THE TRANSMISSION OF DIGITALLY-CODED-SPEECH SIGNALS BY MEANS OF RANDOM ACCESS DISCRETE ADDRESS SYSTEMS

by

ADRIAN PERCY CLARK, M.A.

March 1969

A Thesis submitted for the
Degree of Doctor of Philosophy
in the Faculty of Engineering,
University of London

DEPARTMENT OF ELECTRICAL ENGINEERING,
IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY,
UNIVERSITY OF LONDON.

ABSTRACT

The thesis is concerned with the design and detection of digitally-coded-speech signals, where these are multiplexed in a random access discrete address system. Only single links using binary-coded antipodal signals are considered.

The following four arrangements are studied:-

1) Many transmitters feeding many receivers.
2) Many transmitters feeding a single receiver.
3) A single transmitter feeding many receivers.
4) A single transmitter feeding a single receiver via a multi-channel link.

In the arrangement 1, the systems use asynchronous multiplexing with modulated carriers whose instantaneous frequencies vary continuously with time. In the arrangements 2 to 4, the systems use synchronous multiplexing with baseband signals.

Three systems are proposed for the arrangement 1 and the method of operation of the third of these is analysed theoretically.

The conditions necessary for the unique detectability of the transmitted signals in arrangements 2 to 4 are derived and various methods are proposed for achieving this.

Many different iterative detection processes are proposed for the arrangement 2, and are tested by computer simulation. The more effective of these are tested under various conditions of noise and level variations, and the convergence of some of the latter is analysed theoretically.

Computer simulation tests are carried out with the arrangement 4, to assess the performance of a particular coding and detection process.

ACKNOWLEDGEMENTS

The author would like to express his gratitude to his supervisor, Mr. J. J. Sparkes now at the University of Essex, to Mr. J. M. Howl, Mr. D. Q. Mayne, Mr. J. M. Garduno and Mr. G. R. Selby, of Imperial College, and to Mr. J. L. Smith of British Telecommunications Research Ltd., for many helpful discussions which contributed greatly to his understanding of some of the more difficult problems encountered.

The financial support of British Telecommunications Research Ltd. and of the Science Research Council, which made this work possible, is gratefully acknowledged.

CONTENTS

IMPORTANT DIAGRAMS

GLOSSARY OF SYMBOLS AND TERMS

$\binom{n}{m}$	number of combinations of m out of n
* (on line)	convolution
* (superscript)	conjugate transpose
$\lvert X \rvert$	magnitude (absolute value) of X, if X is a scalar; length (Euclidean norm) of X, if X is a vector
$\{X_i\}$	the set $X_1, X_2, \ldots, X_n$ (n is given in the text)
max y	maximum value of the variable scalar y
max $\{z_i\}$	maximum value of the members of the set
min $\{z_i\}$	minimum value of the members of the set
M^T	transpose of the matrix M
det M	determinant of the matrix M
$\rho(M)$	spectral radius of the matrix M

$\boxed{\Sigma}$	addition	block-diagram symbols
$\ominus$	subtraction	
$\otimes$	multiplication	
$\boxed{\int}$	integration	

FDM	frequency-division multiplex
TDM	time-division multiplex
RADAS	random-access discrete-address system
RASSAS	random-access sequentially-switched address system
AM	amplitude modulation
FM	frequency modulation
PM	phase modulation

A <u>double-modulation</u> signal is one generated in two consecutive and separate modulation processes, where each of these may be either AM, FM or PM.

A <u>PM-FM signal</u> is one in which a PM signal is used to frequency modulate the transmitted carrier.

A <u>signal element</u> is a unit component of a digitally-coded signal.

Two signal-elements are said to be orthogonal when their cross-correlation coefficient is zero.

A frequency-time matrix is the sub-division of the total frequency-time space containing a signal element, into separate frequency-bands and time slots.

A unit area in a frequency-time matrix is the area in the frequency-time domain occupied by the combination of one frequency-band and one time-slot in that matrix.

A varying-frequency carrier is a frequency modulated carrier whose instantaneous-frequency varies continuously with time.

The frequency-time trace of a signal is the path followed by the instantaneous frequency of the carrier in the frequency-time domain.

A synchronous system is one in which the sending and receiving equipments are operating continuously at substantially the same number of signal elements per second and are maintained, by correction, in the desired phase relationship.

In coherent detection of a digital signal the receiver makes use of a prior knowledge of the phase of the received signal carrier in an element detection process.

In incoherent detection of a digital signal the receiver has no prior knowledge of the carrier phase at the start of an element detection process.

Envelope detection is incoherent AM detection.

In differentially-coherent detection of a PM signal the receiver compares the carrier phase in a signal element with that in the preceding element. A detection process here involves two adjacent elements and this is incoherent PM detection.

The terms random access, discrete address, element address, element synchronism, synchronous multiplexing, asynchronous multiplexing, general asynchronous system and channel-synchronized asynchronous system, are defined in Section 1.3.

1.0 INTRODUCTION

1.1 Subject of Thesis

This thesis is concerned with the application of random-access discrete-address techniques and further developments of such techniques, to the multiplexing of digitally-coded-speech signals. The two basic problems studied are the design and detection of the signals fed over the common transmission path.

1.2 Conventional Methods of Multiplexing Signals

The most widely used methods of multiplexing signals are frequency-division multiplex (FDM) and time-division multiplex (TDM). Whereas only FDM may be applied to analogue signals, either FDM or TDM or a combination of the two may be applied to suitable digital signals. The important property of these multiplex methods is that the different transmitted signals are orthogonal.

Consider a combined FDM and TDM system designed to carry n separate digital signals, where each signal comprises a serial stream of binary-coded AM pulses, an element "1" being represented by a pulse and "0" by the absence of a pulse. The time slots and frequency bands for these signals can be arranged as in Fig. 1.

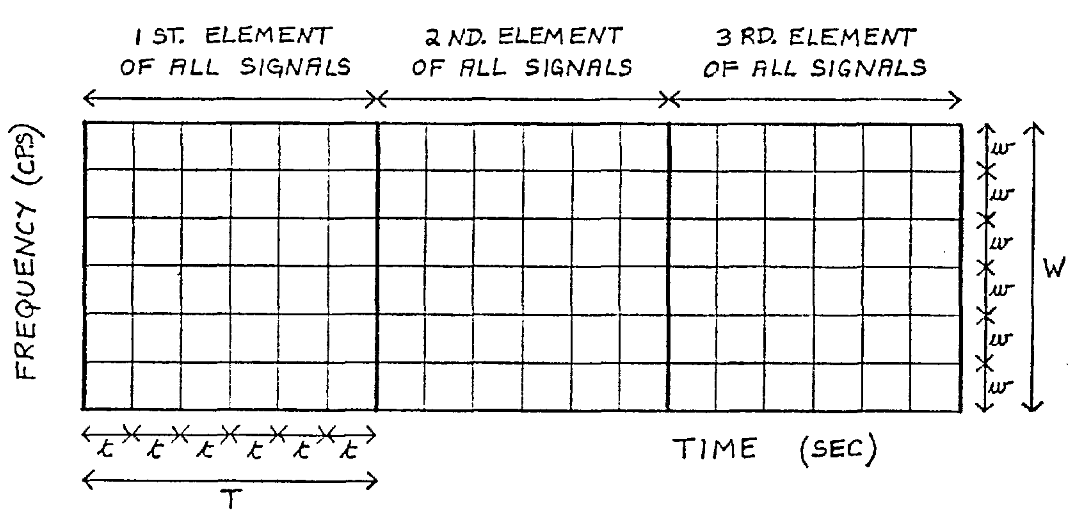


FIG. 1. FREQUENCY-TIME MATRICES.

There are $\frac{W}{w}$ separate frequency-bands and $\frac{T}{t}$ separate time-slots in the frequency-time space allocated to n signal-elements, one from each of the n different signals. This frequency-time space will be referred to as the "frequency-time matrix". Three of these are shown in Fig. 1. Clearly

$$n = \frac{W}{w} \cdot \frac{T}{t}, \qquad (1)$$

so that n is the number of "unit areas" in the frequency-time matrix. Each signal occupies a different unit area in a matrix. The equivalent TDM system would have n time-slots and one frequency-band in the frequency-time matrix, and the equivalent FDM system would have n frequency-bands and one time-slot. In a practical system a unit area in Fig. 1 would have a frequency-time product of approximately 2.

1.3 Random Access Discrete Address Systems

If each unit area of a frequency-time matrix is associated with only one receiver, then a communication system with n subscribers but no central exchange can be designed. The unit area allocated to a subscriber is the "discrete address" of that subscriber. A calling subscriber transmits a signal having the discrete address of the subscriber he wishes to contact. After a sufficient delay to allow the receiver to recognize and become synchronized to this signal, he gives his own address and the return channel using this address is then set up. Arrangements involving only single two-way links will be considered here.

In general there are many more than n subscribers, so that in an arrangement equivalent to that just described, the discrete address of a subscriber must use more than one unit-area of the frequency-time matrix. If each address uses m unit areas, there can ideally be altogether $\binom{n}{m}$ different subscribers. In any signal element here, each of the m unit areas has the binary value of that element. The weakness of this arrangement is that since each unit-area forms part of the signal of many subscribers, interchannel interference is likely to become severe when the number of simultaneous signals approaches n.

Where there are a number of transmitters in different locations, the frequency-time matrices (elements) of a signal will not in general be in phase with those of any other. To enable a receiver to determine the phase of the wanted signal-elements, one of the unit areas used by an address always occupies the first time-slot. In addition, the equipment is simplified if a signal never uses more than one unit-area in any one time-slot. With these two restrictions the number of addresses is

$$\left(\frac{W}{w}\right)^m \cdot \binom{\frac{T}{t} - 1}{m - 1} \qquad \text{where } m \leqslant \frac{T}{t}.$$

Where the discrete address is repeated with every signal-element, as it is here, it is sometimes referred to as the "element address". Because of the lack of synchronism between the different transmitted signals, these are said to be "asynchronously multiplexed". Where the transmitted signals are fed through a single repeater, it is possible to arrange that at the repeater and therefore at each receiver, the signals are in "element synchronism", which means that the elements (frequency-time matrices) of the different signals are in phase as in Fig. 1.[D2] These signals are "synchronously multiplexed".

A communication network in which a subscriber has direct access to any other, without the intervention of a central exchange, is known as a "random-access" system. Hence the general title "random-access discrete-address system" or RADAS.

Arrangements of RADAS using asynchronous multiplexing, can be classified into "general asynchronous systems" and "channel-synchronized asynchronous systems". In the former the various channels of the system are all sensitive to the same general regions of the frequency-time space, whereas in the latter each channel is sensitive to only a fraction of the total frequency-time space used.[D7, D22]

1.4 Published Work on RADAS

The arrangement of RADAS outlined in Section 1.3 is based on the RADEM system developed by Motorola, Inc. Delta modulation is used here to convert the original speech waveform into a binary-coded digital signal and the latter is then converted into a transmitted signal of the type described.[B2, D16, D20]

An alternative arrangement of RADAS, called RACEP and designed by the Martin Company, samples the original speech waveform to produce a pulse-position modulated (PPM) signal, each of whose pulses is then converted into three corresponding PPM signals, occupying different unit-areas in the associated frequency-time matrix.[B2, D12, D15]

Following these two early systems, both of which are channel-synchronized asynchronous systems, a number of different arrangements of RADAS have been designed and tested.[D] Some of these have been studied as systems for achieving multiple access to a communications satellite.[D1,D2,D19,D30,D38,D39,D47] The most interesting of these various systems are the general asynchronous systems, alternatively described as "spread-spectrum" systems, which use digital pseudo-noise signals.[D1,D2,D23,D25]

The general conclusion reached from these investigations is that RADAS with asynchronous multiplexing can sometimes lead to a simpler overall system design than the equivalent FDM or TDM system. It has, however, an appreciable element error probability in the absence of noise and a lower tolerance to additive noise than the latter systems, when the number of simultaneous signals approaches n. Since a much higher element-error-rate can be tolerated with digitally-coded-speech signals than with data signals, RADAS is of most interest as a system for transmitting digitally-coded-speech signals.

In reference D13 the arrangement of RADAS uses synchronous multiplexing and each receiver detects all transmitted signals. It is shown here that if there are always $m+1$ active transmitters out of a larger total number and if each transmitter has an output signal power S, with no attenuation in transmission, then the total channel capacity C_1 is given by

$$C_1 = W \log_2\left(1 + \frac{(m+1)S}{N}\right), \tag{2}$$

where W cps is the total available bandwidth and N is the total white gaussian noise power in this band. C_1 is the same as the channel capacity in the corresponding arrangement, having only a single transmitter which delivers the same total power $(m+1)S$ as the $m+1$ transmitters above.[A1] It is also the same as the channel capacity in the equivalent FDM system, having $m+1$ orthogonal signals each of power S, which together use all the available bandwidth.[D7, D13]

In the large majority of the arrangements of RADAS studied,[D] the transmitted signals are not orthogonal and a receiver only detects the wanted signal, treating the remaining signals as noise. Assuming that a general asynchronous system is used, the total channel capacity is now

$$C_2 = (m+1)W \log_2 \left(1 + \frac{S}{mS + N} \right). \qquad (3)$$

As before there are m+1 signals each with a power S.[D7,D13]

A comparison between the two channel capacities shows an appreciable advantage for C_1 over C_2, when m and $\frac{S}{N}$ are both fairly large. For instance when $10 < m < 100$ and $10 \text{ db} < \frac{S}{N} < 20 \text{ db}$, then $5C_2 < C_1 < 10C_2$.[D7]

A general asynchronous system has in turn an appreciable advantage over a channel-synchronized asynchronous system. For instance when $10 < m < 100$ and $10 \text{ db} < \frac{S}{N} < 20 \text{ db}$, then $4C_3 < C_2 < 40C_3$, where C_3 is the total channel capacity of the channel-synchronized asynchronous system. Each channel is here assumed to use only 0.001 of the total number of degrees of freedom available to the transmitted signals.[D7]

Studies into novel methods of multiplexing and routing signals, not employing RADAS,[E,B4] suggest that over telephone-line links there are a number of alternative techniques for the automatic routing of signals to the called subscriber. These are in general preferable to conventional arrangements of RADAS, since they use orthogonal signals and do not therefore suffer the interchannel interference normally associated with the latter.

Very little work seems to have been done on practical arrangements of RADAS in which a receiver does much more than detect just the wanted signal. The reason for this is probably that the large majority of the systems studied, involve radio links between many transmitters and many receivers. Considerations of equipment economy together with the complex transmitted waveforms, prevent anything but the simplest detection process being considered for the receivers. However, where the transmission medium is a line which permits the transmission of simple baseband signals, and where a single receiver is fed from a number of different transmitters, much more sophisticated detection processes could be used.

Studies into methods for reducing intersymbol and interchannel interference in practical multi-channel systems,[F] show that in general useful reductions in interference can be obtained without the use of unduly complex equipment. Furthermore, where the transmitted signals are linearly independent but not necessarily orthogonal, detection errors due solely to intersymbol and interchannel interference, may be entirely eliminated by means of suitable linear filters in the receiver. The tolerance to noise is however decreased as signals in different channels become less nearly orthogonal. In all these systems a single receiver is used which detects all the received signals, as in reference D13.

If techniques similar in principle to those just mentioned, could be applied to RADAS, a significant improvement in performance could be expected. Such systems should have many useful applications.

1.5 Work on RADAS carried out by the Author before October 1965

1.5.1 Introduction

The author began his work on RADAS in June 1964, when he carried out a feasibility study to determine the basic techniques most likely to lead to an effective yet simple RADAS design. The work was completed in August 1964 and is described in reference B2. The following is a brief outline of this investigation.

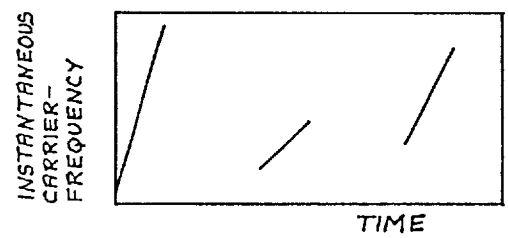
1.5.2 Incoherent Systems

In the early part of 1964 the two most important systems described in the published literature were RADEM and RACEP. It was decided to take RADEM as the starting point and to see what improvements could be applied to this basic system. The application considered was that with radio links between many transmitters and many receivers. Only binary-coded signals were studied, the element address being defined here to be the signal waveforms for the two binary-elements "0" and "1".

Modulation methods using frequency shift and time-shift keying and various combinations of these, were studied as alternatives to the amplitude modulation used in RADEM. The latter modulation method, although making a more efficient use of bandwidth, gives a poor performance under conditions of fading. Two or three of the alternative modulation methods were found to have useful properties. In all these arrangements, an element "1" of a given address is characterized by the presence of signal in m of the n unit-areas of the frequency-time matrix, where $m \ll n$, and an element "0" by the presence of signal in a disjoint set of m unit-areas.

Signals having an instantaneous carrier frequency varying linearly with time, as in "chirp" radar,[C11] were considered next. Each frequency-time matrix here contains a number of separate "varying-frequency" signals each of which occupies a different time-slot, for instance as follows:-

ELEMENT "0" :



ELEMENT "1" :

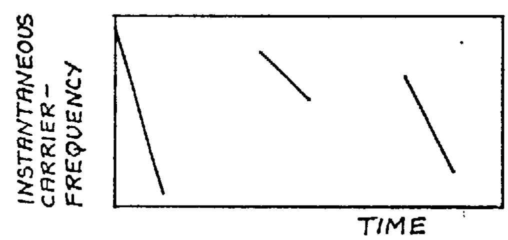


FIG. 2. INCOHERENT VARYING-FREQUENCY SIGNAL.

These signals will probably give an inferior performance when there are large differences between the levels of the various received signals, but when there is a small range of levels they should enable a slightly larger number of signals to be transmitted simultaneously, for a given average interchannel interference level. The reason for this advantage is the reduction in the maximum interference level likely to be caused by any one interfering signal-element. The advantage can alternatively be regarded as being due to a more uniform distribution of interference levels between the different interfering signals.

1.5.3 Adaptive Systems

Adaptive arrangements of "frequency-band switching" and "transmitter-speed control" were studied, as a means of reducing interchannel interference. In the former system, each receiver has a stand-by address, occupying a different region of the frequency-time matrix to the normal address. Whenever the measured interference level at the receiver exceeds a predetermined value, both the transmitter and the receiver automatically switch to the stand-by address. In the arrangement of transmitter-speed control, the phase of the transmitted signal-elements is automatically adjusted to minimize the measured interference level at the receiver. Both these arrangements use feedback control signals via the return speech channel and in their more sophisticated forms are subject to severe instability. They also require complex equipment. They should however under favourable conditions appreciably increase the maximum number of signals which may be transmitted simultaneously.

In view of the disadvantages of the adaptive arrangements of RADAS just considered, it was decided to confine further investigations to non-adaptive systems.

1.5.4 RASSAS

In order to reduce the serious interference effects caused by a high-level unwanted signal, having an element address rather similar to that of the wanted signal, it was proposed that the discrete element address used in all the systems studied so far, should be replaced by a "sequentially-switched" element address. Thus successive elements of a signal have different element addresses, no element address being repeated until all other addresses have been used. A discrete address is used as before to set up a call, and a suitable timing or count-down signal then informs the receiver of the particular signal-element at which the transmitter will begin sequentially switching the element addresses. Assuming that the receiver has prior knowledge of the sequence of element addresses that will then be transmitted, the receiver can remain correctly synchronized to the transmitter throughout the rest of the call. This arrangement will be referred to as a "random-access sequentially-switched address system" or RASSAS.

RASSAS is a special case of RADAS in which the discrete address of a signal changes from element to element. It is a general asynchronous system, whereas the arrangements previously described are channel-synchronized asynchronous systems. Although RASSAS uses a type of pseudo-noise signal, it requires no additional bandwidth relative to the equivalent arrangement of RADAS. Provided that there are at least say 10^7 different element-addresses in a sequence, which could in general be arranged quite easily, there is a negligible probability that any two signals will drift into phase over a period of a few minutes, assuming a timing oscillator stability of about 1 in 10^6.

The useful property of RASSAS is that any two interfering signals of the same level will ideally produce the same average interference level in the detected wanted signal. Furthermore the resultant interference signal has the characteristics of random noise. It will sound as a hiss or rumble instead of the short bursts of intelligible cross-talk obtained with the arrangements of RADAS. An appreciably higher level of interference should therefore be acceptable with RASSAS than with RADAS. With delta modulation used to produce the digitally-coded-speech signal, an element error rate as high as 1 in 10 could probably be tolerated with RASSAS, before the reconstituted speech signal becomes unintelligible.

1.5.5 Coherent System

The final arrangement studied was an arrangement of RASSAS using a continuous signal with no amplitude or phase discontinuities, whose instantaneous carrier frequency varies linearly with time over each time-slot of the frequency-time matrix. A new address is used for each signal-element transmitted, and the binary elements "0" and "1" for a given address could typically appear as follows:-

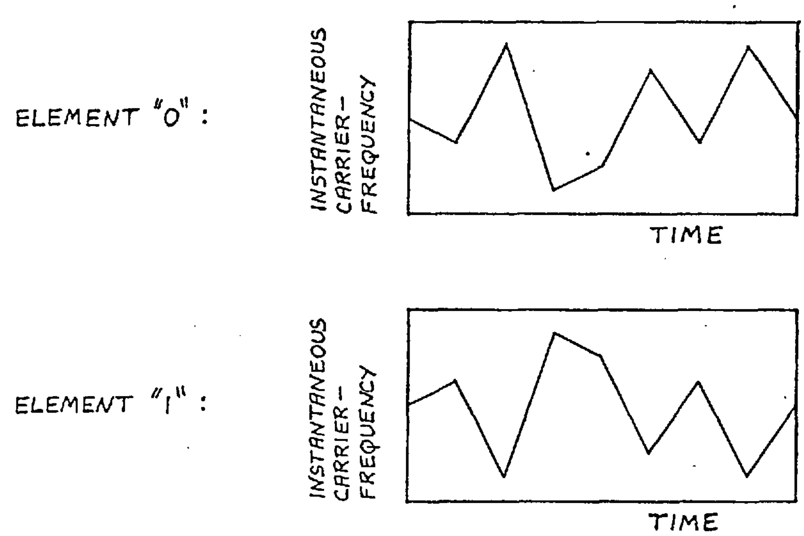


FIG. 3. COHERENT VARYING-FREQUENCY SIGNAL.

The plot of the instantaneous carrier frequency against time is referred to here as the "frequency-time trace" of the signal. Clearly the frequency-time trace is a function both of the sequence of element binary-values as well as of the sequence of element addresses.

Assume for simplicity that the instantaneous carrier frequency at the end of a signal element is always the same as that at the beginning, and that there are p different possible values of the instantaneous carrier frequency at the boundary between two time-slots, there being q time-slots in a signal element. There are now $\frac{1}{2}(p-1)^{(q-1)}$ different possible element-addresses. By a suitable choice of waveforms, for the two binary-elements corresponding to each address, it should be possible to arrange that the two binary-elements are always nearly orthogonal.

The advantage of this arrangement over an incoherent system is that under favourable conditions the receiver can carry out a process of coherent detection over each signal-element, since it can have a prior knowledge both of the exact frequency-time traces of the two binary values of an element, and of the carrier phase at the start of the element. The receiver is therefore ideally sensitive to an area with a frequency-time product of only $\frac{1}{2}$ in each frequency-time matrix.[F4,A2] In practice, however, the receiver cannot generally have so accurate a prior knowledge of a received signal-element, with the result that it will be sensitive to an area with a frequency-time product of say 2 or more.

The arrangement just described is referred to as the "coherent system", the previous arrangements being classed as "incoherent systems". The coherent system will not in general give a good performance when there is a wide range of received signal levels or when the transmission medium introduces significant phase variations over the duration of a signal element. Under favourable conditions, however, an arrangement of coherent RASSAS should give a better performance than any of the other arrangements considered.

1.6 Aims of the Research Project

The intention of this research project, which was started in October 1965, has been to continue the work outlined in Section 1.5 with a view to determining further techniques for reducing the interchannel interference levels in an arrangement of RADAS.

In the general communication network there are four possible basic configurations involving only single links:-

1) Many transmitters feeding many receivers.
2) Many transmitters feeding a single receiver.
3) A single transmitter feeding many receivers.
4) A single transmitter feeding a single receiver via a multi-channel link.

Where there are many transmitters or many receivers, these are assumed to be in separate locations.

Although all four of the above configurations have been studied in some detail, the majority of the time has been spent on the arrangement 2, since the latter appears to offer the most interesting possibilities for an application of RADAS. The introductory work carried out from October 1965 to October 1966, was concerned mainly with the arrangements 1 and 4, and is described in two unpublished reports.[B3,B4] These form an elementary introduction to this thesis.

In the arrangement 1, considerations of equipment economy dictate that a receiver detects only the wanted signal and treats all other signals as noise. The main problem studied here was that of the optimum signal design.

In the arrangements 2, 3 and 4 it is assumed that the transmission medium passes baseband signals and that synchronous multiplexing is used. This is the ideal situation for the economic application of more sophisticated detection processes at the receiver. The main problem studied in the arrangement 2 was that of the optimum detection process.

In view of the wide field covered by this research project, the work has been concentrated into certain well defined topics, which in the view of the author have not yet been adequately studied. The project is thus one of attempting to fill some of the larger gaps in the present state of the art.

In this thesis the subject under investigation is treated as a branch of electrical engineering and not as a branch of applied mathematics. This is because the aim of the investigation has not been the detailed analysis of some known system or rigorous proof of some relationship, but rather the development of new techniques which may lead to useful practical systems.

2.0 RADAS WITH MANY TRANSMITTERS AND MANY RECEIVERS

2.1 Introduction

Communication networks, where there are many transmitter-receivers in separate locations, may either use direct links between the different subscribers or alternatively all signals may be routed via a central repeater or exchange. Whenever the total bandwidth available is considerably greater than that needed to transmit the required maximum number of simultaneous signals as orthogonal waveforms, a suitable arrangement of RADAS may enable useful equipment economies to be achieved, without excessively reducing the received speech quality.[D2]

Where there is no central repeater or exchange, asynchronous multiplexing must in general be used and there is often likely to be a wide range of levels for the different signals reaching a receiver. Under these conditions the signals must be designed in such a way that there is a high probability that any interfering signal is orthogonal to a wanted signal over the duration of an element. The best way of achieving this is to use signals similar to those in RADEM and to arrange that a signal occupies the minimum number of unit areas (say 3) in any frequency-time matrix.[D1,D2] Furthermore, the use of FM (that is FSK) in place of AM, although reducing the probability of orthogonality, has a number of other useful advantages which should more than offset this disadvantage.[C3] An element "0" is here transmitted by a signal occupying a different set of unit areas to the element "1", instead of by no-signal as in the AM system. Systems using signals of this general type have been widely studied and will not be considered further here.[D] Varying-frequency signals would not be very suitable for these applications.

Where there is a central repeater and arrangements can be made to control the transmitted signal powers for approximately equal power levels at the repeater input, or alternatively where there is no repeater and the maximum signal attenuation in transmission is less than say 20 db, continuous varying-frequency signals could be used.

2.2 System A

This is essentially a modification of the coherent varying-frequency system (Section 1.5.5). In System A the same frequency-time trace is used for both binary values of a signal element, and differentially-coded binary PM (PSK) is used for the message modulation. Thus a "1" is represented by an instantaneous phase-change of 180° in the varying-frequency carrier, at the boundary between two adjacent signal-elements, and a "0" as no phase change here. The frequency-time trace of a signal element is now independent of the message modulation and is therefore uniquely determined by the element address.

Coherent detection is used at the receiver, with a locally generated reference-carrier which has the same frequency-time trace as the wanted signal but with no additional phase-modulation. The phase of the reference carrier is adjusted by means of a suitable phase-locked loop to be in phase or in anti-phase with the received carrier.

A discrete element-address, using an FM RADAS signal of the type outlined in Section 2.1, is used for setting up a call, and once a call has been established the system is automatically switched over to a varying-frequency RASSAS signal. This has the useful property that once a subscriber is engaged in a call he will not experience significant interference from another subscriber attempting to contact him.

It is well known that a wide-deviation FM signal with a linear frequency-time trace, similar to that in one time-slot of Fig. 3 (Section 1.5.5), has an energy-density spectrum which is essentially flat over the signal frequency-band and zero outside.[C11] The varying-frequency signal used by System A can for practical purposes be considered to have a power-density spectrum of rather similar shape, so that it is difficult to listen in to or to jam. System A is described in some detail in reference B3.

System A, although simpler than the Coherent System (Section 1.5.5), involves considerable equipment complexity. This is partly due to the complex nature of the varying-frequency signal and partly to the arrangements for switching over to this signal from the discrete address.

2.3 System B

This uses a continuous transmitted signal whose instantaneous carrier frequency varies continuously with time. Each receiver has a discrete address as in RADAS but instead of this being a discrete element-address, it has a duration very much longer than that of one signal-element. The element address is therefore sequentially switched as in the varying-frequency signal in System A. Thus System B has the properties of both

RADAS and RASSAS.

The frequency-time trace of the transmitted signal in System B is the sum of two sine-waves of equal amplitudes. One of these has a frequency, f_1 cps, which is somewhat greater than the signal element rate, and the other has a much lower frequency, f_2 cps. The discrete address of each receiver is the unique pair of values allocated to f_1 and f_2 for that receiver.

Because of the simple form of the transmitted signal, the receiver can be designed to detect the presence of a signal having the correct address and to synchronize onto this signal automatically. No special signal need therefore be transmitted for setting up a call.

As in the case of System A, differentially-coded binary PM is used for the message modulation. This is applied to the varying-frequency carrier at the transmitter, and coherent detection is used at the receiver. System B, like System A, is a general asynchronous system, since over a sufficient number of signal elements the receiver is sensitive to the whole area of the frequency-time matrix. System B is described in some detail in reference B3.

System B is basically much simpler than System A but has the disadvantage that special arrangements must be made to prevent interference in a call between two subscribers, when a third subscriber attempts to contact one of these two. Another disadvantage of the System B is the fact that its signals are not such ideal pseudo-noise signals as those in System A, so that interchannel interference will in general be more serious under equivalent conditions.[B3]

In both systems A and B, the modulation and detection processes used for the binary-coded signal elements are optimum in the sense that, given ideal equipment and no phase variations introduced by the transmission medium, and assuming that a separate detection process is used for each received element, the optimum tolerance to additive white gaussian noise is obtained for a given signal-element energy.[C1-C5]

Systems A and B have one serious weakness. In order to achieve ideal coherent detection of a binary-coded PM signal, where one signal-element is the negative of the other, it is essential that the receiver has exact prior knowledge of the form of one of the two elements. This necessarily means that the generator of the reference carrier in the coherent detector, must have an instantaneous frequency accurate to approximately 1 part in 100c , where c is the number of carrier cycles per signal element. Assume a total signal bandwidth of say 10 Mc and an element duration of 25 μsec, and suppose that the received signal is frequency translated at the receiver input, to occupy the lowest convenient frequency-band, say 10 to 20 Mc. A signal element in System B, whose mean instantaneous carrier frequency has the maximum value, has about 400 cycles. Thus the oscillator used to generate the reference carrier, must have an instantaneous-frequency accurate to 1 part in 40000. Although this could easily be achieved by an oscillator having a constant instantaneous-frequency, considerable practical problems are involved in designing a frequency-modulated oscillator whose instantaneous frequency tracks that of the received signal to this order of accuracy, particularly where its modulating waveform, as in System B, may be subject to phase errors due to the effects of noise.

Errors in the tracking of the wanted varying-frequency signal can of course be reduced, by decreasing the response time (effective integration period) of the phase-locked loop, which is used to synchronize the phase of the reference carrier to that of the received signal. However, since the response time must now be reduced to a fraction of the duration of a signal element, this inevitably reduces the tolerance of the receiver to additive noise, under ideal conditions, thus destroying much of the advantage gained by the use of coherent detection.

Although System B has some interesting possibilities, where the transmission medium does not introduce significant phase variations in the transmitted signal carrier and where the area of the frequency-time matrix is relatively small, it appears to be of limited value where these two conditions are not both satisfied.

2.4 System C

2.4.1 Introduction

Although not capable of as good a performance as System B, under conditions favourable to the latter, System C will operate correctly when neither of the two conditions required by System B is satisfied. Thus it could for instance be used for the transmission of digitally-coded-speech signals over radio links. It has the one important weakness, common to all varying-frequency systems, which is that the different received signal levels must have a range of less than say 20 db.

System C is similar to System B, except that it uses a "double-modulation" method for generating the binary-coded signal elements and it uses a form of incoherent detection at the receiver. It will be assumed that the transmission rate per channel is of the order of 40 k-bits/sec.

2.4.2 Transmitter

The block diagram of the transmitter is shown in Fig. 4 and the waveforms obtained at different points in the circuit are shown in Fig.5.

Two frequencies f_1 and f_2 cps are generated in separate oscillators and the output sine-waves, which are arranged to have exactly equal levels, are added together and then used as the main modulating waveform for the frequency-modulated oscillator. Neglecting for the moment the waveform at the terminal J, the resultant varying-frequency carrier at the terminal D has a frequency-time trace which is the sum of the two sine-waves $\sin 2\pi f_1 t$ and $\sin 2\pi f_2 t$. Different values of f_1 and f_2 are used for every channel and in each case $f_1 \gg f_2$. The combination of f_1 and f_2 used for any channel is the discrete address of that channel.

The element timing waveform generator produces at the terminal F, a series of regularly-spaced short positive pulses, at f_g pps, whose rising edges determine the signal-element boundaries. These are not synchronized in any way to the f_1 and f_2 cps sine-waves. The element timing waveform is fed to the associated digital equipment, where it is used to synchronize the binary-coded-speech signal fed to the transmitter. The latter signal represents a "1" as a negative level and a "0" as a positive level.

The AND and OR gates together with the $\div 2$ stage re-code the binary-coded speech signal into a form where a "1" is represented by a positive or negative transition and a "0" by no change. The resultant signal, at terminal H, is fed as the modulating waveform to the phase modulator. The carrier waveform fed to the phase modulator at terminal E is an f_g cps sine-wave, synchronized to the timing waveform at F. A positive level in the signal at H allows the carrier at E to pass through the phase modulator unchanged, whereas a negative level inverts the carrier, that is it shifts its phase by 180°. The phase modulator is thus a simple switched inverter. The resultant signal at the terminal I is passed through the band-pass filter, which limits the signal frequency band to approximately $\frac{1}{3} f_g$ - $2 f_g$ cps, to give the band-limited signal g(t) at the terminal J. $f_1 > 2 f_g$ and $f_2 < \frac{1}{3} f_g$, so that neither f_1 nor f_2 lie within the frequency band of g(t). The latter is a differentially-coded binary PM signal which carries the message information to be transmitted. Its level is very much lower than that of the sine waves at A and B, and it is added to these so that the modulating waveform at C is the sum of the waveforms at A, B and J. The signal at the output of the frequency-modulated oscillator can therefore be regarded as a varying-frequency carrier, with a frequency-time trace given by the sum of the f_1 and f_2 cps sine-waves, which has been frequency modulated by the binary PM signal at J. Thus the waveform at D is a PM-FM varying-frequency signal. This is fed to the output stage from which it is transmitted. It will be assumed here that the transmitted signal has a constant instantaneous-amplitude.

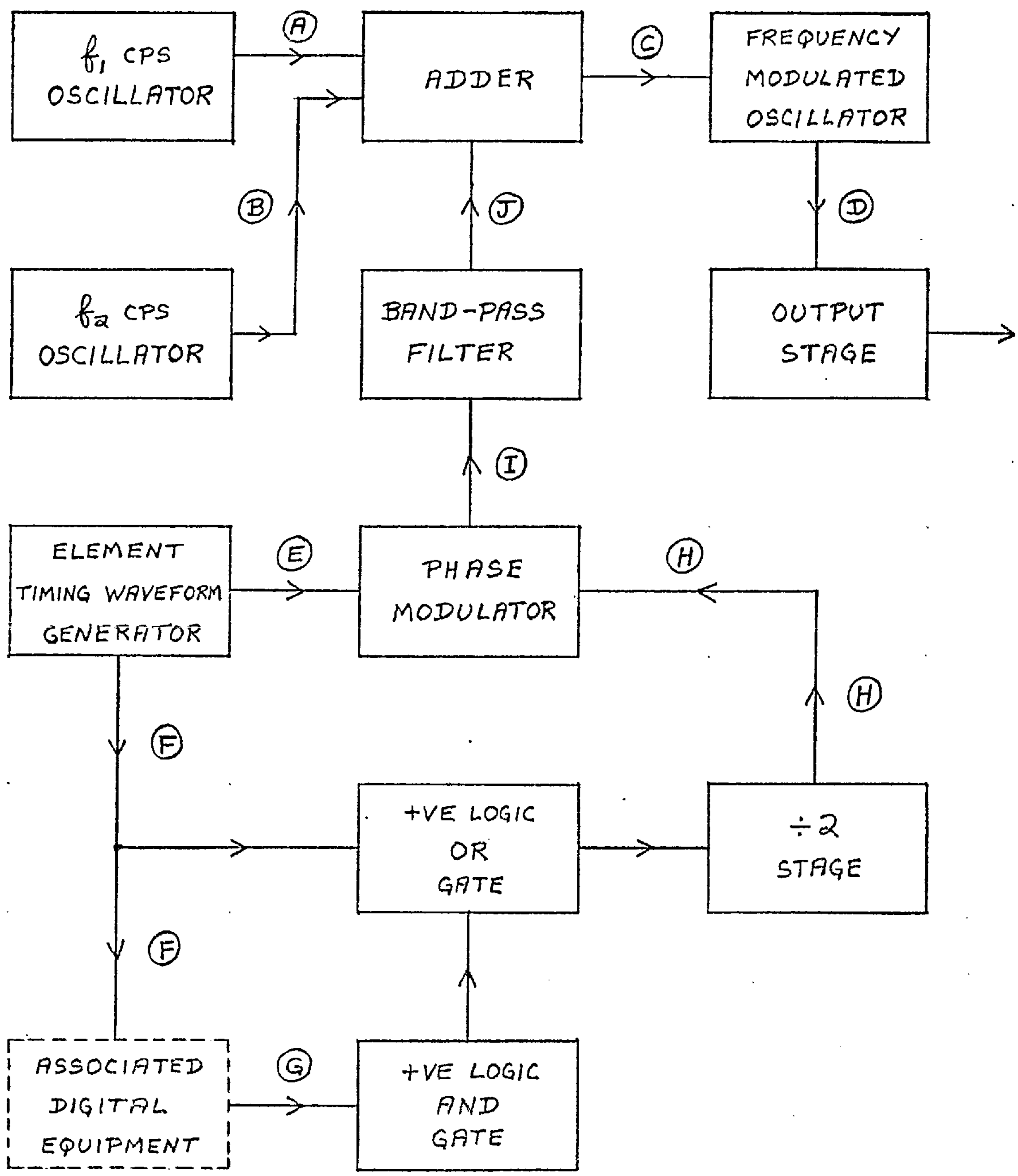


FIG. 4. SYSTEM C : TRANSMITTER BLOCK DIAGRAM.

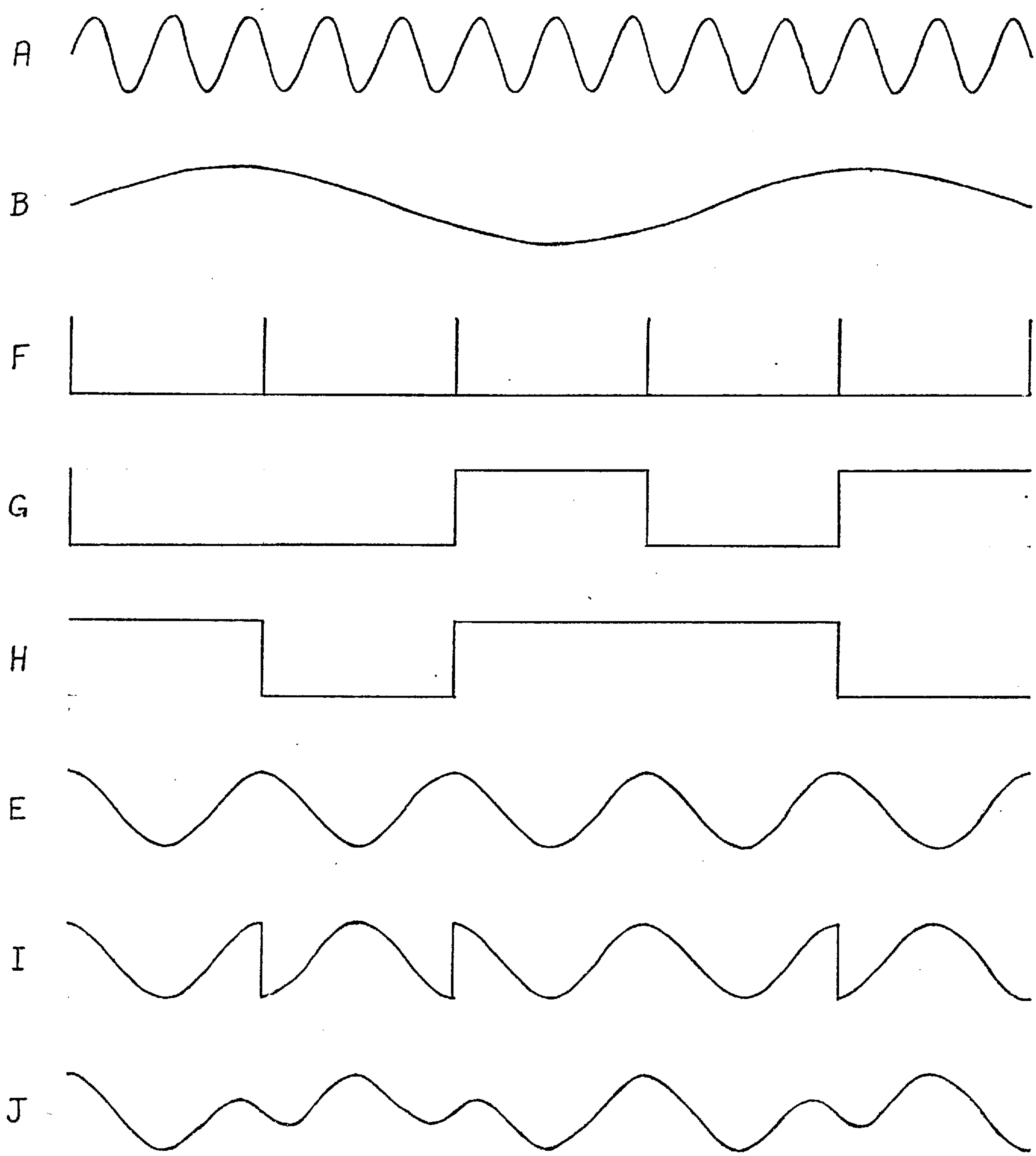


FIG. 5. SYSTEM C : TRANSMITTER WAVEFORMS.

2.4.3 Receiver

The block diagram of the receiver is shown in Fig. 6 and the waveforms obtained at different points in the circuit are shown in Fig. 7.

The received signals are fed to the receiver input stage where they are filtered and then amplified in an automatic gain controlled amplifier. The gain of the latter is controlled by the level of the detected signal at the terminal C, in order to prevent possible overloading at this point.

The output signal from the receiver input stage, at the terminal A, is fed both to an FM discriminator and to a piece-wise linear envelope-detector followed by a squarer. The two resultant output signals are multiplied together in a product modulator to give the detected signal at terminal C. As shown in equation 9 (Appendix 1), this signal contains, in addition to a large number of intermodulation products, a series of waveforms each of whose voltage varies linearly with the instantaneous frequency of a different one of the received signals. Thus when the correct calling signal is received, two sine-waves of frequencies f_1 and f_2 cps appear at C.

The waveforms at C are fed to two correlation detectors, one of which is tuned to f_1 cps and the other to f_2 cps. The first of these contains the f_1 cps oscillator, a -90° phase shifter, the product modulators 2 and 3, and the low-pass filters 1 and 3. The second contains the corresponding circuits associated with the f_2 cps oscillator.

The method of operation of the two correlation detectors is the same and is described in Appendix 2. Once stable equilibrium has been obtained, following the appearance of the f_1 and f_2 cps signals at C, the sine wave at the terminal E has the same phase as the f_1 cps signal at C, and the terminal K has a positive voltage proportional to the level of the f_1 cps signal. Similarly the sine wave at the terminal F has the same phase as the f_2 cps signal at C, and the positive voltage at L is proportional to the level of the f_2 cps signal. These correlation detectors have the properties of very-narrow-band filters and can operate correctly in the presence of high-level interference.[B3,C12]

The output signals from the low-pass filters 3 and 4, at the terminals K and L, are fed to separate level detectors. The output signal from each of these is maintained at zero volts, unless the input signal exceeds a given threshold level, in which case the output signal is set to a given positive voltage. The positive-logic AND gate gives an output of zero volts, unless both input signals are positive, in which case the output has a fixed positive voltage. This signal at the terminal M is fed to the integrator and level detector whose output signal at N goes positive in response to that at M, after a certain time delay, provided the signal at M remains positive during this period. The signal at N does not respond to rapid oscillations in the waveform at M. When the signal at N goes positive it operates the calling indicator, which informs the called subscriber that a received signal, having the correct address, has been detected. At the same time, the bistable circuit fed from N, which at the end of the previous call was set to give a negative signal at P, is now reset to give a positive signal at P, and this signal remains positive for the rest of the call.

The output sine-waves from the f_1 and f_2 cps oscillators, at the terminals E and F, have equal levels and are fed via the adder to the frequency-modulated oscillator. The sum of these two waveforms frequency-modulates the oscillator. The received varying-frequency signals at the terminal A are fed via the delay network to the terminal B, and the delay introduced by this network is equal to that introduced between the terminals A and C by the intervening circuits. Thus under conditions of equilibrium, the sum of the f_1 and f_2 cps frequency components at the terminal C should correspond exactly to the frequency-time trace of the wanted varying-frequency carrier at the terminal B, neglecting the message modulation, $g(t)$, applied to the latter signal. The modulation index and mean frequency in the frequency-modulated oscillator are chosen so that the instantaneous frequency of the output signal at Q follows that of the wanted received carrier at B, with a constant difference of f_d cps between the instantaneous frequencies, again neglecting the modulation of the received carrier by $g(t)$. Thus the signal at Q is given by

$$r(t) = q \cos\left[2\pi \int_0^t (f_c - f_d + f_e \cos 2\pi f_1\tau + f_e \cos 2\pi f_2\tau)\, d\tau\right], \tag{15}$$

and the signal at B is given by

$$s(t) = b(t)\cos\left[2\pi \int_0^t (f_c + g(\tau) + u(\tau) + f_e \cos 2\pi f_1\tau + f_e \cos 2\pi f_2\tau)\, d\tau\right], \tag{16}$$

assuming for convenience that both carriers have a zero phase angle at t=0. q is a constant and $b(t)$ is the amplitude of the wanted received signal at B. f_c is the mean value of the instantaneous carrier frequency at the transmitter output, and f_e is the peak deviation in the instantaneous carrier frequency introduced in the frequency-modulated oscillator by each of the f_1 and f_2 cps modulating waveforms. The latter are taken here for convenience as $f_e \cos 2\pi f_1 \tau$ and $f_e \cos 2\pi f_2 \tau$. $g(\tau)$ is the message modulation applied to the varying-frequency carrier at the transmitter, and $u(\tau)$ is a shift in the instantaneous carrier frequency introduced in the transmission path. t and τ are used interchangeably here for the time variable.

The signal at the output of the product modulator 6 is given by

$$s(t).r(t) = \tfrac{1}{2}b(t)q \cos\left[2\pi \int_0^t (f_d + g(\tau) + u(\tau))\, d\tau\right]$$

$$+ \tfrac{1}{2}b(t)q \cos\left[2\pi \int_0^t (2f_c - f_d + g(\tau) + u(\tau) + 2f_e \cos 2\pi f_1\tau + 2f_e \cos 2\pi f_2\tau)\, d\tau\right]. \qquad (17)$$

$\tfrac{1}{2}f_c > f_e \gg f_d$, and f_d is much less than the lowest value of $(f_c + f_e \cos 2\pi f_1\tau + f_e \cos 2\pi f_2\tau)$. The maximum magnitude of $u(\tau)$ could be of the same order as that of $g(\tau)$, where $\max |g(\tau)| \ll f_d$. f_1, f_2 and the highest frequency component of $g(\tau)$ are much less than f_d. Again the highest frequency components of b(t) and $u(\tau)$ would normally be very much less than f_2 cps.

Thus the frequency band occupied by the signal

$$\tfrac{1}{2}b(t)q \cos\left[2\pi \int_0^t (2f_c - f_d + g(\tau) + u(\tau) + 2f_e \cos 2\pi f_1\tau + 2f_e \cos 2\pi f_2\tau)\, d\tau\right] \qquad (18)$$

is much higher than that occupied by the signal

$$\tfrac{1}{2}b(t)q \cos\left[2\pi \int_0^t (f_d + g(\tau) + u(\tau))\, d\tau\right], \qquad (19)$$

there being negligible overlap between the two. This can be seen from the fact that the expression inside each integral is the instantaneous frequency of the corresponding signal.

The band-pass filter 1 removes the signal component 18 and passes the signal component 19 to the terminal R. The FM discriminator 2 does not respond to the slow variations in the amplitude of the received signal, given by b(t), and gives the output signal

$$g(t) + u(t) , \tag{20}$$

this being the deviation from f_d of the instantaneous frequency at time t, for the signal at R. The constant of proportionality is for convenience omitted here.

Since the lowest frequency component of g(t) is approximately $\frac{1}{3}f_g$ cps and $\frac{1}{3}f_g > f_2$, the highest frequency component of u(t) is normally very much less than the lowest frequency component of g(t).

The band-pass filter 2 has a pass-band extending approximately from $\frac{1}{3}f_g$ to $2f_g$ cps, as for the corresponding band-pass filter at the transmitter, so that it removes all frequency components outside the band occupied by the signal g(t) and passes only this signal to the terminal S.

If the tracking error, u(t), is caused partly or wholly by unintended discrepancies between the varying-frequency carrier at Q in the receiver and the signal at D in the transmitter, instead of being caused entirely by frequency-modulation effects in the transmission path as assumed above, it will probably contain frequency components in the pass band of the band-pass filter 2 and so introduce noise into the signal at S. In order to reduce the effects of this noise, a wider frequency-deviation can be used for the message modulation, thereby increasing the magnitude of g(t).

The f_g cps sine-wave generator, the product modulator 7, the low-pass filter 5 and the slicer, together form a coherent PM detector for the signal at S.

The f_g cps generator is of conventional design. It full-wave rectifies the signal at S, frequency divides the $2f_g$ cps component extracted from this signal and uses the resultant f_g cps signal, suitably phased, to synchronize an f_g cps oscillator. The signal at T is the oscillator output sine-wave. This may be either in phase or in anti-phase with the f_g cps carrier of any particular signal-element at S.

The signals at S and T are multiplied together in the product modulator 7 and the resultant signal at U is filtered in the post-detection low-pass filter 5 to give the waveform at V. In the slicer, this is sliced at zero volts and amplified, to give the square-wave signal at W. The latter is the coherent detector output signal.

The element timing waveform generator uses the transitions in the waveform at W to control a phase-locked oscillator, whose output waveform is shaped to produce the timing waveform at terminal X.[B3] Each timing pulse at X is ideally located at the mid point of the corresponding element in the waveform at W.

In the sampling and bistable circuit, the waveform at W is sampled and regenerated to give the waveform at Y. This is then re-coded in the following bistable and comparator circuit to give an output binary signal at Z, in which an element "1" is represented as a negative level and a "0" as a positive level.

This signal is fed via the gate to the receiver output terminal. The gate only transmits the received signal when the control signal at the terminal P is positive, that is when the received signal has first been identified as a correctly addressed signal.

