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## Third Revision (R3)

## Geometrical Features underlying the Perception of Collinearity

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#### Abstract

The magnitude of the Poggendorff bias in perceived collinearity was measured with a 2 AFC task and roving pedestal, and was found to be in the region of 6-8 deg, within the range of previous estimates. Further measurements dissected the bias into several components: (1) The small ( $\sim 1 \mathrm{deg}$ ) repulsion of the orientation of the pointer from the parallel, probably localized in the part of the line near the intersection (2) A small ( $<1 \mathrm{deg}$ ) location bias affecting the intersection of pointers and inducing lines; and (3) A larger (> 1 deg) bias in the orientation of virtual lines crossing the gap between two parallels, towards the orientation of the parallels, or equivalently (4) An orthogonal bias in actively constructing a virtual line across the gap. We conclude that orientation repulsion by itself is an inadequate explanation of the Poggendorff effect, and that a full explanation must take account of the way in which observers construct virtual lines in visual space in order to carry out elementary geometrical tasks such as extrapolation.


## Introduction

It is not understood how the visual system makes elementary geometrical constructions, such as measuring the collinearity of separated line segments (Morgan, 1999, Ninio, 2014) We should not be surprised, then, that we also fail to understand the causes of biases in perceived collinearity, such as the bias shown in the famous Poggendorff figure (Fig. 1). Most readers will see the two 45 deg pointers more aligned in the right-hand configuration than in the left, although the opposite is actually the case.

Conceptual confusion has resulted when variants of the basic Poggendorff figure are called the 'Poggendorff illusion' and are assumed to have the same mechanism (Hotopf and Hibberd, 1989; Ninio, 2014). Such variants include amputations of lines, replacements of lines by dots, replacement of lines by subjective contours (Tibber, Melmoth, and Morgan, 2008), figures emphasizing perspective cues (Gillam, 1971) and horizontal rotation of the Poggendorff figure itself, which shows a smaller bias than the upright version (Hotopf and Hibberd, 1989). In this paper we renounce the term 'illusion' in favour of 'bias' and we refer to the 'P-bias' as any bias in the perception of collinearity in the same direction as that seen in the traditional, upright 4-line Poggendorff figure. A simple mnemonic for remembering the direction of the P -bias is that it is in the direction expected if the left-hand pointers in Fig. 1 are mentally rotated to appear more orthogonal to the parallel. It must be emphasized that this is merely a convenient description of the bias, not an explanation. An alternative description is that the virtual angle between the two intersection points is mentally rotated in the

## anticlockwise direction, making the right-hand pointer appear

displaced upwards. If the pointers are replaced with circles (c.f. Figs

## 4 and 11 below) this allows us to describe a bias using the same metric as a P-bias.

Fig 1 about here
The figure shows examples of stimuli used to measure the Poggendorff perceptual bias. The observer's task (2AFC) was to decide whether the oblique pointers were more aligned in the left-hand figure or on the right. In the example shown the pointers in the left-hand figure are closer to physical alignment, but a perceptual bias (the Poggendorff effect) makes them appear less aligned. In the experiments both stimuli could be given a pedestal misalignment (the same for both figures) to which was added a test misalignment for one of the figures, randomly left or right. Thus the test stimulus could be either closer to alignment or further away, depending on the pedestal level.


A large variety of values for the P-bias are reported in the literature. Sometimes the effect is reported in terms of the apparent displacement, in units of DVA (degrees of visual angle) of one of the pointers from the point of true collinearity (e.g. Hamburger, Hansen \& Gegenfurtner, 2007). If the origin of the P -bias is a mispointing by one or other of the pointers (e.g. Hotopf \& Hibberd, 1989; Ninio, 2014; Ninio \& O'Regan, 1999) the DVA measure will vary with the pointer angle and the separation of the parallels. An alternative measure is the apparent rotation of one or both of the pointers (in radians or deg) inferred from the shift expressed as DVA. Using this measure, Morgan (1999) reported P-biases in the region of 5 deg. ( 0.0873 rad) Hamburger et al.(2007) report DVA shifts for one pointer in the Method of Adjustment of $\sim 1$ deg DVA. Using the information that the DVA between the verticals was 3.1 deg and the angle of the pointer 52.5 deg (K. Hamburger, personal communication) their shift can be expressed as a mispointing of 6.9 deg, similar to that in Morgan (1999).

The P- bias almost certainly has several distinct causes (Hotopf and Ollerearnshaw, 1972; Hotopf, Ollerearnshaw, and Brown, 1974). Some insight into the possible causes of the bias can be gained by stating the computational requirements of a distant alignment task (Morgan, 1999). These include (1) measuring the orientation of the two obliques and determining that they are the same, (2) locating the proximal terminations of the pointers (i.e. their terminations on the inducing line) (3) measuring the orientation of the virtual line between the two proximal pointer terminations and,finally (4) comparing the results of steps (1) and (3). Biases in (1) may arise
from cross-orientation inhibition (Blakemore, Carpenter, and Georgeson, 1970). Biases in (2) have been predicted from optical (Glass, 1970) and neural (Morgan, 1999) blurring. Biases in (3) could arise from unknown causes, including one that Hotopf and Hibberd (1989) call the 'horizontal bias alignment effect'. Biases in (4) have not been previously considered, and we keep with this tradition.

