

Through-Thickness Velocity Profile Measurements in an Elastohydrodynamic Contact

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This work presents for the first time through-thickness velocity profiles obtained in an EHL contact by photobleached imaging. The velocity profile was inferred by following the evolution of the shape of a photobleached plug formed through the thickness of the fluorescently-doped lubricant, oligomer polybutene (PB), in the contact when shear was applied. The proposed methodology was validated by successfully obtaining the expected linear profile with PB experiencing Couette flow. The methodology was then applied to PB in an EHL contact. The variation of the profiles within the contact area was also investigated.

The velocity profile of PB in an EHL contact severely deviates from the common linear assumption and exhibits inhomogeneous shear: three regions of varying shear rate have been observed. The phenomenon is shown to be neither due to thermal nor diffusion effects. PB also shows significant slip at the glass-liquid interface. The amount of slip varies with position in the contact. Possible causes, such as pressure induced viscosity enhancement, as well as the significance of the findings and the benefits of the technique are discussed.

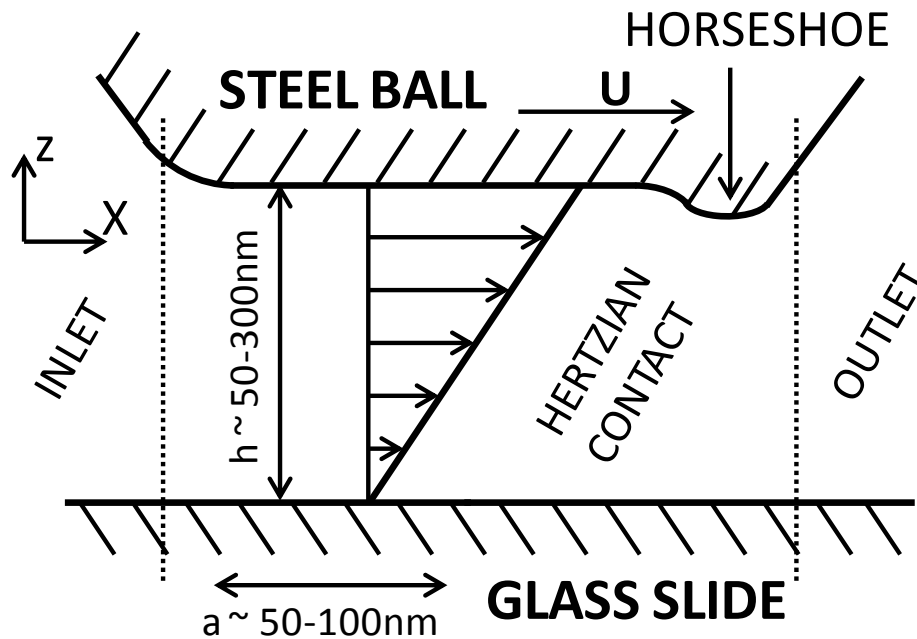
The linear velocity profile in an EHL contact is usually assumed for both the film thickness and friction predictions. The profile has however never been measured experimentally until now. This work enables the validation of conventional assumptions and the study of flow heterogeneity of lubricants in a contact. This facilitates an improved understanding of the rheology of confined lubricant and hence more accurate predictions of tribological properties.

EHL, velocity profile, lubricant flow, fluorescence, interfacial slip, thin film keywords 4-6

EHL:Elastohydrodynamic lubrication;PB:Polybutene

1 Introduction

Lubricants are frequently used in engineering systems to improve efficiency by manipulating their tribological performance. Operating conditions, including viscosity and film thickness, are chosen such that wear rate is minimized and friction can be controlled. The system operates in the elastohydrodynamic lubrication (EHL) regime when the normal stress applied to the contact is high enough such that elastic deformation of the rubbing surfaces occurs. The lubricant film thickness is typically in the sub-micrometer range and predictions of film thickness and friction are crucial for components designed to operate in the EHL regime. Numerical and theoretical expressions used for such purposes have been developed, with varying degrees of success. The derivation of these expressions requires the knowledge of the velocity profile of the fluid [1]. An EHL contact formed by two surfaces sliding relative to each other is shown in **Fig. 1**, together with the most commonly assumed velocity profile of the lubricant. This profile can be described using the Reynolds equation where, assuming the pressure gradient is small, the velocity of the lubricant should vary linearly through the thickness of the film [2]. It should be noted that if the assumed velocity profile is inappropriate, the predictions made based on this profile will be inaccurate. In this work, the goal is to develop a methodology capable of obtaining the through-thickness profile in an EHL contact. It will then be used to verify the universality of the linear velocity profile in a low speed, sliding tribological contact.



20

21 **Fig. 1** Schematic of an EHL contact. The surface of the steel ball is moving at a velocity U . A
22 lubricant film with mostly constant sub-micron thickness is created in the Hertzian contact. A
23 linear (Couette) velocity profile is indicated by arrows. A diminution of the film thickness in the
24 outlet is associated with a pressure spike.

25 The relatively high shear rate and pressure in an EHL contact cause severe changes in
26 the rheology of the lubricant [3]. It can become viscoelastic, and in some cases it may
27 even display glassy behaviour [4]. It has been shown that lubricants can solidify [5,6] at
28 pressures commonly encountered in EHL applications. These observations suggest that
29 the commonly assumed linear velocity profile may be inappropriate. Alternative models
30 describing the velocity profile have been suggested. Ehret et al. [7] assumed that the
31 lubricant was liquid near the two bounding surfaces, but that in the center of the film, it

1 flowed like a solid plug. This has been used to successfully explain some anomalous
2 behaviour seen in EHL with high viscosity lubricants. Meurisse and Espejel [8] argued
3 that there is adhesion between the lubricant and the surfaces at molecular scales. This
4 caused them to develop a theoretical framework, where the lubricant is separated into
5 layers of varying viscosity. The phenomenon is commonly referred to as inhomogeneous
6 shear and it describes a significant localized variation in rheology accompanied by a
7 change in the local shear rate and hence the velocity profile. Such changes could be
8 caused by localized changes in viscosity which in turn could be induced by heating,
9 pressure, shear or confinement. Inhomogeneous shear has been seen in shear flows of
10 foams [9], entangled polymers [10] and yield stress fluids [11]. There are two types of
11 inhomogeneous shear. Thermally induced shear inhomogeneity can appear when there
12 is a mismatch between the heat transfer rates at the two sliding surfaces. A temperature
13 gradient is then induced in the liquid, which causes a variation in viscosity and therefore
14 affects the velocity profile. Mechanically induced shear inhomogeneity, which can be
15 highly localised, occurs when lubricant solidifies and effectively yields under the applied
16 stresses. Bair et al. have developed a high pressure rheometer [12], capable of
17 measuring the rheology of lubricants at conditions comparable to those encountered in
18 EHL. Using particle image velocimetry, they showed that pressure can induce mechanical
19 inhomogeneous shear, although the film thickness studied was much larger than that in
20 an EHL contact. Molecular dynamics has been used to further probe the nature of
21 pressurised shear flows. Heyes et al. [13] studied a pressurised simple Leonard-Jones
22 liquid in plane Couette flow. They showed that the liquid does indeed show significant
23 inhomogeneous shear when conditions similar to EHL are applied. Furthermore the shear
24 inhomogeneity is not always restricted to the surfaces, but in some cases it occurs at the
25 center of the film.

26 Apart from the existence of inhomogeneous shear, Heyes et al. [13] also showed that the
27 particular liquid simulated can slip at the wall. It is well known that polymers [14] and non-
28 wetting liquids [15] slip when subject to a sufficient shear stress. Given the high stresses
29 in an EHL contact, slip is quite likely. Choo et al. [16] showed that a large reduction in
30 friction was observed when a glass surface was treated with a hydrophobic coating. They
31 explained this reduction by claiming that the lubricant must slip at the liquid-solid
32 interface. Since friction was the only measurement made, no comment on the variation of
33 slip within the contact could be made, and an average property was assumed. Guo et al.
34 [17] estimated the slip length of polybutene (PB) in an EHL point contact. A dimple in the
35 film thickness was created by instantaneous loading of the contact. By assuming a linear
36 velocity profile, the motion of the dimple through the contact could be related to the slip
37 length. Again, the slip length found was inferred and was assumed not to vary with
38 position in the contact.

39 The velocity profile in a lubricated tribological contact has not yet been measured
40 experimentally. An experimental scheme that allows the continuous mapping of fluid
41 velocities in the thickness direction is necessary to obtain such information. This is
42 challenging, especially in EHL where films commonly have submicron thickness. The use
43 of velocimetry at scales below the diffraction limit is a difficult task. Nevertheless, some
44 contemporary techniques are capable of performing such measurements. Stimulated
45 emission depletion, combined with photobleaching anemometry [18], has been used to
46 determine the velocity of pressure-driven flow of water through a nanocapillary. The
47 measurements were done in steps of 35 nm through the film thickness. Thus only a few
48 measurements could be performed through the thickness of a 100 nm fluid film typical to
49 EHL using this approach. Nano-particle image velocimetry techniques [19,20] have been
50 implemented yielding through-thickness resolutions of about 50nm, also making them
51 inappropriate for use in EHL velocity profile determination. The necessary size of
52 nanoparticles used (~20 nm) is also prohibitively large due to particle-wall interactions

1 [21]. Smaller probes, such as single molecules, could potentially be used to overcome
2 this problem. However, they also face the challenge of the diffraction limit and offer
3 similar thickness-resolution limitations as micro- and nano-particle image velocimetry with
4 a significant reduction in the signal to noise ratio. Atomic force microscopy [22] can
5 achieve a higher spatial resolution, but its application in an EHL contact is difficult due to
6 access limitations. Fluorescence imaging has been used to study the flow in a contact
7 under rolling conditions [23]. In [23], fluorescently doped lubricant was added at the inlet
8 of the contact which was originally flooded with undoped lubricant. The motion of doped
9 lubricant was monitored as it moved across the contact. It was shown that the pressure-
10 driven flow is negligible and that there is an increase in relative velocity close to the
11 location with minimum film thickness. While this result is unique, as no other direct
12 velocity measurement for an EHL contact exists, it is an average velocity and reveals no
13 information about the variation of velocity through the film thickness. Therefore, the
14 technique would not be appropriate for a sliding contact.

