PHOTONIC AND HADRONIC INTERACTIONS

by

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ABSTRACT

The covariant approach to Regge poles is extended to provide a unique prescription for reggeizing processes involving photons. The Regge contributions to the invariant amplitudes for the processes : $\gamma N \longrightarrow \pi N$, πN^*, VN; $\gamma\pi \longrightarrow V\pi$; $VN \longrightarrow \pi N$; $V\pi \longrightarrow V\pi$ are calculated and tabulated.

We conclude that the pion reggeizes at $t = \mu_\pi^2$ in $\gamma N \longrightarrow \pi N$, πN^*, the rho reggeizes at $t = \mu_\rho^2$ in $\gamma N \rightarrow \rho N$ and that the Pomeron reggeizes in elastic Compton scattering in such a way that no fixed poles are necessary in strong or electromagnetic processes.

Class III pion conspiracy is considered in each process and we conclude that it is consistent with the data for $\gamma N \longrightarrow \pi N$, πN^*, VN, γN provided the formalism is properly interpreted. Other conspiracies are considered as well as evasion.

The fundamental problem of gauge invariance is handled throughout by means of a gauge projection operator and the effect of gauge invariance on kinematic singularities and zeros is critically examined.

The covariant approach is related to helicity formalism throughout and especially for $VN \longrightarrow \pi N$, $\gamma N \longrightarrow \pi N$ where gauge invariance and kinematic factors in the helicity amplitudes are carefully considered.

A covariant technique for calculating Regge contributions to differential cross sections is developed and the results tabulated. The covariant formalism itself is expounded with special emphasis on the differential technique and is applied to example processes.

PREFACE

The work in this thesis was carried out in the Department of Theoretical Physics, Imperial College, London, between October 1966 and June 1969 under the supervision of Professor P.T. Matthews, F.R.S. The author wishes to thank Professor Matthews, Dr. M.D. Scadron and Dr. H. Jones for their help and encouragement.

Except where stated in the text, the work in this thesis is original and has not been submitted in this or any other University for any other degree.

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I INTRODUCTION

1. The Problems

The recent high energy data for the photonic processes[1,2] $\gamma N \longrightarrow \pi N$[3], πN^*[4], ρN[5] along with speculation on the role of the Pomeron in nucleon Compton scattering[6] has forced searching examination of the conventional models of high energy physics and realization that the kinematic behaviour of photonic helicity amplitudes differs from that of massive ones.

All of the conventional models for inelastic scattering are essentially peripheral, assuming that the process is dominated by the pole nearest the physical region in the cross channel (t), and all stem from elementary one particle exchange (OPE). The Pomeron permits the description of diffractive scattering in the Regge model.

The first difficulty arises with pion exchange in $\gamma p \longrightarrow \pi^+ n$ where the process exhibits a sharp forward peak in the differential cross section ($d\sigma/dt$) which, because of its slope, $(d\sigma/dt)/\sigma \sim 1/\mu_\pi^2$, is strongly suggestive of pion exchange dominance. Turning to the models however, one finds that the one amplitude to which the pion contributes (f_{01}^-) behaves as $t/(t-\mu_\pi^2)$ and clearly vanishes at $t = 0$, just outside the physical region, and requires that the cross section dip rather than peak in the forward direction. A way around this is to write the amplitude as [7]

$$\frac{t}{t-\mu_\pi^2} = \frac{\mu_\pi^2}{t-\mu_\pi^2} + 1$$

and to discard the second term which, of course, is isotropic and the resulting prescription is equivalent to total S-wave absorption.

However, Boyarski et al.[3] and Fincham et al.[8] have noted that their attempts to fit the $\gamma p \longrightarrow \pi^+ n$ data with a Gottfried-Jackson[9] peripheral absorption model have not been entirely successful. Along with the OPE and absorption models the Regge model also predicts a vanishing cross section at t = 0, and a forward dip.[10,11,12,13,14]

A similar problem arises in $\gamma p \longrightarrow \pi^- N^{*++}(1236)$ where the data exhibits a sharp near forward peak which falls off rapidly as $|t| \longrightarrow t_{min} \simeq 0$. Here the pion contributes to more than one amplitude and various mechanisms have been suggested[2] to explain the marked difference between the forward t behaviour of $d\sigma/dt$ for $\gamma p \longrightarrow \pi^+ n$ and $\gamma p \longrightarrow \pi^- N^{*++}$.

The second problem is peculiar to the Regge model and also concerns the pion. A question has arisen as to whether the reggeized pion contribution to the differential cross section gives rise to a dynamical pion pole at $\alpha = 0$[15,16]; in other words, does the pion reggeize at the pole? A particle exchange is said 'not to reggeize' if its Regge amplitude

a) does not give rise to a pole in the differential cross section at the quantum numbers of the particle (a right signature point)

b) gives rise to a vanishing contribution at a wrong signature point. The question of the pion reggeizing at $t = \mu_\pi^2$ in $\gamma N \longrightarrow \pi N, \pi N^*$ is identical to that of the rho reggeizing at $t = \mu_\rho^2$ in $\gamma N \longrightarrow \rho N$. A somewhat more physical question is whether the Pomeron reggeizes at t = 0 (a wrong signature point) in nucleon Compton scattering[6]. The consequence of its failure so to do is that the forward (t = 0) cross section will fall off as energy increases rather than remain constant.

Intimately related to the previous problem is that of gauge invariance. Independent of model considerations it has been noted that some kinematic factors for t-channel helicity amplitudes change by factors of k_t, the photon momentum, for processes involving massive and massless photons.[17,18,19,20] Further, gauge invariance has been used to introduce poles into invariant amplitudes at least in $\gamma N \rightarrow \pi N$[11,21]. Such a pole, required essentially by kinematic considerations, is anomolous in a world of physics governed by dynamical poles.

By no means the least problem of photonic processes is that of fixed poles in the J-plane.[1,2,11,22,23,24] Fixed poles are forbidden in pure hadronic processes by unitarity, however, thay are not forbidden in photoproduction which is considered only to first order in the electromagnetic coupling. The question is, of course, are they necessary?

2. The Approach

In considering these problems we will confine ourselves to a Regge context and examine invariant rather than helicity amplitudes. Since the inception of Regge theory[25] ten years ago the tendency has been to reggeize helicity amplitudes because of their amenability to partial wave decomposition and the consequent ease of including spin effects. As well, helicity amplitudes contribute coherently to the differential cross section[26]. They are however subject to constraint equations at thresholds, pseudothresholds and $t = 0$, imposed by analyticity. A further problem arises in the Regge description of unequal mass[27] processes where singularities arise in the partial wave decomposition and have to be cancelled by the exchange of daughter trajectories[28] to preserve the analyticity of the amplitude.

Constraint equations involve two or more helicity amplitudes and when each amplitude individually satisfies the particular constraint (in a given model) such a solution is termed 'evasive'. The alternative solution involving collusion between the amplitudes is termed a 'conspiracy'.[29]

Conspiracy, evasion and daughters were developed mainly in nucleon-nucleon scattering after Volkov and Gribov[30] introduced a conspiring triplet of Regge trajectories to give a finite contribution to the differential cross section at t = 0. Helicity formalism was used to describe the process and Leader[31] classified the solutions to the constraint equations obeyed by the amplitudes in terms of evasion, conspiracy and daughter exchange. Toller[32] and Freedman and Wang[33] have examined these problems from a group theoretical point of view and have, for equal mass scattering, classified the possible conspiracies at t = 0. Comprehensive reviews are given by Bertocchi[29] and ref.(26).

The need for conspiracy arose historically because of the sharp forward peak apparently due to pion exchange in the differential cross section for $N\bar{N}$ scattering, pn charge exchange and $\gamma p \rightarrow \pi^+ n$. In each case the reggeized pion exchange vanished at t = 0 and, in the absence of cuts, implied a forward dip which of course was not seen.

In order to get a forward pion peak a trajectory with the same quantum numbers as the pion, but with opposite parity, the π_c is introduced with $\alpha_\pi(0) = \alpha_{\pi_c}(0)$ and the residues conspire at t = 0 to give a non vanishing forward contribution to the differential cross section.

Using this approach Phillips[7] and Arbab and Dash[34] are able to fit pn charge exchange provided they use a pion residue with a linear t-dependence. The π_c residue is assumed constant and the trajectory to be very flat - to avoid predicting a 0^+ low mass particle. They point out that the conspiring pion in this process belongs to an M = 1 Toller pole, or to a Class III[29] conspiracy.

Ball, Frazer and Jacob[35] (BFJ) take the same π, π_c conspiracy as Arbab and Dash and, using it to leading order only, fit the forward peak in $\gamma p \longrightarrow \pi^+ n$. They do not examine daughter exchange or the third member of the Class III pion conspiracy, a trajectory with the same quantum numbers as the A_1 but with $\alpha_{A_1^c}(0) = \alpha_\pi(0)-1$.

Conspiracy is not without its drawbacks as Le Bellac[36] has shown. Combining factorization and Class III pion conspiracy at the $N\bar{N}$ vertex, he shows that the differential cross section for $\pi^+ p \longrightarrow \rho^0 N^{*++}$ vanishes at t = 0, and this forward dip is <u>not</u> observed.[2,27] A way around this problem is to abandon conspiracy altogether and to consider the cut contribution to the amplitude which is then no longer factorizable. Some success[38] has been had by this approach. However as our purpose is to examine pion (and other) conspiracies in photonic processes we shall not dwell on it. Also, without leaving the fold of conspiracy, Arbab and Brower[39] have successfully avoided the Le Bellac forward dip by considering interference between an A_1 trajectory and the conspiring pion.

The desire to effect our study of conspiracy, evasion and the peculiarities of photonic processes by examining the invariant, rather than helicity amplitudes is motivated by their simple crossing and analyticity properties. There are no constraint equations and the only rule is that the invariant amplitudes be non singular except, of course, where they have dynamical poles. Once the Regge contributions to the invariant amplitudes have been tabulated, conspiracy analysis simply becomes the study of singularity cancellation in the Regge couplings and is straight forward to carry out[40].

There is the point that invariant amplitudes are neither obvious nor, in all cases, unique[41], however minimal sets of amplitudes which are both free of kinematic singularities (KSF) and of kinematic zeros (KZF) have been enumerated for the processes which we propose to study by Bardeen and Tung[42] and by Jones and Scadron[43]. The latter authors have also furnished equivalence theorems[41] which relate any additional covariants to those in the minimal set.

Recent investigations [17,18,19,20] into the asymptotic behaviour of helicity amplitudes for these processes has revealed that extra kinematical threshold factors have been necessary to get the correct KSF behaviour. Henyey[12] has shown that this extra power of photon momentum is essentially a multipole radiation effect due to the masslessness of the photon. The extra momentum power is exactly what allows an additional unit of helicity flip to be transferred to the Reggeon without violating angular momentum conservation. When going over to the cross channel however, the kinematic zero due to the momentum factor becomes absorbed in the high energy asymptotic behaviour. Consequently, a non zero "nonsense" amplitude is allowed in spite of the apparent angular momentum restrictions.

Such subtle crossing arguements of helicity formalism appear in the covariant approach as gauge invariance effects. Our analysis is based on the latter approach as we feel that gauge invariance, handled in both ref (42) and ref (43) by means of a gauge projection operator, is a more straight forward way of treating zero mass complications.

The difficulties of performing a partial wave decomposition for invariant amplitudes have been studied by several authors[44] and we shall follow the approach adopted by Scadron[45] which paves the way for reggeization[46,47].

Once the formalism is set up for photonic processes we find that the question of whether the pion reggeizes in $\gamma N \longrightarrow \pi N$, the rho in $\gamma N \longrightarrow \rho N$ or the Pomeron in $\gamma N \longrightarrow \gamma N$ is clearly answered in the affirmative. We make the usual Regge pole assumption that the Pomeron does dominate the behaviour of high energy elastic processes with $\alpha(0) = 1$, so as to imply spin independent (constant) cross sections even in photonic processes. No fixed poles will be necessary for this result.

II FORMALISM

1. General

We first review the essential points of the method in the case of massive particle reactions. The basic remark is that the projection operator $\mathcal{P}^{J}_{\beta_1\ldots\beta_J;\alpha_1\ldots\alpha_J}(\Delta)$ can be used to select out the J^{th} partial wave of the general t-channel ($t = \Delta^2$) partial wave expansion (J is integer). Because this projection operator is also the numerator of a spin J propagator of mass $\sqrt{t}$, we can think of the $\mathcal{M}$-function's J^{th} partial wave as being the sum over "Reggeon" exchange of mass $\sqrt{t}$ weighted by arbitrary coefficients. Each Reggeon exchange is of the form[45,46]

$$\begin{aligned} \mathcal{M} &= \mathcal{C}(P) : \mathcal{P}^{J}(\Delta) : \mathcal{C}(Q) \\ &= \mathcal{C}_R(P) : \mathcal{P}^{J}(P,Q;\Delta) : \mathcal{C}_R(Q) \end{aligned} \qquad (1)$$

where $\mathcal{C}(P)$ and $\mathcal{C}_R(P)$ are the covariant and "reduced" vertex couplings respectively at the final vertex (see Appendix I for details and Figs 1, 2 for kinematics). $\mathcal{P}^{J}(P,Q;\Delta)$ is the contracted or partially contracted projection operator,[45] $\mathcal{P}^{J}$, $\mathcal{P}^{J}_{\beta;}$, $\mathcal{P}^{J}_{;\alpha}$ ... etc., given in Appendix I where the covariant formalism with explicit emphasis on the differential technique is expounded. In Appendix II it is applied to two example processes ($VN \rightarrow \pi N$, $V\pi \rightarrow V\pi$). As the contracted projection operators are simply combinations of solid Legendre polynomials, threshold and also factorization properties are built into the Regge poles from the outset.

The couplings listed in ref(45) and Appendix I differ according to the 'normality' of the vertex and we denote normal (+) and abnormal (-) couplings by $\mathcal{C}^{\pm}$. A normal (abnormal) particle has parity $(-)^J$ ($(-)^{J+1}$) and the normality of a vertex is the product of the normalities of the particles at the vertex. Abnormal boson vertices involve contractions of momenta and the anti-symmetric Levi-Civita tensor which, when taken between Dirac bispinors, give rise to reduction formulae termed "abnormal reductions"[41]. We present several abnormal reductions in a completely general form and also list expansions of the reductions in terms of the kinematic covariants of the calculated processes (Appendix III).

As we make frequent use of the Regge form of the differential cross section in the covariant formalism we carry out in detail the original scheme for calculating spin sums

$$\sum_{\text{all}\,\lambda} |T|^2 \sim \text{tr}\, \mathcal{M}_{fi}\, \mathcal{P}_{ii'}\, \overline{\mathcal{M}}_{f'i'}\, \mathcal{P}_{f'f}$$

suggested by Scadron[45] (Appendix IV) and tabulate the results.

The KSF decomposition of the covariant $\mathcal{M}$-function can be written in general as[41]

$$\mathcal{M} = \sum_i A_i(s,t)\mathcal{K}^i \tag{2}$$

where the kinematic covariants $\mathcal{K}^i$ carry the covariant spin indices of the $\mathcal{M}$-function and the invariant amplitudes $A_i(s,t)$ are both KSF <u>and</u> KZF. By extracting the covariants $\mathcal{K}^i$ out of the contracted projection operators, we can compare eqn(1) with eqn(2) and thus obtain the partial wave (Regge) expansion of the invariant amplitudes A(s,t).

In the spinless, equal mass case ($\pi\pi$) we have

$$A(s,t) = \sum_J c_J\, A_J\, (t)\, (-PQ)^J\, P_J(\cos\theta_t)$$

along with the Froissart-Gribov continuation through the equation

$$(-PQ)^J\, A_J(t) = \frac{(2J+1)}{c_J} \int dZ_s\, A_s(s,t)\, Q_J\, (Z_s)$$

$$= \int dZ_s A_s(s,t)\, Z^{-(J+1)}\, {}_2F_1(\tfrac{1}{2}(1+J), \tfrac{1}{2}(2+J); \tfrac{3}{2}+J; \tfrac{1}{z^2})\, .$$

As this hypergeometric function is regular at both positive and negative integer J, A_J cannot have fixed "kinematic" poles at these J values. For a given trajectory (with no confluence points) $A_J = g^2$ where g is the spinless coupling on spin J (we shall delete the dependence of g on J).

2. Nonsense Zeros and Regge Prescription

Given that the residue functions g_i (f_i) in the normal (abnormal) vertex functions contain no kinematic poles in J we can pin down their (nonsense) zeros in J which arise when a coupling cannot exist. For example the normal ($N\bar{N}J$) reduced coupling is (Appendix I)

$$\mathcal{C}^+ (\tfrac{1}{2}\tfrac{1}{2}\, J) = \left[g_1\, P_{B1} + g_2 \gamma_{B2} \right]$$

which holds so long as $J \geq 1$. However when $J = 0$ the γ_B coupling cannot exist[47] and g_2 must vanish. Comparing with the spinless case $A = g^2\, c_J \mathcal{P}_J$ where the g's do not have kinematic zeros[48] at $J = 0$, we conclude that $g_2 \sim J$ (not $\sqrt{J}$) at $J = 0$.

If we calculate $\pi N \longrightarrow \pi N$ scattering in the t-channel, we find

$$\mathcal{M} = \left[g_1 P_B + g_2 \gamma_B\right] : \mathcal{P}^J : [g]$$

$$= gg_1 \mathcal{P}^J + gg_2 \gamma_B \mathcal{P}^J_B;$$

$$= gg_1 c_J \mathcal{P}_J - gg_2 \frac{c_J}{J} (\not{Q} \mathcal{P}'_J + Q^2 m \mathcal{P}'_{J-1})$$

$$= (gg_1 c_J \mathcal{P}_J - gg_2 m Q^2 \frac{c_J}{J} \mathcal{P}'_{J-1}) - gg_2 \frac{c_J}{J} \mathcal{P}'_J$$

and the B amplitude ($\mathcal{M} = A + \not{Q}B$) behaves as

$$B = -gg_2 \frac{c_J}{J} \mathcal{P}'_J \qquad (3)$$

B, however, does <u>not</u> contain a fixed pole for finite s as $g_2 \sim J$, $J \rightarrow 0$.

Thus for each free index, the appropriate pole in J due to the factors c_J/J, c_J/J^2, $c_J/(J-1)$ etc. occuring in, for example, eqn (3) will always be exactly cancelled in the related couplings. This "sense choosing" mechanism is quite natural in the covariant approach provided we explicitly keep track of the 1/J factors. Our couplings then, can be taken to be analytic in J with no kinematic poles and containing nonsense zeros at the values of J where the coupling is unattainable.

In the calculations which follow we specifically require g_2, $f_2 \sim J$ in $\mathcal{C}^{\pm}(\frac{1}{2}\frac{1}{2}\ J)$ and $g_{2,3}$, $f_{2,3} \sim J$, g_4, $f_4 \sim J(J-1)$ in $\mathcal{C}^{\pm}(\frac{1}{2}\ \frac{3}{2}\ J)$. The gauge invariant photonic vertices will be treated as they arise.

The Regge prescription is then (Appendix I)

$$c_J \rho_J \longrightarrow (-\nu)^{\alpha} \quad \frac{(1 \pm e^{i\pi\alpha} \quad)}{2 \sin\pi\alpha} \cdot \pi\alpha'$$

For unequal masses $\nu \longrightarrow \nu(\Delta) = \nu - \frac{P.\Delta\, Q.\Delta}{t}$

so the question of singular daughters(28) at t = 0, or dispersed Regge terms(49) coupled with background(26) calculation arises.

For any reggeized exchange of a particle with mass μ, spin J we insist that(50)

(i) there be a pole at $t = \mu^2$ in the Regge amplitude

(ii) the differential cross section behave as

$$(t - \mu^2)^2 \, d\sigma/dt \sim s^{2J-2} \text{ at the pole}$$

(iii) the Regge contribution be identical to the elementary one pole exchange contribution at the pole.

3. Covariant Evasion(47) and t = 0

Since we have insisted that $\Delta_B \rho^J_{B;} = \rho^J{}_{;\alpha} \Delta_\alpha = 0$ to preserve the 2J + 1 multiplicity of the spin J Reggeon we will encounter 1/t singularities in high spin (and unequal mass) reactions due to terms like $g_{B\alpha}(\Delta) = g_{B\alpha} - \frac{\Delta_B \Delta_\alpha}{t}$. In the sense that the 1/t problem of unequal mass and high spin are both due to the boost prescription from the rest (c.m.) frame of the Reggeon, the covariant formalism treats both unequal mass and high spin on the same footing.

As the invariant amplitudes cannot be singular except at dynamical poles these 1/t factors must be cancelled by zeros in individual, or combinations of, Regge residues. If we are considering each Regge exchange separately, the covariant evasion machanism which causes the necessary zeros in the couplings is the spin reduction to only two spin states corresponding to the exchange of a Reggeon with mass $\sqrt{t} = 0$. Quantitatively this is brought about by the 'internal' gauge invariance condition at each vertex

$$\Delta_\alpha \mathcal{C}_\alpha(Q) = 0\,, \quad \Delta_\beta \mathcal{C}_\beta(P) = 0\,, \quad t = 0$$

Questions arise as to the correct interpretation of the $\mathcal{C}\cdot\Delta$ condition.

To take an example, consider $\mathcal{C}^-(\tfrac{1}{2}\tfrac{1}{2}\,J)$ and write out the full (not reduced) coupling

$$\mathcal{C}^-(\tfrac{1}{2}\tfrac{1}{2}\,J) = \mathcal{C}^-_{R\beta_1}(\tfrac{1}{2}\tfrac{1}{2}\,J)\;P_{\beta_2}\ldots P_{\beta_J}$$

$$= \gamma_5\left[f_1 P_{\beta_1} + f_2\gamma_{\beta_1}\right] P_{\beta_2}\ldots P_{\beta_J}\,.$$

$$\Delta_{\beta_1}\mathcal{C}^-_{\beta_1\ldots\beta_J} = \gamma_5\left[f_1\, P.\Delta + f_2 \not{\Delta}\right] P_{\beta_2}\ldots P_{\beta_J}$$

$$= 2m_+\gamma_5\left[f_1 m_- - f_2\right] P_{\beta_2}\ldots P_{\beta_J}$$

So long as $m_- \neq 0$ the $\mathcal{C}.\Delta$ condition unambiguously requires that

$$f_1 m_- - f_2 = 0\,, \quad t = 0$$

However we shall observe that explicit $1/t$ singularities in leading order terms occur only for A_1 type exchange in processes ($\gamma N \to \pi N, \rho N$) where $m_- = 0$, and where this fact has allowed us to involve charge conjugation invariance to split up the abnormal coupling (Section III,1) to permit the A_1 to couple via f_2 only.

Keeping this in mind we establish the following rule to govern the $\epsilon\cdot\Delta$ constraint when $m_- = 0$: it applies only to the reduced coupling and only to those couplings which are not pure momentum couplings (f_1, g_1). Stated in this form it parallels exactly the nonsense zero discussion in the previous section.

Returning then to $\epsilon^-(N\bar{N}J)$ the condition is now $f_2 = 0$, $t = 0$ and from NN[47] and $\gamma N \longrightarrow \pi N$ scattering it is apparent that $f_2 \sim t$, $t \rightarrow 0$. Applying $\epsilon\cdot\Delta = 0$, $t = 0$ to other fermion vertices we find

$$\epsilon^+ (\tfrac{1}{2}\tfrac{1}{2}\, J) \;:\; g_1 m_+ + g_2 = 0 \text{ (no restriction if } m_- = 0)$$

$$\epsilon^+ (\tfrac{1}{2}\,\tfrac{3}{2}\, J) \;:\; m_+ m_- (g_1 m_+ + g_2) - (g_3 m_+ + g_4) = 0$$

$$\epsilon^- (\tfrac{1}{2}\,\tfrac{3}{2}\, J) \;:\; m_+ m_- (f_1 m_- - f_2) - (f_3 m_- - f_4) = 0$$

Boson vertex functions, provided $m_- \neq 0$, present no difficulties and the prescription requires

$$\epsilon^+ (10\; J) \;:\; \mu'^2 g_1 + 4 g_2 = 0$$

$$\epsilon^- (10\; J) \;:\; \text{No restriction}$$

Vertices involving photons, however require careful consideration in order to avoid placing excessively strong constraints on the Regge couplings at $t = 0$. One must keep in mind that we are dealing with on-shell couplings[45] and the presence of an external massless photon along with a massless Reggeon requires that we treat the vertex according to our equal mass rule. Of the photon vertices which we use $\mathcal{C}.\Delta = 0$, $t = 0$ imposes <u>no</u> restriction on $\tilde{\mathcal{C}}^{\pm}(\gamma 0J)$, $\tilde{\mathcal{C}}^{\pm}(\gamma\gamma J)$, $\tilde{\mathcal{C}}^{+}(\gamma 1J)$ and requires that $\tilde{f}_2 = 0$, $t = 0$ in $\tilde{\mathcal{C}}^{-}(\gamma 1J)$.

It is interesting to note that once gauge invariance is imposed upon $\mathcal{C}^{+}_{\mu}(10\,J)$, internal gauge invariance is automatically satified for $\tilde{\mathcal{C}}^{+}_{\mu}(\gamma 0J)$. The same principle applies to $\mathcal{C}_{\mu\nu}(11J)$, $\tilde{\mathcal{C}}_{\mu\nu}(\gamma 1J)$ and $\tilde{\mathcal{C}}_{\mu\nu}(\gamma\gamma J)$ and we say that external gauge invariance implies internal gauge invariance at $t = 0$.

4. Covariant Conspiracy

If two or more trajectories cross at $t = 0$, the resulting confluence destroys the KSF property of the factorized residues and allows certain residues to conspire together in a singular fashion so as to keep the total reggeized invariant amplitudes finite. Conspiratorial exchanges which couple to the $N\bar{N}$ vertex are termed(29) Class II ($M = 0$) or Class III ($M = 1$) conspiracies (Class I, $M = 0$ is evasion).

Such conspiratorial solutions (but not always) allow the cross sections to be non-vanishing at $t = 0$, hence predictions differing from those of the evasive solution can be made. From the covariant point of view, couplings which cause a $1/t$ factor to appear in a leading order contribution to an invariant amplitude, also give a non-vanishing contribution to the cross section at $t = 0$ - unless evasion is chosen. Whereas, if they do not give rise to a $1/t$ factor in the invariant amplitude, they will not contribute to the cross section at $t = 0$ - unless conspiracy is chosen.

5. Gauge Invariance, Kinematic Zeros and Singularities

Covariant decomposition of photonic processes involving invariant amplitudes free of kinematic singularities and zeros are now understood[42,43,51]. The $\mathcal{M}$-function expansions are of the form

$$\tilde{\mathcal{M}}_\mu = \sum_i \tilde{A}_i \tilde{\mathcal{K}}^i_\mu$$

where $k_\mu \tilde{\mathcal{M}}_\mu = k_\mu \tilde{\mathcal{K}}_\mu = 0$. To guarantee gauge invariance $\tilde{\mathcal{K}}$ can be written as $\tilde{\mathcal{K}} = \mathcal{K}.\mathcal{G}$ where $\mathcal{G}$ is the gauge projection operator[42,43]

$$\mathcal{G}_{\mu'\mu} = g_{\mu'\mu} - \frac{k_{\mu'} Q_\mu}{k.Q}$$

Removal of kinematic zeros of the amplitudes is insured if $\tilde{\mathcal{K}}$ contains no singular terms; hence linear combinations and finally multiplication by k.Q will be necessary to cancel such terms induced by the gauge invariance requirement[42,43].

In a manner similar to that of the previous sections covariant Regge poles in processes involving a photon arise from

$$\tilde{\mathcal{M}}_\mu = \mathcal{C}(P) : \mathcal{P}^J(\Delta) : \tilde{\mathcal{C}}_\mu(Q) \qquad (4)$$

where the photon vertex is gauge invariant, $\tilde{\mathcal{C}}_\mu(Q)\, k_\mu = 0$ if

$$\tilde{\mathcal{C}}_\mu(Q) = \mathcal{C}_{\mu'}(Q)\mathcal{G}_{\mu'\mu}$$

We take $Q = \frac{1}{2}(k + k')$ to be the relative momenta at the photon-boson vertex ($k'^2 = \mu'^2$).

We can choose the vertex functions to be KSF (<u>and</u> KZF) in $t = \Delta^2$ from the start thus ensuring a KSF (and KZF) development analagous to that of eqn(4), with the possible exception of 1/t factors.

Significant advantage is gained by factoring out the gauge projection operator from the coupling, because the identities

$$Q'_{\mu} = Q_{\mu'}\mathcal{G}_{\mu'\mu} = 0$$

$$\Delta'_{\mu} = \Delta_{\mu}\mathcal{G}_{\mu'\mu} = 0$$

greatly simplify the structure of the contracted projection operator

$$\tilde{\mathcal{P}}_{\mu;}^{J}\ (P,Q\ ;\Delta) = \mathcal{P}_{\mu'}{}^{J}{}_{;}\ (P,Q\ ;\Delta)\mathcal{G}_{\mu'\mu}\ .$$

III PHOTOPRODUCTION

1. $\gamma N \rightarrow \pi N$

A. Photon-Pion Vertex

The abnormal $\gamma \pi J$ vertex $\tilde{\mathcal{E}}^-_\mu(\gamma\, 0J)$ corresponding to normal exchange, 0^+, 1^-, 2^+, ... is

$$\tilde{\mathcal{E}}^-_\mu(\gamma\, 0J) \equiv \mathcal{E}^-_\mu(\gamma\, 0J) = \tilde{f}(t)\,\mathcal{E}_{\alpha_1\mu}(Q\Delta)\, Q_{\alpha_2} \cdots Q_{\alpha_J}$$

since

$$\left[g_{\mu\mu'} - \frac{k_{\mu'}Q_\mu}{k.Q}\right]\mathcal{E}_{\alpha_1\mu'}(Q\Delta) = \mathcal{E}_{\alpha_1\mu}(Q\Delta) \quad .$$

The gauge invariant normal vertex is

$$\left[g_1 Q_{\mu'} Q_\alpha + g_2 g_{\mu'\alpha}\right] \mathcal{G}_{\mu'\mu} = g_2\, g'_{\mu\alpha}$$

where

$$g_2\, g'_{\mu\alpha} = g_2 \left[g_{\mu\alpha} - \frac{k_\alpha Q_\mu}{k.Q}\right] \quad .$$

Removing the singularity(52),

$$\tilde{\mathcal{E}}^+_\mu(\gamma\, 0J) = \tilde{g}(t)k.Q\left[g_{\mu\alpha_1} - \frac{Q_{\alpha_1}Q_\mu}{k.Q}\right] Q_{\alpha_2} \cdots Q_{\alpha_J}$$

$$= \tilde{g}(t)k.Q\, g'_{\mu\alpha_1} Q_{\alpha_2} \cdots Q_{\alpha_J}$$

where $g_2 = \tilde{g}(t)\, k.Q$.

A more pedestrian way of arriving at this is to require explicitly $k_\mu \mathcal{E}^+_\mu(\gamma\, 0J) = 0$, which establishes the relation $g_2 = -k.Qg_1$. The residue $\tilde{g}(t)$ is now KSF in t (at least at $k.Q = -\frac{1}{4}(t - \mu'^2)$).

B. Pion Reggeization

The $\gamma\pi\alpha$ vertex in conjunction with the pion trajectory near the pole, $\alpha(t) \longrightarrow 0$, as $t \longrightarrow \mu^2$ ($k.Q \longrightarrow 0$ and we have dropped the prime on μ') causes concern as its gauge invariant structure appears to vanish there and to consequently require a nonsense zero in the residue at $t = \mu^2$, $\alpha = 0$ in order to preserve gauge invariance. Were this the case, the cross section which is proportional to $t\tilde{g}(t)^2 \left|\xi_\pi\right|^2$ (Appendix IV) would not have a pole at $\alpha = 0$ and the pion exchange would be said not to reggeize[15,16]. That this is not the case can be seen by noting that the coupling is indeed gauge invariant at the pole and the residue is not obliged to vanish. This is to be expected as the elementary pion pole amplitude is gauge invariant, on its own, at the pole and we expect the reggeized pion exchange to coincide with the pion pole exchange at the pole. To show this we demonstrate that the Regge vertex $\tilde{g}(t)\left[k.Q\; g_{\mu\alpha} - Q_\mu Q_\alpha\right]$ and the elementary pion exchange vertex $-4eQ$ imply

$$\tilde{g}(\mu^2) = 4e \quad .$$

Assuming the nucleons to have equal masses, G - parity conservation at the $N\bar{N}$ vertex demands that[45]

$$\mathcal{C}^{+}(N\bar{N}J) \xrightarrow{G} (-)^{I+J}\left[g_1\, P_{\nu_1} + g_2\, \gamma_{\nu_1}\right] P_{\nu_2} \cdots P_{\nu_J}$$

$$\mathcal{C}^{-}(N\bar{N}J) \xrightarrow{G} (-)^{I+J}\left[f_1\, \gamma_5 P_{\nu_1} - f_2\, \gamma_5\, \gamma_{\nu_1}\right] P_{\nu_2} \cdots P_{\nu_J}$$

which we rewrite as[40]

$$\mathcal{C}^{+}(N\bar{N}J) = (1 + C_n(-)^J)\left[g_1 P_{\rho_1} + g_2 \gamma_{\rho_1}\right] P_{\rho_2} \cdots P_{\rho_J}.$$

$$\mathcal{C}^{-}(N\bar{N}J) = \left[(1 + C_n(-)^J) f_1 \gamma_5 P_{\rho_1} + (1 - C_n(-)^J) f_2 \gamma_5 \gamma_{\rho_1}\right] P_{\rho_2} \cdots P_{\rho_J}$$

where $C_n = G(-)^I$ is the charge conjugation parity of the neutral member of the exchange multiplet. We are also led to define C-normality ($C_n(-)^J = 1$) and C-abnormality ($C_n(-)^J = -1$), which are meaningful when either G or C (I = 0) are conserved; in the same way that P-normality is meaningful when parity is conserved.

Now we calculate the general case of abnormal and normal exchange in $\gamma N \longrightarrow \pi N$ and later extract the specific case of pion exchange, which couples via f_1, the C-normal, P-abnormal coupling. The Regge form of the covariant $\mathcal{M}$-function is[53]

$$\tilde{\mathcal{M}}_{\mu}^{-} = \mathcal{C}^{-}(\bar{N}NJ) : \mathcal{P}^{J} : \tilde{\mathcal{C}}_{\mu}^{+}(\gamma 0 J)$$

$$= k.Q\, \tilde{g}(t)\, \gamma_5 \left[(1 + C_n(-)^J) f_1(t) \tilde{\mathcal{P}}^{J}_{;\mu} + (1 - C_n(-)^J) f_2(t) \gamma_{\rho} \tilde{\mathcal{P}}^{J}_{\rho;\mu}\right]$$

Using the contracted projection operators of Appendix I, we can drop Q'_{μ} and Δ'_{μ} so that

$$\tilde{\mathcal{P}}^{J}_{;\mu} = -\frac{c_J}{J} P'_{\mu} \mathcal{P}'_J$$

$$\tilde{\mathcal{P}}^{J}_{\rho;\mu} = \frac{c_J}{J^2}\left[-g_{\rho\mu} \mathcal{P}'_J + Q_{\rho}(\Delta) P'_{\mu} \mathcal{P}''_J + Q(\Delta)^2 P_{\rho} P'_{\mu} \mathcal{P}''_{J-1}\right].$$

Thus, we can isolate the Regge contributions to the isospin[54] invariant amplitudes $\tilde{A}_i^{(+,o,-)}$ where

$$\tilde{\mathcal{M}}_\mu^{(+,o,-)} = \sum_i \tilde{A}_i^{(+,o,-)} \tilde{\mathcal{K}}_\mu^i \qquad ,$$

and we use the traditional isospin decomposition[55]

$$\tilde{A} = \tilde{A}^{(+)}\delta_{\alpha,3} + \tilde{A}^{(-)} \tfrac{1}{2}\left[\tau_\alpha, \tau_{\alpha_3}\right] + \tilde{A}^{(o)}\tau_\alpha \, .$$

In the t-channel, $\tilde{A}^{(+)}$ corresponds to I = 0 exchange and $\tilde{A}^{(o,-)}$ to I = 1 exchange. Our kinematic covariants are essentially those of CGLN[43,55],

$$\tilde{\mathcal{K}}_\mu^1 = \gamma_5 \not{k} \gamma_\mu' = \gamma_5 \not{k} \gamma_\mu$$

$$\tilde{\mathcal{K}}_\mu^2 = \gamma_5 \, k.Q \, P_\mu' = \gamma_5 \, (k.Q \, P_\mu - k.PQ_\mu)$$

$$\tilde{\mathcal{K}}_\mu^3 = \gamma_5 \, k.Q \gamma_\mu' = \gamma_5 \, (k.Q\gamma_\mu - \not{k} \, Q_\mu)$$

$$\tilde{\mathcal{K}}_\mu^4 = \gamma_5 (k.P \, \gamma_\mu' - \not{k}P_\mu' \;) = \gamma_5 \, (k.P\gamma_\mu - \not{k} \, P_\mu \,).$$

For normal exchange,

$$\tilde{\mathcal{M}}_\mu^+ = \mathcal{C}^+ \, (N\bar{N}J) : \mathcal{P}^J \; ; \; \tilde{\mathcal{C}}^- \; (\gamma o J)$$

$$= \tilde{f}(t) \; (1 + c_n(-)^J \;) \left[g_1 \, \mathcal{P}^J_{;\alpha} \, \epsilon_{\alpha\mu}(Q\Delta) + g_2 \gamma_\beta \mathcal{P}^J_{\beta;\alpha} \epsilon_{\alpha\mu}(Q\Delta) \right].$$

The abnormal decompositions are given in Appendix III, and the extraction of Regge contributions to the invariant amplitudes parallels that in Appendix II. The Regge contributions to the invariant amplitudes for both normal and abnormal exchange are given in Table I.

Returning to pion exchange, the trajectory contributes only to $\tilde{A}_2^{(-)}$, giving

$$\tilde{A}_2^{(-)}(\nu, t) \sim \tilde{g}(t)\, f_1(t)\, (-\nu)^{\alpha_\pi(t)-1} \quad \xi_\pi \pi\alpha' , \quad \nu \to \infty .$$

In the limit $\alpha(t) \to 0$, $t \to \mu^2$,

$$\pi \alpha' \xi_\pi \longrightarrow (t - \mu^2)^{-1}$$

and

$$\tilde{A}_2^{(-)} \sim - \frac{\tilde{g}(\mu^2)\, f_1(\mu^2)}{(t - \mu^2)\,\nu} .$$

Now compare this with the elementary pion pole contribution

$$\mathcal{M}_\mu^{(-)} = \frac{4e\, g_{\pi \bar{N} N}}{(t - \mu^2)} Q_\mu$$

which is not in general gauge invariant. At the pole ($k.Q = 0$) however, the gauge invariant covariant $\tilde{K}_\mu^2$ becomes $-\nu Q_\mu$ and

$$\tilde{A}_2^{(-)}\text{ pole} = - \frac{4e\, g_{\pi \bar{N} N}}{\nu\,(t - \mu^2)}$$

where $\nu = (s - m^2)/2$ at the pole. Comparing the elementary pole contribution to the Regge contribution, at the pole, of the pion exchange and identifying $f_1(\mu^2)$ with $g_{\pi\bar{N}N}$ we extract the desired relation $g(\mu^2) = 4e$. The reggeized pion contributes to the cross section at $t = \mu^2$ and is identical to the elementary pole term there, as required in section II.

Now we examine the kinematic singularity at $t = \mu^2$ in $\tilde{A}_2$ due to gauge invariance as suggested by Ball[21] and show that it does not in fact exist. It led a life of peaceful obscurity for eight years before the current interest in kinematic singularities and zeros in scattering amplitudes resulted in its removal by Ebata and Lassila[56] and Henyey[12]. In the covariant treatment of $\gamma N \longrightarrow \pi N$ such a singularity in $\tilde{A}_2$ would have appeared in other amplitudes as well; it was this difficulty which forced us, in collaboration with Scadron[40] to remove it at about the same time.