An alternative to this Cartesian approach is to consider an analogue process of extrapolation, which bridges the gap between the parallels by linking together local units that have the same orientational specificity as the pointers, and which are preferentially linked in a direction that is similar to that of their local specificity. Such a linking has previously been postulated as an 'association field' (Field, Hayes, and Hess, 1993) or as a 'collector unit' (Morgan and Baldassi, 1997; Morgan and Hotopf, 1989) to explain the Fraser 'twisted cord' effect, and the appearance of 'spiderweb' lines in grids and lattices. This kind of explanation differs from the Cartesian in that it does require spatial position of features to be made explicit or compared, but as we shall see, it is logically difficult to distinguish from the Cartesian model in any particular case with purely psychophysical data.

In this paper, we concentrate on biases in Steps 1, 2 and 3. Biases in location of the intersection points could result from neural blurring in first (Glass, 1970) or second-order filters (Morgan, 1999) that place the centroid of the blurred intersection inside the acute angle. One line of evidence supporting blurring is that increased optical
blurring or low-pass filtering enhances the magnitude of the P-bias in the Poggendorff figure, as well as in its acute-angle and obtuse-angle amputated versions (Morgan, 1999). Evidence for a location shift was also found (Morgan, 1999) using the rather difficult task of matching the perceived orientation of the virtual line between the two intersections to that of a grating.

In the present experiments we measure the P -bias in various configurations using a 2AFC task designed to distinguish a genuine perceptual bias from a response bias or deliberate criterion shift (Morgan, Melmoth, and Solomon, 2013). The task is explained briefly in the legend to Fig. 1. Its essence is that the offset from collinearity in the test figure is added to a pedestal in both test and comparison figure, so that it can either reinforce or counteract any perceptual bias depending on the pedestal level, which is varied over trials and is unknown to the observer. Thereby, the observer is prevented from feigning a perceptual bias by a strategy such as 'response on left button if unsure' or 'respond to test if unsure' (Morgan, Dillenburger, Raphael, and Solomon, 2012). The task is a genuine 2AFC, as opposed to the Method of Single Stimuli (Morgan , Watamanuik, and McKee, 2000), with which it is frequently confused (e.g. Taya, Adams, Graf and Lavie, 2009).

We used the 2AFC task because we thought it important that participants should be unable to infer the true point of collinearity in the figures from repeated trials. Learning of this kind may explain the decrement in biases that is commonly reported with the Geometric Illusions over time (e.g. Predebon, 2006). Since we intended to use
the same participants over a large variety of conditions, we were concerned to avoid this learning. Using the Method of Single Stimuli it is difficult to choose the range of values with which the participant is presented. If the range is centred around true alignment, the observer can soon infer a bias from their distribution of responses between the two buttons and adjust accordingly (Morgan, Watamaniuk and McKee, 2000); if one the other hand, it is centered around the putative Point of Subjective Equality there is a risk of petitio principii. The Method of Adjustment, which is probably the most widely used method in the field (e.g. Ninio \& O’Regan, 1999; Weintraub et al., 1980; Predebon, 2006; Blakemore et al., 1980; Morgan, 1999) avoids this difficulty, but allows the observer some degree of experimentation with the figure, in conjunction with scanning eye movements, which may not be altogether desirable. In our 2AFC Method the observer never knew which of the two figures was in reality 'more aligned', and any perceptual bias would have no effect on the distribution of responses between the two categories 'left more collinear' or 'right more collinear'. Pilot studies (Morgan et al., 2014) showed that the Method produced stable results over repeated testing.

Five Experiments will be reported:

1. Measurement of the basic P-bias by pointer collinearity.
2. Measurement of positional bias in proximal pointer terminations.
3. Measurement of bias in pointer orientation (Blakemore et al., 1970).
4. Measurement of the spatial integration region for orientation at the proximal pointer terminations.
5. Measurement of the P-bias without pointers.

## Materials and Methods

Apparatus and Stimuli. In experiments carried out in City University London, stimuli were presented on the LCD display of a MacBookPro laptop computer with screen dimensions $33 \times 20.7 \mathrm{~cm}$ ( $1440 \times 900$ pixels) viewed at 0.57 m so that 1 pixel subtended 1.25 arcmin visual angle (VA). The background screen luminance was $50 \mathrm{~cd} / \mathrm{m}^{2}$. In Cologne, stimuli were presented on the screen of SONY Trinitron monitor with resolution $1400 \times 1050$ pixels and viewed at 75 cm so that 1 pixel subtended 1.33 arcmin . The background screen luminance was a neutral gray $16 \mathrm{~cd} / \mathrm{m}^{2}$, with average luminance of the stimulus components being $49 \mathrm{~cd} / \mathrm{m}^{2}$. Stimuli were generated by MATLAB and PSYCHTOOLBOX PTB3 software (Brainard, 1997). The dimensions will be stated when describing the individual experiments.

