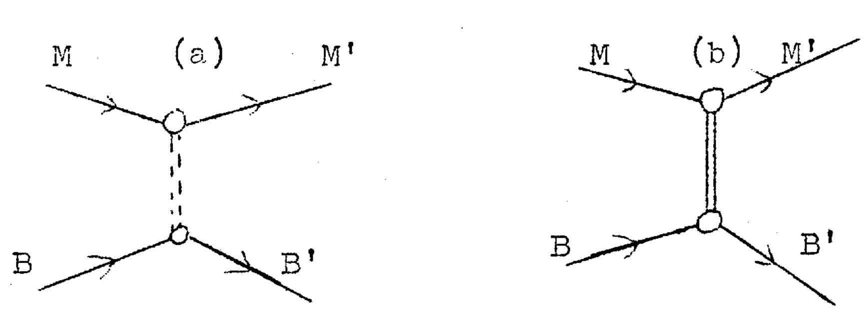


Fig.1.3 (a) t channel meson exchange
(b) u channel Baryon exchange

The task of explaining these features is considerably simplified if we assume that strong interactions are invariant under certain groups of transformations: in other words that they have to satisfy certain conservation laws. For instance before the advent of SU(3) the interaction Lagrangian for the then known hadrons, assuming a three point vertex, was

$$L_{strong} = g_1\bar{N}\,N.\pi + g_2\bar{\Sigma}\Lambda.\pi\ h.c.$$

$$g_3\bar{\Sigma}\Lambda\Sigma.\pi + g_4\bar{\Xi}\tau\Xi.\pi + g_5\bar{N}K\Lambda + h.c.\ g_6\bar{N}\tau K.\Sigma + h.c.$$

$$+g_7\bar{\Xi}K\Lambda + h.c. + g_8\bar{\Xi}K.\Sigma + h.c.$$

with eight independent couplings. Introduction of SU(3) reduces this to a single one[6]. In an analogous way application of U(6) ⊠ U(6) ⊠ O(3) places constraints on the parameters present in Regge pole theory and consequently simplifies our physical interpretation of the theory; again it does so in field theory models which would otherwise be plagued by arbitrary parameters. Theeenergy range of the incident momentum in which group theoretical methods can be most simply and fruitfully applied is the 2-12 Gev, in which range an appreciable number of resonances are formed. It is in this region that most of the 1^-, $3/2^+$, $5/2^+$ resonances are seen[7]. Resonance formation is summarised in Fig.1.1, Fig.1.2, and Fig.1.3. As can be seen from these diagrams the formations show several simple features and the appropriate transition amplitudes can be written fairly simply - hence their amenability to group theoretical treatment. More complicated exchanges either of states with high spins or of complex configurations cannot be excluded; however the simple exchanges of a few particles belonging to the σ^-, $1/2^+$, 1^- nonet or of a one or two Regge trajectories have been highly successful in explaining a large body of data[8]. In fact Jackson, Gottfried and others have made a detailed empirical investigation on the quantum numbers of the exchanged systems and have shown that forward or backward peaking is caused by simple exchanges rather than by more complicated difractive effects[7,9]. Their results are based on the angular distribution of the decay of the resonances which yield information on the production act provided the spin and parity of the resonances have been established by independent methods. The spin population of a resonance is usually described by a density matrix $\rho_{mm'}$ where m and m' are the magnetic quantum numbers of the decay products expressed in the rest frame of the resonance. For J = 1 resonance decaying into two spinless boson e.g. $\rho \to 2\pi$ the general angular distribution of the decay product is

$$W(\theta,\phi) = \frac{3}{4\pi} \left(\rho_{00} \cos^2\theta + \rho_{11} \sin^2\theta - \rho_{1-1} \sin^2\theta \cos 2\phi - \sqrt{2}\,\rho_{10} \sin 2\theta \cos\phi\right) \qquad 1.1$$

and for a J = 3/2 resonance going into spinless boson and baryon of spin 1/2 the relevant expression is

$$W(\theta,\phi) = \frac{3}{4\pi} \left(\rho_{33} \sin^2\theta + \rho_{11}\left(\frac{1}{3} + \cos^2\theta\right) - \left(\frac{2}{\sqrt{3}}\right) \mathrm{Re}\,\rho_{3-1} \sin^2\theta \cos 2\phi - \left(\frac{2}{\sqrt{3}}\right) \mathrm{Re}\,\rho_{31} \sin 2\theta \cos\phi\right) \qquad 1.2$$

Measured in suitable frames these two expressions become

$\sim \cos^2\theta$ and $\sim (1+3\cos^2\theta)$ for spin-zero exchanges. This means that the type of exchange mechanism can be tested from the decay dorrelation independently of the momentum transfer distribution.

1.2 The Peripheral Model

The angular distributions for the productions of resonances is summarised in Fig.1.2 with the forward or backward peaking indicating small momentum transfer; they are almost entirely confined to momentum transfers of less than 0.5 Gev with an average value of 0.2 - 0.4. This predominance of small momentum transfers implies that glancing collisions are most important in these reactions. This fact can be stated in various equivalent ways: collisions with large impact parameters give rise preferentially to quasi-two body reactions; these collisions are dominated by high partial waves or equaivalently that the reaction is mediated by a long-range force corresponding to the exchange of a light particle; nearby singularities in the t-channel dominate the reaction. This strong peaking at very small values of t and the angular distributions seem to be largely independent of the type of exchange occuring and bearing little or no relation to the mass of the possible exchanged particle. Given the peripheral nature of the processes a peripheral model with one particle exchange was naturally the first one to be used for these data. Fig.1.4 shows the essential features of this model

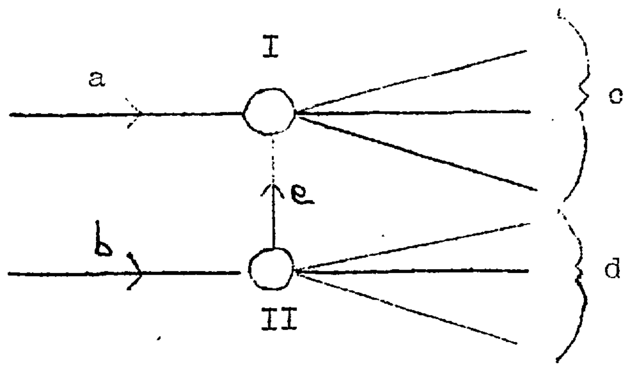


Fig.1.4 General case of peripheral interaction

where c and d could be resonances. The matrix element has the following structure.

$$M_{fi} = M_I(t,m_c^2)\ \frac{1}{m_e^2 - t}\ M_{II}(t,m_d^2) \qquad 1.3$$

where M_I and M_{II} are vertex functions. On the mass-shell they are the matrix elements for the following processes:

$$a + e \longrightarrow c$$
$$b + \bar{e} \longrightarrow d$$

When c and d are quasi or real particles these vertex functions at $-t = m_e^2$ are proportional to the coupling constants. Here t is the momentum transfer and is given by[10]

$$\begin{aligned} -t &= (P_c - P_a)^2 \\ &= m_c^2 + m_a^2 - 2E_cE_a + 2P_cP_a \cos\theta \end{aligned} \qquad 1.4$$

For instance in this model assuming a ONE-PION-EXCHANGE (OPE) then we have for the process $\pi P \longrightarrow \rho P$

$$\frac{d\sigma}{dt} = \frac{\pi}{4m_\rho^2\ m_N^2\ P_{inc}}\ M_I^2\ M_{II}^2\ t^2\ (t - \frac{(m_\rho - m_\pi)^2)(t-(m_\rho+m_\pi)^2)}{t - m_\pi^2}$$

with

$$M_I^2 = \frac{g_{\rho\pi\pi}^2}{4\pi}$$

$$M_{II}^2 = \frac{G_{N\pi\pi}^2}{4\pi} \qquad 1.6$$

Such models have been studied by Drell[11], Salzman and Salzman[12] and by Ferrari and Selleri[13] who introduced ad hoc form factors of the type

$$F(t) = \frac{0.72}{1+(\mu^2-t)/4.73\mu^2} + \frac{0.28}{1+((\mu^2-t)/32\mu^2)^2} \qquad 1.7$$

$$F(t) = \exp(-\lambda t) \qquad 1.8$$

in order to overcome the ~~lamentable~~ short comings of the unadorned OPE model[14]. However most processes considered seem to need their own particular form factor irrespective of the exchanges; further the OPE model with form factors predict the same cross-sections for $MN \to NN^*$ and $\bar{N}N \to NN^*$ contrary to experiment. These form factors modify primarily the medium-to-short range forces which may involve multiple scattering or exchange of massive particles about which we know nothing and so such an arbitrary treatment with no physical interpretation is hardly justifiable. Also some of these factors require a three pion state with mass squared $\simeq 5m^2_\pi$ and particles with same quantum numbers as the ρ and ω but with a lighter mass - particles which are not known in nature. It was mainly for these reasons that this model was rejected in favour of the Absorption Model.

1.3 The Absorption Model

This model seems to be able to cope with the following features:

(i) a particular quasi-two-body production channel only accounts for a small fraction of the total inelastic cross-section on account of the many competing channels;

(ii) as the quasi particles are formed peripherally only the highest partial waves contribute to their formation;

(iii) more complicated reactions at small impact parameters cannot contribute to resonance formation. This model completely removes the lowest partial waves leaving unchanged the higher ones with the result that there is a reduction of the cross-section, important modifications of the angular distribution and an alteration in the decay correlations.

The form of the Absorption Model we shall use is the one proposed by Sopkovitch[15,16] although many others exist[17,18,19]. It has its origin in the low energy nuclear physics Distorted Wave Born Approximation (DWBA) model. It will be recalled that there the transition amplitude T_{fi} is approximated by the matrix element

$$T_{fi} \simeq \langle \phi_f^- | V | \phi_i^+ \rangle \qquad 1.9$$

Where ϕ_f^- and ϕ_i^+ are wave functions of the system in the final and initial state respectively, V is the interaction causing the transition. The wave functions ϕ^+ and ϕ^- are customarily approximated by the wave functionsoof an optical-model potential whose imaginary part simulates the absorptive effects of the many competing channels. How elastic scattering ~~implies~~ can be used to simulate absorption (or vice versa) may be seen by recalling that the transition amplitude for non-relativistic scattering is given by

$$f(\theta) = \sum_l (2l+1) \frac{S_l - 1}{2ip} P_l (\cos \theta) \qquad 1.10$$

where

$$S_l = e^{2i\delta_l} \qquad 1.11$$

and

$$\delta_l = \alpha_l + i\beta_l \qquad \beta > 0 \qquad 1.12$$

and the elastic and inelastic scattering given respectively by

$$\sigma_{el} = \frac{\pi}{p^2} \sum_l (2l+1) |S_l - 1|^2 \qquad 1.13$$

$$\sigma_{in} = \frac{\pi}{p^2} \sum_l (2l+1) (1 - |S_l|^2) \qquad 1.14$$

In the Optical Model it is assumed that $\beta_l \gg \alpha_l$ so that

$$S_l = e^{-\beta_l} \qquad 1.15$$

From this it is clear that absorption ~~implies~~ modifies elastic scattering.

In the same way one may see how absorptive effects may be simulated by elastic scattering. In high energy scattering the wave functions representing the particles may be assumed to be highly localised so that one may represent geometrically, the whole process as happing inside two concentric circles with radii ν and μ if the particles interact inside the inner one there is 'absorption' as far as the inelastic process $|\alpha\rangle \rightarrow |\beta\rangle$ is concerned whereas the outer circle represent the region in which this inelastic or Born exchange occurs. μ is assumed to be of the order of m^{-1} where m is the mass of the exchanged particle whereas ν represents the range of elastic scattering.* See Fig. 1.5.

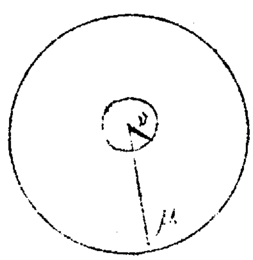


Fig.1.5. Range of interaction of Born term and elastic scattering.

In the Born approximation the scattering amplitude which is given by

$$f(\theta) = \frac{p}{i} \int_0^\infty J_0(tb)\ (e^{i\chi(b)} - 1)\ b\,db \qquad 1.16$$

where t stands for the three momentum and b the impact parameter becomes

$$= 2p^2 \int_0^\infty J_0(tb)\ B(b)\ b\,db \qquad 1.17$$

* We use units of which $\hbar = c = 1$

where we have taken

$$e^{i\chi(b)} - 1 = 2ipB(b) \qquad 1.18$$

this being the usual Born approximation.

To represent the effects of absorption it is usually assumed that the particle is under the influence of a complex potential $U_{\alpha\alpha}$ and $U_{\beta\beta}$ in the initial channel and final channel respectively. As $p >> v$ the appropriate wave function is

$$\phi(z) \simeq e^{ikz} \rho(z)$$

where in the WKB approximation[21]

$$\rho(b+kz) = \exp\left\{\frac{im}{p} \int_{-\infty}^{z} U(b+kz')dz'\right\} \qquad 1.19$$

The phase shift suffered is then given by

$$\delta_{\mu\mu} = -\frac{m}{p} \int_{-\infty}^{0} U_{\mu\mu}(b+kz')dz' \qquad \mu = \alpha, \beta \qquad 1.20$$

If we include these two phase shifts suffered by this particle we have that finally the amplitude $f(\theta)$ is ~~now~~ given by

$$f_{\beta\alpha}(\theta) = 2p^2 \int e^{i\delta_{\alpha\alpha}} \left(J_0(tb)\, B_{\beta\alpha}(b)\right) e^{i\delta_{\beta\beta}}\, bdb \qquad 1.21$$

This result is true only under the conditions we have assumed but bearing it in mind we postulate, has an ansatz, that in the relativistic region the Born term is modified by absorption which may be simulated by elastic scattering between the particles in the initial and final state and is given by

$$(B_{modified})_{\alpha\beta} = S^{1/2}_{\alpha\alpha} \; B_{\alpha\beta} \; S^{1/2}_{\beta\beta} \qquad 1.22$$

This relation is represented diagramatically in Fig. 1.6.

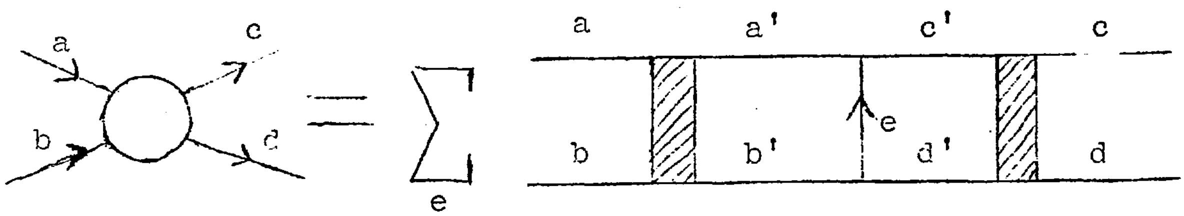


Fig.4. Schematic representation of eq.1.22. The shaded blobs represent elastic scattering in the initial and final states.

It is to be emphasised that eq.1.22 is valid under the conditions

$$p^{-1} << \mu << \nu \qquad 1.22a$$

and that for the situation

$$p^{-1} << \nu << \mu \qquad 1.22b$$

Durand and Chin have shown that the appropriate modification is

$$(B_{modified})_{\alpha\beta} = \frac{1}{2} \; (S_{\alpha\alpha} \; B_{\alpha\beta} \; + \; B_{\alpha\beta} \; S_{\beta\beta}) \qquad 1.22c$$

The use of such a relation in the relativistic region has not been proved at all and its justification lies mainly on the success it has had in explaining a host of experimental data,[8,9,15,22] and in its great simplicity from the computational point of view. It is solely for this reason that we will adopt it as one of our main tools for explaining the production of high spin particles. In Chapter III we shall investigate it further and look at some of its subsequent modifications.

Although the absorption model met great success from early on its drawbacks were soon apparently as it could not cope with exchanges mediated by high spin particles[9,24] and could not explain the decrease of the total Croy section with increasing lab energy. For such cases the amplitudes varied too rapidly over any appreciable energy range and eventually violated unitarity. It was for this reason, among others, that the Regge Pole Model - our second formalism - was revived.

1.4 The Regge Pole Model

We recapitulate here some of the relevant relationships of non-relativistic Regge Theory[25] to show the form of the amplitudes we shall eventually adopt in our subsequent applications, and the assumptions under which these will be deemed to be true. The scattering amplitude is given in its simplest form by

$$F(\text{Cos}\ \Theta,\ E) = \sum_{i} \frac{\beta_i(E)\ P_{\alpha_i}(E)\ (-\text{Cos}\ \Theta)}{\text{Sin}\ \pi\alpha_i(E)} + \begin{matrix}\text{Background}\\ \text{Integral}\end{matrix} \qquad 1.23$$

Where

$$\beta_i(E) = -\frac{\pi}{k}\ (2\alpha_i(E) + 1)\ \ R_i(E) \qquad 1.24$$

with $R_i(E)$ being the residue of the relevant pole. Introduction of space exchange potentials - exchange forces for the relativistic region - modifies this to

$$F\,(\text{Cos}\ \theta, E) = \sum_{i} \frac{\beta_i(E)}{\text{Sin}\ \pi\alpha_i(E)} \left(P\alpha_i(E)(-\text{Cos}\ \theta) + \tau P_{\alpha_i}(E)(\text{Cos}\theta) \right) + BI(\text{Cos}\ \theta,\ E) \qquad 1.25$$

Where τ is the signature. In analogy with this the relevant relativistic amplitude in the t channel will be taken to be

$$\bar{T}(\bar{s},\bar{t}) = \sum_{i} \frac{\beta_i(\bar{s})}{\text{Sin}\ \pi\ \alpha_i(\bar{s})} \left(P\alpha_i(\bar{s})\ (-\text{Cos}\ \theta) + \tau P_{\alpha_i}(\bar{s})(\text{Cos}\ \theta) \right) + BI\ (\text{Cos}\ \theta_t,\ \bar{s}) \qquad 1.26$$

Fig.1.5 explains our convention

s- Channel Reaction

$s = (P_a + P_b)^2 = (P_c + P_d)^2$

$t = (P_c - P_a)^2 = (P_b - P_d)^2$

$T = T(s,t)$

t- Channel Reaction

$\bar{s} = (P_a + P_{\bar{c}})^2 = (P_{\bar{b}} + P_d)^2$

$\bar{t} = (P_a - P_{\bar{b}})^2 = (P_{\bar{c}} - P_d)^2$

$\bar{T} = \bar{T}\ (\bar{s},\bar{t})$

Fig 1.5 Kinematics

To go to the s-channel we use the relation

$$\text{Cos}\ \theta_t = \frac{2(s + \frac{t}{2} - m_1^2 - m^2)}{\sqrt{t-2m^2}\ \sqrt{t-4m^2}}$$

$$\sim -s \quad \text{as} \quad s \longrightarrow \infty \qquad 1.27$$

$$\text{and} \quad t < 0\ , \quad \text{finite}$$

and the asymptotic limit[26]

$$P_{\alpha(t)}\,(-\cos\theta_t) = k(t)\,\left(\frac{s}{s_0}\right)^{\alpha(t)} \qquad \text{for } s \longrightarrow \infty \qquad 1.28$$

It similarly can be shown that

$$BI\,(\cos\theta_t, \bar{s}) \sim O\,(s^{-1/2}) \qquad 1.29$$

We then have

$$P_{\alpha(t)}\,(-\cos\theta_t) + \Upsilon\, P_{\alpha(t)}\,(\cos\theta_t)$$

$$= k(t)\,\left(\frac{s}{s_0}\right)^{\alpha(t)} + \Upsilon\,\left(-\frac{s}{s_0}\right)^{\alpha(t)}$$

$$\equiv k(t)\;\; 1 + \Upsilon \exp\,(-\,i\pi\alpha(t)\;\; \left(\frac{s}{s_0}\right)^{\alpha(t)} \qquad 1.30$$

where Υ is the signature of the Regge Pole. Defining the signature factor as

$$\xi(t) = \frac{1 + \Upsilon \exp(-\,i\pi\alpha(t))}{\sin\pi\,\alpha(t)} \qquad 1.31$$

the amplitude for S-channel becomes[27]

$$T\;(s,t) = \sum \gamma_i(t)\,\xi_i(t)\,\left(\frac{s}{s_0}\right)^{\alpha_i(t)} + O\,(s^{-1/2}) \qquad 1.32$$

We shall assume that $\alpha(t)$ the trajectory of the t-channel Regge Pole continued into the s-channel is real[28]; such an assumption seems to be justified for bosons exchanges Regge trajectories. We shall also assume that the residue function $\gamma(t)$ to be real - this can be proved from general principles of quantum field theory if $\alpha(t)$ is real[28]. Just as for the OPE model a characteristic property of which is the occurance of two coupling constant one for each vertex, we shall assume that the one pole contribution can be factorized toc[29]. More precisely this means that the residue function can be written

$$\gamma(t) = (\text{Known Kinematical factors})\ \gamma_{aRc}(t) \cdot \gamma_{cRd}(t) \qquad 1.33$$

Where $\gamma_{aRc}(t)$ refers to the coupling of the Regge pole R to the t-channel initial state $\bar{a}c$ while γ_{bRd} refers to the coupling of R to $\bar{b}d$. As in the OPE model the same Regge pole coupling $\gamma_{aRc}(t)$ may occur in different reactions and is independent of how the Regge pole couple to the other two particles; and obviously one may use internal symmetry schemes to relate couplings of members of the same multiplet[30]. However it has to be emphasised that factorization is a property of one Regge Pole contribution only and that as soon as several trajectories exchanged there is in general no clear-cut prediction from this principle. It should be emphasised that this relation tacitly assumes that the t-channel amplitude is dominated by bound states or resonances. Experimentally in two body collisions depicted in fig. 5 the s-channel differential cross-section is appreciable and has a peaking for small momentum transfers only if there exist a particle or resonance in the exchange channel. Also we shall ignore the contributions of cuts which are proportional to $\left(\frac{s}{s_o}\right)^{\alpha^c(t)} \left[\log\left(\frac{s}{s_o}\right)\right]^{-\nu}$ where ν is a number depending on the nature of the branch point $\alpha^c(t)$.

The amplitude depends strongly on the exchange of the Regge trajectories $\alpha(t)$ on which all particles have the same quantum numbers. Experimentally there is a strong correlation between the energy-dependence of the cross-sections we have described in fig. 2 and the exchange quantum numbers. Morrison[31] assuming a relation on the form

$$\sigma = \sigma_o P_{Lab}^{-n} * \qquad 1.34$$

has found that the values of n fall naturally into four groups depending on the exchange quantum numbers:

Vacuum exchange	$n \sim 0$	
Charge or isospin exchange	$n \sim 2$	1.35
Strangeness exchange	$n \sim 2.5$	
Baryon number exchange	$n \sim 3\text{-}4$	

The t-channel amplitudes may contain Kinematical branch point, poles or even zeros at the boundary of the physical region at the threshold, psudo-thresholds and at $t = 0$[32]. Many authors, among them Cohen-Tannoudji, Morel and Navelet, CTMN[33], have given general prescriptions for removing these unwanted features. These 'regularised' amplitudes may then be connected to s-channel amplitudes by means of a relation of the form

$$T_\lambda (s,t) = \sum_{\lambda'} X_{\lambda\lambda'} \bar{T}_{\lambda'} (\bar{s},\bar{t}) \qquad 1.36$$

where X is the crossing matrix which is a rational function of s and t. For T(s,t) to be free from singularities there must exist relations between different t channel

* P_{Lab} is momentum in Lab

regularised amplitudes. These give rise to constraint equations at the threshold and pseudo-threshold of our amplitudes; the general consequences of these have been classified as follows by Leader[34]:

(i) there exist relations between trajectories of different Regge Poles with different quantum numbers valid at the constraint points. This is known as conspiracy

(ii) the constraint equations are satisfied by enforcing conditions on different residuum functions but not on the trajectories. This is known as evasion

(iii) the constraints are satisfied by requiring the existence of sequences of Regge poles with the same quantum numbers on different but related trajectories. These trajectories are known as daughters.

Later on we shall invoke certain of these properties to rid ourselves of unwanted singularities.

The problem of constraints may also be viewed from a much more general point of view at $t = 0$[35]. At this point the scattering amplitude is invariant under the little group of the general Poincare group belonging to $P_\mu = 0$ which is isomorphic to the homogeneous Lorentz Group $O(3,1)$. The scattering amplitude can then be expanded into the irreducible representations of this group and a 'Reggeization' performed by means of a sort of Sommerfeld-Watson transform.* The whole procedure is completely analogous to the 3-Dimensional non relativistic case. The asymptotic behaviour of the scattering amplitude for $s \to \infty$ at $t = 0$ is correspondingly dictated by the exhchage of Toller poles - these poles are characterised by

* See Chapter 4

the usual quantum numbers plus the following:

(i) A Lorentz quantum number M which takes the following values

$$M = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$$

with the half integers referring to boson and ~~the other~~ integer to fermions.

(ii) The Lorentz signature $T_L = \pm 1$ and is related to CPT.

(iii) And the complex 4 dimensional angular momentum $\alpha(t)$ which gives the dominant asymptotic behaviour as $s^{\alpha(t)}$.

One can also deduce that:

(i) Any Toller pole gives rise to an infinite family of Regge poles at $j_n = \alpha - n$ the α trajectory is the parent one and the others are known as the daughters.

(ii) All the residues of the daughters satisfy factorization if the parent Toller pole does.

(iii) As one unique pole determines all the properties like trajectories and residuum functions of all the Regge poles in the family one has definite relations existing between different Regge poles corresponding to the same Lorentz pole. This is conspiracy. These relations are identical to those found by CTMN.

There is yet another source of singularities which come this time from the signature factor. For positive signatures this is

$$\xi(t) = \left(\sin \frac{\pi}{2} \alpha(t)\right)^{-1} \exp - i \frac{\pi}{2} \alpha(t) \qquad 1.37$$

and give rise to a pole at $\alpha(t) = 0$. This cannot be accepted for $t \leq 0$. We redefine the residue function in order to get rid of this undesired pole which is known as a 'Ghost'[36]:

$$\gamma(t) = \alpha(t) \quad (\alpha(t) + 1) \; \bar{\gamma} \; (t) \qquad 1.38$$

and it is with this new $\gamma(t)$ that we work. The second factor is introduced as the residue function must vanish at $\alpha(t)$ values which are symmetric with respect to $\alpha(t) = -\frac{1}{2}$.

One further point needs to be mentioned. We shall use mostly straight line Regge trajectories although there is no convincing theoretical argument for or against it[37]. They are not straight line for Yukawa potentials[38] and moreover an infinite linear increase of the trajectory seems to be disagreement with the basic requirements of Regge pole model and strong interaction dynamics in general[39]. Exchange of particles with high spin requires $\alpha(t) \leq 1$ from the general fact that $\frac{d\sigma}{dt} \sim s^{2\alpha-2}$ for a one pole exchange and the fact that the differential cross-section decreases experimentally with energy. However inspite of all the arguments one has almost a complete functional freedom for the trajectories and an appreciable number of parameters can be introduced into the theory to patch up the t dependence at fixed s. The curve fitter can consider not only additional trajectories but cuts, fixed poles and a certain amount of liberty is introduced on account of evasion and conspiracy[40].

However it cannot be over emphasised that one of the doubtful virtues of the Regge Pole model is its flexibility. At best it only affords a convenient parametrization of the scattering amplitude and provides a convenient framework for the phenomenological discussion of the data. Having no theoretical backing for the relativistic regions it is nothing more than that in spite of its great aesthetical appeal.

CHAPTER II

Attempts to find groups which would englobe both the Poincaré Group $\mathcal{P}$ and Internal Symmetry groups have not been successful at all. The great difficulties faced by such an endeavour have been summarised as follows by Feldman and Matthews: [41, 42]

a) The Michel - O'Raifertaigh Theorem which states that any symmetry group which contains $\mathcal{P} \otimes SU(3)$ demands a momentum four vector with more than four components.[43,44]

b) If we combine the four relativistic spinor indices with those of SU(3) it is found that invariance requires the particle multiplets to be infinite dimensional.[44]

c) If these infinite-dimensional multiplets are degenerate then the requirements of causality forbid Fermi Statistics completely irrespective of spin. [41]

Given this pathological situation it is to be wondered why one should continue to use symmetry groups at all. The answer is that we use them inspite of these difficulties, for we have no alternative approach at ~~all~~ present for tackling the actual dynamical problems of scattering experiments such as were listed at the beginning of Chapter I.

Other approaches such as those of current algebra,[45] in which scheme one postulates the commutation rules which hold when the current are given their simplet forms have been successful on a different plane. The most remarkable success of this approach has been the Adler-Weisberg [46]sum rule which gives the absolute value of the ratio between axial-vector and vector coupling constants of beta decay in terms of pion-nucleon total cross-section. Similarly one may obtain other kind of such useful relationship but no help on the dynamical plane.[47]

2.1 The Quark Model

The other main approach is the Quark Model,[48] the most attractive feature of which is that it predicts in a very direct way relations between certain properties of nucleons and mesons. The basic assumption is one of additivity stating that some particular property of a hadron or a meson is a sum of terms belonging to the quarks and antiquarks composing that particle - all such applications are based on properties of bound quarks and are quite independent of what a quark would be if free of or the mechanism binding them together.[49] Mathematically formulated the additivity principle is as follows:

2.1 For the process $A + B \longrightarrow A' + B'$

$$\langle A'B'|T|AB\rangle = \delta(P'_A + P'_B - P_A - P_B) \quad \langle A'B'|T_{ij}|AB\rangle \tag{2.1}$$

Where the operator T_{ij} acts on the i^{th} quark of A and the j^{th} quark of B but leaves the other quarks unaffected; in this model a further assumption is made:

$$\langle A'B'|T_{ij}|AB\rangle = \langle Q_{A'} Q_{B'}|T_{ij}|Q_A Q_B\rangle f_i^{A'A}(t) f_j^{B'B}(-t) \tag{2.2}$$

Here $\langle Q^i_{A'} Q^j_{B'}|T|Q^i_A Q^j_B\rangle$ describes the collision of quarks in the initial spin and SU(3) states $Q^i_A Q^j_B$ to final spin and states $Q^i_A \; Q^j_B$. It is tacitly assumed that A A' and B B' have the same quarks composition. The form factors $f^{AA'}$ and $f^{BB'}$ represent the overlap between quark wave function (A, A') and (B, B') respectively. As a consequence the elastic scattering of hadrons are given as linear superposition of basic scattering of quarks and antiquarks.

Denoting the matrix element of

$A + B \longrightarrow A + B$ by (AB)

and

$$\langle Q_i Q_j|T_{ij}|Q_i Q_j\rangle = (Q_i Q_j)$$

we have

$$(PP) = 4(Q_p Q_p) + 4(Q_p Q_n) + (Q_n Q_n)$$

$$(K^+P) = 2(Q_p Q_p) + (Q_p Q_n) + 2(Q_\lambda Q_p) + (Q_{\bar{\lambda}} Q_n)$$

$$(\pi^+ P) = 2(Q_p Q_n) + (Q_p Q_n) \tag{2.3}$$

Where the mesons and protons are made up of Quarks and anti-Quarks

$$\pi^+ = Q_p \bar{Q}_n , \quad K^+ = Q_p \bar{Q}\lambda \qquad \text{and so on}$$
$$P = Q_p Q_p Q_n \qquad 2.4$$

From these one then obtains relationships for cross-section in the forward direction which can be compared to experiment by applying the Optical Theorem:

$$\sigma_t (PP) - \sigma_t(NP) = \sigma_t(K^+P) - \sigma_t(K^+N)$$

$$\sigma_t(K^+P) - \sigma_t(K^-P) = \sigma_t(\pi^+P) - \sigma_t(\pi^-P) + \sigma_t(K^+N)-\sigma_t(K^-N) \qquad 2.5$$

In deriving these relationships the quark amplitudes were not assumed invariant under SU(3) or any other group. If we invoke the former we regain the Johnson-Treiman relation:

$$\tfrac{1}{2}[\sigma_t(K^+P) - \sigma_t(K^-P)] = \sigma_t(\pi^+P) - \sigma_t(\pi^-P)$$
$$= \sigma_t(K^+N) - \sigma_t(K^-N) \qquad 2.6$$

Invariance under larger groups, except possibly $SU(6)_W$ - which are discussed later - which classifies meson and baryons in different multiplets do not give such simple relationships. The result of eq. 2.5 and 2.6 agree fairly well with experiment.

At high energies ($\bar{Q}$, Q,) and (Q, Q,) become identical so that

$$\langle AB|T|AB\rangle = ig_{12}(t)f^A(t)f^B(t) \qquad 2.7$$

which looks identical to eq. 1.32 for the Regge Pole Model. The great merit of this model is that it predicts a great degree of universality for high energy difraction of all hadrons, as is suggested by present day measurements and their extrapolations. A more direct and more general consequence of the implications of equations 2.1 and 2.2 for $s \to \infty$ and t small is that the quantum numbers in the t channel are those of the $Q\,\bar{Q}$ system but with positive and negative signatures. These are in fact precisely what we find for meson nucleon reactions where nonets are exchanged. In consequence one may view this model as providing a dynamical basis for the exchange of Regge trajectories; it also accounts for the universality of the trajectories and their relation to particles. The basic ideas of this model can be extended to inelastic high energy scattering at small momentum transfers under certain plausible assumptions for the behaviour of the form factors. Most of the predictions are again in fair agreement with experiment. The weak point is that we have no justification whatsoever for the dynamical assumptions made so long as we know nothing about quark forces[50] which by all indications seem to be characterized by a coupling constant of the order of $f^2_{QQ}/4\pi=26$ which is one order of magnitude above the universal coupling $f^2_{\rho\pi\pi}/4\pi=2.5$;

further it should be remembered that we just do not know how to solve the problem of bound states in which the binding energy is of the order of the mass of the bound particles. And although we shall be assuming throughout this thesis that hadron are made up of Quarks this will be done more because it is a convenient group thereoretical concept than a belief in the model we have just outlined; for the same reason we shall not discuss evidence for or against the existence of quarks.[48]

2.2. The Mixing of Internal and Space - Time Symmetries

U (6) ⊗ U (6) ⊗ O (3) and its subgroup will be the main tool we shall be using;[51] we shall not be regarding it as a strict invariance group but rather use it as a symmetry group for three-point functions adding the dynamical assumption that the unitarity relations for these vertex functions is dominated by given two body intermediate states. In this way it is possible to obtain useful, results albeit in very restricted sense, from these dynamical groups. In Chapter 3 unitarity will be enforced in an ad hoc way by assuming absorption of low order partial wave amplitudes.

We now examine in detail how unitarity and other basic precepts of particle physics like crossing ar violated in the framework of these dynamical group and see to what extent our limited use of their applicability is justified. In doing so we shall use both highly sophisticated mathematical arguments as well as down to earth examples. There is a danger in the latter approach in the sense that too naive an approach, although leading to the correct conclusions, might mask the real issue. For instances it has been claimed that adding angular momentum to the Pauli Spin matrices in the construction of the generators of U(6) lead to overtly manifest ridiculous situations.[52] If we concentrate on the one ~~with~~ involving λ_3 of the Gell - Mann generator of SU(3) we would obtain the following generator of U (6)

$$I_{13} = \left(\vec{\sigma} + i\vec{r}\,\frac{\delta}{\delta r}\right)\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix} \qquad 2.8$$

which is an operator which does not act on the λ quark but acts with opposite sign on p's and n's. Thus the orbital part effects a spatial separation of p's and n's, ~~and to an~~ increasing ~~extent~~ with distance from the origin. If for instance we consider two particles in initial state of a sctattering experiment the effect of such an operation would be to separate then beyond the range of the interaction force; or equally it would separate a bound quark-antiquark system composing a meson without any expenditure of energy. This argument is certainly wrong. For would this operator not act on the 'range' of interaction force too? One cannot use this argument without investigating more thoroughly this effect. On the other hand it can be claimed that this description is not manifestly covariant in the sense that for one observer in the rest system of a bound state the particles would be together whereas for another observer they would be arbitrarily separated. Consequently this set of operators cannot be accepted - for this latter reason only and not for the former one. With this in mind we now proceed to give an account of the rigorous theorem forbidding mixing of the Inhomogenous Lorentz Group (ILG) with any internal symmetry group.[53] They then will be illustrated by simple examples.

In any such symmetry, besides obtaining a dynamical group interaction formalism, one would like to have the possibility of obtaining a mass formula for particles belonging to the same representations. Denote such a symmetry group by G. Then the algebra of G would be the direct sum of an internal symmetry algebra U and that of the $\mathcal{P}$ i.e.

$$G = U \oplus \mathcal{P} \qquad 2.9$$

This leads immediately to the conclusion that the particles belonging to the same irreducible reqpresentation of would have the same mass. As a minimum requirement for mass spliting we would like the elements U_i of U not to commute with the translation elements of the algebra of $\mathbb{P}$, i.e.

$$[U_i, P_\mu] \neq 0 \qquad 2.10$$

That this would indeed give us mas-splitting can be seen as follows.[54] Consider a group which contains SU(3) and time translations as non-commutative subgroups, ignoring homogeneous Lorentz invariance L for the time being. The elements for time-translation can be represented by the Hamiltonian H, which we also assume has the eight baryons as eigenvectors with their observed masses:

$$H|N\rangle = M_N|N\rangle \quad , \quad H|\Lambda\rangle = M|\Lambda\rangle$$
$$H|\Sigma\rangle = M_\Sigma|\Sigma\rangle \quad , \quad H|\Xi\rangle = M_\Xi|\Xi\rangle \qquad 2.11$$

The mass splitting Operator in this scheme would then be

$$L = H + a\,Y + b\,[J^2 - Y^2/4] \qquad 2.12$$

where J is the isospin generator.

Then from

$$[L, \tfrac{1}{2}(\lambda_4 - i\lambda_5)]|\Xi\rangle = 0$$

$$[L, \tfrac{1}{2}(\lambda_6 - i\lambda_7)]|\Lambda\rangle = 0 \qquad 2.13$$

we find that a and be give the Gell-Mann mass formula except that it is an exact relation for this representation. For comparison sake we mention that the usual Mass breaking operator of SU(3) is

$$M_\nu = T^1_\nu + T^8_\nu + T^{27}_\nu + T^{64}_\nu$$

where $\nu = (o,o,o,)$ is the vector labelling the I.R. representations and the T's are scalar operators such that $\Delta I = \Delta I_3 = \Delta Y = 0$; keeping the first two terms only we have

$$\left(\begin{matrix}\mu\\ \nu\end{matrix}\middle| M \middle|\begin{matrix}\mu\\ \nu\end{matrix}\right) = M_o + M_1 Y + M_2[J(J+1) - Y^2/4]$$

and from this we obtain an approximate relation for the masses of the baryons.

So in our scheme we would like the elements of $\mathfrak{U}$ not to commute with the P_μ 's but we would still like them to do so with those of L;[56] this last requirment just means that the quantum numbers associated with unitary symmetry do not change when one performs a homogeneous Lorents transormation. The elements of the algebra $\mathfrak{P}$ obey the following commutation relations:[55]

(56) McGlinn

$$[J_{\mu\nu}, J_{\lambda\sigma}] = i[g_{\nu\lambda} J_{\mu\sigma} + g_{\mu\sigma} J_{\nu\lambda} - g_{\mu\lambda} J_{\nu\sigma} - g_{\nu\sigma} J_{\mu\lambda}] \tag{2.14}$$

$$[P_{\mu}, J_{\alpha\beta}] = [g_{\alpha\mu} P_{\beta} - g_{\mu\beta} P_{\alpha}] \tag{2.15}$$

$$[P_{\mu}, P_{\nu}] = 0 \tag{2.16}$$

$$J_{\mu\alpha} = - J_{\alpha\mu} \tag{2.17}$$

If we denote the elements of U by Ui with i = 1, 2, 3,....n the 10 elements of IHL can be denoted by $Un+_4$ to Un+ 10 with the following identification[56]

$$P_i = U_{n+i} \quad , \quad 1 \leq i \leq 4$$

$$J_{12} = U_{n+5} \quad , \quad J_{13} = U_{n+6} \, , \, J_{14} = U_{n+7}$$

$$J_{23} = U_{n+8} \quad , \quad J_{24} = U_{n+9} \tag{2.18}$$

The algebra of the full symmetry group G will then be determined by

$$[U_J, U_K] = C^{\ell}_{JK} \; U_{\ell} \tag{2.19}$$

where the structure constants satisfy

$$C^{\ell}_{JK} = - C^{\ell}_{KJ} \tag{2.20}$$

$$C^{P}_{IS} \; C^{S}_{JK} + C^{P}_{JS} \; C^{S}_{KI} + C^{P}_{KS} \; C^{S}_{IJ} = 0 \tag{2.21}$$

The requirement that the elements of $\mathcal{H}$ commute with those of U immediately implies

$$C^{I}_{JK} = 0 \qquad \text{for} \quad J \leq N,\ K \geq N+4$$

The restriction $G = U \oplus \mathcal{P}$ implies that for $i \leq n$, $j \geq n+4$ $n+1 \leq K \leq n+4$, $C^{P}_{js} = C^{s}_{iJ} = 0$ so that the Jacobi identity reduces to

$$C^{P}_{IS}\, C^{S}_{JK} = 0$$

Continuing on the same line of thought, using relations 2.14 - 2.16 we arrive at the conclusion that

$$C^{I}_{KJ} = 0 \quad \text{for} \quad K \leq N,\ J \geq N+1 \quad \text{all } I \qquad 2.22$$

But this means that $G = U \oplus \mathcal{P}$, in other words if we demand

$$[U_I\,,\ J_{\mu\nu}] = 0 \quad \text{for all} \quad I \qquad 2.23$$

i.e. commutation of the Unitary Symmetry elements with those of $\mathcal{H}$ we automatically obtain

$$[U_I,\ P_\mu] = 0 \qquad 2.24$$

Then any unitary representation of G will be infinite dimensional, being the product of those of U and $\mathcal{P}$ and we shall have infinite towers of particles with complete mass degeneracy - precisely the sort of thing we wanted to avoid. What happens if we weaken the condition of eq. 2.23 and demand

$$[U_I, \; J_{\mu\nu}] \neq 0 \qquad 2.25$$

except for one value of i and identify this operator, with say, the charge operator? Michel and Sudarshah[57] have shown that McGlinn's theorem still follows. This problem has been tackled in a much more general way by O'Raifeartaigh [44, 58] who does not assume that G is the direct sum of U and $\mathcal{P}$ but rather has investigated the way in which both can be imbedded in a Lie Algebra G assuming only that this is of finite order. Then using Levi's Theorem [59] which states that any Lie algebra G can be written as

$$G = F \oplus S \qquad 2.26$$

where F is a semi-simple subalgebra of G and S is an invariant solvable subalgebra [60] of G he obtains the result that:

either (a) $L \subset F$ and $P \subseteq S$

OR (b) $P \cap S = 0$

It will be recalled that $\mathcal{P}$, $\mathcal{L}$ and P are the algebras of the Inhomogeneous, the Homogeneous and translation of the Poincare group respectively. The relevance of this theorem is that it tells us how we can classify the ways in which $\mathcal{P}$ can be imbedded in the larger Algebra G. This classification be made by dividing the case (a) into the following three cases

(1) $S = P$ (ii) $P \subset S$ where S is abelian (iii) $P \subset S$ but S not abelian

Case (b) may be reduced to (iv) $P \cap S$

O'Raifeartaigh then proceeds to show that case (i), up to a redifinition leads to $G = \mathcal{P} \oplus U$ which is just the McGlinn Theorem; case (ii) cannot be reduced to a direct sum in this way but has the disadvantage of introducing a translation algebra of more than four dimensions - this point will be discussed later on as it has some similarities with U(6,6) or $U(6) \otimes U(6)$; case (iii) is unphysical as physicists do not know how to interpret non-Abelian algebras - case (iv) is equivalent to imbedding P in a simple algebra. It would seem that of all the ways in which we can imbed $\mathcal{P}$ and U in G, the direct sum one is the only physically attractive one with consequences already discussed. The case of G being of infinite order has been discussed by Jordan [61] but the final conclusions are not so bright either. His argument is further continued to show how forlorn is our goal of obtaining a scheme which will describe mass-splitting. Without attempting to define exactly what is a multiplet at least one can ask such an object to have the following characteristics:

(a) particles belonging to the same physical 'grouping' should be represented by state vectors belonging to the same I.R. of the gobal algebra G

(b) each such state vector should be an eigenvector of $P_\mu P^\mu$ and

(c) $P_\mu P^\mu$ being an observable should be Hermitian. Then the following theorem precludes any mass-splitting among particles belonging to the same-multiplet.

Let $G \supset P$ and let $\mathcal{H}$ be a Hilbert Space on which the irreducible representations of G operate. If on $\mathcal{H}$, $P^2 = P_\mu P^\mu$ has a discrete eigenvalue m^2 and P^2 is Harmitian then the eigenspace $\mathcal{H}_m$ belonging to m^2 of P is closed. As H is irreducible this means $\mathcal{H} = \mathcal{H}_m$. The mass operator then has either a continuous spectrum or a spectrum consisting of one point. A continuous spectrum cannot constitute a multiplet and if we do have a multiplet then it has only one mass.

In order to circumvent these formidable obstacles Salam et al, Paris and W. Ruhl introduced the auxiliary Group approach.[62] In this scheme the Lagrangians one writes down are 'index invariant' in the sense of SU(2) or SU(3) (for example) In the former formalism an Isopin invariant vertex for NNM can be written as follows:

Starting with

$$N_a = \begin{pmatrix} P \\ n \end{pmatrix} \qquad \bar{N}^a = (\bar{P}, \bar{n}) \qquad 2.27.$$

which transform under I sping r satations through $\vec{\theta}$

as $N_a \longrightarrow (e^{i\vec{\tau}/2.\vec{\theta}})^{a'}_a N_{a'}$, $\bar{N}^a = \bar{N}^{a'}(e^{-i\vec{\tau}/2.\vec{\theta}})^a_{a'}$

and

$$\pi^b_a = \begin{pmatrix} \pi^o/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^o/\sqrt{2} \end{pmatrix} \qquad 2.28$$

which transorm as

$$\pi^b_a \longrightarrow (e^{-i\vec{\tau}/2.\vec{\theta}})^{a'}_a \ \pi^{b'}_{a'} \ (e^{i\vec{\tau}/2.\vec{\theta}})^b_{b'} \qquad 2.29$$

we can write the I spin invariant vertex as

$$L = g_{NN\pi} \ \bar{N}^a \ \pi^b_a \ N_b = g_{NN\pi}[(\bar{P}P-\bar{n}n)\pi^o/\sqrt{2}+\bar{P}n\pi^+ + \bar{n}P\pi^-] \qquad 2.30$$

Such a Lagrangian is then invariant under rotation in the Isospin space as the exponential factors of such a transormation cancel out in pairs. The difficulty with Poincaré invariance, on the other hand, is that the transformations depend explicitly on the momenta of the particles at the vertex and we do not have simple cancellation. We cannot construct, consequently, scalar invariants by a naive saturation of indices as is done for Unitary Symmetries. How this arises and how one usually goes round this can be as follows.

