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Lender Learning and Entry under General Demand Uncertainty[1]

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Abstract

In this paper, we examine the effect of potential entry on learning by a lender when the demand shock has a general distribution. We show that under this type of noise, entry does not lead to any changes in the equilibrium expected signals and therefore, there is no effect on learning by the lender, unlike the case when noise is uniformly distributed. The result holds even when contracts are not observable.

JEL Codes: D4, D8;

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1 Introduction

In this paper, we examine the effect of potential entry on learning by a principal when demand is uncertain. Learning by a principal is important to study since firms enter into various types of contracts, with lenders, employees and other firms, while at the same time facing uncertainty in the market or in their costs. Firms often know more about their costs and/or demand than the contracting party and thus, the third party may be able to learn about private information of the firm through contracting variables and publicly observable information. While there is a vast literature on experimentation by firms under different market structures and using different choice variables (see Aghion, Espinosa and Jullien (1993), Mirman, Samuelson and Urbano (1993) and Belleflamme and Bloch, (2001) for example), the issue of learning by a principal has only been studied recently. Jeitschko and Mirman (2002), in a pioneering paper study learning by a principal when the agent has private information and the outcome is noisy (see also, Jeitschko, Mirman and Salgueiro (2002)). In a series of papers, Jain, Jeitschko and Mirman, JJM henceforth, (in particular, JJMa (2002) and JJMb (2005)), examine the dual problem of learning and entry-deterrence by adding potential entry in the market in which the agent operates, along the lines of Milgrom and Roberts (1983). Their work provides insight into two issues - the effect of an agency relationship on probability of entry and the effect of potential entry on the nature of the equilibrium contracts, in particular, the agent's incentives and the principal's learning about the privately known parameter. JJMa (2002) find that when contracts are publicly observed, and the demand shock is uniformly distributed, potential entry leads to more learning by the lender (strategic experimentation) by weakening the agent's incentive problem as well as by making the benefits of learning higher. JJMb (2005) show that when the contract is not observed by the entrant, the principal's inability to commit via a publicly observed contract leads to less learning and a higher probability of entry than when contracts are publicly observed.

The purpose of our paper is to examine the effect of potential entry on learning by the principal for a general demand shock. We do so by relaxing the main assumption of JJM, namely that the demand shock is uniformly distributed. Interestingly, we find that regardless of observability of the contract, potential entry has no effect on the equilibrium expected outputs in the first period and therefore, no effect on learning, although some contract parameters do change. Under uniform distribution, learning is either complete or zero. That is, the lender either learns the intercept from the price observation or learns nothing from it. When demand shock is generally distributed along the entire real line and satisfies some technical conditions, learning is never zero or complete. Thus, our paper shows that the conclusions of JJMa and JJMb regarding the effect of potential entry on learning by a principal depend critically on the assumption of a uniformly distributed demand shock which leads to extreme learning outcomes. Aghion, Espinosa and Jullien (1993) perform a similar comparison of their results. That is, they analyze learning in a duopoly location game under demand uncertainty mainly under the assumption of a uniformly distributed demand shock and then check whether their results generalize to a general distribution. They find that results under uniform distribution do not necessarily carry over, unless the discount factor is large. While results weaken, their conclusion is not quite as drastic as we find in our model. It is not the general demand shock per se that is driving our result. The result is very much in the context of the model where linearity of demand, the information structure and the contracting environment interact together.

We maintain the other main assumptions of JJMa, namely that demand is linear and contracts are short-term. We also maintain the assumption of publicly observed contracts until later in the paper where we consider hidden contracts as well. However, instead of cost uncertainty, we assume that it is the demand intercept that is private information of the incumbent. We do so for ease of comparing our results with the literature on experimentation as well as with JM. As Jain (2009b) shows, the effect of potential entry on learning under demand uncertainty is quite similar to the case of cost uncertainty, ensuring that we are able to compare our results with JJM. That is, our results are robust to the whether it is the demand parameter or the cost parameter that is private information of the incumbent.

