ASPECTS OF INELASTIC SCATTERING THEORY.

by

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ABSTRACT.

In the last few years one of the most significant new developments in hadron dynamics has been the introduction of the concept of duality. This thesis is concerned with dual models and their phenomenological application to specific reactions.

In chapter 1, a brief account of the assumptions leading to the concept of duality and a discussion of their justification is given. The Veneziano model and its properties which will be needed in later chapters are considered.

In the second chapter a model based on the five point Veneziano function is applied to three particle production processes. One of the difficulties encountered in phenomenological applications of the Veneziano model is that it has poles on the real axis. In chapter 3, we present a simple smoothed Veneziano model for $K^- p \rightarrow \overline{K}^{*0} n$ in which the poles are moved off the real axis. This gives the resonances a finite width without introducing ancestor particles into the amplitude.

Residue recurrence relations for general Veneziano amplitudes are derived in chapter 4.

Finally, in chapter 5, the construction of a general class of dual amplitudes is considered. In particular an amplitude for equal mass scattering in which there are no daughter particles is constructed.

In the appendix the asymptotic properties of Veneziano amplitudes are derived.

PREFACE.

The work presented in this thesis was carried out between October 1968 and April 1971 under the supervision of Professor T.W.B. Kibble to whom the author is grateful for guidance. Except where stated, this work is original and has not been presented for a degree of this or any other university.

The author wishes to express his gratitude to many members of the Theoretical Physics Department at Imperial College, in particular S.A. Adjei, B.J. Hartley, Dr. R.W.Moore and K.J.M. Moriarty with whom he has collaborated on a series of Regge and Veneziano models associated with the work presented in this thesis. Finally, the financial support of the Science Research Council is gratefully acknowledged.

CONTENTS

CHAPTER 1. DUALITY AND THE VENEZIANO MODEL.

1.1 Introduction.

The origin of duality(1) lies in the use of finite energy sum rules(2). These rules are merely a convenient way of stating the analytic and Regge asymptotic properties of amplitudes. As an example we shall consider a two body scattering process described by an amplitude $A(\nu,t)$ where

$$\nu = \frac{s-u}{4m} \tag{1.1}$$

and where m is the mass of the target particle. We shall assume that, for $|\nu| > N$ the amplitude may be written as a sum of Regge pole terms:

$$A(\nu,t) = \sum_i \beta_i(t) \frac{1 \pm e^{-i\pi\alpha_i(t)}}{\sin \pi\alpha_i(t)} \nu^{\alpha_i(t)} \tag{1.2}$$

where $\alpha_i(t)$ and $\beta_i(t)$ are the trajectory and residue functions of the t-channel Regge poles. The $\pm$ sign depends on whether $A(\nu,t)$ is even or odd under $\nu \to -\nu$. Experimentally, amplitudes are found to be approximately Regge behaved for $\nu \gtrsim 3$-5 GeV/c.

Now consider the integral

$$\oint \operatorname{Im} \nu^n A(\nu,t)\, d\nu \tag{1.3}$$

where the contour of integration is as shown in Fig.1. Since all the singularities of the amplitude lie outside the contour the integral vanishes. Substituting eqn.1.2 into eqn.1.3 we obtain

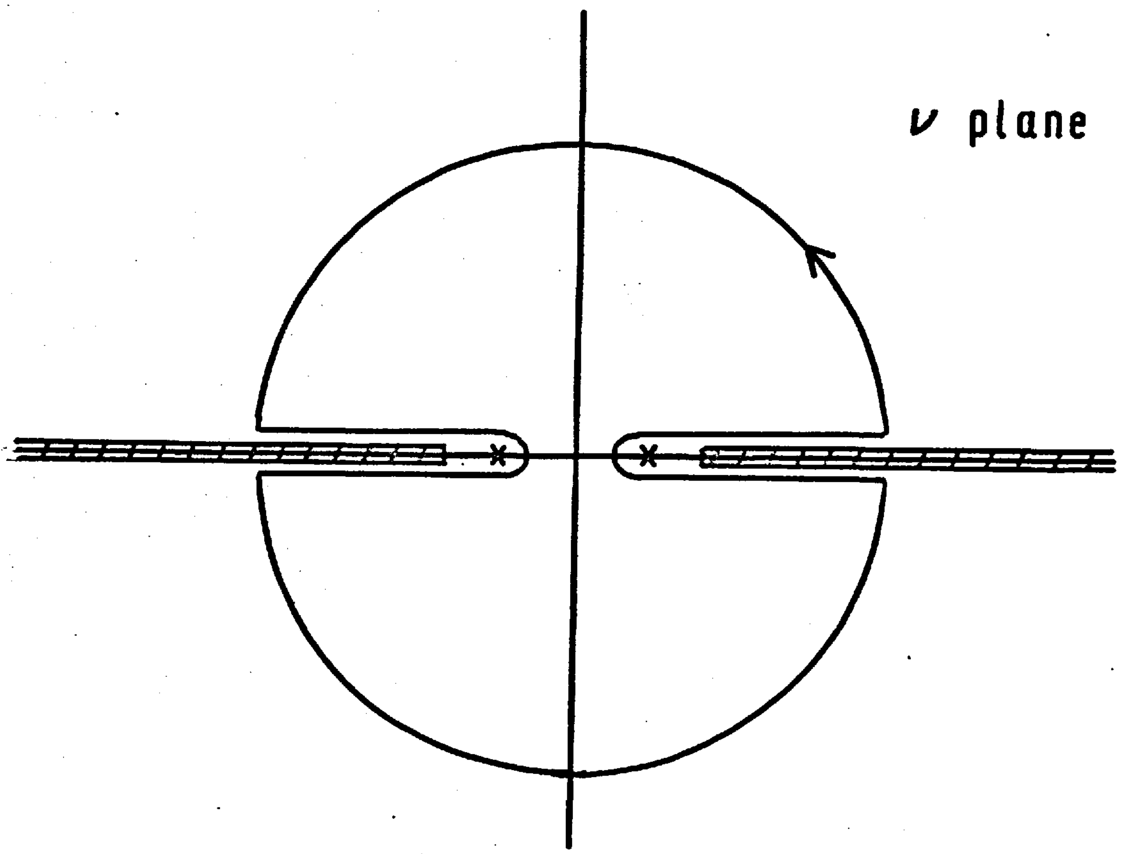


Fig. 1

$$\int_0^N \nu^n \operatorname{Im} A(\nu,t)\,d\nu = \sum_i \frac{\beta_i(t)\, N^{\alpha_i(t)+n+1}}{\alpha_i(t)+n+1} \tag{1.4}$$

where n= even integer if $A(\nu,t) = -A(-\nu,t)$,

n= odd integer if $A(\nu,t) = A(-\nu,t)$.

These relations are examples of higher moment sum rules. They are derived using the assumptions of analyticity and Regge asymptotic behaviour of the scattering amplitude. Physically these rules imply that the Regge amplitude extrapolated to low energies, where it would not normally be assumed to apply, will provide an average description of the amplitude in the resonance region. That this is indeed true was shown in the early application of finite energy sum rules (FESR) to πN scattering[2].

If we now make the additional assumption that at low energies the imaginary part of the amplitude is completely dominated by resonances, FESR indicate that the sum of t-channel Regge poles equals the sum of s and u-channel resonances. This may be stated in another way: <u>either</u> the s and u-channel poles <u>or</u> the t-channel Regge poles determine the complete amplitude. Thus the amplitude may be described in two different ways. This is the basis of what we call duality. If we regard the extrapolated Regge amplitude as only providing an average description of the amplitude in the resonance region we refer to this as "global duality" whereas if it reproduces the detailed structure of the amplitude in the resonance region we refer to this as "local duality". The models we consider in this thesis are locally dual.

Duality is in direct contradiction with interference models[3] in which the Regge amplitude is <u>added</u> to the resonances. As far as duality is concerned interference models involve double counting. It is, however, difficult to make a definite statement on which approach is correct because of the freedom available in the way an amplitude may be divided into resonances and background. Recently interference models have become less popular and duality has gained wide-spread acceptance. Probably one of the best objections to interference models is the awkward resonance properties required to fit the data. For example, to explain the S_{11} πN amplitude it is necessary[4] to introduce a 1440 MeV S_{11} resonance to cancel the Regge contribution in order to explain the physical amplitude which almost vanishes.

In the early days of FESR it was observed by Freund[5] that the narrow resonance approximation failed for the I_t=0 $\pi\pi$ amplitude. This failure was attributed to the pomeron contribution to the I_t=0 amplitude. Freund (and, shortly after, Harari[6]) suggested that the pomeron should be associated with the s-channel background and not with the resonances. That the pomeron must be treated differently from other trajectories may be seen in simply terms by considering K^-p and π^-p elastic scattering. The pomeron contributes to both of these processes, however, each has different s-channel resonances. It is difficult to see how the pomeron can be built out of either N and Δ or Σ and Λ resonances. A more quantitative argument for treating the pomeron separately has been given by Harari[1]:

The pomeron contributes only to the I=0 t-channel amplitude. The crossing matrix tells us that this amplitude has equal projections on all s-channel isotopic spin states. Therefore if the pomeron were associated with any s-channel resonance structure this would have to be independent of the s-channel isospin. Although,for other trajectories, cancellations between trajectories can cause particular s-channel projections to vanish, for the pomeron this is not possible as it is higher lying than all other trajectories.

As a result of the specialtreatment required for the pomeron we are led to a two component amplitude;

1. Resonances $\leftrightarrow$ Regge.
2. Background $\leftrightarrow$ Pomeron.

This exclusion of the pomeron from the usual "resonance $\leftrightarrow$ Regge" duality is exhibited by all present dual models since they do not include pomeron contributions. This thesis, which concerns dual models, will therefore only be concerned with inelastic reactions.

At this stage it seems worthwhile to summarise the assumptions which have gone into the derivation of this two component amplitude.

Firstly, using Cauchy's theorem we were able to relate the high energy behaviour of the amplitude to the low energy behaviour. We then made two assumptions;

1. Resonance dominance of the imaginary part of the amplitude at low energies.
2. Regge asymptotic behaviour.

In 1 we imply that there is no background contribution (except that associated with the pomeron) to the imaginary part of the amplitude. A test of this has been made by examining the phase shifts of the s-channel partial waves for πN scattering using t-channel states of definite isospin(7). The results indicate that for the $I_t=0$ state there is a steadily rising purely imaginary background (due to the pomeron) whereas for the $I_t=1$ state there is no indication of any background. This seems to indicate that, at least for this process, the assumption of resonance dominance is reasonable.

Our second assumption is somewhat more dubious. It is known phenomenologically that simple Regge pole models are often inadequate in describing the data and Regge cuts are important. No account of cut contributions has been taken and this problem is largely ignored in the literature, possibly because the exact details of the form of Regge cuts are not known.

As experimental evidence of the two component amplitude we may quote the case of the K^-p and K^+p total cross sections (see Jackson's review talk at the Lund Conference 1969). The physical region for K^-p is given by $\nu > m_K$ and that for K^+p by $\nu < -m_K$. The pomeron contribution is constant and symmetric on crossing $\nu \to -\nu$. For K^+p (which is exotic) we have only the pomeron contribution whereas for K^-p the total cross section has the resonance contribution superimposed on the pomeron contribution. As $\nu \to \infty$ the K^-p cross section <u>decreases</u> towards a constant value equalling the pomeron contribution.

1.2 Exchange Degeneracy.

Before we start to consider particular dual models, one important consequence of duality which will prove very useful in chapter 2 must be introduced. This is the connection between the absence of resonances in one channel and the exchange degeneracy of trajectories in the dual channel. If, for a particular process, there are no s-channel resonances then, because of our assumption of resonance dominance, the imaginary part of the amplitude must vanish. Duality tells us that the amplitude may be regarded as either a sum of resonances or as a sum of t-channel Regge exchanges. Hence, looking at the amplitude in terms of the t-channel trajectories it should still have no imaginary part. This is only possible if there is a cancellation between trajectories. Such a cancellation between two trajectories can only occur at all energies if both the trajectories have the same $\alpha(t)$ and the same residue functions. Thus duality implies strong exchange degeneracy of the t-channel trajectories when there are no s-channel resonances.

Consider, for example, the process;

$$K^+p \rightarrow K^+p.$$

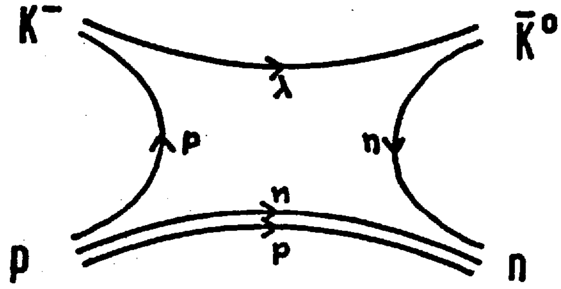
The t-channel trajectories describing this process at high energies are P, f, ω, A_2, ρ. Because of the absence of any resonances in the s-channel, duality implies degeneracy of the ρ and A_2 and of the ω and f. Although exchange degeneracy seems to hold quite well for trajectory functions, especially for the ρ, A_2, ω, f system and K^*, K^{**}, detailed tests[8] of degeneracy indicate that it is,at best, only an approximate property. Alternatively, one could adopt the point of view that because exchange degeneracy appears to be so good in the positive t region the apparent inconsistencies with experiment for negative t are really due to the presence of Regge cuts which have been ignored.

1.3 Exotic Resonances and Duality Diagrams.

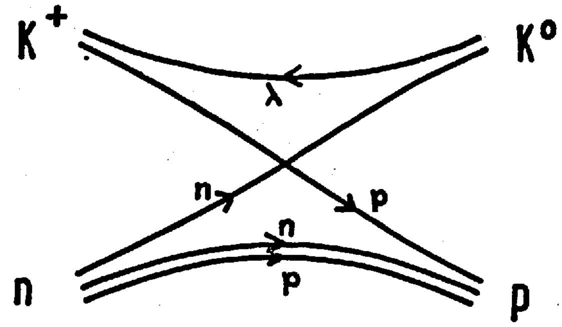
At the present time there is strong experimental evidence supporting the assumption that there are no exotic particles (or, at least,that any such particles must be very weakly coupled). By this we mean that all particles are either $q\bar{q}$ (mesons) or qqq (baryons) combinations of quarks. A graphical description of duality and the absence of exotic particles is provided by duality diagrams[9]. These diagrams show the rearrangement of the quarks during an interaction and are drawn according to the following set of rules:

1. All baryons are made up of three quarks.
2. Mesons are made up of a quark and an antiquark.
3. No quark line can terminate in the same particle as it originated from.
4. Cutting the diagram in a baryon channel involves cutting through three quark lines.
5. Cutting the diagram in a meson channel involves cutting through a quark-antiquark pair.

Quark diagrams constructed as above are said to be



a.



b.

Fig. 2

"legal". Diagrams which disobey the rules are "illegal" and involve the exchange of exotic particles in at least one channel. In Fig.2a we illustrate a legal duality diagram and in Fig.2b an illegal diagram having an exotic s-channel.

Any process for which we cannot draw a legal duality diagram must have a purely real amplitude. This is just a restatement of the fact that illegal diagrams contain exotic channels in which there are no resonances.

1.4 The Veneziano Model.

It was first realised by Veneziano[10] that it is possible to construct an amplitude having many of the properties required by duality. This amplitude is simply Euler's Beta function.

If we consider the scattering of a system of scalar (0^+) particles then the Veneziano ansatz states that this process is described by an amplitude:

$$A = V(s,t) + V(s,u) + V(u,t) \tag{1.5}$$

where

$$V(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} . \tag{1.6}$$

$\alpha(s)$ is a trajectory function which, as we shall see later, must be linear in s. By construction, this amplitude is crossing symmetric. We note that it has been written as a function of s, t and u although only two of these variables are independent. This is a common feature in Veneziano theory and in doing calculations we must usually regard them as independent, not applying the constraint between them until the end.

The amplitude, A, has poles in all three channels for $\alpha = 0, 1, 2 \ldots$ However, it does not have simultaneous poles in any two channels. To prove this it is sufficient to consider the residue of a pole in any one channel since the amplitude is crossing symmetric.

We shall consider a pole in the s-channel. Such poles can only arise from the $\Gamma(-\alpha(s))$ in the first two terms of A. We can write these terms as

$$\Gamma(-\alpha(s))\left[\frac{\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} + \frac{\Gamma(-\alpha(u))}{\Gamma(-\alpha(u)-\alpha(s))}\right] \cdot \qquad (1.7)$$

Noting that

$$\Gamma(-\alpha(s)) = \frac{-\pi}{\Gamma(1+\alpha(s)).\sin\pi\alpha(s)} \qquad (1.8)$$

we see that the residue of a pole at $\alpha(s)=m$ is given by

$$\frac{(-1)^{m+1}}{\Gamma(m+1)}\left[\frac{\Gamma(-\alpha(t))}{\Gamma(-m-\alpha(t))} + \frac{\Gamma(-\alpha(u))}{\Gamma(-m-\alpha(u))}\right] \cdot \qquad (1.9)$$

This expression is just a polynomial in $\alpha(t)$ and $\alpha(u)$ and contains no poles in u or t. Hence simultaneous poles in two variables do not occur. We note that, if the trajectories are linear, the residue is a polynomial of order m in t (since u may be expressed in terms of m and t) and may be expanded in terms of Legendre polynomials. We find that the residue of the pole at $\alpha(s)=m$ contains terms in $P_j(\cos\theta)$ for all j satisfying $m \geqslant j \geqslant 0$. This means that each parent resonance has a complete set of daughters. We should also note that, if the trajectories are not linear then the residue will also contain contributions from terms in $P_j(\cos\theta)$ for $j > m$ i.e. ancestors.

Using eqn.1.8 it is possible to write A as a series of s and u pole terms. For example

$$V(s,t) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,(m-\alpha(s))} \frac{\Gamma(-\alpha(t))}{\Gamma(m-\alpha(t))} \quad . \qquad (1.10)$$

We have to treat these series with caution since they are, in general, divergent (this must certainly be the case for $\alpha(t)=0,1,2\ldots$ as we know that $V(s,t)$ has poles at these points).

It should be noted that the poles of the Veneziano amplitude lie on the real axis. This means that the Veneziano model is a zero width resonance model and hence violates unitarity. Although there are various methods by which the resonances may be moved off the real axis (see chapters 2 and 3) the unitarisation of the model has not yet been achieved.

We have shown that it is possible to write the Veneziano amplitude as a sum of poles. In order to show that it is, in fact, a dual model it is necessary to show that it has Regge asymptotic behaviour. This is proved in the appendix where signature properties of the model are also mentioned.

So far we have only considered the simplest possible Veneziano amplitude for scalar particle scattering. We could, however, add terms like

$$\frac{\Gamma(m-\alpha(s)).\Gamma(n-\alpha(t))}{\Gamma(r-\alpha(s)-\alpha(t))} \tag{1.11}$$

with m and $n \leqslant r \leqslant m+n$

to the amplitude without spoiling the Regge behaviour. Such terms are called satellite terms and lead to an ambiguity in the Veneziano model. If the model is to have any usefulness we must be able to obtain a good approximation to physical amplitudes using only a few satellite terms

1.5 The Five-Point Amplitude.

It was known as long ago as 1905 [11] that generalisations of the Euler function existed. This fact was rediscovered by several authors[12] shortly after Veneziano proposed his model. Here we shall only give a simple derivation of the five-point function (B_5) which will be used in chapter 2. Similar methods may be used to obtain the general N-point function.

If we consider a term of the Veneziano amplitude for two particle scattering, for example V(s,t), we note that this term corresponds to the permutation of the

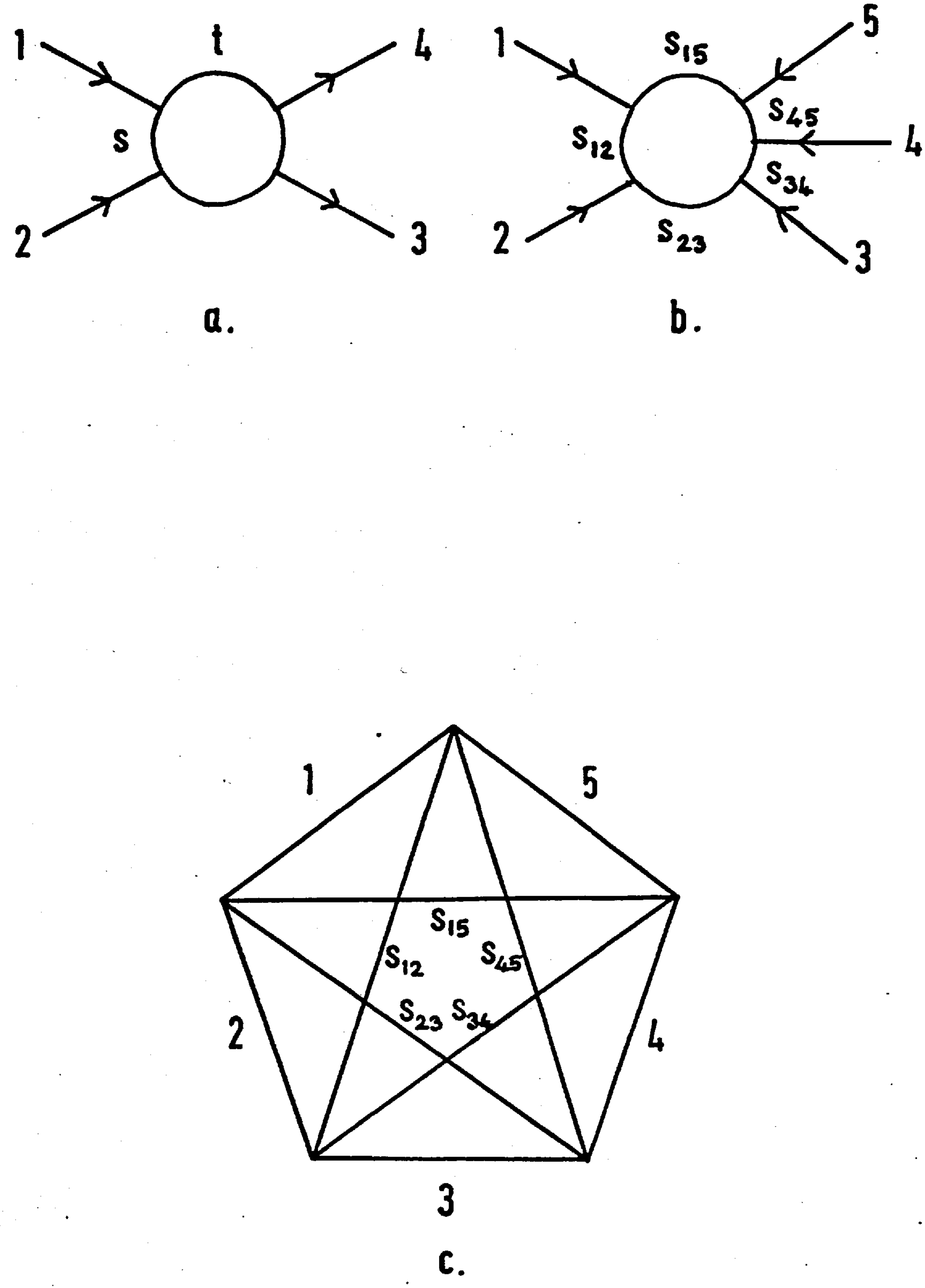


Fig. 3

external particles indicated in Fig.3a. The crossing symmetric amplitude is obtained by summing over the terms corresponding to the other non-cyclic permutations of the external particles. In deriving the five-point function we shall consider the term corresponding to the permutation shown in Fig.3b. As with the four-point function the crossing symmetric amplitude is obtained by summing over the twelve non-cyclic permutations of the external particles. Noting that the term corresponding to Fig.3a has poles in s and t only, the most logical way to generalise the Beta function is to construct a function having poles in all (five) planar Mandelstam channels corresponding to Fig.3b. These channels may be defined generally as

$$s_{ij} = (P_i + P_{i+1} \cdots P_j)^2 \tag{1.12}$$

where P_k is the four momentum of the particle k. These variables may be conveniently illustrated by means of a dual diagram (not to be confused with duality diagrams). This is a polygon whose sides are numbered according to the permutation of the external particles being considered. The Mandelstam variables are then represented by diagonals on the polygon. If the diagonals corresponding to two variables intersect the variables are said to be dual and our generalised Beta function should not have simultaneous poles in these variables (n.b. we cannot draw a pole graph with simultaneous poles in dual channels). To see how we may construct such a function we return to the Beta function and write it as an integral:

$$B(-\alpha(s),-\alpha(t)) = \int_0^1 u^{-\alpha(s)-1}(1-u)^{-\alpha(t)-1} du . \qquad (1.13)$$

This may be written as

$$\int_0^1 u^{-\alpha(s)-1} v^{-\alpha(t)-1} du\, dv \qquad (1.14)$$

with the constraint u+v=1. Therefore

$$B(-\alpha(s),-\alpha(t)) = \int_0^1 u^{-\alpha(s)-1} v^{-\alpha(t)-1} \delta(u+v-1) . \qquad (1.15)$$
$$\times du . dv .$$

Poles in $\alpha(s)$ arise because of the singularity of the integrand at u=0 for $\alpha(s)=0,1,2...$ and poles in $\alpha(t)$ arise because of similar singularities at v=0. Simultaneous poles in s and t are prevented because when u=0 v=1 and vice versa. If we denote the trajectory corresponding to the variable s_{ij} by α_{ij} then we can immediately write down the generalisation of eqn.1.14 as

$$\int_0^1 u_1^{-\alpha_{12}-1} u_2^{-\alpha_{23}-1} u_3^{-\alpha_{34}-1} u_4^{-\alpha_{45}-1}$$
$$\times u_5^{-\alpha_{51}-1} du_1 du_2 du_3 du_4 du_5 \qquad (1.16)$$

with the constraints $u_i = 1 - u_{i-1} u_{i+1}$

The set of constraint equations may be solved in terms of any two of the u_i which correspond to non-dual channels. (For the N-point function there are N-3 independent u_i. This corresponds to the number of mutually non-dual channels).

If we take u_1 and u_3 as independent we obtain

$$\begin{aligned} &u_2 - 1 - u_1 u_3 \\ &u_4 = (1-u_3)/(1-u_1 u_3) \\ &u_5 = (1-u_1)/(1-u_1 u_3) . \end{aligned} \tag{1.17}$$

Thus we may write B_5 as

$$\begin{aligned} B_5(-\alpha_{12},-\alpha_{23},-\alpha_{34},-\alpha_{45},-\alpha_{51}) = \\ \int_0^1 \prod_{i\neq 1,3} \delta(u_i + u_{i-1}u_{i+1} - 1) . u_1^{-\alpha_{12}-1} u_2^{-\alpha_{23}-1} \\ u_3^{-\alpha_{34}-1} u_4^{-\alpha_{45}-1} u_5^{-\alpha_{51}-1} du_1 du_2 du_3 du_4 du_5 \end{aligned} \tag{1.18}$$

$$\begin{aligned} = \int_0^1 \int_0^1 du_1 du_3 \, u_1^{-\alpha_{12}-1} (1-u_1 u_3)^{\alpha_{45}+\alpha_{51}-\alpha_{23}} \\ \times u_3^{-\alpha_{34}-1} (1-u_3)^{-\alpha_{45}-1} (1-u_1)^{-\alpha_{51}-1} \end{aligned} \tag{1.19}$$

If we substitute eqn.1.17 in eqn.1.19 we obtain an alternative form for B_5 in which the pole structure is more easily seen:

$$B_5 = \int_0^1 \int_0^1 \frac{du_1 du_3}{1-u_1 u_3} \cdot \prod_{i=1}^{5} u_i^{-\alpha_{i,i+1}-1} \tag{1.20}$$

We show in the appendix that the five-point function has correct double-Regge asymptotic behaviour. This fact will needed in chapter 2.

In this chapter we have shown how the concept of duality arose through the use of FESR and how a model having many of the properties required of a dual model can be constructed. Finally we have shown how we may generalise this model to reactions involving five external particles. We have only considered those simple properties of the Veneziano model which will be needed later in this thesis. There are several good reviews[13] of Veneziano theory which also give references to the many papers written on the detailed properties of the model. For a more general survey of high energy models contemporary with early work on the Veneziano model, the reviews of Jackson[1] and Jacob[14] may be recommended.