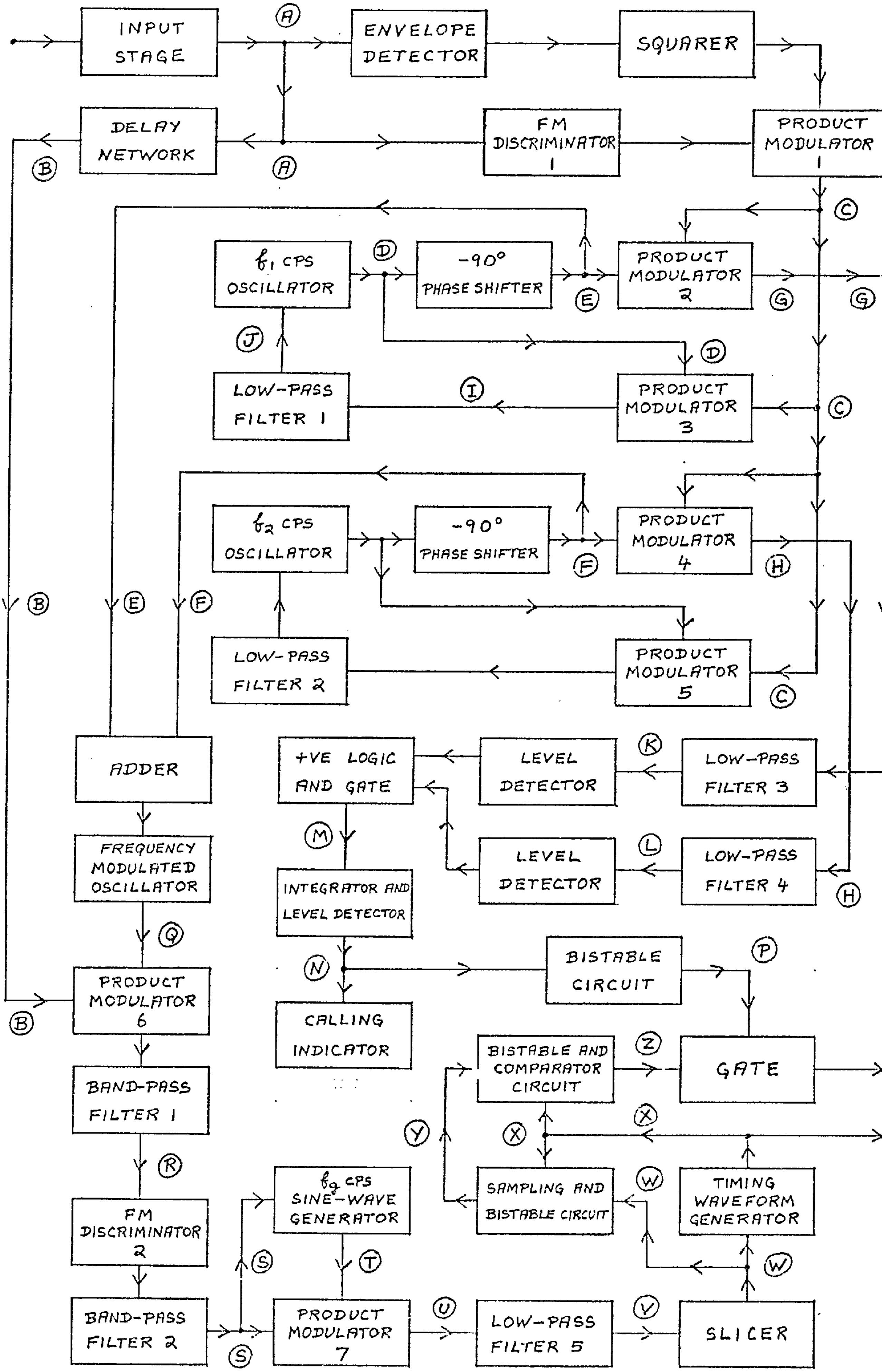


FIG. 6. SYSTEM C : RECEIVER BLOCK DIAGRAM.

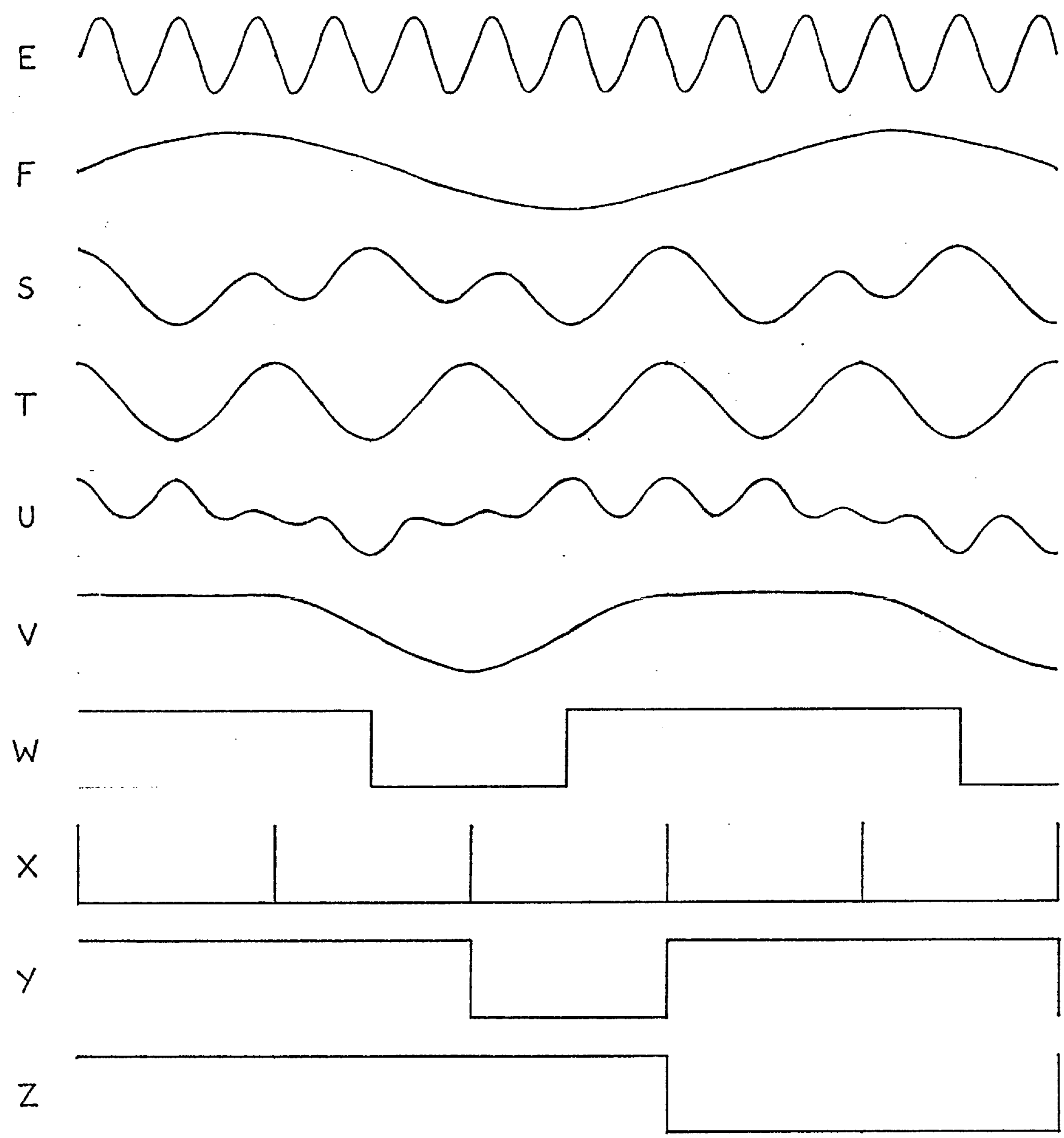


FIG. 7. SYSTEM C: RECEIVER WAVEFORMS.

2.5 Assessment of the Systems B and C

If the message modulation applied to the varying-frequency carrier at the transmitter of System C, uses a relatively wide deviation FM signal, that is if max $|g(t)| \gg f_g$, the two possible binary forms of any transmitted signal-element will be approximately orthogonal. Furthermore the receiver in System C uses a process of incoherent detection on the received signal. In System B, however, the two possible binary forms of any transmitted signal-element are the negatives of each other, and coherent detection is used at the receiver. This suggests that when there are no frequency-modulation effects in the transmission path and the interchannel interference levels are low, the System B should gain an advantage of about 4 db over System C, in tolerance to additive white gaussian noise at high signal/noise ratios.[C1-C5]

When the interchannel interference levels are high, the performance of System C would be appreciably poorer than that of System B, because of the threshold effect in the FM discriminator 2. The signal/noise ratio at the receiver input which marks the beginning of the further degradation in the performance of System C relative to System B as the signal/noise ratio is decreased, may be lowered by reducing the bandwidth of the band-pass filter 1 in the receiver of System C. This reduces the level of the noise entering the FM discriminator 2. The bandwidth of the filter must however be wide enough to handle the tracking errors in the instantaneous frequency of the signal at Q. Furthermore, the frequency deviation used for the message modulation g(t), must be wide enough to give this FM signal an adequate tolerance to these tracking errors. There is therefore a strict limit on the minimum bandwidth which may be used for the band-pass filter 1.

At the expense of an appreciable increase in equipment complexity, the threshold level in the FM discriminator may be reduced by the use of threshold extension techniques. The FM discriminator 2 may be replaced either by a frequency-compressive feedback FM demodulator or alternatively by a phase-locked FM demodulator, either of which should reduce the threshold level by a few decibels.[C13]

Clearly the System B should be used wherever its advantage in tolerance to additive noise can be exploited. However it seems that in perhaps the majority of practical applications the System C should give a better performance, because of its much greater tolerance to errors in the tracking of the instantaneous frequency of the received signal carrier.

2.6 Optimum Performance obtainable with Systems A, B and C

Approximate upper bounds for the best possible performances of Systems A, B and C can be obtained as follows.

Assume that there are $m+1$ statistically independent binary-coded signals, each with the same mean power level S and element duration T, and having a constant element energy

$$E = ST. \qquad (21)$$

Suppose that there is no additive or multiplicative noise in the transmission path and let $m \gg 1$. Assume also that the system is an idealised general-asynchronous-system using continuous varying-frequency signals, in each of which the frequency-time trace follows a random path such that the power-density spectrum is constant over the signal frequency band and essentially zero outside. All signals occupy the same frequency band, with bandwidth W cps. The sum of m of these signals has a mean power mS, with a constant power-density over the signal frequency band, and the resultant signal value is a random variable with an approximately gaussian probability density.[A7,D2,C19] Thus the m interfering signals at a receiver input can be considered as the approximate equivalent of band-limited white gaussian noise, having zero mean and variance mS,[B4] and a noise power per unit bandwidth (defined for positive frequencies only) given by

$$N_o = \frac{mS}{W}, \qquad (22)$$

so that
$$\frac{E}{N_o} = \frac{WT}{m}. \qquad (23)$$

It has been shown that in an ideal binary PM (PSK) system using coherent detection, the probability of an error in a detected signal-element is

$$P_1 = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{2E}{N_o}}}^{\infty} \exp(-\tfrac{1}{2}x^2)dx, \qquad (24)$$

and it has been shown that in an ideal binary FM (FSK) system using incoherent detection, the probability of an error in a detected signal-element is

$$P_2 = \frac{1}{2} \exp \left(- \frac{E}{2N_o} \right) . \qquad (25)$$

See references C2 and C5.

The minimum element error probability of System A or B is therefore approximately

$$P_1 = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{2WT}{m}}}^{\infty} \exp(- \tfrac{1}{2} x^2) dx , \qquad (26)$$

and the minimum element error probability of System C is approximately

$$P_2 = \frac{1}{2} \exp \left(- \frac{WT}{2m} \right) . \qquad (27)$$

In Fig. 8, P_1 and P_2 are plotted against m, for values of m in the range $\frac{2WT}{20}$ to 2WT.

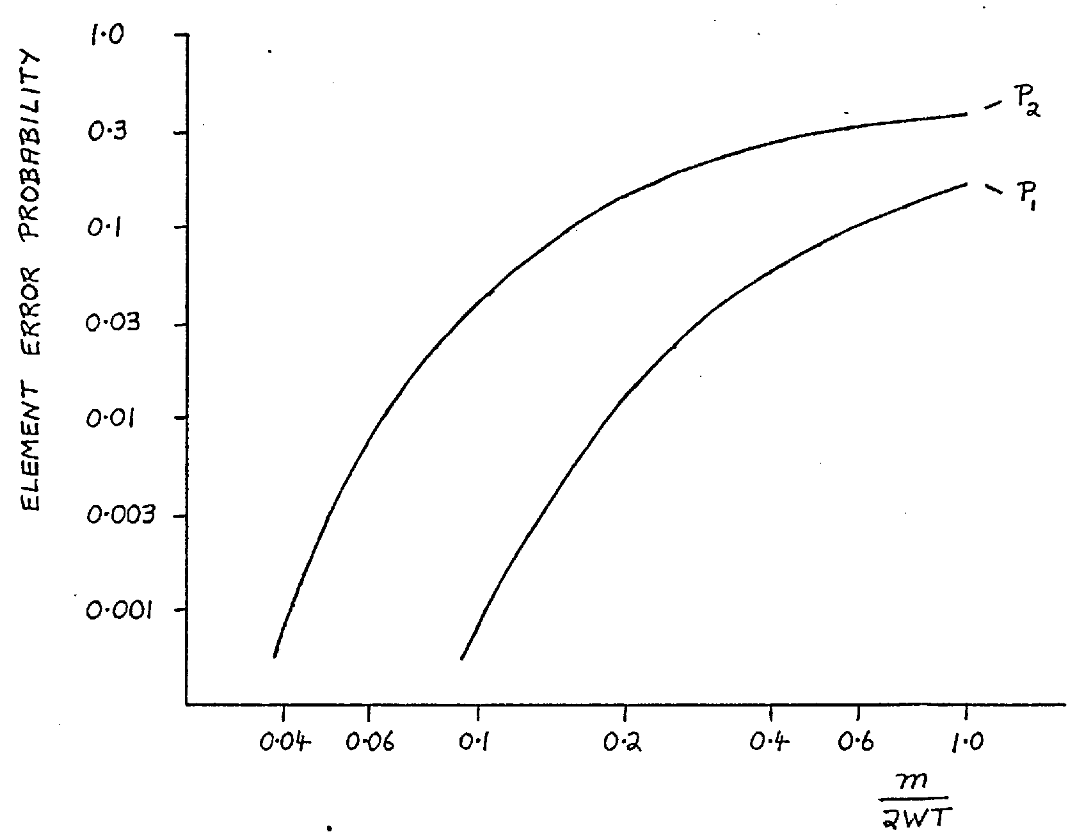


FIG. 8. VARIATION OF P_1 AND P_2 WITH $\frac{m}{2WT}$.

With the differential coding of the binary PM signals in Systems A, B and C, the minimum attainable error probabilities are in fact about twice those given in Fig. 8, at low error probabilities, and somewhat less than twice at error probabilities greater than 0.1. Differential coding need not of course be used, but more complex equipment is then required at the receiver, which must now allocate the correct binary values to the two possible carrier phases in a PM signal element.

An ideal binary-coded PM-FDM or PM-TDM system has zero element error probability for $m \leqslant 2WT-1$, under conditions similar to the above. Thus an arrangement of RADAS of the type being considered, is unlikely to be an attractive alternative to this, except in applications where m is always less than $\frac{2WT}{20}$. In the latter case, an arrangement of RADAS using either System B or System C, has interesting possibilities because it involves no very complex equipment.

3.0 SIGNAL DESIGN FOR RADAS WITH MANY TRANSMITTERS AND A SINGLE RECEIVER

3.1 Influence of the Detection Process on Signal Design

In an arrangement of RADAS using a single receiver fed from many transmitters, it becomes economic to use considerably more sophisticated detection processes than those considered in Section 2.0. However, for these arrangements to be really effective, it is necessary that certain conditions are satisfied by the transmitted signals.

The receiver here must carry out a detection process on the total received signal and determine the most likely binary values of the signal elements in each individual signal. If the individual received signals are not in element synchronism, or worse still if the elements in different signals have different durations which are not simply related to each other, the receiver cannot achieve anything approaching an optimum detection process on any one received signal-element, unless the detection process involving this element is of a much longer duration than that of the element. Any such detection process must generally involve considerable equipment complexity in relation to the standard of performance obtained. For this reason it will be assumed here that all individual signals have the same element duration of T seconds and that all signals are in element synchronism at the receiver. Under these conditions the receiver carries out a separate detection process on each resultant (total) received signal-element, to determine the corresponding binary values of the individual signals.

If the individual received signals are modulated carriers in which the carrier frequency is very much greater than the signal-element rate, as in a typical radio system, then very small changes in the relative transmission delays of the different signals can cause considerable changes in the resultant received signal. Although suitable arrangements of double modulation and incoherent AM and FM detection could probably be designed to tolerate these effects,[B1] any such arrangement would not only involve appreciably greater equipment-complexity than that in the equivalent baseband system, but it would also have a poorer performance, because the receiver can no longer have a knowledge of the carrier phase relationships between the different received signals. For this reason it will be assumed here that only baseband signals are used, the transmission medium being a line or other suitable channel, which passes the signal frequency-band with no frequency modulation or translation effects. A telephone circuit which includes a carrier link would not be a suitable transmission path here.[C3]

Since we are interested primarily in systems having the best performance for an acceptable degree of equipment complexity, it will be assumed that all transmitted signals are binary-coded antipodal signals and that each resultant (total) received signal-element is detected separately. An ideal transmission system using a binary antipodal signal, achieves the best possible tolerance to additive white gaussian noise together with the simplest detection process at the receiver, among the various possible systems using binary-coded signals and a separate detection process for each received signal-element.[B4,C3,C4]

It will also be assumed that a transmitter generates a continuous serial stream of signal elements, each having a nominal duration of T seconds and occupying the (double sided) frequency-band -W to W cps. Consider an element starting at time $t = 0$. This element is produced at the transmitter by first generating a sequence of 2WT very short rectangular pulses, which are regularly spaced at $\frac{1}{2W}$ seconds. The k th pulse here is given by $c_k\delta(t - \frac{k}{2W})$, where $k = 1, \ldots, 2WT$. $\delta(t)$ is the unit impulse at $t=0$ and c_k may have any positive or negative value within some predetermined range.

These pulses are passed through an ideal low-pass filter whose transfer function H(f) is given by

$$H(f) = \begin{cases} 1, & -W < f < W \\ 0, & \text{elsewhere} \end{cases} \qquad (31)$$

where f cps is the frequency.

The signal at the output of the low-pass filter is fed over the transmission path to the receiver, where it is passed through another low-pass filter having a transfer function H(f), and then sampled at the 2WT points given by $t = \frac{k}{2W}$. The delay introduced by the two low-pass filters and the transmission path is neglected here, and the transmission path is assumed to introduce no signal distortion.

As indicated in Appendix 3, all the useful information in the received signal-element can be obtained from the 2WT sample values of this element, at the time instants $t = \frac{k}{2W}$.[A8,A9] The subsequent detection processes in the receiver can therefore operate entirely on the 2WT sample values obtained for each signal-element. This enables a considerable reduction in equipment complexity to be achieved.

It will be assumed that the receiver stores the 2WT sample values of a signal element in 2WT capacitor stores. Two sets of these capacitor stores are required, so that while one holds a signal element for the subsequent detection processes, the other is receiving the sample values of the following element.

The column vector R, representing a received signal-element together with additive white gaussian noise, is defined by the 2WT sample values of the element, so that the k th component of this vector, r_k, is given by

$$r_k = s\left(\frac{k}{2W}\right) + n\left(\frac{k}{2W}\right)$$

$$= s_k + n_k \text{, for } k = 1, \ldots, 2WT \text{,} \qquad (43)$$

where s(t) and n(t) are the received signal and noise waveforms respectively. $s_k = s\left(\frac{k}{2W}\right)$ and $n_k = n\left(\frac{k}{2W}\right)$.

Thus $$R = S + N \text{,} \qquad (44)$$

where $S = \left[s_k\right]$ and $N = \left[n_k\right]$ are the column vectors representing the received signal-element and the noise respectively.

The arrangement studied so far in this section has involved a single transmitter feeding a single receiver. It has been assumed that a separate timing signal is fed from the transmitter to the receiver, so that the receiver can determine the correct time instants at which to sample the received waveform, and so that it also knows the positions of the element boundaries in relation to these time instants.

Consider now the general case where there are many transmitters feeding a single receiver. The timing signal is here generated at the receiver and fed via the common transmission path to all the transmitters in turn, so that it arrives back at the receiver together with all the transmitted signals. The transmitted signal from each transmitter is synchronized to this timing signal, and all signals reach the receiver in element synchronism. The arrangement is as shown in Fig. 9.

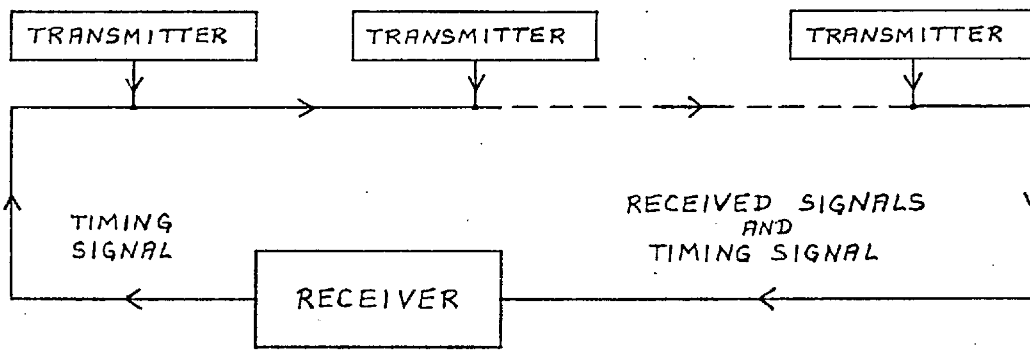


FIG. 9. SYNCHRONIZATION OF THE TRANSMITTED SIGNALS.

A separate transmission path, following the same route as the common transmission path, can if necessary be used for the timing signal, provided that the separate transmission path introduces everywhere the same delay as the common transmission path. The received timing signal informs the receiver of the correct time instants at which to sample the received waveform and also of the positions of the element boundaries in relation to these time instants.

When there are m active transmitters, a received signal-element vector S is the sum of m individual signal elements Q_i, for i = 1, ..., m, so that

$$S = \sum_{i=1}^{m} Q_i \tag{45}$$

and R = S + N as before.

The arrangements just outlined for generating and receiving the signal waveforms, are both simple and effective. Although the transfer function H(f) is not physically realizable, a reasonable approximation to this can normally be obtained in practice. Where the maximum tolerance to errors in the sampling instants is required, it will be necessary to use a wider and suitably rounded frequency spectrum for the transmitted signals. One such arrangement is considered in reference B4. See also the references F1 to F3. For the purposes of this thesis, however, the ideal arrangement described above, will be assumed.

3.2 Unique Detectability

The receiver accepts the total received signal and in the detection of each element it determines the most likely binary values of the m individual signal-elements. It is assumed that the receiver has prior knowledge of the number of received signals and of their element addresses. The element address of an individual signal does not vary over the duration of that signal, so that the arrangement of RADAS considered here is a channel-synchronized synchronous system.

In the ideal case, the levels of the different received signals may be chosen as required and do not vary with time. In this case a total received signal-element cannot be detected uniquely if and only if it has the same vector for two or more different sets of the binary values of the individual signal-elements.

Suppose that for a given set of m received signals, there are at least two sets of binary values giving the same total signal-element. Consider two of these sets. Suppose that x of the signals have different binary values in the two sets and m-x have the same binary values. Since for each signal, an element "1" is the negative of an element "0", the resultant of the x signals in one set must be the negative of the resultant in the other, whereas the resultant of the remaining m-x signals must of course be the same in the two sets. Thus the sum of the x signals in each set must be zero. It follows that if a set of m signals cannot be uniquely detected, then a subset of these signals has a zero resultant vector. The converse is also true, since any set of x signals having a zero sum is unchanged if all x binary values are changed, which necessarily implies that these cannot be uniquely detected.

If for a given set of m element-addresses, the resultant of the corresponding m signal-elements is uniquely detectable for all possible sets of binary values of the m elements, then the set of m element-addresses will be said to be uniquely detectable. Clearly, a necessary and sufficient condition for this is that the resultant signal vector has 2^m different possible positions in the signal space.

From the definition of linear dependence and the fact that the two binary forms of a given element-address are the negatives of each other and therefore linearly dependent, it follows immediately that if a set of element addresses is not uniquely detectable then these addresses are linearly dependent. The converse is however not true. Consider, for instance, two signal vectors which are colinear. Suppose one vector has twice the length of the other. There are four different resultant vectors, corresponding to the four different combinations of the two binary values, so that the element addresses are uniquely detectable. Nevertheless, the vectors are linearly dependent. Linear dependence does not therefore necessarily imply that the signals cannot be uniquely detected.

From the above we have:-

Theorem 1. A necessary and sufficient condition for the unique detectability of the sum of m binary antipodal signals is that there is no combination of binary values for which a subset of these signals adds to zero.

Theorem 2. A sufficient condition for the unique detectability of the sum of m binary antipodal signals is that these are linearly independent.

Another important case to be studied is that where the levels of the different received signals cannot be chosen as required and may have a wide range of values. The levels may for instance vary with time. It is shown in reference F7 that the necessary condition for unique detectability under conditions equivalent to those here, is that the signals are linearly independent. This can also be seen by applying the definition of linear dependence to the received signal vectors $\{Q_i\}$. m signal vectors $\{Q_i\}$ are defined to be linearly dependent if there exist m scalar values $\{u_i\}$ not all zero, such that

$$\sum_{i=1}^{m} u_i Q_i = 0 . \qquad (46)$$

Under these conditions it must always be possible to arrange a subset of these signals to add to zero, by suitably adjusting the signal levels and binary values. Thus to ensure unique detectability the signals must now be linearly independent.

If 2WT = v, so that a signal element has v degrees of freedom and the corresponding vector has v components, then there cannot at any one time be more than v linearly independent element-addresses. Thus where the received signals must be linearly independent, there cannot be more than v signals received simultaneously.

Linear independence is a sufficient condition to ensure unique detectability in the two different situations considered above, but whereas in the second of the two cases it is also a necessary condition, in the first it may often be unduly restrictive.

3.3 Signals of Constant Power Level

In Section 3.2 no particular signal code or waveform was assumed and so no restrictions were imposed on the element addresses on this account. Such an arrangement, although extremely flexible, could involve fairly complex equipment. The arrangement which achieves perhaps the best compromise between flexibility and equipment economy, is that where all the components of a signal vector have the same magnitude, different signals having different arrangements of positive and negative signs in the v components.

Let an individual received signal-element be given by

$$Q_i = z_i C_i , \tag{47}$$

where z_i is a non-zero real scalar whose magnitude is determined by the received signal level and whose sign is determined by the element binary-value. C_i is a column vector whose v components have unit magnitude and signs determined by the element address. C_i will for convenience be considered as the signal-element address, so that it is assumed here that no two addresses are distinguished from each other simply by the signal level. Since z_i is a scalar, two Q_i's are linearly dependent if and only if the corresponding C_i's are also linearly dependent.

If two received signal-elements have the same address, C_i, these are not necessarily uniquely detectable, since with different binary values and the same level, the two signals will add to zero. It will therefore be assumed that for the received signals, $C_i \neq \pm C_j$ for all $i \neq j$. Under these conditions, two signal vectors must always span two dimensions in the v-dimensional signal space, so that these signals must be linearly independent. Thus if there are only two received signals, these will always be uniquely detectable.

Consider three received signals, $\{Q_i\}$, whose resultant (total) element is given by

$$S^T = \sum_{i=1}^{3} z_i C_i^T = Z^T C , \tag{48}$$

where $C = \left[c_{ij}\right]$ is a 3xv matrix whose i th row is the element-address C_i^T, and Z is the column-vector $\left[z_i\right]$ with three components.

The three signals, $\{Q_i\}$, can only be linearly dependent if the rank of C is less than 3. Since $|c_{ij}| = 1$ for all i and j, and since no two of the three addresses $\{C_i\}$ can be linearly dependent, it is always possible by elementary transformations to convert the matrix C into the matrix

$$D = \begin{bmatrix} 1 & 1 & 1 & d_{14} & \cdots & d_{1v} \\ -1 & 1 & 1 & d_{24} & \cdots & d_{2v} \\ 1 & -1 & 1 & d_{34} & \cdots & d_{3v} \end{bmatrix},$$

where $|d_{ij}| = 1$ for all i and j. D clearly has rank 3, so C must also have rank 3. It immediately follows that the addresses $\{C_i\}$ and so the three signals $\{Q_i\}$, must be linearly independent. Thus if there are only three received signals, these will always be uniquely detectable.

Consider four received signals whose resultant (total) element is given by

$$S^T = \sum_{i=1}^{4} z_i C_i^T = Z^T C, \tag{49}$$

where $C = \left[c_{ij}\right]$ is a 4xv matrix whose i th row is C_i^T, and Z is the column-vector $\left[z_i\right]$ with four components.

The rank of C cannot be less than 3. Suppose that it is 3. Since no two or three of the addresses $\{C_i\}$ can be linearly dependent, it is always possible by elementary transformations to convert the matrix C into the matrix

$$D = \begin{bmatrix} 1 & 1 & 1 & d_{14} & \cdots & d_{1v} \\ -1 & 1 & 1 & d_{24} & \cdots & d_{2v} \\ 1 & -1 & 1 & d_{34} & \cdots & d_{3v} \\ 1 & 1 & -1 & d_{44} & \cdots & d_{4v} \end{bmatrix},$$

where the 4×3 matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

and D each have rank 3. $\left|d_{ij}\right| = 1$ for all i and j.

If $$Z^T D = 0 \,, \tag{50}$$

then $$-z_1 = z_2 = z_3 = z_4 \tag{51}$$

gives the values of all Z satisfying equation 50.

Since the elementary transformations carried out on the matrix C, involve only the interchange of rows or columns and changing the signs of all components in a row or column, all vectors Z satisfying the equation $Z^T C = 0$ must also satisfy

$$|z_1| = |z_2| = |z_3| = |z_4| . \tag{52}$$

It follows that if the four received signal-elements are linearly dependent, then the element addresses will not be uniquely detectable for equal received signal-levels, but they will be uniquely detectable if the received signal-levels are not all the same.

Consider now five received signal-elements whose addresses $\left\{ C_i^T \right\}$ are the rows of the matrix C, where

$$C = \begin{bmatrix} -1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 \end{bmatrix} . \tag{53}$$

C is a 5x5 matrix of rank 4, so that the five signal-elements are linearly dependent. The solution of the equation $Z^T C = 0$ is

$$Z^T = u(-1, -1, -1, -1, 2) \,, \tag{54}$$

where u is a real scalar with any positive or negative value. Since $Z^T C$ is the total signal-element received, the five element-addresses are not uniquely detectable when the first four signals all have the same level and the fifth signal has twice this level. They will however be uniquely detectable for any other set of relative signal-levels, say for instance when all the received levels are the same.

Thus for more than four received signals, linear dependence does not necessarily imply that these addresses will not be uniquely detectable for equal received signal-levels.

It may incidentally be shown that for equal received levels no odd number of these signal vectors can add to zero.

If the signals of constant power level considered here are to be of value in an application of RADAS, it is important that an adequate number of possible addresses $\{C_i\}$ should be available. In Appendix 4 an approximate upper bound is derived for the number of available element-addresses, for the important case where there are up to v received signals which have equal levels and are linearly independent. The result is plotted in Fig. 10 below.

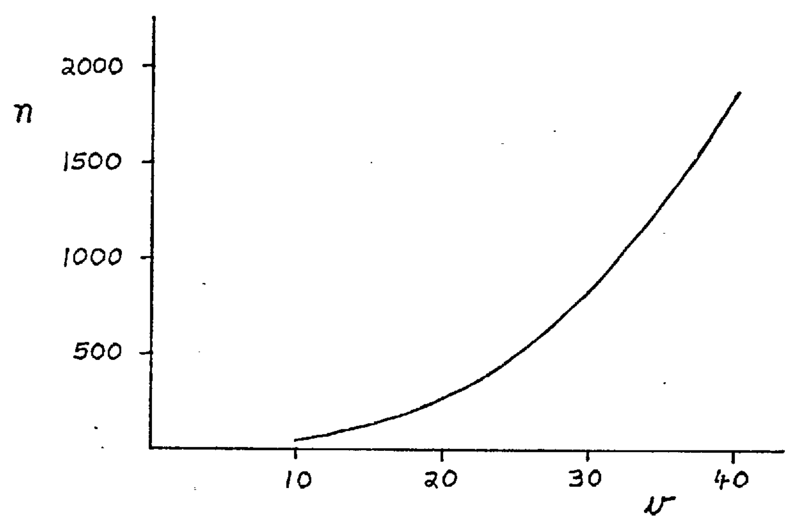


FIG. 10. MAXIMUM NUMBER OF DIFFERENT ADDRESSES, n, FOR v LINEARLY-INDEPENDENT SIGNALS OF CONSTANT POWER LEVEL AND EQUAL RECEIVED LEVELS.

In view of the criteria used to derive this upper bound, it is unlikely to be approached very closely by the maximum number of addresses actually available. The upper bound probably gives a better indication of the maximum number of addresses obtainable when the received addresses are only required to be uniquely detectable for equal received levels, and not necessarily linearly independent.

An important property of the signals of constant power level, when $v \gg 1$, is that so long as the element addresses are uniquely detectable and the received signals have equal levels, the minimum distance between two possible positions of the resultant signal vector, for any number of simultaneously received signals, is $\frac{2}{\sqrt{v}}$ times that when only one signal is received. In the worst case, where there are two received addresses having this minimum distance between two possible positions of their resultant vector, there is a probability of typically 1 in 2 that the resultant vector actually transmitted will be separated from another possible position by this minimum distance. Thus if an optimum detector is used, the tolerance to additive white gaussian noise in the two channels using these addresses, is reduced to nearly $10 \log_{10} \frac{v}{4}$ db below that where only one signal is received. In the latter case the tolerance to additive noise is the same as that in the equivalent FDM or TDM system using antipodal signals.

The optimum signal design here is taken to be that which maximizes the minimum distance between two possible positions of the resultant (total) signal-vector for the m received signals. Where there are many transmitters and many receivers, as in the case of the systems A, B and C, the optimum signal design is taken as that which minimizes the maximum projection of any unwanted signal onto the wanted individual signal-vector. The optimum signal design for the systems A, B and C may clearly result in received signals which are not uniquely detectable and which are therefore unusable here.

3.4 Adaptive Coding to Ensure Linear Independence

Attempts have been made to develop parity-check codes such that unique detectability can be ensured for up to a given number of received signals. p components are added to each signal-element, to give a total of v + p components, in which all components have magnitude unity and the v components can generate $2^{(v-1)}$ different addresses. Although a limited degree of success was achieved with some of the codes investigated, it became evident that this approach did not in general lead to very efficient codes. A better method of determining suitable element-addresses for unique detectability, appears to be by direct selection of the element addresses, using a computer.

Another approach to this problem is to use adaptive coding. The arrangements of RADAS considered here are already in one sense adaptive, since a timing signal is fed from the receiver to the various transmitters, to adjust the transmitted signals to be in element synchronism at the receiver input. Under these conditions it only requires an additional control signal to be fed from the receiver to the various transmitters, to enable each transmitted signal to be modified in some way that is dependent on the other transmitted signals. By this means the transmitted signals can be adjusted to be linearly independent and therefore uniquely detectable for all received levels.

Consider v signal vectors given by the rows of the v x v matrix

$$M = \begin{bmatrix} a_{11} & b_{12} & b_{13} & \cdots & b_{1v} \\ b_{21} & a_{22} & b_{23} & \cdots & b_{2v} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{i1} & b_{i2} & \cdots & a_{ii} \cdot & b_{iv} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{v1} & b_{v2} & \cdots & \cdots & b_{vv} \end{bmatrix}, \qquad (63)$$

where a_{ij} is an even integer and b_{ij} an odd integer, for all i and j. All components of the above matrix are odd, except for the first v-1 components of the main diagonal.

Add the last row of M to each of the other rows, to give the matrix

$$\begin{bmatrix} b_{11} & a_{12} & a_{13} & \cdots & a_{1v} \\ a_{21} & b_{22} & a_{23} & \cdots & a_{2v} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & b_{ii} \cdot & a_{iv} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{v1} & b_{v2} & \cdots & \cdots & b_{vv} \end{bmatrix},$$

where as before a_{ij} is an even integer and b_{ij} an odd integer, for all i and j. All components of this matrix are even except for the main diagonal and the last row.

Add each of the first v-1 rows of the above matrix to the last row, to give the matrix

$$L = \begin{bmatrix} b_{11} & a_{12} & a_{13} & \cdots & a_{1v} \\ a_{21} & b_{22} & a_{23} & \cdots & a_{2v} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & b_{ii} \cdot & a_{iv} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{v1} & a_{v2} & \cdots & \cdots & b_{vv} \end{bmatrix} = \left[\ell_{ij}\right], \qquad (64)$$

where all components are even except for the main diagonal whose components are all odd.

The determinant of this matrix is given by

$$\det L = \sum^{v!} (-1)^e \, \ell_{1k_1} \, \ell_{2k_2} \cdots \ell_{vk_v} \, , \tag{65}$$

where $k_1, k_2, \ldots, k_v$ range over all the $v!$ permutations of the numbers 1, 2, ..., v, taken v at a time, and where e is the number of inversions of $k_1, k_2, \ldots, k_v$ from the normal order 1, 2, ..., v.[A10]

Clearly only one of the v! terms on the right hand side of equation 65 is odd, namely

$$(-1)^e \, \ell_{11} \, \ell_{22} \cdots \ell_{vv} = b_{11} b_{22} \cdots b_{vv} \; . \tag{66}$$

The value of this term cannot be cancelled to zero in equation 65, since the sum of the remaining terms must be even. The components $\{ b_{ii} \}$ here are of course those in equation 64.

Thus $\qquad \det L \neq 0 \; .$ $\qquad$ (67)

Since the transformations by means of which the matrix M is converted into the matrix L, do not affect the value of the determinant of the matrix, it follows that

$$\det M \neq 0 \; . \tag{68}$$

Since both L and M are non-singular, the v signal-vectors given by the rows of L or M must be linearly independent. Thus we have

Theorem 3. A sufficient condition for v signal-vectors, each with v components, to be linearly independent, is that these when multiplied by a real constant are given by the rows of L or M.

Under the most unfavourable conditions, the signal vectors given by the first v-1 rows of the matrix M give a better tolerance to additive white gaussian noise than do the signal vectors given by the v rows of L or M. Thus the former signal-vectors are to be preferred.

If for i = 1, ..., (v-1), the original i th signal-vector has v components d_{ij} such that $|d_{ij}| = 1$ for j = 1, ..., v, then the v-1 signal-vectors can be made linearly independent by arranging that $|d_{ii}|$ is even for i = 1, ..., (v-1). Thus each signal-vector has the magnitude of one component changed. With this arrangement a maximum of v-1 linearly independent signals can be transmitted simultaneously, the total number of available addresses being $2^{(v-1)}$.

The receiver here feeds back to the transmitter the element timing signal and an additional signal to indicate which of the v components must have its magnitude changed in the next new signal to be transmitted. This component must of course not coincide with any of those whose magnitudes have been changed in the signals already being transmitted.

An interesting and basically simple arrangement is that where $d_{ii} = 0$ for $i = 1, \ldots, (v-1)$. With this arrangement the total number of available addresses is $2^{(v-2)}$, since for a received element-address to be uniquely recognizable after one of its components has been set to zero, each original element-address must differ from every other in at least two of the v components. $2^{(v-2)}$ is nevertheless considerably larger than the corresponding value of n in Fig. 10 (Section 3.3). When $v \gg 1$, the minimum distance between two possible positions of the resultant signal-vector for m received signals, is here $\frac{1}{\sqrt{2}}$ times that in the equivalent non-adaptive system, where $|d_{ij}| = 1$ for all i and j and the element addresses are selected to ensure unique detectability. Thus if an optimum detector is used, the minimum tolerance to additive white gaussian noise in a channel of the adaptive system, is about 3 db below that in the equivalent non-adaptive system, and therefore approximately $10 \log_{10} \frac{v}{2}$ db below that of a channel in the equivalent FDM or TDM system.

The alternative to the above approach is to give $|d_{ii}|$ an even value greater than or equal to 2, for $i = 1, \ldots, (v-1)$. Again, the v×v matrix $D = \left[d_{ij}\right]$, where $|d_{ii}| > v-1$ for all i and $|d_{ij}| = 1$ for all $i \neq j$, is strictly diagonally dominant, so that it is non-singular, by a well known theorem.[A12] The v signal-vectors given by the rows of this matrix may therefore be used, since they are linearly independent regardless of the exact value of $|d_{ii}|$.

The most effective of the adaptive systems are clearly those where $|d_{ii}| > v-1$. The transmitted signals here are nearly orthogonal and the tolerance to additive noise approaches that of the equivalent TDM system. An individual transmitted signal-element can here be considered to contain two components: the original RADAS signal in which $|d_{ij}| = 1$ for all i and j, and an adaptive component in which $|d_{ii}| > v-2$ for all i and $|d_{ij}| = 0$ for all $i \neq j$. The arrangement is thus a combination of RADAS and conventional TDM. It does not however appear to have any overall advantage over the equivalent message-switching system, which uses a conventional TDM signal and transmits the message-address separately, before the start of the message. Such systems are considered in some detail in reference B4.

3.5 AM System

This is an alternative arrangement of RADAS, using adaptive coding, which may under certain conditions have useful advantages over the equivalent message-switching system. In this arrangement, instead of each individual signal-element carrying the address of the signal, the duration of the address is extended in time so that it takes several successive elements of the same individual signal to carry the message address, just as in Systems B and C.[B3] The arrangement is based on a conventional TDM system using bipolar signals in which $|d_{ii}| = 1$ for all i and $d_{ij} = 0$ for all $i \neq j$. The successive binary digits which carry an individual signal, that is the successive d_{ii} for a given value of i, are amplitude modulated by the addition to these digits of binary-coded digits carrying the address of the called subscriber. Each signal-digit has added to it one address-digit. The address digits have an amplitude of one tenth to one fifth of that of the signal digits, giving a depth of modulation of 10 to 20°/o . An element "0" of the address digits is given the same sign as that of the signal digit to which it is added, and an element "1" is given the opposite sign. The message address has a fixed length of h digits and is repeated sequentially. Clearly the different message addresses can be selected from an extremely wide range, and the preferred arrangement, which will be considered here, has a minimum Hamming distance of 3 between any two addresses.

The receiver feeds an additional timing signal to the various transmitters in order to synchronize the message addresses received from these, so that the receiver has prior knowledge of which is the first digit in each received message-address. The receiver detects the message address of an individual received signal, by applying AM envelope detection to the appropriate digits, d_{ii}, of the total received signal. That is, the receiver determines the magnitude of each received digit and subtracts from this the magnitude of an unmodulated signal-digit, to give the corresponding detected address digit. A low-pass filter whose impulse response is a single rounded positive pulse with an effective duration 2gt, where g = 100 to 1000 and t is the repetition period of the address, is allocated to each of the h digits in the message address. A separate set of h low-pass filters is allocated to each of the v individual received signals. A current or voltage pulse proportional in value to a detected address digit, is fed into the appropriate low-pass filter, each time this digit is received. The output signal from each filter is compared with both a positive and a negative threshold level, each having half the magnitude of the corresponding maximum signal-level. If the output signal has a value outside the range included between the two threshold levels, the corresponding address digit is detected as being present and as having the appropriate binary-value. A negative output-signal represents an element "1" and a positive signal an element "0". In the absence of noise, g consecutive appearances of the k th address-digit, following its absence for a period somewhat greater than 2gt, will just cause the k th digit to be detected as present. Similarly the disappearance of the k th digit in g consecutive addresses, following its presence for a period somewhat greater than 2gt, will just cause the digit to be detected as absent. Immediately the presence of at least h-1 of the h digits of a valid message-address, is detected at the receiver, the address is recognized and the detected signal is automatically connected to the corresponding receiver output terminal.

It is shown in Appendix 5 that with additive white gaussian noise at the receiver input and with a high signal/noise ratio, the probability of a failure in the detection of a message address, at a given instant, is very small compared with the probability of a signal-element error, so long as $g > \frac{2}{x^2}$, where x is the magnitude of an address digit relative to that of a signal digit. The probability of the wrong message-address being detected is an order of magnitude lower and so should not be important.

Since the depth of amplitude modulation applied to the signal digits, d_{ii}, is only 10 to 20$^{o}/_{o}$, the resultant reduction in tolerance to additive white gaussian noise of a transmitted signal is only about 1 to 2 db. Furthermore, the individual transmitted signals are orthogonal as in a conventional TDM system, so that the tolerance of this arrangement to additive noise is only about 1 to 2 db poorer than that of the equivalent message-switching system which uses a conventional TDM signal. The arrangement is therefore equally suitable for data or digitally-coded-speech signals.

The particular advantage of this arrangement over the equivalent message-switching system, is that since the message address is being transmitted continuously with the signal, temporary loss of transmission caused by "transient interruptions" or long bursts of high-level noise,[C3] will not cause the loss of the remainder of the message. The arrangement is therefore particularly suited for the automatic routing or message switching of long signal-messages. On the other hand, because of the appreciable time delay of say 0.1 to 1 seconds in the detection of the message address at the receiver, the arrangement is not suitable for the transmission of short messages, whose duration is of this order of magnitude. Nor can any effective use be made here of the gaps in a speech signal, by stopping transmission during the gaps and so reducing the occupancy of the common transmission path.

The system will handle up to v simultaneous signals and its tolerance to additive noise will approach the best that can be expected from any arrangement of RADAS.

4.0 SIGNAL DETECTION IN RADAS WITH MANY TRANSMITTERS AND A SINGLE RECEIVER

4.1 Optimum Detection Processes

It is assumed here, as in Section 3.1, that the individual received signals are binary antipodal baseband signals in element synchronism, each total received signal-element being detected separately.

Where the receiver has prior knowledge of the element addresses and levels of the received signals, the detection process which minimizes the element error probability in each channel, is similar to that described on pages 32-35 of reference F24. This is of no great practical interest because of its complexity and so will not be considered further here.

Assume that the individual received signal-elements are statistically independent and equally likely to have either binary value. The signals are received in the presence of additive white gaussian noise, and the receiver has prior knowledge of the element addresses and levels of the received signals. For the minimum probability of error in the detected binary values of an element detection process, the resultant or total vector corresponding to these binary values must be that nearest to the received vector in the signal space.[A8,F24]

In an element detection process here, the receiver generates in turn the resultant vectors corresponding to the different combinations of binary values of the individual signal-elements. Each resultant vector is subtracted from the received vector, whose components are stored in v storage capacitors and remain unchanged throughout the element-detection process. The components of the difference vector are squared and added, to give the square of the distance between the two vectors. In the first subtraction process, the distance measure together with the associated binary-values are stored. In subsequent subtraction processes no action is taken, unless the distance measure is smaller than that stored. When this occurs, the new distance measure together with the associated binary-values replace those stored. Thus at the end of the element-detection process, the receiver has a record of the detected binary-values. Since the separate operations in the detection process are carried out sequentially, these can be performed by a single piece of equipment, so that no great equipment complexity is involved here.

If there are m received signals, the receiver must carry out sequentially 2^m separate subtraction processes. Where the received signal is a 40 k-bps digitally-coded-speech signal, it would not be economic to carry out more than about 100 subtraction processes, so that no more than 6 or 7 simultaneous signals could be handled. Clearly, for a detection process of this type to be of any real practical value, it is important that the number of separate operations in an element-detection process, should increase linearly and not exponentially with the number of signals received.

When the receiver has no prior knowledge of the levels of the received signals and these may vary over a wide range, the above detection process would not in general operate correctly. The received signals must now be linearly independent and the optimum detection process is one of linear filtering.[F7, F23]

Consider the detection process at the receiver for a total received signal-element. Suppose that the receiver knows only the number, m, and the element-addresses of the individual received signals. The element-address of the i th signal is given by the real column-vector Y_i, which has v components and unit length. The m element-addresses $\{Y_i\}$ are assumed to be linearly independent. Let the individual received signals in the element detection process be given by

$$Q_i = z_i Y_i \quad \text{for } i = 1, \ldots, m , \tag{74}$$

where z_i is a scalar having any positive or negative value. The total signal-element is

$$S = \sum_{i=1}^{m} Q_i . \tag{75}$$

Let the received noise-vector be N, whose v components are statistically independent gaussian random variables with zero mean and variance σ_n^2. Thus the resultant received vector is

$$R = S + N = \sum_{i=1}^{m} z_i Y_i + N = YZ + N , \tag{76}$$

where the i th column of the v×m matrix Y is Y_i and Z is the real m-component column-vector $\left[z_i\right]$.

Since the m signals $\{Q_i\}$ are linearly independent, they span an m-dimensional subspace of the v-dimensional signal space. For the given m element-addresses, the signal S is uniquely determined by the m signals $\{Q_i\}$ and vice versa. In the presence of white gaussian noise at the receiver input, and with no prior knowledge of the $\{z_i\}$ or σ_n^2, the best estimate that the receiver can make of S is to take this as being the vector in the m-dimensional subspace, closest to R. Thus, if the projection of R onto the subspace spanned by the m addresses $\{Y_i\}$, is the vector P, then P is the best estimate of S.

The projection of R onto Y_i is $d_i Y_i$, where

$$d_i = Y_i^T R . \tag{77}$$

d_i is the inner product of the vectors R and Y_i. Since the m linearly independent vectors $\{Y_i\}$ form a basis for the subspace spanned by these vectors, the vector P is completely and uniquely defined as the vector in this subspace whose projection onto Y_i, for i=1, ..., m, is $d_i Y_i$. There is a one-to-one relationship between P and the $\{d_i\}$.

If the receiver takes x_i as the estimated value of z_i, for i=1, ..., m, the estimated value of S is

$$\sum_{i=1}^{m} x_i Y_i = YX , \tag{78}$$

where X is the real m-component column-vector $[x_i]$. The vector YX lies in the subspace spanned by the m vectors $\{Y_i\}$.

If now $\quad Y_i^T(YX) = d_i$, for i=1, ..., m , $\qquad$ (79)

the projection of YX onto Y_i, for i=1, ..., m, is $d_i Y_i$.

Thus $\quad YX = P$, $\qquad$ (80)

so that the values of $\{x_i\}$ given by equation 79 are the estimated values of $\{z_i\}$ given by the vector P.

From equation 79,

$$Y^T YX = D , \tag{81}$$

where D is the real m-component column-vector $[d_i]$.

Thus $\quad AX = D$, $\qquad$ (82)

where $\quad A = Y^T Y$. $\qquad$ (83)

So that $\quad X = A^{-1}D$. $\qquad$ (84)

Since the m addresses $\{Y_i\}$ are real and linearly independent,

$A = [a_{ij}]$ is a real m×m symmetric positive-definite matrix, so that its inverse A^{-1} clearly exists. The components of A are the cross-correlation coefficients for the different pairs of element addresses.

d_i is the output signal obtained in response to the received vector R, from a correlation detector whose reference signal is Y_i. This correlation detector multiplies each component of R by the corresponding component of Y_i and adds the products, to give the output signal $Y_i^T R$. Since the v components of R are stored at the receiver input throughout the detection process, the separate operations of multiplication and addition are carried out simultaneously. The received vector R must be fed simultaneously to m correlation detectors tuned to the m element-addresses $\{Y_i\}$. The output signals from the correlation detectors are the m components of the vector D. To obtain the m estimates $\{x_i\}$ of the received signal values $\{z_i\}$, the output signals of the correlation detectors must be fed through a network which performs the linear transformation A^{-1} on these signals. Thus the optimum detector is as shown in Fig. 11.

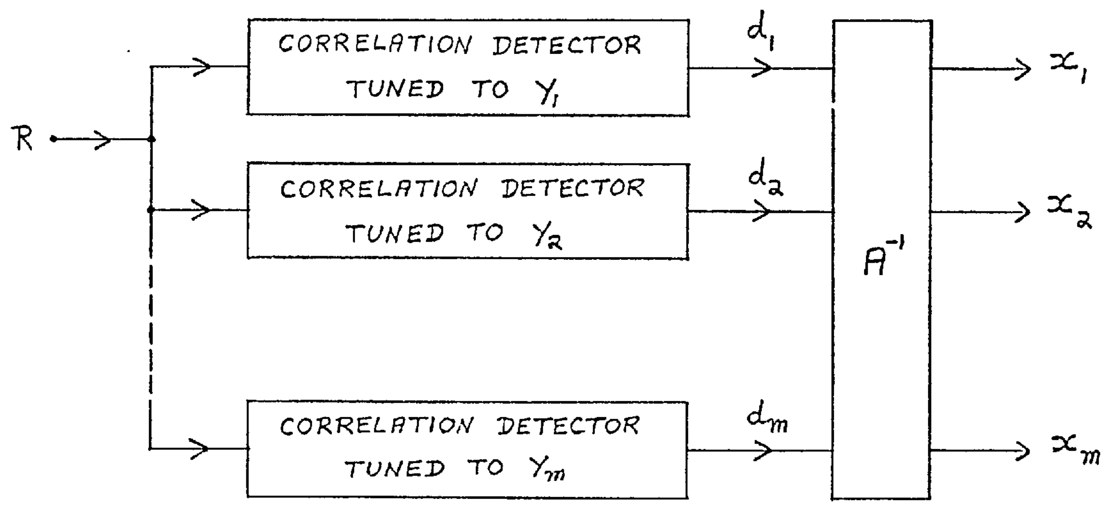


FIG. 11. OPTIMUM DETECTOR FOR m RECEIVED SIGNALS $\{z_i Y_i\}$, WHERE THE $\{Y_i\}$ ARE KNOWN BUT THE RECEIVER HAS NO PRIOR KNOWLEDGE OF THE $\{z_i\}$ OR σ_n^2.

This detector minimizes the probability of error in an element detection process.

Since the transmitted signals are antipodal, the detection process is not affected by a constant attenuation applied to the $\{x_i\}$. Thus the network A^{-1} can under favourable conditions be simply a set of m^2 attenuators together with arrangements for combining the m signals at each output. In the final stage of the detection process, not shown in Fig. 11, the receiver examines the signs of the $\{x_i\}$ and allocates the appropriate binary-values to the corresponding $\{Q_i\}$.