Subjects. A total of 9 subjects participated in the experiments. The subjects in London were AJ, a male PhD student and JS, an experienced psychophysical observer. In Cologne the subjects were one of the authors (BD), two naïve paid subjects (DW and MK), a psychophysically experienced postdoctoral fellow (KS) and an experienced PhD student NN. Two subjects carried out some conditions in Cologne and some in London: MM (author) and JF, a PhD student. Informed consent was obtained prior to inclusion and procedures were in accordance with the Declaration of Helsinki.

Procedure. On each trial the observer was presented simultaneously with two figures, the test and the reference, and had to make a decision, for example, in which of the two figures the pointers were more collinear. The test was randomly positioned on the left or the right. In some conditions, to be specified, the test and reference were preceded by a standard, to show, for example, what collinearity looked like. The decision was indicated by pressing ' 1 ' or ' 2 ' on the computer keyboard. On each trial the relevant physical attribute of the figures, for example the collinearity, was perturbed by one of three pedestal levels applied to both test and standard; and in addition by a cue chosen from 9 present levels and applied to the test figure only. Finally, two different contexts were randomly interleaved (with each trial containing only one context), for example, the presence or absence of vertical lines in the Poggendorff figure. Each combination of pedestal, cue and context was sampled randomly without replacement until there had been 5 trials in each, making a total of 270 trials per block. Every 50 trials the observer was invited by a screen message to take a rest before pressing the space bar to present the next stimulus.

Data fitting to extract bias and precision. Within the context of SignalDetection Theory (Green and Swets, 1966), appearances of the standard and target can be described by normal distributions $S$ and $T$, such that $S \square N\left(p+,{ }^{2} / 2\right)$ and $T \square N\left(p+t+,{ }^{2} / 2\right)$, where $\sigma^{2}$ is the variance of the performance-limiting noise, $p$ and $p+t$ represent the physical tilts of standard and target, and $\mu$ represents any perceptual bias, such as may be induced by the context $f$. Note that all tilts are signed, such that negative values represent clockwise tilts. Given
these definitions, the probability of choosing the standard in our comparison-of-comparisons task is given by (Morgan, Grant, Melmoth, \& Solomon,,2015)

$$
\begin{align*}
\operatorname{Pr}(" \mathrm{~S} ") & =\operatorname{Pr}(|S|<|T|) \\
& =\operatorname{Pr}\left(\frac{S^{2}}{T^{2}}<1\right) . \tag{1}
\end{align*}
$$

Note that $S^{2} / T^{2}$ is a random variable having a doubly non-central $F$ distribution. Its denominator's noncentrality parameter is $2(p++t)^{2} /{ }^{2}$, its numerator's noncentrality parameter is $2(p+)^{2} /{ }^{2}$, and both denominator and numerator have 1 degree of freedom.

The observed response probability density functions from each context were fit separately to (1) using the Matlab function spncf to extract the maximum likelihood estimates of $\mu$ and $\sigma$ under the two contexts.

## Experiments

## Experiment 1. Measurement of the basic P-bias in collinearity

## Experiment 1.1

The purpose of Experiment 1.1.was to obtain a baseline measurement of the P-effect and to validate the 2AFC Method, using a configuration where the right-hand pointer of the traditional vertical Poggendorff figure was replaced by a dot. Results showed no systematic differences over observers between the two different experimental setups ( 1.33 arcmin pixel size in Cologne vs 1.25 in London). The stimuli are illustrated in Fig. 2. Each trial began with a 1 s presentation a reference stimulus with a 45 deg pointer and aligned dot... This was followed by a test and a standard with the test
randomly on the left and the right of the screen. The task was to decide which of the two stimuli was aligned more like the reference.

The pointer orientation was 45 deg . The length of the verticals was 600 pix ( 12.5 deg VA ) and their separation one quarter of this ( 3.125 deg VA). Pointer length was 70 pixels ( 1.47 deg VA). The relative vertical position of the pointers in the left-hand and right-hand stimulus was randomly perturbed over trials in the range $+/-1.47$ deg VA to avoid horizontal alignments being used as cue surrogates. The line thickness for both pointer and inducers (verticals) was 5 pix (6.25 arcmin).

On each of 270 trials the orientation of the virtual line joining the pointers across the gap between the verticals was perturbed by one of three pedestal rotations ( $-4,0$ or +4 deg ) and (in the case of the test figure) by a cue rotation ( 9 values between -4 and +4 deg). Trials with the control (no verticals) and experimental (verticals present) context were randomly interleaved.

The right-hand pointer of the traditional Poggendorff figure was replaced by a filled circle (radius=6.325 arcmin ) centered on the right-hand vertical.

Fig. 2 Caption The figure illustrates the stimuli in Experiment 1.1. The top row shows stimuli for the control condition; the bottom row shows the experimental condition. The reference stimulus is shown on the left; the test and standard on the right. For explanation of the task see the text on Experiment 1.1. Note: in the experiment the reference stimulus was the same size as the test and standard.


The results ( Fig. 3 )showed a large P-bias: -8.6 deg (Mean) and -5.95 $\operatorname{deg}$ (Median); and the probability of the distribution of observed $\Delta$ scores on the null hypothesis that $\Delta=0$ is $\mathrm{p}=0.0215$ (paired t-test; $t(8)=2.85)$.

