Villegas, A. & Haberman, S. (2014). On the Modeling and Forecasting of Socioeconomic Mortality Differentials: An Application to Deprivation and Mortality in England. North American Actuarial Journal, 18(1), pp. 168-193. doi: 10.1080/10920277.2013.866034

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Longevity 8 - On the modelling and forecasting of socio-economic mortality differentials: an application to deprivation and mortality in England

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On the modelling and forecasting of socio-economic mortality differentials: an application to deprivation and mortality in England

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Abstract

In any country, mortality rates and indices such as life expectancy usually differ across sub-populations, for example, defined by gender, geographic area or socio-economic variables (e.g. occupation, level of education, income). These differentials, and in particular those related to socio-economic circumstances, pose important challenges for the design of public policies for tackling social inequalities, as well as for the design of pension systems and the management of longevity risk in pension funds and annuity portfolios. We discuss the suitability for the modelling and forecasting of socio-economic differences in mortality of several multiple population extensions of the Lee-Carter model, including a newly introduced relative model based on the modelling of the mortality in socio-economic subpopulations alongside the mortality of a reference population. Using England mortality data for socio-economic subpopulations defined using a deprivation index, we show that this new relative model exhibits the best results in terms of goodness of fit and ex-post forecasting performance. We then use this model to derive projections of deprivation specific mortality rates and life expectancies at pensioner ages and analyse the impact of socio-economic differences in mortality on the valuation of annuities.

Keywords: Mortality modelling; multipopulation models; socio-economic circumstances; annuity pricing

1. Introduction

In any country, mortality rates and indices such as life expectancy usually differ across subpopulations, for example, defined by gender, geographic area or socio-economic variables. In particular, there is a well established inverse relationship between socio-economic circumstances - whether measured by educational attainment, occupation, income or area deprivation - and mortality, with higher socio-economic subgroups having lower mortality rates and, in most cases, also experiencing faster mortality improvements than lower socio-economic subgroups (see, e.g., Shkolnikov et al. (2006); Johnson (2011); Tarkiainen et al. (2012); Raleigh and Kiri (1997)).

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These socio-economic differences in mortality not only pose significant challenges for the design of public policies for tackling social inequalities, but also for the design of pension systems and the management of longevity risk in pension funds and annuity portfolios. On the one hand, differential mortality can have important consequences on the redistribution properties of both defined benefit and defined contribution pension schemes (Liebman, 2002; Brown, 2002), for example, undermining the equity and solidarity of national pensions systems by inducing an undesirable transfer of wealth away from lower socio-economic groups with shorter life expectancy to higher socio-economic groups with above average longevity. On the other hand, the ignorance of mortality heterogeneity when valuing pension liabilities or pricing annuities could result in an inadequate funding of annuity and pension obligations. Furthermore, the successful development of a market of standardised longevity securities requires a good understanding of socio-economic mortality differentials as they are in most situations the main determinant of the basis risk associated with index-based longevity hedges (Coughlan et al., 2011).

In view of this, there is a need for methods that help us assess the magnitude of socio-economic mortality differentials within a population and that enable us to examine their possible future evolution. In general, to measure and project mortality differentials we require a modelling approach that permits the simultaneous modelling of mortality in a group of subpopulations. Moreover, in the specific context of socio-economic subpopulations, such a model should ideally capture both mortality level differentials and mortality improvement differentials among the subpopulations, that is, differentials in the average level of mortality and differentials in the pace of mortality change.

Some authors have suggested the application of generalised linear models (Madrigal et al., 2011) and survival models (Richards, 2008) in the quantification of socio-economic mortality differentials. These statistical methods have specifically been proposed for the assessment of baseline (level) mortality differentials, ignoring, in part due to the lack of appropriate data, the differences in improvements by socio-economic characteristics and the modelling of their possible future evolution. Provided that data requirements are met, a modelling alternative that does allow the consideration of both level and trend differentials in mortality, as well as the projection of their future evolution, is offered by the numerous stochastic mortality models that have recently been proposed for the simultaneous modelling and forecasting of mortality in a group of populations. Unfortunately, most of these multipopulation models have not been designed with the aim of assessing socio-economic differences in mortality, but with the purpose of comparing the mortality evolution of a group of countries or of genders or regions within a country. As a result, some of these models may lack some of the desirable features of an approach for the modelling and forecasting of mortality in a group of socio-economic subpopulations, which include: transparency for the disentangling of level and improvement differentials in mortality; consistency of subpopulation-specific mortality forecasts with national mortality forecasts; ability to produce adequate interval forecasts of mortality differentials; and ability to produce mortality rates forecast that preserve the inverse relationship between socio-economic circumstances and mortality.

In this paper, we discuss the suitability for the modelling and forecasting of socio-economic differences in mortality of several multiple population extensions of the Lee-Carter model (Lee and Carter, 1992), including a newly introduced relative model based on the modelling of the mortality in socio-economic subpopulations alongside the mortality of a ref-

erence population. This later model has especially been designed taking into consideration the characteristics of socio-economic subpopulations as well as some of the typical issues of mortality data disaggregated by socio-economic circumstances. Although previous studies have investigated the use of multipopulation mortality models in the assessment of mortality differentials among two countries (Li and Hardy, 2011) and regions within a country (Debón et al., 2011), this is the first study to apply such models to socio-economic mortality differentials in particular. More specifically, we use multipopulation mortality models to analysis the extent of mortality differentials across deprivation subgroups in England.

The reminder of this paper is organised as follows. Section 2 describes several alternative methods for the measurement and projection of mortality differentials. Particularly, Section 2.4 introduces our proposed new relative modelling approach. In Section 3, we apply these models in the examination of the relationship between deprivation and mortality in the English population, emphasising on its implications for the valuation of life annuities. Finally, Section 4 concludes with a discussion of our main findings.

2. Modelling socio-economic mortality differentials

In this section, we describe several multipopulation extensions of the Lee-Carter model (Lee and Carter, 1992), and discuss their suitability for the modelling of socio-economic mortality differentials. All these models propose a parametric representation of the central death rate $_n\mu_{xtg}$ in year t for people age $[x, x + n)$ in subpopulation g, based on a cross-classified mortality experience containing the observed number of deaths $_n d_{xtg}$ at ages $[x, x + n)$, $x \in \mathcal{X} := \{x_1, \ldots, x_k\}$, in year t, $t \in \mathcal{T} := \{t_1, \ldots, t_n\}$, for subpopulation g, $g \in \mathcal{G} := \{g_1, \ldots, g_m\}$, with matching exposure $_n e_{xtg}$.

2.1. Independent modelling

The simplest approach for modelling mortality in a set of subpopulations would be to use independent unrelated Lee-Carter models for each subpopulation. Thus, we could model mortality in each subpopulation $g \in \mathcal{G}$ using the specification

$$\log {}_n\mu_{xtg} = \alpha_{xg} + \beta_{xg}\kappa_{tg}, \tag{1}$$

where α_{xg} captures the general age-specific mortality pattern for subpopulation g, k_{tg} is a time varying mortality index representing the overall level of mortality in year t for subpopulation g, and β_{xg} measures the age-specific response to changes in the general level κ_{tg}. The parameters of this model are identifiable only up to a transformation as for any constants c_1 and $c_2 \neq 0$, if we replace α_{xg} by $\alpha_{xg} + c_1\beta_{xg}$, β_{xg} by $\frac{1}{c_2}\beta_{xg}$, and κ_{tg} by $c_2(\kappa_{tg} - c_1)$, equation (1) will produce the same log death rates. Therefore, in order to ensure the identifiability of the model, the constraints

$$\sum_{x \in \mathcal{X}} \beta_{xg} = 1, \quad \sum_{t \in \mathcal{T}} \kappa_{tg} = 0 \tag{2}$$

are imposed.

In this model mortality forecasts are obtained by modelling and forecasting the period indexes using independent univariate ARIMA processes. In most applications, each period index k_{tg}, $g \in \mathcal{G}$, can be modelled using a random walk with drift (ARIMA(0,1,0)):

$$\kappa_{tg} = d_g + \kappa_{t-1,g} + \xi_{tg}, \tag{3}$$

where d_g is the drift term and ξ_{tg} is a normally distributed error term with zero mean and variance σ_g^2. We note that under the independent modelling approach the error terms for each subpopulation ξ_{tg}, $g \in \mathcal{G}$, are assumed to be independent of one another.

The independent modelling approach is straightforward to implement. However, it has several shortcomings. First, it assumes no interdependence among the mortality of the subpopulations, a very unrealistic assumption for socio-economic subpopulations within a country, which are likely to follow similar mortality trends. Second, although mortality level differentials could be assessed by comparing the α_{xg} terms and trend differentials by comparing the β_{xg} and κ_{tg} terms, this is not a straightforward task.

The assumption of complete independence among the subpopulations can be relaxed by, instead of using independent random walks to model the time indexes, using a multivariate random walk with drift so that

$$\boldsymbol{\kappa}_t = \mathbf{d} + \boldsymbol{\kappa}_{t-1} + \boldsymbol{\xi_t}, \quad \boldsymbol{\xi_t} \sim N(\mathbf{0}, \boldsymbol{\Sigma}), \tag{4}$$

where $\boldsymbol{\kappa}_t := (\kappa_{t,1}, \kappa_{t,2} \ldots, \kappa_{t,m})'$, $\mathbf{d}$ is a vector of drift parameters and $\boldsymbol{\Sigma}$ is the variance-covariance matrix. Since the diagonal terms of the variance-covariance matrix $\boldsymbol{\Sigma}$ coincide with the variances σ_g^2, $g \in \mathcal{G}$, of ξ_{tg} in (3), the central and interval projections of mortality rates for each subpopulation derived from the multivariate random walk and the independent random walk with drifts are identical. However, the multivariate random walk yields more precise interval projections of mortality differentials as the non-diagonal terms of the variance-covariance matrix $\boldsymbol{\Sigma}$ capture the dependence among the subpopulations.

A further enhancement of the independent approach would be the use of co-integration methods to model $\boldsymbol{\kappa}_t$ as suggested by Carter and Lee (1992) and described in Li and Hardy (2011) and in Yang and Wang (2013). However, although theoretically sound, mortality data disaggregated by socio-economic variables are typically available for short periods of time, hampering the application of the required econometric techniques.

2.2. The joint k-model

A second alternative for modelling mortality differentials is the joint-κ model proposed by Carter and Lee (1992). This extension of the Lee-Carter model assumes that all the subpopulations are driven by a single period index. Formally, the model can be expressed as

$$\log {}_n\mu_{xtg} = \alpha_{xg} + \beta_{xg}\kappa_t, \tag{5}$$

where α_{xg} describes the average age profile for subpopulation g, κ_t is a period index driving the mortality trend for all subpopulations, and β_{xg} represents the age-subpopulation-specific pattern of mortality change.

In order to facilitate the assessment of mortality differentials, it is convenient to re-parametrised the model as[1]:

$$\log {}_n\mu_{xtg} = \alpha_x + \alpha_{xg} + (\beta_x + \beta_{xg})\kappa_t \tag{6}$$

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[1] Delwarde et al. (2006) have considered a similar parametrisation in the simultaneous modelling of mortality in five developed countries.

with the following constraints

$$\sum_{x \in \mathcal{X}} \beta_x = 1, \qquad \sum_{t \in \mathcal{T}} \kappa_t = 0 \tag{7}$$

$$\sum_{g \in \mathcal{G}} \alpha_{xg} = 0, \qquad \sum_{g \in \mathcal{G}} \beta_{xg} = 0 \quad \text{for all } x \in \mathcal{X} \tag{8}$$

to ensure the identifiability of the model. This parametrisation allows the measurement of subpopulation mortality as deviations from the general mortality pattern of the population. Thus, parameters α_x and α_{xg} capture mortality level differentials, whereas parameters β_x and β_{xg} capture mortality improvement differentials. Specifically, the term $\exp(\alpha_{xg})$ quantifies the average percentage deviation of subpopulation g from the the general level of mortality in the population $\exp(\alpha_x)$. That is, if $\exp(\alpha_{xg}) > 1$ then at ages $[x, x + n)$ mortality in subpopulation g is higher than in the total population, and if $\exp(\alpha_{xg}) < 1$ then mortality is lower. Similarly, if $\beta_{xg} > 0$ then at ages $[x, x + n)$ mortality in subpopulation g is improving at a faster pace than in the total population and if $\beta_{xg} < 0$ then mortality is improving at a slower rate.

Whilst the independent modelling approach includes a period index for each subpopulation, the joint-κ has a single period index. Hence, mortality forecast for all the subpopulations can be derived by modelling this period index using a univariate random walk with drift

$$\kappa_t = d + \kappa_{t-1} + \xi_t \tag{9}$$

Besides the transparency provided by parametrisation (6) for the identification of both level and trend differentials in mortality, there are other compelling statistical and demographics reasons that suggest that the joint-κ may be a very attractive approach. Statistically, a single time index is a very parsimonious way of linking the mortality of multiple populations. Demographically, a single mortality driver may impose greater consistency among the subpopulations, ruling out the possibility that the mortality of the subpopulations evolves in completely different ways. Nevertheless, a single mortality driver also implies that the improvement rates of the subpopulations will be perfectly correlated, resulting in extremely narrow interval forecasts of mortality differentials.

There are two restricted versions of the joint-κ model which are worth exploring as alternatives for modelling mortality differentials: the "common factor" model introduced by Li and Lee (2005) and the stratified or additive Lee-Carter model considered in Butt and Haberman (2009) and in Debón et al. (2011). The common factor model, obtained by setting $\beta_{xg} = 0$, can be expressed as

$$\log {}_n\mu_{xtg} = \alpha_x + \alpha_{xg} + \beta_x \kappa_t \tag{10}$$

and the stratified Lee-Carter, which also sets $\alpha_{xg} = \alpha_g$, is given by

$$\log {}_n\mu_{xtg} = \alpha_x + \alpha_g + \beta_x \kappa_t \tag{11}$$

These two variants of the joint-κ model, although very parsimonious, may be too stringent for some applications. Both models assume the same mortality improvements for all

subpopulations, which implies that improvement differentials in mortality are assumed to be non-existent and makes it impossible to draw any conclusion as to whether relative mortality differentials are increasing or decreasing. In addition, the stratified Lee-Carter model ignores any variation of mortality differentials with age, failing to capture the commonly observed decrease in socio-economic differentials in mortality with rising age (Hoffmann, 2005).

