FOOTBRIDGE RESPONSE: IMPROVED UNDERSTANDING THROUGH PROBABILISTIC INTERPRETATION

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Summary

Design of footbridges is dominated by the structural response under their most common pedestrian loading scenarios. Failures of several structures under these serviceability conditions have highlighted that fundamental assumptions considered when developing load models applied in their design were not realistic. Following these failures, a lot of recent research has focused on the topic of improving the definition of both vertical and lateral loading, included its nonlinear character, as well as on flow movement on platforms. However these improvements have not as yet resulted in improved load models for design purposes. Building upon some of these recent advances, this paper introduces a new methodology for evaluating the response of footbridges under serviceability conditions. This method arises from a probabilistic characterisation of the parameters involved. The main aim of this work is to highlight the advantages of the probabilistic approach as opposed to the deterministic approaches, currently included in design codes and guidelines, when the intention is to obtain the best possible reproduction of reality. With this purpose, the paper presents the framework and underlying assumptions of the probabilistic method, the typical assumptions and methods in deterministic approaches, and the comparison of response of an example structure under diverse loading scenarios obtained from both approaches. Comparison of the results highlights the advantages of the probabilistic method over the other as well as the key parameters to be taken into account in order to obtain the best possible reproduction of real service scenarios and, therefore, responses.

Keywords: Footbridge; Service response; probabilistic approach; vertical load; lateral load

1. Introduction

During late 1970's pedestrian loads were first considered for the fulfilment of serviceability limit states. Thus far, the 21st century has seen many advances in this field published in the academic literature. However, load models prescribed by codes and design guides have not adopted these more sophisticated approaches and instead remain similar to those first introduced (based primarily upon deterministic approaches). In an attempt to rectify this situation, a new proposal is introduced in this paper based on a probabilistic framework. The paper highlights the drawbacks of current deterministic approaches, outlines the probabilistic considerations proposed for a better reproduction of real scenarios (with both individuals and streams of pedestrians), and compares results of the proposed model with those of current methods.

2. Codes: focus on deterministic approaches

Methods currently applied to assess the structural response of footbridges under pedestrian loads have been developed considering mainly deterministic descriptions of the variables involved. This deterministic approach is adopted for the representation of variables used within the pedestrian load models such as: step frequency, pedestrian speed, pedestrian mass, dynamic load amplitude, among others. It is well-known that most of these characteristics possess significant inherent variability for individual pedestrians, let alone for groups.

The deterministic values may be representative if they are calibrated from large datasets. However, the load models into which the parameters are input remain very similar to those proposed decades earlier (based on very limited datasets and analysis tools) and do not incorporate much of the research that has been published over the past decade or so (*e.g.*, multiple experimental measurements of parameters such actions amplitudes, step frequencies and speed or step frequency variability). This fact is outlined throughout the short overview given below regarding the existing load models.

2.1 Description of available methods

One of the first works approaching the definition of dynamic loads to assess structural response in service of footbridges was that developed by Blanchard *et al*. [1]. In 1977, the authors, due to limitations in both computational resources and access to load measurements, adopted single values of load amplitude, speed, step frequency or step length in order to propose a load model (they were one of the first authors adopting a Fourier series approach to define pedestrian loads). This work was later adopted in codes for serviceability assessment of footbridges, such in [2] and [3].

Matsumoto *et al*. in 1978 [4], evaluating the deficiencies of models not including multiple pedestrians crossing simultaneously, proposed a semi-probabilistic approach to take into account such scenarios. Their empirical observations led to them describing the step frequencies of the population according to a normal distribution and arrival times according to a Poisson distribution. From this basis, they proposed the equivalent force corresponding to a continuous stream of pedestrians. This semi-probabilistic approach to assess multiple pedestrian effects is still currently used.

Later, in 1993, similar approaches for multiple pedestrian scenarios were introduced by Grundmann *et al.* [5]. The authors considered a probabilistic description of the variable step frequency (normally distributed) and a form of synchronisation to derive an equivalent load model for groups or continuous streams. Models derived from this work were adopted in guidelines such [6]. Models with similar approaches are still used in codes such [7] or [8] (vertical assessment is similar, while lateral components are derived entirely from experimental observations).