Consider the eight invariant amplitudes B_i for the process $VN \longrightarrow \pi N$ (Appendix III) which Ball[21] showed to be KSF. Two of these amplitudes, B_3, B_7 vanish under the subsidiary condition $\varepsilon_\mu(k).k = 0$. When gauge invariance is imposed in the form $k_\mu M_\mu = 0$, two relations emerge

$$k.P \; B_1 + k.Q \; B_2 = 0$$

$$k.P \; B_5 + k.Q \; B_6 + B_4 = 0$$

and the first one, related to $\gamma N \longrightarrow \pi N$ gives

$$\tilde{A}_2 = \frac{B_1}{k.Q} = - \frac{B_2}{k.P}$$

where B_1 and B_2 are KSF. If we are to preserve $\tilde{A}_2$ as KSF and KZF we are forced to assume that B_1 has a kinematic zero at $t = \mu^2$ and not that $\tilde{A}_2$ has a kinematic singularity as was assumed by Ball. A similar arguement requires such a zero in B_2 at $k.P = 0 = (s - u)/4$. In the covariant formalism a kinematic pole in $\tilde{A}_2$ would have to reside in the coupling $\tilde{g}(t)$ and would appear also in $\tilde{A}_3$ and $\tilde{A}_4$ (see Table I). It was this unexpected requirement which first drew our attention to the problem.

As a final comment on the amplitudes for abnormal exchange we note that they all remain finite at $\alpha = 0$ and with the assistance of the nonsense zero in $f_2(t)$ at $\alpha = 0$ $(f_2 \sim \alpha)$, the term $f_2 \xi_\pm$ is never singular at $\alpha = 0$, regardless of signature. The one exception to this of course is $\tilde{A}_2^{(-)}$ which contains the dynamical pion pole.

The result, that the pion reggeizes, is independent of the masses and spins at the fermion vertex. When we examine the covariant Regge expansion of $\gamma N \longrightarrow \pi N^*$ (Section III, 2) we again find that the invariant amplitudes to which the pion contributes become pole like near $t = \mu^2$, and all others do not as they develop extra zeros. In a similar fashion the rho trajectory reggeizes at the $\gamma \rho J$ vertex (Section III, 3) and in general the J_o particle trajectory reggeizes when coupled to the normal $(\gamma J_o J)$ vertex at $t = m_{J_o}^2$, $J = J_o$.

C. Conspiracy

Unfortunately the contributing of the pion to the differential cross section at $t = \mu^2$ does not solve the problem of charged pion photoproduction in the near forward direction $(t \simeq o)$. A sharp forward peak has been noted in $\gamma p \longrightarrow \pi^+ n$ data up to 16Gev(3) which because of its width and energy dependence is strongly suggestive of pion dominance. However, the Regge differential cross section for single Reggeon exchange (Table IV),[85]

$$\nu^2 \frac{d\sigma}{dt} \sim t\, \tilde{f}^2 \left[(mg_1 + g_2)^2 - \frac{t}{4} g_1^2 \right] \left| \xi''_{\pm} \right|^2 \nu^{2\alpha^{\mp}_{\mp}}$$

$$+ t^2 \tilde{g}^2 f_1^2 \left| \xi_{\pm} \right|^2 \nu^{2\alpha^{\pm}_{\mp}}$$

$$+ t\, \tilde{g}'^2 f_2^2 \left| \xi'_{\pm} \right|^2 \nu^{2\alpha^{-}_{-}}$$

where $\alpha_{\pm}$ $(\alpha^{\pm})$ corresponds to the P(C) normality of the trajectories, clearly vanishes at t = 0 for finite couplings and is inconsistent with pion dominance.

In order to prevent the cross section from vanishing at t = 0 without abandoning the simple model of single trajectory exchanges we are led to consider singular residue functions. Specifically, if the pion couplings of $\tilde{g}f_1$ behave like t^{-1} near t = 0, then the pion indeed contributes to the differential cross section. There are however far reaching consequences of this approach.

In order to preserve the analyticity of the invariant amplitudes we are obliged to exchange a trajectory (π_c) with opposite parity but otherwise identical quantum numbers as the pion and a third trajectory (A_1^c) with quantum numbers identical to those of the A_1 and $\alpha_{A_1^c}(0) + 1 = \alpha_{\pi}(0) = \alpha_{\pi_c}(0)$. This combination of exchanges, along with all of the daughter trajectories is precisely a Class III conspiracy (M = 1 Toller pole) and we shall see that having made the demand 'that the reggeized pion contribute to the differential cross section at t = 0' the enforcement of the basic requirement of analyticity of the invariant amplitudes yields the necessary conspirators.

Consider Table I and the amplitude $\tilde{A}_2$. If we are to keep $\tilde{A}_2$ finite to leading order ($\nu^{\alpha_\pi - 1}$), then the normal π_c exchange must be involved to give

$$\tilde{g}f_1 + 2\tilde{f}\, g_1 = \text{finite}, \ t = 0, \quad \xi_\pi = \xi_{\pi_c}$$

The 1/t singularity in $\tilde{g}f_1$ is cancelled by a 1/t singularity in fg_1 which leads immediately to the requirement

$$\tilde{f}\left[mg_1 + g_2\right] = \text{finite}, \ t = 0$$

to keep $\tilde{A}_1$ finite. However we have now introduced a 1/t into $\tilde{f}g_2$ and in order to keep $\tilde{A}_3$ finite a trajectory with $\alpha(0) = \alpha_{\pi_c}(0) - 1$ has to be involved along with the relation.

$$\tilde{g}' f_2 + \frac{\mu^2}{2} \frac{(J-1)}{J}\tilde{f}g_2 = \text{finite}, \ t = 0, \quad \xi_{A_1^c} = - \xi_{\pi_c} \quad .$$

Once we have reggeized, the $1/\alpha$ will be cancelled by the nonsense zero in g_2 ($g_2 \sim \alpha$, $\alpha \rightarrow 0$).

To pursue this examination to order $\nu^{\alpha_\pi - 3}$ we must consider daughter trajectories which, because of the unequal mass ($\gamma\pi$) vertex will have residues singular in t. Daughters in this context arise from the expansion

$$c_J \mathcal{P}_J' = J\, \nu(\Delta)^{J-1} - \frac{J\,(J-1)(J-2)}{1\,!\;2(2J-1)} P(\Delta)^2\, Q(\Delta)^2\, \nu(\Delta)^{J-3}$$

$$+ \frac{J!\;\; P(\Delta)^4 Q(\Delta)^4 \nu(\Delta)^{J-5}}{(J-5)!\; 2!\; 4(2J-1)(2J-3)} - \frac{J!\, P(\Delta)^6 Q(\Delta)^6 \nu(\Delta)^{J-7}}{(J-7)!\; 3!\; 8(2J-1)(2J-3)(2J-5)}$$

$$+ \;\dots$$

where $P.\Delta = 0$, $\nu(\Delta) = \nu$, $P(\Delta) = P$

and because of the one equal mass vertex the daughter trajectories are each two units of angular momentum apart and have the same signature as the parent.

Examining first $\tilde{A}_1$ we note that only π_c exchange contributes and consequently the form

$$2\tilde{f}\, \frac{c_J}{J^2} \left[P^2 g_1 + m g_2\right] J \mathcal{P}_J'$$

must be finite to leading order $(\nu^{\alpha-1})$ and all daughter exchanges must exactly cancel the singular residues of the lower order terms. This fixes the form of the first daughter residues as

$$P^2 \left[\tilde{f} g_1\right]_{D1} + m \left[\tilde{f} g_2\right]_{D1} =$$

$$\frac{(J-1)(J-2)}{2(2J-1)}\; P^2\, Q(\Delta)^2 \left[P^2\, \tilde{f} g_1 + m \tilde{f} g_2\right]$$

Turning to the normal contribution to $\tilde{A}_4$ and rewriting it as

$$2\tilde{f}\,\frac{c_J}{J^2}\left[\frac{1}{m}\;(P^2g_1 + mg_2)JP_J' \;+\; \frac{t}{4m}\,g_1\,JP_J'\right.$$

$$\left. -\;\frac{t}{4}\,g_2\,Q(\Delta)^2\,P_{J-1}''\right]$$

we see that the first term poses no problem. If however we expand the above and include the abnormal contribution and the first daughter,

$$2\left[\frac{1}{m}\;\tilde{f}\;(P^2g_1 + mg_2)\;\nu^{J-1}\right.$$

$$+\;\frac{t}{4m}\;\tilde{f}g_1\;\left(\nu^{J-1} - \frac{(J-1)(J-2)}{2(2J-1)}\;P^2Q(\Delta)^2\;\nu^{J-3} + \dots\right)$$

$$-\;\frac{t}{4}\;\tilde{f}g_2\,Q(\Delta)^2\;\frac{(J-1)(J-2)}{J(2J-1)}\;\nu^{J-3} + \dots$$

$$\left. +\;\frac{t}{4m}\left[\tilde{f}g_1\right]_{D1}\;\nu^{J-3}\right]$$

$$-\;\frac{(t-\mu^2)}{4}\;\tilde{g}'\;f_2\;\frac{(J-2)}{(J-1)}\;\nu^{J-3} + \dots$$

we determine the form of $\left[\tilde{f}g_1\right]_{D1}$,

$$\left[\tilde{f}g_1\right]_{D1} = \frac{(J-1)(J-2)}{2(2J-1)}\;Q(\Delta)^2\;\left[P^2\tilde{f}g_1 + 2\,\frac{m}{J}\,\tilde{f}g_2\right]$$

$$+\;\frac{m}{2t}\;(t-\mu^2)\,\frac{(J-2)}{(J-1)}\;\left[\tilde{g}'\;f_2\right]$$

which in turn determines $\left[\tilde{f}\,g_2\right]_{D1}$,

$$\left[\tilde{f}g_2\right]_{D1} = \frac{(J-1)(J-2)^2}{J\ 2(2J-1)} \quad P^2\, Q(\Delta)^2 \left[\tilde{f}g_2\right]$$

$$- \ p^2 \ \frac{(t-\mu^2)}{2t} \ \frac{(J-2)}{(J-1)} \left[\tilde{g}' f_2\right]$$

and from the contribution to $\tilde{A}_3$,

$$\left[\tilde{g}'\ f_2\right]_{D1} = P^2 Q(\Delta)^2 \ \frac{(J-3)}{(J-1)} \left[\frac{(J-2)(J-3)}{2\ (2J-1)} + 1\right]$$

Returning to $\tilde{A}_2$ we see that the problem, thanks to the m/t in the abnormal contribution (A_1^c) is identical to that already solved in $\tilde{A}_4$, with the exception of the pion daughters which must cancel all of the singular pion terms exactly and conspire with no other terms.

This straight forward technique of cancelling singularities can easily be continued to all lower orders and we are left with non singular Regge contributions to the invariant amplitudes with leading order terms

$$\tilde{A}_1 \sim + 2\tilde{f} \left[m(mg_1 + g_2) - \frac{t}{4} g_1 \right] (-\nu)^{\alpha_{\pi_c} - 1} \xi_{\pi_c} \pi \alpha'_{\pi_c}$$

$$\tilde{A}_2 \sim - \tilde{g}f_1 (-\nu)^{\alpha_\pi - 1} \xi_\pi \pi \alpha'_\pi - 2\, \tilde{f}g_1 \ (-\nu)^{\alpha_{\pi_c} - 1} \xi_{\pi_c} \pi \alpha'_{\pi_c}$$

$$\tilde{A}_4 \sim - 2\tilde{f}\ (mg_1 + g_2) \ (-\nu)^{\alpha_{\pi_c} - 1} \xi_{\pi_c} \pi \alpha'_{\pi_c}$$

which contributes to the leading order differential cross section

$$\nu^2 \frac{d\sigma}{dt} \sim \Big\{ (\tfrac{t}{4})^2 (\tilde{g}f_1)^2 \; |\xi_\pi|^2 \; \nu^{2\alpha_\pi} + (\tfrac{t}{4})^2 (2\tilde{f}g_1)^2 \; |\xi_{\pi_c}|^2 \; \nu^{2\alpha_{\pi_c}}$$

$$+ (\tfrac{t}{4}) \left[2\tilde{f} (mg_1 + g_2)\right]^2 \; |\xi_{\pi_c}|^2 \; \nu^{2\alpha_{\pi_c}} \Big\}$$

The presence of the 1/t in the Regge contribution to $\tilde{A}_2$ allows as well a Class II conspiracy between $C_n(-)^J = -1$ and $C_n(-)^J = 1$ abnormal trajectories. For example, a conspiracy is clearly possible between an A_1 and that from a π-like (B-like) trajectory π' (B') with $\alpha_{A_1}(0) = \alpha_{\pi'}(0) + 1$. The defining relation is

$$\tilde{g}' f_2 \frac{(J-1)}{J} \frac{m\mu^2}{t} - \tilde{g}f_2 = \text{finite}, \; t = 0, \; \xi_{A_1} = -\xi_{\pi'}$$

where $\hat{g}' f_2 \sim$ cst, $\tilde{g}f_1 \sim 1/t$.
An $A_1 - \pi'$ conspiracy will not produce the sharp forward peak and is not considered in phenomenological fits(10,12,35). It is however of academic interest as the singular pion-like couplings cancel the 1/t in $\tilde{A}_2$ and allow the A_1 couplings to be non vanishing and to consequently contribute to $d\sigma/dt$ through $\tilde{A}_4$. Were the 1/t not cancelled the resulting $\hat{g}' f_2 \sim t$ behaviour would prevent any t = 0 contribution.

D. Evasion

In the process $\gamma p \longrightarrow \pi^0 p$ no forward peak is observed(10) which is consistent with the absence of pion exchange (forbidden by C-invariance) and evasive ω and B amplitudes are used to effect a good fit(10,57). From Table I we see that B and ω amplitudes are non singular to leading order and it is easily shown that daughter exchange removes the lower order singular terms. No evasive constraints are required. The dip in the ω-contribution[10] occurs in the covariant formalism through the vanishing of the $\tilde{\mathcal{C}}^{-}(\gamma 0 \alpha)$ coupling for $\alpha = 0$.

For charged photoproduction it is possible to exchange an A_1 and the apparently singular contribution to $\tilde{A}_2$ is rendered finite by imposing the $\mathcal{C}.\Delta$ condition on the nucleon vertex which forces $f_2 \sim t,\ t \longrightarrow 0$.

E. Helicity Formalism

As $\gamma N \longrightarrow \pi N$ is the most commonly considered photonic process and as most authors prefer to use helicity amplitudes and the constraint equations imposed upon them by analyticity as the context in which to discuss evasion and conspiracy, we here connect their approach with ours.

Following the standard procedure of Gell-Mann et al[58] we write

$$\bar{f}_{\lambda\mu} = \left[\cos\frac{\theta}{2}t\right]^{-|\lambda+\mu|} \left[\sin\frac{\theta}{2}t\right]^{-|\lambda-\mu|} f_{\lambda\mu}$$

where $f_{\lambda\mu} = f^{t}_{\lambda_1 \lambda_3;\ \lambda_2 \lambda_4}(s,t)$

$$\lambda = \lambda_1 - \lambda_3\ ,\quad \mu = \lambda_2 - \lambda_4,$$

θ_t is the t-channel scattering angle and $f^{t}_{\lambda_1 \lambda_3;\ \lambda_2 \lambda_4}(s,t)$ is defined by Jacob and Wick[59]. The $\bar{f}_{\lambda\mu}$ are KSF in s.

It is traditional to define the asymptotically parity conserving helicity amplitudes

$$\bar{f}^{\pm}_{\lambda\mu} = \bar{f}_{\lambda\mu} \pm \bar{f}_{-\lambda\mu} .$$

Actually

$$f^{\pm}_{o1} = \frac{\sin\Theta_t}{2} \bar{f}^{\pm}_{o1} \quad ; \quad f^{\pm}_{11} = \bar{f}^{\pm}_{11} + \cos\Theta_t \, \bar{f}^{\mp}_{11}$$

and we see that $\bar{f}^{\pm}_{o1}$ is parity conserving at all energies and only $\bar{f}^{\pm}_{11}$ contains an opposite ($\mp$) parity contribution one order below leading order. This point is not emphasized in the literature and it came to our attention when we calculated the Regge contributions to the $\bar{f}^{\pm}_{\lambda\mu}$ (Table II). It is also made by Henyey[12].

Expressing the $\bar{f}^{\pm}_{\lambda\mu}$ in terms of the invariant amplitudes,

$$\bar{f}^{-}_{o1} = p_t k_t \left[\tilde{A}_1 - \frac{t}{4} \tilde{A}_2 + m \tilde{A}_4 \right]$$

$$\bar{f}^{+}_{o1} = - \tfrac{1}{2} k_t \sqrt{t} \; \tilde{A}_1$$

$$\bar{f}^{-}_{11} = - \tfrac{1}{2} p_t k_t \sqrt{t} \; \tilde{A}_3$$

$$\bar{f}^{+}_{11} = k_t \left[m \tilde{A}_1 - p_t^2 \tilde{A}_4 \right]$$

enables us to calculate the Regge contributions given in Table II. The kinematic notation is given in Appendix I.

If these equations are inverted we see that the following constraints are necessary if $\tilde{A}_2$ and $\tilde{A}_4$ are to be KSF

$$i\,\bar{f}^{-}_{01} + \bar{f}^{+}_{11} \sim 0\,(\sqrt{t})\ ,\ t \to 0 \tag{5}$$

$$\bar{f}^{+}_{11} + \bar{f}^{+}_{01} \sim 0\,(m^2 - t/4),\ t \to 4m^2$$

$$\bar{f}^{+}_{11} + \frac{2m}{\mu}\,\bar{f}^{+}_{01} \sim 0(t-\mu^2)\ ,\ t \to \mu^2$$

$$\bar{f}^{-}_{01} \sim (t-4m^2)^{\frac{1}{2}}\,(t-\mu^2)\ ,\ t \to \mu^2\ ,\ t \to 4m^2\ .$$

The first constraint relating positive and negative parity exchanges at t = 0 is the traditional $\gamma N \to \pi N$ conspiracy relation[10,60]. It can alternately be satisfied by each amplitude behaving like $\sqrt{t}$ near t = 0 (evasion) or by the combination behaving like $\sqrt{t}$ (conspiracy).

In the case of pion conspiracy the π contributes to $\bar{f}^{-}_{01}$, the π_c and the A_1^c contribute to $\bar{f}^{+}_{11}$ (see Table II). It is readily seen from Table II that eqn (5) requires to leading order that

$$\sqrt{t}\ \left[-m\tilde{g}f_1 + 2\tilde{f}g_2\right] \sim \sqrt{t}\ ,\ t \to 0$$

which is just

$$\left.\begin{aligned} &\tilde{g}f_1 + 2\tilde{f}g_1 \sim \text{cst} \\ &mg_1 + g_2 \sim \sqrt{t} \end{aligned}\right\}\ t \to 0$$

and the conspiracy analysis proceeds exactly as in the previous section. If $\tilde{g}f_1$ and $\tilde{f}g_1$ are non singular, the solution is evasive and if they each behave like 1/t it is conspiratorial.

Although the $\bar{f}^{\pm}_{\lambda\mu}$ have been reggeized[10,11], they still contain kinematic zeros at pseudothresholds $t = 4m^2$, $t = \mu^2$ and kinematic singularities at $t = 0$. Wang[61] has shown how to remove the t singularity by writing

$$\tilde{f}^{\pm}_{\lambda\mu} = K^{\pm}_{\lambda\mu}(t) \; \bar{f}^{\pm}_{\lambda\mu}$$

where the $\tilde{f}^{\pm}_{\lambda\mu}$ are KSF in s and t and are the proper helicity amplitudes to reggeize. Because the $\tilde{f}^{\pm}_{\lambda\mu}$ are not KZF in t a constraint equation is still required to preserve analyticity at $t = 0$.

The problem now arises of the proper form for $K^{-}_{01}(t)$. Looking at the expression of the $\bar{f}^{\pm}_{\lambda\mu}$ in terms of the $\tilde{A}_i$ and keeping in mind that $\tilde{A}_2$ does not contain a kinematic singularity at $t = \mu^2$ due to gauge invariance[21] the proper K factors are

$$K^{-}_{01} = (t - \mu^2)^{-1} \; (t - 4m^2)^{-\frac{1}{2}} \; t^{\frac{1}{2}} \qquad K^{-}_{11} = (t - \mu^2)^{-1} (t - 4m^2)^{-\frac{1}{2}}$$

$$K^{+}_{01} = (t - \mu^2)^{-1} \qquad\qquad K^{+}_{11} = t^{\frac{1}{2}}(t - \mu^2)^{-1} \; .$$

We are in accord with Henyey and differ from BFJ[35] by a factor of $2\,k_t$ which is just necessary to cancel the pole due to gauge invariance (BFJ use $K^{-}_{01} = (t - 4m^2)^{-\frac{1}{2}} \sqrt{t}$).

The $t = 0$ constraint equation is

$$2m\,\tilde{f}^{-}_{01} - \tilde{f}^{+}_{11} \sim O(t) \quad , \quad t \rightarrow 0 \qquad (5)$$

and we note agreement with a recent paper by Daboul[20], BFJ use the constraint

$$2m\,\tilde{f}^{-}_{01} + \mu^2\,\tilde{f}^{+}_{11} \sim O(t) \quad , \quad t \to 0 \quad . \tag{6}$$

Even with pion conspiracy it is necessary to introduce a linear variation of the pion residue with t in order to reconcile the height of the forward peak with the known value of the pion nucleon coupling constant[12,35]. Such a parameterization leads to a zero in the pion residue function which Arbab and Dash[34] suggest is to be expected from O (3,1) considerations. One has complete freedom in such a parameterization provided that the pion pole term is recovered at $t = \mu^2$.

(i) Pion Reggeization

The arguement that the pion reggeizes which we have constructed for $\tilde{A}_2^{(-)}$ applies identically to $\tilde{f}^{-}_{01}$ since, for pion exchange

$$\tilde{f}^{-}_{01} = - \frac{t}{16}^{3/2} \tilde{A}_2^{(-)}$$

This particular sense-nonsense amplitude, for pion exchange, does not vanish for $\alpha_\pi = 0$, it does in fact reproduce the elementary pion exchange at the pole $t = \mu^2_\pi$. Had we used the K^{-}_{01} of BFJ it would not have been pole like at $\alpha = 0$, and would not have reproduced the elementary exchange. We stress again that this remarkable situation occurs when the exchanged particle is identical to an external particle.

Next consider f^{-}_{01} for pion exchange

$$f^-_{o1} = \frac{\sin\Theta_t}{2} \bar{f}^-_{o1} = - p_t k_t \frac{\sin\Theta_t}{2} . \frac{t}{4} \tilde{A}_2^{(-)}$$

Now, from Appendix I and ref (10)

$$- N (s,t)^2 = t(p_t k_t \sin\Theta_t)^2$$

$$= \tfrac{1}{4}[stu - tm^2(m^2 - u^2) - m^2 u^4]$$

So, $$f^-_{o1} = i \frac{\sqrt{t}}{8} N(s,t) \tilde{A}_2^{(-)}$$

Does $N(s,t) = 0, t = \mu^2$? No, since

$$- N(s,\mu^2) = \mu^2 [s(-s + 2m^2) - m^4]$$

$$= -\mu^2 (s - m^2)^2 \neq 0 , s \neq m^2$$

$$= - \mu^2 4 \nu^2$$

and $$f^-_{o1} = \frac{-i\mu^2 \nu}{8} . - 4 \frac{ef_1(\mu^2)}{\nu (t - \mu^2)} = - \frac{ie \mu^2}{2(t - \mu^2)} f_1(\mu^2)$$

the pion pole as expected.

The amplitude $\bar{f}^-_{o1}$ does not exhibit pole like behaviour, however the differential cross section

$$\frac{d}{dt} = \frac{2}{\pi s k_s^2} \left|\bar{f}^-_{o1}\right|^2 \sin^2\Theta t$$

$$= \frac{2}{s k_s^2} \left|f^-_{o1}\right|^2$$

does, provided that $\bar{f}^-_{01}$ does not vanish at $\alpha_\pi = 0$ - which it does not.

(ii) Gauge Invariance

We note that for massive photons (Appendix II, Table III) there are two additional amplitudes $\bar{f}^-_{00}$, $\bar{f}^+_{10}$ which must vanish as $\mu_v \rightarrow 0$. This is effected by requiring,

$$4 g_1 + \mu^2 g_2 = 0 , \quad \mu_v = 0 \quad (g_i \equiv h_i \text{ in Table III})$$

which is just the external gauge condition which we previously derived by requiring that $k_\mu . \mathcal{E}_\mu(10J) = 0$.

F. Superconvergence

An amplitude $A(\nu)$ which satifies a dispersion relation

$$A(\nu) = \frac{1}{\pi} \int_{-\infty}^{+\infty} d\nu' \; \frac{\text{Im } A(\nu')}{\nu' - \nu}$$

and is subject to the bound.

$$\left| A(\nu) \right| < \nu^{\varepsilon} \quad , \quad \varepsilon < -1, \quad \nu \text{ large}$$

satifies the superconvergence relation

$$\int_{-\infty}^{+\infty} d\nu \; \text{Im} \; A(\nu) = 0 \qquad .$$

If $A(\nu)$ is even under crossing $s \leftrightarrow u$ ($\nu \leftrightarrow -\nu$) the superconvergence relation is trivially satisfied. Consider then only amplitudes odd under crossing.

Using CGLN's isospin decomposition and crossing relations we have $\tilde{A}^{(-)}_{1,2,4}$ and $\tilde{A}_3^{(+,o)}$ odd and the rest even under crossing. The amplitudes $\tilde{A}^{(-)}$, $\tilde{A}^{(o)}$ correspond to I = 1 exchange in the t-channel and $\tilde{A}^{(+)}$ corresponds to I = 0 exchange. Treating $\tilde{A}^{(+)}$, $\tilde{A}^{(-)}$ as due to the ρ -like part of the photon (isovector) and $\tilde{A}^{(o)}$ due to the ω-like part (isoscalor) we deduce that in the t-channel B, ρ contribute to $\tilde{A}^{(o)}$, π, π_c, A_1, A_2 to $\tilde{A}^{(-)}$ and ω contributes to $\tilde{A}^{(+)}$.

Looking at the asymptotic behaviour of the $\tilde{A}_i$'s in Table I it is clear that there is a superconvergence relation on $\tilde{A}_3^{(+)}$ which will be dominated by ω-exchange. For $\tilde{A}^{(o)}$, there is a relation for $\tilde{A}_3^{(o)}$ dominated by ρ and a trivial relation for $\tilde{A}_2^{(o)}$. For $\tilde{A}^{(-)}$ there is no immediately obvious relation. If however we take the combination

$$\tilde{A}_1^{(-)} + m\tilde{A}_4^{(-)} - \frac{t}{4}\tilde{A}_2^{(-)}$$

only the pion contributes and the combination is superconvergent at $t = 0$, $\alpha_\pi < 0$.

In summary, we get relations for

$$\tilde{A}_3^{(+,o)}(\nu) \sim \nu^{\alpha-2} \qquad ; \omega, \rho$$

$$\tilde{A}_1^{(-)} + m\tilde{A}_4^{(-)} - \frac{t}{4}\tilde{A}_2^{(-)} \sim \nu^{\alpha-1} \qquad ; \pi$$

all of which have been noted[14,62,63,64,65]. However only ref (64) includes all three. Our point in this discussion is that given the asymptotic behaviour of the invariant amplitudes, the superconvergence relations are obvious.

2. Reggeization in $\gamma_\mu N \rightarrow \pi N^*_\nu$

A. Kinematic Covariants : Calculation

The $\mathcal{M}$ - function is

$$\tilde{\mathcal{M}}_{\mu\nu} = \mathcal{C}^+_\nu \left(\tfrac{1}{2}\tfrac{3}{2} J\right) : \mathcal{P}^J : \tilde{\mathcal{C}}^+_\mu (\gamma\, 0 J)$$

$$+ \mathcal{C}^-_\nu \left(\tfrac{1}{2}\tfrac{3}{2} J\right) : \mathcal{P}^J : \tilde{\mathcal{C}}^-_\mu (\gamma\, 0 J)$$

which we decompose in terms of the kinematic covariants[43]

$$\tilde{K}^1_{\mu\nu} = P_\nu \not{k} \gamma'_\mu \qquad \tilde{K}^5_{\mu\nu} = Q_\nu (k.P\, \gamma'_\mu - \not{k}\, P'_\mu)$$

$$\tilde{K}^2_{\mu\nu} = Q_\nu \not{k} \gamma'_\mu \qquad \tilde{K}^6_{\mu\nu} = Q_\nu P'_\mu - P_\nu P'_\mu - k.P g'_{\mu\nu}$$

$$\tilde{K}^3_{\mu\nu} = k.Q\, P_\nu P'_\mu \qquad \tilde{K}^7_{\mu\nu} = Q_\nu \gamma'_\mu - P_\nu \gamma'_\mu - g'_{\mu\nu} \not{k}$$

$$\tilde{K}^4_{\mu\nu} = k.Q\, P_\nu \gamma'_\mu \qquad \tilde{K}^8_{\mu\nu} = k.Q\, g'_{\mu\nu}$$

$$\tilde{K}^9_{\mu\nu} = P_\nu (k.P\, \gamma'_\mu - \not{k}\, P'_\mu)$$

$$\tilde{K}^{10}_{\mu\nu} = k.Q\, g'_{\mu\nu} \not{k}$$

where the equivalence theorems of Scadron and Jones[41,43] remove the last two (we drop $\mu\nu$ labels)

$$\tilde{K}^{9}_{\mu\nu} = + (m_+ + m_-)\tilde{K}^1 + \tilde{K}^4 - m_+\tilde{K}^6 - \tfrac{1}{4}(t - 4m_+^2)\tilde{K}^7 + m_-\tilde{K}^8$$

$$m_+\tilde{K}^{10}_{\mu\nu} = -(\nu - \tfrac{\mu^2}{2})\tilde{K}^1 - \tfrac{1}{4}(t - 4m_-^2)\tilde{K}^2$$

$$+ (\nu + k.Q)\left[\tilde{K}^6 - m_+\tilde{K}^7 + \tilde{K}^8\right]$$

$$+ m_-\left[\tilde{K}^4 - \tilde{K}^5 + \tfrac{\mu^2}{4}\tilde{K}^7 + (m_+ + m_-)\tilde{K}^8\right]$$

we also require

$$\gamma_\rho \frac{\partial}{\partial P_\rho}\tilde{K}^{3,6} = \tilde{K}^{4,7}$$

where we recall that $\gamma_\nu U^\lambda_\nu(N^*) = 0$ (Appendix I) and consequently $\gamma_\rho \frac{\partial}{\partial P_\rho} P_\nu = 0.$

Proceeding with normal exchange the M-function is,

$$\tilde{M}^+_{\mu\nu} = \gamma_5\left[f_1 P_{\rho_1} P_{\rho_2} P_\nu + f_2 \gamma_{\rho_1} P_{\rho_2} P_\nu + f_3 g_{\rho_1\nu} P_{\rho_2} + f_4 g_{\rho_1\nu}\gamma_{\rho_2}\right]:$$

$$\mathcal{P}^J : \epsilon_{\mu\alpha_1}(Q\Delta)\,\tilde{f}(t)$$

$$= \left[f_1\gamma_5 P_\nu \mathcal{P}^J_{;\alpha_1}\epsilon_{\mu\alpha_1}(Q\Delta) + f_2\gamma_5 P_\nu \gamma_{\rho_1}\mathcal{P}^J_{\rho_1;\alpha_1}\epsilon_{\mu\alpha_1}(Q\Delta)\right.$$

$$\left. + f_3\gamma_5\mathcal{P}^J_{\nu;\alpha_1}\epsilon_{\mu\alpha_1}(Q\Delta) + f_4\gamma_5\gamma_{\rho_2}\mathcal{P}^J_{\nu\rho_2;\alpha_1}\epsilon_{\mu\alpha_1}(Q\Delta)\right]\tilde{f}(t)$$

$$= \frac{c_J\,\tilde{f}(t)}{J^2(J-1)}\left\{ -J(J-1)\, f_1\,\gamma_5 P_\nu N_\mu \mathcal{P}'_J - (J-1)\, f_2\left[\gamma_5 P_\nu T_\mu \mathcal{P}'_J\right.\right.$$

$$- \gamma_5\left(\not{Q}(\Delta)\mathcal{P}''_J + Q(\Delta)^2 \not{P}(\Delta)\mathcal{P}''_{J-1}\right) P_\nu N_\mu \Big]$$

$$+ (J-1)\, f_3\gamma_5 \Big[-\mathcal{E}_{\mu\nu}(Q\Delta)\, \mathcal{P}'_J + Q_\nu(\Delta)\, N_\mu \mathcal{P}''_J$$

$$+ Q(\Delta)^2 P_\nu(\Delta)\, N_\mu \mathcal{P}''_{J-1} \Big]$$

$$+ f_4\, \gamma_5 \Big[\not{Q}\, \mathcal{E}_{\mu\nu}(Q\Delta)\, \mathcal{P}''_J + Q(\Delta)^2(\not{P}\mathcal{E}_{\mu\nu}(Q\Delta) + N_\mu \gamma_\nu(\Delta)\,)\mathcal{P}''_{J-1}$$

$$- T_\mu\ {}^J\mathcal{P}'_{\nu;} + \not{Q}\, N_\mu\ {}^J\mathcal{P}''_{\nu;}$$

$$+ Q(\Delta)^2 \not{P}\, N_\mu\ {}^{J-1}\mathcal{P}''_{\nu;} \Big] \Big\}$$

and using

$$\gamma_5 \not{Q}(\Delta)\, N_\mu = \gamma_5 \not{K} N_\mu + \frac{m_+}{t}t + 2Q.\Delta\,)\, \gamma_5 N_\mu$$

$$\gamma_5 \not{P}(\Delta)\, N_\mu = -\gamma_5 \frac{m_-}{t}\left[t - 4m_+^2\right] N_\mu$$

$$\gamma_5\, Q_\nu(\Delta)\, N_\mu = \gamma_5\, Q_\nu\, N_\mu + \frac{2Q.\Delta}{t}\gamma_5\, P_\nu N_\mu$$

$$\gamma_5\, P_\nu(\Delta)\, N_\mu = \frac{\left[t + 4m_+m_-\right]}{t}\, \gamma_5\, P_\nu\, N_\mu$$

$${}^J\mathcal{P}^{(n)}_{\nu;} = -\, Q_\nu \mathcal{P}_J^{(n-1)} - \frac{P_\nu}{t}\Big[2Q.\Delta\ \mathcal{P}_J^{(n-1)} + Q(\Delta)^2\left[t + m_+m_-\right]\mathcal{P}_{J-1}^{(n-1)}\Big]$$

along with the abnormal $\gamma N \rightarrow \pi N^*$ decompositions in Appendix III we extract the normal contributions to the invariant amplitudes given in Table V.

Abnormal exchange proceeds in a similar manner.

$$\tilde{M}^{-}_{\mu\nu} = \left[g_1 P_{\rho_1} P_{\rho_2} P_\nu + g_2 \gamma_{\rho_1} P_{\rho_2} P_\nu + g_3 g_{\rho_1 \nu} P_{\rho_2} + g_4 g_{\rho_1 \nu} \gamma_{\rho_2}\right] : \mathcal{P}^J : \tilde{g}(t) k.Q g'_{\mu\alpha_1}$$

$$= k.Q \tilde{g}(t) \left[g_1 P_\nu \mathcal{P}^J_{;\alpha_1} g'_{\mu\alpha_1} + g_2 P_\nu \gamma_{\rho_1} \mathcal{P}^J_{\rho_1;\alpha_1} g'_{\mu\alpha_1} \right.$$

$$\left. + g_3 \mathcal{P}^J_{\nu;\alpha_1} g'_{\mu\alpha_1} + g_4 \gamma_{\rho_2} \mathcal{P}^J_{\nu,\rho_2;\alpha_1} g'_{\mu\alpha_1} \right]$$

$$= \frac{\tilde{g}(t) c_J}{J^2(J-1)} \left\{ - J(J-1)\, g_1 k.Q\, P_\nu\, P'_\mu\, \mathcal{P}'_J \right.$$

$$- (J-1)\, g_2 k.Q\, P_\nu \left[\gamma'_\mu \mathcal{P}'_J - P'_\mu (\not{Q}(\Delta) \mathcal{P}''_J + Q(\Delta)^2 \not{P}(\Delta) \mathcal{P}''_{J-1})\right]$$

$$+ (J-1)\, g_3\, k.Q \left[- g'_{\mu\nu} \mathcal{P}'_J + Q_\nu P'_\mu \mathcal{P}''_J + Q(\Delta)^2 P_\nu(\Delta) P'_\mu \mathcal{P}''_{J-1}\right]$$

$$+ g_4 k.Q \left[\not{Q} g'_{\nu\mu} \mathcal{P}''_J + Q(\Delta)^2 (\not{P} g'_{\nu\mu} + P'_\mu \gamma_\nu(\Delta)) \mathcal{P}''_{J-1}\right.$$

$$- \gamma'_\mu\, {}^J\mathcal{P}'_{\nu;} + \not{Q} P'_\mu\, {}^J\mathcal{P}''_{\nu;}$$

$$\left.\left. + Q(\Delta)^2 \not{P} P'_\mu\, {}^{J-1}\mathcal{P}''_{\nu;} \right] \right\}$$

where we use

$$\not{Q}(\Delta) = \not{K} - \frac{m_-}{t}\left[t + 2Q.\Delta\right] \quad ; \quad Q_\nu(\Delta) = Q_\nu + \frac{2Q.\Delta}{t} P_\nu$$

$$\not{P}(\Delta) = \frac{m_+}{t}\left[t - 4m_-^2\right] \quad ; \quad P_\nu(\Delta) = \frac{P_\nu}{t}\left[t + 4m_+ m_-\right]$$

$$\gamma_\nu(\Delta) = \frac{4m}{t} P_\nu$$

and decompositions

$$k.Q\, P'_\mu Q_\nu = \tilde{\mathcal{K}}^3 + k.Q\,\tilde{\mathcal{K}}^6 + \nu\,\tilde{\mathcal{K}}^8$$

$$k.Q\, \gamma'_\mu Q_\nu = \tilde{\mathcal{K}}^4 + k.Q\,\tilde{\mathcal{K}}^7 + \tilde{\mathcal{K}}^{10}$$

$$k.Q\, P'_\mu P_\nu \not{k} = \nu\tilde{\mathcal{K}}^4 - k.Q\,\tilde{\mathcal{K}}^9$$

$$k.Q\, P'_\mu Q_\nu \not{k} = \nu\tilde{\mathcal{K}}^4 - k.Q\,\tilde{\mathcal{K}}^5 + \nu k.Q\,\tilde{\mathcal{K}}^7 + \nu\tilde{\mathcal{K}}^{10}$$

$$k.Q\, P'_\mu Q_\nu(\Delta)\not{k} = -4\frac{k.Q\,\nu}{t}\tilde{\mathcal{K}}^4 - k.Q\tilde{\mathcal{K}}^5 + \nu k.Q\,\tilde{\mathcal{K}}^7$$

$$-2k.Q\,\frac{Q.\Delta}{t}\tilde{\mathcal{K}}^9 + \nu\tilde{\mathcal{K}}^{10}$$

$$k.Q\, \gamma'_\mu Q_\nu(\Delta) = -4\frac{k.Q}{t}\tilde{\mathcal{K}}^4 + k.Q\tilde{\mathcal{K}}^7 + \tilde{\mathcal{K}}^{10}$$

$$k.Q\, P'_\mu Q_\nu(\Delta) = -4\frac{k.Q}{t}\tilde{\mathcal{K}}^3 + k.Q\tilde{\mathcal{K}}^6 + \nu\tilde{\mathcal{K}}^8$$

to get the contributions to the abnormal invariant amplitudes given in Table V.