2.3. The three-way Lee-Carter

The three-way extension of the Lee-Carter model proposed by Russolillo et al. (2011) provides an additional alternative for modelling mortality differentials. This variant of the Lee-Carter model adds a subpopulation parameter that deals with trend differences in mortality. Specifically, the model is given by

$$\log {}_n\mu_{xtg} = \alpha_{xg} + \beta_x\lambda_g\kappa_t, \tag{12}$$

where α_{xg} measures the age-subpopulation-specific pattern of mortality, β_x and κ_t have the same interpretation as in the Lee-Carter model, and λ_g is a new term capturing the variability in the improvement rates of the subpopulations. As with the joint-κ model, the interpretation of mortality differentials is made easier if we consider the re-parametrisation

$$\log {}_n\mu_{xtg} = \alpha_x + \alpha_{xg} + \beta_x\lambda_g\kappa_t, \tag{13}$$

with constraints

$$\sum_{g\in\mathcal{G}} \alpha_{xg} = 0 \quad \text{for all } x \in \mathcal{X} \tag{14}$$

$$\sum_{x\in\mathcal{X}} \beta_x = 1, \quad \sum_{t\in\mathcal{T}} \kappa_t = 0, \quad \sum_{g\in\mathcal{G}} \lambda_g = 1 \tag{15}$$

to ensure identifiability of the model. In this new parametrisation parameters α_x and α_{xg} capture level differentials in mortality and have the same interpretation as in the joint-κ model. Improvement differentials in mortality are captured by parameters λ_g, with $\lambda_g > 1$ ($\lambda_g < 1$) meaning that mortality in subpopulation g improves at a faster (slower) rate than mortality in the general population. As in the joint-κ model mortality forecasts are obtained by modelling the singe mortality driver κ_t using a univariate random walk with drift.

The joint-κ model and the three-way Lee-Carter share similar advantages and disadvantages. As the joint-κ model, the three-way Lee-Carter offers a parsimonious and transparent way of assessing mortality differentials, but, as a result of having a single mortality driver, it also suffers from very narrow interval forecasts of mortality differentials.

2.4. A relative modelling approach

Finally, in order to model socio-economic mortality differentials, we introduce a new relative modelling approach whereby subpopulation mortality is modelled relative to the mortality of a reference population, which would in most cases be the national population of the country from which the subpopulations come from. A relative approach offers several advantages when compared with the previously described approaches that rely exclusively on the mortality data of the socio-economic subpopulations. First, national mortality data are

normally available for a longer period than mortality data disaggregated by socio-economic circumstances, permitting a more precise estimation of the long-run mortality trend. Moreover, modelling the subpopulations alongside the national population will ensure the consistency of the subpopulation-specific mortality forecasts with the national mortality forecasts. Second, as opposed to mortality data for socio-economic subpopulations which are normally available in an age-grouped format, national mortality data are typically available for individual ages, facilitating the consideration of the effect of year of birth (cohort) in mortality which has been identified in some populations (see, e.g., Willets (2004) for the UK).

Thus, we assume that besides the subpopulation mortality experience $(_{n}d_{xtg}, {}_{n}e_{xtg})$, there is available an additional experience containing the number of deaths d'_{xt} at age x, $x \in \mathcal{X}' =: \{x'_1, \ldots, x'_{k'}\}$, in year t, $t \in \mathcal{T}' := \{t'_1, \ldots, t'_{n'}\}$ for a reference population with corresponding exposures e'_{xt}, possibly covering a wider age range and a longer period of time than in the subpopulations' data, i.e., $x'_1 \leq x_1$, $x'_{k'} \geq x_k + n$, $t'_1 \leq t_1$, and $t'_{n'} \geq t_k$.

With this additional data at hand, we follow a modelling approach similar to that of Jarner and Kryger (2011) and model the subpopulations death rates, $_{n}\mu_{xtg}$, relative to the reference population death rates, μ'_{xt}. Specifically, we model the mortality rates of the reference population as

$$\log \mu'_{xt} = \alpha'_x + \beta'_x \kappa'_t + \gamma'_{t-x} \tag{16}$$

and the mortality rates of the subpopulations as

$$\log {}_{n}\mu_{xtg} = \log {}_{n}\bar{\mu}'_{xt} + \alpha_{xg} + \beta_x \kappa_{tg}, \tag{17}$$

where

$$_{n}\bar{\mu}'_{xt} = \left(\prod_{i=0}^{n-1} \mu'_{x+i,t}\right)^{\frac{1}{n}} = \exp\left(\frac{1}{n}\sum_{i=0}^{n-1}\left(\alpha'_{x+i} + \beta'_{x+i}\kappa'_t + \gamma'_{t-x-i}\right)\right) \tag{18}$$

is the geometric average of the mortality rates in the reference population between age x and age $x + n - 1$. The identifiability of the model is ensured by imposing the following parameter constraints

$$\sum_{x \in \mathcal{X}'} \beta'_x = 1 \tag{19}$$

$$\sum_{t \in \mathcal{T}'} \kappa'_t = 0 \tag{20}$$

$$\sum_{\substack{z=t-x \\ t \in \mathcal{T}', x \in \mathcal{X}'}} \gamma'_z = 0 \tag{21}$$

$$\sum_{x \in \mathcal{X}} \beta_x = 1 \tag{22}$$

$$\sum_{t \in \mathcal{T}} \kappa_{tg} = 0 \quad \text{for all } g \in \mathcal{G} \tag{23}$$

The parametric structure defined by (16) was introduced by Renshaw and Haberman (2006) as a generalisation of the Lee-Carter model to allow for the consideration of cohort

effects. In (16), α'_x captures the general age-specific mortality pattern in the reference population, κ'_t represents the overall time trend of mortality in the reference population, β'_x measures the age-specific response to changes in the general level of mortality, and γ'_{t-x} captures the cohort effect. This parametrisation assumes that cohorts effects are the same for all the subpopulations, which might be reasonable for socio-economic subpopulations within a country but might not be appropriate for the modelling of multiple populations of a different nature such as countries. We also note that for some applications the cohort effect might not be significant and one might consider the original Lee-Carter model, omitting γ'_{t-x} from the model.

Equation (17) models mortality in the subpopulations relative to mortality in the reference population. Within this parametric structure α_{xg} captures mortality level differentials, whilst β_x and κ_{tg} capture mortality improvement differentials. As in the joint-k and three-way Lee-Carter models, the term $\exp(\alpha_{xg})$ measures the average percentage deviation of subpopulation g from the pattern of mortality in the reference population. The subpopulation-specific time index κ_{tg} measures the deviations of mortality improvements in population g from the mortality improvements of the reference population. Therefore, a decreasing trend in κ_{tg} implies that mortality in subpopulation g is improving at a faster rate than in the reference population, while on the contrary, an increasing trend means that mortality is improving at a slower pace than in the reference population.

The age-modulating parameter β_x indicates the magnitude of mortality improvement differentials at each particular age. In principle, we could consider a model structure where the age-modulating parameter is subpopulation specific (i.e., β_{xg} as opposed to β_x) as in the augmented common factor model proposed by Li and Lee (2005). However, besides being more parsimonious, a subpopulation-independent specification of the age-modulating parameter is convenient in the forecasting of mortality in socio-economic subpopulations, where an ordering of mortality levels is natural (subpopulations with lower socio-economic conditions tend to have higher mortality than sub-populations with higher socio-economic conditions). Notice that if subpopulation g_1 has historically had lower mortality than subpopulation g_2 (i.e. $\alpha_{x,g_1} < \alpha_{x,g_2}$ for all $x \in \mathcal{X}$), then a sufficient condition for maintaining this ordering in the forecasted mortality rates (i.e. $_n\mu_{x,t_n+h,g_1} < {}_n\mu_{x,t_n+h,g_2}, h > 0$) is $\beta_x > 0$, and $(\kappa_{t_n+h,g_1} - \kappa_{t_n,g_1}) > (\kappa_{t_n+h,g_2} - \kappa_{t_n,g_2}), h > 0$. Hence, the problem of preserving the mortality ordering among the subpopulations reduces to modelling appropriately the multivariate time index κ_{tg}.

Mortality rate extrapolations for the subpopulations require time series forecasts of the multivariate period index $\boldsymbol{\kappa}_t := (\kappa_{t,1}, \kappa_{t,2} \ldots, \kappa_{t,m})'$ as well as projected values of the mortality rates in the general population, which in turn require times series forecasts of κ'_t and possibly of γ'_{t-x}, depending on the projected trajectories of interest. We employ a random walk with drift to model the reference population period index, and extrapolation of the cohort parameter is not required in the case study considered in this paper. For the period index of the subpopulations, we consider a multivariate random walk with drift (see equation (4)) so that any potential dependence among the subpopulations is captured.

2.5. Fitting the models

To fit the models we consider that subpopulation death counts are independent Poisson responses $_nD_{xtg} \sim Poisson(_ne_{xtg}\, {}_n\mu_{xtg})$ and derive parameter estimates by maximising the

log-likelihood

$$\mathcal{L}(_nd_{xtg}, {_n\hat{d}_{xtg}}) = \sum_{x\in\mathcal{X}}\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}}\omega_{xtg}\left\{{_nd_{xtg}}\log{_n\hat{d}_{xtg}} - {_n\hat{d}_{xtg}} - \log{_nd_{xtg}!}\right\}, \quad (24)$$

where $_n\hat{d}_{xtg}$ denotes the expected number of death predicted by the model and ω_{xtg} are 0-1 weights indicating empty or omitted data cells.

For the new relative model we suppose that the reference population death counts are also independent Poisson responses $D'_{xt} \sim Poisson(e'_{xt}\mu'_{xt})$ and estimate simultaneously the parameters of equations (16) and (17) by maximising the total model log-likelihood under the assumption of independence between D'_{xt} and D_{xtg}:

$$\begin{aligned}
\mathcal{L}(_nd_{xtg}, {_n\hat{d}_{xtg}}, d'_{xt}, \hat{d}'_{xt}) &= \mathcal{L}_{ref}(d'_{xt}, \hat{d}'_{xt}) + \mathcal{L}_{sub}(_nd_{xtg}, {_n\hat{d}_{xtg}}) \\
&= \sum_{x\in\mathcal{X}'}\sum_{t\in\mathcal{T}'}\omega'_{xt}\left\{d'_{xt}\log\hat{d}'_{xt} - \hat{d}'_{xt} - \log d'_{xt}!\right\} \\
&\quad + \sum_{x\in\mathcal{X}}\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}}\omega_{xtg}\left\{{_nd_{xtg}}\log{_n\hat{d}_{xtg}} - {_n\hat{d}_{xtg}} - \log{_nd_{xtg}!}\right\},
\end{aligned} \quad (25)$$

where

$$_n\hat{d}_{xtg} = {_ne_{xtg}}\exp\left(\frac{1}{n}\sum_{i=0}^{n-1}\left(\alpha'_{x+i} + \beta'_{x+i}\kappa'_t + \gamma'_{t-x-i}\right) + \alpha_{xg} + \beta_x\kappa_{tg}\right),$$

$$\hat{d}'_{xt} = e'_{xt}\exp\left(\alpha'_x + \beta'_x\kappa'_t + \gamma'_{t-x}\right),$$

and ω'_{xt}, ω_{xtg} are 0-1 weights that indicate empty or omitted data cells. In order to maximise the log-likelihood functions, we employ suitable straightforward extensions of the Newton-Raphson iterative procedure used by Brouhns et al. (2002) in the estimation of the Lee-Carter model.

It is well-known that as a result of the relationship cohort = period - age, age-period-cohort (APC) modelling is problematic (Renshaw and Haberman, 2006). Hence, the inclusion of a cohort effect in the modelling of the reference population of the relative model makes the fitting of this model complicated. Specifically, it has been reported that cohort-based extension of the Lee-Carter model have a slow rate of convergence and a lack of stability in the fitted parameters (Cairns et al., 2009). Cairns et al. (2009) suggest that these issues might be the result of a remaining identifiability problem, with the log-likelihood function being flat or approximately flat in certain dimensions. In fact, Hunt and Villegas (2013) have proved that if the period index κ'_t is approximately linear, as is the case in the mortality experience of most developed countries (Tuljapurkar et al., 2000), then an approximately invariant parameter transformation arises. Therefore, to overcome the problems with the fitting of the relative model we follow Hunt and Villegas (2013) and add to constraints (19)-(23) the constraint

$$\sum_{\substack{c=t-x \\ t\in\mathcal{T}', x\in\mathcal{X}'}}(c - \bar{c})\gamma'_c = 0, \quad (26)$$

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where $\bar{c} = \frac{1}{k+n-1} \sum_{c=t-x, t \in \mathcal{T}', x \in \mathcal{X}'} c$, which combined with constraint (21) ensures that any linear trend in the cohort effect is eliminated, and improves the stability and convergence of the model.

3. Case study: Mortality by deprivation in England

In this section we employ the previously discussed models in the investigation of the relationship between socio-economic circumstances and mortality in England. To this end, we have derived a socio-economic classification of the English population using the Index of Multiple Deprivation 2007 (IMD 2007) which measures socio-economic circumstances at a small area level (Noble et al., 2007). The IMD 2007 is a composite index of deprivation comprising seven deprivation domains[2] and is calculated for each geographically defined Lower Layer Super Output Area (LSOA) in England. There are 32,482 LSOAs in England covering approximately 1,500 people each. In our analysis LSOAs are ranked and then grouped into deprivation quintiles based on their IMD 2007 score. For each deprivation quintile we have population and deaths estimates for the period 1981-2007, classified by sex and age groups 50-54,...,80-84. Throughout this paper, we will refer to the deprivation quintiles as Q1, Q2, Q3, Q4, and Q5, with Q1 being the least deprived quintile of the population and Q5 the most deprived quintile. A particular feature of this dataset is that it addresses the issues of low volume, consistency and credibility that have been encountered in other similar datasets of mortality disaggregated by socio-economic cirmcumstances for the English population. The details of this matter as well as the details of the compilation of this dataset are described in Lu et al. (2012).

For the proposed new relative model, we consider deprivation-specific mortality relative to mortality in England and Wales. Thus, we use as reference population data the England and Wales mortality experience for calendar years 1961-2009 and individual ages $10, 11, \ldots, 99$, obtained from the Human Mortality Database (2012). Despite the fact that the available subpopulation data cover the age range 50-84 and the period 1981-2007, we use for the reference population a wider age range, 10-99, and a longer observation period, 1961-2009, so that more reliable estimates of the cohort effect and of the long-run mortality trend can be produced. We exclude ages below 10 years as infant mortality tends to exhibit a significantly different behaviour than mortality at older ages.

3.1. Comparison of the models

We first compare the models discussed in Section 2 in terms of their ability to fit and forecast the mortality differentials in the deprivation subpopulations. Thus, we have fitted the independent Lee-Carter model, the joint-κ model and its constrained variants the stratified Lee-Carter and common factor models, the three-way Lee-Carter model, and the new relative model using both the Lee-Carter model and the age-period-cohort model (16) for the England and Wales reference population. Due to space constraints, we limit our comparisons

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[2]The seven deprivations domains with their percentage participation in the index are: i) Income deprivation (22.5%), ii) Employment deprivation (22.5%), iii) Health deprivation and disability (13.5%), iv) Education, skills and training deprivation (13.5%), v) Barriers to housing and services (9.3%), vi) Crime (9.3%), and vii) Living Environment deprivation (13.5%).