Fig. 3 legend. Panels 1-9 show the results for individual participants in Experiment 1.1. The bottom right panel ('Mean') shows the means over all participants. Within each panel, the first column ( $\mu 1$ ) shows the maximum-likelihood estimate of the bias in the control condition, the second column ( $\mu 2$ ) represents the bias in the experimental condition; the third column ( $\sigma 1$ ) represents the standard deviation ( $1 /$ slope) of the psychometric function in the control condition; the fourth column ( $\sigma 2$ ) represents the standard deviation( $1 /$ slope) of the psychometric function in the experimental condition; and the final column ( $\Delta$ ) represents $\mu 2-\mu 1$, the net bias. The error bars show the inter-quartile range ( $25 \%-75 \%$ ) of values obtained from parametric bootstrapping. Negative biases are in the direction expected from a P-bias (see text for further explanation). The asterisks indicate a significant difference between $\mu 1$ and $\mu 2$ ( ${ }^{* * *}$ , $\mathrm{p}<0.001$ ).


Experiment 1.2. The purpose of Experiment 1.2.was to examine the vertical version of the Poggendorff figure with two pointers, and to compare the P-bias found in the previous experiment (1.1) with a single pointer. The results show a strong negative bias in all participants, which is in the direction expected from the P-bias. The net effect (experimental - control) was -6.46 deg (mean) and -5.1 deg (median), and the probability of the distribution of observed $\Delta$ scores on the null hypothesis that $\Delta=0$ is 0.0106 (paired t-test, $\mathrm{t}(8)=3.32$ ). Values of $\sigma$ of $\sim 5$ deg are similar to those reported by Morgan et al., (2013) and were not significantly different between experimental and control conditions. Values of $\mu$ and $\sigma$ are the same order of magnitude, as again is typical for geometrical biases such as the Muller-Lyer (Morgan , Hole, and Glennerster, 1990) with $\mu$ in this instance being greater. The bias was not significantly different from that of Experiment 1.1, in agreement with the findings of Weintraub, Kranz \& Olson (1980), so we cannot exclude the possibility that a single pointer is sufficient for the full P-bias. This suggests that one origin of the P-effect is a mispointing or misangulation of the pointer, in agreement with findings and analysis of Ninio \& O'Regan (1999) and Ninio (2014).

## Experiment 1.3

The purpose of Experiment 1.3 .was to check that the results of Experiment 1.1 were not due to having 'upwards' pointers. The experiment was the same as Experiment 1.1 except that the pointers were pointing downwards instead of upwards, and participation was restricted to bd, dw, ks, jf, mm and nn. Results (not illustrated)
showed a net bias ( $\Delta$ ) of -4.5 deg (mean) and -4.62 deg (median).
The difference in $\Delta$ values between Experiment 1.1 and 1.2 was not significant ( $\mathrm{p}=0.76$; $\mathrm{t}(5)=0.318)$ ).

## Experiment 2: Measurement of positional bias in proximal pointer

## termination

The purpose of Experiment 2. was to see if there was a bias in the apparent angle of the virtual line joining the left and right terminations of the pointers on their respective vertical parallels. Such bias was reported by Morgan (1999), who advanced this bias a partial explanation of the P-bias. The stimuli are illustrated in Fig. 4. There was no separate standard stimulus. The pedestal and cues in this case were rotations of the angle between the two inner dots around the center of the figure. The angle between the outer dots/intersections was kept constant at 45 deg.

Fig. 4 caption. Stimuli for Experiment 2 with the control task on the left and the experimental task on the right. In the control the task was to decide in which figure the 4 dots were more aligned. In the experimental case (right) the outer dots were replaced by the intersections between the oblique pointers and the verticals.


The control task was to choose the stimulus (left or right) in which the inner dot pair was more aligned with the outer dot pair. In the experimental stimulus the outer dots were replaced with intersections of pointers and vertical lines. The pointers were parallel, but pointing in different directions, ruling out the use of pointing as a cue. Because there are no pointers in the control task it is not meaningful to refer to a P-bias in this case. We therefore adopt the convention that an apparent rotation of the angle between the outer dots is negative. This is consistent with the notation for the P bias described earlier in the Introduction. Using this convention a displacement of the intersections into the acute angle is predicted to cause a positive bias, opposite the P-effect.

Results (Fig. 5) showed the predicted effect in all but one (ks) of 8 participants. The difference between experimental and control
biases was $2.89 \operatorname{deg}(\mathrm{t}(7)=3.29 ; p=0.013)$ in the direction predicted. However, interpretation is complicated by the negative bias found in the control ( $-1.58 \mathrm{deg}, p=0.013, \mathrm{t}(7)=3.32$ ). The positive effect found in the experimental case is small and barely significant (1.3 deg, $p=0.047, \mathrm{t}(7)=2.40)$.