Most recent models follow similar approaches to those of [4] or [5]. Loads from individual pedestrians are still mainly defined deterministically, but for scenarios with multiple pedestrians, where the importance of variability is most evident, several parameters are described using a probabilistic approach. This is the case for guidelines such as Sétra [9] or Synpex [10]. Sétra [9] proposes load models to represent multiple-pedestrian scenarios that stem from considering that the mean step frequencies and arrival time of each pedestrian in the scenario have a probabilistic definition (as opposed to the rest of the parameters of the load model such load amplitudes, which are deterministically described). Synpex [10] represents a step forward compared to the Sétra guideline. Synpex proposes a method that represents the scenarios with crowd streams similar to the Sétra guideline but considering that step frequency distributions depend on the density of the flow. Furthermore, the deterministic definition of pedestrian load amplitudes is obtained from an extensive experimental data.

Previous models can be easily implemented. Nonetheless, they simplify characteristics that may have influence on results and that are not grasped by these descriptions of pedestrian loads (such the inherent variability of individual pedestrian actions, different characteristics of individuals in a crowd or interactions in the movement of pedestrians in a flow).

2.2 Advantages and drawbacks of the current deterministic models: do they reproduce real scenarios?

Pedestrian load models mentioned above and assessments performed using them have, as an advantage, a relatively straightforward application. However, these models describe phenomena and scenarios involving humans, and we are well-aware of the intrinsic variations among members of this species. This variability dictates that the parameters used within the models should at least attempt to represent well-known effects such as:

1) The inability of pedestrians to perform the same action with exactly the same magnitudes (pedestrians do not walk with constant speed, exact consecutive step lengths or timing of step placement). This phenomenon is named intra-variability.
2) Characteristics of different subjects (the population is described by a wide range of values for mass, walking speed, step frequency, step length, load amplitudes, *etc*.) or the same subjects under different situations (such traffic density or aim of the journey) vary significantly. We refer to this variability as inter-variability.
3) The performance of each subject in a flow depends upon the nature and state of the flow (collective behaviour).

Deterministic models presented before have not considered and are therefore unable to represent many of these aspects of pedestrian loading. However, these phenomena may have important influences upon the response of a given footbridge. Accordingly, the following section presents a model based upon a probabilistic framework that provides a means to account for several of the aspects previously mentioned. By comparing the response predictions from the new model with those of existing approaches we have a basis against which to evaluate how important it is to capture these effects in load models for design.

3. Probabilistic approach

In line with proposals described above, during recent years, alternative models embracing probabilistic approaches trying to overcome the limitations mentioned previously have been published (models based on factors described as random variables). This is the case of articles such as [11], where pedestrian intra-variability is introduced through randomly generated variables (variability is included by defining loads with different harmonics with phases randomly generated). Inter-variability has been partially considered in some of the models mentioned previously (almost exclusively focused on step frequency as random variable). Proposals included in [4], [5], [9], [10] are examples of such considerations. Alternatively, only a few works have included other parameters (*e.g.*, mass, speed, *etc.*), such as [12]. Nonetheless, these parameters are treated as independent although intuition suggests that they are not (*e.g.*, pedestrians with certain anthropomorphic characteristics will tend to choose certain speeds and step frequencies).

Regarding collective behaviour, most works including multiple-pedestrian scenarios do so through Monte Carlo simulations (exclusively focusing the simulation on the arrival time of each subject). However, these simulations can give place to impossible scenarios (such many pedestrians entering the structure at the same time, or pedestrians overtaking others even if it were not physically possible, *etc.*) and disregard interactions that might have an important impact on the structural response (such changes in direction or velocity and step frequency). Conversely, only a few works including accurate modelling of collective pedestrian behaviour have been found [13], despite the fact that it is believed this will have a significant impact upon structural response.

3.1 Description of new load model

In order to include intra-variability, inter-variability and collective behaviour in pedestrian load models, a methodology attempting the best description possible of each of the individuals that comprise the population is proposed hereunder. The method attempts to describe accurately the characteristics of a pedestrian population and the individual loads transmitted by each pedestrian.