15 Photobleached imaging has been used to study slip [24–26], as well as diffusion in
16 electro-osmotic flows [27]. The technique shows many similarities to caged-fluorescence
17 imaging [28,29], which is part of the greater field of molecular tagging velocimetry. Most
18 recently, photobleaching has been used to measure dispersion and diffusion coefficients
19 in submicron geometries [30]. Using an extended analysis, it has also been used to
20 reconstruct the velocity profile in a gravity driven thick film with Poiseuille flow [31].
21 Ultimately, the geometry independence and the velocity reconstruction capability make
22 the technique a suitable candidate for velocimetry in an EHL contact.

23 In this paper, photobleached imaging is applied to determine the through-thickness
24 velocity profile in an EHL contact. A fluorescent dye is dissolved in a test lubricant and a
25 high intensity focussed laser is used to bleach a small column of fluid (photobleached
26 plug) within the lubricated, sliding contact. The movement and shape of this
27 photobleached plug is then monitored by using a low intensity laser to map the
28 fluorescence intensity of the lubricant film over time.

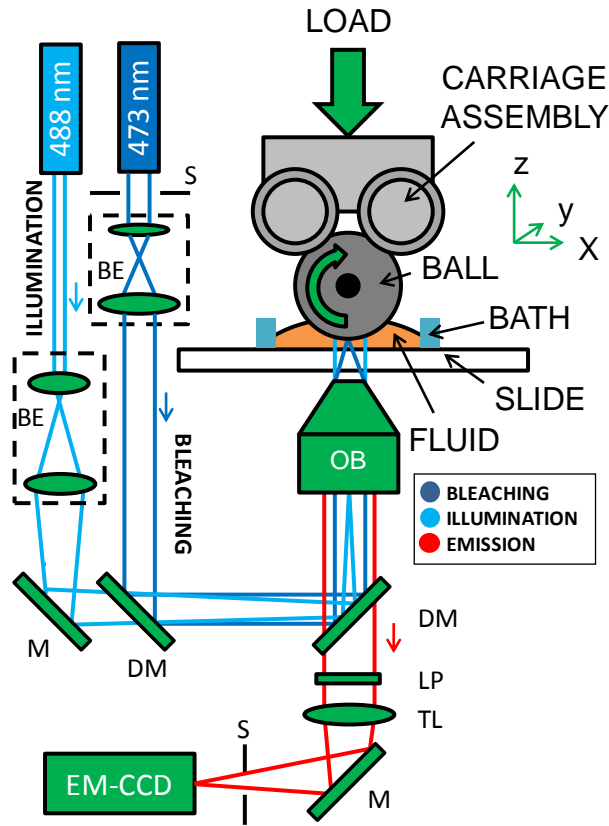
29 The aim of this paper is to demonstrate the ability of the proposed methodology to
30 measure the through-thickness velocity profile in a thin film, and to successfully apply the
31 technique to an EHL contact. This work focuses on obtaining the resolution of the
32 methodology and its sensitivity in detecting potential behavioural heterogeneities in the
33 contact. The effects of load and shear rate on the obtained results, while important, are a
34 topic for further study. First, the experimental setup is described. Then, the theoretical
35 framework used for analysing the data and reconstructing the velocity profile is defined. A
36 submicron plane Couette flow was used to validate the framework, and to show that the
37 correct velocity profile can be reconstructed. The technique was then applied to an EHL
38 point contact, showing that the velocity profile severely deviates from theoretical
39 predictions. The direction of the two-dimensional flow within the contact was determined,
40 showing how the liquid flowed around the contact, and how it was squeezed out due to
41 the pressure gradient. Finally, measurements were made at various positions in the
42 contact, where the slip length was extracted and compared to the pressure distribution.

43 **2 Experimental**

44 Through-thickness velocity profiles of a lubricant were obtained for two flow conditions: a
45 plane Couette flow created by shearing a thin film of liquid between parallel glass slides;
46 and an EHL point contact created by squeezing the lubricant between a rotating steel ball
47 and a stationary glass slide. The confined lubricant was doped with fluorescent dye and
48 was placed on top of an inverted microscope. Photobleached imaging was used to tag a
49 volume of the fluid. Observing and analysing the evolution of the shape of this tagged

1 volume as the lubricant was sheared allowed the reconstruction of the through-thickness
2 velocity profile.

3 2.1 Optical setup

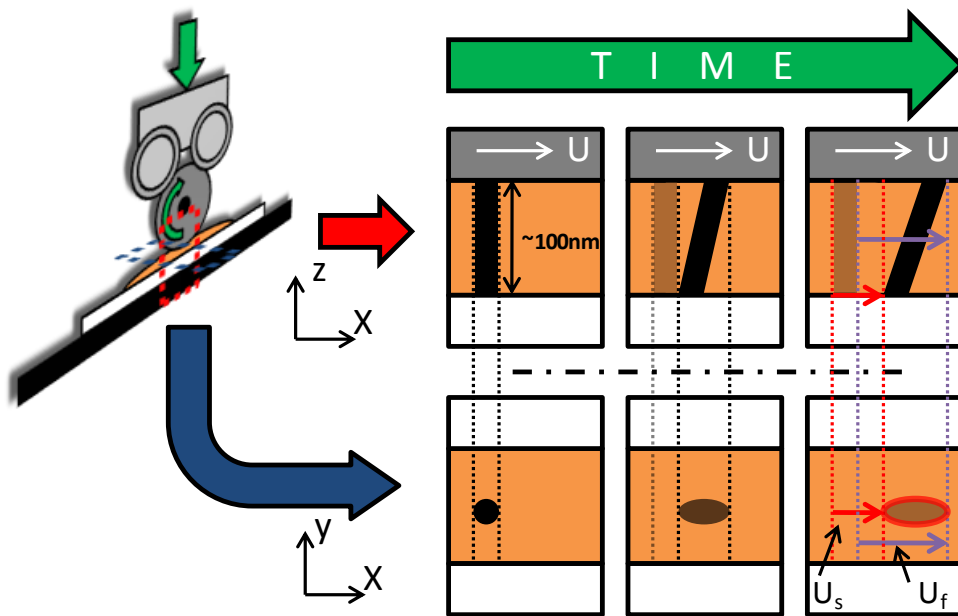


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5 **Fig. 2** Schematic of the photobleached imaging setup. BE: Beam expander, S: Shutter, M: Mirror,
6 DM: Dichroic mirror, OB: Objective, LP: Long pass filter, TL: Tube lens, EM-CCD: Electron
7 multiplying charge coupled device. The EHL contact setup is shown in the figure, which can be
8 replaced with parallel plate glass slides for plane Couette flow (not shown).

9 A schematic of the experimental setup is presented in **Fig. 2**. The sample was either a
10 film suspended between two glass slides (not shown), or a film created using a loaded
11 rotating ball (shown in **Fig. 2**). A Zeiss Axiovert 200M inverted microscope with a 10x
12 (0.25 NA), 20x (0.4 NA) or 50x (0.7 NA) objective was used for imaging. An Andor Ixon3
13 860 electron multiplying charged coupled device camera was used for acquisition.
14 Shutter heads and a shutter control box by Stanford Research Systems controlled the
15 bleach duration and protected the camera during bleaching.

16 Blue (473 nm at 20 mW on stage) and cyan (488 nm at 12 mW on stage) solid state
17 lasers, operating in TEM₀₀ mode, were used in this experiment for creating the tagged
18 volume for flow profile reconstruction through photobleaching and illuminating the
19 observation volume by laser excitation respectively. The power experienced by the
20 sample was around 40 mW and 24 mW from the blue and cyan lasers respectively due to
21 reflection at the steel surface. The cyan illumination beam was expanded and focused
22 onto the back of the objective resulting in epi-fluorescence illumination. Fluorescent
23 probes were dissolved into model lubricant. When illuminated by the cyan laser, the
24 lubricant emitted homogenous fluorescence. The blue laser was used as a bleaching
25 beam to create a small photobleached plug, in the form of a dark cylinder within the
26 fluorescent probe volume, through the thickness of the confined lubricant. This plug was

1 the tagged volume, whose evolution in shape was followed as the model lubricant was
 2 sheared. The plug had an inverse Gaussian intensity distribution in x- and y-directions,
 3 and a homogenous intensity distribution in z-direction. The blue laser passed through a
 4 shutter which controlled the time the lubricant was exposed to the bleach beam, and
 5 hence the diameter of the plug. After the blue laser was expanded and collimated using
 6 a beam expander, it rejoined the cyan laser through a dichroic beam splitter and was
 7 focused onto the sample stage. The fluorescence emission from the sample was
 8 collected through the objective and the dichroic mirror. It then passed through a long pass
 9 filter to remove the residual signal from the cyan laser. Finally, a tube lens focused the
 10 emission onto the chip of the camera. A shutter was installed just in front of the camera to
 11 protect it from the bleaching beam. For the 20x objective, the radius of the diffraction
 12 limited volume was $3.60\ \mu\text{m}$, and the depth of field was $1.52\ \mu\text{m}$. For the 50x objective,
 13 the radius of the diffraction limited volume was $1.50\ \mu\text{m}$, and the depth of field was 480
 14 nm . The film thickness was smaller than the depth of field of the objective for the
 15 experimental conditions used in this work, ensuring that a through-thickness
 16 photobleached plug was generated during this process.