Figure 1: Parameter estimates of the independent Lee-Carter model for the male deprivation subpopulations.
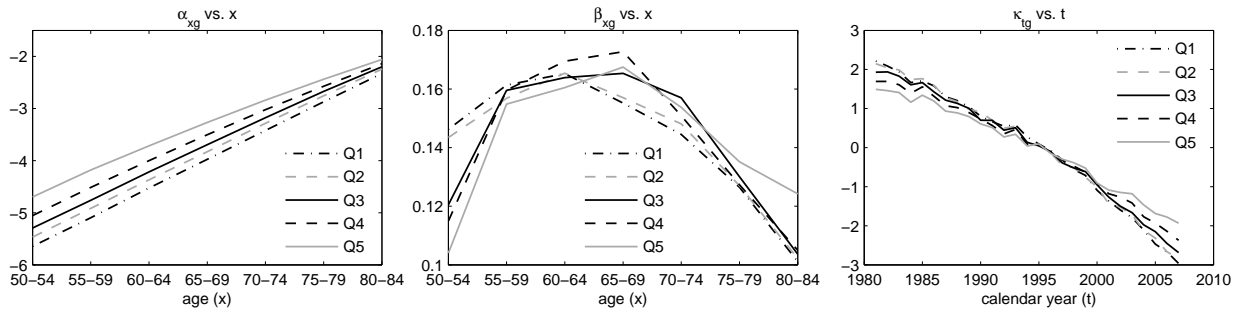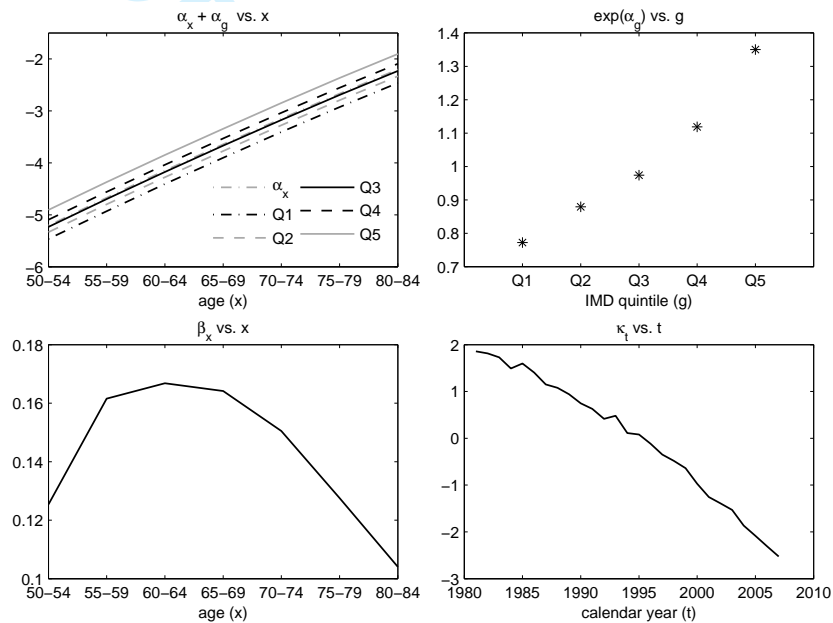

Figure 2: Parameter estimates of the stratified Lee-Carter model for the male deprivation subpopulations.

to the male population, but most of the conclusions of this section also apply to the female population. Figures 1 to 7 present the corresponding parameters estimates for the male population. These parameters plots reveal a clear association between deprivation and mortality in the English population. Specifically, α_g in the stratified Lee-Carter model and α_{xg} in all the other models show a marked inverse relationship between deprivation and mortality, with more deprived subpopulations having considerably higher mortality than less deprived ones. In addition, parameters β_{xg} of the joint-κ model, λ_g of the three-way Lee-Carter model, and κ_{tg} of the relative models indicate that less deprived supbpopulations have experienced faster mortality improvements than most deprived ones. Although this last conclusion could also be drawn from the parameters of the independent Lee-Carter model, it is less obvious as it requires the analysis of the interactions between β_{xg} and κ_{tg}, underscoring one of the main disadvantages of this approach.

A first way of evaluating the goodness of fit of the models is inspecting the standardised

Figure 3: Parameter estimates of the common factor model for the male deprivation subpopulations.
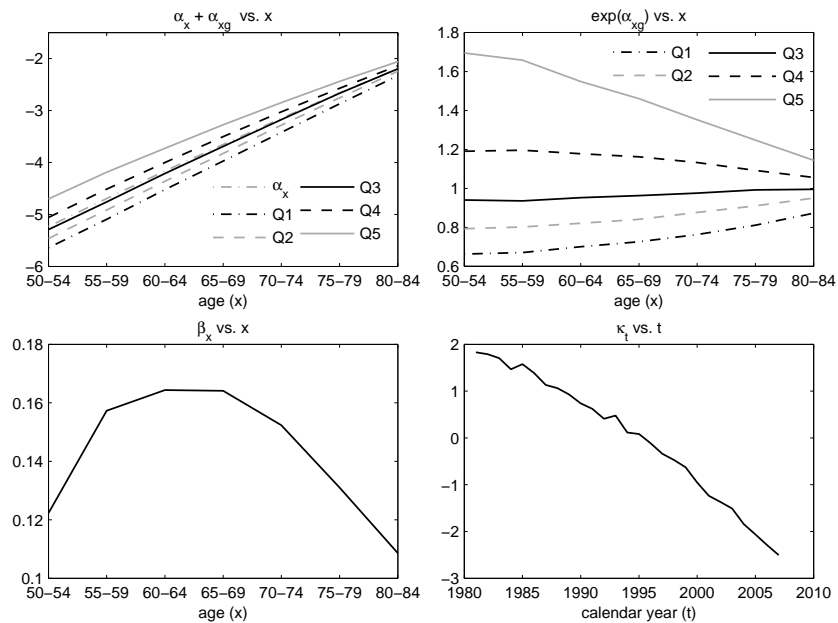

Figure 4: Parameter estimates of the joint-κ model for the male deprivation subpopulations.
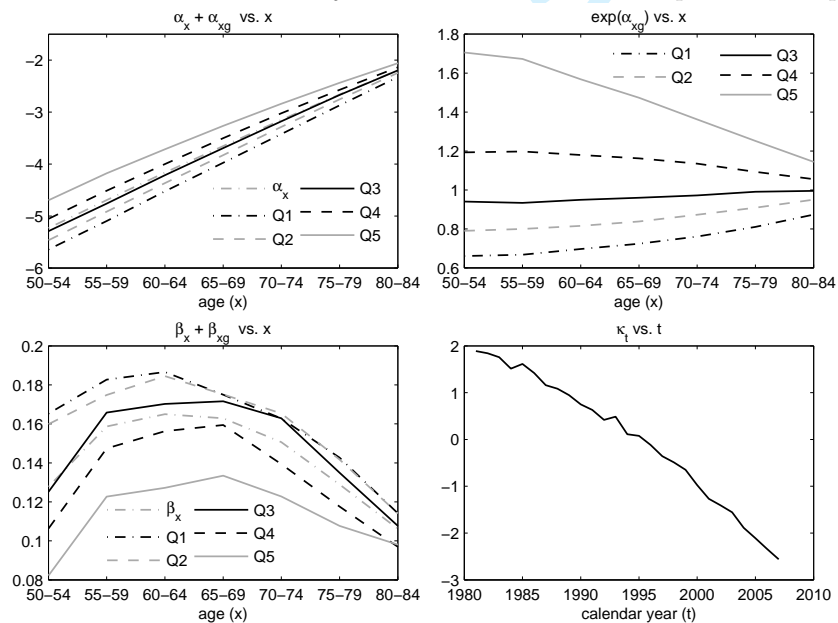
Figure 5: Parameter estimates of the three-way Lee-Carter model for the male deprivation subpopulations.
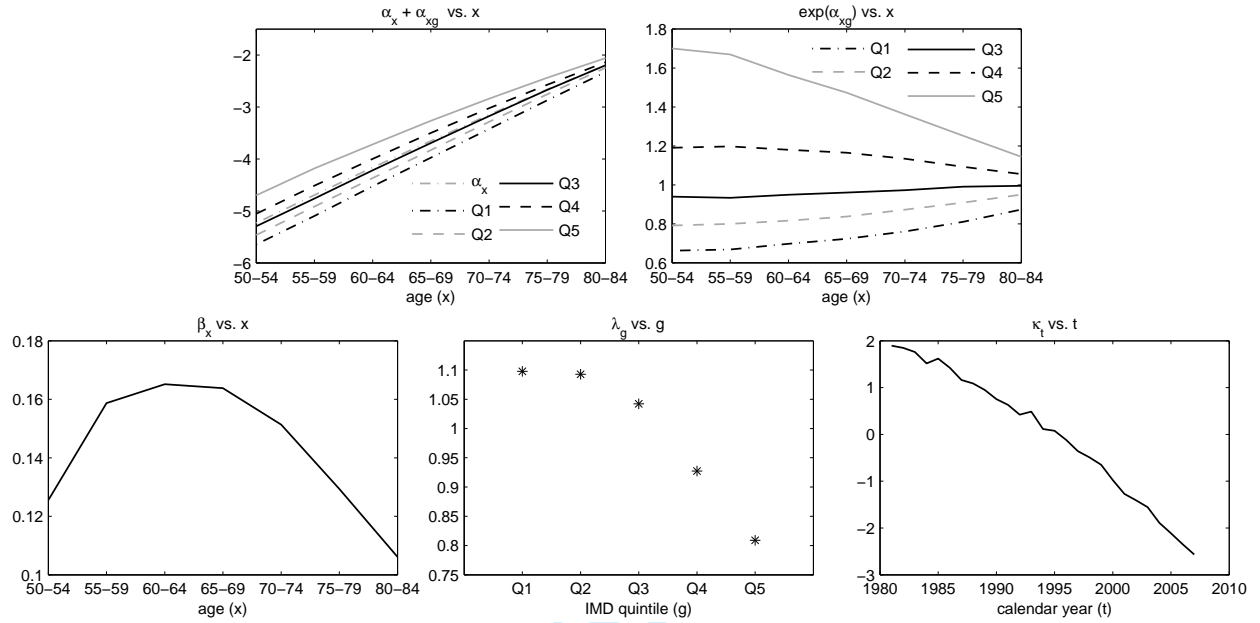

Figure 6: Parameter estimates of the relative model for the male deprivation subpopulations with Lee-Carter reference.
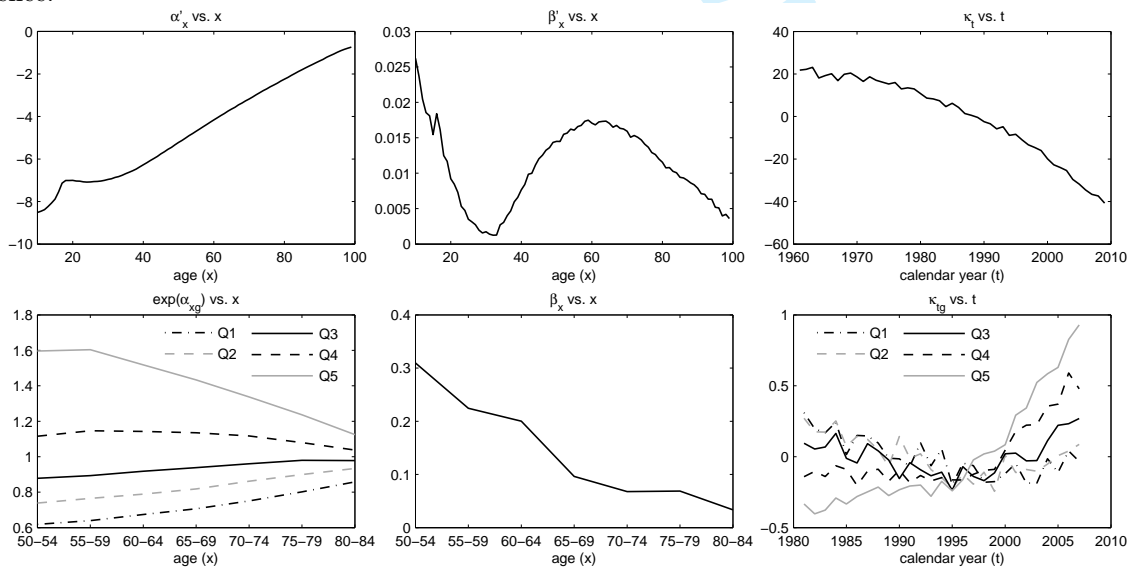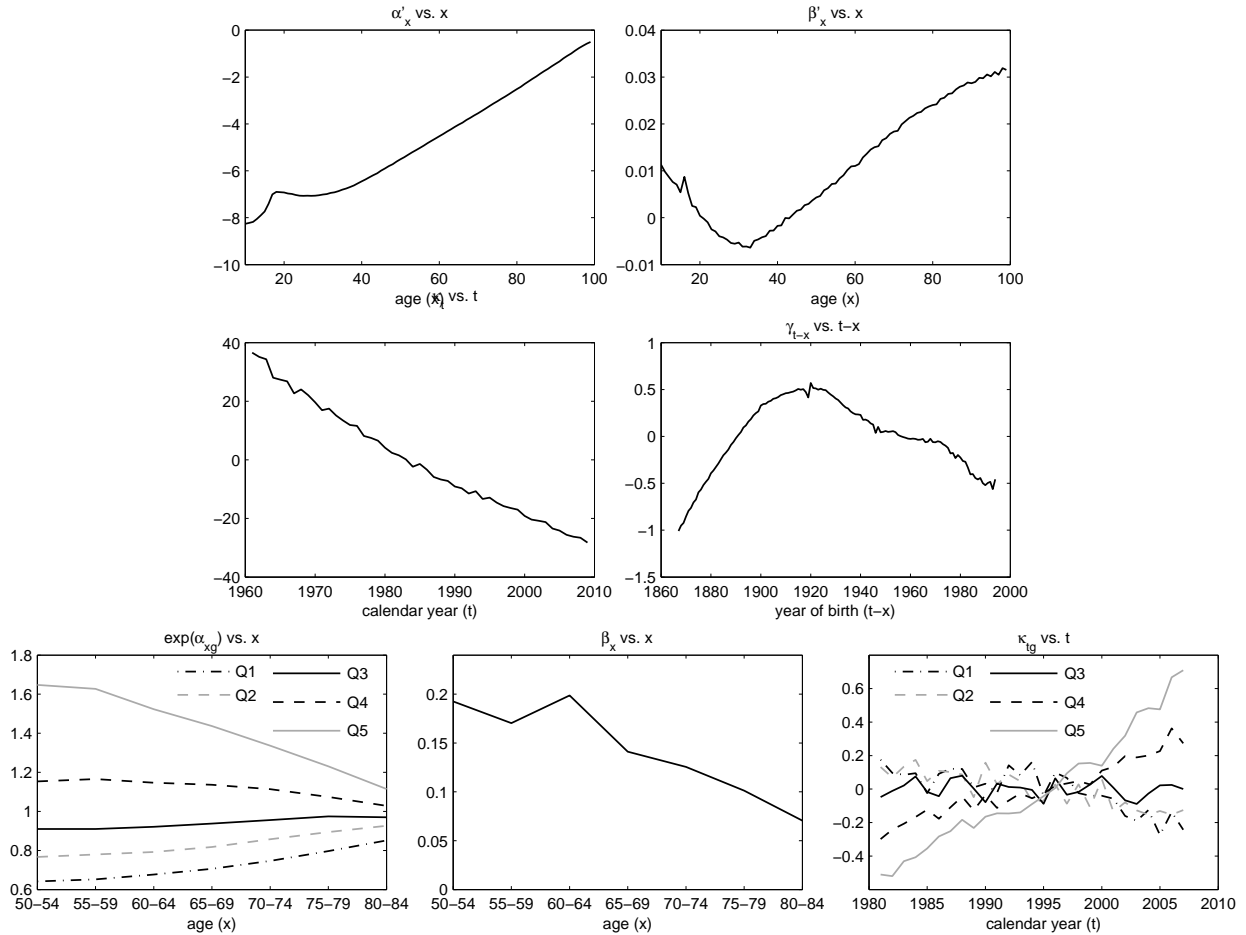