The second point, regarding the loads imparted by individuals, is addressed through a load model that defines temporal sequences of load for each pedestrian foot individually, see Figure 1 (as opposed to loads defined through Fourier series). These loads rely upon parameters such as: step frequency and pedestrian mass for vertical loads; and step frequency, step width, pedestrian height and mass for lateral loads. A good load model is obtained through an accurate description of these parameters, including their random nature. This new model permits the incorporation of:

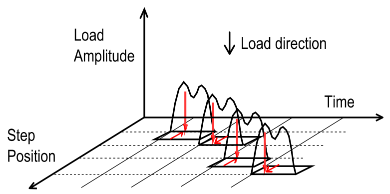

Fig. 1 Vertical and lateral amplitude loads of single pedestrian according to proposed model

1) Intra-variability, as the model defines individual loads not only based on average values of step frequency, speed, *etc.*, but also on specific values of these parameters in time.
2) Inter-variability, creating pedestrian loads including their random nature (and considering correlation between characteristics that may be related such pedestrian height with mass, preferred speed and step frequency).
3) Crowd behaviour, modelling as realistically as possible the response that each pedestrian would have within a crowd stream of different characteristics (such as flow density, *etc.*).

The definition of the vertical load transmitted by an individual foot is based upon data defined in [10], where more than a thousand recordings of vertical loads constitute the base for their accurate description in time (according to step frequency and mass of the pedestrian).

Lateral loads present several challenges compared to vertical loads when attempting to define their amplitude. On one hand, despite the fact that several researchers (e.g. [10], [14]) have tried to define their amplitude, these loads present a large variability. On the other hand, as suggested in multiple research works and seen during the famous response of the London Millennium Bridge [15], lateral loads of pedestrians seem to be more influenced by the perceived movement of the structure, becoming non-linear with structural response. Pointing in this direction, and overcoming the lack of exhaustive amplitude definitions for these lateral loads, the lateral loads considered in this paper are derived from models that define the movement of the Centre of Mass (*CoM*) of each pedestrian [16]. According to this proposal, lateral loads are defined using parameters such as: step frequency, pedestrian mass and height and step width (the resulting amplitude in time resembles a rectangle as depicted in Figure 1).

3.2 Load scenarios with single pedestrians. Adoption of probabilistic approach: application and effects on response

As explained previously, the parameters used to define vertical and lateral loads in the proposed model are: pedestrian mass and step frequency for vertical loads; and pedestrian mass and height, and step frequency and width for lateral loads. Of these parameters, those that can adopt variable values in time within the same pedestrian actions are step frequency and step width. Accordingly, intra-variability in vertical loads is expressed through the variation of step frequency in consecutive steps, whereas for lateral loads intra-variability can be reproduced through variation of step timing and step width of consecutive steps.

3.2.1 Vertical loads

Regarding step frequency, several research works [17] point out that the variability of steps while walking is not entirely random but has a certain correlation. This correlation seems realistic when thinking that a pedestrian will try to maintain a constant speed of movement (long-term correlation), compensating errors of previous steps by adjusting the step frequency correspondingly at consecutive steps (short-term correlation), *i.e.*, if previous steps where shorter in time and total speed was slower, a pedestrian speeds up and tries to compensate for the departure from the 'target' velocity in a gentle but not-random manner.

In order to model the step frequency variation, this short and long-term variability is assumed to be described using the Metropolis-Hastings algorithm (a Markov Chain Monte Carlo method that reproduces values based on previous results and allocating them to include a long-term relationship). The long term correlation of steps is defined by a normal distribution, with the standard deviation the main parameter of the Metropolis-Hastings algorithm and the mean the step frequency for a given pedestrian. According to authors such [10] or [18], the standard deviation of the step frequency adopts values up to 0.15Hz.

Based on these previous considerations and on the load model mentioned above, in the following we present a comparison of the results obtained according to loading scenarios produced by a single pedestrian walking on three simply supported structures with fundamental vertical frequencies of 1.8Hz, 1.89Hz and 3.24Hz. In each loading scenario a single pedestrian walks with loads defined by the characteristics defined in Table 1 (where D and P define a deterministic or probabilistic load model, S a model including a single pedestrian and load amplitudes of C are defined according to code [8] and NM according to the proposed new model).

