

Modelling valve stiction[☆]

M.A.A. Shoukat Choudhury^a, N.F. Thornhill^b, S.L. Shah^{a,*}

^aDepartment of Chemical and Materials Engineering, University of Alberta, Edmonton, Canada, AB T6G 2G6

^bDepartment of Electronic and Electrical Engineering, University College London, London, UK WC1E 7JE

Received 18 January 2003; accepted 13 May 2004

Available online 28 July 2004

Abstract

The presence of nonlinearities, e.g., stiction, and deadband in a control valve limits the control loop performance. Stiction is the most commonly found valve problem in the process industry. In spite of many attempts to understand and model the stiction phenomena, there is a lack of a proper model, which can be understood and related directly to the practical situation as observed in real valves in the process industry. This study focuses on the understanding, from real-life data, of the mechanism that causes stiction and proposes a new data-driven model of stiction, which can be directly related to real valves. It also validates the simulation results generated using the proposed model with that from a physical model of the valve. Finally, valuable insights on stiction have been obtained from the describing function analysis of the newly proposed stiction model.

© 2004 Elsevier Ltd. All rights reserved.

Keywords: Stiction; Stickband; Deadband; Hysteresis; Backlash; Deadzone; Viscous friction; Coulomb friction; Process control; Slip jump

1. Introduction

A typical chemical plant has hundreds or thousands of control loops. Control performance is very important to ensure tight product quality and low cost of the product in such plants. The economic benefits resulting from performance assessment are difficult to quantify on a loop-by-loop basis because each problem loop contributes in a complicated way to the overall process performance. Finding and fixing problem loops throughout a plant shows reduced off-grade production, reduced product property variability, and occasionally lower operating costs and improved production rate (Paulonis & Cox, 2003). Even a 1% improvement either in energy efficiency or improved controller maintenance direction represents hundreds of millions of dollars in savings to the process industries (Desborough & Miller, 2002). Oscillatory variables are one of the main causes for poor performance of control loops and a key

challenge is to find the root cause of distributed oscillations in chemical plants (Qin, 1998; Thornhill, Huang, & Zhang, 2003a; Thornhill, Cox, & Paulonis, 2003b). The presence of oscillations in a control loop increases the variability of the process variables, thus causing inferior quality products, larger rejection rates, increased energy consumption, reduced average throughput and profitability. Oscillations can cause a valve to wear out much earlier than its life period it was originally designed for. Oscillations increase operating costs roughly in proportion to the deviation (Shinskey, 1990). Detection and diagnosis of the causes of oscillations in process operation are important because a plant operating close to product quality limit is more profitable than a plant that has to back away because of variations in the product (Martin, Turpin, & Cline, 1991). Oscillatory feedback control loops are a common occurrence due to poor controller tuning, control valve stiction, poor process and control system design, and oscillatory disturbances (Bialkowski, 1992; Ender, 1993; Miao & Seborg, 1999). Bialkowski (1992) reported that about 30% of the loops are oscillatory due to control valve problems. The only moving part in a control loop is the control valve. If the control valve contains nonlinearities, e.g., stiction, backlash, and deadband,

[☆] A preliminary version of this paper was presented at ADCHEM 2003, Hong Kong, January 11–14, 2004.

*Corresponding author. Tel.: +1-780-492-5162; fax: +1-780-492-2881.

E-mail address: sirish.shah@ualberta.ca (S.L. Shah).

the valve output may be oscillatory, which in turn can cause oscillations in the process output. Among the many types of nonlinearities in control valves, stiction is the most common and one of the long-standing problems in the process industry. It hinders the achievement of good performance of control valves as well as control loops. Many studies (Armstrong-Hélouvy, Dupont, & De Wit, 1994; Aubrun, Robert, & Cecchin, 1995; McMillan, 1995; Taha, Dumont, & Davies, 1996; Wallén, 1997; Horch & Isaksson, 1998; Sharif & Grosvenor, 1998; Horch, Isaksson, & Forsman, 2000; Horch, 2000; Ruel, 2000; Gerry & Ruel, 2001) have been carried out to define and detect static friction or stiction. However, there is lack of a unique definition and description of the mechanism of stiction. This work addresses this issue and modelling of valve friction. The parameters of a physical model, e.g., mass of the moving parts of the valve, spring constants, and forces, are not explicitly known. These parameters need to be tuned properly to produce the desired response of a valve. The effect of changes in these parameters is also not known. Working with such a physical model is therefore often time consuming and cumbersome for simulation purposes. Also, in industrial practice stiction and other related problems are identified in terms of the % of the valve travel or span of the valve input signal. The relationship between the magnitudes of the parameters of a physical model and deadband, backlash or stiction (expressed as a % of the span of the input signal) is not simple. The purpose of this paper is to develop an empirical data-driven model of stiction that is useful for simulation and diagnosis of oscillation in chemical processes. The main contributions of this paper are:

- Clarification of the confusion prevailing in the control literature and in the control community regarding the misunderstanding of stiction and the terms closely related to it.
- A new formal definition of stiction has been proposed using parameters similar to those used in the American National Standard Institution's (ANSI) formal definition of backlash, hysteresis, and deadband. The key feature of these definitions is that they focus on the input–output behaviour of such elements. The proposed definition is also cast in terms of the input–output behaviour.
- A new two-parameter data-driven model of stiction has been developed and validated with a mechanistic model of stiction and also with data obtained from industrial control valves suffering from stiction. The data-driven model is capable of handling stochastic inputs and can be used to perform simulation of stiction in Matlab's Simulink environment in the studies of stiction-relevant control loop problems.
- A describing function analysis of the newly proposed stiction model reveals valuable insights on stiction behaviour. For example, pure deadband or backlash cannot produce limit cycles in the presence of a PI controller unless there is an integrator in the plant under closed-loop feedback configuration.

The paper has been organized as follows: First, a thorough discussion of the terms related to valve nonlinearity has been presented, followed by the proposal of a new formal definition of stiction. Some practical examples of valve stiction are provided to gain true insights of stiction from real-life data. Then the results of a mechanistic model of stiction were used to validate the corresponding subsequent results of the data-driven stiction model. Finally, a describing function analysis of the newly proposed stiction model has been presented.

2. What is stiction?

There are some terms such as deadband, backlash and hysteresis, which are often misused and wrongly used in describing valve problems. For example, quite commonly a deadband in a valve is referred to *backlash* or *hysteresis*. Therefore, before proceeding to the definition of stiction, these terms are first defined for a better understanding of the stiction mechanism and a more formal definition of stiction.

2.1. Definition of terms relating to valve nonlinearity

This section reviews the American National Standard Institution's (ANSI) formal definition of terms related to stiction. The aim is to differentiate clearly between the key concepts that underlie the ensuing discussion of friction in control valves. These definition can also be found in (EnTech, 1998; Fisher-Rosemount, 1999), which also make reference to ANSI. ANSI (ISA-S51.1-1979, Process instrumentation Terminology) defines the above terms as follows:

- Backlash: “*In process instrumentation, it is a relative movement between interacting mechanical parts, resulting from looseness, when the motion is reversed*”.
- Hysteresis: “*Hysteresis is that property of the element evidenced by the dependence of the value of the output, for a given excursion of the input, upon the history of prior excursions and the direction of the current traverse*”.
 - “*It is usually determined by subtracting the value of deadband from the maximum measured separation between upscale-going and downscale-going indications of the measured variable (during a full-range traverse, unless otherwise specified) after transients*”.

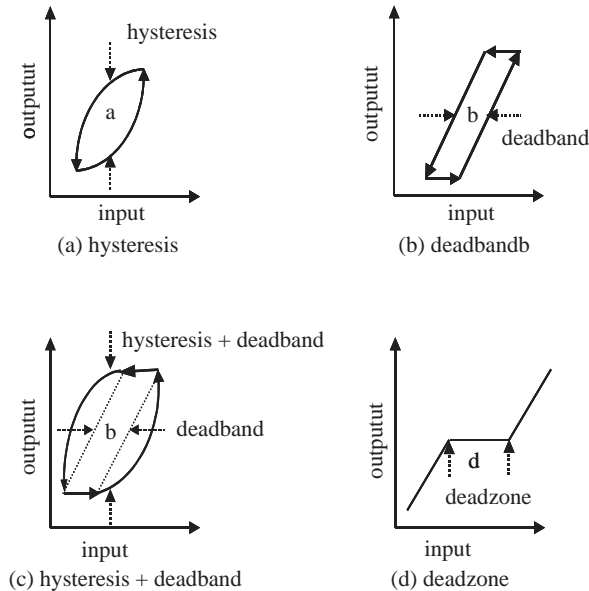


Fig. 1. Hysteresis, deadband, and deadzone (redrawn from ANSI/ISA-S51.1-1979).

have decayed”. Fig. 1(a) and (c) illustrates the concept.

- “Some reversal of output may be expected for any small reversal of input. This distinguishes hysteresis from deadband”.
- **Deadband:** “In process instrumentation, it is the range through which an input signal may be varied, upon reversal of direction, without initiating an observable change in output signal”.
 - “There are separate and distinct input–output relationships for increasing and decreasing signals (see Fig. 1(b))”.
 - “Deadband produces phase lag between input and output”.
 - “Deadband is usually expressed in percent of span”.
- **Dead zone:** “It is a predetermined range of input through which the output remains unchanged, irrespective of the direction of change of the input signal”.
 - “There is but one input–output relationship (see Fig. 1(d))”.
 - “Dead zone produces no phase lag between input and output”.

The above definitions show that the term “backlash” specifically applies to the slack or looseness of the mechanical part when the motion changes its direction. Therefore, in control valves it may only add deadband effects if there is some slack in rack-and-pinion type actuators (Fisher-Rosemount, 1999) or loose connections in rotary valve shaft. ANSI (ISA-S51.1-1979) definitions and Fig. 1 show that hysteresis and deadband

are distinct effects. Deadband is quantified in terms of input signal span (i.e., on the x-axis), while hysteresis refers to a separation in the measured (output) response (i.e., on the y-axis).

