Issues in the Ratings of Active Power Filters

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Abstract

The fitting of an active power filter (APF) to mitigate the effects of a diode or thyristor bridge-rectifier is predicated on the assumption that the rating of the filter is reasonable (*i.e.,* small) compared to the rating of the existing bridge rectifier or of a replacement active rectifier. This paper analyses the ratings of both shunt and series APFs in a variety of operating conditions. Ratings are assessed through peak voltage and mean current as appropriate for junction semiconductor devices. RMS ratings are also given because of their familiarity. It has been shown that the series APF, appropriate for the compensation of harmonic voltage sources, is of a generally higher rating than the shunt APF, appropriate for harmonic current sources. The use of a shunt APF where a series APF is appropriate results in poor device utilisation. The semiconductor ratings of an APF can be reduced by using hybrid schemes. Removing large-amplitude, low-order current harmonics with harmonic traps is effective for the shunt APF. Reducing peak voltages by attenuating high frequency components is effective with the series APF. Arranging rectifiers into multi-pulse arrangements is seen to be effective for both shunt and series APFs. The ratings are approximately halved for the 12-pulse case.

1 Introduction

Standards such as IEC 1000 oblige equipment manufactures to guard against harmonic emissions across a range of power levels. Utility operator's regulations (such as G59 or others based on IEEE 519) oblige customers to examine their total harmonic emissions. The most common harmonically polluting load is a diode (or thyristor) bridge-rectifier in which the semiconductors operate at line frequency. These are commonplace in industrial variable-speed drives as the first stage of an AC/DC/AC power conversion. They were also widespread in consumer electronics until recently and have had a particularly deleterious effect in "off-line" switching regulators in which the first power conversion stage was diode rectification at mains voltage with no utility-side transformer. Recently however, at least some of these power supplies have used "power factor corrected" switch-mode rectifiers with near-sinusoidal current waveforms. However, switch-mode rectifiers (in consumer electronics or industrial drives) are expensive compared to bridge-rectifiers and are resisted unless other advantages are apparent (*e.g.,* universal input voltage range or regeneration).

Active power filters (APFs) have long been touted as the retro-fit solution to harmonic problems of bridge rectifiers [1, 2, 3 and 4] and are even considered as an option in new-build equipment in preference to an active rectifier.

The case for using an APF is that a relatively small inverter can be used to inject cancellation current (or voltage) to compensate for the distortion produced by a non-linear load. It is important not to take this argument at face value but to examine in detail the factors that affect the rating of an APF. Ratings are not the only determinant of costs but they are a very important factor. An APF and an active rectifier are technologically similar and so ratings may well be the deciding factor when choosing between them. Both are considerably more complex (in terms of both control and semiconductor characteristics) than a diode rectifier. The APF will not be adopted where it does not offer a significant ratings advantage.

Several styles of APF exist and have been usefully categorised and discussed in [5] using ideas of duality to explore the similarity between different approaches. Figure 1(a) shows a shunt connected APF injecting current to cancel the distortion of a harmonic current source. Figure 1(b) shows a series APF injecting a voltage to cancel the distortion of a harmonic voltage source. Figure 1(c) shows a shunt APF connected to inject current to cancel current distortion cause indirectly by a harmonic voltage source. Figure 1 (d) to (f) show various hybrid schemes in which passive filters correct some of the distortion and the APF deals with the remainder.

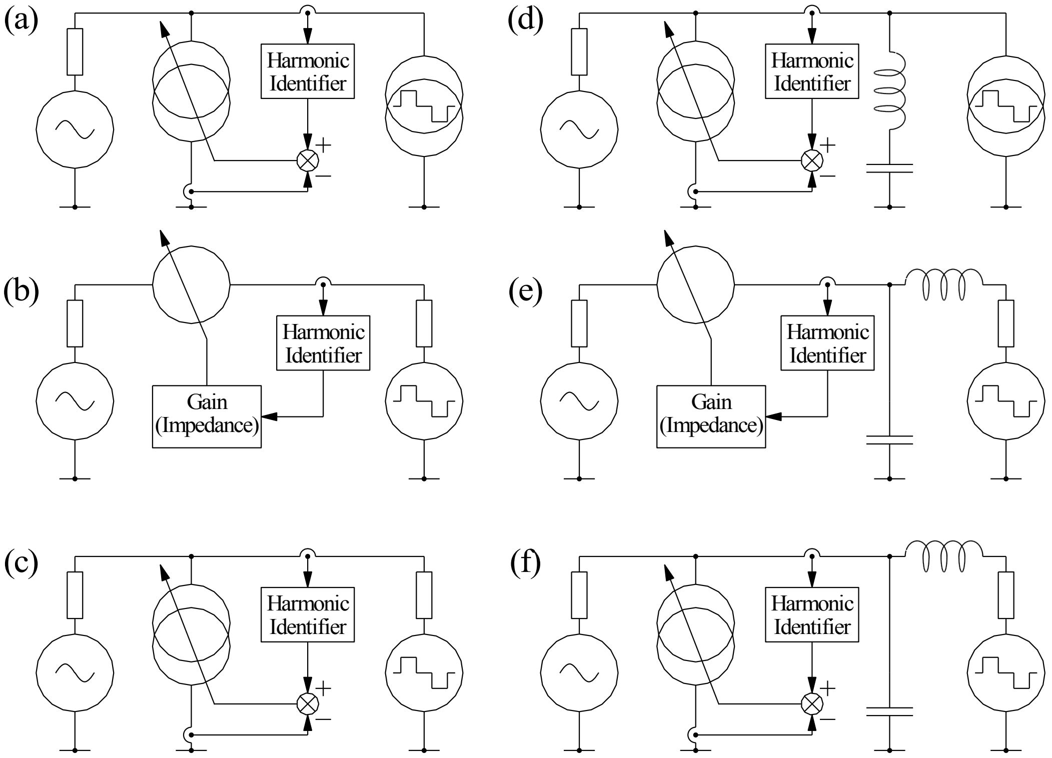

Figure 1 Example power filters systems: (a)-(c) active filters; (d)-(f) hybrid active/passive filters