We assume that the demand intercept can take two values, high and low. The demand shock is distributed according to a general density function on the entire real line that satisfies the monotone likelihood ratio property (MLRP) among other conditions to be made precise below. We find that under these assumptions on the demand shock and linearity of demand, the effect of potential entry is limited to a shift in the information rent to the good-type incumbent as well as in the future benefit from experimentation, and therefore, does not lead to any change in the expected output targets set in the contract. Intuitively, there are two factors behind this surprising result. First, the potential entrant's expected profits are independent of its posterior belief about demand. This is a feature that is well-noted and discussed, especially in JJMb (2005). However, its implications for general demand uncertainty are not fully explored. It turns out that this feature has an important implication for the entry rule, namely, that when learning is never complete nor zero, entry must occur for all beliefs (we refer to this as sure entry) or not occur at all, unlike in the uniform case. Second, the shifts in the information rent term and the expected profits of the principal in the second period are also independent of the posterior belief, due to the entrant's output being independent of the posterior belief. Further, the infinite support of the shock distribution implies that shifts independent of the belief are irrelevant in determining the sensitivity of expected information rent and expected benefits from experimentation to the expected price targets set in the first period contract.

Jain (2009a) shows that under uniform distribution, sure entry affects the choice of outputs either through the ratchet effect or the experimentation effect, depending on whether the cost parameter is private information or the demand parameter is private information. In this paper, on the other hand, we show that sure entry has no effect on either the ratchet effect or experimentation behavior of the lender, once the assumption on the demand shock is relaxed. While actual profits and payments change due to sure entry, choice variables, outputs or expected prices do not. Thus, it is not sure entry per se that causes the effect of entry on learning to disappear. It is the combined effect of sure entry, absence of complete learning and the contracting environment that together lead to our result. For example, Dimitrova and Schlee (2003) show that sure entry in the second period can reduce information acquisition by a monopolist incumbent even when demand shock has a general distribution. The presence of a lender and hence a contracting relationship alters these results.

We assume throughout most of the paper that contracts are publicly observed so that the potential entrant and the lender have the same beliefs. Later in the paper, in Section 5, we show that our results are robust to this assumption. Interestingly, with a generally distributed demand shock, unobservability of contracts does not make a difference, unlike in JJM. This is because the potential entrant's expected profits are conditional on its own posterior belief, not the lender's. And the possibility of divergent beliefs does not impact the fact that the entrant's expected profits are independent of its posterior belief. In particular, the potential entrant does not take into account the possibly different belief of the lender when solving its problem. It can only use its own posterior beliefs in determining its best strategy and this strategy is independent of the posterior. This implies that the lender's value function as well as the good incumbent's information rent remain unchanged from the public contract case.

In Section 2, we present the model; in Section 3, the benchmark model with no-entry; in Section 4, the model with entry and the main results of the paper; in Section 5, the discussion of the hidden contract case and in Section 6, we conclude.

2 Model

There are two time periods with a monopolist incumbent in the first period and Cournot duopolists in the second period, if entry occurs. The inverse demand function in each period is,

$$p = \widetilde{a} - bq + \epsilon,$$

where ϵ is a random, unobservable term that is distributed on the entire real line according to the density function $f(\epsilon)$ and has zero mean, and q is total output. We assume that f satisfies monotone likelihood ratio property (MLRP) and is twice continuously differentiable. The lender believes that $\widetilde{a} \in \{\underline{a}\ ,\overline{a}\}$, $\underline{a} < \overline{a}$, with probability of $a = \overline{a}$ being ρ_0. We similarly use upper bars and lower bars on quantities to denote these variables under high demand and low demand respectively. Note that $\overline{p} = \overline{a} - b\overline{q}$ is the expected price under high demand and $\underline{p} = \underline{a} - b\underline{q}$ is the expected price under low demand. Further, we use $\overline{f}$ and $\underline{f}$ to denote the densities $f(p - (\overline{a} - b\overline{q}))$ and $f(p - (\underline{a} - b\underline{q}))$, respectively. That is, $\overline{f}$ is the density corresponding to high demand and $\underline{f}$ corresponds to low demand. The random component of demand is assumed to be i.i.d. over the two time periods. Costs are assumed to be zero, for simplicity.