By means of a Lorentz boost L one obtains a state with four-vector P^{μ} from one with four-vector K^{μ} which are defined as follows:

$$K^{\mu} = (0,\ 0,\ 0,\ m) \quad ; \quad P^{\mu} = (\underline{P},\ \omega = (\underline{P}+m^2)^{1/2}) \tag{2.31}$$

$$L^{\mu}_{\nu}\ (\underline{P})\ K^{\nu} = P^{\mu} \tag{2.32}$$

$$L\ (\underline{P}) = R(\underline{P})\ B(\ \underline{P}\ )\ R^{-1}\ (\underline{P}) \tag{2.33}$$

B is the boost from rest to momentum p in the z direction and is given by

$$B^{\mu}_{\nu}\ (\underline{P}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \text{Cosh}\ \Theta & \text{Sinh}\ \Theta \\ 0 & 0 & \text{Sinh}\ \Theta & \text{Cosh}\ \Theta \end{pmatrix} \tag{2.34}$$

$$\text{Sin h}\ \Theta = \underline{P}\ /\ m \quad \text{and} \quad \text{Cos h}\ \Theta = P_o/\ m \tag{2.35}$$

R(P) is the rotation about a vector in the x-y plane normal to P which carries the Z axis into the direction P. We then have for state vector transformations

$$|\ P^{\mu}\ \sigma > \ = [\ m/\omega\ (\underline{P})\ ]^{\frac{1}{2}}\ \ U\ [\ L\ (\underline{P})\ ]\ |\ K^{\mu}\ \sigma > \tag{2.36}$$

where $\sigma = (J\ ,\ J_3)$ is the spin label.

The normalisation factor being chosen to give

$$[\ \omega(\underline{P})\ \omega(P')\ ]^{1/2}\ \langle P'\ \sigma'\ |\ P\ \sigma\ \rangle = \delta_{\sigma'\sigma}\ \delta^3(\underline{P} - \underline{P}')\ \omega(P) \qquad 2.37$$

and therefore the following orthogonality relation

$$\langle\ P\ \sigma\ |\ P\ \sigma\ \rangle \ = \ \delta_{\sigma'\ \sigma}\ \ \delta^3\ (\underline{P} - \underline{P}') \qquad 2.38$$

The set of states $\{\ |P, \sigma\rangle\}$ constitute the mass m spin J representation of ILG. Amongst themselves they transform as follows:

$$\begin{aligned} U[\Lambda]\ |P\sigma\rangle &= [\ m/\omega(\underline{P})]^{1/2}\ U[\Lambda\, L(\underline{P})]\ |K\sigma\rangle \\ &= [\ m/\omega(\underline{P})]^{1/2}\ U[L(\Lambda\,\underline{P})]U[L^{-1}(\Lambda\underline{P})\ L(\underline{P})]/K\sigma\rangle \\ &= [\omega(\ \underline{P})/\omega(\underline{P})]^{1/2} \sum_{\sigma'} D_{\sigma'\sigma}[L^{-1}(\Lambda\underline{P})\ L(\underline{P})]\ |\underline{P}\sigma'\rangle \end{aligned} \qquad 2.39$$

$L^{-1}(\Lambda P)\Lambda L(P)$ is the generator of 'Wigner Rotation' which takes K^μ of e.g. 2.31 via P^μ and $(\Lambda P)^\mu$ back to K^μ
The analogue of eq. 2.39 for ordinary rotations is angular momentum or Isopin space is

$$U\ [R]\ |J\ J_3\ \rangle \ = \sum_{J_3'}\ D^J\ [R]\ |J\ J_3'\ \rangle \qquad 2.40$$

[We have not considered translation in the above as this can be easily done by multiplying $|P\sigma\rangle$ by a phase factor $\exp(-iP.a)$ corresponding to $x^\mu \longrightarrow X^\mu + a^\mu$]. ~~Remark~~ Note however that

The Wigner roation depends not only on the parameters $W_{\mu\nu}$ which specify the general Lorentz transformations:

$$L^{\mu}_{\nu}(\underline{P}) = \delta^{\mu}_{\nu} + \omega^{\mu}_{\nu}(\underline{P}) \quad \text{with} \quad \omega_{\mu\nu} = g_{\mu\alpha}\,\omega^{\alpha}_{\mu} \qquad 2.41$$

$$U[\Lambda,a] = \exp\ i(\tfrac{1}{2}\,\omega^{\mu\alpha}\,J_{\mu\alpha} - a^{\mu}\,P_{\mu})$$

$$\text{and} \quad \omega_{\mu\alpha} = -\omega_{\alpha\mu} \qquad 2.42$$

but also on the momentum value P of the operator being transformed. The invariants of Quantum Field theory are usually written in terms of the creation and annihilation operators [41]

$$a^{+}(P\,\sigma)|\,\rangle_{o} = |\,P\,\sigma\,\rangle \quad \text{and} \quad a\,(P\,\sigma)|\,\rangle_{o} = o\,. \qquad 2.43$$

where $|\,\rangle_o$ is the vaccum state. Under a pure Lorentz transformation which takes P^{μ} to $(\Lambda P)^{\mu}$ the creation operator transforms according to

$$U[\Lambda]\,a^{+}(P\sigma)U^{-1}[\Lambda] = \sqrt{\frac{\omega(\Lambda P)}{\omega(P)}}\sum_{\sigma'} D_{\sigma'\sigma}[L^{-1}(\Lambda\underline{P})\Lambda L(\underline{P})]\;a^{+}(\Lambda P,\sigma) \qquad 2.44$$

And again the Wigner rotation depends on the momenta.

In order to construct invariants in the usual way one needs to decouple the spinor index from the momentum. This is done by means of an auxiliary group which contains the homogeneous Lorentz group as a subgroup. As this latter group will play a role in our subsequent discussion we state a few facts about its representations. The irreducible representations are specified by two numbers K_o, C

where

$$\frac{1}{2} J^{\mu\alpha} J_{\mu\alpha} | K_o\, C > \;=\; (J^2 - K^2) | K_o\, C > \;=\; (K_o^2 + C^2 - 1) | K_o\, C >$$

and

$$\frac{1}{8}\, \epsilon_{\mu\beta\alpha\rho} J^{\mu\beta} J^{\alpha\rho} | K_o\, C > \;=\; J.K | K_o\, C > \;=\; i\, K_o\, C | K_o\, C > \qquad 2.45$$

2.46

From these it is also seen that (-K.,-c) also specify an equivalent representation, although by convention we restrict ourselves to representations having $K_o \geq 0$. For both finite and infinite representations

$$K_o = 0, \frac{1}{2}, 1, \frac{2}{3}, \ldots\ldots\ldots \qquad 2.47$$

The component of a representation are labelled by $\sigma = J, J_3$ which can be either integer or half integer satisfying

$$K_o \leq J$$
$$-J \leq J_3 \leq J \qquad 2.48$$

We then have two classes of representations

(1$^{\underline{o}}$) <u>Finite dimensional Non-Unitary Representations</u>[55]

$$|C| = Ko + n \qquad 2.49$$

where n is a positive integer and J has the finite range

$$K_o \leq J \leq |c| - 1 \qquad 2.50$$

(2$^{\underline{o}}$) <u>Infinite Dimensional representations</u>

In this case either

c is pure imaginary

and $K_o = 0, \frac{1}{2}, \frac{3}{2}$ 2.51

or $K_o = o$, C real and within the range
$0 \leq C \leq 1$ 2.52

In both of these cases J has no upper limit and runs over an infinite range of integer or half-integer values.

For both cases the representations are specified by

$$|K_o \ C > \qquad 2.53$$

and individual components by

$$|K_o \ C \ ; \ J \ J_3 > \qquad 2.54$$

The Parity Operator R satisfies the following set of relations with generators of linear transformation of both of the above representation spaces

$$[R, P_i]_+ = 0 \quad 2.55$$

$$[R, P_o]_- = 0 \quad 2.56$$

$$[R, K_i] = 0 \quad 2.57$$

$$[R, J_i] = 0 \quad 2.58$$

$$R^2 = 1 \quad 2.59$$

And for both unitary and non-unitary representations

$$R|K_o\ C\ ;\ J\ J_3 > = \pm\ |\ K_o - C\ ;\ J\ J_3 > \quad 2.60$$

Where

$$K_i = J_{oi} = - J_{io} \quad 2.61$$

and

$$J_i = \epsilon_{ijk}\ J_{ik} \quad 2.62$$

for the generators to $\wp$ defined by eqs. 2.14–2.17

The symbols $[\,]_+$ and $[\,]_-$ denote anti-commutations and commutation relations respectively.

The generators of the Wigner rotation form a set of unitary operators as their net effect is to transform $|K^\mu \sigma\rangle$ = $|0, 0, 0, m\rangle$ = $|M, \sigma\rangle$ among themselves – in fact they constitute the little group spin rotation set. This group of transformation involve the generators K_i of pure Lorentz transformations or boosts. Whatever auxiliary group we consider must include these generators. We denote the representation states for this group - to be specified as we go on - by

By definition $\langle \alpha|\beta\rangle = \delta_{\alpha\beta}$ and $|\alpha\rangle\langle\alpha| = 1$

and

$$\langle m\sigma| U[L^{-1}(\ P)\ L(P)] |m\sigma'\rangle = D_{\sigma'\sigma}[L^{-1}(\Lambda P)\ L(P)]$$

$$= \langle m\sigma\ e^{-i\epsilon'.(\Lambda P)_e i\ -P_e i\ P}|m\sigma'\rangle$$

$$= \langle m\sigma\ e^{-i\epsilon'.(\Lambda P)}|\alpha\rangle\langle\alpha|e^{i\eta.k}|\beta\rangle \langle\beta|e^{i\ .P}|\gamma\rangle\ \langle\gamma|m\sigma'\rangle \qquad 2.63$$

The auxiliary Operator is defined as

$$\Lambda_\alpha\ (P) = \langle\alpha|e^{-i\epsilon.P}|\beta\rangle\ \langle\beta|m\sigma\rangle\ a(P\sigma)$$

$$= U_\alpha(P)\ a(P,\sigma) \qquad 2.64$$

The role of the 'Generalised Spinor' appearing in eq. 2.64 can be easily seen if we take the auxiliary group to be the Homogeneous Lorentz group or SL(2,C) and we take $|\alpha\rangle$ to be either the dotted or undotted representations of HL which in the notation of eq. 2.53 can be labelled

$$|K_o\ C\rangle = |\tfrac{1}{2},\ \pm\ \tfrac{3}{2}\rangle \qquad 2.65$$

The components $|\alpha\rangle$ can be specified by a label taking values $\pm\frac{1}{2}$ and the relations between these and the physical state of the representations of the little group allows us to write

$$\langle\alpha|m\ \sigma\rangle = \delta^{\alpha}_{\sigma} \quad \text{with } \sigma,\ \alpha = 1,2 \qquad 2.66$$

If we want to include eigenvalues of R, the parity operator in all this, we have to take a combination of dotted and undotted spinors combined. For J = 1/2 we recuperate the usual Dirac Spinors and the above relation becomes

$$< \alpha | M \sigma > = \delta^{\alpha}_{\sigma} \quad \text{with} \quad \alpha, \sigma = 1, \ldots\ldots, 4 \qquad 2.67$$

It should be noted at this point that as we are dealing with finite (non unitary) dimensional representations of HL, $< \alpha | K_i | \beta >$ is not Hermitian. In fact it is anti-Hermitian as can be demostrated as follows:

For J = 1/2 and the irreducible representations denoted by ($\frac{1}{2}$, 0) and (0, $\frac{1}{2}$) which correspond to the dotted and undotted case we have

$$< \alpha | K_i | \beta > = - \frac{i}{2} (\sigma_i)^{\beta}_{\alpha} \qquad \alpha, \beta = 1,2 \qquad 2.68$$

$$< \dot{\alpha} | K_i | \dot{\beta} > = \frac{i}{2} (\sigma_i)^{\dot{\alpha}}_{\dot{\beta}} \qquad \dot{\alpha}, \dot{\beta} = 1,2 \qquad 2.69$$

Also for Dirac reducible representation ($\frac{1}{2}$, o) ⊕ (0, $\frac{1}{2}$) we have

$$< \alpha | K_i | \beta > = \frac{1}{2} (\sigma_{oi})^{\beta}_{\alpha} \qquad \alpha, \beta = 1, \ldots, 4 \qquad 2.70$$

where

$$\sigma_{\mu\nu} = \frac{i}{2} [\gamma_{\mu}, \gamma_{\nu}] \qquad 2.71$$

All three are manifestly anti-Hermitian. For Unitary infinite dimensional irreducible representations of LG the corresponding $\langle \alpha | K_i | \beta \rangle$ are Hermitian.

From eq. 2.70 we see that

$$\langle \alpha | e^{-i \cdot k} | \beta \rangle = [\exp(-\frac{i\epsilon_i \sigma_{oi}}{2})]^{\beta}_{\alpha}$$
$$= [\cosh \frac{|\theta|}{2} - i P_i \sigma_{oi} \sinh \frac{|\theta|}{2}]^{\beta}_{\alpha}$$
$$= [\frac{P_{o+m}}{2}]^{1/2} [1 - \frac{i \sigma_{oi} P_i}{P_o + m}]^{\beta}_{\alpha} \qquad 2.72$$

And so for the Spinor we have

$$U_\alpha(P) = [\exp(-\frac{i\epsilon_i \sigma_{oi}}{2})]^{\delta}_{\beta} \langle \delta | m \sigma \rangle$$
$$= [(\frac{P_o + m}{m})]^{1/2} [1 - \frac{i\sigma_{oi} P_i}{P_{o+m}}]^{\delta}_{\alpha} \langle \delta | m\sigma \rangle \qquad 2.73$$

And under a Lorentz transformation, the auxiliary operators become

$$U[\,]A_\alpha(P)U^{-1}[\,] = \langle \alpha | e^{-i\epsilon' P} | \beta \rangle \langle \beta | m\sigma \rangle U[\Lambda] a(P\sigma) U^{-1}[\Lambda]$$
$$= \langle \alpha | e^{-i\epsilon' P} | \beta \rangle \langle \beta | m\sigma \rangle [\frac{\omega(P)}{\omega(P)}]^{\frac{1}{2}} \sum D_{\sigma'\sigma}[L^{-1}(\Lambda P) \times L(P)] a(P\sigma')$$
$$= \langle \alpha | e^{-i\epsilon' P} | \beta \rangle \langle \beta | m\sigma \rangle \langle m\sigma | U[L^{-1}(\Lambda P)\Lambda L(P)] | m\sigma' \rangle \times a(P\sigma')$$
$$= \langle \alpha | e^{-i\epsilon' P} | \beta \rangle \langle \beta | U[L^{-1}(\Lambda P)\Lambda L(P)] | \gamma \rangle \langle \gamma | m\sigma' \rangle$$
$$= \langle \alpha | e^{-i\epsilon' P} | \beta \rangle A_\beta(P)$$
$$= S^{\beta}_{\alpha} A_\beta(P) \qquad 2.74$$

In eq. 2.74 the Spinor indices have replaced $\sigma = (J, J_3)$, the spin labels.

We see then that the spinor ~~label~~ operator A_α undergoes a pure matrix transformation under Lorentz transformation, independent of the momentum p — hence the reason for introducing the auxiliary group.

The duel operator to $A_\alpha \quad (P)$ is

$$A^{\alpha}(P) = a^{+}(P\sigma) \langle m\sigma|\beta\rangle \langle\beta|e^{i \quad P}|\alpha\rangle \qquad 2.75$$

and

$$U\,[\Lambda]\, A^{\alpha}(P)\, U^{-1}\,[\Lambda] = A^{\beta}(\Lambda P)\, (S^{-1})^{\alpha}_{\beta} \qquad 2.76$$

so that $A^{\alpha}(P) \quad A_{\alpha}(P)$ is a scaler, just as for Unitary Symmetries.

For unitary representations $|\alpha\rangle$

$$A^{\alpha}(P) = A^{+}_{\alpha} \quad (P) \qquad 2.78$$

but for non-unitary $|\alpha\rangle$ the relation between A^{α} and A_{α} depends on the particular representations. For the Dirac $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ representation

$$A^{\alpha}(P) = (A_{\beta}(P)\,)^{+}\, (\gamma_{o})^{\alpha}_{\beta} \qquad 2.79$$

and, as we shall see later, in general we need to define a matrix such that

$$A^{\alpha}(P) = (A_{\beta}(P))^{+} \Gamma^{\alpha}_{\beta} \qquad 2.80$$

We also need the following auxiliary operator for the construction of Poincaré invariants:

$$\tilde{B}_{\alpha}(P) = \langle\alpha|e^{-i\epsilon P}|\beta\rangle \langle\beta\, m\sigma\rangle\, b^{+}(P,\sigma) \qquad 2.81$$

$$= (\overline{V}_{\alpha}(P))^{\sigma}\, b^{+}(P,\sigma)$$

where

$$\overline{V}_{\alpha}(P,\sigma) = [\exp(-\frac{i\,\sigma_{0i}\epsilon_{i}}{2})]^{\delta}_{\alpha} \langle\delta|m\sigma\rangle \qquad 2.82$$

$\tilde{B}_{\alpha}(P)$ is a creation operator which transforms like Eq.2.76 under Lorentz transformations. The matrix B of eq.2.81 is defined by the following relationship:

$$\langle m_{\sigma}|U[L^{-1}(\Lambda P)\Lambda L(P)]|m\sigma'\rangle == \langle m\sigma'|B^{-1}U[\quad]B|m\sigma\rangle \qquad 2.83$$

(see footnote 65)

From which we can construct the usual fields

$$\Psi_{\alpha}(x) = (\tfrac{1}{2\pi})^{4}\int(\lambda A_{\alpha}(P)e^{-iPx} + \mu\, B_{\alpha}(P)e^{iPx})\Theta(P_{o})\delta(P^{2}-m^{2})\, d^{4}P \qquad 2.84$$

Weinberg[63] has shown that for particles of spin j we must use the representations (j, o) or (o, j) for the $\alpha\rangle$, in order to obtain the right connection between spin and statistics. That is:

for j an integer and $\lambda = \mu$

we have

$$[\psi_\alpha(x) \ , \psi^+_\beta (y)]_- = 0 \qquad (x-y)^2 < 0 \qquad 2.85$$

and for j one half an odd integer

$$[\psi_\alpha(x) \ , \psi^\dagger_\beta(y)]_+ = 0 \qquad (x-y)^2 < 0 \qquad 2.86$$

He further demonstrates that these causality relations lead to crossing symmetry and CTP invariance.

2.3 The Auxiliary Group Approach

The approach of Salam in his original paper was to exploit the fact that Lorentz transformations on the suffix α ~~is~~ are independent of momentum and and so to extend the auxiliary group to contain internal symmetry groups and $\mathcal{P}$ as subgroups. For his global symmetry group he took U (2,2) ⊗ SU (3) or U (6,6) also commonly called $\tilde{U}(12)$. The Quark fields in this formalism are $\psi_A = \psi_{\alpha a}$ with a = 1,2, 3 and α = 0,. 3 and transform as

$$\delta\psi_{\beta a} = i(\epsilon^j + \epsilon_5^j\gamma_5 + \epsilon_\mu^j\gamma_\mu + i\epsilon_{\mu 5}\gamma_\mu\gamma_5 + \tfrac{1}{2}\epsilon^j_{\mu\nu}\sigma_{\mu\nu})^\beta_\alpha (T)^b_a\psi_{\beta b} \qquad 2.87$$

As can be seen from this equation the Unitary Symmetry indices play a fairly straighforward role so we can ignore it in the following. The auxiliary group now is U(2,2)[64]. It leaves $Z_1^2 + Z_2^2 - Z_3^2 - Z_4^2 \in \mathbb{C}$* invariant. It will therefore also leave $\bar{\psi}^+ \gamma_0 \psi$, where ψ are the usual Dirac Spinirs, invariant. The infinitesimal generators of this group are F^r, r = 1, ... 16 and in its basic representation they are 4 x 4 matrices which are such that

$$\langle\alpha|F^r|\beta\rangle = \frac{1}{2}(\Gamma^r)^{\beta}_{\alpha} \tag{2.88}$$

where

$$\Gamma^r = 1, \quad i\gamma_5, \quad \gamma_\mu, \quad \gamma_5\gamma_\mu, \quad \sigma_{\mu\nu} \tag{2.89}$$

$$\gamma_5 = \frac{1}{4!} \epsilon^{\mu\beta\rho\sigma} \gamma_\mu \gamma_\beta \gamma_\rho \gamma_\sigma \tag{2.90}$$

In order to ensure that the relations between a Hermitian conjugate operator and its dual for U(2,2) are of the type given by eqs. 2.78, 2.79, these Γ's satisfy:

$$\gamma_0 (\Gamma^r)^+ \gamma_0 = \Gamma^r \tag{2.91}$$

* $\mathbb{C}$ = The complex field of Numbers

Just as before we can introduce the relevant auxiliary operators:

$$A_\alpha(P) = \langle\alpha|e^{i\epsilon P}|\beta\rangle \langle\beta|m\sigma\rangle \quad a(P,\sigma) \qquad 2.92$$

where now $|\alpha\rangle$ is a representation of U(2,2); as this latter group contains the Lorentz group LG as a subgroup under a pure Lorentz transformations we have

$$U[\Lambda]\, A_\alpha(P)\, U^{-1}[\Lambda] = \langle\alpha|e^{i\eta P}|\beta\rangle\, A_\beta(P)$$

$$= S_\alpha^\beta \quad A_\beta(P) \qquad 2.93$$

But under a general index transformation of U(2,2) we have:

$$A_\alpha(P) \longrightarrow \langle\alpha|e^{i\eta_r F^r}|\beta\rangle \quad A_\beta(P) \qquad 2.94$$

$$= T_\alpha^\beta \quad A_\beta(P)$$

The dual of A_α (P) is A^α (P) and this transforms as

$$A^\alpha(P) \longrightarrow A^\beta(\Lambda P)\ (T^{-1})_\beta^\alpha \qquad 2.95$$

Actually as in the basic representation $(A_\beta^*)\ (\gamma_o)_\beta^\alpha$ transforms as the dual, it is this operatos which will be used henceforth for construction of invariants. Auxiliary operators representing higher spin particles can be constructed from the product of these basic fields. Such a field might for example be $\phi_{\alpha}^{\beta}(x)$, constructed from products of $A_{\alpha}(P)$, $A^{\beta}(P)$ and will be given in terms of

$$A_{\alpha}^{\beta}(P) = (U(P)^{s})_{\alpha}^{\beta}\ a(P,s) \qquad 2.96$$

where

$$(U(P)^{s})_{\alpha}^{\beta} = \langle\alpha\beta|\,e^{-i\ k}\,|\alpha'\beta'\rangle\ \langle\alpha'\beta'|\,m^{s}\rangle$$

$$= (\exp\{-i\ {}_{i}\sigma_{oi}/2)_{\alpha}^{\alpha'}\ \langle\alpha'\beta'|\,ms\rangle\ (\exp(i\ {}_{i}\sigma_{oi}/2)_{\beta'}^{\beta} \qquad 2.97$$

From these we can define the field $\phi_{\alpha}^{\beta}(x)$ as

$$\phi_{\alpha}^{\beta}(x) = \left(\frac{1}{2\pi}\right)^{4}\int (A_{\alpha}^{B}(P)e^{-iPx} + \tilde{B}_{\alpha}^{\beta}(P)\ e^{iPx})\theta\ (P_{o})\delta(P^{2}-m^{2})d^{4}P \qquad 2.98$$

This procedure may be continued to still higher spin fields such as $\psi_{\alpha\beta}^{\delta}$ and its dual $\psi_{\gamma}^{\alpha\beta}$. These, under eq. 2.94, transforms as

$$\psi_{\alpha\beta}^{\delta}(P) \longrightarrow T_{\alpha}^{\alpha'}\ T_{\beta}^{\beta'}\psi_{\alpha'\beta'}^{\delta'}\ (T^{-1})_{\delta'}^{\delta} \qquad 2.99$$

and

$$\psi_{\delta}^{\alpha\beta}(P) \longrightarrow (T^{-1})_{\alpha'}^{\alpha}\ (T^{-1})_{\beta'}^{\beta}\ \psi_{\delta'}^{\alpha'\beta'}(T)_{\delta}^{\delta'} \qquad 2.100$$

It may further be shown that these fields obey

Bargmann-Wigner[66] equations

$$\phi^{\rho}_{\alpha}\,(i\partial\!\!\!/+m)^{\beta}_{\rho} = 0 \qquad 2.101$$

$$(i\partial\!\!\!/ + m)^{\beta'}_{\beta}\,\psi_{\alpha\beta'\gamma} = 0 \qquad 2.102$$

In this original paper Salam[62] did not consder the whole set of generators of U(2,2) but took $\epsilon_5 = \epsilon_\mu = \epsilon_{\mu 5} = 0$ in eq.2.87. For this subgroup of U(2,2) one can then define and anti-symmetric matrix $(C^{-1})^{\alpha\beta}$ such that $(C^{-1})\psi^T$ transforms as $\psi^+\gamma_0$. Then $(C^{-1})\psi^T\psi$ is an invariant just like $\bar{\psi}\psi$. [The ψ's are Dirac Field here].

Furthermore

$$(\Gamma_R C) = W\,(\Gamma_R C)^T \qquad 2.103$$

where $W = +1$ for $\Gamma = \gamma_\mu, \sigma_{\mu\nu}$ 2.104

$= -1$ for $\Gamma = \gamma_5,\ \gamma_5\gamma_\mu,\ 1$ 2.105

This matrix $(C^{-1})^{\alpha\beta} = -(C^{-1})^{\beta\alpha}$ plays the role of a metric for this subgroup of U(2,2) and is extensively used for the construction of high-rank spinors. For example a second rank symmetric spinor must have the form

$$\phi_{\alpha\beta} = [\,(\gamma_\mu C)\,\phi_\mu + \tfrac{1}{2}(\sigma_{\mu\nu}C)\,\phi_{\mu\nu}\,]_{\alpha\beta} \qquad 2.106$$

and likewise the general antisymmetric spinor has the form

$$\phi_{[\alpha\beta]_-} = [(C\phi) + (\gamma_5 C)\, \phi_5 + i(\gamma_\mu \gamma_\sigma)\, \phi_{\mu 5}]_{\alpha\beta} \qquad 2.107$$

and a fully symmetric spinor of rank 3 has the form

$$\phi_{\alpha\beta\gamma} = \phi_{\alpha\mu} (\gamma_\mu C)_{\beta\gamma} + \frac{1}{2} \psi_{\alpha\mu\nu} (\sigma_\mu C)_{\beta\gamma} \qquad 2.108$$

with ϕ and $\psi_{\alpha\mu\nu}$ obeying certain subsidiary conditions. We shall be using such relationships extensively later on. From these fields then one can construct invariant Lagrangian by index saturation; for instance in Salam's approach the Baryon - Baryon - Meson vertex is given by

$$L = \bar{\psi}^{A'BC} (\gamma_R T^J)^A_{A'} \psi_{ABC} \, \phi^J_R \qquad 2.109$$

which is invariant under index transformation just as the Lagrangian representing the charge independence of very high energy hadron collisions is invariant.

This procedure can be also extended to unitary representations, i.e. to infinite dimensional ones.

We now investigate under what circumstances this requirement of index invariance is consistent with the unitary of the S Matrix which states that

$$\sum |\langle f|S|i\rangle|^2 = 1 \qquad 2.110$$

this may be expressed in terms of the transition matrix T as follows for states characterised by initial p and final p'

$$[1 - i(2\pi)^4\delta^4(P - P')\,T]\,[1+i(2\pi)^4\delta^4(P-P')T^+] = 1 \qquad 2.111$$

2.111

which gives

$$(2\pi)^4\delta^4(P - P')\,(2\pi)^4\delta^4\,(P-P_n)\langle P|T^+|n\rangle\,\langle n|T|P'\rangle \qquad 2.112$$
$$= i\,(2\pi)^4\delta^4(P-P')\,\langle P|T - T^+|P'\rangle$$

where summation over n is understood.

As usual the states $|n\rangle$ can be expressed as an outer product of Fock creation operators on the vacuum and consequently on the L.H.S. it is sufficient to consider the contribution of a single particle in the sum. If T is index invariant the whole expression on L.H.S must be invariant. This means that single particle contributions of the form

$$\sum t^{\alpha}\,U_{\alpha}(P)^{s}\,(U_{\beta}(P)^{s})^{+}\,(\Gamma^{+})^{\delta}_{\beta}\,t_{\delta} \qquad 2.113$$

must be invariant.

See eq. (2-95, 2.91) for meaning of ($U^{+}_{\beta}(P)^{s}\Gamma^{+}$) ; we have lumped together all operators in t. Under an index transformation only these operators are affected and this term goes, formally into

$$\sum_{P,s} t^{-}\lambda_s^{-1}\, U\, U^{+} \Gamma^{+}\, st \qquad 2.114$$

Which means that we must have

$$\sum U\, U^{+} \Gamma = 1 \qquad 2.115$$

In virtue of eqs. 2.73 this may be written as

$$\langle\alpha|e^{-i\epsilon k}|\beta\rangle\langle\beta|M\sigma\rangle \langle M\sigma|\gamma\rangle \langle\gamma|\, e^{i\epsilon k^{+}}|\delta\rangle \langle\delta|\Gamma^{+}|\lambda\rangle \qquad 2.116$$

$$= \delta_{\alpha}^{\lambda} \qquad 2.116$$

We have two cases: Unitary and Non-Unitary representations of U (2, 2)

(a) For non-unitary representations we see from eq. that

$$\langle\alpha|e^{-i\epsilon k}|\beta\rangle = - \langle\alpha|e^{-i\epsilon k}|\beta\rangle^{+} \qquad 2.117$$

which may alternatively be expressed as

$$K = - K^{+} \qquad 2.118$$

If we write

$$H\ (P) = e^{-i\epsilon k} = H^{+} \qquad 2.119$$

eq. 2.116 may be written in operator form as

$$H\ (P)\ \ O\ H^{+}(P)^{+} = 1 \qquad 2.120$$

Where $O = |M\sigma\rangle\langle\sigma M|$, is projection operator 2.121

and $O^2 = 1$ 2.122

Using these last relationships we then get that

$$\Gamma^{+} = [\, H\ (P)\ ]^{-2} \qquad 2.123$$

for all p, which is impossible.

This treatment may be made more trassparent by the following two down-to-earth examples.

Any Lagrangan of the form

$$L = \overline{\psi}(\gamma_{\mu}\ \delta_{\mu} + M)\psi + L_{int} \qquad 2.124$$

with equal time commutation relations

$$[\psi_{\alpha}(x)\ ,\psi^{+}_{\beta}(y)\,] = \delta\ (\vec{x} - \vec{y}) \qquad 2.125$$

need to be invariant under say U (4) which is a subgroup of U (2, 2).

The commutation relations are invariant under transformations with unitary matrices

$$\psi'\ (x) = U\,\psi\,(x)\ \ ;\ \psi^{+} = \psi^{+}\ U^{+} \qquad 2.126$$

but in order that such a group be a symmentry of the group the quadratic part of the Lagrangian should also be invariant; this means that

$$U^{+}\,\gamma_{o}U = \gamma_{o} \quad \text{and} \quad U^{+}\,\gamma_{i}U = \gamma_{i} \qquad 2.127$$

with $\quad U^{+} = U^{-1}$

this may be written

$$[\ U,\ \gamma_{o}] = [U,\ \gamma_{i}] = 0 \qquad 2.128$$

There is only a non-trivial non-vanishing solution unless P=0

$$U = 1 \qquad 2.129$$

Suppose we start our quest for a relativistic theory with the representations of the U6) group of spin invariance. As this is a non-relativistic formalism the quark wave functions are specified by momenta $\vec{K} \geq 0$ and a spinor ψ_A where $A = a\alpha$ and a represents the unitary symmetry indices with values 1, 2, 3 whereas α is the spin indices with value 1, 2. All Dirac states can be obtained by boosting up particles at rest, in other words by boosting up the spinors of the form

$$\begin{pmatrix} \phi \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ \chi \end{pmatrix} \qquad 2.130$$

There ϕ and χ are two-spinors. Once we do this we can ask under what conditions the S matrix elements be spin independent in the ordinary sense i.e. be invariant under U(6) the spin part of

$$\phi \longrightarrow \phi' = (1 + i\,\epsilon.\sigma)\,\phi \qquad 2.131$$

where $\vec{\sigma}$ are the Pauli spin matrices. That such a scheme is not Lorentz invariant can be immediately seen as we start by boosting two-spinors ~~of say~~ at rest in a laboratory frame and this makes the laboratory a preferred system. (see Eq 2.129) In any case the futility of such a scheme may be more clearly demostrated by writing the transition amplitude for any process in this scheme. If for instance we choose π N scattering then from Chew-Low-Nambu-Goldberger we know that the general Lorentz invariant parity conserving element transition matrix is

$$T = A(s,t)\;\bar{U}\,U + B(s,t)\;\bar{U}\;(K+K')_\mu \gamma_\mu\;U \qquad 2.132$$

where Kand K' are the initial and final state momenta of the nucleon and

$$U = \sqrt{E_N + m} \begin{vmatrix} \phi \\ \frac{\sigma.K}{E+m} \phi \end{vmatrix} \qquad 2.133$$

and represents the boost of $\begin{matrix}\phi\\0\end{matrix}$. Expanding T we have

$$T = A(s,t)\, \phi^{*}x\,(1- \frac{\sigma.K'}{E'1m} \frac{\sigma.K}{E + m})\phi$$

$$- B(s,t)(K_0'+K_0)\, \phi^{*}\,(1+ \frac{\sigma.K'}{E'+m} \frac{\sigma.K}{E+m})\phi$$

$$+ B(s,t)\phi^{*}[\sigma.(\vec{K}+ \vec{K'}) \frac{\sigma.K}{E+m} + \frac{\sigma.K'}{E+m} \sigma.(K+K')]\phi \qquad 2.134$$

To meet U(2) spin-independence these should be no σ's between the ϕ's. However this means

$$A(s,t) = B(s,t) = 0 \qquad 2.135$$

We can salvage the above approach if we demand spin-independence only in the C.M. frame for which

$$\vec{K} = \vec{K'} = 0 \qquad 2.136$$

Then we have

$$\frac{1}{2} \frac{T}{(E+m)(E'+m)} = A^{*}(s,t)\phi^{*}\,\phi \qquad 2.137$$

However there are three objections to choosing the C.M. frame:

(a) Crossing Symmetry is violated as the C.M. system is not a crossing invariant concept.[52]

(b) Locality is not satisfied. If we consider two particles, one at rest, the other moving at very high speed and outside their interaction range, then in C.M. frame both are moving very fast and so the velocity of one affects the internal dynamics of the other - thus this symmetry formalism cannot be regarded as a good candidate especially if we demand local causality. See P. 29

(c) This approach make the C.M. a preferred system.

This then means that index invariance for the S matrix elements is not consistent with unitarity for finite dimensional representations of the auxiliary group.

For <u>Unitary infinite dimensional representations</u> the position is completely different.

For such representations we have

$$K = K^{+} \qquad 2.138$$

$$H(P) = e^{-i\epsilon . k} \qquad 2.139$$

$$H^{+}(P) = e^{i\epsilon . k} \qquad 2.140$$

making H(P) unitary.

An equation 2.120 then becomes

$$H(P)\ O\ H^{+}(P) = 1 \qquad 2.141$$

which is obviously true from the relationship $\langle\alpha|M\sigma\rangle = \delta^{\alpha}_{\sigma}$ which gives

$$O = 1 \qquad 2.142$$

This means that index invariance is thus consistent with the unitarity of the S matrix provided we take infinite dimensional representations for the auxiliary group.

For finite dimensional representation the usual connection between spin, statistics and commutation relations exist -as has bee shown by Weinberg.[63] However for infinite dimensional representations we have[67]

$$[\psi_\alpha(x), \psi_\beta^+(y)]_\pm = [\langle\alpha|H(P)\ O\ H^+(P)|\beta\rangle\ e^{-iP(x-y)}$$

$$\pm\ \langle\alpha|H(P)\ \tilde{O}\ H^+(P)|\beta\rangle\ e^{iP(x-y)}]\ (2\pi)^{-3}\theta(P_0)\delta(P^2-m^2) \qquad 2.143$$

where

$$\tilde{O} = B|m\bar{\sigma}\rangle\ \langle m\bar{\sigma}|B^{-1} \qquad 2.144$$

and $\bar{\sigma}$ refers to antiparticles.

If we write

$$P(p) = H(p)\ O\ H^+(p) \qquad 2.145a$$

$$\tilde{P}(p) = H(p)\ \tilde{O}\ H^+(p) \qquad 2.145b$$

we then see we need to have

$$P(p) = \tilde{P}(-p) \qquad \text{for } [\psi_\alpha(x), \psi_\beta^+(y)]_- = 0\ , \quad (x-y)^2 < 0 \qquad 2.146$$

and

$$P(p) = -\tilde{P}(-p) \qquad \text{for } [\qquad\qquad]_+ = 0 \qquad 2.147$$

As $P(p)$ and $\tilde{P}(p)$ are projection operators eq. 2.146 can be satisfied but not eq. 2.147.

The consequences of the introduction of the auxiliary group approach can now be summarised by the following theorems of Matthews-Feldman:[41]

Theorem 1.

Invariance of the S matrix with respect to the index transformations of the auxiliary group is incompatible with its unitary unless all auxiliary operators are unitary representations of the auxiliary group and there is a one-to-one correspondence between the components of the auxiliary infinite dimensional representations 'and the states of the corresponding physical multiplets.

Theorem 2.

A local field constructed from those auxiliary operators satisfying Theorem 1 cannot satisfy anti-commutation relation.

These authors have also considered multimass fields which are characterised by creation and annihilation operators of the form

$$a(mS \, ; \, U \, S_3) \mid >_o = 0 \qquad 2.148$$

$$a^+(mS \, ; \, U \, S_3) \mid >_o = \mid mS \, ; \, U \, S_3> \qquad 2.149$$

where m, S, S_3 have their usual meaning and U is a four Vector ~~velocity label~~. These fields are the most general free field which transforms locally under Lorentz transformations. Such fields are linear combination of the annihilation and creation operators - defined in eqs. 2.14-12.142 - for particles belonging to an infinite tower of irreducible multiplets of Φ. The requirements of causal commutations and anti-commutations can be satisfied simply if one assume Bose statistics for the particles in the towers for both integer and half-odd integer spin. Generalised Fermi fields can still be constructed from such an approach but they do not enable one to construct an index invariant S matrix satisfying Unitarity and the correct statistics. The theorems quoted then also apply to these generalised fields.

It seems then that there is little we can do about these restrictive and powerful theorems. There is no way of going round them for such is their generality. All the same one cannot deny the utilitarian aspect of the approach of Salam and others. Such a fact has been exploited before. The foremost example is the SO(4) symmetry of the relativistic and non relativistic H atom which accounts for the degeneracy of levels with the same principal Quantum number but different angular momentum.[68] The success of such an approach cannot be denied but there the problem of relativistic covariance is more acute. Another example is the generalised spin of Wigner which has been quite successful in Atomic Physics. The history of the theory of hydrogen atom demostrates that progress is possible without developing a formalism that resolves the apparent contradictions between spin symmetries and relativistic covariance. However it may be argued that in all such approaches we were dealing with a coupling constant $\frac{e^2}{4\pi} = \frac{1}{137}$ which is small.

The Hadrom physics where not even a shadow of a theory exist we have to grasp at any recipe which presents itself. The formalism of Salam offers us a framework, not only with the possibility of predicting as yet undiscovered resonances but more important a dynamical mechanism for a variety of decay or collision processes - it allows the calculation of a complete set of vertex functions, for every energy and momentum transfer. Agreement with experiment will be our only criterion of acceptability. But whether they agree poorly or not the transition amplitudes of the formalism have all the features scattering amplitudes should have and therefore they have a much better chance than the predictions of Unitary Symmetry alone.

CHAPTER III

3.1 Critical Survey of the Absorption Model

A cursory glance at experimental data on πN and KN scattering in the 1 - 15 Gev region[69] reveals the existence of a high number of channels, a substantial portion of which show the formation of one or two resonances in the final state. For instance π^+P, at 4.0, Gev has the following distribution:

Main Reactions (The figures indicate cross-section in mb)

(1)	π^+P	6.42	(6)	$\pi^+\pi^+\pi^-P$	3.09
(2)	$\pi^+\pi^oP$	2.31	(7)	$\pi^+\pi^+\pi^-\pi^oP$	3.43
(3)	$\pi^+\pi^+n$	1.44	(8)	$\pi^+\pi^+\pi^+\pi^-n$	0.93
(4)	$\pi^+P(m\pi^o)$ $m \geq 2$	3.04	(9)	$\pi^+\pi^+\pi^+\pi^-P(m\pi^o)$ $m \geq 2$	1.27
(5)	$n\pi^+\pi^+(m\pi^o)$ $m \geq 1$	1.78	(10)	$n\pi^+\pi^+\pi^+\pi^+\pi^-(m\pi^o)$ $m \geq 1$	0.70

and reaction 2 itself has the following features $P\rho^+$ (0.35), $N^{*++}\pi^o$ (0.3), $N^{*+}\pi^+$ (0.2), $P\pi^+\pi^o$ (1.55), $n\pi^+\pi^+$ (1.35) while for reaction 6 we have $N^{*++}\pi^+\pi^-$ (1.1), $N^{*++}\rho^o$ (0.6), $N^{*++}F^o$ (0.1), $N^{*o}\pi^+\pi^+$ (0.25), $P\pi^+\rho^o$ (0.65), $P\pi^+\rho^o$ (no resonance) (0.3), PA_1^+ (0.1), PA_2^+ (0.25), $P\pi^+\pi^+\pi^-$ (0.4). The other channels too show similar distributions.

The predominance of small momentum transfer indicates that 'large impact parameter collisions'[7,8] are most important in these reactions; also the one particle exchange gives a natural explanation to the decay correlations. An analogous situation is also met in low energy Physics, especially as regards the former feature; and there Butler[70] suggested eliminating from the interaction region in configuration space, a sphere with a radius of the order of the sum of radii of the colliding particles and to calculate the contribution from the rest of the configuration by the plane wave Born approximation. In this crude fashion he eliminated the low partial waves which dominate the reaction cross section and which in most cases violate the Unitarity bound. In this way he obtained the necessary collimation of the differential cross section - but if this picture is true there should be considerable effects caused by the shadow of the absorption region. It was for this reason that Sopkovich[15,71] suggested that a modified absorption model to account for the peripheral nature of high energy collisions. The simplest form of this model is, as we have already quoted, given by

$$\langle\lambda_c\lambda_d/T^J/\lambda_a\lambda_b\rangle = (\langle\lambda_c\lambda_d/S_f^J/\lambda_c\lambda_d\rangle)^{1/2} \langle\lambda_c\lambda_d/B^J/\lambda_a\lambda_b\rangle$$
$$\times \quad (\langle\lambda_a\lambda_b/S_i^J/\lambda_a\lambda_b\rangle)^{1/2} \qquad 3.1$$

Where B^J are the partial wave Born helicity amplitudes for the reaction and S_ℓ^J are the partial wave helicity amplitudes for elastic scattering in the initial and final channels, (ℓ = initial, final).

The latter functions are obtained from spinless diffraction analysis of elastic scattering; parametrised in terms of a Gaussian model of radius $R(s)$ and opacity $C(s)$[72] they are

$$\langle\lambda_1\lambda_2/S_i^J/\lambda_1\lambda_2\rangle = 1 - C_1(s)\exp\left(-j(j+1)/R_\ell^2(s)\,K^2\right) \qquad 3.2$$

Where K is the magnitude of the three momentum in the centre-of-mass frame. The Optical Theorem gives the following relation between C(s) and R(s)

$$C(s) = \sigma_{tot}(s)/2\pi R^2(s) \qquad 3.3$$

From plausibility arguments and for consistency C(s) has to lie between 0.8 and 1.0;[72] once we have fixed the value of C - we shall take it to be 1.0 throughout - we can determine the value of R(s) from experimental results on elastic scattering. This is true for the initial state elastic scattering but not for the final one, where there are particles like $f^0 N^{*++}$ on which no elastic data exist, we cannot estimate the value of R. Modest variations of the strength and range of the final-state interaction do not lead to significant changes in the differential cross-section as we shall see; Jackson et al have also shown that they hardly produce changes in the spin density matrix. The factor $S_{\alpha\alpha}$ (Eq 1.22, 3.21) is positive, real and small for low partial waves and increases monotonically towards unity for high partial waves. How far is it realistic? There are three small objections following assumption: (i) our assumption that only the helicity-nonflip elastic amplitudes are essential (ii) our assumption that the elastic scattering is pure imaginary[73,75] (see footnote 74) (iii) the range of peripheral Born exchange is not always smaller than the range of forces in the entrance and exit channel[75].

What about the validity domain of the absorbed Born model itself now? The one-pion exchange theory with absorption is in remarkably good agreement with experimental data for reactions like $\pi P \rightarrow \rho P$, $\pi P \rightarrow \rho N^{*}$, $KP \rightarrow K^{*}N$, $\bar{P}P \rightarrow \bar{N}^{*}N^{*}$ etc. which are all mediated through a π. Furthermore there are no counter examples of two-body hadronic processes in which one-pion exchange is allowed and ~~no reasonable agreement~~ which disagrees with observed data[7,9,71]. Why this is so may perhaps be illustrated by a simple example[77]. If we consider the process $a+b \rightarrow c+d$, where all the particles have equal masses and are spinless the partial wave expansion may be written

$$\phi\,(s,t) = \sum_{l=L}^{l=\infty} \left(\frac{2l+1}{2}\right) T_l(s)\, P_l(\cos\theta) + \sum_{l=0}^{l=L} \left(\frac{2l+1}{2}\right) T_l(s) P_l(\cos\theta) \qquad 3.4$$

where

$$L = P/2m_\pi \gg 1 \qquad 3.5$$

The first term on the R.H.S. expresses the contribution of the pion and the second term that of particles heavier than the pion i.e. exchanges with particles of masses $\geq 2m_\pi$. The pion partial wave contribution is given by

$$T^{l}_{Born} \simeq \left(\frac{M^2}{4}\right) g_1 g_2 \int_{-1}^{+1} \frac{P_l(\cos\theta)\, d\,(\cos\theta)}{m_\pi^2 - t} \qquad 3.6$$

Which using eq.3.5

$$= \frac{M^2}{4}\; g_1 g_2 \; K_o\left(\frac{1}{2L}\right) \qquad 3.7$$

Where K_o denotes the modified Bessel function.