Where the receiver is always fed with m signals having m given addresses, but the receiver has no prior knowledge of the $\{z_i\}$ or σ_n^2, the arrangement of Fig. 11 achieves the best compromise between performance and equipment complexity. The arrangement can handle up to v simultaneous signals.

In the RADAS situation, the number and addresses of the received signals are continuously changing. Since

$$A^{-1} = \left[\frac{\alpha_{ji}}{\det A} \right], \quad \text{for } i, j = 1, \ldots, m, \tag{85}$$

where α_{ij} is the cofactor of a_{ij}, it follows that the addition of a new signal to the m already being received, will not just require 2m+1 additional components for A^{-1} but will in general cause the majority of the existing components to be changed. Clearly any such arrangement could become extremely complex. It is therefore of interest to examine alternative methods of performing the linear transformation A^{-1}.

4.2 Detection with Signal Cancellation

In 1936 L. A. MacColl proposed an ingenious arrangement for increasing the transmission rate of a serial digital transmission system.[F5,F6,B4] The essential feature of his arrangement is as follows. Where each separate pulse of the received digital signal is lengthened, due to a combination of attenuation and delay distortions in the transmission path, the receiver need only use the first portion of each received pulse in order to achieve correct detection of the pulse. Assuming that the transmission path has fixed attenuation and phase characteristics, the remaining portion of each pulse is completely determined for a given amplitude of the sample value obtained at the end of the first portion. It follows therefore that after having sampled a pulse, the receiver can generate an exact replica of the remaining portion and add the inverted replica to the received signal, so as to cancel out the remainder of the pulse. Thus immediately after a pulse has been sampled at the receiver, the pulse is reduced to zero and the receiver is then ready to begin receiving the next pulse. Received signals in which there is appreciable intersymbol interference between neighbouring pulses, can be successfully detected in this way.

It is evident that this basic technique can be applied not only to a serial stream of pulses but also to a number of simultaneously received signals of widely differing levels.[B4] Suppose that there are m received signals $\{z_i Y_i\}$ in element synchronism, where the element addresses $\{Y_i\}$ are real unit vectors which need not here be linearly independent. Let $|z_1| \gg |z_2| \gg \cdots \gg |z_m|$. If the signals $\{z_i Y_i\}$ are binary antipodal signals and the receiver knows the levels $\{|z_i|\}$ and addresses $\{Y_i\}$ of the individual signals, then the receiver can detect the m signals as follows.

The resultant received vector R is given by

$$R = S + N = \sum_{i=1}^{m} Q_i + N = \sum_{i=1}^{m} z_i Y_i + N , \qquad (86)$$

where N is the noise vector, as before. R is fed simultaneously to m correlation detectors tuned to the m element-addresses, as in Fig. 11 but with the network A^{-1} omitted. The output signal from the first detector is sampled and depending upon whether this is negative or positive, the signal $-|z_1| Y_1$ or $|z_1| Y_1$ is subtracted from R at the input. If the output signal is zero, the subtracted signal is selected at random from its two possible values, $\pm |z_1| Y_1$. If $z_1 Y_1$ is correctly detected it is eliminated from the m received signals at the detector input. The output signal from the second correlation detector is now sampled and the signal $z_2 Y_2$ similarly eliminated from the detector input. This process continues until all m signals have been detected and cancelled. In the absence of detection errors, only the noise vector N now remains at the input. Although this is clearly not an optimum detection process, it is very simple to implement.

In the arrangement of MacColl's proposal, it is assumed that the pulses following that being detected, do not cause intersymbol interference in the detection of this pulse, so that incorrect detection cannot be caused on this account. In the arrangement here, however, the signals remaining to be detected in any element detection process, may cause the incorrect detection of an individual signal-element or at least a noticeable reduction in tolerance to additive noise, if the differences between the levels of the various signals are not great enough.

4.3 Iterative Detection Processes using Large Steps

4.3.1 Constraints I, J and K

In the arrangement described towards the end of the previous section, the received vector R is stored throughout the whole of the detection process, so that it is possible to repeat the detection cycle, if necessary several times. Thus after the completion of the first detection cycle, the outputs of the correlation detectors are again examined sequentially and in the same order as before. When the sign of the i th correlation-detector output signal is not in agreement with the sign of the i th signal already subtracted from R, for any value of i, and the magnitude of the correlation-detector output signal exceeds $|z_i|$, then the sign of the subtracted signal is changed. When the signs disagree and the magnitude of the output signal equals $|z_i|$, then the subtracted signal is selected at random from its two possible values, $\pm |z_i| Y_i$. In all other cases the subtracted signal is left unchanged. The detection cycle may be repeated as many times as required. This is of course an iterative detection process.

In this arrangement, at the end of the process of selection and subtraction of the i th signal in the k th detection cycle, the magnitude $|x_{ik}|$ of the i th signal $x_{ik}Y_i$ subtracted from R, is constrained to satisfy the equation

$$|x_{ik}| = |z_i| , \text{ for all i and all } k > 0 , \quad (87)$$

where x_{ik} is a real scalar. $x_{io} = 0$ for all i (Section 4.2).

The constraint K is defined as that where the individual signals $\{x_{ik}Y_i\}$, subtracted from the input, are constrained to satisfy equation 87.

The constraint K may be modified by arranging that in the k th detection cycle, the i th signal subtracted from the input is no longer such that the resultant subtracted i th signal is $\pm |z_i| Y_i$, but is $d_{ik}Y_i$, where d_{ik} is the corresponding correlation-detector output signal at the time immediately preceding the subtraction. Thus, immediately after the subtraction of the i th signal in the k th detection cycle, the total i th signal subtracted from the vector R at the input is

$$x_{ik}Y_i \quad = \quad \sum_{j=1}^{k} d_{ij}\,Y_i \quad , \quad \text{for all } i \text{ and } k, \qquad (88)$$

and there is zero output from the i th correlation detector.

The constraint I is defined as that where the individual signals subtracted from the input are given by equation 88. It represents the absence of any constraints applied to the $\{x_{ik}\}$.

If, with the constraint I, only the i th signal $z_i Y_i$ is received and there is no noise, the corresponding detector gives an output signal z_i at the start of the first detection cycle, so that the signal subtracted from R at the end of this cycle is $z_i Y_i$. This is of course the same as that with the constraint K, under the same conditions.

The constraint I may be modified so that at the end of the process of selection and subtraction of the i th signal in the k th detection cycle, the magnitude $|x_{ik}|$ of the total i th signal $x_{ik}Y_i$ subtracted from R, is constrained to satisfy the equation

$$|x_{ik}| \quad \leqslant \quad |z_i| \quad , \quad \text{for all } i \text{ and } k. \qquad (89)$$

The constraint J is defined as that where the individual signals $\{x_{ik}Y_i\}$, subtracted from the input, are constrained to satisfy equation 89.

The constraint J is clearly a compromise between I and K. Where the constraint I or J is used, it is assumed that $|z_1| \geqslant |z_2| \geqslant \cdots\cdots \geqslant |z_m|$, and the m addresses $\{Y_i\}$ must be linearly independent.

At the start of the first detection cycle, in all the arrangements considered, $x_{io} = 0$ for all i; and when the total number of n detection cycles has been completed, the receiver determines the signs of the $\{x_{in}\}$ and allocates the appropriate binary values to the corresponding $\{Q_i\}$.

4.3.2 Detection Processes 1 to 4

Each of the three constraints I, J and K may be combined with different rules for selecting the order in which the m signals are subtracted in a detection cycle. The different rules will be identified with different detection processes, as follows.

Detection Process 1

In each cycle the m signals are selected and subtracted simultaneously, but the individual subtraction processes are otherwise similar to those described in Section 4.3.1. In the first detection cycle with the constraint K, the subtracted signals are determined as in Section 4.2.

Detection Process 2

In each cycle the m signals are selected and subtracted sequentially, exactly as described in Section 4.3.1. With the constraint K, the first detection cycle is as described in Section 4.2. When two or more signals have equal levels, the order in which they are selected and subtracted is chosen arbitrarily, but remains fixed from one cycle to another.

Detection Process 3

In each cycle the m signals are selected and subtracted sequentially, but not necessarily in the same order in any two cycles. The i th correlation detector here has a reference signal $|z_i| Y_i$ for all i, instead of Y_i used in the detection processes 1 and 2.

Consider the start of the (k+1) th detection cycle. Let the signal at the output of the i th correlation detector be $d_{i(k+1)}$ and let the total i th signal subtracted from the input as a result of the previous k detection cycles be $x_{ik}Y_i$ for all i. The receiver now determines the value of i for which $|d_{i(k+1)} + |z_i| x_{ik}|$ is maximum and carries out the process of subtraction for the corresponding signal. This will in general change the output signals of the m correlation detectors. The receiver then determines the value of i for which $|d_{i(k+1)} + |z_i| x_{ik}|$ is maximum for the remaining m-1 signals, and subtracts the corresponding signal from the input. This process is repeated until all m signals have been subtracted.

With the constraint I,

$$x_{i(k+1)} = x_{ik} + \frac{d_{i(k+1)}}{|z_i|}, \text{ for all } i, \tag{90}$$

where $d_{i(k+1)}$ is the output signal from the i th correlation detector immediately preceding the subtraction of the i th signal, which is

$$\frac{d_{i(k+1)}}{|z_i|} \cdot Y_i , \qquad \text{for all } i. \tag{91}$$

Thus there is zero output signal from the i th detector immediately following the subtraction of the i th signal.

With the constraint J, $x_{i(k+1)}$, determined as in equation 90, is constrained to satisfy

$$\left| x_{i(k+1)} \right| \leq \left| z_i \right| \quad \text{for all } i \text{ and } k. \tag{92}$$

With the constraint K,

$$\left| x_{i(k+1)} \right| = \left| z_i \right| \quad \text{for all } i \text{ and } k. \tag{93}$$

In the first detection cycle and after each process of selection, the output signal of the selected correlation detector (say the i th) is sampled, and depending upon whether this is negative or positive, the signal $-\left|z_i\right| Y_i$ or $\left|z_i\right| Y_i$ is subtracted from R at the input. If the output signal is zero, the subtracted signal is selected at random from $\pm \left|z_i\right| Y_i$. In subsequent detection cycles and after each process of selection, if the sign of the detector output signal (say the i th) is not in agreement with the i th signal already subtracted from R, and if the magnitude of the detector output signal exceeds z_i^2, then the sign of the subtracted signal is changed. When the signs disagree and the magnitude of the output signal equals z_i^2, then the subtracted signal is selected at random from its two possible values, $\pm \left|z_i\right| Y_i$. In all other cases the subtracted signal is left unchanged.

Detection Process 4

The signals here are not selected cyclically. As in 3, the i th correlation detector has a reference signal $\left|z_i\right| Y_i$ for all i.

Consider first the constraint I. In the k th individual process of selection and subtraction, the receiver samples the output signals $\left\{ d_{ik} \right\}$ of the m correlation detectors. It determines the value of i for which $\left| d_{ik} \right|$ is maximum and subtracts the corresponding signal

$$\frac{d_{ik}}{\left|z_i\right|} \cdot Y_i$$

from the input, to give zero output from the corresponding detector. A new set of output signals $\left\{ d_{i(k+1)} \right\}$ results and the receiver repeats the procedure for the (k+1)th process of selection and subtraction, and so on.

Whereas in the detection processes 1 to 3, k increases by 1 for each new detection cycle, in the detection process 4, k is taken to increase by 1 for each new individual process of selection and subtraction.

Consider now the constraint J. At the start of the (k+1)th individual process of selection and subtraction, the total i th signal subtracted from the input is $x_{ik}Y_i$, for all i. If no constraints were applied as with the constraint I, the total i th signal subtracted from the input at the end of the (k+1)th process of selection and subtraction, would be

$$x_{i(k+1)}\,Y_i = \left(x_{ik} + \frac{d_{i(k+1)}}{|z_i|}\right) Y_i \,. \tag{94}$$

With the constraint J,

$$\left|x_{i(k+1)}\right| \leqslant \left|z_i\right| \quad \text{for all i and k,} \tag{95}$$

so that $x_{i(k+1)}$ is the value of

$$x_{ik} + \frac{d_{i(k+1)}}{|z_i|}$$

truncated if necessary so that equation 95 is satisfied. The receiver determines the value of i for which

$$\left| z_i\left(x_{i(k+1)} - x_{ik}\right) \right|$$

is maximum and subtracts the corresponding signal

$$\left(x_{i(k+1)} - x_{ik}\right)Y_i$$

from the input, to make the total i th signal subtracted from the input, $x_{i(k+1)}Y_i$. A new set of output signals, $\left\{d_{i(k+2)}\right\}$, results and the receiver repeats the procedure on these, and so on.

Where the constraint I or J is used, the detection process is automatically terminated when

$$\max\left\{\left|d_{i(k+1)}\right|\right\} < e\left(\min\left\{z_i^2\right\}\right) \quad \text{for all i,} \tag{96}$$

where e is a positive constant such that $e \ll 1$.

Consider now the constraint K, where

$$\left|x_{i(k+1)}\right| = \left|z_i\right| \text{ or } 0 \;, \quad \text{for all i and k,} \tag{97}$$

and consider the (k+1)th individual process of selection and subtraction.

Let $f_i = - d_{i(k+1)} \dfrac{x_{ik}}{|z_i|}$, for all i. (98)

The receiver now determines the value of i for which g_i is a maximum (most positive), where

$$
\begin{aligned}
g_i &= |d_{i(k+1)}| && \text{if } x_{ik} = 0 \\
g_i &= f_i && \text{if } x_{ik} \neq 0 \text{ and } f_i > z_i^2 \\
g_i &= 0 && \text{if } x_{ik} \neq 0 \text{ and } f_i = z_i^2 \\
g_i &= -1 && \text{if } x_{ik} \neq 0 \text{ and } f_i < z_i^2 .
\end{aligned}
$$

If for this value of i, $x_{ik} = 0$, the receiver sets

$$x_{i(k+1)} = 0 \qquad \text{if } g_i = 0$$

or $$x_{i(k+1)} = \frac{d_{i(k+1)}}{|d_{i(k+1)}|} \cdot |z_i| \qquad \text{if } g_i > 0 .$$

Alternatively, if for this value of i, $x_{ik} \neq 0$, the receiver sets

$$x_{i(k+1)} = -x_{ik} \qquad \text{if } g_i \geqslant 0$$

or $$x_{i(k+1)} = x_{ik} \qquad \text{if } g_i < 0 .$$

The (k+1)th process of selection and subtraction generates a new set of output signals $\{d_{i(k+2)}\}$ and the receiver repeats the above procedure on these, and so on.

When, for the selected value of i, $g_i = 0$ and $x_{ik} = 0$, the receiver checks to see if there is another value of i for which $g_i = 0$ and $x_{ik} \neq 0$. If so, the receiver proceeds with this value of i. If not, or alternatively if $g_i < 0$, the detection process is automatically terminated.

Clearly, any of the detection processes 1 to 4 may be combined with any one of the constraints I, J or K, to give altogether 12 different detection processes. These will be known by the combination of the two appropriate symbols, for instance 3J.

Arrangement H

An important modification which may be applied to the eight detection processes using the constraint I or J, and in particular to the detection processes 1 and 2, is to arrange that the individual signal subtracted from the input in a process of subtraction, is multiplied by a positive constant h, where $h \neq 1$. Thus in the process 2I the subtracted signal becomes $h\, d_{ik}Y_i$ for all i and k, instead of $d_{ik}Y_i$. In the process 2J the total subtracted signal, $x_{ik}Y_i$, must still satisfy the equation 89, so that for $h \gg 1$ the process approaches 2K. Similarly for the other processes involving the constraint J. Any process with the constraint I or J in which the subtracted signal is multiplied by a constant h, where $h \neq 1$, will have the letter H added to the symbols identifying the system. Thus the modified detection process 2I, mentioned above, will be known as the process 2IH. Where a general detection process such as 2IH is mentioned, this will for simplicity be taken to include the special case where $h = 1$.

The detection processes, listed in the order of increasing equipment complexity, are 2, 1, 4 and 3, the arrangements 1 and 2 being considerably less complex than 3 and 4. For any particular detection process, the equipment complexity does not appear to be very seriously affected by which of the constraints I, J or K is used. The use of the arrangement H should have a negligible effect on equipment complexity. In a typical application of RADAS all these detection processes should be substantially less complex than the equivalent arrangement of Fig. 11. This is because in these systems the addition or removal of any correlation detector together with its associated circuits, does not affect the circuits associated with the other correlation detectors, each group of circuits being effectively independent of the others.

4.3.3 Convergence of the Detection Processes 1IH, 2IH and 4I

Consider the start of the (k+1)th detection cycle of the process 1IH, with m received signals $\{z_iY_i\}$, where the element addresses $\{Y_i\}$ are v-component real unit-vectors. The output signal from the i th correlation detector is $d_{i(k+1)}$ and the total i th signal already subtracted from the input is

$$x_{ik}Y_i = \sum_{j=1}^{k} h\, d_{ij}Y_i \text{ , for all i,} \tag{100}$$

where h is a positive constant. Thus the resultant input signal to the m correlation detectors is

$$R - \sum_{i=1}^{m} x_{ik}Y_i = R - YX_k \text{ ,} \tag{101}$$

where $$R = S + N = \sum_{i=1}^{m} z_i Y_i + N, \qquad (102)$$

as before. Y is the v×m matrix whose i th column is Y_i, and X_k is the m-component column-vector $[x_{ik}]$. The corresponding signal-vector at the outputs of the m correlation-detectors is

$$D_{k+1} = Y^T(R - YX_k) = D - AX_k, \qquad (103)$$

where D_{k+1} is the m-component column-vector $[d_{i(k+1)}]$, $D = D_1 = Y^T R$ is the correlation-detector output-vector at the start of the iterative detection process, and $A = [a_{ij}] = Y^T Y$ as before.

At the end of the (k+1)th detection cycle,

$$X_{k+1} = X_k + h\,D_{k+1} = X_k + h(D - AX_k), \qquad (104)$$

so that $$X_{k+1} = (I - hA)X_k + hD, \quad k \geqslant 0, \qquad (105)$$

where I is the m×m identity matrix, and $X_o = 0$.

In the special case where h = 1, the process 1IH reduces to the process 1I and equation 105 becomes

$$X_{k+1} = (I - A)X_k + D, \quad k \geqslant 0. \qquad (106)$$

Equation 106 represents the point Jacobi iterative method applied to the solution of the linear simultaneous equations AX = D, in the special case where $a_{ii} = 1$ for all i.[A12] (I - A) is the point Jacobi matrix associated with the matrix A. Equation 105 represents the point Jacobi method with overrelaxation.

Consider now the start of the i th individual process of subtraction in the (k+1)th detection cycle of the process 2IH. For j = 1, ..., (i-1), the total j th signal subtracted from the input is

$$x_{j(k+1)} Y_j = \sum_{\ell=1}^{k+1} h\, d_{j\ell} Y_j, \qquad (107)$$

where $d_{j\ell}$ is the output signal from the j th correlation detector, immediately preceding the ℓ th subtraction of the j th signal. For j = i, ..., m, the total j th signal subtracted from the input is

$$x_{jk} Y_j = \sum_{\ell=1}^{k} h\, d_{j\ell} Y_j. \qquad (108)$$

Thus the resultant input signal to the m correlation detectors at this moment is

$$R - \sum_{j=1}^{i-1} x_{j(k+1)}Y_j - \sum_{j=i}^{m} x_{jk}Y_j \,, \tag{109}$$

giving an output signal from the i th detector,

$$\begin{aligned} d_{i(k+1)} &= Y_i^T\left(R - \sum_{j=1}^{i-1} x_{j(k+1)}Y_j - \sum_{j=i}^{m} x_{jk}Y_j\right) \\ &= d_i - \sum_{j=1}^{i-1} a_{ij}x_{j(k+1)} - \sum_{j=i}^{m} a_{ij}x_{jk} \,, \end{aligned} \tag{110}$$

where $d_i = d_{i1} = Y_i^T R$ and $A = \left[a_{ij}\right] = Y^T Y$, as before.

The signal subtracted from the input in the i th subtraction process of the (k+1)th detection cycle, is $hd_{i(k+1)}Y_i$, where $d_{i(k+1)}$ is given by equation 110. Thus

$$\begin{aligned} x_{i(k+1)} &= x_{ik} + h\, d_{i(k+1)} \\ &= x_{ik} + h\left(d_i - \sum_{j=1}^{i-1} a_{ij}x_{j(k+1)} - \sum_{j=i}^{m} a_{ij}x_{jk}\right) . \end{aligned} \tag{111}$$

Let A = I - B - C, where I is the identity matrix and B and C are respectively strictly lower and upper triangular m×m matrices whose non-zero components are the negatives of the corresponding components in A. Then from equation 111,

$$X_{k+1} = X_k + h\left(D + BX_{k+1} + (C-I)X_k\right) \tag{112}$$

or $$(I-hB)X_{k+1} = \left((1-h)I + hC\right)X_k + hD \,. \tag{113}$$

Since (I-hB) is non-singular for all h,

$$X_{k+1} = (I-hB)^{-1}\left((1-h)I + hC\right)X_k + h(I-hB)^{-1}D \,, \quad k \geqslant 0 \,, \tag{114}$$

where $D = D_1$ and $X_o = 0$, as before.

Equation 114 represents the point successive overrelaxation method or the extrapolated Gauss-Seidel method, applied to the solution of the linear simultaneous equations $AX = D$, in the special case where $a_{ii} = 1$ for all i.[A12]

$$(I - hB)^{-1} . \left((1-h)I + hC \right)$$

is the point successive relaxation matrix.

In the special case where $h = 1$, the detection process 2IH reduces to the process 2I. Equation 114 now becomes

$$X_{k+1} = (I-B)^{-1} C X_k + (I-B)^{-1} D , \qquad k \geqslant 0 . \qquad (115)$$

This is the point Gauss-Seidel iterative method, corresponding to the successive overrelaxation method of equation 114.[A12] $(I-B)^{-1}C$ is the point Gauss-Seidel matrix associated with the matrix A.

It can similarly be shown that the process 4I is the Gauss-Southwell relaxation method,[G7] where this is applied to the solution of the simultaneous equations $EV = D$. E is the $m \times m$ matrix $\left[e_{ij} \right]$ where

$$e_{ij} = \left| z_i \right| Y_i^T Y_j \left| z_j \right| \qquad (116)$$

and V is the m-component column-vector $\left[v_i \right]$. Where a unique solution-vector V exists and in the absence of noise, $\left| v_i \right| = 1$ for all i.

Each of the equations 105, 106, 114 and 115 is the well known representation of the matrix equation $AX = D$ in the corresponding iterative form:-

$$X_{k+1} = J^{-1} K X_k + J^{-1} D , \qquad k \geqslant 0 , \qquad (117)$$

where $A = J - K$. It is well known that the vector X_{k+1} in equation 117 will converge to the solution vector X in the equation $AX = D$, as $k \to \infty$, if and only if the spectral radius of $J^{-1}K$ is less than unity.[A12]

An important theorem by Ostrowski states that, if $A = F - G - G^*$ is an $m \times m$ Hermitian matrix, where F is Hermitian and positive definite, and $(F - hG)$ is non-singular for $0 \leqslant h \leqslant 2$, then the spectral radius of

$$(F - hG)^{-1} . \left((1 - h)F + hG^* \right)$$

is less than unity if and only if A is positive definite and $0 < h < 2$.[A12]

Except where otherwise stated, it is assumed throughout Section 4.0 that the m address-vectors $\{ Y_i \}$ are linearly independent. Clearly $m \leq v$ and the vxm matrix Y is real and of rank m. Since $A = Y^T Y$, A is a real symmetric positive-definite matrix. Also $A = I - B - C$ where $C = B^T = B^*$, and of course I is a real symmetric positive-definite matrix. Furthermore, $I - hB$ is non-singular for all h. Thus by Ostrowski's theorem, the spectral radius of

$$(I - hB)^{-1}. \Big((1-h)I + hC \Big)$$

in equation 114, is less than unity if and only if $0 < h < 2$.

It follows that the iterative detection process 2IH will converge to give the solution vector X for the equation $AX = D$, so long as $0 < h < 2$, and clearly the process 2I will converge. Provided that a sufficient number of detection cycles are used, the detection process 2IH or 2I performs the linear transformation A^{-1} on the vector D, so that it performs the same function as the network A^{-1} in Fig.11 (Section 4.1) and may therefore be used in place of this.

It has been shown that the Gauss-Southwell relaxation process, when applied to the solution of a set of linear simultaneous equations $EV = D$, will converge to the solution so long as the matrix E is symmetric and positive definite.[G7] From equation 116, E is real, symmetric and positive-definite, since the $\{ Y_i \}$ are real and linearly independent. Thus the detection process 4I will converge.

If a matrix A is a strictly or irreducibly diagonally dominant mxm matrix, then both the associated point Jacobi and point Gauss-Seidel matrices are convergent, and the iterative methods given by equations 106 and 115 for the matrix equation $AX = D$, are convergent.[A12]

The conditions which must be satisfied by the element-addresses $\{ Y_i \}$, to ensure that the matrix A is strictly diagonally dominant, are derived in Appendix 6. Under these conditions either of the detection processes 1I or 2I will converge. Adaptive coding of the type described in Section 3.4 can be used to generate the m transmitted signals here.

In any of these various detection processes which converge, the correlation-detector output signal vector, D_k, tends to zero as k increases, so that $AX_k \rightarrow D$.

4.3.4 Detection Processes 5 to 7

There is another class of iterative detection processes, based on the method of Kaczmarz for solving linear simultaneous equations.[G7] These are closely related to the processes just described, although derived in a very different manner.

Consider the set of m simultaneous equations $AX = D$, where $A = \left[a_{ij}\right]$ is an $m \times m$ non-singular matrix which need not now be symmetric or positive definite. The equations can be considered as a set of m hyperplanes in m-dimensional Euclidean space. Since A is non-singular, there is a unique solution vector U for which $AU = D$. This vector is given by the point of intersection of the m hyperplanes.

The i th hyperplane is given by

$$\sum_{j=1}^{m} a_{ij} x_j = d_i \; . \qquad (129)$$

Consider any vector $P = \left[p_j\right]$ and drop a perpendicular from this point onto the i th hyperplane, to give at the foot of the perpendicular the vector $Q = \left[q_j\right]$. Since the vector Q-P is perpendicular to the hyperplane,

$$q_j - p_j = g\, a_{ij} \, , \quad \text{for all } j \, , \qquad (130)$$

where g is a constant.

Also

$$\sum_{j=1}^{m} a_{ij} q_j = d_i \, , \qquad (131)$$

since Q lies on the hyperplane.

Thus

$$g = \frac{d_i - \sum_{j=1}^{m} a_{ij} P_j}{\sum_{j=1}^{m} a_{ij}^2} \; . \qquad (132)$$

Given the vector P, the vector Q can be determined from equation 130.

The triangle whose vertices are given by P, Q and U, is a right-angled triangle, so that the hypotenuse U - P is longer than the side U - Q. The vector Q must therefore always be closer to the solution vector U, than P, unless of course P lies on the i th hyperplane, in which case P and Q coincide.

In the method of Kaczmarz, an arbitrary point P is first selected, and the point is projected onto the first hyperplane. The resulting point is projected onto the second hyperplane, and so on in cyclic order, until a sufficiently close approximation to the solution vector U is obtained.[G7]

This iterative process has the advantage over the Gauss-Seidel process that it does not require the matrix A to be symmetric and positive definite, in order to ensure convergence to the required solution vector. It is only necessary that A be non-singular.

Detection Process 5

Consider first the constraint I. The detection process is an application of the method of Kaczmarz to the basic system represented by Fig. 11. The iterative process here performs the linear transformation A^{-1} on the output signal-vector D, to obtain the solution vector X of the equation AX = D. As in the detection process 2I, the output signals of the m correlation detectors are selected and reduced to zero sequentially and in a fixed cycle. The i th correlation detector has a reference signal Y_i.

At the start of the (k+1)th individual process of selection and subtraction, YX_k is the total signal subtracted from R at the input to the correlation detectors. As before, Y is the v×m matrix whose i th column is the unit address-vector Y_i. Thus the resultant input signal is $R-YX_k$, giving an output signal-vector D_{k+1} from the m correlation detectors, where

$$D_{k+1} = Y^T(R - Y X_k) = D - A X_k . \qquad (133)$$

$D = D_1 = Y^TR$ and $A = \left[a_{ij}\right] = Y^TY$, as before.

The output signal from the i th correlation detector is now

$$d_{i(k+1)} = d_i - \sum_{j=1}^{m} a_{ij}x_{jk} . \qquad (134)$$

From equations 130 and 132, the new estimate X_{k+1} of the solution vector X, is

$$X_{k+1} = X_k + \frac{d_{i(k+1)}}{\sum_{j=1}^{m} a_{ij}^2} \cdot A_i , \qquad (135)$$

where A_i is the m-component column-vector given by the i th column of A^T or A. The new total signal subtracted from R at the input to the detectors, is now YX_{k+1}.

In the process of subtraction associated with the i th correlation detector, signal vectors corresponding to all the m received signals are subtracted from the input, the coefficient $w_{\ell(k+1)}$ of the ℓ th signal-vector $w_{\ell(k+1)}Y_\ell$ subtracted, being

$$w_{\ell(k+1)} = x_{\ell(k+1)} - x_{\ell k} = \frac{d_{i(k+1)}a_{i\ell}}{\sum_{j=1}^{m} a_{ij}^2} . \qquad (136)$$

The constraint J or K may be used here instead of I. Alternatively the constraint IJ or IK may be used.

Constraints IJ and IK

The appropriate constraint J or K is applied to X_k, but only at the end of each complete detection cycle which involves all m correlation detectors. The value of X_k after intermediate processes of selection and subtraction is allowed to adopt any value, as with the constraint I.

The detection process 5I should converge to the required solution vector, but it clearly involves considerable equipment complexity. It has the disadvantage relative to the detection processes 1 to 4, that the circuits associated with any correlation detector are no longer independent of the other detectors.

Detection Process 6

Consider first the constraint I. The process applies Kaczmarz's method directly to the received signals, without the use of correlation detectors. The simultaneous equations to be solved here are given by YX = R, where Y is the v×m matrix $[y_{ji}]$, whose i th column is the unit address-vector Y_i, as before, and R is the v-component column-vector $[r_j]$. X is the solution vector and has m components. The receiver samples the first component of the received vector R and generates the signals w_iY_i for i = 1, ..., m, which are added together and then subtracted from R to give a zero first component. $\{w_i\}$ are scalar values. The process is repeated for the second component of R and so on cyclically.

Consider the (k+1)th individual process of subtraction and suppose that this operates upon the j th component of R. Just before the subtraction, YX_k is the total signal subtracted from R at the input to the correlation detectors, so that the value of the j th component, $d_{j(k+1)}$, of the resultant input signal-vector D_{k+1} is

$$d_{j(k+1)} = r_j - \sum_{i=1}^{m} y_{ji}x_{ik} . \qquad (137)$$

From equations 130 and 132, the new estimate X_{k+1} of the solution vector X, is

$$X_{k+1} = X_k + \frac{d_{j(k+1)}}{\sum_{i=1}^{m} y_{ji}^2} \cdot V_j , \qquad (138)$$

where V_j is the m-component column-vector given by the j th column of Y^T.

In the process of subtraction giving X_{k+1}, the coefficient $w_{\ell(k+1)}$ of the ℓ th signal-vector $w_{\ell(k+1)}Y_\ell$ subtracted, for $\ell = 1, \ldots, m$, is

$$w_{\ell(k+1)} = x_{\ell(k+1)} - x_{\ell k} = \frac{d_{j(k+1)}y_{j\ell}}{\sum_{i=1}^{m} y_{ji}^2} . \qquad (139)$$

This is clearly not an optimum detection process and in the presence of noise the v hyperplanes here will not in general intersect at a point, contrary to the situation in the detection process 5. When v > m there are more simultaneous equations than there are unknown variables $\{ x_i \}$, so that in the presence of noise there are in general inconsistencies between the v equations. Nevertheless, as k increases the vector X_k converges to the vicinity of the optimum solution vector X. Furthermore in the particular case where all the received signals have equal levels and have address vectors $\{ Y_i \}$ all of whose components have equal magnitudes, the arrangement becomes no more complex than the detection process 2.

Any of the constraints J, K, IJ or IK may be used here in place of I.

Detection Process 7

This is a modification of the process 6. The v components of the resultant input vector D_k are selected and subtracted sequentially, but not necessarily in the same order in any two detection cycles. In the first process of selection and subtraction in any detection cycle, the receiver determines which of the v components of D_k has the largest magnitude and selects this component for the process of subtraction. After the subtraction it determines which of the remaining v-1 components has the largest magnitude and selects this for the next process of subtraction, and so on until all v components have been subtracted. The detection cycle is then repeated as often as required.

Any of the constraints I, J, K, IJ or IK may be used here.

4.4 Iterative Detection Processes using Small Steps

4.4.1 Convergence of Detection Process 1IH

In the detection processes considered in Section 4.3, where these do not involve overrelaxation, it is assumed that the magnitude of the subtracted signal in an individual process of subtraction, is such as to reduce to zero the output signal of the appropriate correlation detector, in the absence of any constraint. Where overrelaxation is used, its purpose is normally to increase the rate of convergence and with this in view the subtracted signal is usually greater than that used in the absence of overrelaxation. In all these cases the iterative process can be considered to use large steps.

Consider now the detection process 1IH, as given by equation 105 in Section 4.3.3:-

$$X_{k+1} = (I - hA)\,X_k + hD\ , \qquad k \geqslant 0\ . \tag{140}$$

This iterative process converges to the required solution vector X if and only if

$$\rho(I - hA) \quad < \quad 1 \tag{141}$$

where $\rho(I - hA)$ is the spectral radius of $I - hA$.[A12]

In equation 140, $A = \left[a_{ij}\right] = Y^TY$, where Y is a v×m matrix whose i th column is given by the real unit-vector Y_i. The m address-vectors $\{Y_i\}$ are assumed to be linearly independent. Thus A is an m×m real symmetric positive-definite matrix, where $|a_{ii}| = 1$ for all i and $|a_{ij}| < 1$ for all $i \neq j$. All the eigenvalues of A are positive.

Let P be the orthogonal matrix such that

$$P^{-1}A\,P \;=\; Q\ , \tag{142}$$

where $Q = [q_{ij}]$ is a real diagonal matrix. Clearly $q_{ii} > 0$ for all i.

Also $$P^{-1}(I - hA)P = I - hQ = U, \tag{143}$$

where U is a real diagonal matrix.

Since the spectral radius of any matrix is invariant under similarity transformations,

$$\rho(Q) = \rho(A) \tag{144}$$

and $$\rho(U) = \rho(I - hA). \tag{145}$$

But $$\sum_{j=1}^{m} |a_{ij}| < m, \quad \text{for all } i, \tag{146}$$

so that $$\rho(A) < m, \tag{147}$$

by a well known theorem.[A12]

Thus $$\rho(Q) < m. \tag{148}$$

Since $q_{ii} > 0$ for all i and $U = I - hQ$, a sufficient condition to ensure that $\rho(U) < 1$ is that

$$h \leqslant \frac{2}{m}. \tag{149}$$

From equations 141 and 145, this is therefore also a sufficient condition to ensure the convergence of the iterative detection process 1IH.

Since $m \leqslant v$, the condition that the process 1IH will always converge is given by

$$h \leqslant \frac{2}{v}. \tag{150}$$

Normally $v \gg 1$, so that if equation 150 is satisfied, $h \ll 1$ and the iterative process uses small steps.

4.4.2 Detection Processes 8 to 14

Detection Process 8

Because of its great importance, the detection process 1IH in which $h \leqslant \frac{2}{v}$ will be known as the process 8I.

This process can be modified, as in the previous cases, so that the total i th signal $x_{ik}Y_i$, subtracted from the input, is constrained to satisfy

$$|x_{ik}| \leq |z_i|, \text{ for all i and k.} \qquad (151)$$

This is the detection process 8J. There is no detection process 8K.

Detection Process 9

This is a modification of the detection process 8, in which h is no longer a constant but a function of the number of the detection cycle.

The value of h in the k th detection cycle is

$$h_k = \frac{1}{g - \ell + 1}, \qquad (152)$$

where $$\ell = k - ng \leq g. \qquad (153)$$

g, ℓ and n are positive integers, and g is a constant.

The whole process repeats itself every $g = \frac{1}{h_1}$ cycles. In each detection cycle all signals are selected and subtracted simultaneously.

Where no constraints are applied to the vector X_k, this is the detection process 9I. Where the constraint J is applied, it is the process 9J. Where the constraint K is applied, but only at the end of each set of $\frac{1}{h_1}$ detection cycles, it is the process 9IK.

Detection Process 10

This is a modification of the detection process 8. Let the i th individual signal subtracted in the k th detection cycle be $w_{ik}Y_i$, where w_{ik} is a scalar. If the output signal from the i th detector, immediately preceding the subtraction, is d_{ik}, and $|d_{ik}| < e|z_i|$, where e is a positive constant, then $w_{ik} = 0$. If $|d_{ik}| \geq e|z_i|$, then

$$w_{ik} = \pm c|z_i|, \qquad (154)$$

the sign taken for w_{ik} being the same as that of d_{ik}. c is a positive constant such that $c \ll 1$. When $e = 0$ and $d_{ik} = 0$, w_{ik} is selected at random from its two possible values $\pm c|z_i|$.

In each detection cycle all signals are selected and subtracted simultaneously. At the end of the k th detection cycle, the total i th signal subtracted from the input is

$$x_{ik}Y_i = (x_{i(k-1)} + w_{ik})Y_i, \text{ for all i and k,} \qquad (155)$$

so that $$x_{ik} = x_{i(k-1)} + w_{ik}, \text{ for all i and k.} \qquad (156)$$

Where no constraints are applied to the vector X_k, this is the detection process 10I. Where the constraint J is applied, it is the process 10J. Where the constraint K is applied, but only at the end of each set of $\frac{1}{c}$ detection cycles, it is the process 10IK.

Detection Process 11

This is a modification of the detection process 10. Instead of all signals being selected and subtracted simultaneously in each detection cycle, the signals are selected and subtracted sequentially and in a fixed cycle. The detection process is otherwise as described for the process 10. The arrangement can alternatively be regarded as a modification of the detection process 2K.

Where no constraints are applied to the vector X_k, this is the detection process 11I. Where the constraint J is applied, it is the process 11J.

Detection Process 12

This is a modification of the detection process 11. The ith correlation detector now has a reference signal $|z_i|Y_i$ instead of Y_i used in 10 and 11. At the start of the k th detection cycle, the receiver determines the value of i for which $|d_{ik}|$ is maximum and subtracts the corresponding signal from the input. The receiver then determines the value of i for which $|d_{ik}|$ is maximum among the remaining m-1 detectors, and subtracts the appropriate signal, and so on until all m signals have been subtracted. Thus the signals are selected sequentially and cyclically, but not necessarily in the same order in any two cycles. The value of the subtracted signal is determined as in 10, but the threshold level with which $|d_{ik}|$ is compared, is ez_i^2. $w_{ik} = 0$ or $\pm c|z_i|$. The constraint I or J may be applied here, as for 11.

Detection Process 13

This is a modification of the detection process 12. The signals are selected sequentially but not cyclically. In the k th individual process of selection and subtraction, the receiver determines the value of i for which $|d_{ik}|$ is maximum, and subtracts the corresponding signal, which is determined as in 12. Each selection is made from all m detectors, and the process of selection and subtraction is repeated as often as required. As in 12, the i th correlation detector has a reference signal $|z_i| Y_i$.

The arrangement may be regarded as a modification of the detection process 4K. The constraint I or J may be applied here.

Detection Process 14

This is a modification of the detection process 11. As for the detection processes 8 to 11, the i th correlation detector has a reference signal Y_i, for all i. The value of w_{ik} in the k th process of subtraction of the i th signal, is determined exactly as for the detection processes 10 and 11, and w_{ik} is added to $x_{i(k-1)}$, as before. Immediately following the addition, the sign of the resultant signal $\left(x_{i(k-1)} + w_{ik}\right)$ is determined and to this signal is added the signal

$$u_{ik} = \pm b \left| z_i \right| \tag{157}$$

where b is a positive constant such that $b \ll c$. The sign taken for u_{ik} is the same as that of $\left(x_{i(k-1)} + w_{ik}\right)$, the sign being chosen at random when $x_{i(k-1)} + w_{ik} = 0$. Thus

$$x_{ik} = x_{i(k-1)} + w_{ik} + u_{ik}\,, \quad \text{for all i and k.} \tag{158}$$

Where no constraints are applied to the vector X_k, this is the detection process 14I. Where the constraint J is applied, it is the process 14J.

The interesting property of each of the detection processes 10 to 14, is that these are entirely digital, in the sense that they involve only "yes-no" decisions in determining the signals to be subtracted. If the present steady reduction in the cost of integrated circuits and equivalent devices continues, the cost of making such receivers may in the future compare favourably with that of any of the detection processes 1I to 9I and 1J to 9J.

Although a simple theoretical analysis of the detection processes 10 and 11 indicates that these will converge for extremely small values of c (equation 154), it has not been found possible to show theoretically that convergence will be obtained for useful values of c. Computer simulation has proved to be a much more effective tool for studying these systems. The convergence of the detection processes 10 to 14 has therefore been examined in some detail by means of computer simulation, and is discussed in Section 5.9.

4.5 Continuous Detection Processes

4.5.1 Detection Process 15

The (k+1)th detection cycle of the detection process 8I is given by equation 105 in Section 4.3.3:-

$$X_{k+1} = (I - hA)X_k + hD \,, \qquad k \geqslant 0 \,, \tag{159}$$

so that
$$X_{k+1} - X_k = h(D - AX_k) = hD_{k+1} \,, \tag{160}$$

from equation 103 in Section 4.3.3.

Let $h \to 0$, $X_{k+1} - X_k = \delta X$, $X_k = X$ and $D_{k+1} = E$, where $X = \left[x_i\right]$ and $E = \left[e_i\right]$ are functions of k.

Then
$$\delta X = hE \,. \tag{161}$$

δX is the vanishingly small change in X determined by the value of hE. The subsequent change in E, resulting from the change δX in X, is also vanishingly small.

Consider now the continuous process in which for each i a current ge_i is fed to a storage capacitor whose voltage gives the value of x_i. g is a positive constant. Assuming that all m storage capacitors have the same capacity,

$$\dot{X} = \ell E \,, \tag{162}$$

where ℓ is a positive constant. There are m received signals, as before.

Thus
$$\delta X \simeq \ell \, \delta t \, E = hE \,, \tag{163}$$

where $\ell \, \delta t = h$. This agrees with equation 161.

At any instant during the detection process, the total signal subtracted from R at the input is

$$\sum_{i=1}^{m} x_i Y_i = YX \,,$$

so that the resultant input signal is R - YX. Y_i and Y are as defined at the beginning of Section 4.3.3.

The continuous detection process clearly represents the limiting case of the iterative detection process 8I, as $h \to 0$. Since 8I has been shown to converge for all sufficiently small values of h, it follows that the continuous process will also converge.

The continuous detection process just considered has no constraints

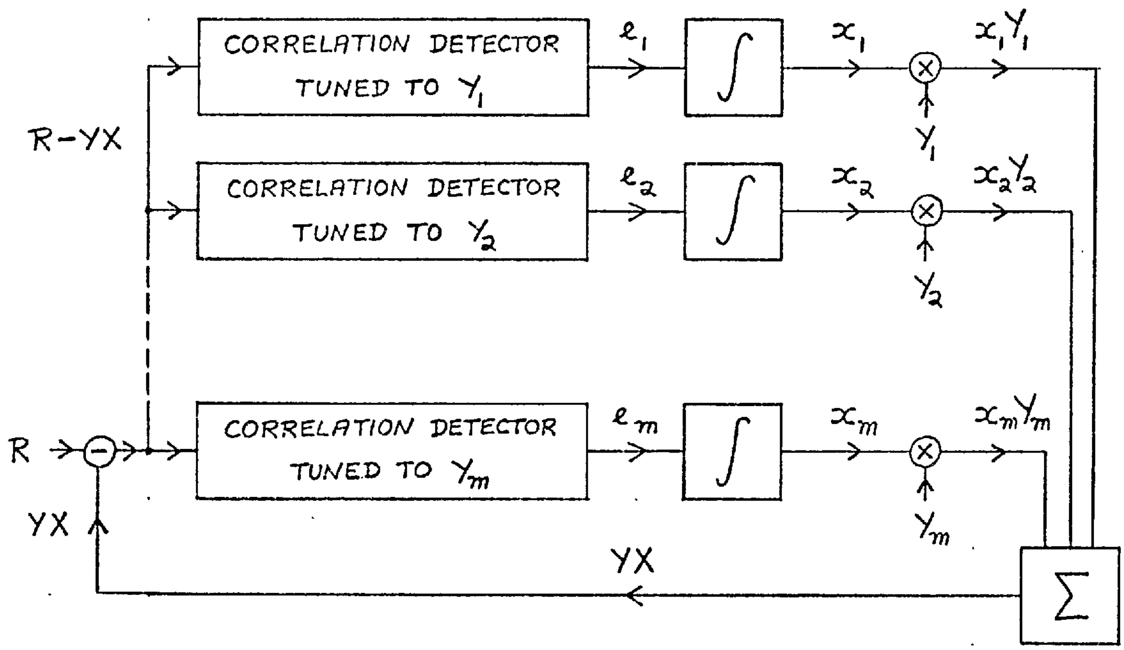


FIG. 12. DETECTION PROCESS 15I.

applied to X and is the process 15I. Where the constraint J is applied, it is the process 15J. The arrangement of 15I is shown in Fig. 12.

At the start of an element detection process, the vector X is set to zero and the received vector R is fed to the input, so that

$$E = Y^T R = D . \qquad (164)$$

At the end of the element detection process, E = 0 and X is the solution vector of the equation AX = D. During the detection process,

$$E = Y^T(R - YX) = D - AX . \qquad (165)$$

It follows from equation 162 that

$$\dot{X} = \ell E = \ell(D - AX) \qquad (166)$$

or

$$\frac{1}{\ell}\dot{X} + AX = D . \qquad (167)$$

Equation 167 describes a well known analogue method for the solution of the linear matrix equation AX = D.[G18,G19] A commonly used analogue method involves high-gain amplifiers in feedback loops with heavy negative feedback.[G2] Such systems are however often difficult to stabilize.[G9,G29] The use of integrators as in Fig. 12, not only removes the need for high-gain amplifiers but serves in addition to mask the characteristics of any amplifiers in the feedback loop.[G9] No difficulty should therefore be experienced in stabilizing the arrangement of Fig. 12, provided only that the matrix A is positive definite.

The circuits for the detection processes 1 to 4 and 8 to 14 are the same as that of 15I in Fig. 12, except that the vector E becomes D_{k+1} and X becomes X_k, and the m integrators are replaced by the appropriate m circuits which determine x_{ik} from d_{ik}, for $i = 1, \ldots, m$. 15I and 15J therefore appear to involve considerably less complex equipment than the other detection processes. In all these arrangements the circuits associated with any correlation detector are independent of any other correlation detector or its associated circuits. Thus when a new call is set up or an existing call is broken down, this only involves the addition or removal of the appropriate correlation detector and its associated circuits, without affecting the other circuits. Such an arrangement is ideally suited to RADAS.

4.5.2 Convergence of Detection Process 15

Consider first the detection process 15I. As before let the received vector be

$$R = S + N , \tag{168}$$

where
$$S = \sum_{i=1}^{m} z_i Y_i = YZ . \tag{169}$$

Y is the real v×m matrix $\left[y_{ij}\right]$ of rank m, whose j th column is the unit address-vector Y_j. Z is the m-component column-vector $\left[z_i\right]$ and N is the v-component column-vector $\left[n_i\right]$.

During the detection process,

$$R - YX = Y(Z - X) + N = G , \tag{170}$$

where the vector G is the error in the estimate of R.

The square of the length of the input error vector G is

$$\begin{aligned} f &= G^TG = \left(Y(Z-X) + N\right)^T.\left(Y(Z-X) + N\right) \\ &= (Z-X)^TY^TY(Z-X) + N^TY(Z-X) + (Z-X)^TY^TN + N^TN \\ &= (Z-X)^TA(Z-X) + 2(Z-X)^TY^TN + N^TN \\ &= Z^TAZ - Z^TAX - X^TAZ + X^TAX + 2Z^TY^TN - 2X^TY^TN + N^TN , \end{aligned} \tag{171}$$

where $A = \left[a_{ij}\right]$ is the real symmetric positive-definite matrix Y^TY.

A, Y, Z and N are constants during the element detection process.

$$\frac{\partial f}{\partial x_i} = -\sum_{j=1}^{m} z_j a_{ji} - \sum_{j=1}^{m} a_{ij} z_j + \sum_{\substack{j=1\\ j\neq i}}^{m} x_j a_{ji} + \sum_{\substack{j=1\\ j\neq i}}^{m} a_{ij} x_j + 2a_{ii} x_i - \sum_{j=1}^{v} 2n_j y_{ji}$$

$$= -2\left(\sum_{j=1}^{m} a_{ij}(z_j - x_j) + \sum_{j=1}^{v} n_j y_{ji}\right), \text{ for all } i. \quad (172)$$

From equations 171 and 172, f is positive definite and is a unimodal function of the $\{x_i\}$. At the minimum of f,

$$\frac{\partial f}{\partial x_i} = 0 \quad \text{for all } i, \quad (173)$$

so that $$\sum_{j=1}^{m} a_{ij}(z_j - x_j) + \sum_{j=1}^{v} n_j y_{ji} = 0 \quad \text{for all } i, \quad (174)$$

or $$A(Z - X) + Y^T N = 0. \quad (175)$$

But $$A(Z - X) + Y^T N = Y^T(YZ + N) - AX$$

$$= Y^T R - AX = D - AX. \quad (176)$$

Thus at the minimum of f, $AX = D$.

From equations 166 and 176,

$$\dot{X} = \ell(D - AX) = \ell\left(A(Z - X) + Y^T N\right), \quad (177)$$

so that

$$\frac{dx_i}{dt} = \ell\left(\sum_{j=1}^{m} a_{ij}(z_j - x_j) + \sum_{j=1}^{v} n_j y_{ji}\right), \text{for all } i. \quad (178)$$

From equations 172 and 178,

$$\frac{dx_i}{dt} = -\frac{\ell}{2}\cdot\frac{\partial f}{\partial x_i}, \quad \text{for all } i. \quad (179)$$

Thus $$\frac{df}{dt} = \sum_{i=1}^{m}\left(\frac{\partial f}{\partial x_i}\cdot\frac{dx_i}{dt}\right) = -\frac{\ell}{2}\sum_{i=1}^{m}\left(\frac{\partial f}{\partial x_i}\right)^2 \leqslant 0. \quad (180)$$

Since $\frac{\partial f}{\partial x_i} = 0$ for all i, only at the minimum of f, f must eventually be reduced to its minimum value, at which $AX = D$.[A14] Furthermore, it follows from equation 179 that the variables $\{x_i\}$ will approach their values at the minimum of f, along the direction of steepest descent with respect to the function f.[A13,G7] As shown in Section 4.1, the solution vector X given by $AX = D$, is the optimum estimate of Z, in the case where the receiver has no prior knowledge of Z and where the received signals are in the presence of additive white gaussian noise whose level is not known.

In the detection process 15J, the values of $\{x_i\}$ are constrained so that at all times

$$|x_i| \leqslant |z_i| \text{ , for all } i . \tag{181}$$

From equation 172,

$$\frac{\partial^2 f}{\partial x_i \partial x_j} = 2a_{ij} \text{ , for all } i \text{ and } j . \tag{182}$$

From equations 172 and 182 it can be shown that f is linearly unimodal (unimodal along any straight line) in the m-dimensional Euclidean space whose m axes represent the $\{x_i\}$. It follows that if the function f is confined to the values of the $\{x_i\}$ which satisfy equation 181, it is still linearly unimodal.

During a period of time when certain of the $\{x_i\}$ are held constant by the constraints of equation 181, $\frac{dx_i}{dt} = 0$ for all corresponding values of i. Suppose that m - k of the $\{x_i\}$ are held constant and re-number the remaining k x_i's, x_1 to x_k. From equation 180,

$$\frac{df}{dt} = -\frac{\ell}{2} \sum_{i=1}^{k} \left(\frac{\partial f}{\partial x_i} \right)^2 \leqslant 0 . \tag{183}$$

If the minimum of f has coordinates $\{x_i\}$ which are all within the constraints applied by equation 181, the system will clearly converge to the vector X for which $AX = D$, although the rate of convergence, $\frac{df}{dt}$, may at times be slower than before.

Consider now the case where the minimum of f has coordinates $\{x_i\}$ some of which are outside the constraints of equation 181. It is clear from equation 183 that the detection process always attempts to minimize the function

$$f = (R - YX)^T.(R - YX) . \tag{184}$$

Thus the vector YX, which is the estimate of the received vector R, is adjusted to have the minimum distance from R, within the constraints applied. These are of course that YX must lie in the subspace spanned by the m element-addresses $\{Y_i\}$ and that $|x_i| \leqslant |z_i|$ for all i. YX will always converge to the required vector V at the minimum distance from

R, since f is a unimodal function of the $\{x_i\}$ within the region bounded by the constraints on the $\{x_i\}$.

