OPTIMAL DESIGN OF PIPES IN SERIES: AN EXPLICIT APPROXIMATION.

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Abstract

This paper introduces a new methodology for the optimum design of pipes in series, named Optimum Hydraulic Grade Line (OHGL). This methodology is explicit and is based on the knowledge of the series topology and the geometrical distribution of water demands on nodes, i.e. the way in which the pipe in series delivers water mass as function of the distance from the entrance. OHGL consists in the pre-determination of that hydraulic grade line which gives the minimum construction cost, in an explicit way. Once this line has been established, calculation of the pipe's continuous diameters is direct; after a round up to commercial diameters is developed. To validate the proposed methodology, several pipes in series were designed both using GA and OHGL. Four hundred series were used in total, each with different topological characteristics and demands.

Keywords:
Pipe in series, optimum design, genetic algorithms, optimum hydraulic grade line.

1. INTRODUCTION

This paper is based on the optimum design of pipes in series to obtain the minimum construction cost. A pipe in series is a system of pipes connected to each other in such a way that one or several of their characteristics vary: diameter, absolute roughness, and discharge. Water demand may or may not exist in each of the system nodes. Pipes in series are very common in irrigation systems but they can also be found in potable water distribution systems. By analyzing hydraulic principles and patterns usually found in the least-cost designs, in this paper a new methodology is proposed to design optimal systems in an explicit, direct way, by assuming beforehand the way in which available energy is spent in the pipes in series.

The I-Pai Wu (1975) Criterion is probably the most important method for drip irrigation main lines design, a typical pipe in series problem. It consists in assigning the minimum pressure value at the end of the line and then calculating the hydraulic gradient line (HGL) in the upstream pipes. Wu used irrigation lines composed by pipes in series in which the single pipes' length was equal or very similar. The water demands in the nodes were uniform. Wu concluded that the least-cost design was produced when using a parabolic HGL with a sag of 15% of the total head-loss in the middle of the series. Several authors have proposed different techniques such as the dynamic programming of algorithms, the analysis of lateral pipes, analytic approximations, hydraulic analysis based on spatial variability, or local loss evaluation.

This paper deals with the Wu methodology and tries to adjust it so that the single pipes' lengths and node demands have no restrictions. An alternative approach to the optimal design of pipes in series is introduced. The research proved that the HGL was in effect a parabola. The sag of the parabola depended on the nodal distribution (series topology) and water demand in each of the pipes. The sag is easily obtained when using the geometrical distribution of the water demand. Once this value is known, the optimum design is explicit and it can be built in a direct way, despite the pipeline configuration. The proposed methodology was verified with a high number of randomly generated pipes in series with varying entrance head, pipe lengths and node demands. The series were first designed using genetic algorithms (GA) with a large number of generations and iterations. Then they were designed by the new approach. The results favored the new method because least-costs designs are obtained with no need of iterations. This is useful, but the proposed methodology does not rely on the optimal design of pipes in series, since metaheuristic algorithms can solve it without much effort. What the proposed methodology does however, is present least cost systems in an explicit way. Not only is this methodology one of a minimum cost design, it also allows the understanding of how the different variables of a given series (demands magnitude and location, pipe lengths, pipeline costs, etc.) affect this design.

2. PROBLEM FORMULATION

Once we know this research deals with series of constructive pipes *without loops*, the optimal design of pipes in series problem is defined as: *Given a system layout (length and slope of each pipe is included) and nodes water demands, find the diameter combination that has the minimum construction cost. This combination must obey the restrictions posed by mass conservation in nodes, energy conservation in pipes, and minimum pressure in nodes. The availability of the diameters in markets must also be taken into account.* Mathematically, the problem objective can be expressed as:

$$Minimize(C) \tag{1}$$

where C is pipe in series construction cost; it is calculated as a diameter potential function:

$$C = \sum_{i=1}^{NT} a \cdot L_i \cdot D_i^b \tag{2}$$

where NT is the number of pipes in the series, L_i is the length of pipe i, D_i is the diameter of pipe i, and a and b are regression parameters taking into account the pipe costs. Problem restrictions are:

– Mass conservation (see Figure 1):

$$Q_T = Q_\alpha + \sum_{i=1}^{\alpha-1} Q_{L_i} \tag{3}$$

where Q_T is total discharge (in series first pipe), Q_α is the discharge in pipe Q_{Li} is the lateral discharge (demand) at the end of pipe i.