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CHAPTER 2. A VENEZIANO MODEL FOR THREE PARTICLE PRODUCTION.

2.1 Introduction.

In chapter 1 we considered the Veneziano amplitude for the scattering of a system of scalar (0^+) particles. Such an amplitude is of little practical use because no data on such processes exist, even the existence of any 0^+ particles is not certain. However, the possibility of being able to construct amplitudes in which one could incorporate in a single term both resonances and Regge asymptotic behaviour led to many attempts to construct phenomenological amplitudes for $2 \to 2$ scattering processes[(1)] for which data existed. Most of these models compared only the asymptotic form of the Veneziano amplitude with the data. Essentially this was the same as using a Regge amplitude with residues determined by the Veneziano model. Because no entirely self-consistent way of constructing a Veneziano model in which the external particles have spin has been found, these phenomenological amplitudes used a Veneziano parametrisation for the invariant amplitudes. However, perhaps the greatest contribution of the Veneziano model to high energy phenomenology has been in three particle production. Previous models for such processes (such as the Chan-Loskiewicz-Allison model[(2)]) which reproduce only the correct Regge behaviour have large numbers of parameters. Shortly after the rediscovery of the five-point function Bardakçi and Ruegg[(3)] showed that, using this function, it was possible to construct Veneziano amplitudes for $2 \to 3$ particle processes in which the external particles were pions and kaons. For such processes the only trajectories coupling to the external particles are ρ, ω, f, A_2, ϕ, f', K^* and K^{**}. Thus the amplitude must not have a resonance for $\alpha=0$. Bardakçi and Ruegg found that this could be achieved by writing their amplitude in the form

$$A = \sum_{\text{perms}} I_i \, \varepsilon_{\alpha\beta\gamma\delta} \, P_1^\alpha P_2^\beta P_3^\gamma P_4^\delta \times B_5(1-\alpha_{12}, 1-\alpha_{23}, 1-\alpha_{34}, 1-\alpha_{45}, 1-\alpha_{51}) \qquad (2.1)$$

where the sum is over the non-cyclic permutations of the external particles, P_i is the four momentum of the external particle i and α_{ij} are linear Regge trajectories coupling to the particles i and j. The kinematic factor is uniquely determined by parity requirements and results in the amplitude having correct Regge asymptotic behaviour in all channels. The factor I_i is an isospin factor given for $K\bar{K} \to 3\pi$ by

$$I_i = K_2^\dagger \tau_{i_3} \tau_{i_4} \tau_{i_5} K_1 \qquad (2.2)$$

where K is an isospinor and τ_i are Pauli matrices. The effect of the I_i is to eliminate terms which would involve exotic exchanges.

The first application of this type of model was made by Petersson and Törnqvist[4] to the process $K^- p \to \pi^- \pi^+ \Lambda$.

This process is essentially of the type considered by Bardakçi and Ruegg except that two of the mesons are replaced by baryons. Petersson and Törnqvist used an amplitude of the form given in eqn.2.1 only, instead of including an isospin factor, I_i, the requirement that there should be no exotic channels was imposed by hand by excluding permutations which would involve exotic exchanges. In order to get the baryon resonances at the correct positions the arguments of the five-point function were shifted by a half integer. Although this model effectively ignores the baryon spin, as a description of the spin average process, it was a remarkable success and many other processes were fitted using similar models(5).

In this chapter we shall construct a model which takes some account of the spins of the external particles and which allows the construction of amplitudes for processes in which pion exchange is possible.

I. THE MODEL FOR $K^-p \rightarrow K^{*-}\pi^+n$.

2.2 The Veneziano Terms.

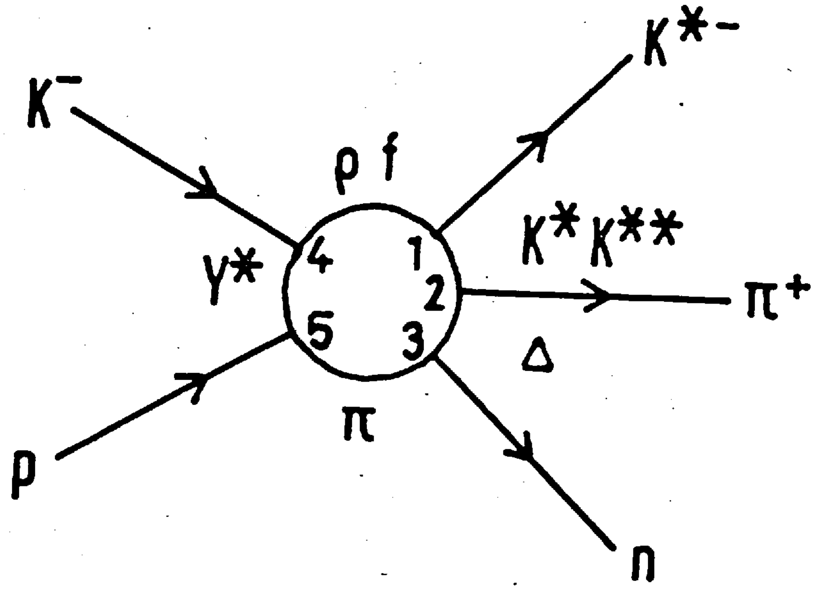
We shall start by considering the process $K^-p \rightarrow K^{*-}\pi^+n$ and assume that it may be described by an amplitude of the form

$$A = K \sum_{\ell=1}^{12} B_{5_\ell}(k_1-\alpha_{12}, k_2-\alpha_{23}, k_3-\alpha_{34}, k_4-\alpha_{45}, k_5-\alpha_{51}). \qquad (2.3)$$

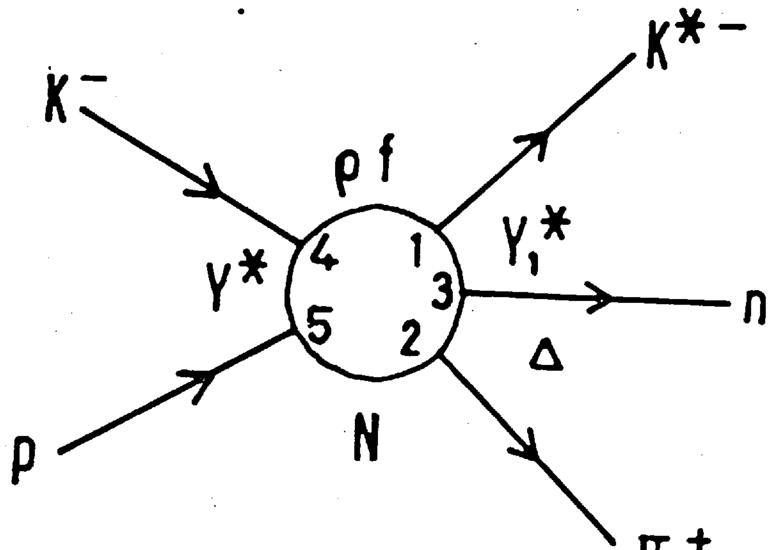
where α_{ij} is a linear Regge trajectory coupling to the external particles i and j and k_i is the spin of the lowest resonance on the trajectory. The sum is over the twelve non-cyclic permutations of the external particles. K is a kinematic factor. By taking the same kinematic factor in front of each B_5 term we ensure that, if all twelve terms are present, each trajectory has a definite signature.

It is necessary for all trajectories to have the same slope as otherwise the amplitude diverges exponentially at fixed angle(6), signature is impossible(7) and ghosts are inevitable(8).

Of the twelve permutations in eqn.2.3 those in which the K^- and n or the K^{*-} and p are adjacent will not contribute because they involve exotic baryon channels. Similarly, permutations with the K^- and π^+ adjacent will not contribute because they have a doubly charged meson channel. This leaves only the two terms corresponding to the permutations shown in Fig.4. Both of these permutations correspond to legal Harari-Rosner duality diagrams as shown



a.



b.

FiG. 4

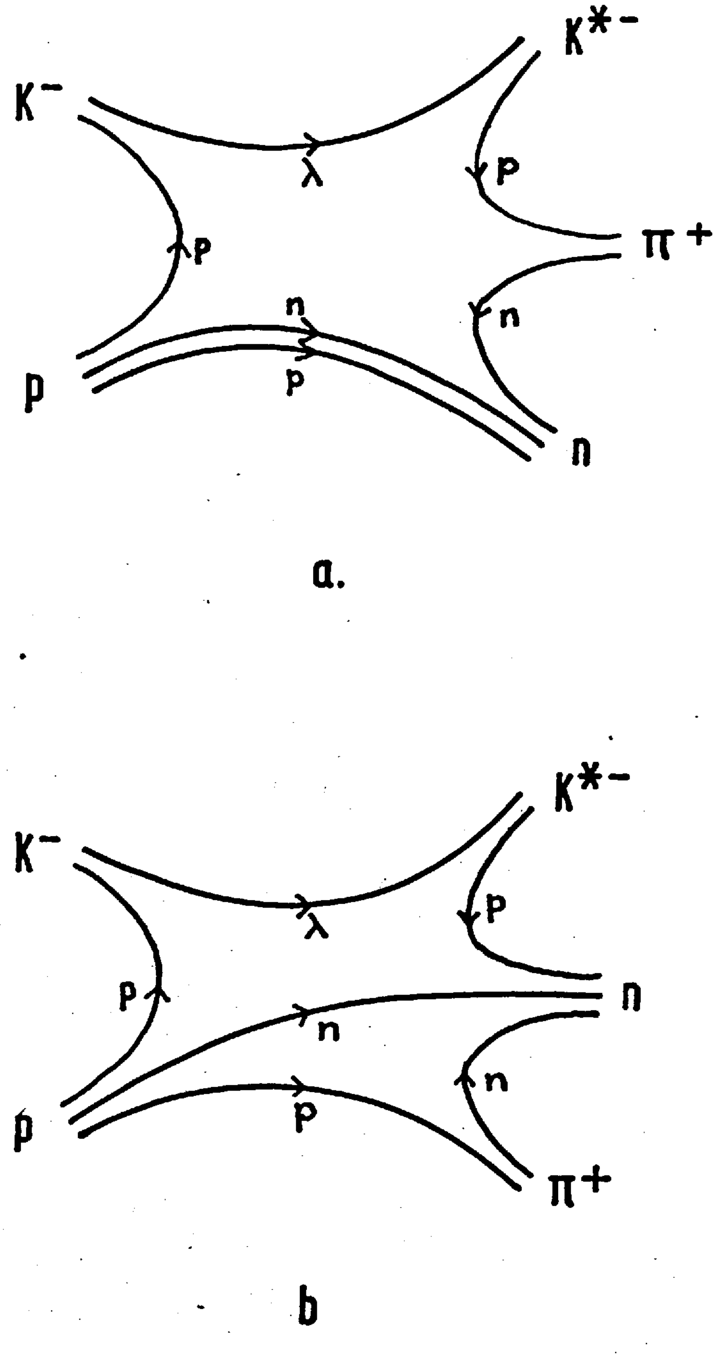


Fig.5

in Fig.5. We note that, as a result of having three exotic channels in this process, duality implies that all the trajectories except the one coupling to the $\pi^+ n$ subsystem are exchange degenerate. Eqn.2.3 implies that we only have one trajectory coupling to each channel. Although for most channels this is true, for some channels (such as $K^- p$) there are a number of possible trajectories. However, in order to make practicable any application of the model we must use experimental evidence to decide which trajectory is dominant in each channel. Although, in theory, one could add additional terms with all possible combinations of trajectories computational difficulties prevent this being practicable. Also, because of the poorness of the data, any determination of parameters to obtain a best fit to the data would be impossible.

The trajectories we have used were determined as follows:

1. For the $K^{*-}\pi^+$ channel the only possibility is a degenerate K^*, K^{**} trajectory.
2. It is known experimentally that the processes $K^- p \to \bar{K}^{*0} n$[9] and $K^- p \to \bar{K}^{**0} n$[10] are dominated by pion exchange. Therefore we have used a pion trajectory for the pn channel since strong K^{**} production is seen in the $K^{*-}\pi^+$ channel.
3. Having chosen a pion trajectory for the pn channel the only possible choice for the $K^- K^{*-}$ channel is a degenerate ρ, f trajectory (ω and A_2 are prevented by G parity conservation).

4. The effective mass distribution for the $\pi^+ n$ channel shows a complicated resonance structure in which the $\Delta(1236)$ is dominant and so we have used this trajectory. However, there is also significant N(1470) production. By omitting the N(1470) trajectory we will, of course, not obtain this resonance in our results. Other than this we find the results are little altered by which trajectory is chosen for this channel.
5. The $K^{*-}n$ channel in Fig.4b can only couple to a Y_1^* trajectory. The experimental data for the $K^{*-}n$ mass distribution shows no structure and provides no clue as to which trajectory we should use. According to Schmid(11) the only Y_1^* trajectory which couples strongly to the $\bar{K}N$ system is the $\Sigma(1385)$. In the hope that this still applies to the $\bar{K}^*N$ system we have used this trajectory.
6. For the K^-p channel we also use a $\Sigma(1385)$ trajectory. In principle one could also use a Y_0^* trajectory. Because the data is at high energies ($s_{Kp} > 10$ GeV^2) this choice makes little difference to the results since all trajectories have the same slopes.
7. Finally, for the π^+p channel there are a number of possible trajectories. We use a N_α trajectory because we would expect it to be dominant in the exchange channel as is the case in πN backward elastic scattering.

Thus our amplitude is

$$A = K\Big[B_5\big(1-\alpha_\rho(t_{KK^*}),1-\alpha_{K^*}(s_{K^*\pi}),\tfrac{3}{2}-\alpha_\Delta(s_{\pi\Lambda}),$$

$$-\alpha_\pi(t_{pn}),\tfrac{3}{2}-\alpha_\Sigma(s_{Kp})\big)+B_5\big(1-\alpha_\rho(t_{KK^*}), \tag{2.4}$$

$$\tfrac{3}{2}-\alpha_\Sigma(s_{K^*\Lambda}),\tfrac{3}{2}-\alpha_\Delta(s_{\pi\Lambda}),\tfrac{3}{2}-\alpha_N(t_{p\pi}),\tfrac{3}{2}-\alpha_\Sigma(s_{Kp})\big)\Big].$$

We note that the argument of the N_α trajectory in the second term has been shifted by 1. As we shall see later, this is to ensure that we get correct asymptotic behaviour.

As we have stated in chapter 1 the Veneziano model is a zero width resonance model. If it were possible to unitarise the Veneziano model we would expect the resonances to occur as second sheet poles and to have a finite width. In order to give the resonances a width in our model we add an imaginary part to the trajectories. This moves the poles off the real axis giving them a width but does not move them onto the second sheet. Lovelace has proposed(12) that the trajectories should be parametrised as

$$\alpha(s) = \alpha_0 + \alpha' s + \alpha_I\sqrt{s_0 - s}$$

$$s_0 = s_{\text{threshold}}. \tag{2.5}$$

This results in the trajectories being real below threshold but complex above. An unfortunate feature of this parametrisation is that it leads to an infinite series of ancestors because the trajectories are not linear in s. An alternative procedure (which we shall use) is to add a linear imaginary part to the trajectories in the channels for which s is above threshold. This does not give rise to ancestors but does lead to complications when we wish to go to crossed processes. We shall return to the problem of finding more acceptable ways of moving the poles of the Veneziano model off the real axis in chapter 3. However in this chapter we will parametrise our trajectories as

$$\alpha(s) = \alpha_0 + \alpha' s + i\alpha_I \tag{2.6}$$

$$\begin{aligned} \alpha_I &= k(s - s_0) \quad , s > s_0 . \\ &= 0 \qquad\qquad , s < s_0 . \end{aligned} \tag{2.7}$$

To evaluate the imaginary part we use the following argument:

Near to a resonance at angular momentum ℓ the partial wave amplitude is of the form[13]:

$$f_\ell \sim \frac{r(s,\ell)}{\ell - \alpha(s)} \tag{2.8}$$

$$= \frac{r(s,\ell)}{\ell - \alpha_0 - \alpha' s - i\alpha_I} . \tag{2.9}$$

This corresponds to a Breit-Wigner resonance of half width

$$\Delta s = \Gamma M = \frac{\alpha_I}{\alpha'} = \frac{k(s - s_0)}{\alpha'} . \tag{2.10}$$

Using eqn.2.10 we can chose k so that the first few resonances on each trajectory have approximately the correct widths. (Strictly, by taking different k for different trajectories, we are breaking the "equal slopes" requirement on the trajectories).

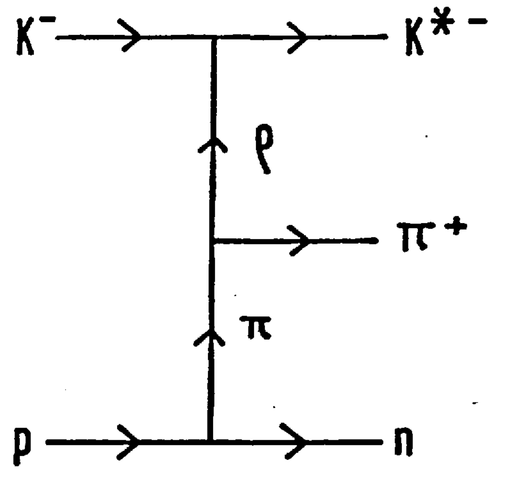
To determine the real part of the trajectories we have taken a slope of 0.9 $(\text{GeV})^{-2}$ for all trajectories and have then determined the intercepts from a Chew Frautschi plot. We obtain

$$\begin{aligned}
\alpha_{K^*}(s) &= 0.18 + 0.9s + i\,0.1(s-s_0)\,, \quad s_0 = (m_{K^*}+m_\pi)^2 \\
\alpha_\rho(t) &= 0.48 + 0.9t\,, \\
\alpha_\Sigma(s) &= -0.22 + 0.9s + i\,0.15(s-s_0)\,, \quad s_0 = (m_p+m_K)^2 \\
\alpha_N(t) &= -0.3 + 0.9t\,, \\
\alpha_\pi(t) &= -0.0175 + 0.9t\,, \\
\alpha_\Delta(s) &= 0.12 + 0.9s + i\,0.25(s-s_0)\,, \quad s_0 = (m_n+m_\pi)^2.
\end{aligned} \tag{2.11}$$

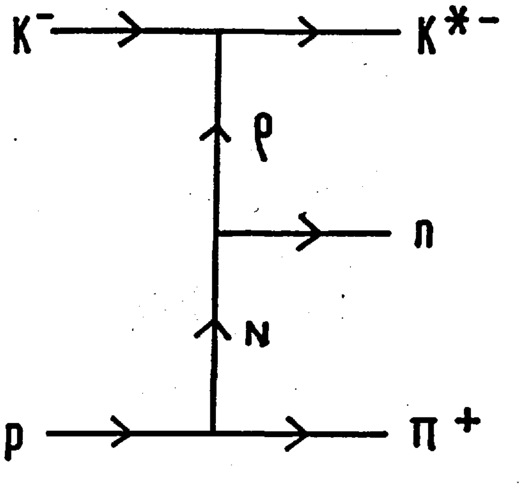
We note that in deriving the amplitude of eqn.2.4 we regard the external K^* as being a stable particle.

2.3 The Kinematic Factor

We now come to the problem of choosing the kinematic factor K. The data for this process shows a strong concentration of events at small t_{KK^*} and t_{pn}. This suggests that the amplitude is dominated by the nearby poles: the ρ and π in t_{KK^*} and t_{pn} respectively as indicated in Fig.6a. In a Regge model we would also expect the term corresponding to Fig.6a to be more important than that corresponding to Fig.6b since it involves only meson exchange whereas Fig.6b has a baryon exchange. Baryon trajectories have lower intercepts than meson trajectories and this leads to terms involving baryon exchange giving smaller contributions to amplitudes. Therefore we base our choice of K on the peripheral diagram of Fig.6a. This diagram corresponds to



a.



b.

Fig. 6

a double pole limit of the first term of our Veneziano amplitude. Near to the π and ρ poles the first term of eqn.2.4 behaves as

$$K \frac{1}{-\alpha_\pi} \cdot \frac{1}{1-\alpha_\rho} . \tag{2.12}$$

We now evaluate the diagram corresponding to Fig.6a using covariant vertex functions and propagators[14,15]. The vertex functions we require are given by

$$\begin{aligned} C_{\alpha\beta}(K, K^*, \rho_{ex}) &= \tfrac{1}{2} g_1\, \varepsilon_{\alpha\beta}(K+K^*, K-K^*) , \\ C_\mu(\rho_{ex}, \pi, \pi_{ex}) &= \tfrac{1}{2} g_2 (\pi + \pi_{ex})_\mu , \\ C(p, \pi, n) &= g_3 \gamma_5 \end{aligned} \tag{2.13}$$

where $\varepsilon_{\alpha\beta}(P, P') \equiv \varepsilon_{\alpha\beta}{}^{\gamma\delta} P_\gamma P'_\delta$

and where we have denoted the momentum of the K^- etc. by K_μ. The subscript "ex" refers to exchanged particles. g_1, g_2 and g_3 are coupling constants.

The propagators are simply

$$\begin{aligned} P_{\mu\nu}(\rho_{ex}) &= \frac{g_{\mu\nu} - (\rho_{ex})_\mu (\rho_{ex})_\nu / m_\rho^2}{t_{KK^*} - m_\rho^2} . \\ P(\pi_{ex}) &= \frac{1}{t_{pn} - m_\pi^2} . \end{aligned} \tag{2.14}$$

The pole graph then gives

$$\begin{aligned} A_{pole} = g_3 \epsilon_\alpha(\lambda)\, C^{\alpha\beta}(K, K^*, \rho_{ex})\, P_{\beta\mu}(\rho_{ex}) \\ C^\mu(\rho_{ex}, \pi, \pi_{ex})\, P(\pi_{ex})\, \bar{u}_n \gamma_5 u_p \end{aligned} \tag{2.15}$$

$$= \frac{g_1 g_2 g_3 \, \epsilon_\alpha(\lambda) . \varepsilon^\alpha(\pi, K^*, K) \bar{u}_n \gamma_5 u_p}{(t_{KK^*} - m^2_{K^*})(t_{pn} - m^2_\pi)} \qquad (2.16)$$

where $\epsilon_\alpha(\lambda)$ is the K^* polarisation vector. Thus if we choose

$$K = \alpha'^2 g_1 g_2 g_3 \, \epsilon_\alpha(\lambda) \varepsilon^\alpha(\pi, K^*, K) \bar{u}_n \gamma_5 u_p$$

$$\equiv C \, \epsilon_\alpha(\lambda) \varepsilon^\alpha(\pi, K^*, K) \bar{u}_n \gamma_5 u_p . \qquad (2.17)$$

our Veneziano model has the correct couplings at the π and ρ poles. For higher poles on the π and ρ trajectories we cannot expect the two models to correspond. This is because; 1) for higher spins more covariant coupling mechanisms become available, 2) the Veneziano model has daughter contributions. It should alsó be pointed out that the vertex functions we have used are strictly only applicable when all the particles are on mass shell. We shall assume that the choice of K given by eqn.2.17 still holds when $t_{KK^*} \neq m^2_\rho$ and $t_{pn} \neq m^2_\pi$.

2.4 Asymptotic Behaviour of the Amplitude

We now show that our choice of K leads to our Veneziano amplitude having correct Regge asymptotic behaviour. To show this it is easiest to consider the spin average $\frac{1}{2} \sum_{\text{spins}} |A|^2$. Firstly we need to evaluate $\sum_{\text{spins}} |K|^2$.

$$\sum_{\text{spins}} |K|^2 = \sum \left| c\, \bar{u}_n \gamma_5 u_p\, \epsilon_\alpha(\lambda)\, \varepsilon^\alpha(\pi, K^*, K) \right|^2 \quad (2.18)$$

$$= c^2\, \mathrm{Tr}\left[(\not{p}_n + m_n)(\not{p}_p - m_p) \right] \varepsilon^\alpha(\pi, K^*, K)$$

$$\left(g_{\alpha\beta} - \frac{K^*_\alpha K^*_\beta}{m^2_{K^*}} \right) \varepsilon^\beta(\pi, K^*, K) \quad (2.19)$$

$$= 8\left(t_{pn} - (m_p - m_n)^2\right) c^2$$

$$\times \begin{vmatrix} m^2_{K^*} & K^*\cdot K & K^*\cdot\pi \\ K^*\cdot K & m^2_K & K\cdot\pi \\ K^*\cdot\pi & K\cdot\pi & m^2_\pi \end{vmatrix} \quad (2.20)$$

$$= 8c^2\left(t_{pn} - (m_p - m_n)^2\right)\left[m^2_{K^*} m^2_K m^2_\pi - m^2_{K^*}(K\cdot\pi)^2 \right.$$

$$\left. - m^2_\pi (K^*\cdot K)^2 - m^2_K (K^*\cdot\pi)^2 + 2(K^*\cdot\pi)(K\cdot\pi)(K^*\cdot K) \right]. \quad (2.21)$$

Using

$$K\cdot K^* = \frac{m^2_K + m^2_{K^*} - t_{KK^*}}{2}, \quad (2.22)$$

$$K^*\cdot\pi = \frac{s_{K^*\pi} - m^2_{K^*} - m^2_\pi}{2}, \quad (2.23)$$

$$K\cdot\pi = (m^2_K + m^2_\pi - t_{K\pi})/2$$

$$= \frac{s_{K^*\pi} + t_{KK^*} - t_{pn} - m^2_{K^*}}{2}. \quad (2.24)$$

We obtain

$$\sum_{\text{spins}} |K|^2 = 8c^2\left(t_{pn}-(m_p-m_n)^2\right)\Big[m_{K^*}^2 m_K^2 m_\pi^2$$
$$-m_{K^*}^2 \frac{(s_{K^*\pi}+t_{KK^*}-t_{pn}-m_{K^*}^2)^2}{4} \tag{2.25}$$
$$-m_\pi^2 \frac{(m_K^2+m_{K^*}^2-t_{KK^*})^2}{4} - m_K^2 \frac{(s_{K^*\pi}-m_{K^*}^2-m_\pi^2)^2}{4}$$
$$+\frac{(m_K^2+m_{K^*}^2-t_{KK^*})(s_{K^*\pi}-m_{K^*}^2-m_\pi^2)(s_{K^*\pi}+t_{KK^*}-t_{pn}-m_{K^*}^2)}{4}\Big]$$

If we now consider the asymptotic behaviour of the amplitude of eqn.2.4 as $s_{Kp}\to\infty$, $s_{K^*\pi}\to\infty$, $s_{\pi n}\to\infty$ with t_{pn} and t_{KK^*} fixed and with

$$\frac{s_{K^*\pi}\cdot s_{\pi n}}{s_{Kp}} \equiv \eta \text{ (finite)}. \tag{2.26}$$

Then using the asymptotic form for B_5 as given in the appendix.

$$B_5\left(1-\alpha_\rho, 1-\alpha_{K^*}, \tfrac{3}{2}-\alpha_\Delta, -\alpha_\pi, \tfrac{3}{2}-\alpha_\Sigma\right) \equiv B_5(a)$$
$$\simeq s_{K^*\pi}{}^{\alpha_\rho(t_{KK^*})-1}\, s_{\pi n}{}^{\alpha_\pi(t_{pn})}$$
$$\times f\left(\eta, 1-\alpha_\rho(t_{KK^*}), -\alpha_\pi(t_{pn})\right). \tag{2.27}$$

$$B_5\left(\tfrac{3}{2}-\alpha_\Sigma, 1-\alpha_\rho, \tfrac{3}{2}-\alpha_\Sigma, \tfrac{3}{2}-\alpha_\Delta, \tfrac{3}{2}-\alpha_N\right) \equiv B_5(b)$$
$$\to 0.$$

Using eqn.2.25

$$\frac{1}{2}\sum_{\text{spins}}|A|^2 \simeq s_{K^*\pi}{}^{2\alpha_\rho}\, s_{\pi n}{}^{2\alpha_\pi}\Big[-c^2 f(\eta,1-\alpha_\rho,-\alpha_\pi)^2 \times \left(t_{pn}-(m_p-m_n)^2\right) t_{KK^*}\Big]$$

$$\text{as } s_{K^*\pi},\, s_{\pi n},\, s_{Kp} \to \infty \qquad (2.28)$$
$$t_{pn},\, t_{KK^*} \text{ fixed}$$
$$\text{with } \frac{s_{K^*n}\, s_{\pi n}}{s_{Kp}} = \eta .$$

Thus our amplitude has the correct asymptotic behaviour for the double-Regge limit corresponding to Fig.6a We may also consider the double-Regge limit corresponding to Fig.6b, i.e.

$$s_{Kp} \to \infty \;,\; s_{K^*n} \to \infty \;,\; s_{\pi n} \to \infty$$
$$t_{KK^*},\, t_{\pi p} \quad \text{fixed} \qquad (2.29)$$
$$\frac{s_{K^*n}\,.\,s_{\pi n}}{s_{Kp}} = \eta \quad \text{(finite)} .$$

To evaluate this limit we need to express eqn.2.25 in terms of the variables used in eqn.2.29. To do this we return to eqn.2.21 and use

$$K^*.\pi = \frac{s_{Kp} - s_{K^*n} - s_{\pi n} + m_n^2}{2} , \qquad (2.30)$$

$$K\cdot\pi = \frac{s_{Kp} + t_{p\pi} - s_{K^*n} - m_p^2}{2} , \qquad (2.31)$$

$$t_{pn} = t_{KK^*} - t_{p\pi} - s_{n\pi} + m_p^2 + m_n^2 + m_\pi^2 . \qquad (2.32)$$

Eqn.2.25 becomes

$$\sum |K|^2 = 8c^2\left(t_{KK^*} - t_{p\pi} - s_{n\pi} + m_\pi^2 + 2m_p m_n\right)$$

$$\times \Big[m_{K^*}^2 m_K^2 m_\pi^2 - m_{K^*}^2 \frac{(s_{Kp} + t_{p\pi} - s_{K^*n} - m_p^2)^2}{4}$$

$$- m_\pi^2 \frac{(m_K^2 + m_{K^*}^2 - t_{KK^*})^2}{4} - m_K^2 \frac{(s_{Kp} - s_{K^*n} - s_{n\pi} + m_n^2)^2}{4} \tag{2.33}$$

$$+ \frac{(m_K^2 + m_{K^*}^2 - t_{KK^*})(s_{Kp} - s_{K^*n} - s_{n\pi} + m_n^2)(s_{Kp} + t_{p\pi} - s_{K^*n} - m_p^2)}{4} \Big] .$$

In the limit given in eqn.2.29 the Veneziano terms behave as

$$B_5(a) \to 0$$

$$B_5(b) \simeq s_{n\pi}^{\alpha_N(t_{p\pi}) - 3/2} \, s_{K^*n}^{\alpha_\rho(t_{KK^*}) - 1} \tag{2.34}$$

$$\times f\left(\eta, \tfrac{3}{2} - \alpha_N(t_{p\pi}), 1 - \alpha_\rho(t_{KK^*})\right) .$$

Using eqn.2.33

$$\frac{1}{2} \sum_{\text{spins}} |A|^2 \simeq s_{n\pi}^{2\alpha_N(t_{p\pi})} \, s_{K^*n}^{2\alpha_\rho(t_{KK^*})}$$

$$\times \left[c^2 f\left(\eta, \tfrac{3}{2} - \alpha_N(t_{p\pi}), 1 - \alpha_\rho(t_{KK^*})\right)^2 \frac{t_{KK^*}}{\eta^2} \right] \tag{2.35}$$

as $s_{n\pi}, s_{K^*n}, s_{Kp} \to \infty$

with t_{KK^*}, $t_{p\pi}$ fixed

$$\frac{s_{\pi n} \, s_{K^*n}}{s_{Kp}} = \eta .$$

Thus we have the correct double-Regge behaviour expected in this limit. We note that if we had not shifted the argument of the N_α trajectory in the second term of eqn.2.4 we would not have obtained correct asymptotic behaviour for the limit of eqn.2.29.