B. Pion Reggeization in $\gamma p \rightarrow \pi^{\pm} N^{*0,++}$

The pion, as in the analysis of $\gamma N \rightarrow \pi N$, can only be exchanged when the external pion is charged. Examining the abnormal amplitudes in Table V and recalling that $g_{2,3,4} = \alpha\bar{g}_{2,3,4} \sim \alpha$, $\alpha \rightarrow 0$ we find that pion pole terms appear in $\tilde{A}_3^-$ and $\tilde{A}_8^-$

$$\tilde{A}_3^- \sim + \tilde{g}g_1 \, (-\nu)^{\alpha_\pi - 1} \, \xi_\pi \pi\alpha' \quad ; \; A_8^- \sim - \frac{\tilde{g}g_3}{\alpha} \, (\alpha - 1)(-\nu)^{\alpha_\pi - 1} \xi_\pi \pi\alpha'$$

and for $\nu \to \infty$, $t \to \mu^2$

$$\tilde{A}_3^- \sim - \frac{\tilde{g}(\mu^2) \, g_1(\mu^2)}{(t - \mu^2)\,\nu} \qquad \tilde{A}_8^- \sim - \frac{\tilde{g}(\mu^2) \, \bar{g}_3(\mu^2)}{(t - \mu^2)\,\nu}$$

and as $g(\mu^2)$ is not constrained to vanish by our arguements in section (III.1.A) the pion again reggeizes at $t = \mu^2$ in $\gamma N \to \pi N^*$. However, once again the presence of the (γOJ) vertex gives rise to a differential cross section for single Reggeon exchange proportional to t (Table IV) which vanishes at t = 0 and does not appear to predict the behaviour of the $d\sigma/dt$ data of ref (4) which rises from small t to a maximum near $-t = \mu^2$, then falls as e^{12t} out to $-t \sim 0.2$ Gev^2, after which it becomes roughly equal in slope and magnitude to the $\gamma p \to \pi^+ n$ cross section (e^{3t}).

C. Conspiracy in $\gamma p \to \pi^{\pm} \; N^{*\,o,++}$

Because the process involves four unequal masses the daughter trajectories are spaced by one unit of angular momentum. Their residues, as in $\gamma N \to \pi N$, are singular in t and are required to cancel the singular terms in the expansion

$$c_J \, \mathcal{P}_J' = J\,\nu(\Delta)^{J-1} - \frac{J(J-1)(J-2)}{1 \cdot 2(2J-1)} \; P(\Delta)^2 \, Q(\Delta)^2 \, \nu\,(\Delta)^{J-3} + \ldots$$

$$= J\nu^{J-1} - J(J-1)\nu^{J-2} \, \frac{(P.\Delta \, Q.\Delta\,)}{t} + \frac{J(J-1)(J-2)}{2} \frac{(P.\Delta \, Q.\Delta\,)^2}{t} \nu^{J-3}$$

$$- \frac{J(J-1)(J-2)}{1 \cdot 2(\;J-1)} \;\; P(\Delta)^2 \, Q(\Delta)^2 \, \nu^{J-3} \ldots$$

The first and all other odd daughters have signature opposite to the parents', even daughters have the same signature as the parent. Removing all of the singular contributions below leading order by daughter exchange and ignoring the rest, we reggeize and arrive at the amplitudes in Table VI. This permits a simplified conspiracy analysis.

Considering Class III pion conspiracy we let $\tilde{g}g_1 \sim 1/t$ in $\tilde{A}_3^-$ and cancel the singularity with the π_c contribution to $\tilde{A}_3^+$

$$2\tilde{f}\, f_1 + \tilde{g}\, g_1 \sim \text{finite},\ t \rightarrow 0, \qquad \tilde{A}_3$$

which implies the following relations

$$\tilde{g}\,\frac{g_4}{m_+} + 2\,\tilde{f}\left[m_+\,(f_1 m_- - f_2) - f_3\right] \sim \text{finite},\quad t \rightarrow 0,\ \tilde{A}_{1,6,7}$$

$$\tilde{g}g_3\,\frac{(\alpha - 1)}{\alpha} + 2\tilde{f}m_-\left[f_1 m_- - f_2\right] \sim \text{finite},\quad t \rightarrow 0,\ \tilde{A}_8$$

$$\tilde{g}\,\frac{g_4}{m_+} + 2\tilde{f}f_4 m_- \sim \text{finite},\ t \rightarrow 0,\ \tilde{A}_{2,5}$$

$$\tilde{g}g_2 - 2\tilde{f}\left[m_+\, f_1 + (f_1 m_- - f_2)\right] \sim \text{finite},\quad t \rightarrow 0,\ \tilde{A}_4$$

The singularities cancel consistently at t = 0 and permit a non vanishing cross section there. The M = 1 pion conspiracy along with t-dependent pion residues, should be able to account for the sharp peak. As for the rapid rise of $d\sigma/dt$ between t_{min} and $-t \sim \mu_\pi^2$ which appears to suggest a vanishing cross section at t = 0 the question arises 'how can a pion conspiracy predict both a sharp forward peak, <u>and</u> a vanishing $d\sigma/dt$ at $t = 0$.? To speculate on this consider the conspiracy relations for $\tilde{A}_3$ and $\tilde{A}_8$ and note the couplings $g_1\tilde{g}$ and $g_3\tilde{g}$ which appear in the pion pole terms are connected through $\tilde{f}f_1$, and could themselves conspire at $t = 0$ to effect the rapid t behaviour.[86] Such behaviour can only happen in a process where the pion contributes to more than one amplitude. In the $\gamma p \longrightarrow \pi^+ n$ situation once one accepts pion conspiracy one has a non vanishing cross section which could only be gotten rid of (should a turn over ever be found in the forward peak) with difficulty.

Finally we note the similarity between the conspiracy constraints and the internal gauge constraint on $\rho^-(\frac{1}{2}\ \frac{3}{2}\ J)$.

D. Evasion in $\gamma N \longrightarrow \pi N^*$

No $1/t$ terms appear in leading order contributions to the amplitudes and lower order $1/t$ occurrences can be dealt with by daughter exchange. At $t = 0$ the internal gauge condition will impose constraints on the fermion couplings but not on the boson couplings.

Pion exchange is forbidden (C-parity) in $\gamma p \longrightarrow \pi^0 N^{*+}$ and we expect the process to behave like $\gamma p \longrightarrow \pi^0 p$ (ω exchange).

E. Fits to Data[87]

For a discussion of attempts to fit $\gamma p \longrightarrow \pi^- N^{*++}$ see Harari in Ref (1) and the recent paper by Gotsman[66]. Gotsman using a vector dominance - Regge model produces a good fit to the data up to 8 Gev and in doing so uses four independent residues for each of normal and abnormal exchange. In the covariant formalism the number of independent residues is given immediately by the vertex function.

3. $\gamma N \to VN$

A. Photon-Vector Particle-Vertex

Consider the normal ($\gamma_\mu 1_\nu J$) vertex. A spin count reveals[45] $\frac{1}{2} \times 2 \times 3 = 3$ normal (or abnormal) couplings because the photon has only two spin states. The vector-vector vertex has five couplings, but application of gauge invariance removes two since $Q'_\mu = 0$, leaving the covariants $g_{\nu\alpha_1}$, $g'_{\mu\alpha_2}$, $g'_{\mu\alpha_1}$, $Q_\nu Q_{\alpha_2}$ and $g'_{\mu\nu} Q_{\alpha_1} Q_{\alpha_2}$. Then, cancellation of the singular parts[42,43] leaves us with the KSF form[40]

$$\tilde{\mathcal{E}}^{+}_{\mu\nu}(\gamma 1 J) = \Big[\tilde{g}_1(t)\, k.Q\, g'_{\mu\nu} Q_{\alpha_1} Q_{\alpha_2} + \tilde{g}_2(t)\,(2g'_{\mu\alpha_1} Q_\nu - g'_{\mu\nu} Q_{\alpha_1}) Q_{\alpha_2} + \tilde{g}_3(t)\, k.Q\, g'_{\mu\alpha_1} g_{\nu\alpha_2} \Big] Q_{\alpha_3} \cdots Q_{\alpha_J}$$

where $k_\nu \longrightarrow 2Q_\nu$, and $k_\alpha \longrightarrow Q_\alpha$ since $\Delta_\alpha P^J_{\alpha_i} = 0$.

Similarly, the abnormal vertex can be written in the KSF form

$$\tilde{\mathcal{E}}^{-}_{\mu\nu}(\gamma 1 J) = \Big[\tilde{f}_1(t)\, \mathcal{E}_{\mu\nu}(Q\Delta)\, Q_{\alpha_1} Q_{\alpha_2} + \tilde{f}_2(t) \mathcal{E}_{\mu\nu\alpha_1}(k) Q_{\alpha_2} + \tilde{f}_3(t)\, k.Q\, (g'_{\mu\alpha_1} \mathcal{E}_{\nu\alpha_2}(Q\Delta) + g_{\nu\alpha_1} \mathcal{E}_{\mu\alpha_2}(Q\Delta)) \Big] Q_{\alpha_3} \cdots Q_{\alpha_J}$$

We do not need the covariant $\mathcal{E}_{\mu\nu\alpha_1}(k')$ as

$$\tilde{\mathcal{E}}_{\mu\nu\alpha_1}(k') = \mathcal{E}_{\mu\nu\alpha_1}(k') - \frac{1}{k'.k}\, k'_\mu\, \mathcal{E}_{\nu\alpha_1}(k'k)$$

$$= -\frac{1}{(k'.k)} \left(\epsilon_{\mu\nu(Q\Delta)} Q\alpha_1 - \mu_v^2 \epsilon_{\mu\nu\alpha_1(k)} \right)$$

from the identity A2 of ref (41).

B. Reggeization in $\gamma N \longrightarrow VN$

The covariant $\mathcal{M}$-function is

$$\tilde{\mathcal{M}}_{\mu\nu} = \sum_{i=1}^{12} \tilde{A}_i \tilde{K}^i_{\mu\nu}$$

where(43)

$$\tilde{K}^1_{\mu\nu} = P_\nu \not{k} \gamma'_\mu \qquad \tilde{K}^7_{\mu\nu} = P_\nu (k.P \gamma'_\mu - \not{k}P'_\mu)$$

$$\tilde{K}^2_{\mu\nu} = Q_\nu \not{k} \gamma'_\mu \qquad \tilde{K}^8_{\mu\nu} = [\gamma_\nu, (k.P \gamma'_\mu - \not{k}P'_\mu)]$$

$$\tilde{K}^3_{\mu\nu} = k.Q\, P_\nu P'_\mu \qquad \tilde{K}^9_{\mu\nu} = [\gamma \not{k} \gamma']_{\mu\nu} = \gamma_\nu \not{k} \gamma'_\mu - \gamma'_\mu \not{k} \gamma_\nu$$

$$\tilde{K}^4_{\mu\nu} = k.Q(\gamma_\nu P'_\mu - P_\nu \gamma'_\mu) \qquad \tilde{K}^{10}_{\mu\nu} = 2Q_\nu P'_\mu - \nu g'_{\mu\nu}$$

$$\tilde{K}^5_{\mu\nu} = k.Q(\gamma_\nu P'_\mu + P_\nu \gamma'_\mu) \qquad \tilde{K}^{11}_{\mu\nu} = 2Q_\nu \gamma'_\mu - g'_{\mu\nu} \not{k}$$

$$\tilde{K}^6_{\mu\nu} = k.Q(\gamma_\nu \gamma'_\mu - \gamma'_\mu \gamma_\nu) \qquad \tilde{K}^{12}_{\mu\nu} = k.Q\, g'_{\mu\nu}$$

and also

$$\tilde{K}^{13}_{\mu\nu} = Q_\nu (k.P \gamma'_\mu - \not{k} P'_\mu)$$

$$\tilde{K}^{14}_{\mu\nu} = k.Q\, g'_{\mu\nu} \not{k}$$

where the equivalence theorems[41,43] remove the last two (we drop $\mu\nu$ labels)

$$\tilde{K}^{13} = -m\,\tilde{K}^{1} + \tilde{K}^{4} + \tfrac{1}{2}m\tilde{K}^{8} + \tfrac{1}{2}\,P^{2}\tilde{K}^{9}$$

$$m\,\tilde{K}^{14} = k.P(-\tilde{K}^{1} + \tfrac{1}{2}\tilde{K}^{8} + \tfrac{1}{2}m\,\tilde{K}^{9} + \tilde{K}^{12})$$

$$+ k.Q(-\tfrac{1}{2}\tilde{K}^{6} + \tilde{K}^{10} - m\,\tilde{K}^{11}) - \tfrac{1}{4}(t+\mu^{2})\tilde{K}^{2} \; .$$

We also require,

$$\gamma_{\beta}\frac{\partial}{\partial P_{\beta}}\,\tilde{K}^{1} = \tilde{K}^{11} + \tfrac{1}{2}\,\tilde{K}^{9}$$

$$\gamma_{\beta}\frac{\partial}{\partial P_{\beta}}\,\tilde{K}^{3} = \tilde{K}^{5}\;,\quad \gamma_{\beta}\frac{\partial}{\partial P_{\beta}}\tilde{K}^{4} = -\tilde{K}^{6}\;,\quad \gamma_{\beta}\frac{\partial}{\partial P_{\beta}}\tilde{K}^{5} = 2\,\tilde{K}^{12}$$

$$\gamma_{\beta}\frac{\partial}{\partial P_{\beta}}\,\tilde{K}^{10} = \tilde{K}^{11}$$

Proceeding with normal exchange the $\mathcal{M}$-function is

$$\tilde{\mathcal{M}}^{+}_{\mu\nu} = \left[g_1\,P_{\beta} + g_2\gamma_{\beta} \right] : \mathcal{P}^{J} : \tilde{C}^{+}_{\mu\nu}(\gamma\,1\,J)$$

$$= g_1\Big\{ \tilde{g}_1\,\mathcal{P}^{J}\,k.Q g'_{\mu\nu} + \tilde{g}_2(2\mathcal{P}^{J}_{;\alpha_1}g'_{\mu\alpha_1}Q_{\nu} - \mathcal{P}^{J}g'_{\mu\nu})$$

$$+ \tilde{g}_3\,k.Q\,\mathcal{P}^{J}_{;\nu\alpha_2}\,g'_{\alpha_2\mu}\Big\}$$

$$+ g_2\,\frac{1}{J}\;\gamma_{\beta}\frac{\partial}{\partial P_{\beta}}\Big\{\quad\Big\}$$

and we are deliberately emphasizing the differential technique.

The first term (g_1 coupling) also gives the $\mathcal{M}$-function for the process $\gamma\pi \rightarrow V\pi$ which we pause to consider. The only covariants available are $\tilde{\mathcal{K}}^3$, $\tilde{\mathcal{K}}^{10}$, $\tilde{\mathcal{K}}^{12}$ and we write the $\mathcal{M}$-function expansion as

$$\tilde{\mathcal{M}} = g_1 \sum_{i=3,10,12} \tilde{D}_i \tilde{\mathcal{K}}^i$$

and with the help of Appendix I

$$g_1 \tilde{D}_3 = g_1 \frac{c_J}{J(J-1)} \left[\tilde{g}_3 \, \mathcal{P}_J'' \right]$$

$$g_1 \tilde{D}_{10} = g_1 \frac{c_J}{J(J-1)} \left[-(J-1)\, \tilde{g}_2 \, \mathcal{P}_J' + \tilde{g}_3 \, k.Q \frac{(t+\mu_v^2)}{2t} \, P^2 \, \mathcal{P}_{J-1}'' \right]$$

$$g_1 \tilde{D}_{12} = g_1 \frac{c_J}{J(J-1)} \left[J(J-1)\tilde{g}_1 \, \mathcal{P}_J - (J-1)\tilde{g}_2 \frac{4k.Q}{t} P^2 \mathcal{P}_{J-1}' \right.$$

$$\left. - \tilde{g}_3 \, P^2 (\mathcal{P}_{J-1}' - \nu \frac{(t+\mu_v^2)}{2t} \mathcal{P}_{J-1}'') \right]$$

where also

$$Q_\nu \,(\Delta) = Q_\nu \frac{(t+\mu_v^2)}{t} \quad ; \quad Q(\Delta)^2 = -\frac{1}{4t}\,(t-\mu_v^2) = -4\frac{(k.Q)^2}{t}$$

and $m' = m$.

C. Normal Exchange in $\gamma\pi \rightarrow V\pi$; Pomeron and Rho Reggeization

The $(\pi\pi J)$ vertex permits only normal exchange and eliminates any possibility of conspiracy. The 1/t terms in $\tilde{D}_{10}$, $\tilde{D}_{12}$ are two or more orders below leading order and are cancelled by daughter exchanges which in this process (one equal mass vertex) differ by two units of angular momentum and the internal gauge condition imposes no restrictions.

Now we write out the full $(\gamma 1 J)$ coupling in order to discuss nonsense zeros.

$$\tilde{C}^{+}_{\mu\nu}(\gamma 1 J) = \Big\{\Big[\tilde{g}_1(t)(k.Qg_{\mu\nu} - 2Q_\mu Q_\nu)Q\alpha_1\, Q\alpha_2$$

$$\tilde{g}_2(t)(2g_{\mu\alpha_1}Q_\nu - g_{\mu\nu}\, Q\alpha_1)\, Q\alpha_2$$

$$\tilde{g}_3(t)(k.Q\, g_{\mu\alpha_1} - Q_\mu Q\alpha_1)\, g_{\nu\alpha_2}\Big] Q\alpha_3 \cdot\cdot\, Q\alpha_J \Big\}$$

At J = 0,

$$\tilde{C}^{+}_{\mu\nu}(\gamma 10) = \Big\{\Big[\tilde{g}_1(to)k.Qg'_{\mu\nu} - \tilde{g}_2(to)g_{\mu\nu}\Big] Q\alpha_1 \cdots Q\alpha_J \Big\}$$

to preserve gauge invariance $\tilde{g}_2(to) = 0$, $J = 0$ and $\tilde{g}_3(to) \equiv 0$ as the coupling does not exist at J = 0.

At J = 1,

$$\tilde{C}_{\mu\nu}(\gamma 11) = \Big\{\Big[\tilde{g}_1(t_1)\; k.Q\; g'_{\mu\nu}\;\; Q\alpha_1 Q\alpha_2$$

$$+ \tilde{g}_2(t_1)\;(2g'_{\mu\alpha_1}\, Q_\nu - g'_{\mu\nu}\;\, Q\alpha_1)\, Q\alpha_2$$

$$+ \tilde{g}_3(t_1)\;(- Q_\mu Q\alpha_1 g_{\nu\alpha_2})\Big] Q\alpha_3 \cdots Q\alpha_J \Big\}$$

where, to preserve gauge invariance

$$g_3\;(t)\; k.Q = 0,\; t = t_1$$

which implies that if $k.Q = 0$, $t = t_1 = \mu_v^2$ a nonsense zero is <u>not</u> required in $g_3(t)$ at $t = t_1$, $J = 1$.

Next consider the reggeized differential cross section (Table IV)

$$\nu^2 \frac{d}{dt} \sim \left[g_{\pi}^2 \right] \left[\left((t-\mu_v^2)\tilde{g}_1 + 4\hat{g}_2 \right)^2 + \tfrac{1}{2} t\hat{g}_1\tilde{g}_3 - \frac{t}{4\mu_v^2}(2\tilde{g}_2 + \hat{g}_3)^2 \right] \nu^{2\alpha} |\xi_{\pm}|^2$$

For $\rho^{\pm}$ exchange in $\gamma\pi^{\pm} \longrightarrow \rho^{\pm}\pi^{0}$ at $t = \mu_\rho^2$ $\tilde{g}_3(\mu_\rho^2) \neq 0$. Since t, $\tilde{g}_2$, $\tilde{g}_1 \neq 0$ also, the cross section develops a rho pole at $t = \mu_\rho^2$, as expected. For Pomeron exchange in $\gamma\pi \longrightarrow (\rho^{0}, \omega, \phi)\pi$, $\tilde{g}_3(0) = 0$, $t = 0$ and only the first term in the cross section, $(-\mu_v^2 \; \tilde{g}_1 + 4\tilde{g}_2)^2$, remains. Since the internal gauge condition imposes no constraints on these couplings, the Pomeron contribution is non vanishing, in other words, the Pomeron reggeizes at $t = 0$ in $\gamma\pi \longrightarrow (\rho^{0}, \omega, \phi)\pi$.

The arguement that $\rho^{\pm}$ reggeizes in $\gamma p \rightarrow \rho^{\pm} p$ and the Pomeron reggeizes in $\gamma p \longrightarrow (\rho^{0}, \omega, \phi)p$ is essentially the same as above. Since $t = 0$ is just outside the physical region for $\gamma p \rightarrow \rho^{0} p$ we would expect $d\sigma/dt \sim \nu^{2\alpha_P(t) - 2}$ to fall off slowly with increasing energy in the intermediate energy region and less slowly as energy increases and $t = 0$ becomes very close to the physical region and $\alpha(t_{foward}) \rightarrow 1$. Harari in ref (1) points out that for $\gamma p \longrightarrow \rho^{0} p$ the cross section approaches a constant for high energies and this would appear to confirm Pomeron reggeization at the (γ 1 J) vertex(67).

D. Normal Exchange in $\gamma N \longrightarrow VN$

Once the differentiation by $\gamma_\beta \frac{\partial}{\partial P_\beta}$ is carried out we require the following decompositions

$$\not{\!\emptyset}\, \tilde{\mathcal{K}}^{3} = -\, k.Q\tilde{\mathcal{K}}^{7} + \tfrac{1}{2}\, k.P(\tilde{\mathcal{K}}^{5} - \tilde{\mathcal{K}}^{4})$$

$$\not{k}\,\tilde{\mathcal{K}}^{10} = k.P\,\tilde{\mathcal{K}}^{11} - 2\,\tilde{\mathcal{K}}^{13}$$

$$= 2m\,\tilde{\mathcal{K}}^{1} - 2\,\tilde{\mathcal{K}}^{4} - m\,\tilde{\mathcal{K}}^{8} - P^2\,\tilde{\mathcal{K}}^{9} + k.P\,\tilde{\mathcal{K}}^{11}$$

$$m\not{k}\,\tilde{\mathcal{K}}^{12} = \tilde{\mathcal{K}}^{14}$$

$$= k.P(-\tilde{\mathcal{K}}^{1} + \tfrac{1}{2}\tilde{\mathcal{K}}^{8} + \tfrac{1}{2}m\tilde{\mathcal{K}}^{9} + \tilde{\mathcal{K}}^{12})$$

$$+ k.Q(-\tfrac{1}{2}\tilde{\mathcal{K}}^{6} + \tilde{\mathcal{K}}^{10} - m\,\tilde{\mathcal{K}}^{11})$$

$$- \tfrac{1}{4}(t + \mu'^2)\,\tilde{\mathcal{K}}^{2}$$

where $m_- = 0$ and the $\tilde{\mathcal{K}}^{13}$, $\tilde{\mathcal{K}}^{14}$ decomposition are taken from ref (43).

Performing the $\gamma_\rho \frac{\partial}{\partial P_\rho}$ differentiation and denoting $\gamma_\rho \frac{\partial}{\partial P_\rho} \tilde{D}_i$ by $\tilde{D}_i'$,

$$\tilde{\mathcal{M}}^{+}_{\mu\nu} = \frac{c_J}{J^2(J-1)} \Big\{ J\, g_1 \Big[\tilde{D}_{12}\,\tilde{\mathcal{K}}^{12} + \tilde{D}_3\,\tilde{\mathcal{K}}^{3} + \tilde{D}_{10}\,\tilde{\mathcal{K}}^{10} \Big]$$

$$+ g_2 \Big[\tilde{D}_3\,\tilde{\mathcal{K}}^{5} + \tilde{D}_{10}\,\tilde{\mathcal{K}}^{11} + \tilde{D}_{12}'\,\tilde{\mathcal{K}}^{12}$$

$$+ \tilde{D}_3'\,\tilde{\mathcal{K}}^{3} + \tilde{D}_{10}'\,\tilde{\mathcal{K}}^{10} \Big] \Big\}$$

and using

$$\gamma_\rho \frac{\partial}{\partial P_\rho}\, \mathcal{P}_J^{(n)} = - \Big[\not{k}\, \mathcal{P}_J^{(n-1)} + mQ(\Delta)^2\, \mathcal{P}_{J-1}^{(n-1)} \Big] , \quad m_- = 0$$

we find that

$$\tilde{D}_3' = -\, m\, Q(\Delta)^2\, \mathcal{P}_{J-1}'''\, \tilde{g}_3 \;-\; \not{k}\, \mathcal{P}_J'''\, \tilde{g}_3$$

$$\tilde{D}_{10}' = -mQ(\Delta)^2 \left[-\,(J-1)\, \tilde{g}_2\, \mathcal{P}_{J-1}'' + \tilde{g}_3 k\cdot Q\, \frac{\left[t+\mu_v^2\right]}{2t}\, \mathcal{P}_{J-2}''' \right]$$

$$+\; 2m\, \tilde{g}_3 k\cdot Q\, \frac{\left[t+\mu_v^2\right]}{2t}\, \mathcal{P}_{J-1}''$$

$$-\; \not{k} \left[-\,(J-1)\, \tilde{g}_2\, \mathcal{P}_J'' + \tilde{g}_3 k\cdot Q\, \frac{\left[t+\mu_v^2\right]}{2t}\, p^2\, \mathcal{P}_{J-1}''' \right]$$

$$\tilde{D}_{12}' = -mQ(\Delta)^2 \left[J(J-1)\tilde{g}_1 \mathcal{P}_{J-1}' - (J-1)\tilde{g}_2\, \frac{4k\cdot Q}{t}\, p^2\, \mathcal{P}_{J-2}'' \right.$$

$$\left. -\; \tilde{g}_3\, p^2 \left(\mathcal{P}_{J-2}'' - \nu \frac{(t+\mu_v^2)}{2t}\, \mathcal{P}_{J-2}''' \right) \right]$$

$$-\; 2m \left[\tilde{g}_3 \left(\mathcal{P}_{J-1}' - \nu \frac{(t+\mu_v^2)}{2t}\, \mathcal{P}_{J-1}'' \right) + (J-1)\tilde{g}_2\, \frac{4k\cdot Q}{t}\, \mathcal{P}_{J-1}' \right]$$

$$-\; \not{k} \left[(J)(J-1)\tilde{g}_1\, \mathcal{P}_J' - (J-1)\tilde{g}_2\, \frac{4k\cdot Q}{t}\, p^2\, \mathcal{P}_{J-1}'' \right.$$

$$\left. -\; \tilde{g}_3\, p^2 \left(\mathcal{P}_{J-1}'' - \frac{(t+\mu_v^2)}{2t} \left(\nu\, \mathcal{P}_{J-1}''' + \mathcal{P}_{J-1}'' \right) \right) \right]$$

Using the $\not{k}\, \tilde{\mathcal{K}}^i$ decompositions we can extract the contributions to the invariant amplitudes for normal exchange (Table VII). The discussion of Pomeron reggeization is identical to that for $\gamma\pi \to \rho^\circ\pi$, as is the arguement that the $\rho^\pm$ exchange reggeizes at $t = \mu_\rho^2$ for $\gamma p \to \rho^+ n$, $\gamma n \to \rho^- p$.

F. Abnormal Exchange in $\gamma N \to VN$

The $\mathcal{M}$-function for abnormal exchange is

$$\tilde{M}_{\mu\nu} = \gamma_5 \left[f_1 P_\nu + f_2 \gamma_\nu \right] : \mathcal{P}^J : \left[\tilde{f}_1 \, \epsilon_{\mu\nu}(Q\Delta) \, Q_{\alpha_1} Q_{\alpha_2} \right.$$

$$+ \tilde{f}_2 \, \epsilon_{\mu\nu\alpha_1}(k) \, Q_{\alpha_2}$$

$$\left. + \tilde{f}_3 k.Q (g'_{\mu\alpha_1} \epsilon_{\nu\alpha_2}(Q\Delta) + g_{\nu\alpha_1} \epsilon_{\mu\alpha_2}(Q\Delta)) \right]$$

where the $(N\bar{N}J)$ coupling is split into C-normal (f_1) and C-abnormal (f_2) parts; the latter to be derived from the former by differentiation as in the previous section. Proceeding then with C-normal (i.e. pion exchange) coupling.

$$\tilde{M}^{-,+}_{\mu\nu} = \gamma_5 f_1 \left[\tilde{f}_1 \mathcal{P}^J \epsilon_{\mu\nu}(Q\Delta) + \tilde{f}_2 \, \mathcal{P}^J_{;\alpha_1} \epsilon_{\mu\nu\alpha_1}(k) \right.$$

$$+ \tilde{f}_3 \, k.Q \, \mathcal{P}^J_{;\mu\alpha_2} \epsilon_{\nu\alpha_2}(Q\Delta) - \tilde{f}_3 \mathcal{P}^J_{;\alpha_2} \epsilon_{\nu\alpha_2}(Q\Delta) Q_\mu$$

$$\left. + \hat{f}_3 \, k.Q \, \mathcal{P}^J_{;\nu\alpha_2} \epsilon_{\mu\alpha_2}(Q\Delta) \right]$$

and, using Appendix I,

$$\tilde{M}^{-,+}_{\mu\nu} = \frac{\gamma_5 f_1 \, c_J}{J(J-1)} \left\{ J(J-1) \, \mathcal{P}_J \, \tilde{f}_1 \, \epsilon_{\mu\nu}(Q\Delta) - (J-1) \tilde{f}_2 \left[\mathcal{P}'_J \epsilon_{\mu\nu}(Pk) \right. \right.$$

$$\left. + \mathcal{P}'_{J-1} \, P^2 \, \frac{(t-\mu_V^2)}{t} \, \epsilon_{\mu\nu}(Qk) \right]$$

$$+ \tilde{f}_3 \, k.Q \left[- \epsilon_{\nu\mu}(Q\Delta) \, P^2 \, \mathcal{P}'_{J-1} + P_\mu N_\nu \mathcal{P}''_J \right.$$

$$\left. + Q_\mu \, (\Delta) \, N_\nu P^2 \, \mathcal{P}''_{J-1} \right]$$

$$- \tilde{f}_3 \left[\nu \mathcal{P}_J'' + P^2 Q\,(\Delta)^2\, \mathcal{P}_{J-1}'' \right] N_\nu Q_\mu$$

$$+ \tilde{f}_3\, k.Q \left[- \mathcal{E}_{\mu\nu(Q\Delta)} P^2 \mathcal{P}_{J-1}' + P_\nu N_\mu \mathcal{P}_J'' \right.$$

$$\left. + Q_\nu(\Delta)\, N_\mu\, P^2\, \mathcal{P}_{J-1}'' \right] \quad \Bigg\}$$

where we have used

$$Q_\alpha(\Delta) = Q_\alpha \frac{(t-\mu_v^2)}{t} \quad ; \quad \mathcal{E}_{\mu\nu(Q\Delta)} = 2\, \mathcal{E}_{\mu\nu(Qk)} \; ; \; m_- = 0.$$

Finally,

$$\tilde{\mathcal{M}}_{\mu\nu}^{-,+} = \frac{\gamma_5\, f_1\, c_J}{J(J-1)} \Big[F_1 \mathcal{E}_{\mu\nu(kQ)} + F_2\, \mathcal{E}_{\mu\nu(kP)} + F_3 k.Q\, P_\mu' N_\nu$$

$$+ F_4\, P_\nu\, N_\mu + F_5\, Q_\nu\, N_\mu \Big]$$

where

$$F_1 = -(J-1)(2\tilde{f}_1 J \mathcal{P}_J + \tilde{f}_2\, P^2\, \frac{4k.Q}{t}\, \mathcal{P}_{J-1}')$$

$$F_2 = (J-1)(\tilde{f}_2 \mathcal{P}_J')$$

$$F_3 = \tilde{f}_3 \mathcal{P}_J''$$

$$F_4 = k.Q\, \tilde{f}_3 \mathcal{P}_J''$$

$$F_5 = \tilde{f}_3\, k.Q\, P^2\, \frac{(t+\mu_v^2)}{t}\, \mathcal{P}_{J-1}''$$

Using the abnormal decompositions and related relations in Appendix III we get the contributions to the invariant amplitudes given in Table VII.

Differentiating the C-normal result by $\gamma_\beta \frac{\partial}{\partial P_\beta}$ gives us the following $\mathcal{M}$-function for C-abnormal exchange.

$$\tilde{\mathcal{M}}^{-,-}_{\mu\nu} = \frac{\gamma_5 f_2 c_J}{J^2(J-1)} \Big[F_2 \, \epsilon_{\mu\nu}(k\gamma) + F_3 k.Q \, \gamma'_\mu N_\nu$$

$$+ F_3 k.Q \, P'_\mu T_\nu + F_4 \, \gamma_\nu N_\mu$$

$$+ F_4 P_\nu T_\mu + F_5 Q_\nu T_\mu$$

$$+ F'_1 \, \epsilon_{\mu\nu}(kQ) + F'_2 \, \epsilon_{\mu\nu}(kP) + F'_3 \, k.Q \, P'_\mu N_\nu$$

$$+ F'_4 P_\nu N_\mu + F'_5 Q_\nu N_\mu \Big]$$

where again, with the decompositions in Appendix III we get the contributions to the amplitudes in Table VII. To compress the table we define $\bar{F}_i$, $\bar{\bar{F}}_i$ such that

$$F'_i = \not{k} \bar{F}_i + \bar{\bar{F}}_i$$

The asymptotic behaviour of either $\bar{F}$, $\bar{\bar{F}}$ is one below that of F since

$$\gamma_5 \gamma_\beta \frac{\partial}{\partial P_\beta} \mathcal{P}^{(n)}_J = - \gamma_5 \not{k} \, \mathcal{P}^{(n-1)}_J + 4 \gamma_5 m \frac{kQ}{t} \mathcal{P}^{(n-1)}_J$$

giving,

$$\bar{F}_1 = +(J-1)(2\tilde{f}_1 J \mathcal{P}_J' + 4\frac{k.Q}{t}p^2 \tilde{f}_2 \mathcal{P}_{J-1}'') \quad , \quad \bar{\bar{F}}_1 = -\frac{4m}{t}k.Q(J-1)(2\tilde{f}_1 J \mathcal{P}_J' + 4\frac{k.Q}{t}p^2 \tilde{f}_2 \mathcal{P}_{J-1}'')$$

$$\bar{F}_2 = -(J-1)\tilde{f}_2 \mathcal{P}_J'' \quad , \quad \bar{\bar{F}}_2 = (J-1)\tilde{f}_2 \, 4\frac{m}{t} k.Q \, \mathcal{P}_J''$$

$$\bar{F}_3 = -\tilde{f}_3 \mathcal{P}_J''' \quad , \quad \bar{\bar{F}}_3 = \tilde{f}_3 \, 4\frac{m}{t} k.Q \, \mathcal{P}_J'''$$

$$\bar{F}_4 = -\tilde{f}_3 k.Q \, \mathcal{P}_J''' \quad , \quad \bar{\bar{F}}_4 = \tilde{f}_3 \, 4\frac{m}{t} (k.Q)^2 \, \mathcal{P}_J'''$$

$$\bar{F}_5 = -\tilde{f}_3 k.Q \, p^2 \frac{(t+\mu_v^2)}{t} \mathcal{P}_{J-1}''' \, , \quad \bar{\bar{F}}_5 = \tilde{f}_3 \, 4\frac{m}{t} (k.Q)^2 \, p^2 \frac{(t+\mu_v^2)}{t} \mathcal{P}_{J-1}''' \, .$$

Examination of the nonsense zeros in $\tilde{C}^{-}_{\mu\nu}(\gamma 1 J)$ reveals that for $J = 0$, $\tilde{f}_2$, $\tilde{f}_3$ vanish while $\tilde{f}_1$ remains and for $J = 1$, $\tilde{f}_1$, $\tilde{f}_2$ remain while $\tilde{f}_3$ vanishes provided that $k.Q \neq 0$, $J = 1$. As there are no known abnormal particles with mass and spin identical to that of ρ, ω, ϕ we consider $\tilde{f}_3$ to have a nonsense zero at $J = 1$.

Pion reggeization presents no problem in $\gamma N \rightarrow VN$ as $\tilde{f}_1$ is non vanishing at $\alpha = 0$, $t = \mu_\pi^2$ and the pion pole appears in the differential cross section (Table IV),

$$\nu^2 \frac{d\sigma}{dt} \sim \left[-2t f_1^2\right]\left[((t-\mu_v^2)\tilde{f}_1 + 4\tilde{f}_2) - \frac{t}{4\mu_v^2}(4\tilde{f}_2 - (t-\mu_v^2)\tilde{f}_3)^2\right]\nu^{2\alpha_\pi(t)} \left|\xi_\pi\right|^2$$

which vanishes (along with all other abnormal exchange contributions) at $t = 0$.

Pion exchange does not play the dominant role in $\gamma N \rightarrow (\rho, \omega, \phi)N$

that it does in $\gamma N \rightarrow \pi N$, πN^* nor are its difficulties the same. As $\gamma N \rightarrow V^0 N$ is a 'pseudoelastic' process[1] the diffraction mechanism (Pomeron exchange in a Regge model) takes over in the near forward direction and pion exchange, although possible, becomes less important. For example, the behaviour of the $\gamma p \rightarrow \rho^0 p$ data is almost entirely consistent with diffraction[1,68]. However, the explanation of $\gamma p \rightarrow \omega p$ data requires both diffraction and pion exchange[1,68]. Maor and Yock[69] suggest that this can be explained by the conservation of Bronzan and Low's[70] A-quantum number which forbids the vertex $(\gamma \pi \rho)$ while allowing $(\gamma \pi \omega)$. Authors[71,72,73] who do Regge fits to the $\gamma p \rightarrow \rho^0 p$, $\gamma p \rightarrow \omega p$ data, although their approaches vary, treat the former process as diffractive and the latter as a combination of diffraction and pion exchange.

As we are examining a formalism and not dynamics we offer no solution to the $\gamma p \rightarrow \phi p$ problem. Harari[68] points out that the dominance of a diffraction mechanism is consistent with the data on ρ, ω production but predicts a ϕ production rate too large by an order of magnitude. He also comments that no version of the diffraction picture is capable of predicting the correct value for both $\sigma_\rho ; \sigma_\omega$ and $\sigma_\rho ; \sigma_\phi$ and that a combination of diffraction and pion exchange does not help.

F. Conspiracy in $\gamma N \rightarrow VN$

Conspiracy at t = 0 has not been invoked to fit $\gamma N \rightarrow VN$ data[88] as the reggeized pion exchange in conjunction with Pomeron exchange has proved adequate[72,73]. We show that a Class III pion conspiracy is possible at t = 0 (that is, it gives a non vanishing contribution to $d\sigma/dt$ at t = 0), consistent with factorization which requires that if the pion conspires at the $\bar{N}N$ vertex in $\gamma N \rightarrow \pi N$, so must it also in $\gamma N \to V^0 N$. The absence of the pion peak in $\gamma p \to \rho^0 p$ can be explained by the dominance of the Pomeron exchange or perhaps by evasion at the $(\gamma\rho^0\pi)$ vertex. That is, the couplings at the $(\gamma\rho^0\pi)$ vertex would eliminate any singularities introduced by the conspiring $(N\bar{N}\pi)$ vertex and the conspiracy, like evasion would result in a vanishing $d\sigma/dt$ at $t = 0$. This suggestion is prompted by the discovery in section IV.3 that the conspiring pion cannot give $d\sigma/dt \neq 0$, $t = 0$ in $\gamma N \to \gamma N$. For $\gamma p \to \omega^0 p$ we would expect a fit similar to that resulting from a non conspiring, reggeized pion.

The equal mass $(N\bar{N}J)$ vertex gives rise to daughter terms separated by two units of angular momentum and adjusting the singular daughter residues gives us freedom to cancel all singular contributions to the invariant amplitudes two orders below the leading order contributions. This done, we reggeize and present the results in Table VIII.

From our study of $\gamma N \to \pi N$ we know that the abnormal nucleon couplings f_1, $f_2 \sim t^{-\frac{1}{2}}$ for pion conspiracy. The relations required to keep the invariant amplitudes finite at $t = 0$ are

$$\frac{g_2}{m}\tilde{g}_1 - f_1\left[2\tilde{f}_1 + \tilde{f}_3\right] \sim \text{cst}, \; t \to 0 \qquad \tilde{A}_{1,8,9}$$

$$\frac{\mu_v^2}{4}\,\frac{g_2}{m}\,\tilde{g}_1 + f_1\left[\tilde{f}_2 + \frac{\mu_v^2}{2}\tilde{f}_3\right] \sim \text{cst}, \; t \to 0 \qquad \tilde{A}_{2,6}$$

$$g_1\tilde{g}_3 + \frac{\mu_v^2}{2} f_1\tilde{f}_3 \sim \text{cst}, \; t \to 0, \qquad \tilde{A}_{3,4,5}$$

and

$$g_2\,(k.Q\tilde{g}_1) + m g_1\tilde{g}_2 \sim \text{cst}, \; t \to 0, \qquad \tilde{A}_{10}$$

$$(\tilde{g}_1 k.Q - \tilde{g}_2) \sim O(\sqrt{t}), \; t \to 0, \qquad \tilde{A}_{11}$$

$$(m g_1 + g_2) \sim O(\sqrt{t}), \; t \to 0, \qquad \tilde{A}_{12}$$

The residue of $\tilde{A}_7$ and part, $(2(\alpha -1)\ g_2\tilde{g}_2 - \alpha g_2 g_3)$, of the $\tilde{A}_4$ residue remain uncancelled, however the introduction of an A_1^c effects the cancellation as it did in $\gamma N \longrightarrow \pi N$ and the Class III conspiracy is complete.