Then

$$\phi\,(s,t) = \sum_{l} \left(\frac{2l+1}{2}\right) \frac{M^2}{4}\; g_1 g_2\, K_o\left(\frac{1}{2L}\right) P_l(\cos\theta) + \sum_{l} \left(\frac{2l+1}{2}\right) T_l P_l\,(\cos\theta) \qquad 3.8$$

From the Unitary relation for the partial wave amplitudes:

$$2\, I_m T^l_{if} = \sum (T^l_{in})^* (T_{nf}) \qquad 3.9$$

we see that $T_\ell \leq 1$ so that we may write

$$\left| \sum_\ell \left(\frac{2l+1}{2}\right) T_l P_l \, (\cos\theta) \right| \leq \sum_{l=0}^{l=L} \frac{2l+1}{2} = \frac{1}{2} L^2 \qquad 3.10$$

If we consider forward scattering we may transform the summation of the other contribution to $\phi(s,t)$ to an integral:

$$\sum_\ell \left(\frac{2l+1}{2}\right) K_o\left(\frac{l}{2L}\right) \simeq \int_L^\infty l K_o\left(\frac{l}{2L}\right) dl \simeq L^2 \int_1^\infty X \, dX \, K_o\left(\frac{X}{2}\right)$$

$$= \frac{7}{2} L^2 \qquad 3.11$$

Finally we have

$$\phi\,(s,t) \simeq \frac{7}{2} L^2 \frac{M^2 g_1 g_2}{4\,P^2} + \frac{1}{2} L^2 \qquad 3.12$$

The second term on the R.H.S. must be small compared to the first one for us to 'absorb' it; this means that

$$\frac{M^2}{4P^2} g_1 g_2 \geq \frac{1}{7} \qquad 3.13$$

which is well satisfied in the few Gev region - hence the success of the Absorption model for exchange reactions dominated by one pion.

What are the disadvantages of this model itself? Basically it comes from the fact that it is a hybrid model combining potential theory and field theory. By introducing absorption we are giving the hadron a structure which we do not take into account afterwards in our use of the Born term[78]. If we assume that the hadrons are made up of Quarks we could presumably assume that some of these entities play the role of spectators and so provide absorption and that the Born term accounts for the strong interaction. Although this would still be a hybrid model it has a physical appeal and is completely analogous to the description of Hadron-Nuclei reactions; but so long as one uses " left-hand " negative real axis singularities only the introduction of absorption is artificial.

Also there are two main reasons against the presence of absorption[78]. The first one is that a form-factor for the reaction $P+P \rightarrow N^{*++}n$ determined at 0.970 Gev, where only one inelastic channel is practically open ($\sigma_{tot} = 40mb$, $\sigma_{el} = 20mb$, $\sigma_{N^*n} = 19mb$), accounts very well for the reaction at higher energies although other inelastic channels take now a considerable share of the total cross-section. The second one is based on a unitarity-analyticity argument and may be illustrated by the process $\pi N \rightarrow \rho N$ and let $\frac{1}{2}\phi_\ell$ be its amplitudes. Denote by $\phi_{1\ell}$ and $\phi_{2\ell}$ respectively the elastic partial wave amplitudes for $\pi N, \rho N$ scattering and let $(\Theta_{1n})_\ell$ and $(\Theta_{2n})_\ell$ be the amplitudes for other final states n which can be reached from these initial states. Then Unitarity gives us the following relationships:

$$\mathrm{Im}\,\phi_{1\ell} - |\phi_{1\ell}|^2 = \frac{1}{4}\phi_\ell^2 + \sum_n (\Theta_{1n})_\ell^2 \quad 3.14$$

$$\mathrm{Im}\,\phi_{2\ell} - |\phi_{2\ell}|^2 = \frac{1}{4}\phi_\ell^2 + \sum_n (\Theta_{2n})_\ell^2 \quad 3.15$$

$$I_m \phi_1 = \phi_{11}^* \phi_1 + \phi_1^* \phi_{21} + 2\sum_n (c_{1n}^* \phi_{2n})_1 \qquad 3.16$$

Assuming that

$$\phi_{j1} = \frac{1}{2i} (\exp 2i\delta_{11} - 1) \quad j = 1,2 \qquad 3.17$$

Then the equations 3.14 - 3.16 are satisfied provided $|\phi_\ell|^2 \leq 1$. Both the Form-factor and the absorption formalism make sure that this is so. The sum in eqs. 3.14 - 3.16 being the sum of a large number of complex quantities, each small in number, may be equated to zero - this is called the random phase approximation (RPA) and will be used later. Then we have that

$$\frac{I_m \phi_1}{Re\phi_1} = tg \quad (\alpha_{11}+\alpha_{21}) \qquad 3.18$$

This means that the only restriction on ϕ_ℓ is on its phase; at high energies the α's are near zero. For the Born term $Im\phi_\ell = 0$, and, as a first approximation it may then be accepted. The main conclusion is that this unitarity relation does not give rise to a reduction of the modulus of the Born term and so contradict the idea that there must be absorption effects of other channels. Unitarity gives rise to absorption for the elastic channels due to the presence of the squared moduli and forces the phase shifts to become complex but this is not so for the inelastic case. However this argument holds true for the 1-5 Gev region and at higher energies one must take account of other channels wich cannot be simply lumped off to zero in the Random Phase Approximation (RPA).

From the experimental point of view the energy dependence of the model and its predictions for high spin exchanges are its main failures. Form factors both in s and t have been introduced but these suffer from indeterminancy of parameters. Dar et al[71] have suggested modifying the model by the inclusion of an energy dependent factor in the Born term partial wave amplitude:

$$\langle \lambda_c \lambda_d | T^J(s,t) | \lambda_a \lambda_b \rangle = \alpha^J(s,t) \langle \lambda_c \lambda_d | B^J(s) | \lambda_a \lambda_b \rangle \tag{3.19}$$

Where

$$\alpha^J(s) = \frac{\sigma_{tot}(s)}{\sigma_{el}(s)} \frac{1 - |\eta_J|^2}{\sum_{f\,\lambda} \langle \lambda_c \lambda_d | B(s,t) | \lambda_a \lambda_b \rangle^2} \tag{3.20}$$

Here η_J is the phase shift for elastic scattering and $\sum_f$ represents the maximum set of final inelastic channels for which one can define consistently helicity Born amplitudes. It has been very successful but as it rests on shaky foundations we shall not adopt it.

It should also be remembered that for high energy the optical radius is of the order of 1 fermi (fm) over a wider energy range while $m_\pi^{-1} = 2\text{fm}$ and $m_\rho^{-1} = \frac{1}{2}\text{fm}$. Thus the conditions of eqs. 1.22 - 1.22d are not satisfied.

The other alternative which attempts to take into account the effect of Unitarity and the many inelastic channels is the K matrix approach.

3.2 The K Matrix Approach[79]

In this approach one starts from the relationship

$$S = \frac{1 + i\rho K'}{1 - i\rho K'} \tag{3.21a}$$

and

$$T = K' - i\rho K' \qquad 3.21b$$

where ρ's are density-of-state factors.

This guarantees that S is unitary if K is symmetric and real; this condition is automatically satisfied if we take K' = Born term.

If we define $K' = (\rho)^{1/2} K (\rho)^{1/2}$ we get a simpler formal relation

$$T = K - iKT \qquad 3.22a$$

or

$$T_{\beta\alpha} = K_{\beta\alpha} - K_{\beta\gamma}\left(\frac{1}{1+iK}\right)_{\gamma\gamma'} K_{\gamma'\alpha} \qquad 3.22b$$

The first term represents the Born term and the second one unitarity corrections. The entire philosophy of this K matrix approach is to estimate this correction. Watson[80] for instance approximates it by two contributions. Using the RPA principle one may say that of all terms of the form

$$\sum_{\gamma} K_{\beta\gamma}\left(\frac{1}{1+iK}\right)_{\gamma\gamma'} = T_{\beta\gamma'} \quad \text{and} \quad \sum_{\gamma'}\left(\frac{1}{1+iK}\right)_{\gamma\gamma'} K_{\gamma'\alpha} = T_{\gamma\alpha}$$

the largest will be given by when $\gamma' = \beta$ and $\gamma = \alpha$ respectively. By setting $\gamma' = \beta$ and $\gamma = \alpha$ we obtain the following two correction terms: $iT_{\beta\beta}K_{\beta\alpha}$ and $iK_{\beta\alpha}T_{\alpha\alpha}$. Inserting these in eq. 3.22 we have

$$T_{\beta\alpha} = K_{\beta\alpha} - i\,(T_{\beta\beta} K_{\beta\alpha} + K_{\beta\beta} T_{\beta\alpha})$$

$$= B_{\beta\alpha} - i\,(T_{\beta\beta} B_{\beta\alpha} + B_{\beta\alpha} T_{\beta\beta})$$

$$= \frac{1}{2}\ \left((1 - 2iT_{\beta\beta}) B_{\beta\alpha} + B_{\beta\alpha} (1 - 2i\,T_{\alpha\alpha})\right)$$

$$= \frac{1}{2}\,(S_{\beta\beta} B_{\beta\alpha} + B_{\beta\alpha} S_{\alpha\alpha}) \qquad 3.23$$

If $S_{\alpha\alpha} = S_{\beta\beta}$ i.e. if the elastic corrections are equal in the initial and final states we then ~~reouperate~~ recover eq. 1.22. This derivation it should be observed, makes no mention of absorption and is derived solely on the assumption that the total number of states with three or more uncorrelated particles is very large and that their matrix elements are all small[22,23].

This argument has been carried further by Dietz and Pilkuhn[31] who start their approach from eq. 3.22. (In the following the first and second subscripts refer to the number of particles in the initial state and second state respectively.) If we expand this equation in a formal way we obtain relationships of the form

$$T_{22} = 2\,K_{22} + i\rho_2\,K_{22}\,T_{22} + i\rho_2'\,K_{23}\,T_{23} + \ldots\ldots\ldots \qquad 3.24$$

$$T_{32} = 2\,K_{32} + i\rho_3\,K_{32}\,T_{22} + i\rho_3'\,K_{32}\,T_{32} + \ldots\ldots\ldots$$

$$= (1 - iK)^{-1}\,K_{32}\,(2 + i\rho\,T_{22}) \qquad 3.25$$

Taking $T = 2K + iKT$

If we substitute the last relationship in eq. 3.24 we get

$$T_{22} = \frac{2\,K_{22} + B_{22}}{1 - \rho_2(K_{22} + B_{22})} \qquad 3.26$$

where

$$B_{22} = i K_{23} (1 - i K_{33})^{-1} K_{32} \qquad 3.27$$

It can be shown T_{22} will satisfy unitarity provided Im B_{22} is positive. We assume that all the K's except those for two particles reaction are statistically independent and are given by a distribution function P

$$p(K_{ij}) = \frac{\Upsilon}{\sqrt{2\pi}} \exp\left(\frac{-K_{ij}^2}{2\,\cdot^2}\right) \qquad 3.28$$

one then obtains that (See 31)

$$\sum_{j=1}^{N} K_{ij}^2 = C_i \simeq (N-1)\Upsilon^2 = C \qquad 3.29a$$

$$\sum \Upsilon_{ij} K_{jk} \simeq 0 \qquad 3.29b$$

From this one can show that putting $B_{22} = ib$. ~~Then~~

$$T_{22} = 2\left(\frac{K + i K^2 + ib(1+b)}{(1+b)^2 + K^2}\right) \qquad 3.30$$

b can be determined from elastic scattering.

The damping of this amplitude is due to two particles effects and three particle channels.

In this derivation assuming the number of channels N = 200 the authors deduce ~~one~~ makes errors of the order of 10% in the approximations of K_{ij}

Following on these lines and using the methods of Van Hove[82], Fincham et al[83] have suggested an approach which does not contain any parameter and in which one only uses the elastic scattering of the initial state only. Usually in practical applications of eq. 1.22 some room for manoeuvre is always available as the final state elastic scattering is usually unknown. In this method the channel space is divided into the experimentally important two-body and quasi-two-body channels and the background set of all other channels - we denote the former by Latin letters and the latter by Greek ones. Also Operators which have elements only between significant channels will be denoted by a bar. The Unitarity equation now reads:

$$\sum_c S_{bc} S_{ca} + \sum_\gamma S_{b\gamma} S_{\gamma a} = \delta_{ba} \qquad 3.31$$

Denoting by F the second sum or overlap term on the L.H.S. this may be rewritten formally as

$$\bar{S}\,\bar{S}^+ = 1 - F \qquad 3.32$$

Now we write

$$S = G\Omega \qquad 3.33$$

with

$$GG^+ = 1 - F \qquad 3.34a$$

and

$$\Omega^* = \frac{1 + i\bar{K}}{1 - i\bar{K}} \qquad 3.34b$$

Where $\bar{K}$ is Hermitian.

According to the R.P.A., $\sum_\gamma T_{b\gamma} T_{\gamma a}$ for $a \neq b$ can be neglected compared to $\Sigma\, T_{a\gamma} T_{\gamma a}$.

This means that G is a diagonal matrix and can be shown to be a complex multiple of the unit matrix. So we can write

$$\bar{S} = G \frac{1 + i\bar{K}}{1 - i\bar{K}} \qquad 3.35$$

Eliminating the real and imaginary parts of G between the diagonal and off diagonals elements of eq. 3.33 we obtain

$$T_{ba} = \frac{i\,((1+i\bar{K})/(1-i\bar{K}))_{ba}}{((1+i\bar{K})/(1-i\bar{K}))_{aa}} \times S_{aa} \quad , \quad S = 1 - iT \qquad 3.36$$

One obvious advantage here is that the unknown final-state elastic scattering is not included explicitly in the equation. We can take K_{ba} to be the Born term; however there is an arbitrariness in the choice of the diagonal elements of $\bar{K}$. We choose the simplest case: $\bar{K}_{aa} = 0$ which corresponds to an entirely imaginary amplitude in a two-channel model. For a two-channel model with $K_{11} = K_{22} = 0$ eq. 3.36 becomes

$$T^J = B^J . S^J_{11} . \frac{1}{1-(\frac{1}{4}B)^2} \qquad K_{21} = \frac{1}{2} B_{21}, \quad K_{12} = \frac{1}{2} B_{12} \qquad 3.37$$

If we let $S_{ii} = S_{ff}$ in eq. 1.22 we then see that the factor in the denominator will give extra absorption. This will result in differential cross-section which is reduced in magnitude and has a greater chance of being ~~hope feature which we hope will be~~ adequate for an A_2 (Spin 2) exchange in the process $\pi^- P \rightarrow \eta n$.

3.3 Orbital, Global and Regge trajectory Resonance Classifications

There are, in principle, three approaches on the question of classification of higher resonances into multiplets of symmetry groups: the 'Orbital', 'Global' and 'Regge' [84]. Here we discuss the first two only, leaving discussion of the third one for the last chapter.

In the Orbital framework one believes all particles, both meson and baryons, are made up of Quar-AntiQuark and three Quarks systems respectively. The spin of all quarks $\vec{S}_q$ and antiquarks $\vec{S}_{\bar{q}}$ couple to a total quark spin S. These may or may not have an orbital excitation L; in case they have this angular momentum is then coupled to S to give the total angular momentum J which is the spin of the physical particle: L + S = J. The resonances so generated will have the same internal quantum numbers as the basic particles but may have different spins and parities. In the language of SU(3) this would mean that one needs nothing more than octets, nonets, decuplets. Radial excitation, SU(3) breaking, L - S splitting have been introduced in this model but we shall ignore these and keep to the simple basic one.

The resulting multiplets would then have parity $P = (\text{Parity of } Q\,\bar{Q} \text{ system})\,(-1)^S$, charge conjugation number $C = (-1)^{L+S}$. The evidence for this scheme is overwhelming[84]. All the presently known mesons are naturally classified according to it. Further all natural parity mesons, according to this model must have normal change conjugation C = P: experimentally we do not have a single established meson which violates this rule. This model however conflicts with the

Regge one which classifies particles on parent and daughters trajectories. All the odd daughters of all natural parity trajectories as well as many other daughter states are not allowed by the quark model. To get out of this difficulty Gell-Mann and Zweig have suggested we should classify mesons according to $U(6) \otimes U(6) \otimes O(3,1)$ as the existence of duaghters originates from the $O(3,1)$ group. This however introduces an additional quantum giving more families of particles which so far have not been found. Overall this scheme has been extremely successful from the experimental point of view; also the wave functions have a simple form in this scheme as we shall see further on.

In the Global scheme the resonances are built up by addition of more quarks and antiquark all piled up in an overall S state. The more Q and $\bar{Q}$ we add the higher are the spins and internal symmetry quantum numbers so the resonances may belong $\{8\}$, $\{10\}$, $\{27\}$ etc. of $SU(3)$ or equivalently to the $\{35\}$, $\{56\}$, $\{405\}$, $\{700\}$ etc. of $SU(6)$. The fact that most of these high number multiplets are empty and that no esoteric particles (i.e. those with high Isotopic-hypercharge numbers) exist counts heavily against this model. Also the wave functions in this approach are just too long and complicated. It should be mentioned, however, that the discovery of a single esoteric particle with, say $I = 1$, $Y = 2$, which is allowed in peripheral collision of Kaons or Pions on nucleons would be a fatal blow to the Supermultiplet scheme but quite a success for the Global one.

It is convenient to subdivide the exotic states into two kinds; the first have values of isospin, hypercharge and baryon numbers not found in the quark-antiquark $(Q\bar{Q})$ and three quark systems (QQQ). The second kind are non-strange mesons which do not appear in the $Q - \bar{Q}$ system; namely $J^{PC} = 0^{--}$ and natural parity $P = (-1)^L$ states, with odd CP, abnormal conjugation.

One reason for missing exotic states of the second kind is the suppression of their coupling to two pseudoscalor mesons. Conservation of angular momentum, parity and charge conjugation forbid $\pi\pi, K\bar{K}$, and $\eta\pi$ couplings for all exotic states of the second kind, and π couplings for all even J exotic states. SU(3) forbids the $\eta\pi$ coupling for exotic states of the second kind which are in SU(3) octets[85]. This means then that exotic states of the second kind would not be produced by pseudoscalar exchange or decay into two psudoscalar mesons. There are SU(6)w selection rules[86] which forbid the decay of exotic states into common two body channels and this may perhaps explain why they may have been missed experimentally. Other theoretical arguments suggest the existence of exotic boson states coupled to baryons only and not to mesons, and observable as baryon-anti baryon resonances. Experimentally[87] there seems to be some evidence for positive strangeness baryon resonances and exotic t-channel exchanges at low energies but such evidence is not strong. Furthermore, one must remember that analysis of peripheral reactions indicate that objects appearing as s-channel resonances are also exchanged in t or u channels; and what is particularly striking is the absence of a forward or backward peak in those cases when the exchanged particle would have to have quantum numbers which do not correspond to known resonances - a fact which has been fruitfully exploited by the exponent of the Regge Theory.

The absence of these exotic states has also been quite successfully exploited in the applications of super convergence and finite energy sum rules. Sum rules[88] of the form

$$\frac{1}{2}\int_{-N}^{N}\nu^n \, \mathrm{Im}\, A(\nu,t)\, d\nu = f_n(N,t) = \frac{\beta(t)N^{\alpha(t)+n+1}}{\alpha(t)+n+1} \qquad 3.38$$

where $f_n(N,t)$ represents an integral over a contour at high energy $|s| = N$ of the asymptotic limit of Regge amplitude, are assumed to be dominated by resonances[89]; one also assumes the non-resonant background to the left is balanced by the Pomeranchuk contribution to the right hand side and that these two contributions can be subtracted from the sum rule. This leaves only resonances on the left hand side and only Regge trajectories other than the Pomeranchuk on the right. The absence of exotic states is introduced into this sum rule by setting the left and right hand sides of this equation equal to zero for channels having s and t channel exotic resonances respectively.

This remark may be illustrated by looking at the particular case of $\pi\pi$ scattering where $I = 2$ is exotic[90]. A sum rule with $I = 2$ in the s-channel has zero on the left hand side and contributions on the right hand side from $I = 0$ and $I = 1$ exchanges but not $I = 2$ exchange. The contributions of isoscalar and isovector exchanges must therefore cancel one another unless each of them vanishes identically. Since this cancellation must hold as a function of the energy N which can be varied in the sum rule, the isoscalar and isovector must cancel one another unless each of them vanishes identically. For $\pi\pi$, $K\pi$ and KK scattering this would then imply the existence of nonets with the standard mixing angles[90,91]. The requirements of the absence of exotic states also leads to relations between masses and coupling constants on Regge trajectories and in certain cases implies the degeneracy of trajectories, for instance that of the f^o and the A_2 [92].

The application of this approach is not without its difficulties, an example of which is encountered in baryon anti baryon scattering[93] which requires contributions from exotic resonances. Also in those reactions which are mediated mainly by pseudoscalar exchanges applications of the F.E.S.R. of eq. 3.38 requires the degeneracy of isoscalar and isovector trajectories but there is no isoscalar degenerate with the pion[94]. It has also been argued that some of observed low I - Y, $J^P = \frac{3}{2}^-$ may possesstthe group structure of the {27} of SU(3) or correspondingly of the {700} of SU(6); the group structure was tested by analysis of their decay widths[95]. Overall then we can say that although the evidence is strongly in favour of the Supermultiplet scheme it is not conclusively so; at the currently available accelator energies the issue cannot be decided[96,97].

3.4 Production of $J^P = 2^+$ and 1^+ Mesons

The transition amplitudes for such processes may be written in either the Kinetic Supermultiplet formalism[51] or else in the Global Symmetry one[62]. The Dimensions of the representations which will accommodate these spin 2^+ and 1^+ mesons are 4212 and 5940 in U(6,6) theory; the corresponding SU(6) representations are 189 and 405 respectively. The wave functions which describe these mesons can then be extracted from:

(a) For the 4212

$\Psi^{(A\ B)}_{(C\ D)}(P)$ which is completely antisymmetric in lower and upper indices respectively.

(b) For the 5940

$\Psi^{\{A\ B\}}_{\{C\ \}}(P)$ Completely symmetric in lower and upper indices respectively.

Here $A, B, \ldots = \alpha a, \beta b, \ldots,$ where $\alpha, \beta, \ldots,$ are the U(2,2) indices and range from 0 to 3 while $a, b, \ldots,$ are the Unitary Symmetry indices and take values 1,2,3.

Of these two representations we can choose either one, as there are no convincing arguements for eliminating the other. Using their $SU(2,2) \otimes SU(3)$ decompositions, which are given below we can extract those parts which describe a 2^+ and 1^+ particle:

(i) $SU(2,2) \otimes SU(3)$ decomposition[98] of the 4212 is

$$\begin{aligned} 4212 = & (84,8) \oplus (84,1) \oplus (45,10) \oplus (\overline{45},10) \oplus (45,8) \oplus (\overline{45},8) \\ & \oplus (20,27) \oplus (20,8) \oplus (20,1) \oplus (15,27) \oplus (15,10) \oplus (15,10) \\ & \oplus 3 \times (15,8) \oplus (15,1) \oplus (1,27) \oplus (1,8) \oplus (1,1) \end{aligned} \qquad 3.39$$

Here the first numbers of the brackets refer to the SU(2,2) multiplets whereas the second ones refer to those of SU(3).

(ii) For the 5940 decomposition into $SU(2,2) \otimes SU(3)$ multiplets we have

$$5940 = (84,27) \oplus (84,27) \oplus (84,1) \oplus (45,10) \oplus (\overline{45},\overline{10}) \oplus (45,8) \oplus (\overline{45},8) \oplus (20,8) \oplus (20,1) \oplus (15,27) \oplus (15,10) \oplus (15,\overline{10}) \oplus 3 \times (15,8) \oplus (15,1) \oplus (1,27) \oplus (1,27) \oplus (1,8) \oplus (1,1) \qquad 3.40$$

Their decompositons look much more familiar in terms of their SU(3) ⊠ SU(2) decompositions which are respectively:

$$189 = (8,5) \oplus (1,5) \oplus (10,3) \oplus (\overline{10},3) \oplus 2 \times (8,3) \oplus (27,1) \oplus (8,1) \oplus (1,1) \qquad 3.41$$

and

$$405 = (27,5) \oplus (8,5) \oplus (1,5) \oplus (27,3) \oplus (10,3) \oplus (\overline{10},3) \oplus 2 \times (8,3) \oplus (27,1) \oplus (8,1) \oplus (1,1) \qquad 3.42$$

And it should be recalled that these representations make their appearance in the following way in the SU(6) x SU(6) decomposition of SU(6,6)

$$4212 = (15,\overline{15}) \oplus (\overline{15},15) \oplus (6,\overline{6}) \oplus (\overline{6},6) \oplus (35,35) \oplus (1,1) \oplus (84,\overline{6}) \oplus (6,\overline{84}) \oplus (\overline{84},6) \oplus (\overline{6},84) \oplus (35,1) \oplus (1,35) \oplus (189,1) \oplus (1,189) \qquad 3.43$$

$$5940 = (21,\overline{21}) \oplus (\overline{21},21) \oplus (6,\overline{6}) \oplus (\overline{6},6) \oplus (35,35) \oplus (1,1) \oplus (120,\overline{6}) \oplus (6,\overline{120}) \oplus (\overline{120},6) \oplus (\overline{6},120) \oplus (35,1) \oplus (1,35) \oplus (405,1) \oplus (1,405) \qquad 3.44$$

Such a decomposition comes from the fact that under $SU(6) \otimes SU(6)$ the basic spinors ψ_A and ψ^A reduce in the following way

$$\psi_A = (6,1)_+ \oplus (1,6)_- \qquad 3.45a$$

$$\psi^A = (\bar{6},1)_+ \oplus (1,\bar{6})_- \qquad 3.45b$$

where the appended signs indicate the value taken by γ_0 in the respective subspaces. This decomposition becomes more familiar if we remember that in the Pauli representation of the Dirac matrices which are

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad 3.46$$

one can separate the Dirac spinor ψ into $\begin{pmatrix} \phi \\ \chi \end{pmatrix}$

Bearing these decompositions in mind we can then extract those parts of the wavefunctions which describe Spin 2, 1 and 0 particles; after application of the Bargmann-Wigner equation the wave functions belonging to the 4212 are given as follows:

$$\begin{aligned}\psi^{(A\ B)}_{(C\ D)}(P) = {} & \frac{1}{2}\left[\frac{1}{2M}(\gamma.P+M)\gamma_5 C\right]_{\gamma\delta}\left[\frac{1}{2M}C^{-1}\gamma_5(\gamma.P-M)\right]^{\alpha\beta}\eta^{\{a\ b\}}_{\{c\ d\}}(P) \\ & + \frac{1}{\sqrt{2}}\left[-\frac{1}{2M}(\gamma.P+M)\gamma_5 C\right]_{\gamma\delta}\left[\frac{1}{2M}C^{-1}\gamma_\mu(\gamma.P-M)\right]^{\alpha\beta}V^{(a\ b)}_{\mu\{c\ d\}}(P) \\ & + \frac{1}{\sqrt{2}}\left[\frac{1}{2M}(\gamma.P+M)\gamma_5 C\right]_{\gamma\delta}\left[\frac{1}{2M}C^{-1}\gamma_\mu(\gamma.P-M)\right]^{\alpha\beta}V^{a\ b}_{\mu(c\ d)}(P) \\ & + \left[\frac{1}{2M}(\gamma.P+M)\gamma_\mu C\right]_{\gamma\delta}\left[\frac{1}{2M}C^{-1}\gamma_\nu(\gamma.P-M)\right]^{\alpha\beta}\phi^{(a\ b)}_{\mu\nu(c\ d)}(P)\end{aligned}$$

3.47

where $(P^2-M^2)\phi_{\mu\nu}(P) = 0 = (P^2-M^2)V_\mu(P) = (P^2-M^2)\quad(P) = 0$

and $$P_\mu \phi_{\mu\nu}(P) = P_\mu V_\mu(P) = 0$$

The η describes 0^+ particles, V_μ describes 1^+ ones whereas $\phi_{\mu\nu}$ describes the 2^+, 1^+ and 0^+ jointly. It may be decomposed further into

$$\phi_{\mu\nu}(P) = S_{\{\mu\nu\}}(P) + A_{\mu\nu}(P) + \frac{1}{3} g_{\mu\nu}\phi$$

where the S, A and ϕ describe 2^+, 1^+ and 0 particles respectively.

The corresponding Unitary Spin decomposition is symbolically given by

$$\eta^{\{ab\}}_{\{cd\}} = \eta(27)^{\{ab\}}_{\{cd\}} + \frac{1}{2\sqrt{5}} [\delta^a_c \ \eta(8)^b_d + \text{permutations}] + \frac{1}{2\sqrt{6}} (\delta^a_c\delta^b_d + \delta^a_q\delta^b_p)\ \eta(1) \qquad 3.48$$

$$V^{[ab]}_{\{cd\}} = \frac{1}{\sqrt{2}} U(10)^{[ab]}_{\{cd\}} + \frac{1}{2\sqrt{3}} \Big[\delta^a_c\ U^b_d(8) \quad \delta^b_c U \quad \delta^a_d U^b(8) - \delta^b_d V(8) \Big] \qquad 3.49$$

$$\phi^{[ab]}_{[cd]} = \epsilon^{abt}\epsilon_{cdu}\ \phi(8)^u_t + \frac{1}{\sqrt{3}} (\delta^a_c\delta^b_d - \delta^a_d\delta^b_c)\phi(1) \qquad 3.50$$

The U(6,6) invariant coupling of <u>4212</u> with two <u>143</u> mesons is unique and is given by

$$L = G\ \phi^{[AB]}_{[CD]}(P_3)\ \phi^C_A\ (P_1)\ \phi^D_B\ (q) \qquad 3.51$$

See eq 2.109 for definitions of $\phi^c_A(P)$ etc

Since however strict U(6,6) forbids the decays $2^+ \rightarrow 0^-0^-$ and $2^+ \rightarrow 1^-0^-$ we have to introduce symmetry breaking through the use of the momentum spurion of the type

$$P^A_B \doteq (P\gamma)^\alpha_\beta\, \delta^a_b \qquad \delta^a_b \text{ is a Kronecker delta} \qquad 3.52$$

in order to accommodate such vertices in the theory. Once we do this we have two Lagrangians involving two couplings

$$L_1 = G_1 q^C_A \phi^{[AB]}_{[CD]}(P_3)\quad \phi^E_B(P_1)\phi^D_E(q) - \phi^E_B(q)\phi^D_E(P_1) \qquad 3.53a$$

$$L_2 = G_2 q^C_A q^D_B \phi^{[AB]}_{[CD]}(P_3)\phi^E_F(P_1)\phi^F_E(q) \qquad 3.53b$$

Unfortunately we do not know how to relate these couplings and this would introduce parameters into our theory - precisely the sort of situation we ~~have~~ wanted to avoid by introducing dynomial groups to ~~eliminate~~. Furthermore the introduction of spurions is dubious. It should however be mentioned that Delbourgo[99] has used such an approach to study the decay of the f^o, in which case only one of the Lagangians is needed. Although such a procedure may be used for certain decays it cannot however be used consistently for production reactions when both Lagrangians have to be used. For these reasons then we prefer the Kinetic Supermultiplet formalism which as we shall see is much simpler in its philosophy in addition to being on firmer ground from the experimental point of view[84].

In this P wave Quark-anti Quark model these entities are so tightly bound that they form, to all intents and purposes, a single object, as is implied by its group structure

$U(6,6) \otimes O(3)$. This means that these mesons are described by a 143 component object belonging to $U(6,6)$ and moreover they possess an independent four-vector character which represents their P wave behaviour. More explicitly they are represented by $\phi^{A}_{B\mu}(P)$ as follows. The Unitary symmetry part of this is completely similar to that of the ϕ^{A}_{B} which has been described by Salem[62] et al and used by others[83]. Suppressing these Unitary Spin indices then, these mesons are described by $\phi^{\alpha}_{\beta\mu}(\underline{P})$ which obey the following conditions:

$$(\gamma P)^{\beta'}_{\beta} \phi^{\alpha}_{\beta'\mu} (P) = M \phi^{\alpha}_{\beta\mu} (P) \qquad 3.54a$$

$$(\gamma P)^{\alpha}_{\alpha'} \phi^{\alpha'}_{\beta\mu} (P) = M \phi^{\alpha}_{\beta\mu} (P) \qquad 3.54b$$

and $$P_{\mu} \phi^{\alpha}_{\beta\mu} = 0 \qquad 3.55$$

The field $\Phi^{A}_{B\mu}(P)$ can then be expanded in terms of the Dirac matrices as follows

$$\phi^{\alpha}_{\beta\mu}(P) = [S^{i}_{\mu}(P) + \gamma_{\lambda}\phi^{i}_{\lambda,\mu}(P) + \tfrac{1}{2} \sigma_{\nu\lambda} T^{i}_{\nu\lambda\mu}(P) + i\gamma_{\lambda}\gamma_{5} \phi^{i}_{\lambda\mu 5}(P) + \gamma_{5}\phi^{i}_{\mu 5}(P)]^{\alpha}_{\beta}(T^{i})^{a}_{b} \qquad 3.56$$

The constraints of eqs. 3.53 - 3.54 then give us the following relations among the components of the wave function:

$$S_\mu = 0$$
$$P_\nu \phi_{\nu\mu 5} = -im\, \phi_{\mu 5}$$
$$P_\nu \phi_{\mu 5} = im\, \phi_{\nu\mu 5}$$
$$P_\mu T_{\mu\sigma\nu} = -im\, \phi_{\sigma\nu}$$
$$i(P_\nu \phi_{\mu\sigma} - P_\mu \phi_{\nu\sigma}) = m\, T_{\nu\mu\sigma}$$

3.57

(all these relations follow immediately from the Bargman - Wigner eq.)

We also have the following condition, which is somet mes refered to as the divergenc-less condition:

$$P_\mu \phi_{\nu\mu}(P) = P_\mu T_{\nu\sigma}(P) = P_\mu \phi_{\mu 5}(P) = P_\mu \phi_{\nu\mu 5}(P) = 0$$

3.58

Using these relations the wave function simplifies to

$$\phi^A_{B\mu}(P) = [(\frac{M+P\gamma}{M})\, \gamma_\nu \phi^i_{\nu\mu}(P) + (\frac{m+\not{P}}{m})\, \gamma_5 \phi^i_{\mu 5}(P)]\, [T^i]^a_b$$

3.59

Here the first term describes a 2^+ particle whereas the second term represents a 1^+ particle. The parity is obtained from the relation

$$\begin{aligned} P &= (\text{Parity of Quark K - Anti-Quark K})\, (-1)^L \\ &= (-1)\, (-1)^L = 1 \\ &= + \end{aligned}$$

3.60

For the production of these mesons the U(6,6) O(3) Lagrangian is given by

$$L(P_3 P_1) = G \, \phi^{A}_{B\mu}(P_3)\,[\phi^{C}_{A}(P_1)\phi^{B}_{C}(q) - \phi^{C}_{A}(q)\phi^{B}_{C}(P_1)]\,\frac{q^{\mu}}{S} \qquad 3.61$$

and corresponds to the vertex shown in Fig. 3.1

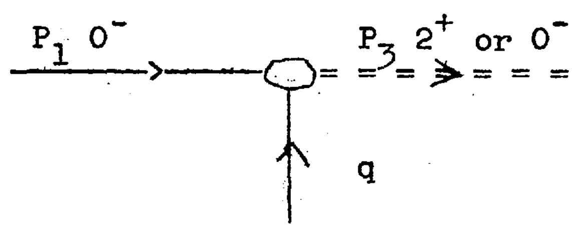


Fig. 3.1 Meson Resonance Production. Definitions of symbols in Eq 3.78 etc

This Lagrangian must conserve parity and charge conjugation; that it does so may be shown as follows. The basic U(6,6) quarks from which it is made transform as

$$\delta \psi_{\alpha a} = i(\epsilon^{j} + \epsilon^{j}\gamma_5 + \epsilon^{j}_{\mu}\gamma_{\mu} + \epsilon^{j}_{\mu 5} i \gamma_{\mu}\gamma_5 + \tfrac{1}{2}\epsilon^{j}_{\mu\nu}\,\sigma_{\mu\nu})^{\beta}_{\alpha}(T^{j})^{b}_{a}\,\psi_{\beta b}$$

which contains the parity transformation $P: \psi \to \gamma_o P$. And to see that charge conjugation is preserved we only have to remember that the charge conjugation antisymmetrical matrix $C_{\alpha\beta}$, which is defined by

$$C^{-1}\Gamma_R C = \omega\,\Gamma_R^{T} \qquad \omega = + \text{ for } \Gamma = \gamma_\mu, \sigma_{\mu\nu}$$

$$C^{\alpha\beta} C_{\alpha\sigma} = \delta^{\beta}_{\sigma} \qquad \omega = - \text{ for } R = 1, \gamma_5, \gamma_\mu \cdot \gamma_5 \qquad 3.62$$

acts as a lowering operator and $(C^{-1})^{\alpha\beta}$ as a raising operator one and that under charge conjugation

$$\psi_\alpha \longrightarrow C_{\alpha\alpha'} \bar{\psi}^{\alpha'} \qquad 3.63$$

$$\phi_\alpha^\beta \longrightarrow \pm \; C_{\alpha\alpha'} (C^{-1})^{\beta\beta'} \phi_{\beta'}^{\alpha'} \qquad 3.64$$

The scalar expression given by eq. 3.6● is then trivially seen to be C invariant.

This Lagrangian may be written formally as

$$L = G\,(J_5\phi_5 + J_\mu\phi_\mu) \qquad 3.65$$

where ϕ_5 and ϕ_μ are the pseudoscalor and vector fields corresponding to the 0^- and 1^- nonets. In this section these will be the only mediators of our Yukawa exchanges. The $U(6,6) \otimes O(3)$ prediction for those parts of the pseudoscalar and vector currents relevant to the interaction of the 2^+ meson nonet with the 0^- and 1^- mesons are (neglecting SU(3) indices)

All symbols defined after Eq. 3.72

$$J_5^1 = \frac{\delta L}{\delta\phi_5} = \frac{4G}{S}\left(\frac{1}{MW} + \frac{1}{S} + \frac{M}{S.MW}\right) \{\bar{\phi}_{\mu\lambda}(P_3)\,\phi_5(P_1)\}_D \; P_1^\mu P_1^\lambda \qquad 3.66$$

$$J_\mu^1 = \frac{\delta L}{\delta\phi_\mu} = \frac{4G}{S}\left(\frac{1}{M.S} - \frac{1}{MW.M} - \frac{1}{MW.S}\right) \epsilon_{\mu\alpha\beta\nu} \; P_1^\rho P_1^\beta P_3^\alpha \; x$$

$$x \; \{\bar{\phi}_{\mu\rho}(P_3)\,\phi\,(P_1)\}_F \qquad 3.67$$

For the production of the 1^+ the relevant current is

$$J_\mu^1 = G\,A_\mu \,\frac{P_1^\lambda}{S}\, [\,\phi_\lambda\,(P_3)\,\phi(P_1)\,]_F \qquad 3.68$$

where

$$A_{\mu} = \frac{4}{M_1 S^2 . MW} [(M_1 S - MW.S - S^2)P_3^{\mu} + (M.S - M.M_o W - M_1^2)P_1^{\mu}] \tag{3.69}$$

The corresponding expressions for the bottom vertices which are symbolically represented in Fig.3.2a are given by

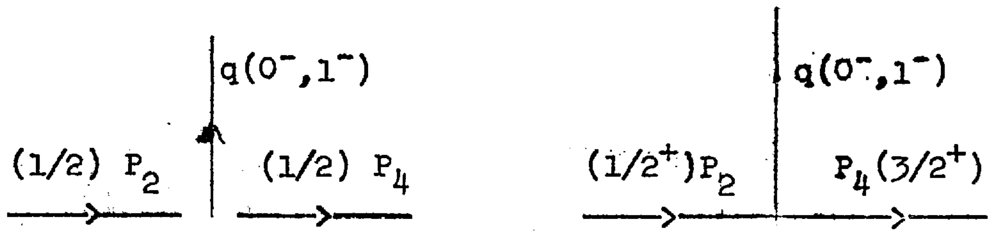


Fig. 3.2a $\frac{1}{2}^+(0^-,1^-)\ \frac{1}{2}^+$ and $\frac{1}{2}^+(0^-)\frac{3}{2}^+$ Vertices

$$J_5^2 = G_2(1 + \frac{2m}{s})\ \frac{P^2}{4m^2} \left[N(P_4)\gamma_5 N(P_a) \right]_{D+\frac{2}{3}F}$$

$$J_{\mu}^2 = G_2 [\ \frac{P_{\mu}}{2m}(1 + \frac{q^2}{2mV}) \left[\bar{N}(P_4)N(P_2) \right]_F + (1 + \frac{2m}{V}) \times \left[\bar{N}(P_4)\frac{r_{\mu}}{4m^2} N(P_2) \right]_{D+\frac{2}{3}F}] \tag{3.70}$$

$$P_{\mu} = (P_2 + P_4)_{\mu} \tag{3.71}$$

In the case the bottom vertex involves the formation of a resonance the current is

$$J_5^2 = G_2\left(1 + \frac{2MO}{s}\right) \frac{P_2^\lambda}{MO}\ \bar{D}_\lambda(P_4)N(P_2) \tag{3.72}$$

The masses appearing in these expressions are taken ad hoc to be the average of the octet, nonent or decuplet to which the particles belong: S = 417 Mev, average of the 0^-, MO = 1.1 MeV MV = 850, average of the 1^-, MW = 549 Mev, average of both 0^- and 1^-, M = 1375 Mev, average of the 2^+, M_1 = 1244, the average of the 1^+. These last two nonets not being too familiar as the others are shown in Fig.

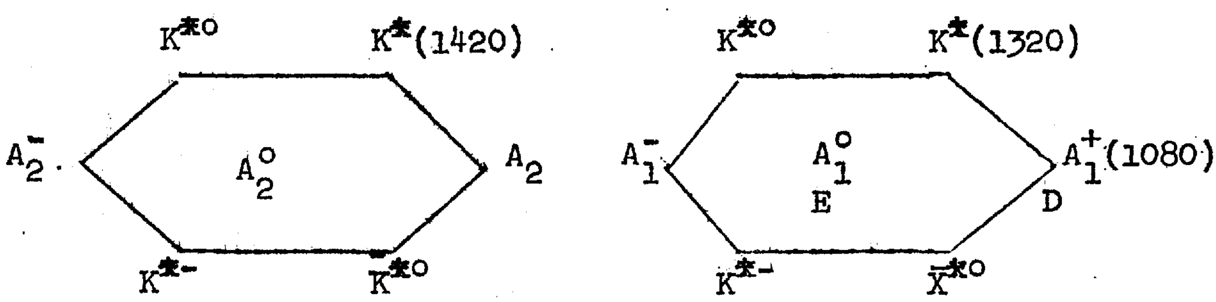


Fig. 3.3. The 2^+ and $\bar{1}^+$ Nonets.[84]

The relations expressed by eqs. 3.69 - 3.71 which are derived from the original U(6,6) Lagrangian of Salam[62]

$$L(P_2P_4) = \psi^{A'BC}(P_4)\phi^A_{A'}(9)\psi_{ABC}(P_2) \tag{3.73}$$

may obviously be regarded as being obtained from $U(6,6) \otimes O(3)$ with L = 0. P_μ and r_μ of eq. 3.70 are the conventional morms for the 'electric' and 'magnetic' interactions and divided by 2m and $4m^2$ respectively they are the coefficients of the Sach form factors. As far as the algebraic work is concerned it is more convenient to rewrite J_μ in terms of P_μ and γ_μ as follows

$$J_\mu = \left[M_1(q^2) \frac{P_\mu}{2m} \bar{N}(P_4)N(P_2) \right]_F + M_2(q^2) \left[\bar{N}(P_4)\gamma_\mu N(P_2) \right] D + \frac{2}{3}F \qquad 3.74$$

and this will be the form we shall be working with.

$$\begin{aligned} M_1 &= F_c - F_M \\ M_2 &= [1-q^2/4m^2]\, F_M \end{aligned} \qquad 3.75$$

The transition amplitudes for the production of these mesons may then be symbolically written as

$$\phi(s,t) = \sum_i J_5^1 \frac{1}{m_i^2 - t} J_5^2 + \sum_j J_\mu^{\prime} \left(\frac{g_\mu - q_\mu q_\mu / m_j^2}{m_j^2 - t} \right) J^2 \qquad 3.76$$

where the summation over i and j represent the number of allowed exchanges of 0^- and 1^- particles respectively. The masses in the propagators are taken to be that of the physical particle exchanged. More explicitly the amplitude for the production of a 2^+ via the exchange of a vector for example will be

$$\phi(s,t) = \frac{4G_1 G_2}{S} \left(\frac{1}{MWS} - \frac{1}{MW.M} - \frac{1}{MW.S} \right) \epsilon_{\mu\alpha\beta\nu} P_3^\alpha P_1^\beta P_1^\rho$$

$$\times\ \phi_{\mu\rho}(P_3)\phi(P_1) \left[- \frac{g_{\mu\sigma} + q_\mu q_\sigma/m_e^2}{m_e^2 - t} \right] [F_1(P_2+P_4)_\sigma \bar{N}(P_4)N(P_2)$$

$$+ F_2 \bar{N}(P_4)\gamma_\sigma N(P_2)] \qquad 3.77$$

where

$$F_1 = 1 + \frac{q^2}{2MV.m} - \frac{5}{3}(1 + \frac{2m}{MV}) \qquad 3.78a$$

$$F_2 = \frac{5}{3}(1 + \frac{2m}{MV})\ (1 + \frac{P_2 \cdot P_4}{m^2}) \qquad 3.78b$$

The contribution coming from the second term of the propagator will be zero as $\overline{N}(f-m) = (f_2-m)n(P_2) = 0$. Similarly one writes the other terms. The moments appearing in these expressions are diagramatically shown in Fig. and are given by

$$P_1 = \begin{bmatrix} E_1 \\ K \operatorname{Sin} \Theta \\ 0 \\ K \operatorname{Cos} \Theta \end{bmatrix} ; \quad P_2 = \begin{bmatrix} E_2 \\ -K \operatorname{Sin}\Theta \\ 0 \\ -K \operatorname{Cos}\Theta \end{bmatrix} ; \quad P_3 = \begin{bmatrix} E_3 \\ 0 \\ 0 \\ Q \end{bmatrix} ; \quad P_4 = \begin{bmatrix} E_4 \\ 0 \\ 0 \\ -Q \end{bmatrix} \qquad 3.79$$

which are their forms in the Centre of Mass frame; Q and K are the 3-momenta of the particles after an before collision (see footnote 100).

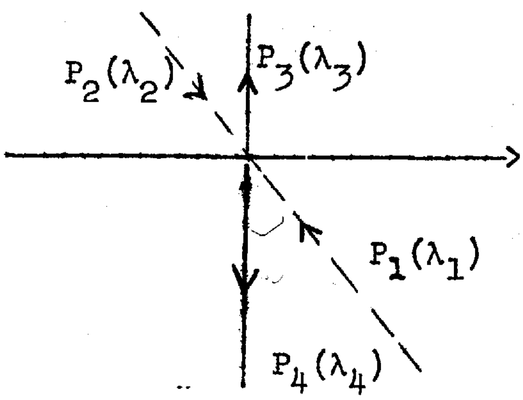


Fig. 3.3 Four Momenta and helicities in the C.M. frame.