Table 1 Definition of load model scenarios

Name of scenario	Characteristics		
	Amplitude	Mean step frequency	Standard deviation step frequency
DSC	Code [8]	1.8 Hz	-
DSNM	New Model	1.8 Hz	-
PSNM1	New Model	1.8 Hz	0.025 Hz
PSNM2	New Model	1.8 Hz	0.050 Hz
PSNM3	New Model	1.8 Hz	0.075 Hz
PSNM4	New Model	1.8 Hz	0.100Hz
PSNM5	New Model	1.8 Hz	0.125Hz
PSNM6	New Model	1.8 Hz	0.150 Hz

In all cases, simulated pedestrians step on the structure the same number of times, with a step length of approximately 0.7m and have a mass proportional to that of the structure (Pedestrian mass / Bridge mass = constant).

Results produced for the scenarios listed in Table 1 for a structure with fundamental vertical frequency f_{sv} of 1.8Hz are illustrated in Figure 2a, those for the structure with f_{sv} = 1.89 Hz in Figure 2b and those for the structure with f_{sv} = 3.24 Hz in Figure 2c. For scenarios PSNM1 to PSNM6 these figures represent the results of 20 simulations, which are depicted with box plots that represent data statistically. In these box plots, the whiskers or vertical lines represent data between the lower extreme and the lower 25th percentile or 75th percentile and higher extreme and results in the box data between the lower 25th and the higher 75th percentile (where the horizontal line is the median).

Results produced on the structure with fundamental vertical frequency of 1.80 Hz are represented in Figure 2a. This figure shows how accelerations with loads defined in [8] and those of the new model are considerably different (code results are at least 1.6 times larger than those of the new model). However the most important outcome of the figure is the clear impact of variability on response (scenarios PSNM1 to PSNM6). Comparing results of the scenario DSNM with those scenarios with small variability (PSNM1-PSNM3), maximum results of the cases with variability are similar to those produced with constant step frequency (DSNM); however results equal to the median or below have values 0.8 times or smaller than those for DSNM. For larger variability (PSNM4-PSNM6), intra-variability has an even larger impact on response, with results between the median and the lower quartile being between 0.7 and 0.3 times those of DSNM and results between the lower quartile and the lower extreme between 0.5 and 0.2 times those of DSNM.

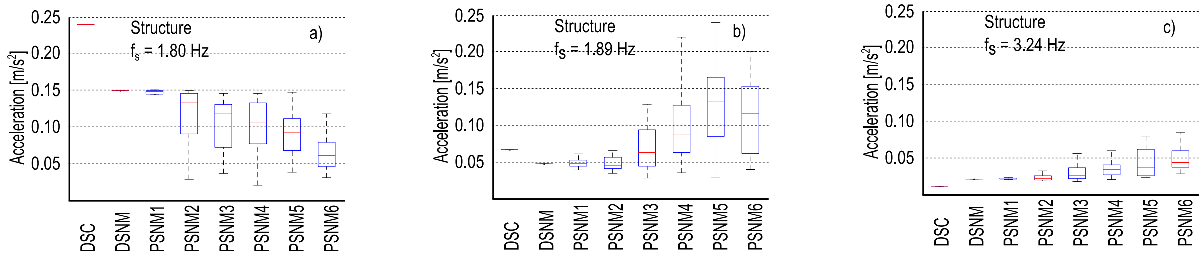

Fig. 2 Accelerations caused by pedestrian with constant or variable step (see Table 1) on structure with f_s = 1.8Hz (Figure 2a), f_s = 1.89Hz (Figure 2b) or f_s = 3.24Hz (Figure 2c)