It is interesting to note that the normal (π_c) contribution to $d\sigma/dt$ vanishes at t = 0 because of the constraint on $\tilde{A}_{11}$. Were we to carry this further and impose our suggestion of evasion at the ($\gamma V \pi$) vertex the following conditions would be required

$$\tilde{g}_1 \sim \sqrt{t} \ , \quad 2\tilde{f}_1 + \tilde{f}_3 \sim \sqrt{t} \ , \quad t \to 0 \qquad \tilde{A}_{1,8,9}$$

$$\tilde{f}_2 + \frac{\mu_v^2}{2}\tilde{f}_3 \sim \sqrt{t} \ , \quad t \to 0 \qquad \tilde{A}_{2,6}$$

$$g_1\tilde{g}_3 + \frac{\mu_v^2}{2} f_1\tilde{f}_3 \sim \text{cst} \ , \quad t \to 0 \qquad \tilde{A}_{3,4,5}$$

$$\tilde{g}_2 \sim \sqrt{t} \ , \quad t \to 0 \qquad \tilde{A}_{10}$$

G. <u>Evasion in $\gamma N \longrightarrow VN$</u>

For any $\gamma N \longrightarrow VN$ process there is a 1/t in the leading order contribution to $\tilde{A}_3$ resulting from A_1 exchange which can be removed by imposing internal gauge invariance on the nucleon vertex as in $\gamma N \longrightarrow \pi N$.

As we have already pointed out the data for $\gamma N \longrightarrow VN$ can be fitted with evasive amplitudes[72,73] the only draw back being that a consistent fit for all of the processes involved ($\gamma p \longrightarrow \rho^o p$,

$\gamma p \rightarrow \omega p$, $\gamma p \rightarrow \phi p$) has not been achieved[68].

Evasive solutions fit the data,[72,73] but not all the data consistently[68].

IV COMPTON SCATTERING

1. Photon-Photon Vertex[40]

For the $\gamma\,\gamma$ J vertex there are two couplings for normal or abnormal exchanges. C-parity further divides these vertices into J-even or J-odd classes. That is, from the point of view of analytic continuation to complex J, an even J coupling implies a factor $\frac{1}{2}(1 + (-)^J)$ and an odd coupling implies a factor $\frac{1}{2}(1 - (-)^J)$. Hence, together with a positive signature trajectory, $\frac{1}{2}(1 + (-)^J)$, for integer J we have $\frac{1}{2}(1 + (-)^J)\,\frac{1}{2}(1 + (-)^J) = \frac{1}{2}(1 + (-)^J) \rightarrow \frac{1}{2}(1 + e^{-i\pi\alpha})$ and $(1 + (-)^J)(1 - (-)^J) = 0$.

Normal exchanges lead to the covariants $g'_{\mu\nu}$ and $g'_{\mu\alpha_1} g'_{\nu\alpha_2}$ where

$$g'_{\mu\alpha} = g_{\mu\alpha} - \frac{k'_\mu\, k_\alpha}{k'.k}$$

$$g'_{\mu\nu} = g_{\mu\nu} - \frac{k'_\mu\, k_\nu}{k'.k}$$

and $t = -2k'.k$. Cancellation of the $1/t^2$ and $1/t$ terms leads to a form of vertex which does not induce kinematic singularities into the Regge contributions to the invariant amplitudes.

$$\tilde{C}^{+}_{\mu\nu} = \frac{1}{2}(1 + (-)^J)\Big[t\tilde{g}_1(t)\, g'_{\mu\nu}\, Q_{\alpha_1} Q_{\alpha_2} + \tilde{g}_2(t)\,(t g'_{\mu\alpha_1} g'_{\nu\alpha_2} - 2 g'_{\mu\nu} Q_{\alpha_1} Q_{\alpha_2})\Big] Q_{\alpha_3} \cdots Q_{\alpha_J}$$

For abnormal exchanges the couplings split, one for J even and one for J odd. The final abnormal vertices are

$$\tilde{C}^{-}_{\mu\nu}\ (\gamma\gamma\ J) = \tfrac{1}{2}(1 + (-)^{J}\ )\ \tilde{f}_1(t)\ \mathcal{E}_{\mu\nu}(Q\Delta)\ Q\alpha_1 \ldots Q\alpha_J$$

for J even, and

$$\tilde{C}^{-}_{\mu\nu}\ (\gamma\gamma\ J) = \tfrac{1}{2}(1 - (-)^{J}\ )\Big[\tilde{f}_2(t)\ \ g'_{\nu\alpha_1}\mathcal{E}_{\mu\alpha_2}(Q\Delta)$$

$$+\ g'_{\mu\alpha_1}\mathcal{E}_{\nu\alpha_2}(Q\Delta)\Big]\ Q\alpha_3 \ldots Q\alpha_J$$

for J odd. For the J odd case we need not add a $\mathcal{E}_{\mu\nu\alpha_1}(Q)$ term as

$$\tilde{\mathcal{E}}_{\mu\nu\alpha_1}(Q) = \mathcal{E}_{\mu\nu\alpha_1}(Q) + \frac{1}{t}\,(k_\mu\mathcal{E}_{\nu\alpha_1}(Q\Delta) + k'_\nu\mathcal{E}_{\mu\alpha_1}(Q\Delta))$$

$$= 0$$

by identity A2 of ref (41). Further, no factor of t in $f_2(t)$ is necessary because of the above equation.

We note in passing that the structure of these normal and abnormal couplings forbids the decays of a $1^{\pm}$ particle into two photons, a well known selection rule.

2. Pomeron Reggeization : Pion Compton Scattering

In the same manner that the pion contributes to the differential cross section at $t = \mu_\pi^2$ in one photon processes, so also does the Pomeron contribute at t = 0 in two photon processes.

Consider Compton scattering off a spinless target, $\gamma + 0 \rightarrow \gamma + 0$

(e.g. pion compton scattering). The $\mathcal{M}$-function for this process can be written as[74]

$$\tilde{M}_{\mu\nu} = \tilde{A}_1 t g'_{\mu\nu} + \tilde{A}_2 t \left(P'_\mu P'_\nu - \tfrac{1}{2} P'^2 g'_{\mu\nu} \right)$$

where $g'_{\mu\nu} = g_{\mu\nu} + \dfrac{2k_\mu k'_\nu}{t}$, $P'_\nu = P_\nu + \dfrac{4\nu Q_\nu}{t}$

and $P'^2 = P^2 + \dfrac{4\nu^2}{t}$. It is well known that $\tilde{A}_1$ and $\tilde{A}_2$ are KSF in both ν and t.

Since only normal exchanges in the t-channel are allowed in this process, we have

$$\begin{aligned}
\tilde{M}_{\mu\nu} &= \mathcal{C}^+(00J;\, P) \;:\; \mathcal{P}^J \;:\; \tilde{\mathcal{C}}^+_{\mu\nu}(\gamma\gamma J\,;\,Q) \\
&= \tfrac{1}{2}(1 + (-)^J)\, g(t) \Big[t\tilde{g}_1(t) g'_{\mu\nu} \mathcal{P}'_J \\
&\quad + \tilde{g}_2(t) \left(t\mathcal{P}^J_{;\mu\nu} - 2g'_{\mu\nu}\mathcal{P}^J \right) \Big]
\end{aligned}$$

using $Q'_\mu = \Delta'_\mu = Q'_\nu = \Delta'_\nu = 0$ in the above along with the recursion relations in Appendix I, it is easy to show that

$$\hat{\mathcal{P}}^J_{;\mu\nu} \equiv t\mathcal{P}^J_{;\mu\nu} - 2g'_{\mu\nu}\mathcal{P}^J = t\left(P'_\mu P'_\nu - \tfrac{1}{2} P'^2 g'_{\mu\nu}\right) c_J \mathcal{P}''_J$$

and consequently

$$\tilde{A}_1 = g(t)\, \tilde{g}_1(t)\, c_J \mathcal{P}_J\, \tfrac{1}{2}(1 + (-)^J)$$

$$\tilde{A}_2 = \frac{g(t)\tilde{g}_2(t)}{J(J-1)} \; c_J \, \mathcal{P}''_J \, \tfrac{1}{2}(1 + (-)^J)$$

where the C-parity factor $\frac{1}{2}(1 + (-)^J)$ serves to select only the positive signature Regge trajectories. It is clear that a KSF $\tilde{g}_{1,2}(t)$ leads to a KSF $\tilde{A}_{1,2}(\nu, t)$ (in t). As the $(\pi\pi J)$ vertex precludes Class II and III conspiracies, and the $(\gamma\gamma J)$ vertex is gauge invariant we expect to see no 1/t terms in the Regge decomposition of $\tilde{A}_{1,2}$ (Note that there were 1/t terms in the $\rho\pi \rightarrow \rho\pi$ calculation in Appendix II, however covariant evasion or internal gauge invariance could be applied to remove them). As external and internal gauge invariance are equivalent for $\tilde{\mathcal{C}}(\gamma\gamma J)$ at t = 0 we expect any conspiracy (or 1/t) complication in Compton scattering to be associated with the other vertex.

The form of $\tilde{A}_{1,2}$ is consistent with the fact that they correspond exactly to the spin nonflip and spin flip helicity amplitudes in the t-channel; that is $\tilde{A}_1 \sim d^J_{00} \sim P_J$ and $\tilde{A}_2 \sim d^J_{20} \sim P''_J$. As for the J-factors in $\tilde{A}_2$ the nonsense zero in $\tilde{g}_2(t) \sim \alpha$, $\alpha \rightarrow 0$, cancels the 1/J term and the C-parity factor $\frac{1}{2}(1 + (-)^J)$ cancels $(J-1)^{-1}$.

In order to complete the arguement that the Pomeron reggeizes at t = 0, J = 1 we must show that there is <u>not</u> a nonsense zero in $\tilde{g}_2(t)$ at J = 1. Of course were there such a zero, both $\tilde{A}_2$ and $\tilde{A}_1$ would vanish at $\alpha = 1$ and the Pomeron would not reggeize. We proceed by writing the $\tilde{g}_2$ coupling out in full

$$\tilde{g}_2(t) \quad (t g'_{\mu\alpha_1} g'_{\nu\alpha_2} - 2 g'_{\mu\nu} Q_{\alpha_1} Q_{\alpha_2})$$

$$= \tilde{g}_2(t) \Big[t \, g_{\mu\alpha_1} g_{\nu\alpha_2} + 2(k_\mu g_{\nu\alpha_1} + k'_\nu \, g_{\mu\alpha_1}) Q_{\alpha_2} - 2 g_{\mu\nu} Q_{\alpha_1} Q_{\alpha_2} \Big]$$

At $J = 0$, $t = t_o$, the only remaining coupling term is $\tilde{g}_2(t_o)(-2g_{\mu\nu})Q\alpha_1 Q\alpha_2$ which can satisfy gauge invariance if and only if $\tilde{g}_2(t_o) = 0$ at $J(t_o) = 0$. At $J = 1$, $t = 0$ the coupling becomes

$$2\tilde{g}_2(0)\left[k_\mu g_{\nu\alpha_1} + k'_\nu g_{\mu\alpha_1} - 2g_{\mu\nu}Q\alpha_2\right]Q\alpha_1$$

which is indeed gauge invariant <u>at t = 0</u> and consequently $\tilde{g}_2(0)$ is not required to vanish[75] at $t = 0$ in the same way that $\tilde{g}(\mu^2) \neq 0$ for $\gamma N \longrightarrow \pi N$.

Upon reggeizing, $J \longrightarrow \alpha$, $\nu \longrightarrow \infty$ we get

$$\tilde{A}_1 \longrightarrow g\tilde{g}_1(-\nu)^{\alpha}\, \xi_+ \pi \alpha'$$

$$\tilde{A}_2 \longrightarrow g g_2(-\nu)^{\alpha-2}\, \xi_+ \pi \alpha'$$

and the cross section (Table IV) is

$$\nu^2 \frac{d\sigma}{dt} \sim 2g^2\left[t^2\tilde{g}_1^2 + 4\tilde{g}_2^2\right]\nu^{2\alpha}\left|\xi_+\right|^2$$

and because $g(o)$, $\tilde{g}_2(o)$ and $\xi_+(o)$ are not zero at $\alpha(o) = 1$, the Pomeron reggeizes in pion Compton scattering. Factorization of the residues ensures that the Pomeron reggeizes for any Compton target and within the framework of the model as discussed in Section I the constancy of total photo absorption cross sections at high energies is guaranteed. The arguement for the Pomeron reggeizing is logically equivalent to that for the pion and rho; if the one holds, the other surely must.

In terms of helicity amplitudes the t-channel helicity flip amplitudes is

$$f^{t}_{1,-1;o,o} \sim t\, p_t^2 \sin^2\Theta_t\, \tilde{A}_2 = 4k_t^2 p_t^2 \sin^2\Theta_t\, \tilde{A}_2$$

and again, the angular momentum factor $k_t^2 = t/4$ is absorbed in the high energy (cross channel) limit of $\sin\Theta_t$ and the asymptotic behaviour of $\tilde{A}_2 \sim \nu^{\alpha-2}$ is converted to $f^{t}_{1,-1;o,o} \sim \nu^{\alpha}$ and <u>no</u> nonsense zero at t = 0 is required in the amplitude.

In the more academic case of isovector photons, the I = 1 , odd signature, ρ trajectory does not reggeize at J = 1, $t = \mu_\rho^2$ in the same way the Pomeron does at J = 1, t = 0. The spin flip helicity amplitude is obliged to vanish at $t = \mu_\rho^2$ in order to conserve angular momentum and there is no way for this to happen unless $\tilde{A}_2$ contains a nonsense zero at $t = \mu_\rho^2$. In the covariant language this means that the $\tilde{g}_2$ coupling cannotbe gauge invariant at J = 1, $t = \mu_\rho^2$ unless $g_2(\mu_\rho^2) = 0$.

Thus, in accordance with the analysis of refs (23) and (24) the Fubini weak amplitude, $App(\nu,\, t,\, k^2 = k'^2 = 0) = t\tilde{A}_2(\nu, t)$ must contain a fixed pole at J = 1 if the Fubini weak sum rule(76) is to have any meaning in the Regge sense.

3. <u>Nucleon Compton Scattering</u>

Factorization allows the Pomeron to contribute to the double photon vertex, and consequently to the forward cross section in the same manner for $\gamma_\mu + N \longrightarrow \gamma_\nu + N$ as $\gamma_\mu + \pi \longrightarrow \gamma_\nu + \pi$. However we consider the process in detail both to demonstrate that the method works for higher spin reactions and to analyse possible conspiracies.

First we enumerate the six covariants[43]

$$\tilde{K}^1_{\mu\nu} = t g'_{\mu\nu}$$

$$\tilde{K}^2_{\mu\nu} = t\left[P'_\mu \; P'_\nu \; - \tfrac{1}{2}P'^2 \, g'_{\mu\nu}\right]$$

$$\tilde{K}^3_{\mu\nu} = t\left\{P', \gamma'\right\}_{\mu\nu} - 4\nu g'_{\mu\nu} \; \not{Q}$$

$$\tilde{K}^4_{\mu\nu} = 4P'_\mu P'_\nu \; \not{Q} + \nu\left(\left[\gamma' \not{Q} \gamma'\right]_{\mu\nu} - 2\left\{P', \gamma'\right\}_{\mu\nu} \right) + \tfrac{1}{2}t \, g'_{\mu\nu} \; \not{Q}$$

$$\tilde{K}^5_{\mu\nu} = t\left[\gamma'_\mu , \gamma'_\nu\right] + 8(m\not{Q} - \nu) \, g'_{\mu\nu}$$

$$\tilde{K}^6_{\mu\nu} = t\left[\gamma' \not{Q} \gamma'\right]_{\mu\nu} = 4m\gamma_5 \, \epsilon_{\mu\nu}(Q\Delta)$$

where $\left\{P', \gamma'\right\}_{\mu\nu} = P'_\mu \gamma'_\nu + \gamma'_\mu P'_\nu$ and $\left[\gamma' \not{Q} \gamma'\right]_{\mu\nu} = \gamma'_\mu \not{Q} \gamma'_\nu - \gamma'_\nu \not{Q} \gamma'_\mu$. That this set does indeed lead to invariant amplitudes $\tilde{A}_i (i = 1, \dots 6)$ which are KSF and KZF in s and t is shown in great detail ref (41). Other choices are given by Yamamoto[51] and Bardeen and Tung[42]. Two other seemingly independent covariants exist, one of which is related to this set by

$$2m^2 t g'_{\mu\nu} \; \not{Q} = 2m\nu \tilde{K}^1_{\mu\nu} + \tfrac{1}{4}mt\tilde{K}^5_{\mu\nu} - \nu\tilde{K}^6_{\mu\nu}$$

and another useful relation is given by

$$4t \left(P'_\mu P'_\nu - \tfrac{1}{2} P'^2 \, g'_{\mu\nu}\right) \not{Q} \equiv \hat{K}^4_{\mu\nu}$$

$$= 2\nu\left(\tilde{K}^3_{\mu\nu} - m\tilde{K}^1_{\mu\nu}\right) + t\left(\tilde{K}^4_{\mu\nu} - \tfrac{1}{4}\, mt\, \tilde{K}^5_{\mu\nu}\right).$$

Next we reggeize the corresponding invariant amplitudes. For normal exchanges

$$\tilde{\mathcal{M}}^{+}_{\mu\nu} = \mathcal{C}^{+}(\tfrac{1}{2}\tfrac{1}{2}J:\,P) : \mathcal{P}^{J}(\Delta) : \tilde{\mathcal{C}}^{+}_{\mu\nu}(\gamma\gamma J;\,Q)$$

$$= \tfrac{1}{2}(1 + (-)^{J}) \Big\{ g_1\tilde{g}_2 \mathcal{P}^{J} \tilde{\mathcal{K}}^{1}_{\mu\nu} + g_1\tilde{g}_2 \hat{\mathcal{P}}^{J}_{;\mu\nu}$$

$$+ t g_2\tilde{g}_1 \, \gamma_{\beta}\mathcal{P}^{J}_{\beta;} \, g'_{\mu\nu} + g_1\tilde{g}_2 \mathcal{P}^{J}_{\beta\,;\mu\nu} \Big\}$$

where $\hat{\mathcal{P}}^{J}_{;\mu\nu}$ is defined above, so that

$$\gamma_{\beta}\hat{\mathcal{P}}^{J}_{\beta;\mu\nu} = \frac{c_J}{J}\gamma_{\beta}\frac{\partial}{\partial P_{\beta}}\Big\{t\,(P'_{\mu}\;P'_{\nu} - \tfrac{1}{2}P'^{2}g'_{\mu\nu})\,\mathcal{P}''_{J}\Big\} .$$

Then using

$$\frac{\partial}{\partial P_{\beta}}P'_{\mu} = g'_{\beta\mu} \;,\; \frac{\partial}{\partial P_{\beta}}P'^{2} = 2\,P'_{\beta} \;,\; Q^{2} = -\frac{t}{4}$$

$$\gamma'_{\beta}\;P'_{\beta} = \gamma_{\beta}P'_{\beta} = \not{P} + \frac{4\nu}{t}\not{\Delta} \;,\; P^{2} = m^{2} - \tfrac{1}{4}t$$

we get

$$\gamma_{\beta}\hat{\mathcal{P}}^{J}_{\beta;\mu\nu} = \frac{c_J}{J}\Big\{\Big[t\,\{\gamma'P'\}_{\mu\nu} - t(m + \tfrac{4\nu}{t}\not{\Delta})\,g'_{\mu\nu}\Big]\;\mathcal{P}''_{J}$$

$$+ t\,(P'_{\mu}P'_{\nu} - \tfrac{1}{2}P'^{2}g'_{\mu\nu})\gamma_{\beta}{}^{J}\mathcal{P}''_{\beta;}\Big\}$$

$$= \frac{c_J}{J}\Big\{\Big[\tilde{\mathcal{K}}^{3}_{\mu\nu} - m\tilde{\mathcal{K}}^{1}_{\mu\nu}\Big]\mathcal{P}''_{J}$$

$$-t(P'_{\mu}\;P'_{\nu} - \tfrac{1}{2}P'^{2}g'_{\mu\nu})\Big[\not{\Delta}\,\mathcal{P}'''_{J} + Q^{2}\not{P}\,\mathcal{P}'''_{J-1}\Big]\Big\}$$

$$= \frac{c_J}{J}\left\{\left[\tilde{K}^3_{\mu\nu} - m\tilde{K}^1_{\mu\nu}\right]P''_J - \hat{K}^4_{\mu\nu} P'''_J + \tfrac{1}{4}mt\tilde{K}^2_{\mu\nu}\right\}.$$

Using this result and Appendix I it is straight forward to find the Regge contributions to $\tilde{A}_i$ for normal exchange. They are listed in Table IX.

For abnormal exchanges, the contributions again separate according to C-normality. For the C-normal case (π-exchange), we have

$$\tilde{M}^{-,+}_{\mu\nu} = C^{-}(\tfrac{1}{2}\tfrac{1}{2}\, J\ ;\ P) : P^{J}(\Delta) : \tilde{C}^{+}_{\mu\nu}(\gamma\gamma J\ ;\ Q)$$

$$= f_1\tilde{f}_1\, \gamma_5\, \varepsilon_{\mu\nu}(Q\Delta)\, P^{J}$$

$$= \frac{f_1\tilde{f}_1}{4m}\, c_J\, P_J\, \tilde{K}^6_{\mu\nu} \quad .$$

In the C-abnormal case (A_1 exchange)

$$\tilde{M}^{-,-}_{\mu\nu} = f_2\tilde{f}_2\, \gamma_5\, \gamma_\beta \left\{ P^{J}_{\beta;\alpha_1\alpha_2}\, g'_{\mu\alpha_1}\varepsilon_{\nu\alpha_2}(Q\Delta) + P^{J}_{\beta;\alpha_1\alpha_2}\, g'_{\nu\alpha_1}\varepsilon_{\mu\alpha_2}(Q\Delta) \right\}$$

$$= f_2\tilde{f}_2\, \gamma_5\gamma_\beta \left\{ \tilde{P}^{J}_{\beta;\mu\alpha_2}\varepsilon_{\nu\alpha_2}(Q\Delta) + \tilde{P}^{J}_{\beta;\nu\alpha_2}\varepsilon_{\mu\alpha_2}(Q\Delta) \right\}$$

Using

$$\tilde{P}^{J}_{;\mu\alpha_2} = \frac{c_J}{J(J-1)}\left[P_{\alpha_2} P'_{\mu} P''_{J} - P^2 g'_{\mu\alpha_2} P'_{J-1} + P^2 Q_{\alpha_2} P'_{\mu} P''_{J-1} \right]$$

and

$$\tilde{\mathcal{P}}^{J}_{;\mu\alpha_2}\,\mathcal{E}_{\nu\alpha_2}(Q\Delta) = \frac{c_J}{J(J-1)}\left[N_\nu P'_\mu\ \mathcal{P}''_J - P^2\,\mathcal{E}_{\nu\mu}(Q\Delta)\,\mathcal{P}'_{J-1}\right]$$

we see that

$$\tilde{\mathcal{P}}^{J}_{;\mu\alpha_2}\,\mathcal{E}_{\nu\alpha_2}(Q\Delta) + \tilde{\mathcal{P}}^{J}_{;\nu\alpha_2}\,\mathcal{E}_{\mu\alpha_2}(Q\Delta) = \frac{c_J}{J(J-1)}\,\mathcal{P}''_J\left\{P', N\right\}_{\mu\nu}$$

and consequently

$$\tilde{M}^{-,-}_{\mu\nu} = f_2\tilde{f}_2\,\frac{c_J}{J^2(J-1)}\ \gamma_5\gamma_\beta\,\frac{\partial}{\partial P_\beta}\,\mathcal{P}''_J\,\left\{P', N\right\}_{\mu\nu} ,$$

where we need

$$\begin{aligned}\gamma_5\gamma_\beta\ \frac{\partial}{\partial P_\beta}\left\{P', N\right\}_{\mu\nu}\mathcal{P}''_J &= \mathcal{P}''_J\,\gamma_5\,\left\{\gamma'\ N\right\}_{\mu\nu}\\ &+ \mathcal{P}''_J\,\gamma_5\left\{P'\ \gamma_\beta\frac{\partial}{\partial P_\beta}\ N\right\}_{\mu\nu}\\ &- \mathcal{P}'''_J\ \left\{P'\ \gamma_5 \not{Q} N\right\}_{\mu\nu}\end{aligned}$$

and Appendix III, to get

$$\begin{aligned}\tilde{M}^{-,-}_{\mu\nu} = -\,f_2\tilde{f}_2\,\frac{c_J}{J^2(J-1)}\Big[&-2(\mathcal{K}^4 - \tfrac{1}{4}m\mathcal{K}^5)_{\mu\nu}\,\mathcal{P}''_J\\ &+ (\tfrac{1}{2}mP^2\,\mathcal{K}^1 + m\mathcal{K}^2 - \tfrac{1}{2}P^2\mathcal{K}^3\\ &+ \nu\mathcal{K}^4 - \tfrac{1}{4}m\,\nu\mathcal{K}^5)_{\mu\nu}\ \mathcal{P}'''_J\ \Big]\end{aligned}$$

Now we are prepared to investigate possible conspiracies. From Table IX we see that π exchange contributes only to $\tilde{A}_6$ whereas A_1 exchange does not. This precludes any Class II conspiracy from giving a finite contribution to the cross section at t = 0. Moreover we see that no invariant amplitude has a singular 1/t factor in its Regge decomposition and consequently, although Class III conspirators can be exchanged, they cannot give rise to a non vanishing cross section at t = 0. That is, if we let the pion couplings become singular as $f_1 \sim t^{-\frac{1}{2}}$, $\tilde{f}_1 \sim t^{-\frac{1}{2}}$ then the normal pion conspirator, π_c, must have $g_{1,2} \sim t^{-\frac{1}{2}}$, $mg_1 + g_2 \sim t^{\frac{1}{2}}$ as in NN scattering[47] and $\gamma N \longrightarrow \pi N$, along with $\tilde{g}_1 \sim t^{-\frac{1}{2}}$, $\tilde{g}_2 \sim t^{\frac{1}{2}}$ and no correlation with an A_1- type conspirator. In other words from the covariant form of the cross section (Table IV),

$$\nu^2 \frac{d\sigma}{dt} \sim 2\left[(mg_1 + g_2)^2 - \frac{t}{4} g_1^2\right]\left[t\tilde{g}_1 + 4\tilde{g}_2^2\right]\nu^{2\alpha^+_+}\left|\xi''_\pm\right|^2$$

$$+ \frac{1}{2} t^3 f_1^2 \tilde{f}_1^2 \nu^{2\alpha\pm}\left|\xi_\pm\right|^2 + 2 f_2^2 \tilde{f}_2^2 \nu^{2\alpha^-_-}\left|\xi'_\pm\right|^2 ,$$

we see that the existence of a Class II or Class III conspiracy in

NN $\longrightarrow$ NN, or $\gamma N \longrightarrow \pi N$, πN^*, ρN does not give rise to a non vanishing forward cross section for $\gamma N \longrightarrow \gamma N$. This is certainly consistent with the elastic nature of the process.

In the helicity analysis, the t-channel conspiracy relation[77] at t = 0 can be expressed in terms of our invariant amplitudes[43] as

$$\tilde{f}^-_{0;0} - \nu \tilde{f}^+_{0;1} = 4t\, \tilde{A}_6 \longrightarrow O(t) \ , \ t \longrightarrow 0$$

where

$$f^{+}_{o;1} = t^{-1} (f_{11;\frac{1}{2}-\frac{1}{2}} + f_{11;-\frac{1}{2}\frac{1}{2}})$$

$$f^{-}_{o;o} = t^{-\frac{1}{2}} (f_{11;\frac{1}{2}\frac{1}{2}} + f_{11;-\frac{1}{2}-\frac{1}{2}}) \quad .$$

This leads to the obvious condition that $\tilde{A}_6$ remains finite. Moreover, because the normalities of the parity conserving combinations are opposite a Class II conspiracy is ruled out. Finally charge conjugation on $\tilde{f}^{-}_{o;o}$ prevents A_1-type trajectories from contributing to this Class III conspiracy condition. Thus the conclusions drawn are exactly the same as in the covariant approach.

Actually, the absence of 1/t terms in both $\gamma\pi \to \gamma\pi$ and $\gamma N \to \gamma N$ could have been inferred from the $\gamma\gamma J$ coupling. That is, 1/t terms arise from either unequal mass vertices or from higher spin vertices and they are removed in evasive cases by imposing internal gauge invariance at t = 0 which constrains the couplings there. As internal and external gauge invariance are equivalent at t = 0 no restrictions can be imposed upon $g_{1,2}$ or f_{12} which implies that no 1/t terms can be associated with the ($\gamma\gamma$ J) vertex. This is consistent with the presence of a 1/t due to higher spin in $\rho\pi \to \rho\pi$ and its absence in $\gamma\pi \to \gamma\pi$. In $\gamma N \to \gamma N$ a singular term would have to be associated with f_2 in $\bar{\rho}(\frac{1}{2}\frac{1}{2}J)$ in order to be removed by $\rho^{-}.\Delta = 0$, t = 0, the only available constraint and it is interesting that no singular term develops. In $\gamma N \to \gamma N^{*}$ however, 1/t contributions would appear associated with the unequal mass ($N\bar{N}^{*}J$) vertex.

V CONCLUSIONS

Starting with the covariant formalism of Scadron(45) and Jones(46,47) we have combined it with the gauge projection operator(41,42) and established a formalism for reggeizing invariant amplitudes in photonic processes(40). We then calculated the Regge contributions to the invariant amplitudes for the processes : $\gamma N \longrightarrow \pi N, \pi N^*$, VN, $\gamma N; \gamma\pi \longrightarrow V\pi, \gamma\pi$; $VN \longrightarrow \pi N$; $V\pi \longrightarrow V\pi$ and tabulated them for conspiracy analyses and for possible sum rule calculations(43). To facilitate our examination of conspiracy and evasion we have developed the method, suggested by Scadron(45), for calculating Regge contributions to $d\sigma/dt$ and tabulated the results.

We conclude that the pion reggeizes at $t = \mu_\pi^2$ in every process in which it can be exchanged as does the rho at $t = \mu_\rho^2$ and that the Pomeron reggeizes at t = 0, $\alpha = 1$ in Compton scattering and in the pseudoelastic processes $\gamma N \longrightarrow V^0 N$, $\gamma\pi \longrightarrow V^0\pi$. The condition of internal gauge invariance, part of the prescription for covariant evasion, must be carefully defined for the (γ1J) vertex, however no fixed poles in J had to be introduced to ensure these results.

Examining Class III pion conspiracy in the covariant formalism we noted that it implied singular residue functions for the amplitudes of the conspirators if a forward peak was to be effected and that in the process $\gamma N \longrightarrow \gamma N$ conspiracy did not give rise to a non vanishing differential cross section at t = 0. We introduced the possibility of one vertex evading while the other conspires in an attempt to solve the difficulties of $\gamma N \longrightarrow \pi N^*$, $V^0 N$. The success of such an approach of course can only be determined by a fit to the data for the processes involved and those related to them through factorization.

We examined the prescription for covariant evasion[47] and presented the internal gauge invariance constraints on the externally gauge invariant photonic vertices $\tilde{C}(\gamma 0J)$, $\tilde{C}(\gamma 1J)$, $\tilde{C}(\gamma\gamma J)$.

In general however, we conclude that, in the absence of cuts, pion conspiracy is more consistent with the data than evasion. Whether the pion really is a member of an M = 1 Toller pole or whether the neglected cut contributions are significant - as recent papers would have us believe, or whether absorptive corrections are the answer can only be resolved in time by the best fit of the most processes with the least parameters.

Careful examination of gauge invariance and kinematic zeros in the processes $VN \longrightarrow \pi N$, $\gamma N \longrightarrow \gamma N$ showed that no kinematic singularity at $t = \mu_\pi^2$ due to gauge invariance is contained in the amplitude $\tilde{A}_2$ for $\gamma N \longrightarrow \pi N$ in agreement with recent papers[12,56].

For the processes $VN \longrightarrow \pi N$, $\gamma N \longrightarrow \pi N$ we related the helicity and covariant formalisms and examined the problems of gauge invariance as $\mu_v \longrightarrow 0$. We also examined constraint equations imposed by analyticity as well as kinematic zeros and singularities and critically compared our results with the literature.

For both our own work and for future reference we have provided unequal mass abnormal decompositions.

We critically examined the literature from the covariant point of view for $\gamma N \longrightarrow \pi N$ superconvergence relations and conclude that they appear more readily in the covariant approach.

Generally, we regard the covariant formalism as a convenient method of obtaining reggeized invariant amplitudes for both photonic and massive processes. Once the asymptotic crossing behaviour is known, sum rules can be derived. The simple analyticity requirements on invariant amplitudes carry over to the Regge residues and studies of conspiracy and evasion become straight forward when compared with similar problems in helicity formalism. No fixed poles in the J plane have to be added to the formalism to ensure the proper asymptotic behaviour of differential cross sections even though they are permitted by unitarity in photonic process.

APPENDICES

APPENDIX I

Covariant Formalism

1. Kinematic Notation

For an s-channel process, $A(p) + B(k) \longrightarrow C(p') + D(k')$, involving fermions with momenta p, p' and bosons with momenta k, k and with $p + k = p' + k'$, we define $P = \frac{1}{2}(p + p')$, $Q = \frac{1}{2}(k + k')$, $\Delta = (p' - p) = (k - k')$ and $\nu = P.Q = \frac{1}{4}(s - u)$, $t = \Delta^2$, $N_\mu = \mathcal{E}_\mu(PQ\Delta)$, $T_\mu = \mathcal{E}_\mu(\gamma Q\Delta)$. As well, $p^2 = m^2$, $k^2 = \mu^2$ and for unequal mass cases $m_\pm = \frac{1}{2}(m' \pm m)$, $\mu_\pm = \frac{1}{2}(\mu' \pm \mu)$. In photonic processes $\mu = 0$ and $\mu' = \mu_B$ where B represents the photo-produced boson and frequently $\mu_\pi = \mu$.

The square of the Kibble boundary curve is $-N^2$ so that $N^2 = -t(p_t k_t \sin\Theta_t)^2$ in the t-channel c.m. frame. When boosted to the general frame it can be written as $N^2 = t(\nu(\Delta)^2 - P(\Delta)^2 Q(\Delta)^2)$ where $\nu(\Delta) = \nu - \frac{1}{t}(P.\Delta\ Q.\Delta)$, $P(\Delta)^2 = (P^2 - \frac{1}{t}(P.\Delta)^2)$, $Q(\Delta)^2 = Q^2 - \frac{1}{t}(Q.\Delta)^2$ and $p_t k_t = -P(\Delta)Q(\Delta)$.