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Figure 7: Parameter estimates of the relative model for the male deprivation subpopulations with age-period-cohort reference.

deviance residuals of the fitted subpopulation model

$$r_{xtg} = \text{sign}(_n d_{xtg} - _n \hat{d}_{xtg}) \sqrt{\frac{\text{dev}(x,t,g)}{\hat{\phi}}}, \qquad \hat{\phi} = \frac{D(_n d_{xtg}, _n \hat{d}_{xtg})}{N - \nu} \qquad (27)$$

and, when applicable, the deviance residuals of the reference population model

$$r'_{xt} = \text{sign}(d'_{xt} - \hat{d}'_{xt}) \sqrt{\frac{\text{dev}'(x,t)}{\hat{\phi}}}, \qquad \hat{\phi}' = \frac{D'(d'_{xt}, \hat{d}'_{xt})}{N' - \nu'}, \qquad (28)$$

where

$$D(_n d_{xtg}, _n \hat{d}_{xtg}) = \sum_{x \in \mathcal{X}} \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} \text{dev}(x,t,g) = \sum_{x \in \mathcal{X}} \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} 2\omega_{xtg} \left\{ _n d_{xtg} \log \frac{_n d_{xtg}}{_n \hat{d}_{xtg}} - (_n d_{xtg} - _n \hat{d}_{xtg}) \right\},$$

$$D'(d'_{xt}, \hat{d}'_{xt}) = \sum_{x \in \mathcal{X}'} \sum_{t \in \mathcal{T}'} \text{dev}'(x,t) = \sum_{x \in \mathcal{X}'} \sum_{t \in \mathcal{T}'} 2\omega'_{xt} \left\{ d'_{xt} \log \frac{d'_{xt}}{\hat{d}'_{xt}} - (d'_{xt} - \hat{d}'_{xt}) \right\},$$

Figure 8: Parameter estimates of the relative model for the female deprivation subpopulations with age-period-cohort reference.
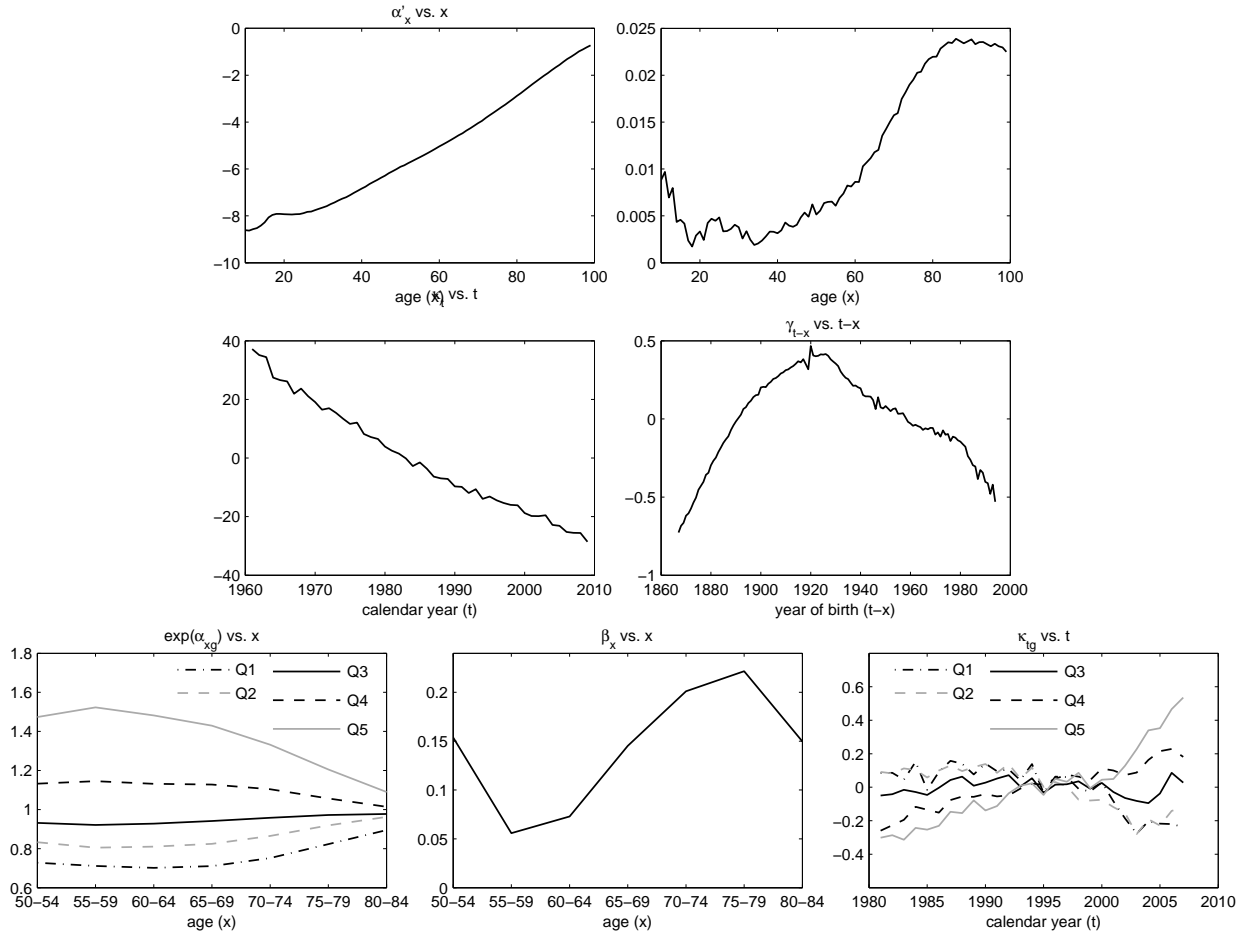

$N = \sum_{xtg} \omega_{xtg}$, $N' = \sum_{xt} \omega'_{xt}$ are the number of observations in the subpopulation and reference population data, respectively, ν is the effective number of parameters in the subpopulation part of the model, and ν' is the effective number of parameters in the reference population part of the model[3]. Regular patterns in the residuals are a sign of the inability of the model to describe all of the phenomena appropriately.

Figures 9 plots the standardised deviance residuals of the models without a reference population, whereas Figure 10 plots the residuals of the relative models with a reference population. The residuals of the stratified Lee-Carter model show systematic patterns by age and calendar time, indicating that deprivation subpopulations do not satisfy the under-

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[3]For example, the stratified Lee-Carter model (11) requires the estimation of k values of α_x, m values of α_g, k values of β_x and n values of β_x, totalling $2k + m + n$ parameters, but since the parameters have to satisfy the constraints $\sum_x \beta_x = 1$, $\sum_t \kappa_t = 0$ and $\sum_g \alpha_g = 0$, the effective number of parameters is $\nu = 2k + m + n - 3$. For the relative model with age-period-cohort reference we take v' as the number of parameters in the reference population equation (16) less 4 to account for constraints (19)-(21), (26) and v as the number of parameters that appear exclusively in the subpopulation equation (17) less 2 for constraints (22) and (23).
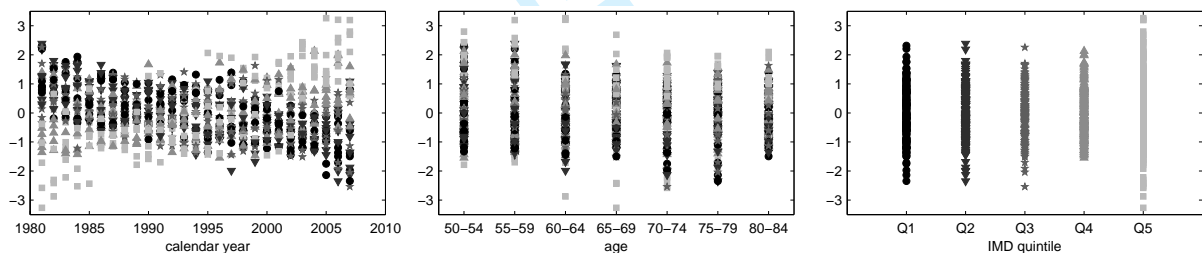
Figure 9: Deviance residuals for the models without a reference population applied to the male deprivation subpopulations.
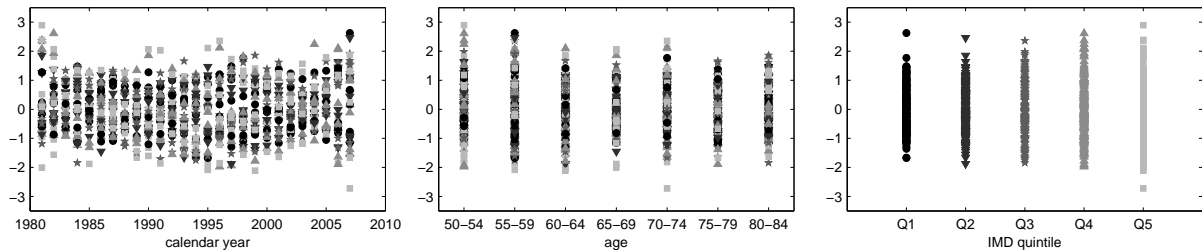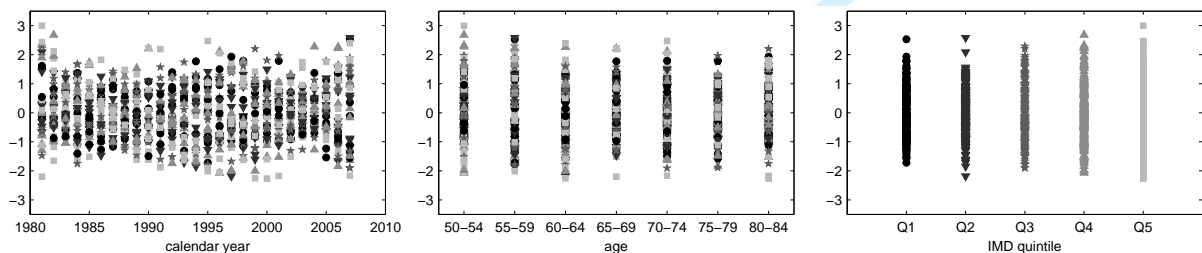

(a) Lee-Carter: $\log{}_n\mu_{xtg} = \alpha_{xg} + \beta_{xg}\kappa_{tg}$
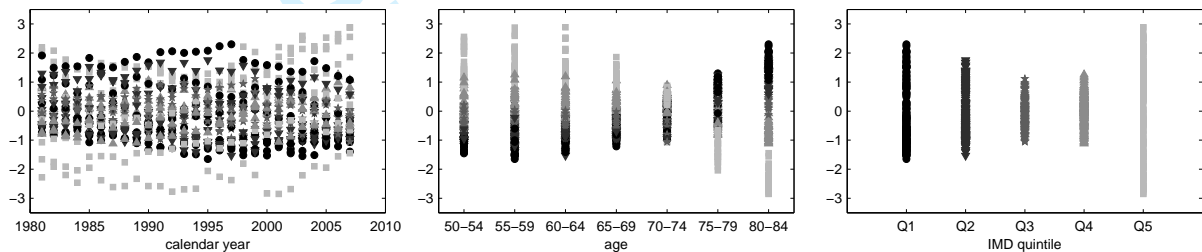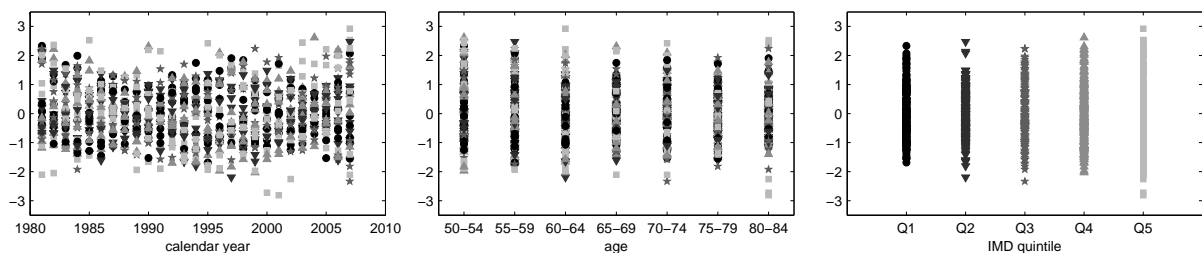
(b) Stratified Lee-Carter: $\log{}_n\mu_{xtg} = \alpha_x + \alpha_g + \beta_x\kappa_t$

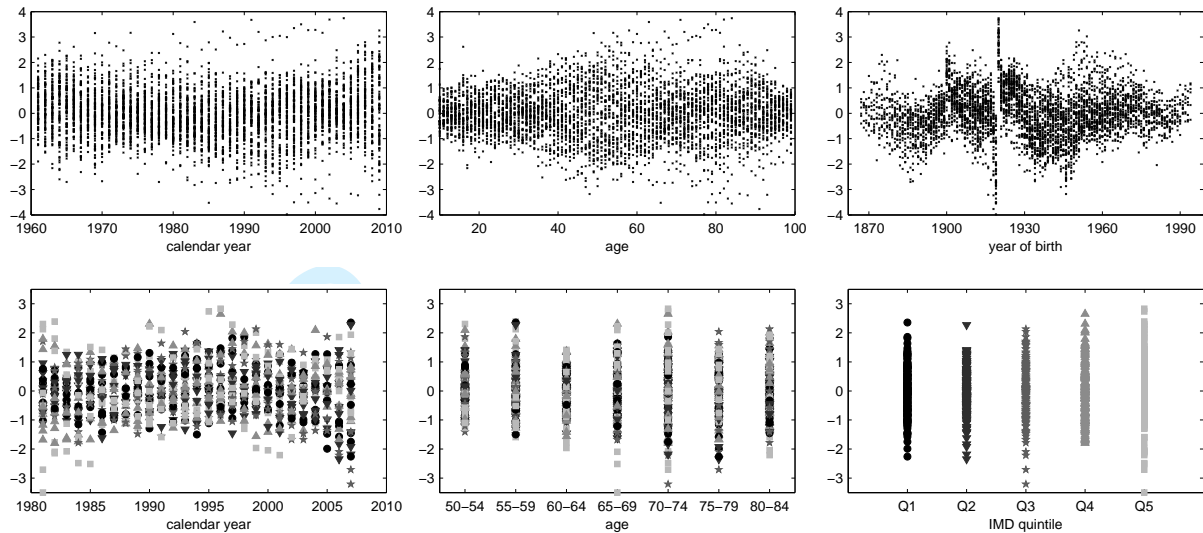(c) Common factor: $\log{}_n\mu_{xtg} = \alpha_x + \alpha_{xg} + \beta_x\kappa_t$

