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Institutional Underpinnings of Trustworthiness in Infrastructure Contracts: Trust, Institutions and Contract Design*

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Abstract

This paper discusses trust and trust perceptions in infrastructure contracts. We focus on perceptions of the trustworthiness of the government purchasers of infrastructure services by the supplying companies and by the governments themselves. In particular, we allow for trust misalignments which may give rise to 'undertrusting' and 'overtrusting'. The core of the paper sets out a game theoretic model of contracts which we use to explore the impact of trust misalignment both on economic efficiency (measured by expected welfare) and on investment levels. We explore flexible contracts with and without pre-payments, rigid contracts (which do not allow for post-investment renegotiation) and hybrid contracts. We then compare the efficiency of the flexible contracts to that of hybrid contracts using as a criterion the expected welfare implications of each contract. The model is used to shed light on current issues on the sustainability of private investment infrastructure contracts in developed and in developing countries.

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1 INTRODUCTION

In this paper, we investigate trust issues in infrastructure contracts, the viability of such contracts and the role of regulation and institutions in general in affecting these contracts[1]. More specifically, we focus on how well aligned are trust perceptions between buyers and sellers of infrastructure services, as well as on absolute levels of trust. It turns out from our analysis, that the mutual alignment and evolution of trust perceptions are both of major importance. Infrastructure assets are very long-lived, sunk assets and the services that they provide are typically highly politically sensitive (electricity, water, transport, etc.). In consequence, the critical issue for effective and sustained delivery is how to ensure adequate trust between the supplying entity and the government which is purchasing the output and providing the legal underpinnings, including possible regulatory arrangements. These issues are always important for private infrastructure investment in general, but are particularly important, firstly, for riskier investments; and, secondly, for infrastructure investment in difficult institutional environments as are frequently found in developing and transition countries.

A wide range of institutional arrangements is observed in practice to support private investment in infrastructure. This range includes, at one extreme licensed suppliers operating on infinite length contracts supervised by a regulator (classic electricity, water and telecom regulation) through to, at the other end of the spectrum, fixed length concession contracts with no external supervisory body other than the courts (e.g. many toll road contracts). There are also many hybrid models which combine contracts with regulatory or other forms of external regulation/arbitration etc. in various ways (e.g. UK railways, the London Underground and many others). As argued later, contracts and regulation are better regarded as complements rather than substitutes[2]. It is worth pointing out that infrastructure contracts – like long-term contracts between companies - can be found with various degrees of external contract resolution and with varying degrees of renegotiating flexibility both in terms of tariff and similar changes and for post-investment renegotiation[3].

In this paper, we explicitly introduce a measure for the quality of the institutional and regulatory conditions that prevail in the environment where the contract is to be implemented. This is a variable that covers the aspects of country governance that most directly impact on the likelihood that the contract will be fairly administered and enforced and that any contract disputes

[1] Contracts in our paper refer to legally binding agreements that involve investment as well as operation and management. This includes all concession contracts with investments as well as US infinite duration franchises, but excludes French aftermage contracts. It also excludes PPPs except for those where there is a regulatory entity in place (like the London Underground PPPs).

[2] See Stern (2003), Bolt (2003 and 2007). These both discuss the issues arising from the perspective of a regulated industry. Athias and Saussier (2006) discuss these issues arising from the perspective of concession contract design.

[3] See Athias and Saussier (2006) for evidence on this for toll road concessions and Menard and Saussier (2002) for water supply arrangements.

will be resolved in an impartial manner. Apart from the presence of a regulator, this most obviously concerns issues to do with the rule of law, the reliability and timeliness of law courts, and levels of probity (and corruption) in public life[4].

We present and report the results of a game theoretic model which includes not just the contract and the relevant institutional framework, but also the potential for renegotiation (including regulatory review). Hence, following Menard and Saussier (2002), we consider the relative merits of different types of contractual arrangements. In addition, we explicitly consider the role of term revision and renegotiation clauses, following the evidence of Athias and Saussier (2006) on the way that they are used (and not used) in toll road concession contracts.

We adopt a game theoretic approach to these issues starting from a consideration of alternative types of contract. We first develop a typology of contracts building on Athias and Saussier who distinguished between:

(a) *Flexible contracts,* which explicitly allow for contract renegotiations after investments have been made; and

(b) *Rigid contracts*, which set fixed contract terms before the investments are made and do not allow for subsequent term changes or renegotiation.

We then develop hybrid models which are constructed by introducing a variable for the probability of renegotiating a fixed contract after the investment has taken place. We integrate the discussion of contracts with that of regulation on the basis that external regulators can allow simpler contracts, easier dispute resolution and, in particular, more readily agreed contract renegotiation. This perspective arises from Laffont (2005), Guasch and Straub (2006) as well as Stern (2003).

The measure of trustworthiness on which we focus is the probability that the contract between the buyer and the seller will be enforced, taking the simplest dimension of enforcement, i.e. that of the buyer paying the firm. Hence the probability for dishonest behaviour exists from the side of the seller. We explore a range of contractual and institutional arrangements that can reduce this perception gap and/or help guarantee payment. These may take a variety of forms from (a) insurance type of arrangements (for example World Bank guarantees against regulatory risk) to (b) some form of explicit pre-payment contracts (see, for instance, Braynov and Sandholm, 2002). Essentially in our model, the term 'prepayment' is interpreted broadly and can mean any facility that provides an almost "as good as in your pocket money" to the buyer and/or the seller.

The paper sets out the formal modeling relationship between trust perceptions and different contractual arrangements. A bargaining model between the

[4]Specialist regulatory or similar external agencies may be given some of the responsibilities for these issues, but within a legal framework under which appeals and possibly implementation would be done by the local courts. Of course, countries may decide to establish independent regulatory or similar agencies as a way of *signalling* to investors their commitment to fair dealing on infrastructure contracts and investment. Such policy signalling devices were advocated by the World Bank and others in the 1990s but actually go back to medieval times for trade courts, as shown in Greif (2006).

buyer and the seller is developed using a framework based on Nash bargaining. We compare the expected welfare in each of these models with those from an incentive-compatible benchmark contract and use these expected welfare comparisons to evaluate the appropriateness in different circumstances of different contract forms, or external regulator and pre-payment/guarantee arrangements.

In section 2 we address the question of whether concession contracts are substitutes or complements. This is followed by section 3 where the framework for our model is set out. We attempt to tackle the problem of misalignment of trust perceptions regarding the buyer by using alternative forms of flexible contracts, followed by an analysis of rigid contracts. Then the welfare implications of rigid contracts with no commitment not to renegotiate are compared to those of flexible contracts. In section 4 the issue of trust and trust perceptions is discussed in more depth in the light of the results derived from the model in section 3. Finally, we draw our conclusions in section 5.

2 CONCESSION CONTRACTS AND REGULATION: SUBSTITUTES OR COMPLEMENTS?

From around the mid-1980s, there was a strong push towards developing regulatory agencies as the key way in which trust could be established for countries privatising their utilities or wishing to expand private investment. This policy was heavily influenced by developments in the 1980s in the UK and in Chile as well as some other countries. Indeed, World Bank policy advice suggested that for developing countries, establishing an independent regulator would signal that they were trustworthy for supporting private investment (including allowing a reasonable rate of return). Hence, it was suggested that establishing such agencies was a way of assuring infrastructure investors that countries were now trustworthy i.e. eliminating the 'undertrusting' problem.

Proponents of this view failed to give sufficient weight to:

(i) the degree to which governments would intervene into regulatory decisions;

(ii) the degree to which regulatory laws and institutions provided discretionary powers (which enabled governments to intervene arbitrarily); and

(iii) the time it takes to establish regulators and the volume of specialist resources required.

In consequence, by the late 1990s, the optimistic view of regulators was seriously battered by major regulatory failures, in particular after the Asian financial crisis in 1997-98 when new regulators were effectively discarded and many investments (or at least debt contracts for the investments) became unviable.

Spiller (2004) argues that concession contracts provide individualized regulation with contracts that are rigid by origin rather than 'relational' as typically found in long-term contracts between private sector entities.

The problem with this argument is that, as is now well-known, tight contracts are very brittle in the face of shocks and renegotiation can be difficult. Renegotiation rates are typically very high for developing country concession contracts - particularly for toll road and water concessions which Guasch (2004) report at (respectively) 55% and 74% for Latin America over the period 1989-2000. Of course, far from all renegotiations lead to project collapse. Nevertheless, according to the World Bank PPI database, over the period 1990-2004, 160 infrastructure projects accounting for 9% of investment flows were canceled or in distress. For water, 7% of projects accounting for 37% of investment flows in the sector were canceled or became distressed, i.e. a disproportionate number of high value concession projects.

The counter-argument to the case for rigid contracts in Stern (2005) is that, where country governance is sufficiently supportive, trust is better achieved for infrastructure investment by establishing a separate regulatory entity, which has been assigned legal powers to act of its own volition. This agency has the authority, in consultation with regulated companies and their consumers, to modify existing regulatory obligations (for example, tariffs and quality of service) and to establish new regulatory rights and obligations. In particular, it has the right to review and revise regulatory obligations according to some defined process. Hence, it operates as a full regulator, including a degree of *bounded and accountable discretion.*

Such a mechanism provides a way in which contracts can be reappraised and revised in the light of changing circumstances according to a pre-agreed and impartial process. Hence it allows simpler and more transparent initial contracts and better enforcement. Of course, although regulators may behave in the way recommended, there is no guarantee that they *will actually* do so. Hence, the search for other mechanisms to help establish regulatory capacity and reputation like initial regulatory risk guarantees, etc. to help underpin agreements and induce more trust earlier i.e. what we have described as 'prepayment' agreements.

In many cases contracts without external regulatory support will at best require major renegotiation and in many cases will fail. However, there are also cases where contracts with little or no regulatory support may well be sufficient. The difference depends firstly on the nature of the contracted service and its associated investment (e.g. whether it is a straightforward and/or previously successfully delivered investment); and, secondly, on the degree of trust between the purchaser and the seller and/or on whether the parties do or do not have a previous history of successfully managing such contracts. We discuss this further with examples in the next section and again in section 4 following the results from our bargaining model.

In the model we introduce the concepts of 'undertrusting' and 'overtrusting'. These can be understood as follows:

Consider the typical case where the government is the buyer of infrastructure services via some type of long duration contract or equivalent[5] and the seller is a private company, typically an infrastructure company. We define "under-

[5] A fixed period UK-style infrastructure regulatory licence and an infinite US utility authorisation would both be included in this categorisation.

trusting" as the case where the selling firm's belief that the buyer will honour the contract is less than the buyer himself perceives it to be. Similarly we define as "overtrusting" the case where seller's estimate that it will receive full payment under the contract is higher than the buyer believes it to be.

With undertrusting, the key problem is how to motivate and sustain ongoing and agreed levels of investment in the face of unforeseen developments and incomplete contracts. Conversely with overtrusting, the key problem is whether or not companies, having made particular investments, will receive payments that they think they are owed under the contract when unanticipated changes are needed and/or unforeseen developments occur. Hence, one would expect contracts with undertrusting to break down relatively slowly but contracts with overtrusting to collapse rapidly.

Trust alignment occurs when the beliefs of the seller and buyer are the same. Of course, this may be at a high level of trust (as in countries in the top 5% of country governance scores) or at a very low level of trust (as in countries with very low country governance scores). In what follows, we show that, while the best outcome – particularly for consumers – involves contracts at high levels of mutual trust; infrastructure contracts involving quite large amounts of privately financed investment may be sustainable in circumstances of low trust (e.g. Paraguay in the 1960s and '70s, some central African states), even if highly suboptimal.

Note that this paper is primarily concerned with expected welfare comparisons arising from perceived trust variations of the buyer government. In consequence, we consider neither (a) government beliefs of the trustworthiness of the seller; nor (b) issues arising because a corrupted buyer government may not be interested in expected welfare. We leave these for subsequent work.

Similar trust and trust perception issues exist over the commitment of the seller (i.e. the infrastructure company). We do not explicitly discuss such issues in what follows but the analysis should be similar, albeit this time referring to the probability of investment rather than the probability of full payment. We leave this for future research - as well as combining buyer and seller trust and trust perception issues.

3 THE MODEL

The model draws on game-theoretic bargaining models that have been developed in related contexts. In particular, we draw attention to McMillan and Waxman (2007) which explores the importance of trust in terms of the way it influences the bargaining power of governments and multi-national companies. We also draw attention to the paper by Braynov and Sandholm (2002) which has a technical discussion of how trust can be integrated and dealt within different types of Nash bargaining solution modelling environments. The general issue of government and company reputation in infrastructure concession contracts is discussed in Guasch and Straub (2006) paper on concession contract renegotiation.

Weakness in contract enforcement is one of the main reasons why developing countries generally find it more difficult to attract both international trade as well as infrastructure investment. The dynamic process by which firms engaged in international trade build trustworthiness because contracts are not completely enforceable, is discussed in Araujo and Ornelas (2007) regarding short term international trade contracts. Baraynov and Sandholm discuss contracting with uncertain levels of trust in a static model and the importance of the extent to which the seller's trust equals the buyer's actual trustworthiness. We do combine both of these characteristics in this paper.

The institutional environment

We define an institutional parameter $\lambda \in [0, 1]$ that measures the country's ability to enforce concession contracts. This measure can simply be the proportion of contracts that a legal system enforces as Anderson and Young (2006) suggest, or can also include other parameters in a country's institutional set up such as the degree of international indebtedness of the country concerned. Alternatively it can be considered as an indicator of the probability that contract violations will be detected and punished, as well as whether or not there are effective adjudicating procedures for disputes. Finally the value of λ may be also determined by whether or not there is some type of external regulatory agency in place, including agencies whose role is to monitor and enforce such contracts, or an external body arbitration agency/facility. The efficiency of such a body will determine how close λ is to one.