2 The basis of semiconductor ratings

Electrical machines are rated according to their RMS current and voltage. The limit on current is a thermal one and, since the losses are normally ohmic in nature, the root-mean-square is the form of averaging that gives the correct indication of the heating effect irrespective of wave-shape. Strictly, the voltage rating depends on the ability of the insulation to withstand the peak voltage but since the voltage waveform is normally close to sinusoidal, RMS voltage can be used as a surrogate for peak voltage.

Although in many ways a series APF (which injects a correcting voltage) is the dual of a shunt APF (which injects a correcting current), one should not expect this duality to extend to the ratings because of the asymmetry between the over-voltage and over-current failure mechanisms of semiconductors.

The effect of excess voltage on a semiconductor is near-instantaneous failure due to avalanche or punch-through causing very high power dissipation and rapid local melting [6]. Thus, the correct indication of the voltage ratings is given by the peak voltage. An APF in series connection develops a complex wave-shape with a high crest-factor (as discussed in section 3.2). The RMS voltage can not be used as a reliable indication of the peak voltage in these circumstances.

The reaction to excess current is more gradual because the extra power loss gradually raises the temperature of the die according to the thermal time-constant of the system. Thus, the current rating can be based on an average current that reflects the dominant power loss mechanism [6]. For a MOSFET, the conduction loss mechanism is ohmic and an RMS average is appropriate. For an IGBT or GTO-thyristor, the conduction loss mechanism is a voltage drop and the mean of the current gives the correct indication of losses irrespective of wave-shape. Regardless of the type of device, switching power loss (and reverse recovery power loss) is proportional to the current being switched. The mean current correctly indicates the average loss. Overall, mean current is the best indicator of semiconductor current ratings for arbitrary current waveforms. For the special case of a MOSFET, some weighted combination of mean and RMS current could be used.

A rating based on the mean current and peak voltage will be calculated for all the example systems. However, because of its familiarity, a conventional RMS-based VA rating of the various APFs will also be given. A *per-unit* system will be used: for conventional ratings this is to a base of RMS phase-voltage and power (per phase). The impedance base is derived in the normal way and, by incorporating the system frequency, can be used to quote inductors and capacitors too. For semiconductor ratings the bases are the peak phase voltage and the half-cycle mean of the fundamental current.

Ratings will be assessed analytically assuming ideal conditions such as instantaneous diode commutation, operation at unity displacement-factor and constant values of current/voltage in smoothing inductor/capacitor. The effects of relaxing these conditions are assessed through circuit simulation.

3 Rectifier Waveforms

There are several classifications of bridge-rectifiers but the most important distinction here is between those with an inductance dominated DC-side (which are current-stiff) and those with a capacitance dominated DC-side (which are voltage-stiff), figure 2. In an inductive rectifier the current in the DC-side inductor defines the magnitude of the AC-side currents. In a capacitive rectifier, the voltage on the DC-side capacitor defines the magnitude of the AC-side voltage. A common arrangement in high power rectifiers is an *LC* smoothing filter on the DC-side. This presents an inductive characteristic to the AC-side and a capacitive characteristic to the DC-load.

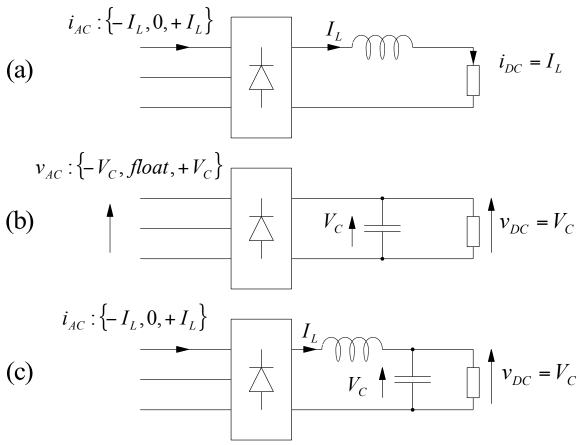

Figure 2 Inductive, capacitive and inductive-capacitive DC-side filters and the AC-side conditions they impose.

The choice of component values for rectifiers and the dependence of DC-side ripple and AC-side distortion on those component values are discussed in [7].

3.1 Inductive Rectifier

Figure 3 shows the AC-side current waveform taken from a SPICE simulation of an inductive rectifier. For this and other simulations the conditions in table 1 were set and used as the base quantities for *per-unit* values:

Line Voltage	400 V		
Power (per phase)	3 kW		
Frequency	50 Hz		
Phase Voltage	230.9 V^{RMS}	326.6 V^{peak}	
Phase Current	13.0 A^{RMS}	18.34 A^{peak}	11.68 A^{mean}
Base Impedance	17.77 Ω		
Base Inductance	56.6 mH		
Base Capacitance	179 µF		

Table 1 Simulation conditions and bases of the *pe- unit* quantities

The inductive rectifier contained a 1 *pu* DC-side inductor and 0.01 *pu* AC-side inductors. The effect of these can be seen in the top plot as, respectively, a slight ripple on the tops of the current pulses and the finite commutation time of the diodes (seen as sloped edges of the current pulses). The middle plot confirms the amplitude of the fundamental as 18.34 A^{peak} and the bottom plot shows the waveform necessary to cancel the harmonic content of the phase current. The fundamental current had a slight phase-lag of 4º (a displacement factor of 0.998).