The only expressionswe need to know are the components of the wave functions $\phi_\mu(P_3)$ and $\phi(P_3)$ for the various values of the helicity amplitude. They may be directly computed as follows, starting from the fact that the Spin One wave function are simply given by[101]

$$Y_{1,0} = \sqrt{\frac{3}{4\pi}}\,\mathrm{Cos}\,\Theta \quad , \quad Y_{1,\pm 1} = \mp \sqrt{\frac{3}{8\pi}}\; e^{\pm\phi}\,\mathrm{Sin}\,\Theta \qquad 3.80$$

in the rest frame of the particle. If we represent the 4 dimensional coordinate system by

$$x = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} ; \quad y = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} ; \quad z = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} ; \quad t = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad 3.81$$

then the wave functions of eq. 3.79 may be written, on a unit sphere, as

$$\phi_\mu\,[S_z = \pm 1;\ P = (m,0)\,] = \mp \frac{(x \pm iy)}{\sqrt{2}} = \frac{-1}{\sqrt{2}} = \frac{-1}{\sqrt{2}} \begin{bmatrix} 1 \\ \pm i \\ 0 \\ 0 \end{bmatrix} \qquad 3.82$$

$$\phi_\mu\,[S_z = o;\ P = (m,o)\,] = z = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad 3.83$$

Then by means of the following Lorentz boost[102]

$$L_{(z)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{E}{m} & \frac{P}{m} \\ 0 & 0 & \frac{P}{m} & \frac{E}{m} \end{pmatrix} \qquad 3.84$$

we obtain the wave functions for a particle of Spin 1, helicity $\lambda = \pm 1,0$ moment along the z axis and energy E:

$$\phi_\mu\,[\lambda,\ P = (E,0,0,P)\,] = L_\mu^\nu\ \phi_\nu[\ \lambda,\ P = m,0,0,0\ ] \qquad 3.85$$

The two wave functions of eq. 3.82 do not change under this boost but that of eq. 3.83 now becomes

$$\phi_\mu\ [\lambda,\ P = (m,0,0,P)] = \begin{bmatrix} 0 \\ 0 \\ E/m \\ P/m \end{bmatrix} \qquad 3.86$$

Should one wish to obtain the wave functions at an angle Θ to the z axis one only has to apply the rotation R(0,Θ,0) to this wave function:[101]

$$R(0,0,0) = = \begin{bmatrix} \text{Cos}\ \Theta & 0 & \text{Sin}\ \Theta & 0 \\ 0 & 1 & 0 & 0 \\ -\text{Sin}\ \Theta & 0 & \text{Cos}\ \Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad 3.87$$

The wave functions for a spin 2 particle can be easily obtained by suitable combinations of spin one wave functions:[103]

$$\phi_{\mu\nu}[\ s = \pm 2,\ P = (m,o)\ ] = \phi_\mu\ (s=\pm 1)\ \phi\ (s=\pm 1) \qquad 3.88$$

$$\phi_{\mu\nu}[s = \pm 1,\ P = (m,o)] = \frac{1}{\sqrt{2}}\ [\phi_\mu(s=o)\ \phi_\nu(s=\pm 1) \oplus \phi_\mu(s=\pm 1)\ x \\ x\ \phi_\nu\ (s=o)\ ] \qquad 3.89$$

$$\phi_\mu\ [s = o,\ P=(m,o)] = \frac{1}{\sqrt{6}}\ [\phi_\mu(s=1)\phi_\nu(s=-)\oplus\phi_\mu(s=-1)\phi_\nu(s=1) \\ \oplus\ 2\phi_\mu\ (s=o)\ \phi_\nu\ (s=o)\ ] \qquad 3.90$$

All the wave functions of the R.H.S. are rest-frame ones. To obtain the wave function for spin 2 with momentum P along the z axis we must Lorentz boost each index individually e.g.

$$\phi_{\mu\nu}\ [\ s=2,\ P = (m,0,0,P)] = [\ L(P)\ ]^\sigma_\mu\ \phi_\sigma[s=2,\ P=(m,0)] \otimes \\ [\ L(P)\ ]^\rho\ \phi_\rho\ [s=2,\ P=(m,0)] \qquad 3.91$$

It should be emphasized that on the R.H.S. of this equation we have an outer matrix multiplication and not an ordinary one, and is done as follows:[104]

$$A \otimes B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \otimes B = \begin{bmatrix} a_{11}B & a_{12}B \\ a_{12}B & a_{22}B \end{bmatrix}$$

In a similar way one obtains the spin $\frac{3}{2}$ and $\frac{5}{2}$ wave functions. Denoting the Dirac Spin $\frac{1}{2}$ wave functions by

$$\psi_+ = N\binom{E+m}{P} \otimes \chi_+ \qquad \psi_- = N\binom{E+m}{-P} \otimes \chi_- \qquad 3.91a$$

where

$$N = 1 \quad [2m(E+m)]^{1/2}, \; \chi_+ = \binom{1}{0} \text{ and } \chi_- = \binom{0}{1} \qquad 3.91b$$

then the $\frac{5}{2}$ wave function may be formally written as

$$\left|\tfrac{5}{2}, \pm\tfrac{5}{2}\right> = |2, \pm 2>\psi_\pm \qquad 3.92$$

$$\left|\tfrac{5}{2} \pm \tfrac{5}{2}\right> = \frac{1}{\sqrt{5}}[|2, \pm 2>\psi_\mp \oplus |2, \pm 1>\psi_\pm] \qquad 3.93$$

$$\left|\tfrac{5}{2}, \pm \tfrac{1}{2}\right> = \frac{2}{\sqrt{10}}[|2, \pm>\psi_\mp \oplus |2, 0>\psi_+] \qquad 3.94$$

Below we list the explicit forms of the wave functions for different helicity states:

Spin 1

<u>$\lambda = \pm 1$</u>: $\phi_1 = \pm 1/\sqrt{2}$, $\phi_2 = i/\sqrt{2}$, $\phi_2 = \phi_o = 0$

<u>$\lambda = 0$</u>: $\phi_1 = 0$, $\phi_2 = 0$, $\phi_3 = -E/m$, $\phi_o = -P/m$

Spin 2

$$\phi_{\mu\nu} = \phi_{\nu\mu}$$

$\underline{\lambda = \pm 2}$: $\phi_{11} = 1/2, \quad \phi_{12} = \pm i/2, \quad \phi_{22} = -\frac{1}{2}$,

$\phi_{13} = \phi_{23} = \phi_{33} = \phi_{oo} = \phi_{01} = \phi_{02} = \phi_{03} = 0$

$\underline{\lambda = \pm 1}$: $\phi_{13} = -\frac{E}{2m}, \quad \phi_{23} = -\frac{iE}{2m}, \quad \phi_{10} = \pm\frac{P}{2m}, \quad \phi_{20} = -\frac{iP}{2m}$,

$\phi_{11} = \phi_{22} = \phi_{12} = \phi_{33} = \phi_{30} = \phi_{00} = 0$

$\underline{\lambda = 0}$: $\phi_{11} = -1/\sqrt{6}, \quad \phi_{22} = -1/\sqrt{6}, \quad \phi_{33} = (\frac{2}{\sqrt{6}})\frac{E^2}{m^2}$,

$\phi_{30} = (2/\sqrt{6})PE/m^2, \quad \phi_{00} = (2/\sqrt{6})\frac{P^2}{m^2}$,

$\phi_{12} = \phi_{13} = \phi_{23} = \phi_{10} = \phi_{20} = 0$

Spin 3/2

$\underline{\lambda = \pm 3/2}$: $\psi_1 = \pm\frac{1}{\sqrt{2}}\psi_\pm, \quad \psi_2 = \frac{i}{2}\psi_\pm, \quad \psi_3 = \psi_o = 0,$

$\underline{\lambda = \pm 3/2}$: $\psi_1 = \pm\frac{1}{\sqrt{6}}\psi_\pm, \quad \psi_2 = \frac{i}{\sqrt{6}}\psi_\mp, \quad \psi_3 = -\frac{2}{\sqrt{6}}\frac{E}{m}\psi_\pm,$

$\psi_o = -\frac{2}{\sqrt{6}}\frac{P}{m}\psi_\pm$

Spin 5/2

$\underline{\lambda = \pm 5/2}$: $\psi_{11} = \frac{1}{2}\psi_\pm, \quad \psi_{12} = \frac{i}{2}\psi_\pm, \quad \psi_{22} = -\frac{1}{2}\psi_\pm,$

$\psi_{13} = \psi_{23} = \psi_{33} = \psi_{00} = \psi_{01} = \psi_{02} = \psi_{03} = 0$

$\underline{\lambda = \pm 3/2}$: $\psi_{11} = \frac{1}{2\sqrt{5}}\psi_\mp, \quad \psi_{12} = \frac{i}{2\sqrt{5}}\psi_\mp, \quad \psi_{22} = -\frac{1}{2\sqrt{5}}\psi_\mp$,

$\psi_{13} = -\frac{1}{\sqrt{5}}\frac{E}{m}\psi_\mp, \quad \psi_{23} = -\frac{i}{\sqrt{5}}\frac{E}{m}\psi_\pm, \quad \psi_{10} = -\frac{1}{\sqrt{5}}\frac{P}{m}\psi_\pm,$

$\psi_{20} = -\frac{i}{\sqrt{5}}\frac{P}{m}\psi_\pm, \quad \psi_{33} = \psi_{30} = \psi_{00} = 0$

$\lambda = \pm 1/2$: $\psi_{11} = -\frac{1}{\sqrt{10}} \psi_{\pm}$, $\psi_{22} = -\frac{1}{\sqrt{10}} \psi_{\pm}$, $\psi_{13} = \pm \frac{1}{\sqrt{10}} \frac{E}{m} \psi_{\mp}$

$$\psi_{23} = \frac{2}{\sqrt{10}} \frac{E^2}{m^2} \psi_{\pm}, \quad \psi_{10} = \mp \frac{1}{\sqrt{10}} \frac{P}{m} \psi_{\mp}, \quad \psi_{20} = -\frac{i}{\sqrt{10}} \frac{P}{m} \psi_{\mp}$$

$$\psi_{30} = \frac{2}{\sqrt{10}} \frac{PE}{m^2} \psi_{\pm}, \quad \psi_{00} = \frac{2}{\sqrt{10}} \frac{P^2}{m^2} \psi_{\pm}, \quad \psi_{12} = 0$$

We now have everything needed to evaluate our amplitudes except the coupling constant. For 2^+ production the basic couplings from which all the others are obtained by U(6,6) is $\frac{G^2_{A\rho\pi}}{4\pi}$. This obtained from[105]

$$\left[\frac{4}{S} \left(\frac{1}{MW} + \frac{1}{S} + \frac{M}{S.MW} \right) \right]^2 = \frac{G^2_{A\rho\pi}}{4\pi} \qquad 3.95a$$

and

$$\Gamma = \frac{G^2_{A\rho\pi}}{4\pi} \cdot \frac{1}{10} \cdot \frac{Q^5}{M^4} \qquad 3.95b$$

where Q is the three momenta of the vector meson in the rest frame of the decay $2^+ \rightarrow 1^- \, 0^-$, M is the mass of the A_2 and Γ is the width for the decay. Γ was tkaen to be equal to 0.28 Gev. The basic U(6,6) coupling $\frac{g^2_{NN\pi}}{4\pi}$ for the bottom vertex is obtained from

$$\frac{g^2_{NN\pi}}{4\pi} \left(1 + \frac{2m}{S}\right) \frac{25}{9} = 14.9 \qquad 3.96$$

For the case of the 1^+ production the basic coupling was derived from the relationship

$$\Gamma = \frac{G^2_{A_1\rho\pi}}{4\pi} \cdot \frac{1}{3} \cdot \frac{Q^3}{m^2} \qquad 3.97$$

$$\left[\frac{4}{M_1 S^2 MW} \right]^2 \frac{g^2_{A_1\rho\pi}}{4\pi} = \frac{G^2_{A_1\rho\pi}}{4\pi} \qquad 3.98$$

In writing down the couplings we have to take into account mixing between the eighth member of the octet and the singlet. If W is the former and $\emptyset$ the latter then the respective physical particles will be given by

$$W_{phy} = \text{Sin}\ \theta \,|\, W\rangle + \frac{1}{\sqrt{3}}\ \text{Cos} \,|\, \emptyset\rangle$$

$$\emptyset_{phy} = \frac{1}{\sqrt{3}}\ \text{Sin}\ \theta \,|\, \emptyset\rangle - \text{Cos} \,|\, W\rangle$$

The D type Lagrangian involving 'mixed' particles will be of the form

$$\text{Tr}\,[\,\overline{M}\,\lambda_i\, M + \overline{M}\, M\, \lambda_i\,]\, M^i$$

where we sum over i from 0 to 8. As $\lambda_o = 1$ this trace can be written as

$$\sum_{i=1}^{8} \text{Tr}\,[\,\overline{M}\,\lambda_i\, M + \overline{M}\, M\, \lambda_i\,]\, M^i + \text{Tr}(\overline{M}\, M + \overline{M}\, M)$$

It is the last term which involves mixing. It should be noted that for F type Lagrangian this last term is zero. The F^o for example will be given, for a mixing angle $\theta = 30^o$, by

$$F^o_{phy} = \frac{1}{2}\,|\,F^o\rangle + \frac{\sqrt{3}}{2} \quad \frac{1}{\sqrt{3}}\,|\,F^*\rangle$$

where F^o and the F^* are the 1220 Mev and 1500 Mev 2^+ mesons respectively. The couplin will then be

$$g_{\pi^+\overline{F}^o\pi^-} = \frac{1}{2}\, g_{\pi^+\overline{F}^o\pi^-} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{3}}\, g_{\pi^+\overline{F}^*\pi^-}$$

From the Lagrangians we have written down we have the following general types of couplings

$2^+ \rightarrow 1^- \; 0^-$ F type

$2^+ \rightarrow 0^- \; 0^-$ D type

$1^+ \rightarrow 0^- \; 0^-$ F type

$1^+ \rightarrow 1^- \; 0^-$ D type

Assuming a mixing angle of 10°, 40° and 30° respectively for the 0^-, 1^- and 2^+ the various couplings of interest are given in terms of the basic ones as follows:

2^+ Production Vertex 1

$$g_{6\pi\pi} = g_{A_2\rho\pi} \; ; \quad g_{A_2\eta\pi} = 1.286 \; ; \quad g_{K^{*o}\pi K} = 1.0 \; ;$$

$$g_{K^*\pi K} = \frac{1}{\sqrt{2}} g_{A_2\rho\pi} \; ; \quad g_{K^*\phi K} = g_{A_2\rho\pi} \; ; \quad g_{K^*\omega K} = \frac{1}{\sqrt{2}} g_{A_2\rho\pi}$$

$$g_{K^*\rho K} = \frac{1}{\sqrt{2}} g_{A_2\rho\pi} \; ; \quad g_{K^{*-}\eta K} = 0.964$$

For 1^+ Production

$$g_{K^*\omega K} = \frac{2}{\sqrt{3}} g_{A_1\rho\eta} \; ; \quad g_{K\rho K} = \frac{1}{\sqrt{2}} g_{A_1\rho\pi}$$

Vertex 2

$g_{N\eta N} = 0.214 \, g_{N\pi N}$ for F type

$\phantom{g_{N\eta N}} = 0.984 \, g_{N\pi N}$ for D type

$g_{N\phi N} = \frac{1}{\sqrt{2}} g_{N\pi N}$ both for F and D type

$g_{N\rho N} = \frac{1}{\sqrt{2}} g_{N\pi N}$ both for F and D type

$g_{N\omega N} = \frac{1}{\sqrt{2}} g_{N\pi N}$ both for F and D type

$g_{N^*\pi N} = g_{N\pi N}$

The amplitudes are decomposed in the helicity formalism of Jacob and Wick[106]

$$\langle P_3\lambda_3, P_4\lambda_4 | T | P_1\lambda_1, P_2\lambda_2\rangle = \phi_{\lambda\mu}(s,t) \qquad 3.99$$

$$\phi_{\lambda\mu}(s,t) = \sum_J (2J+1)\ T^J(s)\ d^J_{\lambda\mu}(t) \qquad 3.100$$

where

$$\lambda = \lambda_2 - \lambda_1, \qquad \mu = \lambda_4 - \lambda_3 .$$

Then making use of the orthogonality of the $d^J_{\lambda\mu}(\Theta)$ functions

$$\int_{-1}^{+1} d^J_{\lambda\mu}(\text{Cos }\Theta)\ d^{J'}_{\lambda\mu}(\text{Cos }\Theta) = [2/(2J+1)]\delta_{JJ'} \qquad 3.101$$

we obtain the partial wave helicity amplitudes

$$T^J(s) = \frac{1}{2}\int_{-1}^{+1} \phi(s,t) d^J_{\lambda\mu}(\text{Cos }\Theta)\ d(\text{Cos }\Theta) \qquad 3.102$$

Once this partial wave amplitude has been modified according to the prescription of eq. 3.1

$$T'(s) = (S)^{1/2}\ \ T(s)\ (S)^{1/2} \qquad 3.1$$

the unitarised amplitude may be written in the form

$$\phi'_{\lambda\mu}(s,t) = \sum_J (2J+1)\ T'(s)\ d^J_{\lambda\mu}(\text{Cos }\Theta) \qquad 3.103$$

The $d^J_{\lambda\mu}(\text{Cos}\Theta)$ functions may be written in terms of Legendre polynomials. In terms of the Jacobi d_N polynomials they assume the form[107]

$$d^J_{\lambda\mu}\ (\mathrm{Cos}\ \theta) = \left[\frac{(J+\lambda)!\,(J-\lambda)!}{(J+\mu)!\,(J-\mu)!}\right]^{1/2} (\mathrm{Cos}\tfrac{1}{2}\theta)^{\lambda+\mu}\ (\mathrm{Sin}\tfrac{1}{2}\theta)^{\lambda-\mu}\ \times$$

$$\times \quad P^{J-\lambda}_{\lambda-\mu,\,\lambda+\mu}\ (\mathrm{Cos}\ \theta)$$

3.104

$$P^J_{oo}(\mathrm{Cos}\ \theta) = P_j(\mathrm{Cos}\ \theta)$$

3.105

The other relevant Jacobi Polynomials may be written as linear combinations of Legendre polynomials by making use of these two recurrence relations:

$$\tfrac{1}{2}\ (2 + \alpha + \beta + 2n)\ (x + 1)\ \ P^n_{\alpha\beta+1}\ (x)$$

$$= (n + 1)\ P^{n+1}_{\alpha\beta}\ (x) + (1 + \beta + n)\ P^n_{\alpha,\beta}(x)$$

3.106

$$\tfrac{1}{2}\ (2 + \alpha + \beta + 2n)\ (x - 1)\ P^n_{\alpha+1,\beta}(x)$$

$$= (n + 1)\ P^{n+1}_{\alpha\beta}(x) - (1 + \alpha + n)\ P^n_{\alpha,\beta}(x)$$

3.107

The derivation is simplified if we use the property

$$d^J_{\lambda\mu}(\theta) = (-1)^{\mu}\ d^J_{\lambda\mu}(\theta) = (-1)^{\lambda}\ d^J_{-\mu-\lambda}(\theta)$$ 3.108

There 20 and 12 amplitudes respectively for 2^+ and 1^+ productions but by means of eq. 3.125 these numbers are halved.

3.5 Production of $5/2^+$ Baryon

This Baryon is usually regarded as the first excited state of the proton and in the Kinetic Supermultiplet scheme it is clssified in an SU(6) octet characterised by (56,1;L=2). The reason why we choose the above multiplet and not the (56,1;L=1) one is that the latter, if we write $L=\alpha(t)=\alpha_B(t)-\frac{1}{2}$, α_B being the Regge trajectory, will not be the first recurrence of the basic particle if we take into consideration the signature rule Δ L=2 [108]. Specifically the $5/2^+$ octet and the $7/2^+$ decimet are the first recurrences of the $1/2^+$ octet and $3/2^+$ decimet contained in the (56,1;L=0) whereas the remaining multiplets of the (56,1;L=1) lie on a Regge trajectory that becomes nonsense at L=0.

The wave function describing this particle is given in the terminology of Salam by $\psi_{ABC,\mu\nu}(P)$ on which we impose the following constraints so that it describes a $5/2^+$ particle amongst others

$$\psi_{ABC,\mu\nu}(P) = \psi_{ABC,\nu\mu}(P) \tag{3.109}$$

$$g^{\mu\nu}\psi_{ABC,\mu\nu}(P) = 0 \quad , \quad g^{\mu\nu} \text{ is the metric,} \tag{3.110}$$

$$P^{\mu}\psi_{ABC,\mu\nu}(P) = 0 \tag{3.111}$$

The wavefunction is completely symmetric in A,B,C and obeys the generalised Bargmann-Wigner equation introduced by Salam et al [62,66,109]

$$(\gamma.P - M)_{\alpha}^{\alpha'}\psi_{\alpha'\beta\gamma,\mu\nu}(P) = 0 \tag{3.112}$$

We then can decompose it as follows

$$\Psi_{ABC,\mu\nu}(P) = \Big[(\gamma_\rho C)_{\alpha\beta}\ D_{\rho\gamma\ \mu\nu}(P) + \frac{i}{2m}(\sigma_{\rho\lambda}C)(P_\rho D_{\lambda\gamma\mu\nu}(P) - P_\lambda D_{\rho\gamma\mu\nu}(P)\ )\Big]_{abc}$$

$$+ \Big[(\gamma\underline{P}+M)\gamma_5 C(\Psi^{S}_{\gamma\mu\nu}(P))_a \quad sbc + \text{cyclic permutation of abc}\Big] + \ldots\ldots\ldots \qquad 3.113$$

where the D's and Ψ's are generalised Rarita-Schwinger wave-functions describing the $7/2^+$ decimet and the $5/2^+$ octet respectively; the dots stand for the remaining multiplets contained in the (56,1;L=2).

On the other hand the simplest object describing a 5/2+ resonance correctly i.e. with the right parity, Baryon number 1, charge e etc is $\Psi^{FG}_{ABCDE}(P)$ in the Global formalism. Its U(6,6) decomposition is

$$12\otimes12\otimes12\otimes12\otimes12\otimes\overline{12}\otimes\overline{12}$$

$$= \underline{56,056} \oplus 4 \times \underline{370656} \oplus 5 \times \underline{623700} \oplus 6 \times \underline{876096} \oplus 5 \times \underline{880880} \oplus \underline{324324} \oplus 10 \times \underline{5720} \oplus 30 \times \underline{35100} \oplus 10 \times \underline{16016} \oplus \underline{46332} \oplus 4 \times \underline{308308} \oplus 5 \times \underline{520520} \oplus 6 \times \underline{731808} \oplus 5 \times \underline{737100} \oplus 4 \times \underline{741312} \oplus \underline{272272} \oplus 20 \times \underline{220} \oplus 40 \times \underline{576} \oplus 20 \times \underline{364} \qquad 3.114$$

Picking the parts describing a 5/2 spin particle from this will be long and tedious a task and the Lagrangian involving them will be almost impossible to manipulate. So in the name of simplicity we have to reject this scheme.

The U(6,6) $\otimes$O(3) interaction Logragian for the vertex containing this particle is

$$L\ (P_1 P_3) = G_1 \overline{\Psi}^{A'BCD\mu\nu}(P_3)\ \phi^{A}_{A'}(q)\ \Psi_{ABC}(P_1)\ \frac{q_\mu q_\nu}{M^2} \qquad 3.115$$

Corresponding to the vertex shown in Fig. 3.2b.

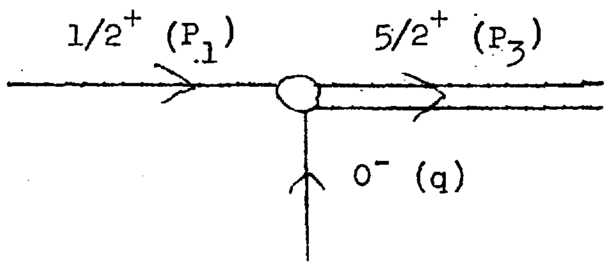


Fig. 3.2b Vertex with resonance formation

This may be written formally as

$$L = G_1\ (J^{I}_5\ \phi_5 + J^{I}_\mu\ \phi_\mu) \qquad 3.116$$

The only current we will need will be J^{I}_5 as we consider a 0^- exchange only

$$J^{I}_5 = G_1 \left[1 + \left(\frac{m + M}{S}\right)\right] \left[\frac{(m+M)^2 - q^2}{2mM}\right] \Psi_{\mu\nu}(P_3)\gamma_5 N(P_1)\frac{q^\mu q^\nu}{M^2}$$

(see pp 93. equ. 3.72) 3.117

The transition amplitude will be given as before by the first term of eq. 3.75. G_2, the U(6,6) coupling for NN_π is given as before and we make the simple assumption that G_1 is equal to it. As we have no way at present of relating couplings of particles belonging to the multiplets (56,1;L) for different values of L this assumption is not unreasonable. (see sect. A.5. of App. A.)

It should be noted that again owing to parity conservation the number of independent helicities is halved to 24. See eq. 3.125.

3.6 Exchange of 2^+ Meson

In this section we look at the exchange of a spin 2 particle in the reaction $\pi^- P \longrightarrow \eta\ n$.

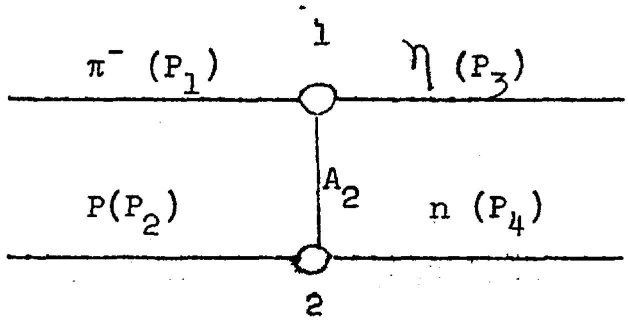


Fig. 3.3 Exchange of a Spin-2 particle.

The U(6,6) ⊗ O(3) Lagrangian describing the top-vertex is the one given by eq. 3.60 with slight alterations of the arguments:

$$L = G_1 \phi^A_{B\mu}(q) \left[\bar{\phi}^C_A(P_3)\phi^B_C(P_1) - \phi^C_A(P_1)\bar{\phi}^B_C(P_3) \right] \frac{(P_1+P_3)^\mu}{S}$$

3.118

The current involved in the vertex 1 is obtained by functional differentiation of L and is given by

$$J = \frac{\delta L}{\delta\phi_{\mu\nu}} = \frac{4G_1}{M^2} \left(\frac{1}{MW} + \frac{1}{M} + \frac{S}{M.MW}\right) \left[\phi_5(P_3)\phi_5(P_1)\right]_D P_1^\mu(P_1+P_3)^\nu$$

3.119

The Lagrangian involving vertex 2 is given by

$$L\ (P_2P_4) = G_2 \Psi^{A'BC}\ (P_4)\ \phi_{A'\sigma}(q) \Psi_{ABC}\ (P_2)\ \left(\frac{P_2+P_4}{M}\right)^\sigma$$

3.120

and the current obtained by functional differentiation

$$J^2_{\sigma\lambda} = \frac{\delta L}{\delta\phi_{\sigma\lambda}} = \left[(P_2+P_4)_\lambda\ F_1\left[\bar{N}(P_4)N(P_2)\right]_F + F_2[\bar{N}(P_4)\gamma_\lambda N(P_2)]\right] \times \left(\frac{P_2+P_4}{M}\right)_\sigma$$

3.121

where F_1 and F_2 are given by eqs. 3.77a and 3.77b.

The transition amplitude for this reaction is then given by

$$\phi\ (s,t) = J^{1}_{\mu\nu} \frac{P_{\mu\nu\sigma\lambda}}{M^2-t} \quad J^{2}_{\sigma\lambda} \tag{3.122}$$

where $P_{\mu\nu\sigma\lambda}$ the spin 2 propagator is given by

$$P_{\mu\nu\sigma\lambda} = \frac{1}{2} \quad (d_{\mu\sigma}\, d_{\nu\lambda} + d_{\mu\lambda}\, d_{\nu\sigma} \quad - \quad \frac{2}{3}\, d_{\mu\nu}\, d_{\sigma\lambda}) \tag{3.123}$$

and

$$d_{\mu\nu} = g_{\mu\nu} - q_{\mu}\, q_{\nu} \,/\, M^2 \tag{3.124}$$

As we have mentioned before it would be rash to expect the Absorptive peripheral model to be able to cope with a spin 2 exchange when we know it already fails for spin 1 exchanges. So we shall not modify the partial wave Born term as prescribed by eq. 3.1 i.e. not by

$$T^{J'}(s) = (S^J)^{1/2} \quad T^J(s) \quad (S^J)^{1/2} \tag{3.1}$$

but rather by that of eq. 3.37

$$T^{J'}(s) = B^J \frac{1}{1-1/4\,(B^J)^2}\, S^J_{ii} \tag{3.37}$$

where S_{ii} represents elastic scattering in the initial state.

For this reaction there are four helicity amplitudes which can be written formally as $\langle\lambda_3\lambda_4/T/\lambda_1\lambda_2\rangle$ where $\lambda_1 = \lambda_3 = 0$ and $\lambda_2 = \lambda_4 = \pm 1/2$. However from the following relationship, which comes from Parity invariance, we can reduce the number of independent amplitudes from 4 to 2:

$$\langle\lambda_3\lambda_4/T/\lambda_1\lambda_2\rangle = \frac{\eta_3\eta_4}{\eta_1\eta_2}(-1)^{S_3+S_4\ -\ S_1 S_2}\ (-1)^{\lambda-\mu}\langle-\lambda_3-\lambda_4/T/-\lambda_1-\lambda_2\rangle \tag{3.125}$$

Here the η's are the basic parities of particles and s their ~~parities~~ spins. Written explicitly these two amplitudes, are, in the Born approximation of eq. 3.122, as follows

$$B_{++} = \langle o+/T/0+\rangle \qquad 3.126$$

$$B_{+-} = \langle o+/T/0-\rangle \qquad 3.127$$

As modified by the prescription of eq. 3.37[83] they become

$$(T^J_{++})' = \left[B^J_{++} \quad X^J_{++} + B^J_{+-} \quad X^J_{+-}\right]\left[(S^J_{11})_{++}\right] \qquad 3.128$$

$$(T^J_{+-})' = \left[B^J_{+-} \quad X^J_{++} + B^J_{++} \quad X^J_{+-}\right]\left[(S_{11})_{++}\right] \qquad 3.129$$

$$X^J = \left[1 - \tfrac{1}{4}(B^J)^2\right] \qquad 3.130$$

It is these modified partial waves which are then used in eq. 3.103 to obtain our absorbed amplitudes. Our normalization is such that the differential cross section for unpolarised particles in the initial state is given by[110]

$$\frac{d\sigma}{dt} = \frac{1}{(2S_1 + 1)(2S_2 + 1)} \frac{1}{16\pi SQ} \sum_i |\phi_i|^2 \qquad 3.131$$

where the summation is over the independent helicity amplitudes. The density matrix elements will only be plotted for those which appear in the decay $2^+ \longrightarrow 0^- 0^-$ and $2^+ \longrightarrow 1^- 0^-$. They are obtained from[111]

$$\rho_{\lambda_3'\lambda_3} = N \sum_{1,2,4} \langle\lambda_3' \lambda_4/T/\lambda_1\lambda_2\rangle\langle\lambda_3\lambda_4/T/\lambda_1\lambda_2\rangle^* \qquad 3.132$$

where $N = \dfrac{1}{2\sum|\phi_i|^2}$ 3.133

This density matrix is $(2S_3+1)^2$ but the elements have the following properties:

i) Trace property: $T_r(\rho) = 1$

ii) Parity: $\rho_{mm'} = (-1)^{m-m'} \rho_{-m-m'}$

iii) Hermiticity: $\rho_{mm'} = \rho^*_{m'm}$

3.7 Comparison With Experiment and Conclusions

i) The results for the case $PP(\pi^0)N(5/2)P$ is amazing. This relatively simple model successfully describes the formation of the $5/2^+$ resonance. We have no normalisation factor and no parameter either: the differential cross-section is well accounted for both in shape and in absolute value, for the region of validity of the peripheral model.

ii) Again this approach successfully describes the formation of the f^0 and the f^0N^{*++} double resonance. These above three productions, it should be remembered, are mediated by 0^- particles.

iii) The results for $\pi^+P(\eta,\rho)A_2^+ P$, $K^-P(\pi,\eta,\rho,\phi,\omega)K^{*-} P$ $K^-P(\pi^-,P^-)K^{*0}$, , $\pi^+P(\rho)A_1$, $\pi^+P(\omega,\rho)BP$, and $K^-P(\rho,\omega)\, K_1^{*-}P$, where the exchanged particles are indicated in the brackets, are bad. The contribution coming from the 0^- particles are in agreement with experimental data but the contribution of the 1^- exchanges are two to three orders of magnitude too large and they also have the wrong t dependence but this behaviour seems to improve with increasing lab momentum. This is in contrast to the results of the earlier U(6,6) applications[112] where the vector.constributions were found, to be smaller than those of the pseudoscalar ones and so, thereone had good agreement for a wide variety of processes. The spin density for some of the processes are given with the 0^- and 1^- contributions shown separately. Forthcoming experimental results will no doubt indicate which are the dominant contributions. However this will not count in favour of $U(6,6)\otimes O(3)$ The present failure would seem, overall, to be symptomatic of the absorbed peripheral model's inability to prevent the well-known wrong vector particles t dependence.

iv) The modification of the usual peripheral model as expressed by eq. 3.37 does not work for the spin 2 exchange. This model does not predict either the shape or the absolute value of the differential cross-section. As there are only two partial waves here we have included them to show how they are modified by Absorption.

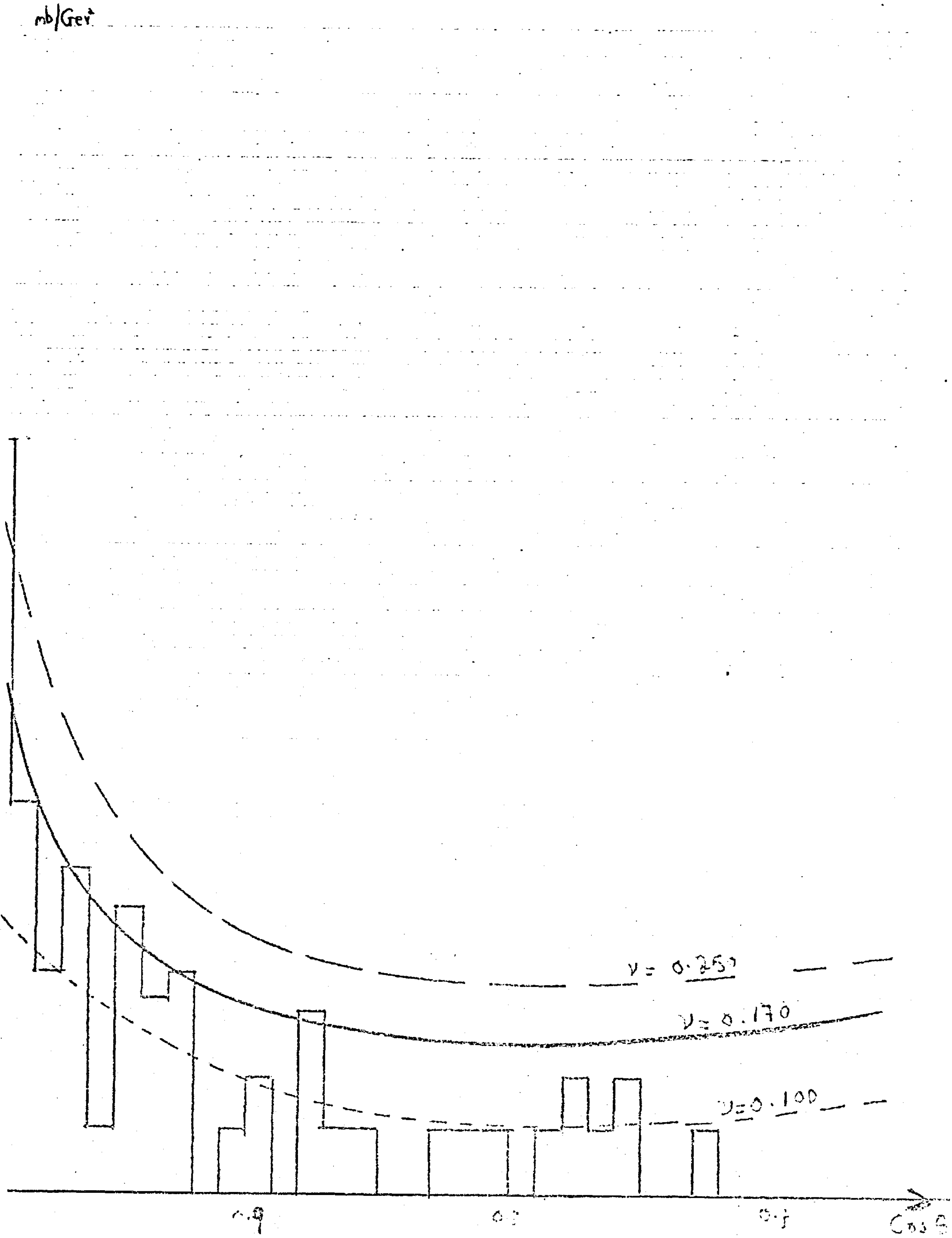
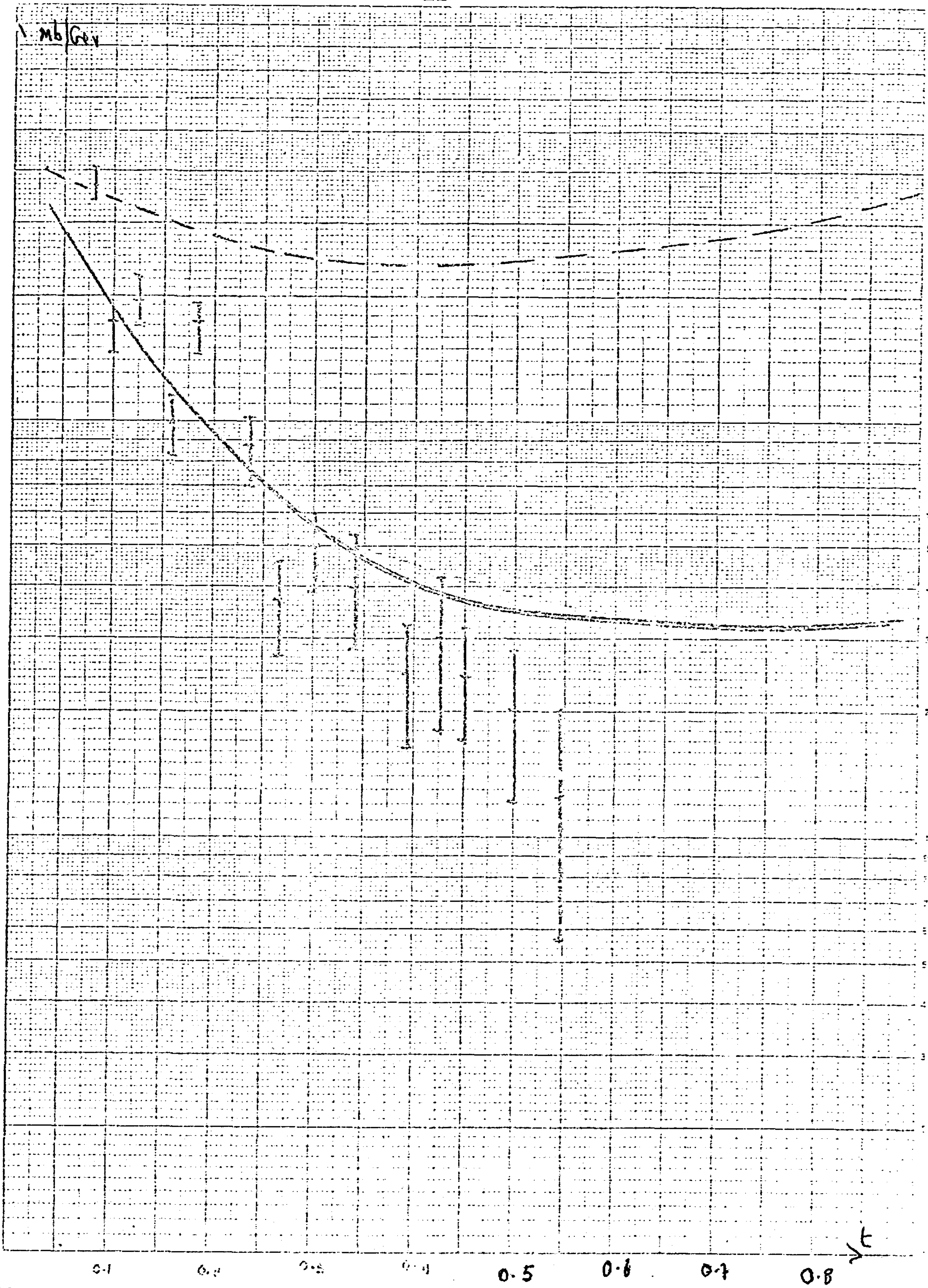


Fig. 3.4 Differential Cross-section against Cos θ for pp ⟶ $N^*(\frac{5}{2}^+)P$ at 5.5 Gev/c for different values of $\nu = R_f^{-1}(s)$. Data from Ref. 116. (R_f is radius of interaction in final state)



ig. 3.5 Differential Cross-section against t for $\pi^- P \longrightarrow F^o n$ at 4.0 Gev. —with absorption; - - - - without absorption. Data from Ref. 113.

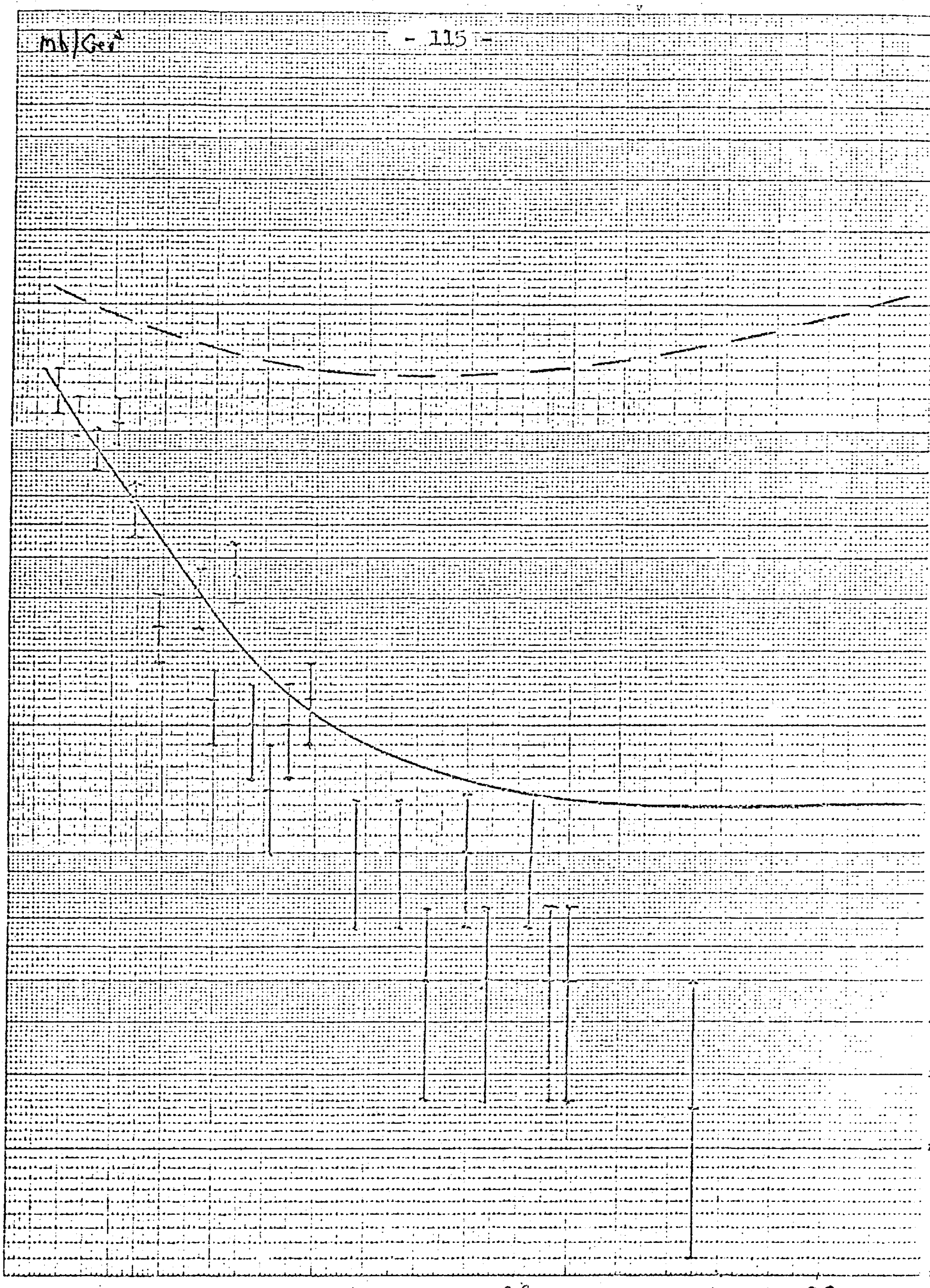


Fig. 3.6 Differential Cross-section against t for $\pi^+ n \longrightarrow F^0 p$ at 6.0 Gev. —— with absorption; - - - - without absorption. Data from Ref. 113

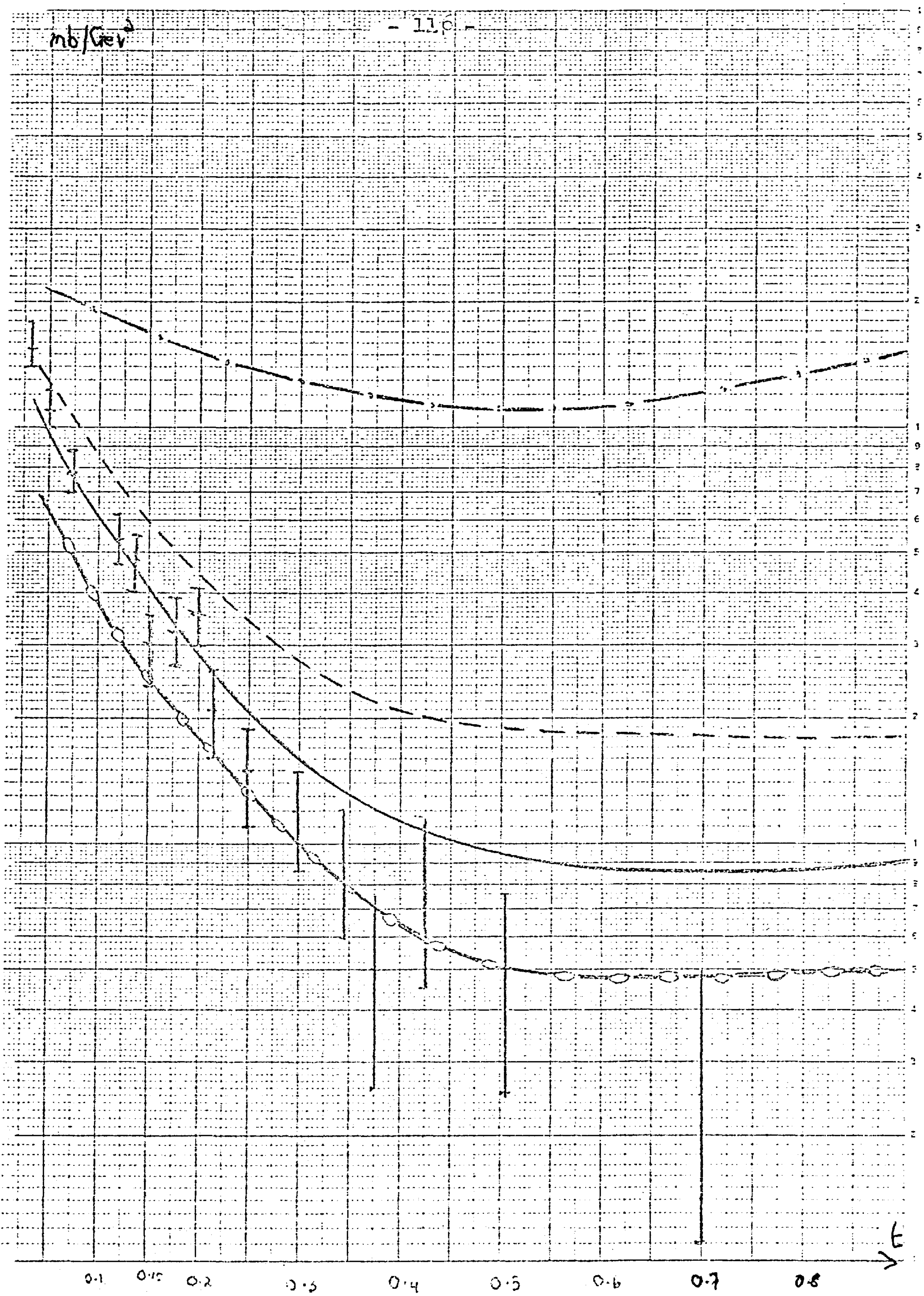
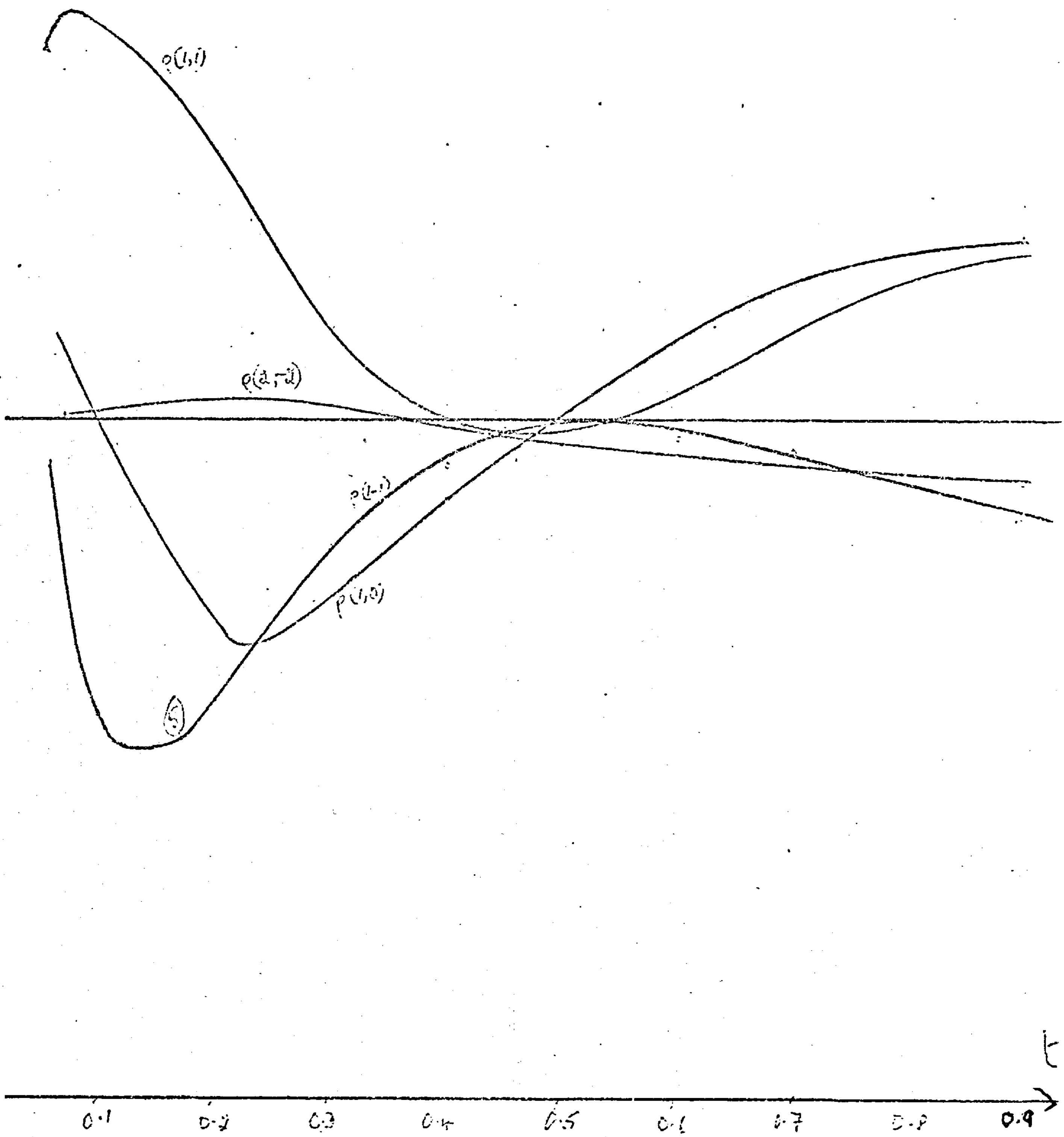


Fig. 3.7 Differential Cross-section for $\pi^- P \longrightarrow F^0 n$ at 8.0 Gev/c for different values of $R_f^{-1}(s) = \nu$; —— ν = 0.270; ---- ν = 0.4; -o-o-o- γ = 0.1; -·-·-·- without absorption. Data from Ref. 114.



ig. 3.8 Density-Matrix elements for decay of F^{0} at 8.0 Gev/c.