17

18 Fig. 3 Schematic of the photobleached imaging setup with a sketch of the spatiotemporal
 19 evolution of the photobleached plug. U_s is the slip velocity and U_f is the velocity of the front of
 20 the plug.

21 2.2 Photobleached imaging for flow profiling

22 **Fig. 3** shows a schematic of the spatiotemporal response of a photobleached plug, in a
 23 fluorescently doped lubricant, due to Couette flow in the x-direction, with interfacial slip
 24 (slip velocity of U_s) at the bottom interface. The photobleached plug has much lower
 25 fluorescence intensity than its surrounding fluorescent lubricant. The x-z plane depicts the
 26 through-thickness view of the confined lubricant, which cannot be visualised
 27 experimentally due to the submicron thickness of the film. The x-y plane corresponds to
 28 the image plane, which was observed in the experiment. The intensity distribution within
 29 the photobleached plug is initially Gaussian, as it is produced by a diffraction-limited
 30 focused laser beam. The convection of the fluid causes the shape of the plug to change
 31 with time. In the case presented in **Fig. 3**, the observed plug elongates in the x-direction

1 and the back of the spot moves with a velocity U_s . Other velocity profiles due to Poiseuille
2 flow and inhomogeneous shear would produce distinctly different results where the
3 intensity distribution in x becomes distorted. Both the elongation of the plug and the
4 interfacial velocity of the lubricant can be measured experimentally. The evolution of the
5 spot can be used to reconstruct the unknown through-thickness velocity profile.

6 **2.3 Acquisition**

7 Acquisition of images was achieved using a Labjack U6 digital acquisition system. Four
8 background images were taken prior to bleaching. The shutter controlling the bleaching
9 beam was then opened for a set duration. Subsequently, 16 frames were acquired at a
10 given exposure time, capturing the response of the bleached plug. Synchronous
11 averaging was employed to reduce noise [32]. Due to the high viscosity and the small film
12 thickness of the model lubricant used in this study, the characteristic time for through
13 thickness flow profile development is of the order of 10^{-16} second. Thus steady state
14 conditions required for synchronous averaging can be assumed. While the flow is
15 developed, it is possible that there are transient changes in the microstructure of the
16 liquid. Such changes would not be resolved by the technique but an average result would
17 still be acquired. A delay between each sequential acquisition was employed to allow
18 most of the photobleached dye to exit the observation volume. The camera background
19 I_b was removed from the image sequences and the final result was the normalised
20 photobleached intensity I_N given by $I_N(t) = (I_0 - I(t))/I_0$ where I_0 is the pre-bleach
21 intensity and $I(t)$ is the intensity at time t .

22 **2.4 Fluid and tracer**

23 The fluid investigated in this work was PB H-300 (supplied by Ineos Oligomers). PB is an
24 oligomer which has high viscosity at relatively low molecular weight. With a number
25 average molecular weight of 1300 g/mol, PB H-300 has a low shear rate viscosity, μ_0 , of
26 100 Pa s at 20°C and 0.62 Pa s at 100°C (measured using a TA AR2000Ex rheometer).
27 A fluorescent probe, Nile red (Sigma-Aldrich), was dissolved into the PB at a
28 concentration of 1 mM by magnetic stirring at 150°C. No surface adsorption of the dye to
29 either glass or steel could be seen during the experiments.

30 PB is commonly considered to be a Newtonian fluid, but at higher molecular weights, and
31 more importantly, at higher pressures, it is known to shear thin at the shear rates
32 encountered in this work [3]. Here, a peak pressure, p , of approximately 0.315 GPa is
33 applied when PB is confined in an EHL contact. Using the Barus equation ($\mu = \mu_0 e^{\alpha p}$,
34 where $\alpha = 32 \text{ GPa}^{-1}$ is the pressure viscosity coefficient measured using interferometry),
35 the viscosity, μ , of PB H-300 at such pressure is about $2.4 * 10^6$ Pa s. With a glass
36 viscosity, μ_g , of 10^7 Pa s [3], PB can be very viscous, or even partially or completely
37 solidify under the experimental EHL conditions. While the pressure applied in this work
38 may not cause common lubricants such as mineral oil ($\mu_g = 10^7 \text{ Pa s}$, $\alpha = 23.1 \text{ GPa}^{-1}$)
39 [33] and polyalphaolefin ($\mu_g = 10^6 \text{ Pa s}$, $\alpha = 12.6 \text{ GPa}^{-1}$) [33] to solidify, pressures in
40 rolling element bearings readily reach up to 4 GPa, at which point enhanced viscosity has
41 been observed [5,6]. Hence the behaviour observed with PB in this work is relevant to
42 common lubricants. The potential of PB to deviate from commonly assumed flow
43 behaviour in the applied conditions allows the effectiveness of the proposed methodology
44 to be demonstrated: if PB displays a non-linear velocity profile, the detection of such a
45 profile would show that the proposed methodology is appropriate.

46 Nile red is usually considered to be a photostable fluorophore [34]. When dissolved in PB,
47 a relatively fast photobleaching rate constant of 20 ms was achieved using 473 nm

1 excitation (40 mW on stage) with a 50x objective. A typical bleach time of 10 ms resulted
 2 in a 40% peak reduction in intensity for the PB-nile red solution. Heating during
 3 photobleaching needs to be considered as it could cause a change in the viscosity of the
 4 fluid [35]. Finite element simulations of heat transfer were performed for an axisymmetric
 5 Gaussian heat source in PB using Elmer (<http://www.csc.fi/english/pages/elmer>). An
 6 analytical heat source equivalent to the heating experienced by PB due to photobleaching
 7 under the experimental conditions was generated based on an infinite cylindrical
 8 Gaussian heat source surrounded by PB (unbound case) using the approach outlined in
 9 [36]. The simulated temperature rise in the unbound case agreed well with the analytical
 10 solution [36]. A simulation was then performed using the experimental EHL conditions,
 11 with PB bounded by steel and glass (bounded case). The resultant steady state
 12 temperature rise was 3 K. This would cause an approximate change in viscosity of 5%.
 13 The temperature gradient would however decay within 1 ms after photobleaching. Hence
 14 over the experimental time scale (about 100 ms), the viscosity change due to
 15 photobleaching would be negligible. Control experiments were conducted with various
 16 combinations of Nile red concentrations and laser bleaching powers to lower the amount
 17 of heat experienced by PB. No significant change in the determined velocity profiles was
 18 observed, confirming that the results obtained are not affected by heating due to
 19 photobleaching.

20 The determination of the through-thickness velocity profile by the observation of the
 21 change in the shape of the photobleached plug with time requires the Brownian diffusion
 22 coefficient of the fluorescent probe, D , in the contact to be quantified. Due to the squeeze
 23 film effect, D is not a trivial quantity to measure in a tribological contact. A submicron film
 24 of PB H-300 was prepared between two glass slides. Fluorescence recovery after
 25 photobleaching [37] was used to measure D in the PB film. It was found to be $3.8 \times 10^{-15} \pm$
 26 $1.1 \times 10^{-15} \text{ m}^2/\text{s}$. This value can be compared to the prediction from the two-dimensional
 27 Stokes-Einstein diffusion equation $D = k_b T / (4\pi\mu_0 r)$ where k_b is the Boltzmann constant,
 28 T is the temperature and r is the hydrodynamic radius. At a temperature of 20°C, a
 29 kinematic viscosity of 100 Pa s and a radius of 0.5 nm [38], the theoretical diffusion
 30 coefficient of Nile red in PB H-300 is $6.4 \times 10^{-15} \text{ m}^2/\text{s}$, which is comparable to the
 31 experimental value. The measured D is the upper bound value for the diffusion of Nile red
 32 in PB. In an EHL contact, D would be significantly smaller due to the pressure
 33 experienced by PB. The effect of dye diffusion on data interpretation will be discussed in
 34 the next section.

35 **3 Analysis**

36 **3.1 Experimental framework**

37 The observation volume was homogeneously illuminated. The photobleaching of the dye
 38 by the bleaching beam in the contact caused the appearance of a photobleached plug.
 39 Consider the x-z plane through the center of the plug (Fig. 3). Due to the beam shape, the
 40 initial concentration of excited unbleached fluorophores, c_0 , within the plug immediately
 41 after the photobleaching process, i.e. at $t = 0$, is given by the Gaussian distribution
 42 presented in equation (1)

$$c_0(x) = 1 - Ae^{-\frac{(x-x_0)^2}{2\sigma_x^2}} \quad (1)$$

43 where A is the fraction of bleached fluorophores, x_0 is the initial central position of the
 44 plug and σ_x is the radius of the bleaching beam. Assuming steady state, homogeneous

1 diffusion and incompressible flow, equation (2) describes the response of the initial
 2 distribution of unbleached fluorophores using a simplified convection-diffusion equation

$$\frac{\partial c}{\partial t} = D\nabla^2 c - \vec{u} \cdot \nabla c \quad (2)$$

3 where c is the spatiotemporal concentration of unbleached fluorophores, t is time, ∇ is the
 4 gradient function, and \vec{u} is the velocity field. The one-dimensional Reynolds equation can
 5 be used to describe the flow in a thin film under isothermal conditions. A solution, using
 6 no-slip boundary conditions, as presented in equation (3), shows that the through-
 7 thickness velocity profile, $u(z)$, is a combination of a Poiseuille pressure-driven term (left)
 8 and a Couette shear driven term (right)

$$u(z) = \frac{1}{2\mu} \frac{\partial P}{\partial x} (z^2 - zh) + U \frac{z}{h} \quad (3)$$