2.2. Discussion of the term “stiction”

Different people or organizations have defined stiction in different ways. A few of the definitions are reproduced below:

- According to the Instrument Society of America (ISA) (ISA Subcommittee SP75.05, 1979), “stiction is the resistance to the start of motion, usually measured as the difference between the driving values required to overcome static friction upscale and downscale”. The definition was first proposed in 1963 in American National Standard C85.1-1963, “Terminology for Automatic Control”, and has not been updated. This definition was adopted in ISA 1979 Handbook (ISA Subcommittee SP75.05, 1979) and remained exactly the same in the revised 1993 edition.
- According to EnTech (1998), “stiction is a tendency to stick-slip due to high static friction. The phenomenon causes a limited resolution of the resulting control valve motion. ISA terminology has not settled on a suitable term yet. Stick-slip is the tendency of a control valve to stick while at rest, and to suddenly slip after force has been applied”.
- According to Horch (2000), “The control valve is stuck in a certain position due to high static friction. The (integrating) controller then increases the set point to the valve until the static friction can be overcome. Then the valve breaks off and moves to a new position (slip phase) where it sticks again. The new position is usually on the other side of the desired set point such that the process starts in the opposite direction again”. This is the extreme case of stiction. On the contrary, once the valve overcomes stiction it might travel smoothly for some time and then stick again when the velocity of the valve is close to zero.
- In a recent paper, Ruel (2000) reported “stiction as a combination of the words stick and friction, created to emphasize the difference between static and dynamic friction. Stiction exists when the static (starting) friction exceeds the dynamic (moving) friction inside the valve. Stiction describes the valve’s stem (or shaft) sticking when small changes are attempted. Friction of a moving object is less than when it is stationary. Stiction can keep the stem from moving for small control input changes, and then the stem moves when there is enough force to free it. The result of stiction is that the force required to get the stem to move is more than is required to go to the desired stem position. In presence of stiction, the movement is jumpy”.

This definition is close to the stiction as measured online by the people in process industries—putting the control loop in manual and then increasing the valve input in little increments until there is a noticeable change in the process variable.

- In Olsson (1996), stiction is defined as *short for static friction as opposed to dynamic friction. It describes the friction force at rest. Static friction counteracts external forces below a certain level and thus keeps an object from moving.*

The above discussion reveals the lack of a formal and general definition of stiction and the mechanism(s) that causes it. All of the above definitions agree that stiction is the static friction that keeps an object from moving and when the external force overcomes the static friction the object starts moving. But they disagree in the way it is measured and how it can be modelled. Also, there is a lack of clear description of what happens at the moment when the valve just overcomes the static friction. Some modelling approaches described this phenomena using a Stribeck effect model (Olsson, 1996). These issues can be resolved by a careful observation and a proper definition of stiction.

2.3. A proposal for a definition of stiction

The motivation for a new definition of stiction is to capture the descriptions cited earlier within a definition that explains the behaviour of an element with stiction in terms of its input–output behaviours, as is done in the ANSI definitions for backlash, hysteresis, and deadband. The new definition of stiction is proposed by the authors based on careful investigation of real process data. It is observed that the phase plot of the input–output behaviour of a valve “suffering from stiction” can be described as shown in Fig. 2. It consists of four components: deadband, stickband, slip jump and the moving phase. When the valve comes to rest or changes the direction at point A in Fig. 2, the valve sticks. After

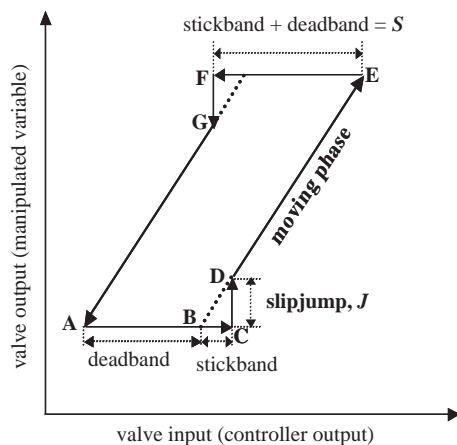


Fig. 2. Typical input–output behaviour of a sticky valve.

the controller output overcomes the deadband (AB) and the stickband (BC) of the valve, the valve jumps to a new position (point D) and continues to move. Due to very low or zero velocity, the valve may stick again in between points D and E in Fig. 2 while travelling in the same direction (EnTech, 1998). In such a case, the magnitude of deadband is zero and only stickband is present. This can be overcome if the controller output signal is larger than the stickband only. It is usually uncommon in industrial practice. The deadband and stickband represent the behaviour of the valve when it is not moving, though the input to the valve keeps changing. Slip jump represents the abrupt release of potential energy stored in the actuator chambers due to high static friction in the form of kinetic energy as the valve starts to move. The magnitude of the slip jump is very crucial in determining the limit cyclic behaviour introduced by stiction (McMillan, 1995; Piipponen, 1996). Once the valve slips, it continues to move until it sticks again (point E in Fig. 2). In this moving-phase, dynamic friction is present which may be much lower than the static friction. As depicted in Fig. 2, this section has proposed a rigorous description of the effects of friction in a control valve. Therefore, “*stiction is a property of an element such that its smooth movement in response to a varying input is preceded by a sudden abrupt jump called the slip-jump. Slip-jump is expressed as a percentage of the output span. Its origin in a mechanical system is static friction which exceeds the friction during smooth movement*”. This definition has been exploited in the next and subsequent sections for the evaluation of practical examples and for modelling of a control valve suffering from stiction in a feedback control configuration.

3. Practical examples of valve stiction

The objective of this section is to observe effects of stiction from the investigation of data from industrial control loops. The observations reinforce the need for a rigorous definition of the effects of stiction. This section analyses four data sets. The first data set is from a power plant, the second and third are from a petroleum refinery and the other is from a furnace. To preserve the confidentiality of the plants, all data are scaled and reported as mean-centred with unit variance. In order to facilitate the readability of the paper by practising industrial people, the notations followed by the industrial people have been used. For example, pv is used to denote the process variable or controlled variable. Similarly, op is used to denote the controller output, mv is used to denote valve output or valve position, and sp is used to denote set point.

- Loop 1 is a level control loop which controls the level of condensate in the outlet of a turbine by

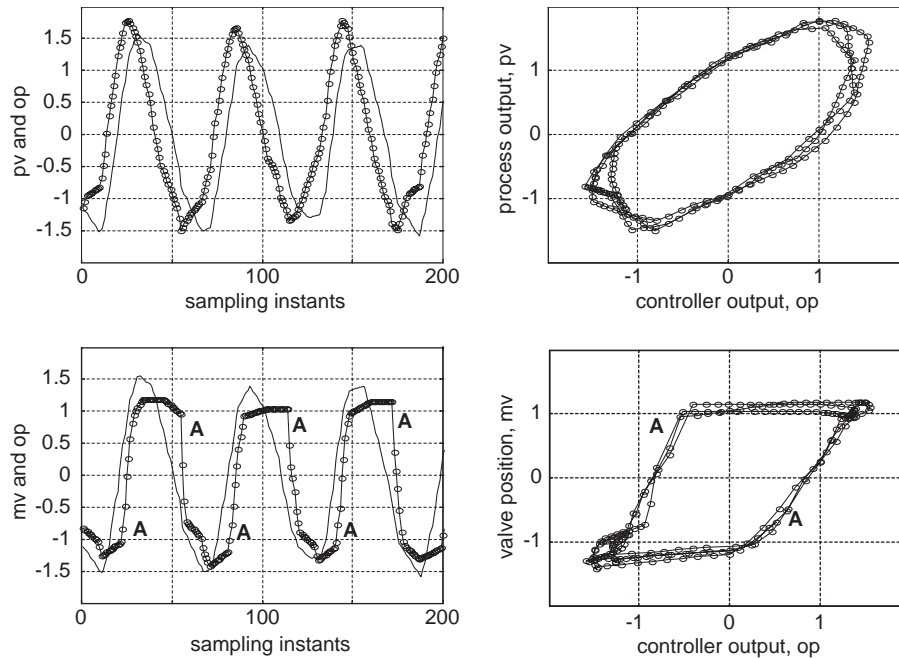


Fig. 3. Flow control cascaded to level control in an industrial setting, the line with circles is pv and mv , the thin line is op .

manipulating the flow rate of the liquid condensate. In total, 8640 samples for each tag were collected at a sampling rate of 5 s. Fig. 3 shows a portion of the time domain data. The left panel shows time trends for level (pv), the controller output (op) which is also the valve demand, and valve position (mv) which can be taken to be the same as the condensate flow rate. The plots in the right panel show the characteristic $pv-op$ and $mv-op$ plots. The bottom figure clearly indicates both the stickband plus deadband and the slip jump effects. The slip jump is large and visible from the bottom figure, especially when the valve is moving in a downward direction. It is marked as 'A' in the figure. It is evident from this figure that the valve output (mv) can never reach the valve demand (op). This kind of stiction is termed as undershoot case of valve stiction in this paper. The $pv-op$ plot does not show the jump behaviour clearly. The slip jump is very difficult to observe in the $pv-op$ plot because the process dynamics (i.e., the transfer function between mv and pv) destroys the pattern. This loop shows one of the possible types of stiction phenomena clearly. The stiction model developed later in the paper based on the control signal (op) is able to imitate this kind of behaviour.

- Loop 2 is a liquid flow slave loop of a cascade control loop. The data were collected at a sampling rate of 10 s and the data length for each tag was 1000 samples. The left plot of Fig. 4 shows the time trend of pv and op . A closer look of this figure shows that the pv (flow rate) is constant for some period of 8 time though the op changes over that period. This is the period during which the valve was stuck. Once the

valve overcomes deadband plus stickband, the pv changes very quickly (denoted as 'A' in the figure) and moves to a new position where the valve sticks again. It is also evident that sometimes the pv overshoots the op and sometime it undershoots. The $pv-op$ plot has two distinct parts—the lower part and the upper part extended to the right. The lower part corresponds to the overshoot case of stiction, i.e., it represents an extremely sticky valve. The upper part corresponds to the undershoot case of stiction. These two cases have been separately modelled in the data-driven stiction model. This example represents a mixture of undershoot and overshoot cases of stiction. The terminologies regarding different cases of stiction have been made clearer in Section 5.

- Loop 3 is a slave flow loop cascaded with a master level control loop. A sampling rate of 6 s was used for the collection of the data and a total of 1000 samples for each tag were collected. The top panel of Fig. 5 shows the presence of stiction with a clear indication of stickband plus deadband and the slip jump phase. The slip jump appears as the control valve just overcomes stiction (denoted as point 'A' in Fig. 5). This slip jump is not very clear in the $pv-op$ plot of the closed-loop data (top right plot), because both pv and op jump together due to the probable presence of a proportional only controller. But it shows the presence of deadband plus stickband clearly. Sometimes it is best to look at the $pv-sp$ plot if it is a cascaded loop and the slave loop is operating under proportional control only. The bottom panel of Fig. 5 shows the time trend and phase plot of sp and pv where the slip jump behaviour is clearly

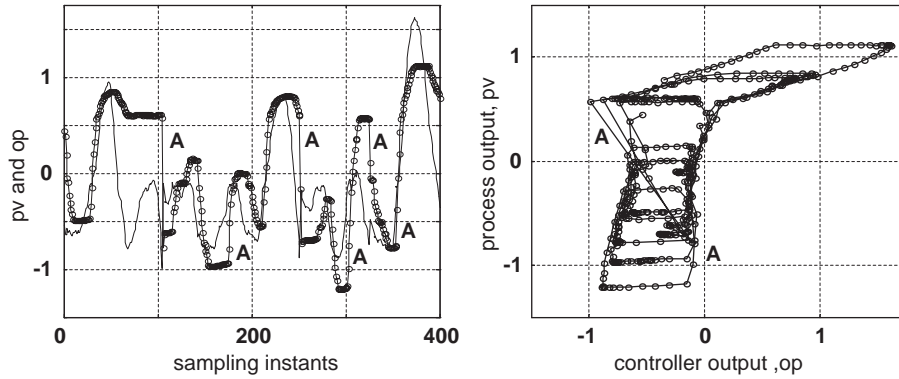


Fig. 4. Data from a flow loop in a refinery, time trend of *pv* and *op* (left)—the line with circles is *pv* and the thin line is *op*, and the *pv-op* plot (right).