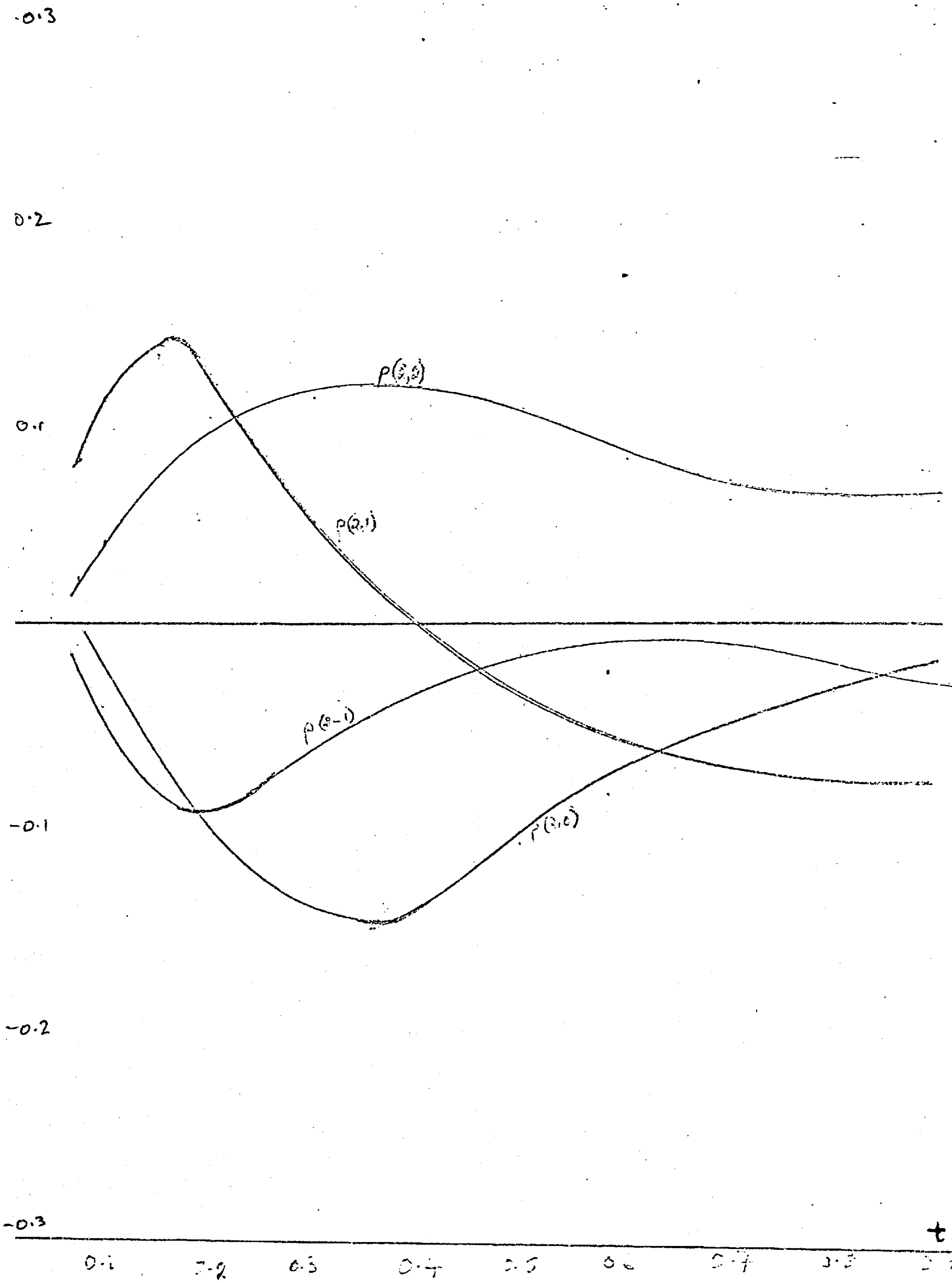


Fig. 3.9 Density-Matrix elements for decay of F^{O} at 8.0 Gev/c.

ig. 3.10 Differiential cross-section for $\pi^+ p \longrightarrow f^0 N^{*++}$ at 8.0 Gev/c
——with absorption; - - - - without absorption.

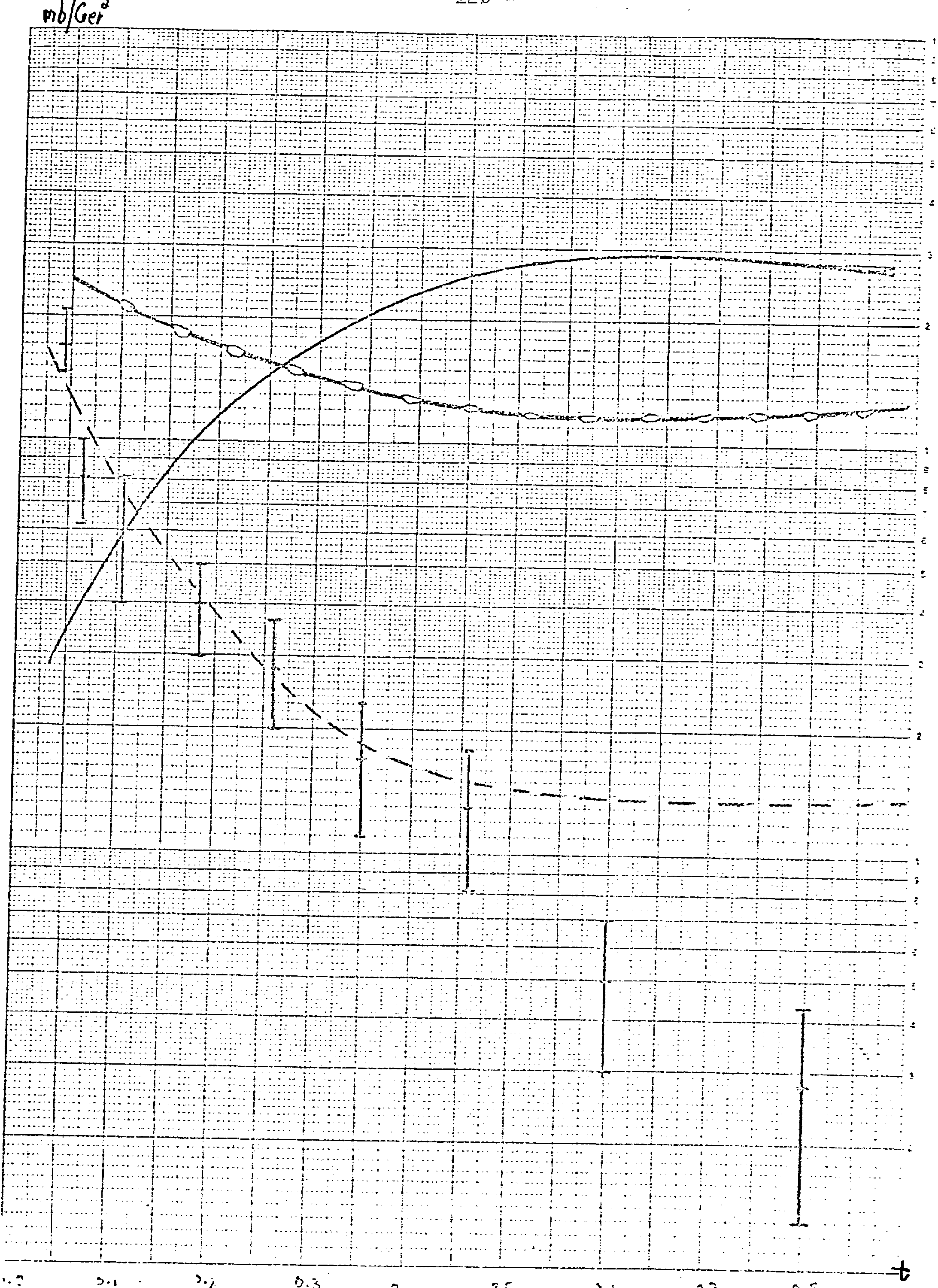


Fig. 3.11 Differential cross-section for $\pi^+ p \longrightarrow A_2 P$ at 8.0 Gev/c

- - - - - η exchange; -o-o-o-o- ρ exchange; ——— total.

Data from Ref. 114.

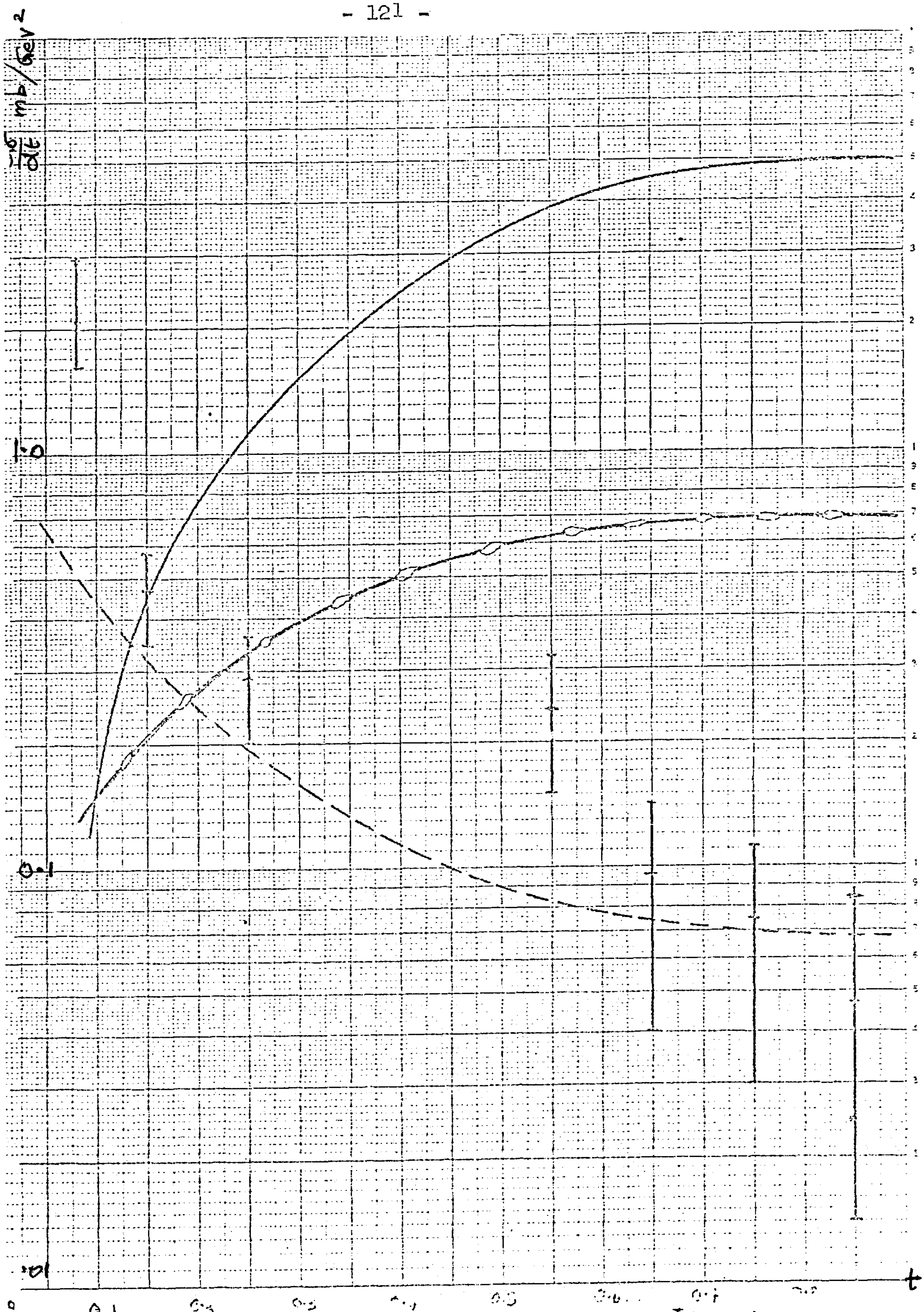


Fig. 3.12 Differential cross-section for $K^-P \longrightarrow K^*(1420)n$ at 4.1 Gev/c - - - π^- exchange; -o-o-o-o- ρ exchange; —— total
Data from Ref. 115

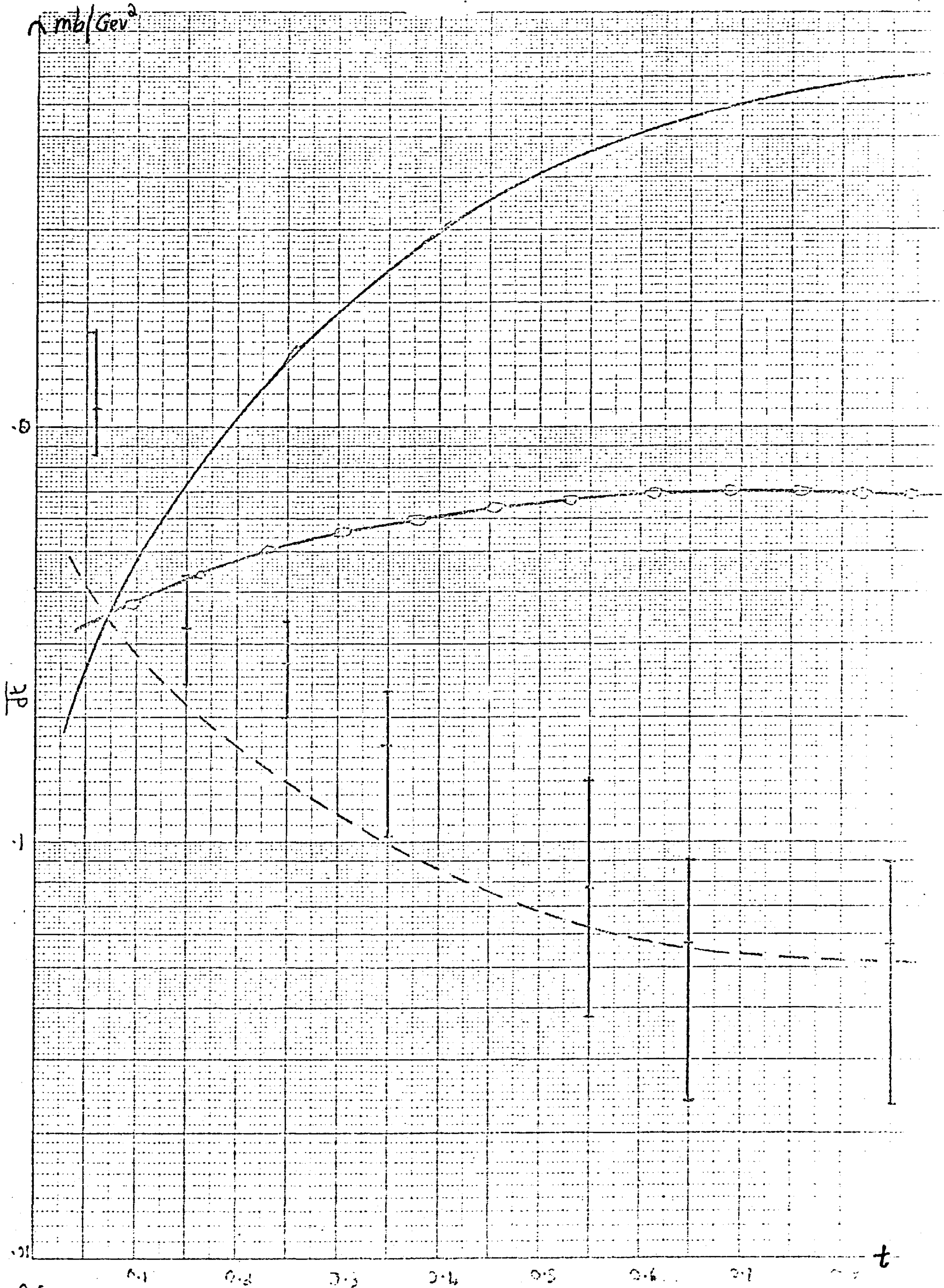


Fig. 3.13 Differential cross-section for $K^-P \longrightarrow K^*(1420)m$ at 5.5. Gev/c. - - - - π^- exchange; -o-o-o- ρ exchange $\times 10^{-1}$ —— total $\times 10^{-1}$ Data from Ref. 115.

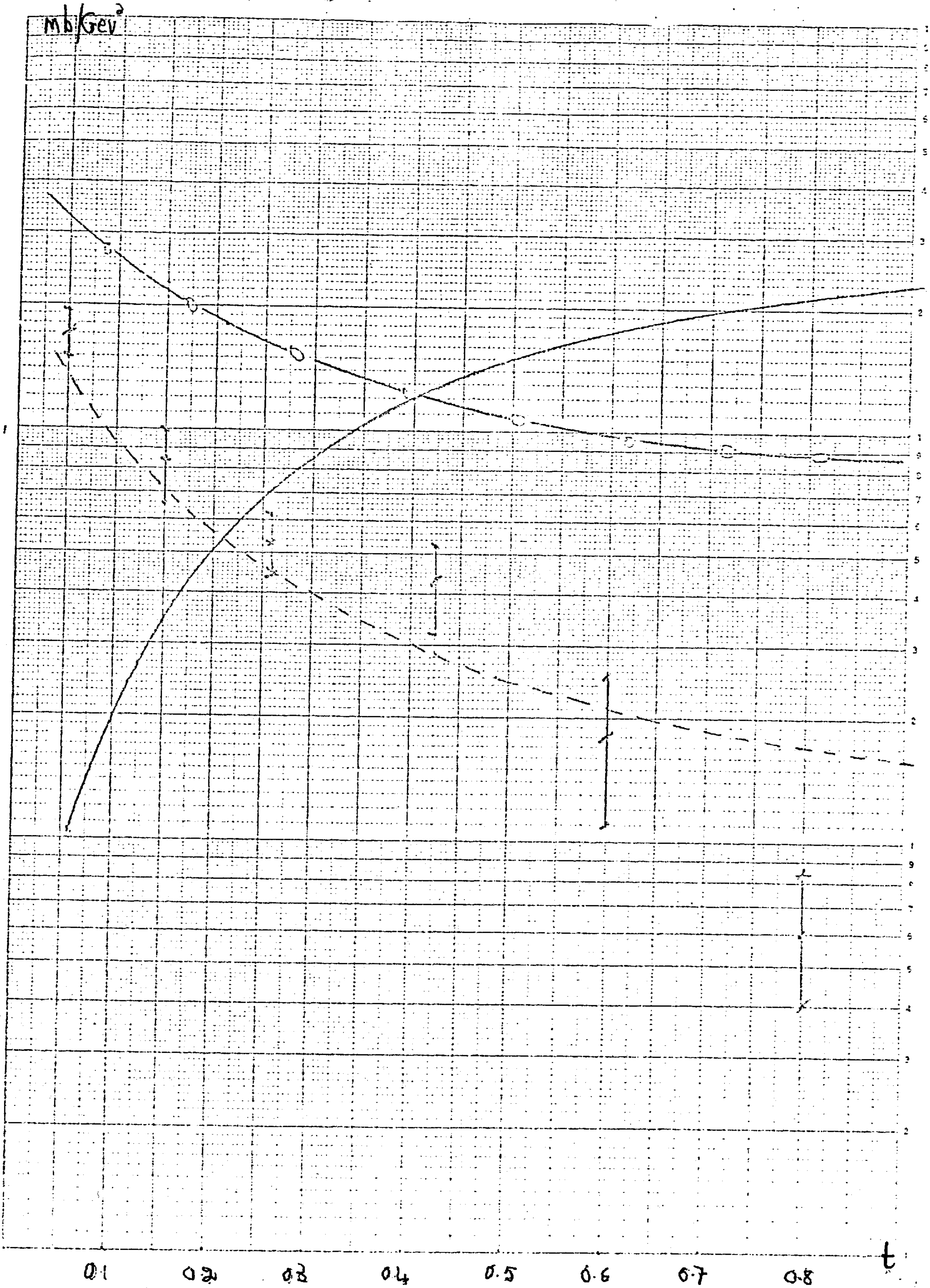
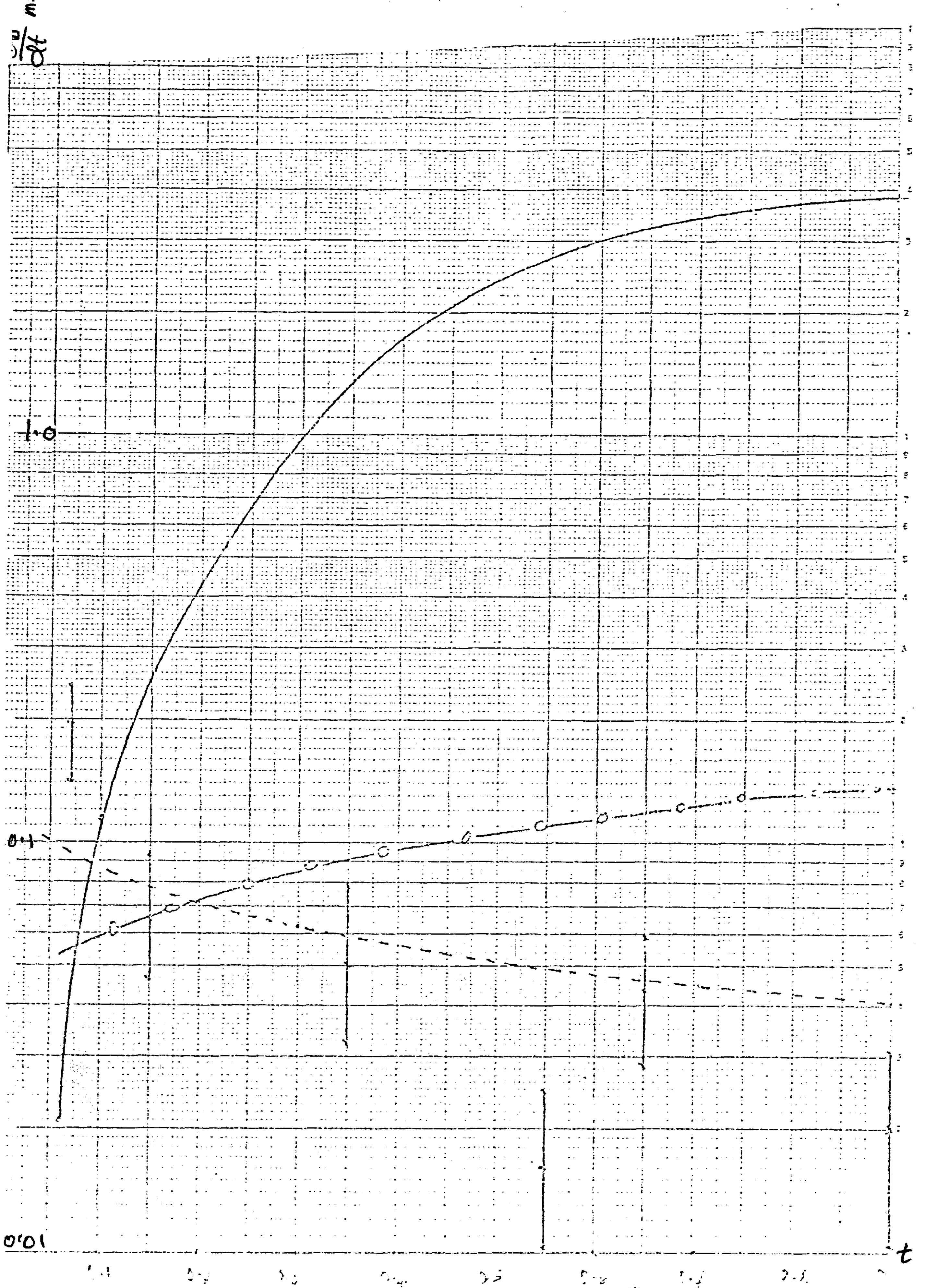


Fig. 3.14 Differential cross-section for $K^-P \longrightarrow K^*(1420)n$ at 10.0 Gev/c. - - - - π^- exchange: -o-o-o-o- ρ exchange ×10⁻¹: ---- total ×10⁻. Data from Ref. 114.



ig. 3.15 Differential cross-section for $K^-P \longrightarrow K^*(1420)$ P at 5.5. Gev/c.
– – – – $\pi^0\eta$ exchange; -o-o-o- $\rho, \emptyset, \omega$ exchange at 5.5 Gev/c $\times 10^{-1}$
Data from Ref. 115

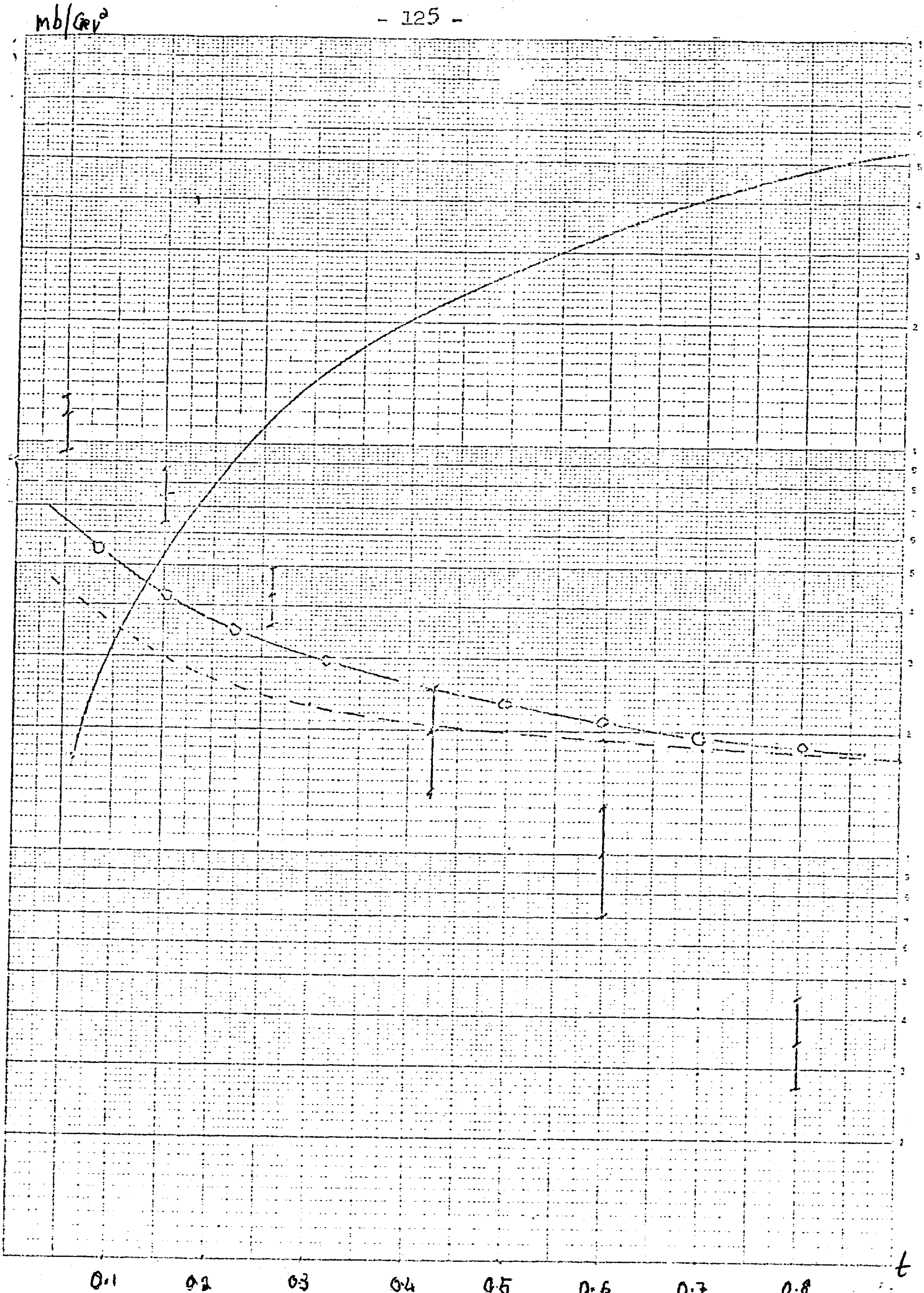


Fig. 3.16 Differential cross-section for $K^-P \longrightarrow K^*(1420)P$ at 10.0 Gev/c. - - - - - - $\pi^0+\eta$ exchange; -o-o-o- $\rho, \emptyset, \omega$ exchange at 10.0 Gev/c. Data from Ref. 114.

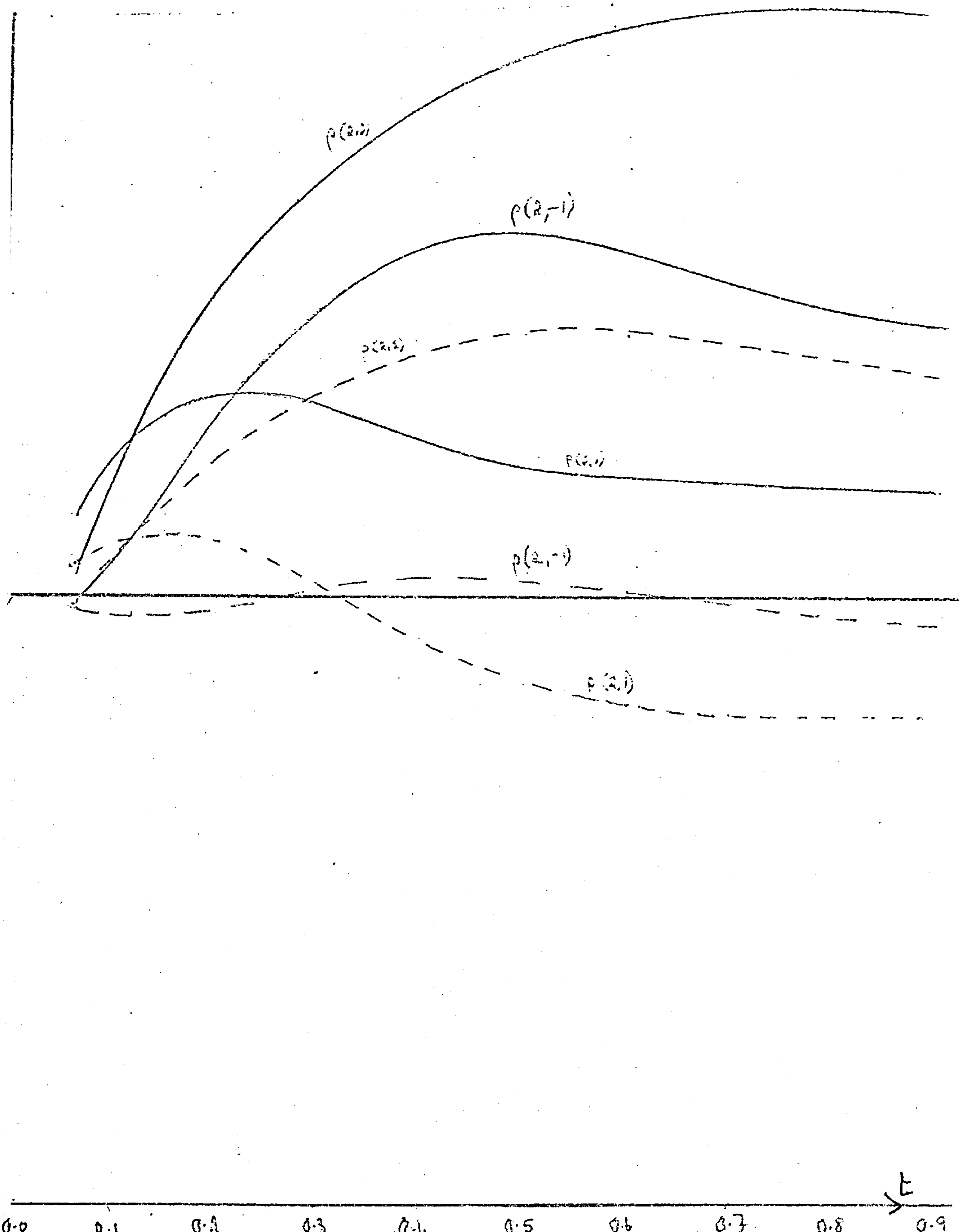


Fig. 3.17 Density matrix elements for decay of $K^{*-}(1420)$ at 10.0 Gev/c.
——pseudoscalar; - - - - vector exchange.

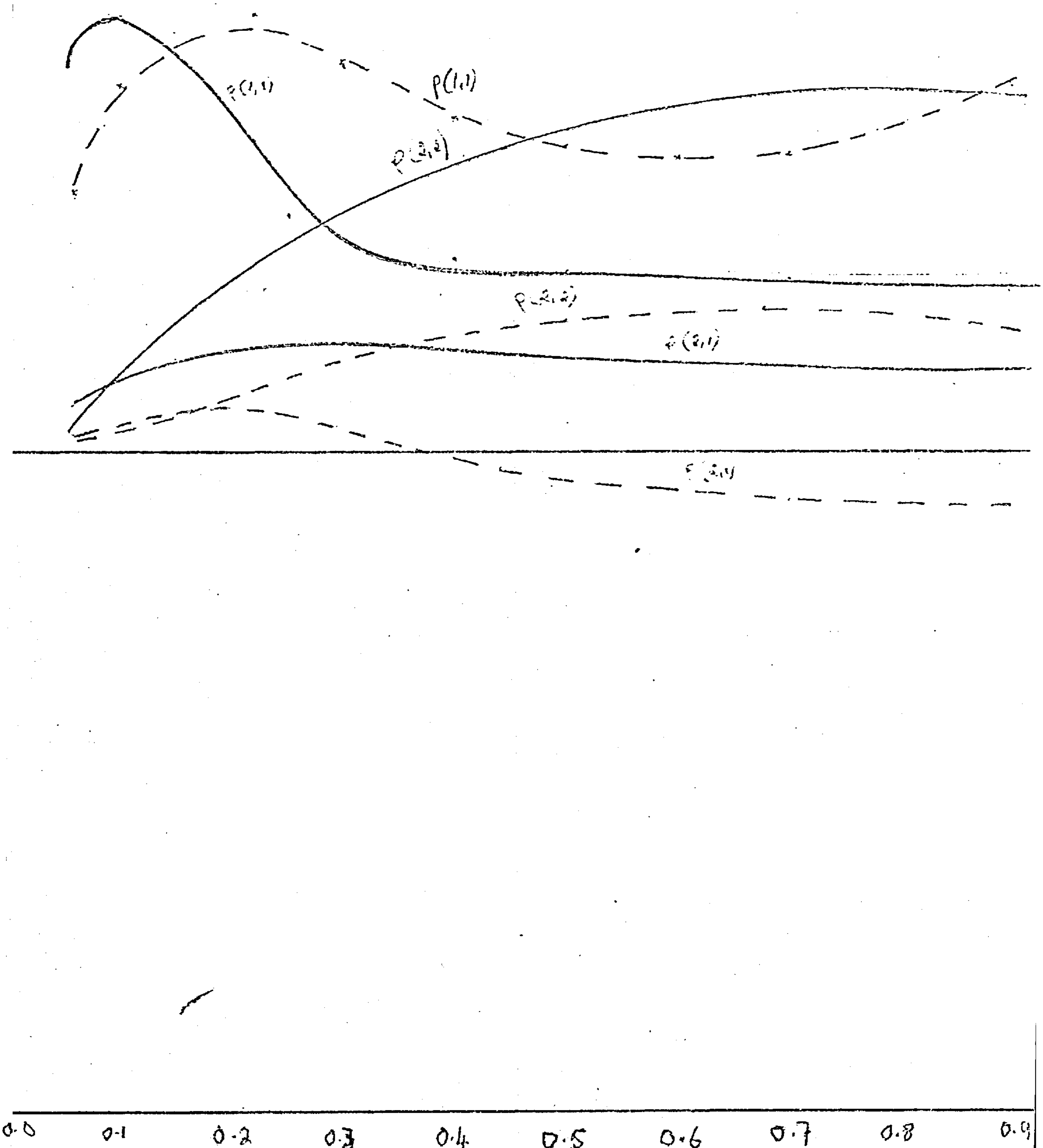


Fig. 3.18 Density matrix elements for decay of $K^*(1420)$ at 10.0 Gev/c.
—— pseudoscalar; - - - - vector exchange.

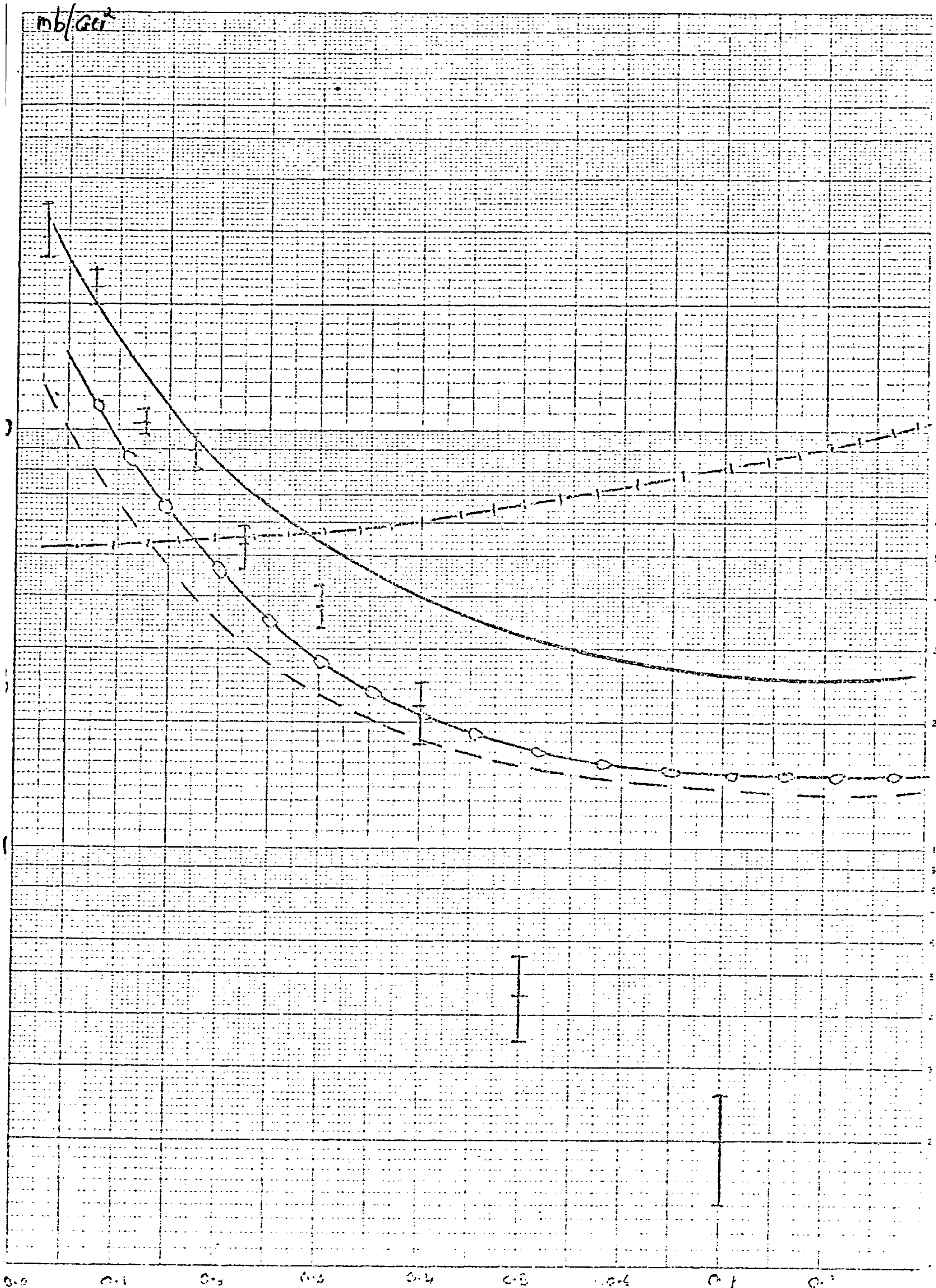


Fig. 3.19 Differential cross-section for $K^-P \longrightarrow K^*(1320)$ at 10.0 Gev/c. - - - - ρ exchange x 10^{-2}; -o-o-o- ω x 10^{-2}; - - - - - total without absorption x 10^{-3}. Data from Ref.114

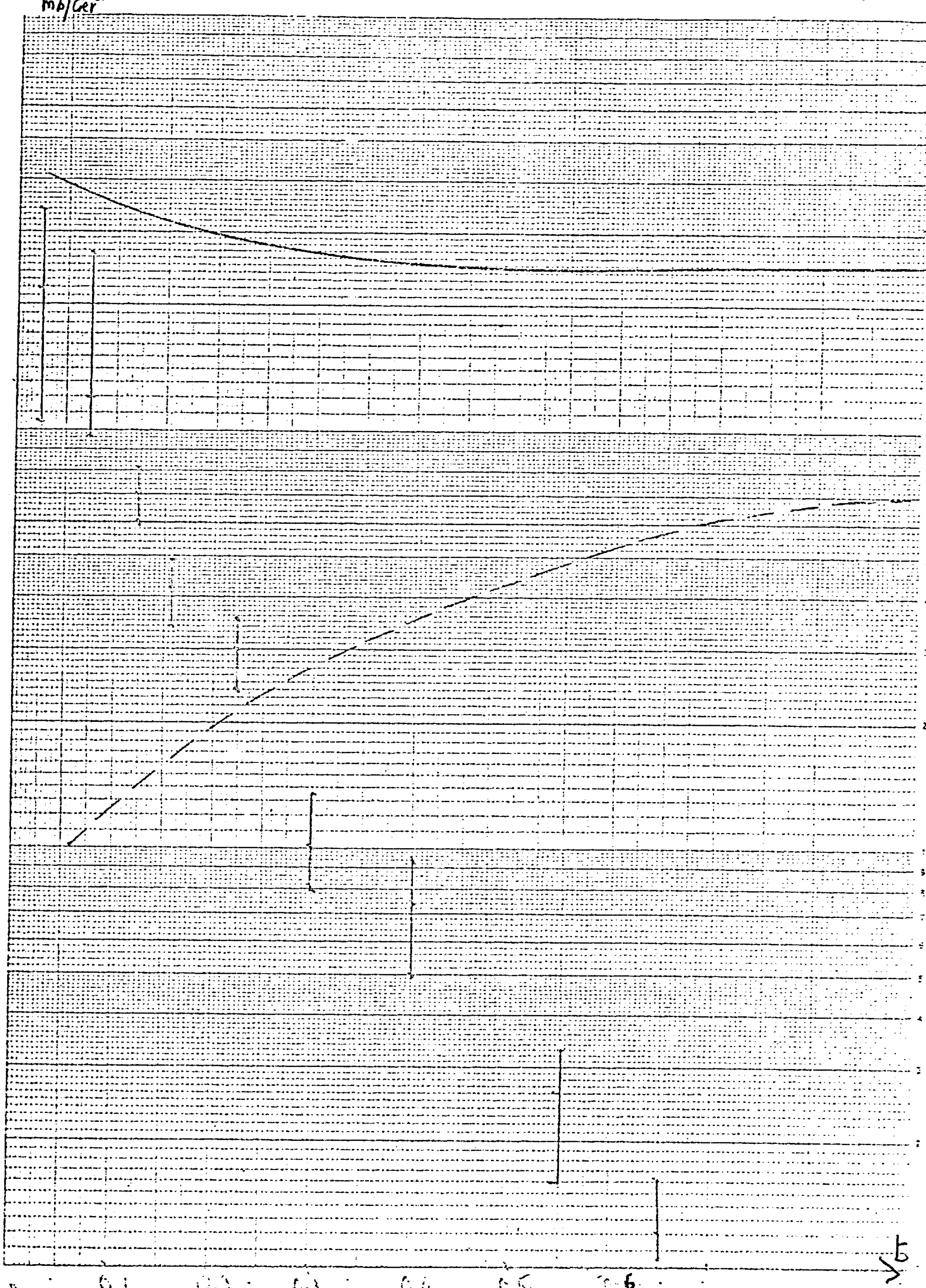


Fig. 3.20 Differential cross-section for $\pi^+P \xrightarrow{} A_T(1070)$ at 8.0 Gev/c. ——— with absorption x 10⁻, - - I - without absorption x 10^{-4}, Data from Ref. 114.