9 where μ is the viscosity, $\partial P / \partial x$ is the pressure gradient, h is the film thickness and U is
 10 the relative sliding speed. In the region of the contact where the film thickness is almost
 11 constant, the flow is modelled as shear driven plane Couette with a pressure gradient.
 12 The Poiseuille term is relatively small due to the high viscosity [23], so that

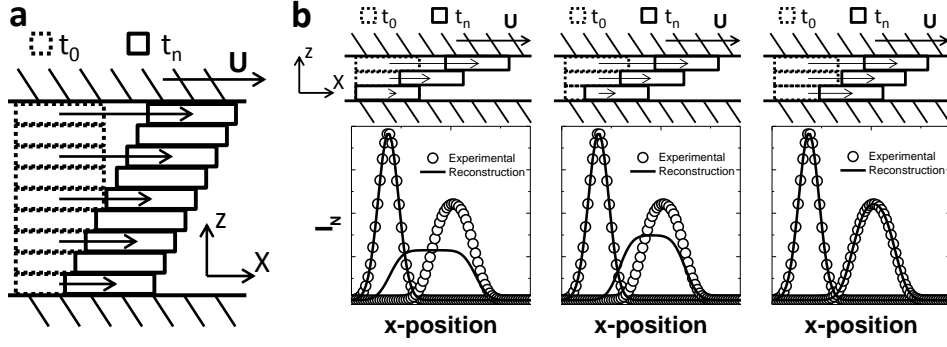
$$u(z) \approx Uz/h \quad (4)$$

13 If a slip boundary condition is used instead at the stationary surface, the resulting velocity
 14 profile is

$$u(z) = U(z + b)/(h + b) \quad (5)$$

15 where b is the slip length.

16 The Peclet number $Pe = LU/D$, where L is the characteristic length and D is the diffusion
 17 coefficient of the fluorescent dye in PB, describes the relative effects of convection and
 18 diffusion. For the z direction, where diffusion effects are largest in a thin film, the Peclet
 19 number is $Pe_z = hU/D$. The evolution of the intensity distribution is heavily dependent on
 20 this number. If convection is favoured, the intensity distribution will vary in z . If diffusion
 21 dominates, the distribution will be invariant in z . For experiments in this work the Peclet
 22 number was above 1000. Molho et al. [27] concluded that if $Pe \gg 100$, diffusion and
 23 dispersion effects are negligible. As $Pe_x = Pe_y \gg Pe_z$, diffusion can be neglected
 24 altogether. This enables the velocity profile reconstruction scheme to be simplified. The
 25 fluid can be modelled as infinitesimal layers of concentration c_0 at $t = 0$. Each layer
 26 moves independently with velocity $u(z)$ and there is no mixing between layers. This is
 27 shown schematically in Fig. 4a.



1

2 Fig. 4 Schematic of the framework used for the velocity profile reconstruction. (a) The fluid in the
 3 x-z plane, modelled as layers moving independently of each other. (b) Top: Three successive
 4 guesses for the velocity profile at times t_0 and t_n . Bottom: Corresponding comparison between
 5 typical experimental (circles) and numerical (line) intensity distributions I_N .

6 Under the assumption that the photobleaching occurs relatively quickly, c_0 will be
 7 equivalent for all layers. The appropriateness of this assumption can be confirmed by
 8 studying the symmetry of the intensity distribution of the photobleached plug right after
 9 the photobleaching process. If the bleach time is relatively long compared to the velocity
 10 of the lubricant flow, the distribution will be distorted and appears non-Gaussian.
 11 Simulations performed using the conditions in the experimental work confirm that the
 12 effect of the bleach times used in this work on the reconstructed velocity profile was
 13 negligible.

14 Taking $\vec{u} = (u, 0, 0)^T$ and using the assumption of negligible diffusion, equation (2)
 15 reduces to

$$\partial c / \partial t = \vec{u} \cdot \nabla c = u \partial c / \partial x \quad (6)$$

16 The solution to this hyperbolic partial differential equation is

$$c(x, t) = c_0(x - ut) \quad (7)$$

17 Using equation (1) and (7) the response of each layer can be described as

$$c(x, z, t) = 1 - Ae^{-\frac{(x-(ut+x_0))^2}{2\sigma^2}} \quad (8)$$

18 Given a velocity profile $u(z)$ the theoretical spatiotemporal intensity distribution I^* ,
 19 comparable to the experimental results, is the integral of these layers

$$I^*(x, t) = \int_0^h c(x, z, t) dz \quad (9)$$

20 This is in contrast with earlier experiments for similar submicron geometries [26,30],
 21 where significant diffusion reduces the technique resolution to two dimensions due to the
 22 equilibration of intensity in the z-direction. Variations of the intensity distribution in z have
 23 previously been studied [29,31] for larger dimensions, with the expected non-Gaussian
 24 response. The benefit of negligible diffusion in this work is that effects such as slip and
 25 inhomogeneous shear can be directly visualised, despite the narrow geometry of a
 26 contact.

1 3.2 Velocity profile reconstruction algorithm

2 The experimental technique described in section 2 captures the evolution of the intensity
3 distribution of a photobleached plug as the lubricant is being sheared. The analysis
4 presented in section 3.1 correlates such evolution with the velocity profile of the lubricant
5 through the determination of through-thickness concentration of unbleached probes. Most
6 of the confined lubricant film, especially in regions away from the minimum film thickness
7 region (horseshoe region), has constant film thickness, as confirmed in this work using
8 interferometry. If the observation volume is small relative to the dimensions of the
9 contact, the velocity profile can be assumed to be locally invariant in the x-direction.
10 Given that the total fluorophore concentration is constant and fluid motion is dominated
11 by convection, the intensity distribution is directly coupled to the velocity profile such that
12 $I(x, y, t) = f(I_0, u(z))$. Unfortunately the coupling is non-trivial. It has previously been
13 solved numerically by assuming an n^{th} order velocity profile [39]. It has also been solved
14 by constraining the velocity profile [31]. A similar approach to [31] is used in this work.
15 $u(z)$ is solved numerically by a least squares minimization of the difference between the
16 theoretical intensity distribution I^* as shown in equation (9) and the experimental
17 distribution I . For the discretized intensity distribution, as is the case for the experimental
18 data, the Cartesian axes x , y and z are referred to as i , j and k respectively and time t is
19 referred to as frame number n . To simplify the problem and to reduce noise, the intensity
20 is averaged in the y -direction over $2w$ pixels such that $I(i, n) = \sum_{j=-w}^w I(i, j, n) / (2w)$. The
21 minimization effectively varies the velocity profile until I^* matches I . This is shown
22 schematically in figure Fig. 3b, where the theoretical intensity distribution for slip Couette
23 flow is to be reconstructed. Three successive iterations of the minimization are shown
24 culminating in a match of the intensities and a determined velocity profile.

25 The discrete minimization takes the form

$$26 \sum_{i=1}^{128} \sum_{n=1}^N \|I^* - I\|^2 < \delta_I \quad (10)$$

27 where N is the number of images in the sequence and δ_I is the convergence criterion for
28 the intensity minimization. It is assumed that the velocity of the fluid at the glass surface
29 is not negative and, at the steel surface, the fluid does not flow faster than the surface
30 speed U . This sets the boundary conditions $u(1) \geq 0$ and $u(h) \leq U$. While this scheme
31 effectively finds the velocity profile that results in the best agreement between I^* and I ,
32 the solution may not be unique. A second constraint is applied, which states that the
33 velocity profile must be continuous and minimises the difference between velocities of
34 adjacent lubricant layers. A second order finite difference scheme is chosen to impose
35 this constraint

$$36 \sum_{k=2}^{K-1} \|u(k+1) - 2 * u(k) + u(k-1)\| < \delta_u \quad (11)$$

37 where K is the number of discrete layers used in the simulation and δ_u is the
38 convergence criterion for the continuity minimization. Given δ_I and δ_u , the $u(z)$ obtained
39 is unique. Thus the solution is affected by the minimization ratio δ_I/δ_u . After imposing a
40 convergence criterion of 10^{-8} the complete minimization scheme is given by

$$\sum_{i=1}^{128} \sum_{n=1}^N \|I^* - I\|^2 + \frac{\delta_I}{\delta_U} \sum_{k=2}^{K-1} \|u(k+1) - 2 * u(k) + u(k-1)\| < 10^{-8} \quad (12)$$

1 This problem is solved using a non-linear least squares solver ‘lsqnonlin’ in Matlab. The
 2 minimisation ratio was chosen based on two parameters which quantify the accuracy of
 3 the reconstructed velocity profiles. The coefficient of determination R^2 can be used to
 4 describe the goodness of fit that the simulated intensity I^* , produced by the reconstructed
 5 velocity profile, has with the experimental intensity I . The standard error σ_s of the
 6 determined velocity of each layer is calculated based on the distribution of velocities
 7 obtained in each iteration during the reconstruction of a velocity profile and can be
 8 obtained by using the residuals and jacobian of the minimization problem [40]. The best
 9 trade-off between a good intensity fit and a continuous profile is made when the ratio of
 10 the two parameters R^2/σ_s is maximised. This approach is consistent with other numerical
 11 solutions investigating similar problems in that they either specify a velocity profile [39], or
 12 a user defined minimization parameter [31]. The method of velocity profile reconstruction
 13 employed in this study means that the number of layers m does not affect the resolution
 14 of the technique. Increasing m only determines the number of points describing the
 15 velocity profile, hence m was kept constant for all cases in this study. A Matlab script
 16 was written to implement the minimisation scheme described.