Trustworthiness beliefs held by the firm

Imperfect contract enforcement coupled with opportunistic behaviour by the host government and the presence of very large sunk costs typically found in the majority of infrastructure contracts means that all three factors for substantial transactions costs are present. These factors may prevent such contracts from happening in the first place. On the other hand, the more the government abides by the terms of a contract the more convinced the investing firm becomes of the trustworthiness of the purchasing government.

a' is the selling firm's belief that the government will honour the terms of the contract and that it will be paid on delivering the product or service of the contract on the terms stipulated in the contract. There is little theoretical analysis of the dynamic process by which trustworthiness is built as a response to the lack of perfect enforceability of contracts. However anecdotal evidence of how trust can be used to compensate for the lack of formal legal agreements or other relevant features of the institutional set up abound. Grief (1993) analyses the formation of a coalitions by medieval merchants to compensate for limited contract enforceability, while McMillan and Woodruff (1999) show how relationships based on trust arise and develop in environments where where there is virtually no contract enforcement as is the case of Vietnam. Perhaps the most striking example comes from De Brux (2008). The author discusses the first worldwide airport concession which happened in the Kingdom of Cambodia, a country with a very weak judicial system, widespread corruption and no previous history of concession contracts. The French group had a signed a concession contract in 1995 to extensively modernise and run the existing airport and also built a new one. During the summer of 1997 the Asian economic crisis started to spread over Cambodia. At the same time, a military insurrection took place in the capital of the country. However both parties, after months of renegotiation, successfully amended the contract. The success of this contract came from the willingness of both parties to allow the spirit of the contract to prevail, rather than the letter of the contract which allowed termination of the agreement by either side under such extreme circumstances. Cooperation through renegotiation prevailed over opportunism in this case.[6]

The firm updates is beliefs about the purchaser's type according to Bayes rule:

$$a'(\{paid\}, a') = pr(m \mid \{the\ government\ pays\ the\ seller\} \cap a') = \frac{a'}{a' + \lambda(1 - a')} > a'$$

Hence the adjustment in a' is upwards. This allows for the fact that the firm may get paid even if the current government (purchaser) is not trustworthy - in other words he would like to engage in opportunistic behaviour but is prevented from doing so by the institutional environment which successfully enforces the contract. Hence we have a separation of the institutional environment from a', the reputation (trustworthiness) of the government of the day, where the former plays a role in the updating mechanism of the latter.

While Araujo and Ornelas discuss a series of one-period trading contracts, the outcome of which feeds into the reputation of the buyer with whom a long term relationship is established, we instead wish to apply this approach into a typically long term infrastructure contract where trustworthiness tends to grow incrementally over time. However, there is no doubt that a history of previously successfully concession contracts in a country has a positive externality effect on future concession contracts by increasing the primal estimate by the seller (investor) of the buyer's trustworthiness. The strength of this externality will of course depend on whether this buyer is the same or a different government. We discuss this in the next subsection.

The probability that the firm will receive its payment increases with a' as well as λ. More specifically the firm has an initial prior estimate (belief) $a'_\tau > 0$ and we allow this to increase up to a maximum of 1 during the length of the contract. In other words we define a history according to which the firm updates its estimate regarding the trustworthiness of the buyer during the life of the contract as and when further information regarding the government

[6] Macaulay (1963), argued that a key virtue of relational contracting is that parties can count on each other to abide buy the spirit of the contract and therefore do not waste as much in specifying its letter. This outcome has also appeared here, despite the absence of a previous relationship between the two contracting parties.

becomes available. Then we label as $a_\tau'^k(C, a')$, the value of the estimate by the seller of the probability that the purchaser is trustworthy given a starting date τ for the concession contract after k periods (defined in years, or months, or even days as appropriate) during which experience is cumulated applying a cardinality C, where $C = \sum_{j=\tau}^{\tau+k-1} h_j$, $h_j \in \{0, 1\}$. Hence during the length of the contract (which, if in infrastructure, will typically tend to run in double digit years) estimates of trustworthiness of the trading partner will be updated through the life of the contract. Events (news) if non existent or positive are indicated by $h_j = 1$ (no news is good news[7]) and if negative are indicated by $h_j = 1 - j$. Under this mechanism information regarding the trustworthiness of the government by the firm will be cumulated under time. The estimate by the firm at time $\tau + k$ of the government's trustworthiness on a contract that started at time τ is given by the following Bayesian updating process:

$$a_\tau'^k(C, a_\tau') = \frac{a_\tau'}{a_\tau' + \lambda^C(1 - a_\tau')} \tag{1}$$

Clearly if there is a bad event, $C = 0$, and trustworthiness will revert in the next period back to its prior value of a_τ' from where it will start increasing once more. In the absence of bad events C will increase and $a_\tau'^k$ will increase towards its upper maximum value of 1. Hence $a_\tau'^k \in [a_\tau', 1]$

The justification for this uneven treatment is that humans perceive trust destroying events as more noticeable than trust building events; hence the former tend to carry more weight than the latter. Once trust is lost it is costly to rebuilt and it will take time for this to happen. The institutional environment plays a role since payment will be enforced even if the government is untrustworthy if the institutional factors will ensure that payment will occur in any case. However in our model enforcement is separated from trust as the latter can compensate for the absence of the former (Greif, 1993), although ideally the one should act as a complement to the other as discussed in the previous section.

Hence the probability that the firm will receive payment is $a = a' + \lambda(1 - a')$ which is clearly increasing both in a' and in λ. If the institutional setting of the country improves, then the firm will expect to receive its payment with a higher probability and a' will become less important in determining the probability of payment. If a' is updated upwards, then the institutional parameter becomes increasingly less important in determining the probability of payment.

Updating delays and the importance of prior beliefs

Note that a purchaser's reputation at time τ is not affected by the increase in λ at that time (denoted by λ_τ), as a is a function of the events that have occurred

[7] Tirole (2008) argues that "parties to a contract tend to specialize in identifying bad news for themselves/good news for the other party". However in our model, the buyer faces no participation constrains as it is the seller who pays for the investment. Hence it is in the seller's interest to unveil both bad as well as good news that may affect his expected profit.

so far during the contract, not the current one. However it will affect its future reputation at time $\tau + k$. This means that to some extent an improvement in λ slows down the updating process regarding the trustworthiness the trading partner because it makes it more difficult for the firm to determine whether the government is complying with the terms of the contract voluntarily or because of the institutional restrictions (including the threat of a legal or regulatory challenge, or the commencing of external arbitration, etc.). Hence if a change in λ occurs at time τ, then after k periods:

$$\frac{\partial a'^{k}_{\tau}}{\partial \lambda_{\tau}} = \frac{-C\lambda^{C-1} a'_{\tau}(1-a'_{\tau})}{\left(a'_{\tau} + \lambda^{C}(1-a'_{\tau})\right)^{2}} < 0 \tag{2}$$

And the impact on the overall probability of payment is

$$\frac{da^{k}_{\tau}}{d\lambda_{\tau}} = \frac{\partial(a^{k}_{\tau} + \lambda(1-a'^{k}_{\tau}))}{\partial \lambda_{\tau}} = (1-a'^{k}_{\tau}) + (1-\lambda)\frac{\partial a'^{k}_{\tau}}{\partial \lambda_{\tau}} \tag{3}$$

$\frac{da^{k}_{\tau}}{d\lambda_{\tau}}$ will be positive provided that:

$$\frac{\lambda}{1-\lambda}\left(1 + \frac{1-a'_{\tau}}{a'_{\tau}}\lambda^{C}\right) > C \tag{4}$$

The left hand side in the above is an increasing function of λ. This implies that in countries with an already strong enforcement environment (UK, France, etc.), (4) will hold, and a further increase in λ because it only has a limited role in the trustworthiness updating mechanism will lead to increase in the probability of payment as estimated by the seller.

However, in a country with a low initial λ, its increase may reduce the probability of payment as estimated by the seller by significantly delaying the establishment of a reputation for the buying government (i.e. delaying a'^{k}_{τ} becoming substantially higher than the prior belief).

On the other hand experiences such as the one described by DeBrux regarding Cambodia and the successful outcome in the case of the airport contract result in a substantial increase in the prior belief a'_{τ} that will be the starting point in a future contract. Hence there is an important externality that the history of a previously successful contract will confer on a new contract in terms of increasing a'_{τ}, thus making further updates desirable, but less crucial than before. If this is combined with a subsequent increase in λ it will lead to a substantial upward revision in the probability of payment a^{k}_{τ}.

The above does not negate what was discussed in section 2. It is clearly ideal to have a combination of a high λ with a high a'_{τ} complementing each other within a contract. However it does explain why within the historical context of each country, the establishment of high trustworthiness by a government usually precedes strong enforcement institutions, rather that the other way round. Initially it takes a genuinely honest government within a developing country that wishes to establish trustworthiness and 'break' with the past to do so. Then this will be followed by the establishment of legal and regulatory institutions (typically by that same government) that will strengthen enforcement institutions (increase λ) and make the success of future contracts less government specific, i.e. less dependent on the willingness of subsequent governments to honor infrastructure contracts. This point is further discussed below.

The buying government's type

The government's own assessment of its trustworthiness is defined as a random variable b representing the probability of the favourable event $\varepsilon = \{$*the government pays the seller*$\}$ occurring, which depends on the degree of commitment by the government to honour the contract and on the level of contract enforcement in that country. Following Myerson (1979), the level of commitment by the government is a variable state of nature, which is defined broadly enough to include all subjective unknowns which might influence it.

A country where b is markedly less than 1 is most likely to arise in circumstances where a government has entered into arrangements with private (particularly foreign) investors but without total commitment. This introduces major uncertainties concerning the performance of the contract in the future. For instance, this may be so because of insufficient tax revenue to fund preferred public sector options, as a result of political and economic pressure from a higher level of government, or as a condition for international lending or aid assistance. Another case is where there is a clear possibility that a change of regime/government would probably lead to a major renegotiation or suspension of the contract (viz. Venezuela pre and post the Chavez presidency and the Chad government's actions to suspend the Future Generations Fund to collect earmarked savings from oil sales). A further possibility is where political opposition to private investment in infrastructure increases over time so that the political costs to the government of maintaining the private investment contracts gradually increase, making the government's commitment to pay low. This will lead to a decrease in the value of b.

We would normally expect that government commitment to an infrastructure contract would be affected by the level of enforcement at time τ, λ_τ, since a government would be unlikely to enter a contract with a low probability of paying if it knew that it is very likely that it would be forced to pay the contract by the mechanisms in place. Hence we would expect b to be an increasing function of λ.

The government has a strong incentive to inflate the true actual value of b. But even if the government truthfully reveals this value - and we shall assume initially that this is the case - clearly a and b differ even when they are both common knowledge. Disagreement on mutual beliefs can occur among rational agents if the agents have different priors (Aumann 1976) and they follow a different process of updating or forming such beliefs.

Hybrid contracts

Following Athias and Saussier (2006), a hybrid contract reflects the inability of the investor in a rigid contract to reliably predict investment outcomes (both investment costs and revenues arising). Major forecasting errors and/or major shocks cause significant maladaptation costs, which can be positive (e.g. where demand is much higher than predicted for a toll road) as well as negative (e.g. in substantive investment cost overruns or unexpectedly low demand).

Our paper considers flexible contracts, rigid contracts and hybrid contracts. We denote the flexibility of the contract by η. We define fully flexible contracts (i.e. contracts where a post-signing change in contract terms, and/or a renegotiation is certain) as those where $\eta = 0$. For fully rigid contracts (i.e. those where post-investment changes in contract terms and/or renegotiations are fully excluded), $\eta = 1$. We also consider *hybrid* contracts, i.e. those where there is some positive expectation of post-investment changes in contract terms and/or renegotiations. For such contracts, $0 < \eta < 1$, with a greater degree of rigidity as $\eta \to 1$.

In what follows, we first consider three types of fully flexible contracts. We then look at rigid contracts and finally, at the hybrid contract, where there is a positive probability of ex post renegotiation. The three types of flexible contracts that we consider are:

1. The Athias and Saussier flexible contract model, but including our trustworthiness parameters a and b.

2. The same model with a guaranteed pre-payment mechanism.

3. A benchmark, incentive compatible "F-contract" model with prepayments.

We show that the third model sets such prepayment terms, that the total surplus split between the infrastructure company and the government is identical to the one found in A&S where trustworthiness terms were not included in the model. We conclude by comparing the efficiency of the hybrid model to the first flexible contract model.

We now turn to the formal analysis where, in each of the subsequent discussions we analyse the bargaining model that corresponds to each of the above models. The time line of the models is as follows:

$< \tau$	τ	$\tau + k, k > 0$
Type of contract	Investments and pay are set	Updating of investment and pay
and payment provisions	on the basis of the	decisions in flexible and flexible
are chosen and signed	realised values of	with prepayments contracts
	a, b, and λ at time τ.	Renegotiation may occur $(1 - \eta)$
	Prepayments (if any) are also set	in rigid contracts

3.1 Flexible contracts

Let us start with the flexible contracts where we include into the expected payoff functions the parameters a and b as announced respectively by the seller (investing firm) and the buyer (government).

The firm's expected profit function and the expected consumer surplus[8] are given by the following functions respectively:

$$\begin{aligned}\Pi^f &= P_0 - C_0 + at - i \\ CS^f &= B_0 - P_0 + \overline{f}R(i) - bt\end{aligned}$$

where B_0 and C_0 are positive constants representing, firstly, the social benefits and, secondly, the costs of providing the basic service without any investment. t denotes the amount of payment going to the firm following renegotiation between the company and the government on how the surplus $R(i)$ $(R' > 0, R'' < 0, R''' < 0)$ created by the investment i undertaken by the firm will be shared between the two parties. $\overline{f}$ and α are inverse measures of the cost of renegotiation and of the degree of asset specificity respectively. If $\alpha = 0$ then the investment is wholly sunk and hence has no opportunity cost. Therefore, $r(i) = \alpha R(i)$ is the proportion of the surplus $R(i)$ which is not sunk and hence has an opportunity cost. For infrastructure industries, $r(i)$ is likely to have a low value. For notational simplicity we shall henceforth refer to expected profits and expected consumer surplus simply as profits and consumer surplus.