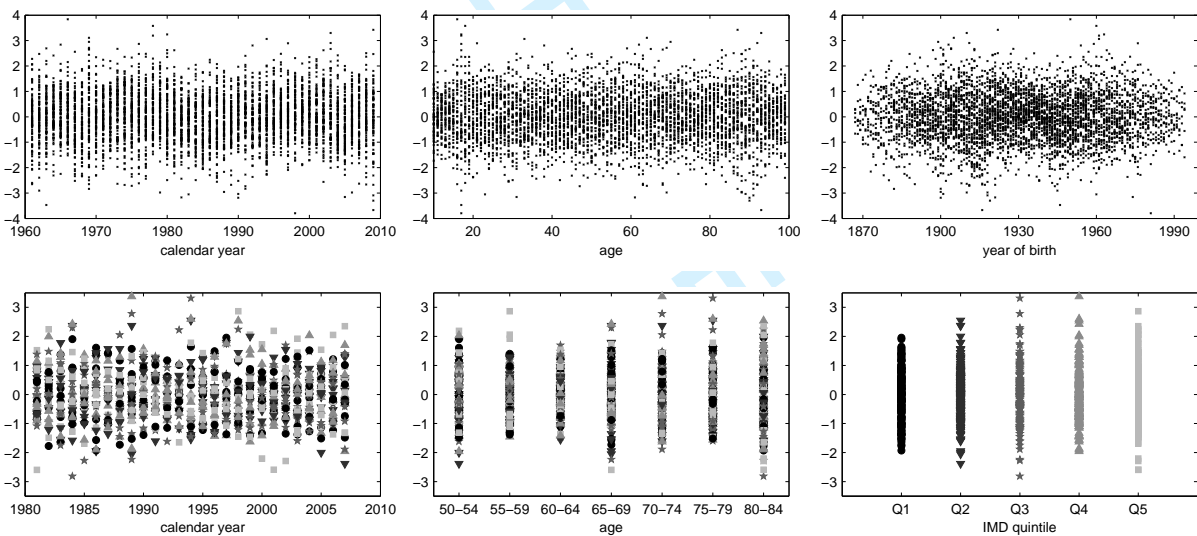(d) Joint-κ: $\log{}_n\mu_{xtg} = \alpha_x + \alpha_{xg} + (\beta_x + \beta_{xg})\kappa_t$

(e) Three way Lee-Carter: $\log{}_n\mu_{xtg} = \alpha_x + \alpha_{xg} + \beta_x\lambda g\kappa_t$

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Figure 10: Deviance residuals for the models with a reference population applied to the male deprivation subpopulations.

(a) Lee-Carter reference: $\log \mu'_{xt} = \alpha'_x + \beta'_x \kappa'_t$

(b) Age-period-cohort reference: $\log \mu'_{xt} = \alpha'_x + \beta'_x \kappa'_t + \gamma_{t-x}$

Note: The first row of each subfigure displays the residuals for the reference population and the second row of each subfigure displays the residuals for the deprivation subpopulations.

lying model assumption that on a log scale the age-specific mortality profile of the subgroups defined by the extra variate (g) deviate parallelly from the population age-specific mortality profile. The common factor model does not exhibit any clear pattern by age and deprivation quintile, signifying that it captures satisfactorily mortality level differentials among the deprivation subpopulations. However, this model assumes that mortality level differentials are constant over time, failing to capture differences in the rate of mortality improvement as revealed by the diagonal patterns observed in the calendar year residual plots. The independent Lee-Carter model, the joint-κ model and the three-way Lee-Carter model do not

exhibit any clear pattern in their residual plots, suggesting that all three models capture successfully both mortality level and mortality improvement differentials among the deprivation subgroups. In Figure 10 we observe a distinctive ripple effect in the year-of-birth residual plots under the Lee-Carter model for the reference population, indicating a failure to capture cohort effects. This contrasts with the absence of any major distinctive pattern in both the reference population and subpopulation residual plots under age-period-cohort modelling, indicating the success of this model in capturing the main effects of the reference population as well as in capturing mortality differentials in the deprivation subpopulations.

When assessing the goodness of fit of different models, it is natural that models with more parameters provide a better fit to the data. Therefore, we use information criteria to rule out the possibility that the better fit observed in a model is a result of overparametrisation. Information criteria modify the maximum likelihood criterion by including a penalty function so that overparametrised models are penalised. Here, we consider the Bayes Information Criterion (BIC) which has been proposed by Cairns et al. (2009) for the quantitative comparison of mortality projection models. This criterion is given by $BIC = \mathcal{L} - 0.5\nu \log N$, where $\mathcal{L}$ denotes the maximum log-likelihood of the model and ν is the effective number of parameters of the model. From this definition, it is clear that models with higher BIC values are preferable. In order to allow the comparison between models with and without a reference population, for models with a reference population we take the total model log-likelihood as being comprised of a reference population part, $\mathcal{L}_{ref}$, and a subpopulation part, $\mathcal{L}_{sub}$, as in equation (25), and report thus the two corresponding BIC values $BIC_{ref} = \mathcal{L}_{ref} - 0.5\nu' \log N'$ and $BIC_{sub} = \mathcal{L}_{sub} - 0.5\nu \log N$.

Table 1 contains BIC values for each of the seven adjusted models, together with their respective rankings (in brackets). The small BIC values for the stratified Lee-Carter model and the common factor model confirm the inability of these models to capture adequately the differentials in mortality of the deprivation subpopulations. We observe that the relative model with age-period-cohort reference, the only model capturing the cohort effect, is by far on top of the subpopulation BIC ranking. This model is followed by the joint-κ model, the relative model with Lee-Carter reference and the three-way Lee Carter, all of them with very close matching statistics. We also note that among models with a reference population, the age-period-cohort model performs much better than the Lee-Carter model, providing evidence of the existence of a cohort effect.

It is possible that a model has good in-sample-fit but produces poor ex-post forecast, that is, forecasts that deviate significantly from the realised outcomes. Therefore, in addition to the assessment of goodness of fit, it is necessary to evaluate the ex-post forecasting performance of the models using suitable backtesting methods. For that purpose, we apply a expanding horizon backtest as proposed by Dowd et al. (2010). This backtesting procedure entails the following steps:

(a) Select a metric of interest, that is, the forecasted variable which is the focus of the backtest. Since our interest is forecasting both individual subpopulation mortality rates and mortality rates differentials among the subpopulations, we base our backtest exercise on the subpopulation-specific probability of dying between ages 50 and 85, $_{35}q_{50,t,g}$, as well as on the ratio of this probabilities in the most and least deprived subpopulations, $_{35}q_{50,t,Q5}/_{35}q_{50,t,Q1}$, and in the fourth and second IMD quintiles of the population,

Table 1: Log-likelihood $\mathcal{L}$, effective number of parameters ν, and BIC values for the different models applied to the male deprivation subpopulations.

Model	Subpopulations			Reference population		
	ν	$\mathcal{L}$	BIC	ν'	$\mathcal{L}$	BIC
Independent LC	195	-6 448	-7 116(5)	-	-	-
Stratified LC	43	-22 770	-22 918(7)	-	-	-
Common factor	66	-7 827	-8 053(6)	-	-	-
Joint-k	94	-6 538	-6 860(2)	-	-	-
Three way LC	71	-6 670	-6 913(4)	-	-	-
Relative (LC reference)	171	-6 310	-6 896(3)	227	-31 025	-31 997(2)
Relative (APC reference)	171	-5 630	-6 216(1)	353	-23 251	-24 001(1)

$_{35}q_{50,t,Q4}/_{35}q_{50,t,Q2}$, with $_{35}q_{50,t,g}$ calculated on a period basis using the expression

$$_{35}q_{50,t,g} = 1 - \left(1 - {}_5q_{50,t,g}\right)\left(1 - {}_5q_{55,t,g}\right)\cdots\left(1 - {}_5q_{80,t,g}\right)$$

where

$$_5q_{x,t,g} = \frac{5\,{}_5\mu_{x,t,g}}{1 + 2.5\,{}_5\mu_{x,t,g}}, \quad x = 50, 55, \ldots, 80.$$

(b) Select the historical "lookback" period used to estimate the parameters of the model and select the time horizon over which the forecasts will be made. We fit the models over the restricted period 1981-2000 and perform forecasts for the period 2001-2007.

(c) Compare graphically the forecasts against realised outcomes of the metrics of interest. We plot 95% prediction intervals (fan charts) of the metrics of interest obtained by simulating 10 000 paths of the period indexes of each of the models with realised outcomes of the metrics superimposed. We note that these prediction intervals ignore any provision for parameter uncertainty.

We exclude the stratified Lee-Carter model, the common factor model, and the relative model with Lee-Carter reference from the backtesting exercise as these models show a poor performance in terms of in-sample-fit. For the independent Lee-Carter model, we include both the forecasted intervals using independent random walks with drift for each subpopulation's period index and using a multivariate random walk with drift. Figure 11 shows the comparison of the prediction intervals and the realised outcomes of the metrics of interest. From this figure we note the following:

- The consistent underestimation of mortality decline by all the models with the exception of the relative model with an age-period-cohort reference, giving evidence that the possibility of capturing cohort effects results in substantial improvements in forecasting performance.

- The exact agreement between prediction intervals of subpopulation mortality rates of the two alternative forecasting methods for the independent Lee-Carter model, but the difference in the prediction intervals of the mortality differentials, with the independent random walk approach resulting in excessively wide prediction intervals.

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Figure 11: Predicted and realised outcomes of the backtesting metrics for the period 2001-2007.
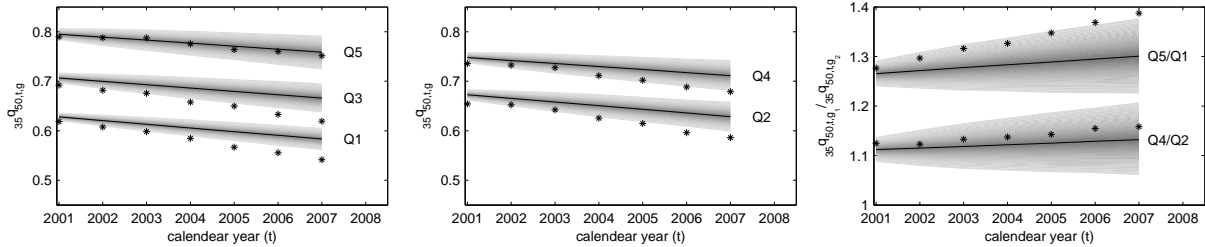
(a) Lee-Carter with independent random walk with drift
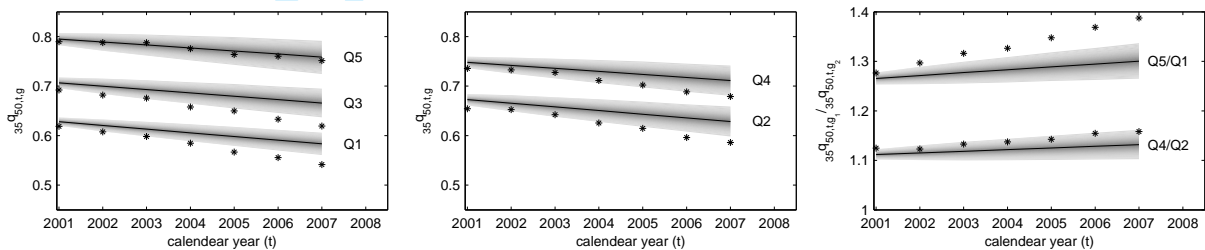
(b) Lee-Carter with multivariate random walk with drift
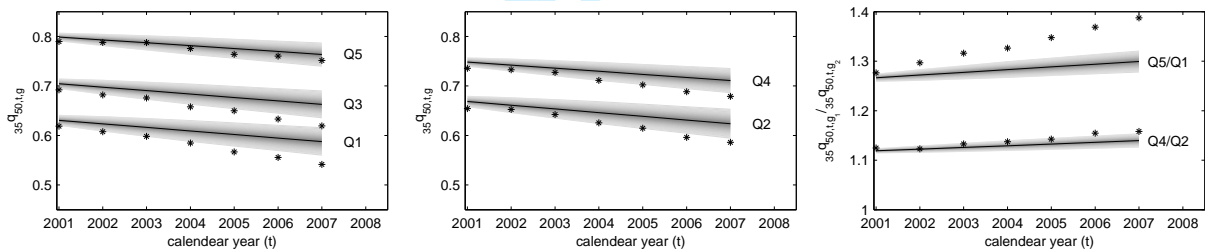
(c) Joint-κ

(d) Three-way Lee-Carter

(e) Relative model with APC reference

Note: Left and central frames depict the values for $_{35}q_{50,t,g}$ and right frames depict the values for $_{35}q_{50,t,Q5}/_{35}q_{50,t,Q1}$ and $_{35}q_{50,t,Q4}/_{35}q_{50,t,Q2}$

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- The extremely narrow prediction intervals of mortality differentials produced by the joint-κ model and the three-way Lee-Carter model, which is a result of the perfect correlation of subpopulation mortality improvements induced by the use of a single mortality driver for all the subpopulations. Therefore, although th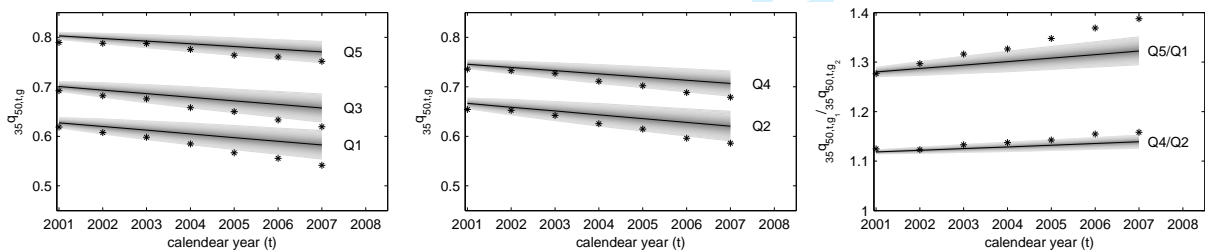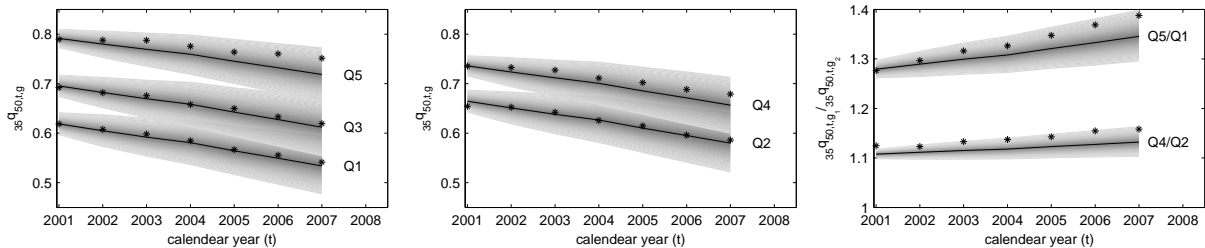ese two models show reasonable goodness of fit and provide a transparent framework for quantifying historical mortality differentials, they are likely to underestimate the uncertainty associated with the predictions of mortality differentials.