For comparison, Figure 4 shows similar waveforms but for a rectifier with a 0.01 *pu* DC-side inductor. The phase current shows greater ripple and the cancellation reference has changed accordingly.

In both simulations the fundamental component of current is 13.0 A^{rms}. With 1 *pu* inductance the AC-side current is 13.50 A^{rms} and the cancellation reference is 3.76 A^{rms}. With 0.01 *pu* inductance the AC-side current is 13.58 A^{rms} and the cancellation reference is 3.77 A^{rms}.

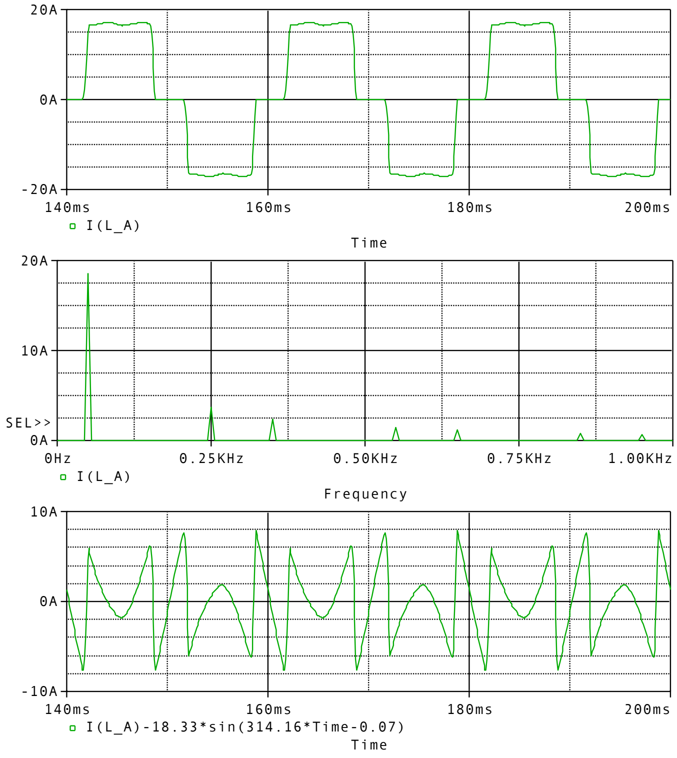

Figure 3 Simulation of an inductive rectifier with a 1 *pu* DC-side inductance. Top: phase-current; centre: spectrum of phase-current; bottom: distortion content of phase-current.

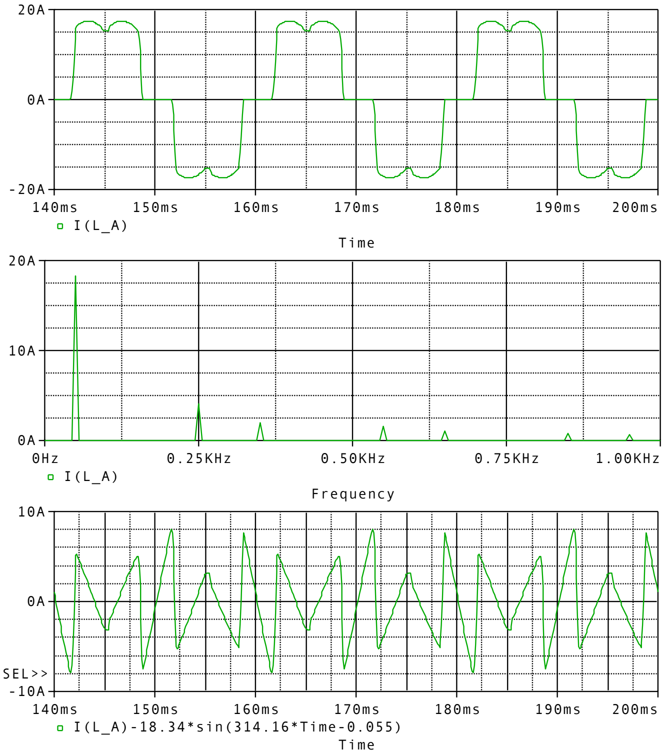

Figure 4 Simulation of an inductive rectifier with a 0.01 *pu* DC-side inductance. Top: phase-current; centre: spectrum of phase-current; bottom: distortion content of phase-current.

3.2 Capacitive Rectifier

The distortion mechanism in a capacitive rectifier is different to that in an inductive rectifier. The DC-side capacitor imposes a line-voltage at the AC terminals of the rectifier according to which diode pair is carrying current. The third phase, that with no forward biased diode, caries no current and the connection floats. A voltage drop is imposed across the line impedance of the conducting phases. This voltage drop drives both fundamental and harmonic currents.

The designer of an APF for a capacitive rectifier is faced with the alternatives of using a shunt filter to cancel the harmonic current or a series filter to cancel directly the harmonic voltage that drives the current. The value of the line impedance between the grid and the rectifier is an important parameter in the design of the APF.

A capacitive rectifier was simulated with a DC-side capacitance of 1 *pu* (which is much smaller than commonly used in practice) and an AC-side inductance of 0.01 *pu* (to represent the line impedance). The voltage drop across the line impedance and the phase current are shown in figure

5. The phase current shows the familiar double-pulse form expected of a three-phase (6-pulse) bridge-rectifier. The spectrum of the voltage drop reveals a fundamental component and the characteristic distortion components of a 6-pulse rectifier (harmonics of order $6k\pm1$). The voltage on the DC-side capacitor was 518V (1.586 *pu* on a base of the peak phase-voltage).