Our choice for the factor K is based on consideration of the dominant peripheral diagram for the process $K^- p \to K^{*-}\pi^+ n$. The amplitude of eqn.2.4 may also be used to describe the processes obtained by crossing the external particle lines. In general, for processes which only involve crossing meson lines, we would have obtained the same kinematic factor if we had started with the crossed process and applied our prescription for determining K. In the case of processes in which baryon lines are crossed (e.g. $p\bar{n} \to K^{*-}\pi^+ K^+$) we would obtain different kinematic factors. This is partly a consequence of our inability to include fermions in the Veneziano model in an internally consistent way. However, we do note that the amplitude of eqn.2,4 does have correct asymptotic behaviour in all double-Regge limits for all crossed processes including those in which the baryon lines are crossed.

2.5 K^{**}(1420) Production.

The most significant feature of the data for the process being considered is the K^{**}(1420) production in the $K^{*-}\pi^+$ channel. We must check that the kinematic factor we have chosen is a reasonable choice for describing the

K^{**} production. In order to do this we consider the diagram corresponding to Fig.7a. This diagram may be evaluated using exactly the same techniques as were used before. In addition we shall need the spin-2 propagator numerator:

$$\begin{aligned} P_{\alpha\beta\gamma\delta}(K^{**}) &= g_{\alpha\gamma}g_{\beta\delta} + g_{\alpha\delta}g_{\beta\gamma} - \tfrac{2}{3}g_{\alpha\beta}g_{\delta\gamma} \\ &+ \frac{1}{m^2}\Big[\tfrac{2}{3}g_{\alpha\beta}P_\gamma P_\delta + \tfrac{2}{3}g_{\delta\gamma}P_\alpha P_\beta - g_{\alpha\gamma}P_\beta P_\delta \\ &- g_{\alpha\delta}P_\beta P_\gamma - g_{\beta\gamma}P_\alpha P_\delta - g_{\beta\delta}P_\alpha P_\gamma\Big] \\ &+ \frac{4}{3m^4}P_\alpha P_\beta P_\gamma P_\delta \end{aligned} \tag{2.36}$$

where $P = P_{K^{**}}$, $m = m_{K^{**}}$.

The vertex functions are

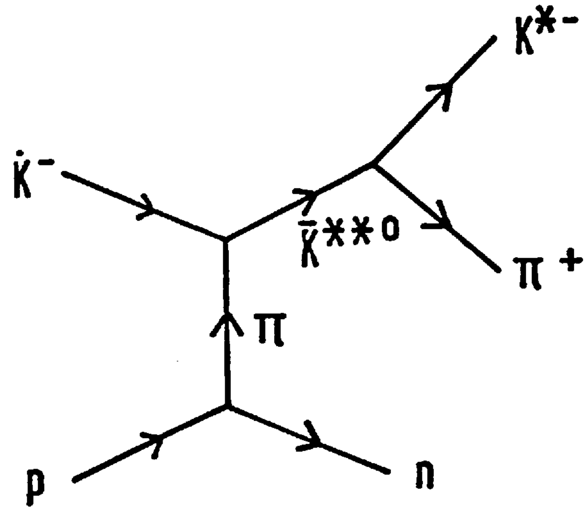
$$C_{\gamma\delta\eta}(K^*,\pi,K^{**}) = \tfrac{1}{4}g_4(K^*-\pi)_\gamma\,\mathcal{E}_{\delta\eta}(K^*-\pi,K^*+\pi), \tag{2.37}$$

$$C_{\alpha\beta}(K,\pi,K^{**}) = \tfrac{1}{4}g_5(K-\pi_{ex})_\alpha(K-\pi_{ex})_\beta\,. \tag{2.38}$$

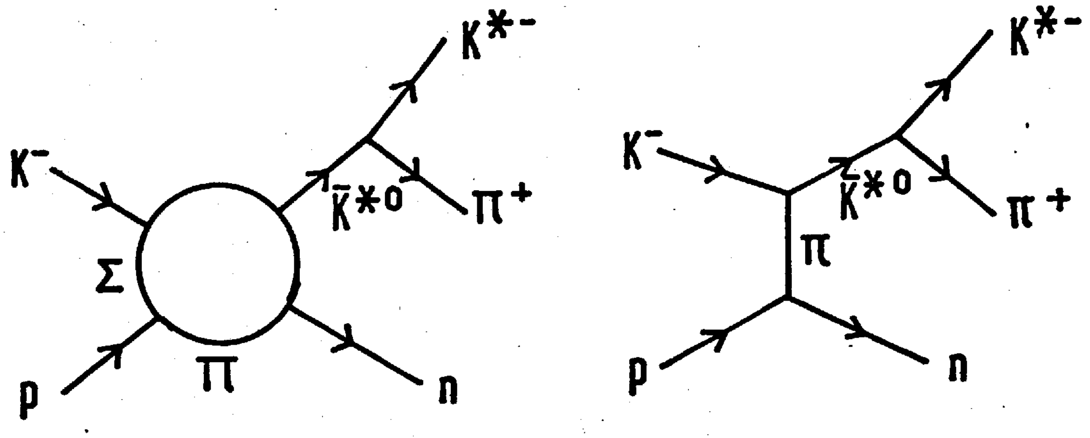
The K^{**} production diagram then gives

$$\begin{aligned} A_{K^{**}} &= g_3\,\bar{u}_n\gamma_5 u_p\,C_{\alpha\beta}(K,\pi,K^{**})\,P^{\alpha\beta\gamma\delta}(K^{**}) \\ &\quad\times\frac{C_{\gamma\delta\eta}(K^*,\pi,K^{**})\,\epsilon^\eta(\lambda)}{(s_{K^*\pi}-m^2)(t_{pn}-m_\pi^2)} \end{aligned} \tag{2.39}$$

$$= \frac{g_3g_4g_5\,\bar{u}_n\gamma_5u_p\,X^\eta\epsilon_\eta(\lambda)}{16(t_{pn}-m_\pi^2)(s_{K^*\pi}-m^2)} \tag{2.40}$$



a.



b.

Fig. 7

where

$$X_\eta = (K-\Pi_{ex})_\alpha (K-\Pi_{ex})_\beta \Big(g^{\alpha\gamma} g^{\beta\delta} + g^{\alpha\delta} g^{\beta\gamma} - \frac{1}{m^2}\left[g^{\alpha\delta} p^\beta p^\gamma + g^{\beta\delta} p^\alpha p^\gamma \right]\Big) (K^* - \pi)_\gamma \tag{2.41}$$

$$\begin{aligned} &\varepsilon_{\delta\eta}(K^*-\pi, p) \\ &= 8\, \varepsilon_\eta (K, K^*, \pi)\Big[2K\cdot K^* - m_{K^*}^2 - 2K\cdot\pi + m_\pi^2 \\ &\quad - \frac{1}{m^2}\left(2K\cdot K^* + 2K\cdot\pi - m_{K^*}^2 - 2\pi\cdot K^* - m_\pi^2\right)\left(m_{K^*}^2 - m_\pi^2\right)\Big] . \end{aligned} \tag{2.42}$$

We obtain

$$\begin{aligned} A_{K^{**}} = {} & \varepsilon_\eta (K, K^*, \pi)\, \epsilon^\eta(\lambda)\, \bar{u}_n \gamma_5 u_p \cdot g_3 g_4 g_5 \\ & \times \Big[m_K^2 + m_{K^*}^2 + m_\pi^2 - 2t_{KK^*} + t_{pn} - s_{K^*\pi} \\ & - \frac{(m_{K^*}^2 - m_\pi^2)}{m^2} (m_K^2 - t_{pn}) \Big] \Big/ \left(2(s_{K^*\pi} - m^2)(t_{pn} - m_\pi^2)\right) . \end{aligned} \tag{2.43}$$

The residue at the double pole is

$$\frac{g_3 g_4 g_5}{2}\, \epsilon_\eta(\lambda)\, \bar{u}_n \gamma_5 u_p\, \varepsilon^\eta (K, K^*, \pi) \Big[m_K^2 + m_{K^*}^2 + 2m_\pi^2 - m^2 - 2t_{KK^*} - \frac{(m_{K^*}^2 - m_\pi^2)(m_K^2 - m_\pi^2)}{m^2} \Big] . \tag{2.44}$$

We may compare this with the residue of our Veneziano amplitude at the double pole $\alpha_\pi = 0$, $\alpha_{K^*} = 2$. This gives

$$\text{Res } A(\alpha_\pi = 0, \alpha_{K^*} = 2) = C\epsilon^{\eta}(K,\pi,K^*)\epsilon_\eta(\lambda) \\ \alpha_\rho(t_{KK^*})\,\bar{u}_n\gamma_5 u_p . \tag{2.45}$$

We see that both eqns.2.44 and 2.45 have the same form. Thus our kinematic factor may be regarded as being consistent with the K^{**} production mechanism. The residues given in eqns.2.44 and 2.45 are not expected to be identical as the Veneziano amplitude also contains a 1^- daughter at the K^{**} mass.

Thus we have shown that the amplitude for $K^- p \to K^{*-}\pi^+ n$ defined by eqns.2.4 and 2.17 has correct Regge asymptotic behaviour and is consistent with the K^{**} production mechanism.

2.6 The Amplitude for $K^- p \to \bar{K}^{*o} n$.

We now show that the five-point amplitude for $K^- p \to K^{*-}\pi^+ n$ may be consistently reduced to obtain an amplitude for $K^- p \to \bar{K}^{*o} n$. We note that only the first term of our amplitude has a K^* resonance in the $K^{*-}\pi^+$ channel. In order to see how the five-point amplitude should be factorised we make use of the experimental fact that the K^* is produced by pion exchange for this reaction. Therefore we consider the diagram of Fig.7b. We evaluate this diagram as before using vertex functions:

$$C_\alpha(K,\pi_{ex},K^*_{ex}) = \tfrac{1}{2} g_6 (K^- - \pi_{ex})_\alpha$$

$$C_{\alpha\beta}(K^*_{ex},K^*,\pi) = \tfrac{1}{2} g_7 \,\mathcal{E}_{\alpha\beta}(K^* + \pi, K^* - \pi) . \tag{2.46}$$

We obtain

$$\frac{1}{2} g_3 g_6 g_7 \, \bar{u}_n \gamma_5 u_p \frac{(K-\pi_{ex})^\alpha}{t_{pn}-m_\pi^2} \cdot \frac{g_{\alpha\beta}-(K^*_{ex})_\alpha (K^*_{ex})_\beta / m^2_{K^*}}{s_{K^*\pi}-m^2_{K^*}}$$

$$\times \frac{1}{2} \varepsilon^{\beta\gamma}(K^*+\pi, K^*-\pi)\, \epsilon_\gamma(\lambda) \tag{2.47}$$

$$= \frac{g_3 g_6 g_7 \, \bar{u}_n \gamma_5 u_p \, \varepsilon^\gamma(K,\pi,K^*)\, \epsilon_\gamma(\lambda)}{(t_{pn}-m_\pi^2)(s_{K^*\pi}-m^2_{K^*})} . \tag{2.48}$$

The numerator of this expression is the same as the residue of our Veneziano amplitude at the π and K^* poles (it is, in fact, just the kinematic factor K). We can now see how to factorise the five-point amplitude. Near to the K^* pole our Veneziano amplitude may be written as

$$A_{K^*} = -C \frac{\bar{u}_n \gamma_5 u_p \; \varepsilon^\gamma(K,\pi,K^*)\epsilon_\gamma(\lambda)}{\alpha'(s_{K^*\pi}-m^2_{K^*})} B(-\alpha_\pi, \tfrac{3}{2}-\alpha_\Sigma) \tag{2.49}$$

$$= \frac{-C}{2\alpha'} \bar{u}_n \gamma_5 u_p (K-\pi_{ex})^\alpha \frac{[g_{\alpha\beta}-(K^*_{ex})_\alpha (K^*_{ex})_\beta / m^2_{K^*}]}{s_{K^*\pi}-m^2_{K^*}}$$

$$\times \frac{1}{2} \varepsilon^{\beta\gamma}(K^*+\pi, K^*-\pi)\, \epsilon_\gamma(\lambda)\, B(-\alpha_\pi, \tfrac{3}{2}-\alpha_\Sigma) . \tag{2.50}$$

We now replace the spin 1 propagator numerator by a complete set of spin 1 states (i.e. we put the K^* on mass shell)

$$A_{K^*} = \frac{-C}{2\alpha' g_7} \bar{u}_n \gamma_5 u_p (K-\pi_{ex})^\alpha B(-\alpha_\pi, \tfrac{3}{2}-\alpha_\Sigma) \epsilon_\alpha(\lambda')$$

$$\sum_{\lambda'} \epsilon^*_\beta(\lambda') \frac{\varepsilon^{\beta\gamma}(K^*+\pi, K^*-\pi)}{s_{K^*\pi}-m^2_{K^*}} \frac{g_7}{2} \epsilon_\alpha(\lambda) \tag{2.51}$$

Here the three point $K^*_{ex}K^*\pi$ vertex has been separated from the five-point amplitude and we may therefore write the four-point amplitude as

$$A_{K^-p\to\bar{K}^{*0}n} = \frac{c\,\bar{u}_n\gamma_5 u_p}{2\alpha' g_7}(K-\pi_{ex})^\alpha \epsilon_\alpha(\lambda')\,B(-\alpha_\pi, \tfrac{3}{2}-\alpha_\Sigma) \tag{2.52}$$

$$= \frac{c\,\bar{u}_n\gamma_5 u_p}{\alpha' g_7} K^\alpha\cdot\epsilon_\alpha(\lambda')\,B(-\alpha_\pi(t), \tfrac{3}{2}-\alpha_\Sigma(s)). \tag{2.53}$$

We must now check that this amplitude has the correct Regge asymptotic behaviour. Considering first the spin average:

$$\frac{1}{2}\sum_{\text{spins}}|A|^2 = -c'^2\,K^\mu\left(g_{\mu\nu} - \frac{K^*_\mu K^*_\nu}{m^2_{K^*}}\right)K^\nu$$

$$B(-\alpha_\pi, \tfrac{3}{2}-\alpha_\Sigma)^2\,.\,(t-(m_p-m_n)^2) \tag{2.54}$$

where $c' = \dfrac{c}{\alpha' g_7}$

$$\frac{1}{2}\sum|A|^2 = -c'^2\left(m_K^2 - \frac{(m_K^2+m^2_{K^*}-t)^2}{4m^2_{K^*}}\right)(t-(m_p-m_n)^2) \tag{2.55}$$

$$\times\, B(-\alpha_\pi, \tfrac{3}{2}-\alpha_\Sigma)^2.$$

For $s\to\infty$, t fixed

$$\frac{1}{2}\sum_{\text{spins}}|A|^2 \simeq (-s)^{2\alpha_\pi(t)}\Big[-\alpha'^{2\alpha_\pi}c'^2\Big(m_K^2$$

$$-\frac{(m_K^2+m^2_{K^*}-t)^2}{4m^2_{K^*}}\Big)(t-(m_p-m_n)^2)\,\Gamma(-\alpha_\pi(t))^2\Big]. \tag{2.56}$$

For the crossed process $K^{*0}p\to K^+n$ (obtained by crossing $s\leftrightarrow u$) as $s\to\infty$, u fixed

$$B(-\alpha_\pi(t), \tfrac{3}{2}-\alpha_\Sigma(u)) \sim (\alpha' s)^{\alpha_\Sigma - 3/2}\,\Gamma(\tfrac{3}{2}-\alpha_\Sigma) \tag{2.57}$$

$$\frac{1}{2}\sum_{\text{spins}}\left|A_{K^{*o}p\to K^+n}\right|^2 \simeq -s^{2\alpha_\Sigma(u)}\left[\frac{c'^2\alpha'^{2\alpha_\Sigma-3}}{4m_{K^*}^2}\right. \tag{2.58}$$
$$\left.\times\,\Gamma\left(\tfrac{3}{2}-\alpha_\Sigma(u)\right)^2\right].$$

Both these asymptotic behaviours have the correct Regge form.

For computational purposes it is most convenient to express our four-point amplitude (eqn.2.53) in terms of helicity amplitudes. Adopting the convention that the beam direction is along the positive z axis we may write (in the s-channel c.m. frame)

$$K_\mu^- = (E_1, 0, 0, k),$$
$$K_\mu^* = (E_3, q\sin\theta, 0, q\cos\theta). \tag{2.59}$$

where θ is the scattering angle and where the particles are labelled: $K^-\equiv 1$, $p\equiv 2$, $\bar{K}^{*o}\equiv 3$, $n\equiv 4$. In this frame the polarisation vector has components(16)

$$\epsilon_\mu(1) = -\frac{1}{\sqrt{2}}(0, \cos\theta, i, -\sin\theta),$$
$$\epsilon_\mu(0) = \left(\frac{q}{m_3}, \frac{E_3}{m_3}\sin\theta, 0, \frac{E_3}{m_3}\cos\theta\right), \tag{2.60}$$
$$\epsilon_\mu(-1) = -\frac{1}{\sqrt{2}}(0, -\cos\theta, i, \sin\theta).$$

The helicity spinors are

$$u_p(\lambda_2) = \sqrt{E_2+m_2}\left(1+\frac{i\underline{\sigma}\cdot\underline{p}\,\gamma_5}{E_2+m_2}\right)\psi^{(\lambda_2)}(\theta) \tag{2.61}$$
$$\gamma_5^2 = -1$$

$$\psi^{(1/2)}(\theta=0)=\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad , \psi^{(-1/2)}(\theta=0)=\begin{pmatrix}0\\1\\0\\0\end{pmatrix} ,$$

$$\bar{u}_n(\lambda_4) = \sqrt{E_4+m_4}\ \psi^{+(\lambda_4)}(\theta)\left(1-\frac{i\,\underline{\sigma}.\underline{n}\,\gamma_5}{E_4+m_4}\right).$$

$$\psi^{+(1/2)}(\theta) = \left(\cos\theta/_2,\ \sin\theta/_2, 0, 0\right), \tag{2.62}$$

$$\psi^{+(-1/2)}(\theta) = \left(-\sin\theta/_2,\ \cos\theta/_2, 0, 0\right).$$

Using eqns.2.59-62 we obtain

$$\bar{u}_n(1/2)\gamma_5 u_p(1/2) = -\bar{u}_n(-1/2)\gamma_5 u_p(-1/2) \tag{2.63}$$

$$= -i\cos\theta/_2\ \frac{(E_4+m_4)k-(E_2+m_2)q}{\sqrt{(E_4+m_4)(E_2+m_2)}},$$

$$\bar{u}_n(1/2)\gamma_5 u_p(-1/2) = \bar{u}_n(-1/2)\gamma_5 u_p(1/2) \tag{2.64}$$

$$= -i\sin\theta/_2\ \frac{(E_4+m_4)k+(E_2+m_2)q}{\sqrt{(E_4+m_4)(E_2+m_2)}}.$$

$$\epsilon(\lambda_3)\cdot K^- = \frac{-k\sin\theta}{\sqrt{2}} \quad , \lambda_3=1$$

$$\frac{E_1 q}{m_3}-\frac{kE_3\cos\theta}{m_3} \quad , \lambda_3=0 \tag{2.65}$$

$$\frac{k\sin\theta}{\sqrt{2}} \quad , \lambda_3=-1 .$$

From eqns.2.63-65 it is easy to construct the helicity amplitudes:

$$\langle\lambda_3\lambda_4|\phi|\lambda_1\lambda_2\rangle \equiv C\,\epsilon^{\alpha}(\lambda_3)\cdot K^{-}_{\alpha}\,\bar{u}_n(\lambda_4)\gamma_5 u_p(\lambda_2) \times B\left(-\alpha_\pi(t), \tfrac{3}{2}-\alpha_\Sigma(s)\right). \tag{2.66}$$

Parity invariance implies

$$\langle\lambda_3\lambda_4|\phi|\lambda_1\lambda_2\rangle = \frac{\eta_3\eta_4}{\eta_1\;\eta_2}(-1)^{s_3+s_4-s_1-s_2} \times(-1)^{\lambda-\mu}\langle-\lambda_3,-\lambda_4|\phi|-\lambda_1,-\lambda_2\rangle \tag{2.67}$$

where $\lambda=\lambda_1-\lambda_2$, $\mu=\lambda_3-\lambda_4$. s_i is the spin and η_i the parity of the particle i.

Thus only six of the twelve helicity amplitudes are independent. In terms of the helicity amplitudes the differential cross section is given by

$$\frac{d\sigma}{dt} = \frac{\sum_i^{12}|\phi_i|^2}{(2s_1+1)(2s_2+1)64\pi k^2 s}. \tag{2.68}$$

The density matrix elements for the $\bar{K}^{*o}$ are given by

$$\rho_{\lambda_3'\lambda_3} = \sum_{\lambda_1\lambda_2\lambda_4}\frac{\langle\lambda_3'\lambda_4|\phi|\lambda_1\lambda_2\rangle\langle\lambda_3\lambda_4|\phi|\lambda_1\lambda_2\rangle^*}{\sum|\phi_i|^2}. \tag{2.69}$$

However, the experimental results for the density matrix are given in the Jackson frame. This is the rest frame of the $\bar{K}^{*o}$ chosen such that the z axis is parallel to the incident particle's (K^-) momentum and the y axis is perpendicular to the production plane. The density matrix elements, $\rho_{\lambda'\lambda}^{(\mathrm{Jackson})}$, in the new frame are given by

$$\rho^{(\text{Jackson})}_{\lambda'\lambda} = \sum_{\lambda'_3\lambda_3} d^1_{\lambda'\lambda'_3}(\Psi_3)\, d^1_{\lambda\lambda_3}(\Psi_3)\, \rho_{\lambda'_3\lambda_3} \qquad (2.70)$$

where $d^1_{\lambda\lambda'}$ are the usual spin 1 rotation matrices and Ψ_3 is the angle between the K^- and n as seen in the Jackson frame. We may evaluate this angle using

$$\operatorname{Tan}\Psi_3 = \frac{m_3\sqrt{1-\cos^2\theta}}{E_3\left(\cos\theta - \frac{qE_1}{kE_3}\right)} \qquad (2.71)$$

2.7 Normalisation of the Amplitudes.

Up to now we have not attempted to interpret the couplings g_i which appear in the vertex functions we use to construct kinematic factors. In the amplitude for $K^-p \to K^{*-}\pi^+n$ they only affect the overall normalisation of the amplitude and, since we are only comparing the model with percentage distributions we do not need to know this. However, for $K^-p \to \bar{K}^{*o}n$, we compare the model with the differential cross section data which requires us to know the normalisation constant (c') in the amplitude of eqn.2.53. In our results (Fig.12) we have used c' as a parameter to fit the data.

Our kinematic factors have been determined by evaluating pole graphs using vertex functions which are strictly on mass shell. Therefore the g_i can, in theory, be related to physical coupling constants. Unfortunately, in many cases these are very poorly determined experimentally and it is therefore more appropriate to use a suitable symmetry scheme to relate the coupling constants we require to those which are well determined (e.g. $g_{\rho\pi\pi}$ and $g_{NN\pi}$). One such symmetry scheme which has had considerable success[17] is U(6,6) [18]. This allows us to determine all meson-meson-meson coupling constants in terms of $g_{\rho\pi\pi}$ and all meson-baryon-baryon coupling constants in terms of $g_{NN\pi}$ For example, writing eqn.2.53 as

$$A = \alpha' g_{KK^*\pi}\, g_{n p\pi}\, \bar{u}_n \gamma_5 u_p K^\alpha \epsilon_\alpha(\lambda') \, B\left(-\alpha_n(t), \tfrac{3}{2} - \alpha_\Sigma(s)\right) \tag{2.72}$$

and using U(6,6) we may obtain (see Ref.17)

$$\begin{aligned} g_{\Lambda p\pi} &= 19.1 \\ g_{K^*K\pi} &= 7.1 \end{aligned} \tag{2.73}$$

(n.b. for our particular example we have only really used SU(3) to relate the required coupling constants to $g_{\rho\pi\pi}$ and $g_{NN\pi}$).