The Nash Bargaining solution will be used in the flexible framework to determine the payment going to the firm

$$(\overline{f}R(i) - bt)(at - r(i)) \tag{5}$$

The first parenthesis shows the net gain of the investment to the buyer, while the second parenthesis shows the yield to the seller after subtracting from the expected payment the opportunity cost of its investment.

The participation constraint for the firm to enter in a contract with the government in this country is

$$(a'_\tau + \lambda_\tau(1 - a'_\tau))\, t > i - P_0 + C_0 \tag{6}$$

The above clearly shows that the firm's decision to enter a contract with the government is **dependent on the prior belief** this company has regarding the government's trustworthiness at the time τ when the contract is to be signed and the state of the institutional environment at that time. As we have already discussed this prior may be the product of a history of previous concession contracts in that country the success of which will lead to the upward revision

[8] Here we assume that the government fully represents the interests of the consumers. If this full alignment hypothesis is dropped, then this can be easily reflected by the model by assigning appropriate weights to the consumer and producer surplus within the government's objective function.

of this prior, i.e. a positive externality of such contracts is internalised in the decision of whether to participate in this new infrastructure contract. As long as $a_\tau > \overline{a}$, where $\overline{a} = \frac{i - P_0 + C_0}{t}$ the firm will invest.

On the other hand since it is the firm that makes the investment and pays for it, the gain to the government is always positive. Therefore the contract never violates the government's participation constraint.

The payment solution[9] at time $\tau + k$ is:

$$t^{f,k} = \frac{a_\tau^k \overline{f} R(i) + b\alpha R(i)}{2a_\tau^k b} = \frac{\overline{f} R(i)}{2b} + \frac{\alpha R(i)}{2a_\tau^k} \tag{7}$$

The equation above suggests that the lower a and b are, the higher is t, the value of the payment to the firm. However since the contract is flexible, this payment is updated in line with the values of trustworthiness perceptions. We have inserted the time superscript of the elapsed periods since commencement at time τ, in order to denote the stage in the update mechanism of the seller's belief regarding the buyer's trustworthiness. For brevity we have removed the subscript of the starting time in all variables except for a, where it is retained to distinguish a_τ^k from the prior belief of the probability of payment, $a_\tau = a'_\tau + \lambda_\tau(1 - a'_\tau)$. As we have already discussed λ and b may also change from time to time.

Conclusion 1 *Better trustworthiness of the government buyer as estimated by the seller and as perceived by the government for itself will lead to a lower t paid, and hence a better deal for the country in terms of its share of the revenue from the project.*

This result is confirmed econometrically in a recent paper by McMillan and Waxman (2007), where their evidence indicates that higher quality of institutions will lead to a larger share of the revenues from the investment accruing to the country. In a sense an increase in a and/or b corresponds to a reduction in the political risk premium and the cost of capital for a firm to accept a long duration contract with the government of a particular country. *It is also possible that this reduction may also increase the government's bargaining power (as McMillan and Waxman argue), rather than just its share in the rents. (This case is studied later, through the F-contract analysis.)*

Substituting the above result back into the expected profit and consumer surplus functions gives:

$$\Pi^{f,k} = P_0 - C_0 + \frac{a_\tau^k \overline{f} R(i)}{2b} + \frac{\alpha R(i)}{2} - i$$
$$CS^{f,k} = B_0 - P_0 + \frac{\overline{f} R(i)}{2} - \frac{b}{a_\tau^k} \frac{\alpha R(i)}{2}$$

Hence these expectations are continuously revised over time as the seller updates its beliefs regarding the trustworthiness of the buyer, as well as a result

[9] As both expressions in the product are positive we can apply a monotonic transformation of the expression into logarithms and easily check that both the FOC as well as the SOC are satisfied.

of any changes in b. Also note that the lower are sunk costs (i.e. the higher is α), the higher are expected profits and the lower is the expected consumer surplus. The profit maximising level of investment for the firm is:

$$i^{f,k} \mid R'(i^{f,k}) = \frac{2b}{a_\tau^k \overline{f} + b\alpha} \tag{8}$$

Hence investment decisions taken by the firm are updated in accordance to his beliefs regarding the probability of payment by the buying government. It follows that as $R'' < 0$, $\frac{\partial i^{f,k}}{\partial b} < 0$ while $\frac{\partial i^{f,k}}{\partial \alpha}$, $\frac{\partial i^{f},k}{\partial a_\tau^k}$, $\frac{\partial i^{f,k}}{\partial \overline{f}} > 0$. Total expected welfare at time $\tau + k$ is:

$$W^{f,k} = \Pi + CS = B_0 - C_0 + \left(\frac{a_\tau^k + b}{2b}\right)\overline{f}R(i^f) + \left(\frac{a_\tau^k - b}{2a_\tau^k}\right)\alpha R(i^{f,k}) - i^{f,k} \tag{9}$$

From (8) if $b > a_\tau^k$, then:

$$i^{f,k} \mid R'(i^{f,k})_{b>a_\tau^k} = \frac{2b}{a_\tau^k \overline{f} + b\alpha} > \frac{2b}{b\overline{f} + b\alpha} = \frac{2}{\overline{f} + \alpha} = i^f \mid R'(i^f)_{b=a_\tau^k<1} = i^f \mid R'(i^f)_{b=a_\tau^k=1} \tag{10}$$

Since $R' = \frac{2b}{a_\tau^k \overline{f} + b\alpha}$, it is easy to conclude that R' is an increasing function of b and a decreasing function of a_τ^k. As R' is an inverse function of investment (since $R'' < 0$), this means that investment and surplus are decreasing functions of b and increasing functions of a_τ^k. The above results indicate that

$$i^{f,k}_{b=a_\tau^k<1} = i^{f,k}_{b=a_\tau^k=1} \text{ and } R(i^{f,k}_{b=a_\tau^k<1}) = R(i^{f,k}_{b=a_\tau^k=1})$$

For high levels of investment and corresponding surplus what matters is that $b = a_\tau^k$ irrespective of whether absolute trustworthiness levels are high or low. Hence high levels of trustworthiness ($b = a_\tau^k = 1$) are not required, rather just matching values of b and a_τ^k.

Conclusion 2 *Untrustworthy agents can transact as efficiently as trustworthy agents provided that they hold similar estimates of the buyer's commitment to the payment agreement in the contract, provided that these are above a minimum level $\overline{a}$. Investment levels will be as high as when the probability of payment perceived by each side is one, if the update mechanism of trustworthiness of the beliefs of the seller results into the buyer being trusted (by the seller) to carry off the payment to the degree that it deserves to be trusted (in terms of its commitment). Hence alignment of beliefs, rather than whether the buyer government is per se a trustworthy contracting party, is important.*

Conclusion 2 explains why some countries which are ruled by a tight and corrupt elite where a_τ^k and b are both low but matching, can still sustain private investment in sunk assets through renegotiation with a monopoly supplier (as in the case of some sub-Saharan African countries). Note that this model of matching expectations only works in flexible contracts as it depends on continuous updating and renegotiation, or within a relational contract arrangement.

Undertrusting adds a deadweight loss by making the expected welfare function directly dependent on α as indicated in (9). The higher α is (the lower sunk costs are) then the higher this loss is. Moreover the surplus following renegotiation is now multiplied by a factor $\frac{a_\tau^k+b}{2b}$ less than one, that measures the degree of undertrusting.

Conclusion 3 *Undertrusting is damaging not only because it reduces the investment and corresponding surplus accruing to the society, but also because it reduces the welfare expectations by which a decision authority would rank this contractual choice.*

Note that the opposite conclusion holds for overtrusting. Overtrusting directly increases expected welfare in the two ways mentioned above. This means that flexible contracts where overtrusting is present will lead to big expected welfare gains in the short run. However they are likely to end in rapid contract collapse as soon as there is a realignment of expectations held by the investing companies to more realistic values (viz. Argentina in the late 1990s).

Theorem 1 *When renegotiation costs are sufficiently low and asset specificity sufficiently high, expected welfare is a decreasing function of the government's (buyer's) perception of its trustworthiness and an increasing function of the selling firm's perception of the government's trustworthiness.*

This holds when $\frac{\overline{f}}{\alpha} > \frac{b}{a_\tau^k} > 1$. *In other words for* $a_\tau^k\overline{f} > b\alpha$ $(b > a_\tau^k)$, b (a_τ^k) *has a negative (positive) indirect impact on expected welfare, reinforcing the negative (positive) direct impact of the same parameter on the function.*

Proof. Please refer to the appendix. ■

As was shown earlier, if $b > a_\tau^k$, expected welfare W^f is smaller than it would be if b was reduced to equal a_τ^k (and vice versa). *Hence undertrusting reduces expected welfare, just as overtrusting increases it (albeit temporarily as mentioned earlier).* Such a result does not require complete trustworthiness, but rather only that $b = a_\tau^k$. We now turn out attention to an attempt to tackle the problem of trust misalignment with the use of prepayment contracts.

3.2 Prepayment contracts

As mentioned in page 5, prepayment contracts can be viewed broadly so as to include arrangements with partial risk guarantees, where the guarantee is against the opportunism that may arise from the side of the government/regulator.

For flexible contracts with prepayments, the Nash bargaining problem is:

$$(\overline{f}R(i) - P_0 - bt)(P_0 + a_\tau^k t - r(i))$$

which gives a payment solution:

$$t^{P_0,k} = \frac{a_\tau^k \overline{f}R(i) + b\alpha R(i) - (a_\tau^k + b)P_0}{2a_\tau^k b} \tag{11}$$

Substituting this back into the expected profit function and consumer surplus functions gives:

$$\begin{aligned}\Pi^{P_0} &= P_0 - C_0 + \tfrac{a_\tau^k \overline{f}R(i)+b\alpha R(i)-(a_\tau+b)P_0}{2b} - i = \\ &\tfrac{(b-a_\tau)P_0}{2b} + \tfrac{a_\tau^k \overline{f}}{2b}R(i) + \tfrac{b\alpha}{2b}R(i) - C_0 - i \\ CS^{P_0} &= B_0 - P_0 - \tfrac{a_\tau^k \overline{f}R(i)+b\alpha R(i)-(a_\tau+b)P_0}{2a_\tau^k} + \overline{f}R(i) = \\ &= B_0 + \tfrac{(b-a_\tau^k)P_0}{2a_\tau^k} + \tfrac{a_\tau^k \overline{f}}{2a_\tau^k}R(i) - \tfrac{b\alpha}{2a_\tau^k}R(i)\end{aligned}$$

A prepayment will be set on the basis of the values at time τ. The existence of a prepayment will clearly affect the participation constraint of the firm as $\overline{a}$ will now become $\overline{a} = \frac{C_0+i-\frac{(b-a_\tau)P_0}{2b}}{t^{P_0}}$. As both expected profit as well as expected consumer surplus are increasing functions of the prepayment, this can be set at a maximum when there is an issue of undertrusting $(b > a)$:

$$P_0 = \tfrac{a_\tau \overline{f}R(i)+b\alpha R(i)}{a_\tau+b}$$

which if replaced into the above functions they become

$$\begin{aligned}\Pi^{P_0,k} &= \tfrac{(a_\tau+a_\tau^k)\overline{f}R(i)+\left[2b+(a_\tau-a_\tau^k)\right]\alpha R(i)}{2(a_\tau+b)} - C_0 - i \\ CS^{P_0,k} &= B_0 + \tfrac{(a_\tau+a_\tau^k)\left[b\overline{f}R(i)-b\alpha R(i)\right]}{2a_\tau^k(a_\tau+b)}\end{aligned}$$

The profit maximising level of investment at time $\tau + k$ is:

$$i^{P_0,k} \mid R'(i^{P_0,k}) = \frac{2(a_\tau + b)}{(a_\tau + a_\tau^k)\overline{f} + \left[2b + (a_\tau - a_\tau^k)\right]\alpha} \tag{12}$$

It is easy to check that $R'(i^{P_0,k})$ is decreasing in a_τ^k (for $\overline{f} > \alpha$), which combined with $a_\tau^k \geqq a_\tau$ means that it takes its maximum value for $R'(i^{P_0,k=0}) = \frac{a_\tau+b}{a_\tau\overline{f}+b\alpha}$. Clearly as $R' > 0$, $R'' < 0$ and $R'' < 0$, then since $b > a_\tau^k \geqq a_\tau$:

$$R'(i^{P_0,k>0}) \leqslant R'(i^{P_0,k=0}) = \frac{a_\tau + b}{a_\tau\overline{f} + b\alpha} < R'(i^{f,k=0}) = \frac{2b}{a_\tau\overline{f} + b\alpha}. \tag{13}$$

It follows that time at time $\tau + k$ both the investment and the corresponding surplus in a prepayment contract are higher than at time τ, which in turn are higher than the investment and surplus in a flexible contract at time τ. The inequality reads as:

$$i^{P_0,k>0} \geqslant i^{P_0,k=0} > i^{f,k=0} \text{ and } R(i^{P_0,k>0}) \geqslant R(i^{P_0,k=0}) > R(i^{f,k=0}). \qquad (14)$$

It is worth noting, that some of the appeal of a flexible contract may be restored over time if the trustworthiness of the government, as updated by the seller during the life of the contract, increases upwards over time (albeit in a snakes and ladders form). At sufficiently high levels of a_τ^k, the level of investment $i^{f,k>0}$ (which $\geqslant i^{f,k=0}$) may come to exceed first $i^{P_0,k=0}$, and then eventually even $i^{P_0,k>0}$ at the same time period. Therefore, at its later stages and if upwards trustworthiness updating has not been interrupted, a flexible contract with no prepayments may look preferable to the one with prepayments in terms of the investment incentives the former provides.