4.5.3 Relative Tolerance of 15I and 15J to Additive Gaussian Noise

A useful insight into the effects of the constraints I and J on the tolerance to additive gaussian noise, can be gained from the following simple example. Assume that two binary antipodal signals $Q_1 = z_1Y_1$ and $Q_2 = z_2Y_2$ are received in the presence of white gaussian noise. The possible positions of the individual signal-vectors Q_1 and Q_2 in the plane containing Y_1 and Y_2, are shown by the points marked $\odot$ in Fig. 13. Q_1 may be either U_1 or U_3 and Q_2 may be either U_2 or U_4. The four possible positions of the total received signal

$$S = Q_1 + Q_2 = z_1Y_1 + z_2Y_2 , \tag{185}$$

are shown by the points marked $\boxdot$. It is assumed that U_1 and U_2 correspond to the positive values of z_1 and z_2, and that Q_1 and Q_2 have equal levels, so that $|z_1| = |z_2|$. The total received vector, R = S + N, will not in general lie on the plane containing S, and the projection of R onto this plane is the vector P.

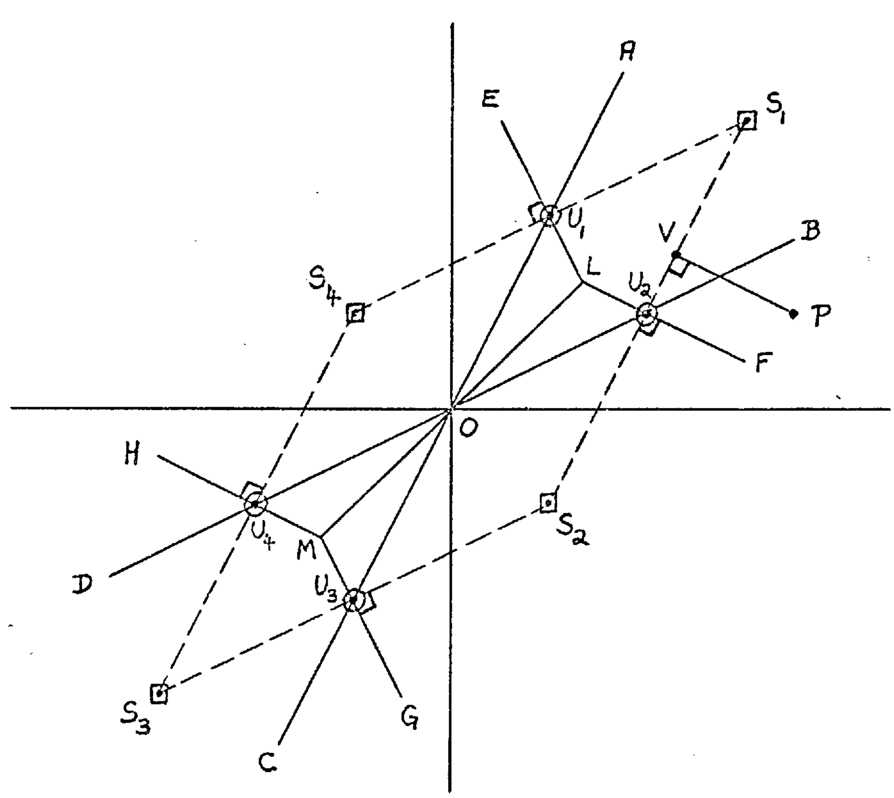


FIG. 13. DECISION REGIONS FOR THE DETECTION OF Q_1 AND Q_2.

The detection process 15I determines the values of x_1 and x_2 as those which satisfy the equation

$$x_1Y_1 + x_2Y_2 = P . \tag{186}$$

This is a process of linear filtering. The receiver then allocates to Q_1 and Q_2 the binary values corresponding to the signs of x_1 and x_2. The two dimensional signal-space is here divided into four decision regions whose boundaries are given by AOB, DOA, COD and BOC. For the particular position of P shown in Fig. 13, it follows from equation 186 that $x_1 < 0$ and $x_2 > 0$, so that the binary values allocated to Q_1 and Q_2 are "1" and "0" respectively. Thus R is detected as the particular $S_i (i=1,\ldots,4)$ in the decision region containing P. This is the optimum detection process in the case where the receiver has prior knowledge only of Y_1 and Y_2.

Suppose now that the receiver knows $|z_1|$ and $|z_2|$ exactly, but has no prior knowledge of their signs, which are assumed to be equally likely and statistically independent. In this case the optimum decision regions in Fig. 13 are separated by the boundaries ELF, HMLE, GMH and FLMG.[A8] As before, the receiver detects the received vector R as being the particular S_i $(i=1,\ldots,4)$ in the decision region containing P. Thus in Fig. 13, R is detected as S_1, so that $x_1 = +|z_1|$ and $x_2 = +|z_2|$, the binary value "0" being allocated to both Q_1 and Q_2.

When Q_1 and Q_2 are orthogonal (at right angles), the decision regions here are the same as those where linear filtering is used. However, when the angle between Q_1 and Q_2 approaches 0° or 180°, there is a considerable difference between the two sets of decision regions. In an arrangement of RADAS the angle between Q_1 and Q_2 would not normally be 90°, so that if the receiver knows $|z_1|$ and $|z_2|$, linear filtering would have a noticeably lower tolerance to additive white gaussian noise than would the optimum detection process just described.

Assuming that the receiver knows $|z_1|$ and $|z_2|$, the detection process 15J determines the vector V which is confined to the parallelogram $S_1S_2S_3S_4$ and which has the minimum distance from R and therefore also from P. When P is inside $S_1S_2S_3S_4$ this is clearly P itself, whereas when P is outside $S_1S_2S_3S_4$, it is the foot of the perpendicular from P onto the nearest side of $S_1S_2S_3S_4$. The detection process determines the values of x_1 and x_2 as those which satisfy the equation

$$x_1Y_1 + x_2Y_2 = V . \tag{187}$$

The four decision regions here are separated by the boundaries EU_1OU_2F, HU_4OU_1E, GU_3OU_4H and FU_2OU_3G. As before, R is detected as the particular $S_i (i=1,\ldots,4)$ in the decision region containing P. Thus in Fig. 13 R is detected as S_1 so that the binary value "0" is allocated to both Q_1 and Q_2.

The decision boundaries in the detection process 15J, where these are not the same as those in 15I, correspond more closely to those of the optimum detector. Thus where the received signal levels $|z_1|$ and $|z_2|$ are equal and are known by the receiver, the detection process 15J should give a better tolerance to additive white gaussian noise than the detection process 15I. It appears furthermore that this conclusion may be extended to the case where there are more than two received signals and probably also to the more general case where these do not have equal levels, although the detailed analysis here becomes difficult.

Since the detection processes 2I, 4I, 8I and 15I all converge to the vector X for which AX = D, these should all have the same tolerance to additive white gaussian noise. By a similar argument, if it is assumed that the detection processes 2J, 4J, 8J and 15J all converge, these should all have a given and in general somewhat better tolerance to additive white gaussian noise, in the case where the received signal levels are equal and are known by the receiver.

4.6 Other Iterative Detection Processes

The majority of the detection processes studied have been selected as those which lead to the simplest receiver design, on the assumption that the receiver is designed specifically to detect the m received signals. Where there is available at the receiver a general purpose computer which could be involved in the detection processes, then a very much wider range of iterative processes becomes available.[G] Of particular interest is the method of conjugate gradients,[G4,G26] and other equivalent arrangements.[G15,G20,G27,G32,G38,G39] These have the useful property that when the matrix A is symmetric and positive definite, the detection process involves only m iterations, one for each signal received. As before, it is assumed that the iterative process replaces the network A^{-1} in Fig. 11.

4.7 Arrangements for Setting-up and Breaking-down a Call

It has been assumed here that the receiver has prior knowledge, not only of the number of signals received, but also of the addresses of the received signals. Thus during the setting up of a call, the calling transmitter must inform the receiver of the element address of its transmitted signal.

When the receiver has an accurate knowledge of the received signal levels, the following arrangement can be used. To set up a call, a transmitter sends a continuous stream of the elements "0" having the address of the called subscriber. The signal is transmitted for a fixed total period and at a much lower level than that used for a signal during a call. No serious interference should therefore be caused in the other transmitted signals. At the end of each element detection process, when the receiver has determined the binary values of the different received signals, the receiver removes the total subtracted signal YX (see Fig. 12, Section 4.5.1) and replaces this by the signal $Y\hat{Z}$, which is then subtracted from R at the input to the detectors. The i th component of $\hat{Z}$ is $\pm |z_i|$, the sign being positive or negative depending upon whether the detected binary value of z_iY_i is "0" or "1" respectively. If there are no errors in the detected signals, the signal remaining at the detector input is the calling signal together with the noise vector. This signal is now fed to an "address detector", similar to that which follows the process of AM detection in the AM System (Section 3.5). Immediately the presence of an address is recognized here, a correlation detector having the given reference signal, together with the associated circuits, is automatically switched into operation. At the same time, the output signal from the circuits associated with the correlation detector, is fed to the appropriate receiver output terminal, and an acknowledgement signal, bearing the address just detected, is fed back to the transmitters. The calling transmitter will only begin transmitting at the full signal level, when it has detected this acknowledgement signal. When two transmitters attempt to set up a call simultaneously, neither address will in general be detected. By staggering the intervals used by different transmitters, before they attempt to set up a call the second time, it can be arranged that the two transmitters will not normally interfere with each other in the second attempt.

When the receiver does not have an accurate knowledge of the received signal levels, which would most often be the case, the above arrangement of signal cancellation cannot be used to isolate the calling signal from the others. In this case the calling signal must be transmitted over a separate channel. In other respects, however, this arrangement is similar to that previously described.

When a separate channel is not used for setting up a call, or where a detection process with the constraint K is used, a unique signal must be transmitted at the end of the call to indicate to the receiver the particular signal-element at the end of which the transmitted signal will be cut off. When this moment comes, the appropriate correlation detector and its associated circuits are disconnected.

When a separate channel is used for setting up a call, and with any detection process not using the arrangement K, it is not necessary for the receiver to disconnect the correlation detector and its associated circuits, at the exact instant when the received signal is cut off. No special signal need therefore be transmitted here to indicate the end of a call. This is now detected by monitoring the levels of the different x_i's at the end of each element detection process. When the average value of $|x_i|$ over say 100 successive elements is below a certain threshold level, for any value of i, it is assumed that the corresponding signal has been cut off, and the appropriate correlation detector and its associated circuits are disconnected. This is clearly the simplest arrangement.

5.0 COMPUTER SIMULATION OF THE DETECTION PROCESSES 1 TO 15

5.1 Introduction

For an iterative detection process to be of practical value, not only must the process converge to the required solution vector, but it must do so within a reasonable number of detection cycles. Since it appeared unlikely that a theoretical analysis of this problem, starting from known techniques,[A12,A13,A14] would lead to any very useful results for an application of RADAS, it was decided to study the problem by means of computer simulation.

The detection processes 1 to 15 have been tested under various conditions, the method of operation of these processes being as described in Section 4.0. The computers used for the tests were the I.B.M. 7094 at Imperial College and the University of London, Atlas computer. All programs were written in Fortran IV, a total of about 70 hours computing time being used.

It is assumed here that the transmitted signals and the transmission medium have the basic properties described in Section 3.1. It is also assumed, as in Section 4.0, that the receiver has prior knowledge of the number and element-addresses of the received signals.

5.2 General Signals

An element of the i th received signal is given by $Q_i = z_i Y_i$, where Y_i is the 10-component column-vector $\left[y_{ji} \right]$, in which j=1,...,10. $|y_{ji}| = \frac{1}{\sqrt{10}}$ for all i and j. Thus $|Y_i| = 1$ for all i, as before. These are the signals of constant power level considered in Section 3.3. Each element-address Y_i of the m received signals, is selected at random from the 2^{10} possible element-addresses, subject to the restriction that no two element-addresses are the same or the negatives of each other. In each selection, all permissible element-addresses are equally likely. The selection of the m element-addresses is repeated for each new element received, so that this is an arrangement of RASSAS. RASSAS is used here as a convenient means of obtaining a measure of the overall average performance of the system. It would not in general be suitable for a practical application, because of the excessive equipment complexity involved.

Two different situations are studied for the received signal levels $\left\{ |z_i| \right\}$. In the first, the levels are all equal such that $|z_i| = \sqrt{10}$

for all i. In the second, each $|z_i|$ is selected at random from 200 different values spaced at 0.1 db and covering a 20 db range of levels. The level selected for any one of the m signals is equally likely to have any one of the 200 possible values, different selections being statistically independent. The selection of the signal levels is repeated for each new element received. A range of levels of 20 db is about as great as could be handled accurately by practical detection processes of the various types studied.

The binary value of an element of the i th signal is given by the sign of z_i. An element is equally likely to have either binary value. With the exception of the restrictions on the m element-addresses in any total-element, any one selection of an address, level or binary value is statistically independent of any other.

In each test, ℓ total-elements are received for each of the values of m from 2 to 11. For $m \geqslant 4$, the received signals may not only be linearly dependent but also not uniquely detectable (Section 3.3). These signals therefore represent the most general case where neither linear independence nor unique detectability can be ensured.

5.3 Linearly Independent Signals

An element of the i th signal is given by $Q_i = z_i Y_i$, where Y_i is the 11-component column-vector $[y_{ji}]$, in which j=1,...,11. $|y_{ii}| = 0$ for all i and $|y_{ji}| = \frac{1}{\sqrt{10}}$ for all $j \neq i$. Thus $|Y_i| = 1$ for all i, as before. The element address of the i th signal is generated by selecting the address Y_i, for which $|y_{ji}| = \frac{1}{\sqrt{10}}$ for j=1,...,11, at random from the 2^{11} possible addresses. All addresses are equally likely and no restrictions are imposed on the selection of any address. y_{ii} is then set to zero. An arrangement of RASSAS is used as before, but only the case of equal received levels, where $|z_i| = \sqrt{10}$ for all i, is studied here.

In each test, ℓ total-elements are received for each of the values of m from 2 to 10. The m vectors $\sqrt{10}\ Y_i^T$ are the first m rows of the matrix M (equation 63, Section 3.4), in the particular case where the even integers are zero, the odd integers are $\pm$ 1 and v = 11. From Theorem 3 in Section 3.4, the m received signals are always linearly independent, and therefore from Theorem 2 in Section 3.2, they are uniquely detectable.

5.4 Level Variations and Additive Gaussian Noise

In order to determine the effects of variations in the received signal levels from their nominal values, the received signal-elements, $\{ z_i Y_i \}$, in an element detection process, are replaced by $\{ z_i(1 + f_i)Y_i \}$, where the $\{ f_i \}$ are m sample values of a random variable which is uniformly distributed between g_1 and g_2. $g_1 \leqslant 0$ and $g_2 \geqslant 0$ are constants which determine respectively the lower and upper limits of the possible received signal levels. A new set of values is selected for the $\{ f_i \}$ for each new total-element received, the different sample values of the random variable being statistically independent. The detection process at the receiver operates on the total received element, on the assumption that the received signal levels are $\{ | z_i | \}$.

Tests with level variations have been carried out on the more effective detection processes and for the case where all received signal levels are nominally equal, that is where the $\{ | z_i | \}$ are all equal. This arrangement simulates the situation where the different transmitters were initially adjusted to give equal signal levels at the receiver, but the received levels have since drifted from their nominal values, due to variations both in the transmitted levels and in the attenuations over the transmission path. The term "level variations" will be used here to refer to this situation, as opposed to that where the $\{ | z_i | \}$ may vary over a range of 20 db and the receiver has an accurate knowledge of these.

The tests with level variations are aimed at comparing the performances of the different detection processes, under conditions where the receiver has an inaccurate knowledge of the received signal levels. The tests are not intended to give an absolute measure of the performance of any detection process under practical conditions of fading or variations in transmitted levels, since the received signal levels would not normally have a rectangular probability density nor would they in general vary independently.

The more effective detection processes have also been tested for their tolerance to additive white gaussian noise, with and without level variations, in the case where all received signal levels are nominally equal. As before, in an element detection process the resultant received vector R is given by

$$R = \sum_{i=1}^{m} Q_i + N . \tag{188}$$

Whereas in all the other tests the noise vector, N, is zero, its v components are now sample values of statistically independent gaussian random variables, each having zero mean and a standard deviation of

$$\frac{|z_i|\,\sigma_n}{\sqrt{10}} \tag{189}$$

where $\frac{|z_i|}{\sqrt{10}}$ is the nominal magnitude of a non-zero component of any of the m individual received signal-elements.

5.5 Detection Processes

For each total received signal-element, the receiver performs the iterative detection process under test. This is one of the detection processes 1 to 14, with one of the different constraints applied. The detection process 15 is tested as the detection process 8. The signs of the $\{z_i\}$ are compared with the signs of the corresponding estimates $\{x_{ik}\}$, at suitable values of k. k and x_{ik} are here as defined in Section 4.3.1. If for n of the m individual signal-elements, the signs of z_i and x_{ik} are different, then the fraction of the signals wrongly detected is $\frac{n}{m}$. For each value of m tested, the average value of $\frac{n}{m}$ is determined for ℓ total-elements received. ℓ remains constant throughout the complete test. p is the resultant average value of $\frac{n}{m}$, over all values of m. Thus p is an estimate of the long term element error probability per channel in an arrangement of RASSAS, where at any instant m is equally likely to have any of its different possible values. p is also a weighted estimate of the fraction of the total number of possible signal combinations which will be correctly detected, so that p is a measure of the degree of convergence for the given value of k. The variation of p with k gives an indication of the rate of convergence of the iterative detection process.

For the detection processes 8 to 15, the term "detection cycle" will now be re-defined as follows. For the processes 8 and 9, a detection cycle will be taken to involve respectively $\frac{1}{h}$ and $\frac{1}{h_1}$ cycles of the iterative process (Section 4.4.2). For the processes 10, 11, 12 and 14, a detection cycle will be taken to involve $\frac{1}{c}$ cycles of the iterative process (Section 4.4.2). The detection process 13 is of course non-cyclic and the process 15 is tested as the process 8. The relative values of p after k detection cycles, for different detection processes and for different values of k, enable both the degrees and rates of convergence of these processes to be compared.

5.6 Tests with General Signals and No Noise

The more important results obtained from these tests are shown in Figs. 14 to 20. The confidence limits for these results are given in Table 1 (Section 5.10). The value of p at the end of the stated number of detection cycles, is the value determined for ℓ total-elements and for the ten values of m from 2 to 11. The value of ℓ normally used is shown in each figure. Wherever a different value is used, this is shown in brackets against the appropriate graph. Where no values of ℓ are shown, these are given in the text.

Some of the better detection processes have been tested with the number of components in a signal vector changed first to 9 and then to 11, but no very significant change in performance is obtained for any of these, so long as the different processes are compared for similar values of $\frac{m}{v}$, where v is the number of components in a signal-vector R.

With general signals, a small fraction of the total received elements are not uniquely detectable, so that p cannot decay to zero. From Sections 3.2 and 3.3 it appears that the minimum attainable value of p should be much smaller when the received signal levels have a range of 20 db than when they are equal. The accurate determination of the minimum value of p is difficult in either case and has not been attempted here, since no direct use is made of this quantity.

Some of the detection processes have been tested with the minimum Hamming distance between the signal-elements $\{ \pm | z_i | Y_i \}$, increased from 1 to 3. In each case the results show an increase in both the rate and degree of convergence.

In Figs. 14 and 15, $h = 0.1$, $h_1 = 0.1$, $c = 0.1$ and $e < 0.00001$, wherever they apply. The only exceptions are 4I and 4J for which $e = 0.01$. The detection process 4K has an upper limit of 30 subtraction processes but normally performs only as many subtraction processes as there are received signals. 4I and 4J each have an upper limit of 100 subtraction processes, an upper limit of 200 giving values of p respectively 0.0099 and 0.0012, with $\ell = 300$. 13I and 13J have an upper limit of 300 subtraction processes.

Some of the better detection processes have been tested with the possible signal levels spaced at intervals of 0.09 and 0.11 db instead of 0.1 db, but there are no noticeable differences in the results. This suggests that the results in Figs. 14 and 15 apply also to the general case where the signal levels are statistically independent random variables, with a uniform probability density on the decibel scale over a given range of 20 db, and zero outside this range.

The value of p obtained for the detection processes 2J and 3J, after 10 detection cycles, is shown by the appropriate circled point in Fig. 14. These processes appear to be converging. An appreciably more rapid rate of convergence is however shown by 4J. 3K and 4K appear to be the best detection processes here. Of the remaining detection processes, only 9J is converging at a useful rate.

In Figs. 16 and 17, $h = 0.1$, $h_1 = 0.1$, $c = 0.1$ and $b = 0.01$, wherever they apply. $e = 0.01$ for the detection processes 4, 10, 11 and 13 and $e = 0.05$ for 14J. The detection processes 4 and 13 have the same upper limits to the number of subtraction processes as in Figs. 14 and 15. 4K normally performs as many subtraction processes as there are received signals, and in no case more. The best detection processes here appear to be 14J, 11J and 12J.

In Figs. 14 to 17, the detection process achieving the best overall performance is 4J. 4I and 4J are tested with a threshold level $e = 0.01$, throughout Figs. 14 to 25. Additional tests with $e = 0.001$ and $e = 0.1$, show no change in performance with the former and a small degradation with the latter. Thus for equal received signal levels, an accuracy of about $1^{\circ}/_{\circ}$ in the detector output signals should be more than adequate.

With the exception of the detection processes 3K, 4K, 3J and 4J, a detection process which shows a significant rate of convergence, converges more rapidly when the received levels are equal than when they are not. Since the very good performance of the detection processes 3K and 4K under the conditions of Fig. 14, requires a more accurate knowledge of the received signal levels than is likely to be achieved in practice, and since there are other obvious practical difficulties in operating with signals of widely differing levels, the most interesting situation is that where the received levels are nominally equal.

The detection processes 1, 5, 10 and 13, and any process using the constraint K, have a poor performance with equal received signal levels. Although 9I and 9J have a more rapid rate of convergence than 8I and 8J, they would involve more complex equipment. These various arrangements are not therefore considered further.

In Figs. 18 and 19, values of p are plotted for up to 20 detection cycles of the more promising of the arrangements in Figs. 16 and 17. Detection processes considered here, but not in Figs. 16 and 17, are various arrangements of 2JH, 8J, 11J and 14J. In Fig. 18, the numbers in brackets immediately below 2J and 3J give the number of detection cycles, and the corresponding numbers for 4I and 4J give the maximum number of subtraction processes permitted. The values of ℓ used in Fig. 18 are: ℓ = 300 for 2JH(h=1.375), 2JH(h=1.875), 3J(50), 4I(100), 7J, 8I, 8J(h=0.05) and 8J(h=0.1); ℓ=600 for 2JH(h=1.625), 4I(200), 6I and 6J; ℓ=1000 for 2I, 2J, 2J(50), 2JH(h=1.25), 2JH(h=1.5) and 2JH(h=1.75); ℓ=1200 for 4J(200); ℓ=1500 for 3J and 4J(100).

In Fig. 19, the parameters associated with the different graphs for 11J are as follows:-

	c	e	ℓ
(1)	0.2	0.01	1000
(2)	0.1	0.05	1000
(3)	0.1	0.01	1500
(4)	0.1	0.0	1000
(5)	0.05	0.025	1000

and the parameters associated with the different graphs for 14J are as follows:-

	b	c	e	ℓ
(1)	0.005	0.1	0.05	600
(2)	0.01	0.1	0.05	1200
(3)	0.015	0.1	0.05	1500
(4)	0.02	0.1	0.05	600
(5)	0.033	0.1	0.05	300
(6)	0.01	0.1	0.01	1000
(7)	0.015	0.1	0.01	1200
(8)	0.02	0.1	0.01	1000

The standard deviation for the value of p for the graph 14J(2) after the fifth detection cycle, estimated from four sample values each with ℓ=300, is σ=0.00023. Although Student's t-test cannot be applied here, since the total number of errors is too small for this number to have an approximately normal distribution, it is reasonable to expect that the 95% confidence limits for the value of p should lie in the range $\pm$0.0004 to $\pm$0.0008 about the given value of p. No great significance can therefore be attached to the different degrees of convergence of the graphs for 14J.

In Fig. 20, the limits to the signal level variations are given by g_1 = -0.1 and g_2 = 0.1 (Section 5.4). The values of ℓ in the left half of Fig. 20 are: ℓ =300 for 3J, 7J and 8J; ℓ =1000 for 2J, 2JH and 6J. The parameters associated with the graphs in the right half of Fig. 20 are as follows:-

	b	c	e	ℓ
11J	-	0.1	0.01	1000
14J(1)	0.01	0.1	0.05	300
14J(2)	0.015	0.1	0.05	300
14J(3)	0.02	0.1	0.05	300
14J(4)	0.01	0.1	0.01	600
14J(5)	0.015	0.1	0.01	600
14J(6)	0.02	0.1	0.01	600

A comparison of the systems tested here with the same systems in the absence of level variations (Figs. 18 and 19), shows only small differences in their performances. This suggests that the performance of a detection process with the constraint J is not critically dependent on an accurate knowledge of the signal levels at the receiver.

The effect of the level variations is to reduce the probability that the received signal-elements are not uniquely detectable. At the same time the minimum distance between two possible values of the total received vector, corresponding to two sets of binary values of the individual signal-elements, for the case where these are uniquely detectable, can now be much smaller than that where there are no level variations. It is interesting to observe that of the detection processes 14J, those with e=0.01 are the most effective in Fig. 20, contrary to the situation in Fig.19.

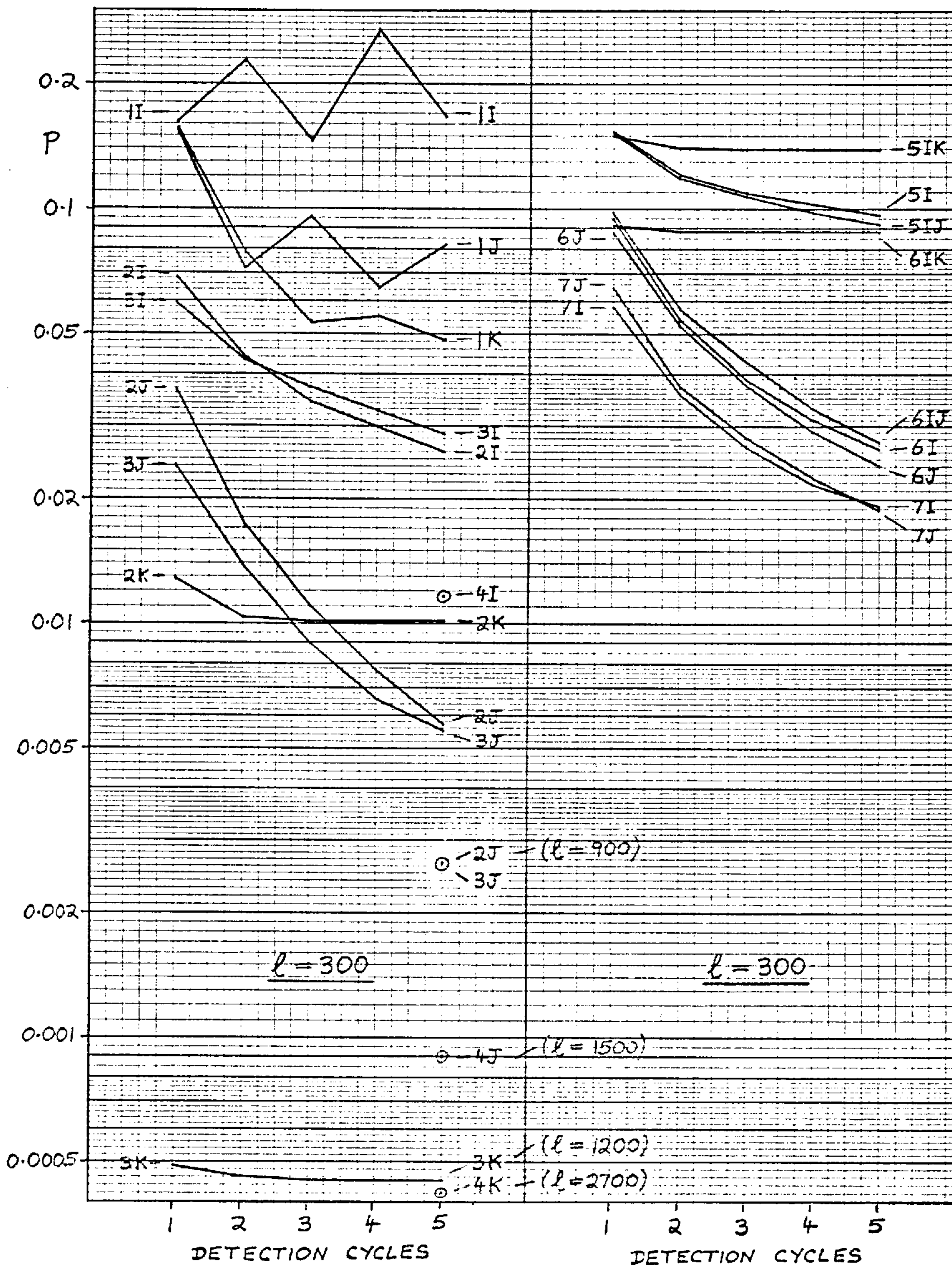


FIG. 14. GENERAL SIGNALS WITH NO NOISE OR LEVEL VARIATIONS. 20 DB RANGE OF LEVELS.
DETECTION PROCESSES USING LARGE STEPS.

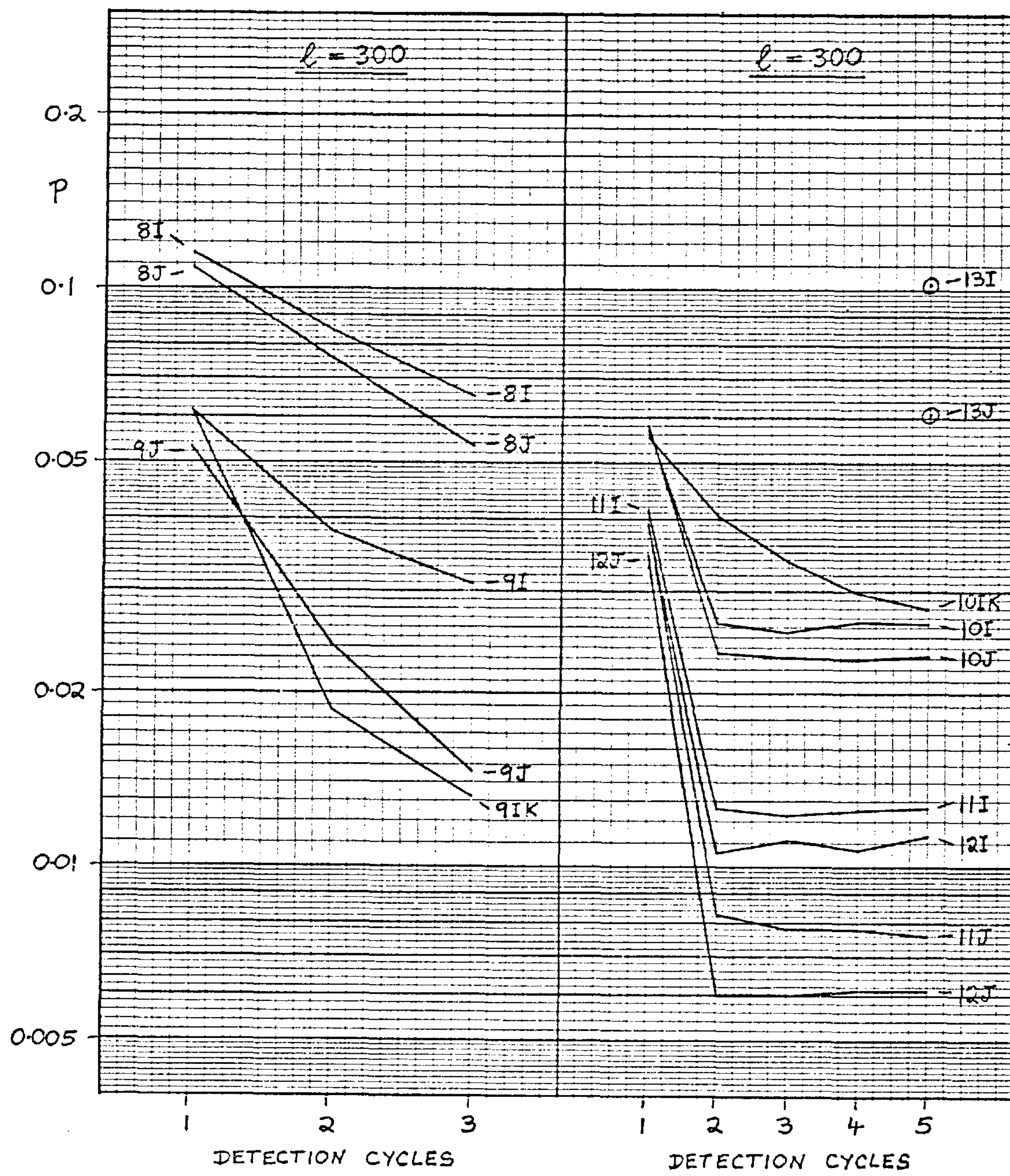


FIG. 15. GENERAL SIGNALS WITH NO NOISE OR LEVEL VARIATIONS.
20 DB RANGE OF LEVELS.
DETECTION PROCESSES USING SMALL STEPS.

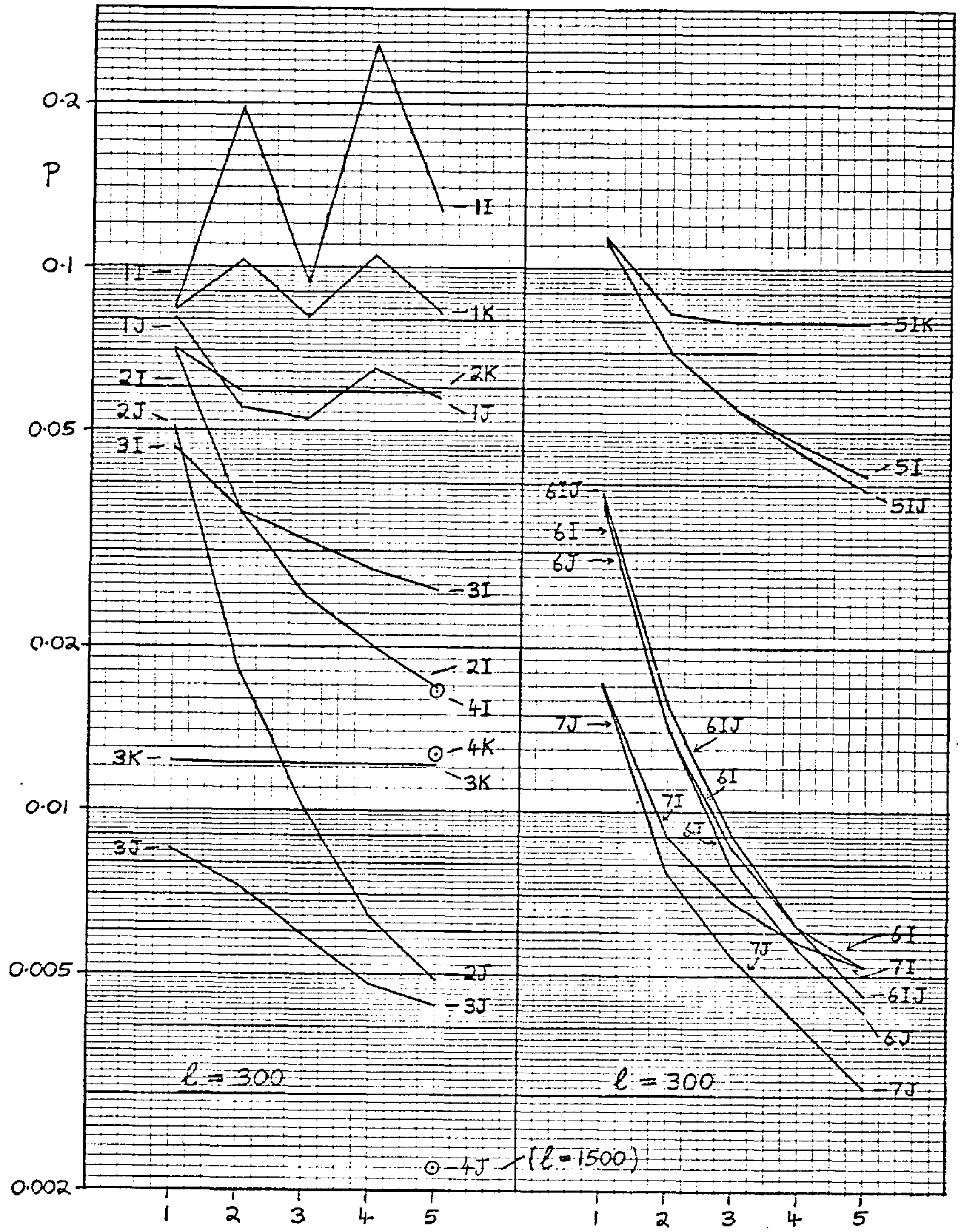


FIG. 16. GENERAL SIGNALS WITH NO NOISE OR LEVEL VARIATIONS.
EQUAL LEVELS.
DETECTION PROCESSES USING LARGE STEPS.

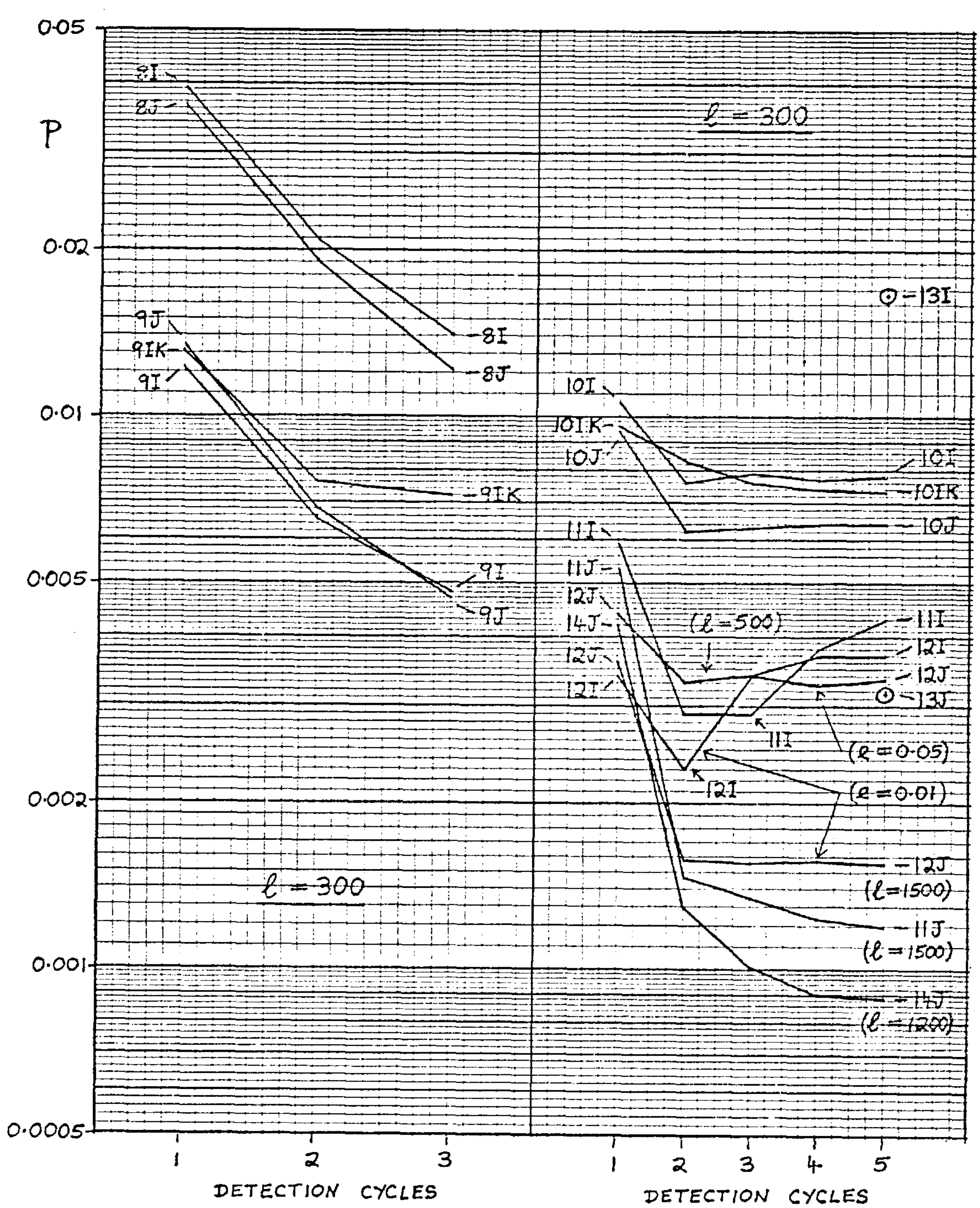


FIG. 17. GENERAL SIGNALS WITH NO NOISE OR LEVEL VARIATIONS. EQUAL LEVELS. DETECTION PROCESSES USING SMALL STEPS.

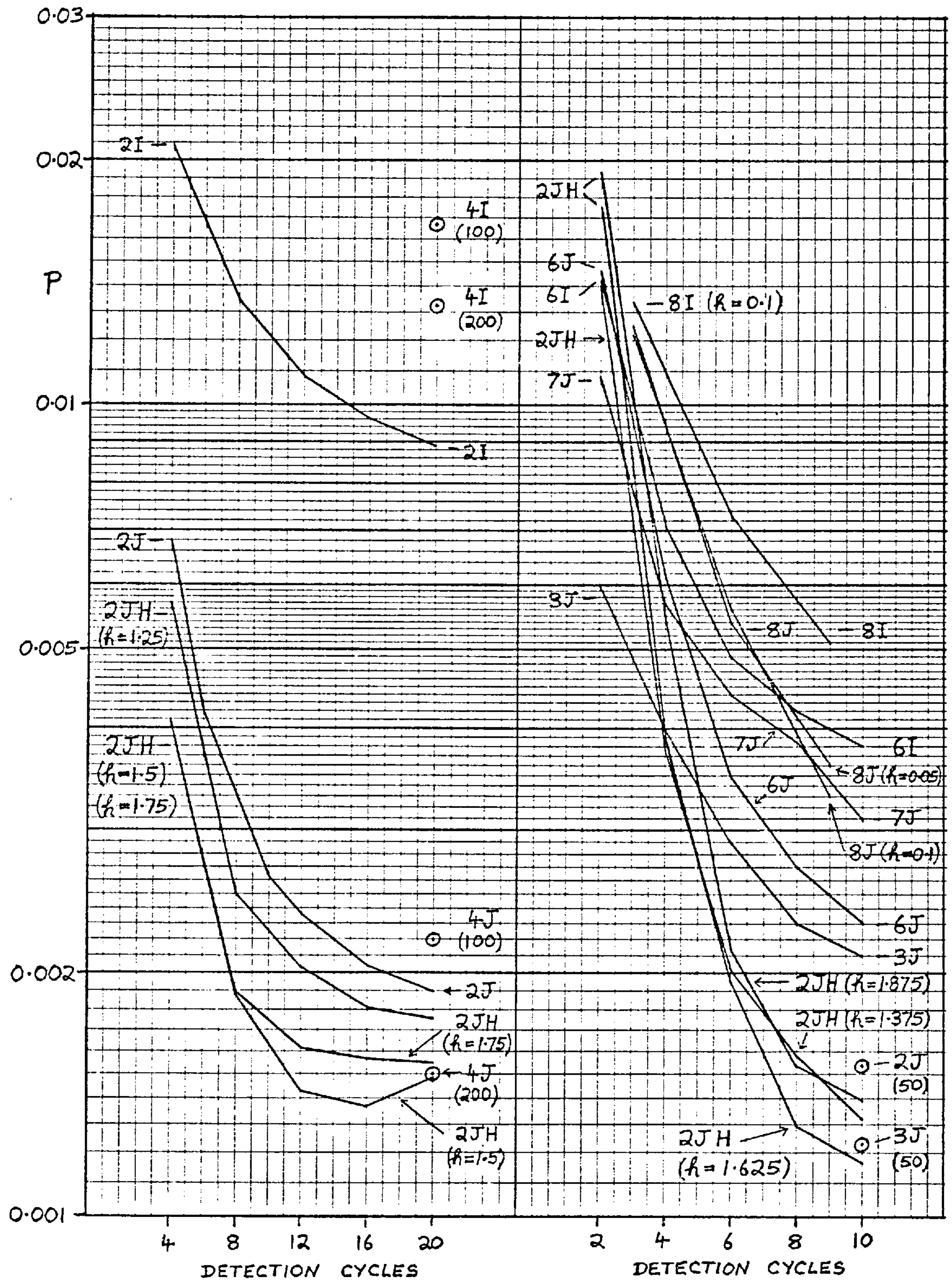


FIG. 18. GENERAL SIGNALS WITH NO NOISE OR LEVEL VARIATIONS. EQUAL LEVELS.

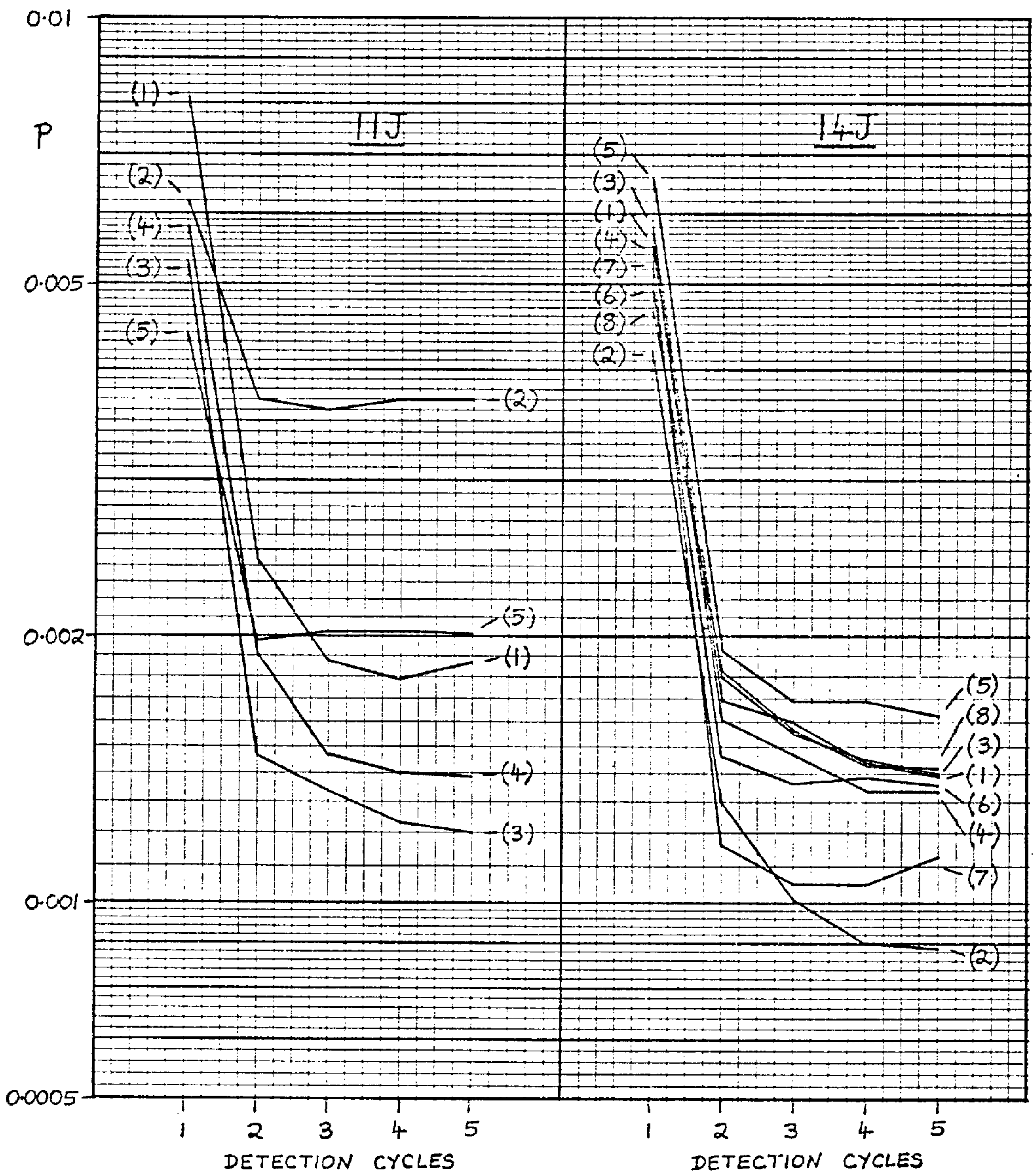


FIG. 19. GENERAL SIGNALS WITH NO NOISE OR LEVEL VARIATIONS.
EQUAL LEVELS.
DETECTION PROCESSES 11J AND 14J.

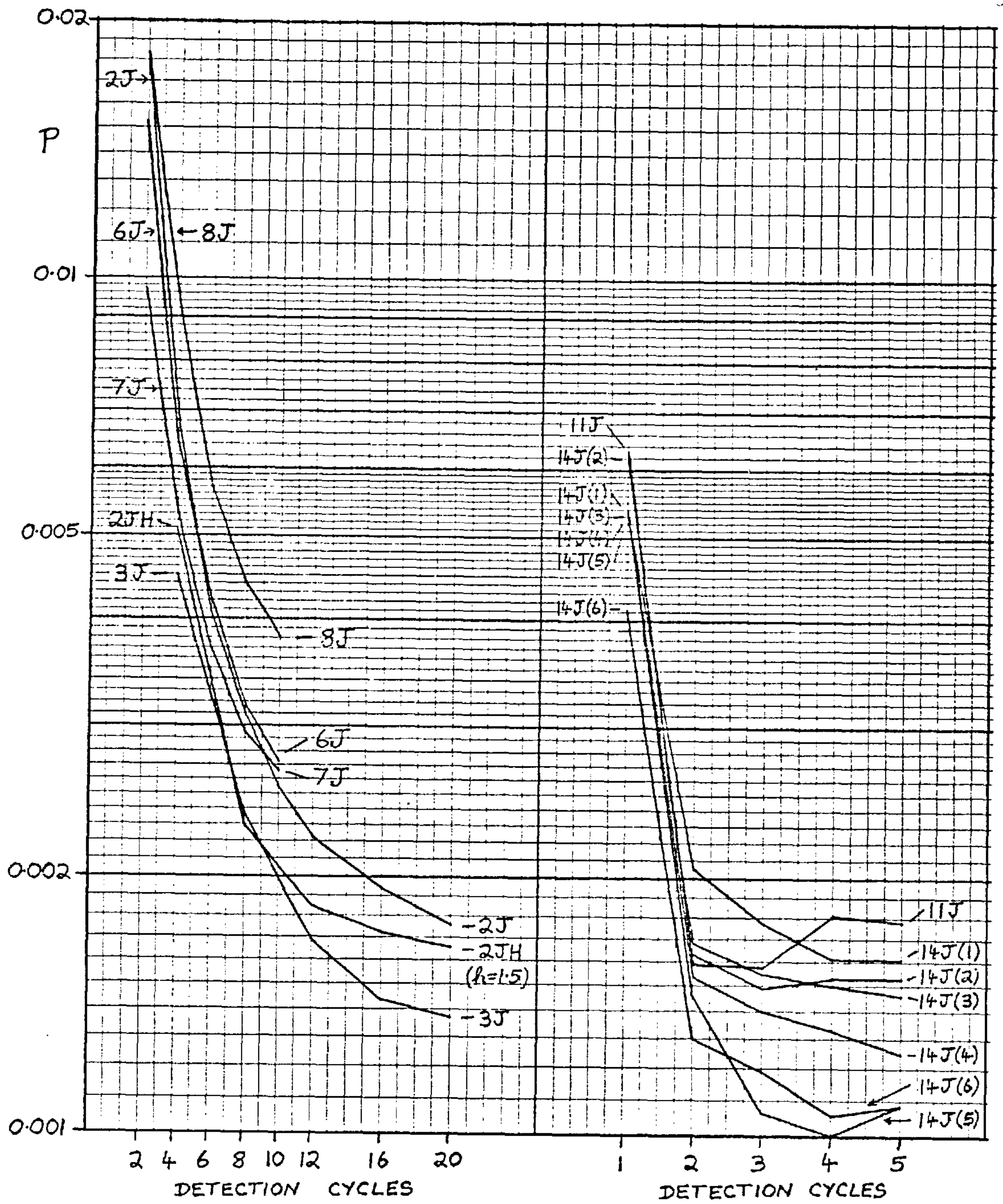


FIG. 20. GENERAL SIGNALS WITH LEVEL VARIATIONS BUT NO NOISE.

5.7 Tests with Linearly Independent Signals and No Noise

The more important results obtained from these tests are shown in Figs. 21 to 25. The confidence limits for these results are given in Table 1 (Section 5.10). The value of p at the end of the stated number of detection cycles, is the value determined for ℓ total-elements and for the nine values of m from 2 to 10. The value of ℓ normally used is shown in each figure. Wherever a different value is used, this is shown in brackets against the appropriate graph. When no values of ℓ are shown, these are given in the text. In all cases $|z_i| = \sqrt{10}$, for all i.