The boosted four momentum(45) written explicitly is $P_\mu(\Delta) = P_\mu - \frac{1}{t}(P.\Delta)\Delta_\mu$. The following relations are useful in calculation :

$$P = p' - \tfrac{1}{2}\Delta = p + \tfrac{1}{2}\Delta \quad ; \quad Q = k' + \tfrac{1}{2}\Delta = k - \tfrac{1}{2}\Delta$$

$$4P^2 + \Delta^2 = 2(m^2 + m'^2) = 4(m_+^2 + m_-^2)$$

$$4Q^2 + \Delta^2 = 2(\mu^2 + \mu'^2) = 4(\mu_+^2 + \mu_-^2)$$

$$P(\Delta)^2 = -\frac{1}{4t}\left[t - 4m_+^2\right]\left[t - 4m_-^2\right] = -p_t^2$$

$$Q(\Delta)^2 = -\frac{1}{4t}\left[t - 4\mu_+^2\right]\left[t - 4\mu_-^2\right] = -k_t^2$$

$$P.\Delta = 2m_+m_- \quad ; \quad Q.\Delta = -2\mu_+\mu_-$$

$$\nu(\Delta) = \nu + 4\frac{m_+m_- \mu_+\mu_-}{t}$$

$$\nu = |P||Q|\cos\theta_t \; .$$

We define the s-channel variables as $\Lambda = \frac{1}{2}(p - k)$, $\Lambda' = \frac{1}{2}(p' - k')$, $K = (p + k)$ and $s \leftrightarrow t$ channel crossing by $p \leftrightarrow -k'$. This gives $\Lambda \leftrightarrow -Q$, $\Lambda' \leftrightarrow P$, $K \leftrightarrow \Delta$ and $\Lambda(K) \leftrightarrow -Q(\Delta)$, $\Lambda'(K) \leftrightarrow P(\Delta)$, $\Lambda(K).\Lambda'(K) \leftrightarrow -\nu(\Delta)$. (See Figs 1,2)

2. High Spin Wave Function

Following Scadron[45] we represent the free boson of spin J, helicity λ and mass μ by a covariant tensor wave function $\epsilon^{\lambda}_{\alpha_1 \dots \alpha_J}(k)$ where

$$\epsilon^{\lambda}_{\alpha_1 \dots \alpha_J}(k) = \sum_{\Lambda_1 \dots \Lambda_J} \langle \Lambda_1 \dots \Lambda_J | J\lambda \rangle \epsilon^{\Lambda_1}_{\alpha_1}(k) \dots \epsilon^{\Lambda_J}_{\alpha_J}(k)$$

and[78,79]

$$\langle \Lambda_1 \dots \Lambda_J | J\lambda \rangle = \left[2^{J - \Sigma|\Lambda_\alpha|} \frac{(J+\lambda)!\,(J-\lambda)!}{(2J)!} \right]^{\frac{1}{2}} \delta_{\Sigma\Lambda_\alpha, \lambda}$$

The $\epsilon^{\lambda}_{\alpha}(k)$ ($\alpha = 0,1,2,3$, Λ is helicity) are spin one covariant polarization vectors[80] subject to the subsidiary condition $k_\alpha \epsilon^{\Lambda}_{\alpha}(k) = 0$. The covariant spin J wave function obeys the following constraints

$$(k^2 - \mu^2)\, \mathcal{E}^{\lambda}_{\alpha_1 \cdots \alpha_J}(k) = 0$$

$$k_{\alpha_1}\, \mathcal{E}^{\lambda}_{\alpha_1 \cdots \alpha_J}(k) = 0$$

$$g_{\alpha_1 \alpha_2}\, \mathcal{E}^{\lambda}_{\alpha_1 \cdots \alpha_J}(k) = 0 \, .$$

For a high spin fermion with $s = J + \frac{1}{2}$ we use $\mathcal{U}^{\lambda}_{\alpha_1 \cdots \alpha_J}(p)$ where

$$\mathcal{U}^{\lambda}_{\alpha_1 \cdots \alpha_J}(p) = \sum_{\Lambda,\sigma} \langle J, \tfrac{1}{2} \Lambda, \sigma \,|\, J + \tfrac{1}{2}, \lambda \rangle\, \mathcal{E}^{\lambda}_{\alpha_1 \cdots \alpha_J}(p)\, \mathcal{U}^{\sigma}(p)$$

and $\mathcal{U}^{\sigma}(p)$ is a dirac spin $\frac{1}{2}$ bispinor. The subsidiary conditions are[81]

$$(\not{p} - m)\, \mathcal{U}^{\lambda}_{\alpha_1 \cdots \alpha_J}(p) = 0$$

$$\gamma_{\alpha_1}\, \mathcal{U}^{\lambda}_{\alpha_1 \cdots \alpha_J}(p) = 0$$

$$p_{\alpha_1}\, \mathcal{U}_{\alpha_1 \cdots \alpha_J}(p) = 0 \, .$$

We use the following normalization;

$$\mathcal{E}^{*\lambda}_{\alpha_1 \cdots \alpha_J}(k)\, \mathcal{E}^{\lambda'}_{\alpha_1 \cdots \alpha_J}(k) = \delta_{\lambda\lambda'} (-)^J$$

$$\overline{\mathcal{U}}^{\lambda}_{\alpha_1 \cdots \alpha_J}(p)\, \mathcal{U}^{\lambda'}_{\alpha_1 \cdots \alpha_J}(p) = 2m\, \delta_{\lambda\lambda'} (-)^J \, .$$

4. Projection Operators

The traceless, symmetric projection operator on O(3) helicity labels is

$$\mathcal{P}^{s}_{\beta_1 \ldots \beta_J ; \alpha_1 \ldots \alpha_J}(\Delta) = \sum_{\lambda} \Psi^{\lambda}_{\beta_1 \ldots \beta_J}(\Delta)\, \overline{\Psi}^{\lambda}_{\alpha_1 \ldots \alpha_J}(\Delta)$$

where $\Psi^{\lambda}_{\beta_1 \ldots \beta_J}$ is either $\mathcal{E}^{\lambda}_{\beta_1 \ldots \beta_J}$ or $\mathcal{U}^{\lambda}_{\beta_1 \ldots \beta_J}$ and $\overline{\Psi}^{\lambda}_{\alpha_1 \ldots \alpha_J}$ is either $\mathcal{E}^{*\lambda}_{\alpha_1 \ldots \alpha_J}$ or $\overline{\mathcal{U}}^{\lambda}_{\alpha_1 \ldots \alpha_J}$ according to whether bosons or fermions are considered. This spin sum we take as the numerator of the high spin propagator, 'on-shell'. That is, we identify it with the second term on the r.h.s. of the following equation

$$\frac{\bar{\mathcal{P}}^{s}}{t-m^2-i\epsilon} = \frac{\bar{\bar{\mathcal{P}}}^{s}}{t-m^2} + i\pi\,\delta(t-m^2)\,\mathcal{P}^{s}$$

The projection operator has the following properties for spin J bosons[82]

$\mathcal{P}^{J}_{\beta_1 \ldots \beta_J ; \alpha_1 \ldots \alpha_J}$ is symmetric and traceless in the α and β labels.

$$\Delta_{\beta_1} \mathcal{P}^{J}_{\beta_1 \ldots \beta_J ; \alpha_1 \ldots \alpha_J} = \Delta_{\alpha_1} \mathcal{P}^{J}_{\beta_1 \ldots \beta_J ; \alpha_1 \ldots \alpha_J} = 0$$

$$\mathcal{P}^{J}_{\beta_1 \ldots \beta_J ; \alpha_1 \ldots \alpha_J} \mathcal{P}^{J}_{\alpha_1 \ldots \alpha_J ; \alpha'_1 \ldots \alpha'_J} = (-)^J \mathcal{P}^{J}_{\beta_1 \ldots \beta_J ; \alpha'_1 \ldots \alpha'_J}$$

$$g_{\alpha_1 \beta_1} \mathcal{P}^{J}_{\beta_1 \ldots \beta_J ; \alpha_1 \ldots \alpha_J} = -\frac{(2J+1)}{(2J-1)} \mathcal{P}^{J-1}_{\beta_2 \ldots \beta_J ; \alpha_2 \ldots \alpha_J} ,$$

and for spin $s = J + \frac{1}{2}$ fermions

$$\mathcal{P}^{s}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J}(\Delta) = \frac{(J+1)}{(2J+3)} \gamma_\lambda (\not{p}-m)\gamma_\rho \mathcal{P}^{s+1}_{\lambda\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J}(\Delta)$$

$\mathcal{P}^{s}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J}$ is symmetric and traceless in the α and β lables.

$$\Delta_{\beta_1}\mathcal{P}^{s}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J} = \Delta_{\alpha_1}\mathcal{P}^{s}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J} = 0$$

$$\mathcal{P}^{s}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J}\mathcal{P}^{s}_{\alpha_1 \dots \alpha_J; \alpha'_1 \dots \alpha'_J} = 2m(-)^J\mathcal{P}^{s}_{\alpha_1 \dots \alpha_J; \alpha'_1 \dots \alpha'_J}$$

$$g_{\alpha_1 \beta_1} \mathcal{P}^{s}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J} = -\frac{(J+1)}{J}\mathcal{P}^{s-1}_{\beta_2 \dots \beta_J; \alpha_2 \dots \alpha_J}$$

$$(\not{\Delta}-m)\mathcal{P}^{s}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J}(\Delta) = \mathcal{P}^{s}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J}(\Delta)(\not{\Delta}-m) = 0$$

$$\gamma_{\beta_1}\mathcal{P}^{s}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J} = \mathcal{P}^{s}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J}\gamma_{\alpha_1} = 0$$

5. Contracted Projection Operators

Contracting the projection operator with initial (Q_α) and final (P_β) t-channel momenta, we get

$$\mathcal{P}^{J}(P,Q;\Delta) = P_{\beta_1} \dots P_{\beta_J}\mathcal{P}^{J}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J} Q_{\alpha_1} \dots Q_{\alpha_J}$$

where(45) .

$$\mathcal{P}^{J}(P,Q:\Delta) = c_J \mathcal{P}_J (P(\Delta).Q(\Delta))$$

and

$$c_J = 2^J \frac{J!J!}{(2J)!} = \frac{\sqrt{\pi}}{2^J} \frac{\Gamma(J+1)}{\Gamma(J+\frac{1}{2})} .$$

$\mathcal{P}_J$ is the solid Legendre polynomial with arguement $\nu(\Delta) = P(\Delta).Q(\Delta)$

$$\mathcal{P}^J(\nu(\Delta)) = |P(\Delta)|^J |Q(\Delta)|^J P_J(\cos\Theta_t)$$

and

$$\nu(\Delta) = |P(\Delta)||Q(\Delta)| \cos\Theta_t .$$

We prefer to use $\mathcal{P}^J(P,-Q;\Delta) = c_J \mathcal{P}_J(-\nu(\Delta))$ to facilitate regg-eization.

A spin J, on-shell, propagator (numerator) coupled to spin zero particles is completely specified by $\mathcal{P}^J(P,-Q;\Delta)$ [45,83]. However, when the external particles have spin the covariant projection operator must carry covariant labels which are 'freed' by a differential technique due to Scadron[45] which we consider in detail to establish notation and to present some new results.

To recover the initial and final labels[45]

$$\mathcal{P}^J_{;\alpha} = \frac{1}{J}\frac{\partial}{\partial Q_\alpha}\mathcal{P}^J(P,-Q;\Delta) = -\frac{c_J}{J}\left[P_\alpha(\Delta)\,\mathcal{P}'_J + P(\Delta)^2 Q_\alpha(\Delta)\,\mathcal{P}'_{J-1}\right]$$

$$\mathcal{P}^J_{\beta;} = \frac{1}{J}\frac{\partial}{\partial P_\beta}\mathcal{P}^J(P,-Q;\Delta) = -\frac{c_J}{J}\left[Q_\beta(\Delta)\,\mathcal{P}'_J + Q(\Delta)^2 P_\beta(\Delta)\,\mathcal{P}'_{J-1}\right]$$

where J^{-1} arises from the J possible ways of performing the differen-tiation

$$\frac{\partial}{\partial P_{\beta_1}} (P_{\beta_1} \cdots P_{\beta_J} \mathcal{P}^J_{\beta_1 \cdots \beta_J; \alpha_1 \cdots \alpha_J} Q_{\alpha_1} \cdots Q_{\alpha_J})$$

$$= J\, P_{\beta_2} \cdots P_{\beta_J} \mathcal{P}^J_{\beta_1 \cdots \beta_J; \alpha_1 \cdots \alpha_J} Q_{\alpha_1} \cdots Q_{\alpha_J}$$

$$= J\, \mathcal{P}^J_{\beta_1;} \qquad ,$$

continuing

$$\frac{\partial}{\partial P_{\beta_2}} \quad \frac{\partial}{\partial P_{\beta_1}} \mathcal{P}^J = (J-1)\, J\, \mathcal{P}^J_{\beta_1 \beta_2;}$$

$$\frac{\partial}{\partial P_{\beta_J}} \cdots \frac{\partial}{\partial P_{\beta_1}} \mathcal{P}^J = J!\, \mathcal{P}^J_{\beta_J \cdots \beta_1;}$$

where we required

$$\frac{\partial}{\partial P_\beta} \quad P_\gamma(\Delta) = g_{\beta\gamma}(\Delta) = g_{\beta\gamma} - \frac{\Delta_\beta \Delta_\gamma}{t}$$

$$\frac{\partial}{\partial P_\beta} \quad P(\Delta)^2 = 2P_\beta(\Delta) \qquad .$$

The recursion relations for $_J(-\nu(\Delta))$ are

$$J\, \mathcal{P}_J = -\nu(\Delta)\, \mathcal{P}'_J - P(\Delta)^2 Q(\Delta)^2 \mathcal{P}'_{J-1}$$

$$(J-1)\mathcal{P}'_J = -\nu(\Delta)\, \mathcal{P}''_J - P(\Delta)^2 Q(\Delta)^2 \mathcal{P}''_{J-1}$$

$$(J+1)\mathcal{P}_J = \nu(\Delta)\, \mathcal{P}'_J + \mathcal{P}'_{J+1}$$

$$(J+2)\mathcal{P}'_J = \nu(\Delta)\, \mathcal{P}''_J + \mathcal{P}''_{J+1}$$

$$(2J-1)\mathcal{P}'_{J-1} = \mathcal{P}''_J - P(\Delta)^2 Q(\Delta)^2 \mathcal{P}''_{J-2}$$

where one must recall that

$$\frac{\partial}{\partial(-\mathcal{V}(\Delta))} \mathcal{P}_J(-\mathcal{V}(\Delta)) = + \mathcal{P}'_J(-\mathcal{V}(\Delta))$$

$$\frac{\partial}{\partial(-\mathcal{V}(\Delta))} \mathcal{V}(\Delta) = -1 \quad .$$

For actual calculation we define

$$\frac{\partial}{\partial P_\beta} \mathcal{P}_J^{(n)} = {}^J\mathcal{P}^{(n)}_{\beta;} = -\left[Q_\beta(\Delta)\mathcal{P}_J^{(n-1)} + Q(\Delta)^2 P_\beta(\Delta)\mathcal{P}_{J-1}^{(n-1)}\right]$$

and similarly for ${}^J\mathcal{P}^{(n)}_{;\alpha}$ where (n) refers to the n^{th} derivative with respect to the arguement of the solid Legendre polynomial. As an example we take $\mathcal{P}^J_{\beta;}$ and derive $\mathcal{P}^J_{\beta;\alpha}$

$$\mathcal{P}^J_{\beta;\alpha} = \frac{1}{J}\frac{\partial}{\partial Q_\alpha}\mathcal{P}^J_{\beta;} = -\frac{c_J}{J^2}\left[g_{\alpha\beta}(\Delta)\mathcal{P}'_J + 2Q_\alpha(\Delta)P_\beta(\Delta)\mathcal{P}'_{J-1}\right.$$

$$\left. + Q_\beta(\Delta)\,{}^J\mathcal{P}'_{;\alpha} + Q(\Delta)^2 P_\beta(\Delta)\,{}^{J-1}\mathcal{P}'_{;\alpha}\right]$$

$$= \frac{c_J}{J^2}\left\{ - g_{\alpha\beta}(\Delta)\mathcal{P}'_J - 2Q_\alpha(\Delta)P_\beta(\Delta)\mathcal{P}'_{J-1}\right.$$

$$+ Q_\beta(\Delta)\left[P_\alpha(\Delta)\mathcal{P}''_J + P(\Delta)^2 Q_\alpha(\Delta)\mathcal{P}''_{J-1}\right]$$

$$\left. + Q(\Delta)^2 P_\beta(\Delta)\left[P_\alpha(\Delta)\mathcal{P}''_{J-1} + P(\Delta)^2 Q_\alpha(\Delta)\mathcal{P}''_{J-2}\right]\right\}$$

The recursion relations convert this expression to that found in ref(45) and we note that for J = 1 this is the conventional spin 1 propagator

$$\mathcal{P}^{1}_{\beta;\alpha} = - g_{\beta\alpha}(\Delta) = - \left[g_{\beta\alpha} - \frac{\Delta_\beta \Delta_\alpha}{t} \right] .$$

As well, $P_\beta \; \mathcal{P}^{J}_{\beta;\alpha} = \mathcal{P}^{J}_{;\alpha} \; ; \mathcal{P}^{J}_{;\alpha} \; Q_\alpha = \mathcal{P}^{J}$

Pulling off more labels we get

$$\mathcal{P}^{J}_{;\alpha_1\alpha_2} = \frac{c_J}{J(J-1)} \Big\{ - g_{\alpha_1\alpha_2}(\Delta)\, P(\Delta)^2\, \mathcal{P}'_{J-1}$$

$$+ P_{\alpha_1}(\Delta) \left[P_{\alpha_2}(\Delta)\, \mathcal{P}''_{J} + P(\Delta)^2\, Q_{\alpha_2}(\Delta)\, \mathcal{P}''_{J-1} \right]$$

$$+ Q_{\alpha_1}(\Delta)\, P(\Delta)^2 \left[P_{\alpha_2}(\Delta)\, \mathcal{P}''_{J-1} + P(\Delta)^2 Q_{\alpha_2}(\Delta)\, \mathcal{P}''_{J-2} \right] \Big\}$$

$$\mathcal{P}^{J}_{\beta_2\beta_1;\alpha_1} = \frac{c_J}{J^2(J-1)} \Big\{ \left(g_{\beta_1\beta_2}(\Delta) Q_{\alpha_1} + Q_{\beta_1} g_{\beta_2\alpha_1}(\Delta) \right) \mathcal{P}''_{J}$$

$$+ \left(Q(\Delta)^2 \left[P_{\beta_1} g_{\beta_2\alpha_1}(\Delta) + P_{\alpha_1} g_{\beta_1\beta_2}(\Delta) \right] + 2Q_{\alpha_1} Q_{\beta_1} P_{\beta_2} \right) \mathcal{P}''_{J-1}$$

$$- g_{\beta_1\alpha_1}(\Delta)^J \mathcal{P}'_{\beta_2;} - (2J+1)\, g_{\beta_1\beta_2}(\Delta)\, Q_{\alpha_1} \mathcal{P}'_{J-1}$$

$$+ (P_{\beta_1} Q_{\alpha_1} + Q_{\beta_1} P_{\alpha_1})^J \mathcal{P}''_{\beta_2;}$$

$$+ (Q(\Delta)^2 P_{\beta_1} P_{\alpha_1} + P(\Delta)^2 Q_{\beta_1} Q_{\alpha_1})^{J-1} \mathcal{P}''_{\beta_2;}$$

$$- (2J+1)\, P_{\beta_1} Q_{\alpha_1}{}^{J-1} \mathcal{P}'_{\beta_2;} \Big\}$$

$$\wp^{J}_{\beta_2\beta_1;\alpha_1\alpha_2} = \frac{c_J}{J^2(J-1)^2}\Big\{\big(g_{\beta_1\beta_2}(\Delta)g_{\alpha_1\alpha_2}(\Delta) + g_{\beta_1\alpha_2}(\Delta)g_{\beta_2\alpha_1}(\Delta)\big)\wp''_J$$

$$+ 2\Big(Q_{\alpha_2}\big[P_{\beta_1}\, g_{\beta_2\alpha_1}(\Delta) + P_{\alpha_1}\, g_{\beta_1\beta_2}(\Delta)\big]$$

$$+ P_{\beta_2}\big[Q_{\alpha_1}\, g_{\alpha_2\beta_1}(\Delta) + Q_{\beta_1}\, g_{\alpha_2\alpha_1}(\Delta)\big]\Big)\wp''_{J-1}$$

$$- g_{\beta_1\alpha_1}(\Delta)\; {}^{J}\wp'_{\beta_2;\alpha_2} - (2J+1)\, g_{\beta_1\beta_2}(\Delta)\, g_{\alpha_1\alpha_2}(\Delta)\wp'_{J-1}$$

$$+ \big(P_{\beta_1}g_{\alpha_1\alpha_2}(\Delta) + g_{\beta_1\alpha_2}(\Delta)\, P_{\alpha_1}\big)\; {}^{J}\wp''_{\beta_2;}$$

$$- (2J+1)\, P_{\beta_1}\, g_{\alpha_1\alpha_2}(\Delta)\; {}^{J-1}\wp'_{\beta_2;}$$

$$+ \big(2Q_{\alpha_2}P_{\beta_1}P_{\alpha_1} + P(\Delta)^2\big[g_{\alpha_2\beta_1}(\Delta)Q_{\alpha_1} + Q_{\beta_1}g_{\alpha_1\alpha_2}(\Delta)\big]\big)\; {}^{J-1}\wp''_{\beta_2;}$$

$$+ \big(g_{\beta_1\beta_2}(\Delta)\, Q_{\alpha_1} + Q_{\beta_1}\, g_{\beta_2\alpha_1}(\Delta)\big)\; {}^{J}\wp''_{;\alpha_2}$$

$$+ \Big(Q(\Delta)^2\big[P_{\beta_1}g_{\beta_2\alpha_1}(\Delta) + P_{\alpha_1}g_{\beta_1\beta_2}(\Delta)\big] + 2Q_{\alpha_1}Q_{\beta_1}P_{\beta_2}\Big)\; {}^{J-1}\wp''_{;\alpha_2}$$

$$- (2J+1)\, g_{\beta_1\beta_2}(\Delta)\, Q_{\alpha_1}\; {}^{J-1}\wp'_{;\alpha_2}$$

$$+ \big(P_{\beta_1}Q_{\alpha_1} + Q_{\beta_1}P_{\alpha_1}\big)\; {}^{J}\wp''_{\beta_2;\alpha_2}$$

$$+ \big(Q(\Delta)^2P_{\beta_1}P_{\alpha_1} + P(\Delta)^2Q_{\beta_1}Q_{\alpha_1}\big)\; {}^{J-1}\wp''_{\beta_2;\alpha_2}$$

$$- (2J+1)\, P_{\beta_1} Q_{\alpha_1}\, {}^{J-1}\mathcal{P}'_{\beta_2;\,\beta_2} \Big\}$$

For reference, the spin 2 and 3 propagators derived by this method are

$$\mathcal{P}^2_{\beta_2\beta_1;\alpha_1\alpha_2} = \frac{1}{6}\Big\{ 3\,\big[(\beta_1\alpha_2)(\beta_2\alpha_1) + (\beta_1\alpha_1)(\beta_2\alpha_2)\big] - 2\big[(\beta_1\beta_2)(\alpha_1\alpha_2)\big]\Big\}$$

$$\begin{aligned}\mathcal{P}^3_{\beta_3\beta_2\beta_1;\alpha_1\alpha_2\alpha_3} = \frac{1}{30}\Big\{ & 2(\beta_1\beta_2)\big[(\alpha_1\alpha_2)(\beta_3\alpha_3)+(\alpha_1\alpha_3)(\beta_3\alpha_2)+(\alpha_2\alpha_3)(\beta_3\alpha_1)\big] \\ & + 2(\beta_1\beta_3)\big[(\alpha_1\alpha_2)(\beta_2\alpha_3)+(\alpha_1\alpha_3)(\beta_2\alpha_2)+(\alpha_2\alpha_3)(\beta_2\alpha_1)\big] \\ & + 2(\beta_2\beta_3)\big[(\alpha_1\alpha_2)(\beta_1\alpha_3)+(\alpha_1\alpha_3)(\beta_1\alpha_2)+(\alpha_2\alpha_3)(\beta_1\alpha_1)\big] \\ & - 5\big[(\beta_1\alpha_1)(\beta_2\alpha_2)(\beta_3\alpha_3)+(\beta_1\alpha_3)(\beta_2\alpha_1)(\beta_3\alpha_2)+(\beta_1\alpha_2)(\beta_2\alpha_3)(\beta_3\alpha_1) \\ & + (\beta_1\alpha_1)(\beta_1\alpha_2)(\beta_3\alpha_2)+(\beta_1\alpha_3)(\beta_2\alpha_2)(\beta_2\alpha_3)+(\beta_1\alpha_2)(\beta_2\alpha_1)(\beta_2\alpha_2)\big]\Big\}\end{aligned}$$

where the labels (α, β) represents $g_{\alpha\beta}(\Delta)$.

Fermion propagators are listed in ref(45).

6. Reduced Regge Couplings[45]

(i) Boson-Boson-Boson

$$\mathcal{C}^+(00J) = g$$

$$\mathcal{C}^-(00J) = 0$$

$$\mathcal{C}^{+}_{\mu}(01J) = g_1 Q_\alpha Q_\mu + g_2\, g_{\alpha\mu}$$

$$\mathcal{C}^{-}_{\mu}(01J) = f \cdot \mathcal{E}_{\mu\alpha(Q\Delta)}$$

$$\mathcal{C}^{+}_{\mu\nu}(11J) = \Big[g_1 Q_{\alpha_1} Q_{\alpha_2} Q_\mu Q_\nu + g_2\, g_{\mu\nu}\, Q_{\alpha_1} Q_{\alpha_2}$$

$$+ g_3 g_{\alpha_1\nu} Q_{\alpha_2} Q_\mu + g_4 g_{\alpha_1\mu} Q_{\alpha_2} Q_\nu + g_5 g_{\alpha_1\mu} g_{\alpha_2\nu} \Big]$$

$$\mathcal{C}^{-}_{\mu\nu}(11J) = \Big[f_1 \mathcal{E}_{\mu\nu(Q\Delta)} + f_2 \mathcal{E}_{\mu\nu\alpha_1(Q)}\, Q_{\alpha_2}$$

$$+ f_3 \mathcal{E}_{\mu\nu\alpha_1(\Delta)}\, Q_{\alpha_2} + f_4 \mathcal{E}_{\alpha_1\mu(Q\Delta)}\, g_{\alpha_2\nu} \Big]$$

(ii) <u>Fermion-Fermion-Boson</u>

$$\mathcal{C}^{+}(\tfrac{1}{2}\tfrac{1}{2}J) = \Big[g_1 P_{\beta_1} + g_2 \gamma_{\beta_1} \Big]$$

$$\mathcal{C}^{-}(\tfrac{1}{2}\tfrac{1}{2}J) = \gamma_5 \Big[f_1 P_{\beta_1} + f_2 \gamma_{\beta_1} \Big]$$

$$\mathcal{C}^{+}_{\mu}(\tfrac{1}{2}\,\tfrac{3}{2}\,J) = \Big[g_1 P_{\beta_1} P_{\beta_2} P_\mu + g_2 \gamma_{\beta_1} P_{\beta_2} P_\mu$$

$$+ g_3 g_{\beta_1\mu} P_{\beta_2} + g_4 g_{\beta_1\mu} \gamma_{\beta_2} \Big]$$

$$\mathcal{C}^{-}_{\mu}(\tfrac{1}{2}\,\tfrac{3}{2}\,J) = \gamma_5 \Big[f_1 P_{\beta_1} P_{\beta_2} P_\mu + f_2 \gamma_{\beta_1} P_{\beta_2} P_\mu$$

$$+ f_3\, g_{\beta_1\mu} P_{\beta_2} + f_4\, g_{\beta_1\mu} \gamma_{\beta_2} \Big]$$

In all cases couplings are for massive particles.

APPENDIX II

Example Calculation : $VN \longrightarrow \pi N$, $V\pi \longrightarrow V\pi$

1. $V_\mu N \longrightarrow \pi N$

Tune reversal invariance makes this process equivalent to $\pi N \longrightarrow VN$ where V represents ρ, ω, ϕ . We calculate $VN \longrightarrow \pi N$ as it readily converts to $\gamma N \longrightarrow \pi N$.

The $\mathcal{M}$-function expanded in terms of kinematic covariants is

$$\mathcal{M}_\mu = \sum_{i=1}^{8} B_i \mathcal{K}_\mu^i$$

where

$$\mathcal{K}_\mu^1 = \gamma_5 P_\mu \qquad \mathcal{K}_\mu^5 = \gamma_5 \not{k} P_\mu$$

$$\mathcal{K}_\mu^2 = \gamma_5 Q_\mu \qquad \mathcal{K}_\mu^6 = \gamma_5 \not{k} Q_\mu$$

$$\mathcal{K}_\mu^3 = \gamma_5 k_\mu \qquad \mathcal{K}_\mu^7 = \gamma_5 \not{k} k_\mu$$

$$\mathcal{K}_\mu^4 = \gamma_5 \gamma_\mu \qquad \mathcal{K}_\mu^8 = \gamma_5 \not{k} \gamma_\mu \quad .$$

The subsidiary condition $k_\mu \epsilon_\mu(k) = 0$ is applied to eliminate B_3 and B_7.

Noting that the vertices of $VN \longrightarrow \pi N$ are of opposite normality we write the contribution to the $\mathcal{M}$-function as

$$\mathcal{M}_\mu = \mathcal{C}^+(\tfrac{1}{2}\tfrac{1}{2}J) : \mathcal{P}^J : \mathcal{C}_\mu^-(10J)$$

$$+ \mathcal{C}^-(\tfrac{1}{2}\tfrac{1}{2}J) : \mathcal{P}^J : \mathcal{C}_\mu^+(10J).$$

$$\mathcal{M}_\mu = \left[g_1 P_\beta + g_2 \gamma_\beta\right] : \mathcal{P}^J : \left[f\, \epsilon_{\mu\alpha(Q\Delta)}\right]$$

$$+ \gamma_5 \left[f_1 P_\beta + f_2 \gamma_\beta\right] : \mathcal{P}^J : \left[h_1 Q_\mu Q_\alpha + h_2 g_{\mu\alpha}\right]$$

$$= f\left[g_1 \mathcal{P}^J_{;\alpha} + g_2 \gamma_\beta \mathcal{P}^J_{\beta;\alpha}\right] \epsilon_{\mu\alpha(Q\Delta)}$$

$$+ \gamma_5 \left[f_1 h_1 \mathcal{P}^J Q_\mu + f_1 h_2 \mathcal{P}^J_{;\mu} + f_2 h_1 \gamma_\beta \mathcal{P}^J_{\beta;} Q_\mu + f_2 h_2 \gamma_\beta \mathcal{P}^J_{\beta;\mu}\right]$$

Then, considering normal exchange ($\mathcal{M}^+_\mu$) and abnormal exchange ($\mathcal{M}^-_\mu$) separately, we get

$$\mathcal{M}^+_\mu = \frac{c_J}{J^2}\, f\Big\{ - J g_1 N_\mu + g_2 \Big[-T_\mu\, \mathcal{P}'_J + \not{Q}(\Delta) N_\mu\, \mathcal{P}''_J$$

$$+ Q(\Delta)^2 \not{P}(\Delta)\, N_\mu\, \mathcal{P}''_{J-1} \Big] \Big\}$$

Using

$$\not{P}(\Delta) = \not{P} - \frac{P.\Delta}{t}\not{\Delta} = \frac{m_+}{t}\left[t - 4m_-^2\right]$$

$$\not{Q}(\Delta) = \not{K} + \frac{\not{\Delta}}{2}\left[1 - \frac{2Q.\Delta}{t}\right] = \not{K} + m_-\left[1 + \frac{4\mu_+\mu_-}{t}\right]$$

and Appendix III,

$$\mathcal{M}^+_\mu = \frac{c_J}{J^2}\, f\Big\{ \Big[- g_1 J \mathcal{P}'_J + g_2 \frac{(m_-}{t}\left[t + 4\mu_+\mu_-\right] \mathcal{P}''_J$$

$$+ \frac{m_+}{t} Q(\Delta)^2 \left[t - 4m_-^2\right]\, \mathcal{P}''_{J-1}) \Big] N_\mu$$

$$- g_2\, T_\mu\, \mathcal{P}'_J + g_2 \not{K} N_\mu \mathcal{P}''_J \Big\}$$

where we retain unequal nucleon mass terms in order to observe the appearance of 1/t factors associated with unequal fermion and boson masses. From the abnormal reductions in Appendix III, we can write down the contribution to the invariant amplitudes arising from normal particle exchange.

$$B_1^+ = 2f \frac{c_J}{J^2} \left\{ -(k.Q - \mu^2)\, X_J - \mu^2 m_- g_2\, \mathcal{P}''_J \right\}$$

$$B_2^+ = 2f \frac{c_J}{J^2} \left\{ -k.P\, X_J + \mu^2 m_+ g_2\, \mathcal{P}''_J \right\}$$

$$B_3^+ = 0$$

$$B_4^+ = 2f \frac{c_J}{J^2} \left\{ \left[m_+ k.P - m_-(k.Q - \mu^2) \right] X_J - k.P\, g_2\, \mathcal{P}'_J + \left[(k.P)^2 - (k.Q - \mu^2)^2 + \frac{\mu^2}{4}(t - 4m_-^2) \right] g_2\, \mathcal{P}''_J \right\}$$

$$B_5^+ = 2f \frac{c_J}{J^2} \left\{ -m_+ X_J + g_2\, \mathcal{P}'_J - k.P\, g_2\, \mathcal{P}''_J \right\}$$

$$B_6^+ = 2f \frac{c_J}{J^2} \left\{ +m_- X_J + (k.Q - \mu^2)\, g_2\, \mathcal{P}''_J \right\}$$

$$B_7^+ = 0$$

$$B_8^+ = 2f \frac{c_J}{J^2} \left\{ -\tfrac{1}{4}(t - 4m_+^2)\, X_J - m_+ g_2\, \mathcal{P}'_J + \left[m_+ k.P - m_-(k.Q - \mu^2) \right] g_2\, \mathcal{P}''_J \right\}.$$

where $X_J = \Big[- g_1 J \mathcal{P}'_J + g_2 \big(\frac{m_-}{t} - (t + 4\mu_+\mu_-\big) \mathcal{P}''_J$

$$+ \frac{m_+}{t} Q(\Delta)^2 \left[t - 4m_-^2 \right] \mathcal{P}''_{J-1} \Big]$$

and $k.P = \nu = Q.P - m_+m_-$.

For abnormal exchange,

$$\mathcal{M}_\mu = \frac{c_J}{J^2} \gamma_5 \Big\{ J^2 f_1 h_1 \mathcal{P}_J - J f_1 h_2 (P_\mu(\Delta) \mathcal{P}'_J + P(\Delta)^2 Q_\mu(\Delta) \mathcal{P}'_{J-1}$$

$$- J f_2 h_1 (\not{Q}(\Delta) \mathcal{P}'_J + Q(\Delta)^2 \not{P}(\Delta) \mathcal{P}'_{J-1}) Q_\mu$$

$$+ f_2 h_2 \Big(- \gamma_\mu(\Delta) \mathcal{P}'_J - Q_\mu(\Delta) \not{P}(\Delta) \mathcal{P}'_{J-1}$$

$$+ \not{Q}(\Delta) \left[P_\mu(\Delta) \mathcal{P}''_J + P(\Delta)^2 Q_\mu(\Delta) \mathcal{P}''_{J-1} \right]$$

$$+ Q(\Delta)^2 \not{P}(\Delta) \left[P_\mu(\Delta) \mathcal{P}''_{J-1} + P(\Delta)^2 Q_\mu(\Delta) \mathcal{P}''_{J-2} \right] \Big) \Big\} .$$

Using

$$\gamma_5 P_\mu(\Delta) = \mathcal{K}^1_\mu - \frac{4m_+m_-}{t} (\mathcal{K}^2_\mu - \mathcal{K}^3_\mu)$$

$$\gamma_5 Q_\mu(\Delta) = \frac{1}{t} \left[t - 4\mu_+\mu_- \right] \mathcal{K}^2_\mu + \frac{4\mu_+\mu_-}{t} \mathcal{K}^3_\mu$$

$$\gamma_5 \gamma_\mu(\Delta) = \mathcal{K}^4_\mu - \frac{4m_+}{t} (\mathcal{K}^2_\mu - \mathcal{K}^3_\mu)$$

$$\gamma_5 \not{P}(\Delta) = - \frac{m_-}{t} \left[t - 4m_+^2 \right]$$

$$\gamma_5 \not{Q}(\Delta) = \gamma_5 \not{K} + \frac{m_+}{t} \left| t - 4\mu_+\mu_- \right|$$

we have

$$B_1^- = \frac{c_J}{J^2}\Big\{ - f_1h_1 J \wp'_J + f_2h_2 \frac{m_+}{t} \quad t - 4\mu_+\mu_- \quad \wp''_J$$

$$- Q(\Delta)^2 \quad \frac{m_-}{t} t - 4m_+^2 \wp''_{J-1} \Big\}$$

$$B_2^- = \frac{c_J}{J^2}\Big\{ f_1h_1 J^2 \wp_J - f_1h_2 J \Big[- \frac{4m_+m_-}{t} \wp'_J + P(\Delta)^2 \frac{[t-4\mu_+\mu_-]}{t} \wp'_{J-1} \Big]$$

$$+ f_2h_1 J \Big[- \frac{m_+}{t} (t-4\mu_+\mu_-) \wp'_J + Q(\Delta)^2 \frac{m_-}{t} (t-4m_+^2) \wp'_{J-1} \Big]$$

$$+ f_2h_2 \Big[+ \frac{4m_+}{t} \wp'_J + \big[t-4\mu_+\mu_-\big] \big[t-4m_-^2\big] \frac{m_-}{t^2} \wp'_{J-1}$$

$$+ \frac{m_+}{t} \big[t-4\mu_+\mu_-\big] \Big[- \frac{4m_+m_-}{t} \wp''_J + \frac{[t-4\mu_+\mu_-]}{t} P(\Delta)^2 \wp''_{J-1} \Big]$$

$$- \frac{m_-}{t} \big[t-4m_-^2\big] Q(\Delta)^2 \Big[- \frac{m_+m_-}{t} \wp''_J + \frac{[t-4\mu_+\mu_-]}{t} P(\Delta)^2 \wp''_{J-2} \Big] \Big] \Big\}$$

$$B_4^- = - \frac{c_J}{J^2} f_2h_2 \wp'_J$$

$$B_5^- = \frac{c_J}{J^2} f_2h_2 \wp''_J$$

$$B_6^- = \frac{c_J}{J^2} \Big\{ - f_2h_1 J \wp'_J + f_2h_2 P(\Delta)^2 \quad \frac{t-4\mu_+\mu_-}{t} \wp''_{J-1} \Big\}$$

$$B_8^- = 0$$

In order to discuss gauge invariance in the limit $\mu_\nu \to 0$ in section III.1.E.(ii) we relate the invariant amplitudes to the asymptotically parity conserving amplitudes $\bar{f}^{\pm}_{\lambda\mu}$ and present the Regge contributions to them in Table III. Our amplitudes are essentially those of Högassen and Salin[60]

$$\bar{f}^{-}_{o1} = p_t \left[\frac{t^{\frac{1}{2}}}{2} B_1 - m\, k_o B_5 - k_o B_8 \right]$$

$$\bar{f}^{+}_{o1} = - k_t \frac{t^{\frac{1}{2}}}{2} B_8$$

$$\bar{f}^{-}_{11} = p_t \left[B_4 + \nu B_5 \right]$$

$$\bar{f}^{+}_{11} = - k_t \left[p_t^2 B_5 - m B_8 \right]$$

$$\bar{f}^{-}_{oo} = - \frac{k_o \cos\Theta_t}{\mu_\nu \sqrt{2}} \bar{f}^{-}_{o1} + \frac{k_t}{\mu_\nu \sqrt{2}} \left[+ \nu B_8 - \frac{t}{4} B_2 - m B_4 \right.$$
$$\left. + m\, k_o \frac{t^{\frac{1}{2}}}{2} B_6 \right]$$

$$\bar{f}^{+}_{1o} = - \frac{2p_t}{\mu_\nu \sqrt{2}} \left[- k_o (B_4 + \nu B_5) + k_t^2 \frac{t^{\frac{1}{2}}}{2} B_6 \right]$$

where $k_o^2 = k_t^2 + \mu_\nu^2$, $k.Q - \mu_\nu^2 = - \frac{\sqrt{t}}{2} k_o$, $m_- = 0$.

The Wang $\mathcal{K}(t)$ factors for writing the KSF(in s and t) amplitude

$$\tilde{f}^{\pm}_{\lambda\mu} = K(t)\, \bar{f}^{\pm}_{\lambda\mu}$$

are

$$\bar{K}_{o1} = p_t^{-1} t^{\frac{1}{2}} \qquad \bar{K}_{11} = p_t^{-1}$$

$$K^{+}_{o1} = (k_t t^{\frac{1}{2}})^{-1} \qquad K^{+}_{11} = k_t^{-1}$$

$$K^{-}_{oo} = tk_t \qquad K^{+}_{1o} = p_t^{-1}\, t^{\frac{1}{2}} \; .$$

2. $V_\mu \pi \longrightarrow V_\nu \pi$

As before we keep the four masses unequal until the end

$$\mathcal{M}_{\mu\nu} = \sum_{i=1}^{4} B_i \, \mathcal{K}^{i}_{\mu\nu}$$

$$\mathcal{K}^{1}_{\mu\nu} = P_\mu P_\nu \qquad \mathcal{K}^{3}_{\mu\nu} = \tfrac{1}{2}\left[P_\mu Q_\nu + Q_\mu P_\nu\right]$$

$$\mathcal{K}^{2}_{\mu\nu} = Q_\mu Q_\nu \qquad \mathcal{K}^{4}_{\mu\nu} = g_{\mu\nu} \; .$$

As $\mathcal{C}^{-}(00J) = 0$, only normal exchange is permitted and

$$\mathcal{M}_{\mu\nu} = \mathcal{C}^{+}(00J) : \mathcal{P}^{J} : \mathcal{C}^{+}_{\mu\,\nu}(11J)$$

$$= g_\pi \Big[(g_1 Q_\mu Q_\nu + g_2\, g_{\mu\nu})\, \mathcal{P}^{J} + g_3\, \mathcal{P}^{J}_{;\nu} Q_\mu + g_4\, \mathcal{P}^{J}_{;\mu} Q_\nu + g_5\, \mathcal{P}^{J}_{;\mu\nu} \Big]$$

$$= \frac{c_J g_\pi}{J(J-1)} \Big\{ (g_1 Q_\mu Q_\nu + g_2 g_{\mu\nu})\, J(J-1)\, \mathcal{P}_J - (J-1)\frac{g_3}{2}\Big[(P_\mu(\Delta)Q_\nu + P_\nu(\Delta)Q_\mu)\, \mathcal{P}'_J + P(\Delta)^2 (Q_\mu(\Delta)Q_\nu + Q_\nu(\Delta)Q\ \ )\, \mathcal{P}'_{J-1}\Big]$$

$$+ g_5 \Big| - g_{\mu\nu}(\Delta)\, P(\Delta)^2\, \wp'_{J-1}$$

$$+ P_{\mu}(\Delta) \Big| P_{\nu}(\Delta)\, \wp''_{J} + P(\Delta)^2 Q_{\nu}(\Delta)\, \wp''_{J-1} \Big|$$

$$+ P(\Delta)^2 Q_{\mu}(\Delta) \Big| P_{\nu}(\Delta)\, \wp''_{J-1} + P(\Delta)^2 Q_{\nu}(\Delta)\, \wp''_{J-2} \Big|$$

where $C^{+}_{\mu\nu} = C^{+}_{\nu\mu}$. Using the following relations

$$P_{\substack{\mu \\ \nu}}(\Delta) = P_{\substack{\mu \\ \nu}} \pm \frac{4m_+m_-}{t}\, Q_{\substack{\mu \\ \nu}}$$

$$Q_{\substack{\mu \\ \nu}}(\Delta) = Q_{\substack{\mu \\ \nu}} \left[1 \mp \frac{4\mu_+\mu_-}{t}\right]$$

and the subsidiary conditions

$$k_{\mu} \cdot \epsilon_{\mu}(k) = 0 , \qquad k'_{\nu}\, \epsilon_{\nu}(k') = 0$$

we get

$$B_1 = \frac{c_J}{J(J-1)}\, g_{\pi} \left\{ g_5\, \wp''_{J} \right\}$$

$$B_2 = \frac{c_J}{J(J-1)}\, g_{\pi} \Bigg\{ g_1 J(J-1)\, \wp_{J} - g_3 (J-1) P(\Delta)^2\, \wp'_{J-1}$$

$$+ g_5 \Bigg(- \frac{1}{4t} P(\Delta)^2\, \wp'_{J-1}$$

$$+ \frac{4m_+m_-}{t} \left[- \frac{4m_+m_-}{t}\, \wp''_{J} + \frac{P(\Delta)^2}{t} \left[t + 4\mu_+\mu_- \right] \wp''_{J-1} \right]$$

$$+ \frac{P(\Delta)^2}{t}\left[t+4\mu_+\mu_-\right]\left[-\frac{4m_+m_-}{t}\wp''_{J-1} + \frac{P(\Delta)^2}{t}\left[t+4\mu_+\mu_-\right]\wp''_{J-2}\right]\Big)\Bigg\}$$

$$B_3 = \frac{c_J g_\pi}{J(J-1)}\left\{ -(J-1)g_3\wp'_J + g_5\frac{P(\Delta)^2}{t}\left[t+4\mu_+\mu_-\right]\wp''_{J-1} \right\}$$

$$B_4 = \frac{c_J g_\pi}{J(J-1)}\left\{ J(J-1)\, g_2\wp_J - g_5 P(\Delta)^2\wp'_J \right\} \quad .$$

Setting $m_- = 0$, $\mu_- = 0$ we note that the 1/t in the first term of the g_5 bracket of B_2 remains(46). It arises from the consideration of spin.