Results produced on a structure with f_s = 1.89 Hz are depicted in Figure 2b. In this figure, results obtained under the load scenario DSC are slightly larger than those with small variability but considerably smaller than those with large variability. For scenarios PSNM4-PSNM6, Figure 2b shows how results equal or above the median are at least 2.1 times those of constant step DSNM, and results between the higher quartile and the higher extreme are at least 3.1 times those for DSNM. Comparing the results of Figures 2a and 2b we can see how in some cases variable step with a non-resonant mean step frequency can produce larger responses than those of a pedestrian with resonant mean step frequency (compare DSNM of Figure 2a with PSNM5 of Figure 2b). This phenomenon is explained by results depicted in Figure 3. This figure shows accelerations produced by a single pedestrian walking at a constant step frequency equal to that of the structure (in all cases pedestrians step the same number of times on the structure and the ratio pedestrian mass-to-bridge mass is constant). Figure 3 shows how the results of loads with frequency up to 2.2Hz are larger than results with smaller step frequency, which is due to the fact that the load shape and amplitude changes with the step frequency. For Figure 2b, although the pedestrian aims to walk at a mean step frequency of 1.8Hz, with relatively large variability he may transmit loads at frequencies coinciding with the vertical structural frequency (in resonance), and as seen in Figure 3 resonant loads with larger step frequency produce larger response.

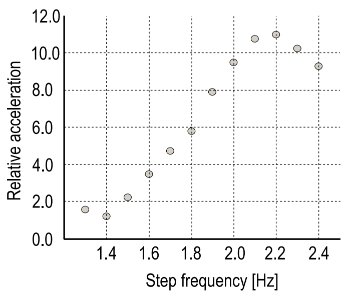

Fig. 3 Relative accelerations (relative to acceleration for f_p = 1.3Hz) caused by pedestrian with constant step on structure with f_s = f_p, f_p [1.3-2.4Hz]

Results corresponding to accelerations of a structure with f_s = 3.24 Hz are depicted in Figure 2c. This figure depicts the impact of non-constant step frequency when pedestrian mean step frequency is far from the frequency of the structure. Whereas the constant step frequency scenarios (DSC and DSNM) predict a modest acceleration, loads of pedestrians with non-constant step frequency produce results that may be non-negligible. For the largest variability (PSNM6), results equal or above the median are at least 2.9 times those produced with constant step (DSNM) and results between the higher quartile and the higher extreme are at least 4.2 times the results for DSNM.

3.2.2 Lateral loads

Similar to vertical loads, the same algorithm (Metropolis-Hastings) and ranges of variability (σ_d = 0-0.15Hz) are considered to appraise the effect of step frequency variability on lateral structural response. Based on these aspects and on the load model mentioned above, the impact of step variability is assessed under loading scenarios defined by a single pedestrian walking on three simply supported structures with fundamental lateral frequencies of 1.15Hz, 3.45Hz and 3.15Hz and pedestrian loads defined by the characteristics of Table 1 (except scenario DSC).

Results presented in Figures 4a, 4b and 4c show how lateral response is quite sensitive to step frequency variability. In those cases where the target step frequency of the pedestrian is resonant with the structural frequency, even a small variability (PSNM2 of Figures 4a and 4b) show how median accelerations are 0.5 times or less of those caused with constant step frequency (DSNM). Figure 4c, showing results for single pedestrians walking on a platform where structural lateral frequencies are very dissimilar to those of the pedestrian, depicts how accelerations become larger for non-constant step (in almost all degrees of variability the median is around 1.5 times larger than the result with constant step, DSNM). Another important observation stems from comparison of results with variable step frequency (when the mean step frequency is resonant or non-resonant with the structural frequency). For constant step frequency (DSNM) results of Figures 4a and 4b are much larger than those of Figure 4c. However, once there is variability (PSNM2 to PSNM6), results of the three Figures are similar in magnitudes, which implies that the impact on response of pedestrians with non-resonant mean step frequency is important as well as those with mean resonant step frequency.

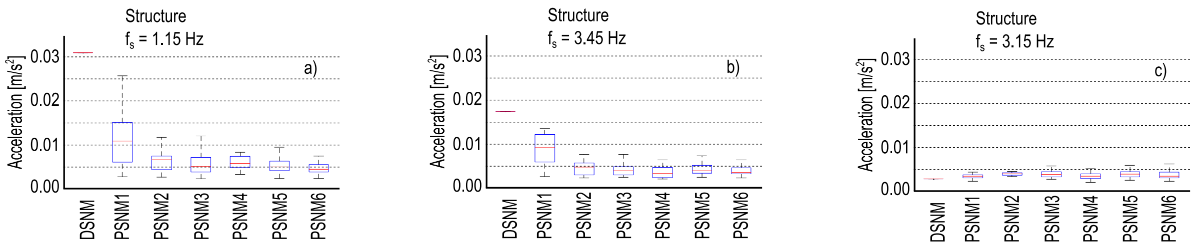

Fig. 4 Lateral accelerations caused by pedestrian with constant or variable step frequency (mean = 2.3Hz) on structure with lateral f_s = 1.15Hz (Figure 4a), f_s = 3.45Hz (Figure 4b) or f_s = 3.15Hz (Figure 4c)

