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RECTANGULAR STEEL SILOS: FINITE ELEMENT PREDICTIONS OF FILLING WALL PRESSURES

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Abstract: The pressures exerted on the walls of rectangular planform steel flexible-walled silos by several different stored granular bulk solids are investigated using a validated finite element model that has been used in several previous studies. These pressures and the state of stress in the bulk solid are explored for a range of silo geometries and stored bulk solids. The results show that the horizontal pressure distribution across a silo wall is generally not uniform. This demonstrates that widely used theories may be adequate for stiff concrete silos, are far from suited to flexible-walled steel silos, and the differences can be used to produce much lighter structures. These findings match previously published experimental and analytical results for square planform silos where much larger pressures develop in the corners. The present analyses show that rectangular silos differ from those of square section, in that the mean pressure and degree of pressure variation is different on the two walls. The mechanisms causing these changes are investigated. The results further demonstrate that relatively small changes in the properties of a stored solid can produce significant changes in the pressure magnitudes and patterns, and hence greatly influence the silo structural design. The paper concludes that existing design guidance is seriously deficient and leads to metal silos that are considerably more expensive than is necessary.

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Introduction

Steel silos are constructed with a variety of planforms of which circular is the most common due to the apparently simple internal pressure regime which leads to shell structures predominantly under membrane stresses [1]. The structural design process appears to be simple and exploits the full strength of the thin shell. However, alternative structural forms such as rectangular planform can offer significant advantages in terms of both ease and cost of fabrication, as well as efficient use of space [2,3]. While stiffeners, or corrugated-sheet rectangular silos are also used in industrial practice, only the simpler and more common rectangular silos constructed from planar panels are considered here.

The stress resultants that support the loads from the stored bulk solid are different in each planform type. In circular planform silos, the high stiffness against radial deformations lead to small wall deformations [4,5] with the solid constrained to retain the same shape. By contrast, rectangular silos support the loads by a combination of bending and membrane actions, and the bending deformations may be relatively large if the wall is reasonably thin [2]. The larger deflections lead to reduced wall pressures on significant parts of the silo walls, so that more economic structural designs can be achieved by using moderately flexible walls. Finally, increased membrane action in these walls may reduce bending effects [6,7].

Pressure predictions in current silo design standards [e.g. 8-10] mostly use the theory of Janssen [11] as the reference condition. The vertical and horizontal pressures acting on the wall are derived from the equilibrium of a conceptual horizontal slice of stored bulk solid at each depth leading to a mean normal pressures against the wall given by (Eq. 1)

$$p_h = p_o\left[1 - e^{-z/z_o}\right] \tag{1}$$

in which z is the depth below the equivalent surface, γ is the stored solid unit weight and p_o is the asymptotic pressure at great depth given by

$$p_o = \frac{\gamma}{\mu}\frac{A}{U} \tag{2}$$

and the characterising depth z_o is given by

$$z_o = \frac{1}{K\mu}\frac{A}{U} \tag{3}$$

where the wall perimeter is U, the plan cross-sectional area is A, the wall friction coefficient is μ and the mean lateral pressure ratio is K. The hydraulic radius is given by A/U.

The mean pressure on the silo wall is deemed to only vary with depth, but depends on the parameters that characterise the bulk solid. However, Eq. 1 gives no information about any horizontal variation of pressure, and by default, it is assumed to be constant, though Janssen's [11] original paper proposed that the pressure at the mid-side of each wall in a rectangular silo might be higher [12]. The assumption of invariant horizontal pressure around the circumference of a circular silo is widely adopted, though significant asymmetry has been shown to develop even in symmetrically filled and discharged silos [e.g. 13-16]. By contrast, rectangular planform silos have a systematic asymmetry which can lead to large variations in horizontal pressure [17]. Experimental observations on a pilot scale square planform silo showed that a substantial reduction in wall pressure occurs at the mid-side if the wall is flexible [18,19]. These experimental observations were reinforced and accurately modelled by finite element calculations using a validated constitutive model for the stored bulk solid [20,7].

Many authors [21-28] have used the finite element method to predict the pressures in circular silos during both filling and discharge, but studies of rectangular silos are rare. A major review and comparative study of the finite element method was carried out by Rotter *et al* [29]. The effect of wall flexibility in circular silos was studied by Ooi and Rotter [24] where the critical role of the relative stiffness of the solid and the containing structure was demonstrated, and by Goodey *et al* [7] for square planform silos which made the same finding. The reduction in pressures at the mid-side of each wall in a rectangular silo was also shown by Goodey *et al* [30] in a preliminary study.

This paper presents a wider-ranging study of pressures on the walls of rectangular planform silos using the same validated finite element model [20].

Current design guidance

The notation and conventions used in this paper to describe rectangular planform silos are shown in Fig. 1 taken from Eurocode EN 1991-4 [8]. Pressures in rectangular silos with plan aspect ratios (a/b with $a>b$) of up to 2 are investigated. Silos with very high plan aspect ratios are generally classed as bunkers by design guides [31] and are often treated theoretically using the assumption of plane strain [32]. This implies that the pressure on the long wall is independent of the pressure on the short wall of a bunker, but this may not be the case for silos.

Reimbert and Reimbert [33] tested model-scale silos in which they measured the force on each wall, and assumed the pressure to be uniform, thus evaluating only the mean pressure. They drew the practical conclusion from their experiments that the pressure on the small wall, *b*, is close to that which would act on a square silo of side length *b*. By considering the vertical equilibrium of a slice of stored material, the pressure on the longer walls was deduced. This led to the pressure on the long walls as that acting on a square silo of wall side length b':

$$b' = b\left(2 - \frac{b}{a}\right) \tag{4}$$

where *a* and *b* are as indicated in Fig. 1. These dimensions were then applied using Janssen's equation with a constant value of lateral pressure ratio to deduce the mean pressure on each wall. Gaylord and Gaylord [31] reported a similar derivation of pressures based on the same assumptions but using different notation.

The Eurocode EN 1991-4 [8] only gives pressure predictions for rigid walled silos. For slender silos, these are based upon the Janssen equation and use the area to circumference ratio, A/U, which leads to the equivalence:

$$\frac{A}{U} = \frac{ab}{2(a+b)} = \frac{b}{2}\left(\frac{1}{1+b/a}\right) \tag{5}$$

A constant lateral pressure ratio, K, is assumed at all depths, which implies that the pressure on all walls is identical.