– Energy conservation (see Figure 1):

$$\Delta H = \sum_{i=1}^{NT} h_{fi} + \sum_{i=1}^{m} h_{mi} \tag{4}$$

where m is number of fittings causing minor losses, hf_i is friction loss in pipe i, hm_i is minor loss in fitting i. Friction losses are calculated with Darcy-Weisbach equation in conjunction with Colebrook-White equation.

– Minimum pressure in demand nodes:

$$H_j \geq H_j^{min} \tag{5}$$

where H_j is piezometric head in node j and H_{jmin} is minimum pressure required in node j.

– Pipe diameters can only assume discrete values belonging to commercial diameters setΦD:

$$D_i \in \Phi_D, \quad \forall_i \tag{6}$$

3. OPTIMUM HYDRAULIC GRADE LINE ANALYSIS FOR A PIPE IN SERIES

As it was mentioned in Section 1, the first step in this research is to analyze the shapes of the HGLs corresponding to minimum costs designs of several pipes in series. The first researcher to suggest that the HGL of minimum cost of pipes in series has a particular shape was I-Pai Wu (1975). Wu established that minimum cost series (considering construction and materials costs only) usually has an HGL that is concave up and closet to the straight line between the hydraulic grade level at entrance (point A, Figure 1) and the hydraulic grade at the series end (point B). Wu also established that the OGHL, in the mid section of the series, has a sag of 15% of ΔH regarding the straight line previously described, where ΔH is the available total head.

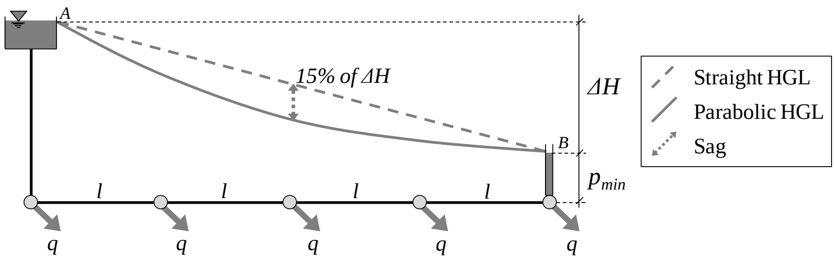
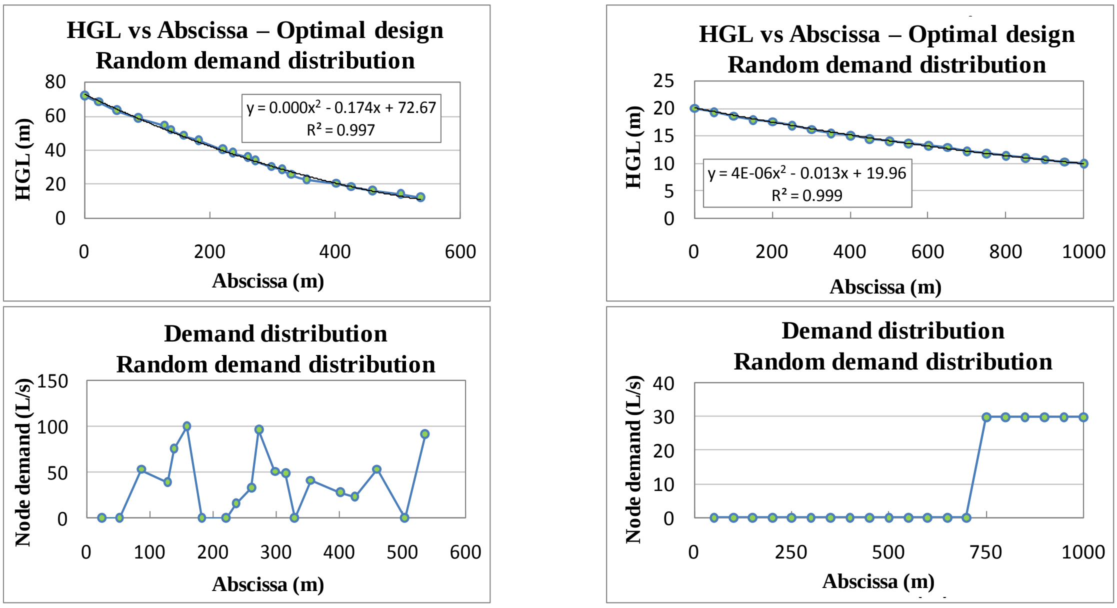