The expected welfare function at time τ ($k = 0$) is:

$$W^{P_0,k=0} = B_0 - C_0 + \overline{f} R(i^{P_0,k=0}) - i^{P_0,k=0} \qquad (15)$$

We next calculate the impact of b on $W^{P_0,k=0}$, again through the split of the total derivative into a direct and an indirect effect. As the direct effect on expected welfare is clearly equal to zero at time τ, we get:

$$\frac{dW^{P_0}}{db} = \frac{\partial W^{P_0}}{\partial i^{P_0}} \frac{\partial i^{P_0}}{\partial b} = \frac{b(\overline{f}-\alpha)}{(a_\tau \overline{f}+b\alpha)} \frac{a_\tau(\overline{f}-\alpha)}{(a_\tau \overline{f}+b\alpha)^2 R''(i^{P_0,k=0})} = \frac{b a_\tau (\overline{f}-\alpha)^2}{(a_\tau \overline{f}+b\alpha)^3 R''(i^{P_0,k=0})}$$

which clearly is negative as $R''(i^{P_0}) < 0$. Hence again b has a negative impact on welfare in the case of a prepayment contract.

Similarly,

$$\frac{\partial W^{P_0,k=0}}{\partial a_\tau} = \frac{\partial W^{P_0,k=0}}{\partial i^{P_0,k=0}} \frac{\partial i^{P_0,k=0}}{\partial a_\tau} = \frac{b(\overline{f}-\alpha)}{(a_\tau \overline{f}+b\alpha)} \frac{b(a_\tau-\overline{f})}{(a_\tau \overline{f}+b\alpha)^2 R''(i^{P_0,k=0})} = \frac{-b^2(\overline{f}-\alpha)^2}{(a_\tau \overline{f}+b\alpha)^3 R''(i^{P_0,k=0})} > 0$$

Conclusion 4 *Under prepayment contracts, expected welfare is a decreasing function of the buying state's probability of payment, and an increasing function of the seller's estimate of receiving payment. The impact of both at time τ is only limited to their impact on the level of investment and neither affects the expected welfare function directly. As already noted the preference for a prepayment contract relative to a flexible one is decreasing in $a_\tau^{k=0}$.*

Theorem 2 *The higher the degree of undertrusting, the more efficient are prepayment contracts compared to flexible contracts.*

Proof. Please refer to the appendix. ■

This gives us a preference relation for the decision maker between the two types of contracts, defined in a similar way to that of Myerson relating prizes (here welfare) to variable states of nature (here pay occurring), where the latter encompasses all subjective unknowns which might influence the prize to be received.

Issues of incentive compatibility may exist in flexible contracts. In particular, it may be beneficial for the buyer to declare an overestimate of his commitment to pay given that this will reduce the share of the payment that goes to the firm. Even with the context of a prepayment contract the buyer may still have an incentive to overstate b.

Theorem 3 *Both the flexible payment contracts and the flexible prepayment contracts are not always incentive compatible as buyer governments have an incentive to exaggerate their stated trustworthiness to a considerable extent.*

Proof. Please refer to the appendix. ■

Therefore in both types of contracts the buyer has an incentive to overstate his trustworthiness as long as the adverse impact that an inflated b has on investment is not so detrimental for it to more than offset any direct gains accruing to the buyer by overstating his trustworthiness.

3.3 F-contracts

If we drop the assumption that the buyer will always honestly declare its estimate of its own trustworthiness, then in this case it will be more appropriate to use an F-prepayment contract of the form:

$$F = (\overline{f}R(i) - P_0 - bt)^{a_\tau^k}(P_0 + a_\tau^k t - r(i))^b$$

Such a contract allows the trustworthiness declared by each party to affect the payoff of the other party. *This approach coupled up with prepayments makes the payoffs going to both parties independent of the trustworthiness parameters.* As we will show below, the impact of undertrusting on investment and welfare can be eliminated by establishing an F-contract.

The relevance of this model is not as a real world possible contract but in its role as a hypothetical benchmark. As we have seen flexible contracts become more difficult to agree and sustain as $\frac{b}{a_\tau^k}$ increases, because such contracts are no longer incentive compatible. The level of efficiency as measured by welfare, achieved by the benchmark F-contract model is only possible in the pure flexible and flexible prepayment contracts when $a_\tau^k = b$.

For F-contracts, the payment solution is determined by maximising F with respect to t:

$$t = \frac{\overline{f}R(i) + \alpha R(i) - 2P_0}{a_\tau^k + b} \qquad (16)$$

$\Pi^{PF} = P_0 - C_0 + a_\tau^k \frac{\overline{f}R(i)+\alpha R(i)-2P_0}{a_\tau^k+b} - i = \frac{(b-a)P_0+a_\tau^k\overline{f}R(i)+a_\tau^k\alpha R(i)}{a+b} - C_0 - i$
$CS^{PF} = B_0 - P_0 - b\frac{\overline{f}R(i)+\alpha R(i)-2P_0}{a_\tau^k+b} + \overline{f}R(i) = B_0 + \frac{(b-a_\tau^k)P_0+a_\tau^k\overline{f}R(i)-b\alpha R(i)}{a_\tau^k+b}$

As both of the above functions are increasing functions of the prepayment amount, the latter needs to be increased as the level of undertrusting ($b > a_\tau^k$) increases. This eliminates the incentive for the state to announce a trustworthiness higher than its true one in order to improve its share of consumer surplus as the prepayment is:

$$P_0 = \frac{\overline{f}R(i)+\alpha R(i)}{2}$$

which is independent of both a and b. This means that the prepayment is time independent. If replaced into the expected profit and consumer surplus functions these also become time independent and respectively equal to:

$$\Pi^{PF} = \frac{(a_\tau^k+b)(\overline{f}R(i)+\alpha R(i))}{2(a_\tau^k+b)} - C_0 - i = \frac{\overline{f}R(i)}{2} + \frac{\alpha R(i)}{2} - C_0 - i$$
$$CS^{PF} = B_0 + \frac{(a_\tau^k+b)\overline{f}R(i)-(a_\tau^k+b)\alpha R(i)}{2(a_\tau^k+b)} = B_0 + \frac{\overline{f}R(i)}{2} - \frac{\alpha R(i)}{2}$$

The prepayment profit maximising level of investment is:

$$i^{PF} \mid R'(i^{PF}) = \frac{2}{\overline{f}+\alpha} \qquad (17)$$

$$W^{PF} = B_0 - C_0 + \overline{f}R(i^{PF}) - i^{PF} \qquad (18)$$

We see that the investment decision within an F-contract is identical to the one in the A&S model, and similarly the payments to the seller and the buyer are independent of the trustworthiness parameters. *The F-contracts fully avoid the implications of trustworthiness by introducing a system of prepayments such that both the direct as well as the indirect effects of such parameters are eliminated. This also means that the investment decisions taken by the firm are not dependent on the updating mechanism of the seller's beliefs regarding the trustworthiness of the buyer.* The key point is that the payments in the A&S model have now become prepayments in the F-contracts.

Theorem 4 *The higher the degree of undertrusting, the more efficient an F-contract compared to a flexible one and to a flexible prepayment one.*

Proof. Please refer to the appendix. ■

Using expected welfare as a criterion, within undertrusting an F-contract will rank as superior to the other two types of flexible contracts.

3.4 Rigid contracts

We define as rigid a contract that specifies the main contract terms (e.g. prices, payments, etc.) in advance of the investment – and for the duration of the contract. In addition, the contract permanently specifies the level of the investments to be made.

Renegotiation is excluded from the theoretical model. However, in practice, renegotiation cannot be excluded and is common – not least to rescue projects where one or both parties finds emerging outcomes becoming unacceptable. Hence, the pure model is, to some extent, a hypothetical reference model.

The model incorporates some of this via the introduction of 'maladaptation' costs, which are defined as the difference between expected surplus levels and actual (outcome) surplus levels. The impact of maladaptation costs, $\underline{f}$, falls on investment levels; the way in which this happens is explained directly below. However, higher than expected maladaptation costs will inevitability increase the probability to renegotiate a rigid contract. This latter issue is discussed in the 3.5, while in this section we temporarily assume that such a probability is equal to zero.

In this model, the expected payoffs are:

$$\Pi^r = P_0 - C_0 + a_\tau \underline{f} R(i) - i$$
$$CS^r = B_0 - P_0 + (1 - \underline{f} b) R(i)$$

The value of the maladaptation parameter, $\underline{f} < 1$, is an inverse measure of the potential size of the investor's loss over the distribution of outcomes; e.g. actual versus expected traffic flows for toll roads. This parameter in our model is multiplied by the investor's expectation a_τ that the buyer will pay him the surplus agreed in the contract. Correspondingly, the surplus received by the purchasing government is increased by this investor's loss.

The firm will choose an investment level i^r such that:

$$i^r \mid R'(i^r) = \frac{1}{a_\tau \underline{f}} \tag{19}$$

So in this case the level of investment *only depends on the prior belief held by the seller regarding the trustworthiness of the buyer,*a_τ.

In this model, expected welfare is:

$$W^r = \Pi^r + CS^r = B_0 - C_0 + \left[1 + (a_\tau - b)\underline{f}\right] R(i^r) - i^r \tag{20}$$

As in the case of flexible uncertain contracts, W^r is a decreasing function of b, and an increasing function of a. For the case of $a = b$, the maladaptation costs only influence welfare indirectly (through their impact on investment). If $b > a$ (undertrusting) then the direct impact of $\underline{f}$ is negative and constitutes a deadweight loss, while in the reverse case of overtrusting its direct impact on welfare is positive.

The government's own perception of its trustworthiness only has a direct effect on welfare, as investment is not affected by b. Hence:

$$\frac{dW^r}{db} = -\underline{f}R(i^r) < 0$$

The impact of a_τ on welfare still retains both a direct and an indirect effect:

$$\frac{dW^r}{da_\tau} = \frac{\partial W^f}{\partial a_\tau} + \frac{\partial W^r}{\partial i^r}\frac{\partial i^r}{\partial a_\tau} = \underline{f}R(i^f) + \left[\frac{1+(a_\tau - b)\underline{f}}{a_\tau \underline{f}} - 1\right]\frac{\partial i^r}{\partial a} \Leftrightarrow$$
$$\frac{dW^r}{da_\tau} = \underline{f}R(i^f) + \frac{1-b\underline{f}}{a_\tau \underline{f}}\frac{\partial i^r}{\partial a_\tau} > 0$$

In other words, with undertrusting W^r is an increasing function of a_τ, i.e. an increasing function of the seller's prior belief regarding the trustworthiness of the buying state. Unlike the flexible uncertain payment contracts, here the *actual* size of the parameter a_τ does matter on investment since if $a_\tau = b < 1$:

$$i^r \mid R'(i^r)_{b=a_\tau<1} = \frac{1}{a_\tau \underline{f}} > i^r \mid R'(i^r)_{b=a_\tau=1} = \frac{1}{\underline{f}} \Leftrightarrow i^r_{a_\tau=b<1} < i^r_{a_\tau=b=1} \qquad (21)$$

This implies that $R(i^r)_{b=a_\tau<1} < R(i^r)_{b=a_\tau=1}$.

In summary, for rigid contracts matching estimates of trustworthiness are still important, but in this case, unlike the flexible contract case, the absolute level of the prior belief a_τ is important and needs to be high. Good outcomes on efficiency and investment require both a_τ to be close to one, as well as aligned values between b and this prior belief.

3.5 Hybrid contracts

We finally analyse the hybrid model, where there is always the probability that subsequent to investment taking place, rigid contracts will be renegotiated, and/or key terms reset. Following this, the key issues we explore in this section is the relative efficiency (in terms of welfare) between flexible to hybrid models in terms of the values key parameters: a) maladaptation costs, b) the probability of renegotiation of an *ex ante* rigid contract, c) sunk costs and d) renegotiation costs.

We follow the terminology of A&S and denote by $(1-\eta)$ the probability to see an *ex ante* rigid contract renegotiated. We calculate the profit function of the firm:

$$\Pi^H = \eta\Pi^r + (1-\eta)\Pi^f =$$
$$= \eta\left(P_0 - C_0 + a_\tau \underline{f}R(i) - i\right) + (1-\eta)\left(P_0 - C_0 + \frac{a_\tau^k \overline{f}R(i)}{2b} + \frac{\alpha R(i)}{2} - i\right) \Leftrightarrow$$
$$\Pi^H = P_0 - C_0 + a_\tau R(i)\left(\eta\underline{f} + \frac{(1-\eta)a_\tau^k\overline{f}}{2ba_\tau} + \frac{(1-\eta)\alpha}{2a_\tau}\right) - i \qquad (22)$$

The profit maximising level of investment at time $\tau + k$ will be:

$$i^{H,k} \mid R'(i^{H,k}) = \frac{2}{\eta(2a_\tau \underline{f} - \frac{a_\tau^k}{b}\overline{f} - \alpha) + \frac{a_\tau^k}{b}\overline{f} + \alpha} = \frac{2b}{2\eta a_\tau b\underline{f} + (1-\eta)(a_\tau^k \overline{f} + b\alpha)} \tag{23}$$

So when $\eta = 1$, the results are identical to those of a rigid contract as the government can credibly commit not to renegotiate the contract, while for $\eta = 0$ they coincide with those in the flexible model discussed in the beginning of this paper. We focus on the hybrid case where $0 < \eta < 1$.