Since the received signals are linearly independent, the minimum attainable value of p is zero. In order to show zero values of p, where necessary, the vertical scales in Figs. 21 to 24 are modified so that these are linear over the range 0.0 to 0.00005. The vertical scale of Fig. 25 is linear over the range 0.0 to 0.0001. p is only shown as zero if there are no errors in detection.

A detection process is considered to converge completely in a given test with linearly independent signals, if the value of p decays to zero and remains at zero as the detection process proceeds. It is of course assumed here that no noise is present.

In Fig. 21 it appears that the detection process 2IH converges more rapidly for h=1 than for h > 1. When h=1, p converges to zero after somewhat more than 100 detection cycles. When h=2, p remains above 0.25, indicating that the system does not converge.

The detection process 2JH has the maximum rate of convergence when h is in the range 1.75 to 2.0. The graph plotted for h=1.75 to 2.0 is the average value of p for the six values of h spaced at intervals of 0.05 from 1.75 to 2.0. Convergence is not obtained when h $\geq$ 2.25.

2JH converges over a wider range of values of h and at a considerably faster rate than 2IH. It is clearly the preferable system here.

In Fig. 22, the figures in brackets immediately below 4I and 4J show the maximum number of subtraction processes permitted. The detection processes 2, 3, 4 and 8 all appear to be converging. The constraint J introduces a considerable increase in the rate of convergence of each of the processes 3 and 4, a smaller increase for 2 and a reduction in the final rate of convergence for 8. It is shown theoretically in Section 4.3.3 that the detection processes 2I and 4I will converge, and from the analysis in Sections 4.4.1 and 4.5.2, both 8I and 8J should converge for sufficiently small values of h, the latter because it approximates to 15J. Furthermore 15J should in general converge at a slower rate than 15I. Thus the results in Fig. 22 are consistent with the theoretical analysis.

The parameters for the four graphs for the detection process 12 in Fig. 22, are as follows:-

	c	e	ℓ
12I(1)	0.1	0.05	300
12I(2)	0.1	0.01	600
12J(1)	0.1	0.05	300
12J(2)	0.1	0.01	300

None of these systems converges completely.

In Fig. 23, the parameters associated with the different graphs for 11I are as follows:-

	c	e	ℓ
I(1)	0.1	0.05	1000
I(2)	0.1	0.01	1000
I(3)	0.05	0.025	1000
I(4)	0.05	0.01	1000
I(5)	0.05	0.005	1000

and the parameters associated with the different graphs for 11J are as follows:-

	c	e	ℓ
J(1)	0.1	0.05	1000
J(2)	0.1	0.01	1000
J(3)	0.1	0.0	1000
J(4)	0.05	0.025	1000
J(5)	0.05	0.01	1000
J(6)	0.05	0.005	1000
J(7)	0.05	0.0	1000

None of the arrangements of 11I or 11J converges completely, although the better of these systems, as for 12J(2) in Fig. 22, only fail to converge for a very small fraction of the signal combinations tested.

In Fig. 24, the parameters associated with the two graphs for 14I are as follows:-

	b	c	e	ℓ
I(1)	0.01	0.1	0.05	500
I(2)	0.01	0.1	0.01	500

and the parameters associated with the different graphs for 14J are as follows:-

	b	c	e	ℓ
J(1)	0.005	0.1	0.05	300
J(2)	0.01	0.1	0.05	600
J(3)	0.015	0.1	0.05	600
J(4)	0.02	0.1	0.05	300
J(5)	0.01	0.1	0.01	1000
J(6)	0.015	0.1	0.01	1000
J(7)	0.02	0.1	0.01	1000
J(8)	0.01	0.2	0.01	1000
J(9)	0.02	0.2	0.01	1000
J(10)	0.01	0.5	0.01	1000

14J is completely convergent for c = 0.1, e = 0.01 and b = 0.01 or 0.015. It is probably also convergent for c = 0.2, e = 0.01 and b = 0.01, after a sufficient number of detection cycles.

In Fig. 25, the parameters associated with the different graphs for 14J are as follows:-

	b	c	e	g_1	g_2	ℓ
(1)	0.01	0.1	0.05	-0.1	0.1	300
(2)	0.015	0.1	0.05	-0.1	0.1	300
(3)	0.02	0.1	0.05	-0.1	0.1	300
(4)	0.01	0.1	0.01	-0.1	0.1	600
(5)	0.01	0.1	0.01	-0.2	0.2	600

As with the general signals in Fig. 20, level variations over a range of $\pm\ 10^{o}/_{o}$ of the nominal signal level $|z_i|$, reduce the degree of convergence of 14J when e = 0.05 but not when e = 0.01.

The detection process 14J, with b = 0.01, c = 0.1 and e = 0.01, and the detection process 2JH, with $1.75 \leqslant h \leqslant 2.0$, have the best overall performances in this series of tests.

The detection process 12 is appreciably more complex than 11 and appears to achieve no advantage in performance. It is not therefore considered further.

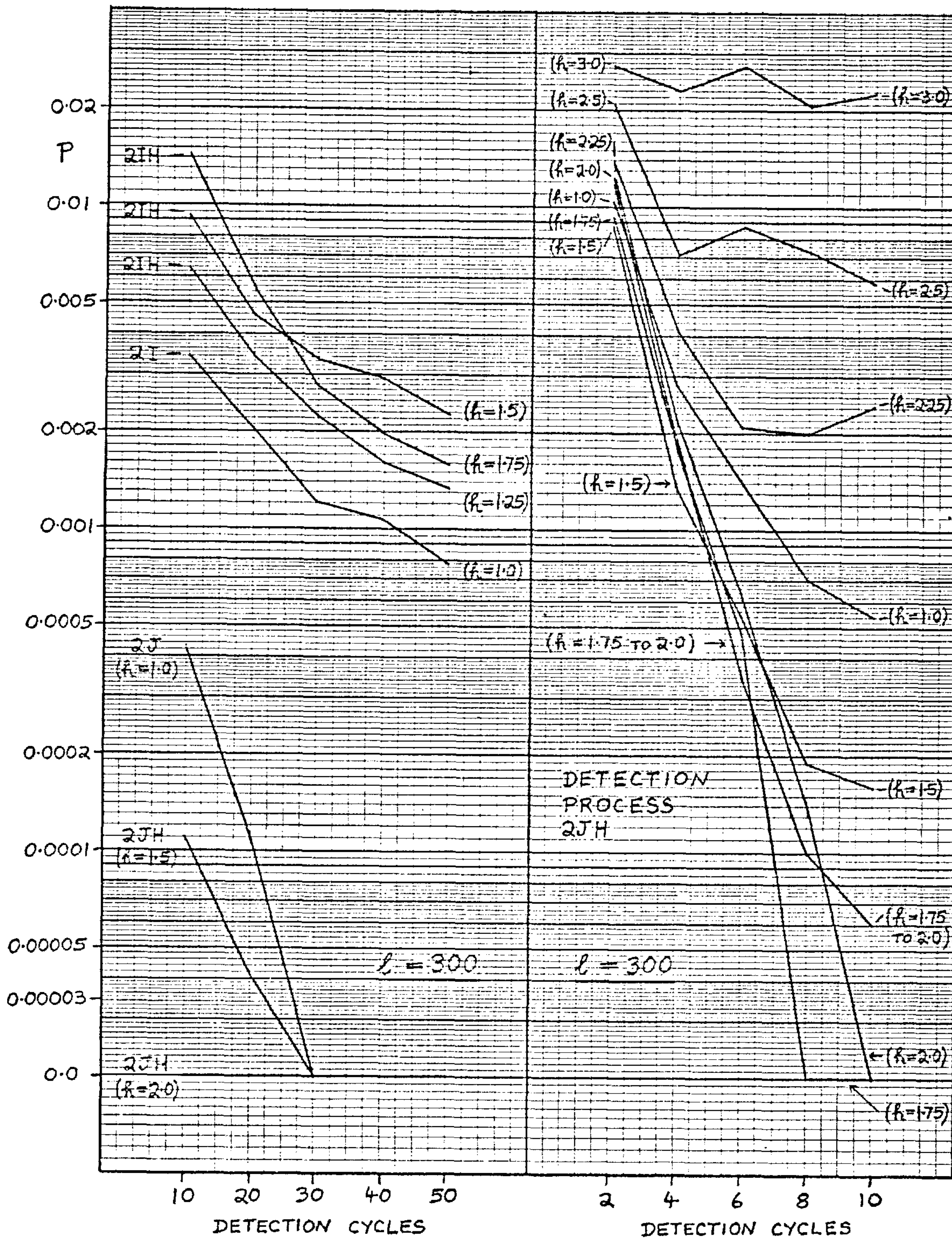


FIG. 21. LINEARLY INDEPENDENT SIGNALS WITH NO NOISE OR LEVEL VARIATIONS. DETECTION PROCESS 2.

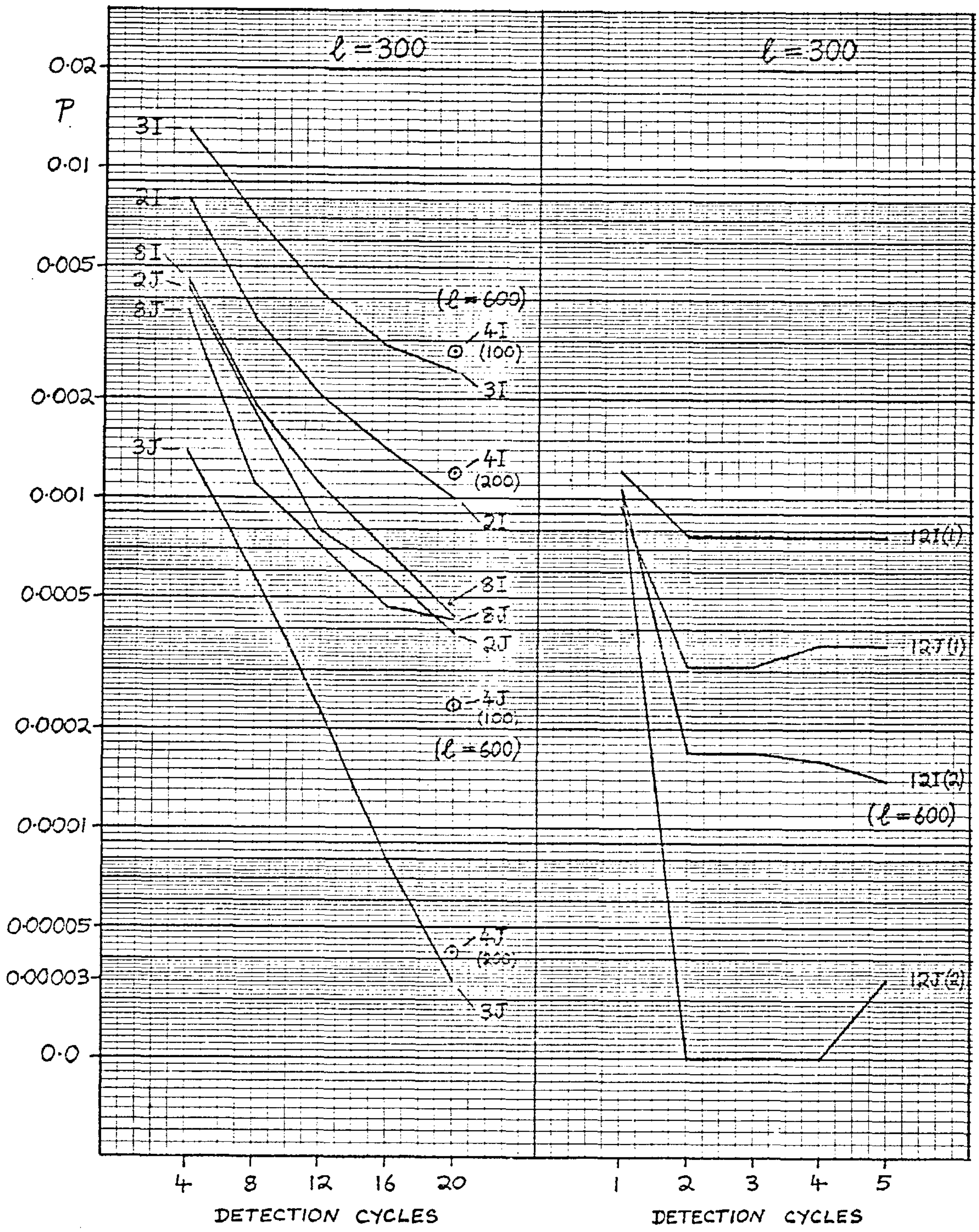


FIG. 22. LINEARLY INDEPENDENT SIGNALS WITH NO NOISE OR LEVEL VARIATIONS.

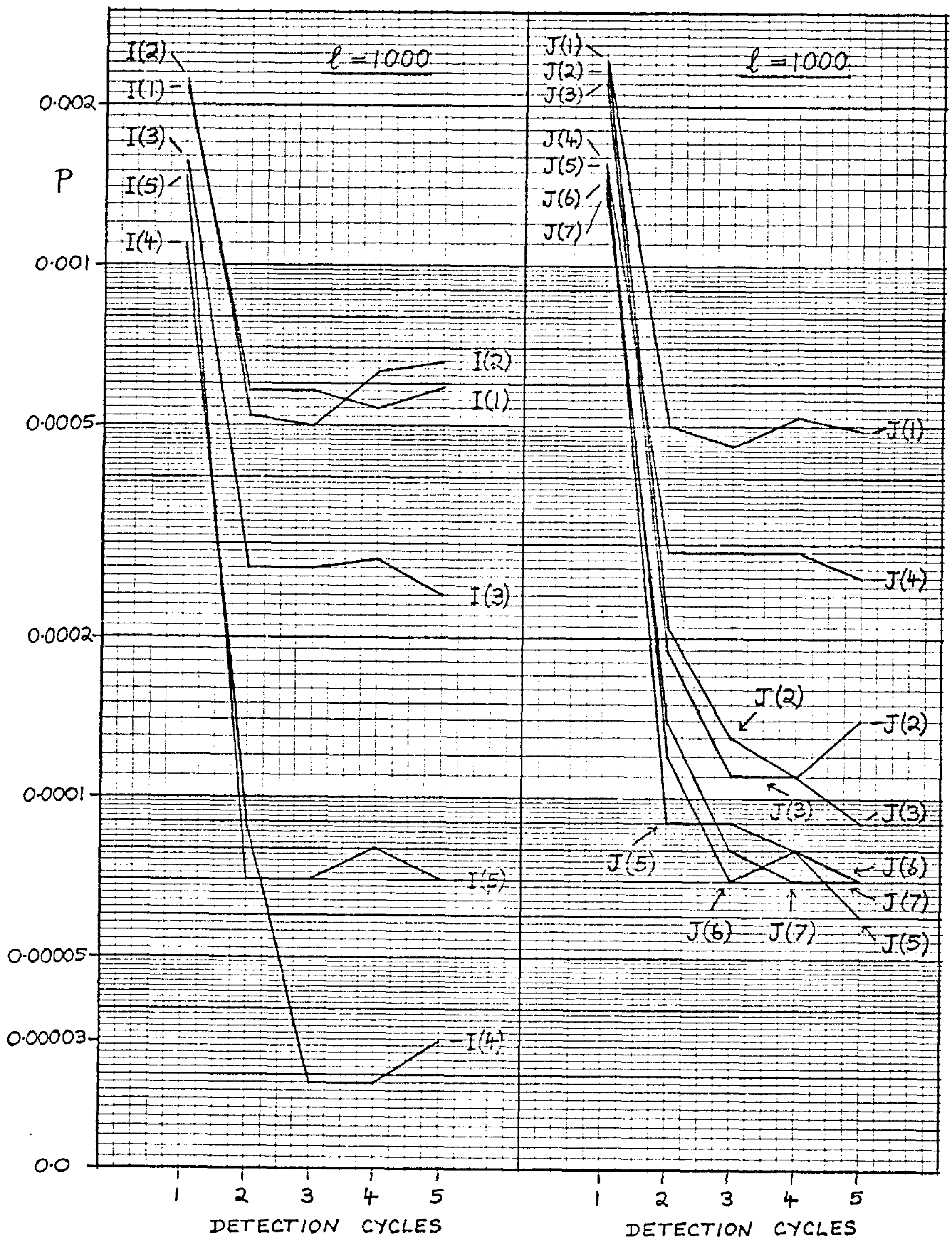


FIG. 23. LINEARLY INDEPENDENT SIGNALS WITH NO NOISE OR LEVEL VARIATIONS. DETECTION PROCESS II.

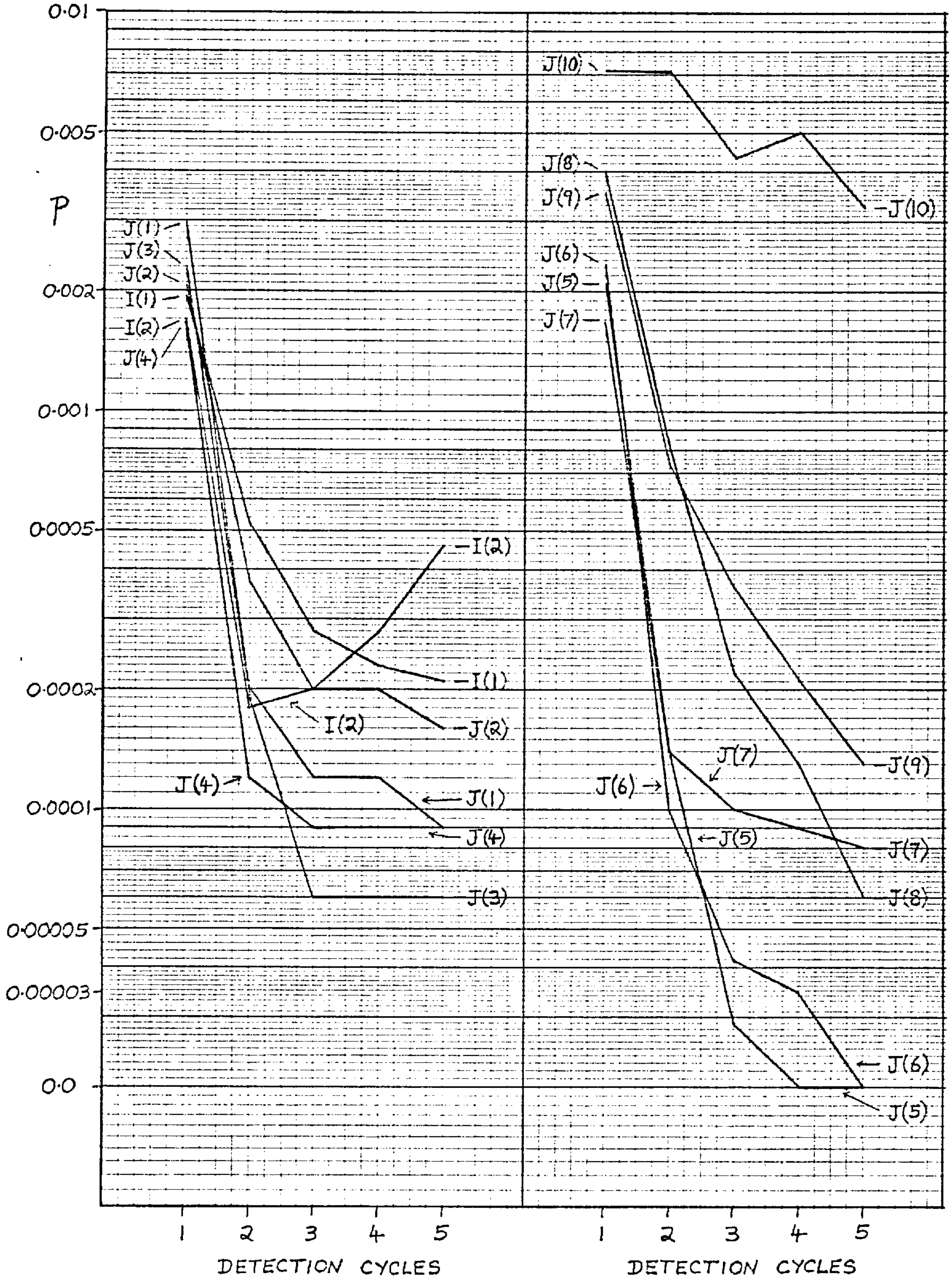


FIG. 24. LINEARLY INDEPENDENT SIGNALS WITH NO NOISE OR LEVEL VARIATIONS. DETECTION PROCESS 14.

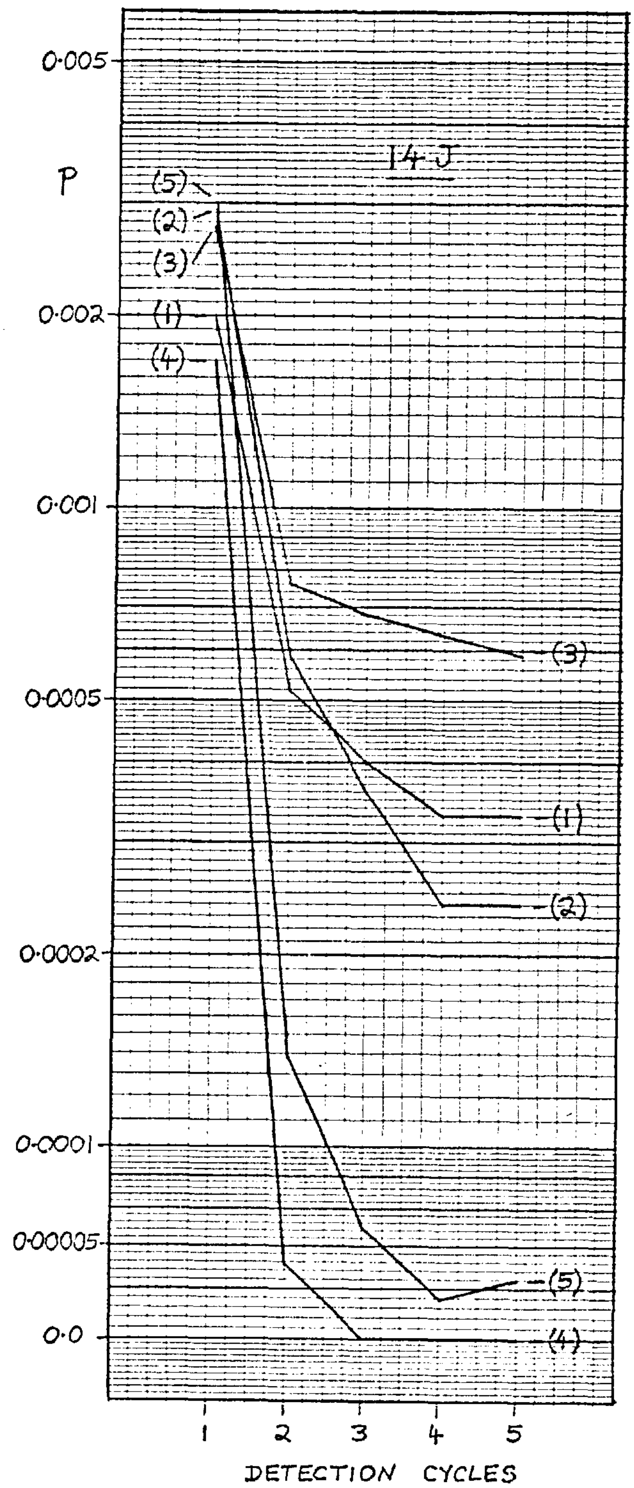


FIG. 25. LINEARLY INDEPENDENT SIGNALS WITH LEVEL VARIATIONS BUT NO NOISE. DETECTION PROCESS 14J.

5.8 Tests with Noise and Level Variations

The more important results obtained from these tests are shown in Figs. 26 to 39. The confidence limits for these results are given in Table 2 (Section 5.10). For all except Fig. 39, linearly independent signals are used (Section 5.3). The value of p at the end of the stated number of detection cycles, is the value determined for ℓ total-elements and for the five values of m from 2 to 6. The values of ℓ are shown as before, the only exception being that ℓ = 1200 for the graph marked "1", for each of 2I, 2J, 3I, 3J, 4I and 4J. In all cases $|z_i| = \sqrt{10}$, for all i.

The tests here with no noise or level variations represent a considerably less severe test of the convergence of the detection process, than do the corresponding tests in Section 5.7. This has been arranged to provide somewhat more realistic signals for the tests with noise and level variations.

The vertical scale of each of the Figs. 26 to 38 is linear for values of p greater than 0.002. For $p < 0.002$ the vertical scale is linear between each adjacent pair of values marked. p is only shown as zero when there are no errors in detection.

The parameters associated with the different graphs in each of the Figs. 26 to 38, are as follows:-

	σ_n	g_1	g_2
1	1.0	0.0	0.0
2	1.0	-0.1	0.1
3	1.0	-0.2	0.2
4	1.0	-0.3	0.3
5	1.0	-0.5	0.5
6	0.75	-0.5	0.0
7	0.8414	0.0	0.0
8	0.6685	-0.5	0.0
11	0.0	0.0	0.0
12	0.0	-0.1	0.1
13	0.0	-0.2	0.2
14	0.0	-0.3	0.3
15	0.0	-0.5	0.5
16	0.0	-0.5	0.0

For all graphs in Fig. 39: $\sigma_n = 1.0$, $g_1 = 0.0$ and $g_2 = 0.0$.

The parameter values given for 6 and 16 above, are those where the signal levels vary over a range of 6 db and the constraint J, where applied, is correct for the received signal levels having the maximum value in this range. This is probably the best arrangement of the constraint J, when the received signal levels are not accurately known but may vary over a known range, because in no case is the estimate, x_{ik}, of a received signal value, z_i, prevented from reaching its correct value. For signals not having the maximum level, the arrangement represents a relaxation of the constraint J and so becomes a compromise between the constraints J and I.

The tests with no noise or level variations (graphs 11 in Figs. 26 to 38), show convergence for all the detection processes tested, except for 11J with c = 0.1 and e = 0.0 or 0.05 (Fig. 36). 8I and 8J with h = 0.025, in Fig. 32, are clearly converging even though p has not reached zero (see Figs. 30 and 31).

Tests with no noise (graphs 11 to 16 in Figs. 26 to 38) show that 2I, 4I, 8I and 11I, with suitable values of h, c and e, where appropriate, converge under all the conditions of level variations tested. Tests with no noise also show that 2JH, 3J, 4J, 8J, 11J and 14J, with suitable values of h, b, c and e, where appropriate, converge under all the conditions of level variations except 15 ($g_1 = -0.5$, $g_2 = 0.5$). 8J with h = 0.05 (Fig. 31) has been shown to achieve this result, if the maximum number of detection cycles is increased to 20.

As might be expected, the constraint I leads to a greater tolerance to level variations than the constraint J. Correct operation of the various detection processes with the constraint J, can however be obtained in the absence of noise, even when the levels of one or two of the received signals have drifted by nearly 3 db above their nominal value. The constraint J does not therefore require an unduly accurate knowledge of the received signal levels.

The results of the tests with additive gaussian noise (graphs 1 to 8) give a useful general guide to the relative performances of the different detection processes, but they must be treated with some caution. The results of any test approximate to the overall average performance of the corresponding arrangement of RADAS, for all possible combinations of element-addresses and signal-levels within the specified range, when m is equally likely to have any value from 2 to 6. The fact that a given arrangement shows a better tolerance to additive gaussian noise than another, under these conditions, does not necessarily imply that the same advantage will be maintained for any given combination of element-addresses and signal-levels. For instance, the detection processes 2I and 2J each show a slightly poorer performance with general signals (Fig. 39) than with linearly independent signals (Fig. 26, graphs 1), even though the worst-case tolerance to additive gaussian noise with the former signals, when uniquely detectable, is ideally about 3 db better than that with the latter signals, when $v \gg 1$ and an optimum detector is used (Section 3.4). The number of detection errors caused by the non-uniquely detectable general-signals, is small compared with the total number of errors counted in any of these tests, and if taken into account suggests that each detection process has a similar tolerance to noise with either type of signal. When $m \ll v$, there is a high probability that the m element-addresses selected at random will be nearly orthogonal,[D2] so that on theoretical grounds the tests with gaussian noise should give the performance of a detection process under typical or average conditions, when the individual received signal-vectors are more nearly orthogonal than colinear. The results of the tests will not therefore be greatly influenced by the minimum tolerance to noise of the possible received signals. Further experimental confirmation of this is given in Section 5.11.

Fig. 39 shows that the detection processes 6I and 6J have appreciably lower tolerances to additive gaussian noise than 2I and 2J. This is in agreement with the theoretical analysis of these systems (Section 4.3.4). The detection processes 3 and 4 achieve no advantage in tolerance to noise over the detection process 2, showing that the ordering of the individual processes of subtraction is not very important here. The detection process 7, however, achieves an appreciable advantage over 6. Clearly the ordering of the individual processes of subtraction in 7, can to some degree make up for the inferior performance of the basic system. Since the detection processes 6 and 7 would involve appreciably more complex equipment when used with linearly independent signals, than would the majority of the other detection processes, the processes 6 and 7 are not studied further here.

For certain of the detection processes using the constraint I, the value of p tends to increase with the number of detection cycles, over part of the range tested. This tendency is particularly marked in 11I. However 11I could not be used successfully with only one detection cycle, in order to exploit the advantage in tolerance to noise, because at this stage of the detection process it has non-zero values of p in the absence of noise, under most of the conditions of level variations tested (Figs. 33 and 34).

The most striking result from Figs. 26 to 38 is the general similarity between the performances of the different detection processes, when these all have the same constraint I or J. The constraint J clearly gives a better tolerance to additive gaussian noise than the constraint I, for all the detection processes and under all conditions of level variations tested. This effect is considered in more detail in Section 5.11. Even when due allowance is made for the failure of some detection processes with the constraint J to converge in the absence of noise, under the more severe conditions of level variations, the constraint J appears to have a useful overall advantage over the constraint I.

The detection processes 3J and 4J require more complex equipment than the other detection processes, and 2J is rather slow to converge. 2JH with h = 1.25 to 1.5, however, has very good convergence properties, even under the most extreme conditions tested (Figs. 18, 20 and 21), and it is a relatively simple system. 2JH with h = 1.25 to 1.5 is therefore one of the preferred detection processes.

Fig. 38 shows that a serious degradation in performance results in the detection process 14J, if c is increased from 0.1 to 0.2. Tests with 11J (Fig. 19) similarly indicate that for correct operation c should not exceed 0.1. On balance, the arrangements of 11J and 14J showing the best overall tolerance to noise and level variations, are as follows:-

11J:
1) c = 0.05, e = 0.01
2) c = 0.05, e = 0.025
3) c = 0.1, e = 0.01

14J:
1) b = 0.015, c = 0.1, e = 0.05
2) b = 0.01, c = 0.1, e = 0.01

With neither detection process is there a significant difference between the performances of the different arrangements listed, when taking into account the confidence limits of these results (Table 2, Section 5.10).

It is reasonable to assume, although requiring further confirmation, that with a particular combination of received element-addresses, having a small minimum distance between possible values of the resultant vector, the arrangements of 11J may have a lower tolerance to additive noise than those of 14J, because of the better convergence properties of the latter (Figs. 19, 20, 23 and 24). Under these conditions the best arrangement of 14J appears to be that with b = 0.01, c = 0.1 and e = 0.01, and the best arrangement of 11J that with c = 0.05 and e = 0.01. However, where an adaptive coding system is used to keep the received signals nearly orthogonal (Section 3.4), the relative performances of the different arrangements would be approximately as predicted by Figs. 33 to 38. Under these conditions the best arrangement of 14J appears to be that with b = 0.015, c = 0.1 and e = 0.05, and the best arrangement of 11J that with c = 0.1 and e = 0.01, the latter because it requires fewer separate processes of subtraction per detection cycle than the alternative arrangements. Although the detection process 14J is more complex than 11J, it appears on balance to be the preferable arrangement, because of its much better convergence properties.

The detection process 8J is of considerable interest because for small values of h the performance of the system approximates to that of 15J (Section 4.5.1), which is by far the simplest of all the arrangements considered (Fig. 12, Section 4.5.1). The combined evidence of Figs. 30 to 32 confirms that the performance of the detection process 15J should be similar to that of 8J. It should therefore be similar to that of the preferred arrangements of 2JH, 11J and 14J, under any of the conditions of noise and level variations considered in Figs. 26 to 38. 15J is clearly the preferred detection process.

In Figs. 30 to 32, 8J has a lower rate of convergence than 8I. This is consistent with Fig. 22 and with the theoretical analysis of 15I and 15J in Section 4.5.2. The main weakness of 15J is probably that under difficult conditions as in Fig. 22, it may be necessary to use a very small effective time-constant for the integrators of Fig. 12, in order to complete the detection process in the available time, and this may lead to instability. No difficulty should however be experienced under the much less stringent conditions which apply in Figs. 30 to 32.

The preferred detection processes, listed in the order of their apparent overall merits, are 15J, 14J, 2JH and 11J.

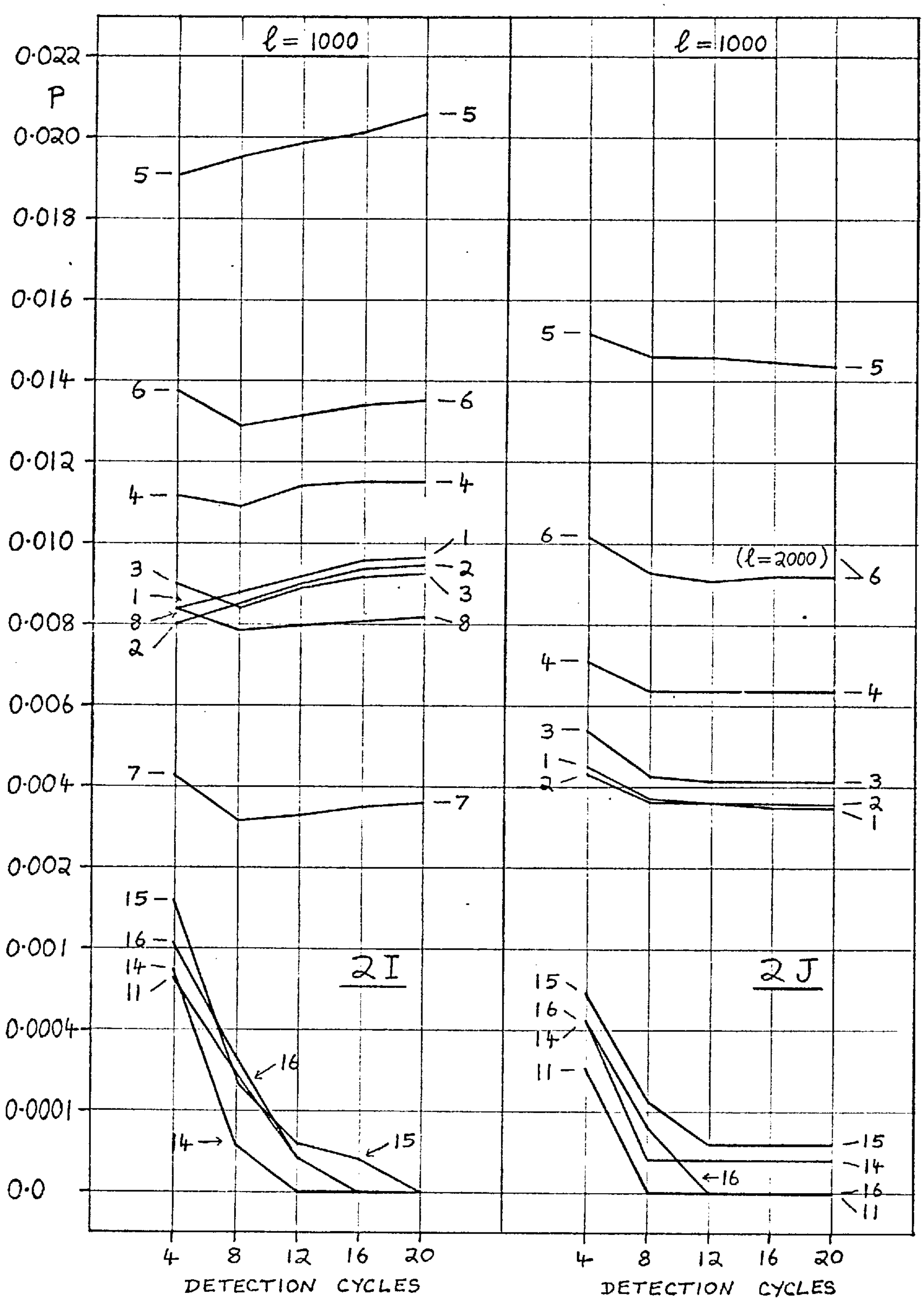


FIG. 26. LINEARLY INDEPENDENT SIGNALS WITH NOISE AND LEVEL VARIATIONS.
DETECTION PROCESSES 2I AND 2J.

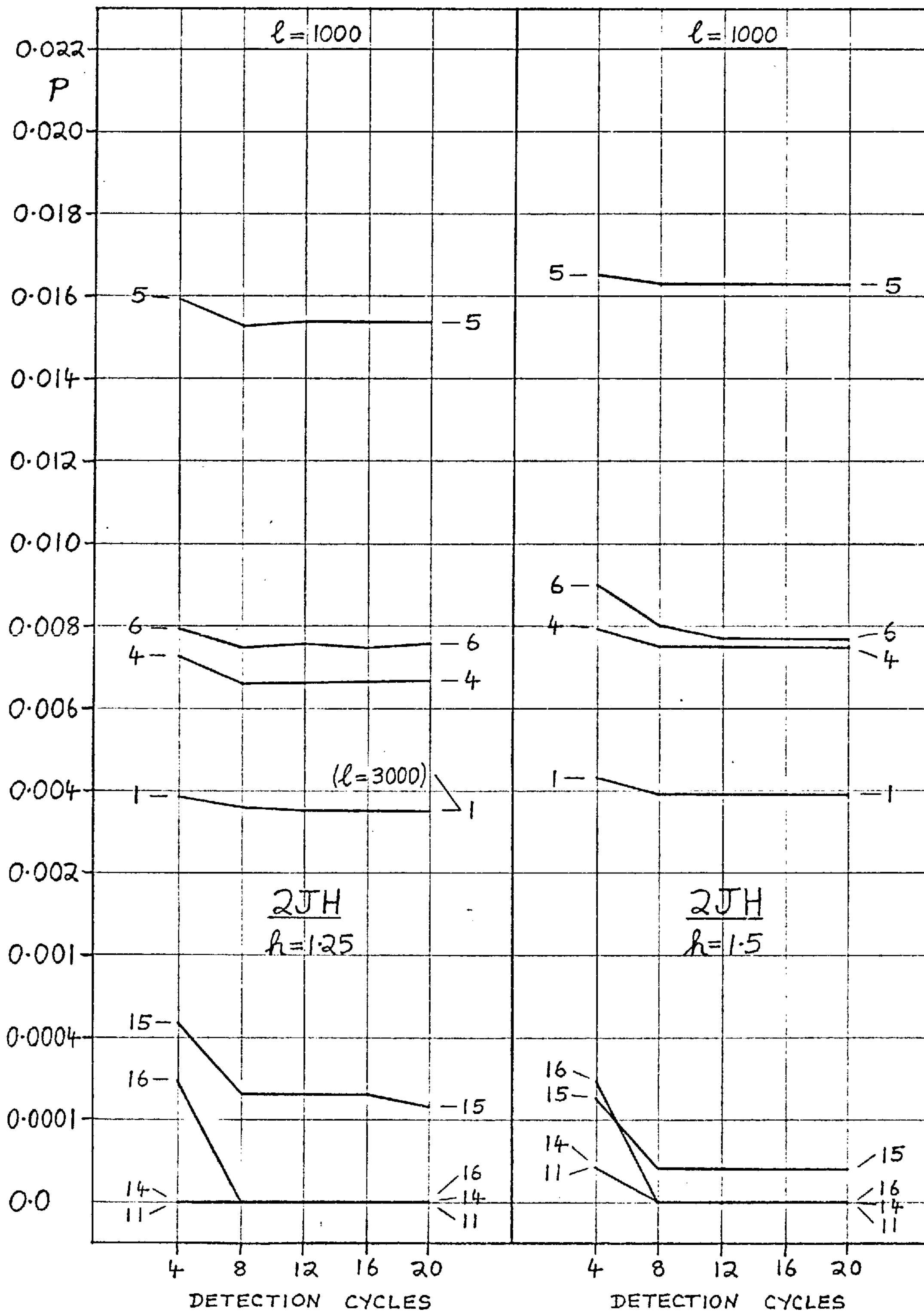


FIG. 27. LINEARLY INDEPENDENT SIGNALS WITH NOISE AND LEVEL VARIATIONS.
DETECTION PROCESS 2JH.

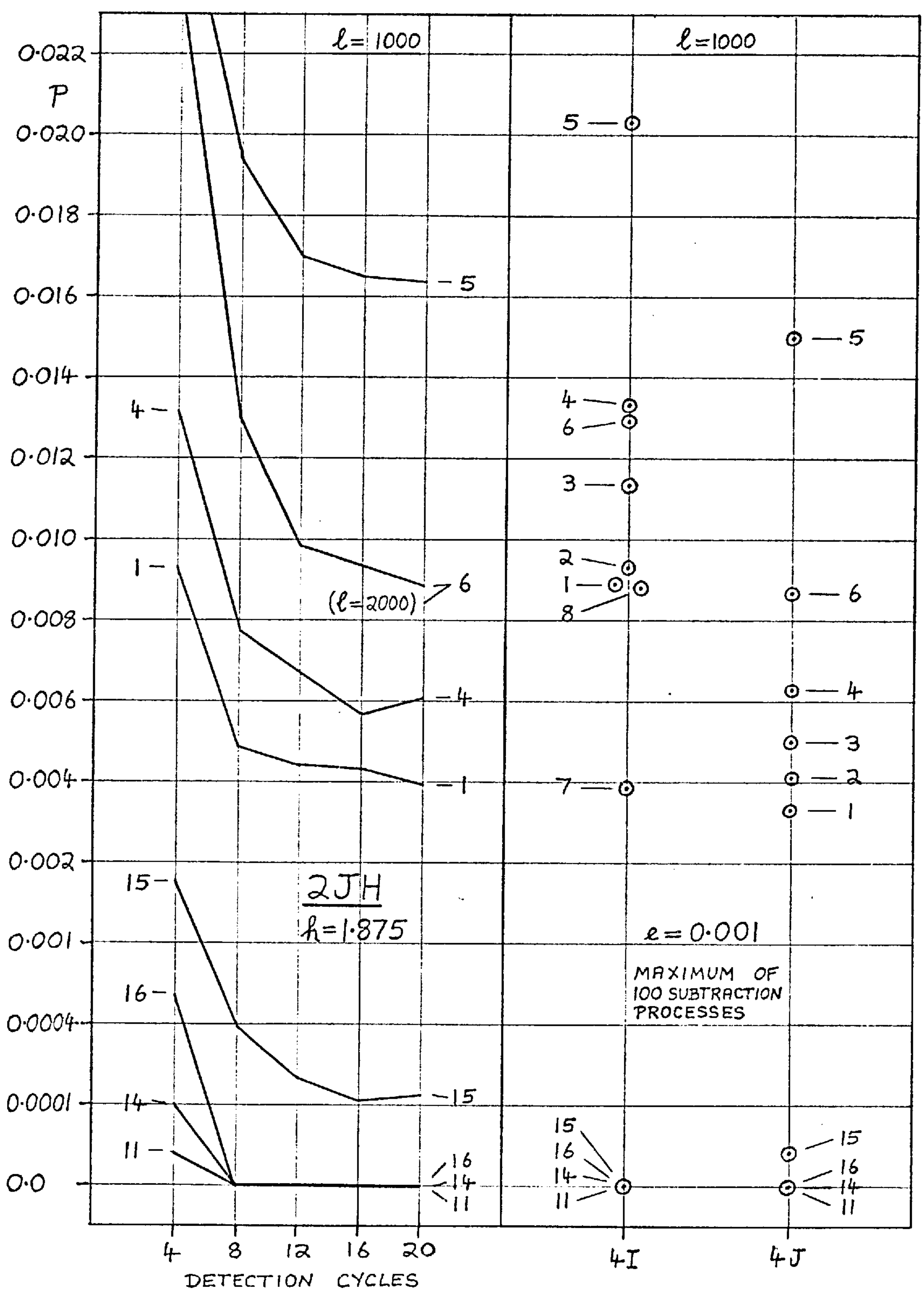


FIG. 28. LINEARLY INDEPENDENT SIGNALS WITH NOISE AND LEVEL VARIATIONS.
DETECTION PROCESSES 2JH, 4I AND 4J.

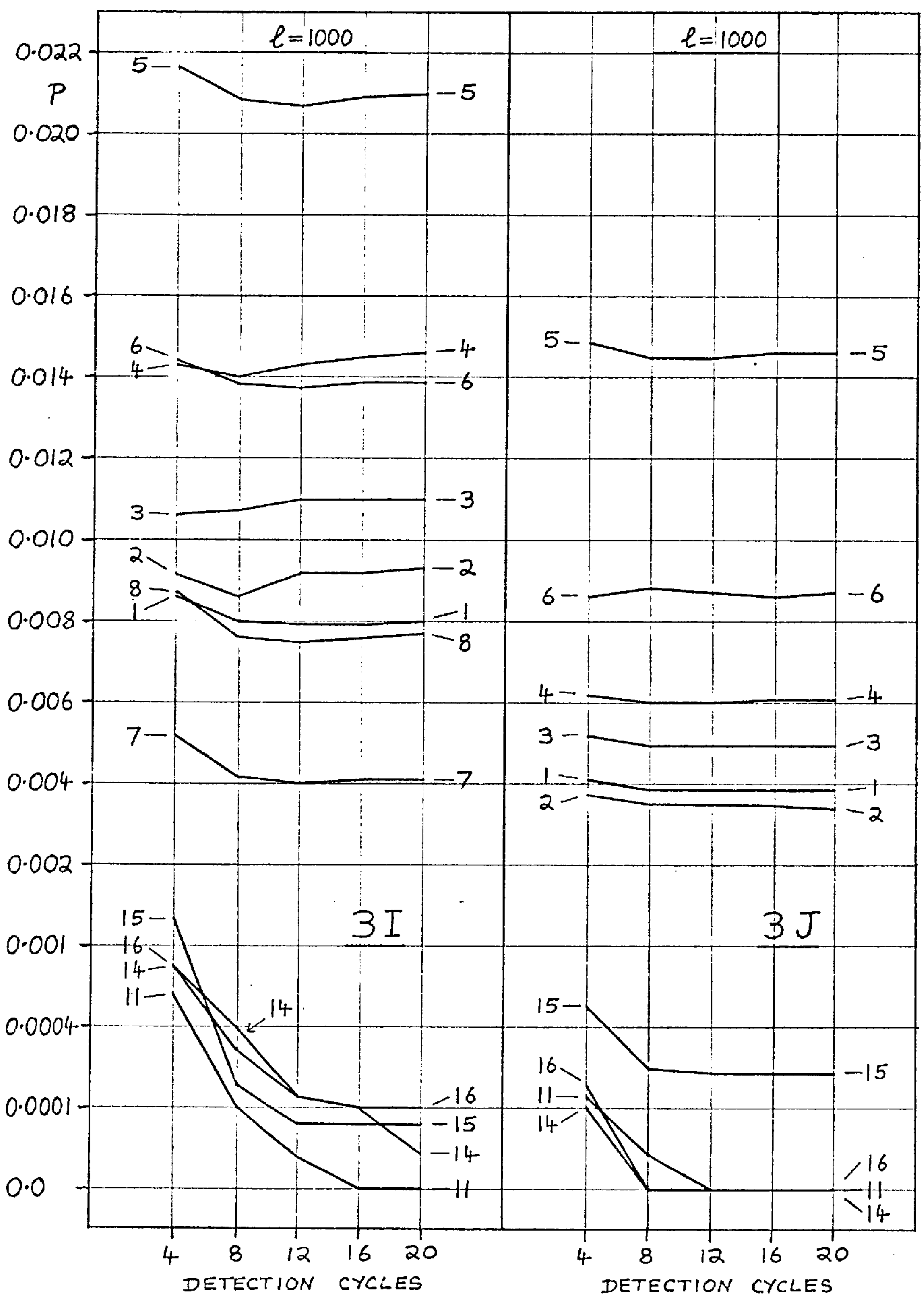


FIG. 29. LINEARLY INDEPENDENT SIGNALS WITH NOISE AND LEVEL VARIATIONS.
DETECTION PROCESSES 3I AND 3J.

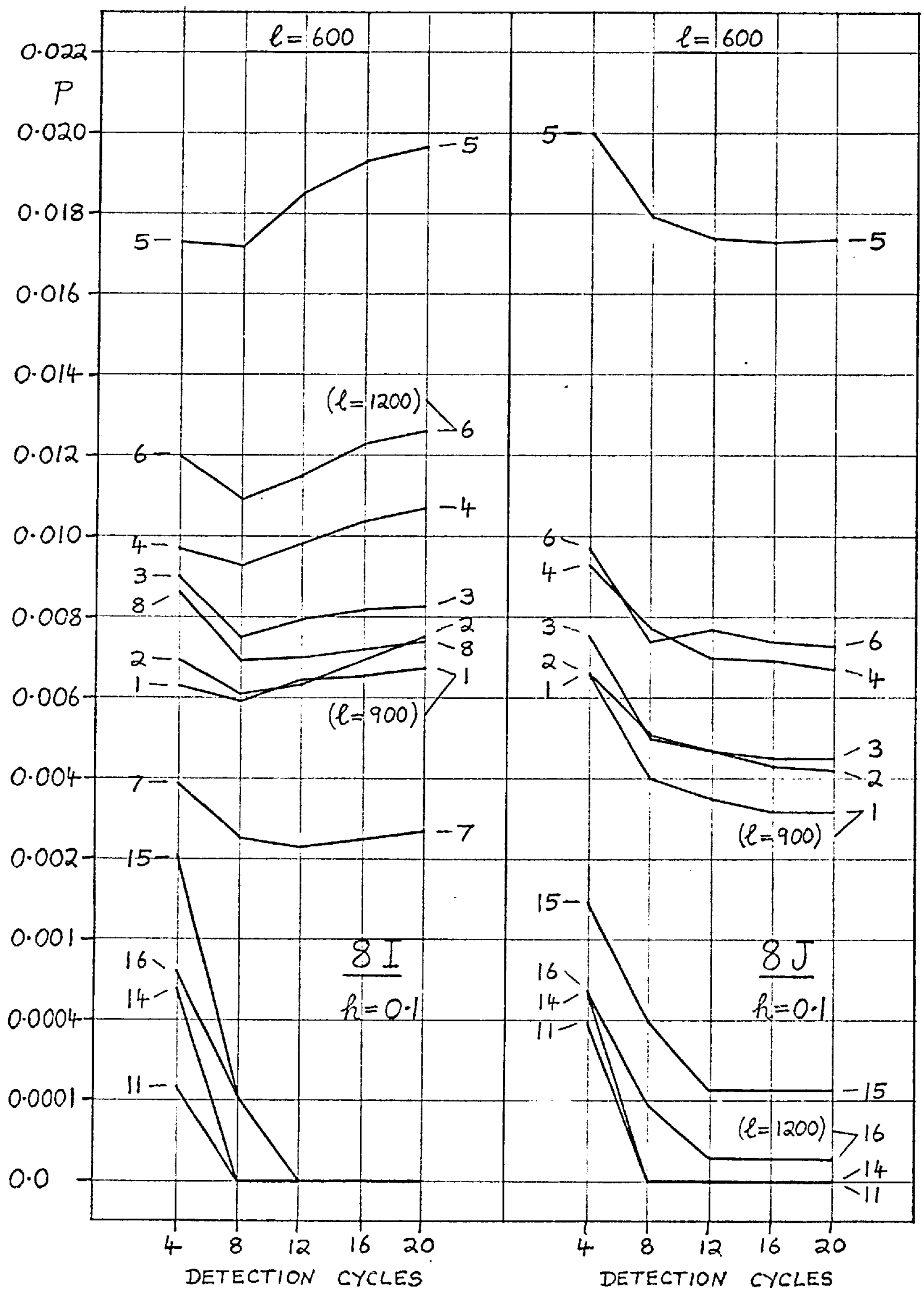


FIG. 30. LINEARLY INDEPENDENT SIGNALS WITH NOISE AND LEVEL VARIATIONS.
DETECTION PROCESSES 8I AND 8J.