The Australian standard, AS 3774 [10] uses the Janssen equation with different horizontal dimensions for the long and the short wall, leading to different pressures on them. The adopted dimension of each wall varies with the wall ratio a/b, but there are clearly typographical errors in the associated table, making the result difficult to apply. It is unclear whether global equilibrium is maintained when this empirical device is used.

Pressures predicted by several codes and theories for a sample silo with wall ratio $a/b = 2$ (Fig. 2) show that a silo designer obtains quite different pressure regimes when using different sources. All the above current design codes imply that the wall of the silo is rigid and that the lateral pressure ratio is constant throughout. This may be acceptable for the stiffer walls of concrete silos, but for flexible walled steel silos such pressure regimes are inaccurate. Since the structural design is dominated by wall normal pressures, this assumption has significant implications both for structural continuity at the corners and for the wall strength requirements. If the longer wall is even slightly flexible, it moves outwards. When the corners are structurally stiff (i.e. identical wall rotations about a vertical axis through the corner) the shorter wall must move inwards pressing into the stored bulk solid. There is then an inevitable increase in short wall pressures and a decrease in long wall pressures. Thus, the flexibility of the walls can play a critical role in determining the differences in mean pressure on the long and short walls.

Existing finite element models

Since experimentally measured pressures are only known at discrete points and their interpretation is often difficult [34] and analytical pressure prediction involves very considerable simplifying assumptions, it is desirable to use the finite element method to devise reliable design rules. A number of research groups have applied the finite element method to silo pressure predictions with varying degrees of success when compared with both the theoretical and experimental evidence. Experimental observations have shown that a 3D model is required for a square silo, due to the variation of wall pressure across each silo wall [6,35]. An adequate description of the stored bulk solid behaviour is particularly difficult [36] and a number of authors have implemented acceptable limitations in order to produce tractable models [24,7]. Most previous finite element work on silo pressures has assumed the silo wall to be a rigid boundary, but in their analysis of circular silos Ooi and Rotter [24] demonstrated that the pressures could be governed by the relative stiffness of the granular solid and the wall, and that k (the *local* value of horizontal to vertical pressure) could vary throughout the stored bulk solid. However, they also showed that a circular silo wall had to be very thin and the granular solid rather stiff for the effect to make a significant change in the wall pressures.

For non-circular planforms, three-dimensional silo analyses demonstrate the complex patterns of wall pressure that occur in a silo. Guines *et al*, [25] investigated a 3D silo and showed that wall flexibility influenced both the predicted normal pressures and the location of the maximum wall normal pressure. Goodey [37] showed that the variation of k throughout the stored bulk solid could be quite large for filling pressures in both square and rectangular planform silos.

Finite Element Modelling

Validated Finite element model

Finite element models can produce predictions for both filling and discharge pressures, but the work presented here investigates only filling pressures. As a result the granular bulk solid has been modelled as a continuum, using a non-linear elasto-plastic stress-dependent constitutive law for the bulk solid. The validation was based on two major series of experiments on pilot scale silos [17,34] and extensive comparisons drawn to validate the finite element model by Goodey *et al* [20]. This validation also used a range of different stored bulk solids and silo geometries to guarantee its wide applicability.

Description of the FE model for a rectangular planform silo and stored bulk solid

The finite element model was constructed using the commercially-verified package, ABAQUS [38]. Taking advantage of symmetry, only one quarter of the structure was modelled (Fig. 3). The silo walls were modelled using 4-noded quadrilateral shell elements and the transition corner, where the box and hopper walls meet, was restrained against vertical displacement, with symmetry conditions on the vertical boundaries. The box was modelled as supported on columns at the corners with a pyramidal hopper below. The linear elastic structure was treated as mild steel with $E = 210$ GPa and $\nu = 0.3$.

The stored granular solid was modelled using 8-noded brick continuum elements. A Coulomb friction model was used for the interface between the solid and the wall, with a constant wall friction coefficient μ. Values are given in Table 1 for the constitutive model of the solids studied here. These solids were Leighton Buzzard sand (a widely-studied stiff sand), pea gravel and wheat. Values for the sand and pea gravel were taken from simple tests [35], while the values for wheat were taken from the triaxial tests of Ooi [39].

Five planform ratios were considered: $a/b = 1.0$, 1.1, 1.3, 1.5 and 2.0. The plan dimension of the short wall was maintained at 1.5m and the planform ratio varied by altering the length of the long wall. The corresponding hydraulic radii are respectively 0.25, 0.26, 0.28, 0.30 and 0.33. The height of these silos was fixed at 10m for this initial study, resulting in a relatively slender silo with vertical aspect ratio $h/b = 6.67$ [8].

The models all had a hopper below the box, with the transition junction between them. The boundary condition of a hopper or flat base can be represented in different ways, but the choice can have a major effect on pressure predictions above the transition [40]. A hopper is chosen here to minimise the influence on the box pressures. To avoid serious distortion of the adjacent elements, the hopper angle was set at approximately 45° with the outlet centrally located with respect to the planform. The mesh density was the same as that verified by the convergence tests in previous studies [37].

Findings from the FE calculations

Rigid walled rectangular silos - comparison to current design guidance

As current design codes assume the silo to have rigid walls, this condition was analysed first using a finite element model in which the wall nodes were fully restrained against displacement and rotation in all directions. The chosen bulk solid was Leighton Buzzard sand in a planform ratio $a/b = 2$.

The mean normal pressure on each of the long and short walls (Fig. 4) was calculated from the finite element output using Simpson's rule, and compared with the prediction of EN 1991-4 [8]. For rigid walls, the mean horizontal pressure on the long and short walls was the same, indicating that the pressure is invariant across each wall. The values were quite accurately predicted by this standard. Some end effects are apparent near the box/hopper transition where the average pressure deviates from that predicted by EN 1991-4 [8]. These end-effects have previously been noted by many other researchers, e.g. [41].