Calculating the difference between the hybrid and the flexible contracts,

$$\begin{aligned} CS^{H,k} = \eta CS^r + (1-\eta)CS^f = \eta\left(B_0 - P_0 + (1-\underline{f}b)R(i)\right) + \\ +(1-\eta)\left(B_0 - P_0 + \tfrac{\overline{f}R(i)}{2} - \tfrac{b}{a_\tau^k}\tfrac{\alpha R(i)}{2}\right) = \\ B_0 - P_0 + \left(\eta(1-\underline{f}b) + (1-\eta)\tfrac{\overline{f}}{2} - (1-\eta)\tfrac{b}{a_\tau^k}\tfrac{\alpha}{2}\right)R(i) \end{aligned}$$

Hence the expected welfare outcome in the hybrid contract at time $\tau + k$, $W^{H,k}$, is:

$$\begin{aligned} W^{H,k} = B_0 - C_0 + \left[\eta + \eta\underline{f}(a_\tau - b)\right]R(i^{H,k}) + (1-\eta)\alpha\tfrac{a_\tau^k - b}{2a_\tau^k}R(i^{H,k}) + \\ +(1-\eta)\overline{f}\tfrac{a_\tau^k + b}{2b}R(i^{H.k}) - i^{H,k} \end{aligned}$$

If we calculate the difference between the expected welfare in a hybrid contract and the expected welfare in a flexible contract we get:

$$W^{H,k} - W^{f,k} = \eta + \eta\underline{f}(a_\tau - b)R(i^{H,k}) + \left[\overline{f}\frac{a_\tau^k + b}{2b} + \alpha\frac{a_\tau^k - b}{2a_\tau^k}\right]\left[(1-\eta)R(i^{H,k}) - R(i^{f.k})\right] +$$

$$+ i^{H,k} - i^{f,k} \tag{24}$$

We next calculate the impact on this difference of all the parameters, namely $\overline{f}, \underline{f}, \alpha$ and η. We present the direct and indirect effects (through investment) of all these parameters in the appendix. The results lead us to the following conclusions:

Proposition 1 *For $a_\tau^k \overline{f} > b\alpha$ and $\eta > 0$, the lower are maladaptation costs (the higher is $\underline{f}$), the more efficient is the hybrid model relative to the flexible one. But if the negative direct (welfare) effect comes to dominate the positive indirect one (on investment), then the flexible model is more efficient relative to the hybrid one.*

Proof. Please refer to the appendix. ■

Notice how the existence of undertrusting gives rise to the requirement that sunk costs are sufficiently high and renegotiation costs sufficiently low for $a_\tau^k \overline{f} > b\alpha$ to hold; a condition more demanding than in corresponding proposition in the A&S paper, where the necessary condition was that $\overline{f} > \alpha$. Our analysis in the appendix shows that the assumption $a_\tau^k \overline{f} > b\alpha$ implies that the impact of investment on welfare is positive for both types of contracts $\left(\frac{\partial W^H}{\partial i^H}, \frac{\partial W^f}{\partial i^f} > 0 \right)$.

Specifically, the gap between $\overline{f}$ and α needs to be sufficiently high so that when the former is multiplied by the updated belief of the firm regarding the probability of pay by the government, $a_\tau^k < 1$ and the latter by b $(> a_\tau^k)$, the inequality does not change its direction. The more pronounced undertrusting becomes (the higher the gap between b and a_τ^k is), the higher the difference between $\overline{f}$ and α needs to be for the proposition to hold.

In other words, the higher the level of undertrusting, the less likely is the hybrid contract to be more efficient. This is because an increase in the gap between b and a will make the inequality $a_\tau^k \overline{f} > b\alpha$ increasingly more difficult to sustain. If it becomes unsustainable and $a_\tau^k \overline{f} < b\alpha$, then the positive impact of $\underline{f}$ on welfare will be far smaller and, possibly, even negative. Hence substantial undertrusting may turn the tables in favour of the flexible contract!

The next three propositions, which are all formally set out, proved and analysed in the appendix, set the preference relations for a decision maker between the hybrid and the flexible contract based on expected welfare.

Proposition 2 argues that the lower the probability to renegotiate is, the more efficient a hybrid contract compared to a flexible one. Undertrusting lessens the positive impact of η on the expected welfare superiority of the hybrid contract. The direct effect of η may be negative; if this is the case it will reduce or even dominate the positive indirect effect of η on welfare. So extensive undertrusting reduces the strength of this proposition and may even come to reverse it.

Proposition 3 argues that the lower the asset specificity the more efficient a flexible contract compared to a hybrid one. As long as we restrict the direct negative impact of a higher α on the superiority of the flexible contract to be larger (in absolute terms) that the same impact on flexible contract (this can be done by assuming a sufficiently low η),the direct effect further reinforces the welfare superiority of the flexible contract as established in terms of the indirect effect.

Finally, according to proposition (4), provided that η is sufficiently high for the positive direct effect of $\overline{f}$ on expected welfare to be larger under a flexible rather than hybrid contract, this direct effect will reinforce the indirect effect in supporting the argument that the expected welfare superiority of the flexible model will increase as $\overline{f}$ increases.

4 DISCUSSION AND IMPLICATIONS OF THE MODEL

In the literature to date, there is no distinction between trust levels between investors and (buyer) governments. Indeed, the implicit assumption is that there is no misalignment between these two. However, this may be an unfortunate simplification.

The reality is not only that contracts are most likely to break down when there is such misalignment, but also that as we have shown it is unlikely to achieve incentive compatible contracts unless perceptions can be aligned. That, in turn, may help explain whether infrastructure contracts that run into difficulties can be renegotiated between the parties. Our conjecture is that in circumstances where the seller's perception of the reliability of the buyer is much lower than the buyer's, the contract is most likely to break down irrevocably. For contracts where the seller's updated perception of the buyer's reliability overshoots (is higher than justified given the buyer's type, e.g. there is overtrusting), contracts may continue satisfactorily until the misalignment is revealed at which point they are likely to fall apart rapidly. Conversely, contracts can survive where the levels of trust are low but perceptions are correctly aligned.

Most of the discussion on trust and most of the theoretical models in this field accept that a and b can be high or low depending on the country or project but, implicitly or explicitly assume that $a = b$. However, there are circumstances where a and b are both low (e.g. under 0.5) but both parties have the same perspective and hence private investment in infrastructure may be sustainable. Conversely, there may be circumstances where a and b are both relatively high (e.g. above 0.5) but a is sufficiently less than b so as to create a significant degree of mistrust.

Table 1

Perceptions of Trustworthiness

Case (A)	Case (C)
Repeat project and/or contract	New type of project or contract
Country with strong institutions	Country with strong institutions
and high trustworthiness reputation	and high trustworthiness reputation
$a = b,\ \ a, b \longrightarrow 1$ (*large*)	$a < b,\ \ a, b \gtrsim 0.5$ (*moderate*)
Case (B)	Case (D)
Repeat project and/or contract	New type of project or contract
Country with weak and/or	Country with past history of weak
corrupt institutions	and/or corrupt institutions but trying to
	establish reputation for trustworthiness
$a = b,\ \ a, b \longrightarrow 0$	$a < b,\ \ a, b < 0.5$ (*small*)

These issues were fully explored in the theoretical model set out in the previous sections. We can summarise these by setting out a simple typology, based on a 4 quadrant table as in Table 1 above we then set out and discuss some illustrative examples on each case as set in this table. The table and the discussion focuses primarily on undertrusting, but we will say something further on overtrusting afterwards. Let us briefly consider each quadrant in turn.

Case (A) represents well-established types of project in high reputation countries. For many types of infrastructure concession contracts, trust is likely to be high and the contracting parties may well be able to monitor, enforce and revise their contracts straightforwardly, without a need for external assistance (other than occasionally for arbitration/dispute resolution or similar) or for 'pre-payment' arrangements. Hence, private investment is readily forthcoming and at a reasonable cost of capital as perceived risk is low.Examples include repeat water supply management contracts in easy to access and process water (as in Menard and Saussier), repeat UK PPP contracts in politically uncontentious sectors, electricity distribution in Chile. This is a sustainable and efficient process that is potentially welfare maximising.

Case (B) represents infrastructure contracts in countries with low quality institutions. Supplying companies can and do have supply contracts with governments in Latin America, Sub-Saharan Africa and elsewhere where government is arbitrary and/or corrupt – and liable to change on the replacement of the current autocrat. These contracts can support investment.

The key point is that they are *relational* contracts where the monitoring, enforcement and revision is done between the two parties who know one another well. The contracts will only be sustainable if the supplier can expect to receive a return adjusted for higher risks and for any (potentially large) corruption payments – both buyer and seller may receive returns from corrupt practices. Clearly, these contracts are far from optimal for the consumers or taxpayers in the countries concerned and are likely to be monopoly contracts that do not allow any competitive entry (e.g. no independent power producers or competing telecom operators). But, given aligned beliefs on each party's trustworthiness of the regime over the life of the contract, they do allow a low-level, incentive compatible equilibrium with positive private investment and without the need for a 'pre-payment' arrangement. This process is sustainable (at least while the current parties continue) but it is highly inefficient and far from welfare maximizing.

Case (C) represents early and potentially difficult contracts in high reputation countries. In the UK, the London Underground PPPs were in this category (and one – the Tube Line PPP - appears to be progressing well, even if the Metronet PPP has failed) as were the NATS air traffic control contract and early PPPs in hospitals and prisons. In these cases, to sustain the contract, it helps to have an external regulator – the PPP Arbiter for the London Underground, the Civil Aviation Authority for NATS.

Note that in most of these cases, there has been an explicit or implicit government guarantee providing a 'pre-payment' facility. There clearly are potential incentive compatibility problems so that breakdown is likely in the case of major disputes. But, provided the first contract (or first few years of the contract) go well and any major problems are addressed by successful renegotiation (as with NATS), subsequent contracts or periods should go well as updating may mean that a^k_τ becomes equal to or close to b.

Most of these contracts faced not only significant political risk, but most also had serious construction, technology, demand and/or demand risks. The London Underground contracts also had major issues of ongoing rather than front-loaded investment. Interestingly, an early major success for UK PPPs was the Dartford bridge crossing of the River Thames. For that project, an effective and robust 'pre-payment' mechanism was put in place through allowing the duration of the concession to correspond to that necessary for the contractor to recover his principal and earn an agreed rate of return (i.e. an NPV contract).

Finally, Case (D) represents countries that start with poor reputations but are trying hard to obtain private investment into infrastructure e.g. into roads, power generation, sometimes water. To attract that investment, governments may pass concession laws, introduce independent regulators or allow external arbitration or similar. Examples of such countries include Uganda, Nigeria, Mozambique and Romania.

However, supplying companies are likely to want a demonstrable record of achievement for those institutions before reducing their cost of capital risk premia. Hence, the private investment may not be possible – at least not at a cost that would be acceptable in terms of the final tariff. In these circumstances, private investment will only be forthcoming and sustainable if there is external support in place e.g. from an effective 'pre-payment' agreement. That is where transitional regulatory risk guarantees and other forms of external underpinning (e.g. on-demand guarantees, bilateral investment treaties, comprehensive credit insurance, etc.) can help align perceptions or relative trustworthiness.

If this process is successful and the country floats away from the (hopefully unused) pre-payment support, the result is an efficient equilibrium with realigned trust perceptions.

The table omits overtrusting. The latter raises more difficult issues and seems to be much more difficult to anticipate. The non-alignment of trust perceptions is only revealed after some time. Typically it is not only some period after pre-agreed investments have been made, but later when unanticipated problems have arisen which require an increase in the revenue requirement if the supplying entity under the contract is not to be forced into bankruptcy. Sometimes, as in the Argentinian case, the misperception is on the powers of any regulatory agency, particularly at times of crisis.

Note that sometimes trust perception problems clearly exist, but it is unclear whether we are observing undertrusting or overtrusting. The collapse of the London Underground Metronet PPPs initially looks like an example of undertrusting – and certainly some of the features of the contract reflect undertrusting via the political risks from the hostility of the London authorities to a PPP model. However, it is also possible to argue that the failure is a case of overtrusting where the investor believed that the contract was closer to a cost-plus contract than was actually the case. This example points to the need, in future work, to extend the model to encompass perceptions of sellers as well as of buyers.

5 CONCLUSIONS

The key conclusions of this paper are:

(i) How far governments and investors share close perceptions of government trustworthiness in contract enforcement is at least as important for the sustainability of infrastructure contracts and investment as whether absolute levels of trust are high or low.

(ii) Undertrusting (i.e. where the company has a lower perception of government trustworthiness than the government has) is the more frequently observed case. This occurs when investing infrastructure firms have a lower expectation of full payment under the contract than the government has. Updating of the former with no interuptions may allow undertrusting to be eventually resolved at some point during the duration of the contract.

(iii) The negative effects of undertrusting increase the higher the degree of trust misalignment.

(iv) Undertrusting is a transitional state of imbalance, primarily associated with innovative contracts or first-time contracting parties. To rectify it, requires the use of external measures such as pre-payment arrangements, regulatory and/or other guarantees, or other specific actions to realign initially different perceptions.

(v) Overtrusting is theoretically less likely and is also less frequently observed in practice. It is not easy to rectify, and when revealed is likely to lead to rapid contract breakdown. The revelation is likely to arise when the buyer approaches the seller for a post-investment revenue increase relative to what was expected and/or specified in the original contract .

(vi) Rigid contracts appear particularly unattractive in our model as they are dependent for sustainability not only on closely aligned trust perceptions, but also on high absolute levels of trust, whereas flexible relational infrastructure concession contracts are potentially sustainable with low but aligned levels of trust, typically on a relational basis.

(vii) Under conditions of undertrusting, flexible contracts are not incentive compatible so that successful renegotiation in situations of serious problems is likely to be very difficult.

(viii) Hybrid contracts tend to be more efficient than either pure rigid or flexible contracts except at higher levels of undertrusting when flexible contracts dominate – assuming they are sustainable.

(ix) The potential for trust misalignments e.g. over future investment requirements and costs, technology uncertainties, political sensitivities, etc. provides additional support for the role of external regulatory or similar agencies, guarantees and similar mechanisms. These also support the use of hybrid contracts (rigid contracts but with a strictly positive probability of renegotiation.)

The work reported here is, as far as we are aware, a first attempt to seriously model the role of potential trust estimates and trust misalignments on investment and expected welfare estimates used to evaluate infrastructure contracts. Such an analysis helps bring out the potential role of regulatory agencies, of external guarantee mechanisms and of contract features that reduce uncertainty (e.g. NPV contracts).

We look forward to seeing whether our proposed framework and approach can be usefully extended. More importantly, we would like to be able to test its predictions using real world data on infrastructure contracts in developing as well as in OECD countries.