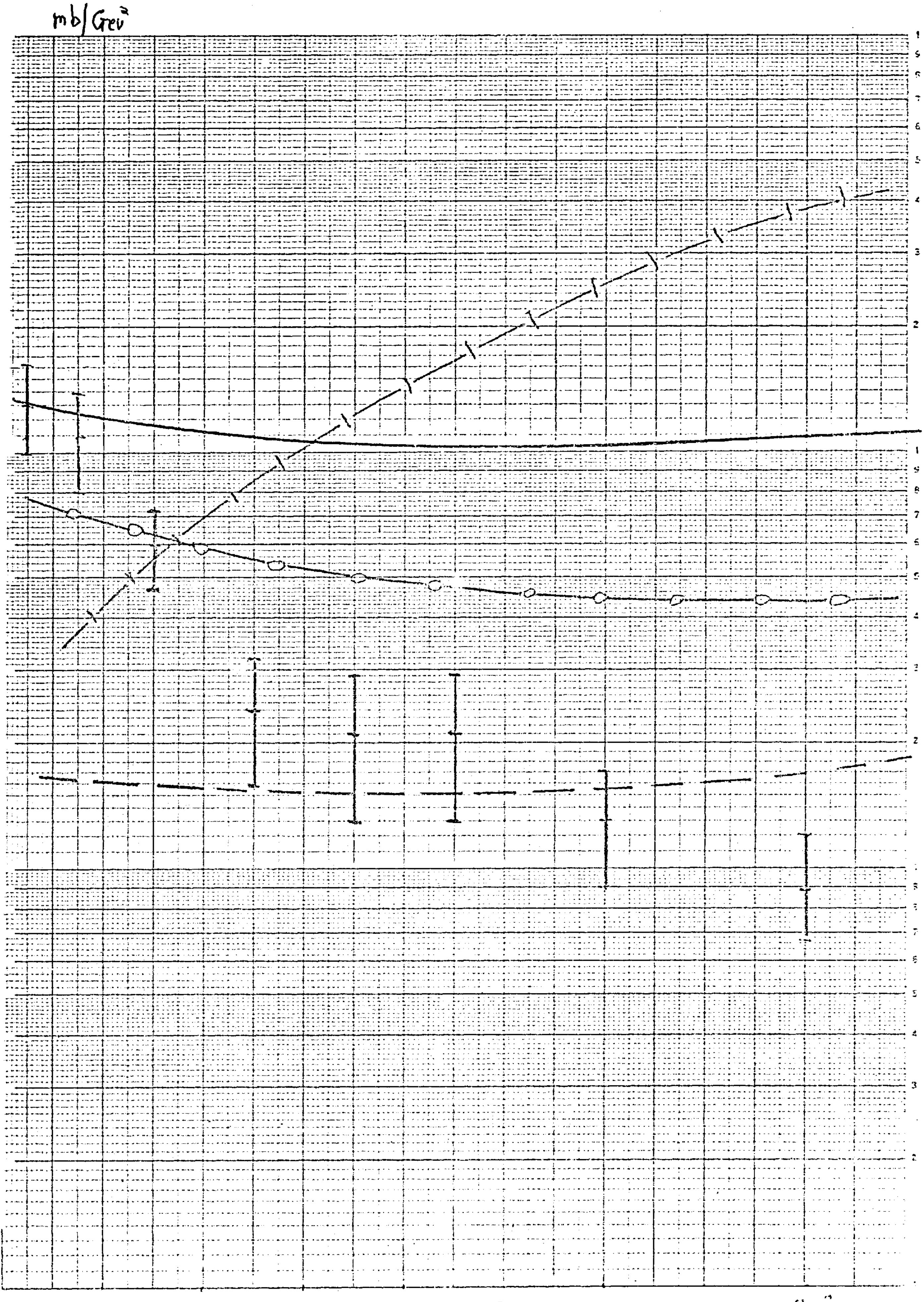


Fig. 3.21 Differential cross-section for $\pi^+ p \longrightarrow B_1(1220)p$ at 8.0 Gev/c, -o-o-o- $\mu\omega$ exchange x 10^{-2}; - - - - ρ exchange, x 10^{-2}; ——— total x 10^{-2}; - - - - total without x 10^{-2}.

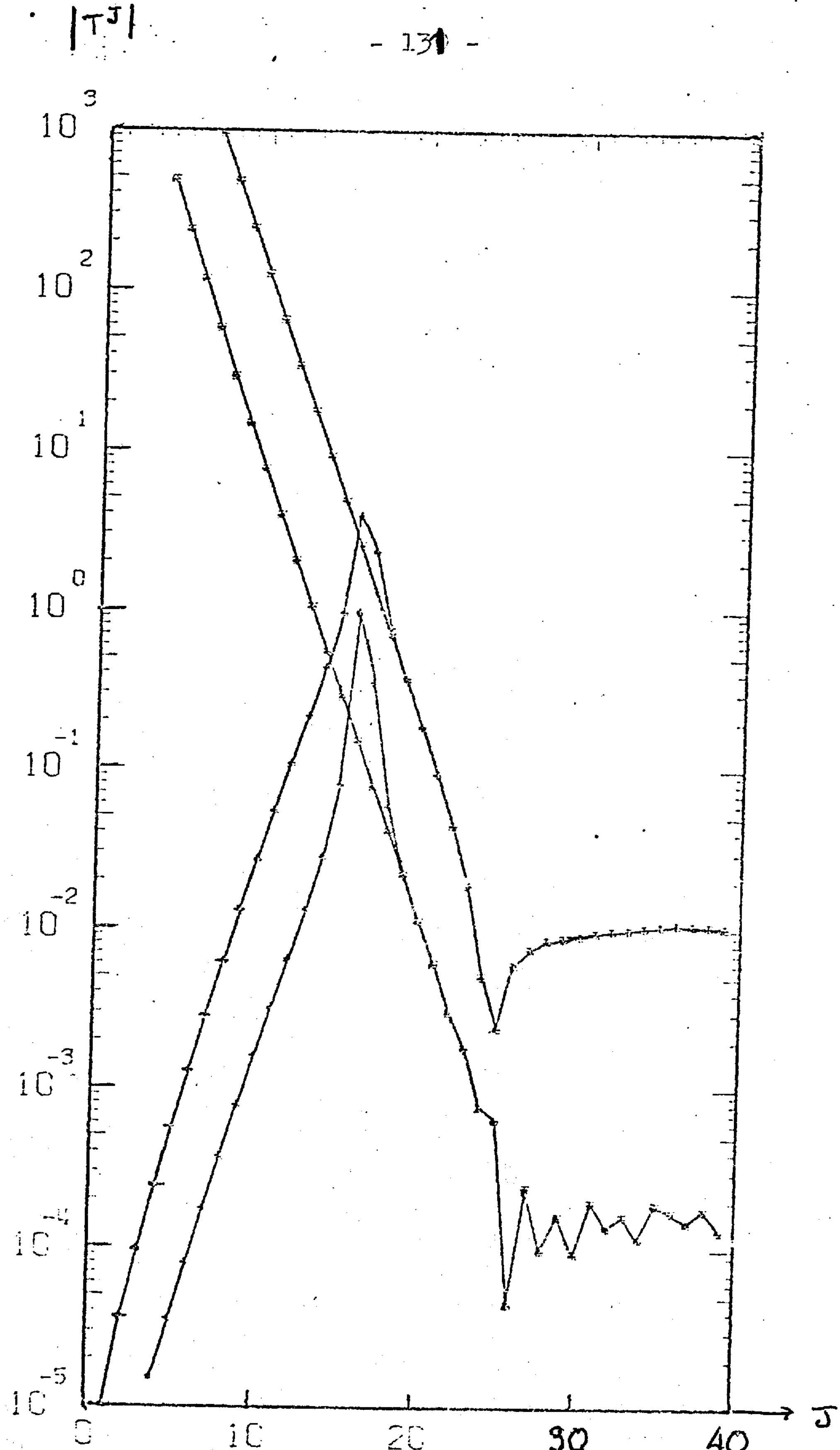


Fig. 3.21 Partial Waves for $\pi p \longrightarrow \eta n$

see eqn. 3.102

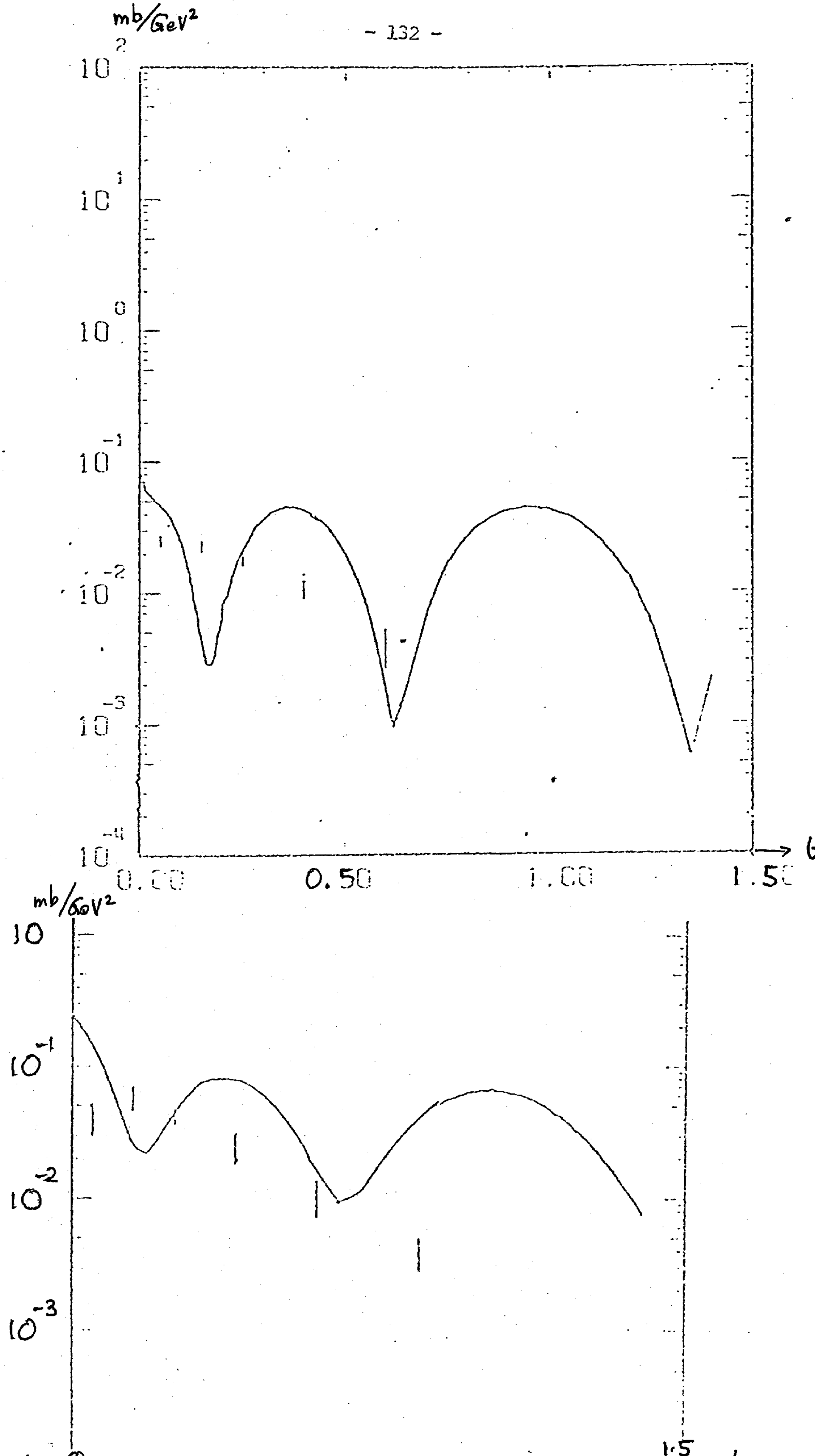
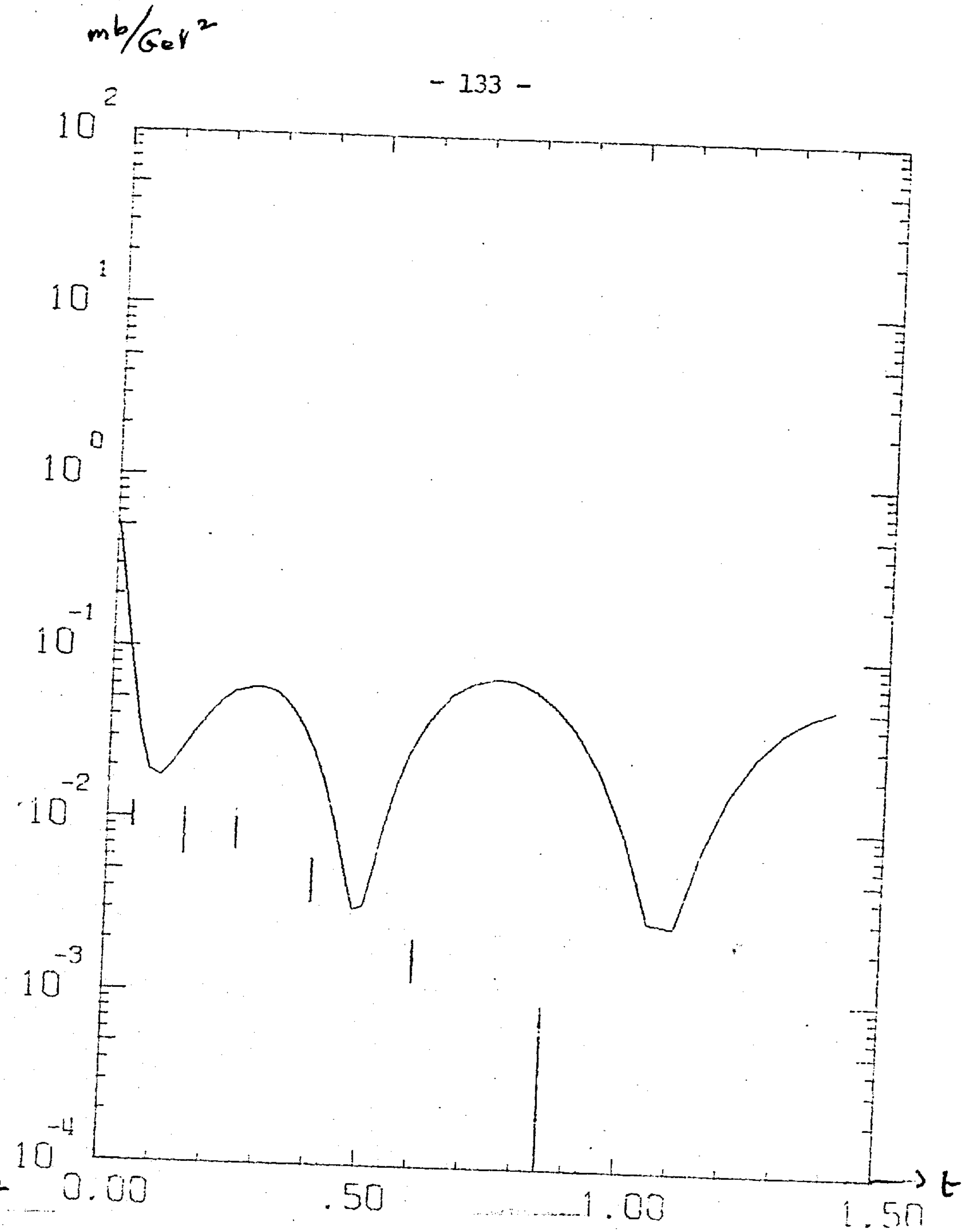


Fig 3.23 DIfferential cross-section for $\pi^- P \longrightarrow \eta n$ at 5.5 and 9.8 Gev/c data from Ref. 135



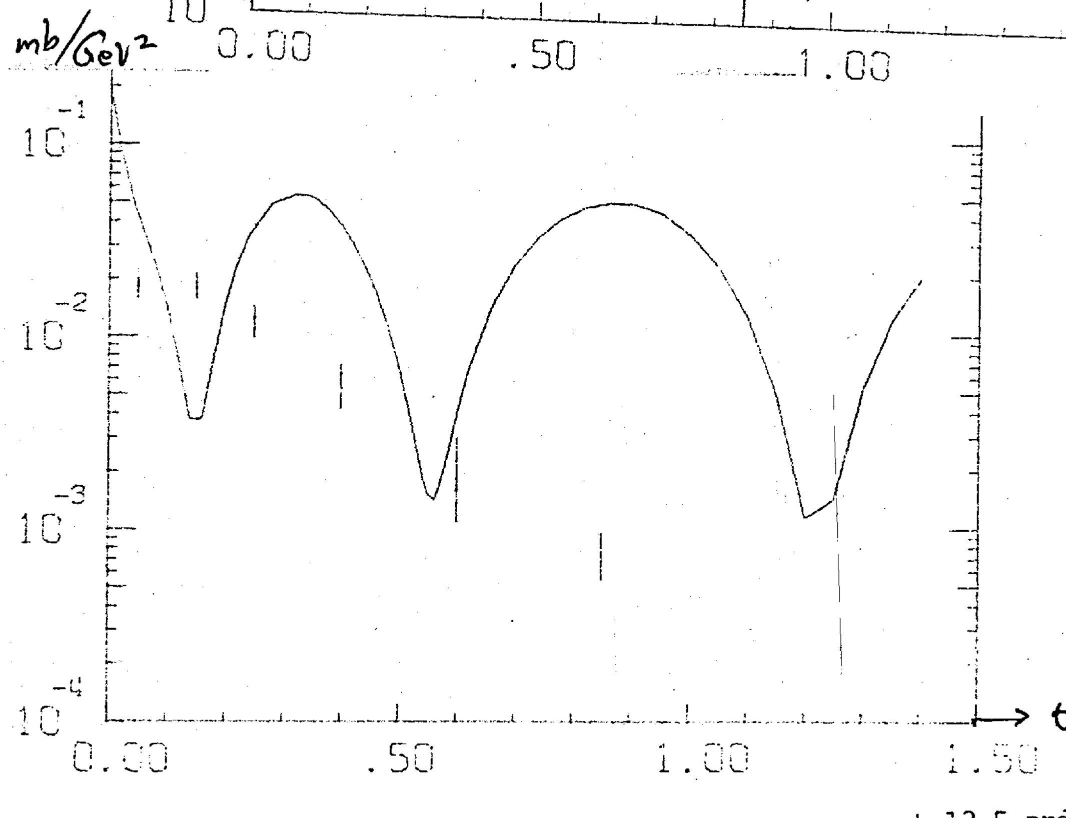


Fig. 3.24 Differential cross-section for $\pi P \rightarrow \eta n$ at 13.5 and18.8 Gev/c
Data from Ref.135

CHAPTER IV

4.1 Introduction

As we have already pointed out the Regge Pole Model is the other main alternative to the Absorption Model for the description of High Energy data. Although it has enjoyed considerable success in explaining the characteristic features of meson-baryon charge exchange processes, an unfortunate feature of the present status of the theory is that its quantitative fits to the experimental data are plagued by an embarrasingly large number of phenomenological parameters. The compilations of R.J.N.Phillips et al[118] illustrate the wide divergence of parameters from author to author. The use of SU(3) symmetry, assumption of exchange degeneracy reduces some of the parameters but a certain number survive: quantities like D/F ratios for varying degrees of spin-flip in the vertices, the relative magnitudes of the vertex couplings, the possibility of different exponential damping factors in each of these and a choice of various kinds of dip mechanism at nonsense points. As before the mechanism which cuts down the number of parameters is the use of orbitally excited supermultiplet theories of the $U(6) \otimes U(6)$ type. By the manner in which spin in such theories is combined with unitary spin and owing to the operation of a $U(6)_W$ generalised helicity conservation at a 3-point vertex, these theories, as we have shown, predict satisfactorily the nature and relative magnitudes of couplings.

The formalism to write such couplings was first ~~made~~ introduced by Freund and Arnold[119] who applied their work to near forward scattering. However the properly Reggeized version of this was given only by Salam and his co-workers;[120] they go much deeper into the question, discussing among others the criteria for assigning the positive parity mesons to one multiplet and not to another.

4.2 Reggeised Symmetry Schemes

Reggeisation is conventionally performed through a partial wave expansion of the S matrix. This expansion can be understood either as a consequence of the rotation invariance of the S matrix or alternatively as a mathematical expansion in terms of an appropriately chosen set of functions. In the ordinary Regge Analysis the role of the two approaches blend themselves and there is a one-to-one correspondence between the two. This can be illustrated easily as follows. Let as consider a two particles state in the CM, $|P_1\ P_2\rangle$; it has only 3 independent components, which we may denote by P_θ, Θ, ϕ. Making use of the invariance of T we can write

$$\begin{aligned}\langle P_1P_2|T|P_3P_4\rangle &= \delta(P-P')\ \langle P_oRq|T|P_oR'q\rangle \\ &= \sum_{lml'm'} \langle P_oR''q\ P_olm\rangle\langle P_olm|T|P_ol'm'\rangle\ \langle l'm'\ Pq\rangle \\ &= \sum_{l,l',m,m'} D^l_{om}(R)\ D^l_{m'o}(I)\delta_{ll'}\ \delta_{mm'}\ \langle P_oL|T|P_{ol}\rangle \\ &= \sum_{l} T^l(E)\ \ P_l(\cos\Theta)\end{aligned} \qquad 4.1$$

From the mathematical point of view however we could say that the amplitude being square integrable it can be expanded in terms of representation functions of the rotation group:

$$F(\phi_1,\Theta,\phi_2) = \sum_{jmm'} f^j_{mm'}\qquad D^j_{mm'}(\phi_1,\Theta,\phi_2) \qquad 4.2a$$

Where

$$f^{j}_{mm'} = \int_G F\, e^{i\phi_1 m}\, e^{i\phi_2 m} \quad d^{j}_{mm'}(\theta)\ d\cos\theta\ d\phi_1\ d\phi_2 \qquad 4.2b$$

For scalar scattering $f_{mm'} = d_{m'o}\ d_{mo}$ so this decomposition reduces to the former. However this analogy is far from being so simple for higher symmetries so we shall adopt the mathematical approach, drawing on the invariance of the S matrix for our choice of complete sets of functions.

The rotation symmetries of the S matrix we shall be invoking are given below; they are based on the following:

a) Particles at rest group themselves into (2J+1) components multiplets of $SU(2)_J$

b) Three point functions with all particles confined to the 0-3 plane show helicity conservation:

$$\langle\lambda|T(E)|\lambda_1\lambda_2\rangle = \delta_{\lambda,\lambda_1-\lambda_2}\ \mathcal{T}_{\lambda_1\lambda_2}(E) \qquad 4.3$$

c) Four point functions with all particles confined collinearly (forward scatt·ring) show net helicity conservation

$$\langle\lambda_3\lambda_4|T(E)|\lambda_1\lambda_2\rangle = \delta_{\lambda_3-\lambda_2,\lambda_1-\lambda_2}\ T_{\lambda_3\lambda_4,\lambda_1\lambda_2}(E) \qquad 4.4$$

We can extract the angular dependence of $T(E,\theta)$ —— the amplitude for scattering with the final particles rotated through an angle θ out of the 0-3 plane —— by a purely mathematical procedure which consist in expanding this function in a complete set of orthonormal, square integrable functions as follows:

$$T(E,\theta) = \sum_n T_n(E) f_n(\theta) \qquad 4.5$$

Bearing b) and c) in mind it pays to choose the set $d^{J}_{\mu\lambda}(\theta)$ of the rotation functions, so that we have

$$\langle\lambda_3\lambda_4|T(E,\theta)|\lambda_1\lambda_2\rangle = \sum_J (2J+1)\quad T^J(E)\quad d^{J}_{\lambda\mu}(\theta) \qquad 4.6$$

One may tie in all the three conditions by assuming that $T^J(E)$ exhibits poles in the expansion parameter J according to

$$\langle\lambda_3\lambda_4|T|\lambda_1\lambda_2\rangle \to= \sum_{J\lambda\lambda'} \delta_{\lambda_3-\lambda_4,\lambda^1} G^J_{\lambda_3\lambda_4} \frac{d^J_{\lambda'\lambda}(\Theta)}{E^2 \; m_J^2} G_{\lambda_1\lambda_2} \times$$

$$\times \; \delta_{\lambda,\lambda_1-\lambda_2} \qquad 4.7$$

Replacing the summation by the Sommerfeld-Watson integral we can then obtain the usual Regge amplitude:

$$\lim_{\cos\Theta\to\infty} \langle\lambda_3\lambda_4|T(E,\Theta)|\lambda_1\lambda_2\rangle$$

$$\sim \sum_{\lambda\lambda'} \delta_{\lambda_3-\lambda_4,\lambda'} G_{\lambda_3\lambda_4} \frac{d^\alpha_{\lambda\lambda^1}(-\Theta)}{\sin\pi\alpha(E)} G_{\lambda_1\lambda_2} \times$$

$$\times \; \delta_{\lambda,\lambda_1-\lambda_2} \qquad 4.8$$

For the higher symmetries too the notion of helicity plays an important role and is variously known as W spin or generalised helicity. For the group U(6,6) of Chapter III this group is generated by $(1,\gamma_3, \gamma_0\gamma_2)T^i$ (*), and these leave invariant the T matrix for collinear processes confined to the 0-3 plane. The W group is defined as that part of the rest symmetry which is left invariant by a particular Lorentz transformation J_{03}, which includes only one component of spin, J_{12}. Since the Lorentz transformation $e^{-i\alpha J_{03}}$ can be used to boost a rest state into motion along the 3 axis one can see that W symmetry leaves the 3-component of momentum invariant. Two particle states, which form the basic entities of quantum scattering, with vanishing total momentum and with relative momentum directed along the 3-axis can be classified into W representations. These states constitute a

(*) (see equ. 2.87 for def. of T^i)

manifold which is invariant under the W group.

In complete analogy with the familiar situation we have just outlined, on generalizing we would like to have the following criteria:

a') Physical particles group themselves in $U(6) \otimes U(6)$ multiplets

b') Three point functions exhibit W-spin conservation

$$\langle W|T(E)|W_1W_2\rangle = \sum \langle \xi W|W_1W_2\rangle \, T_{W_1W_2}(E) \tag{4.9}$$

Where $\langle \xi W W_1W_2\rangle$ denotes the $U(6)_W$ Clebsch-Gordon coefficients which couples $D^{W_1} \otimes D^{W_2}$ to D^W. As in general we have more than one-independent coupling, we have to include the parameter ξ to distinguish among them.

c') Collinear scattering processes also exhibit W conservation:

$$\langle W_3W_4| T |W_1W_2\rangle = \sum \langle W_3W_4|\xi' W\rangle \quad T^W_{\xi\xi'}(E) \langle \xi W|W_1W_2\rangle \tag{4.10}$$

d') Non-collinear four point functions show conservation of coplanar symmetry $U(3) \otimes U(3)$ which has no analogue for the smaller rest symmetry $SU(2)_J$

If now we wish to expand our amplitude we choose a complete set of suitable $d^N_{WW'}(\Theta)$ functions defined as follows

$$\langle W_3W_4|T(E,\Theta)|W_1W_2\rangle = \sum_{N\xi W \xi' W'} \langle W_3W_4|\xi' W'\rangle \, d^N_{WW'}(\Theta) \times$$

$$\times \, T^N_{\xi\xi'}(E) \, \langle \xi W|W_1W_2\rangle \tag{4.11}$$

N stands for a relevant Casimir invariant of $U(6) \otimes U(6)$; as we have 12 such choices the one we shall eventually adopt will be discussed later on. Since we are eliminating a single angle Θ, the completness notion requires that we sum over a one-parameter family of U(6) ⊗ U(6) representations D^N. These functions are well defined if they are nondegenerate in their $U(6)_W$ content. In other words if we have a complete set of basic vectors which can be unambiguously labelled $|NW\rangle$ then the functions

$$d^N_{WW'}(\theta) = \langle NW'|e^{i\theta J_2}|NW\rangle \qquad 4.12$$

are well defined. Different choices of N'S and W'S will give completely different Reggeisation schemes. For example if we choose N to be the Quark Number, which is defined as half the sum of Quarks and antiQuarks, we may characterize the representations D^N by U(6) ⊗ (6) tensors $\phi^{\beta_1 \dots \beta_{N-(1/2)B}}_{\alpha_1 \dots \alpha_{N+(1/2)B}}$, where B denotes the Baryon number and N takes the values $\frac{3}{2}$ B, $\frac{3}{2}$ B+1 $\frac{3}{2}$ B+2,...,; one may then show that any square integrable functio defined over the interval $0 \leq \theta \leq \pi$ and satisfying the appropriate boundary conditions at $\theta = 0, \pi$ may be expanded interms of the $d^N_{WW'}(\theta)$. These remarks may be futher clarified if we consider the O(4) group; its generalized helicity group is O(3) while O(2) plays the role of the coplanar group. Imposing the conditions a' to d' we encounter only flipless amplitudes $T_{S\lambda S'}(\theta)$ which can then be expanded as follows

$$T_{J\lambda J'}(E,\theta) = \sum_{j\sigma} T^{j\sigma}(E)\; d^{j\sigma}_{J\lambda J'}(\theta) \qquad 4.13$$

Where the d(θ) are the complete set of 'rotation' functions for O(4). For O(ν) the expansion theorem reads:

$$T_{N_{\nu-1}\; N_{\nu-2}\; N'_{\nu-1}}(E,\theta) = \sum_{N_\nu} T^{N_\nu}(E)\; d^{N_\nu}_{N_{\nu-1}\; N_{\nu-2}\; N'_{\nu-3}}(\theta)$$

4.14

Assuming we have found the appropriate quantum number N which we wish to Reggeise and we have settled the ambiguities of W spin labelling we may then proceed as before: we assume T(E,Θ) is meromorphic in the E plane

$$\langle W_3W_4|T(E,\Theta)|W_1W_2\rangle = \sum_{N\zeta W\zeta' W'} \langle W_3W_4|\zeta' W'\rangle\, G^N_{\zeta' W' W_3 W_4} \times$$

$$\times \frac{d^{N(\Theta)}_{W'W}}{E^2 - m_N^2}\, G^N_{\zeta W W_1 W_2}\, \langle \zeta W|W_1W_2\rangle \qquad 4.15$$

(G's are coupling-constants)

and then pass to the Regge amplitude in completely analogy to eq. 4.8,

$$\lim_{\cos\Theta \to \infty} \langle W_3W_4|T(E,\Theta)|W_1W_2\rangle$$

$$\sim \langle W_3W_4|\zeta' W'\rangle\, G^\alpha_{\zeta' W' W_3 W_4}\, \frac{d^\alpha_{W'W}(\Theta)}{\sin\pi\alpha(E)}\, G^\alpha_{\zeta W W_1 W_2}\, \langle \zeta W|W_1W_2\rangle \qquad 4.16$$

Where $\alpha(M_N^2) = N$ is the master trajectory function and represents for example, the complexification of Quark number.

As before we have two choices of Symmetry scheme, the super Multiplet or the Global one. With the latter it happens that an irreducible representation of the rest symmetry will contain some W representations more than once, and this will lead to labelling problems; it will be necessary to introduce extraneous operators into the system in order to obtain a complete set of quantum numbers with which to label the states. These operators will generally not commute with J_{03} —— otherwise they would belong to the W algebra —— and so for example will not be conserved in forward scattering where W spin is conserved. This may be illustrated as follows for the groups SU(6,6) and SL(6,c). We tabulate the generators of the group itself together with those of the rest symmetry from which it arose and then those of the W group.

i) SL(6,c) : $J_{\mu\nu}$, J^i, $J^i_{\mu\nu}$, J^i_5 , i=1,2,....8

SU(6): J_{ab}, J^i, J^i_{ab} a,b=1,2,3

SU(3) ⊗ SU(3) ⊗ U(1): J_{12}, J^i, J^i_{12}

ii) SU(6,6): $J_{\mu\nu}$, $J^i_{\mu\nu}$, $J_{\mu 5}$, $J^i_{\mu 5}$, J^i_5 , J_μ, J^i_μ, J^i

SU(6) ⊗ SU(6) ⊗ U(1): J_{ab}, J^i_{ab}, J_{a5}, J_0, J^i_0, J^i

SU(6): J_{12}, J^i_{12}, J_{15}, J_{25}, J^i_{15}, J^i_{25}, J^i

The rest symmetry SU(6) requires 20 labels. Five of these are provided by the SU(6). In the W chain 11 are provided by SU(3)⊗SU(3) ⊗ U(1) and its subgroups. It is necessary to supplement these with 4 constructs $H_1 \ldots H_4$ in order to fill out the W chain. The SU(6,6) rest symmetry requires 41 labels. The Casimir operators give 5+5+1 = 11 of these. The W chain yields 20 leaving another 10 to be made up. Choosing suitable labels is not an easy task but once it has been done we can then proceed to calculate our generalised d^J functions. The groups we shall be dealing with in our analytic continuation will be:

Relativistic Symmetry	Rest Symmetry	W Symmetry	Crossed Channel Symmetry
SO(ν,1)	SO(ν)	SU(ν-1)	SO(ν-1,1)
SL(2ν,C)	SU(2ν)	SU(ν)⊗SU(ν)⊗U(1)	SU(ν,ν)
SU(ν,ν)	SU(ν)⊗SU(ν)⊗U(1)	SU(ν)	SL(ν,C)⊗O(1,1)

The rotation function $d^N_{WW}(\Omega)$ associated with the groups is a sum of derivatives of a basic function d^N_{11}, which appears in superscalar scattering with the exchange of a multiplet labelled with the quantum number N. This is analogous to the statement that the $d^J_{\lambda\lambda'}(\theta)$ in three dimensions canbe expressed as sums of derivatives of $P_J(\theta)$. Let G denote the multiplet symmetry at rest, G_W the generalised helicity subgroup, G the embedding covariant group, and N the label of class representation; then the functions are as follows:[121]

(a) For $G = U(\nu) \otimes U(\nu)$, $G_W = U(\nu)$, $\mathcal{G} = U(\nu, \nu)$

See footnote 127

$$d^N(\theta) \sim C_N^{1/2\nu} (\mathrm{Cos}\ \theta)$$

(b) For $G = U(2\nu)$, $G_W = U(\nu) \otimes U(\nu)$, $\mathcal{G} = SL(2\nu, C)$

$$d^N(\theta) \sim C^{\nu-1/2} (\mathrm{Cos}\ \theta)$$

(c) For $G = U(\nu)$, $G_W = U(\nu-1)$, $\mathcal{G} = U(\nu,1)$

$$d^N(\theta) = (\mathrm{Cos})^N$$

(d) For $G = O(\nu)$, $G_W = O(\nu-1)$, $\mathcal{G} = O(\nu,1)$

$$d^N(\theta) \quad C_N^{1/2\nu} (\mathrm{Cos}\ \theta)$$

$C_N^{\nu}(\mathrm{Cos}\ \theta)$ stand for the Gegenbauer polynomials.

These general remarks may be clarified by looking at the Reggeisation of $O(6) \approx SU(4)$. The set of one particle states at rest may be labelled $|\lambda_m\ P_m\ q_m\ \alpha\rangle$; these labels form the so called Gelfand pattern[122]

$$\begin{pmatrix} \lambda_m & P_m & q_M \\ a & & b \\ p & & q \\ & I & \\ & I_3 & \end{pmatrix} \equiv \begin{pmatrix} \lambda_m & P_m & q_m \\ & \alpha & \end{pmatrix}$$

defined relative to the chain of subgroups

$$O(6) \supset O(5) \supset O(4) \supset O(3) \supset O(2)$$

In a six dimensional space x_1, x_2, x_3 correspond to the physical dimensions and x_4 x_5 x_6 to internal isospin coordinates. We can label the representations of the members of the chain with Casimir invariants as follows:

GROUP	COORDINATES AFFECTED	LABELS
O(6)	x_1 x_2 x_3 x_4 x_5 x_6	λ_m P_m q_m
O(5)	x_1 x_2 x_4 x_5 x_6	a b
O(4)	x_2 x_4 x_5 x_6	P q
O(3)	x_4 x_5 x_6	I
O(2)	x_5 x_6	I_3

The parameters which enter into the labelling of an irreducible representation of O(6) satisfy the inequalities

$$\lambda_m \geq a \geq P_m \geq b \geq q_m$$

$$a \geq P \geq b \geq -q$$

$$P \geq I \geq q$$

$$I \geq I_3 \geq -I$$

They are all integers or half-integers; the dimensionality of the representation is

$$\frac{1}{2!3!}\{(P_m+q_m+1)(P_m-q_m+1)(\lambda_m-P_m+1)(\lambda_m-q_M+2) \times$$

$$\times (q_m+\lambda_m+2)(\lambda_m+P_m+3)\}$$

In this pattern we could associate λ_m and P_m with third components of spin and isospin respectively. Some typical representations corresponding to (λ_m P_m q_m) with their (2s+1, 2I+1) content are

$\underline{15}$ = (110) = (13) (31) (33)

$\underline{64}$ = (210) = (13), (31), $(33)^2$, (15), (51), (35), (53)

$\underline{20}$ = $(\frac{3}{2}\,\frac{3}{2}\,\frac{3}{2})$ = (22), (44)

With these we can now construct two particles states to give eventually our scattering amplitudes:

$$\langle ab|T(\theta, E)|ab'\rangle$$

$$= \delta_{I_3 I_3'}\, \delta_{II'}\, \delta_{PP'}\, \delta_{qq'} \sum_{\lambda_m P_m q_m} T^{\lambda_m P_m q_m}(E)\, d^{\lambda_m P_m q_m}_{ab(Pq)a'b'}(\theta)$$

(*Analogue of equ. 3.99, 3.100.*) 4.17

subject to the following restriction

$$\lambda_m \geq a \geq P_m \geq b \geq q_m$$

$$\lambda_m \geq a' \geq P_m \geq b' \geq q_m$$ 4.18

so that P_m and q_m cover a finite range while λ_m varies from max (a, a') to $+\infty$

We now complexify λ_m for example:

$$\langle ab|T(\theta)|a'b'\rangle = \frac{1}{2i}\int \frac{d\lambda_m}{\sin\pi\lambda_m} \sum_{P_m q_m} T^{\lambda_m P_m q_m}(E)\, d^{\lambda_m P_m q_m}_{ab(Pq)a'b'}(\theta)$$

+ pole contributions 4.19

See equ. 1.23

Disregarding the signature complications, the contribution of a pole at $\lambda_m = \alpha(t)$ would be

$$\frac{\langle ab|\beta^{P_m q_m}|a'b'\rangle}{\sin\pi\alpha}\; d^{\alpha P_m q_m}_{ab(Pq)a'b'}(\theta)$$ 4.20

See equ. 1.24

The trajectory $\lambda_m = \alpha(t)$ would then tie together a sequence of O(6) representations $D^{\alpha P_m q_m}$ with

$$\alpha = \alpha_0+1,\ \alpha_0+2, \ldots\ldots$$

The high energy behaviour of the amplitude should be dominated by the λ_m with the largest value of Re λ_m and they would be of the form

$$\langle ab| T |a'b'\rangle \sim \langle ab| \beta^{P_m q_m} |a'b'\rangle \; (\text{Cos}\ \theta)^{\lambda_m - |P_m - P| - |q_m - q|} \qquad 4.21$$

From this viewpoint one sees that the dominant pole is no longer classified by the I value of its trajectory but rather by the new quantum numbers P_m, q_m. Such a trajectory gives rise to supermultiplets generally combining several I values as well as spin values, when it passes through integer values of λ_m. The formalism in a sense Reggeises both spin and isospin, treating them both on an equal footing. However physical unitarity must act as a breaker of this exact O(6) symmetry, for the S matrix is a submatrix operating within the 0123 subspace and diagonal within 456.

The model we have just presented was based on the rest symmetry O(6); it would be more realistic to consider a rest symmetry of the form U(6) ⊗ U(6) which would lead to a relativistic symmetry U(6,6). In connection with the Reggeisation of this group Salam et al[120] have proved the following theorem:

Generalized partial-wave expansions of an invariant S matrix into $U(\nu) \otimes U(\nu)$ components provide at most a ν-fold infinity of Casimir operators which can be continued into the complex plane. The remaining ν Casimir operators possess fixed finite ranges, fixed by the W spin Casimir operators of incoming and outgoing states.

On Reggeisation we have to proceed to the crossed channel where the little group is $GL(\nu, C)$.[123] The unitary representations are characterized by at most ν continous Casimir Operators ρ_i, with $i = 1, 2, \ldots \nu$, spanning an infinite range $-\infty < \rho_i < \infty$ In the Gelfand and Naimark[124] classification there are ten series of irreducible representations: one nondegenerate and nine degenerate ones which differ from each other physically in their degree of degeneracy. If the multiplicities are perfectly general, then only the nondegenerate series of representations of $GL(\nu, C)$ must be considered for which all ν continuous Casimir operators are independent. However if the multiplicities are fixed at certain low values we need to consider as well more degenerate classes of representations. From the mathematical point of view all classes should presumably be included in the crossed-channel expansion, with meromorphy assumptions for the amplitude giving rise to ten distinct classes of Regge poles. The most degenerate series leads to the Reggeization of just the quark number whereas for other series not only the quark number but also other Casimir operators are Reggeised. There exist no criteria for choosing anyone of them; we know nothing about the asymptotic behaviour of these functions so that we cannot invoke mathematical simplicity either.

The simplest possible case, belonging to the most degenerate series, to Reggeize is the Quark number. This class of $U(6) \otimes U(6)$ states is characterised by a single Casimir label N and there is no degeneracy of W states:

$$N = \quad 0 \qquad 1 \qquad 2$$

$$(W_1, W_2) = \quad (1,1) \;,\; (6,\bar{6}) \;,\; (21,\overline{21}), \ldots\ldots \quad B = o$$

$$(W_1, W_2) = \quad (56,1) \;,\; (126,\bar{6}) \;,\; (252,\overline{21}), \ldots\ldots \quad B = 1$$

One makes use of the multispinorial representation of $U(6) \otimes U(6)$ particle fields

$$\phi(P) \;,\; \phi^A_B(P) \;,\; \ldots\ldots \;,\; \phi^{(B_1 B_2 B_3 \ldots\ldots B_N)}_{(A_1 A_2 A_3 \ldots\ldots A_N)}(P)$$

for mesons

(B is Baryon number)

$$\psi^{(A_1A_2A_3)}(P),\ \psi_{(A_1A_2A_3A_4)}(P),\ldots\ldots,\psi^{(B_1B_2\ldots\ldots B_N)}_{(A_1A_2\ldots\ldots A_{N+3})}(P)$$

for Baryons

to evaluate our relevant $d^N_{WW'}$ functions. For elastic scattering the amplitude is

$$T = \frac{G_N^2(|q|\,|q'|)^N}{(2m_N)^{2N}(P^2 - m_N^2)} \quad C_N^{1/2\nu}(q.q') \tag{4.22}$$

which may be continued analytically to give the Reggeised amplitude

$$T = \left(\frac{|q|\,|q'|}{4m^2}\right)^{\alpha(p^2)} \frac{\beta(p^2)}{\text{Sin}\ \pi\alpha(p^2)} \quad C^{1/2\nu}_{\alpha(p^2)}(q.q') \tag{4.23}$$

(see fig 4.2)

The Reggeisation technique here consists in making the replacement

$$\frac{G_N^{\pm}\ G_N'\ C_N^{\mu}}{P - M_N^2} \longrightarrow \frac{\gamma^{\pm}(\alpha,t)\gamma'(\alpha,t)(1+e^{-i\pi\alpha})\ C^{\mu}_{\alpha}}{\text{Sin}\ \pi\alpha} \tag{4.23b}$$

$C_N^{(1/2)\nu}(\Theta)$ are Gegenbaner polynomials. (For definition of γ's see equ. 1.32.)