The predicted vertical stresses on a horizontal plane through the bulk solid are quite invariant, which confirms the assumption that the value of k may be treated as constant at any level. Because the stored solid is completely restrained by the rigid walls, very little plastic straining is observed and it is consequently reasonable that a single ratio of lateral to vertical pressure (such as K as assumed in EN 1991-4) is adopted in design.

The predicted wall pressures from the finite element model with rigid walls have shown good agreement with EN 1991-4 (Fig. 2) but indicate that both the simple theory of Reimbert and Reimbert [33] and that of AS 3774 [10] are probably in error, since both predict different pressures on the long and short walls. AS 3774 predicts a smaller difference between the long and the short walls, and for design purposes this difference may be insignificant, but Reimbert and Reimbert predict quite a large difference. It may be noted that Reimbert and Reimbert based their calculations on a small number of small scale experiments, and these tests may have been significantly affected by scale effects that can be very influential [42].

Flexible walled rectangular silos

In rectangular silos with flexible walls, the flexibility of the wall plays a significant role [17]. In a square planform silo, the normal wall pressure is higher in structurally stiff areas, such as the vertical corner of the box or at the transition. In the structurally flexible areas, near the mid-side of each wall, the pressure is lower. An extensive explanation of the mechanics of this load transfer was given elsewhere [19,20], but in simple terms an arching mechanism develops within the stored solid across the diagonals of the box, transferring vertical load from the flexible wall midsides to the structurally stiffer corners. This leads to much reduced pressures against the wall at the midsides. Initial studies [30] showed that a similar form of pressure distribution to that of a square planform should exist in rectangular planform silos. Using the planform ratio $a/b = 2$, the effect of varying the wall stiffness was explored by altering the wall thickness. To provide a clear exploration of the mechanics of wall flexibility, unstiffened plates with a very wide range of thicknesses were explored ($15 \leq b/t \leq 300$) for a silo filled with Leighton Buzzard sand. Initial predictions are shown in Fig. 5, where even a very high wall stiffness ($b/t = 15$) leads to a clearly evident disparity between the average wall pressure on the two walls, and only when the walls are rigid is there no disparity at all. The mean wall pressure on the long wall tends to be less than that predicted by EN 1991-4 [8]. It may also be noted that, by contrast with the predictions of both Reimbert and Reimbert and AS 3774, the normal pressure on the long wall is lower than that on the short wall. This is caused by the phenomenon outlined above, where the horizontal bending continuity of the corners of the silo structure reduce long wall pressures and increase short wall pressures. The outward deformation of the flexible long wall (shown schematically in Fig. 6) induces corner rotations about the vertical axis that lead to inward deformation of the short wall. Quite large displacements may exist when soft solids are stored in silos with thin walls. For example, for a ratio a/b of 2 and a/t of 100, displacements of $\delta/a = 1/455$ may develop in the longer wall and $\delta/b = 1/1606$ in the shorter wall. This results in lower (tending towards active) pressures on the long wall and higher (tending towards passive) pressures on the short wall.

An additional analysis, in which the flexural continuity at the corner was removed, modelling each wall as a separate plate, demonstrated that both walls then experienced outward deformations, producing lower pressures on the short wall than in the original calculation.

When the wall thickness is decreased, the variation of pressure across each wall at the depth of 5m in the box (Fig. 7) is exaggerated. For clarity, the distance along the short wall is plotted in the negative direction. At the box corner, the two orthogonal horizontal stresses in the solid differ in this flexible-walled silo. Since the value of K in silos is always far below unity, it is reasonable to assume that the major principal stress is vertical. The high pressures in the corner are consistent with the arching mechanism between diagonally opposing corners presented by Rotter *et al* [19].

Variation of stored material

The analyses were repeated using the properties of wheat and pea gravel. The variation of horizontal pressure across the wall at a depth of 5m is shown in Figs 7b and 7c respectively. Wheat is a significantly softer material than Leighton Buzzard sand, and the result is that whilst the same phenomena seen in sand are still to be observed, the magnitude of these effects is greatly reduced in the lighter, softer solid. The absolute magnitude of wall pressures is also reduced because the density of wheat is lower. By contrast, pea gravel, with the same density and other classical bulk solids properties as sand, behaves quite differently. Peak wall pressures are higher for the pea gravel that has very similar bulk properties to the sand, and the arching effect is more pronounced. It is clear that other parameters, such as the wall friction coefficient, have a significant effect on the wall pressures, and further investigation is needed. No predictive model of this effect exists in current design guidance.

Variation of planform ratio

The different planform ratios defined above were modelled using a flexible wall of thickness b/t = 100. This relatively thin wall might lead to geometrically nonlinear plate bending effects, but an exploration of this additional phenomenon is beyond the scope of the current paper.

The average wall normal pressures on the long and the short walls in sand are shown in Fig. 8, represented as a percentage deviation from the appropriate EN 1991-4 value. Again, the average pressure on the short wall is seen to be systematically higher than that on the long wall. With increasing planform ratio there is an increasing difference between the average pressures predicted on the long and short walls. As the ratio a/b increases, there is a consistent but smaller percentage decrease in average pressure on the long walls, but pressure increases on the short wall.

Comparison with predictive model for square silos of Rotter *et al* [19]

Rotter *et al* [19] proposed an empirical model based on experimental data that enables the redistribution of pressure at a horizontal level to be determined. This takes the form of a two-parameter hyperbolic function:

$$p = p_m \left(\frac{\alpha}{\sinh \alpha} \right) \cosh\left(\frac{2\alpha x}{L} \right) \qquad (6)$$

in which p_{m} is the mean wall pressure at any level, x is the horizontal distance from the wall centreline, L is the width of the silo side and α is a coefficient to be determined. The mean wall pressure p_{m} may be compared with the Janssen pressure and the value of α determines the level of pressure redistribution. Goodey *et al* [7] showed that the finite element method produced results that also closely followed this functional form in square planform silos.

Table 2 shows the values of α computed for a range of rectangular silos of planform ratio *a/b* from 1 to 2. The values of α all rise as the wall stiffness is reduced. An increase in effective wall stiffness is caused by either increasing the wall thickness or reducing the planform ratio. Even with a large wall thickness, the values of α may not converge for the short and long walls, and the values for the longer wall are always greater than those for the short wall.