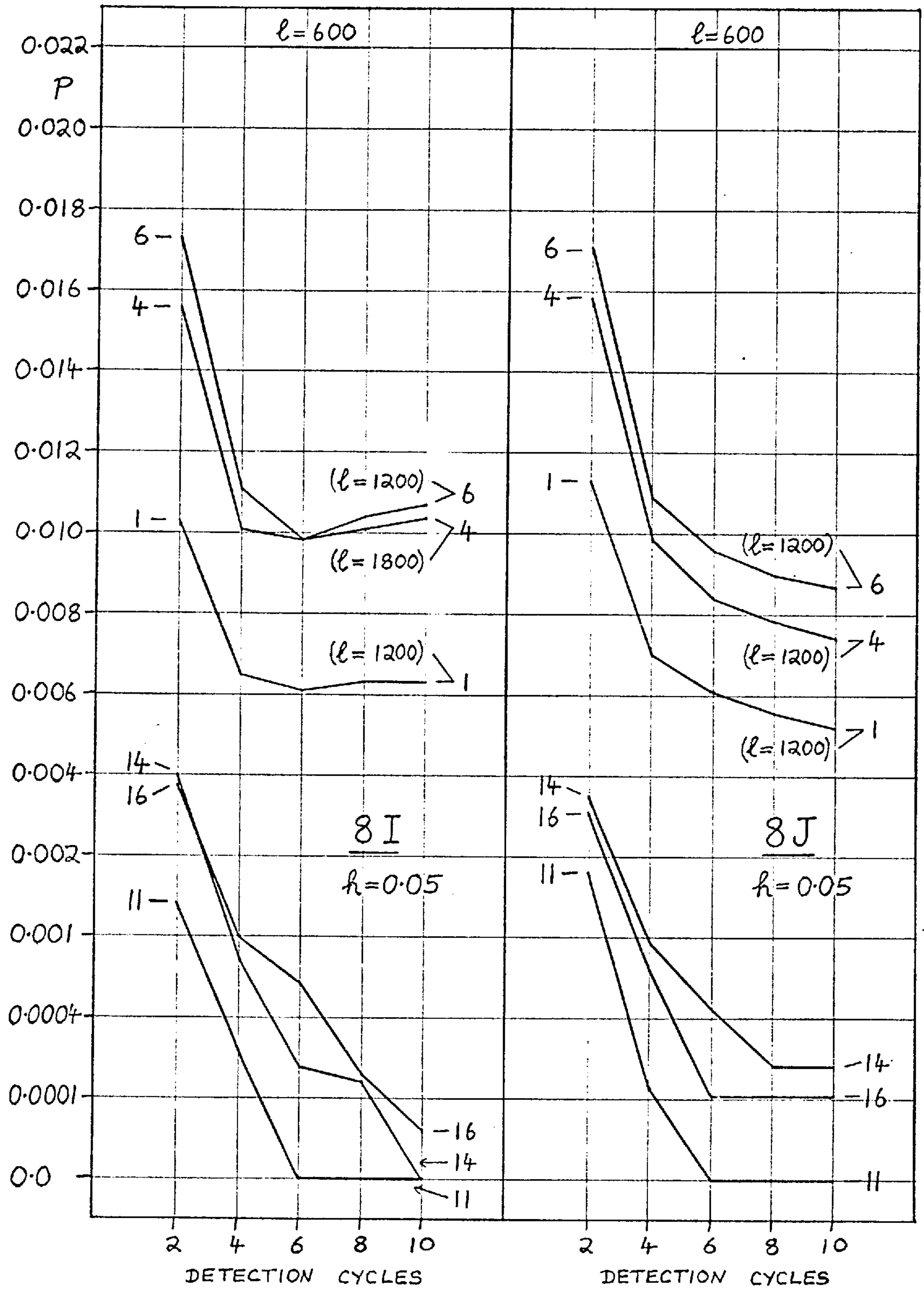


FIG. 31. LINEARLY INDEPENDENT SIGNALS WITH NOISE AND LEVEL VARIATIONS.
DETECTION PROCESSES 8I AND 8J.

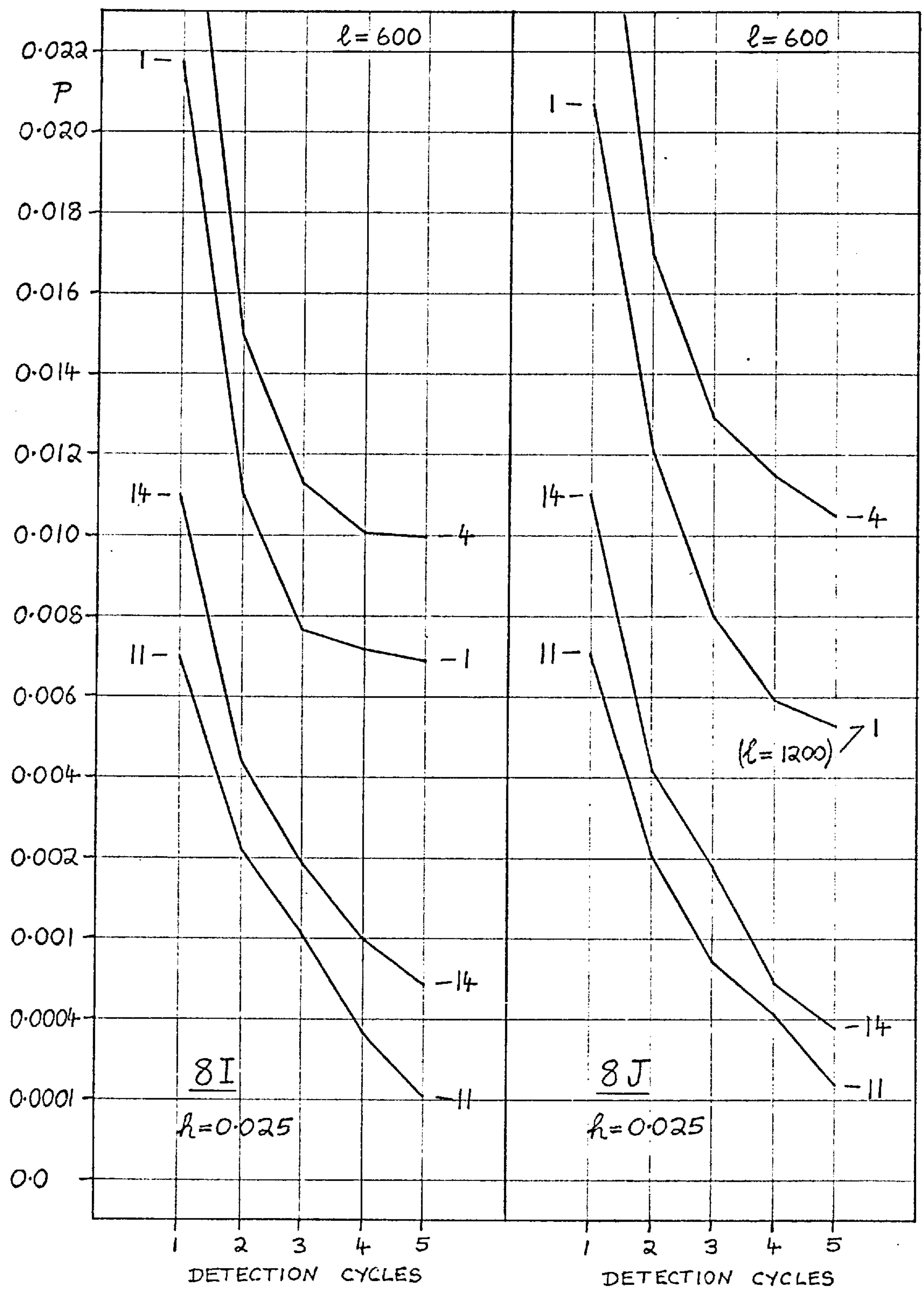


FIG. 32. LINEARLY INDEPENDENT SIGNALS WITH NOISE AND LEVEL VARIATIONS.
DETECTION PROCESSES 8I AND 8J.

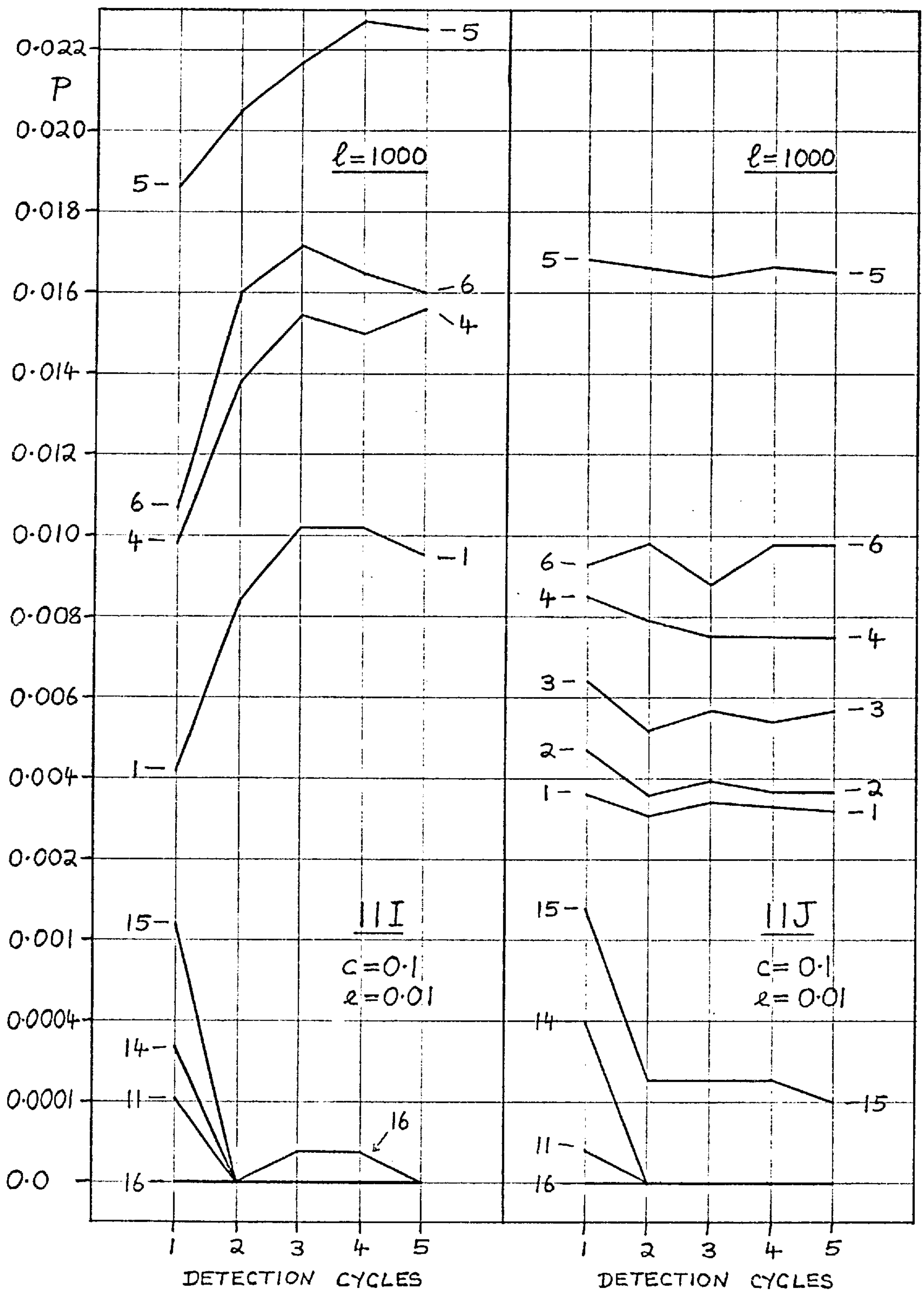


FIG. 33. LINEARLY INDEPENDENT SIGNALS WITH NOISE AND LEVEL VARIATIONS.
DETECTION PROCESSES III AND IIJ.

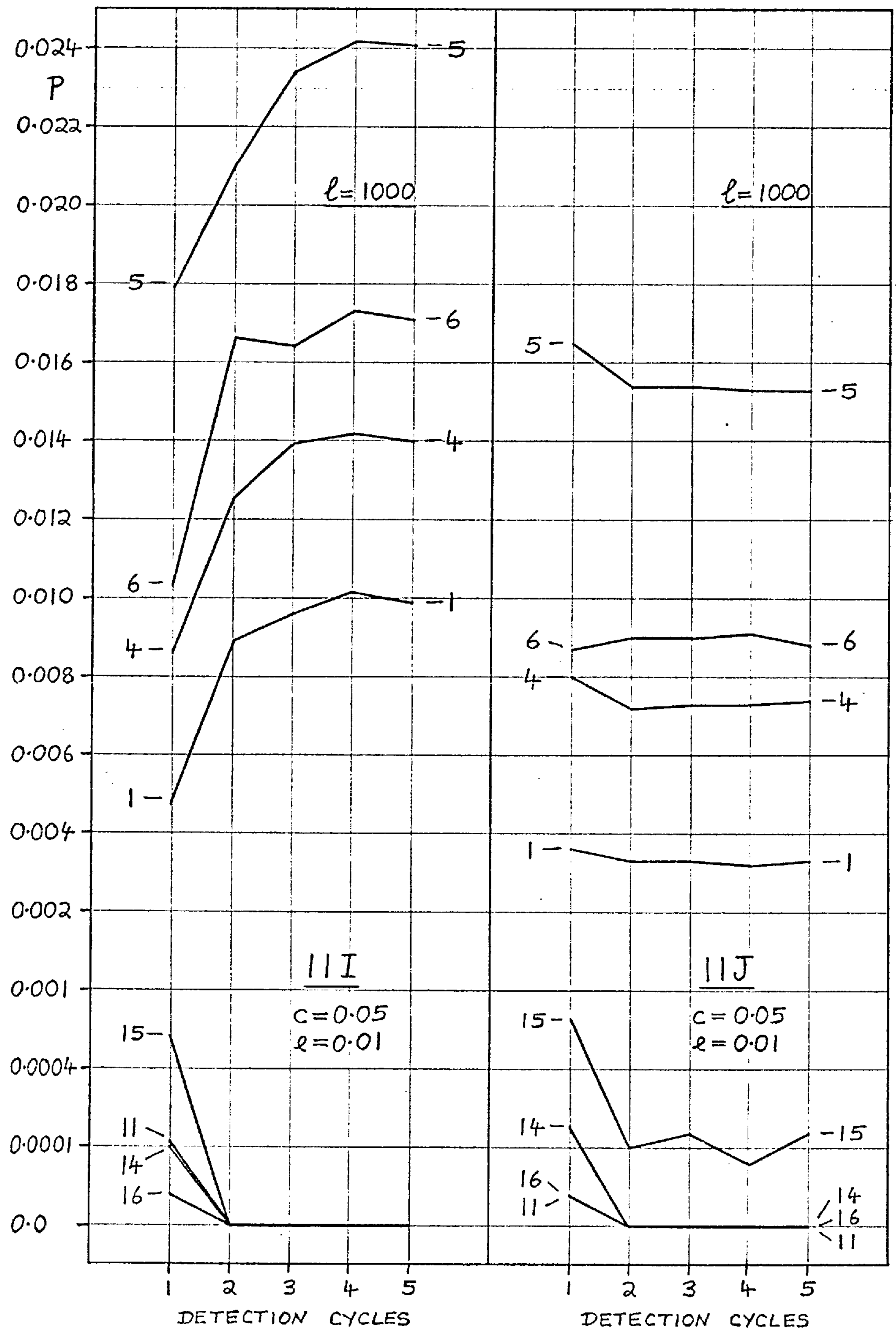


FIG. 34. LINEARLY INDEPENDENT SIGNALS WITH NOISE AND LEVEL VARIATIONS.
DETECTION PROCESSES 11I AND 11J.

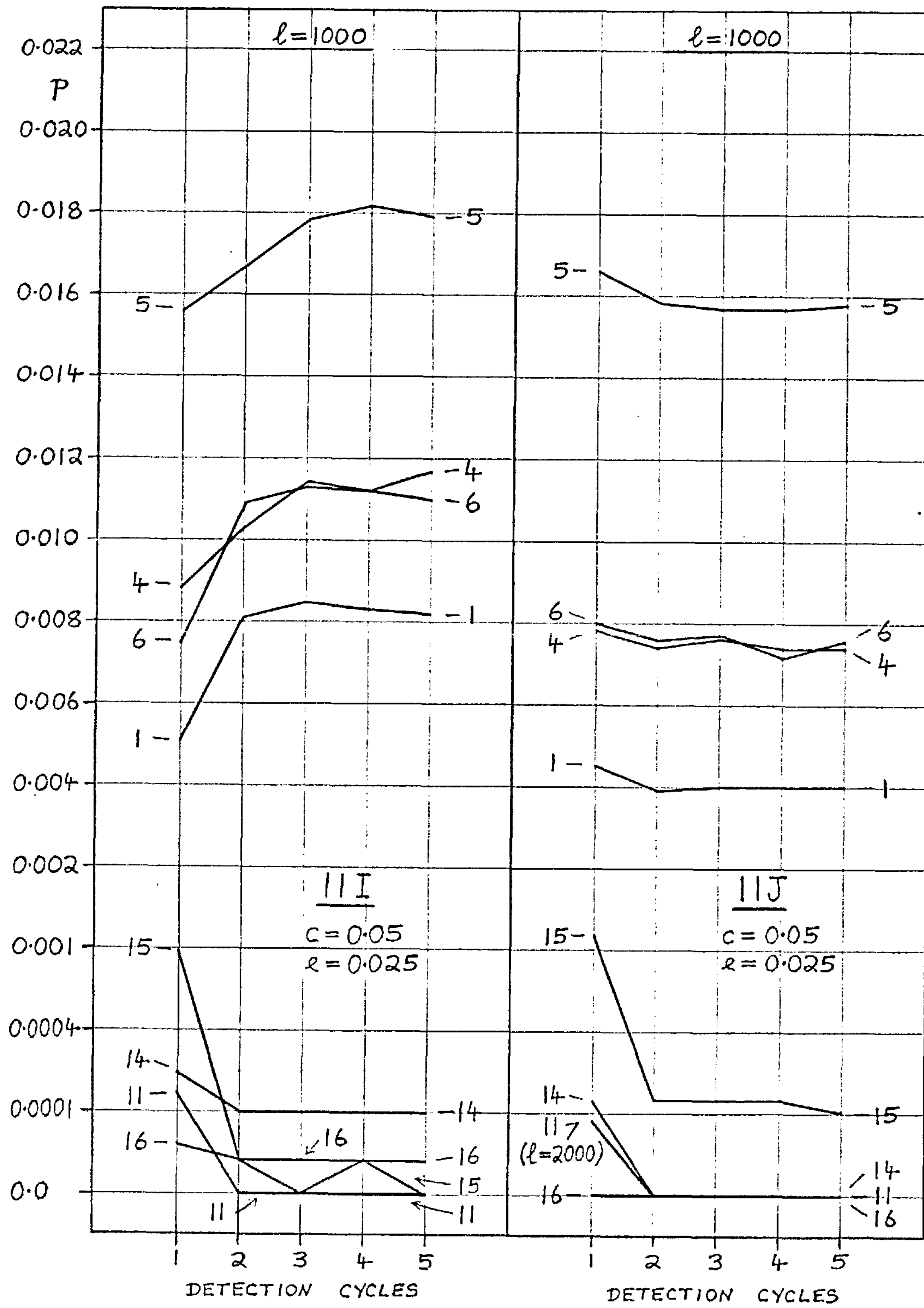


FIG. 35. LINEARLY INDEPENDENT SIGNALS WITH NOISE AND LEVEL VARIATIONS. DETECTION PROCESSES 11I AND 11J.

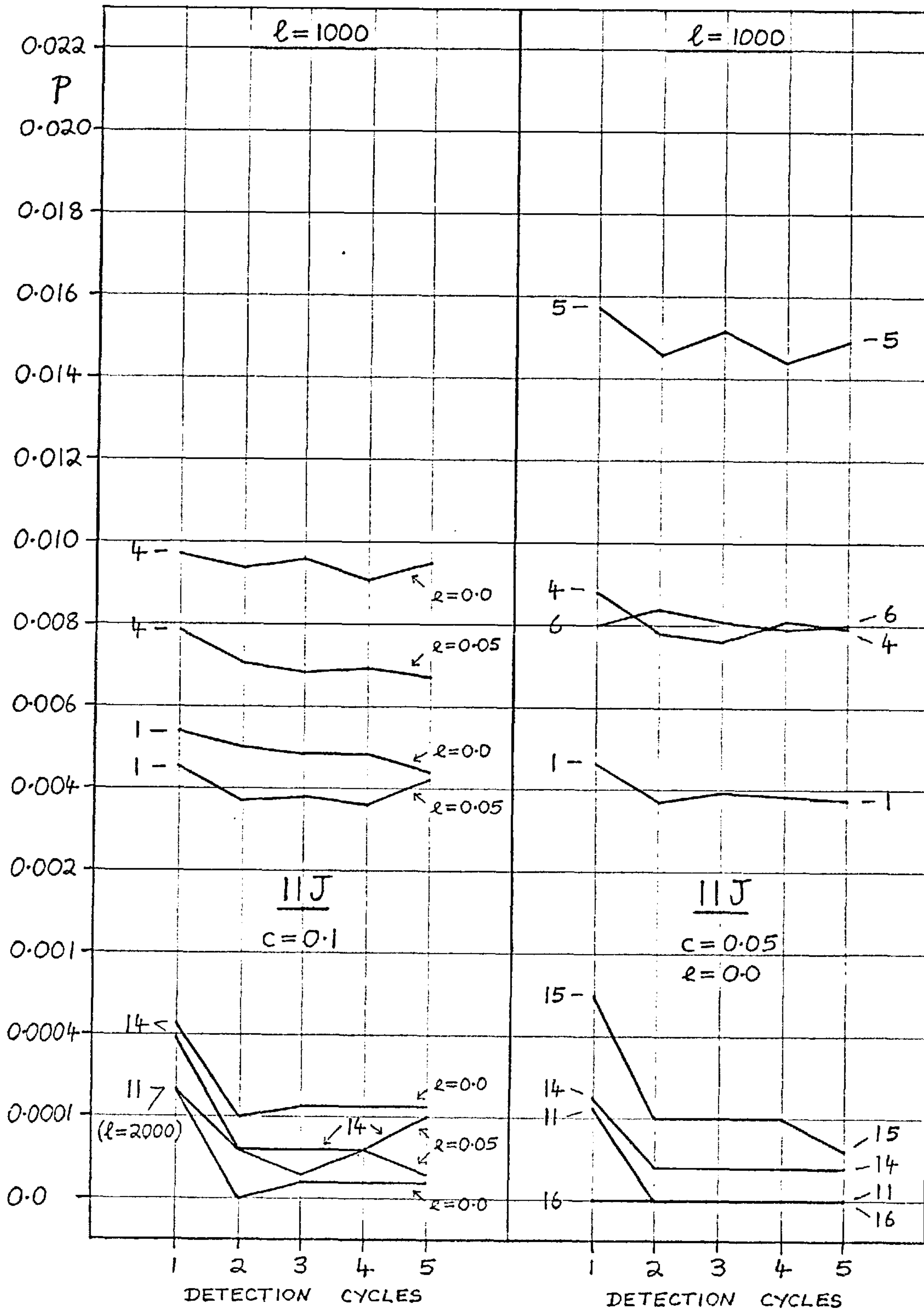


FIG. 36. LINEARLY INDEPENDENT SIGNALS WITH NOISE AND LEVEL VARIATIONS.
DETECTION PROCESS 11J.

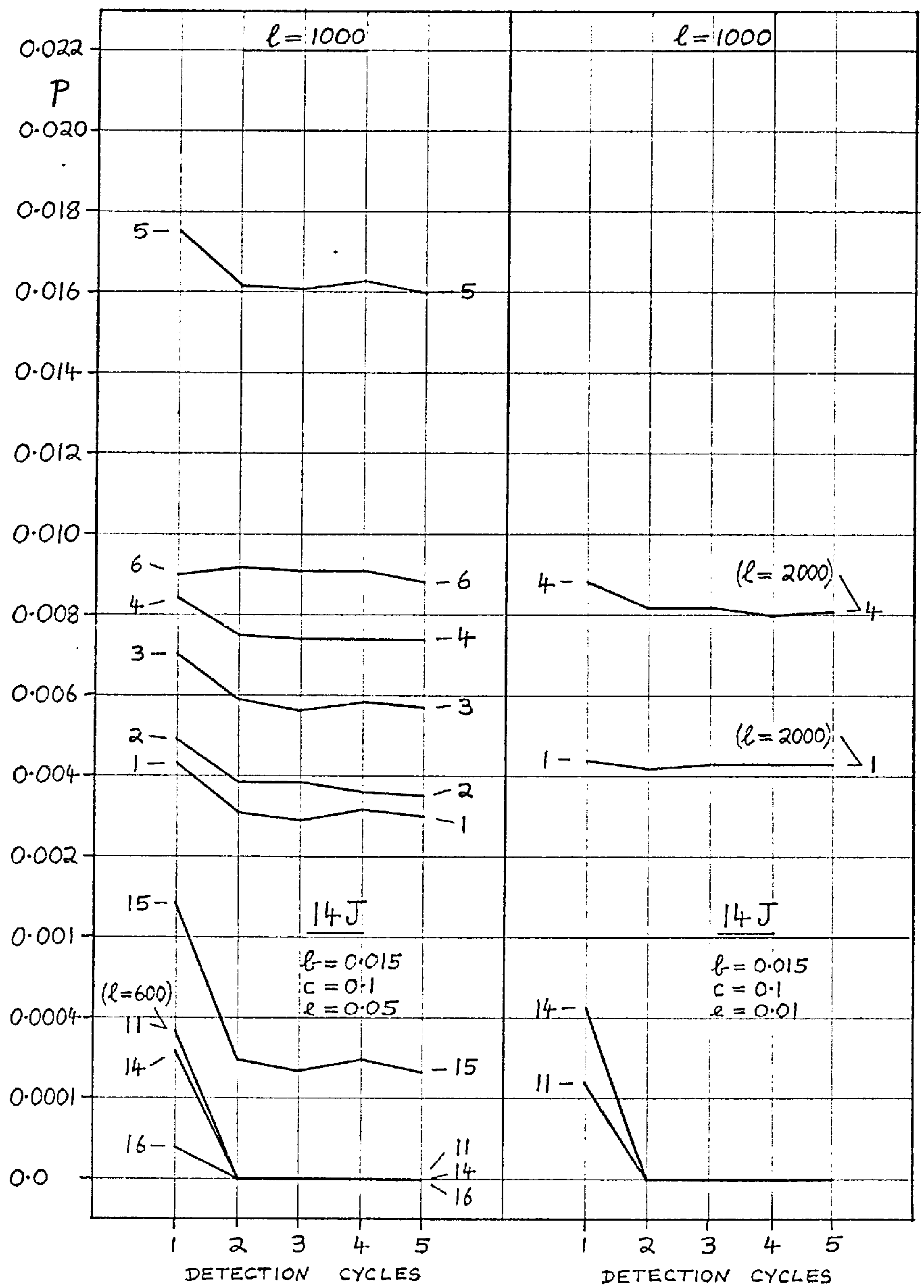


FIG. 37. LINEARLY INDEPENDENT SIGNALS WITH NOISE AND LEVEL VARIATIONS.
DETECTION PROCESS 14J.

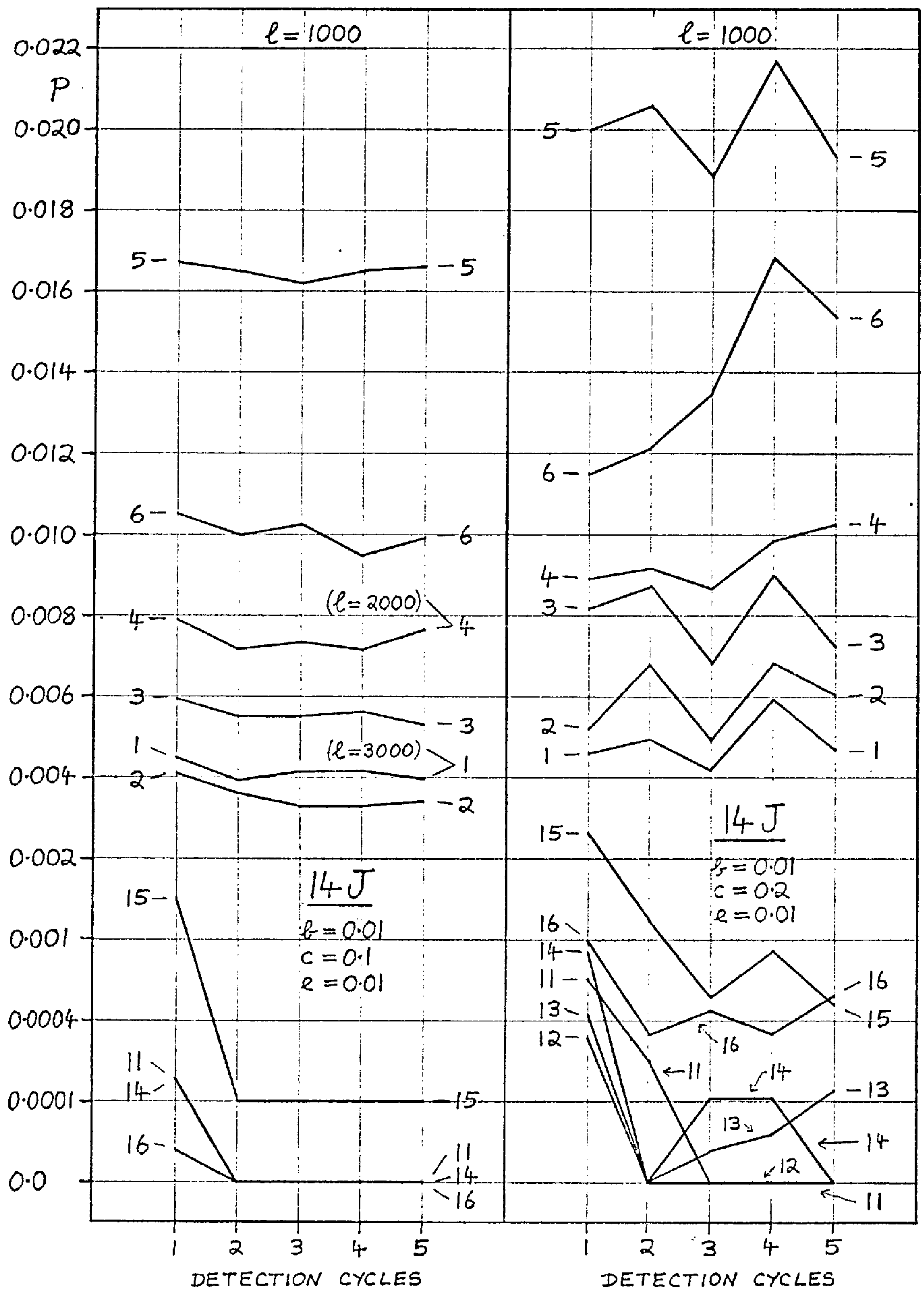


FIG. 38. LINEARLY INDEPENDENT SIGNALS WITH NOISE AND LEVEL VARIATIONS.
DETECTION PROCESS 14J.

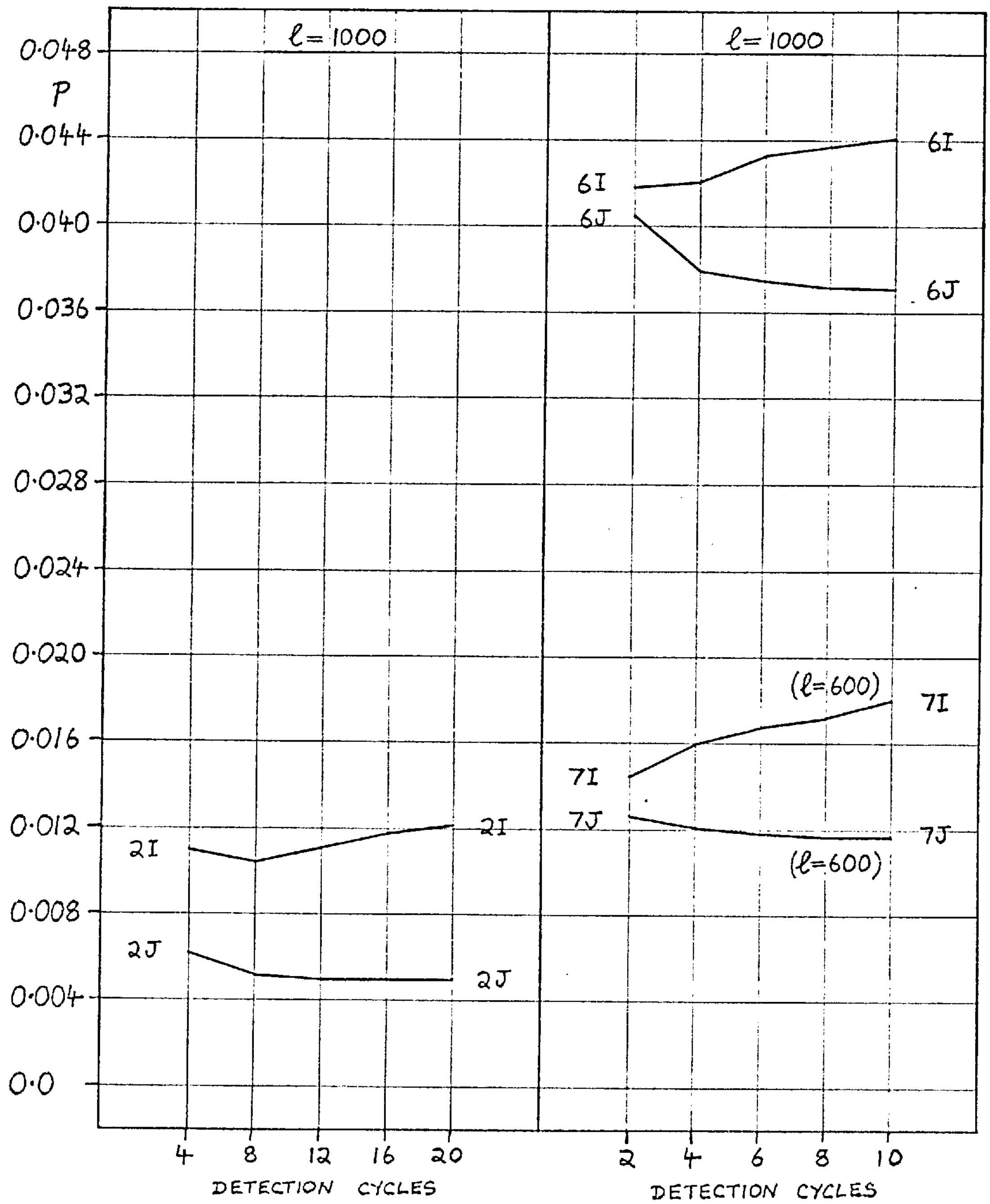


FIG. 39. GENERAL SIGNALS WITH NOISE BUT NO LEVEL VARIATIONS. EQUAL LEVELS. DETECTION PROCESSES : 2I, 2J, 6I, 6J, 7I, 7J.

5.9 Convergence of the Detection Processes 10 to 14

The convergence properties of these systems are considered in detail in Sections 5.6 to 5.8, and are summarised here.

The results of the computer simulation tests without noise, show that under the less extreme conditions tested (Figs. 33 to 38), the detection processes 11, 12 and 14 will converge with either of the constraints I or J and with suitable parameter values. Under the most extreme conditions tested, only 14J converges (Figs. 22 to 25). 10 and 13 do not in general converge with the parameter values tested (Fig. 17), although 13J will converge under the more favourable of the conditions tested. The above conclusions concerning the detection processes 12 and 13, are derived from details of the computer simulation results not shown in Figs. 14 to 39.

The constraint J in general gives a better degree of convergence than the constraint I, for the detection processes 10 to 14, and of these the detection process 14J has by far the best convergence properties.

5.10 Confidence Limits

In Table 1 below, the 95°/$_o$ confidence limits for the values of p in Figs. 14 to 25 are given for different values of p and ℓ, on the assumption that the dependence between the individual element errors in an element detection process, is equivalent to that where the errors always occur in groups of 5 for general signals and in groups of 4 for linearly independent signals. Where p decreases or increases steadily with the number of detection cycles, the confidence limits apply to the value of p at the end of the last detection cycle.

Analysis of the results of the tests by computer simulation, shows that the total number of errors obtained in a test, is approximately given by

$$n = t\,p\,\ell \qquad (190)$$

where t = 100 for Figs. 14 to 20 and t = 80 for Figs. 21 to 25. If the errors are statistically independent, $n > 30$, $p \ll 1$ and an accuracy of no better than say 20°/$_o$ is required for the confidence limits, then it can be assumed that n has a gaussian probability density with a mean $\mu = n$ and a standard deviation $\sigma = \sqrt{n}$. For given values of $p > 0$ and ℓ, the 95°/$_o$ confidence limits for the value of p are approximately

$$\pm \frac{2\sigma}{\mu}p = \pm \frac{2p}{\sqrt{n}} = \pm 2\sqrt{\frac{p}{t\ell}}\ , \qquad (191)$$

$p \backslash \ell$	300	600	1000	1500	2000	3000
0.1	± 0.0082	± 0.0058	± 0.0045	± 0.0037	± 0.0032	± 0.0026
0.03	± 0.0045	± 0.0032	± 0.0024	± 0.0020	± 0.0017	± 0.0014
0.01	± 0.0026	± 0.0018	± 0.0014	± 0.0012	± 0.0010	± 0.00082
0.003	+0.0018 -0.0012	± 0.0010	± 0.00077	± 0.00063	± 0.00055	± 0.00045
0.001	+0.0015 -0.0007	+0.00083 -0.00047	+0.00054 -0.00041	± 0.00037	± 0.00032	± 0.00026
0.0003	+0.00101 -0.00028	+0.00063 -0.00025	+0.00044 -0.00020	+0.00033 -0.00016	+0.00025 -0.00014	+0.00018 -0.00012
0.0001	+0.00050 -0.00010	+0.00050 -0.00010	+0.00032 -0.00009	+0.00024 -0.00009	+0.00020 -0.00008	+0.00015 -0.00007
0.0	0.00040	0.00020	0.00012	0.00008	0.00006	0.00004

TABLE 1 Approximate 95% confidence limits to the value of p, expressed as deviations from the given value of p, for different values of p and ℓ in Figs. 14 to 25.

where the limits are expressed as deviations from the given value of p.

In any test with general or linearly independent signals, where p has not fallen to zero by the end of the last detection cycle and is not steadily decreasing to zero, there is a high degree of dependence between the individual element errors in an element detection process. The result of this dependence is to reduce the effective number of independent errors, n, obtained in the test, and so to widen the confidence limits. For instance, if errors always occur in independent groups of d, with complete dependence within a group, then the effective number of independent errors is $\frac{1}{d}\times$(total number of errors). Thus the confidence limits are multiplied by $\sqrt{d}$, assuming that the effective number of independent errors is not less than 30.

If due account is taken of the effects of non-unique detectability with general signals, it appears that a reasonable approximation for the effective number of independent errors in a test, is $n = 20p\ell$ for both general and linearly independent signals. Thus $t = 20$.

Where $n > 30$ in Table 1, the confidence limits are determined from equation 191, with $t = 20$. Where $n < 30$, the confidence limits are estimated from the correct binomial distribution, assuming $20p\ell$ independent errors and using the results of reference C22. For $p = 0$, it is assumed that the equivalent number of independent elements received in a test is 25ℓ. This number has been estimated from the computer simulation results, which show that for small values of p the error probability only becomes significant for the two or three largest values of m.

It is unlikely that the true confidence limits for any detection process exceed the values given in Table 1, except possibly the confidence limits for 14J and 2JH, which may be slightly wider.

In Figs. 14 to 25, the $95^{\circ}/_{\circ}$ confidence limits for the difference between the values of p for two different detection processes, where these have similar values of p, are approximately 1.4 times the $95^{\circ}/_{\circ}$ confidence limits for the mean value of p.

In Table 2 below, the upper and lower bounds for the $95^{\circ}/_{\circ}$ confidence limits of p are given for Figs. 26 to 39, assuming that $\ell = 1000$. The lower bounds assume that the individual element errors are all statistically independent. In Figs. 26 to 39 the total number of errors obtained in a test is approximately given by $n = 25p\ell$, so that $t = 25$. Thus from equation 191, the lower bounds for the $95^{\circ}/_{\circ}$ confidence limits are approximately given by $\pm 2\sqrt{\frac{p}{25\ell}}$. The upper bounds for the confidence limits are determined by assuming that in an element detection process the individual elements are all detected correctly or incorrectly. Under these conditions, the effective number of independent errors in a test is $n = 5p\ell$, so that $t = 5$. Thus from equation 191, the upper bounds for the confidence limits are given by $\pm 2\sqrt{\frac{p}{5\ell}}$. Where $n < 30$, the correct binomial distribution is assumed for n. For $p = 0$, it is assumed that the equivalent number of independent elements received in a test is bounded by 15ℓ and 3ℓ.

Since the value of t varies somewhat from one detection process to another, the confidence limits in Tables 1 and 2 are only accurate to about $-40^{\circ}/_{\circ}$ to $+60^{\circ}/_{\circ}$, even where the effect of the dependence between element errors is as estimated. They are however adequate for our purposes.

The mean and standard deviation for p, determined from seven graphs

p	0.0	0.0033	0.0067	0.0100	0.0150	0.0200
UPPER BOUNDS	0.0010	+0.0020 -0.0014	$\pm$0.0023	$\pm$0.0028	$\pm$0.0035	$\pm$0.0040
LOWER BOUNDS	0.0002	$\pm$0.0007	$\pm$0.0010	$\pm$0.0013	$\pm$0.0015	$\pm$0.0018

TABLE 2 Upper and lower bounds for the 95% confidence limits to the value of p, expressed as deviations from the given value of p, for different values of p in Figs. 26 to 39.

No. 1 in Figs. 26 to 38, one each for the detection processes 2J, 3J, 4J and 8J, and three for 11J, are $\mu = 0.0035$ and $\sigma = 0.00032$. The corresponding values determined from four graphs No. 1 of each of 2JH and 14J, are respectively $\mu = 0.0038$, $\sigma = 0.00101$ and $\mu = 0.0037$, $\sigma = 0.00102$. The differences between the three mean values of p are not significant. The standard deviation for 14J and 2JH is some three times that for the other detection processes. This is a greater increase than that observed in the absence of noise. If $p = 0.0037$, with statistical independence between the individual element errors, the standard deviation is 0.00038 (assuming $t = 25$ and $\ell = 1000$). It appears therefore that in the presence of additive gaussian noise there is appreciable dependence between the individual element errors in an element detection process for 2JH and 14J, whereas there is little or no dependence for 2J, 3J, 4J, 8J and 11J.

5.11 Effects of the Constraints I and J on the Tolerance to Additive Gaussian Noise

For the detection processes 2, 3, 4 and 8, with no level variations (graphs No. 1), the constraint J improves the tolerance to additive gaussian noise by about $1\frac{1}{2}$ db over that obtained with the constraint I. Where the limits of the level variations are $g_1 = -0.5$ and $g_2 = 0.0$ (graphs No. 6), the constraint J improves the tolerance to gaussian noise by about 1 db. For the other cases tested, the improvement varies somewhere within or very close to the range 1 to $1\frac{1}{2}$ db. The one exception appears to be graph 5 for 8I and 8J in Fig. 30. For the detection process 11, the constraint J achieves a slightly greater advantage over I than that described above, when e = 0.01 (Figs. 33 and 34), and a slightly smaller advantage when c = 0.05 and e = 0.025 (Fig. 35).

The tolerance to additive gaussian noise for all the detection processes 2J, 2JH, 3J, 4J, 8J, 11J and 14J, is essentially the same for any given range of signal level variations, provided only that suitable values are used for h, b, c and e, where appropriate.

For the graph J in Fig. 40 below, the value of p_a is the average value of p at the end of the complete detection process, for the arrangements of the detection processes 2J, 2JH, 3J, 4J, 8J, 11J and 14J in Figs. 26 to 38, which converge. For each value of p_a, the results of altogether 11 different tests have been used, one each for 2J, 3J, 4J and 8J, three for 11J and four each for 2JH and 14J. For the graph I, the values of p_a are determined as before but now for the detection processes 2I, 3I, 4I and 8I, the result of one test for each of these processes being used for each value of p_a. 11I converges to slightly different values of p, and so is not included in these results. The value of ℓ for any value of p used here, is in the range 900 to 1200, being 1000 in most cases.

The estimated 95°/o confidence limits for the values of p_a in the graphs I and J, are approximately ± 0.0018 for graph I and ± 0.0005 for graph J. The 95°/o confidence limits for the values of p corresponding to any value of p_a, are approximately ± 0.0037 for graph I and ± 0.0018 for graph J. For each graph, a gaussian probability density having a given

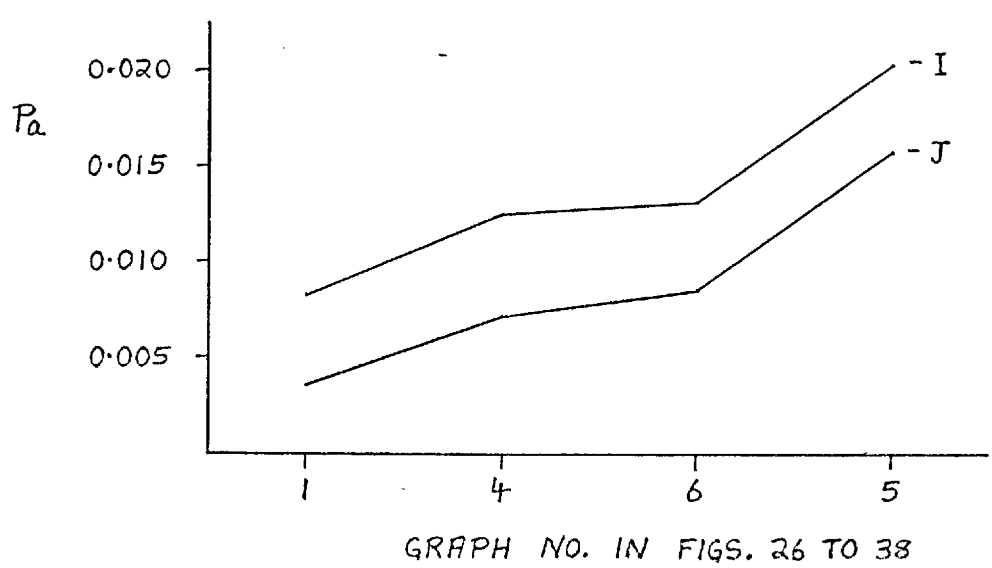


FIG. 40. VALUES OF p_a FOR DIFFERENT CONDITIONS OF NOISE AND LEVEL VARIATIONS, USING LINEARLY INDEPENDENT SIGNALS.

mean and standard deviation is assumed for the values of p corresponding to any value of p_a, and Student's t-test is applied to these.

For the graph J in Fig. 41 below, the value of p_m for each value of m, is the average value of the error probability per channel, determined at the end of the complete detection process, for the arrangements of the detection processes 2J, 2JH, 3J, 4J, 8J, 11J and 14J in Figs. 26 to 38, which converge. For each value of p_m, the results of altogether 19 different tests have been used, one each for 2J, 3J, 4J and 8J, four each for 2JH and 11J and seven for 14J. For the graph I, the values of p_m are determined as before but now for the detection processes 2I, 3I, 4I and 8I, the result of one test for each of these processes being used for each value of p_m. The value of ℓ for any test is in the range 600 to 1200, being 1000 in most cases.

The graph O is the theoretical error-probability per channel, calculated for m orthogonal signals having the same form and signal/noise ratio as the signals for the graphs I and J. The orthogonal signals are assumed to be detected in ideal correlation detectors. The error probability of 0.00078 for these signals, is a little lower than the minimum error probability obtainable for any of the signals used for graphs I and J, since $m \geqslant 2$ and the minimum cross-correlation coefficient for any two of the latter signals is 0.1 and not zero.

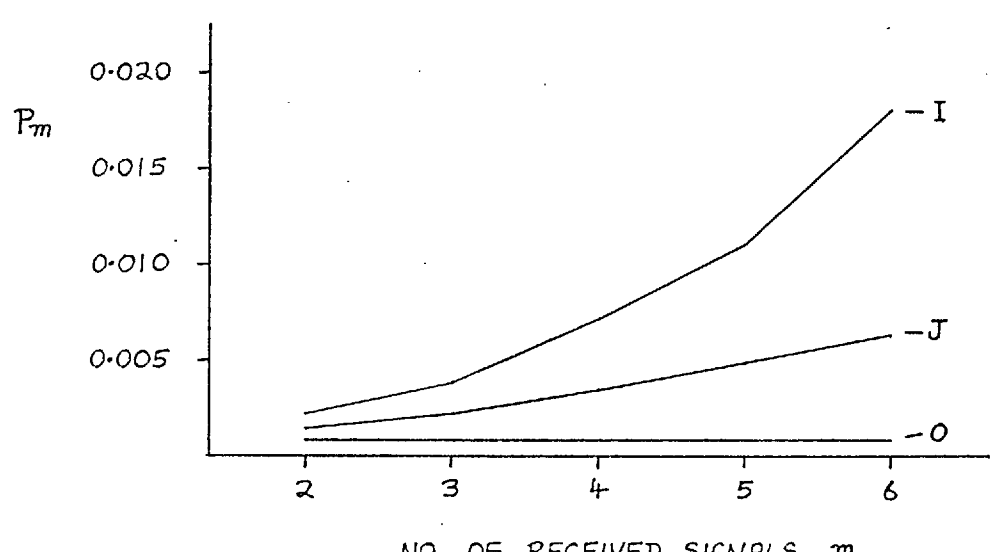


FIG. 41. VARIATION OF p_m WITH m, USING LINEARLY INDEPENDENT SIGNALS WITH NOISE BUT NO LEVEL VARIATIONS. $\sigma_n = 1$.

When m $\ll$ v there is a high probability that the m individual signal-elements selected for a transmitted total-element, will be nearly orthogonal.[D2] Thus it is to be expected that as m is reduced to 2, the value of p_m will fall to a value of the same order as but slightly greater than 0.00078.

The estimated 95°/o confidence limits for the values of p_m in the graphs I and J are approximately ± 0.0039 for graph I and ± 0.0005 for graph J. The 95°/o confidence limits for the individual values of the estimated error-probability per channel, from which any value of p_m is determined, are approximately ± 0.0079 for graph I and ± 0.0021 for graph J. For each graph, a gaussian probability density having a given mean and standard deviation, is assumed for the individual values of the estimated error probability corresponding to any value of p_m, and Student's t-test is applied to these.

Figs. 40 and 41 suggest strongly that the different detection processes with the constraint J, will in general converge to the same solution vector, for the same received signals and noise, that is the same resultant vector R. With the constraint I there is an appreciably wider variation between the performances of the different detection processes than there is with the constraint J, but the agreement is still quite good. Figs. 40 and 41 show a significant and consistent advantage for the constraint J over the constraint I, for all the different conditions tested. This confirms the previous conclusions drawn from Figs. 26 to 39 as well as the simple theoretical analysis in Section 4.5.3.

The analysis in Section 4.5.3 suggests that the constraint J achieves no advantage over the constraint I when the individual received signals are orthogonal, both arrangements being optimum here, but the constraint J should achieve a steadily increasing advantage over I, as the individual signals become less nearly orthogonal. The relatively small advantage in tolerance to additive gaussian noise shown by J over I in Figs. 26 to 38, is consistent with the fact that in these tests the individual signal-vectors are generally nearly orthogonal.

Under the conditions most favourable to the constraint J (graphs 1 and 6 in Figs. 26 to 38), it is therefore likely that for any particular combination of element addresses and signal levels, J will never have an inferior performance to I and may have an advantage in tolerance to additive gaussian noise appreciably greater than that obtained in Figs. 26 to 38. On the other hand, under the most severe conditions of level variations (graphs 4 and 5), J may well result in a poorer performance than I, for certain combinations of element addresses and signal levels. These effects clearly require more detailed investigation.

5.12 Tests on Other Detection Processes

Computer simulation tests were carried out, using general signals with no noise or level variations, on a number of detection processes not so far considered here. The symbols used in this section are as defined in Section 4.0.

In the first of these arrangements studied, the detection process does not use correlation detectors but uses the input error signal

$$G = R - \sum_{i=1}^{m} x_i Y_i \quad = \quad R - YX \ , \qquad (192)$$

which is the difference between the received vector and the estimate of this vector. Before each process of subtraction, a search procedure must be carried out to determine the value of the additional subtracted signal, $w_{ik}Y_i$, which minimizes the length of the error vector, G. The arrangements tested are digital systems using small steps, equivalent to 11I, 11IJ and 11IK with c = 0.1 and e = 0.0, and 11J with c = 0.1 and e = 0.05. In every case only a limited degree of convergence is obtained. The detection process is here attempting to determine the vector X in the matrix equation YX = R. The matrix Y is in general neither square nor symmetric and positive definite, so that complete convergence of the iterative process is not to be expected. The application of Kaczmarz's method to the same situation (detection process 6, Section 4.3.4) has however been shown to converge.

In the second group of detection processes, the magnitude of x_{ik} is increased by a fixed small amount after each process of subtraction involving the i th signal, for all i. This arrangement is of course used with great effect in 14J. It has been applied also to 8J but no improvement in performance appears to be obtained here.

In the third group of detection processes, the constraint J is used but its magnitude instead of being fixed at $|z_i|$ for all i, increases steadily to $|z_i|$ from a very much smaller value, during the detection of each total-element. . This modification has been applied to 8J and 11J. In each case the performance is at least as good as that when the constraint is fixed. More detailed tests are needed to determine whether or not any useful improvement in performance can be obtained by this arrangement.