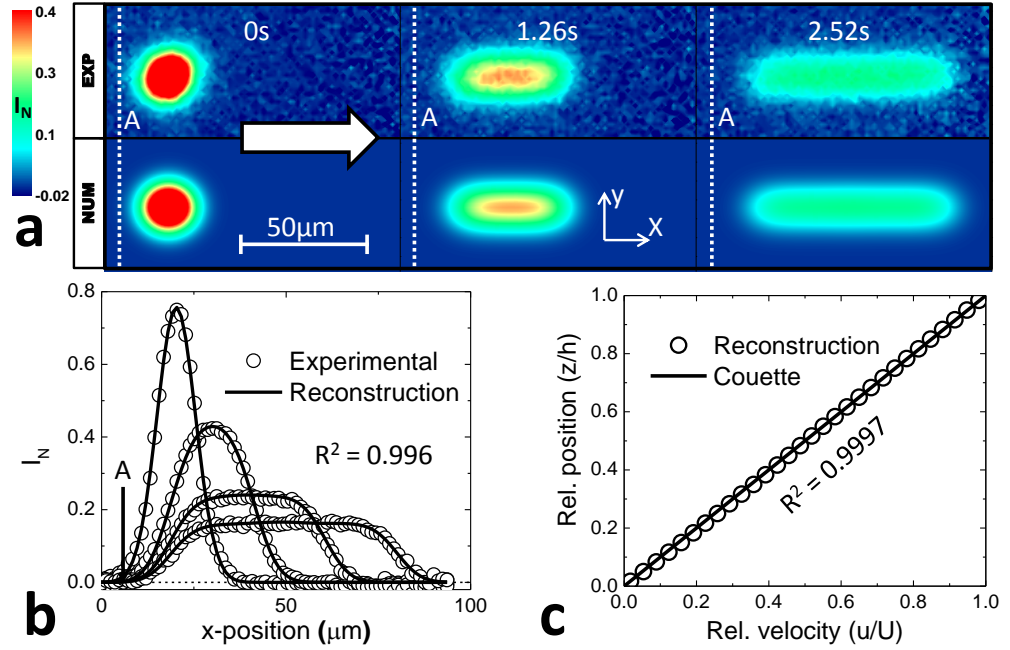
17 4 Validation

18 4.1 Setup

19 To assess the suitability of the use of photobleached imaging and the implementation of
 20 the velocity profile reconstruction algorithm for velocity profile determination, a flow with a
 21 known velocity profile was studied. Plane Couette flow was applied to a thin film of PB H-
 22 300 between two glass microscope slides. Experiments were carried out for a film with 1
 23 μm thickness. This film thickness is below the depth of focus of the optical system when a
 24 magnification of 20x is used, ensuring a through-thickness photobleached plug can be
 25 formed by photobleaching. A constant velocity was applied to the top slide while the
 26 bottom slide remained stationary, creating the Couette flow in the PB film.

27 4.2 Results

28 The experimental and reconstructed spatiotemporal intensity distribution of plane Couette
 29 flow in a 1 μm PB film is presented in **Fig. 5a**. The velocity of the top slide was 26.3 $\mu\text{m/s}$
 30 while the bottom slide remained stationary. The bleach time was 25 ms, the exposure
 31 time was 180 ms. The resulting photobleached plug had an initial diameter of 20 μm . 10x
 32 synchronous averaging was applied to improve the signal to noise ratio of the result.
 33 Single acquisitions were also carried out, yielding similar results.



1

2 **Fig. 5** Plane Couette validation results. (a) Experimental (top) and reconstructed numerical
 3 (bottom) spatiotemporal intensity distribution. The white arrow indicates the shearing direction
 4 of the fluid. (b) Comparison between averaged experimental (circles) and numerical (line)
 5 intensity distributions. (c) Reconstructed (circles) and Couette (line) relative velocity profiles.

6 Compare the simulated image sequence (the ‘NUM’ row in **Fig. 5a**) with the experimental
 7 result (the ‘EXP’ row in **Fig. 5a**), the reconstructed distribution agrees well with the
 8 experimental results in the evolutions of both intensity and shape of the photobleached
 9 plug. The integral of the intensity distribution varies less than 2% over the sequence. To
 10 better quantify the goodness of fit between I^* (numerical) and I (experimental), the data
 11 was analyzed further by averaging the intensity in y over 5 pixels. The resulting intensity
 12 distribution is shown in **Fig. 5b**. The initial intensity distribution of the photobleached plug
 13 is well approximated by a Gaussian distribution ($R^2 > 0.99$), showing that the bleaching
 14 process can be assumed to be instantaneous. The average coefficient of determination of
 15 all fits is $R^2 = 0.996$, supporting a good agreement between the numerical reconstruction
 16 and the experimental data. Both **Fig. 5a** and **Fig. 5b** show no change in the position at the
 17 back of the photobleached plug (point A in **Fig. 5b**) with time, suggesting no measurable
 18 slip can be observed. The intensity distribution flattens out at longer times, which is
 19 expected for Couette flow.

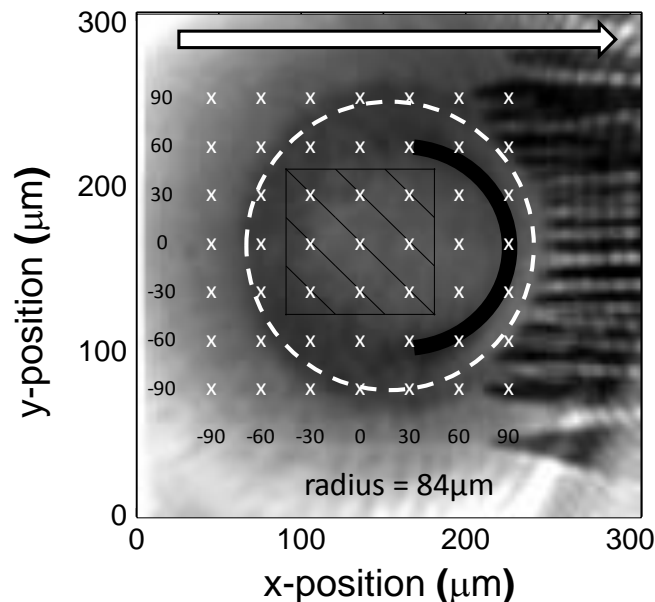
20 The reconstructed velocity profile (shown as circles) in **Fig. 5c** matches very well with the
 21 Couette flow (black line) applied to the PB film. In addition, correct surface velocities for
 22 both top and bottom plates are found. Thus it justifies the capability of the velocity profile
 23 reconstruction algorithm in obtaining accurate velocity profiles. These results also confirm
 24 that the experimental framework and the assumption of negligible diffusion (stated in
 25 section 3.1) are applicable to these experimental conditions.

26 Conclusively, using appropriate assumptions, it is possible to use the two-dimensional
 27 spatiotemporal intensity distribution to probe velocity variations through the thickness of a
 28 thin film. Having validated the use of the proposed technique for a simple flow, it was
 29 applied to the case of EHL.

1 5 EHL flow profiling

2 5.1 Setup

3 A pure sliding contact was created by loading a 3/4" rotating steel ball (PCS instruments)
4 onto a microscope glass slide, with a polydimethylsiloxane bath containing the PB (see
5 **Fig. 2**). The roughness of the ball and glass slide was measured using a Veeco optical
6 profilometer in phase-shifting interferometry mode. The arithmetic mean value Ra , was
7 found to be less than 5 nm for the steel ball and less than 0.5 nm for the glass slide. The
8 location of the ball was fixed using a ball carriage assembly (PCS instruments) that
9 constricted the motion in the z-direction. A load of 5 N was applied using a dead weight.
10 This produced a theoretical Hertzian contact of radius 87 μm which was confirmed
11 experimentally by fluorescence imaging and interferometry. **Fig. 6** shows the EHL contact
12 visualised using fluorescence imaging at 10x magnification. The maximum theoretical
13 Hertzian pressure at this load was 315 MPa. The surface velocity was calibrated by
14 tracking features on the steel ball surface and was found to be accurate within 2%. The
15 central x- and y-position of the contact was maintained within 5 μm over the revolution of
16 the steel ball. Compare the experimental time frame (~ 100 ms) to the characteristic time
17 of contact drift (~ 166 s), such drift will be ignored in this study. An EHL ultra thin film
18 measurement system (PCS instruments) was used to measure the film thickness using
19 interferometry. Most of the film within the contact had a constant thickness, allowing the
20 flow in the hatched region in **Fig. 6** to be analysed as plane Couette flow. The steel ball
21 was driven by a stepper motor. The sliding velocity was 360 $\mu\text{m/s}$, giving a film thickness
22 of 170 nm. The sliding speed and load were chosen such that the contact operated within
23 the piezoviscous-elastic region. The mapping coordinates, indicating the positions at
24 which the fluid was probed, are indicated by white crosses.