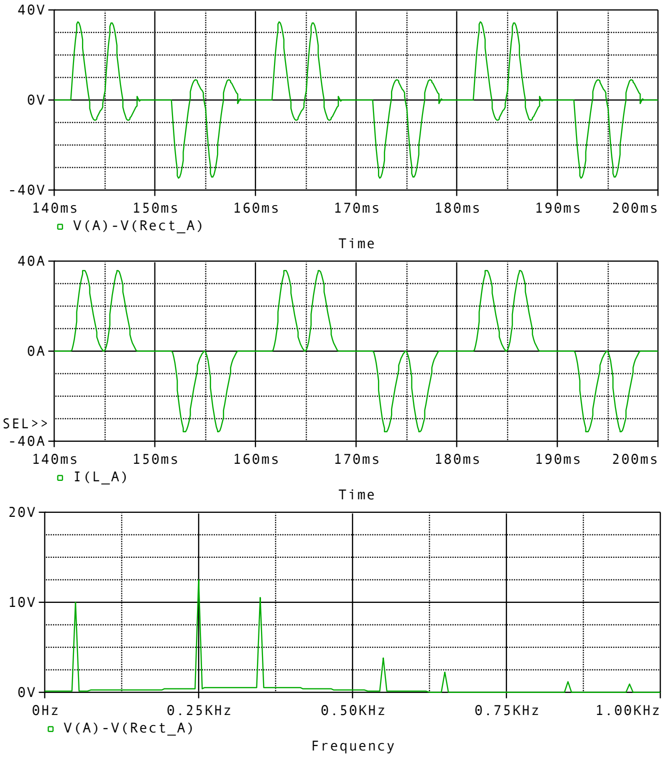

Figure 5 Simulation of a capacitive rectifier with 1 *pu* DC-side capacitance and 0.01 *pu* AC-side inductance. Top: voltage across line impedance; centre: phase current; bottom: spectrum of voltage across line impedance.

If the AC-side inductance is increased then the phase-current distortion is reduced but high amplitude harmonics appearing in the voltage drop across the inductance as shown in figure 6. The voltage on the DC-side capacitor reduces to 498.8 V (1.527 *pu*) because of the increased voltage drop across the line inductance.

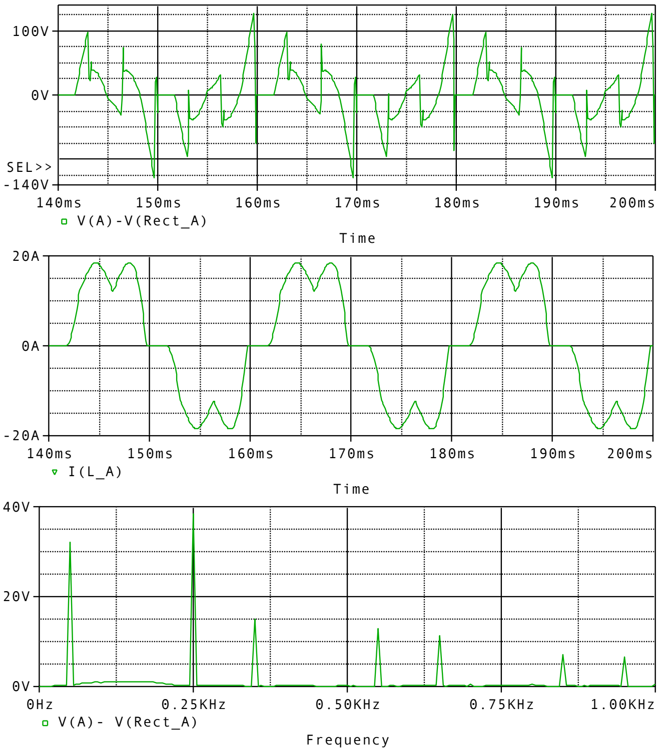

Figure 6 Simulation of a capacitive rectifier with 1 *pu* DC-side capacitance and 0.1 *pu* AC-side inductance. Top: voltage across line impedance; centre: phase current; bottom: spectrum of voltage across line impedance.

If sinusoidal currents are imposed on the rectifier, by the action of a series APF for instance, then all phases carry current at all times and the rectifier terminal voltages become piecewise constant. Figure 7 shows waveforms from a simulation in which a series APF was used to cancel almost all of the voltage distortion with the result that the phase current is substantially sinusoidal. The APF used a second-order Butterworth band-pass filter as the harmonic identifier. The pass-band was set at 200 Hz to 2 kHz. This frequency range includes the most important harmonics and recognises the upper frequency limit on waveforms that can be synthesised by the inverter within the APF. The APF was used to present an impedance of 80Ω (4.5 *pu*) in the line for frequencies in the pass-band of the harmonic identifier (the stop-band of the APF). An additional control-loop ensured that there was no real power exchange between the APF and the line.

Under these conditions, the DC-side capacitor voltage was 514 V (1.574 *pu*). The peak rectifier phase-voltage was 345.6 V (1.058 *pu*) and the peak voltage required of the APF was 159.0 V (0.487 *pu*). When the APF was set to absorb a real power of 0.02 *pu* (to simulate inverter power losses) the voltages changed as follows: DC-side capacitor voltage 481V (1.472 *pu*); peak rectifier voltage 324 V (0.992) and peak APF voltage 124 V (0.380 *pu*).