Other results obtained by means of computer simulation, are as follows. The introduction of a threshold level, e, into the detection processes 1K and 2K, where e steadily decreases from 1 to zero, has little effect on 2K but gives a significant improvement in the degree of convergence of 1K. The improvement is however not sufficient to make 1K an effective system. If 1K and 2K are modified so that the magnitude of the total i th subtracted signal, $x_{ik}Y_i$, instead of being fixed at $|z_i|$ for all i, steadily increases to $|z_i|$ from a very much smaller value, during the detection of each total-element, a significant increase in the degree of convergence is obtained for both detection processes. Since the improvement is obtained during the first two or three detection cycles, this further illustrates the importance of using small steps in purely digital systems.

2JH with h = 0.1 has an appreciably lower rate of convergence than 11J with c = 0.1 and e = 0.01.

6.0 RADAS WITH A SINGLE TRANSMITTER AND MANY RECEIVERS

6.1 Introduction

As in Sections 3.0 to 5.0, only baseband signals are considered and the transmission medium is assumed to introduce no frequency-modulation effects. A total transmitted signal-element, S, is the sum of m individual binary signal-elements, which are in element synchronism and each of which is addressed to a different receiver. At any receiver, a resultant received signal-vector R is given by R = S + N, where N is a noise vector and each vector has v components, as before. The attenuation in transmission is neglected here, since it does not affect the basic operation of the system so long as it is the same for all signals.

6.2 System using Simple Detection Processes

For the most economical arrangement, the design of the receivers should be kept as simple as possible, if necessary at the expense of a more complex transmitter. The simplest receiver design is that where the receiver has only one correlation detector, this being tuned to its element-address Y_i. In order that this detector should detect the received signal-vector R correctly, in the absence of noise, each transmitted signal-element S should be arranged so that the corresponding output-signal from the correlation detector in the i th receiver, is given by

$$d_i = Y_i^T S = z_i , \quad \text{for } i = 1, \ldots, m , \qquad (193)$$

where

$$z_i = \pm k , \qquad \text{for } i = 1, \ldots, m . \qquad (194)$$

k is a constant. The binary value of an individual element is given by the sign of z_i, as before.

In the presence of noise,

$$d_i = Y_i^T R = z_i + Y_i^T N , \quad \text{for } i = 1, \ldots, m . \qquad (195)$$

Except where all the element-addresses $\{ Y_i \}$ are orthogonal,

$$S \neq \sum_{i=1}^{m} z_i Y_i . \qquad (196)$$

Instead, the projection of S onto Y_i is equal to $z_i Y_i$, for all i.

From equation 193,

$$Y^T S = Z , \tag{197}$$

where Y is the v×m real matrix whose i th column is the unit-vector Y_i, and $Z = \left[z_i \right]$ is an m-component column-vector.

From equations 194 and 197, $Y^T S$ has 2^m different possible values, which form the vertices of an m-dimensional hypercube and so span an m-dimensional space. Since the matrix Y remains unchanged for these different values of $Y^T S$, Y must be of rank m. Thus the m element-addresses, $\{Y_i\}$, must be linearly independent. It follows that up to v receivers but no more, can be fed simultaneously from the single transmitter.

One problem in generating the signal S at the transmitter is that S is not uniquely determined by Z for a given Y in equation 197, except when v different element-addresses are being transmitted. When this occurs, Y is non-singular, so that

$$S = (Y^T)^{-1} Z . \tag{198}$$

Thus the difficulty can be overcome by always making up the number of element addresses to v. The v element-addresses must always be linearly independent and the values of z_i for the additional addresses can be set to zero.

The best method of generating S at the transmitter, appears to be as follows. From equation 197,

$$YY^T S = HS = YZ , \tag{199}$$

where Z now has v components, Y is a v×v non-singular matrix and $H = YY^T$ is a real symmetric positive-definite matrix. Thus

$$S = H^{-1} YZ . \tag{200}$$

Clearly $$YZ = \sum_{i=1}^{v} z_i Y_i = \sum_{i=1}^{m} z_i Y_i , \tag{201}$$

since only the first m of the $\{ z_i \}$ are non-zero.

The transmitter generates YZ and performs the linear transformation H^{-1} on the resultant vector. Since the majority of the components of H^{-1} will in general change with any change in the combination of element-addresses $\{Y_i\}$, the use of a linear network for H^{-1} would lead to considerable equipment complexity in an application of RADAS. A better arrangement is to use an iterative process, similar to 151. Instead of the process performing the linear transformation A^{-1} on the vector $Y^T R$ to give the vector X, as in Fig. 12 (Section 4.5.1), it now performs the linear transformation H^{-1} on the vector YZ to give the vector S (equation 200). Thus in Fig. 12, R is replaced by Z, X by S, m by v, and Y_i^T by the i th row of the matrix Y, for i = 1, ..., v. The i th column of Y is the element address of the i th signal, as before. Since H is real, symmetric and positive-definite, the process will always converge (Section 4.5.2). The iterative process could not be used to achieve the simpler linear-transformation $(Y^T)^{-1}$ in equation 198, since Y is not in general positive definite, so that convergence could not be ensured.

The arrangement just described is an adaptive system. The levels of the different individual signals, comprising a total signal-element S, are adjusted to eliminate the effects of interchannel interference. This would otherwise occur whenever these signals are not orthogonal, since the individual signals are detected by means of simple correlation detectors.

The main weakness of this arrangement is that an unduly high transmitted level may be required for S, whenever a resultant vector of a subset of the vectors $\{z_i Y_i\}$ is small compared with the magnitudes of the individual vectors in the subset. This can be avoided if the v element-addresses are always nearly orthogonal. Where the total number of addresses is not much greater than v, or where some form of adaptive coding is used at the transmitter to keep the element-addresses nearly orthogonal, the maximum transmitted level required for S need not be excessive, so that a useful system could be designed.

6.3 AM System

Since the arrangement described in the previous section is an adaptive system, a preferable arrangement is the AM System, described in Section 3.5. The latter can be modified for working with a single transmitter, with only minor changes. Compared with the arrangement in Section 6.2, this involves an appreciably simpler transmitter but more complex receivers. It should in general achieve a useful advantage in tolerance to additive noise, for a given mean transmitted power level.

7.0 MULTI-CHANNEL LINK BETWEEN A SINGLE TRANSMITTER AND A SINGLE RECEIVER

7.1 Introduction

The same basic assumptions are made here as in Section 3.1, concerning the transmitted signals and the transmission medium. If the transmitted signals can be arranged to be linearly independent and of equal level, the detection process 15J (Fig. 12, Section 4.5.1) can be used at the receiver. Alternatively one of the detection processes 14J, 2JH or 11J could be used. The arrangement achieving the best compromise between performance and equipment economy, is probably the AM System (Section 3.5) when suitably adapted for working with a single transmitter and a single receiver.

In the particular case where the total number of different addresses is only twice the number of components in a signal vector, the coding and detection process 16, to be described here, leads to a most effective and simple arrangement of RADAS. As in Sections 4.0 and 5.0, it is assumed that the receiver has prior knowledge of the number and element-addresses of the received signals, and as in Section 6.0, the attenuation in transmission is neglected.

7.2 Orthogonal Sets B and C

The 2v element-addresses $\{ Y_i \}$ are selected from two sets of v orthogonal signals, where v is the number of components in a signal vector. It is assumed that the element energies of all received signals are equal. For the signals in the orthogonal set B, $y_{ii} = 1$ for all i and $y_{ji} = 0$ for all $j \neq i$, where $Y = \left[y_{ji} \right]$ is the v×v matrix whose columns, Y_i, are the element addresses in the orthogonal set B. Thus Y is the v×v identity matrix I. For the signals in the orthogonal set C, the corresponding v×v matrix Y is the orthogonal matrix such that $\left| y_{ji} \right| = \frac{1}{\sqrt{v}}$ for all j and i. When this matrix is multiplied by $\sqrt{v}$, so that $\left| y_{ji} \right| = 1$ for all j and i, it becomes a Hadamard matrix. Such matrices have been discovered for all values of v which are multiples of 4 in the range 4 to 200, with the exception of 116, 156 and 188.[C17] When $v = 2^n$, where n is a positive integer, the rows or columns of the Hadamard matrix become the corresponding Walsh functions, which may be generated in a particularly simple manner.[D2,C18]

It can readily be shown that the orthogonal sets B and C are such that the minimum distance of every member of one set from the v members of the other set, is maximized, this distance being the same for all pairs of signals with one in each set. The optimum detection process for the m received signals, where these are linearly independent and the receiver has no prior knowledge of the values of the $\{|z_i|\}$, is as shown in Fig. 11 (Section 4.1). This detection process determines the value of the vector X in the equation

$$AX = D \,, \tag{202}$$

where the m×m matrix A is given in partitioned form by

$$A = \begin{bmatrix} I_1 & F \\ F^T & I_2 \end{bmatrix} . \tag{203}$$

I_1 and I_2 are identity matrices and all components of F have magnitude $\frac{1}{\sqrt{v}}$. This matrix is an example of a matrix with property A,[G5] and is alternatively described as a consistently ordered 2-cyclic matrix.[A12,G13] The matrix A is of course also real, symmetric and positive definite. It has been shown that for the matrix equation 202, with the matrix A given by equation 203, both the point Gauss-Seidel and the point Jacobi iterative processes will converge.[A12,G5,G13] Thus the detection processes 1 and 2 should both be effective here. Furthermore, any detection process determining the vector X in equation 202, is appreciably simplified by the fact that the received signals belong to two orthogonal sets.

The weakness of the arrangement, when $v = 2^{2n}$ where n is a positive integer, is that when there are many more than $\sqrt{v}$ signals in each orthogonal set, there is a high probability that the signals are linearly dependent and there is an appreciable probability that they are not uniquely detectable. For other values of $v \gg 1$, such that the signals are always uniquely detectable, there is an appreciable probability of a very low tolerance to additive noise, so that the guarantee of unique detectability is of no real practical advantage.

Various sets of orthogonal signals suitable for applications of RADAS are considered in some detail in reference B4.

7.3 Coding and Detection Process 16

A better way of using the signals in the two orthogonal sets is to transmit the signals in set C as the envelope of the signals in set B. For each total-element transmitted, the signals in sets B and C are first generated as before, the sum of the signals in set B being stored separately from the sum of the signals in set C. If the j th digit (component) of the sum of the signals in set C is negative, its sign is arranged to be the opposite to that of the coincident signal in set B, and if it is positive its sign is arranged to be the same. If there is no coincident signal in set B, the j th digit is left unchanged. The above procedure is carried out for $j = 1, \ldots, v$. All individual signal-elements have equal levels, such that $|z_i| = \sqrt{v}$ for all i in each orthogonal set.

The receiver uses correlation detectors as in Fig. 11 and the detection process is a modification of the process 2K. In the first detection cycle for a total element at the receiver, the received signals in set B are detected first, by determining the signs of the appropriate digits. Since these signals are orthogonal they may be detected sequentially or simultaneously, whichever is the most convenient. If any of these digits are negative, their signs are now changed to make them positive. The magnitude, $\sqrt{v}$, of a signal in set B, is then subtracted from each of them. If all received signals in set B have been correctly detected, the resultant signal is the sum of all received signals in set C as first generated at the transmitter, together with a noise vector. This signal is fed to the appropriate correlation detectors, where the individual signals are detected.

In the second detection cycle, the detected binary value of each received signal in set C is used to generate the corresponding signal vector, and these vectors are then added together to regenerate the sum of the received signals in set C, as used to amplitude modulate the signals in set B. If, coincident with any received signal in set B, there is a negative digit in this regenerated signal, with a magnitude greater than $\sqrt{v}$, then the detected binary value of the corresponding signal in set B is changed from its value determined in the first detection cycle. At the same time, in the circuit which changes the sign of a negative received digit where this coincides with a received signal in set B, the sign of the received digit is now left unchanged. As before, $\sqrt{v}$ is subtracted from the value of each of these digits and the resultant digits together with the remaining received digits are fed to the correlation detectors for the received signals in set C. The outputs of the latter are then sampled to determine their binary values. The above procedure is repeated for each subsequent detection cycle. In practice only two detection cycles would probably be used, since no useful reduction in the error probability per channel appears to be achieved by the subsequent detection cycles.

A signal in set B is not uniquely detectable if the coincident digit of the sum of all received signals in set C, has the opposite sign and same magnitude. If this condition is not satisfied by the sum of all received signals in set C but only by a subset of these, the coincident signal in set B is in general uniquely detectable. By suitably selecting the number of components in a signal vector, for instance $v = 12$ or 20, and maintaining the signal levels equal, it is possible to ensure that the unique detectability of the received signals in set B is not prevented by the effect described. However a better minimum tolerance to additive noise is obtained if $v = 2^{2n}$ and the total number of signals in set C is arranged always to be odd. This is achieved by transmitting an additional signal when necessary.

When the coding and detection process 16 is not used and signals in the orthogonal sets B and C are transmitted, with $v = 16$, a received signal is not uniquely detectable if it is in a subset of received signals in the same orthogonal set, which together with a subset of received signals in the other set, add to give a zero resultant vector (Section 3.2). Clearly this cannot be prevented by using only an odd number of signals in the set B or C, or by some other equivalent arrangement. However it can be avoided by changing v to say 12 or 20, but at the expense of a low minimum tolerance to additive noise. With $v = 16$, the probability of a signal in either orthogonal set here being not uniquely detectable, appears in general to be considerably higher than for the coding and detection process 16 with an odd number of signals in set C.

7.4 Tests by Computer Simulation

Figs. 42 and 43 compare the performance of the coding and detection process 16 with the performances of various detection processes, where the latter use signals in the orthogonal sets B and C as described in Section 7.2. In every case $|z_i| = 4$ for all i in each set.

In these and all subsequent tests, $v = 16$, so that there are altogether 16 different element-addresses in each orthogonal set. Furthermore, an arrangement of RASSAS is used, so that when there are m_B and m_C received signals respectively in the orthogonal sets B and C, a random selection is made of these numbers of element addresses from the two orthogonal sets, for each new total signal-element transmitted. The $\{z_i\}$ of the different individual signal-elements are statistically independent and equally likely to have either binary value.

For each pair of values of m_B and m_C, ℓ total signal-elements are transmitted, and the value of p, determined independently for the signals in each orthogonal set, is the average of the estimates of the error probability per channel, obtained for the different combinations of m_B and m_C. In Fig. 42 there are 25 such combinations and in Fig. 43 only 4. The values of ℓ used, are shown as for Figs. 14 to 39.

It is clear from Figs. 42 and 43 that the coding and detection process 16 not only converges to much lower values of p than the other arrangements tested, but it does so much more rapidly. It is also interesting to observe here that for a given constraint I, J or K, the detection processes 1 and 2 have similar convergence properties.

In Fig. 44 the parameters associated with the different graphs are as follows:-

	Set B		Set C		ℓ
	m_B	$\lvert z_i \rvert$	m_C	$\lvert z_i \rvert$	
1	odd	4	odd	4	200
2	even	4	even	4	200
3	even	4	odd	4	100
4	odd	4	even	4	100
5	odd	6	odd	4	100
6	even	6	even	4	100
7	odd	8	odd	4	100
8	even	8	even	4	100

Where m_B is marked "odd", p is determined from all combinations of m_B and m_C such that m_B has an odd value in the range 7 to 15. Where m_B is marked "even", p is determined from all combinations of m_B and m_C such that m_B has an even value in the range 8 to 16. Similarly for m_C.

These tests, together with more detailed tests with ℓ = 1000 whose results are not shown here, suggest that when the levels of the signals in set C are 6 db or more below the levels of the signals in set B, the levels in each set being equal, then in the absence of noise there are no errors in detection for the process 16, so long as $\lvert z_i \rvert$ for each set is even and m_C is always odd.

In Figs. 45 and 46 the letter B or C against a graph indicates the orthogonal set to which the values of p apply. The parameters associated with the different graphs here are as follows:-

	Set B m_B	Set B $\lvert z_i \rvert$	Set C $\lvert z_i \rvert$	σ_n
B1, C1	16	1.0	0.4000	0.125
B2, C2	8	1.0	0.4000	0.125
B3, C3	16	1.0	0.3636	0.125
B4, C4	8	1.0	0.3636	0.125
B5, C5	16	1.0	0.4000	0.000
B6, C6	16	1.0	0.3636	0.000

σ_n^2 is the variance of the sample values of the additive white gaussian noise.

Figs. 45 and 46 give the values of p at the end of the first and second detection cycles respectively. The values of p plotted here are the estimates of the error probability per channel for given values of m_B and m_C. The values of p originally determined for each of the graphs C1 to C4 remain approximately the same, regardless of the number of received signals in set C. Thus to simplify Figs. 45 and 46, the graphs plotted for C1 to C4 show in each case a constant value of p, which is its average value for all values of m_C from 1 to 16.

In Table 3 below, the 95°/o confidence limits for the values of p in Figs. 42 to 46 are given for different values of p and ℓ . The confidence limits are calculated in the same way as those in Tables 1 and 2 (Section 5.10), so that these are only approximate estimates and assume that in the detection of a total element, errors always occur in groups of 4. With the degree of dependence likely to be obtained between the individual element errors, it is unlikely that the true confidence limits will exceed or even reach the values given in Table 3. The latter may therefore be taken as approximate upper bounds to the true confidence limits. Alternatively they may be taken as the 95°/o confidence limits for the difference between the values of p, for two different detection processes having similar values of p.

The confidence limits for the graphs C1 to C4 in Table 4 below, have been estimated from the individual measured values, assuming that these have a gaussian probability density.

It can readily be shown that for the signal/noise ratio which applies to the graphs C1 and C2, the probability of an element error in the detection of a signal in set C, assuming that no other signals are received, is 0.00069.

152

p	FIG. 42		FIG. 43	FIG. 44		FIGS.45 & 46	
	ℓ =100 (t=82.5)	ℓ =200 (t=82.5)	ℓ =1000 (t=8.5)	ℓ =100 (t=82.5)	ℓ =200 (t=82.5)	B1, B3, B5 & B6 (t=4)	B2 & B4 (t=2)
0.1	±0.0070	±0.0049	±0.0069	±0.0070	±0.0049	-	-
0.03	±0.0038	±0.0027	±0.0038	±0.0038	±0.0027	-	-
0.01	±0.0022	±0.0016	±0.0022	±0.0022	±0.0016	±0.0032	+0.0054 -0.0041
0.003	+0.0014 -0.0011	±0.00085	+0.0014 -0.0011	+0.0014 -0.0011	±0.00085	+0.0025 -0.0014	+0.0044 -0.0020
0.001	-	-	+0.0011 -0.0005	+0.0012 -0.0006	+0.00070 -0.00042	+0.0020 -0.0008	+0.0032 -0.0009
0.0003	-	-	-	+0.00081 -0.00027	+0.00051 -0.00021	+0.0015 -0.0003	+0.0015 -0.0003
0.0001	-	-	-	+0.00050 -0.00010	+0.00035 -0.00010	+0.00050 -0.00010	+0.00050 -0.00010
0.0	-	-	0.00019	0.00038	0.00019	0.00019	0.00038

TABLE 3 Approximate 95°/₀ confidence limits to the value of p, for different values of p and ℓ in Figs. 42 to 46.

GRAPH	FIG. 45	FIG. 46
C1, C2	±0.00012	±0.00010
C3, C4	±0.00023	±0.00022
C5, C6	0.00008	0.00008

TABLE 4 ± 95°/₀ confidence limits for the values of p in the graphs C1 to C6 in Figs. 45 and 46.

For C3 and C4 it is 0.00181. The signals are here assumed to be detected in ideal correlation detectors. For the conditions tested and within the confidence limits for the results of these tests, it therefore appears that in the coding and detection process 16, the tolerance to noise of the signals in set C is not significantly different from that of these signals when transmitted and detected with no other signals present.

Fig. 46 suggests that when there are normally around 8 received signals in set C, the best overall tolerance to additive gaussian noise should be obtained with $|z_i| = 0.4$ for all i in set C, and an odd number of signals in this set. When there are normally around 12 signals in set C, the best overall tolerance to additive gaussian noise should be obtained with $|z_i| = 0.3636$ for all i in set C, and an even number of signals in this set. It is assumed that $|z_i| = 1.0$ for all i in set B, and that there are normally as many or more signals in set B as there are in set C.

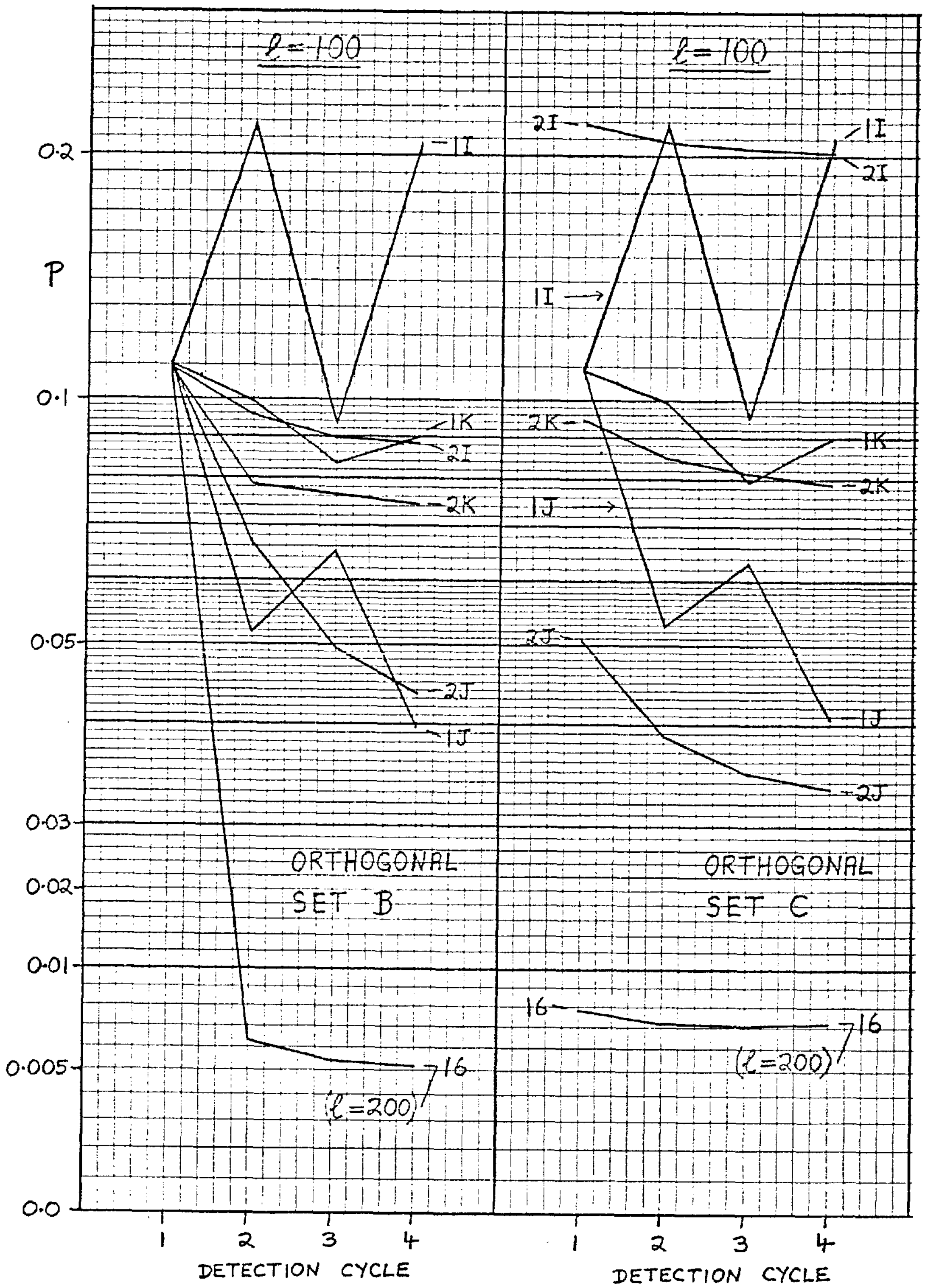


FIG. 42. ORTHOGONAL SETS B AND C WITH NO NOISE OR LEVEL VARIATIONS. EQUAL LEVELS. P AVERAGED OVER ALL COMBINATIONS OF ODD NUMBERS IN THE TWO SETS, BETWEEN 7 AND 15.

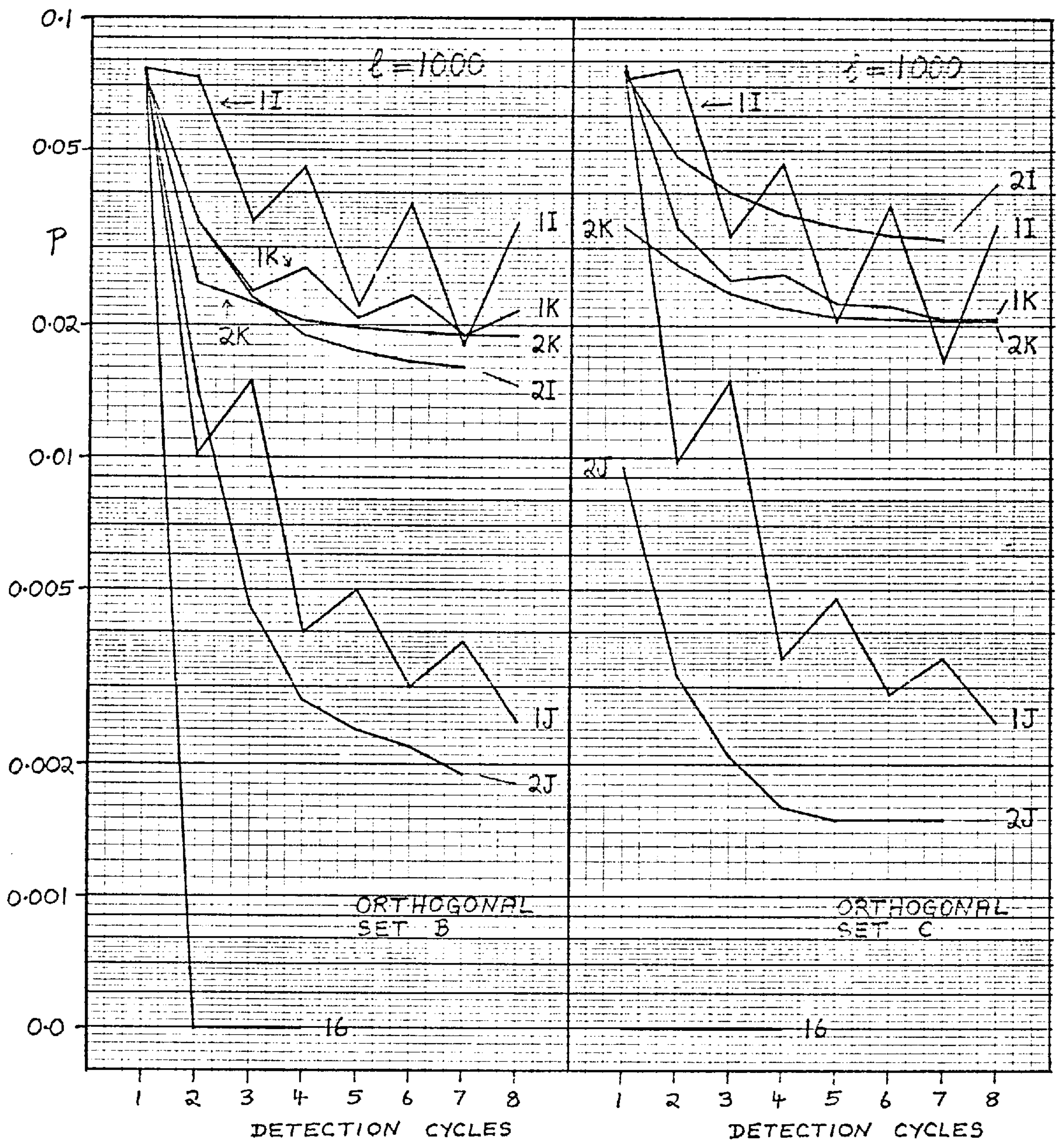


FIG. 43. ORTHOGONAL SETS B AND C WITH NO NOISE OR LEVEL VARIATIONS. EQUAL LEVELS.

P AVERAGED OVER ALL COMBINATIONS OF ODD NUMBERS IN THE TWO SETS, BETWEEN 7 AND 9.

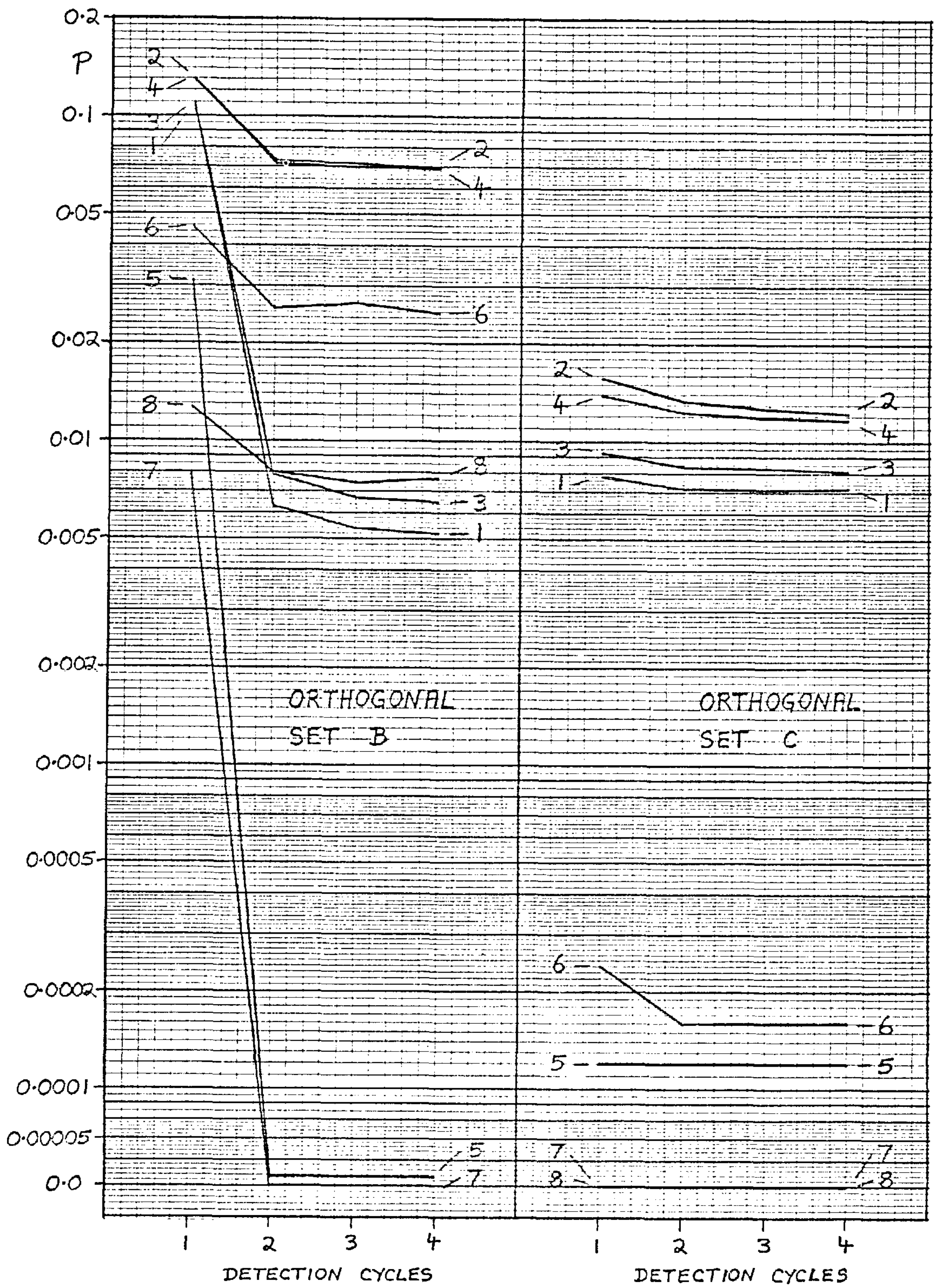


FIG. 44. ORTHOGONAL SETS B AND C WITH NO NOISE OR LEVEL VARIATIONS.
CODING AND DETECTION PROCESS 16.

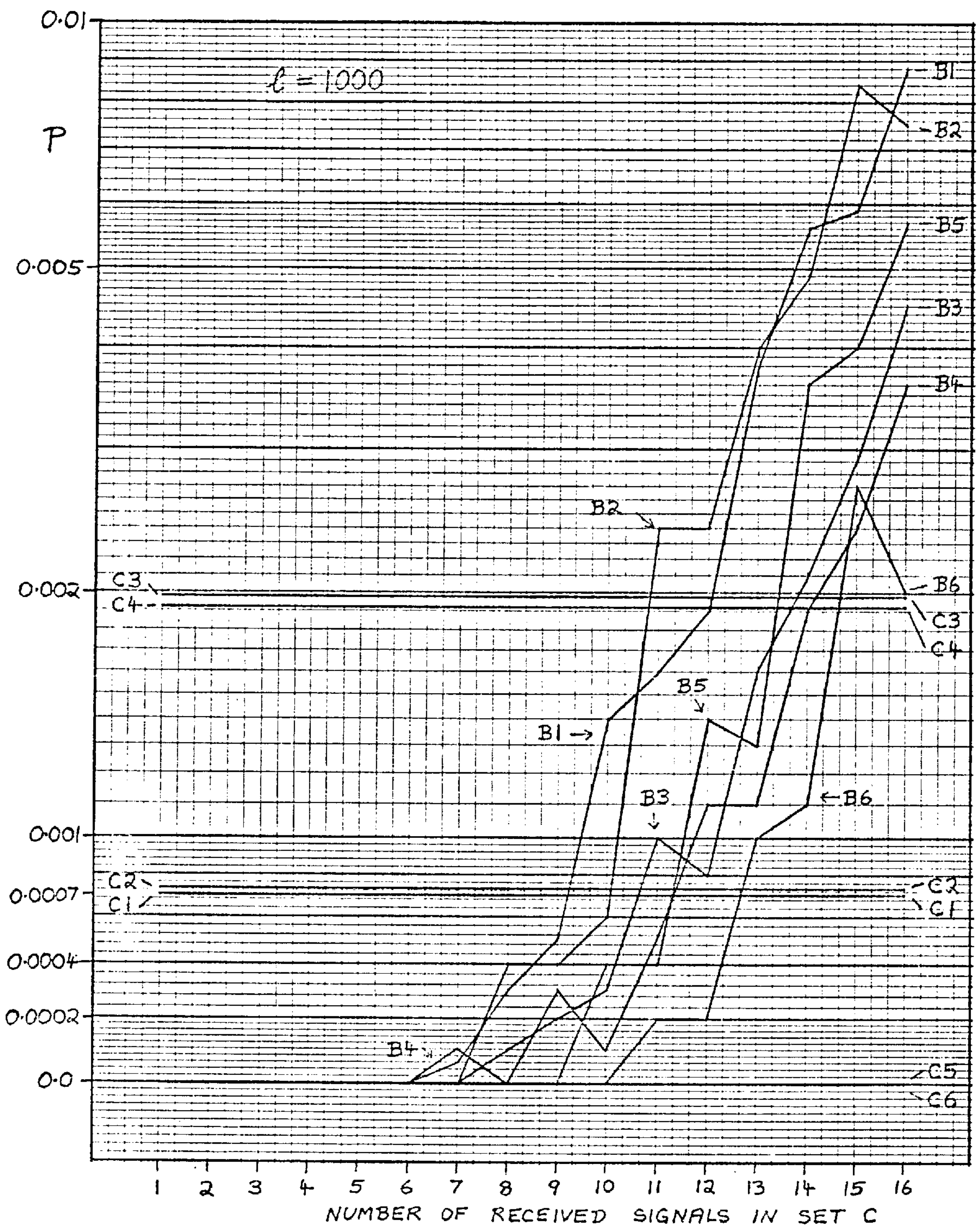


FIG. 45. ORTHOGONAL SETS B AND C WITH NOISE BUT NO LEVEL VARIATIONS.
CODING AND DETECTION PROCESS 16.
FIRST DETECTION CYCLE.

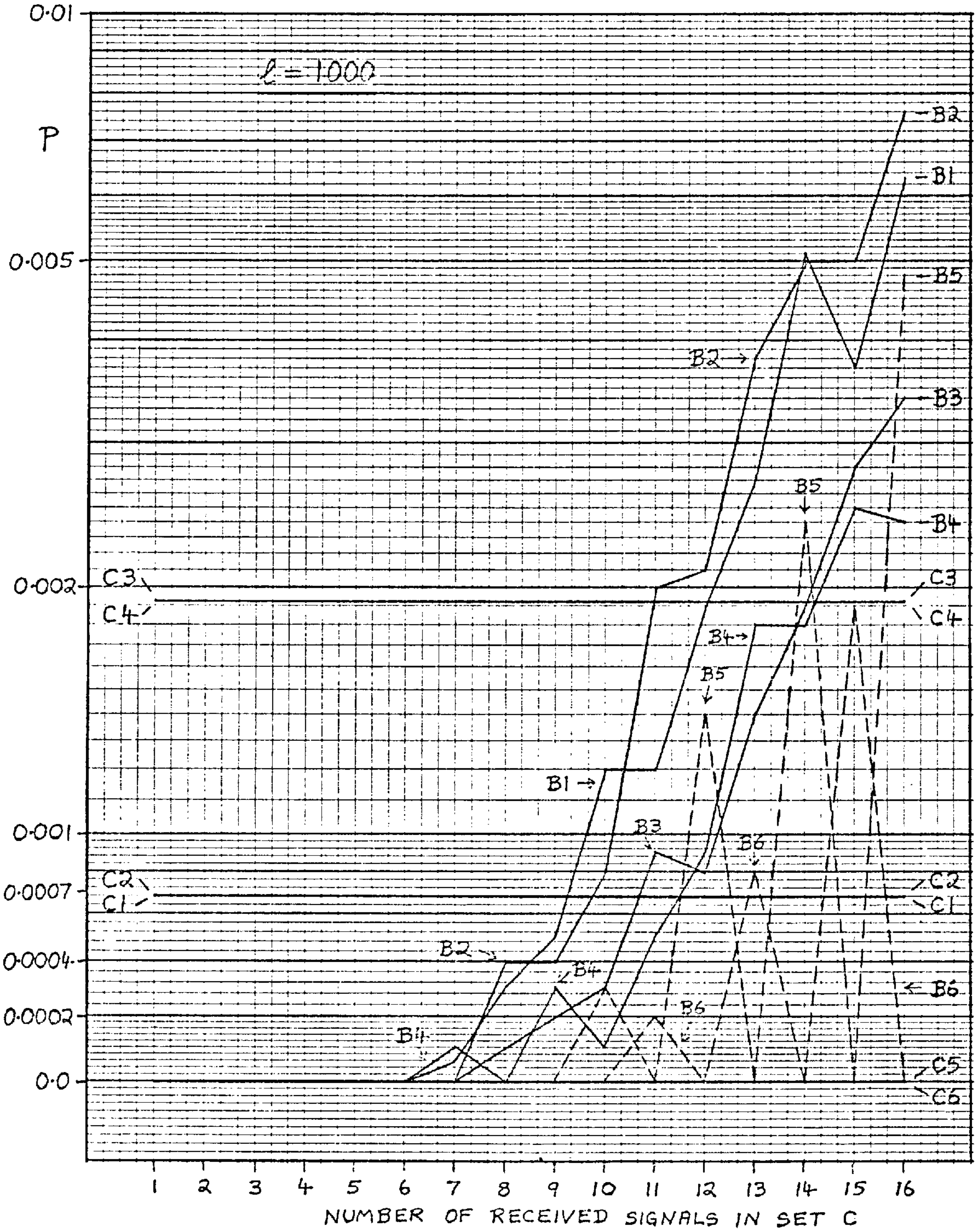


FIG. 46. ORTHOGONAL SETS B AND C WITH NOISE BUT NO LEVEL VARIATIONS.

CODING AND DETECTION PROCESS 16.

SECOND DETECTION CYCLE.

8.0 COMMENTS ON THE RESEARCH PROJECT

8.1 Originality

All the work described in this thesis which is not specifically ascribed to others, usually by quoting the appropriate reference, is original to the best of the author's knowledge.

The following are the more important of the contributions which are believed to be original. The design and analysis of the Systems A, B and C (Section 2.0). The analysis of the conditions for unique detectability (Sections 3.2 and 3.3). The design of the adaptive coding systems for linear independence (Section 3.4). The AM System (Section 3.5). The design and analysis of the optimum detector for several non-orthogonal signals, for the case where the receiver has no prior knowledge of their levels (Section 4.1). The application of MacColl's proposal to the detection of several signals received simultaneously (Section 4.2). The development of MacColl's proposal leading to a unified treatment of the various detection processes 1 to 15 (Section 4.0). The application of the constraints I, J and K to these processes (Section 4.0). The application of the point Jacobi, Gauss-Seidel and Gauss-Southwell iterative methods to the detection processes 1, 2 and 4 respectively (Sections 4.3.2 and 4.3.3). The application of Kaczmarz's method to the detection processes 5, 6 and 7 (Section 4.3.4). The detection processes 3 and 9 to 14 (Sections 4.3.2 and 4.4.2). The application of conventional analogue techniques for solving linear simultaneous equations, to the detection process 15 (Section 4.5). The further modifications of the detection processes 1 to 15 (Section 5.12). The system using simple detection processes in an arrangement with a single transmitter and many receivers (Section 6.2). The use of sets of orthogonal signals for RADAS (Section 7.2). The coding and detection process 16 (Section 7.3). The theoretical analysis of the different coding and detection processes, with the exception of the elegant proof for the non-singularity of the matrix M (Section 3.4), which is due to Mr. G. R. Selby of Imperial College. All computer-simulation tests and all computer programs. The use of RASSAS signals and the quantity p, to determine the degree of convergence of an iterative detection process when operating with a general class of signals.

8.2 Possible Further Investigations

The constraint J is most effective in improving both the rate and degree of convergence of the majority of the detection processes tested. It does not however necessarily guarantee the correct detection in the absence of noise, of received signals which are uniquely detectable but linearly dependent. This can be seen by considering the five received signals whose element-addresses are given by the columns of the matrix

$$Y = \begin{bmatrix} -1 & 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & -1 & -1 \end{bmatrix}, \tag{204}$$

where Y is of rank 4. The sum of the five signals is

$$S = YZ, \tag{205}$$

where Z is the 5-component column-vector $[z_i]$. If $Z^T = (1,1,1,1,1)$, then $S^T = (1,1,1,1,1)$. If an iterative detection process with the constraint J is used at the receiver, such that the estimate $X = [x_i]$ of the vector Z is constrained to satisfy the equation

$$|x_i| \leqslant |z_i| = 1, \quad \text{for } i = 1, \ldots, 5, \tag{206}$$

then the solution vector X obtained at the receiver in response to the received vector S, must satisfy equation 206 and

$$S = YX. \tag{207}$$

There is an infinite number of these solution vectors, between the two extreme values given by $X^T = (0,0,0,0,-1)$ and $X^T = (1,1,1,1,1)$, each of them satisfying the equation

$$x_i = \tfrac{1}{2}(1 + x_5), \quad \text{for } i = 1, \ldots, 4. \tag{208}$$

To ensure correct detection ($X = Z$) in a case such as this it is essential that the receiver accepts only those binary values which together with the correct signal levels, give the best approximation to the total received signal-vector S. One approach to this is to use the constraint J and to introduce a continuous tendency for the $\{|x_i|\}$ to increase in magnitude towards the $\{|z_i|\}$, during the detection process. This is achieved automatically in the detection processes 14J and 2JH. It is therefore of some interest to determine whether or not the better arrangements of 14J and 2JH achieve an advantage over the other detection processes, when general signals of equal levels are being received. In the tests carried out here (Figs. 16 to 20) the confidence limits are too wide for any definite conclusions to be reached on this.

Further tests are also needed to check whether or not the apparent dependence between element errors in a detection process of 14J or 2JH, is accompanied by a reduction in tolerance to additive gaussian noise, for certain combinations of element addresses and signal levels. There is clearly a need to examine the performance of all the more promising detection processes, that is 15J, 14J, 2JH and 11J, when using RADAS rather than RASSAS signals.

It would finally be of interest to examine the effects on the detection process 14J, of a constraint J which instead of being fixed at $|z_i|$, increases steadily to $|z_i|$ from a very much smaller value, during the detection of each total-element. On the basis of the available evidence this appears to be the most promising of all the detection processes, for applications where the received signals are uniquely detectable but linearly dependent.

8.3 Possible Applications

The arrangements of greatest potential value studied in this thesis are the System C, the AM System, the detection process 15J and the coding and detection process 16. The detection processes 14J, 2JH and 11J also have some interesting possibilities.

The System C and the AM System would, under suitable conditions which are of course quite different for the two systems, provide useful alternatives to existing or proposed message-switching systems.

The coding and detection process 16 could be used to provide an increase of around $50^{o}/_{o}$ in the channel capacity of a conventional TDM system, where this uses binary-coded bipolar baseband signals. When the number of simultaneous signals transmitted, that is the number of calls, does not exceed the number of components v of a signal vector, only signals of the orthogonal set B are transmitted. When more than v signals are transmitted, the additional signals are selected from the orthogonal set C. The levels of the signals in an orthogonal set are equal, the signals in set C having the appropriate level to maximize the overall tolerance to additive noise. Alternative arrangements enabling the capacity of the TDM system to be increased by a factor of 2 or 3 times, are considered in some detail in reference B4.

The detection process 15J, or one of the processes 14J, 2JH or 11J, may be used in many applications where it is required to find the vector X from a known matrix A and vector D in the equation AX = D, given that A is an mxm real, symmetric, positive-definite matrix and varies slowly with time. This is the situation where separate groups of m orthogonal binary-coded signals are transmitted over a channel which introduces slowly-time-varying intersymbol or interchannel interference. By transmitting known test signals at suitable intervals, a reasonably accurate knowledge of A can be maintained at the receiver. From the vector D obtained in response to a set of m received signals, the receiver determines the vector X and hence the individual binary values. A possible application for this arrangement is, for instance, an adaptive filter used to reduce the effects of multi-path propagation in a digital H.F. radio or other equivalent transmission link.

9.0 CONCLUSIONS

In an arrangement of RADAS with many transmitters and many receivers, the interfering signals in the detection of an individual signal-element, must be treated as noise in the interests of equipment economy. Under these conditions some errors are inevitably caused by interchannel interference. Where there is only a single receiver, errors due to interchannel interference may be entirely eliminated by detecting the sum of the received binary antipodal signals as a single multi-level signal, provided that the received signals are baseband, in element synchronism and linearly independent. Although in the latter case the maximum transmission rate over the common channel is much greater, for an acceptable tolerance to additive noise, than it is in the former case, it is always inferior to that of the equivalent TDM system using orthogonal signals. Where suitable adaptive coding is used with RADAS, as in the AM System, its performance is however only slightly inferior to that of the TDM system.

Where the required transmission rate over a common channel with many transmitters and many receivers, is always small compared with that available with the equivalent FDM or TDM system using orthogonal signals, RADAS may sometimes permit a useful reduction in equipment complexity to be achieved, without suffering an excessive degradation in performance, provided only that digitally-coded-speech and not data signals are transmitted. The most promising of the transmission systems studied for this application, is the System C.

In the arrangement of RADAS using many transmitters and a single receiver, 15J appears to be by far the simplest of the different detection processes studied, and it has in general as good a performance as that of any of the other systems. Other detection processes which may achieve a useful compromise between performance and equipment economy are 14J, 2JH and 11J.

Where there is a single transmitter feeding a single receiver via a multi-channel link, and the total number of different addresses is only twice the maximum number of orthogonal signal-elements, then the simplest and most effective of the systems studied, appears to be the coding and detection process 16. An arrangement of this type could be used to increase the capacity of an equivalent TDM system by about $50^{o}/_{o}$, while at the same time minimizing the reduction in tolerance to noise of the original TDM signals.

Perhaps the most useful outcome of the investigations carried out here, is the development of coding and detection processes, which may under certain conditions be used to eliminate or at least greatly reduce the intersymbol and interchannel interference between binary-coded digital signals reaching a single receiver, in applications where this interference varies with time.

APPENDIX 1

DETECTION OF THE FREQUENCY-TIME TRACES OF A NUMBER OF VARYING-FREQUENCY SIGNALS

Suppose that at the terminal A of the receiver of System C there are m varying-frequency signals, each of which has no discontinuities in amplitude or instantaneous frequency.

Let the amplitude and phase of the k th signal at time t be $a_k(t)$ and $\theta_k(t)$ respectively, for $k = 1, \ldots, m$. Assume for convenience that when $t = 0$, $\theta_k(t) = 0$ for all k.

Let the resultant signal obtained by adding together the m varying-frequency signals, be given by the real part of $V(t)$, where

$$V(t) = v(t)e^{j\phi(t)} = \sum_{k=1}^{m} a_k(t)e^{j\theta_k(t)} \quad . \tag{4}$$

Thus the amplitude of the resultant signal at A is given by $v(t)$ and its phase angle by $\phi(t)$.

From equation (4),

$$v(t)\cos\phi(t) = \sum_{k=1}^{m} a_k(t)\cos\theta_k(t) \quad , \tag{5}$$

$$v(t)\sin\phi(t) = \sum_{k=1}^{m} a_k(t)\sin\theta_k(t) \quad . \tag{6}$$

$$\therefore \quad \tan\phi(t) = \frac{\sum_{k=1}^{m} a_k(t)\sin\theta_k(t)}{\sum_{k=1}^{m} a_k(t)\cos\theta_k(t)} \quad , \tag{7}$$

and $\sec^2\phi(t).\ \dot{\phi}(t)$

$$= \left(\sum_{k=1}^{m} a_k(t)\dot{\theta}_k(t)\cos\ \theta_k(t)\ + \sum_{k=1}^{m} \dot{a}_k(t)\sin\ \theta_k(t)\right)$$

$$\cdot \left(\sum_{k=1}^{m} a_k(t)\ \cos\ \theta_k(t)\right) \div \left(\sum_{k=1}^{m} a_k(t)\cos\ \theta_k(t)\right)^2$$

$$+ \left(\sum_{k=1}^{m} a_k(t)\dot{\theta}_k(t)\sin\ \theta_k(t)\ - \sum_{k=1}^{m} \dot{a}_k(t)\cos\ \theta_k(t)\right)$$

$$\cdot \left(\sum_{k=1}^{m} a_k(t)\sin\ \theta_k(t)\right) \div \left(\sum_{k=1}^{m} a_k(t)\cos\ \theta_k(t)\right)^2$$

$$= \left(\sum_{k=1}^{m} a_k^2(t)\dot{\theta}_k(t)\cos^2\theta_k(t) + \sum_{k=1}^{m}\ \sum_{\substack{\ell=1\\ \ell\neq k}}^{m} a_k(t)a_\ell(t)\dot{\theta}_\ell(t)\cos\ \theta_k(t).\cos\ \theta_\ell(t)\right.$$

$$+ \sum_{k=1}^{m} a_k(t)\dot{a}_k(t)\cos\ \theta_k(t).\sin\ \theta_k(t)\ + \sum_{k=1}^{m}\ \sum_{\substack{\ell=1\\ \ell\neq k}}^{m} a_k(t)\dot{a}_\ell(t)\cos\ \theta_k(t).\sin\ \theta_\ell(t)$$

$$+ \sum_{k=1}^{m} a_k^2(t)\dot{\theta}_k(t)\sin^2\theta_k(t) + \sum_{k=1}^{m}\ \sum_{\substack{\ell=1\\ \ell\neq k}}^{m} a_k(t)a_\ell(t)\dot{\theta}_\ell(t)\sin\ \theta_k(t).\sin\ \theta_\ell(t)$$

$$\left.- \sum_{k=1}^{m} a_k(t)\dot{a}_k(t)\sin\ \theta_k(t).\cos\ \theta_k(t) - \sum_{k=1}^{m}\ \sum_{\substack{\ell=1\\ \ell\neq k}}^{m} a_k(t)\dot{a}_\ell(t)\sin\ \theta_k(t).\cos\ \theta_\ell(t)\right)$$

$$\div \left(\sum_{k=1}^{m} a_k(t)\cos\ \theta_k(t)\right)^2$$

$$= \left(\sum_{k=1}^{m} a_k^2(t)\dot{\theta}_k(t) + \sum_{k=1}^{m}\ \sum_{\substack{\ell=1\\ \ell\neq k}}^{m} a_k(t)a_\ell(t)\dot{\theta}_\ell(t)\cos(\theta_\ell(t) - \theta_k(t))\right.$$

$$\left.+ \sum_{k=1}^{m}\ \sum_{\substack{\ell=1\\ \ell\neq k}}^{m}\ a_k(t)\dot{a}_\ell(t)\sin(\theta_\ell(t) - \theta_k(t))\right) \div v^2(t)\cos^2\phi(t)\ .$$

$$\therefore\ v^2(t)\dot{\phi}(t)\ = \sum_{k=1}^{m}\ a_k^2(t)\dot{\theta}_k(t)$$

$$+ \sum_{k=1}^{m} a_k(t) \sum_{\substack{\ell=1\\ \ell\neq k}}^{m}\Big(a_\ell(t)\dot{\theta}_\ell(t)\cos(\theta_\ell(t)-\theta_k(t)) + \dot{a}_\ell(t)\sin(\theta_\ell(t)-\theta_k(t))\Big)\ . \qquad (8)$$

At any value of t for which either $\sec^2\phi(t).\dot{\phi}(t)$ or $\dot{\phi}(t)$ is infinite, $v^2(t)\dot{\phi}(t)$ is zero, the latter function being continuous and bounded.