APPENDIX III

Abnormal Reductions

1. γ - Matrices

Our metric is $g_{oo} = g_{ii} = -1$ and the γ matrices are defined by $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$ so that $\gamma_5^2 = -1$.

As the literature is written both in the above convention and in that adopted by CGLN[55] we use the following conversions (c denotes CGLN) : $\gamma_o^c = \gamma_o$, $\gamma_i^c = -i\gamma_i$, $\gamma_5^c = -i\gamma_5$. The 4-vector is $a^c = (ia_o, a_i)$ and $a^c.b^c = -a.b$, $\not{a}^c = i\not{a}$, $\gamma_5^c\not{a}^c = \gamma_5\not{a}$.

2. Covariant Identities

The Levi-Civita tensor is

$$\begin{aligned}\varepsilon_{\alpha\mu\nu\delta} = -\gamma_5\Big[\gamma_\alpha\gamma_\mu\gamma_\nu\gamma_\delta \quad &- g_{\alpha\mu}\gamma_\nu\gamma_\delta + g_{\nu\delta}\gamma_\alpha\gamma_\mu \\ &- g_{\alpha\delta}\gamma_\mu\gamma_\nu - g_{\mu\nu}\gamma_\alpha\gamma_\delta + g_{\alpha\nu}\gamma_\mu\gamma_\delta \\ &+ g_{\mu\delta}\gamma_\alpha\gamma_\nu + g_{\alpha\delta}\, g_{\mu\nu} - g_{\alpha\nu}\, g_{\mu\delta} \\ &+ g_{\alpha\mu}\, g_{\nu\delta}\Big]\end{aligned}$$

and we use the abbreviation $\varepsilon(ABCD) = \varepsilon_{\alpha\mu\nu\delta}\, A_\alpha\, B_\mu\, C_\nu\, D_\delta$.

Evaluating various ε products between spinors $\bar{U}(p')$, $U(p)$ we find

$$\begin{bmatrix} 1 \\ -\gamma_5 \end{bmatrix} \varepsilon_\mu(\gamma Q \Delta) = 2\begin{bmatrix} \gamma_5 \\ 1 \end{bmatrix}\Big[\pm m_\pm (\not{K}\gamma_\mu - k_\mu) + k.P\gamma_\mu - \not{K} P_\mu\Big]$$

$$\begin{bmatrix} 1 \\ -\gamma_5 \end{bmatrix} \mathcal{E}_{\mu(PQ\Delta)} = 2\begin{bmatrix} \gamma_5 \\ 1 \end{bmatrix} \Big[-\tfrac{1}{4}(t - m_{\pm}^2)\, \not{k}\, \gamma_\mu$$

$$\pm (m_{\pm}\, k.P + m_{\mp}\, (k.Q - \mu^2))\, \gamma_\mu$$

$$+ (k.Q - \mu^2)\, P_\mu \mp m_{\pm}\, \not{k}\, P_\mu$$

$$- k.P\, Q_\mu \mp m_{\mp}\, \not{k}\, Q_\mu$$

$$+ (\tfrac{1}{4}(t - 4m_{\pm}^2) + k.P)\, k_\mu$$

$$\pm m_{\mp}\, \not{k}\, k_\mu \Big]$$

$$\begin{bmatrix} 1 \\ -\gamma_5 \end{bmatrix} \gamma_\nu \mathcal{E}_{\mu(PQ\Delta)} = 2\begin{bmatrix} \gamma_5 \\ 1 \end{bmatrix} \Big[-\gamma_\nu \Big\{ -\tfrac{1}{4}(t - 4m_{\pm}^2)(\not{k}\gamma_\mu - k_\mu)$$

$$\pm m_{\mp}\, P_\mu \not{k} \pm m_{\mp}\, (Q_\mu - k_\mu)\, \not{k}$$

$$\mp (m_{\mp}\, k.P + m_{\pm}\, (k.Q - \mu^2))\, \gamma_\mu$$

$$- (k.P(Q_\mu - k_\mu) - (k.Q - \mu^2)\, P_\mu) \Big\}$$

$$- \Big\{ (P_\nu - Q_\nu + k_\nu)(-(m_+ - m_-)(\not{k}\gamma_\mu - k_\mu)$$

$$+ (P_\mu \not{k} - k.P\, \gamma_\mu)$$

$$+ (Q_\mu - k_\mu)\, \not{k} - (k.Q - \mu^2)\gamma_\mu \Big\} \Big]$$

$$\begin{bmatrix} 1 \\ -\gamma_5 \end{bmatrix} \not{k}\ \mathcal{E}_\mu(PQ\Delta) = 2 \begin{bmatrix} \gamma_5 \\ 1 \end{bmatrix} \Big[+\Big((k.P)^2 - (k.Q - \mu^2)^2 + \tfrac{2}{4}(t - 4m_{\pm}^2) \Big) \gamma_\mu$$

$$\pm (m_{\pm}\ k.P + m_{\mp}(k.Q - \mu^2))\ \not{k}\,\gamma_\mu$$

$$\mp m_{\mp}\,\mu^2 P_\mu \ - k.P\ \not{k}\ P_\mu$$

$$\mp m_{\pm}\,\mu^2 Q_\mu \ + (k.Q - \mu^2)\ \not{k}\ Q_\mu$$

$$-((m_{+} - m_{-})(k.P - (k.Q - \mu^2) \pm m_{\pm}\mu^2)\ k_\mu$$

$$- \big((k.Q - \mu^2) + \tfrac{1}{4}(t - 4m_{\mp}^2) \big)\ \not{k}\ k_\mu \ \Big]$$

$$\begin{bmatrix} 1 \\ -\gamma_5 \end{bmatrix} \not{P}\ \mathcal{E}_\mu(PQ\Delta) = \pm m_{\pm} \begin{bmatrix} 1 \\ \gamma_5 \end{bmatrix} \mathcal{E}_\mu(PQ\Delta)$$

$$\begin{bmatrix} 1 \\ -\gamma_5 \end{bmatrix} \not{\Delta}\ \mathcal{E}_\mu(PQ\Delta) = \pm 2m_{\mp} \begin{bmatrix} 1 \\ \gamma_5 \end{bmatrix} \mathcal{E}_\mu(PQ\Delta)$$

also

$$\gamma_\rho \frac{\partial}{\partial P_\rho}\ \mathcal{E}_\mu(PQ\Delta) = \mathcal{E}_\mu(\gamma Q \Delta)$$

For photons $\mu = 0$ and terms involving k_μ vanish - provided the photon carries the μ label.

3. $\mathcal{E}$ - Products

Contracting momenta with the determinant(41)

$$\varepsilon_{\mu\beta'\gamma'\delta'}\varepsilon_{\mu\beta\gamma\delta} = - \begin{vmatrix} g_{\mu\mu} & g_{\mu\beta} & g_{\mu\gamma} & g_{\mu\delta} \\ g_{\beta'\mu} & g_{\beta'\beta} & g_{\beta'\gamma} & g_{\beta'\delta} \\ g_{\gamma'\mu} & g_{\gamma'\beta} & g_{\gamma'\gamma} & g_{\gamma'\delta} \\ g_{\delta'\mu} & g_{\delta'\beta} & g_{\delta'\gamma} & g_{\delta'\delta} \end{vmatrix}$$

we find the following relations of use in Appendix IV

$$\varepsilon_{\mu\beta'\gamma'}(A)\ \varepsilon_{\mu\beta\gamma}(B) =$$

$$-\Big\{4\,(g_{\beta'\beta}\, g_{\gamma'\gamma}\, A.B + g_{\beta'\alpha}\, B_{\gamma'} A_{\beta} + g_{\gamma'\beta}\, A_{\gamma}\, B_{\beta'}$$

$$- g_{\gamma'\gamma}\, A_{\beta}\, B_{\beta'} - g_{\beta'\beta}\, A_{\gamma}\, B_{\gamma'} - A.B\, g_{\gamma'\beta}\, g_{\beta'\gamma}\,)$$

$$- g_{\mu\beta}\,(g_{\beta'\mu}\, g_{\gamma'\gamma}\, A.B + g_{\beta'\gamma}\, B_{\gamma'} A_{\mu} + B_{\beta'} A_{\gamma}\, g_{\gamma'\mu}$$

$$- g_{\gamma'\gamma}\, A_{\mu}\, B_{\beta'} - g_{\beta'\mu}\, B_{\gamma'} A_{\gamma} - A.B\, g_{\gamma'\mu}\, g_{\beta'\gamma}\,)$$

$$+ g_{\mu\gamma}\,(g_{\beta'\mu}\, g_{\gamma'\beta}\, A.B + g_{\beta'\beta}\, B_{\gamma'} A_{\mu} + B_{\beta'} A_{\beta}\, g_{\gamma'\mu}$$

$$- g_{\gamma'\beta}\, A_{\mu}\, B_{\beta'} - g_{\beta'\mu}\, A_{\beta}\, B_{\gamma'} - A.B\, g_{\gamma'\mu}\, g_{\beta'\beta}\,)$$

$$- B_{\mu}\,(g_{\beta'\mu}\, g_{\gamma'\beta}\, A_{\gamma} + g_{\beta'\beta}\, g_{\gamma'\gamma}\, A_{\mu} + g_{\beta'\gamma}\, g_{\gamma'\mu}\, A_{\beta}$$

$$- g_{\gamma'\beta}\, g_{\beta'\gamma}\, A_{\mu} - g_{\gamma'\gamma}\, g_{\beta'\mu}\, A_{\beta} - g_{\beta'\beta}\, g_{\gamma'\mu}\, A_{\gamma}\,)\Big\}$$

$$\mathcal{E}_{\mu\beta'}(AC)\;\mathcal{E}_{\mu\beta\gamma}(B) =$$

$$-\Big[g_{\beta\beta'}(A.BC_{\gamma} - C.BA_{\gamma}) + g_{\beta'\gamma}(C.BA_{\beta} - A.BC_{\beta})$$

$$+ C_{\beta}A_{\gamma}B_{\beta'} - C_{\gamma}A_{\beta}B_{\beta'}\Big]$$

$$\mathcal{E}_{\mu\beta'}(AC)\;\mathcal{E}_{\mu\beta}(BD) =$$

$$-\Big[g_{\beta'\beta}\big((A.B)(C.D) - (C.B)(A.D)\big) + D_{\beta'}(C.BA_{\beta} - A.BC_{\beta}.)$$

$$+ (A.D)\,C_{\beta}B_{\beta'} - C.D\,A_{\beta}B_{\beta'}\Big] \quad .$$

Specific examples are

$$\mathcal{E}_{\mu\beta}(Q\Delta)\;\mathcal{E}_{\mu\beta'}(Q\Delta) =$$

$$-\Big[g_{\beta\beta'}\,tQ(\Delta)^2 + Q^2\Delta_{\beta}\Delta_{\beta'} - tQ_{\beta}Q_{\beta'}$$

$$+ (Q\Delta)\left[Q_{\beta}\Delta_{\beta'} + \Delta_{\beta}Q_{\beta'}\right]\Big]$$

$$\mathcal{E}_{\beta\beta'}(P\Delta)\;\mathcal{E}_{\beta\alpha'}(Q\Delta) =$$

$$-\Big[g_{\beta'\alpha'}\,t\,V(\Delta) - P.\Delta\,\Delta_{\beta'}\Delta_{\alpha'}$$

$$+ (Q.\Delta\,\Delta_{\alpha'}P_{\beta'} + P.\Delta\,\Delta_{\beta'}Q_{\alpha'}) - t\,P_{\beta'}Q_{\beta'}\Big]$$

4. Decompositions in terms of Kinematic Covariants

A. $\gamma N \rightarrow \pi N$

$$T_\mu = 2m\,\tilde{K}^1_\mu + 2\,\tilde{K}^4_\mu$$

$$N_\mu = 2\,P^2\,\tilde{K}^1_\mu + 2\,\tilde{K}^2_\mu + 2m\,\tilde{K}^4_\mu$$

$$\not{k}\,N_\mu = 2k.Pm\,\tilde{K}^1_\mu - 2k.Q\,\tilde{K}^3_\mu + 2k.P\,\tilde{K}^4_\mu$$

B. $\gamma_\mu N \rightarrow \pi N^*_\nu$

$$-\gamma_5 P_\nu N_\mu = 2\Big[-\tfrac{1}{4}\left[t+4m_+m_-\right]\tilde{K}^1_{\mu\nu} + \tilde{K}^3_{\mu\nu} - (m_+ + m_-)\,\tilde{K}^4_{\mu\nu} + m_+m_-\,\tilde{K}^6_{\mu\nu} + \frac{m_-}{4}(t-4m_-^2)\,\tilde{K}^7_{\mu\nu} - m_-^2\,\tilde{K}^8_{\mu\nu}\Big]$$

$$-\gamma_5 Q_\nu N_\mu = 2\Big[(\nu - \frac{\mu^2}{4})\,\tilde{K}^1_{\mu\nu} + \tilde{K}^3_{\mu\nu} - (m_+ + m_-)\tilde{K}^4_{\mu\nu} - \nu\,\tilde{K}^6_{\mu\nu} + (\nu m_+ - m_-\frac{\mu^2}{4})\,\tilde{K}^7_{\mu\nu} + (k.Q - m_-^2 - m_+m_-)\,\tilde{K}^8_{\mu\nu}\Big]$$

$$-\gamma_5 P_\nu \not{k}\,N_\mu = 2\Big[m_+(\nu - k.Q)\,\tilde{K}^1_{\mu\nu} + (\nu - k.Q)\,\tilde{K}^4_{\mu\nu} - m_+\nu\,\tilde{K}^6_{\mu\nu} - \frac{\nu}{4}(t - 4m_+^2)\,\tilde{K}^7_{\mu\nu} + m_-\nu\,\tilde{K}^8_{\mu\nu}\Big]$$

$$-\gamma_5 Q_\nu \not{k}\,N_\mu = \frac{2}{m_+}\Big[k.Q(\nu - \frac{\mu^2}{4})\,\tilde{K}^1_{\mu\nu} - (m_+m_-\nu + k.Q(m_+^2 + m_-^2 - t/4))\,\tilde{K}^2_{\mu\nu} - k.Q(m_+ + m_-)\,\tilde{K}^4_{\mu\nu} + (m_+\nu - k.Q\,m_-)\,\tilde{K}^5_{\mu\nu}.$$

$$- k.Q(\nu + k.Q)\tilde{K}^{6}_{\mu\nu} + k.Q(\nu m_{+} - \frac{\mu^2}{4} m_{-})\tilde{K}^{7}_{\mu\nu}$$

$$- k.Q(\nu + k.Q + m_{+}m_{-} + m_{-}^{2})\tilde{K}^{8}_{\mu\nu} \Big]$$

$$-\gamma_5 P_\nu T_\mu = 2\Big[m_{+}\tilde{K}^{1}_{\mu\nu} + \tilde{K}^{4}_{\mu\nu} - m_{+}\tilde{K}^{6}_{\mu\nu}$$

$$- (t - 4m_{+}^{2})\tilde{K}^{7}_{\mu\nu} + m_{-}\tilde{K}^{8}_{\mu\nu} \Big]$$

$$-\gamma_5 Q_\nu T_\mu = 2\Big[- m_{-}\tilde{K}^{2}_{\mu\nu} + \tilde{K}^{5}_{\mu\nu}\Big]$$

$$-\gamma_5 \epsilon_{\mu\nu(Q\Delta)} = 2\Big[m_{+}\tilde{K}^{7}_{\mu\nu} + \tilde{K}^{1}_{\mu\nu} - \tilde{K}^{6}_{\mu\nu} \Big]$$

$$-\gamma_5 \not{k}\, \epsilon_{\mu\nu(Q\Delta)} = 2\Big[m_{+}\tilde{K}^{1}_{\mu\nu} + m_{-}\tilde{K}^{2}_{\mu\nu} + \tilde{K}^{4}_{\mu\nu} - \tilde{K}^{5}_{\mu\nu} - m_{+}\tilde{K}^{6}_{\mu\nu}$$

$$- (k.Q + \tfrac{1}{4}(t - 4m_{+}^{2}))\tilde{K}^{7}_{\mu\nu} + m_{-}\tilde{K}^{8}_{\mu\nu} \Big]$$

c. $\gamma_\mu N \longrightarrow V_\nu N$

$$-\gamma_5 \epsilon_{\mu\nu(kQ)} = 2\Big[\frac{m_{+}}{2}\tilde{K}^{9}_{\mu\nu} + \tfrac{1}{4}\tilde{K}^{8}_{\mu\nu} - \tfrac{1}{2}\tilde{K}^{1}_{\mu\nu} \Big]$$

$$-\gamma_5 \epsilon_{\mu\nu(kP)} = 2\Big[\tfrac{1}{2}\tilde{K}^{2}_{\mu\nu} + \tfrac{1}{4}\tilde{K}^{6}_{\mu\nu} - \frac{m_{-}}{4}\tilde{K}^{9}_{\mu\nu} \Big]$$

$$-\gamma_5 P_\nu N_\mu = 2\Big[- \tfrac{1}{4}(t - 4m_{-}^{2})\tilde{K}^{1}_{\mu\nu} + \frac{m_{+}}{2}(\tilde{K}^{4}_{\mu\nu} - \tilde{K}^{5}_{\mu\nu}) - m_{-}\tilde{K}^{7}_{\mu\nu} + \tilde{K}^{3}_{\mu\nu} \Big]$$

$$-\gamma_5 Q_\nu N_\mu = 2\Big[- \tfrac{1}{4}(t - 4m_{-}^{2})\tilde{K}^{2}_{\mu\nu} - \frac{m_{+}}{2}(k.Q\,\tilde{K}^{11}_{\mu\nu} + \tilde{K}^{14}_{\mu\nu}) - m_{-}\tilde{K}^{13}_{\mu\nu}$$

$$+ \tfrac{1}{2}(k.Q\,\tilde{K}^{10}_{\mu\nu} + k.P\,\tilde{K}^{12}_{\mu\nu}) \Big]$$

$$(m_- = 0) \quad = 2\left[\frac{\nu}{2}\tilde{K}^1_{\mu\nu} + \frac{1}{2}\, k.Q\, \tilde{K}^2_{\mu\nu} + \frac{1}{4}\, k.Q\, \tilde{K}^6_{\mu\nu} - \frac{\nu}{4}\tilde{K}^8_{\mu\nu} - \frac{m\nu}{4}\tilde{K}^9_{\mu\nu} \right]$$

$$-\gamma_5\, k.Q\, P'_\mu N_\nu = \Big[(\nu^2 - 2m^2 k.Q)\tilde{K}^1_{\mu\nu} + \frac{\nu}{4}(t+\mu_v^2)\tilde{K}^2_{\mu\nu} + 2k.Q\,\tilde{K}^3_{\mu\nu}$$

$(m_- = 0)$

$$+ mk.Q(\tilde{K}^4_{\mu\nu} - \tilde{K}^5_{\mu\nu}) + \frac{\nu}{4}(2k.Q + t)\tilde{K}^6_{\mu\nu}$$

$$-\left(\frac{\nu^2}{2} - P^2\, k.Q\right)\tilde{K}^8_{\mu\nu} - \frac{m}{2}(\nu^2 - 2k.Q\, P^2)\tilde{K}^9_{\mu\nu}$$

$$+ \frac{t}{4}(k.Q\,\tilde{K}^{10}_{\mu\nu} + \nu\tilde{K}^{12}_{\mu\nu}) \Big]$$

$$-\gamma_5\, k.Q\, \gamma'_\mu N_\nu = \Big[-\frac{2P^2\nu}{m}\tilde{K}^1_{\mu\nu} - \frac{2P^2}{m}\frac{(t+\mu_v^2)}{4}\tilde{K}^2_{\mu\nu} + 2\nu\tilde{K}^4_{\mu\nu} + 2k.Q\,\tilde{K}^7_{\mu\nu}$$

$(m_- = 0)$

$$- P^2\frac{k.Q}{m}\tilde{K}^6_{\mu\nu} + \frac{P^2\nu}{m}\tilde{K}^8_{\mu\nu} + P^2\nu\,\tilde{K}^9_{\mu\nu} - mk.Q\,\tilde{K}^{10}_{\mu\nu}$$

$$- \left(m + \frac{t}{2m}\right)\nu\,\tilde{K}^{12}_{\mu\nu} \Big]$$

$$-\gamma_5 \quad \varepsilon_{\mu\nu(k\gamma)} = \frac{1}{2}\tilde{K}^9_{\mu\nu}$$

$$-\gamma_5 k.Q\, P'_\mu\, T_\nu = m_-\,(k.Q\,\tilde{K}^8_{\mu\nu} - \nu\tilde{K}^6_{\mu\nu} - k.Q\,\tilde{K}^{10}_{\mu\nu} - \nu\tilde{K}^{12}_{\mu\nu})$$
$$- 2\,(\nu\tilde{K}^4_{\mu\nu} + k.Q\,\tilde{K}^7_{\mu\nu})$$

$$(m_- = 0) \qquad = -2\nu\tilde{K}^4_{\mu\nu} - 2k.Q\,\tilde{K}^7_{\mu\nu}$$

$$-\gamma_5\, \gamma_\nu N_\mu = -2\tilde{K}^4_{\mu\nu} - 2\tilde{K}^5_{\mu\nu} - m\tilde{K}^8_{\mu\nu} + 2m\tilde{K}^{10}_{\mu\nu}$$
$$- P^2\tilde{K}^9_{\mu\nu} + 2P^2\tilde{K}^{11}_{\mu\nu}$$

$$-\gamma_5 P_\nu T_\mu = 2m_- \tilde{K}^1_{\mu\nu} - 2\tilde{K}^7_{\mu\nu}$$

$$-\gamma_5 Q_\nu T_\mu = 2m_- \hat{K}^2_{\mu\nu} + 2m\tilde{K}^1_{\mu\nu} - 2\tilde{K}^4_{\mu\nu} - m\tilde{K}^8_{\mu\nu} - P^2\tilde{K}^9_{\mu\nu}$$

$$-\gamma_5 \not{k}\, \mathcal{E}_{\mu\nu}(kQ) = 2m_- \tilde{K}^2_{\mu\nu} + \frac{k.Q}{2}\tilde{K}^9_{\mu\nu} - 2\tilde{K}^{13}_{\mu\nu}$$

$$(m_- = 0) \qquad = +2m\tilde{K}^1_{\mu\nu} - 2\tilde{K}^4_{\mu\nu} - m\tilde{K}^8_{\mu\nu} - (P^2 - \frac{k.Q}{2})\tilde{K}^9_{\mu\nu}$$

$$-\gamma_5 \not{k}\, \mathcal{E}_{\mu\nu}(kP) = +2m_+ \tilde{K}^2_{\mu\nu} + \frac{\nu}{2}\tilde{K}^9_{\mu\nu} - k.Q\,\tilde{K}^{11}_{\mu\nu} - \tilde{K}^{14}_{\mu\nu}$$

$$(m_- = 0) \qquad = +\frac{\nu}{m}\tilde{K}^1_{\mu\nu} + \frac{(t+\mu_v^2 - 8m^2)}{4m}\tilde{K}^2_{\mu\nu} + \frac{k.Q}{2m}\tilde{K}^6_{\mu\nu}$$

$$-\frac{\nu}{2m}\tilde{K}^8_{\mu\nu} - \frac{k.Q}{m}\tilde{K}^{10}_{\mu\nu} - \frac{\nu}{m}\tilde{K}^{12}_{\mu\nu}$$

$$-\gamma_5\, k.Q\, P'_\mu \not{k} N_\nu = \frac{1}{m}(P^2 - \frac{k.Q}{2})(\nu^2 - 2m^2\, k.Q)\tilde{K}^1_{\mu\nu}$$

$$+\frac{\nu}{4m}(P^2 - \frac{k.Q}{2})(t + \mu_v^2)\ \tilde{K}^2_{\mu\nu}$$

$$+2(\nu^2 - (k.Q)^2 + P^2\, k.Q)\tilde{K}^4_{\mu\nu} - (k.Q)^2\tilde{K}^5_{\mu\nu}$$

$$+\nu\, k.Q\left(m + \frac{1}{2m}(P^2 - \frac{k.Q}{2})\right)\tilde{K}^6_{\mu\nu} + 2\,k.Q\,\nu\,\tilde{K}^7_{\mu\nu}$$

$$+\left(-m(k.Q)^2 - \frac{1}{2m}(\nu^2 - 2m^2 k.Q)(P^2 - \frac{k.Q}{2})\right)\tilde{K}^8_{\mu\nu}$$

$$-\frac{1}{2}(P^2 - \frac{k.Q}{2})(\nu^2 - 2k.Q\,P^2)\tilde{K}^9_{\mu\nu}$$

$$+\left(m(\nu - 2k.Q) - \frac{\nu}{m}(P^2 - \frac{k.Q}{2})\right)(k.Q\,\tilde{K}^{10}_{\mu\nu} - \nu\tilde{K}^{12}_{\mu\nu})$$

$$-\gamma_5 \not{k}\, P_\nu\, N_\mu = -2mk.Q\, \tilde{K}^1_{\mu\nu} + k.Q(\tilde{K}^4_{\mu\nu} - \tilde{K}^5_{\mu\nu}) + 2\nu \tilde{K}^7_{\mu\nu}$$

$$-\gamma_5 \not{k}\, Q_\nu\, N_\mu = 2\nu \tilde{K}^{13}_{\mu\nu} - 2mk.Q\, \tilde{K}^2_{\mu\nu} - k.Q(k.Q\, \tilde{K}^{11}_{\mu\nu} + \tilde{K}^{14}_{\mu\nu})$$

$$= \left(\frac{k.Q}{m} - 2m\right)\tilde{K}^1_{\mu\nu} - \frac{k.Q}{4m}(t+\mu_v^2)\tilde{K}^2_{\mu\nu} + 2\nu \tilde{K}^4_{\mu\nu} + \frac{(k.Q)^2}{2m}\tilde{K}^6_{\mu\nu}$$

$$+ \nu\left(m - \frac{k.Q}{2}\right)\tilde{K}^8_{\mu\nu} + \nu\left(P^2 - \frac{k.Q}{2}\right)\tilde{K}^9_{\mu\nu} - \frac{(k.Q)^2}{m}\tilde{K}^{10}_{\mu\nu} - \nu\frac{k.Q}{m}\tilde{K}^{12}_{\mu\nu}$$

D. $\gamma_\mu N \longrightarrow \gamma_\nu N$[84]

$$\gamma_5\ \epsilon_{\mu\nu(Q\Delta)} = \frac{\tilde{K}^6}{4m}$$

$$\{P'\ \gamma_5 \not{Q}\, N\}_{\mu\nu} = \frac{mP^2}{2}\tilde{K}^1_{\mu\nu} + m\tilde{K}^2_{\mu\nu} - \frac{P^2}{2}\tilde{K}^3_{\mu\nu} + \nu\tilde{K}^4_{\mu\nu} - \frac{1}{4}m\tilde{K}^5_{\mu\nu}$$

$$\{P'\ \gamma_5\, T\}_{\mu\nu} = \tilde{K}^4_{\mu\nu} + \frac{\nu}{t}\tilde{K}^6_{\mu\nu} + \frac{1}{2}\tilde{K}^7_{\mu\nu}$$

$$\gamma_5\{\gamma N\}_{\mu\nu} = 4\left(-P^2 g'_{\mu\nu}\not{Q} + 4P'_\mu P'_\nu \not{Q} - 2\nu\{P\gamma\}_{\mu\nu} + 4m\nu g'_{\mu\nu}\right)$$

and

$$\gamma_5\{P'T\}_{\mu\nu} + \gamma_5\{\gamma N\}_{\mu\nu} = 2\left(\tilde{K}^4_{\mu\nu} - \frac{m}{4}\tilde{K}^5_{\mu\nu}\right)$$

where [41,43],

$$[\gamma'\not{Q}\gamma']_{\mu\nu} = \frac{t}{4}m[\gamma',\gamma']_{\mu\nu}\ .$$

APPENDIX IV

Reggeized Cross Sections

1. Introduction

The spin averaged differential cross section in the asymptotic region is written as[45]

$$\nu^2 \frac{d\sigma}{dt} \sim I$$

where

$$I = \mathrm{tr}\, \mathcal{M}_{fi}\, \rho_{ii'}\, \bar{\mathcal{M}}_{f'i'}\, \rho_{ff'}$$

and

$$\mathcal{M}_{fi} = \mathcal{C}_{f\beta}\,(B)\,\mathcal{P}^{J}_{\beta;\alpha}\,\mathcal{C}_{i\alpha}\,(A) \quad .$$

$\rho_{ii'}$, $\rho_{ff'}$ are the projection operators for the incoming and out going particles respectively, A, B, represent the spins at the vertices and α , β refer to all of the internal labels on the vertex functions.

If the $\mathcal{M}$-function is split into normal (+) and abnormal (-) exchange parts

$$\mathcal{M} = \mathcal{M}^{+} + \mathcal{M}^{-}$$

the cross section becomes

$$\nu^2 \frac{d\sigma}{dt} \sim I^{++;\pm\pm} + I^{--;\mp\mp} + I^{+-;\pm\mp} + I^{-+;\mp\pm}$$

where,

$$I^{++;\pm\pm} = \mathrm{tr}\, \mathcal{M}^{+}_{fi}\, \rho_{ii'}\, \overline{\mathcal{M}}^{\pm}_{f'i'}\, \rho_{ff'}$$

$$= \mathrm{tr}\, \mathcal{C}^{+}_{f\beta}(B)\, \rho^{J}_{\beta;\alpha}\, \mathcal{C}^{\pm}_{i\alpha}(A)\, \rho_{ii'}\, \overline{\mathcal{C}}^{+}_{f'\beta'}(B)\, \rho^{J}_{\beta';\alpha'}\, \overline{\mathcal{C}}^{\pm}_{i'\alpha'}(A)\, \rho_{ff'}$$

The double normality subscripts allow for the possibilty of identical or opposite normality at the vertices. Factoring the above equation gives

$$I^{++;\pm\pm} = \mathrm{tr}\, \mathcal{C}^{+}_{f\beta}(B)\, \rho_{ff'}\, \overline{\mathcal{C}}^{+}_{f'\beta'}(B)\, \rho^{J}_{\beta;\alpha}\, \rho^{J}_{\beta';\alpha'}\, \mathrm{tr}\, \mathcal{C}^{\pm}_{i\alpha}(A)\, \rho_{ii'}\, \overline{\mathcal{C}}^{\pm}_{i'\alpha'}(A)$$

$$= \mathrm{tr}\, (B)^{++}_{\beta\beta'}\, \rho^{J}_{\beta;\alpha}\, \rho^{J}_{\beta';\alpha'}\, \mathrm{tr}\, (A)^{\pm\pm}_{\alpha\alpha'}$$

Since

$$\Delta_{\beta}\rho^{J}_{\beta;\alpha} = \rho^{J}_{\beta;\alpha}\Delta_{\alpha} = 0$$

$$P_{\beta}\, \rho^{J}_{\beta;\alpha} = \rho^{J}_{;\alpha} \;,\; \rho^{J}_{;\alpha}\, Q_{\alpha} = \rho^{J} \;,\; \rho^{J} = c_J \rho_J \;,$$

only terms of the form

$$P_{\beta}\, P_{\beta'}\, \rho^{J}_{\beta;\alpha}\, \rho^{J}_{\beta';\alpha'}\, Q_{\alpha}\, Q_{\alpha'} = (c_J\, \rho_J)^2 \sim \nu^{2J}$$

will contribute to leading order in the differential cross section. This requires that traces which give leading order contributions are of the form

$$\mathrm{tr}\, (A)_{\alpha\alpha'} = \left[\qquad\qquad \right] Q_{\alpha} Q_{\alpha'} \tag{6}$$

$$\mathrm{tr}\, (B)_{\beta\beta'} = \left[\qquad\qquad \right] P_{\beta} P_{\beta'}$$

We now show that traces which contain vertices of the same normality will contribute to leading order, whereas those which mix normality contribute terms which are, at best, one order below leading order and can be ignored in asymptopia.

2. Traces Involving Same Normality

For FFB (F=fermion, B=boson) traces of either normality, inspection of the vertex functions in Appendix I reveals that there is always one pure momentum coupling (g_1, f_1) capable of giving a trace in the form of eqn(6). The γ_5 present in the abnormal vertex function appears twice in the trace and consequently gives rise only to sign problems. For example, consider the trace.

$$\mathrm{tr}(\tfrac{1}{2}\tfrac{1}{2}J)^{--} = \mathrm{tr}\, \mathcal{C}^{-}_{\beta}(\tfrac{1}{2}\tfrac{1}{2}J)(\not{p} + m)\overline{\mathcal{C}}^{-}_{\beta'}(\tfrac{1}{2}\tfrac{1}{2}J)(\not{p}' + m')$$

$$= \mathrm{tr}\ \gamma_5(f_2\gamma_\beta + f_1 P_\beta)(\not{p} + M)\gamma_5(-f_2\gamma_{\beta'} + f_1 P_{\beta'})(\not{p}' + m')$$

$$= 8P_\beta\ P_{\beta'}\left[(f_2 - m_- f_1)^2 - \frac{t}{4}\ f_1^2\right]$$

$$+ 8g_{\beta\beta'}\left[\frac{t}{4} - m_+^2\right]$$

The first term clearly contributes to leading order, however the second term, assuming that the α, α' labels have been contracted away, gives

$$g_{\beta\beta'}\,\mathcal{P}^J_{\beta;}\mathcal{P}^J_{\beta;} = -\mathcal{P}^J_{\beta;}\frac{c_J}{J}\left(Q_\beta(\Delta)\,\mathcal{P}'_J + Q(\Delta)^2 P_\beta(\Delta)\,\mathcal{P}'_{J-1}\right)$$

$$= \frac{(c_J)^2}{J}\,Q(\Delta)^2\left((\mathcal{P}'_J)^2 + P(\Delta)^2(\mathcal{P}'_{J-1})^2 + 2V(\Delta)\,\mathcal{P}'_J\,\mathcal{P}'_{J-1}\right).$$

The best asymptotic behaviour of the previous is two orders below leading order, consequently we shall ignore metric tensor contributions.

For normal BBB traces the arguement is the same as that for the FFB case. However, for abnormal calculations the coupling depends on the anti-symmetric tensor $\epsilon_{\alpha\eta\gamma\delta}$ and the trace will be a product of anti-symmetric tensors and momenta as in the following example

$$\mathrm{tr}(10J)^{--} = e^{-}_{\alpha\mu}(10J)\left[g_{\mu\mu'} - \frac{k_\mu k_{\mu'}}{\mu^2}\right]e^{-}_{\alpha'\mu'}(10J)$$

$$= f^2\,\epsilon_{\mu\alpha}(Q\Delta)\,\epsilon_{\mu\alpha'}(Q\Delta)$$

$$= f^2\, t\left[Q_\alpha Q_{\alpha'} - Q(\Delta)^2 g_{\alpha\alpha'}\right]$$

$$\approx \left[t\ f^2\right]$$

Other products of tensors and momenta which occur in traces of more complicated vertex functions do not contribute to leading order.

3. Traces Involving Mixed Normality

The mixed normality FFB traces contain a γ_5 term from the abnormal vertex which gives rise to an anti-symmetric tensor. The corresponding BBB traces also contain an anti-symmetric tensor which comes explicitly from the abnormal vertex. This is illustrated in the following two examples

$$\mathrm{tr}(\tfrac{11}{22}J)^{+-} = \mathrm{tr}\, e^{+}_{\rho}(\tfrac{11}{22}J)(\not p + m)\,\bar{e}^{-}_{\rho}(\tfrac{11}{22}J)(\not p' + m')$$

$$= \mathrm{tr}\left[g_2\gamma_\rho + g_1 P_\rho\right](\not p + m)\,\gamma_5\left[-f_2\gamma_{\rho'} + f_1 P_{\rho'}\right](\not p' + m')$$

$$= - g_2 f_2 \, \mathrm{tr}\, \gamma_5 \gamma_\beta \not{p} \gamma_{\beta'} \not{p}'$$

$$= - 4 g_2 f_2 \, \epsilon_{\beta' \beta} (p\, p')$$

$$= - 4 g_2 f_2 \, \epsilon_{\beta \beta'} (P \Delta)$$

$$\mathrm{tr}(10J)^{+-} = e^{+}_{\alpha\mu}(10J) \left[g_{\mu\mu'} - \frac{k_\mu k_{\mu'}}{\mu^2} \right] e^{-}_{\alpha'\mu'}(10J)$$

$$= g \, \epsilon_{\alpha\mu}(Q\Delta) \left[f_2 \, g_{\alpha'\mu} + f_1 Q_{\alpha'} Q_\mu \right]$$

$$= g \, f_2 \, \epsilon_{\alpha\alpha'} (Q\Delta)$$

Any mixed normality contribution to the differential cross section, then, is of the form

$$\epsilon_{\beta\beta'}(P\Delta) \, \mathcal{P}^{J}_{\beta;\alpha} \, \mathcal{P}^{J}_{\beta';\alpha'} \, \epsilon_{\alpha\alpha'}(Q\Delta)$$

which behaves, at best, as $t\nu^{2J-1}$. We shall ignore mixed normality contributions.

4. Prescription for Reggeized Cross Sections

The traces which contribute to leading order only have been calculated and are listed in Table IV. For the exchange of a given trajectory, the differential cross section is simply the product of the required traces. Thus,

$$\nu^2 \frac{d\sigma}{dt} \sim \left[\qquad \right] \left[\qquad \right] \nu^{2\alpha(t)} \left| \xi_{\pm} \right|^2$$

Minor complications arise if the couplings at a vertex are split up by selection rules. As an example, consider $\gamma p \to \pi^+ n$ with equal proton and neutron masses. As a result of G-parity invariance the pion exchange contribution to the cross section is

$$\nu^2 \frac{d\sigma}{dt} \sim \left| -\frac{t}{4} f_1^2 \right| \left| -\frac{t}{4} \tilde{g} \right| \nu^{2\alpha_\pi(t)} \left| \xi_\pi \right|^2$$

and rho exchange gives

$$\nu^2 \frac{d\sigma}{dt} \sim 8 \left[(g_2 + m_+ g_1)^2 - \frac{t}{4} g_1^2 \right] \left[t \tilde{f}^2 \right] \nu^{2\alpha_\rho(t)} \left| \xi''_\rho \right|^2 \quad .$$

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81. Our γ-algebra is discussed in Appendix III.

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85. From this point on ξ includes the factor $(\pi\alpha')$ unless it appears explicity. Also, we adopt the convention of using ξ'' for normal exchange and ξ , ξ' for abnormal, C-normal, C-abnormal exchange. When no C-normality distinction is made, ξ is used for a abnormal exchange.

86. Gilman in ref(2) has remarked on this problem and suggested that as the pion contributes to two amplitudes one may vanish while the other contains a relatively constant cut contribution. This could account for the turn over. Our suggestion that the $(NN^{*}\pi)$ vertex evades at $t = 0$ to ensure a vanishing $M = 1$ pion contribution has two drawbacks 1) factorization would

require identical behaviour in $\pi N \rightarrow \rho N^*$ which is not seen and 2) to produce both the peak and the turn over the residues would have to be strongly t-dependent.

87. A very recent fit to $\gamma N \rightarrow V^0 N$, πN^* has been carried out by J.P. Ader, M. Capdeville, Bordeaux (July 1969) Preprint, in which they point out that, neglecting cuts, conspiracy (M =1, pion) is consistent with the data while evasion, (not to be confused with evasion at a vertex) is not. They carry out their fit to $\gamma N \rightarrow \pi N^*$ data without strongly t-dependent residues which would suggest that evasion at the ($NN^*\pi$) vertex is not in fact necessary.