Figure 1. I-Pai Wu criterion.

Wu's criterion is a methodology for the design of irrigation systems and is applicable only to pipes in series with uniform node demands, i.e. with the same magnitude and spacing. The aim of this research was to develop a criterion applicable to systems of any hydraulic and topological characteristics. A study was done to find the optimum HGL shape to pipes in series with non-uniform demands. Here one hundred twenty series with different demand magnitudes and special distribution, a variable topography, different entrance piezometric head, and different pipe lengths were generated. For each one of the series, a minimum cost design was calculated (using AGs), and it was found that the HGLs always were quadratic curves (obtained R^2 always were higher than 98%). As examples, Figure 2 shows minimum cost HGLs for two non uniform demands series. The figure shows that HGL corresponding to minimum cost designs is, actually, a parabola o quadratic curve; however, the shape of this curve is different for each of the series.



(A) (B)

Figure 2. (A) Optimum HGL and demand distribution for a series with demand concentration at the end. (B) Optimum HGL and demand distribution for a series with random demands.

As shown by the analysis, optimum HGL is a function of three factors: *demand distribution, relation of total demand discharge and series total length, and costs function*. Additionally, it was observed that system's total available head (ΔH) does not affect the HGL shape significantly. Since the optimum HGL has a parabolic shape, three points must be known in order to determine and equation. For any pipes in series, the HGL's initial and final points are known:

– At series initial point, zero abscissa, HGL is the entrance piezometeric head (tank, reservoir, pump), which means that initial point is known: $P_{initial}(0, LGH_{entrance})$.

– In the final node, abscissa equal to series total length, HGL is minimum and is equal to last node elevation plus the minimum required pressure head ($LGH_{mín} = Z + P_{min}$); therefore, final point in the curve is also known: $P_{final}(L_{total}, LGH_{mín})$.

So that the equation is determined, a third point is needed. An easily identifiable point with a known abscissa is a maximum in the HGL curve sag, which happens to be in half the total length. Using the 120 series analysis, the way in which the three factors mentioned earlier (i.e. demand distribution, relation between total discharge and total length, and costs function exponent) affect HGL maximum sag was established. The results obtained by the analysis are shown later on. It is important to note that maximum sag is given as a percentage of system's total available head (ΔH).

3.1. Effect of demand distribution on the maximum sag of optimum HGL

To analyze the effect of the demand distribution in the magnitude of the HGL sag, 50 sets of pipes were generated with the same total length (1000 m), the same HGL in the supply source (50 m) and equal total demand (1 m^3/s) but with different demand patterns; in this way is possible analyze the effect of the demand in the optimum sag magnitude. To measure the distribution of demands for pipes in series, two indicators were developed: Demands Centroid ($\overline{X}$) and Uniformity Coefficient (UC). The first is a measure of the general location of the demands along the series of pipes and the second is a measure of the dispersion of the demands about the Demands Centroid. To calculate them, the following expressions were established:

- *Demands Centroid ($\bar{x}$):*

$$\bar{x} = \frac{\left(\frac{\sum_{i=0}^{NN} q_i \cdot d_i}{Q_{total}} \right)}{L_{total}} \tag{7}$$

where NN is the number of demand nodes, q_i is the demand flow at node i, d_i is the distance from node i to the source of supply, Q_{total} is the total flow demanded by the system and L_{total} is the total length of the series. In general, if the *Demands Centroid* is large, the magnitude of the maximum sag is small. The demands dispersion, that affects this value, is explained by the UC.

- *Uniformity Coefficient (UC):* To calculate UC it is necessary calculate independently, the demands centroid of each of two sections in which the general demands centroid ($\overline{x}$) divides the series of pipes; it should be noted that these two centroids are calculated with respect to the general centroid ($\overline{x}$). Once each centroid is calculated, a weighted average of both, based on the length of each of the two sections, is calculated:

$$UC = \overline{X_1}.\left(\frac{L_{Section\ 1}}{L_{Total}}\right) + \overline{X_2}.\left(\frac{L_{Section\ 2}}{L_{Total}}\right) \tag{8}$$

where $\bar{x}_1$ is the demands centroid of section 1, $\bar{x}_2$ is the demands centroid of the section 2, $L_{\ section\ 2}$ is the length of the first section and $L_{\ section\ 2}$ of the section 2. The centroid of each section is calculated as follows:

$$\overline{X_t} = \frac{\frac{\sum_{i=1}^{NNt} d_{nodei\ -centroid}}{\sum_{i=1}^{NNt} q_i}}{Ltotal} \tag{9}$$

where NN_t is the number of nodes in the section t, $d_{nodei\text{-}centroid}$ is the distance from node i to general demands centroid ($\bar{x}$). Using a statistical adjustment made in the program DataFit ®, was obtained the following expression to estimate the Optimum sag based on $\bar{x}$ and UC.

$$Sag = a + b \cdot \bar{X} + c \cdot UC + d \cdot (UC^2) \tag{10}$$

The values of the coefficients in Equation 10 are presented below:

Table 1. Values of the coefficients of Equation 10 obtained by regression.

Regression Variable Results				
Variable	Value	Standard Error	t-ratio	Prob(t)
a	0.435521465	3.64E-02	11.96423962	0
b	-0.176612805	2.08E-02	-8.49833046	0
c	-0.977366227	0.252627675	-3.868801108	0.00034
d	0.906254447	0.459064981	1.974131079	0.05439

3.2 Effect of the relation between total discharge and total length over the optimum HGL maximum sag.

To analyze the effect of the relation between the total demand flow and total length of the pipes in series on the HGL maximum sag, 5 different demand patterns were analyzed; for each one of them, 24 series of pipes with the same entrance HGL (50 m), the same costs function exponent (1.46) and same demands distribution, but with different demands magnitude and pipe lengths (in each of the series generated, the demands and pipe lengths were multiplied by a different factor, and accordingly the demands pattern, the *Demands Centroid* and the *Uniformity Coefficient* remain the same, despite the difference between the magnitude of demands and the total length). Statistical analysis determined that HGL sag as function of discharge (Q) and total length (L) is:

$$f(Q,L) = \frac{Q^2}{L^3} \tag{11}$$

For each of the 5 analyzed demand patterns, a total of 24 series were obtained. For each one of them, the value of this function was calculated (Equation 11). The results are plotted against the optimum sag value in each case (see Figure 3).