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7 APPENDIX

Proof of Theorem 1

We first show that $W^{f,k}$ is a decreasing function of b. To calculate the impact of b on $W^{f,k}$ we split this into a direct and an indirect effect; the latter evaluates the impact of b on $W^{f,k}$ via the impact of b on the level of investment. Hence:

$$\frac{dW^{f,k}}{db} = \frac{\partial W^{f,k}}{\partial b} + \frac{\partial W^{f,k}}{\partial i^f}\frac{\partial i^{f,k}}{\partial b} = -\frac{2a_\tau^k}{(2b)^2}\overline{f}R(i^{f,k}) - \frac{1}{2a_\tau^k}\alpha R(i^{f,k}) + \left[\frac{a_\tau^k+b}{2b}\overline{f}\frac{2b}{a_\tau^k\overline{f}+b\alpha} + \frac{a_\tau^k-b}{2a_\tau^k}\alpha\frac{2b}{a_\tau^k\overline{f}+b\alpha} - 1\right]\frac{\partial i^{f,k}}{\partial b} =$$
$$-\frac{2a_\tau^k}{(2b)^2}\overline{f}R(i^{f,k}) - \frac{1}{2a_\tau^k}\alpha R(i^{f,k}) + \frac{b(a_\tau^k\overline{f}-b\alpha)}{a_\tau^k(a_\tau^k\overline{f}+b\alpha)}\frac{\partial i^{f,k}}{\partial b} =$$
$$= -\frac{2a_\tau^k}{(2b)^2}\overline{f}R(i^{f,k}) - \frac{1}{2a_\tau^k}\alpha R(i^{f,k}) + \frac{b(a_\tau^k\overline{f}-b\alpha)}{a_\tau^k(a_\tau^k\overline{f}+b\alpha)}\frac{\frac{2a_\tau^k\overline{f}}{(a_\tau^k\overline{f}+b\alpha)^2}}{R''(i^{f,k})} \Leftrightarrow$$

$$\frac{dW^{f,k}}{db} = -\frac{2a_\tau^k}{(2b)^2}\overline{f}R(i^{f,k}) - \frac{1}{2a_\tau^k}\alpha R(i^{f,k}) + \frac{2\overline{f}b(a_\tau^k\overline{f}-b\alpha)}{(a_\tau^k\overline{f}+b\alpha)^3R''(i^{f,k})} \tag{25}$$

If $b > a_\tau^k$ (undertrusting) and $\frac{\overline{f}}{\alpha} > \frac{b}{a_\tau^k} > 1$, $\frac{\partial W^{f,k}}{\partial i^{f,k}} = \frac{b(a_\tau^k\overline{f}-b\alpha)}{a_\tau^k(a_\tau^k\overline{f}+b\alpha)} > 0$, and $\frac{\partial R'(i^{f,k})}{\partial b} = \frac{2a_\tau^k\overline{f}}{\left(a_\tau^k\overline{f}+b\alpha\right)^2} > 0$. The latter implies a negative impact of b on the level of investment $i^{f,k}$ since $\frac{\partial i^{f,k}}{\partial b} = \frac{\partial i^{f,k}}{\partial R'(i^{f,k})}\frac{\partial R'(i^{f,k})}{\partial b} = \frac{\frac{\partial R'(i^{f,k})}{\partial b}}{R''(i^{f,k})} < 0$. Therefore, b has a negative indirect impact on expected welfare, reinforcing the negative direct impact.

Similarly, we calculate the impact of a_τ^k on W^f:

$$\frac{dW^{f,k}}{da_\tau^k} = \frac{\partial W^{f,k}}{\partial a_\tau^k} + \frac{\partial W^{f,k}}{\partial i^{f,k}}\frac{\partial i^{f,k}}{\partial a_\tau^k} = \frac{1}{2b}\overline{f}R(i^{f,k}) + \frac{2b}{(2a_\tau^k)^2}\alpha R(i^{f,k}) + \left[\frac{a_\tau^k+b}{2b}\overline{f}\frac{2b}{a_\tau^k\overline{f}+b\alpha} + \frac{a_\tau^k-b}{2a_\tau^k}\alpha\frac{2b}{a_\tau^k\overline{f}+b\alpha} - 1\right]\frac{\partial i^{f,k}}{\partial a_\tau^k} =$$
$$\frac{1}{2b}\overline{f}R(i^{f,k}) + \frac{2b}{(2a_\tau^k)^2}\alpha R(i^{f,k}) + \frac{b(a_\tau^k\overline{f}-b\alpha)}{a_\tau^k(a_\tau^k\overline{f}+b\alpha)}\frac{\partial i^{f,k}}{\partial a_\tau^k}$$

As $\frac{\partial R'(i^{f,k})}{\partial a_\tau^k} = \frac{-2b\overline{f}}{\left(a_\tau^k\overline{f}+b\alpha\right)^2} < 0$, this implies a positive impact of a_τ^k on the level of investment $i^{f,k}$ since $\frac{\partial i^{f,k}}{\partial a_\tau^k} = \frac{\partial i^{f,k}}{\partial R'(i^{f,k})}\frac{\partial R'(i^{f,k})}{\partial a_\tau^k} = \frac{\frac{\partial R'(i^{f,k})}{\partial a_\tau^k}}{R''(i^{f,k})} > 0$.

Proof of Theorem 2

Taking the difference between $W^{P_0} - W^f$ at time τ $(k = 0)$ and differentiating with respect to b gives:

$$\frac{\partial(W^{P_0}-W^f)}{\partial b} = \frac{2a_\tau}{(2b)^2}\overline{f}R(i^f) + \frac{1}{2a_\tau}\alpha R(i^f) - \frac{b(a_\tau\overline{f}-b\alpha)}{a_\tau(a_\tau\overline{f}+b\alpha)}\frac{\partial i^f}{\partial b} + \frac{b(\overline{f}-\alpha)}{(a_\tau\overline{f}+b\alpha)}\frac{\partial i^{P_0}}{\partial b} =$$
$$\frac{2a_\tau}{(2b)^2}\overline{f}R(i^f) + \frac{1}{2a_\tau}\alpha R(i^f) - \frac{2\overline{f}b(a_\tau\overline{f}-b\alpha)}{(a_\tau\overline{f}+b\alpha)^3R''(i^f)} + \frac{ba_\tau(\overline{f}-\alpha)^2}{(a_\tau\overline{f}+b\alpha)^3R''(i^{P_0})} =$$
$$= \frac{2a_\tau}{(2b)^2}\overline{f}R(i^f) + \frac{1}{2a_\tau}\alpha R(i^f) + \frac{b}{(a_\tau\overline{f}+b\alpha)^3}\frac{a_\tau(\overline{f}-\alpha)^2R''(i^f)-2\overline{f}(a_\tau\overline{f}-b\alpha)R''(i^{P_0})}{R''(i^f)R''(i^{P_0})}$$

where since $R'', R''' < 0$ and $i^{P_0} > i^f$ we have that $R''(i^{P_0}) < R''(i^f) < 0$.
Analogously,

$$\frac{\partial(W^{P_0}-W^f)}{\partial a_\tau} = \frac{-b^2(\overline{f}-\alpha)^2}{(a_\tau\overline{f}+b\alpha)^3 R''(i^{P_0})} - \frac{1}{2b}\overline{f}R(i^f) - \frac{2b}{(2a_\tau)^2}\alpha R(i^f) + \frac{2\overline{f}b^2(a_\tau\overline{f}-b\alpha)}{a_\tau(a_\tau\overline{f}+b\alpha)^3 R''(i^f)} =$$
$$= -\frac{1}{2b}\overline{f}R(i^f) - \frac{2b}{(2a_\tau)^2}\alpha R(i^f) - \frac{b^2}{a_\tau(a_\tau\overline{f}+b\alpha)^3}\frac{a_\tau(\overline{f}-\alpha)^2R''(i^f)-2\overline{f}(a_\tau\overline{f}-b\alpha)R''(i^{P_0})}{R''(i^f)R''(i^{P_0})}$$

Hence unless $\frac{\overline{f}^2-\alpha^2}{2\overline{f}\alpha} < \frac{b}{a_\tau}-1$[10] and $-\frac{b^2}{a_\tau(a_\tau\overline{f}+b\alpha)^3}\frac{a_\tau(\overline{f}-\alpha)^2R''(i^f)-2\overline{f}(a_\tau\overline{f}-b\alpha)R''(i^{P_0})}{R''(i^f)R''(i^{P_0})} > \frac{1}{2b}\overline{f}R(i^f)+\frac{2b}{(2a_\tau)^2}\alpha R(i^f)$, we will have that $\frac{\partial(W^{P_0}-W^f)}{\partial b}$ is positive and $\frac{\partial(W^{P_0}-W^f)}{\partial a_\tau}$ negative. Hence as undertrusting increases because a_τ decreases and/or b decreases, a prepayment contract becomes more efficient in relation to a flexible contract.

Proof of Theorem 3

Let us assume a falsely declared beta (say b^{dl}) higher that the true beta (say b^{tr}). By totally differentiating the payment solution in (8) we get that:

$$\frac{dt^{f,k}}{db^{dl}} = \frac{\partial t^{f,k}}{\partial b^{dl}} + \frac{\partial t^{f,k}}{\partial i^{f,k}}\frac{\partial i^{f,k}}{\partial b^{dl}} = \frac{-\overline{f}R(i^{f,k})}{2(b^{dl})^2} + \left(\frac{1}{a_\tau^k}\right)\frac{\partial i^{f,k}}{\partial b^{dl}} < 0$$

Hence as

$$CS^{f,k} = B_0 - P_0 + \overline{f}R(i^{f,k}) - b^{tr}t^{f,k} \Leftrightarrow$$
$$\frac{dCS^{f,k}}{db^{dl}} = \overline{f}\frac{2b^{dl}}{a_\tau^k\overline{f}+b^{dl}\alpha}\frac{\partial i^{f,k}}{\partial b^{dl}} - b^{tr}\frac{dt^{f,k}}{db^{dl}} \Leftrightarrow$$

$$\frac{dCS^{f,k}}{db^{dl}} = b^{tr}\frac{\overline{f}R(i^{f,k})}{2(b^{dl})^2} + \frac{1}{a_\tau^k\overline{f}+b^{dl}\alpha}\frac{(2\overline{f}a_\tau^kb^{dl} - \overline{f}a_\tau^kb^{tr} - b^{tr}b^{dl}\alpha)}{a_\tau^k}\frac{\partial i^{f,k}}{\partial b^{dl}} \quad (26)$$

As $\overline{f} > \alpha$ and $1 \geqslant b^{dl} > b^{tr}$, the term in the brackets is positive and hence the overall indirect effect is negative. Hence the buyer has an incentive to overstate his trustworthiness as long as the impact of an inflated b on investment is not so detrimental as to substantially reduce both i and hence R (which are determined by the firm given its profit maximising investment decision) to such an extent that more than offsets any direct gains for the buyer.

Similarly, for the case of prepayments, by totally differentiating $P_0 = \frac{a_\tau\overline{f}R(i)+b^{dl}\alpha R(i)}{a_\tau+b^{dl}}$, we get that:

$$\frac{dP_0}{db^{dl}} = \frac{\partial P_0}{\partial b^{dl}} + \frac{\partial P_0}{\partial i^{P_0}}\frac{\partial i^{P_0}}{\partial b^{dl}} =$$
$$\frac{-a_\tau\overline{f}R(i)}{(a_\tau+b^{dl})^2} + \frac{\alpha R(i)(a+b^{dl})-b^{dl}\alpha R(i)}{(a_\tau+b^{dl})^2} + \left(\frac{a_\tau+b^{dl}}{a_\tau\overline{f}+b^{dl}\alpha}\right)\frac{a_\tau\overline{f}+b^{dl}\alpha}{a_\tau+b^{dl}}\frac{\partial i^{P_0}}{\partial b^{dl}} \Leftrightarrow$$
$$\frac{dP_0}{db^{dl}} = \frac{a_\tau(\alpha-\overline{f})R(i)}{(a_\tau+b^{dl})^2} + \frac{\partial i^{P_0}}{\partial b^{dl}} < 0$$

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[10] This is rather unlikely: clearly $\frac{\overline{f}^2-\alpha^2}{2\overline{f}\alpha}$ is an increasing function of $\overline{f}$ and a decreasing function of α, hence unless the former is small and the latter large this inequality is unlikely to hold. If $\frac{\overline{f}}{\alpha} > \frac{b}{a_\tau} > 1$, it is more likely than not that $\frac{\overline{f}^2-\alpha^2}{2\overline{f}\alpha} > \frac{b-a_\tau}{a_\tau}$ rather than the other way round.

Hence as

$$\begin{gathered} CS^{P_0} = B_0 - P_0 + \overline{f}R(i) - 0b^{tr} \Leftrightarrow \\ \frac{dCS^{P_0}}{db^{dl}} = \overline{f}\frac{a_\tau + b^{dl}}{a_\tau \overline{f} + b^{dl}\alpha}\frac{\partial i^{P_0}}{\partial b^{dl}} - \frac{dP_0}{db^{dl}} \Leftrightarrow \end{gathered}$$

$$\frac{dCS^{P_0}}{db^{dl}} = \frac{a_\tau(\overline{f} - \alpha)R(i)}{(a_\tau + b^{dl})^2} + \frac{b^{dl}(\overline{f} - \alpha)}{a_\tau \overline{f} + b^{dl}\alpha}\frac{\partial i^{P_0}}{\partial b^{dl}} \tag{27}$$

Hence again the buyer has an incentive to overstate his trustworthiness provided that the impact of an inflated b on investment is not so detrimental that the reduction in investment as denoted by the second term in the relation above more than offsets any direct gains to the buyer (as denoted by the first term). However, unlike $\frac{dCS^f}{db^{dl}}$, $\frac{dCS^{P_0}}{db^{dl}}$ does not depend on the magnitude of the true value of b, b^{tr}.