Apart from the step frequency, it has been mentioned that lateral actions of a pedestrian can vary according to the width of consecutive steps. It is considered that the effects of its variability should be studied in order to include or disregard it in an accurate load model; nonetheless the results presented in this paper do not include this effect.

3.3 Multiple pedestrian scenarios: probabilistic implementation and effects on response

This section includes a description of the procedures that should be adopted in a load model that strives to describe in detail the pedestrian population (including its random nature). For some of these factors, the paper describes a possible method to include them in the load models and it describes an appraisal of their effects on response.

3.3.1 Inter-variability

There are several characteristics that are necessary to obtain an optimal description of crowds through probabilistic approach. However, only the random nature of step frequency has been considered in available models. This factor has a significant impact on response; therefore service appraisal credibility relies upon this factor enormously. Accordingly, in order to evaluate correctly this factor, it is of utmost importance to describe accurately the parameters influencing this frequency.

Several researchers provide relatively different probabilistic representations of this factor (see [12]), that were obtained from experimental measurements. These descriptions have been largely used in guidelines and codes (mainly their proposed mean step frequency). However, a more meticulous description can be obtained from considerations introduced in [19] and [20]. According to [19] there is a clear correlation between speed and frequency, and according to [20] experimental data shows how speed chosen by pedestrians is determined by anthropometric (such height),age, sociological, density flow and travel purpose characteristics, among others. Therefore, a good description of the distribution of step frequencies used by crowds crossing a structure will be founded on an accurate definition of such relationships.

A detailed correlation between pedestrian characteristics and chosen speed, and speed and frequency can be found in [21], where there is a probabilistic description of these relationships based on data from more than 1500 and 900 subjects respectively. According to these relations, and considering data from UK population (in terms of age and height), the step frequency distributions depicted in Figure 5 can be defined, according to the density of the crowd and general aim of the journey. Results proposed by other authors [12] reasonably coincide with those in Figure 5 for crowds commuting (pedestrians with specific purpose while travelling) or during leisure time (foot traffic in shopping areas, *etc.*) and relatively low densities. Rush hour (pedestrians heading to work) proposed values are slightly larger.

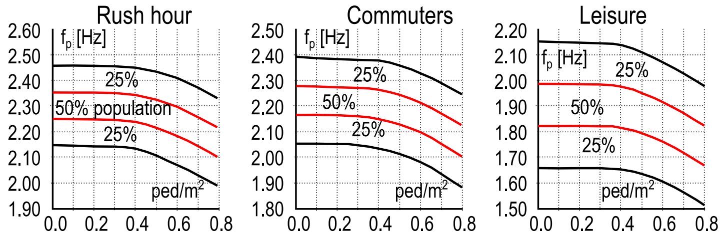

Fig. 5: Distribution of mean step frequencies (based on UK population characteristics) according to aim of journey and density of the flow

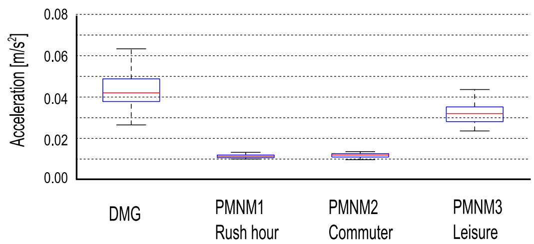

Fig. 6 Maximum vertical response under different multiple pedestrian scenarios on structure with f_s = 1.8Hz (P or D describes probabilistic or deterministic approaches, M the consideration of multiple pedestrians and G or NM loads according to Sétra [9] or the new model).

The advantage of this description is evident, compared to those proposed so far (with normal distributions fixed regardless of the crowd likely to use the structure). Structures can be evaluated according to the possible flow that will cross the structure.