Discussion

It is now widely accepted that the filling pressures in a rigid-walled silo are reasonably well predicted by Janssen’s equation, as it appears in various modified forms in different Codes of Practice and standard texts [1,8-10]. In flexible-walled silos its application is much less certain. Pressure variations in circular silos caused by ring stiffeners have been found (e.g. [24]), and the authors have shown that in square planform silos, the relative stiffness of the stored solid and the silo wall can play a major role in determining the pressure distribution. The predictions of a finite element model using an appropriate constitutive law have shown that the wall pressures and the stresses within the stored bulk solid are far from uniform in a rectangular planform silo.

Since the horizontal bending moments that develop in rectangular silo walls dominate the structural requirements for strength, and these moments are very sensitive to the mid-side pressure, it is clear that significant design savings can be obtained for these structures. The wall plate thickness can be reduced, leading to lower weight and more structurally efficient forms.

The development of design guidance to permit engineers to exploit these findings requires the changing distribution of wall pressures, characterised by α, to be identified for a wide range of solids, aspect ratios, wall thicknesses and depths in the silo. Many current industrial silos have horizontal stiffeners or corrugated walls, and their effect may also need to be addressed. The values of α must be related to the stiffness of the stored solid, together with advice on how that stiffness can be determined. These questions are far beyond the scope of this paper, but further work has already been undertaken which should lead to a full description of appropriate design calculations.

Conclusions

The predictions of a 3-D finite element model have been presented for filling pressures in a rectangular planform silo with walls having a range of systematically-chosen flexibilities containing three different stored bulk solids. The state of stress in the stored solid and the pressures imposed on the silo walls again confirm that the horizontal pressure distribution at any given depth is likely to be far from uniform.

The relative elastic stiffness of stored material and structural elements determines the distribution of horizontal wall pressures. The systematic FE-based study of non-circular silos enables the importance of this relative stiffness to be identified.

Two apparently similar granular solids have shown somewhat different behaviour and this indicates that the values used in models need to incorporate parameters that distinguish these stiffness characteristics.

The model presented for the prediction of pressure distribution shows that values of α, a simple expression to identify the pressure variation, give a good comparison to the data.

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Notation

A	cross-section plan area
a	width of the long silo wall
b	width of the short silo wall
b'	equivalent side length (Reimbert)
$\mathrm{d_c}$	characteristic diameter
e	initial voids ratio
E	modulus of elasticity
h	height of silo
k	local ratio of horizontal wall pressure to mean vertical stress in solid
K	ratio of mean horizontal wall pressure to mean vertical stress in solid
L	Length of silo side
t	wall thickness
p	normal pressure against silo wall
p_{h}	mean normal wall pressure
p_{o}	asymptotic wall pressure at great depth
$\mathrm{p}^{\mathrm{el}}_{\mathrm{t}}$	elastic tensile strength of stored material
U	wall circumference
x	horizontal distance from the wall centreline
Z	depth below surface
$\mathrm{Z_0}$	maximum depth below surface
α	coefficient of pressure non-uniformity
γ	stored solid unit weight
ψ	angle of dilation
κ	log bulk modulus
ν	Poisson’s ratio

μ	wall friction coefficient
φ_i	internal angle of friction
σ_c	initial yield stress of stored material

Table 1 Parameters required for constitutive law, the assumed values and the source of these values

Parameter	Leighton Buzzard sand (Lahlouh *et al*, 1995)	Pea Gravel (Lahlouh *et al*, 1995)	Wheat (Ooi, 1990)
Logarithmic bulk modulus, κ	0.002	0.003	0.015
Poisson's ratio, ν	0.316	0.306	0.369
Initial voids ratio, e	0.67	0.555	0.80
Elastic tensile strength, P^{el}_{t} (kPa)	0	0	0
Internal angle of wall friction, ϕ_i (°)	45.1	46.1	39.1
Angle of dilation, ψ (°)	0	0	0
Initial yield stress, σ_c (kPa)	0.25	0.25	0.25
Initial Bulk density, γ (kg/m^3)	1587	1704	761
Coefficient of wall friction, μ	0.445	0.392	0.440

Table 2 – Values of α for rectangular planform silos of different ratios; stored material Leighton Buzzard sand

Wall Thickness (mm)	1:1	1.1:1 Short wall	1.1:1 Long wall	1.3:1 Short wall	1.3:1 Long wall	1.5:1 Short wall	1.5:1 Long wall	2:1 Short wall	2:1 Long wall
20	1.79	1.78	2.01	1.64	2.39	1.61	2.46	1.05	2.62
30	1.19	1.19	1.36	1.22	1.74	1.25	2.07	1.24	2.46
50	0.63	0.62	0.72	0.67	0.97	0.71	1.23	0.87	1.83
100	0.30	0.29	0.32	0.38	0.41	0.37	0.51	0.45	0.86