Hence we obtain

$$\alpha' g_{\Lambda p\pi}\, g_{K^*K\pi} = \cancel{135}\ 122. \tag{2.74}$$

We may compare this with the value obtained by fitting c' to the data:

$$\alpha' g_{\Lambda p\pi}\, g_{K^*K\pi} = 96 \tag{2.75}$$

2.8 Comparison with the Data.

The amplitude for $K^- p \to K^{*-} \pi^+ n$ has been compared with data [15] at 6.0 and 10.0 GeV/c using the phase space program FOWL[19] and the program[20] for B_5 written by Hopkinson. The $K^- p \to \bar{K}^{*o} n$ amplitude was compared with data[21] at 4.1, 5.5 and 10.1 GeV/c using the program FCN[22]. The results are shown in Figs.8-13.

Agreement with the main features of the three particle production data is good. Because of the large statistical errors on the experimental bins it is not possible to make a detailed comparison with the theory for the mass squared distributions. We do, however, get approximately the correct coupling for the K^{**} resonance in the $K^{*-}\pi^+$ channel. Because of the exchange degeneracy of this trajectory we also get a resonance corresponding to a 3^- recurrence of the K^*. There is no indication of such a resonance in the data. For the $\pi^+ n$ channel the $\Delta(1236)$ appears to be too strongly coupled at 6.0 GeV/c (the data is too poor to draw any conclusion at 10.0 GeV/c). However, this is probably due to the fact that we have not included terms with the N(1470) and other possible N and Δ trajectories in this channel. Each of these terms would be expected to give a similar background contribution to the Δ trajectory term we have used and would lead to the Δ peak not appearing so high. The $K^{*-}n$ distribution is featureless; the general shape is reproduced correctly by the theory. The agreement for the momentum transfer distributions is good.

For the $K^- p \to \bar{K}^{*o} n$ amplitude the differential cross section results agree well with the data. We obtain the correct t dependence. The s dependence is also correct - as we expect since the amplitude is known to have correct Regge asymptotic behaviour. For the density matrices our amplitude predicts

$$\rho_{oo}(t) = 1 \quad , \; Re\, \rho_{1o}(t) = 0 \; , \; \rho_{1,-1}(t) = 0. \qquad (2.76)$$

These are the values expected for simple pion exchange and the values obtained from Regge models with pion trajectories. However, since the Regge exchange includes contributions from high spin particles we expect the density matrices to vary with t as is seen in the data. If we take absorptive corrections into account the results are somewhat improved. However, since such corrections are not included in the five-point amplitude we are not justified in including them just in the four-point amplitude.

Figure Captions

Figure 8 Mass-squared distributions at 6.0 GeV/c for $K^-p \to K^{*-}\pi^+n$.

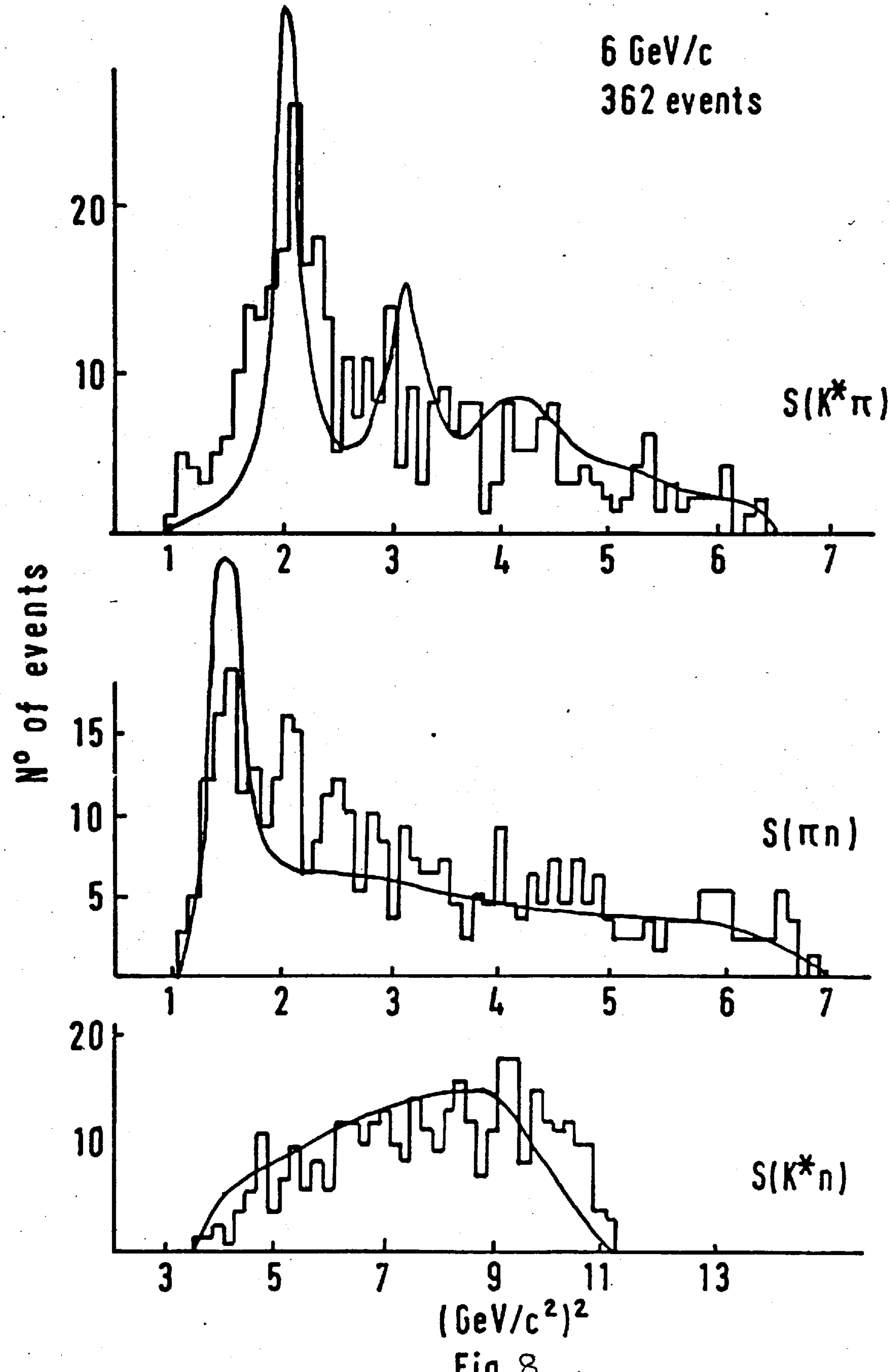
Figure 9 Momentum transfer distributions at 6.0 GeV/c for $K^-p \to K^{*-}\pi^+n$.

Figure 10 Mass-squared distributions at 10.0 GeV/c for $K^-p \to K^{*-}\pi^+n$.

Figure 11 Momentum transfer distributions at 10.0 GeV/c for $K^-p \to K^{*-}\pi^+n$.

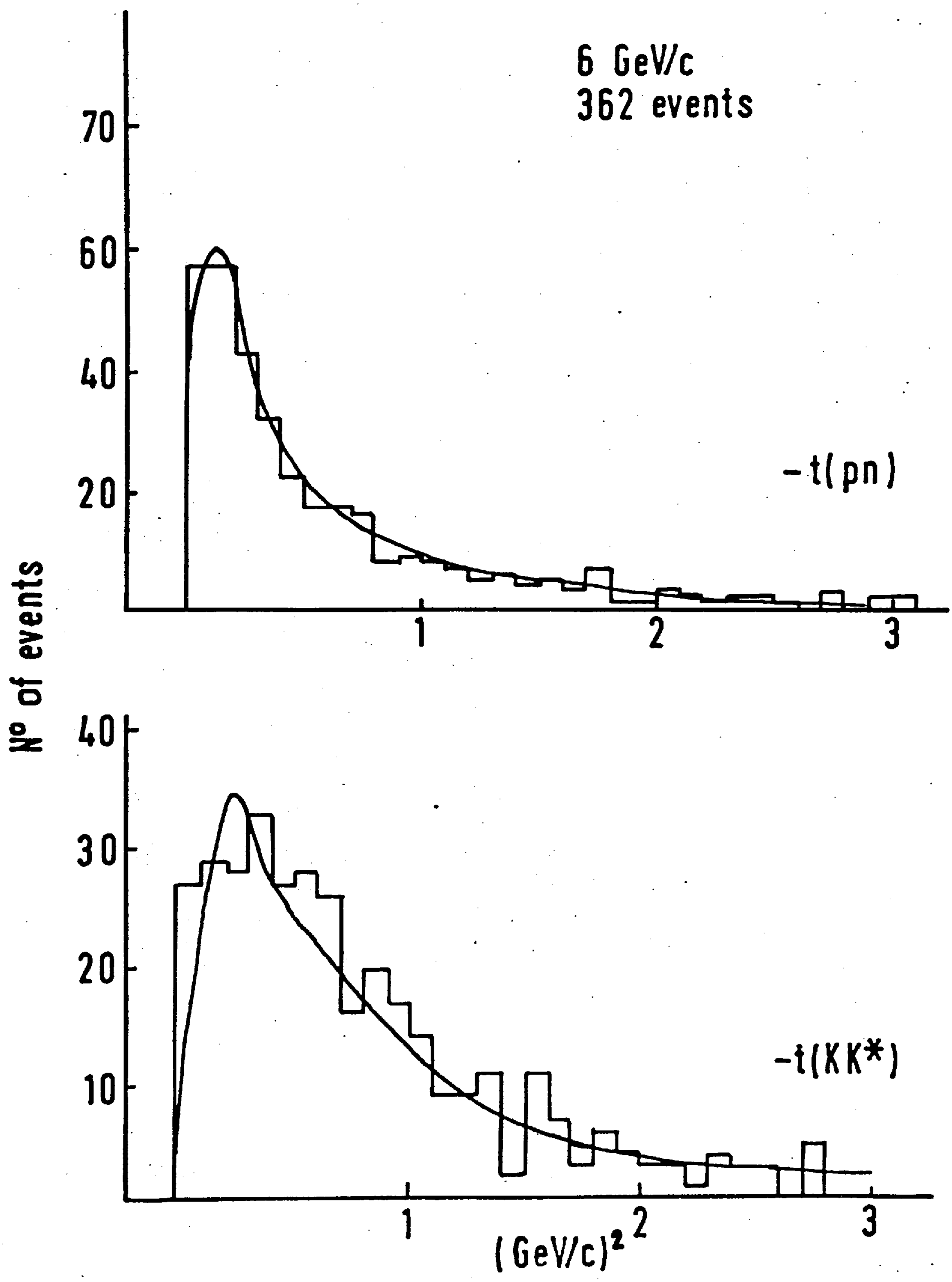
Figure 12 Differential cross sections for $K^-p \to \overline{K}^{*o}n$.

Figure 13 Density matrix for $K^-p \to \overline{K}^{*o}n$ at 4.1 GeV/c.



6 GeV/c
362 events
20
10
S(K*π)
1
2
3
4
5
6
7
N° of events
15
10
5
S(πn)
1
2
3
4
5
6
7
20
10
S(K*n)
3
5
7
9
11
13
(GeV/c²)²

Fig. 8



6 GeV/c
362 events
70
60
40
20
1
2
3
-t(pn)
Nº of events
40
30
20
10
-t(KK*)
1
2
3
(GeV/c)²

Fig. 9

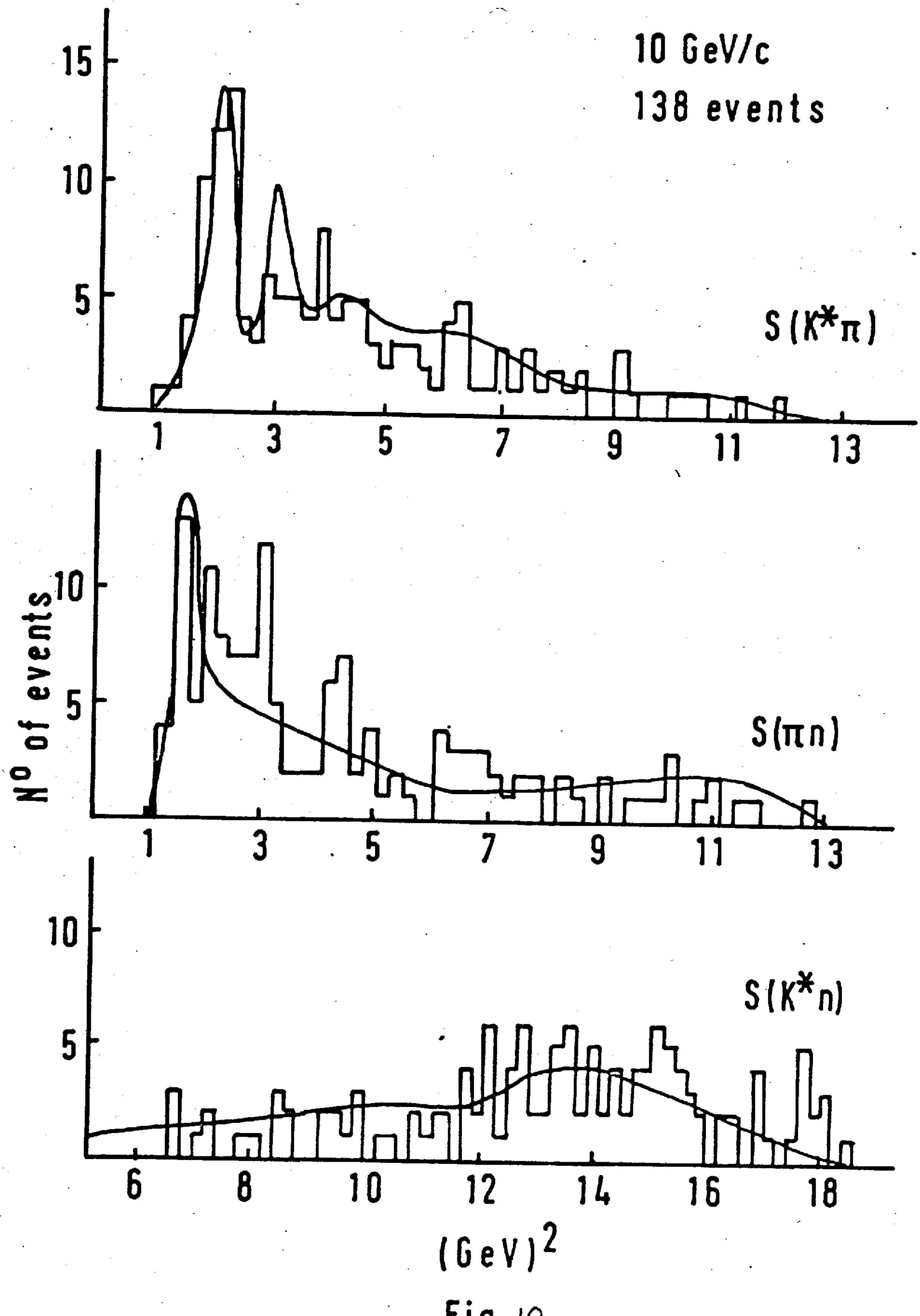


Fig. 10

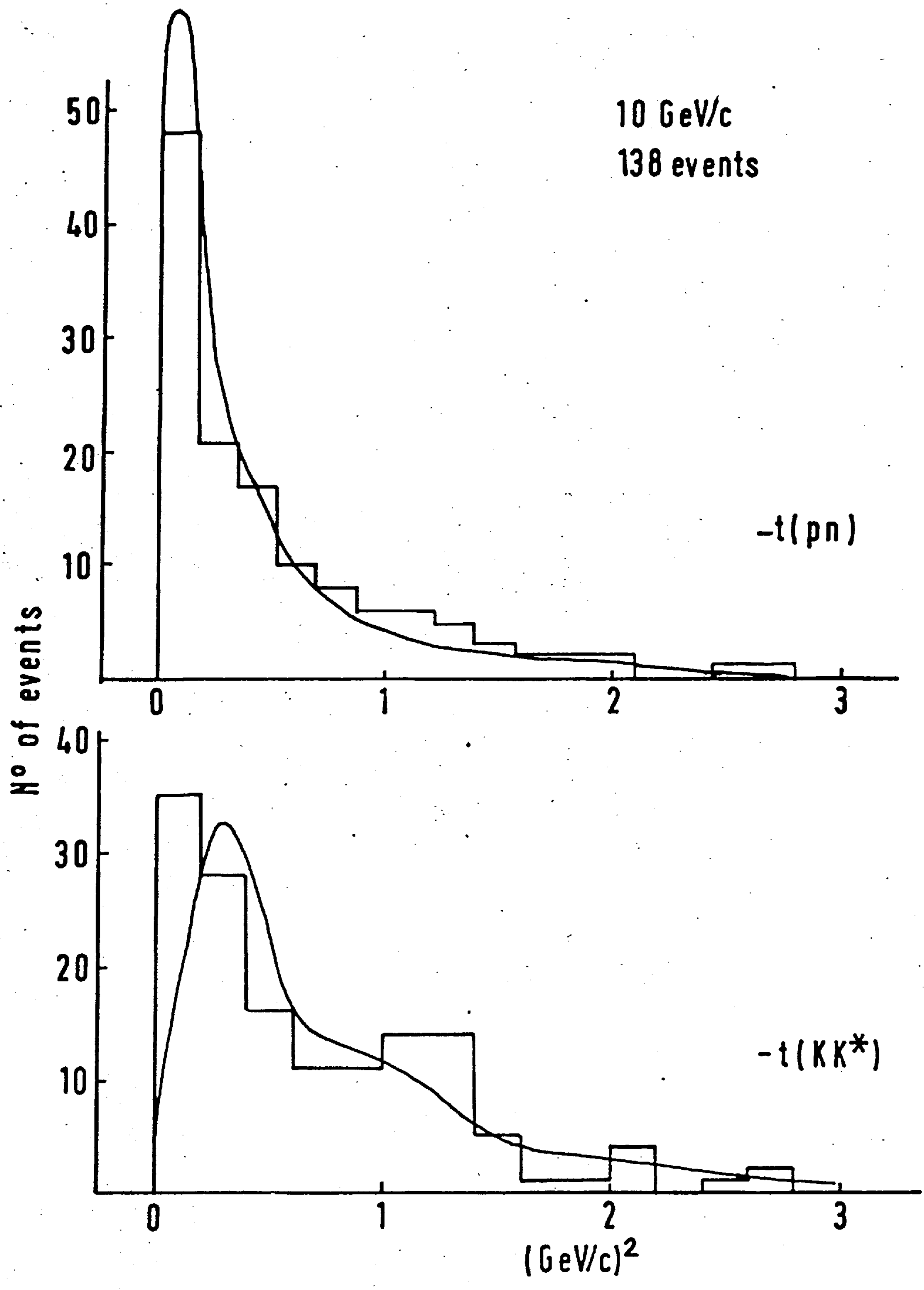
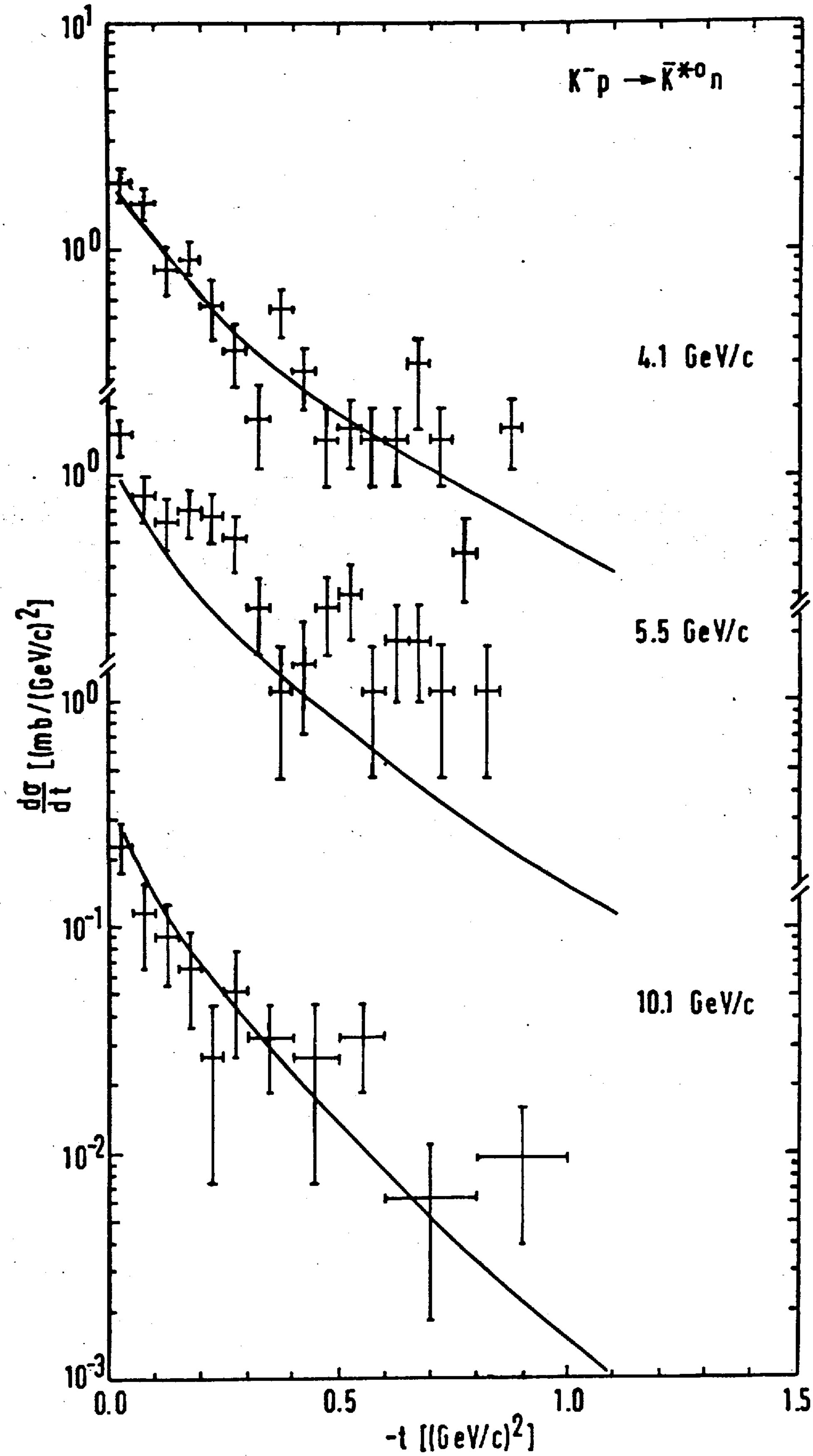


Fig. 11



K⁻p → K̄*⁰n
4.1 GeV/c
5.5 GeV/c
10.1 GeV/c
$\frac{d\sigma}{dt}$ [(mb/(GeV/c)^2]
10^1
10^0
10^0
10^0
10^{-1}
10^{-2}
10^{-3}
0.0
0.5
1.0
1.5
$-t$ [(GeV/c)^2]

Fig. 12

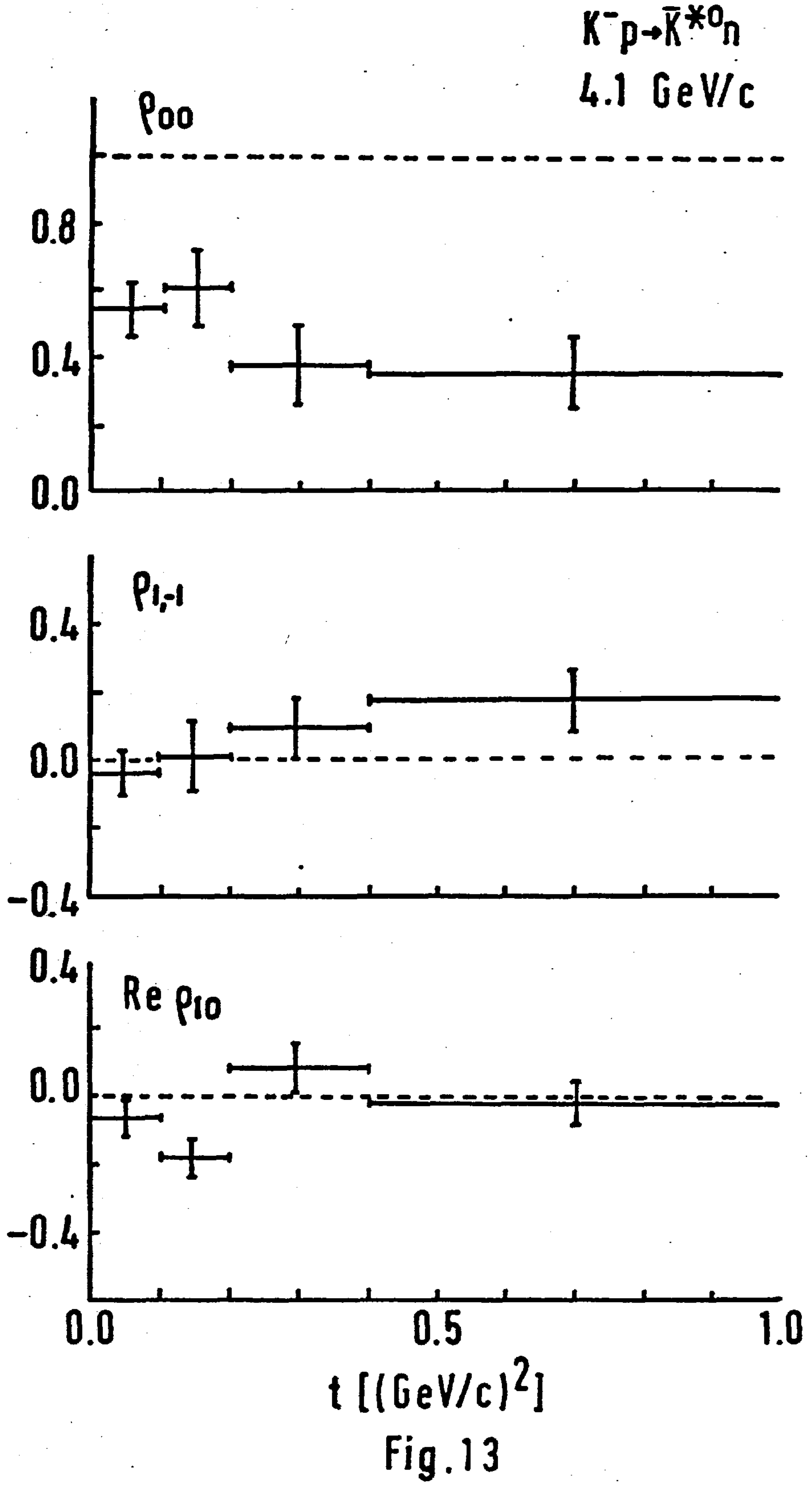

K⁻p→K̄*⁰n
4.1 GeV/c
ρ_{00}
0.8
0.4
0.0
$\rho_{1,-1}$
0.4
0.0
-0.4
0.4
Re ρ_{10}
0.0
-0.4
0.0
0.5
1.0
t [(GeV/c)^2]

Fig.13

II. THE MODEL FOR $\pi^- p \to K^0 \overline{K}^0 n$.

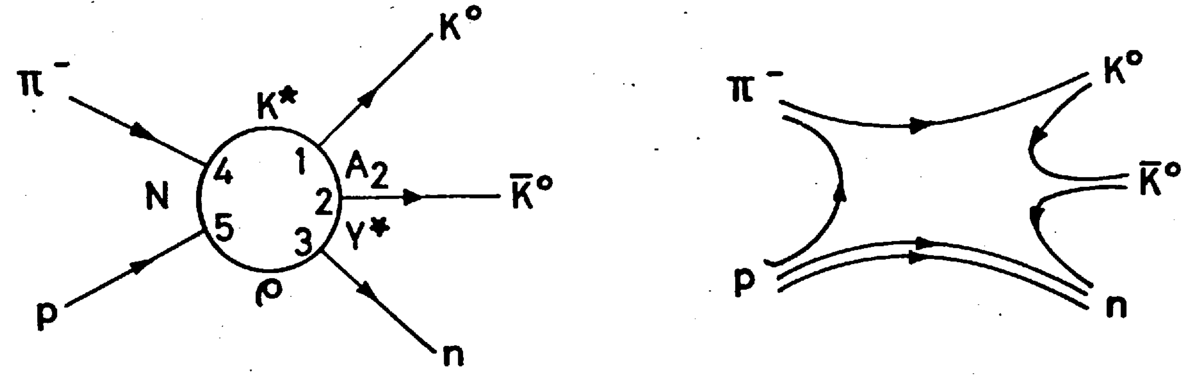
2.9 The Veneziano Amplitude.