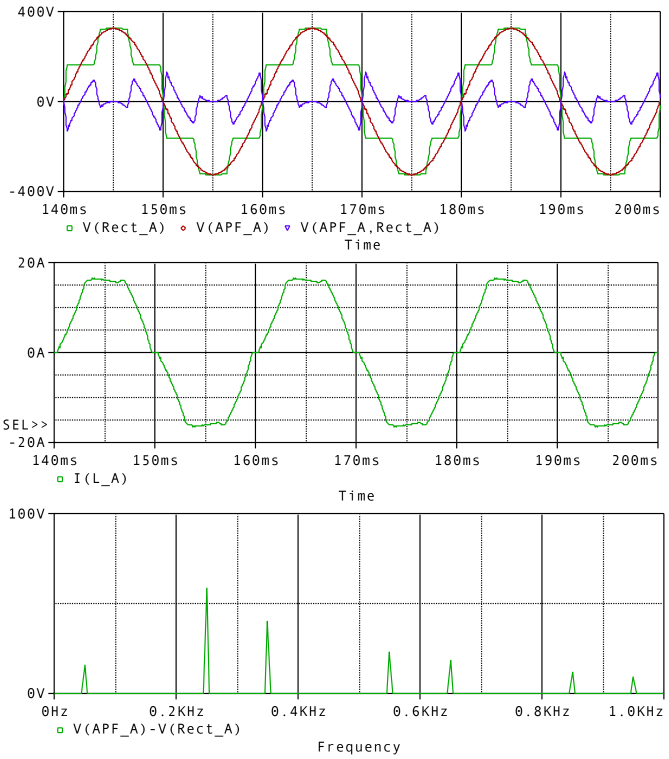

Figure 7 Simulation of a capacitive rectifier with near sinusoidal current (DC-side capacitance: 1 *pu*; AC-side inductance: 0.01 *pu*). Top: phase-voltage of rectifier, grid and voltage introduced by APF; centre: phase current; bottom: spectrum of voltage introduced by APF.

4 Active Power Filter Ratings

4.1 Inductive Rectifier and Shunt APF

The inductive rectifier draws harmonic currents from the grid and so the natural solution is a shunt APF that will inject a cancelling current. The inverter of the APF will need to operate at slightly above the grid voltage in order to force current into the grid but for convenience it is taken that the voltage rating of the shunt APF is 1 *pu*. The idealised waveforms for the inductive rectifier are piece-wise constant [8] and are defined over the first quarter cycle as:

$$\begin{aligned} i_{AC}(\omega t) &= 0 \qquad 0 \le \omega t < \tfrac{\pi}{6} \\ &= I_L \qquad \tfrac{\pi}{6} \le \omega t < \tfrac{\pi}{2} \end{aligned}$$

Fourier Series Analysis identifies the fundamental component of this waveform as:

$$I_{AC}^{1rms} = \frac{\sqrt{6}}{\pi} I_L$$

Subtracting the fundamental component from the AC-side current yields the cancellation reference:

$$i_{AC}\left(\omega t\right) = I_L\left(0 - \frac{2\sqrt{3}}{\pi}\sin\left(\omega t\right)\right) \qquad 0 \le \omega t < \tfrac{\pi}{6}$$

$$= I_L\left(1 - \frac{2\sqrt{3}}{\pi}\sin\left(\omega t\right)\right) \qquad \tfrac{\pi}{6} \le \omega t < \tfrac{\pi}{2}$$

The RMS value of the cancellation current can be calculated and expressed as a fraction of the fundamental current:

$$\frac{I_{APF}^{RMS}}{I_{AC}^{1RMS}} = \sqrt{\frac{\pi^2}{9} - 1} = 0.311$$

The standard RMS current rating of the APF is therefore 0.311 *pu* in idealised form. This compares with values found by simulation of 0.289 and 0.290 *pu* for rectifiers with inductors of 1 and 0.01 *pu*. The idealised case appears to over-estimate slightly the current rating of a realistic rectifier.

As discussed in section 2, the mean current is more appropriate than the RMS current when rating semiconductor switches. The half-cycle mean is normally used with AC signals to avoid a zero result. With a highly distorted waveform this is not sufficient and the absolute value of the relevant current should be used for calculation of the mean. Whether positive or negative, the current will flow through a semiconductor that produces a voltage drop that causes power dissipation. Strictly, the average should be adjusted to account for the difference in voltage drop (and dissipation) that occurs with forward current through a switching device and reverse current through a diode.

The cancellation reference for the inductive rectifier changes sign when the fundamental component of the AC-side current crosses the DC-side current. For the idealised inductive rectifier, this occurs at a first quadrant angle, φ defined by:

$$\sin\left(\phi\right) = \frac{I_L}{I_{AC}^{1peak}} = \frac{\pi}{2\sqrt{3}}$$

The mean of the cancellation current can then be found:

$$I_{APF}^{mean} = \frac{2}{\pi}\left[\int_0^{\frac{\pi}{6}} I_{AC}^{1peak}\sin\left(\theta\right)d\theta - \int_{\frac{\pi}{6}}^{\phi}\left(I_{AC}^{1peak}\sin\left(\theta\right) - I_{DC}\right)d\theta + \int_{\phi}^{\frac{\pi}{2}}\left(I_1\sin\left(\theta\right) - I_{DC}\right)d\theta\right]$$

$$= \frac{2}{\pi} I_{AC}^{1peak}\left(1 - \sqrt{3} + 2\cos(\phi) + \frac{\pi}{\sqrt{3}}\left(\phi - \frac{\pi}{3}\right)\right)$$

The mean of a sinusoid is 2/π of its peak:

$$\frac{I_{APF}^{mean}}{I_{AC}^{1mean}} = \left(1-\sqrt{3}+2\cos(\phi)+\frac{\pi}{\sqrt{3}}\left(\phi-\frac{\pi}{3}\right)\right) = 0.272$$

Thus the APF current is 0.2715 *pu* on a mean basis. The simulations of figure 3 and figure 4 yield values of 0.253 and 0.265 *pu* respectively. Again, the idealised case slightly over-estimates the current required in a realistic case.