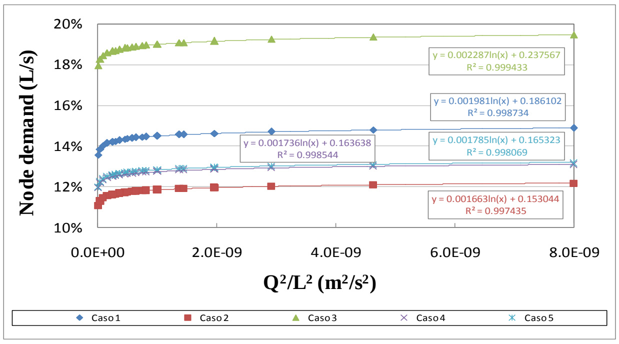
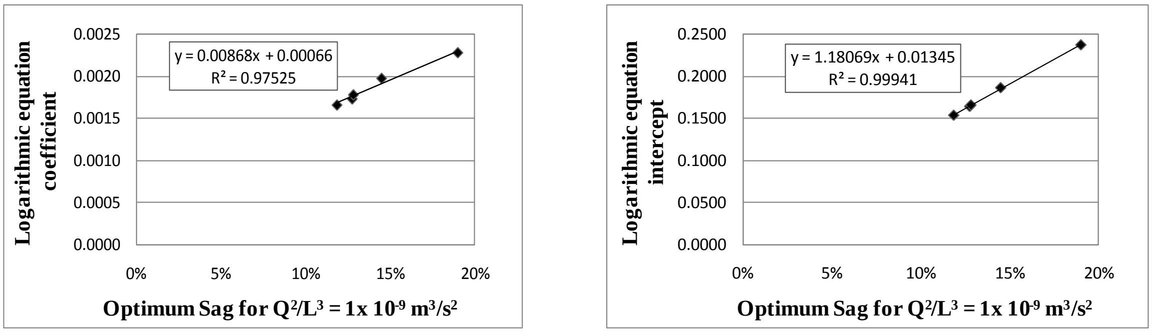


Figure 3. Combined effect of total length and total demand in HGL optimum sag vs. abscissa

In the figure, it is clear that the function f (Q, L) has a logarithmic relation with respect to the HGL optimum sag of the tested series of pipes. The coefficients of multiple determination obtained in all cases are above 99%, which means that the defined function f (Q, L) explains 99% of the optimum sag variation (if the demands distribution and HGL reservoir remain constant). In addition, the logarithmic curves obtained are approximately parallel, suggesting a relation between the optimum sag for a fixed value of the function f (Q, L) and the coefficients and intercepts of the logarithmic equations adjusted to data. The following graphs were made to determine the existence of that relation:



(A) (B)

Figure 4. A) Coefficients of logarithmic equations in Figure 4 vs. optimum sag with $Q^2/L^3 = 1 \times 10^{-9}$ m^3/s^2. B) Intercepts of logarithmic equations in Figure 4 vs. optimum sag with $Q^2/L^3 = 1 \times 10^{-9}$ m^3/s^2.

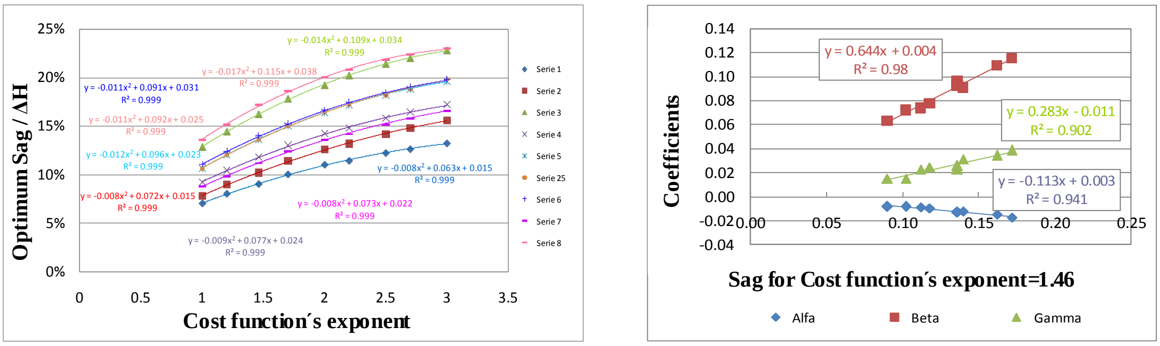
In the Figure 4, there is a linearly relation between the Optimum sag for a fixed value of $Q^2/L^3 = 1 \times 10^{-9}$ m^3/s^2 and the parameters of adjusted logarithmic equations. The reason of developing these graphs for a value of f (Q, L) = 1×10^{-9} m^3/s^2 is that all the series, used in the statistical analysis to explain optimum sag as function of *Demands Centroid* and *Uniformity Coefficient*, have a total length of 1000 m and a total demand of 1 m^3/s; evaluating the function **f** to these values, the following is obtained:

$$f(Q,L) = \frac{Q^2}{L^3} = \frac{(1m^3/s)^2}{(1000\,m)^3} = 1 \cdot 10^{-9}\ \text{m}^3/\text{s}^2 \tag{12}$$

Knowing both the *Demands Centroid* and the *Uniformity Coefficient* for a given set of pipes, Equation 10 is used to get the optimum sag for a value of the function f (Q, L) = 1×10^{-9} m^3/s^2, which is the condition under the equation was developed. With this value, Figure 5 is used to determine the coefficient and intercept value of the logarithmic function describing the variation of the optimum sag in terms of Q^2/L^3; later this equation can be used to calculate the optimum sag for actual relation Q^2/L^3 in the pipes in series that are being analyzed. This procedure is explained below.

3.3. Effect of the costs function exponent on optimum HGL maximum sag.

The costs that were generated by the previous series were calculated with a cost function similar to Equation 2, with a costs exponent of 1.46 and a coefficient of 0.015. However, the optimum design and, consequently, the optimum sag in a series of pipes can change when the exponent of the costs function changes. To analyze the effect of the costs function exponent on HGL optimum sag, 9 series of pipes were generated; for each one of them the optimum sag was calculated for costs function exponent values between 1 and 3. The following figure shows the results obtained:



(A) (B)

Figure 5. A) Analysis of the effect of the costs function exponent in HGL Optimum sag. B) Coefficients of quadratic equations in Figure 6-A vs. optimum sag for 1.46 as costs exponent.

In Figures 5-A and 5-B, the relation between the optimum sag and the exponent of the costs function fits perfectly to a parabola. In addition, all curves are approximately parallel and the values of the coefficients in the parabolas are large if the value of the sag is large for a given costs exponent. To verify this, there is a graph with optimum sag for a given costs exponent against the coefficients α, β and γ of quadratic equations fitted to the previous data. Figure 5-B shows the graph obtained for 1.46 as the value of the exponent of the costs function, the value that was used in this study to estimate the cost of the pipes. The relation between the optimum sag (for a fixed exponent of costs) and the value of the coefficients of adjusted quadratic equations in the figure is linear. Thus, if the optimum sag for an exponent of 1.46 is known, the equations obtained from Figure 5 can be used to determine the optimum sag for any value of the costs function exponent.

From the analysis of the factors that determine the HGL optimum sag, a procedure for estimating this sag for a system of series of pipes has been defined for any set of topological, hydraulic and commercial properties; the procedure is described below:

4. DESIGN METHODOLOGY

The steps to estimate the optimum sag size are:

1. Calculate the Demands Centroid using Equation 7.
2. Calculate the Uniformity Coefficient (CU) using Equation 8.
3. Using Equation 10 to estimate the optimum sag size according to Demands Centroid and Uniformity Coefficient. The sag calculated with this equation corresponds to a ratio $Q^2/L^3 = 1 \times 10^{-9}$ m^3/s^2 and an exponent of the costs function of 1.46.
4. From the sag obtained in Step 3, estimate the optimum sag for the exponent of the costs function (n) you have, using the following expression:

$$Optimum\ Sag = \alpha . n^2 + \beta . n + \gamma \tag{13}$$

n: exponent of the costs function
$\alpha = -0.1134 + 0.0032 * F_{1.46}$ (From Figure 5)
$\beta = 0.6443 * F_{1.46} - 0.0043$ (From Figure 5)
$\gamma = 0.2835 + 0.0111 * F_{1.46}$ (From Figure 5)
$F_{1.46}$: Optimum sag for an exponent of 1.46 (obtained in Step 3).