In the last section we showed that it was possible to construct a Veneziano model for three particle production having correct Regge asymptotic behaviour. However, our model does not correspond to a conventional double-Regge model such as that proposed by Chan et al.[23]. Therefore it seems worthwhile to compare our Veneziano model with data in the double-Regge region. The process we shall consider is $\pi^- p \to K^0 \overline{K}^0 n$ at 12 GeV/c. This process has already been treated using a double-Regge model[24].

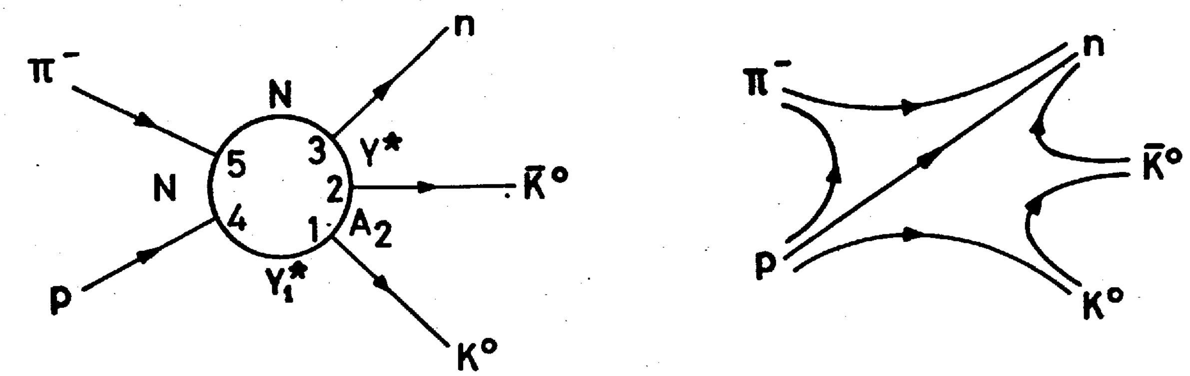
To determine the five-point Veneziano amplitude for this process we follow exactly the same procedure as for $K^- p \to K^{*-} \pi^+ n$. Because of the requirement that there should be no exotic exchanges all permutations where p and $\overline{K}^0$, n and K^0 or π^- and $\overline{K}^0$ are adjacent do not contribute to the amplitude. The two remaining permutations are shown in Fig.14 together with their corresponding duality diagrams which are both legal. As a consequence of there being three exotic channels for this process, all the trajectories, except the one coupling to the $\pi^- p$ subsystem, will be exchange degenerate.

The trajectories we have used were determined as follows:

1. The only possibility for the $\pi^- K^0$ channel is a degenerate K^*, K^{**} trajectory.
2. The experimental data[25] for the $K_1^0 K_1^0 n$ decay mode of $K^0 \overline{K}^0 n$ shows strong resonances in the $K_1^0 K_1^0$ effective mass



a.



b.

Fig. 14

distributions corresponding to the f and A_2. Therefore we have used a degenerate ρ, ω, f, A_2 trajectory for the $K^o\overline{K}^o$ channel.

3. For the pn channel we have used the same trajectory as for the $K^o\overline{K}^o$ channel. We make this choice because we expect the ρ trajectory to be dominant at high energies because of its high intercept. Also experimental evidence[26] suggests that the A_2 in the $K\overline{K}$ channel is produced by ρ exchange. The situation is not so clear for the production mechanism for the f. However, it is consistent with A_2 exchange.
4. The $\pi^-p \rightarrow K_1^oK_1^on$ data show no structure in the K_1^on mass distribution and in the absence of any preference we choose a $\Sigma(1385)$ trajectory for the $n\overline{K}^o$ channel. This is the only Y_1^* trajectory coupling strongly to the $\overline{K}N$ system. We could, however, use a Y_0^* trajectory. Because we shall only be considering events where $s_{n\overline{K}} > 3.5$ GeV^2 we do not expect the results to be sensitive to this choice (This has been checked using a $\Lambda(1520)$ trajectory in the $\overline{K}n$ channel. To within the statistical errors on the data the distributions are not significantly altered except for the $s_{K\overline{K}}$ distribution for which a poorer fit is obtained for $s_{ij} > 5$ GeV^2).
5. For the π^-p channel we use a N_α trajectory. Again the choice is rather arbitrary but because we are using high energy data ($s_{\pi p} = 23$ GeV^2) the results are insensitive to this choice. We note that the trajectory coupling to the π^-p subsystem has a definite signature in our

model. This is fortunate as there is no experimental evidence for the N_α trajectory being exchange degenerate.

6. For the pK^0 channel we use a $\Sigma(1385)$ trajectory because it is the only trajectory which couples strongly to this subsystem.

Thus we may write our five-point amplitude as:

$$A = K\Big(B_5\big(1-\alpha_{K^*}(t_{\pi K}),\, 1-\alpha_\rho(s_{K\bar{K}}),\, \tfrac{3}{2}-\alpha_\Sigma(s_{\bar{K}n}),\, 1-\alpha_\rho(t_{pn}),\, \tfrac{1}{2}-\alpha_N(s_{\pi p})\big) + B_5\big(\tfrac{3}{2}-\alpha_N(t_{\pi n}),\, \tfrac{3}{2}-\alpha_\Sigma(s_{n\bar{K}}),\, 1-\alpha_\rho(s_{K\bar{K}}),\, \tfrac{3}{2}-\alpha_\Sigma(t_{pK}),\, \tfrac{1}{2}-\alpha_N(s_{\pi p})\big)\Big). \qquad (2.77)$$

We note that the argument of the N_α in the πn channel in the second term of this expression has been shifted as before. Again this is to ensure correct asymptotic behaviour. We do not shift the N_α argument in the $\pi^- p$ channel as this would result in there being no $N_\alpha(938)$ resonance on this trajectory for the process being considered. Unfortunately as a result of doing this some of the crossed processes will have incorrect asymptotic behaviour but this will not interfere with our study of the Regge limit of the process $\pi^- p \to K^0 \bar{K}^0 n$.

Our trajectories are given by

$$\begin{aligned}
\alpha_N(s) &= -0.3 + 0.9s + i\,0.12\,(s-s_0), \quad s_0 = (m_\pi + m_p)^2, \\
\alpha_\rho(s) &= 0.48 + 0.9s + i\,0.15\,(s-s_0), \quad s_0 = 4m_K^2, \\
\alpha_\Sigma(s) &= -0.22 + 0.9s + i\,0.15\,(s-s_0), \quad s_0 = (m_n + m_K)^2, \qquad (2.78) \\
\alpha_{K^*}(t) &= 0.3 + 0.9t.
\end{aligned}$$

In order to determine the kinematic factor for this process we consider the two peripheral diagrams of Fig.15. We expect the diagram of Fig.15a to give the dominant contribution to the amplitude because the diagram of Fig.15b involves double baryon exchange (numerically we find that the contribution to the amplitude from diagram 15b is about 10% of that from diagram 15a). Therefore we determine our kinematic factor by evaluating diagram 15a according to our usual prescription. The following couplings are required:

$$\begin{aligned}
C_\alpha(p, \rho_{ex}, n) &= g\gamma_\alpha + g'\frac{(p+n)_\alpha}{2}, \\
C_{\alpha\beta}(\rho_{ex}, \bar{K}^0, K^*_{ex}) &= \frac{g''}{2}\,\varepsilon_{\alpha\beta}(\rho_{ex}+\bar{K}, \rho_{ex}-\bar{K}), \qquad (2.79) \\
C_\alpha(K^*_{ex}, \pi, K^0) &= \frac{g'''}{2}(\pi+K^0)_\alpha,
\end{aligned}$$

we obtain

$$\frac{1}{2}g''g'''(\pi+K)_\alpha\,\frac{\left(g^{\alpha\beta}-(K^*_{ex})^\alpha(K^*_{ex})^\beta/m^2_{K^*}\right)\varepsilon_{\beta\gamma}(\bar{K},\rho_{ex})}{t_{\pi K}-m^2_{K^*}}$$

$$\frac{\left(g^{\gamma\delta}-\rho_{ex}^\gamma\rho_{ex}^\delta/m^2_\rho\right)}{t_{pn}-m^2_\rho}\,\bar{u}_n\left(g\gamma_\delta+g'\frac{(p+n)_\delta}{2}\right)u_p \qquad (2.80)$$

$$= -g''g'''\,\frac{\varepsilon^\delta(K,\bar{K},\pi)\,\bar{u}_n\left(g\gamma_\delta+g'\frac{(p+n)_\delta}{2}\right)u_p}{(t_{\pi K}-m^2_{K^*})(t_{pn}-m^2_\rho)}. \qquad (2.81)$$

Near to the K^* and ρ poles the Veneziano amplitude may be written as

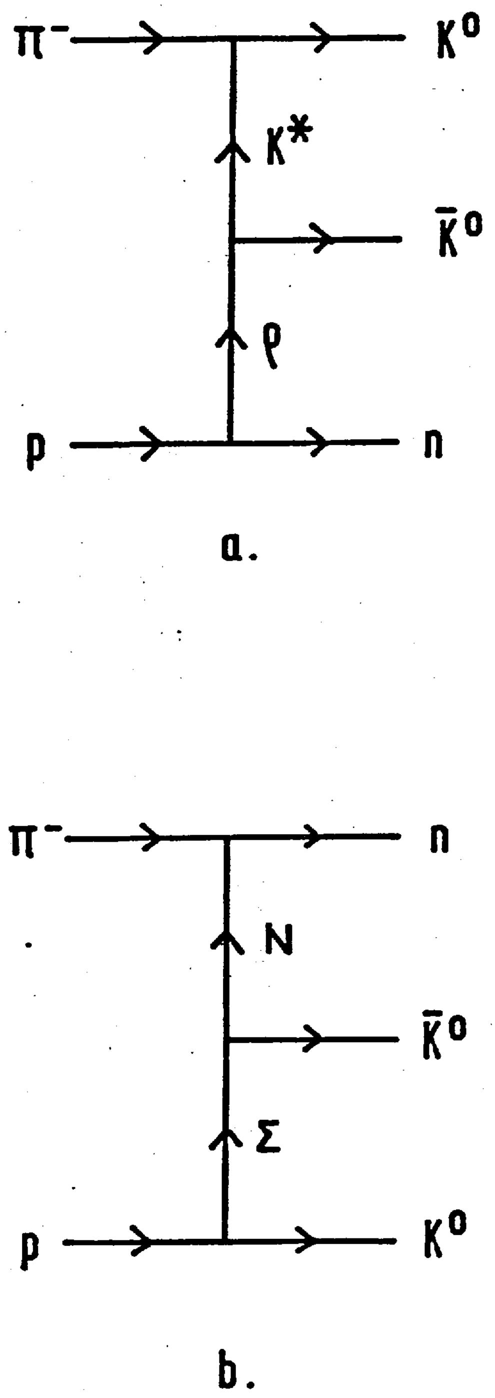


Fig. 15

$$A \simeq \frac{K}{\alpha'^2 (t_{\pi K} - m^2_{K^*})(t_{pn} - m^2_{\rho})} . \tag{2.82}$$

Comparing eqns.2.81 and 2.82 we write

$$K = C\, \varepsilon^{\delta}(K,\bar{K},\pi)\, \bar{u}_n \left(g\gamma_\delta + g' \frac{(p+n)_\delta}{2}\right) u_p$$
$$C = -g'' g''' \alpha'^2 . \tag{2.83}$$

Although the absolute normalisation of the amplitude is irrelevant in comparing the model with the data the relative magnitudes of the couplings g and g' are required. Rather than use this as a parameter we have used the value predicted by U(6,6) (see Ref.17). This gives

$$\frac{g'}{g} = -.72 . \tag{2.84}$$

2.10 Asymptotic Behaviour.

We now have to check that our amplitude has correct Regge asymptotic behaviour. To do this we need to evaluate the trace

$$\sum_{\text{spins}} |K|^2 = C^2\, \mathrm{Tr}\Big[(\not{p}_p + m_p)\left(g\gamma_\delta + g'\frac{(p+n)_\delta}{2}\right)(\not{n} + m_n)$$
$$\times \left(g\gamma_\mu + g' \frac{(p+n)_\mu}{2}\right)\Big]\, \varepsilon^{\delta}(K,\bar{K},\pi)\, \varepsilon^{\mu}(K,\bar{K},\pi)$$

$$= c^2\Big[\Big(2g'^2\big((m_p+m_n)^2-t_{pn}\big)+8gg'(m_p+m_n)$$

$$+8g^2\Big)\,\varepsilon(n,K,\bar{K},\pi)^2+2\big(t_{pn}-(m_p-m_n)^2\big)g^2$$

$$\varepsilon_\mu(K,\bar{K},\pi)\,\varepsilon^\mu(K,\bar{K},\pi)\Big]. \qquad (2.85)$$

Now

$$\varepsilon(n,K,\bar{K},\pi)^2 = \varepsilon(n,p,K,\pi)^2$$

$$= -\begin{vmatrix} m_n^2 & p\cdot n & K\cdot n & \pi\cdot n \\ p\cdot n & m_p^2 & p\cdot K & p\cdot\pi \\ K\cdot n & p\cdot K & m_K^2 & K\cdot\pi \\ \pi\cdot n & p\cdot\pi & K\cdot\pi & m_\pi^2 \end{vmatrix} \qquad (2.86)$$

and

$$K\cdot n = \frac{s_{\pi p}-s_{K\bar{K}}-s_{\bar{K}n}+m_{\bar{K}}^2}{2}$$

$$p\cdot K = \frac{s_{\pi p}+t_{\pi K}-s_{n\bar{K}}-m_\pi^2}{2} \qquad (2.87)$$

$$\pi\cdot n = \frac{s_{\pi p}+t_{pn}-s_{K\bar{K}}-m_p^2}{2}$$

$$p\cdot n = \frac{m_p^2+m_n^2-t_{pn}}{2} \qquad K\cdot\pi = \frac{m_K^2+m_\pi^2-t_{\pi K}}{2}$$

$$p\cdot\pi = \frac{s_{\pi p}-m_\pi^2-m_p^2}{2}$$

Using these relations we can see that, in the limit

$$s_{K\bar{K}} \Rightarrow \infty\ ,\ s_{\bar{K}\Lambda} \Rightarrow \infty\ ,\ s_{\pi p} \Rightarrow \infty$$

such that $\dfrac{s_{K\bar{K}}\, s_{\bar{K}\Lambda}}{s_{\pi p}} = \eta$ (finite)

$t_{\pi K}, t_{p\Lambda}$ fixed.

$$\varepsilon(\Lambda, K, \bar{K}, \pi)^2 \sim s_{K\bar{K}}^2\, s_{\bar{K}\Lambda}^2\, f_n(\eta, t_{\pi K}, t_{p\Lambda}) \tag{2.88}$$

We have already shown (p.40) that in this limit

$$\varepsilon_\mu(K,\bar{K},\pi)\,\varepsilon^\mu(K,\bar{K},\pi) \simeq +\ t_{\pi K}\, s_{K\bar{K}}^2 \tag{2.89}$$

and

$$B_5\left(1-\alpha_{K^*}, 1-\alpha_\rho, \tfrac{3}{2}-\alpha_\Sigma, 1-\alpha_\rho, \tfrac{1}{2}-\alpha_N\right)$$
$$\simeq s_{K\bar{K}}^{\alpha_{K^*}(t_{\pi K})-1}\, s_{\bar{K}\Lambda}^{\alpha_\rho(t_{p\Lambda})-1}\, f(\eta, 1-\alpha_\rho, 1-\alpha_{K^*})$$

$$B_5\left(\tfrac{3}{2}-\alpha_N, \tfrac{3}{2}-\alpha_\Sigma, 1-\alpha_\rho, \tfrac{3}{2}-\alpha_\Sigma, \tfrac{1}{2}-\alpha_N\right) \Rightarrow 0\,. \tag{2.90}$$

$$\therefore\quad \frac{1}{2}\sum_{\text{spins}} |A|^2 \sim s_{K\bar{K}}^{2\alpha_{K^*}(t_{\pi K})}\, s_{\bar{K}\Lambda}^{2\alpha_\rho(t_{p\Lambda})}\, \text{func.}(\eta, t_{\pi K}, t_{p\Lambda}). \tag{2.91}$$

Thus our amplitude has correct Regge behaviour in this limit. Similarly it can be shown that the amplitude has correct Regge behaviour in the double-Regge limit corresponding to Fig.15b. We note that the asymptotic form of our amplitude contains an explicit dependence on η . Here our model differs from the assumptions of conventional double-Regge models. For example, in Chan's model for this process Fig.15b is ignored and the amplitude for Fig.15a is written as

$$\gamma_{K^*}(t_{\pi K})\,\gamma_\rho(t_{pn})\,\gamma(t_{\pi K},t_{pn},\phi)\,s_{K\bar{K}}^{\alpha_{K^*}(t_{\pi K})}\,s_{\bar{K}n}^{\alpha_\rho(t_{pn})} \tag{2.92}$$

where γ_{K^*} and γ_ρ describe the Reggeon-π-K and Reggeon-p-n couplings and γ describes the Reggeon-Reggeon-$\bar{K}$ coupling. We note that the latter coupling depends on some azimuthal angle, ϕ , describing the relative orientations of the "top and bottom halves" of the double-Regge graph. ϕ may be taken as the Treiman Yang angle defined in the rest frame of the $\bar{K}$ as

$$\cos\phi = \frac{(\underline{p}_\pi \times \underline{p}_K)\cdot(\underline{p}_p \times \underline{p}_n)}{|\underline{p}_\pi \times \underline{p}_K|\;|\underline{p}_p \times \underline{p}_n|}$$

$$= \frac{\varepsilon_\mu(\pi,K,\bar{K})\;\varepsilon^\mu(p,n,\bar{K})}{|\varepsilon_\mu(\pi,K,\bar{K})|\,|\varepsilon_\mu(p,n,\bar{K})|}\,. \tag{2.93}$$

When $s_{K\bar{K}}$, s_{Kn} and $s_{\bar{K}n} > |t_{\pi K}|$, $|t_{pn}|$ and m_i^2, we can write[23]

$$\cos\phi \simeq \frac{1}{2\sqrt{t_{pn}}\sqrt{t_{\pi K}}}\Big(t_{\pi K}+t_{pn}-m_{\bar{K}}^2+\big(t_{pn}^2+t_{\pi K}^2 \tag{2.94}$$

$$-2t_{\pi K}t_{pn}-2m_{\bar{K}}^2(t_{\pi K}+t_{pn})+m_{\bar{K}}^4\big)/\eta\Big)\,.$$

Therefore γ may be expressed as a function of $t_{\pi K}$, t_{pn} and η. However, in applying the double-Regge model it is assumed that the variation of γ with ϕ can be ignored and the couplings are parametrised as

$$\gamma_{K^*}(t_{\pi K})\gamma_{\rho}(t_{pn})\gamma(t_{\pi K},t_{pn},\phi) = e^{\Omega_{K^*}t_{\pi K}}\, e^{\Omega_{\rho}t_{pn}} \tag{2.95}$$

where Ω_{K^*} and Ω_{ρ} are used as free parameters in fitting the data. Thus in this model the amplitude is assumed to be independent of η. The Veneziano model has far less freedom than double-Regge models. We have no parameters with which to modify the dependence of the amplitude on $t_{\pi K}$ and t_{pn}. This dependence is already built into the model. Chan et al. obtained good fits to the data with their model. However, in order to do so they found it necessary to take a K^* trajectory slope of almost zero.

2.11 A Comment on the Data.

Experimentally it is only possible to observe the $K_1^oK_1^o$ decay mode of the $K^o\overline{K}^o$ final state. The $K_1^oK_2^o$ and $K_2^oK_2^o$ decays are not seen. This experimental limitation is equivalent to projecting out the $K^o\overline{K}^o$ states with positive C parity and G parity $(-1)^I$. This can be seen as follows:

Writing the K and $\overline{K}$ as spinors

$$|K\rangle = \begin{pmatrix} K^+ \\ K^o \end{pmatrix}, \quad |\overline{K}\rangle = \begin{pmatrix} \overline{K}^o \\ -K^- \end{pmatrix} \tag{2.96}$$

we may construct $I=1, I_3=1$ states of definite C parity.

$$|1,1\rangle^{\pm} = \frac{1}{\sqrt{2}}\left(|K^+\bar{K}^o\rangle \pm |\bar{K}^oK^+\rangle\right) \tag{2.97}$$

where the $\pm$ sign refers to the C parity. Using the lowering operator we may obtain

$$|1,0\rangle^{\pm} = \frac{1}{2}\left(|K^o\bar{K}^o\rangle - |K^+K^-\rangle \mp |K^-K^+\rangle \pm |\bar{K}^oK^o\rangle\right) \tag{2.98}$$

and the orthogonal I=0 states

$$|0,0\rangle^{\pm} = -\frac{1}{\sqrt{2}}\left(|K^o\bar{K}^o\rangle + |K^+K^-\rangle \pm |K^-K^+\rangle \pm |\bar{K}^oK^o\rangle\right). \tag{2.99}$$

The K^o and $\bar{K}^o$ are only observed through the weak decays of the definite CP states K_1^o and K_2^o (we ignore CP violation which is a small effect).

$$K^o = \frac{K_1^o + iK_2^o}{\sqrt{2}}, \quad \bar{K}^o = \frac{K_1^o - iK_2^o}{\sqrt{2}}. \tag{2.100}$$

Substituting eqn.2.100 in 2.99 we obtain

$$\begin{aligned}
|1,0\rangle^{+} &= \tfrac{1}{2}\left(|K_1^oK_1^o\rangle + |K_2^oK_2^o\rangle - |K^+K^-\rangle - |K^-K^+\rangle\right)\\
|1,0\rangle^{-} &= \tfrac{1}{2}\left(i|K_2^oK_1^o\rangle - i|K_1^oK_2^o\rangle - |K^+K^-\rangle + |K^-K^+\rangle\right)\\
|0,0\rangle^{+} &= -\tfrac{1}{2}\left(|K_1^oK_1^o\rangle + |K_2^oK_2^o\rangle + |K^+K^-\rangle + |K^-K^+\rangle\right)\\
|0,0\rangle^{-} &= -\tfrac{1}{2}\left(i|K_2^oK_1^o\rangle - i|K_1^oK_2^o\rangle + |K^+K^-\rangle - |K^-K^+\rangle\right)
\end{aligned} \tag{2.101}$$

Thus we see that the $K_1^oK_1^o$ final state can only arise from C=+1 $K^o\overline{K}^o$ states. Also, because the $K_1^oK_1^o$ state consists of two identical bosons and the K and $\overline{K}$ both have intrinsic parity -1, any resonance coupling to $K_1^oK_1^o$ must have $J^P = 0^+, 2^+, 4^+ \ldots$ and G parity $(-1)^I$.

Therefore some resonances (e.g. $\rho_N(1650)$) which can couple to the $K^o\overline{K}^o$ channel will not be seen in the data. At high energies we do not expect the resonances to be important and, providing we can distinguish which K_1^o came from the K^o and which came from the $\overline{K}^o$, we can use the $\pi^- p \to K_1^oK_1^o n$ data to test our model.

In order to distinguish the K^o and $\overline{K}^o$ we note that the c.m. longitudinal momenta in the direction of the beam are given by(23)

$$
\begin{aligned}
p_K^\ell &\simeq \frac{1}{2\sqrt{s_{\pi p}}}\left(s_{\pi p} - s_{\bar{K}n} + 2t_{\pi K}\right) \\
p_{\bar{K}}^\ell &\simeq \frac{1}{2\sqrt{s_{\pi p}}}\left(s_{\bar{K}n} - s_{K\bar{K}} + 2t_{pn} - 2t_{\pi K}\right) \qquad (2.102) \\
p_n^\ell &\simeq -\frac{1}{2\sqrt{s_{\pi p}}}\left(s_{\pi p} - s_{K\bar{K}} + 2t_{pn}\right)
\end{aligned}
$$

$$\text{for} \quad s_{\pi p}, s_{\bar{K}n}, s_{Kn}, s_{K\bar{K}} \gg m_i^2$$

As explained earlier we expect Fig.15a to give the dominant contribution to the amplitude. In the Regge limit for this diagram we have

$$s_{\pi p}, s_{K\bar{K}}, s_{\bar{K}n} \gg |t_{\pi K}|, |t_{pn}|$$

$$\eta = \frac{s_{\bar{K}n}\, s_{K\bar{K}}}{s_{\pi p}} \sim t_{\pi K}, t_{pn}, m_i^2 \qquad (2.103)$$

Eqns.2.102 and 2.103 imply

$$\rho_K^\ell > \rho_{\bar{K}}^\ell > \rho_n^\ell \tag{2.104}$$

All the experimental events satisfy

$$\rho_{K_1^0}^\ell > \rho_{K_1^0}^\ell > \rho_n^\ell \tag{2.105}$$

This confirms our assumption that the diagram of Fig.15a gives the dominant contribution to the amplitude. The Regge limit of the term corresponding to Fig.15b would imply

$$\rho_K^\ell < \rho_{\bar{K}}^\ell < \rho_n^\ell \tag{2.106}$$

No experimental events were found with

$$\rho_{K_1^0}^\ell < \rho_{K_1^0}^\ell < \rho_n^\ell \tag{2.107}$$

To within the accuracy permitted by eqn.2.104 and eqn.2.105 we can use these equations to distinguish the K^0 and $\bar{K}^0$. Unfortunately due to the flatness of the $t_{\pi K}$ distribution the estimated fraction of wrongly ordered mesons is about 20%.[24]

2.12 Discussion of the Results.

The predictions of our Veneziano amplitude are shown in Figs.16-18. We have made a comparison with two sets of experimental events:

1. Those with s_{Kn}, $s_{\overline{K}n}$, $s_{K\overline{K}} > 3.5$ GeV^2,
2. Those with s_{Kn}, $s_{\overline{K}n}$, $s_{K\overline{K}} > 5.0$ Gev^2.

In both cases reasonable agreement with the data for the mass squared distributions is obtained. However, the agreement with the momentum transfer data is not good. In particular the model fails to reproduce the shape of the $t_{\pi K}$ distribution - the Veneziano model does not provide a solution to the difficulty found by Chan et al. in fitting this distribution (except by assuming the K^* trajectory slope is approximately zero). The t_{pn} distribution is not sufficiently peaked in the forward direction. This is probably caused by our neglect of pion exchange in this channel which may still have a significant effect at the energy we are considering.

We also show the Treiman Yang angle distribution. Our model gives satisfactory agreement with the data. We note that there is a peak near to $\phi = 180^\circ$. This is more a kinematic consequence of the model producing forward peaks in the momentum transfer distributions rather than the explicit dependence of the amplitude on η (ϕ).

Figure Captions

Figure 16 Mass-squared and Treiman-Yang angle (ϕ) distributions for the 224 events for which $s_{K\overline{K}}$, $s_{\overline{K}n}, s_{Kn} > 3.5$ GeV^2 at 12.0 GeV/c.

Experimental data: ——————
Theoretical prediction: ----------

Figure 17 Momentum transfer distributions for the 224 events for which $s_{\overline{K}K}, s_{\overline{K}n}, s_{Kn} > 3.5$ GeV^2 at 12.0 GeV/c.

Figure 18 $s_{\overline{K}K}$ and $-t_{K\pi}$ distributions at 12.0 GeV/c for the 73 events for which $s_{\overline{K}K}, s_{\overline{K}n}, s_{Kn} > 5.0$ GeV^2.