The ordinary J plane trajectory reduction of the generalised N-plane trajectory can be obtained from the series

$$C_{\alpha}^{(1/2)\nu}(\Theta) = \sum_k A_{k\alpha}\ P_{\alpha-2k}(\Theta) \tag{4.24}$$

where

$$A^{\nu}_{k\alpha} = -\frac{2\alpha-4k+1}{8(\Gamma(1/2))^2}\sum_{r=0}^{k}\frac{\Gamma(r+1/2)\,\Gamma(\alpha+1/2-r)}{\Gamma(r+1)\,\Gamma(\alpha-r+1)}\ \times$$

$$\times\ (\alpha-2r)\ \frac{\Gamma(k-r-1/2)\,\Gamma(\alpha-k-r)}{\Gamma(k-r+1)\,\Gamma(\alpha-k-r+3/2)} \tag{4.25}$$

The Regge N trajectories are as shown in Fig. 4.1

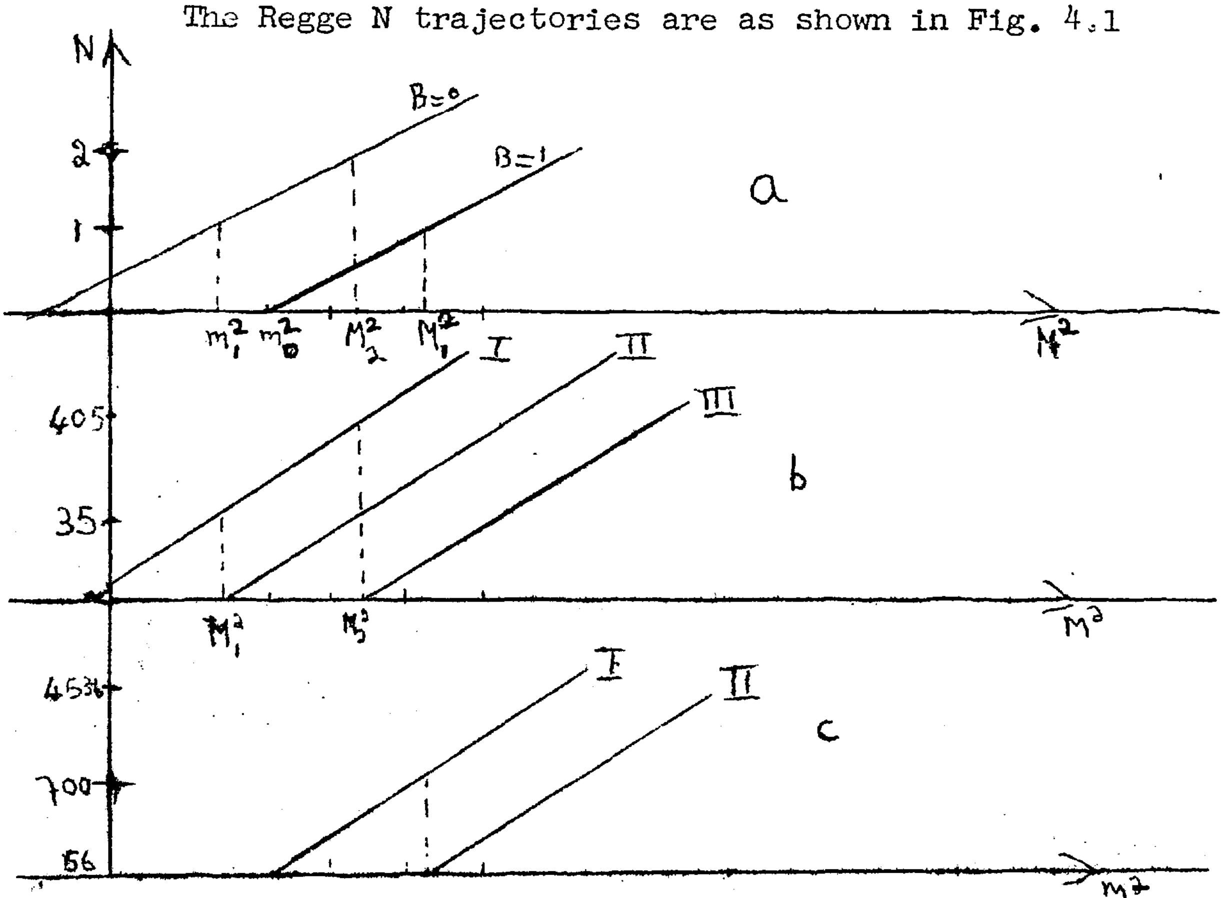


Fig 4.1

(a) Quark number Regge plot

(b) SU(6) meson decomposition of (a)

(c) SU(6) Baryon decomposition of (a)

From the U(6) and $U(2)_J \otimes U(3)_F$ (*) content of the $U(6) \otimes U(6)$ we can interpret more clearly the meaning of the physical content of the trajectories:

$$(6,\overline{6}) = 1 \oplus 35 \quad 4.26$$

$$(21,\overline{21}) = 1 \oplus 35 \oplus 405 \quad 4.27$$

and

$$35 = (1,3) \oplus (8,3+1) \quad 4.28$$

$$405 = (1,5) \oplus (8,5+3) \oplus (10+\overline{10},3) \oplus (27,5+3+1) \oplus (1,1) \oplus (8,3+1) \quad 4.29$$

(*) Label J is spin and F unitary Spin

From these decompositions we can trace out generations n=0,1,2,.... of parallel J trajectories for each F. These will have the following properties (i) From external couplings all negative J residues must vanish (ii) for sufficiently large F external couplings must ensure that all residues occuring below a critical mass should vanish e.g. with the 27 fold no particles with masses M_2 should materialize (iii) the number of members in suceeding n generations of trajectories increases.

The basic consequences we can draw from this scheme are:

i) We have an infinity of trajectories whose characteristic are interrelated; thus one will have a single master formula $m = m(N)$

ii) Residue functions of different J poles are related.

iii) One will be automatically summing over the contributions from all J trajectories by proceeding directly with the generalised $d^N_{WW'}$ functions belonging to the supermultiplet group.

As can be seen this scheme does not resolve the difficulties associated with the Global symmetry schemes whereas, one suspects, that its advantages might also be present in the Supermultiplet scheme.

4.3 Reggeised Supermultiplets And Applications

The rest symmetry for this scheme is $SU(6)\otimes O_L(3)$. Even though the physical ideas behind the two types of models A and B (*) are different the techniques for applying Regge ideas to the high-energy behaviour of scattering amplitudes are very similar. In this model it is the orbital quantum number L which will be Reggeised.

(*) A is the global model and B the supermultiplet.

Unlike our previous application we now work in the M-function approach of Salam et al (120) using a multispinor formalism; this approach has the merits of exhibiting manifest covariance, of allowing crossing to be performed with ease, of automatically incorporating the threshold and other mass-dependent Kinematic factors. Our Kinematics too will be different and is described in Fig 4.2

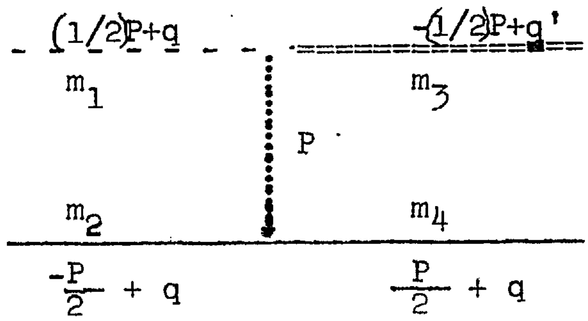


Fig 4.2 Kinematics

The multispinors which will appear in the Lagrangians are:

$\phi^{A\,(\beta_1\ldots\beta_N)}_{B\,(\beta^1\ldots\beta^n)}$ for $(6,\bar{6};N)$ particles

$\Psi^{(\beta_1\ldots\beta_N)}_{(ABC)\,(\alpha^1_1\ldots\alpha_N)}$ for $(56,1;N)$ particles

These are subject to the constraints we have already met; see eqs. 3.53-3.54

The two Lagrangians we shall need are:

for the $(6,\bar{6};1)_{(1/2)P+q}\ (6,\bar{6};0)_{(1/2)P-q}\ (6,\bar{6};N)_P$ vertex .

$$L = \frac{1}{\mu^{N-1}}{}^{(*)}\ \phi^A_B\,((1/2)P+q)\ \phi^D_C\,((1/2)P-q)\ \{\mu q\ [h^+_{10}(\delta^C_B \frac{\delta}{\delta q^D_A} - \delta^A_D \frac{\delta}{\delta q^B_C}) +$$

$$h^-_{10}\ (\delta^C_D \frac{\delta}{\delta q^D_A} + \delta^A_D \frac{\delta}{\delta q^B_C})]\} +$$

(*) μ is the mass of the exchanged particle., see eqn. 3.52 for definition of q^B_C etc.:

$$\mu^3 \frac{\delta}{\delta q} h_{11}^{+} \left(\delta_B^C \frac{\delta}{\delta q_A^D} - \delta_D^A \frac{\delta}{\delta q_C^B}\right) +$$

$$\mu^3 \frac{\delta}{\delta q} h_{11}^{-} \left(\delta_B^C \frac{\delta}{\delta q_A^D} + \delta_D^A \frac{\delta}{\delta q_C^B}\right)\Big] - \Phi_N (P,q) \qquad 4.30$$

and for the $(56,1;0)_{(1/2)P-q}$ $(\overline{56}, ;0)_{(1/2)P+q}$ $(6,\overline{6};N)$ vertex

$$(*) \quad L = \frac{1}{M^N} \overline{U}^{(ACD)}((1/2)P+q)\, U_{(BCD)}((-1/2)P+q)$$

$$[G_o \delta_A^B + mG_1 \not\delta q_B^A]\, \Phi_N (P_1 q) \qquad 4.31a$$

where we have used the abbreviation

$$\Phi_N(P_1 q) = q_{\mu_1} q_{\mu_2} \cdots\cdots q_{\mu_N} q_B^A \varnothing_A^B{}_{\mu_1 \cdots\cdots \mu_N} (P) \qquad 4.31b$$

The superscripts $\pm$ on the couplings h for the meson couplings of eq 4.30 refer to the even and odd values of the meson. Bose statistics tells us that $h^{+} = o$ when N is odd and $h^{-} = N$ ~~is~~ (even).

The meson multispinors describing 2^{+}, 1^{+} mesons are now decomposed as —— we have suppresed Unitary spin indices-:

$$\varnothing_{\mu\nu}(P) = \varnothing_{(\mu\nu)} + \frac{1}{2} E_{\lambda\mu\nu\rho} \frac{P\lambda \varnothing\rho}{\mu} + 1/3\, G_{\mu\nu} Tr(\varnothing) \qquad 4.32$$

which correspond to an O(2,1) decomposition.

It is always possible to express the covariant M functions in the form $M = \bar{U}\ldots T\ldots U$ (120) where T for example may be

$$T_{AB}^{A'B'} = D_B^A(q)\, D_{B'}^{A'}(q')\, \Delta_N \quad (P; qq') \qquad 4.33$$

where the D's stands for various differentials whose order is governed by the external excitation numbers and

(*) M is mass of exchanged particle
m is mass of in (out) going particles

$$\triangle_N = (q \cdot q')^{N+1} \{ (\cos \theta)^{N+1} \quad (P^2 - M_N^2) \} \qquad 4.34$$

The first few derivatives we shall need are:

$$\frac{\delta}{\delta q_A^B} \triangle_N = \frac{(q.q')^N}{P^2 - M^2} \frac{\{(M+P)q'(M-P)\}_B^A}{4M^2} \qquad 4.35a$$

$$\frac{\delta}{\delta q_\mu} \triangle_N = \frac{N(q.q')^N q'}{P^2 - M^2} \left(-q'_\mu + \left(\frac{P.q'}{M^2}\right) P_\mu\right) \qquad 4.35b$$

$$\frac{\delta^2}{\delta q_\mu \delta q_A^B} \triangle_N = \frac{N(q.q')^{N-1} q'^2}{P^2 - M^2} \left(-q\mu + \left(\frac{P.q'}{M^2}\right) P_\mu\right) \qquad 4.36$$

$$\frac{\delta^2}{\delta q_B^B \delta q_{B'}^{'A'}} \triangle_N = \frac{(q.q')^N}{P^2 - M^2} (M+P)_B^{B'} \quad (M-P)_{A'}^A \qquad 4.37$$

These derivative couplings and the methods of differentiation we have just outlined were developed by Zemach[125] and Scadron[126].

Reggeisation consists in the replacement

$$\frac{(q.q')^N}{P^2 - M^2} \longrightarrow \frac{(q.q')^{\alpha-1}}{\sin \pi(\alpha-1)} \qquad 4.38a$$

$$N \longrightarrow \alpha - 1 \qquad 4.38b$$

(See eq. 4.23b also)

This is just the conventional Reggeisation which gives poles at "nonsense" values of N ($\alpha = 0,1,2,\ldots\ldots$). Gell-Mann has suggested that to avoid these poles one should have a ghost(*) eliminating mechanism. Representing natural and unnatural parity exchange by n and u respectively we then make the following replacements for natural parity

$$\frac{1}{P^2 - M^2} \longrightarrow \frac{1}{\sin \pi(\alpha_n - 1)} = - \frac{\Gamma(\alpha_n)\,\Gamma(1-\alpha_n)}{\pi} \qquad 4.39a$$

(*) see equ. 1.37, 1.38.

$$N \longrightarrow \alpha_n - 1 \qquad 4.39b$$

and for unnatural parity

$$\frac{1}{P^2-M^2} \longrightarrow \frac{1}{\operatorname{Sin}\pi\alpha_u} = -\frac{\Gamma(1+\alpha_u)\Gamma(-\alpha_u)}{\pi} \qquad 4.39c$$

$$N \longrightarrow \alpha_u \qquad 4.39d$$

The Gell-Mann ghost-eliminating mechanism is introduced by dividing by $\Gamma(\alpha_n)$ and by $\Gamma(1+\alpha_u)$ for the natural and unnatural parity exchanges respectively.

The argument so far does not introduce the signature factor at all. In what follows, whenever we write $G^{\pm}$, we shall assume that N - signature projections $(1/2)(1\pm e^{i\pi N})$ are to be included. see equ. 1.37, 1.38.

The amplitude which describes the production of $2^+, 1^+$ and 0^+ is then

$$T = \overline{U}^{ACD}(1/2P+q)U_{BCD}(-1/2P+q)\phi^{B'}_{A'}(1/2P+q')\overline{\phi}^{D'}_{C'\nu}(1/2P-q') .$$

$$(M\mu)^{-N+1} \{ M G_1 \frac{\delta}{\delta q^A_B} \} .$$

$$\{\mu^3 \frac{\delta}{\delta q'_\nu} \{ h^+_{11} (\delta^{C'}_{B'} \frac{\delta}{\delta q'^{D'}_{A'}} - \delta^{A'}_{D'} \frac{\delta}{\delta q'^{B'}_{C'}}) + h^-_{11}(\delta^{C'}_{B'} \frac{\delta}{\delta q'^{D'}_{A'}} \quad x$$

$$x \quad \delta^{A'}_{B'} \frac{\delta}{\delta q^{B'}_{C'}})\}\} \quad \Delta_N \qquad 4.40$$

Upon Reggeisation à la Gell-Mann we obtain the an amplitude, from which one has to pick the parts describing the production of 2^+, 1^+ and 0^+ particles; obviously we also have to decompose $\overline{U}^{ACD}$ and U_{BCD} into octet and decuplet pieces. In writing it we have used the abbreviation

$$(\Gamma^{+}_{-})^B_A = \frac{1}{2M} (M \pm \not{p})^B_A , \quad \Gamma = \text{Gamma Function}$$

The Reggeised amplitude written fully is then

$$T = (1+\frac{M}{2_\mu}) \bar{U}^{ACD} (\Gamma_+ q' \Gamma_- T^i \mu)^B_A U_{BCD} q'_\mu$$

$$\Big[\tfrac{1}{2}\beta^{(10)}_{(-)} (1-e^{+i\pi\alpha-}) \Gamma(1-\alpha-) (\not{0}_5,\not{0})^i_D (\underline{q}.\underline{q}/\mu M)^{\alpha.-1}$$

$$+ \tfrac{1}{2}\beta^{(10)}_{(+)} (1+e^{i\pi\alpha_+}) \Gamma(1-\alpha_+) (\not{0}_5,\not{0}_{5\mu})^i_F (\underline{q}.\underline{q}'/\mu M)^{\alpha_+-1} \Big]$$

$$+ (1+\frac{M}{2\mu}) \bar{U}^{ACD} (\Gamma_+ q' \Gamma_- Ti/\mu)^B_A U_{BCD} q_\mu (\mu/M)$$

$$\Big[\tfrac{1}{2}\beta^{(11)}_{(+)} (1+e^{i\pi\alpha_+}) \Gamma(1-\alpha_+)(\alpha_+1) (\not{0}_5,\not{0}_{5\mu})^i_D (\underline{q}.\underline{q}'/\mu M)^{\alpha_+-2}$$

$$+ \tfrac{1}{2}\beta^{(11)}_{(-)} (1-e^{i\pi\alpha_-}) \Gamma(1-\alpha_-)(\alpha_--1)(\not{0}_5,\not{0}_{5\mu})^i_F (\underline{q}.\underline{q}'/\mu M)^{\alpha_--2} \Big]$$

$$+ (1+\frac{M}{2\mu}) \bar{U}^{ACD} (\Gamma_+ \{q/,\gamma_\lambda\}\gamma_5 \Gamma_- T^i/2\mu)^B_A U_{BCD} q'_\mu$$

$$\Big[\tfrac{1}{2}\beta^{(10)}_{(-)} (1-e^{i\pi\alpha_-}) \Gamma(1-\beta_-)(\not{0}_5,\not{0}_{\lambda,\mu})^i_F (\underline{q}.\underline{q}'/\mu M)^{\alpha_--1}$$

$$+ \tfrac{1}{2}\beta^{(10)}_{(+)} (1+e^{i\pi\alpha_+}) \Gamma(1-\alpha_+) (\not{0}_5,\not{0}_{\lambda,\mu})^i_D (\underline{q}.\underline{q}'/\mu M)^{\alpha_+-1} \Big]$$

$$+ (+\frac{M}{2\mu}) \bar{U}^{ACD} (\Gamma_+ \{q',\gamma_\lambda\}\gamma_5 \Gamma_- T^i/2\mu)^B_A U_{BCD} q_\mu (\mu/M)$$

$$\Big[\tfrac{1}{2}\beta^{(11)}_{(+)} (1+e^{i\pi\alpha_+}) \Gamma(1-\alpha_+) (\alpha_+-1)(\not{0}_5,\not{0}_{\lambda\mu})^i_F (\underline{q}.\underline{q}'/\mu M)^{\alpha_+-2}$$

$$+ \tfrac{1}{2}\beta^{(11)}_{(-)} (1-e^{i\pi\alpha_-}) \Gamma(1-\alpha_-)(\alpha_--1)(\not{0}_5,\not{0}_{\lambda,\mu})^i_D (\underline{q}.\underline{q}'/\mu M)^{\alpha_--2} \Big]$$

$$+ (1+\frac{M}{2\mu}) \bar{U}^{ACD} (\Gamma_+ \gamma_\lambda\gamma_\sigma \Gamma_-)^B_A U_{BCD} q'_\mu \qquad x$$

$$\times \left[\tfrac{1}{2}\beta^{(10)}_{(-)} (1-e^{i\pi\alpha_-})\Gamma(1-\alpha_-)(\phi_5,\phi_{\lambda,\mu})^i_D (q.q'/\mu M)^{\alpha_- -1} \right.$$

$$\left. + \tfrac{1}{2}\beta^{(10)}_{(+)} (1+e^{i\pi\beta_+})\Gamma(1-\alpha_+)(\phi_5,\phi_{\lambda,\mu})^i_F (q.q'/\mu M)^{\alpha_+ -1} \right]$$

$$+ \ (1+\frac{M}{2\mu})\ \bar{U}^{ACD}(\Gamma_+ \ \gamma_\lambda\gamma_5\Gamma_-)^B_A \ U_{BCD}\ q_\mu(\mu/M)$$

$$\left[\tfrac{1}{2}\beta^{(11)}_{(+)} (1+e^{i\pi\alpha_+})\Gamma(1-\alpha_+)(\alpha_+-1)(\phi_5,\phi_{\lambda,\mu})^i_D (q.q'/\mu M)^{\alpha_+ -2} \right.$$

$$\left. \tfrac{1}{2}\beta^{(11)}_{(-)} (1-e^{i\pi\alpha_-})\Gamma(1-\alpha_-)(\alpha_--1)(\phi_5,\phi_{\lambda,\mu})^i_F (q.q'/\mu M)^{\alpha_- -2} \right] \qquad 4.41$$

This formidable expression may be made more physically transparent if we pick out the various parts of interest and write the differential cross-section in the following way:

For $0^- \ \frac{1}{2}^+ \longrightarrow 2^+ \ \frac{1}{2}^+$

$$\frac{d\sigma}{dt} = \alpha.\ \mathrm{Tr}\ \{(\tfrac{\not P}{2} + \not q + m)\ J_{\lambda\mu}\ (-\tfrac{\not P}{2} + \not q + m)\ \bar{J}_{\lambda'\mu'}\}$$

$$\times \sum_{\mathrm{Spins}} \phi_{\lambda\mu}\ (-\tfrac{P}{2} + q)\ \bar{\phi}_{\lambda'\mu'}\ (-\tfrac{P}{2} + q) \qquad 4.4$$

where

$$\alpha = \frac{1}{64\pi^2 P_L^2}\ \frac{1}{M_N^2}\ \frac{1}{(2S_1+1)(2S_2+1)}\ \frac{1}{10\times(0.197316)^2} \qquad 4.43$$

and

$$\sum \phi_{\lambda\mu}\ \bar{\phi}_{\lambda'\mu'} = d_{\lambda\lambda'}\ d_{\mu\mu'} + d_{\lambda\mu'}\ d_{\mu\lambda'} - \tfrac{2}{3}\ d_{\lambda\mu}\ d_{\lambda'\mu'} \qquad 4.44a$$

$$d_{\lambda\mu} = -\ g_{\lambda\mu} + (-\tfrac{1}{2}\ P + q')_\lambda(-\tfrac{1}{2}\ P + q')_\mu \qquad 4.44b$$

For $0^- \ \frac{1}{2}^+ \longrightarrow 1^+ \ \frac{1}{2}^+$

$$\frac{d\sigma}{dt} = \alpha \, \mathrm{Tr}\{(\frac{\not P}{2} + \not q + m) J_{\lambda\mu} (-\frac{\not P}{2} + \not q + m) \bar{J}_{\lambda'\mu'}\}$$

$$\times \; \{\frac{1}{2} \in_{\lambda\mu\sigma\rho} \in_{\lambda'\mu'\sigma'\rho'} \; P_\sigma P_{\sigma'} \{- g_{\rho\rho'} + \frac{(-\frac{P}{2} + q')(-\frac{P}{2}+q')}{\mu^2}\}\} \qquad 4.45$$

The currents $J_{\lambda\mu}$ appearing in these expressions are

$$J_{\lambda\mu} = \frac{A_\mu}{\mu} \; P_\lambda \gamma_5 + i \, \frac{B_\mu}{\mu} \; \sigma_{k\lambda} \, q'_k \, \gamma_5 \, \frac{-\in k\lambda\rho\nu}{2\mu^2 m} \, P_\rho q'_\nu \{C_\mu q_k + D_\mu \gamma_k\} \qquad 4.46$$

where for the sake of computation we can write

$$A_\mu = a' q'_\mu + a \, q_\mu \qquad 4.47a$$

$$B_\mu = b' q'_\mu + b \, q_\mu \qquad \text{etc.} \qquad 4.47b$$

In more details these are

$$A_\mu = - \frac{S}{2t} \; (1 - \frac{t}{4m^2})_{D + \frac{2}{3} F} \qquad \times$$

$$\times \Big[\{q'_\lambda \beta^{10} \Gamma(1-\alpha) \; (\frac{S}{2\mu m})^{\alpha-1} \{\frac{1}{2}(1-e^{-i\pi\beta})(\phi_5,\phi_{\lambda\mu})_F + \frac{1}{2}(1+e^{-i\pi\alpha})(\phi_5,\phi_{\lambda\mu})_D$$

$$+ q_\lambda (\frac{\mu}{m}) \beta^{11} \Gamma(2-\alpha) (\frac{S}{2\mu m})^{\alpha-2} \{\frac{1}{2}(1+e^{-i\pi\alpha})(\phi_5,\phi_{\lambda\mu})_F + \frac{1}{2}(i-e^{-i\pi\alpha})$$

$$\times \; (\phi_5,\phi_{\lambda\mu})\}\} \Big]$$

$$+ \frac{m}{t} \mu \; (1+\frac{t}{4\mu m})(1-\frac{t}{4m^2})_{D+\frac{2}{3} F} \qquad \times$$

$$\times \Big[q'_\lambda \beta^{10} \; (1-\alpha)(\frac{S}{2\mu m})^{\alpha-1} \{\frac{1}{2}(1-e^{-i\pi\alpha})(\phi_5,\phi_{\lambda\mu})_D + \frac{1}{2}(1+e^{-i\pi\alpha})(\phi_5 \phi_{\lambda\mu})_F\}$$

$$+ q_\lambda (\frac{\mu}{m}) \beta^{11} \; (2-\alpha)(\frac{S}{2\mu m})^{\alpha-2} \{\frac{1}{2}(1+e^{-i\pi\alpha})(\phi_5 \phi_{\lambda\mu})_D + \frac{1}{2}(1-e^{-i\pi\alpha})(\phi_5 \phi_{\lambda\mu})_F$$

$$4.48$$

$$B_{\mu} = -\frac{1}{2}\left(1-\frac{t}{4m^2}\right)_{D+\frac{2}{3}F}\Big[\qquad\Big] \qquad 4.49$$

$$C_{\mu} = \frac{1}{2}\left(1+\frac{m}{\mu}\right)_{\frac{1}{3}F-D}\Big[\qquad\Big] \qquad 4.50$$

$$D_{\mu} = \frac{1}{2}\left(1-\frac{t}{4m^2}\right)_{D+\frac{2}{3}}\Big[\qquad\Big] \qquad 4.51$$

The Square brackets in the last three expressions stand for the first term in the square brackets of eq. 4.48. The particles written on top of the field operators are those associated with the trajectories i.e. these terms will represent the contributions of these particles when they are exchanged.

In deriving these expressions we have already removed Kinematic singularities which make their appearance whenever spins are involved. Typical expressions are products of the form

$$\left(1+\frac{2m}{\sqrt{t}}\right)\left(1+\frac{\sqrt{t}}{2\mu}\right)\beta(t)$$

which is not an analytic function of t, near $t = o$. There are two mechanism for removing these singularities: the first is evasion, which means that $\beta(t)$ must have a compensating zero; the second is the addition of conspiring trajectories. As we pointed out in Chapter I, Toller has proposed an elegant ~~formulation~~ solution of this problem; however an alternative solution is to introduce conspiring trajectories following Gribov. This is the solution ~~most~~ best suited to the multispinor formalism for hadrons as it springs from the "doubling", first introduced by Gribov, which finds a natural explanation in terms of quarks within a multispinor framework. We then have to add to the amplitudes extra terms with the sign of M reversed. For this amplitude we typical~~ly~~ meet the combination

$$\frac{1}{2}\,(1+\frac{2m}{M})(1+\frac{M}{2\mu})\beta(M)\Gamma(1-\alpha)s^{\alpha-1}+\frac{1}{2}(1-\frac{2m}{M})(1-\frac{M}{2\mu})\beta'(M)\,\Gamma(1-\alpha')s^{\alpha'-1}$$

Taking $\alpha(0) = \alpha'(0)$, $\beta(0) = \beta'(0)$, the $M = \sqrt{t}$ singularity dissapears. The whole procedure boils down to the following prescription

$$T_{effective} = \frac{1}{2}(T + T_{conspirator}) \qquad 4.52$$

There is no reliable theory to take account of mass-splitting between members of a supermultiplet, so the only course open at present is to take the positions of the trajectories as empirical inputs.

Before we proceed to consider the applications of this model we must discuss the role of the Reggeised pion.[128] This particle has stubbornly resisted attempts at Reggeisation. One of the reasons is that the contribution of pion exchange in NN collisions is given by

$$\frac{G^2}{t-m_\pi^2}\,\bar{U}\gamma_5 U\bar{U}\gamma_5 U$$

Now summation over the nucleon spins gives a factor t for each $\bar{U}\gamma_5 U$ so the cross-section is given by (100 footnote)

$$\frac{d\sigma}{dt} = \frac{\pi}{2k^2 s}\;\frac{G^4\;t^2}{(t^2-m_\pi^2)} \qquad 4.53$$

and for a Reggeised pion one obtains

$$\frac{d\sigma}{dt} = \frac{\pi}{2k^2 s}\;G^4 t^2\left[\pi\,\alpha\frac{1+e^{-i\pi\alpha}}{\mathrm{Sin}\;\pi\alpha}\;(\frac{s}{s_0})^{\alpha}\right]^2 \qquad 4.54$$

G is the nucleon-nucleon-pion coupling. This cross-section is zero in the forward direction —— in direct contradiction with experiments which imply that pion exchange is responsible for the forward peaking of the differential cross-section.

To reconcile theory with experiment conspiracy and evasion have been put forward.[129] These explanations however have met with some setbacks quite aside from the rather artificial character of the fits as exemplified by the widely varying residues,[130] obtained by different authors.
For instance it has been shown by Le Bellac[129] that conspiracies predict dips in the forward directions for reactions such as

$$\pi N(\pi)\rho\Delta\ ,\quad KN(\pi)K\Delta\ ,\quad \pi N(\pi)F^{o}\Delta$$

whereas experimentally no such dips are observed. It has also been suggested that one should interpret the pion as a J_o+= 1 Toller pole and using conspiracy to obtain a non-vanishing contribution.[131] In the present formalism Delbourgo,[132] regarding the pion as a fermion-antifermion composite, and using symmetry breaking, has obtained an amplitude which apparently has the right behaviour but the whole scheme has been violently criticised[128] and so we shall not use it. Another scheme which has been successfully exploited by Moriarty[133] et al is to 'absorb' the Reggeised amplitude. As can be seen from eq. 4.52 the amplitude $\emptyset$ is given by

$$\emptyset \sim \frac{t}{t-m_\pi^2}$$

$$= 1 + \frac{m_\pi^2}{t-m_\pi^2}$$

The first term is an s wave contribution which is removed by absorption corrections giving

$$\emptyset\ (\text{absorbed}) \sim \frac{m_\pi^2}{t-m_\pi^2}$$

Given these difficulties we cannot study all the reactions which were considered in Chapter III; only those not involving the exchange of a 0^- particle will be considered. These are:

$\pi^+P(\rho)A_2P$, $\pi^+P(\rho)A_1P$, $K^-P(\omega,\rho,A_2)K^-(1320)P$, $\pi^+P(\omega,\rho)B_1P$

where the exchanged particles are indicated in the brackets. The couplings for each of these exchanges have been given in Chapter III. We assume, as is customary, that the residues are just numbers and that all the trajectories are linear, $\alpha = \alpha_0 + \alpha_1 t$. We fix the value of the constant α_o by making our line go through the mass of the particle it represents. We then have the following sets of parameters. The parameters which are allowed to vary are lebelled by X and the figures after them show the range of values between which they were constrained.

(i) For the A_2 trajectory

$\alpha_1 = X_1$	$0.7 \leq X_1 \leq 1.0$
$\alpha_o = 2.0 - 1.745x\alpha_1$	
$\beta = X_2$	$-\infty < X_2 < \infty$

(ii) For the RHO trajectory

$\alpha_1 = X_3$	$0.8 \leq X_3 \leq 1.0$
$\alpha_o = 1.0 - 0.582 + \alpha_1$	
$\beta = X_4$	$-\infty < X_4 < \infty$

(iii) For the OMEGA trajectory

$\alpha_1 = X_5$	$0.1 \leq X_5 \leq 2.0$
$\alpha_o = 1.0 - 0.613x\alpha_1$	
$\beta = X_6$	$-\infty < X_6 < \infty$

(iv) For the A_1 trajectory

$\alpha_1 = X_7$	$0.1 \leq X_7 \leq 2.0$
$\beta = X_8$	$\infty < X_8 < \infty$

(v) For the PHI trajectory

$\alpha_1 = X_9$ $\quad 0.1 \leq X_9 \leq 2.0$

$\alpha_o = 1.0 - 1.04 x \alpha_1$

$\beta = X_{10}$ $\quad -\infty < X_{10} < \infty$

We then see that overall we have 10 parameters, half of which are severly constrained between narrow ranges. In performing the CHI square[134] minimisation the search was made at intervals of 0.1 for the residues, and 0.01 for the slopes.

4.4 Comparison with Experiments and Conclusions

The results, as can be seen from the figures are not exactly brilliant. For 58 experimental data points the minimised CHI square, was 2500. The value of χ^2 characterises the goodness of the fit. Ideally with data, free from systematic error and a perfect theory, the expected value is the number of data points less the number of adjusted parameters. However, when the quoted accuracy of data becomes less than the systematic errors of experiment, the value of χ^2 soars. And this is precisely what is happening here. The number of data is low and the errors on them are large. So that, although the χ^2 is very large, one cannot immediately conclude that the results are poor. With the errors we have, no theory with a smooth energy variation can give an ideal fit.

The final results for the trajectories and the residues were as follows:

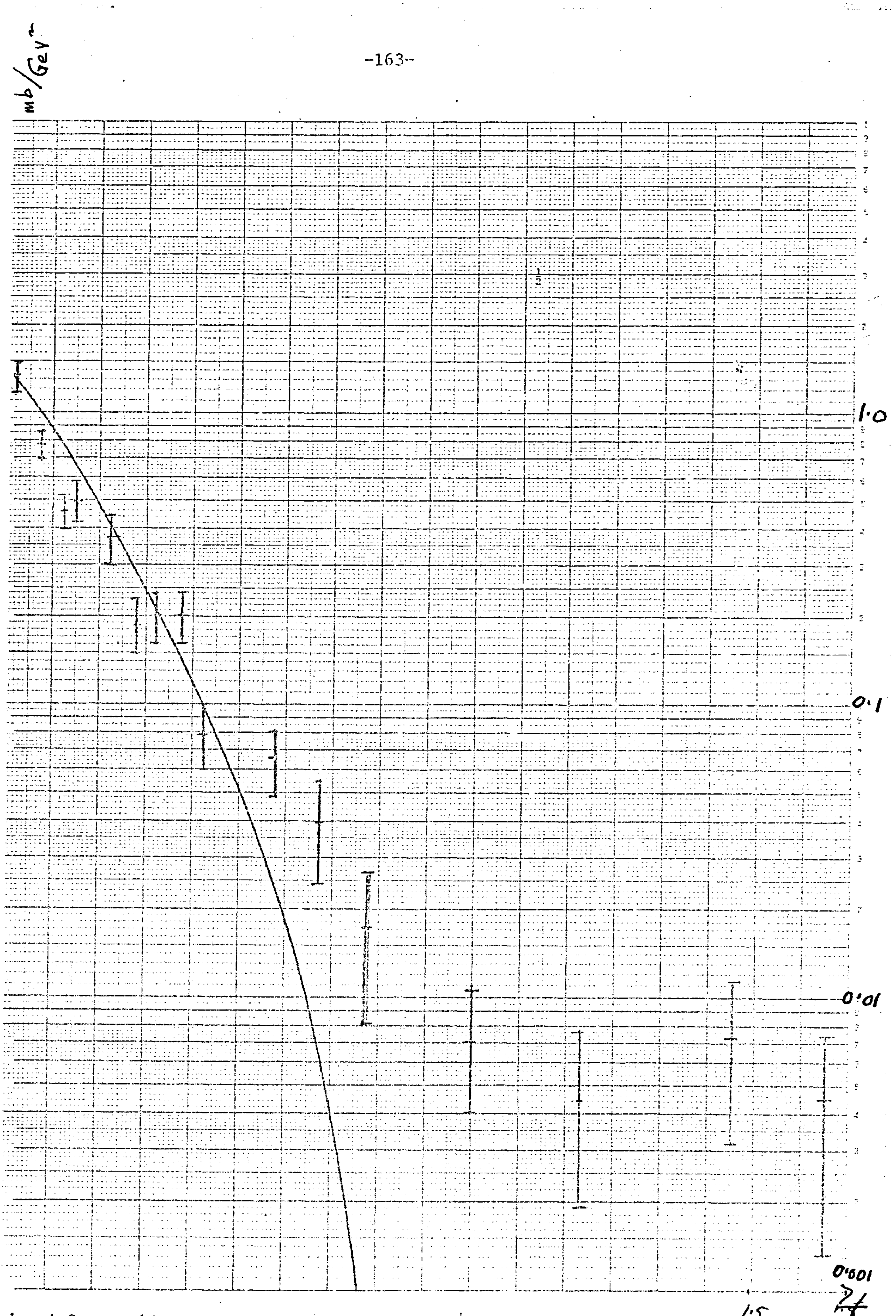
for the A_2 the slope was 0.88 and the residue was 50.3 respectively; for the ρ they were 0.8 and 1.02 respectively; for ω they were 151.1; for the A_1 they were 0.8 and 1.0 respectively;

for the ϕ 0.9 and 1.0 respectively; the slope and the residues are in units of $(Gev/c)^{-2}$ and $(Gev/c)^{-1}$ respectively. The slopes have the generally accepted values. As for the residues, we cannot compare them directly with values of R.J.N Phillips et al[19,135] or with thos of K.Moriarty et al[133]; the reason being that the former has too many parameters -for instance he has 18 parameters for KN scattering- so that comparison is meaningless; the latter uses a slope $\alpha=\alpha_0+\alpha_1 e^{\alpha_2 t}$ which means there are four parameters for each non-linear trajectory. However we can compare the relative values of the ρ and A_2 residues: from SU_3 the ratio of the process $\pi^- P(\rho)\pi^0 n/\pi^- P(A_2)\eta n$, where the exchanged trajectories are indicated in bracket, is $(\frac{2}{2/\sqrt{3}})^2=3$ whereas we get 2500. The only explanation we can put forward for this huge discrepancy is that we had a very poor statistics at our disposal; the total number of points for all the processes was a meagre 58 with the errors on them being large. Again when we assumed exchange degeneracy i.e. $\alpha_\rho=\alpha_{A_2}$, $\beta_\rho=\beta_{A_2}$ etc, our χ^2 was 8,300; values of the slopes were $\alpha_\rho=\alpha_{A_2}=0.9$; $\alpha_\omega=0.3$; $\alpha_{A_1}=0.7$; $\alpha_\phi=0.1$ all are in units of $(Gev/c)^{-2}$. The residues were $\beta_\rho=\beta_{A_2}=45.6$; $\beta_\omega=81$; $\beta_{A_1}=11$; $\beta_\phi=7$; all are in units of $(Gev/c)^{-2}$. Given the large χ^2 one cannot attach much significance to these values.

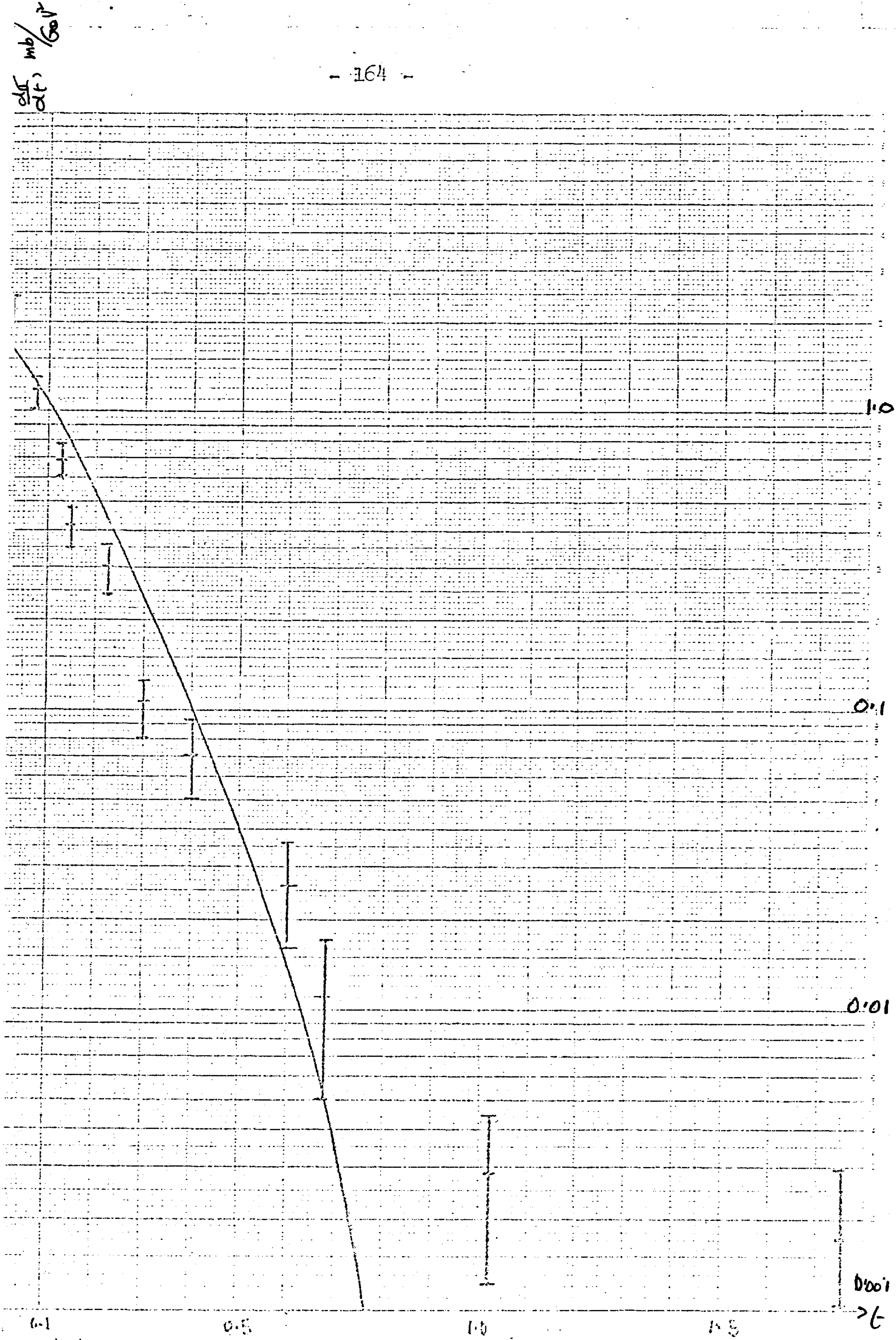
The theory does not solve the problem of the Reggeised pion— one of its main drawbacks. Another way to incorporate pion exchange is to absorb the Reggeised amplitude:[139]

$$T' = T_{Regge} + (S)^{1/2}\ T\ (S)^{1/2}$$

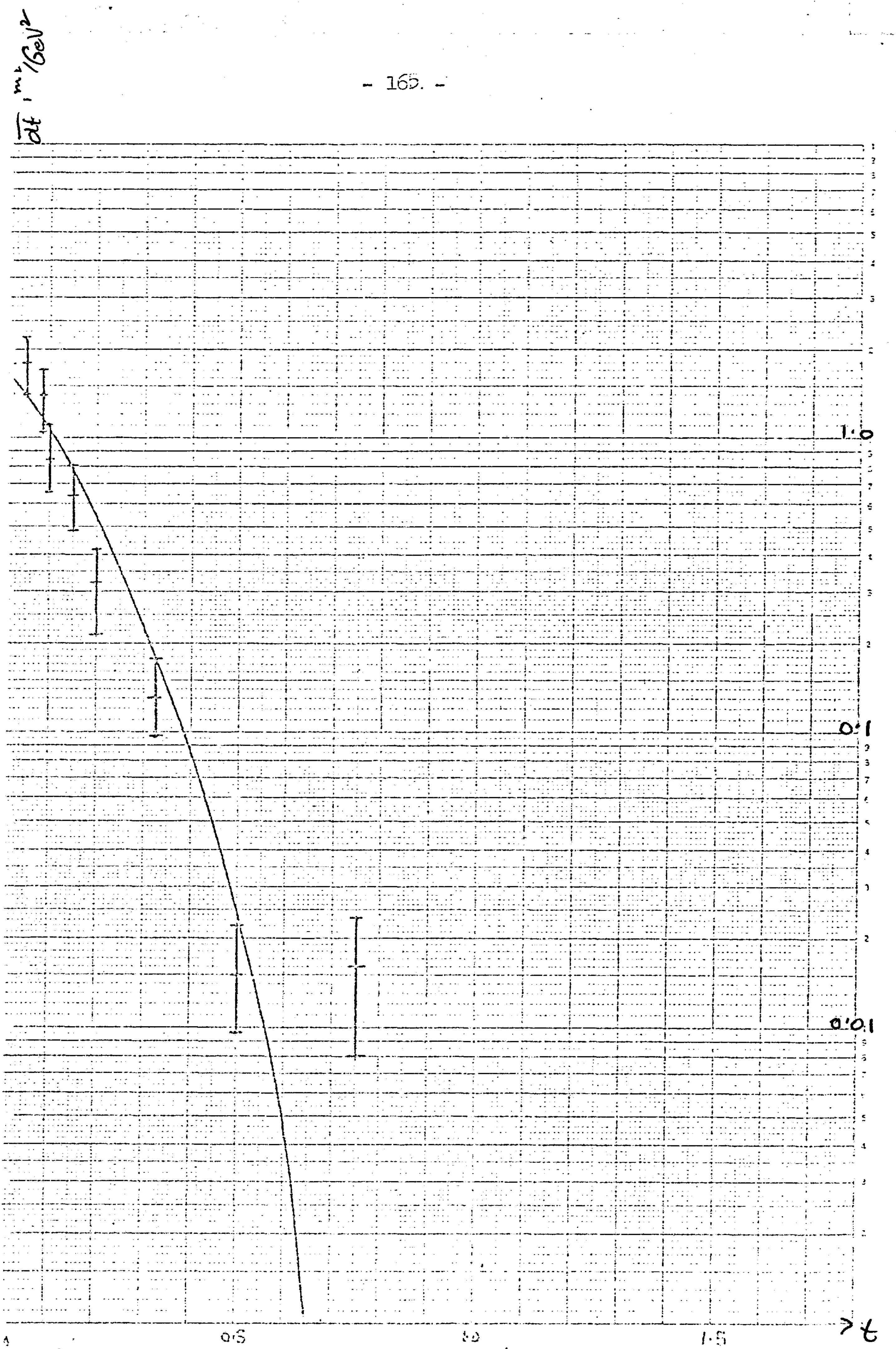
The Absorbed Reggeised amplitudes overcomes some of the difficulties of the Regge pole approach[140] —such as exchange of 0^- particles — and also overcomes the difficulties of the Absorption model such as wrong energy dependence and the problem of the exchange of high spin particles.



ig. 4.3 Differential cross-section for $\pi^+p \longrightarrow A_2P$ at 8.0 Gev/c. Data from Ref. 114.



ig. 4.4 Differential cross-section for $\pi^{+}p \longrightarrow A_1(1070)P$ at 8.0 Gev/c. Data from Ref. 114(1st.)



ig. 4.5 Differential cross-section for $\pi^+ p \longrightarrow A_1(1070)p$ at 8.0 Gev/c. Data from Ref. 114 (2nd.)