The ratings of the APF can be compared to the ratings of the bridge-rectifier itself and of the active rectifier that could be used to replace the APF/bridge-rectifier combination, table 2.

	V^{peak} pu	I^{RMS} pu	I^{mean} pu
APF	1.0	0.311	0.272
Bridge Rectifier	1.0	1.039	0.951
Active Rectifier	1.0	1.0	1.0

Table 2 Comparison of APF rating and rectifier rating for the case of an inductive rectifier

4.2 Capacitive Rectifier and Series APF

The near piecewise-constant phase-voltage waveform of the capacitive rectifier, as seen in figure 7, can be idealised (for the first quadrant) as:

$$\begin{aligned} v_{AC}(\omega t) &= \tfrac{1}{3}V_C \quad 0 \le \omega t < \tfrac{\pi}{3} \\ &= \tfrac{2}{3}V_C \quad \tfrac{\pi}{3} \le \omega t < \tfrac{\pi}{2} \end{aligned}$$

The peak of the fundamental component of this waveform is:

$$V_{AC}^{1peak} = \frac{2}{\pi}V_C$$

This equation predicts a DC-side capacitor voltage of $\pi/2=1.571$ *pu* which is a slight under-estimate of the value found in simulation (1.574 *pu*).

Subtracting the fundamental component from the AC-side voltage yields the cancellation reference as:

$$v_{AC}^{harm}(\omega t) = V_C\left(\frac{1}{3} - \frac{2}{\pi}\sin(\omega t)\right) \qquad 0 \le \omega t < \tfrac{\pi}{3}$$

$$= V_C\left(\frac{2}{3} - \frac{2}{\pi}\sin(\omega t)\right) \qquad \tfrac{\pi}{3} \le \omega t < \tfrac{\pi}{2}$$

The peaks of this waveform occur at angles which are multiples of π, and that peak value is:

$$V_{APF}^{peak} = \frac{1}{3}V_C = \frac{\pi}{6}V_{AC}^{1\,peak} = 0.524\,pu$$

The value found in simulation was 0.487 *pu*. The difference is accounted for by the fact that the high frequency response of the APF was deliberately limited and this will have reduced the sharpness of the peaks of the voltage waveform. Thus, the theoretical value represents an over-estimate of the voltage rating required in practice.

4.3 Capacitive Rectifier and Shunt APF

There are many reported studies, such as [9, 10], of using a shunt APF to cancel the current distortion that results from the voltage distortion of a capacitive APF. The disadvantages of such an approach have been explored by Peng in [3, 5]. The shunt APF is operated so that it supplies any harmonic current demanded by the rectifier. The rectifier therefore has a source that exhibits very low impedance in the harmonic frequency range. This is will be a significantly lower impedance than that of the grid and so the rectifier will draw greater amplitude harmonic currents than from the grid alone (albeit that this is drawn from the APF rather than from the grid).

The harmonics drawn by the rectifier and supplied by the APF will depend, principally, on the impedance connected between them. Thus it is difficult to isolate a useful general statement of APF current rating. Instead, example simulations will be used to show the variation of rating with coupling impedance.

If the APF perfectly cancels the harmonic currents, then the voltage at the point where the APF couples to the line will be sinusoidal (provided the grid voltage is sinusoidal). For rating purposes, it is sufficient to examine the harmonic current drawn by a rectifier under these conditions. These were the conditions used to generate figure 5 and figure 6. A further set of simulations was used to generate the additional data points for figure 8.

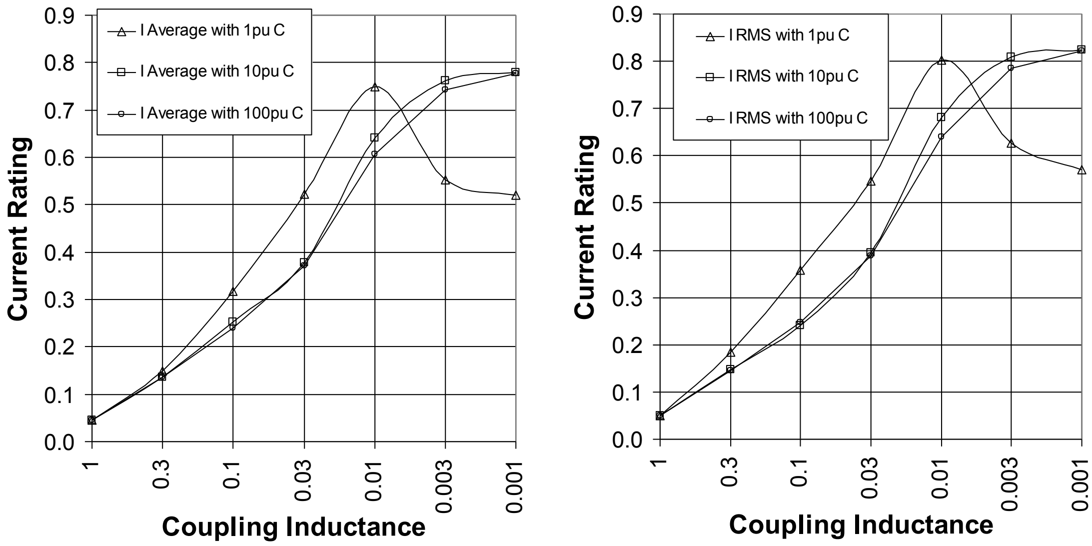

Figure 8 Mean current ratings (left) and RMS current ratings (right), expressed in *pu*, of a shunt APF operating with a capacitive rectifier as a function of the coupling inductance between them. Results are for DC-side capacitance of 1 *pu,* 10*pu* and 100 *pu*.