Figure 6 compares results of maximum vertical accelerations obtained in a structure with first vertical frequency of 1.8Hz under the action of a pedestrian flow of 0.2 ped/m^2 density with loads and movement variables (speed, frequency and step length) defined according to [9] (DMG in Figure 6) or according to the proposed new model (loads with amplitudes such as those in Figure 1, individual step frequencies with intra-variability and mean step frequencies of each pedestrian following the distributions in Figure 5 for each traffic class, corresponding to PMNM1 to PMNM3).

Results of Figure 6 show how the predictions of [9] are considerably larger than those of the other cases. Differences among PMNM1, PMNM2 and PMNM3 are also considerable. Both PMNM1 and PMNM2 have maximum response values that are 0.5 times the minimum response produced by [9] (explained by the distributions from Figure 5). However, for PMNM3 (leisure purpose traffic), responses are more similar, although results equal of above the median for DMG are larger than all the results of PMNM3.

From Figure 6, it is evident that step frequencies of pedestrians need to be accurately described. As well, it suggests that it is worthwhile evaluating structural response according to the class of traffic expected (instead of using generalised frequency distributions), since a structural configuration that is valid in particular locations might not be adequate for other locations.

Apart from the probabilistic description of speed and mean step frequency, other parameters whose inclusion in a probabilistic model should be considered are the different masses of pedestrians (mass not randomly assigned but according to anthropometric characteristics) or the variability of the load amplitudes (in this study load shapes, Figure 1, have been defined with mean data). These represent future lines of work to improve the probabilistic model proposed.

3.3.2 Crowd behaviour

In addition to the characteristics described above probabilistically, collective crowd modelling has been mentioned as another factor that would have influence. Frequency (and velocity of movement) of each pedestrian represents one of the most influential factors in response, and in cases with relatively high density of pedestrians Monte Carlo simulations may not be adequate since interactions among pedestrians are more likely (changing direction, overtaking, *etc*.). Currently, there are multiple methods that model pedestrians as particles, such as that applied in [13]. Despite the fact that these are computationally demanding it is believed that these should be considered to provide a more accurate service response (a comparison of results including these approaches is beyond the scope of the present paper).

4. Conclusions

The paper proposes a new load model based on recent advances in pedestrian load descriptions and on the incorporation of the random nature of parameters involved in the definition. Several of them are accurately described in the new model and their impact is evaluated by comparison with results from more traditional deterministic methods.

The model is able to reproduce the variability of the actions transmitted by each pedestrian intrinsically. For both vertical and lateral loads, the effects on response of non-constant step frequency are very significant. For vertical loads, it has been seen how large responses could be obtained even with non-resonant mean step frequency (step frequency relatively similar to the structure frequency) and that pedestrian effects with mean step frequencies well away from the structural frequency produce effects that should not be dismissed. For lateral loads, even small variability in step frequencies has a large impact on the response, showing that pedestrians with mean step frequency equal or very different from the lateral structural frequency can generate very similar effects. Due to these similar results (between pedestrians with variable step and mean step frequency resonant or not resonant with the structural frequency), all should be considered in structural analyses.

Regarding scenarios with multiple pedestrians, the model includes a proposal for the distributions of mean step frequencies to be used by pedestrians according the travel purpose and crowd density (for further detail, refer to [21]). This proposal is based on a large dataset, since the reliability of the model depends, to a large degree, on the correct and accurate representation of the parameters included in its definition.

Comparison of results obtained with loads defined with proposed speeds, frequencies and inter-variability of pedestrians and those obtained with models currently used show large and inconsistent differences (both larger and smaller). Therefore, in order to accurately predict response in different situations it is proposed to evaluate the serviceability limit state of vibration according to the type of traffic likely to use the structure.

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6. References

[1] BLANCHARD, J., DAVIES, B. L., and SMITH, J. W., “Design criteria and analysis for dynamic loading of footbridges”, *Symposium on Dynamic Behaviour of Bridges*. Crowthorne, Berkshire, Transport and Road Research Laboratory, 1977.

[2] BRITISH STANDARDS INSTITUTE, “BS 5400-Part 2: 1978, Steel, concrete and composite bridges. Part 2. Appendix C: Vibration serviceability requirements for foot and cycle track bridges”, 1978.