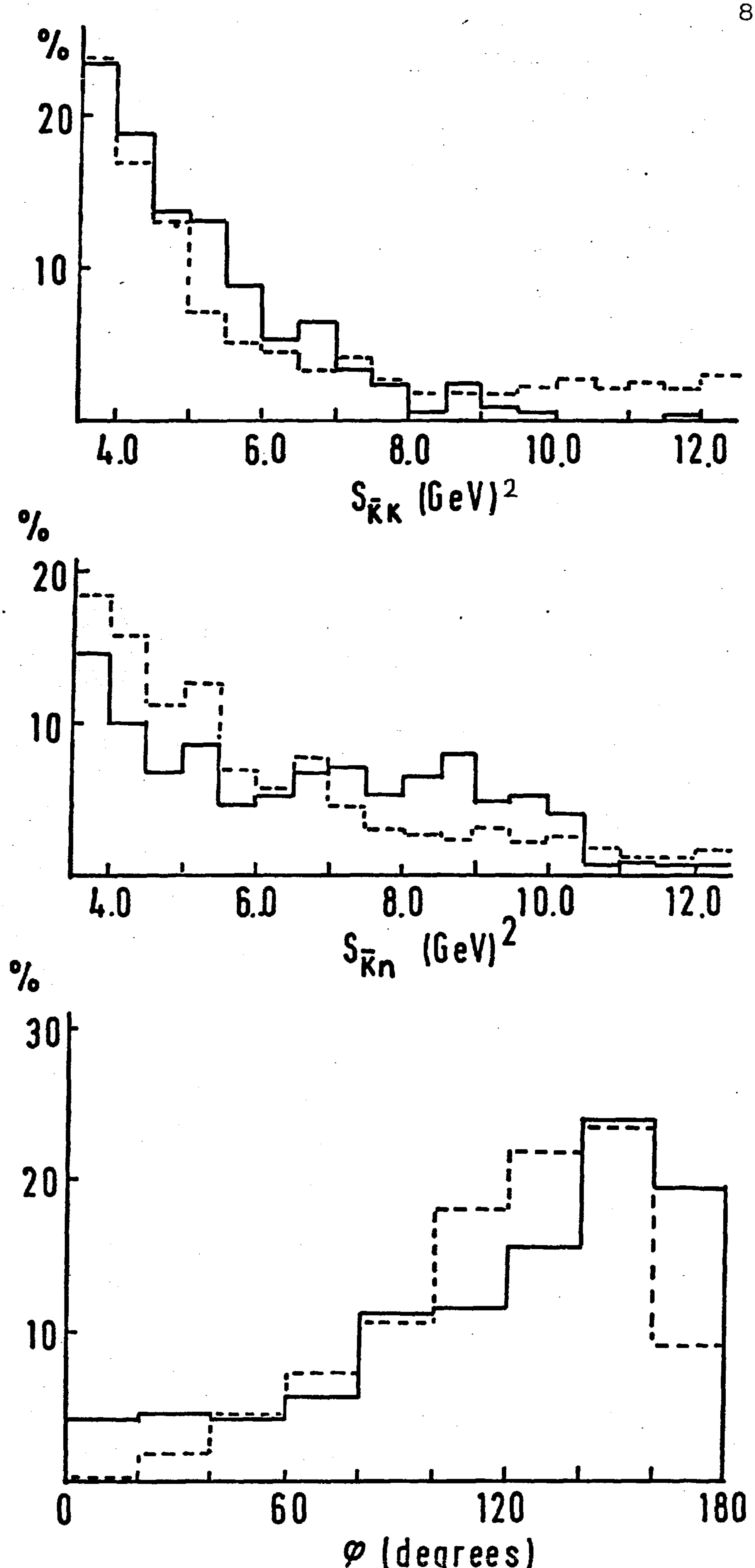


Fig.16 224 events with $S_{ij} > 3.5$ (GeV)^2

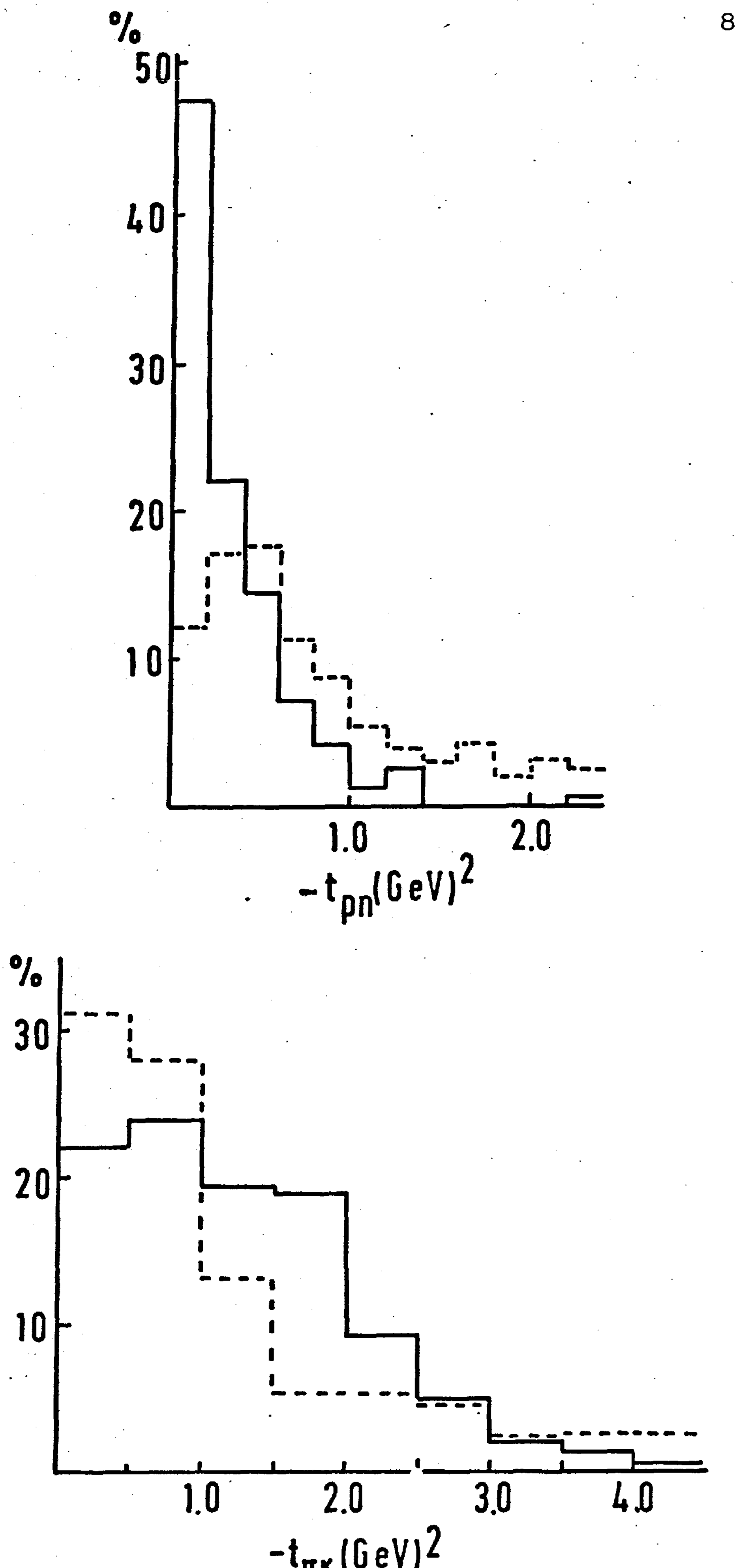


Fig.17 224 events with $S_{ij} > 3.5\ (GeV)^2$

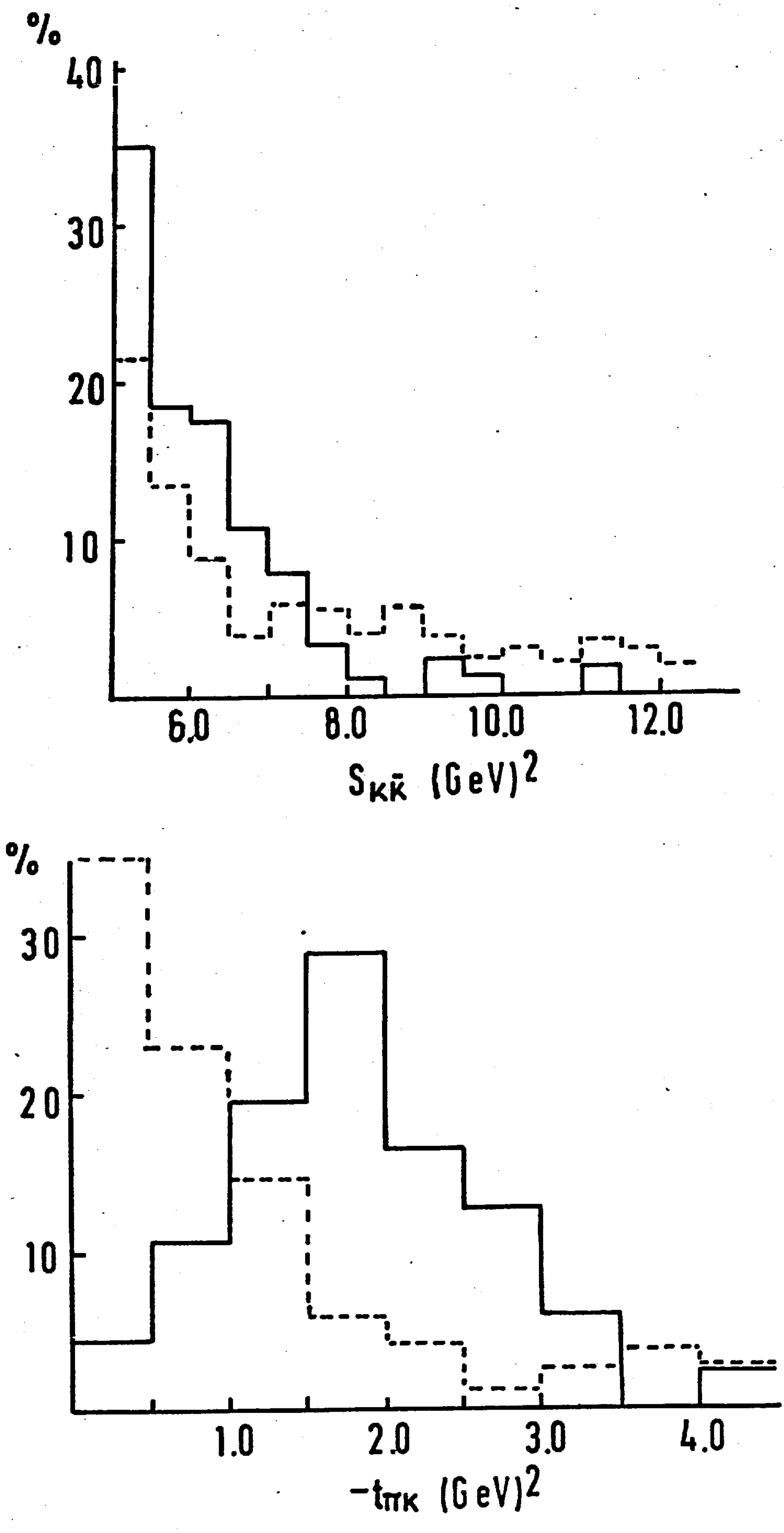


Fig.18 73 events with $S_{ij} > 5.0$ (GeV)^2

III. CONCLUSIONS.

In this chapter we have presented a Veneziano model for three particle production in which an attempt has been made to take account of the spins of the external particles. In doing so we have been able to avoid having the unphysical fermion resonances in crossed processes which were an undesirable feature of the earlier "spinless" model. The $K^- p \to K^{*-} \pi^+ n$ amplitude has correct asymptotic behaviour for all crossed processes and the $\pi^- p \to K^0 \overline{K}^0 n$ amplitude for processes which do not involve crossing the baryons. The model provides a prescription for determining the kinematic factor for any process including those involving pion exchange for which the choice made in earlier Veneziano models has been rather arbitrary.

Against these advantages of the model we must set its weaknesses. The most important of these is the fact that the prescription for determining the kinematic factor is based on consideration of what we consider to be the most important peripheral diagram for the process being considered. This means that the choice is made by refering to only one of the permutations contributing to the amplitude. If we were to evaluate every possible double peripheral graph corresponding to our five-point blob diagrams we would, in general, find that our kinematic factor was only one of many possibilities. However, since our choice is based on physical arguments concerning which term we expect to be most important for the process under consideration it is not unreasonable to assume that our amplitude should provide a reasonable description of that process. It is also quite often the case (e.g. for $K^- p \Rightarrow K^{*-} \pi^+ n$) that our prescription gives the same kinematic factor for all processes involving only crossing the mesons and therefore we would expect the amplitude to provide a reasonable description of these processes on crossing.

Although often still having correct asymptotic behaviour our Veneziano model is not, in general, self consistent on crossing the baryons. This difficulty is associated with the fact that it is not possible to include fermions in the Veneziano model in a fully satisfactory manner. Some progress towards constructing a Veneziano model which is consistent under baryon crossing has been made recently by making use of the U(6,6) symmetry scheme and duality diagrams[27].

In applying the model proposed in this chapter we find that we obtain good agreement with the main features of the data but when we look at more detailed properties of the model (such as the double-Regge limit) we find the agreement is not so impressive. This is probably due to the simplifying assumptions made in order to make the application of the model practicable. Perhaps, in the future, improved data and numerical techniques will allow some of these difficulties to be overcome.

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CHAPTER 3. A SMOOTHED VENEZIANO MODEL FOR $K^- p \to \bar{K}^{*0} n$.

3.1 The Zero Width Resonance Problem.

One of the main difficulties encountered in phenomenological applications of the Veneziano model is the problem of how to give the resonances a finite width. The simplest method of achieving this is to add an imaginary part to the trajectories thus moving the poles off the real axis and resulting in the resonances having a finite width (however, the poles still remain on the physical sheet). We have already discussed two possible parametrisations for the imaginary part in chapter 2. Both of these lead either to the introduction of ancestors or to the loss of crossing symmetry of the amplitude. However, since it is possible to construct phenomenological amplitudés in which these disadvantages are not manifestly apparent this method of giving the resonances a width has gained wide spread acceptance among phenomenologists.

A far more elegant technique for moving the poles off the real axis and onto the second sheet is to perform a convolution of the Veneziano amplitude with a suitable "smoothing" function. Models of this type have been proposed by Martin[1] and Burkhardt et al.[2]. These authors write the smoothed amplitude in the form

$$\tilde{V}(s,t) = \int_a^b \phi(x) . V(sx,tx)\, dx \tag{3.1}$$

where $\phi(x)$ is a suitably chosen smoothing function and $V(s,t)$ the original Veneziano term. This particular method of smoothing the amplitude has several disadvantages:

1. Because of the form of the integral of eqn.3.1 the Veneziano amplitude is effectively averaged over a line in the s-t plane. This implies some correlation between the independent variables s and t.
2. One finds that this smoothing technique leads to all resonances of the same mass having the same width. This is a serious disadvantage when one wishes to construct phenomenological amplitudes with different trajectories in different channels.
3. The evaluation of integrals of the type given in eqn.3.1 is generally difficult.

Instead of using an amplitude of the type given in eqn.3.1 we shall consider a smoothed amplitude consisting of terms of the form

$$\tilde{V}(s,t) = \int_a^b \phi(x).\theta(y).V(xs,yt)\,dx\,dy. \qquad (3.2)$$

Although at first sight this appears even more complicated than eqn.3.1 we shall see that it enables us to overcome the disadvantages listed above. It is immediately obvious that, since the variables s and t are smoothed independently, the first of these is removed.

3.2 The Model.

We have shown in chapter 1 that the Veneziano amplitude may be written as a sum of pole terms. For example the s-t term of the simple scalar (0^+) particle amplitude may be written as

$$V(s,t) = B(-\alpha(s),-\alpha(t)) = \sum_{m=0}^{\infty} \frac{(-1)^m \; \Gamma(-\alpha(t))}{m!\,(m-\alpha(s))\,\Gamma(-m-\alpha(t))} \tag{3.3}$$

Consider the term containing the pole at $\alpha(s)=k$. We may write this term as

$$\frac{R_k(t)}{k-\alpha(s)} \tag{3.4}$$

where $R_k(t)$ is a polynomial of order k in t. We now take

$$\phi(x) = \frac{h(x)}{(x-1)^2+\gamma_s^2} \quad , \quad \theta(y) = \frac{g(y)}{(y-1)^2+\gamma_t^2} \tag{3.5}$$

where h(x) and g(y) are analytic functions in the range of integration. The contribution of the pole term to the smoothed amplitude will be

$$\int_{1-\epsilon}^{1+\epsilon} \frac{h(x)\,g(y)\,R_k(ty)\,dx\,dy}{\left[(x-1)^2+\gamma_s^2\right]\left[(y-1)^2+\gamma_t^2\right](k-\alpha(sx))} \tag{3.6}$$

Here we have chosen the range of integration so that we are, in effect, averaging the Veneziano amplitude over a rectangle of side 2ϵ centred on the point s,t in the s-t plane. If we choose h(x) and g(y) so that

$$\int_{1-\epsilon}^{1+\epsilon} \frac{g(y)\,dy}{(y-1)^2+\gamma_t^2} = \int_{1-\epsilon}^{1+\epsilon} \frac{h(x)\,dx}{(x-1)^2+\gamma_s^2} = 1 \qquad (3.7)$$

then, providing ϵ is sufficiently small, on performing the integral over y we obtain

$$\int_{1-\epsilon}^{1+\epsilon} \frac{g(y)\cdot R_k(ty)\,dy}{(y-1)^2+\gamma_t^2} \equiv \tilde{R}_k(t) \simeq R_k(t). \qquad (3.8)$$

However, even without making any assumptions about ϵ or g(y), $R_k(t)$ will still be a polynomial of order k in t. We must now consider the integral over x. We can rewrite eqn.3.6 as

$$\tilde{R}_k(t)\cdot\int_{1-\epsilon}^{1+\epsilon} \frac{h(x)}{((x-1)^2+\gamma_s^2)}\cdot\frac{1}{(k-\alpha_0-\alpha' s x)}\,dx \qquad (3.9)$$

where $\alpha(s)=\alpha_0+\alpha' s$

$$= \tilde{R}_k(t)\left[\frac{1}{2i\gamma_s(k-\alpha_0-\alpha' s(1+i\gamma_s))}\right.$$

$$\times\int_{1-\epsilon}^{1+\epsilon}\frac{h(x)\,dx}{x-1-i\gamma_s} \qquad (3.10)$$

$$- \frac{1}{2i\gamma_s(k-\alpha_0-\alpha' s(1-i\gamma_s))} \int_{1-\epsilon}^{1+\epsilon} \frac{h(x)}{x-1+i\gamma_s} \, dx$$

$$+ \frac{(\alpha' s)^2}{(k-\alpha_0-\alpha' s)^2+(\alpha' s\gamma_s)^2} \int_{1-\epsilon}^{1+\epsilon} \frac{h(x)\, dx}{(k-\alpha_0-\alpha' s x)} \Bigg] .$$

Hence the x integration is reduced to the evaluation of integrals of the form

$$\int_{1-\epsilon}^{1+\epsilon} \frac{h(x)}{x-1+c} \, dx \tag{3.11}$$

where c is a constant.

When c lies on the line of integration we interpret the integral as its principal value.

The first two terms of eqn.3.10 give rise to poles at

$$k = \alpha_0 + \alpha' s \pm i\gamma_s \alpha' s . \tag{3.12}$$

Thus the poles are moved off the real axis in the same way as would result from adding a linear imaginary part to the trajectory. However, in our model we do not destroy the crossing symmetry of the amplitude nor are ancestors introduced since $R_k(t)$ is a polynomial of order k in t. We shall also see later that the poles now only occur on unphysical sheets.

The third term in eqn.3.10 gives rise to a cut from

$s=\frac{k-\alpha_0}{\alpha'(1+\epsilon)}$ to $s=\frac{k-\alpha_0}{\alpha'(1-\epsilon)}$ in the s plane. This cut does not correspond to any physical cut and is an unfortunate feature of this type of model. Burkhardt et al. have proposed(2) a method of smoothing in which the resulting cut has some of the properties of the threshold cut required by unitarity. However, their method is only applicable to four-point amplitudes and suffers from the same disadvantages as were mentioned earlier.

In choosing h(x) and g(y) we must ensure that h(x)=0 and g(y)=0 at the ends of the range of integration in order that the integral shall not diverge. The simplest choice for h(x) and g(y) satisfying this is

$$\begin{aligned} h(x) &= A\left[(x-1)^2-\epsilon^2\right]^{m/2} \\ g(y) &= A'\left[(y-1)^2-\epsilon^2\right]^{m/2} \end{aligned} \tag{3.13}$$

where A and A' are chosen so that eqn.3.7 is satisfied. In general, one finds that for m= odd integer the cut term has a square root branch point(3) and for m= even integer the cut is logarithmic. As m increases the relative magnitude of the cut contribution to the amplitude decreases. The first two terms in eqn.3.10 are found to be independent of h(x) but the third is not. By choosing m sufficiently large in eqn.3.13 we can make the cut contribution negligibly small. For m=1 (as used by Burkhardt et al.) numerical tests suggest that the cut contribution is of about the same order of magnitude as that from the pole terms. For m=2 the cut contribution is a few orders of magnitude smaller than the pole terms. Although we could make the cut contribution even smaller by taking a larger value for m, we shall use m=2 as this allows the integral of eqn.3.11 to be evaluated easily.

Therefore eqn.3.11 becomes

$$I = \int_{1-\epsilon}^{1+\epsilon} A\,\frac{[(x-1)^2-\epsilon^2]}{x-1+c}\,dx. \qquad (3.14)$$

Let $x-1=\epsilon u$

$$I = \int_{-1}^{1} A\epsilon^3\,\frac{u^2-1}{\epsilon u+c}\,du \qquad (3.15)$$

$$= \int_{-1}^{1} A\epsilon\left[\epsilon u - c + \frac{c^2-\epsilon^2}{\epsilon u+c}\right]du. \qquad (3.16)$$

Assuming $\left|\frac{c}{\epsilon}\right| > 1$ if c is real, we obtain

$$I = -2A\epsilon c + A(c^2-\epsilon^2)\left[\log(\epsilon+c) - \log(c-\epsilon)\right]. \qquad (3.17)$$

Care must be taken when evaluating the logarithm. We shall take the physical sheet as corresponding to the principal value of the logarithmic function. Thus we obtain

$$\int_{1-\epsilon}^{1+\epsilon} \frac{h(x)}{x-1\pm i\gamma_s}\,dx = A\Big[\mp 2\epsilon i\gamma_s - (\gamma_s^2+\epsilon^2) \tag{3.18}$$

$$\times\left[\operatorname{Log}(\epsilon \pm i\gamma_s) - \operatorname{Log}(\pm i\gamma_s - \epsilon)\right]\Big]$$

$$= \mp A\left\{ 2\epsilon i\gamma_s + (\gamma_s^2+\epsilon^2)\left[\operatorname{Log}(\epsilon+i\gamma_s)\right.\right.$$

$$\left.\left. - \operatorname{Log}(i\gamma_s-\epsilon)\right]\right\}. \tag{3.19}$$

Using eqn.3.7 we may determine A:

$$\int_{1-\epsilon}^{1+\epsilon} \left(\frac{h(x)}{x-1-i\gamma_s} - \frac{h(x)}{x-1+i\gamma_s} \right) \frac{dx}{2i\gamma_s} = 1 \tag{3.20}$$

$$= \frac{A}{i\gamma_s}\left\{ 2\epsilon i\gamma_s + (\gamma_s^2+\epsilon^2)\left[\operatorname{Log}(\epsilon+i\gamma_s)\right.\right.$$

$$\left.\left. - \operatorname{Log}(i\gamma_s-\epsilon)\right]\right\}.$$

Therefore

$$\int_{1-\epsilon}^{1+\epsilon} \frac{h(x)}{x-1\pm i\gamma_s}\,dx = \mp i\gamma_s \,. \tag{3.21}$$

The third term of eqn.3.10 has a singularity on the line of integration when $\frac{k-\alpha_0}{\alpha'(1+\epsilon)} < s < \frac{k-\alpha_0}{\alpha'(1-\epsilon)}$. The integral of eqn.3.15 must then be interpreted as a principal value taking the contour of integration above the real axis as we go round the pole. Thus, when $\left|\frac{c}{\epsilon}\right| < 1$ and c real, we obtain:

$$I = -2A\epsilon c + A(c^2-\epsilon^2)\left[\operatorname{Log}(\epsilon+c)\right. \tag{3.22}$$

$$\left. - \operatorname{Log}|c-\epsilon| - i\pi\right].$$

Therefore we obtain our final result for the smoothed pole term of eqn.3.4:

$$\tilde{R}_k(t)\left[\frac{1}{2(k-\alpha_0-\alpha' s(1+i\gamma_s))}+\frac{1}{2(k-\alpha_0-\alpha' s(1-i\gamma_s))}\right. \tag{3.23}$$

$$+\ \frac{\alpha' s\left[2\epsilon b+(\epsilon^2-b^2)\left(\mathrm{Log}(\epsilon+b)-\mathrm{Log}(b-\epsilon)\right)\right]}{\left((k-\alpha_0-\alpha' s)^2+(\alpha' s\gamma_s)^2\right)}$$

$$\times\ \frac{i\gamma_s}{2i\epsilon\gamma_s+(\gamma_s^2+\epsilon^2)\left[\mathrm{Log}(\epsilon+i\gamma_s)-\mathrm{Log}(i\gamma_s-\epsilon)\right]}$$

where $b = 1-\dfrac{k-\alpha_0}{\alpha' s}$.

In this expression the cut term has been evaluated for $s+i\delta$, $\delta\to 0_+$ and is therefore the correct contribution for the physical region. It can be shown that on the physical sheet the pole terms in eqn.3.23 are cancelled by similar poles in the third term of the expression. Thus the poles of the smoothed amplitude only occur on unphysical sheets. To see this more clearly we return to eqn.3.6 and look at the singularity structure of the integrand in the complex x plane. The integrand has fixed poles at $x=1\pm i\gamma_s$ and a moving pole corresponding to each pole of the Veneziano amplitude at

$$x_k = \frac{k-\alpha_0}{\alpha' s}\ , \quad k=0,1,2\ldots \tag{3.24}$$

The contour of integration required for evaluating the amplitude in the physical region lies just above the real axis (see curve C in Fig.19a). The poles in s will only occur when the contour is pinched against one of the fixed poles by one of the moving poles. The physical amplitude in the region of the cut is defined by approaching the cut from above on the physical sheet of the s plane (e.g. along path P_1 in Fig.19b). Now consider the moving pole at x_k as we follow the path P_2 or P_3 in the s plane. On the x plane x_k will follow the corresponding curves shown in Fig.19a. In neither case will the contour be pinched and therefore the poles will not occur on the physical sheet. In order to pinch the contour x_k must follow a path such as P_4. However, in doing so we must pass through the cut on the s plane thereby going onto another sheet. The pole occuring at

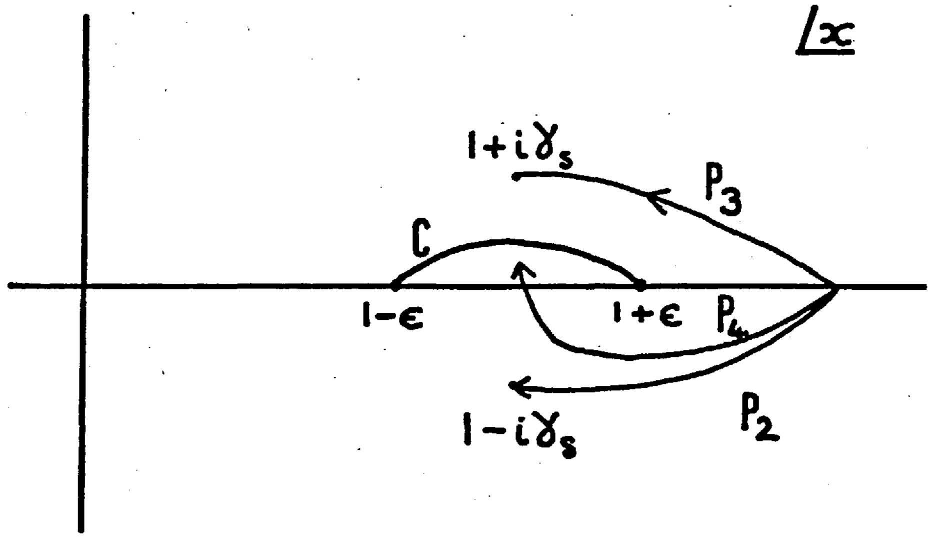
$$k = \alpha_0 + \alpha'(1 + i\gamma_s)s \tag{3.25}$$

will lie on the second sheet and be reached by going through the cut from above. It therefore lies near to the physical region and will correspond to a resonance. The pole at

$$k = \alpha_0 + \alpha'(1 - i\gamma_s)s \tag{3.26}$$

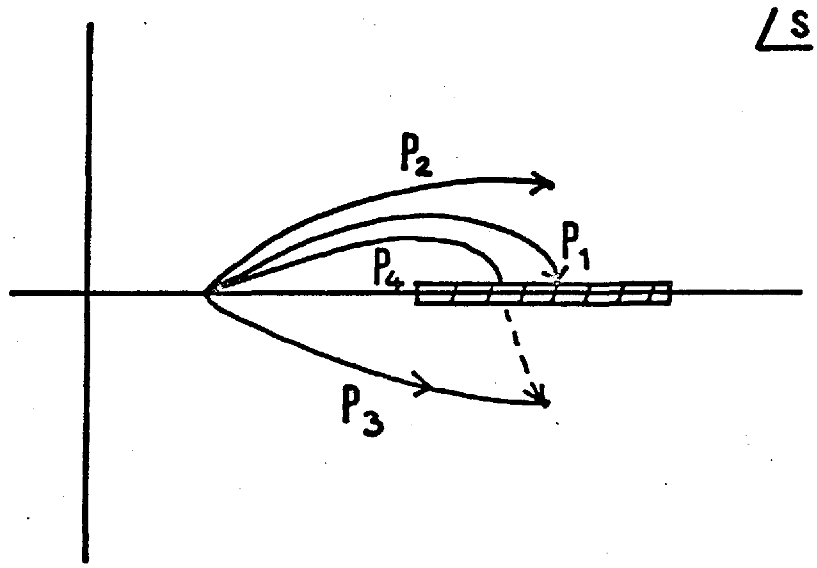
lies on a third sheet reached by going through the cut from below and is far removed from the physical region and its contribution there may be ignored.