List of figures

Figure 1 – Notation used for rectangular planform silos

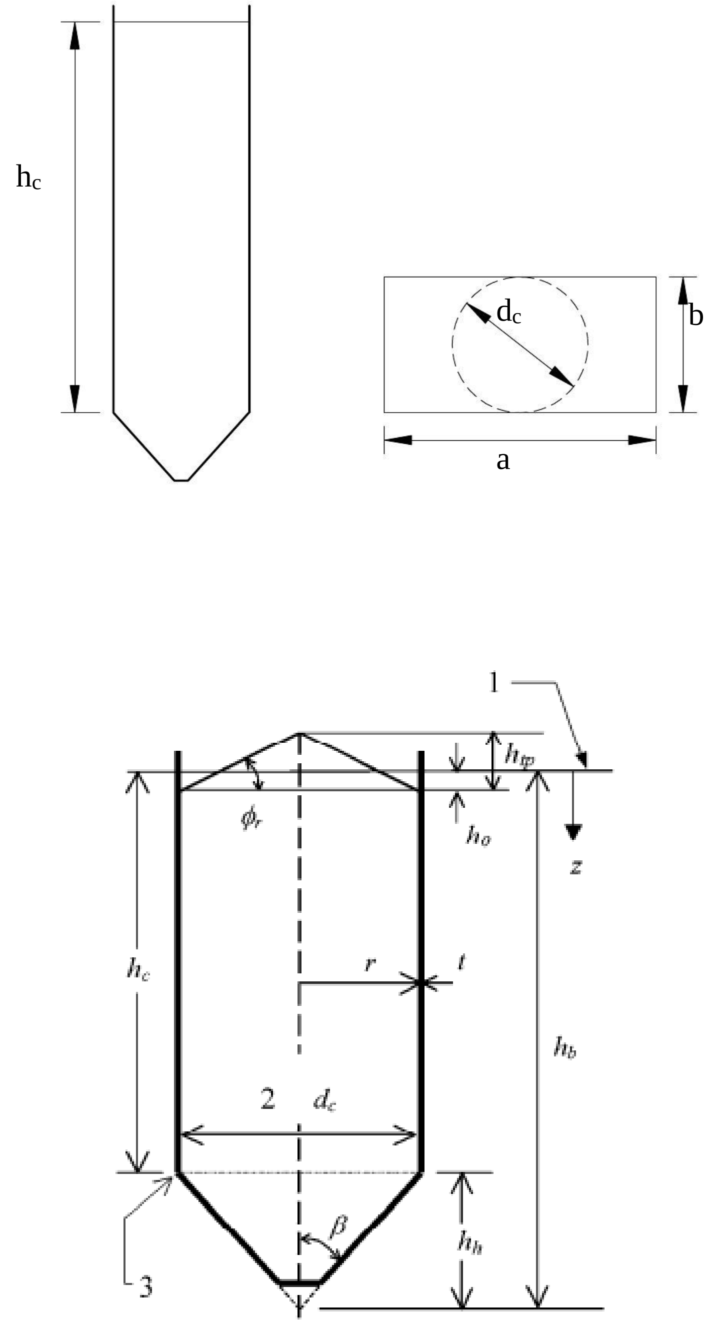

Figure 2 - Pressure predictions from codes and Reimberts' (R&R) design guides

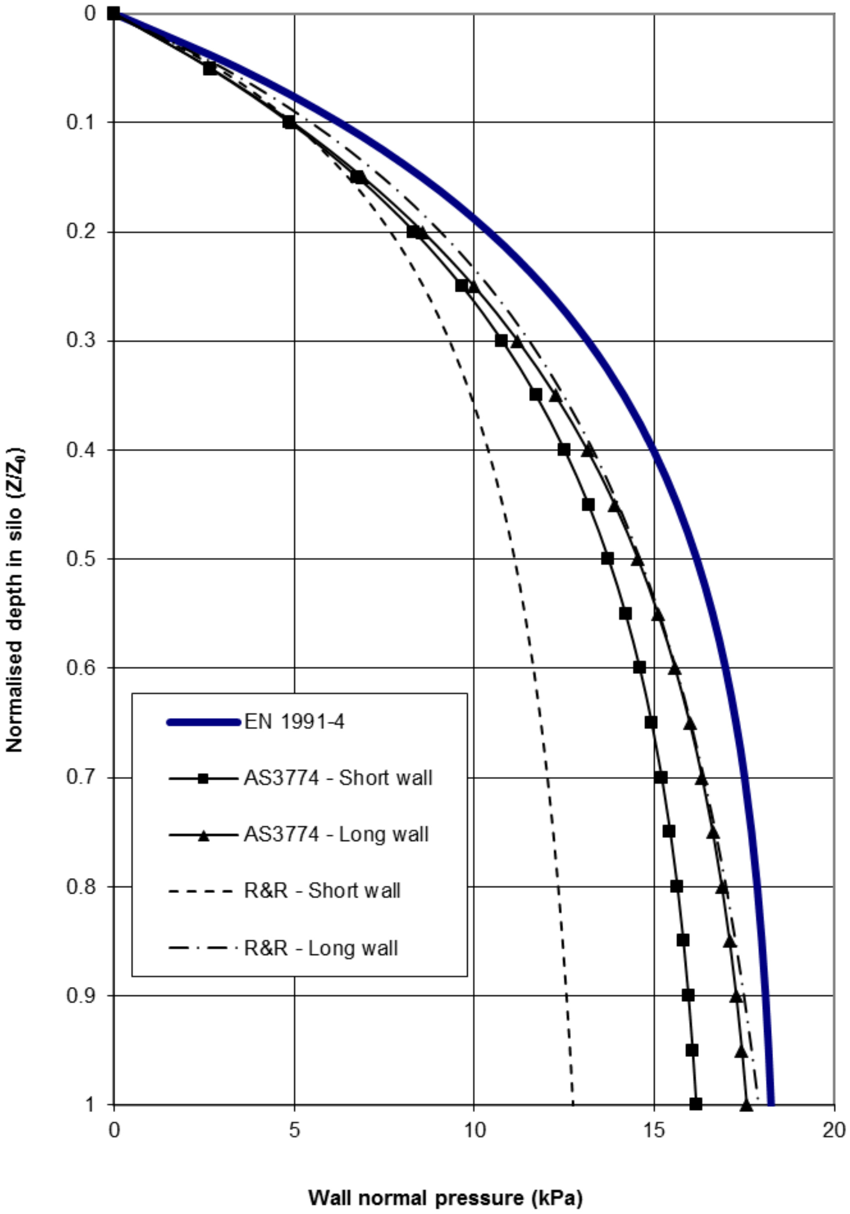

Figure 3 – View of finite element model