The incumbent observes the realization of $\widetilde{a}$ and then chooses quantity q and thereafter ϵ and p are realized. The price p is publicly observable, but the quantity, q, is the private information of the producing firm(s) and unverifiable to others. The incumbent requires outside funding in the amount of F in each period. Thus, contracts are assumed to be short term. A financial intermediary, i.e., a bank, provides these funds in exchange for later repayment of R. For simplicity, and without loss of generality, F is normalized to be zero, and the entrant is assumed not to need any funding. The contract between the bank and the incumbent takes the form of a repayment schedule that maps the observed market price p into an amount $R(p)$ that the incumbent must pay to the bank. However, it is the expected profit and thus, expected repayment, corresponding to a choice of output or expected price, that matters in calculating optimal choices. While, the bank cannot directly target these unobserved variables, it can and does provide incentives to the incumbent through the repayment schedule to target them. Thus, as in JJM, we only analyze the expected repayment, and expected prices (outputs).[1]

3 Benchmark Model - No Entry

In this section, we consider the case where there is only one firm in both periods. As is standard, the equilibrium of this model is derived by solving the second period problem first. Since the contracts are short-term, the lender must write a contract in the second period to

[1] Jeitschko and Mirman (2002) and Jeitschko, Mirman and Salgueiro (2002) explain how the optimal contract is determined in this noisy environment. Since price is the only observable to the principal, the contract must be based on that. However, the principal can construct this schedule in a way that induces the agent to target a certain expected price, the actual price being random. These expected prices, or price targets, for each type, are determined first and then the implementation of these targets is determined. In general, there are many repayment schedules that can implement the targets. Since output choice is what drives learning as well as distribution of prices, we focus on this variable, and equivalently the price target, and refer the reader to JM for further details regarding implementation.

maximize its expected profit $u_b = (1-\rho)\underline{R} + \rho\overline{R}$ subject to the incentive compatibility and participation constraints of the two types of incumbent.[2] Here, $\underline{R}$ and $\overline{R}$ denote the expected repayment from the incumbent in the low demand and high demand states respectively, and ρ is the posterior belief about high demand, based on the observation of the first period price and obtained using Bayes' rule as follows:

$$\rho \equiv \frac{\rho_0 \overline{f}}{\rho_0 \overline{f} + (1-\rho_0)\underline{f}}.$$

As is standard, only two constraints bind: the incentive compatibility constraint of the high demand incumbent requiring that the expected net profits (net of repayment) from targeting $\overline{p}$ be at least as high as expected net profits from targeting $\underline{p}$:[3]

$$(\overline{a} - b\overline{q})\overline{q} - \overline{R} \geq \left(\frac{\overline{a} - \underline{a}}{b} + \underline{q} \right) (\underline{a} - b\underline{q}) - \underline{R},$$

and participation constraint of the low-demand incumbent:

$$(\underline{a} - b\underline{q})\underline{q} - \underline{R} \geq 0.$$

[2] We use the same notation for the second period variables as for the first period, for convenience.

[3] We assume that expected prices are targeted, leading of course, to corresponding output levels. To target $\underline{p} = \underline{a} - b\underline{q}$, q on the right hand side of the constraint needs to be $\frac{\overline{a}-\underline{a}}{b} + \underline{q}$. In JJM, targeting outputs and prices is equivalent because, there, the cost parameter is the source of information asymmetry. Under demand uncertainty, they are not, but in terms of learning, the variable targeted is irrelevant. Since price is the observable, it seems natural to set expected prices as the targets.