The second sheet resonance given by eqn.3.25 will



$\lfloor x$
$1+i\gamma_s$
P_3
C
$1-\epsilon$
$1+\epsilon$
P_4
$1-i\gamma_s$
P_2

a.



$\lfloor s$
P_2
P_1
P_4
P_3

b.

Fig. 19

have a width given by

$$\Gamma = m\gamma_s \tag{3.27}$$

where m is the mass of the resonance.

Hence the width will increase linearly with mass along the trajectory. For most trajectories this is not inconsistent with the experimental data on resonance widths. We may also note that since we can choose γ differently for each trajectory we are not forced to give all resonances of the same mass the same width.

The asymptotic behaviour of the Veneziano amplitude will of course be modified by the smoothing integration. As well as Regge pole terms, the asymptotic form of the amplitude will also contain Regge cut terms of the form:

$$\tilde{V}(s,t) \underset{\substack{s\to\infty \\ t \text{ fixed}}}{\sim} p(s)\, s^{\alpha_0 + \alpha'(1-\epsilon)t} \tag{3.28}$$

where p(s) increases slower than a power of s (i.e. p(s) will contain terms in logs). Providing the range of smoothing is small (ϵ of the same order of magnitude as the experimental error in determining the trajectory slopes or smaller) then we would not expect this modification to be discernible in a phenomenological application. Because we have chosen our smoothing function $\phi(x)$ so that it vanishes at the ends of the range of integration the real axis poles of the Veneziano amplitude will become smeared out for large s. This will give rise to a smooth asymptotic behaviour even along the positive real axis (i.e. in the physical region).

3.3 Application of the Model to $K^- p \to \bar{K}^{*o} n$.

In our smoothed Veneziano model the poles acquire a similar parametrisation for their widths as is obtained by adding linear imaginary parts to the trajectories. One expects the contributions from the poles at $k = \alpha_o + \alpha' s(1 - i\gamma_s)$ which are far removed from the physical region to be negligible. However, it is not immediately obvious that the second sheet poles at $k = \alpha_o + \alpha' s(1 + i\gamma_s)$ will dominate the amplitude since, in addition to the poles, we also have a cut contribution which may be significant. Therefore we shall test our smoothing technique by applying it to a simple Veneziano model.

In chapter 2 we have already obtained a simple amplitude for $K^- p \to \bar{K}^{*o} n$ and have used it to calculate differential cross sections. In this application a linear imaginary part was added to the s-channel trajectory. Hence this provides us with an ideal choice for testing our smoothing technique.

Our Veneziano amplitude for $K^- p \to \bar{K}^{*o} n$ is given by

$$A = N \bar{u}_\Lambda \gamma_5 u_p K^\alpha . \epsilon_\alpha(\lambda) B\left(-\alpha_\pi(t), \tfrac{3}{2} - \alpha_\Sigma(s)\right) \tag{3.29}$$

where $\alpha_\Sigma(s) = -0.22 + 0.9s$

$\alpha_\pi(t) = -0.0175 + 0.9t$

and where N is a normalisation parameter.

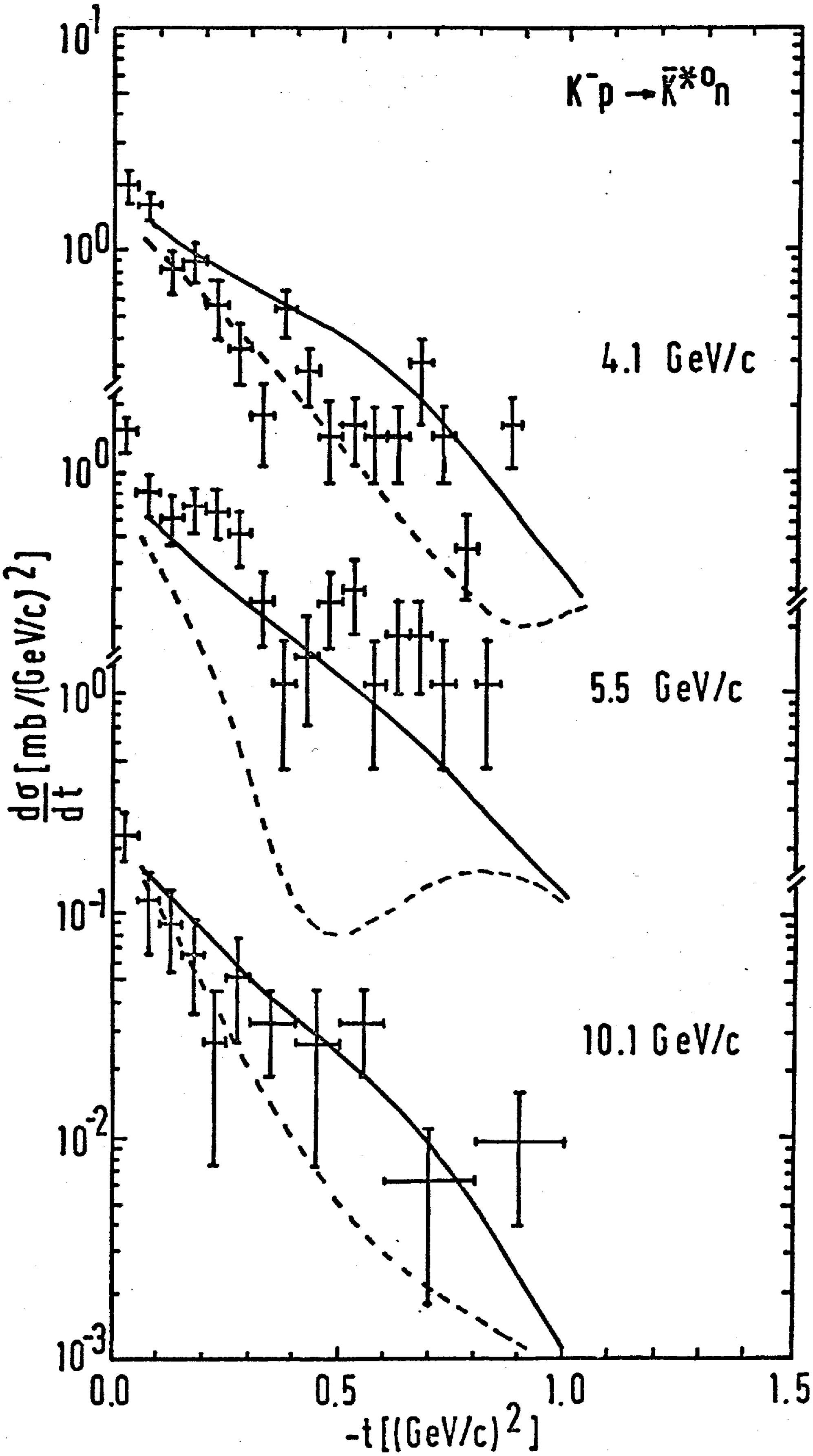
In this amplitude we replace the Veneziano term by its smoothed form obtaining:

$$A_{smooth} = N\bar{u}_n\gamma_5 u_p K^\alpha \epsilon_\alpha(\lambda)$$

$$\int_{1-\epsilon}^{1+\epsilon}\int_{1-\epsilon}^{1+\epsilon} \phi(x)\theta(y)\, B\left(\tfrac{3}{2}-\alpha_\Sigma(sx), -\alpha_\pi(ty)\right)dx\,dy. \tag{3.30}$$

We have chosen γ_s and γ_t so that we obtain the correct width for the pion and approximately the correct widths for the resonances on the $\Sigma(1385)$ trajectory. We use γ_s=.05, $\gamma_t=5.10^{-5}$. The integral of eqn.3.30 cannot be evaluated explicitly. However, the integration over x may be carried out by using eqn.3.23 and the expansion of eqn.3.3. The y integration may be carried out numerically by computer.

In Fig.20 we show the differential cross section predictions at 4.1, 5.5 and 10.1 GeV/c for ϵ=.05 (dashed line) and ϵ=0.1 (solid line). For small ϵ the results are obviously sensitive to the range of integration. For $\epsilon \lesssim .03$ the contribution from the third term in eqn.3.23 is very small and the first two terms combine to give an amplitude which is essentially a sum of the real parts of a series of Breit-Wigner resonances. As ϵ increases the imaginary part of the amplitude becomes more significant and cancellations between the contributions from the first two



$K^-p \to \bar{K}^{*o}n$
4.1 GeV/c
5.5 GeV/c
10.1 GeV/c
$\frac{d\sigma}{dt}$ [mb/(GeV/c)^2]
10^1
10^0
10^0
10^0
10^{-1}
10^{-2}
10^{-3}
0.0
0.5
1.0
1.5
-t [(GeV/c)^2]

Fig.20

and the third terms of eqn.3.23 give rise to a complicated cross section structure for $\epsilon \sim .05$. As ϵ is increased further ($\epsilon \gtrsim .08$) the cross sections become smoother changing little as ϵ is increased. This is what we would expect from viewing the smoothing procedure as averaging the Veneziano amplitude over a small area in the s-t plane. In simple terms we expect the averaged amplitude to become "smooth" when the range of s over which we integrate is of the same order of magnitude as the distance between the poles (i.e. ~ 1 GeV^2). Our data is for s lying between 8 and 20 GeV^2. Therefore we would expect our predictions to be "smooth" for $\epsilon \gtrsim .10$. This is what we find on doing the detailed calculations.

3.4 Conclusions.

In this chapter we have seen how the poles of the Veneziano model can be moved off the real axis onto the second sheet without destroying the crossing symmetry of the amplitude or introducing ancestor particles. This provides a great improvement on the "imaginary parts" method. As a result of the smoothing technique the amplitude contains unphysical cuts. However, these cuts are essential in order that the poles should be removed from the physical sheet. We find that our results are sensitive to the range of integration in the smoothing integral. However, as the range is increased this sensitivity diminishes.

We also note that there is considerable freedom in choosing our smoothing function. This may possibly be used to advantage by trying to find smoothing methods which lead to more realistic cut terms and which are less sensitive to the range over which we average the Veneziano amplitude.

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CHAPTER 4. RECURRENCE RELATIONS FOR POLE RESIDUES IN GENERAL VENEZIANO AMPLITUDES.

4.1 The Four-Point Amplitude.

In this chapter we derive relationships between the residues of neighbouring poles in a general class of Veneziano amplitudes.

We start by considering the simple Veneziano amplitude for scalar (0^+) particle scattering:

$$A(s,t) = \int_0^1 x^{-\alpha(s)-1}(1-x)^{-\alpha(t)-1}\,dx + (s,u\text{ term}) + (u,t\text{ term}), \tag{4.1}$$

where $\alpha(s) = \alpha_0 + \alpha' s$.

As it was pointed out in chapter 1 we may add satellite terms of the form

$$\frac{\Gamma(m-\alpha(s)).\Gamma(n-\alpha(t))}{\Gamma(r-\alpha(s)-\alpha(t))}, \quad m,n \leq r \leq m+n, \tag{4.2}$$

to this amplitude without affecting the asymptotic behaviour. The extra terms simply alter the residues of the poles. We shall consider the class of Veneziano amplitudes given by

$$A = \int_0^1 x^{-\alpha(s)-1}(1-x)^{-\alpha(t)-1} f(x)\,dx + \int_0^1 x^{-\alpha(s)-1}(1-x)^{-\alpha(u)-1} g(x)\,dx + \int_0^1 x^{-\alpha(t)-1}(1-x)^{-\alpha(u)-1} h(x)\,dx. \tag{4.3}$$

where f(x), g(x), and h(x) are arbitary functions analytic in the range $0 \leqslant x \leqslant 1$. By expanding f, g and h as Taylor series in x we may write the amplitude, A, as

$$A = \sum_{m=0}^{\infty} b_m B(m-\alpha(s), -\alpha(t)) + \sum_{n=0}^{\infty} c_n B(n-\alpha(s), -\alpha(u))$$
$$+ \sum_{p=0}^{\infty} d_p B(p-\alpha(t), -\alpha(u)). \tag{4.4}$$

If f, g and h are symmetric under $x \Leftrightarrow 1-x$, then A is crossing symmetric in s, t and u. The amplitude given by eqn.4.3 is the most general Veneziano amplitude which can be written as a sum of Beta functions.

Consider the first term V(s,t) of eqn.4.3. In the left hand plane of both s and t $V(s,t)$, $V(s-\frac{1}{\alpha'},t)$ and $V(s,t-\frac{1}{\alpha'})$ are analytic and we may define

$$F(s,t) = V(s,t-1/\alpha') + V(s-1/\alpha',t) - V(s,t). \tag{4.5}$$

F(s,t) will be analytic in this region and from eqn.4.3 we see that

$$F(s,t) = 0. \tag{4.6}$$

However, if eqn.4.6 is true in the left half planes of s and t, by Liouville's theorem it will be true everywhere that F(s,t) is analytic.

Now consider a pole at $\alpha(s)=k$. Near to this pole we may expand V(s,t) and $V(s-\frac{1}{\alpha'},t)$ as

$$V(s,t) = \frac{\Gamma_k(t)}{k-\alpha(s)} + \text{entire function}. \tag{4.7}$$

$$V(s-\tfrac{1}{\alpha'},t) = \frac{r_{k-1}(t)}{k-\alpha(s)} + \text{entire function.} \tag{4.8}$$

where $r_k(t)$ is the residue of the pole of $V(s,t)$ at $\alpha(s)=k$. Near to the pole we may write eqn.4.6 as

$$F(s,t) = \frac{r_{k-1}(t)+r_k(t-1/\alpha')-r_k(t)}{k-\alpha(s)} + \text{entire function} = 0. \tag{4.9}$$

Thus

$$r_{k-1}(t)+r_k(t-1/\alpha')-r_k(t)=0. \tag{4.10}$$

This relation between the residues of neighbouring poles of $V(s,t)$ in the s-channel applies for any function, $f(x)$, in the amplitude of eqn.4.3. The same result may also be obtained by considering each term of the expansion of eqn.4.4. The residue from the term with coefficient b_m, $m \leq k$ for the pole at $\alpha(s)=k$ is

$$r_{k,m}(t) = \frac{(-1)^{k-m}\, b_m\, \Gamma(-\alpha(t))}{\Gamma(k+1-m)\,\Gamma(m-k-\alpha(t))}. \tag{4.11}$$

and
$$r_{k+1,m}(t) = \frac{(-1)^{k-m+1}\, b_m\, \Gamma(-\alpha(t))}{\Gamma(k+2-m)\,.\,\Gamma(m-k-1-\alpha(t))} \tag{4.12}$$

$$r_{k+1,m}(t) - r_{k,m}(t) = \frac{(-1)^{k-m+1}\, b_m\, \Gamma(-\alpha(t))}{\Gamma(k+2-m)\,\Gamma(m-k-\alpha(t))} \times \left[(m-k-1-\alpha(t))+(k+1-m)\right] \tag{4.13}$$

$$= \frac{(-1)^{k-m+1}\,\Gamma(-\alpha(t)+1)\,b_m}{\Gamma(k+2-m)\;\Gamma(m-k-\alpha(t))} = r_{k+1,m}(t-1/\alpha').$$

Summing over m we obtain eqn.4.10. A similar relation will connect the residues of the s-channel poles arising from the (s,u) term. Relations of the type given in eqn.4.10 provide a means of checking whether it is possible to construct a Veneziano amplitude of the type given in eqn.4.3 having some specified residue structure. Using eqn.4.10 it is simple to see whether, having constructed an amplitude having the required residue structure at one pole, it will still have the required structure at the next.

4.2 The Five-Point Amplitude.

We shall consider a general five-point term of the form

$$V(s_{12},s_{23},s_{34},s_{45},s_{51}) = \int_0^1\int_0^1 \frac{du_1\,du_3}{1-u_1u_3}\, u_1^{-\alpha(s_{12})-1} u_2^{-\alpha(s_{23})-1} u_3^{-\alpha(s_{34})-1} u_4^{-\alpha(s_{45})-1} u_5^{-\alpha(s_{51})-1} f(u_1,u_3) \tag{4.14}$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{mn}\, B_5\big(m-\alpha(s_{12}),-\alpha(s_{23}),n-\alpha(s_{34}), -\alpha(s_{45}),-\alpha(s_{51})\big). \tag{4.15}$$

As described in chapter 1 there is a constraint on the variables u_j given by

$$u_j = 1-u_{j-1}\cdot u_{j+1}. \tag{4.16}$$

Thus V will satisfy equations of the form

$$V\left(s_{12}, s_{23}-1/\alpha', s_{34}, s_{45}, s_{51}\right)+V\left(s_{12}-1/\alpha', s_{23}, s_{34}-1/\alpha', s_{45}, s_{51}\right)$$

$$-V\left(s_{12}, s_{23}, s_{34}, s_{45}, s_{51}\right) = 0 \tag{4.17}$$

with corresponding residue relations of the form

$$\Gamma_{k-1}\left(s_{12}, s_{34}, s_{45}, s_{51}\right) = \Gamma_k\left(s_{12}, s_{34}, s_{45}, s_{51}\right)$$

$$-\Gamma_k\left(s_{12}-1/\alpha', s_{34}-1/\alpha', s_{45}, s_{51}\right) \tag{4.18}$$

for poles in s_{23}.

For the N-point case V will be a function of the $\frac{N}{2}.(N-3)$ planar Mandelstam channels s_i corresponding to the permutation being considered. We should note that, for $N>5$, only $3N-10$ of the s_i are independent. Thus, in general, V will not be on mass shell. In using N-point functions the s_i may be regarded as independent in doing calculations and the constraints between the s_i only applied to the final results.

The constraints on the integration variables u_i of the N-point function are

$$u_i = 1 - \prod_k u_k \tag{4.19}$$

where k runs over all channels dual to i. Thus we obtain

$$V\left(s_i \ldots, s_j - 1/\alpha', \ldots\right) + V\left(s_i - \epsilon_i \frac{1}{\alpha'}\right) - V\left(s_i\right) = 0 \tag{4.20}$$

for poles in s_j..

where $\epsilon_i = 1$ if s_i is dual to s_j
$= 0$ otherwise.

This leads to residue relations

$$\Gamma_{k-1}(s_1,\ldots,s_{j-1},s_{j+1},\ldots) + \Gamma_k(s_i - \epsilon_i \tfrac{1}{\alpha'})$$
$$-\Gamma_k(s_1,\ldots,s_{j-1},s_{j+1},\ldots) = 0. \qquad (4.21)$$

Since for $N > 5$ not all terms in eqn.4.21 can be simultaneously put on mass shell the usefulness of these relations will be reduced.

CHAPTER 5. A GENERAL CLASS OF DUAL AMPLITUDES.

5.1 Introduction.

As we have mentioned earlier the asymptotic behaviour and pole positions in the simple Veneziano amplitude for scalar (0^+) particle scattering are unaffected by the addition of sattelite terms. The residues of the poles are, however, altered. The most general crossing symmetric Veneziano amplitude for such processes may be written as

$$\sum_{m=0}^{\infty} \sum_{r=m}^{2m} c_{mr} \frac{\Gamma(m-\alpha(s)) . \Gamma(m-\alpha(t))}{\Gamma(r-\alpha(s)-\alpha(t))} \tag{5.1}$$

$$+ (s \Leftrightarrow u) + (u \Leftrightarrow t).$$

If we consider the pole at $\alpha(s)=k$ then the residue will be a polynomial of order k in $\alpha(t)$. There will, in general, be k+1 terms with m=k which do not have poles for $\alpha(s)<k$. The choice of the coefficients c_{kr} for these terms will not affect the residues of the lower poles. These terms will have residues of order k,k-1,..0 in $\alpha(t)$. By making suitable choices for c_{kr} we can give the coefficients of the polynomial residue of the pole at $\alpha(s)=k$ any values. Thus we see that it is possible to construct Veneziano amplitudes of arbitrary polynomial residue structure in terms of an infinite series of Veneziano terms. The construction of such series and their convergence has been investigated in great detail by Khuri(1). Although

the existence of such series may be established the actual construction of an amplitude with some required residue structure will usually be a formidable task.

In this chapter we shall investigate the explicit construction of dual amplitudes having any desired residue structure. In order to do this we shall not adopt the type of procedure discussed above but instead construct an amplitude by modifying Euler's Beta function integral.

A feature of the Veneziano model is the presence of infinite series of daughter particles accompanying the the resonances on the parent trajectory. Theoretically daughters must be present in unequal mass scattering in order to preserve the analyticity of the amplitude. However, for $2 \to 2$ equal mass scattering they are theoretically unnecessary. As a specific application of our technique we shall construct an amplitude for such processes in which there are no daughter particles. Although the technique may be straightforwardly extended to construct amplitudes for unequal mass scattering without daughters such amplitudes would have unphysical singularities. If the residues of the poles in the unequal mass case are to be entire functions of the Mandelstam variables then daughters must be present.

5.2 <u>General Four-Point Amplitudes</u>.

For $2 \to 2$ scattering of scalar (0^+) particles we may write the fully crossing symmetric amplitude as a sum of terms each of which corresponds to a different ordering of the external particles. We may write each of these terms as integrals of the form

$$F(\mu,\lambda) = \int_0^1 x^{\mu-1}(1-x)^{\lambda-1} f(x,\mu,\lambda)\, dx \tag{5.2}$$

where f is any function having continuous derivatives to all orders in x in the range of integration. We have written the amplitude in terms of the variables μ and λ, where these are linear trajectories (e.g. $-\alpha(s)$ and $-\alpha(t)$), in order to simplify the notation later on. The amplitude of eqn.5.2 is more general than that considered in chapter 4.

If f can be expanded as a Taylor series in x of the type

$$f(x,\mu,\lambda) = a_0(\mu,\lambda) + a_1(\mu,\lambda).x(1-x)\ldots \tag{5.3}$$

where, assuming the amplitude is crossing symmetric in μ and λ, the coefficients a_r may be written as

$$a_r = \sum_{j=0}^{r} b_j\,(\mu+\lambda)^j \tag{5.4}$$

then we may write $F(\mu,\lambda)$ as a series of the type given in eqn.5.1.

In general the amplitude of eqn.5.2 will have poles for μ and λ =0,1,2,... The pole at μ=-k will have a residue given by

$$\frac{1}{k!}\,\partial^{k}_{x=0}(1-x)^{\lambda-1}f(x,-k,\lambda). \tag{5.5}$$

Our aim is to construct an amplitude with any required residue structure. Therefore we shall require that the residue given in eqn.5.5 should be $h_1(k, \lambda)$ where h_1 is any analytic function of λ. This may be achieved if

$$\partial^{k}_{x=0}\,f(x,-k,\lambda) = h_1(k,\lambda).k!$$

and

$$\partial^{m}_{x=0}\,f(x,-k,\lambda) = 0, \quad m=0,1,..k-1. \tag{5.6}$$

These conditions may be satisfied by writing

$$f(x,\mu,\lambda) = \left[1+\phi(1-x).(g_1(\mu,x)-1)\right] \times\left[1+\phi(x).(g_2(\lambda,1-x)-1)\right] \tag{5.7}$$

where

$$\phi(1)=1 \quad , \; \phi(0)=0$$
$$\partial^{r}_{x=0,1}\,\phi(x)=0 \quad , \; r=1,2\ldots \tag{5.8}$$

and

$$\partial^{r}_{x=0}\;g_1(-k,x)=0 \quad , r=0,1..k-1$$
$$=k!\,h_1(k,\lambda), \; r=k \tag{5.9}$$
$$g_1(0,0)=h_1(0).$$

Eqn.5.7 has been constructed so that the residues of the poles in μ and λ are equal to h_1 and h_2 respectively. The functions defined by eqn.5.8 are called van der Corput neutralisers[2]. These functions have already proved useful in the removal of parity doublets in the supermultiplet Veneziano model[3]. An example of these functions is given by

$$\phi(x) = \frac{\theta(x)}{\theta(1)}$$

where
$$\theta(x) = \int_0^x (-\ln v)^{\ln v} (-\ln(1-v))^{\ln(1-v)} \, dv. \tag{5.10}$$

We shall discuss choices for the functions $g_i(u,x)$ in the next section. For the present we shall assume that functions satisfying eqn.5.9 exist.

With $f(x, \mu, \lambda)$ defined as in eqns.5.7-5.9 we obtain an amplitude with any required residue structure. As a particular example we shall construct an amplitude for equal mass scalar particle scattering in which there are no daughter particles. In order to do this the residue at the pole at $\mu\ (=-\alpha(s)) = -k$ must be proportional to $P_k(\cos\theta_s)$ where P_k is a Legendre polynomial and θ_s is the s-channel c.m. scattering angle. Similarly the residues of the poles at $\lambda\ (=-\alpha(t)) = -j$ must be proportional to $P_j(\cos\theta_t)$. As we are considering equal mass scattering we can write the scattering angles as

$$\cos\theta_s = 1 + \frac{2t}{s-4m^2} \tag{5.11}$$

where m is the mass of the external particles.

If
$$\begin{aligned} \mu &= -\alpha(s) = -\alpha_0 - \alpha' s \\ \lambda &= -\alpha(t) = -\alpha_0 - \alpha' t \end{aligned} \tag{5.12}$$

then
$$\cos\theta_s = 1 + \frac{2(\alpha_0+\lambda)}{\alpha_0+\mu+4m^2\alpha'}$$
$$= 1 + \frac{2(\alpha_0+\lambda)}{\alpha_0-k+4m^2\alpha'} \equiv \cos\theta_s^{(k)} \tag{5.13}$$

at the pole $\mu=-k$.

Similarly

$$\cos\theta_t = 1 + \frac{2(\alpha_0+\mu)}{\alpha_0-j+4m^2\alpha'} \equiv \cos\theta_t^{(j)} \tag{5.14}$$

at the pole $\lambda=-j$

If we choose

$$h_1(k,\lambda) = c_k P_k(\cos\theta_s^{(k)}),$$
$$h_2(j,\lambda) = c_j P_j(\cos\theta_t^{(j)}). \tag{5.15}$$

where c_k gives the coupling strength of the spin k particle, then the residue at the pole $\mu=-k$ is $c_k P_k(\cos\theta_s)$ and at the pole $\lambda=-j$ the residue is $c_j P_j(\cos\theta_t)$.

Hence we obtain an amplitude for 2→2 equal mass scattering with no daughter particles. It should be emphasised that this does not mean that there are no daughter trajectories. These are in fact still present but are empty. It should also be noted that there will also be poles in s in the (s,u) term of the full amplitude. This term must also be constructed so that it has daughterless residues.