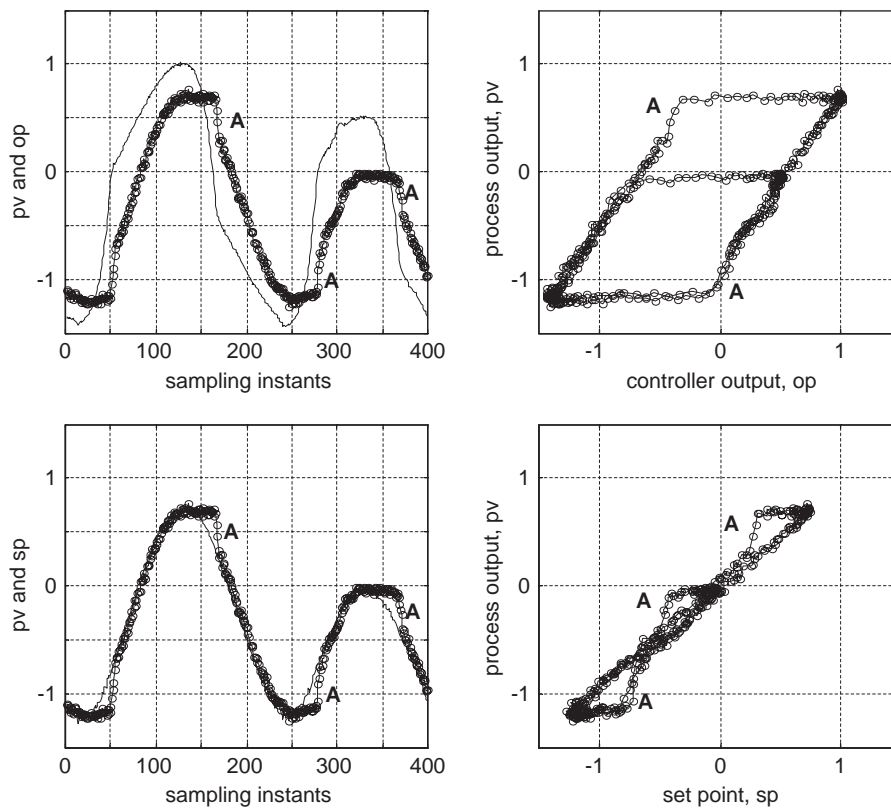


Fig. 5. Data from a flow loop in a refinery, time trend of *pv* and *op* (top left)—the line with circles is *pv* and the thin line is *op*, the *pv-op* plot (top right), time trend of *pv* and *sp* (bottom left), line with circles is *pv* and thin line is *sp*, and the *pv-sp* plot (bottom right).

visible. This example represents a case of pure stick-slip or stiction with no offset.

- Loop 4 is a temperature control loop on a furnace feed dryer system at the Tech-Cominco mine in Trail, British Columbia, Canada. The temperature of the dryer combustion chamber is controlled by manipulating the flow rate of natural gas to the combustion chamber. A total of 1440 samples for each tag were collected at a sampling rate of 1 min. The top plot of the left panel of Fig. 6 shows time trends of

temperature (*pv*) and controller output (*op*). It shows clear oscillations both in the controlled variable (*pv*) and the controller output. The presence of distinct cycles is observed in the characteristic *pv-op* plot (see Fig. 6 top right). For this loop, there is a flow indicator close to this valve and these indicator data were available. In the bottom figure this flow rate is plotted versus *op*. The flow rate data looks quantized but the presence of stiction in this control valve was confirmed by the plant engineer. The bottom plots

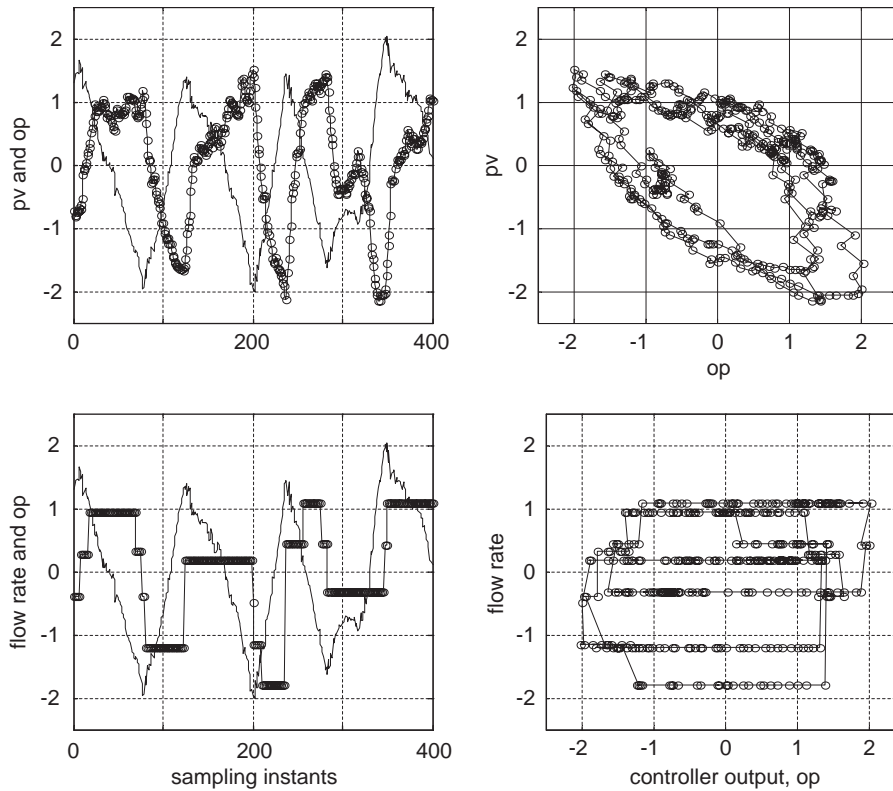


Fig. 6. Industrial dryer temperature control loop data, lines with circles are *pv* and *flowrate*, the thin line is *op* (the bottom left panel).

clearly show the stickband and the slip jump of the valve. Note that the moving phase of the valve is almost absent in this example. Once the valve overcomes stiction, it jumps to the new position and sticks again.

4. A physical model of valve friction

4.1. Model formulation

The purpose of this section is to understand the physics of valve friction and reproduce the behaviour seen in real plant data. For a pneumatic sliding stem valve, the force balance equation based on Newton’s second law can be written as

$$M \frac{d^2x}{dt^2} = \sum Forces = F_a + F_r + F_f + F_p + F_i, \quad (1)$$

where M is the mass of the moving parts, x is the relative stem position, $F_a = Au$ is the force applied by pneumatic actuator where A is the area of the diaphragm and u is the actuator air pressure or the valve input signal, $F_r = -kx$ is the spring force where k is the spring constant, $F_p = -\alpha\Delta P$ is the force due to fluid pressure drop where α is the plug unbalance area and ΔP is the fluid pressure drop across the valve, F_i is the extra force required to force the valve to be into the seat and F_f is the friction force (Whalen, 1983;

Fitzgerald, 1995; Kayihan & Doyle III, 2000). Following Kayihan and Doyle III, F_i and F_p will be assumed to be zero because of their negligible contribution in the model.

The friction model is from (Karnopp, 1985; Olsson, 1996) and was used also by (Horch & Isaksson, 1998). It includes static and moving friction. The expression for the moving friction is in the first line of Eq. (2) and comprises a velocity-independent term F_c known as Coulomb friction and a viscous friction term vF_v that depends linearly upon velocity. Both act in opposition to the velocity, as shown by the negative signs.

$$F_f = \begin{cases} -F_c \operatorname{sgn}(v) - vF_v & \text{if } v \neq 0, \\ -(F_a + F_r) & \text{if } v = 0 \text{ and } |F_a + F_r| \leq F_s, \\ -F_s \operatorname{sgn}(F_a + F_r) & \text{if } v = 0 \text{ and } |F_a + F_r| > F_s. \end{cases} \quad (2)$$

The second line in Eq. (2) is the case when the valve is stuck. F_s is the maximum static friction. The velocity of the stuck valve is zero and not changing, therefore the acceleration is zero also. Thus, the right-hand side of Newton’s law is zero, so $F_f = -(F_a + F_r)$. The third line of the model represents the situation at the instant of breakaway. At that instant, the sum of forces is $(F_a + F_r) - F_s \operatorname{sgn}(F_a + F_r)$, which is not zero if $|F_a + F_r| > F_s$. Therefore, the acceleration becomes non-zero and the valve starts to move.

A disadvantage of a physical model of a control valve is that it requires several parameters to be known. The mass M and typical friction forces depend upon the design of the valve. Kayihan and Doyle III (2000) used manufacturer’s values suggested by Fitzgerald (1995) and similar values have been chosen here apart from a slightly increased value of F_s and a smaller value for F_c in order to make the demonstration of the slip jump more obvious (see Table 1). Fig. 7 shows the friction force characteristic in which the magnitude of the moving friction is smaller than that of the static friction. The friction force opposes velocity (see Eq. (2)), thus the force is negative when the velocity is positive.

The calibration factor of Table 1 is introduced because the required stem position x_r is the input to the simulation. In the absence of stiction effects, the valve moving parts come to rest when the force due to air pressure on the diaphragm is balanced by the spring force. Thus, $Au = kx$ and so the calibration factor relating air pressure u to x_r is k/A . The consequences of miscalibration are discussed below.