Since the two constraints are binding, we obtain,

$$
\begin{aligned}
\underline{R} &= (\underline{a} - b\underline{q})\underline{q}, \overline{R} = (\overline{a} - b\overline{q})\overline{q} - \frac{\overline{a} - \underline{a}}{b}(\underline{a} - b\underline{q}), \\
\overline{u} &= \frac{\overline{a} - \underline{a}}{b}(\underline{a} - b\underline{q}). \qquad (1)
\end{aligned}
$$

Here $\overline{u}$ denotes the information rent of the high demand incumbent in the second period.

Next, we show that the equilibrium quantity $\underline{q}$ is a function of the second period belief, so that the information rent is a function of posterior beliefs. The lender chooses $\underline{q}$ and $\overline{q}$, to maximize,

$$
\rho\left((\overline{a} - b\overline{q})\overline{q} - \frac{1}{b}(\overline{a} - \underline{a})(\underline{a} - \underline{bq})\right) + (1 - \rho)(\underline{a} - b\underline{q})\underline{q}.
$$

The first–order–conditions are sufficient and yield,

$$
\begin{aligned}
\underline{q} &= \frac{\underline{a}}{2b} + \frac{\rho}{1 - \rho}\frac{\overline{a} - \underline{a}}{2b}, \qquad (2) \\
\overline{q} &= \frac{\overline{a}}{2b}.
\end{aligned}
$$

$$
V(\rho) \equiv u_b(\rho) = \frac{\underline{a}^2}{4b} + \frac{\rho}{1 - \rho}\frac{(\overline{a} - \underline{a})^2}{4b}, \qquad (3)
$$

Substituting (2) into (1) yields $\overline{u}(\rho)$. We next examine the properties of this function:

$$
\overline{u}' = -(\overline{a} - \underline{a})\frac{d\underline{q}}{d\rho} = -\frac{1}{2b}(\overline{a} - \underline{a})^2\left(\frac{1}{1 - \rho}\right)^2 < 0. \qquad (4)
$$

This result is consistent with the existing literature on contracts: information rent of the high type falls as the posterior belief that the demand is high increases. As we will see below, the second-period information rent of the high-type represents the ratchet effect payment to the high-demand incumbent. Our interest is in examining how this derivative changes when there is entry. We also want to examine the effect of entry on the derivative of the value function of the lender, $u_b(\rho)$, given by Equation (3), with respect to ρ. This derivative captures the experimentation effect.

Having analyzed the second period problem, we now turn to the first period. The lender chooses the first period quantities $\overline{q}$ and $\underline{q}$ to maximize,

$$\rho_0 \overline{R} + (1-\rho_0)\underline{R} + \int u_b(\rho(\overline{q}, \underline{q}, p))(\rho_0 \overline{f} + (1-\rho_0)\underline{f})dp,$$

subject to the first period incentive compatibility and individual rationality constraints. The binding individual rationality constraint of the low-demand continues to be the same as the static one because the second period expected profit of the low-type is zero (because of binding second period individual rationality constraint). On the other hand, the incentive compatibility constraint of the high demand incumbent needs to be modified to incorporate the expected information rent from the second period, $\int \overline{u}(\rho) f(p) dp$. Incorporating this, the binding incentive compatibility constraint of the high-demand incumbent is:

$$
\begin{aligned}
(\overline{a} - b\overline{q})\overline{q} - \overline{R} + \int \overline{u}(\rho)\overline{f}(p)dp &= (\underline{a} - b\underline{q})(\frac{\overline{a} - \underline{a}}{b} + \underline{q}) - \underline{R} + \int \overline{u}(\rho)\underline{f}(p)dp, \\
(\overline{a} - b\overline{q})\overline{q} - \overline{R} - \int \overline{u}(\underline{f} - \overline{f})dp &= (\underline{a} - b\underline{q})(\frac{\overline{a} - \underline{a}}{b}), \\
\overline{R} &= (\overline{a} - b\overline{q})\overline{q} - \frac{1}{b}(\overline{a} - \underline{a})(\underline{a} - b\underline{q}) - \int \overline{u}(\underline{f} - \overline{f})dp.
\end{aligned}
$$