- The relatively poor performance of all the models in the forecasting of mortality differentials with a consistent underestimation of the widening of mortality differentials. Nevertheless, the relative model with a age-period-cohort reference shows the best performance, with central predictions that deviate the least from the realised outcomes and predictions intervals that match the observed variability.

In summary, the independent Lee-Carter model, the joint-κ model, the three-way Lee-Carter model and the relative model with age-period-cohort reference all succeed in fitting historical mortality differentials in the English male population, with the latter model outperforming the others according to the BIC criterion. In addition, the relative model with age-period-cohort reference outperforms the others in terms of ex-post evaluation of the model predictions. Equivalent analyses - not shown here - for the female population lead to similar conclusions. For these reasons, our subsequent discussions on the differentials in mortality across deprivations quintiles will be based on the relative model with age-period-cohort reference.

3.2. Historical mortality differentials: 1981-2007

We now use the new relative model with age-period-cohort reference to analyse the socio-economic differentials in mortality observed in the English male and female populations during the period 1981-2007. Figures 7 and 8 depict the corresponding parameter estimates for the male and female populations, respectively.

We first turn our attention to the parameter estimates of the England and Wales reference populations. The main age-effects plots (α'_x vs. x) for both sexes have the usual features of static life tables, with a more pronounced accident hump in males and lighter mortality in females. For both males and females, the period index plot (κ'_x vs. t) exhibits a steady decline in mortality with a mild curvature. For both genders, the plot of the age-modulating parameter β'_x reveals that the ages in the range 20-40 have experienced the slowest mortality decline, with β'_x being even negative for the male experience. This result coincides with the pronounced decrease in the rate at which mortality improved in the last quarter of the past century reported by Renshaw and Haberman (2003) for both males and females in the age band 20-40. Finally, the plots of the cohort effect (γ'_{t-x} vs. $t - x$) for both sexes are similar and reveal the impact of the 1919 influenza pandemic and the rapid mortality improvement experienced by the generation born between 1925 and 1945 as reported by Willets (2004). It is worth mentioning that the cohort effect for the generations born after 1945 need to be interpreted with care as they only reflect the behaviour of mortality at young and middle ages, and it is not totally clear whether these effects will still hold at older ages.

The subpopulation parameters reveal very different subpopulation mortality rate levels as well as very different evolutions of these rates over time. In particular, the plots of $\exp(\alpha_{xg})$

show a steep increase in mortality rate levels with increasing deprivation. For instance, during the period 1981-2007 males age 50-54 in the least deprived quintile experienced 36% lower mortality than the reference population, which contrasts with the 65% higher mortality experienced by males age 50-54 in the most deprived quintile. Significant differences in the level of mortality are observed for both genders and all ages, though with varying magnitudes. Males show higher mortality level differentials than females. For instance, while for males age 60-64 the ratio between mortality in the most deprived and the least deprived quintile, $\exp(\alpha_{x,Q5})/\exp(\alpha_{x,Q1})$, is 2.25, for females this ratio reduces to 2.11. In addition, mortality level differentials narrow with age. At age 50-54 males and females in the most deprived quintile have respectively 2.57 and 2.03 higher mortality than persons of the same sex and age in the least deprived quintile. By contrast, at age 80-84 males and females in the most deprived quintile have respectively 1.31 and 1.22 higher mortality than persons of the same sex and age in the least deprived quintile. This decrease in socio-economic mortality differences with rising age is commonly reported in mortality research (Hoffmann, 2005).

The plots of κ_{tg} indicate that, in spite of the overall decrease in mortality levels, the more deprived quintiles have experienced slower mortality reductions than the less deprived ones, evidencing a widening of the mortality gap between the least and the most deprived areas of England. The sharp deceleration after 1997 in the pace of mortality improvement of the most deprived quintile of the female population indicated by $\kappa_{t,Q5}$ is particularly noticeable. For both males and females, it is also worth noticing the close alignment between the period indexes for the two least deprived quintiles, $\kappa_{t,Q1}$ and $\kappa_{t,Q2}$, which contrasts with the marked differences in the corresponding level parameters $\alpha_{x,Q1}$ and $\alpha_{x,Q2}$. This indicates that despite the fact that there are material level differences between these two subpopulations, improvement differences are negligible.

The plots of β_x show that in the male population the highest differentials in the rate of mortality improvement are observed at ages 50-54 and 60-64, whereas in the female population the greatest mortality improvement differentials are achieved at ages 70-74 and 75-79. We also notice that at ages 55-59 and 60-64 females from all deprivation quintiles have experienced more or less the same mortality reduction. For males, we see a decline in mortality improvement differentials with rising age similar to that observed in mortality level differentials. Nonetheless, this inverse relationship between age and mortality improvement differential does not hold for females. In fact, from age group 55-59 until age group 75-79 improvement differences increase with age.

3.3. Mortality differential projections

In order to project age-subpopulation-specific mortality rates and examine the possible future evolution of mortality differentials, we extrapolate the period indexes κ'_t and κ_{tg}. The presence of a mild curvature in κ'_t complicates its forecasting. Although second order ARIMA models should produce good time series fits, Haberman and Renshaw (2009) argue against the use of this approach as it tends to produce excessively wide prediction intervals. Therefore, we instead follow Haberman and Renshaw (2009) and curtail the time series at a perceived point of departure from linearity. To assist with this subjective task, we monitor the profile of the R^2 linear regression goodness-of-fit statistic, constructed backwards following the approach proposed by Denuit and Goderniaux (2005). For both males and females, the R^2 statistic attains a maximum around 1980: thus, we model κ'_t post 1980 using

Figure 12: Period index time series for the England and Wales reference population (modelled post 1980).

<div align="center">(a) Male population   (b) Female population</div>
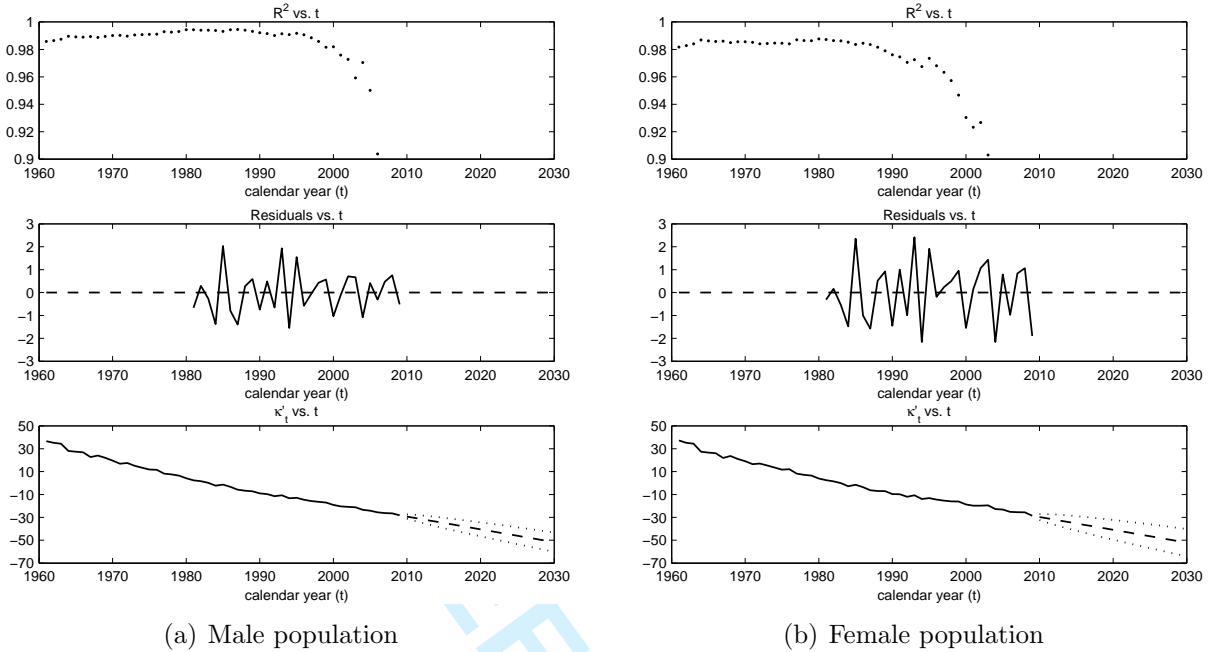
Note: First row: R^2 goodness of fit statistics. Second row: random walk with drift time series residuals. Third row: time series with predictions and with 95% prediction intervals.

a random walk with drift. The corresponding residual and time series plots, together with R^2 profiles are presented in Figure 12.

Since κ_{tg} does not show any significant departure from linearity, we use a multivariate random walk with drift to model its dynamics. We recall from Section 2 that this approach permits the consideration of correlations in the mortality evolution of the different subpopulations which is a necessary feature for the adequate estimation of prediction intervals of mortality differentials.

We use the forecasted values of κ_t' and κ_{tg} together with the estimated α_x', β_x', γ_{t-x}', α_{xg}, and β_x to obtain forecasts of subpopulation specific mortality rates for ages 50-84 up to year 2030. We note that forecasts for this period and age range do not require the extrapolation of the cohort component.

These projected mortality rates are then employed to compute: the ratio between the mortality rates of the most and least deprived quintiles $_5\mu_{x,t,Q_5}/_5\mu_{x,t,Q_1}$, the difference between the mortality rates of the most and least deprived quintiles $_5\mu_{x,t,Q_5} - _5\mu_{x,t,Q_1}$, subpopulation specific period life expectancies

$$\mathring{e}_{xtg} = 0.5 + \sum_{h=1}^{110-x} {}_hp_{xtg}, \tag{29}$$

and the life expectancy gap between the most and least deprived quintiles $\mathring{e}_{x,t,Q_1} - \mathring{e}_{x,t,Q_5}$, where $_hp_{xtg} = p_{xtg}p_{x+1,t,g}\cdots p_{x+h-1,t,g}$ and $p_{xtg} = \exp(-\mu_{xtg})$.

The implementation of the period life expectancy formula (29) requires subpopulation mortality rates, μ_{xtg}, for individual ages $x, x+1, \ldots, 110$, which are not available due to the

<div align="center">23</div>

Figure 13: Time series of fitted and forecasted mortality rates $_5\mu_{xtg}$ for the deprivation subpopulations.

(a) Male population

(b) Female population
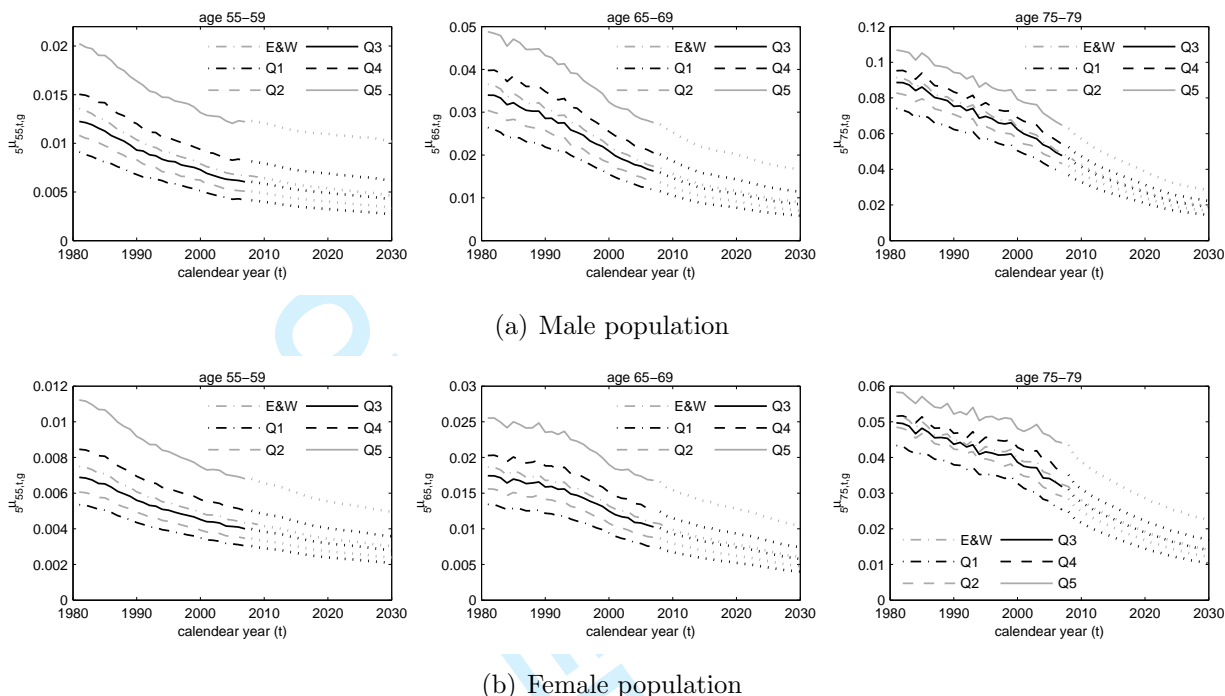
Note: The lines labelled "E&W" correspond to the fitted and projected values of $_5\bar{\mu}'_{xt}$ for the England and Wales reference population.

age-grouped format of our subpopulation dataset and its truncation at age 84. To tackle this problem, we set $\mu'_{110,t} = 0.7$ and extrapolate the England and Wales mortality rates along the age axis up to age 110 using the topping-out by age technique proposed by Haberman and Renshaw (2009). This technique, which is a variant of the widely used demographic method introduced by Coale and Kisker (1990), uses the quadratic differencing formula

$$u_j = \log q'_{99+j,t+j} = a + bj + cj(j+1),\tag{30}$$
$$q'_{99+j,2009+j} = 1 - \exp(-\mu'_{99+j,2009+j}), \quad j = -1, 0, \dots, 11,$$

which requires $u_{-1} = \log q'_{98,t-1}$, $u_0 = \log q'_{99,t}$ and $u_{11} = \log q'_{110,t+11}$.

These extrapolated mortality rates are then used as a reference to expand the abridged subpopulation mortality rates under the assumptions that $_5\mu_{xtg}/_5\bar{\mu}'_{xt} \approx \mu_{x+2,t,g}/\mu'_{x+2,t}$ and that mortality differences vanish at age 100. More specifically, for ages $x \leq 82$ we interpolate log-linearly the mortality ratio $\mu_{x,t,g}/\mu'_{x,t}$ after setting $\mu_{x+2,t,g}/\mu'_{x+2,t} = {_5\mu_{xtg}}/{_5\bar{\mu}'_{xt}}$, $x \in \{50, 55, \dots, 80\}$; for ages $82 < x \leq 99$, which are outside the observable age range, we modify (30) and use the differencing formula

$$u_j = \log\left|\log\frac{\mu_{82+j,t,g}}{\mu'_{82+j,t}}\right| = a + bj + cj(j+1), \quad j = -1, 0, \dots, 17,\tag{31}$$

with $u_{-1} = \log\left|\log\mu_{81,t,g}/\mu'_{81,t}\right|$, $u_0 = \log\left|\log\mu_{82,t,g}/\mu'_{82,t}\right|$ and $u_{17} = \log 0.01$; and for ages $100 \leq x \leq 110$ we set $\mu_{xtg}/\mu'_{xt} = 1$.