5.3 The Functions g_i.

The functions $g_i(u,x)$ are defined by eqn.5.9. We may write them as a Taylor series in x:

$$g_1(\mu,x) = \sum_{n=0}^{\infty} x^n q_n(\mu)\, h_1(n,\lambda) \tag{5.16}$$

where $q_n(\mu)$ satisfies

$$q_n(-r) = 0 \qquad r = n+1,\ldots$$
$$q_n(-n) = 1. \tag{5.17}$$

Therefore the functions $q_n(\mu)$ must have zeros at all integers less than $-n$. They must also be entire functions of μ as they must not introduce additional singularities in μ. The simplest choice satisfying these conditions is

$$q_n(\mu) = \frac{1}{\Gamma(n+1+\mu)}. \tag{5.18}$$

Choosing $q_n(\mu)$ as above will ensure that we get the required residue structure. However, asymptotically this expression will increase faster than any finite power of μ as $|\mu|\to\infty$, $\pi > |\arg\mu| > \pi/2$. This will prevent us from obtaining Regge behaviour in this region. Therefore we may redefine $q_n(\mu)$ as

$$q_n(\mu) = \frac{p_n(\mu)}{\Gamma(n+1+\mu)} \tag{5.19}$$

$$p_n(-n) = 1 \tag{5.20}$$

where $p_n(\mu)$ is such that $q_n(\mu)$ is asymptotically constant or decreasing with $|\mu|$.

Noting that[4]

$$\Gamma(z+a) \simeq z^{z+a-1/2}\, e^{-z}\sqrt{2\pi} \qquad \text{as } |z|\to\infty \quad |\arg z|<\pi. \tag{5.21}$$

We shall consider two choices for $p_n(\mu)$. Firstly

$$p_n(\mu) = \exp\left[-(\mu+n)\left(1-\ln(\mu-A)\right)\right]. \tag{5.22}$$

This satisfies eqn.5.20. With this choice $q_n(\mu)\sim\mu^{-1/2}$ as $|\mu|\to\infty$. It should be noted that this does not result in an extra $\mu^{-\frac{1}{2}}$ factor in the asymptotic behaviour of our amplitude which would spoil the Regge behaviour. This will become obvious when we consider the asymptotic behaviour of the amplitude in the next section. We have written eqn.5.22 in exponential form so that, together with the condition $|\arg\mu|\leq\pi$, q_n is single valued. We note that, as defined by eqns.5.19 and 5.22, $q_n(\mu)$ has a branch point at $\mu=A$. Below this point $g(\mu,x)$ is real whereas above $\mu=A$ $g(\mu,x)$ becomes complex. As we expect amplitudes to be real below threshold and complex above we shall choose A to be the threshold point in the μ channel.

One of the assumptions leading to the concept of duality is that, at low energies, it is the imaginary part of the amplitude which is dominated by resonances. With this in mind it is interesting to consider a second choice for $p_n(\mu)$ and take

$$q_n(\mu) = \frac{\exp\left(-(\mu+n)(1-\ln(\mu-A)) + \frac{1}{2}(\ln(\mu-A) - \ln|n+A|)\right)}{\Gamma(n+1+\mu)} \tag{5.23}$$

With this choice $q_n(\mu)$ is asymptotically constant for all $|\arg \mu| < \pi$. Above threshold the resonances occur in the imaginary part of the amplitude. This appears to be more in keeping with the concept of duality than the original Veneziano model. However, in a unitarised model the poles would occur on the second sheet and until one knows how to construct such a model it is not possible to tell what should happen to the poles as the width of the resonances tends to zero.

At this stage we may note that there is considerable ambiguity in the definition of the $g_i(\mu, x)$. Just as the addition of satellites does not affect the asymptotic behaviour in the Veneziano model, so we find in cur model there is an arbitrariness in the choice of the functions g_i if we only insist on the amplitude having a certain residue structure and Regge asymptotic behaviour.

Finally, in order to show that the functions g_i do exist, we must show that the series of eqn.5.16 converges. By D'Alembert's ratio test the series will converge if

$$\left|\frac{u_{n+1}}{u_n}\right| = \left|\frac{x \; q_{n+1}(\mu) \; h_1(n+1, \lambda)}{q_n(\mu) \; h_1(n, \lambda)}\right| < 1 \tag{5.24}$$

$$n \to \infty$$

Taking $q_n(\mu)$ as in eqn.5.22

$$\left|\frac{u_{n+1}}{u_n}\right| = \left|\frac{x \exp(-1+\ln(\mu-A)).h_1(n+1,\lambda)}{(n+1+\mu)\; h_1(n,\lambda)}\right| \tag{5.25}$$

$$= \left|\frac{x \;(\mu-A)\; h_1(n+1,\lambda)}{e\;(n+1+\mu)\, h_1(n,\lambda)}\right|.$$

As $n \to \infty$, $\frac{u_{n+1}}{u_n}$ will be less than 1 for all finite μ and λ providing

$$\left|\frac{h_1(n+1,\lambda)}{n.h_1(n,\lambda)}\right| \xrightarrow{n\to\infty} 0. \tag{5.26}$$

5.4 Asymptotic Behaviour of the Amplitude.

To start with we shall consider the asymptotic behaviour of our amplitude as $\mu \to \infty$ (i.e. $s \to -\infty$) along the real axis. As a specific example we consider our 'daughterless' amplitude. This is given by

$$F(\mu,\lambda) = \int_0^1 x^{\mu-1}(1-x)^{\lambda-1}\Big[1+\phi(1-x).$$

$$\Big(\sum_{n=0}^{\infty} x^n q_n(\mu)\, c_n P_n(\cos\theta_\mu^{(n)})-1\Big)\Big] \tag{5.27}$$

$$\times\Big[1+\phi(x)\Big(\sum_{j=0}^{\infty}(1-x)^j q_j(\lambda) c_j P_j(\cos\theta_\lambda^{(j)})-1\Big)\Big]dx$$

Substituting $x=e^{-y}$

$$F(\mu,\lambda) = \int_0^\infty e^{-\mu y}(1-e^{-y})^{\lambda-1}\Big[1+\phi(1-e^{-y})$$
$$\times\Big(\sum_{n=0}^\infty e^{-yn}q_n(\mu)\,c_n P_n(\cos\theta_\mu^{(n)})-1\Big]$$
$$\times\Big[1+\phi(e^{-y})\Big(\sum_{j=0}^\infty(1-e^{-y})^j q_j(\lambda)\,c_j$$
$$\times P_j(\cos\theta_\lambda^{(j)})-1\Big]dy. \tag{5.28}$$

As $\mu\to\infty$ the integral is dominated by the contribution from the region near to $y=0$. Since $\phi(1-e^{-y})\to 0$ and $\phi(e^{-y})\to 1$ as $y\to 0$ faster than any finite power of y

$$F(\mu,\lambda) \overset{\mu\to\infty}{\sim} \int_0^\infty e^{-\mu y}\sum_{j=0}^\infty(1-e^{-y})^{j+\lambda-1}$$
$$\times q_j(\lambda)\,c_j P_j(\cos\theta_\lambda^{(j)})\,dy. \tag{5.29}$$

Now let $\mu y=w$

$$F(\mu,\lambda) \overset{\mu\to\infty}{\sim} \int_0^\infty \frac{e^{-w}}{\mu}\sum_{j=0}^\infty(1-e^{-w/\mu})^{j+\lambda-1}$$
$$\times q_j(\lambda)\,c_j P_j(\cos\theta_\lambda^{(j)})\,dw. \tag{5.30}$$

We may expand

$$(1-e^{-w/\mu})^{j+\lambda-1} = \left(\frac{w}{\mu}\right)^{j+\lambda-1} +$$
$$(1-j-\lambda)\sum_{n=0}^\infty \varphi_n(-j-\lambda)\left(\frac{w}{\mu}\right)^{n+j+\lambda} \tag{5.31}$$

where the Stirling polynomials, $\Psi_n(x)$, are defined by the generating function[5]:

$$\left(\frac{1-e^{-y}}{y}\right)^{-x-1} = 1+(x+1)\sum_{n=0}^{\infty}\Psi_n(x)y^{n+1}, \tag{5.32}$$

Then

$$F(\mu,\lambda) \overset{\mu\to\infty}{\sim} \mu^{-\lambda}\sum_{j=0}^{\infty}\mu^{-j}\Gamma(j+\lambda)\,q_j(\lambda) \times c_j\,P_j\left(\cos\theta_\lambda^{(j)}\right). \tag{5.33}$$

As $\mu\to\infty$

$$P_j\left(\cos\theta_\lambda^{(j)}\right) \sim \left(\frac{2\mu}{\alpha_0+4m^2\alpha'-j}\right)^j\frac{(2j)!}{(j!)^2\,2^j} \tag{5.34}$$

$$\therefore\ F(\mu,\lambda) \overset{\mu\to\infty}{\sim} \mu^{-\lambda}\sum_{j=0}^{\infty}\Gamma(j+\lambda)\,q_j(\lambda) \times\left(\frac{2}{\alpha_0+4m^2\alpha'-j}\right)^j\frac{(2j)!}{2^j(j!)^2}\,c_j \tag{5.35}$$

$$= \mu^{-\lambda}\sum_{j=0}^{\infty}\frac{(2j)!\;c_j\;e^{-(j+\lambda)(1-\ln(\lambda-A))}}{(j+\lambda)(\alpha_0+4m^2\alpha'-j)^j\;(j!)^2} \tag{5.36}$$

with q_n chosen as in eqn.5.22.

This series converges providing λ is not a negative integer and c_j does not increase with j. Therefore we obtain Regge asymptotic behaviour along the positive real μ axis.

The integral of eqn.5.28 is well defined for $|\arg \mu| < \frac{\pi}{2}$ and the same arguments as above can be used to establish Regge asymptotic behaviour everywhere in the right half plane of μ.

In order to prove that the amplitude is Regge behaved in the left half plane we must first show that the integral of eqn.5.28 can be analytically continued into this region. In order to do this we rotate the range of integration in the complex y plane.

To illustrate how this may be done consider the integral

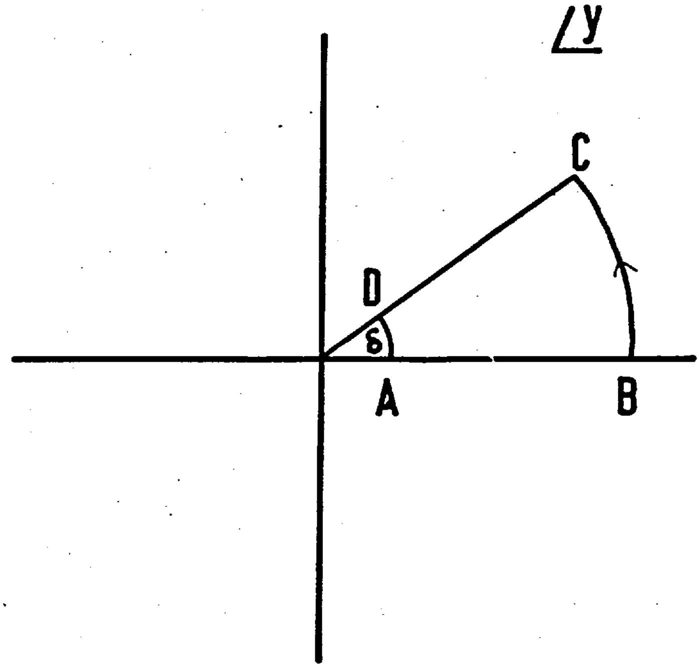
$$\int_A^B T(y)\,dy. \tag{5.37}$$

Now consider the integral of T(y) around the contour ABCD as shown in Fig.21a. Providing there are no singularities of T(y) either on or inside the contour we have

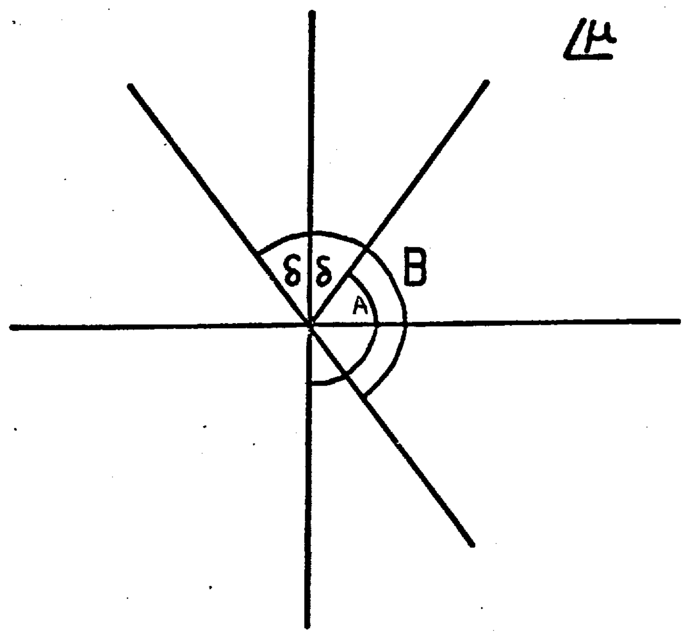
$$\int_A^B T(y)\,dy = \int_D^C T(y)\,dy - \int_B^C T(y)\,dy - \int_D^A T(y)\,dy. \tag{5.38}$$

If we now allow $A \to 0$ and $B \to \infty$ and, if the integrals over BC and DA vanish, we have

$$\int_A^B T(y)\,dy = \int_D^C T(y)\,dy \tag{5.39}$$



a.



b.

Fig. 21

Hence, providing the contributions from the arcs traced out by the ends of the range of integration are zero and the line of integration passes over no singularities of the integrand, we may rotate the line of integration through an angle δ as shown in Fig.21a.

Considering the integral of eqn.5.28 we see that, on rotation, the line of integration passes over no singularities of the integrand providing $|\arg y| < \frac{\pi}{2}$ and $|\arg \mu y| < \pi/2$ throughout the rotation. Also, because of the factor e^{-y^μ} the contribution from the arc BC vanishes as $B \to \infty$ subject to the same conditions. The contribution from AD also vanishes as $A \to 0$ since we still have $|\phi(e^{-y})| \to 0$ and $|\phi(1-e^{-y})| \to 1$ faster than any finite power of y. We note that this would have not been the case if we had chosen the more familiar van der Corput neutraliser:

$$\phi(x) = \theta(x) / \theta(1)$$

where $$\theta(x) = \int_0^x e^{-\frac{1}{u} - \frac{1}{1-u}} du. \tag{5.40}$$

Therefore we can make the rotation through the angle δ providing μ lies in the segment A shown in Fig.21b. However, the rotated integral will be well defined for all μ satisfying $-\frac{\pi}{2} + \delta < \arg \mu < \pi/2 + \delta$ i.e. in segment B. If $\delta < \pi/2$ the two segments overlap and provide an analytic continuation of $F(\mu, \lambda)$ for $-\frac{\pi}{2} + \delta < \arg \mu < \pi/2 + \delta$. We can now follow through the arguments of eqns.5.29-5.35 and prove Regge asymptotic behaviour in this region. Thus we are able to prove that our amplitude is Regge behaved for $|\arg \mu| < \pi$.

This is the same region for which Regge behaviour of the Veneziano model can be established. Finally, we note that in order to show that the full amplitude is Regge behaved we must show that the (s-u) term tends to zero exponentially as $s\to\infty$, t fixed. This may be achieved by using the method employed by Suzuki in a model[6] which uses van der Corput neutralisers to construct a Veneziano model with non-linear trajectories.

5.5 Conclusions.

We have shown that, by using van der Corput neutralisers, it is possible to construct dual amplitudes with any required residue structure.

If we consider the integral of $\mathrm{Im}F(\mu,\lambda)$ with q_n defined as in eqn.5.23 around the contour shown in Fig.22 we see that, since the contour encloses no singularities, the integral is zero. The contribution from the circular arc as $R\to\infty$ will be a sum of Regge terms. As $\epsilon\to 0$ we expect the contribution from the straight portion of the contour to be dominated by the poles on the real axis. However, our amplitude will also have a background term. In order to evaluate this background term as $R\to\infty$ with $\epsilon=0$ we would require the asymptotic behaviour of the amplitude along the negative real μ axis. If our amplitude is to be consistent with "Resonance $\equiv$ Regge" duality and with the experimental evidence supporting the concept of resonance dominance

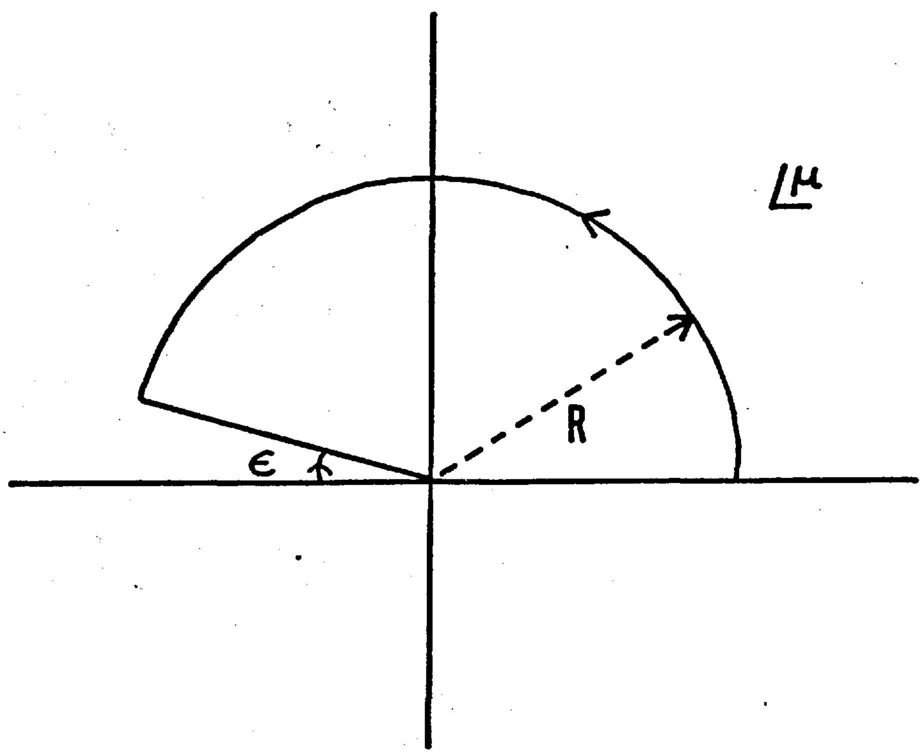


Fig. 22

the background must be small. It is possible that this requirement could remove some of the arbitrariness in the choice of the g_i. The cuts starting at threshold in each channel in our amplitude do not correspond to the threshold cuts of unitarity. The construction of amplitudes with more realistic cuts provides scope for further investigations of this type of model.

Finally, it is interesting to speculate that, since it is possible[7] to write propagator numerators (this is essentially what the h_i are) for continuous spin J exchanges coupling to any external particles, it may be possible to construct an amplitude for processes involving fermions.

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APPENDIX.

Asymptotic behaviour of the Veneziano model.

In this appendix we shall prove that the Veneziano model has Regge asymptotic behaviour. Firstly we consider a simple four-point amplitude of the form:

$$A = \frac{\Gamma(-\alpha(s)).\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} + (s \Leftrightarrow u) + (t \Leftrightarrow u). \tag{A.1}$$

In order to determine the asymptotic behaviour of this amplitude we use Stirling's asymptotic form for the Gamma function(1):

$$\Gamma(z+a) = z^{z+a-1/2} e^{-z} \sqrt{2\pi} \left(1 + O\left(\tfrac{1}{z}\right)\right) \tag{A.2}$$
$$\text{for } |z| \to \infty, \ |\arg z| < \pi, \ |\arg(z+a)| < \pi.$$

Therefore as $|s| \to \infty$, $\arg s > 0$, t fixed we obtain:

$$A \simeq \Gamma(-\alpha(t))\left[(-\alpha(s))^{\alpha(t)} + (-\alpha(u))^{\alpha(t)}\right] \tag{A.3}$$

The second term in eqn.A.1 may be shown to vanish exponentially as $|s| \to \infty$, t fixed providing $\alpha(s)$ and $\alpha(t)$ are linear trajectories of the same slope. The asymptotic form of eqn.A.3 only corresponds to Regge behaviour if the trajectories are linear. If both trajectories have the same slope we have

$$A \simeq \Gamma(-\alpha(t))\,(\alpha' s)^{\alpha(t)}\left[e^{-i\pi\alpha(t)} + 1\right] \tag{A.4}$$

Here we see that the (s,t) and (u,t) terms combine to give the trajectory a definite signature. If the s-channel was exotic then the (s,t) and (s,u) terms would be absent. As a result we would no longer have a signature factor and the t-channel trajectory would be degenerate.

We note that, because of the infinite series of poles on the real axis, it is not possible to show that the Veneziano amplitude is Regge behaved along the positive real axis.

Because of the well known asymptotic properties of the Gamma function it is relatively easy to prove the Regge behaviour of the four-point amplitude. We shall now consider the five-point amplitude and show briefly how it has Regge behaviour in the double-Regge limit. To do this we shall follow the method of Bardakci and Ruegg(2). As in the four-point case it can be shown(3) that by adding all twelve terms of the crossing symmetric amplitude each trajectory is given a definite signature. However, we shall only consider a single term of the five-point amplitude. From eqn.1.19 we may write

$$B(-\alpha_{12}, -\alpha_{23}, -\alpha_{34}, -\alpha_{45}, -\alpha_{51})$$

$$= \int_0^1\int_0^1 du_1\, du_3\; u_1^{-\alpha_{12}-1}(1-u_1u_3)^{\alpha_{45}+\alpha_{51}-\alpha_{23}} \qquad (A.5)$$

$$\times\; u_3^{-\alpha_{34}-1}(1-u_3)^{-\alpha_{45}-1}(1-u_1)^{-\alpha_{51}-1}.$$

Substituting

$$u_1 = \exp\left(\frac{-xy}{\alpha_{34}\alpha_{45}}\right) \quad , \quad u_3 = \exp\left(\frac{y}{\alpha_{34}}\right) \tag{A.6}$$

$$B = -\int_0^\infty\int_0^\infty dx\, dy \; \frac{y}{\alpha_{34}^2\alpha_{45}} \; e^{-y+\frac{xy}{\eta}}$$

$$\times \left(1 - e^{\frac{-xy}{\alpha_{34}\alpha_{45}} + \frac{y}{\alpha_{34}}}\right)^{\alpha_{45}+\alpha_{51}-\alpha_{23}} \left(1 - e^{\frac{y}{\alpha_{34}}}\right)^{-\alpha_{45}-1}$$

$$\times \left(1 - e^{-xy/\alpha_{34}\alpha_{45}}\right)^{-\alpha_{51}-1} \tag{A.7}$$

where $\eta = \dfrac{\alpha_{34}\alpha_{45}}{\alpha_{51}}$,

Now let $\quad x' = -\dfrac{x}{\alpha_{45}} \quad , \quad y' = -\dfrac{y}{\alpha_{34}}$.

$$B = (-\alpha_{45})^{\alpha_{51}}(-\alpha_{34})^{\alpha_{23}} \int_0^\infty\int_0^\infty dx\, dy\, e^{-y+\frac{xy}{\eta}}\, x^{-\alpha_{51}-1}$$

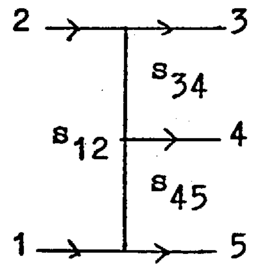
$$y^{-\alpha_{23}-1}\; K^{-\alpha_{51}-1}\, M^{-\alpha_{23}-2}\, L .$$

$$\text{where} \quad K = \left(\frac{1-e^{-x'y'}}{1-e^{-x'y'-y'}}\right)\frac{1}{x'} \tag{A.8}$$

$$M = \frac{1-e^{-x'y'-y'}}{y'} \qquad L = \left(\frac{1-e^{-y'}}{1-e^{-x'y'-y'}}\right)^{-\alpha_{45}-1}$$

We shall consider the limit $\alpha_{34}, \alpha_{45}, \alpha_{12} \to -\infty$ such that η remains finite and negative. However, our final result will be true for the limit $|\alpha_{34}|, |\alpha_{45}|, |\alpha_{12}| \to \infty$,

$|\arg \alpha_{ij}| > 0$. This limit corresponds to the double-Regge diagram:



In the integration x and y, and x' and y' lie between 0 and $+\infty$. It can be shown that there exists a constant, C, such that $|K|$, $|L|$, $|M| < C$ in the range of integration. We can also show that the following limits, uniform in x' and y' exist.

$$\lim_{x',y'\to 0} K = 1 \,, \quad \lim_{x',y'\to 0} M = 1 \,, \quad \lim_{\substack{x',y'\to 0 \\ x,y \text{ fixed} \\ \alpha_{45}\to -\infty}} L = e^{-x} \tag{A.9}$$

We can write eqn.A.8 as

$$B = (-\alpha_{45})^{\alpha_{51}} (-\alpha_{34})^{\alpha_{23}} \times$$
$$\left(\int_0^P\!\!\int_0^P + \int_P^\infty\!\!\int_0^P + \int_0^P\!\!\int_P^\infty + \int_P^\infty\!\!\int_P^\infty \right) (\text{Integrand})\, dx\, dy \tag{A.10}$$

Because L, M and N are bounded we can make the last three of these integrals arbitrarily small by choosing P sufficiently large. Keeping P fixed we can then make $|s_{34}|$ and $|s_{45}|$ sufficiently large so that

$$\left| \int_0^P\!\!\int_0^P (\text{Integrand})\, dx\, dy - \int_0^P\!\!\int_0^P e^{-x-y+\frac{xy}{\eta}}\, x^{-\alpha_{51}-1}\, y^{-\alpha_{23}-1}\, dx\, dy \right|$$
$$< \epsilon \tag{A.11}$$

where ϵ is any arbitrarily small positive constant. This is possible because of the existence of the uniform limits for K, L and M. By choosing P sufficiently large initially we can also make

$$\left|\left(\int_0^\infty\int_0^\infty - \int_0^P\int_0^P\right) dx\,dy\, e^{-x-y+xy/\eta}\, x^{-\alpha_{51}-1}\, y^{-\alpha_{23}-1}\right| < \epsilon \tag{A.12}$$

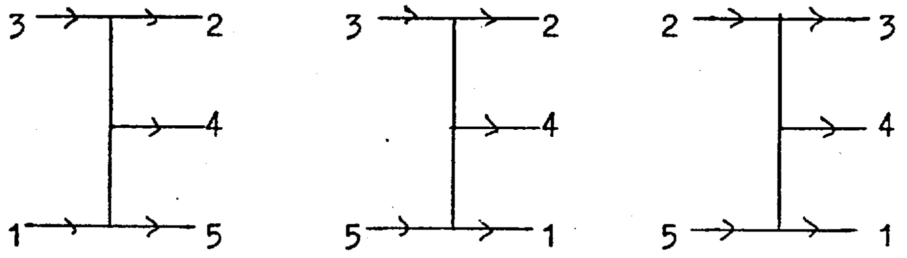
Therefore we obtain finally

$$\lim_{\substack{\alpha_{34},\alpha_{45}\to-\infty \\ \eta \text{ finite} \\ \alpha_{23},\alpha_{15} \text{ fixed}}} B(-\alpha_{12},-\alpha_{23},-\alpha_{34},-\alpha_{45},-\alpha_{51}) = (-\alpha_{45})^{\alpha_{51}}(-\alpha_{34})^{\alpha_{23}} \times \int_0^\infty\int_0^\infty e^{-x-y+\frac{xy}{\eta}}\, x^{-\alpha_{51}-1}\, y^{-\alpha_{23}-1}\, dx\,dy. \tag{A.13}$$

$$\equiv (s_{45})^{\alpha_{51}}(s_{34})^{\alpha_{23}}\, f(\eta,-\alpha_{23},-\alpha_{15})$$

providing the trajectories are linear.

In the limit considered in eqn.A.13 the terms in the full five-point amplitude giving rise to the double-Regge diagrams:



will combine with the term we have considered to give the α_{23} and α_{15} trajectories a definite signature. The other eight terms in the amplitude will vanish exponentially in this limit.

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