5. From the sag obtained in Step 4, which corresponds to a ratio $Q^2/L^3 = 1 \times 10^{-9}$, calculate the sag to the terms of Q^2/L^3. For this is used the following expression (from Figure 4):

$$Optimum\ Sag = a \cdot ln\left(\frac{Q^2}{L^3}\right) + b \tag{14}$$

a = 0.00868 * $F_{1x10\text{-}9}$ + 0.00066 (from Figure 4-A)
b = 1.18069 * $F_{1x10\text{-}9}$ + 0.01345 (from Figure 4-B)
$F_{1x10\text{-}9}$: Optimum sag for a ratio $Q^2/L^3 = 1 \times 10^{-9}$ (Obtained on step 4).

By knowing the magnitude of the maximum deflection of the optimal HGL, the third point of this curve can be known and it is possible to determine the quadratic equation that describes its trajectory. This equation is as follows.

$$HGL(x) = \alpha \cdot x^2 + \beta \cdot x + \gamma \tag{15}$$

where HGL(x) is the ideal HGL at the point x and the coefficients α, β and γ depend on the entrance HGL, the minimum HGL, the maximum length of the pipes in series and the optimal sag. The methodology to design a series of pipes, by previously determining the optimal HGL is described below:

1. Set the design parameters, topological and hydraulic characteristics of the pipes in series (i.e. minimum required pressure, pipe´s length, pipe's roughness, head at the source tank, base demands and the cost function).
2. Estimate the optimal sag value of the HGL.
3. Calculate the ideal HGL at each node of the pipes on series using Equation 15.
4. At each section of the series an objective energy loss is assign, as the difference between the ideal HGL values of its upstream and downstream nodes, estimated on step 3.

$$Objective\ Energy\ Loss_{ij} = HGL_{ideali} - HGL_{idealij} \tag{16}$$

where i is the upstream node and j is the downstream node of the pipe. With the objective energy loss and the flow rate in each pipe, the optimal diameter size is calculated using the Darcy-Weisbach equation with the Colebrook-White equation. It is clear that the result achieved is a preliminary configuration of the series that fulfills the hydraulic restrictions of the design problem; however, since the pipe diameters are continuous values, the commercial requirements are not accomplished. In order to round off diameters to commercial available values, restriction programming procedures were implemented, which are low complexity and require a small number of hydraulic simulations, a great advantage for the design of large series of pipes.

5. PROPOSED METHODOLOGY VS. METAHEURISTIC (GA) DESIGN

In order to verify the OHGL design methodology, a total of 400 different series with random topological characteristics (node demand, pipe lengths and source head) and horizontal topography were used. The series were classified into three types: 160 series with constant node demand and constant pipe length, 120 series with constant node demand but random pipe lengths and 120 series with random node demand and pipe lengths. The series were designed using both methodologies OHGL and genetic algorithms. The genetic algorithm used in the design has the following characteristics: simple crossover operators with breeding probability inversely proportional to the cost function and roulette selection where the actual number of descendants of an individual varies considerably and is not equal to the expected number of these.

The initial topological and topographical characteristics were: number of pipes at each series (t): between 3 and 30; pipe length (l): between 10 and 100 m; node demand (q): between 5 and 150 L/s; source tank elevation (h): between 20 and 50 m. The minimum head is 15 m. The pipe material selected was PVC, with an absolute roughness of 0.0000015 m; and the available pipe size diameters were 50, 75, 100, 150, 200, 250, 300, 350, 400, 450, 500, 600, 750, 800, 1000, 1200, 1400, 1500 and 1800 mm. Some other parameters of the design were the kinematic viscosity equal to 1.141×10^{-6} m^2/s; the coefficient and exponent of the cost function were 0.015 and 1.46 respectively.