With a large coupling inductance, the current distortion is small and the APF current rating is therefore small also. A value of 1 *pu* coupling inductance is, however, too large to be useful since it will cause a large voltage drop at line fundamental frequency and reduce the DC voltage output and the power available.

With a small value of coupling inductance, the current distortion is large and the APF current rating rises almost to match that of the rectifier itself. With a small value of DC-side capacitor, the current rating is seen to peak and then reduce again. This occurs because the DC-voltage almost follows the line voltage shape (*i.e.*, a large ripple develops). In normal practice a large DC-side capacitor, greater even than 10 *pu*, is used to provide low voltage ripple and a degree of mains-loss ride-through. With a large DC-side capacitor, peak in current rating occurs at very low values of coupling inductance. The lowest coupling inductance shown here, 0.001 *pu*, represents the inductance of the supply wires alone without any additional commutation inductance in the rectifier.

4.4 Comparison of Shunt and Series Ratings

The susceptibility of an inverter to peak voltage means that its voltage rating will need to be high when synthesising voltage waveforms with a high crest factor. This is not the case for the current rating. Thus, a series APF compensating a capacitive rectifier should be rated at 0.487 whereas a shunt APF compensating an inductive rectifier has should be rated at 0.270.

It is tempting then to examine a shunt APF compensating a capacitive rectifier. Figure 8 demonstrated that there is not a single answer to the ratings question but that the value of the coupling inductance is important. If the impedance is less than 0.03 *pu* then the shunt APF will require a higher rating than the series APF for the same capacitive rectifier.

Comparing a shunt APF compensating a capacitive rectifier with one compensating an inductive rectifier, the inductive rectifier will require the lower APF rating unless the coupling impedance of the capacitive rectifier is above 0.1 *pu*.

A capacitive rectifier may include some AC-side inductance to control the diode commutation (and its associated commutation notches) but it is likely that extra inductance will need to be added to achieve acceptable APF ratings for the shunt compensation case.

5 Approaches to Rating Reduction

Hybrid-APFs formed of standard APFs and passive filters have been proposed as methods of reducing the rating of the semiconductors required. The many possible arrangements and operating regimes are reviewed in [5]. For a shunt APF, the approach should be to remove through passive means those harmonics making the greatest contribution to the average current. These are normally taken to be the low order harmonics. Harmonic traps or multi-pulse arrangements of rectifiers can be considered. For a series APF, the approach should be to attenuate the peak voltage. This could be achieved through passive low-pass filters or through multi-pulse arrangements.

5.1 Multi-pulse Rectifiers

A 12-pulse inductive rectifier can be formed from two 6-pulse rectifiers with their DC-connections in series and the AC-connections phase-shifted 30º with respect to each other. The form of analysis used in section 4.1 will be followed. The idealised form of rectifier phase current is:

$$i_{AC} = \begin{cases} I_L \dfrac{1}{\sqrt{3}} & 0 \le \theta < \dfrac{\pi}{6} \\ I_L\left(\dfrac{1}{\sqrt{3}}+1\right) & \dfrac{\pi}{6} \le \theta < \dfrac{\pi}{3} \\ I_L\left(\dfrac{2}{\sqrt{3}}+1\right) & \dfrac{\pi}{3} \le \theta < \dfrac{\pi}{2} \end{cases}$$

From which the fundamental component can be identified as:

$$I_{AC}^{1\,peak} = \frac{4\sqrt{3}}{\pi} I_L$$

The required current of a shunt APF is found by subtracting this fundamental component. Taking the mean (of the absolute value) of the APF current and dividing through by the mean of the fundamental component gives:

$$\frac{I_{APF}^{mean}}{I_{AC}^{1\,mean}} = 2\left(\cos\phi_1 + \cos\phi_2 + \cos\phi_3 - 1 - \frac{\sqrt{3}}{2}\right)$$
$$+ \frac{\pi}{4\sqrt{3}}\left(\frac{1}{\sqrt{3}}\left(2\phi_1 - \frac{\pi}{6}\right) + \left(1+\frac{1}{\sqrt{3}}\right)\left(2\phi_2 - \frac{\pi}{2}\right) + \left(1+\frac{2}{\sqrt{3}}\right)\left(2\phi_3 - \frac{5\pi}{6}\right)\right)$$

$$= 0.132$$

where $\phi_1 = \sin^{-1}\left(\frac{\pi}{12}\right)$, $\phi_2 = \sin^{-1}\left(\frac{\pi}{4\sqrt{3}} + \frac{\pi}{12}\right)$, $\phi_3 = \sin^{-1}\left(\frac{\pi}{4\sqrt{3}} + \frac{\pi}{6}\right)$

Thus, the shunt APF operating with a 12-pulse inductive rectifier has a current rating of 0.132 *pu* which is approximately half of the rating required for the 6-pulse rectifier (0.272). Half of the harmonics (order 6(2*k*+1)±1) are cancelled in the 12-pulse case and so a halving of the APF rating might be expected.

The expression for the RMS is:

$$\frac{I_{APF}^{RMS}}{I_{AC}^{1\,RMS}} = \sqrt{\left(\frac{\pi}{6}\right)^2\left(2+\sqrt{3}\right) - 1} = 0.152$$

This gives a rating of 0.152 *pu* compared to 0.311 *pu* for the 6-pulse case.