88. Since the writing of this section Ader and Capdeville(87) have fitted $\gamma N \rightarrow V^0 N$ data with pion conspiracy. They deprecate the evasive fit to $\gamma p \rightarrow \omega^0 p$ because of the strong t-dependence of the Pomeron residue required and suggest that the only consistent solution arises from pion conspiracy with constant residues. The constant residue presents a problem due to factorization as the parameterization of the conspiring $N\bar{N}\pi$ vertex required to fit $pn \rightarrow np$, $\gamma p \rightarrow \pi^+ n$ was t-dependent and vanished at $-t \simeq \mu^2$. Such a parameterization, say Ader and Capdeville, is inconsistent with the data for $\gamma p \rightarrow \omega^0 p$. As the pion contribution to $\gamma p \rightarrow \rho^0 p$ is small, the Ader and Capdeville fit is due mainly to Pomeron exchange.

TABLES AND FIGURES

TABLE I : PION PHOTOPRODUCTION, REGGE CONTRIBUTION TO INVARIANT AMPLITUDES.

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS.	ABNORMAL EXCHANGE		NORMAL EXCHANGE
	A_1 , A^c_1 $C_n(-)^J=-1$	π, B, $C_n(-)^J=1$	$\rho, \omega, \phi, A_2, \pi_c$, $C_n(-)^J=1$
$\tilde{A}_1$			$+2\tilde{f}\,\frac{c_J}{J^2}\left[m(mg_1+g_2) - \frac{t}{4}g_1\right] J\,\mathcal{P}'_J$
$\tilde{A}_2$	$\tilde{g}' f_2 \frac{c_J}{J^2}\,\frac{m}{t}\,(t-\mu^2)\,\mathcal{P}''_J$	$-\tilde{g} f_1\,\frac{c_J}{J}\,\mathcal{P}'_J$	$+2\tilde{f}\,\frac{c_J}{J^2}\left[m g_2 Q^2(\Delta)\,\mathcal{P}''_J - g_1 J\,\mathcal{P}'_J\right]$
$\tilde{A}_3$	$-\tilde{g}'\, f_2\,\frac{c_J}{J^2}\left[\mathcal{P}'_J - \nu\mathcal{P}''_J\right]$		$+2\tilde{f} g_2\,\frac{c_J}{J^2}\,\frac{(t-\mu^2)}{4}\,\mathcal{P}''_J$
$\tilde{A}_4$	$\tilde{g}'\, f_2\,\frac{c_J}{J^2}\,\frac{(t-\mu^2)}{4}\,\mathcal{P}''_J$		$-2\tilde{f}\,\frac{c_J}{J^2}\left[(mg_1+g_2)\,J\,\mathcal{P}'_J - \frac{t}{4}Q^2(\Delta)\,\mathcal{P}''_{J-1}g_2\right]$

TABLE II : $\gamma N \longrightarrow \pi N$, Regge Contributions to Helicity Amplitudes.

HELICITY AMPLITUDES REGGE CONTRIBUTION	ABNORMAL ECHANGE		NORMAL EXCHANGE
	A_1 , A^c_1, $C_n(-)^J=-1$	π , B, $C_n(-)^J= 1$	A_2, ρ , ω , ϕ , π_c, $c_n(-)^J= 1$
$\bar{f}^-_{o1}$		$P_t k_t \frac{t}{4} (\frac{c_J}{J} \tilde{g} f_1) \mathcal{P}'_J$	
$\bar{f}^+_{o1}$			$-\frac{c_J}{J} \tilde{f} \frac{k_t}{2} \sqrt{t} \left[m(mg_1 + g_2) - \frac{t}{4} g_1 \right] \mathcal{P}'_J$
$\bar{f}^-_{11}$	$+ P_t k_t \frac{\sqrt{t}}{2} (\frac{c_J}{J^2} \tilde{g}' f_2)(\mathcal{P}'_J - \nu \mathcal{P}''_J)$		$- P_t k^2_t \frac{t}{4} (2\frac{c_J}{J^2} \tilde{f} g_2) \mathcal{P}''_J$
$\bar{f}^+_{11}$	$- P^2_t k^2_t \frac{\sqrt{t}}{2} (\frac{c_J}{J^2} \tilde{g}' f_2) \mathcal{P}''_J$		$+ k_t \frac{t}{4} (2\frac{c_J}{J^2} \tilde{f} g_2)(\mathcal{P}'_J - \nu \mathcal{P}''_J)$

TABLE III : $VN \rightarrow \pi N$, REGGE CONTRIBUTIONS TO HELICITY AMPLITUDES.

HELICITY AMPLITUDES REGGE CONTRIBUTIONS	ABNORMAL EXCHANGE		NORMAL EXCHANGE
	$A_1,\ A^c{}_1,\ C_n(-)^J=-1$	$\pi,\ B,\ C_n(-)^J=1$	$\rho,\ \omega,\ \phi,\ \pi_c,\ C_n(-)^J=1,\ A_2$
$\bar{f}_{o1}^{\,-}$		$-P_t\frac{\sqrt{t}}{2}(f_1h_1\frac{c_J}{J})\wp'_J$	
$\bar{f}_{o1}^{\,+}$			$-k_t\frac{\sqrt{t}}{2}(2f\frac{c_J}{J^2})(-g_1P_t^2+mg_2)\,J\wp'_J$
$\bar{f}_{11}^{\,-}$	$-P_t\,(\frac{c_J}{J^2}h_2f_2)(\wp'_J-\nu\wp''_J)$		$-P_t(\frac{t}{4}k_t^2+m^2\mu_v^2)(2fg_2\frac{c_J}{J^2})\,\wp''_J$
$\bar{f}_{11}^{\,+}$	$P_t^2\,k_t(\frac{c_J}{J^2}\,f_2h_2)\,\wp''_J$		$+\frac{k_t t}{4}(2fg_2\,\frac{c_J}{J^2})(\wp'_J-\nu\wp''_J)$
$\bar{f}_{oo}^{\,-}$		$\frac{\sqrt{t}}{2\sqrt{2}\mu_v}(f_1\frac{c_J}{J})\Big\{\Big[h_2k_oP_t\cos\theta_t+\frac{\sqrt{t}}{2}k_t\nu h_1\Big]\wp'_J$ $-k_tP_t^2\Big[h_2\,k_o-\frac{\sqrt{t}}{2}k_t^2h_1\Big]\wp'_{J-1}\Big\}$	
$\bar{f}_{1o}^{\,+}$	$-\frac{2P_t}{\mu_v\sqrt{2}}(\frac{c_J}{J^2}f_2)(k_oh_2-k_t^2\frac{\sqrt{t}}{2}\,h_1)J\,\wp'_J$		$-2P_t\,k_o\mu_v m^2(2fg_2\,\frac{c_J}{J^2})\,\wp''_J$

TABLE IV : VERTEX TRACES.

VERTEX	TRACE
$(00J)^{++}$	$\left[\ g^2\ \right]$
$(00J)^{--}$	$\left[\ 0\ \right]$
$(01J)^{++}$	$\left[\ 4\,(g_2 - (\mu_+\mu_- + \frac{t}{4})g_1)^2\,\mu^{-2} - (\frac{t}{4})\ g_1{}^2\ \right]$
$(01J)^{--}$	$\left[\ tf^2\ \right]$
$(0\gamma J)^{++}$	$\left[-\frac{t}{4}\tilde{g}^2\right]$
$(0\gamma J)^{--}$	$\left[\ t\tilde{f}^2\ \right]$
$(1\gamma J)^{++}$	$\left[\ ((t-\mu^2)\tilde{g}_1+4\tilde{g}_2)^2+\frac{1}{2}t\tilde{g}_1\tilde{g}_3-\frac{t}{4\mu^2}(2\tilde{g}_2+\tilde{g}_3)^2\ \right]$
$(1\gamma J)^{--}$	$\left[\ ((t-\mu^2)\tilde{f}_1+4\tilde{f}_2)^2-\frac{t}{4\mu^2}(4\tilde{f}_2-(t-\mu^2)\tilde{f}_3)^2\ \right]$
$(\gamma\gamma J)^{++}$	$\left[\ \tilde{g}_2^2 + \frac{1}{4}t^2\tilde{g}_1^2\ \right]$
$(\gamma\gamma J)^{--}$	$\left[\ 2t^2\tilde{f}_1{}^2\ \right]$ or $\left[\ 8\tilde{f}_2{}^2\ \right]$
$(\frac{1}{2}\frac{1}{2}J)^{++}$	$\left[\ 8\,(m_+g_1+g_2)^2 - \frac{1}{4}t^2g_1^2\right]$
$(\frac{1}{2}\frac{1}{2}J)^{--}$	$\left[\ 8\,(m_-f_1-f_2)^2 - \frac{1}{4}t\ g_1^2\right]$
$(\frac{1}{2}\frac{1}{2}J)^{--}$ (self conjugate fermions)	$\left[\ -2tf_1{}^2\right]$ or $\left[\ 8f_2{}^2\ \right]$

TABLE V : $\gamma N \longrightarrow \pi N^*$, REGGE CONTRIBUTIONS TO INVARIANT AMPLITUDES.

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	NORMAL EXCHANGE
	$\rho, \omega, \phi, A_2, \pi_c, \quad C_n(-)^J = 1$
$\tilde{A}_1^+$	$\frac{-2c_J\tilde{f}}{J^2(J-1)}\Big\{ J(J-1)\frac{f_1}{4}\left[t+4m_+m_-\right]\wp_J' + (J-1)f_2\left[-m_+(\wp_J' - \nu\wp_J'') - \frac{4m_+^2m_-}{t}k.Q\,\wp_J'' + Q(\Delta)^2\frac{m_-}{4t}(t-4m_+^2)(t+4m_+m_-)\wp_{J-1}''\right]$ $+(J-1)f_3\left[-(\wp_J' - \nu\wp_J'') - \frac{Q(\Delta)^2}{4t}(t+4m_+m_-)^2\wp_{J-1}''\right] + f_4\left[m_+(1-\frac{4k.Q}{t})\wp_J'' - m_+(\nu + \frac{k.Q\,\nu}{m_+^2} - \mu^2 - \frac{4\mu^2m_+m_-}{t})\wp_J'''\right.$ $+((t+4m_+m_-)\frac{Q(\Delta)^2}{t}2m_+ - Q(\Delta)^2m_-)\wp_{J-1}'' + m_+\frac{Q(\Delta)^2}{t}\left[t+4m_+m_-\right]\left[\frac{m_+m_-}{4} + \frac{\mu^2}{4}\right]\wp_{J-1}'''$ $\left. + m_-Q(\Delta)^2(-\frac{2}{4t}(t+4m_+m_-) + (\nu - \frac{\mu^2}{4}))\wp_{J-1}''' - \frac{m_-}{4t}Q(\Delta)^4(t+4m_+m_-)^2\wp_{J-2}'''\right]\Big\}$
$\tilde{A}_2^+$	$\frac{-2c_J\tilde{f}}{J^2(J-1)}\Big\{\frac{f_4}{m_+}\left[m_+m_-\nu + k.Q(m_+^2 + m_-^2 - \frac{t}{4})\right]\wp_J'''\Big\}$
$\tilde{A}_3^+$	$\frac{-2c_J\tilde{f}}{J^2(J-1)}\Big\{-J(J-1)f_1\,\wp_J' - (J-1)f_2\left[\frac{4m_+}{t}k.Q\,\wp_J'' + Q(\Delta)^2\frac{m_-}{t}(t-4m_+^2)\wp_{J-1}''\right] + (J-1)f_3\left[-\frac{4k.Q}{t}\wp_J'' + \frac{Q(\Delta)^2}{t}(t+4m_+m_-)\wp_{J-1}''\right]$ $+f_4\left[\frac{4k.Q}{t}m_+\wp_J''' - Q(\Delta)^2\frac{4m_+}{t}\wp_{J-1}'' - \frac{m_+}{t}Q(\Delta)^2(t+4m_+m_-)\wp_{J-1}''' - \frac{4k.Q}{t}m_-Q(\Delta)^2\wp_{J-1}''' + \frac{m_-}{t}Q(\Delta)^4(t+4m_+m_-)\wp_{J-2}'''\right]\Big\}$

TABLE V (Cont'd.) : $\gamma N \longrightarrow \pi N^*$

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	NORMAL EXCHANGE
	$\rho, \omega, \phi, A_2, \pi_c \quad C_n(-)^J = 1$
$\tilde{A}_4^+$	$\frac{-2c_J\tilde{f}}{J^2(J-1)}\Big\{ J(J-1)f_1\left[m_+ + m_-\right]\mathcal{P}'_J + (J-1)f_2\Big[-(\mathcal{P}'_J - \nu\mathcal{P}''_J) - (m_+^2 + m_+m_- - t)\frac{k.Q}{t}\mathcal{P}''_J + (m_+ + m_-)\frac{m_-}{t}Q(\Delta)^2(t-4m_+^2)\mathcal{P}''_{J-1}\Big]$ $+(J-1)f_3\Big[(m_+ + m_-)\frac{4k.Q}{t}\mathcal{P}''_J + (m_+ + m_-)\frac{Q(\Delta)^2}{t}(t+4m_+m_-)\mathcal{P}''_{J-1}\Big] + f_4\Big[-\frac{4k.Q}{t}\mathcal{P}''_J + \frac{4m_+}{t}Q(\Delta)^2(2m_- - m_+)\mathcal{P}''_{J-1}$ $-4k.Q\left(-\frac{m_+}{t}(m_+m_-) + \frac{4(k.Q)^2}{t} + \mu^2\nu + \frac{m_-}{m_+}k.Q\right)\mathcal{P}'''_J + (t+4m_+m_-)(m_+^2 + m_+m_- - k.Q - \nu)\frac{Q(\Delta)}{t^2}\mathcal{P}'''_{J-1}$ $+\frac{4k.Q}{t}m_-(m_+ + m_-)Q(\Delta)^2\mathcal{P}'''_{J-1} + \frac{m_-}{t}Q(\Delta)^4(t+4m_+m_-)\mathcal{P}'''_{J-2}\Big]\Big\}$
$\tilde{A}_5^+$	$\frac{-2c_J\tilde{f}}{J^2(J-1)}\Big\{ -(m_+\nu - k.Qm_-)\frac{f_4}{m_+}\mathcal{P}'''_J\Big\}$
$\tilde{A}_6^+$	$\frac{-2c_J\tilde{f}}{J^2(J-1)}\Big\{ -J(J-1)f_1m_+m_-\mathcal{P}'_J + (J-1)f_2\Big[m_+(\mathcal{P}'_J - \nu\mathcal{P}''_J) - \frac{4m_+^2}{t}k.Qm_-\mathcal{P}''_J - Q(\Delta)^2\frac{m_-}{t}(t-4m_+^2)\mathcal{P}''_{J-1}\Big]$ $+(J-1)f_3\Big[(\mathcal{P}'_J - \nu\mathcal{P}''_J) - \mu^2\frac{m_+m_-}{t}\mathcal{P}''_J + \frac{Q(\Delta)^2}{t}(t+4m_+m_-)m_+m_-\mathcal{P}''_{J-1}\Big] + f_4\Big[4m_+k.Q\mathcal{P}''_J - m_+\mathcal{P}''_J$

TABLE V (Cont'd.) : $\gamma N \longrightarrow \pi N^*$

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	NORMAL EXCHANGE $\rho,\ \omega,\ \phi,\ A_2,\ \pi_c \qquad C_n(-)^J = 1$
$\tilde{A}_6^+$	$- m_+^2 \frac{m_-}{t}(2Q\cdot\Delta)\,\mathcal{P}_J''' - m_+\nu\,\mathcal{P}_J''' - \frac{2Q\cdot\Delta}{t} m_+\nu\,\mathcal{P}_J''' - \frac{k\cdot Q}{m_+}(\nu + k\cdot Q)\,\mathcal{P}_J''' - Q(\Delta)^2 \frac{4m_+^2 m_-}{t}\mathcal{P}_{J-1}'' + m_- Q(\Delta)^2\,\mathcal{P}_{J-1}''$ $-\left(-\frac{m_+^2 m_-}{t} Q(\Delta)^2(t+4m_+m_-) + m_+m_-^2 Q(\Delta)^2 \frac{2Q\cdot\Delta}{t} - m_-Q(\Delta)^2\nu - \frac{Q(\Delta)^2}{t}(t+4m_+m_-)m_+\nu\right)\mathcal{P}_{J-1}'''$ $+ \frac{m_+m_-^2}{t}\, Q(\Delta)^4\,(t+4m_+m_-)\,\mathcal{P}_{J-2}''' \,\Big]\Big\}$
$\tilde{A}_7^+$	$\frac{-2c_J\tilde{f}}{J^2(J-1)}\Big\{ -J(J-1)f_1\frac{m_-}{4}(t-m_+^2)\,\mathcal{P}_J' + (J-1)f_2\Big[\frac{(t-4m_+^2)}{4}(\mathcal{P}_J' - \nu\mathcal{P}_J'') - \frac{k\cdot Q\, m_+ m_-}{t}(t-4m_+^2)\,\mathcal{P}_J''$ $- Q(\Delta)^2\, \frac{m_-^2}{4t}\big[t-4m_+^2\big]^2\,\mathcal{P}_{J-1}''\Big] + (J-1)f_3\Big[-m_+(\mathcal{P}_J' - \nu\mathcal{P}_J'') - \frac{m_-\mu^2}{4t}(2t-4m_+^2)\,\mathcal{P}_J''$ $+ \frac{m_-}{4}(t-4m_+^2)Q(\Delta)^2(t+4m_+m_-)\,\mathcal{P}_{J-1}''\Big] + f_4\Big[(m_+^2 - k\cdot Q)\,\mathcal{P}_J'' + \Big(\frac{-m_+m_-}{4t}\mu^2(t-4m_+^2) - m_+(\nu m_+ - \frac{m_-}{4}\mu^2)$ $- \frac{k\cdot Q}{m_+}(\nu m_+ - \mu^2\frac{m_-}{4})\Big)\mathcal{P}_J''' + \Big((-\frac{m_+m_-}{t} - Q(\Delta)^2)(t-4m_+^2) - \frac{Q(\Delta)^2}{4}(t+4m_+m_-))(t-4m_+^2) - m_+m_-Q(\Delta)^2\Big)\mathcal{P}_{J-1}''$

TABLE V (Cont'd.) : $\gamma N \longrightarrow \pi N^*$

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	NORMAL EXCHANGE $\rho, \omega, \phi, A_2, \pi_c \qquad C_n(-)^J = 1$
$\tilde{A}_7^+$	$+ \left(-\frac{m_- m_+}{4t}(t-4m_+^2)(t+4m_+m_-)\, Q(\Delta)^2 - \frac{m_-^2}{4t}\mu^2 Q(\Delta)^2 (t-4m_+^2) + m_- Q(\Delta)^2(\nu m_+ - \frac{m_-}{4}\mu^2) \right) \mathcal{P}'''_{J-1}$ $+ \frac{m_-^2}{4t}(t-4m_+^2) Q(\Delta)^4 (t+4m_+m_-)\, \mathcal{P}'''_{J-2} \Big] \Big\}$
$\tilde{A}_8^+$	$\frac{-2c_J \tilde{f}}{J^2(J-1)} \Big\{ J(J-1) f_1 m_-^2 \mathcal{P}'_J + (J-1) f_2 \Big[-m_-(\mathcal{P}'_J - \nu \mathcal{P}''_J) + \frac{m_-^2 m_+}{t} 4k.Q\, \mathcal{P}''_J + \frac{m_-^3}{t} Q(\Delta)^2 (t-4m_+^2)\, \mathcal{P}''_{J-1} \Big] + (J-1) f_3 \Big[\frac{(m_-^2 \mu^2}{t} - m_-^2 - m_+ m_- + k.Q) \mathcal{P}''_J$ $- \frac{m_-}{t} Q(\Delta)^2 (t+4m_+m_-)\, \mathcal{P}''_{J-1} \Big] + f_4 \Big[-\frac{4k.Q}{t} \mathcal{P}''_J + (\frac{k.Q}{m_+}(\nu + k.Q + m_+ m_- + m_-^2) + \frac{\mu^2 m_- \nu}{t} - (k.Q - m_-^2 - m_- m_+)\, m_+$ $- \mu^2 \frac{m_+ m_-^2}{t}) \mathcal{P}'''_J + (-\frac{m_-^2 m_+}{t}(t+4m_+m_-) Q(\Delta)^2 - \frac{m_- \mu^2}{t} Q(\Delta)^2 + m_- Q(\Delta)^2 (k.Q - m_-^2 - m_- m_+) - \frac{m_-}{t} \nu Q(\Delta)^2 (t+4m_+m_-)\, \mathcal{P}'''_{J-1}$ $- \frac{m_-^3}{t} Q(\Delta)^4 (t+4m_+m_-)\, \mathcal{P}'''_{J-2} \Big] \Big\}$

TABLE V (Cont'd.) : $\gamma N \longrightarrow \pi N^*$

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	ABNORMAL EXCHANGE
	π, B, $C_n(-)^J = 1$; A_1, A_1^c, $C_n(-)^J = -1$
$\tilde{A}_1^-$	$\frac{c_J}{J^2(J-1)}\tilde{g}\left\{-(\nu-\frac{\mu^2}{4})\frac{g_4}{m_+}(2\wp''_J-\nu\wp'''_J) - (J-1)(m_+ + m_-)k.Qg_2\wp''_J+(\frac{m_+ + m_-}{t})k.Qg_4(-\mu^2\wp'''_J+Q(\Delta)^2(t+4m_+m_-)\wp'''_{J-1}\right\}$
$\tilde{A}_2^-$	$\frac{c_J}{J^2(J-1)}\tilde{g}\left\{\frac{g_4}{m_+}(2\wp''_J-\nu\wp'''_J)\right\}$
$\tilde{A}_3^-$	$\frac{c_J}{J^2(J-1)}\tilde{g}\left\{-J(J-1)g_1\wp'_J+(J-1)g_2\left[\frac{4m_-}{t}k.Q\wp''_J+\frac{Q(\Delta)^2}{t}(t+4m_+m_-)\wp''_{J-1}\right]+(J-1)g_3\left[-\frac{4k.Q}{t}\wp''_J+\frac{Q(\Delta)^2}{t}(t+4m_+m_-)\wp''_{J-1}\right]\right.$
	$\left.+g_4\left[Q(\Delta)\frac{4m_-}{t}\wp''_{J-1}+\frac{4k.Q}{t}(\frac{4m_-}{t}k.Q\wp'''_J+m_+Q(\Delta)^2\wp'''_{J-1})-\frac{Q(\Delta)^2}{t}(t+4m_+m_-)(\frac{4m_-}{t}k.Q\wp'''_{J-1}+m_+Q(\Delta)^2\wp'''_{J-2})\right]\right\}$
$\tilde{A}_4^-$	$\frac{c_J}{J^2(J-1)}\tilde{g}\left\{-(J-1)g_2(\wp'_J-\nu\wp''_J)-(J-1)k.Qg_2\wp''_J+g_4\left[-\frac{4k.Q}{t}(\wp''_J-\nu\wp'''_J)+\frac{Q(\Delta)^2}{t}(t+4m_-m_+)(\wp''_{J-1}-\nu\wp'''_{J-1})\right.\right.$
	$\left.\left.+\frac{k.Q}{t}(2Q.\Delta\,\wp'''_J+Q(\Delta)^2(t+4m_-m_+)\wp'''_{J-1}\right]+\frac{m_-}{m_+}g_4(2\wp''_J-\nu\wp'''_J)\right\}$
$\tilde{A}_5^-$	$\frac{c_J}{J^2(J-1)}\tilde{g}\left\{g_4k.Q\wp'''_J-\frac{m_-}{m_+}g_4(2\wp''_J-\nu\wp'''_J)\right\}$

TABLE V (Cont'd.) : $\gamma N \longrightarrow \pi N^*$

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	ABNORMAL EXCHANGE
	π , B, $C_n(-)^J = 1$; A_1, A_1^c , $C_n(-)^J = -1$
$\tilde{A}_6^-$	$\frac{c_J}{J^2(J-1)} \tilde{g} \Big\{ +(J-1) g_3 k.Q\, \wp''_J + (J-1) m_+ g_2 \wp''_J + g_4 \Big(Q(\Delta)^2 (m_- \wp'''_J - m_+ \wp'''_{J-1} k.Q) - k.Q \frac{m_+}{t} (2Q.\Delta\, \wp'''_J + Q(\Delta)^2 (t+4m_- m_+) \wp'''_{J-1}$ $+ \frac{1}{m_+} (\nu + k.Q)(2\wp''_J - \nu \wp'''_J) \Big) \Big]$
$\tilde{A}_7^-$	$\frac{c_J \tilde{g}}{J^2(J-1)} \Big\{ (J-1) \frac{g_2}{4} (t-4m_+^2) \wp''_J + g_4 \Big[k.Q(\wp''_J - \nu \wp'''_J) - \frac{1}{4}(t-4m_+^2) k.Q (2Q.\Delta\, \wp'''_J + Q(\Delta)^2 (t+4m_+ m_-) \wp'''_{J-1} \quad)$ $- (\nu + k.Q - \frac{m_- \mu^2}{4m_+})(2\wp''_J - \nu \wp'''_J \quad) \Big] \Big\}$
$\tilde{A}_8^-$	$\frac{c_J \hat{\tilde{g}}}{J^2(J-1)} \Big\{ - (J-1) g_2 m_- \wp''_J + (J-1) g_3\, \nu \wp''_J + g_4 \Big[Q(\Delta)^2 m_+ (\wp''_{J-1} - \nu \wp'''_{J-1} \quad) - \frac{4m_-}{t} k.Q\, \nu \wp'''_J$ $+ \frac{k.Q}{t} (2Q.\Delta\, \wp'''_J + Q(\quad)^2 (t+4m_+ m_-) \wp'''_{J-1}) + \frac{(m_+ + m_-)}{m_+} \quad (2\wp''_J - \nu \wp'''_J \quad) \Big] \Big\}$

TABLE VI : $\gamma N \longrightarrow \pi N^*$, REGGEIZED INVARIANT AMPLITUDES (LEADING ORDER).

REGGEIZED INVARIANT AMPLITUDES	ABNORMAL EXCHANGE	NORMAL EXCHANGE
	π, B, $C_n(-)^J = 1$; A_1, A_1^c, $C_n(-)^J = -1$	ρ, ω, ϕ, A_2, π_c, $C_n(-)^J = 1$
$\tilde{A}_1$	$\tilde{g}\frac{g_4}{m_+}(-\nu)^{\alpha-1}\xi_\pm$	$-2\tilde{f}\left[\frac{f_1}{4}(t+4m_+m_-) - m_+f_2 - f_3\right](-\nu)^{\alpha-1}\xi''_\pm$
$\tilde{A}_2$	$\tilde{g}\frac{g_4}{m_+}(-\nu)^{\alpha-2}\xi_\pm$	$+2\tilde{f}\, f_4 m_-(-\nu)^{\alpha-2}\xi''_\pm$
$\tilde{A}_3$	$-\tilde{g}g_1(-\nu)^{\alpha-1}\xi_\pm$	$-2\tilde{f}f_1(-\nu)^{\alpha-1}\xi''_\pm$
$\tilde{A}_4$	$-\tilde{g}g_2(-\nu)^{\alpha-1}\xi_\pm$	$-2\tilde{f}\left[(m_+ + m_-) f_1 - f_2\right](-\nu)^{\alpha-1}\xi''_\pm$
$\tilde{A}_5$	$-\tilde{g}g_4\frac{m_+}{m_-}(-\nu)^{\alpha-2}\xi_\pm$	$-2\tilde{f}f_4(-\nu)^{\alpha-2}\xi''_\pm$
$\tilde{A}_6$	$-\tilde{g}\frac{g_4}{m_+}(-\nu)^{\alpha-1}\xi_\pm$	$-2\tilde{f}\left[-m_+m_-f_1 + m_+f_2 + f_3\right](-\nu)^{\alpha-1}\xi''_\pm$

TABLE VI : $\gamma N \longrightarrow \pi N^*$, REGGEIZED INVARIANT AMPLITUDES (LEADING ORDER).

REGGEIZED INVARIANT AMPLITUDES	ABNORMAL EXCHANGE	NORMAL EXCHANGE
	π, B, $C_n(-)^J = 1$; A_1, A_1^c, $C_n(-)^J = -1$	$\rho, \omega, \phi, A_2, \pi_c, C_n(-)^J = 1$
$\tilde{A}_7$	$+\tilde{g}g_4(-\nu)^{\alpha-1}\,\xi_{\pm}$	$-2\tilde{f}\left[-f_1 \frac{m_-}{4}(t-4m_+^2) + \frac{f_2}{4}(t-4m_+^2) - m_+f_3\right](-\nu)^{\alpha-1}\,\xi''_{\pm}$
$\tilde{A}_8$	$-\tilde{g}g_3 \frac{(\alpha-1)}{\alpha}(-\nu)^{\alpha-1}\,\xi_{\pm}$	$-2\tilde{f}\left[m_-^2 f_1 - m_- f_2\right](-\nu)^{\alpha-1}\,\xi''_{\pm}$

TABLE VII : $\gamma N \longrightarrow VN$, REGGE CONTRIBUTIONS TO THE INVARIANT AMPLITUDES.

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	ABNORMAL	NORMAL
	π, B, $C_n(-)^J = 1$	ρ, ω, ϕ, A_2, π_c, Pomeron $C_n(-)^J = 1$
$\tilde{A}_1$	$\frac{-c_J}{J(J-1)} f_1 \Big\{ J(J-1)2\tilde{f}_1 \mathcal{P}_J + (J-1)\tilde{f}_2 \frac{4k.Q}{t} P^2 \mathcal{P}'_{J-1} + \tilde{f}_3 \Big[(\nu^2 - (2m^2 + \frac{t}{2})k.Q)\mathcal{P}''_J + k.Q P^2 \frac{(t+\mu_v^2)}{t} \mathcal{P}''_{J-1} \Big]\Big\}$	$-\frac{c_J}{J^2(J-1)} \frac{g_2}{m} \Big\{ -J(J-1)\tilde{g}_1 \nu \mathcal{P}'_J + (J-1)\tilde{g}_2 \Big\vert \frac{4k.Q}{t} P^2 \nu \mathcal{P}''_{J-1} - 2m^2 \mathcal{P}''_J \Big\vert + \tilde{g}_3 P^2 \Big[\frac{(t+\mu_v^2)}{2t} (2m^2 k.Q \mathcal{P}'''_{J-1} - \nu^2 \mathcal{P}'''_{J-1} - \nu \mathcal{P}''_{J-1}) + \nu \mathcal{P}''_{J-1} \Big]\Big\}$
$\tilde{A}_2$	$\frac{-c_J}{J(J-1)} f_1 \Big\{ (J-1)\mathcal{P}'_J \big(\tilde{f}_2 + (t+\mu_v^2)\frac{\tilde{f}_3}{4}\big) + \tilde{f}_3 \frac{\nu}{2}(t+\mu_v^2)\mathcal{P}''_J \Big\}$	$\frac{+c_J}{J^2(J-1)} \frac{g_2}{4m}(t+\mu_v^2) \Big\{ J(J-1)\tilde{g}_1 \mathcal{P}'_J - (J-1)\tilde{g}_2 \frac{4k.Q}{t}\mathcal{P}''_{J-1} + \tilde{g}_3 P^2 \Big[\frac{(t+\mu_v^2)}{2t}(\nu \mathcal{P}'''_{J-1} + \mathcal{P}''_{J-1}) - \mathcal{P}''_{J-1}\Big]\Big\}$
$\tilde{A}_3$	$\frac{-c_J}{J(J-1)} f_1 \tilde{f}_3 \, 2k.Q \, \mathcal{P}''_J$	$\frac{c_J}{J^2(J-1)} \Big\{ - g_2\tilde{g}_3 mQ(\Delta)^2 \mathcal{P}'''_{J-1} + Jg_1\tilde{g}_3 \mathcal{P}''_J \Big\}$
$\tilde{A}_4$	$\frac{-c_J}{J(J-1)} m \, f_1\tilde{f}_3 k.Q \, \mathcal{P}''_J$	$\frac{-c_J}{J^2(J-1)} 2g_2 \Big\{ (J-1)\tilde{g}_2 \mathcal{P}''_J - \hat{g}_3 \Big[k.Q \frac{(t+\mu_v^2)}{2t} P^2 \mathcal{P}'''_{J-1} + \frac{\nu}{4}\mathcal{P}'''_J \Big]\Big\}$

TABLE VII (Cont'd.) : $\gamma N \to VN$, REGGE CONTRIBUTIONS TO THE INVARIANT AMPLITUDES.

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	ABNORMAL EXCHANGE	NORMAL EXCHANGE
	π, B, $C_n(-)^J = 1$	ρ, ω, ϕ, A_2, π_c, Pomeron $C_n(-)^J = 1$
$\tilde{A}_5$	$\frac{+c_J}{J(J-1)}\, m f_1 \tilde{f}_3\, k.Q\, P''_J$	$\frac{c_J}{J^2(J-1)}\, \frac{g_2}{2} \tilde{g}_3 \left\{ 2P''_J - \nu P'''_J \right\}$
$\hat{A}_6$	$\frac{-c_J}{J(J-1)}\, \frac{f_1}{2} \left\{ (J-1)\, P'_J (\tilde{f}_2 + (t+\mu^2)\frac{\tilde{f}_3}{4}) + \tilde{f}_3 \frac{\nu}{2}(t+\mu^2) P''_J \right\}$	$\frac{c_J}{J^2(J-1)}\, g_2 \frac{k.Q}{m} \left\{ J(J-1)\tilde{g}_1 P'_J - (J-1)\tilde{g}_2 \frac{4k.Q}{t} P^2 P''_{J-1} + \tilde{g}_3 \left[\frac{(t+\mu^2)}{2t} \cdot (\nu P''_{J-1} + P''_{J-1}) - P^2 P''_{J-1} \right] \right\}$
$\hat{A}_7$		$\frac{c_J}{J^2(J-1)}\, g_2 \hat{g}_3\, k.Q\, P'''_J$
$\tilde{A}_8$	$\frac{+c_J}{J(J-1)}\, \frac{f_1}{2} \left\{ J(J-1)\, 2\hat{f}_1 P_J + (J-1)\hat{f}_2 P^2 \frac{4k.Q}{t} P'_{J-1} \right.$ $\left. \tilde{f}_3 \left[\nu k.Q\, P^2 \frac{(t+\mu^2)}{t} P''_{J-1} + (\nu^2 - 2P^2 k.Q) P''_J \right] \right\}$	$-\frac{1}{2} \tilde{A}^{+}_1$

TABLE VII (Cont'd.) : $\gamma N \longrightarrow VN$, REGGE CONTRIBUTIONS TO THE INVARIANT AMPLITUDES.

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	ABNORMAL EXCHANGE	NORMAL EXCHANGE
	π, B, $C_n(-)^J = 1$	ρ, ω, ϕ, A_2, π_c, Pomeron, $C_n(-)^J = 1$
$\tilde{A}_9$	$m\, A_8^{-,+}$	$\frac{c_J}{J^2(J-1)} \frac{g_2}{2}\Big\{ - J(J-1)\tilde{g}_1 \nu \mathcal{P}'_J + (J-1)\tilde{g}_2 P^2 \Big[\frac{4k.Q}{t}\nu \mathcal{P}''_{J-1} - \mathcal{P}''_J\Big]$ $+ \tilde{g}_3 P^2 \Big[\frac{(t+\mu_v^2)}{2t}(P^2 k.Q\, \mathcal{P}'''_{J-1} - \nu^2 \mathcal{P}'''_{J-1} - \nu \mathcal{P}''_{J-1}) + \nu \mathcal{P}''_{J-1}\Big]\Big\}$
$\tilde{A}_{10}$	$\frac{-c_J}{J(J-1)}\, f_1 \tilde{f}_3 \frac{t}{4} k.Q\, \mathcal{P}''_J$	$\frac{c_J}{J^2(J-1)}\Big\{\frac{g_2}{m}\Big[-J(J-1)k.Q\tilde{g}_1 \mathcal{P}'_J + (J-1)\hat{g}_2 Q(\Delta)^2 \frac{t}{4}\mathcal{P}''_{J-1}$ $+ \tilde{g}_3 \Big[\frac{(t+\mu_v^2)}{2t} k.Q(2m^2 \mathcal{P}''_{J-1} - Q(\Delta)^2 P^2 \mathcal{P}''_{J-1} - P^2(\nu \mathcal{P}'''_{J-1} + \mathcal{P}''_{J-1})$ $+ k.QP^2 \mathcal{P}''_{J-1}\Big]\Big] + Jg_1 \Big[- (J-1)\hat{g}_2 \mathcal{P}'_J + g_3 k.Q \frac{(t+\mu_v^2)}{2t} P^2 \mathcal{P}''_{J-1}\Big]\Big\}$
$\tilde{A}_{11}$		$\frac{c_J}{J^2(J-1)}\, g_2 \Big\{ J(J-1)(k.Q\tilde{g}_1 - \hat{g}_2)\mathcal{P}'_J + \frac{\mu^2}{t} g_3 k.QP^2\, \mathcal{P}''_{J-1} \Big\}$

TABLE VII (Cont'd.) : $\gamma N \longrightarrow VN$, REGGE CONTRIBUTIONS TO THE INVARIANT AMPLITUDES.