The output signal from the FM discriminator 1 is $\dot{\phi}(t)$ and the output signal from the squarer is $v^2(t)$. Thus the signal at terminal C is $v^2(t)\dot{\phi}(t)$.

The value of $a_\ell(t)$ would normally vary only slowly with time, so that $\dot{a}_\ell(t) \simeq 0$ for $\ell=1,\ldots,m$.

$$\therefore \; v^2(t)\dot{\phi}(t) \simeq \sum_{k=1}^{m} a_k^2(t)\dot{\theta}_k(t) + \sum_{k=1}^{m} a_k(t) \sum_{\substack{\ell=1 \\ \ell\neq k}}^{m} a_\ell(t)\dot{\theta}_\ell(t)\cos\left(\theta_\ell(t)-\theta_k(t)\right) \qquad (9)$$

The first term on the right hand side of equation 9 is the sum of m separate terms, each of which is proportional to the frequency-time trace of a different one of the m varying-frequency signals.

$$\dot{\theta}_k(t) = 2\pi\left(f_c + g_k(t) + f_e\cos 2\pi f_{k1}t + f_e\cos 2\pi f_{k2}t\right), \quad \text{for } k=1,\ldots,m. \qquad (10)$$

The components of $\dot{\theta}_k(t)$ here are as defined for equation 16 (Section 2.4.3). $f_c > 2f_e \gg f_{k1} \gg f_{k2} \gg 1$, $g_k(t)$ is the message modulation applied to the k th signal and the two frequencies f_{k1} and f_{k2} cps define the discrete address of this signal.

$$\frac{d}{dt}\left(\theta_\ell(t)-\theta_k(t)\right) = \dot{\theta}_\ell(t)-\dot{\theta}_k(t) = 2\pi f_e\left(\cos 2\pi f_{\ell 1}t + \cos 2\pi f_{\ell 2}t - \cos 2\pi f_{k1}t - \cos 2\pi f_{k2}t\right) + 2\pi\left(g_\ell(t) - g_k(t)\right), \qquad (11)$$

where $f_e \gg \max\left|g_k(t)\right| = \max\left|g_\ell(t)\right|$.

Thus the signal $\cos\left(\theta_\ell(t)-\theta_k(t)\right)$ is a wide-deviation FM signal, whose instantaneous frequency varies continuously over a range from zero to a little over $4f_e$ cps. In the absence of message modulation, this has a line spectrum with frequency components which are harmonics of f_{k1}, f_{k2}, $f_{\ell 1}$ and $f_{\ell 2}$, together with the sums and differences of these harmonics. The frequency components are spread over a frequency range only a little wider than that swept out by the instantaneous carrier frequency.[A7] Under normal conditions, the message modulation signal $g_k(t)$ or $g_\ell(t)$, has a continuous power-density spectrum extending from about $\frac{1}{3}f_g$ to $2f_g$ cps. Thus in the presence of message modulation and with any reasonable modulation-index, the line spectrum becomes effectively blurred into a continuous spectrum. Since $g_k(t)$

or $g_\ell(t)$ is not synchronized or simply related to any of the frequency components f_{k1}, f_{k2}, $f_{\ell 1}$ or $f_{\ell 2}$, the signal $\cos\left(\theta_\ell(t)-\theta_k(t)\right)$ normally has a smooth power-density spectrum. The power spectral density is maximum at zero cps and decreases slowly as the frequency rises to $4f_e$ cps, falling rapidly to zero as the frequency increases above $4f_e$ cps.

The second term on the right hand side of equation 9 represents the intermodulation products at the terminal C. Assuming equal received signal levels and statistically independent message-signals, the intermodulation products have a total power level of m(m-1) times that of any one f_{k1} or f_{k2} cps component, say for instance that f_1 cps component in Fig. 6. They have a resultant spectrum which is continuous and extends from zero cps to a little above $4f_e$ cps. Since normally $f_e > 10^6$ and the effective bandwidth of a correlation detector is less than 10 cps, it appears that for values of m around 10, very little interference should ideally be caused in a correlation detector by the intermodulation products.

f_{k1} and f_{k2}, for all k, should lie outside the frequency band of $g_k(t)$, which extends from about $\frac{1}{3} f_g$ to $2 f_g$ cps. Otherwise appreciable interference may be caused by the message modulation in the corresponding correlation detectors. Furthermore, in order to prevent the intermodulation products at the terminal C from developing line spectra and so possibly causing excessive interference in a correlation detector, the message modulation should be applied continuously to all transmitted signals, and the transmission of regular signal patterns for extended periods should be avoided.

APPENDIX 2

CORRELATION DETECTOR FOR AN f_1 CPS SINE-WAVE

The feedback control loop for this detector involves the product modulator 3 and the low-pass filter 1. The function of this circuit is to adjust the sine wave at the terminal E to be exactly in phase with the f_1 cps sine-wave component at C. The sine wave at D multiplies the signals at C, in the product modulator 3. If there is no f_1 cps frequency component at C, then there is no d.c. component at the output of the product modulator, at the terminal I. The low-pass filter 1 has a very-low-frequency cut-off and removes essentially all the a.c. components from its input signal, to give therefore zero volts at J. The signal here is the control signal for the f_1 cps oscillator, which now oscillates at its natural frequency.

Suppose that the signal at C contains an f_1 cps component $a(t)\sin 2\pi f_1 t$, where $a(t)$ is the amplitude of the signal component. Let the signal at E be $\sin(2\pi f_1 t + \theta(t))$, where $\theta(t)$ is its phase angle relative to the f_1 cps sine-wave at C. The signal at D is now $\cos(2\pi f_1 t + \theta(t))$ and the resultant output signal from the product modulator 3, at the terminal I, is given by

$$a(t)\sin 2\pi f_1 t \cdot \cos(2\pi f_1 t + \theta(t))$$

$$= \tfrac{1}{2}a(t)\sin(4\pi f_1 t + \theta(t)) - \tfrac{1}{2}a(t)\sin\theta(t), \qquad (12)$$

assuming unity gain in the modulator.

With correct system design and in the absence of rapid fading, both $a(t)$ and $\theta(t)$ should only change slowly with time and in such a way that the signal $-\frac{1}{2}a(t)\sin\theta(t)$ contains only very-low-frequency components. Under these conditions the low-pass filter 1 removes the signal $\frac{1}{2}a(t)\sin(4\pi f_1 t + \theta(t))$ and passes the signal $-\frac{1}{2}a(t)\sin\theta(t)$ to the terminal J.

The f_1 cps oscillator is so designed that a positive voltage at the terminal J increases the instantaneous frequency of the oscillator by an amount proportional to the voltage magnitude, and a negative voltage at J similarly decreases its frequency. Taking f_1 as a constant,

$$\dot{\theta}(t) = -k\,a(t)\sin\theta(t), \qquad (13)$$

where k is a positive constant and $a(t) \geqslant 0$.

The condition where $\theta(t) = 0$ is thus the point of stable equilibrium and the instantaneous frequency of the f_1 cps oscillator is automatically adjusted until the sine wave at the terminal E is exactly in phase with the f_1 cps component in the signal at C. The condition where $\theta = \pi$ is a point of unstable equilibrium and the oscillator is very unlikely to remain in this condition for long.

The general conditions for stability of a phase-locked oscillator of this type as well as its transient behaviour and response to noise, are analysed in some detail in reference C12. These will not be considered further here.

When stable equilibrium has been obtained, the signal at G, resulting from the component $a(t)\sin 2\pi f_1 t$ at C, is

$$\begin{aligned} & a(t) \sin^2 2\pi f_1 t \\ = {} & \tfrac{1}{2}a(t) - \tfrac{1}{2}a(t)\cos 4\pi f_1 t \, . \end{aligned} \qquad (14)$$

The low-pass filter 3 has a very-low-frequency cut-off and filters out the second component of this signal, that is $-\frac{1}{2}a(t)\cos 4\pi f_1 t$. It also effectively removes the other a.c. components present at G. If the impulse response of the filter is $h(t)$, the signal at the filter output is $h(t) * \frac{1}{2}a(t)$. This is proportional to the appropriate weighted average of $a(t)$ over the period $t-T$ to t, where T is the total period over which $h(t)$ has a significant value. Thus the magnitude of the positive voltage at K is a measure of the level of the f_1 cps sine-wave at C.

APPENDIX 3

ANALYSIS OF BASEBAND SIGNAL WAVEFORMS

The transmitter low-pass filter, the transmission path and the receiver low-pass filter may together be represented by a single low-pass filter with a transfer function H(f) (equation 31, Section 3.1).

The impulse response h(t) of this filter is given by

$$h(t) = \int_{-\infty}^{\infty} H(f)\exp(j2\pi ft)\,df$$

$$= \int_{-W}^{+W} \cos 2\pi ft\,df$$

$$= 2W.\frac{\sin 2\pi Wt}{2\pi Wt}, \qquad (32)$$

neglecting the time delay in the filter which is ideally infinite.

The waveform s(t) at the output of the filter is given by

$$s(t) = \sum_{k=-\infty}^{\infty} c_k.2W.\frac{\sin\pi(2Wt-k)}{\pi(2Wt-k)}, \qquad (33)$$

so that $$s\left(\frac{k}{2W}\right) = 2Wc_k, \qquad (34)$$

for all k.

Let $$u_k(t) = \sqrt{2W}\cdot\frac{\sin\pi(2Wt-k)}{\pi(2Wt-k)}. \qquad (35)$$

The Fourier transform of $u_k(t)$ is

$$U_k(f) = \int_{-\infty}^{\infty} u_k(t)\exp(-j2\pi ft)\,dt$$

$$= \exp\left(-\frac{j2\pi fk}{2W}\right)\int_{-\infty}^{\infty}\sqrt{2W}\,\frac{\sin 2\pi Wt}{2\pi Wt}\exp(-j2\pi ft)\,dt$$

$$= \exp\left(-\frac{j2\pi fk}{2W}\right)\frac{1}{\sqrt{2W}}H(f). \qquad (36)$$

From Parseval's theorem,

$$\int_{-\infty}^{\infty} u_k(t)u_\ell(t)\,dt = \int_{-\infty}^{\infty} U_k(f)U_\ell^*(f)\,df$$

$$= \frac{1}{2W}\int_{-W}^{+W}\exp\left(\frac{j2\pi f(\ell-k)}{2W}\right)df = \delta_{k\ell}, \qquad (37)$$

where k and ℓ are integers, $\delta_{k\ell}$ is the Kronecker delta and $*$ here represents the complex conjugate. Thus the waveforms $\{u_k(t)\}$ are orthonormal functions.

The signal element represented at the input of the low-pass filter by the sequence of 2WT pulses

$$\sum_{k=1}^{2WT} c_k \,\delta\left(t - \frac{k}{2W}\right) \quad , \tag{38}$$

appears at the output of the low-pass filter, whose delay is again neglected, as

$$\sum_{k=1}^{2WT} c_k \cdot 2W \cdot \frac{\sin\pi(2Wt-k)}{\pi(2Wt-k)}$$

$$= \sum_{k=1}^{2WT} \sqrt{2W} \; c_k \, u_k(t)$$

$$= \sum_{k=1}^{2WT} \frac{1}{\sqrt{2W}} \; s\left(\frac{k}{2W}\right) u_k(t) \quad , \tag{39}$$

from equation 34, so that it contains 2WT of the orthonormal functions $u_k(t)$. These may be taken as unit vectors to specify the 2WT orthogonal axes of the signal space in which the transmitted signal-element is defined. The signal-element clearly has 2WT degrees of freedom,[A1,A2] and the k th component of the vector defining the signal element is

$$\frac{1}{\sqrt{2W}} \quad s\left(\frac{k}{2W}\right) \quad . \tag{40}$$

The response to s(t), at $t = \infty$, of a correlation detector or matched filter tuned to $u_k(t)$, is given by

$$\int_{-\infty}^{\infty} s(t)u_k(t) \;\; dt \;\; = \int_{-\infty}^{\infty} S(f)U_k^*(f) \;\; df$$

(where S(f) is the Fourier transform of s(t), and $*$ here represents the complex conjugate)

$$= \int_{-\infty}^{\infty} S(f) \left(\frac{1}{\sqrt{2W}} \;\; H(f) \exp\left(- \frac{j2\pi fk}{2W}\right) \right)^* \;\; df$$

(from equation 36)

$$= \frac{1}{\sqrt{2W}} \cdot \int_{-W}^{+W} S(f)\exp\left(\frac{j2\pi fk}{2W}\right) \;\; df$$

$$= \frac{1}{\sqrt{2W}} \cdot s\left(\frac{k}{2W}\right) \quad . \tag{41}$$

Thus $s\left(\frac{k}{2W}\right)$, the sample value of the waveform at the output of the low-pass filter at the time $t = \frac{k}{2W}$, is $\sqrt{2W}$ times the k th component of the vector defining the signal element. It is also $\sqrt{2W}$ times the response to the total waveform s(t) at $t = \infty$, of a filter matched to $u_k(t)$. The signal element at the output of the low-pass filter given by equation 39, is therefore determined completely and in an optimum manner, by the 2WT sample values for k = 1,...,2WT.[A8,A9]

Whereas the signal element at the input to the low-pass filter has a duration of T seconds and infinite bandwidth, the corresponding signal-element at the output of the low-pass filter has infinite time duration but a bandwidth -W to W cps. A band-limited signal-element is however for convenience associated entirely with its 2WT sample values, so that it is described as having a duration of T seconds.

If the transmission path introduces into the signal at the receiver input, gaussian noise having zero mean and a constant power density of $\frac{1}{2} N_o$ over the frequency band -W to W, it can be shown that the resulting noise components at the sample values of the signal, at the output of the receiver low-pass filter, are statistically independent zero-mean gaussian random variables with variance WN_o.[A8] Furthermore, by the sampling theorem, the noise signal n(t) at this point is given by

$$\begin{aligned} n(t) &= \sum_{k=-\infty}^{\infty} n\left(\frac{k}{2W}\right) \frac{\sin\pi(2Wt-k)}{\pi(2Wt-k)} \\ &= \sum_{k=-\infty}^{\infty} \frac{1}{\sqrt{2W}} n\left(\frac{k}{2W}\right) u_k(t) , \end{aligned} \tag{42}$$

so that the noise signal is completely determined by its sample values, $n\left(\frac{k}{2W}\right)$, for all k.[A8,A9]

APPENDIX 4

APPROXIMATE UPPER BOUND TO THE NUMBER OF DIFFERENT ADDRESSES, FOR v LINEARLY-INDEPENDENT SIGNALS OF CONSTANT POWER LEVEL AND EQUAL RECEIVED LEVELS

The receiver is assumed to have prior knowledge of the number and element addresses of the received signals. Thus if there are never more than m received signals, where $m \leqslant v$ and the signals are always uniquely detectable, one set of p signal-elements may have the same resultant (total) vector as another set of q signal-elements, which need not be a disjoint set, so long as there are more than m different addresses in the union of the two sets.

Consider any two subsets g and h of a given set of m received element-addresses, where m is assumed even. g and h may or may not be disjoint. Suppose that for suitable sets of binary values in g and h, the two resultant vectors are equal. A signal element common to both sets need not of course have the same binary value in the two sets. Since the two binary forms of any signal-element are linearly dependent, it necessarily follows that under these conditions the element addresses in the union of g and h are linearly dependent, so that the m received element-addresses are also linearly dependent. Thus no two subsets of the m element-addresses may have the same possible resultant vector.

It follows that no set of $\frac{1}{2}$ m element-addresses selected from the n different addresses, can have the same possible resultant vector as any other set of $\frac{1}{2}$ m element-addresses, whether or not these are disjoint. Each of these sets must also have $2^{\frac{1}{2}m}$ different resultant vectors. There are $\binom{n}{\frac{1}{2}m}$ different sets of $\frac{1}{2}$ m addresses, selected from the n addresses, so that there must be altogether at least

$$\binom{n}{\frac{1}{2}m} 2^{\frac{1}{2}m} \qquad (55)$$

different possible resultant vectors.

A set of $\frac{1}{2}$ m signals of the type being considered, has at the most $(\frac{1}{2}m + 1)^v$ different possible resultant vectors. Thus

$$\binom{n}{\frac{1}{2}m} 2^{\frac{1}{2}m} \leqslant (\tfrac{1}{2}m + 1)^v . \qquad (56)$$

Each of the v components of an individual received signal-element is assumed to have a magnitude of unity, so that each of the v resultant components of the sum of k signal-elements, where $k \leqslant m$, will be odd or even depending upon whether k is odd or even. Thus the resultant vector of the sum of $(\frac{1}{2}m - e)$ signals, where e is an even integer less than $\frac{1}{2}m$, is a possible resultant vector for $\frac{1}{2}m$ signals. But no set of up to $\frac{1}{2}m$ element-addresses selected from the n different addresses, can have the same possible resultant vector as any other set of up to $\frac{1}{2}m$ element-addresses, whether or not these are disjoint or have the same number of members. Thus

$$\sum_{e=0}^{f} \binom{n}{\frac{1}{2}m-e} 2^{\left(\frac{1}{2}m-e\right)} \leqslant (\tfrac{1}{2}m + 1)^{v}, \qquad (57)$$

where f is the largest even integer less than $\frac{1}{2}m$. This reduces approximately to

$$\binom{n}{\frac{1}{2}m} 2^{\frac{1}{2}m} < (\tfrac{1}{2}m + 1)^{v}, \qquad (58)$$

if a maximum error of a few parts in a hundred is permitted.

Consider the important case where v is even and m=v.

Now $$\binom{n}{\frac{1}{2}v} 2^{\frac{1}{2}v} < (\tfrac{1}{2}v + 1)^{v}, \qquad (59)$$

or $$\binom{n}{\frac{1}{2}v} < \left(\tfrac{1}{2}(\tfrac{1}{2}v + 1)^2\right)^{\frac{1}{2}v}. \qquad (60)$$

But $$\binom{n}{\frac{1}{2}v} = \frac{n!}{(\frac{1}{2}v)!(n-\frac{1}{2}v)!}$$

$$> \frac{(n-\frac{1}{2}v + 1)^{\frac{1}{2}v}}{(\frac{1}{2}v)!}. \qquad (61)$$

And by Stirling's formula, so long as $v \gg 1$,

$$(\tfrac{1}{2}v)! \simeq (2\pi)^{\frac{1}{2}} \left(\tfrac{1}{2}v\right)^{\frac{1}{2}(v+1)} e^{-\frac{1}{2}v},$$

so that $$\frac{(n-\frac{1}{2}v+1)^{\frac{1}{2}v}}{(2\pi)^{\frac{1}{2}}(\frac{1}{2}v)^{\frac{1}{2}(v+1)} e^{-\frac{1}{2}v}} < \left(\tfrac{1}{2}(\tfrac{1}{2}v + 1)^2\right)^{\frac{1}{2}v},$$

or $$(n - \tfrac{1}{2}v+1)^{\frac{1}{2}v} < (\pi v)^{\frac{1}{2}} \left(\frac{v}{4e} (\tfrac{1}{2}v + 1)^2 \right)^{\frac{1}{2}v},$$

or $$n < \tfrac{1}{2}v-1 + (\pi v)^{\frac{1}{v}} \cdot \frac{v}{4e} (\tfrac{1}{2}v+1)^2. \qquad (62)$$

Maximum values of n, satisfying the above inequality for $10 \leqslant v \leqslant 40$, are plotted in Fig. 10 (Section 3.3).

APPENDIX 5

PROBABILITY OF A FAILURE IN THE DETECTION OF A MESSAGE ADDRESS IN THE AM SYSTEM

Assume a high signal/noise ratio at the receiver input, with additive white gaussian noise such that the noise components at the sample values of the received signal are statistically independent zero-mean gaussian random variables with variance σ_1^2. Under these conditions the noise signal at the output of a low-pass filter can very approximately be represented by a gaussian signal with zero mean and a variance of $2g\sigma_1^2$. It is assumed here that the output signal at any instant is the sum of the preceding 2g input signal-pulses. With the equivalent practical filter, the output signal at any instant would be dependent on rather more than the 2g preceding input-pulses, and their individual contributions to the output signal would be suitably weighted. As a result, the variance of the output noise signal would be typically 1 or 2 db greater than $2g\sigma_1^2$. The error is however not excessive for our purposes and so will be neglected.

Suppose that the amplitude of a received unmodulated signal-digit is 1 and that of an address digit is x. After somewhat more than 2g consecutive appearances of the k th address digit of the i th received signal, the signal at the output of the corresponding low-pass filter will have the maximum amplitude of 2gx and will be positive or negative depending upon whether the address digit is a "0" or a "1". The corresponding threshold has half the value of this signal. Thus the probability that the presence of the k th address digit will not be detected at a given instant is

$$p_1 = \frac{1}{\sqrt{4\pi g\sigma_1^2}} \int_{gx}^{\infty} \exp\left(-\frac{y^2}{4g\sigma_1^2}\right) dy$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{gx^2}{2\sigma_1^2}}}^{\infty} \exp\left(-\frac{1}{2}y^2\right) dy \quad . \qquad (69)$$

The probability of an error in the detection of the binary value of an unmodulated signal-digit, in the presence of gaussian noise with zero mean and variance σ_2^2, is

$$p_2 = \frac{1}{\sqrt{2\pi\sigma_2^2}} \int_1^{\infty} \exp\left(-\frac{y^2}{2\sigma_2^2}\right) dy$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{1}{\sigma_2^2}}}^{\infty} \exp\left(-\frac{1}{2}y^2\right) dy . \qquad (70)$$

Thus $p_1 = p_2$, if $\sqrt{\frac{gx^2}{2\sigma_1^2}} = \sqrt{\frac{1}{\sigma_2^2}}$

or $$\frac{\sigma_1^2}{\sigma_2^2} = \frac{1}{2} gx^2 . \qquad (71)$$

The presence of the signal address will not be detected so long as there are at least two address digits not detected. Since the noise signals at the outputs of the different low-pass filters are statistically independent, the probability of the address not being detected at a given instant is

$$p_3 = \sum_{n=2}^{h} \binom{h}{n} p_1^{\,n} (1-p_1)^{(h-n)} \simeq \frac{1}{2} h^2 p_1^2 \ll p_1 , \qquad (72)$$

for the normal situation where

$$p_1^{-1} \gg h^2 \gg h \gg 1 . \qquad (73)$$

When $\frac{1}{2} gx^2 > 1$, then for a given received signal/noise ratio, $p_1 < p_2$ so that $p_3 \ll p_2$.

Under these conditions, the detection of the message address should only begin to be affected by additive white gaussian noise at the receiver input, when there is already a high element error-rate in the detected signal, since the element error probability of a detected signal is greater than p_2.

APPENDIX 6

CONDITIONS WHICH MUST BE SATISFIED BY THE ELEMENT ADDRESSES TO ENSURE THAT THE MATRIX A IS STRICTLY DIAGONALLY DOMINANT

Suppose that the m columns $\{Y_i\}$ of the v×m matrix $Y = [y_{ji}]$ are no longer vectors of unit length but are such that $|y_{ji}| = 1$ for all $j \neq i$ and $|y_{ii}| = g > 1$ for all i. g is a positive constant and no restrictions are placed on the component signs of any vector Y_i.

The condition for the matrix $A = [a_{ij}] = Y^TY$ to be strictly diagonally dominant is that

$$|a_{ii}| > \sum_{\substack{j=1\\ j\neq i}}^{m} |a_{ij}| \qquad \text{for } i = 1,\ldots,m, \qquad (118)$$

or

$$|Y_i^TY_i| > \sum_{\substack{j=1\\ j\neq i}}^{m} |Y_i^TY_j| \qquad \text{for } i = 1,\ldots,m, \qquad (119)$$

so that

$$g^2 + (v-1) > (m-1)(2g + v - 2), \qquad (120)$$

or

$$g^2 - 2(m-1)g > (m-1)(v-2)-(v-1), \qquad (121)$$

or

$$\begin{aligned}\big(g - (m-1)\big)^2 &> (m-1)(v-2)-(v-1)+(m-1)^2\\ &= \left((m-1)+ \tfrac{1}{2}(v-2)\right)^2 - \tfrac{1}{4}(v-2)^2-(v-1)\\ &= \left((m-1)+ \tfrac{1}{2}(v-2)\right)^2 - \tfrac{1}{4}v^2. \qquad (122)\end{aligned}$$

Clearly the value of g needed to satisfy the inequality 122, increases with m. Since $m \leqslant v$, the value of g required to ensure that A is strictly diagonally dominant for all permissible values of m, is given by

$$\big(g - (v-1)\big)^2 > \left((v-1)+ \tfrac{1}{2}(v-2)\right)^2 - \tfrac{1}{4}v^2, \qquad (123)$$

or

$$\begin{aligned}\big(g - (v-1)\big)^2 &> (\tfrac{3}{2}v-2)^2 - \tfrac{1}{4}v^2\\ &= 2v^2 - 6v + 4\\ &= 2(v-1)(v-2). \qquad (124)\end{aligned}$$

The inequality 124 is satisfied if

$$\big(g - (v-1)\big)^2 > 2(v-1)^2, \qquad (125)$$

or $\quad g - (v-1) > \sqrt{2}\,(v-1),$ (126)

or $\quad g > (1 + \sqrt{2}\,)\,(v-1).$ (127)

Let $\quad f = \sqrt{g^2 + v - 1}\ .$ (128)

If $|y_{ji}| = \frac{1}{f}$ for all $j \neq i$ and $|y_{ii}| = \frac{g}{f}$ for all i, where g satisfies the inequality 127, then the matrix A is strictly diagonally dominant for all $m \leq v$. The element addresses here are unit vectors, as before.

REFERENCES

A) BACKGROUND THEORY

A1 C. E. Shannon and W. Weaver, "The Mathematical Theory of Communication", The University of Illinois Press, Urbana, 1964.

A2 P. M. Woodward, "Probability and Information Theory, with Applications to Radar", Pergamon Press Ltd., London, 1953.

A3 L. S. Schwartz, "Principles of Coding, Filtering and Information Theory", Spartan Books, Inc., Baltimore, 1963.

A4 M. Schwartz, "Information Transmission, Modulation and Noise", McGraw-Hill Book Co., Inc., New York, 1959.

A5 Y. W. Lee, "Statistical Theory of Communication", John Wiley and Sons, Inc., New York, 1960.

A6 A. Papoulis, "Probability, Random Variables, and Stochastic Processes", McGraw-Hill Book Co., Inc., New York, 1965.

A7 P. F. Panter, "Modulation, Noise and Spectral Analysis", McGraw-Hill Book Co., Inc., New York, 1965.

A8 J. M. Wozencraft and I. M. Jacobs, "Principles of Communication Engineering", John Wiley and Sons, Inc., New York, 1965.

A9 J. C. Hancock and P. A. Wintz, "Signal Detection Theory", McGraw-Hill Book Co., Inc., New York, 1966.

A10 E. T. Browne, "Introduction to the Theory of Determinants and Matrices", The University of North Carolina Press, 1958.

A11 L. J. Paige and J. D. Swift, "Elements of Linear Algebra", Ginn and Co., New York, 1961.

A12 R. S. Varga, "Matrix Iterative Analysis", Prentice-Hall, Inc., New Jersey, 1962.

A13 D. J. Wilde, "Optimum Seeking Methods", Prentice-Hall, Inc., New Jersey, 1964.

A14 J. La Salle and S. Lefschetz, "Stability by Liapunov's Direct Method with Applications", Academic Press, New York, 1961.

B) UNPUBLISHED REPORTS FORMING AN ELEMENTARY INTRODUCTION TO THIS THESIS

B1 A. P. Clark, "Various arrangements of phase modulation for use in digital transmission systems", B.T.R. Technical Report 65/038/TR, June 1965.

B2 A. P. Clark, "Possible modulation methods for RADAS and RASSAS", B.T.R.Technical Report 65/020/TR, February 1965.

B3 A. P. Clark, "The transmission of digitally-coded-speech signals by means of RADAS and RASSAS. Part 1: separate transmitters and receivers", Imperial College, April 1966. Also B.T.R.Technical Report 66/052/TR, April 1966.

B4 A. P. Clark, "The transmission of digitally-coded-speech signals by means of RADAS and RASSAS. Part 2: a common transmitter and a common receiver", Imperial College, October 1966. Also B.T.R. Technical Report 66/067/TR, October 1966.

C) TOPICS IN COMMUNICATION THEORY RELEVANT TO RADAS

C1 G. L. Turin, "Error probabilities for binary symmetric ideal reception through nonselective slow fading and noise", Proc.IRE, Vol.46, pp.1603-1619, September 1958.

C2 A. B. Glenn, "Comparison of PSK vs. FSK and PSK-AM vs. FSK-AM binary-coded transmission systems", IRE Trans. Commun. Systems, Vol. CS-8, No.2, pp.87-100, June 1960.

C3 A. P. Clark, "Considerations in the choice of the optimum data transmission systems for use over telephone circuits", J.Brit.IRE, Vol.23, No.5, pp.331-355, May 1962.

C4 A. P. Clark, "A phase modulation data transmission system for use over the telephone network", The Radio and Electronic Engineer, Vol. 27, No.3, pp.181-195, March 1964.

C5 E. Arthurs and H. Dym, "On the optimum detection of digital signals in the presence of white Gaussian noise - a geometric interpretation and a study of three basic data transmission systems", IRE Trans. Commun. Systems, Vol. CS-10, No.4, pp.336-372, December 1962.

C6 R. O. Rowlands, "The relative efficiencies of various binary detection systems", IRE Intern. Conv. Record, Vol. 10, Part 4, pp.185-189, 1962.

C7 I. Jacobs, "Comparison of m-ary modulation systems", BSTJ, Vol.46, No.5, pp.843-864, May-June 1967.

C8 T. Murakami, "Analysis of a multiple signal f.m. detection system", RCA Rev., Vol.27, No. 3, pp.425-455, September 1966.

C9 P. D. Shaft, "Hard limiting of several signals and its effect on communication system performance", IEEE Intern.Conv.Record, Vol.13, Part 2, pp.103-113, 1965.

C10 A. G. Reed, "Effects of band limiting the R.F. spectrum in a communication satellite system employing wide deviation frequency modulation", Electronics Letters, Vol.3, No.3, pp.95-97, March 1967.

C11 C. E. Cook and M. Bernfield, "Radar Signals", pp.1-17, Academic Press, New York, 1967.

C12 A. J. Viterbi, "Principles of Coherent Communication", pp.3-120, McGraw-Hill Book Co., Inc., New York, 1966.

C13 P. F. Panter, "Modulation, Noise and Spectral Analysis", pp.478-504, McGraw-Hill Book Co., Inc., New York, 1965.

C14 J. W. Emling and D. Mitchell, "The effects of time delay and echoes on telephone conversations", BSTJ, Vol.42, No.6, pp.2869-2891, November 1963.

C15 J. K. Wolf, "Effects of channel errors on delta modulation", IEEE Trans. Commun. Technol., Vol.COM-14, No.1, pp.2-7, February 1966.

C16 T. G. Birdsall, M. P. Ristenbatt and S. B. Weinstein, "Analysis of asynchronous time multiplexing of speech sources", IEEE Trans. Commun.Systems, Vol.CS-10, No.4, pp.390-397, December 1962.

C17 S. W. Golomb, "Digital Communications", pp.53-62, Prentice-Hall,Inc., New Jersey, 1964.

C18 D. A. Bell, "Walsh functions and Hadamard matrices", Electronics Letters, Vol.2, No.9, p.340, September 1966.

C19 F. M. Reza, "An Introduction to Information Theory", pp.254-263, McGraw-Hill Book Co., Inc., New York, 1961.

C20 W. W. Harman, "Principles of the Statistical Theory of Communication", pp.68-71, McGraw-Hill Book Co., Inc., New York, 1963.

C21 A. D. Wyner, "Random packings and coverings of the unit n-sphere", BSTJ, Vol.46, No.9, pp.2111-2118, November 1967.

C22 S. E. Portny, "Large sample confidence limits for binary error probabilities", Proc.IEEE, Vol.54, No.12, p.1993, December 1966.

D) PRINCIPLES AND PRACTICE OF RADAS

D1 J. Kaiser, J. W. Schwartz and J. M. Aein, "Multiple access to a communication satellite with a hard limiting repeater - Vol.1", Institute for Defense Analyses, U.S.A., Report R-108, April 1965.

D2 J. M. Aein and J. W. Schwartz (editors), "Multiple access to a communication satellite with a hard limiting repeater - Vol.2", Institute for Defense Analyses, U.S.A., Report R-108, April 1965.

D3 W. D. White, "Theoretical aspects of asynchronous multiplexing", Proc.IRE, Vol.38, No.3, pp.270-275, March 1950.

D4 A. L. Hopper, "Non-synchronous pulse multiplex system", Electronics, Vol.25, No.8, pp.116-120, August 1952.

D5 J. R. Pierce and A. L. Hopper, "Non-synchronous time-division with holding and with random sampling", Proc.IRE, Vol.40, No.9, pp.1079-1088, September 1952.

D6 J. P. Costas, "Poisson, Shannon and the radio amateur", Proc.IRE, Vol.47, No.12, pp.2058-2068, December 1959.

D7 J. E. Taylor, "Asynchronous multiplexing", Communication and Electronics, No. 46, pp.1054-1062, January 1960.

D8 J. H. Weber, "Traffic efficiencies in congested band radio systems", Proc.IRE, Vol.48, No.11, pp.1910-1911, November 1960.

D9 D. H. Hamsher, "System concepts for address communication systems", IRE Trans.Vehic.Commun., Vol.VC-9, No.3, pp.72-76, December 1960.

D10 H. Magnuski, "Wideband channel for emergency communication", IRE Intern. Conv. Record, Vol.9, Part 8, pp.80-84, March 1961.

D11 D. P. Harris, "Unscheduled spectrum sharing in communications", 5th Global Commun. Symp., pp.273-276, May 1961.

D12 "RACEP-discrete address system makes possible clear voice transmission", Electrical Engineering, pp.730-731, September 1961.

D13 F. F. Fulton, "Efficient multiplexing of intermittent transmitters", IRE WESCON Conv. Record, Paper No.29/3, 1961.

D14 J. Klapper and B. Rabinovici, "A synchronous mode of RADAS communication", IRE Trans.Vehic.Commun., Vol.VC-11, No.1, pp.50-55, August 1962.

D15 H. McGuire, "Combined coding increases spectrum usage", Electronics, Vol.35, No.43, pp.64-66, 26th October 1962.

D16 H. Magnuski and W. D. Dehart, "Analysis of random access discrete address systems", IRE 8th Natl. Commun. Symp., 1962.

D17 J. J. Kepcke, "Maximum spectrum utilization by wide band transmission", IEEE Trans.Vehic.Commun., Vol. VC-12, No.1, pp.81-87, 1963.

D18 L. Schuchman, "Analysis of random access communication systems", IEEE Conference Paper, No.CP63-1094, June 1963.

D19 J. A. Stewart and E. A. Huber, "Comparison of modulation methods for multiple access synchronous satellite communication systems", Proc.IEE, Vol.111, No.3, pp.535-546, March 1964.

D20 "RADA - new concept in radio communications", International Electronics, Vol.7, No.5, pp.19-22, May 1964.

D21 R. C. Sommer, "On the optimization of random access discrete address communications", Proc.IEEE, Vol.52, No.10, p.1255, October 1964.

D22 C. H. Dawson, "An introduction to random access discrete address systems", IEEE Intern.Conv.Record, Vol.12, Part 6, pp.154-158, 1964.

D23 A. B. Glenn, "Code division multiplex system", IEEE Intern.Conv. Record, Vol.12, Part 6, pp.53-61, 1964.

D24 W. Bedsole and J. Lomax, "Propagation measurements for a frequency-time coded pulse communication system", IEEE Intern.Conv.Record, Vol.12, Part 6, pp.170-181, 1964.

D25 H. Blasbalg, D. Freeman and R. Keeler, "Random-access communications using frequency shifted PN (pseudo noise) signals", IEEE Intern. Conv.Record, Vol.12, Part 6, pp.192-216, 1964.

D26 C. H. Dawson and H. Sklar, "False addresses in a random access system employing discrete time-frequency addressing", IEEE Intern. Conv. Record, Vol.12, Part 6, pp.217-230, 1964.

D27 R. C. Sommer, "An improved detection technique for random access discrete address (RADA) communications", Proc.IEEE, Vol.53, No.2, pp.185-186, February 1965.

D28 A. M. McCalmont, "Multiple access discrete address concepts", IEEE Trans. Vehic.Commun., Vol.VC-14, No.1, pp.39-42, March 1965.

D29 D. Chesler, "M-ary RADA system", Proc.IEEE, Vol.53, No.4, pp.390-391, April 1965.

D30 H. Blasbalg, "A comparison of pseudo-noise and conventional modulation for multiple access satellite communications", IBM J., Vol.9, No.4, pp.241-255, July 1965.

D31 F. Corr, R. Crutchfield and J. Marchese, "A pulsed pseudo-noise V.H.F. radio set", IBM J., Vol.9, No.4, pp.256-263, July 1965.

D32 R. Van Blerkom, R. E. Sears and D. G. Freeman, "Analysis and simulation of a digital matched filter receiver of pseudo-noise signals", IBM J., Vol. 9, No. 4, pp.264-273, July 1965.

D33 A. M. McCalmont, "Design concepts for multiple access discrete address (MADA) communication systems", IEEE 10th Natl.Commun.Symp.

D34 S. B. Dayton and S. Bernstein, "Co-channel interference, signal detection and SOMADA systems", IEEE 10th Natl.Commun.Symp.

D35 R. C. Sommer, "Asynchronously multiplexed channel capacity", Proc.IEEE, Vol.54, No.1, pp.79-80, January 1966.

D36 R. C. Sommer, "Asynchronously multiplexed binary channel capacity", Proc.IEEE, Vol.54, No.1, pp.80-81, January 1966.

D37 G. Einarsson, "Time-frequency coded pulse communication systems", Ericsson Technics, No. 1, pp.53-80, 1966.

D38 J. W. Schwartz, J. M. Aein and J. Kaiser, "Modulation techniques for multiple access to a hard limiting satellite repeater", Proc.IEEE, Vol.54, No.5, pp.763-777, May 1966.

D39 F. Assadourian and D. L. Jacoby, "Multiple-access considerations for communication satellites", RCA Rev., Vol.27, No.2, pp.179-198, June 1966.

D40 J. W. Schwartz, "A note on asynchronous multiplexing", IEEE Trans. Inform. Theory, Vol.IT-12, No.3, pp.396-397, July 1966.

D41 D. Chesler, "Performance of a multiple address RADA system", IEEE Trans. Commun. Technol., Vol.COM-14, No.4, pp.369-372, August 1966.

D42 R. C. Sommer, "A coded RADA system", Proc.IEEE, Vol.54, No.9, pp.1196-1197, September 1966.

D43 B. W. Beals and F. R. Zitzmann, "Simulation for RADAS", IEEE Trans. Commun.Technol., Vol.COM-14, No.5, pp.553-562, October 1966.

D44 R. C. Sommer, "The noisy, asynchronously multiplexed, binary channel", IEEE Trans.Inform.Theory, Vol.IT-13, No.1, pp.140-142, January 1967.

D45 R. B. Ward, "Digital communications on a pseudonoise tracking link using sequence inversion modulation", IEEE Trans.Commun.Technol., Vol.CM-15, No.1, pp.69-78, February 1967.

D46 J. S. Engel, "The distribution of traffic in a nonblocking random access discrete address communication system", IEEE Trans.Commun. Technol., Vol.CM-15, No.1, pp.130-131, February 1967.

D47 J. H. Wittman, "Categorization of multiple access/random access modulation techniques", IEEE Trans.Commun.Technol., Vol.CM-15, No.5, pp.724-725, October 1967.

D48 R. C. Sommer, "Time-frequency (RADA) signalling with a random number of active subscribers", Proc.IEEE, Vol.56, No.2, pp.212-213, February 1968.

D49 R. C. Sommer, "High efficiency multiple access communications through a signal processing repeater", IEEE Trans.Commun.Technol., Vol.COM-16, No.2, pp.222-232, April 1968.

D50 J. H. Wittman, "Analysis of a hybrid frequency-time hopping random-access satellite communication system", IEEE Trans.Commun.Technol., Vol.COM-16, No.2, pp.303-310, April 1968.

E) SOME RECENT METHODS OF MULTIPLEXING AND ROUTING SIGNALS, NOT EMPLOYING RADAS

E1 J. M. Fraser, D. B. Bullock and N. G. Long, "Over-all characteristics of a TASI system", BSTJ, Vol.41, No.4, pp.1439-1454, July 1962.

E2 H. Miedema and M. G. Schachtman, "TASI quality-effect of speech detectors and interpolation", BSTJ, Vol.41, No.4, pp.1455-1473, July 1962.

E3 P. Baran, "On distributed communications networks", IEEE Trans. Commun.Systems, Vol.COM-12, No.1, pp.1-9, January 1964.

E4 J. Otterman, "A proposed dispersed control for a communication system", IEEE Intern.Conv.Record, Vol.13, Part 2, pp.177-180, 1965.

E5 L. H. Graham, "Automatic message relay and sequential search techniques", IEEE Intern.Conv.Record, Vol.13, Part 1, pp.252-255, 1965.

E6 L. B. Farabee, "Computer simulation for communications system design", IEEE Intern.Conv.Record, Vol.13, Part 1, pp.256-259, 1965.

E7 V. I. Johannes and R. H. McCullough, "Multiplexing of asynchronous digital signals using pulse stuffing with added-bit signalling", IEEE Trans.Commun.Technol., Vol.COM-14, No.5, pp.562-568, October 1966.

E8 J. Le Corre and A. Pirotte, "Pulse-code modulation for automatic switching of a military network", Electrical Communication, Vol.42, No.3, pp.357-367, 1967.

E9 G. H. Hartley and J. H. Dejean, "Potentialities of an integrated digital network", Electrical Communication, Vol.42, No.3, pp.392-401, 1967.

E10 J. H. Dejean, D. Cohen, C. Grandjean and P. Seneque, "Dispersed telecommunication network structure", Electrical Communication, Vol.42, No.3, pp.402-414, 1967.

E11 H. Hasegawa, Y. Tezuka, and Y. Kasahara, "Digital data dynamic transmission systems", IEEE Trans.Commun.Systems, Vol.COM-12, No.3, pp.58-65, September 1964.

E12 T. Hasegawa, Y. Tezuka and Y. Kasahara, "Transmission delay and channel loading in digital data dynamic transmission systems", IEEE Trans.Commun.Technol., Vol.COM-14, No.2, pp.94-101, April 1966.

E13 P. A. Wing, "WISDOM - self synchronizing delta-modulation orthogonal-channel modulator", Electronics Letters, Vol.4, No.11, pp.211-212, 31st May 1968.

F) METHODS OF REDUCING EITHER INTERSYMBOL OR INTERCHANNEL INTERFERENCE

F1 H. Nyquist, "Certain topics in telegraph transmission theory", Trans.AIEE, Vol.47, pp.617-644, April 1928.

F2 E. D. Sunde, "Theoretical fundamentals of pulse transmission", BSTJ, Vol.33, pp.721-788 and 987-1010, May and July 1954.

F3 W. R. Bennett and J. R. Davey, "Data Transmission", pp.53-66 and 94-114, McGraw-Hill Book Co.,Inc., New York, 1965.

F4 D. Gabor, "Theory of communication", J.IEE, Vol.93, Part 3, No.26, pp.429-457, November 1946.

F5 Le Roy A. MacColl, United States Patent 2,056,284, 6th October 1936.

F6 H. S. Black, "Modulation Theory", pp.107-110, D. Van Nostrand, Princeton, New Jersey, 1953.

F7 L. A. Zadeh and K. S. Milier, "Fundamental aspects of linear multiplexing", Proc.IRE, Vol.40, No.9, pp.1091-1097, September 1952.

F8 H. W. Farris, "Alternative detection of cochannel FM signals", Proc.IEEE, Vol.46, No.11, pp.1876-1877, November 1958.

F9 E. J. Baghdady, "Alternative detection of cochannel FM signals", Proc.IEEE, Vol.47, No.5, p.994, May 1959.

F10 E. J. Baghdady, "Linear cancellation technique for suppressing impulse noise", IRE WESCON Conv.Record, Vol.4, Part 7, pp.27-35, 1960.

F11 R. J. Kavanagh, "A note on optimum linear multivariable filters", Proc.IEE, Vol.108, Part C, Monograph No. 439M, pp.412-417, April 1961.

F12 R. W. Lucky and J. C. Hancock, "On the optimum performance of N-ary systems having two degrees of freedom", IEEE Trans.Commun.Systems, Vol.CS-10, No.2, pp.185-192, June 1962.

F13 G. Lacks, "Optimization of signal waveforms", IEEE Trans.Inform. Theory, Vol.IT-9, No.2, pp.95-97, April 1963.

F14 J. E. Bridges and R. A. Zalewski, "Orthogonal detection to reduce common channel interference", Proc.IEEE, Vol.52, No.9, pp.1022-1028, September 1964.

F15 D. W. Tufts, "Nyquist's problem - the joint optimization of transmitter and receiver in pulse amplitude modulation", Proc.IEEE, Vol.53, No.3, pp.248-259, March 1965.

F16 T. H. Crystal, "Orthogonal detection to reduce common channel interference", Proc.IEEE, Vol.53, No.7, pp.742-743, July 1965.

F17 M. J. Wiggins, "Pulse error reduction in pulse position modulation systems", IEEE Intern.Conv.Record, Vol.13, Part 2, pp.14-19, 1965.

F18 L. Kurz and S. S. Rappaport, "Reliable digital signalling in the presence of additive gaussian noise and interchannel interference", Proc.Natl.Electronics Conference, Vol.21, pp.307-312, 1965.

F19 C. L. Weber, "New solutions to the signal design problem for coherent channels", IEEE Trans.Inform.Theory, Vol.IT-12, No.2, pp.161-167, April 1966.

F20 P. D. Deighton, "Optimization of a realisable receiver in pulse amplitude modulation", Electronics Letters, Vol.3, No.3, pp.129-130, March 1967.

F21 P. D. Deighton, "Joint optimization of realisable receiver and transmitter in data transmission systems", Electronics Letters, Vol.3, No.7, pp.342-344, July 1967.

F22 R. W. Chang, "Precoding for multiple speed data transmission", BSTJ, Vol.46, No.7, pp.1633-1649, September 1967.

F23 D. A. Shnidman, "A generalized Nyquist criterion and an optimum linear receiver for a pulse modulation system", BSTJ, Vol.46, No.9, pp.2163-2177, November 1967.

F24 L. Schiff, "On the transmission of information over the Gaussian and related channels", Ph.D. Thesis, Polytechnic Institute of Brooklyn, June 1968.

G) TECHNIQUES FOR SOLVING LINEAR SIMULTANEOUS EQUATIONS

G1 W. A. Adcock, "An automatic simultaneous equation computer and its use in solving secular equations", Rev.Sci.Instr., Vol.19, pp.181-187, March 1948.

G2 G. A. Korn, "Stabilization of simultaneous equation solvers", Proc.IRE, Vol.37,. No.9, pp.1000-1002, September 1949.

G3 M. L. Stein, "Gradient methods in the solution of systems of linear equations", J.Res.N.B.S., Vol.48, No.6, pp.407-413, June 1952.

G4 M. R. Hestenes and E. Stiefel, "Methods of conjugate gradients for solving linear systems", J.Res.N.B.S., Vol.49, No.6, pp.409-436, December 1952.

G5 D. Young, "Iterative methods for solving partial difference equations of elliptic type", Trans.Amer.Math.Soc., Vol.76, pp.92-111, 1954.

G6 P. M. Honnel and R. E. Horn, "Analogue computer synthesis and error matrices", AIEE Trans.Commun.Electronics, Vol.75, Part 1, pp.26-32, March 1956.

G7 E. F. Beckenbach (editor), "Modern Mathematics for the Engineer", First Series, pp.428-479, McGraw-Hill Book Co., Inc., New York, 1956.

G8 V. N. Faddeeva, "Computational Methods of Linear Algebra", pp.63-145, Dover Publications,Inc., New York, 1959.

G9 R. E. Horn and P. M. Honnel, "Electronic network synthesis of linear algebraic matrix equations", AIEE Trans.Commun.Electronics, Vol.78, Part 1, pp.1028-1032, January 1960.

G10 H. H. Rosenbrock, "An automatic method for finding the greatest or least value of a function", Computer J., Vol.3, No.3, pp.175-184, October 1960.

G11 A. W. O. Firth, "Optimization problems: solution by an analogue computer", Computer J., Vol.4, No.1, pp.68-72, April 1961.

G12 B. A. Carré, "The determination of the optimum accelerating factor for successive over-relaxation", Computer J., Vol.4, No.1, pp.73-78, April 1961.

G13 D. W. Martin and G. J. Tee, "Iterative methods for linear equations with symmetric positive definite matrix", Computer J., Vol.4, No.3, pp.242-254, October 1961.

G14 N. S. Bromberg, "Maximization and minimization of complicated multivariable functions", AIEE Trans.Commun.Electronics, Vol.80, No.58, pp.725-730, January 1962.

G15 M. J. D. Powell, "An iterative method for finding stationary values of a function of several variables", Computer J., Vol.5, No.2, pp.147-151, July 1962.

G16 R. M. Baer, "Note on an extremum locating algorithm", Computer J., Vol.5, No.3, p.193, October 1962.

G17 H. A. Spang, "A review of minimization techniques for non-linear functions", SIAM Rev., Vol.4, No.4, pp.343-365, October 1962.

G18 J. A. Knudson, "Iterative procedures", Proc.IRE, Vol.50, No.11, p.2377, November 1962.

G19 T. Kasvand, "Comments on iterative procedures", Proc.IEEE, Vol.51, No.7, p.1064, July 1963.

G20 R. Fletcher and M. J. D. Powell, "A rapidly convergent descent method for minimization", Computer J., Vol.6, No.2, pp.163-168, July 1963.

G21 D. J. Evans, "The extrapolated modified Aitken iteration method for solving elliptic difference equations", Computer J., Vol.6, No.2, pp.193-201, July 1963.

G22 D. J. Evans and C. V. D. Forrington, "An iterative process for optimizing symmetric successive over-relaxation", Computer J., Vol.6, No.3, pp.271-273, October 1963.

G23 E. D'Sylva and G. A. Miles, "The SSOR iteration scheme for equations with σ_1 ordering", Computer J., Vol.6, No.4, pp.366-367, January 1964.

G24 M. S. Lynn, "On the equivalence of SOR, SSOR and USSOR as applied to σ_1-ordered systems of linear equations", Computer J., Vol.7, No.1, pp.72-75, April 1964.

G25 D. J. Evans, "The extrapolated modified Aitken iteration method applied to σ_1-ordered systems of linear equations", Computer J., Vol.7, No.2, pp.137-140, July 1964.

G26 R. Fletcher and C. M. Reeves, "Function minimization by conjugate gradients", Computer J., Vol.7, No.2, pp.149-153, July 1964.

G27 M. J. D. Powell, "An efficient method for finding the minimum of a function of several variables without calculating derivatives", Computer J., Vol.7, No.2, pp.155-162, July 1964.

G28 P. M. Honnell, R. J. Mulholland and E. V. Krishnamurthy, "Further comments on iterative procedures", Proc.IEEE, Vol.52, No.8, pp.976-978, August 1964.

G29 E. V. Krishnamurthy, "Solutions of linear systems as eigenvalue problems", Proc.IEEE, Vol.52, No.9, pp.1065-1066, September 1964.

G30 J. M. McCormick and M. G. Salvadori, "Numerical Methods in Fortran", pp.71-91, Prentice-Hall,Inc., New Jersey, 1964.

G31 C. P. Gilbert and B. J. J. McHugh, "Economical circuits for the analogue solution of algebraic equations", IEEE Trans. Electronic Computers, Vol.EC-14, No.2, pp.217-229, April 1965.

G32 R. Fletcher, "Functional minimization without evaluating derivatives-a review", Computer J, Vol.8, No.1, pp.33-41, April 1965.

G33 W. E. Langlois, "Conditions for termination of the method of steepest descent after a finite number of iterations", IBM J., Vol.10, No.1, pp.98-99, January 1966.

G34 S. Mitter, L. S. Lasdon and A. D. Warren, "The method of conjugate gradients for optimal control problems", Proc.IEEE, Vol.54, No.6, pp.904-905, June 1966.

G35 P. C. Young, "Parameter estimation and the method of conjugate gradients", Proc.IEEE, Vol.54, No.12, pp.1965-1967, December 1966.

G36 L. S. Lasdon, S. K. Mitter and A. D. Warren, "The conjugate gradient method for optimal control problems", IEEE Trans.Automatic Control, Vol.AC-12, No.2, pp.132-138, April 1967.

G37 B. T. Fang, "Newton's method and steepest ascent", IEEE Trans. Automatic Control, Vol.AC-12, No.2, p.203, April 1967.

G38 W. I. Zangwill, "Minimizing a function without calculating derivatives", Computer J., Vol.10, No.3, pp.293-296, November 1967.

G39 W. C. Davidon, "Variance algorithm for minimization", Computer J., Vol.10, No.4, pp.406-410, February 1968.