Fig. 5 legend. Results of Experiment 2 presented with the same conventions as in Fig. 3


We shall return later to the P-bias found in the control condition and its interpretation (Experiment 5 ). In the meantime, the conclusion from the Experiments described so far is that a large ( $\sim 6 \mathrm{deg}$ ) P-bias can be found in all cases where it can be interpreted in terms of an apparent rotation of the pointer(s) away from the orientation of the inducing lines (Blakemore et al., 1970). The alternative
interpretation, a shift of the location of the intersections, is ruled out for being too small (Experiment 2).

These conclusions leave us with a problem, because the orientationrepulsion explanation of the P-bias has been tested directly and found wanting. Hotopf and Ollerearnshaw (1972) measured the apparent orientation of the pointer (which they call the 'traversal') by matching to a neutral line, and found that even with a 30 deg traversal the bias was less than half of that of the P-bias measured by adjusting the angle between the traversals. Unaware of this previous work, Morgan (1999) measured the apparent orientation of a 30 arcmin pointer using a 2 -dot comparison stimulus and found no significant bias, although there was a P-bias with a very short (6 arcmin) pointer, which was proposed to be due to neural blurring. Wenderoth, White, and Beh (1978) used a pointer-dot alignment task (similar to our Experiment 1.3) but varied the position of the dot on either side of the pointer. Their Fig. 10 shows that there was no significant misalignment when the dot was on the unattached side of the pointer, but the normal P-bias when it was on the attached side. We attempt to illustrate this important but neglected finding in Fig. 6. Finally, the orientation repulsion reported by Blakemore et al. (1970) was only in the region of 1 deg, far too small to account for the P-bias in the Poggendorff figure. To confirm this discrepancy in the same set of observers, and to determine key parameters, we performed the following Experiments, using a Blakemoresque figure.

Fig. 6 caption. The figure attempts to illustrate the effect described by Wenderoth et al., (1978). The 45 deg pointer appears to point lower than the dot on the right (the P-bias) but to be collinear with the dot on the left. This has the consequence that the orientation of the virtual line between the two dots appears more vertical than that of the pointer.

## Experiment 3: Measurement of bias in pointer orientation

The purpose of Experiment 3 ('Blakemoresque') was to measure the orientation repulsion effect reported by Blakemore et al.(1970) with our 2AFC Method, and to compare values of angle repulsion found by this method with those inferred from the P-bias (earlier experiments). Examples of the stimuli are shown in Fig. 7. The observer was presented simultaneously with two figures, the test and the reference, and had to decide in which of them (left or right) the upper two lines were more parallel. The test was randomly positioned on the left or the right. Line width was 2 pixels (2.5
arcmin). Each figure was randomly and independently rotated around its vertex on each trial in the range $+/-5.7296$ deg ( 0.1 radians; uniform PDF). The upper line in each array is a standard. The orientation of the lower line relative to the standard was 18.43 deg. On each of 270 trials the middle line was perturbed from the standard orientation by one of three pedestal rotations ( $-3.5,0$ or +3.3 deg ) and (in the case of the test figure) by a cue rotation ( 9 values between -3.5 and +3.5 deg ). The length of all the lines was 70.7 pixels ( 1.47 deg VA). The separation between the reference line (top) and the left-hand end of the middle (test) line was 25 pixels (0.52 deg VA).

On half the trials the lower line was present (Experimental Condition) and on the other half it was absent (Control Condition).

Fig. 7 caption. Example of stimuli used in Experiment 3 . In 3.1, 3.2 and 3.3 the task is to decide in which of the two figures (left or right) the top two lines are more parallel. Both stimuli had a pedestal rotation from parallel which could be $-3,0$ or 3 deg (randomly interleaved) and in addition the test (randomly left or right) had an additional rotation from the pedestal. In experiment 3.4 the task is to decide in which of the two stimuli the dot is more collinear with the upper line.


Experiment 3.1 The configuration is shown in the top row of Fig.7. This is conceptually similar to the stimulus used by Blakemore et al. Results (Fig. 8) showed a small ( 0.99 deg) but highly significant $(\mathrm{t}(8)=4.3 ; \mathrm{p}=0.026)$ bias in the P-direction. This confirms the findings of Blakemore et al. and shows that the orientation-repulsion effect is indeed smaller than the P-bias by almost an order of magnitude.

Fig. 8 Caption. Results from Experiment 3.1 using same conventions as in Fig. 3.


## Experiment 3.2.

The same 9 participants as in Experiment 3.1.1 took part in this experiment, which was the same as 3.1.2 except that a small (12.5 arcmin) gap was introduced between the left-hand terminations of the bottom and middle lines. It is known that such a gap reduces the P-bias (Day, 1988 and earlier work). Results (not illustrated) showed a net P-bias of 0.42 , which was not significant (paired t-test; $\mathrm{t}(8)=1.43 ; p=0.19)$.

## Experiment 3.3

8 of the participants in Experiment 3.1.1 took part in this experiment in which the middle line was extended leftwards to make a T-
junction with the lower line (see Figure 9). This was an attempt to make the configuration more similar to the traditional Poggendorff figure. Results (not illustrated) showed a net P-bias of 0.7 deg, which just failed to reach significance $(\mathrm{t}(7)=2.18 ; p=0.065)$. Once again, the effect is too small to explain the P-bias in the Poggendorff figure by almost an order of magnitude.