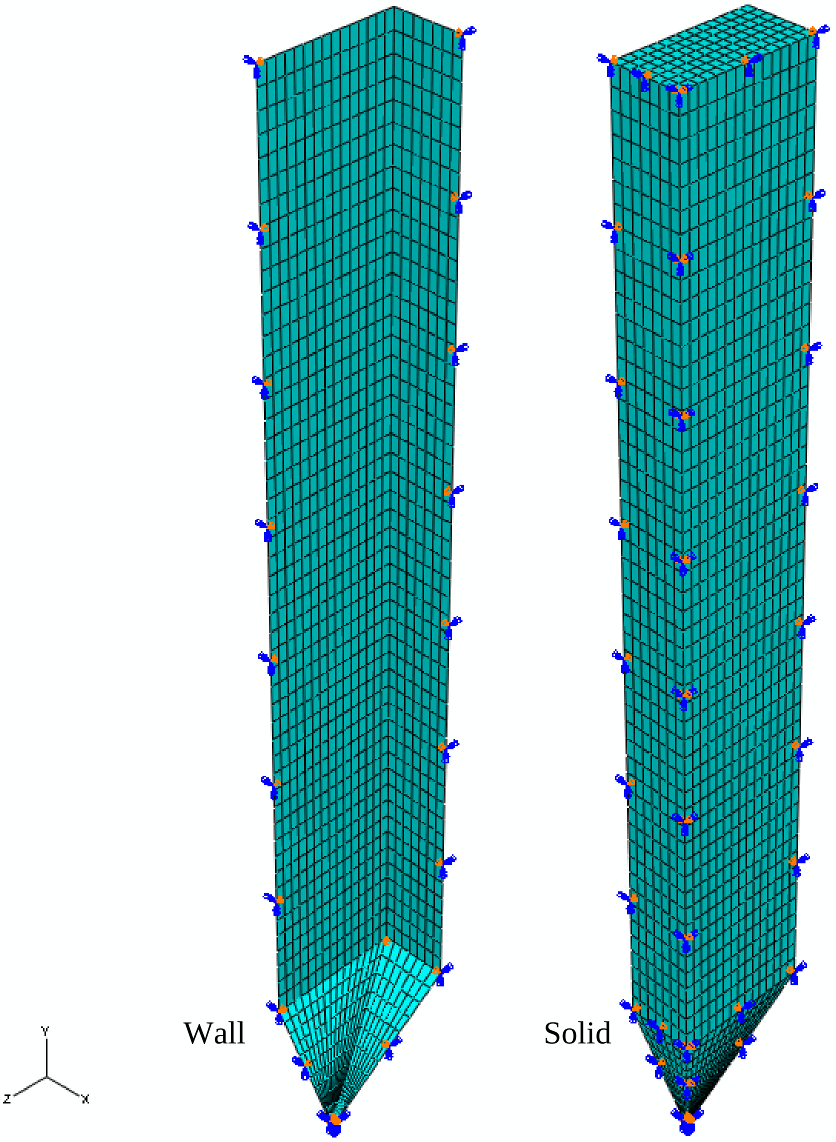

Figure 4 – Pressure predictions in rigid wall silos

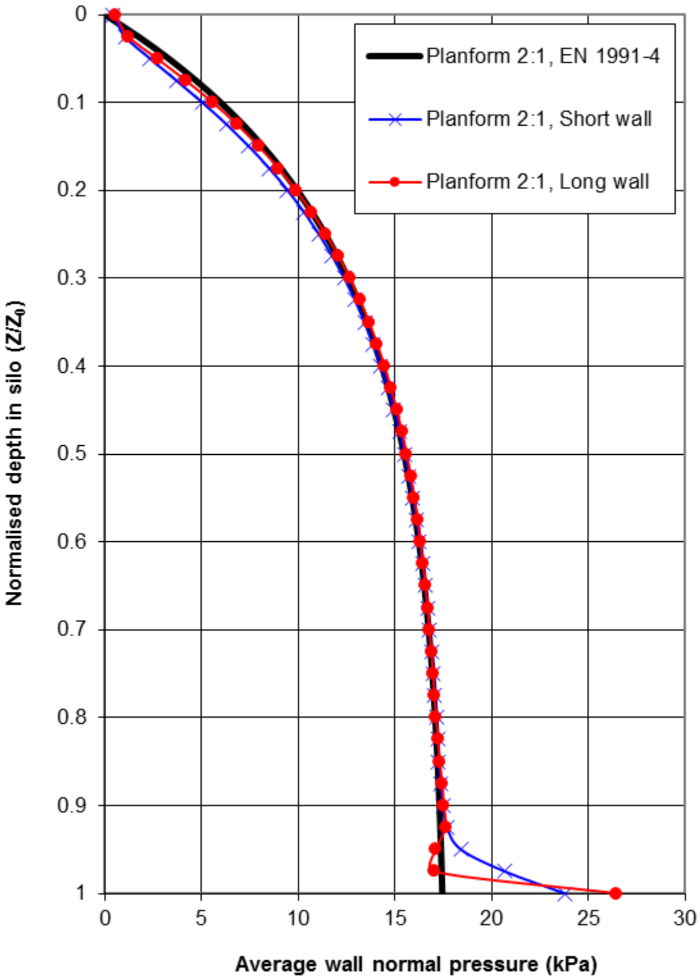

Figure 5 – 2:1 Ratio flexible walls, sand fill

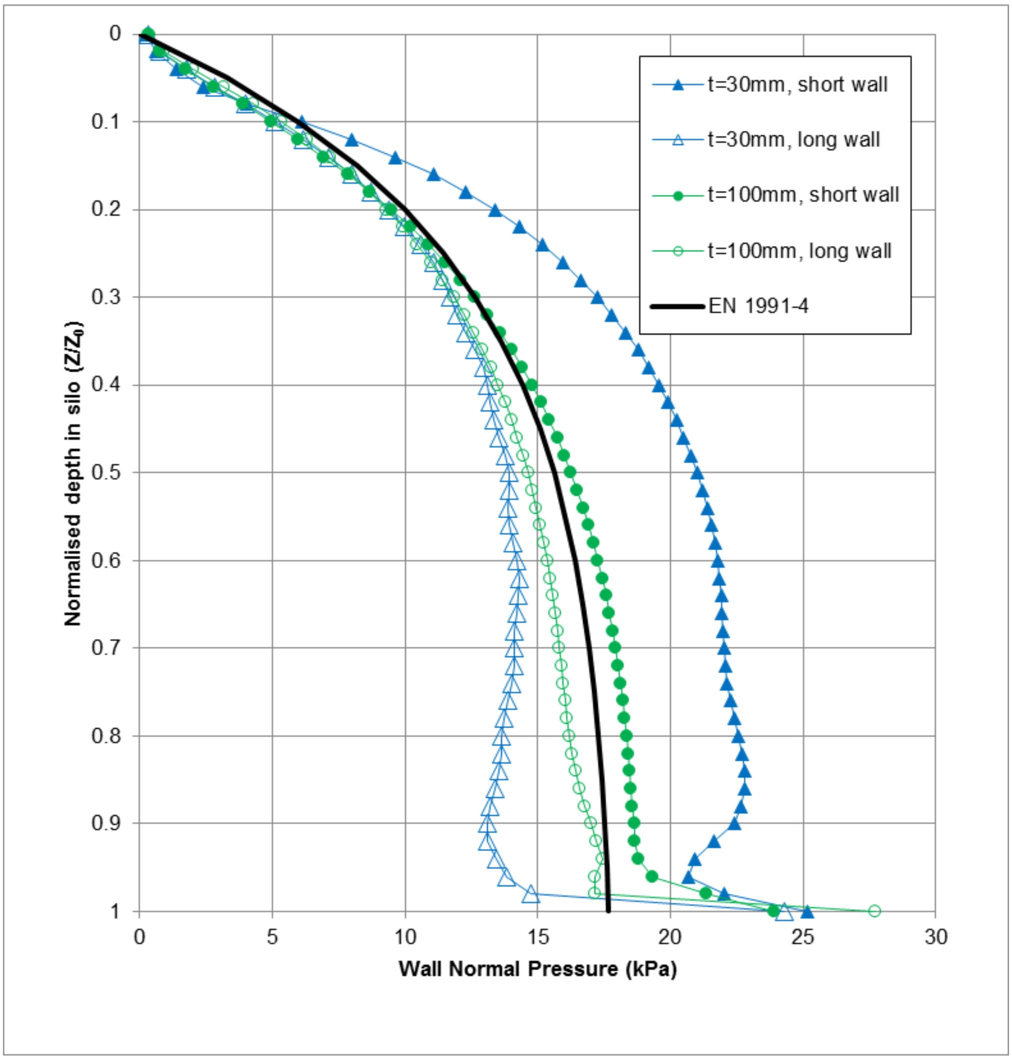

Figure 6 – Wall normal displacements at 5m depth in a 2:1 planform ratio bin