That is, the first period repayment to the lender falls by $\int \overline{u}(\underline{f} - \overline{f})dp$, a positive term, since $\overline{u}$ is a decreasing function of ρ and by MLRP, of p, and $\overline{f}$ lies to the right of $\underline{f}$, implying a higher weight on higher prices. In other words, $\overline{f}$ first-order stochastically dominates $\underline{f}$ and therefore, expected value of any continuous and decreasing function under $\overline{f}$ is smaller than under $\underline{f}$.

We consider the effect of sure entry on the maximization problem of the lender next.

4 Entry

In the second period, having observed the price at the end of the first period, both the entrant and the lender update their beliefs about demand using Bayes' rule to ρ, given above. If entry occurs, the incumbent and the entrant engage in Cournot duopoly in each realized state. We first show that entry must occur either for all beliefs or not occur at all, depending on the size of entry cost, assumed to be constant. It suffices to show that the entrant's expected profits are independent of its posterior belief ρ.[4] The entrant maximizes, by choosing q_e,

$$\pi_e = \left(\widehat{a} - b\left(q_e + \widehat{q}\right)\right) q_e$$

where $\hat{q} \equiv \rho\overline{q}_e + (1-\rho)\underline{q}_e$ is the expected output of the incumbent. We similarly define $\widehat{a}$.

The necessary and sufficient first order condition yields:

$$\widehat{a} - 2bq_e - b\widehat{q} = 0 \Rightarrow q_e = \frac{\widehat{a} - b\widehat{q}}{2b}. \tag{5}$$

The lender maximizes $u_{be} = (1-\rho)\underline{R} + \rho\overline{R}$ subject to the two binding constraints as discussed in Section 3:

$$\underline{R} = (\underline{a} - b\underline{q}_e - bq_e)\underline{q}_e,$$

and,

$$(\overline{a} - b\overline{q}_e - bq_e)\overline{q}_e - \overline{R} = (\overline{a} - bq - bq_e)q - \underline{R},$$

where q is such that the expected price is the same as $\underline{p}$, which is obtained by setting $\overline{a} - bq - bq_e = \underline{a} - b\underline{q}_e - bq_e$, so that $q = \frac{1}{b}(\overline{a} - \underline{a}) + \underline{q}_e$. Substituting, we obtain,

$$\overline{R} = (\overline{a} - b\overline{q}_e - bq_e)\overline{q}_e - \frac{1}{b}(\overline{a} - \underline{a})(\underline{a} - b\underline{q}_e - bq_e).$$

[4] The analysis that follows is standard and used in JJM for the cost parameter case and in Jain (2009b) for the demand parameter case.

The necessary and sufficient first order conditions yield:

$$\begin{aligned} \underline{q}_e &= \frac{\underline{a} - bq_e}{2b} + \frac{\rho}{1-\rho}\frac{\overline{a} - \underline{a}}{2b}, \\ \overline{q}_e &= \frac{\overline{a} - bq_e}{2b}. \end{aligned} \tag{6}$$

and therefore,

$$\hat{q} = \frac{\widehat{a} - bq_e}{2b} + \rho\frac{\overline{a} - \underline{a}}{2b}. \tag{7}$$

It can be verified that the price expected by the entrant is the same as if it were facing a low-demand incumbent[5]. Solving (5) and (7) simultaneously, and then substituting in (6), yields:

$$\begin{aligned} q_e &= \frac{\underline{a}}{3b}, \\ \underline{q}_e &= \frac{\underline{a}}{3b} + \frac{\rho}{1-\rho}\frac{\overline{a} - \underline{a}}{2b}, \\ \overline{q}_e &= \frac{3\overline{a} - \underline{a}}{6b} \end{aligned} \tag{8}$$