Table 1
Nominal values used for physical valve simulation

Parameters	Kayihan and Doyle III (2000)	Nominal case
M	3 lb (1.36 kg)	1.36 kg
F_s	384 lbf (1708 N)	1750 N
F_c	320 lbf (1423 N)	1250 N
F_v	3.5 lbf s in ⁻¹ (612 N s m ⁻¹)	612 N s m ⁻¹
Spring constant, k	300 lbf in ⁻¹ (52,500 N m ⁻¹)	52,500 N m ⁻¹
Diaphragm area, A	100 in ² (0.0645 m ²)	0.0645 m ²
Calibration factor, k/A	—	807,692 Pa m ⁻¹
Air pressure	10 psi (68,950 Pa)	68,950 Pa

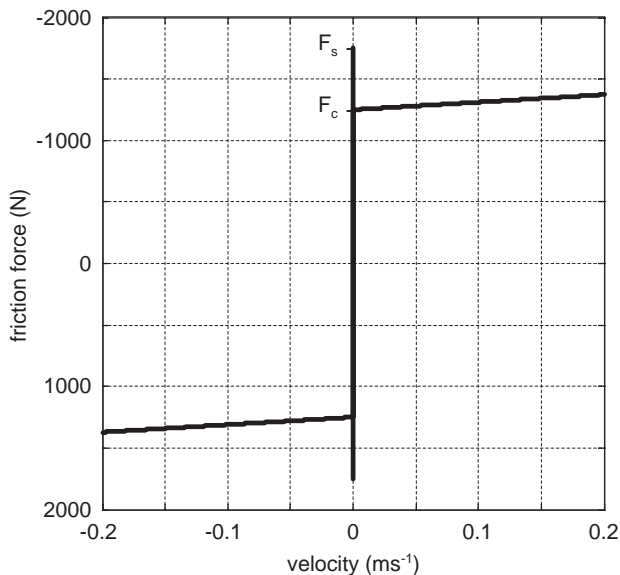


Fig. 7. Friction characteristic plot.

4.2. Valve simulation

The purpose of simulation of the valve was to determine the influence of the three friction terms in the model. The nonlinearity in the model is able to induce limit cycle oscillations in a feedback control loop, and the aim is to understand the contribution of each friction term to the character and shape of the limit cycles.

4.2.1. Open loop response

Fig. 8 shows the valve position when the valve model is driven by a sinusoidal variation in op in open loop in the absence of the controller. The left-hand column shows the time trends and the right-hand panels are plots of valve demand (op) versus valve position (mv). Several cases are simulated using the parameters shown in Table 2. The “linear” values are those suggested by Kayihan and Doyle III for the best case of a smart valve with Teflon packing requiring an air pressure of about 0.1 psi (689 Pa) to start moving.

In the first row of Fig. 8, the Coulomb friction F_c and static friction F_s are small and linear viscous friction dominates. The input and output are almost in phase in the first row of Fig. 8 because the sinusoidal input is of low frequency compared to the bandwidth of the valve model and is on the part of the frequency response function where input and output are in phase.

Valve deadband is due to the presence of Coulomb friction F_c , a constant friction which acts in the opposite direction to the velocity. In the deadband simulation case the static friction is the same as the Coulomb

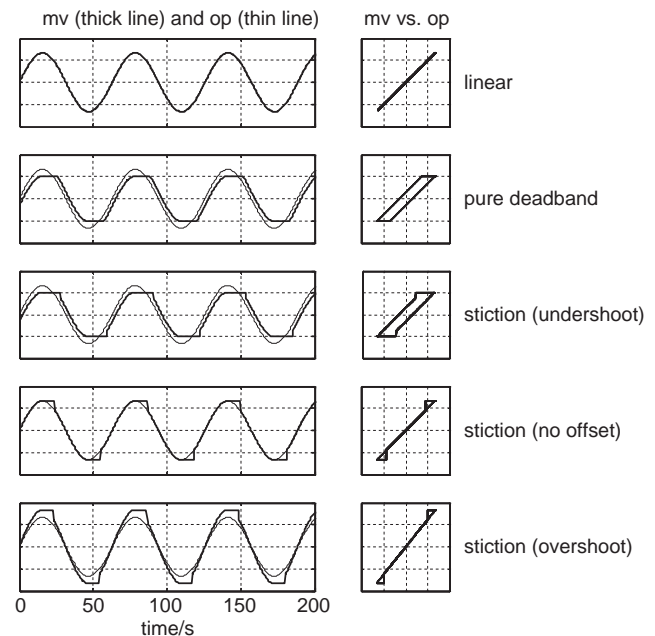


Fig. 8. Open-loop response of mechanistic model. The amplitude of the sinusoidal input is 10 cm in each case.

Table 2
Friction values used in simulation of physical valve model

Parameters	Linear	Deadband	Stiction		
			Undershoot		No offset
			Open loop	Closed loop	
F_s (N)	45	1250	2250	1000	1750
F_c (N)	45	1250	1250	400	0
F_v (N s m ⁻¹)	612	612	612	612	612

friction, $F_s = F_c$. The deadband arises because, on changing direction, the valve remains stationary until the net applied force is large enough to overcome F_c . The deadband becomes larger if F_c is larger.

A valve with high initial static friction such that $F_s > F_c$ exhibits a jumping behaviour that is different from a deadband, although both behaviours may be present simultaneously. When the valve starts to move, the friction force reduces abruptly from F_s to F_c . There is therefore a discontinuity in the model on the right-hand side of Newton’s second law and a large increase in acceleration of the valve moving parts. The initial velocity is therefore faster than in the $F_s = F_c$ case, leading to the jump behaviour observed in the third row of Fig. 8. If the Coulomb friction F_c is absent, then the deadband is absent and the slip jump allows the mv to catch up with the op (fourth row). If the valve is miscalibrated, then swings in the valve position (mv) are larger than swings in the demanded position (op). In that case, the gradient of the op – mv plot is greater than unity during the moving phase. The bottom row of Fig. 8 shows the case when the calibration factor is too large by 25%. A slip jump was also used in this simulation.

4.2.2. Closed-loop dynamics

For assessment of closed-loop behaviour, the valve output drives a first-order plus dead time process $G(s)$ and receives its op reference input from a PI controller $C(s)$ where:

$$G(s) = \frac{3e^{-10s}}{10s + 1}, \quad C(s) = 0.2 \left(\frac{10s + 1}{10s} \right). \quad (3)$$

Fig. 9 shows the limit cycles induced in this control loop by the valve, together with the plots of valve position (mv) versus valve demand (op). The limit cycles were present even though the set point to the loop was zero. That is, they were internally generated and sustained by the loop in the absence of any external setpoint excitation.

There was no limit cycle in the linear case dominated by viscous friction or in the case with deadband only when $F_s = F_c$. It is known that deadband alone cannot induce a limit cycle unless the process $G(s)$ has integrating dynamics, as will be discussed further in Section 5.3.1.

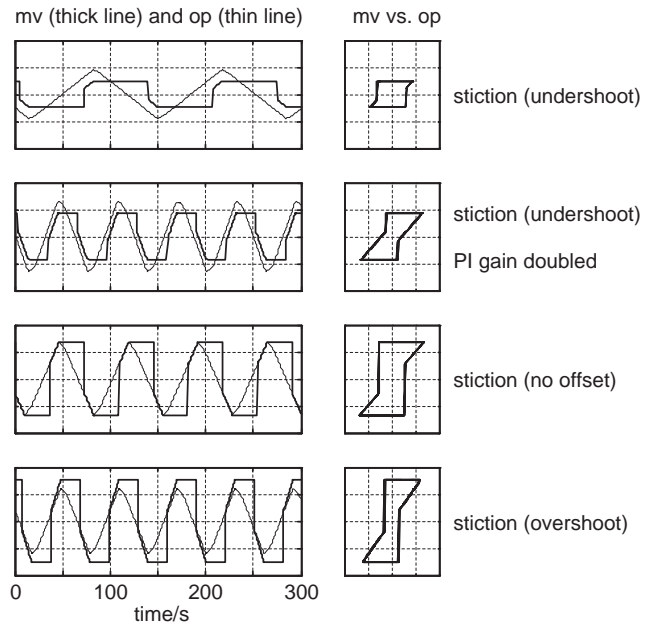


Fig. 9. Closed-loop response of mechanistic model.

The presence of stiction ($F_s > F_c$) induces a limit cycle with a characteristic triangular shape in the controller output. Cycling occurs because an offset exists between the set point and the output of the control loop while the valve is stuck which is integrated by the PI controller to form a ramp. By the time the valve finally moves in response to the controller op signal, the actuator force has grown quite large and the valve moves quickly to a new position where it then sticks again. Thus, a self-limiting cycle is set up in the control loop.

If stiction and deadband are both present, then the period of the limit cycle oscillation can become very long. The combination $F_s = 1750$ N and $F_c = 1250$ N gave a period of 300 s while the combination $F_s = 1000$ N and $F_c = 400$ N had a period of about 140 s (top row, Fig. 9), in both cases much longer than the time constant of the controlled process or its cross-over frequency. The period of oscillation can also be influenced by altering the controller gain. If the gain is increased the linear ramps of the controller output signal are steeper, the actuator force moves through the deadband more quickly and the period of the limit cycle becomes shorter (second row, Fig. 9). The technique of changing the controller gain is used by industrial control engineers to test the hypothesis of a limit cycle induced by valve nonlinearity while the plant is still running in closed loop.

In the pure stick-slip or stiction with no offset case shown in the third row of Fig. 9, the Coulomb friction is negligible and the oscillation period is shorter because there is no deadband. The bottom row in Fig. 9 shows that miscalibration causes an overshoot in closed loop.

5. Data-driven model of valve stiction

The proposed data-driven model has parameters that can be directly related to plant data and it produces the same behaviour as the physical model. The model needs only an input signal and the specification of deadband plus stickband and slip jump. It overcomes the main disadvantages of physical modelling of a control valve, namely that it requires the knowledge of the mass of the moving parts of the actuator, spring constant, and the friction forces. The effect of the change of these parameters cannot be determined easily analytically because the relationship between the values of the parameters and the observation of the deadband/stickband as a percentage of valve travel is not straightforward. In a data-driven model, the parameters are easy to choose and the effects of these parameter change are simple to realize.

5.1. Model formulation

The valve sticks only when it is at rest or it is changing its direction. When the valve changes its direction, it comes to rest momentarily. Once the valve overcomes stiction, it starts moving and may keep on moving for sometime depending on how much stiction is present in the valve. In this moving phase, it suffers only dynamic friction, which may be smaller than the static friction. It continues to do so until its velocity is again very close to zero or it changes its direction.

In the process industry, stiction is generally measured as a % of the valve travel or the span of the control signal (Gerry & Ruel, 2001). For example, a 2% stiction means that when the valve gets stuck it will start moving only after the cumulative change of its control signal is greater than or equal to 2%. If the range of the control signal is 4–20 mA, then a 2% stiction means that a change of the control signal less than 0.32 mA in magnitude will not be able to move the valve.

In our modelling approach, the control signal has been translated to the percentage of valve travel with the help of a linear look-up table. In order to handle stochastic inputs, the model requires the implementation of a PI(D) controller including its filter under a full industrial specification environment. Alternatively, an exponentially weighted moving average (EWMA) filter placed right in front of the stiction model can be used to reduce the effect of noise. This practice is very much consistent with industrial practices. The model consists of two parameters—namely the size of deadband plus stickband S (specified in the input axis) and slip jump J (specified on the output axis). Note that the term ' S ' contains both the deadband and stickband. Fig. 10 summarizes the model algorithm, which can be described as:

- First, the controller output (mA) is provided to the look-up table where it is converted to valve travel %.