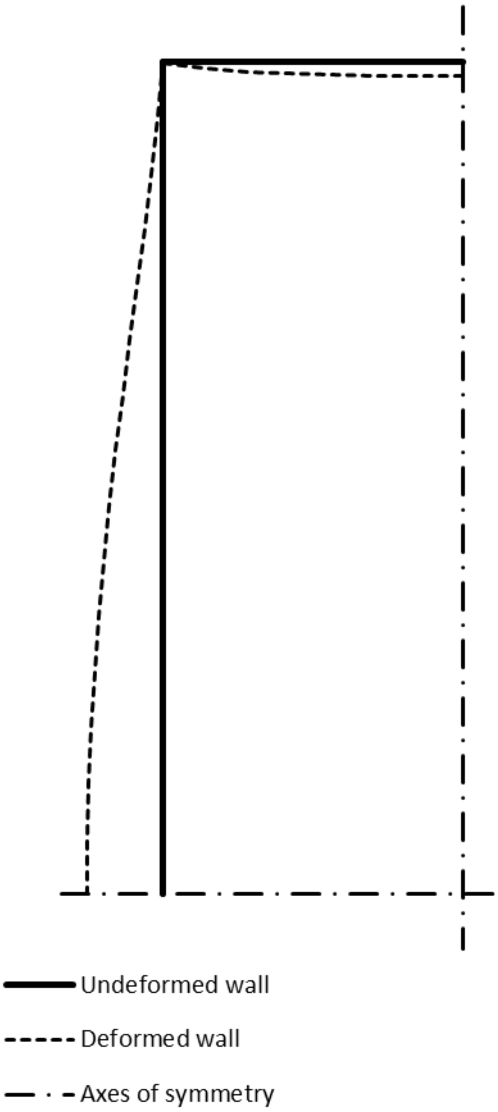

Figure 7a – Variation of pressure across walls 5m depth, 2:1 ratio, different t, sand fill

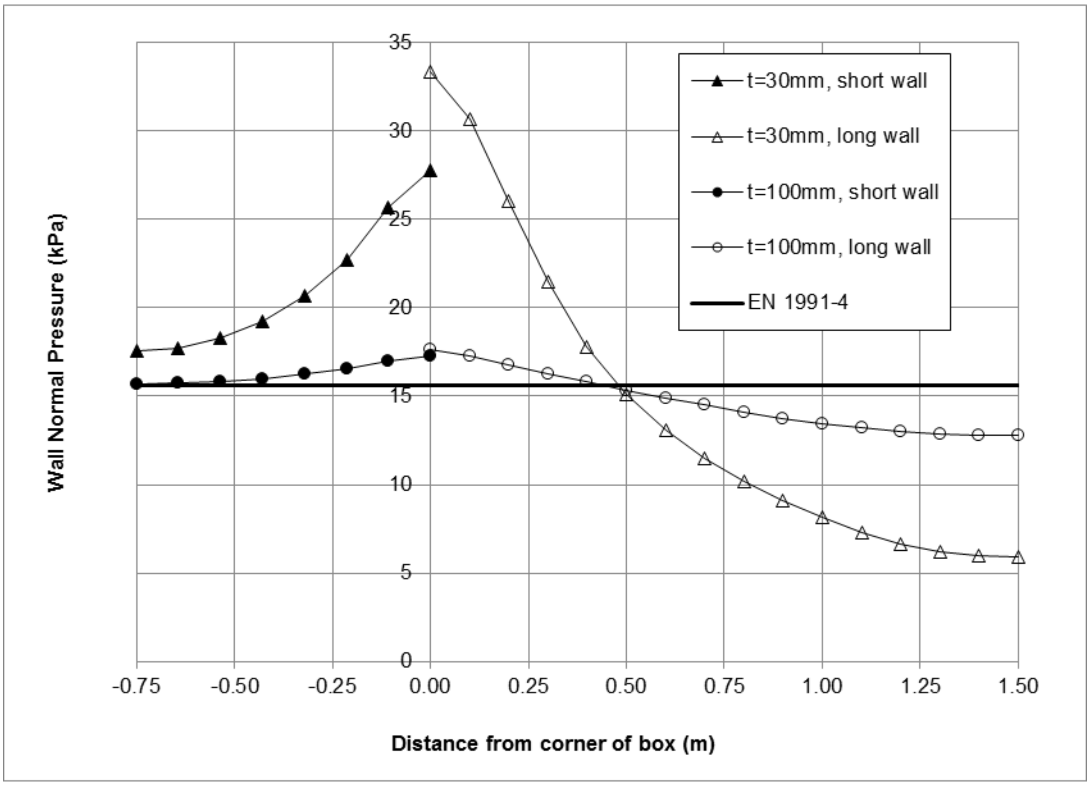

Figure 7b – Variation of pressure across walls 5m depth, 2:1 ratio, different t, wheat fill