Series Type 1: The 88.75% (142 series) of the 160 designed series had a lower cost with the OHGL methodology, 6.25% (10 series) had the same cost for both OHGL and genetic algorithms methodologies. Finally, 5% (8 series) had a lower cost when designed with GA; although, the cost disparity was always under 1%, demonstrating the goodness of the explicit design. On the other side, in some cases the series designed by OHGL reached costs considerably lower than the GA designs, with differences up to 33%.

Series Type 2: The 80.83% (97 series) of the 120 series had a lower cost with the OHGL methodology, 10% (12 series) had the same cost for both OHGL and GA methodologies, and 9.17% (11 series) had a lower cost when designed with genetic algorithms. In this series type, the same cost pattern as in Series Type 1 was observed.

Series Type 3: The 79.17% (95 series) of the 120 designed series had a lower cost with the OHGL methodology, 12.50% (15 series) had the same cost for both OHGL and GA methodologies, and 8.33% (10 series) had a lower cost when designed with GA.

Table 2. Results for cost comparison for the two methodologies.

TYPE	TOTAL SERIES	# OF SERIES	%	COMPARISSION
1	160	152	95,00	OHGL≤GA
		8	5,00	OHGL>GA
2	120	109	90,83	OHGL≤GA
		11	9,17	OHGL>GA
3	120	110	91,67	OHGL≤GA
		10	8,33	OHGL>GA

6. CONCLUSIONS

- An explicit methodology for optimal design of pipes in series was successfully tested and developed. Unlike most existing design methodologies, based on the imitation of natural and physical phenomena (e.g. genetic algorithm, harmony search, particle swarm, ant colony), this methodology was based on the understanding of hydraulics and the system's topology.
- It was found that it is possible to design least-cost series of pipes if it is previously known the optimal hydraulic grade line, which consists of a set of points (X, Y, HGL_{ideal}), where X and Y are the plane coordinates corresponding to each node of the series and the HGL_{ideal} is the head each node should have to achieve the least-cost configuration. The shape of this line fits a quadratic function which curvature depends on the hydraulic characteristics, topological and commercial restrictions such as spatial demand distribution, relationship between the total demand flow and the total pipes length, and the cost function.
- The relationship between the HGL and the series characteristics were studied; besides, a methodology to estimate the parabola equation for the optimal HGL was proposed, which represents the optimal manner to use the available power within the pipes series.
- The cost differences between the series design, where GA costs were under the OHGL costs, are quite low; no more than 1% for Series Type 1. Also, the difference of costs between Type 2 and Type 3 series reached 5%. These kinds of series are atypical because of the inconsistent energy dissipation. When the OHGL is more economical than the GA designs, the cost differences can reach 33%. The series that had equal or lower costs when designed with GA are those where the mayor demand is localized downstream, so the demand´s centroid moves downwards.
- Based on the results found, it can be concluded that the OHGL methodology is effective and efficient to achieve optimal pipes in series design. The low complexity and efficiency are the main advantages of the OHGL methodology over metaheuristics algorithms; moreover, the OHGL methodology has an explicit nature and no dependence whatsoever on the initial pipes diameter configuration.
- The OHGL methodology can be expand to the case of water distribution system (WDS) designs, where the metaheuristic algorithms used have a significant random component and require a huge number of hydraulic simulations to explore the search space to achieve an approximate least-cost design. Additionally, due to its randomness, the results accomplished at each run are not always the same, which is the reason why a certain number of runs must be made (where a big number of hydraulics simulations are performed) until a good design is obtained and is usually the unique value that can be published.
- The OHGL methodology allows the designer to understand the optimal design´s hydraulics, something that can become a useful tool for existing WDS optimization, using the optimal available power use concept.

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