Substituting these results into the entrant's profit function yields:

$$\pi_e = bq_e^2 = \frac{\underline{a}^2}{9b}, \tag{9}$$

[5]The expected price $\widehat{p} = \widehat{a} - b\hat{q} = \frac{\underline{a} + bq_e}{2}$.

and into the lender's expected profit function yields:

$$V_e(\rho) \equiv u_{be}(\rho) = \frac{\underline{a}^2}{9b} + \frac{\rho}{1-\rho}\frac{(\overline{a}-\underline{a})^2}{4b}, \tag{10}$$

Thus, the entrant's expected profits, given in (9), do not depend on the posterior belief, ρ, and therefore, the entry decision must also be independent of it. If these profits exceed the cost of entry, the entrant enters for all values of ρ, otherwise, it doesn't enter. This means that entry is either for all possible beliefs or none. We summarize this in the following proposition.

Proposition 1 *When demand is linear and the price intercept is private information of the incumbent, the optimal lending contract in the second period induces the entrant to produce an output as if demand is low and therefore, to either enter for all beliefs (if the cost of entry is low) or stay out for all beliefs.*

Note that when the demand shock is uniformly distributed, the entrant's expected profits are given by (9) only when there is no learning or when demand is revealed to be low. If the demand is revealed to be high, the entrant earns higher profits. However, when demand shock has a general distribution, so that learning is never at the extreme, Equation (9) describes the entrant's profits for all posterior beliefs.[6]

[6] This is an important feature of the model and results from the interaction of the principal-agent structure on one hand and the duopoly game on the other, given linear demand and the fact that the price intercept is the unknown parameter.

Next, we discuss the information rent of the high demand incumbent with entry. The binding incentive compatibility constraint of the high demand incumbent shows that the incumbent receives an information rent in the second period. Due to entry, this rent now equals:

$$\overline{u}_e = \frac{\overline{a} - \underline{a}}{b}(\underline{a} - b\underline{q}_e - bq_e).$$

Substituting for $\underline{q}_e$ and q_e (from equation system (8)) into this equation yields the information rent under entry to the high type incumbent. The main feature of this rent is the dependence on ρ. Notice that the only way ρ affects the information rent is through the low-type incumbent's output, which in turn is simply the shifted version of the output under no-entry (see equations (2) and (8)). That is, the term involving ρ is exactly the same as in the no-entry model. So, while the information rent under entry is lower, its derivative with respect to ρ is exactly the same as when there is no entry. This feature leads to the following lemma:

Lemma 1 $\overline{u}' = \overline{u}_e' < 0$

Proof. $\overline{u}_e' = -(\overline{a} - \underline{a})\frac{d\underline{q}_e}{d\rho} = -(\overline{a} - \underline{a})\frac{d\underline{q}}{d\rho} < 0$ ■

So far, we analyzed the second period variables under entry. We now discuss the first period maximization problem of the lender. As in the previous section, the lender maximizes the sum of the two period profits subject to incentive compatibility and individual rationality constraints, restated for the first period. That is, the lender chooses the first period quantities

$\overline{q}$ and $\underline{q}$ to maximize,

$$\rho_0 \overline{R} + (1-\rho_0)\underline{R} + \int u_{be}(\rho(\overline{q}, \underline{q}, p))(\rho_0 \overline{f} + (1-\rho_0)\underline{f})dp,$$

subject to the first period incentive compatibility and individual rationality constraints. The individual rationality constraint of the low demand incumbent does not change as a result of entry because the second period expected rent of the low-demand incumbent continues to be zero. The first period incentive compatibility constraint of the high type in the first period changes and can be derived in the same way as for the no-entry case:

$$\overline{R} = (\overline{a} - b\overline{q})\overline{q} - \frac{1}{b}(\overline{a} - \underline{a})(\underline{a} - b\underline{q}) - \int \overline{u}_e(\underline{f} - \overline{f})dp.$$

We observe two changes in the maximization problem due to entry. One is that the future profits of the lender are now given by u_{be} rather than u_b. This is the term that drives the experimentation effect. The second change is in the information rent of the high demand incumbent, where $\overline{u}_e$ replaces $\overline{u}$. This term drives the ratchet effect. We now analyze both of these terms to understand the effect of entry, starting with the ratchet effect.