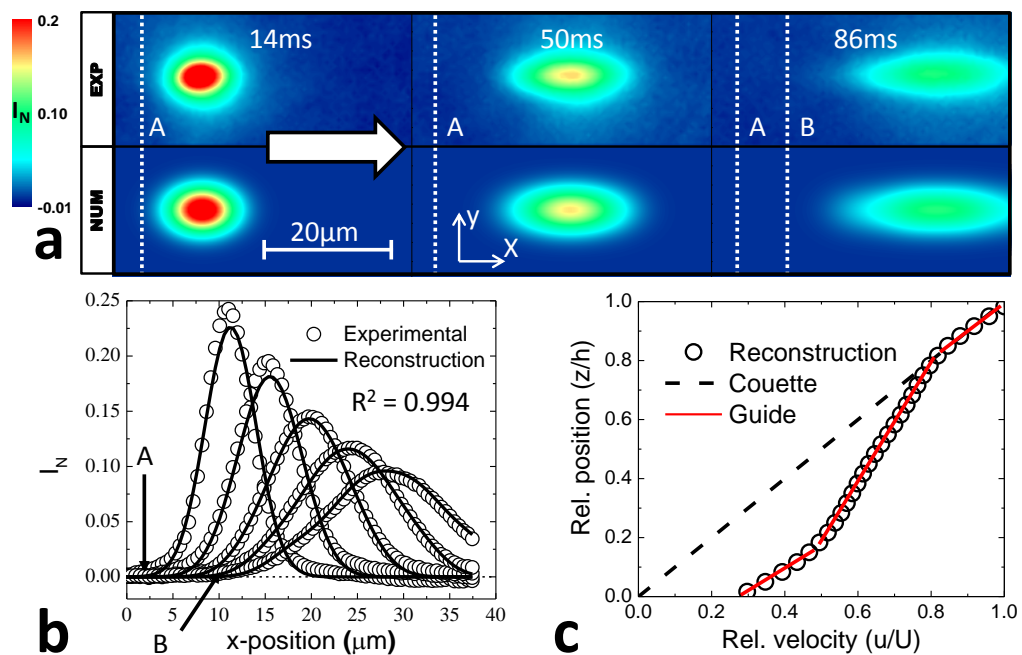
25

26 **Fig. 6** Fluorescence image of the EHL contact. Brighter intensity generally corresponds to a
27 thicker film. The flow direction is indicated by the white arrow. The calculated Hertzian contact
28 area is marked by the dotted white line. A grid used for mapping the slip length and flow
29 direction in the contact is shown overlaid with distance referenced to position [0, 0], given in
30 microns. The horseshoe is indicated by a black arc.

1 The temperature of the fluid needs to be considered as a change in just a few degrees
 2 significantly alters the viscosity, and therefore the velocity profile. Heat can be generated
 3 due to (1) friction; and (2) photobleaching. In contacts, temperature rises due to friction of
 4 up to 50°C have been measured experimentally [41]. The expected temperature rise can
 5 be determined using analysis described in [1]. Using a conservative friction coefficient of
 6 0.1 [42] and the thermal properties for PB [43], the temperature rise due to friction in the
 7 applied experimental condition was estimated to be less than 0.01 K; the low value being
 8 primarily due to the very low sliding speeds employed. The temperature rise for the same
 9 system has been assumed to be negligible by others [44]. It should also be noted that, if
 10 the temperature rise was significant, there would be a distinct difference in the velocity
 11 profile near the two surfaces due to the difference in heat transfer. This was not seen
 12 experimentally. The effect of the temperature rise due to photobleaching can be
 13 neglected, as shown in section 2.4. Thus isothermal conditions were assumed in this
 14 study.

15 5.2 Results

16 Photobleached imaging with 100x synchronous averaging was applied to an EHL point
 17 contact. The bleach time was 8.5 ms, the exposure time was 4 ms. A 50x objective was
 18 used for imaging, resulting in a photobleached plug with an initial diameter of 6 μm . The
 19 first two images acquired for each set of experiments are not analyzed due to the finite
 20 opening time of the shutter protecting the camera from the bleach beam. Therefore,
 21 results are shown after this shutter has completely opened. In all experiments, the
 22 integral of the experimental intensity distribution varied no more than 2% while the whole
 23 plug was contained within the observation area. A typical experimental result obtained at
 24 a position close to the center of the EHL contact (position [0, 0] in Fig. 6), is presented in
 25 Fig. 7. The numerical intensity distribution computed based on the velocity profile
 26 reconstruction algorithm is also included for comparison. The averaged intensity
 27 distribution over 5 pixels in the y-direction is shown in Fig. 7b.



28

29 **Fig. 7** Photobleached imaging velocimetry results for an EHL contact. (a) Experimental (EXP) and
 30 reconstructed numerical (NUM) spatiotemporal intensity distribution. The white arrow indicates

1 the shearing direction of the fluid. (b) Comparison between averaged experimental (circles) and
2 numerical (line) intensity distributions. (c) Reconstructed (circles) and Couette (dashed line)
3 relative velocity profiles. The solid lines are for guidance and indicate that the lubricant was
4 separated into three distinct layers.

5 The initial intensity distribution, as shown in **Fig. 7b**, was symmetric, which confirms that
6 the assumption of instantaneous bleaching is appropriate. A comparison between results
7 obtained in an EHL contact (**Fig. 7**) and under the no-slip Couette condition (**Fig. 5**), as
8 applied in the validation experiment, reveals several anomalies. Significant motion of the
9 back of the spot, from point A to point B as indicated in **Fig. 7b**, was observed, suggesting
10 that slip has taken place at the bottom surface. This is confirmed by the reconstructed
11 profile in **Fig. 7c** with PB having a relative velocity of about 0.3 at the stationary glass
12 surface. Secondly, at longer times, the photobleached plug only elongated by a small
13 amount and the average intensity distribution did not show a plateau as in the case of the
14 Couette flow (**Fig. 5b**). Therefore, the use of a slip Couette fit to describe the velocity of
15 the system studied under EHL condition would be inappropriate.

16 The reconstructed velocity profile based on the proposed velocity profile reconstruction
17 scheme is presented in **Fig. 7c**. This is the first published result of a through-thickness
18 velocity profile measurement in an EHL contact. It shows that the velocity profile at this
19 particular location is non-linear. The resulting numerical intensity distribution matched the
20 experimental results well, with an average goodness of fit $R^2 = 0.994$ (shown in **Fig. 7b**).
21 Therefore, the reconstructed, nonlinear velocity profile provides a satisfactory account of
22 the experimental intensity response. This non-linear velocity profile contradicts the profile
23 commonly assumed in EHL theory [1,2], and those found using isothermal simulations
24 [45]. It suggests that the fluid experiences different shear rates depending on the distance
25 from the rubbing surfaces. While the physical cause for the shape of the profile is not yet
26 known, such that the mathematical form that should be adopted for the profile is unclear;
27 one can still clearly identify three regions through the thickness of the film, as indicated by
28 the overlaid solid lines. Layers with localized shear rates appear to have developed within
29 the fluid, with the surface regions experiencing higher shear rates than the lubricant in the
30 central region.

31 **6 Mapping**

32 With a method for determining velocity profiles established, the technique is applied to
33 various positions in an EHL contact to investigate flow heterogeneities. The one-
34 dimensional theoretical normal pressure experienced by the lubricant in an EHL contact
35 can be determined using Hertzian contact mechanics. Since a point contact is used, there
36 is also a significant pressure gradient in the y-direction resulting in two-dimensional flow.
37 The position-dependent pressure and the geometric effect may cause the flow profile of
38 the lubricant to vary throughout the contact. It is possible that non-linear velocity profiles
39 observed in this study may result from a pressure induced viscosity change in the liquid.
40 If this is the case, there should be significant variations in both slip length and velocity
41 gradient with pressure. This can be verified by mapping the velocity profiles in the
42 contact. Measurements of the through-thickness velocity profile and the 2-D x-y plane
43 average velocity were performed at positions marked as 'x' in **Fig. 6** within a 7x7 grid. Six
44 tests were carried out at each position with different specimen, and average values are
45 reported.

1

2 6.1 Two-dimensional flow map in the x-y plane

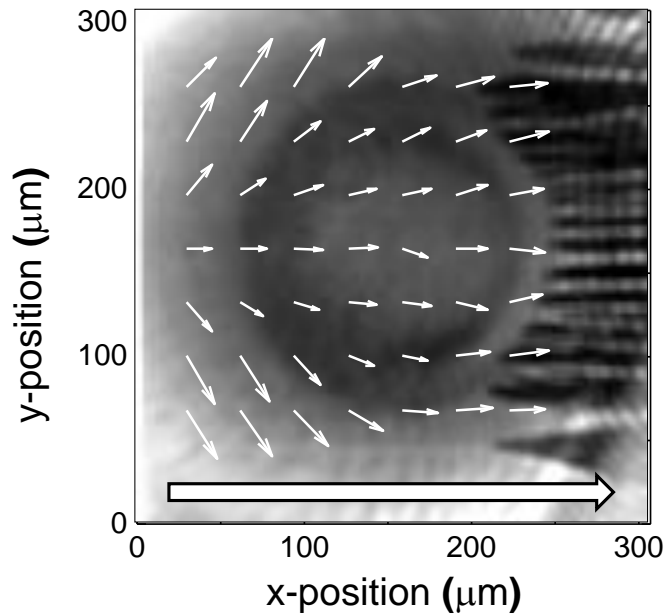
3 The two-dimensional flow of a lubricant in a contact is complicated, due to the unknown
4 pressure distribution and the rheology of the liquid. There are simultaneous effects of
5 geometry, cavitation and pressure gradients. The spatiotemporal intensity maps can be
6 used to characterise the z-averaged two-dimensional flow of the liquid. The y-component
7 of velocity, at time t , was determined by finding the position of the center of mass of
8 intensity y_{CM} using

$$\int_{y=0}^W \left(y \int_{x=0}^L I(x, y, t) dx \right) dy = (y_{CM} + y_0) \int_{y=0}^W \int_{x=0}^L I(x, y, t) dx dy \quad (13)$$

9 where W is the width (in the y-direction) of the observation area, L is the length (in the x-
10 direction) of the observation area and y_0 is the central position of the initial bleach spot in
11 y. y_0 is found by fitting a two-dimensional Gaussian intensity distribution to the initial spot
12 using

$$I_0(x, y) = 1 - Ae^{-\left(\frac{(x-x_0)^2}{2\sigma_x^2} + \frac{(y-y_0)^2}{2\sigma_y^2}\right)} \quad (14)$$

13 where σ_y is the radius in y of the bleaching beam. Equations (13) and (14) are
14 discretized to allow the use of the pixelated experimental data. The average velocity of
15 the lubricant is then given by y_0/t . A similar expression was used to determine the x-
16 component of velocity. **Fig. 8** shows a vector plot of the two-dimensional velocity profile
17 within the EHL contact and in the inlet region. For clarity, the y-component of the vectors
18 shown has been scaled up tenfold.