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	ABNORMAL EXCHANGE	NORMAL EXCHANGE
	π, B, $C_n(-)^J = 1$	ρ, ω, ϕ, A_2, π_c, Pomeron, $C_n(-)^J = 1$
$\tilde{A}_{12}$	$\frac{-c_J}{J(J-1)} f_1 \tilde{f}_3 \frac{t}{4} \nu \wp''_J$	$\frac{c_J}{J^2(J-1)} \Big\{ \frac{g_2}{m} \Big[-J(J-1)\tilde{g}_1 (\nu \wp'_J + m^2 Q(\Delta)^2 \wp'_{J-1})$ $+(J-1)\tilde{g}_2 \frac{4k.Q}{t} (\nu P^2 \wp''_{J-1} + m^2 Q(\Delta)^2 \wp''_{J-2} - 2m^2 \wp'_{J-1})$ $-\tilde{g}_3 \Big[\frac{(t+\mu_v^2)}{2t} \big(\nu P^2 (\nu \wp'''_{J-1} + \wp''_{J-1}) + m^2 Q(\Delta)^2 P^2 \nu \wp'''_{J-2} + 2m^2 \nu \wp''_{J-1}\big)$ $- (\frac{t\nu}{4} \wp''_{J-1} + m^2 J \wp'_{J-1}) \Big]\Big]$ $+J g_1 \Big[J(J-1)\tilde{g}_1 \wp_J - (J-1)\tilde{g}_2 \frac{4k.Q}{t} P^2 \wp'_{J-1} - \tilde{g}_3 P^2 \wp'_{J-1}$ $+ \tilde{g}_3 \frac{(t+\mu_v^2)}{2t} \nu \wp''_{J-1} \Big] \Big\}$

TABLE VII (Cont'd.) : $\gamma N \longrightarrow VN$

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	ABNORMAL EXCHANGE $A_1, \quad A_1^c, \quad C_n(-)^J = -1$
$\tilde{A}_1^{-,-}$	$\frac{-c_J f_2}{J^2(J-1)} \Big\{ -\frac{2P^2 \nu}{m} F_3 + 2mF_5 + 2m\bar{F}_1 + \frac{\ }{m} \bar{F}_2 + \frac{1}{m}(P^2 - \frac{k.Q}{2})(\nu^2 - 2m^2 k.Q)\bar{F}_3 - 2mk.Q\bar{F}_4 + (\frac{k.Q}{m} - 2m)\bar{F}_5$ $- \bar{\bar{F}}_1 + (\nu^2 - 2m^2 k.Q)\bar{\bar{F}}_3 - \frac{t}{2}\bar{\bar{F}}_4 + \nu\bar{\bar{F}}_5 \Big\}$
$\tilde{A}_2^{-,-}$	$\frac{-c_J f_2}{J^2(J-1)} \Big\{ -\frac{2P^2}{m} \frac{(t+\mu_v^2)}{4} F_3 + \frac{(t+\mu_v^2 - 8m^2)}{4m} \bar{F}_2 + \frac{\nu}{4m}(P^2 - \frac{k.Q}{2})(t+\mu_v^2)\bar{F}_3 - \frac{k.Q}{4m}(t+\mu_v^2)\bar{F}_5 + \bar{\bar{F}}_2 + \frac{\nu}{4}(t+\mu_v^2)\bar{\bar{F}}_3$ $+ k.Q\bar{\bar{F}}_5 \Big\}$
$\tilde{A}_3^{-,-}$	$\frac{-c_J f_2}{J^2(J-1)} \Big\{ 2k.Q\bar{\bar{F}}_3 + 2\bar{\bar{F}}_4 \Big\}$
$\tilde{A}_4^{-,-}$	$\frac{-c_J f_2}{J^2(J-1)} \Big\{ -2F_4 - 2F_5 - 2\bar{F}_1 + 2(\nu^2 - (k.Q)^2 + P^2 k.Q)\bar{F}_3 + k.Q\bar{F}_4 + 2\nu\bar{F}_5 + mk.Q\bar{\bar{F}}_3 + m\bar{\bar{F}}_4 \Big\}$
$\tilde{A}_5^{-,-}$	$\frac{-c_J f_2}{J^2(J-1)} \Big\{ -2F_4 - (k.Q)^2\bar{F}_3 - k.Q\bar{F}_4 - mk.Q\bar{\bar{F}}_3 - m\bar{\bar{F}}_4 \Big\}$

TABLE VII (Cont'd.) : $\gamma N \longrightarrow VN$

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	ABNORMAL EXCHANGE A_1, A_1^c, $C_n(-)^J = -1$
$\tilde{A}_6^{-,-}$	$\frac{-c_J f_2}{J^2(J-1)} \left\{ -\frac{P^2}{m} k.QF_3 + k.Q\bar{F}_2 + \nu k.Q(m + \frac{1}{2}m(P^2 - \frac{k.Q}{2})\bar{F}_3 + \frac{(k.Q)^2}{2m}\bar{F}_5 + \frac{1}{2}\bar{\bar{F}}_2 + \frac{}{4}(2k.Q+t)\bar{\bar{F}}_3 + \frac{1}{2}k.Q\bar{\bar{F}}_5 \right\}$
$\tilde{A}_7^{-,-}$	$\frac{-c_J f_2}{J^2(J-1)} \left\{ -2F_4 + 2k.Q\,\nu\bar{F}_3 + 2\nu\bar{F}_4 \right\}$
$\tilde{A}_8^{-,-}$	$\frac{-c_J f_2}{J^2(J-1)} \left\{ \frac{P^2\nu}{m} F_3 - mF_4 - mF_5 - m\bar{F}_1 - \frac{\nu}{2m}\bar{F}_2 + (-m(k.Q)^2 - \frac{1}{2m}(\nu^2 - 2m^2 k.Q))(P^2 - \frac{k.Q}{2})\bar{F}_3 + \nu(m - \frac{k.Q}{2})\bar{F}_5 \right.$ $\left. + \frac{1}{2}\bar{\bar{F}}_1 - (\frac{\nu^2}{2} - P^2 k.Q)\bar{\bar{F}}_3 - \frac{\nu}{2}\bar{\bar{F}}_5 \right\}$
$\tilde{A}_9^{-,-}$	$\frac{-c_J f_2}{J^2(J-1)} \left\{ \frac{1}{2}F_2 + P^2\nu F_3 - P^2F_4 - P^2F_5 - P^2\bar{F}_1 - \frac{1}{2}(P^2 - \frac{k.Q}{2})(\nu^2 - 2k.Q\,P^2)\bar{F}_3 + \nu(P^2 - \frac{k.Q}{2})\bar{F}_5 + m\bar{\bar{F}}_1 \right.$ $\left. - (\frac{\nu^2}{2} - P^2 k.Q)\bar{\bar{F}}_3 - \frac{\nu}{2}\bar{\bar{F}}_5 \right\}$
$\tilde{A}_{10}^{-,-}$	$\frac{-c_J f_2}{J^2(J-1)} \left\{ -mk.Q\,F_3 + 2mF_4 - k.Q\bar{F}_2 + k.Q(m(\nu - 2k.Q) - \frac{\nu}{m}(P^2 - \frac{k.Q}{2}))\bar{F}_3 - \frac{(k.Q)^2}{m}\bar{F}_5 + \frac{t}{4}k.Q\bar{\bar{F}}_3 \right\}$

TABLE VII (Cont'd.) : $\gamma N \longrightarrow VN$

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	ABNORMAL EXCHANGE
	$A_1, \quad A_1^c, \quad C_n(-)^J = -1$
$A_{11}^{-,-}$	$\frac{-c_J f_2}{J^2(J-1)} \left\{ 2P^2 F_4 \right\}$
$A_{12}^{-,-}$	$\frac{-c_J f_2}{J^2(J-1)} \left\{ -(m + \frac{t}{2m})\nu F_3 - \frac{\nu}{m}\bar{F}_2 - \nu(m(\nu - 2k.Q) - \frac{\nu}{m}(P^2 - \frac{k.Q}{2}))\bar{F}_3 - \frac{\nu k.Q}{m}\bar{F}_5 + \frac{t\nu}{4}\bar{\bar{F}}_3 \right\}$

TABLE VIII : $\gamma N \longrightarrow VN$, REGGEIZED INVARIANT AMPLITUDES (LEADING ORDER).

REGGEIZED INVARIANT AMPLITUDES	ABNORMAL EXCHANGE		NORMAL EXCHANGE
	A_1, A_1^c, $C_n(-)^J = -1$	π, B, $C_n(-)^J = 1$	ρ, ω, ϕ, A_2, π_c, Pomeron, $C_n(-)^J = 1$
$\tilde{A}_1$	$-f_2\left[\frac{4m}{t}(t+2k.Q)\tilde{f}_1+\tilde{f}_2\frac{(\alpha-1)}{\alpha}+\frac{\tilde{f}_3}{m}\left(-P^2(\alpha-4)+(\alpha-2)\frac{k.Q}{2t}(t+8m^2)\right)\right](-\nu)^{\alpha-1}\xi'$	$-f_1(2\tilde{f}_1+\tilde{f}_3)(-\nu)^{\alpha}\xi$	$\frac{g_2}{m}\tilde{g}_1(-\nu)^{\alpha}\xi''$
$\tilde{A}_2$	$-f_2\left\{\left(\tilde{f}_2\frac{(\alpha-1)}{\alpha}\left(-\frac{(t+\mu_v^2-8m^2)}{4}+\frac{8m^2}{t}k.Q\right)+\frac{\tilde{f}_3}{\alpha m}\frac{(t+\mu_v^2)}{4}\left(P^2+\frac{k.Q}{2t}(t-8m^2)(\alpha-2)\right)\right)\times(-\nu)^{\alpha-2}\xi'\right\}$	$-f_1\left(\tilde{f}_2-\frac{(t+\mu_v^2)}{2}\tilde{f}_3\right)(-\nu)^{\alpha-1}\xi$	$\frac{g_2}{4m}\tilde{g}_1(t+\mu_v^2)(-\nu)^{\alpha-1}\xi''$
$\tilde{A}_3$	$-f_2\left\{16\frac{m}{t}(k.Q)^2\frac{(\alpha-2)}{\alpha}\tilde{f}_3\right\}(-\nu)^{\alpha-3}\xi'$	$-2k.Qf_1\tilde{f}_3(-\nu)^{\alpha-2}\xi$	$-g_1\tilde{g}_3(-\nu)^{\alpha-2}\xi''$
$\tilde{A}_4$	$-f_2\left\{-4\tilde{f}_1+2\frac{\tilde{f}_3}{\alpha}(\alpha-2)\right\}(-\nu)^{\alpha-1}\xi'$	$-mk.Qf_1\tilde{f}_3(-\nu)^{\alpha-2}\xi$	$+\frac{g_2}{2\alpha}\left\{(\alpha-1)\tilde{g}_2+(\alpha-2)\frac{\tilde{g}_3}{4}\right\}(-\nu)^{\alpha-2}\xi''$

TABLE VIII : $\gamma N \rightarrow VN$, REGGEIZED INVARIANT AMPLITUDES (LEADING ORDER).

REGGEIZED INVARIANT AMPLITUDES	ABNORMAL EXCHANGE		NORMAL EXCHANGE
	A_1, A_1^c, $C_n(-)^J = -1$	π, B, $C_n(-)^J = 1$	ρ, ω, ϕ, A_2 π_c, Pomeron, $C_n(-)^J = 1$
$\tilde{A}_5$	$-f_2\left\{-2\frac{\tilde{f}_3}{\alpha}k.Q\right\}(-\nu)^{\alpha-2}\,\xi'$	$mk.Qf_1\tilde{f}_3(-\nu)^{\alpha-2}\,\xi$	$-\frac{g_2}{2}\tilde{g}_3(-\nu)^{\alpha-2}\,\xi''$
$\tilde{A}_6$	$-f_2\left\{\tilde{f}_2\frac{(\alpha-1)}{\alpha}\frac{4k.Q}{mt}P^2+\frac{\tilde{f}_3 k.Q}{\alpha\, m}\cdot\left(+\frac{P^2}{2}(\alpha-4)+(m^2-\frac{k.Q}{2})(\alpha-2)\right)\right\}(-\nu)^{\alpha-2}$ $\times\,\xi'$	$-\frac{f_1}{2}\left(\tilde{f}_2-\frac{(t+\mu_v^2)}{2}\tilde{f}_3\right)(-\nu)^{\alpha-1}\,\xi$	$\frac{k.Q}{m}g_2\tilde{g}_1(-\nu)^{\alpha-1}\,\xi''$
$\tilde{A}_7$	$-f_2\left\{-2k.Q\frac{\tilde{f}_3}{\alpha}(-2\alpha+5)\right\}(-\nu)^{\alpha-2}\,\xi'$		$+k.Qg_2\tilde{g}_3(-\nu)^{\alpha-3}\frac{(\alpha-2)}{\alpha}(-\nu)^{\alpha-3}\,\xi''$
$\tilde{A}_8$	$-f_2\left\{-\frac{2m}{t}\tilde{f}_1(t+2k.Q)-f_2\frac{(\alpha-1)}{2\alpha m}+\frac{\tilde{f}_3}{2m\alpha}\left(P^2(\alpha-4)-\frac{k.Q}{2}(\alpha-2)\right)\right\}(-\nu)^{\alpha-1}$ $\times\,\xi'$	$\frac{f_1}{2}(\tilde{f}_1+\tilde{f}_3)(-\nu)^{\alpha}\,\xi$	$-\frac{g_2}{2m}\tilde{g}_1(-\nu)^{\alpha}\,\xi''$

TABLE VIII : $\gamma N \rightarrow VN$, REGGEIZED INVARIANT AMPLITUDES (LEADING ORDER).

REGGEIZED INVARIANT AMPLITUDES	ABNORMAL EXCHANGE		NORMAL EXCHANGE
	A_1, A_1^c, $C_n(-)^J = -1$	π, B, $C_n(-)^J = 1$	ρ, ω, ϕ, A_2, π_c, Pomeron, $C_n(-)^J = 1$
$\tilde{A}_9$	$-f_2\left\{-2\tilde{f}_1\left(P^2-4m^2\frac{k.Q}{t}\right)+\tilde{f}_2\frac{(\alpha-1)}{2\alpha}+\frac{\tilde{f}_3}{2\alpha}\left(P^2(\alpha-4)-\frac{k.Q}{4t}(\alpha-2)(t+16k.Q)\right)\right\}(-\nu)^{\alpha-1}\xi'$	$\frac{m}{2}f_1(2\tilde{f}_1+\tilde{f}_3)(-\nu)^{\alpha}\xi$	$-g_2\tilde{g}_1(-\nu)^{\alpha}\xi''$
$\tilde{A}_{10}$	$-f_2\left\{\frac{k.Q}{m}\tilde{f}_2\frac{(\alpha-1)}{\alpha}+k.Q\frac{\tilde{f}_3}{\alpha}\left(m(\alpha-1)-\frac{P^2}{m}(\alpha-2)\right)\right\}(-\nu)^{\alpha-2}\xi'$	$f_1\tilde{f}_3t\frac{k.Q}{4}(-\nu)^{\alpha-2}\xi$	$-\left\{k.Q\frac{g_2}{m}\tilde{g}_1+g_1\tilde{g}_2\right\}(-\nu)^{\alpha-1}\xi''$
$\tilde{A}_{11}$	$-f_2\left\{2P^2\frac{\tilde{f}_3}{\alpha}k.Q\right\}(-\nu)^{\alpha-2}\xi'$		$g_2(\tilde{g}_1k.Q-\tilde{g}_2)(-\nu)^{\alpha-1}\xi''$
$\tilde{A}_{12}$	$-f_2\left\{-\frac{\tilde{f}_2}{m}\frac{(\alpha-1)}{\alpha}+\frac{f_3}{\alpha m}\left(m^2+\frac{t}{4}(\alpha+4)\right)\right\}(-\nu)^{\alpha-1}\xi'$	$f_1\tilde{f}_3\frac{t}{4}(-\nu)^{\alpha-1}\xi$.	$-\left\{\frac{g_2}{m}\tilde{g}_1+g_1\tilde{g}_1\right\}(-\nu)^{\alpha}\xi''$

TABLE IX : NUCLEON COMPTON SCATTERING, REGGE CONTRIBUTIONS TO INVARIANT AMPLITUDES.

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	ABNORMAL EXCHNAGE		NORMAL EXCHANGE
	A_1, A_1^c, $C_n(-)^J = -1$	π, $C_n(-)^J = 1$	ρ, ω, ϕ, A_2, π_c, Pomeron, $C_n(-)^J = 1$
$\tilde{A}_1$	$-f_2\tilde{f}_2 \dfrac{c_J}{J^2(J-1)} \tfrac{1}{2} P^2 \mathcal{P}'''_J$		$\tilde{g}_1 g_1 c_J \;{}_J - \dfrac{g_1 g_2}{m} \dfrac{c_J}{J} (\nu \mathcal{P}'_J + m^2 Q^2 \mathcal{P}'_{J-1})$ $+ \tfrac{1}{2}\tilde{g}_2 g_2 \dfrac{c_J}{J^2(J-1)} m(\nu \mathcal{P}'''_J - 2\mathcal{P}''_J)$
$\tilde{A}_2$	$-f_2\tilde{f}_2 \dfrac{c_J}{J^2(J-1)} m \mathcal{P}'''_J$		$\tilde{g}_2 g_1 \dfrac{c_J}{J^2(J-1)} \mathcal{P}''_J - \tilde{g}_2 g_2 \dfrac{c_J}{J^2(J-1)} mQ^2 \mathcal{P}'''_J$
$\tilde{A}_3$	$+f_2\tilde{f}_2 \dfrac{c_J}{J^2(J-1)} \tfrac{1}{2}P^2 \mathcal{P}'''_J$		$-\tfrac{1}{2}\tilde{g}_2 g_2 \dfrac{c_J}{J^2(J-1)} (\nu \mathcal{P}'''_J - 2\mathcal{P}''_J)$
$\tilde{A}_4$	$-f_2\tilde{f}_2 \dfrac{c_J}{J^2(J-1)} (\nu \mathcal{P}'''_J - \mathcal{P}''_J)$		$-\tilde{g}_2 g_2 \dfrac{c_J}{J^2(J-1)} \dfrac{t}{4} \mathcal{P}'''_J$
$\tilde{A}_5$	$+f_2\tilde{f}_2 \dfrac{c_J}{J^2(J-1)} \tfrac{1}{4}m(\nu \mathcal{P}'''_J - \mathcal{P}''_J)$		$\tilde{g}_1 g_2 \dfrac{c_J}{J} \dfrac{Q^2}{2m} \mathcal{P}'_J - \tilde{g}_2 g_2 \dfrac{c_J}{J^2(J-1)} \dfrac{mQ^2}{4} \mathcal{P}'''_J$

TABLE IX (Cont'd.) ; NUCLEON COMPTON SCATTERING, REGGE CONTRIBUTIONS TO INVARIANT AMPLITUDES.

INVARIANT AMPLITUDES REGGE CONTRIBUTIONS	ABNORMAL EXCHANGE		NORMAL EXCHANGE
	A_1, A_1^c, $C_n(-)^J = -1$	π, $C_n(-)^J = 1$	ρ, ω, ϕ, A_2, Pomeron, $C_n(-)^J = 1$
$\tilde{A}_6$		$\frac{f_1 \tilde{f}_1}{4m} c_J \; J$	$\tilde{g}_1 g_2 \frac{c_J}{J} \frac{\nu}{2m^2} \mathcal{P}'_J$

FIGURE CAPTIONS

Fig. 1 : Kinematics, s-channel

Fig. 2 : Kinematics, t-channel

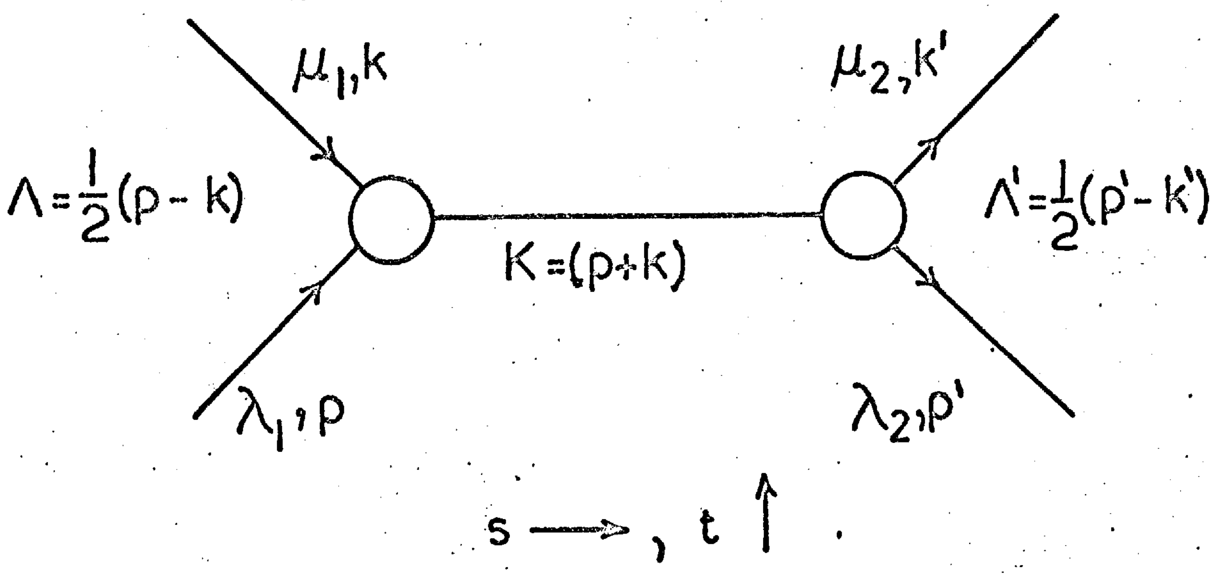
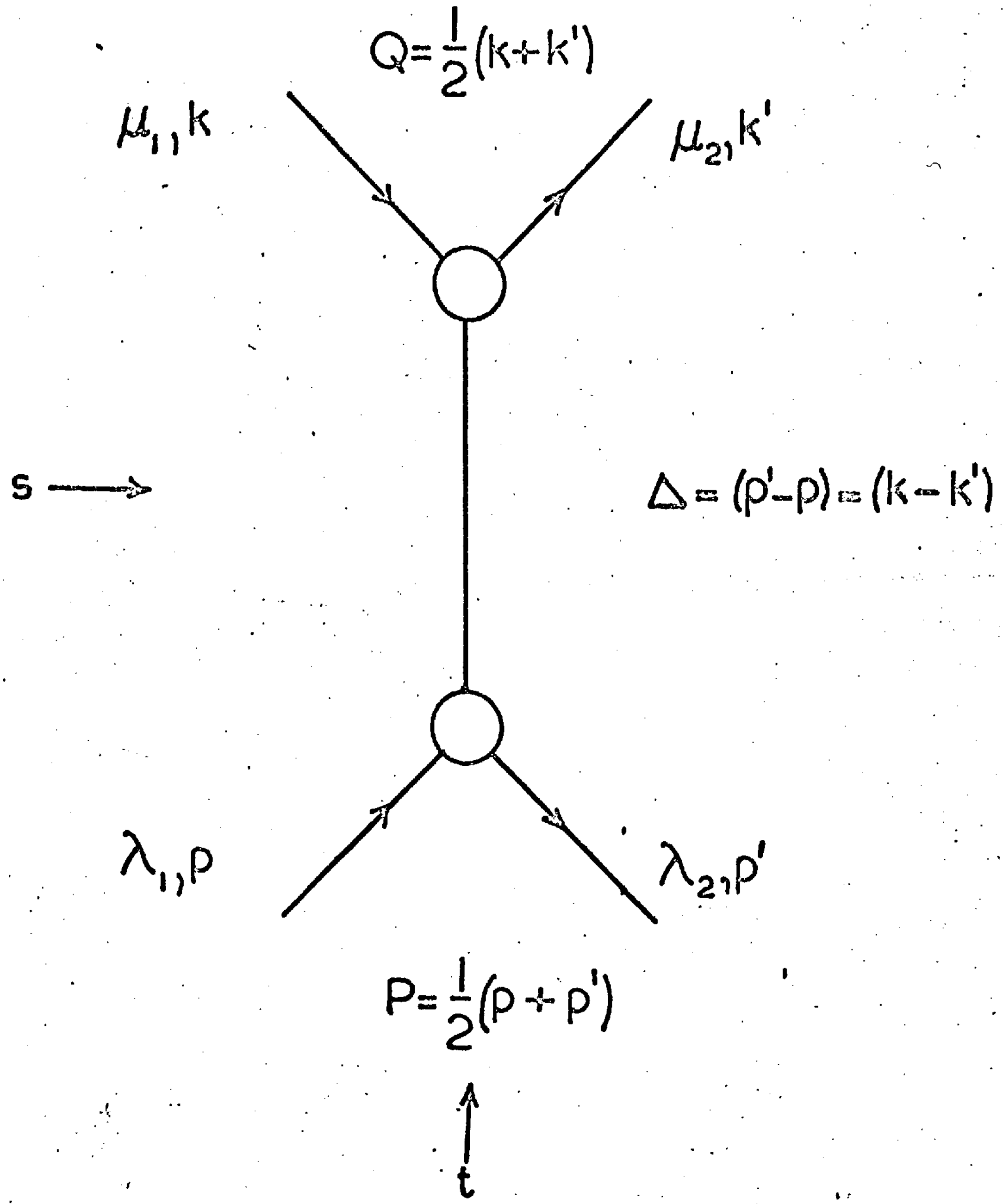

μ_1, k
μ_2, k'
$\Lambda = \frac{1}{2}(p - k)$
$\Lambda' = \frac{1}{2}(p' - k')$
$K = (p + k)$
λ_1, p
λ_2, p'
$s \longrightarrow, \; t \uparrow$

Fig. I.

Fig. 2.


$Q = \frac{1}{2}(k + k')$
μ_1, k
μ_2, k'
s
$\Delta = (p' - p) = (k - k')$
λ_1, p
λ_2, p'
$P = \frac{1}{2}(p + p')$
t

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Pion-Nucleon Backward Scattering and a Modification to the Peripheral Absorption Model (*).

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(ricevuto il 14 Febbraio 1969)

Summary. — The peripheral absorption model with $U_{6,6}$ symmetry imposed at the vertices and a recently proposed modification to the model are applied to backward $\pi^{\pm}p$ scattering and the results compared.

In this note we apply the peripheral absorption model with $U_{6,6}$ symmetry imposed at the vertices, and a recently proposed modification (1) of the model to $\pi^{\pm}p$ backward scattering.

The absorption model with the addition of $U_{6,6}$ symmetry to relate coupling constants has been successful in fitting processes dominated by 0^- exchange (2); however, for processes dominated by higher spin exchange such as 1^- in $\pi^-p \to \pi^0 n$ the model has predicted neither the right order of magnitude for the differential cross-section nor the momentum-transfer dependence. FINCHAM *et al.* (1) (FMM) have attempted to improve upon this by explicitly relating the dominant inelastic channels to each other through the overlap matrix (3)

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(1) D. G. FINCHAM, J. H. R. MIGNERON and K. J. MORIARTY: *Nuovo Cimento*, 57 A, 588 (1968).

(2) J. D. JACKSON: *Rev. Mod. Phys.*, 37, 484 (1965). Other references to the absorption model can be found in this paper.

(3) A. BIALAS and L. VAN HOVE: *Nuovo Cimento*, 38, 1385 (1965); A. BIALAS and K. ZALEWSKI: *Nuovo Cimento*, 46 A, 425 (1966).

and parametrizing the remaining background channels in terms of the initial-state elastic scattering.

We examine $\pi^{\pm}p$ backward scattering because of the limited number of inelastic channels and the variation in structure in the differential cross-sections. The peripheral absorption model has had little success [4] in fitting the rapid fall-off with increased momentum transfer (u) and subsequent recovery of the $\pi^+p \to p\pi^+$ differential cross-section and the relatively flat behaviour of the $\pi^-p \to p\pi^-$ cross-section. As well, peripheral absorption calculations based on nucleon and $\mathcal{N}^*(1236)$ exchange predict cross-sections too large by several orders of magnitude.

The modified amplitude from the absorption model in the Sopkovich [5] form is

$$T^J_{ba} = \sqrt{S^J_{bb}}\, V^J_{ba} \sqrt{S^J_{aa}}\,, \tag{1}$$

where a, b are channel labels, S^J_{aa}, S^J_{bb} are elastic-scattering matrix elements and V^J_{ba} is the inelastic Born amplitude. The usual parametrization of the elastic scattering is used for both models:

$$(S^J_{aa})_{++} = 1 - C\exp[-l(l+1)/R^2k^2]\,, \qquad (S^J_{aa})_{+-} = 0\,, \tag{2}$$

with $J = l + \frac{1}{2}$.

In FMM the S-matrix is

$$\bar{S}^J_{ab} = g[(1 + i\bar{K}^J)/(1 - i\bar{K}^J)]_{ab}\,, \tag{3}$$

where g is a complex number in channel space and a bar on a matrix indicates that it has elements only between «significant channels» [1]. The modified amplitude is then

$$T^J_{ab} = \frac{i[(1 + i\bar{K}^J)/(1 - i\bar{K}^J)]_{ba}}{[(1 + i\bar{K}^J)/(1 - i\bar{K}^J)]_{aa}}\, S^J_{aa}\,, \tag{4}$$

where g has been eliminated by dividing eq. (3) by $\bar{S}^J_{aa}$, an operation permitted only when $[(1+i\bar{K}_J)/(1-i\bar{K}_J)]_{aa}$ is not zero. The diagonal $\bar{K}$ elements are taken to be zero and

$$\bar{K}^J_{ab} = \tfrac{1}{2}\langle \lambda_{b_1}, \lambda_{b_2} | V^J | \lambda_{a_1}, \lambda_{a_2} \rangle\,. \tag{5}$$

[4] J. S. Trefil: *Phys. Rev.*, 148, 1452 (1966).
[5] N. J. Sopkovich: *Nuovo Cimento*, 26, 186 (1962).

Considering meson-baryon scattering, eq. (4) is treated as a 2×2 matrix equation in channel space to get

$$T^J_{21} = V^J_{21} S^J_{11} [1 - \tfrac{1}{4} V^J_{21}]^{-1} . \tag{6}$$

Then, including spin effects by taking V^J_{21} as a 2×2 matrix in spin space,

$$\begin{cases} (T^J_{21})_{++} = [(V^J_{21})_{++} X^J_{++} - (V^J_{21})_{+-} X^J_{+-}] (S^J_{11})_{++} , \\ (T^J_{21})_{+-} = [(V^J_{21})_{+-} X^J_{++} + (V^J_{21})_{++} X_{+-}] (S^J_{11})_{++} , \end{cases} \tag{7}$$

where the matrix $X^J = [1 - \frac{1}{4} V^{J2}_{21}]^{-1}$. We note that eq. (6) does not hold for $(V^J_{21})^2_{++} - (V^J_{21})^2_{+-} = 4$.

We consider the intermediate energy region, beyond that of direct-channel interference, and evaluate only the u-channel graph (Fig. 1) for nucleon and $\mathcal{N}^*$ exchange. For $\pi^+ p \to p\pi^+$ neutron and $\mathcal{N}^{*0}$ are exchanged whereas only $\mathcal{N}^{*++}$ is exchanged in $\pi^- p \to p\pi^-$.

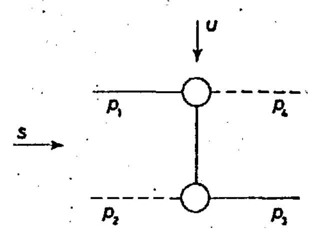


Fig. 1. – Pole graph for baryon exchange.

To determine the Born amplitude we use the $U_{6,6}$ currents to fix the couplings (6). For nucleon exchange,

$$V'_{\lambda_1\lambda_3} = S\left[g\left(1 + \frac{2m}{\mu}\right)\frac{PP'}{4m^2}\right]^2 \overline{U}_{\lambda_3}(p_3)\,\frac{\Delta - M_{\mathcal{N}}}{u - M^2_{\mathcal{N}}}\, U_{\lambda_1}(p_1) , \tag{8}$$

where $\Delta = (p_4 - p_1)$; $P = (2p_1 - p_4)$; $P' = (2p_3 - p_2)$ and S is a combination of F/D and SU_3 coupling factors equal to $2(\frac{5}{3})^2$ for $\pi^\pm p \to p\pi^\pm$. The coupling constant, g, is given in terms of the known $G_{\pi\mathcal{N}\mathcal{N}}$ constant

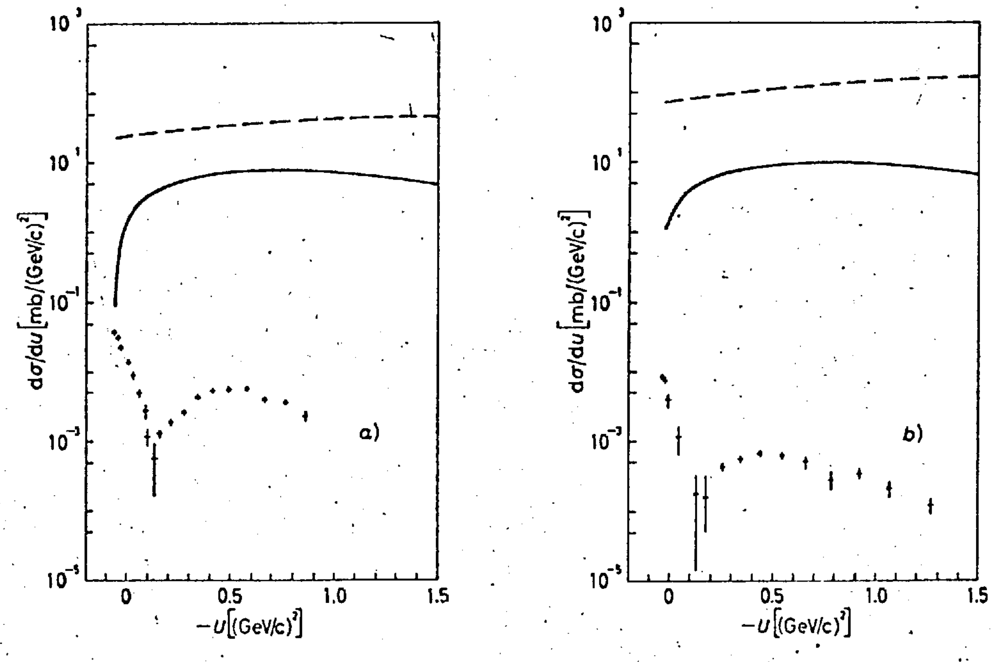
$$\frac{G^2_{\pi\mathcal{N}\mathcal{N}}}{4\pi} = \frac{g^2}{4\pi}\left(\frac{5}{3}\right)^2\left(1 + \frac{2m}{\mu}\right)^2 = 14.9 .$$

For the $U_{6,6}$ masses we take $m = 0.939$ GeV, $\mu = 0.417$ GeV and $M_{\mathcal{N}} = 0.939$ GeV.

For $\mathcal{N}^*$ exchange (6),

$$V'_{\lambda_1\lambda_3} = -\sqrt{2}\, S\left[\frac{g}{m}\left(1 + \frac{2m}{\mu}\right)\right]^2 \overline{U}_{\lambda_3}(p_3)\,\frac{p_{2\mu}\nabla_{\mu\nu}p_{4\nu}}{u - M^2_{\mathcal{N}^*}}\, U_{\lambda_1}(p_1) , \tag{9}$$

(6) A. Salam, R. Delbourgo and J. Strathdee: *Proc. Roy. Soc.*, A 284, 146 (1965); M. A. Beg and A. Pais: *Phys. Rev. Lett.*, 14, 267 (1965); B. Sakita and K. C. Wali: *Phys. Rev. Lett.*, 14, 404 (1965); *Phys. Rev.*, 139, B 1355 (1965); R. Delbourgo, *et al.*: *Seminar on High-Energy Physics and Elementary Particles, Trieste* (Vienna, 1965), p. 455.



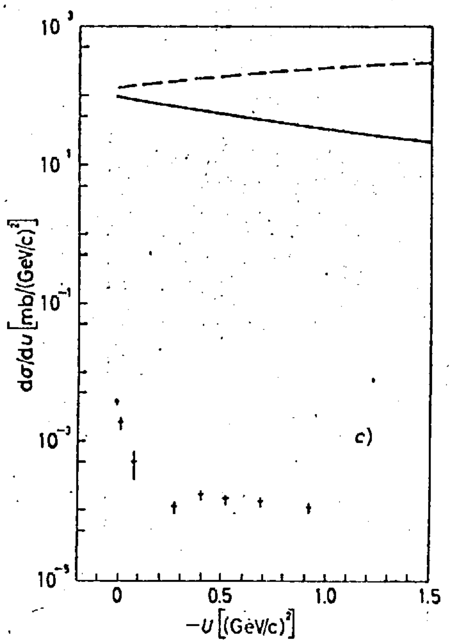


Fig. 2. – π^+p backward scattering: differential cross-section data (7) and predictions of i) the peripheral absorption model (— — —) and ii) the FMM model (———) at *a*) 5.9 GeV/c ($C = 0.81$, $R = 0.27\,(\text{GeV})^{-1}$), *c*) 9.9 GeV/c ($C = 0.76$, $R = 0.27\,(\text{GeV})^{-1}$), *b*) 13.7 GeV/c ($C = 0.745$, $R = 0.27\,(\text{GeV})^{-1}$).

(7) D. P. Owen, F. C. Peterson, J. Orear, A. L. Read, D. G. Ryan, D. H. White, A. Ashmore, C. J. S. Damerell, W. R. Frisken and R. Rubinstein: *Phys. Rev.* (to be published).

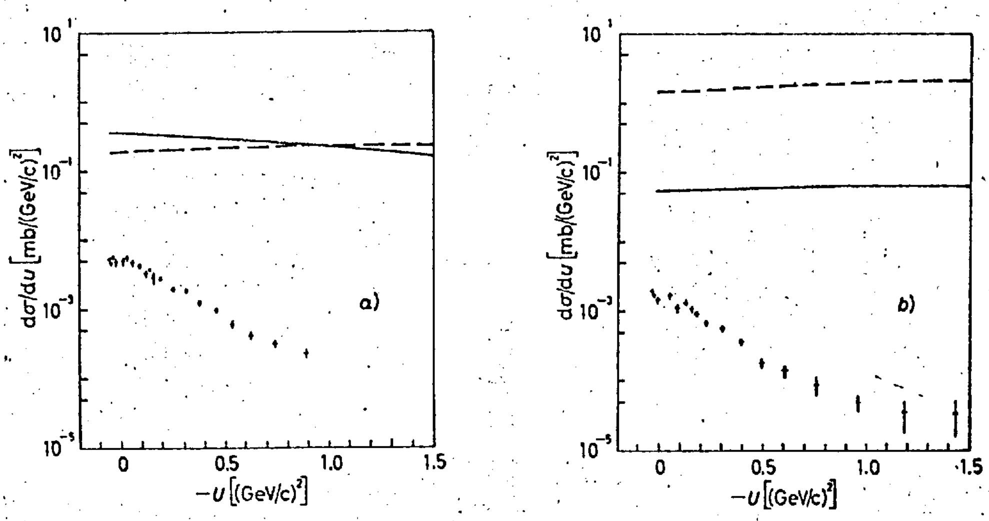
where

$$\nabla_{\mu\nu} = (\not{\Delta} + M_{\mathcal{N}^*})\left[-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3M^2_{\mathcal{N}^*}}(\gamma_\mu \Delta_\nu - \gamma_\nu \Delta_\mu) + \frac{2\Delta_\mu \Delta_\nu}{3M^2_{\mathcal{N}^*}}\right] - \frac{2}{3M^2_{\mathcal{N}^*}}(\Delta^2 - M^2_{\mathcal{N}^*})[\gamma_\mu \Delta_\nu - \gamma_\nu \Delta_\mu + (\not{\Delta} - M_{\mathcal{N}^*})\gamma_\mu\gamma_\nu]$$

and S is $1/\sqrt{3}$ for $\mathcal{N}^{*0}$ and 1 for $\mathcal{N}^{*++}$.

Primed amplitudes are related to unprimed ones by a density of states factor $\varrho = k/8\pi\sqrt{s}$, where k is the centre-of-mass momentum in initial and final channels, such that

$$S^J = 1 - iT^J, \qquad T^J = \varrho T'^J.$$



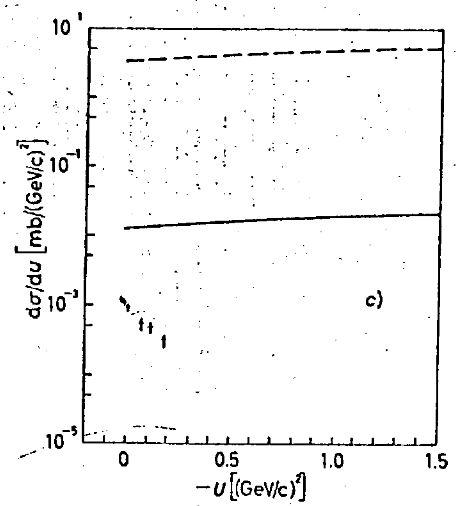


Fig. 3. – π^-p backward scattering: differential cross-section data ([7]) and predictions of i) the peripheral absorption model (— — —) and ii) the FMM model (———) at *a*) 5.9 GeV/c ($C = 0.79$, $R = 0.26\ (\text{GeV})^{-1}$), *b*) 9.9 GeV/c ($C = 0.74$, $R = 0.26\ (\text{GeV})^{-1}$), *c*) 13.7 GeV/c ($C = 0.715$, $R = 0.26\ (\text{GeV})^{-1}$).

The spin averaged differential cross-section is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}u} = \frac{\pi}{k^2} \frac{1}{(8\pi\sqrt{s})^2} [|T'_{++}|^2 + |T'_{+-}|^2].$$

The results of eqs. (1) and (6), applied to the $\pi^{\pm}p \to p\pi^{\pm}$ data of OWEN *et al.* (7) at 5.9, 9.9, 13.7 GeV/c are presented in Fig. 2, and 3. The conventional absorption model results are poor, as expected (4). The FMM model gives improved predictions for $\pi^{+}p \to p\pi^{+}$ data with the exception of the 13.7 GeV/c fit. This results from an unphysical singularity in the approximation which is usually innocuous, but for $l = 4$, at this energy, it approaches the physical region.

As eqs. (1), (6) contain no free parameters we are denied the freedom of an adjustable residue open to Regge fits which permits a several order of magnitude variation between forward and backward predictions. We take the improved but by no means good fit of the FMM model as a further indication of the efficacy of explicit consideration of inelastic channels in the context of the peripheral absorption model.

* * *

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RIASSUNTO (*)

Si applicano allo scattering all'indietro $\pi^{\pm}p$ il modello di assorbimento periferico, con la simmetria $U_{6,6}$ imposta ai vertici, ed una modifica al modello proposta recentemente e si confrontano i risultati.

(*) *Traduzione a cura della Redazione.*

Пион-нуклонное рассеяние назад
и модификация периферической абсорбционной модели.

Резюме (*). — Периферическая абсорбционная модель с $U_{6,6}$ симметрией, наложенной на вершины, и недавно предложенное видоизменение модели применяются к $\pi^{\pm}p$ рассеянию назад, и полученные результаты сравниваются.

(*) *Переведено редакцией.*