We need to examine how information rent changes as the first period outputs change. This proof turns out to be similar to the proof of Theorem 2 in Jeitschko and Mirman (2002), where they show that,

$$\frac{d}{d\overline{x}}\int \overline{u}(\underline{f}-\overline{f})dX = \int \overline{u}'c(\overline{f}'\underline{f}^2 - \underline{f}'\overline{f}^2)dX,$$

where $c = -\frac{\rho_0(1-\rho_0)}{D^2}$, for $D = \rho_0\overline{f} + (1-\rho_0)\underline{f}$.

This equation continues to hold in our model, with $\overline{x}$ replaced with the expected price $\overline{p}$ and X replaced with the observed price p. However, then, by Lemma 1 above, introducing entry has no effect on the ratchet effect, since $\overline{u}'$ does not change with entry.

Proposition 2 *The lender chooses the same expected signals in the first period with entry as without entry, in order to minimize the ratchet effect payment.*

Another way to see this is to note that the only difference between the information rent under no-entry and under entry is a term that is independent of the posterior belief ρ. Then the expected information rent from the second period under entry equals,

$$\int \overline{u}_e(\underline{f}-\overline{f})dp = \int (\overline{u}-\alpha)(\underline{f}-\overline{f})dp,$$

where α is the term that is independent of the posterior belief. Now,

$$\int (\overline{u}-\alpha)(\underline{f}-\overline{f})dp = \int \overline{u}(\underline{f}-\overline{f})dp - \int \alpha(\underline{f}-\overline{f})dp = \int \overline{u}(\underline{f}-\overline{f})dp.$$

Thus, potential entry has no effect on the first period because the difference in the information rent of the high type is a constant and the high-demand incumbent earns it with probability one regardless of whether it reports its type truthfully.

Next we look at the experimentation effect in the presence of entry. This is the counterpart of Theorem 3 in Jeitschko and Mirman (2002). They show that,

$$\frac{d}{d\overline{x}}\int V(\rho)\left(\rho_0\overline{f}+(1-\rho_0)\underline{f}\right)dX=\rho_0\int V''(1-\rho)\frac{d\rho}{dX}\overline{f}dX.$$

This expression continues to hold for the demand model as well with X replaced with p and $\overline{x}$ replaced with expected price, $\overline{p}$. But then, entry changes the experimentation effect only if the second derivative V'' is different. However, it is not different as is obvious from equations (3) and (10). We state the result in the following proposition.

Proposition 3 *Entry has no effect on strategic experimentation by the principal.*

Another way to see this is to note that the value function under entry is simply the value function under no-entry plus a constant term. Since the constant term is known, potential entry does not lead to any additional experimentation.

Overall then, entry has no effect on the expected price targets chosen in the first period by the lender, whether through the ratchet effect or through the experimentation effect, unlike the existing results for uniform distribution. That is, entry has no effect on incentives or experimentation. The following proposition contains this overall result:

Proposition 4 *When the price intercept is private information of the incumbent, and the* *demand shock has a general distribution that has support on the entire real line, potential entry, if it occurs, has no effect on learning by the principal.*

This is a striking result compared to what is found when noise is uniformly distributed, as in JJMa. The only substantive difference between that model and here is the assumption that noise has support on the entire real line, thereby ruling out complete learning. Thus, the property that no price observation reveals the state of demand, given the model and the assumption of MLRP, is driving results in this paper. However, the intuition for this stark difference between results lies in the interplay of three factors: linear demand structure, the demand intercept being the unknown parameter and the presence of contracting. Because of this interplay, the entrant's expected profits are independent of posterior belief, as long as this posterior is stictly between 0 and 1, which is the case when complete learning is not allowed. As a result, the entrant either enters for all beliefs or does not, depending on the cost of entry and the lender cannot deter entry, unlike the case of uniform distribution. Further, because of the interplay of the three factors, given that entry occurs, outputs and profits are shifted by a constant, leading to our result.