- If this is less than 0 or more than 100, the valve is saturated (i.e., fully closed or fully open).
- If the signal is within 0–100% range, the algorithm calculates the slope of the controller output signal.
- Then the change of the direction of the slope of the input signal is taken into consideration. If the 'sign' of the slope changes or remains zero for two consecutive instants, the valve is assumed to be stuck and does not move. The 'sign' function of the slope gives the following
 - If the slope of input signal is positive, the sign (slope) returns '+1'.
 - If the slope of input signal is negative, the sign(slope) returns '-1'.
 - If the slope of input signal is zero, the sign (slope) returns '0'.

Therefore, when sign(slope) changes from '+1' to '-1' or vice versa, it means the direction of the input signal has been changed and the valve is in the beginning of its stick position (points A and E in Fig. 2). The algorithm detects stick position of the valve at this point. Now, the valve may stick again while travelling in the same direction (opening or closing direction) only if the input signal to the valve does not change or remains constant for two consecutive instants, which is usually uncommon in practice. For this situation, the sign(-slope) changes to '0' from '+1' or '-1' and vice versa. The algorithm again detects here the stick position of the valve in the moving phase and this stuck condition is denoted with the indicator variable ' $I' = 1$ '. The value of the input signal when the valve gets stuck is denoted as x_{ss} . This value of x_{ss} is kept in memory and does not change until the valve gets stuck again. The cumulative change of input signal to the model is calculated from the deviation of the input signal from x_{ss} .

- For the case when the input signal changes its direction (i.e., the sign(slope) changes from '+1' to '-1' or vice versa), if the cumulative change of the input signal is more than the amount of the deadband plus stickband (S), the valve slips and starts moving.
- For the case when the input signal does not change direction (i.e., the sign(slope) changes from '+1' or '-1' to zero, or vice versa), if the cumulative changes of the input signal is more than the amount of the stickband (J), the valve slips and starts moving. Note that this takes care of the case when the valve sticks again while travelling in the same direction (EnTech, 1998; Kano, Maruta, Kugemoto, & Shimizu, 2004).
- The output is calculated using the equation:

$$\text{output} = \text{input} - \text{sign}(\text{slope})(S - J)/2 \quad (4)$$

and depends on the type of stiction present in the valve. It can be described as follows:

- *Deadband*: If $J = 0$, it represents pure deadband case without any slip jump.

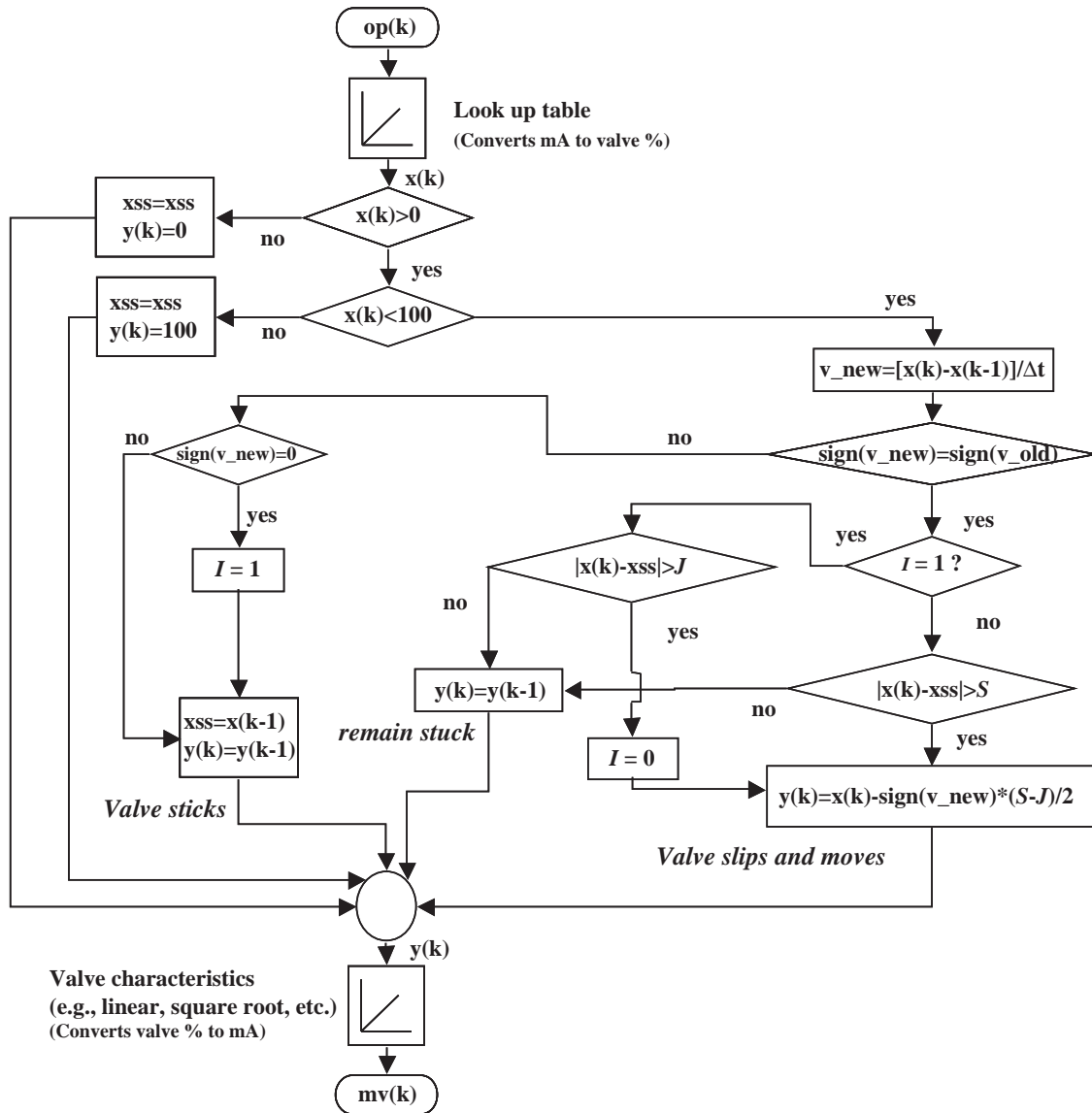


Fig. 10. Signal and logic flow chart for the data-driven stiction model.

- *Stiction (undershoot)*: If $J < S$, the valve output can never reach the valve input. There is always some offset. This represents the undershoot case of stiction.
- *Stiction (no offset)*: If $J = S$, the algorithm produces pure stick-slip behaviour. There is no offset between the input and output. Once the valve overcomes stiction, valve output tracks the valve input exactly. This is the well-known “stick-slip case”.
- *Stiction (overshoot)*: If $J > S$, the valve output overshoots the valve input due to excessive stiction. This is termed as overshoot case of stiction.

Recall that J is an output (y -axis) quantity. Also, the magnitude of the slope between input and output is 1.

- The parameter J signifies the slip jump start of the control valve immediately after it overcomes the deadband plus stickband. It accounts for the offset between the valve input and output signals.
- Finally, the output is again converted back to a mA signal using a look-up table based on the valve characteristics such as linear, equal percentage or square root, and the new valve position is reported.

5.2. Open-loop response of the model under a sinusoidal input

Fig. 11 shows the open-loop behaviour of the new data-driven stiction model in the presence of various types of stiction. Plots in the left panel show the time trend of the valve input op (thin solid line) and the

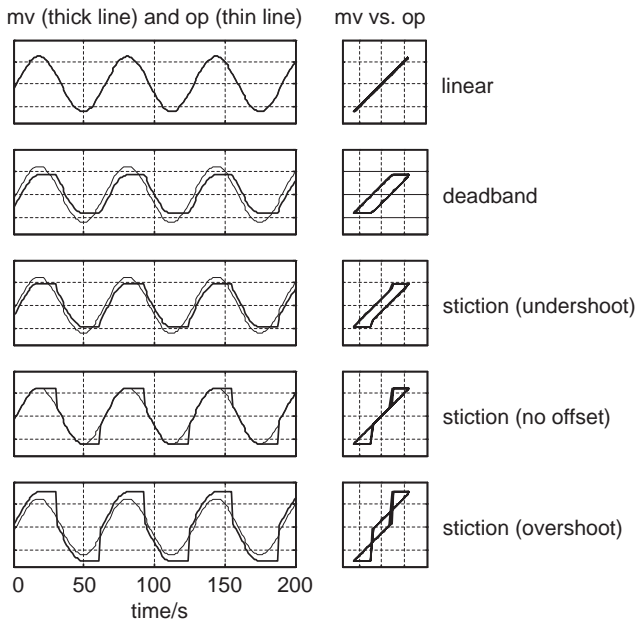


Fig. 11. Open-loop simulation results of the data-driven stiction model.

output mv (thick solid line). The right panel shows the input–output behaviour of the valve on an X – Y plot.

- The first row shows the case of a linear valve without stiction.
- The second row corresponds to pure deadband without any slip jump, i.e., $J = 0$. Note that for this case, the magnitude of stickband is zero and deadband itself equals to ‘ S ’.
- The third row shows the undershoot case of a sticky valve where $J < S$. This case is illustrated in the first and second examples of industrial control loops. In this case, the valve output can never reach the valve input. There is always some offset.
- The fourth row represents pure stick-slip behaviour. There is no offset between the input and output. Once the valve overcomes stiction, valve output tracks the valve input accurately.
- In the fifth row, the valve output overshoots the desired set position or the valve input due to excessive stiction. This is termed as overshoot case of stiction.

In reality, a composite of these stiction phenomena may be observed. Although this model is not directly based on the dynamics of the valve, the strength of the model is that it is very simple to use for the purpose of simulation and can quantify stiction as a percentage of valve travel or span of input signal. Also, the parameters used in this model are easy to understand, realize and relate to real stiction behaviour. Though this is an empirical model and not based on physics, it is observed

that this model can correctly reproduce the behaviour of the physics-based stiction model. This can be observed by comparing Fig. 12 with Fig. 9. The data for these figures are obtained from the simulation of the same process and controller, but with different stiction models. The notable features are:

- For a first-order plus time delay model, both stiction models show no limit cycle for the case of pure deadband. Both models show that for limit cycles a certain amount of slip jump is required.
- Both models show limit cycle even in the presence of pure deadband if the process contains an integrator in closed loop.
- Both models produce identical results for other cases of stiction.

The open-loop simulation results for both models look very similar in Figs. 11 and 8. Note that a one-to-one comparison of these figures cannot be made because there is no direct one-to-one relation among the parameters of the empirical data-driven model and that of the physics-based model.

5.3. Closed-loop behaviour of the model

Closed loop behaviour of the stiction model has been studied for two different cases, namely, a concentration loop and a level loop. The concentration loop has slow dynamics with a large dead time. The level loop has only an integrator. The transfer functions, controllers and parameters used in simulation are shown in Table 3. The magnitudes of S and J are specified as a percentage (%) of valve input span and output span, respectively. Results are discussed in a separate section for each of the loops.