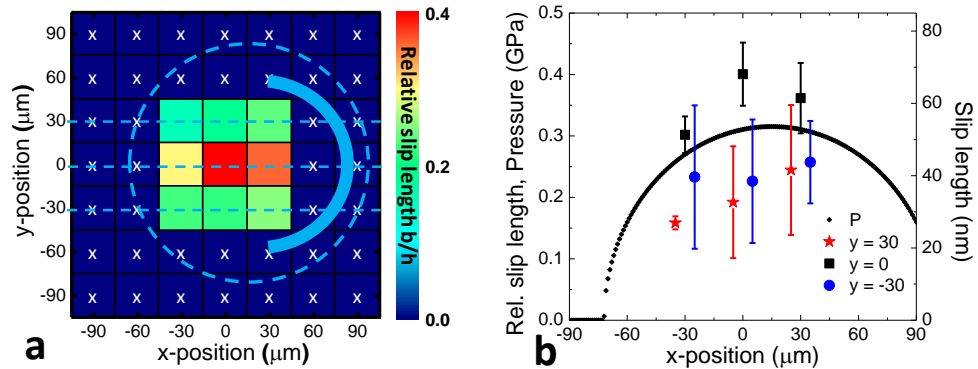
19

20 **Fig. 8** Fluorescence image of the contact. Vector plot of velocity field overlaid with white arrows.
21 Velocity in y has been scaled up 10x for clarity. The large white arrow indicates the flow direction.

1 The x-component was much larger than the y-component in all positions. Within the
 2 contact, the y-component of velocity was less than 5% of the x-component. Due to the
 3 high viscosity of pressurised PB, $\frac{h^2}{\mu} \frac{\partial P}{\partial x}$ is relatively small. Thus, based on equation (3), the
 4 Poiseuille component of flow is effectively negligible. In the central region of the contact,
 5 the flow in the y-direction was minimal. Near the inlet however, geometry effects play a
 6 much larger role, causing the liquid to flow around the contact. At the outlet, the results
 7 are distorted due to cavitation.

8 6.2 Slip measurements

9 The existence of 2-D flow, while small, does have an effect on the quality of the
 10 reconstructed velocity profile. Furthermore, near the edges of the contact, the film
 11 thickness is not constant. Only reconstructed profiles with a goodness of fit, R^2 , higher
 12 than 0.99 are used for further analysis. In the central region of the contact (hatched area
 13 in **Fig. 9**), flow in the y-direction was minimal, and the film thickness was constant to give
 14 reconstructed velocity profiles with $R^2 > 0.99$. The glass-PB interfacial velocity was
 15 determined using a linear fit. The fit was made using the first three points of the velocity
 16 profile from $z = 0$ (as indicated by the bottom solid line in **Fig. 7c**), and the slip velocity
 17 was determined by extrapolation to $z = 0$. The slip length was then found using equation
 18 **Error! Reference source not found.**. The slip length as a function of position is shown in
 19 **Fig. 9a** and **Fig. 9b**.



20

21 **Fig. 9** Variation of slip length with position in the contact. (a) Color map of relative slip length
 22 against the position in the contact. The Hertzian contact area (dotted circle) and the horseshoe
 23 (thick arc) are overlaid. Crosses indicate positions with $R^2 < 0.99$. The center lines of the
 24 selected rows are indicated by straight dotted lines. (b) Mean relative slip length along the three
 25 center lines in (a) with respect to position in x. All data points are at $x = -30, 0$ or 30 μm . The
 26 data for $y = 30$ and -30 μm has been shifted in x for clarity. The pressure for the central line
 27 ($y = 0$) is also plotted against x-position in the contact. The error bars indicate the standard
 28 deviation of the measured slip length for the results with $R^2 \geq 0.99$.

29 There is a significant variation of the relative slip length (measured slip length divided by
 30 the film thickness) with respect to the probing position in the contact, as shown in figure
 31 **Fig. 9**. Slip lengths of 30 - 70 nm have been measured at the PB-glass interface. Since
 32 the theoretical Hertzian pressure also varies with position in the contact, the normal
 33 pressure experienced by the fluid and the slip length measured may be correlated. As
 34 discussed in section 5.2, the typical flow profile as shown in **Fig. 7** can be divided into
 35 three regions: two regions next to the rubbing surfaces with high shear rate, and the

1 central region with a lower shear rate. Apart from the changing slip length, a variation in
2 shear rate experienced by the central region of the flow profiles against x-position is also
3 observed. Since both a change in viscosity due to normal pressure and a change in
4 interfacial slip can affect the relative shear rate, it makes the two effects difficult to
5 decouple. More work is in progress to establish their relationship.

6 **7 Discussion**

7 The methodology presented in this work uses photobleached imaging and a minimization
8 scheme to reconstruct the through-thickness velocity profile of a liquid film under EHL
9 conditions. Slip lengths for ambient pressure conditions have been measured previously
10 using photobleaching [24–26]. However the methodology developed in this study is the
11 first that allows capturing velocity variations through submicron film thicknesses. It offers
12 several distinct benefits. The depth of focus of the bleaching beam, which determines the
13 thickest fluid film the bleaching beam is able to form a through-thickness photobleached
14 plug for velocity profile reconstruction, is about 1 μm . This makes the technique
15 particularly suited for the study of submicron EHL films. The method can also be
16 extended to investigate the flow of confined liquid films with a similar range of
17 thicknesses. The through-thickness velocity profiles obtained contain information about
18 the flow properties of the sheared lubricant. The relative shear rate at every z-position
19 can be obtained using the slope of the velocity profile. If the shear stress is known, the
20 through thickness viscosity variation of the liquid can be determined. Hence the technique
21 would enable in situ rheometry [46]. The non-monotonic stress-strain relationship as
22 indicated by the existence of multiple shear rates at a given shear stress would not have
23 been captured by conventional friction measurements where only average observations
24 can be made.

25 The approach developed in this study is not able to resolve sharp velocity gradients near
26 the wall, although molecular dynamics results suggest that such variations are not
27 prevalent [13,47–49]. This is because the experimental result is an average of the
28 intensity within the film, and the relative intensity of one or two molecular layers at the
29 proximity to the wall would be insignificant. In addition, the velocity profile reconstruction
30 requires the fluid to be composed of layers of finite thickness which should not be thinner
31 than the size of an individual fluid molecule. Thus the interfacial velocity will be measured
32 at minimum one fluid layer away from the surface, however thin this layer is. Although it is
33 possible to measure the velocity of the fluid very close to the interface, the true interfacial
34 slip velocity cannot be resolved unless the correct mathematical form of the flow profile is
35 known. The nature and the amount of the slip, whether it is apparent or real, are therefore
36 not precisely determined in this work. While this is a drawback, it is also a distinction
37 other techniques are unable to make. A continuity criterion imposed on the velocity profile
38 reconstruction scheme, as stated in equation (12), limits the relative shear rates that can
39 be measured with the proposed technique. The proposed approach will still capture
40 discontinuities in velocity profiles when the shear rate approaches zero, although such
41 profiles will be smoothed. Thus the technique is capable of detecting shear localization at
42 the interface (slip) and at the center of the film. Implementation of other criteria to
43 alleviate this issue is possible and is in progress. Nevertheless the profile obtained is
44 valuable for better understanding the rheological behaviour of the fluids which is essential
45 in developing the necessary model for obtaining the interfacial velocity and viscosity, and
46 hence the local friction.

47 The technique works best when the diffusion of the fluorescent probe through the
48 thickness of the film is small, i.e when the Peclet number is large. Low Peclet number
49 conditions can be accommodated to an extent by incorporating the effect of the through-
50 thickness diffusion of fluorescent probes in the velocity profile reconstruction scheme.

1 This will be implemented in future work. If the Peclet number is small, the technique can
2 also be used to measure diffusion coefficients using an alternative analysis [30].

3 The interfacial slip length determination presented in this work, unlike most slip length
4 measurements carried out in EHL contacts [16,17] which assume a linear velocity profile,
5 does not require knowledge *a priori* of the through-thickness flow profile. Based on the
6 results found here, the assumption of a linear velocity profile in EHL contacts can be
7 inappropriate and hence the estimations of slip velocity, and the slip length erroneous.
8 The relaxation on the flow profile assumption opens up the possibility of applying other
9 possible profiles in an EHL contact and is crucial to the understanding of rheology in such
10 conditions.

11 The proposed methodology allows point-wise measurements, such that velocity profiles
12 can be measured at different positions in the contact. This effectively creates a three-
13 dimensional velocity profile map and allows local rheological properties of the lubricant to
14 be investigated. This differs from commonly-used friction-based and interferometry-based
15 techniques which infer the nature of the lubricant flow properties from ensemble
16 measurements. The mean slip length across the contact has been previously measured
17 using interferometry [17] by observing the motion of the dimple created in the EHL film.
18 Yet, the successful mapping of flow profiles in the contact, and the location dependent
19 velocity profile and slip length presented in this study demonstrate that heterogeneous
20 behaviour can exist in a contact. Hence the constant slip length usually assumed for
21 modelling friction reduction by interfacial slip in a contact [16,17] can be an inappropriate
22 assumption and requires verification. The relationship between local slip and friction,
23 being an ensemble average measurement, is more complicated than the conventional
24 treatment might suggest. This, along with the non-linear profile, could possibly explain the
25 anomalous Stribeck curves found [42] for the same system. More experiments are
26 required to understand the relationship between slip length, normal pressure and shear
27 rate. This is not a trivial undertaking as the viscosity varies non-linearly with pressure and
28 shear rate [3], which in turn influences the shear stress experienced by the sheared fluid.
29 Furthermore, the interfacial shear stress affects the slip length for polymers in a non-
30 linear manner [24].