Figure 14: Time series of mortality rate differentials between the most and least deprived quintiles of the population.

(a) Male population   (b) Female population
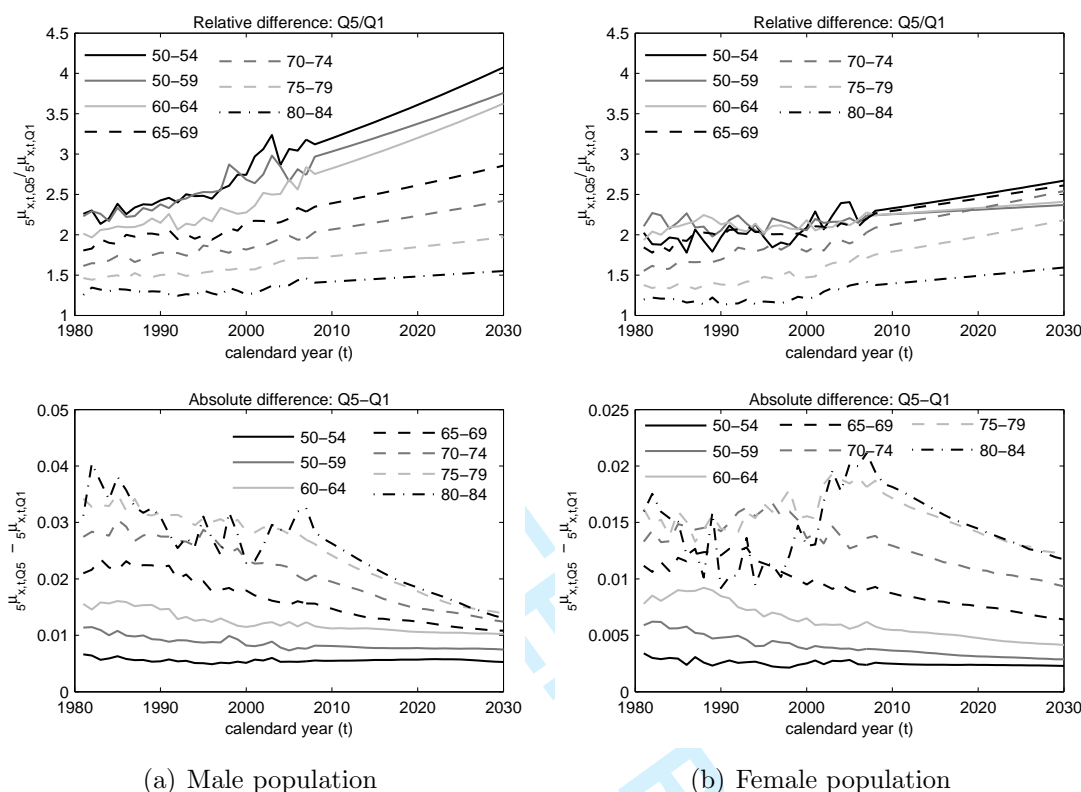
Note: Values prior to 2007 are observed differentials and values post 2007 are projected differentials.

Figure 13 depicts the projected evolution of deprivation-specific mortality rates at selected ages. A key feature of note is that these projected mortality rates exhibit a coherent behaviour, in the sense that they are consistent with the England and Wales forecasted rates and that there are no cross-overs between the subpopulations. This is a major characteristic of mortality projections produced using the relative modelling approach proposed in this paper. The forecasted fast decline of mortality rates at age 75-79, especially in the female population, is also particularly noteworthy. This feature is a reflection of the faster mortality decline experienced by those born between 1925-1945 captured by the cohort parameters of the reference population.

Figure 13 suggests a tendency to convergence in the mortality rates of the deprivation subpopulations. To investigate further this feature, we display in Figure 14 the relative mortality difference between the most and least deprived quintiles, $_5\mu_{x,t,Q_5}/_5\mu_{x,t,Q_1}$, as well as the corresponding absolute mortality difference, $_5\mu_{x,t,Q_5} - _5\mu_{x,t,Q_1}$. In relative terms, we observe a widening of the mortality differences between the extreme deprivation quintiles, reflecting the slowest mortality improvements experienced by the most deprived subpopulation. By contrast, in absolute terms, with the exception of some old ages in the female population, a general narrowing in mortality differences is observed. Therefore, when drawing conclusions on the widening or narrowing of mortality differences, the reference measure needs to be clearly stated.

Figure 15: Central projections and 95% prediction intervals of mortality rate differences at age 65-69.

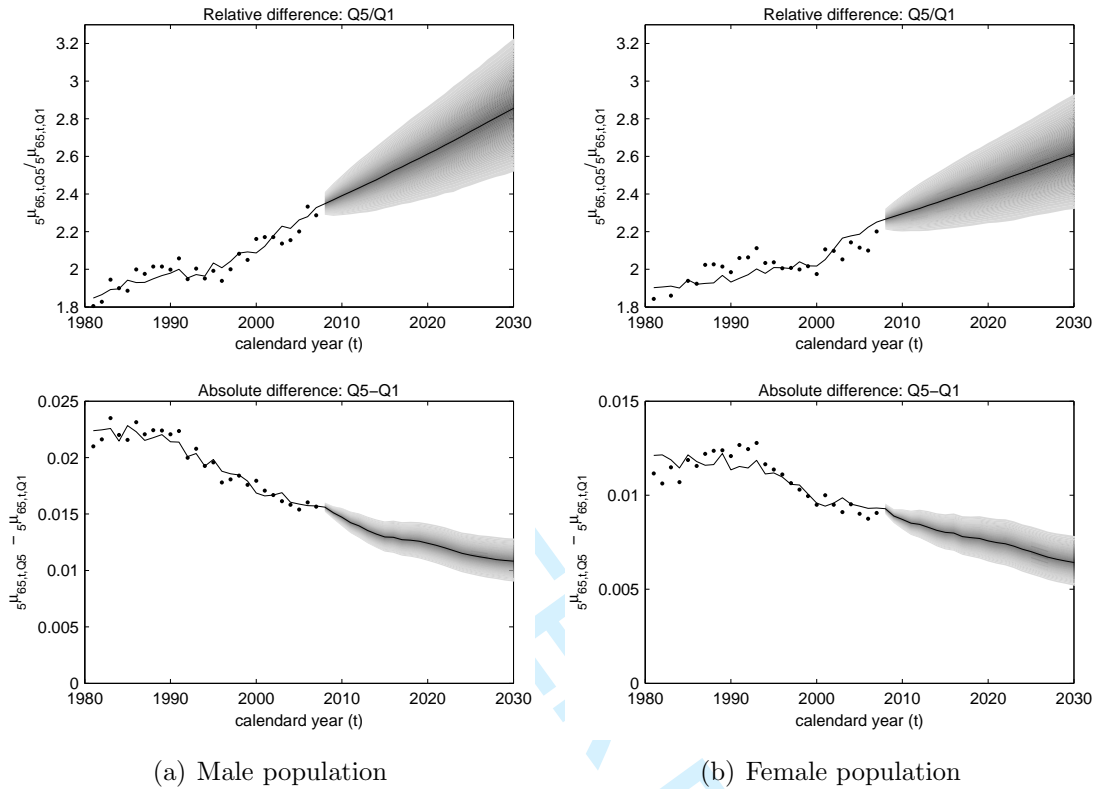(a) Male population   (b) Female population

Figure 15 presents a closer look at mortality differentials projections at age 65-69: central predictions are accompanied with 95% prediction intervals (fan charts) obtained by simulating 10 000 paths of the random walk model of κ'_t and the multivariate random walk model of κ_{tg}. For both sexes a widening of mortality relative differentials is projected, although with a steeper trend and sligthly wider prediction intervals for males. Whereas for men the mortality rate at age 65-69 in 2030 of the most deprived quintile is forecasted to be between 2.5 and 3.2 times the mortality rate of the least deprived quintile, for women the ratio between the mortality rate at age 65-69 of the the most deprived quintile and the least deprived quintile is projected to range between 2.3 and 2.9.

Forecasted period life expectancies at age 65 together with the life expectancy gap between the most and least deprived quintiles are displayed in Figure 16. We note that despite the overall increase in life expectancy, the most deprived quintile has gained fewer years in life expectancy than the least deprived quintile. Specifically, the life expectancy gap between the most and the least deprived quintile increased from 2.9 years for males and 2.3 for females in 1981 to 3.8 and 3.2 years in 2007, respectively. For males, the life expectancy gap is projected to reach a value of 4 years in 2030 with a 95% confidence interval of 3.5-4.5 years. For females, the life expectancy gap is forecasted to remain practically unchanged at 3.1 years, but with a considerably wide prediction interval of 2.6-3.9 years in 2030.

Figure 16: Historical and projected period life expectancies and life expectancy gap at age 65.

<div align="center">(a) Male population</div>

<div align="center">(b) Female population</div>
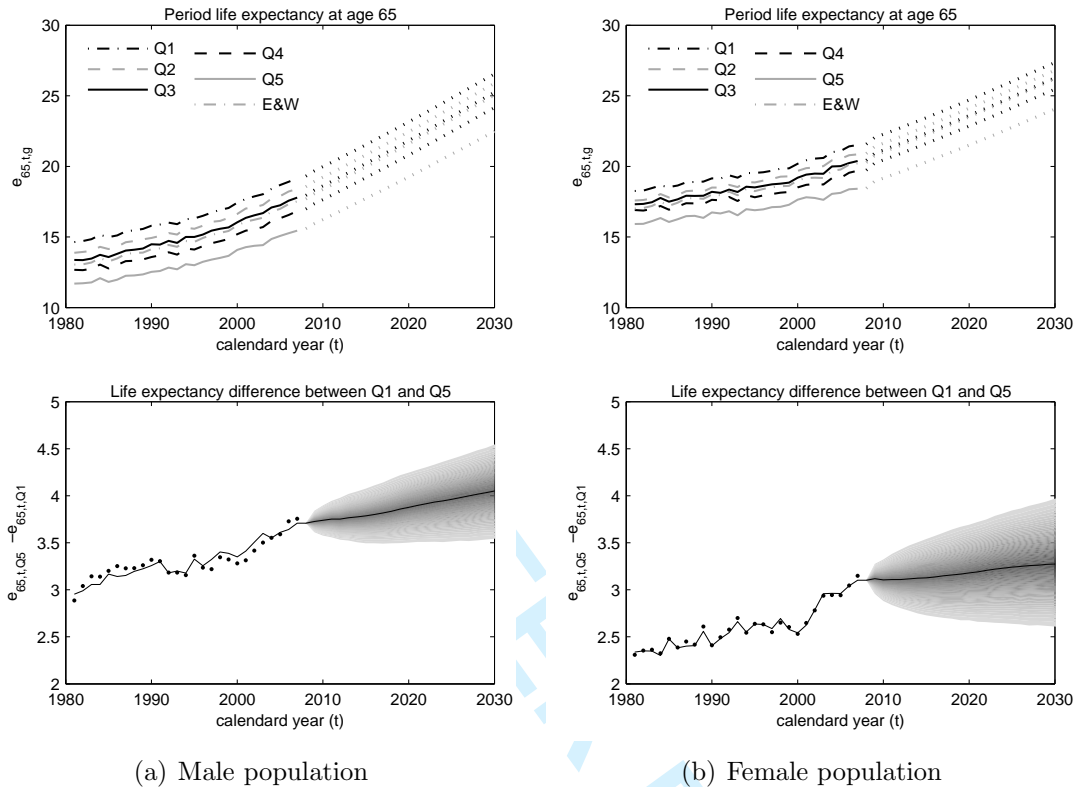
Note: The lines labelled "E&W" correspond to period life expectancies for the England and Wales reference population. Central projections of the life expectancy gap $\mathring{e}_{x,t,Q_1} - \mathring{e}_{x,t,Q_5}$ are accompanied by 95% prediction intervals.

3.4. Implications for life annuities

In order to assess the financial implications of socio-economic mortality differentials, in this section we investigate the variability of annuities values across deprivation subgroups.

A typical practice in the valuation of annuities is to vary the baseline mortality assumptions according to socio-economic characteristics, but to assume the same future mortality improvements for all individuals, regardless of their socio-economic characteristics (Madrigal et al., 2011; Lu et al., 2012). Therefore, to examine the extent to which these assumptions are reasonable, we consider two alternative mortality scenarios. In a first scenario we suppose that there are both level and improvement differences in mortality and compute projected mortality rates using the full forecasting model:

$$ {}_n\mu_{x,2007+j,g} = {}_n\bar{\mu}'_{x,2007+j} \exp\left(\alpha_{xg} + \beta_x \kappa_{2007+j,g}\right), \quad j > 0. \tag{32} $$

In a second scenario we assume that improvement differentials are non-existent and take mortality differences as being fixed at their 2007 level. Thus, we project mortality rates using the expression

$$ {}_n\mu_{x,2007+j,g} = {}_n\bar{\mu}'_{x,2007+j} \exp\left(\alpha_{xg} + \beta_x \kappa_{2007,g}\right), \quad j > 0, \tag{33} $$

Table 2: Percentage increase/decrease in annuity rates over the England and Wales annuity rate

Males

| | | Scenario 1: Level and improvement differences | | | | | Scenario 2: Only level differences | | | | |
Age	E&W	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
50	18.71	4.5%	2.7%	0.8%	-2.7%	-8.8%	4.1%	2.5%	0.8%	-2.1%	-7.2%
55	17.23	5.3%	3.1%	0.9%	-3.1%	-9.7%	5.0%	2.9%	1.0%	-2.5%	-8.2%
60	15.55	6.1%	3.5%	1.0%	-3.4%	-10.3%	5.8%	3.3%	1.0%	-2.8%	-8.9%
65	13.53	6.9%	3.9%	1.1%	-3.5%	-10.4%	6.6%	3.7%	1.1%	-3.0%	-9.2%
70	11.37	7.4%	3.9%	1.1%	-3.4%	-10.0%	7.1%	3.8%	1.1%	-3.0%	-9.1%
75	8.93	7.7%	3.9%	1.1%	-3.0%	-9.0%	7.4%	3.8%	1.1%	-2.7%	-8.4%
80	6.58	7.0%	3.4%	1.3%	-2.0%	-6.8%	6.9%	3.4%	1.3%	-1.8%	-6.4%

Females

| | | Scenario 1: Level and improvement differences | | | | | Scenario 2: Only level differences | | | | |
Age	E&W	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
50	19.23	2.9%	1.7%	0.4%	-1.7%	-5.4%	2.6%	1.5%	0.5%	-1.2%	-4.2%
55	17.99	3.4%	1.9%	0.4%	-1.9%	-6.1%	3.1%	1.7%	0.5%	-1.4%	-4.9%
60	16.28	4.2%	2.3%	0.6%	-2.3%	-7.1%	3.9%	2.1%	0.6%	-1.7%	-5.8%
65	14.59	4.6%	2.5%	0.5%	-2.4%	-7.6%	4.2%	2.2%	0.5%	-1.9%	-6.4%
70	12.44	5.1%	2.5%	0.5%	-2.4%	-7.8%	4.7%	2.3%	0.6%	-1.9%	-6.8%
75	9.99	5.0%	2.2%	0.5%	-2.2%	-7.3%	4.8%	2.0%	0.5%	-1.8%	-6.5%
80	7.42	4.4%	1.6%	0.5%	-1.5%	-5.7%	4.2%	1.5%	0.4%	-1.2%	-5.1%

Note: The column "E&W" presents the annuity rate for a person in England and Wales age x in 2007 at a 4% interest rate.

which assumes that the mortality improvements of all the subpopulations follow the same behaviour of the improvements of the England and Wales reference population. Table 2 presents for these two mortality scenarios the percentage deviation of the annuity rate of each deprivation quintile with respect to the comparable England and Wales annuity rate. The values presented correspond to level immediate annuities for individuals age $50, 55, \ldots, 80$ in 2007, computed under the cohort trajectory[4] and assuming an interest rate of 4%. Referring to Table 2 we note the following:

- As expected, annuity values decrease significantly as the level of deprivation increases. For example, under the assumption of both level and improvement differentials in mortality, the annuity rate at age 65 for males in the most deprived quintile is 10.4% less than the corresponding rate for the England and Wales population. This contrasts with a 6.9% excess for males age 65 in the least deprived quintile compared to the England and Wales population.