5.3.1. Concentration loop

The transfer function model for this loop was obtained from (Horch & Isaksson, 1998). This transfer function with a PI controller in a feedback closed-loop configuration was used for the simulation. Steady-state results of the simulation for different stiction cases are presented in Figs. 12 and 13. In both figures, thin lines are the controller output. The triangular shape of the time trend of controller output is one of the characteristics of stiction (Horch, 2000). Note that Fig. 12 looks really similar to Fig. 9, for the same process and controller, but with the physics-based valve model. In all cases, the presence of stiction causes limit cycling of the process output. In the absence of stiction, there are no limit cycles, which is shown in the first row of Fig. 12. The presence of pure deadband also does not produce a limit cycle. It only adds dead time to the process. This conforms with the findings of McMillan (1995) and Piipponen (1996), where they clearly stated that the presence of pure deadband only adds dead time to the

Table 3
Process model and controller transfer function, and parameter values used for closed-loop simulation of the data-driven model

Loop type	Process	Controller	Stiction							
			Deadband		Undershoot		No offset		Overshoot	
			S	J	S	J	S	J	S	J
Concentration	$\frac{3e^{-10s}}{10s+1}$	$0.2\left(\frac{10s+1}{10s}\right)$	5	0	5	2	5	5	5	7
Level	$\frac{1}{s}$	$0.4\left(\frac{2s+1}{2s}\right)$	3	0	3	3.5	3	3	3	4.5

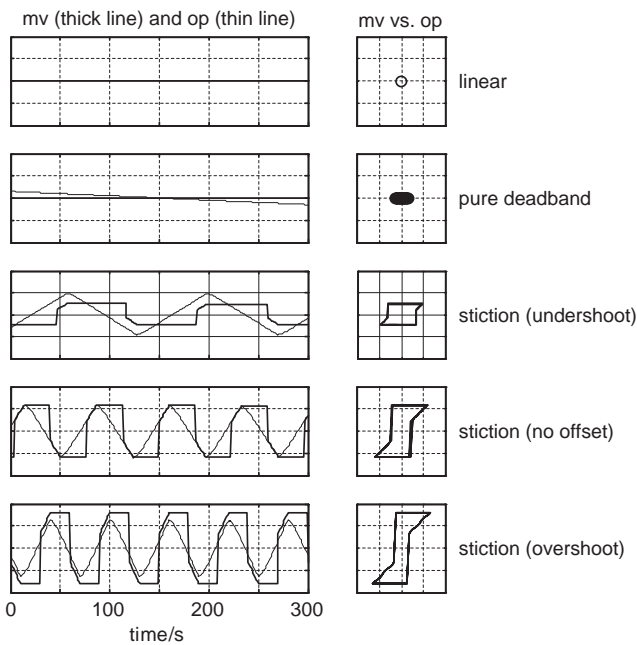


Fig. 12. Closed-loop simulation results of a concentration loop in the presence of the data-driven stiction model.

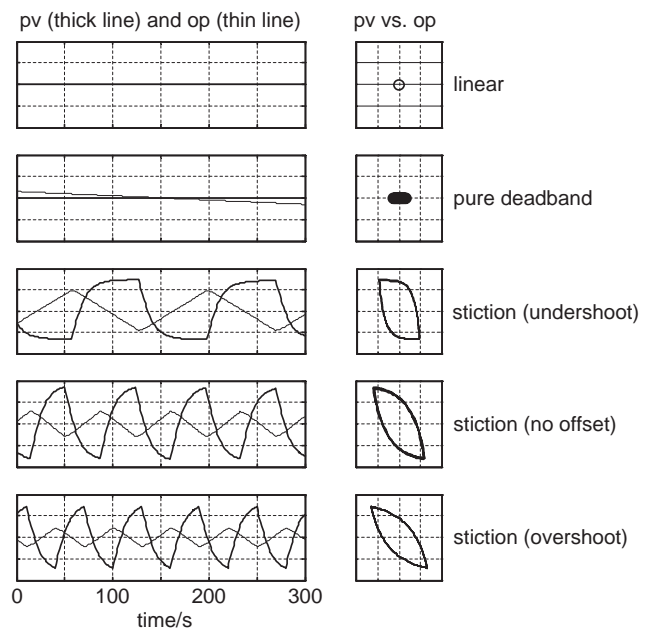


Fig. 13. Closed-loop simulation results of a concentration loop in the presence of the data-driven stiction model.

process and the presence of deadband together with an integrator produces a limit cycle (discussed further in level control loop case). Fig. 12 shows the controller output (*op*) and valve position (*mv*). Mapping of *mv* vs. *op* clearly shows the stiction phenomena in the valve. It is common practice to use a mapping of *pv* vs. *op* for valve diagnosis (see Fig. 13). However, in this case, such a mapping only shows elliptical loops with sharp turn around points. The reason is that the *pv*–*op* map captures not only the nonlinear valve characteristic but also the dynamics of the process, $G(s)$, which in this case is a first-order lag plus deadtime. Therefore, if the valve position data are available, one should plot valve position (*mv*) against the controller output (*op*). The *pv*–*op* maps should be used with caution except for in-liquid flow low loops where the flow through the valve (*pv*) can be taken to be proportional to valve opening (*mv*).

5.3.2. A level control loop

The closed-loop simulation of the stiction model using only an integrator as the process was performed to investigate the behaviour of a typical level loop in the presence of valve stiction. Results are shown in Figs. 14 and 15. The second row in both Figs. 14 and 15 shows that the deadband can produce oscillations. Again, it is observed that if there is an integrator in the process dynamics, then even a pure deadband can produce limit cycles, otherwise the cycle decays to zero. The *mv*–*op* mappings clearly show the various cases of valve stiction. The *pv*–*op* plots show elliptical loops with sharp turn around. Therefore, as was noted also in an earlier example, the *pv*–*op* map is not a very reliable diagnostic for valve faults in a level loop. A diagnostic technique, developed by the authors (Choudhury, Shah, & Thornhill, 2004), based on higher order statistical analysis of data is able to detect and diagnose the presence of stiction in control loops.

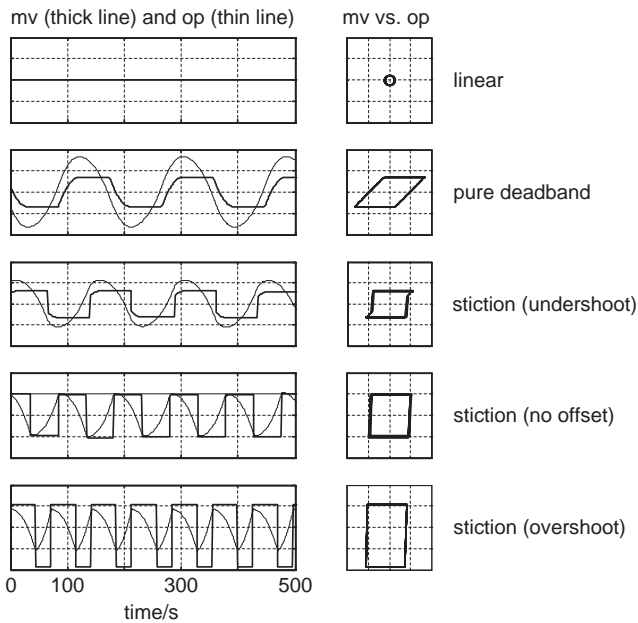


Fig. 14. Closed-loop simulation results of a level loop in the presence of the data-driven stiction model.

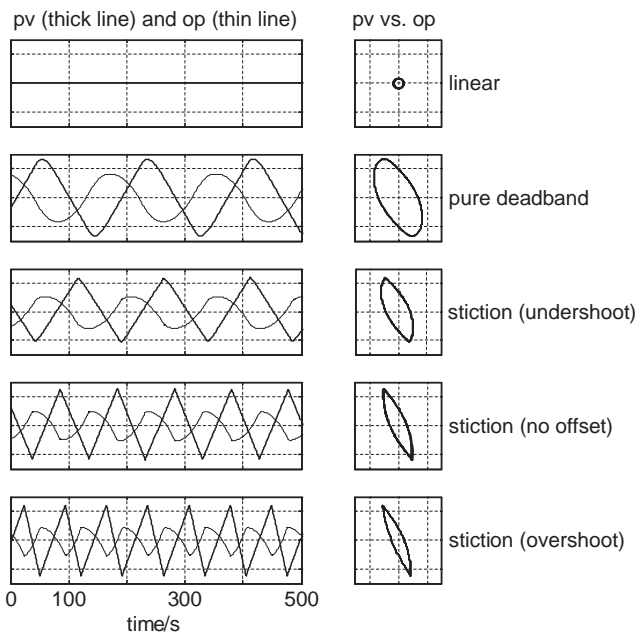


Fig. 15. Closed-loop simulation results of a level loop in the presence of the data-driven model.

6. Describing function analysis

6.1. Introduction

A non-linear actuator with a stiction characteristic may cause limit cycling in a control loop. Further insights into the behaviour of such systems may be achieved through a describing function analysis (Cook, 1986). The nonlinearity is modelled by a nonlinear gain N . The assumptions inherent in the approximation are

that there are periodic signals present in the system and that the controlled system is low pass and responds principally to the fundamental Fourier component. The conditions for oscillation in a negative feedback loop arise when the loop gain is -1 :

$$G_0(i\omega) = -\frac{1}{N(X_m)} \quad (5)$$

where $G_0(i\omega)$ is the open-loop frequency response which includes the controlled system and the proportional plus integral controller, and $N(X_m)$ is the describing function which depends on the magnitude of the controller output X_m . When the condition $G_0(i\omega) = -1/N(X_m)$ is met, the system will spontaneously oscillate with a limit cycle. The variation of the quantity $-1/N(X_m)$ with signal amplitude means that signals initially present in the loop as noise can grow until they are big enough to satisfy the equality and hence provide a self-starting oscillation. The solution to the complex equation $G_0(i\omega) = -1/N(X_m)$, if one exists, may be found graphically by superposing plots of $G_0(i\omega)$ and $-1/N$ on the same set of axes.

The aims of describing function analysis are to gain insights into the simulation results and industrial observations presented in the paper.

6.2. An expression for the describing function

The describing function of a nonlinearity is

$$N = \frac{Y_f}{X} \quad (6)$$

where X is a harmonic input to the nonlinearity of angular frequency ω_0 and Y_f is the fundamental Fourier component angular frequency ω_0 of the output from the nonlinearity. Thus, a Fourier analysis is needed on the output signals shown as bold lines in Fig. 11. The quantity N depends upon the magnitude of the input X_m . N is complex for the stiction nonlinearity because the output waveform has a phase lag compared to the input. The describing function is derived in the appendix, where it is shown that:

$$N = -\frac{1}{\pi X_m} (A - iB), \quad (7)$$

where

$$A = \frac{X_m}{2} \sin 2\phi - 2X_m \cos \phi - X_m \left(\frac{\pi}{2} + \phi \right) + 2(S - J) \cos \phi, \quad (8)$$

$$B = -3\frac{X_m}{2} + \frac{X_m}{2} \cos 2\phi + 2X_m \sin \phi - 2(S - J) \sin \phi, \quad (9)$$

$$\phi = \sin^{-1} \left(\frac{X_m - S}{X_m} \right). \quad (10)$$

6.3. Asymptotes of the describing function

Fig. 10 indicates that there is no output from the nonlinearity if $X_m < S/2$. Therefore, the two extreme cases are when $X_m = S/2$ and $X_m \gg S$.