31 The finding that the velocity profile is non-linear as presented in **Fig. 7c**, in other words,
32 the existence of inhomogeneous shear for an isothermal case, is of significance. Similar
33 profiles have been observed for flows with significant temperature gradients [50,51].
34 Inhomogeneous shear has also been observed for heterogeneous fluids [9–11]. The
35 pressure sensitive rheology of PB, as described in section 2.4, may contribute to its
36 peculiar velocity profiles. It is conjectured that the pressure causes the lubricant to
37 solidify. Energy is then somehow dissipated and causes temperature rises sufficient to
38 melt the layers near the two boundaries and thus the lubricant flows like a plug in the
39 center. In the current work, the existence of a definite shear rate in the central region of
40 the lubricant, as shown by the velocity profile in **Fig. 7c**, indicates that the photobleached
41 plug did not completely solidify. One explanation for the behaviour is that rather than
42 complete solidification, the glass transition temperature of the fluid is increased under the
43 experimental conditions and hence the viscosity increases substantially. Indeed in some
44 instances, liquid can undergo a second order phase transition [52]. Surface heating can
45 reduce the viscosity of the fluid near the surfaces, yet the mechanism of this effect is
46 unknown. The heat from photobleaching is low (3 K) and decays too quickly (in 1 ms) to
47 have an effect. While friction induced heating is a candidate, the insignificant theoretical
48 temperature rise of 0.01 K suggests the contrary. Hence the formation of a temperature
49 gradient is unlikely. In addition, if the heating were friction induced, the heat transfer at
50 the two surfaces would be significantly different and so would the shear rates, which is
51 not the case. The source of possible surface heating and hence the origin of displayed

1 variations of shear rate and/or viscosity across the thickness of the film remains unclear.
2 There is currently no theoretical description of the velocity profile encountered in this
3 work, making a comparison to other work cumbersome. Similar profiles to those
4 presented in this work have been observed experimentally in systems with pressures and
5 shear rates comparable to EHL [13,53] and in molecular dynamics simulations of
6 amorphous systems [54]. They have also been suggested for the EHL condition [7].
7 Phase-changes can cause inhomogeneous shear and would be expected for a liquid of
8 the viscosity used in this work [6]. Under the effect of shear, solidification should cause
9 significant changes in the stress and viscosity field and thus the velocity profile might
10 deviate from the linear shape predicted by the Reynolds equation. To further clarify the
11 effect of solidification on velocity profiles, a study investigating the influences of imposed
12 shear rate and normal pressure on the formation of inhomogeneous layers should be
13 carried out. This is a topic for future work. Experiments with low viscosity lubricants to
14 investigate the existence of similar phenomena at high shear rates will be carried out.

15 In EHL theory [1,2], the friction at any point in the contact is given by the through-
16 thickness integral of the shear stress. It is normally assumed that the shear rate is
17 constant (i.e. Couette flow). This allows the determination of the shear stress through
18 many of the available relationships that relate shear stress τ to shear strain $\dot{\gamma}$ using
19 viscosity μ : $\tau = f(\mu, \dot{\gamma})$. If the shear stress experienced by the lubricant is constant
20 throughout the film, the variation of the local shear rate through the thickness of the film
21 presented in this work implies that local viscosity heterogeneity exists. This means that
22 current EHL theory, which uses an average shear rate, is inadequate in accurately
23 predicting the viscosity in situations where inhomogeneous shear is prevalent. Thus, the
24 film thickness and friction predictions are invalidated as well. This also applies to any
25 analysis requiring knowledge of the flow profile, such as infrared temperature
26 measurements [12] used to determine surface stresses.

27 Data on through-thickness velocity profiles in thin films are scarce due to the
28 experimental challenge imposed by the thickness of EHL films. Through-thickness flow
29 profiles have been examined using similar molecular tagging velocimetry techniques
30 under Poiseuille flow conditions [31,39]. These results were however obtained with films
31 thicker than those encountered in EHL conditions, and thus may have little direct
32 relevance in EHL. In addition, the rate of diffusion in their systems could potentially be
33 prohibitively fast [26,30] to be applied to the determination of flow profiles in a thin film.
34 Therefore the results presented in this paper, while performed with a model system, are
35 unique and show that the methodology developed is capable of studying the flow of an
36 EHL film. While the details of the profile will obviously be governed by properties of the
37 fluid observed, the applicability of the approach is independent of the materials system
38 used.

39 The spatial resolution of the technique is on the order of 30 μm in the direction of flow (x-
40 direction and about 2 μm normal to the flow (y-direction). These values are defined as the
41 minimum observation area required for the velocity profile reconstruction scheme to make
42 good fits ($R^2 > 0.99$). For the slip velocity measurement, the resolution is approximately
43 20 $\mu\text{m/s}$ (determined by simulations testing the ability of the reconstruction scheme to
44 reproduce known velocity profiles) and thus about 10 nm for the slip length. The z-
45 resolution for velocity measurements is estimated to be at minimum two molecular layers
46 (~ 5 nm). It should be noted that the slip- and z-resolution cannot be directly measured
47 due to the use of a reconstruction scheme to obtain the velocity profile. The real
48 resolution could be found if known sub-micron velocity profiles were available for
49 comparison. The temporal resolution of the technique currently allows capturing transient
50 phenomena of about 15 ms. With image accumulation and averaging, the signal to noise
51 ratio of images increases and that allows better fitting of flow profiles. The temporal

1 resolution can be significantly improved with the use of shorter photobleaching times
2 (made possible with the use of mechanical chopper, and a more photosensitive dye [55]).
3 This would allow flow profiles of lower viscosity lubricants to be examined. The
4 resolutions of the technique compare well to other techniques. Both local slip length and
5 velocity profile can be obtained by conducting point-wise measurements. The average
6 slip length has been measured in an EHL contact by the dimple method [17]. Due to the
7 large difference in lubricant film thickness it is difficult to compare the resolution of the slip
8 length measurements between [17] and this work. Compared to slip measurements
9 performed by fluorescence recovery after photobleaching [25] and by double-focus cross
10 correlation [20] in other geometries, the slip length resolution is higher in this work, mainly
11 due to the use of a thin lubricant film. In regards to stimulated emission depletion
12 photobleaching anemometry [18] the spatial resolution in the x-direction and the temporal
13 resolution for this work is relatively low at the moment but can be improved as discussed
14 above. The resolution in z is however enhanced, mainly because no z-positioning is
15 required for this technique.

16 With the experimental and analytical protocol established, both the global and local
17 effects of various parameters, such as base oil properties, operating conditions (pressure,
18 entrainment speed and slide-roll ratio), and the use of additives on flow properties of the
19 lubricant system can be studied. The knowledge obtained will be helpful for developing a
20 new theoretical framework for EHL with inhomogeneous shear and will be useful for thin
21 film lubrication applications as well as other nanoscopic flows.

22 **8 Conclusions**

23 A novel methodology, based on the use of photobleached imaging, is presented to allow
24 the local through-thickness velocity profile and the slip length to be measured in an EHL
25 contact. The first measurement of the through-thickness velocity profile in an EHL contact
26 has been obtained in this work. The technique was first validated by successfully
27 reconstructing the expected linear velocity profile without slip for a micron thick film of PB
28 under Couette flow. It was then applied to an EHL contact where it was observed that PB
29 slips on the glass surface and takes on a non-linear velocity profile. The drastic change of
30 the velocity profile in the EHL contact cannot be caused by diffusion based on the large
31 Peclet number for the EHL case. Assuming that the shear stress experienced by the
32 lubricant remains constant throughout its thickness, the non-linear profiles obtained
33 suggest a variation in through-thickness viscosity. The large normal pressure
34 experienced by the lubricant causes an increase in the viscosity of the lubricant and
35 possibly a phase transition and hence the lubricant flows almost like a plug in the central
36 region of the film. Near the boundaries, the lubricant possesses lower viscosity which
37 could be due to energy dissipation and/or surface heating. This causes the velocity profile
38 to separate into three layers of varying shear rate, commonly referred to as
39 inhomogeneous shear. Pressure induced inhomogeneous shear has been shown to be
40 possible in theory and in molecular dynamics. It has also been seen in experimental
41 measurements of thicker pressurised films and complex fluids.

42 Since the technique can obtain the local velocity profile in the contact with point-wise
43 measurements, a velocity profile mapping was carried out. The results show that the local
44 slip length in the contact varies with probing position. This can be related to the variation
45 of pressure with location, and therefore the variation of the solid-liquid interfacial shear
46 stress.

47 The effects of slip and inhomogeneous shear on film thickness, friction and the general
48 analysis in EHL applications are significant and therefore the study and development of
49 theories describing these phenomena are necessary. This can only be done with the

1 assistance of experimental data which has until now been unavailable. While a viscous
2 fluid is used as model lubricant, the technique can be extended to study lower viscosity
3 lubricants by using a more photosensitive system and incorporating the effect of diffusion
4 in the velocity reconstruction scheme. Ultimately, the technique will be useful for in situ
5 rheology in EHL thin films and for velocimetry in submicron flows of viscous liquids.

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