We next relax the assumption of publicly observable contracts to allow for different posterior beliefs of the entrant and the principal.

5 Hidden Contract

JJMb (2005) show that observability of the contract matters when the demand shock is uniformly distributed. We find that this is not true when the demand shock has a general distribution with support on the entire real line. Suppose that the contract is not observed by the entrant. Then the entrant must conjecture the price targets set in the contract before updating beliefs. While in equilibrium, these conjectures must be the same as the actual targets, to determine equilibrium, one must examine the consequence of incorrect conjectures. Suppose that the entrant believes that the first period outputs set in the contract are $\overline{q}^e$ and $\underline{q}^e$, for high-demand and low-demand states respectively. Let the resulting posterior belief be denoted by ρ^e. The entrant maximizes its expected profits conditional on entry in the same way as for the public contract, except for using a possibly different posterior belief. Thus, as in Equation 5, $q_e = \frac{\widehat{a}-b\widehat{q}}{2b}$, except that $\widehat{q}$ is calculated using ρ^e, not ρ. Next, the entrant solves the lender's second period problem in the same way as before except it must use its own posterior ρ^e in the lender's problem as well. While the entrant is aware that its posterior is not necessarily correct, it has no way of computing the lender's posterior. Thus, the incumbent's output corresponding to high demand and low demand continue to be given by Equation 6, except that ρ^e is used instead of ρ. However, then, the expected price faced by the entrant continues to be independent of its posterior belief and therefore, its profit-maximizing output continues to be given by $\frac{a}{3b}$.Thus, not observing the contract makes no difference to the entrant, in terms of what output it should be producing. Given that the entrant produces $\frac{a}{3b}$, the lender, while using a different posterior, namely ρ, also continues to choose the same optimal output levels as in the public contract case, since the entrant's optimal output is independent of its posterior belief. Incorporating the entrant's optimal output in its solution yields the lender exactly the same outputs as given by (8). This in turn implies that the lender's value function remains unaffected by the divergence of beliefs and is given by (10). Similarly, the first period incentive compatibility constraint is unaffected as well because the information rent in the second period is the same as in the observable contract case. Thus, unlike the case where the demand shock is uniformly distributed, entry has no effect on the first period price targets even when the potential entrant does not observe the first period contract. This result is formally stated in the following Proposition.

Proposition 5 *When the price intercept as well as the first period contract are private information of the incumbent, and the demand shock has a general distribution that has support on the entire real line, potential entry, if it occurs, has no effect on learning by the principal.*

6 Conclusion

In this paper, we have shown that the assumption of uniform distribution is critical in the work of JJMa and JJMb where they show that potential entry changes the extent of learning by the lender. We have shown that these results fail to generalize when demand is linear, the demand intercept is unknown to the lender and the demand shock is distributed along the entire real line. We have also shown that observability of the contract by the entrant does not matter when the demand shock is general. However, it is not the infinite support of the random term per se or the linearity of demand per se that causes experimentation to be ineffective. Indeed, the lender experiments in the absence of entry under these conditions. It is the combination of infinite support of the random shock, the nature of the unknown demand parameter and the contracting environment that leads to additional experimentation being ineffective. Indeed, preliminary results show that if slope of the demand curve is the source of private information, then the potential entrant's expected profits depend on the posterior belief and therefore, both the cost and the benefit of experimentation are likely to change as a result of potential entry even when the demand shock is distributed along the entire real line. That said, the result for the price intercept being unknown is important, given the pervasive use of this information structure in the experimentation literature as well as the contracting literature.

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