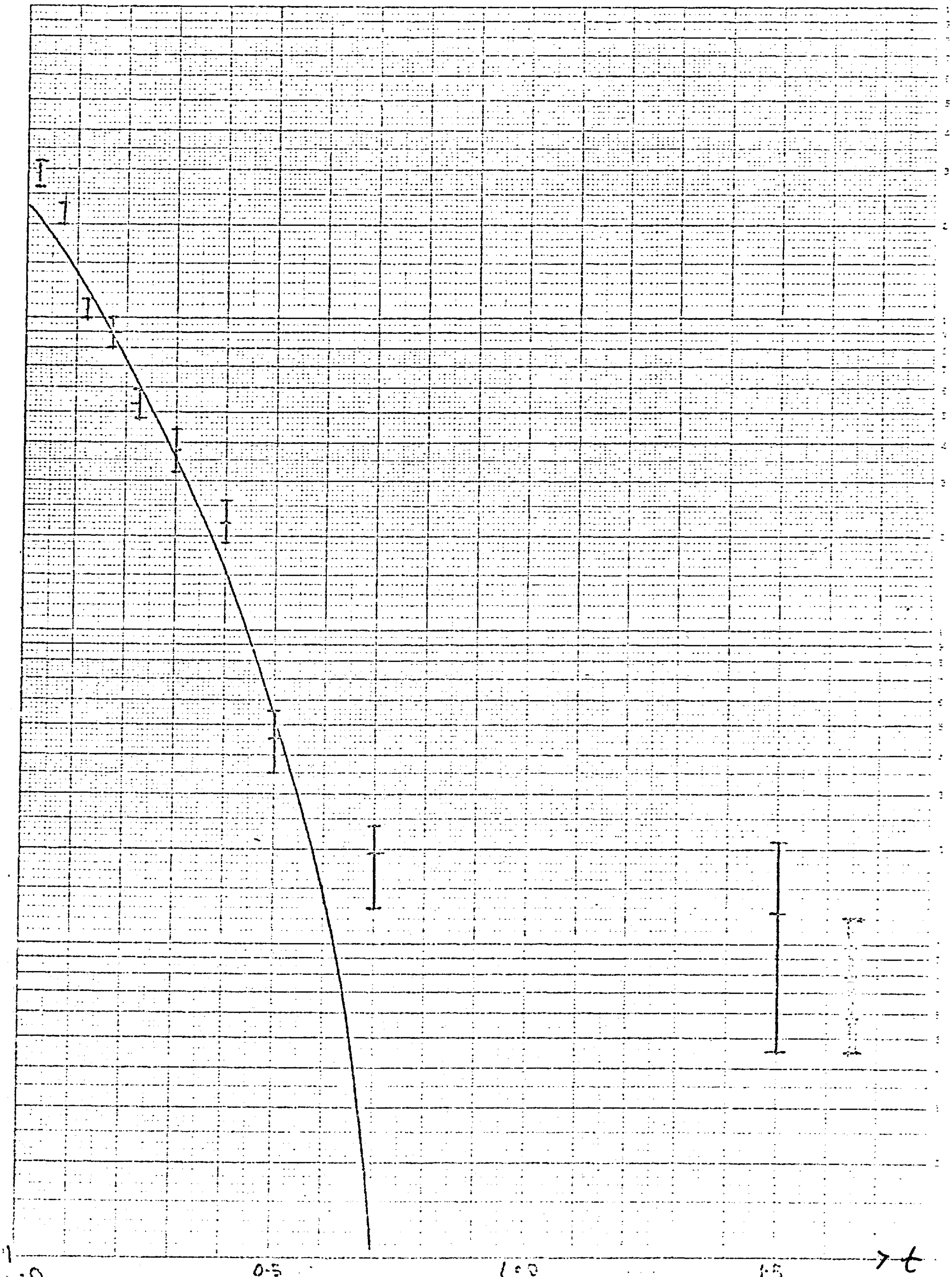


Fig. 4.6 Differential cross-section for $K^-P \longrightarrow K^*(1320)\ P$ at 10.0 Gev/c. Data from Ref. 114.

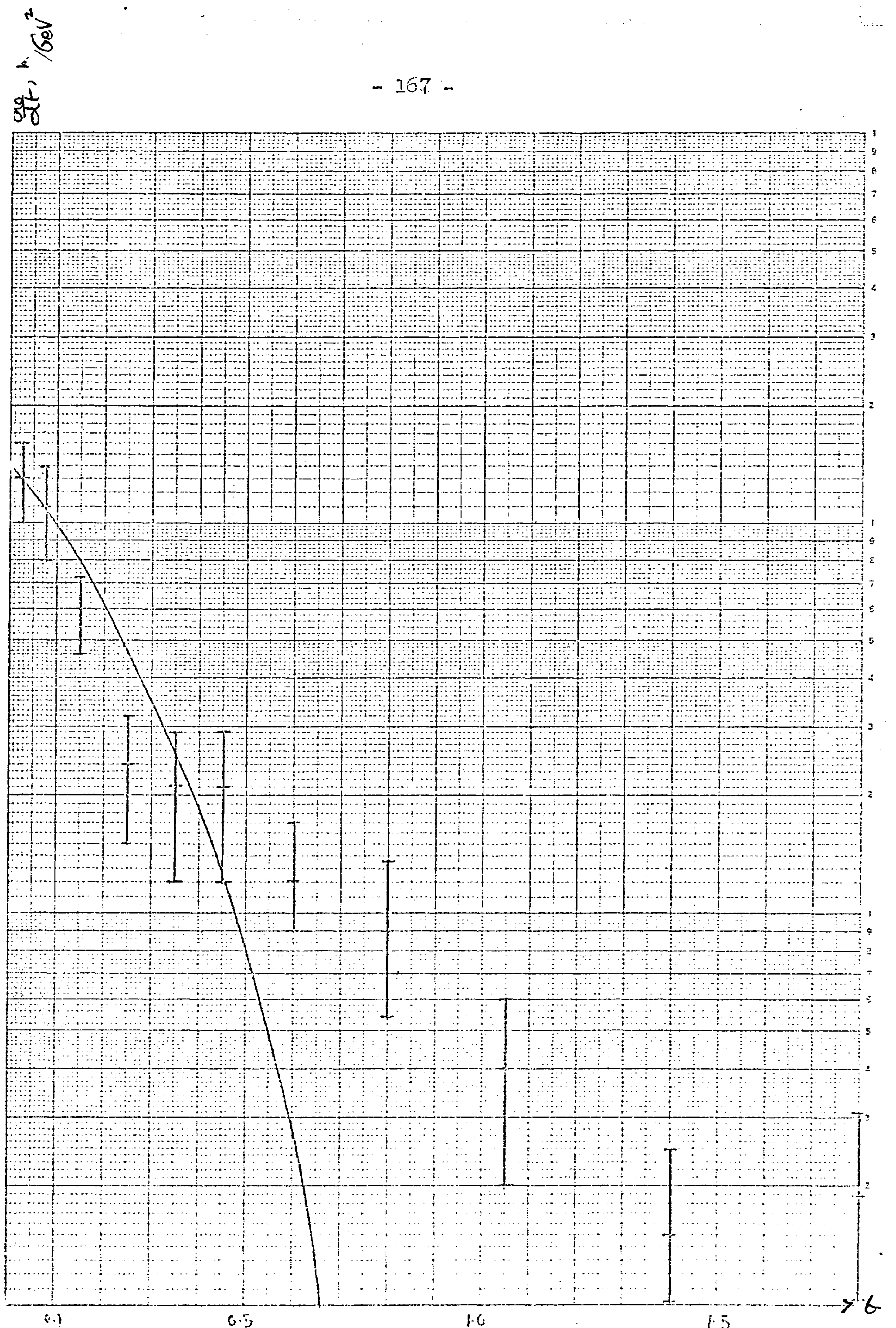


Fig. 4.7 Differential cross-section for $\pi^+P \longrightarrow B_1(1220)P$ at 8.0 Gev/c. Data from Ref. 114.

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$$E^{(1)} = (E_1 E)$$

form an invariant subalgebra g E and is called the first derived algebra of E. If E is abelian $E^{(1)} = o$; if it is semisimple then $E^{(1)} = E$. Let

$$E^{(2)} = (E^{(1)}, E^{(1)}) ;$$

then $E^{(2)}$ is called the second derived algebra of E and is an invariant subalgebra of $E^{(1)}$ and of E. A Lie Algebra is said to be solvable if for some integer k

$$E^{(k)} = o$$

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$$(J_i, J_i) = i\, f_{iik}\, J_k$$

one can construct the dual representations

$$J' = -\, i\, J^T$$

The corresponding representations of the group elements are

$$U = \exp\,(i \epsilon J)$$

and

$$U' = (U^{-1})^T$$

For ordinary rotations there exist a matrix B which relates these two representations

$$D = B D' B^{-1}$$

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74. The scattering amplitude for πN scattering may be written formally as

$$F = D + i A$$

where

$$A^{\pm} = \frac{k\sigma^{\pm}}{4\pi}$$

$$D^{-}(k) = D^{+}(o) + \frac{f^2 k^2}{M(1 - 1/4\ M^2)(\omega - 1/2\ M)^2}$$

$$+ \frac{k^2}{2\pi^2} P \int_1^{\infty} \frac{\omega'}{k'} \frac{\sigma^{+}(\omega')}{\sigma'^2 - \sigma^2} d\sigma'$$

$$D^{-}(k) = \frac{2f^2\omega}{\omega^2 - 1/4\ M^2} + \frac{\omega}{2\pi} P \int_1^{\infty} \frac{\sigma^{-}(\omega')d\omega'}{\omega'^2 - \omega^2}$$

Here k and $\omega = \sqrt{1+k^2}$ are the momentum and energy of the pion i $M = 6.72$ is the nucleon mass; $f^2 = 0.081$; $D^{+}(o)=o$. The superscripts denote combinations of isotopic $\frac{1}{2}$ and $\frac{3}{2}$ states in $\pi^{\pm}P$— $\pi^{\pm}P$ scattering. The contribution from the Dispersion integral is found to be important in such a analysis.

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89. Provided the amplitude satisfies analyticity and crossing, and has Regge asymptotic behaviour, its discontinuity in $\omega = \frac{1}{2}(s - u)$ of $A_\omega(\omega, t)$, at fixed t satisfies

$$\frac{1}{2}\int_{-N}^{+N} A_{\omega}(\omega,t)\,\omega^{n}\,d\omega = \int_{-N}^{+N}\Big(\text{Background integral} + \sum_{\text{Regge cuts}} + \sum_{\text{Regge poles}}\Big)\,\omega^{n}\,d\omega$$

If we drastically truncate the right side keeping only the leading Regge pole, we then obtain eq. 3.38.

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Let $E = \sqrt{PL^2+M_1^2}$ where PL is the lab momentum;

$S = M_1^2 + M_2^2 + 2.M_2.E$; then $k = PLxM_2/\sqrt{s}$

$E_1 = \sqrt{k^2 + M_1^2}$; $E_2 = \sqrt{s} - E_1$; $E_3 = (s+M_3^2 - M_4^2) /2\sqrt{s}$;

$Q = \sqrt{E_3^2 - m_3^2}$

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127. All rest symmetry groups G must contain the subgroups $SU_J(2) \otimes SU_F(3)$ where $SU_J(2) = SU_L(2) \oplus SU_S(2)$. the $SU_J(2)$ into the Lorentz structure, one identifies the conventional helicity subgroup as that subgroup of $SU_J(2)$ whose elements commute with the Lorentz boosting operator J_{03} .On top of examples given we give the following examples to illustrate the above-mentioned rule:

(i) $G = SU(6) \otimes O_L(3)$, $G_W = U(3) \otimes U(3)_W \otimes O(2)$

$$\mathcal{G} = SL_S(6,C) \otimes O_L(3,1)$$

(ii) $G = U(6) \otimes U(6)_{F,S} \otimes O_N(4)$, $G_W = U_W(6) \otimes O(3)$

$$\mathcal{G} = U_S(6,6) \otimes O_N(4,1), \quad \text{or}$$

$$U_S(6,6) \otimes O_N(4,2) = U(6,6) \otimes U_N(2,2)$$

$$\text{(iii) } G = U(6) \otimes U(6)_{F,S} \otimes SU_N(3), \quad G_W = U_W(6) \otimes U(2)$$

$$\mathcal{G} = U_S(6,6) \otimes U_N(3,1)$$

128. These difficulties have been summarised by Ph.Salin in 'Topical Conference on High Energy Collisions of Hadrons' Pp 478, CERN 68-7; E.Leader, invited paper, Institute of Physics and the Physical Society, Conference on Elementary Particles Physics, Cambridge University, 1969.(Unpublished) J.H. Weiss Phys. Rev. 176, 1822, 1968.

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130. R.J.N.Phillips Nuclear Physics B2, 394, 1967; J.S.Ball, W.Frozer and M.Jacob Phy. Rev. Letters 20, 518, 1968; F. Arbab and J.W.Dash, UCRL - 17585.

131. M. Toller Nuovo Cimento 41, 167, 1966.

132. R.Delbourgo 'Pion Exchange in Reggeised Supermultiplet Theory' Imperial College Prepint ICTP/68/21.

133. K.Moriarty et al Phy. Rev. 9D, 2619, 1970; ibid 187,1921, 1969; ibid 11D, 3195, 1970, ibid 1D, 954,1970.

134. Chi Square is defined as

$$X^2 = \sum \left(\frac{X_{exp.} - X_{theo}}{1/2 \text{ error}}\right)^2$$

135. R.J.N.Phillips and W.Rorita Phy. Rev. 139, B1336, 1965; W.Rarita and B.Schwarzschild, ibid 162, 1378, 1967; R.J.N. Phillips et al ibid 165, 1615, 1958.

136. It has been shown that for certain classes of reasonable

potentials, e.g. a Yukawa one, the resultant Regge trajectory is such that

$$\alpha(t) \longrightarrow \frac{1}{2} N - \frac{1}{2} \quad \text{as } t \longrightarrow -\infty$$

where N is a positive integer. The leading trajectory behaves as

$$\alpha(t) \longrightarrow -1 \quad \text{as } t \longrightarrow -\infty$$

In Chap 12 of Ref. 137 non-linear trajectories for various potentials are shown. In the relativistic domain, perturbation theory shows that the typical trajectory obtained by summing Feynmann diagrams is of the form

$$\alpha(t) = -N + \sum_{n=1} g^{2n} \; K_n(t)$$

where $K_n(t) \longrightarrow 0$ as $t \longrightarrow -\infty$ Again for the leading trajectory $\alpha(t) \longrightarrow -1$ as $t \longrightarrow -\infty$ See ref. 138; S.Klein Phy. Rev. 1D, 609, 1970

137. R.G.Newton: 'The Complex J Plane' Benjamin 1964.

138. R.J.Eden, P.V.Landshoff, D.I.Olive and J.C.Polkinghorne: 'The analytic S Matrix' C.U.P. 1966.

139. R.C.Arnold Phy. Rev. 140B, 1022, 1965; R.C. Arnold and M.L.Blackmon ibid 176, 2082, 1968; M.L.Blackmon and G.R. Goldstein ibid 179, 1480, 1969; F.S.Henyey et al Phy. Rev. Letters 21, 946, 1968; F.Schremp Nucl. Phys. B6,487,1968; J.N.J.White Phy. Letters 27B, 92, 1968; M.L.Blackmon Phy. Rev. 178, 2385, 1969; M.L.Blackmon et al ibid 183,1452,1969

140. Some of the problems associated with the simple Regge Pole approach are discussed by R.C.Arnold, Argonne National Lab. Report No. ANL/HEP 6804, 1968.

APPENDIX A

Evaluation of One- Particle-Exchange Graphs.

Section A.1

In the computation of these amplitudes we chose the Pauli representation of the γ matrices:

$$\gamma_o = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

$$i\gamma_5 = \gamma_o\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

where

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

As we have pointed out in Chapter III, e.g. eq. 3.99, these amplitudes may be written formally as

$$\langle P_3\ \lambda_3,\ P_4\ \lambda_4 / T / P_1\ \lambda_1,\ P_2\ \lambda_2 \rangle$$
$$= \langle \lambda_3/\lambda_1 \rangle \langle \lambda_2/\lambda_4 \rangle$$

This result follows directly from eq. 3.76 and essentially it means that we can evaluate the vertices appearing in these diagrams independently of each other . See figs. 3.1 and 3.2.

The bottom vertex, $\langle\lambda_2/\lambda_4\rangle$, in the case ~~this~~ where it involves pseudo-scalar exchange is given explicity by 3.70 of Pp. 92.

$$\langle\lambda_2/\lambda_4\rangle = \bar{N}_{\pm}(P_4) i\gamma_5 N(P_2)$$
$$= N_4 N_2 \left[\begin{pmatrix} E_4+m_4 \\ \pm P_4 \end{pmatrix} \otimes \chi_{\mp} \right]^{\dagger} \gamma_o i\gamma_5 \left[\begin{pmatrix} E_2+m_2 \\ \pm P_2 \end{pmatrix} \otimes \chi_{\mp} \right]$$

Evaluation of the above expression and those similar to it is considerably simplified if we write the γ matrices appearing in them as the outer product of two 2 x 2 matrices

$$\gamma_0 = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} \otimes \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} ; \quad \gamma_5 = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \otimes \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

Then using the identity $(A\otimes B)(C\otimes D) = (A\,C) \otimes (B\,D)$ several times we have nothing more than the multiplication of 2 x 2 matrices to deal with.

N_4 and N_2 are normalization factors given by equation 3.916.

The net result is:

$$\bar{N}(P_4;\lambda_4=\pm)\gamma_5 N(P_2;\lambda_2=+) = -\frac{1}{D^{1/2}}\,[BQ\mp AK]\begin{pmatrix}C_1\\S_1\end{pmatrix}$$

$$\bar{N}(P_4;\lambda_4=\pm)\gamma_5 N(P_2;\lambda_2=-) = \frac{1}{D^{1/2}}\,[BQ\pm AK]\begin{pmatrix}-S_1\\C_1\end{pmatrix}$$

where $A = E_4+m_4$; $B = E_2+m_2$; $C = (E_1+m_1)(E_3+m_3)$; $D = AxB$

$S_1 = \text{Sin}\,\frac{\theta}{2}$; $C_1 = \text{Cos}\,\frac{\theta}{2}$; $K = \underline{P}_{Lab}*\,M_2/S^{1/2}$; $Q = \sqrt{E_3^2 - M_3^2}$

See also footnote 100.

For the exchange of a vector particle we need the following expressions which appear in eq. 3.74:

$$\bar{N}(P_4;\lambda_4=+)N(P_2;\lambda_2=\pm) = \frac{1}{D^{1/2}}\,(D\mp QK)\begin{pmatrix}C_1\\-S_1\end{pmatrix}$$

$$\bar{N}(P_4;+)\gamma_0 N(P_2;\pm) = \frac{1}{D^{1/2}}\,(D\pm QK)\begin{pmatrix}C_1\\-S_1\end{pmatrix}$$

$$\bar{N}(P_4;+)\gamma_1 N(P_2;\pm) = \frac{1}{D^{1/2}}\,(D\pm QK)\begin{pmatrix}C_1\\-S_1\end{pmatrix}$$

$$\bar{N}(P_4;+)\gamma_2 N(P_2;\pm) = \frac{1}{D^{1/2}}\,(BK\pm AQ)\begin{pmatrix}-S_1\\-C_1\end{pmatrix}$$

$$\bar{N}(P_4;+)\gamma_3 N(P_2;\pm) = \frac{1}{D^{1/2}}(BK\pm AQ)\binom{-C_1}{S_1}$$

The only other term we need to know for the bottom vertex is the one involving the formation of a $\frac{3}{2}^+$ resonance

$P_2^\lambda \; \bar{D}_{\lambda_4}(P_4)N_{\lambda_2}(P_2)$. The values of $D_{\lambda_4}(P_4)$ for $\lambda_4 = \pm\frac{3}{2}, \pm\frac{1}{2}$ are listed on page 99.

Using these we then have

$$\langle\tfrac{3}{2} / \pm \tfrac{1}{2}\rangle = \frac{-KSin\theta}{(2D)^{1/2}}(D\mp QK)\binom{C_1}{S_1}$$

for instance;

its value for the other helicity amplitudes are listed at the end of section A.4.

Section A.2

For the production of 2^+ particles one needs to evaluate $\bar{\phi}_{\mu\lambda}(P_3)\,\phi_5(P_1)\,P_1^\mu\,P_1^\lambda$ and $\epsilon_{\mu\alpha\beta\nu}\,P_1^\rho P_1^\beta P_3^\alpha\;\bar{\phi}_{\mu\rho}(P_3)\phi(P_1)$ which correspond to the $2^+0^-0^-$ and $2^+1^-0^-$ vertices respectively. See eqs. 3.66 and 3.67. The momenta are given by eq. 3.78 and the various values of the 2^+ wavefunction corresponding to $\lambda_3 = \pm 2, \pm 1, 0$ are listed on page 99. Evaluation of these terms is fairly straightforward and then to obtain the ten independent amplitudes we only have to multiply these by the relevant expressions of the bottom vertex. Their values are given at the end of Section A.4. Then the amplitudes are:

All symbols retain their meaning unless and until they are redefined.

For 0^- exchange: $\langle\lambda_3\lambda_4/T/\lambda_1\lambda_2\rangle$ *

ϕ_1 = SQ6. T_1. V_2PP. VC = $\langle\lambda_3=0/\lambda_1=0\rangle\langle\lambda_4=+/\lambda_2=+\rangle$

ϕ_2 = SQ6. T_1. V_2PM. VC = <0/0><+/->

ϕ_3 = K. SN. CC. V_2PP.VC = <1/0><+/+>

ϕ_4 = K. SN. CC. V_2PM. VC = <1/0><+/->

ϕ_5 = K. SN. CC. V_2MP. VC = <1/0><-/+>

ϕ_6 = K. SN. CC. V_2MM. VC. = <1/0><-/-)

ϕ_7 = CC1. V_2PP. VC = <2/0><+/+>

ϕ_8 = CC1. V_2PM. VC = <2/0><+/->

ϕ_9 = CC1. V_2MP. VC = <2/0><-/+>

ϕ_{10}= CC1. V_2MM. VC = <2/0><-/->

where

VC = $G_1G_2/(t - W_2)$, W_2 = mass of the 0^- particle and G_1 , G_2 are the couplings at the two vertices their explicit form being given on Pp. 102.

$$V2PP = -\frac{1.0}{D^{1/2}}(QB - KA)\ C_1$$

$$V2PM = -\frac{1.0}{D^{1/2}}(QP+KA)S_1$$

$$V2MP = \frac{1.0}{D^{1/2}}(QP+KA)S_1$$

$$V2MM = \frac{1.0}{D^{1/2}}(QB - KA)\ C_1$$

$$SQ6 = 1/6^{1/2}$$

$$T_1 = \frac{1}{M_3^2}\ (2(E_1Q)^2 - (M_3K.\text{Sin }\theta)^2 + 2.0(E_3\text{Cos }\theta)^2 - 4.E_1E_3.Q.K.\text{Cos }\theta)$$

$$CC = (E_1Q - E_3K\ \text{Cos }\theta)/M_3$$

$CC1 = 0.5.(K.Sin\ \Theta)^2$

$SN = Sin\ \Theta$, $CS = Cos\ \Theta$

For 1^- exchange (with the previous notation)

the helicity amplitudes are: ~~as before~~.

$\phi_1 = 2SQ6.\ VC.\ FS2.\ K.\ SN.\ CC\phi.2Qp.\ S_1/M_3^2$

$\phi_2 = 2SQ6.\ VC.\ FS2.\ K.\ SN.\ CC\phi.2QM.\ C_2/M_3^2$

$\phi_3 = CC.\ VC(FS1.\ K.\ SN.\ ESM.\ DM.\ C_2 + FS2.(E_3.K.Sin\ \Theta\ .$

$(QP.C_2 - DP.S_1\ )) + 2.\ CC.\ QP.\ S_1)/2.$

$\phi_4 = CC.\ VC.(FS1.\ K.\ ESM.\ DP.\ S_1 + FS2(E_3.\ K.\ Sin\ \Theta$

$(QM.\ S_1 + DM.\ C_1) - 2.\ CC.\ QM.\ C_1)/2.$

$\phi_5 = -\ CC.\ VC.\ K.\ SN.\ S_1(FS1.\ ESM.\ DP + FS2.\ E_3(QM + DM)/2.$

$\phi_6 = CC.\ VC.\ K.\ SN.\ C_2(FS1.\ ESM.\ DM - FS2.E_3(QP - DP)/2.$

$\phi_7 = VC.\ K.\ SN.\ (K.\ SN.\ FS1.\ ESM.\ DM.\ C_2 + FS2(E_3.\ K.\ SN$

$(QP.\ C_1 - DP.\ S_1) + 2.\ CC.\ QP.\ S_1)/2.0$

$\phi_8 = VC.\ K.\ SN.\ (K.\ SN.\ FS1.\ ESM.\ DP.\ S_1 + FS2(E_3K.\ SN.$

$(QM.\ S_1 + DM.\ C_1) - 2.\ CC.\ QM.\ C_2)/2.0$

$\phi_9 = VC.\ (K.\ SN)^2.\ S_1\ (FS1.\ ESM.\ DP - FS2.\ E_3(QM + DM)/2.0$

$\phi_{10} = VC.\ (K.\ SN)^2.\ C_2(FS1.\ ESM.\ DM - FS2.\ E_3(QP - DP)/2.0$

where

$FS1 = 1.0 + t/(2MV.\ MO) - \frac{5}{3}(1. + \frac{2MO}{MV})$

$FS2 = \frac{5.}{3.}\ (1.0 + \frac{2MO}{MV})(1.0 + 2.\ MO^2 - t)/MO^2$

$ESM = E_3(K.\ CS + Q + E_2 + E_4) + CC$

DP = D + Q.K

DM = D - Q.K

QP ∓ QB + K. A

QM = QB - KA

Definitions of MO and MV on *Pp. 93.*

Section A.3

For the production of 1^- particles we only have to evaluate the expression $P_1^\lambda \, \phi_\lambda(P_3)$ as shown by eq. 3.68. The six independent amplitudes are

ϕ_1 = CC. (V2PP + FSPM). VC = <0/0><+/+>

ϕ_2 = CC. (V2PM + FSPP). VC = <0/0><+/->

ϕ_3 = CC. (V2PP + FSPM). VC = <1/0><+/+>

ϕ_4 = CC. (V2PM + FSPP)VC = <1/0><+/->

ϕ_5 = CC. (V2MP + FSPP). VC = <1/0><-/+>

ϕ_6 = CC. (V2MM + FSPM). VC = <1/0><-/->

where

FSE = $E_1 + E_3$

FSK = K. SN

FSQK = K. Cos Θ + Q

P1 = $E_1(E_2 + E_4) + K^2$ + K. Q

P2 = $E_3(E_2 + E_4)$ + Q. K. Cos Θ + Q^2

FSP = $(P_1 + P_2)$FS1/$D^{1/2}$

FSPP = FSP. DP. S_1

FSPM = FSP. DM. C_1

CC = $(E_3 \cdot K \cdot \cos\Theta - M_3 \cdot E_1 Q)/M_3$

$KSN2 = - \frac{K.SN.}{2^{1/2}}$

$V2MM = FS2. (FSE. DP. C_1 + FSK. QP. S_1 + FSQK. QP.C_1)/D^{1/2}$

$V2PP = FS2. (FSK. DP. S_1 + FSQK. QP. C_1 - FSE. DP. S_1)/D^{1/2}$

$V2PM = FS2 (FSE. DM. C_1 - FSK. QM. C_1 + FSQK. QM. S_1)/D^{1/2}$

$V2MP = FS2(FSK.QP. S_1 - FSE. DM. S_1 - FSQK. QM. S_1)D^{1/2}$

Section A.4

For the production of the double resonance F^0N^{*++} in the process $\pi^+P \longrightarrow F^0N^{*++}$ the 20 independent amplitudes are as follows:

	$\langle\lambda_3\lambda_4/ /\lambda_1\lambda_2\rangle$	$\equiv$	$\langle\lambda_3/\lambda_1\rangle\langle\lambda_4/\lambda_2\rangle$
ϕ_1	= V10. V22PP. VC	=	$\langle 0/0\rangle\langle\frac{1}{2}/+\rangle$
ϕ_2	= V10. V22PM. VC	=	$\langle 0/0\rangle\langle\frac{1}{2}/-\rangle$
ϕ_3	= V10. V23PP. VC	=	$\langle 0/0\rangle\langle\frac{3}{2}/+\rangle$
ϕ_4	= V10. V22 PM. VC	=	$\langle 0/0\rangle\langle\frac{3}{2}/-\rangle$
ϕ_5	= V11. V22PP. VC	=	$\langle 1/0\rangle\langle\frac{1}{2}/+\rangle$
ϕ_6	= V11. V22PM. VC	=	$\langle 1/0\rangle\langle\frac{1}{2}/-\rangle$
ϕ_7	= V11. V22MP. VC	=	$\langle 1/0\rangle\langle-\frac{1}{2}/+\rangle$
ϕ_8	= V11. V22MM. VC	=	$\langle 1/0\rangle\langle-\frac{1}{2}/-\rangle$
ϕ_9	= V11. V23PP. VC	=	$\langle 1/0\rangle\langle\frac{3}{2}/+\rangle$
ϕ_{10}	= V11. V23PM. VC	=	$\langle 1/0\rangle\langle\frac{3}{2}/-\rangle$
ϕ_{11}	= V11. V23MP. VC	=	$\langle 1/0\rangle\langle-\frac{3}{2}/+\rangle$
ϕ_{12}	= V11. V23MM. VC	=	$\langle 1/0\rangle\langle-\frac{3}{2}/-\rangle$
ϕ_{13}	= V12. V22PP. VC	=	$\langle 2/0\rangle\langle\frac{1}{2}/+\rangle$
ϕ_{14}	= V12. V23PM. VC	=	$\langle 2/0\rangle\langle\frac{1}{2}/-\rangle$
ϕ_{15}	= V12. V23MP. VC	=	$\langle 2/0\rangle\langle-\frac{1}{2}/+\rangle$
ϕ_{16}	= V12. V23MM. VC	=	$\langle 2/0\rangle\langle-\frac{1}{2}/-\rangle$

ϕ_{17} = V12. V23PP. VC = <2/0><$\frac{3}{2}$/+>
ϕ_{18} = V12. V23PM. VC = <2/0><$\frac{3}{2}$/->
ϕ_{19} = V12. V23MP. VC = <2/0><-$\frac{3}{2}$/+>
ϕ_{20} = V12. V23MM. VC = <2/0><-$\frac{3}{2}$/->

where

$CC = 20(Q.\ E_2 - E_4.K.CS)/M_4$

$V10 = SQ6.\ (2\ Q^2E_1^2 - (M_3K.\ SN)^2 + 4\ E_3.E_1Q)/M_3^2$

$V11 = K.SN.(E.1Q - E_3.K.\ \mathrm{Cos}\ \theta)/M_3$

$V12 = (K.\ SN.)^2/2.0$

$V23PP = K.\ SN.\ DM.\ C_1/(2D)^{1/2}$
$V23PM = K.\ SN.\ DP.\ S_1/(2D)^{1/2}$
$V23MP = K.\ SN.\ DP.\ S_1/(2D)^{1/2}$

$V23MM = -K.\ SN.\ DM.\ C_1/(2D)^{1/2}$
$V22PP = (-K.\ SN.\ DP.\ S_1 - 2.0.\ CC.\ DM.\ C_1)/(6D)^{1/2}$
$V22PM = (+K.\ SN.\ DM.\ C_1 - CC.\ DP.\ S_1)/(6D)^{1/2}$

$V22MP = -(K.\ SN.\ DM.\ C_1 - CC.\ DP.\ S_1)/(6D)^{1/2}$

$V22MM = -(K.\ SN.\ DP.\ S_1 + CC.\ DM.\ C_1)/(6D)^{1/2}$

See also end of Section A.2.

Section A.5

And finally for the production of the $\frac{5}{2}^+$ in the reaction $P + P \longrightarrow \frac{5}{2}^+ + P$ the 24 independent helicity amplitudes are given below. These expressions are obtained from eqs. 3.117, 3.70 and multiplying them together as indicated by the first term of eq. 3.76. Written explicitly the transition amplitude is

ϕ_3 = SQ10. SC. QM. ((RHO. FP. SN) - (FM. K. SN. C2. CC))

$= \langle \frac{1}{2}/\frac{1}{2} \rangle \langle -\frac{1}{2}/\frac{1}{2} \rangle$

ϕ_4 = SQ10. SC. QP. (RHO. FP. S2 - FM. K. SN2. CC)

$= \langle \frac{1}{2}/\frac{1}{2} \rangle \langle -\frac{1}{2}/-\frac{1}{2} \rangle$

ϕ_5 = -SQ10. SC. QP. (RHO. FM. SN) + FP. K. SN. S2. CC)

$= \langle \frac{1}{2}/-\frac{1}{2} \rangle \langle \frac{1}{2}/\frac{1}{2} \rangle$

ϕ_6 = SQ10. SC. QM. (RHO. FM. C2 + FP. K. SSN2. CC)

$= \langle \frac{1}{2}/-\frac{1}{2} \rangle \langle \frac{1}{2}/-\frac{1}{2} \rangle$

ϕ_7 = SQ10. SC. QP.(RHO. FM. C2 + FP. K. SN2. CC)

$= \langle \frac{1}{2}/-\frac{1}{2} \rangle \langle -\frac{1}{2}/+\frac{1}{2} \rangle$

ϕ_8 = -SQ10. SC. QM (RHO. FP. SN - FM. K. SN. C2. CC)

$= \langle \frac{1}{2}/-\frac{1}{2} \rangle \langle -\frac{1}{2}/-\frac{1}{2} \rangle$

ϕ_9 = SQ5. SC QM. K. SN.(K. SN2. FP. 0.25 + C2. FM. CC)

$= \langle \frac{3}{2}/\frac{1}{2} \rangle \langle \frac{1}{2}/\frac{1}{2} \rangle$

ϕ_{10}= SQ5. SC. QP. K. SN2 (K.S2. FP. 0.25 + CC. FM.)

$= \langle \frac{3}{2}/\frac{1}{2} \rangle \langle \frac{1}{2}/-\frac{1}{2} \rangle$

ϕ_{11}= SQ5. SC. QM. K. SN2 (K. C2. FM. 0.25 - CC. FP)

$= \langle \frac{3}{2}/\frac{1}{2} \rangle \langle -\frac{1}{2}/\frac{1}{2} \rangle$

ϕ_{12}= SQ5. SC. QP. K. SN(K.SN2. FM. 0.25 - S2. FP. CC)

$= \langle \frac{3}{2}/\frac{1}{2} \rangle \langle -\frac{1}{2}/-\frac{1}{2} \rangle$

ϕ_{13}= SQ5. SC. QP. K. SN2.(K. S2. FP. 0.25 + CC. FM)

$= \langle \frac{3}{2}/-\frac{1}{2} \rangle \langle \frac{1}{2}/\frac{1}{2} \rangle$

ϕ_{14} = -SQ5. SC. QM. K. SN2 (K. S2. FP. 0.25 + CC. FM)

= $\langle\frac{3}{2}/-\frac{1}{2}\rangle\langle+\frac{1}{2}/-\frac{1}{2}\rangle$

ϕ_{15} = SQ5. SC. QP. K. SN (K. SN2. FM. 0.25 - S2. CC. FP)

= $\langle\frac{3}{2}/-\frac{1}{2}\rangle\langle-\frac{1}{2}/+\frac{1}{2}\rangle$

ϕ_{16} = -SQ5. SC. QM. K. SN2. (K. C2. FM. 0.25 - CC. FP)

= $\langle\frac{3}{2}/-\frac{1}{2}\rangle\langle-\frac{1}{2}/-\frac{1}{2}\rangle$

ϕ_{17} = 0.25. SC. QM. K^2. SN2. FM. C2

= $\langle\frac{5}{2}/\frac{1}{2}\rangle\langle\frac{1}{2}/\frac{1}{2}\rangle$

ϕ_{18} = 0.25. S.C. QP. K^2. SN2. SN. FM.

= $\langle\frac{5}{2}/\frac{1}{2}\rangle\langle\frac{1}{2}/-\frac{1}{2}\rangle$

ϕ_{19} = -0.25. SC. QM. K^2. SN2. SN. FP

= $\langle\frac{5}{2}/\frac{1}{2}\rangle\langle-\frac{1}{2}/\frac{1}{2}\rangle$

ϕ_{20} = -0.25. SC. QP. S2. K^2. SN2. FP

= $\langle\frac{5}{2}/\frac{1}{2}\rangle\langle-\frac{1}{2}/-\frac{1}{2}\rangle$

ϕ_{21} = 0.25.SC. QP. K^2. SN2. SN. FM

= $\langle\frac{5}{2}/-\frac{1}{2}\rangle\langle\frac{1}{2}/\frac{1}{2}\rangle$

ϕ_{22} = -0.25.SC. QM. K^2. SN2. C2. FM

= $\langle\frac{5}{2}/-\frac{1}{2}\rangle\langle+\frac{1}{2}/-\frac{1}{2}\rangle$

ϕ_{23} = -0.25. SC. QP. K^2. SN2. FP. S2

= $\langle\frac{5}{2}/-\frac{1}{2}\rangle\langle-\frac{1}{2}/\frac{1}{2}\rangle$

ϕ_{24} = 0.25. SC.QM. K^2. SN2. SN. FP

= $\langle\frac{5}{2}/-\frac{1}{2}\rangle\langle-\frac{1}{2}/-\frac{1}{2}\rangle$

where
SQ10 = $1/\sqrt{10}$ SQ5 = $1/\sqrt{5}$

C2 = 1 + Cos Q , S2 = 1 - Cos Q

SN2 = Sin^2 Q , CS2 = Cos^2 Q

CC = $(E1.\ Q - E_3.K.CS)/M$

RHO = $QEEK.\ CS - Q2E_1 + K^2\ \text{Sin}^2\ Q/2.0 - E31K.\ CS2$

QEEK = $4.0.Q.E_3.\ E_1.\ K/(2.0.M_3.M_3)$

Q2E1 = $Q^2.E_1^2/M^2$

E31K = $E_3^2.\ K^2/M^2$

FM = $(E_1 + M_1)\ Q - (E_3 + M_3)\ K$

FP = $(E_1 + M_1)\ Q + (E_3 + M)K$

See also end of Section A.2.

2^+ Reggeized Differential Cross-Section

Section A.6

$$\frac{d\sigma}{at} \propto A^2[-\frac{2t^2}{\mu^2 m^2}(1-t/4\mu^2)]$$

$$+ (A.q')(A^*.q')[\frac{2t^2}{m^2\mu^4}(1 - t/3\mu^2)]$$

$$+ B^2 x \frac{2}{m^2}[S^2(\frac{t}{4\mu^4} - \frac{7}{3m^2}) - 4m^2(1 - t/4\mu^2)(1 + t/3m^2)]$$

$$+ (B.q)(B^*.q)\{-\frac{8}{3m^2}(1 - t/4\mu^2)\}$$

$$+ (B.q')(B^*.q')\{\frac{2}{3m^2\mu^2}[\frac{2S^2}{\mu^2} + t + 12m^2(1 - t/4m^2)\ .$$

$$.\ (1 - t/4\mu^2)]\}$$

$+ [(B.q')(B^*.q) + (B.q)(B^*.q')(B^*.q')] \{\frac{4S}{3m^2\mu^2}\}$

$+ [A.B^* + B.A^*]\{\frac{4St}{m^2\mu^2} (1 - t/4\mu^2)\}$

$+ [(A.q')(B^*.q') + (B.q')(A^*.q')] \{\frac{-2St}{m^2\mu^4} (1 - t/3\mu^2)\}$

$+ C^2\{\frac{2}{m^2} (1 - \frac{t}{4m^2}) t.S2Bu\}$

$+(C.q')(C^*.q')\{\frac{t}{2\mu^4 m^4} (1 - t/4m^2) S2Bu\}$

$+ [(B.C^*) + (B^*.C)] \{\frac{-2t}{\mu^2 m^4} S2Bu\}$

$+ [(B.q')(C^*.q') + (B^*.q')(C.q')] \{\frac{2t}{\mu^4 m^4} S2Bu\}$

$+ D^2\{\frac{2}{m^2\mu^2} [S^2 - \frac{t^2}{4} + \frac{28}{3} \mu^2 m^2 (1 - t/7m^2)] \}$

$+ (D.q)(D^*.q)\{\frac{8}{3m^2}\}$

$+ (D.q')(D^*.q')\{\frac{32}{3m^2\mu^4} [\frac{S^2}{2} - \frac{t^2}{16} - m^2\mu^4(1-t/4m^2)(1-t/4\mu^2$
$+ 2m^2\mu^3(1-t/11m^2)]\}$

$+ [(D.q')(D^*.q)+(D.q)(D^*.q')] \{- \frac{45}{3\mu^2 m^2}\}$

$+ i[A.D^* - D.A^*]\{- \frac{4t}{\mu m} (1 - t/4\mu^2)\}$

$+ i[(A.q')(D.q')^-(A^*.q')]\{\frac{4t}{m\mu^3}(1 - \frac{t}{3\mu^2})\}(-1)$

$+ F^2\{\frac{-2}{m^2\mu^6} [(S^2 - 4\mu^2 m^2 (1 - t/4m^2)S2Bu\}$

$+ (F.q)(F^*.q)[\frac{12}{m^2\mu^4} S2Bu]$

$$+ (F.q')(F^{*}.q')\{ \frac{8}{3m^2\mu^8} [S^2 - 3m^2\mu^2(1-t/4\mu^2).S2Bu \}$$

$$+ [(F.q')(F^{*}.q)+(F^{*}.q')(F.q)]\{- \frac{4S}{3m^2\mu^6} \; S2Bu\}$$

$$+ i[-B.D^{*} - D.B^{*}] \{\frac{S}{\mu m} (\frac{40}{3} - \frac{t}{\mu^2})\}$$

$$+i[(B.q')(D^{*}.q)-(B^{*}.q')(D.q)] \{\frac{-2t}{\mu^3 m}\}$$

$$+ i[(B.q)(D^{*}.q')-(B^{*}.q)(D.q')] \{\frac{4t}{3m\mu^3}\}$$

$$+ i[(B.q')(D^{*}.q')-(B^{*}.q')(D-q')]\frac{4S}{3m\mu^3} (10 - \frac{t}{\mu^2})$$

$$+ [(D.q)(F^{*}.q)+(D^{*}.q)(F.q)] \{\frac{4S}{3m^2\mu^2}\}$$

$$+ [(D.q')(F^{*}.q')+(D^{*}.q')(F.q')] \{\frac{4S}{3m^2\mu^6} (S^2 - 3m^2\mu^2(1-t/4\mu^2) \times (1-t/3\mu^2))\}$$

$$+ [(D.q)(F^{*}q')+(D^{*}.q)(F.q')] \{\frac{-2}{m^2\mu^4} (\frac{S^2}{3}+m^2+(1-t/4m^2))\}$$

$$+ [(D.q')(F^{*}.q)+(D^{*}.q')(F.q)] \{\frac{-4}{3m^2\mu^4}(\frac{S^2}{2} - m^2 t(1-\frac{t}{4m^2}))\}$$

$$+ [(D.F^{*})+(F.D^{*})] \{\frac{-4S}{m^2\mu^4} \; S2Bu\}$$

$$+ i[B.F^{*} + F.B^{*}] \{\frac{8}{m\mu^3} \; S2Bu\}$$

$$+ i[(B.q)(F^{*}.q)-(B^{*}.q)(F.q)] \{\frac{-8}{3\mu m} (1 - t/4\mu^2)\}$$

$$+ i[(B.q')(F^{*}.q')-(B^{*}.q')(F.q')] \frac{-8}{m\mu^5}[\frac{S^2}{3} - m^2\mu^2(1-t/4m^2)(1-t/4\mu^2)]$$

$$+ i[(B.q)(F^{*}.q')-(B^{*}.q)(F.q')] \{\frac{4S}{3\mu^3}(1+t/2\mu^2)\}$$

$$+ i[(B.q')(F^{*}.q)-(B^{*}.q/)(F.q)] \{\frac{-4S}{3m\mu^3}(1-3t/4\mu^3)\}$$

1^+ Reggeized Differential Cross-Section

Section A.7

$$\frac{d\sigma}{dt} \propto B^2[\frac{1}{m^2\mu^2}\{(S^2+\mu^2 t(1-t/4\mu^2))t(1+t/4\mu^2)-2S^2+-t^2\mu^2(1-t/4\mu^2) + 4t(1-t/4m^2)(1-t/4\mu^2)]$$

$$+ [(B.q')(C^{*}.q')+(C.q')(B^{*}.q')]\{\frac{t^2}{m^4\mu^2}(\frac{S^2}{4\mu^2} - m^2(1-t/4m^2))$$

$$+ [(B.q)(C^{*}.q)+(B^{*}.q)(C.q)]\{\frac{-t^3}{4m^4\mu^2}(1-t/4\mu^2)\}$$

$$+ [(B.q)(C^{*}.q')+(B.q')(C^{*}.q)+(C.q)(B^{*}.q')+(C.q')(B^{*}.q)] \times \{\frac{St^3}{8m^4\mu^4}\}$$

$$+ (C.q)(C^{*}.q)\ \{\frac{-t^2}{4m^4\mu^2}(1-t/4m^2)\}$$

$$+ (C.q')(C.q')\{\frac{t^2(1-t/4m^2)}{4m^4\mu^4} . 4\ S2Bu\}$$

$$+ [(C.q)(C^{*}.q')+(C.q')(C^{*}.q)]\{\frac{St^3}{8m^4\mu^4}(1-\frac{t}{4\mu^2})\}$$

$$+ [(B.q')(B^*.q')] \{\frac{t^2}{m^2\mu^2}\}$$

$$+ [(B.q)(B^*.q)][- \frac{4t}{m^2}(1-t/4\mu^2)]$$

$$+ [(B.q)(B^*.q')+(B^*.q)(B.q')][\frac{2St}{m^2\mu^2}]$$

$$+ D^2 \frac{t}{\mu^2} (1 - \frac{t}{4m^2})$$

$$+ [(D.q)(D^*.q)] \{\frac{4t}{m^2}\}$$

$$+ (D.q')(D^*.q')\{\frac{4t}{\mu^2} (1 - t/4m^2)\}$$

$$+ i[B.D^* - D.B^*] \{\frac{- st^2}{2m\mu^3}\}$$

$$+ i[(B.q')(D^*.q') - (B^*.q')(D.q')] \{\frac{2t}{m\mu^3} S\}$$

$$+ F^2[- \frac{4t}{\mu^4} (1 - t/4m^2) \text{ S2Bu}]$$

$$+ (F.q)(F^*.q)\{\frac{4t}{m^2\mu^4} \text{ S2Bu}\}$$

$$+ i[B^*F-B.F^*] \{- \frac{4t}{m\mu^3} \text{ S2Bu}\}$$

$$+ i[(B.q)(F^*.q)-(F.q)(B^*.q)] \{\frac{4t}{\mu m} (1 - t/4m^2)\}$$

$$+ i[(B^*.q)(F.q')-(B.q)(F^*.q')] \frac{2St}{m\mu^3}$$

$$+ [(D.q)(F^*.q)+(F.q)(D^*.q)] \frac{2St}{m^2\mu^2}$$

$$+ [(D.q)(F^*.q')+(D^*.q)(F.q')] \{\frac{-4t}{\mu^2}(1 - t/4m^2)\}$$

$$+ i[(B.q')(D^*.q)-(B^*.q')(D.q)]\{\frac{4t}{\mu m}\}$$

The values of A, B, C, D, F are given by eqs. 4.47a, 4.47b, 4.48 to 4.51.

$$S2Bu = \frac{1}{4} S^2 - m_2^2(1 - \frac{t}{4m^2})\mu^2(1 - \frac{t}{4\mu^2})$$

It should be emphasised that in these two expressions not all the terms contribute to all the different processes, depending on the trajectories being exchanged, some of the A's, B's, etc. are zero.