When $X_m \gg S$, the effects of the deadband and slip-jump are negligible and the nonlinearity in Fig. 17 becomes a straight line at 45° . The output is in phase with the input and $N = 1$. Thus, $-1/N(X_m) = -1$ when $X_m \gg S$.

In the limit when $X_m \rightarrow S/2$, the output is as shown in Fig. 16. The left-hand plot shows the output for a slip-jump with no deadband ($S = J$), while the right-hand plot shows a magnified plot of a deadband with no slip jump ($J = 0$). In both cases, the output lags the input by one quarter of a cycle. The output is a square wave of magnitude X_m in the $S = J$ case and the describing function is $N = \frac{4}{\pi} e^{-i\pi/2}$. For the deadband with no slip jump ($J = 0$) case, the output magnitude becomes very small. The describing function is $N = \epsilon e^{-i\pi/2}$ where $\epsilon \rightarrow 0$ as $X_m \rightarrow S/2$. Appendix A provides detailed calculations of these results and also shows for the general case that

the describing function limit when $X_m = S/2$ is

$$N = \frac{4}{\pi} \times \frac{J}{2} e^{-i\pi/2}. \tag{11}$$

6.4. Insights gained from the describing function

Fig. 17 shows graphical solutions to the limit cycle equation $G_0(i\omega) = -1/N(X_m)$ for the composition control loop (left panel) and level control loop (right panel) presented earlier. The describing function is parameterized by X_m and the open-loop frequency response function of the controller and controlled system is parameterized by ω . Both systems are closed-loop stable and thus intersect the negative real axis between 0 and -1 . The plots explain the behaviour observed in simulation.

It is clear from the left-hand panel of Fig. 17 that there will be a limit cycle for the composition control loop if a slip-jump is present. The slip-jump forces the $-1/N$ curve onto the negative imaginary axis in the $X_m = S/2$ limit. Thus, the frequency response curve of

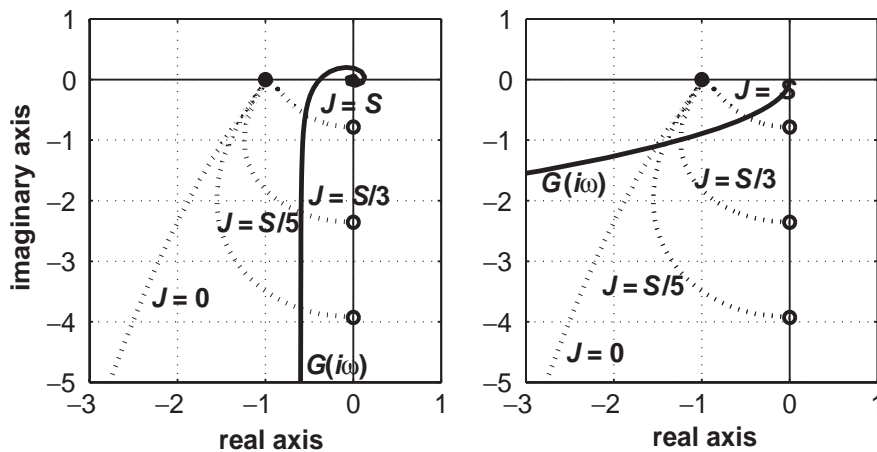


Fig. 17. Graphical solutions for limit cycle oscillations. Left panel: composition control loop. Right panel: level control loop. Dotted lines are the $-1/N$ curves and the solid line is the frequency response function.

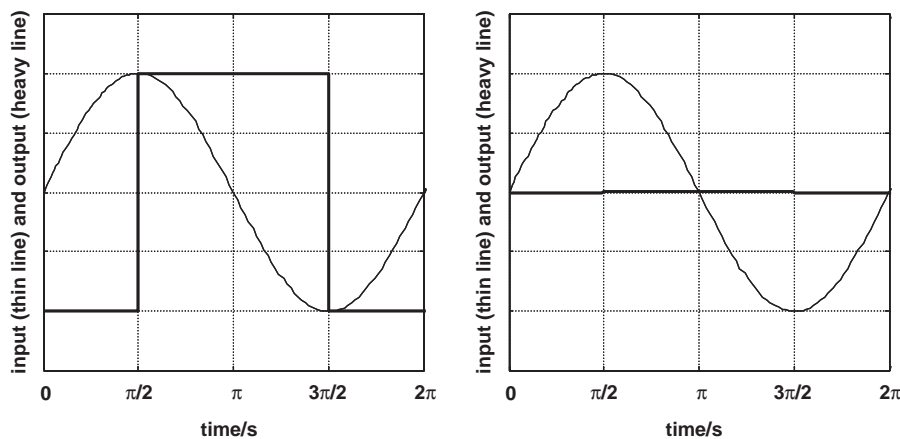


Fig. 16. Input (thin line) and output (heavy line) time trends for the limiting case as $X_m = S/2$. Left panel: slip-jump only with $S = J$. Right panel: deadband only with $J = 0$. The output in the left plot has been magnified for visualization; its amplitude becomes zero as X_m approaches $S/2$.

the FOPTD composition loop and its proportional plus integral controller is guaranteed to intersect with the describing function because the integral action means open-loop phase is always below $-\pi/2$ (i.e. it is in the third quadrant of the complex plane at low frequency).

The figure also shows the $-1/N$ curve for the deadband limit cycle. In the $X_m = S/2$ limit, the curve becomes large, negative and imaginary. The composition loop does not have a limit cycle if the nonlinearity is a pure deadband, because the frequency response curve does not intersect the $-1/N$ curve. The lack of a limit cycle in this case has been noted by other authors (McMillan, 1995; Piipponen, 1996).

The level loop with proportional plus integral control has a frequency response for which the phase becomes $-\pi$ at low frequency. The right-hand panel of Fig. 17 shows that it will intersect the $-1/N$ curves for the slip-jump cases and also for the pure deadband case. Therefore, a valve with a deadband and no slip-jump can cause a limit cycle oscillation for an integrating process with a P+I controller. The frequency of oscillation is higher and the period of oscillation shorter when the slip-jump is present because the $-1/N$ curves with the slip-jump intersect the frequency response curve at higher frequencies than the $-1/N$ curve for the deadband.

The above insights from describing function analysis indicate that for online compensation of stiction it can be beneficial to use a PI controller where the integral action has a variable strength (Gerry and Ruel, 2001). If absolute error ($sp - pv$) is smaller than some value, then take out the integral action, otherwise use it. For example, if error is less than some value (say, x), then $k_i = 0$, if not, $k_i = \text{normal value}$, where k_i is the integral constant of the controller. Using this method, when the valve is within the stiction band, the integral is missing from the controller. The controller output will not integrate having the end of removing the stiction cycle from the loop. Alternatives are special control algorithms that identify and compensate for the friction characteristics of the valve, for instance as described in Armstrong-Hélouvy et al. (1994); Hatipoglu and Ozguner (1998); Kayihan and Doyle III (2000); Hagglund (2002) and Tao, Chen, and Joshi (2002).

7. Conclusion

A generalized definition of valve stiction based on the investigation of the real plant data has been proposed. Since the physics-based model of stiction is difficult to use because of the requirement of knowledge of mass and forces, a simple yet powerful data-driven empirical stiction model has been developed. Both closed- and open-loop results have been presented and validated to show the capability of the model. It is recommended

that when using an X - Y plot to analyse valve problems one should use mv - op plot instead of pv - op .

Acknowledgements

Financial support to the first author in the form of a Canadian International Development Agency (CIDA) scholarship is gratefully acknowledged. The project has also been supported by the Natural Sciences and Engineering Research Council of Canada (NSERC), Matrikon Consulting Inc., and the Alberta Science and Research Authority (ASRA) in the form of an NSERC-Matrikon-ASRA Industrial Research Chair Program at the University of Alberta. The authors are also grateful to BP Oil Ltd. and Tech-Cominco Inc. in Trail, B.C., Canada, for permitting us to use their data.

Appendix A. Outline derivation of describing function

A.1. Expression for the output of the nonlinearity

The output from the stiction nonlinearity (e.g. the bold trend in the third panel of Fig. 11) is not analytic. It is convenient to consider a sine wave input with angular frequency of 1 rad s^{-1} and period 2π . The output then is

$$y(t) = \begin{cases} k \left(X_m \sin(t) - \frac{S-J}{2} \right), & 0 \leq t \leq \frac{\pi}{2}, \\ k \left(X_m - \frac{S-J}{2} \right), & \frac{\pi}{2} \leq t \leq \pi - \phi, \\ k \left(X_m \sin(t) + \frac{S-J}{2} \right), & \pi - \phi \leq t \leq \frac{3\pi}{2}, \\ k \left(-X_m + \frac{S-J}{2} \right), & \frac{3\pi}{2} \leq t \leq 2\pi - \phi, \\ k \left(X_m \sin(t) - \frac{S-J}{2} \right), & 2\pi - \phi \leq t \leq 2\pi, \end{cases}$$

where X_m is the amplitude of the input sine wave, S is the deadband plus stickband, J is the slip-jump, $\phi = \sin^{-1}((X_m - S)/X_m)$ and k is the slope of the input-output characteristic in the moving phase ($k = 1$ is assumed for a valve).