- Although mortality differentials decrease with rising age, the impact of these differen-

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[4]As opposed to a period approach, a cohort approach makes full allowance for the future evolution of mortality rates. Thus, the annuity rate for an individual of subpopulation g age x in year t is computed using mortality rates $\mu_{xtg}, \mu_{x+1,t+1,g}, \mu_{x+2,t+2,g} \cdots, \mu_{110,t+110-x,g}$

tials on annuity values does not decrease significantly with age, and, in some cases, can even increase. For instance, at age 80 the annuity rate for males in the least deprived quintile is 7.0% higher than the corresponding annuity rate for the whole of the England and Wales population, which is an even greater difference than the 4.5% observed at age 50.

- The impact of improvement differentials on the valuation of annuities is in general of second order when compared to that of level differentials. In fact, for most ages and deprivation subgroups the difference between annuity rates computed under the two alternative mortality scenarios is less than 0.7%. This difference is only significant for the youngest ages of the most deprived quintile, reaching a maximum of 1.6% at age 50 in the male population. These results suggest that assuming the absence of improvement differentials in mortality is in principle reasonable for the valuation of annuities. In contrast, the correct estimation of initial socio-economic differentials in mortality is critical in the pricing and reserving of annuities and pensions.

- The variability of annuity rates by socio-economic characteristics can be more significant than the variability of annuity rates by gender. For instance, at age 65 the percentage difference between the annuity rate for females and males in England and Wales is $14.59/13.53 - 1 = 7.8\%$, whereas the percentage difference between the annuity rate for the two extreme deprivations quintiles is $6.9\% - (-10.4\%) = 17.3\%$ and $4.6\% - (-7.6\%) = 12.2\%$, for males and females, respectively. Although in this simple comparison the significance of socio-economic circumstances is exaggerated by the fact that deprivation quintile is a multilevel factor while gender is a binary factor, this result is still interesting in view of the recent European Court of Justice ruling that insurance premiums and benefits after 21 December 2012 should be gender neutral (Court of Justice of the European Union, 2011), and, hence, socio-economic characteristics might be a candidate for substituting part of the role of gender in the pricing of annuities.

4. Discussion

In this paper we have examined a number of alternatives for the modelling and forecasting of socio-economic differentials in mortality, including several existing multipopulation extensions of the Lee-Carter model and a newly proposed relative model. An application to deprivation subpopulations in England showed that in the presence of both level and improvement differentials, the new relative model exhibits the best results in terms of goodness of fit and forecasting performance. A key feature for the success of this model is the possibility of relying on the wider mortality experience of a reference population which enables the consideration of cohort effects and a more reliable estimation of the long-run mortality trend.

We note, however, that in other applications with different aims and using datasets with other characteristics, a different model might be appropriate. For instance, if the focus of the study is on the assessment of historical mortality differentials rather than on their forecasting, the joint-κ model and the three-way Lee-Carter model deserve serious consideration. Given

the commonly observed widening of relative mortality differentials and the decrease in socio-economic mortality differentials with rising age, models that are not able to capture any of these features, such as the common factor model and the stratified Lee-Carter, are likely to be too rigid for most applications. In spite of that, these models may still be applied successfully in situations where there are data limitations and more parsimonious models are preferred, or in the study of mortality differentials in populations of a different nature. An example of the later is the application of the stratified Lee-Carter in the modelling of mortality in Spanish regions discussed in Debón et al. (2011).

A noticeable advantage of the Lee-Carter based methods discussed in this paper is that they offer a simple methodology for projecting subpopulation-specific mortality rates. However, since these methods are based on pure extrapolation of past trends, they assume that socio-economic mortality improvement differentials will remain constant in the future, ignoring the fact that policy interventions for reducing health inequalities will likely change the future mortality gradient. Nevertheless, many of the factors behind health inequalities change very slowly and, hence, the assumption that relative mortality differentials will not narrow seems a reasonable starting point for forecasting (Wanless et al., 2012).

A simplifying assumption of the relative model introduced in this paper is that cohort effects are the same between socio-economic subpopulations. However, there is some evidence that cohort effects may vary across socio-economic subgroups. For instance, it has been reported that the cohort effect for assured lives, who are more likely to belong to higher socio-economic subgroups, is centred in a slightly earlier generation than seen in the general population of England and Wales (CMI Bureau Mortality Sub-Committee, 2002; Willets, 2004). Consequently, the development of models that allow for socio-economic variations in cohort effects deserves further investigation.

In the second part of this paper we have applied the new proposed relative model to analyse the extent of mortality differentials across deprivation subgroups in England during 1981 through 2007. This analysis reveals a clear association between area deprivation and mortality rates, with people living in more deprived areas having higher mortality rates than those living in less deprived areas. The mortality differentials found in this study are substantial. In fact, at some ages the mortality rates of the most deprived quintile can be more than twice the mortality rates of the least deprived quintile. In addition, our analysis indicates a widening of the relative mortality gap between more and less deprived areas of England, mainly as a result of the slower mortality improvements experienced by the lowest socio-economic subgroups.

It has been shown that socio-economic differences in mortality have a significant impact on the pricing of life annuities. Moreover, despite the fact that socio-economic differences in mortality tend to decrease with age, it was found that their financial impact is still very significant at old ages. These results become more relevant in light of the European Court of Justice ban on the use of gender as an underwriting variable. With gender removed from the list of admissible rating factors, socio-economic related rating factors gain relative importance in the modelling of longevity risk.

Finally, we recognise some data issues that might distort our results on the association between mortality and deprivation. First, our quintile groups are defined using the IMD 2007, implicitly assuming that the relative ranking by deprivation of small areas in England remained unchanged over the 1981-2007 period. Second, it is plausible that healthier people

will tend to move from more deprived areas to less deprived ones and vice versa, resulting in a potential bias toward higher mortality inequalities. However, previous studies have shown that the majority of small areas in England have stayed in the same deprivation quintile over our period of study (Norman, 2009; Lu et al., 2012), and that selective migration has a negligible impact on the analysis of trends in mortality inequalities (Norman et al., 2005). A detailed discussion of these two issues with particular reference to the dataset used in this paper is provided by Lu et al. (2012, Appendix A).

5. Acknowledgments

References

Brouhns, N., Denuit, M., Vermunt, J., Dec. 2002. A Poisson log-bilinear regression approach to the construction of projected lifetables. Insurance: Mathematics and Economics 31 (3), 373–393.

Brown, J. R., Dec. 2002. Differential Mortality and the Value of Individual Account Retirement Annuities. In: Feldstein, M., Liebman, J. B. (Eds.), The Distributional Aspects of Social Security and Social Security Reform. University of Chicago Press.

Butt, Z., Haberman, S., 2009. Ilc: A collection of R functions for fitting a class of Lee-Carter mortality models using iterative fitting algorithms. Actuarial Research Paper, Cass Business School.

Cairns, A. J., Blake, D., Dowd, K., Coughlan, G. D., Epstein, D., Ong, A., Balevich, I., 2009. A quantitative comparison of stochastic mortality models using data from England and Wales and the United States. North American Actuarial Journal 13 (1), 1–35.

Carter, L. R., Lee, R. D., 1992. Modeling and forecasting US sex differentials in mortality. International Journal of Forecasting 8 (3), 393–411.

CMI Bureau Mortality Sub-Committee, 2002. Working Paper 1: An interim basis for adjusting the '92 Series' mortality projections for cohort effects. CMI, London.

Coale, A., Kisker, E., 1990. Defects in data on old age mortality in the United States: New procedures for calculating approximately accurate mortality schedules and life tables at the highest ages. Asian and Pacific Population Forum 4, 1–31.

Coughlan, G. D., Khalaf-Allah, M., Ye, Y., Kumar, S., Cairns, A. J., Blake, D., Dowd, K., 2011. Longevity hedging 101: A framework for longevity basis risk analysis and hedge effectiveness. North American Actuarial Journal 15 (2), 150–176.

Court of Justice of the European Union, 2011. Taking the gender of the insured individual into account as a risk factor in insurance contracts constitutes discrimination. Press Release, No 12/11.

Debón, A., Montes, F., Martínez-Ruiz, F., Dec. 2011. Statistical methods to compare mortality for a group with non-divergent populations: an application to Spanish regions. European Actuarial Journal 1 (2), 291–308.

Delwarde, A., Denuit, M., Guillén, M., Vidiella-i Anguera, A., 2006. Application of the Poisson log-bilinear projection model to the G5 mortality experience. Belgian Actuarial Bulletin 6 (1), 54–68.

Denuit, M., Goderniaux, A.-C., 2005. Closing and projecting life tables using log-linear models. Bulletin of the Swiss Association of Actuaries (1), 29–48.

Dowd, K., Cairns, A. J., Blake, D., Coughlan, G. D., Epstein, D., Khalaf-Allah, M., 2010. Backtesting stochastic mortality models: An ex-post evaluation of multi-period-ahead density forecasts. North American Actuarial Journal 14 (3), 281–298.

Haberman, S., Renshaw, A., 2009. On age-period-cohort parametric mortality rate projections. Insurance: Mathematics and Economics 45 (2), 255–270.

Hoffmann, R., Aug. 2005. Do socioeconomic mortality differences decrease with rising age? Demographic Research 13, 35–62.

Human Mortality Database, 2012. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). www.mortality.org or www.humanmortality.de.

Hunt, A., Villegas, A., 2013. Robustness and stability in cohort-based extensions of the Lee-Carter model of mortality. Working paper.

Jarner, S. F., Kryger, E. M., 2011. Modelling adult mortality in small populations: The Saint Model. ASTIN Bulletin 41 (2), 377–418.

Johnson, B., 2011. Deriving trends in life expectancy by the National Statistics Socio-economic Classification using the ONS Longitudinal Study. Health Statistics Quarterly (49), 9–52.

Lee, R. D., Carter, L. R., Sep. 1992. Modeling and forecasting U.S. mortality. Journal of the American Statistical Association 87 (419), 659–671.

Li, J. S., Hardy, M. R., 2011. Measuring basis risk in longevity hedges. North American Actuarial Journal 15 (2), 177–200.

Li, N., Lee, R. D., 2005. Coherent mortality forecasts for a group of populations: An extension of the Lee-Carter method. Demography 42 (3), 575–594.

Liebman, J., 2002. Redistribution in the current US social security system. In: Feldstein, M., Liebman, J. (Eds.), The Distributional Aspects of Social Security and Social Security Reform. University of Chicago Press.

Lu, J., Wong, W., Bajekal, M., 2012. Mortality improvements by socio-economic circumstances in England (1982 to 2006). Institute and Faculty of Actuaries Sessional Research Paper. (Presented Sept. 2012).

Madrigal, A., Matthews, F., Patel, D., Gaches, A., Baxter, S., 2011. What longevity predictors should be allowed for when valuing pension scheme liabilities. British Actuarial Journal 16 (1), 1–38.

Noble, M., Mclennan, D., Wilkinson, K., Whitworth, A., Exley, S., Barnes, H., Dibben, C., 2007. The English indices of deprivation 2007. Department of Communities and Local Government, London.

Norman, P., Aug. 2009. Identifying change over time in small area socio-economic deprivation. Applied Spatial Analysis and Policy 3 (2-3), 107–138.

Norman, P., Boyle, P., Rees, P., Jun. 2005. Selective migration, health and deprivation: a longitudinal analysis. Social Science and Medicine 60 (12), 2755–71.

Raleigh, V. S., Kiri, V. a., Dec. 1997. Life expectancy in England: variations and trends by gender, health authority, and level of deprivation. Journal of Epidemiology and Community Health 51 (6), 649–58.

Renshaw, A., Haberman, S., Jan. 2003. Lee-Carter mortality forecasting: a parallel generalized linear modelling approach for England and Wales mortality projections. Journal of the Royal Statistical Society: Series C (Applied Statistics) 52 (1), 119–137.

Renshaw, A., Haberman, S., Jun. 2006. A cohort-based extension to the Lee-Carter model for mortality reduction factors. Insurance: Mathematics and Economics 38 (3), 556–570.

Richards, S., 2008. Applying survival models to pensioner mortality data. British Actuarial Journal 14 (2), 257–303.

Russolillo, M., Giordano, G., Haberman, S., 2011. Extending the LeeCarter model: a three-way decomposition. Scandinavian Actuarial Journal (2), 96–117.

Shkolnikov, V. M., Andreev, E. M., Jasilionis, D., Leinsalu, M., Antonova, O. I., McKee, M., Oct. 2006. The changing relation between education and life expectancy in central and eastern Europe in the 1990s. Journal of Epidemiology and Community Health 60 (10), 875–81.

Tarkiainen, L., Martikainen, P., Laaksonen, M., Valkonen, T., Mar. 2012. Trends in life expectancy by income from 1988 to 2007: decomposition by age and cause of death. Journal of Epidemiology and Community Health 66 (7), 573–578.

Tuljapurkar, S., Li, N., Boe, C., 2000. A universal pattern of mortality decline in the G7 countries. Nature 405 (6788), 789–792.

Wanless, D., Pattison, J., McPherson, K., Haberman, S., Blakemore, C., Wong, W., Lu, J., Jan. 2012. Life expectancy past and future variations by socio-economic group in England and Wales. Longevity Science Advisory Panel. Available at www.longevitypanel.co.uk.

Willets, R., 2004. The cohort effect: Insights and explanations. British Actuarial Journal 10 (4), 833–877.

Yang, S. S., Wang, C.-W., Nov. 2013. Pricing and securitization of multi-country longevity risk with mortality dependence. Insurance: Mathematics and Economics 52 (2), 157–169.