[3] BRITISH STANDARDS INSTITUTION, “BS 5400-Part 2: 2006, Steel, concrete and composite bridges. Part 2: Specification for loads”, 2006.

[4] MATSUMOTO, Y., NISHIOKA, T., SHIOJIRI, H., and MATSUZAKI, K., “Dynamic Design of Footbridges”, *IABSE Proceedings*, 1978, pp. 1-15.

[5] GRUNDMANN, H., KREUZINGER, H. and SCHNEIDER, M., “Schwingungsuntersuchungen für Fußgängerbrücken”, *Bauingenieur*, Vol. 68, 1993, pp. 215-225.

[6] FEDERATION INTERNATIONALE DU BETON, “Guidelines for the design of footbridges, Fib Bulletin 32”, 2005.

[7] BRITISH STANDARDS INSTITUTE, “BS EN 1995-2: 2004, Eurocode 5: Design of timber structures. Part 2: Bridges”, 2004.

[8] BRITISH STANDARDS INSTITUTE, “NA to BS EN 1991-2: 2003: UK National Annex to Eurocode 1: Actions on structures. Part 2: Traffic loads on bridges”, 2008.

[9] SETRA, “Assessment of vibrational behaviour of footbridges under pedestrian loading”, Paris, October 2006.

[10] BUTZ, C., FELDMANN, M., HEINEMEYER, C., ET AL. “Advanced load models for synchronous pedestrian excitation and optimised design guidelines for steel footbridges (SYNPEX)”, European Commission, Research Fund for Coal and Steel, 2008.

[11] ZIVANOVIC, S., PAVIC, A., and REYNOLDS, P., “Probability-based prediction of multi-mode vibration response to walking excitation”, *Engineering Structures*, Vol. 29, 2007, pp. 942-954.

[12] PEDERSEN, L., and FRIER, C., “Sensitivity of footbridge vibrations to stochastic walking parameters”, *Journal of Sound and Vibration,* Vol.329, 2010, pp. 2683-2701.

[13] CARROLL, S.P., OWEN, J.S., and HUSSEIN, M.F., “A coupled biomechanical/discrete element crowd model of crowd-bridge dynamic interaction and application to the Clifton Suspension Bridge”, *Engineering Structures*, Vol.49, 2013, pp.58-75.

[14] INGOLFSSON, E.T., GEORGAKIS, C.T., and JONSSON, J., “Pedestrian-induced lateral vibrations of footbridges: A literature review”, *Engineering Structures*, Vol. 45, 2012, pp.21-52.

[15] DALLARD, P., FITZPATRICK, A.J., FLINT, A., LE BOURVA, S., LOW, A., RIDSDILL SMITH, R.M., and WILLFORD, M., “The London Millennium footbridge”, *The Structural Engineer*, 2001, pp.17-33.

[16] MACDONALD, J.H.G., “Lateral excitation of bridges by balancing pedestrians”, *Proceedings of the Royal Society a-Mathematical Physical and Engineering Sciences*, Vol. 465, 2009, pp.1055-1073.

[17] HAUSDORFF, J.M., PENG, C.K., LADIN, Z., WEI, J.Y., and GOLDBERGER, A.L., “Is walking a random walk? Evidence for long-range correlations in stride interval of human gait”, *Journal of Applied Physiology*, Vol. 78, 1995, pp.349-358.

[18] MARUYAMA, H., and NAGASAKI, H., “Temporal variability in the phase durations during treadmill walking”, *Human Movement Science*, Vol. 11, 1992, pp.335-348.

[19] BERTRAM, J. E. A., and RUINA, A., “Multiple walking speed–frequency relations are predicted by constrained optimization”, *Journal of Theoretical Biology*, Vol. 209, 2001, pp.445-453.

[20] WEIDMANN, U., *Transporttechnik der Fußgänger - Transporttechnische Eigenschaften des Fußgängerverkehrs (Literaturstudie)*, Institut für Verkehrsplanung, Transporttechnik, Strassen- und Eisenbahnbau IVT an der ETH Zurich, 1993, 109 pp.

[21] RAMOS, C., STAFFORD, P.J., and RUIZ-TERAN, A.M., “Pedestrian actions: relevance of stochastic representation of reality”, Manuscript pending publication.