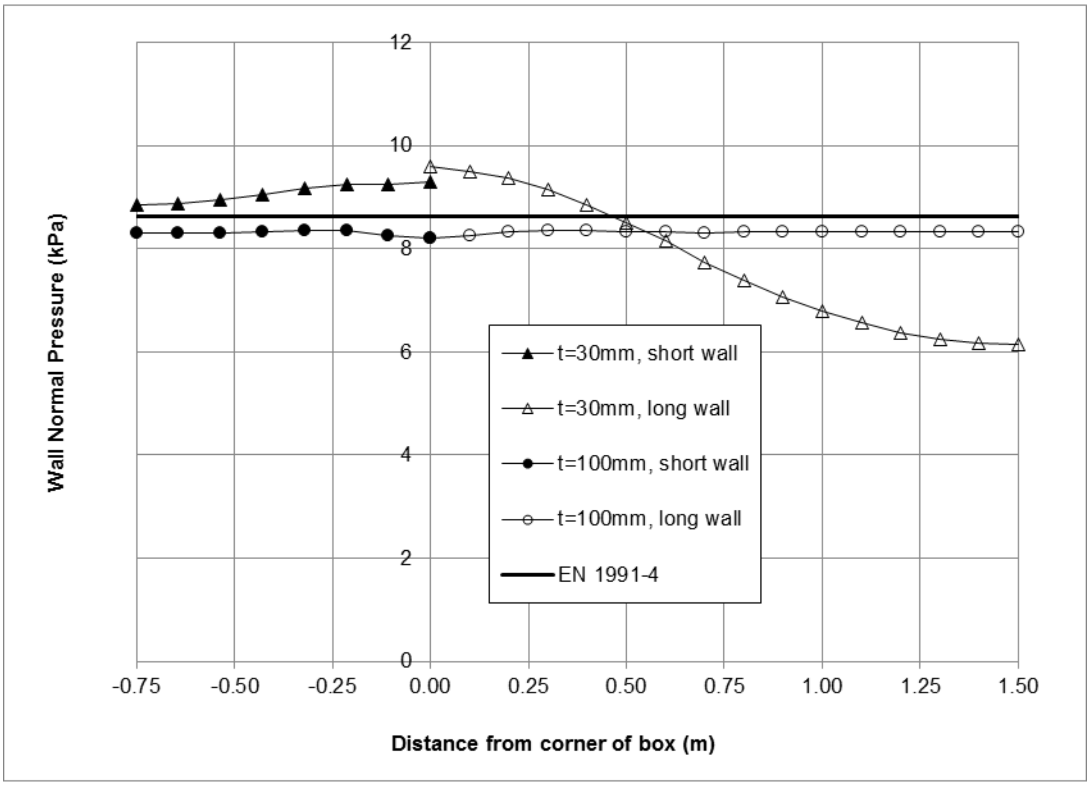

Figure 7c – Variation of pressure across walls 5m depth, 2:1 ratio, different t, gravel fill

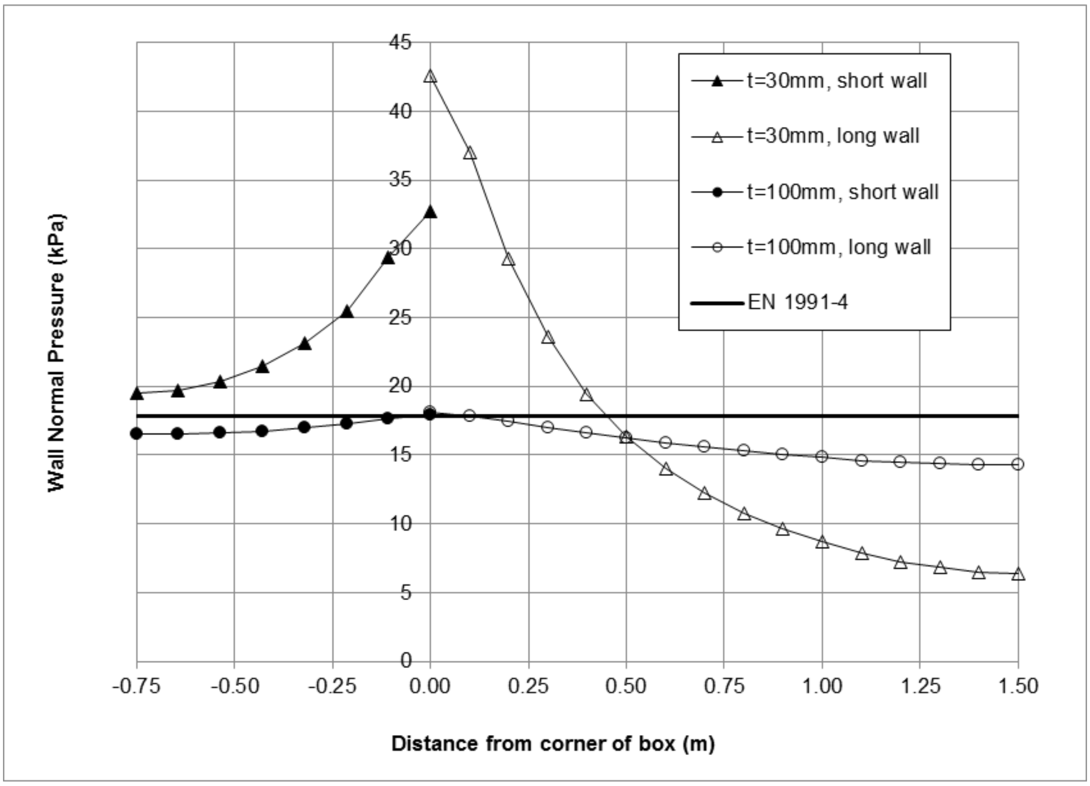

Figure 8 – Different planform ratios, t=30mm, deviation from EN prediction