Proof of Theorem 4

Comparing the PF solution (prepayment F-contracts) to those of pure flexibility contracts f or prepayment ones, it is easy to show that for $b > a_\tau > a_\tau^k$:

$$\begin{gathered} R'(i^{PF}) = \frac{2}{\overline{f}+\alpha} \leqslant R'(i^{P_0,k}) = \frac{2(a_\tau + b)}{(a_\tau + a_\tau^k)\overline{f} + [2b + (a_\tau - a_\tau^k)]\alpha} \\ R'(i^{PF}) = \frac{2}{\overline{f}+\alpha} \leqslant R'(i^{f,k}) = \frac{2b}{a_\tau \overline{f} + b\alpha} \end{gathered}$$

it follows that $i^{PF} > i^{P_0,k}$, $i^{PF} > i^{f,k}$ and $R(i^{PF}) > R(i^{P_0,k})$, $R(i^{PF}) > R(i^{f,k})$.

As $\frac{\partial W^{PF}}{\partial i^{PF}} = \overline{f}R'(i^{PF}) - 1 = \frac{2\overline{f} - \overline{f} - \alpha}{\overline{f}+\alpha} = \frac{\overline{f}-\alpha}{\overline{f}+\alpha}$, $\frac{\partial i^{PF}}{\partial b} = \frac{\partial i^{PF}}{\partial R'(i^{PF})}\frac{\partial R'(i^{PF})}{\partial b} = \frac{\frac{\partial R'(i^{PF})}{\partial b}}{R''(i^{PF})} = 0$ and $\frac{\partial R'(i^{PF})}{\partial b} = 0$, both the direct as well as the indirect effects of b on welfare are zero. Hence taking the difference between $W^{PF} - W^f$ and differentiating with respect to b gives:

$$\frac{\partial\left(W^{PF} - W^{f,k}\right)}{\partial b} = \frac{2a_\tau^k}{(2b)^2}\overline{f}R(i^{f,k}) + \frac{1}{2a_\tau^k}\alpha R(i^{f,k}) - \frac{b(a_\tau^k \overline{f} - b\alpha)}{a_\tau^k(a_\tau^k \overline{f} + b\alpha)}\frac{\partial i^{f,k}}{\partial b} > 0$$

Hence the higher b is, the more efficient is the F- prepayment contract as compared to the pure flexible contract of A&S.

Similarly both the direct as well as the indirect effect of a on W^{PF} is zero, and hence the lower a_τ^k is, the more efficient an F- prepayment contract as compared to a flexible contract.

In an analogous manner, we compare the expected welfare implications of an F-contract to that of a prepayment contract. Calculating:

$$W^{PF} - W^{P_0,k} = \overline{f}R(i^{PF}) - i^{PF} - \overline{f}R(i^{P_0,k}) + i^{P_0,k}$$

and differentiating $W^{P_0,k}$ with respect to b gives:

$$\frac{dW^{P_0,k}}{db}=\frac{\partial W^{P_0,k}}{\partial b}+\frac{\partial W^{P_0}}{\partial i^{P_0,k}}\frac{\partial i^{P_0,k}}{\partial b}=\frac{\partial W^{P_0,k}}{\partial b}+\frac{\partial W^{P_0}}{\partial i^{P_0,k}}\frac{\partial R'(i^{P_0,k})}{\partial b}\frac{\partial i^{P_0,k}}{\partial R'(i^{P_0,k})}$$

The direct effect of b on the expected welfare in a prepayment contract is:

$$\frac{\partial W^{P_0,k}}{\partial b}=\frac{(a_\tau+a_\tau^k)\left(\overline{f}-\alpha\right)R(i^{P_0,k})}{2(a_\tau+b)^2}+$$

$$\frac{2\left(a_\tau^k\right)(a_\tau+a_\tau^k)\left(\overline{f}-\alpha\right)R(i^{P_0,k})(a_\tau+b)-2\left(a_\tau^k\right)(a_\tau+a_\tau^k)\left(\overline{f}-\alpha\right)R(i^{P_0,k})b}{4(a_\tau^k)^2(a_\tau+b)^2}=$$

$$=\frac{\left(a_\tau^k\right)(a_\tau+a_\tau^k)\left(\overline{f}-\alpha\right)R(i^{P_0,k})}{2(a_\tau^k)(a_\tau+b)^2}+\frac{(a_\tau+a_\tau^k)\left(\overline{f}-\alpha\right)R(i^{P_0,k})(a_\tau+b)-(a_\tau+a_\tau^k)\left(\overline{f}-\alpha\right)R(i^{P_0,k})b}{2(a_\tau^k)(a_\tau+b)^2}\Leftrightarrow$$

$$\frac{\partial W^{P_0,k}}{\partial b}=\frac{(a_\tau+a_\tau^k)^2\left(\overline{f}-\alpha\right)R(i^{P_0,k})}{2(a_\tau^k)(a_\tau+b)^2}>0$$

The indirect effect of b on the expected welfare in a prepayment contract will the product of the follwing three terms:

$\frac{\partial W^{P_0,k}}{\partial i^{P_0,k}}=\frac{\left(\overline{f}-\alpha\right)(a_\tau+2b-a_\tau^k)}{(a_\tau+a_\tau^k)\overline{f}+(2b+a_\tau-a_\tau^k)\alpha}$, $\frac{\partial R'(i^{P_0,k})}{\partial b}=\frac{\left(\overline{f}-\alpha\right)(a_\tau+a_\tau^k)}{\left[(a_\tau+a_\tau^k)\overline{f}+(2b+a_\tau-a_\tau^k)\alpha\right]^2}$ and $\frac{\partial i^{P_0,k}}{\partial R'(i^{P_0,k})}=\frac{1}{R''(i^{P_0,k})}$

So,

$$\frac{\partial W^{P_0,k}}{\partial i^{P_0,k}}\frac{\partial R'(i^{P_0,k})}{\partial b}\frac{\partial i^{P_0,k}}{\partial R'(i^{P_0,k})}=\frac{\left(\overline{f}-\alpha\right)^2(a_\tau+2b-a_\tau^k)(a_\tau+a_\tau^k)}{\left[(a_\tau+a_\tau^k)\overline{f}+(2b+a_\tau-a_\tau^k)\alpha\right]^3R''(i^{P_0,k})}<0$$

At time τ the direct effect is equal to zero and we have that:

$$\frac{\partial\left(W^{PF}-W^{P_0,k}\right)}{\partial b}=-\frac{\left(\overline{f}-\alpha\right)^2ba_\tau}{\left[(a_\tau\overline{f}+b\alpha\right]^3R''(i^{P_0,k})}>0$$

Hence the higher b is, the more efficient the F-prepayment contract is as compared to the prepayment contract, and this effect tends to be intensified towards the beginning of the contract where a is lower. If later on during the life of the contract there is upwards updating in the trust beliefs held by the firm, then this will reduce the superiority of such F contracts.

The opposite result will hold for a_τ as:

$$\frac{\partial(W^{PF}-W^{P_0,k})}{\partial a_\tau}=0+\frac{b^2(\overline{f}-\alpha)^2}{(a_\tau\overline{f}+b\alpha)^3R''(i^{P_0,k})}<0.$$

■

Next, we set out in detail the proofs for the four propositions referred to in section 3.5. For $\eta>0$, $i^{H,k}>i^{f,k}$, $R(i^{H,k})>R(i^{f,k})$ we require that $R'(i^{H,k})<R'(i^{f,k})$, which holds if:

$$2a_\tau b\underline{f}-a_\tau^k\overline{f}>b\alpha \tag{28}$$

It then follows that $R''(i^{H,k})<R''(i^{f,k})<0$.

We now look at the impact of parameters on investment. First we look at **the impact of** α :

$$
\begin{aligned}
\frac{\partial i^{f,k}}{\partial \alpha} &= \frac{\frac{\partial R'(i^{f,k})}{\partial \alpha}}{R''(i^{f,k})} = \frac{\frac{-2b^2}{(a_\tau^k \overline{f}+b\alpha)^2}}{R''(i^{f,k})} > 0 \\
\frac{\partial i^{H,k}}{\partial \alpha} &= \frac{\frac{\partial R'(i^{H,k})}{\partial \alpha}}{R''(i^{H,k})} = \frac{\frac{-2b^2(1-\eta)}{\left(2\eta a_\tau b\underline{f}+(1-\eta)(a_\tau^k \overline{f}+b\alpha)\right)^2}}{R''(i^{H,k})} > 0 \\
2b^2 > 2b^2(1-\eta), &\ \frac{1}{(a_\tau^k \overline{f}+b\alpha)^2} > \frac{1}{\left(2\eta a_\tau b\underline{f}+(1-\eta)(a_\tau^k \overline{f}+b\alpha)\right)^2}
\end{aligned}
$$

and

$$
0 > R''(i^{f,k}) > R''(i^{H,k}) \Leftrightarrow -\frac{1}{R''(i^{f,k})} > -\frac{1}{R''(i^{H,k})} > 0,
$$

it follows that

$$
\frac{\partial i^{H,k}}{\partial \alpha} < \frac{\partial i^{f,k}}{\partial \alpha} \tag{29}
$$

Also, for the **impact on investment of renegotiation costs,** $\overline{f}$:

$$
\begin{aligned}
\frac{\partial i^{f,k}}{\partial \overline{f}} &= \frac{\frac{\partial R'(i^{f,k})}{\partial \overline{f}}}{R''(i^{f,k})} = \frac{\frac{-2ba}{(a_\tau^k \overline{f}+b\alpha)^2}}{R''(i^{f,k})} > 0 \\
\frac{\partial i^{H,k}}{\partial \overline{f}} &= \frac{\frac{\partial R'(i^{H,k})}{\partial \overline{f}}}{R''(i^{H,k})} = \frac{\frac{-2ba_\tau^k(1-\eta)}{\left(2\eta a_\tau b\underline{f}+(1-\eta)(a_\tau^k \overline{f}+b\alpha)\right)^2}}{R''(i^{H,k})} > 0
\end{aligned}
$$

Hence as above,

$$
\frac{\partial i^{H,k}}{\partial \overline{f}} < \frac{\partial i^{f,k}}{\partial \overline{f}} \tag{30}
$$

The **impact of the maladaptation parameter,** $\underline{f}$ is:

$$
\begin{aligned}
\frac{\partial i^{f,k}}{\partial \underline{f}} &= 0 \\
\frac{\partial i^{H,k}}{\partial \underline{f}} &= \frac{\frac{\partial R'(i^{H,k})}{\partial \underline{f}}}{R''(i^{H,k})} = \frac{\frac{-4\eta a_\tau b^2}{\left(2\eta a_\tau b\underline{f}+(1-\eta)(a\overline{f}+b\alpha)\right)^2}}{R''(i^{H,k})} > 0
\end{aligned}
$$

Therefore:

$$
\frac{\partial i^{H,k}}{\partial \underline{f}} > \frac{\partial i^{f,k}}{\partial \underline{f}} = 0 \tag{31}
$$

The impact of b **:**

$$
\begin{aligned}
\frac{\partial i^{f,k}}{\partial b} &= \frac{\frac{\partial R'(i^{f,k})}{\partial b}}{R''(i^{f,k})} = \frac{\frac{2a_\tau^k \overline{f}}{(a_\tau^k \overline{f}+b\alpha)^2}}{R''(i^{H,k})} < 0 \\
\frac{\partial i^{H,k}}{\partial b} &= \frac{\frac{\partial R'(i^{H,k})}{\partial b}}{R''(i^{H,k})} = \frac{\frac{2(1-\eta)a_\tau^k \overline{f}}{\left[2\eta a_\tau b\underline{f}+(1-\eta)(a_\tau^k \overline{f}+b\alpha)\right]^2}}{R''(i^{H,k})} < 0 \\
\frac{2a\overline{f}}{(a\overline{f}+b\alpha)^2} &> \frac{2(1-\eta)a_\tau^k \overline{f}}{\left[2\eta a_\tau b\underline{f}+(1-\eta)(a_\tau^k \overline{f}+b\alpha)\right]^2} \\
0 > R''(i^{f,k}) > R''(i^{H,k}) &\Leftrightarrow -\frac{1}{R''(i^{f,k})} > -\frac{1}{R''(i^{H,k})} > 0 \\
-\frac{\frac{2a_\tau^k \overline{f}}{(a\overline{f}+b\alpha)^2}}{R''(i^{f,k})} &> -\frac{\frac{2(1-\eta)a_\tau^k \overline{f}}{\left[2\eta a_\tau b\underline{f}+(1-\eta)(a\overline{f}+b\alpha)\right]^2}}{R''(i^{H,k})} \Leftrightarrow
\end{aligned}
$$

$$0 > \frac{\partial i^{H,k}}{\partial b} > \frac{\partial i^{f,k}}{\partial b} \tag{32}$$

Also the impact of a_τ^k:

$$\frac{\partial i^{f,k}}{\partial a_\tau^k} = \frac{\frac{\partial R'(i^{f,k})}{\partial a_\tau^k}}{R''(i^{f,k})} = \frac{\frac{-2b\overline{f}}{(a_\tau^k\overline{f}+b\alpha)^2}}{R''(i^{H,k})} > 0$$

$$\frac{\partial i^{H,k}}{\partial a_\tau^k} = \frac{\frac{\partial R'(i^{H})}{\partial a_\tau^k}}{R''(i^{H,k})} = \frac{\frac{-2b(1-\eta)\overline{f}}{\left[2\eta a_\tau b\underline{f}+(1-\eta)(a_\tau^k\overline{f}+b\alpha)\right]^2}}{R''(i^{H,k})} > 0$$

We have that,

$$2b\overline{f} > 2b(1-\eta)\overline{f}$$

and,

$$\frac{1}{(a_\tau^k\overline{f}+b\alpha)^2} > \frac{1}{\left[2\eta a_\tau b\underline{f}+(1-\eta)(a_\tau^k\overline{f}+b\alpha)\right]^2}$$

Moreover,

$$0 > R''(i^{f,k}) > R''(i^{H,k}) \Leftrightarrow -\frac{1}{R''(i^{f,k})} > -\frac{1}{R''(i^{H,k})} > 0$$

Therefore,

$$-\frac{\frac{-2b\overline{f}}{(a_\tau^k\overline{f}+b\alpha)^2}}{R''(i^{f,k})} > -\frac{\frac{-2b(1-\eta)\overline{f}-4b^2\eta\underline{f}}{\left[2\eta a_\tau b\underline{f}+(1-\eta)(a_\tau^k\overline{f}+b\alpha)\right]^2}}{R''(i^{H,k})} \Leftrightarrow$$

$$\frac{\partial i^{H,k}}{\partial a_\tau^k} < \frac{\partial i^{f,k}}{\partial a_\tau^k} \tag{33}$$

Finally, we examine the impact on investment on the probability or renegotiation, η:

$$\frac{\partial i^{f,k}}{\partial \eta} = 0$$

and

$$\frac{\partial i^{H,k}}{\partial \eta} = \frac{\frac{\partial R'(i^{H,k})}{\partial \eta}}{R''(i^{H,k})} = \frac{\frac{-2b\left[2a_\tau b\underline{f}-a_\tau^k\overline{f}-b\alpha\right]}{\left[2\eta a_\tau b\underline{f}+(1-\eta)(a_\tau^k\overline{f}+b\alpha)\right]^2}}{R''(i^{H,k})} > 0$$

Hence,

$$\frac{\partial i^{H,k}}{\partial \eta} > \frac{\partial i^{f,k}}{\partial \eta} = 0 \tag{34}$$

To derive the indirect impact of the above parameters on welfare, we first need to calculate the investment derivatives $\frac{\partial W^{H,k}}{\partial i^{H,k}}$ and $\frac{\partial W^{f,k}}{\partial i^{f,k}}$ respectively.