A 12-pulse capacitive rectifier can be formed of two 6-pulse units with the DC connections in parallel. The phase voltage of the rectifier is:

$$v_{AC} = \begin{cases} V_C \dfrac{1}{3} & 0 \le \theta < \dfrac{\pi}{6} \\ V_C\left(\dfrac{1}{3} + \dfrac{1}{\sqrt{3}}\right) & \dfrac{\pi}{6} \le \theta < \dfrac{\pi}{3} \\ V_C\left(\dfrac{2}{3} + \dfrac{1}{\sqrt{3}}\right) & \dfrac{\pi}{3} \le \theta < \dfrac{\pi}{2} \end{cases}$$

The fundamental component of this waveform is:

$$V_{AC}^{1\,peak} = \frac{4}{\pi} V_C$$

Subtracting the fundamental component from the complete waveform yields the voltage required of a series APF. By inspection, the peak voltage of the APF occurs at $\theta = k\pi$ and its value is:

$$V_{APF}^{peak} = \frac{1}{3} V_C = \frac{\pi}{12} V_{AC}^{1\,peak} = 0.262$$

Thus a series APF compensating a 12-pulse capacitive rectifier requires a rating of $0.262\,pu$ compared to a rating of $0.487\ pu$ for the 6-pulse rectifier.

5.2 Passive Filters

The cancellation reference for the inductive rectifier can be represented in the frequency domain through Fourier series analysis. The standard filters can then be applied to establish how a passive filter may be used to reduce the effort required from the APF. For the purpose of this study, harmonic traps are assumed to cancel perfectly the quoted harmonic component and low-pass filters are assumed to be 2nd order filters with a well-damped characteristic. These of these are both ideal conditions that are unlikely to be met in practice. The design of passive filters for power system applications is discussed in [11].

5.3 Comparison of Passive Filter and Multi-Pulse Methods

The harmonic mitigation techniques in sections 5.1 and 5.2 lead to the current ratings for a shunt APF summarised in Table 3.

	I_{APF}^{mean} *pu*	I_{APF}^{RMS} *pu*
6-pulse	0.272	0.311
6-pulse with harmonic trap for 5th and 7th	0.137	0.191
6-pulse with harmonic trap for 5th, 7th, 11th and 13th	0.095	0.149
6-pulse with harmonic trap for 5th and 7th and 2 kHz LPF	0.073	0.089
12-pulse	0.132	0.152
12-pulse with harmonic trap for 11th and 13th	0.069	0.094
12-pulse with harmonic trap for 5th and 7th and 2 kHz LPF	0.071	0.078

Table 3 Comparison of current rating of a shunt APF compensating an inductive rectifier with a variety of additional harmonic mitigation techniques

It can be seen that the harmonic trap for 5th and 7th harmonic is almost as effective in reducing the current rating as using a 12-pulse rectifier. Adding further harmonic traps for the 11th and 13th harmonic makes the passive filter more effective than the 12-pulse rectifier. This is because the 11th and 13th harmonics contribute more to the average current than the 17th, 19th, 29th and 31st harmonics that are absent from the 12-pulse system. Adding a harmonic trap for the 11th and 13th harmonics to a 12-pulse rectifier cancels the largest of its remaining harmonics and almost halves the rating of the required APF.

The low-pass filter has only a moderate effect because the high-order harmonics it attenuates make only a small contribution to the average (mean or RMS) current.

Adding passive filters to a series APF presents some issues for the stability of the system but in principle the technique can reduce the voltage rating of the APF. Table 4 summarises the results.

	V_{APF}^{peak} *pu*
6-pulse	0.487
6-pulse with harmonic trap for 5th and 7th	0.487
6-pulse with 2 kHz LPF	0.277
12-pulse	0.262

Table 4 Comparison of voltage rating of a series APF compensating an capacitive rectifier with a variety of additional harmonic mitigation techniques

Removing low-order harmonics has little effect on the peak voltage of the cancellation waveform. The peaks are largely contributed to by the high-order harmonics. Adding a low-pass filter cutting off at 2 kHz reduces the peak voltage almost as effectively as moving to a 12-pulse rectifier.

In the case of hybrid active-passive power filters, the active element can assume two roles: cancellation of components not dealt with by the passive element and also the provision of damping for the passive element. There are several reported configurations and control schemes for hybrid filters: a summary of these appears in [5]. An example of a control scheme in which the active element provides damping of the passive element is [].

6 Conclusions

The basis of rating APFs has been established as the combination of the peak voltage and mean current that the inverter within the APF must supply. Analytical methods of assessing the ratings have been established for the series-APF/capacitive-rectifier (a rating of $0.487\,pu$) and shunt-APF/inductive rectifier ($0.272\,pu$). The analysis makes several simplifying assumptions but these have been shown through circuit simulation to err on the side of caution. Although the two cases discussed are duals of each other, the ratings are not equivalent because of the different responses to over-voltage and over-current.

The third (and commonly cited) case is a shunt-APF/capacitive-rectifier. This attempts to compensate a harmonic voltage source with a controlled current source. The current that must be supplied depends critically on the (inductive) impedance between rectifier and APF. An inductance of larger than $0.03\,pu$ must be present for this combination to result in a lower APF rating than the series-APF compensating the same load.

Compensation of a multi-pulse rectifier requires significantly lower APF ratings than for a 6-pulse rectifier. For instance, a 12-pulse rectifier requires an APF with approximately half the rating of that for a 6-pulse rectifier. Where it is possible to aggregate 6-pulse loads into multi-pulse arrangements, a single APF of reduced rating could be used.

As expected, the APF in a hybrid filter arrangement requires a lower rating than for an APF alone. For a shunt APF, harmonic traps for low-order current harmonics are effective. For a series APF, attenuation of voltage peaks with a low pass filter is effective.

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