## Experiment 3.4

The same 9 participants as in Experiment 3.1.1 took part in this experiment in which they chose the stimulus in which the pointer and dot were more aligned (Fig. 9 bottom row). Results (not illustrated) showed a net P-bias of $-1.057 \mathrm{deg}(\mathrm{t}(8)=4.15 ; p=0.003)$, not significantly different from Experiment 3.1.

## Experiment 4: Measurement of the spatial integration region for orientation at the proximal pointer terminations

Orientation repulsion effects, measured in several different ways and by several different experimenters, are unable to explain the P-bias by almost an order of magnitude (Experiment 3). But the alternative mechanism of a positional shift of the pointer fares even worse, being either non-existent or too small (Experiment 2). This seems to leave us without a clear mechanism for the P-bias. A possible resolution of the problem was suggested by Morgan (1999) and by Day (1988), who showed that the P-bias could be reduced by distorting the inducing line so that the pointer met it locally at a right angle. The suggestion is that orientational repulsion is a local process, confined to a small area where the two mutually-repulsing lines meet or nearly (Experiment 3.2) meet. Pointing uses information at the line
end in the direction of pointing, while estimates of line orientation use the line as a whole, and are therefore little affected by repulsion at the tip. The hypothesis is consistent with several experiments showing that small gaps between pointer and inducer can reduce or even abolish the P-bias (Day, 1988); Day does not give the baseline Pbias in deg but we estimate it at about 5 deg. Morgan (1999) further tested this idea by introducing small near-threshold 'bends' in the pointer just before they meet the inducing line. As predicted, bends in the same direction as putative orientational repulsion increased the P-bias while those in the opposite direction decreased it. A Method of Adjustment was used, varying the position of one of the pointers to be collinear with the other. We now attempt to see if this effect can be confirmed using a more rigorous 2AFC procedure.

## Experiment 4.1

Examples of the stimuli are shown in Fig. 9. Line thickness of the verticals was 5 pixels ( 6.25 arcmin) as in Experiments 1 and 2. Each pointer consisted of a single line, which was subsequently blurred by an isotropic Gaussian filter with a space constant of 5 pixels in order to smooth the bend and the edges. The angle of the pointer was 45 deg at its free tip but at some point before it reached the vertical it was given a new angle specified by its slope $(y / x)$. A slope of 0.5 meant that its slope was halved. Three different values of final slope were used, $0.8,1.0$ and 1.2. In terms of their difference from 45 deg these corresponded to $6.3 \mathrm{deg}, 0$ deg and -5 deg respectively. These values were chosen to be near the threshold for detecting a slope difference after the stimulus was Gaussian blurred. The x position at which the bend began was 0.7 units of pointer $x$ distance from the
start, and thus 0.3 units of pointer x distance from the vertical (18.75 arcmin). In other words, if the bent segment were removed there would have been a gap between the proximal pointer end and the vertical. We express the distance in this way to facilitate comparison with experiments where a gap has been introduced between proximal pointer end and the vertical (e.g. Day, 1988).

Fig. 9 caption. The figure shows examples of stimuli used in Experiment 4.1. Each of the two panels shows a particular condition of pointer 'bend'. In the left-hand panel the pointer bends towards the vertical (slope $y / x$ 1.2). On the right it bends towards the horizontal (slope $\mathrm{y} / \mathrm{x} 0.8$ ). Irrespective of pointer bend the task was to decide in which stimulus (left or right) the two pointers were more collinear.


Results (Fig. 10) showed that bends towards the horizontal increased the P-bias (-7.46 deg) and those away from the horizontal decreased it ( -4.1 deg ) , relative to the control condition ( -5.67 deg ),
as would be expected if the P-bias is due to orientational repulsion.
To remove variance due to overall level of bias between subjects, the biases in the two 'bendy' conditions were divided by the control bias, and subjected to a t-test with the null hypothesis that the ratio was unity. Using this test, the effect of the bend in the same direction as the P-bias fell just short of significance (-7.46 deg; $\mathrm{t}(8)=2.1$; $p=$ 0.068 ); the opposite bend had a significant effect ( -4.1 deg ; $\mathrm{t}(8)=2.47$; $p=0.039$ ) and the comparison between the two standardized experimental biases was also significant ( $-7.46 \mathrm{vs} . ~-4.1 \mathrm{deg}$; $\mathrm{t}(8)=2.47 ; p=0.047)$. That these effects were only marginally significant was due to the obvious differences between subjects, only four of whom (BD, KS, MM and NN) showed a convincing effect. Considerable individual differences are also evident in the data reported by Morgan (1999; Figure 6). This suggests that subjects could use different strategies, for example, by attending to one or the other end of the pointer, or averaging the orientation. It should also be noted that the bend opposite to the P-bias failed to reverse the latter. Indeed, the effect of the bend on the P-bias was numerically much smaller than the bend magnitude itself. Thus bends of - 6.3 deg and +5 deg produced changes of 1.79 deg and 1.57 deg respectively. This confirms the findings of Morgan (1999) where the effects were in the region of 1 deg only, although the bends in that experiment were also much smaller ( $\sim 1 \mathrm{deg}$ ).