When calculating the indirect effect on expected welfare in the flexible and hybrid rigid models we impose the condition:

$$0 < \frac{\partial W^{H,k}}{\partial i^{H,k}} = \frac{b}{a_\tau^k} \frac{2\eta a_\tau(1-\underline{f}b) + (1-\eta)(a_\tau^k\overline{f} - b\alpha)}{\left(2\eta a_\tau b\underline{f} + (1-\eta)(a_\tau^k\overline{f} + b\alpha)\right)} < \frac{\partial W^{f,k}}{\partial i^{f,k}} = \frac{b}{a_\tau^k} \frac{(a_\tau^k\overline{f} - b\alpha)}{(a_\tau^k\overline{f} + b\alpha)} \tag{35}$$

For the above to hold, it suffices to show that:

$$\begin{gathered}\left[2\eta a_\tau(1-\underline{f}b) + (1-\eta)(a_\tau^k\overline{f} - b\alpha)\right](a_\tau^k\overline{f} + b\alpha) < \\ \left[\left(2\eta a_\tau b\underline{f} + (1-\eta)(a_\tau^k\overline{f} + b\alpha)\right)\right](a_\tau^k\overline{f} - b\alpha) \Leftrightarrow \\ 2\eta a_\tau(1-\underline{f}b)(a_\tau^k\overline{f} + b\alpha) < 2\eta a_\tau b\underline{f}(a_\tau^k\overline{f} - b\alpha) \Leftrightarrow \\ (a_\tau^k\overline{f} + b\alpha) < b\underline{f}(a_\tau^k\overline{f} - b\alpha) + \underline{f}b(a_\tau^k\overline{f} + b\alpha) \Leftrightarrow \\ (1 - b\underline{f})(a_\tau^k\overline{f} + b\alpha) - b\underline{f}(a_\tau^k\overline{f} - b\alpha) < 0 \Leftrightarrow\end{gathered}$$

$$b\alpha < a_\tau^k\overline{f}(2b\underline{f} - 1) \tag{36}$$

If $\frac{a_\tau}{a_\tau^k} > \overline{f}$, then

$$a_\tau^k\overline{f}(2b\underline{f} - 1) < 2a_\tau b\underline{f} - a_\tau^k\overline{f}.$$

This means that inequality (36) implies inequality (28), while if $\frac{a_\tau}{a_\tau^k} < \overline{f}$, (28) implies (36). Remember that what has also been assumed so far is that $a_\tau^k\overline{f} > b\alpha$. Therefore, (36) can be true provided that $2b\underline{f} - 1$ is positive and sufficiently close to one for the direction of the inequality to be retained.

Proof of Proposition 1

Obviously both the direct as well as the indirect effect of $\underline{f}$ on welfare under a flexible contract is zero. Hence $\frac{\partial i^{H,k}}{\partial \underline{f}} > \frac{\partial i^{f,k}}{\partial \underline{f}} = 0 \Leftrightarrow \frac{\partial W^{H,k}}{\partial i^H}\frac{\partial i^{H,k}}{\partial \underline{f}} > \frac{\partial W^f}{\partial i^f}\frac{\partial i^f}{\partial \underline{f}} = 0$. Moreover as the direct effect of $\underline{f}$ on $W^{H,k}$ is equal to $\eta(a-b)R(i^{H,k}) < 0$ this means that the efficiency of the hybrid model relative to the flexible one is eroded by the existence of a deadweight loss in the case of undertrusting. If the negative direct effect on $W^{H,k}$ is dominated by the positive indirect effect on $W^{H,k}$, then the higher $\underline{f}$ (i.e. the lower the misalignment cost is), the more efficient the hybrid contract is compared to a flexible one. On the other hand, if the direct effect dominates the indirect effect the reverse will be the case. However the latter is unlikely to happen for as long as $\frac{\overline{f}}{\alpha} > \frac{b}{a_\tau^k} > 1$ as the gap in the value of b and a_τ^k will be exceeded by the gap in the values of $\overline{f}$ and α. ■

For the next proposition, we shall add to the assumption $a_\tau^k\overline{f} > b\alpha$, assumption (28) where $2a_\tau b\underline{f} - \overline{f}a_\tau^k > b\alpha$. As already mentioned, this latter inequality assumption implies that:

$$\begin{aligned} i^{H,k} &> i^{f,k}, \\ R(i^{H,k}) &> R(i^{f,k}), \\ R'(i^{H,k}) &< R'(i^{f,k}), \\ R''(i^{H,k}) &< R''(i^{f,k}). \end{aligned}$$

Proposition 2 *For $a_\tau^k\overline{f} > b\alpha$, $2ba_\tau\underline{f} - \overline{f}a_\tau^k > b\alpha$ and $\eta > 0$, the lower the probability to renegotiate a rigid contract (the higher η), the more efficient a hybrid contract compared to a flexible one.*

Proof. If $b\alpha < 2ba_\tau\underline{f} - a_\tau^k\overline{f}$, then $\frac{\partial i^{H,k}}{\partial\eta} > \frac{\partial i^{f,k}}{\partial\eta} = 0$. This means that given (28):

$$\frac{\partial W^H}{\partial i^{H,k}}\frac{\partial i^{H,k}}{\partial\eta} > \frac{\partial W^{f,k}}{\partial i^{f,k}}\frac{\partial i^{f,k}}{\partial\eta} = 0$$

Moreover in the hybrid contract there is a direct effect of the same parameter equal to:

$$[1+\underline{f}(a_\tau - b)]R(i^{H,k}) - \alpha\frac{a_\tau^k - b}{2a_\tau^k}R(i^{H,k}) - \overline{f}\frac{a_\tau^k + b}{2b}R(i^{H,k}) \Leftrightarrow$$
$$\frac{2R(i^{H,k})}{4a_\tau^k b}[2a_\tau^k b + 2a_\tau^k b\underline{f}(a_\tau - b) - b(a_\tau^k - b)\alpha - a_\tau^k(a_\tau^k + b)\overline{f}]$$

For the direct effect to reinforce the indirect one, the former should be positive, which is the case as:

$$2ba_\tau^k > a_\tau^k(a_\tau^k + b)\overline{f} + b(a_\tau^k - b)\alpha - 2a_\tau^k b\underline{f}(a_\tau - b) \Leftrightarrow$$
$$2ba_\tau^k > a_\tau^k(a_\tau^k\overline{f} + b\alpha - 2b\underline{f}a_\tau) + b(a_\tau^k\overline{f} - b\alpha + ba_\tau^k\underline{f})$$
$$ba_\tau^k(1-\overline{f}) + ba_\tau^k(1-\underline{f}) + b\alpha > a_\tau^k(a_\tau^k\overline{f} + b\alpha - 2b\underline{f}a_\tau)$$

Given (28) $a_\tau^k\overline{f} + b\alpha - 2b\underline{f}a_\tau < 0$, and hence the above inequality always holds. ■

For the remaining two propositions, we shall start from the requirement that the impact of investment on expected welfare in a hybrid contract is smaller that the impact of investment on expected welfare in a flexible model ($0 < \frac{\partial W^{H,k}}{\partial i^{H,k}} < \frac{\partial W^{f,k}}{\partial i^{f,k}}$). As already shown, this requires inequality (36), which is re-written below, to hold:

$$a_\tau^k\overline{f}(2b\underline{f} - 1) > b\alpha \Longleftrightarrow$$

$$\underline{f} > \frac{a_\tau^k\overline{f} + b\alpha}{2a_\tau^k b\overline{f}}\left(> \frac{a_\tau^k\overline{f} + b\alpha}{2a_\tau^k b} = \frac{\overline{f}}{2b} + \frac{\alpha}{2a_\tau^k} > \frac{\overline{f} + \alpha}{2}\right)$$

Hence inequality (36) imposes further size boundaries for $\overline{f}$, $\underline{f}$ and α. In particular, the lower boundary for the value of $\underline{f}$, which is an inverse measure of the misalignment costs, becomes even more restrictive than in Proposition 2. More simply, the maladaptation costs are smaller (i.e. $\underline{f}$ is higher) than the level needed to ensure that as the probability of renegotiation decreases, the hybrid contract becomes more efficient relative to the flexible one. Assuming that $a\overline{f} > b\alpha$, (28) and (36) all hold we have the following two propositions:

Proposition 3 *For $0 < \frac{\partial W^{H,k}}{\partial i^{H,k}} < \frac{\partial W^{f,k}}{\partial i^{f,k}}$ and $0 < \eta < \frac{R(i^{H,k}) - R(i^{f,k})}{R(i^{H,k})}$, the lower the level of asset specificity (i.e. the higher α), the more efficient the flexible contract compared to the hybrid one.*

Proof. Concerning the indirect effects, given that $0 < \frac{\partial i^{H,k}}{\partial \alpha} < \frac{\partial i^{f,k}}{\partial \alpha} \Leftrightarrow$

$$\frac{\partial W^{H,k}}{\partial i^{H,k}} \frac{\partial i^{H,k}}{\partial \alpha} < \frac{\partial W^{f,k}}{\partial i^{f,k}} \frac{\partial i^{f,k}}{\partial \alpha},$$

if (36) and (28) both hold. As already discussed the direct effect of α on welfare is negative in both models given the introduction of a deadweight loss if $a_{\tau}^{k} < b$. Hence for this negative impact to be of a smaller absolute size for the flexible compared to the hybrid one, so that the above inequality is preserved, the condition is that

$$(1-\eta)R(i^{H,k}) > R(i^{f,k}) \Leftrightarrow \eta < \frac{R(i^{H,k}) - R(i^{f,k})}{R(i^{H,k})}$$

The upper boundary set for η reflects the requirement that the direct effect (deadweight loss) of α on welfare is (in absolute terms) smaller in the flexible contract than in hybrid one. This combined with the indirect effect of α will imply that $\frac{d(W^{H,k} - W^{f,k})}{d\alpha} < 0$, and therefore proposition 3 applies. ■

Proposition 4 *For* $0 < \frac{\partial W^{H,k}}{\partial i^{H,k}} < \frac{\partial W^{f,k}}{\partial i^{f,k}}$ *and* $\eta > \frac{R(i^{H,k}) - R(i^{f,k})}{R(i^{H,k})}$, *the lower the renegotiation costs (i.e. the higher* $\overline{f}$*), the more efficient a flexible contract compared to a hybrid one.*

Proof. Given that $0 < \frac{\partial i^{H,k}}{\partial \overline{f}} < \frac{\partial i^{f,k}}{\partial \overline{f}} \Leftrightarrow$

$$\frac{\partial W^{H,k}}{\partial i^{H,k}} \frac{\partial i^{H,k}}{\partial \overline{f}} < \frac{\partial W^{f,k}}{\partial i^{f,k}} \frac{\partial i^{f,k}}{\partial \overline{f}},$$

if (36) is satisfied which means that (35) holds. Therefore there is a larger indirect effect for the flexible as compared to the hybrid model. This will be strengthened by the direct effect if $(1-\eta)R(i^{H,k}) < R(i^{f,k}) \Leftrightarrow \eta > \frac{R(i^{H,k}) - R(i^{f,k})}{R(i^{H,k})}$ as then $\frac{\partial W^{H,k}}{\partial \overline{f}} < \frac{\partial W^{f,k}}{\partial \overline{f}}$. In other words, the lower boundary for η reflects the requirement that the direct positive effect of $\overline{f}$ on welfare is greater in the flexible contract than in hybrid rigid one. Given this boundary, proposition 4 of the A&S paper, that the lower the renegotiation costs (the higher $\overline{f}$), the more efficient the flexible contract relative to the hybrid model (i.e. $\frac{d(W^{H} - W^{f})}{d\overline{f}} < 0$), is reinforced under conditions of undertrusting. ■

The actual size of the commitment not to renegotiate matters in both propositions (3) and (4). This is once more the result of the existence of the direct effect that both parameters α and $\overline{f}$ have on welfare, but in an opposite manner. As far as proposition (4) is concerned, the higher η is (the higher the commitment not to renegotiate) the more similar the hybrid model becomes to the pure rigid one. All other things being equal, the higher $\overline{f}$ is (the lower renegotiation costs are), the more advantageous the flexible contract is. This result is the same as the A&S proposition (4). On the other hand, the lower η is (the higher the probability to renegotiate), then the hybrid model becomes increasingly similar to the flexible one. Hence the latter contract loses some of its advantage in terms of low renegotiation costs, but gains an advantage in terms of low asset specificity terms, as it further strengthens the argument that a flexible contract is to be preferred if sunk costs are low, as proposition (3) indicates.