A.2. Evaluation of the fundamental Fourier component

The fundamental component of the complex Fourier series is

$$\frac{1}{2\pi} \int_{t=0}^{2\pi} y(t) e^{-it} dt$$

where, after substitution of $\sin(t) = \frac{1}{2i}(e^{-it} - e^{-it})$:

$$\begin{aligned} & \int_{t=0}^{2\pi} y(t)e^{-it} dt \\ &= \int_{t=0}^{\pi/2} k \left(\frac{X_m}{2i}(e^{it} - e^{-it}) - \frac{S-J}{2} \right) e^{-it} dt \\ &+ \int_{t=\pi/2}^{\pi-\phi} k \left(X_m - \frac{S-J}{2} \right) e^{-it} dt \\ &+ \int_{t=\pi-\phi}^{3\pi/2} k \left(\frac{X_m}{2i}(e^{it} - e^{-it}) + \frac{S-J}{2} \right) e^{-it} dt \\ &+ \int_{t=3\pi/2}^{2\pi-\phi} k \left(-X_m + \frac{S-J}{2} \right) e^{-it} dt \\ &+ \int_{t=2\pi-\phi}^{2\pi} k \left(\frac{X_m}{2i}(e^{it} - e^{-it}) - \frac{S-J}{2} \right) \\ &\times e^{-it} dt. \end{aligned} \tag{A.1}$$

Writing it compactly:

$$\begin{aligned} & \int_{t=0}^{2\pi} y(t)e^{-it} dt = T1 + T2 + T3 + T4 + T5, \\ & \text{where } T1 = \int_{t=0}^{\pi/2} k \left(\frac{X_m}{2i}(e^{it} - e^{-it}) - \frac{S-J}{2} \right) e^{-it} dt, \text{ and so on.} \\ & \text{Evaluation term by term gives:} \\ & T1 = \frac{k}{2}(X_m - S + J) + ik \left(\frac{S-J}{2} - \frac{\pi}{4} \right), \\ & T2 = -k \left(X_m - \frac{S-J}{2} \right) (1 - \sin \phi) - ik \\ & \quad \times \left(X_m - \frac{S-J}{2} \right) \cos \phi, \\ & T3 = k \left(\frac{X_m}{4}(1 + \cos 2\phi) - \frac{S-J}{2}(1 + \sin \phi) \right) \\ & \quad + ik \left(\frac{X_m}{4} \sin 2\phi - \frac{X_m}{2} \left(\frac{\pi}{2} + \phi \right) + \frac{S-J}{2} \cos \phi \right), \\ & T4 = -k \left(X_m - \frac{S-J}{2} \right) (1 - \sin \phi) - ik \\ & \quad \times \left(X_m - \frac{S-J}{2} \right) \cos \phi, \\ & T5 = -k \left(\frac{X_m}{4}(1 - \cos 2\phi) + \frac{S-J}{2} \sin \phi \right) \\ & \quad - ik \left(\frac{X_m \phi}{2} + \frac{d}{2} - \frac{X_m a}{4} \sin 2\phi - \frac{S-J}{2} \cos \phi \right). \end{aligned} \tag{A.2}$$

Collecting terms gives the wanted fundamental Fourier component of the output:

$$\frac{1}{2\pi} \int_{t=0}^{2\pi} y(t)e^{-it} dt = \frac{1}{2\pi} (B + iA),$$

where

$$\begin{aligned} A &= \left(k \frac{X_m}{2} \sin 2\phi - 2kX_m \cos \phi \right. \\ &\quad \left. - kX_m \left(\frac{\pi}{2} + \phi \right) + 2k(S - J) \cos \phi \right) \end{aligned}$$

and

$$\begin{aligned} B &= -3k \frac{X_m}{2} + k \frac{X_m}{2} \cos 2\phi \\ &\quad + 2kX_m \sin \phi - 2k(S - J) \sin \phi. \end{aligned}$$

The fundamental component of the complex Fourier series of the input sine wave is $X_m/2i$. Therefore, the describing function is

$$N = \frac{B + iA}{2\pi} \times \frac{2i}{X_m} = -\frac{1}{\pi X_m} (A - iB).$$

A.3. Evaluation of limiting cases

There is no output from the nonlinearity when $X_m < S/2$. The limiting cases considered are therefore $X_m = S/2$ and $X_m \gg S$.

When $X_m \gg S$ then $\phi = \sin^{-1}((X_m - S)/X_m) = \frac{\pi}{2}$, $A = -k\pi X_m$, $B = 0$ and thus $N = k$. This result is to be expected because the influence of the stickband and jump are negligible when the input has a large amplitude and the output approximates a sinewave of magnitude kX_m . The slope of the moving phase for a valve with a deadband is $k = 1$ when the input and output to the nonlinearity are expressed as a percentage of full range. Therefore, for a valve with stiction, $N = 1$, when $X_m \gg S$.

When $X_m = S/2$ the result depends upon the magnitude of the slip-jump, J . For the case with no deadband ($S = J$), $\phi = -\frac{\pi}{2}$, $A = 0$, $B = -4kX_m$ and $N = -ik \frac{4}{\pi} = k \frac{4}{\pi} e^{-i\pi/2}$. For a valve with $k = 1$, $N = \frac{4}{\pi} e^{-i\pi/2}$. This result describes the situation where the output is a square wave of amplitude X_m lagging the input sine wave by one quarter of a cycle, as shown in Fig. 16.

For intermediate cases where both deadband and slip jump are present such that $|S - J| > 0$, then the $X_m = S/2$ limit gives $\phi = -\frac{\pi}{2}$, $A = 0$, $B = -2kJ$ and $N = -ik2J/\pi X_m = k2J/\pi X_m e^{-i\pi/2}$. For instance, if $J = S/2$ and $k = 1$ then the $X_m = S/2$ limit gives $N = \frac{2}{\pi} e^{-i\pi/2}$ and the output is a square wave of amplitude $X_m/2$ lagging the input sine wave by one quarter of a cycle.

When the nonlinearity has a deadband only and no slip-jump ($J = 0$), the describing function has a limit given by $N = \epsilon e^{-i\pi/2}$ where $\epsilon \rightarrow 0$ as $X_m \rightarrow S/2$.

References

Armstrong-Hélouvy, B., Dupont, P., & De Wit, C. C. (1994). A survey of models, analysis tools and compensation methods for the control of machines with friction. *Automatica*, 30(7), 1083–1138.

- Aubrun, C., Robert, M., & Cecchin, T. (1995). Fault detection in control loops. *Control Engineering Practice*, 3, 1441–1446.
- Bialkowski, W. L. (1992). Dreams vs. reality: A view from both sides of the gap. In *Control systems* (pp. 283–294). Canada: Whistler BC.
- Choudhury, M. A. A. S., Shah, S. L., & Thornhill, N. F. (2004). Diagnosis of poor control loop performance using higher order statistics. *Automatica* to appear in October 2004 issue.
- Cook, P. A. (1986). *Nonlinear dynamical systems*. Englewood cliffs, NJ: Prentice-Hall.
- Desborough, L., & Miller, R. (2002). Increasing customer value of industrial control performance monitoring—honeywell's experience. In *AIChE Symposium Series 2001*, No. 326 (pp. 172–192).
- Ender, D. (1993). Process control performance: Not as good as you think. *Control Engineering*, 40, 180–190.
- EnTech, . (1998). *EnTech control valve dynamic specification (version 3.0)*.
- Fisher-Rosemount, . (1999). *Control valve handbook*. Marshalltown, IW, USA: Fisher Controls International Inc.
- Fitzgerald, B. (1995). *Control valve for the chemical process industries*. New York: McGraw-Hill, Inc.
- Gerry, J., & Ruel, M. (2001). *How to measure and combat valve stiction online*. Instrumentation, systems and automated society. Houston, TX, USA. <http://www.expertune.com/articles/isa2001/StictionMR.htm>.
- Hagglund, T. (2002). A friction compensator for pneumatic control valves. *Journal of Process Control*, 12(8), 897–904.
- Hatipoglu, C., & Ozguner, U. (1998). Robust control of systems involving non-smooth nonlinearities using modified sliding manifolds. *American control conference* (pp. 2133–2137). Philadelphia, PA.
- Horch, A. (2000). Condition monitoring of control loops. *Ph.D. thesis*. Royal Institute of Technology, Stockholm, Sweden.
- Horch, A., & Isaksson, A. J. (1998). A method for detection of stiction in control valves. In *Proceedings of the IFAC workshop on line fault detection and supervision in the chemical process industry*. Session 4B. Lyon, France.
- Horch, A., Isaksson, A. J., & Forsman, K. (2000). Diagnosis and characterization of oscillations in process control loops. In *Proceedings of the control systems 2000* (pp. 161–165). Victoria, Canada.
- ISA Subcommittee SP75.05, (1979). Process instrumentation terminology. *Technical Report ANSI/ISA-S51.1-1979*. Instrument Society of America.
- Kano, M., Maruta, H., Kugemoto, H., & Shimizu, K. (2004). Practical model and detection algorithm for valve stiction. In *Proceedings of the Seventh IFAC-DYCOPS Symposium*, Boston, USA, to appear.
- Karnopp, D. (1985). Computer simulation of stick-slip friction in mechanical dynamical systems. *Journal of Dynamic Systems, Measurement, and Control*, 107, 100–103.
- Kayihan, A., & Doyle III, F. J. (2000). Friction compensation for a process control valve. *Control Engineering Practice*, 8, 799–812.
- Martin, G. D., Turpin, L. E., & Cline, R. P. (1991). Estimating control function benefits. *Hydrocarbon Processing*, 68–73.
- McMillan, G. K. (1995). Improve control valve response. *Chemical Engineering Progress: Measurement and Control*, 77–84.
- Miao, T., & Seborg, D.E. (1999). Automatic detection of excessively oscillatory control loops. In *Proceedings of the 1999 IEEE international conference on control applications*. Kohala Coast-Island of Hawai'i, Hawai'i, USA.
- Olsson, H. (1996). Control systems with friction. *Ph.D. thesis*. Lund Institute of Technology, Sweden.
- Paulonis, M. A., & Cox, J. W. (2003). A practical approach for large-scale controller performance assessment, diagnosis, and improvement. *Journal of Process Control*, 13(2), 155–168.
- Piipponen, J. (1996). Controlling processes with nonideal valves: Tuning of loops and selection of valves. In *Preprints of Control Systems '96* (pp. 179–186). Halifax, Nova Scotia, Canada.
- Qin, S. J. (1998). Control performance monitoring—a review and assessment. *Computers and Chemical Engineering*, 23, 173–186.
- Ruel, M. (2000). Stiction: The hidden menace. *Control Magazine*. <http://www.expertune.com/articles/RuelNov2000/stiction.html>.
- Sharif, M. A., & Grosvenor, R. I. (1998). Process plant condition monitoring and fault diagnosis. *Proceedings of the Institute of Mechanical Engineering*, 212(Part E), 13–30.
- Shinskey, F. G. (1990). How good are our controllers in absolute performance and robustness. *Measurement and Control*, 23, 114–120.
- Taha, O., Dumont, G. A., & Davies, M. S. (1996). Detection and diagnosis of oscillations in control loops. In *Proceedings of the 35th conference on decision and control*. Kobe, Japan.
- Tao, G., Chen, S., & Joshi, S. M. (2002). An adaptive control scheme for systems with unknown actuator failures. *Automatica*, 38, 1027–1034.
- Thornhill, N. F., Cox, J. W., & Paulonis, M. A. (2003b). Diagnosis of plant-wide oscillation through data-driven analysis and process understanding. *Control Engineering Practice*, 11(12), 1481–1490.
- Thornhill, N. F., Huang, B., & Zhang, H. (2003a). Detection of multiple oscillations in control loops. *Journal of Process Control*, 13, 91–100.
- Wallén, A. (1997). Valve diagnostics and automatic tuning. In *Proceedings of the American control conference* (pp. 2930–2934). Albuquerque, New Mexico.
- Whalen, B. R. (1983). *Basic instrumentation* (3rd ed.). Austin, TX: Petroleum Extension Service (PETEX).