Fig. 10 caption. The figure shows the results of Experiment 4.1. Within each panel, the first column ( $\mu 1$ ) shows the maximumlikelihood estimate of the bias in the condition where the pointer bends towards the horizontal, the second column ( $\mu 2$ ) represents the bias in the control condition (no bend); the third column ( $\mu 3$ ) shows the bias when the pointer bends towards the vertical; columns 4-6
( $\sigma 1 \sigma 2 \sigma 3$ ) represents the standard deviations (1/slope) of the psychometric functions in the three conditions. Column 7 shows the difference ( $\Delta=\mu 2-\mu 1$ ). The error bars show the inter-quartile range ( $25 \%-75 \%$ ) of values obtained from parametric bootstrapping. Negative biases are in the direction expected from a P-bias (see text for further explanation).


## Experiment 4.2

This experiment used the same participants and was identical to the previous Experiment (4.1) except that the bend was nearer to the inducer ( 0.1 in units of pointer-distance) equivalent to a gap of 6.25 arcmin. As in the previous experiment, to remove variance due to overall level of bias between subjects, the biases in the two 'bendy' conditions were divided by the control bias, and subjected to a t-test with the null hypothesis that the ratio was unity. Using this test, the effect of the bend in the same direction as the P-bias was not significant $(\mathrm{t}(8)=1.37 ; \mathrm{p}=0.21)$; nor was the opposite bend effect $(\mathrm{t}(8)=0.06 ; \mathrm{p}=0.96)$ and the comparison between the two
standardized experimental biases was also not significant $(\mathrm{t}(8)=1.05$; $\mathrm{p}=0.33$ ).

## Experiment 4.3

This experiment was identical to the previous Experiment (4.1) except that the bend was in the middle of the inducer ( 0.5 in units of pointer $x$-distance) equivalent to a gap of 31.25 arcmin. The participants ( $\mathrm{N}=5$ ) were BD,DW,KS,MM and TP. Results (not illustrated) showed mean P-biases of -11.3, -5.3 and -0.5 deg in the bend-to-horizontal, the control and the bend-to-vertical conditions respectively. These are almost exactly what would be expected from the bends of $-6,0$ and +5 in the three conditions, if the bend were added to the control P-bias.

These results confirm the conjecture that the direction of pointing is determined by the orientation of the proximal pointer segment over a finite integration region. We can estimate the size of this region as greater than 18.75 and smaller than 31.25 arcmin. Since this region is small, orientational repulsion (OR) need not affect the whole of the pointer to produce a P-bias. However, the present experiment does not directly demonstrate a limited range of OR. Previous experiments showing that a gap at the proximal pointer termination reduces or abolishes the P-bias do not directly demonstrate a limited range either, because they could be explained by a segmentation process that prevents OR between separate objects.

## Experiment 5. Measurement of the P-bias without pointers.

In the search for further causes, we return to the effect found in the control task of Experiment 2.1 (Fig. 4 left-hand panel) involving dots alone, and no pointer. The orientation of the virtual line joining the dots on the inducers was apparently steeper than that joining the inner dot probe. The following experiments investigate this effect with differing techniques.

It may seem paradoxical to talk of a P-bias without pointers, but as already noted, one interpretation of the P-bias is that it involves the construction of a virtual line spanning the space between the pointers. Only if this has the same orientation as the pointers themselves can the pointers be collinear. If the line that is constructed is more orthogonal to the inducing lines than the pointer, then a P-bias will result.

## Experiments 5.1 and 5.2

Examples of the stimuli used in Experiments 5.1 and 5.2 are shown in Fig. 11. In Experiment 5.1 the configuration was vertical; in 5.2 it was horizontal. In both cases the task was to decide in which of the two figures the three dots were more collinear. The angle between the two dots on the parallels was adjusted by varying the position of the rightmost dot. In the control condition the parallels were absent.

Fig. 11 caption. Examples of stimuli used in Experiments 5.1 (left-hand pair) and 5.2 (right-hand pair.) In both cases the task was to decide in which of the two figures the three dots were more collinear.


Results for Experiment 5.1 (vertical configuration) are shown in Fig, 12. The data showed a significant difference $(\mathrm{t}(7)=2.82 ; p=0.026)$ between the control condition ( 0.15 deg ) and the experimental (1.7579) with a net difference of -1.9 deg.

Results for Experiment 5.2 with 6 participants (horizontal configuration) showed a significant difference ( $\mathrm{t}(5)=2.86$ ); $p=0.036$ ) between the control condition ( -0.47 deg ) and the experimental (2.50 ) with a net difference of -2.03 deg.

Fig. 12 caption. The figure shows the Results for Experiment 5.1, with the same conventions as Fig. 3.


The bias could result either from the left hand pair of points (the virtual pointer) or the right-hand pair. To distinguish these possibilities the next experiment used only the left-hand pair and dispensed with the virtual pointer.

## Experiment 5.3

Examples of the stimuli are shown in Fig. 13. The task was to decide in which of the two figures the polar angle between the two dots was nearer to 45 deg. Because there is only a weak natural standard for 45 deg (c.f. Morgan, 1990) experimental trials with the parallels present were interleaved with control trial with parallels absent, and in the latter, the observer was given veridical feedback. (A large square to indicate a correct response and a small square to indicate
an error). As in previous experiments, three pedestals were randomly interleaved in both control and experimental tasks.

Fig. 13 caption. Examples of stimuli used in Experiment 5.3. The task was to decide in which of the two figures the polar angle between the dots was closer to 45 deg. In the control task the parallels were absent and feedback was given..


Results (Fig. 14) showed that the control condition was successful in reducing the bias, except for AJ who had a large positive bias.

However, all participants had a negative bias in the experimental condition, in the direction of the P-bias. There was a significant difference $(\mathrm{t}(8)=5.23 ; \mathrm{p}=0.003)$ between the control condition $(0.69$ $\mathrm{deg})$ and the experimental (-2.27) with a net difference of -2.97 deg . These results confirm the findings of the previous experiment (5.1), without the complication of an additional dot.

Fig. 14 caption. The figure shows the Results for Experiment 5.3, with the same conventions as Fig. 3.


There are two interpretations of the bias found in these experiments (5.1-5.3). The more obvious of the two is that the perceived polar angle between two dots superimposed on two parallels is biased towards the orientation of the parallels themselves. This assumes, however, that the observer has an unbiased representation of a 45 deg standard crossing the parallels. If this virtual 45 deg line were biased in the direction orthogonal to the parallels, the same bias would result. It has already been argued that one interpretation of the P-bias is that the observer constructs a biased representation of a virtual line by an analogue process. To account for the P-bias, it must
be assumed that this bias is in the direction orthogonal to the parallels. A mnemonic for this bias is that it is a form of 'least effort' or shortest path bias. These two possibilities cannot be distinguished by the psychophysical methods we have used.

## General Discussion

These experiments have confirmed that there is a robust P-bias, using a 2 AFC procedure, when the task is aligning the pointers in a traditional vertical Poggendorff figure. There is a bias of at least the same magnitude when the right-hand pointer is replaced with a dot (Experiment 1.1), suggesting that the alignment of two pointers is not necessary for the effect. Rather, the data are consistent with a misangulation of a single pointer (Ninio, 2014; Ninio \& O'Regan, 1999).

The experiments also find, in agreement with Blakemore et al.(1970) that there is a bias when the task is to match the orientation of a pointer with a neutral line, or with a dot (Experiment 3). These results are consistent with angular repulsion based on crossorientation inhibition (Blakemore et al., 1970). However, the magnitude of this effect, ( $\sim 1 \mathrm{deg}$ ) which is similar to that reported by Blakemore et al., is too small to account for the P-bias found in the traditional Poggendorff figure.

Morgan (1999) argued that one cause of the P-bias is a mislocation of the intersection between pointer and inducing line, based upon
neural blurring in large, second-order filters. The evidence came from an experiment in which observers matched the apparent orientation of the virtual line joining the two junctions in a Poggendorff figure, when that angle was varied between - 60 deg and 60 deg. Observers rotated a patch of sinusoidal grating to match the perceived orientation of the virtual line. Results indicated that the apparent end points of the virtual line were locate not at the junction, but at a point displaced into the acute angle. However, the effects reported by Morgan (1999) were too small to account for the P-bias in the traditional Poggendorff figure. Morgan (1999) concluded that a mislocation bias could at best account for only part of the P-effect, and the present results support this conclusion.

The experiments have also revealed a bias in comparing the orientation of the virtual line crossing the gap between two dots placed on two parallel lines (Experiment 5.3). This is not the Hotopf and Hibberd (1989) 'Horizontal-vertical assimilation tendency' because the reference orientation was 45 deg. Nor is it their 'horizontal bias alignment effect', since it applies to both horizontal and to vertical parallels. The virtual line bias effect that we report here is, to the best of our knowledge, novel The bias can be interpreted either as a bias in the perceived orientation of the virtual line joining two dots; or as a bias in constructing a reference across the gap. In the latter case, the bias is in the direction of constructing a line that is biased to the shortest distance between the two parallels, that is, an orthogonal tendency, in the same direction as the 'orthogonal orientation bias' affecting short line segments crossing small gaps (Morgan, Medford, and Newsome, 1995). The two
interpretations of the bias cannot be distinguished by the present data.

Our results confirm the idea that the P-bias combines a number of distinct biases that work in the same direction (Hotopf and Hibberd, 1989; Hotopf and Ollerearnshaw, 1972; Hotopf, Ollerearnshaw and Brown, 1974; Morgan, 1999; Gallace et al., 2012). Like other illusions the so-called 'Poggendorff illusion' has evolved in the literature to be conspicuous rather than informative (Morgan and Casco, 1990). The upright version that is normally presented combines all the known effects to produce a conspicuous effect. These include (1) The repulsion of the pointer orientation from the parallel, probably localized quite near the intersection (2) A location bias affecting the intersection of pointers and parallels, (3) Hotopf and Hibberd's (1989) horizontal bias alignment effect, (4) a general bias in the orientation of virtual lines crossing the gap between two parallels.

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